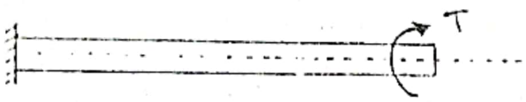
TORSION

Torque :

Torque is defined as the one kind of moment with respect to the longitudinal axis of an object. The unit of torque is N m or ft.lb.



Torsion :

Torsion is the twisting of an object due to an applied torque. Torsion is the action of twisting or the state of being twisted specially one end of object relative to the others.

Torsional stress :

Torsional stress is defined as the shearing stress due to torsion.

Torsional strain: Torsional strain is defined as the deformation produced by the application of torque.

Assumptions for deriving torsion formula:

1. Circular sections remain circular.

ii. Plane sections remain plane and do not warp

iii. The projection upon a transverse section of straight radial lines in the section remain straight.

iv. Shaft is loaded by twisting couples in planes that are perpendicular to the axis of the shaft.

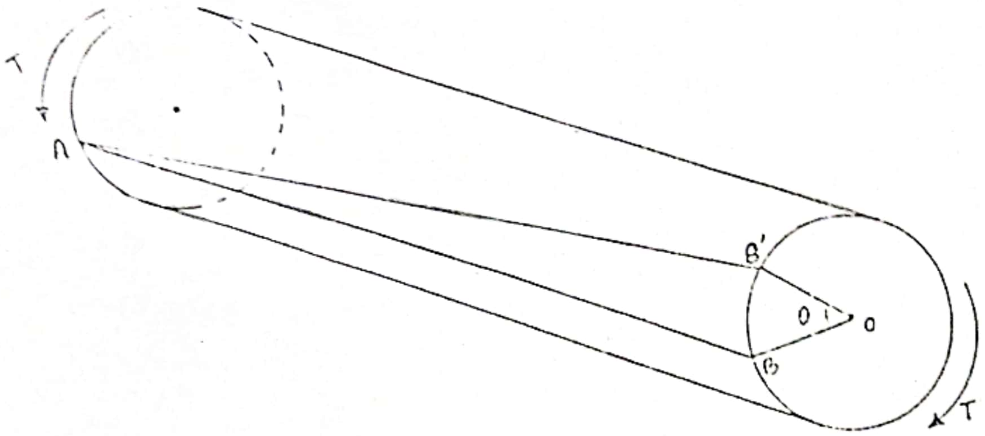
v. Stresses do not exceed the proportional limit.

SHAFT

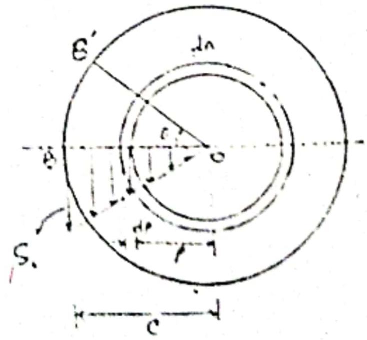
Fahim .

Shaft : A shaft is a rotating machine element usually in circular cross section which is used to transmit power from one part to another.

Derive torsional formula to determine maximum stress in a solid circular shaft :



If a torque T is applied at the ends of the solid circular shaft, a fiber AB on the outside surface which is normally straight it will be twisted into a helix AB' as the shaft is twisted through the angle θ



for the equilibrium of the free body diagram,

Resisting torque = External torque

Let the maximum unit stress at outer fiber in the surface is S_s.

dA = elementary area in the form of a narrow ring of radius ρ.

$d\rho$ = the thickness of the narrow strip.

Then from the law of stress variation,

Unit stress at $dA = \frac{\rho}{c} S_s$

The force exerted by the stress over the area dA

$$= \frac{\rho}{c} S_s . dA$$

The moment of this force with respect to the axis of the shaft = force x moment arm

$$= \frac{\rho}{c} . S_s . dA \times \rho$$

$$= \frac{\rho^2}{c} S_s dA .$$

The sum of the moment of all the stresses on the entire cross section is the resisting torque,

$$T_{resisting} = \int_0^c \frac{\rho^2}{c} S_s dA$$

$$= \frac{S_s}{c} \int_0^c \rho^2 dA$$

$$= \frac{S_s J}{c}$$

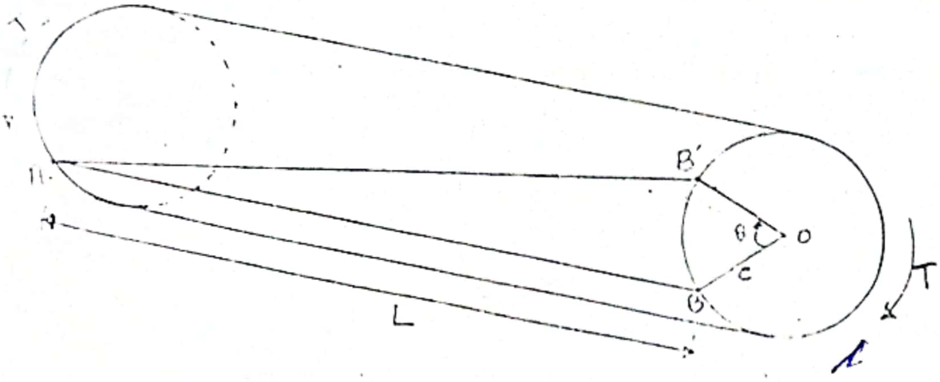
where $J = \int_0 \rho^2 dA$ is the polar moment of inertia of the cross section.

Hence resisting torque T = applied torque T

$$\Rightarrow \frac{S_s J}{c} = T$$

$$\Rightarrow \boxed{S_s = \frac{Tc}{J}}$$

Determination of torsional formula for angle of twist.



AB represents an element of cylindrical surface of the untwisted shaft and AB' represents the curve (part of a helix) which is the same element assume after the torque is applied.

∠BOB' is the angle of twist for the fiber represented by the element AB. The total angular deformation in the length L is BB'.

So the unit deformation, $\delta_s = \frac{BB'}{L}$

and $BB' = c\theta$

From the Hook's law, $\delta_s = \frac{S_s}{G}$,

$$\Rightarrow \frac{BB'}{L} = \frac{S_s}{G}$$

$$\Rightarrow \frac{c\theta}{L} = \frac{S_s}{G}$$

$$\Rightarrow \theta = \frac{S_s L}{Gc}$$

But the torsional stress, $S_s = \frac{Tc}{J}$

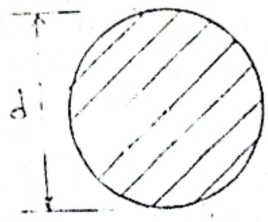
Hence, $\theta = \frac{(Tc/J)L}{Gc}$

$$\boxed{\theta = \frac{TL}{JG}}$$

Derive an expression for solid and hollow circular shaft for maximum shearing stress:

For solid circular shaft:

Polar moment of inertia,



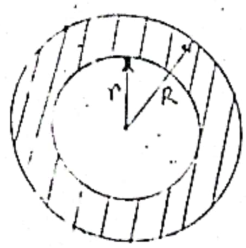
$$J = \frac{\pi r^4}{2} = \frac{\pi (d/2)^4}{2} = \frac{\pi d^4}{32}$$

Maximum shearing stress,

$$S_s = \frac{Tc}{J} = \frac{T \cdot d/2}{\pi d^4/32} = \frac{16T}{\pi d^3}$$

For hollow circular shaft:

Polar moment of inertia,



$$J = \frac{\pi}{2}(R^4 - r^4) = (\pi/32)(D^4 - d^4)$$

Maximum shearing stress

$$S_s = \frac{Tc}{J} = \frac{TR}{\pi/2 (R^4 - r^4)} = \frac{2TR}{\pi(R^4 - r^4)} = \frac{16TD}{\pi(D^4 - d^4)}$$

Modulus of rigidity :

The ratio of shear stress to the shear strain is called the modulus of rigidity. It is also known is modulus of elasticity in shear.

Symbolically.

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

Torsional stiffness

The torsional stiffness (k) is defined as the torque per radia. twist.

ie.

$$k = \frac{T}{\theta}$$

Power transmitted by a shaft:

If T is the applied torque T and ω is the angular velocity of the shaft then the power transmitted by the shaft is

$$P = T.\omega$$

$$= T.\frac{2\pi N}{60} \quad wt$$

$$= \frac{2\pi NT}{60\times10^3} \quad kw \; ; \text{ where } N = rpm.$$

$$T = N\cdot m.$$

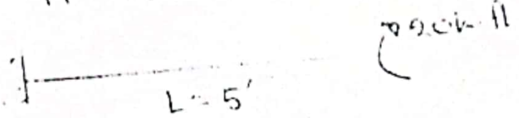
and

$$P = \frac{TN}{63000} \; HP \; ; \text{ where } T = lb.in$$

$$N = rpm$$

1 Hz = 1 1/s
1 wt = 1 N·m/s
1 hp = 550 lb. ft/sec.
1 hp = 745.7 wt

01. Calculate the maximum torsional stress and angle of twist if $G = 12 \times 10^6$ psi. and $\phi = 3$ in.



Solution:

Maximum torsional stress, $S_s = \dfrac{TC}{J}$

$$= \frac{20 \times 10^3 \times 12 \times (3/2)}{\frac{\pi \times 3^4}{32}}$$

$= 45270.74$ psi

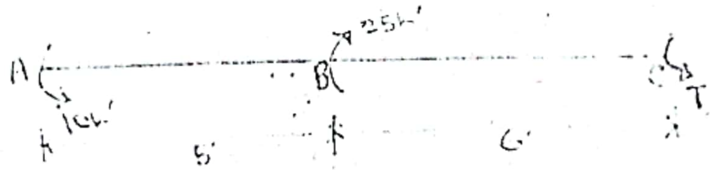
$= 45.27$ ksi. $\boxed{\text{Ans}}$

Angle of twist, $\theta = \dfrac{TL}{JG}$

$$= \frac{20 \times 12 \times (5 \times 12)}{\left(\frac{\pi \times 3^4}{32}\right) \times 12 \times 10^3}$$

$= 0.15$ rad. $\boxed{\text{Ans}}$

02. Calculate maximum torsional stress and angle of twist between A & C. $G = 12 \times 10^6$ psi. and $\phi = 4"$

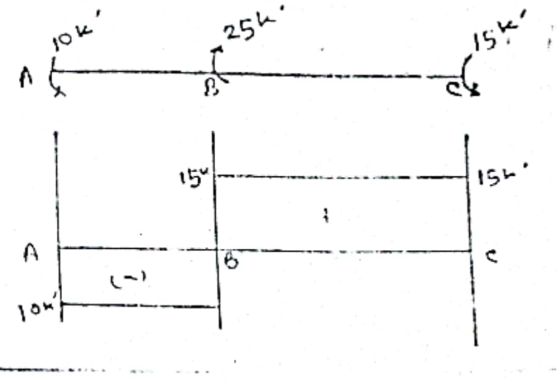


Solution:

$\Sigma T = 0$

$\Rightarrow -10 + 25 - T = 0$

$\Rightarrow T = 15k'$



As the diameter is same and maximum torque is 15k' acts on BC. So maximum stress occurs on BC.

So maximum torsional stress

$$\boxed{\begin{aligned} S_s &= \frac{Tc}{J} \\ &= \frac{15\times12\times(4/2)}{\frac{\pi\times(4)^4}{32}} \\ &= 14.32\text{ ksi} \quad \boxed{\text{Ans}} \end{aligned}}$$

Angle of twist between AC

Considering clockwise deformation as positive

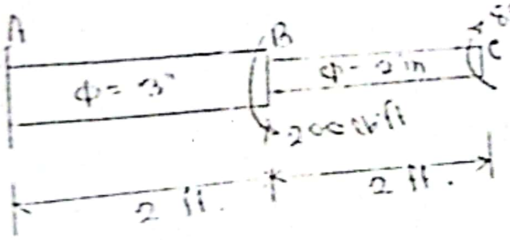
$$\theta_{AC} = \theta_{BC} - \theta_{AB}$$

$$= \left(\frac{TL}{JG}\right)_{BC} - \left(\frac{TL}{JG}\right)_{AB}$$

$$= \frac{15\times12\times5\times12}{\frac{\pi\times4^4}{32}\times12\times10^3} - \frac{10\times12\times5\times12}{\frac{\pi\times4^4}{32}\times12\times10^3}$$

$$= 0.019\text{ rad} \quad \boxed{\text{Ans}}$$

Q3. A solid aluminium shaft is subjected to two torques as shown. Determine the maximum shearing stress and the angle of twists at the free end. Use $G = 4\times10^6$ psi. [2008]

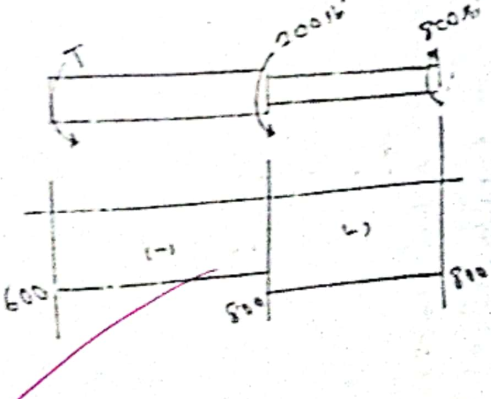


Solution:

$\Sigma T = 0$

$\Rightarrow 800 - 200 - T = 0$

$\Rightarrow T = 600$ lb.ft.



Shearing stress at AB, $S_s = \frac{Tc}{J}$

$$= \frac{(600\times12)\times 3/2}{\frac{\pi\times 3^4}{32}}$$

$$= 1358.12 \text{ psi.}$$

Shearing stress at BC,

$$\boxed{S_s = \frac{Tc}{J} = \frac{(800\times12)\times(2/2)}{\frac{\pi\times 2^4}{32}}} = 6111.5$$

Maximum shearing stress $= 6111.55$ psi. $\boxed{\text{Ans}}$

Angle of twist, $\theta = \theta_{AB} + \theta_{BC}$

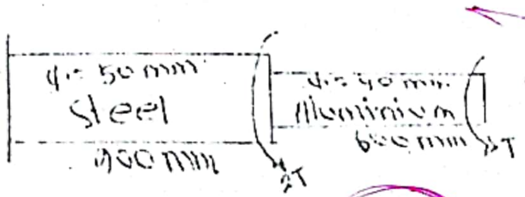
$$= \left(\frac{TL}{JG}\right)_{AB} + \left(\frac{TL}{JG}\right)_{BC}$$

$$= \frac{600\times12\times2\times12}{\frac{\pi\times3^4}{32}\times4\times10^6} + \frac{800\times12\times2\times12}{\frac{\pi\times2^4}{32}\times4\times10^6}$$

$$= 0.042 \text{ rad. } \boxed{\text{Ans}}$$

$\gamma = S_s$

Q4. Determine the maximum safe value of T subjected to the following conditions $\tau_{st} \le 83$ MPa, $\tau_{al} \le 65$ MPa and the angle of rotation of the free end is limited to 6°. $G_{st} = 83$ GPa, $G_{al} = 28$ GPa



$\frac{2T}{\pi r^3}$ $\quad \frac{TR}{\frac{\pi r^3}{2}}$

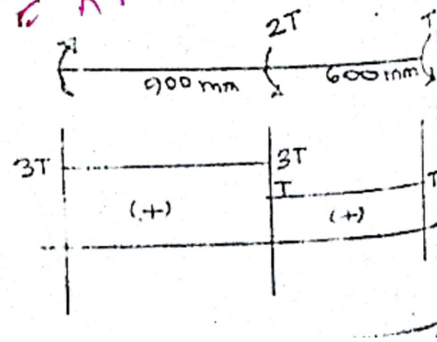
Solution:

$\frac{16T}{\pi d^3}$

For steel; $\tau_{st} = \frac{16T_{st}}{\pi d^3}$

$\Rightarrow 83 = \frac{16\times3T}{\pi\times(50)^3}$

$\Rightarrow T = 679042.16$ N·mm



$\Rightarrow T = 679$ N-m.

For aluminium :

$$\tau_{al} = \frac{16 T_{al}}{\pi d^3}$$

$$\Rightarrow 55 = \frac{16 \times T}{\pi \times (40)^3}$$

$$\Rightarrow T = 691150.38 \text{ N.mm}$$

$$= 691.15 \text{ N.m}$$

Angle of twist,

$\boxed{1 \text{ GPa} = 10^3 \text{ N/mm}^2}$

$\boxed{1 \text{ rad} = 57.3 \text{ degree}}$

$$\theta = \theta_{st} + \theta_{al}$$

$$= \left(\frac{TL}{JG}\right)_{st} + \left(\frac{TL}{JG}\right)_{al}$$

$$\Rightarrow \frac{6}{57.3} = \frac{3T \times 900}{\frac{\pi \times (50)^4}{32} \times 83 \times 10^3} + \frac{T \times 600}{\frac{\pi \times (40)^4}{32} \times 28 \times 10^3}$$

$$\Rightarrow \frac{6}{57.3} = T\left(\frac{3 \times 900 \times 32}{\pi \times (50)^4 \times 83 \times 10^3} + \frac{600 \times 32}{\pi \times (40)^4 \times 28 \times 10^3}\right)$$

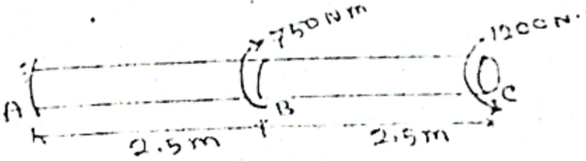
$$\Rightarrow T = 757260.54 \text{ N-mm}$$

$$= 757.26 \text{ N-m}.$$

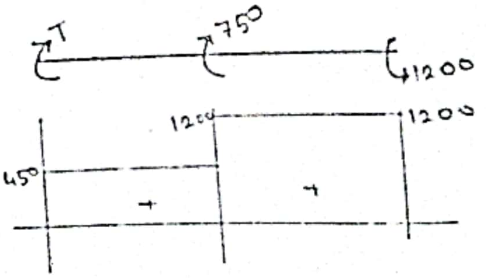
The safe value of T is 679 N-m.

Ans: T = 679 N.m.

Q5. Using G = 83 GPa determine the required diameter of the shaft if the shearing stress is limited to 60 MPa and the angle of rotation at the free end is not to exceed 4°.



Solution:



$\Sigma T = 0$

$\Rightarrow T + 750 - 1200 = 0$

$\Rightarrow T = 450$ N-m

Angle of twist, $\theta = \theta_{AB} + \theta_{BC}$

$$\Rightarrow \frac{4}{57.3} = \frac{1200\times 2.5}{\frac{\pi \times d^4}{32}\times 83\times 10^9} + \frac{450\times 2.5}{\frac{\pi d^4}{32}\times 83\times 10^9}$$

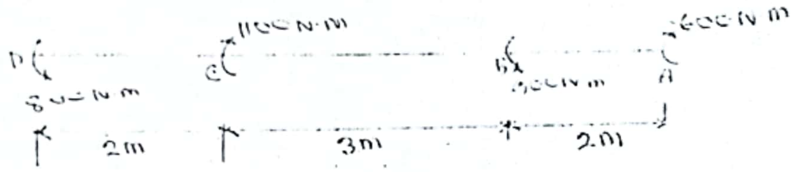
$$\Rightarrow \frac{1}{d^4} = 137898.46$$

$$\Rightarrow d = 0.05189 \text{ m}$$

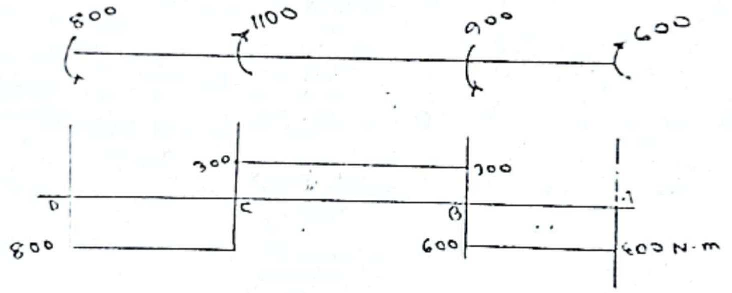
$$\therefore d = 51.89 \text{ mm}. \quad \boxed{\text{Ans}}$$

Kip/ft

06. Determine the relative angle of twist of gear D relative to A. Use $G = 28$ GPa and $\phi = 50$mm.



Solution:



Angle of twist,

$$\theta_{D/A} = \theta_{AB} - \theta_{BC} + \theta_{CD}$$

$$= \left(\frac{TL}{JG}\right)_{AB} - \left(\frac{TL}{JG}\right)_{BC} + \left(\frac{TL}{JG}\right)_{CD}$$

$$= \{(600\times 2) - (300\times 3) + (800\times 2)\} \times \frac{1}{\frac{\pi\times(0.05)^4}{32}\times 28\times 10^9}$$

$$= 0.1106 \text{ rad}$$

$$= (0.1106\times 57.3) \text{ degree}$$

$$= 6.34^\circ. \quad \boxed{\text{Ans}}$$

Kip = psi

pould / sq inch

Ksp = Kpof / sq

07. What is the maximum diameter of a solid steel shaft that will not twist through more than 3° in a 6 m length when subjected to a torque of 12 kN m? What maximum shearing stress is developed? Use G= 83 GPa.

Solution:

Given,

$$\theta = 3^\circ = \frac{3}{57.3} \text{ rad.}$$

$$L = 6 m$$

$$T = 12 \text{ KN.m}$$

$$G = 83 \text{ GPa} = 83\times10^6 \text{ KN/m}^2$$

Now,

$$\theta = \frac{TL}{JG}$$

$$\Rightarrow \frac{3}{57.3} = \frac{12\times6}{\left(\frac{\pi\times d^4}{32}\right)\times83\times10^6}$$

$$\Rightarrow d^4 = \frac{12\times6\times32\times57.3}{\pi\times83\times10^6\times3}$$

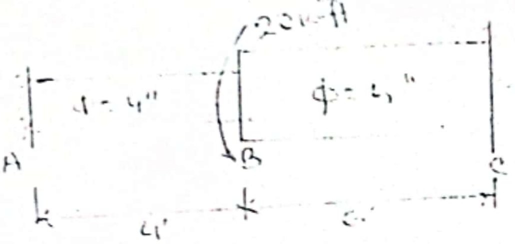
$$\Rightarrow d = 0.114 \text{ m} = 114 \text{ mm}.$$ Ans

Maximum shearing stress,

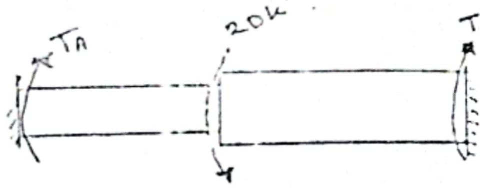
$$S_s = \frac{16T}{\pi d^3} = \frac{16\times12\times10^3\times10^3}{\pi\times(114)^3} = 41.25 \text{ N/mm}^2 = 41.25 \text{ MPa}$$ Ans

E.P.

05. Calculate maximum shearing stress which is developed in the following shaft. $G = 12 \times 10^6$ psi.



Solution :



$T_A + T_c = 20$ ——— (i)

and $\theta_{AB} = \theta_{BC}$

$\Rightarrow \left(\frac{TL}{JG}\right)_{AB} = \left(\frac{TL}{JG}\right)_{BC}$

$\Rightarrow \dfrac{T_A \times 4}{\frac{\pi \times \left(\frac{4}{12}\right)^4}{32} \times G} = \dfrac{T_c \times 6}{\frac{\pi \times \left(\frac{6}{12}\right)^4}{32} \times G}$

$\Rightarrow \dfrac{T_A \times 4}{(4)^4} = \dfrac{T_c \times 6}{(6)^4}$

$\Rightarrow T_A - 0.6144\, T_c = 0$ ---------- (ii)

Solving equation (i) & (ii)

$T_A = 7.61$ k-ft.

$T_c = 12.39$ k-ft.

Shearing stress in AB, $S_s = \dfrac{16 T_A}{\pi d^3}$

$= \dfrac{16 \times 7.61 \times 12000}{\pi \times (4)^3}$

$= 7267.01$ psi.

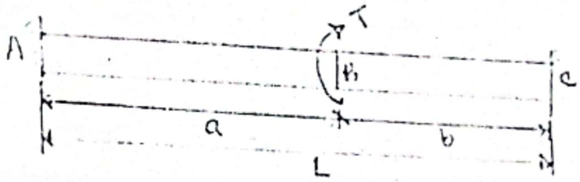
Shearing stress in BC, $S_s = \dfrac{16\, T_c}{\pi d^3}$

$= \dfrac{16 \times 12.39 \times 12000}{\pi \times (6)^3}$

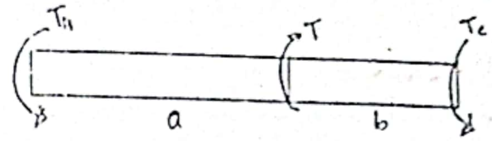
$= 6546/69$ psi.

$\therefore$ Maximum shearing stress, $S_{s(AB)} = 7267.01$ psi $\boxed{\text{Ans}}$

9. A torque is applied to a solid shaft. Prove that the resisting torques at the walls are $T_A = \frac{Tb}{L}$ and $T_C = \frac{Ta}{L}$. How would these values be changed if the shaft were hollow?



Solution:



$\Sigma T = 0$

$\Rightarrow \quad T_A + T_C = T \quad \cdots\cdots \text{(I)}$

and $\quad \theta_{AB} = \theta_{CB}$

$$\Rightarrow \quad \left(\frac{TL}{JG}\right)_{AB} = \left(\frac{TL}{JG}\right)_{CB}$$

$$\Rightarrow \quad \frac{T_A \times a}{JG} = \frac{T_C \times b}{JG}$$

$$\Rightarrow \quad T_A = \frac{T_C\, b}{a} \quad \cdots\cdots\cdots \text{(II)}$$

from equation (I)

$$\frac{T_C\, b}{a} + T_C = T$$

$$\Rightarrow T_C \left(\frac{b+a}{a}\right) = T$$

$$\Rightarrow T_C = \frac{Ta}{a+b}$$

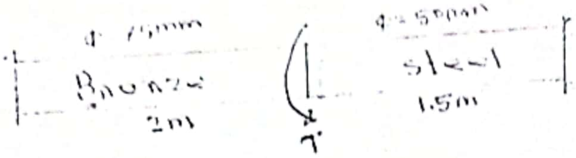
$$\boxed{\therefore T_C = \frac{Ta}{L}}$$

from equation (II) $\quad T_A = \frac{Ta}{L} \cdot \frac{b}{a}$

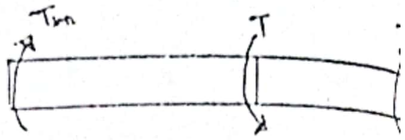
$$\boxed{T_A = \frac{Tb}{L}}$$

Since these values of T_A and T_C only the result of length ratio. No matter if the shaft is hollow.

10. Compute the maximum torque T that can be applied $\tau_{bn} \leq 60$ MPa, $\tau_{st} \leq 80$ MPa and $G_{bn} = 35$ GPa, $G_{st} = 83$ GPa.



Solution:



$\Sigma T = 0$

$\Rightarrow T_{bn} + T_{st} = T$ ------- (i)

and $\theta_{bn} = \theta_{st}$

$$\Rightarrow \left(\frac{TL}{JG}\right)_{bn} = \left(\frac{TL}{JG}\right)_{st}$$

$$\Rightarrow \frac{T_{bn} \times 2}{\frac{\pi (0.075)^4}{32} \times 35\times10^9} = \frac{T_{st} \times 1.5}{\frac{\pi \times (0.05)^4}{32} \times 83\times10^9}$$

$$\Rightarrow T_{bn} = \frac{(0.075)^4 \times 35 \times 1.5}{(0.05)^4 \times 83 \times 2} T_{st}$$

$\Rightarrow T_{bn} = 1.601\, T_{st}$ -------- (ii)

For bronze:

$$\tau_{bn} = \left(\frac{Tc}{J}\right)_{bn}$$

$$\Rightarrow 60\times10^6 = \frac{(1.601 T_{st}) \times \frac{0.075}{2}}{\frac{\pi \times (0.075)^4}{32}}$$

$\Rightarrow T_{st} = 3104.37$ N-m.

From equation (ii) $T_{bn} = 1.601 \times 3104.37$

$= 4970.1$ N-m.

Now $\tau_{st} = \left(\frac{Tc}{J}\right)_{st}$

$$\Rightarrow \tau_{st} = \frac{3104.37 \times \frac{0.05}{2}}{\frac{\pi \times (0.05)^4}{32}}$$

$= 126.48\times10^6$ N/m²

$= 126.48$ MPa.

which is out of limit so this is not acceptable.

Again,

<u>For steel:</u>

$$\tau_{st} = \left(\frac{Tc}{J}\right)_{st}$$

$$\Rightarrow 80 \times 10^6 = \frac{T_{st} \times \left(\frac{0.05}{2}\right)}{\frac{\pi \times (0.05)^4}{32}}$$

$$\Rightarrow T_{st} = 1963.5 \text{ N·m.}$$

From equation (1) $T_{br} = 1.601 \times 1963.5$

$$= 3143.56 \text{ N·m.}$$

Now

$$\tau_{br} = \left(\frac{Tc}{J}\right)_{br}$$

$$= \frac{3143.56 \times \left(\frac{0.075}{2}\right)}{\frac{\pi \times (0.075)^4}{32}}$$

$$= 37.95 \times 10^6 \text{ N/m}^2.$$

$$= 37.95 \text{ MPa} < 60 \text{ MPa}.$$

This is acceptable.

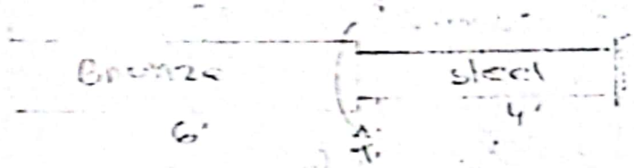
Hence the maximum torque, $T = T_{br} + T_{st}$

$$= 3143.56 + 1963.5$$

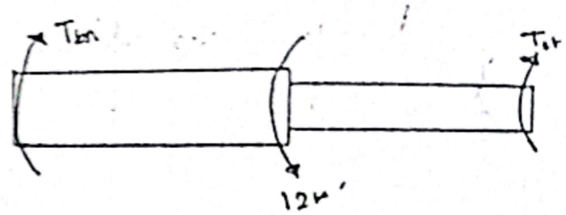
$$= 5107.06 \text{ N·m.}$$

$\boxed{\text{Ans}}$

11. Determine the diameters of each segment so that each material will be simultaneously stressed to its permissible limit when a torque $T = 12$ k.ft is applied. For bronze $\tau \leq 8$ ksi, $G = 6\times10^6$ psi and for steel $\tau \leq 12$ ksi, $G = 12\times10^6$ psi. [2013]



Solution:



$\Sigma T = 0$

$\Rightarrow T_{bn} + T_{st} = 12\times12$

$= 144 \ \ldots\ldots\ (\mathrm{I})$

and $\quad \theta_{bn} = \theta_{st}$

$\Rightarrow \left(\frac{TL}{JG}\right)_{bn} = \left(\frac{TL}{JG}\right)_{st}$

$\Rightarrow \dfrac{T_{bn}\times 6\times12}{\frac{\pi\times(d_{bn})^4}{32}\times 6\times10^3} = \dfrac{T_{st}\times 4\times12}{\frac{\pi\times(d_{st})^4}{32}\times 12\times10^3}$

$\Rightarrow \dfrac{T_{bn}}{(d_{bn})^4} = \dfrac{T_{st}}{3\,(d_{st})^4}$

$\Rightarrow \left(\dfrac{d_{st}}{d_{bn}}\right)^4 = \dfrac{1}{3}\,\dfrac{T_{st}}{T_{bn}} \ \ldots\ldots\ldots\ (\mathrm{II})$

For bronze:

$$\tau_{bn} = \left(\frac{Tc}{J}\right)_{bn}$$

$\Rightarrow 8 = \dfrac{T_{bn}\times d_{bn}/2}{\frac{\pi(d_{bn})^4}{32}}$

$\Rightarrow T_{bn}\times d_{bn}\times 32 = 2\pi\times 8(d_{bn})^4$

$\Rightarrow \dfrac{T_{bn}}{(d_{bn})^3} = \pi/2 \ \ldots\ldots\ldots\ (\mathrm{III})$

For steel:

$$\tau_{st} = \left(\frac{Tc}{J}\right)_{st}$$

$$\Rightarrow 12 = \frac{T_{st} \times d_{st}/2}{\frac{\pi \times (d_{st})^4}{32}}$$

$$\Rightarrow T_{st} \cdot d_{st} \times 32 = 2\pi \times 12 (d_{st})^4$$

$$\Rightarrow \frac{T_{st}}{(d_{st})^3} = \frac{3\pi}{4} \quad \text{.........} \quad (iv)$$

$$(iii) \div (iv) \Rightarrow \frac{T_{bn}}{(d_{bn})^3} \Big/ \frac{T_{st}}{(d_{st})^3} = (\pi/2)/(3\pi/4)$$

$$\Rightarrow \frac{T_{bn}}{T_{st}} \times \left(\frac{d_{st}}{d_{bn}}\right)^3 = \frac{4}{6}$$

$$\Rightarrow \frac{T_{bn}}{T_{st}} \cdot \left(\frac{d_{st}}{d_{bn}}\right)^4 \cdot \frac{d_{bn}}{d_{st}} = \frac{4}{6}$$

$$\Rightarrow \frac{T_{bn}}{T_{st}} \cdot \frac{1}{3} \frac{T_{st}}{T_{bn}} \cdot \frac{d_{bn}}{d_{st}} = \frac{4}{6}$$

$$\Rightarrow \frac{d_{bn}}{d_{st}} = 2 \quad \text{.........} \quad (v)$$

from equation (ii) $\quad (1/2)^4 = \frac{1}{3} \frac{T_{st}}{T_{bn}}$

$$\Rightarrow \frac{3}{16} T_{bn} = T_{st} \quad \text{.........} \quad (vi)$$

from equation (i);

$$T_{bn} + \frac{3}{16} T_{bn} = 144$$

$$\Rightarrow T_{bn} = \frac{144}{1 + (3/16)}$$

$$= 121.26 \text{ k-in.}$$

from equation (v);

$$T_{st} = \frac{3}{16} \times 121.26$$

$$= 22.74 \text{ k-in.}$$

From equation (iii);

$$\frac{121.26}{(d_{br})^3} = \pi/2$$

$$\Rightarrow (d_{br})^3 = \frac{121.26}{\pi/2}$$

$$\Rightarrow d_{br} = 4.26 \text{ in.}$$

From equation (v); $d_{st} = \frac{d_{br}}{2}$

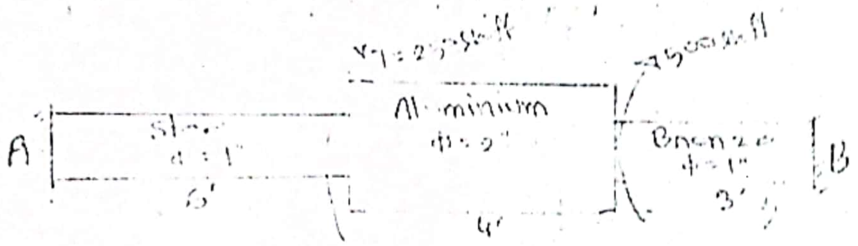
$$= \frac{4.26}{2}$$

$$= 2.13 \text{ in.}$$

Ans: $d_{br} = 4.26$ in.

$d_{st} = 2.13$ in.

12. Calculate maximum stress of the following shaft where $G_{st} = 12\times10^6$ psi, $G_{al} = 4\times10^6$ psi and $G_{br} =$ [illegible] psi. [2005, 2009, 2012, 2017]

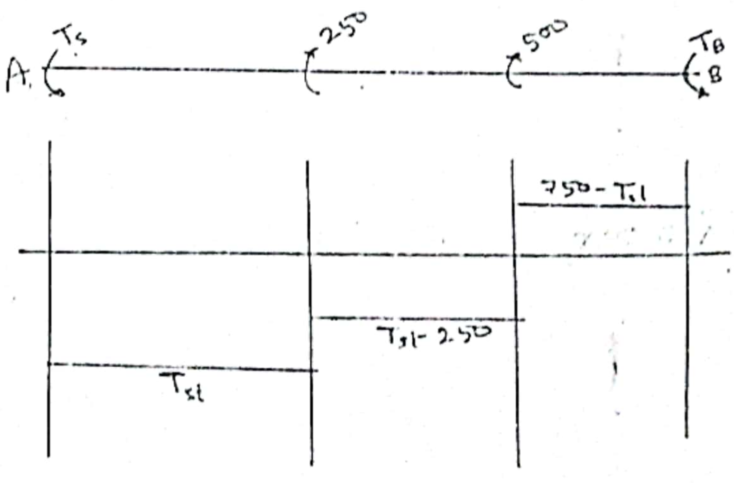


Solution:

$\Sigma T = 0$

$\Rightarrow T_{st} + T_{br} = 750$

$\Rightarrow T_{Br} = 750 - T_{st}$



and $\theta_{A/B} = 0$

$$\Rightarrow \left(\frac{TL}{JG}\right)_{st} + \left(\frac{TL}{JG}\right)_{al} - \left(\frac{TL}{JG}\right)_{br} = 0$$

$$\Rightarrow \frac{T_{st} \times 6}{\frac{\pi \times (1/12)^4}{32} \times (12 \times 10^6 \times 144)} + \frac{(T_{st} - 250) \times 4}{\frac{\pi \times (\frac{2}{12})^4}{32} \times (4 \times 10^6 \times 144)} - \frac{(750 - T_{st})\, 3}{\frac{\pi \times (1/12)^4}{32} \times 6 \times 10^6 \times 144} = 0$$

$$\Rightarrow \frac{6 T_{st}}{(1/12)^4 \times 12} + \frac{(T_{st} - 250)4}{(1/6)^4 \times 4} - \frac{(750 - T_{st})\, 3}{(1/12)^4 \times 6} = 0$$

$$\Rightarrow 10368\, T_{st} + 1296\,(T_{st} - 250) - 10368\,(750 - T_{st}) = 0$$

$$\Rightarrow (10368 + 1296 + 10368)\, T_{st} = 1296 \times 250 + 10368 \times 750$$

$$\Rightarrow T_{st} = 367.65 \text{ lb. ft.}$$

$$\therefore T_{al} = T_{st} - 250$$

$$= 367.65 - 250$$

$$= 117.65 \text{ lb.ft.}$$

and $T_{Br} = 750 - T_{st}$

$$= 750 - 367.65$$

$$= 382.35 \text{ lb.ft.}$$

Maximum stress in steel, $\tau_{st} = \frac{16 T_{st}}{\pi d^3}$

$$= \frac{16 \times 367.65 \times 12}{\pi \times 1^3}$$

$$= 22469.11 \text{ psi}$$

$$= 22.47 \text{ ksi.}$$

Maximum stress in aluminium, $\tau_{al} = \frac{16 T_{al}}{\pi d^3}$

$$= \frac{16 \times 117.65 \times 12}{\pi \times 2^3}$$

$$= 898.78 \text{ psi.}$$

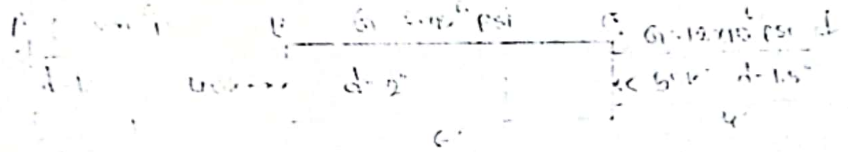
Maximum stress in bronze, $\tau_{br} = \dfrac{16 T_{br}}{\pi d^3}$

$$= \frac{16 \times 382.35 \times 12}{\pi \times 1^3}$$

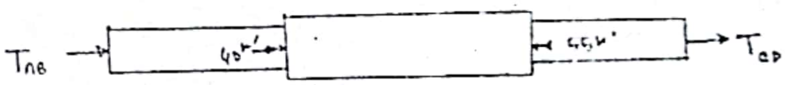
$$= 23367.51 \text{ psi}.$$

Hence maximum stress in the shaft 23367.51 psi. $\boxed{\text{Ans}}$

13. A shaft composed of segments AB, BC & CD is fastened to rigid supports and loaded as shown in figure below. Determine the maximum shearing stress developed in the shaft. [2007]



solution :



$\theta_{A/D} = 0$

$$\Rightarrow \left(\frac{TL}{JG}\right)_{CD} - \left(\frac{TL}{JG}\right)_{BC} - \left(\frac{TL}{JG}\right)_{AB} = 0$$

$$\Rightarrow \frac{(15 - T_{AB}) \times 4 \times 12}{\frac{\pi \times (1.5)^4}{32} \times 12 \times 10^3} - \frac{(T_{AB}+40) \times 6 \times 12}{\frac{\pi \times (2)^4}{32} \times 6 \times 10^3} - \frac{T_{AB} \times 5 \times 12}{\frac{\pi \times (1.5)^4}{32}, 8 \times 10^3} = 0$$

$$\Rightarrow \frac{16}{243}(15 - T_{AB}) - \frac{1}{16}(T_{AB}+40) - \frac{10}{81} T_{AB} = 0$$

$$\Rightarrow \left(\frac{16}{243} + \frac{1}{16} + \frac{10}{81}\right) T_{AB} = \frac{16}{243} \times 15 - \frac{40}{16}$$

$$\Rightarrow \frac{979}{3888} T_{AB} = -\frac{245}{162}$$

$$\Rightarrow T_{AB} = -6 \text{ k-ft}$$

Maximum shearing stress:

$$\text{In } AB,\ \tau_{AB} = \frac{16\,T_{AB}}{\pi d^3}$$

$$= \frac{16 \times 6 \times 12}{\pi \times (1.5)^3}$$

$$= 108.65 \text{ ksi.}$$

$$\text{In } BC,\ \tau_{BC} = \frac{16\,(-6+40) \times 12}{\pi \times (2)^3}$$

$$= 259.74 \text{ ksi.}$$

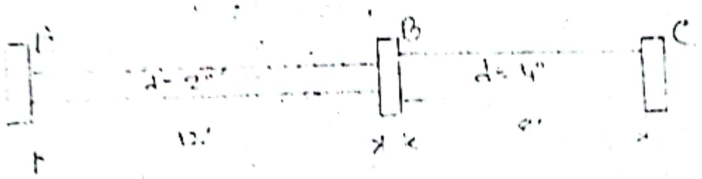
$$\text{In } CD,\ \tau_{CD} = \frac{16\,[15-(-6)] \times 12}{\pi \times (1.5)^3}$$

$$= 380.27 \text{ ksi.}$$

Maximum shearing stress in the shaft = 380.27 ksi.

$\boxed{\text{Ans}}$

14 The shaft shown in figure below rotates at 170 rpm with 50 hp taken off at A, 40 hp removed at B, and 90 hp applied at C. Determine the maximum shearing stress and the angle of twist of gear A relative to gear C. Assume G = 10×10⁶ psi. [2006]



Solution:

$$T_A = -\,63000 \times \frac{P}{N}$$

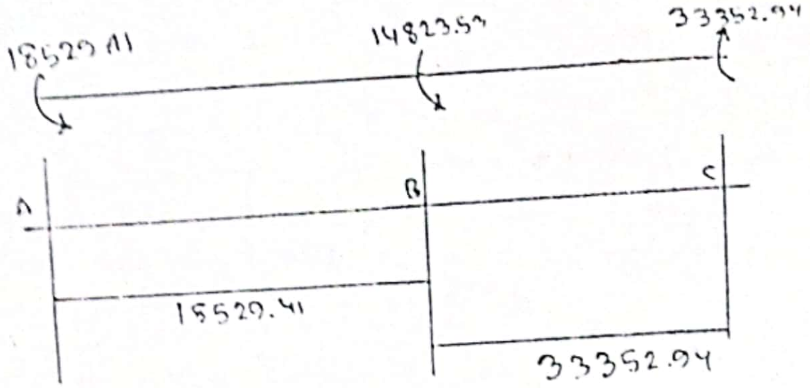
$$= -\,63000 \times \frac{50}{170}$$

$$= -18529.41 \text{ lb.in}$$

$$T_B = -\,63000 \times \frac{40}{170}$$

$$= -14823.53 \text{ lb.in}$$

$$T_C = 63000 \times \frac{90}{170} = 33352.94 \text{ lb.in}$$



shearing stress in AB, $\tau_{AB} = \frac{16T}{\pi d^3}$

$$= \frac{16 \times 18529.41}{\pi \times (2)^3}$$

$$= 11796.19 \text{ psi}.$$

shearing stress in BC, $\tau_{BC} = \frac{16T}{\pi d^3}$

$$= \frac{33352.94 \times 16}{\pi \times (4)^3}$$

$$= 2654.14 \text{ psi}.$$

Angle of twist. $\theta_{A/C} = \theta_{AB} + \theta_{BC}$

$$= \left(\frac{TL}{JG}\right)_{AB} + \left(\frac{TL}{JG}\right)_{BC}$$

$$= \frac{18529.41 \times 12 \times 12}{\frac{\pi \times (2)^4}{32} \times 10 \times 10^6} + \frac{33352.94 \times 8 \times 12}{\frac{\pi \times (4)^4}{32} \times 10 \times 10^6}$$

$$= 0.1826 \text{ rad}.$$

$$= 10.46^\circ.$$

Ans: $\tau_{max} = 11796.19$ psi.

$\theta_{A/C} = 10.46^\circ$.

1. A hollow steel shaft with an 18 inch outside diameter and a 10 inch inside diameter is to transmit 15000 hp. The shearing stress is not to exceed 8000 psi. Determine the permissible speed of the shaft. [2011]

Solution : Given. $D_o = 18$ in.

$D_i = 10$ in.

$P = 15000$ hp.

$S_s = 8000$ psi.

For hollow circular shaft maximum shearing stress.

$$S_s = \frac{16 T D_o}{\pi (D_o^4 - D_i^4)}$$

$$\Rightarrow 8000 = \frac{16 T \times 18}{\pi \{(18)^4 - (10)^4\}}$$

$$\Rightarrow T = 8288219.552 \text{ lb.in}.$$

and power, $P = \frac{TN}{63000}$

$$\Rightarrow N = \frac{63000 \times 15000}{8288219.552}$$

$$= 114.02 \text{ r.p.m.}$$

speed, $\omega = \frac{2\pi N}{60}$

$$= \frac{2\pi \times 114.02}{60}$$

$$= 11.94 \text{ rad/sec}.$$

$\boxed{\text{Ans}}$

1[illegible] maximum shearing stress [illegible] diameter solid circular [illegible] carrying a torque [illegible] which [illegible] the angle of twist per unit length [illegible] material [illegible] for which $G = 12\times10^6$ psi [2018]

Solution: Given,

diameter, $d = 4$ in.

Torque, $T = 228000$ lb.in.

$G = 12\times10^6$ psi.

Maximum shearing stress for solid circular shaft,

$$S_s = \frac{16T}{\pi d^3}$$

$$= \frac{16\times228000}{\pi\times(4)^3}$$

$$= 18143.66 \text{ psi.} \quad \boxed{\text{Ans}}$$

Angle of twist per unit length,

$$\frac{\theta}{L} = \frac{T}{JG}$$

$$= \frac{228000}{\frac{\pi\times(4)^4}{32}\times12\times10^6}$$

$$= 7.56\times10^{-4} \text{ rad}$$

$$= 0.0433 \text{ degree.} \quad \boxed{\text{Ans}}$$

GEAR

Gear:

A gear is a rotating machine part having cut projections called teeth which mesh with another toothed part to transmit power or force.

Purposes:

I. Transmitting force, power or torque.

II. Change the direction of rotation of a shaft.

Gear ratio:

The ratio of the angular velocity of the input or driving gear to the output or driven gear.

The velocity v of the point of contact on the pitch circles is the same on both gears.

i.e. $v = r_1\omega_1 = r_2\omega_2$.

where r_1, ω_1 are the radius and angular velocity of input gear and r_2, ω_2 for the output gear.

Hence.

$$GR = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} = \frac{N_2}{N_1} = \frac{T_2}{T_1}$$

where N and T are no. of teeth and Torque.

Mechanical advantage: The ratio of no. of teeth of output gear to the input gear.

$$MA = \frac{N_2}{N_1}$$

If the output gear has more teeth than the input gear then the gear increases the input torque and vice-versa.

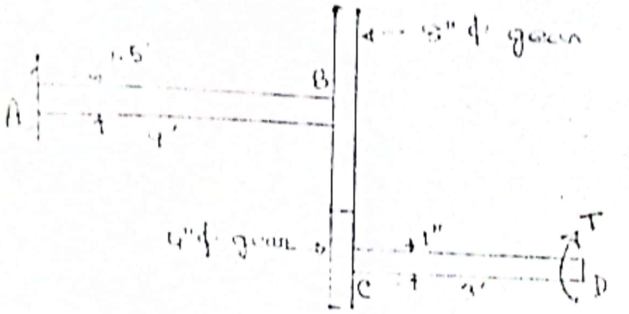
* when two gears mesh together the second one always turns in the opposite direction.

Driving gear: The driving gear or drive gear is the source of power or rotation

Driven gear: The driven gear is turned or moved by the drive gear.

* By moving smaller gear to the larger gear the torque increases.

Q. Two long shafts are gear connected as shown in the following figure. Find the angle of twist at end D if shaft CD is subjected to a twisting moment of 10,000 lb.in, $G = 12\times10^6$ psi. [2015]



Solution :

$$T_{CD} = 10000 \text{ lb.in}$$

$$\text{Gear ratio, } GR = \frac{D_{output}}{D_{input}}$$

$$= \frac{8}{4}$$

$$= 2$$

$$T_{AB} = -GR \cdot T_{CD}$$

$$= -2\times10000$$

$$= -20000 \text{ lb.in}$$

$$\theta_{AB} = \left(\frac{TL}{JG}\right)_{AB}$$

$$= \frac{-20{,}000\times4\times12}{\frac{\pi\times1.5^4}{32}\times12\times10^6}$$

$$= -0.16 \text{ rad}$$

$$\theta_B = \theta_A + \theta_{AB}$$

$$= -0.16 \text{ rad} \quad [\because \theta_A = 0]$$

$$\theta_B = -GR\cdot\theta_C$$

$$\Rightarrow \theta_c = -\tfrac{1}{2}\times(-0.16)$$

$$= 0.08 \text{ rad}$$

$$\theta_{CD} = \left(\frac{TL}{JG}\right)_{CD}$$

$$= \frac{10000 \times 3 \times 12}{\frac{\pi \times 1^4}{32} \times 12 \times 10^6}$$

$$= 0.306 \text{ rad.}$$

Angle of twist of end D, $\theta_D = \theta_{CD} + \theta_C$

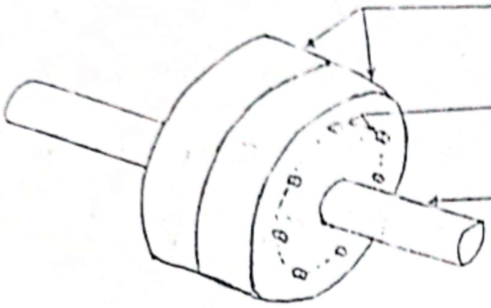
$$= 0.306 + 0.08$$

$$= 0.386 \text{ rad.}$$

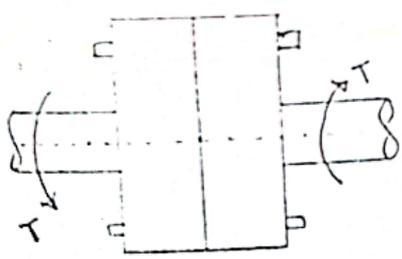
$\boxed{\text{Ans}}$

FLANGED BOLT COUPLINGS

Flanged bolt coupling: Flanged bolt coupling is a type of coupling used between rotating shafts that consists of flanges one of which is fixed at the end of the each shaft, the two flanges being bolted together with bolts fitted circumferentially to complete the drive.



Derive the relationships of stresses in two or more concentric circles in a flanged and bolted coupling. 2011



In flanged bolt coupling the torque is transmitted by the shearing force P created in the bolts that is assumed to be uniformly distributed.

For any number of bolts n, the torque capacity

$$T = Pn \times R$$
$$= \left(\frac{\pi}{4} d^2 * S_s\right) n \times R$$

where, P = shearing force

d = diameter of each bolt

S_s = shearing stress

n = number of bolts in a circle.

R = radius of bolt circle.

If the coupling has two concentric circles of bolts then the torque capacity is

$$T = P_1 R_1 n_1 + P_2 R_2 n_2$$

For rigid flanges the shear deformations in the bolts are proportional to their radial distances from the shaft axis. the shearing strains are related by

$$\frac{\gamma_1}{R_1} = \frac{\gamma_2}{R_2}$$

modulus of rigidity, $G = \dfrac{\text{shearing stress}}{\text{shearing strain}}$

$$\Rightarrow G = \frac{S_s}{\gamma}$$

Hence $\dfrac{S_{s1}}{G_1 R_1} = \dfrac{S_{s2}}{G_2 R_2}$

$$\Rightarrow \frac{P_1/A_1}{G_1 R_1} = \frac{P_2/A_2}{G_2 R_2}$$

The bolts have same area and same material

then $A_1 = A_2$ and $G_1 = G_2$

Therefore $$\boxed{\frac{P_1}{R_1} = \frac{P_2}{R_2}}$$

Q) In a rivet group subjected to a twisting couple T. show that the torsion formula $\tau = \frac{T\rho}{J}$ can be used to find the shearing stress τ at the center of any rivet. Let $J = \Sigma A\rho^2$ where A is the area of a rivet at the radial distance ρ from the centroid of the rivet group.

Solution.

The shearing stress at the center of any rivet

is $\frac{P}{A}$.

The torsion formula $\tau = \frac{T\rho}{J}$

where, $T = PRn$

$\rho = R$

and $J = \Sigma A\rho^2$

$= AR^2n$.

Hence $\tau = \frac{PRn \cdot R}{AR^2n}$

$= \frac{P}{A}$

This shows that the torsion formula $\tau = \frac{T\rho}{J}$ can be used to find the shearing stress at the center of any rivet.

02. A flanged bolt coupling consists of 10 20mm diameter bolts spaced evenly around a bolt circle 400mm in diameter. Determine the torque capacity of the coupling if the allowable shearing stress in the bolts is 40 MPa.

Solution.

Here, Diameter of bolt, $d = 20mm$
$= 0.02m$

No. of bolts, $n = 10$

Radius of bolt circle, $R = \frac{400}{2}$
$= 200 mm$
$= 0.2 m$

Shearing stress, $S_s = 40$ MPa
$= 40 \times 10^6 N/m^2$.

Now Torque, $T = PRn$

$$= \left(\frac{\pi d^2}{4} \times S_s\right) Rn$$

$$= \frac{\pi \times (0.02)^2 \times 40 \times 10^6 \times 0.2 \times 10}{4}$$

$$= 25132.74 \text{ N-m}$$

$$= 25.13 \text{ KN-m}. \quad \boxed{\text{Ans}}$$

03. A flanged bolt coupling consists of ten steel $\frac{1}{2}$in dia. bolts spaced evenly around a bolt circle 14 in. in dia. Determine the torque capacity of the coupling if the shearing stress in the bolts is 6,000 psi

Solution.

Here, No. of bolts, $n = 10$

dia. of bolt, $d = 0.5$ in.

radius of bolt circle, $R = \frac{14}{2}$
$= 7$ in.

Shearing stress, $S_s = 6000$ psi

Now Torque, $T = \left(\frac{\pi d^2}{4} S_s\right) nR$

$$= \frac{\pi \times (0.5)^2}{4} \times 6000 \times 10 \times 7$$

$$= 82466.81 \text{ lb.in.}$$

$$= 6872.23 \text{ lb.ft.}$$ Ans

04. A flanged bolt coupling consists of eight 10mm dia steel bolts on a bolt circle 400mm in dia and six 10mm dia steel bolts on a concentric bolt circle 300mm in dia. What torque can be applied without exceeding a shearing stress of 60 MPa in the bolts?

Solution:

For one bolt in outer circle, $P_1 = \frac{\pi d^2}{4} S_s$

$$= \frac{\pi \times \left(\frac{10}{1000}\right)^2}{4} \times 60 \times 10^6$$

$$= 4712.39 \text{ N.}$$

For one bolt in inner circle, $\frac{P_1}{R_1} = \frac{P_2}{R_2}$

$$\Rightarrow P_2 = \frac{P_1 R_2}{R_1}$$

$$= \frac{4712.39 \times \left(\frac{0.3}{2}\right)}{\frac{0.4}{2}}$$

$$= 3534.29 \text{ N.}$$

Torque, $T = P_1 R_1 n_1 + P_2 R_2 n_2$

$$= (4712.39 \times 0.2 \times 8) + (3534.29 \times 0.15 \times 6)$$

$$= 7539.82 + 3180.86$$

$$= 10720.68 \text{ N.m}$$

$$= 10.72 \text{ kN.m.}$$ Ans

05. A torque of 700 lb.ft is to be carried by a flanged bolt coupling that consists of eight $\frac{1}{2}$ in dia steel bolts on a circle of diameter 12 in and six $\frac{1}{2}$ in dia steel bolts on a circle of dia 9 in. Determine the shearing stress in the bolts.

Solution :

We know $\frac{P_1}{R_1} = \frac{P_2}{R_2}$

$$\Rightarrow \frac{(\pi d^2/4)\, S_{S_1}}{R_1} = \frac{(\pi d^2/4)\, S_{S_2}}{R_2}$$

$$\Rightarrow S_{S_1} = \frac{R_1}{R_2} S_{S_2}$$

$$\therefore S_{S_1} = \frac{6}{4.5} S_{S_2}$$

And $T = P_1R_1n_1 + P_2R_2n_2$

$$\Rightarrow 700\times12 = \frac{\pi}{4} d^2 \times S_{S_1} \times R_1 n_1 + \frac{\pi d^2}{4} \times S_{S_2} \times R_2 n_2$$

$$= \frac{\pi}{4} \times (1/2)^2 \times \frac{6}{4.5} S_{S_2} \times 6\times8 + \frac{\pi\times(1/2)^2}{4} \times S_{S_2} \times 4.5\times6$$

$$\Rightarrow S_{S_2} = \frac{700\times12}{91\pi/16}$$

$= 470.12$ psi. (in the inner bolts)

$$\therefore S_{S_1} = \frac{6}{4.5} \times 470.12$$

$= 626.83$ psi. (in the outer bolts)

Ans: $S_{S_1} = 626.83$ psi

$S_{S_2} = 470.12$ psi

Determine the number of 10mm dia steel bolts that must be used on the 400mm dia bolt circle of the coupling described in problem 04, to increase the torque capacity to 14 kN·m.

Solution:

From problem 04:

$P_1 = 4712.39$ N

$P_2 = 3534.29$ N.

and $R_1 = 200$ mm $= 0.2$ m

$n_1 = ?$

$R_2 = 150$ mm $= 0.15$ m

$n_2 = 6$

$T = 14000$ N·m.

Now $T = P_1 R_1 n_1 + P_2 R_2 n_2$

$\Rightarrow 14000 = 4712.39 \times 0.2 \times n_1 + 3534.29 \times 0.15 \times 6$

$\Rightarrow n_1 = 11.48$ Nos.

≈ 12 bolts $\boxed{\text{Ans}}$

07. A flange bolt coupling consists of six ½ in steel bolts evenly spaced around a bolt circle 12 in. in dia and four 3/4 in aluminium bolts on a concentric bolt circle 8 in. in dia. What torque can be applied without exceeding 9000 psi in the steel or 6000 psi in the aluminium? Assume $G_{st} = 12 \times 10^6$ psi and $G_{al} = 4 \times 10^6$ psi.

Solution:

We know, $\left(\frac{S_s}{GR}\right)_{st} = \left(\frac{S_s}{GR}\right)_{al}$

$$\Rightarrow \frac{S_{s(st)}}{12\times10^6 \times (12/2)} = \frac{S_{s(al)}}{4\times10^6 \times (8/2)}$$

$$\Rightarrow S_{s(st)} = 4.5\, S_{s(al)}$$

Again, $T = (PR\tau)_{st} + (PR\tau)_{al}$

$= \frac{\pi \times (1/2)^2}{4} \times S_{s(st)} \times 6 \times 6 + \frac{\pi \times (3/4)^2}{4} \times S_{s(al)} \times 4 \times 4$

$= \frac{9\pi}{4} S_{s(st)} + \frac{9\pi}{4} S_{s(al)}$

$= \frac{9\pi}{4} (4.5\, S_{s(al)} + S_{s(al)})$

$= \frac{99\pi}{8} \times 6000$

$= 233263.25$ lb.in

$= 233.26$ k.in.

And $T = \frac{9\pi}{4} (S_{s(st)} + S_{s(al)})$

$= \frac{9\pi}{4} (S_{s(st)} + \frac{2}{9} S_{s(st)})$

$= \frac{11\pi}{4} S_{s(st)}$

$= \frac{11\pi}{4} \times 9000$

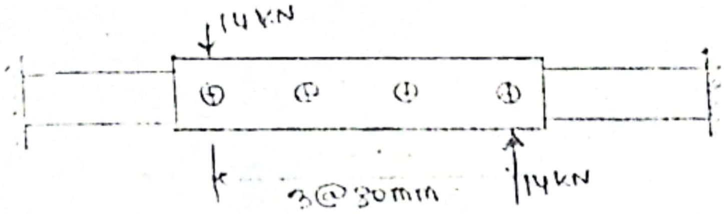
$= 77754.42$ lb.in.

$= 77.75$ kip.in.

∴ The safe value is 77.75 kip.in.

$\boxed{\text{Ans}}$

Q8. A plate is fastened to a fixed member by four 20mm dia rivets arranged as shown in fig. compute the maximum and minimum shearing stress developed.



Solution:

Here, $T = 14000 \times 240$

$= 336 \times 10^4$ N.mm.

$J = \Sigma A \rho^2$

$= A_1 \rho_1^2 n_1 + A_2 \rho_2^2 n_2$

$= \frac{\pi \times (20)^2}{4} (40^2 \times 2 + 120^2 \times 2)$

$= 32 \times 10^5 \pi$ mm^4.

Maximum stress, $\tau = \frac{T\rho}{J}$

$= \frac{336 \times 10^4 \times 120}{32 \times 10^5 \pi}$

$= 40.11$ N/mm^2

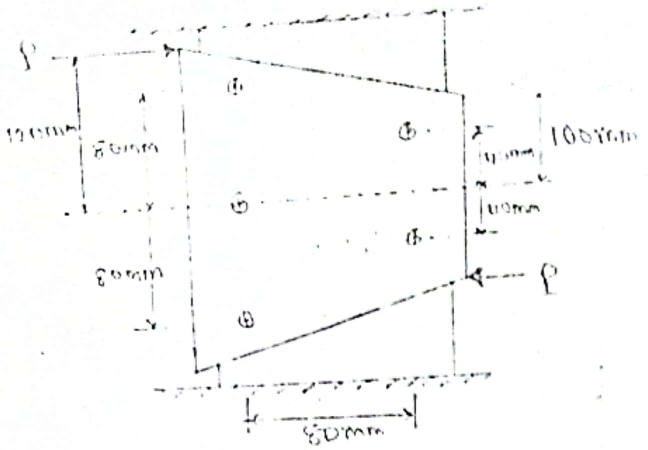
$= 40.11$ MPa. $\boxed{\text{Ans}}$

Minimum stress, $\tau = \frac{336 \times 10^4 \times 40}{32 \times 10^5 \pi}$

$= 13.37$ N/mm^2

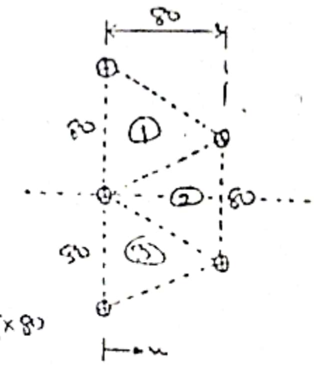
$= 13.37$ MPa. $\boxed{\text{Ans}}$

09. The plate is fastened to the fixed member by five 10mm dia rivets. Compute the value of the load P so that the average shearing stress in any rivet doesn't exceeded 70 MPa.



Solution.

The location of centroid of the rivet group:



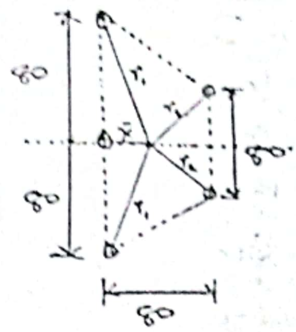
$$\Sigma A = \tfrac{1}{2}(160+80)\,80$$
$$= 9600 \text{ mm}^2$$

$$\Sigma Ax = A_1x_1 + A_2x_2 + A_3x_3$$
$$= (\tfrac{1}{2}\times 80\times 80)\,\tfrac{80}{3} + (\tfrac{1}{2}\times 80\times 80)\times\tfrac{2}{3}\times 80 + (\tfrac{1}{2}\times 80\times 80)\times(\tfrac{80}{3})$$
$$= \frac{1024000}{3} \text{ mm}^3.$$

$$\therefore \bar{x} = \frac{\Sigma Ax}{\Sigma A}$$
$$= \frac{1024000/3}{9600}$$
$$= \frac{320}{9}$$
$$= 35.55 \text{ mm}.$$

Here, $r_1 = \sqrt{80^2 + 35.55^2}$

$= 87.54$ mm.



and $r_2 = \sqrt{(40)^2 + (80-35.55)^2}$

$= 59.80$ mm.

$$J = \Sigma A \rho^2$$

$$= A(2r_1^2 + 2r_2^2 + \bar{x}^2)$$

$$= \frac{\pi \times 10^2}{4}(2 \times 87.54^2 + 2 \times 59.8^2 + 35.55^2)$$

$$= 1864722.612 \text{ mm}^4.$$

The critical rivets are at distance r_1 from the centroid.

$$\tau = \frac{T\rho}{J}$$

$$\Rightarrow 70 = \frac{P(120+100) \times 87.54}{1864722.612}$$

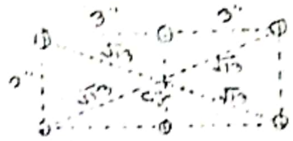
$$\Rightarrow P = 6777.71 \text{ N}.$$

$$= 6.78 \text{ KN}. \quad \boxed{\text{Ans}}$$

10. Six 7/8 in rivets fasten the plate as in the figure below to the fixed member. Find the average shearing stress caused in each rivet by 14 kip loads. What additional load P can be applied before the average shearing stress in any rivet exceeds 15000 psi? 2016

Solution:

The location of centroid of the rivet group:



$$J = \Sigma A \rho^2$$

$$= \frac{\pi d^2}{4}(4\rho_1^2 + 2\rho_2^2)$$

$$= \frac{\pi \times (7/8)^2}{4}\left\{4\times(\sqrt{13})^2 + 2\times\left(\frac{2}{2}\right)^2\right\}$$

$$= 32.47 \text{ in}^4.$$

Determination of average shearing stress:

$T = 14 \times 10 = 140$ k.in.

$$\tau_{max} = \frac{T\rho_1}{J}$$

$$= \frac{140 \times \sqrt{13}}{32.47}$$

$= 15.55$ ksi. $\boxed{\text{Ans}}$

$$\tau_{min} = \frac{T\rho_2}{J}$$

$$= \frac{140 \times 1}{32.47}$$

$= 4.31$ ksi. $\boxed{\text{Ans}}$

Determination of additional load P:

1st case (P<14 k):

$$T = 14\times10 - P\times6$$
$$= 140 - 6P \text{ k-in.}$$

$$\tau = \frac{T\rho}{J}$$

$$\Rightarrow 8000 = \frac{(140-6P)\times1000\times\sqrt{13}}{32.47}$$

$$\Rightarrow P = 11.33 \text{ kips.} \boxed{\text{Ans}}$$

2nd case (P>14 k):

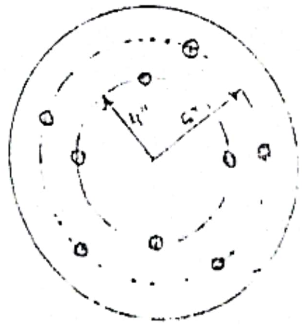
$$T = 6P - 140 \text{ k-in.}$$

$$\tau = \frac{T\rho}{J}$$

$$\Rightarrow \frac{8000}{1000} = \frac{(6P-140)\times\sqrt{13}}{32.47}$$

$$\Rightarrow P = 35.34 \text{ kips.} \boxed{\text{Ans}}$$

11. Determine the maximum shearing stress developed in the bolts of the coupling shown in the following figure. The coupling transmit 400 hp at 630 rpm, and all bolts have equal diameters of 3/8 inch. [2010, 2012]



Solution: Given, power, P = 400 hp.

N = 630 rpm.

We know Power, $P = \frac{TN}{63000}$

$$\Rightarrow T = \frac{63000P}{N}$$

$$= \frac{63000 \times 400}{630}$$

$$= 40000 \text{ lb.in.}$$

$$T = P_1R_1n_1 + P_2R_2n_2$$

$$\Rightarrow 40000 = P_1 \times 6 \times 5 + P_2 \times 4 \times 4$$

$$\Rightarrow 30P_1 + 16P_2 = 40000 \text{ .... (I)}$$

and $\frac{P_1}{R_1} = \frac{P_2}{R_2}$

$$\Rightarrow P_1 = \frac{R_1}{R_2}P_2$$

$$= \frac{6}{4}P_2$$

$$\Rightarrow P_1 - 1.5P_2 = 0 \text{ .... (II)}$$

Solving equations (I) & (II);

$P_1 = 983.61$ lb.

$P_2 = 655.74$ lb.

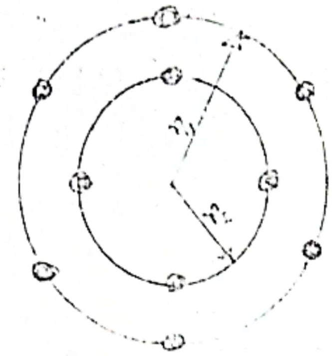
Maximum shearing stress, $S_s = \frac{P_1}{A}$

$$= \frac{983.61}{\frac{\pi \times (3/8)^2}{4}}$$

$$= 8905.75 \text{ psi}$$

$\boxed{\text{Ans}}$

12. Twelve ½ inch full diameter bolts are arranged in two concentric circles in flanged couplings similar to the figure shown below. Find the maximum horsepower that can be transmitted by the coupling if the shaft speed is 315 rpm and the maximum permissible shearing stress is 5000 psi. Here: $R_1 = 8''$, $R_2 = 4''$. [2011, 2015]



Solution: Given,

$n_1 = 8$

$n_2 = 4$

$R_1 = 8''$

$R_2 = 4''$

$d = ½''$

$N = 315$ rpm.

$S_s = 5000$ psi.

We know.

$$S_s = \frac{P_1}{A}$$

$$\Rightarrow 5000 = \frac{P_1}{\frac{\pi \times (1/2)^2}{4}}$$

$$\Rightarrow P_1 = 981.75 \text{ lb}.$$

$$\frac{P_1}{R_1} = \frac{P_2}{R_2}$$

$$\Rightarrow P_2 = \frac{4}{8} \times 981.75$$

$$= 490.87 \text{ lb}.$$

Torque, $T = P_1 R_1 n_1 + P_2 R_2 n_2$

$= 981.75 \times 8 \times 6 + 490.87 \times 4 \times 4$

$= 54977.92$ lb.in

Horsepower, $P = \dfrac{TN}{63000}$

$= \dfrac{54977.92 \times 315}{63000}$

$= 274.89$ hp.

$\boxed{\text{Ans}}$

HELICAL SPRING

Spring: A spring may be defined as an elastic member whose primary function is to deflect or distord under the action of applied load, and it recovers its original shape when load is released.

Helix: A curve that lies on the surface of a cylinder or cone and cuts the object at a constant angle.

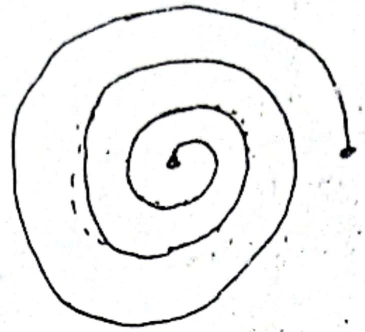
Helix angle: Helix angle is the angle between any helix and an axial line on its right circular cylinder or cone. Helix angle of the closed coiled helical spring is limited to 10° maximum. For open coiled helical spring helix angle is larger than 10°.

Pitch of a helix: The pitch of a helix is defined as the axial distance between two similar points on the adjacent coils, ie.

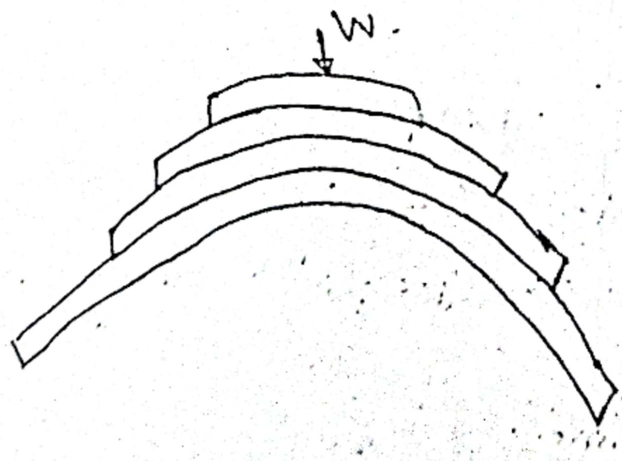
Types of spring:

1. Spiral spring: The are made of flat strip of metal wound in the form of spiral and

loaded in torsion. In this spring the major stresses are tensile and compression due to bending.



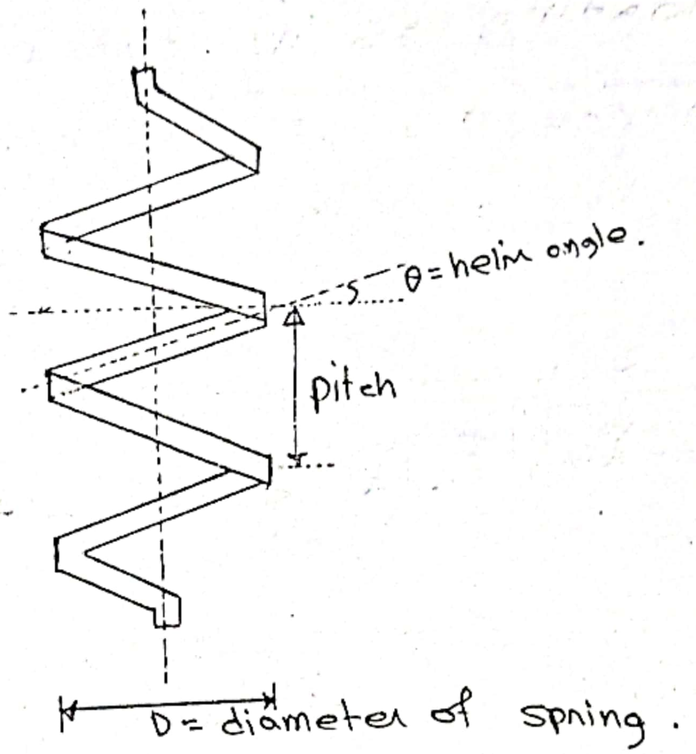
2. Leaf spring: They are composed of flat bars of varying lengths clamped together so as to obtain greater efficiency. It may be full elliptic, semi elliptic or cantilever types. In these type of springs the major stresses which come into picture are tensile and compressive. These type of springs are used in the automobile suspension system.



3. Helical Spring :

They are made of wire coiled into a helical form, the load being applied along the axis of the helix. In these type of springs the major stresses is torsional shear stress due to twisting.

The are both used in tension and compression.



3.1. closed coil helical spring :

In helical spring if coil is wound tightly providing no visible gap between two adjacent coils then the spring is called close coiled helical spring.

3.2. Open coil helical spring :

when sufficient gap is provided between two adjacent coils then the spring is called open coiled helical spring.

Uses of spring:

1. To apply forces and to control motions as in brakes.

2. To measure forces as in spring balance.

3. To store energy as in clock spring.

4. To reduce the effect of shock or impact loading as in carriage springs.

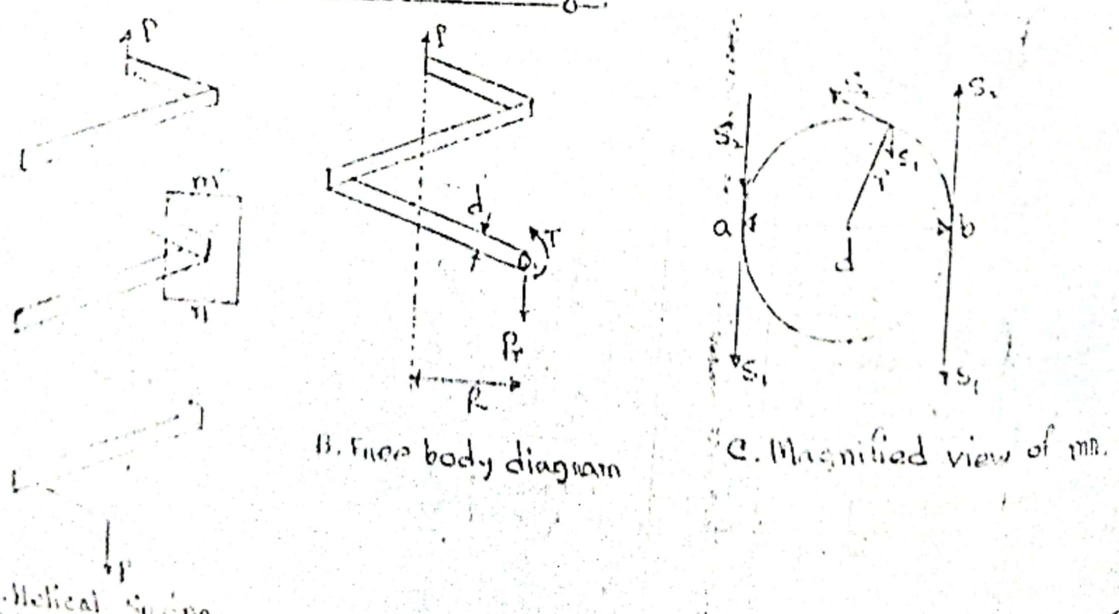
5. To change the vibrating characteristics of a member as inflexible mounting motors.

Assumptions in deriving the formula:

1. The bending and shear effects may be neglected.

2. The helix angle may be neglected

Derive an expression for the maximum shearing stress of a helical spring:



A. Helical Spring

B. Free body diagram

C. Magnified view of mn.

The closed coil helical spring is shown in fig. A. is elongated by an axial load P. To determine the stresses produced by P a plane is considered through any typical section mn.

To balance the applied axial load P the spring must provide the resistance P_r equal to P being equal, opposite and parallel create a couple of magnitude PR which can be balanced only by an opposite couple. This resisting couple is created by a torsional shearing stress distributed over the cross section of the spring and is represented by $T = PR$.

Two types of shearing stress are produced–

1. Direct shearing stress S_1 and
2. Variable torsional shearing stress S_2.

At point b the stresses are oppositely directed and at point a the two shearing stresses are colinear and in the same sense.

The maximum shearing stress occurs at the inside element and is given by the sum of the direct shearing stress ($S_1 = P/A$) and the maximum value of torsional shearing stress ($S_2 = \frac{Tr}{J}$).

$$S_s = S_1 + S_2$$

$$= P/A + \frac{Tr}{J}$$

$$= \frac{P}{\frac{\pi d^2}{4}} + \frac{T(d/2)}{\frac{\pi d^4}{32}}$$

$$= \frac{4P}{\pi d^2} + \frac{16 PR}{\pi d^3} \qquad [\because T = PR]$$

$$= \frac{16 PR}{\pi d^3} \cdot \frac{d}{4R} + \frac{16 PR}{\pi d^3}$$

$$\boxed{S_{s(max)} = \frac{16 PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)}$$

Use this formula when the spring is not curvature

This formula neglects the curvature of the spring. This is used for light spring where the ratio $\frac{d}{4R}$ is small.

For heavy spring and considering the curvature of the spring R.A.M. Whal formula is more precise. and it is given by

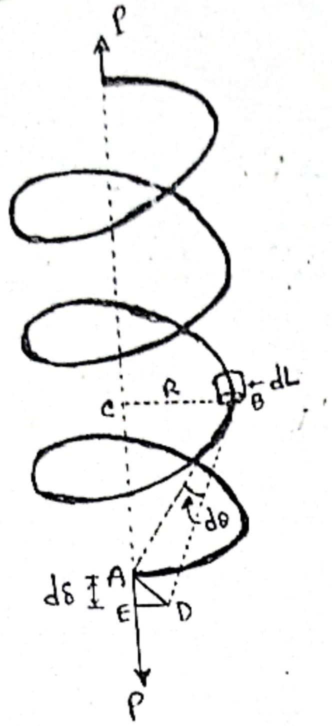
$$\boxed{S_s = \frac{16 PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)}$$

Where m is called the spring index and $\frac{4m-1}{4m-4}$ is called the Wahl Factor.

Where $m = \frac{2R}{d} = \frac{D}{d}$

Hence $$\boxed{S_{s(max)} = \frac{16 PR}{\pi d^3}\left(1 + \frac{0.5}{m}\right)}$$

Derive an expression for deflection of spring:



consider the spring in figure to be rigid except the small length dL.

The end A will rotate to D through the small angle $d\theta$. As $d\theta$ is small angle so that the arc AD is considered as a straight line.

From triangle ABD;

$$AD = AB.d\theta$$

From the similar triangle ADE and BAC

$$\frac{AE}{AD} = \frac{BC}{AB}$$

$$\Rightarrow \frac{d\delta}{AB.d\theta} = \frac{BC}{AB}$$

$$\Rightarrow d\delta = R\,d\theta$$

We know $\theta = \frac{TL}{JG}$

Differentiating w.r.t. L

$$\frac{d\theta}{dL} = \frac{T}{JG}$$

$$\Rightarrow d\theta = \frac{T}{JG}dL$$

$$\Rightarrow d\theta = \frac{PR}{JG} dL$$

Hence $\quad d\delta = R \cdot \frac{PR}{JG} dL$

Total elongation on deflection can be obtained by integrating all the elements of spring.

$$\int d\delta = \frac{PR^2}{JG} \int dL$$

$$\Rightarrow \delta = \frac{PR^2 L}{JG}$$

Replacing L by $2\pi R n$ which is the length of n coils of radius R, and J by $\frac{\pi d^4}{32}$

$$\delta = \frac{P \cdot R^2 \cdot 2\pi R n}{\frac{\pi d^4}{32} G}$$

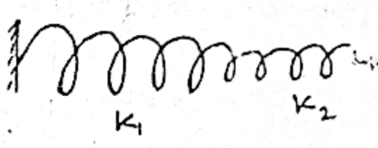
$$\boxed{\delta = \frac{64 P R^3 n}{G d^4}}$$

Spring constant (k): The ratio of applied load P to the deflection δ is called the spring constant.

$$\boxed{k = \frac{P}{\delta}}$$

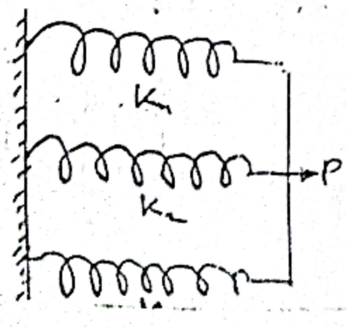
Spring in series:

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$



Spring in parallel:

$$k = k_1 + k_2 + k_3$$



Q) Determine the maximum shearing stress and elongation in a helical steel spring composed of 20 turns of 20mm dia. wire on a mean radius of 90mm when the spring is supporting a load of 1.5 kN. G = 83 GPa.

Solution : Given,

$n = 20$

$d = 20mm$

$R = 90mm$

$G = 83\ GPa$

$= 83 \times 10^3\ N/mm^2$

$P = 1.5k$

$= 1500N.$

We know,

$$S_{s(max)} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

$$= \frac{16 \times 1500 \times 90}{\pi \times (20)^3}\left(1 + \frac{20}{4 \times 90}\right)$$

$$= 90.72\ N/mm^2.$$

$$= 90.72\ MPa. \quad \boxed{Ans}$$

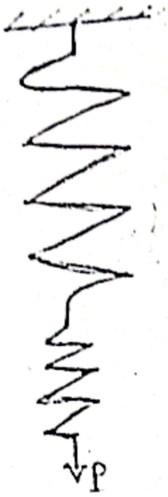
and

$$\delta = \frac{64PR^3n}{Gd^4}$$

$$= \frac{64 \times 1500 \times 90^3 \times 20}{83 \times 10^3 \times (20)^4}$$

$$= 105.40\ mm. \quad \boxed{Ans}$$

02. Two steel springs arranged in series and support a load P. The upper spring has 12 turns of 25mm diameter wire on a mean radius of 100mm. The lower spring consists of 10 turns of 20mm diameter wire on a mean radius of 75mm. If the maximum shearing stress in either spring must not exceed 200 MPa, compute the maximum value of P and the total elongation of the assembly. Use R.A.M. Wahl formula and G = 83GPa. Compute the equivalent spring constant by dividing the load by the total elongation.



Solution:

For upper spring:

Given,

$n = 12$

$d = 25m$

$R = 100\,mm$

$\therefore m = \dfrac{2R}{d}$

$= \dfrac{2 \times 100}{25}$

$= 8$

$$S_{s(max)} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

$$\Rightarrow 200 = \frac{16P \times 100}{\pi \times (25)^3}\left(\frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8}\right)$$

$$\Rightarrow P = 5182.29\ N$$

For lower spring:

$$n = 10$$
$$d = 20 \text{ mm}$$
$$R = 75 \text{ m}$$
$$m = \frac{2R}{d} = \frac{75 \times 2}{20} = 7.5$$

$$\therefore \quad S_{s(max)} = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

$$\Rightarrow 200 = \frac{16 \times P \times 75}{\pi \times (20)^3}\left(\frac{4 \times 7.5 - 1}{4 \times 7.5 - 4} + \frac{0.615}{7.5}\right)$$

$$\Rightarrow P = 3498.28 \text{ N}.$$

For safe value, $P = 3498.28$ N. $\boxed{\text{Ans}}$

Total elongation, $\delta = \delta_1 + \delta_2$

$$= \left(\frac{64PR^3n}{Gd^4}\right)_1 + \left(\frac{64PR^3n}{Gd^4}\right)_2$$

$$= \frac{64 \times 3498.28 \times 100^3 \times 12}{83 \times 10^3 \times (25)^4} + \frac{64 \times 3498.28 \times 75^3 \times 10}{83 \times 10^3 \times (20)^4}$$

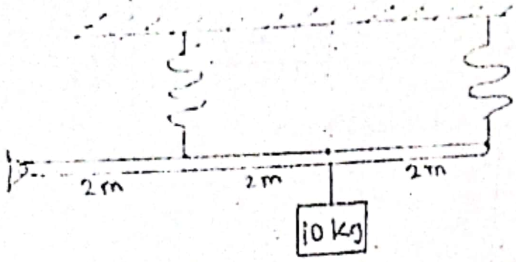
$$= 153.99 \text{ mm}. \quad \boxed{\text{Ans}}$$

Equivalent spring constant, $K_{equivalent} = \frac{P}{\delta}$

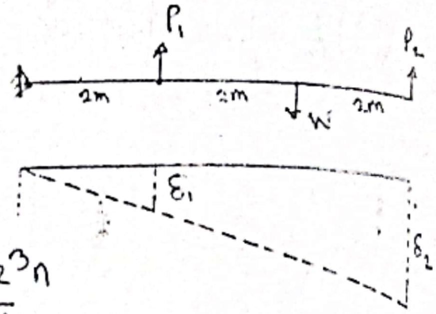
$$= \frac{3498.28}{153.99}$$

$$= 22.72 \text{ N/mm}. \quad \boxed{\text{Ans}}$$

03. A rigid bar, hinged at one end, is supported by two identical springs. Each spring consists of 20 turns of 10mm wire having a mean diameter of 150 mm. Compute the maximum shearing stress in the springs. Neglect the mass of the rigid bar.



Solution:



$$\frac{\delta_1}{2} = \frac{\delta_2}{6}$$

$$\Rightarrow \delta_1 = \frac{1}{3}\delta_2$$

$$\Rightarrow \frac{64 P_1 R^3 n}{G d^4} = \frac{1}{3}\,\frac{64 P_2 R^3 n}{G d^4}$$

$$\Rightarrow P_1 = \frac{P_2}{3}$$

$\Sigma M_{\text{at hinge}} = 0$

$$\Rightarrow W \times 4 - P_1 \times 2 - P_2 \times 6 = 0$$

$$\Rightarrow 2P_1 + 6P_2 = 4W$$

$$\Rightarrow P_1 + 3P_2 = 2 \times (10 \times 9.81)$$

$$\Rightarrow \frac{P_2}{3} + 3P_2 = 2 \times 98.1$$

$$\Rightarrow P_2 = 58.86\ \text{N}.$$

and $P_1 = \frac{58.86}{3}$

$= 19.62\ \text{N}.$

For spring at left, $S_{s1} = \frac{16 P_1 R}{\pi d^3}\left(1 + \frac{d}{4R}\right)$

$$= \frac{16 \times 19.62 \times 75}{\pi \times (10)^3}\left(1 + \frac{10}{4 \times 75}\right)$$

$= 7.744\ \text{N/mm}^2$

$= 7.744\ \text{MPa}$ $\boxed{\text{Ans}}$

for spring at right, $S_{s_2} = \frac{16P_2R}{\pi d^3}\left(1+\frac{d}{4R}\right)$

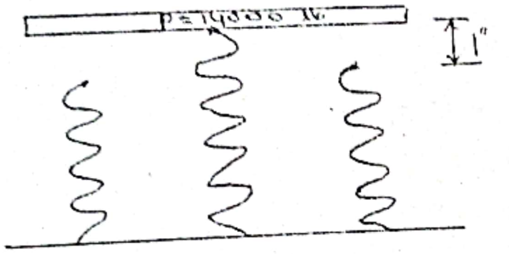
$= \frac{16\times 58.86\times 75}{\pi\times(10)^3}\left(1+\frac{10}{4\times 75}\right)$

$= 23.232\ N/mm^2$

$= 23.232\ MPa$ $\boxed{Ans}$

4. A rigid plate rests on the central spring as shown in figure is 1 inch higher than the symmetrically located outer springs. Each of the outer spring consists of 18 turns of 0.5 inch dia of wire of mean dia of 4 in each. The central spring has 24 turns of 0.75 inch dia wire of mean dia 6 inch. Determine maximum shearing stresses using R.AM Wahl formula in each spring for P = 14000 lb. G = 30×10⁶ psi.

[2005, 2012]



Solution:

For inner spring:
$n = 24$
$d = 0.75$ in
$D = 6$ inch.

For outer springs:
$n = 18$
$d = 0.5$ in;
$D = 4$ in.

$$\delta = \frac{64P'R^3n}{Gd^4}$$

$$\Rightarrow 1 = \frac{64\times P'\times(3)^3\times 24}{30\times 10^6\times(0.75)^4}$$

$$\Rightarrow P' = 228.88\ lb.$$

Now

$$P_{in} + 2P_{out} = 14000 - 228.88$$

$$= 13771.12 \quad \cdots\cdots (1)$$

and $\delta_{in} = \delta_{out}$

$$\Rightarrow \left(\frac{64P'R^3n}{Gd^4}\right)_{in} = \left(\frac{64PR^3n}{Gd^4}\right)_{out}$$

$$\Rightarrow \left(\frac{P'R^3 n}{d^4}\right)_{in} = \left(\frac{PR^3 n}{d^4}\right)_{out}$$

$$\Rightarrow \frac{P'_{in} \times 3^3 \times 24}{(0.75)^4} = \frac{P_{out} \times 2^3 \times 18}{(0.5)^4}$$

$$\Rightarrow P'_{in} - 1.125 P_{out} = 0 \text{ ---------- (ii)}$$

Solving equations (i) & (ii);

$P'_{in} = 4957.6$ lb.

$P_{out} = 4406.76$ lb.

$$\therefore P_{in} = P'_{in} + P'$$
$$= 4957.6 + 228.88$$
$$= 5186.48 \text{ lb.}$$

Stress in inner spring $= \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$

$$= \frac{16 \times 5186.48 \times 3}{\pi \times (0.75)^3}\left(1 + \frac{0.75}{4 \times 3}\right)$$

$$= 199576.42 \text{ psi.}$$

$\boxed{\text{Ans}}$

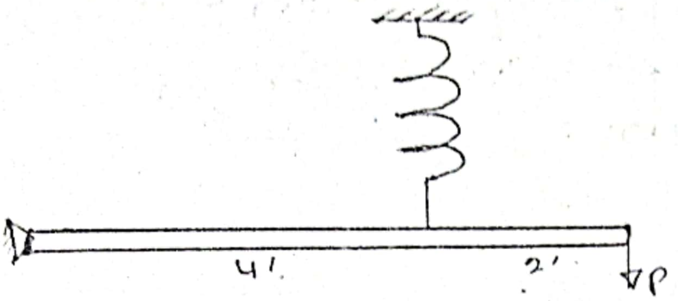
Stress in outer spring $= \frac{16PR}{\pi d^3}\left(1 + d/4R\right)$

$$= \frac{16 \times 4406.76 \times 2}{\pi \times (0.5)^3}\left(1 + \frac{0.5}{4 \times 2}\right)$$

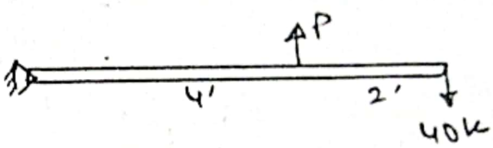
$$= 381538.55 \text{ psi.}$$

$\boxed{\text{Ans}}$

05. The rigid beam shown in the figure is hinged at one end and supported by a helical spring. Determine the stress and deflection in the spring if $P = 40$ kips. The spring consists of 24 turns of $\frac{1}{2}$ in diameter wire on a mean coil diameter of 10 inch. Given $G = 12 \times 10^6$ psi. [2007]



Solution :



$\Sigma M_{\text{at hinge}} = 0$

$\Rightarrow 40 \times 6 - P \times 4 = 0$

$\Rightarrow P = 60$ kip.

Stress in spring, $S_s = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$

$= \frac{16 \times 60 \times 10^3 \times 5}{\pi \times (1/2)^3}\left(1 + \frac{1/2}{4 \times 5}\right)$

$= 12528677.12$ psi.

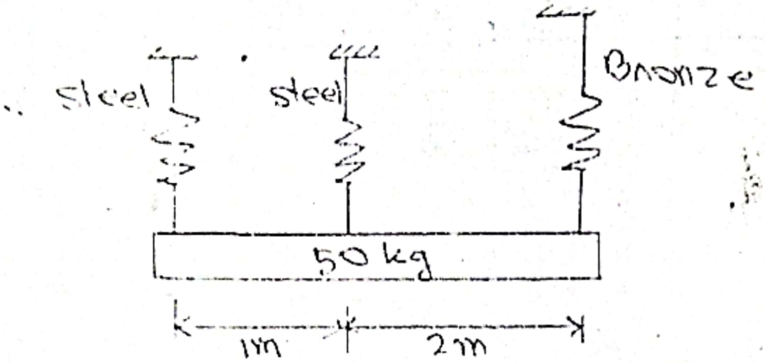
$= 12528.68$ ksi $\boxed{\text{Ans}}$

Deflection, $\delta = \frac{64 P R^3 n}{G d^4}$

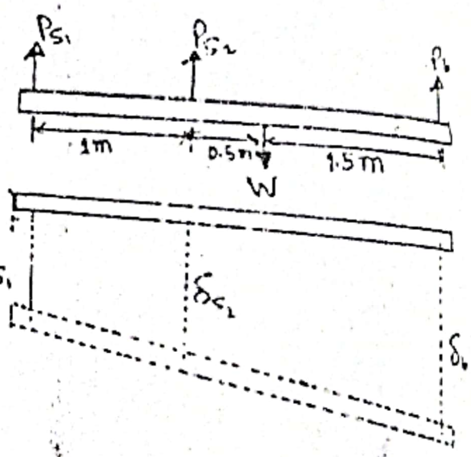
$= \frac{64 \times 60 \times 10^3 \times 5 \times 24}{12 \times 10^6 \times (1/2)^4}$

$= 614.4$ in $\boxed{\text{Ans}}$

06. A homogeneous 50 kg rigid block is supported by the three springs whose lower ends were originally at the same level. Each steel spring has 24 turns of 10mm dia wire on a mean dia of 100mm and G = 83 GPa. The bronze spring has 48 turns of 20mm dia wire on a mean dia of 150 mm and G = 42 GPa. Compute the maximum shearing stress in each spring. [2008, 2016, 2018]



Solution:



$\Sigma F_y = 0$

$\Rightarrow P_{s_1} + P_{s_2} + P_b = W$

$= 50 \times 9.81$

$\Rightarrow P_{s_1} + P_{s_2} + P_b = 490.5 \quad \text{.......(I)}$

and $\dfrac{\delta_{s_2} - \delta_{s_1}}{1} = \dfrac{\delta_b - \delta_{s_1}}{3}$

$\Rightarrow 3\delta_{s_2} - 3\delta_{s_1} + \delta_{s_1} - \delta_b = 0$

$\Rightarrow 3\delta_{s_2} - 2\delta_{s_1} - \delta_b = 0$

$\Rightarrow 3\left(\dfrac{64PR^3n}{Gd^4}\right)_{s_2} - 2\left(\dfrac{64PR^3n}{Gd^4}\right)_{s_1} - \left(\dfrac{64PR^3n}{Gd^4}\right)_b = 0$

$\Rightarrow \dfrac{3 \times P_{s_2} \times 50^3 \times 24}{83 \times 10^3 \times 10^4} - \dfrac{2 \times P_{s_1} \times 50^3 \times 24}{83 \times 10^3 \times 10^4} - \dfrac{P_b \times 75^3 \times 48}{4[illegible]} = 0$

$\Rightarrow \dfrac{9P_{s_2}}{830} - \dfrac{3P_{s_1}}{415} - \dfrac{27P_b}{8960} = 0 \quad \text{.......(II)}$

And $\Sigma M_{s_1} = 0$

$\Rightarrow W \times 1.5 - P_{s_2} \times 1 - P_b \times 3 = 0$

$\Rightarrow P_{s_2} + 3 P_b = 490.5 \times 1.5$

$= 735.75$ .... (iii)

Solving equations (i), (ii) & (iii);

$P_{s_1} = 144.78$ N

$P_{s_2} = 150.71$ N.

$P_b = 195.01$ N.

stress in outer steel spring $= \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$

$= \frac{16 \times 144.78 \times 50}{\pi \times (10)^3}\left(1 + \frac{10}{4 \times 50}\right)$

$= 38.71$ N/mm²

$= 38.71$ MPa. $\boxed{\text{Ans}}$

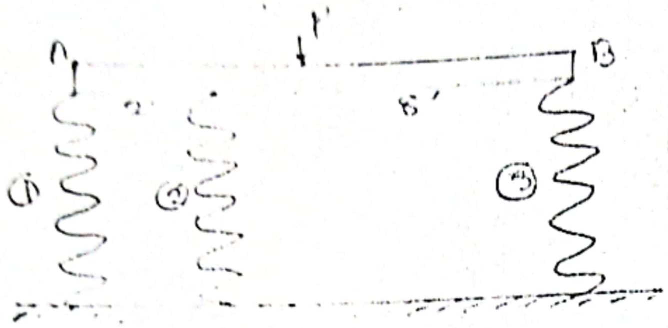
stress in inner steel spring $= \frac{16 \times 150.71 \times 50}{\pi \times 10^3}\left(1 + \frac{10}{4 \times 50}\right)$

$= 40.30$ MPa $\boxed{\text{Ans}}$

stress in bronze spring $= \frac{16 \times 195.01 \times 75}{\pi \times (20)^3}\left(1 + \frac{20}{4 \times 75}\right)$

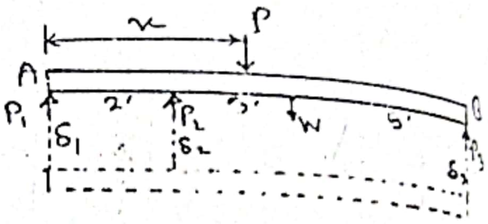
$= 9.93$ MPa $\boxed{\text{Ans}}$

07. The rigid bar AB weights 400 lb and supports a load P = 1000 lb. If these free lengths of the springs are equal to prior to loading, where should be the load 'P' placed if the bar is to remain horizontal? $G = 12\times10^6$ psi. [2010]



Spring-1	Spring-2	Spring-3
$n = 28$	$n = 24$	$n = $ [illegible]
$d = 1/2"$	$d = 3/4"$	$d = $ [illegible]
$D = 4"$	$D = 6"$	$D = $ [illegible]

Solution:



$\Sigma F_y = 0$

$\Rightarrow P_1 + P_2 + P_3 = P + W$

$= 1000 + 400$

$= 1400$ -------- ①

$\Sigma M_A = 0$

$\Rightarrow P_2 \times 2 + P_3 \times 10 - W \times 5 - P \times x = 0$

$\Rightarrow Px - 2P_2 - 10P_3 = -4 \times 500$

$= -2000$ ------- ⑪

And $\delta_1 = \delta_2 = \delta_3$

$\therefore \delta_1 = \delta_2$

$\Rightarrow \left(\frac{64PR^3n}{Gd^4}\right)_1 = \left(\frac{64PR^3n}{Gd^4}\right)_2$

$\Rightarrow \frac{P_1 \times 2^3 \times 28}{(1/2)^4} = \frac{P_2 \times 3^3 \times 24}{(3/4)^4}$

$\Rightarrow P_1 = \frac{4}{7} P_2$

and $\delta_2 = \delta_3$

$$\Rightarrow \left(\frac{64PR^3n}{Gd^4}\right)_2 = \left(\frac{64PR^3n}{Gd^4}\right)_3$$

$$\Rightarrow \frac{P_2 \times 3^3 \times 24}{(3/4)^4} = \frac{P_3 \times 4^3 \times 20}{(1)^4}$$

$$\Rightarrow P_3 = \frac{8}{5} P_2$$

from equation (I)

$$\frac{4}{7} P_2 + P_2 + \frac{8}{5} P_2 = 1400$$

$$\Rightarrow P_2 = 441.44 \text{ lb}.$$

$$\therefore P_3 = \frac{8}{5} \times 441.44$$

$$= 706.3 \text{ lb}.$$

From equation (II),

$$Px = -2000 + 2 \times 441.44 + 10 \times 706.3$$

$$\Rightarrow Px = 5945.92$$

$$\Rightarrow x = \frac{5945.92}{P}$$

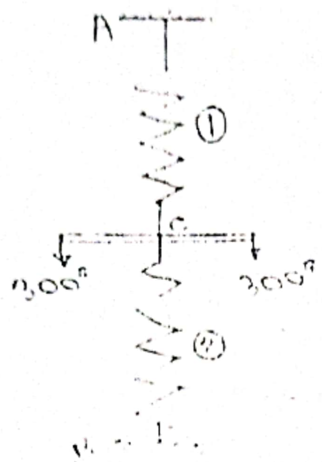
$$= \frac{5945.92}{1000}$$

$$= 5.95 \text{ ft}.$$

The load P should be placed at 5.95 ft from A end.

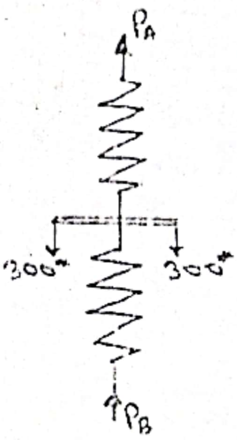
$\boxed{\text{Ans}}$

08. Two springs as shown in the following figure, are joined at C and then placed unstressed, below the supports A and B. Find the reactions at the supports if a force of 600 lb acts as shown. G = 12×10⁶ psi [2013]



Spring-1	Spring-2
$n = 20$	$n = 10$
$d = 1/4''$	$d = 1/2''$
$D = 5''$	$D = 6''$

Solution:



$\Sigma F_y = 0$

$\Rightarrow P_A + P_B - 600 = 0$

$\Rightarrow P_A + P_B = 600$ ------- (I)

and $\delta_A = \delta_B$

$\Rightarrow \left(\dfrac{64PR^3n}{Gd^4}\right)_A = \left(\dfrac{64PR^3n}{Gd^4}\right)_B$

$\Rightarrow \dfrac{P_A \times 2.5^3 \times 20}{(1/4)^4} = \dfrac{P_B \times 3^3 \times 10}{(1/2)^4}$

$\Rightarrow P_A = \dfrac{27}{500} P_B$

$\Rightarrow P_A - 0.054 P_B = 0$ .... (II)

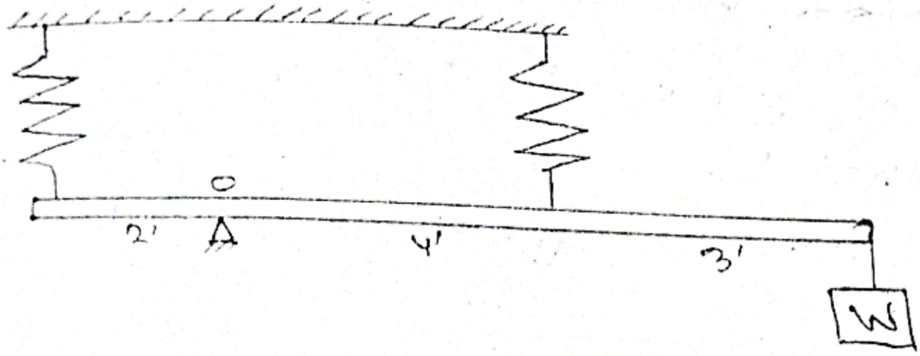
Solving equations (I) & (II);

$P_A = 30.74$ lb.

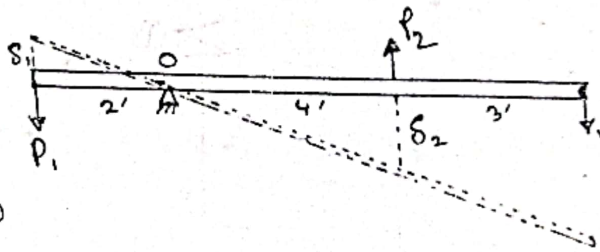
$P_B = 569.26$ lb.

Ans

09. A rigid bar, pinned at point O, is supported by two identical springs as shown in figure below. Each spring consists of 20 turns of 3/4 in diameter wire having a mean diameter of 6 in. Determine the maximum load W that may be supported if the shearing stress in the springs is limited to 20 ksi. [2013, 2017]



Solution:



$\Sigma M_o = 0$

$\Rightarrow P_1 \times 2 + P_2 \times 4 - W \times 7 = 0$

$\Rightarrow 2P_1 + 4P_2 = 7W$ ------ (I)

From the similar triangle;

$$\frac{\delta_1}{2} = \frac{\delta_2}{4}$$

$$\Rightarrow 2\left(\frac{64PR^3n}{Gd^4}\right)_1 = \left(\frac{64PR^3n}{Gd^4}\right)_2$$

$\Rightarrow 2P_1 = P_2$ ------- (II)

Maximum stress $= \frac{16P_2R}{\pi d^3}\left(1 + \frac{d}{4R}\right)$ [$\because P_2 > P_1$]

$$\Rightarrow 20 \times 10^3 = \frac{16 \times P_2 \times 3}{\pi \times (3/4)^3}\left(1 + \frac{3/4}{4 \times 3}\right)$$

$\Rightarrow P_2 = 519.75$ lb.

From equation (II); $P_1 = \frac{1}{2} \times 519.75$

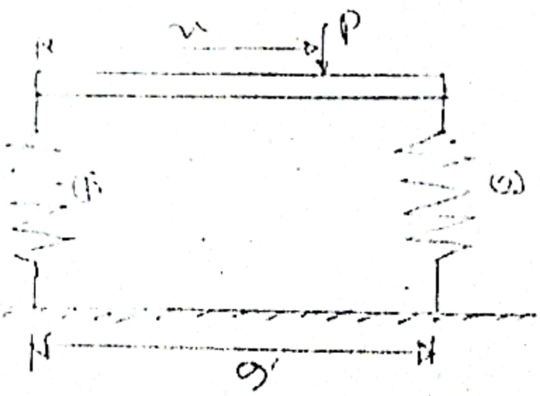
$= 259.87$ lb.

From equation ①

$$2\times 259.87 + 4\times 519.75 = 7W$$

$$\Rightarrow W = 371.25 \text{ lb.} \quad \boxed{\text{Ans}}$$

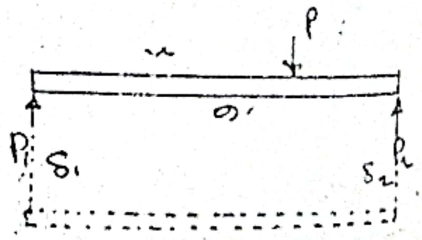
10. A rigid horizontal bar of negligible weight is supported by two springs as shown in the following figure. Determine the distance x in order that the bar remains horizontal after a load P is applied. [2015]



Spring-1	Spring-2
$d = 0.25''$	$d = 0.40''$
$R = 1''$	$R = 2''$
$n = 25$	$n = 10$

$G = 12\times10^6$ psi.

Solution:



$\Sigma F_y = 0$

$\Rightarrow P_1 + P_2 = P \; \ldots\ldots$ ①

and $\delta_1 = \delta_2$

$$\Rightarrow \left(\frac{64PR^3n}{Gd^4}\right)_1 = \left(\frac{64PR^3n}{Gd^4}\right)_2$$

$$\Rightarrow \frac{P_1\times1\times25}{(0.25)^4} = \frac{P_2\times2^3\times10}{(0.40)^4}$$

$$\Rightarrow 2.048P_1 = P_2$$

From equation (1)

$$P_1 + 2.048 P_1 = P$$

$$\Rightarrow P_1 = \frac{P}{3.048}$$

$$\Sigma M_2 = 0$$

$$\Rightarrow P_1 \times 9 - P(9 - x) = 0$$

$$\Rightarrow \frac{P}{3.048} \times 9 = P(9 - x)$$

$$\Rightarrow \frac{9}{3.048} = 9 - x$$

$$\Rightarrow x = 9 - \frac{9}{3.048}$$

$$\therefore x = 6.05 \text{ ft.} \quad \boxed{\text{Ans}}$$

COMBINED STRESSES

Combined stress:

Combined stress is defined as any possible combinations of direct stress and indirect stress developed inside the body.

Three basic types of loadings and corresponding stresses:

1. Axial ⟶ $\sigma_a = P/A$
2. Torsional and ⟶ $\tau = \frac{Tr}{J}$
3. Flexural ⟶ $\sigma_f = \frac{My}{I}$

There are four possible combinations of these loadings:

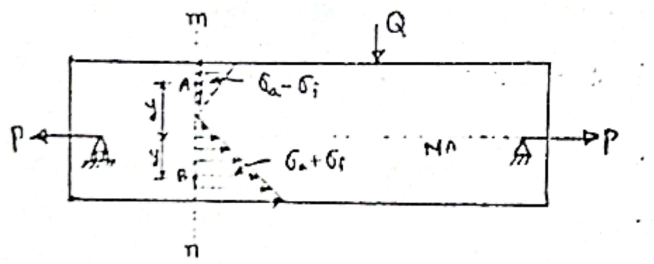
1. Axial and torsional
2. Torsional and flexural
3. Axial and flexural
4. Axial, torsional and flexural.

Direct stress: Stress which are normal to the plane on which the act are called direct stresses and they are either tensile or compressive.

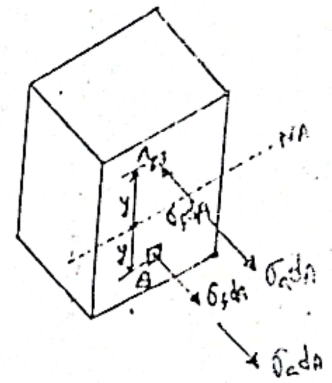
Indirect stress: If the stress is developed due to moment or torsion then it is called as direct stress. Flexural stress and torsional stress are indirect stress.

Derive an illustration for combined axial and bending stress. 2010, 2017.

Derivation:



combined axial and flexure stress.



Section mn

Consider a simply supported beam carries a concentrated load Q and an axial load P. At point B, the flexural stress $\sigma_f = \frac{My}{I}$. It is tensile and is directed normal to the surface of the cross section. The axial stress $\sigma_a = P/A$ which is also tensile and directed normal to the cross section.

The fonce exented on the element at B is $P_1 = \sigma_f dA$. due to flexune and

The fonce exented on the element at B is $P_2 = \sigma_a dA$ due to axial load.

The nesultant fonce at A is equal to the vecton sum of the fonces P_1 and P_2:

$$P = P_1 + P_2$$

Dividing the above equation by dA we get the nesultant stness,

$$\frac{P}{dA} = \frac{P_1}{dA} + \frac{P_2}{dA}$$

$$\Rightarrow \sigma = \frac{\sigma_f dA}{dA} + \frac{\sigma_a dA}{dA}$$

$$\therefore \boxed{\sigma = \sigma_f + \sigma_a}$$

Similanly at a point A in the same section at a distance y above the neutnal axis the nesultant stness is the diffenence between the axial and flexunal stnesses.

$$\sigma = \sigma_a - \sigma_f .$$

If tensile stness is denoted by a positive sign and compnessive stness by a negetive sign, the nesultant stness at any point of the beam is

$$\sigma = \sigma_a \pm \sigma_f$$

$$\boxed{\sigma = \pm \frac{P}{A} \pm \frac{My}{I}}$$

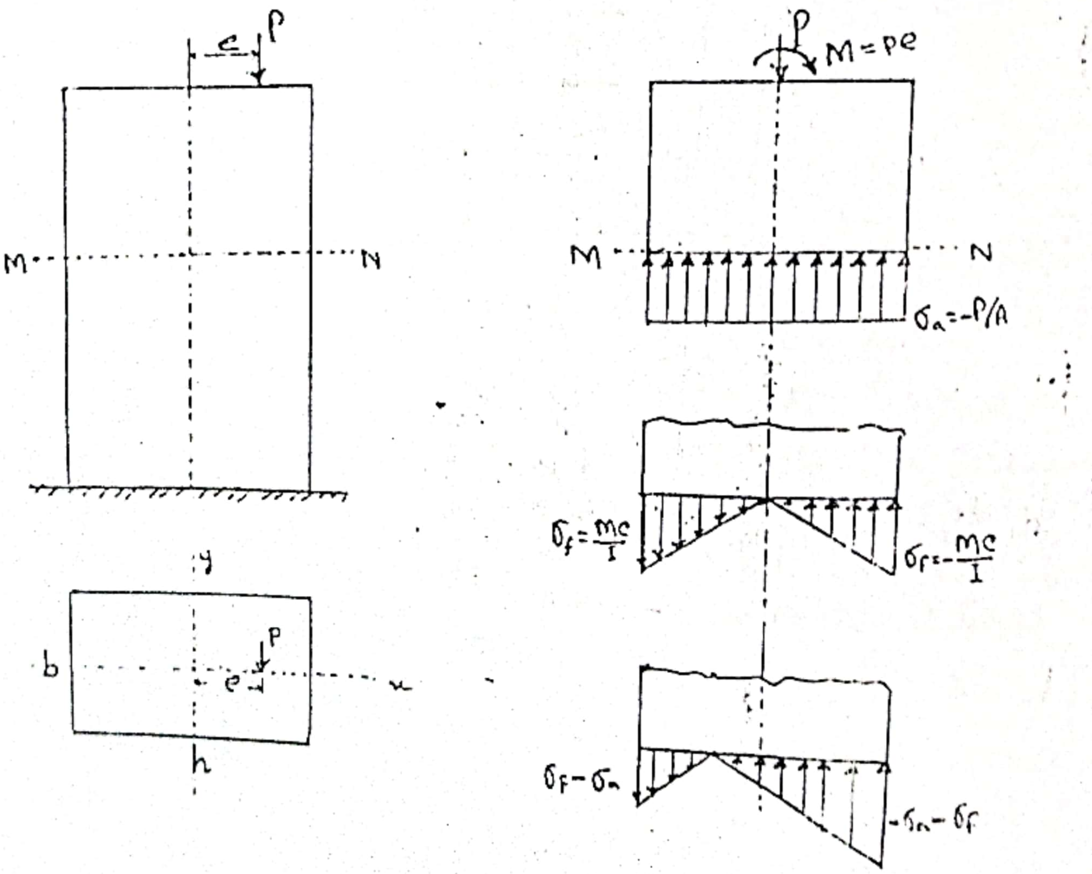
Kern section:

Kern section is defined as the region in which a compressive point load may be applied without producing any tensile stress on the cross section.

Use of kern concept:

The kern concept is widely used in the design of pre stressed concrete beams, footings and concrete dams.

Show that the area of a kern of a square section is 1/18th of the total cross sectional area.



The tensile stress is $\sigma_t = -\sigma_a + \sigma_s$

$$= -P/A + \frac{Mc}{I}$$

For kern section the tensile stress is zero.

$$\sigma_t = 0$$

$$\Rightarrow -P/A + \frac{Mc}{I} = 0$$

$$\Rightarrow P/A = \frac{Mc}{I}$$

$$\Rightarrow \frac{P}{bh} = \frac{Pec}{I}$$

$$\Rightarrow \frac{P}{bh} = \frac{Pe.h/2}{\frac{bh^3}{12}}$$

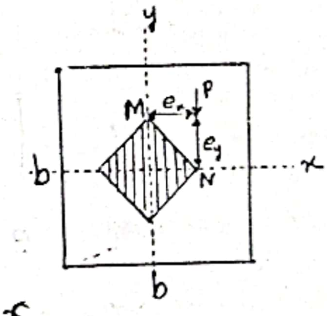
$$\Rightarrow \frac{P}{bh} = \frac{12Peh}{2bh^3}$$

$$\Rightarrow 1 = \frac{6e}{h}$$

$$\Rightarrow e = h/6$$

This is the maximum eccentricity to avoid tension.

For square section:



The tensile stress,

$$\sigma_t = -P/A + \frac{M_y c}{I} + \frac{M_x c}{I}$$

For kern section the tensile stress, $\sigma_t = 0$

$$\Rightarrow P/A = \frac{M_y c}{I} + \frac{M_x c}{I}$$

$$\Rightarrow \frac{P}{b \times b} = \frac{Pe_x \times b/2}{\frac{bb^3}{12}} + \frac{Pe_y \times b/2}{\frac{bb^3}{12}}$$

$$\Rightarrow 1 = \frac{6e_x}{b} + \frac{6e_y}{b}$$

$$\Rightarrow \frac{e_x}{b/6} + \frac{e_y}{b/6} = 1$$

which is the equation of straight line MN. It intersects x and y axes at b/6 and b/6.

Area of kern section $= \{\frac{1}{2} \times e_y \times (e_x + e_x)\} 2$

$= b/6 \times (b/6 + b/6)$

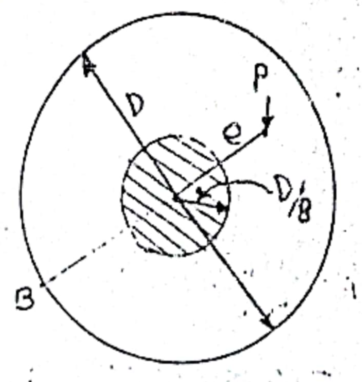
$= b/6 \times b/3$

$= \frac{b^2}{18}$

$= \frac{\text{Total cross-sectional area}}{18}$

$\therefore$ The area of a kern of a section is $1/18^{th}$ of the total cross-sectional area. $\boxed{\text{showed}}$

Show that the kern of a section is a circle whose diameter is one quarter the diameter of the section.



For a circular section of diameter D, the tensile stress is most likely to develop at a point B on the perimeter diametrically opposed to the point of application of P.

The stness at B is

$$\sigma_t = -P/A + \frac{My}{I}$$

$$= -\frac{P}{\frac{\pi D^2}{4}} + \frac{(Pe)\times D/2}{\frac{\pi D^4}{64}}$$

$$= -\frac{4P}{\pi D^2} + \frac{64PeD}{2\pi D^4}$$

for kenn section $\sigma_t = 0$

$$\Rightarrow -4P/\pi D^2 + \frac{64PeD}{2\pi D^4} = 0$$

$$\Rightarrow 1 = \frac{8e}{D}$$

$$\Rightarrow e = D/8$$

$\therefore$ Diameter of the kenn section $= 2\times e = D/4$

$= \frac{\text{Diameter of section}}{4}$

$\therefore$ The kenn of a section is a cincle whose diameter is one quarter the diameter of the cinculan section.

<u>Nonmal stness:</u> Nonmal stness is defined as the nesisting axial fonce per unit anea. It is two types; tensile stness and compressive stness.

<u>Shean stness :</u> Shean stness is defined as the nesisting shean fonce per unit shearing anea. It is also known as tangential stness.

Principal stresses: The maximum and minimum normal stresses are called the principal stresses. They occur on planes of zero shearing stress.

Principal planes: The planes on which the principal stresses act are called the principal planes.

Analytical derivation of variation of stresses at a point

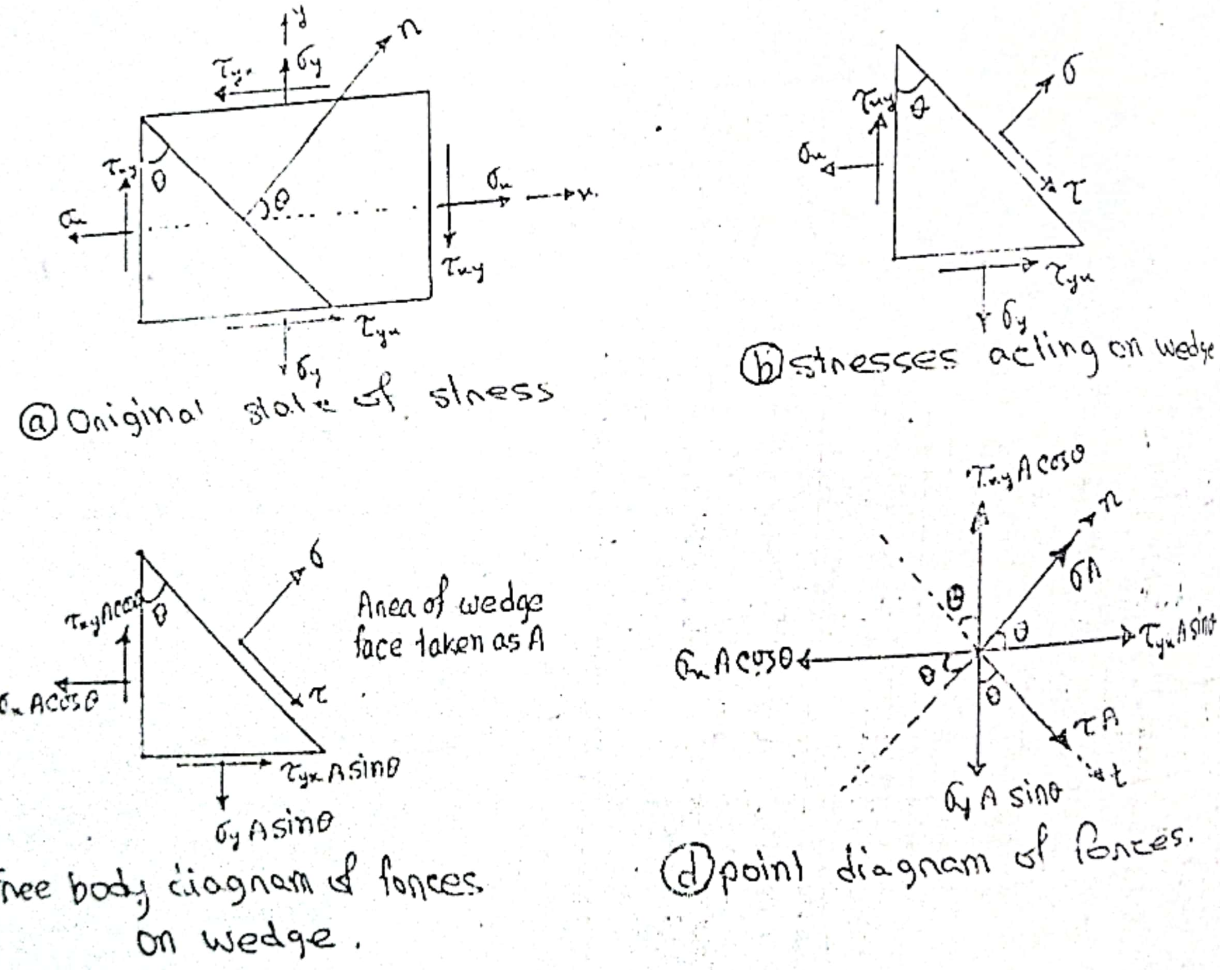


Fig: Variation of stress components.

In determining the stress variation analytically a plane is assumed to pass that cuts the original element into two parts and the conditions of equilibrium are applied to either part.

Fig.ⓐ shows the normal and shearing stress component acting on the plane whose normal n makes an angle θ with the x-axis. The triangular element in fig ⓑ is in equilibrium under the action of the forces arising from the stresses that act over its faces. The area of the inclined face being denoted by A, these forces are shown in the free body diagram in fig.ⓒ.

Applying the conditions of equilibrium to axes chosen in fig.ⓓ, we obtain

$\Sigma F_n = 0$

$$\Rightarrow \sigma A = \sigma_x A\cos^2\theta + \sigma_y A\sin^2\theta - \tau_{xy} A\cos\theta.\sin\theta - \tau_{yx} A\sin\theta.\cos\theta \cdots (I)$$

and $\Sigma F_t = 0$

$$\Rightarrow \tau A = \tau_{xy} A\cos^2\theta + \sigma_x A\cos\theta.\sin\theta - \sigma_y A\sin\theta.\cos\theta - \tau_{yx} A\sin^2\theta \cdots\cdots (II)$$

Since τ_{xy} is numerically equal to τ_{yx} so the above two equations can be written as:

$$\sigma = \sigma_x\left(\frac{1+\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1-\cos 2\theta}{2}\right) - 2\tau_{xy}\frac{\sin 2\theta}{2}$$

$$\Rightarrow \boxed{\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta} \cdots\cdots\cdots (III)$$

And

$$\tau = \tau_{xy}\left(\frac{1+\cos 2\theta}{2}\right) + (\sigma_x - \sigma_y)\cdot\frac{\sin 2\theta}{2} - \tau_{xy}\left(\frac{1-\cos 2\theta}{2}\right)$$

$$= \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\left(\tfrac{1}{2} + \frac{\cos 2\theta}{2} - \tfrac{1}{2} + \frac{\cos 2\theta}{2}\right)$$

$$\boxed{\tau = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta} \cdots\cdots\cdots (IV)$$

The planes for maximum or minimum normal stresses (i.e. principal planes) can be found by differentiating equation (iii) with respect to and setting the derivative equal to zero;

$$\frac{d}{d\theta}(\sigma) = \frac{d}{d\theta}\left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\frac{d}{d\theta}(\cos 2\theta)$$

$$- \tau_{xy}\frac{d}{d\theta}(\sin 2\theta)$$

$$\Rightarrow 0 = 0 + \frac{\sigma_x-\sigma_y}{2}.(-\sin 2\theta).2 - \tau_{xy}\cos 2\theta.2$$

$$\Rightarrow \frac{\sigma_x-\sigma_y}{2}\sin 2\theta = -\tau_{xy}\cos 2\theta$$

$$\Rightarrow \boxed{\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x-\sigma_y}} \quad \text{------------------ (V)}$$

which gives two values of 2θ at differ by 180°.

Hence the planes on which the maximum and minimum normal stresses occur are 90° apart.

The planes of zero shearing stress may be found by setting $\tau = 0$ in equation (iv);

$$0 = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\Rightarrow \boxed{\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x-\sigma_y}} \quad \text{------------------ (VI)}$$

which is identical with equation (V).

Hence the maximum and minimum normal stresses occur on planes of zero shearing stresses.

The planes of maximum and minimum inclined shearing stress can be found by differentiating equation (iv) w.r.t. θ and setting equal to zero;

$$\frac{d}{d\theta}(\tau) = 0$$

$$\Rightarrow \left(\frac{\sigma_x - \sigma_y}{2}\right)\frac{d}{d\theta}(\sin 2\theta) + \tau_{xy}\frac{d}{d\theta}(\cos 2\theta) = 0$$

$$\Rightarrow \left(\frac{\sigma_x - \sigma_y}{2}\right).\cos 2\theta . 2 - \tau_{xy}\sin 2\theta . 2 = 0$$

$$\Rightarrow \boxed{\tan 2\theta_s = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}} \quad \text{------------} \quad (\text{VII})$$

which gives two values of $2\theta_s$ at differ by $180°$.

Hence the planes on which the maximum and minimum shearing stresses occur are $90°$ apart.

Substituting the values of 2θ from equation (v) into equation (iii) and we obtain the principal stresses.

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum normal stress, $\boxed{\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$

Minimum normal stress, $\boxed{\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$

Adding these two equations we get,

$$\boxed{\sigma_{max} + \sigma_{min} = \sigma_x + \sigma_y}$$

which shows that the algebraic sum of the two normal unit stresses on any pair of mutually perpendicular planes at a point equals the algebraic sum of the principal stresses at that point.

Substituting the values of $2\theta_s$ from equation (vii) into equation (iv) and we obtain the maximum shearing stresses;

$$\boxed{\tau_{max} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

Mohr's circle:

German Engineer Otto Mohr in 1882 developed visual interpretation of two dimensional stress using a circle which is known as Mohr's circle. If this construction is plotted to scale the results can be obtained graphically.

From the equation of normal stress (σ) and shearing stress (τ) we get,

$$\sigma = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\Rightarrow \sigma - \frac{\sigma_x+\sigma_y}{2} = \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{......(i)}$$

and $$\tau = \frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad \text{..........(ii)}$$

By squaring and adding equations (i) & (ii) we get,

$$\left(\sigma - \frac{\sigma_x+\sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2 \quad \text{......(iii)}$$

where σ_x, σ_y and τ_{xy} are known constant defining the specified state of stress and σ & τ are variables.

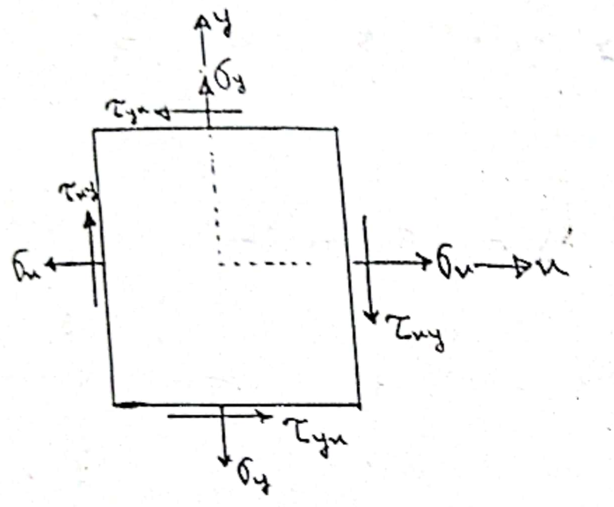
consequently $\frac{\sigma_x + \sigma_y}{2}$ is a constant say c and right side of the equation (ii) is another constant say R. Using these substitution in equation (iii) we get,

$$(\sigma - c)^2 + \tau^2 = R^2$$

which is the equation of a circle of radius R and center $(c, 0)$.

where $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

and $c = \frac{\sigma_x + \sigma_y}{2}$



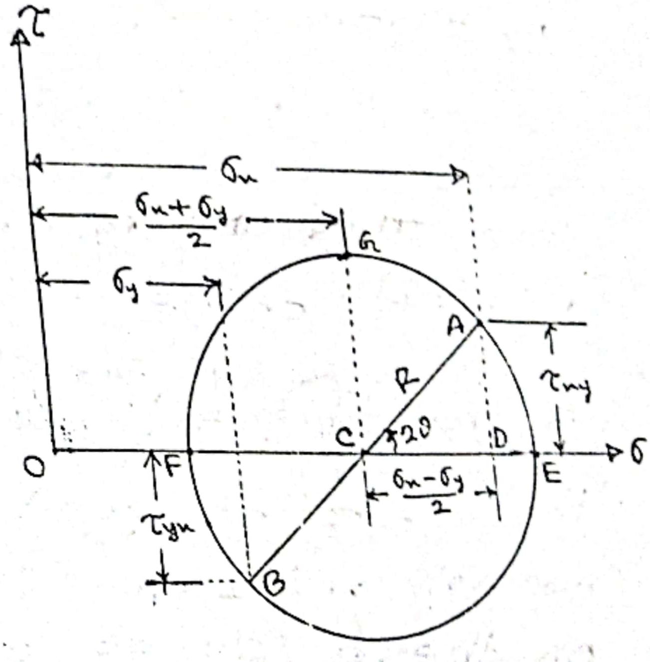


Fig: Mohr's circle for general state of plane stress.

Rules for applying Mohr's circle to combined stresses

1. On rectangular σ-τ axes, plot points having the coordinates (σ_x, τ_{xy}) and (σ_y, τ_{yx}). These points represent the normal and shearing stresses acting on the x and y faces of an element for which the stresses are known. In plotting these points, assume tension as plus, compression as minus and shearing stress as plus when its moment about the center of the element is clockwise.

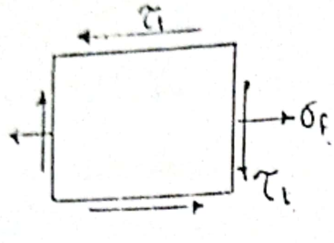
2. Join the points just plotted by a straight line. This line is the diameter of a circle whose center is on the σ-axis.

3. As different planes are passed through the selected point in a stressed body, the normal and shearing stress components on these planes are represented by the coordinates of points whose position shifts around the circumference of Mohr's circle.

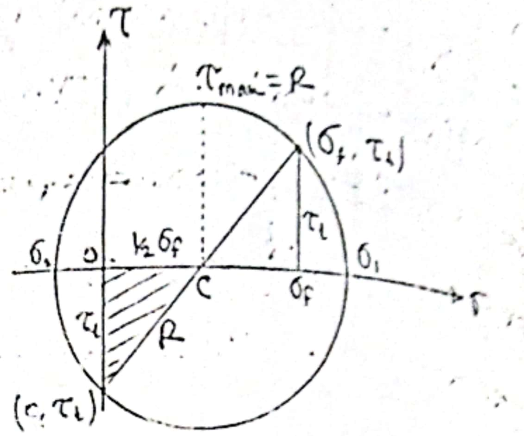
4. The radius of the circle to any point on its circumference represents the axis directed normal to the plane whose stress components are given by the coordinates of that point.

5. The angle between the radii to selected points on Mohr's circle is twice the angle between the normals to the actual planes represented by these points or to twice the space angularity between the planes so represented. The rotational sense of this angle corresponds to the rotational sense of the actual angle between the normals to the planes. i.e. if the n axis is actually at a counterclockwise angle θ from the x-axis, then on Mohr's circle the n radius is laid off at a counterclockwise angle 2θ from the x radius.

Draw the state of stress of an element subjected to simultaneous flexure and torsion and derive illustration for equivalent torque and equivalent moment.



a) The state of stress of an element subjected to simultaneous flexure and torsion



b) Equivalent Mohr's circle.

Since σ_1 and σ_2 have opposite signs, the maximum shearing stress will equal the maximum in plane shearing stress.

Therefore the maximum shearing stress is equal to the radius R.

From the shaded triangle,

$$\tau_{max} = R = \sqrt{(\tfrac{1}{2}\sigma_f)^2 + \tau_1^2} \quad \text{---}($$

Flexure formula as applied to a circular shaft is

$$\sigma_f = \frac{Mr}{I}$$

$$= \frac{Mr}{\frac{\pi r^4}{4}}$$

$$= \frac{4M}{\pi r^3}$$

Torsion formula as applied to a circular shaft is $\tau = \frac{Tr}{J}$

$$= \frac{Tr}{\frac{\pi r^4}{2}}$$

$$= \frac{2T}{\pi r^3}$$

Substituting the values of σ_f and τ in equation ①

$$\tau_{max} = \sqrt{\left(\frac{2M}{\pi r^3}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2}$$

$$= \frac{2}{\pi r^3}\sqrt{M^2+T^2}$$

$$\boxed{\tau_{max} = \frac{2T_e}{\pi r^3}}$$

where equivalent torque $\boxed{T_e = \sqrt{M^2+T^2}}$

Maximum normal stress, $\sigma_{max} = \sigma_1$

$$= \tfrac{1}{2}\sigma_f + R$$

$$= \frac{2M}{\pi r^3} + \frac{2T_e}{\pi r^3}$$

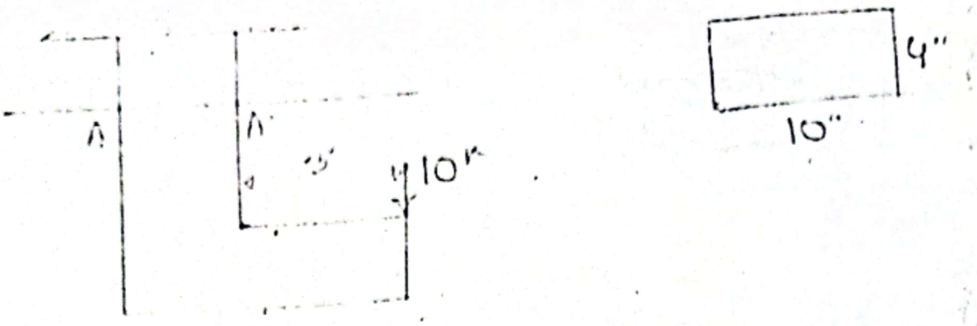
$$= \frac{2}{\pi r^3}(M+T_e)$$

$$= \frac{4}{\pi r^3}\cdot\frac{1}{2}(M+T_e)$$

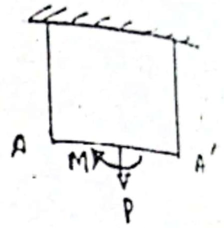
$$\boxed{\sigma_{max} = \frac{4M_e}{\pi r^3}}$$

where equivalent moment, $\boxed{M_e = \frac{1}{2}(M+T_e)}$.

01. Draw the stress diagram of section AA'.



Solution :



Axial stress, $\sigma_a = \frac{P}{A}$

$= \frac{10\times10^3}{4\times10}$

$= 250$ psi.

Flexural stress, $\sigma_f = \pm \frac{Mc}{I}$

$= \pm \frac{10\times10^3 \times (\frac{10}{2} + 3\times12) \times 10/2}{\frac{4\times10^3}{12}}$

$= \pm 6150$ psi.

stress at point A, $\sigma_A = P/A - \frac{Mc}{I}$

$= 250 - 6150$

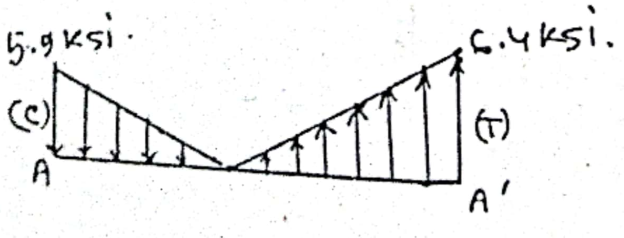
$= -5900$ psi. (c)

stress at point, A', $\sigma_{A'} = P/A + \frac{Mc}{I}$

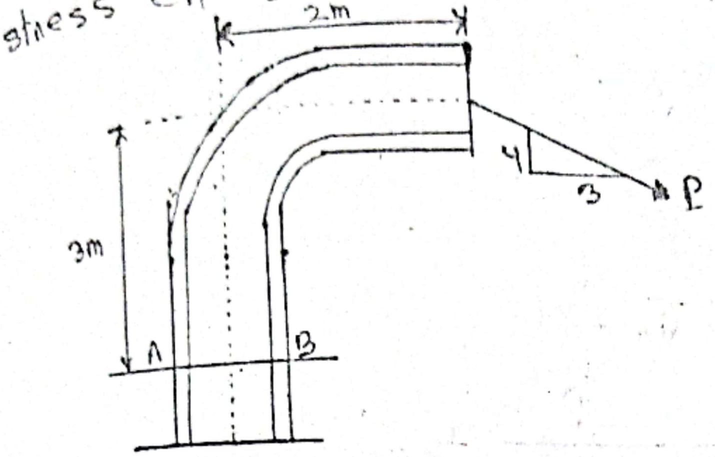
$= 250 + 6150$

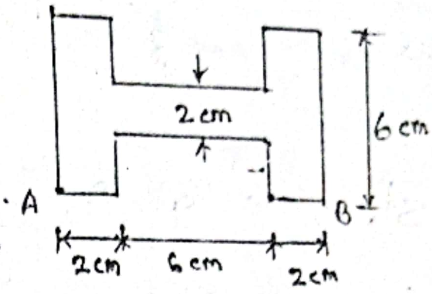
$= 6400$ psi. (T)

stress diagram:

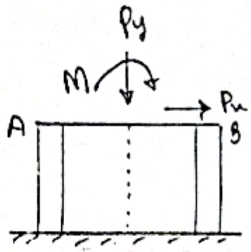


2. Determine the maximum load P that can be applied to the crane boom so that the normal stress on section AB is limited to 80 MPa. [2008]





Solution :



$P_x = \frac{3}{5}P$ and $P_y = \frac{4}{5}P$

$\Sigma M = 0$

$\Rightarrow -M + P_x \times 3 + P_y \times 2 = 0$

$\Rightarrow M = \frac{9}{5}P + \frac{8}{5}P$

$= \frac{17}{5}P$

Moment of inertia, $I = \frac{6^3 \times 2}{12} + 2\left\{\frac{2^3 \times 6}{12} + (2\times 6)\left(\frac{2}{2}+\frac{6}{2}\right)^2\right\}$

$= 428 \text{ cm}^3 = 428 \times 10^3 \text{ mm}^3$

and $c = \frac{10}{2} = 5 \text{ cm} = 50 \text{ mm}$

$A = (10\times 6) - (6\times 2)\times 2 = 36 \text{ cm}^2 = 3600 \text{mm}^2$

Axial stress at A-B, $\sigma_a = -P/A$

$\Rightarrow \sigma_a = -\frac{(4/5)P}{3600}$

$\Rightarrow \sigma_a = -\frac{P}{4500}$

Flexure stress at AB, $\sigma_f = \pm \frac{Mc}{I}$

$= \pm \frac{(17/5)P \times 50}{428 \times 10^3}$

$= \pm \frac{17P}{42800}$

Stress at B, $\sigma_B = -\sigma_a - \sigma_f$

$$\Rightarrow -80 = -\frac{P}{4500} - \frac{17P}{42800}$$

$$\Rightarrow P = 129153.39 \text{ N}$$
$$= 129.15 \text{ KN}$$

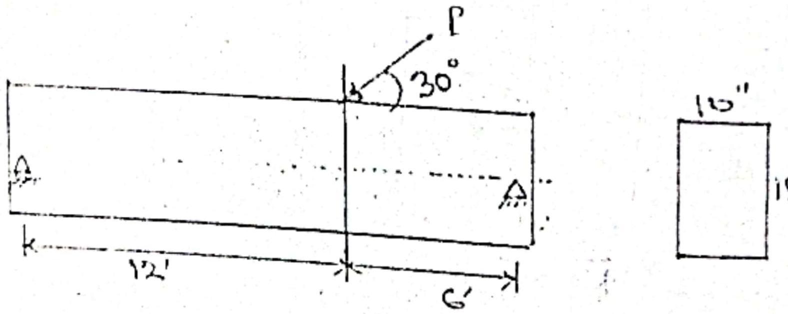
Stress at A; $\sigma_A = -\sigma_a + \sigma_f$

$$\Rightarrow 80 = -\frac{P}{4500} + \frac{17P}{42800}$$

$$\Rightarrow P = 457210.68 \text{ N}$$
$$= 457.21 \text{ KN}.$$

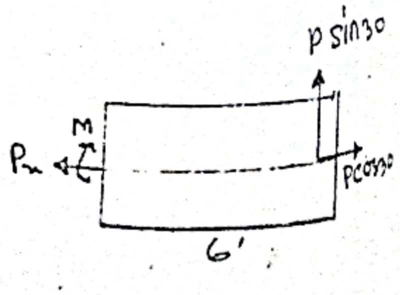
Hence the safe load is $P = 129.15$ KN. $\boxed{\text{Ans}}$

03. A wooden beam 10"x15" is shown in figure below and carries a load P. What is the value of P if the maximum stress is not exceed 2000 psi? [2006]



Solution :

$$M = P\sin 30 \times 6 \times 12$$
$$= 36P \text{ lb.in}.$$



$$P_x = P\cos 30$$
$$= 0.866P \text{ lb}.$$

stress at the bottom of the section

$$\sigma_{max} = P/A + \frac{Mc}{I}$$

$$\Rightarrow 2000 = \frac{0.866P}{10\times15} + \frac{36P\times 15/2}{\frac{10\times15^3}{12}}$$

$$\Rightarrow P\left(\frac{433}{75000} + \frac{12}{125}\right) = 2000$$

$$\Rightarrow P = 19651.51 \text{ lb.}$$

check for stress at the top of the section,

$$\sigma_{max} = P/A - \frac{Mc}{I}$$

$$= \frac{19651.52}{150} - \frac{36\times19651.52\times 15/2}{\frac{10\times15^3}{12}}$$

$$= 131.01 - 1886.55$$

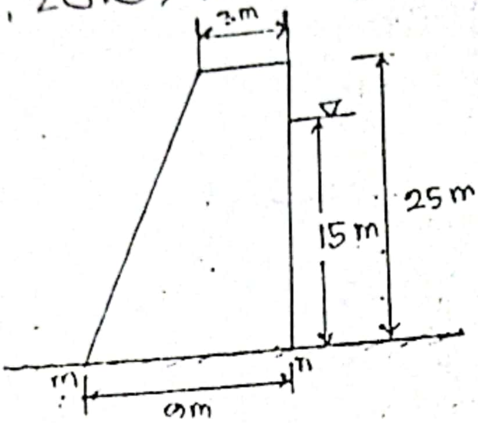
$$= -1755.54 \text{ psi.}$$

$$= 1755.54 \text{ psi (c)}$$

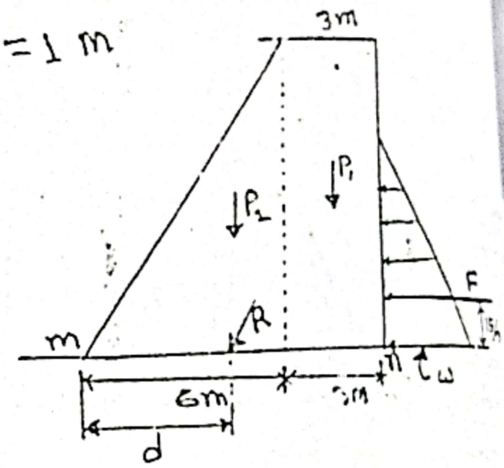
which is less than 2000 psi.

Hence the safe load $P = 19651.51$ lb. $\boxed{\text{Ans}}$

04. A concrete dam has the profile as shown in figure below. If the density of concrete is 2400 kg/m³ and that of water is 1000 kg/m³, determine the maximum compressive stress on section m-n if the depth of water behind the dam is h=15m. [2013, 2015, 2016, 2017]



Solution: Assume length of dam = 1 m



Axial load exerted by dam,

$P = P_1 + P_2$

$= (3\times25\times1)\times2400 + (\frac{1}{2}\times6\times25\times1)\times2400$

$= 180000 + 180000$

$= 360000$ kg.

Pressure intensity due to water, $\omega = \gamma_w h$

$= 1000\times15$

$= 15000$ kg/m².

Force exerted by water, $F = \frac{1}{2}\omega h$

$= \frac{1}{2}\times15000\times15$

$= 112500$ kg

Overturning moment about m, $M_o = F\times 15/3$

$= 112500\times5$

$= 562500$ kg·m.

Resisting moment about m, $M_R = P_1(6+3/2) + P_2 \times \frac{2}{3} \times 6$

$= 180000 \times 7.5 + 180000 \times 4$

$= 2070000$ kg-m.

Location of resultant force on the base of the dam from m,

$$d = \frac{M_R - M_o}{P}$$

$$= \frac{2070000 - 562500}{360000}$$

$= 4.19$ m.

Eccentricity, $e = \frac{b}{2} - d = \frac{9}{2} - 4.19 = 0.31$ m

Moment, $M = Pe = 360000 \times 0.31 = 111600$ kg-m.

Stress, $\sigma = -\frac{P}{A} \pm \frac{Mc}{I}$

$$= -\frac{360000}{9 \times 1} \pm \frac{111600 \times 9/2}{\frac{1 \times 9^3}{12}}$$

$= -40000 \pm 8266.67$

$\sigma_{max} = -40000 - 8266.67$

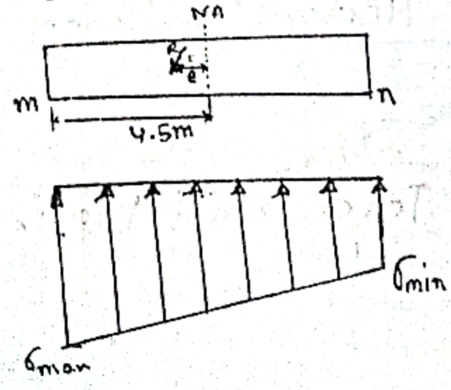
$= -48266.67$ kg/m²

$\therefore \sigma_{max} = 48266.67$ kg/m² (c)

$\sigma_{min} = -40000 + 8266.67$

$= -31733.33$ kg/m²

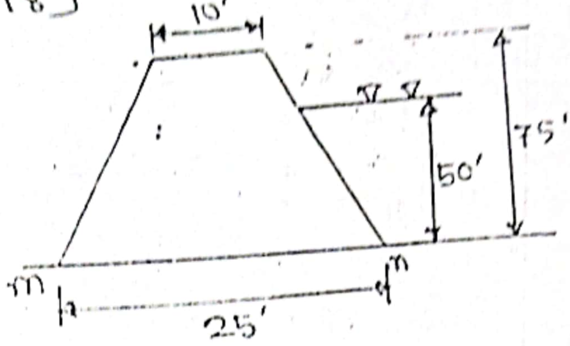
$\sigma_{min} = 31733.33$ kg/m² (c)



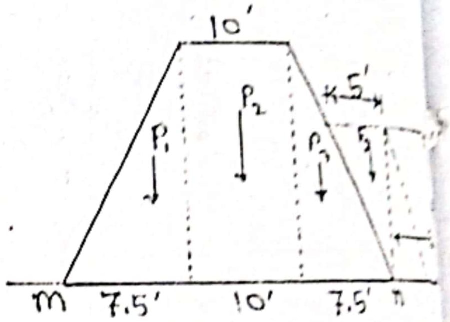
$\therefore$ Maximum compressive stress $= 48266.67$ kg/m².

$\boxed{\text{Ans}}$

05. A concrete dam has the profile shown in figure below. If concrete weighs 150 lb/ft³ and water weighs 62.5 lb/ft³, determine the maximum compressive stress on section mn if the depth of water behind the dam is 50 ft.
[Backlog. 2018]



Solution: Assume length of dam = 1' and the same slope on both sides ——



Axial load exerted by dam,

$P' = P_1 + P_2 + P_3$

$= (\frac{1}{2} \times 7.5 \times 75 \times 1) \times 150 + (10 \times 75 \times 1) \times 150 + (\frac{1}{2} \times 7.5 \times 75 \times 1) \times 150$

$= 42187.5 + 112500 + 42187.5$

$= 196875$ lb.

Axial load exerted by water, $F_y = (\frac{1}{2} \times 5 \times 50 \times 1) \times 62.5$

$= 7812.5$ lb.

Horizontal force exerted by water, $F_x = \frac{1}{2} \times \omega h^2 \times 1$

$= \frac{1}{2} \times 62.5 \times 50^2 \times 1$

$= 78125$ lb.

Total axial load, $P = P' + F_y$

$= 196875 + 7812.5$

$= 204687.5$ lb.

Overturning moment about m, $M_o = F_x \times 50/3$
$= 78125 \times 50/3$
$= 1302083.33$ lb.ft

Resisting moment about m, $M_R = \frac{2}{3} \times 7.5 \times P_1 + P_2\left(7.5 + \frac{10}{2}\right) + P_3\left(17.5 + \frac{7.5}{3}\right) + F_y(25 - 5/3)$

$= 42187.5 \times 5 + 112500 \times 12.5 + 42187.5 \times 20 + 7812.5 \times \frac{70}{3}$

$= 2643229.167$ lb.ft.

Location of resultant force on the base of the dam from m,

$$d = \frac{M_R - M_o}{P} = \frac{2643229.167 - 1302083.33}{204687.5} = 6.55 \text{ ft.}$$

Eccentricity, $e = \frac{25}{2} - 6.55 = 5.95$ ft.

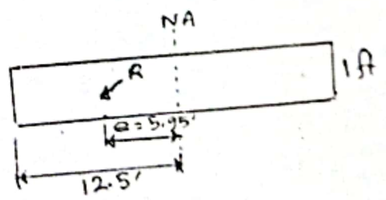
$\therefore$ Stress, $\sigma = -\frac{P}{A} \pm \frac{Mc}{I}$

$$= -\frac{204687.5}{25 \times 1} \pm \frac{204687.5 \times 5.95 \times 25/2}{\frac{1 \times 25^3}{12}}$$

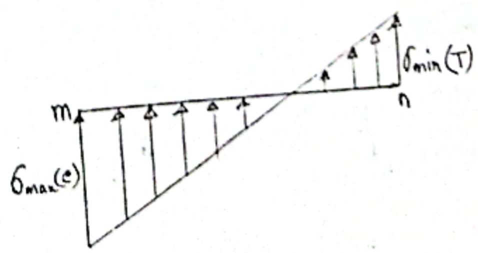
$= -8187.5 \pm 11691.75$

$\sigma_{max} = -8187.5 - 11691.75 = 19879.25$ lb/ft^2 (C)

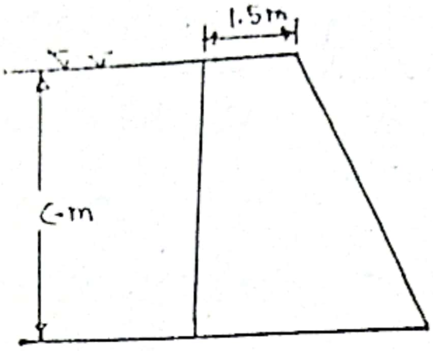
$\sigma_{min} = -8187.5 + 11691.75 = 3504.25$ lb/ft^2 (T)



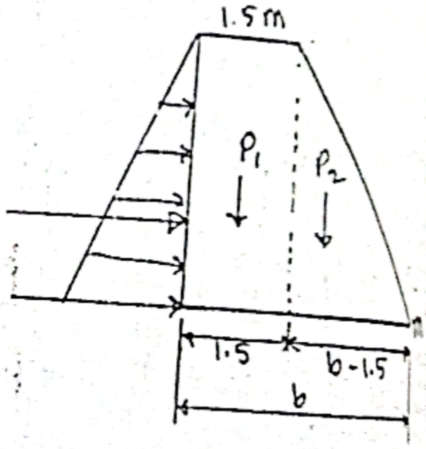
Maximum compressive stress = 19879.25 lb/ft^2 $\boxed{\text{Ans}}$



06. A masonry dam 6m high and 1.5m top width shown in the figure is subjected to water pressure. Find the bottom width of the dam if no tension is to develop at the base. Density of masonry is 20 kN/m³. [2007]



Solution: consider the length of the dam = 1 m. and the base width = b'm.



Axial load exerted by dam, $P = P_1 + P_2$

$= (1.5 \times 6 \times 1)20 + (\frac{1}{2}(b-1.5)6 \times 1) \times 20$

$= 180 + 60(b-1.5)$

$= 90 + 60b$

Force exerted by water, $F = \frac{1}{2}\gamma_w h_1^2 \times 1$

$= \frac{1}{2} \times 9.81 \times 6^2 \times 1$

$= 176.58$ kN.

Overturning moment about M; $M_o = F \times 6/3$

$= 176.58 \times 2$

$= 353.16$ kN-m.

Resisting moment about m,

$$M_R = P_1\left\{(b-1.5)+\frac{1.5}{2}\right\}+P_2\left\{\frac{2}{3}(b-1.5)\right\}$$

$$=180\left\{(b-1.5)+\frac{1.5}{2}\right\}+\left\{60(b-1.5)\right\}\left\{\frac{2}{3}(b-1.5)\right\}$$

$$=180b-135+40(b-1.5)^2$$

$$=180b-135+40(b^2-2b\times1.5+1.5^2)$$

$$=40b^2-120b+90+180b-135$$

$$=40b^2+60b-45$$

Location of resultant load on the base of the dam from m,

$$d=\frac{M_R-M_O}{P}$$

$$=\frac{40b^2+60b-45-353.16}{90+60b}$$

$$=\frac{40b^2+60b-398.16}{90+60b}.$$

Maximum eccentricity to avoid tension,

$$e=b/6$$

$$\Rightarrow b/2-d=b/6$$

$$\Rightarrow b/2-b/6=d$$

$$\Rightarrow b/3=\frac{40b^2+60b-398.16}{90+60b}$$

$$\Rightarrow 90b+60b^2=120b^2+180b-1194.48$$

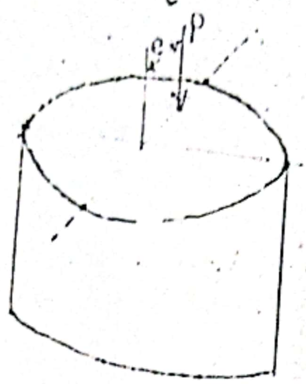
$$\Rightarrow 60b^2+90b-1194.48=0$$

$\therefore b=3.77$ ft. and $b\neq-5.27'$

$\therefore$ Minimum base width of the dam $=3.77$ ft.

$\boxed{\text{Ans}}$

07. Determine the permissible eccentricity, as shown in the following figure, of a load P that acts on a 24 inch diameter post; if the maximum tensile stress is not to exceed 10% of the permissible compressive stress. [2010]



Solution:

Maximum tensile stress is equal to permissible compressive stress.

Hence $\sigma_T = 10\% \, \sigma_c$

$$\Rightarrow -P/A + \frac{Mc}{I} = \frac{10}{100}\left(-P/A - \frac{Mc}{I}\right)$$

$$\Rightarrow -10\,P/A + \frac{10Mc}{I} = -P/A - \frac{Mc}{I}$$

$$\Rightarrow 9\,P/A = \frac{11Mc}{I}$$

$$\Rightarrow \frac{9P}{\frac{\pi d^2}{4}} = \frac{11\times(Pe)\,d/2}{\frac{\pi d^4}{64}}$$

$$\Rightarrow \frac{4\times 9P}{\pi d^2} = \frac{11\times P e\times d\times 64}{\pi d^4\times 2}$$

$$\Rightarrow 36 = \frac{11\times 32\times e}{d}$$

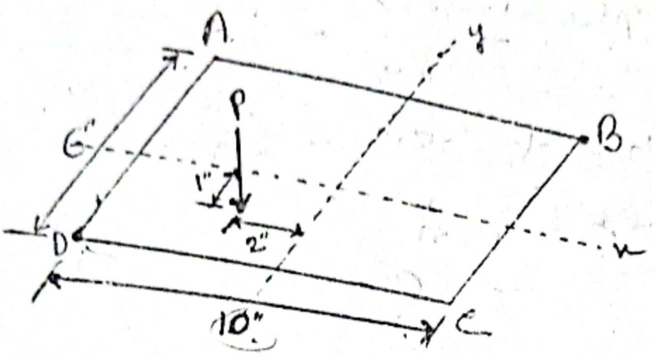
$$\Rightarrow e = \frac{36d}{11\times 32}$$

$$= \frac{36\times 24}{11\times 32}$$

$$= 2.45 \text{ inc} < (d/8 = 3 \text{ inch})$$

$\therefore$ Permissible eccentricity $e = 2.45$ inch

08. A compressive load P = 15000 lb is applied as shown in figure below. Compute the stress at each corner and the location of the neutral axis.
[2012, 2013, 2015, 2016, 2018]



Solution: Here, $P = 15000$ lb.

$x = 10$ in.

$y = 6$ in.

$e_x = 2$ in

$e_y = 1$ in.

$A = 10 \times 6 = 60 \text{ in}^2$

$I_x = \dfrac{10 \times 6^3}{12} = 180 \text{ in}^4$

$I_y = \dfrac{6 \times 10^3}{12} = 500 \text{ in}^4$.

$M_y = Pe_x = 15000 \times 2 = 30000$ lb.in.

$M_x = Pe_y = 15000 \times 1 = 15000$ lb.in.

The resultant stress at any point on the section,

$$\sigma = -P/A \pm \frac{M_y \cdot x}{I_y} \pm \frac{M_x \cdot y}{I_x} = -P/A \pm \frac{(Pe_x)x}{I_y} \pm \frac{(Pe_y)y}{I_x}$$

At point A, $\sigma_A = -P/A - \dfrac{M_y \cdot x/2}{I_y} + \dfrac{M_x \cdot y/2}{I_x}$

$$= -\frac{15000}{60} - \frac{30000 \times 10/2}{500} + \frac{15000 \times (6/2)}{180}$$

$= -250 - 300 + 250$

$= -300$ psi.

$= 300$ psi (c)

At point B, $\sigma_B = -P/A + \dfrac{M_y \, x/2}{I_y} + \dfrac{M_x \, y/2}{I_x}$

$= -250 + 300 + 250$

$= 300$ psi (T)

At point C, $\sigma_C = -P/A + \dfrac{M_y (x/2)}{I_y} - \dfrac{M_x (y/2)}{I_x}$

$= -250 + 300 - 250$

$= -200$ psi.

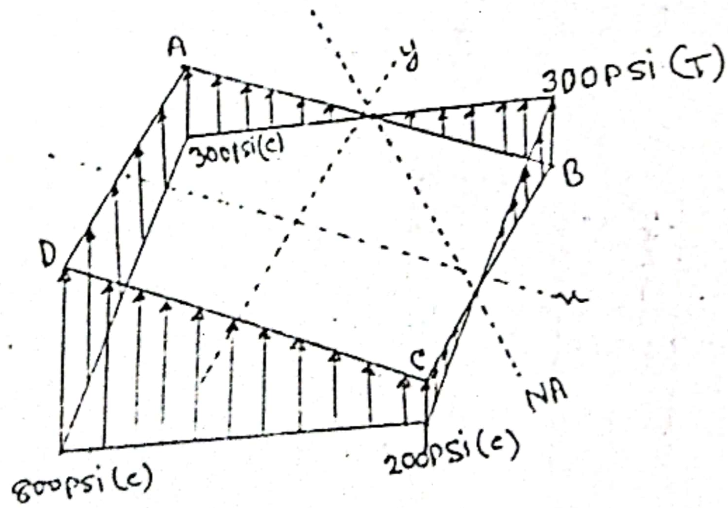
$= 200$ psi (C)

At point D, $\sigma_D = -P/A - \dfrac{M_y (x/2)}{I_y} - \dfrac{M_x (y/2)}{I_x}$

$= -250 - 300 - 250$

$= -800$

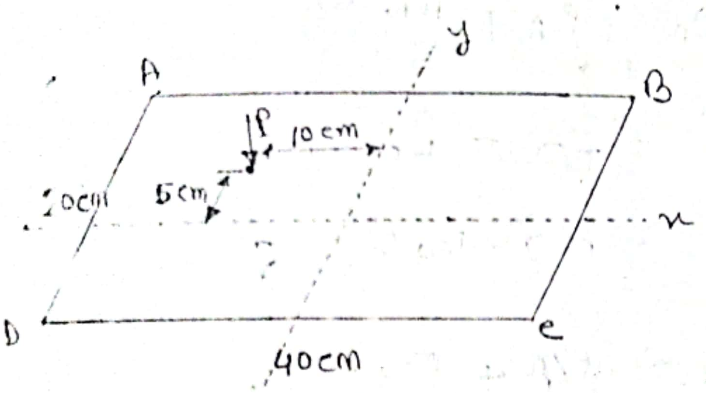
$= 800$ psi (C)



Stress distribution diagram.

A compressive load $P = 200$ kg is applied on a rectangular section ABCD as shown in figure below (i) What additional minimum load, acting normal to the x-section at its centroid, will eliminate tensile stress any where over the cross section. [2009]

(ii) What are the stresses at the corners with the additional load in the center?



Solution: Here, $P = 200$ kg.

$e_x = 10$ cm

$e_y = 5$ cm

$x = 40$ cm

$y = 20$ cm

(i) Now, $A = 40 \times 20 = 800 \text{ cm}^2$

$M_x = Pe_y = 200 \times 5 = 1000$ kg-cm.

$M_y = Pe_x = 200 \times 10 = 2000$ kg-cm.

$I_x = \frac{40 \times 20^3}{12} = 26666.67 \text{ cm}^4$.

$I_y = \frac{20 \times 40^3}{12} = 106666.67 \text{ cm}^4$.

$P/A = \frac{200}{800} = 0.25 \text{ kg/cm}^2$

$\frac{M_y \cdot x/2}{I_y} = \frac{2000 \times 20}{106666.67} = 0.375 \text{ kg/cm}^2$

$\frac{M_x \cdot y/2}{I_x} = \frac{1000 \times 10}{26666.67} = 0.375 \text{ kg/cm}^2$

Resultant stress at any point on the section.

$$\sigma = -P/A \pm \frac{M_y \cdot x}{I_y} \pm \frac{M_x \cdot y}{I_x}$$

Stress at A, $\sigma_A = -P/A - \frac{M_y \cdot x/2}{I_y} - \frac{M_x \cdot y/2}{I_x}$

$= -0.25 - 0.375 - 0.375$

$= -1 \ kg/cm^2$

$= 1 \ kg/cm^2 (C)$

Stress at B, $\sigma_B = -P/A + \frac{M_y \cdot x/2}{I_y} - \frac{M_x \cdot y/2}{I_x}$

$= -0.25 + 0.375 - 0.375$

$= 0.25 \ kg/cm^2 (T)$

Stress at C, $\sigma_C = -P/A + \frac{M_y \cdot x/2}{I_y} + \frac{M_x \cdot y/2}{I_x}$

$= -0.25 + 0.375 + 0.375$

$= 0.50 \ kg/cm^2 (T)$

Stress at D, $\sigma_D = -P/A - \frac{M_y \cdot x/2}{I_y} + \frac{M_x \cdot y/2}{I_x}$

$= -0.25 - 0.375 + 0.375$

$= 0.25 \ kg/cm^2 (C)$

Let a additional load W is placed at the center of the cross section and it produced a uniform compressive stress $\left(\frac{W}{A}\right)$ across the section.

For no tensile stress anywhere in the cross section, the tensile stress σ_c will be equal to W/A.

$$\text{Hence } \frac{W}{A} = \sigma_c$$

$$\Rightarrow \frac{W}{800} = 0.50$$

$$\Rightarrow W = 0.5 \times 800$$

$$\therefore W = 400 \text{ kg. } \boxed{\text{Ans}}$$

(ii) The uniform compressive stress due to additional load W at the center,

$$\sigma = \frac{W}{A}$$

$$= \frac{400}{800}$$

$$= 0.5 \text{ kg/cm}^2 \text{ (c)}$$

Hence the stresses at each corners will be equal to the summation of stress due to additional load and the existing stresses.

Now.

$$\sigma_A = 1 + 0.5$$

$$= 1.5 \text{ kg/cm}^2 \text{ (c)}$$

$$\sigma_B = 0.25 + 0.5$$

$$= 0.75 \text{ (c)}$$

$$\sigma_c = 0.5 - 0.5$$

$$= 0$$

$$\sigma_D = 0.25 + 0.5$$

$$= 0.75 \text{ (c)}$$

10. A state of stress is defined by $S_x = 10000$ psi. and $S_y = 6000$ psi and $S_{xy} = 4000$ psi. Determine the maximum shearing stress on any plane through the stressed point. [2008, 2016]

Solution:

Given, Normal stresses

$S_x = 10000$ psi

$S_y = 6000$ psi

and shearing stress, $S_{xy} = 4000$ psi.

Maximum shearing stress on any plane,

$$\tau_{max} = \pm\sqrt{\left(\frac{S_x - S_y}{2}\right)^2 + S_{xy}^2}$$

$$= \pm\sqrt{\left(\frac{10000 - 6000}{2}\right)^2 + 4000^2}$$

$$= \pm\sqrt{2000^2 + 4000^2}$$

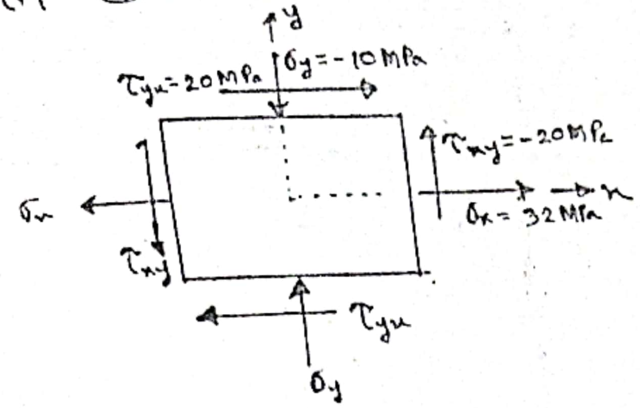
$$= \pm\sqrt{5 \times 4 \times 10^6}$$

$$= \pm 2000\sqrt{5}$$

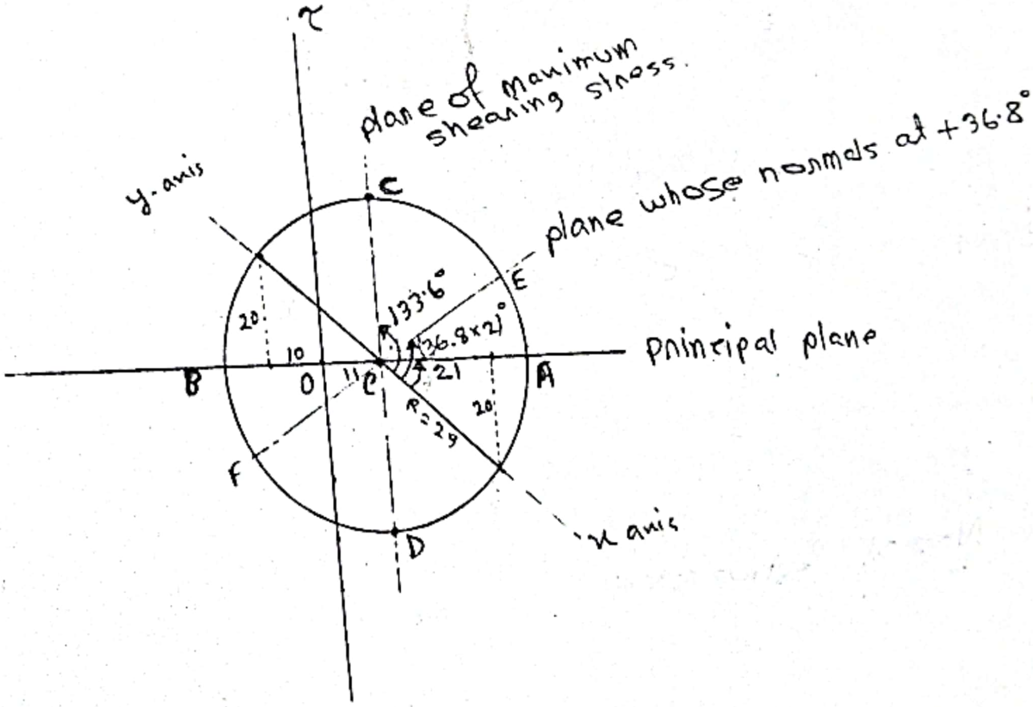
$$= \pm 4472.14 \text{ psi}$$

$\boxed{\text{Ans}}$

11. A state of stress is shown in the following figure. Determine the normal and shearing stresses on ⓐ the principal planes ⓑ the planes of maximum in-plane shearing stress, and ⓒ the planes whose normals are at +36.8° and +126.8° with the x-axis. Show the results of parts ⓐ, ⓑ and ⓒ on complete sketches of differential elements.



Solution:



$$\text{center} = \frac{\sigma_x + \sigma_y}{2} = \frac{32-10}{2} = 11 \text{ MPa}$$

Radius, $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{32+10}{2}\right)^2 + 20^2}$

$= 29$

a) Normal stresses on principal planes AB;

$\sigma_1 = OA = OC + CA = 11 + 29 = 40$ MPa

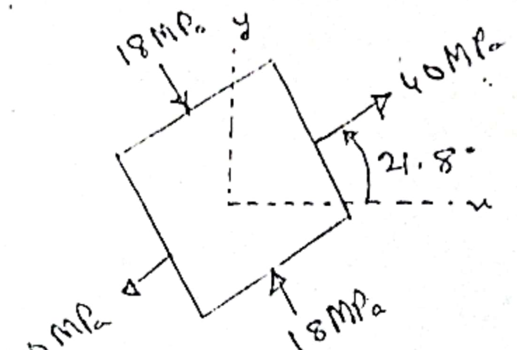
$\sigma_2 = OB = BC - OC = 29 - 11 = -18$ MPa

shearing stresses $= 0$

principal plane, $\tan 2\theta = \frac{20}{21}$

$\Rightarrow 2\theta = 43.6^\circ$

$\Rightarrow \theta = 21.8^\circ$ (ccw).



Principal stresses

b) Plane of maximum shearing stress $2\theta' = 90 + 2\theta$

$= 90 + 43.6$

$= 133.6^\circ$ (ccw)

$\Rightarrow \theta' = 66.8^\circ$ (ccw)

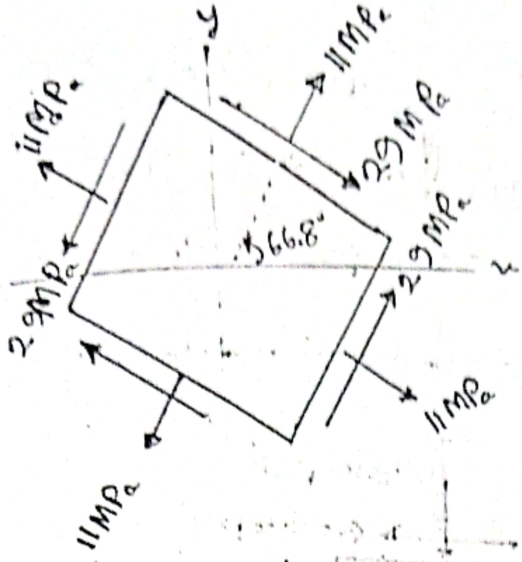
Normal stresses; $\sigma_1 = OC = 11$ MPa

$\sigma_2 = OC = 11$ MPa

Shearing stresses; $\tau_1 = CC = 29$ MPa

$\tau_2 = CD$

$= -29$ MPa



Maximum shear stresses

c) plane whose normal is at $+36.8°$.

Normal stress, $\sigma = 11 + 29\cos\{(36.8*2) - 43.6\}$

$= 11 + 25.11 \cos 30$

$= 36.11$ MPa.

shear stress, $\tau = 29 \sin 30 = 14.5$ MPa.

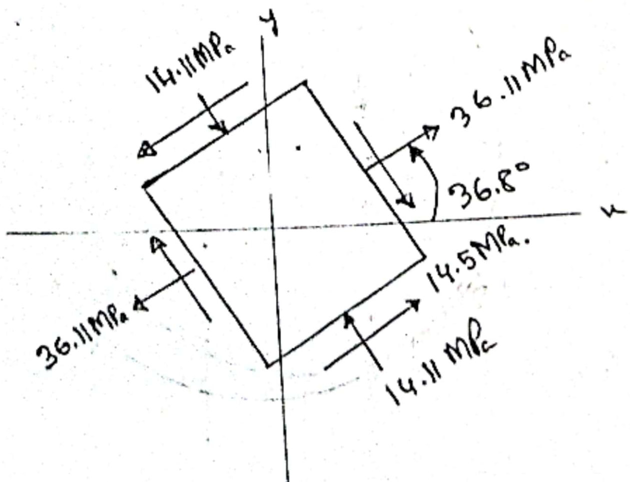
Plane whose normal is at $+126.8°$.

Normal stress, $\sigma' = 11 - 29\cos 30$

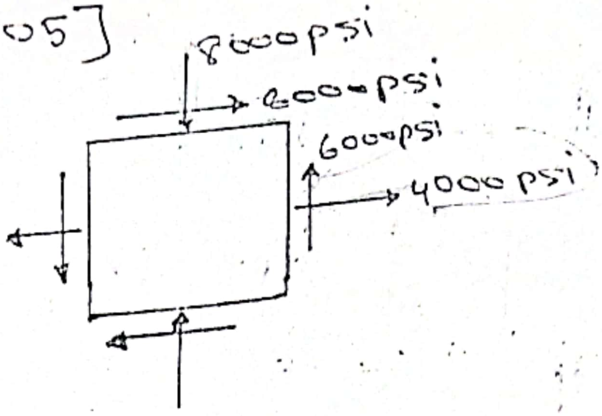
$= -14.11$ MPa

Shear stress, $\tau' = -29 \sin 30$

$= -14.5$ MPa.



12. If an element is subjected to the state of stress shown in figure below. Find the principal stresses. Also compute the stress component on a plane at $\boxed{30^\circ}$ counterclockwise from the x-face. [2005]



Solution:

$$\text{center, } c = \frac{\sigma_x + \sigma_y}{2}$$

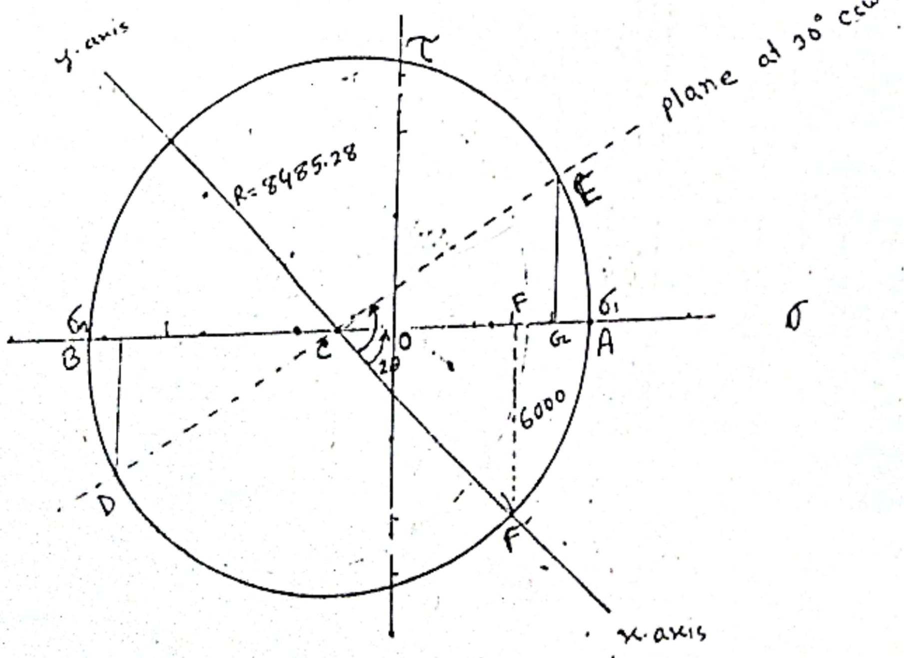
$$= \frac{4000 - 8000}{2}$$

$$= -2000 \text{ psi.}$$

$$\text{Radius, } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}}$$

$$= \sqrt{(6000)^2 + 6000^2}$$

$$= 8485.28 \text{ psi.}$$



principal stresses;

$$\sigma_1 = OA = CA - OC$$
$$= 8485.28 - 2000$$
$$= 6485.28 \text{ psi}. \quad \boxed{\text{Ans}}$$

$$\sigma_2 = OB = OC + CB$$
$$= -2000 - 8485.28$$
$$= -10485.28 \text{ psi}. \quad \boxed{\text{Ans}}$$

principal plane, $\tan 2\theta = \dfrac{FF'}{CF} = \dfrac{FF'}{CO + OF}$

$$= \frac{6000}{2000 + 4000}$$
$$\Rightarrow 2\theta = 45^\circ$$
$$\Rightarrow \theta = 22.5^\circ.$$

stress component on a plane of 30° ccw:

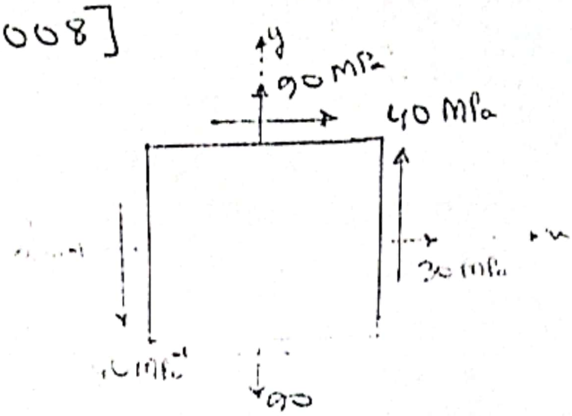
Normal stress, $\sigma = CG$

$$= CE \cos\{(30 \times 2) - 45\}$$
$$= 8485.28 \cos 15$$
$$= 8196.15 \text{ psi}. \quad \boxed{\text{Ans}}$$

shear stress, $\tau = GE$

$$= 8485.28 \sin 15$$
$$= 2196.15 \text{ psi}. \quad \boxed{\text{Ans}}$$

13 If an element is subjected to the state of stress shown in the following figure, determine the principal stresses and the maximum in-plane shearing stress. Also determine the stress components on planes whose normals are at 45° and 135° with the x-axis. Show all results on complete sketches of the appropriate elements. [2008]

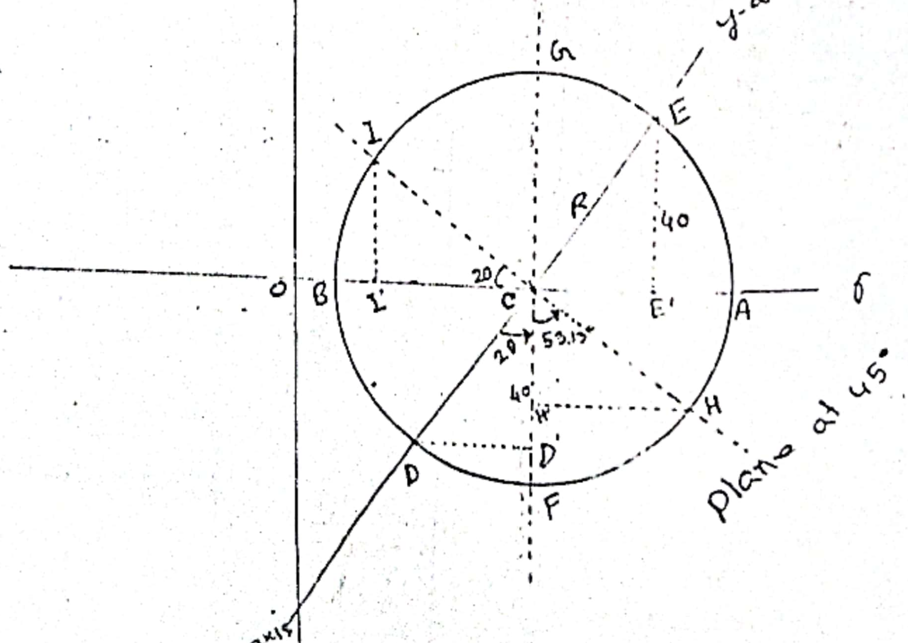


Solution : center, $c = \frac{\sigma_x + \sigma_y}{2} = \frac{30+90}{2} = 60$ MPa

Radius, $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-30)^2 + 40^2}$

$= 50$ MPa.



principal stresses :

$\sigma_1 = OA = OC + CA = 60 + 50 = 110$ MPa

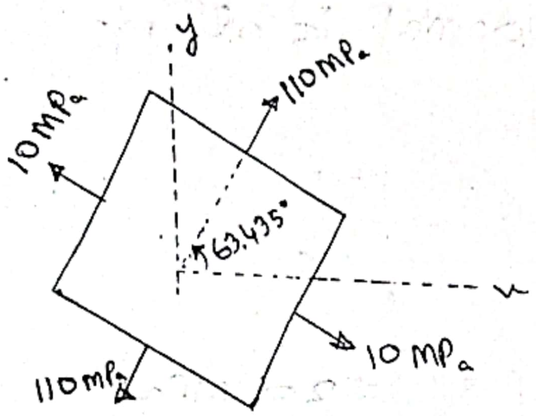
$\sigma_2 = OB = OC - BC = 60 - 50 = 10$ MPa

shearing stresses = 0

principal plane, $2\theta = \tan^{-1}\left(\frac{60-30}{40}\right) + 90$

$\Rightarrow 2\theta = 36.87 + 90 = 126.87°$

$\Rightarrow \theta = 63.435°$



principal stresses.

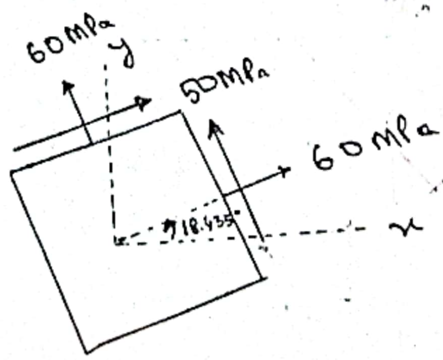
plane of maximum shearing stress, $2\theta = \tan^{-1}\left(\frac{60-30}{40}\right)$

$\Rightarrow \theta = 18.435°$

Normal stresses, $\sigma_1 = \sigma_2 = OC = 60$ MPa

shearing stresses: $\tau_1 = CF = -50$ MPa

$\tau_2 = CG = 50$ MPa



Maximum in plane shearing stress.

plane whose normal is at $45°$ with x-axis.

Normal stress; $\sigma' = OC + HH'$

$= 60 + 50 \sin 53.13$

$= 100$ MPa

shear stress. $\tau' = CH'$

$= -50 \cos 53.13$

$= -30$ MPa

plane whose normal is at $135°$ with x-axis.

Normal stress, $\sigma'' = OI'$

$= OC - CI'$

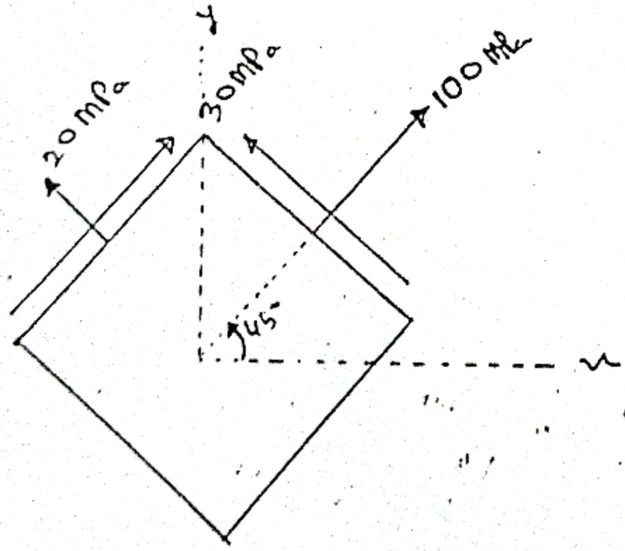
$= 60 - 50 \cos(36.87)$

$= 20$ MPa

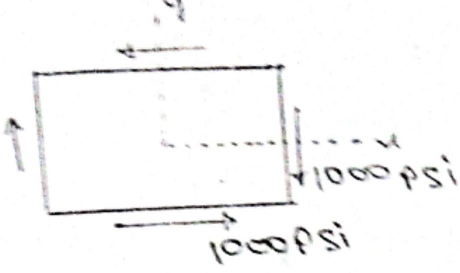
shear stress. $\tau'' = II'$

$= 50 \sin(36.87)$

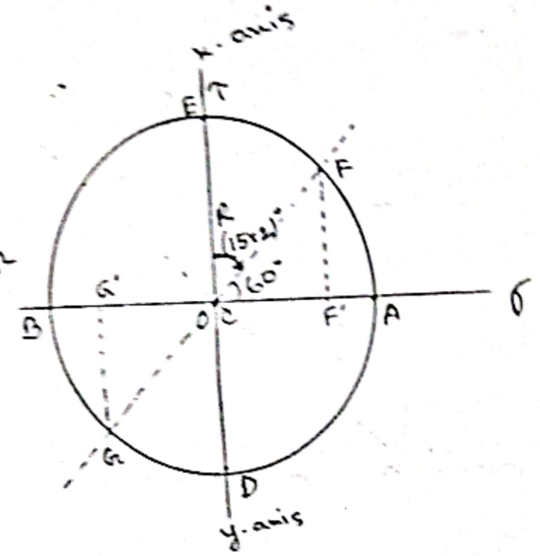
$= 30$ MPa



A state of stress of an element is shown in the following figure. (i) Determine the stress components that act on faces of the element rotated 15 degree clockwise. (ii) Find the value of the principal stresses. [2010, 2018]



Solution:



center, $c = \frac{0+0}{2}$

$= 0$

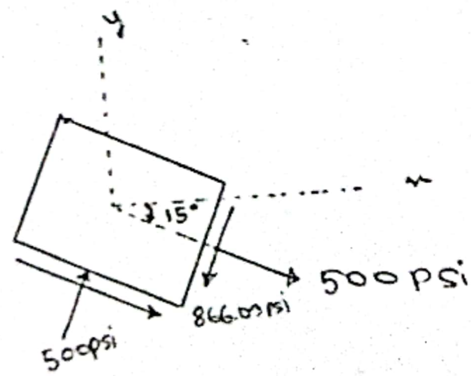
Radius, $R = \sqrt{\left(\frac{0-0}{2}\right)^2 + 1000^2}$

$= 1000$ psi

(i) stress components:

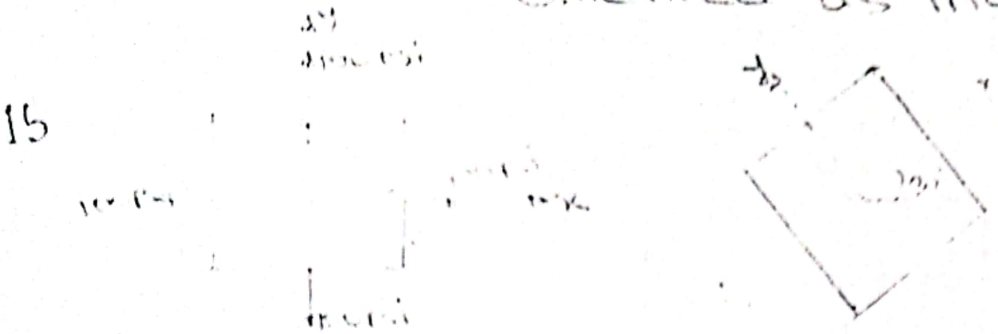
Normal stress, $\sigma = OF' = R\cos 60° = 1000 \cos 60$
$= 500$ psi

Shear stress, $\tau = FF' = R\sin 60 = 1000 \sin 60$
$= 866.03$ psi.



psi.

The state of stress is indicated in the following figure. Determine by means of Mohr's circle components of stress that act on the faces of an element oriented as indicated.

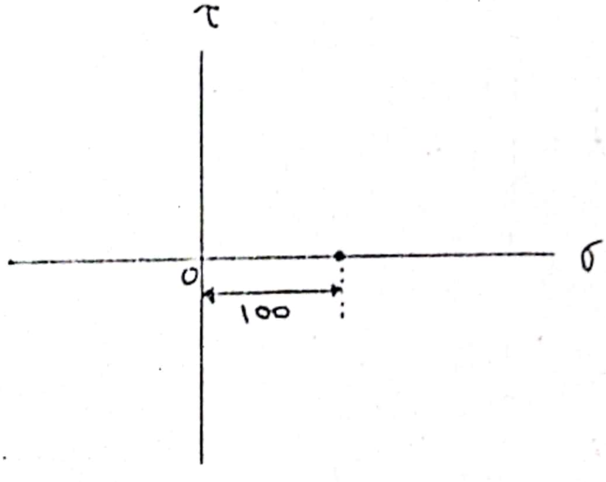


[2011]

Solution:

$$\text{center, } c = \frac{\sigma_x + \sigma_y}{2} = \frac{100+100}{2} = 100\,psi$$

$$\text{Radius, } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0+0} = 0$$

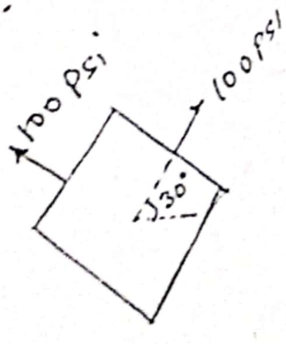


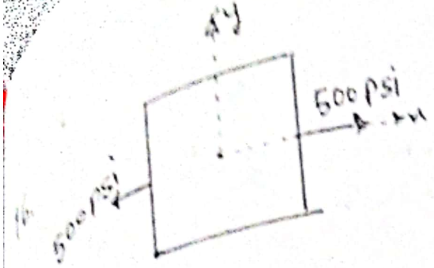
stress components on the oriented face:

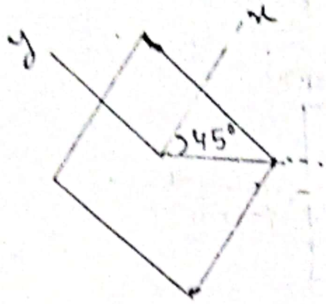
Normal stress $\sigma_1 = 100$ psi.

$\sigma_2 = 100$ psi.

and shear stresses = 0.







[2011, 2012]

solution:

center, $c = \dfrac{\sigma_x + \sigma_y}{2}$

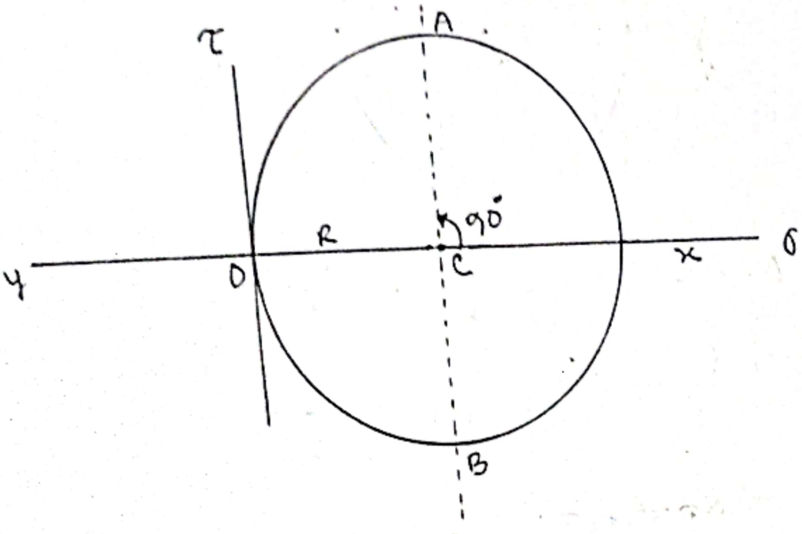
$= \dfrac{500+0}{2}$

$= 250$ psi.

Radius, $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{500-0}{2}\right)^2 + 0}$

$= 250$ psi.

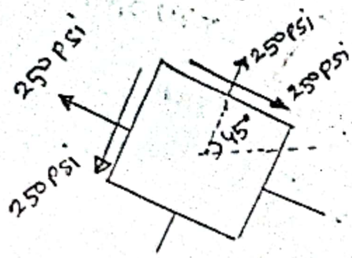


Normal stress at AB: $\sigma_A = OC = 250$ psi.

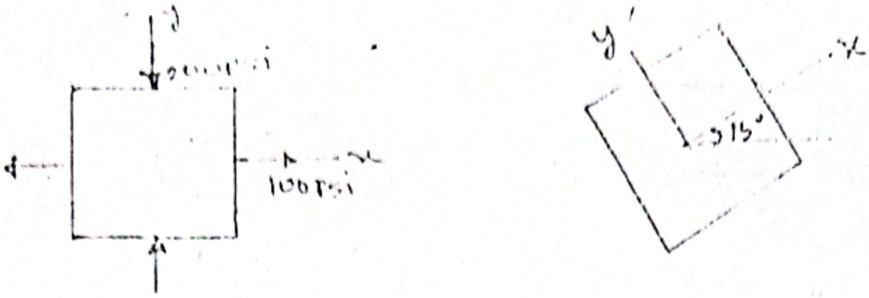
$\sigma_B = OC = 250$ psi.

shear stress, $\tau_A = AC = 250$ psi.

$\tau_B = BC = -250$ psi.



17.



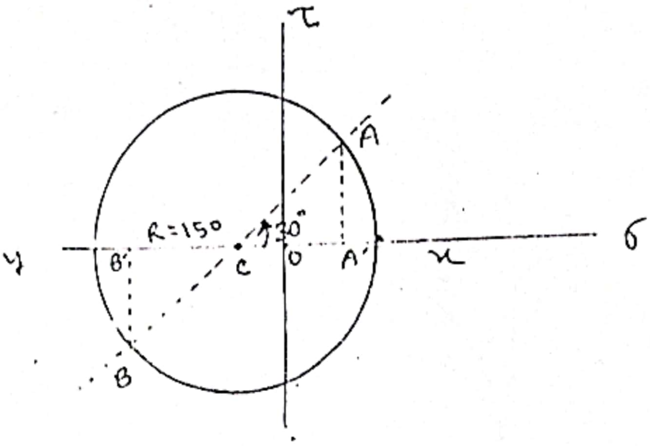
[2012]

Solution:

center, $c = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{100-200}{2} = -50 \text{ psi}$

Radius, $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left\{\dfrac{100-(-200)}{2}\right\}^2 + 0}$

$= 150 \text{ psi}$



From $\Delta AA'C$;

$\sin 30 = \dfrac{AA'}{AC} \Rightarrow AA' = 150 \sin 30 = 75 \text{ psi}$

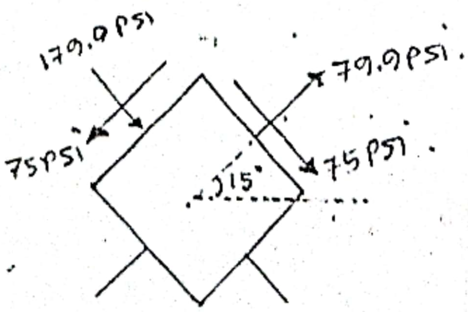
$\cos 30 = \dfrac{A'C}{AC} \Rightarrow A'C = 150 \cos 30 = 129.9 \text{ psi}$

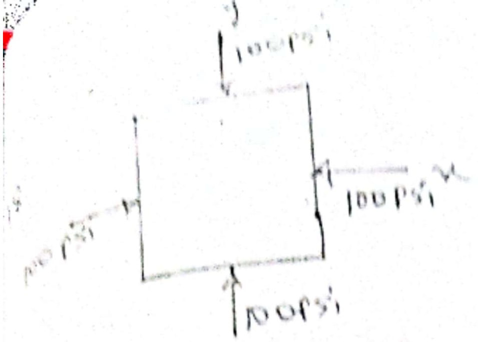
Normal stresses; $\sigma_1 = OA' = A'C - OC = 129.9 - 50 = 79.9 \text{ psi}$

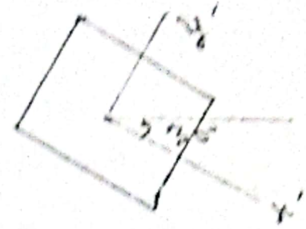
$\sigma_2 = OB' = -50 - 129.9 = -179.9 \text{ psi}$.

Shear stresses; $\tau_1 = AA' = 75 \text{ psi}$.

$\tau_2 = BB' = -75 \text{ psi}$.







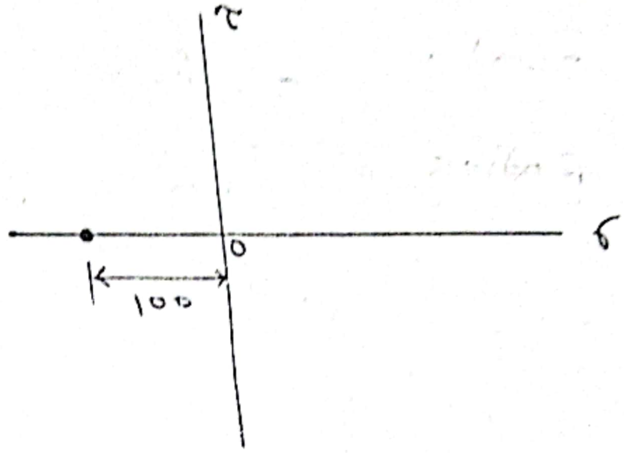
[2012]

Solution:

$$\text{center, } c = \frac{\sigma_x + \sigma_y}{2} = \frac{-100 - 100}{2} = -100$$

$$\text{Radius, } R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-100+100}{2}\right)^2 + 0}$$

$$= 0$$

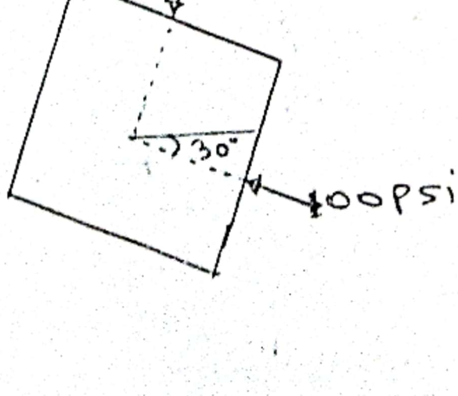


Normal stresses; $\sigma_1 = -100$ psi.

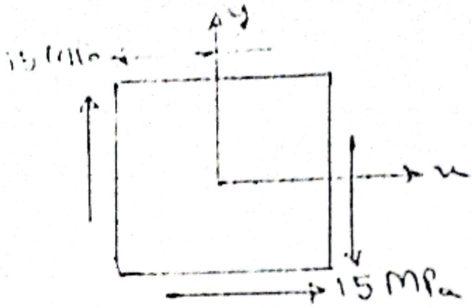
$\sigma_2 = -100$ psi.

Shear stresses; $\tau_1 = 0$

$\tau_2 = 0$



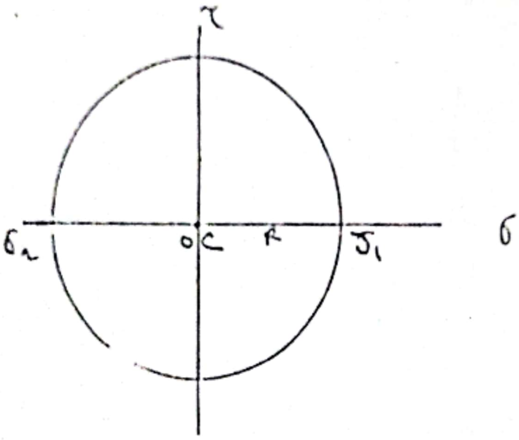
A plane element is subjected to the stress state shown in figure below. Using Mohr's circle determine the principal stresses and show the principal planes. [2017]



Solution:

center, $c = \frac{\sigma_x + \sigma_y}{2} = 0$

Radius, $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 15^2} = 15$



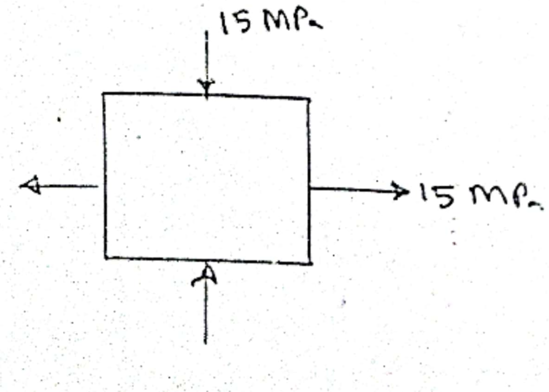
principal stresses;

$\sigma_1 = 15$ psi.

$\sigma_2 = -15$ psi.

shear stresses $= 0$

principal plane $= 0°$



20. The state of stress at a point is the result of the two seperate actions that produce the two states of stress shown in figure below. Determine the principal stresses and principal planes caused by the superposition of two stress states. [2006]

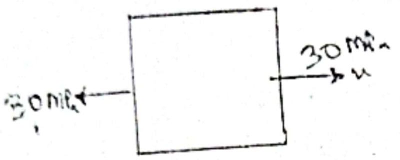


fig.-a

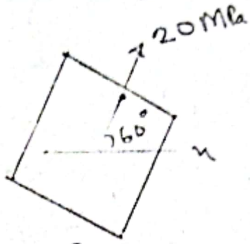


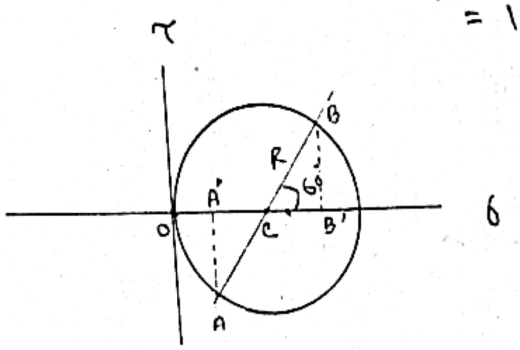
fig.-b

Solution:

From figure-b:

$$\text{center}, c = \frac{\sigma_x+\sigma_y}{2} = \frac{20+0}{2} = 10$$

$$\text{Radius}, R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{20-0}{2}\right)^2 + 0} = 10$$



Normal stresses;

$$\sigma_1 = OB' = OC + R\cos 60 = 10 + 10\cos 60 = 15\text{ MPa}$$

$$\sigma_2 = OA' = OC - A'C = OC - R\cos 60 = 10 - 10\cos 60 = 5\text{ MPa}$$

-Shear stresses; $\tau_1 = BB' = R\sin 60 = 10\sin 60 = 8.66\,MPa$

$\tau_2 = -8.66\,MPa$.

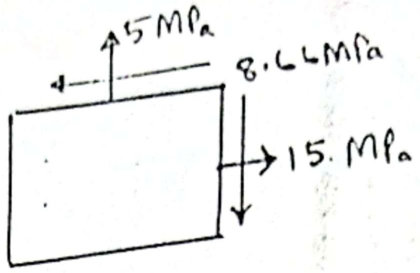
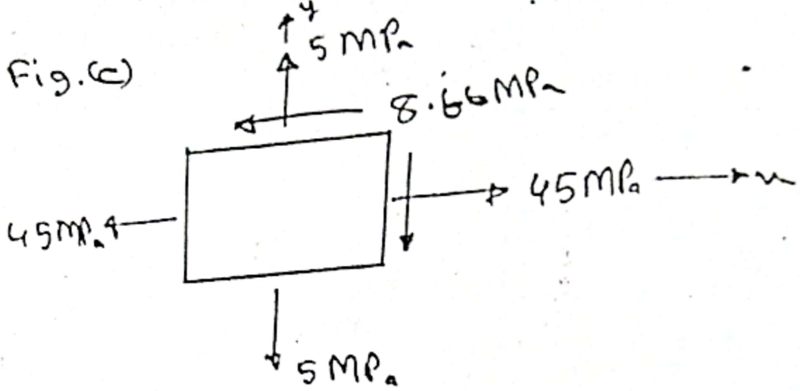


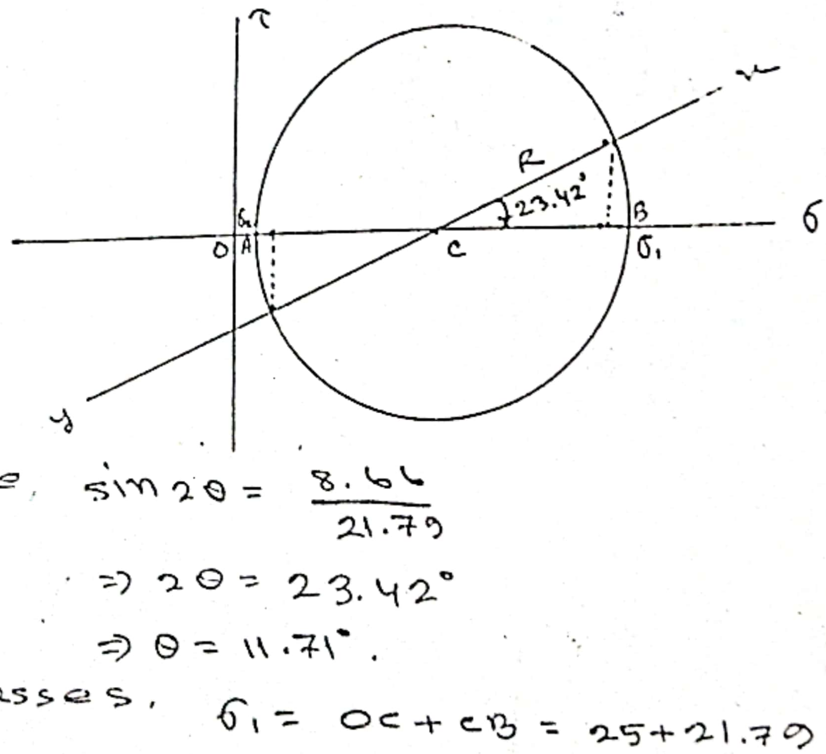
Fig-e.

Adding fig (a) and Fig.(e)



center, $c = \frac{\sigma_x + \sigma_y}{2} = \frac{45+5}{2} = 25\,MPa$

Radius, $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45-5}{2}\right)^2 + 8.66^2} = 21.79\,MPa$



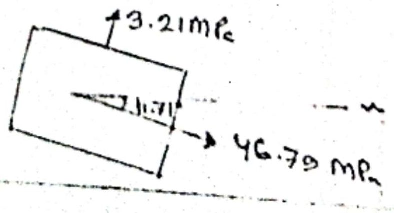
principal plane, $\sin 2\theta = \frac{8.66}{21.79}$

$\Rightarrow 2\theta = 23.42°$

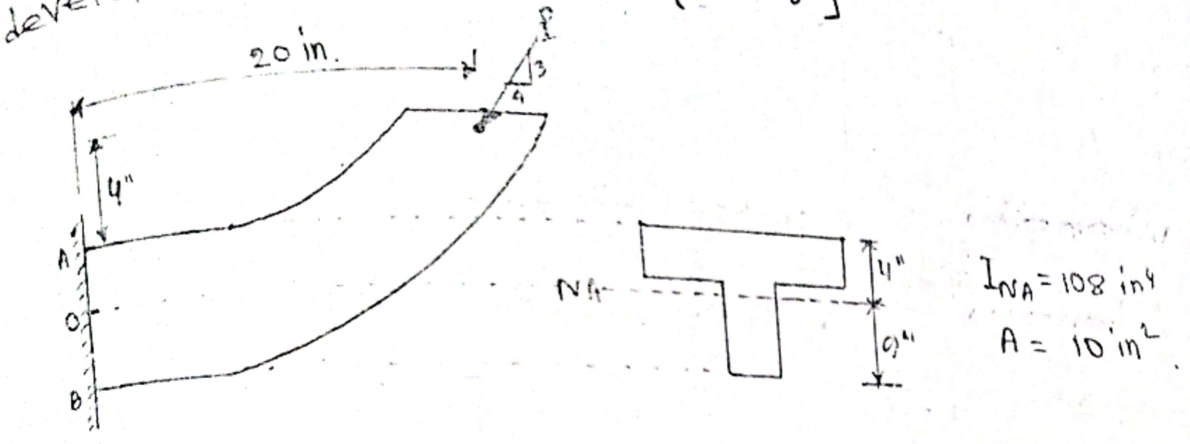
$\Rightarrow \theta = 11.71°$.

principal stresses, $\sigma_1 = OC + CB = 25 + 21.79 = 46.79\,MPa$

$\sigma_2 = OA = OC - AC = 25 - 21.79 = 3.21\,MPa$



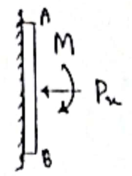
1. If $P = 20{,}000$ lb for bracket shown in figure below, compute the maximum values of S_t and S_c developed at section AB. [2005]



Solution:

$$\frac{P_x}{4} = \frac{P}{5} \quad \Rightarrow P_x = \frac{4}{5} \times 20000 = 16000\ lb.$$

$$\frac{P_y}{3} = \frac{P}{5} \quad \Rightarrow P_y = \frac{3}{5} \times 20000 = 12000\ lb.$$



$$\Sigma M_O = 0$$

$$\Rightarrow -M + P_y \times 20 - P_x \times (4+4) = 0$$

$$\Rightarrow M = 12000 \times 20 - 16000 \times 8 = 0$$

$$= 112000\ lb.in.$$

Stress at B, $S_c = -\frac{P_x}{A} - \frac{Mc}{I}$

$$= -\frac{16000}{10} - \frac{112000 \times 9}{108}$$

$$= -10933.33\ psi.$$

Stress at A, $S_t = -\frac{16000}{10} + \frac{112000 \times 4}{108}$

$$= 2548.15\ psi.$$

Ans: $S_c = 10933.33\ psi.$

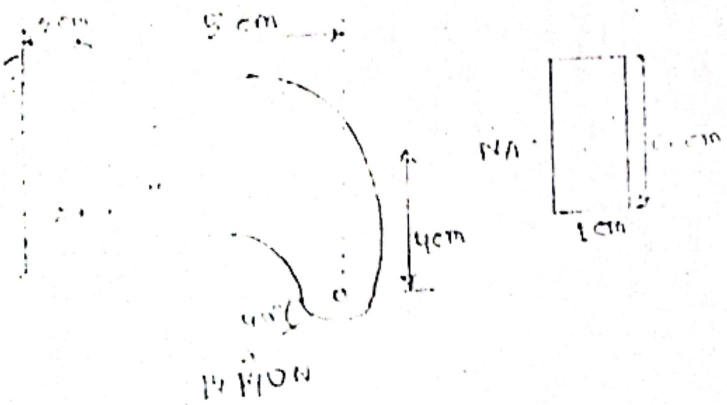
$S_t = 2548.15\ psi.$

2. The cantilever bracket shown in the figure is pulled with a force of 14,140 N. inclined at an angle of 40° to the horizontal. Find for the point A,

i. intensity of shear stress.

ii. intensity of normal stress and

iii. the principal stresses. [2007]



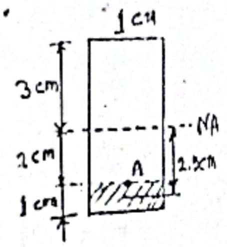
Solution:

i. Shear force, $V = 14140 \sin 40 = 9089.02$ N.

Moment of inertia, $I_{NA} = \frac{1 \times 6^3}{12} = 18$ cm^4.

Area moment, $Q = (1 \times 1)\, 2.5 = 2.5$ cm^3.

width, $b = 1$ cm.



Shear stress, $S_s = \frac{VQ}{Ib}$

$= \frac{9089.02 \times 2.5}{18 \times 1}$

$= 1262.36$ N/cm^2.

$\boxed{\text{Ans}}$

II. Moment, $M = 14140 \cos 40 \times 4 + 14140 \sin 40 \times 5$
$= 88772.56$ N-cm.

Axial force, $P = 14140 \cos 40$
$= 10831.87$ N.

Normal stress at A, $\sigma_A = -\frac{P}{A} - \frac{Mc}{I}$

$= -\frac{10831.87}{1 \times 6} - \frac{88772.56 \times 2}{18}$

$= -1805.31 - 9863.62$

$= -11668.93$ N/cm^2.

$\therefore \sigma_A = 11668.93$ N/cm^2 (C) $\boxed{\text{Ans}}$

III. $\sigma_x = -11668.93$ N/cm^2

$\sigma_y = 0$

$\tau_{xy} = 1262.36$ N/cm^2

Principal stresses; $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{-11668.93 + 0}{2} - \sqrt{\left(\frac{-11668.93 - 0}{2}\right)^2 + (1262.3}$

$= -5834.465 - 5969.47$

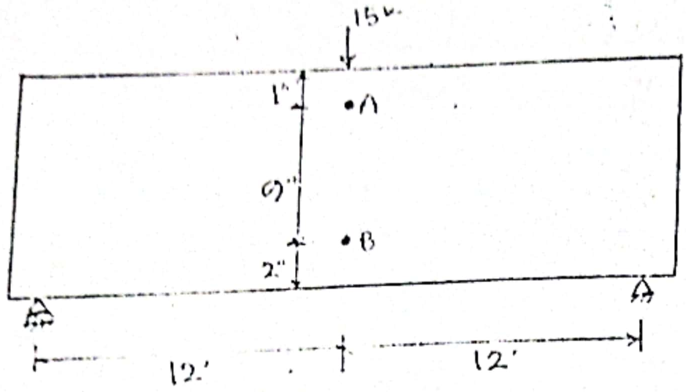
$= -11803.93$ N/cm^2. $\boxed{\text{Ans}}$

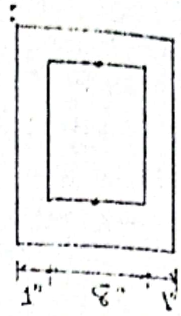
$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= -5834.465 + 5969.47$

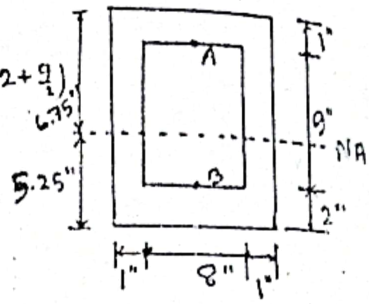
$= 135$ N/cm^2 $\boxed{\text{Ans}}$

23. Compute the principle stress and maximum shearing stress at point A and B of the beam shown in figure below. [2011, 2017]





Solution:



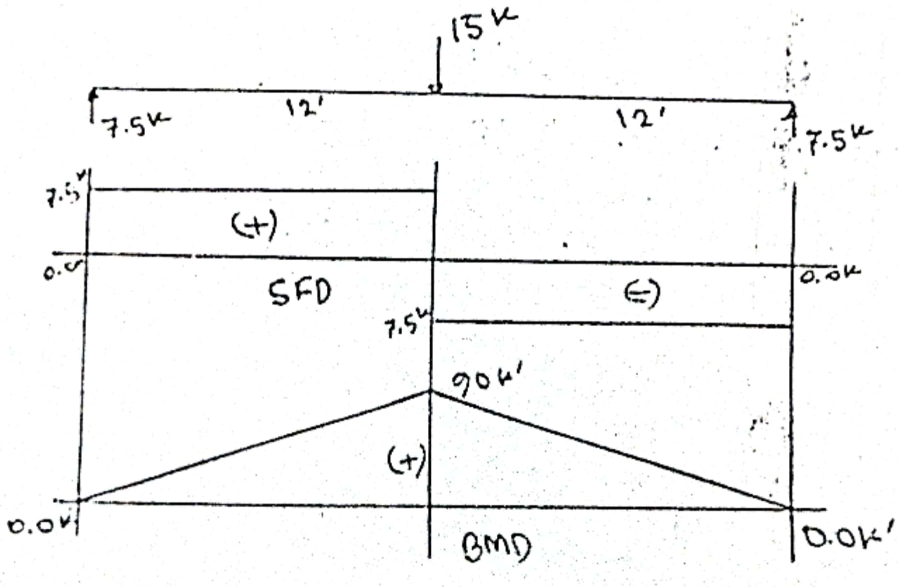
$$\bar{y}\{(10\times12)-(8\times9)\} = 10\times12\times\frac{12}{2} - 8\times9\times(2+\frac{9}{2})$$

$$\Rightarrow \bar{y} = 5.25 \text{ in.}$$

$$I_{NA} = \left\{\frac{10\times12^3}{12} + (10\times12)\times\left(\frac{12}{2}-5.25\right)^2\right\}$$
$$-\left\{\frac{8\times9^3}{12} + (8\times9)\times\left(\frac{9}{2}-3.25\right)^2\right\}$$
$$= 1507.5 - 598.5$$
$$= 909 \text{ in}^4.$$

$$Q_A = (10\times1)(5.75+\tfrac{1}{2}) = 62.5 \text{ in}^3$$

$$Q_B = (10\times2)(3.25+\tfrac{2}{2}) = 85 \text{ in}^4.$$



shear and point A & B, $V = 15k$

moment at A & B, $M = 90k'$

$= 1080 \text{ k.in}$.

Normal stress at A;

$\sigma_A = P/A + \frac{Mc}{I}$

$= 0 + \frac{1080 \times 5.75}{909}$

$= 6.832 \text{ ksi}$.

shear stress at A;

$\tau_A = \frac{VQ_A}{Ib}$

$= \frac{15 \times 62.5}{909 \times 10}$

$= 0.103 \text{ ksi}$.

Principle stress at A; $\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{6.832+0}{2} \pm \sqrt{\left(\frac{6.832-0}{2}\right)^2 + (0.103)^2}$

$= 3.416 \pm 3.418$

$\therefore \sigma_1 = 6.834 \text{ ksi}$.

$\sigma_2 = 0 \text{ ksi}$. $\boxed{\text{Ans}}$

Maximum shearing stress, $\tau = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \pm \sqrt{\left(\frac{6.832-0}{2}\right)^2 + 0.103^2}$

$= \pm 3.418 \text{ ksi}$. $\boxed{\text{Ans}}$

Normal stress at B.

$$\sigma_B = \frac{Mc}{I}$$

$$= \frac{1080 \times 3.25}{909}$$

$$= 3.86 \text{ ksi}.$$

Shear stress at B,

$$\tau_B = \frac{VQ_B}{Ib}$$

$$= \frac{15 \times 85}{909 \times 10}$$

$$= 0.14 \text{ ksi}.$$

Principle stress at B;

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{3.86+0}{2} \pm \sqrt{\left(\frac{3.86-0}{2}\right)^2 + 0.14^2}$$

$$= (1.93 \pm 1.93) \text{ ksi}$$

$$\left.\begin{aligned} \sigma_1 &= 3.86 \text{ ksi}. \\ \sigma_2 &= 0 \text{ ksi}. \end{aligned}\right\} \boxed{\text{Ans}}$$

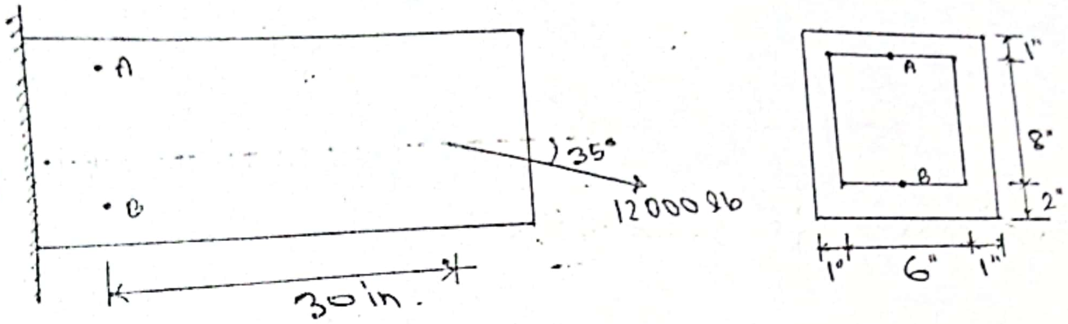
Maximum shearing stress at B;

$$\tau = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm \sqrt{\left(\frac{3.86-0}{2}\right)^2 + 0.14^2}$$

$$= \pm 1.93 \text{ ksi}. \quad \boxed{\text{Ans}}$$

24. Compute the principal stresses and maximum shearing stress at point A & B in the following fig. [2013]



Solution:

Axial force, $P = P\cos 35 = 12000 \cos 35 = 9829.82$ #

Moment, $M = (12000 \sin 35) \times 30 = 206487.52$ lb-in.

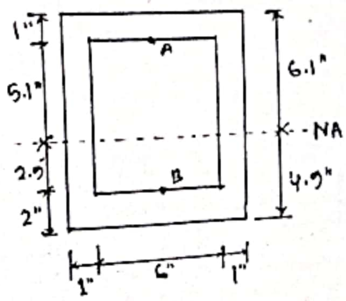
shear force, $V = 12000 \sin 35 = 6882.92$ lb

Location of NA:

$\bar{y}\{(8\times 11) - (6\times 8)\} = (8\times 11 \times \frac{11}{2}) - \{6\times 8 \times (2 + 8/2)\}$

$\Rightarrow \bar{y} = 4.9"$

$I_{NA} = \left\{\frac{8\times 11^3}{12} + (8\times 11)\times\left(\frac{11}{2} - 4.9\right)^2\right\} - \left\{\frac{6\times 8^3}{12} + 6\times 8 \times (4 - 2.9)^2\right\}$



$= 919.01 - 314.08$

$= 604.93 \text{ in}^4$.

$Q_A = 8\times 1 \times (5.1 + 1/2) = 44.8 \text{ in}^3$

$Q_B = 8\times 2 (2.9 + 2/2) = 62.4 \text{ in}^3$.

$A = (8\times 11) - (6\times 8) = 40 \text{ in}^2$

Normal stress at A; $\sigma_A = P/A + \frac{Mc}{I}$

$= \frac{9829.82}{40} + \frac{206487.52\times 5.1}{604.93}$

$= 245.75 + 1740.84$

$\Rightarrow \sigma_A = 1986.59$ psi (T)

Shear stress at A; $\tau_A = \frac{VQ_A}{Ib}$

$= \frac{6882.92 \times 44.8}{604.93 \times 8}$

$= 63.72$ psi.

Principal stresses; $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \frac{1986.59 + 0}{2} + \sqrt{\left(\frac{1986.59 - 0}{2}\right)^2 + 63.72^2}$

$= 993.295 + 995.337$

$= 1988.632$ psi. (T) $\boxed{\text{Ans}}$

$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 993.295 - 995.337$

$= -2.042$ psi (C) $\boxed{\text{Ans}}$

Maximum shearing stress at A;

$\tau = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \pm \sqrt{\left(\frac{1986.59 - 0}{2}\right)^2 + 63.72^2}$

$= \pm 995.337$ psi.

$\boxed{\text{Ans}}$

Normal stress at B; $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = 245.75 - \frac{206487.52 \times 2.9}{604.93}$

$= 245.75 - 989.89$

$= -744.14$ psi.

Shear stress at B, $\tau_B = \frac{VQ_B}{Ib}$

$= \frac{6882.92 \times 62.4}{604.93 \times 8}$

$= 88.75$ psi

Principal stresses at B,

$\sigma_1 = \frac{-744.14 + 0}{2} + \sqrt{\left(\frac{-744.14 - 0}{2}\right)^2 + 88.75^2}$

$= -372.07 + 382.51$

$= 10.44$ psi. (T) $\boxed{\text{Ans}}$

$\sigma_2 = -372.07 - 382.51$

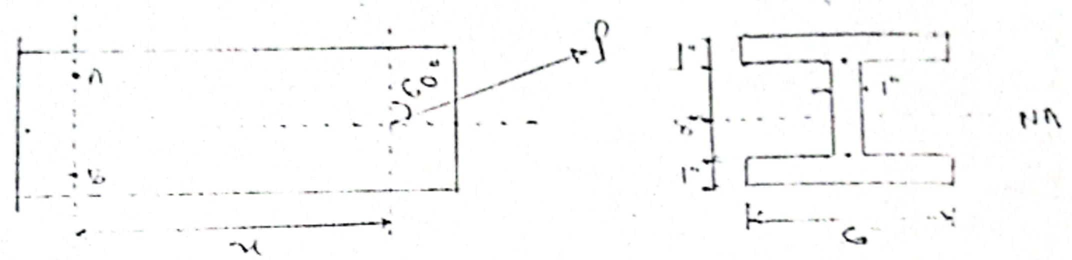
$= -754.58$ psi. (c) $\boxed{\text{Ans}}$

Maximum shearing stresse at B;

$\tau = \pm\sqrt{\left(\frac{-744.14 - 0}{2}\right)^2 + 88.75^2}$

$= \pm\ 382.51$ psi. $\boxed{\text{Ans}}$

25. Calculate the principal stresses and maximum shearing stress at point A and B at the following figure at section x=12" due to P=12000 lb. [2015]



Solution :

Axial force, $P = P\sin 60 = 12000 \sin 60 = 10392.3$

Moment, $M = 12000 \cos 60 \times 12 = 72000$ lb.in.

Shear force, $V = 12000 \cos 60 = 6000$ lb.

$$I_{NA} = \frac{6\times10^3}{12} - \frac{5\times 8^3}{12} = 286.67 \text{ in}^4$$

$$A = 6\times10 - 5\times8 = 20 \text{ in}^2$$

$$Q_A = 1\times6\times(4+\tfrac{1}{2}) = 27 \text{ in}^3$$

$$Q_B = 1\times6\times(4+\tfrac{1}{2}) = 27 \text{ in}^3$$

Normal stress at A;

$$\sigma_A = P/A - \frac{Mc}{I}$$

$$= \frac{10392.3}{20} - \frac{72000\times4}{286.67}$$

$$= 519.615 - 1004.639$$

$$= -485.024$$

Shear stress at A;

$$\tau_A = \frac{VQ_A}{Ib}$$

$$= \frac{6000\times27}{286.67\times6}$$

$$= 94.185$$

Principal stresses at A;

$$\sigma = \frac{-485.024+0}{2} \pm \sqrt{\left(\frac{-485.024-0}{2}\right)^2 + 94.185^2}$$

$$= -242.512 \pm 260.159$$

$$\left.\begin{aligned}\sigma_1 &= -502.67 \text{ psi.}\\ \sigma_2 &= 17.65 \text{ psi.}\end{aligned}\right\} \boxed{\text{Ans}}$$

Maximum shearing stress at A;

$$\tau = \pm \sqrt{\left(\frac{-485.024-0}{2}\right)^2 + 94.185^2}$$

$$= \pm 260.159 \text{ psi.} \quad \boxed{\text{Ans}}$$

Normal stress at B;

$$\sigma_B = P/A + \frac{Mc}{I}$$

$$= 519.615 + 1004.639$$

$$= 1524.254 \text{ psi.}$$

Shear stress at B;

$$\tau_B = \frac{VQ_B}{Ib}$$

$$= \frac{6000 \times 27}{286.67 \times 6}$$

$$= 94.185 \text{ psi.}$$

Principal stresses at B;

$$\sigma = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{1524.254+0}{2} \pm \sqrt{\left(\frac{1524.254-0}{2}\right)^2 + 94.185^2}$$

$$= 762.127 \pm 767.925$$

$$\left.\begin{aligned} \sigma_1 &= 1530.052 \text{ psi} \\ \sigma_2 &= -5.798 \text{ psi.} \end{aligned}\right\} \boxed{\text{Ans}}$$

Maximum shearing stress at B;

$$\tau = \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm \sqrt{\left(\frac{1524.254-0}{2}\right)^2 + 94.185^2}$$

$$= \pm 767.925 \text{ psi} \quad \boxed{\text{Ans}}$$

26. A solid circular shaft is used to transmit simultaneously a torque of 3000 N·m and a maximum bending moment of 2400 N·m. Determine the radius of the smallest shaft that can be used if maximum $\sigma \leq 120$ MPa and maximum $\tau \leq 60$ MPa. [2005]

Solution:

Given,

Torque, $T = 3000$ N-m

Moment, $M = 2400$ N-m.

Now equivalent Torque, $T_e = \sqrt{M^2+T^2}$

$= \sqrt{2400^2+3000^2}$

$= 3841.87$ N-m.

and equivalent moment, $M_e = \frac{1}{2}(M+T_e)$

$= \frac{1}{2}(2400+3841.87)$

$= 3120.935$ N-m.

Maximum shearing stress, $\tau_{max} = \frac{2T_e}{\pi r^3}$

$\Rightarrow 60 = \frac{2 \times 3841.87 \times 10^3}{\pi r^3}$

$\Rightarrow r = 34.42$ mm

Maximum normal stress, $\sigma_{max} = \frac{4M_e}{\pi r^3}$

$\Rightarrow 120 = \frac{4 \times 3120.935 \times 10^3}{\pi r^3}$

$\Rightarrow r = 32.11$ mm.

Hence the radius of the smallest shaft $= 34.42$ mm. $\boxed{\text{Ans}}$

27. A solid shaft is subjected to a maximum bending moment of 2000 lb.ft and a maximum torque of 4000 lb.ft. If allowable normal stress is 10000 psi and shearing stress is 8000 psi, design the shaft. [2007]

Solution: Given, Torque, $T = 4000$ lb.ft.

Moment, $M = 2000$ lb.ft.

Now equivalent torque, $T_e = \sqrt{T^2 + M^2}$

$= \sqrt{4000^2 + 2000^2}$

$= 4472.14$ lb.ft.

equivalent moment, $M_e = \frac{1}{2}(M + T_e)$

$= \frac{1}{2}(2000 + 4472.14)$

$= 3236.07$ lb.ft.

Allowable normal stress $= \frac{4M_e}{\pi r^3}$

$\Rightarrow 10000 = \frac{4 \times 3236.07 \times 12}{\pi r^3}$

$\Rightarrow r = 1.704$ in.

Allowable shearing stress $= \frac{2T_e}{\pi r^3}$

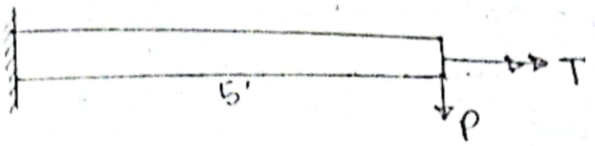
$\Rightarrow 8000 = \frac{2 \times 4472.14 \times 12}{\pi r^3}$

$\Rightarrow r = 1.622$ in.

Hence the proper radius of the shaft $= 1.702$ in.

Ans

28. A solid shaft carries a load of, P = 5k and a torque of T = 10 k.ft as shown in figure below. If the allowable normal stress and shearing stress are 4ksi and 2 ksi respectively what will be the diameter of the shaft? [2006]



Solution :

Moment, $M = P \times 5 = 5 \times 5 = 25$ k.ft.

Torque, $T = 10$ k.ft :

Equivalent Torque, $T_e = \sqrt{M^2 + T^2} = \sqrt{25^2 + 10^2} = 26.93$k'

Equivalent moment, $M_e = \frac{1}{2}(M + T_e) = \frac{1}{2}(25 + 26.93)$

$= 25.96$ k.ft.

Allowable normal stress $= \frac{4M_e}{\pi r^3}$

$\Rightarrow 4 = \frac{4 \times 25.96 \times 12}{\pi r^3}$

$\Rightarrow r = 4.63$ in.

Allowable shearing stress $= \frac{2T_e}{\pi r^3}$

$\Rightarrow 2 = \frac{2 \times 26.93 \times 12}{\pi r^3}$

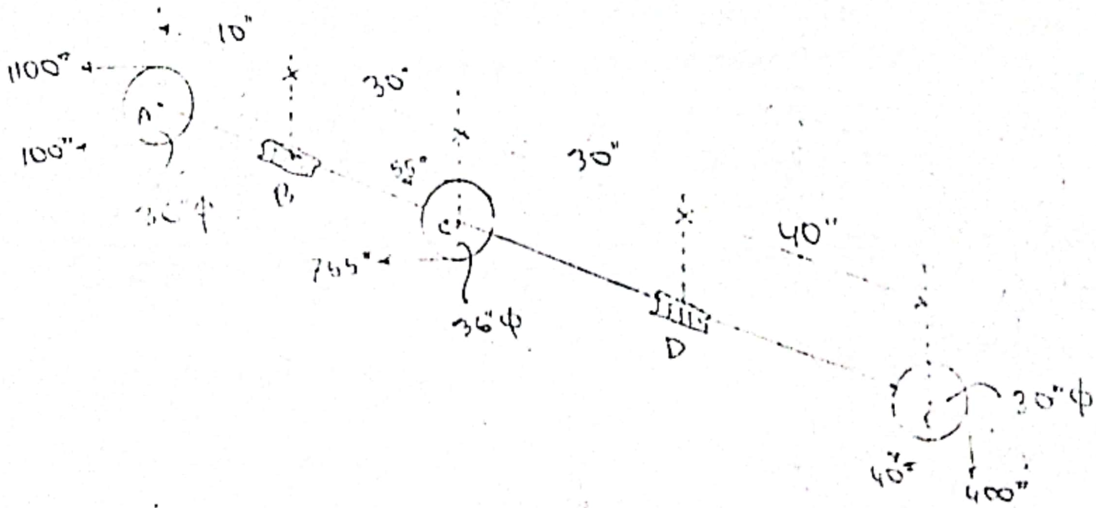
$\Rightarrow r = 4.69$ in.

The proper diameter of the shaft $= 2 \times 4.69$

$= 9.38$ in.

$\boxed{\text{Ans}}$

29. Design a solid shaft to carry the loads shown as in the following figure. The maximum $S_s \leq 8000$ psi and maximum $S_n \leq 12000$ psi; and the belt pulls on pulleys A and C are horizontal and those on pulley E are vertical. [2009, 2016]



Solution:

considering the horizontal plane;

$\Sigma M_D = 0$

$\Rightarrow (755+55)\times 30 + (1100+100)\times 70 - B_h \times 60 = 0$

$\Rightarrow B_h = 1805^{\#}$

$\Sigma F_h = 0$

$\Rightarrow 755+55+1100+100-1805 - D_h = 0$

$\Rightarrow D_h = 205^{\#}$

considering the vertical plane;

$\Sigma M_D = 0$

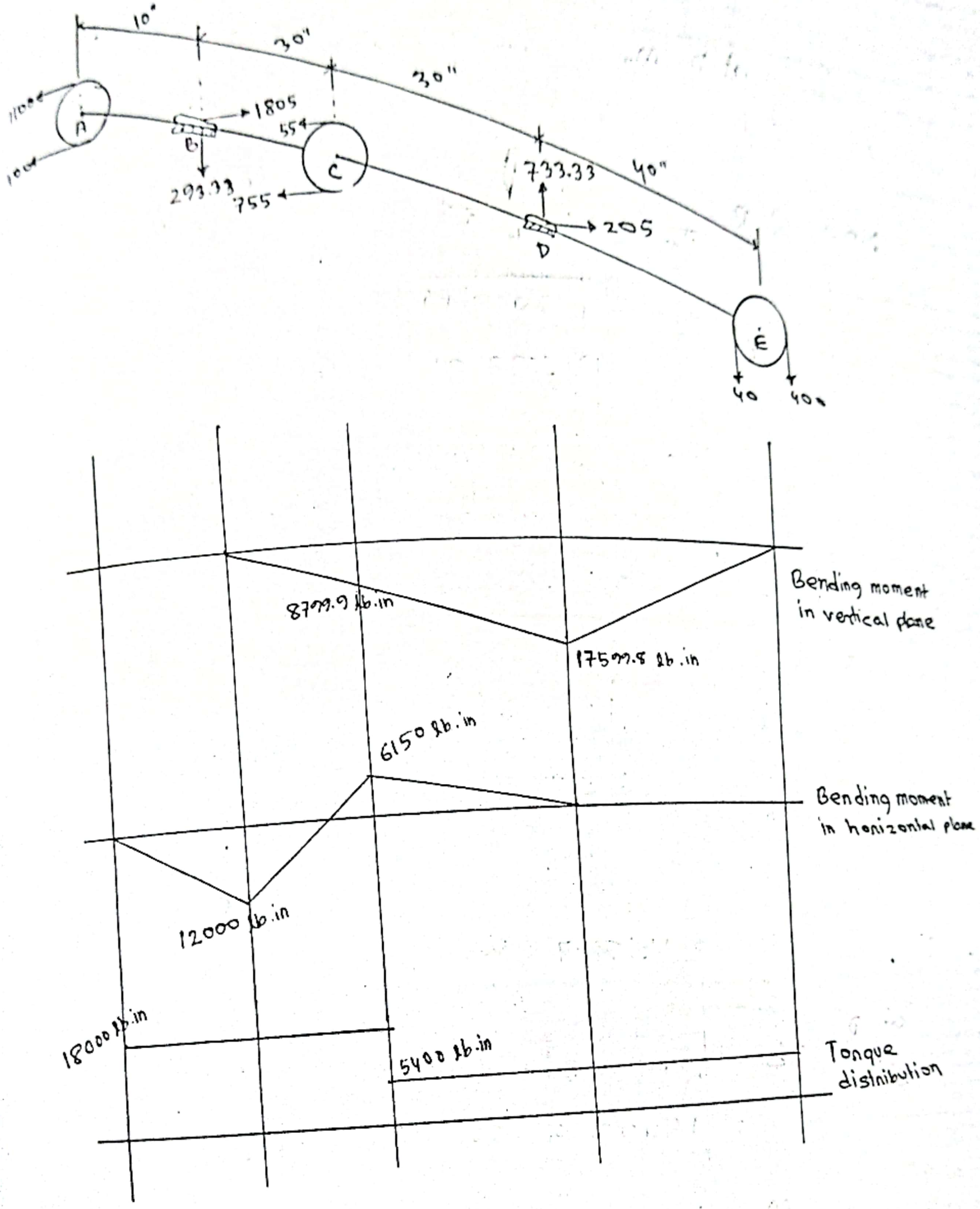
$\Rightarrow (40+400)\times 40 - B_v \times 60 = 0$

$\Rightarrow B_v = 293.33^{\#}$

$\Sigma F_v = 0$

$\Rightarrow D_v = 440 + 293.33$

$= 733.33^{\#}$



The resulting moment at any section is given by, $M = \sqrt{M_h^2 + M_v^2}$

Therefore the resulting moments at B; $M_B = \sqrt{0^2 + 12000^2}$

$= 12000$ lb.in

at c, $M_c = \sqrt{8799.9^2 + 6150^2}$

$= 10735.96$ lb.in.

at D, $M_D = \sqrt{17599.8^2 + 0^2}$

$= 17599.8$ lb.in.

Now at B,

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{12000^2 + 18000^2}$$

$$= 21633.31 \text{ lb.in.}$$

$$M_e = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(12000 + 21633.31)$$

$$= 16816.65 \text{ lb.}$$

at C,

$$T_e = \sqrt{10735.96^2 + 18000^2} = 20958.55 \text{ lb.in.}$$

$$M_e = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(10735.96 + 20958.55)$$

$$= 15847.26 \text{ lb.in.}$$

at D,

$$T_e = \sqrt{17599.8^2 + 5400^2}$$

$$= 18409.59 \text{ lb.in.}$$

$$M_e = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(17599.8 + 18409.59)$$

$$= 18004.69 \text{ lb.in.}$$

Hence largest value of T_e is 21633.31 lb.in.

and M_e is 18004.69 lb.in.

Maximum shearing stress $= \frac{2T_e}{\pi r^3}$

$$\Rightarrow 8000 = \frac{2 \times 21633.31}{\pi r^3}$$

$$\Rightarrow r = 1.20 \text{ in.}$$

Maximum normal stress $= \frac{4M_e}{\pi r^3}$

$$\Rightarrow 12000 = \frac{4 \times 18004.69}{\pi r^3}$$

$$\Rightarrow r = 1.24 \text{ in.}$$

Hence the proper diameter of the shaft

$$d = 2r$$
$$= 2 \times 1.24$$
$$= 2.48 \text{ in.}$$

$\boxed{\text{Ans}}$

BEAM DEFLECTIONS

Beam deflection : Beam deflection is defined as the vertical displacement of a point on a loaded beam.

There are many methods to find out the slope and deflection at a section in a loaded beam.

The maximum deflection occurs where slope is zero.

Some methods to find out the beam deflections:

1. Double integration method.
2. Area moment method.
3. Conjugate beam method.
4. Method of superposition.

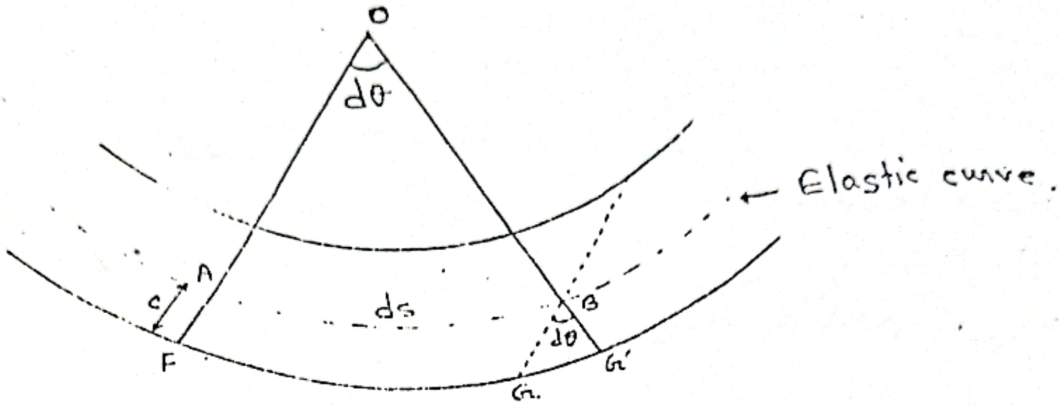
DOUBLE INTEGRATION METHOD

Elastic curve: The edge view of a neutral surface of a deflected beam is called the elastic curve of the beam.

Slope of a beam: Slope at any section in a deflected beam is defined as the angle in radians which the tangent at the section makes with the original axis of the beam.

Flexural rigidity of a beam: The product of the modulus of elasticity and moment of inertia (EI) is called the flexural rigidity of a beam. It is usually constant along the beam.

Relation between radius of curvature and bending moment.



consider (a beam which was straight when it was unloaded).

AB is a short part of the elastic curve of the beam which has been bent by loads. Plane through A and B which was originally parallel and vertical now meet at O.

It can be assumed that bending moments at A and B are equal and radius of curvature OA and OB are equal if A and B are very close and ds being infinitesimal.

The unit deformation of the extreme fiber is GG'

Hence $\boxed{\dfrac{GG'}{FG} = \dfrac{GG'}{AB}}$

$$E = \frac{S_s}{S_t}$$

$$\Rightarrow E = \frac{S_s}{\cancel{AB}} \cdot \frac{AB}{GG'}$$

The unit stress, $S = \frac{GG'}{AB} \cdot E$

$\Delta GBG'$ and ΔAOB may consider as similar triangle. If AB is very small and radius ($OA = OB = \rho$)

is very large compare to c.

$$\boxed{\begin{aligned} \frac{GG'}{AB} &= \frac{c}{\rho} \\ \Rightarrow \frac{S}{E} &= \frac{c}{\rho} \\ \Rightarrow S &= \frac{c}{\rho}E \end{aligned}} \quad \text{------ (i)}$$

But we know the bending stress, $S = \frac{Mc}{I}$ ...... (ii)

From equation (i) & (ii) we get,

$$\frac{Mc}{I} = (c/\rho)E$$

$$\Rightarrow \frac{M}{I} = E/\rho$$

$$\Rightarrow \boxed{M = \frac{EI}{\rho}}$$

which is the required relation.

Q. Following assumptions are commonly made for calculating deflection:

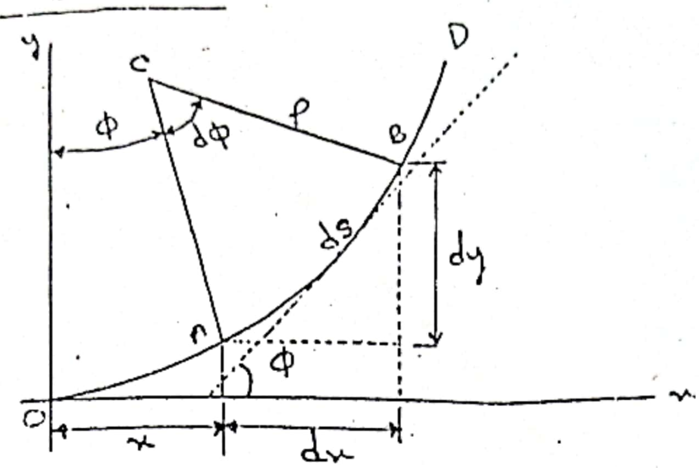
1. Stress caused by bending moment are below proportional limit so Hooks law holds.

ii. Plane section remains plane after the beam bent.

iii. Length of elastic curve is the same as the length of its horizontal projection.

iv. Deflection due to shear are negligible.

General equation of an elastic curve of a deflected beam.



OD is the part of elastic curve of the beam originally straight and horizontal.

consider two points A and B. The distance between them along the curve being ds and the horizontal distance between them being dx.

Let ϕ be the angle between a vertical line and a normal to the curve at A and dϕ be the angle subtended by the arc AB.

Hence $$\frac{ds}{\rho} = d\phi$$

$$\Rightarrow \frac{1}{\rho} = \frac{d\phi}{ds}$$

At any point on the curve where the slope is small then difference between ds and its horizontal projection dx is extremely small. So dx may be substituted from ds.

Hence $$1/\rho = \frac{d\phi}{dx}$$

The slope of the curve is $\frac{dy}{dx}$ at point A. Again $\frac{dy}{dx} = \phi$ in radian if ϕ is small angle.

since $\phi = \frac{dy}{dx}$.

$$\therefore \frac{d\phi}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right).$$

$$\Rightarrow \frac{1}{\rho} = \frac{d^2y}{dx^2}.$$

$$\Rightarrow \boxed{\frac{M}{EI} = \frac{d^2y}{dx^2}}$$

This equation is commonly called the general equation of an elastic curve of the deflected beam which is an expression for the rate of change of slope with respect to x

<u>Sign rule</u>:

* If left side upward then shear is positive.
* If left side clockwise then moment is positive.

Sequence for determining maximum deflection:

step 1: Draw elastic curve.

step-2: Make equation of moment. [upto 70% marks]

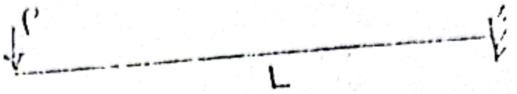
step 3: Make equation of slope $(\frac{dy}{dx})$:

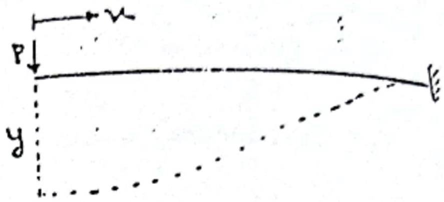
step-4: Make equation of deflection (y).

step-4: Calculate the values of constants C_1 and C_2 by putting boundary condition.

step-5: Get the desire deflection or slope.

01. Compute the maximum deflection of the following beam.





Solution:

Moment, $M = -Px$

$$\Rightarrow EI\frac{d^2y}{dx^2} = -Px \;\cdots\cdots (1)$$

By integration equation (1) w.r. to x,

$$EI\frac{dy}{dx} = -\frac{Px^2}{2} + C_1 \;\cdots\cdots (ii)$$

Again, $$EIy = -\frac{Px^3}{6} + C_1x + C_2 \;\cdots\cdots (iii)$$

when $x = L$ then $\frac{dy}{dx} = 0$

$\therefore$ from equation (ii) $\Rightarrow$ $0 = -\frac{PL^2}{2} + C_1$

$$\Rightarrow C_1 = \frac{PL^2}{2}$$

Again when $x = L$ then $y = 0$

From equation (iii); $0 = -\frac{PL^3}{6} + \frac{PL^2}{2}\cdot L + C_2$

$$\Rightarrow C_2 = -\frac{PL^3}{3}$$

Now from equation (iii);

$$EIy = -\frac{Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3}$$

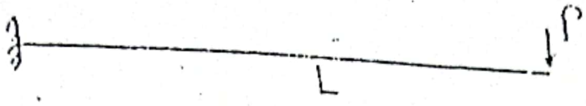
when $x = 0$ then $y = y_{max}$;

$$\therefore EIy_{max} = -\frac{PL^3}{3}$$

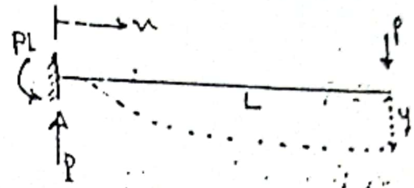
$$\Rightarrow y_{max} = \frac{PL^3}{3EI} (\downarrow)$$

Ans

02. Compute the maximum deflection of the structure shown below.



solution:



$M = -PL + Px$

$\Rightarrow EI \frac{d^2y}{dx^2} = -PL + Px$ .... (I)

By integration;

$$EI \frac{dy}{dx} = -PLx + \frac{Px^2}{2} + C_1 \text{ .... (II)}$$

Again, $EIy = -\frac{PLx^2}{2} + \frac{Px^3}{6} + C_1x + C_2$ .... (III)

when $x = 0$ then $\frac{dy}{dx} = 0$,

from equation (II); $C_1 = 0$

when $x = 0$ then $y = 0$

from equation (III); $0 = C_2$

when $x = L$ then $y = y_{max}$

Then from equation (III);

$$EIy_{max} = -\frac{PL.L^2}{2} + \frac{PL^3}{6}$$

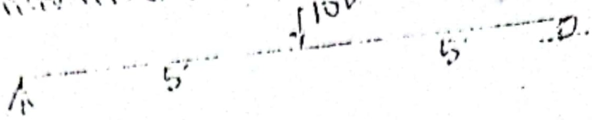
$$= -\frac{PL^3}{2} + \frac{PL^3}{6}$$

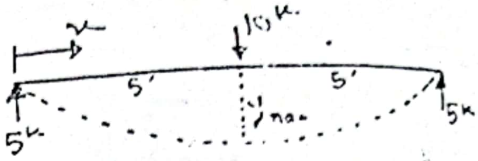
$$= -\frac{PL^3}{3}$$

$$\Rightarrow y_{max} = \frac{PL^3}{3EI} (\downarrow)$$

$\boxed{\text{Ans}}$

Q3. Calculate maximum deflection of the beam.





Solution:

$$M = 5x - 10\langle x-5\rangle$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 5x - 10\langle x-5\rangle \text{ ------- } (1)$$

By integrating (1):

$$EI\frac{dy}{dx} = 5x^2/2 - 10\frac{\langle x-5\rangle^2}{2} + c_1 \text{ ------- } (ii)$$

Again,

$$EI\,y = \frac{5x^3}{6} - 5\frac{\langle x-5\rangle^3}{3} + c_1x + c_2 \text{ ------- } (iii)$$

When $x=0$ then $y=0$

$\therefore$ From equation (iii); $0 = c_2$

when $x=10$ then $y=0$.

$\therefore$ from equation (iii); $0 = \frac{5\times10^3}{6} - \frac{5}{3}(10-5)^3 + 10c_1 + 0$

$$\Rightarrow c_1 = -62.5$$

$\therefore$ From equation (iii);

$$EIy = \frac{5x^3}{6} - \frac{5}{3}\langle x-5\rangle^3 - 62.5x$$

When $x=5$ then $y = y_{max}$

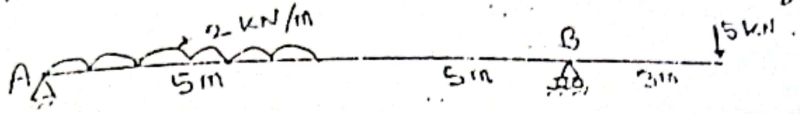
Hence

$$EIy_{max} = \frac{5\times5^3}{6} - \frac{5}{3}\times0 - 62.5\times5$$

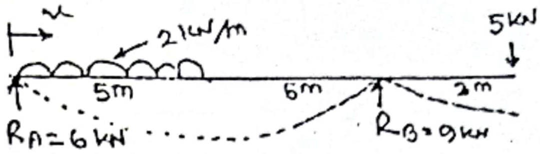
$$= -\frac{625}{3}$$

$$\Rightarrow y_{max} = \frac{625}{3EI}\ (\downarrow)$$

4. Compute location and magnitude of maximum deflection of the following beam.



Solution:



$\Sigma M_A = 0$

$\Rightarrow 2\times5\times5/2 + 5\times(5+5+3) - R_B\times10 = 0$

$\Rightarrow R_B = 9$ kN

$\Sigma F_y = 0$

$\Rightarrow R_A = (2\times5) + 5 - 9$

$= 6$ kN.

$$M = 6x - 2x.x/2 + \frac{2\langle x-5\rangle^2}{2} + 9\langle x-10\rangle$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 6x - x^2 + \langle x-5\rangle^2 + 9\langle x-10\rangle \text{ ------- (i)}$$

Integrating (i) w.r.t. x.

$$EI\frac{dy}{dx} = \frac{6x^2}{2} - \frac{x^3}{3} + \frac{\langle x-5\rangle^3}{3} + \frac{9\langle x-10\rangle^2}{2} + c_1 \text{ ------ (ii)}$$

Again, $EIy = x^3 - \frac{x^4}{12} + \frac{\langle x-5\rangle^4}{12} + \frac{9\langle x-10\rangle^3}{6} + c_1x + c_2 \text{ ..... (iii)}$

when $x = 0$, then $y = 0$

from equation (iii) ; $0 = 0 + c_2 \Rightarrow c_2 = 0$

When $x = 10$ then $y = 0$

from equation (iii) ; $0 = 10^3 - \frac{10^4}{12} + \frac{5^4}{12} + 0 + 10c_1 + 0$

$\Rightarrow c_1 = -21.875$

At maximum deflection slope $\frac{dy}{dx} = 0$

From equation (ii);

$$0 = 3x^2 - \frac{x^3}{3} + \frac{\langle x-5\rangle^3}{3} + \frac{9\langle x-10\rangle^2}{2} - 21.875$$

$$\Rightarrow \frac{x^3}{3} - 3x^2 - \frac{x^3 - 15x^2 + 75x - 125}{3} + 21.875 = 0$$

$$\Rightarrow \frac{x^3}{3} - 3x^2 - \frac{1}{3}x^3 + 5x^2 - \frac{75x}{3} + \frac{125}{3} + 21.875 = 0$$

$$\Rightarrow 2x^2 - \frac{75x}{3} + \frac{125}{3} + 21.875 = 0$$

$$\Rightarrow 6x^2 - 75x + 125 + 65.625 = 0$$

$$\Rightarrow 6x^2 - 75x + 190.625 = 0$$

$$\therefore x = 8.95, \ 3.55$$

At $x = 3.55$ and $x = 8.95$, $y = y_{max}$.

Hence $y_{3.55} = \frac{1}{EI}\left(3.55^3 - \frac{3.55^4}{12} - 21.875 \times 3.55\right)$

$$\therefore y_{3.55} = -\frac{46.15}{EI}$$

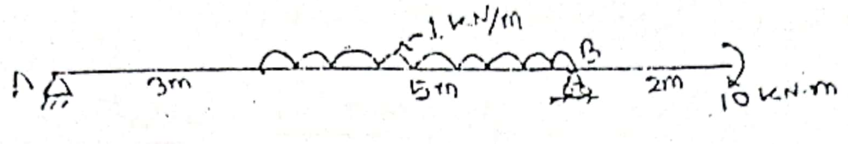
and $y_{8.95} = \frac{1}{EI}\left(8.95^3 - \frac{8.95^4}{12} + \frac{(8.95-5)^4}{12} - 21.875 \times 8.95\right)$

$$= \frac{6.72}{EI}$$

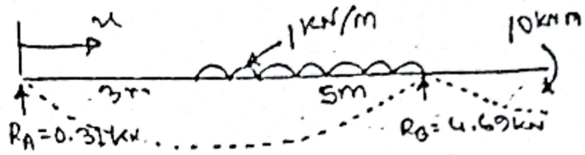
Maximum deflection will occur at $x = 3.55\,m$ from the left support and

The maximum deflection $y_{max} = \frac{46.15}{EI}$ (↓) $\boxed{\text{Ans}}$

05. Calculate location and magnitude of maximum deflection of the following beam.



Solution:



$\Sigma M_A = 0$

$\Rightarrow (1\times 5)(3+5/2) + 10 - R_B \times 8 = 0$

$\Rightarrow R_B = 4.69$ KN.

$\Sigma F_y = 0$

$\Rightarrow R_A = 5 - 4.69 = 0.31$ KN.

$$M = 0.31x - \frac{1\langle x-3\rangle^2}{2} + \frac{1\langle x-8\rangle^2}{2} + 4.69\langle x-8\rangle$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 0.31x - \frac{\langle x-3\rangle^2}{2} + \frac{\langle x-8\rangle^2}{2} + 4.69\langle x-8\rangle \text{ ------ (I)}$$

Integrating (I) w.r.t. x,

$$EI\frac{dy}{dx} = \frac{0.31x^2}{2} - \frac{\langle x-3\rangle^3}{6} + \frac{\langle x-8\rangle^3}{6} + \frac{4.69\langle x-8\rangle^2}{2} + c_1 \text{ ---- (II)}$$

Again,

$$EI.y = \frac{0.31x^3}{6} - \frac{\langle x-3\rangle^4}{24} + \frac{\langle x-8\rangle^4}{24} + \frac{4.69\langle x-8\rangle^3}{6} + c_1x + c_2 \text{ ---- (III)}$$

When $x=0$ then $y=0$,

from equation (III); $0 = c_2$

and when $x = 8$, $y=0$ then from equation (III)

$$0 = \frac{0.31\times 8^3}{6} - \frac{5^4}{24} - 0 + 0 + 8c_1 + 0$$

$\Rightarrow c_1 = -0.0515$

At maximum deflection $\frac{dy}{dx} = 0$

Hence from equation (ii);

$$0 = \frac{0.31x^2}{2} - \frac{\langle x-3\rangle^3}{6} + \frac{\langle x-8\rangle^3}{6} + \frac{4.69\langle x-8\rangle^2}{2} - 0.0515$$

$$\Rightarrow \frac{0.31x^2}{2} - \frac{x^3 - 9x^2 + 27x - 27}{6} - 0.0515 = 0$$

$$\Rightarrow 0.93x^2 - x^3 + 9x^2 - 27x + 27 - 0.309 = 0$$

$$\Rightarrow x^3 - 9.93x^2 + 27x - 26.691 = 0$$

$$\therefore x = 6.33$$

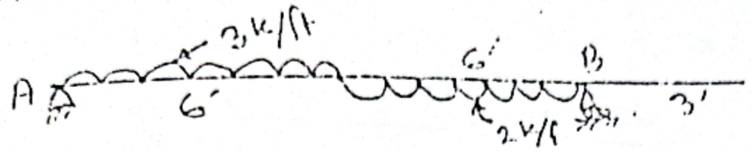
At $x = 6.33$, $y_{6.33} = \frac{1}{EI}\left(\frac{0.31 \times 6.33^3}{6} - \frac{3.33^4}{24} - 0.0515 \times 6.33\right)$

$$= \frac{7.655}{EI}$$

Hence the maximum deflection will occur at $x = 6.33$ m from the left support.

And the maximum deflection $y_{max} = \frac{7.655}{EI}$ $\boxed{\text{Ans}}$

06. calculate the magnitude and location of maximum deflection of the following figure.



solution:

$\Sigma M_A = 0$

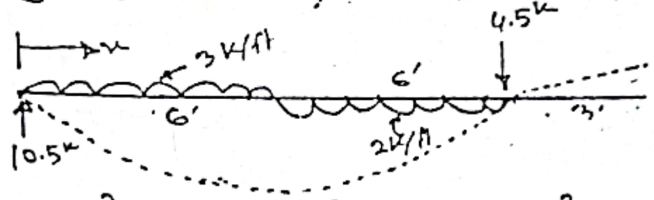
$\Rightarrow 3 \times 6 \times \frac{6}{2} - 2 \times 6 \times (6 + 6/2) + R_B \times 12 = 0$

$\Rightarrow R_B = 4.5\,k$.

$\Sigma f_y = 0$

$\Rightarrow R_A - 3 \times 6 - 4.5 + 2 \times 6 = 0$

$\Rightarrow R_A = 10.5\,k$.



$$M = 10.5x - 3x^2/2 + \frac{3\langle x-6\rangle^2}{2} + \frac{2\langle x-6\rangle^2}{2} - \frac{2\langle x-12\rangle^2}{2} - 4.5\langle x-12\rangle$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 10.5x - \frac{3x^2}{2} + \frac{3\langle x-6\rangle^2}{2} + \langle x-6\rangle^2 - \langle x-12\rangle^2 - 4.5\langle x-12\rangle.$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 10.5x - \frac{3x^2}{2} + \frac{5}{2}\langle x-6\rangle^2 - 5.5\langle x-12\rangle^2 \text{ -------(I)}$$

Integrating (I) w.r.t. x;

$$EI\frac{dy}{dx} = \frac{10.5x^2}{2} - \frac{3x^3}{6} + \frac{5}{2}\frac{\langle x-6\rangle^3}{3} - \frac{5.5\langle x-12\rangle^3}{3} + c_1 \text{ ------(II)}$$

Again,

$$EI.y = \frac{10.5x^3}{6} - \frac{x^4}{8} + \frac{5\langle x-6\rangle^4}{24} - \frac{5.5\langle x-12\rangle^4}{12} + c_1x + c_2 \text{ ----(III)}$$

when $x=0$ then $y=0$ and equation (III) is

$0 = 0 + c_2 \Rightarrow c_2 = 0$

when $x=12$ then $y=0$

$$\therefore 0 = \frac{10.5 \times 12^3}{6} - \frac{12^4}{8} + \frac{5 \times 6^4}{24} + c_1 \times 12$$

$\Rightarrow c_1 = -58.5$

At maximum deflection deflection $\frac{dy}{dx} = 0$

From equation (ii);

$$0 = \frac{10.5x^2}{2} - \frac{x^3}{2} + \frac{5}{2}\frac{\langle x-6\rangle^3}{3} - \frac{5.5\langle x-12\rangle^3}{3} - 58.5$$

$$\Rightarrow \frac{10.5x^2}{2} - \frac{x^3}{2} + \frac{5}{6}(x^3 - 18x^2 + 108x - 216) - 58.5$$

$$\Rightarrow \frac{x^3}{3} - \frac{39}{4}x^2 + 90x - 238.5 = 0$$

$\therefore x = 4.525$

Maximum deflection will occur at $x = 4.525$ ft.

$$EIy_{max} = \frac{10.5}{6} \times 4.525^3 - \frac{4.525^4}{8} - 58.5 \times 4.525$$

$$= -154.98$$

$$\Rightarrow y_{max} = \frac{-154.98}{EI}$$

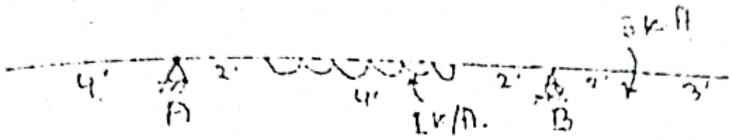
$$\Rightarrow y_{max} = \frac{155}{EI} (\downarrow)$$

Hence the maximum deflection,

$$y_{max} = \frac{155}{EI} (\downarrow)$$

Ans

Q. Compute the location and magnitude of maximum EIy of the following loaded beam.
[CT 2014 series]



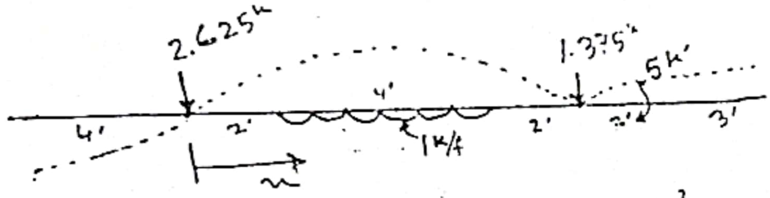
Solution: $\Sigma M_A = 0$

$\Rightarrow 5 - 1 \times 4 \times (2 + 4/2) + R_B \times 8 = 0$

$\Rightarrow R_B = 1.375\ k.$

$\Sigma F_y = 0$

$\Rightarrow R_A = 1 \times 4 - 1.375 = 2.625\ k.$



$$M = -2.625x + \frac{1\langle x-2\rangle^2}{2} - \frac{1\langle x-6\rangle^2}{2} - 1.375\langle x-8\rangle + 5\langle x-10\rangle^0$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = -2.625x + \frac{\langle x-2\rangle^2}{2} - \frac{\langle x-6\rangle^2}{2} - 1.375\langle x-8\rangle + 5\langle x-10\rangle^0 \quad \cdots\cdots (i)$$

Integrating (i) w.r.t. x,

$$EI\frac{dy}{dx} = -\frac{2.625x^2}{2} + \frac{\langle x-2\rangle^3}{6} - \frac{\langle x-6\rangle^3}{6} - \frac{1.375\langle x-8\rangle^2}{2} + 5\langle x-10\rangle + c_1 \quad \cdots\cdots (ii)$$

Again,

$$EIy = \frac{-2.625x^3}{6} + \frac{\langle x-2\rangle^4}{24} - \frac{\langle x-6\rangle^4}{24} - \frac{1.375\langle x-8\rangle^3}{6} + \frac{5\langle x-10\rangle^2}{2} + c_1x + c_2 \quad \cdots\cdots (iii)$$

At $x = 0$, $y = 0$ then from equation (iii)

$0 = c_2$

At $x=8$, $y=0$ then from equation (iii):

$$0 = \frac{-2.625\times 8^3}{6} + \frac{6^4}{24} - \frac{2^4}{24} + 8C_1$$

$\Rightarrow C_1 = 21.33$

At maximum deflection slope $= \frac{dy}{dx} = 0$

From equation (ii);

$$0 = \frac{-2.625x^2}{2} + \frac{\langle x-2\rangle^3}{6} - \frac{\langle x-6\rangle^3}{6} - \frac{1.375\langle x-8\rangle^2}{2} + .5\langle x-10\rangle + 21.33$$

$$\Rightarrow \frac{2.625x^2}{2} - \frac{1}{6}(x^3 - 6x^2 + 12x - 8) + \frac{1}{6}(x^3 - 18x^2 + 108x - 216) - 21.33 = 0$$

$$\Rightarrow \frac{2.625x^2}{2} + x^2 - 2x + 8/6 - 3x^2 + 18x - 36 - 21.33 = 0$$

$$\Rightarrow -0.6875x^2 + 16x - 56 = 0$$

$\therefore x = 4.29$, $x \neq 18.98$.

From equation (iii);

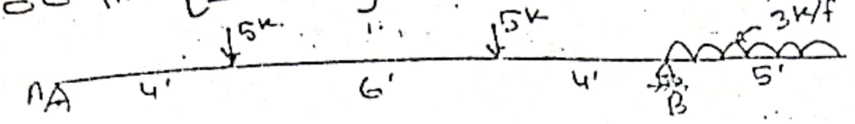
$$EIy = \frac{-2.625\times 4.29^3}{6} + \frac{2.29^4}{24} + 21.33\times 4.29$$

$$= 58.11$$

$$\Rightarrow y = \frac{58.11}{EI}$$

Maximum deflection will occur at $x = 4.29$ ft and the maximum $EIy = 58.11$ ($\uparrow$).

08. An overhanging beam is loaded as shown in figure below. calculate the value of maximum deflection of the beam. $E = 30000 \times 10^3$ ksi and $I = 200$ in^4. [2005]



Solution : $\Sigma M_A = 0$

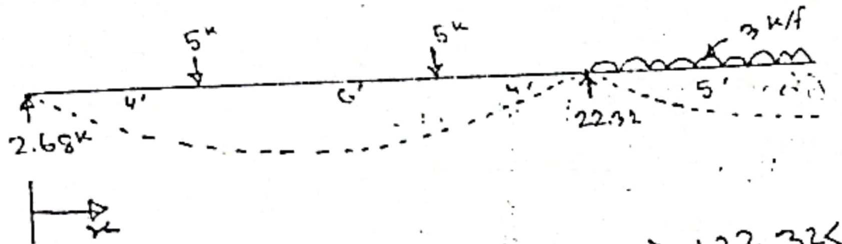
$\Rightarrow 5 \times 4 + 5 \times 9 + 3 \times 5 \times (14 + 2.5) - R_B \times 14 = 0$

$\Rightarrow R_B = 22.32k.$

$\Sigma f_y = 0$

$\Rightarrow R_A = 5 + 5 + 15 - 22.32$

$= 2.68k.$



$$M = 2.68x - 5\langle x-4\rangle - 5\langle x-10\rangle + 22.32\langle x-14\rangle - 3\frac{\langle x-14\rangle^2}{2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 2.68x - 5\langle x-4\rangle - 5\langle x-10\rangle + 22.32\langle x-14\rangle - \frac{3\langle x-14\rangle^2}{2} \quad \text{.......... (I)}$$

Integrating w.r.t. x,

$$EI\frac{dy}{dx} = \frac{2.68x^2}{2} - \frac{5\langle x-4\rangle^2}{2} - \frac{5\langle x-10\rangle^2}{2} + \frac{22.32\langle x-14\rangle^2}{2} - 1.5\langle x-14\rangle^3/3 + C_1 \quad \text{.......... (II)}$$

Again, $$EI.y = \frac{2.68x^3}{6} - \frac{5\langle x-4\rangle^3}{6} - \frac{5\langle x-10\rangle^3}{6} + \frac{11.16\langle x-14\rangle^3}{3} - \frac{1.5\langle x-14\rangle^4}{12} + C_1x + C_2 \quad \text{.......... (III)}$$

when $x=0$, $y=0$ then from equation (iii);

$0 = C_2$

Again when $x=14$, $y=0$ then from equation (iii);

$$0 = \frac{2.68 \times 14^3}{6} - \frac{5 \times 10^3}{6} - \frac{5}{6} \times 4^3 + 14C_1 + 0$$

$\Rightarrow C_1 = -24.21$

At maximum deflection slope $\frac{dy}{dx} = 0$

Hence from equation (ii);

$$0 = \frac{2.68x^2}{2} - \frac{5}{2}\langle x-4\rangle^2 - \frac{5}{2}\langle x-10\rangle^2 - 24.21$$

$\Rightarrow 1.34x^2 - 2.5(x^2 - 8x + 16) - 2.5(x^2 - 20x + 100) - 24.21 = 0$

$\Rightarrow 1.34x^2 - 5x^2 + 70x - 290 - 24.21 = 0$

$\Rightarrow 3.66x^2 - 70x + 314.21 = 0$

$\therefore x = 11.93, 7.20$

At $x = 11.93$, $EIy = \frac{2.68 \times 11.93^3}{6} - \frac{5}{6} \times 7.93^3 - 5/6 \times 1.93^3 - 24.21 \times 11.93$

$= 48.03$

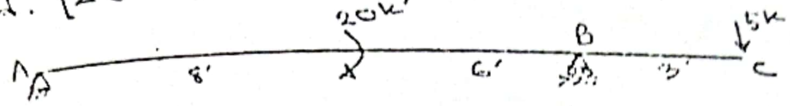
At $x = 7.20$, $EIy = \frac{2.68 \times 7.2^3}{6} - 5/6 \times 3.2^3 - 24.21 \times 7.2$

$= -34.9$

Hence the maximum deflection will occur at $x = 11.93$ ft from the left support.

Maximum deflection, $y_{max} = \frac{48.03}{EI}$ ($\uparrow$)

Q. An overhanging beam is shown in figure below. Calculate the value of maximum linear deflection for the beam. EI constants use double integration method. [2009]



Solution :

$\Sigma M_A = 0$

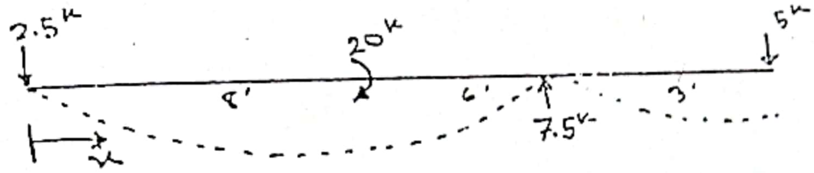
$\Rightarrow 20 + 5(8+6+3) - R_B * 14 = 0$

$\Rightarrow R_B = 7.5k$

$\Sigma F_y = 0$

$\Rightarrow R_A + R_B = 5$

$\Rightarrow R_A = -2.5k$.



$M = -2.5x + 2\langle x-8\rangle^{0} + 7.5\langle x-14\rangle$

$\Rightarrow EI\frac{d^2y}{dx^2} = -2.5x + 2\langle x-8\rangle^{0} + 7.5\langle x-14\rangle$ ------ (I)

Integrating (I) w.r.t. x,

$EI\frac{dy}{dx} = -2.5\frac{x^2}{2} + 2\langle x-8\rangle + \frac{7.5\langle x-14\rangle^2}{2} + C_1$ ------ (II)

Again, $EI.y = -\frac{2.5x^3}{6} + \frac{2\langle x-8\rangle^2}{2} - \frac{7.5\langle x-14\rangle^3}{6} + C_1x + C_2$ ...... (III)

when $x=0, y=0$ then from equation (III);

$0 = C_2$

when $x=14, y=0$ then from equation (III);

$0 = \frac{-2.5\times14^3}{6} + 6^2 + 14C_1$

$\Rightarrow C_1 = 79.1$

At maximum deflection slope $\frac{dy}{dx} = 0$

from equation (ii)

$$0 = \frac{-2.5x^2}{2} + 2\langle x-8\rangle + 79.1$$

$$\Rightarrow 1.25x^2 - 2x + 16 - 79.1 = 0$$

$$\Rightarrow 1.25x^2 - 2x - 63.1 = 0$$

$\therefore x = 7.95$ and $x \neq -6.35$

At $x = 7.95$, $EIy = \frac{-2.5}{6}(7.95)^3 + 79.1 \times 7.95$

$$= 419.49$$

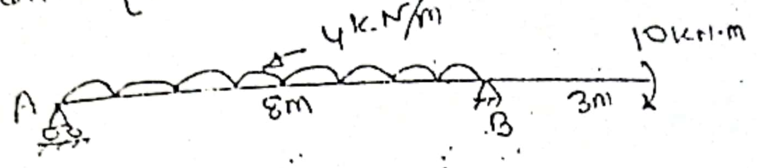
$$\Rightarrow y = \frac{419.49}{EI} (\uparrow)$$

Hence the maximum deflection will occur at $x = 7.95$ ft from the left support.

Maximum deflection $y_{max} = \frac{419.49}{EI} (\uparrow)$

$\boxed{\text{Ans}}$

An overhanging beam is shown in figure below. Calculate the value of maximum linear deflection and rotational deflection for the beam. Use double integration method. EI is constant. [2010]



Solution: $\Sigma M_A = 0$

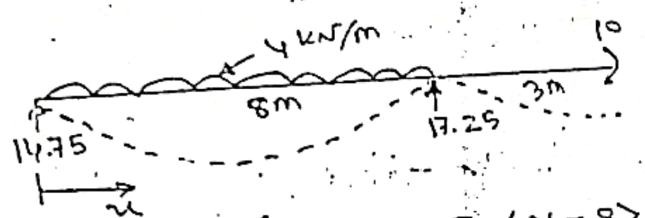
$\Rightarrow 4 \times 8 \times 8/2 + 10 - R_B \times 8 = 0$

$\Rightarrow R_B = 17.25$ KN.

$\Sigma F_y = 0$

$\Rightarrow R_A = 32 - 17.25$

$= 14.75$ KN.



$$M = 14.75x - 4x^2/2 + 17.25\langle x-8\rangle + \frac{4\langle x-8\rangle^2}{2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 14.75x - 2x^2 + 17.25\langle x-8\rangle + 2\langle x-8\rangle^2 \text{ ------ (i)}$$

Integrating (i) w.r.t. x,

$$EI\frac{dy}{dx} = \frac{14.75x^2}{2} - \frac{2x^3}{3} + \frac{17.25\langle x-8\rangle^2}{2} + \frac{2\langle x-8\rangle^3}{3} + C_1 \text{ ------ (ii)}$$

Again, $$EI\,y = \frac{14.75x^3}{6} - \frac{2x^4}{12} + \frac{17.25\langle x-8\rangle^3}{6} + \frac{2\langle x-8\rangle^4}{12} + C_1x + C_2 \text{ ------ (iii)}$$

At $x = 0$, $y = 0$ then from equation (iii);

$0 = C_2$

and at $x = 8$, $y = 0$ then from equation (iii);

$$0 = \frac{14.75}{6} \times 8^3 - \frac{8^4}{6} + \frac{17.25}{6} \times 0 + C_1 \times 8 + 0$$

$\Rightarrow C_1 = -72$

At maximum linear deflection slope $\frac{dy}{dx} = 0$

from equation (ii);

$$0 = \frac{14.75x^2}{2} - \frac{2x^3}{3} + \frac{17.25\langle x-8\rangle^2}{2} + \frac{2\langle x-8\rangle^3}{3} - 72$$

$$\Rightarrow \frac{2}{3}x^3 - 7.375x^2 + 72 = 0$$

$\therefore x = 3.88$ and $x \neq -2.79, -9.98$

At $x = 3.88$, $yEI = \frac{14.75}{6} \times 3.88^3 - \frac{3.88^4}{6} - 72 \times 3.88$

$$= -173.54$$

Hence the maximum deflection will occur at $x = 3.88$ m from the left support.

Maximum linear deflection, $y_{max} = \frac{173.54}{EI}$ ($\downarrow$) $\boxed{\text{Ans}}$

Rotational deflection (slope) at $x = 0$,

$$\frac{dy}{dx} = \frac{-72}{EI}$$

at $x = 8$, $\frac{dy}{dx} \cdot EI = \frac{14.75 \times 8^2}{2} - \frac{2}{3} \times 8^3 - 72$.

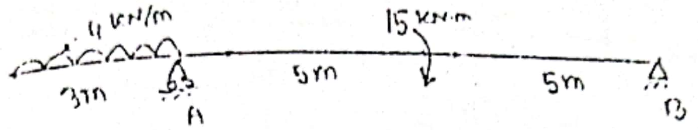
$$\Rightarrow \frac{dy}{dx} = \frac{58.67}{EI}$$

at $x = 11$, $\frac{dy}{dx} \cdot EI = \frac{14.75 \times 11^2}{2} - \frac{2}{3} \times 11^3 + \frac{17.25 \times 3^2}{2}$
$+ \frac{2}{3} \times 3^3 - 72$

$$\Rightarrow \frac{dy}{dx} = \frac{28.67}{EI}$$

Maximum rotational deflection $\left(\frac{dy}{dx}\right)_{max} = \frac{-72}{EI}$ $\boxed{\text{Ans}}$

An overhanging beam ABC is shown in figure below. Using double integration method calculate the value of maximum rotational deflection of the beam. EI is constant. [2011, 2015]



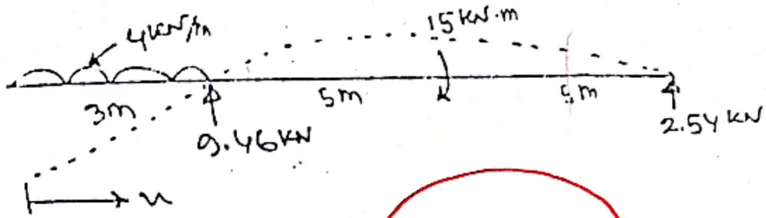
Solution: $\Sigma M_A = 0$

$\Rightarrow 3\times 4\times 3/2 + 15 - R_B \times 13 = 0$

$\Rightarrow R_B = 2.54$ kN.

$\Sigma f_y = 0$

$\Rightarrow R_A = 12 - 2.54 = 9.46$ kN.



$M = -4x^2/2 + 9.46\langle x-3\rangle + 15\langle x-8\rangle^0$

$\Rightarrow EI \frac{d^2y}{dx^2} = -2x^2 + 9.46\langle x-3\rangle + 15\langle x-8\rangle^0$ ---- (i)

Integrating (i) w.r.t. x,

$EI.\frac{dy}{dx} = -\frac{2x^3}{3} + \frac{9.46\langle x-3\rangle^2}{2} + 15\langle x-8\rangle^1 + C_1$ ---- (ii)

Again, $EI.y = -\frac{2x^4}{12} + \frac{9.46\langle x-3\rangle^3}{6} + \frac{15\langle x-8\rangle^2}{2} + C_1x + C_2$ ---- (iii)

when $x=3$, $y=0$ then from equation (iii);

$0 = -\frac{3^4}{6} + \frac{9.46\times 0}{6} + 3C_1 + C_2$ .... (A)

$\Rightarrow 3C_1 + C_2 = 13.5$ ....

when $x=13$, $y=0$ then from equation (iii);

$0 = -\frac{13^4}{6} + \frac{9.46\times 10^3}{6} + \frac{15\times 5^2}{2} + 13C_1 + C_2$

$\Rightarrow 13C_1 + C_2 = 2996$ ...... (B)

Solving equation (A) and (B)

$$C_1 = 298.25$$

$$C_2 = -881.25$$

At $x = 0$, $\left(\frac{dy}{dx}\right) = \frac{1}{EI} \times 298.25$

$$\Rightarrow \left(\frac{dy}{dx}\right)_0 = \frac{298.25}{EI}$$

At $x = 3$, $\left(\frac{dy}{dx}\right)_3 = \left(-\frac{2}{3} \times 3^3 + 298.25\right) 1/EI$

$$= \frac{280.25}{EI}$$

At $x = 13$, $\left(\frac{dy}{dx}\right)_{13} = \left(-\frac{2}{3} \times 13^3 + \frac{9.46}{2} \times 10^2 + 15 \times 5 + 298.2\right.$

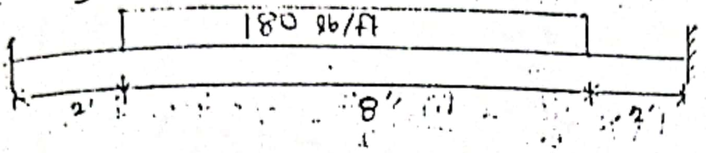
$$= -\frac{618.42}{EI}$$

Hence maximum rotational deflection is

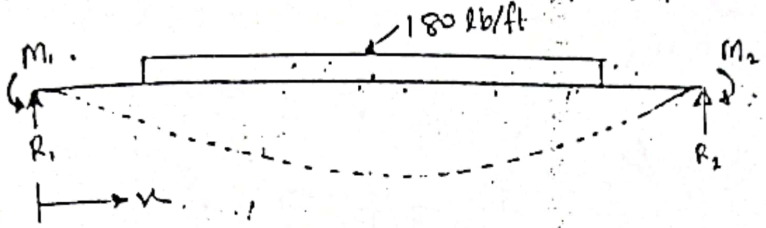
$$\left(\frac{dy}{dx}\right)_{max} = \frac{-618.42}{EI}$$

$\boxed{\text{Ans}}$

For the restrained beam shown in the following figure, compute the end moments and maximum EIS. [2012]



Solution :



$$M = R_1 x - M_1 - \frac{180\langle x-2\rangle^2}{2} + \frac{180\langle x-10\rangle^2}{2}$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = R_1 x - M_1 - 90\langle x-2\rangle^2 + 90\langle x-10\rangle^2 \text{ ------ (i)}$$

Integrating (i) w.r.t. x;

$$EI\frac{dy}{dx} = R_1 x^2/2 - M_1 x - \frac{90\langle x-2\rangle^3}{3} + \frac{90\langle x-10\rangle^3}{3} + C_1 \text{ ----- (ii)}$$

Again,

$$EI\,y = \frac{R_1 x^3}{6} - \frac{M_1 x^2}{2} - \frac{30\langle x-2\rangle^4}{4} + \frac{30\langle x-10\rangle^4}{4} + C_1 x + C_2 \text{ --- (iii)}$$

At origin (support) slope and deflection are zero.

Since C_1 and C_2 represent slope and deflection at origin respectively so $C_1 = C_2 = 0$.

$$\therefore EI\frac{dy}{dx} = \frac{R_1 x^2}{2} - M_1 x - 30\langle x-2\rangle^3 + 30\langle x-10\rangle^3 \text{ ------ (iv)}$$

$$\text{and } EI\,y = \frac{R_1 x^3}{6} - \frac{M_1 x^2}{2} - 7.5\langle x-2\rangle^4 + 7.5\langle x-10\rangle^4 \text{ ----- (v)}$$

At $x = 12'$ slope and deflection both are zero.

Hence. $0 = R_1 \frac{12^2}{2} - M_1 \times 12 - 30 \times 10^3 + 30 \times 2^3$

$\Rightarrow 72R_1 - 12M_1 = 29760$ .... (VI)

and $0 = R_1 \times \frac{12^3}{6} - \frac{M_1 \times 12^2}{2} - 7.5 \times 10^4 + 7.5 \times 2^4$

$\Rightarrow 288R_1 - 72M_1 = 74880$ .... (VII)

Solving equation (VI) & (VII);

$R_1 = 720\,lb.$

$M_1 = 1840\,lb.ft.$

$\Sigma F_y = 0$

$\Rightarrow R_1 + R_2 - 180 \times 8 = 0$

$\Rightarrow R_2 = 720\,lb.$

$\Sigma M_1 = 0$

$\Rightarrow -M_1 + (180 \times 8)(2 + 8/2) - R_2 \times 12 + M_2 = 0$

$\Rightarrow -1840 + 8640 - 720 \times 12 + M_2 = 0$

$\Rightarrow M_2 = 1840\,lb.ft.$

Maximum deflection will occur at midspan, i.e. $x = 6'$

Hence, $EIy = \frac{720 \times 6^3}{6} - \frac{1840 \times 6^2}{2} - (6-2)^4 \times 7.5$

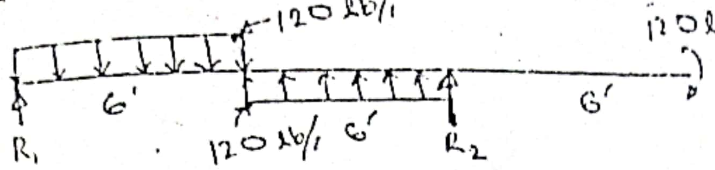
$= -9120$

Ans: $M_1 = 1840\,lb.ft$ (↺)

$M_2 = 1840\,lb.ft$ (↻)

$EI\delta = 9120\,lb.ft^3$ (↓)

Calculate the maximum value of EIδ for the beam loaded as shown in the figure below by double integration method. [2013]



Solution: $\Sigma M_1 = 0$

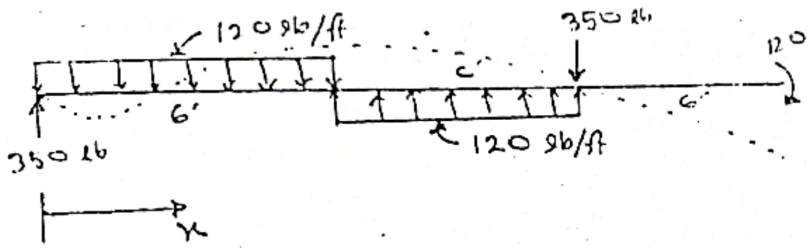
$\Rightarrow 6 \times 120 \times 6/2 + 120 - 120 \times 6 \times (6 + 6/2) - R_2 \times 12 = 0$

$\Rightarrow R_2 = -350\,lb$.

$\Sigma F_y = 0$

$\Rightarrow R_1 + R_2 - 120 \times 6 + 120 \times 6 = 0$

$\Rightarrow R_1 = -R_2 = 350\,lb$.



$$M = 350x - 120x^2/2 + \frac{120\langle x-6\rangle^2}{2} + \frac{120\langle x-6\rangle^2}{2} - \frac{120\langle x-12\rangle^2}{2} - 350\langle x-12\rangle$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 350x - 60x^2 + 120\langle x-6\rangle^2 - 60\langle x-12\rangle^2 - 350\langle x-12\rangle \quad \text{......(i)}$$

Integrating (i) w.r.t. x,

$$EI\frac{dy}{dx} = \frac{350x^2}{2} - \frac{60x^3}{3} + \frac{120\langle x-6\rangle^3}{3} - \frac{60\langle x-12\rangle^3}{3} - \frac{350\langle x-12\rangle^2}{2} + c_1 \quad \text{......(ii)}$$

Again, $$EIy = \frac{350x^3}{6} - 5x^4 + 10\langle x-6\rangle^4 - 5\langle x-12\rangle^4 - \frac{350\langle x-12\rangle^3}{6} + c_1x + c_2 \quad \text{......(iii)}$$

When $x=0$, $y=0$ then from equation (iii);

$C_2 = 0$

When $x=12$, $y=0$ then from equation (iv);

$$0 = \frac{350 \times 12^3}{6} - 5\times 12^4 + 10\times 6^4 - 5\times 6^4 + 12C_1$$

$\Rightarrow C_1 = -300$

At maximum deflection slope $\frac{dy}{dx} = 0$ then from equation (ii);

$$0 = 175x^2 - 20x^3 + 60\langle x-6\rangle^3 - 20\langle x-12\rangle^3 - 175\langle x-12\rangle^2 - 350$$

$\Rightarrow 175x^2 - 20x^3 + 60(x^3 - 18x^2 + 108x - 216) - 350 = 0$

$\Rightarrow 40x^3 - 905x^2 + 6480x - 13310 = 0$

$\therefore x = 3.50$

At $x = 3.5$, $EIy = \frac{350}{6}\times 3.5^3 - 5\times 3.5^4 - 350\times 3.50$

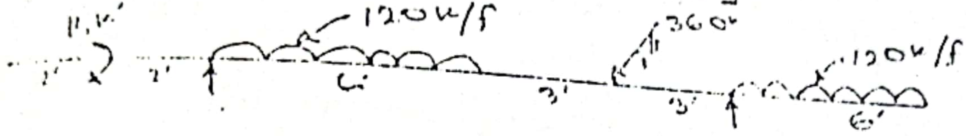
$= 525.73$

At $x = 3.50$ ft from the left end maximum deflection occur and the maximum value of

$EI\delta = 525.73$ lb.ft^3

$\boxed{\text{Ans}}$

Using double integration method compute the value of 'EIy' midway between the supports for the following beam. [2016]



Solution:

$\Sigma M_1 = 0$

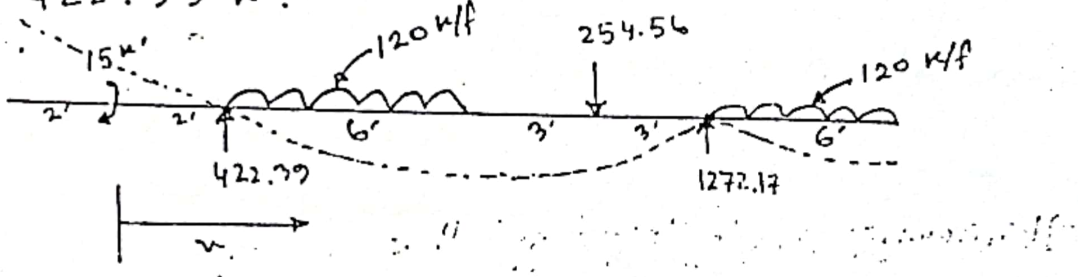
$\Rightarrow 15 + 120 \times 6 \times 3 + \frac{360}{\sqrt{2}} \times 1 (6+3) + 120 \times 6 \times (6+3+3+6/2)$
$- R_2 \times 12 = 0$

$\Rightarrow R_2 = 1272.17\,k$

$\Sigma F_y = 0$

$\Rightarrow R_1 = 6 \times 120 + \left(\frac{360}{\sqrt{2}} \times 1\right) + 6 \times 120 - 1272.17$

$= 422.39\,k.$



$M = 15 + 422.39\langle x-2\rangle - \frac{120\langle x-2\rangle^2}{2} - 254.56\langle x-11\rangle$
$+ 1272.17\langle x-14\rangle - \frac{120\langle x-14\rangle^2}{2} + \frac{120\langle x-8\rangle^2}{2}$

$\Rightarrow EI\frac{d^2y}{dx^2} = 15 + 422.39\langle x-2\rangle - 60\langle x-2\rangle^2 + 60\langle x-8\rangle^2 + 1272.17\langle x-14\rangle - 60\langle x-14\rangle^2$ .... (I)
$- 254.56\langle x-11\rangle$

Integrating (I) w.r.t. x,

$EI\frac{dy}{dx} = 15x + \frac{422.39\langle x-2\rangle^2}{2} - \frac{60\langle x-2\rangle^3}{3} + \frac{60\langle x-8\rangle^3}{3}$
$- \frac{254.56\langle x-11\rangle^2}{2} + \frac{1272.17\langle x-14\rangle^2}{2} - \frac{60\langle x-14\rangle^3}{3} + C_1$
............(II)

Again, $EI.y = \frac{15x^2}{2} + \frac{422.39\langle x-2\rangle^3}{6} - \frac{60\langle x-2\rangle^4}{12} + \frac{60\langle x-8\rangle^4}{12}$
$- \frac{254.56\langle x-11\rangle^3}{6} + \frac{1272.17\langle x-14\rangle^3}{6} - \frac{60\langle x-14\rangle^4}{12} + C_1x + C_2$ .... (iii)

when $x=2$; $y=0$ then from equation (iii)

$0 = \frac{15\times 2^2}{2} + 2C_1 + C_2$

$\Rightarrow 2C_1 + C_2 = -30$ .... (IV)

when $x=14$. $y=0$ then from equation (iii);

$0 = \frac{15\times 14^2}{2} + \frac{422.39\times 12^3}{6} - \frac{60\times 12^4}{12} + \frac{60(6)^4}{12} - \frac{254.56\times 3^3}{6} + 14C_1 + C_2$

$\Rightarrow 14C_1 + C_2 = -24772.8$ .... (V)

Solving equation (IV) and (V);

$C_1 = -2061.9$

$C_2 = 4093.8$

Deflection midway between between the supports i.e.

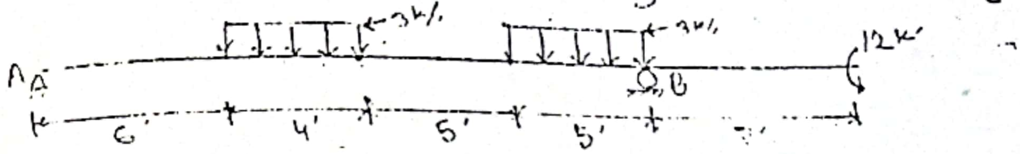
$x = 2 + \frac{12}{2}$
$= 2 + 6$
$= 8$ ft

$EIy = \frac{15\times 8^2}{2} + \frac{422.39\times 6^3}{6} - \frac{60\times 6^4}{12} - 2061.9\times 8 + 4093.8$

$= -3195.36$ k. ft^3

$\boxed{\text{Ans}}$

6. Determine the position of maximum deflection and compute the value of EIY at the same location of the beam loaded as shown in figure below. (Use double Integration method) [2018]



Solution:

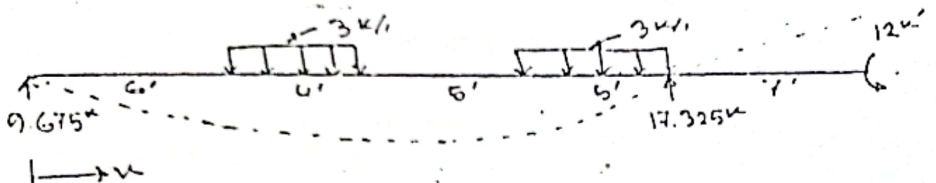
$\Sigma M_A = 0$

$\Rightarrow 3\times4\times8 + 3\times5\times17.5 - 12 - R_B\times20 = 0$

$\Rightarrow R_B = 17.325^k$

$\Sigma F_y = 0$

$\Rightarrow R_A = 12 + 15 - 17.325 = 9.675^k$.



$$M = 9.675x - \frac{3\langle x-6\rangle^2}{2} + \frac{3\langle x-10\rangle^2}{2} - \frac{3\langle x-15\rangle^2}{2} + \frac{3\langle x-20\rangle^2}{2} + 17.325\langle x-20\rangle$$

$$\Rightarrow EI\frac{d^2y}{dx^2} = 9.675x - 1.5\langle x-6\rangle^2 + 1.5\langle x-10\rangle^2 - 1.5\langle x-15\rangle^2 + 1.5\langle x-20\rangle^2 + 17.325\langle x-20\rangle \text{ - - - - - - - - (i)}$$

Integrating (i) w.r.t x,

$$EI\frac{dy}{dx} = \frac{9.675x^2}{2} - \frac{1.5\langle x-6\rangle^3}{3} + \frac{1.5\langle x-10\rangle^3}{3} - \frac{1.5\langle x-15\rangle^3}{3} + \frac{1.5\langle x-20\rangle^3}{3} + \frac{17.325\langle x-20\rangle^2}{2} + c_1 \text{ - - - - (ii)}$$

Again,

$$EI\,y = \frac{9.675x^3}{6} - \frac{1.5\langle x-6\rangle^4}{12} + \frac{1.5\langle x-10\rangle^4}{12} - \frac{1.5\langle x-15\rangle^4}{12} + \frac{1.5\langle x-20\rangle^4}{12} + \frac{17.325\langle x-20\rangle^3}{6} + c_1x + c_2 \text{ - - - - - - (iii)}$$

At $x=0$, $y=0$ then from equation (iii);

$c_2 = 0$.

At $x=20$, $y=0$ then from equation (iii);

$$0 = \frac{9.675\times20^3}{6} - \frac{1.5\times14^4}{12} + \frac{1.5\times10^4}{12} - \frac{1.5\times5^4}{12} + 20c_1$$

$$\Rightarrow 20C_1 = -9269.875$$

$$\Rightarrow C_1 = -463.49$$

At maximum deflection slope $\frac{dy}{dx} = 0$. Hence from equation (ii);

$$0 = \frac{9.675x^2}{2} - \frac{\langle x-6\rangle^3}{2} + \frac{\langle x-10\rangle^3}{2} - 463.49$$

$$\Rightarrow \frac{9.675x^2}{2} - \frac{1}{2}(x^3 - 18x^2 + 108x - 216) + \frac{1}{2}(x^3 - 30x^2 + 300x - 1000) - 463.49 = 0$$

$$\Rightarrow \frac{9.675x^2}{2} - \frac{x^3}{2} + 9x^2 - 54x + 108 + \frac{x^3}{2} - 15x^2 + 150x - 500 - 463.49 = 0$$

$$\Rightarrow -1.1625x^2 + 96x - 855.49 = 0$$

$$\therefore x = 10.16 \text{ ft}.$$

Hence the maximum deflection will occur at $x = 10.16$ ft from left support. and

The maximum deflection

$$EIy = \frac{9.675 \times 10.16^3}{6} - \frac{1.5 \times 4.16^4}{12} + \frac{1.5 \times 0.16^4}{12} - 463.49 \times 10.16$$

$$= -4884.15$$

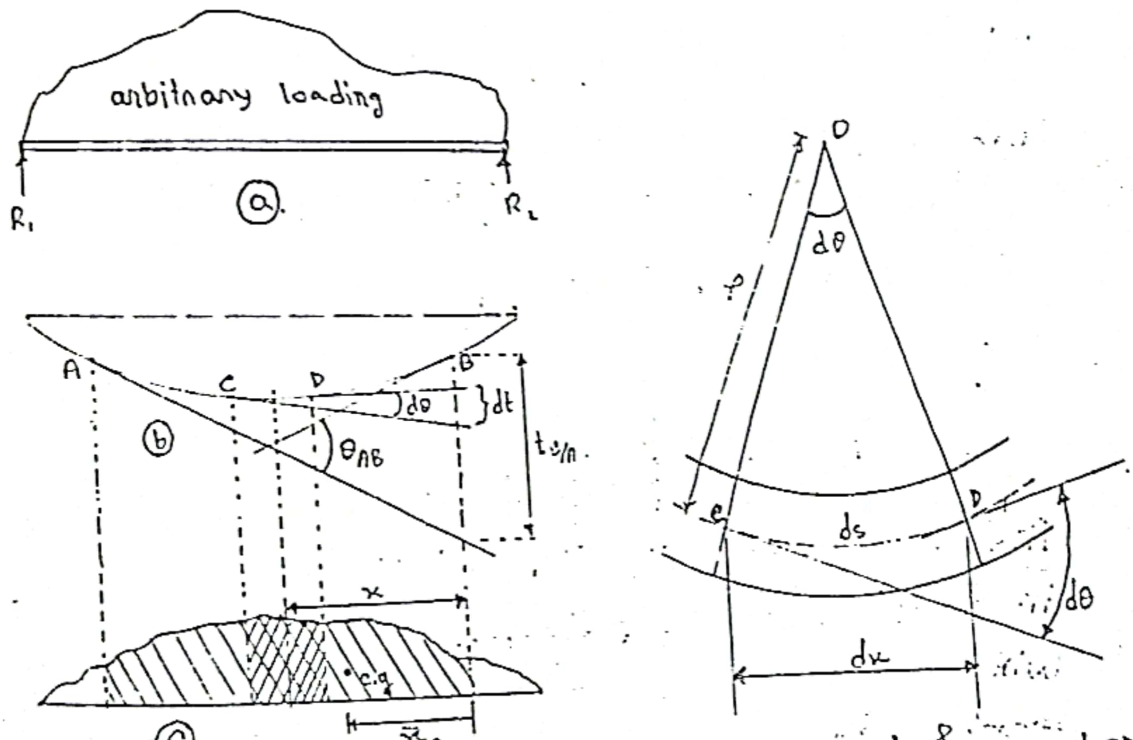
$$\therefore (EIy)_{max} = -4884.15 \text{ k.ft}^3$$

$\boxed{\text{Ans}}$

AREA MOMENT METHOD

Area moment method is a useful and simple method of determining slopes and deflections in beams. It involves the area of the moment diagram and also the moment of that area.

Theorems :



Enlargement of segment CD

Consider a simple beam that supports any types of loading as in fig.(a).

The elastic curve is the edge view of the neutral surface and is shown with greatly exaggerated deflections as in fig (b).

The moment diagram is assumed to be as in fig. (c)

From the enlarged detail of section CD the arc distance ds measured along the elastic curve between the two sections equal $\rho d\theta$ where ρ is the radius of curvature of elastic curve at the given section.

We know $1/\rho = M/EI$ and since $ds = \rho d\theta$

we can write $\frac{1}{\rho} = \frac{M}{EI} = \frac{d\theta}{ds}$

$$\therefore d\theta = \frac{M}{EI} ds$$

So considering ds equal to dx we obtain,

$$d\theta = \frac{M}{EI} dx$$

Tangents drawn to the elastic curve at C and D in fig (b) are separated by an angle $d\theta$ by which sections OC and OD rotate relative to each other.

Hence the change in slope between tangents drawn to the elastic curve at any points A & B will equal the sum of such small angles.

we can say $\theta_{AB} = \int_{\theta_A}^{\theta_B} d\theta$

$$= \frac{1}{EI}\int_{x_A}^{x_B} M\,dx \quad \text{.............} \quad (A)$$

Hence again in fig (b) the distance from the point B on the elastic curve (measured perpendicular to the original position of the beam) that will intersect a tangent drawn to this curve at any other point A is the sum of the intercepts dt created by tangents drawn to the curve at adjacent points.

Each of these intercepts may be considered as the arc of circle of radius x subtended by the angle $d\theta$.

$$dt = x\,d\theta$$

Hence $t_{B/A} = \int dt$

$$= \int x\,d\theta$$

Replacing $d\theta$ we obtained,

$$t_{B/A} = \frac{1}{EI}\int_{x_A}^{x_B} x(M\,dx) \quad \text{.............} \quad (B)$$

The length $t_{B/A}$ is known as the deviation of B from a tangent drawn at A or tangential deviation of B with respect to A.

From the moment diagram in fig ⓒ we see that Mdx is the area of shaded element located a distance x from the ordinate through B. Since $\int Mdx$ means a summation of such elements so equation Ⓐ may be expressed as

$$\boxed{\theta_{AB} = \frac{1}{EI}(Area)_{AB}} \quad \cdots\cdots \text{Ⓒ}$$

This is the algebraic expression of Theorem. I which is stated as follows:

Theorem I :

The change in slope between tangents drawn to the elastic curve at any two points A and B is equal to the product of $\frac{1}{EI}$ multiplied by the area of the moment diagram between these two points.

Again from figure ⓒ the expression $x(Mdx)$ which appears under the integral sign in equation Ⓑ is the moment of area of the shaded element about the ordinate at B.

Hence the geometric significance of the integral $\int x(Mdx)$ is that the integral is equivalent to the moment of area about the ordinate at B of that part of the moment diagram between A and B.

Thus we obtain the algebraic form of Theorem-II:

$$t_{B/A} = \frac{1}{EI} (Area)_{AB} \cdot \bar{x}_B$$

This is stated more formally as:

Theorem-II :

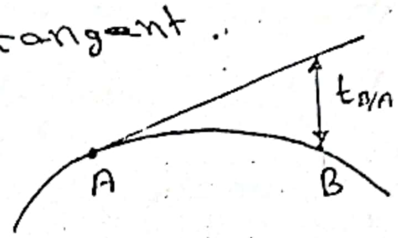
The deviation of any point B relative to a tangent drawn to the elastic curve at any other point A. in a direction perpendicular to the original position of the beam. is equal to the product of $\frac{1}{EI}$ multiplied by the moment of area about B of that part of the moment diagram between points A and B.

Sign rule:

1. Sign of deviation:

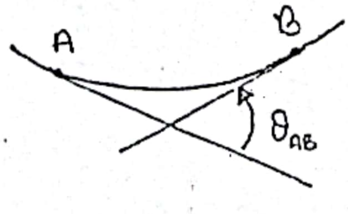
a. Positive deviation: The deviation at any point is positive if the point lies above the reference tangent from which the deviation is measured.

b. Negetive deviation: The deviation at any point is negative if the point lies below the reference tangent.

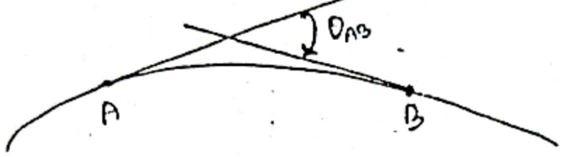


2. Sign of change of slope:

a. Positive change of slope: Positive change of slope θ_{AB} means that the tangent at the rightmost point B is measured in a counter-clockwise direction from the tangent at the leftmost point.



b. Negative change of slope:
Negative change of slope θ_{AB} means that the tangent at the rightmost point B is measured in a clockwise direction from the tangent at the leftmost point.



* Slope $= \frac{1}{EI}$ [Area of the moment diagram]

* Deviation $= \frac{1}{EI}$ [Moment of that moment diagram]

* Deviation $t_{B/A} \neq$ Deviation $t_{A/B}$

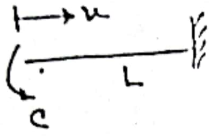
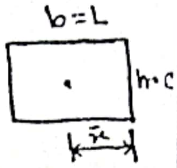
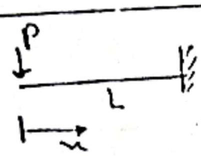
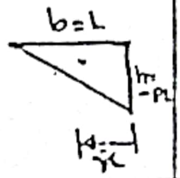
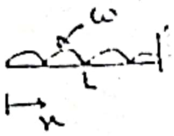
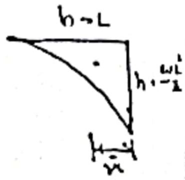
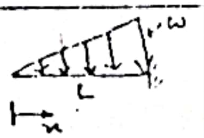
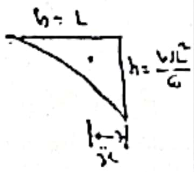
* Sequence for drawing moment diagram:

step-1: Select a reference axis on the most acceptable point on a beam.

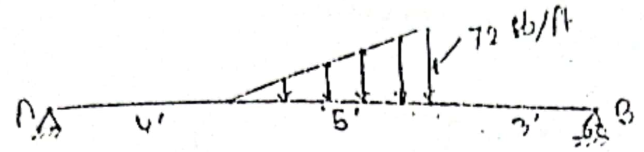
Step-2: consider all applied load as equivalent cantilever load from the reference axis.

Step-3: Draw the moment diagrams for all cantilever loads from the reference axis.

Cantilever loading:

Types of loading	cantilever beam	Moment equation $y = Kx^n$	Degree of moment equation (n)	Moment diagram	Area of moment diagram $\frac{1}{n+1}$ bh	centroid of moment diagram $\frac{1}{n+2} \cdot b$
Couple		$M = -Cx^0$	zero		$\frac{1}{1}bh$	$\frac{1}{2}b$
Concentrated		$M = -Px^1$	1^{st}		$\frac{1}{2}bh$	$\frac{1}{3}b$
Uniformly distributed		$M = -\frac{Px^2}{2}$	2^{nd}		$\frac{1}{3}bh$	$\frac{1}{4}b$
Uniformly varying		$M = -\frac{W}{6L}x^3$	3^{rd}		$\frac{1}{4}bh$	$\frac{1}{5}b$

Q. Compute the moment of area of the moment diagram about the left end.



Solution:

$$\Sigma M_A = 0$$

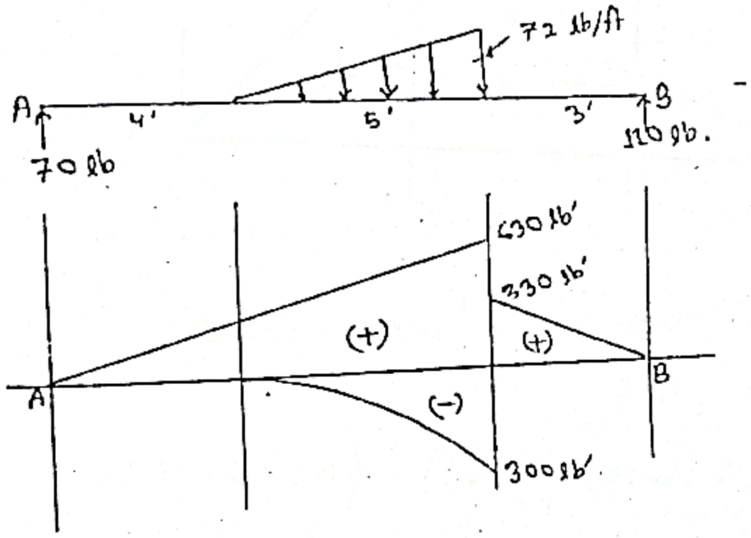
$$\Rightarrow \left(\frac{1}{2}\times 5\times 72\right)\times\left(4+\frac{2}{3}5\right) - R_B\times 12 = 0$$

$$\Rightarrow R_B = 110 \text{ lb.}$$

$$\Sigma F_y = 0$$

$$\Rightarrow R_A = \frac{1}{2}\times 5\times 72 - 110$$

$$= 70 \text{ lb.}$$



Moment of area of the moment diagram,

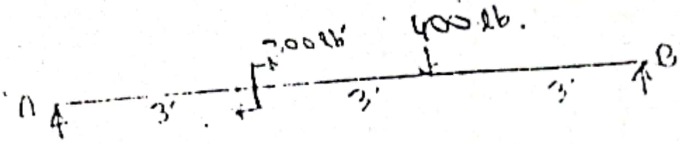
$$(\text{Area})_{AB}\cdot\bar{x}_A = \left\{\left(\frac{1}{2}\times 9\times 630\right)\times\left(\frac{2}{3}\times 9\right)\right\} + \left\{\left(\frac{1}{2}\times 3\times 330\right)\times\left(9+3/3\right)\right\}$$

$$- \left\{\left(1/4\times 5\times 300\right)\times\left(9-5/5\right)\right\}$$

$$= 17010 + 4950 - 3000$$

$$= 18960 \text{ lb.ft}^3 \quad \boxed{\text{Ans}}$$

02.. Compute the moment of area of the moment diagram between the reactions about both the left and the right supports.



Solution: $\Sigma M_A = 0$

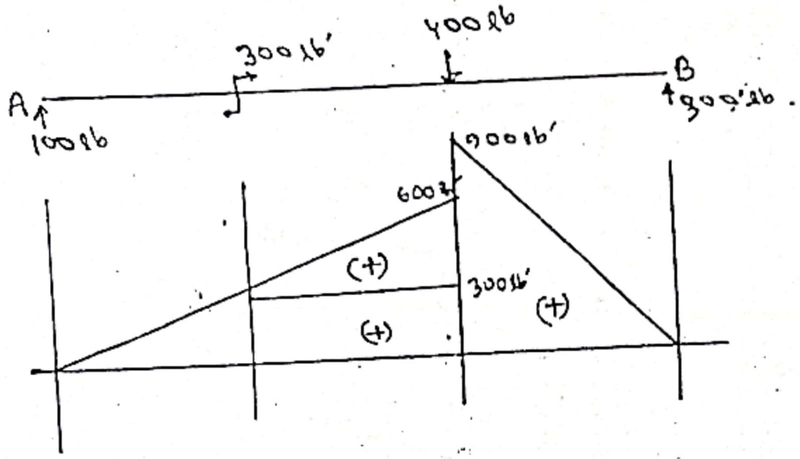
$\Rightarrow 300 + 400 \times 6 - R_B \times 9 = 0$

$\Rightarrow R_B = 300$ lb.

$\Sigma F_y = 0$

$\Rightarrow R_A = 400 - 300$

$= 100$ lb.



The moment of area of the moment diagram:

(i) About left support,

$(Area)_{AB} \cdot \bar{x}_A = \{(\frac{1}{2} \times 6 \times 600) \times (\frac{2}{3} \times 6)\} + \{(300 \times 3)(3 + 3/2)\} + \{(\frac{1}{2} \times 3 \times 900) * (6 + 3/3)\}$

$= 7200 + 4050 + 9450$

$= 20700$ lb.ft³. $\boxed{Ans}$

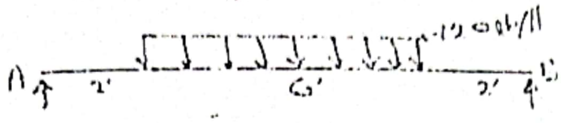
(ii) About right support,

$(Area)_{AB} \cdot \bar{x}_B = \{(1/2 \times 6 \times 600)(3 + 6/3)\} + \{(300 \times 3)(3 + 3/2)\} + \{(1/2 \times 3 \times 900)(\frac{2}{3} \times 3)\}$

$= 9000 + 4050 + 2700$

$= 15750$ lb.ft³. $\boxed{Ans}$

Q3. Calculate $(\text{Area})_{AB} \cdot \bar{x}_B$.



Solution:

$\Sigma M_A = 0$

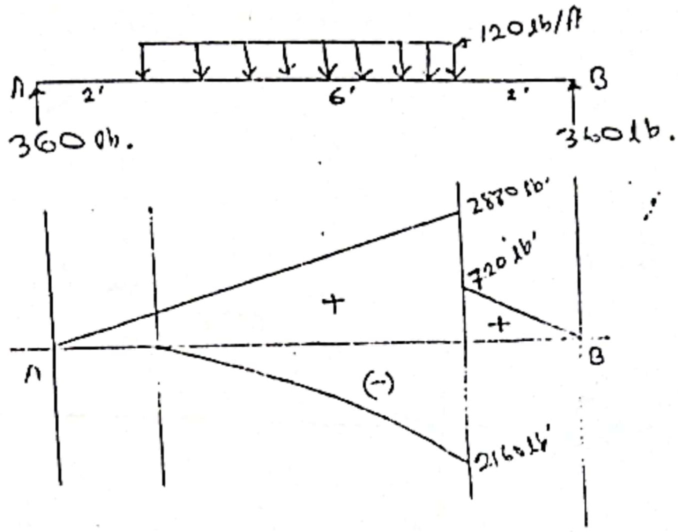
$\Rightarrow 120 \times 6 \times (6/2 + 2) - R_B \times 10 = 0$

$\Rightarrow R_B = 360 \text{ lb}.$

$\Sigma F_y = 0$

$\Rightarrow R_A = 720 - 360$

$= 360^{\#}$



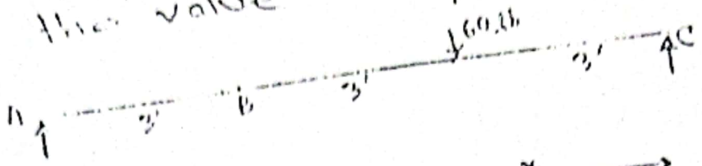
$$(\text{Area})_{AB} \cdot \bar{x}_B = \left\{\left(\tfrac{1}{2} \times 2 \times 720\right) \times \left(\tfrac{2}{3} \times 2\right)\right\} + \left\{\left(\tfrac{1}{2} \times 8 \times 2880\right) \times \left(2 + 8/3\right)\right\}$$

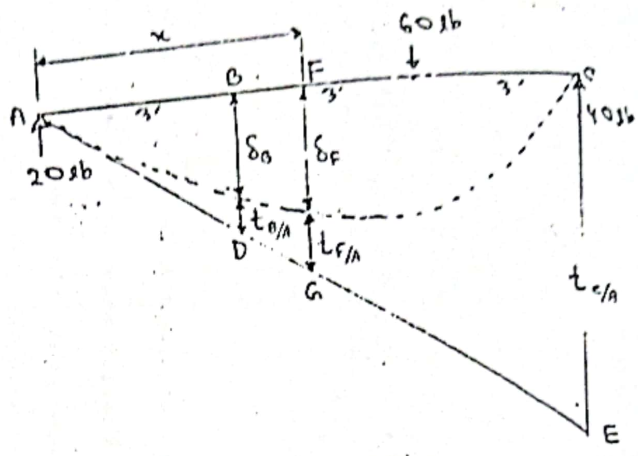
$$- \left\{\left(\tfrac{1}{3} \times 6 \times 2160\right)\left(2 + 6/4\right)\right\}$$

$$= 960 + 53760 - 15120$$

$$= 39600 \text{ lb.ft}^3. \quad \boxed{\text{Ans}}$$

1. compute the value of (i) $EI\delta_B$ and θ_A & (ii) maximum deflection.



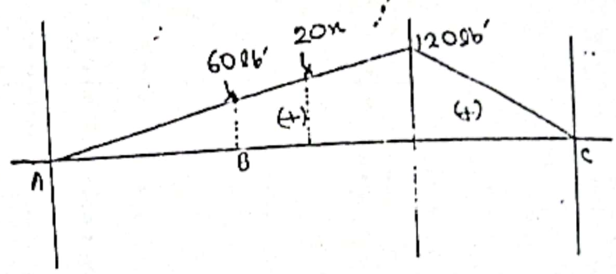


Solution : (i)

$$t_{C/A} = \frac{1}{EI}(\text{Area})_{AC} \cdot \bar{x}_C$$

$$= \frac{1}{EI}\left[\left(\tfrac{1}{2}\times 6\times 120\right)\left(3+6/3\right) + \left(\tfrac{1}{2}\times 3\times 120\right)\left(\tfrac{2}{3}\times 3\right)\right]$$

$$= \frac{2160}{EI}$$



and $t_{B/A} = \frac{1}{EI}(\text{Area})_{AB} \cdot \bar{x}_B$

$$= \frac{1}{EI}\left[\left(\tfrac{1}{2}\times 3\times 60\right)\left(3/3\right)\right]$$

$$= \frac{90}{EI}$$

from the similar triangle ABD & ACE:

$$\frac{BD}{CE} = \frac{AB}{AC}$$

$$\Rightarrow BD = \frac{3}{9}\times t_{C/A} = \frac{2160}{EI}\times \frac{1}{3} = \frac{720}{EI}$$

Now $BD = \delta_B + t_{B/A}$

$$\Rightarrow \frac{720}{EI} = \delta_B + \frac{90}{EI}$$

$$\Rightarrow \delta_B = \frac{630}{EI}$$

$$\Rightarrow EI\,\delta_B = 630 \text{ lb.ft}^3 \quad \boxed{\text{Ans}}$$

and $\theta_A = \frac{CE}{AC}$

$$= \frac{t_{C/A}}{9}$$

$$= \frac{2160}{9\times EI}$$

$$\therefore \theta_A = \frac{240}{EI} \quad \boxed{\text{Ans}}$$

Assume at point F maximum deflection occurs.

Now $t_{F/A} = \frac{1}{EI}(\text{Area})_{AF} \cdot \bar{x}_F$

$= \frac{1}{EI}\left(\frac{1}{2} \times x \cdot 20x\right) \cdot x/3$

$= \frac{10x^3}{3EI}$

From ΔACE and ΔAFG:

$$\frac{FG}{AF} = \frac{CE}{AC}$$

$$\Rightarrow FG = \frac{t_{C/A} \cdot AF}{AC}$$

$$= \frac{2160}{EI} \cdot \frac{x}{9}$$

$$= \frac{240x}{EI}$$

But $FG = \delta_F + t_{F/A}$

$\Rightarrow \frac{240x}{EI} = \delta_F + \frac{10x^3}{3EI}$

$\Rightarrow \delta_F = \frac{240x}{EI} - \frac{10x^3}{3EI}$

$\Rightarrow \delta_{max} \cdot EI = \delta_F \cdot EI = 240x - \frac{10x^3}{3}$

At maximum deflection slope $\frac{d\delta_{max}}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left(240x - \frac{10x^3}{3}\right) = 0$$

$$\Rightarrow 240 - \frac{3 \times 10x^2}{3} = 0$$

$$\Rightarrow x = \sqrt{24}$$

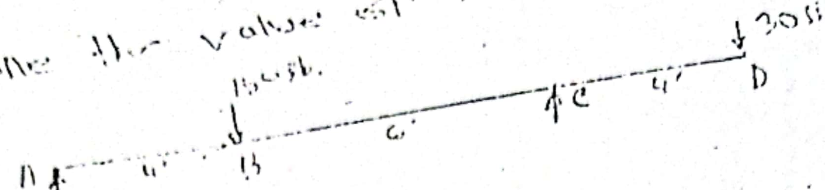
$$= 4.9 \text{ ft.}$$

Maximum deflection, $\delta_{max} = \frac{1}{EI}\left(240 \times 4.9 - \frac{10 \times 4.9^3}{3}\right)$

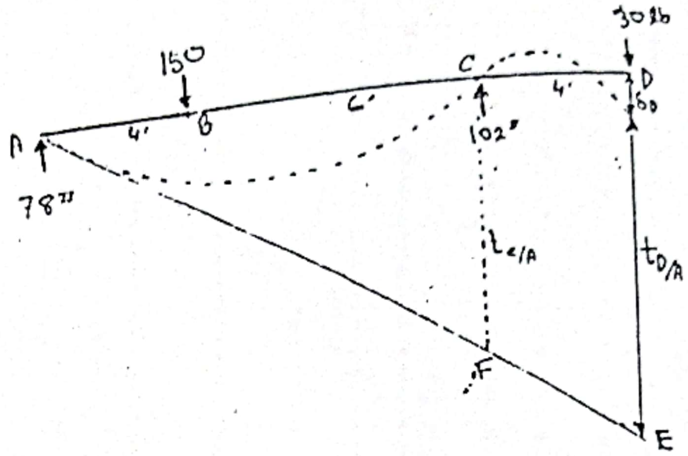
$\therefore \delta_{max} = \frac{783.84}{EI}$. unit.

$\boxed{\text{Ans}}$

Determine the value of deflection at point D.



Solution:



$\Sigma M_A = 0$

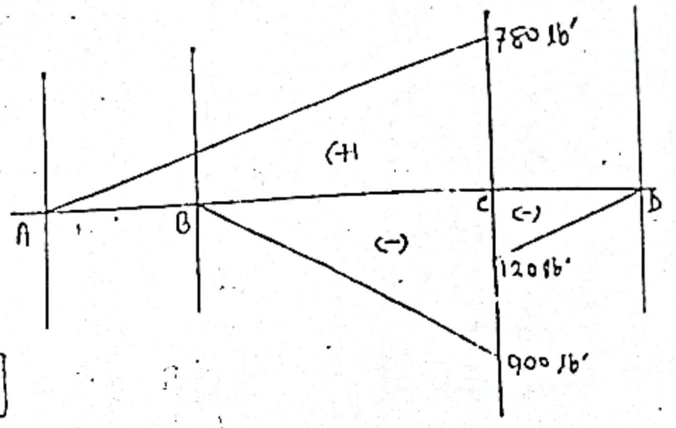
$\Rightarrow 150 \times 4 + 30 \times 14 - R_C \times 10 = 0$

$\Rightarrow R_C = 102$ lb.

$\Sigma F_y = 0$

$\Rightarrow R_A = 150 + 30 - 102$

$= 78$ lb.



Now,

$t_{C/A} = \{(Area)_{AC} \cdot \bar{x}_C\} \frac{1}{EI}$

$= \frac{1}{EI}\left[\left\{\left(\frac{1}{2} \times 10 \times 780\right) \times \frac{10}{3}\right\}\right.$

$\left. - \left\{\left(\frac{1}{2} \times 6 \times 900\right)\left(\frac{6}{3}\right)\right\}\right]$

$= \frac{7600}{EI}$

and $t_{D/A} = \left[(Area)_{AD} \cdot \bar{x}_D\right] \frac{1}{EI}$

$= \frac{1}{EI}\left[\left\{\left(\frac{1}{2} \times 10 \times 780\right)\left(4 + \frac{10}{3}\right)\right\} - \left\{\left(\frac{1}{2} \times 6 \times 900\right)\left(4 + \frac{6}{3}\right)\right\}\right.$

$\left. - \left\{\left(\frac{1}{2} \times 4 \times 120\right)\left(\frac{2}{3} \times 4\right)\right\}\right]$

$= \frac{11760}{EI}$

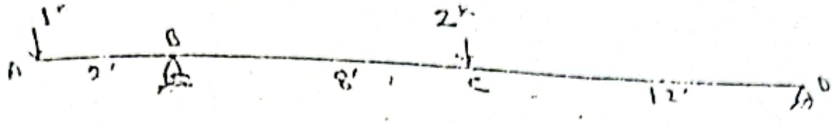
From ΔACF & ΔADE; $DE = 14 \times \frac{t_{C/A}}{10}$

$\Rightarrow \delta_D + \frac{11760}{EI} = 1.4 \times \frac{7600}{EI}$

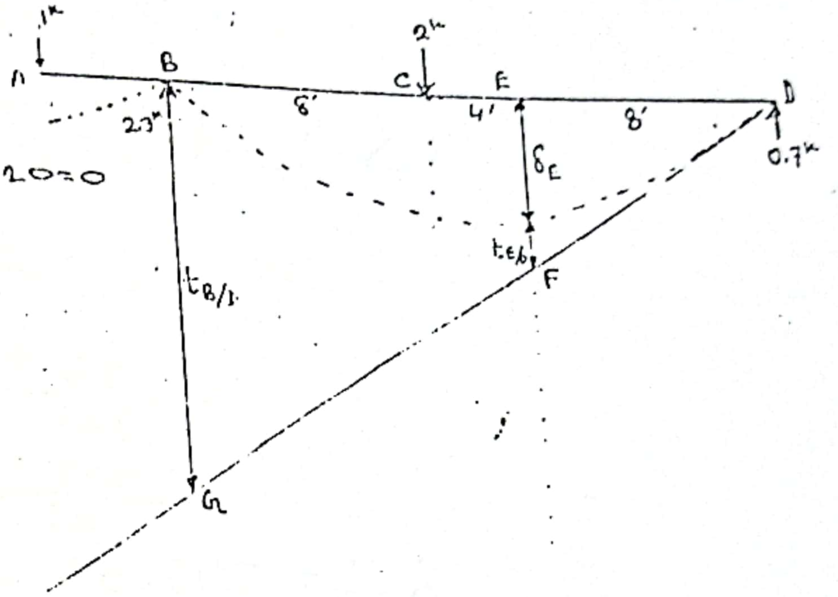
$\Rightarrow \delta_D = \frac{-1120}{EI}$

$\Rightarrow \delta_D = \frac{1120}{EI}$ (↓) $\boxed{\text{Ans}}$

Q. Find EIδ at 8' from right support.



Solution:



$\Sigma M_B = 0$

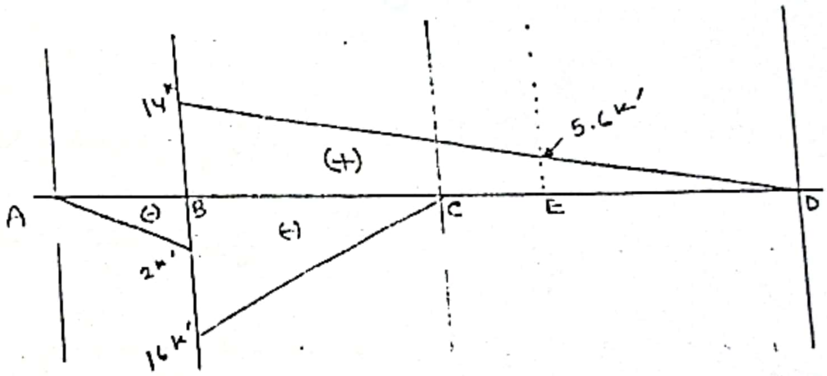
$\Rightarrow -1\times 2 + 2\times 8 - R_D \times 20 = 0$

$\Rightarrow R_D = 0.7^k$

$\Sigma F_y = 0$

$\Rightarrow R_B = 1+2-0.7$

$= 2.3^k$



Now $t_{B/D} = \left[(\text{Area})_{BD} \cdot \bar{x}_B\right] \frac{1}{EI}$

$= \left[\left(\frac{1}{2}\times 20\times 14\times 20/3\right) - \left(1/2 \times 8\times 16 \times 8/3\right)\right] \frac{1}{EI}$

$= \frac{2288}{3EI}$

and $t_{E/D} = \left[(\text{Area})_{DE} \cdot \bar{x}_E\right] \frac{1}{EI}$

$= \left[\left(1/2 \times 8\times 5.6\right)\times 8/3\right] \frac{1}{EI}$

$= \frac{896}{15\,EI}$

From ΔDEF & ΔDBG;

$$\frac{BG}{BD} = \frac{EF}{DE}$$

$$\Rightarrow EF = \frac{DE}{BD} BG$$

$$\Rightarrow \delta_E + t_{E/D} = \frac{8}{20} \times t_{B/D}$$

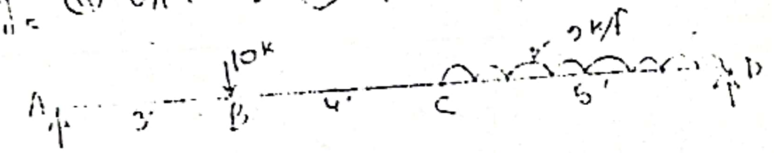
$$\Rightarrow \delta_E = \frac{8}{20} \times \frac{2288}{3EI} - \frac{896}{15EI}$$

$$= \frac{736}{3EI}$$

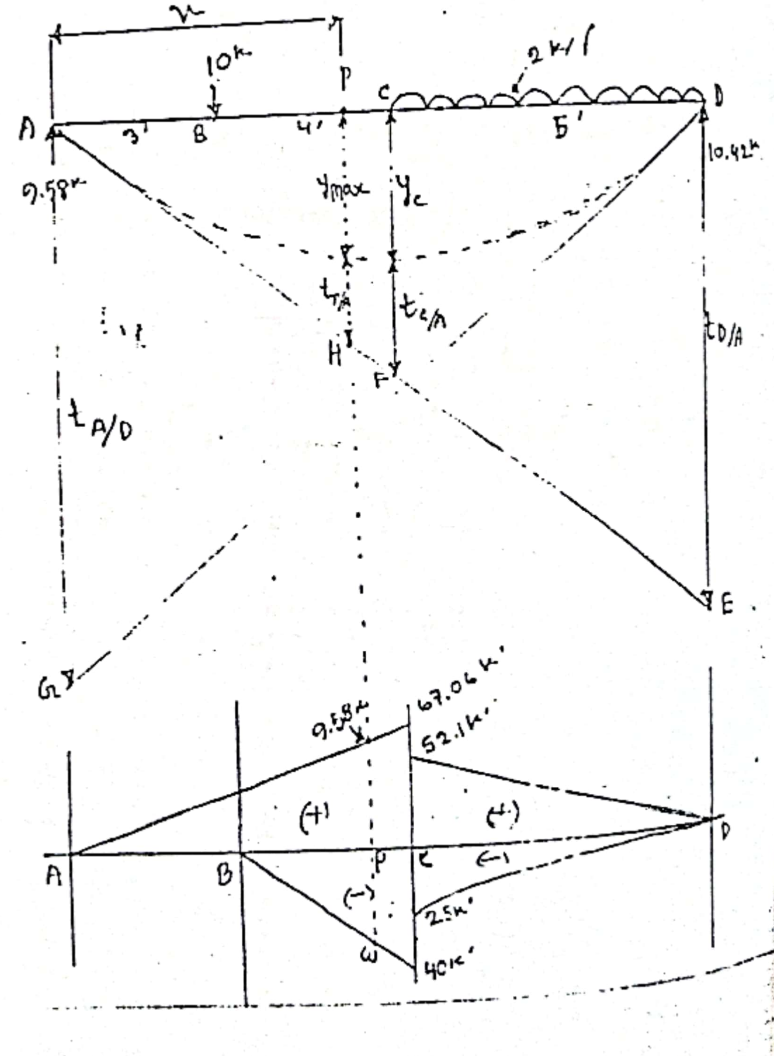
$$= \frac{245.33}{EI}$$

$\therefore \delta EI = 245.33$ $\boxed{\text{Ans}}$

07. Calculate (i) y_c (ii) θ_A & θ_D (iii) maximum deflection.



Solution:



$\Sigma M_A = 0$

$\Rightarrow 10 \times 3 + (2 \times 5)(7 + 5/2) - R_D \times 12 = 0$

$\Rightarrow R_D = 10.42\,k$

$\Sigma F_y = 0$

$\Rightarrow R_A = 10 + (2 \times 5) - 10.42$

$= 9.58^k$

Here,

$$\frac{\omega}{x-3} = \frac{40}{4}$$

$\Rightarrow \omega = 10(x-3)$

Now $t_{D/A} = \frac{1}{EI}\left[(Area)_{AD} \cdot \bar{x}_D\right]$

$$= \frac{1}{EI}\left[\left\{(1/2 \times 7 \times 67.06) \times (5 + 7/3)\right\} + \left\{(1/2 \times 5 \times 52.1)(\tfrac{2}{3} \times 5)\right\} - \left\{(1/2 \times 4 \times 40)(5 + 4/3)\right\} - \left\{(1/3 \times 5 \times 25)(5 - 5/4)\right\}\right]$$

$$= \frac{1492.46}{EI}$$

and $t_{C/A} = \left[(Area)_{AC} \cdot \bar{x}_C\right]\frac{1}{EI}$

$$= \frac{1}{EI}\left[\left\{(1/2 \times 7 \times 67.06)(7/3)\right\} - \left\{(1/2 \times 4 \times 40)(4/3)\right\}\right]$$

$$= \frac{440.99}{EI}$$

Again $t_{AD} = \frac{1}{EI}\left[(Area)_{AD} \cdot \bar{x}_A\right]$

$$= \frac{1}{EI}\left[\left\{(1/2 \times 7 \times 67.06)(\tfrac{2}{3} \times 7)\right\} + \left\{(1/2 \times 5 \times 52.1) \times (7 + 5/3)\right\} - \left\{(1/2 \times 4 \times 40)(3 + \tfrac{2}{3} \times 4)\right\} - \left\{\left(\frac{5 \times 25}{3}\right)(7 + 5/4)\right\}\right]$$

$$= \left[1095.31 + \frac{6773}{6} - \frac{1360}{3} - \frac{1375}{4}\right]\frac{1}{EI}$$

$$= \frac{1427.06}{EI}$$

and $t_{P/A} = \frac{1}{EI}\left[\left\{Area\right)_{AP} \cdot \bar{x}_P\right]$

$$= \frac{1}{EI}\left[\left\{(1/2 \times x \times 9.58x) \times (x/3)\right\} - \left\{(1/2 \times (x-3) \times 10(x-3)) \times \tfrac{x}{3}\right\}\right]$$

$$= \frac{1}{EI}\left[\frac{479x^3}{300} - \frac{5(x-3)^3}{3}\right]$$

(I) From ΔADE & ΔACF;

$$\frac{CF}{AC} = \frac{DE}{AD}$$

$$\Rightarrow CF = \frac{DE}{AD} \cdot AC$$

$$\Rightarrow y_c + t_{C/A} = \frac{t_{D/A}}{12} \times 7$$

$$\Rightarrow y_c = \frac{7}{12} \times \frac{1492.46}{EI} - \frac{440.99}{EI}$$

$$\therefore y_c = \frac{429.61}{EI} \quad \boxed{\text{Ans}}$$

(II) $$\theta_A = \frac{DE}{AD}$$

$$= \frac{t_{D/A}}{AD}$$

$$= \frac{1492.46}{12EI}$$

$$= \frac{124.37}{EI} \quad \boxed{\text{Ans}}$$

and $$\theta_D = \frac{AG}{AD}$$

$$= \frac{t_{A/D}}{AD}$$

$$= \frac{1427.06}{12EI}$$

$$= \frac{118.92}{EI} \quad \boxed{\text{Ans}}$$

(iii) Assume maximum deflection occurs at P.

Hence,

$$\frac{PH}{AP} = \frac{DE}{AD}$$

$$\Rightarrow PH = \frac{DE}{AD} \cdot AP$$

$$\Rightarrow y_{max} + t_{P/A} = \frac{t_{D/A}}{12} \times x.$$

$$\Rightarrow y_{max} = \frac{1492.46x}{12EI} - \frac{1}{EI}\left(\frac{479x^3}{300} - \frac{5(x-3)^3}{3}\right)$$

$$= \frac{1}{EI}\left[\frac{1492.46x}{12} - \frac{479x^3}{300} + \frac{5(x-3)^3}{3}\right]$$

At maximum deflection, $\frac{dy_{max}}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left[\frac{1}{EI}\left(\frac{1492.46x}{12} - \frac{479x^3}{300} + \frac{5(x-3)^3}{3}\right)\right] = 0$$

$$\Rightarrow \frac{1}{EI}\left[\frac{1492.46}{12} - \frac{3\times479x^2}{300} + \frac{5}{3}\times3(x-3)^2\right] = 0$$

$$\Rightarrow 5(x-3)^2 - 4.79x^2 + \frac{1492.46}{12} = 0$$

$$\Rightarrow 5(x^2 - 6x + 9) - 4.79x^2 + \frac{1492.46}{12} = 0$$

$$\Rightarrow 0.21x^2 - 30x + 169.37 = 0$$

$$\therefore x = 5.89 \text{ ft} \quad \text{and} \quad x \neq 136.97'$$

At $x = 5.89$ ft from left end maximum deflection occurs.

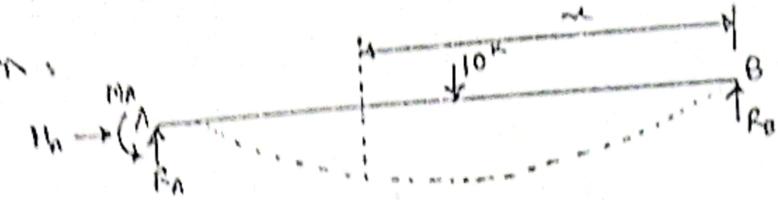
Hence maximum deflection, $y_{max} = \frac{1}{EI}\left[\frac{1492.46\times5.89}{12} - \frac{479\times5.89^3}{300} + \frac{5\times2.89^3}{3}\right]$

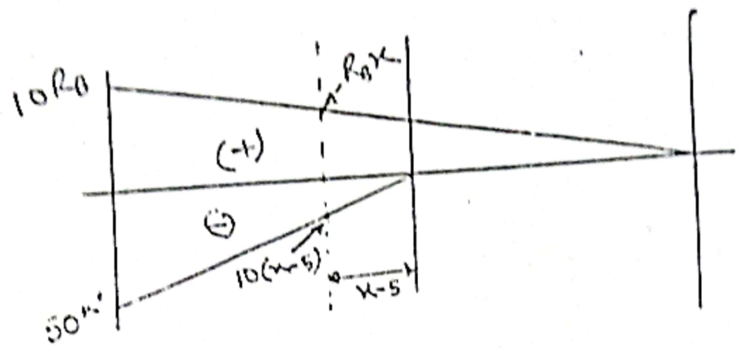
$$= \frac{446.52}{EI}$$

$\boxed{\text{Ans}}$

18. Calculate maximum deflection.

Solution,





But, $t_{B/A} = 0$

$\Rightarrow \frac{1}{EI}\left[(Area)_{AB} \cdot \bar{x}_B\right] = 0$

$\Rightarrow \left\{\left(\frac{1}{2} \times 10 \times 10R_B\right) \times \left(\frac{2}{3} \times 10\right)\right\} - \left\{\left(\frac{1}{2} \times 50 \times 5\right)\left(5 + \frac{2}{3} \times 5\right)\right\} = 0$

$\Rightarrow \frac{1000 R_B}{3} - \frac{3125}{3} = 0$

$\Rightarrow R_B = 3.125\,k.$

Assume at x ft from right support maximum deflection occurs and at maximum deflection slope, $\frac{dy}{dx} = 0$.

Slope, $\frac{dy}{dx} = \frac{1}{EI}(\text{Area of moment diagram})_x$

$\Rightarrow 0 = \frac{1}{EI}\left[\frac{1}{2} \times x \cdot R_B x - \frac{1}{2} \times (x-5) \times 10(x-5)\right]$

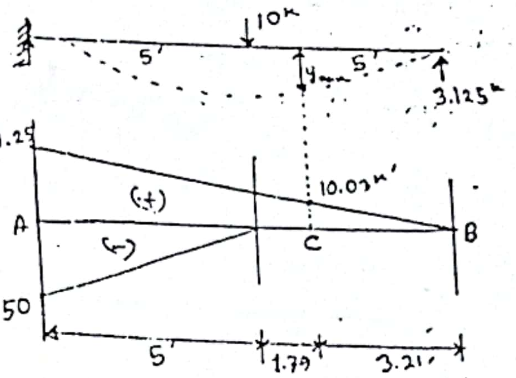
$\Rightarrow \frac{3.125x^2}{2} = 5(x-5)^2$

$\Rightarrow 10(x^2 - 10x + 25) = 3.125x^2$

$\Rightarrow 6.875x^2 - 100x + 250 = 0$

$\therefore x = 3.21$ ft

Here $y_{max} = -t_{c/A}$



$$= -\frac{1}{EI}(Area)_{AC} \cdot \bar{x}_c$$

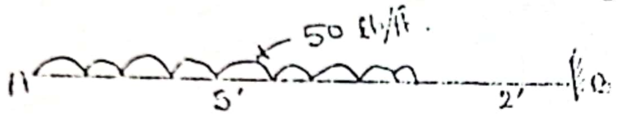
$$= -\frac{1}{EI}\Big[\Big\{(10.03\times6.79)\times\frac{6.79}{2}\Big\}$$
$$+\Big\{(\tfrac{1}{2}\times6.79\times21.22)\times\tfrac{2}{3}\times6.79\Big\}$$
$$-\Big\{(\tfrac{1}{2}\times5\times50)(1.79+\tfrac{2}{3}\times5)\Big\}\Big]$$

$$= -\frac{1}{EI}[231.21 + 326.11 - 640.42]$$

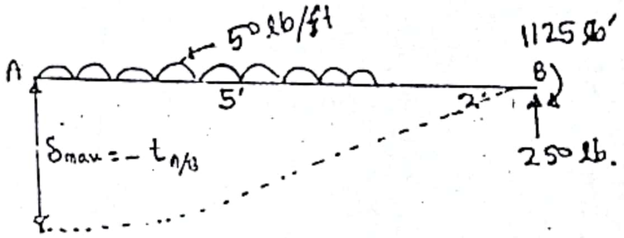
$$= \frac{83.1}{EI}$$

$\boxed{\text{Ans}}$

09. Calculate maximum deflection. [CT-2014 series]



Solution:



$\Sigma F_y = 0$

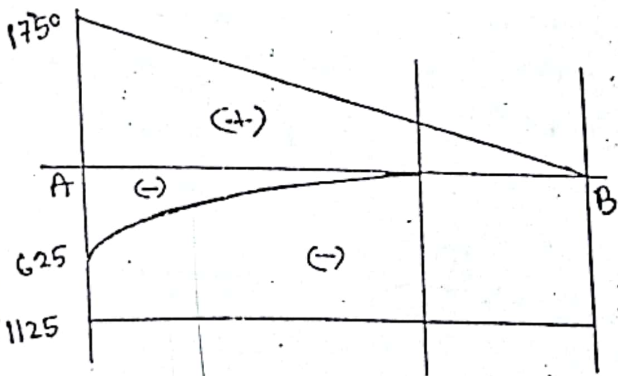
$\Rightarrow 50\times5 - R_B = 0$

$\Rightarrow R_B = 250$ lb.

$\Sigma M_B = 0$

$\Rightarrow M = (5\times50)\times(2+5/2)$

$= 1125$ lb·ft



But $\delta_{max} = -t_{A/B}$

$$= -\frac{1}{EI}\left[(Area)_{AB}\cdot\bar{x}_A\right]$$

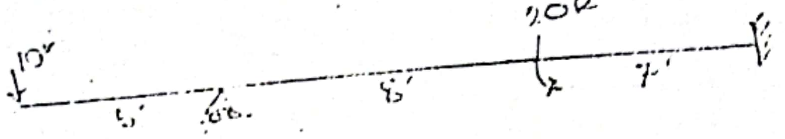
$$= -\frac{1}{EI}\Big[\Big\{(\tfrac{1}{2}\times7\times1750)(7/3)\Big\} - \Big\{\Big(\frac{625\times5}{3}\Big)(5/4)\Big\} - \Big\{(1125\times7)\times7/2\Big\}\Big]$$

$$= -\frac{1}{EI}\Big[\frac{42875}{3} - \frac{15625}{12} - \frac{55125}{2}\Big]$$

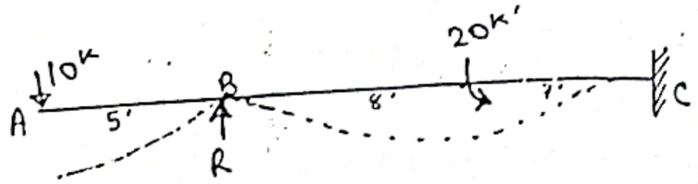
$$\therefore \delta_{max} = \frac{14572.92}{EI}$$

$\boxed{\text{Ans}}$

Draw shear force and bending moment diagram in the following beam is shown in figure. [2005]



Solution:



$t_{A/C} = 0$

$\frac{1}{EI}\left[(Area)_{AC} \cdot \bar{x}_A\right] = 0$

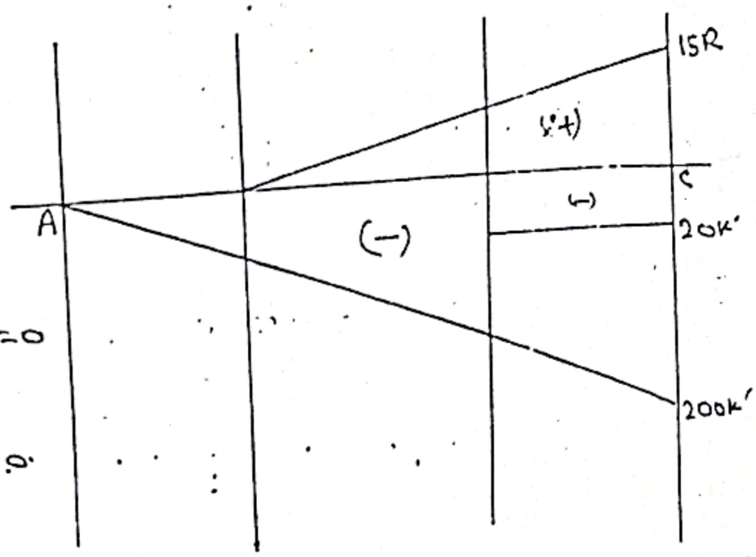
$\{(\frac{1}{2} \times 15 \times 15R) \times ((\frac{2}{3} \times 15) + 5)\}$

$- \{(7 \times 20) \times (13 + 7/2)\}$

$- \{(\frac{1}{2} \times 20 \times 200)(\frac{2}{3} \times 20)\} = 0$

$\Rightarrow 1687.5R - 2310 - \frac{80000}{3} = 0$

$\Rightarrow R = 17.17^k$



$\Sigma F_y = 0$

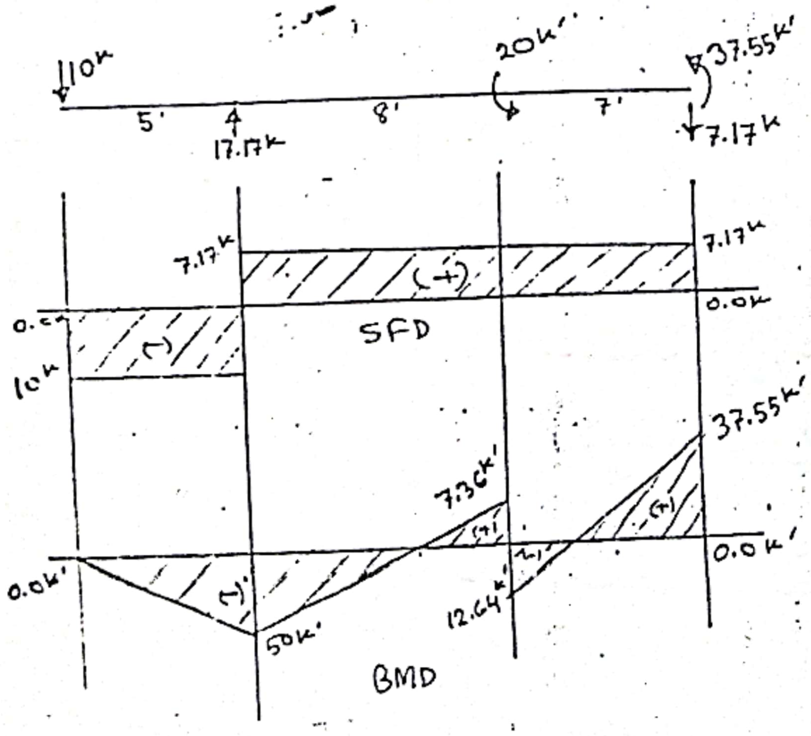
$\Rightarrow R + R_c - 10 = 0$

$\Rightarrow R = -7.17^k$

$\Sigma M_c = 0$

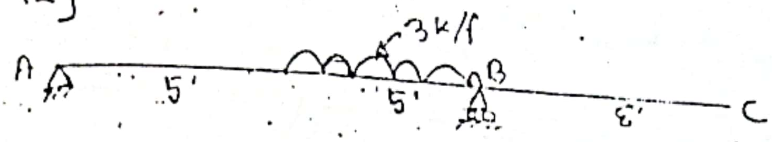
$\Rightarrow M - 20 - (10 \times 20) + 17.17 \times 15 = 0$

$\Rightarrow M = -37.55\,k'$



2Q

An overhanging beam ABC shown in figure below. Using Area moment method calculate the value of maximum deflection of the beam. EI constant. [2006, 2012]



Solution:

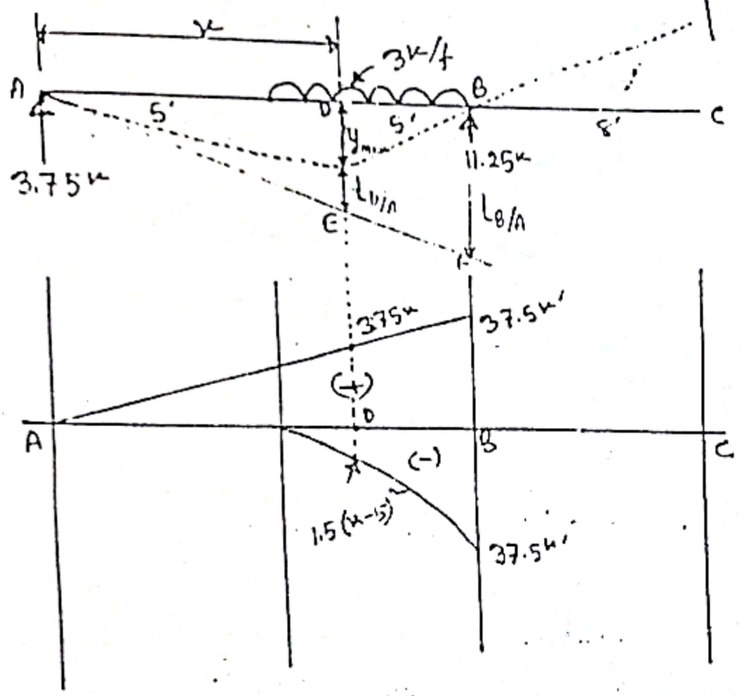
$$\Sigma M_A = 0$$

$$\Rightarrow 3 \times 5 \times (5 + 5/2) - R_B \times 10 = 0$$

$$\Rightarrow R_B = 11.25$$

$$\Sigma F_y = 0$$

$$\Rightarrow R_A = 15 - 11.25 = 3.75^k$$



Here
$$t_{B/A} = \frac{1}{EI} (Area)_{AB} \cdot \bar{x}_B$$

$$= \frac{1}{EI}\left[\left\{\left(\tfrac{1}{2} \times 10 \times 37.5\right) \times \tfrac{10}{3}\right\} - \left\{\left(\frac{5 \times 37.5}{3}\right) \times \left(5/4\right)\right\}\right]$$

$$= \frac{546.875}{EI}$$

Assume maximum deflection occurs at D.

Now, $$t_{D/A} = \frac{1}{EI} (Area)_{AD} \cdot \bar{x}_D$$

$$= \frac{1}{EI}\left[\left\{\left(\tfrac{1}{2} \times x \times 3.75x\right)\left(x/3\right)\right\} - \left\{\left(\frac{1.5(x-5)^2 \times (x-5)}{3}\right) \times \frac{x-5}{4}\right\}\right]$$

$$= \frac{1}{EI}\left(0.625x^3 - \frac{(x-5)^4}{8}\right]$$

From ΔADE & ΔABF;

$$\frac{DE}{AD} = \frac{BF}{AB}$$

$$\Rightarrow y_{max} + t_{D/A} = x \cdot \frac{t_{B/A}}{10}$$

$$\Rightarrow y_{max} = \frac{546.875x}{10EI} - \frac{1}{EI}\left[0.625x^3 - \frac{(x-5)^4}{8}\right]$$

$$= \left(54.69x - 0.625x^3 + \frac{(x-5)^4}{8}\right)\frac{1}{EI}$$

At maximum deflection $\frac{dy_{max}}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left[54.69x - 0.625x^3 + \frac{(x-5)^4}{8}\right]\frac{1}{EI} = 0$$

$$\Rightarrow 54.69 - 1.875x^2 + \frac{(x-5)^3}{2} = 0$$

$$\Rightarrow 109.38 - 3.75x^2 + x^3 - 15x^2 + 75x - 125 = 0$$

$$\Rightarrow x^3 - 18.75x^2 + 75x - 15.62 = 0$$

$$\therefore x = 13.13,\ 5.40,\ 0.22.$$

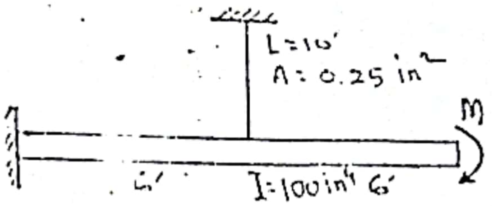
Maximum deflection will occur at $x = 5.40$ ft.

Maximum deflection, $y_{max} = \left(54.69 \times 5.4 - 0.625 \times 5.4^3 + \frac{(0.4)^4}{8}\right)\frac{1}{EI}$

$$\therefore y_{max} = \frac{196.91}{EI}\ (\downarrow)$$

$\boxed{\text{Ans}}$

The midpoint of the steel beam shown in figure below is connected to the vertical steel rod. Determine the maximum value of M if the stress in the rod is not to exceed 10 ksi. Draw the bending moment diagram of the steel beam. [2006]



Solution: Assume $E_{steel} = 30000$ ksi.

For steel rod,

$$\sigma = \frac{P}{A}$$

$$\Rightarrow P = \sigma . A$$

$$= 10 \times 0.25$$

$$= 2.5 \text{ k}.$$

$$\Delta_{rod} = \frac{PL}{AE}$$

$$= \frac{2.5 \times 10 \times 12}{0.25 \times 30 \times 10^3}$$

$$= 0.04 \text{ inch.}$$

$t_{C/A} = \Delta_{rod}$

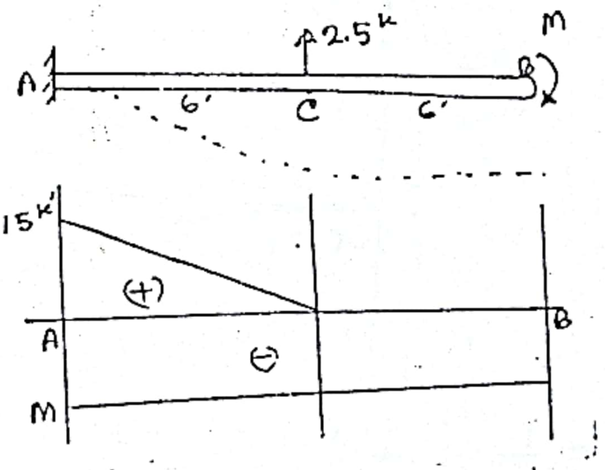
$$\Rightarrow \frac{1}{EI}\left[(Area)_{AC} \cdot \bar{x}_C\right] = 0.04$$

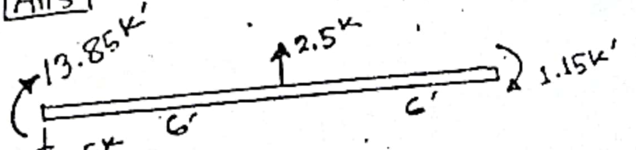
$$\Rightarrow \left[\left\{\left(\tfrac{1}{2} \times 6 \times 15\right) \times \left(6 * \tfrac{2}{3}\right)\right\} - 6M \times 6/2\right] = 0.04\, EI$$

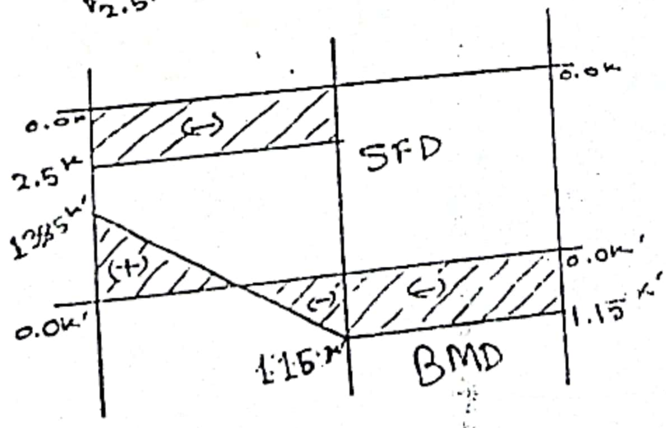
$$\Rightarrow (-180 + 18M)12^3 = 0.04 \times 30 \times 10^3 \times 100$$

$$\Rightarrow M = 13.858 \text{ k-in}$$

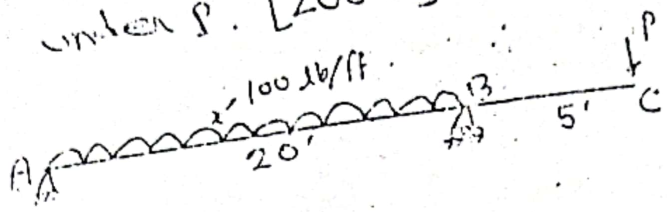
$$= 1.15 \text{ k-ft} \quad \boxed{Ans}$$







13. Determine the value of P to cause a zero deflection under P. [2006]



Solution: $\Sigma M_A = 0$

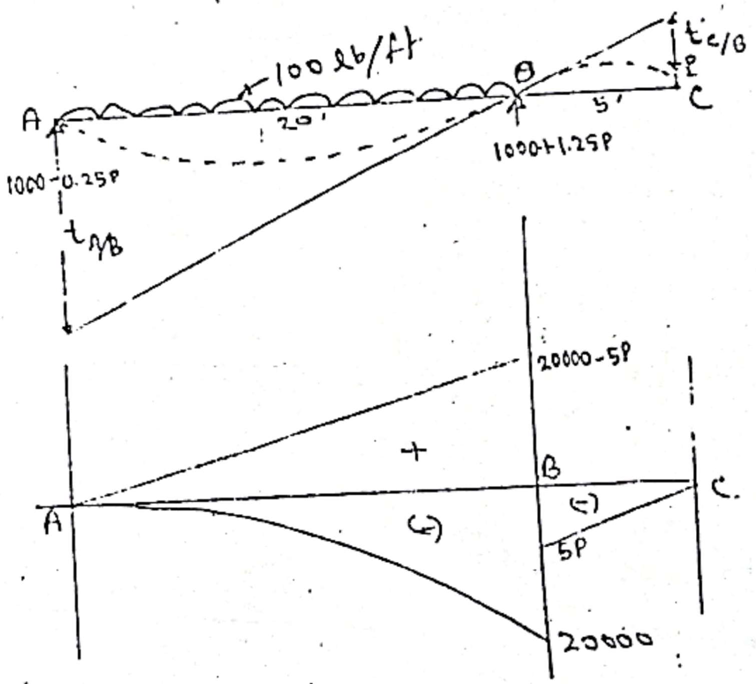
$\Rightarrow 100 \times 20 \times \frac{20}{2} + P \times 25 - R_B \times 20 = 0$

$\Rightarrow R_B = 1000 + 1.25P$

$\Sigma F_y = 0$

$\Rightarrow R_A = P + (100 \times 20) - 1000 - 1.25P$

$= 1000 - 0.25P$



$t_{A/B} = \frac{1}{EI} (Area)_{AB} \cdot \bar{x}_A$

$= \frac{1}{EI} \left[\left\{ \left(\frac{1}{2} \times 20 \times (20000 - 5P) \right) \left(\frac{2}{3} \times 20 \right) \right\} - \left\{ \left(\frac{20 \times 20000}{3} \right) \right. \right.$

$\left. \left. * \left(20 - \frac{20}{4} \right) \right\} \right]$

$= \frac{1}{EI} \left[\frac{400}{3} (20000 - 5P) - 2 \times 10^6 \right]$

$$t_{C/B} = \frac{1}{EI}\left[(Area)_{BC} \cdot \bar{x}_C\right]$$

$$= \frac{1}{EI}\left[\left\{-\left(\tfrac{1}{2} \times 5 \times 5P\right) \times \tfrac{2}{3} \times 5\right\}\right]$$

$$= -\frac{125}{3EI}$$

Now $\dfrac{t_{A/B}}{AB} = \dfrac{t_{C/B}}{BC}$

$$\Rightarrow \frac{1}{EI}\left[\frac{400}{3}(20000 - 5P) - 2\times10^6\right] = \frac{20}{5} \times \frac{125}{3EI}$$

$$\Rightarrow \frac{400}{3}(20000 - 5P) - 2\times10^6 = \frac{500}{3}$$

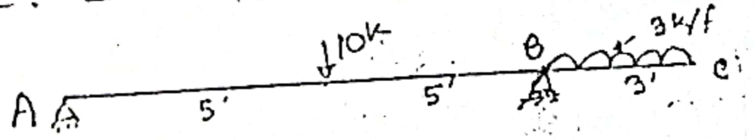
$$\Rightarrow \frac{8\times10^6}{3} - \frac{2000P}{3} = \frac{500}{3} + 2\times10^6$$

$\Rightarrow$

$$\Rightarrow P = \frac{3}{2000}\left(\frac{8\times10^6}{3} - \frac{500}{3} - 2\times10^6\right)$$

$\therefore P = 999.75$ lb. $\boxed{\text{Ans}}$

Using area moment theorem, calculate the maximum deflection of the beam shown in the figure. $E = 29\times10^6$ psi, $I = 250$ in⁴. [2007]



Solution:

$\Sigma M_A = 0$

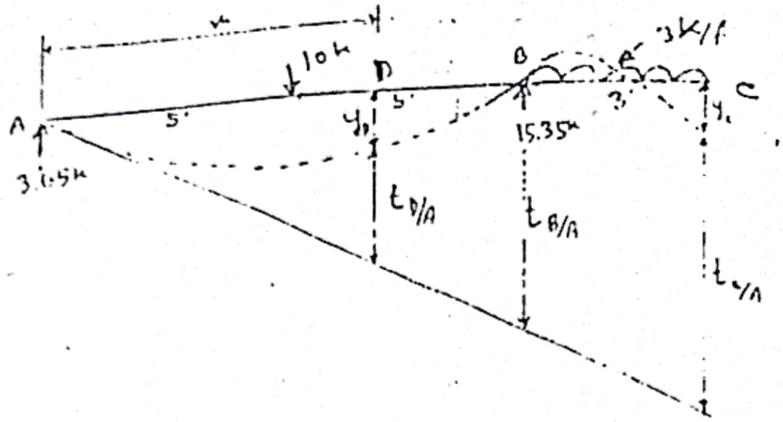
$\Rightarrow 10\times5 + 3\times3(10 + 3/2) - R_B \times 10 = 0$

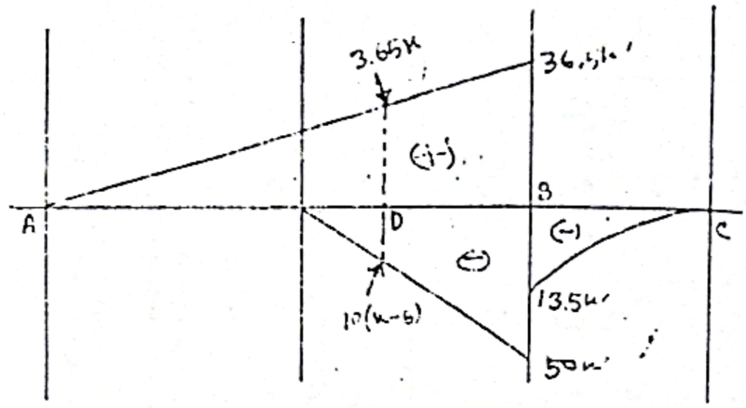
$\Rightarrow R_B = 15.35^k$

$\Sigma F_y = 0$

$\Rightarrow R_A = 10 + 9 - 15.35$

$= 3.65^k$.





$$t_{B/A} = \frac{1}{EI}\left[(\text{Area})_{AB} \cdot \bar{x}_B\right]$$

$$= \frac{1}{EI}\left[\left\{(1/2 \times 10 \times 36.5)(10/3) - \{(1/2 \times 5 \times 50)(5/3)\right\}\right]$$

$$= \frac{400}{EI}$$

$$t_{D/A} = \frac{1}{EI}\left[(\text{Area})_{AD} \cdot \bar{x}_D\right]$$

$$= \frac{1}{EI}\left[\left\{(1/2 * x * 3.65x)(x/3)\right\} - \left\{(1/2 * (x-5) * 10(x-5))\left(\frac{x-5}{3}\right)\right\}\right]$$

$$= \frac{1}{EI}\left\{\frac{73x^3}{120} - \frac{5(x-5)^3}{3}\right]$$

$$t_{C/A} = \frac{1}{EI}\left[(\text{Area})_{AC} \cdot \bar{x}_C\right]$$

$$= \frac{1}{EI}\left[\{(1/2 \times 10 \times 36.5)(3 + 10/3)\} - \{(1/2 \times 5 \times 50)(3 + 5/3)\} - \left\{\left(\frac{3 \times 13.5}{3}\right)\left(3 - \frac{3}{4}\right)\right\}\right]$$

$$= \frac{542.125}{EI}$$

...me at x, maximum deflection occurs. At maximum ...ction slope $\frac{dy}{dx} = 0$.

$$\frac{y_D + t_{D/A}}{x} = \frac{t_{B/A}}{10}$$

$$\Rightarrow y_D = \frac{x}{10} \cdot \frac{400}{EI} - \frac{1}{EI}\left[\frac{73x^3}{120} - \frac{5(x-5)^3}{3}\right]$$

$$= \frac{40x}{EI} - \frac{73x^3}{120EI} + \frac{5(x-5)^3}{3EI}$$

$$\frac{dy_D}{dx} = 0$$

$$\frac{40}{EI} - \frac{1.825x^2}{EI} + \frac{5(x-5)^2}{EI} = 0$$

$$5(x^2 - 10x + 25) - 1.825x^2 + 40 = 0$$

$$3.175x^2 - 50x + 165 = 0$$

$$x = 4.71$$

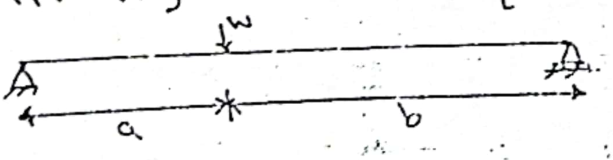
...nce maximum deflection between the supports:

$$y_{max} = \frac{40 \times 4.71}{EI} - \frac{73 \times 4.71^3}{120\,EI} = \frac{124.84}{EI} = \frac{124.84 \times 12^3}{29 \times 10^6 \times 250}$$

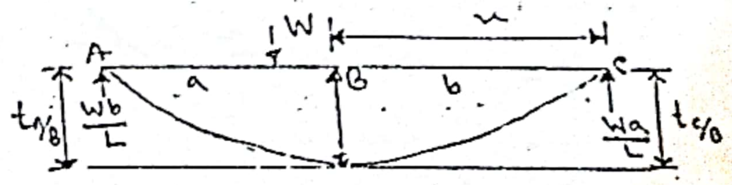
$$= 2.98 \times 10^{-5}\,in$$

$y_{max} = 2.98 \times 10^{-5}\,in$ $\boxed{Ans}$

...erive an expression for the location of maximum deflection ... a beam on two supports due to a single concentrated ...oad as shown in figure below. [2006]



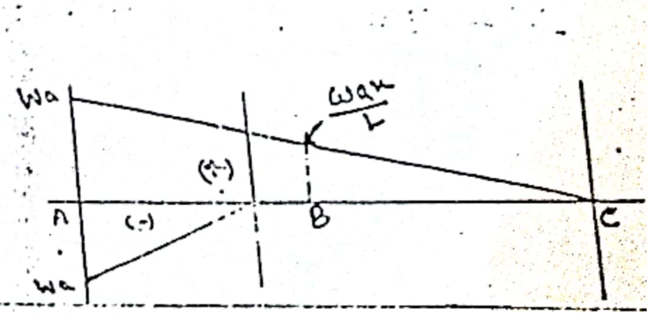
...olution:



$$t_{C/B} = \frac{1}{EI}(\text{Area})_{CB} \cdot \bar{x}_C$$

$$= \frac{1}{EI}\left[\frac{1}{2} \times x \times \frac{Wax}{L} \times \frac{2}{3}x\right]$$

$$= \frac{Wax^3}{3LEI}$$



$t_{A/B} = \frac{1}{EI}(Area)_{AB} \cdot \bar{x}_A$

$= \frac{1}{EI}\left[\left\{\frac{1}{2} * (L-x) * \left(wa - \frac{wax}{L}\right) * \frac{L-x}{3}\right\} + \left\{\frac{wax}{L}(L-x) \times \frac{L-x}{2}\right\}\right.$

$\left. - \left\{\frac{1}{2} \times wa \times a \times a/3\right\}\right]$

$= \frac{1}{EI}\left[\frac{(L-x)^2}{6}\left(wa - \frac{wax}{L}\right) + \frac{wax}{2L}(L-x)^2 - \frac{wa^3}{6}\right]$

$= \frac{1}{EI}\left[\frac{(L-x)^2}{6} * wa\left(1 - \frac{x}{L}\right) + \frac{wax}{2L}(L-x)^2 - \frac{wa^3}{6}\right]$

$= \frac{1}{EI}\left[\frac{(L-x)^2}{6} * wa\left(\frac{L-x}{L}\right) + \frac{wax}{2L}(L-x)^2 - \frac{wa^3}{6}\right]$

$= \frac{1}{EI}\left[\frac{wa}{6L}(L-x)^3 + \frac{wax}{2L}(L-x)^2 + \frac{wa^3}{6}\right]$

At maximum deflection slope will be zero

i.e. $t_{A/B} = t_{C/B}$

$\Rightarrow \frac{1}{EI}\left[\frac{wa}{6L}(L-x)^3 + \frac{wa}{2L}(L-x)^2 x - \frac{wa^3}{6}\right] = \frac{1}{EI} * \frac{wax^3}{3L}$

$\Rightarrow \frac{(L-x)^3}{6L} + \frac{(L-x)^2 x}{2L} - \frac{a^2}{6} = \frac{x^3}{3L}$

$\Rightarrow \frac{(L-x)^3 + 3x(L-x)^2 - a^2L}{6L} = \frac{x^3}{3L}$

$\Rightarrow (L-x)^3 + 3x(L-x)^2 - a^2L - 2x^3 = 0$

$\Rightarrow L^3 - 3L^2x + 3Lx^2 - x^3 + 3x(L^2 - 2Lx + x^2) - a^2L - 2x^3 = 0$

$\Rightarrow -\cancel{3x^3} + 3Lx^2 - 6Lx^2 + \cancel{3x^3} - \cancel{3L^2x} + \cancel{3L^2x} + L^3 - a^2L = 0$

$\Rightarrow -3Lx^2 + L^3 - a^2L = 0$

$$-L(3x^2 + a^2 - L^2) = 0$$

$$\Rightarrow 3x^2 + a^2 - L^2 = 0$$

$$\Rightarrow 3x^2 = L^2 - a^2$$

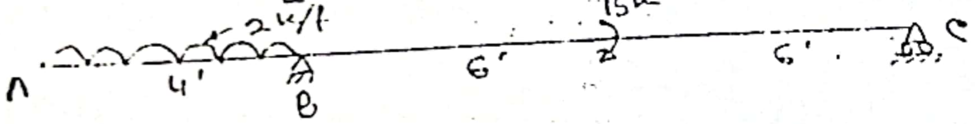
$$\Rightarrow x^2 = \frac{L^2 - a^2}{3}$$

$$\Rightarrow x = \pm\sqrt{\frac{L^2-a^2}{3}}$$

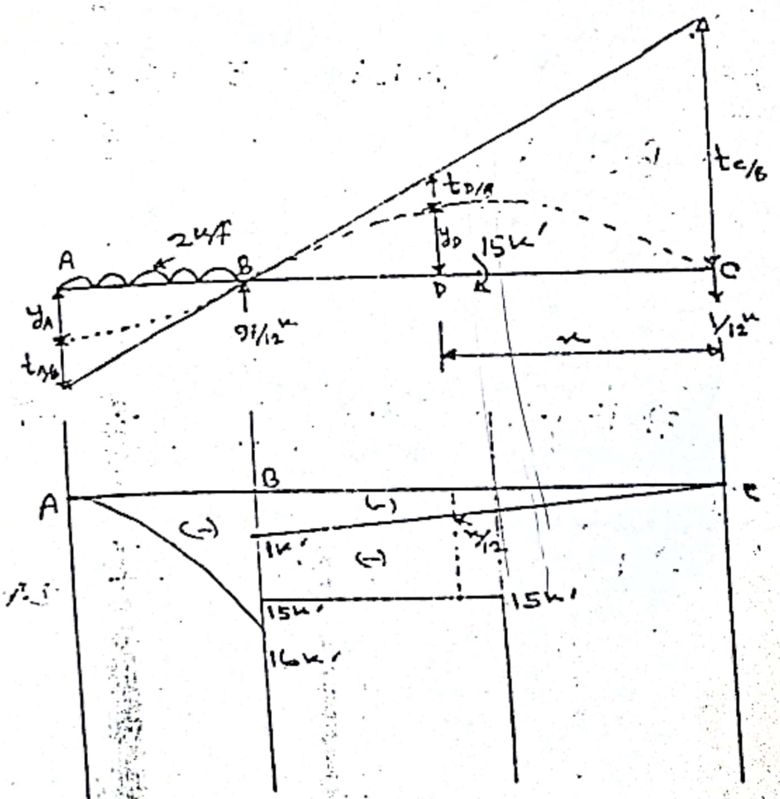
$$\therefore x = \sqrt{\frac{L^2-a^2}{3}}$$

Maximum deflection will occur at $x = \sqrt{\frac{L^2-a^2}{3}}$ from the right support.

Using area moment proposition calculate the value of maximum linear deflection and rotational deflection for the beam shown in figure below. EI is constant. [2008]



Solution:



$\Sigma M_C = 0$

$\Rightarrow R_B \times 12 - 2\times4(12+2) + 15 = 0$

$\Rightarrow R_B = \frac{97}{12}$ k

$\Sigma F_y = 0$

$\Rightarrow R_C + R_B - 2\times4 = 0$

$\Rightarrow R_C = 8 - 97/12$

$= -1/12$ k

$t_{c/B} = \frac{1}{EI}\,(Area)_{CB}\cdot\bar{x}_c$

$= \frac{1}{EI}\left[-\{(1/2\times12\times1)(2/3\times12)\} - \{(15\times6)(6+6/2)\}\right]$

$= -\frac{858}{EI}$

$t_{A/B} = \frac{1}{EI}\,(Area)_{AB}\cdot\bar{x}_A$

$= \frac{1}{EI}\left[-\{(1/3\times4\times16)\times(4-1/4)\}\right]$

$= -\frac{64}{EI}$

$t_{D/B} = \frac{1}{EI}\,(Area)_{DB}\cdot\bar{x}_D$

$\Rightarrow t_{D/B} = \frac{1}{EI}\left[-\{15(12-x)\left(\frac{12-x}{2}\right)\} - \{x/12\,(12-x)\left(\frac{12-x}{2}\right)\} - \{1/2(1-x/12)\right.$

$\left.\times(12-x)\times\frac{2}{3}(12-x)\}\right]$

$= \frac{-1}{EI}\left[7.5(12-x)^2 + \frac{x(12-x)^2}{24} + \frac{(12-x)^3}{36}\right]$

Assume maximum deflection will occur at D.

Now $\frac{y_D + t_{D/B}}{12-x} = \frac{t_{c/B}}{12}$

$\Rightarrow y_D - \frac{1}{EI}\left[7.5(12-x)^2 + \frac{x(12-x)^2}{24} + \frac{(12-x)^3}{36}\right] = \frac{12-x}{12}\left(-\frac{858}{EI}\right)$

$\Rightarrow y_D EI = 7.5(12-x)^2 + \frac{x(12-x)^2}{24} + \frac{(12-x)^3}{36} - 71.5(12-x)$

But $\frac{dy_D}{dx} = 0$.

$\Rightarrow 15(12-x)(-1) + \frac{1}{24}\{x\cdot2(12-x)(-1) + (12-x)^2\} + \frac{3}{36}(12-x)^2(-1)$

$-71.5(-1) = 0$

$\Rightarrow 15(-12+x) + \frac{1}{24}\{2x(x-12) + (12-x)^2\} - \frac{1}{12}(12-x)^2 + 71.5 = 0$

$\Rightarrow 15x - 180 + \frac{x^2}{12} - x + 1/24\,(12-x)^2 - 1/12(12-x)^2 + 71.5 = 0$

$\Rightarrow 15x - 180 + \frac{x^2}{12} - x - \frac{1}{24}(144 - 24x + x^2) + 71.5 = 0$

$\Rightarrow \frac{x^2}{12} + 14x - 108.5 - 6 + x - \frac{x^2}{12} = 0$

$\Rightarrow 15x = 114.5$

$\Rightarrow x = 7.63$

Maximum deflection between supports;

$$y_D \cdot EI = 7.5(12-7.63)^2 + \frac{7.63(12-7.63)^2}{24} + \frac{(12-7.63)^3}{36} - 71.5(12-7.63)$$

$$= -160.84$$

Deflection at free end;

$$\frac{y_A + t_{A/D}}{4} = \frac{t_{C/D}}{12}$$

$$\Rightarrow y_A = \frac{4}{12} \times \left(-\frac{858}{EI}\right) - \left(-\frac{64}{EI}\right)$$

$$= \frac{-286}{EI} + \frac{64}{EI}$$

$$= \frac{-222}{EI}$$

Hence maximum linear deflection, $y = \frac{-222}{EI}$ $\boxed{\text{Ans}}$

Now, $t_{A/C} = \frac{1}{EI}[(\text{Area})_{AC} \cdot \bar{x}_A]$

$$= \frac{1}{EI}\left[-\left\{\frac{4 \times 16}{3} \times (4 - 4/4)\right\} - \left\{(1/2 \times 12 \times 1)(4 + 12/3)\right\} - \left\{(15 \times 6)(4 + 6/2)\right\}\right]$$

$$= \frac{-718}{EI}$$

and $t_{c/A} = \frac{1}{EI} (Area)_{AC} \cdot \bar{x}_c$

$$= \frac{1}{EI}\left[-\left\{\left(\frac{4\times 16}{3}\right)\left(12 + 4/4\right)\right\} - \left\{(1/2 \times 12 \times 1)\left(\frac{2}{3}\times 12\right)\right\} - \left\{(5\times 6)\left(6 + 5/2\right)\right\}\right]$$

$$= -\frac{1135.33}{EI}$$

Now,

$$\theta_A = \frac{t_{c/A}}{AC}$$

$$= \frac{-1135.33}{16\,EI}$$

$$= -\frac{70.96}{EI}$$

$$\theta_c = \frac{t_{A/c}}{AC}$$

$$= \frac{-718}{16EI}$$

$$= -\frac{44.88}{EI}$$

$$\theta_B = \frac{t_{c/B}}{BC}$$

$$= \frac{-858}{12\,EI}$$

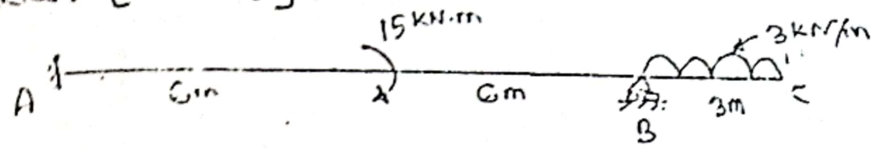
$$= -\frac{71.5}{EI}$$

Hence maximum rotational deflection (rotation)

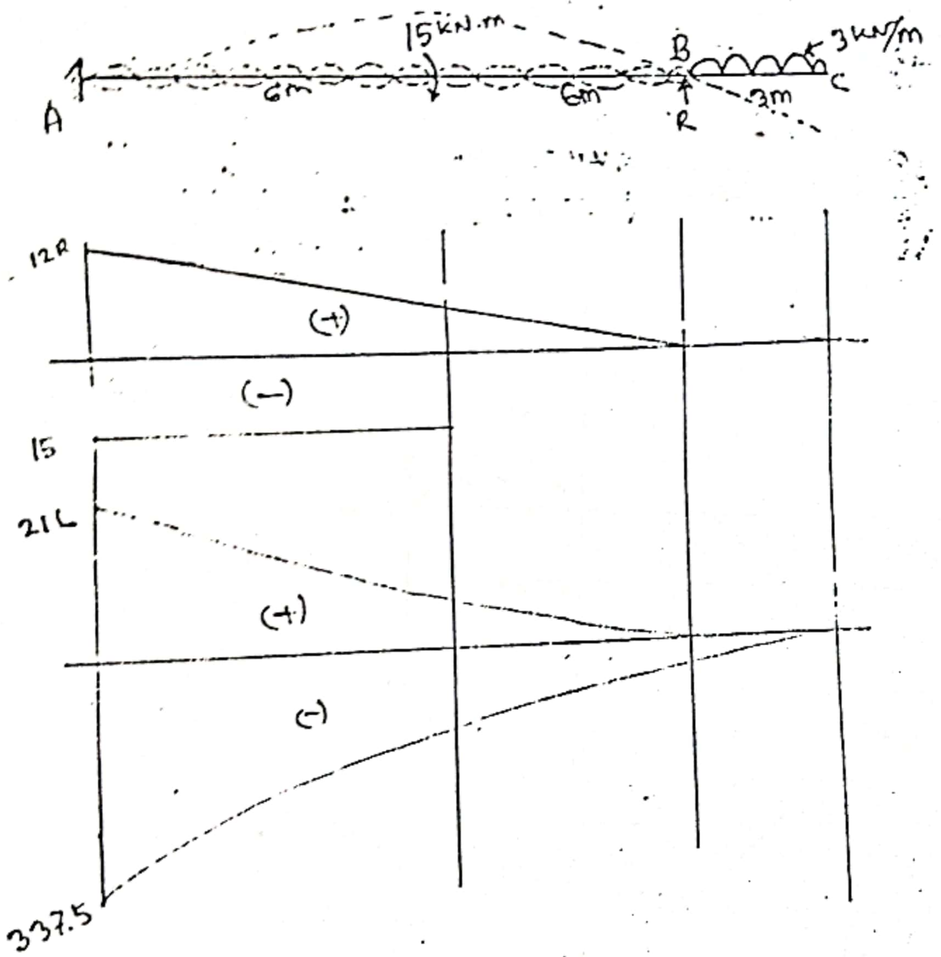
$$\theta_B = -\frac{71.5}{EI}$$

$\boxed{\text{Ans}}$

Draw shear force and bending moment diagram for the beam shown in figure below. [2008]



solution :



Here $t_{C/A} = 0$

$$\Rightarrow \frac{1}{EI}(\text{Area})_{CA}\cdot \bar{x}_C = 0$$

$$\Rightarrow \left\{\left(\tfrac{1}{2}\times 12\times 12R\right)\left(3+\tfrac{2}{3}\times 12\right)\right\} + \left[\left(\frac{216\times 12}{3}\right)\left\{3+\left(12-\frac{12}{4}\right)\right\}\right]$$
$$-\left\{15\times 6\times\left(9+6/2\right)\right\} - \left\{\left(\frac{337.5\times 15}{3}\right)\left(15-\frac{15}{4}\right)\right\} = 0$$

$$\Rightarrow 792R + 10368 - 1080 - 18984.375 = 0$$

$$\Rightarrow 792R = 9696.375$$

$$\Rightarrow R = 12.24 \text{ kN.}$$

$\Sigma F_y = 0$

$\Rightarrow R_A + R - 3 \times 3 = 0$

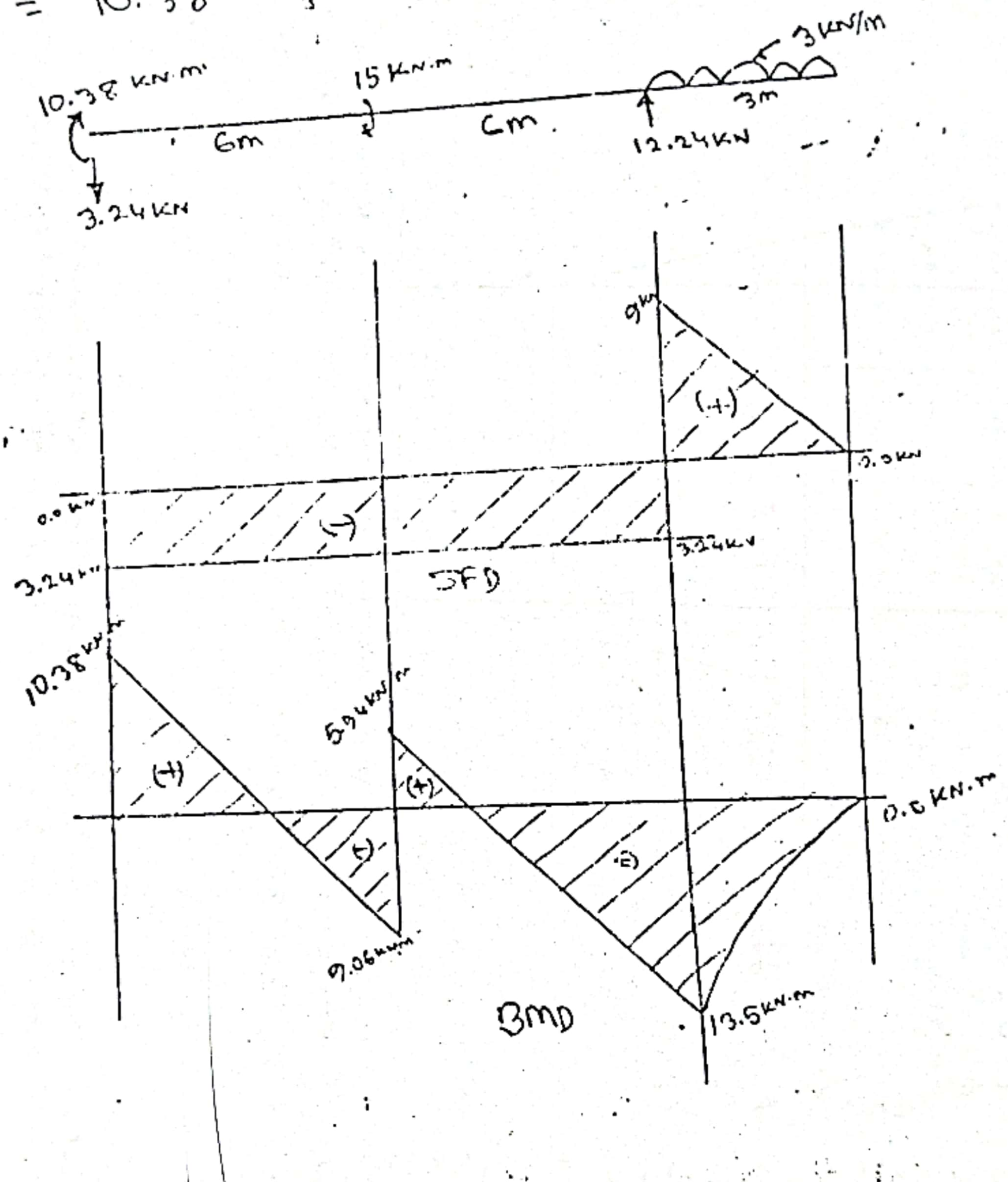
$\Rightarrow R_A = 9 - 12.24$

$= -3.24 \text{ kN.}$

$\Sigma M_A = 0$

$\Rightarrow M_A + 15 + (3 \times 3)(12 + 3/2) - 12.24 \times 12 = 0$

$\Rightarrow M_A = 10.38 \text{ kN.m}$



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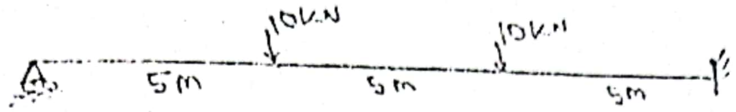
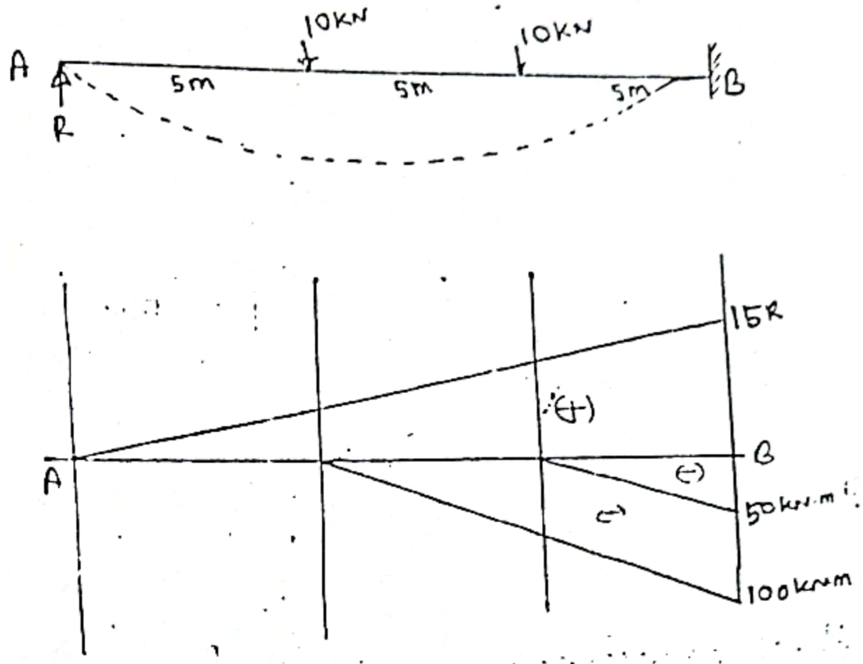
Draw S.F.D & BMD for the following figure shown in below. [2010]



solution:



Here,

$t_{A/B} = 0$

$\Rightarrow \frac{1}{EI}\left[(Area)_{AB} \cdot \bar{x}_A\right] = 0$

$\Rightarrow \{(\frac{1}{2} \times 15 \times 15R) \times (\frac{2}{3} \times 15)\} - \{(\frac{1}{2} \times 10 \times 100)(5 + \frac{2}{3} \times 10)\}$

$- \{(\frac{1}{2} \times 5 \times 50)(10 + \frac{2}{3} \times 5)\} = 0$

$\Rightarrow 1125R = \frac{17500}{3} + \frac{5000}{3}$

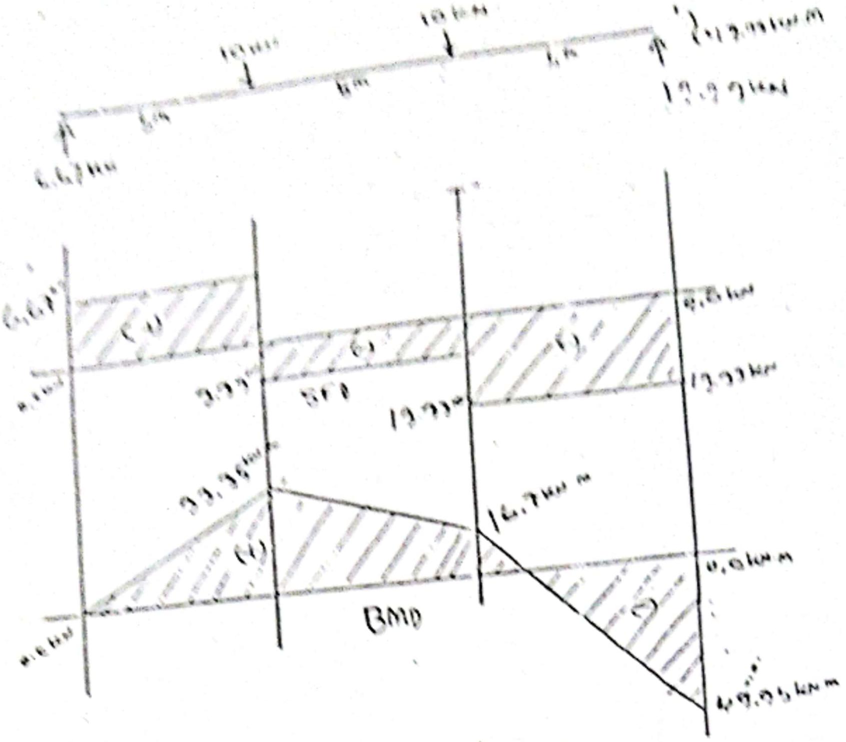
$\Rightarrow 1125R = 7500$

$\Rightarrow R = 6.67$ kN.

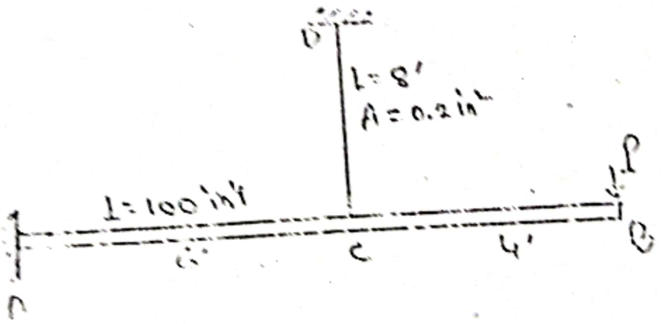
$R_B = 10 + 10 - 6.67 = 13.33$ kN.

$M = 50 + 100 - 6.67 \times 15$

$= 49.95$ kN.m.



12. The steel beam AB is connected to the vertical steel rod CD as shown. Determine the maximum value of P if the stress in the rod is not to exceed 15 ksi. Draw BMD. [2011]



Solution: $E_{steel} = 30 \times 10^3$ ksi.
For steel rod:

Stress, $\sigma = P/A$

$\Rightarrow P = \sigma A$

$= 15 \times 0.2$

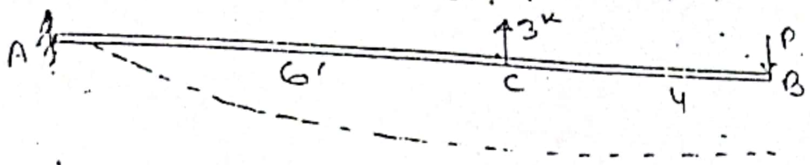
$= 3.0$ k.

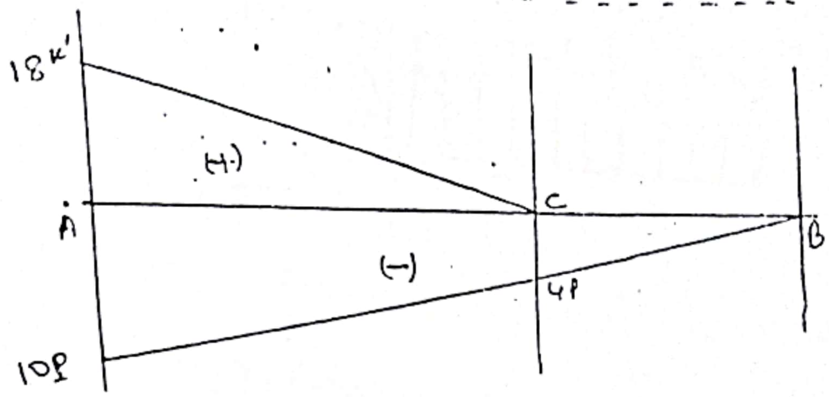
and $\Delta_{rod} = \dfrac{PL}{AE}$

$= \dfrac{3 \times 8 \times 12}{0.2 \times 30 \times 10^3}$

$= 0.048$ in.

consider the beam AB :





Here $-t_{C/A} = \Delta_{rod}$

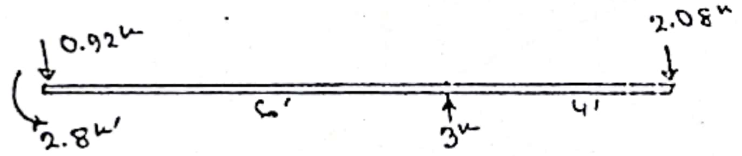
$\Rightarrow -\frac{1}{EI}\left[(Area)_{CA} \cdot \bar{x}_c\right] = \Delta_{rod}$

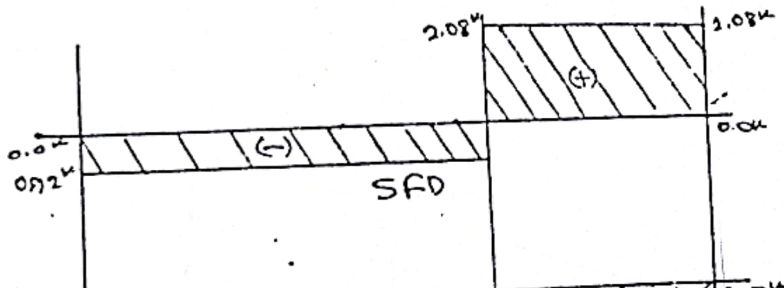
$\Rightarrow -\left[\left(\frac{1}{2} \times 6 \times 18 \times \frac{2}{3} \times 6\right) - (4P \times 6 \times 6/2) - \left(\frac{1}{2} \times 6P \times 6 \times \frac{2}{3} \times 6\right)\right] = \Delta_{rod} EI$

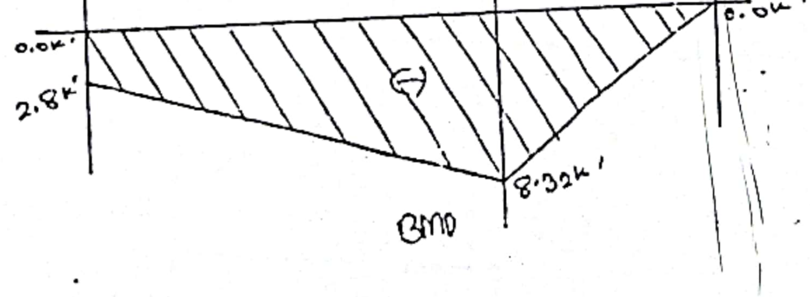
$\Rightarrow -\left[216 - 72P - 72P\right] = \Delta_{rod} EI$

$\Rightarrow (144P - 216)\, 12^3 = 0.048 \times 30 \times 10^3 \times 100$

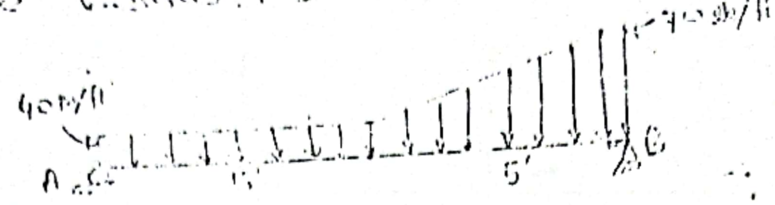
$\Rightarrow P = 2.08^k$.







20. Using area moment method, calculate the value of maximum EIδ for the beam shown in figure below. EI is constant. [2013, 2017]



Solution :

$\Sigma M_A = 0$

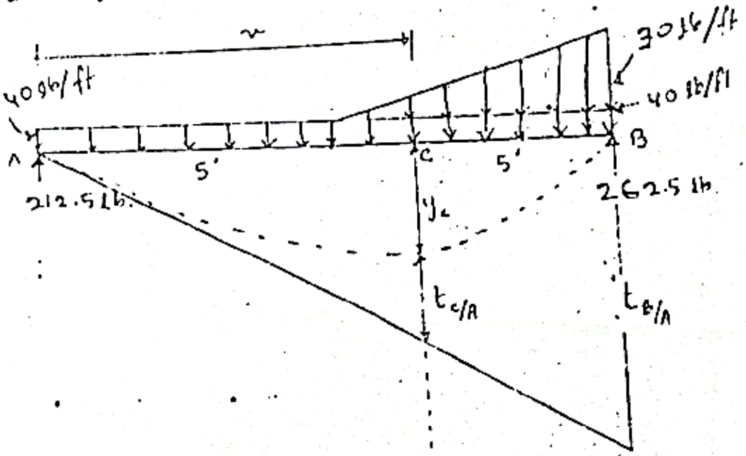
$\Rightarrow 40 \times 10 \times 5 + \frac{1}{2} \times (70 - 40) \times 5 \times (5 + \frac{2}{3} \times 5) - R_B \times 10 = 0$

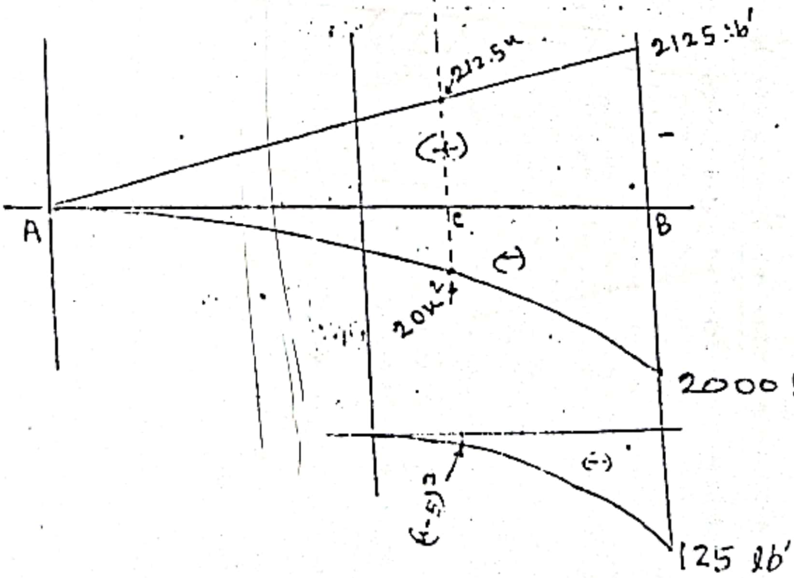
$\Rightarrow R_B = 262.5$ lb.

$\Sigma F_y = 0$

$\Rightarrow R_A = 40 \times (5+5) + \frac{1}{2} \times 5 \times 30 - 262.5$

$= 212.5$ lb.





Here, $t_{B/A} = \frac{1}{EI}[(Area)_{AB} \cdot \bar{x}_B]$

$$= \frac{1}{EI}\left[\left\{\left(\frac{1}{2}\times 10\times 2125\right)\times \frac{10}{3}\right\} - \left\{\left(\frac{10\times 2000}{3}\right)\times \frac{10}{4}\right\} - \left\{\left(\frac{5\times 125}{4}\right)\left(\frac{5}{5}\right)\right\}\right]$$

$$= \frac{18593.75}{EI}$$

Assume at x from left support maximum deflection occurs.

$$t_{C/A} = \frac{1}{EI}[(Area)_{AC} \cdot \bar{x}_C]$$

$$= \frac{1}{EI}\left[\left\{\frac{1}{2}\times x * 212.5x * \frac{x}{3}\right\} - \left\{\frac{20x^2 \cdot x}{3} * \frac{x}{4}\right\} - \left\{\frac{(x-5)^3(x-5)}{4} * \frac{(x-5)}{5}\right\}\right]$$

$$= \frac{1}{EI}\left[\frac{425x^3}{12} - \frac{5x^4}{3} - \frac{(x-5)^5}{20}\right]$$

Now

$$\frac{y_c + t_{c/A}}{AC} = \frac{t_{B/A}}{AB}$$

$$\Rightarrow y_c = \frac{18593.75x}{10EI} - \frac{1}{EI}\left[\frac{425x^3}{12} - \frac{5x^4}{3} - \frac{(x-5)^5}{20}\right]$$

At maximum deflection $dy/dx = 0$

$$\Rightarrow \frac{dy_c}{dx} = 0$$

$$\Rightarrow 1859.375 - \frac{425}{12}\times 3x^2 + \frac{5}{3}\times 4x^3 + \frac{1}{20}[5(x-5)^{4}] = 0$$

$$\Rightarrow 1859.375 - 106.25x^2 + \frac{20x^3}{3} + \frac{1}{4}[x^4 + {}^4C_1 x^3(-5) + {}^4C_2 x^2(-5)^2 + {}^4C_3 x(-5)^3 + {}^4C_4(-5)^4]$$

$\Rightarrow 1859375 - 106.25x^2 + \frac{20}{3}x^3 + \frac{1}{4}x^4 - 5x^3 + 37.5x^2$
$- 125x + 156.25 = 0$

$\Rightarrow 0.25x^4 + \frac{5}{3}x^3 - 68.75x^2 - 125x + 2015.625 = 0$

$\therefore x = 13.35,\ 5.06,\ -6.37,\ -18.71$

Hence the maximum deflection will occur at $x = 5.06$ ft

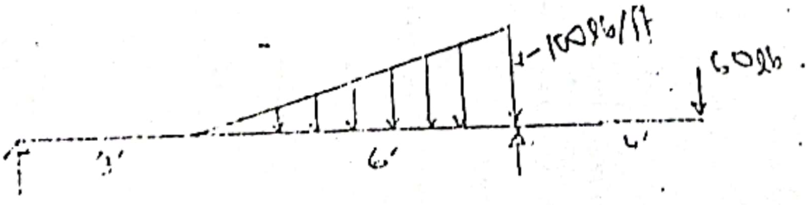
Maximum deflection, $y_{max} = \frac{1859.375 \times 5.06}{EI} - \frac{1}{EI}\left[\frac{425}{12}(5.06)^3 - 5/3(5.06)^4 - \frac{(0.06)^5}{20}\right]$

$= \frac{9408.44}{EI} - \frac{3495.8}{EI}$

$= \frac{5912.64}{EI}$

$\therefore EIy_{max} = 5912.64$ lb-ft³ $\boxed{\text{Ans}}$

21. Determine the value of $EI\delta$ at the right end of the overhanging beam shown in the following figure. [2015]



Solution:

$\Sigma M_2 = 0$

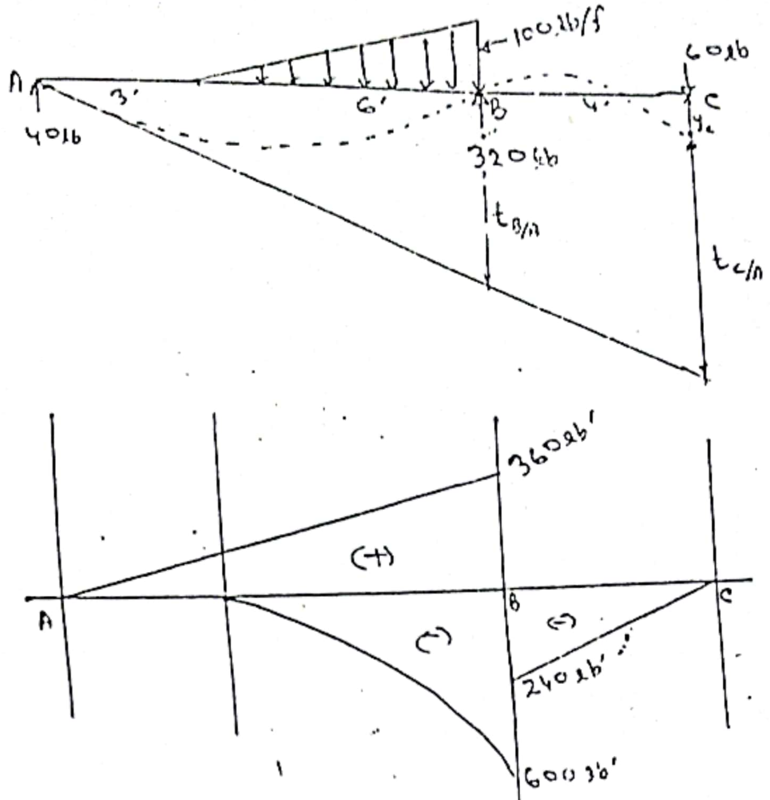
$\Rightarrow R_1 \times 9 + 60 \times 4 - \frac{1}{2} \times 6 \times 100 \times 6/3 = 0$

$\Rightarrow R_1 = 40$ lb.

$\Sigma F_y = 0$

$\Rightarrow R_2 = 60 + \frac{1}{2} \times 6 \times 100 - 40$

$= 320$ lb.



Here.. $t_{B/A} = \frac{1}{EI}\left[(\text{Area})_{AB} \cdot \bar{x}_B\right]$

$$= \frac{1}{EI}\left[\left\{\tfrac{1}{2} \times 9 \times 360 \times 9/3\right\} - \left\{\frac{6 \times 600}{4} \times \frac{6}{5}\right\}\right]$$

$$= \frac{3780}{EI}$$

and $t_{C/A} = \frac{1}{EI}\left[(\text{Area})_{AC} \cdot \bar{x}_C\right]$

$$= \frac{1}{EI}\left[\left\{\tfrac{1}{2} \times 9 \times 360 \times (4 + 9/3)\right\} - \left\{(\tfrac{1}{2} \times 4 \times 240) \times (\tfrac{2}{3} \times 4)\right\} - \left\{\frac{6 \times 600}{4} \times (4 + \tfrac{6}{5})\right\}\right]$$

$$= \frac{5380}{EI}$$

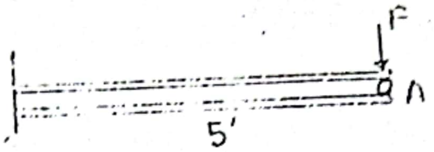
Now

$$\frac{y_c + t_{C/A}}{13} = \frac{t_{B/A}}{9}$$

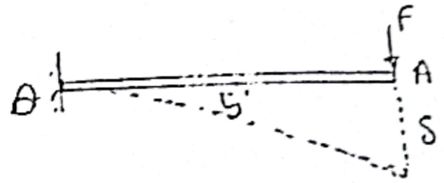
$$\Rightarrow y_c = \frac{13}{9} \times \frac{3780}{EI} - \frac{5380}{EI}$$

$$\therefore y_c = \frac{80}{EI} \quad \boxed{\text{Ans}}$$

1.2. Two steel cantilever beams hold a concentrated load F as shown in the following figure. Find F if the deflection at A is to be 1/2 inch. The moment of inertia of each member is 1.5 in⁴. [2016]



Solution: The load F at the roller may be determined by the condition that at A both cantilevers have the same deflection i.e. ½".

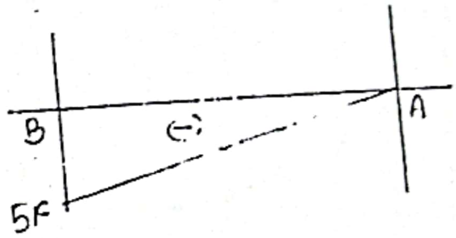


$$\delta = -t_{A/B}$$

$$= -[\text{Area}]_{AB} \cdot \bar{x}_A \cdot \frac{1}{EI}$$

$$= -\frac{1}{EI}\left[-\tfrac{1}{2} \times 5F \times 5 \times \tfrac{2}{3} \times 5\right]$$

$$= \frac{F \times 5^3}{3EI}$$

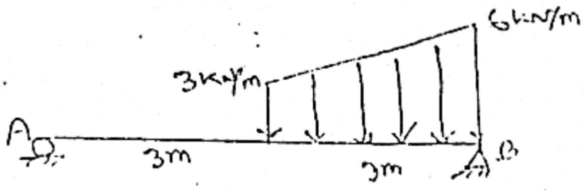


$$\boxed{E_{steel} = 20000 \text{ ksi}}$$

$$\Rightarrow \quad \tfrac{1}{2} = \frac{F \times 125}{3 \times 20000 \times 1.5}$$

$$\Rightarrow \quad F = 522 \text{ kips} \quad \boxed{\text{Ans}}$$

23. Using area moment method, calculate the value of maximum EIδ for the beam shown in figure below. EI is constant. [2018]



Solution:

$\Sigma M_A = 0$

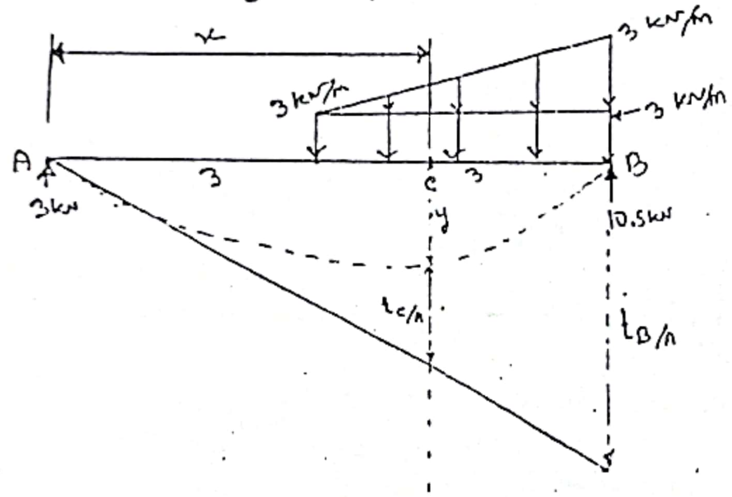
$\Rightarrow 3\times3\times(3+3/2)+(1/2\times3\times3)(3+\frac{2}{3}\times3) - R_B\times6 = 0$

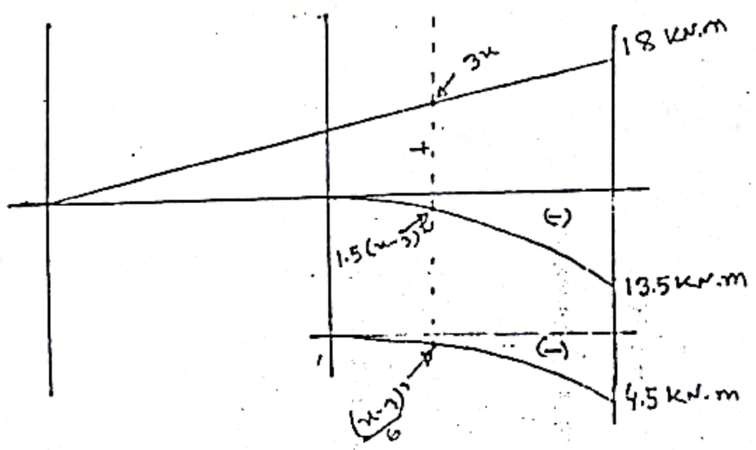
$\Rightarrow R_B = 10.5$ kN.

$\Sigma F_y = 0$

$\Rightarrow R_A = \frac{1}{2}(3+6)\times3 - 10.5$

$= 3$ kN.





Here $t_{B/A} = \frac{1}{EI}\left[(Area)_{BA} \cdot \bar{x}_B\right]$

$$= \frac{1}{EI}\left[\left\{\left(\tfrac{1}{2}\times 6\times 18\right)\times(6/3)\right\} - \left\{\frac{3\times 13.5}{3}\times\frac{3}{4}\right\} - \left\{\frac{3\times 13.5}{4}\times\frac{3}{5}\right\}\right]$$

$$= \frac{95.85}{EI}$$

Assume at x maximum deflection occurs.

$$t_{C/A} = \frac{1}{EI}\left[\{Area\}_{CA} \cdot \bar{x}_C\right]$$

$$= \frac{1}{EI}\left[\left\{\tfrac{1}{2}\cdot x \cdot 3x \cdot x/3\right\} - \left\{\frac{1.5(x-3)^2(x-3)}{3}\cdot\frac{x-3}{4}\right\} - \left\{\frac{(x-3)^3 \cdot (x-3)}{6 * 4}\cdot\frac{x-3}{5}\right\}\right]$$

$$= \frac{1}{EI}\left[\frac{x^3}{2} - \frac{(x-3)^4}{8} - \frac{(x-3)^5}{120}\right]$$

Now $\frac{y + t_{C/A}}{x} = \frac{t_{B/A}}{6}$

$$\Rightarrow y = \frac{95.85x}{6EI} - \frac{1}{EI}\left[\frac{x^3}{2} - \frac{(x-3)^4}{8} - \frac{(x-3)^5}{120}\right]$$

$$= \left[15.975x - x^3/2 + \frac{(x-3)^4}{8} + \frac{(x-3)^5}{120}\right]\frac{1}{EI}$$

At maximum deflection $\frac{dy}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left[15.975x - \frac{x^3}{2} + \frac{(x-3)^4}{8} + \frac{(x-3)^5}{120}\right]\frac{1}{EI} = 0$$

$$\Rightarrow 15.975 - \frac{3x^2}{2} + \frac{4(x-3)^3}{8} + \frac{5(x-3)^4}{120} = 0$$

$$\Rightarrow 15.975 - 1.5x^2 + \frac{1}{2}(x^3 - 9x^2 + 27x - 27) + \frac{1}{24}\left(x^4 - 4x^3 \cdot 3 + \frac{4\cdot 3}{2!}x^2\cdot 3^2 - \frac{4\cdot 3\cdot 2}{3!}x\cdot 3^3 + 3^4\right) = 0$$

$$\Rightarrow 15.975 - 1.5x^2 + x^3/2 - 4.5x^2 + \frac{27x}{2} - 27/2 + x^4/24 - x^3/2 + 2.25x^2 - \frac{108}{24}x + 81/24 = 0$$

$$\Rightarrow \frac{x^4}{24} - 3.75x^2 + 9x + 5.85 = 0$$

$$\therefore x = 87.51,\ 3.02,\ 0,\ -0.53.$$

Hence maximum deflection will occur at $x = 3.02\,m$ from left support.

Maximum deflection, $y_{max} = \left[15.075 \times 3.02 - \frac{(3.02)^3}{2} + \frac{(0.02)^4}{8} + \frac{(0.02)^5}{120}\right]\frac{1}{EI}$

$$= \frac{34.473}{EI}$$

$$\therefore EIy_{max} = 34\,473\ KN.m^3$$

$\boxed{\text{Ans}}$

CONJUGATE BEAM METHOD

conjugate beam:

Conjugate beam is defined as the imaginary beam with the same length as that of the original beam but load at any point on the conjugate beam is equal to the bending moment at that point divided by EI.

Properties of conjugate beam:

1. The span of the conjugate beam is equal to the span of the real beam.

2. The load of the conjugate beam is equal to the moment (M/EI) diagram of the real beam.

3. Shear at any section of the conjugate beam is equal to the slope of the corresponding section of the real beam.

4. Moment at any section of the conjugate beam is equal to the deflection of the corresponding section of the real beam.

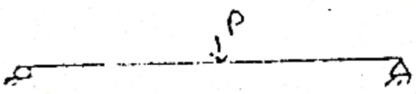
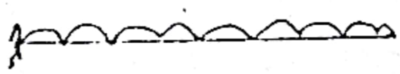
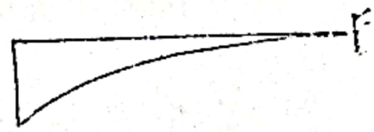
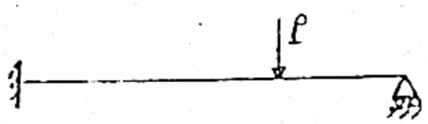
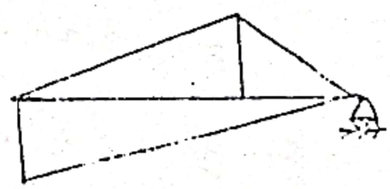
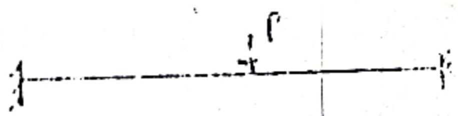
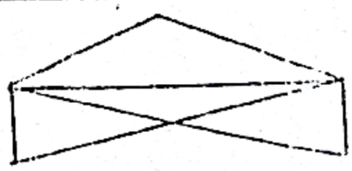
* Maximum deflection occurs at section of zero shear of conjugate beam.

* A fixed end for the real beam becomes free end for the conjugate beam. and vice versa.

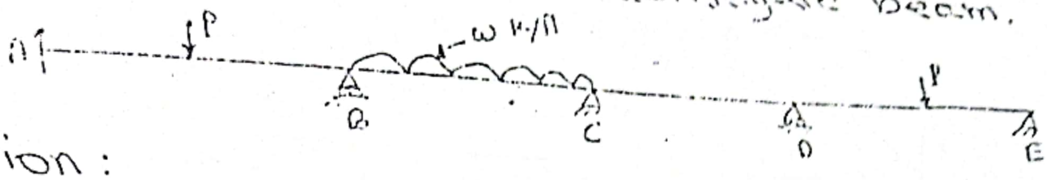
* Interior support (hinge, roller) for the real beam becomes internal link for the conjugate beam. and vice-versa.

* simple support for the real beam becomes simple support for the conjugate beam.

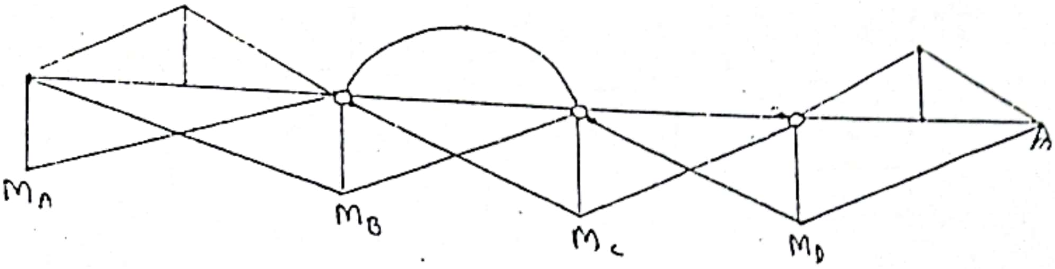
Transfer mechanism :

Real beam	Conjugate beam
	
	
	
	

01. Transfer from real beam to conjugate beam.

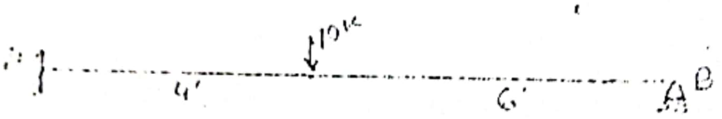


Solution :

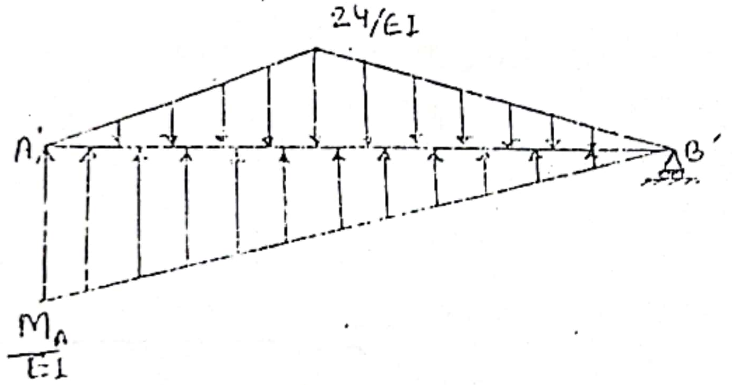


conjugate beam.

02. [illegible] of the following beam.



Solution:



$\Sigma M_{B'} = 0$

$\Rightarrow \frac{1}{2} \times \frac{M_A}{EI} \times 10 \times \frac{2}{3} \times 10 - \frac{1}{2} \times 6 \times \frac{24}{EI} \times \frac{2}{3} \times 6 - \frac{1}{2} \times 4 \times \frac{24}{EI} \times (6 + \frac{4}{3}) = 0$

$\Rightarrow \frac{M_A \cdot 100}{3} = 288 + 352$

$\Rightarrow M_A = 19.2$ k-ft.

$\Sigma M_A = 0$

$\Rightarrow 10 \times 4 - 19.2 - R_B \times 10 = 0$

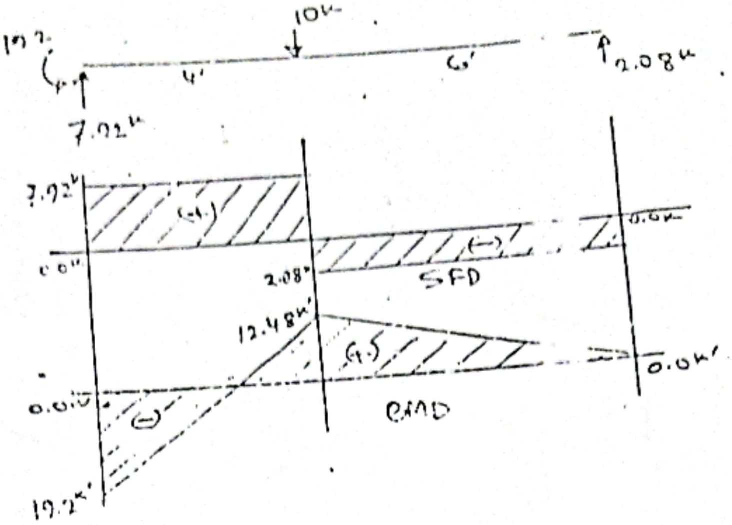
$\Rightarrow R_B = 2.08$k

$\Sigma F_y = 0$

$\Rightarrow R_A + R_B = 10$

$\Rightarrow R_A = 10 - 2.08$

$= 7.92$k.



03. Using conjugate beam method find. $E = 30000$ ksi, $I = 200$ in⁴

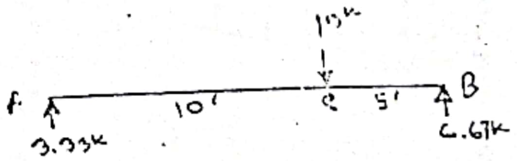
a) Slope at A

b) Deflection at C.

c) The section of maximum deflection.

d) The value of maximum deflection.

Solution:



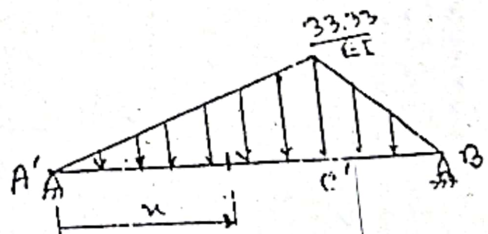
$\Sigma M_{B'} = 0$

$\Rightarrow R_{A'} \times 15 - \frac{1}{2} \times 5 \times \frac{33.33}{EI} \times \frac{2}{3} \times 5$

$- \frac{1}{2} \times 10 \times \frac{33.33}{EI} \times (5 + 10/3) = 0$

$\Rightarrow 15 R_{A'} = \frac{277.75}{EI} + \frac{1388.75}{EI}$

$\Rightarrow R_{A'} = \frac{111.1}{EI}$ k.ft²



ⓐ Slope at A = shear at A'

$= R_{A'}$

$= \frac{111.1 \times 144}{30000 \times 200}$

$= 0.00267$ rad. $\boxed{\text{Ans}}$

(b) Deflection at c = moment at c'

$$= R_A' \times 10 - \frac{1}{2} \times 10 \times \frac{33.33}{EI} \times \frac{10}{3}$$

$$= \frac{111.1}{EI} \times 10 - \frac{3333}{6EI}$$

$$= \frac{555.5}{EI} \text{ k.ft}^3$$

$$= \frac{555.5 \times 12^3}{30000 \times 200}$$

$= 0.16$ in. $\boxed{\text{Ans}}$

(c) Section of maximum deflection = section of zero shear in conjugate beam

Assume x distance from left support shear will be zero in conjugate beam.

$$R_A' - \frac{1}{2} \left(\frac{3333}{10EI} \cdot x\right) \times x = 0$$

$$\Rightarrow \frac{111.1}{EI} - \frac{33.33x^2}{20EI} = 0$$

$$\Rightarrow x = \sqrt{\frac{111.1 \times 20}{33.33}}$$

$\therefore x = 8.16$ ft. $\boxed{\text{Ans}}$

(d) Value of maximum deflection = moment at 8.16' from left support.

$$= R_A' \times 8.16 - \frac{1}{2} \times \frac{33.33 \times 8.16^2}{10EI} \times \frac{8.16}{3}$$

$$= \frac{111.1}{EI} \times 8.16 - \frac{33.33 \times 8.16^3}{30EI}$$

$$= \frac{604.75}{EI} \text{ k.ft}^3$$

$$= \frac{604.75 \times 12^3}{30000 \times 200}$$

$= 0.17$ in. $\boxed{\text{Ans}}$

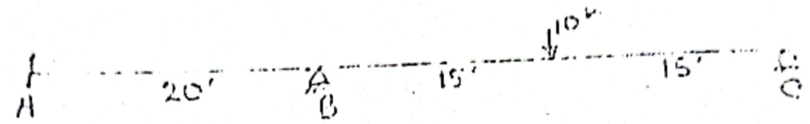
04. Calculate a) M_A and M_B.

b) Distance from c to the point of maximum deflection in span BC.

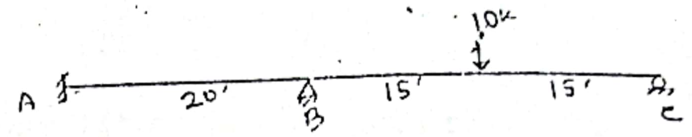
c) Maximum deflection in span BC.

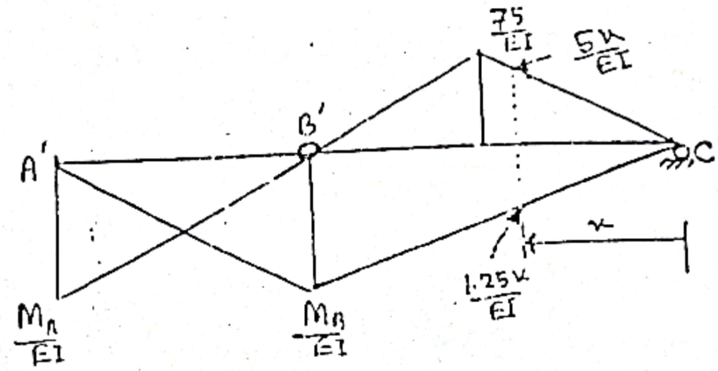
d) The slope at B.

[illegible]



Solution:





ⓐ $\Sigma M_{c'} = 0$

$$\Rightarrow \left\{\frac{1}{2}\times 20\times \frac{M_A}{EI}\times\left(30+\frac{2}{3}\times 20\right)\right\}+\left\{\frac{1}{2}\times 20\times\frac{M_B}{EI}\times\left(30+\frac{20}{3}\right)\right\}+\left\{\frac{1}{2}\times 30\times\frac{M_B}{EI}\times\frac{2}{3}\times 30\right\}-\left\{\frac{1}{2}\times 15\times\frac{75}{EI}\times\left(15+\frac{15}{3}\right)\right\}-\left\{\frac{1}{2}\times 15\times\frac{75}{EI}\times\frac{2}{3}\times 15\right\}=0$$

$$\Rightarrow \frac{1300M_A}{3}+\frac{1100M_B}{3}+300M_B = 11250+5625$$

$$\Rightarrow 1300M_A + 2000M_B = 50625 \quad \text{--------- (i)}$$

considering A'B' part:

$\Sigma M_{B'} = 0$

$$\Rightarrow \frac{1}{2}\times 20\times\frac{M_A}{EI}\times\frac{2}{3}\times 20+\frac{1}{2}\times 20\times\frac{M_B}{EI}\times\frac{20}{3}=0$$

$$\Rightarrow 400M_A + 200M_B = 0 \quad \text{--------- (ii)}$$

solving equation (I) & (II):

$M_A = -18.75\,k'$
$M_B = 37.5\,k'$ $\boxed{\text{Ans}}$

(b) considering B'C' part:

$\Sigma M_{a}' = 0$

$\Rightarrow \frac{1}{2} \times 15 \times \frac{75}{EI} \times \frac{2}{3} \times 15 + \frac{1}{2} \times 15 \times \frac{75}{EI} \times (15 + \frac{15}{3}) - \frac{1}{2} \times 30 \times$

$\frac{M_B}{EI} \times 30/3 - R_c' \times 30 = 0$

$\Rightarrow R_c' \times 30 = \frac{5625}{EI} + \frac{11250}{EI} - \frac{5625}{EI}$

$\Rightarrow R_c' \times EI = 375\,k\text{-}ft^2$

$\Rightarrow R_c' = \frac{375}{EI}$

Distance of maximum deflection = Distance of zero shear in conjugate beam.

Let x distance from right support shear will be zero in conjugate beam.

$R_c' + \frac{1}{2} \times x \times \frac{1.25x}{EI} - \frac{1}{2} \times x \times \frac{5x}{EI} = 0$

$\Rightarrow \frac{375}{EI} + \frac{0.625x^2}{EI} - \frac{2.5x^2}{EI} = 0$

$\Rightarrow x = \sqrt{200}$

$\therefore x = 14.14$ ft. $\boxed{\text{Ans}}$

(c) Maximum deflection = moment at 14.14 ft from right support

$= R_c' * 14.14 + \left(\frac{1}{2} * 14.14 * \frac{1.25 * 14.14}{EI}\right) \times \frac{14.14}{3}$

$- \frac{1}{2} \times 14.14 \times \frac{5 \times 14.14}{EI} \times \frac{14.14}{3}$

$= \frac{375}{EI} \times 14.14 + \frac{588.99}{EI} - \frac{2355.95}{EI}$

$EI.\Delta_{max} = 3535.54 \text{ k.ft}^3$

$\Rightarrow \Delta_{max} = \frac{3535.54 \times 12^3}{30 \times 1000 \times 500}$

$= 0.41$ inch. $\boxed{\text{Ans}}$

(d) Slope at B = shear at B'

$\Sigma F_y = 0$ [considering A'B' part]

$\Rightarrow \frac{1}{2} \times 20 \times \frac{M_A}{EI} + \frac{1}{2} \times 20 \times \frac{M_B}{EI} - R_B' = 0$

$\Rightarrow R_B' = 10 \times \frac{-18.75}{EI} + 10 \times \frac{37.5}{EI}$

$\Rightarrow R_B' EI = 187.5 \text{ k.ft}^2$

$\Rightarrow R_B' = \frac{187.5 \times 144}{30000 \times 500}$

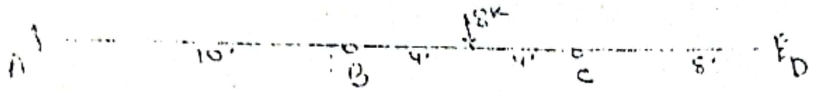
$\Rightarrow R_B' = \frac{9}{5000}$

$\therefore$ Slope at B, $\theta_B = 0.0018$ rad. $\boxed{\text{Ans}}$

05. a. Draw conjugate beam, SFD & BMD.

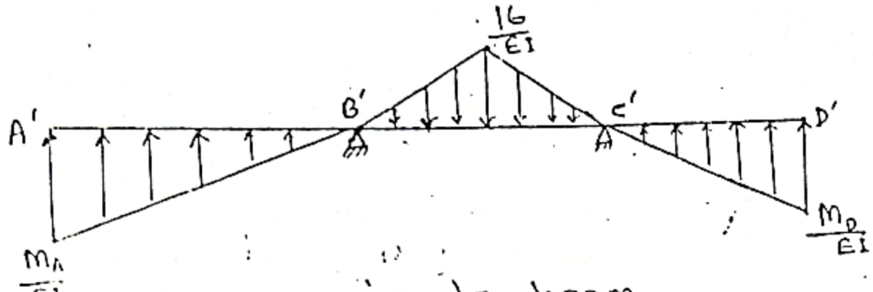
b. Calculate the deflection at point B and C.

E = 30000 ksi and I = 300 in^4.

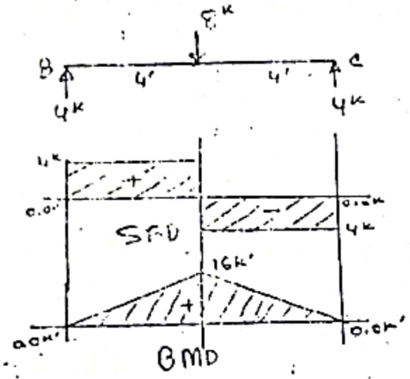


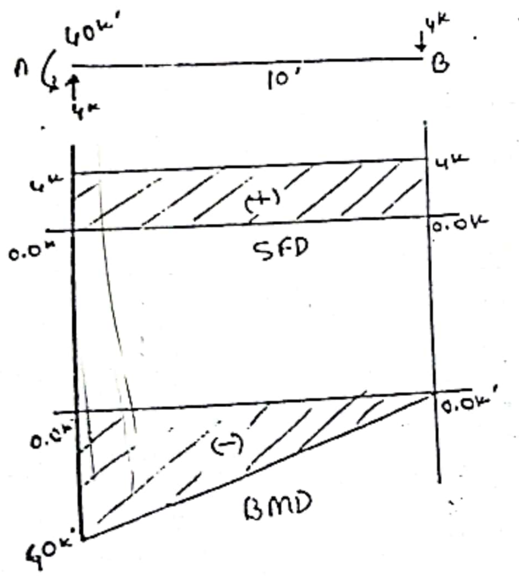
Solution:

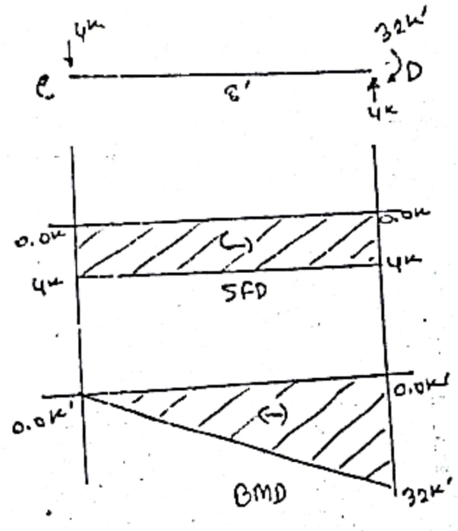
a).



conjugate beam.







b. From BMD: $M_A = 40k'$

$M_D = 32\ k'$

Deflection at B = moment at B'

$= \frac{1}{2} \times 10 \times \frac{M_A}{EI} \times \frac{2}{3} \times 10$ [considering A'O']

$= \frac{100}{3EI} \times 40$

$= \frac{4000 \times 12^3}{3 \times 30000 \times 500}$

$= 0.1536$ in.

$= 0.154$ $\boxed{\text{Ans}}$

Deflection at C = moment at C'

$= \frac{1}{2} \times 8 \times \frac{32}{EI} \times \frac{2}{3} \times 8$ [considering C'D']

$= \frac{2048}{3EI}$

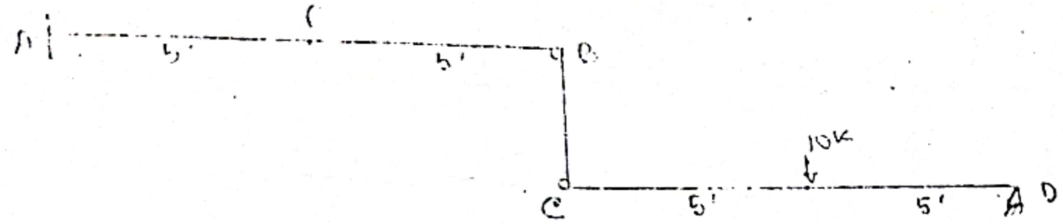
$= \frac{2048 \times 12^3}{3 \times 30000 \times 500}$

$= 0.079$ in.

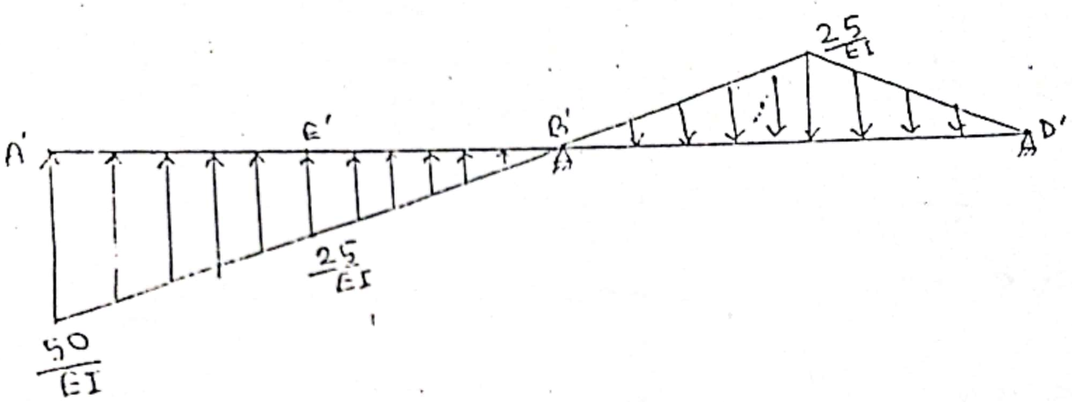
$\boxed{\text{Ans}}$

06 Calculate a. Deflection at B and C & E.
b. Slope at B and D.

E = 30000 ksi and I = 200 in⁴.



Solution:



a)

Deflection at B and C = moment at B' (considering A'B')

$$= \frac{1}{2} \times 10 \times \frac{50}{EI} \times \frac{2}{3} \times 10$$

$$= \frac{5000}{3EI}$$

$$= \frac{5000 \times 12^3}{3 \times 30000 \times 200}$$

$$= 0.48 \text{ inch.} \quad \boxed{\text{Ans}}$$

Deflection at E = moment at E' (considering A'E')

$$= \frac{25}{EI} \times 5 \times 5/2 + \frac{1}{2} \times 5 \times \left(\frac{50}{EI} - \frac{25}{EI}\right) \times \frac{2}{3} \times 5$$

$$= \frac{625}{2EI} + \frac{625}{3EI}$$

$$= \frac{3125 \times 12^3}{6 \times 30000 \times 200}$$

$$= 0.15 \text{ inch} \quad \boxed{\text{Ans}}$$

b. (i) Slope at B = shear at B'

$\Sigma M_{D'} = 0$

$\Rightarrow R_{B'} \times 10 + \{ \frac{1}{2} \times 10 \times \frac{50}{EI} \times (10 + \frac{2}{3} \times 10) \} - (\frac{1}{2} \times 5 \times \frac{25}{EI} \times \frac{2}{3} \times 5)$
$- \{ \frac{1}{2} \times 5 \times \frac{25}{EI} \times (5 + 5/3) \} = 0$

$\Rightarrow R_{B'} \times 10 \cdot EI = \frac{625}{3} + \frac{1250}{3} - \frac{12500}{3}$

$= - \frac{10625}{3}$ k.ft^2

$\Rightarrow R_{B'} = - \frac{10625 \times 12^2}{30 \times 30000 \times 200}$

$= -0.0085$ rad.

$\therefore$ Slope at B, $\theta_B = -0.0085$ rad. $\boxed{\text{Ans}}$

(ii) Slope at D = shear at D'

$\Sigma M_{B'} = 0$ [considering B'D']

$\Rightarrow R_{D'} \times 10 = \frac{1}{2} \times 10 \times \frac{25}{EI} \times 5$

$\Rightarrow R_{D'} = \frac{625}{10EI}$

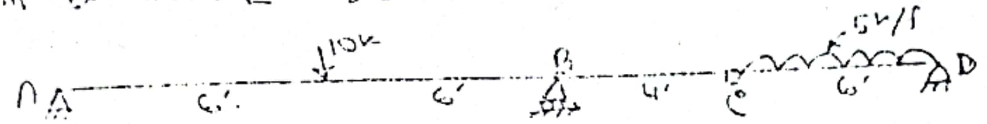
$= \frac{625}{10EI}$

$= \frac{625 \times 12^2}{10 \times 30000 \times 200}$

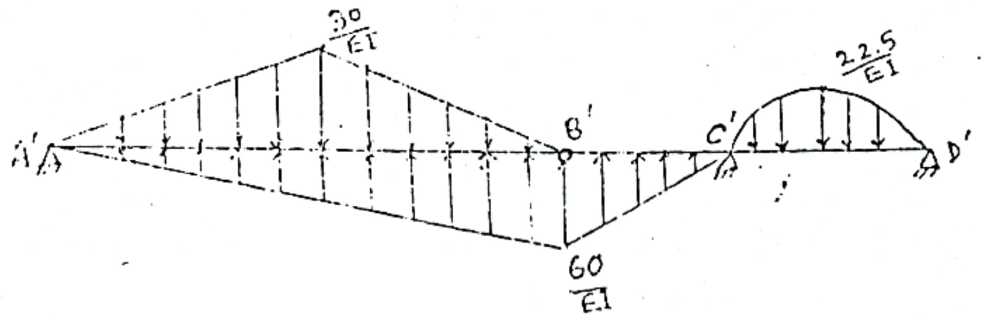
$= 0.0015$ rad.

Slope at D = 0.0015 rad. $\boxed{\text{Ans}}$

Q. Using conjugate beam method find the deflection of point C and rotational deflection at point A and C of the following beam is shown below. $E = 30 \times 10^3$ ksi & $I = 100$ in^4. [2005]



Solution:



$\Sigma M_{B'} = 0$ [considering A'B']

$\Rightarrow R_{A'} \times 12 + \frac{1}{2} \times 12 \times \frac{60}{EI} \times \frac{12}{3} - \frac{1}{2} \times 6 \times \frac{30}{EI} \times (6 + 6/3) - \frac{1}{2} \times 6 \times \frac{30}{EI} \times \frac{2}{3} \times 6 = 0$

$\Rightarrow 12 R_{A'} = \frac{720}{EI} + \frac{360}{EI} - \frac{1440}{EI}$

$\Rightarrow R_{A'} = -\frac{30}{EI}$

$\Sigma F_y = 0$ [considering A'B']

$\Rightarrow R_{A'} + R_{B'} + \frac{1}{2} \times 12 \times \frac{60}{EI} - \frac{1}{2} \times 12 \times \frac{30}{EI} = 0$

$\Rightarrow R_{B'} = \frac{180}{EI} - \frac{360}{EI} + \frac{30}{EI}$

$\Rightarrow R_{B'} = -\frac{150}{EI}$

Deflection at C = moment at C' [considering B'C']

$= \frac{1}{2} \times 4 \times \frac{60}{EI} \times \frac{2}{3} \times 4 - R_{B'} \times 4$

$= \frac{320}{EI} - \left(-\frac{150}{EI}\right) 4$

$= \frac{920}{EI}$

$$= \frac{920 \times 12^3}{30000 \times 100}$$

$\therefore$ Deflection at $C = 0.53$ in $\boxed{\text{Ans}}$

$\Sigma F_y = 0$ [considering $B'C'$]

$$\Rightarrow R_c' - R_B' + \frac{1}{2} \times 4 \times \frac{60}{EI} = 0$$

$$\Rightarrow R_c' = \frac{120}{EI} + \left(-\frac{150}{EI}\right)$$

$$= -\frac{30}{EI}$$

Rotational deflection at A = shear at A'

$$= R_A'$$

$$= -\frac{30}{EI}$$

$$= -\frac{30 \times 12^2}{30000 \times 100}$$

$= -0.0014$ rad $\boxed{\text{Ans}}$

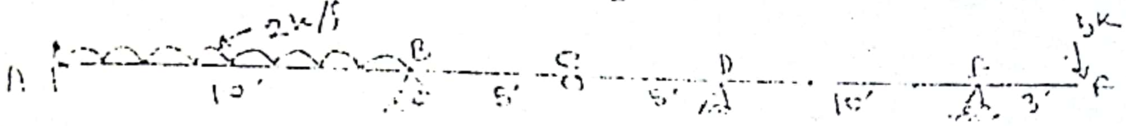
Rotational deflection at c = shear at c'

$$= -\frac{30}{EI}$$

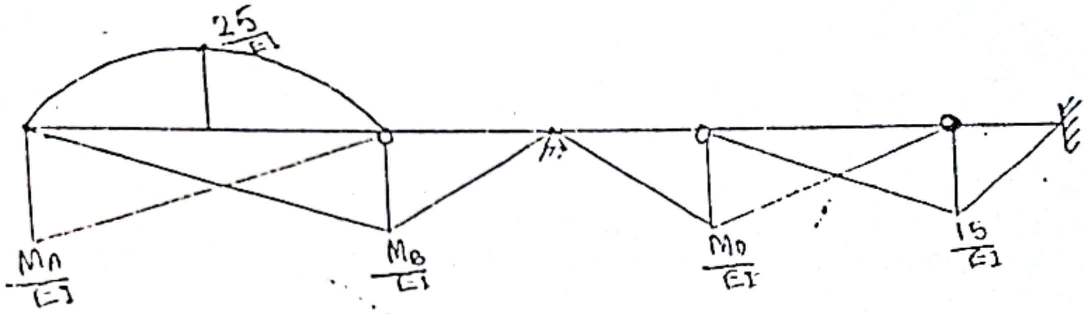
$= -0.00144$ rad

$\boxed{\text{Ans}}$

13. A loaded beam is shown in figure below. Draw the corresponding conjugate beam with load diagram. [2006]

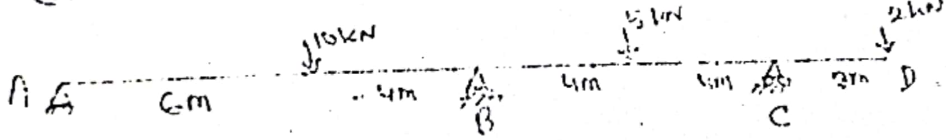


Solution:

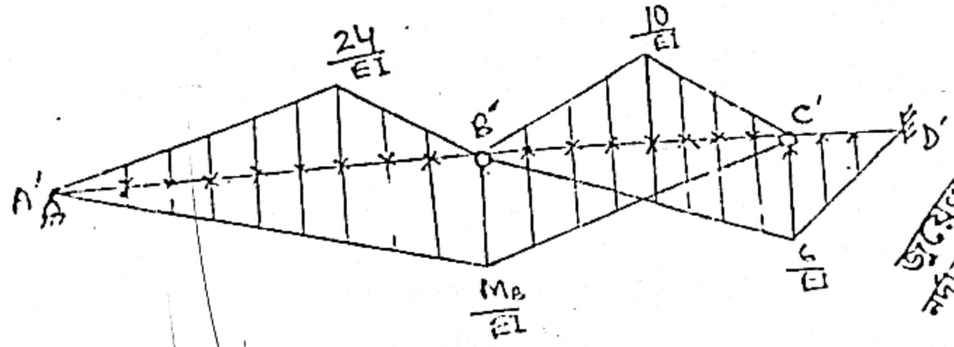


conjugate beam.

14. A continuous beam ABCD is loaded as shown in figure below. Using conjugate beam method find the deflection at point D and rotational deflection at point A of the beam. [2007]



Solution:



conjugate beam.

$\Sigma M_{B'} = 0$ [considering $A'B'$]

$\Rightarrow R_A' \times 10 + \frac{1}{2} \times 10 \times \frac{M_B}{EI} \times \frac{10}{3} - \frac{1}{2} \times 6 \times \frac{24}{EI} \times (4 + 6/3)$

$- \frac{1}{2} \times 4 \times \frac{24}{EI} \times \frac{2}{3} \times 4 = 0$

$\Rightarrow 10R_A' + \frac{50 M_B}{3EI} = \frac{432}{EI} + \frac{128}{EI}$

$\Rightarrow 10\,EI\,R_A' + \frac{50}{3} M_B = 560$ .... (i)

$\Sigma M_{C'} = 0$ [considering $A'C'$]

$\Rightarrow R_A' \times 18 + \left\{\frac{1}{2} \times 10 \times \frac{M_B}{EI} \times \left(8 + \frac{10}{3}\right)\right\} + \left\{\frac{1}{2} \times 8 \times \frac{M_B}{EI} \times \frac{2}{3} \times 8\right\}$

$+ \left\{\frac{1}{2} \times 8 \times \frac{6}{EI} \times 8/3\right\} - \left\{\frac{1}{2} \times 6 \times \frac{24}{EI} \times (12 + 6/3)\right\} - \left\{\frac{1}{2} \times 4 \times \frac{24}{EI} \times \left(8 + \frac{2}{3} \times 4\right)\right\}$

$- \left\{\frac{1}{2} \times 8 \times \frac{10}{EI} \times \frac{8}{2}\right\} = 0$

$\Rightarrow 18 R_A' + \frac{170}{3} \frac{M_B}{EI} + \frac{64}{3} \frac{M_B}{EI} + \frac{64}{EI} - \frac{1008}{EI} - \frac{512}{EI} - \frac{160}{EI} = 0$

$\Rightarrow 18 R_A' + \frac{78 M_B}{EI} = \frac{1616}{EI}$ .... (ii)

$\Rightarrow 18 R_A' EI + 78 M_B = 1616$ ....

Solving equations (i) & (ii);

$R_A' EI = \frac{314}{9}$

$M_B = \frac{38}{3}$

Deflection at D = moment at D' (considering $A'D'$)

$= R_A' \times 21 + \frac{1}{2} \times 10 \times \frac{28}{3EI} \times (11 + 10/3) + \frac{1}{2} \times 8 \times \frac{28}{3EI} \times$

$\left(3 + \frac{2}{3} \times 8\right) + \frac{1}{2} \times 8 \times \frac{6}{EI} \times (3 + 8/3) + \frac{1}{2} \times 3 \times \frac{6}{EI} \times \frac{2}{3} \times 3$

$- \frac{1}{2} \times 6 \times \frac{24}{EI} \times (15 + 6/3) - \frac{1}{2} \times 4 \times \frac{24}{EI} \times \left(1 + \frac{2}{3} \times 4\right)$

$- \frac{1}{2} \times 8 \times \frac{10}{EI} \times \left(3 + \frac{8}{2}\right)$

$$= \frac{314}{9EI} \times 21 + \frac{6020}{9}\frac{1}{EI} + \frac{2800}{9}\frac{1}{EI} + \frac{136}{EI} + \frac{18}{EI}$$

$$- \frac{1224}{EI} - \frac{656}{EI} - \frac{280}{EI}$$

$$= \frac{5600}{3EI} - \frac{2160}{EI}$$

$$= -\frac{880}{3EI}$$

$$\therefore \Delta_D . EI = -\frac{880}{3} \text{ kN.m}^3 \quad \boxed{\text{Ans}}$$

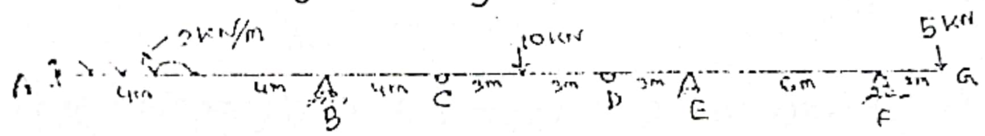
Rotational deflection at A = shear at A'

$$= R_A'$$

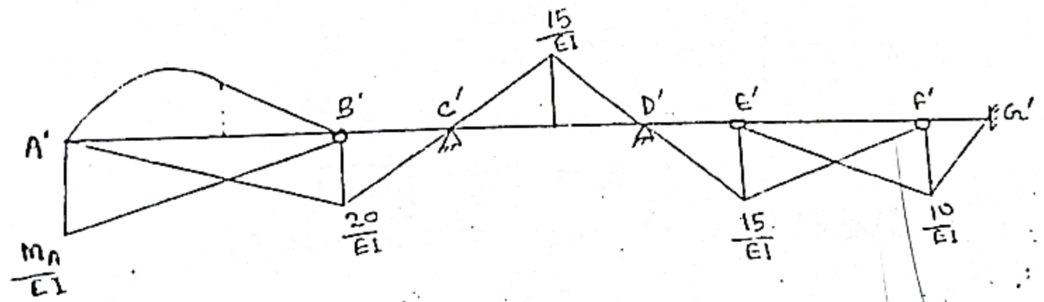
$$= \frac{314}{9EI}$$

$$\Rightarrow \theta_A . EI = \frac{314}{9} \text{ kN.m}^2 \quad \boxed{\text{Ans}}$$

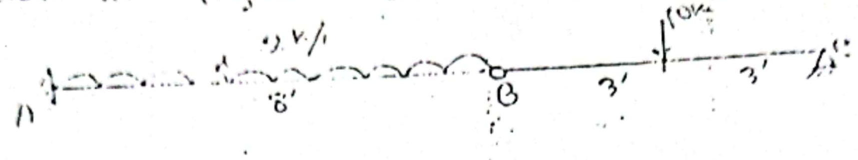
2. A continuous beam is shown in figure below. Draw corresponding conjugate beam with load diagram. [2008]



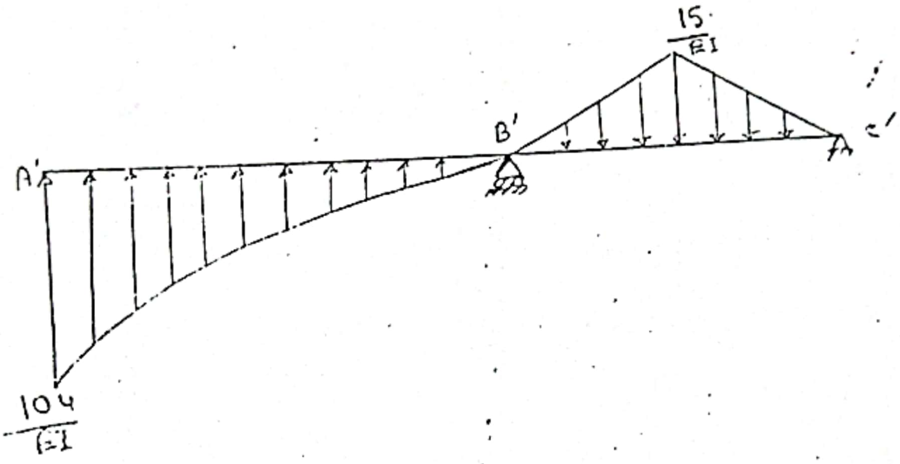
Solution:



1. Using conjugate beam method calculate the value of linear deflection and rotational deflection at point B of the beam ABC as shown in figure below. [2009]



Solution:



Linear deflection at B = moment at B' [considering A'B']

$$= \frac{8 \cdot \frac{104}{EI}}{3} \cdot \left(8 - \frac{8}{4}\right)$$

$$= \frac{1664}{EI}$$

$$\therefore \Delta_B \cdot EI = 1664 \text{ k.ft}^3$$

$$\therefore \Delta_B = \frac{1664}{EI} \quad \boxed{\text{Ans}}$$

Rotational deflection at B = shear at B'

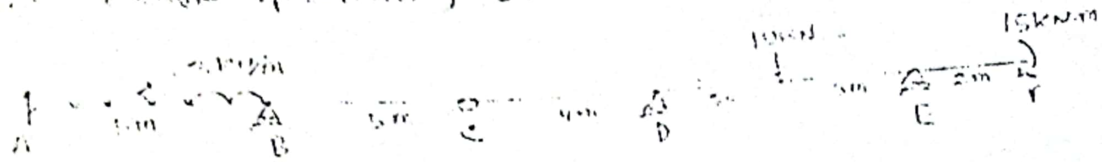
$\Sigma M_{C'} = 0$ [considering A'C']

$$\Rightarrow \frac{\frac{104}{EI} \times 8}{3} \times \left\{6 + \left(8 - \frac{8}{4}\right)\right\} + R_{B'} \times 6 - \frac{1}{2} \times 6 \times \frac{15}{EI} \times 3 = 0$$

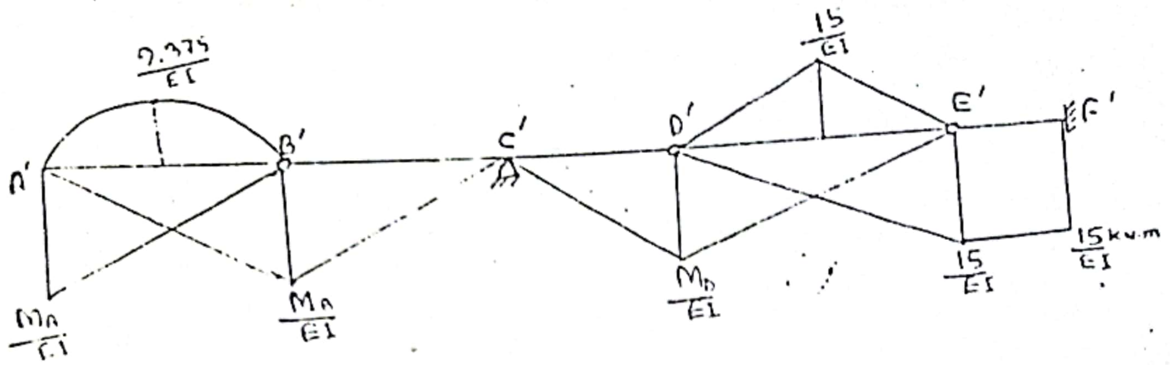
$$\Rightarrow R_{B'} = -\frac{3193}{6EI}$$

Rotational deflection at B, $\theta_B = -\frac{3193}{6EI}$ $\boxed{\text{Ans}}$

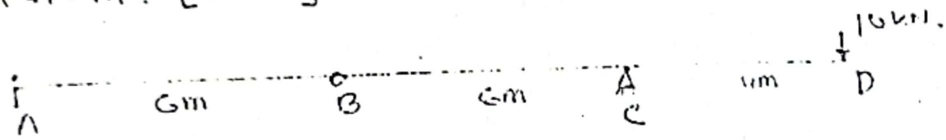
12. A continuous beam is shown in figure below. Draw the corresponding beam with load diagram. [2010, 2013]



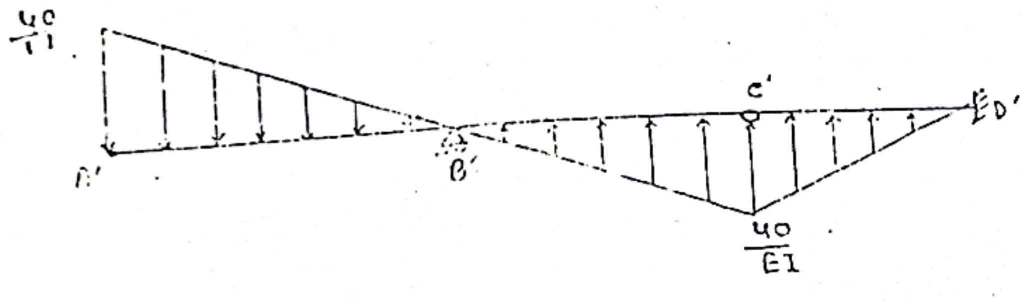
Solution:



3. Using conjugate beam method, find the deflection at B & D and the rotational deflection at B & D. EI is constant. [2011]



Solution:



$\Sigma M_{C'} = 0$ (considering A'C')

$\Rightarrow R_{B'} \times 6 + 1/2 \times 6 \times \frac{40}{EI} \times 6/3 - 1/2 \times 6 \times \frac{40}{EI} \times (6 + \frac{2}{3} \times 6) = 0$

$\Rightarrow 6R_{B'} = \frac{1200}{EI} - \frac{240}{EI}$

$\Rightarrow R_{B'} = \frac{160}{EI}$ (↑)

$\Sigma F_y = 0$

$\Rightarrow R_{B'} + R_{D'} + \frac{1}{2} \times 10 \times \frac{40}{EI} - \frac{1}{2} \times 6 \times \frac{40}{EI} = 0$

$\Rightarrow R_{D'} = \frac{120}{EI} - \frac{200}{EI} - \frac{160}{EI}$

$= \frac{-240}{EI}$

Deflection at B = moment at B' (considering A'B')

$= -\frac{1}{2} \times 6 \times \frac{40}{EI} \times \frac{2}{3} \times 6$

$= -\frac{480}{EI}$

$= \frac{480}{EI}$ (↑) $\boxed{\text{Ans}}$

Deflection at D = moment at D' (considering A'D')

$= \frac{1}{2} \times 4 \times \frac{40}{EI} \times \frac{2}{3} \times 4 + \frac{1}{2} \times 6 \times \frac{40}{EI} \times (4 + 6/3)$

$+ \frac{160}{EI} \times 10 - \frac{1}{2} \times 6 \times \frac{40}{EI} (10 + \frac{2}{3} \times 6)$

$= \frac{640}{3EI} + \frac{720}{EI} + \frac{1600}{EI} - \frac{1680}{EI}$

$= \frac{2560}{3EI}$ (↓) $\boxed{\text{Ans}}$

Rotational deflection at B = shear at B'

$= R_{B'}$

$= \frac{160}{EI}$ $\boxed{\text{Ans}}$

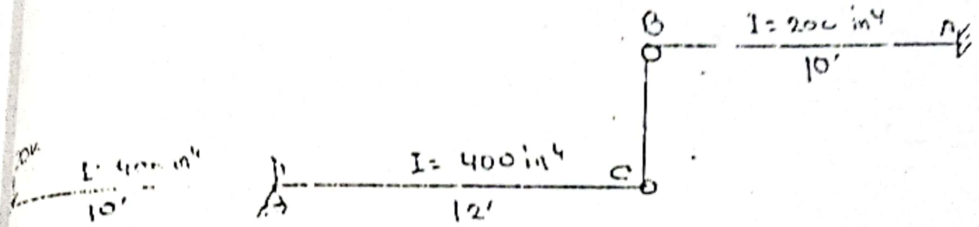
Rotational deflection at D = shear at D'

$= R_{D'}$

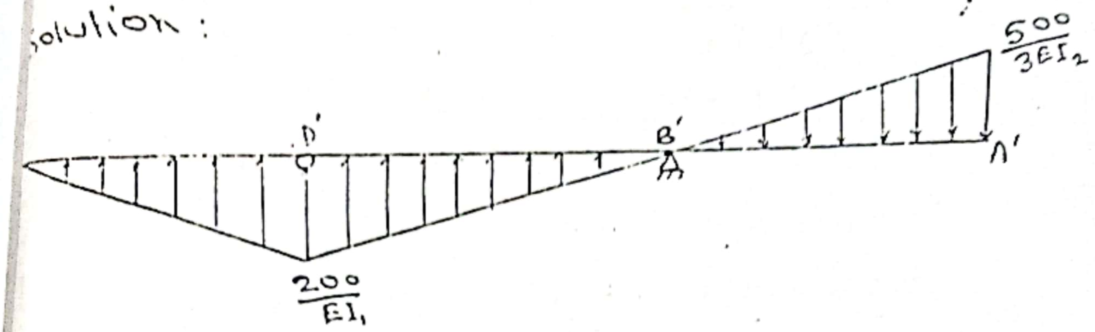
$= \frac{-240}{EI}$

$\boxed{\text{Ans}}$

Using conjugate beam method find
(i) the deflection of E.
(ii) the deflection of B and
(iii) slope of B.
Given E = 30000 ksi. [2012, 2015]



Solution:



Assume, $I_1 = 400 \text{ in}^4$ and $I_2 = 200 \text{ in}^4$

$\sum M_{D'} = 0$ (considering D'A')

$$\Rightarrow -\frac{1}{2}\times 12 \times \frac{200}{EI_1} \times 12/3 - R_{B'}\times 12 + \frac{1}{2}\times 10 \times \frac{500}{3EI_2}\times\left(12+\frac{2}{3}\times 10\right) = 0$$

$$\Rightarrow 12R_{B'} = \frac{140000}{9EI_2} - \frac{4800}{EI_1}$$

$$\Rightarrow R_{B'} = \frac{35000}{27EI_2} - \frac{400}{EI_1} \ (\uparrow)$$

$R \longrightarrow k.ft^2$

$M \longrightarrow k.ft^3$

$EI = k.in^2$

Deflection at E = moment at E'

$$= \frac{1}{2}\times 10 \times \frac{200}{EI_1}\times\frac{2}{3}\times 10 + \frac{1}{2}\times 12\times\frac{200}{EI_1}\times(10+12/3) + \left(\frac{35000}{27EI_2} - \frac{400}{EI_1}\right)\times 22 - \frac{1}{2}\times 10\times\frac{500}{3EI_2}\times\left(22+\frac{2}{3}\times 10\right)$$

$$= \frac{20000}{3EI_1} + \frac{16800}{EI_1} + \frac{770000}{27EI_2} - \frac{8800}{EI_1} - \frac{215000}{9EI_2}$$

$$= \frac{44000}{3EI_1} + \frac{125000}{27EI_2}$$

$$= \left(\frac{44000}{3\times 30000\times 400} + \frac{125000}{27\times 30000\times 200}\right)\times 12^3$$

$$= 3.445 \text{ in}(\downarrow)$$

$\boxed{\text{Ans}}$

(ii) Deflection of B = moment at B'

$$= -\tfrac{1}{2}\times 10\times \frac{500}{3EI_2}\times \frac{2}{3}\times 10$$

$$= -\frac{50000}{9EI_2}$$

$$= -\frac{50000}{9\times 30000\times 200}\times 12^3$$

$$= -1.6 \text{ in}.$$

$$= 1.6 \text{ in}. (\uparrow)$$

$\boxed{\text{Ans}}$

(iii) Slope of B = shear at B'

$$= R_{B'}$$

$$= \frac{35000}{27EI_2} - \frac{400}{EI_1}$$

$$= \left(\frac{35000}{27\times 30000\times 200} - \frac{400}{30000\times 400}\right)12^2$$

$$= 0.026 \text{ rad}.$$

$\boxed{\text{Ans}}$

Using the conjugate beam method, find

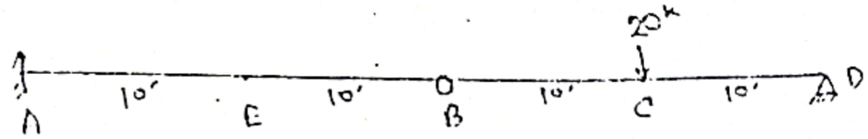
a) The deflection of B and C

b) The deflection of E

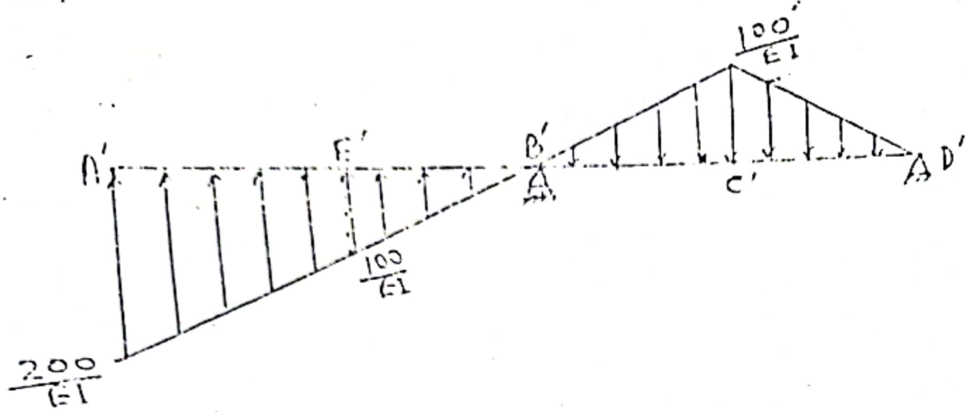
c) The slope of B

d) The slope of D.

Given that: $E = 30000$ ksi, $I = 100$ in^4. [2016]



Solution:



$\Sigma M_{B'} = 0$ (considering A'D')

$\Rightarrow \frac{1}{2} \times 20 \times \frac{200}{EI} \times \frac{2}{3} \times 20 + \frac{1}{2} \times 20 \times \frac{100}{EI} \times \frac{20}{2} - R_D' \times 20 = 0$

$\Rightarrow R_D' = \frac{5500}{3EI}$ (↑)

$\Sigma F_y = 0$

$\Rightarrow R_B' + R_D' + \frac{1}{2} \times 20 \times \frac{200}{EI} - \frac{1}{2} \times 20 \times \frac{100}{EI} = 0$

$\Rightarrow R_B' = \frac{1000}{EI} - \frac{2000}{EI} - \frac{5500}{3EI}$

$= -\frac{6500}{EI}$

ⓐ Deflection of B = moment at B' (considering A'B')

$= \frac{1}{2} \times 20 \times \frac{200}{EI} \times \frac{2}{3} \times 20$

$= \frac{80000}{3EI}$

$$= \frac{80000}{3\times 30000\times 100}\times 12^3$$

$= 1.536$ in $(\downarrow)$ [Ans]

Deflection of C = moment at C' (considering $C'B'$)

$$= R_B'\times 10 - \tfrac{1}{2}\times 10\times \frac{100}{EI}\times \frac{10}{3}$$

$$= \frac{5500}{3EI}\times 10 - \frac{5000}{3EI}$$

$$= \frac{50000}{3EI}\ \text{k.ft}^3$$

$$= \frac{50000\times 12^3}{3\times 30000\times 100}$$

$= 9.6$ in. $(\downarrow)$ [WTH!!!!]

[I know a sad story for this deflection]

b) Deflection of E = moment at E' (considering $A'E'$)

$$= 10\times \frac{100}{EI}\times \frac{10}{2} + \tfrac{1}{2}\times \frac{100}{EI}\times 10\times \tfrac{2}{3}\times 10$$

$$= \frac{25000}{3EI}$$

$$= \frac{25000}{3\times 30000\times 100}\times 12^3$$

$= 4.8$ in $(\downarrow)$ [Ans]

c) Slope of B = shear at B'

$$= R_B'$$

$$= -\frac{6500}{EI}$$

$$= -\frac{6500}{30000\times 100}\times 12^2$$

$= -0.312$ rad [Ans]

d) Slope of D = shear at D'

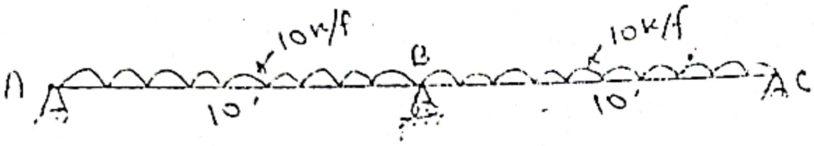
$= R_{D'}$

$= \frac{5500}{3EI}$

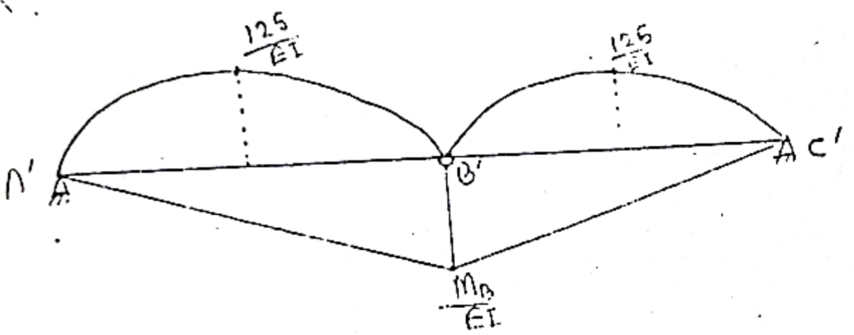
$= \frac{5500}{3 \times 30000 \times 100} \times 12^2$

$= 0.088$ rad. $\boxed{Ans}$

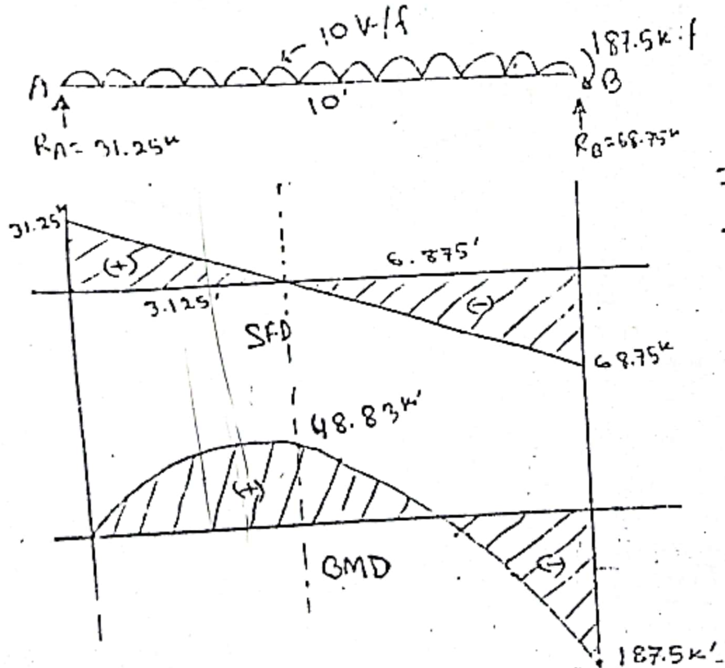
6. Transform the following beam into a conjugate beam and draw SFD and BMD if M_B is -187.5 k.ft.
[2017]



Solution:



conjugat beam.



$\Sigma M_A = 0$

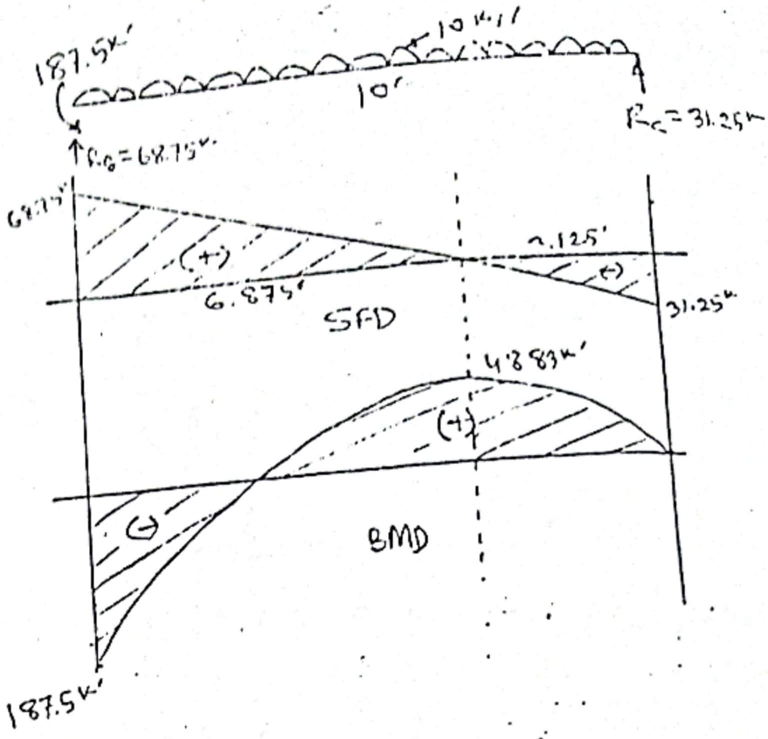
$\Rightarrow 10 \times 10 \times 5 + 187.5 - R_B \times 10 = 0$

$\Rightarrow R_B = 68.75k$

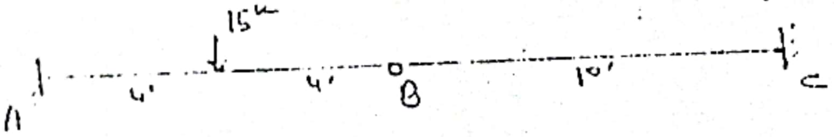
$\Sigma F_y = 0$

$\Rightarrow R_A = 10 \times 10 - 68.75$

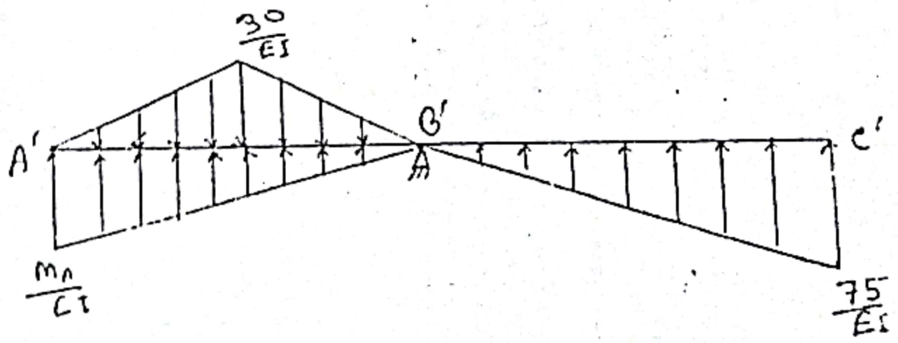
$= 31.25k.$



17. Using conjugate beam method, calculate
i. M_A and M_B and
ii. Deflection at B.
Given E = 30000 k/in² and I = 200 in⁴. [2018]



Solution :



①

$\Sigma M_{B'} = 0$ [considering A'B']

$\Rightarrow \frac{1}{2} \times \frac{M_A}{EI} \times 8 \times \frac{2}{3} \times 8 - \frac{1}{2} \times 8 \times \frac{30}{EI} \times 4 = 0$

$\Rightarrow M_A \times \frac{64}{3} = 480$

$\Rightarrow M_A = 22.5$ k'

and $M_B = 0$

$\boxed{\text{Ans}}$

11. Deflection of B = moment at B' [considering B'C']

$= \frac{1}{2} \times 10 \times \frac{75}{EI} \times \frac{2}{3} \times 10$

$= \frac{2500}{EI}$

$= \frac{2500 \times 12^3}{30000 \times 200}$

$= 0.06$ in ($\downarrow$)

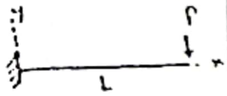
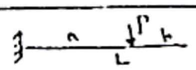
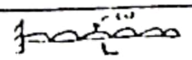
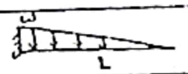
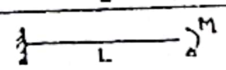
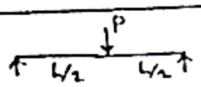
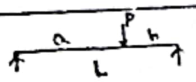
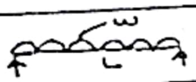
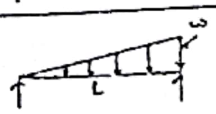
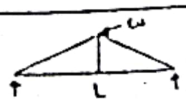
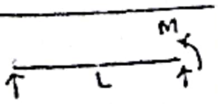
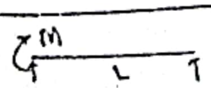
$\boxed{\text{Ans}}$

METHOD OF SUPERPOSITION

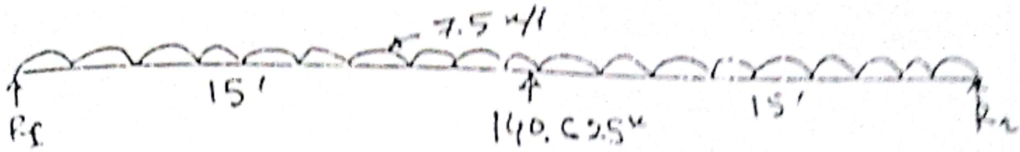
Method of superposition:

The slope or deflection at any point on the beam is equal to the resultant of the slopes or deflections at that point caused by each of the load acting separately.

Summary of beam loadings:

Type of load	M_{max}	slope at end	Deflection equation (y is positive downward)	Δ_{max}
	$-PL$	$\frac{PL^2}{2EI}$	$EIy = \frac{Px^2}{6}(3L-x)$	$\frac{PL^3}{3EI}$
	$-Pa$	$\frac{Pa^2}{2EI}$	$EIy = \frac{Px^2}{6}(3a-x)$ $[0<x<a]$ $EIy = \frac{Pa^2}{6}(3x-a)$ $[a<x<L]$	$\frac{Pa^2}{6EI}(3L-a)$
	$-\frac{wL^2}{2}$	$\frac{wL^3}{6EI}$	$EIy = \frac{wx^2}{24}(6L^2-4Lx+x^2)$	$\frac{wL^4}{8EI}$
	$-\frac{wL^2}{6}$	$\frac{wL^3}{24EI}$	$EIy = \frac{wx^2}{120L}(10L^3-10L^2x+5Lx^2-x^3)$	$\frac{wL^4}{30EI}$
	$-M$	$\frac{ML}{EI}$	$EIy = \frac{Mx^2}{2}$	$\frac{ML^2}{2EI}$
	$\frac{PL}{4}$	$\frac{PL^2}{16EI}$	$EIy = \frac{Px}{12}\left(\frac{3L^2}{4}-x^2\right)$ $[0<x<L/2]$	$\frac{PL^3}{48EI}$
	$\frac{Pab}{L}$	$\theta_L = \frac{Pb(L^2-b^2)}{6EIL}$ $\theta_R = \frac{Pa(L^2-a^2)}{6EIL}$		
	$\frac{wL^2}{8}$	$\frac{wL^3}{24EI}$	$EIy = \frac{wx}{24}(L^3-2Lx^2+x^3)$	$\frac{5wL^4}{384EI}$
	$\frac{wL^2}{9\sqrt{3}}$	$\theta_L = \frac{7wL^3}{360EI}$ $\theta_R = \frac{8wL^3}{360EI}$	$EIy = \frac{wx}{360L}(7L^4-10L^2x^2+3x^4)$	$\frac{2.5wL^4}{384EI}$ at $x=0.519L$
	$\frac{wL^2}{12}$	$\theta = \frac{5wL^3}{192EI}$	$EIy = \frac{wx}{960L}(25L^4-40L^2x^2+16x^4)$ for $0<x<L/2$	$\frac{wL^4}{120EI}$
	M	$\theta_L = \frac{ML}{6EI}$ $\theta_R = \frac{ML}{3EI}$	$EIy = \frac{MLx}{6}\left(1-\frac{x^2}{L^2}\right)$	$\frac{ML^2}{9\sqrt{3}EI}$ at $x = L/\sqrt{3}$
	M	$\theta_L = ML/3EI$ $\theta_R = ML/6EI$	$EIy = \frac{Mx}{6L}(L-x)(2L-x)$	$\frac{ML^2}{9\sqrt{3}EI}$ at $(L-L/\sqrt{3})$

Now.



$\Sigma M_1 = 0$

$\Rightarrow 7.5 \times 30 \times 15 - 140.625 \times 15 - R_2 \times 30 = 0$

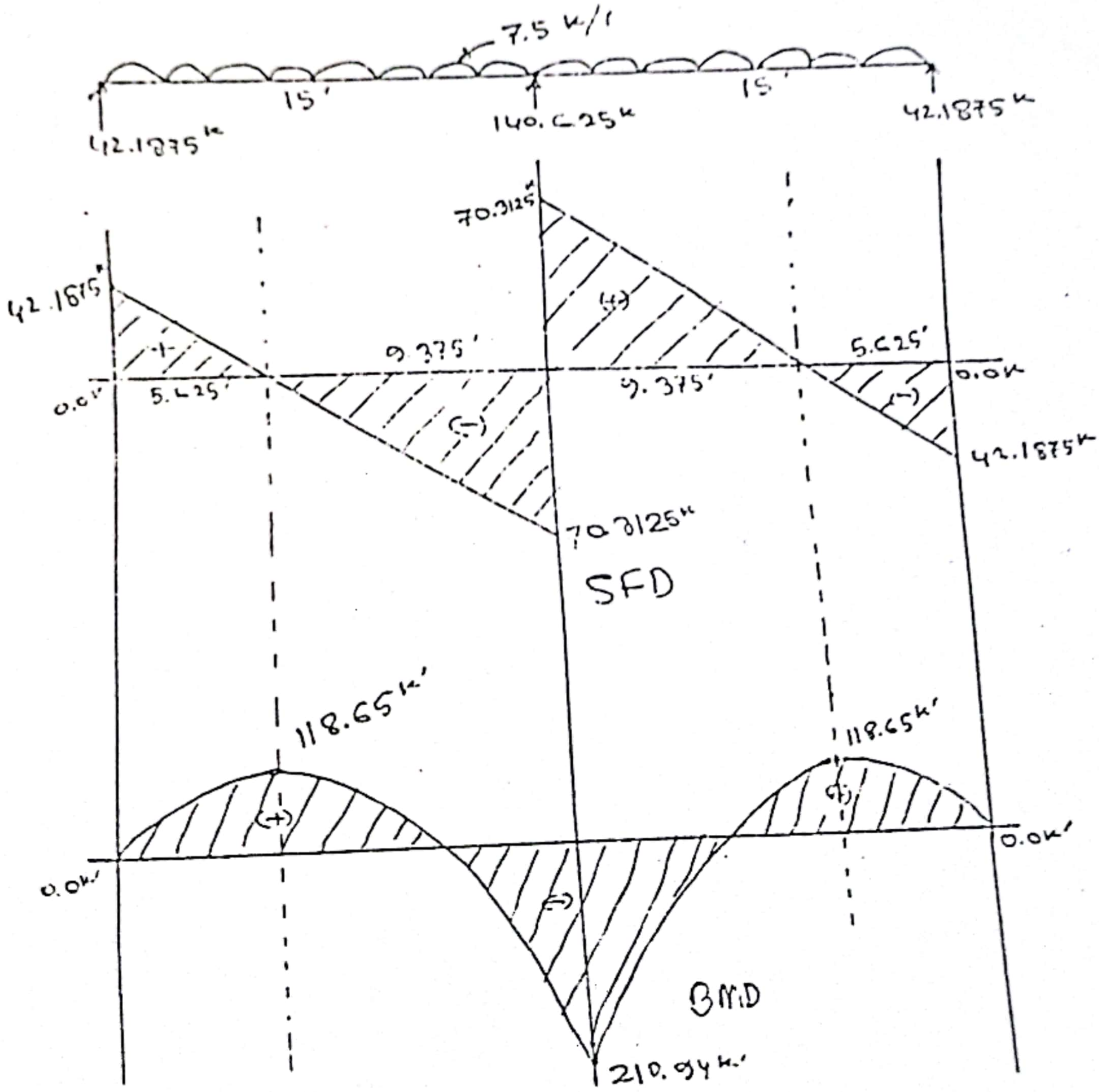
$\Rightarrow R_2 = 42.1875\,k.$

$\Sigma F_y = 0$

$\Rightarrow R_1 + R_2 + 140.625 - 30 \times 7.5 = 0$

$\Rightarrow R_1 = 30 \times 7.5 - 140.625 - 42.1875$

$= 42.1875^k.$



COLUMNS

columns :

If the member of the structure is vertical and both of its ends are fixed rigidly while subjected to axial compressive load, the member is known as column.

A compression member is generally considered as a column when its unsupported length is more than 10 times its least lateral dimension.

Example : A vertical pillar between the roof and floor.

Strut :

If the member of the structure is not vertical and one or both of its ends are hinged or pin joined then the bar is known as strut.

Example : connecting rods, piston rods etc.

Difference between short column and long column:

Short column	Long column
Fails by crushing	Fails by buckling.
Slenderness ratio less than 45	Slenderness ratio greater than 45.
Subjected to compressive stress	Subjected to buckling stress
Radius of gyration is more	Radius of gyration is less
More load capacity.	Less load capacity

Critical load: A critical load is defined as the maximum axial load to which a column can be subjected and still remain straight.

Critical stress: Critical stress is defined as the critical load per unit area. $\sigma_{cr} = \frac{P_{cr}}{A}$

Classification of column:

Depending on the slenderness ratio column can be classified as the following types:

1. Short column:

Those columns have slenderness ratio less than 32 are called short column.

Short column fails by crushing.

2. Intermediate column:

Those columns have slenderness ratio between 32 to 120 are known as intermediate column.

It fails by a combination of crushing and buckling

3. Long column:

Columns having slenderness ratio more than 120 are called long columns.

Long columns fail by buckling or excessive lateral bending...

* The length of short column less than or equal to 10 times the least lateral dimension.

Effective length of a column:

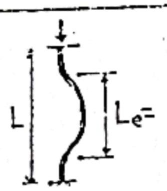
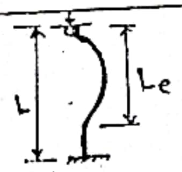
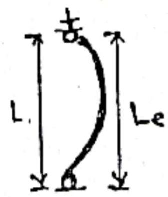
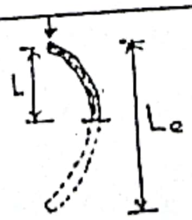
The effective or equivalent length is defined as the distance between two adjacent points of contraflexure on the column.

The shortest distance between the top and bottom most points of the column when it is bended is termed as its effective length.

This will be the length which will be under buckling action.

Effective length depends on the actual length & end condition of the column.

End condition and effective length:

End condition	column	n = No. of times strength of hinged column	Le = Effective length
Both ends fixed.		4	0.5L
One end fixed and other end hinged.		2	$L/\sqrt{2}$
Both ends hinged		1	L
One end free and other end fixed		1/4	2L

Euler's formula for long column :

A theoretical analysis of critical load for long columns was made by the greatest Swiss mathematician Leonhard Euler in 1757.

The illustration is as follows:

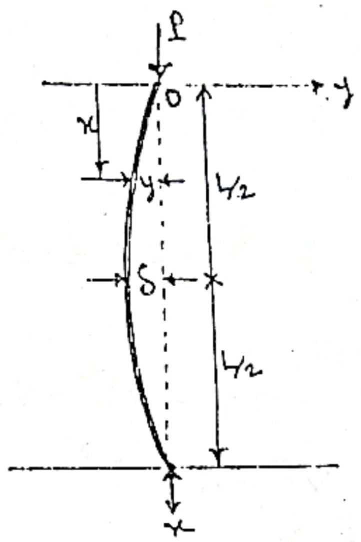


Figure shows the centerline of a column in equilibrium under the action of its critical load P.

The column is assumed to have hinged ends restained against lateral movement.

The maximum deflection δ is so small that there is no appreciable difference between the original length of the column and its projection on a vertical plane.

Under these conditions the slope $\frac{dy}{dx}$ is so small that we may apply the approximate differential equation of the elastic curve of a beam.

$EI \frac{d^2y}{dx^2} = M = P(-y) = -Py$

$\Rightarrow EI \frac{d}{dx}\left(\frac{dy}{dx}\right) = -Py$ .... (i)

Multiplying this by $2dy$ to obtain perfect differential we get by integration,

$$2EI \int \left[\frac{d}{dx}\left(\frac{dy}{dx}\right)\right] dy = -2P\int y\, dy$$

$\Rightarrow \cancel{2}EI \left(\frac{dy}{dx}\right)^2 = -Py^2 + C_1$ .... (ii)

According to figure:

At $y = \delta$, $\frac{dy}{dx} = 0$

From equation (ii) we get, $0 = -P\delta^2 + C_1$

$\Rightarrow C_1 = P\delta^2$

Now $\cancel{2}EI \left(\frac{dy}{dx}\right)^2 = -Py^2 + P\delta^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{P}{EI}(\delta^2 - y^2)$

$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{P}{EI}(\delta^2 - y^2)}$

$\Rightarrow \frac{dy}{\sqrt{\delta^2 - y^2}} = \sqrt{\frac{P}{EI}}\, dx$

By integration; $\int \frac{dy}{\sqrt{\delta^2 - y^2}} = \int \sqrt{\frac{P}{EI}}\, dx$

$\Rightarrow \sin^{-1}(y/\delta) = x\sqrt{\frac{P}{EI}} + C_2$

At $x = 0$, $y = 0$; Then $\sin^{-1}(0) = 0 + C_2$

$\Rightarrow C_2 = 0$

Hence $\sin^{-1}(y/\delta) = x \cdot \sqrt{\frac{P}{EI}}$

$\Rightarrow y = \delta \cdot \sin\left(x\sqrt{P/EI}\right)$

At $x = L$, $y = 0$; then,

$$0 = \delta \sin\left(L\sqrt{P/EI}\right)$$

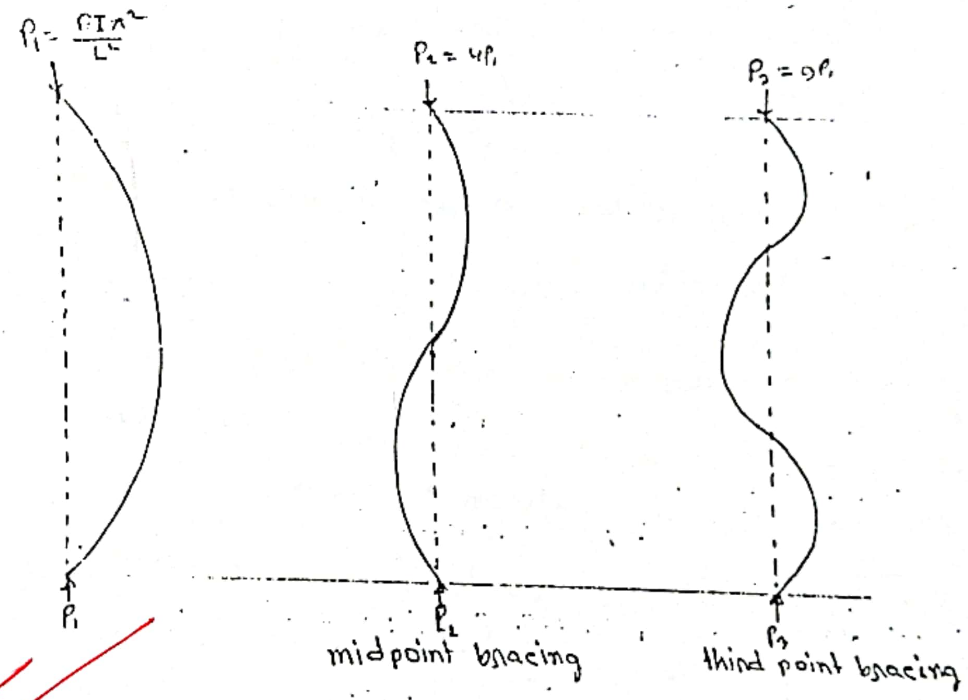
$$\Rightarrow \sin\left(L\sqrt{P/EI}\right) = 0$$

$$\Rightarrow L\sqrt{\frac{P}{EI}} = n\pi \quad [n = 0, 1, 2 \ldots]$$

$$\Rightarrow L^2 \cdot \frac{P}{EI} = n^2\pi^2$$

$$\Rightarrow \boxed{P = n^2 \frac{EI\pi^2}{L^2}}$$

The critical load for both ends hinged column is $P_{cr} = \frac{EI\pi^2}{L^2}$

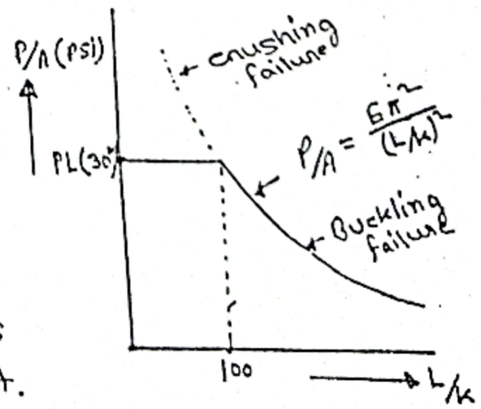


Limitations of Euler curve:

Slenderness ratio $= \frac{L}{k}$

and $k = \sqrt{I/A}$

1. For $\frac{L}{k} < 100$ Euler formula is not valid.
2. Limiting value of critical stress (P/A) is the stress at the proportional limit.



1. An aluminium strut 6′ long has rectangular section 3/4" by 2". A bolt through each end secures the strut so that it acts as a hinged column about an axis perpendicular to the 2" dimension and as a fixed ended column about an axis perpendicular to the 3/4" dimension. Determine the safe value of central load using a factor of safety 2 and $E = 10.3 \times 10^6$ psi.

Solution:

Given, $E = 10.3 \times 10^6$ psi.

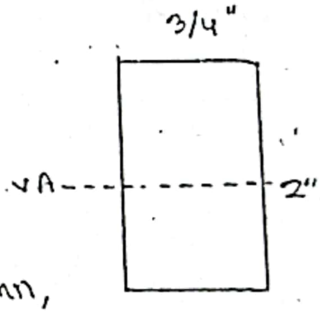
$L = 6' = 72$ in.

Factor of safety = 2.

1.

Moment of inertia, $I = \dfrac{3/4 \times 2^3}{12}$

$= 1/2 \text{ in}^4$.



Critical load for a hinged ended column,

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

$$= \frac{\pi^2 \times 10.3 \times 10^6 \times 1/2}{(72)^2}$$

$$= 9804.87 \text{ lb}.$$

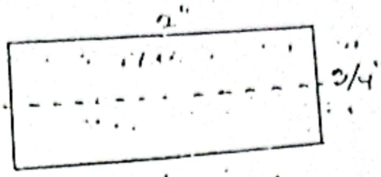
Safe load $= \dfrac{P_{cr}}{2}$

$= \dfrac{9804.87}{2}$

$= 4902.435$ lb. $\boxed{\text{Ans}}$

01. $I = \frac{2\times(3/4)^3}{12}$

$= \frac{9}{128}$ in^4



Critical load for a fixed ended column,

$$P_{cr} = \frac{\pi^2 EI}{(L/2)^2}$$

$$= \frac{4\pi^2 \times 10.3\times10^6 \times \frac{9}{128}}{72^2}$$

$= 5515.24$ lb.

Safe load, $P = \frac{5515.24}{2}$

$= 2757.62$ lb. $\boxed{\text{Ans}}$

02. A 40WF87 section is used as a column 40' long with built in ends.

a) Using AISC formula compute the safe load that can be applied if the effective length is 3/4 of the given length.

b) Compute the safe load if the column is also braced in its midpoint.

Solution:

From table-B to Appendix-3 for 40WF87 section:

$A = 25.56$ in^2

$K = 3.70$ in (least)

a) Effective length, $L_e = (3/4 \times 40) \times 12$

$= 360$ in.

slenderness ratio, $r = \frac{L_e}{k}$

$= \frac{360}{3.70}$

$= 97.297$

Using AISC formula;

$\frac{P}{A} = 17000 - 0.485 (L/k)^2$

$= 17000 - 0.485 \times 97.297^2$

$= 12408.65$ psi.

$\Rightarrow P = 12408.65 \times 25.56$

$= 317165.094$ lb $\boxed{\text{Ans}}$

b) If the column is braced in its midpoint,

Effective length $= \frac{40}{2}$

$= 20 \times 12$

$= 240$ in.

slenderness ratio, $r = L_e/k$

$= \frac{240}{3.70}$

$= 64.865$

Using AISC formula; $P/A = 17000 - 0.485 (L_e/k)^2$

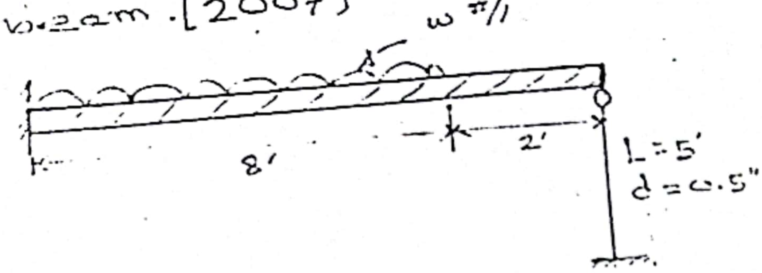
$= 17000 - 0.485 \times 64.865^2$

$= 14959.38$ psi.

$\Rightarrow P = 14959.38 \times 25.56$

$= 382361.75$ lb. $\boxed{\text{Ans}}$

04. A wooden propped cantilever beam carries a distributed load and is supported by a steel column as shown in figure below. Calculate the safe value of w that can be carried by the beam. [2007]



$E_w = 10\times10^6$ psi
$I = 5.0$ in^4.

Solution:

$$E_{steel} = 30\times10^6 \text{ psi}$$

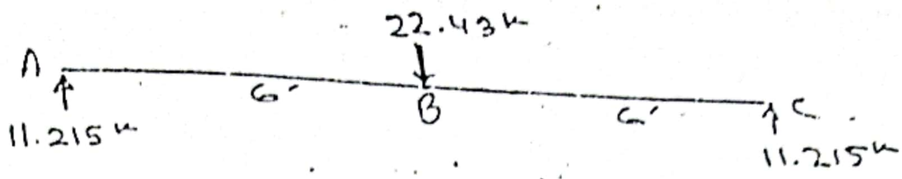
$$I_{steel} = \frac{\pi d^4}{64} = \frac{\pi\times(0.5)^4}{64} = \frac{\pi}{1024}\ \text{in}^4$$

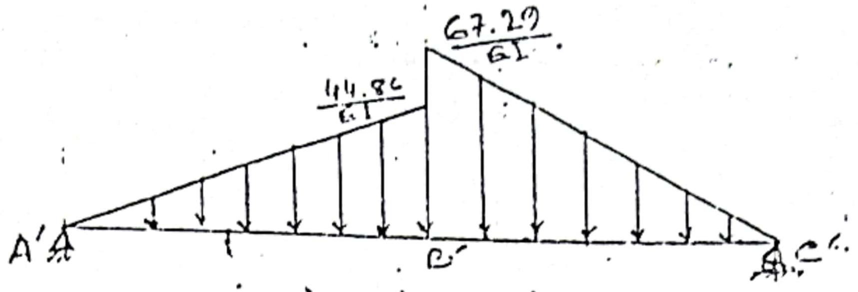
$$L_e = L/\sqrt{2} = 5/\sqrt{2}\ \text{ft} = \frac{60}{\sqrt{2}}\ \text{in}$$

For one end hinged and other end fixed column;

$$\text{critical load};\ P = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2\times30\times10^6\times\frac{\pi}{1024}}{\left(\frac{60}{\sqrt{2}}\right)^2}$$

$$= 504.66\ \text{lb.}$$





conjugate beam

$\Sigma M_{c'} = 0$

$\Rightarrow R_{A'} \times 12 - \frac{1}{2} \times 6 \times \frac{44.86}{EI} \times (6 + 6/3) - \frac{1}{2} \times 6 \times \frac{67.29}{EI} \times \frac{2}{3} \times 6 = 0$

$\Rightarrow R_{A'} = \frac{157.01}{EI}$

Deflection at B = moment at B'

$= R_{A'} \times 6 - \frac{1}{2} \times 6 \times \frac{44.86}{EI} \times 6/3$

$= \frac{157.01}{EI} \times 6 - \frac{269.16}{EI}$

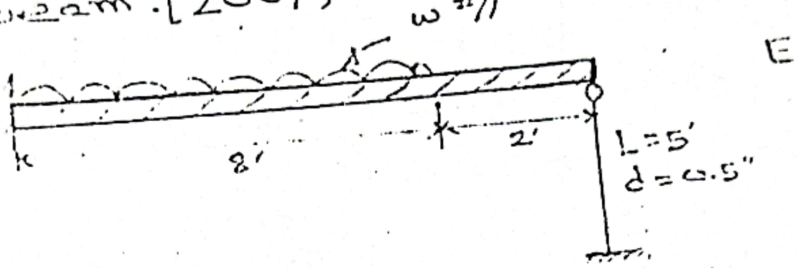
$= \frac{672.9}{EI}\ k\,ft^3$

$= \frac{672.9 \times 12^3}{30 \times 10^3 \times 100}$

$= 0.388$ in (↓)

Ans

04. A wooden propped cantilever beam carries a distributed load and is supported by a steel column as shown in figure below. Calculate the safe value of w that can be carried by the beam. [2007]



$E_w = 10 \times 10^6$ psi
$I = 5.0$ in^4.

Solution: $E_{steel} = 30 \times 10^6$ psi.

$$I_{steel} = \frac{\pi d^4}{64}$$

$$= \frac{\pi \times (0.5)^4}{64}$$

$$= \frac{\pi}{1024} \text{ in}^4.$$

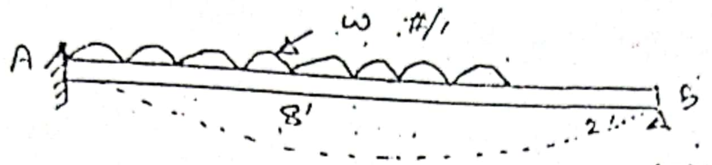
$$L_e = L/\sqrt{2} = 5/\sqrt{2} \text{ ft} = \frac{60}{\sqrt{2}} \text{ in}.$$

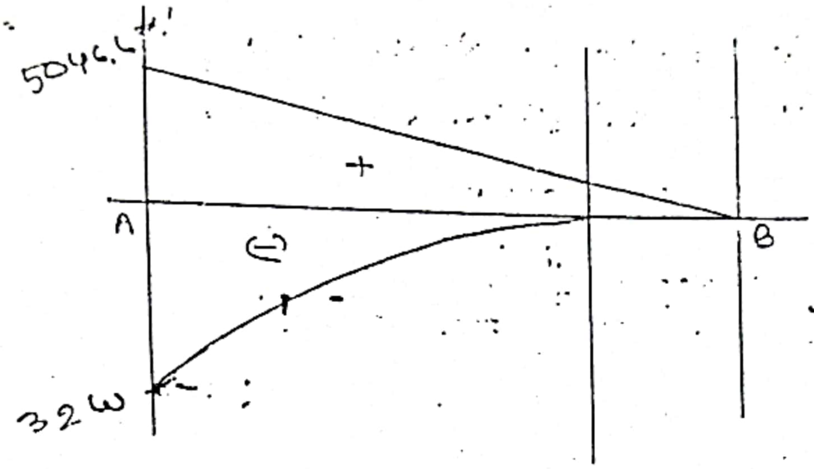
For one end hinged and other end fixed column;

$$\text{critical load}; P = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 \times 30 \times 10^6 \times \frac{\pi}{1024}}{\left(\frac{60}{\sqrt{2}}\right)^2}$$

$$= 504.66 \text{ lb}.$$





Here $t_{B/A} = 0$

$$\Rightarrow \frac{1}{EI}\left[(Area)_{BA} \cdot \bar{x}_B\right] = 0$$

$$\Rightarrow \left(1/2 \times 10 \times 5046.6 \times \frac{2}{3} \times 10\right) - \left\{\frac{8 \times 32w}{3} \times \left(10 - 8/4\right)\right\} = 0$$

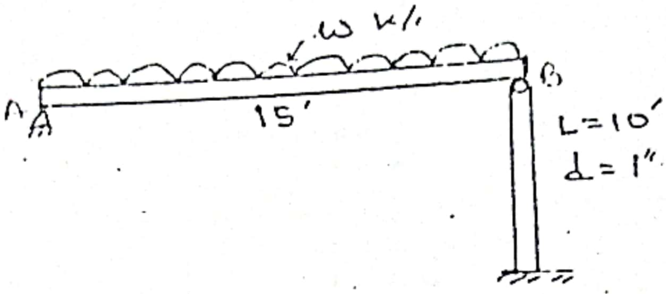
$$\Rightarrow 168220 = \frac{2048w}{3}$$

$$\Rightarrow w = 168220 \times \frac{3}{2048}$$

$$= 246.416 \text{ lb/ft.}$$

$\therefore$ Safe value, $w = 246.42$ lb/ft. $\boxed{\text{Ans}}$

06. A beam is supported by a steel column as shown figure below. Calculate the safe value of w carried by the beam. [2009]



Solution : For steel column:

$E_{steel} = 30\times10^3$ ksi.

$L = 10'$

$d = 1"$

$I = \frac{\pi d^4}{64}$

$= \pi/64 \text{ in}^4$.

$L_e = L/\sqrt{2} = 10\times12/\sqrt{2} = 60\sqrt{2}$ in.

For a one end hinged and other end fixed column;

Critical load, $P = \frac{\pi^2 EI}{L_e^2}$

$= \frac{\pi^2 \times 30\times10^3 \times \pi/64}{(60\sqrt{2})^2}$

$= 2.02$ K.

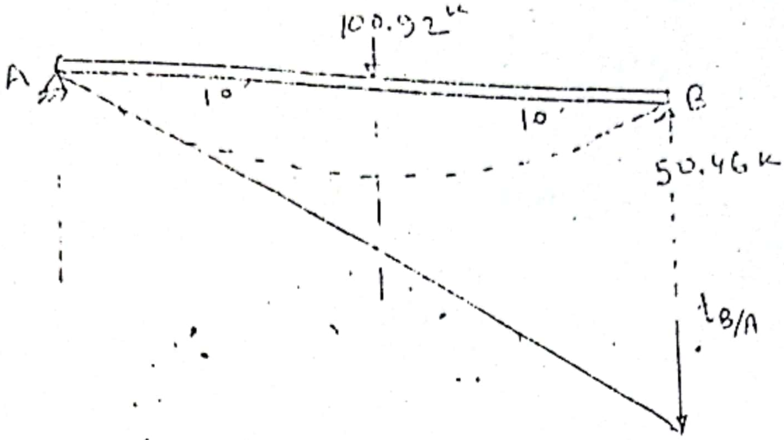
$\Sigma M_A = 0$

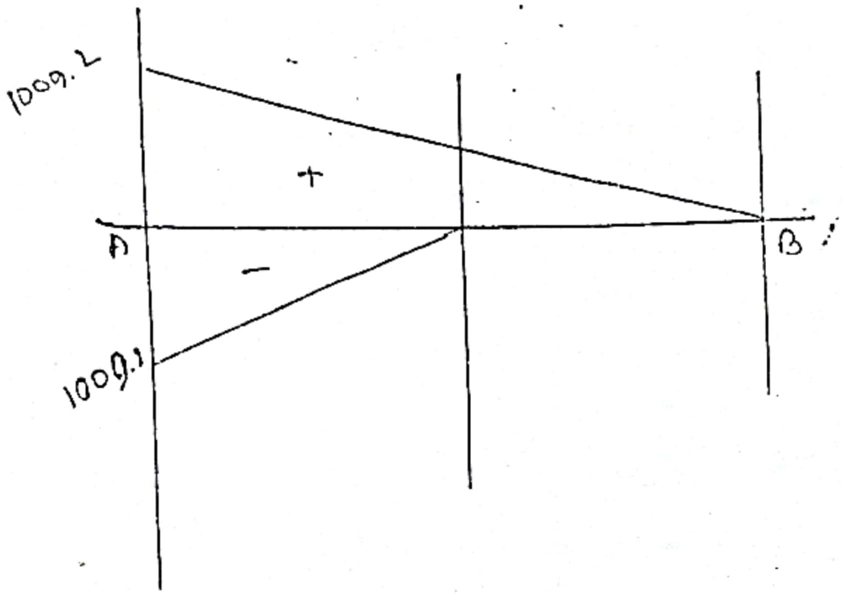
$\Rightarrow w.15\times15/2 - 2.02\times15 = 0$

$\Rightarrow w = 0.26933$ k/f

$\therefore w = 269.33$ lb/ft

$\boxed{\text{Ans}}$





$$t_{B/A} = \frac{1}{EI}\,(\text{Area})_{AB}\cdot \bar{x}_B$$

$$= \frac{1}{EI}\left[\left\{\tfrac{1}{2}\times 20\times 1009.2\times \tfrac{2}{3}\times 20\right\} - \left\{\tfrac{1}{2}\times 10\times 1009.2\times \left(10+\tfrac{2}{3}\times 10\right)\right\}\right]$$

$$= \frac{1}{EI}\left(134560 - 84100\right)$$

$$= \frac{50460\times 12^3}{30\times 10^3\times 50}$$

$$= 58.13 \text{ in.}$$

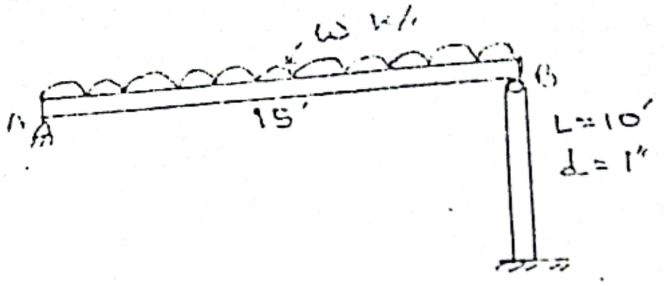
Rotational deflection at A, $\theta_A = \frac{t_{B/A}}{AB}$

$$= \frac{58.13}{20\times 12}$$

$$= 0.242 \text{ rad.}$$

$\boxed{\text{Ans}}$

06. A beam is supported by a steel column as shown figure below. Calculate the safe value of w carried by the beam. [2009]



Solution: For steel column:

$E_{steel} = 30\times10^3$ ksi.

$L = 10'$

$d = 1"$

$I = \frac{\pi d^4}{64}$

$= \pi/64$ in^4.

$L_e = L/\sqrt{2} = 10\times12/\sqrt{2} = 60\sqrt{2}$ in.

For a one end hinged and other end fixed column;

Critical load, $P = \frac{\pi^2 EI}{L_e^2}$

$= \frac{\pi^2 \times 30\times10^3 \times \pi/64}{(60\sqrt{2})^2}$

$= 2.02$ K.

$\Sigma M_A = 0$

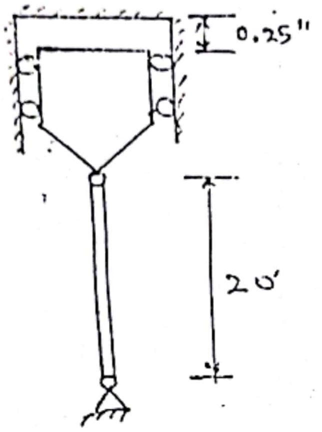
$\Rightarrow w.15\times15/2 - 2.02\times15 = 0$

$\Rightarrow w = 0.26933$ k/f

$\therefore w = 269.33$ lb/ft

$\boxed{\text{Ans}}$

. The column is shown in figure below is pinned at both ends and is free to expand into the opening at the upper end. The bar is steel, 1" in diameter and occupies the position shown at 150°F. Determine the highest temperature to which the column may be heated before it will buckle, $\alpha = 6.5\times10^{-6}/°F$ and $E = 30\times10^6$ psi. [2011]



Solution: for column: $E = 30\times10^6$ psi.

$$I = \frac{\pi d^4}{64} = \frac{\pi\times1^4}{64} = \pi/64 \text{ in}^4$$

$$A = \frac{\pi d^2}{4} = \frac{\pi\times1^2}{4} = \pi/4 \text{ in}^2$$

$$L_e = L = 20' = 20\times12 = 240 \text{ in}.$$

critical load, $P_{cr} = \dfrac{\pi^2 EI}{L_e^2} = \dfrac{\pi^2\times30\times10^6\times\pi/64}{240^2} = 252.33 \text{ lb}.$

Now $\alpha L\Delta t = \dfrac{P_{cr}L}{AE}$

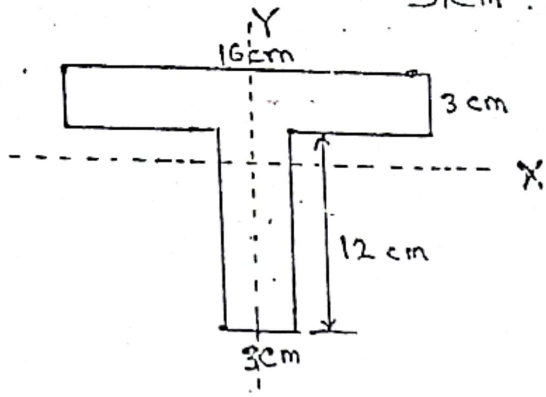
$$\Rightarrow \Delta t = \frac{252.33}{\pi/4\times30\times10^6\times6.5\times10^{-6}}$$

$$\Rightarrow \Delta t = 1.65\ °F$$

$$\Rightarrow T - 150 = 1.65$$

$$\Rightarrow T = 151.65\ °F. \quad \boxed{\text{Ans}}$$

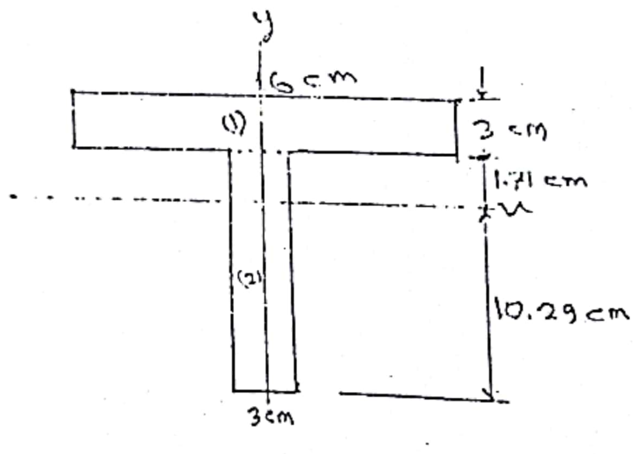
Q. A T section 16 cm × 12 cm × 3 cm is used as a strut of 5 m long with hinged at its both ends. Calculate the crippling load if Young's modulus for the material is 2×10^6 kg/cm². [2015, 2016, 2017]



Solution:

Location of centroid, $\bar{y} = \dfrac{3\times12\times{}^{12}/_2 + 16\times3\times(12+{}^{3}/_2)}{3\times12+16\times3}$

$= 10.29$ cm.



$I_y = I_{y_1} + I_{y_2}$

$= \dfrac{3\times16^3}{12} + \dfrac{12\times3^3}{12}$

$= 1051$ cm⁴.

$I_x = I_{x_1} + I_{x_2}$

$= \left\{\dfrac{16\times3^3}{12} + (16\times3)\times({}^{3}/_2+1.71)^2\right\} + \left\{\dfrac{3\times12^3}{12} + (3\times12)(10.29 - {}^{12}/_2)^2\right\}$

$= 530.6 + 1094.55$

$= 1625.15$ cm⁴.

For both ends hinged, $L_e = L$

$= 5 \text{ m}$

$= 500 \text{ cm}$

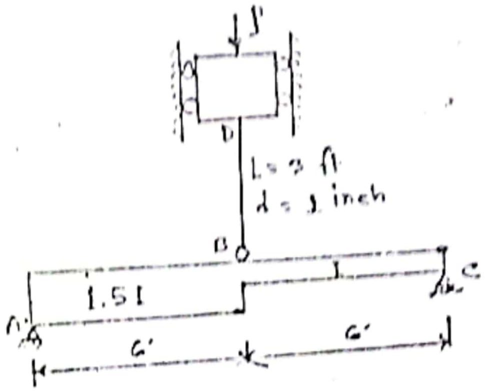
Crippling load, $P = \frac{\pi^2 E I_y}{L_e^2}$ $[\because I_y < I_x]$

$= \frac{\pi^2 \times 2\times10^6 \times 1051}{500^2}$

$= 82983.63 \text{ kg}$

Ans

03. A steel column BD supported by a non-prismatic steel beam ABC as shown in figure below. A concentrated load P is applied at the upper end of the column as shown. Calculate the maximum value of P and deflection at point B. I = 100 in⁴. [2005, 2010]



Solution : $E_{steel} = 30 \times 10^6$ psi.

Length of column, $L = 3$ ft
$= 36$ in.

diameter of column, $d = 1$ in.

moment of inertia, $I = \frac{\pi r^4}{4}$

$= \frac{\pi \times (d/2)^4}{4}$

$= \frac{\pi \times (1/2)^4}{4}$

$= \pi/64$ in⁴.

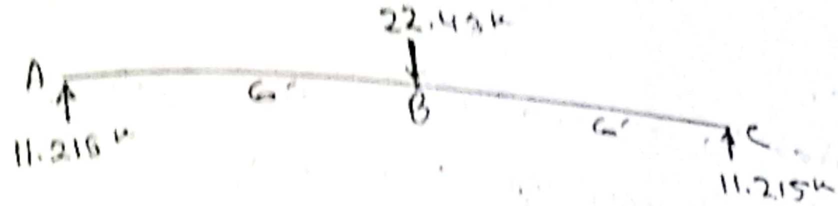
For one end hinged and other end fixed column ;

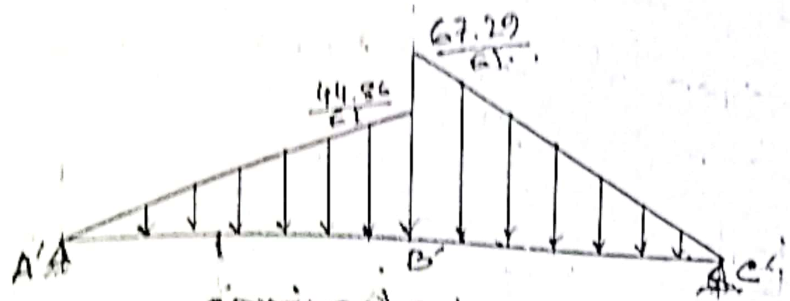
The critical load, $P_{cr} = \frac{\pi^2 EI}{(L/\sqrt{2})^2}$

$= \frac{2\pi^2 \times 30 \times 10^6 \times \pi/64}{(36)^2}$

$= 22429.31$ lb.

$= 22.43$ k. $\boxed{\text{Ans}}$





conjugate beam

$\Sigma M_{c'} = 0$

$\Rightarrow R_{A'} \times 12 - \frac{1}{2} \times 6 \times \frac{44.86}{EI} \times (6 + 6/3) - \frac{1}{2} \times 6 \times \frac{67.29}{EI} \times \frac{2}{3} \times 6 = 0$

$\Rightarrow R_{A'} = \frac{157.01}{EI}$

Deflection at B = moment at B'

$= R_{A'} \times 6 - \frac{1}{2} \times 6 \times \frac{44.86}{EI} \times 6/3$

$= \frac{157.01}{EI} \times 6 - \frac{269.16}{EI}$

$= \frac{672.9}{EI}$ k-ft^3

$= \frac{672.9 \times 12^3}{30 \times 10^3 \times 100}$

$= 0.388$ in (↓)

$\boxed{\text{Ans}}$