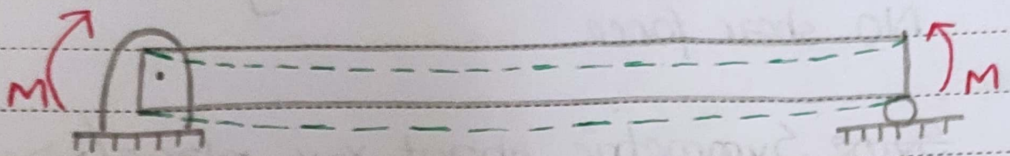
Beam Bending

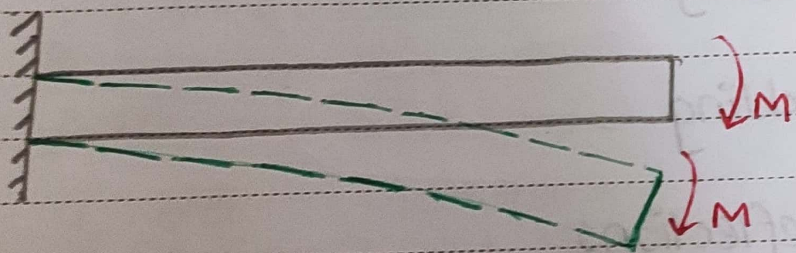
Beam ≡ Member loaded perpendicular to its longitudinal axis

Simple supported beams – pins/rollers at ends



pure bending, flexure under constant B.M., No shear force

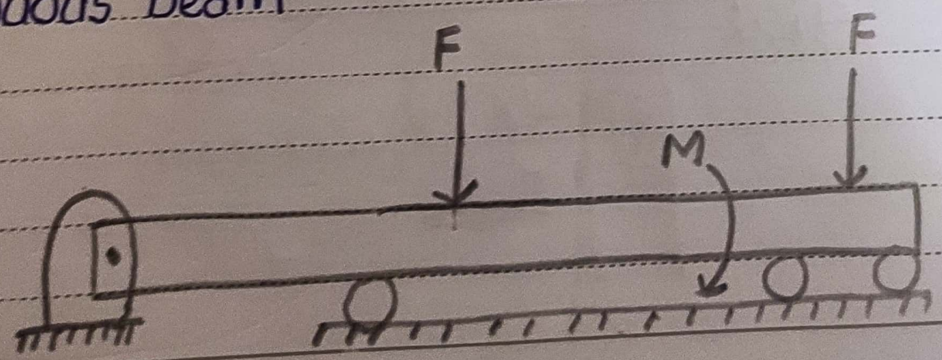
Cantilever Beam – One fixed end.



pure bending, no shear force.

Non Uniform beam bending

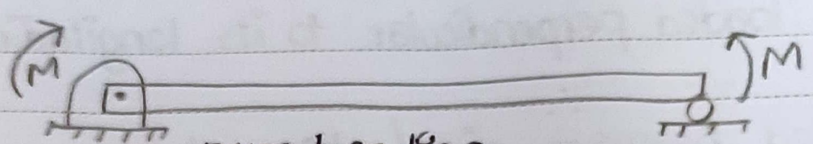
Continuous beam



Note's

Beam bending / flexure with shear force

Derivation of strain curvature relationship for beam bending



pure bending,
Flexure under constant bending moment,
No shear force

Assumptions:

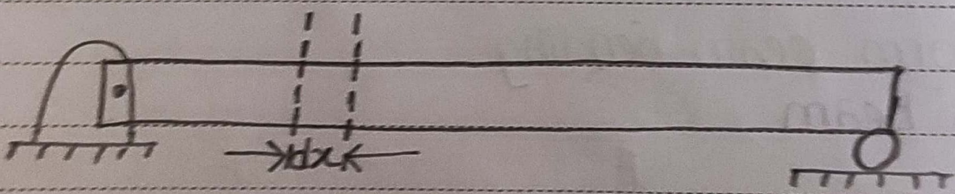
~~Syme~~ Symmetric about x-y plane (plane of bending)

Plane sections remain plane

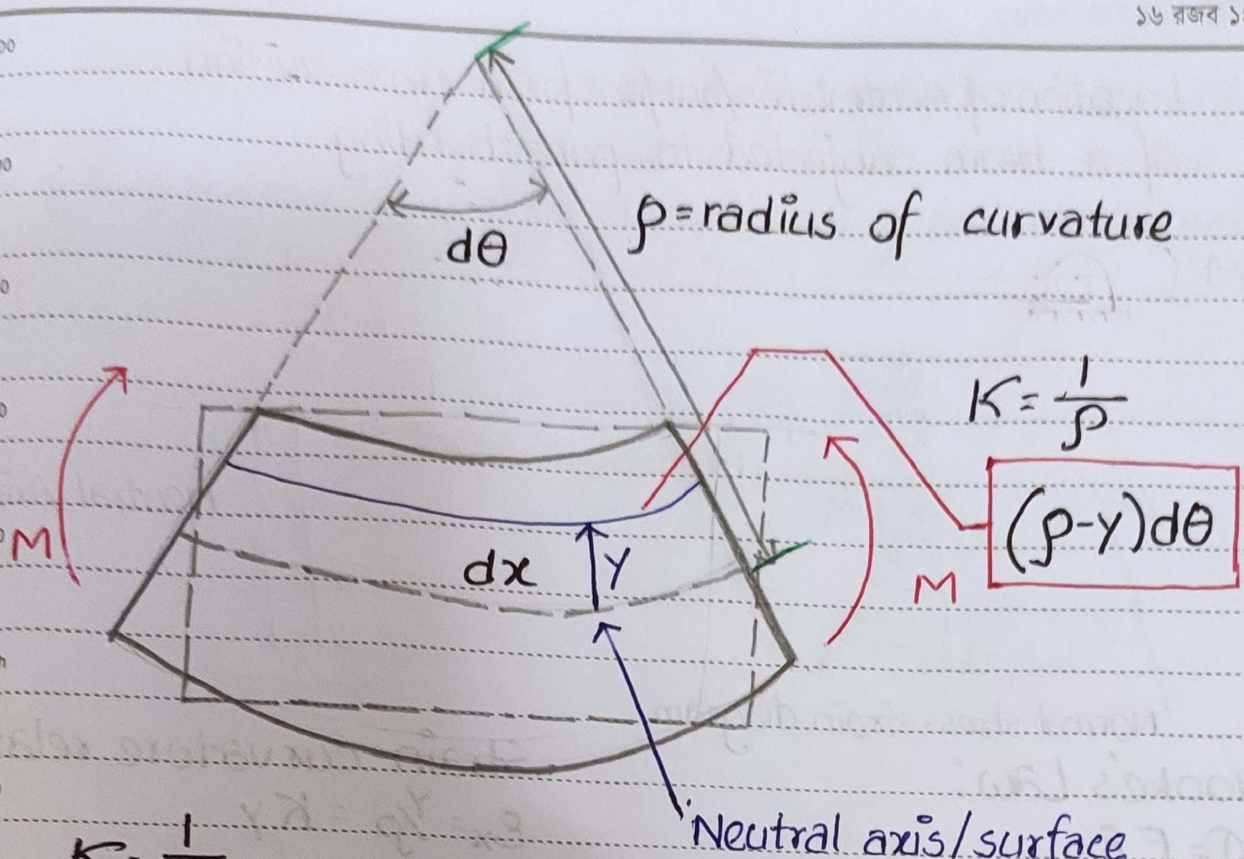
No twisting

No buckling

Small deflections



Note's



$$K = \frac{1}{\rho}$$

$$dx = \rho\, d\theta$$

$$\Rightarrow K = \frac{1}{\rho} = \frac{d\theta}{dx}$$

$$(\rho - y)d\theta = dx - \frac{y}{\rho}dx.$$

~~Stra~~ Strain curvature Relationship

$$\varepsilon_x = \frac{-\frac{y}{\rho}dx}{dx} = -\frac{y}{\rho} = -Ky$$

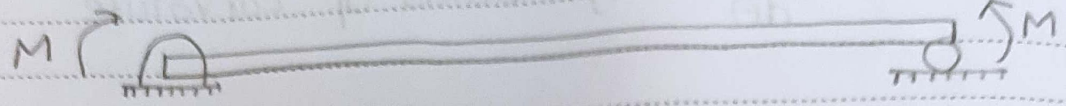
Strain is proportional to curvature and varies linearly with y from neutral axis.

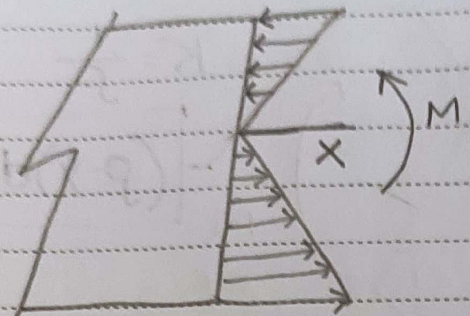
Elongation(+) shortening(-)

Note's

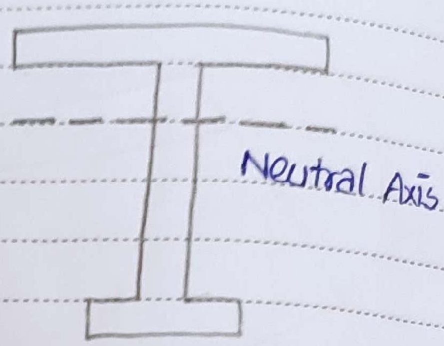
Independent of material

Location of neutral axis/surface for a cross section of a beam subjected to pure bending





Normal stress strain diagram



Hooke's Law:

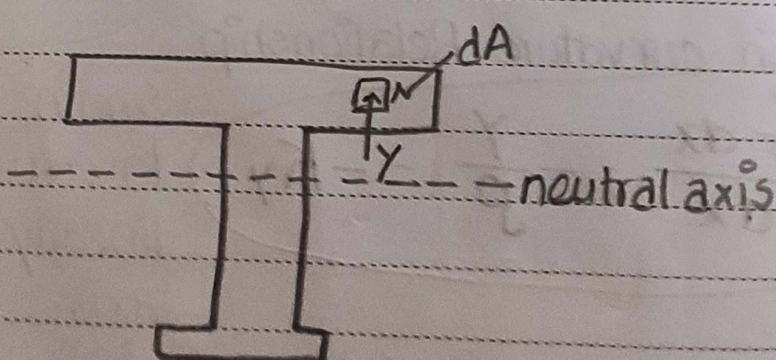
$\sigma = E\epsilon$

Strain curvature relation

$\epsilon_x = y/\rho = Ky$

$$\sigma_x = -\frac{Ey}{\rho} = -EKy$$

Note: For linear elastic material, stress is proportional to curvature and varies linearly with distance y from the neutral axis



08.00 $\Sigma F_x = 0$

09.00 $\int_A \sigma_x \, dA = 0$

Note: The neutral axis coincides with the centroidal axis of the cross section (for flexural Loading and elastic action)

10.00 $\Rightarrow -\int_A E K y \, dA = 0$

11.00 $\Rightarrow \boxed{\int_A y \, dA = 0} \rightarrow$ 1st moment of area w.r.to z axis

12.00

Lunch

02.00

03.00

04.00

05.00

06.00

07.00

08.00

Note's

08.00 Moment Curvature Relationship

09.00

10.00 $\circlearrowleft^{+} M_z = 0$

~~dm + σxydA~~ $dm + \sigma_x y dA = 0$

11.00

$\Rightarrow dM = -\sigma_x y dA$.

12.00

$\Rightarrow M = -\int_A \sigma_x y dA$.

Lunch

$\Rightarrow M = -\int_A -E K y^2 dA$.

02.00

$\Rightarrow M = E K_0 \underbrace{\int_A y^2 dA}$

03.00

$\rightarrow$ Area moment of inertia (2nd moment)

04.00 $\Rightarrow M = EKI$

05.00 So, $K = \frac{1}{\rho} = \frac{M}{EI}$ $\rightarrow$ Curvature is proportional to moment

06.00

07.00 EI = Flexural Rigidity [Resistance of the beam to bending for a given curvature]

08.00

Note's

08.00 Elastic Flexural Formula

09.00 $\sigma_x = -\frac{Ey}{\rho} = -EKy$ $K = \frac{1}{\rho} = \frac{M}{EI}$.

10.00 $\sigma_x = -\frac{My}{I}$ → Flexural formula

11.00 Notes: Stress is directly proportional to B.M., M

12.00 Stress is inversely proportional to the area moment of inertia, I

Lunch

Stress varies linearly with the distance from the neutral axis, y.

02.00

Flexural formula is analogous to: $\tau = \frac{T\rho}{J}$

03.00

Maximum stress:

04.00

$\sigma_{max} = \frac{Mc}{I}$ → c is the furthest distance on the cross section from the neutral axis

05.00

06.00

07.00

08.00

Note's

08.00 Area moment of inertia

09.00 $I = I_{\text{standard shape}} + Ad^2$

10.00

11.00 Section Modulus, S

12.00 $\sigma_{max} = \frac{Mc}{I}$

Lunch $S = \frac{I}{c}$

02.00 $S \geq \frac{M_{max}}{\sigma_{actual\,(allowed)}}$

03.00

Why is an I beam used in construction?

04.00

The more area further from the neutral axis provides

05.00

greater resistance to bending.

06.00

This is why I beams are used in construction.

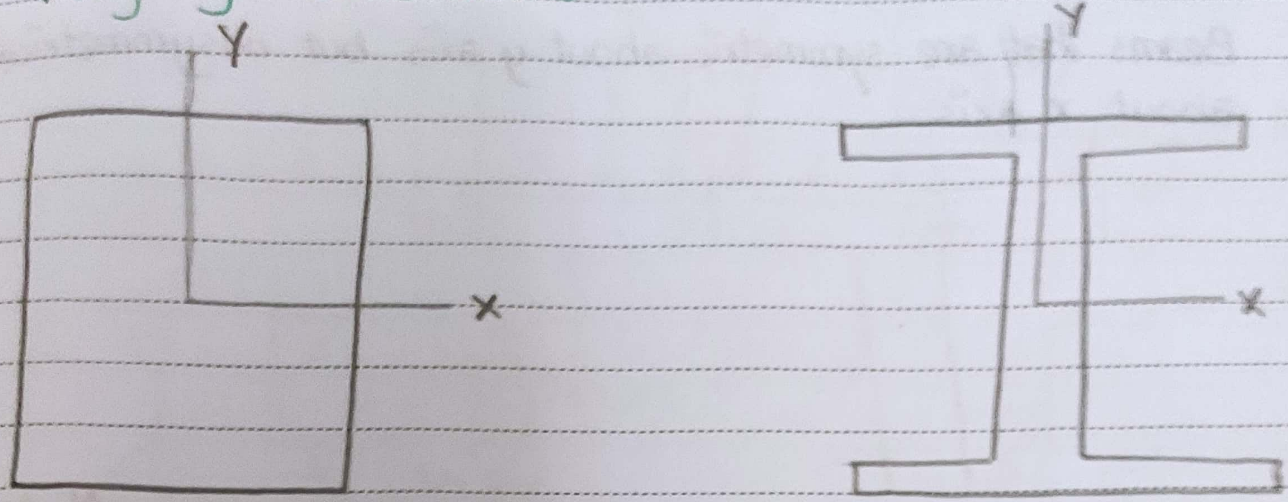
07.00

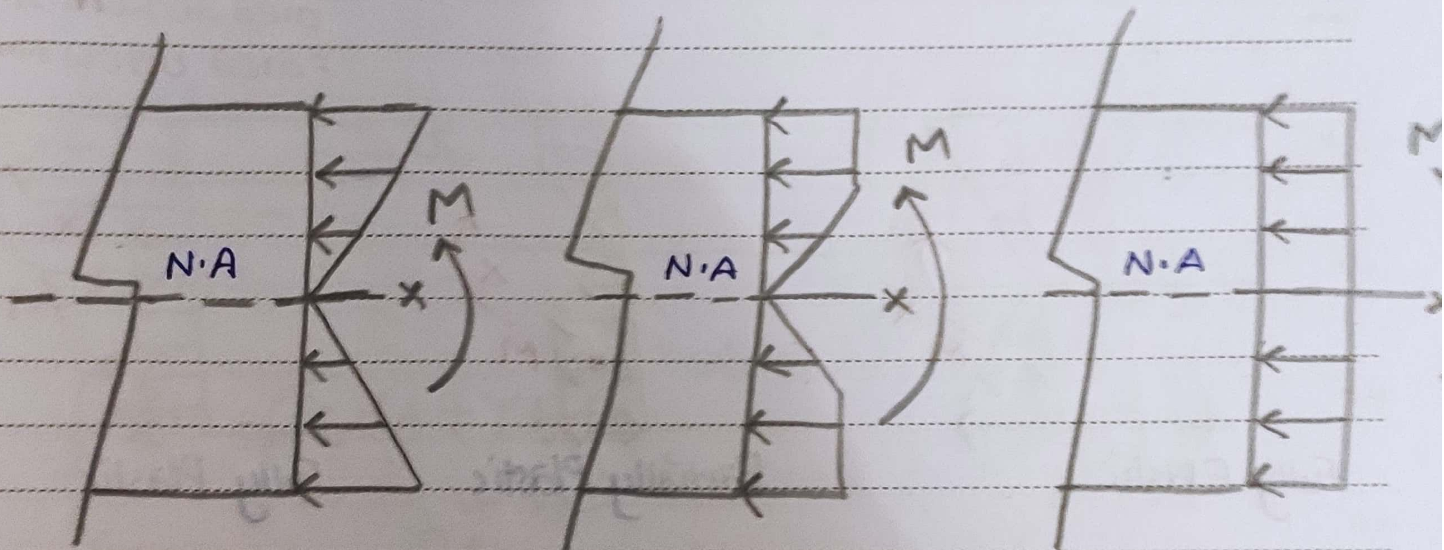
08.00

Note's

Inelastic bending for symmetric cross section

Fully Symmetric Beams





Fully Elastic — Partially Plastic — Fully Plastic

Assumptions:

plane section remains plane

No twisting, no buckling

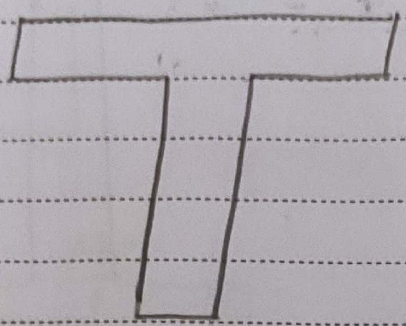
Small Deflections

Note's

08.00 Inelastic bending (unsymmetrical cross sections)

09.00 Beams that are symmetric about y axis but unsymmetrical about x axis

10.00



11.00

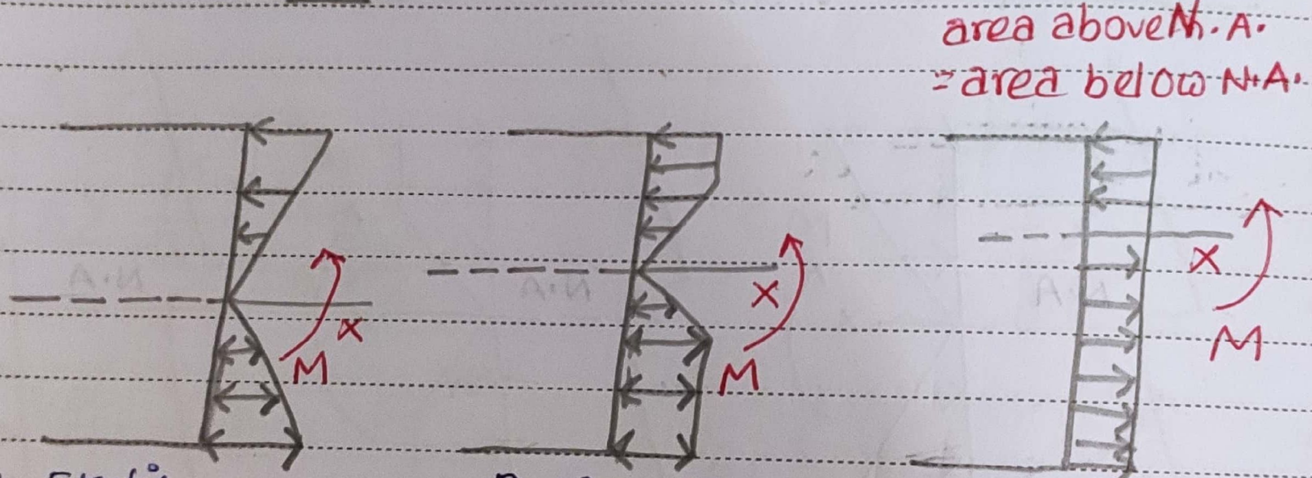
12.00

Lunch

02.00

03.00

04.00



Fully Elastic   Partially Plastic   Fully Plastic

05.00

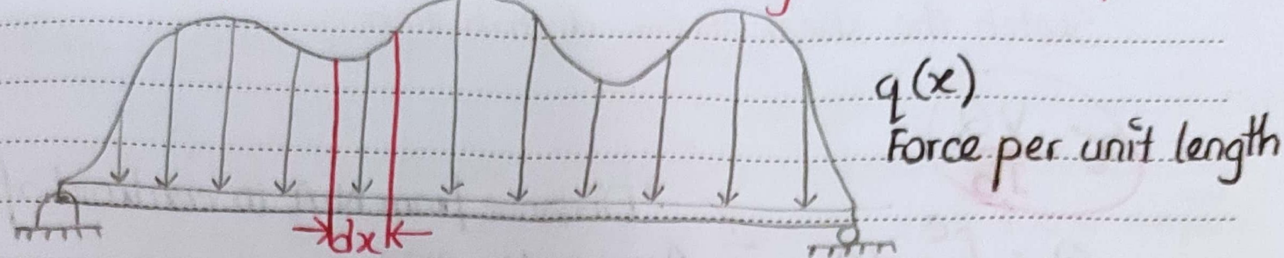
06.00 Neutral axis shifs away from the fibres that first experience inelastic action.

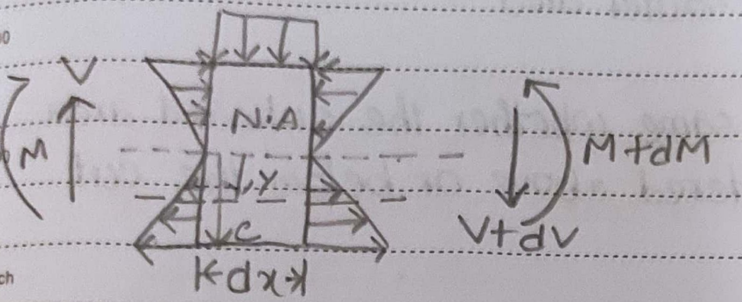
07.00 Stress strain diagram may differ in the inelastic region for

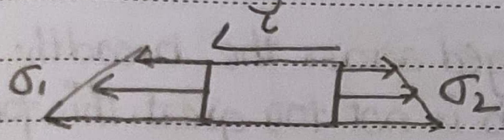
08.00 tension & compression. But these differences maybe reasonably

Note's neglected for real world problems.

Expression for Shear stress in beams subjected to non uniform Bending







$$\Sigma F_x = 0$$

$$\int_y^c \sigma_2 \, dA = \int_y^c \sigma_1 \, dA + \tau \overbrace{b}^{\text{width}} dx$$

$$\Rightarrow \int_y^c \frac{(M+dM)y}{I} dA = \int_y^c \frac{My}{I} dA + \tau b dx.$$

$$\Rightarrow \frac{M}{I} \cancel{\int_y^c y\,dA} + \frac{dM}{I} \underbrace{\int_y^c y\,dA}_{Q} = \frac{M}{I} \cancel{\int_y^c y\,dA} + \tau b dx.$$

Q = first moment of outward area

Note's $\Rightarrow \boxed{\frac{dM}{I} Q = \tau b dx}$ $\Rightarrow \tau = \frac{dM}{dx} \frac{Q}{Ib}$ $\frac{dM}{dx} = V$

$$\boxed{\tau = \frac{VQ}{Ib}}$$

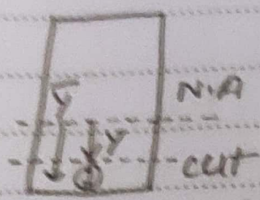
First moment of outward area
Sketch the shear stress distribution

$$\tau = \frac{VQ}{Ib}$$

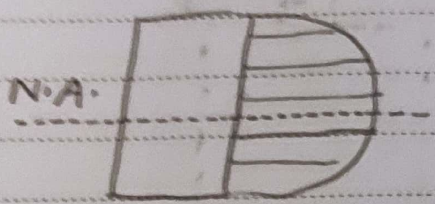
$\bar{y}$ → Distance from N·A to centroid of outer area

$$Q = \int_{y}^{c} y\,dA = \bar{y}A$$

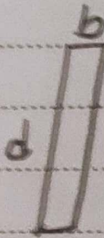
A → outer area



Q is the same whether the outward area is considered above or below the cut



Stress is averaged across the breadth. Hence, if width b is not too great, this formula is accurate



$d/b > 10$ excellent

$d/b \approx 2$ <3% error

$d/b \approx 1$ 12-13% error

Formula can't be used for flanges of I-beams/T-beams
But, in general, the maximum shear stress occurs at the neutral surface

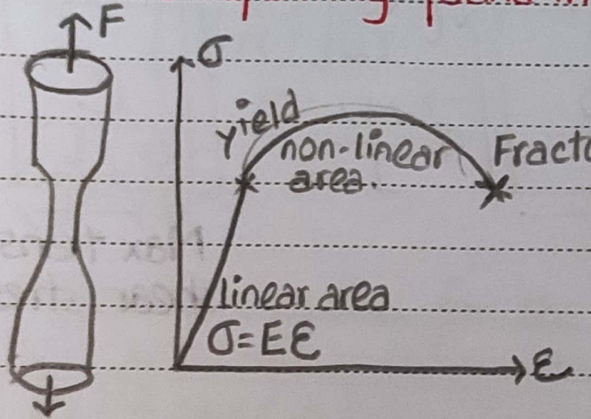
Assumptions:
Linear Elastic material
Prismatic Beams (no Taper)
Small deflection (
~~Plastic Beams (No taper)~~
Cross section edges must be parallel to y axis (not good for triangles, circles & semi circles)
Uniform shear stress across the width of the cross section

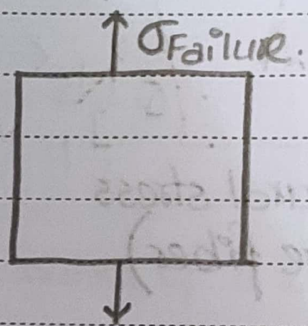
Max Shear stress failure (Tresca's yield criterion)

Failure occurs when: $\tau > \tau_{Failure}$

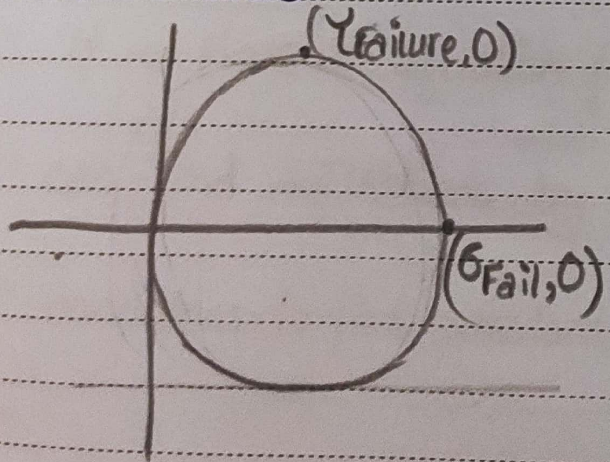
This theory is good for ductile materials (steel, aluminium, plastic)
Because yield in ductile materials is usually caused by the slippage of crystal planes along the max shear stress surface.

We will assume that in a complex loading condition (non-uniform beam bending with flexure & shear), the material has the same capability found in simple torsion test (STT)





Mohr's circle:



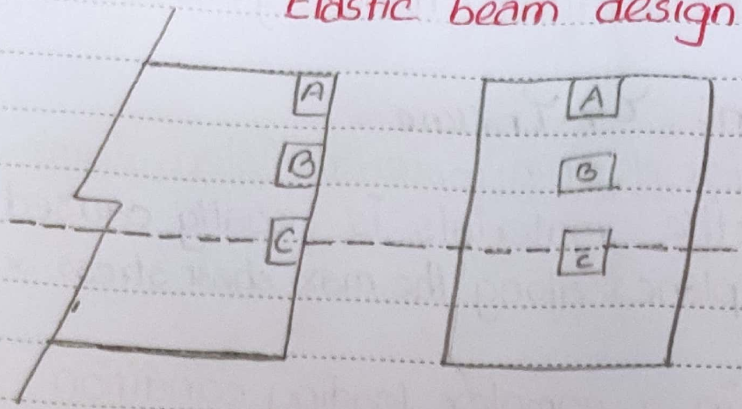
$$\tau_{Failure} = \tau_{yield} = \frac{\sigma_{yield}}{2}$$

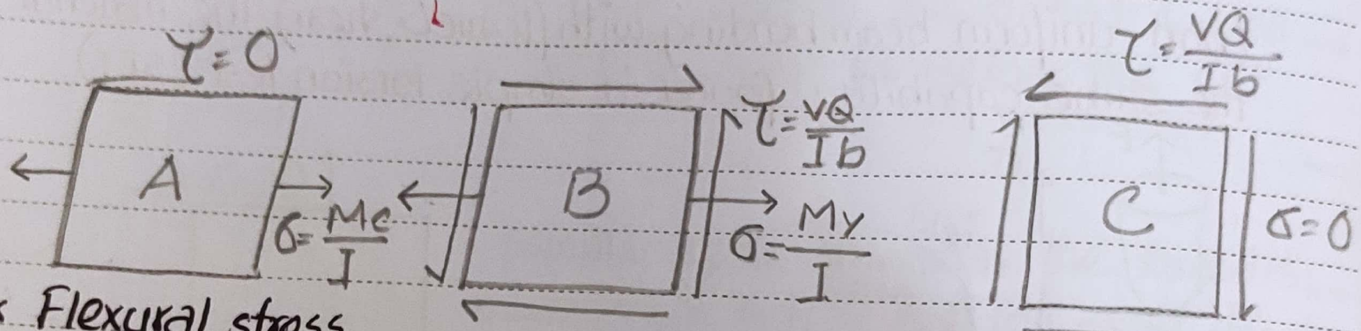
$$\text{Factor of safety} = FoS = \frac{\tau_{yield}}{\tau_{allowed}}$$

FoS > 1 ⟶ Avoid failure.

Note's

Elastic beam design.





Max Flexural stress (At extreme fiber)

Max transverse shear stress

08.00 Differential equation for the elastic curve of a beam

09.00 Moment curvature Relationship

$$\boxed{K = \frac{1}{\rho} = \frac{M}{EI}}$$

10.00 → Flexural Rigidity
curvature is proportional to moment

11.00 Curvature Equation

12.00
$$\frac{1}{\rho} = \frac{\left(d^2y/dx^2\right)}{\left(1+\cancel{\left(\frac{dy}{dx}\right)^2}\right)^{3/2}} \quad \rightarrow 0$$

assume small deformations
square of $\frac{dy}{dx} << 1$

Lunch

02.00
$$\Rightarrow \frac{1}{\rho} = \frac{d^2y}{dx^2}$$

03.00 Differential equation for the elastic curve of a beam

04.00
$$\boxed{EI\frac{d^2y}{dx^2} = M(x)}$$

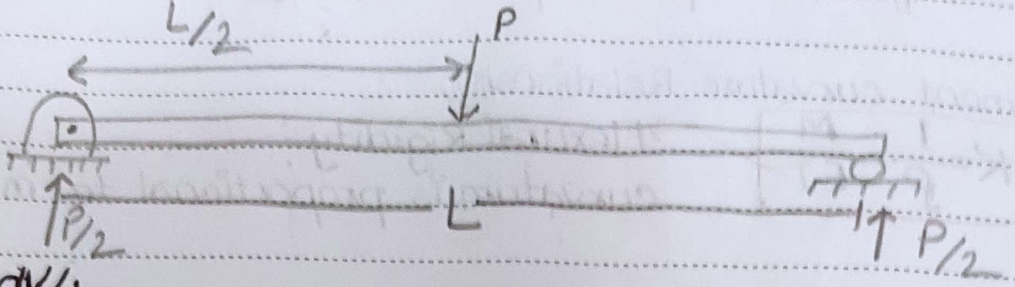
05.00 If we have an equation for the moment along a beam, we
06.00 can find deflections by integrating twice & using boundary
07.00 conditions to find constants of integration

08.00

Note's

08.00

09.00

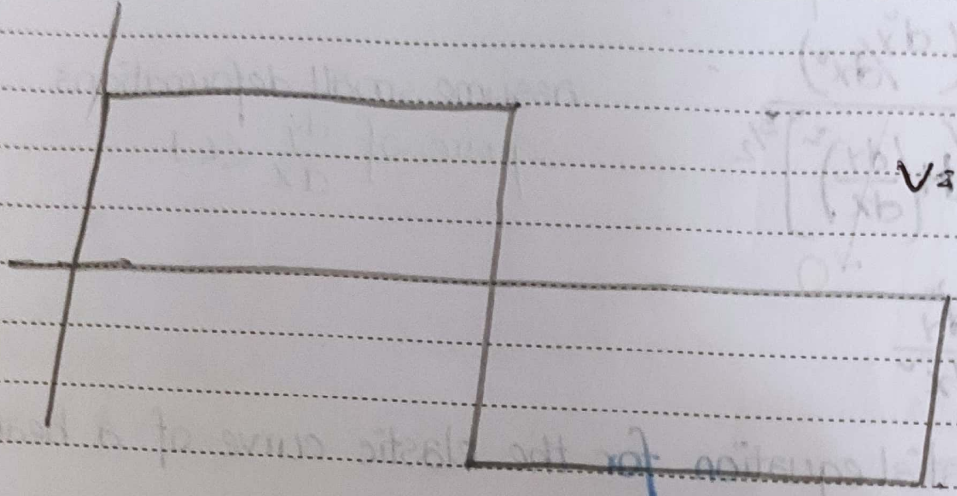


10.00

11.00

$$-q = dV/dx$$

12.00



Lunch

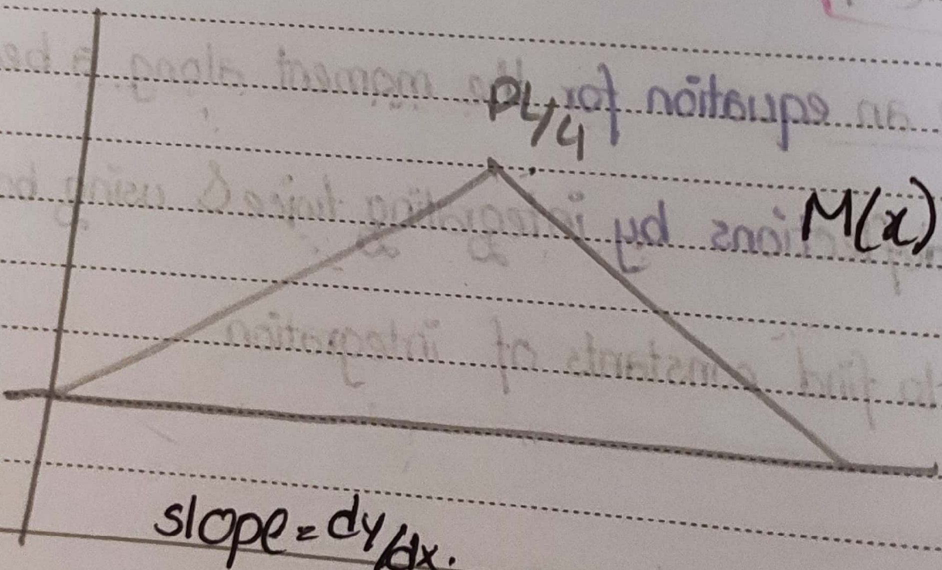
$$V = \frac{dM}{dx} = EI\frac{d^3y}{dx^3}$$

02.00

03.00

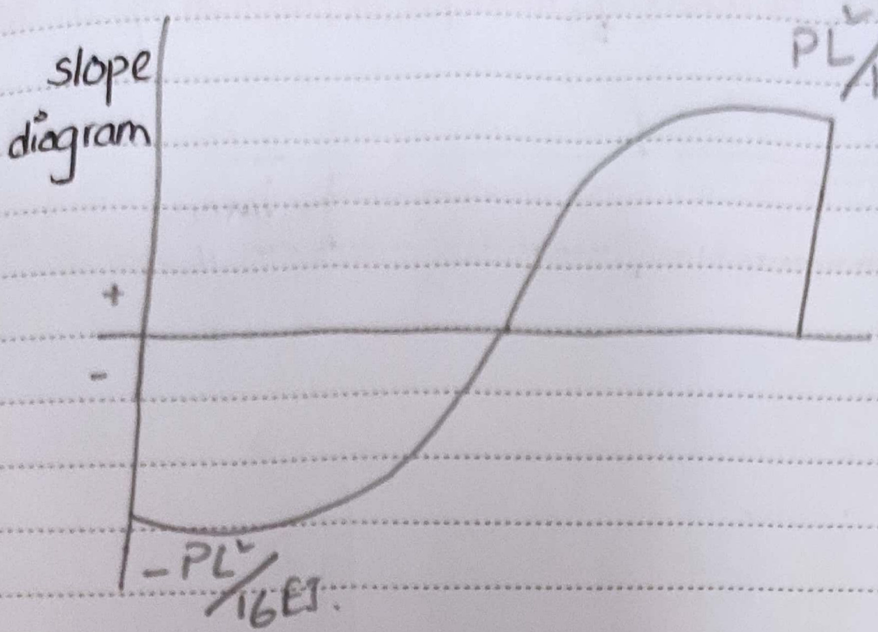
04.00

00



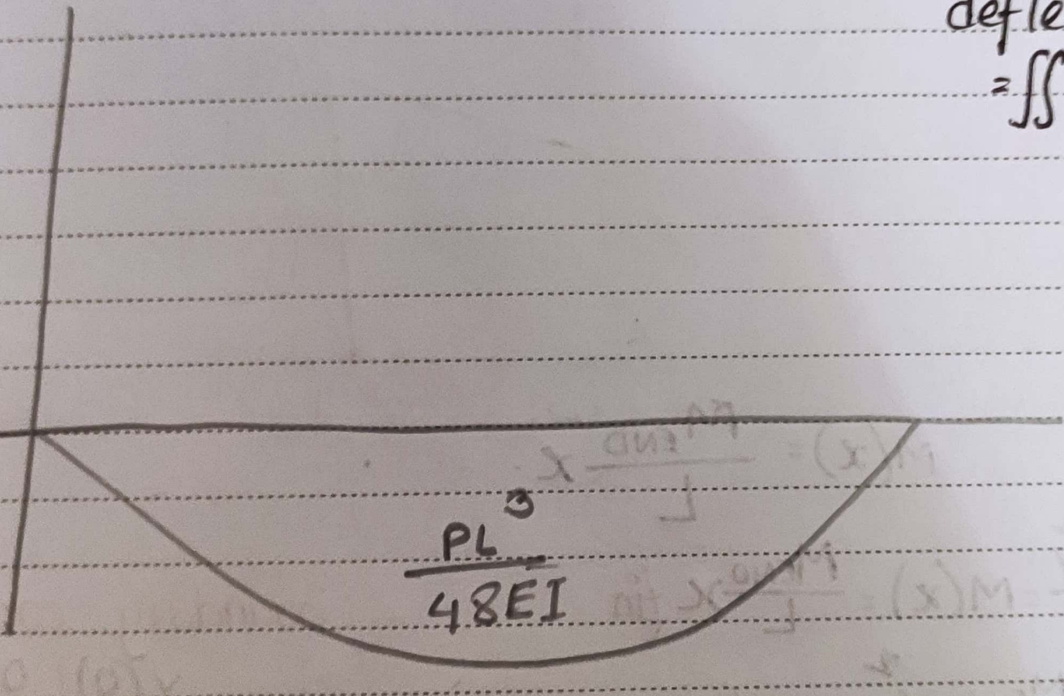
$$M(x) = EI\frac{d^2y}{dx^2}$$

$$\text{slope} = dy/dx.$$



$$\text{Slope} = \frac{dy}{dx} = \frac{PL^2}{16EI}$$

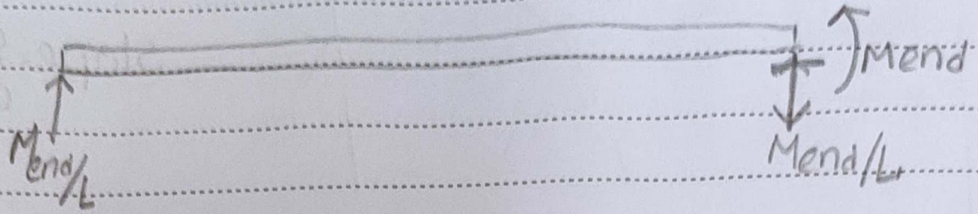
$$\text{deflection} = y = \iint \frac{M}{EI}\,dx\,dx.$$

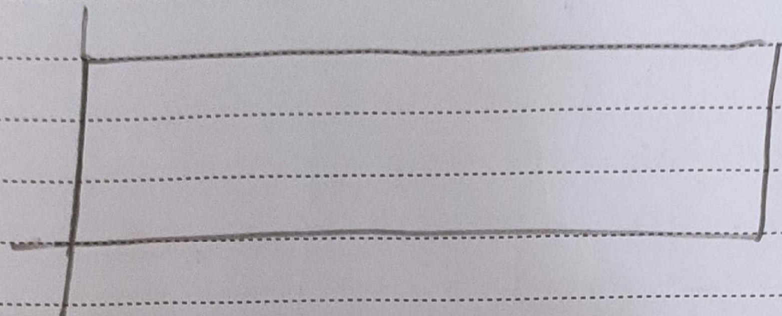


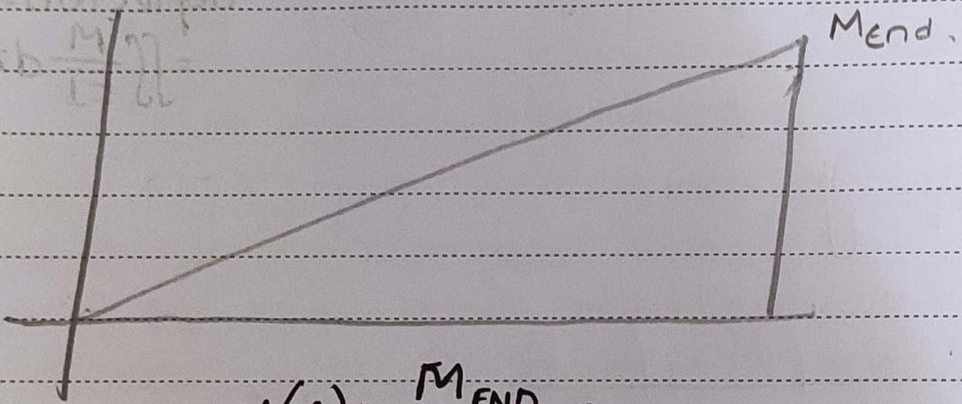
Maximum deflection of a simply supported beam with a concentrated load at the center is $\frac{PL^3}{48EI}$ and occurs at the centre of the beam.

Note's

Detemination of the ~~method~~ equation for deflection of a beam







$$M(x) = \frac{M_{END}}{L}x.$$

$$EI\frac{d^2y}{dx^2} = M(x) = \frac{M_{END}}{L}x \; \cancel{fin}$$

Boundary conditions

$y(0) = 0$

$y(L) = 0$

$$\Rightarrow EI\frac{dy}{dx} = \frac{M_{End}}{L}\cdot\frac{x^2}{2} + C_1 \text{ (integrating once)}$$

$$\Rightarrow EIy = \frac{M_{End}}{2L}\cdot\frac{x^3}{3} + C_1x + C_2 \text{ (double integration)}$$

$y(0) = C_2 = 0$;

$$EI\,y(L) = 0 = \frac{M_{END}}{6L}L^3 + C_1L \Rightarrow C_1 = -\frac{M_{End}}{6}L$$

$$\boxed{EI\,y(x) = \frac{M_{END}\,x^3}{6L} - \frac{M_{END}\,Lx}{6}}$$

Determine max deflection of a simply supported beam (moment at right end)

$$\boxed{EI\,y(x) = \frac{M_{END}}{2L}\frac{x^3}{3} - \frac{M_{END}L}{6}x}$$

Mathematically the max deflection occurs where $\frac{dy}{dx} = 0$, or physically where the slope of beam is zero.

$$EI\frac{dy(x)}{dx} = 0 = \frac{M_{END}}{2L}x^2 - \frac{M_{END}L}{6}$$

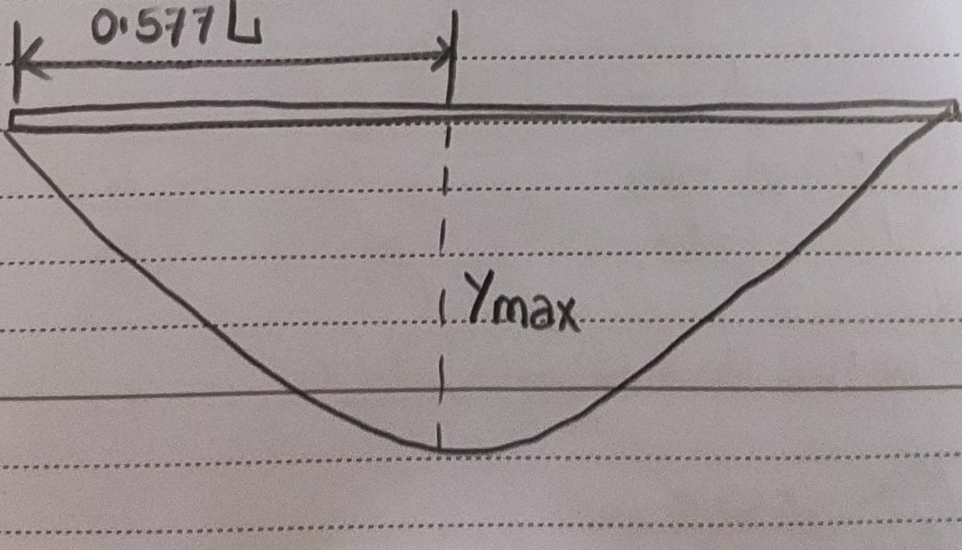
$$\Rightarrow x^2 = L^2/3 \Rightarrow \boxed{x = L/\sqrt{3} = 0.5577L}$$

$$\cancel{EI\frac{dy\left(\frac{L}{\sqrt{3}}\right)}{dx} = \frac{M_{END}L^2}{2L}}$$

$$\cancel{EI\frac{dy\left(\frac{L}{\sqrt{3}}\right)}{dx}} \Rightarrow EI\,y\left(\frac{L}{\sqrt{3}}\right) = \frac{M_{END}}{2L}\frac{L^3}{3\sqrt{3}} - \frac{M_{END}L}{6}\cdot\frac{L}{\sqrt{3}}.$$

$$\Rightarrow y\left(\frac{L}{\sqrt{3}}\right) = \frac{M_{END}}{EI}\left(\frac{L^2}{18\sqrt{3}} - \frac{L^2}{6\sqrt{3}}\right) = \frac{M_{END}}{EI}\left(\frac{-2L^2}{18\sqrt{3}}\right)$$

$$\Rightarrow y\left(\frac{L}{\sqrt{3}}\right) = -\frac{M_{END}L^2}{9\sqrt{3}EI}$$



Note's

08.00 ## Singularity Functions

09.00 Singularity functions are used to write a single moment
10.00 equation using discontinous functions without the need for
11.00 matching conditions.

12.00

$$(x-x_0)^n = \begin{cases} 0 & \text{for } x<x_0 \\ 1 & \text{for } n=0 \\ (x-x_0) & \text{for } n\geq 1 \ \& \ x\geq x_0 \end{cases}$$

Lunch

02.00

3.00

$$\frac{d}{dx}(x-x_0)^n = n(x-x_0)^{n-1} \qquad \text{for } n\geq 1$$

.00

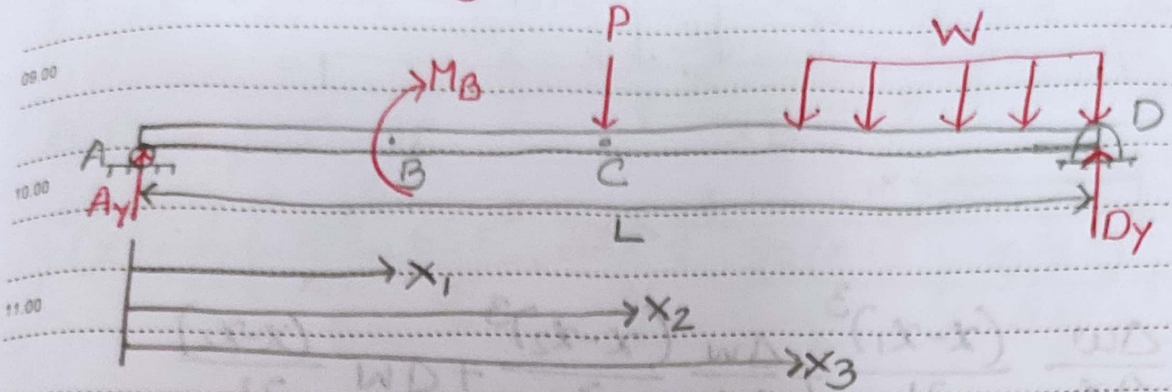
$$\int (x-x_0)^n dx = \frac{1}{n+1}(x-x_0)^{n+1} + C \qquad \text{for } n\geq 0$$

00

0

s

Employ Singularity Function



$M_1 = A_y x$ $\qquad 0 < x < x_1$

$M_2 = A_y x + M_B$ $\qquad x_1 < x < x_2$

$M_3 = A_y x + M_B - P(x - x_2)$ $\qquad x_2 < x < x_3$

$M_4 = A_y x + M_B - P(x - x_2) - W(x - x_3)\frac{(x - x_3)}{2}$ $\qquad x_3 < x < L$

$M = A_y x + M_B (x - x_1)^0 - P(x - x_2)^1 - \frac{W}{2}(x - x_3)^2$ $\qquad 0 \le x \le L$

x where load applied

integer describing load

$F \frac{(x - x_{load})^n}{n!}$

Magnitude of load

Applied Moment: $M_a \frac{(x-a)^0}{0!}$

Point load: $P_b \frac{(x-b)^1}{1!}$

Distributed load: $w \frac{(x-c)^2}{2!}$

Note's

Ramp load: $\frac{\Delta w}{\Delta x} \frac{(x-d)^3}{3!}$

08.00 Beam Deflection by Superposition

09.00 For common beam configurations, when the beam bending
10.00 remains in the linear elastic regions, we can employ
11.00 superposition techniques to determine the beam deflection
12.00 from tables.

Lunch Using the superposition method we can find the
02.00 resultant effect of several loads acting on a member
03.00 at the same time by adding the contribution of each
04.00 load applied individually. There must be a linear relation
05.00 between the applied loads, the stresses & the resulting
06.00 directions.

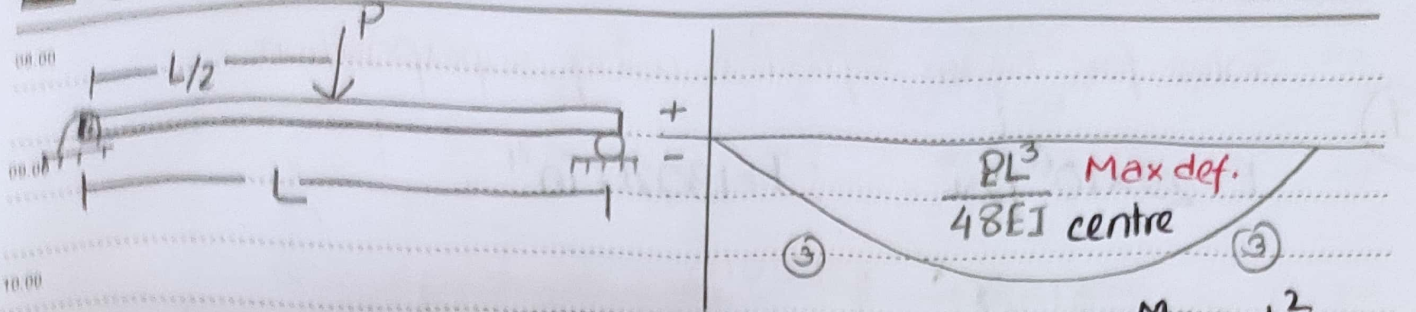
07.00

08.00

Note's

25 April Wednesday **2018**

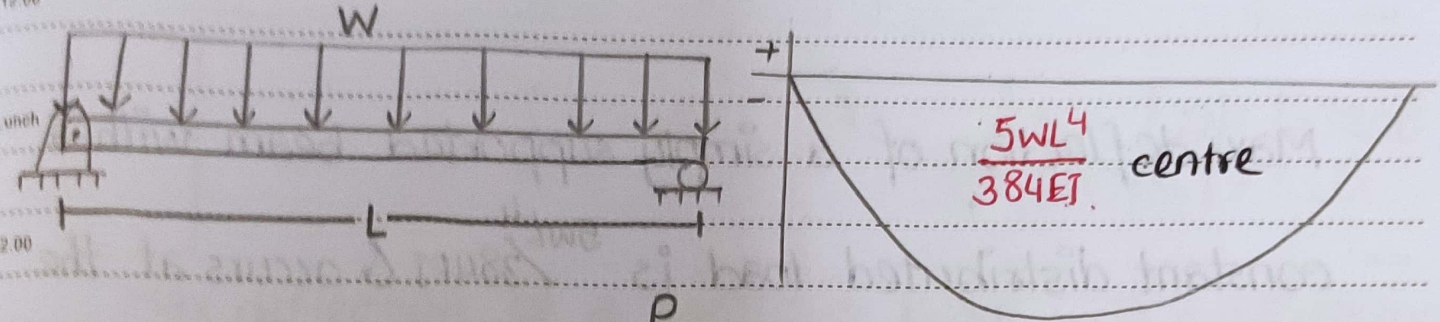
বুধবার
১২ বৈশাখ ১৪২৫ বাংলা
৮ শাবান ১৪৩৯ হিজরি



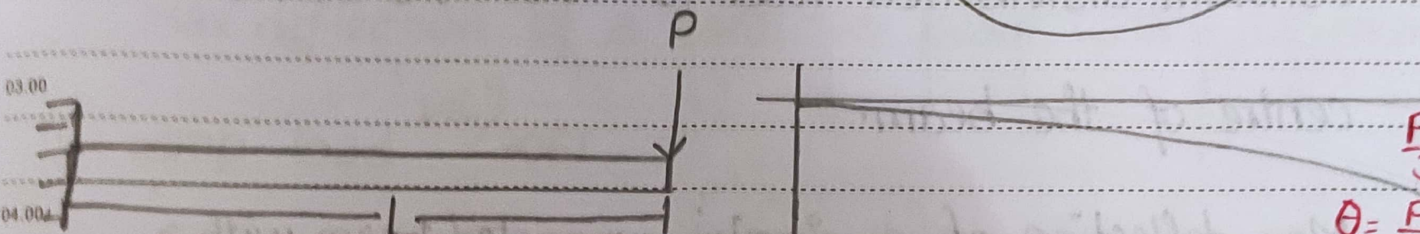
$\frac{PL^3}{48EI}$ Max def. centre

Max deflection $= \frac{-M_{END}}{9\sqrt{3}} \frac{L^2}{EI}$

occurs at $x = 0.577L = L/\sqrt{3}$

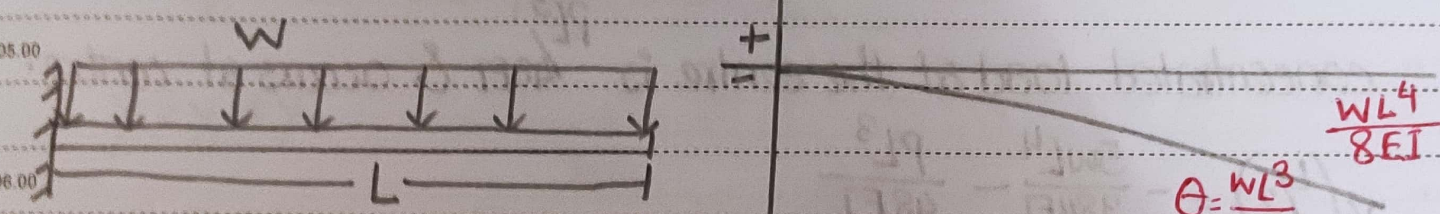


$\frac{5wL^4}{384EI}$. centre



$\frac{PL^3}{3EI}$ (end)

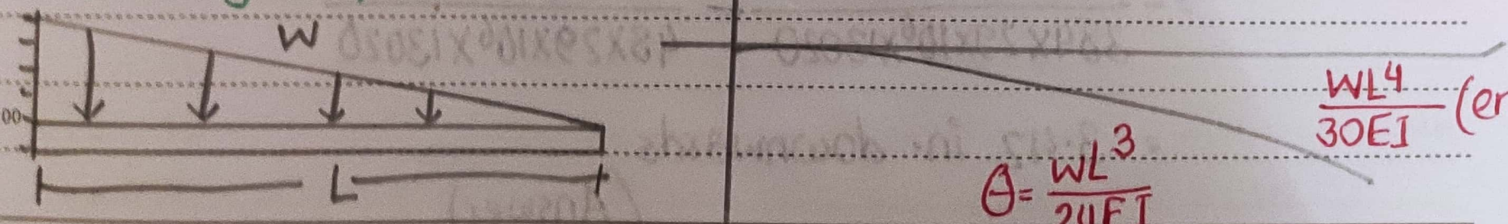
$\theta = \frac{PL^2}{2EI}$



$\frac{wL^4}{8EI}$ (end)

$\theta = \frac{wL^3}{6EI}$

decreasing ramp load.



$\frac{wL^4}{30EI}$ (end)

$\theta = \frac{wL^3}{24EI}$

Note's

26 April 2018 Thursday

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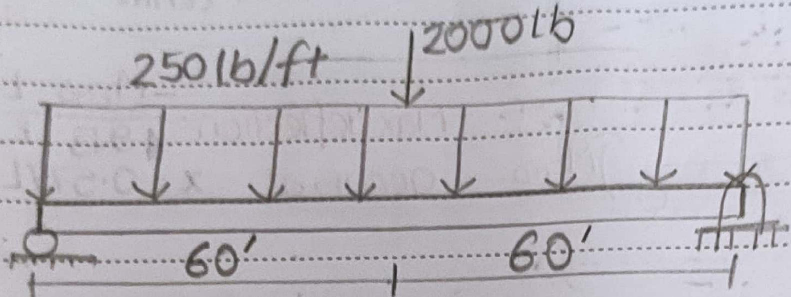
Solve for beam deflection using superposition

1

$E = 29\times10^6$ psi $I = 13020$ in^4



Max deflection of a simply supported beam with a constant distributed load is $5wL^4/384EI$ & occurs at the centre of the beam.

Max deflection of a simply supported beam with a concentrated load at the centre is $PL^3/48EI$ & occurs at centre.

$$y(L/2) = -\frac{5wL^4}{384EI} - \frac{PL^3}{48EI}$$

$$= -\frac{5\times250\times120\times12^3}{384\times29\times10^6\times13020} - \frac{2000\times120^3\times12^3}{48\times29\times10^6\times13020}$$

$= 3.412$ in. downwards

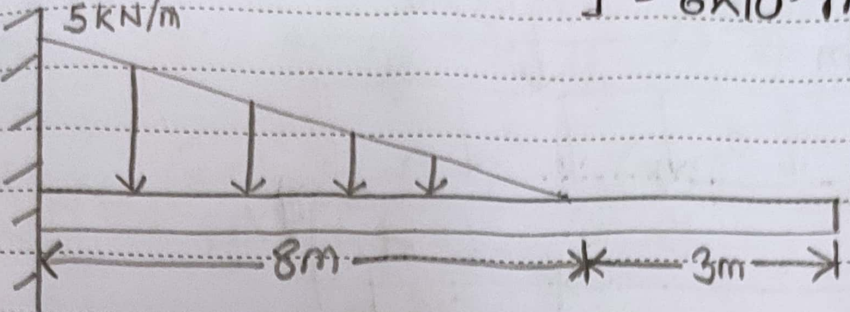
(Answer)

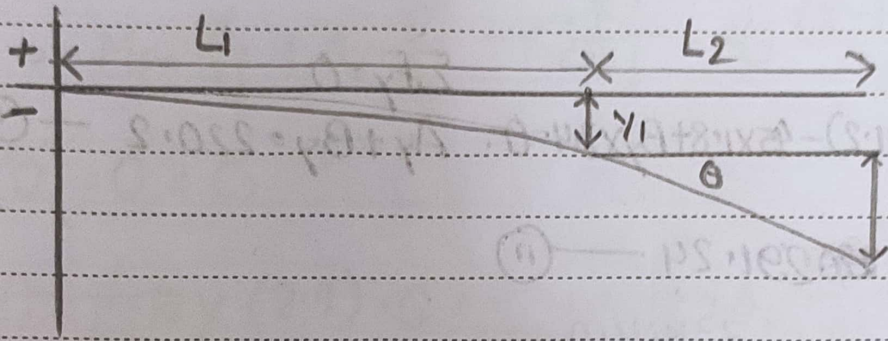
Note's

(2)

$E = 200 \times 10^3$ MPa.

$I = 6 \times 10^9$ mm^4





$|Y_2| = L_2 \tan\theta$

$\Rightarrow |Y_2| = L_2 \theta$

Max deflection of a cantilever beam with a decreasing ramp load $wL^4/30EI$

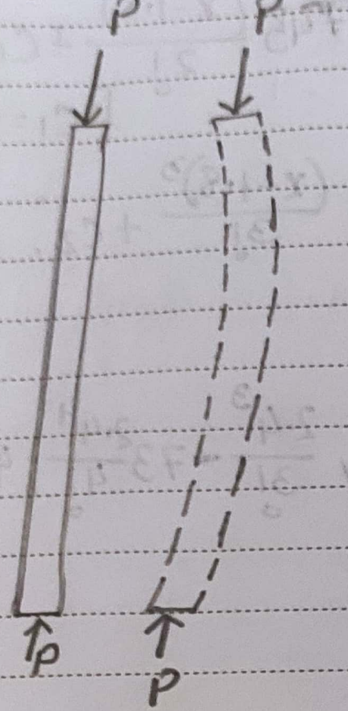
slope at the free end, $\theta = wL^3/24EI$

$y = y_1 + y_2$

$= 0.836$ mm down.

Differential Equation for Column Buckling

A simple column is a long, straight, prismatic bar subjected to compressive, axial loads.



If beams remain straight, analyze using normal methods

Buckling occurs if the column begins to deform laterally. The deflection can become large and lead to catastrophic failure.

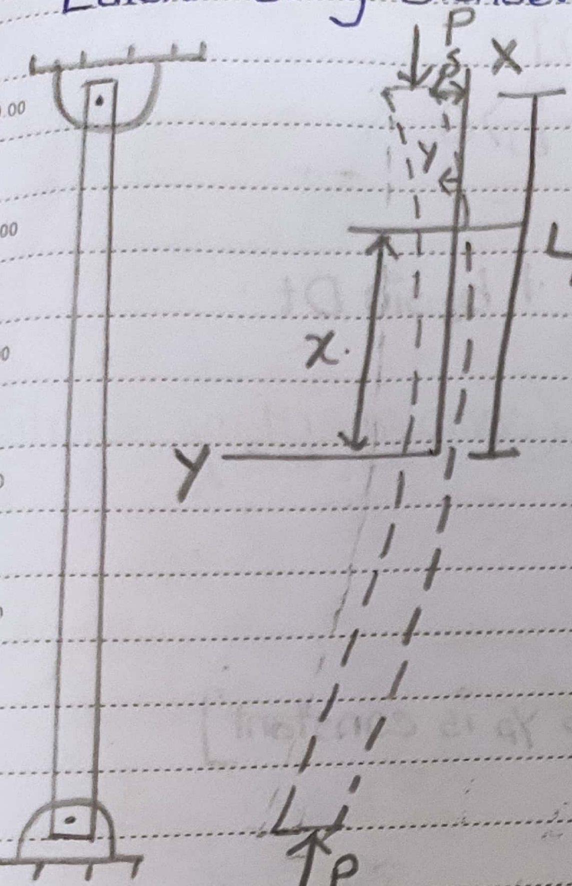
Buckling is a large sudden deformation of a structure due to small increase of existing load.

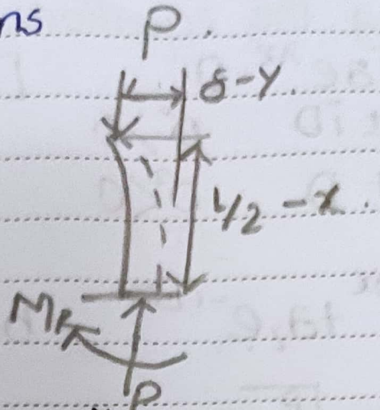
Buckling is when a stable equilibrium becomes unstable.

During initial compression, if a slight pertubation is laterally induced & the load is removed, the column returns to its straight configuration. When buckling occurs, a critical value is reached at which, when perturbed laterally, the column wouldn't return to its straight configuration.

Note's
For long slender columns, critical buckling occurs at stress levels below the proportional limit of material. This is an elastic phenomenon.

Euler's Long Slender Columns





$$EI\frac{d^2y}{dx^2} = M_R = P(\delta - y).$$

$$\boxed{\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P\delta}{EI}}$$ → diff. eq^n for column buckling

$$\frac{d^2y}{dx^2} + \frac{P}{EI}y = \frac{P\delta}{EI} \qquad \text{Let, } D^2 = \frac{P}{EI}.$$

$$\Rightarrow \frac{d^2y}{dx^2} + D^2y = D^2\delta$$

Method of Undetermined co-efficients.

$$y(x) = y_{complementary}(x) + y_{Particular}(x)$$

Complementary Solution

$$\frac{d^2y_c}{dx^2} + D^2y_c = 0$$

Assume solution

$$y_c = ae^{\lambda x}$$

Substituting the equation, we get,

$(\lambda^2+D^2)ae^{\lambda x}=0$ $\quad [ae^{\lambda x}\neq 0]$

$\lambda = \begin{cases} \pm iD & D^2>0 \\ \pm D & D^2<0 \end{cases}$ $\quad$ But, $D=\frac{P}{EI}>0$

$y_c = a_1 e^{iDx} + a_2 e^{-iDx} = b_1 \cos Dt + b_2 \sin Dt$

$y_c(x) = b_1 \cos\sqrt{\frac{P}{EI}}\,x + b_2 \cos\sqrt{\frac{P}{EI}}\,x$ (complementary solution)

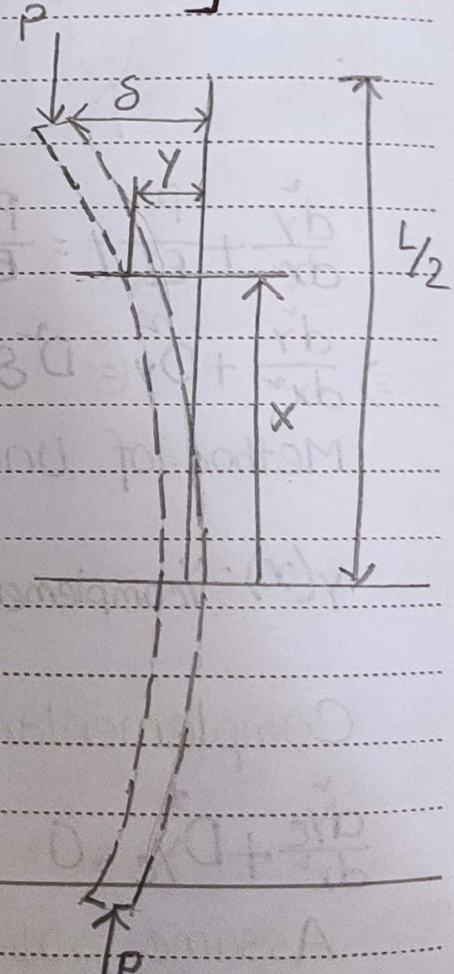
Particular Solution

Let $y_p(x) = C_1$

$\frac{d^2y_p}{dx^2} + D^2 y_p = D^2\delta$ $\quad \left[\frac{d^2y_p}{dx^2}=0\text{; as } y_p \text{ is constant}\right]$

$\Rightarrow C_1 = \delta$

$\Rightarrow y_p(x) = \delta$



$y(x) = b_1 \cos\sqrt{P/EI}\,x + b_2 \sin\sqrt{P/EI}\,x + \delta$

For pinned-pinned connection.

$y(0) = 0 = b_1 \cos 0 + b_2 \sin 0 + \delta$

$\Rightarrow b_1 = -\delta$

$\frac{dy}{dx} = -\sqrt{\frac{P}{EI}}\, b_1 \sin\sqrt{\frac{P}{EI}}\,x + \sqrt{\frac{P}{EI}}\, b_2 \cos\sqrt{\frac{P}{EI}}\,x$

Note's

$\frac{dy(0)}{dx} = 0 = b_2$

~~δ~~ =

$$y(x) = \delta\left(1 - \cos\sqrt{\tfrac{P}{EI}}\, x\right)$$

at $x(L/2)$, $y = \delta$

$$\delta = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}\frac{L}{2}\right)$$

$$\therefore 1 - \cos\sqrt{\frac{P}{EI}}\,\frac{L}{2} = 1$$

$$\Rightarrow \cos\sqrt{P/EI}\; L/2 = 0$$

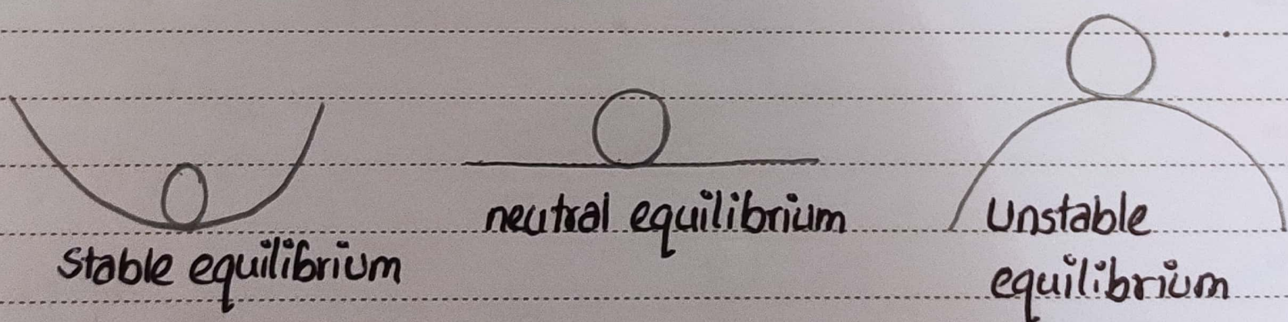
$$\sqrt{P/EI}\; L/2 = \pi/2,\ 3\pi/2,\ 5\pi/2 \ldots\ldots$$

Only the first value has physical significance since it determines the minimum value of P for a non-trivial solution.

$$P_{cr}/EI \cdot L^2/4 = \pi^2/4$$

$$\boxed{P_{CR} = \frac{\pi^2 EI}{L^2}}$$

Stages of column buckling



Note's

06 May 2018 Sunday

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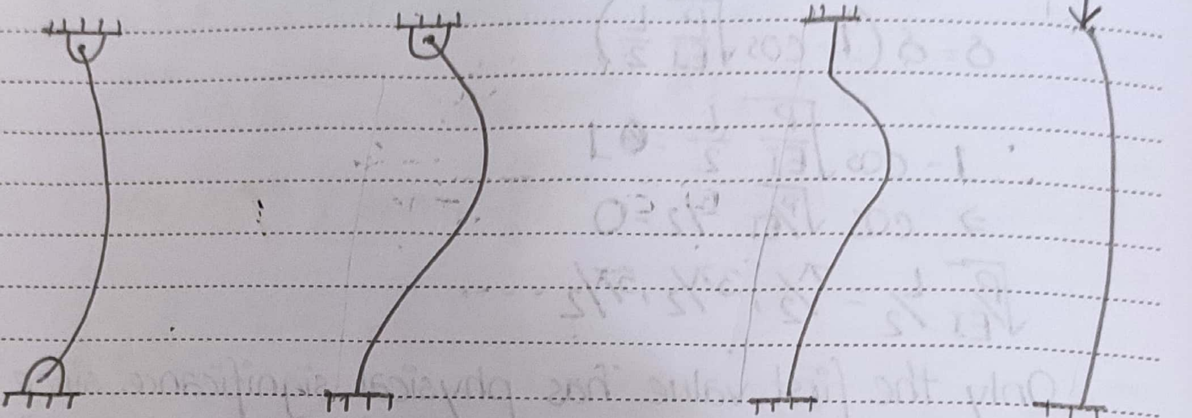
Critical Buckling Loads (Euler Buckling loads)

Pin-pinned Fix pinned Fixed-Fixed fixed-Free



$$P_{CR}=\frac{\pi^2 EI}{L^2}$$

$$P_{CR}=\frac{\pi^2 EI}{(0.7L)^2}$$

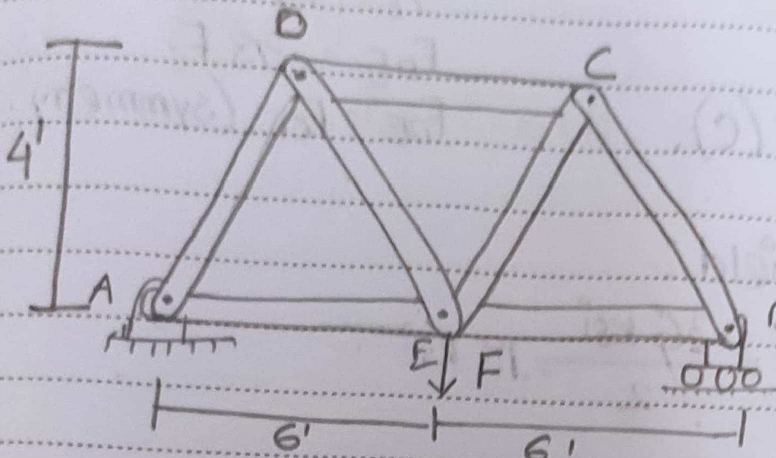
$$P_{CR}=\frac{\pi^2 EI}{(0.5L)^2}$$

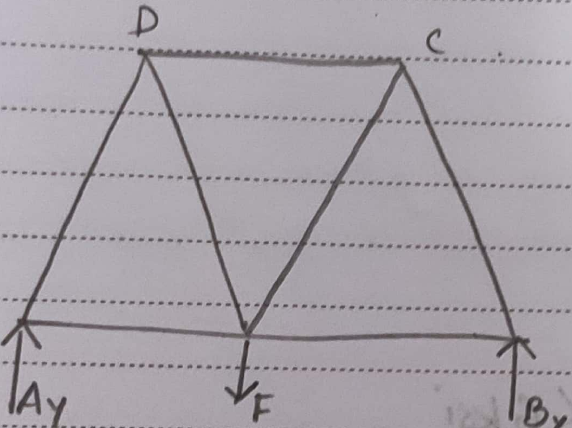
$$P_{CR}=\frac{\pi^2 EI}{(2L)^2}$$

$$P_{CR}=\frac{\pi^2 EI}{(L_{Effective})^2}$$

Note's

Solve an actual Column Buckling





$\Sigma M_A = 0$

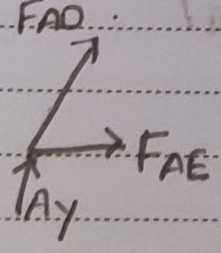
$B_y \times 12 - 6F = 0.$

$\Rightarrow B_y = F/2$ —(i).

$\Sigma F_y = 0$

$A_y = F/2$ —(ii).

(a) FBD of joint A.



$\Sigma F_y = 0$

$F_{AD} = \frac{-5}{8} F.$

$\Rightarrow F_{AD} = 0.625F$ (C)

$\Sigma F_x = 0$

$\frac{3}{\cancel{5}} \times (-\cancel{5}/8) F + F_{AE} = 0.$

$\Rightarrow F_{AE} = 3/8 F.$

Note's

$\Sigma M_E = 0$.

$-\frac{F}{2}(6) - F_{cd}(4) = 0$.

$F_{CD} = -\frac{3}{4}F = 0.75F\,(C)$.

$\Sigma F_y = 0$.

$\frac{F}{2} - \frac{4}{5}F_{DE} = 0$.

$F_{DE} = \frac{5}{8}F$.

$F_{DE} = F_{CE}$ (symmetry.

For axial load yield:

$\tau_{Failure} = \frac{\sigma_{Failure}}{2} = \frac{36\ ksi}{2} = 18\ ksi$.

$FoS = 3 = \frac{18\ ksi}{\tau_{allowed}}$

$\tau_{actual} \le 6\ ksi$

$\sigma_{actual}/2 = \tau_{actual}$.

$= \frac{P_{actual}}{2A} = \tau_{actual} \le 6\ ksi$

$P_{allowed} = 0.75F_{allowed} \le 6\ ksi \times 2 \times \pi \times \left(\frac{1}{4}\right)^2$

$F_{allowed} \le 3.142\ kips$.

Note's

08.00 Slenderness Ratio.

$$P_{CR} = \frac{\pi^2 EI}{(L_{Effective})^2}$$ where the area moment of inertia, I is

09.00 about the axis of bending (min. ~~mo~~ I for buckling)

10.00 Let, $I = Ar^2$ → radius of gyration about axis of bending

11.00
$$\sigma_{Failure} = \frac{P_{CR}}{A} = \frac{\pi^2 E}{(L_{effective}/r)^2}$$

12.00
$$\boxed{\frac{L_{effective}}{r} = \text{Slenderness ratio}}$$

Lunch

Emphirically, the Euler Buckling load only agrees well with

02.00 experimental data if for steel columns, $L_{eff}/r > 140$.

03.00 From previous example,

04.00
$$\sqrt{\frac{I}{A}} = \sqrt{\frac{\pi/4 \left(\frac{1}{4}\right)^4}{\pi \left(1/4\right)^2}} = 0.125$$

05.00
$$L_{eff}/r = \frac{6 \times 12}{0.125} = 575.$$

06.00

07.00

08.00

Note's

08.00 Graph modes of failure between yielding & Buckling

09.00 Modes of Failure

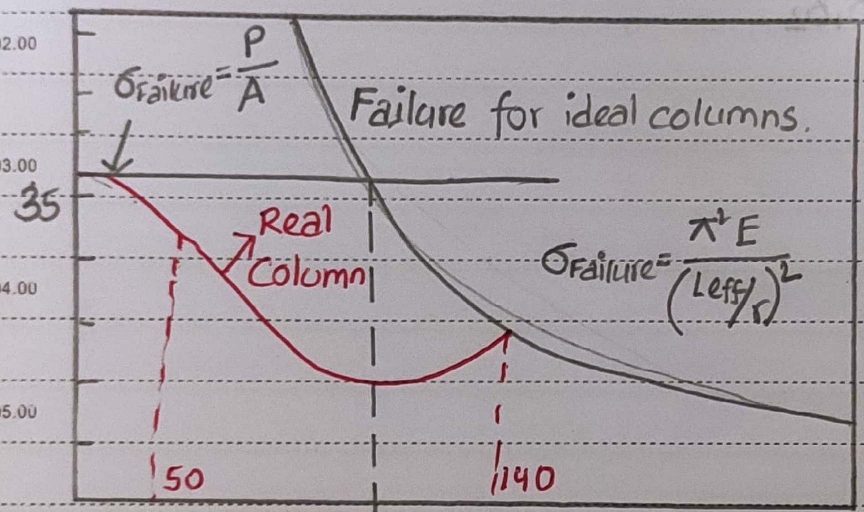
10.00 For Buckling

11.00 $$\sigma_{Failure} = \frac{P_{CR}}{A} = \frac{\pi^2 E}{(L_{eff}/r)^2}$$

12.00 For yielding

Lunch $$\sigma_{Failure} = \frac{P}{A}$$

TYPICAL STEEL COLUMNS



06.00 Yielding short compressive members | ⟷ | Buckling long compressive member.

07.00 For slendeness ratio ≥ 140; Euler's buckling load can be applied

08.00 For slenderness ratio < 50; Yielding

Note's For slenderness ratio $= 50 \sim 140$; medium length compressive membe
use emphirical formula/industry codes.