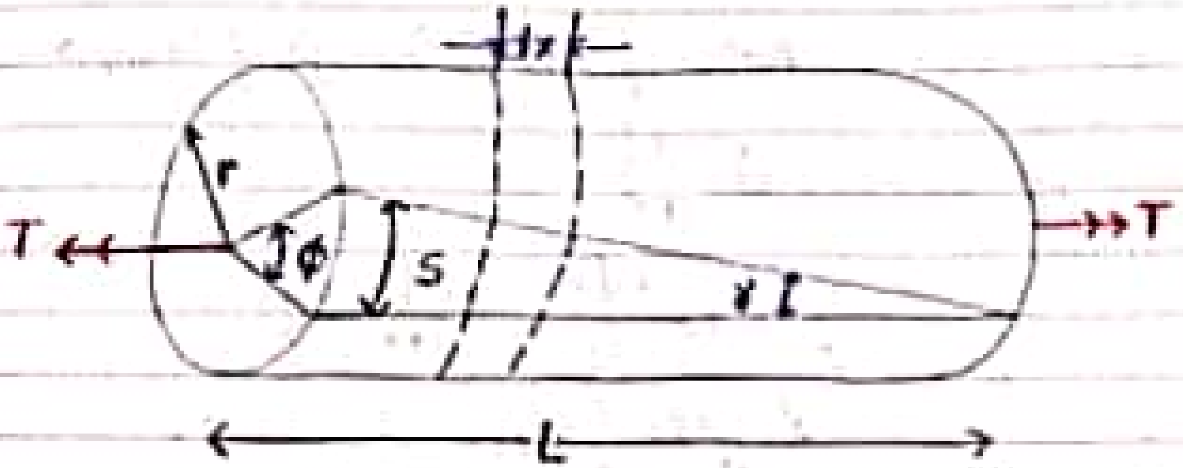
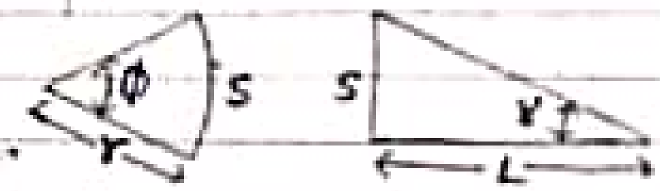
Torsion

Torsion is the twisting of an object due to an applied torque or moment. Units : N·m / ft-lb





Assumptions:

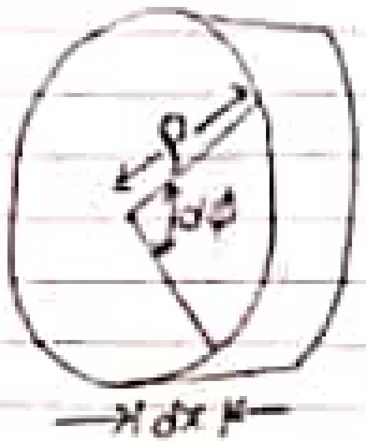
→ Pure torsion

→ Circular cross section (other cross sections warp)

→ Small angles

$s = r\phi$ $\quad \tan\gamma = \frac{s}{L}$ $\quad \Rightarrow \gamma = \frac{s}{L} \Rightarrow s = L\gamma$

$$\gamma_{max} = \frac{r\phi}{L}$$



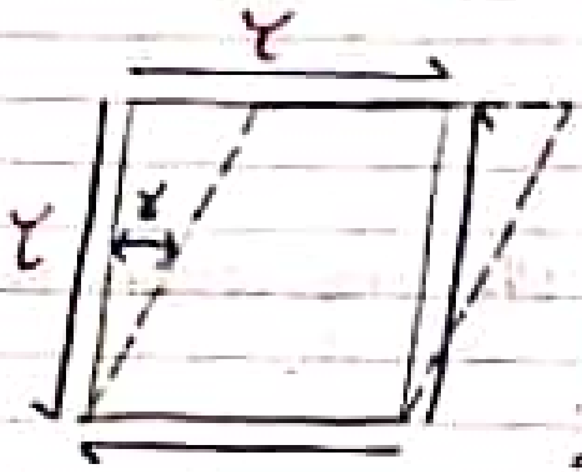
Rate of Twist, θ (angle of twist/unit leng

$$\theta = \frac{d\phi}{dx}$$

$$\boxed{\gamma_{max} = \frac{r\phi}{L} = r\frac{d\phi}{dx} = r\theta}$$

$$\boxed{\gamma = \rho\theta = \frac{\rho}{r}\gamma_{max}}$$

ρ → radial distance from centre



Hooke's Law:

$\tau = G\gamma$ → G: Rigidity modulus

$$\boxed{\tau_{max} = G\gamma_{max}}$$

$$\Rightarrow \boxed{\tau_{max} = Gr\theta}$$

$$\Rightarrow \boxed{\tau = G\rho\theta}$$

$$\Rightarrow \boxed{\tau = \frac{\rho}{r}\tau_{max}}$$

(last two) → At any point

Relation between Shear stress & Applied Torque

$$dT = (dF)\rho = \tau \rho \, dA \qquad \left[\tau = \frac{\rho}{r}\tau_{max}\right]$$

$$= \rho^2 \times \frac{\tau_{max}}{r} dA$$

$$T = \int_A dT = \frac{\tau_{max}}{r}\int_A \rho^2 dA \qquad J = \int_A \rho^2 dA$$

J: Polar moment of inertia

$$T = \frac{\tau_{max}}{r} J$$

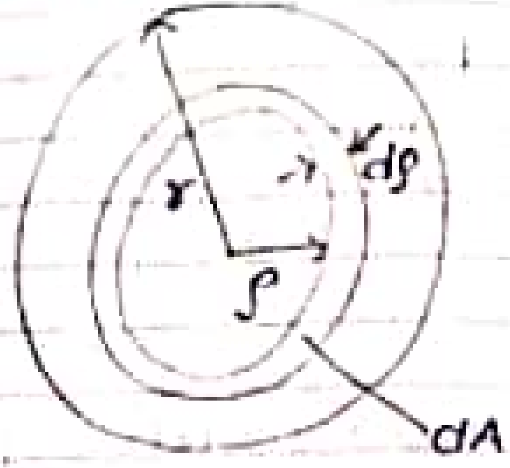
$$\Rightarrow \tau_{max} = \frac{Tc}{J}$$

$$\tau = \frac{T\rho}{J}$$ shear stress at any point / at a distance ρ from center axis.

Note

- T is greater for larger J, polar moment of inertia.
- J is larger when we have more area further from axis of rota

Calculation of polar moment of inertia, J



$$J = \int_A \rho^2 \, dA$$

$$dA = 2\pi\rho \, d\rho$$

$$J = \int_0^r \rho^2 \, 2\pi\rho \, d\rho$$

$$= 2\pi \int_0^r \rho^3 \, d\rho$$

$$= \pi r^4/2$$

For solid circular cross section:

$$J = \pi r^4/2 = \pi D^4/32$$

For hollow circular cross section:

$$J = \frac{\pi}{2}\left(r_{out}^4 - r_{in}^4\right) = \frac{\pi}{32}\left(D_{out}^4 - D_{in}^4\right)$$

→ more efficient for torsion

Angle of Twist

$\theta = \frac{d\phi}{dx} = \phi/L$

$\tau_{max} = G\gamma_{max}$

$\Rightarrow \frac{Tr}{J} = Gr\theta$

$\Rightarrow T = G\theta J$

$\Rightarrow T = \frac{G\phi J}{L}$

$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$ → Angle of Twist

Factor of safety

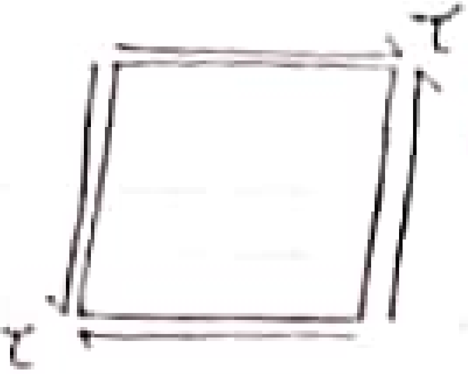
$FoS = \frac{\text{Failure stress}}{\text{Actual stress}} = \frac{\text{Strength of material}}{\text{Actual computed stress}}$

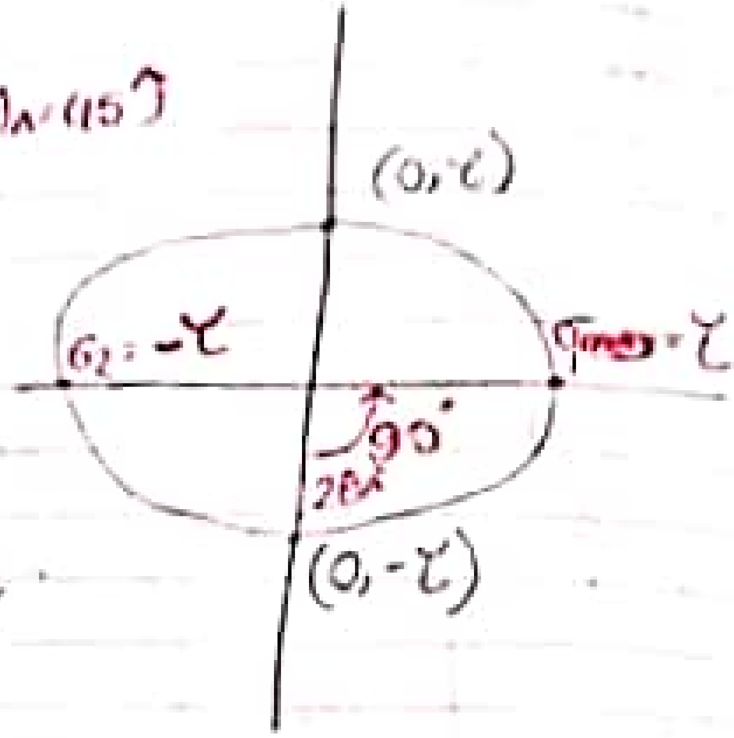
$\boxed{FoS > 1}$

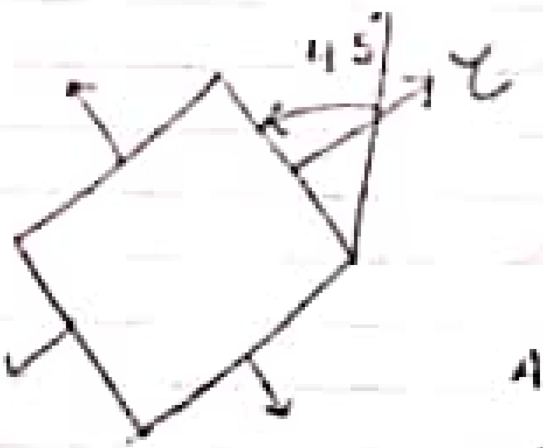
Note:

Buildings $FoS \geq 2$	Boilers/pressure vessels $FoS \geq 8 \cdot 5$
Automobiles $FoS \geq 3$	lifting equipments/Hooks $FoS \geq$
Air/Spacecraft $FoS \geq 1 \cdot 2$ to $2 \cdot 5$	Bolts $FoS \geq 8 \cdot 5$

Circular Bar Torsion



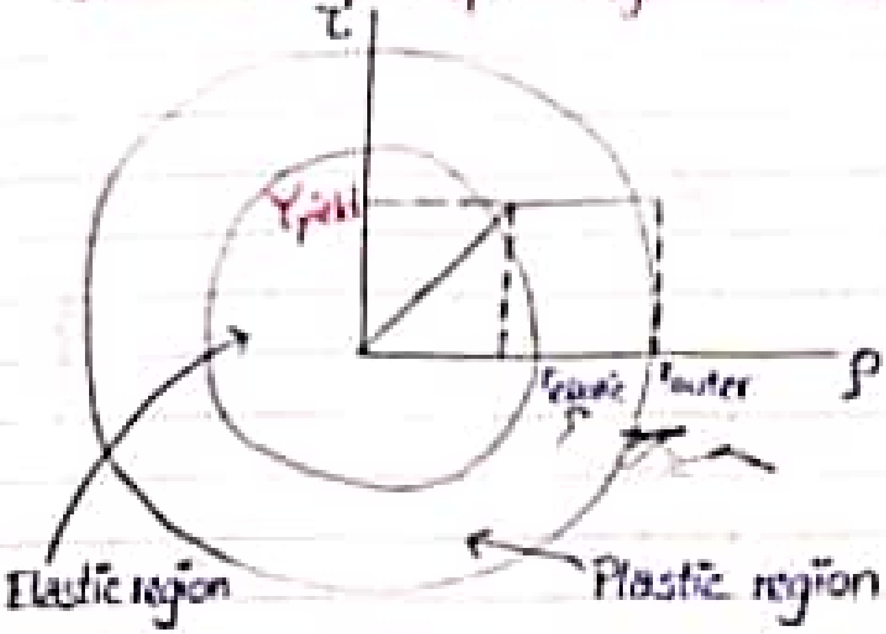




A circular tuble (brittle e.g. chalk) is strong in comp. weak in tension.

Therefore, chalk would break on 45° helical surface.

Inelastic torsion of st. cylindrical shafts



$\gamma_{max} = r_{outer}\theta$

$\frac{\gamma_{max}}{r_{outer}} = \frac{\gamma}{\rho} = \frac{\gamma_{yield}}{r_{elastic}}$

$\gamma = \rho\theta$

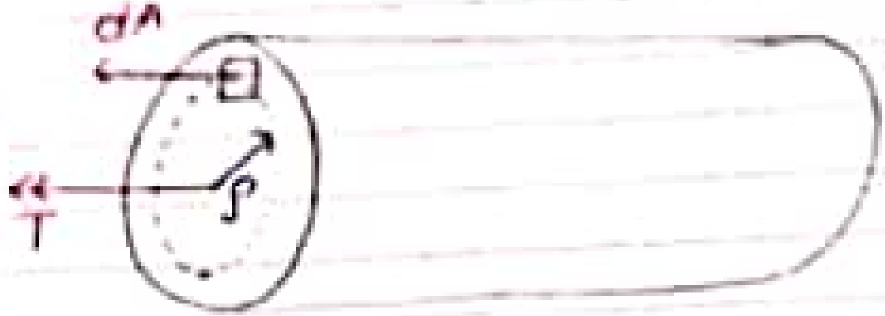
$\tau_{elastic} = G\gamma_{elastic}$

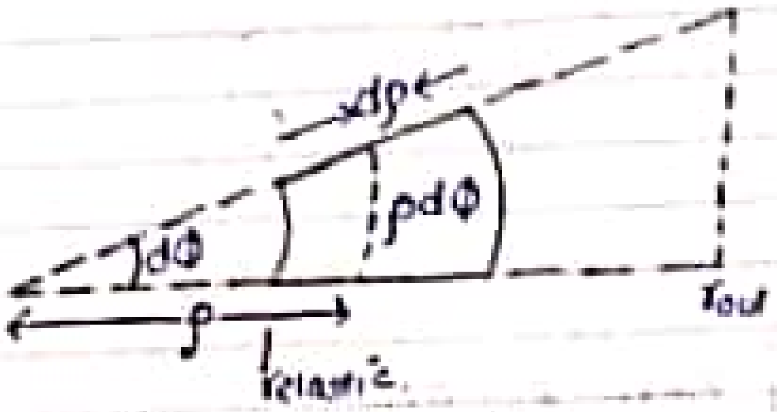
$\tau_{plastic} = \tau_{yield}$

$T_{t} = T_{elastic} + T_{plastic}$

$T_{elastic} = \frac{\tau_{yield} J_{elastic}}{r_{elastic}}$

Note :





$$T_{plastic} = \int_{plastic\ area} \rho \tau_{yield} \, dA$$

$$T_{plastic} = \int_0^{2\pi} \int_{r_{elastic}}^{r_{out}} \rho \tau_{yield} \rho \, d\rho \, d\phi$$

$$\Rightarrow T_{plastic} = 2\pi \tau_{yield} \int_{r_{elastic}}^{r_{out}} \rho^2 \, d\rho$$

$$\Rightarrow T_{plastic} = 2\pi \tau_{yield} \frac{\rho^3}{3}$$

$$\boxed{T_{plastic} = \frac{2}{3} \pi \tau_{yield} \left(r_{out}^3 - r_{elastic}^3 \right)}$$