Heaven 's Light is Our Guide
DEPARTMENT OF CIVIL ENGINEERING
RAJSHAHI UNIVERSITY OF ENGINEERING &TECHNOLOGY

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Combined Stresses

➢ Introduction

- Basic types of loading: axial, torsional and flexural

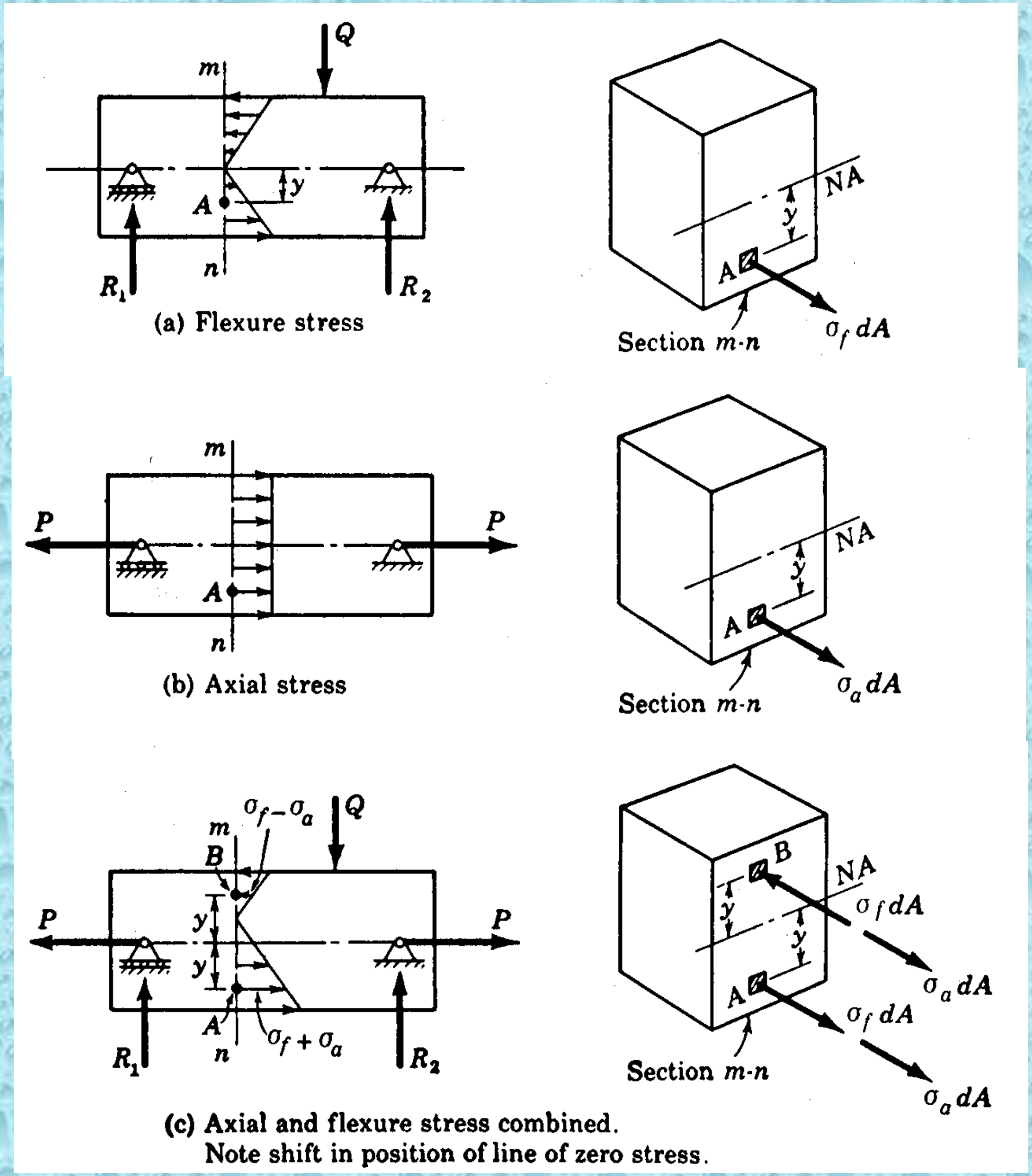
- Stress formulas:

Axial loading - $\sigma_a = \frac{P}{A}$

Torsional loading - $\tau = \frac{T\rho}{J}$

Flexural loading - $\sigma_f = \frac{My}{I}$

➤ Combined Axial & Flexural Loads



(a) Flexure stress

(b) Axial stress

(c) Axial and flexure stress combined.
Note shift in position of line of zero stress.

$$\sigma_f = \frac{My}{I}$$

$$\sigma_a = \frac{P}{A}$$

$$\sigma = \sigma_f + \sigma_a$$

$$\sigma = \frac{\oplus}{\ominus}\frac{P}{A} \pm \frac{My}{I}$$

901. A cantilever beam (Fig. 9-3) has the profile shown so that it will provide sufficient clearances for large pulleys mounted on the line shaft it supports. The reaction of the line shaft is a load $P = 25$ kN. Determine the resultant normal stresses at A and B at the wall.

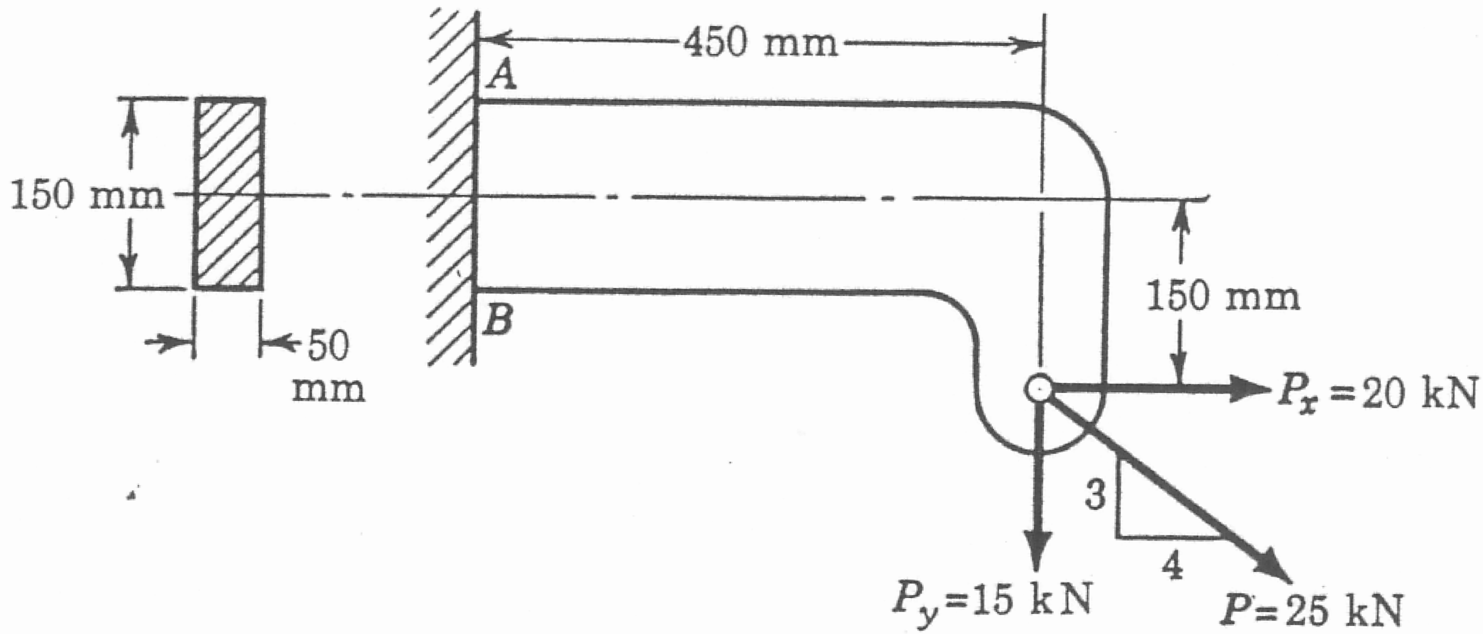
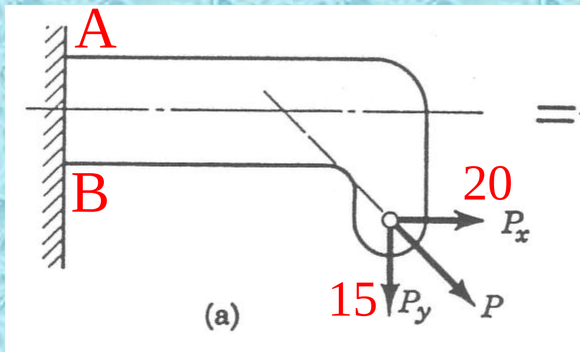


Figure 9-3



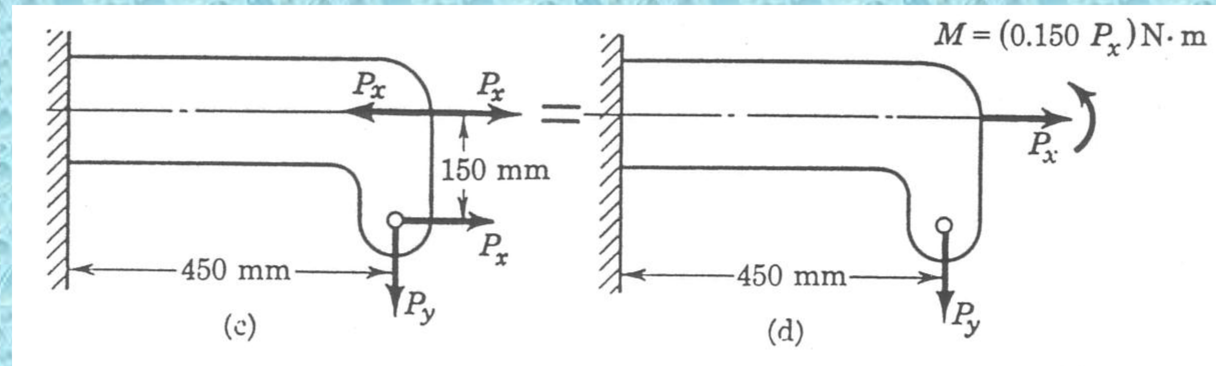


Figure 9-4

$$\left[\sigma = \frac{P}{A} + \left(\frac{Mc}{I} = \frac{6M}{bh^2}\right)\right]$$

$$\sigma_A = \frac{20\times10^3}{(0.05)(0.150)} + \frac{6(0.45\times15\times10^3 - 0.15\times20\times10^3)}{(0.05)(0.150)^2}$$

$$= (2.67\times10^6) + (20.00\times10^6) = 22.67\ \text{MPa}$$

$$\sigma_B = \frac{20\times10^3}{(0.05)(0.150)} - \frac{6(0.45\times15\times10^3 - 0.15\times20\times10^3)}{(0.05)(0.150)^2}$$

$$= (2.67\times10^6) - (20.00\times10^6) = -17.33\ \text{MPa}$$

906. For the 2-in. by 6-in. wooden beam shown in Fig. P-906, determine the normal stresses at A and B. Are these the points of maximum normal stress? If not, where are they located and what are their values?

Ans. $\sigma_A = -921$ psi; $\sigma_B = 599$ psi; max. $\sigma_n = 852$ psi and -1174 psi

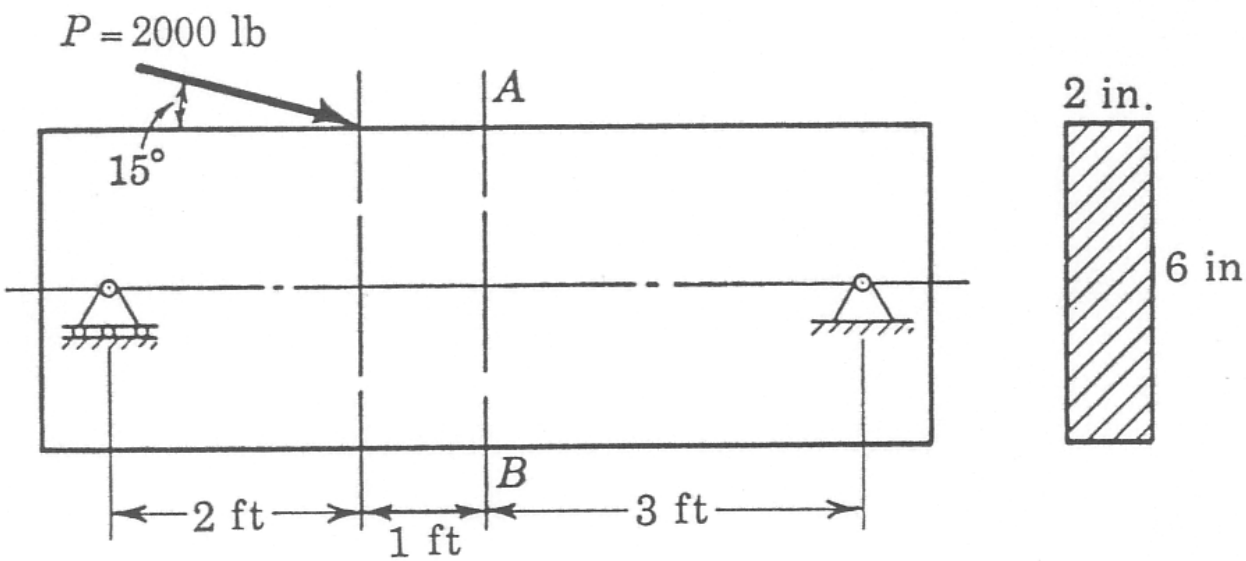
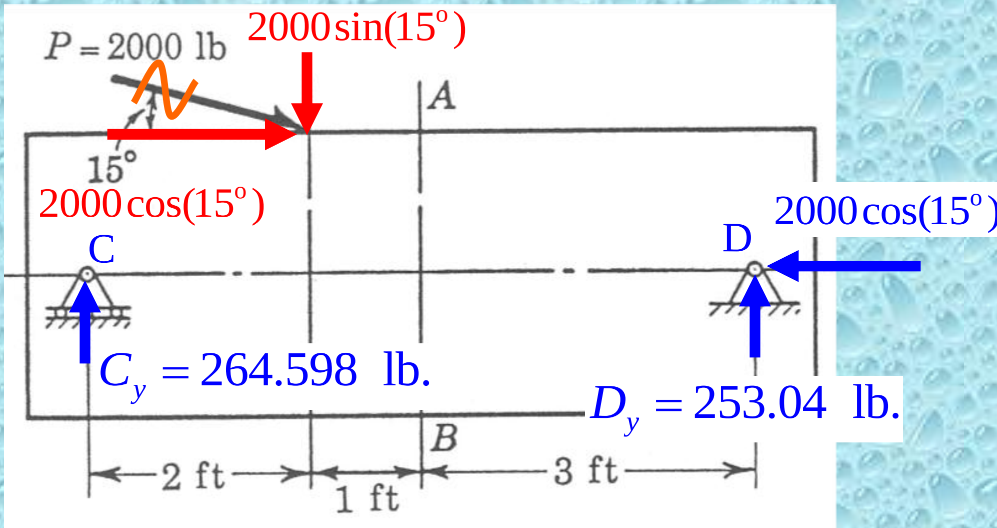


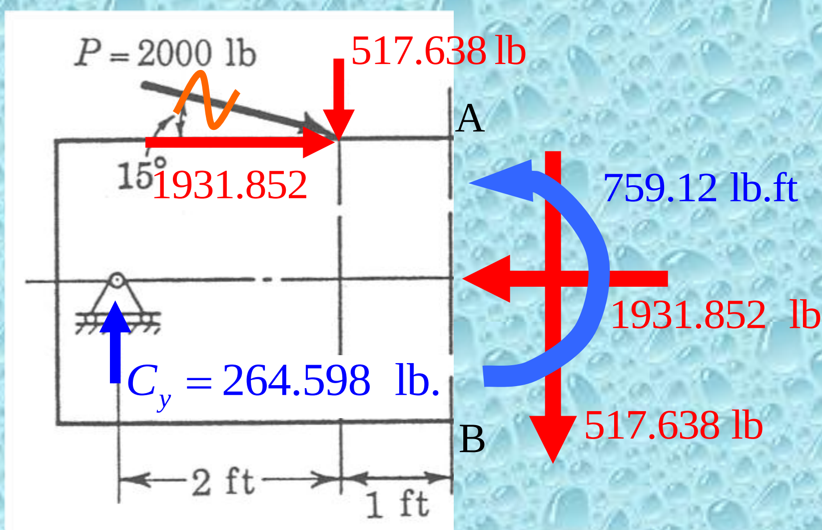
Figure P-906



$\sum F_x = 0:$

$$D_x = 2000\cos(15^{O})$$
$$= 1931.852 \text{ lb.}$$

$\sum M_D = 0:$

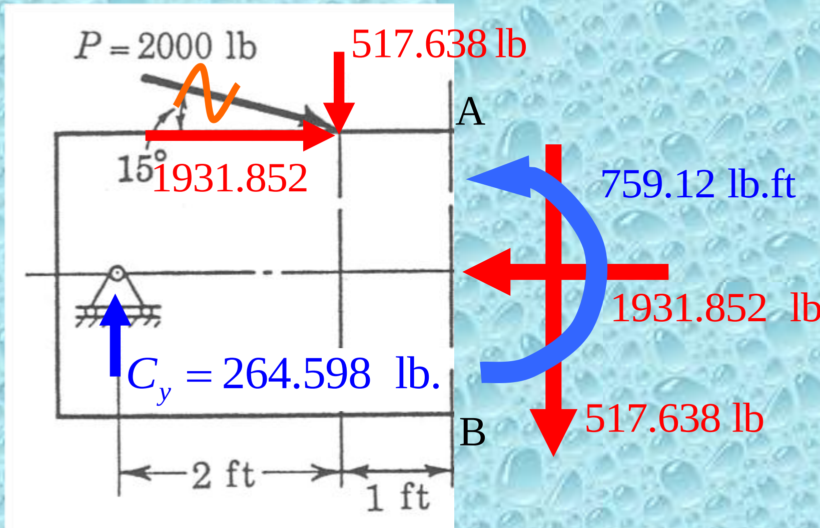


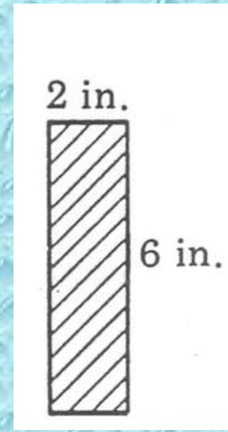
$\sum F_y = 0:$

$$D_y + C_y = 2000\sin(15^{O}) \text{ lb.}$$
$$D_y = 253.04 \text{ lb.}$$

Section AB:

$$M = (3)C_y + (\frac{3}{12})1931.852 - (1)517.638$$
$$= 759.12 \text{ lb.ft}$$





Normal Stresses

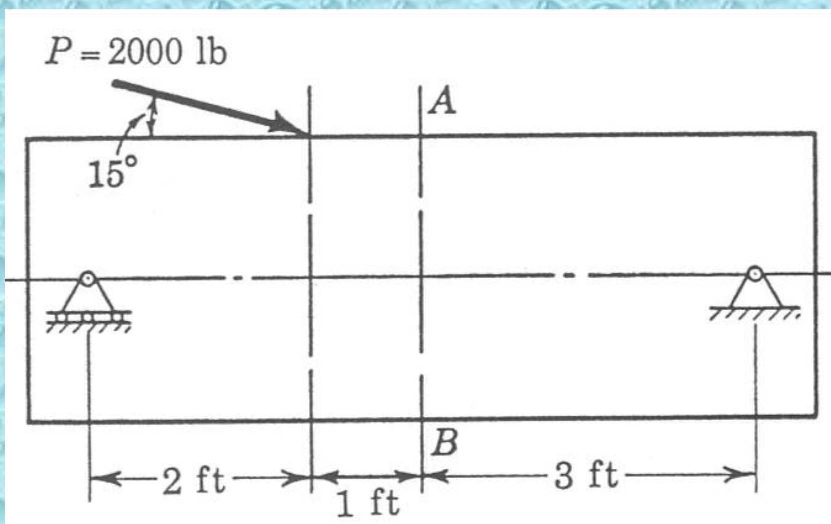
$$\left[\sigma = \frac{P}{A} \pm \left(\frac{Mc}{I} = \frac{6M}{bh^2}\right)\right]$$

$$\sigma_A = -\frac{1931.852}{2\times 6} - \frac{6\times 759.12\times 12}{2\times 6^2}$$

$$= -920.1 \text{ lb/in}^2$$

$$\sigma_B = -\frac{1931.852}{2\times 6} + \frac{6\times 759.12\times 12}{2\times 6^2}$$

$$= 598.1 \text{ lb/in}^2$$



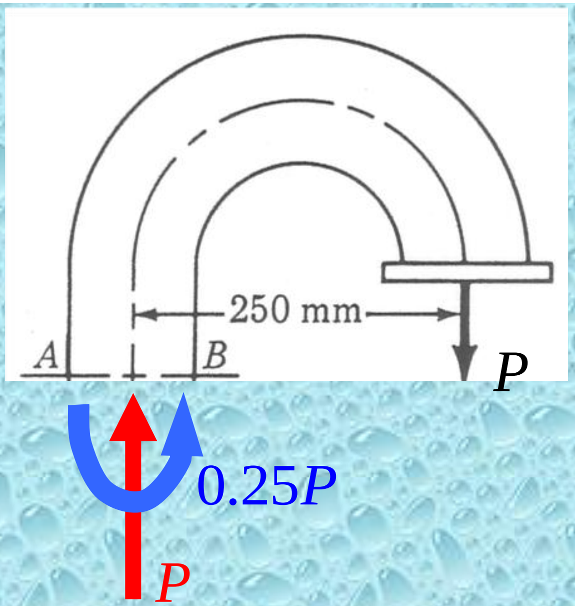
$$\sigma_{\min} = -\frac{1931.852}{2\times 6} - \frac{6\times 1012.16\times 12}{2\times 6^2}$$

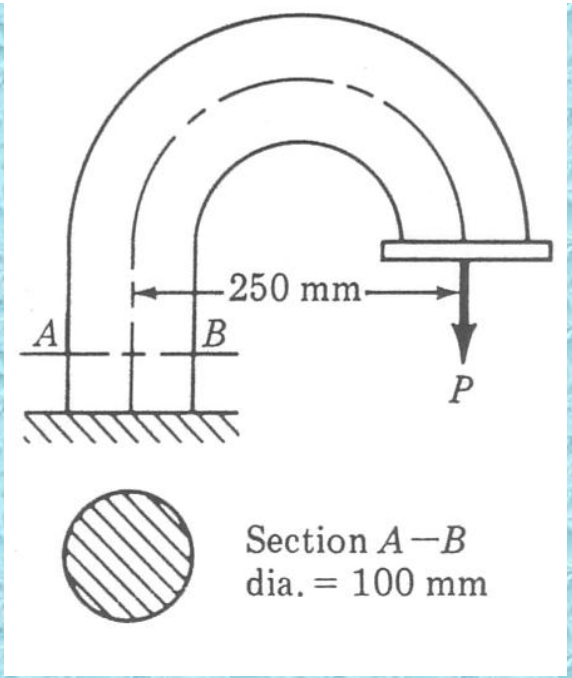
$$= -1173.15 \text{ lb/in}^2$$

$$\sigma_{\max} = -\frac{1931.852}{2\times 6} + \frac{6\times 1012.16\times 12}{2\times 6^2}$$

$$= 851.17 \text{ lb/in}^2$$

907. Determine the largest load P that can be supported by the circular steel bracket shown in Fig. P-907 if the normal stress on section A–B is limited to 80 MPa.





$$A = \tfrac{\pi}{4} D^2 = 0.0025\pi = 0.07854 \text{ m}^2$$

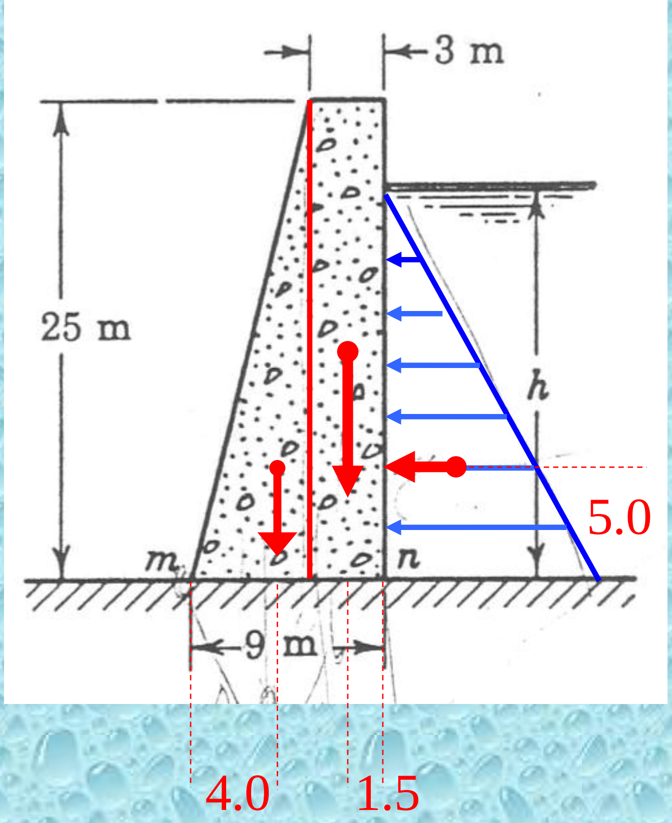
$$I = \tfrac{\pi}{64} D^4 = \pi (0.1)^4 = 1.5625\pi \times 10^{-6} \text{ m}^4$$

$$\sigma = -\frac{P}{A} - \frac{Mc}{I}$$

$$= -\frac{P}{0.0025\pi} - \frac{0.25P \times 0.05}{1.5625\pi}$$

$$= -\frac{8400P}{\pi} \geq -80 \text{ MPa} \quad \Rightarrow \quad P \leq \frac{80 \times 10^6 \times \pi}{8400} = 29.92 \text{ kN}$$

911. A concrete dam has the profile shown in Fig. P-911. If the density of concrete is 2400 kg/m³ and that of water is 1000 kg/m³, determine the maximum compressive stress on section *m–n* if the depth of the water behind the dam is $h = 15$ m.



$$P = 2400 \cdot [\tfrac{1}{2} \times 25 \times 6 + 25 \times 3]$$

$$= 180,000 + 180,000 = 360,000 \text{ kg.}$$

$$M = 180,000 \times 0.5 - 180,000 \times 3.0$$

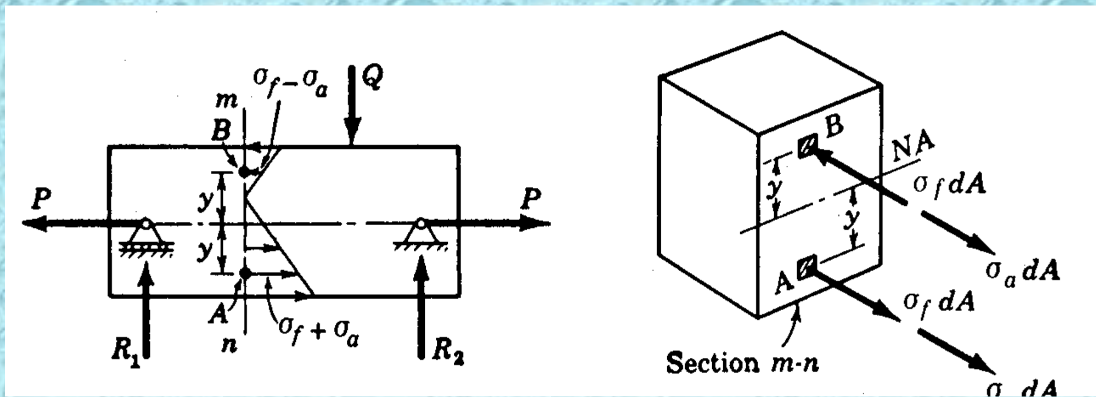
$$+ \tfrac{1}{2} \times (1000 \times 15) \times 15 \times 5$$

$$= 90,000 - 540,000 + 562,500$$

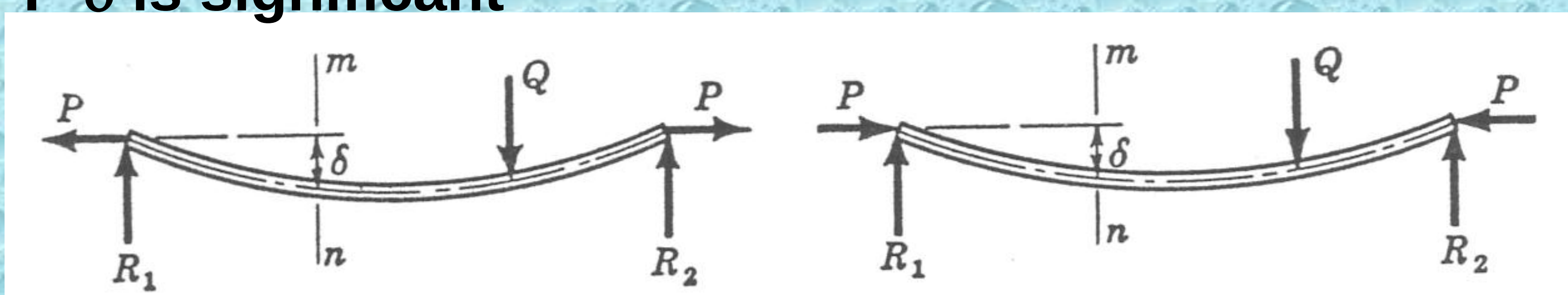
$$= 112,500 \text{ kg-m.}$$

$$\left[\sigma = \frac{P}{A} \pm \left(\frac{Mc}{I} = \frac{6M}{bh^2}\right) \right] \quad \sigma_{\min} = -\frac{360,000}{1 \times 9} - \frac{6 \times 112,500}{1 \times 9^2} = -48,333.33 \text{ kg/m}^2$$

For stiff members the formula $\sigma = \frac{\oplus P}{\ominus A} \pm \frac{My}{I}$ **is appropriate**



For long slender members or columns, the effect of P-δ is significant



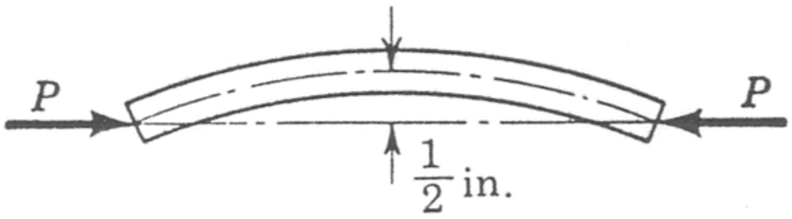
(a) Axial tension

(b) Axial compression

902. Compare the maximum stress in a bent rod $\frac{1}{2}$ in. square, where the load P is $\frac{1}{2}$ in. off center as shown in Fig. P-902, with the maximum stress if the rod were straight and the load applied axially. This problem illustrates why lateral deflection in columns is so dangerous.

Ans. 7 to 1

Fig.(a)



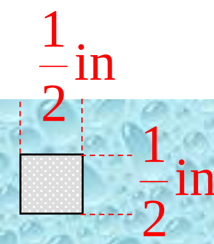


Fig.(b)

$$A = \left(\frac{1}{2}\right)^2 = \frac{1}{4}\ \text{in}^2$$

$$I = \tfrac{1}{12}\left(\frac{1}{2}\right)^4 = \frac{1}{192}\ \text{in}^4$$

max. compressive stress in Fig.(a)

$$\sigma_{\max,(a)} = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{(\frac{1}{4})} - \frac{(\frac{1}{2}P)(\frac{1}{4})}{(\frac{1}{192})} = -28P$$

max. compressive stress in Fig.(b)

$$\sigma_{\max,(b)} = -\frac{P}{A} = -\frac{P}{(\frac{1}{4})} = -4P$$

$$\frac{\sigma_{\max,(a)}}{\sigma_{\max,(a)}} = \frac{28P}{4P} = 7:1$$

Hw10

A punch press has the cast-steel frame shown in Fig. P-908. Determine the greatest force P that can be exerted at the jaws of the punch without exceeding a stress of σ_{allow} ksi at section A–B. The properties of the area are as shown and 1–1 is the centroidal axis.

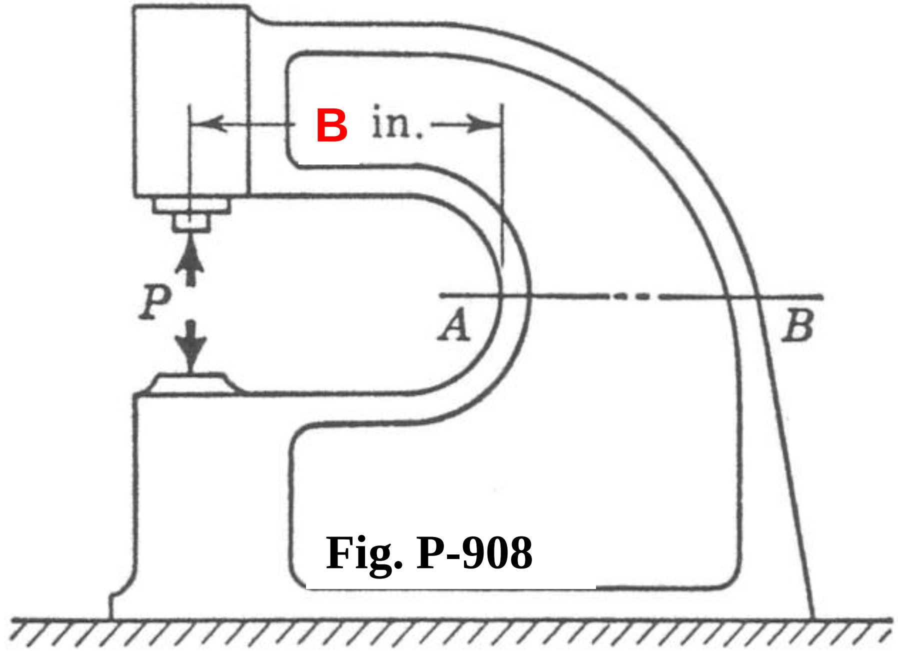
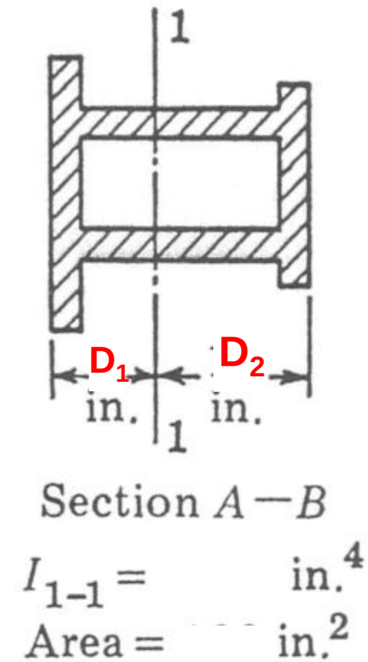


Fig. P-908



ค่า z_1-z_6 ได้จากเลขประจำตัวนิสิต ดังต่อไปนี้

$46z_1z_2z_3z_4z_5z_6$

หมายเหตุ $D_2 = D_1(1+z_2)$ in.

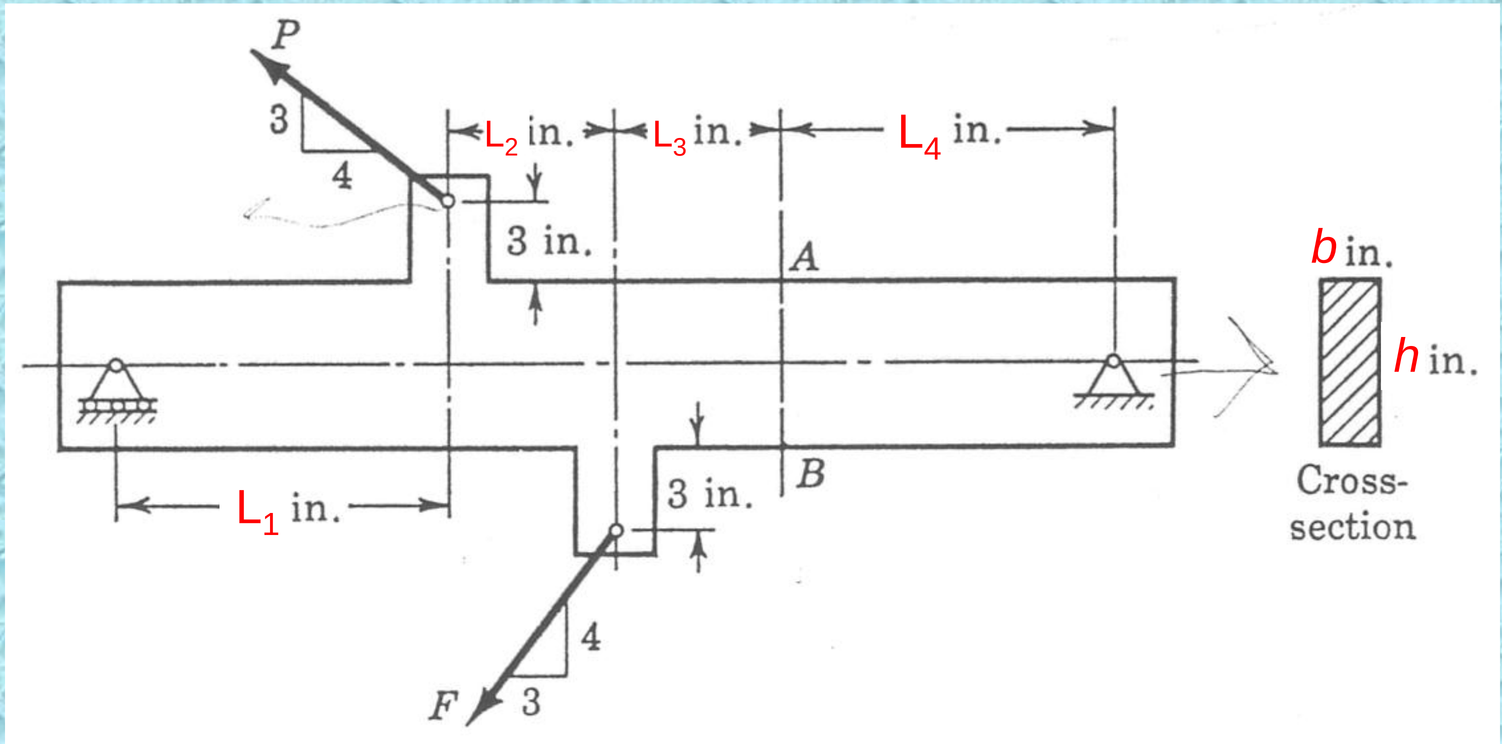
เพื่อให้หน้าตัดมีประสิทธิภาพดีในการรับหน่วยแรง

$D_1 = (1+z_1)$ in. $D_2 = D_1(1+z_2)$ in.

$I_{1\text{-}1} = 1000(1+z_3)$ in^4 Area $= 10(1+z_4)$ in^2

B $= 10(1+z_5)$ in. $\sigma_{allow} = 10(1+z_6)$ ksi.

Hw11 Compute the stresses at *A* and *B* on the link loaded as shown in Fig. P-912



ค่า z_1-z_6 ได้จากเลขประจำตัวนิสิต ดังต่อไปนี้

$46z_1z_2z_3z_4z_5z_6$

<u>หมายเหตุ</u> $h = b(1+z_6)$ in.

เพื่อให้คานมีความลึกไม่น้อยกว่าความกว้างเสมอ

$L_1 = (1+z_1)$ in.	$L_2 = (1+z_2)$ in.
$L_3 = (1+z_3)$ in.	$L_4 = (1+z_4)$ in.
$b = 0.2(1+z_5)$ in.	$h = b(1+z_6)$ in.
$P = (1+z_5)$ kips.	$F = (1+z_6)$ kips.

9-3 Kern of Section: Loads Applied off Axes of Symmetry

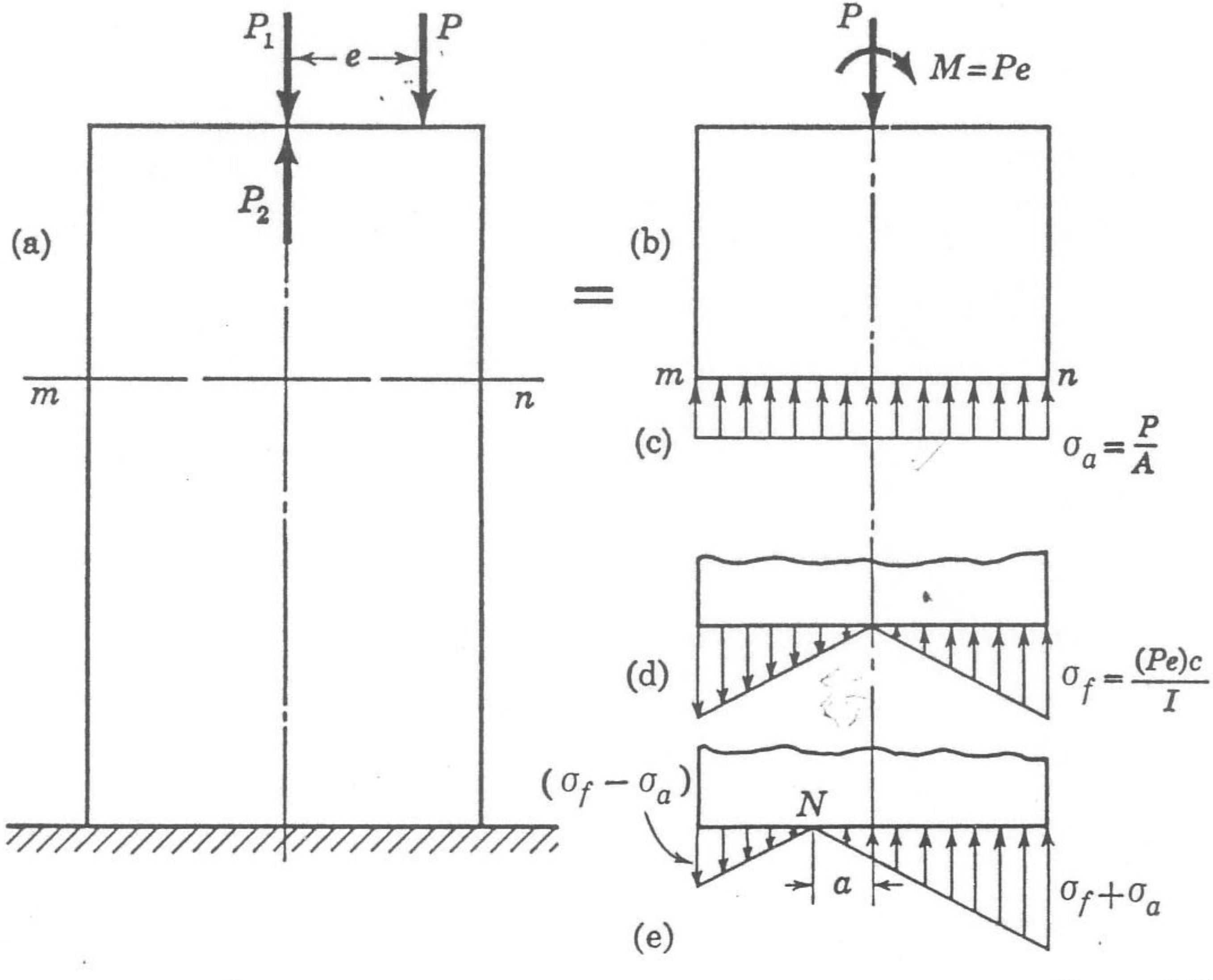


Figure 9-5

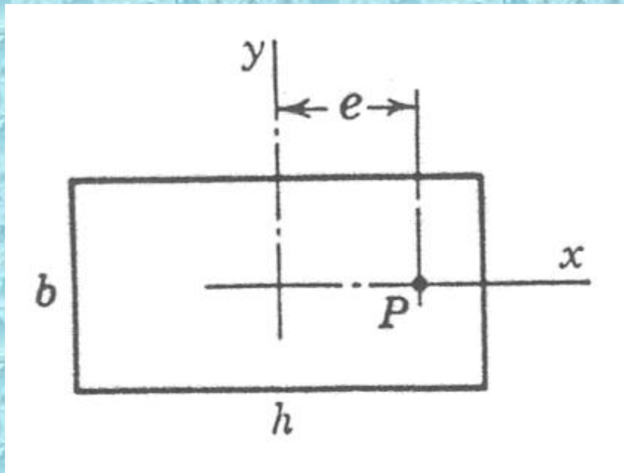
$$\frac{P}{A} = \frac{My}{I} = \frac{(Pe)a}{I}$$

$$a = \frac{I}{Ae}$$

for b × h section

$$\frac{h}{2} = \frac{(bh^3/12)}{bh \times e}$$

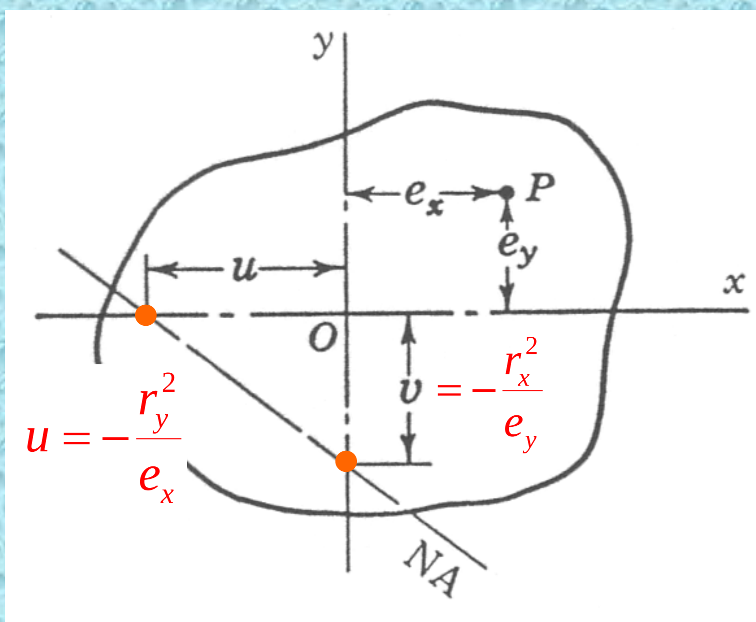
$$e = \frac{h}{6}$$



The maximum eccentricity to avoid tension

$$e = \frac{h}{6}$$

That is in designing of masonry or other structures weak in tension, the resultant load should fall in the middle third of the section.



The general case:

$$\sigma = -\frac{P}{A} - \frac{(Pe_x)x}{I_y} - \frac{(Pe_y)y}{I_x}$$

The position of neutral axis (line of zero stress)

$$0 = -\frac{P}{A} - \frac{(Pe_x)x}{Ar_y^2} - \frac{(Pe_y)y}{Ar_x^2} \quad \Rightarrow \quad 0 = -1 - \frac{e_x}{r_y^2}x - \frac{e_y}{r_x^2}y$$

$$I_x = Ar_x^2$$

$$I_y = Ar_y^2$$

Rectangular section: $0=-\frac{P}{bh}-\frac{(Pe_x)x}{bh^3/12}-\frac{(Pe_y)y}{hb^3/12}$

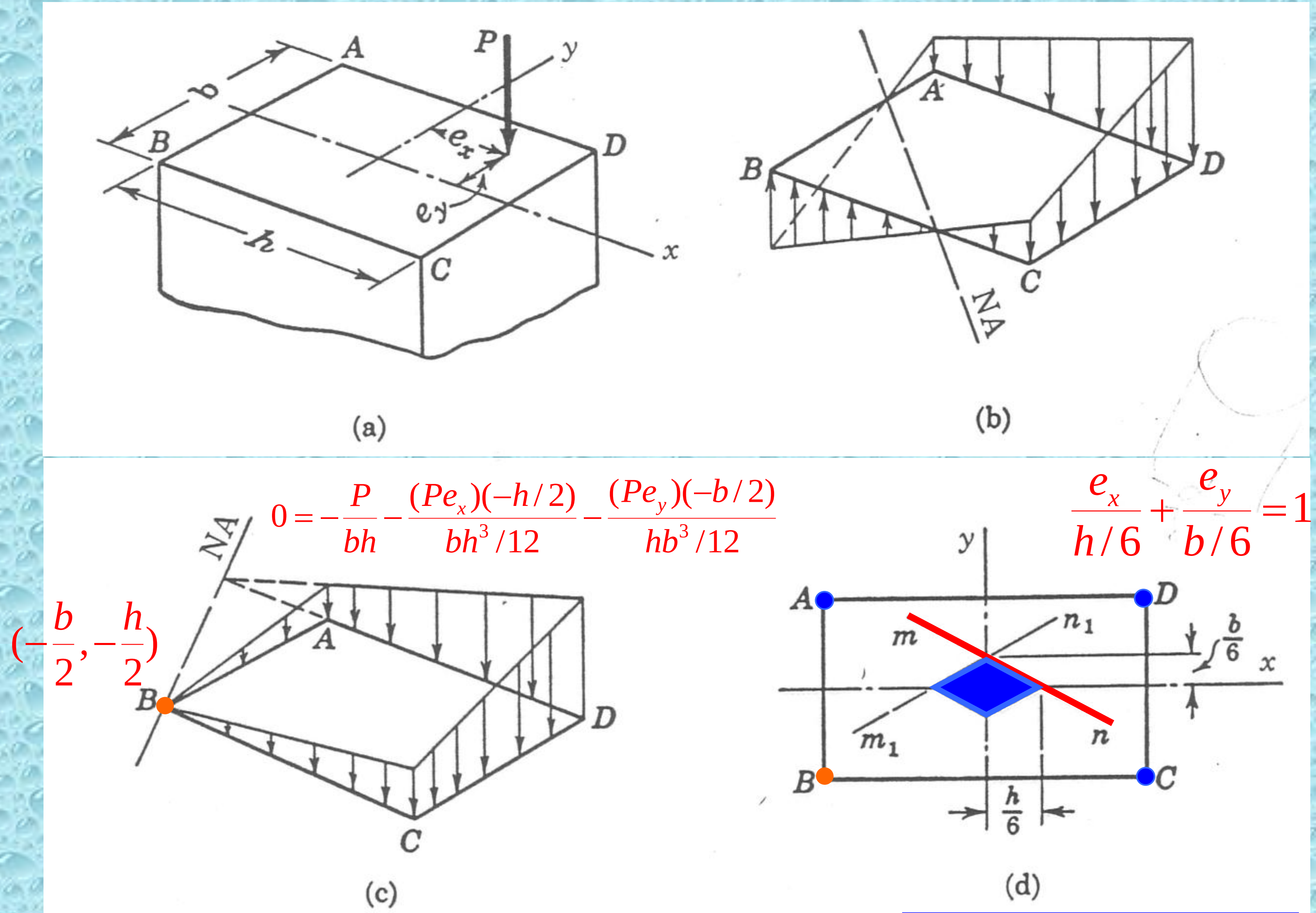
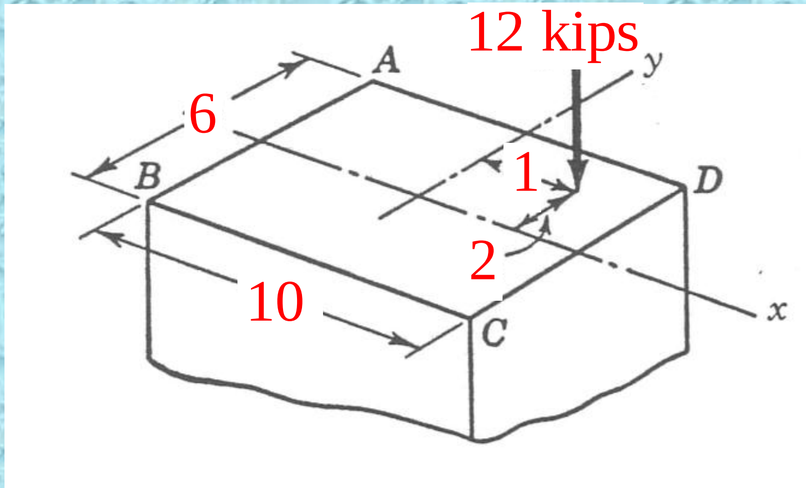


Figure 9-8 Neutral axis for load *P* eccentrically applied and $\boxed{\text{kern of rectangular section.}}$

918 A compressive load P= 12 kips is applied, as in Fig. 9-8a, at a point 1 in. to the right and 2 in. above the centroid of a rectangular section for which h=10 in. and b=6 in. Compute the stress at each corner and the location of the neutral axis. Illustrate the answers with a sketch similar to Fig. 9-8b.



$$\sigma = -\frac{P}{A} - \frac{(Pe_x)x}{I_y} - \frac{(Pe_y)y}{I_x}$$

Rectangular section:

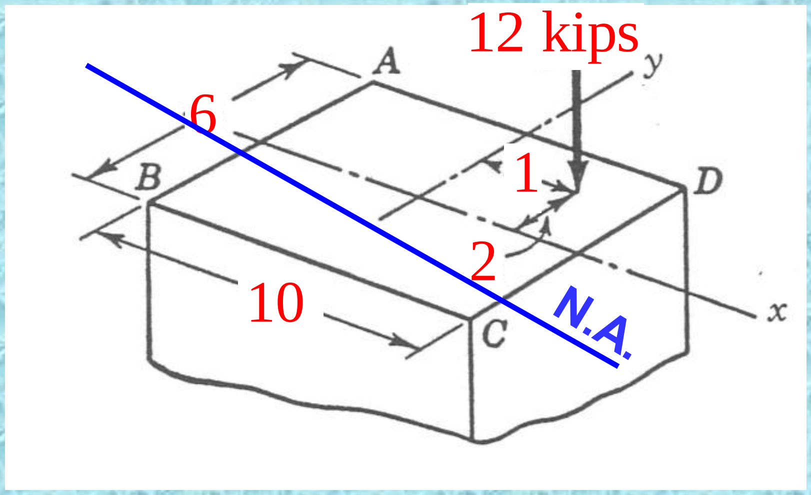
$$\sigma = -\frac{P}{bh} - \frac{(Pe_x)x}{bh^3/12} - \frac{(Pe_y)y}{hb^3/12}$$

$$\sigma_A = -\frac{12}{6\times10} - \frac{(12\times1)(-5)}{6\times10^3/12} - \frac{(12\times2)(3)}{10\times6^3/12} = -0.08 \text{ ksi}$$

$$\sigma_B = -\frac{12}{6\times10} - \frac{(12\times1)(-5)}{6\times10^3/12} - \frac{(12\times2)(-3)}{10\times6^3/12} = 0.72 \text{ ksi}$$

$$\sigma_C = -\frac{12}{6\times10} - \frac{(12\times1)(5)}{6\times10^3/12} - \frac{(12\times2)(-3)}{10\times6^3/12} = 0.48 \text{ ksi}$$

$$\sigma_D = -0.32 \text{ ksi}$$



Position of Neutral Axis:

$$0 = -\frac{P}{bh} - \frac{(Pe_x)x}{bh^3/12} - \frac{(Pe_y)y}{hb^3/12}$$

$$0 = -\frac{12}{6\times 10} - \frac{(12\times 1)(x)}{6\times 10^3/12} - \frac{(12\times 2)(y)}{10\times 6^3/12}$$

$$\frac{3x}{25} + \frac{2y}{3} = -1$$

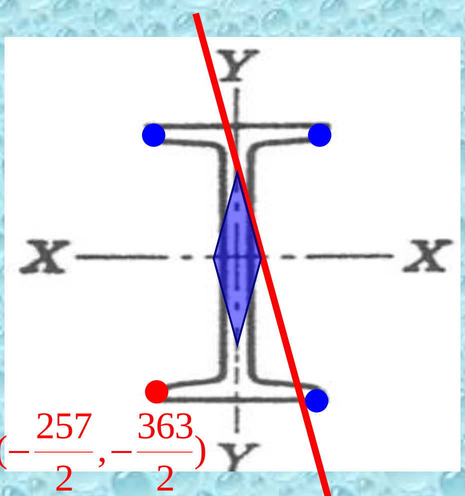
on x axis (y=0) $\Rightarrow x = -25/3 = -8.33$

on y axis (x=0) $\Rightarrow y = -3/2 = -1.5$

921 Calcualte and sketch the kern of a W360 X 122 section.

TABLE B-2 PROPERTIES OF WIDE-FLANGE SECTIONS (W SHAPES): SI UNITS (*Continued*)

Designation	Theoretical mass (kg/m)	Area (mm²)	Depth (mm)	Flange Width (mm)	Flange Thickness (mm)	Web thickness (mm)	Axis X–X I (10^6 mm⁴)	Axis X–X $S = \frac{I}{c}$ (10^3 mm³)	Axis X–X $r = \sqrt{I/A}$ (mm)	Axis Y–Y I (10^6 mm⁴)	Axis Y–Y $S = \frac{I}{c}$ (10^3 mm³)	Axis Y–Y $r = \sqrt{I/A}$ (mm)
W360 ×262	262.7	33 500	387	398	33.3	21.1	894	4 620	163	350	1 760	102
×237	236.3	30 100	380	395	30.2	18.9	788	4 150	162	310	1 570	101
×216	216.3	27 600	375	394	27.7	17.3	712	3 790	161	283	1 430	101
×196	196.5	25 000	372	374	26.2	16.4	636	3 420	159	229	1 220	95.7
×179	179.2	22 800	368	373	23.9	15.0	575	3 120	159	207	1 110	95.3
×162	162.0	20 600	364	371	21.8	13.3	516	2 830	158	186	1 000	95.0
×147	147.5	18 800	360	370	19.8	12.3	463	2 570	157	167	904	94.2
×134	134.0	17 100	356	369	18.0	11.2	415	2 330	156	151	817	94.0
×122	121.7	15 500	363	257	21.7	13.0	365	2 010	153	61.5	478	63.0
×110	110.2	14 000	360	256	19.9	11.4	331	1 840	154	55.7	435	6[illegible]
×101	101.2	12 900	357	255	18.3	10.5	302	1 690	153	50.6	397	[illegible]
×91	90.8	11 600	353	254	16.4	9.5	267	1 510	152	44.8	353	62..
×79	79.3	10 100	354	205	16.8	9.4	227	1 280	150	24.2	236	48.9



$A\left(-\frac{257}{2}, -\frac{363}{2}\right)$

Position of Neutral Axis: $0 = -1 - \frac{e_x}{r_y^2}x - \frac{e_y}{r_x^2}y$

At corner A: $\boxed{0 = -1 + \frac{e_x}{63^2}\frac{257}{2} + \frac{e_y}{153^2}\frac{363}{2}}$

on x-axis (e_y=0): $e_x = \frac{2\times 63^2}{257} = 30.89$ mm

on y-axis (e_x=0): $e_y = \frac{2\times 153^2}{363} = 129.0$ mm

➢ Variation of Stress with Inclination of Element

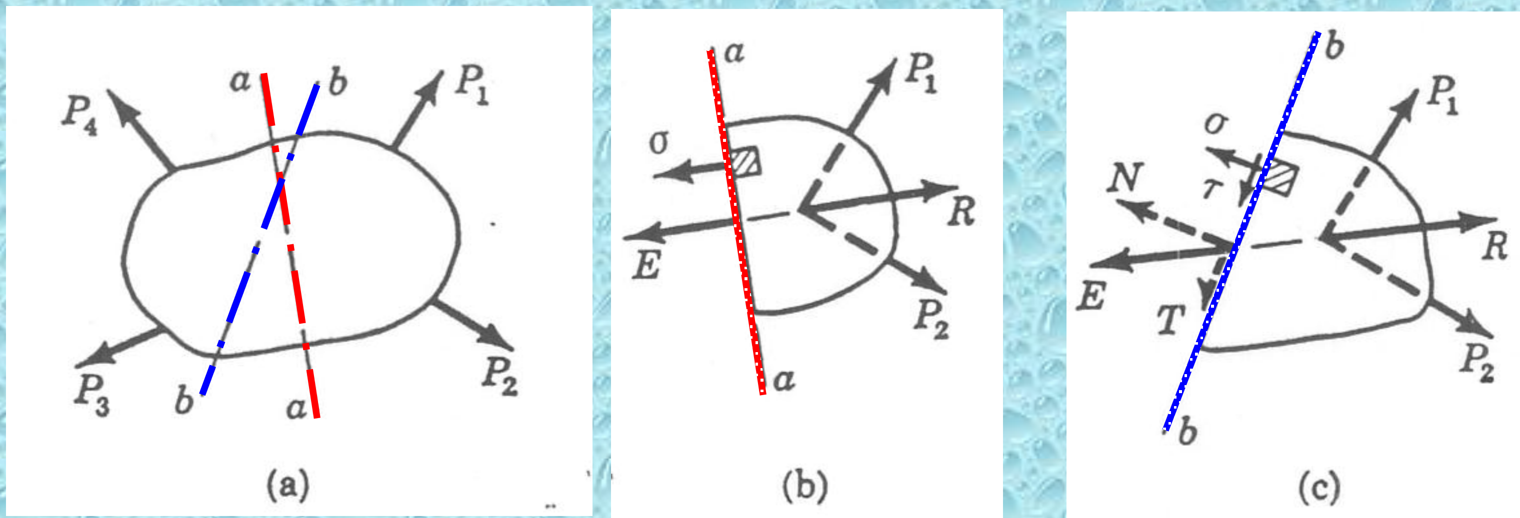
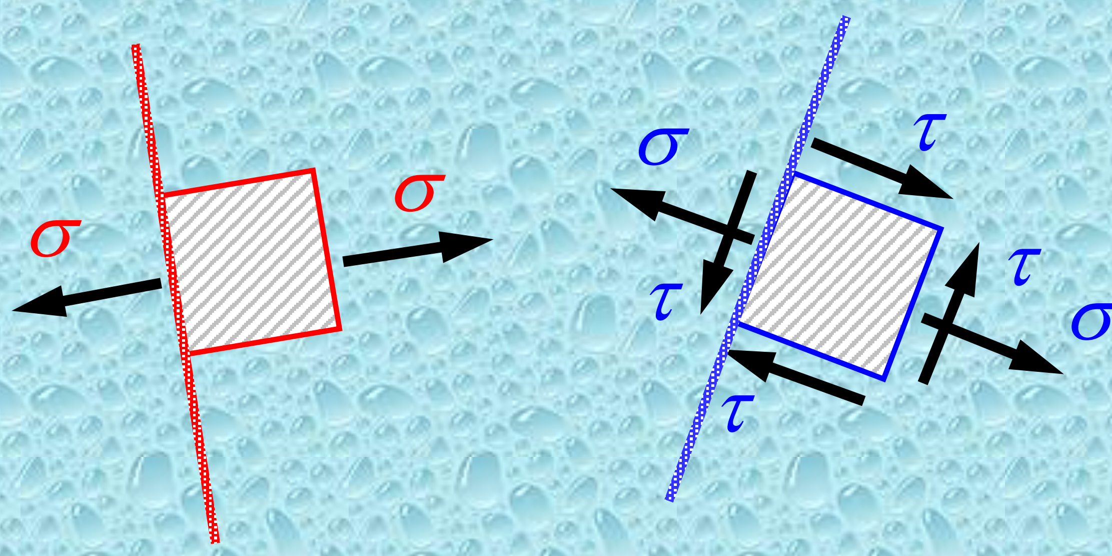
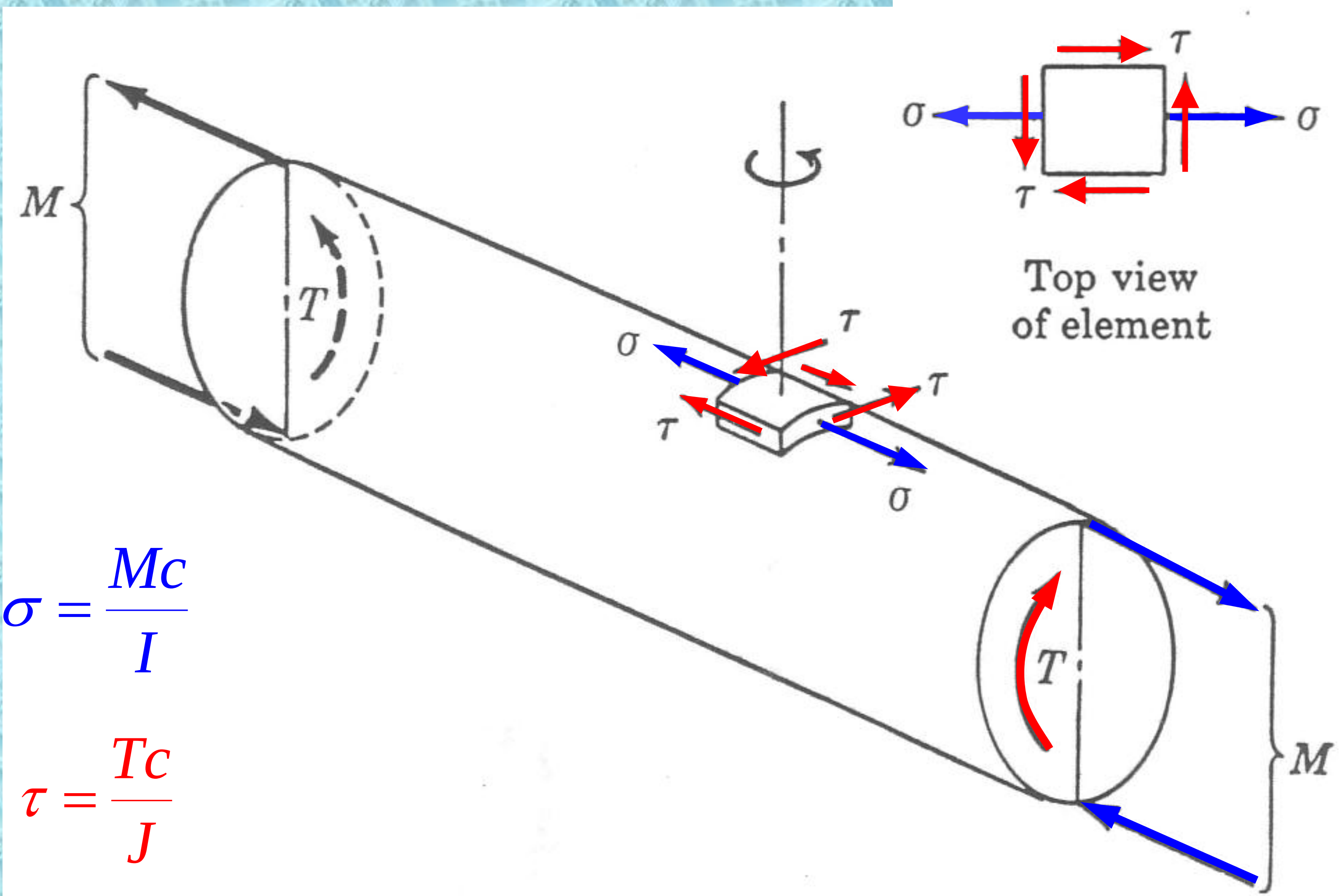


Figure 9-9 Stress at a point varies with inclination of plane through point.



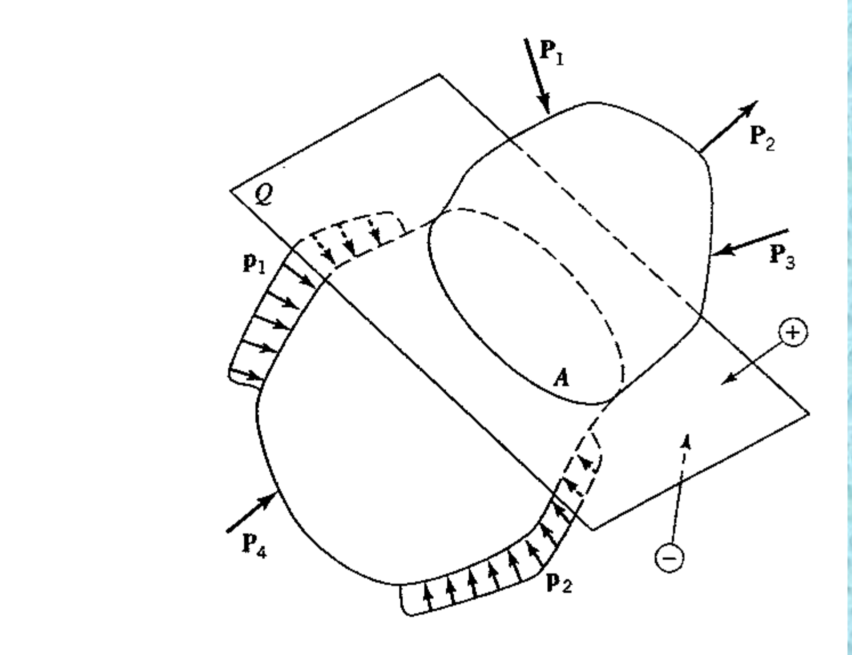


$$\sigma = \frac{Mc}{I}$$

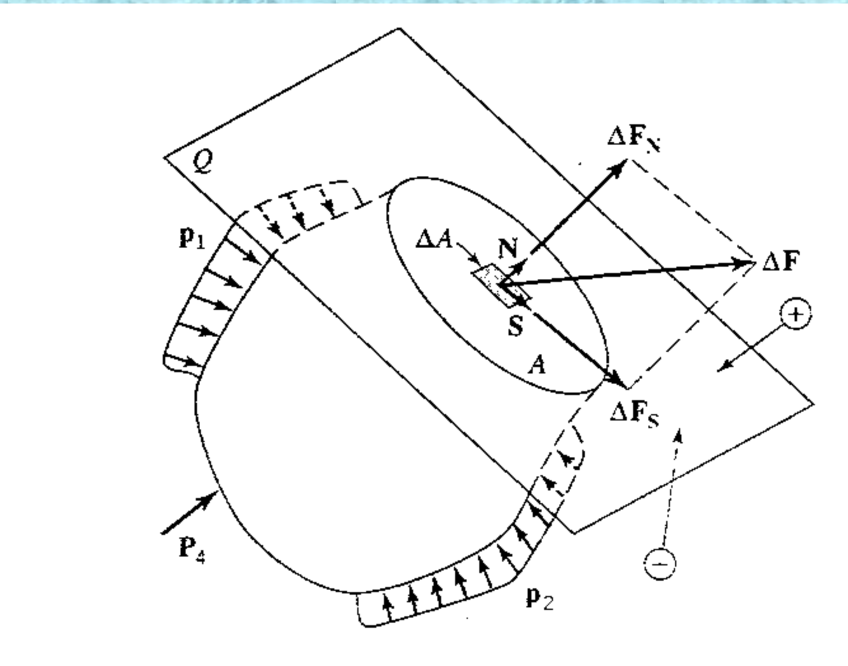
$$\tau = \frac{Tc}{J}$$

Figure 9-10 Stresses caused by simultaneous flexure and torsion.

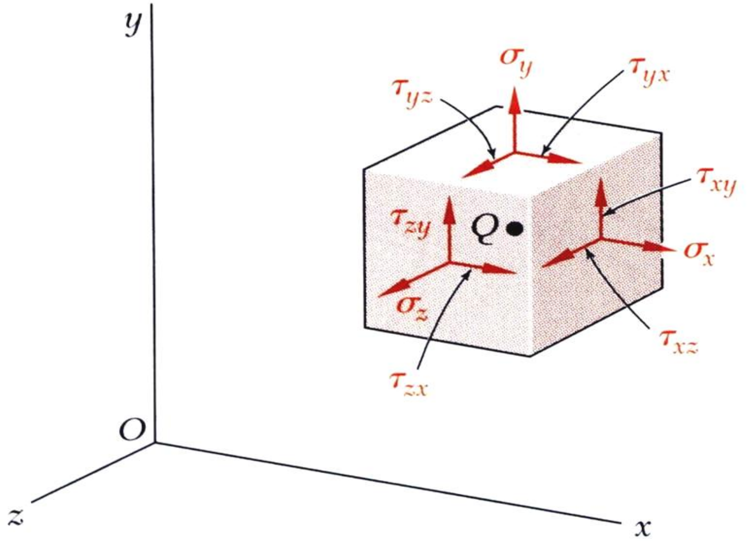
➢ Stress at A Point



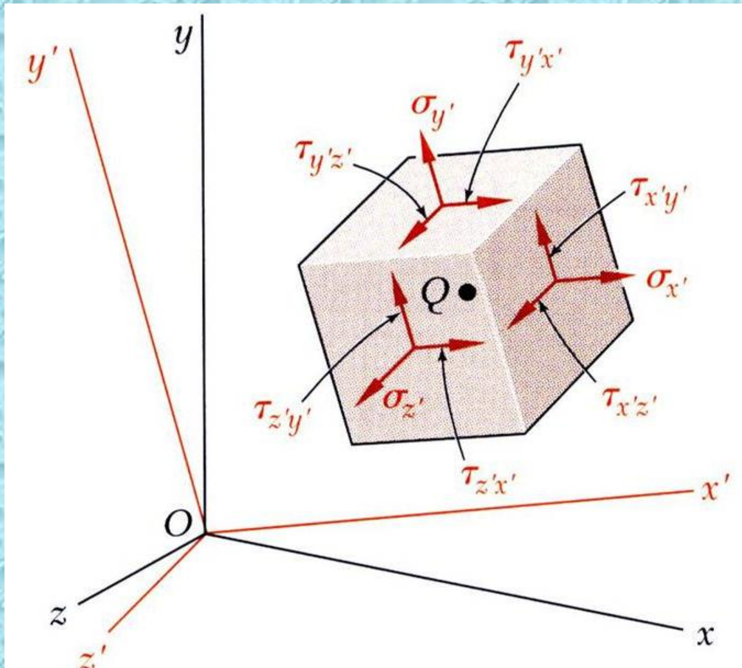
A general loaded body cut by plane Q.



Force transmitted through incremental area of cut body.



Stress at a point really defines the uniform stress distributed over a differential area.



- The most general state of stress at a point may be represented by 6 components,

$\sigma_x, \sigma_y, \sigma_z$ normal stresses

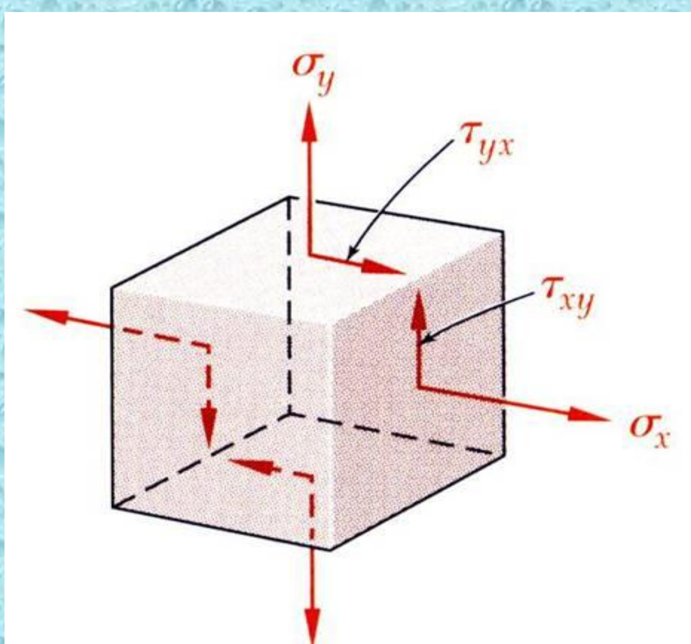
$\tau_{xy}, \tau_{yz}, \tau_{zx}$ shearing stresses

$(\text{Note}: \tau_{xy} = \tau_{yx}, \tau_{yz} = \tau_{zy}, \tau_{zx} = \tau_{xz})$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

symmetry

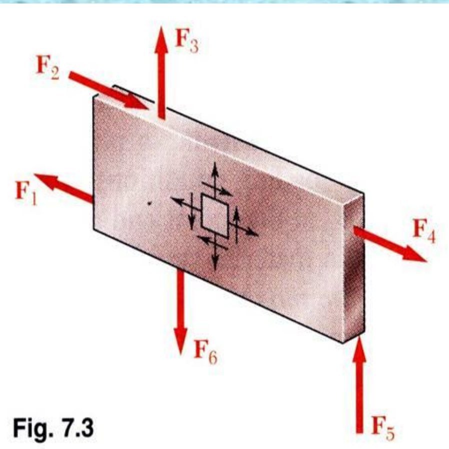
state of stress เมื่อแสดงด้วยระบบโคออร์ดิเนต (xyz)



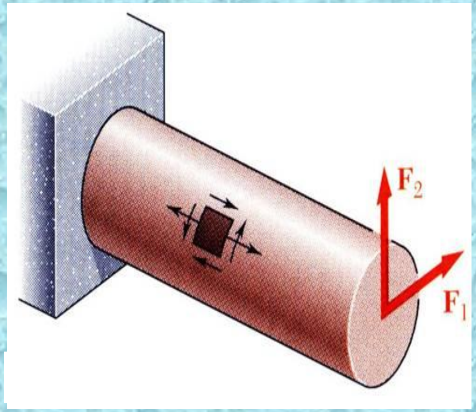
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma'_{xx} & \sigma'_{xy} & \sigma'_{xz} \\ \sigma'_{yx} & \sigma'_{yy} & \sigma'_{yz} \\ \sigma'_{zx} & \sigma'_{zy} & \sigma'_{zz} \end{bmatrix} = \begin{bmatrix} \sigma'_x & \tau'_{xy} & \tau'_{xz} \\ \tau'_{yx} & \sigma'_y & \tau'_{yz} \\ \tau'_{zx} & \tau'_{zy} & \sigma'_z \end{bmatrix}$$

symmetry

state of stress เมื่อแสดงด้วยระบบโคออร์ดิเนต (xyz)



- *Plane Stress* - state of stress in which two faces of the cubic element are free of stress. For the illustrated example, the state of stress is defined by
  $\sigma_x, \sigma_y, \tau_{xy}$ and $\sigma_z = \tau_{zx} = \tau_{zy} = 0.$



- State of plane stress occurs in a thin plate subjected to forces acting in the midplane of the plate.

$$(\sigma_n, \tau_n)$$

- State of plane stress also occurs on the free surface of a structural element or machine component, i.e., at any point of the surface not subjected to an external force.

Plane Stress

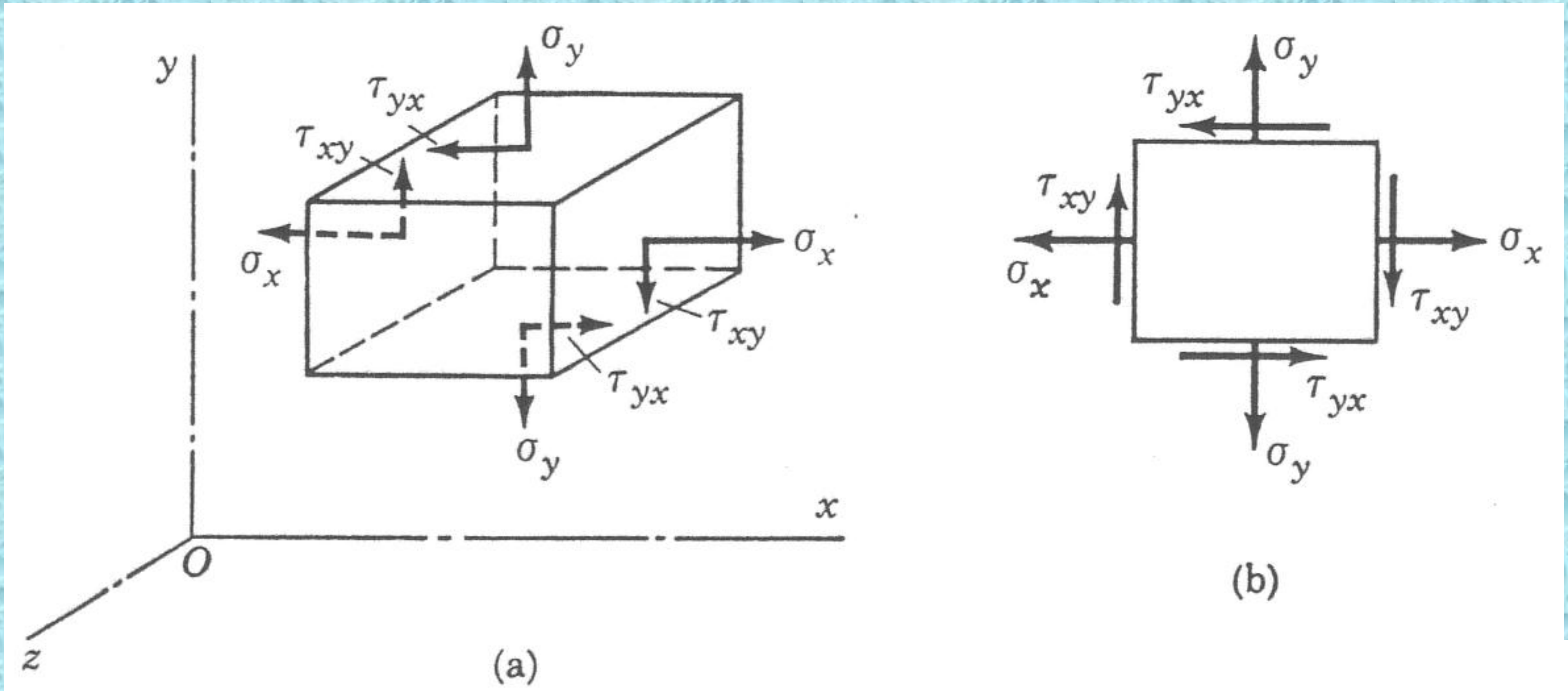
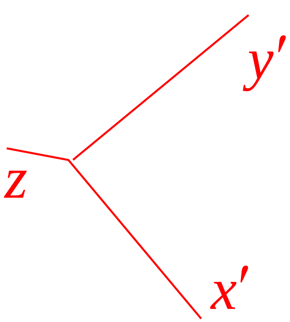
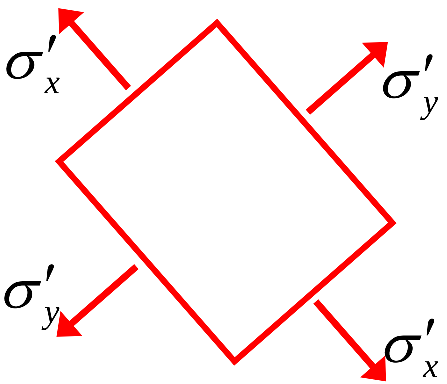


Figure 9-12 Stress components.



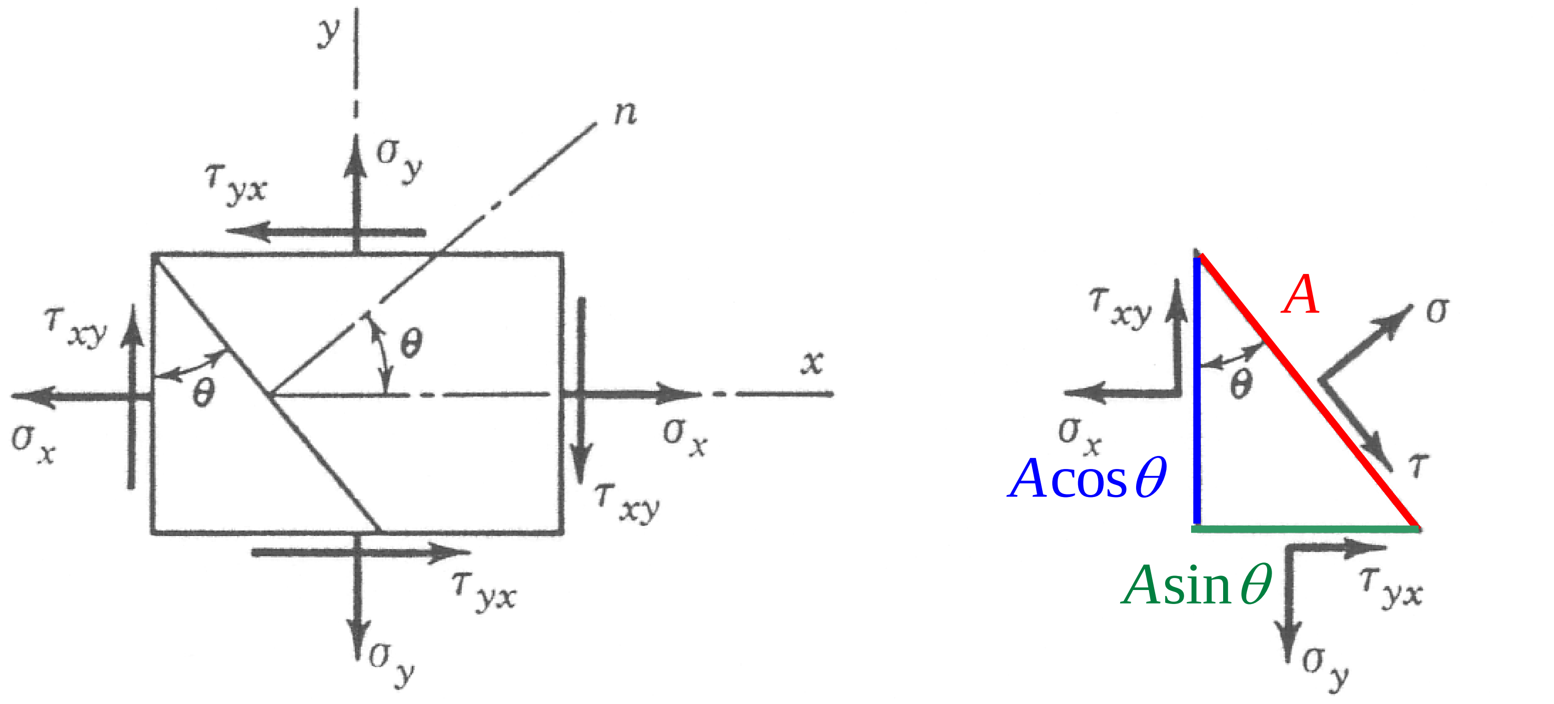


Two methods to compute the maximum stresses i.e.,

(1) Analytical approach

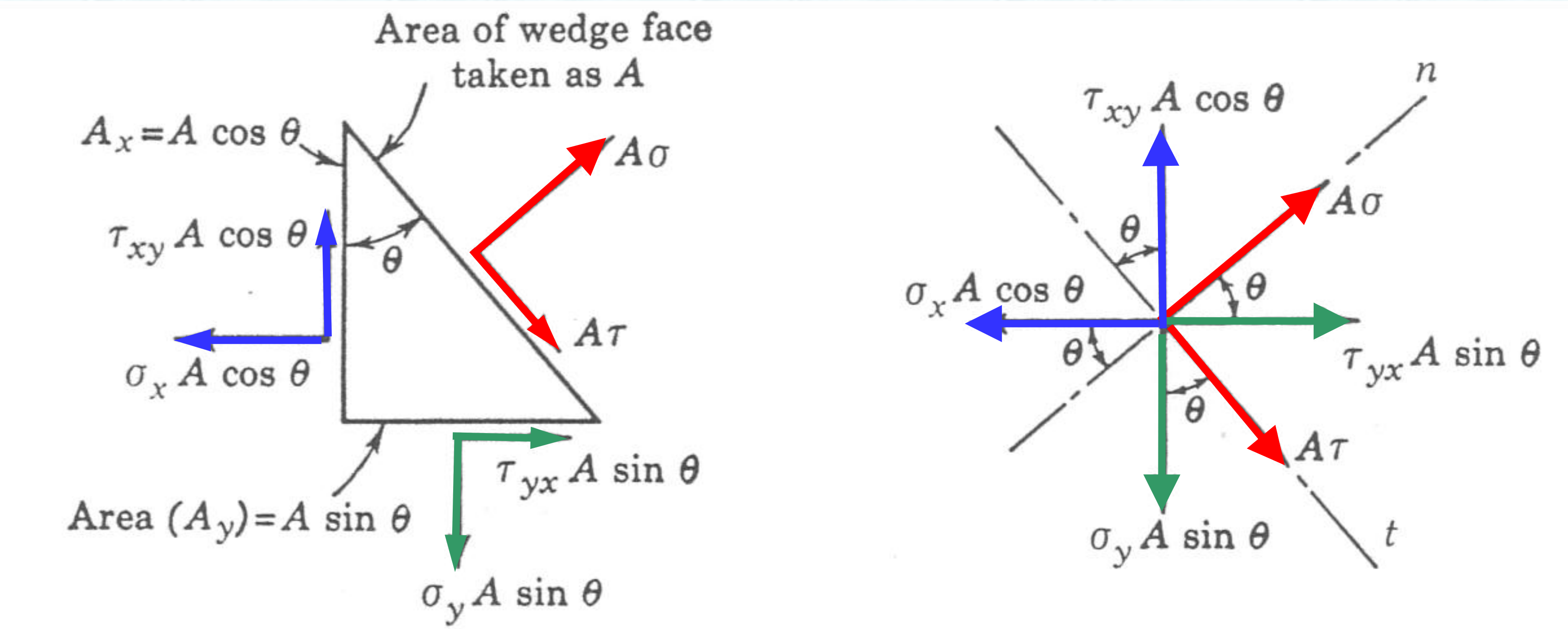
(2) Using of Mohr's circle

➢ Variation of Stress at A Point: Analytical Derivation



(a) Original state of stress

(b) Stresses acting on wedge



(c) Free-body diagram of forces on wedge

(d) Point diagram of forces

$$\boxed{\sum F_n = 0} \quad \sigma A = (\sigma_x A\cos\theta)\cos\theta + (\sigma_y A\sin\theta)\sin\theta - (\tau_{xy} A\cos\theta)\sin\theta$$
$$-(\tau_{yx} A\sin\theta)\cos\theta$$

$$\boxed{\sum F_t = 0} \quad \tau A = (\sigma_x A\cos\theta)\sin\theta - (\sigma_y A\sin\theta)\cos\theta + (\tau_{xy} A\cos\theta)\cos\theta$$
$$-(\tau_{yx} A\sin\theta)\sin\theta$$

$$\boxed{\sum F_n = 0} \quad \sigma A = (\sigma_x A\cos\theta)\cos\theta + (\sigma_y A\sin\theta)\sin\theta - (\tau_{xy} A\cos\theta)\sin\theta$$

$$- (\tau_{yx} A\sin\theta)\cos\theta$$

$$\sigma = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

$$\boxed{\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta}$$

$$\boxed{\sum F_t = 0} \quad \tau A = (\sigma_x A\cos\theta)\sin\theta - (\sigma_y A\sin\theta)\cos\theta + (\tau_{xy} A\cos\theta)\cos\theta$$

$$- (\tau_{yx} A\sin\theta)\sin\theta$$

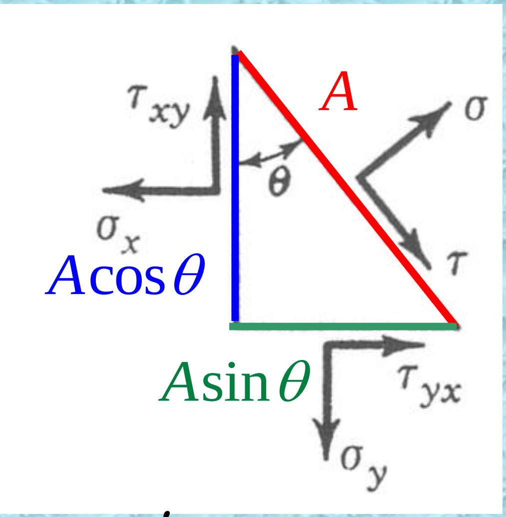
$$\tau = \sigma_x \cos\theta\sin\theta - \sigma_y \sin\theta\cos\theta + \tau_{xy}\cos^2\theta - \tau_{yx}\sin^2\theta$$

$$\boxed{\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta}$$

Note: $\tau_{xy} = \tau_{yx}$, $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$, $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$, $\cos\theta\sin\theta = \dfrac{\sin 2\theta}{2}$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

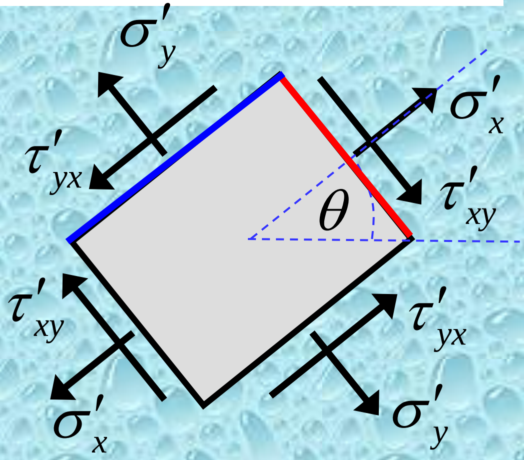
$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau'_{xy} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

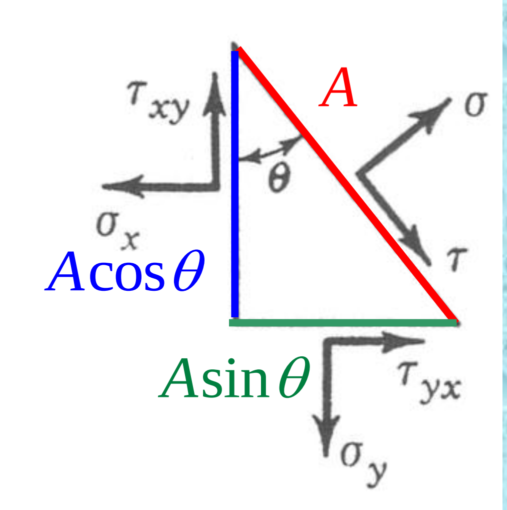


$$\cos 2(\tfrac{\pi}{2} + \theta) = \cos(\pi + 2\theta) = -\cos 2\theta$$

$$\sin 2(\tfrac{\pi}{2} + \theta) = \sin(\pi + 2\theta) = -\sin 2\theta$$

Eq.(9-5) $$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Eq.(9-6) $$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



Find maximum or minimum σ differentiating Eq.(9-5) w.r.t. θ and setting the derivative equal to zero

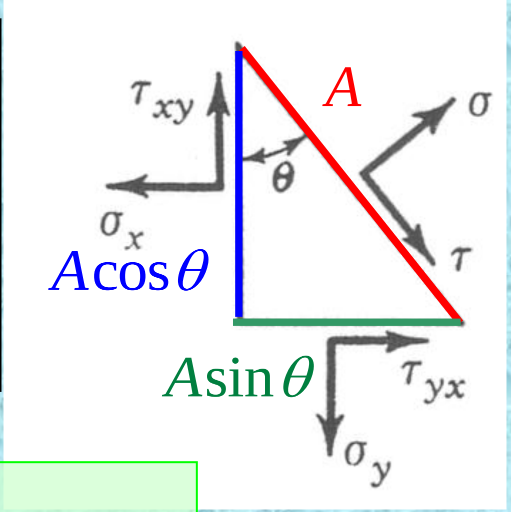
$$\frac{d\sigma}{d\theta} = -2\frac{\sigma_x - \sigma_y}{2}\sin 2\theta - 2\tau_{xy}\cos 2\theta = 0 \quad \Rightarrow \quad \tan 2\theta = -\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Find maximum or minimum τ differentiating Eq.(9-6) w.r.t. θ and setting the derivative equal to zero

$$\frac{d\tau}{d\theta} = 2\frac{\sigma_x - \sigma_y}{2}\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0 \quad \Rightarrow \quad \tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Eq.(9-5) $$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Eq.(9-6) $$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



At zero shearing stress $\tau = 0$

$$0 = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \rightarrow \tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y}$$

ซึ่งเป็นมุมเดียวกับสมการ Eq.(9-7) ดังนั้น ค่า maximum or minimum σ จะเกิดขึ้นเมื่อ $\tau = 0$

$$\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x - \sigma_y}$$

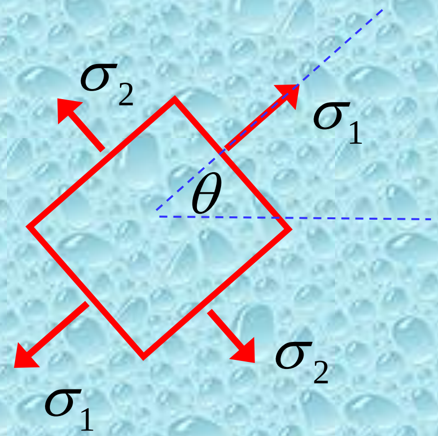
$$\sin 2\theta_1 = \frac{-\tau_{xy}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}}, \quad \cos 2\theta_1 = \frac{\sigma_x - \sigma_y}{2\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}}$$

$$\sin 2\theta_2 = \frac{\tau_{xy}}{\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}}, \quad \cos 2\theta_2 = \frac{\sigma_y - \sigma_x}{2\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}}$$

Maximum or minimum σ (Principal stresses)

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\tan 2\theta = \frac{-2\tau_{xy}}{\sigma_x-\sigma_y}$$

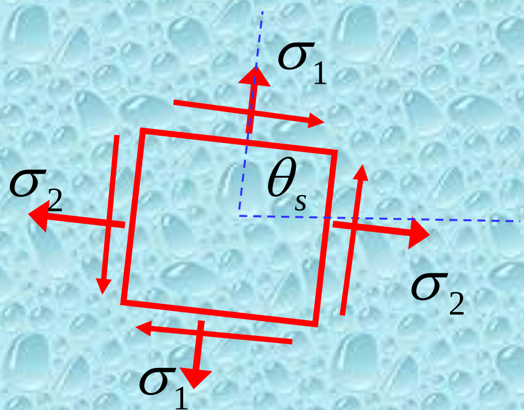


มุม θ และ θ_s ต่างกัน 45°

Maximum or minimum τ

$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \pm\frac{\sigma_1-\sigma_1}{2}$$

$$\tan 2\theta_s = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}$$



925. Two wooden joists 50 mm $\times$ 100 mm are glued together along the joint AB as shown in Fig. P-925. Determine the normal stress and shearing stress in the glue if $P = 200$ kN.

Ans. $\sigma = 16.5$ MPa

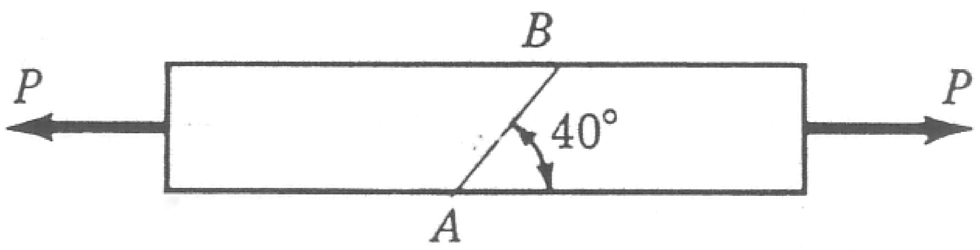


Figure P-925

$$\sigma_x = \frac{P}{A} = \frac{200}{50\times100} = 0.04 \text{ kN/mm}^2 = 40 \text{ MPa}, \quad \sigma_y = 0, \quad \tau_{xy} = 0$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2(-40^{\circ}) - \tau_{xy}\sin 2(-40^{\circ})$$

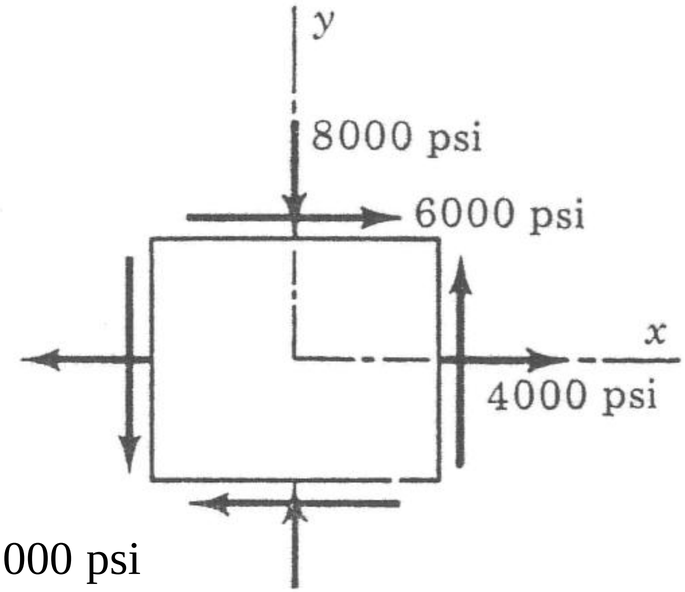
$$= \frac{40+0}{2} + \frac{40-0}{2}\cos 2(-40^{\circ}) - 0\times\sin 2(-40^{\circ}) = 16.5 \text{ MPa}$$

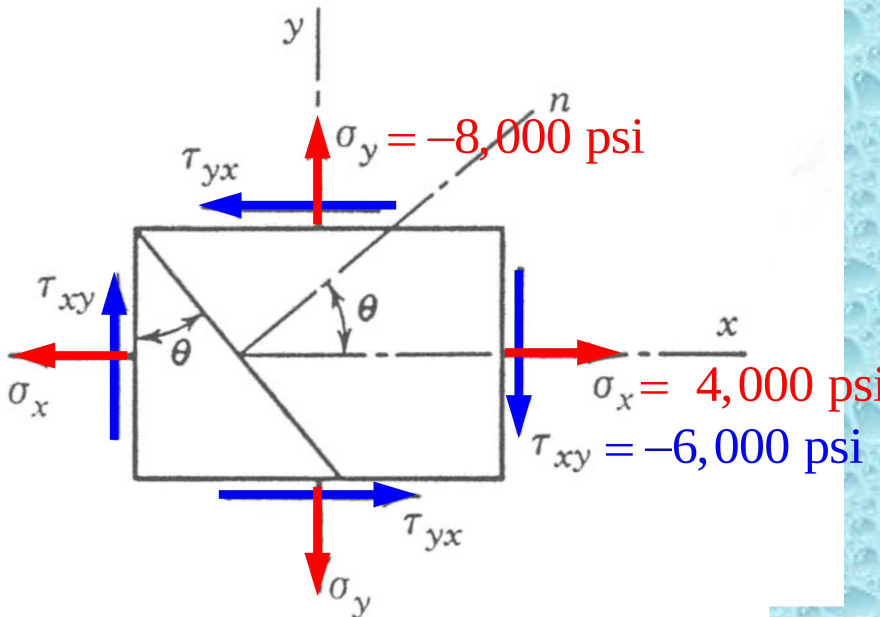
$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{20-0}{2}\sin 2(-40^{\circ}) + 0\times\cos 2(-40^{\circ}) = 9.85 \text{ MPa}$$

926. If an element is subjected to the state of stress shown in Fig. P-926, find the principal stresses. Also, compute the stress components on a plane at 30° counterclockwise from the x face.

Ans. $\sigma_1 = 6480$ psi; $\sigma_2 = -10\,480$ psi; $\sigma = 6200$ psi; $\tau = 2190$ psi





$$\sigma_x = 4{,}000 \text{ psi}$$

$$\sigma_y = -8{,}000 \text{ psi}$$

$$\tau_{xy} = -6{,}000 \text{ psi}$$

$$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right\} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \frac{4000+(-8000)}{2} \pm \sqrt{\left(\frac{4000-(-8000)}{2}\right)^2+(-6000)^2}$$

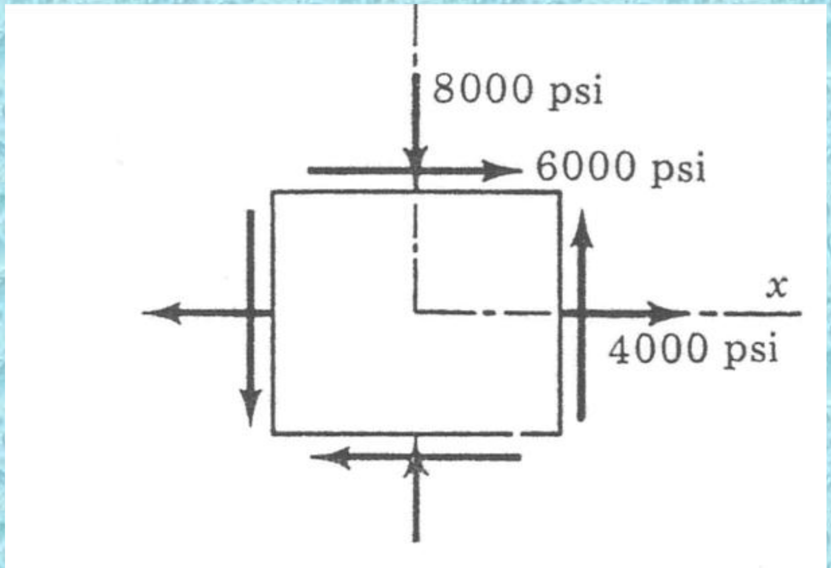
$$= -2000 \pm \sqrt{(6000)^2+(-6000)^2} = -10485.3,\ 6485.3 \text{ psi}$$

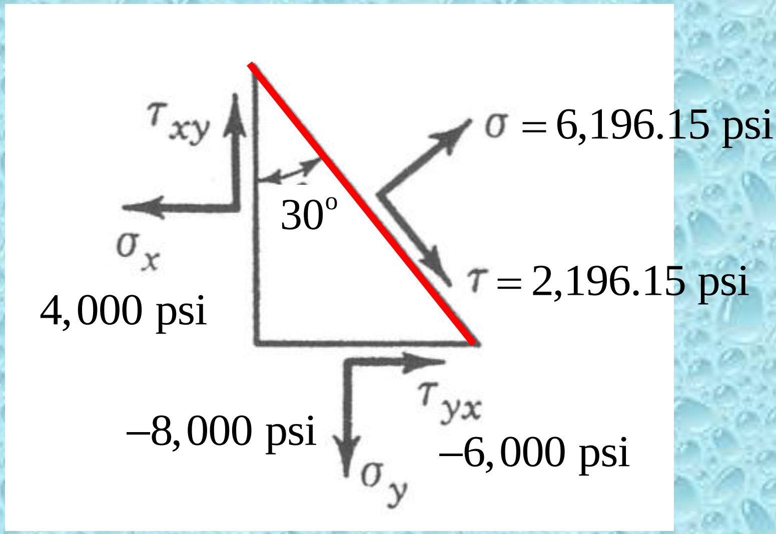
$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2(30^{O}) - \tau_{xy}\sin 2(30^{O})$$

$$= \frac{4000 + (-8000)}{2} + \frac{4000 - (-8000)}{2}\cos 2(30^{O}) - (-6000)\times \sin 2(30^{O}) = 6{,}196.15 \text{ psi}$$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= \frac{4000 - (-8000)}{2}\sin 2(30^{O}) + (-6000)\times \cos 2(30^{O}) = 2196.15 \text{ psi}$$





9-7 Variation of Stress at A Point: Mohr's Circle

Otto Mohr (1882)

Eq.(9-5)
$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Eq.(9-6)
$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (a)$$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (b)$$

Eq.(a)^2 + Eq.(b)^2

$$\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2 \qquad (c)$$

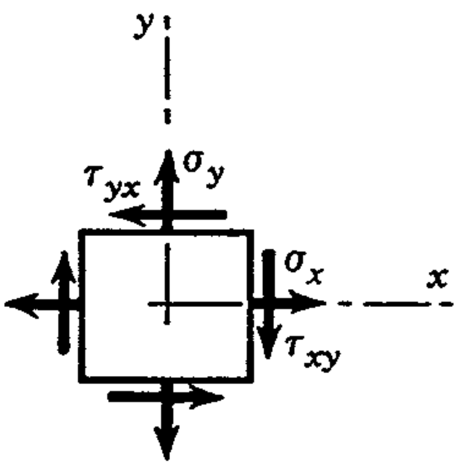
$$(\sigma - C)^2 + \tau^2 = R^2$$

$$\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2 \qquad (c)$$

$$(\sigma - C)^2 + \tau^2 = R^2$$

$$\boxed{C = \frac{\sigma_x + \sigma_y}{2}}$$

$$\boxed{R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$



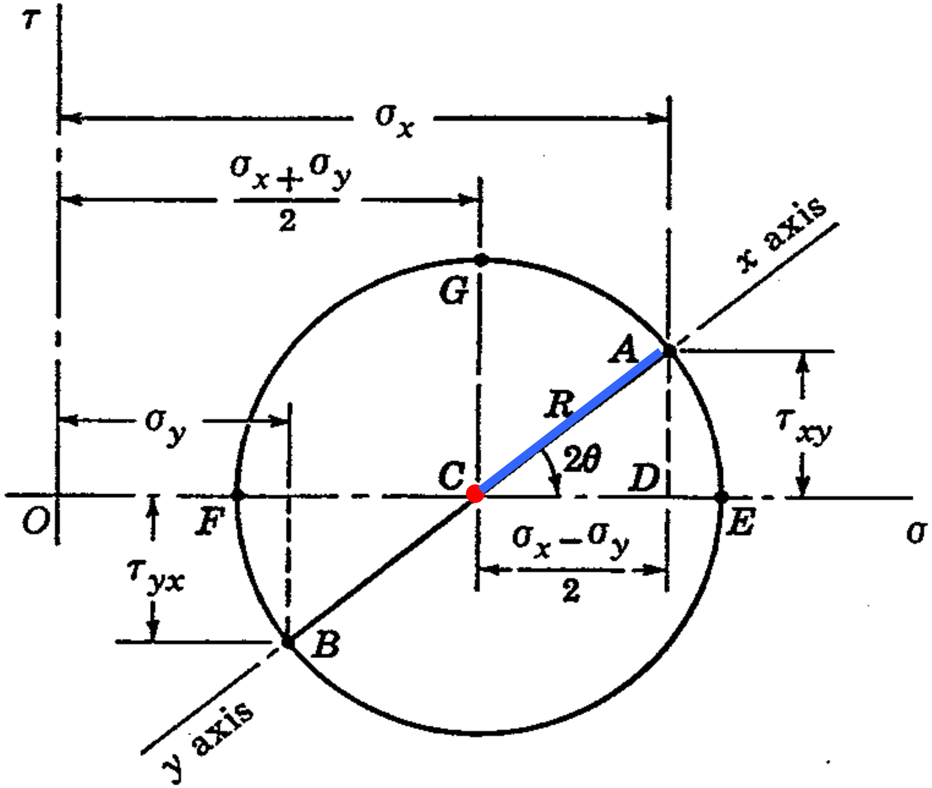
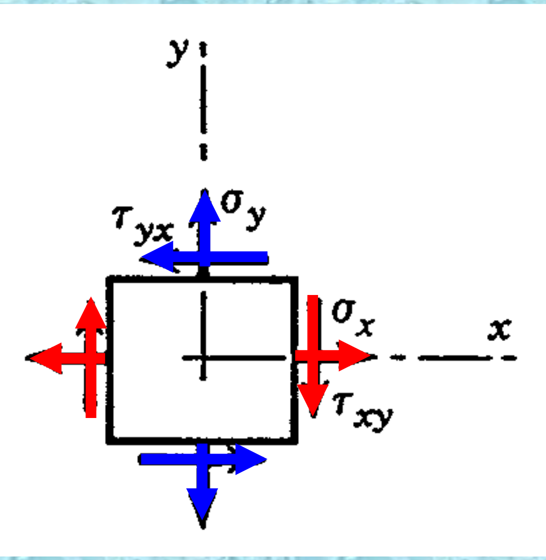
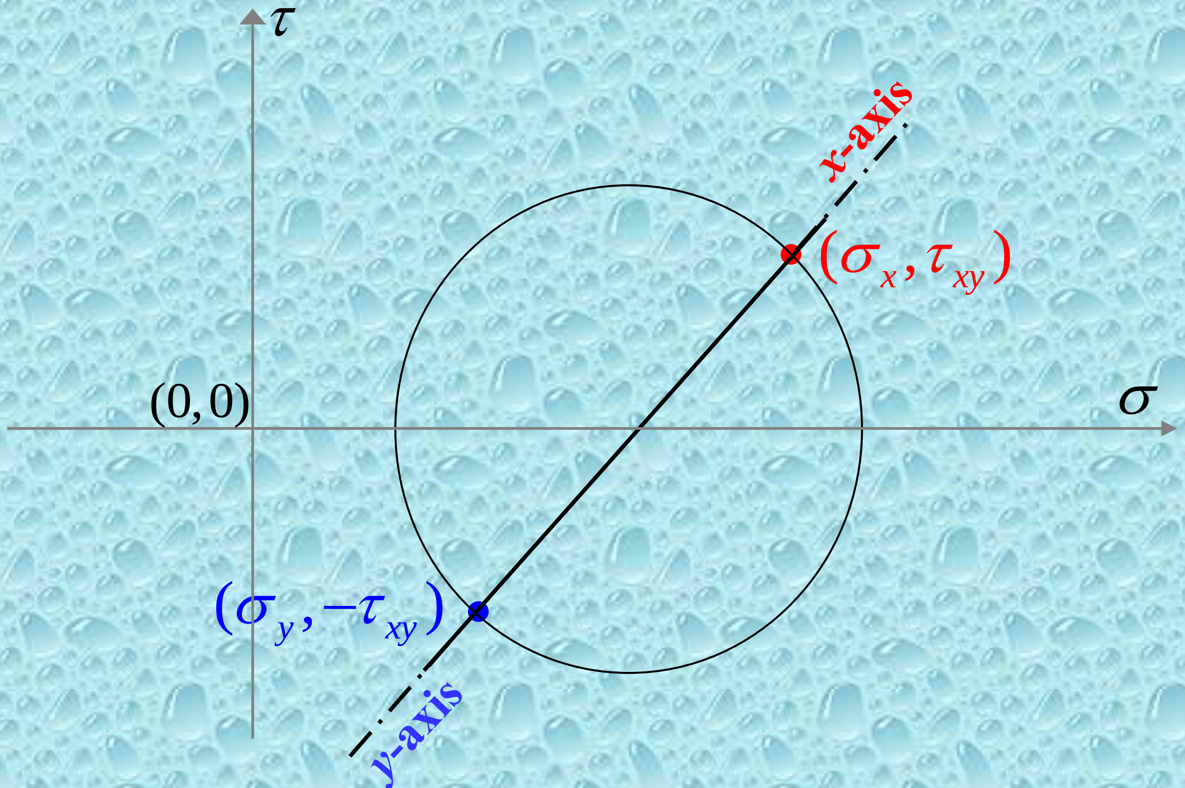


Figure 9-14 Mohr's circle for general state of plane stress.

Rule for Applying Mohr Circle to Combined Stresses

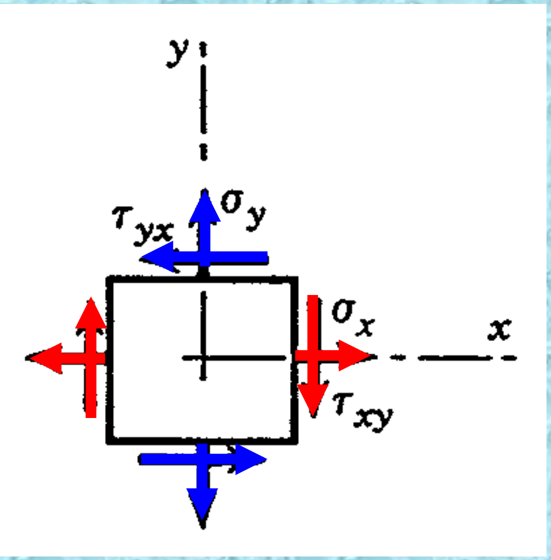
1. On rectangular σ–τ axes, plot points having the coordinates (σ_x, τ_{xy}) and (σ_y, τ_{yx}). These points represent the normal and shearing stresses acting on the x and y faces of an element for which the stresses are known. In plotting these points, assume tension as plus, compression as minus, and shearing stress as plus when its moment about the center of the element is clockwise.*

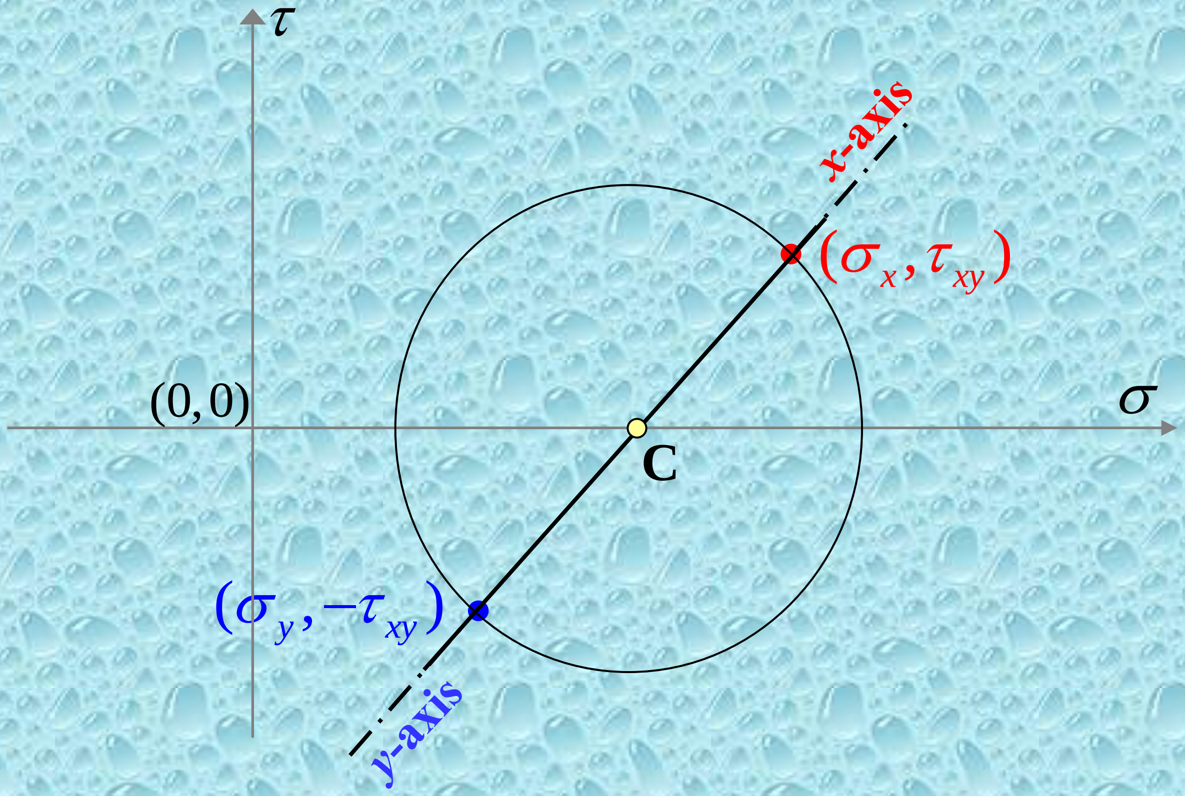




2. Join the points just plotted by a straight line. This line is the diameter of a circle whose center is on the σ axis.

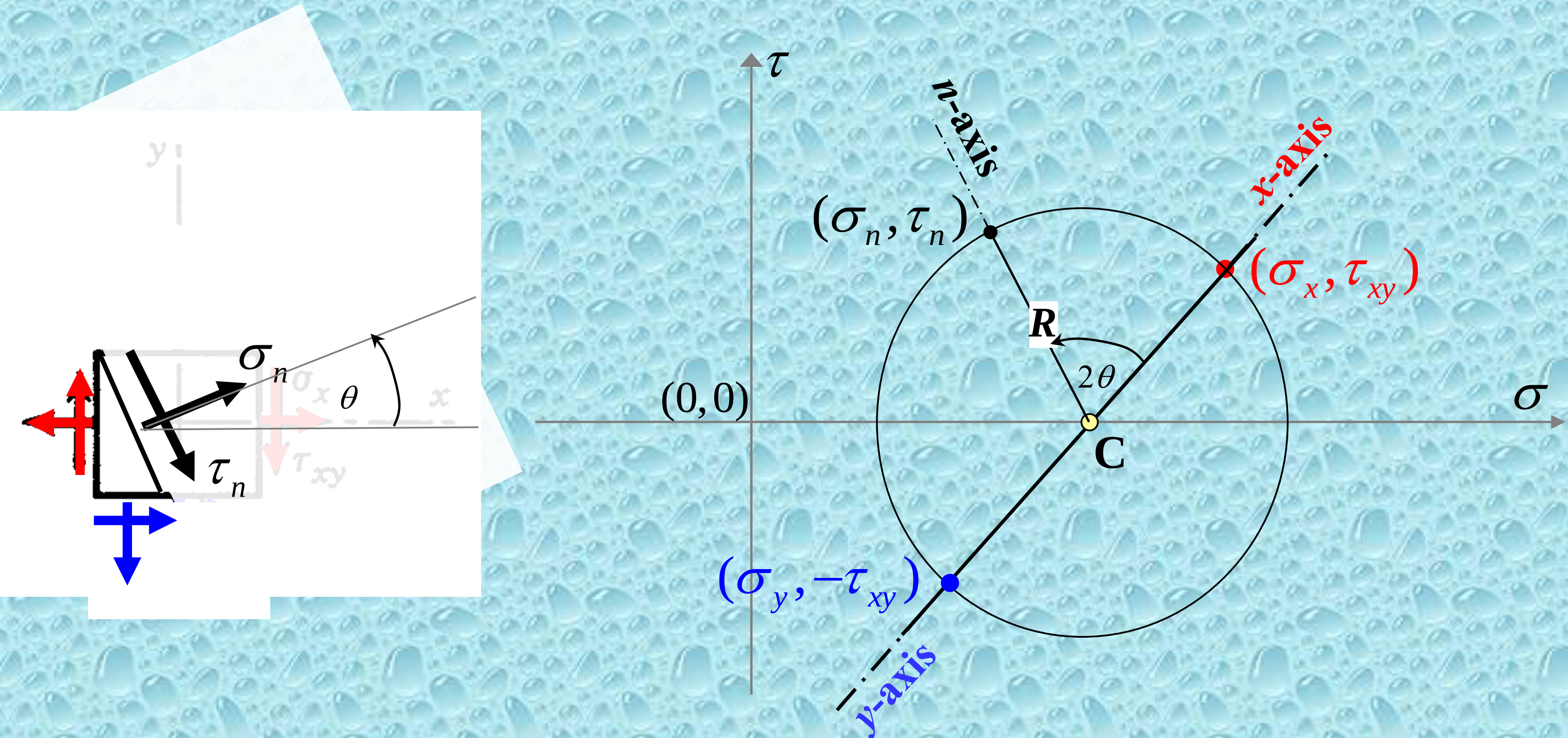
3. As different planes are passed through the selected point in a stressed body, the normal and shearing stress components on these planes are represented by the coordinates of points whose position shifts around the circumference of Mohr's circle.



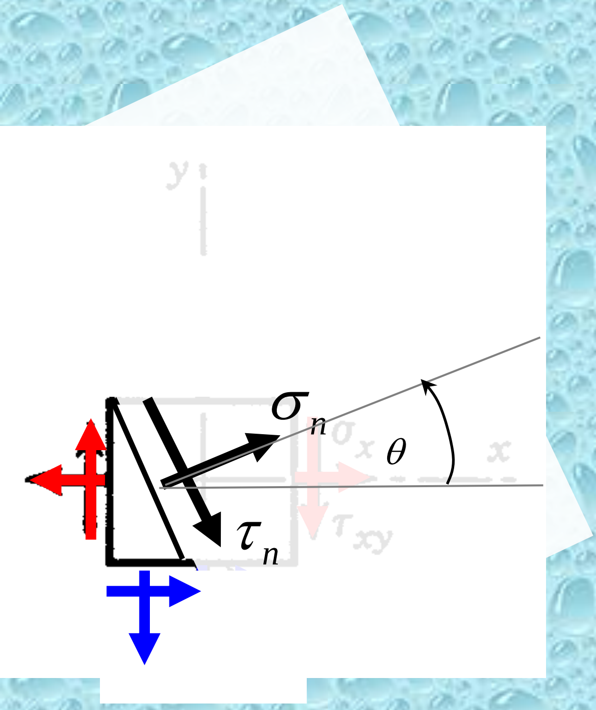


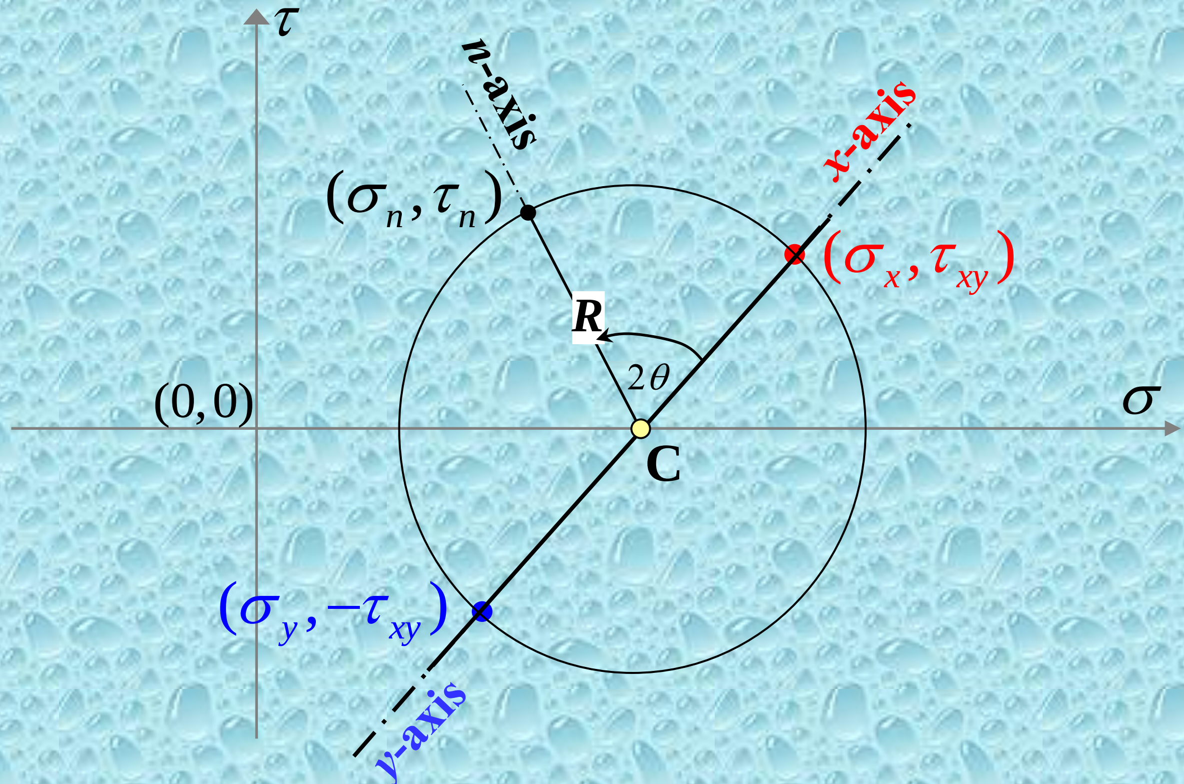
4. The radius of the circle to any point on its circumference represents the axis directed normal to the plane whose stress components are given by the coordinates of that point.

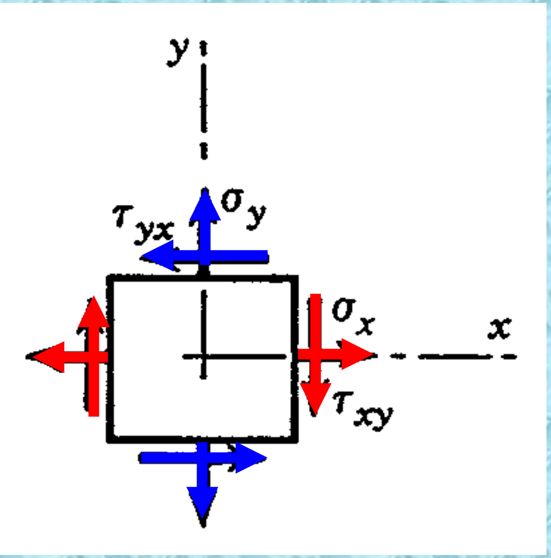
5. The angle between the radii to selected points on Mohr's circle is twice the angle between the normals to the actual planes represented by these points, or to twice the space angularity between the planes so represented.

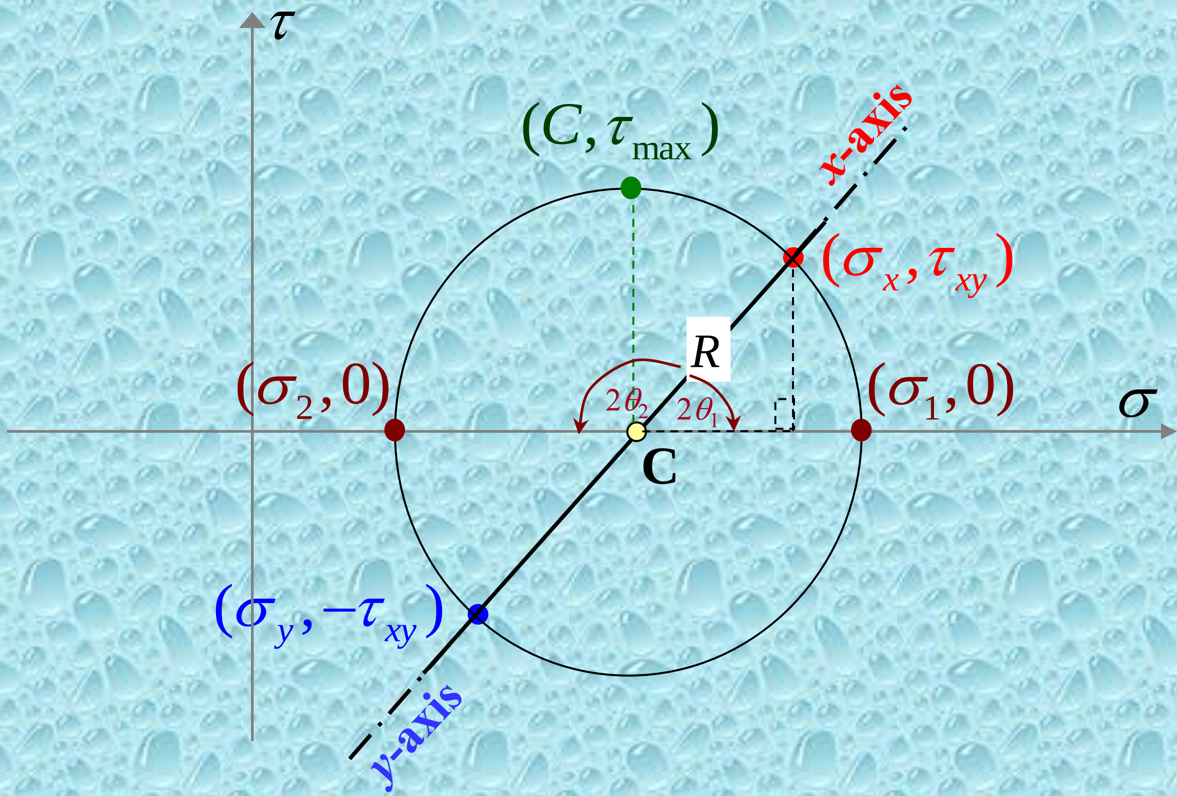


The rotational sense of this angle corresponds to the rotational sense of the actual angle between the normals to the planes; that is, if the n axis is actually at a counterclockwise angle θ from the x axis, then on Mohr's circle the n radius is laid off at a counterclockwise angle 2θ from the x radius.









$$\mathbf{C} = (C, 0) = (\frac{\sigma_x + \sigma_y}{2}, 0)$$

$$\boxed{\begin{aligned} \sigma_1 &= C + R \\ \sigma_2 &= C - R \end{aligned}}$$

$$\boxed{\sin 2\theta_1 = \frac{\tau_{xy}}{R} \quad \text{or} \quad \tan 2\theta_1 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

$$R = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

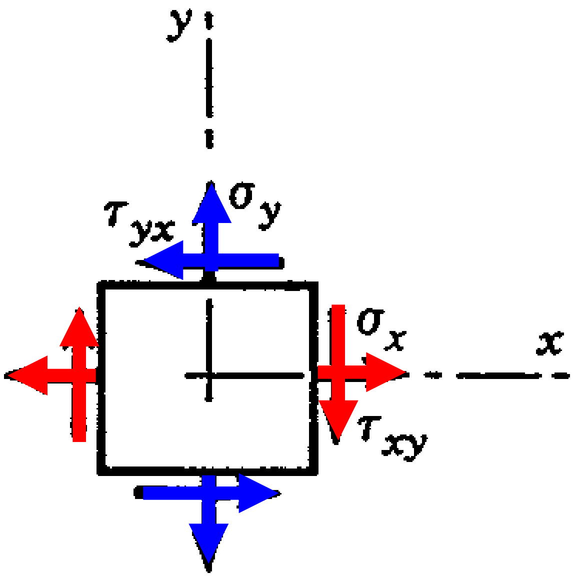
$$\boxed{\tau_{max} = R}$$

$$2\theta_2 = 180^\circ - 2\theta_1$$

$$(\sigma - C)^2 + \tau^2 = R^2$$

$$\boxed{C = \frac{\sigma_x + \sigma_y}{2}}$$

$$\boxed{R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}}$$



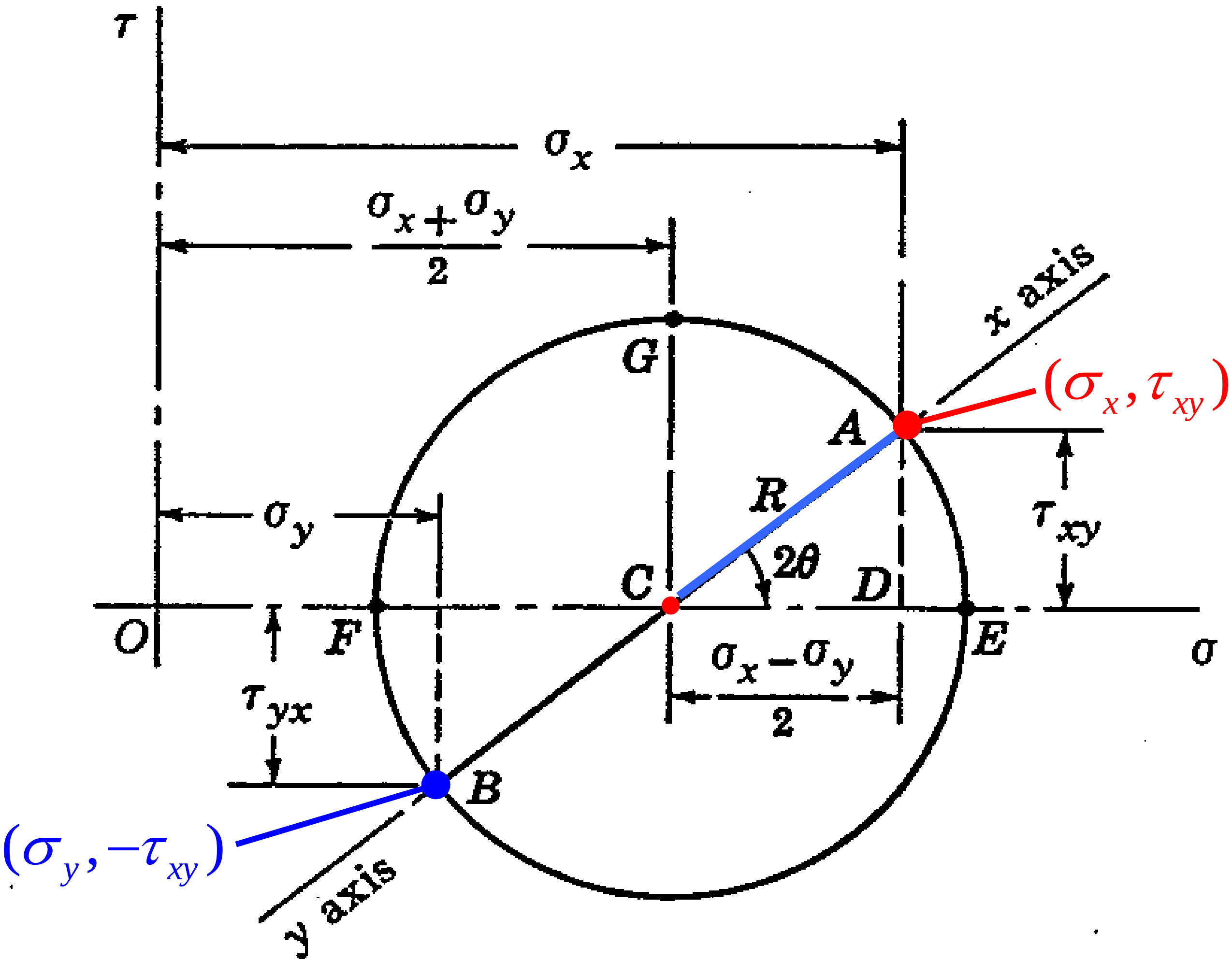
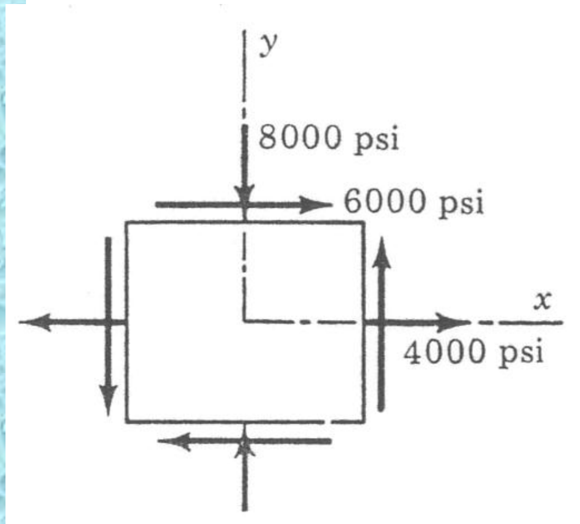
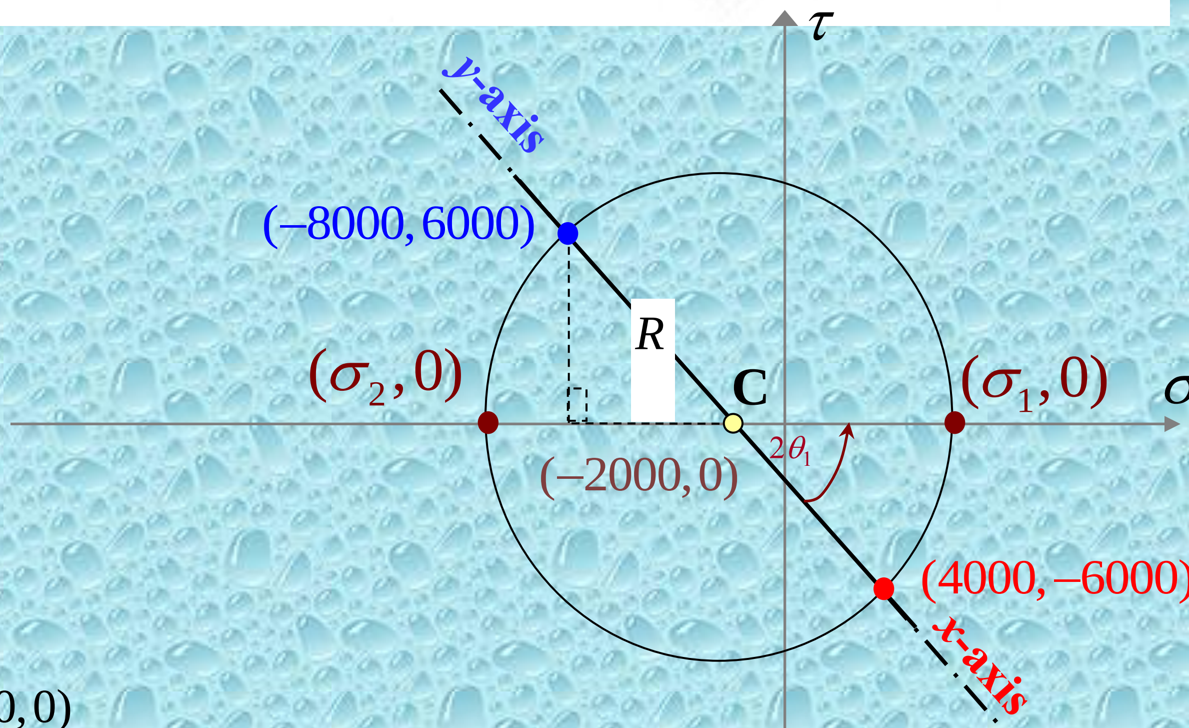


Figure 9-14 Mohr's circle for general state of plane stress.

926. If an element is subjected to the state of stress shown in Fig. P-926, find the principal stresses. Also, compute the stress components on a plane at 30° counterclockwise from the x face.

Ans. $\sigma_1 = 6480$ psi; $\sigma_2 = -10\,480$ psi; $\sigma = 6200$ psi; $\tau = 2190$ psi





$$\mathbf{C} = (C, 0) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

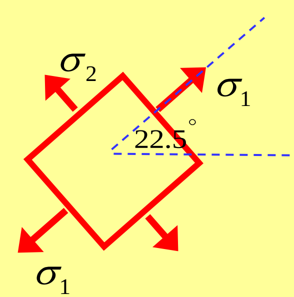
$$= \left(\frac{-8000 + 4000}{2}, 0\right) = (-2000, 0)$$

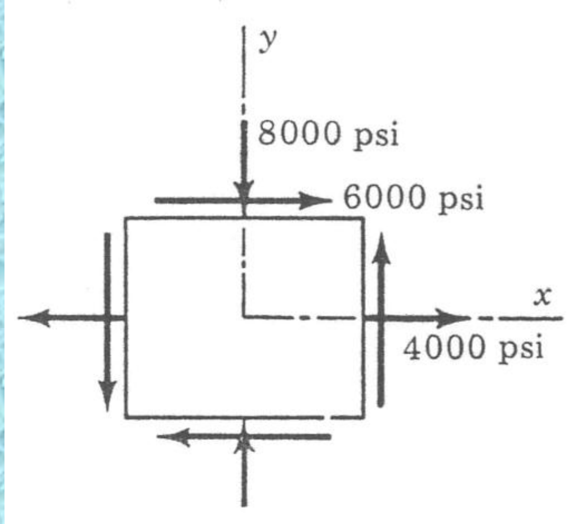
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{4000 + 8000}{2}\right)^2 + 6000^2} = 6000\sqrt{2} \text{ psi}$$

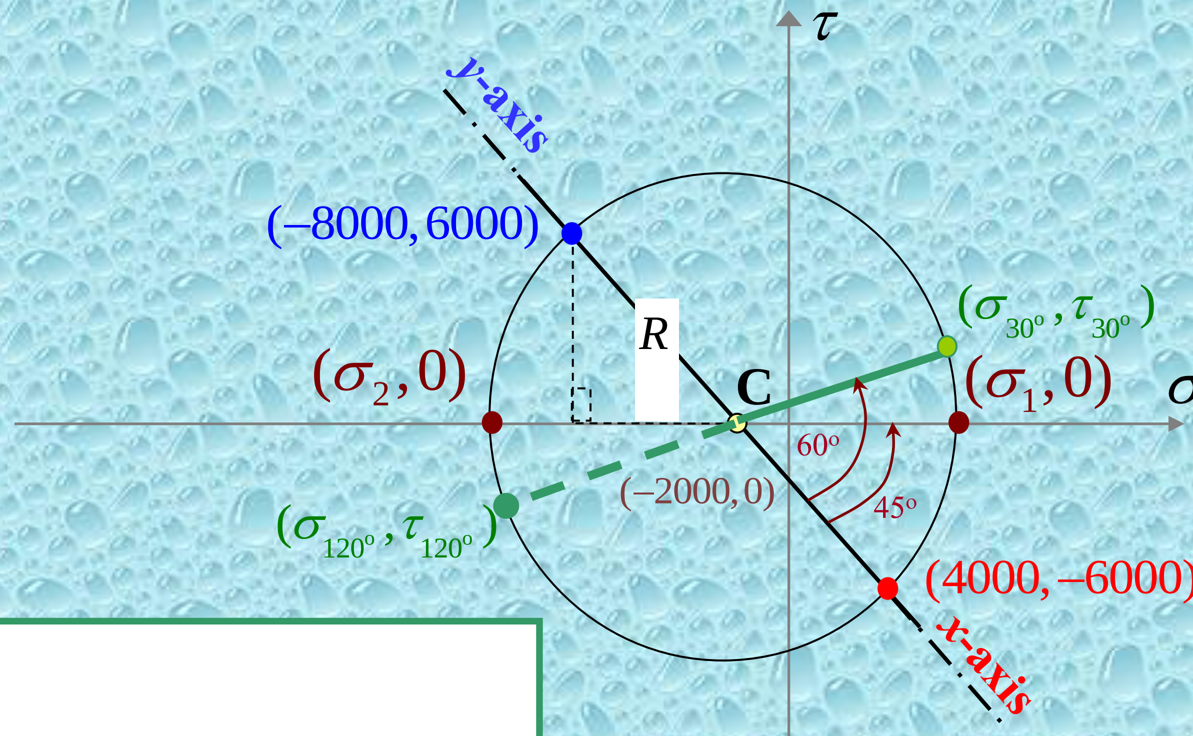
$$\boxed{\sigma_1, \sigma_2 = C \pm R = -2000 \pm 6000\sqrt{2} = 4485.3, -10485.3 \text{ psi}}$$

$$\sin 2\theta_1 = \frac{\tau_{xy}}{R} = \frac{6000}{6000\sqrt{2}}$$

$$\theta_1 = 22.5^{\mathrm{O}}$$



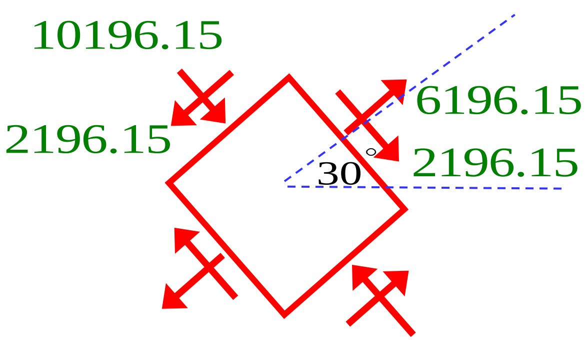




$$\sigma_{30^o} = C + R\cos(15^o)$$

$$= -2000 + 6000\sqrt{2}\cos(15^o) = 6196.15 \text{ psi}$$

$$\tau_{30^o} = R\sin(15^o) = 6000\sqrt{2}\sin(15^o) = 2196.15 \text{ psi}$$



$$\sigma_{120^o} = C - R\cos(15^o)$$

$$= -2000 - 6000\sqrt{2}\cos(15^o) = -10196.15 \text{ psi}$$

$$\tau_{120^o} = -R\sin(15^o) = -6000\sqrt{2}\sin(15^o) = -2196.15 \text{ psi}$$

923. At a certain point in a stressed body, the principal stresses are $\sigma_x = 80$ MPa and $\sigma_y = -40$ MPa. Determine σ and τ on the planes whose normals are at $+30°$ and $+120°$ with the x axis. Show your results on a sketch of a differential element.

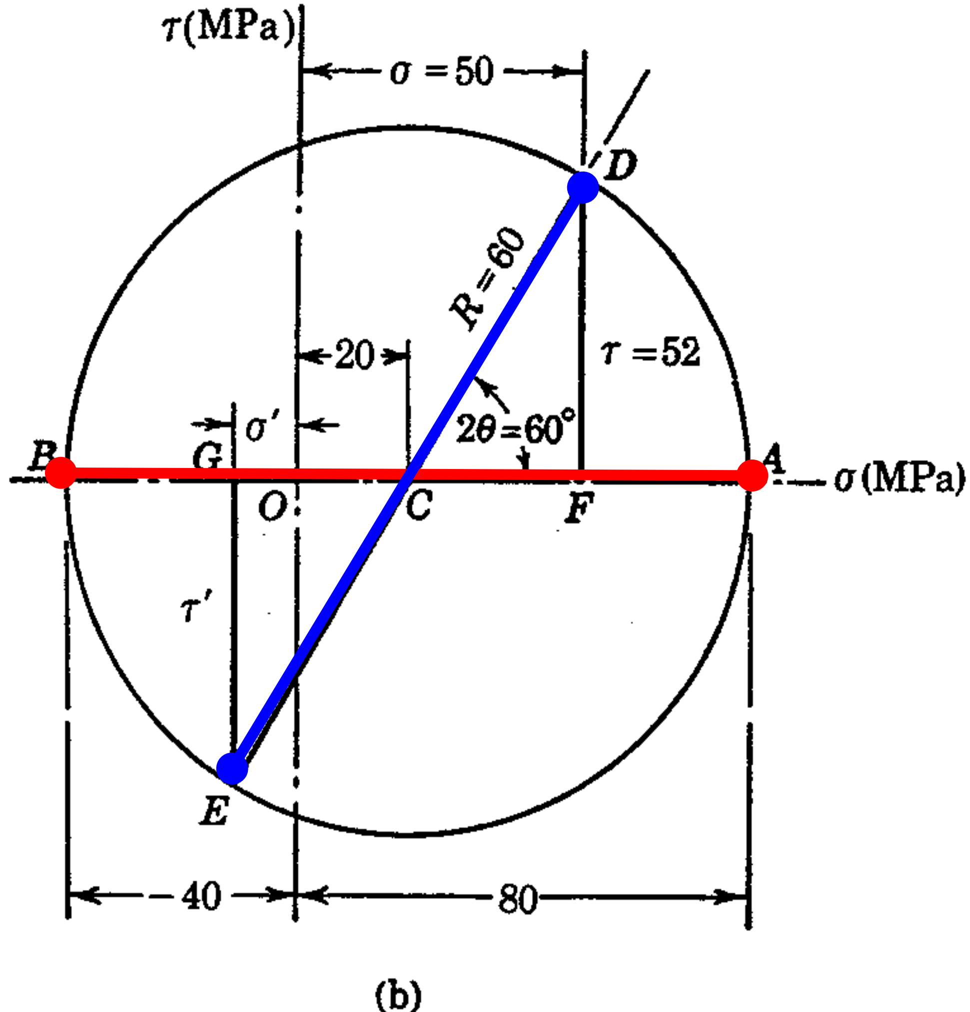
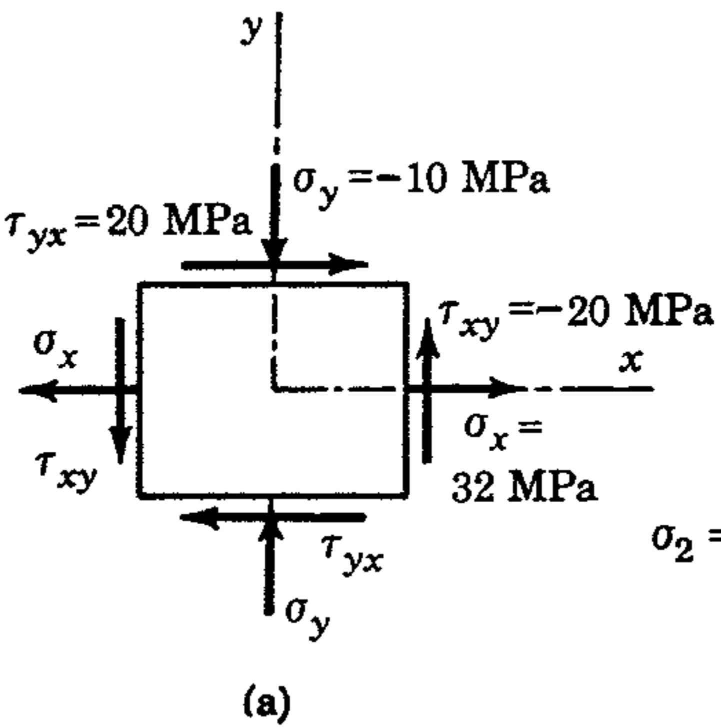


Figure 9-15

924. A state of stress is specified in Fig. 9-17a. Determine the normal and shearing stresses on (a) the principal planes, (b) the planes of maximum in-plane shearing stress, and (c) the planes whose normals are at +36.8° and +126.8° with the x axis. Show the results of parts (a) and (b) on complete sketches of differential elements.



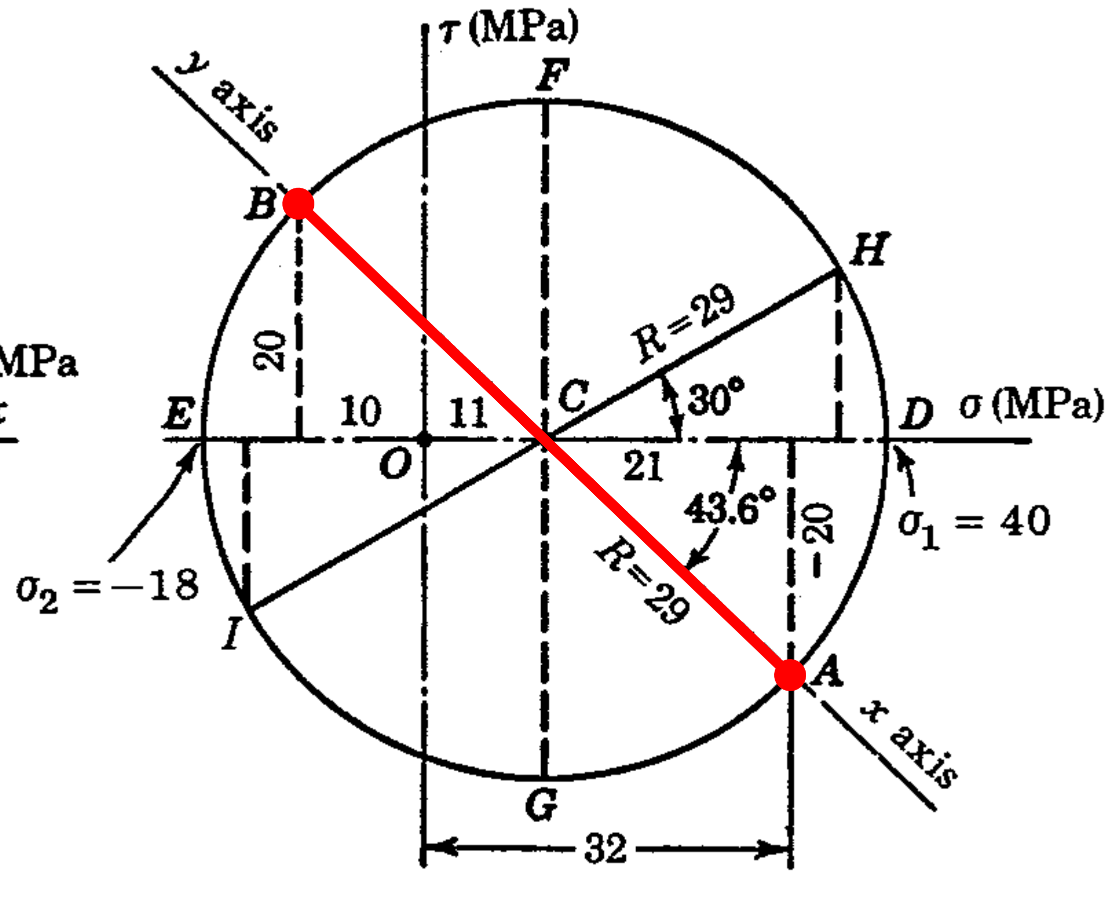
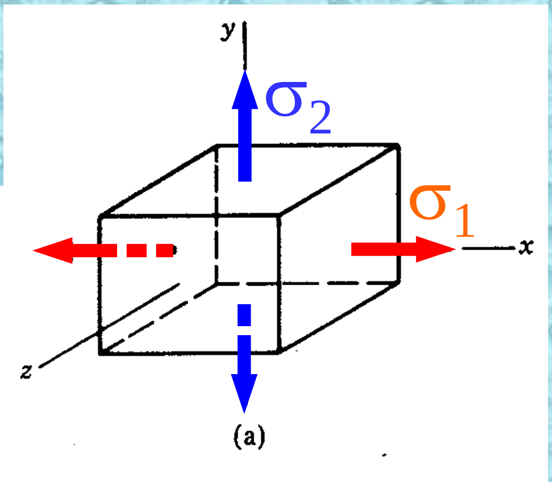
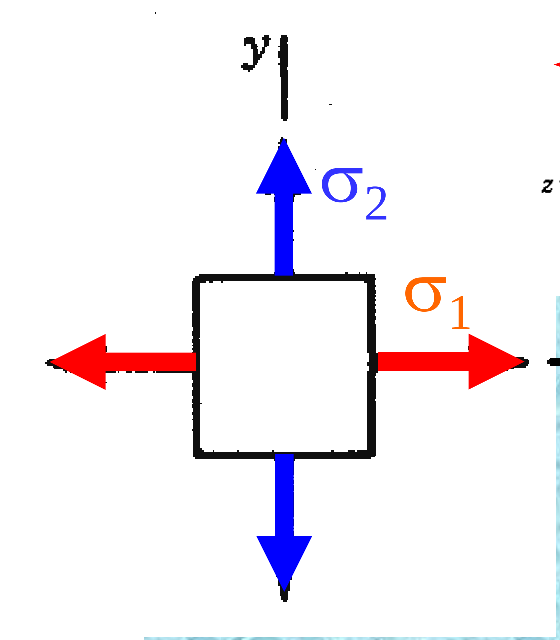
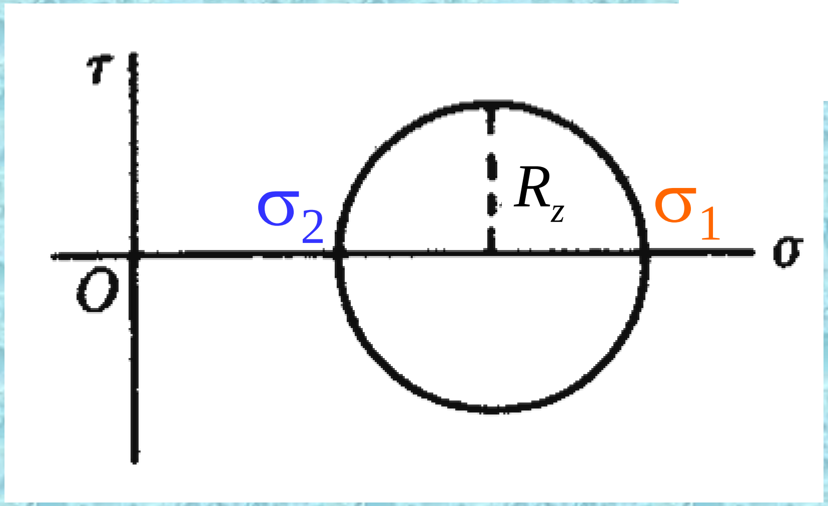


Figure 9-17

➢ Absolute Maximum Shearing Stress

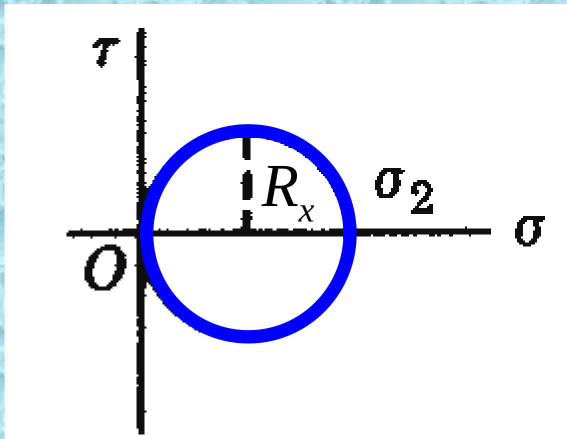


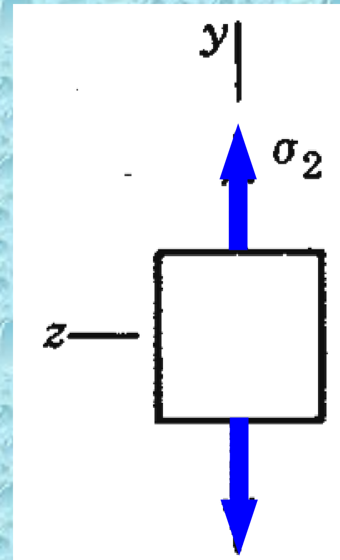


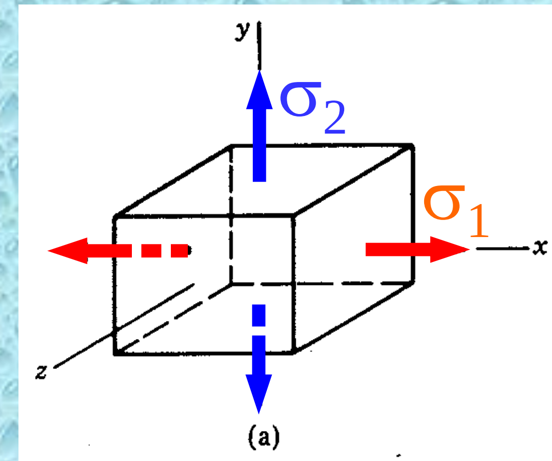


$$R_z = \frac{|\sigma_1 - \sigma_2|}{2}$$

Mohr's circle: Rotation around *z*-axis

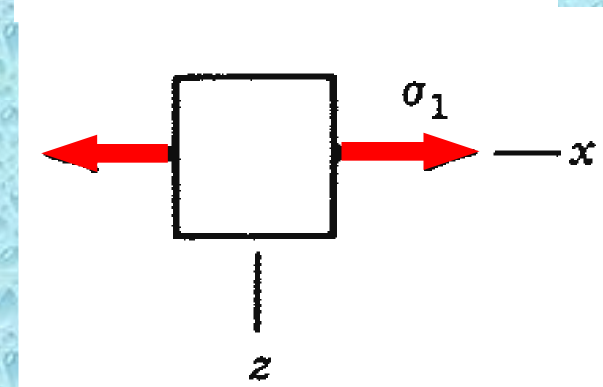




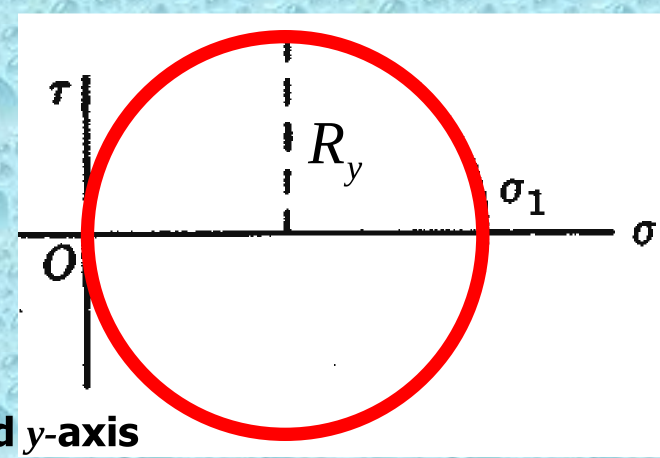


Mohr's circle: Rotation around x-axis

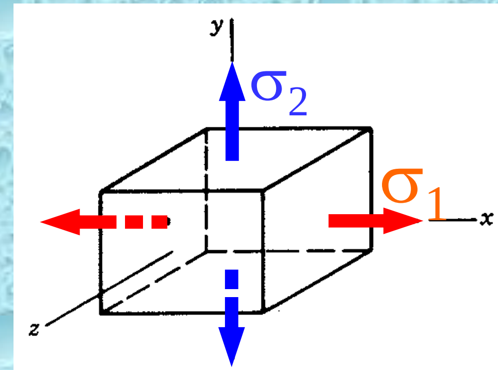
$$R_x = \frac{|\sigma_2|}{2}$$



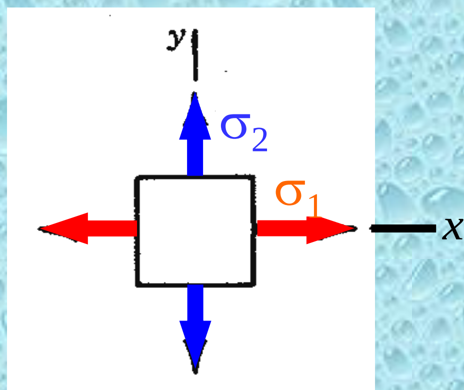
$$R_y = \frac{|\sigma_1|}{2}$$

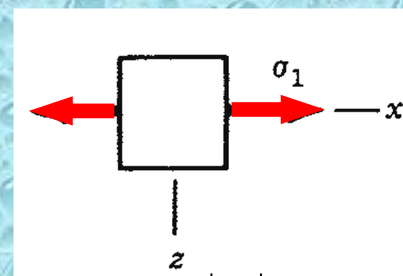


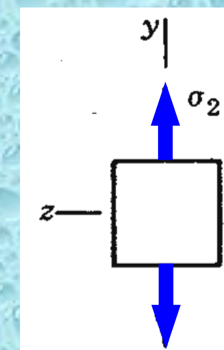
Mohr's circle: Rotation around y-axis



(a)



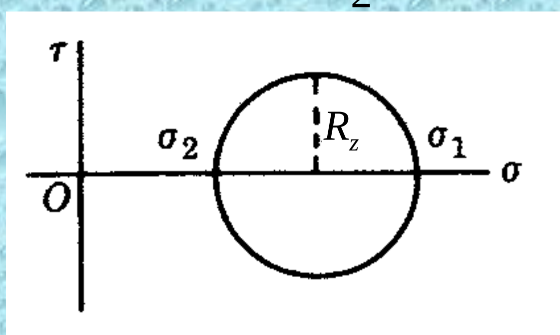




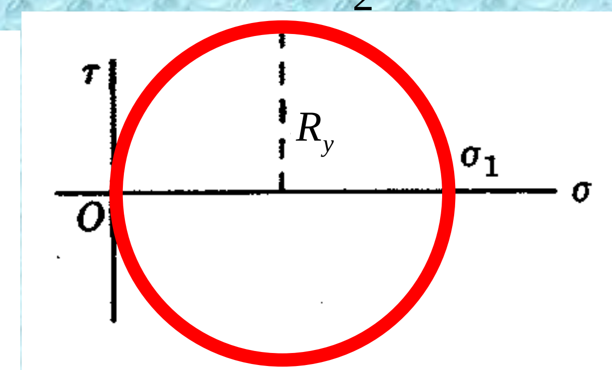
$$R_z = \frac{|\sigma_1 - \sigma_2|}{2}$$

$$R_y = \frac{|\sigma_1|}{2}$$

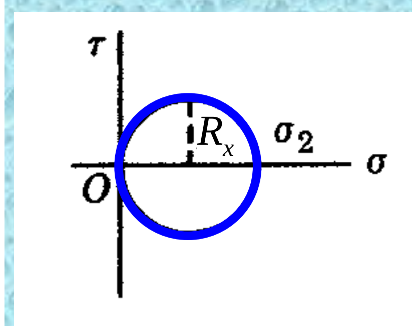
$$R_x = \frac{|\sigma_2|}{2}$$



(b) Rotations around z axis



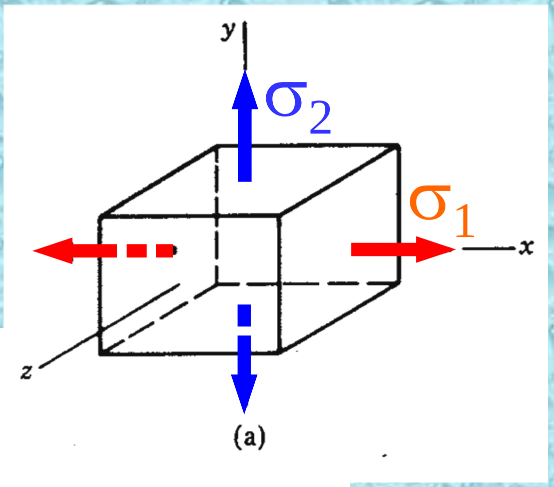
(c) Rotations around y axis

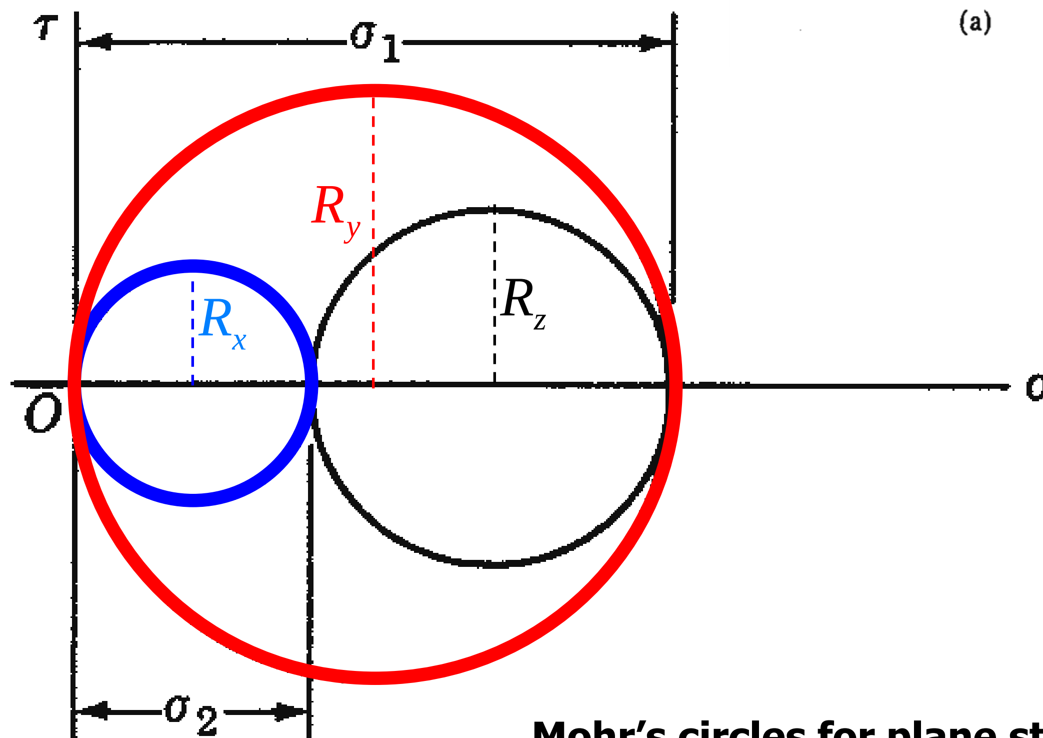


(d) Rotations around x axis

Absolute maximum shearing stress for <u>plane stress</u> is equal to the largest of the following three values

$$R_z = \frac{|\sigma_1 - \sigma_2|}{2}, \; R_z = \frac{|\sigma_1|}{2}, \; R_x = \frac{|\sigma_2|}{2}$$

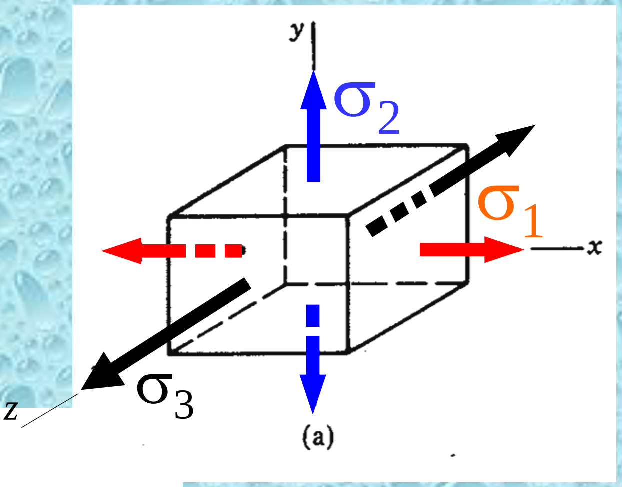


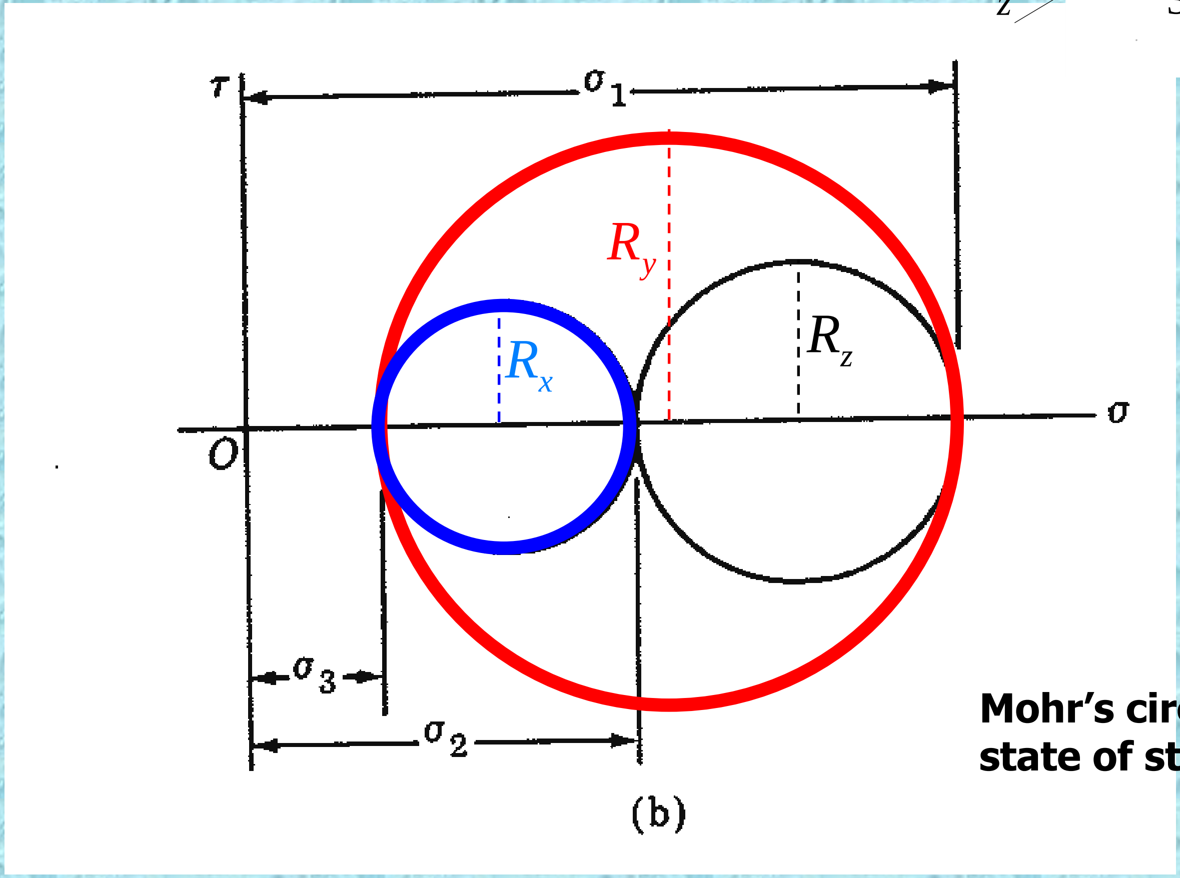


Mohr's circles for plane stress

Absolute maximum shearing stress for general state of stress is equal to the largest of the following three values

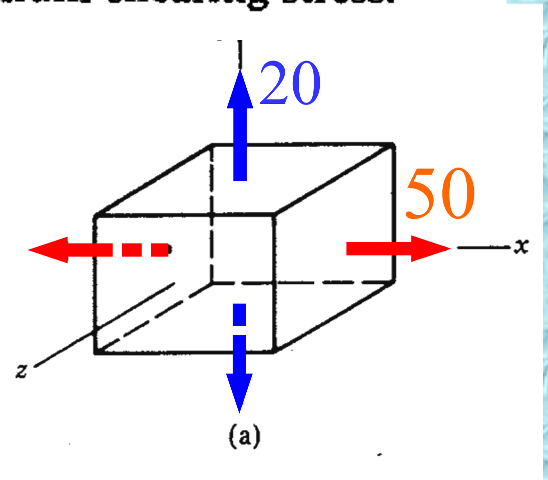
$$R_z = \frac{|\sigma_1 - \sigma_2|}{2},\ R_z = \frac{|\sigma_1 - \sigma_3|}{2},\ R_x = \frac{|\sigma_2 - \sigma_3|}{2}$$





Mohr's circles for general state of stress

941. For a state of plane stress, $\sigma_1 = \sigma_x = 50$ ksi and $\sigma_2 = \sigma_y = 20$ ksi. Determine the maximum in-plane shearing stress and the absolute maximum shearing stress.



Maximum in-plane shearing stress =

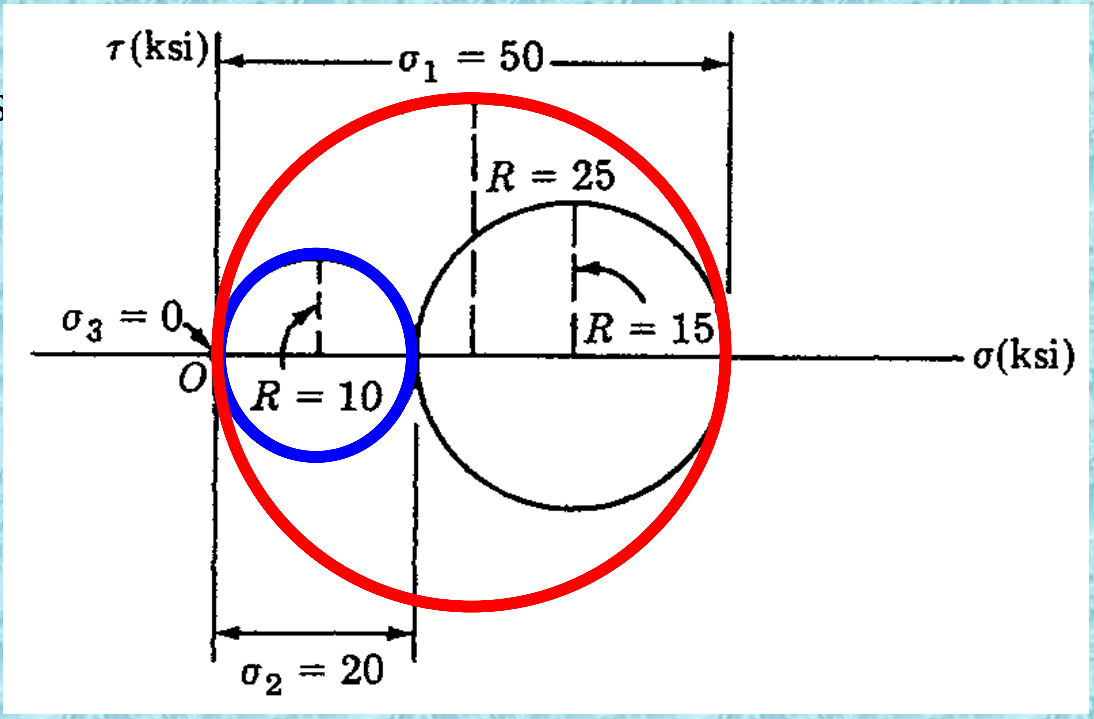
$$\pm\frac{\sigma_1 - \sigma_2}{2} = \frac{50 - 20}{2} = \pm 15 \text{ ksi}$$

Absolute maximum shearing stress is the largest of

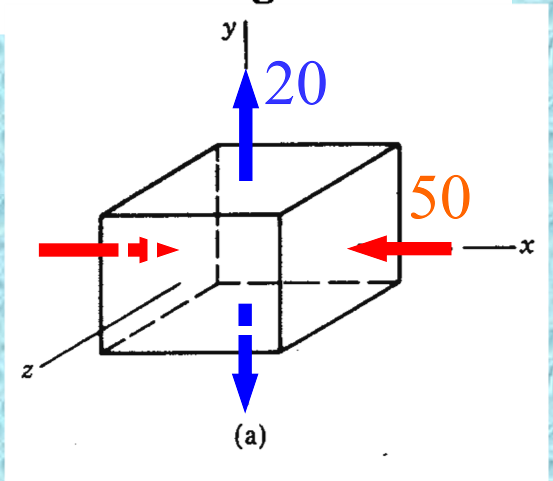
$$\frac{|\sigma_1 - \sigma_2|}{2} = \frac{|50 - 20|}{2} = 15 \text{ ks}$$

$$\frac{|\sigma_1|}{2} = \frac{|50|}{2} = 25 \text{ ksi,}$$

$$\frac{|\sigma_2|}{2} = \frac{|20|}{2} = 10 \text{ ksi,}$$



Ex. For a state of plane stress, $\sigma_1 = \sigma_x = -50$ ksi and $\sigma_2 = \sigma_y = 20$ ksi. Determine the maximum in-plane shearing stress and the absolute maximum shearing stress.



(a)

Maximum in-plane shearing stress =

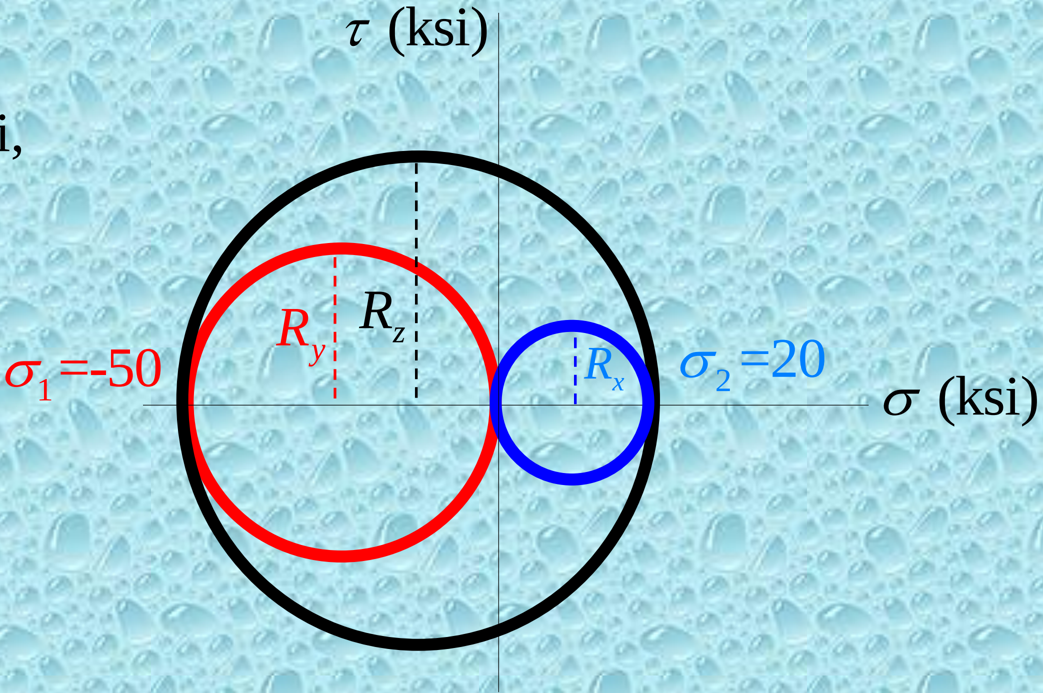
$$\pm\frac{\sigma_1-\sigma_2}{2}=\frac{-50-20}{2}=\mp 35\text{ ksi}\neq$$

Absolute maximum shearing stress is the largest of

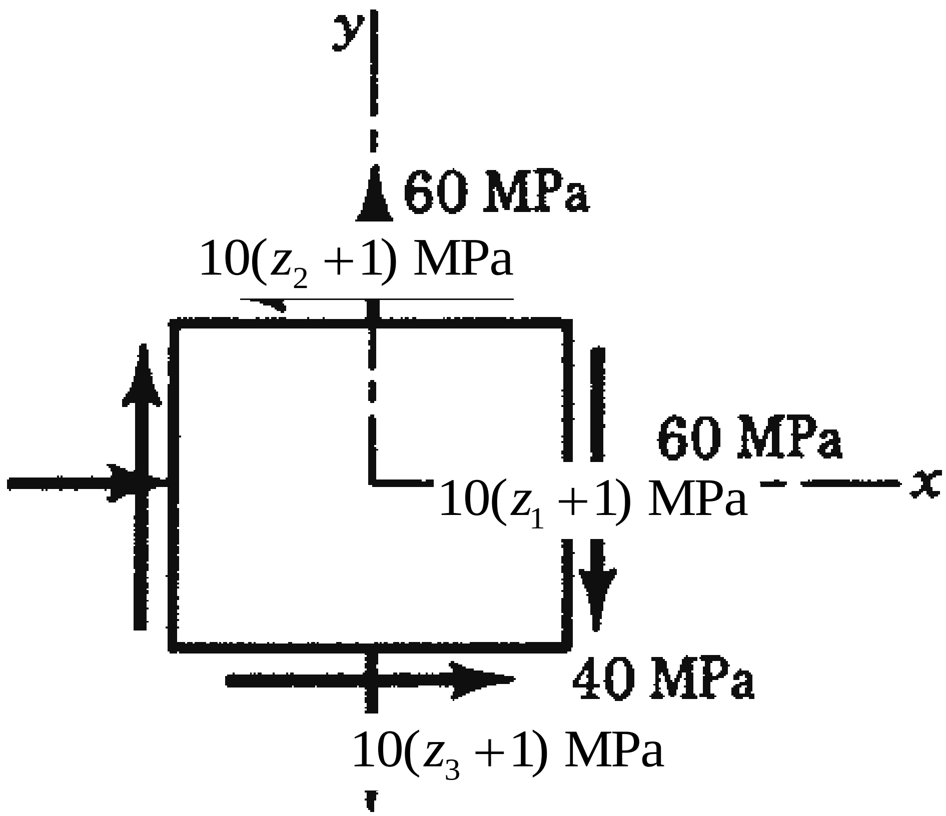
$$\frac{|\sigma_1-\sigma_2|}{2}=\frac{|-50-20|}{2}=35\text{ ksi},$$

$$\frac{|\sigma_1|}{2}=\frac{|-50|}{2}=25\text{ ksi},$$

$$\frac{|\sigma_2|}{2}=\frac{|20|}{2}=10\text{ ksi},$$



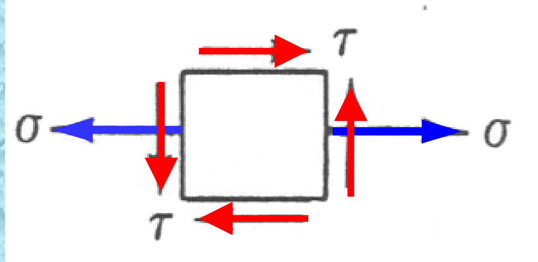
If an element is subjected to the state of stress shown in Fig. P-933, find the principal stresses and the maximum in-plane shearing stresses. Also, determine the stress components on planes whose normals are at 45° and 135° v the figure axis. Show all results on complete sketches of the appropriate elements.



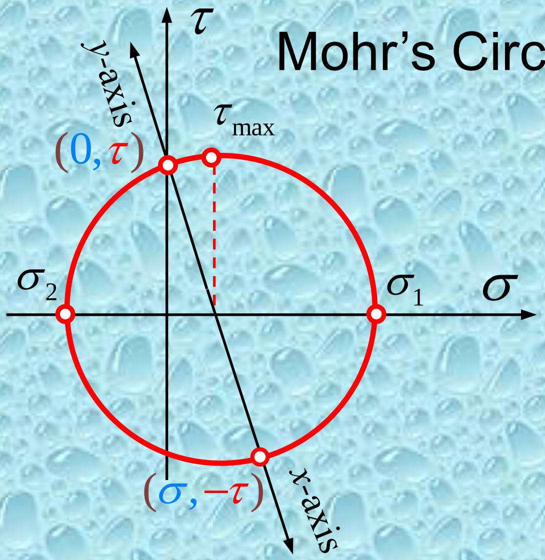
➢ Application of Mohr's Circle to Combined Loadings

Combined Loadings
(axial, torsional, flexural)

Combined stresses

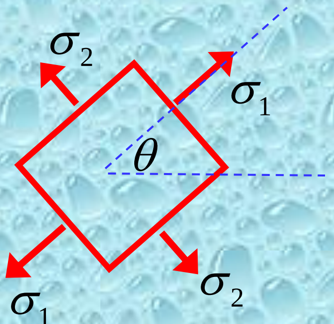


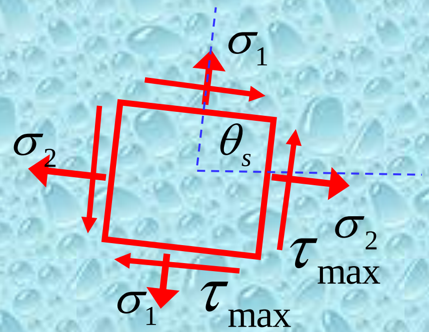
Mohr's Circle



Design Criteria, $\sigma_{allow}, \ \tau_{allow}$

Principal stresses and,
Maximum shearing stress





Stress Trajectories

$$\tau_{\max} = \frac{Tc}{J}$$

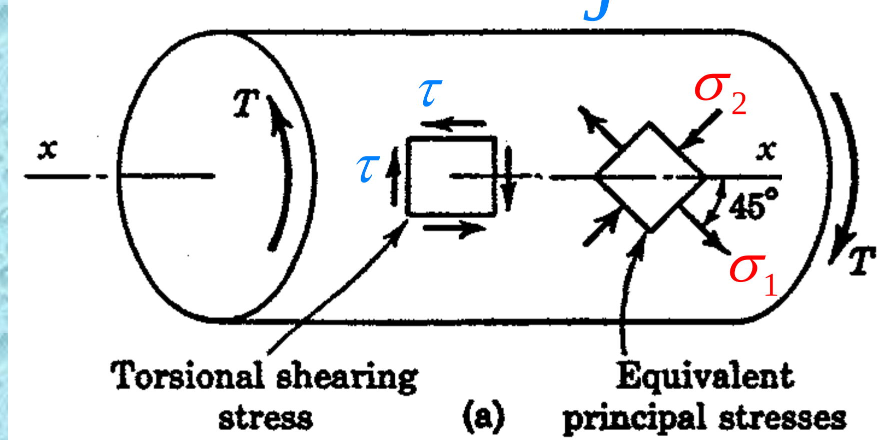
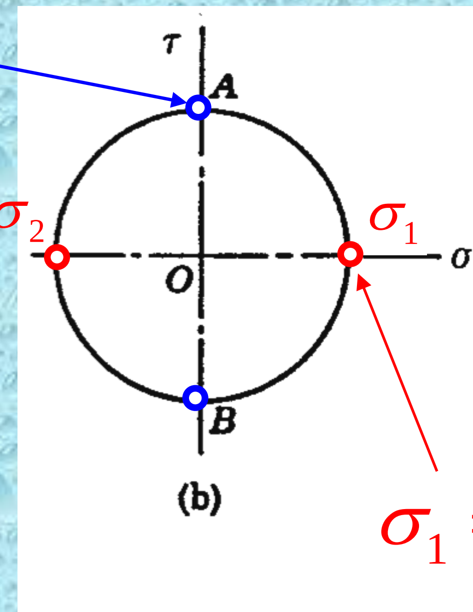
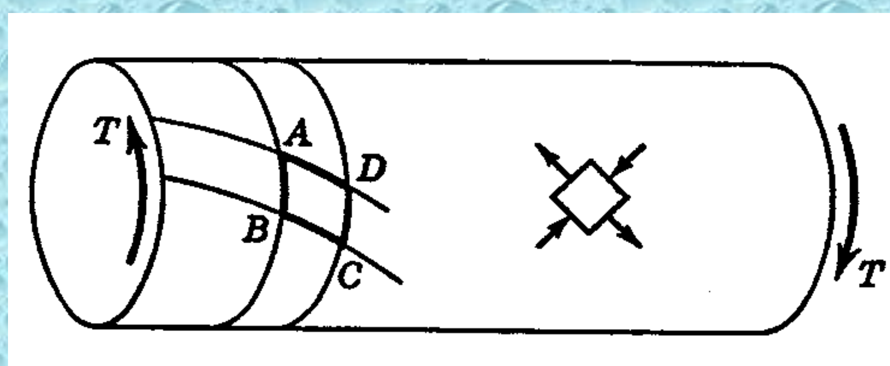


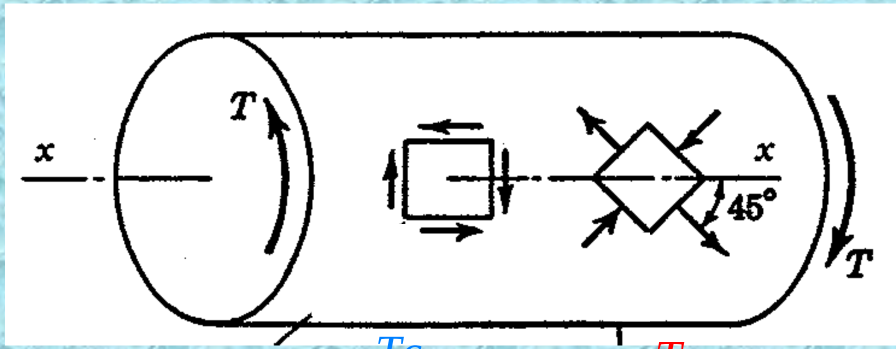
Figure 9-23 Cylinder subjected to torsion.



$$\sigma_1 = \frac{Tc}{J}$$

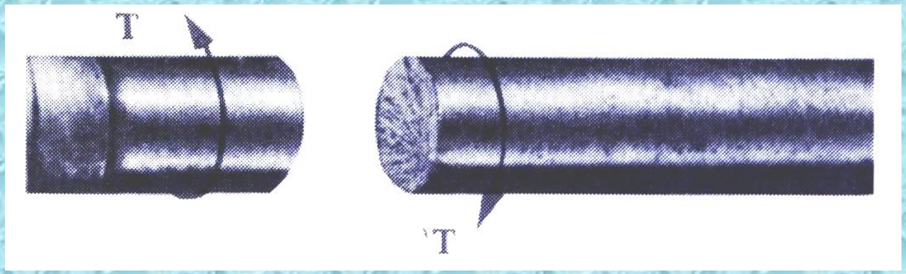


Torsional Failure Modes

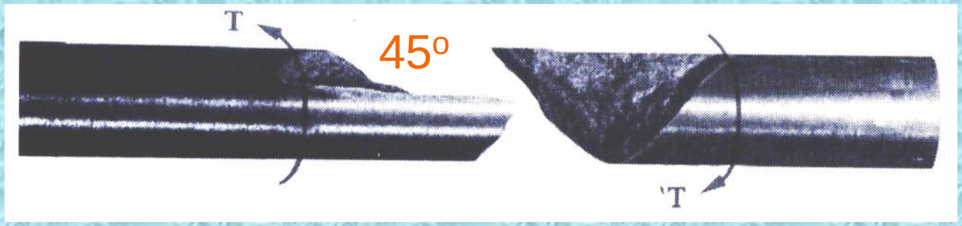


$$\tau_{\max} = \frac{Tc}{J} \qquad \sigma_1 = \frac{Tc}{J}$$

- Ductile materials generally fail in shear. Brittle materials are weaker in tension than shear.

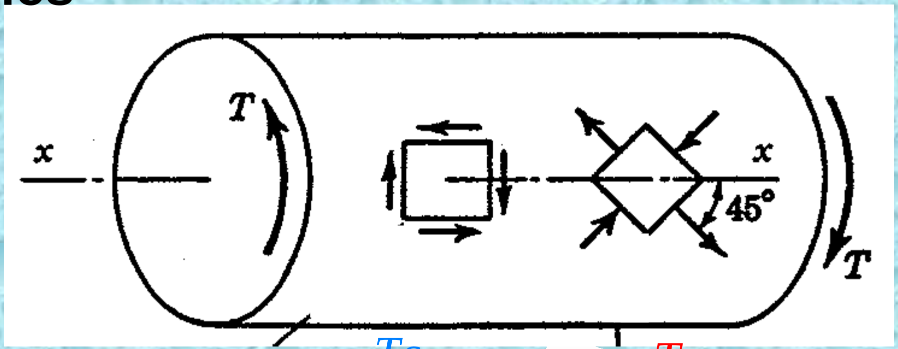


- A ductile specimen breaks along a plane of maximum shear



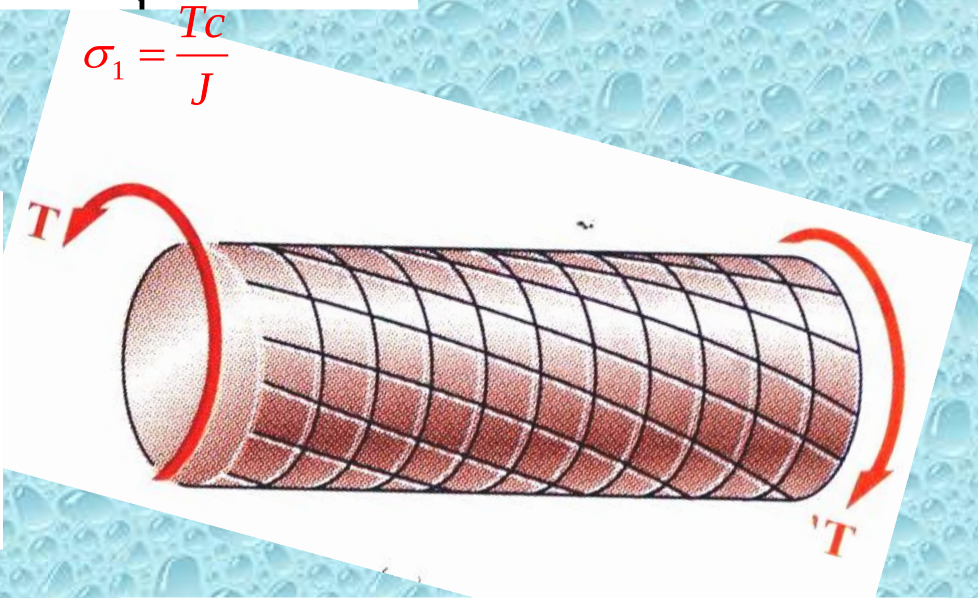
- A brittle specimen breaks along planes perpendicular to σ_1

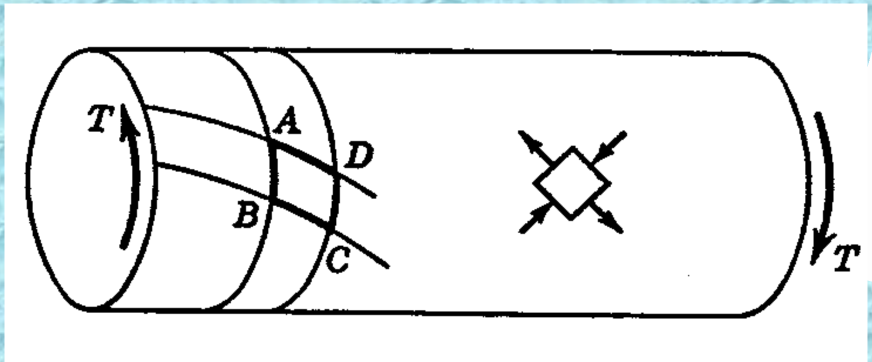
Stress Trajectories for Torsion



$$\tau_{max} = \frac{Tc}{J}$$

$$\sigma_1 = \frac{Tc}{J}$$





Stress Trajectories: lines of principal stress direction but of variable stress intensity

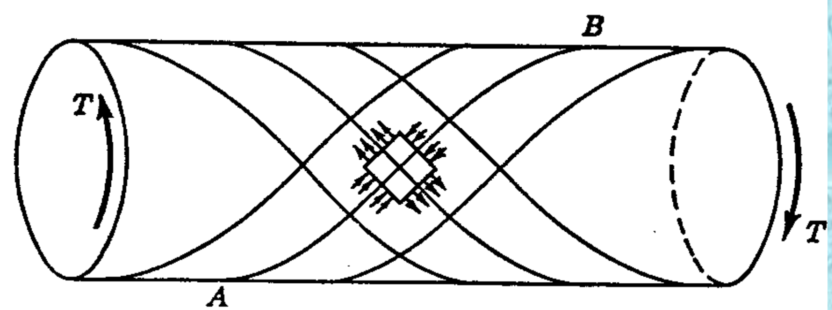
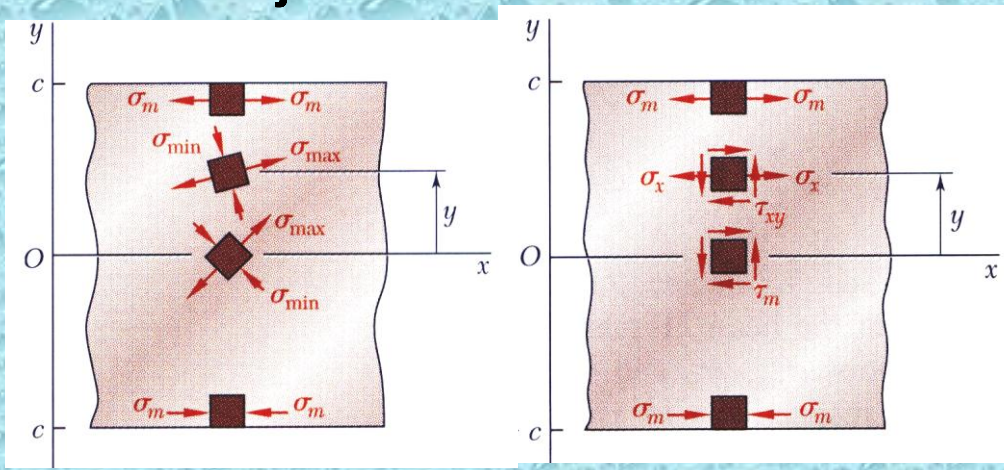
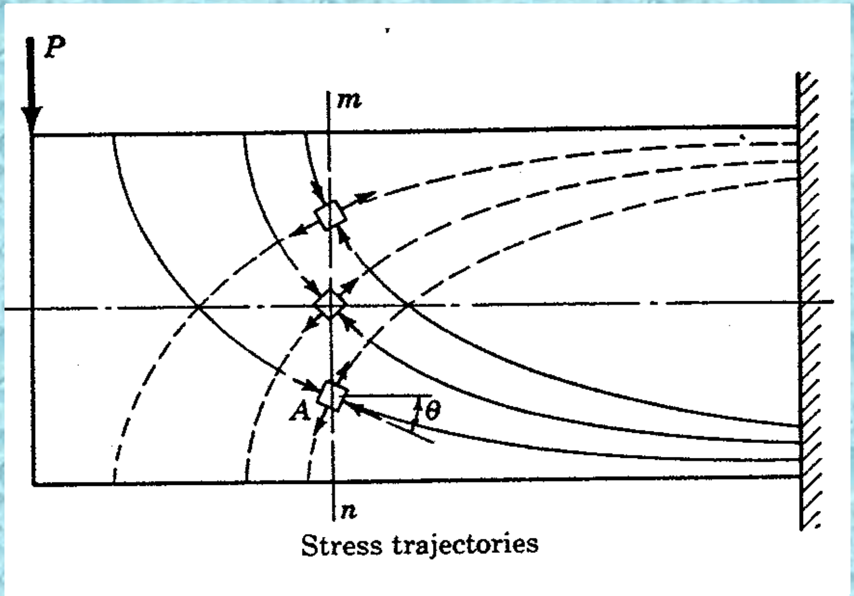


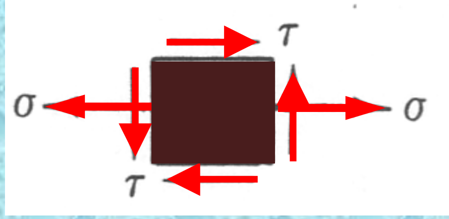
Figure 9-24 Stress trajectories for torsion.

Stress Trajectories for Beam

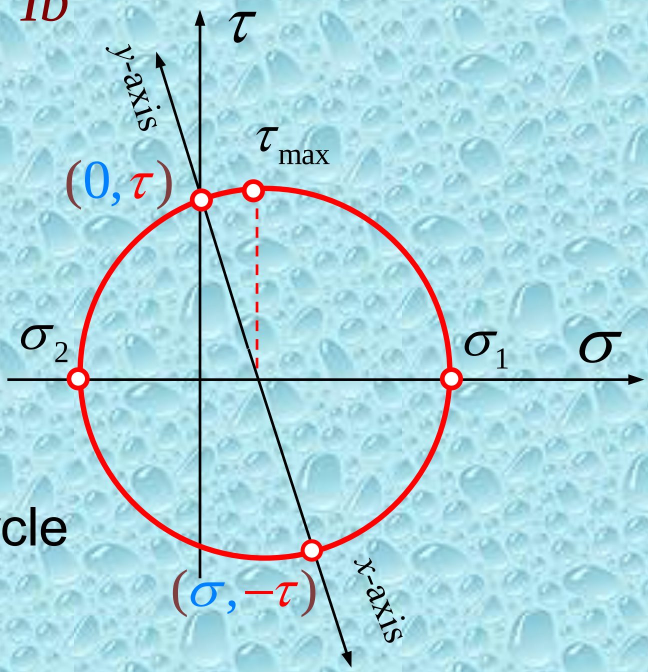




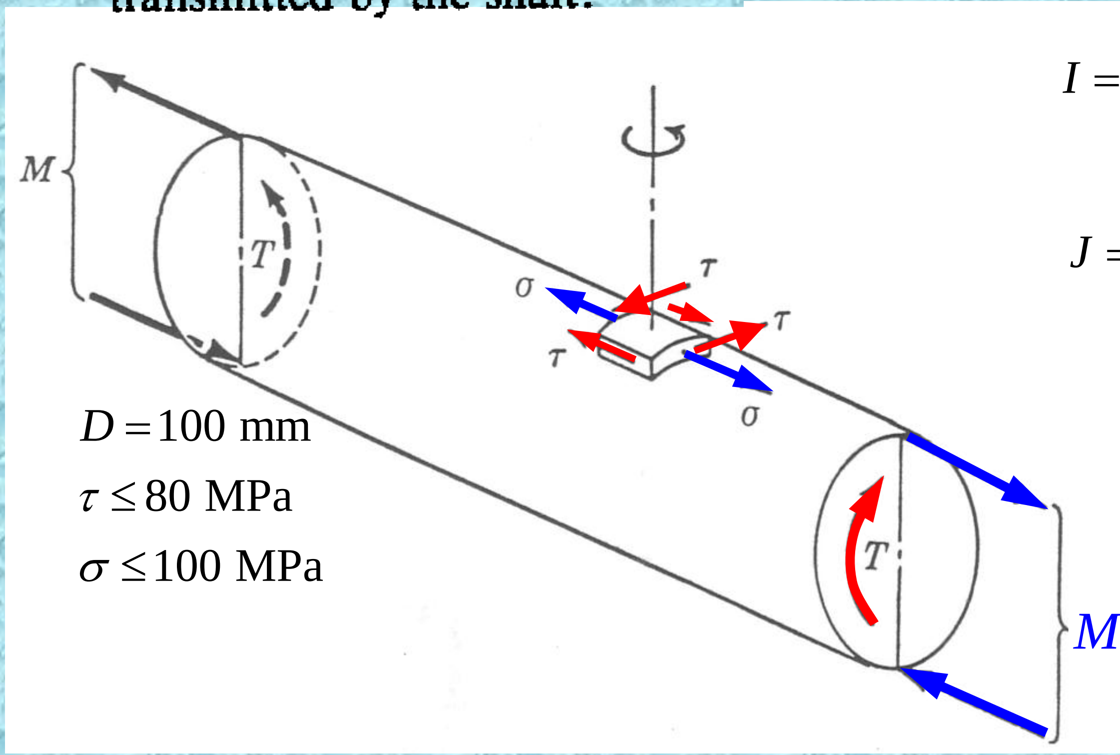
$$\sigma = \frac{My}{I} \qquad \tau = \frac{VQ}{Ib}$$



Mohr's Circle



950. A shaft 100 mm in diameter that rotates at 30 Hz is subjected to bending loads that produce a maximum bending moment of 2500π N · m. Determine the torque that can also act simultaneously on the shaft without exceeding a shearing stress $\tau = 80$ MPa or a normal stress $\sigma = 100$ MPa. What is the maximum power that can be transmitted by the shaft?



$D = 100$ mm

$\tau \le 80$ MPa

$\sigma \le 100$ MPa

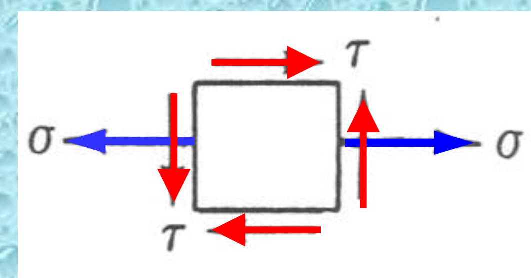
$M = 2500\pi$ N.m

$$I = \frac{\pi D^4}{64} = \frac{\pi (0.1)^4}{64} = 1.5625\times10^{-6}\pi \ \text{m}^4$$

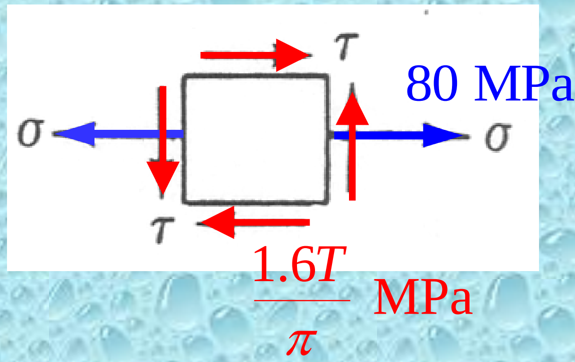
$$J = \frac{\pi D^4}{32} = \frac{\pi (0.1)^4}{32} = 3.125\times10^{-6}\pi \ \text{m}^4$$

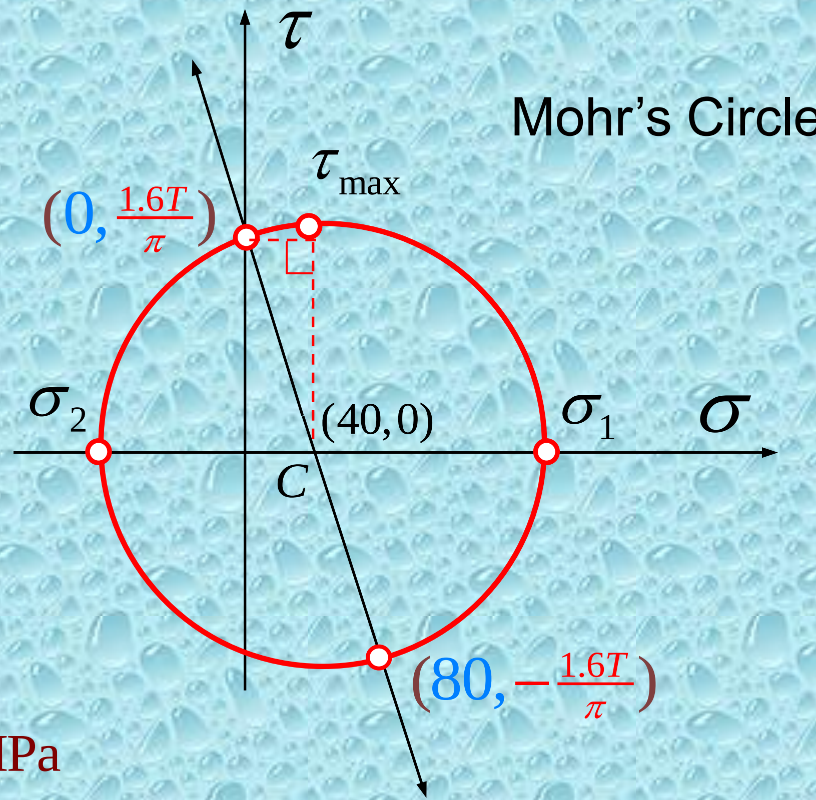
$$\sigma = \frac{Mc}{I} = \frac{(2500\pi)(0.05)}{1.5625\times10^{-6}\pi} = 8\times10^7 \ \text{N/m}^2 = 80 \text{ MPa}$$

$$\tau = \frac{Tc}{J} = \frac{T(0.05)}{3.125\times10^{-6}\pi} = \text{T}(\frac{1.6\times10^6}{\pi}) \ \text{N/m}^2 = \frac{1.6T}{\pi} \text{ MPa}$$



Top view of element





$$C = 40 \text{ MPa}$$

$$\tau_{\max} = R = \sqrt{40^2 + (\frac{1.6T}{\pi})^2} \quad \leq 80 \text{ MPa}$$

$$\sigma_1 = C + R = 40 + \sqrt{40^2 + (\frac{1.6T}{\pi})^2} \quad \leq 100 \text{ MPa}$$

$$\boxed{\sqrt{40^2 + (\frac{1.6T}{\pi})^2} \leq 60 \text{ MPa}}$$

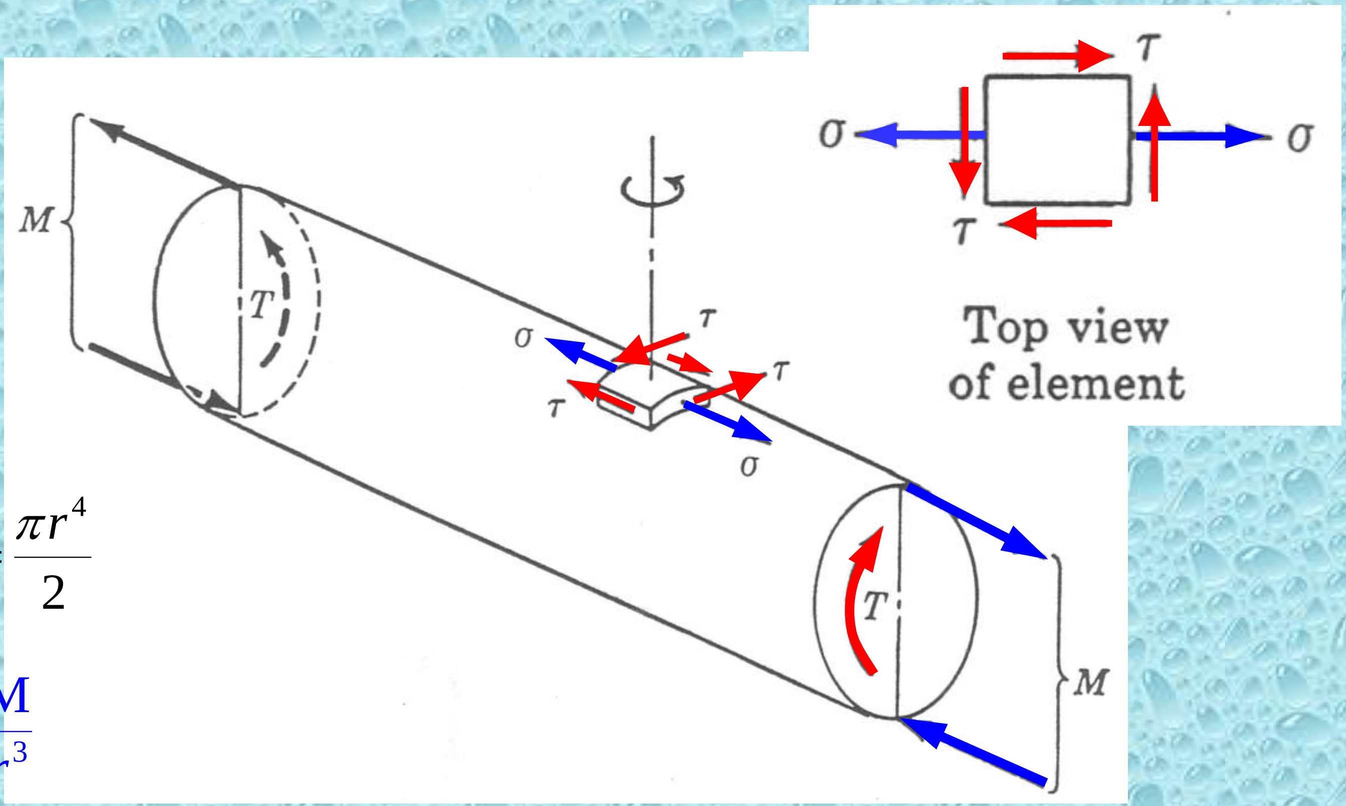
$$T \leq 87.81 \text{ N.m}$$

$$P = 2\pi f \cdot T$$

$$P \leq 2\pi (30)(87.81)$$

$$P \leq 16{,}551.8 \text{ watt}$$

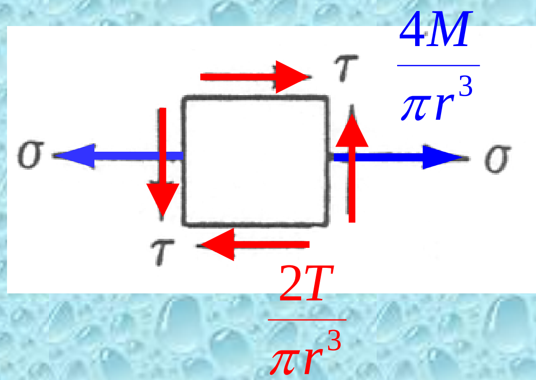
951. A solid shaft is subjected to simultaneous twisting and bending due to a torque T and a maximum bending moment M. Express the maximum shearing stress τ and the maximum normal stress σ in terms of T, M, and the radius r of the shaft. By means of these relations, determine the proper diameter of a solid shaft to carry simultaneously $T = 900$ lb·ft and $M = 600$ lb·ft if $\tau \leq 10$ ksi and $\sigma \leq 16$ ksi.



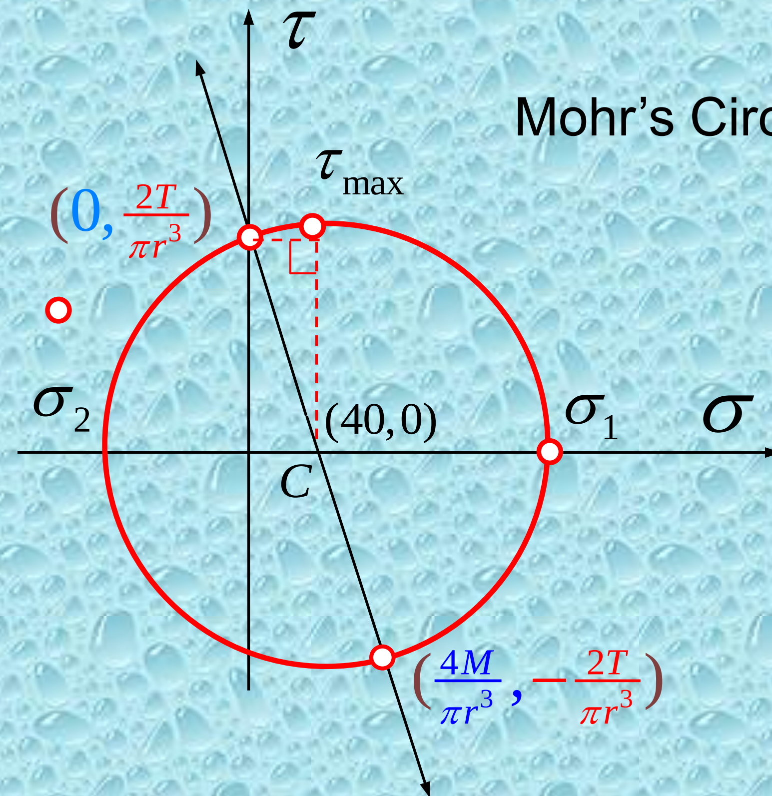
$$I = \frac{\pi r^4}{4}, \quad J = \frac{\pi r^4}{2}$$

$$\sigma = \frac{Mc}{I} = \frac{4M}{\pi r^3}$$

$$\tau = \frac{Tc}{I} = \frac{2T}{\pi r^3}$$



Mohr's Circle



$$C = 2M/(\pi r^3)$$

$$\tau_{\max} = R = \sqrt{\left(\frac{2M}{\pi r^3}\right)^2 + \left(\frac{2T}{\pi r^3}\right)^2} = \frac{2}{\pi r^3}\sqrt{M^2 + T^2}$$

$$\sigma_1 = C + R = \frac{2}{\pi r^3}\left\{M + \sqrt{M^2 + T^2}\right\}$$

$$\text{If} \quad T = 900 \text{ lb-ft} = \frac{900 \times 12}{1000} = 10.8 \text{ kips-in}$$

$$M = 600 \text{ lb-ft} = \frac{600 \times 12}{1000} = 7.2 \text{ kips-in}$$

$$\tau_{\max} \le 10 \text{ ksi}$$

$$\sigma_{\max} \le 16 \text{ ksi}$$

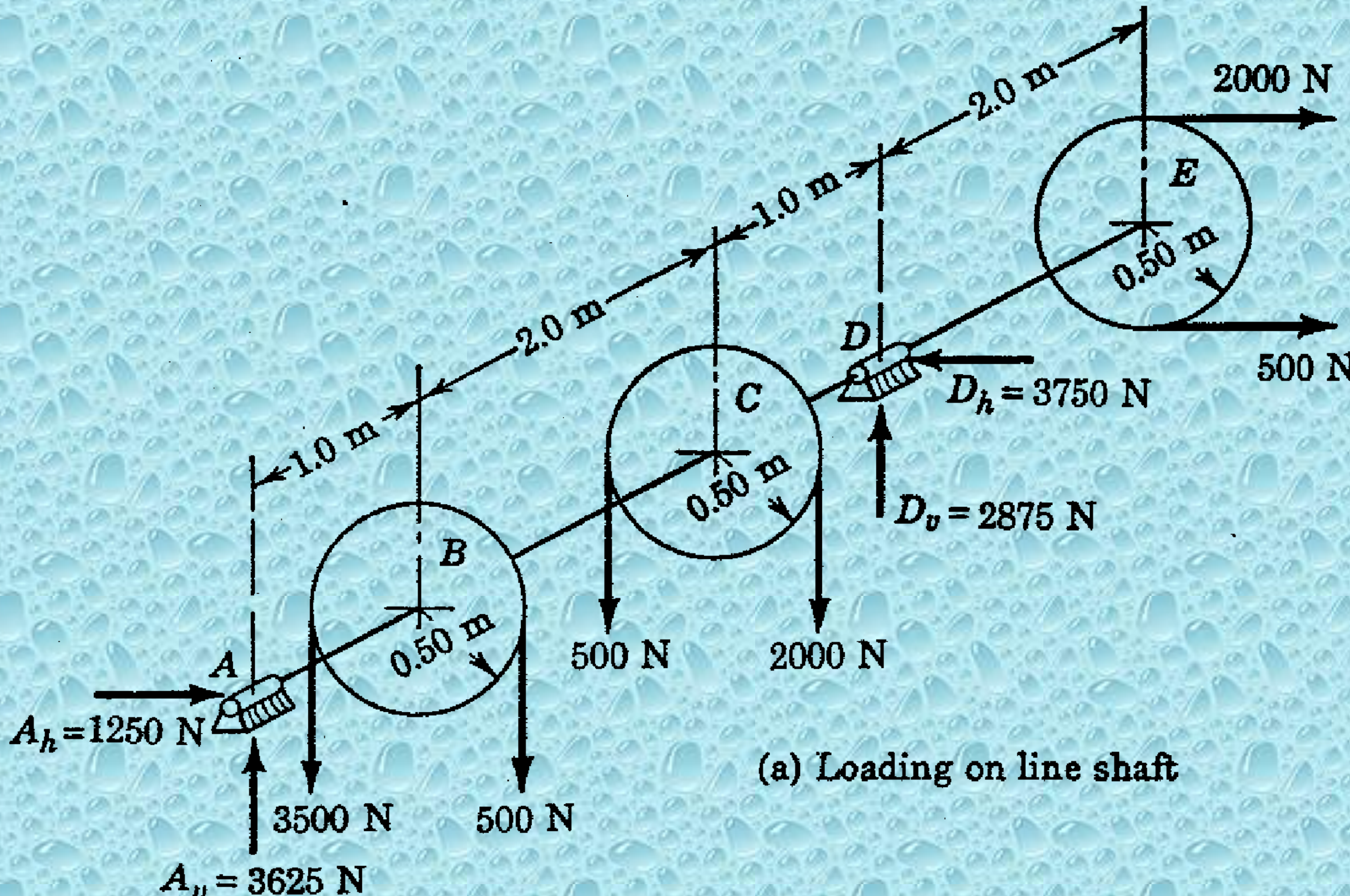
$$\tau_{\max} = \frac{2}{\pi r^3}\sqrt{M^2 + T^2}$$

$$= \frac{2}{\pi r^3}\sqrt{7.2^2 + 10.8^2} = \frac{8.263}{r^3} \text{ ksi} \le 10 \text{ ksi} \quad \Rightarrow \quad \boxed{r \ge 0.938 \text{ in.}}$$

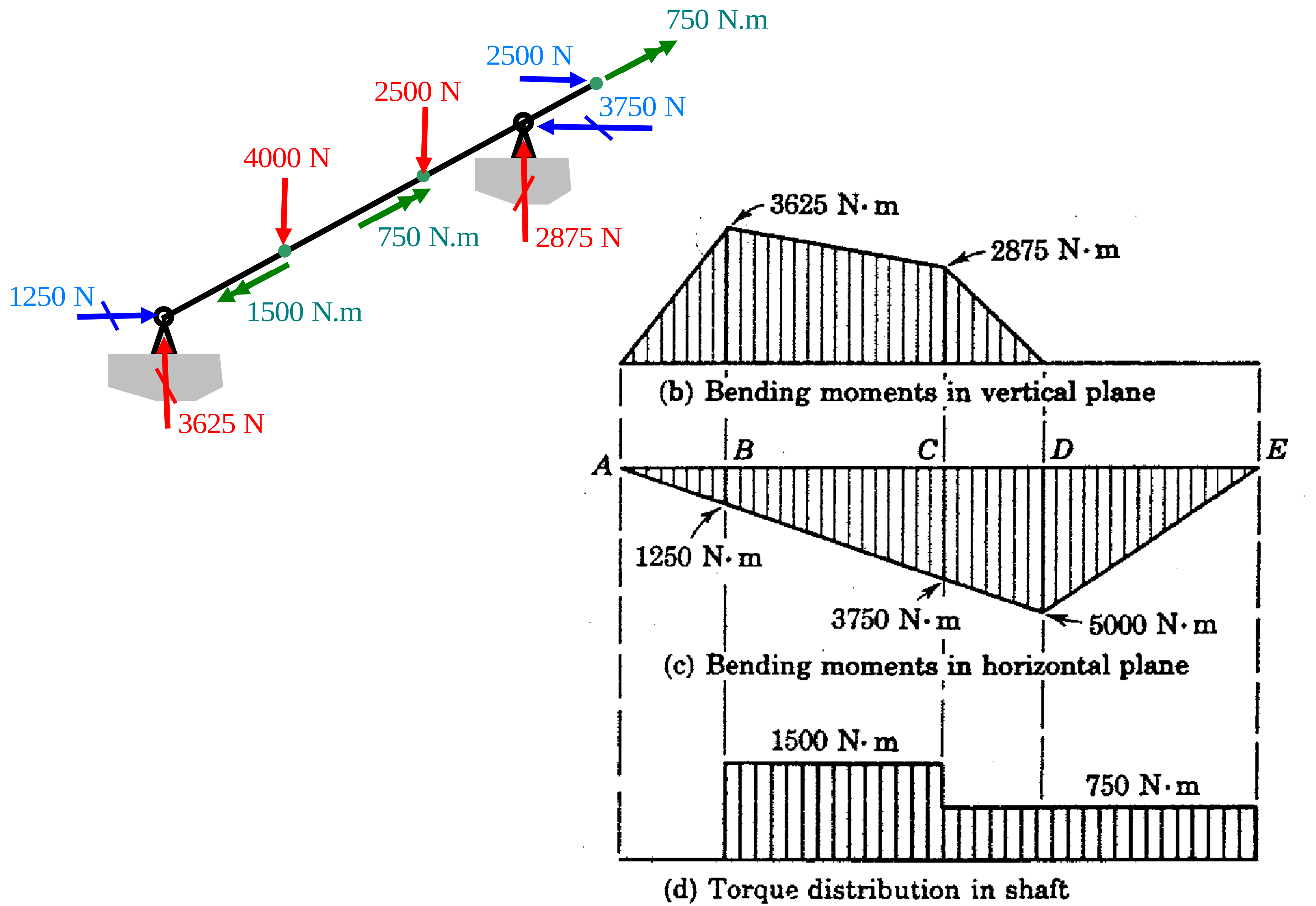
$$\sigma_1 = \frac{2}{\pi r^3}\left\{M + \sqrt{M^2 + T^2}\right\}$$

$$= \frac{2}{\pi r^3}\left\{7.2 + \sqrt{7.2^2 + 10.8^2}\right\} = \frac{12.847}{r^3} \text{ ksi} \le 16 \text{ ksi} \quad \Rightarrow \quad r \ge 0.929 \text{ in.}$$

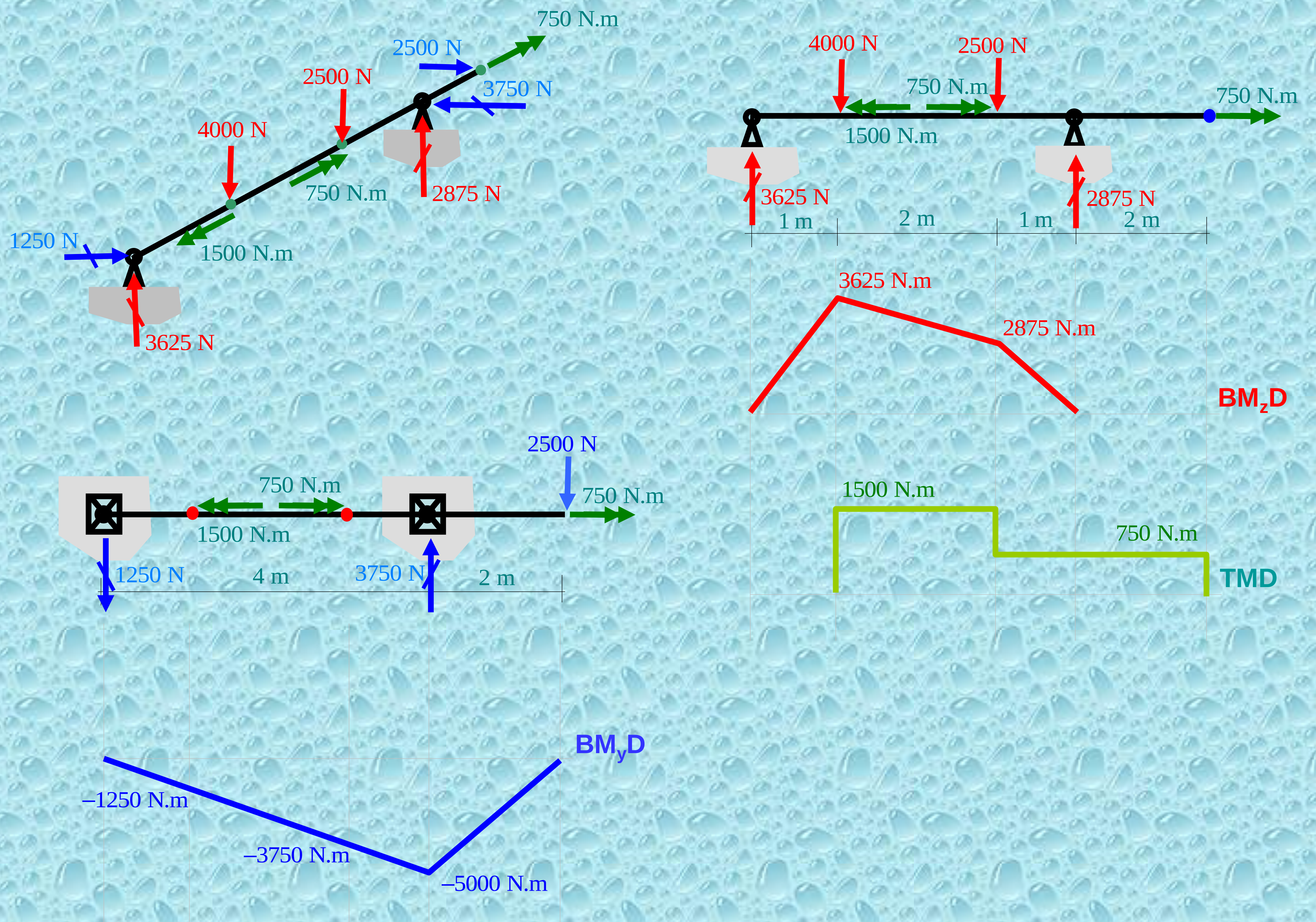
952. Design a solid shaft to carry the loads shown in Fig. 9-29, if max. $\tau \leq 70$ MPa and max. $\sigma \leq 120$ MPa. The belt pulls on pulleys B and C are vertical, and those on pulley E are horizontal. Neglect the masses of the pulleys and shaft.



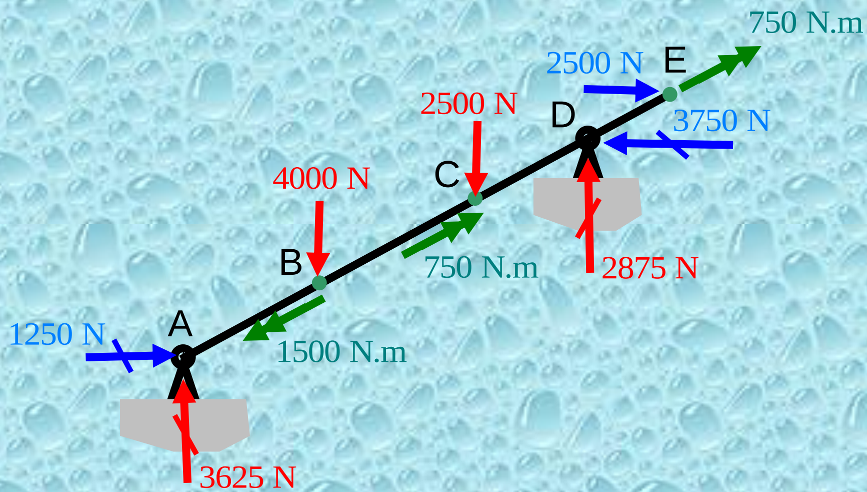
(a) Loading on line shaft



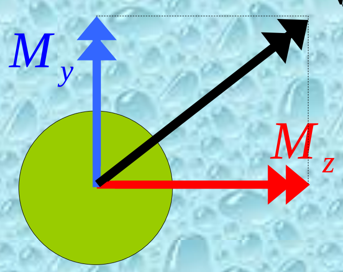
750 N.m
2500 N
2500 N
3750 N
4000 N
750 N.m
2875 N
1250 N
1500 N.m
3625 N
3625 N·m
2875 N·m
(b) Bending moments in vertical plane
A
B
C
D
E
1250 N·m
3750 N·m
5000 N·m
(c) Bending moments in horizontal plane
1500 N·m
750 N·m
(d) Torque distribution in shaft



750 N.m
2500 N
3750 N
2500 N
4000 N
750 N.m
2875 N
1250 N
1500 N.m
3625 N
4000 N
2500 N
750 N.m
750 N.m
1500 N.m
3625 N
2875 N
1 m
2 m
1 m
2 m
3625 N.m
2875 N.m
BMzD
1500 N.m
750 N.m
TMD
2500 N
750 N.m
750 N.m
1500 N.m
1250 N
4 m
3750 N
2 m
BMyD
–1250 N.m
–3750 N.m
–5000 N.m

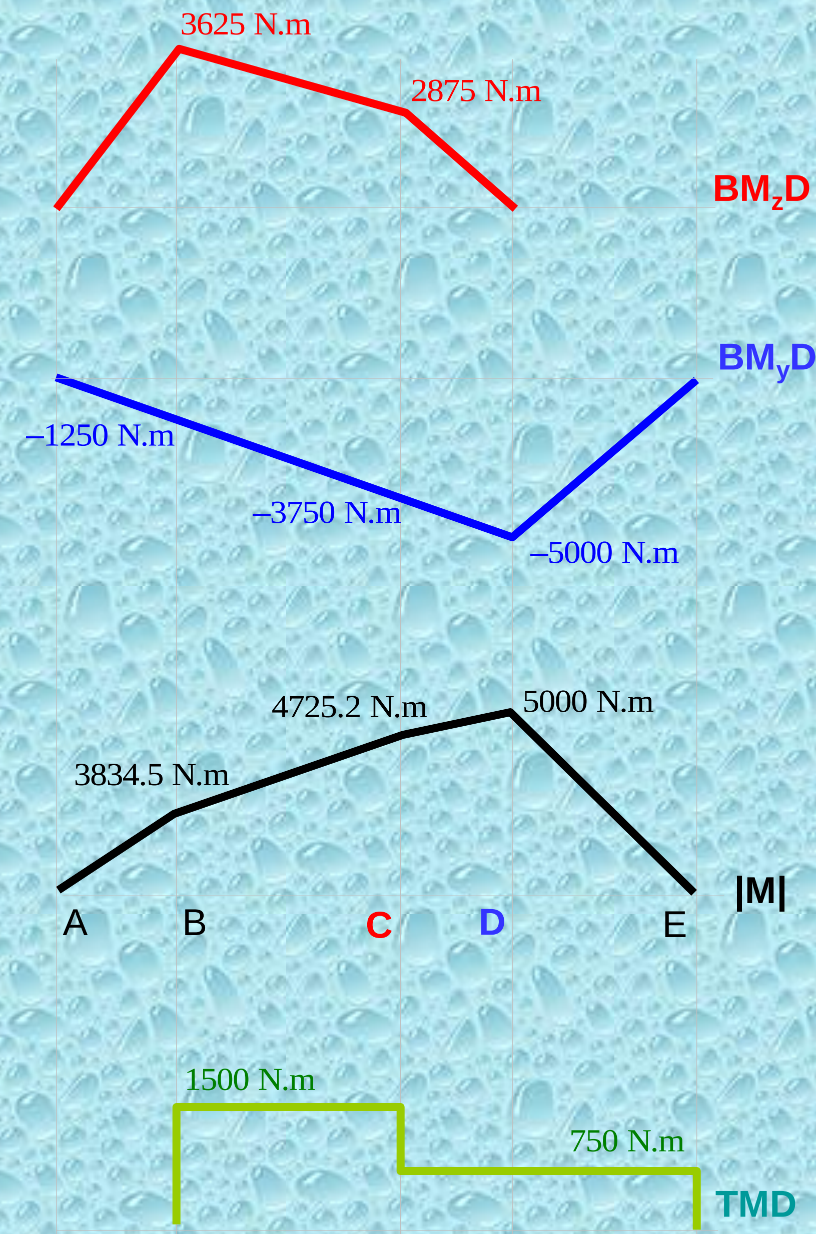


$$|M| = \sqrt{M_z^2 + M_y^2}$$



Cross section of solid shaft

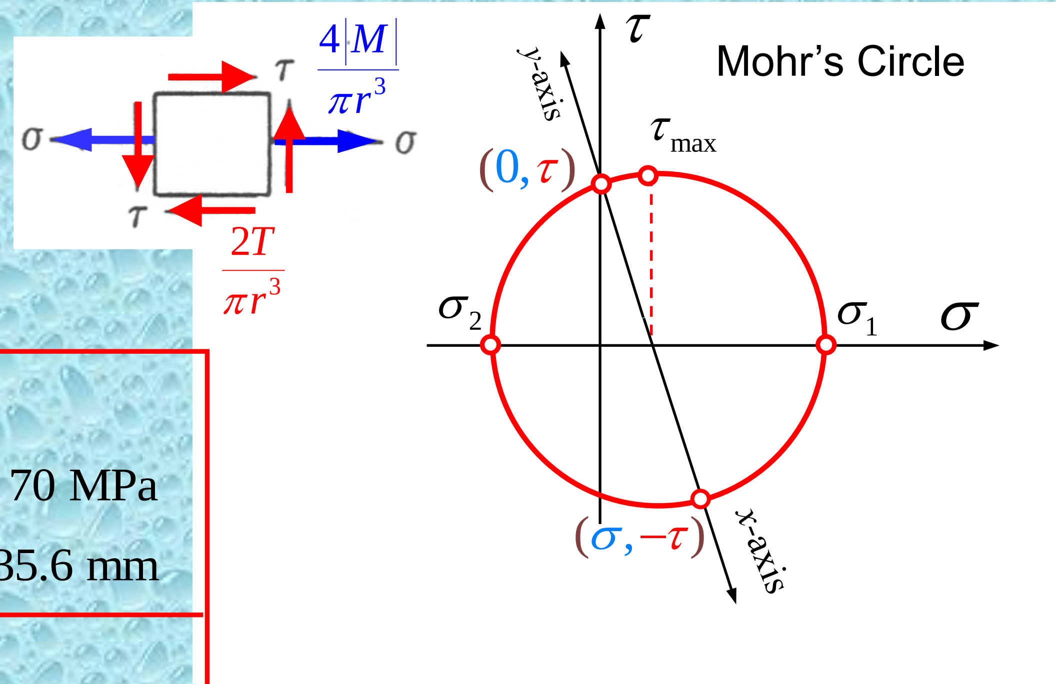
and the resultant moment



From Prob. 951 and this problem.

$$\tau_{max} = \frac{2}{\pi r^3}\sqrt{|M|^2 + T^2} \le 70 \text{ MPa}$$

$$\sigma_1 = \frac{2}{\pi r^3}\left\{|M| + \sqrt{|M|^2 + T^2}\right\} \le 120 \text{ MPa}$$



At section C

$$\tau_{max} = \frac{2}{\pi r^3}\sqrt{4725.2^2 + 1500^2} \times 1000 \text{ mm} \le 70 \text{ MPa}$$

$$r \ge 35.6 \text{ mm}$$

$$\sigma_1 = \frac{2}{\pi r^3}\left\{4725.2 + \sqrt{4725.2^2 + 1500^2}\right\} \times 1000 \le 120 \text{ MPa}$$

$$r \ge 37.2 \text{ mm}$$

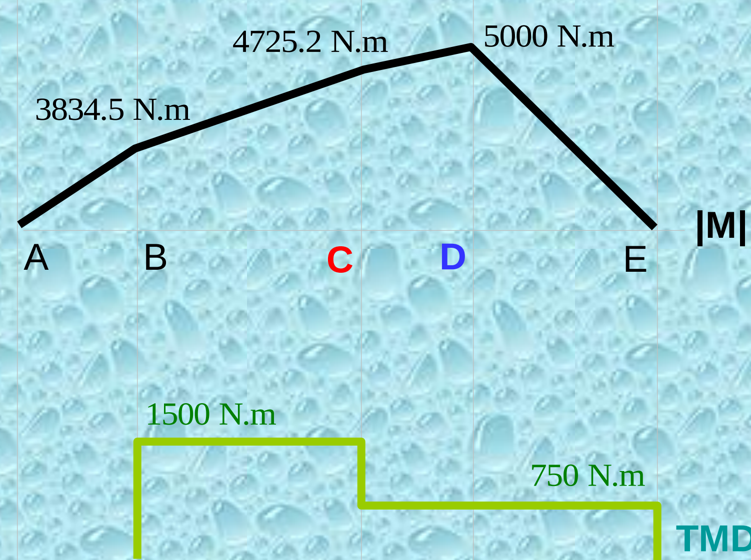
At section D

$$\tau_{max} = \frac{2}{\pi r^3}\sqrt{5000^2 + 750^2} \times 1000 \text{ mm} \quad \le 70 \text{ MPa}$$

$$r \ge 35.8 \text{ mm}$$

$$\sigma_1 = \frac{2}{\pi r^3}\left\{5000 + \sqrt{5000^2 + 750^2}\right\} \times 1000 \le 120 \text{ MPa}$$

$$r \ge 37.7 \text{ mm}$$



$r \ge 37.7$ mm

953. A thin-walled cylindrical pressure vessel with closed ends is 900 mm in diameter and has a wall thickness of 10 mm. If the internal pressure is p = 2.0 MPa, what is the largest torque that can also be applied if the shearing stress is limited to 50 MPa? Assume that the wall of the vessel is suitably braced against buckling.

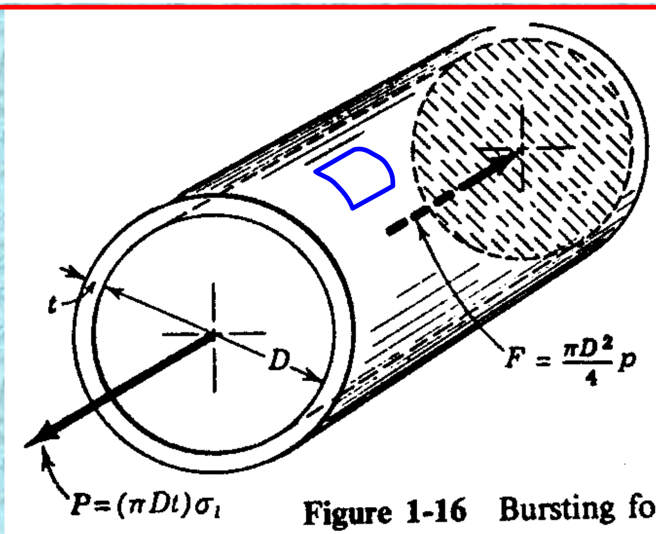
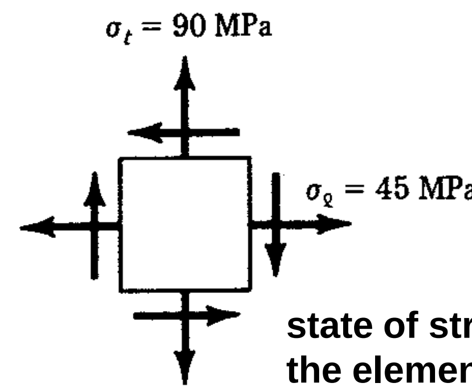


Figure 1-16 Bursting force on a transverse section.

$$\sigma_l = \frac{pD}{4t} = \frac{(2.0 \times 10^6)(0.900)}{4(0.010)} = 45.0 \text{ MPa}$$



state of stress on the element on the surface of vessel

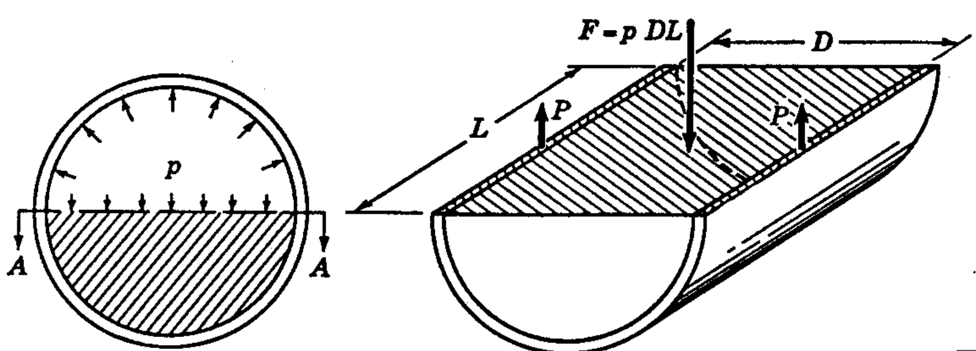
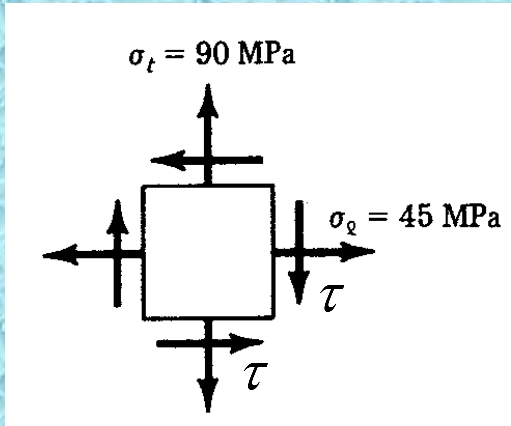
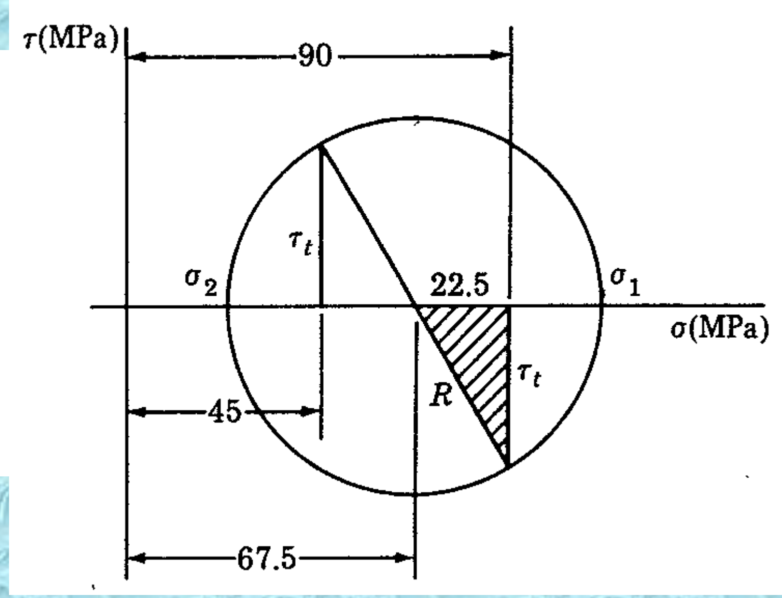


Figure 1-15 Direct evaluation of bursting force F.

$$\sigma_t = \frac{pD}{2t} = 2\sigma_l = 90.0 \text{ MPa}$$





$$\sigma_1 = 67.5 + R$$

$$\sigma_2 = 67.5 - R$$

$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

<u>**Absolute maximum shearing stress**</u> ≤ 50 MPa

$$\left.\begin{aligned} \frac{|\sigma_1 - \sigma_2|}{2} &= R \leq 50\ \text{MPa} \\ \frac{|\sigma_1|}{2} &= \frac{67.5 + R}{2} \leq 50\ \text{MPa} \\ \frac{|\sigma_2|}{2} &= \frac{67.5 - R}{2} \leq 50\ \text{MPa} \end{aligned}\right\} R \leq 32.5\ \text{MPa}$$

$$R^2 = 22.5^2 + \tau_{xy}^2 \leq 32.5^2$$

$$\tau_{xy}^2 \leq 32.5^2 - 22.5^2 = 550$$

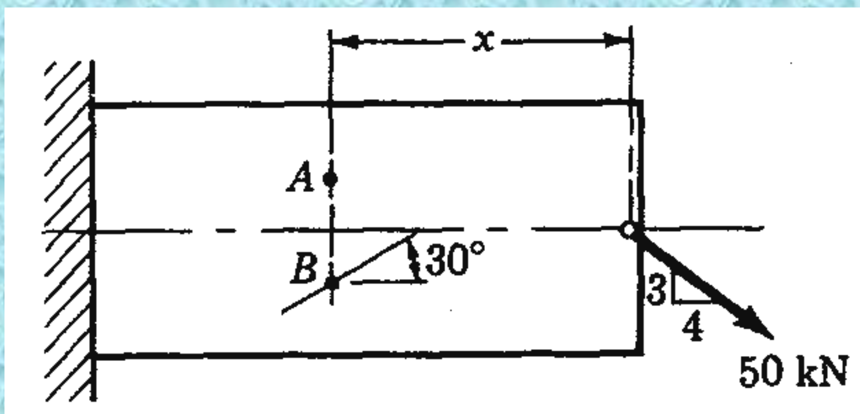
$$\tau_{xy} \leq 23.45\ \text{MPa}$$

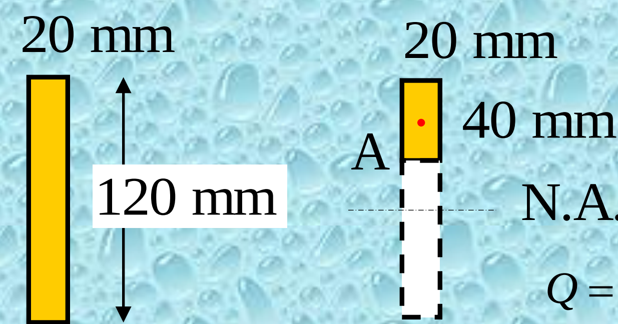
$$\frac{Tc}{J} \leq 23.45\ \text{MPa}$$

$$\frac{T(455\ \text{mm})}{\frac{\pi}{32}\left[920^4 - 900^4\right]} \leq 23.45\ \text{MPa}$$

$$T \leq 301.8\ \text{kN.m}$$

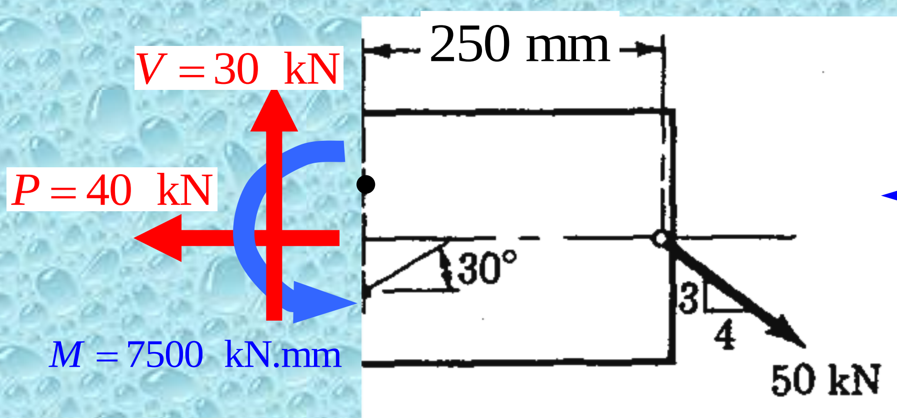
972. Compute the principal stresses and maximum shearing stress at point A in Fig. P-972 at the section $x = 250$ mm. The beam is rectangular, 20 mm wide by 120 mm deep, and point A is 20 mm above the centerline of the beam. Assume the 50-kN load acts at the centroid of the cross section. Show your answers on complete sketches of appropriate differential elements. (*Hint:* Be sure to include the shearing stress caused by the applied load.)

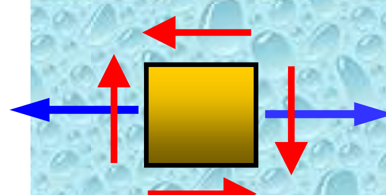




$$Q = (20 \times 40) \times 40 = 3.2 \times 10^4 \ \text{mm}^3$$

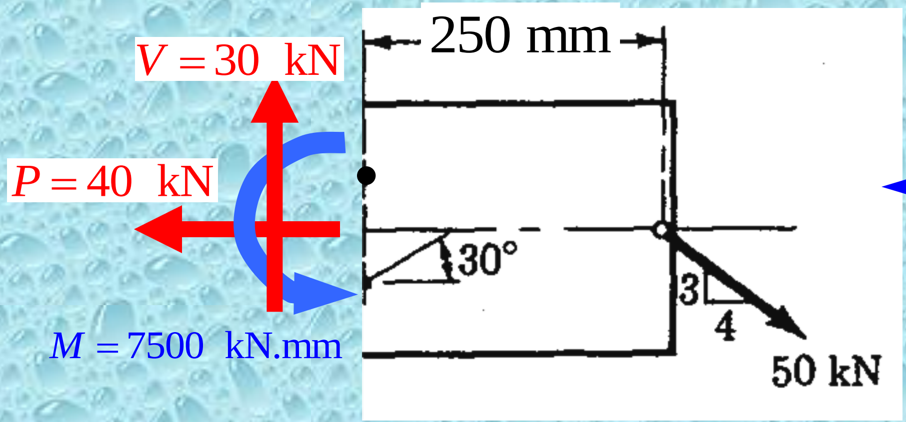
$$I = \frac{20 \times 120^3}{12} = 2.88 \times 10^6 \ \text{mm}^4$$

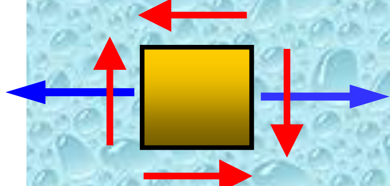




$$\sigma = \frac{P}{A} + \frac{My}{I} = \frac{40}{20 \times 120} + \frac{7500 \times 20}{2.88 \times 10^6} = 68.75 \ \text{MPa}$$

$$\frac{VQ}{Ib} = \frac{30 \times 3.2 \times 10^4}{2.88 \times 10^6 \times 20} = 16.67 \ \text{MPa}$$





$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{40}{20\times 120} + \frac{7500\times 20}{2.88\times 10^{6}} = 68.75\ \text{MPa}$$

$$\frac{VQ}{Ib} = \frac{30\times 3.2\times 10^{4}}{2.88\times 10^{6}\times 20} = 16.67\ \text{MPa}$$

$$\mathbf{C} = (C,0) = \left(\frac{\sigma_x+\sigma_y}{2},0\right) = \left(\frac{68.75+0}{2},0\right) = (34.375,0)$$

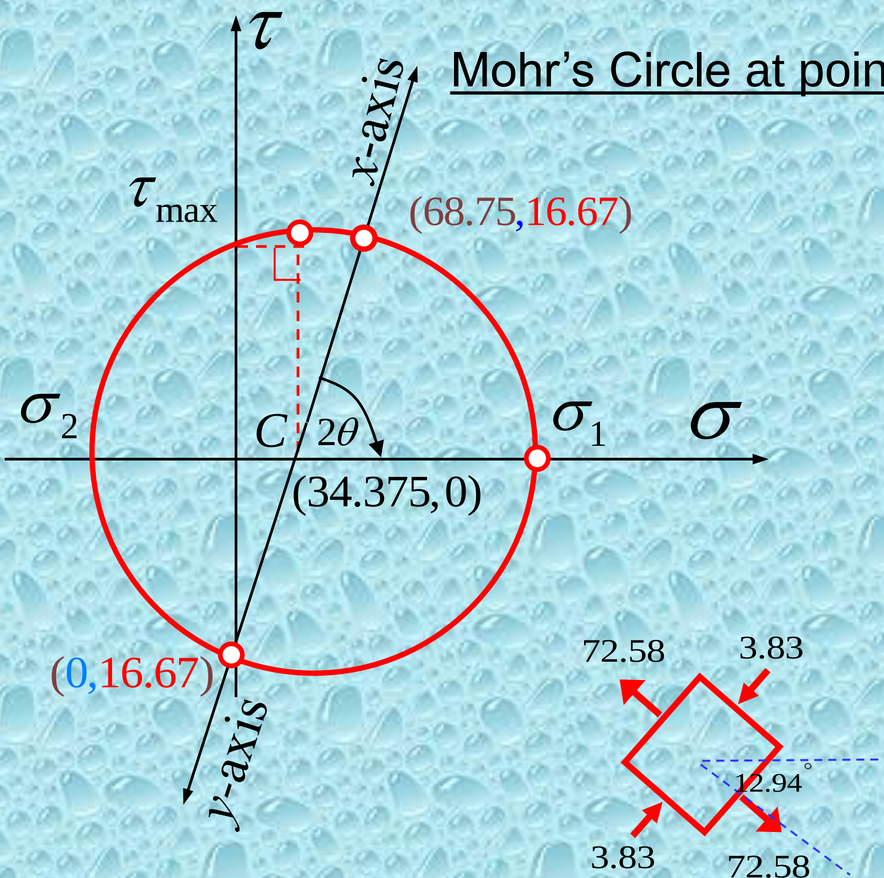
$$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = \sqrt{\left(\frac{68.75}{2}\right)^2+16.67^2} = 38.20\ \text{MPa}$$

$$\sigma_1,\sigma_2 = C\pm R = 34.375\pm 38.20 = 72.578, -3.825\ \text{MPa}$$

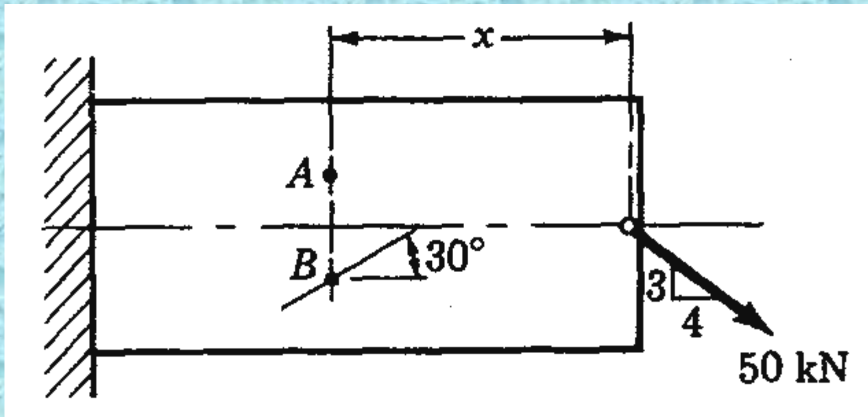
$$\sin 2\theta = \frac{\tau_{xy}}{R} = \frac{16.67}{38.20}$$

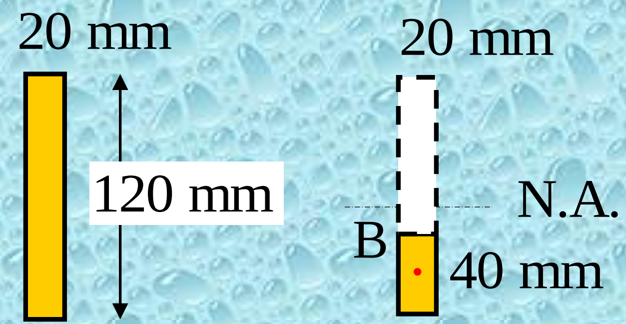
$$\theta = 12.94^{\circ}$$

Mohr's Circle at point A



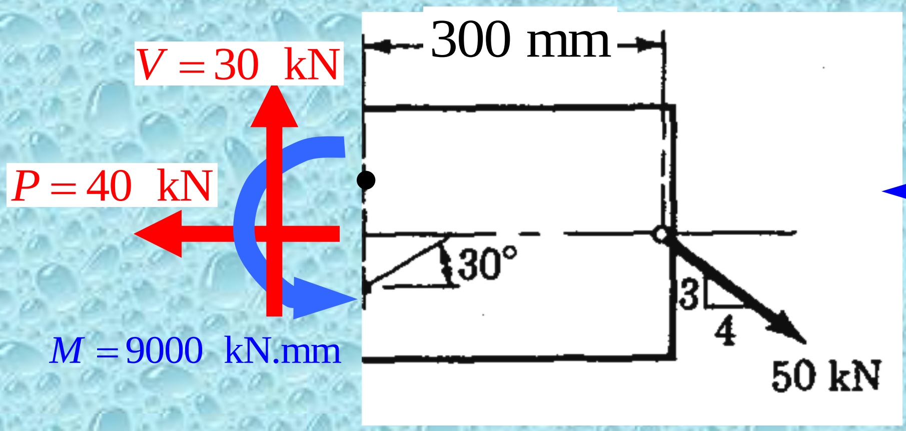
973. For the 20 mm by 120 mm beam described in Prob. 972, determine the stress components on the 30° plane at point B. Assume that x = 300 mm and that B is 20 mm below the centerline of the beam. Show your answers on a complete sketch of a differential element.

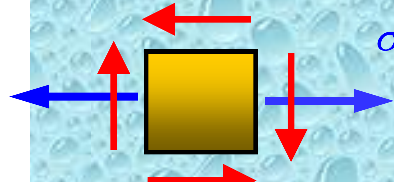




$$I=\frac{20\times 120^3}{12}=2.88\times 10^6\ \text{mm}^4$$

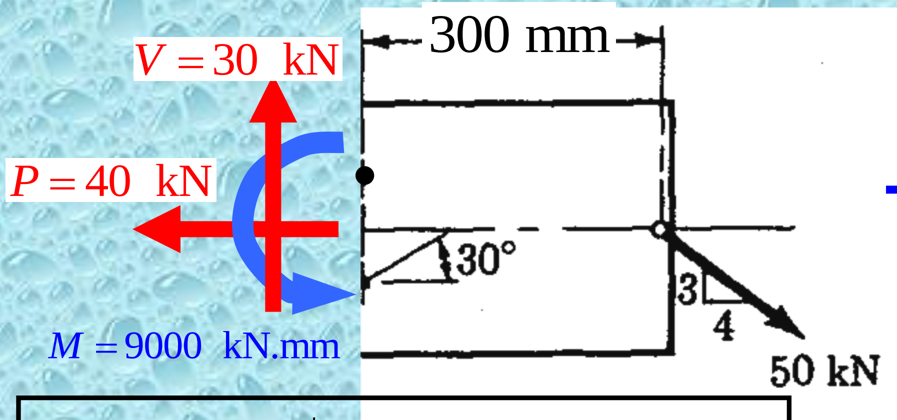
$$Q=(20\times 40)\times 40$$
$$=3.2\times 10^4\ \text{mm}^3$$

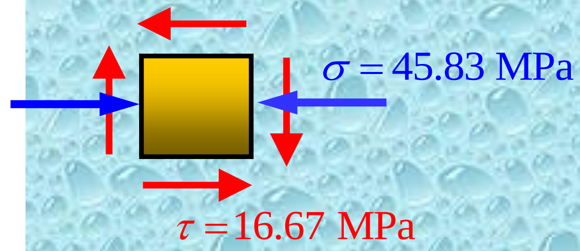




$$\sigma=\frac{P}{A}+\frac{My}{I}=\frac{40}{20\times 120}-\frac{9000\times(-20)}{2.88\times 10^6}$$
$$=-45.83\ \text{MPa}$$

$$\frac{VQ}{Ib}=\frac{30\times 3.2\times 10^4}{2.88\times 10^6\times 20}=16.67\ \text{MPa}$$





$$\mathbf{C} = (C,0) = \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$$= \left(\frac{-45.83 + 0}{2}, 0\right) = (-22.915, 0)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{45.83}{2}\right)^2 + 16.67^2} = 28.34 \text{ MPa}$$

$$\sin 2\theta = \frac{\tau_{xy}}{R} = \frac{16.67}{28.34} \qquad 2\theta = 36.03^{\circ}$$

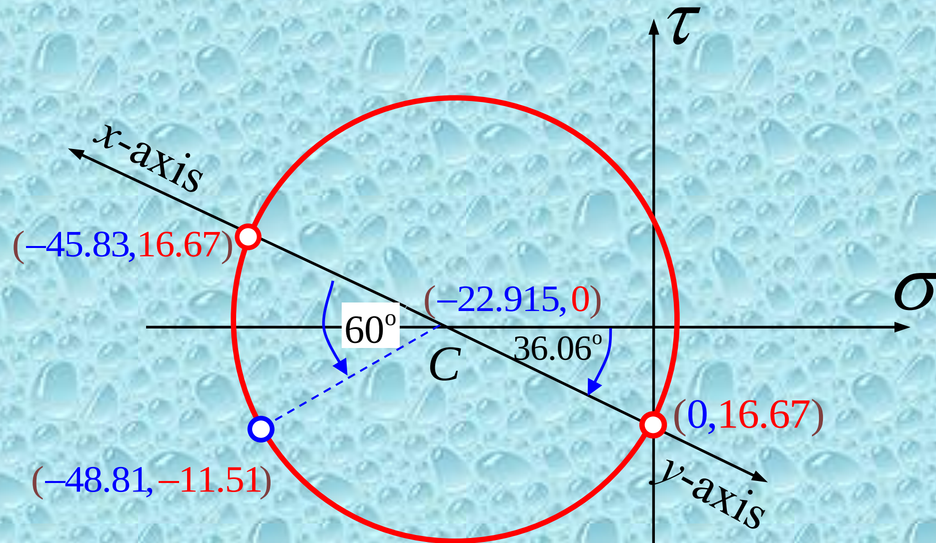
$$\sigma_{30^0} = C - R\cos(60^{\circ} - 36.03^{\circ})$$

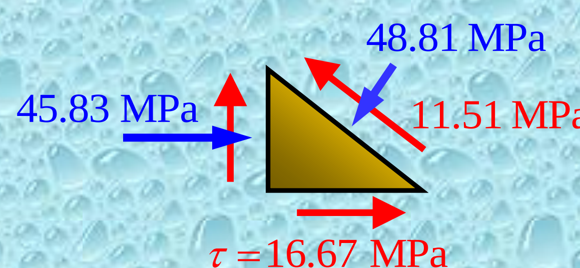
$$= -22.915 - 28.34\cos(23.97^{\circ})$$

$$= -48.81 \text{ MPa}$$

$$\tau_{30^0} = -28.34\sin(23.97^{\circ}) = -11.51 \text{ MPa}$$

Mohr's Circle at point B





Hw18 Design a solid shaft to carry the loads shown in Fig. P-970 if the maximum normal stress and maximum shearing stress are limited to 12 ksi and 8 ksi, respectively. The belts on pulleys *A* and *C* are horizontal, and those on pulley *E* are vertical.

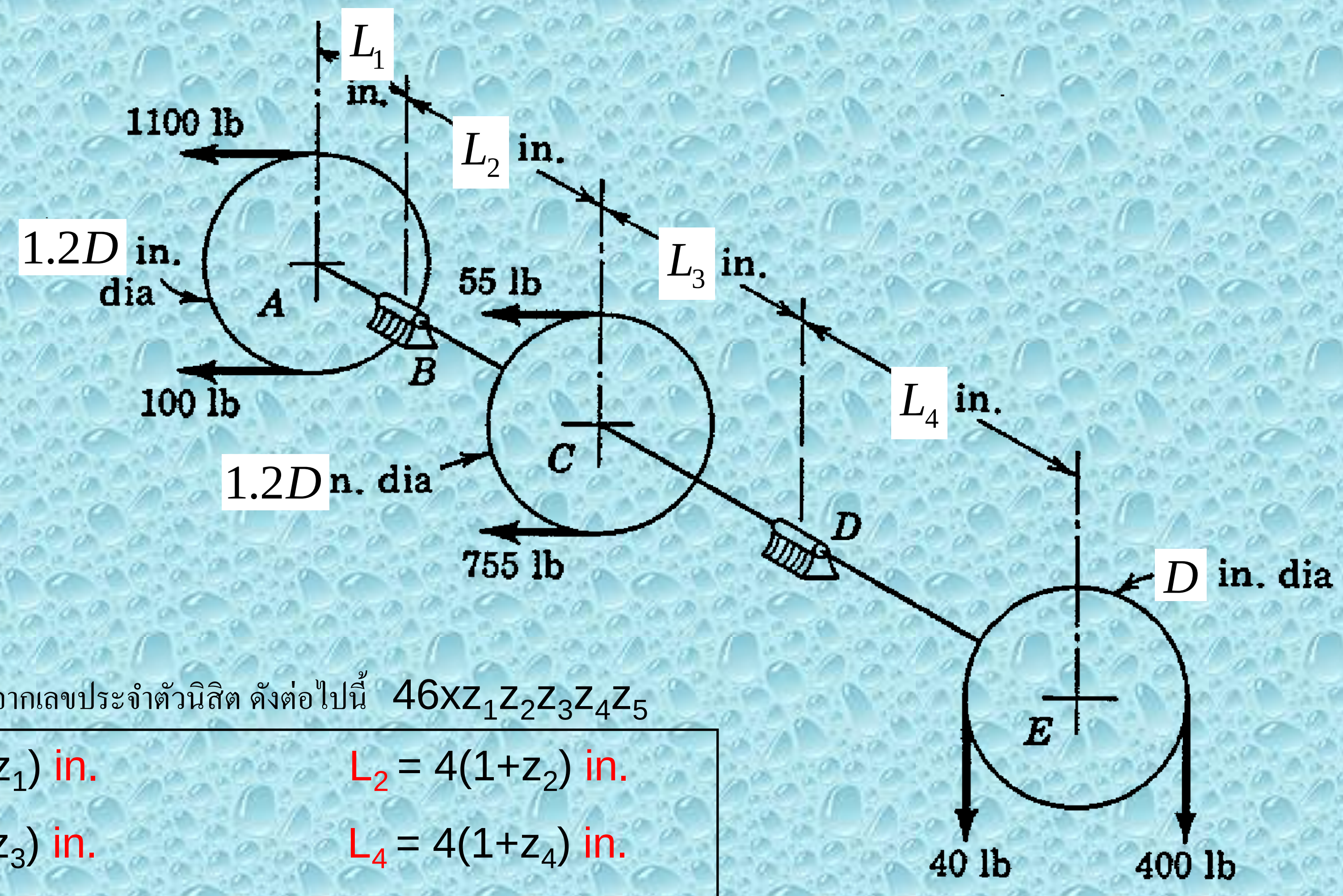
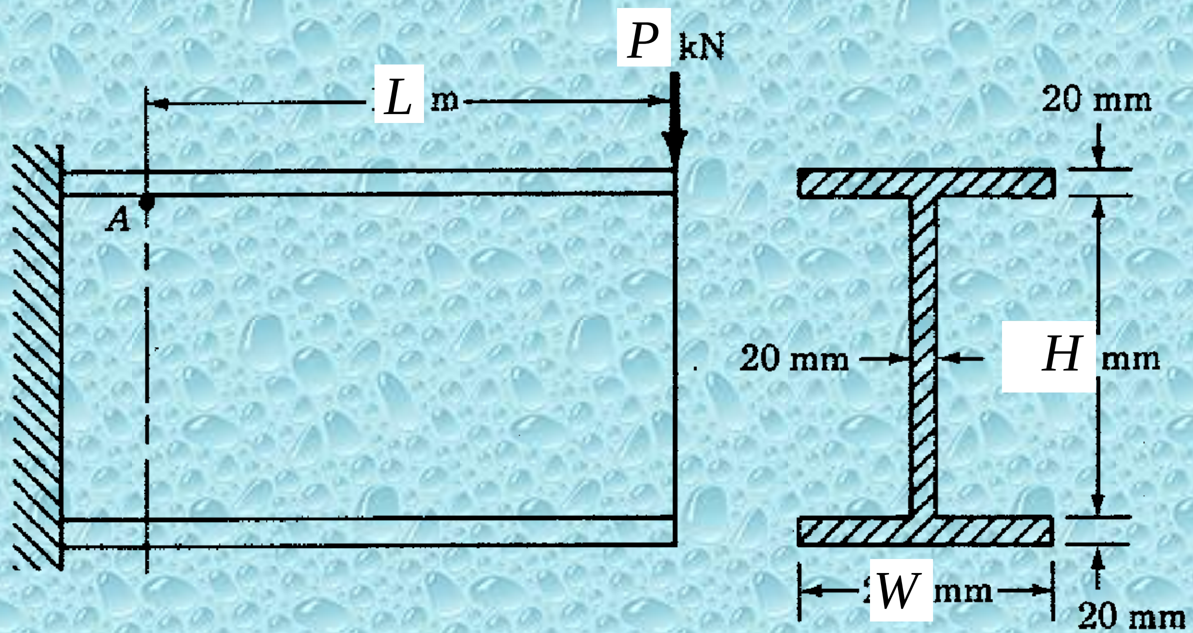


Figure P-970

ค่า z_1-z_5 ได้จากเลขประจำตัวนิสิต ดังต่อไปนี้ $46xz_1z_2z_3z_4z_5$

$L_1 = 4(1+z_1)$ in.	$L_2 = 4(1+z_2)$ in.
$L_3 = 4(1+z_3)$ in.	$L_4 = 4(1+z_4)$ in.
$D = 4(1+z_5)$ in.	

Hw19 For the cantilever beam shown in Fig. P-975, determine the principal stresses at point *A* that is just below the flange. Show your results on a complete sketch of a differential element. Assume the vertical load passes through the centroid of the cross section. Also find the maximum shearing stress at point A. Show your results on a complete sketch of a differential element.



ค่า z_1-z_4 ได้จากเลขประจำตัวนิสิต ดังต่อไปนี้ 46xx$z_1z_2z_3z_4$

L= 0.4(1+z_1) m.	P = 4(1+z_2) kN
H= 40(1+z_3) mm.	W = 40(1+z_4) mm

9-12 RELATION BETWEEN MODULUS OF RIGIDITY AND MODULUS OF ELASTICITY

$$\epsilon_x = \frac{\tau(1+\nu)}{E}, \qquad \epsilon_y = -\frac{\tau(1+\nu)}{E}$$

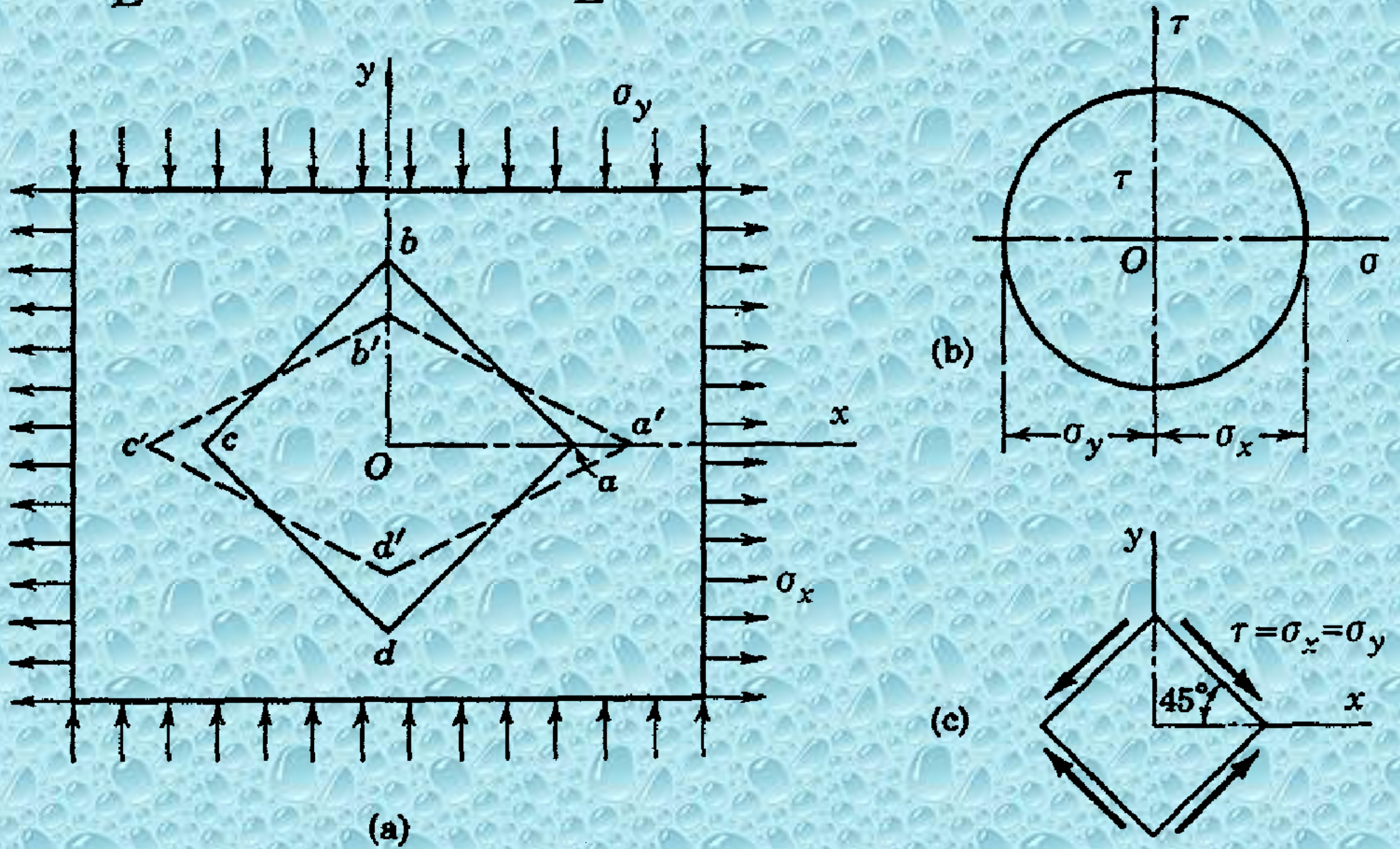


Figure 9-38 Pure shear and shearing strain.

$$\overline{oa'} = \overline{oa}\left[1 + \frac{\tau(1+\nu)}{E}\right], \qquad \overline{ob'} = \overline{ob}\left[1 - \frac{\tau(1+\nu)}{E}\right]$$

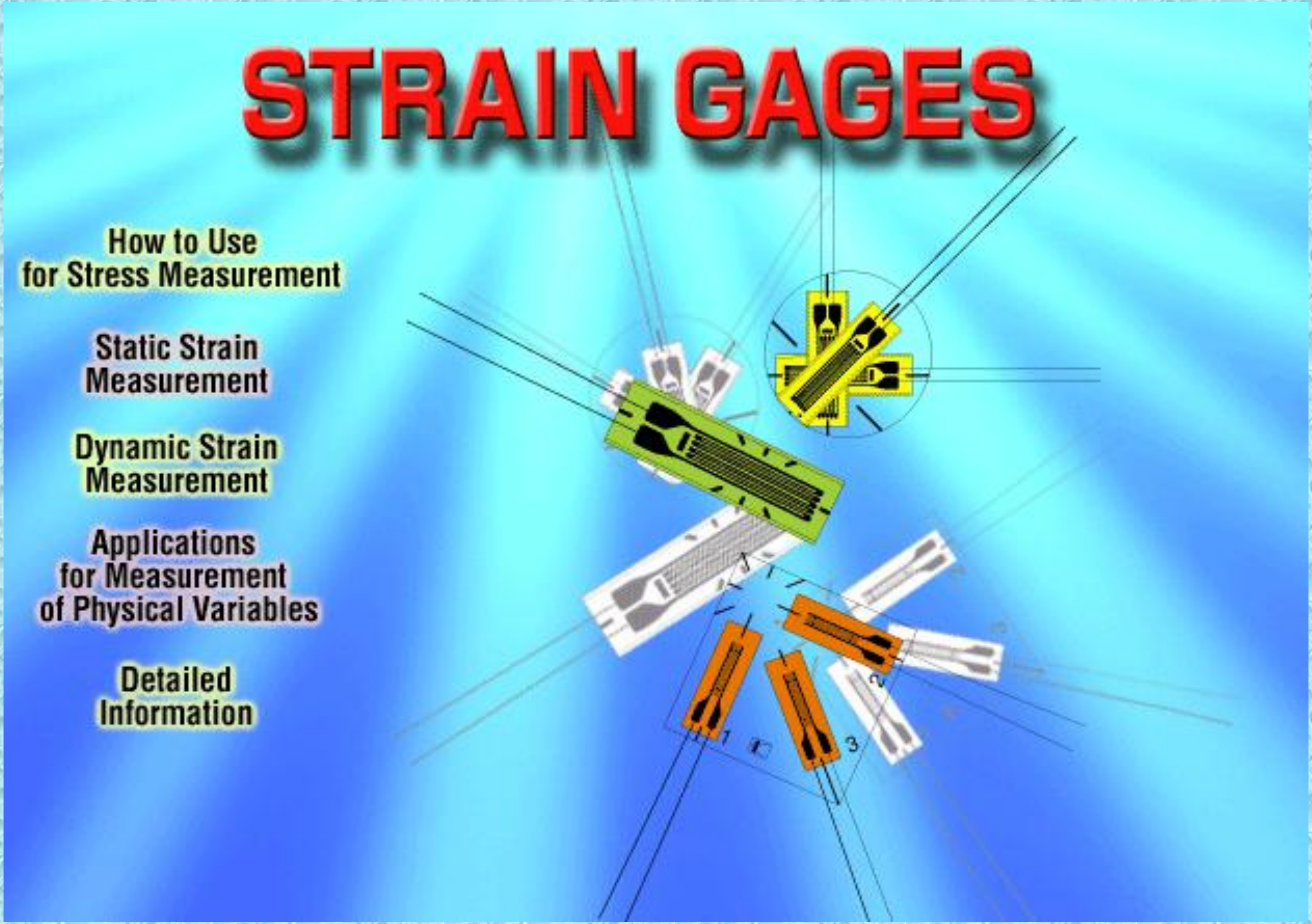
$$\tan oa'b' = \tan\left(45° - \frac{\gamma}{2}\right) = \frac{\overline{ob'}}{\overline{oa'}} = \frac{1 - \tau(1+\nu)/E}{1 + \tau(1+\nu)/E} \qquad (a)$$

$$\tan\left(45° - \frac{\gamma}{2}\right) = \frac{\tan 45° - \tan\frac{\gamma}{2}}{1 + \tan 45° \tan\frac{\gamma}{2}} = \frac{1 - \frac{\gamma}{2}}{1 + \frac{\gamma}{2}}$$

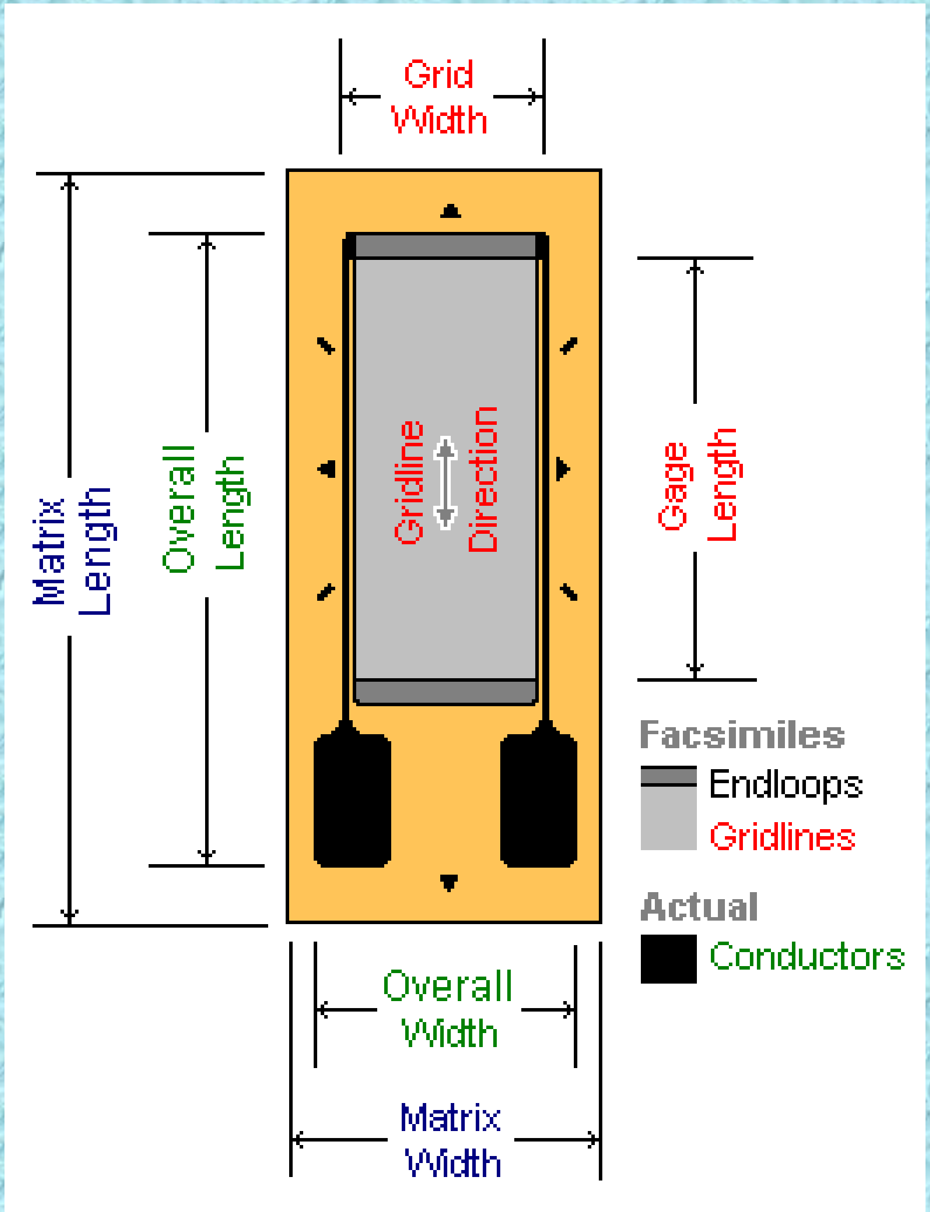
$$\frac{1 - \gamma/2}{1 + \gamma/2} = \frac{1 - \tau(1+\nu)/E}{1 + \tau(1+\nu)/E}$$

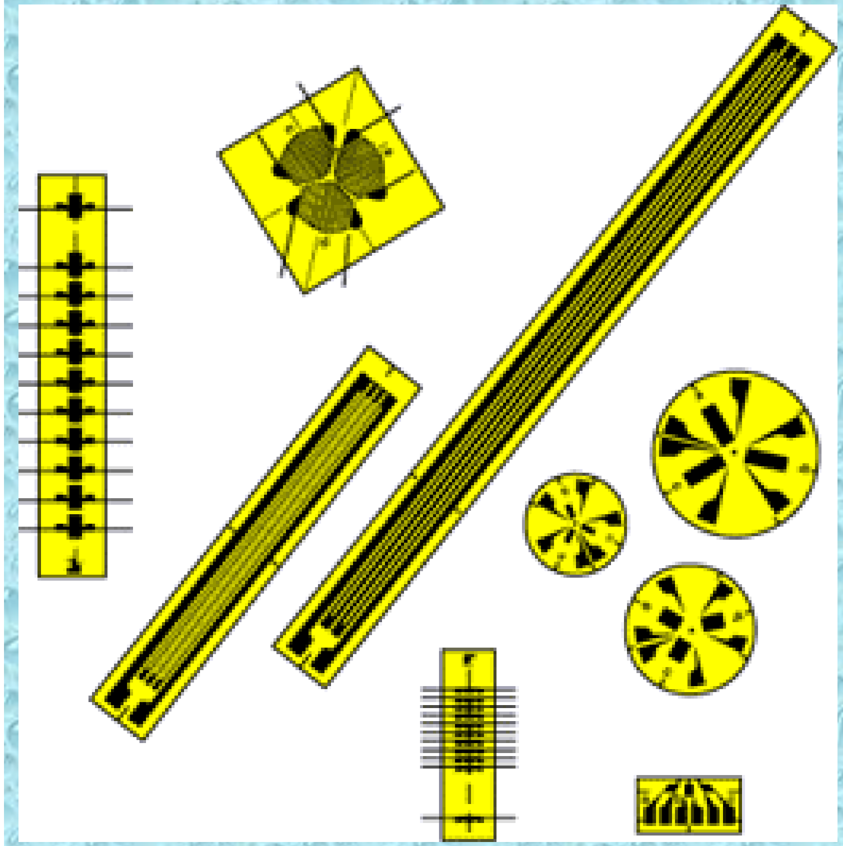
$$\gamma = \frac{2\tau(1+\nu)}{E} \quad \text{or} \quad \frac{\tau}{\gamma} = \frac{E}{2(1+\nu)}$$

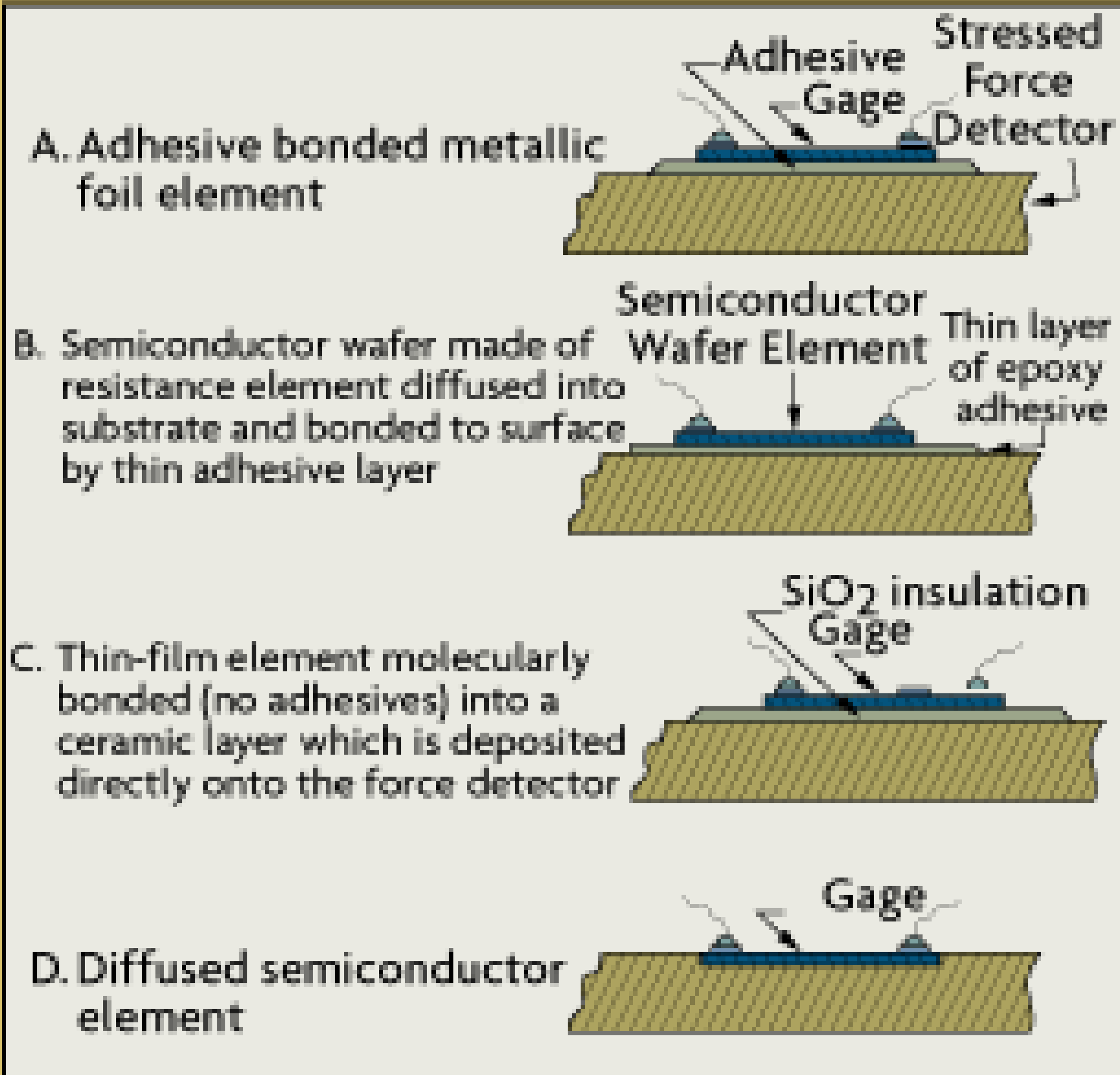
$$\boxed{G = \frac{E}{2(1+\nu)}}$$



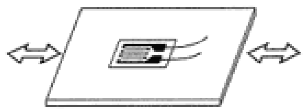
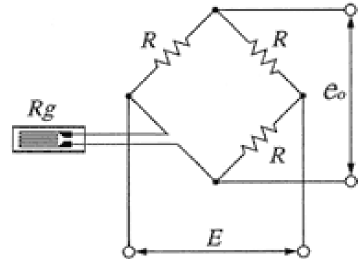
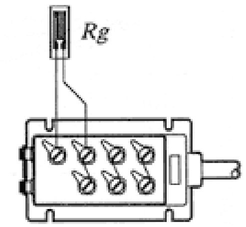
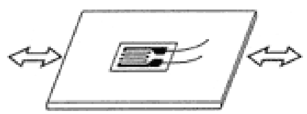
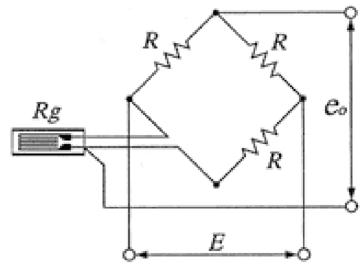
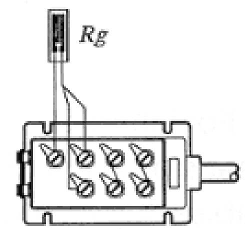
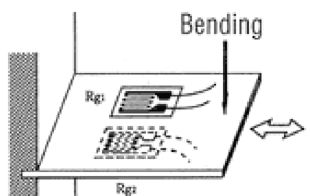
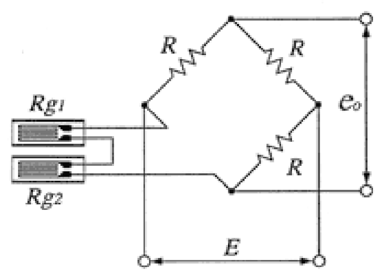
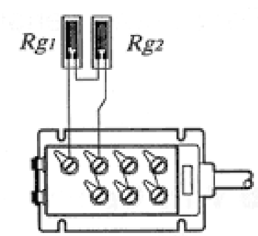
http://www.kyowa-ei.co.jp/english/products.htm


Grid Width
Matrix Length
Overall Length
Gridline Direction
Gage Length
Facsimiles
Endloops
Gridlines
Actual
Conductors
Overall Width
Matrix Width




Adhesive
Gage
Stressed
Force
Detector
A. Adhesive bonded metallic foil element
Semiconductor
Wafer Element
Thin layer of epoxy adhesive
B. Semiconductor wafer made of resistance element diffused into substrate and bonded to surface by thin adhesive layer
SiO_2 insulation
Gage
C. Thin-film element molecularly bonded (no adhesives) into a ceramic layer which is deposited directly onto the force detector
Gage
D. Diffused semiconductor element

How to form strain gage bridges

	Gage method Connection system	Application	Circuit	Output	Remarks	Bridge boxes DB-120P, 350P
1	**1-active-gage 2-wire** Nr. of gage: 1	 Monoaxial stress (uniform tension or compression)		$e_o = \frac{E}{4} K_s \cdot \varepsilon_o$ K_s : Gage factor ε_o : Strain E : Bridge voltage e_o : Output voltage Rg : Gage resistor R : Fixed resistor	No temp. compensation; x1 output; non-linearity correction needed for large strain	
2	**1-active-gage 3-wire** Nr. of gage: 1	 Monoaxial stress (uniform tension or compression)		$e_o = \frac{E}{4} K_s \cdot \varepsilon_o$	No temp. compensation; thermal effect on leadwires cancelled; x1 output; non-linearity correction needed for large strain	
3	**1-active-gage (2 in series) 2-wire (Cancelling bending strain)** Nr. of gages: 2	 Monoaxial stress (uniform tension or compression)		$e_o = \frac{E}{4} K_s \cdot \varepsilon_o$ Rg_1 …… Strain : ε_1 Rg_2 …… Strain : ε_2 $\varepsilon_o = \frac{\varepsilon_1 + \varepsilon_2}{2}$ R : Fixed resistor $Rg_1 + Rg_2 = R$	No temperature compensation; bending strain cancelled; x1 output	

9-10 TRANSFORMATION OF STRAIN COMPONENTS

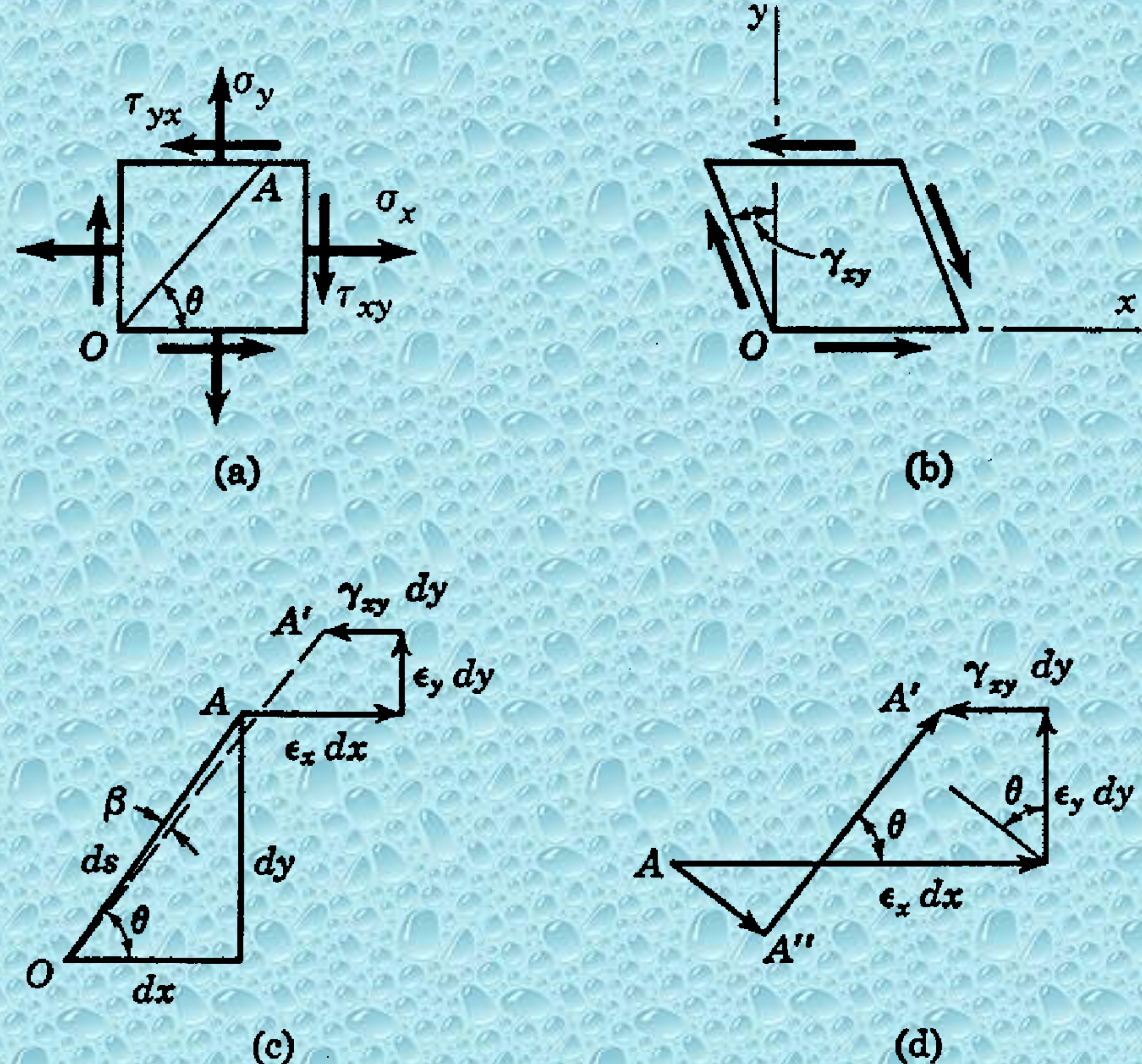
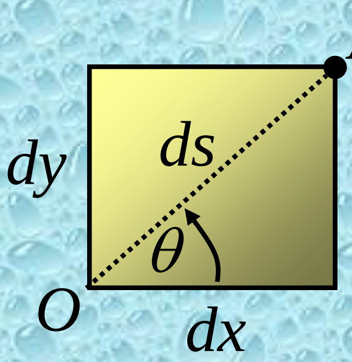
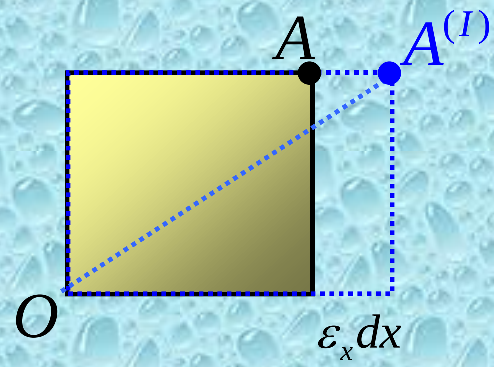


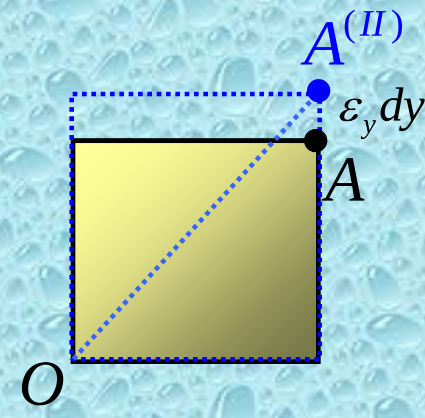
Figure 9-31

Strain and deformation of line element

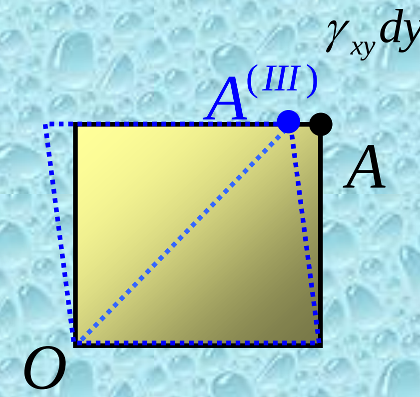




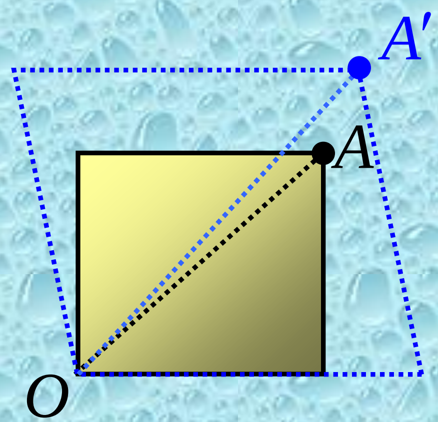
$\varepsilon_x \neq 0, \varepsilon_y = 0, \gamma_{xy} = 0$



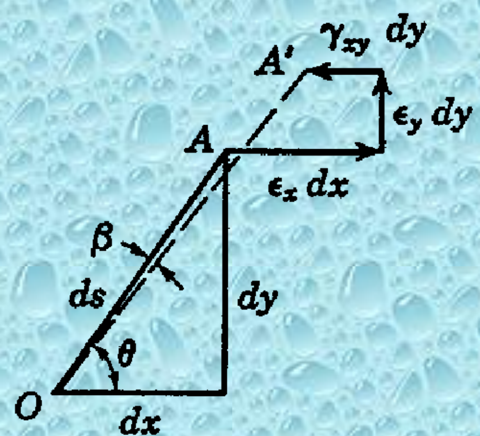
$\varepsilon_x = 0, \varepsilon_y \neq 0, \gamma_{xy} = 0$

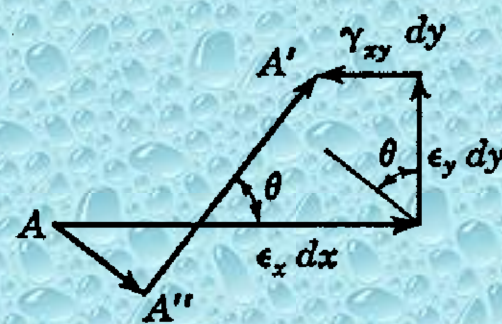


$\varepsilon_x = 0, \varepsilon_y = 0, \gamma_{xy} \neq 0$



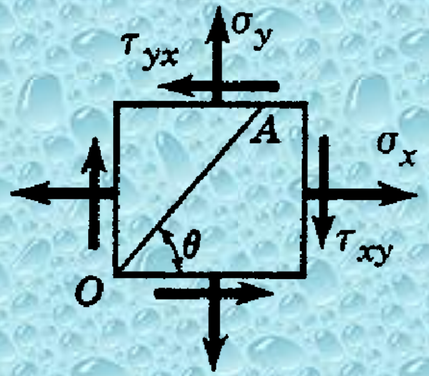
$\varepsilon_x \neq 0, \varepsilon_y \neq 0, \gamma_{xy} \neq 0$

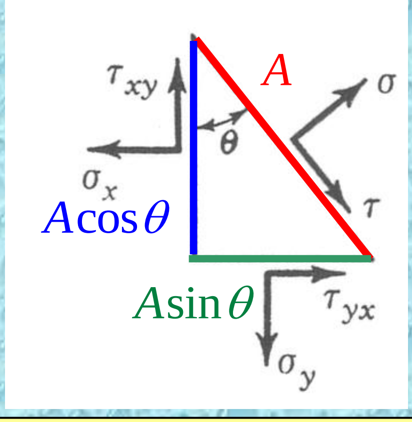




$$\epsilon_a = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta - \frac{1}{2}\gamma_{xy}\sin 2\theta \qquad (9\text{-}17)$$

$$\frac{1}{2}\gamma_{ab} = \frac{(\epsilon_x - \epsilon_y)}{2}\sin 2\theta + \frac{1}{2}\gamma_{xy}\cos 2\theta \qquad (9\text{-}18)$$





$$R_\sigma = R_\epsilon \frac{E}{1+\nu} \qquad (9\text{-}19)$$

$$(OC)_\sigma = (OC)_\epsilon \frac{E}{1-\nu} \qquad (9\text{-}20)$$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad \textbf{Eq.(9-5)}$$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \textbf{Eq.(9-6)}$$

$$R_\sigma = R_\epsilon \frac{E}{1+\nu} \qquad (9\text{-}19)$$

$$(OC)_\sigma = (OC)_\epsilon \frac{E}{1-\nu} \qquad (9\text{-}20)$$

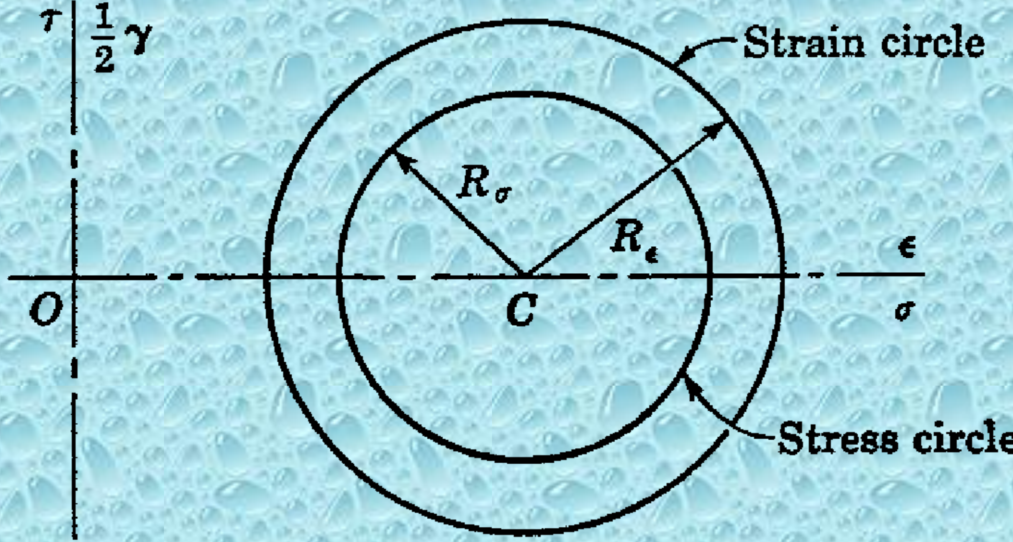
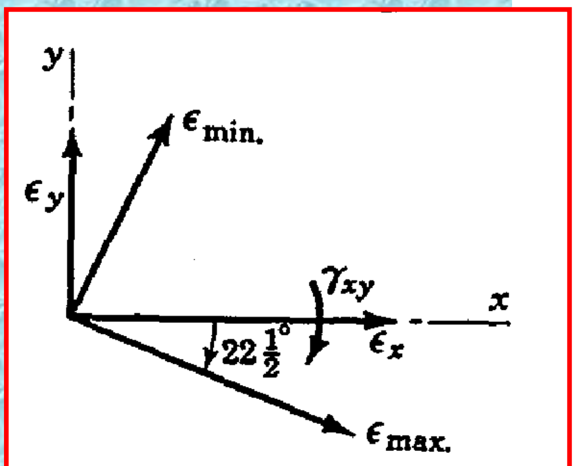


Figure 9-32 Transformation of Mohr's circle of strains to Mohr's circle of stress.

977. In a body subjected to plane strain, there act at a certain point $\epsilon_x = 800 \times 10^{-6}$ m/m, $\epsilon_y = 200 \times 10^{-6}$ m/m, and $\gamma_{xy} = 600 \times 10^{-6}$ rad. Compute (a) the principal strains and the principal strain axes; also (b) the strain ϵ_a in a direction of 60° with the x axis, the strain ϵ_b perpendicular to ϵ_a, and the shearing strain γ_{ab}. Further, if $E = 200$ GPa and $\nu = 0.30$, determine the principal stresses and the normal and shearing stresses on t



$$C = 500 \times 10^{-6} \text{ rad}$$

$$R = 300\sqrt{2} \times 10^{-6} \text{ rad}$$

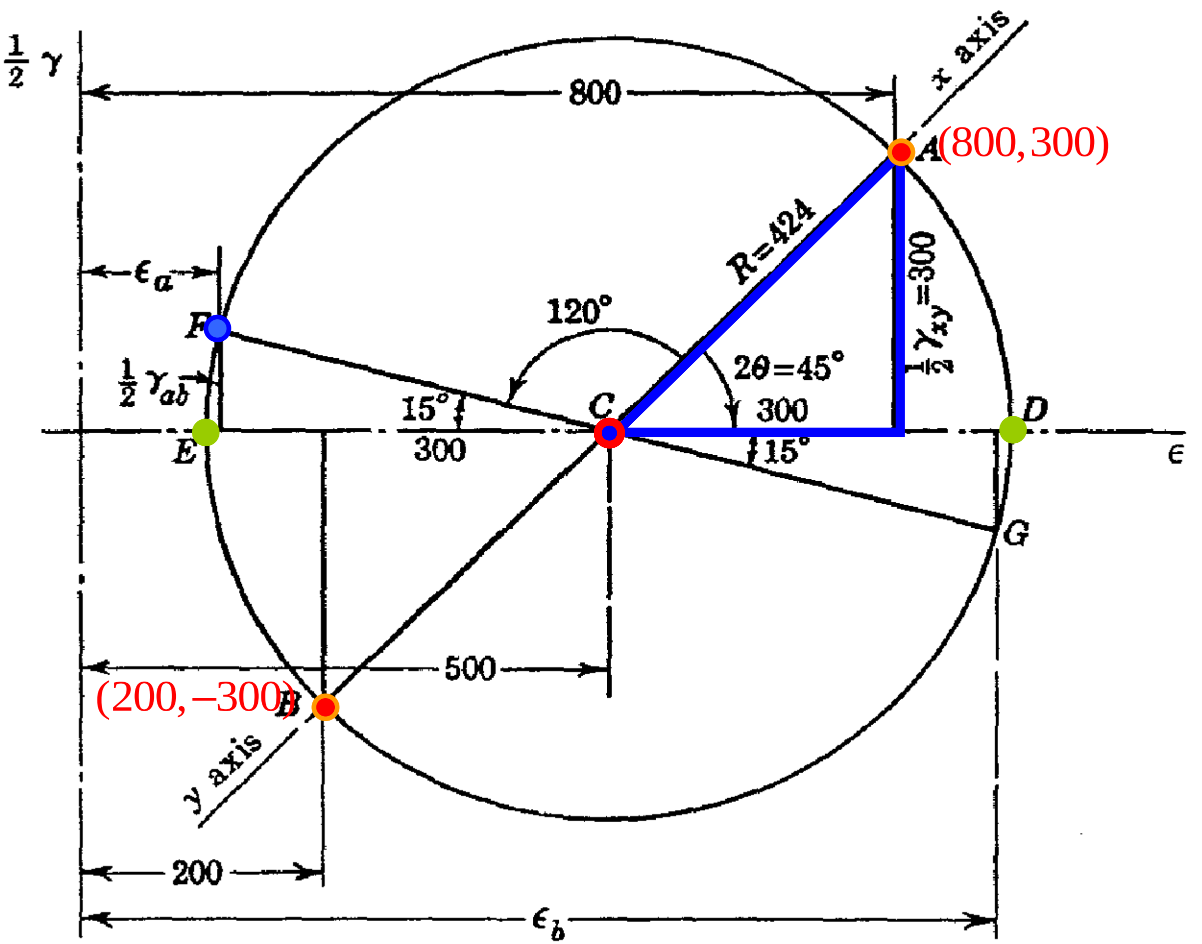


Figure 9-33 Strain circle.

$$\left[R_\sigma = R_\epsilon \frac{E}{1+\nu}\right] \qquad R_\sigma = (424 \times 10^{-6})\frac{200 \times 10^9}{1+0.30} = 65.2 \text{ MPa}$$

$$\left[(OC)_\sigma = (OC)_\epsilon \frac{E}{1-\nu}\right] \qquad (OC)_\sigma = (500 \times 10^{-6})\frac{200 \times 10^9}{1-0.30} = 143 \text{ MPa}$$

at D: $\sigma_1 = 143 + 65.2 = 208$ MPa

at E: $\sigma_2 = 143 - 65.2 = 77.8$ MPa

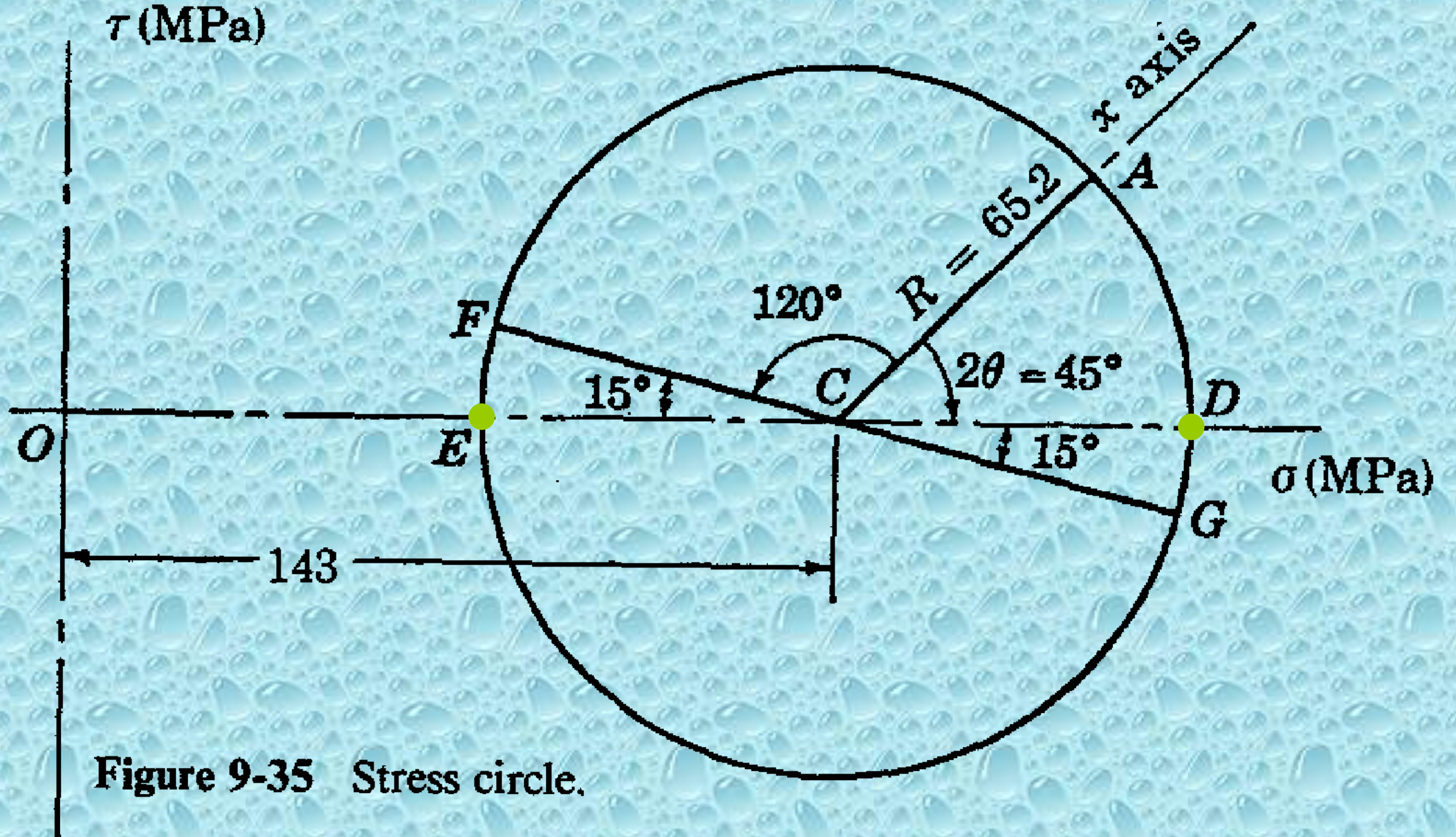


Figure 9-35 Stress circle.

at F: $\sigma = 143 - 65.2 \cos 15° = 80.0$ MPa

$\tau = 65.2 \sin 15° = 16.9$ MPa

at G: $\sigma = 143 + 65.2 \cos 15° = 206$ MPa

If we use the stress-strain relation directly the same answer can be obtained

$$\sigma_x = \frac{E(\epsilon_x + \nu\epsilon_y)}{1 - \nu^2}; \qquad \sigma_y = \frac{E(\epsilon_y + \nu\epsilon_x)}{1 - \nu^2}; \qquad \tau_{xy} = G\gamma_{xy} = \frac{E}{2(1 + \nu)}\gamma_{xy}$$

$$\sigma_1 = \frac{(200 \times 10^9)(924 + 0.30 \times 76)(10^{-6})}{1 - (0.30)^2} = 208 \text{ MPa}$$

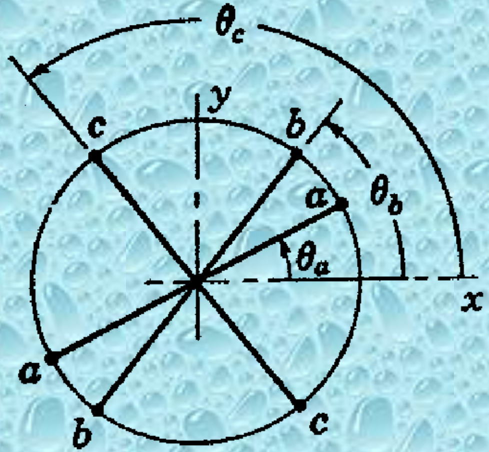
$$\sigma_2 = \frac{(200 \times 10^9)(76 + 0.30 \times 924)(10^{-6})}{1 - (0.30)^2} = 77.6 \text{ MPa}$$

$$\sigma_a = \frac{(200 \times 10^9)(90 + 0.30 \times 910)(10^{-6})}{1 - (0.30)^2} = 79.8 \text{ MPa}$$

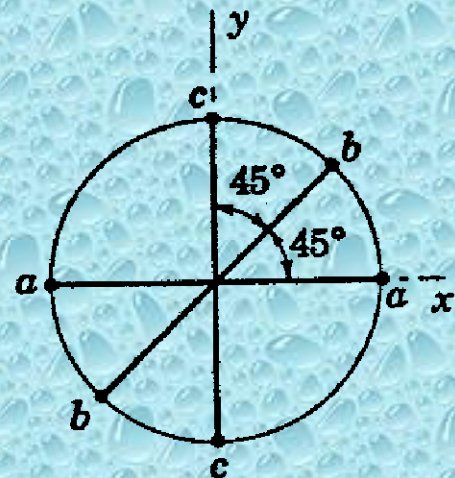
$$\tau_{ab} = \frac{(200 \times 10^9)(220 \times 10^{-6})}{2(1 + 0.30)} = 16.9 \text{ MPa}$$

9-11 THE STRAIN ROSETTE

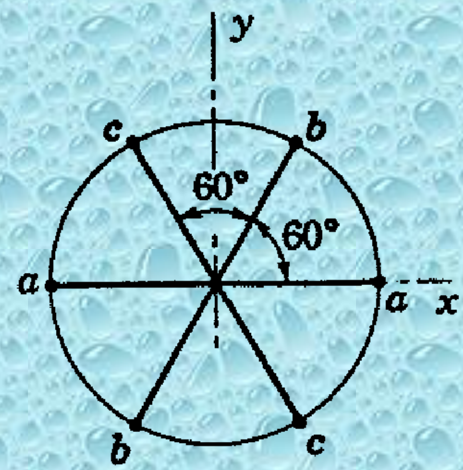
$$\left.\begin{aligned}
\epsilon_a &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_a - \frac{\gamma_{xy}}{2}\sin 2\theta_a \\
\epsilon_b &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_b - \frac{\gamma_{xy}}{2}\sin 2\theta_b \\
\epsilon_c &= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos 2\theta_c - \frac{\gamma_{xy}}{2}\sin 2\theta_c
\end{aligned}\right\}$$



(a) General strain rosette



(b) 45° strain rosette



(c) 60° strain rosette

Figure 9-36 Strain rosettes.

The 45° or Rectangular Strain Rosette

By substituting $\theta_a = 0°$, $\theta_b = 45°$, and $\theta_c = 90°$ in Eq. (*a*) and solving, we obtain

$$\epsilon_x = \epsilon_a, \qquad \epsilon_y = \epsilon_c, \qquad \frac{1}{2}\gamma_{xy} = \frac{\epsilon_a + \epsilon_c}{2} - \epsilon_b \tag{9-21}$$

$$AE = \frac{\epsilon_a + \epsilon_c}{2} - \epsilon_b \tag{b}$$

and

$$CE = \frac{\epsilon_a - \epsilon_c}{2} \tag{c}$$

Therefore the radius $R = CA$ is determined from

$$R = \sqrt{(CE)^2 + (AE)^2} \tag{d}$$

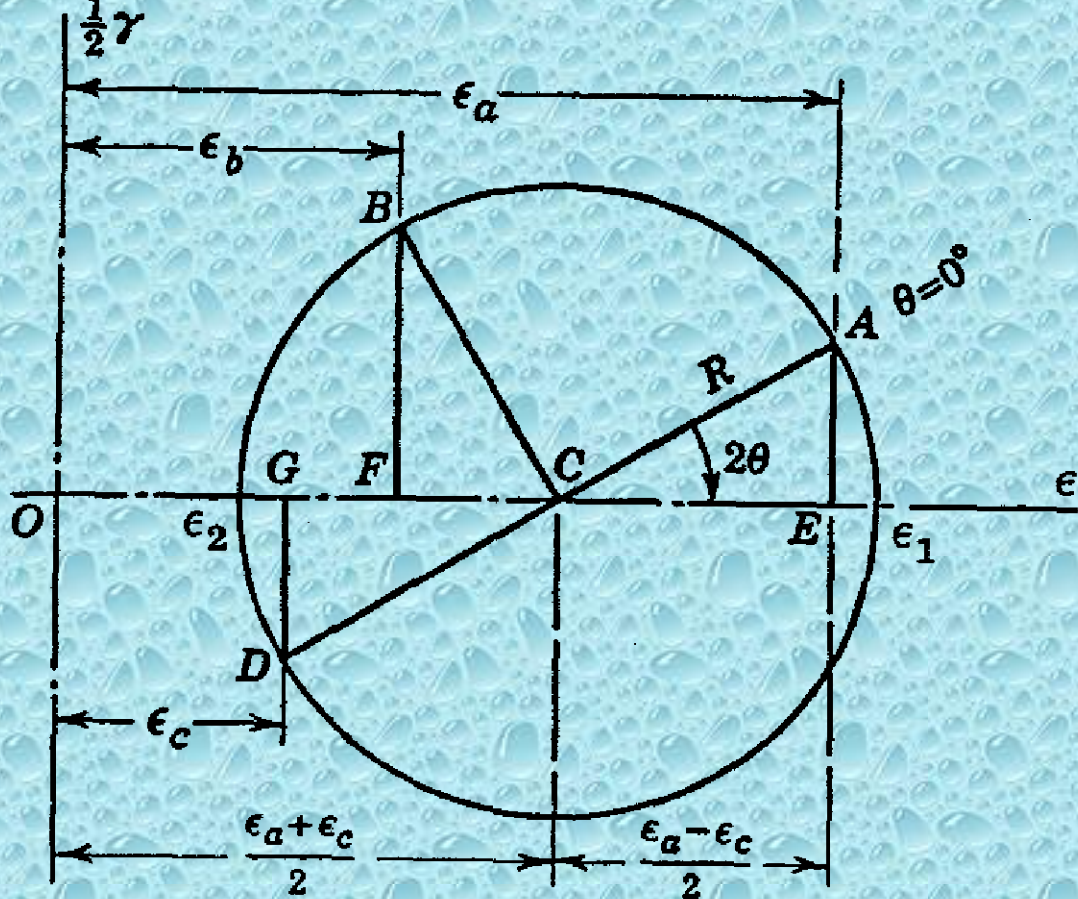


Figure 9-37 Mohr's circle for 45° strain rosette.

The 60° or Equiangular Rosette

In the 60° rosette, the reference angles are $\theta_a = 0°$, $\theta_b = 60°$, and $\theta_c = 120°$. On substituting these values in Eq. (a) and solving, we obtain

$$\left.\begin{aligned} \epsilon_x &= \epsilon_a \\ \epsilon_y &= \tfrac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a) \\ \frac{1}{2}\gamma_{xy} &= \frac{1}{\sqrt{3}}(\epsilon_c - \epsilon_b) \end{aligned}\right\} \qquad (9\text{-}22)$$

987. A 60° strain rosette attached to the aluminum skin of an airplane fuselage measures the following strains in micro-inches per inch: $\epsilon_a = 100$, $\epsilon_b = -200$, and $\epsilon_c = 400$. If $E = 10 \times 10^6$ psi and $\nu = \frac{1}{3}$, compute the principal stresses and the maximum shearing stress.

Ans. Max. $\tau = 2600$ psi; $\sigma_1 = 4100$ psi at $\theta = -45°$; $\sigma_2 = -1100$ psi.

$$\left.\begin{aligned} \epsilon_x &= \epsilon_a \\ \epsilon_y &= \tfrac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a) \\ \frac{1}{2}\gamma_{xy} &= \frac{1}{\sqrt{3}}(\epsilon_c - \epsilon_b) \end{aligned}\right\} \qquad (9\text{-}22)$$

Hw20a จงพิสูจน์ สมการ (9-19) (9-20) ด้วยภาษาของตัวเอง

Hw20b A 60° strain rosette attached to the aluminum skin of an airplane fuselage measures the following strains in micro-inches per inch: ϵ_a , ϵ_b and ϵ_c
If $E = 10 \times 10^6$ psi and $\nu = \frac{1}{3}$, compute the principal stresses and the maximum shearing stress.

Hw21 The three readings on a 45° strain rosette are ϵ_a $\times 10^{-6}$, ϵ_b $\times 10^{-6}$, and ϵ_c $\times 10^{-6}$. If $E = 200$ GPa and $\nu = 0.30$, determine the principal stresses and their directions.

ค่า z_1-z_3 ได้จากเลขประจำตัวนิสิต ดังต่อไปนี้ 46xxx$z_1z_2z_3$

$\varepsilon_a = 100(1+z_1)$ $\varepsilon_b = -100(1+z_2)$

$\varepsilon_c = 100(1+z_3)$

<u>ปริมาณทาง **Physics** สามารถแทนด้วย **Tensor**</u>

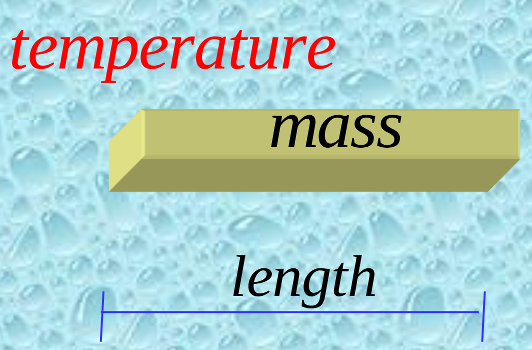
Order 0 = zero order Tensor (Scalar) – Magnitude (มวล, ความหนาแน่น)

Order 1 = first order Tensor (Vector) – Magnitude, Direction (ความเร็ว, แรง)

Order 2 = second order Tensor – Magnitudes, Directions (stress, strain)

… Higher order ….

ปริมาณทาง **Physics** ไม่เปลี่ยนแปลงไปตามระบบโคออร์ดิเนตที่ใช้ในการวัด



$$mass = 2\ \text{kg.} = ??\ \text{lb.}$$

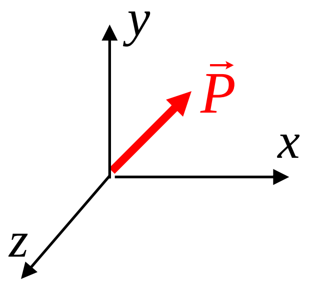
$$length = 5\ \text{in.} = 12.7\ \text{cm.}$$

$$temperature = 50^{O}\,\text{C} = 122^{O}\,\text{F}$$

ปริมาณทาง **Physics** ไม่เปลี่ยนแปลงไปตามระบบโคออร์ดิเนตที่ใช้ในการวัด

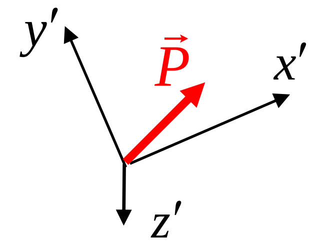
แรง $\vec{P}$ ยังคงมีขนาดและทิศทางเท่าเดิม ไม่ว่าจะแสดง **component** ของเวคเตอร์ด้วยระบบโคออร์ดิเนตอื่น

$$\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix}$$



$$\text{manitude} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

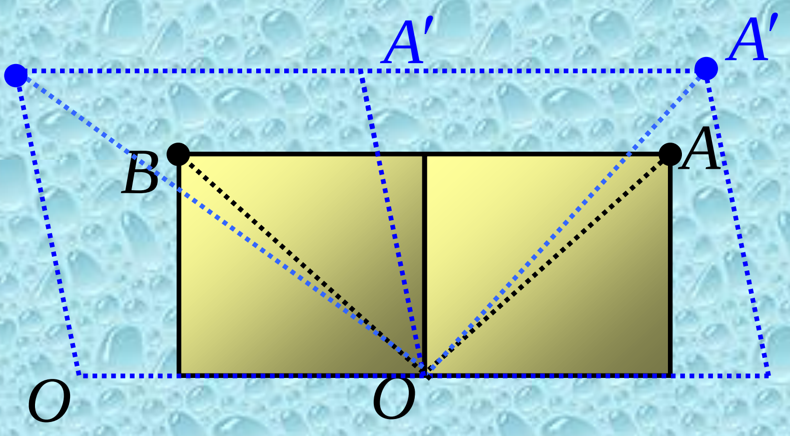
$$\begin{Bmatrix} 0.6 \\ 0.8 \\ 1 \end{Bmatrix}$$



$$\text{manitude} = \sqrt{0.6^2 + 0.8^2 + 1^2} = \sqrt{2}$$

สถานะของหน่วยแรง (**state of stress**) ยังคงมีคุณสมบัติเหมือนเดิม ไม่ว่าจะแสดงด้วยระบบโคออร์ดิเนตอื่น

$$\boldsymbol{\sigma} = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.5 & 3 & -1 \\ 0.2 & -1 & 4 \end{bmatrix}$$



$\varepsilon_x \neq 0, \varepsilon_y \neq 0, \gamma_{xy} \neq 0$ $\varepsilon_x \neq 0, \varepsilon_y \neq 0, \gamma_{xy} \neq 0$

Thanks All