Heaven 's Light is Our Guide

DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING &TECHNOLOGY

Prepared by

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Course Title : Mechanics of Materials-II

Course Code: CE 2213

Lecture: 3 hrs/ week , **Credit: 3.00**

Prereq. CE 211

- **Content of Course:**
- Torsional stresses in shafts and tubes, helical springs, combined stresses, transformation of stresses. Deflection of beam by direct integration, moment area and conjugate beam methods. Buckling of columns.

Reference Books

- 1. Strength of Materials by Ferdinand L. Singer
- 2. Engineering Mechanics of Solids by Egor P. Popov
- 3. Mechanics of Materials by by Laurson and Cox
- 4. Mechanics of Materials by R.C. Hibbeler
- 5. Strength of Materials by William Nash
- 6. Indeterminate Structural Analysis by J. Sterling Kinney

Topics:

Helical Springs

Topics to be Covered

- Introduction
- Define Helical Springs
- Example of Helical Springs
- Assumption for Helical Springs Formula
- Derives Helical Springs Formula for stresses and deflections
- Sample Example Problem on Helical (Analysis and Design Problem)
- Conclusions

Introduction

What is helical spring

Helical spring is a spiral wound wire with a constant coil diameter and uniform pitch

WHAT IS SPRING?

•Springs are elastic bodies (generally metal) that can be twisted, pulled, or stretched by some force. They can return to their original shape when the force is released.

•In other words it is also termed as a resilient member.

Introduction

HELICAL SPRING CLASSIFICATION

1) Extension helical spring
2) Compression helical spring
3) Torsion spring
4) Spiral spring

CLASSIFICATION OF SPRINGS

1) Helical springs
2) Leaf springs

Introduction

Function of Helical spring

- Used to store energy and subsequently release it
- To absorb shock
- To maintain a force between contacting surfaces

EXTENSION HELICAL SPRING

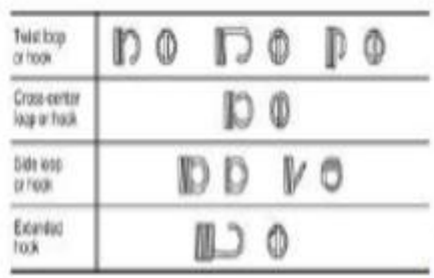
- It has some means of transferring the load from the support to the body by means of some arrangement.
- It stretches apart to create load.
- The gap between the successive coils is small.

Introduction

EXTENSION HELICAL SPRING

- The wire is coiled in a sequence that the turn is at right angles to the axis of the spring.
- The spring is loaded along the axis.
- By applying load the spring elongates in action

EXTENSION SPRINGS AND ITS END HOOKS

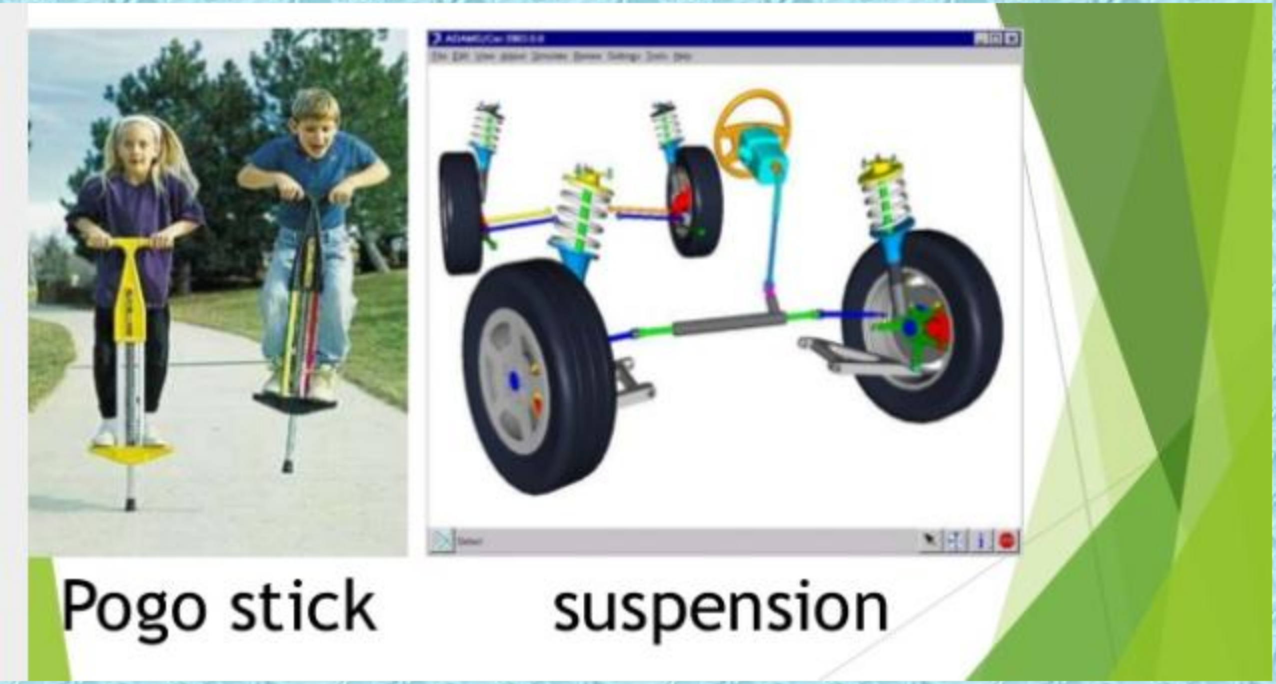


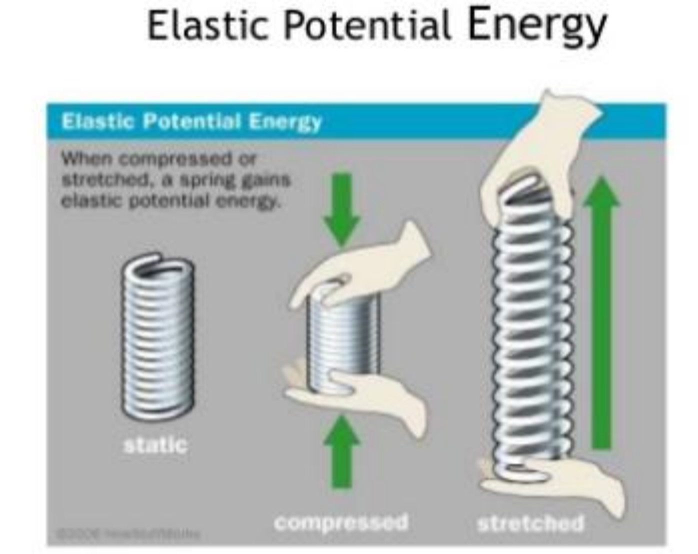
Introduction

APPLICATIONS OF SPRINGS

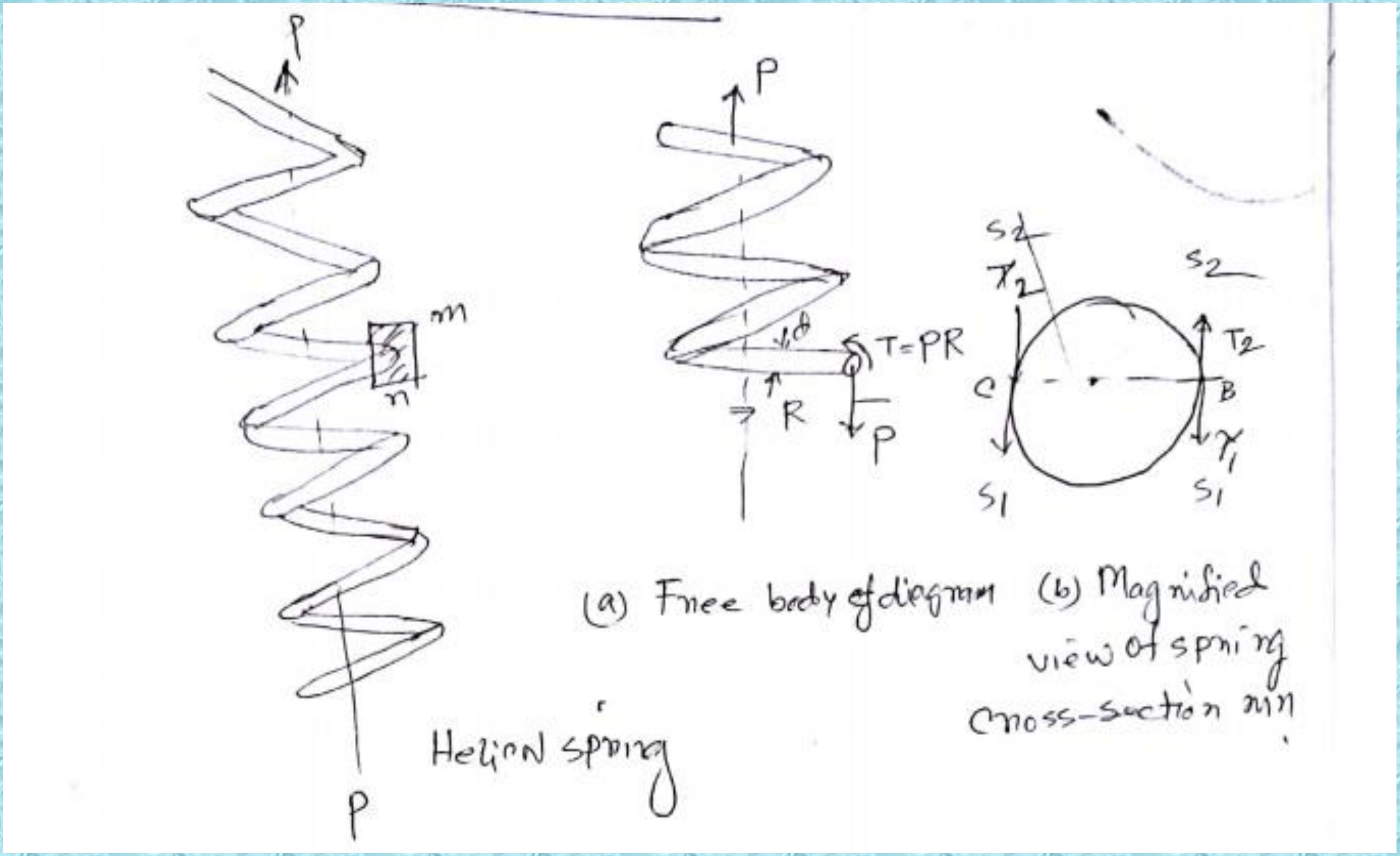
1) To apply forces and controlling motion, as in brakes and clutches.
2) Measuring forces, as in the case of a spring balance.
3) Storing energy, as in the case of springs used in watches and toys.
4) Reducing the effect of shocks and vibrations in vehicles and machine foundations.

Application of Spring

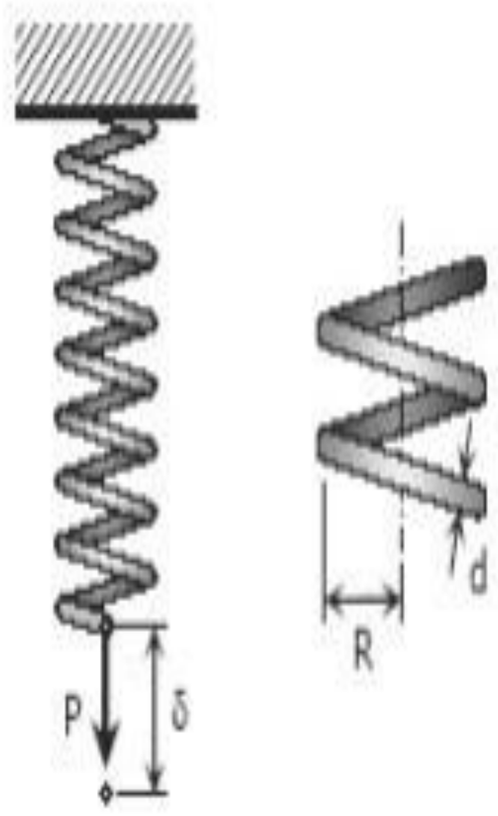




Derive an expression for the maximum shearing stress of a helical spring



When close-coiled helical spring, composed of a wire of round rod of diameter d wound into a helix of mean radius R with n number of turns, is subjected to an axial load P produces the following stresses and elongation:



The closed coil helical spring as shown in the fig is elongated by axial load P . To determine the stress produced by P . we consider a plane mn . Through any typical section as in the fig.

To balance the applied axial load P , The spring must provided the persistence P_r =P opposite equal and parallel Create a couple of magnitude P_r which can be balanced only a opposite couple .

This resisting couple is create by a torsional shearing stress misattributed over the cross section of the spring.

Two types of shearing stress an produced

Direct shearing like , S_1

Variable Torsional shearing stress , like S_2 coupled by the twisting couple $T = PR$

At point C the two shearing stress one collinear and in the same sense.

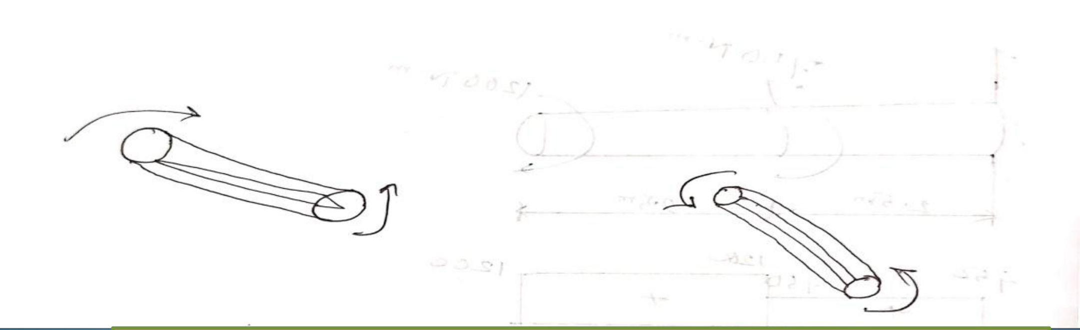
The max , shearing stress Occurs at The inside element and in given by the same of direct shearing stress , $S_1 = \frac{P}{A}$, and max value of torsional shearing stress , $S_2 = \frac{T_r}{J}$

$$Ss = S_1 + S_2 = \frac{P}{A} + \frac{T_r}{J}$$

$$= \frac{P}{\frac{\pi d^2}{4}} + \frac{T.r}{\frac{\pi d^4}{32}}$$

$$= \frac{4P}{\pi d^2} + \frac{16PR}{\pi d^3} = \frac{16PR}{\pi d^3} \left(1 + \frac{d}{4R}\right)$$

$$Ss = \frac{16PR}{\pi d^3} \left(1 + \frac{d}{4R}\right)$$



Torsion of straight and of curved segments

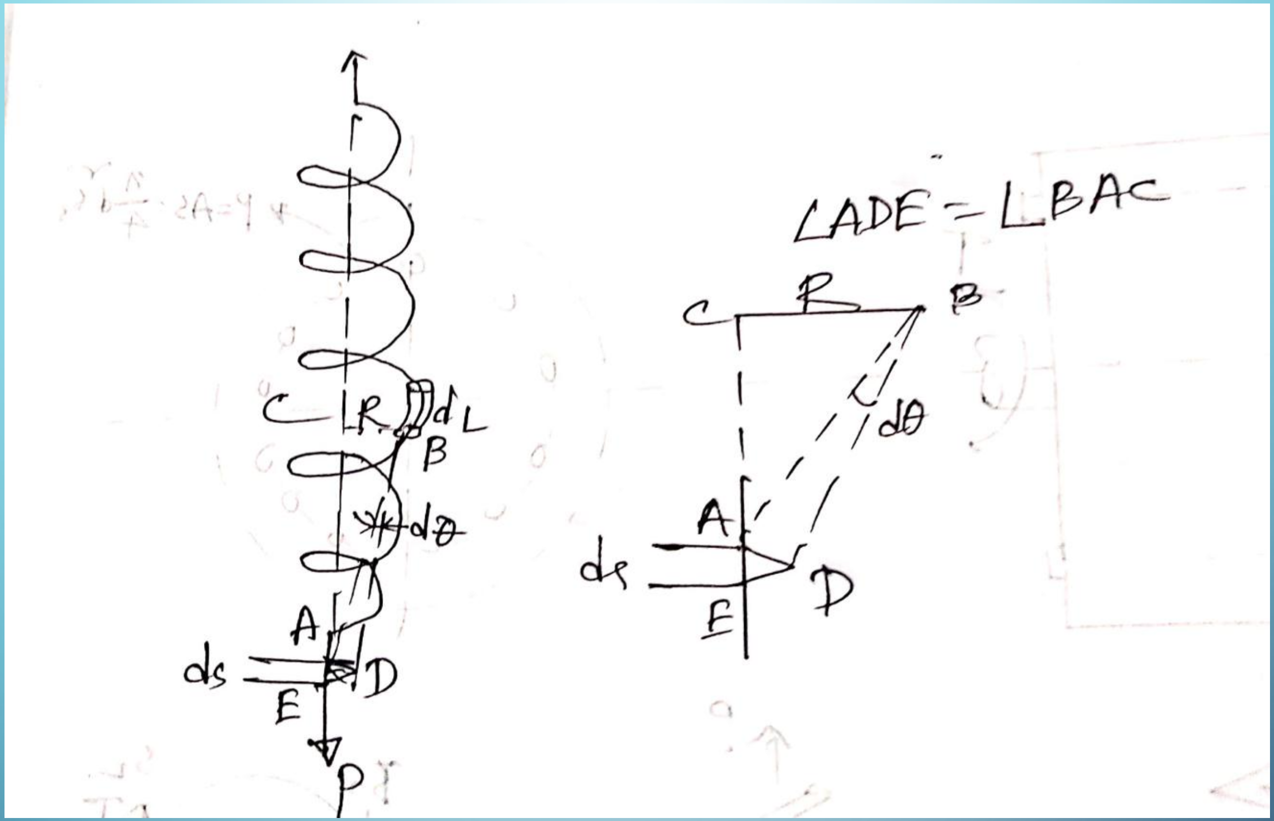
The following formula that takes account of the initial Curvature of the spring wire

$$\text{Max } Ss = \frac{16PR}{\pi d^3} \left(\frac{4m-1}{4m-4} + \frac{0.615}{m} \right)$$

$$m = \frac{2R}{d} = \frac{D}{d}$$

$$Ss = \frac{16PR}{\pi d^3} \left(1 + \frac{0.5}{m}\right).$$

Derive an expression for deflection of spring.



Consider, The spring in the fig inn rigid accept the small

Length dL, assume . The end A rotation to D through the small angle $d\theta$. As $d\theta$ is small the are AD = AB. $d\theta$ may be consider

Now From the similar triangle ADE and BAC

$$\frac{AE}{AD} = \frac{BC}{AB}$$

$$\frac{ds}{AB.d\theta} = \frac{R}{AB}$$

ds = R . dθ.

ds = R. $\frac{(PR).dL}{JG}$ Assume dθ $= \frac{(PR).dL}{JG}$

The total elongation deflection can be obtained by integration all the element of the spring contribute to elongation

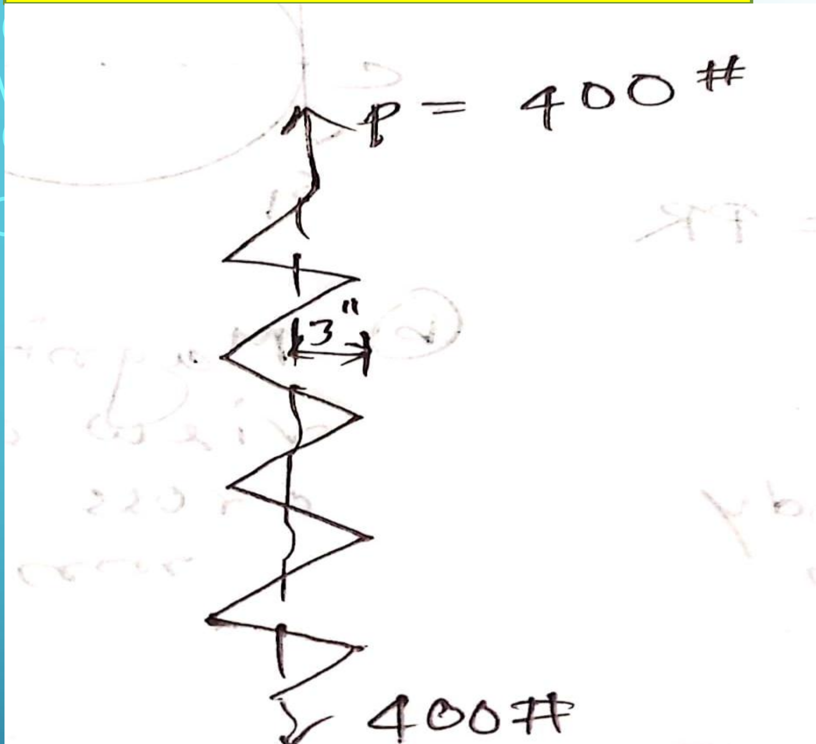
$$\delta = \frac{PR^2.L}{JG}$$

Replacing 1 by 2πrn which is the length of n coil and J by $\frac{\pi d^4}{32}$ we obtain:

$$\delta = \frac{64PR^3n}{Gd^4}$$

This expression for spring deflection

Prob 1. Determine Ss



n = 20

G = 12 X 10^6 psi

$d = \frac{3}{4}$

Find maximum Ss and Δ

$$S = \frac{64PR^3n}{Gd^4}$$

$$= \frac{64\ X\ 40\ X\ 3^3\ X\ 20}{12\ X\ 10^6\ X\ (\frac{3}{4})^4}$$

= 3.64"

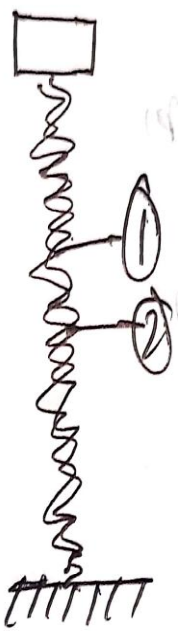
$$Ss = \frac{16PR}{\pi d^3}(1 + \frac{d}{4R})$$

$$= \frac{16\ X\ 400\ X\ 3}{\pi(\frac{3}{4})^3}(1 + \frac{\frac{3}{4}}{4\ X\ 3})$$

$$= \frac{16\ X\ 400\ X\ 3\ X\ 4^3}{\pi\ X\ 3^3}(1 + \frac{3}{4\ X\ 4\ X\ 3})$$

= 15392 psi

Prob. 2



spring – 1

n = 24

$d = \frac{3''}{4}$

D = 6"

spring – 2

n =20

d = 1"

D = 8"

Maximum , Ss = 20000 psi

$G = 16 \times 10^6$ psi ,

Find P?

sol^n: $P_1 + P_2 = P$ --------------(1)

$\Delta_1 = \Delta_2$ --------------(2)

$\delta_1 = \delta_2$

$$\frac{64\ P_1 R_1^3\ n}{G d_1^4} = \frac{64\ P_2 R_2^3\ n}{G d_2^4}$$

$$\frac{64\ X\ P_1\ X\ 3^3 X\ 24}{(\frac{3}{4})^4} = \frac{64\ X\ P_2\ X\ 4^3 X\ 20}{(1)^4}$$

$131072\ P_1 = 131072\ P_2$

$P_1 = 0.625\ P_2$ -----------------(3)

$P_1 + P_2 = P$

$0.625\ P_2 + P_2 = P$

$P_2 = 0.62$ P

From eqn(3) $P_1 = 0.625$ X 0.62 P

= 0.38 P

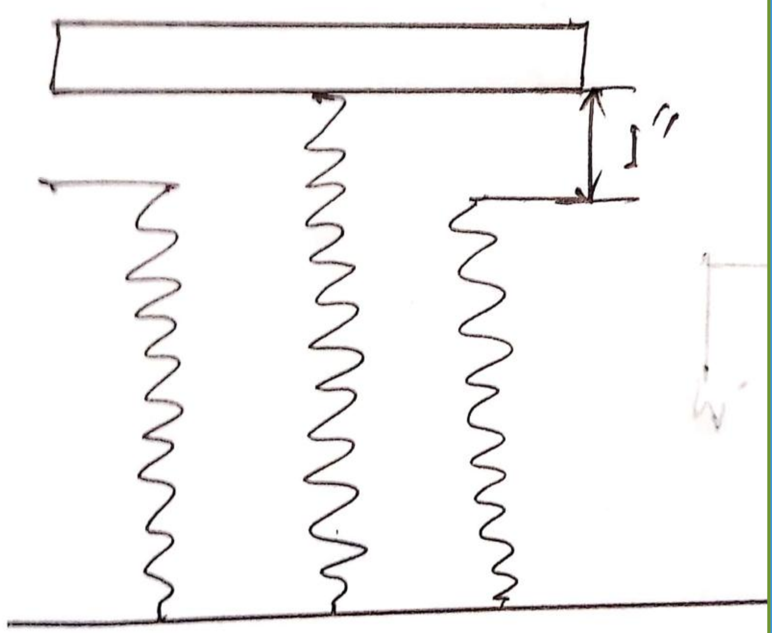
Prob 3. Determine Max Shearing (Ss) Stress in Each Spring.

Inner spring

n = 24

$d = \frac{3''}{4}$

D = 6



outer spring

n = 18

$d = \frac{1''}{2}$

D = 4"

$G = 12 \times 10^6$ psi

P = 1200 lb

$$\delta = \frac{64PR^3n}{Gd^4}$$

$$1 = \frac{64\ X\ P\ X\ 3^3\ X\ 24}{12\ X\ 10^6\ X\ (\frac{3}{4})^4}$$

$$P = \frac{12\ X\ 10^6\ X\ (\frac{3}{4})^4}{64\ X\ (3)^3\ X\ 24} = 91.5516$$

$$P_{in} + 2P_{out} = 1200 - 91.55$$

$$P_{in} + 2P_{out} = 1108.45 \text{ ------------------(1)}$$

and

$$\delta_{in} = \delta_{out}$$

$$\frac{64R^3n}{Gd^4} = \frac{64PR^3n}{Gd^4}$$

$$\frac{64\ X\ P_{in}\ x\ 3^3 X\ 24}{12\ X\ 10^6\ X(\frac{3}{4})^4} = \frac{64\ X\ P_{out}\ X\ 2^3 X\ 18}{12\ X\ 10^6\ X(\frac{1}{2})^4}$$

$$P_{in} = \frac{P_{out}\ X\ 2^3\ X\ 18\ X\ (3/4)^4\ X\ 12}{3^3\ X\ 24\ X\ 12\ X\ (\frac{1}{2})^4}$$

$P_{in} = 1.68\ P_{out}$ / $P_{in} = 1.125 P_0$

$1.125 P_0 + 2 P_{out} = 1108.45$

$3.125 P_0 = 1108.45$

$P_{in} = 399.04$

$P_{in} = 399.04 + 91.55 = 490.6$

$$Ss_{in} = \frac{16PR}{\pi d^3} \left(1 + \frac{d}{4R}\right)$$

$$Ss_{out} = \frac{16PR}{\pi d^3} \left(1 + \frac{d}{4R}\right)$$

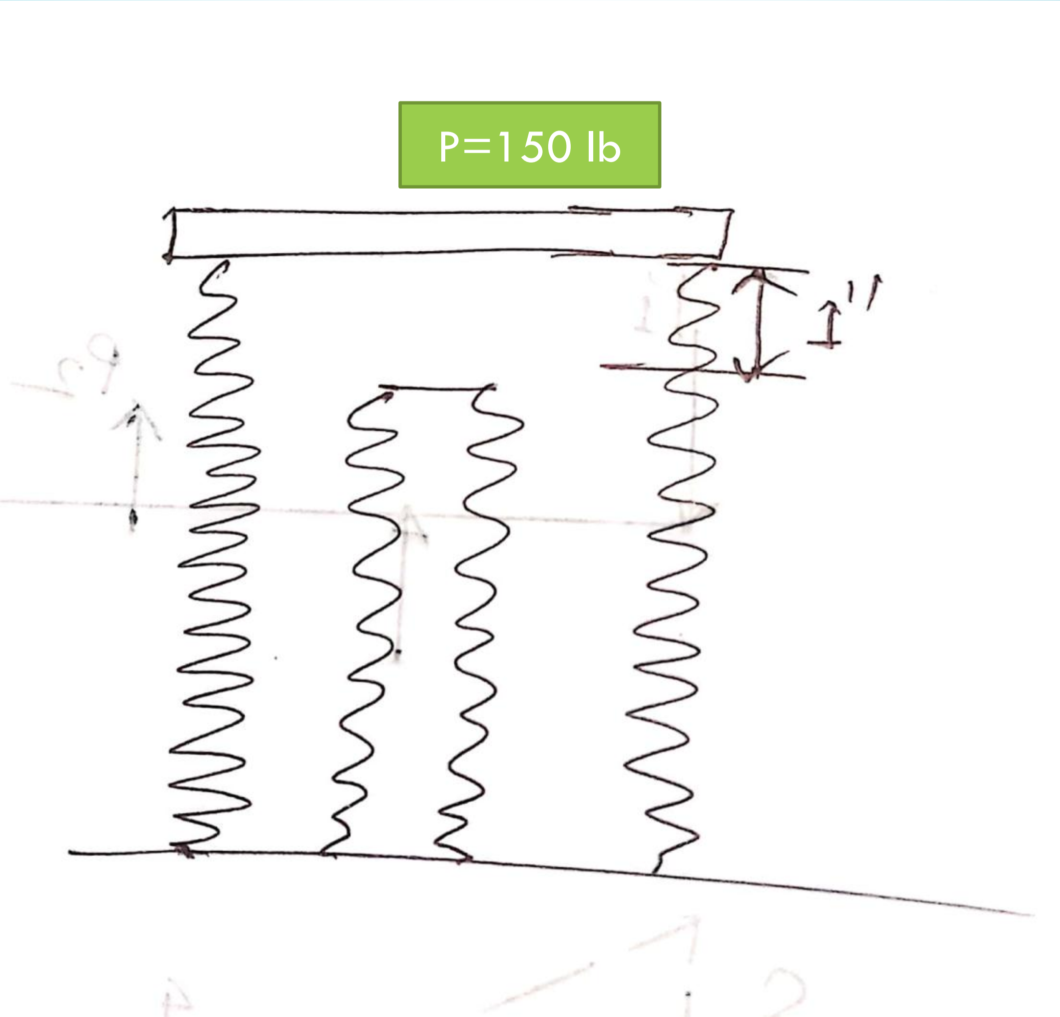
$$= \frac{16\ X\ 490.6\ X\ 3}{\pi (\frac{3}{4})^3} \left(1 + \frac{\frac{3}{4}}{4\ X\ 3}\right)$$

$= 18887$ psi

$$= \frac{16\ X\ 354.7\ X\ 2}{\pi\ X\ 0.5^3} \left(1 + \frac{\frac{1}{2}}{4\ X\ 2}\right)$$

$= 30795.6$ psi

Prob. 4. Determine Max Shearing (Ss) Stress in Each Spring. (Inner and outer same as previous)



$$\delta = \frac{64PR^3n}{Gd^4}$$

$$1 = \frac{64P_{out}R^3n}{Gd^4}$$

$$2P_0 \quad + 2P_{in} \quad = P - P_n$$

and

$$\delta_{in} = \delta_{out}$$

$$\delta_c \quad = 1 + \delta_0 \qquad \text{--------------(1)}$$

$$p = 2P_0 + P_c \quad \text{--------------------(2)}$$

form (1) $\delta_c \quad = 1 + \delta_0$

$$\frac{64 \; X \; P_c \; X \; 3^3 \; X \; 24}{12 \; X \; 10^6 \; X \; (.75)^4} = 1 + \frac{64 \; X \; P_0 \; X \; 2^3 \; X \; 18}{12 \; X \; 10^6 \; X \; (.5)^4}$$

$$0.0109922 \; P_c \quad = 1 + 0.012288 \;\; P_0$$

$$P_c = 91.55 + \;\; 1.125068 \;\; P_0$$

from (2) ,

$$2P_0 + 91.55 + \;\; 1.125068 \;\; P_0 = 1200$$

$$P_0 \quad = 354.7$$

$$P_c = 490.60$$

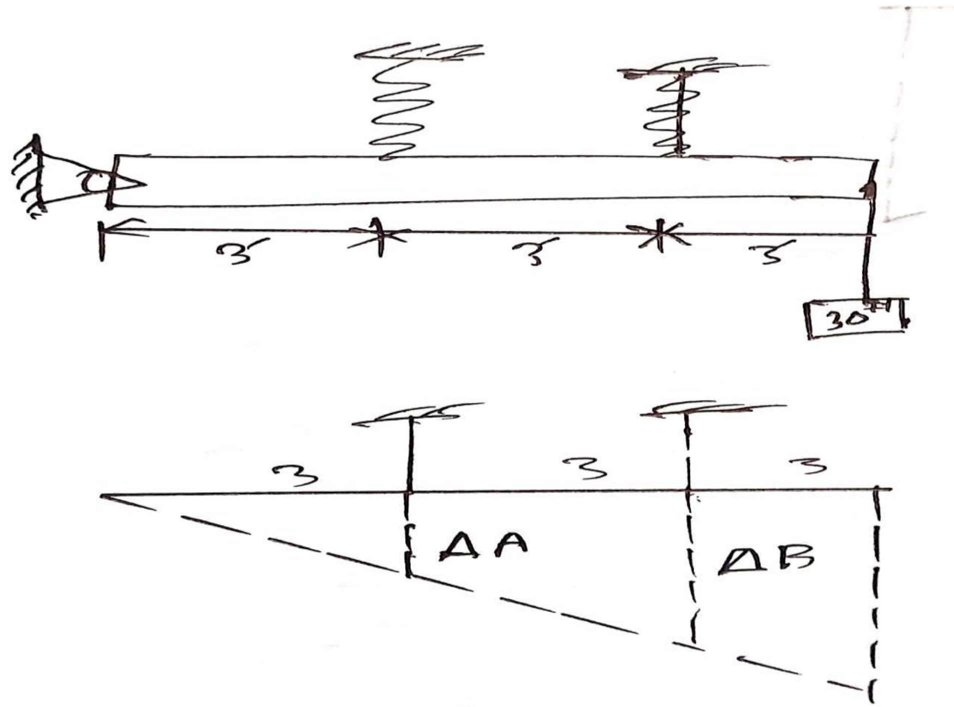
Prop 5. Find maximum Stress of the spring

n= 20

D=5in.

d= ¼ in.

Ss =?



From similar triangle,

$$\frac{\Delta A}{\Delta B} = 3/6 \quad \text{.......(i)}$$

$$\sum Mo = 0$$

$P_A *3+ P_B*6 = 30*9$

$P_A+2P_B =90$ ....(ii)

From eqn. (i)

$2\ \Delta A = \Delta B$

$$64P_An\frac{R^3}{Gd^4} = 64P_Bn\frac{R^3}{Gd^4}$$

$2P_A = P_B$ ....(iii)

From (ii)

$P_A + 2.2PA = 90$

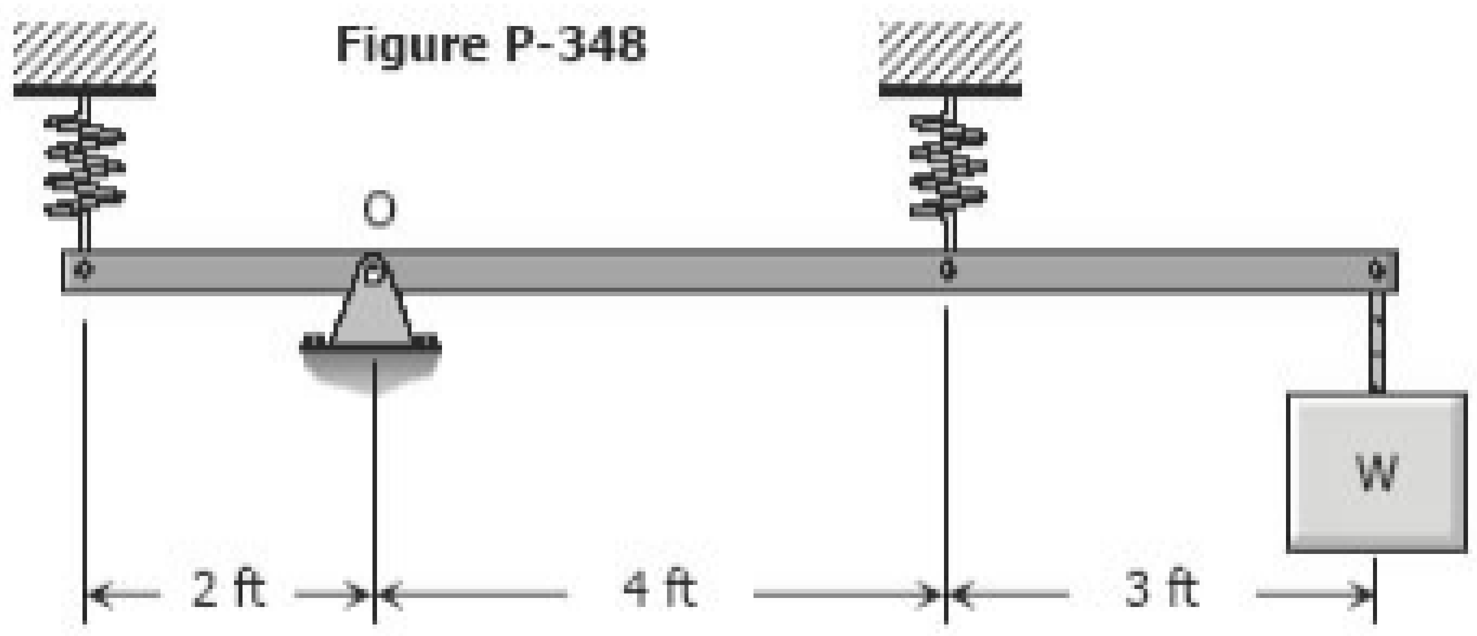
$5P_A = 90$

$P_A = 18$

$P_B = 2*18$

$$Ss = \frac{16PBR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

Prob 6. A rigid bar, pinned at O, is supported by two identical springs as shown in Fig. P-348. Each spring consists of 20 turns of ¾-in-diameter wire having a mean diameter of 6 in. Determine the maximum load W that may be supported if the shearing stress in the springs is limited to 20 ksi.

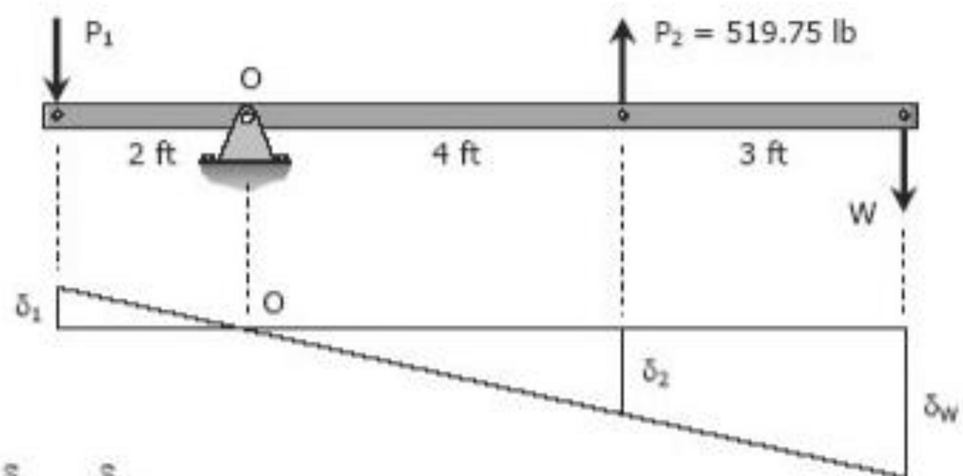


$$\tau_{max} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$ → Equation (3-9)

$$20\,000 = \frac{16P(3)}{\pi(3/4)^3}\left[1 + \frac{3/4}{4(3)}\right]$$

$P = 519.75$ lb

For this problem, the critical spring is the one subjected to tension. Use $P_2 = 519.75$ lb.



$$\frac{\delta_1}{2} = \frac{\delta_2}{4}$$

$\delta_1 = \frac{1}{2}\delta_2$

$$\frac{64P_1R^3n}{Gd^4} = \frac{1}{2}\left(\frac{64P_2R^3n}{Gd^4}\right)$$

$P_1 = \frac{1}{2}P_2 = \frac{1}{2}(519.75)$

$P_1 = 259.875$ lb

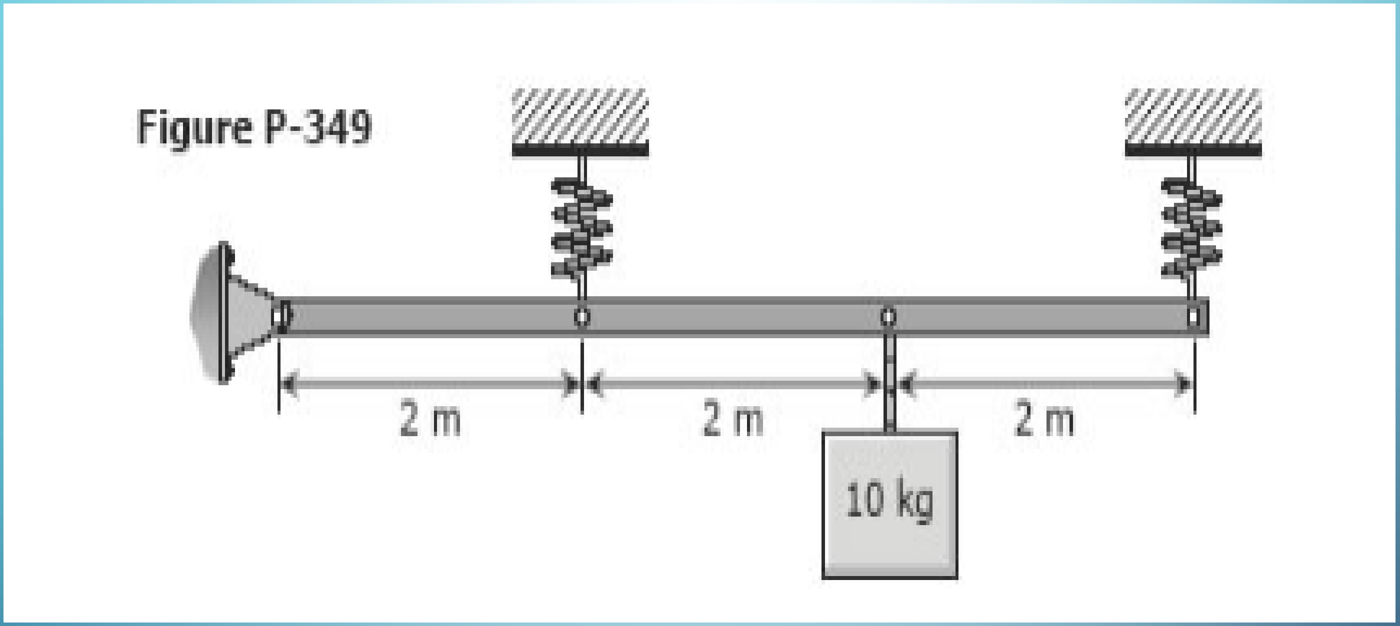
$\Sigma M_O = 0$

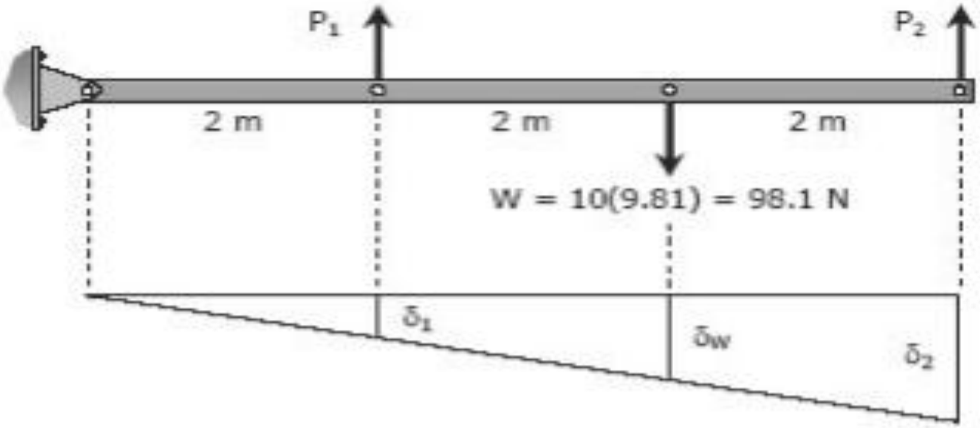
$7W = 2P_1 + 4P_2$

$7W = 2(259.875) + 4(519.75)$

$W = 371.25$ lb

Prob7. A rigid bar, hinged at one end, is supported by two identical springs as shown in Fig. P349. Each spring consists of 20 turns of 10-mm wire having a mean diameter of 150 mm. Compute the maximum shearing stress in the springs, using Eq. (3-9). Neglect the mass of the rigid bar.





$$\frac{\delta_1}{2} = \frac{\delta_2}{6}$$

$$\delta_1 = \tfrac{1}{3}\,\delta_2$$

$$\frac{64 P_1 R^3 n}{G d^4} = \frac{1}{3}\left(\frac{64 P_2 R^3 n}{G d^4}\right)$$

$$P_1 = \tfrac{1}{3}\,P_2$$

$$\Sigma M_{\text{at hinged support}} = 0$$

$$2P_1 + 6P_2 = 4(98.1)$$

$$2(\tfrac{1}{3}\,P_2) + 6P_2 = 4(98.1)$$

$$P_2 = 58.86 \text{ N}$$

$$P_1 = \tfrac{1}{3}\,(58.86) = 19.62 \text{ N}$$

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$ → Equation (3-9)

For spring at left:

$$\tau_{max1} = \frac{16(19.62)(75)}{\pi(10^3)}\left[1+\frac{10}{4(75)}\right]$$

$$\tau_{max1} = 7.744 \text{ MPa}$$

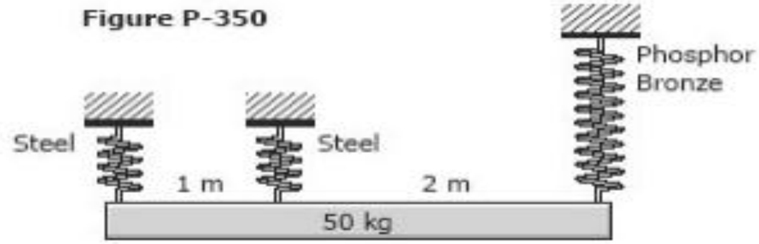
For spring at right:

$$\tau_{max2} = \frac{16(58.86)(75)}{\pi(10^3)}\left[1+\frac{10}{4(75)}\right]$$

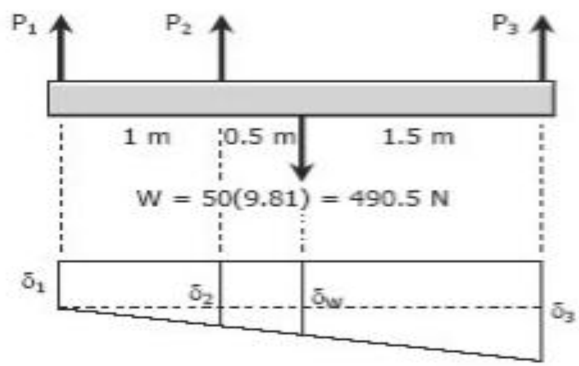
$$\tau_{max2} = 23.232 \text{ MPa}$$

Problem 8: As shown in Fig. P-350, a homogeneous 50-kg rigid block is suspended by the three springs whose lower ends were originally at the same level. Each steel spring has 24 turns of 10-mm-diameter on a mean diameter of 100 mm, and G = 83 GPa. The bronze spring has 48 turns of 20-mm-diameter wire on a mean diameter of 150 mm, and G = 42 GPa. Compute the maximum shearing stress in each spring using Eq. (3-9).

Figure P-350



Solution 350



$\Sigma F_V = 0$

$P_1 + P_2 + P_3 = 490.5$ → Equation (1)

$\Sigma M_1 = 0$

$P_2(1) + P_3(3) = 490.5(1.5)$

$P_2 + 3P_3 = 735.75$ → Equation (2)

$$\frac{\delta_2 - \delta_1}{1} = \frac{\delta_3 - \delta_1}{3}$$

$\delta_2 = \frac{1}{3}\delta_3 + \frac{2}{3}\delta_1$

$$\frac{64P_2(50^3)(24)}{83000(10^4)} = \frac{1}{3}\left[\frac{64P_3(75^3)(48)}{42000(20^4)}\right] + \frac{2}{3}\left[\frac{64P_1(50^3)(24)}{83000(10^4)}\right]$$

$\frac{3}{830}P_2 = \frac{9}{8960}P_3 + \frac{1}{415}P_1$

$\frac{3}{166}P_2 = \frac{9}{1792}P_3 + \frac{1}{83}P_1$ → Equation (3)

From Equation (1)

$$P_1 = 490.5 - P_2 - P_3$$

Substitute P_1 to Equation (3)

$$\frac{3}{166} P_2 = \frac{9}{1792} P_3 + \frac{1}{83} (490.5 - P_2 - P_3)$$

$$\frac{3}{166} P_2 = \frac{9}{1792} P_3 + \frac{981}{166} - \frac{1}{83} P_2 - \frac{1}{83} P_3$$

$$\frac{5}{166} P_2 = \frac{981}{166} - \frac{1045}{148\,736} P_3 \qquad \rightarrow \text{Equation (4)}$$

From Equation (2)

$$P_2 = 735.75 - 3P_3 = \frac{2943}{4} - 3P_3$$

Substitute P_2 to Equation (4)

$$\frac{5}{166} \left(\frac{2943}{4} - 3P_3 \right) = \frac{981}{166} - \frac{1045}{148\,736} P_3$$

$$\left(\frac{1045}{148\,736} - \frac{15}{166} \right) P_3 = \frac{981}{166} - \frac{14\,715}{664}$$

$$P_3 = 195.01 \text{ N}$$

$$P_2 = 735.75 - 3(195.01) = 150.72 \text{ N}$$

$$P_1 = 490.5 - 150.72 - 195.01 = 144.77 \text{ N}$$

$$\tau_{max} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right) \qquad \rightarrow \text{Equation (3-9)}$$

For steel at left:

$$\tau_{max1} = \frac{16(144.77)(50)}{\pi(10^3)}\left[1 + \frac{10}{4(50)}\right] = 38.709 \text{ MPa}$$

For steel at right:

$$\tau_{max1} = \frac{16(150.72)(50)}{\pi(10^3)}\left[1 + \frac{10}{4(50)}\right] = 40.300 \text{ MPa}$$

For phosphor bronze:

$$\tau_{max3} = \frac{16(195.01)(75)}{\pi(20^3)}\left[1 + \frac{20}{4(75)}\right] = 9.932 \text{ MPa}$$

Conclusion

CONCLUSION

- Springs produce a large deflection and used for a number of applications.
- Most springs are made of steel.
- Stress and deflection in coil springs was derived.
- Springs can be connected in series and parallel.
- There are a number of other spring configurations used in engineering.

Thanks All