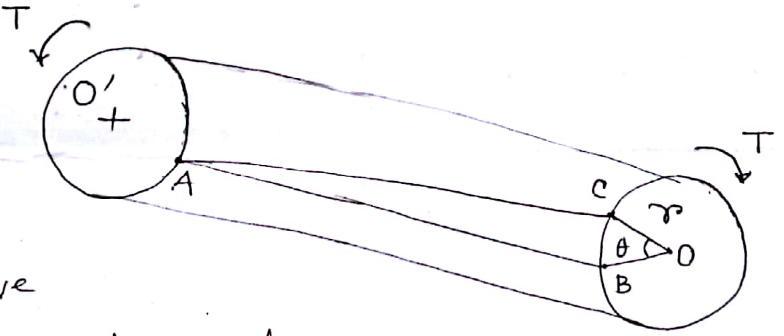
Torsion

Strength of materials
by Singer.

Assumptions :

(1) Circular sections remain circular,
2. plane section remain plane and do not warp
3. The ~~pro~~ projection upon a transverse section of straight radial lines in the section remains straight.
4. Shaft is loaded by twisting couples in planes that are perpendicular to the axis of the shaft.
5. Stresses do not exceed the propotional limit.

Derive torsion formula to determine maximum stress in a circular shaft.



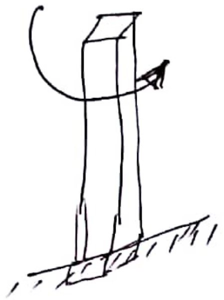
The above Fig showes of a solid circular shaft if a torque T is applied at the end of the shaft. A fibre AB on the outside surface which is normally straight, It will be twisted into a helix AC, as the shaft is twisted through an angle θ.

For the equilibrium of the free body deigram

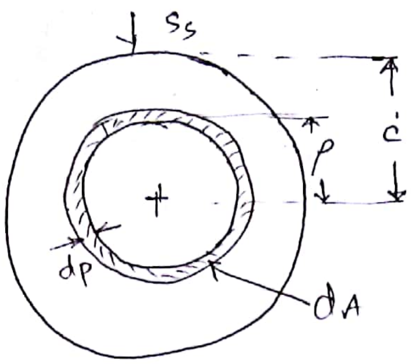
Resisting torque = the external torque.

* torsion is the twisting of an object due to an applied torque. it is expressed in newton-meters. / ft-lb.f

In sections perpendicular to torque axis, the resultant shear stress in this section is perpendicular to the radius.



torsion is the action of twisting or the state of being twisted. especially of one end ~~to~~ of an object relative to the other.



Let, the unit stressed at any fibre in the surface of fibre is S_s.

dA = the elementary area in the form of a narrow ring of radius.

Then, From the low of stress variation unit stress at

$dA = \frac{\rho}{c} . S_s .$

The force exerted by the stress over the area dA is equal to $\frac{\rho}{c} . S_s . dA .$

The moment of this force with respect to the axis of the shaft is equal to,

$$= \text{Force} \times \text{moment arm.}$$

$$= \left\{ \frac{\rho}{c} . S_s . dA \right\} . \rho$$

$$= \frac{\rho^2}{c} . S_s . dA$$

The sum of the moment of all stress on the entire cross section is

$$\text{Total Torque, } T = \int_0^c \frac{\rho^2}{c} . S_s . dA$$

$$= \frac{S_s}{c} \int_0^c \rho^2 dA$$

$$= \frac{S_s}{c} . J. \quad [J = \text{polar moment of intertia of the cross section}]$$

$$T = \frac{S_s}{c} . J.$$

$$\boxed{S_s = \frac{TC}{J}}$$

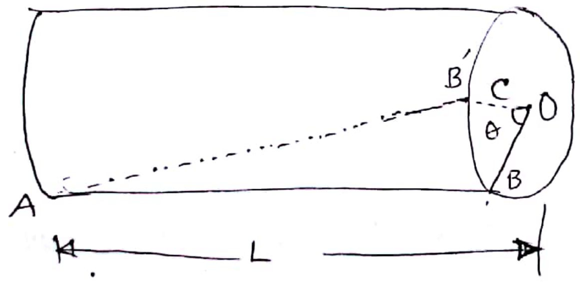
Where, c = out side radius of the shaft.

T = Torque of the shaft.

J = Polar moment of inertia of the cross-section.

S_s = Maximum shearing unit stress in x direction, unit is psi, ksi,

Determination of torsion formula for angle of Twist.



AB represent an ~~element of cylind~~ an element of cylindrical surface of the un-twisted shaft and AB', the curve (part of a helix), which is the same elements assumes after the tork is applied.

∠BOB' is the angle of twist. For the fibre represented by AB. The total angular deformation in the length L is BB'

So, the unit deformation $\delta_s = \frac{BB'}{L}$

and $BB' = C\theta$

From Hook's Low,

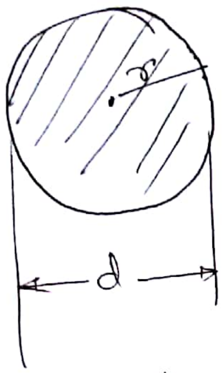
$$\delta_s = \frac{S_s}{E_s}$$

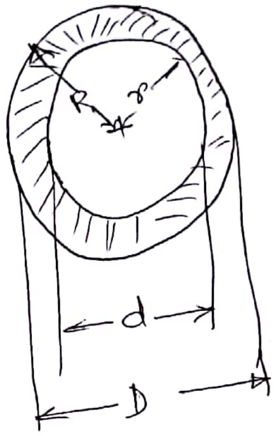
$$\Rightarrow \delta_s = \frac{BB'}{L} = \frac{C\theta}{L} = \frac{S_s}{E_s}.$$

$$\frac{C\theta}{L} = \frac{S_s}{E_s}$$

$$\theta = \frac{S_s L}{E_s c} = \frac{Tc}{J} \cdot \frac{L}{E_s c}$$

$$\boxed{\theta = \frac{TL}{E_s J}}$$





$$J = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

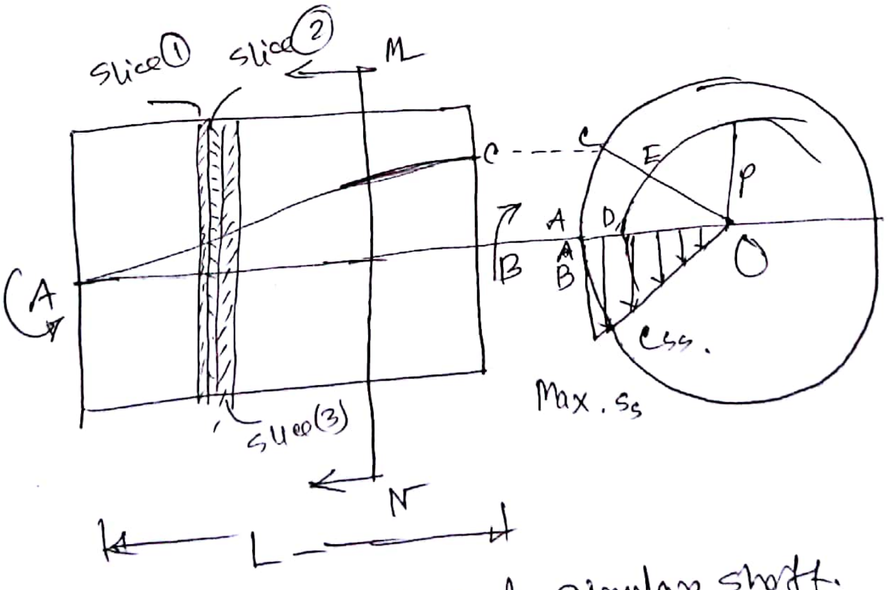
polar moment of inertia.

$$J = \frac{\pi}{2}(R^4 - r^4) = \frac{\pi}{32}(D^4 - d^4)$$

$[r = c]$

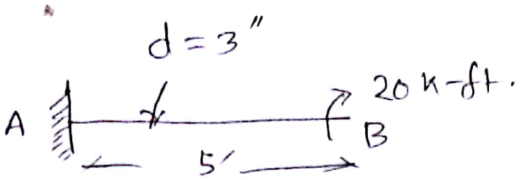
Solid shaft: Max. $S_s = \frac{2T}{\pi r^3} = \frac{16T}{\pi d^3}$

Hollow shaft, $S_s = \frac{2TR}{\pi(R^4 - r^4)} = \frac{16TD}{\pi(D^4 - d^4)}$.



Deformation of circular shaft.

Calculate maximum torsional stress and angle of twist, $E = 12\times10^6$ psi



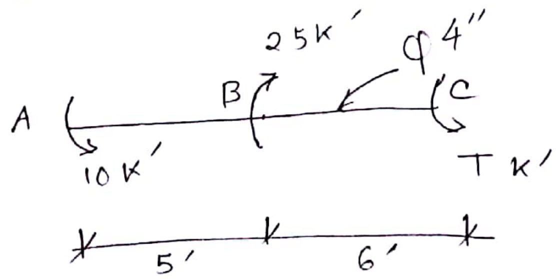
(i) We know, $S_s = \frac{Tc}{J} = \frac{20\times 12{,}000 \times 1.5}{\frac{\pi\times 3^4}{32}} = \frac{}{7.952}$

$= 45270$ psi $= 45.27$ ksi

(ii) $\theta = \frac{TL}{E_s J} = \frac{20\times 12000\times 5\times 12}{12\times10^6\times \frac{\pi\times 34}{32}}$

$= \frac{20\times 12000\times 5}{100000\times 7.952} = 0.151$ rad

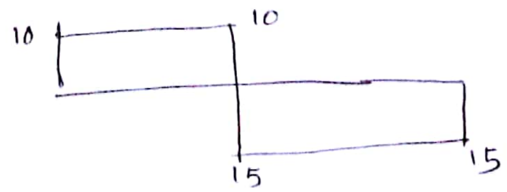
Calculate maximum torsional stress and angle of twist between A and C, $E = 12\times10^6$ psi.



$\Sigma T = \Sigma M = 0$, $10 - 25 + T = 0$

$T = 15$ k'

Here maximum moment is on BC



So, maximum T = 15 at BC

$S_{s\,BC} = \frac{15\times 12000\times 2}{\frac{\pi\times 4^4}{32}} = \frac{15\times 12000\times 2}{25.13} = 14.32$ ksi

We Know,

$$\theta_{AC} = \theta_{AB} - \theta_{BC}$$

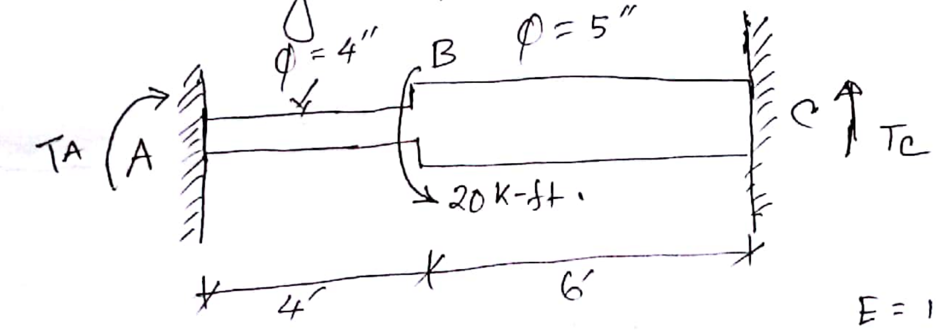
$$= \frac{T_{AB} L_{AB}}{EJ} - \frac{T_{BC} L_{BC}}{EJ}$$

$$= \frac{10 \times 12{,}000 \times 5 \times 12 - 15 \times 12000 \times 6 \times 12}{12 \times 10^6 \times \frac{\pi \times 4^4}{32}}$$

$$= \frac{7200000 - 12960000}{12 \times 10^6 \times \frac{\pi \times 4^4}{32}}$$

$$= \frac{5.76}{12 \times 25.13} = 0.0191 \text{ rad.}$$

Calculate max - S_s ~~and~~ which is developed in the following shaft.



$E = 12 \times 10^6$ psi.

Sol^n: $T_A + T_C = 20$ ——— (i)

and $\theta_{AB} = \theta_{BC}$ ——— (ii)

$$\Rightarrow \frac{T_{AB} L_{AB}}{E J_{AB}} = \frac{T_{BC} L_{BC}}{E J_{BC}}$$

$$\frac{T_A \times 4 \times 12 \times 12000}{12 \times 10^6 \times \frac{\pi \times 4^4}{32}} = \frac{(20 - T_A) \times 6 \times 12 \times 12000}{12 \times 10^6 \times \frac{\pi \times 5^4}{32}}$$

$$\frac{4 T_A}{4^4} = \frac{(20 - T_A) \cdot 6}{5^4}$$

$$625 T_A = 384 (20 - T_A)$$

$1009 T_A = 7680$

$T_A = 7.6$ K-ft.

$T_C = 20 - 7.6 = 12.4$ K-ft.

Max. torsional stress,

$$S_{sAB} = \frac{T_A c}{J} = \frac{7.6 \times 2 \times 12{,}000}{\frac{\pi \times 4^4}{32}} \quad ; \quad J = \frac{\pi \times 4^4}{32} = 25.133$$

$$= 7257.5 \text{ psi}.$$

and

$$S_{sBC} = \frac{T_C \cdot c}{J} = \frac{12.4 \times 12{,}000 \times 2.5}{\frac{\pi \times 5^4}{32}} \quad , \quad J = \frac{\pi \times 5^4}{32} = 61.36$$

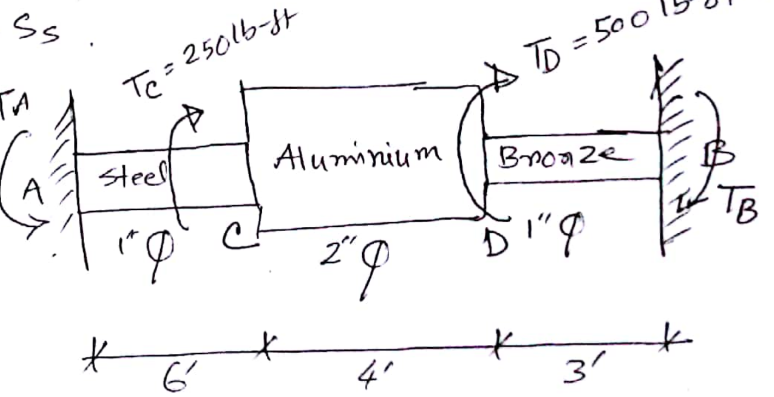
$$= 6062.6 \text{ psi}.$$

Calculate maximum S_s.

Steel, $E \rightarrow 12 \times 10^6$ psi

Al, $E \rightarrow 4 \times 10^6$ psi

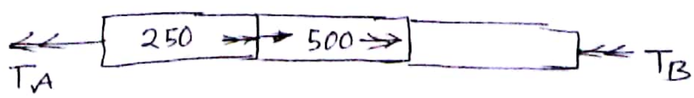
Bronze, $E \rightarrow 6 \times 10^6$ psi



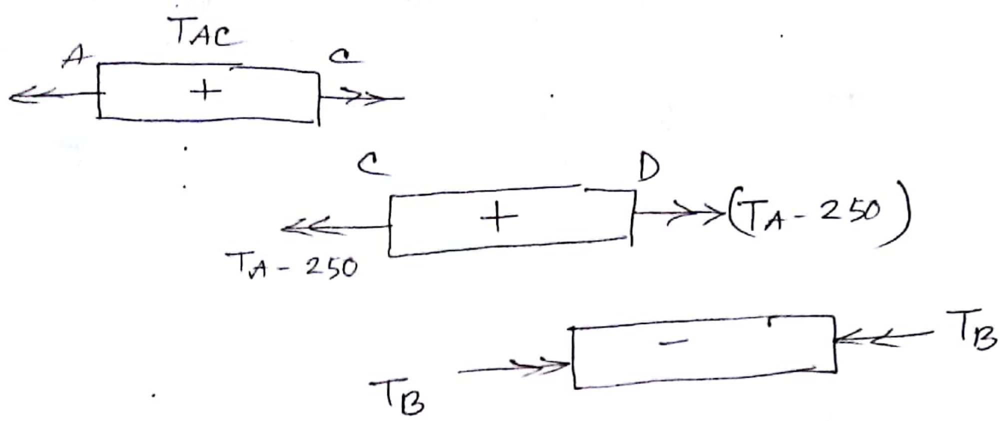
$T_A + T_B = 250 + 500$ —— (1)

$T_A = 750 - T_B$

$\theta_{AB} = 0$



$\theta_{AC} + \theta_{CD} - \theta_{DB} = 0$ —— (2)



$$\frac{T_A \times 6 \times 12}{12\times10^6 \times \frac{\pi\times1^4}{32}} + \frac{(T_A - 250)\times 4\times 12}{4\times10^6 \times \frac{\pi\times 2^4}{32}} - \frac{T_B \times 3\times12}{6\times10^6 \times \frac{\pi\times1^4}{32}} = 0$$

$$\Rightarrow 0.5T_A + \frac{(T_A - 250)}{16} - 0.5T_B = 0$$

$$T_B = 1.125\,T_A - 31.25 \quad \text{—— (iii)}$$

From (I) & (iii)

$$T_A = 750 - T_B.$$

$$T_B = 1.125(750 - T_B) - 31.25$$

$$2.125\,T_B = 812.5$$

$$T_B = 382.35 \text{ lb-ft.}$$

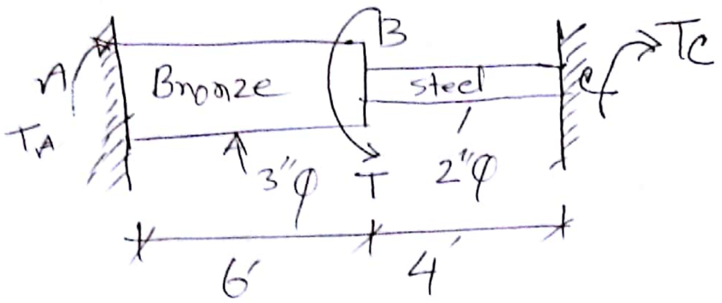
$$T_A = 367.64 \text{ lb-ft.}$$

$$S_{sAC} = \frac{T_{AC}\,c}{J} = \frac{367.64\times.5}{\frac{\pi\times1^4}{32}} = 1872.37$$

$$S_{sCD} = \frac{(367.64 - 250)\times1}{\frac{\pi}{32}\times2^4} = 74.89$$

$$S_{sBD} = \frac{382.3\times0.5}{\frac{\pi}{32}\times(1)^4} = 1947.03$$

Calculate T applied in the shaft.



$E_S = 12\times10^6$ psi
$E_B = 6\times10^6$ psi.

For Bronze $S_S \leq 8000$ psi
Steel $S_S \leq 12{,}000$ psi

Soln. $T_A + T_C = T$ —— (1)

$\theta_{AB} = \theta_{BC}$

$\Rightarrow \dfrac{T_A \times L_A}{E_B \times J_B} = \dfrac{T_C \times L_C}{E_S \times J_S}$

$\Rightarrow \dfrac{T_A \times 6}{6\times10^6 \times \frac{\pi \times 3^4}{32}} = \dfrac{T_C \times 4}{12\times10^6 \times \frac{\pi \times 2^4}{32}}$

$\dfrac{T_A}{3^4} = \dfrac{T_C \times 4}{12 \times 2^4}$

$T_A = \dfrac{4\,T_C \times 3^4}{12 \times 2^4} = 1{\cdot}68\,T_C$

$T_A = 1{\cdot}68\,(T - T_A)$

$T_A + 1{\cdot}68\,T_A = 1{\cdot}68\,T$

$2{\cdot}62\,T_A = 1{\cdot}68\,T$

$\Rightarrow T = \dfrac{2{\cdot}68}{1{\cdot}62}\,T_A.$

$\Rightarrow T = 1{\cdot}59\,T_A$

For Bronze, $S_S = \dfrac{T_A C_A}{J_A}$

$8000 = \dfrac{\frac{T}{1{\cdot}59} \times 1{\cdot}5}{\frac{\pi \times 3^4}{32}}$

$8000 \times 3^4 \times \pi \times 1{\cdot}59 = T \times 1{\cdot}5 \times 32$

$T = 67434{\cdot}3$ lb-in

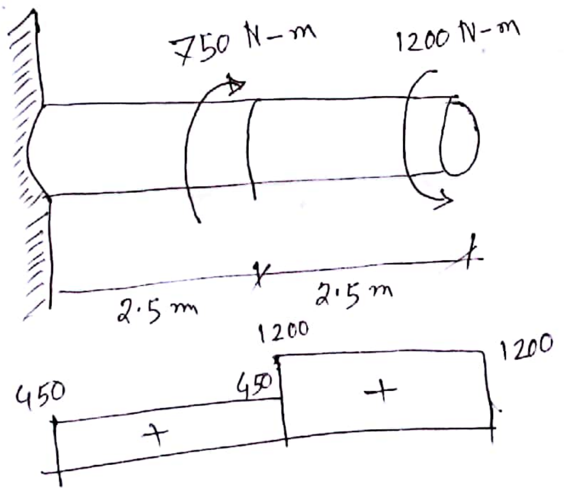
Steel $S_s = \frac{T_c \cdot C_c}{J_c}$

$12{,}000 = \frac{.37T \times 2}{\pi \times \frac{2^4}{32}}$

$12000 \times \pi \times 16 = .37T \times 2 \times 32$

$T = 25472$

A solid shaft is loaded as shown in Fig below. using G = 83 GPa. determine the required diameter of the shaft if the shearing stress is limited to 60 MPa and the angle of rotation at the free end is not to exceed 4 deg.



Given: $\tau_{max} = 60$ MPa $= 60 \times 10^6$ Pa.

* MPa = N/mm²

$\theta = 4^\circ = \frac{4\pi}{180}$

$G = 83$ GPa $= 83 \times 10^9$ Pa → ~~N/mm²~~ Pa = N/m²

$T_{max} = T_{BC} = 1200$ N-m

We know $\tau_{max} = \frac{16 T_{max}}{\pi d^3}$

$\Rightarrow d^3 = \frac{16 T_{max}}{\tau_{max} \pi} = \frac{16 \times 1200 \times 1000}{\pi \times 60 \times \frac{1}{1000} \times 10^6}$

$d^3 = 101859.16$

$d = 46.7$ mm.

Again $\theta_{C/A} = \sum \frac{TL}{JG}$

$$= \sum \frac{32TL}{\pi d^4 G}$$

$$= \frac{32}{\pi d^4 G}\left(T_{AB}L_{AB} + T_{BC}L_{BC}\right)$$

$$\Rightarrow d^4 = \frac{32}{\pi \theta G}\left(T_{AB}L_{AB} + T_{BC}L_{BC}\right)$$

$$= \frac{32 \times 180}{\pi \times 4\pi \times 83 \times 10^3}(450 \times 2.5 + 1200 \times 2.5) \times 10^3$$

$$= 7251.17 \times 10^3$$

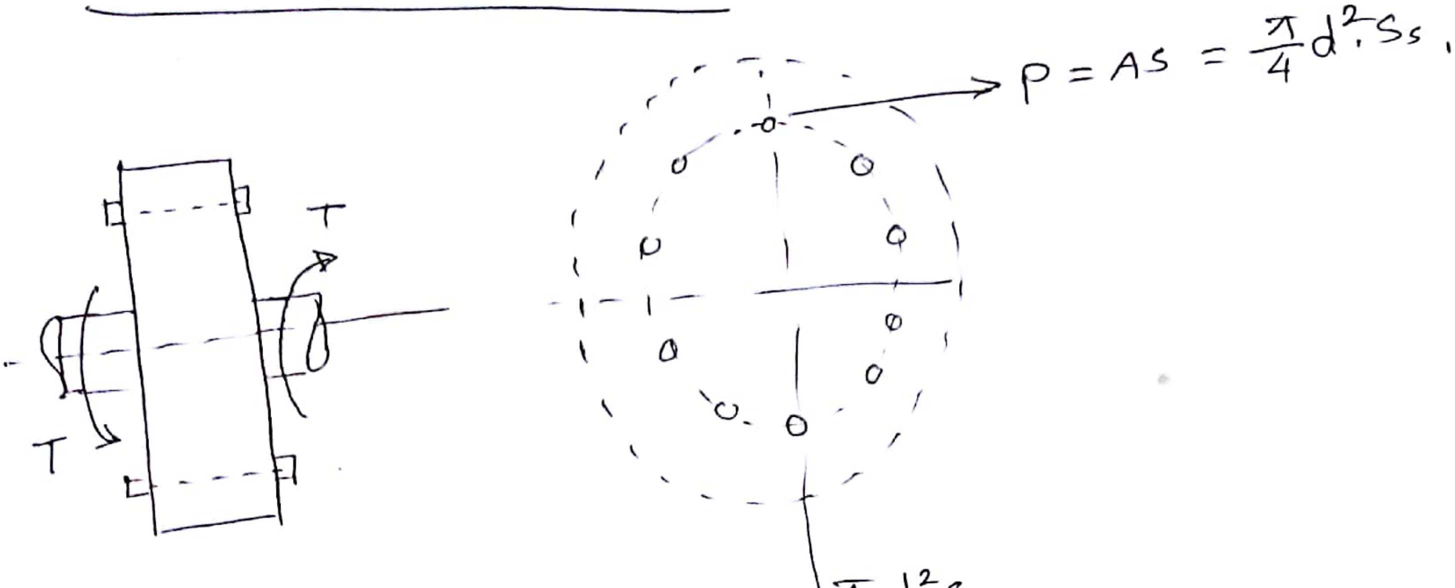
$d^4 = 7251178.68$

$d^2 = 2692.80$

$d = 51.89$ mm.

$\therefore$ required diameter is 51.89 mm.

Flange Bolt coupling :



Load in any bolt $P = As = \frac{\pi}{4} d^2 . S_s$

Torque's capacity of the coupling is $T = PRn = \frac{\pi}{4} d^2 S_s . Rn$

Here, P = Load in an rivet
R = Radius of the bolt circle
n = No. of bolt

Torque capacity the coupling is

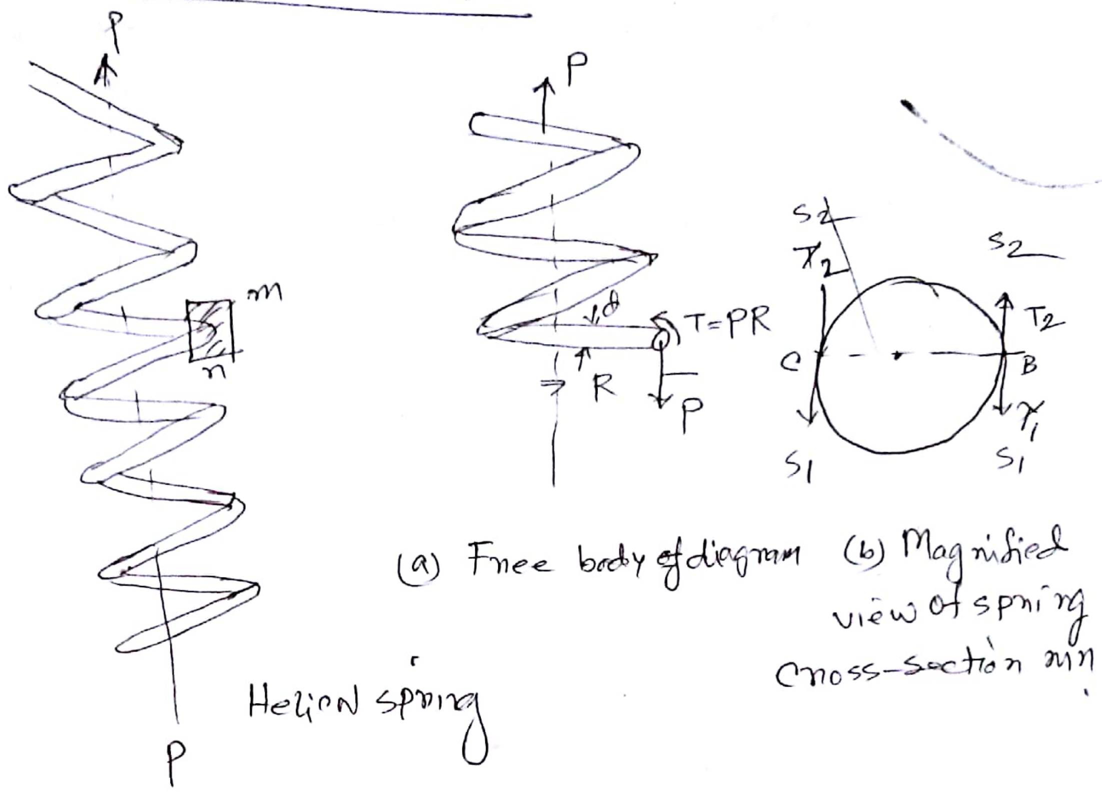
$$T = P_1 R_1 n_1 + P_2 R_2 n_2$$

Relation between P_1 & P_2 is

$$\frac{P_1}{R_1} = \frac{P_2}{R_2}$$

Scanned by CamScanner

Helical spring



(a) Free body of diagram (b) Magnified view of spring cross-section mn

Helical spring

※ Derive an expression for the maximum shearing stress of a helical spring.

The closed coil helical spring as shown in the fig. is elongated by axial Load P. To determine the stress produced by P, we consider a plane mn through any typical section as in the fig.

To balance the applied axial load P, the ~~spring~~ spring must provide the resistance $P_r = P$ ~~being~~ being opposite equal and parallel create a couple of magnitude P_r which can be balanced only a opposite couple.

This resisting couple is created by a torsional shearing stress distributed over the cross section of the spring.

Two types of shearing stress are produced

(I) Direct shearing stress like, S_1

(II) Variable Torsional shearing stress, like S_2

coupled by the twisting couple T = PR.

At point C the two shearing stress ~~are~~ are collinear and in the same sense.

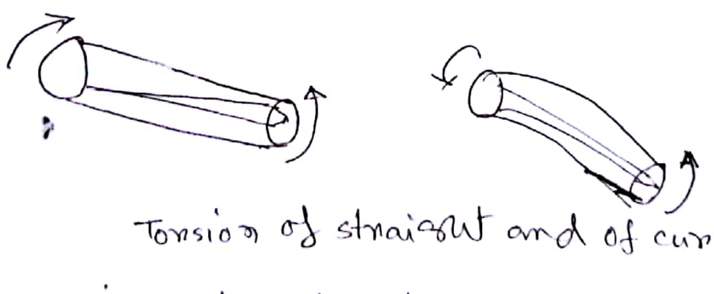
The max. shearing stress occurs at the inside element and is given by the same of direct shearing stress, $S_1 = \frac{P}{A}$, and max value of torsional shearing stress, $S_2 = \frac{Tr}{J}$

$$S_s = S_1 + S_2 = \frac{P}{A} + \frac{Tr}{J}$$

$$= \frac{P}{\frac{\pi}{4}d^2} + \frac{T \cdot r}{\frac{\pi d^4}{32}}$$

$$= \frac{4P}{\pi d^2} + \frac{16PR}{\pi d^3} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

$$S_s = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$



Torsion of straight and of curved segments.

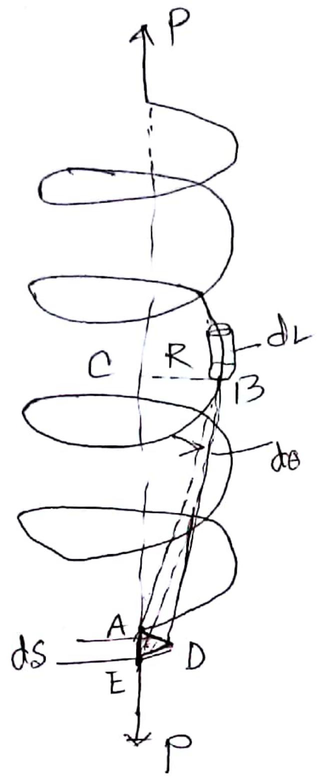
the following formular that takes account of the initial curvature of the spring wire

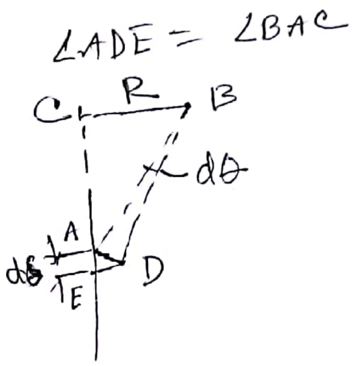
$$\text{Max } S_s = \frac{16PR}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right)$$

$$m = \frac{2R}{d} = \frac{D}{d}$$

$$S_s = \frac{16PR}{\pi d^3}\left(1 + \frac{0.5}{m}\right).$$

Derive an expression for deflection of spring





Consider, the spring in the Fig is rigid accept the small Length dL, assume. The end A rotates to D through the small angle dθ. As dθ is so small the arc $AD = AB.d\theta$ may be consider

Now From the similar triangle ADE and BAC

$$\frac{AE}{AD} = \frac{BC}{AB}$$

$$\frac{ds}{AB.d\theta} = \frac{R}{AB}$$

$$ds = R.d\theta.$$

$$ds = R.\frac{(PR).dL}{JG} \qquad \text{Assume } d\theta = \frac{(PR).dL}{JG}.$$

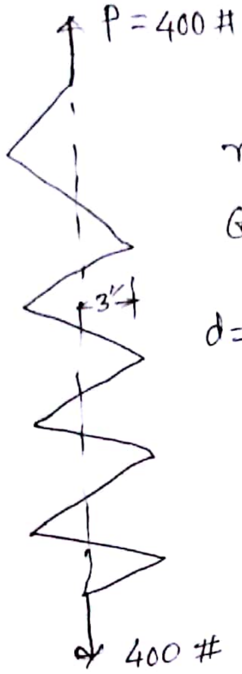
The total elongation deflection can be obtained by intigration all the element of the spring contribute to elongation $\delta = \frac{PR^2 L}{JG}$

Replacing L by $2\pi r n$ which is the length of n coil and J by $\frac{\pi d^4}{32}$, we obtaine:

$$\delta = \frac{64PR^3 n}{Gd^4}$$

This expression for spring deflection.

Prob. 1.



$n = 20$

$G = 12 \times 10^6$ psi

$d = \frac{3}{4}$"

Find maximum S_s and Δ

$$\delta = \frac{64PR^3 n}{Gd^4}$$

$$= \frac{64 \times 400 \times 3^3 \times 20}{12 \times 10^6 \times (\frac{3}{4})^4}$$

$$= 3.64"$$

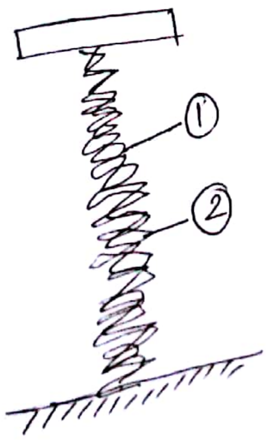
$$S_s = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

$$= \frac{16 \times 400 \times 3}{\pi \times (\frac{3}{4})^3}\left(1 + \frac{3/4}{4 \times 3}\right)$$

$$= \frac{16 \times 400 \times 3 \times 4^3}{\pi \times 3^3}\left(1 + \frac{3}{4 \times 4 \times 3}\right)$$

$$= 15392 \text{ psi}$$

Prob. 2.



spring-1.
$n = 24$
$d = \frac{3}{4}$"
$D = 6$"

spring-2
$n = 20$
$d = 1$"
$D = 8$"

Maximum, $S_s = 20{,}000$ psi

$G = 12 \times 10^6$ psi, Find P ?

Soln: $P_1 + P_2 = P$ ——— (i)

$\Delta_1 = \Delta_2$ ——— (ii)

$\delta_1 = \delta_2$

$$\delta_1 = \delta_2$$

$$\Rightarrow \frac{64 P_1 R_1^3 n}{G d_1^4} = \frac{64 P_2 R_2^3 n}{G d_2^4}$$

$$\Rightarrow \frac{64 \times P_1 \times 3^3 \times 24}{\left(\frac{3}{4}\right)^4} = \frac{64 \times P_2 \times (4)^3 \times 20}{(1)^4}$$

$$131072 P_1 = 81920 P_2$$

$$P_1 = 0.625 P_2 \quad \text{—— (iii)}$$

$$P_1 + P_2 = P$$

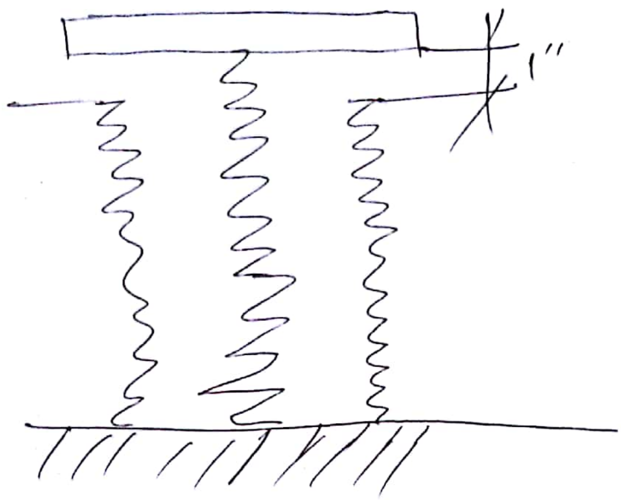
$$0.625 P_2 + P_2 = P$$

$$P_2 = 0.62 P$$

From Eqn. (iii) $P_1 = 0.625 \times 0.62 P$

$$= 0.38 P$$

Determine max. shearing stress in each spring



Inner spring

$n = 24$

$d = 3/4''$

$D = 6.$

outer spring

$n = 18$

$d = 1/2''$

$D = 4''$

$G = 12 \times 10^6$ psi

$P = 1200$ lb.

$$\delta = \frac{64PR^3n}{Gd^4}$$

$$1 = \frac{64 \times P \times (3)^3 \times 24}{12 \times 10^6 \times \left(\frac{3}{4}\right)^4}$$

$$P = \frac{12 \times 10^6 \times \left(\frac{3}{4}\right)^4}{64 \times (3)^3 \times 24} = 91.55 \text{ lb.}$$

$P_{in} + 2P_{out} = 1200 - 91.55$

$P_{in} + 2P_{out} = 1108.45$ —— (1)

and $\delta_{in} = \delta_{out}$

$$\frac{64PR^3n}{Gd^4} = \frac{64PR^3n}{Gd^4}$$

$$\Rightarrow \frac{64 \times P_{in} \times 3^3 \times 24}{12 \times 10^6 \times \left(\frac{3}{4}\right)^4} = \frac{64 \times P_{out} \times 2^3 \times 18}{12 \times 10^6 \times \left(\frac{1}{2}\right)^4}$$

$$P_{in} = \frac{P_{out} \times 2^3 \times 18 \times \left(\frac{3}{4}\right)^4 \times \cancel{12}}{3^3 \times \cancel{24} \times \cancel{12} \times \left(\frac{1}{2}\right)^4}$$

24

$P_{in} = 1'68\, P_{out}$. / $P_{in} = 1'125\, P_o$

1'125 P_o

~~1'68 P_{out}~~ + 2 P_{out} = 1108'45

3·125 P_o

~~3'68 P_{out}~~ = 1108'45

P_{out} = ~~301·20~~ 354'7

∴ P_{in} = ~~506·03~~ 399'04

399'04

P_{in} = ~~506·03~~ + 91'55 = ~~597·58~~ 490'6

$$S_{sin} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right) \quad \text{—— ?}$$

$$S_{sout} = \frac{16PR}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

~~399~~ 490'6

$$S_{s\,inner} = \frac{16 \times \cancel{597.58} \times 3}{\pi \times \left(\frac{3}{4}\right)^3}\left(1 + \frac{3/4}{4 \times 3}\right)$$

= ~~23'006 ksi~~, 18887 ~~ksi~~ psi

354'7

$$S_{sout} = \frac{16 \times \cancel{301.20} \times 2}{\pi \times .5^3}\left(1 + \frac{1/2}{4 \times 2}\right)$$

354'7

= ~~26.091 ksi~~

30725'6 psi

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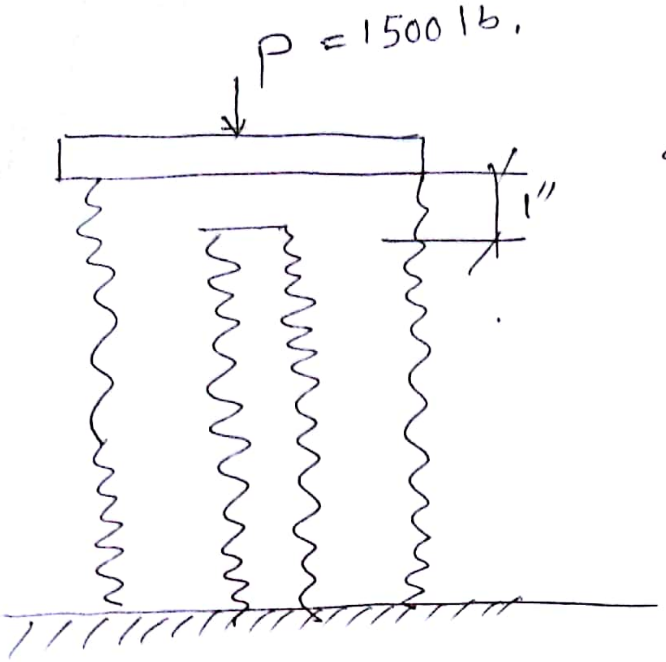

$S_{s\,in} = ?$

$S_{s\,out} = ?$

inner and outer same as previous

$$\delta = \frac{64 P R^3 n}{G d^4}$$

$$2P_o + 2P_{in} = P - P_n$$

$$1 = \frac{64 \times P_{out} \times R^3 n}{G d^4}$$

and

$$\delta_{in} = \delta_{out}.$$

$\delta_c = 1 + \delta_o$ ——— (I)

$P = 2P_o + P_c$ ——— (II)

From (I), $\delta_c = 1 + \delta_o$

$$\Rightarrow \frac{64 \times P_c \times 3^3 \times 24}{12 \times 10^6 \times (.75)^4} = 1 + \frac{64 \times P_o \times 2^3 \times 18}{12 \times 10^6 \times (.5)^4}$$

$$0.010922 P_c = 1 + 0.012288 P_o$$

$$P_c = 91.55 + \sout{1.124336}\ 1.125068 P_o$$

From (II), $2P_o + 91.55 + 1.125068 P_o = 1200$

$$P_o = 354.697$$

$$P_c = 490.60.$$

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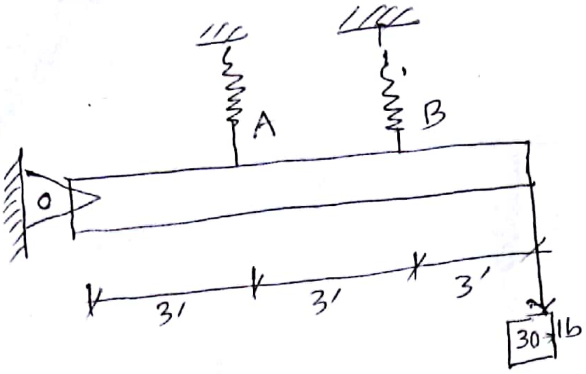
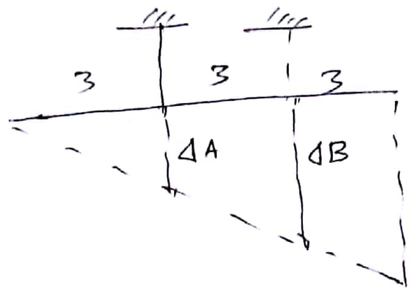


$n = 20$

$D = 5''$

$d = \frac{1}{4}''$

$S_s = ?$



From similar triangle

$$\frac{\Delta A}{\Delta B} = \frac{3}{6} \quad \text{--- (I)}$$

$$\sum M_O = 0$$

$$P_A \times 3 + P_B \times 6 = 30 \times 9$$

$$P_A + 2P_B = 90 \quad \text{--- (II)}$$

From Eq. (I).

$$2\Delta A = \Delta B$$

$$\frac{2 \times 64 \times P_A R^3 n}{Gd^4} = \frac{64 \times P_B \times R^3 n}{Gd^4}$$

$$\Rightarrow \quad 2P_A = P_B \quad \text{--- (III)}$$

From Eqn. (II), $P_A + 2 \cdot 2P_A = 90$

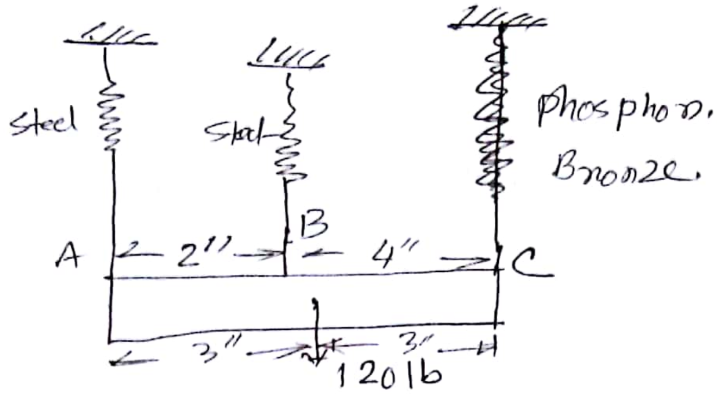
$$5P_A = 90$$

$$P_A = 18$$

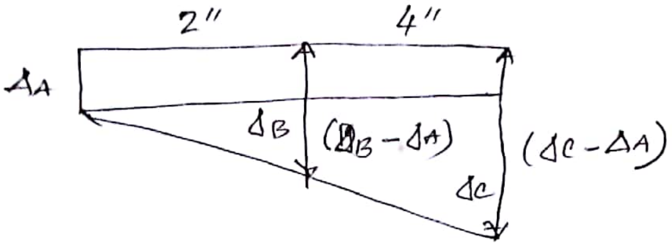
$$P_B = 2 \times 18 = 36$$

$$S_s = \frac{16 P_B \times R}{\pi d^3}\left(1 + \frac{d}{4R}\right)$$

Find Max^m S_s.



	Steel	Bronze
	$n = 24$	$n = 24$
	$d = \frac{1}{4}$"	$d = \frac{1}{2}$"
	$D = 4$"	$D = 6$"
	$G = 12\times10^6$ psi	$G = 6\times10^6$ psi



From similar triangle;

$$\frac{\Delta_B - \Delta_A}{\Delta_C - \Delta_A} = \frac{2}{6} \quad \text{———— (i)}$$

$$P_A + P_B + P_C = 120 \quad \text{———— (ii)}$$

$\Sigma M_A = 0$,

$$P_B \times 2 + P_C \times 6 = 120 \times 3 \quad \text{———— (iii)}$$

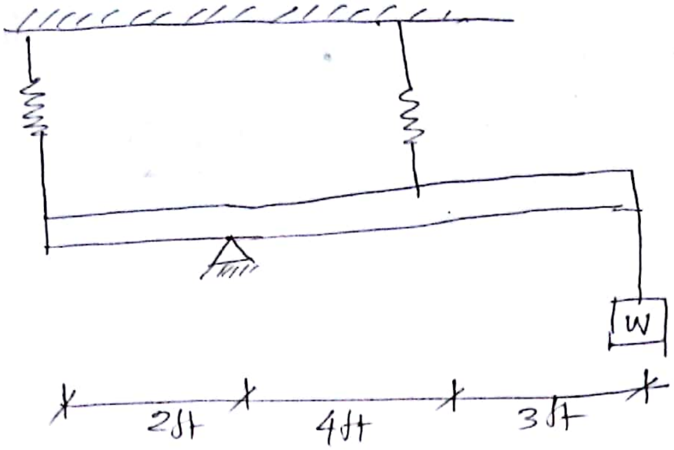
From (i) we get.

$$3(\Delta_B - \Delta_A) = \Delta_C - \Delta_A$$

$$3\Delta_B - 3\Delta_A = \Delta_C - \Delta_A$$

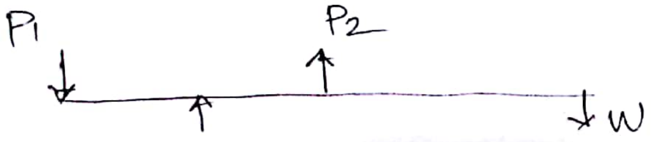
$$3\Delta_B = \Delta_C + 2\Delta_A \quad \text{———— (iv)}$$

#

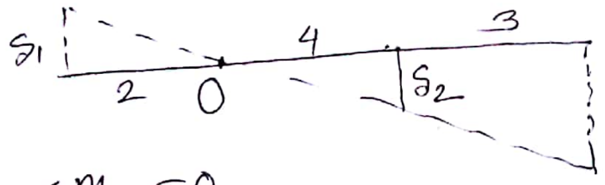


A rigid bar, pinned at point O, is supported by two identical springs as shown in Fig below. Each spring consists of 20 turns of ($\frac{3}{4}$ in) diameter wire having a mean diameter of 6 in. Determine the maximum load W that may be supported if the shearing stress in the springs is limited to 20 ksi.

taking free body diagram



The Linear diagram of deflection is



$\Sigma M_o = 0$

$4P_2 + 2P_1 = 7W$ —— (I)

from similar triangle

$\frac{\delta_1}{\delta_2} = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \delta_2 = 2\delta_1$

$\Rightarrow \frac{64 P_2 R_2^3 n_2}{d_2^4 G} = \frac{64 P_1 R_1^3 n_1}{d_1^4 G} \times 2$

$\Rightarrow P_2 = 2P_1$ —— (II) [as $R_1 = R_2 = R$, $n_1 = n_2 = n$, $d_1 = d_2 = d$

from eq. (I) →

maximum shearing stress will occur by P_2

$$\tau_{max} = \frac{16 P_2 R}{\pi d^3}\left(\frac{4m-1}{4m-4} + \frac{0.615}{m}\right) \quad \text{---} (iii)$$

Given $\tau_{max} = 20\ ksi = 20 \times 10^3\ psi$

$R = \frac{6}{2} = 3\ in,\quad d = \frac{3}{4}\ in$

$m = \frac{D}{d} = \frac{6}{3/4} = 8$

from (iii)

$$20 \times 10^3 = \frac{16 \times P_2 \times 3}{\pi \times \left(\frac{3}{4}\right)^3}\left(\frac{8\times4-1}{8\times4-4} + \frac{0.615}{8}\right)$$

$\Rightarrow \quad 20 \times 10^3 = 42.88 P_2$

$P_2 = 466.42\ lb$

$P_1 = \frac{P_2}{2} = \frac{466.42}{2} = 233.21\ lb.$

$4 \times 466.42 + 2 \times 233.21 = 7W.$

$W = 333.16\ lb.$

using $\tau_{max} = \frac{16 P_2 R}{\pi d^3}\left(1 + \frac{R}{4R}\right)$

$P_2 = 519.75\ lb.$

$P_1 = 259.87\ lb.$

$W = \frac{1}{7}(4P_2 + 2P_1) = 371.25$

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