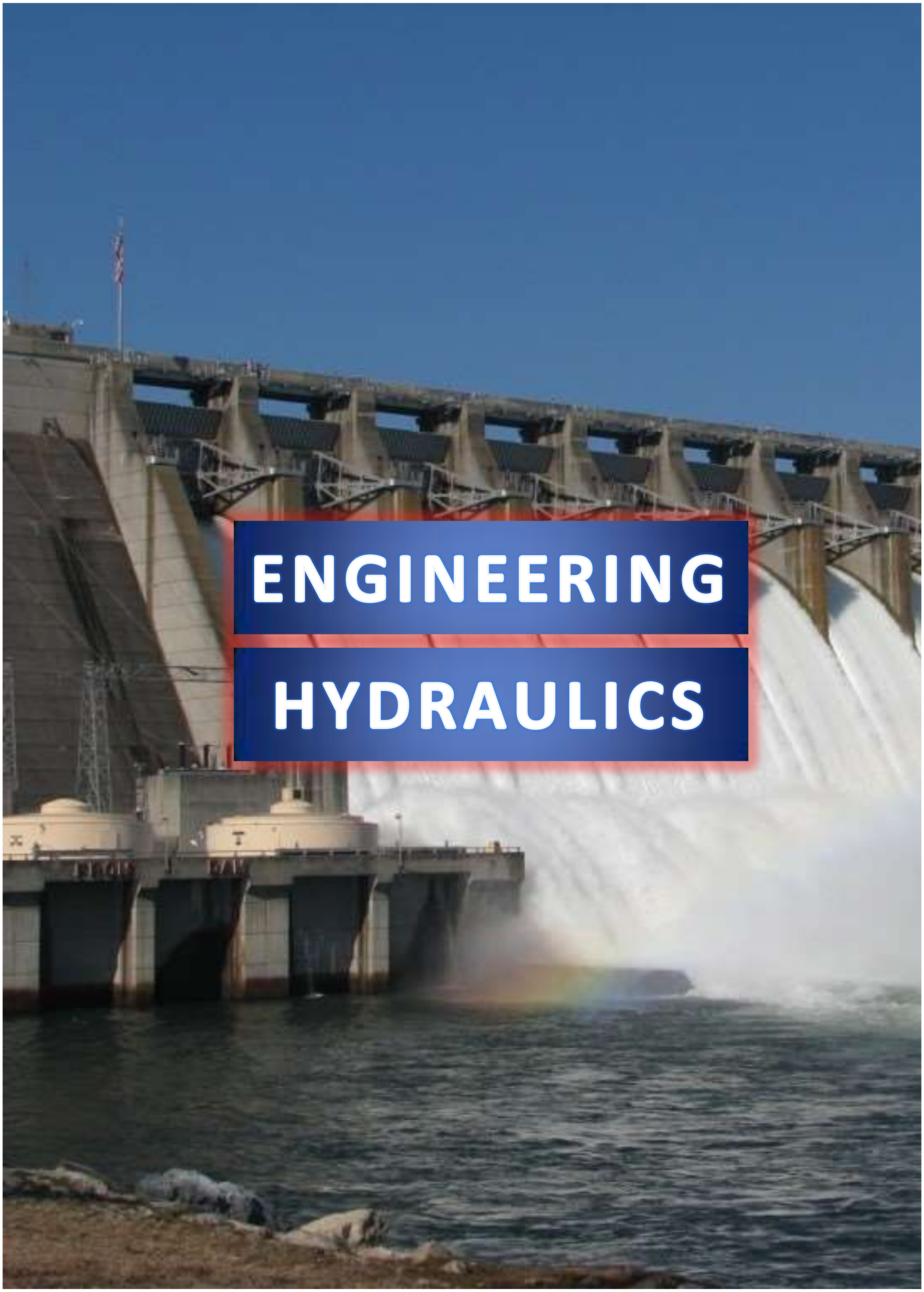

ENGINEERING HYDRAULICS

HandNote On

ENGINEERING HYDRAULICS

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THEORY

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FARHAD
#1500045

Open Channel Flow

⊞ Open Channel: An open channel is a conduit in which liquid flows in a free surface. The free surface is actually an interface between the moving liquid and an overlying fluid medium having constant pressure.

⊞ Pipe: Pipe is a closed conduit in which flow exerts no direct ~~pressu~~ atmospheric pressure but hydraulic pressure only.

16, 15

⊞ Open channel flow: Open channel flow is a type of liquid flow with in a conduit with a free surface that is subject to atmospheric pressure.

⊞ Pipe flow: Pipe flow is a type of liquid flow with in a closed conduit, exerts no direct atmospheric pressure but hydraulic pressure only. It has no free surface, since the water must fill the whole conduit.

⊞ Difference between open channel flow and pipe flow: 16

Open Channel Flow	Pipe Flow
1. Open channel flow must have a free surface	1. Pipe flow has no free surface.
2. It exerts atmospheric pressure.	2. It exerts hydraulic pressure.
3. Flow condition is more complicated than pipe flow.	3. Flow condition is comparetively simple than open channel flow.
4. The physical condition of it varies more widely than pipe flow.	4. The physical condition of pipe flow less wider than that of pipes.
5. Reliable experiement data on flow in open channels are difficult to obtain.	5. Reliable experiment data on pipe flows are easier ~~tha~~ to obtain than open channel flow.
6. The cross section of an open channel flow may be of any shape.	6. The cross section of pipe flow is generally round.

Open channel flow	Pipe flow
7. Maximum velocity in open channel flow usually appears to occur just below the free surface.	7. Maximum velocity occurs at the middle in pipe flow.
8. The roughness in an open channel flow varies with the position of the free surface.	8. In pipes, The interior surface ordinarily ranges in roughness.

15, 13 # Why open channel flow is complicated than pipe channel flow? [Difference: 3,4,5,6,8]

Classification of open channel:

According to the origin a channel may be classified as:

1. Natural channel 2. Artificial channel

1. Natural channel: The channel which is developed in a natural way and which has irregular cross section and shape is known as Natural channel. Example: River, Stream.

2. Artificial channel: The channel which is built artificially i.e. constructed or developed by human effort for carrying water for various purposes is known as Artificial channel. Example: canal, chute etc. Artificial channel must have a regular geometric shape.

According to the geometry of the channel, it may be classified as:

1. Prismatic 2. Non-prismatic channel

1. Prismatic channel: A channel built with unvarying cross-section and constant bottom slope is called prismatic channel.

2. Non prismatic channel: A channel built with varying cross-section and varying bottom slope is called non-prismatic channel.

According to bed material of channel, it may be classified as:

1. Rigid bed channel 2. Movable bed channel.

1. Rigid bed channel: The channel in which bed materials remain rigid i.e. erosion of bed is not easy is called Rigid bed channel.

2. Movable bed channel: The channel in which bed materials moves with the flow i.e. erosion of bed is easy is called Movable bed channel.
It is also called erodable channel.
In this type of channel, when velocity of flow is high, scouring take place and when velocity of flow become low, deposition take place.

Different Artificial open channels:

Under various circumstances in engineering practice the artificial open channel is given different Names. Such as: canal, Flume, chute, drop, culvert, open-flow tunnel. etc.

14 1. Canal: canal is usually a long and mild sloped channel built in the ground, which may be lined or unlined with stone masonry, concrete, cement, wood or bituminous materials.

14 2. Flume: Flume is a channel of wood, metal, concrete or masonry, usually supported on or above the surface of the ground to carry water acaross a depression.

14 3. Chute: The chute is channel having steep slope.

4. Drop: The Drop is similar to chute, but the change in elevation is effected in a short distance.

5. Culvert: The culvert flowing partly full is a covered channel of comparatively short length installed to drain water through highway and rail road embankments.

6. Open flow tunnel: The open-flow-tunnel is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.

Classification of open-channel flow: 12

Open-channel flow is classified as:

1. Steady flow:
   (i) uniform flow
   (ii) varied flow:
      a. Gradually varied flow
      b. Rapidly varied flow

2. Unsteady flow
   (i) Unsteady uniform flow (rare)
   (ii) Unsteady varied flow
      a. Gradually varied unsteady flow
      b. Rapidly varied unsteady flow

Steady flow: Flow in a channel is said to be steady if flow characterestics at any point do not change with time.

For steady flow, $\frac{\partial V}{\partial t} = 0$; $\frac{\partial y}{\partial t} = 0$; $\frac{\partial Q}{\partial t} = 0$ Example:

Unsteady flow: Flow in a channel is said to be unsteady if flow characterestics at any point changes with time.

For unsteady flow, $\frac{\partial V}{\partial t} \neq 0$; $\frac{\partial y}{\partial t} \neq 0$

Uniform flow: Flow in a channel is said to be uniform if cross-section, depth, slope and velocity remain constant over a given length of channel.

Non-uniform flow: Flow in a channel is said to be non-uniform if cross-section, depth, slope and velocity do not remain constant over a given length of channel.

Gradually varied flow: If depth of flow and mean velocity change gradually along the length of the channel, it is called gradually varied flow.

Rapidly varied flow: If depth of flow and mean velocity change abruptly over a comparatively short distance of the channel, it is called rapidly varied flow.

14 State of flow: The state or behavior of open-channel flow is governed basically by the effects of viscosity and gravity relative to the inertial force of flow.

14 Effect of viscosity: Depending on the effect of viscosity relative to inertia, the flow may be:

1. Laminar flow. ($R \leq 500-600$)
2. Turbulent flow. ($R > 2000$)
3. Transitional flow. ($R = 600-2000$)

Laminar flow: The flow is laminar if the viscous force are so strong relative to inertial force. In laminar flow, water particles appear to move in definite smooth paths or streamlines.

Turbulent flow: The flow is turbulent if the viscous forces are weak relative to the inertial forces. In turbulent flow, water particles move in irregular paths.

Transition flow: Between the laminar and turbulent states, there is a mixed or Transitional ~~flow~~ states which is known as Transitional flow.

Reynolds Number: The effect of viscosity relative to inertia can be represented by the reynolds number, define as,

$$R = \frac{VL}{\nu} \doteq \frac{\text{inertia force}}{\text{viscous force}} = \frac{VR}{\mu/\rho} = \frac{VR\rho}{\mu}$$

where, V = velocity of flow

L = Length (R = hydraulic Radius of a conduit)

ν = Kinematic viscosity $= \frac{\mu}{\rho} = \frac{\text{dynamic viscosity}}{\text{mass density}}$

14 16 Effect of gravity: The effect of gravity upon the state of flow is represented by a ratio of inertial forces to gravity forces. The ratio is given by the froude number, defined as,

$$F = \frac{V}{\sqrt{gL}} = \frac{\text{inertial forces}}{\text{gravity forces}} = \frac{V}{\sqrt{gD}}$$

where,

V = mean velocity of flow

L = characterestic length (D, hydraulic depth)

If $F = 1$, the flow is said to be critical $(V = \sqrt{gD})$

if, $F < 1$, the flow is subcritical $(V < \sqrt{gD})$

if, $F > 1$, the flow is supercritical. In this state the inertial forces become dominant; so the flow has a high velocity and usually described as rapid, shooting and torential. $(V > \sqrt{gD})$

16 Celerity of gravity waves: In the mechanics of water waves, the critical velocity $\sqrt{gD}$ is identified as the celerity of the small gravity waves that occur in shallow water in channels as a result of any momentary change in the local depth of water.

It should be noted that a gravity wave can be propagated upstream in water of subcritical flow but not in water of supercritical flow.

Continuity equation:

 # For Steady flow: In a steady flow the volumetric rate of flow passed through various section must be same. Hence,

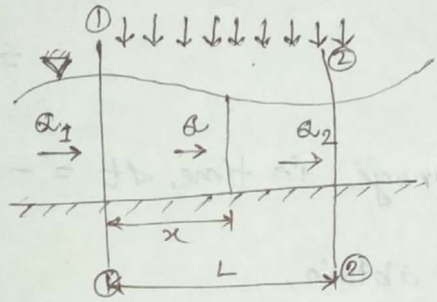
$$Q = A_1 V_1 = A_2 V_2 = \cdots\cdots = A_n V_n$$

 # For steady spatially varied flow:

In a steady spatially varied flow, the discharge at various sections will not be the same.

Consider a SVF with increasing discharge as shown in figure:

The rate of addition dischare, $\frac{dQ}{dx} = q$



The dischare at any section at a distance 'x' from section ①-①,

$$Q = Q_1 + \int_0^x q\,dx$$

$\Rightarrow Q = Q_1 + qx$ $\quad [\because q = \text{constant}]$

and, $Q_2 = Q_1 + qL$ $\quad \left[\because q = \frac{dQ}{dx}\right]$

 # For unsteady flow: 15, 13, 12, 10

Let the flow shown in figure is unsteady and incompressible.

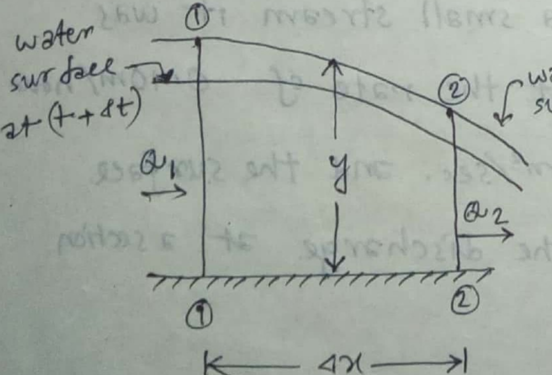


Fig. 1

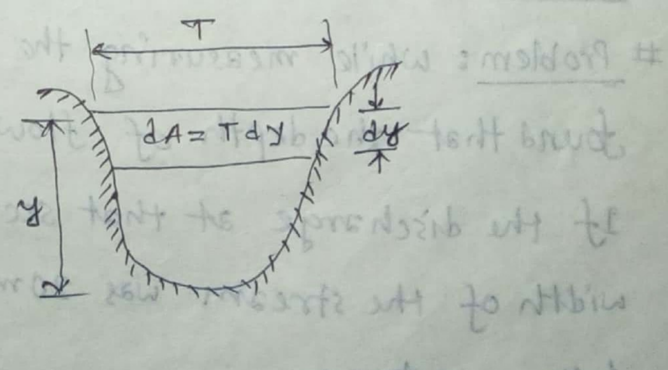


Fig. 2

Let, the discharge in section ②-②, $Q_2 > Q_1$

and the distance between section ①-① and ②-② is Δx

$\therefore$ Excess volume of out flow, $Q_2 - Q_1 = \frac{\partial Q}{\partial x} \Delta x$

Excess volume of out flow in time Δt is $= \frac{\partial Q}{\partial x} . \Delta x . \Delta t$ ----- (I)

If 'T' is the top width of the canal at any depth 'y'

then, $\frac{\partial A}{\partial y} = T$

Rate of decrease of storage at depth 'y' $= -\Delta x . \frac{\partial A}{\partial y} . \frac{\partial y}{\partial t}$

$= -\Delta x . T . \frac{\partial y}{\partial t} \quad \left[\because \frac{\partial A}{\partial y} = T \right]$

$\therefore$ Rate of decrease of storage in time Δt $= - T . \Delta x . \Delta t . \frac{\partial y}{\partial t}$ ----- (II)

Equating (I) and (II) we obtain,

$$\frac{\partial Q}{\partial x} . \Delta x . \Delta t = - T . \Delta x . \Delta t . \frac{\partial y}{\partial t}$$

$$\Rightarrow \frac{\partial Q}{\partial x} + T \frac{\partial y}{\partial t} = 0$$

This is the basic equation of continuity for unsteady incompressible flow.

15 **Statement:** The net change in discharge plus the change in storage should be zero.

\# Problem: While measuring the discharge in a small stream it was found that the depth of flow increases at the rate of 0.10 m/hour. If the discharge at that section was 25 m³/sec. and the surface width of the stream was 20 m, estimate the discharge at a section 1 km upstream.

Solution: Given that,

$$\frac{\partial y}{\partial t} = 0.10\ m/h = \frac{0.10}{60\times60} = 2.778\times10^{-5}\ m/sec$$

$T = 20m$; $\Delta x = 1km = 1000m$; $Q_2 = 25\ m^3/s$

We know,

$$\frac{\partial Q}{\partial x} + T\cdot\frac{\partial y}{\partial t} = 0$$

$$\Rightarrow \frac{\partial Q}{\partial x} = -20\times2.778\times10^{-5}$$

$$\Rightarrow \frac{Q_2-Q_1}{\Delta x} = -5.556\times10^{-4}$$

$$\Rightarrow Q_1 = 25+(5.556\times10^{-4}\times1000) = 25.556\ m^3/s$$ (Ans.)

Velocity distribution of a section:

Due to the presence of free surface and friction along the channel wall, the velocities in a channel are not uniformly distributed in the channel section.

The maximum velocity in ordinary channels usually appears to occur below the free surface at a distance of 0.05 to 0.25 of the depth.

The velocity distribution in a channel section depends on the following factors:

1. Unusual shape of section.
2. Roughness of channel.
3. Presence of bends.
4. Surface wind.

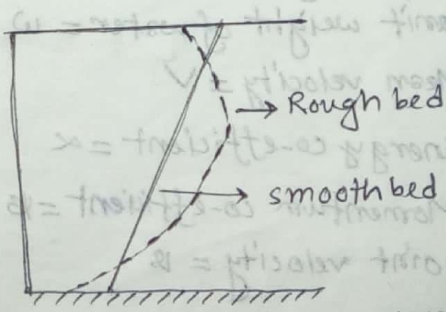


Fig. Effect of roughness on V.D.

In a broad, rapid and shallow stream or in a very smooth channel, the maximum velocity may often be found at the free surface.

The effect of roughness on velocity distribution in an open channel is shown is Figure.

The roughness of the channel will cause the curvature of the vertical velocity distribution curve to increase. On a bend, the velocity increases greatly at the convex side, owing to the centrifugal action of flow. contray to usual belief, surface wind has little effect on velocity distribution.

Velocity distribution co-efficient: 11

When the energy principle is used in computation, the true velocity head may be expressed as $\alpha \frac{V^2}{2g}$, where α is known as energy co-efficient or coriolis co-efficient. α varies from about 1.03 to 1.36 for fairly straight prismatic channel. The value is generally higher for small channels and lower for large streams of considerable depth.

Momentum of fluid passing through a channel section per unit time is expressed by $\beta \frac{wQV}{g}$, where β is known as the momentum co-efficient or Boussinesq co-efficient. β varies from 1.01 to 1.12 for fairly straight prismatic channel.

Determination of velocity distribution co-efficients:

Let, whole water area $= A$

Elementary area $= \Delta A$

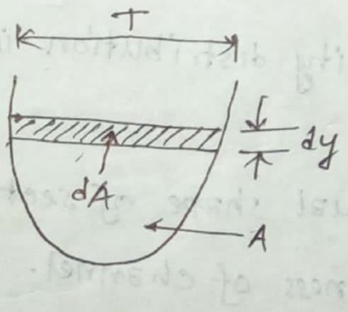
unit weight of water $= w$

Mean velocity $= V$

Energy co-efficient $= \alpha$

Momentum co-efficient $= \beta$

point velocity $= v$



weight of water passing through ΔA per unit time with a velocity v is $= w v \Delta A$

Kinetic energy of water passing ΔA per unit time

$$= \frac{1}{2}\left(\frac{w v \Delta A}{g}\right) \times v^2 = \frac{w v^3 \Delta A}{2g}$$

This is equivalent to the product of the weight $w v \Delta A$ and the velocity head $\frac{v^2}{2g}$.

The total Kinetic energy for the whole water area is equal to $\sum \frac{w v^3 \Delta A}{2g}$ .... (I)

corrected velocity head for whole area $= \alpha \frac{V^2}{2g}$

Total Kinetic energy is $= \frac{\alpha w V^3 A}{2g}$ .... (II)

Equating (I) & (II), we obtain,

$$\frac{\alpha w V^3 A}{2g} = \sum \frac{w v^3 \Delta A}{2g}$$

$$\Rightarrow \alpha = \frac{\sum v^3 \Delta A}{V^3 A}$$

The momentum of water passing δA per unit time

$$= \frac{w \Delta A v}{g} \times v = \frac{w v^2 \Delta A}{g}$$

Total momentum per unit time $= \sum \frac{w v^2 \Delta A}{g}$ ..... (I)

corrected momentum for the whole area $= \frac{\beta w V^2 A}{g}$ ..... (II)

Equating (I) & (II), we obtain,

$$\frac{\beta w V^2 A}{g} = \sum \frac{w v^2 \Delta A}{g}$$

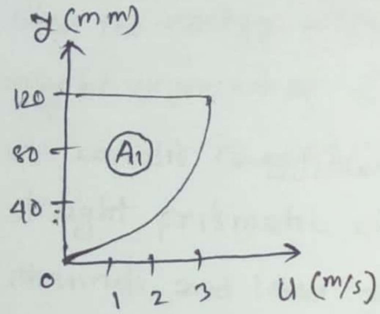
$$\therefore \beta = \frac{\sum v^2 \Delta A}{V^2 A}$$

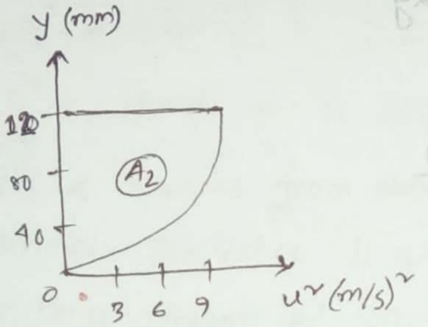
 # problem: The following velocities were measured at the centre line of a very wide rectangular channel of depth equal to 120 mm. Find U, α and β.

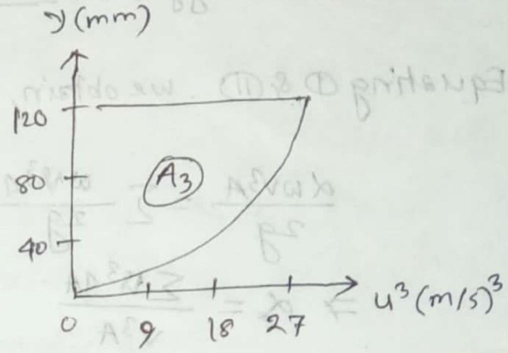
y (mm) →	0	3	10	15	20	40	60	80	100	120
u (m/s) →	0	1.25	1.75	2.05	2.2	2.55	2.75	2.85	2.9	3

Solution:

If here u vs. y, u^2 vs. y and u^3 vs. y graph is plotted then,







$A_1 = .3032\ m^2/s$ → [Simpson's rule]

$A_2 = 0.811\ m^3/s^2$

$A_3 = 2.20\ m^4/s^3$

we know, $U = \frac{1}{h}\int_0^h u\,dy = \frac{A_1}{h} = \frac{.3032}{.12} = 2.53\ m/s$

$$\alpha = \frac{1}{U^3 h}\int_0^h u^3 dy = \frac{A_3}{U^3 h} = \frac{2.20}{(2.53)^3 \times .12} = 1.132$$

$$\beta = \frac{1}{U^2 h}\int_0^h u^2 dy = \frac{A_2}{U^2 h} = \frac{.811}{(2.53)^2 \times .12} = 1.056$$

(Ans)

Help:

$$A_1 = \frac{20}{3} \times \left[0+3+4\times(2.2+2.75+2.90)+2\times(2.55+2.85)\right] = .3013$$

$$A_3 = \frac{20}{3} \times \left(0+3^2+4\times(2.2^2+2.75^2+2.90^2)+2\times(2.55^2+2.85^2)\right] = .81$$

$A_2 =$ (similarly)

4 ** 16

Problem: The velocity distribution in a ~~white~~ wide rectangular section channel may be approximate by the equation, $u = 0.4 + 0.6\frac{y}{h}$ m/s. Find U, α and β if $h = 1.0$ m.

Solution: Given equation, $u = 0.4 + 0.6\frac{y}{h}$

Here, $U = \frac{1}{h}\int_0^h u\,dy = \frac{1}{1}\int_0^1 (0.4 + 0.6\frac{y}{1})\,dy = 0.4\,[y]_0^1 + 0.6 \times [\frac{y^2}{2}]_0^1$

$\therefore U = 0.4 + \frac{0.6}{2} = 0.7$ m/s

Now, $\alpha = \frac{1}{U^3 h}\int_0^h u^3\,dy = \frac{1}{(0.7)^3 \cdot 1}\int_0^1 (0.4 + 0.6\frac{y}{1})^3\,dy = \frac{1}{0.343} \times [0.406]$

$\therefore \alpha = 1.18$

And, $\beta = \frac{1}{U^2 h}\int_0^h u^2\,dy = \frac{1}{(0.7)^2 \cdot 1}\int_0^1 (0.4 + 0.6\frac{y}{1})^2\,dy = \frac{1}{0.343} \times [0.52]$

$\therefore \beta = 1.06$

(Ans)

13 ## Pressure Distribution of parallel flow, convex flow, concave flow in a channel section:

The pressure at any point on the cross section of flow in a channel of small slope can be measured by the height of water column in a piezometer tube installed at that point. Ignoring minor disturbances due to turbulence, it is apparent that this water column should rise from the point of measurement up to the hydraulic grade line or the water surface. Therefore the pressure at any point on the section is directly proportional to the depth of the point below the free surface and equal to the hydrostatic pressure corresponding to this depth. In other words, the distribution of pressure over cross section of the channel is same as the distribution of hydrostatic pressure; That is, the distribution is linear and can be represented by a straight line. This is known as hydrostatic law of pressure distribution.

The application of this law to the pressure distribution in a cross section of a flowing channel is valid, if the flow filaments have no acceleration components in the plane of section. This type of flow is theoretically known as parallel flow. That is, the streamlines have neither substantial curvature nor divergence.

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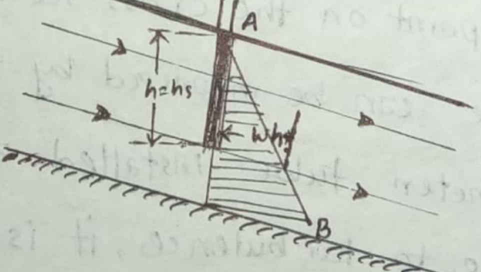


Fig. Pressure distribution in parallel flow

In actual problems, uniform flow is practically parallel flow. Gradully varied flow may also be regarded as parallel flow.

If the curvature of streamline is substantial, the flow is theoretically known as curvilinear flow. The effect of curvature is to produce appreciable acceleration components or centrifugal force normal to the direction of flow. Thus the pressure distribution over the section deviates from hydrostatic if curvilinear flow occurs in the vertical plane. Such curvilinear flow may be convex or concave.

In $\boxed{\text{concave}}$ flow the centrifugal forces are pointing downward to reinforce the gravity action; so the resulting pressure is greater than the other wise hydrostatic pressure of a parallel flow.

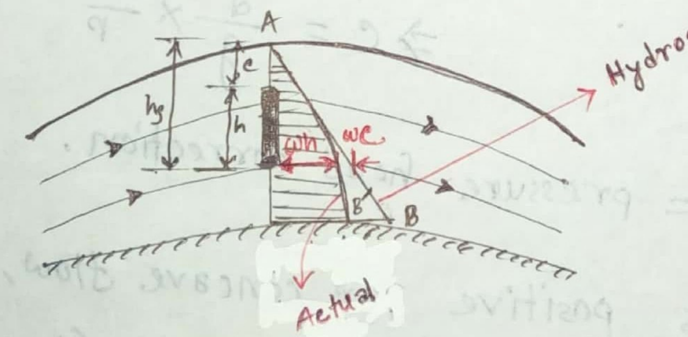


Fig. Pressure distribution in convex flow

In $\boxed{\text{convex}}$ flow, the centrifugal forces are acting upward against the gravity action. consequently the resulting pressure is less than the other wise hydrostatic pressure of a parallel flow.

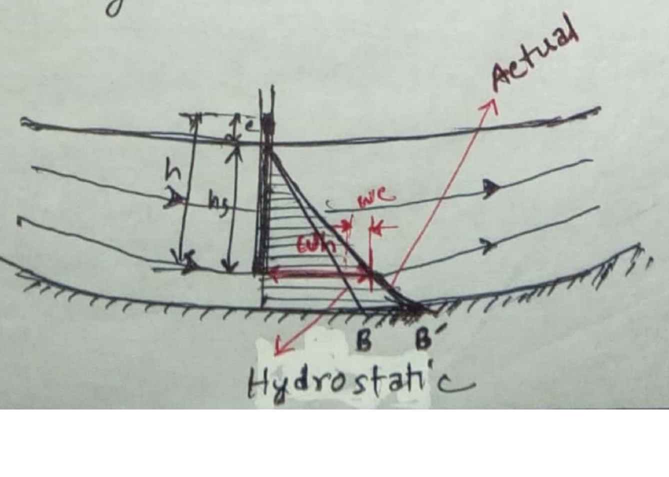


Fig. Pressure distribution in concave flow

Let, deviation from hydrostatic pressure $= c$

centrifugal acceleration $= \frac{v^2}{r}$

gravitational acceleration $= g$

unit weight of water $= w$

velocity of flow $= v$

radius of curvature $= r$

depth of flow $= d$

According to Newton's law of acceleration,

$$\text{centrifugal pressure, } P = \frac{wd}{g} \times \frac{v^2}{r} \quad (\text{cross section, } a = 1 ft^2)$$

$$\Rightarrow \frac{P}{w} = \frac{d}{g} \times \frac{v^2}{r}$$

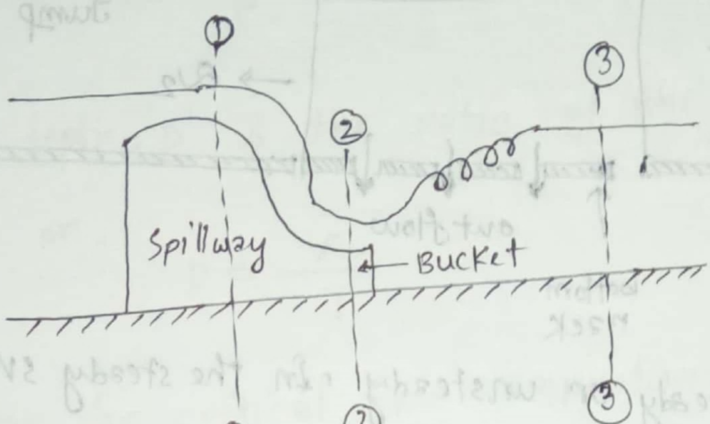
$$\Rightarrow c = \frac{d}{g} \times \frac{v^2}{r} \quad \left[c = \frac{P}{w}\right]$$

where, c = pressure head correction.

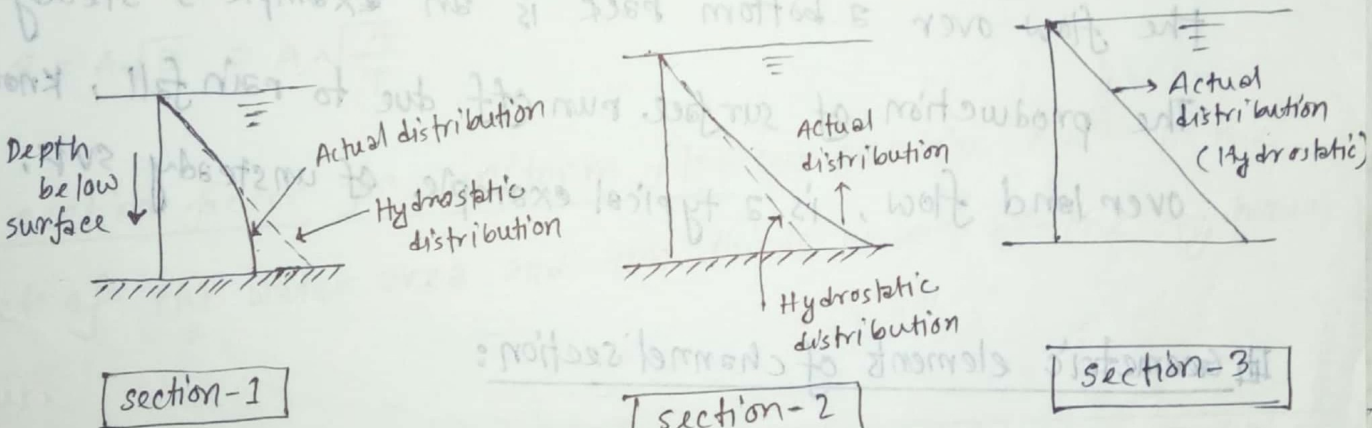
Apparently, c is positive for concave flow, negative for convex flow, and zero for parallel flow.

5.

Draw the pressure distribution diagram of the following flow over spillway in different sections as shown in figure:

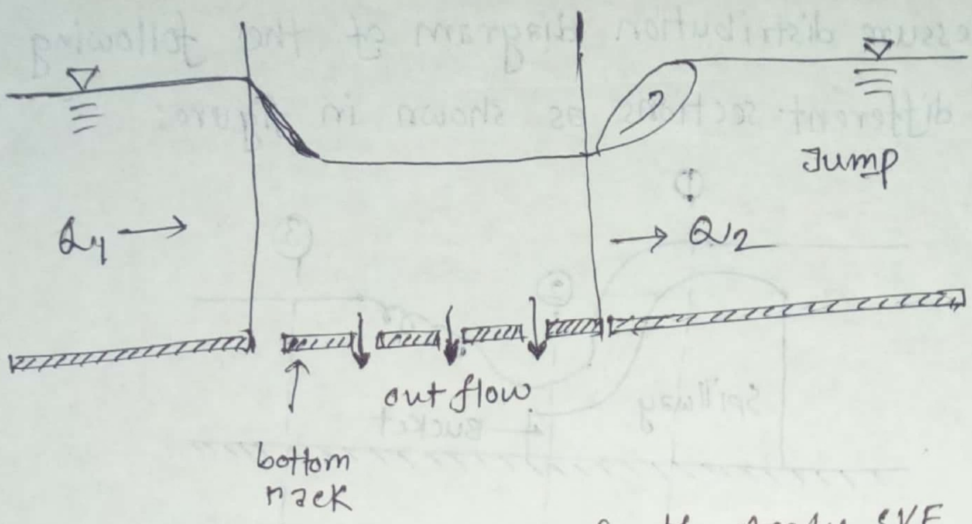


Answer:



Spatially varied flow: 12

Varied flow classified as GVF and RVF assumes that no flow is externally added to or taken out of the channel system. The volume of water in a known time interval is conserved in the channel system. In steady varied flow the discharge is constant at all sections. How ever if some flow is added to or abstruct from the system the resulting varied flow is known as a spatially varied flow.



SVF can be steady or unsteady. In the steady SVF, the discharge while being steady varies ~~flow~~ along the channel length. The flow over a bottom rack is an example of steady SVF. The production of surface run off due to rain fall, known as over land flow, is a typical example of unsteady SVF.

Geometric elements of channel section:

* The depth of flow, 'y' is the vertical distance of the lowest point of a channel section from free surface.

* The top width, 'T' is the width of channel section at the free surface.

* The water area, 'A' is the cross-sectional area of the flow normal to the direction of flow.

* The wetted perimeter, 'P' is the length of the line of intersection of the channel wetted surface with a cross sectional plan normal to the direction of flow.

* The hydraulic radius, 'R' is the ratio of the water area to its wetted perimeter, or

$$R = \frac{A}{P}$$

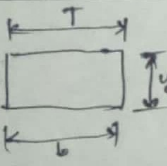
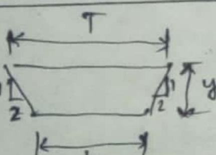
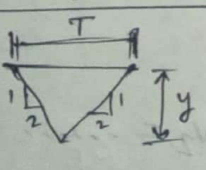
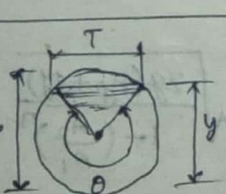
* The hydraulic depth, 'D' is the ratio of the water area to the top width or,

$$D = \frac{A}{T}$$

* The section factor for critical flow computation Z is the product of the water area and the square root of the hydraulic depth

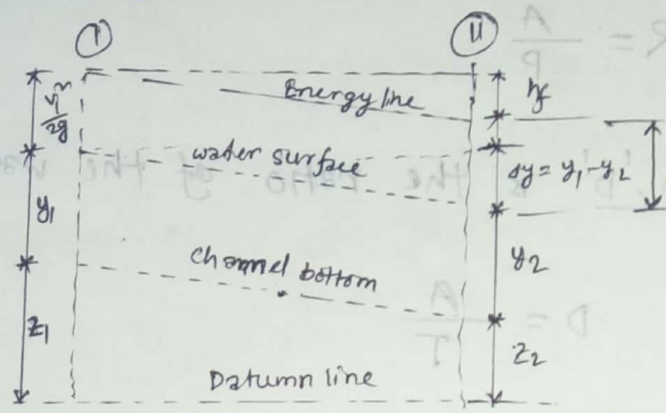
or, $Z = A\sqrt{D} = A\sqrt{\frac{A}{T}}$

* The section factor for uniform flow computation $AR^{\frac{2}{3}}$ is the product of the water area and two thirds power of the hydraulic radius.

Section	Area A	Wetted perimeter P	Hydraulic Radius R	Top width T	Hydraulic Depth D	Section factor Z
Rectangle 	by	$b+2y$	$\frac{by}{b+2y}$	b	y	$by^{1.5}$
Trapezoidal 	$(b+zy)y$	$b+2y\sqrt{1+z^2}$	$\frac{(b+zy)y}{b+2y\sqrt{1+z^2}}$	$b+2zy$	$\frac{(b+zy)y}{b+2zy}$	$\frac{[(b+zy)y]^{1.5}}{\sqrt{b+2zy}}$
Triangle 	zy^2	$2y\sqrt{1+z^2}$	$\frac{zy}{2\sqrt{1+z^2}}$	$2zy$	$\frac{1}{2}y$	$\frac{\sqrt{2}}{2}zy^{2.5}$
Circle 	$\frac{1}{8}(\theta-\sin\theta)d_0^2$	$\frac{1}{2}\theta d_0$	$\frac{1}{4}\left(1-\frac{\sin\theta}{\theta}\right)d_0$	$(\sin\frac{1}{2}\theta)d_0$ or $2\sqrt{y(d_0-y)}$	$\frac{1}{8}\left(\frac{\theta-\sin\theta}{\sin\frac{1}{2}\theta}\right)d_0$	$\frac{\sqrt{2}(\theta-\sin\theta)^{1.5}}{32(\sin\frac{1}{2}\theta)^{0.5}}d_0^{2.5}$

Theoretical discharge of open channel flow: $Q = A_2\sqrt{\frac{2g(\Delta y - h_f)}{1-\left(\frac{A_2}{A_1}\right)^2}}$

Let us consider the figure:



Total energy at section (I), $E_1 = z_1 + y_1 + \frac{v_1^2}{2g}$ ...... (I)

Total energy at section (II), $E_2 = z_2 + y_2 + \frac{v_2^2}{2g} + h_f$ ...... (II)

Equating, $E_1 = E_2$

$$z_1 + y_1 + \frac{v_1^2}{2g} = z_2 + y_2 + \frac{v_2^2}{2g} + h_f$$

$$\Rightarrow z_1 - z_2 + y_1 - y_2 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} + h_f$$

For, small channel slope, $z_1 - z_2 = 0$ &

From figure, $y_1 - y_2 = \Delta y$ $\quad \therefore \Delta y - h_f = \frac{v_2^2 - v_1^2}{2g}$

$$\Rightarrow v_1 = \sqrt{v_2^2 + 2g(h_f - \Delta y)}$$

From continuity equation,

$v_1 A_1 = v_2 A_2 \Rightarrow v_1 = \frac{v_2 A_2}{A_1}$ ...... (IV)

Equating (III) & (IV) we get,

$$v_2^2 + 2g(h_f - \Delta y) = \frac{v_2^2 A_2^2}{A_1^2}$$

$$\Rightarrow 2g(h_f - \Delta y) = v_2^2\left(\frac{A_2^2}{A_1^2} - 1\right)$$

$$\Rightarrow v_2 = \sqrt{\frac{2g(h_f + \Delta y)}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

discharge,

$Q = A_1 v_1 = A_2 v_2 = \ldots\ldots$

$$\therefore Q = A_2 v_2 = A_2\sqrt{\frac{2g(h_f + \Delta y)}{1-\left(\frac{A_2}{A_1}\right)^2}}$$

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ENERGY & MOMENTUM PRINCIPLES

Energy in Open channel Flow:

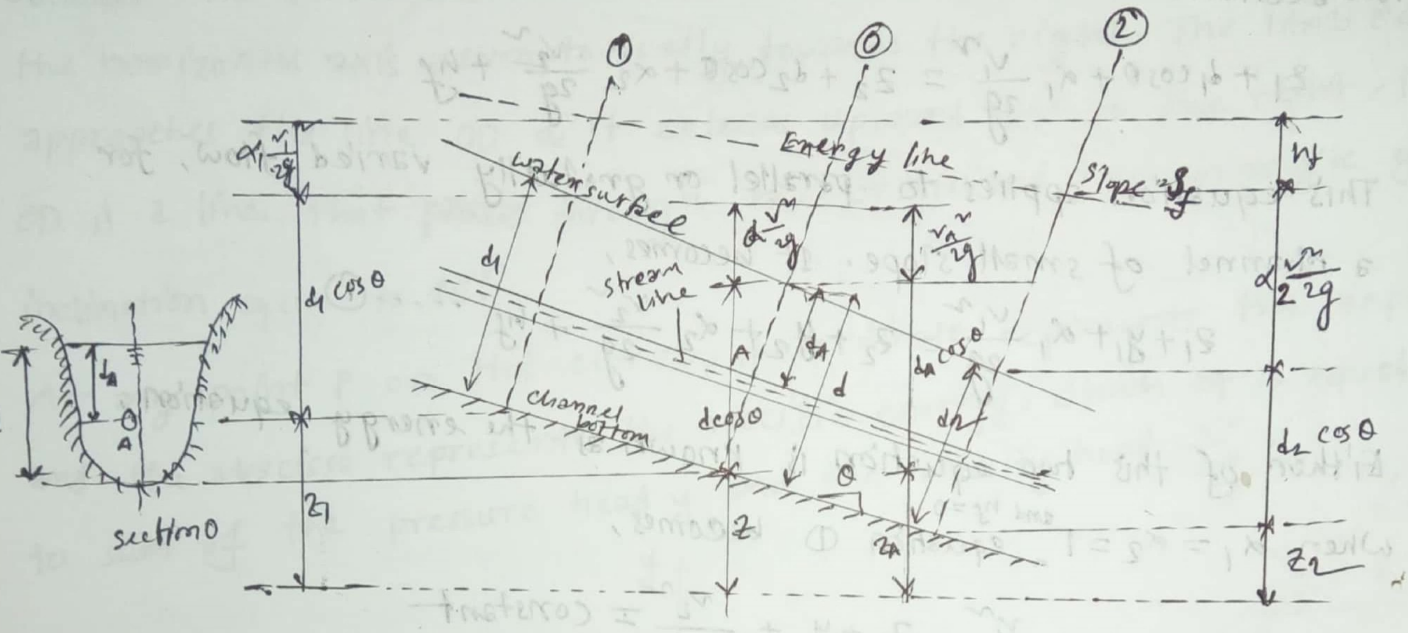

Fig. Energy in gradually varied open channel flow.

with respect to the datum plane, the total head H at a section 0 may be written as,

$$H = Z_A + d_A \cos\theta + \frac{v_A^2}{2g}$$

where, Z_A is the elevation of point A above the datum plane, d_A is the depth of point A, θ is the slope angle, $\frac{v_A^2}{2g}$ is the velocity head.

Total energy at the channel section is,

$$H = Z + d\cos\theta + \alpha \cdot \frac{v^2}{2g}$$

For channel of small slope, $\theta = 0$. Thus the total energy at the channel at channel section is,

$$H = Z + d + \alpha \cdot \frac{v^2}{2g}$$

According to the principle of conservation of energy, the energy head at the upstream section should be equal to the total energy head at the downstream section 2 + the loss of energy 'h_f' between two section.

$$z_1 + d_1 \cos\theta + \alpha_1 \frac{v_1^2}{2g} = z_2 + d_2 \cos\theta + \alpha_2 \frac{v_2^2}{2g} + h_f$$

This equation applies to parallel or gradually varied flow, for a channel of small slope. It becomes,

$$z_1 + y_1 + \alpha_1 \frac{v_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{v_2^2}{2g} + h_f \quad \cdots\cdots (1)$$

Either of this two equation is known as the energy equation.

When $\alpha_1 = \alpha_2 = 1$ and $h_f = 0$ equation (1) becomes,

$$z_1 + y_1 + \frac{v_1^2}{2g} = z_2 + y_2 + \frac{v_2^2}{2g} = \text{constant}$$

This is known as Bernoulli equation.

00, 17, 16

Specific Energy:

Specific energy in a channel section is defined as the energy per pound of water at any section of a channel measured with respect to the channel bottom.

we know,

$$H = z + d\cos\theta + \alpha \frac{v^2}{2g}$$

if $z = 0$, it becomes, $E = d\cos\theta + \alpha \frac{v^2}{2g}$

or, for a channel of small slope and $\alpha = 1$,

$E = y + \frac{v^2}{2g}$; which indicates that the specific energy is equal to the sum of depth of water and the velocity head.

According to the principle of conservation of energy, the energy head at the upstream section should be equal to the total energy head at the downstream section2 + the loss of energy 'h_f' between two section.

$$z_1 + d_1\cos\theta + \alpha_1 \frac{v_1^2}{2g} = z_2 + d_2\cos\theta + \alpha_2 \frac{v_2^2}{2g} + h_f$$

This equation applies to parallel or gradually varied flow, for a channel of small slope. It becomes,

$$z_1 + y_1 + \alpha_1 \frac{v_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{v_2^2}{2g} + h_f \quad \cdots\cdots (1)$$

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Specific energy curve: 2007, 2010, 14, 16, 17

When the depth of flow is plotted against the specific energy for a given channel section and discharge, a specific-energy curve is obtained. The curves has two limbs AC and BC. The limb AC approaches the horizontal axis asymptotically towards the right. The limb BC approaches the line OD as it extends upward and to the right, line OD is a line that passes through the origin and has an angle of inclination equal to 45°.

At any point P on this curve, the ordinate represents the depth and the abscissa represents the specific energy, which ~~eq~~ is equal to sum of the pressure head y and the velocity head $\frac{v^2}{2g}$.

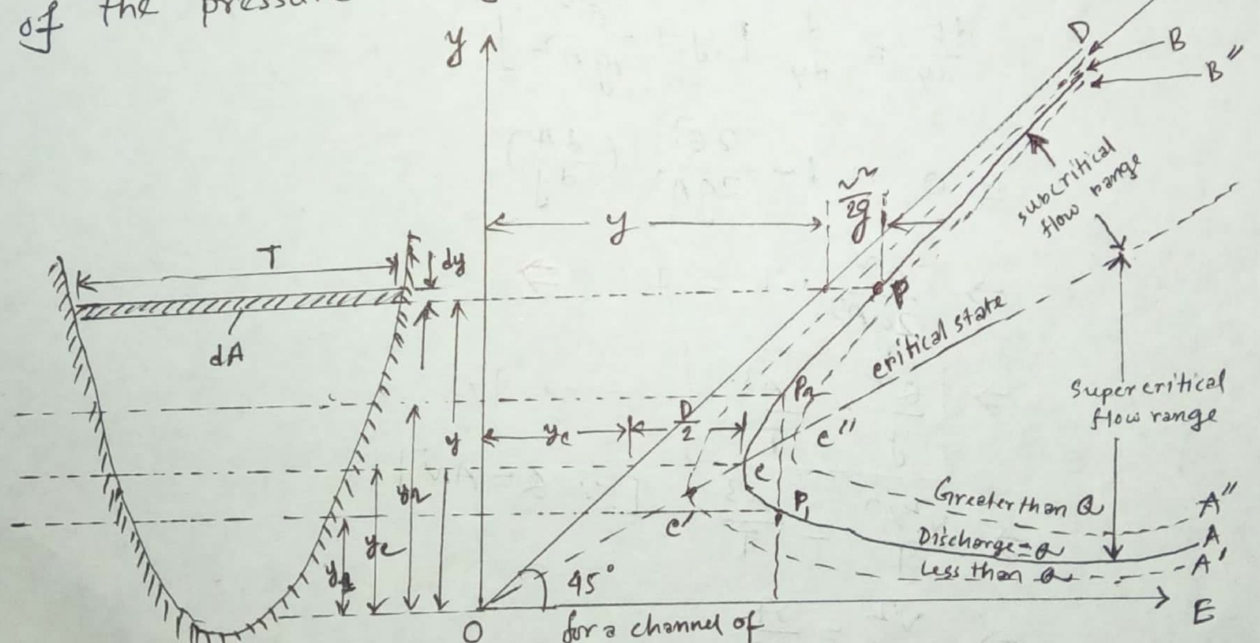
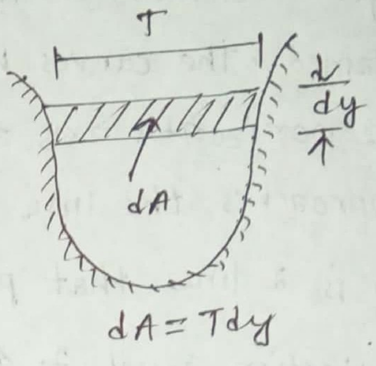


Fig. specific energy curve

The curve shows that for a given specific energy there are two possible depth, for instance, the low stage y_1 and the high stage y_2. The low stage is called the alternate depth of the high stage and vice versa. At the critical state the two alternate depths apparently become one, which is known as the critical depth y_c.

Show that at the critical state of flow the specific energy is a minimum for the given discharge. 03, 14, 16.

Let us, consider a flow in critical state as shown in figure:



$dA = T\,dy$

$\therefore \frac{dA}{dy} = T$

We know,

specific energy,

$$E = y + \frac{v^2}{2g}$$

$$= y + \frac{Q^2}{2gA^2} \quad [\because V = \frac{Q}{A}]$$

For a given discharge, condition for maximum specific energy, $\frac{dE}{dy} = 0$

$$\frac{dE}{dy} = \frac{d}{dy}\left[y + \frac{Q^2}{2gA^2}\right]$$

$$\Rightarrow 0 = 1 - \frac{2Q^2}{2gA^3}\left(\frac{dA}{dy}\right)$$

$$\Rightarrow \frac{Q^2}{gA^3} \cdot T = 1$$

$$\Rightarrow \boxed{\frac{Q^2}{g} = \frac{A^3}{T}} \longrightarrow 17$$

$$\Rightarrow \frac{V^2A^2}{g} = \frac{A^3}{T} \quad [\because Q = AV]$$

$$\Rightarrow \frac{V^2}{g} = \frac{A}{T}$$

$$[\because D = \frac{A}{T}]$$

$$\Rightarrow \frac{V^2}{g} = D \longrightarrow \boxed{\frac{V^2}{2g} = \frac{D}{2}}$$ This is the criterion for critical flow.

$$\Rightarrow V^2 = gD$$

$$\Rightarrow V = \sqrt{gD}$$

$$\Rightarrow \frac{V}{\sqrt{gD}} = 1$$

(13) $\boxed{\therefore Fr = 1}$ which is the definition of critical flow.

Hence, at the critical state of flow the specific energy is a minimum for a given discharge.

⊞ Prove that, Froude Number for critical state of flow, $F = \dfrac{V}{\sqrt{gD\cos\theta/\alpha}}$

$\boxed{\text{First}}$ Derive $\boxed{\dfrac{V^2}{2g} = \dfrac{D}{2}}$ Go (previous question)(↑) 04, 06, 14

$\boxed{\text{Then}}$ If the energy co-efficient is not assumed to be unity, the critical flow criterion is,

$$\alpha \frac{V^2}{2g} = \frac{D}{2}$$

For a channel of large slope angle θ and energy co-efficient α,

$$\alpha \frac{V^2}{2g} = \frac{D\cos\theta}{2}$$

where, D is the hydraulic depth of water area normal to the channel bottom

$$\frac{V^2}{gD\cos\theta/\alpha} = 1$$

$$\Rightarrow \frac{V}{\sqrt{gD\cos\theta/\alpha}} = 1 \quad , \text{ At critical state, } F_r = 1$$

Hence,

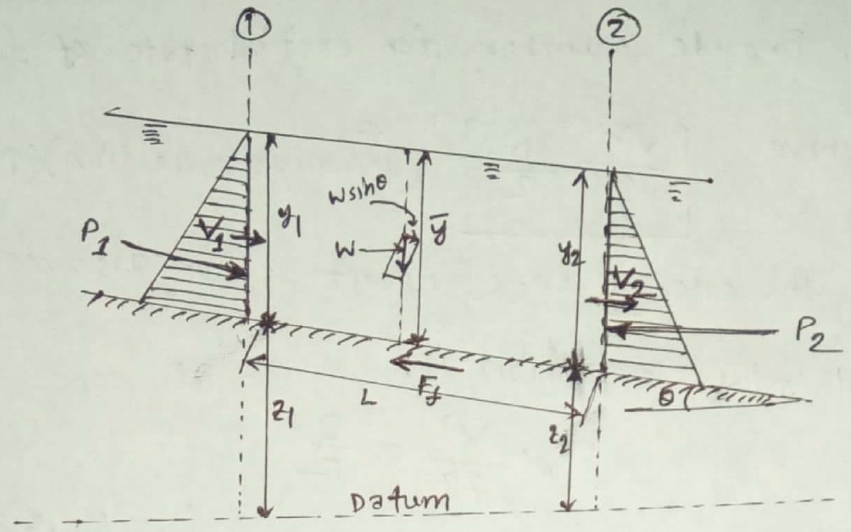
Froude Number may be defined as,

$$F = \frac{V}{\sqrt{gD\cos\theta/\alpha}}$$

⊞ Momentum in open channel flow:

According to Newton's second law of motion, the change of momentum per unit time in the body of water in a flowing channel is equal to the resultant of all the external forces that acting on the body.

Fig.
Application of Momentum principle.



Applying the principle to a channel of ~~ch~~ large slope,
The following expression for the momentum change per unit time in the body of water enclosed between section ① an ② may be written;

$$\frac{Qw}{g}\left(B_2 V_2 - B_1 V_1\right) = P_1 - P_2 + W\sin\theta - F_f$$

where, Q = discharge in cfs.
w = unit weight of water in lb/ft^3
V = mean velocity in fps.
P_1 and P_2 = resultants of pressures acting on the two section
W = weight of water enclosed between the sections.
F_f = Total external force of friction and resistance acting along the surface of contact between water and the channel.

This above equation is known as the momentum equation.

Show that, the momentum equation is similar to the energy equation when applied to certain flow problems,

In this case, gradually varied flow is considered; Accordingly the pressure distribution in the sections may be assumed hydrostatic. and $B' = 1$. Also the slope of the channel is assumed relatively small.

Thus, in the short reach of a rectangular channel of small slope and width b.

$$P_1 = \frac{1}{2} w b y_1^2$$

$$\text{and, } P_2 = \frac{1}{2} w b y_2^2$$

Assume, $F_f = w h_f' b \bar{y}$ where, h_f' = friction head

$\bar{y}$ = average depth or $\frac{y_1 + y_2}{2}$

The discharge through the reach,

$$Q = \frac{1}{2} (V_1 + V_2) b \bar{y}$$

Also, it is evident that the weight of body of water is,

$$W = w b \bar{y} L$$

$$\text{and, } \sin\theta = \frac{z_1 - z_2}{L}$$

substituting all above expression in momentum equation and simplifying,

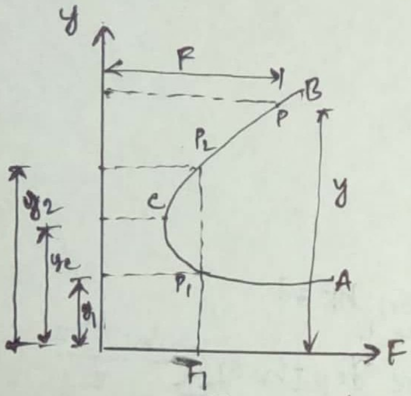
$$z_1 + y_1 + B_1 \frac{V_1^2}{2g} = z_2 + y_2 + B_2 \frac{V_2^2}{2g} + h_f'$$

This equation appears to be practically the same as the energy equation.

Specific force: In applying the momentum principle to a short horizontal reach of a prismatic channel, The external force of friction and weight effect of water can be ignored.

Thus, $\theta = 0$, $F_f = 0$ and Assuming also $\beta_1 = \beta_2 = 1$

Momentum equation becomes,



$$\frac{Qw}{g}(v_2 - v_1) = P_1 - P_2$$

$$\Rightarrow \frac{Qw}{g}(v_2 - v_1) = w\bar{z}_1 A_1 - w\bar{z}_2 A_2$$

$$\Rightarrow \frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \quad \left[\because V = \frac{Q}{A}\right]$$

Two side of this equation are analogous. Hence It may be expressed by a general equation,

$$F = \frac{Q^2}{gA} + \bar{z}A$$

This is called specific force. This function consists of two terms. (i) momentum of the flow passing through the channel section per unit time per unit weight of water. (ii) force per unit weight of water.

Show that At critical state of flow the specific force is a minimum for a given discharge.

We know,
$$F = \frac{Q^2}{gA} + \bar{z}A$$

$$\Rightarrow \frac{dF}{dy} = -\frac{Q^2}{gA^2} \cdot \frac{dA}{dy} + \frac{d}{dy}(\bar{z}A)$$

For minimum specific force $\frac{dF}{dy} = 0$

$$\therefore -\frac{Q^2}{gA^2} \cdot \frac{dA}{dy} + \frac{d}{dy}(\bar{z}A) = 0$$

For a change dy in the depth, the corresponding change $d(\bar{z}A)$ in the static moment of water area about the free surface is equal to, $[A(\bar{z}+dy) + T(dy)^2/2 - \bar{z}A]$

Ignoring differential higher degree, that is $(dy)^2 \simeq 0$ (assuming)

Hence, $d(\bar{z}A) = A\,dy$

Now, $\frac{dF}{dy} = -\frac{Q^2 dA}{gA^2 \cdot dy} + A = 0$

$$\Rightarrow \frac{Q^2}{gA^2} \cdot T = A \qquad \left[\because \frac{dA}{dy} = T\right]$$

$$\Rightarrow \frac{V^2}{g} = \frac{A}{T} \qquad [\because Q = AV]$$

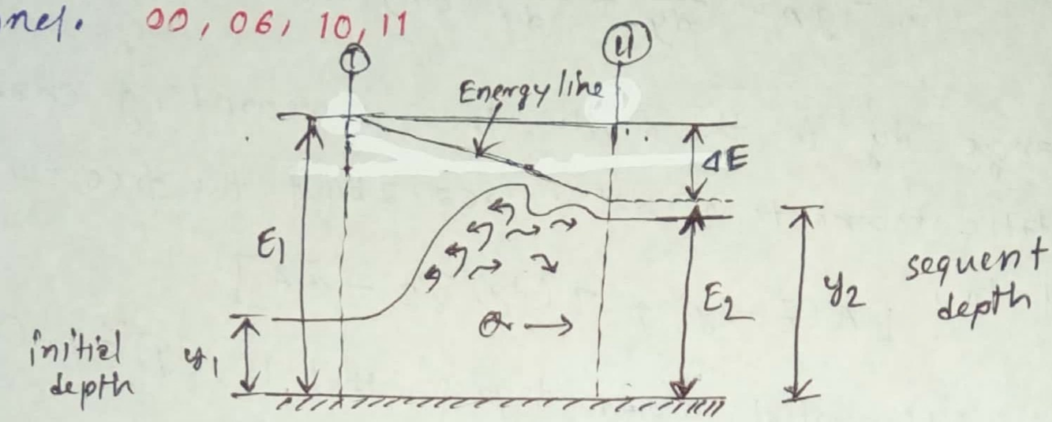
$$\Rightarrow \frac{V^2}{g} = D \qquad \left[\because \frac{A}{T} = D\right]$$

$$\Rightarrow \frac{V^2}{2g} = \frac{D}{2}$$

which is the criterion for critical state of flow.

Therefore is proved that the specific force is a minimum for the given discharge at critical state of flow.

☐ Derive the relationship between the initial depth and the sequent depth of hydraulic jump on a horizontal floor in a rectangular channel. 00, 06, 10, 11



In the hydraulic jump on a horizontal floor,

The external forces of friction and the weight effect of water are negligible.

The specific forces of section I & II respectively, before and after the jump can be considered equal. That is,

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$$

For a rectangular channel of width b, $Q = V_1 A_1 = V_2 A_2$,

$A_1 = by_1$, $A_2 = by_2$, $\bar{z}_1 = \frac{y_1}{2}$ and $\bar{z}_2 = \frac{y_2}{2}$, and $F_1 = \frac{V_1}{\sqrt{gy_1}}$

Substituting these relations in above equation and simplifying,

$$\left(\frac{y_2}{y_1}\right)^3 - (2F_1^2 + 1)\left(\frac{y_2}{y_1}\right) + 2F_1^2 = 0$$

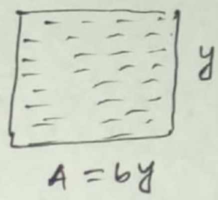
Factoring, $\left[\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2F_1^2\right]\left(\frac{y_2}{y_1} - 1\right) = 0$

Then, let $\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2F_1^2 = 0$

The solution of this quadratic equation is,

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1 + 8F_1^2} - 1\right)$$

▣ Show that at critical state of flow the specific energy head in a rectangular channel is equal to 1.5 times the depth of flow, assuming zero slope and $\alpha = I$. 02



We know, $E = y + \frac{v^2}{2g}$

$\Rightarrow E = y + \frac{Q^2}{2gA^2}$ $[\because Q = AV]$

$\Rightarrow Q = A\sqrt{2g(E-y)}$

If Q is the discharge per unit width of the channel, then $A = (1 \times y) = y$

$\therefore Q = y\sqrt{2g(E-y)}$

$\Rightarrow \frac{dQ}{dy} = \sqrt{2g}\frac{d}{dy}[y\sqrt{E-y}]$

$\Rightarrow \frac{dQ}{dy} = \sqrt{2g}\left[\sqrt{E-y} - \frac{y}{2\sqrt{E-y}}\right]$

For critical state of flow, $\frac{dQ}{dy} = 0$ and $y = y_c$

$\sqrt{E-y_c} = \frac{y_c}{2\sqrt{E-y_c}}$

$\Rightarrow 2(E-y_c) = y_c$

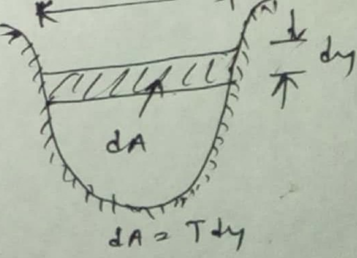
$\Rightarrow 2E = 3y_c$

$\therefore E = 1.5\, y_c$

(Showed)

▣ Prove that the discharge in a given channel section is a maximum for a given specific energy at the critical state of flow. 08

Let us, consider a flow in critical state as shown in figure:



specific energy, $E = y + \frac{v^2}{2g}$

$\Rightarrow E = y + \frac{Q^2}{2gA^2}$ ...① $[\because Q = AV]$

$\Rightarrow Q = A\sqrt{2g(E-y)}$

$dA = T\,dy$

$\therefore \frac{dA}{dy} = T$

For maximum discharge, $\frac{dQ}{dy} = 0$

$$\frac{dQ}{dy} = \sqrt{2g}\left[\frac{d}{dy}\left(A\sqrt{E-y}\right)\right]$$

$$\frac{dQ}{dy} = \sqrt{2g}\left(\frac{dA}{dy}\cdot\sqrt{E-y} - \frac{A}{2\sqrt{E-y}}\right) = 0$$

$$\therefore T\sqrt{E-y} = \frac{A}{2\sqrt{E-y}} \qquad \left[\because \frac{dA}{dy} = T\right]$$

$$\Rightarrow 2T(E-y) = A$$

$$\Rightarrow E = \frac{1}{2}\frac{A}{T} + y \quad \cdots\cdots (11)$$

From (1) & (11), we obtain, $\frac{1}{2}\cdot\frac{A}{T} + y = y + \frac{Q^2}{2gA^2}$

$$\Rightarrow \frac{D}{2} = \frac{v^2}{2g}$$

$$\therefore \frac{v^2}{2g} = \frac{D}{2}$$

which is the criterion for critical state of flow.

Hence, At critical state of flow, the discharge is a maximum for a given specific energy.

Show that the relation between two alternative depth y_1 and y_2 in a rectangular channel can be expressed as $\frac{2y_1^2 y_2^2}{y_1+y_2} = y_c^3$

We know, For critical state of flow, $F = \frac{V_c}{\sqrt{gD}} = 1$

$$\therefore V_c = \sqrt{gD}$$

$$\Rightarrow V_c^2 = gD$$

$$\Rightarrow \frac{Q^2}{A_c^2} = g y_c \quad [\because D = y_c]$$

$$\Rightarrow \frac{Q^2}{b^2 y_c^2} = g y_c \qquad [\because A = b y_c]$$

$$\Rightarrow y_c^3 = \frac{Q^2}{b^2 g} \quad \ldots\ldots (1)$$

For same specific enery,

$$E = y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g}$$

$$\Rightarrow y_1 - y_2 = \frac{v_2^2 - v_1^2}{2g}$$

$$\Rightarrow y_1 - y_2 = \frac{1}{2g}\left[\left(\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_1}\right)^2\right]$$

$$\Rightarrow y_1 - y_2 = \frac{Q^2}{2g}\left[\frac{1}{b^2 y_2^2} - \frac{1}{b^2 y_1^2}\right]$$

$$\Rightarrow y_1 - y_2 = \frac{Q^2}{2g b^2}\left(\frac{y_1^2 - y_2^2}{y_2^2 y_1^2}\right)$$

$$\Rightarrow y_1 - y_2 = \frac{y_c^3}{2} \times \frac{(y_1 + y_2)(y_1 - y_2)}{y_2^2 y_1^2}$$

$$\therefore y_c^3 = \frac{2 y_1^2 y_2^2}{y_1 + y_2}$$

(showed)

Applying the momentum principal equation and continuity equation to the analysis of a submerged hydraulic jump which occurs at the sluice outlet in a rectangular channel, prove that 2010

$$\frac{y_s}{y_1} = \sqrt{1 + 2F_2^2\left(1 - \frac{y_2}{y_1}\right)}$$

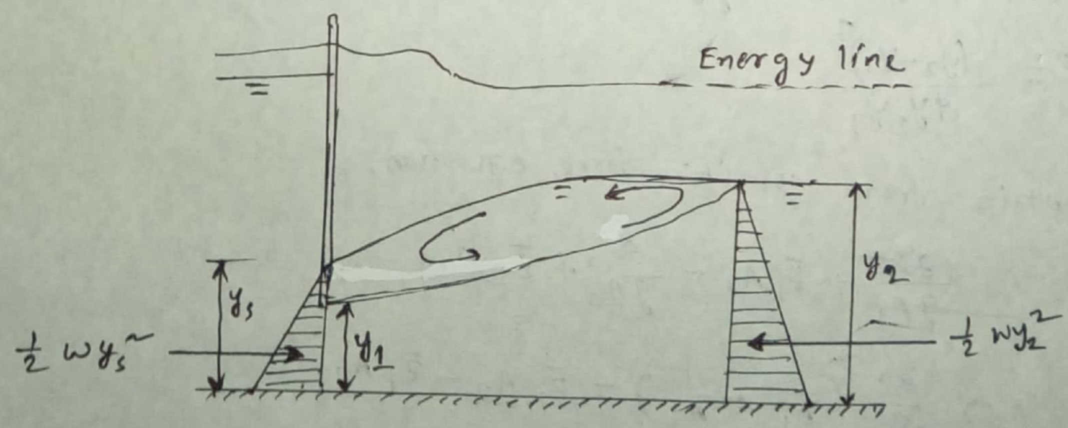


Fig. A submerged hydraulic jump at sluice outlet considering, θ = small, $\beta_1 = \beta_2 = 1$ and $F_f = 1$

we obtain from momentum equation

$$\frac{wQ}{g}(V_2 - V_1) = P_s - P_1$$

$$\Rightarrow \frac{wQ}{g}(V_2 - V_1) = \frac{1}{2} w y_s^2 - \frac{1}{2} w y_2^2$$

$$\Rightarrow \frac{wQ}{g}\left(\frac{Q}{A_2} - \frac{Q}{A_1}\right) = \frac{1}{2} w (y_s^2 - y_2^2)$$

$$\Rightarrow \frac{wQ^2}{g}\left(\frac{1}{by_2} - \frac{1}{by_1}\right) = \frac{1}{2} w (y_s^2 - y_2^2)$$

$$\Rightarrow \frac{2Q^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_1}\right) = y_s^2 - y_2^2 \quad [\text{considering } b = 1]$$

$$\Rightarrow \frac{2Q^2}{y_2^2 g}\left(\frac{1}{y_2} - \frac{1}{y_1}\right) = \left(\frac{y_s}{y_2}\right)^2 - 1$$

$$\Rightarrow \frac{2Q^2}{y_2^3 g}\left(1 - \frac{y_2}{y_1}\right) = \left(\frac{y_s}{y_2}\right)^2 - 1$$

$$\left[\because \text{Specific force}, F_2^2 = \frac{Q^2}{g y_2^3}\right]$$

$$\Rightarrow 2F_2^2\left(1 - \frac{y_2}{y_1}\right) = \left(\frac{y_s}{y_2}\right)^2 - 1$$

$$\Rightarrow \left(\frac{y_s}{y_2}\right)^2 = 1 + 2F_2^2\left(1 - \frac{y_2}{y_1}\right)$$

$$\therefore \frac{y_s}{y_2} = \sqrt{1 + 2F_2^2\left(1 - \frac{y_2}{y_1}\right)}$$

(Proved)

⊞ Prove that the energy loss in a horizontal hydraulic jump is

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_2 y_1}$$

we obtain from specific force equation,

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2$$

$$\Rightarrow \frac{Q^2}{g}\left(\frac{1}{A_1} - \frac{1}{A_2}\right) = \bar{z}_2 A_2 - \bar{z}_1 A_1$$

$$\Rightarrow \frac{Q^2}{g}\left(\frac{1}{y_1} - \frac{1}{y_2}\right) = \frac{1}{2} y_2^2 - \frac{1}{2} y_1^2 \quad \left[\text{considering}, A = y \,\&\, z = \frac{y}{2}\right]$$

$$\Rightarrow \frac{Q^2}{g}\left(\frac{y_2-y_1}{y_1y_2}\right) = \frac{1}{2}(y_2^2-y_1^2)$$

$$\Rightarrow \frac{Q^2}{g} = \frac{1}{2}y_1y_2 \times \frac{(y_2+y_1)(y_2-y_1)}{y_2-y_1}$$

$$\Rightarrow \frac{Q^2}{2g} = \frac{1}{4}y_1y_2(y_1+y_2) \quad \ldots\ldots (1)$$

Now, Loss of energy, $\Delta E = E_1 - E_2$

$$= \left(y_1 + \frac{Q^2}{2gA_1^2}\right) - \left(y_2 + \frac{Q^2}{2gA_2^2}\right)$$

$$= (y_1-y_2) + \frac{Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right)$$

$$= (y_1-y_2) + \frac{Q^2}{2g}\left(\frac{1}{y_1^2} - \frac{1}{y_2^2}\right)$$

$$= (y_1-y_2) + \frac{1}{4}y_1y_2\cdot(y_1+y_2) \times \frac{(y_2^2-y_1^2)}{y_1^2y_2^2} \quad \text{[From eq}^n\text{ (1)]}$$

$$= \frac{4y_1y_2(y_1-y_2) + (y_1+y_2)^2(y_2-y_1)}{4y_1y_2}$$

$$= \frac{(y_2-y_1)\left[(y_1+y_2)^2 - 4y_1y_2\right]}{4y_1y_2}$$

$$= \frac{(y_2-y_1)\times(y_2-y_1)^2}{4y_1y_2}$$

$$\therefore \Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$$

(Proved)

Show that, the head loss in a hydraulic jump formed in a rectangular channel may be expressed as, 2009

$$\Delta E = \frac{(v_1 - v_2)^3}{2g(v_1 + v_2)}$$

First Derive Eqn ① (Go previous question) (↑)

Then,
$$\frac{Q^2}{2g} = \frac{1}{4} y_1 y_2 (y_1 + y_2)$$

$$\Rightarrow \frac{Q^2}{2g} = \frac{1}{4} \cdot \frac{Q}{v_1} \cdot \frac{Q}{v_2} \cdot \left(\frac{Q}{v_1} + \frac{Q}{v_2}\right) \qquad \left[\text{considering, } y = \frac{Q}{v},\ b = 1\right]$$

$$\Rightarrow \frac{Q^2}{2g} = \frac{1}{4} \cdot \frac{Q^3}{v_1 v_2} \left(\frac{v_2 + v_1}{v_1 v_2}\right)$$

$$\Rightarrow Q = \frac{2 v_1^2 v_2^2}{g(v_1 + v_2)} \quad \cdots\cdots ①$$

Now,
$$\Delta E = \left(y_1 + \frac{v_1^2}{2g}\right) - \left(y_2 + \frac{v_2^2}{2g}\right)$$

$$= (y_1 - y_2) + \frac{v_1^2 - v_2^2}{2g}$$

$$= \frac{Q}{v_1} - \frac{Q}{v_2} + \frac{v_1^2 - v_2^2}{2g}$$

$$= Q\left(\frac{v_2 - v_1}{v_1 v_2}\right) + \frac{v_1^2 - v_2^2}{2g}$$

$$= \frac{2 v_1^2 v_2^2}{g(v_1 + v_2)} \times \frac{(v_2 - v_1)}{v_1 v_2} + \frac{v_1^2 - v_2^2}{2g} \qquad [\text{From eq}^n\text{-}①]$$

$$= \frac{(v_1 - v_2)}{2g}\left[(v_1 + v_2) - \frac{4 v_1 v_2}{v_1 + v_2}\right]$$

$$= \frac{(v_1 - v_2)\left[(v_1 + v_2)^2 - 4 v_1 v_2\right]}{2g(v_1 + v_2)}$$

$$\therefore \Delta E = \frac{(v_1 - v_2) \times (v_1 - v_2)^2}{2g(v_1 + v_2)} = \frac{(v_1 - v_2)^3}{2g(v_1 + v_2)}$$

(showed)

Critical Flow:
Its Computation and Applications

Farhad
#1500045

Critical flow: When the depth of flow of water over a certain reach of a given channel is equal to the critical depth, The flow is defined as critical flow.

If the critical state of flow exists throughout the entire length of the channel or over a reach of the channel, the flow in the channel is a critical flow.

Channel slope: Channel slope may be three types:

(i) **critical slope:** The slope of channel that sustains a given discharge at a uniform and critical depth is called the critical slope S_c.

(ii) **Mild or subcritical slope:** A slope of channel less than the critical slope will cause a slower flow of subcritical state for the given discharge is called a mild or subcritical slope.

(iii) **Steep or super critical slope:** A slope of channel greater than the critical slope will result in a faster flow of super critical state, is called steep or super critical slope.

Characterised conditions of critical state of flow:

(1) The specific energy is a minimum for a given discharge.
(2) The discharge is a maximum for a given specific energy.
(3) The specific force is a minimum for a given discharge.
(4) The velocity head is equal to half the hydraulic depth in a channel of small slope.

(5) The froude number is equal to unity.

(6) The velocity of flow in a channel of small slope with uniform velocity distribution is equal to the celerity of small gravity waves in shallow water caused by local disturbances.

Section factor for critical flow computation:

We know that, for critical state of flow velocity head is equal to half of the hydraulic depth i.e.

$$\frac{v^2}{2g} = \frac{D}{2}$$

$$\Rightarrow \frac{v^2}{g} = D$$

$$\Rightarrow \frac{Q^2}{A^2 g} = D \quad \left[\text{substituting } v = \frac{Q}{A}\right]$$

$$\Rightarrow \frac{Q^2}{g} = A^2 D$$

$$\Rightarrow \frac{Q}{\sqrt{g}} = A\sqrt{D}$$

$$\Rightarrow Z = \frac{Q}{\sqrt{g}} \quad \left[\text{where, } Z = A\sqrt{D}\right]$$

When the co-efficient is not unity, $Z = \frac{Q}{\sqrt{\frac{g}{\alpha}}}$

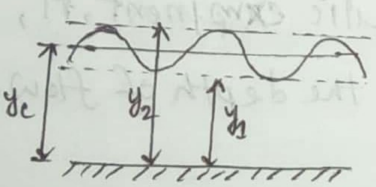
Since the section factor Z is a function of the depth, the equation indicates that there is only one possible critical depth for maintaining the given discharge in a channel and similarly that, when the depth is fixed, there can be only one discharge that maintains a critical flow and makes the depth critical in the given channel section.

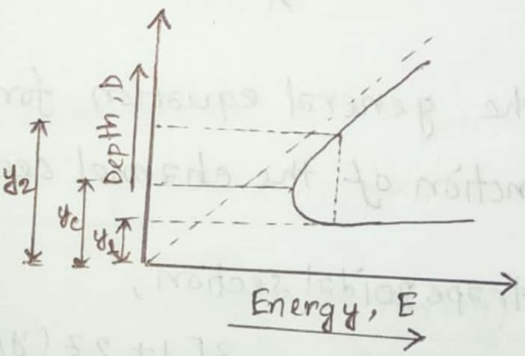
A flow at or near the critical state is unstable – Explain it

A flow at or near the critical state is unstable. This is because a minor change in specific energy at or near to critical state will cause a major change in depth.

It can be also observed that when the flow is near the critical state, the water surface appears unstable and wary. Such phenomenon is generally caused by minor change in energy due to variation in channel roughness, cross-section, slope or deposits of sediments or debris.

In design of a channel, if the depth is found at near the critical depth for a great length of the channel, the slope or slope of the channel should be altered for greater stability.





Hydraulic exponent for critical flow computation:

Since the section factor Z is a function of the depth of flow y, It may be assumed that;

$$Z^2 = Cy^M$$ where, C = co-efficient, M = parameter called hydraulic exponent

Taking logarithm on both sides,

$$\ln Z^2 = \ln(Cy^M)$$

$$\Rightarrow 2\ln Z = \ln C + M \ln y$$

differentiating with respect to y,

$$2\frac{d(\ln Z)}{dy} = 0 + M\frac{d(\ln y)}{dy}$$

$$\Rightarrow \frac{d(\ln Z)}{dy} = \frac{M}{2y} \quad \text{......(i)}$$

We know that, $Z = A\sqrt{D}$

$$\Rightarrow Z = A\sqrt{\frac{A}{T}}$$

Taking logarithms on both sides,

$$\ln Z = \ln\left(A\sqrt{\frac{A}{T}}\right)$$

$$\Rightarrow \ln Z = \frac{3}{2}\ln A - \frac{1}{2}\ln T$$

differentiating with respect to y we obtain,

$$\frac{d(\ln Z)}{dy} = \frac{3T}{2A} - \frac{1}{2T}\frac{dT}{dy} \quad \text{......(ii)} \quad \left[\frac{dA}{dy} = T\right]$$

Equating (i) & (ii) and solving for M,

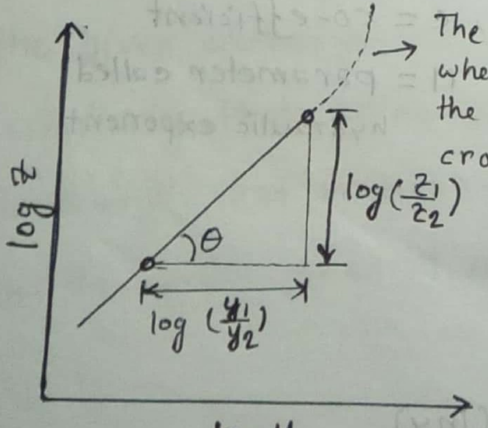
$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right)$$

This is the general equation for hydraulic exponent, M, which is a function of the channel section and the depth of flow.

But for trapezoidal section,

$$Z = \frac{3[1+2z(y/b)]^2 - 2z(y/b)[1+z(y/b)]}{[1+2z(y/b)][1+z(y/b)]}$$

Graphical determination of M:



The plot becomes curve when the depth approaches the gradually converging crown of a closed conduit

Approximate values of M for any channel section,

$$M = 2\tan\theta$$

$$\Rightarrow M = 2\,\frac{\log(Z_1/Z_2)}{\log(y_1/y_2)}$$

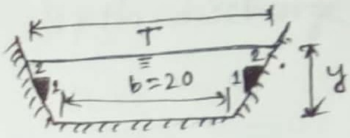
where Z_1 and Z_2 are section factors for any two depth y_1 and y_2 of given section.

Computation of critical flow:

Computation of critical flow involves the determination of critical depth and velocity when the discharge and the channel section are known. There are three methods for computation of critical flow:

1. Algebraic Method: For a simple geometrical channel section.

Example: Compute the critical depth and velocity of the trapezoidal channel carrying a discharge of 400 cfs.



Solution:

The hydraulic depth of the trapezoidal section,

$$D = \frac{A}{T} \Rightarrow D = \frac{y(10+y)}{10+2y}$$

and the water area is, $A = y(20+2y)$

The velocity, $V = \frac{Q}{A} = \frac{200}{y(20+2y)}$

We know that, for critical state of flow,

$$\frac{V^2}{2g} = \frac{D}{2}$$

$$\Rightarrow \frac{(200)^2}{y^2(20+2y)^2 g} = \frac{y(10+y)}{(10+2y)}$$

$$\Rightarrow 4y^3(10+y)^3 g = 80000(5+y)$$

$$\Rightarrow [y(10+y)]^3 = 2484.47(5+y)$$

Solving for y, we obtain, $y = 2.15$ ft. This is the critical depth.

$$\therefore A = 2.15 \times (20 + 2 \times 2.15) = 52.245 \text{ ft}^2$$

and the critical velocity, $V_c = \frac{400}{52.245} = 7.66$ fps.

(Ans.)

2. Graphical Method: For a complicated or natural channel section

Example: A 36 in concrete circular culvert carries a discharge of 20 cfs. Determine the critical depth.

Solution: construct a curve of y vs. Z as shown in figure

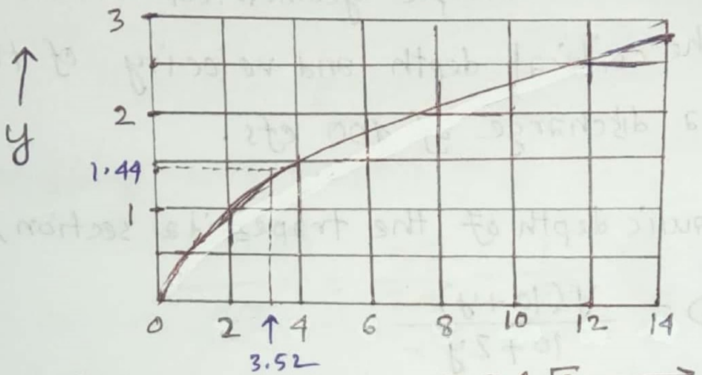


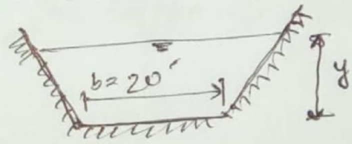
Fig. Curve of y vs. Z for a circular section

Then, $Z = \frac{Q}{\sqrt{g}} = \frac{20}{\sqrt{32.2}} = 3.52$

For $Z = 3.52$, $y_c = 1.44$ from the curve.

3. Method of Design chart: (Fig-4.1-chow)

Example: compute the critical depth of the trapezoidal channel carring a discharge of 400 cfs.



Solution:

$$Z = \frac{Q}{\sqrt{g}} = \frac{400}{\sqrt{32.2}} = 70.5.$$

The value of $\frac{Z}{b^{2.5}} = 0.0394$ [b=20]

For this value, the design chart gives $\frac{y}{b} = 0.108$

$$\therefore y_c = (.108 \times 20) = 2.16$$

10

Channel control: It is defined as a channel feature, Natural or Artifical, which results in the establishment of a unique relation between discharge and depth for a given cross-section.

Artifical controls: (i) weirs (ii) over fall (iii) spillways (iv) Sluices.

Natural control: A rock outcrop is a natural control if its dimensions are such as to result in a unique depth-discharge relation at all stages.

10

Channel Transition:

A transition is a portion of a channel with varing cross section which connects one uniform channel to another, which may or may not have the same cross sectional form.

Where the variation of the channel cross section can be done by:

(i) reducing or increasing width

(ii) rising or lowering the bottom of the channel by sudden or gradually change.

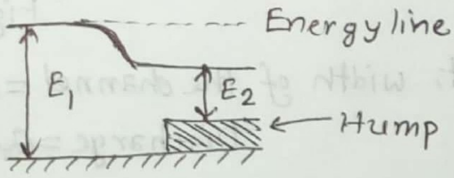


Fig. Channel Transition

Types of channel transition:

Channel transition may be two types:

1. Sudden Transition: Sudden transition are those in which the cross-sectional dimension occur in a relatively short length.

2. Gradual Transition: Gradual transition are those in which the change of cross sectional area takes gradually in a relatively long length of channel.

06, 07, 11, 12, 15

Functions of channel transition:

1. Metering of flow.
2. Dissipation of energy i.e reduce of energy.
3. Reduction or increase of velocity.
4. change in channel section or, Alignment with a minimum energy dissipation.

2009, 2015

Discuss with diagram, the variation of y_1 and y_2 with Δz in the subcritical regime of a horizontal frictionless rectangular channel of width B and discharge Q.

Solution: Let us, consider a horizontal, frictionless rectangular channel as shown below:

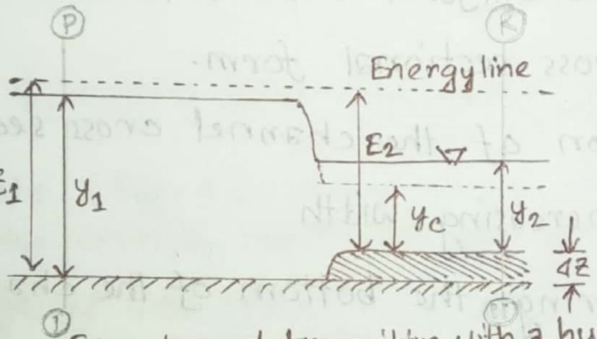


Fig. channel transition with a hump.

Let, width of the channel = B

Discharge = Q

Height of hump = Δz.

Let the flow be subcritical. Since there are no energy losses between section ① an ⑪, construction of a hump causes the specific energy at section ⑪ to decrease by Δz. Thus the specific energies at section ① and ⑪ are given by,

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$E_2 = E_1 - \Delta z$$

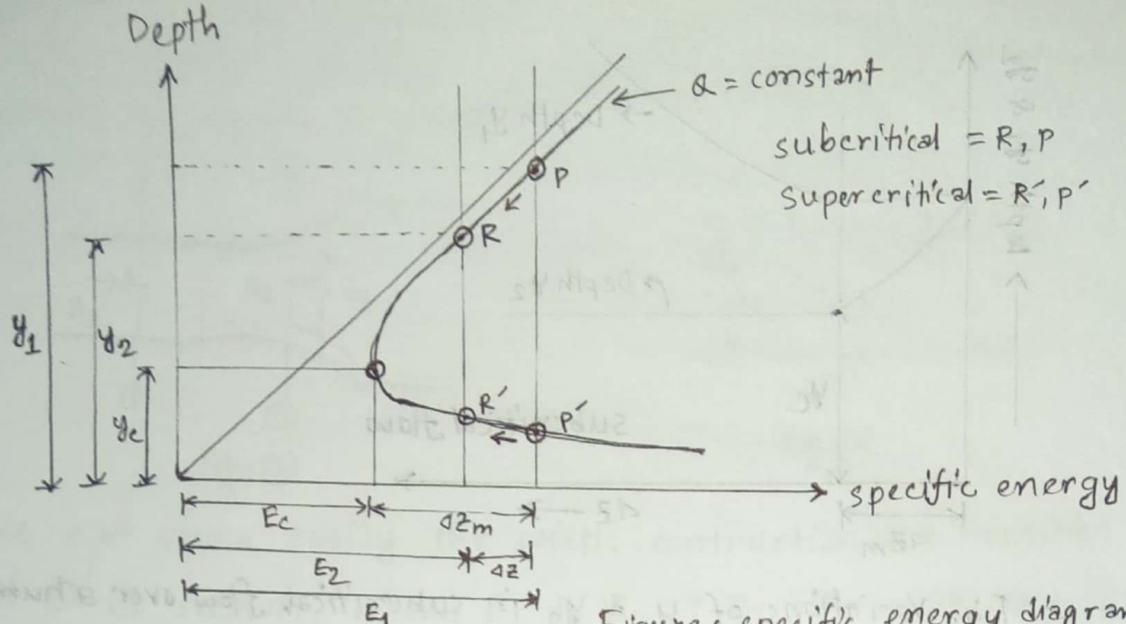


Figure: specific energy diagram.

Since the flow is subcritical, water surface will drop due to a decrease in the specific energy. In above figure, the water surface which was at P at section ① will come down to point R at section ⑪. If the value of ΔZ is increased, the depth at section ⑪ that is y_2 decreased. The minimum depth is reached when point R coinsides with c, the critical depth point. At this point, the hump height will be maximum, that is,

when, $\Delta Z = \Delta Z_m$ then,

$$y_2 = y_c \quad \& \quad E_2 = E_c$$

The condition at ΔZ_m is given by the relation.

$$E_1 - \Delta Z = E_2 = E_c = y_c + \frac{Q^2}{2g B^2 y_c^2}$$

When $0 < \Delta Z < \Delta Z_m$, the upstream water level remains stationary at y_1, while the depth of flow at section ⑪ decreases with ΔZ reaching a minimum value of y_c at $\Delta Z = \Delta Z_m$

The variation of y_1 and y_2 with ΔZ in the subcritical regime can be clearly noticed in the following figure:

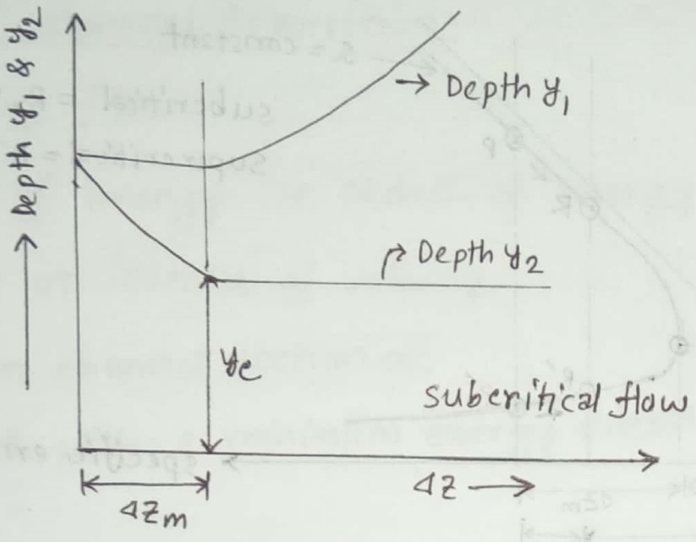


Fig. Variations of y_1 & y_2 in subcritical flow over a hump.

When flow is supercritical :

If y_1 is the super critical flow regime, Figure of specific energy diagram, shows that the depth of flow increases due to the reduction of specific energy.

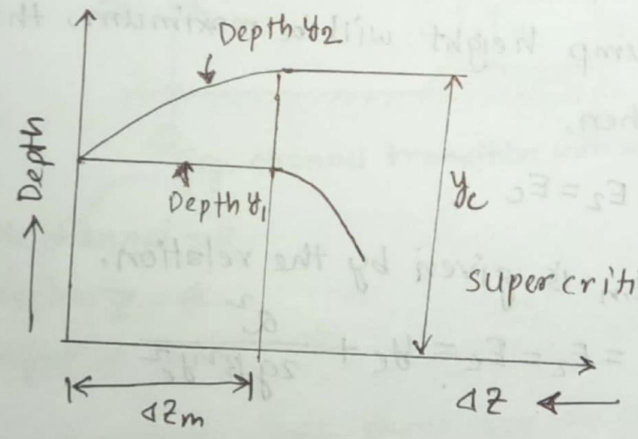


Fig. variations of y_1 & y_2 in supercritical flow over a hump

In figure of specific energy diagram,
point P′ corresponds to y_1 and Point R′ to depth y_2 at the section (1) up to the critical depth y_2 increase to reach y_c at $\Delta Z = \Delta Z_m$. For $\Delta Z > \Delta Z_m$ the depth over the hump $y_2 = y_c$ will remain constant and the upstream depth y_1 will change. It will decreases to have a higher specific energy E_1'.

Explain Channel Transition due to width contraction:

Considering figure (a) and (b)

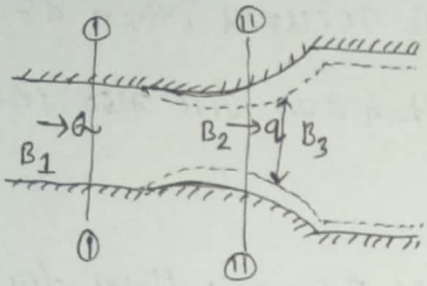


Fig. (a)

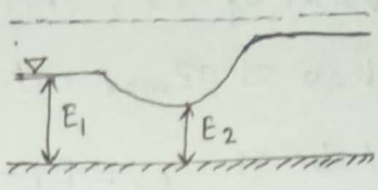


Fig. (b)

We can show easily the width contruction and transtion will form at I–II section, where B_1 to B_2 as B_3 the width are shown and Q as for I–I and q is for II–II sections discharge.

q = Discharge per unit width.

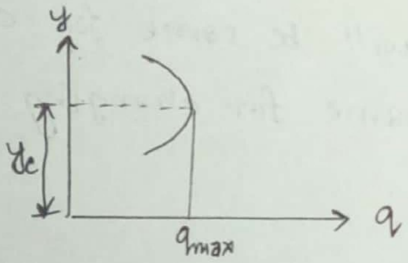


Fig. (c)

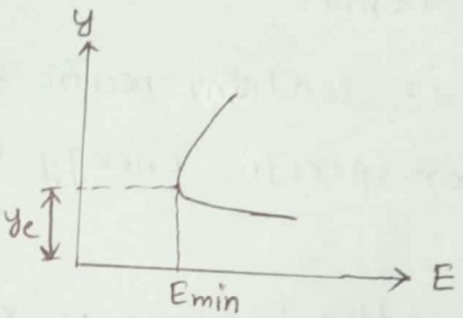
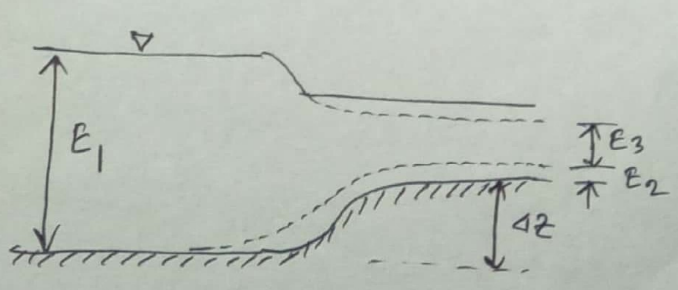


Fig. (d)

E_1 and E_2 are the energy for the initial and critical condition. At the limit or in critical condition, width is less, then the discharge will maximum as q_{max}. For q_{max} the flow depth of upstream head will increase.

Explain channel transition due to the raise of bottom channel:

Considering the figure,



we can explain it very easily.

Here Δz = Hump = the raising of the channel bottom.

At limiting value of Δz, critical flow is occured, then $\Delta z = \Delta z_{max}$

If it reaches at Δz_{max}, the fluid, then Δz will also increases

and E_2 will be as $E_2 = E_1 - \Delta z$

And then if Δz increases after reaches Δz_{max}, then for keeping specific energy decreases then hump will excreate

If no change in upstream head. At Δz_{max} if change of rise bottom can be acquired then flow will be changed then

$E = E_c + \Delta z_{max}$

At then Δz condition specific energy will be same for changing discharge or specific energy will be same for changing discharge.

Afflux: sudden increases of flow depth due to the hump in a rectangular channel is called the afflux.

Hump: sudden increases of width of a rectangular channel is known as the hump of a rectangular channel.

Development of Uniform flow and Its formulas

Qualifications for Uniform flow:

The uniform flow to be considered has the following main features:

(i) the depth, water area, velocity and discharge at every section of the channel reach are constant.

(ii) the energy line, water surface, and channel bottom are all parallel, that is, their slopes are all equal, or $S_f = S_w = S_o = S$

Establishment of uniform flow:

When flow occurs in an open channel, then resistance is encountered by the water as it flows downstream. This resistance is generally countered by the components of gravity force acting on the body of water in the direction of motion.

A uniform flow will be developed, if the resistances is balanced by the gravity forces. The magnitudes of the resistance, when other physical factors of the channel are kept unchanged, depends on the velocity of flow.

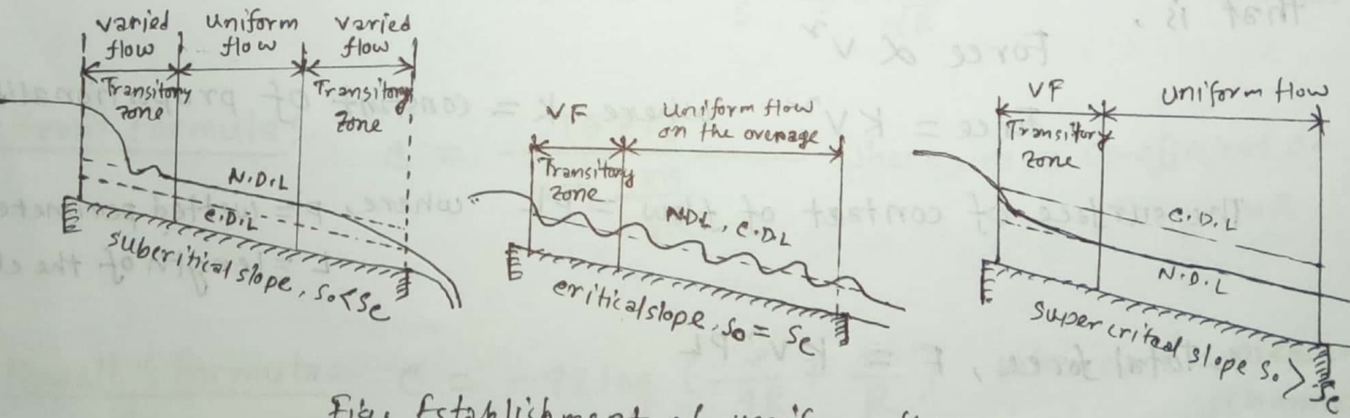


Fig. Establishment of uniform flow.

If the water enters slowly, the velocity and hence the resistance are small and the resistance is outbalanced by the gravity forces, resulting in an accelerating flow in the upstream reach. The velocity and the resistance will gradually increase until a balance between resistance and gravity force is reached. At this moment and after ward the flow becomes uniform.

The upstream reach that is required for establishment of uniform flow is known as transitory zone.

05, 08, 12, 16

Chezy's Formula and its assumption:

It is the first uniform flow formula and usually expressed as follows:

$$V = C\sqrt{RS}$$

where, V = Mean velocity in fps

R = Hydraulic radius in ft

S = slope of energy line

C = Factor of flow resistance

Chezy's formula is based on two assumptions:

First Assumption: The force resisting the flow per unit area of the stream bed is proportional to the square of the velocity that is,

$$\text{Force} \propto V^2$$

$$\text{Force} = KV^2 \quad \text{where } K = \text{constant of proportionality}$$

The surface of contact of flow $= PL$ where, P = wetted perimeter

L = length of the channel

$\therefore$ Total force, $F = KV^2PL$

Second Assumption: The effective component of the gravity force causing the flow must be equal to the total force of resistance. that is,

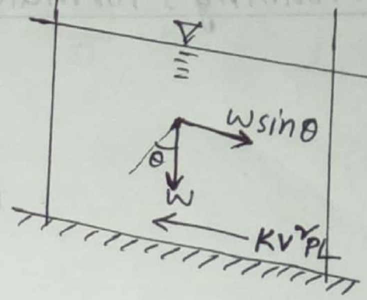
Effective component of gravity force, $w \sin\theta \cdot AL = KV^2PL$

where, w = unit weight of water

A = water area

θ = slope angle

S = channel slope = $\sin\theta$



~~If θ is very small then $\sin\theta = \tan\theta = \theta$~~

So, $wSAL = KV^2PL$

$$\Rightarrow V^2 = \frac{wAS}{KP}$$

$$\Rightarrow V = \sqrt{\left(\frac{A}{P}\right)\left(\frac{w}{K}\right)S}$$

Hydraulic radius, $R = \frac{A}{P}$

Factor, $C = \sqrt{\left(\frac{w}{K}\right)}$

$$\boxed{\therefore V = C\sqrt{RS}}$$

and, Discharge, $Q = AV = AC\sqrt{RS}$

Determination of chezy's resistance factor:

1. G.K Formula: $$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{S}\right) \times \frac{n}{\sqrt{R}}}$$ where, n = co-efficient of Kutter

2. Bazin Formula: $$C = \frac{157.6}{1 + \frac{m}{\sqrt{R}}}$$ where, m = co-efficient of roughness

3. Powell Formula: $$C = -42\log\left(\frac{C}{4R} + \frac{\epsilon}{R}\right)$$ where, ϵ = measure of channel roughness

Manning's Formula: well known form,

$$V = \frac{1.49}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

where, R = Hydraulic radius

S = Slope of energy line

n = co-efficient of roughness

05, 13

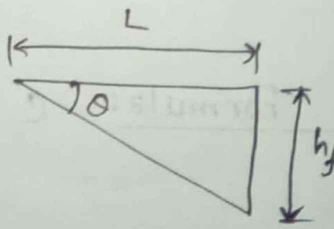
Factor affecting Manning's roughness co-efficient:

1. surface roughness.
2. vegetation.
3. channel irregularity.
4. Channel alignment.
5. Silting and scouring.
6. Obstruction.
7. Size and shape of channel.
8. Stage and discharge.
9. suspended materials and bed load.

08

Prove that, the friction in the Darcy weisbach formula is related to Manning's n by $f = \frac{116 n^2}{R^{\frac{1}{3}}}$

From darcy-weisbach formula,



$$h_f = \frac{f L V^2}{2gD}$$

$$\Rightarrow \frac{h_f}{L} = \frac{f V^2}{2gD}$$

$$\Rightarrow S = \frac{f V^2}{2gD} \quad \left[\because S = \frac{h_f}{L}\right]$$

$$\Rightarrow S = \frac{f}{2gD}\left[\frac{1.49}{n}\cdot R^{\frac{2}{3}}\cdot S^{\frac{1}{2}}\right]^2$$ [From manning's formula, $V = \frac{1.49}{n}\cdot R^{\frac{2}{3}}\cdot S^{\frac{1}{2}}$]

$$\Rightarrow S = \frac{f}{2gD}\cdot\frac{(1.49)^2}{n^2}\cdot R^{\frac{4}{3}}\cdot S$$

$$\Rightarrow 2gD\cdot n^2 = f R^{\frac{4}{3}} \times (1.49)^2$$

$$\Rightarrow 2g \times 4R \times n^2 = f R^{\frac{4}{3}} \times (1.49)^2$$ [For circular section, $D = 4R$]

$$\Rightarrow f = \frac{8gRn^2}{R^{\frac{4}{3}} \times (1.49)^2}$$

$$\therefore f = \frac{116 n^2}{R^{\frac{1}{3}}}$$

(Proved)

☐ Show that chezy's co-efficient, $c = \frac{R^{\frac{1}{6}}}{n}$, where R = hydraulic radius and n = Manning's roughness co-efficient.

The manning's formula may be expressed as,

$$V = \frac{1.49}{n}\cdot R^{\frac{2}{3}}\cdot S^{\frac{1}{2}} \quad \text{in FPS unit}$$

but, $V = \frac{1}{n}\cdot R^{\frac{2}{3}}\cdot S^{\frac{1}{2}}$ in MKS unit

Where, V = mean velocity, R = hydraulic radius, S = slope of energy line and n = co-efficient of roughness.

Now, from chezy's formula, $V = C\sqrt{RS}$ .... (I)

and from manning's formula, $V = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$ .... (II)

from (I) & (II) we obtain,

$$C\sqrt{RS} = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$\Rightarrow C = \frac{1}{n} R^{\frac{1}{6}}$$

(Showed)

05,08

☐ Control section: when the control of flow is achieved at a certain section of the channel, this section is called control section.

05,08

☐ Discharge rating curve: control section holds a definitive stage-discharge relationship. It is always a suitable site for a gaging station. A curve representing the depth-discharge relationship at the gaging station is known as discharge rating curve.

computation of Uniform flow

Farhad
#1500045

conveyance of a channel section:

The measure of the carrying capacity of the channel section is called the conveyance of a channel section. It is denoted by K.

The discharge of uniform flow, $Q = VA = CAR^x S^y = KS^y$

$$\therefore Q = KS^y \quad \text{.... (i)}$$

$$\text{where, } K = CAR^x \quad \text{..... (ii)}$$

when $y = \frac{1}{2}$, the discharge, $Q = K\sqrt{S}$ ..... (iii)

$$\Rightarrow K = \frac{Q}{\sqrt{S}} \quad \text{..... (iv)}$$

when chezy's formula is used,

$K = CAR^{\frac{1}{2}}$ ... (v) where, C = chezy's resistance factor

Similarly,

when manning's formula is used,

$$K = \frac{1.49}{n} A \cdot R^{\frac{2}{3}} \quad \text{..... (vi)}$$

section factor for uniform flow computation:

We Know from manning's formula,

conveyance, $K = \frac{1.49}{n} A \cdot R^{\frac{2}{3}}$

The expression $AR^{\frac{2}{3}}$ is called the section factor for uniform flow computation.

$$AR^{\frac{2}{3}} = \frac{Kn}{1.49} = \frac{nQ}{1.49\sqrt{S}} \qquad [\because K = \frac{Q}{\sqrt{S}}]$$

$$\Rightarrow Q = \frac{1.49}{n} AR^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$\Rightarrow \ln K = \ln\left(\frac{1.49}{n}\right) + \ln(A) + \frac{2}{3}\ln(R)$$

differentiating with respect to y under assumption that n is independent to y:

$$\frac{d(\ln K)}{dy} = 0 + \frac{1}{A}\frac{dA}{dy} + \frac{2}{3}\frac{1}{R}\frac{dR}{dy}$$

$$\Rightarrow \frac{d(\ln K)}{dy} = \frac{T}{A} + \frac{2}{3}\cdot\frac{P}{A}\cdot\frac{d}{dy}\left(\frac{A}{P}\right) \qquad \left[\because \frac{dA}{dy} = T \text{ and } R = \frac{A}{P}\right]$$

$$\Rightarrow \frac{d(\ln K)}{dy} = \frac{1}{3A}\left(5T - 2R\frac{dP}{dy}\right) \quad \text{.....(II)}$$

help:

$$\frac{T}{A} + \frac{2}{3}\cdot\frac{P}{A}\cdot\frac{\left(P\cdot\frac{dA}{dy} - A\frac{dP}{dy}\right)}{P^2}$$

$$\frac{T}{A} + \frac{2}{3A}\frac{\left(P\frac{dA}{dy} - AP\frac{dP}{dy}\right)}{P^2}$$

$$\frac{T}{A} + \frac{2}{3A}\left(T - R\cdot\frac{dP}{dy}\right)$$

$$\boxed{\frac{5T}{3A} - \frac{2R}{3A}\cdot\frac{dP}{dy}}$$

Equating (I) & (II) we obtain,

$$\frac{N}{2y} = \frac{1}{3A}\left(5T - 2R\cdot\frac{dP}{dy}\right)$$

$$\Rightarrow N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right)$$

This is the general equation for hydraulic exponent **N**.

But for trapezoidal section,

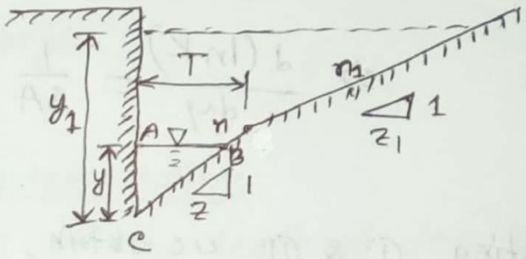
$$N = \frac{10}{3}\,\frac{1+2z(y/b)}{1+z(y/b)} - \frac{8}{3}\,\frac{\sqrt{1+z^2}\,(y/b)}{1+2\sqrt{1+z^2}\,(y/b)}$$

Prove that, the following equation for the discharge in a triangular Highway gutter having one side vertical, one side sloped at 1 on z. Manning's n, depth of flow y and longitudinal slope S.

$$Q = \frac{0.47}{5} f(z) \cdot y^{\frac{8}{3}} \cdot S^{\frac{1}{2}} \quad \text{where } f(z) = \frac{z^{\frac{5}{3}}}{\left[1+\sqrt{1+z^2}\right]^{\frac{2}{3}}}$$

From manning's formula we get,

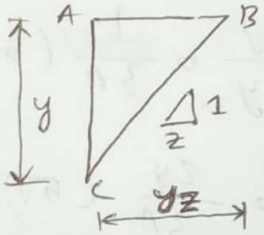
$$\text{Discharge, } Q = \frac{1.49}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}} \quad \text{---- (1)}$$



Here, Area $A = \frac{1}{2} \times AB \times AC$

$$\Rightarrow A = \frac{1}{2} \times yz \times y$$

$$\Rightarrow A = \frac{1}{2} y^2 z$$



wetted perimeter, $P = y + y\sqrt{1+z^2}$

$$\therefore AR^{\frac{2}{3}} = A\left(\frac{A}{P}\right)^{\frac{2}{3}} = \frac{A^{\frac{5}{3}}}{P^{\frac{2}{3}}}$$

$$\Rightarrow AR^{\frac{2}{3}} = \frac{\left(\frac{y^2 z}{2}\right)^{\frac{5}{3}}}{\left(y + y\sqrt{1+z^2}\right)^{\frac{2}{3}}} = \frac{y^{\frac{10}{3}} z^{\frac{5}{3}}}{2^{\frac{5}{3}}} \times \frac{1}{y^{\frac{2}{3}}\left(1+\sqrt{1+z^2}\right)^{\frac{2}{3}}}$$

$$\Rightarrow AR^{\frac{2}{3}} = \frac{z^{\frac{5}{3}}}{\left(1+\sqrt{1+z^2}\right)^{\frac{2}{3}}} \times y^{\frac{8}{3}} \times \frac{1}{2^{\frac{5}{3}}} = f(z) \cdot \frac{y^{\frac{8}{3}}}{2^{\frac{5}{3}}}$$

Now,

$$Q = \frac{1.49}{n} f(z) \frac{y^{\frac{8}{3}}}{2^{\frac{5}{3}}} S^{\frac{1}{2}}$$

$$\therefore Q = \frac{0.47}{n} f(z)\, y^{\frac{8}{3}} \cdot S^{\frac{1}{2}}$$

(Proved)

Determination of equivalent roughness:

1. Horton and Einstein assumed that, each part of the area has the same velocity, which at the same time is equal to the mean velocity of the whole section. By this assumption, equivalent co-efficient ~~coefficient~~ of roughness,

$$n = \left[\frac{\sum_1^N (P_N n_N^{1.5})}{P} \right]^{\frac{2}{3}}$$

2. Pavlovskii and also Muhlhofer assumed that, the total force resisting the flow is equal to the sum of the forces resisting the flow developed in the subdivided areas. By this assumption, equivalent co-efficient of roughness,

$$n = \frac{\left[\sum_1^N (P_N n_N^2) \right]^{\frac{1}{2}}}{P^{\frac{1}{2}}}$$

3. Lotter assumed that, the total discharge of flow is equal to the sum of discharges of the subdivided areas. By this assumption equivalent co-efficient of roughness,

$$n = \frac{P R^{\frac{5}{3}}}{\sum_1^N \left(\frac{P_N R_N^{\frac{5}{3}}}{n_N} \right)}$$

Design of Channel For Uniform Flow

Farhad
#1500045

09, 13, 15

Factors to be considered in the design of channel:

(i) The kind of material forming the channel body which determines the roughness co-efficient.

(ii) The minimum permissible velocity, to avoid deposition if the water carries silt or debris.

(iii) The channel bottom slope and side slope.

(iv) The free board

(v) The most efficient section, either hydraulically or emperically determined.

Types of channel:

1. Erodible Channel: Unlined channels are generally erodible, except those excavated in firm foundations such as rock bed.

2. Non-erodible channel: Most lined channels and built up channels can with stand erosion satisfactory and are therefore considered non-erodible.

Non erodible materials:
These materials used to form the lining of a channel and the body of a built up channel include concrete, stone, masonry, steel, cast iron, timber, glass, plastic etc. The selection of the materials depends mainly on availability and cost of the material, the method of construction and the purpose for which the channel to be used.

Lining:
By lining the channel we mean that the earth surface of channel is linked with stable lining surface as concrete. The purpose of linining a channel is in most cases to prevent erosion but it may be to check seepage losses.

The best hydraulic section: It is known that the conveyance of a channel section increases with increase in the hydraulic radius or with decrease in the wetted perimetter. From a hydraulic view point, therefore, the channel section having the least wetted perimeter for a given area has the maximum conveyance; such a section is known as the best hydraulic section.

The semicircle has the least perimeter among all sections with the same area; hence it is the most hydraulically efficient of all sections.

10

The best hydraulic section is not necessary to the most economic section — why?

The following are the reasons:

(i) The area to be excavated to achieve the area for the hydraulic section may be significantly higher.

(ii) The type of material forming the channel body may not permit to adoption of slope required by best hydraulic section.

(iii) The sharp corner in a cross section which are virtually the zones of stagnation, may lead to deposition if the water carries silt.

(iv) For instance, the type of soil through which the channel is carried may not permit the adoption of 1:1 side slope required for a triangle section.

09, 12, 14, 16

Three conditions for most efficient trapezoidal channel. Prove any one of them:

1. The water surface width is equal to twice the length of the sloping side. i.e $b + 2yz = 2y\sqrt{1+z^2}$

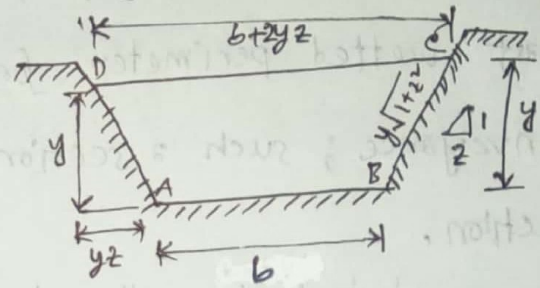


Fig. Cross section of Trapezoidal channel

2. Hydraulic radius, R is equal to the half of the depth of flow.
i.e $R = \frac{y}{2}$

3. A semicircle drawn from O with radius equal to depth of flow touch the three sides of the trapezoidal channel.

16 1st condition:

According to figure, Area of flow, $A = \frac{1}{2} \times (b + b + 2yz)y = (b + yz)y$

$$\therefore b = \frac{A}{y} - yz$$

and, wetted perimeter, $P = b + 2y\sqrt{1+z^2}$

$$\Rightarrow P = \frac{A}{y} - yz + 2y\sqrt{1+z^2}$$

The section of the channel will be most efficient when its wetted perimeter is minimum i.e $\frac{dP}{dy} = 0$

$$\therefore \frac{d}{dy}\left(\frac{A}{y} - yz + 2y\sqrt{1+z^2}\right) = 0$$

$$-\frac{A}{y^2} + 2\sqrt{1+z^2} - z = 0$$

$$\Rightarrow \frac{A}{y^2} + z = 2\sqrt{1+z^2}$$

$$\Rightarrow \frac{A}{y} + yz = 2y\sqrt{1+z^2}$$

$$\Rightarrow b + 2yz = 2y\sqrt{1+z^2}$$

(Proved)

2nd Condition:

Hydraulic Radius, $R = \frac{A}{P}$

$$\Rightarrow R = \frac{(b+yz)y}{b + 2y\sqrt{1+z^2}}$$

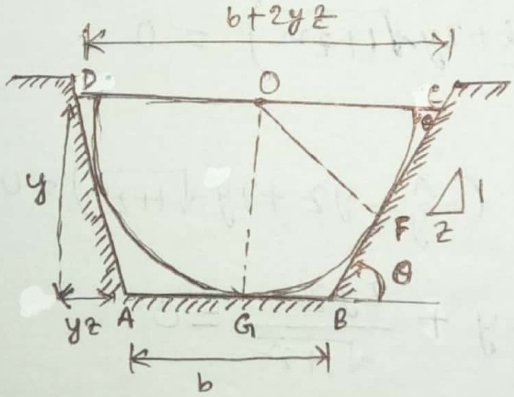
$$\Rightarrow R = \frac{(b+yz)y}{b + (b+2yz)} \quad [\because b+2yz = 2y\sqrt{1+z^2}]$$

$$\Rightarrow R = \frac{(b+yz)y}{2(b+yz)}$$

$$\therefore R = \frac{y}{2}$$

(Proved)

3rd Condition:



$$\sin\theta = \frac{1}{\sqrt{1+z^2}}$$

Let, angle made by the sloping side with the horizontal $= \theta$

centre of top width $=$ O

and, OF $\perp$ BC

From $\triangle$OCF, $\sin\theta = \frac{OF}{OC}$

$$\Rightarrow OF = OC\sin\theta$$

$$\Rightarrow OF = \frac{b+2yz}{2} \times \frac{1}{\sqrt{1+z^2}}$$

$$\Rightarrow OF = y\sqrt{1+z^2} \times \frac{1}{\sqrt{1+z^2}} \quad [\because b+2yz = 2y\sqrt{1+z^2}]$$

$$\therefore OF = y$$

(Proved)

05, 08, 17

Show that the best hydraulic trapezoidal section is one-half of a hexagon.

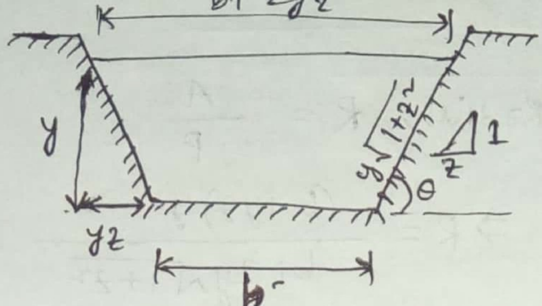


Fig: cross section of trapezoidal channel

According to figure,

Area of flow, $A = (b+yz)y \quad \Rightarrow b = \frac{A}{y} - yz$

and wetted perimeter, $P = b + 2y\sqrt{1+z^2}$

∴ For most efficient section, $\frac{dP}{dz} = 0$

$$\therefore \frac{d\left(b+2y\sqrt{1+z^2}\right)}{dz} = 0$$

$$\Rightarrow \frac{d}{dz}\left(\frac{A}{y} - yz + 2y\sqrt{1+z^2}\right) = 0$$

$$\Rightarrow 0 - y - \frac{2yz}{\sqrt{1+z^2}} = 0$$

$$\Rightarrow -y = \frac{2yz}{\sqrt{1+z^2}}$$

$$\Rightarrow -2z = \sqrt{1+z^2}$$

$$\Rightarrow 4z^2 = 1+z^2$$

$$\Rightarrow z^2 = \frac{1}{3}$$

$$\Rightarrow z = \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$\therefore z = \tan 30^\circ$, This means that the section is a half hexagon.

Most efficient triangular section:

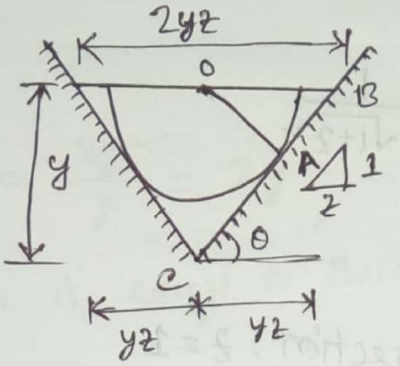


Fig. cross section of triangular channel

Here, $OA \perp BC$

$\sin\theta = \frac{1}{\sqrt{1+z^2}}$

According to figure,

Area of flow, $A = \frac{1}{2} \times 2yz \times y = y^2 z$

$\Rightarrow y = \sqrt{\frac{A}{z}}$

wetted perimeter, $P = 2y\sqrt{1+z^2}$

$\Rightarrow P = 2\sqrt{\frac{A}{z}}\sqrt{1+z^2}$

$\Rightarrow P^2 = 4 \cdot \frac{A}{z} \cdot (1+z^2)$

$\Rightarrow P^2 = 4A\left(z+\frac{1}{z}\right) \quad \ldots (1)$

For most efficient section, wetted perimeter should be minimum.

i.e. $\frac{dP}{dz} = 0$

differentiating the equation (1)

$2P\frac{dP}{dz} = 4A\left(1-\frac{1}{z^2}\right)$

$\Rightarrow 0 = 4A\left(1-\frac{1}{z^2}\right)$

$\Rightarrow z^2 = 1$

$\Rightarrow z = 1$

The hydraulic radius, $R = \frac{A}{P} = \frac{y^2 z}{2y\sqrt{1+z^2}} = \frac{y}{2\sqrt{2}} = \frac{1}{4}\sqrt{2}y$

From $\triangle OAB$, $\frac{OA}{OB} = \sin\theta$

$$\Rightarrow OA = OB \sin\theta$$

$$\Rightarrow OA = \frac{2yz}{2} \cdot \frac{1}{\sqrt{1+z^2}}$$

$$\Rightarrow OA = \frac{yz}{\sqrt{1+z^2}}$$

For most efficient section, $z = 1$

$$\therefore OA = \frac{y}{\sqrt{2}}$$

If a perpendicular OA is drawn on one of the sloping side such that $OA = \frac{y}{\sqrt{2}}$, the obtain semicercle is the most efficient section.

Most efficient rectangular section:

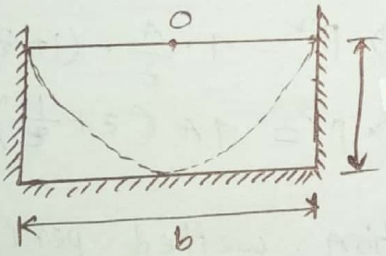


Fig. Cross section of rectangular channel

According to figure,

Area of flow, $A = by \Rightarrow b = \frac{A}{y}$

wetted perimeter, $P = b + 2y$

For most efficient section, $\frac{dP}{dy} = 0$

$$\frac{d}{dy}(b + 2y) = 0$$

$$\frac{d}{dy}\left(\frac{A}{y} + 2y\right) = 0$$

$$\Rightarrow -\frac{A}{y^2} + 2 = 0$$

$$\Rightarrow A = 2y^2$$

$$\therefore b = \frac{A}{y} = \frac{2y^2}{y} = 2y$$

Thus, The width is equal to twice the depth of flow.

The hydraulic radius, $R = \frac{A}{P} = \frac{by}{b+2y} = \frac{2y^2}{2y+2y} = \frac{y}{2}$

06.05

Conditions for maximum velocity and discharge through a circular channel

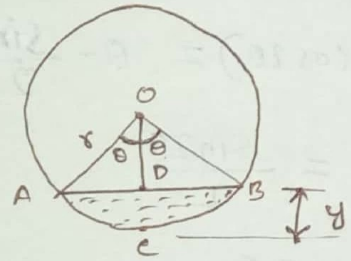


Fig. Cross section of circular channel

According to figure,

wetted perimeter, $P = 2r\theta$

Area of flow, A = Area of ABCA

= Area of OACBO − Area of ΔOAB

$$= \frac{\pi r^2}{2\pi} \times 2\theta - \frac{1}{2} \times 2r\sin\theta \times r\cos\theta$$

$$= r^2\theta - \frac{r^2}{2} \cdot \sin 2\theta$$

2π Angle $\rightarrow \pi r^2$

1 " $\rightarrow \frac{\pi r^2}{2\pi}$

2θ " $\rightarrow \frac{\pi r^2}{2\pi} \times 2\theta$

condition for maximum velocity: $\frac{dR}{d\theta} = 0$

$$\Rightarrow \frac{d}{d\theta}\left(\frac{A}{P}\right) = 0$$

$$\Rightarrow \frac{P\frac{dA}{d\theta} - A\frac{dP}{d\theta}}{P^2} = 0$$

$$\therefore P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0 \quad \cdots\cdots (1)$$

$$\frac{dA}{d\theta} = r^2 - \frac{r^2}{2}\cos 2\theta \cdot 2 = r^2 - r^2\cos 2\theta$$

$$\frac{dP}{d\theta} = 2r$$

$\therefore$ From equation (1) we obtain,

$$2r\theta \times (r^2 - r^2\cos 2\theta) - (r^2\theta - \frac{r^2}{2}\sin 2\theta) \times 2r = 0$$

$$\Rightarrow \theta(1 - \cos 2\theta) = \theta - \frac{\sin 2\theta}{2}$$

$$\Rightarrow \theta\cos 2\theta = \frac{\sin 2\theta}{2}$$

$$\Rightarrow \tan 2\theta = 2\theta$$

Solution of the equation, $2\theta = 257.5^\circ \Rightarrow \theta = 128.75^\circ$

Now, Depth of flow, $y = r - r\cos\theta$

$$\Rightarrow y = r - r\cos 128.75^\circ$$

$$\Rightarrow y = r - r(-0.626)$$

$$\Rightarrow y = r + 0.626r$$

$$\therefore y = 1.626r$$

$$\therefore y = 0.81 \times \text{Diameter of the section}$$

So, maximum velocity occurs when depth of flow is 0.81 times the diameter of circular section

17

Condition for maximum discharge:

Discharge, $Q = AC\sqrt{RS} = AC\sqrt{\frac{A}{P} \cdot S} = C\sqrt{S}\sqrt{\frac{A^3}{P}}$

Discharge will be maximum when, $\frac{d}{d\theta}\left(\frac{A^3}{P}\right) = 0$

$$\Rightarrow \frac{P \times 3A^2 \times \frac{dA}{d\theta} - A^3 \times \frac{dP}{d\theta}}{P^2} = 0$$

$$\Rightarrow 3A^2P\frac{dA}{d\theta} - A^3\frac{dP}{d\theta} = 0$$

$$\Rightarrow 3P\frac{dA}{d\theta} - A \cdot \frac{dP}{d\theta} = 0 \quad \text{.... (1)}$$

Here,

$\frac{dA}{d\theta} = r^2 - r^2\cos 2\theta$ and $\frac{dP}{d\theta} = 2r$

Now From equation (1),

$$3 \times 2r\theta\,(r^2 - r^2\cos 2\theta) - (r^2\theta - \frac{r^2}{2}\sin 2\theta) \cdot 2r = 0$$

$$\Rightarrow 3\theta(1 - \cos 2\theta) = \theta - \frac{\sin 2\theta}{2}$$

$$\Rightarrow 2\theta - 3\theta\cos 2\theta + \frac{\sin 2\theta}{2} = 0$$

$$\Rightarrow 4\theta - 6\theta\cos 2\theta + \sin 2\theta = 0$$

By solving, $2\theta = 308^\circ \Rightarrow \theta = 154^\circ$

Hence, Depth of flow, $y = r - r\cos\theta = r(1 - \cos 154) = r(1 + 0.899)$

$\therefore y = 1.899r$

$\therefore y = 0.95 \times$ Diameter of the ~~channel~~ circular section.

Hydraulic radius, $R = \frac{A}{P} = \frac{r^2\theta - \frac{r^2}{2}\sin 2\theta}{2r\theta} = \frac{r(2\theta - \sin 2\theta)}{4\theta}$

$$\Rightarrow R = \frac{r\left(2\times\frac{\pi}{180}\times 154 - \sin 308\right)}{4\times\frac{\pi}{180}\times 154}$$

$$\Rightarrow R = 0.57r$$

$$\therefore R = 0.29d$$

Method of design of Erodible channel:

Two methods of Approach to the proper design of erodible channels are:

1. Method of permissible velocity.
2. Method of tractive force.

08

Tractive force: When water flows in a channel, a force is developed that acts in the direction of flow on the channel bed. This force, which is simply the pull of water on the wetted area, is known as the tractive force.

In a uniform flow the tractive force is apperantly equal to the effective component of gravity force acting on the body of water, parallel to the channel bottom and equal to WALS.

$$\therefore \tau = WALS$$

Where, W = unit weight of water
A = wetted Area
L = Length of channel reach
S = Slope.

This is also known as shear force or drag force.

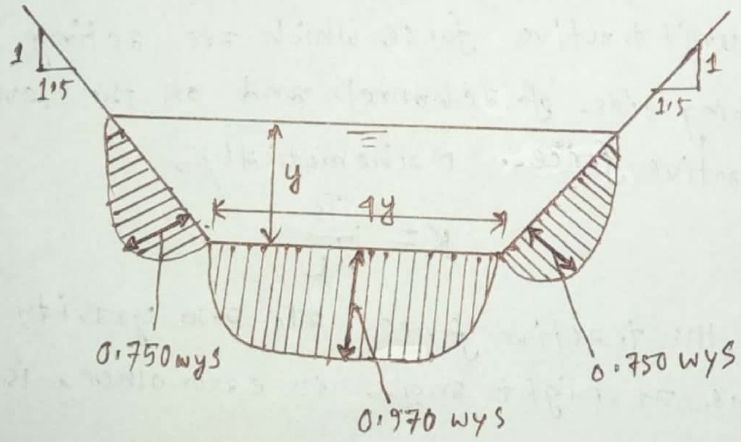
Unit tractive force:
The average value of the tractive force per unit wetted area is called unit tractive force.

$$\tau_0 = \frac{WALS}{PL} = WRS$$

where, P = wetted perimeter
R = Hydraulic Radius

In a wide open channel,
the hydraulic radius, R = depth of flow (y)
hence, $\tau_0 = wyS$

08
Distribution of tractive force in trapezoidal channel section:



Permissible tractive force:
The permissible tractive force is the maximum unit tractive force will not cause serious erosion of the material forming the channel bed on a level surface.

15
critical tractive force:
The unit tractive force which is determined by the laboratory experiment and thus the value obtained is known as critical tractive force.

☐ Tractive force ratio, $K = \sqrt{1 - \frac{\sin^2\phi}{\sin^2\theta}}$

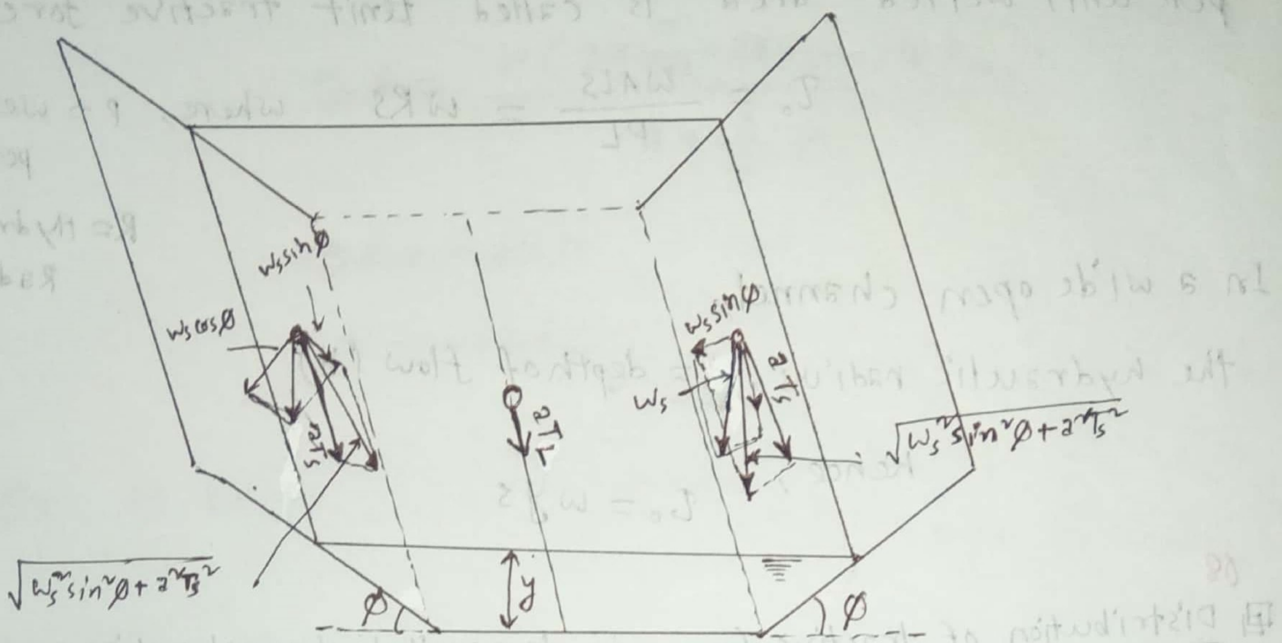


Fig. Analysis of forces acting on a particle resting on the surface of channel bed.

The ratio of two unit tractive force which are acting on a soil particle resting on the sloping side of channel and on the level surface respectively is known as tractive ratio. Mathematically,

$$K = \frac{T_s}{T_L}$$

The resultant of the tractive force aT_s and gravity force component $W_s \sin\phi$ which are at right angle to each other, is $\sqrt{W_s^2 \sin^2\phi + a^2 T_s^2}$

By the principle of frictional motion in mechanics, when the motion is impending, the resistance to motion of the particle is equal to the force tending to cause the motion.

$$\therefore W_s \cos\phi \tan\theta = \sqrt{W_s^2 \sin^2\phi + a^2 T_s^2}.$$

$$\Rightarrow T_s^2 = \frac{1}{a^2}\left(W_s^2 \cos^2\phi \tan^2\theta - W_s^2 \sin^2\phi\right)$$

$$\Rightarrow T_s^2 = \frac{1}{a^2} \times W_s^2 \cos^2\phi \cdot \tan^2\theta \left(1 - \frac{\tan^2\phi}{\tan^2\theta}\right)$$

$$\therefore T_s = \frac{W_1 \tan\theta \cdot \cos\phi}{a} \sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}}$$

Similarly, when the motion of particle on the level surface is impending owing to the tractive force $2T_L$

$$\therefore W_s \tan\theta = 2T_L$$

$$\Rightarrow T_L = \frac{W_s \tan\theta}{2}$$

Hence,

$$K = \frac{T_s}{T_L} = \cos\phi \sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}}$$

$$\therefore K = \sqrt{1 - \frac{\sin^2\phi}{\sin^2\theta}}$$

(Proved)

Gradually Varied Flow

05

Gradually varied flow: The flow in which depth of flow and mean velocity change gradually along the length of the channel, is called gradually varied flow.

Example: 1. Flow upstream of weir or dam.

2. Flow in channels with break in bottom slope.

05

Rapidly varied flow: The flow in which depth of flow and mean velocity change abruptly over a comparatively short distance of the channel, is called Rapidly varied flow.

Example: 1. Hydraulic jump

2. Hydraulic drop

Conditions of Gradually varied flow: 06 characteristics of

1. The flow is steady, that is hydraulic under consideration. flow remain constant for the time interval the hydr

2. The streamlines are partially parallel, that is over the section. distribution of pressure prevails

Assumptions of dynamic equation of GVF:

1. The uniform flow formulae may be used to evaluate the energy slope of GVF. Thus,

$$(S_f)_{G.V.F} = \left(\frac{V_n}{R^{\frac{2}{3}}}\right)^2$$

$$(S_f)_{G.V.F} = \left(\frac{V}{C\sqrt{R}}\right)^2$$

2. The bottom slope of the channel is very small.

3. The channel is prismatic.

4. The energy correction factor α is unity.

5. The pressure distribution in any vertical is hydrostatic

6. Roughness co-efficient is independent of the depth of flow.

Dynamic equation for G.V.F: 05, 07, 10

Total head at any section,

$$H = z + y + \frac{v^2}{2g}$$

$$\Rightarrow \frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right)$$

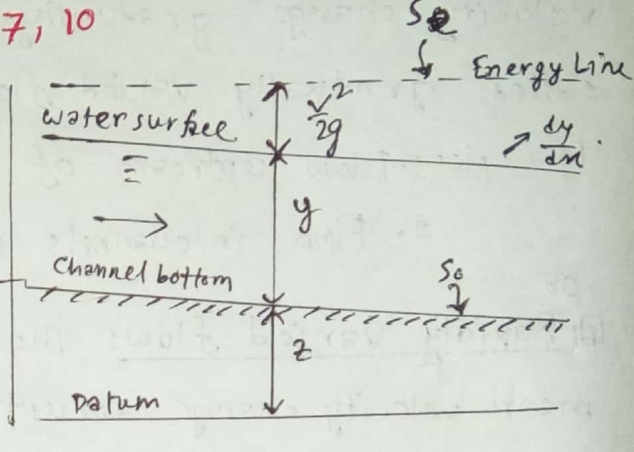


Fig. G.V.F

Here,

$\frac{dH}{dx}$ = the slope of energy line $(-S_e)$

$\frac{dz}{dx}$ = the bed slope $(-S_o)$

$$\therefore -S_e = -S_o + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) = -S_o + \frac{dy}{dx} + \frac{dy}{dx} \cdot \frac{d}{dy}\left(\frac{v^2}{2g}\right)$$

$$\Rightarrow S_o - S_e = \frac{dy}{dx}\left[1 + \frac{d}{dy}\left(\frac{v^2}{2g}\right)\right]$$

$$\therefore \frac{dy}{dx} = \frac{S_o - S_e}{1 + \frac{d}{dy}\left(\frac{v^2}{2g}\right)} \quad \ldots\ldots (1)$$

Here, $\frac{d}{dy}\left(\frac{v^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = -\frac{Q^2}{gA^3}\left(\frac{dA}{dy}\right) = -\frac{Q^2T}{gA^3}$

from eq^n (1) we obtain, $\frac{dy}{dx} = \frac{S_o - S_e}{1 - \frac{Q^2T}{gA^3}}$

This equation is known as dynamic equation of G.V.F

Deduce the dynamic equation for G.V.F in following form

$$\frac{dy}{dx} = S_0 \frac{1-\left(\frac{K_n}{K}\right)^2}{1-\left(\frac{Z_c}{Z}\right)^2}$$

05, 07, 10, 14, 16

$\boxed{\text{First}}$ Derive – $\frac{dy}{dx} = \frac{S_0 - S_e}{1 - \frac{Q^2 T}{g A^3}}$ ...... ①

$\boxed{\text{Then}}$

Let. conveyance at any depth $y \Rightarrow K = \frac{Q}{\sqrt{S_e}}$

conveyance at depth $y_0 \Rightarrow K_0 = \frac{Q}{\sqrt{S_0}}$

$$\therefore \frac{K_0}{K} = \sqrt{\frac{S_e}{S_0}} \Rightarrow \left(\frac{K_0}{K}\right)^2 = \frac{S_e}{S_0}$$

Similarly, section factor ~~for~~ at depth $y \Rightarrow Z = \frac{A^3}{T}$

section factor at critical depth $y_c \Rightarrow Z_c = \frac{A_c^3}{T_c} = \frac{Q^2}{g}$

$$\therefore \frac{Z_c^2}{Z^2} = \frac{Q^2}{g} \times \frac{T}{A^3} = \frac{Q^2 T}{A^3 g}$$

From eqn ① we obtain, $\frac{dy}{dx} = \frac{S_0\left(1 - \frac{S_e}{S_0}\right)}{1 - \frac{Q^2 T}{g A^3}}$

$$\therefore \frac{dy}{dx} = \frac{S_0\left[1-\left(\frac{K_0}{K}\right)^2\right]}{\left[1-\left(\frac{Z_c}{Z}\right)^2\right]}$$

Explain the behavior of flow profile when (i) $y \to y_0$ (ii) $y \to y_c$ (iii) $y \to \alpha$ 10

1. As $y \to y_0$, $V \to V_0$, $S_e = S_0$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - Fr^2} = \frac{0}{1 - Fr^2} = 0$$

The water surfaces approaches the normal depth line asymptotically.

2. As $y \to y_c$, $Fr^2 = 1$, $1 - Fr^2 = 0$

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - Fr^2} = \frac{S_0 - S_e}{0} = \alpha$$

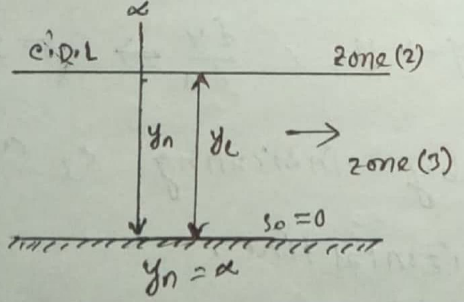
The water surface meets the critical depth line vertically.

3. A $y \to \alpha$, $V = 0 \to Fr = 0 \to S_e \to 0$

$$\lim_{y \to \alpha} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - Fr^2} = \frac{S_0}{1} = S_0$$

The water surface meets a large depth as a horizontal asymtote.

Explain why H_1 and A_1 profile are not practically possible. 16, 15, 10



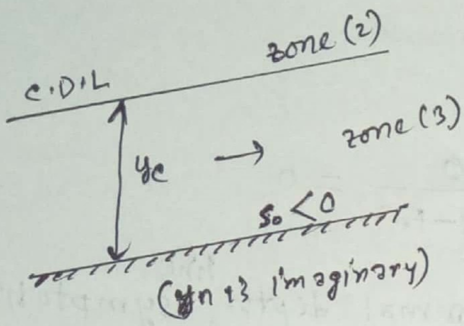
For horizontal channels, $S_0 = 0$

so, $K_n = \frac{Q}{\sqrt{S_0}} = \alpha$ and $y_n = \alpha$

Zone(1) : space above upper line

$y > y_n$ and $y > y_c$

so, zone 1 does not exist and H 1 profile is not practically possible.



For adverse slope channel, $S_0 < 0$

So K_n^2 is negative and y_n is imaginary

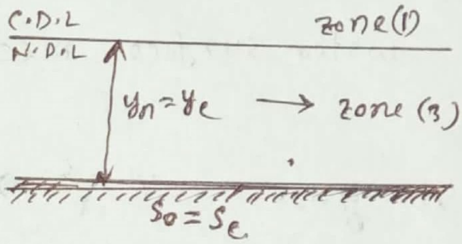
Zone(1): $y > y_n > y_c$

So, Zone 1 cannot exist and A_1 profile is not practically possible.

Show that C_1 and C_3 profiles are approximately horizontal.

2011

in a critical-sloped channel, only two zones (1) and (3) exist and Hence C_1 and C_3 curves will be formed.



In the first zone,

when $y > y_c = y_n$, $\frac{dy}{dx} = (+ve)$ and $y \to y_c$, $\frac{dy}{dx} \to S_0 = S_c$

and as $y \to \infty$, $\frac{dy}{dx} \to S_0 = S_c$ there by indicating that C_1 curve will be more or less a horizontal line.

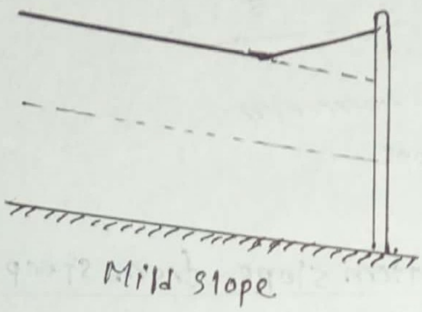
In third zone,

when $y < y_c = y_n$, $\frac{dy}{dx} = (-ve)$ and $y \to y_c$, $\frac{dy}{dx} \Rightarrow S_0 = S_c$

and as $y \to 0$, $\frac{dy}{dx} \to S_0 = S_c$, thereby indicating C_3 curve will also be more or less a horizontal line.

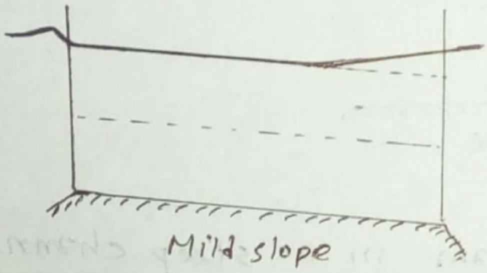
Flow profile:

1. The profile behind a dam in a natural river: 06, 07, 09, 12



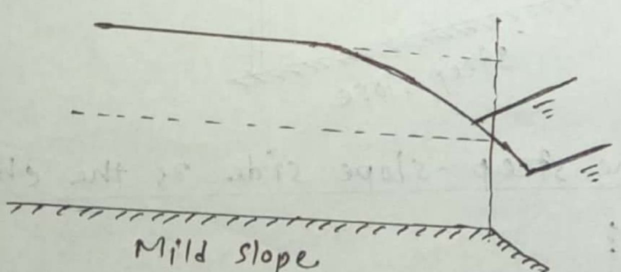
Type: M_1

2. The profile in a canal joining to a reservoir: 05



Type: M_1

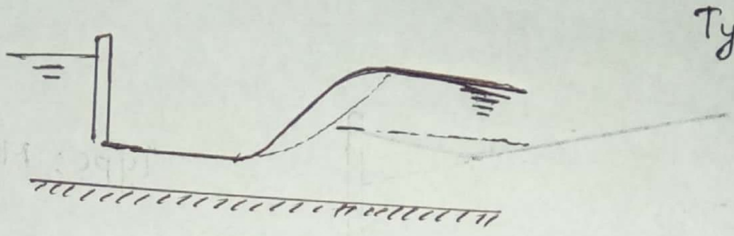
3. The profile in a canal leading to a reservoir: 05



Type: M_2

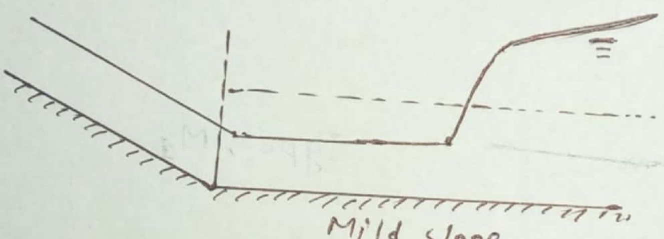
4. The profile in a stream below a sluice: 08, 09, 12, 17

Type: M_3



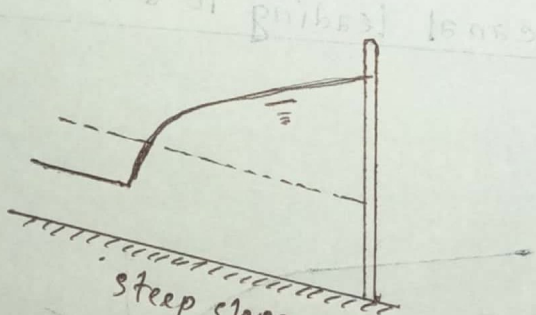
5. The profile after the change in bottom slope from steep to Mild: 05

Type: M_3



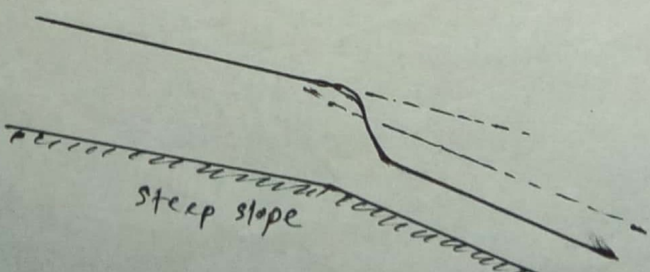
6. The profile flow behind a dam in a steep channel: 08, 09

Type: S_1

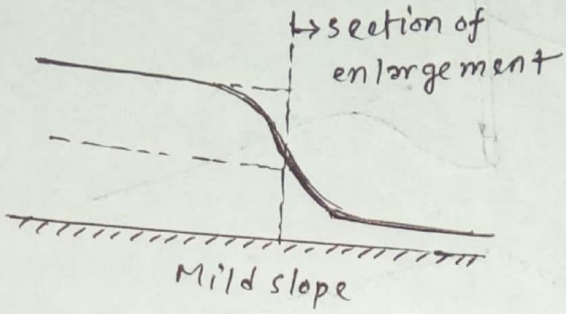


7. The profile on the steep-slope side as the channel slope: 08
changes to steeper:

Type: S_2

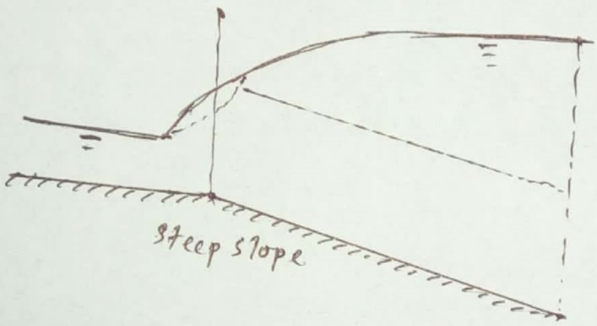


8. The profile at the upstream side of a sudden enlargement of a canal cross-section: 14.05



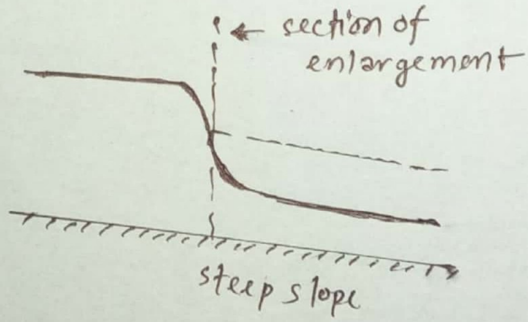
Type: M_2

9. The profile in a steep canal emptying into a pool of high elevation:



Type: S_1

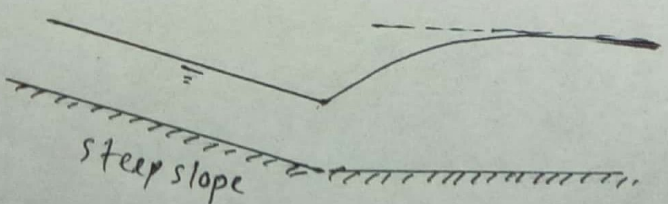
10. The profile formed on the downstream side of an enlargement of channel section:



Type: S_2

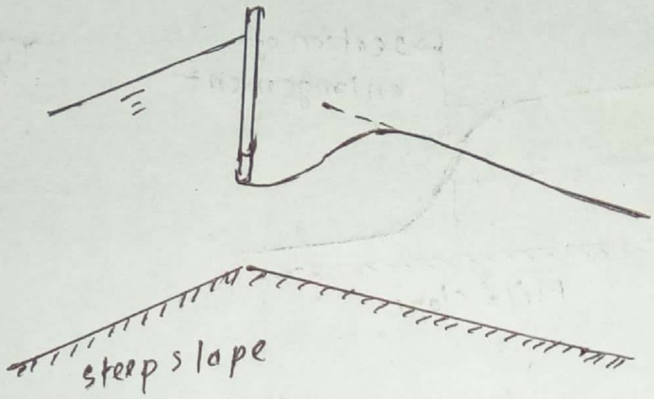
11. The profile on the steep-slope as the channel slope changes from steep to milder steep:

Type: S_3

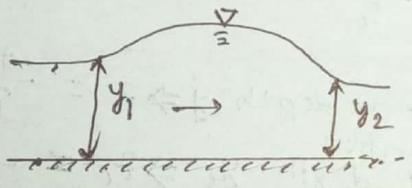
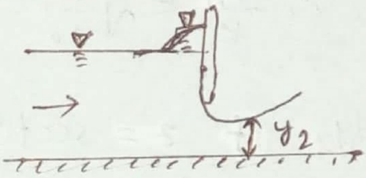


18. The profile below a sluice gate with the depth of the entering flow less than the normal depth on a steep slope:

Type: S_3



Difference between G.V.F and R.V.F: 13, 11

G.V.F	R.V.F
(i) Depth of flow will be changed gradually.	(i) Depth of flow will be changed rapidly.
(ii) Frictional resistance plays an important role in G.V.F	(ii) Frictional resistance is in-significant for R.V.F
(iii) Example: Flow upstream of dam.	(iii) Examples: Hydraulic jump below a sluice gate
(iv) 	(iv) 

MATH

(SUM SIR)

2017, 2014

Velocity distribution in a semi-circular channel of diameter $2R_o$ follows the law of $\frac{u}{U_o} = \left(\frac{y}{R_o}\right)^{\frac{1}{2}}$ in which y is the distance normal to the surface at which the velocity is u and U_o is the velocity at center of the semicircle if $R_o = 2m$ and $U_o = 2 m/sec$, find U, α and β, where U, α, and β indicate usual meaning.

Solution: Given,

$$\frac{u}{u_o} = \left(\frac{y}{R_o}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{u}{2} = \left(\frac{y}{2}\right)^{\frac{1}{2}}$$

$$\Rightarrow u = \sqrt{2}\, y^{\frac{1}{2}}$$

Now,

$$U = \frac{1}{R_o}\int_0^{R_o} u\,dy = \frac{1}{2} \times \sqrt{2}\int_0^2 y^{\frac{1}{2}}\,dy = \frac{\sqrt{2}}{2} \times \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2$$

$$\therefore U = \frac{\sqrt{2}}{2} \times \frac{2}{3} \times 2^{\frac{3}{2}} = \frac{4}{3}$$

$$\alpha = \frac{1}{U^3 R_o}\int_0^{R_o} u^3\,dy = \frac{1}{2\times\left(\frac{4}{3}\right)^3} \times (\sqrt{2})^3 \int_0^2 y^{\frac{3}{2}}\,dy = \frac{3^3}{4^3} \times \frac{(\sqrt{2})^3}{2} \times \frac{2}{5} \times \left[2^{\frac{5}{2}}\right]$$

$$\therefore \alpha = 1.35$$

$$\beta = \frac{1}{U^2 R_o}\int_0^{R_o} u^2\,dy = \frac{1}{\left(\frac{4}{3}\right)^2 \times 2} \times (\sqrt{2})^2 \int_0^2 y\,dy = \left(\frac{3}{4}\right)^2 \times 2 = 1.125$$

(Ans.)

2016

The velocity distribution in a wide rectangular channel may be approximated, by the equation $u = 0.4 + \frac{0.6y}{h}$ (m/s). Find U, α and β, if $h = 1$ m.

Solution:

$$U = \frac{1}{h}\int_0^h u\,dy = \frac{1}{1}\int_0^1 \left(0.4 + \frac{0.6y}{h}\right) dy = 0.7$$

$$\alpha = \frac{1}{U^3 h}\int_0^h u^3 dy = \frac{1}{(.7)^3} \times \int_0^1 \left(0.4 + \frac{0.6y}{1}\right)^3 dy = 1.18$$

$$\beta = \frac{1}{U^2 h}\int_0^h u^2 dy = \frac{1}{(.7)^2} \times \int_0^1 (0.4 + 0.6y)^2 dy = 1.06$$

(Ans.)

2015, 12

The following pressures were measured on a wall. Find the force per unit length of wall, pressure co-efficient at the base of wall and force in excess of the hydrostatic value.

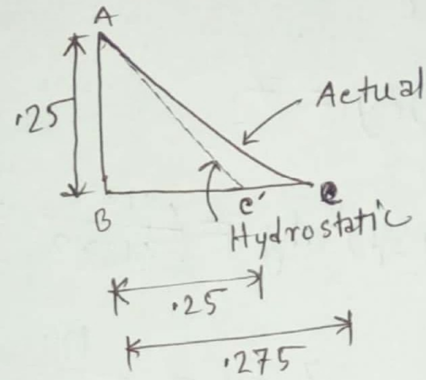
Distance below free surface (mm)	pressure of water (mm)
0	0
50	51
100	104
150	159
200	216
250 (base of wall)	275

Solution: $p = \gamma h = (9810 \times 0.275) = 2697.75\ N/m^2$

we know, $p = k \rho g h \Rightarrow k = \frac{p}{\rho g h} = \frac{2697.75}{1000 \times 9.81 \times .250}$

$\therefore k = 1.1$

$k > 1$, Hence, The flow is concave flow.



Force per unit length = Area of ABC $= \frac{1}{2} \times (9810 \times .275) \times .25$

$= 337.22\ N/m$

Excess force of hydrostatic value = Area of ABC − Area of ABC'

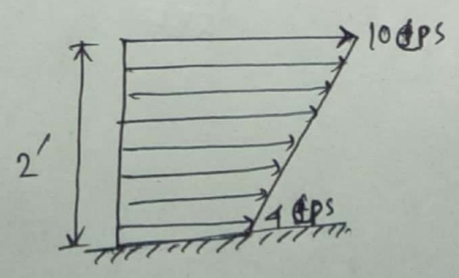
$= 337.22 - \frac{1}{2} \times (9810 \times .25) \times .25$

$= 30.65\ N/m$

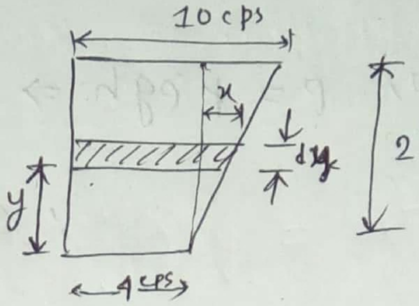
(Ans.)

2012

Determine velocity distribution co-efficient α and β for a wide open channel flow with velocity distribution shown in figure;



Solution: considering an elementary part of velocity distribution :



$$\frac{x}{y} = \frac{10-4}{2}$$

$$\Rightarrow x = 3y$$

$$\therefore u = 4 + x = 4 + 3y$$

$$\therefore U = \frac{1}{2}\int_0^2 (4+3y)\,dy = 7$$

$$\alpha = \frac{1}{7^3 \times 2} \times \int_0^2 (4+3y)^3\,dy = \frac{812}{343 \times 2} = 1.184$$

and $$\beta = \frac{1}{7^2 \times 2} \times \int_0^2 (4+3y)^2\,dy = \frac{104}{49 \times 2} = 1.06$$

(Ans.)

2017, 2015

A low dam 5 ft high having a board horizontal crest built with a rectangular channel 20 ft wide. Assuming that a depth of 2.5 ft measured on the crest is the critical depth. compute the discharge and depth of flow upstream from the dam.

Solution: Here, $h = 5$ ft

$b = 20$ ft

$y_c = 2.5$ ft

we know, $V_c = \sqrt{g y_c}$ (∵ for rectangular channel, $D = y_c$)

$V_c = \sqrt{(32.2 \times 2.5)} = 8.972$ ft/sec.

Now,

$Q = AV_c = b y_c V_c$

$\therefore Q = (20 \times 2.5 \times 8.972) = 448.6$ ft^3/sec. (Ans.)

$E_1 = (E_c + h) = 1.5 y_c + h = (1.5 \times 2.5) + 5 = 8.75$ ft

Again,

$$E_1 = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}$$

$$\Rightarrow 8.75 = y + \frac{(448.6)^2}{2 \times 32.2 \times y^2 \times 20^2}$$

$$\Rightarrow y + \frac{7.81}{y^2} = 8.75$$

$$\Rightarrow y^3 - 8.75y^2 + 7.81 = 0$$

$$\therefore y = 8.6455 \text{ ft}$$

(Ans.)

2015

Find the critical depth for a specific energy head of 1.5 m in a trapezoidal channel 2.0 m bottom width and side slope of 1:1.

<u>solution:</u> We know,

$$E_c = y_c + \frac{V_c^2}{2g}$$

$$\Rightarrow E_c = y_c + \frac{Q^2}{2g A_c^2}$$

$$\Rightarrow E_c = y_c + \frac{A_c}{2T_c} \qquad \left[\because \frac{Q^2}{g} = \frac{A_c^3}{T_c}\right]$$

$$\Rightarrow 1.5 = y_c + \frac{(2+2+2y_c) \times y_c \times .5}{2 \times (2+2y_c)}$$

$$\Rightarrow 1.5 = y_c + \frac{(2+y_c)\, y_c}{2 \times (2+2y_c)}$$

$$\Rightarrow 3 \times (2+2y_c) = 2y_c(2+2y_c) + 2y_c + y_c^2$$

$$\Rightarrow 6 + 6y_c = 4y_c + 4y_c^2 + 2y_c + y_c^2$$

$$\Rightarrow 5y_c^2 = 6$$

$$\therefore y_c = 1.095 \text{ m}$$

(Ans:)

2014

A rectangular channel 8 ft wide, carrying 100 cfs at a depth of 0.5 ft is connected by a straight wall transition to a channel 10 ft wide, flowing at a depth of 4 ft. Determine the flow profile in the transition if the frictional loss through the transition is negligible. If a ump occurs in the transition how can it be eliminated.

Solution: In upstream, $E = y + \frac{Q^2}{2gA^2} = 0.5 + \frac{100^2}{2 \times 32.2 \times (8 \times .5)^2}$

$\therefore E = 10.207$ ft

In down stream, $E = 4 + \frac{100^2}{2 \times 32.2 \times (4 \times 10)} = 4.097$ ft

$\therefore$ Energy difference, $\Delta E = (10.207 - 4.097) = 6.110$ ft

For upstream, $Fr = \frac{V}{\sqrt{gD}} = \frac{100/(8 \times .5)}{\sqrt{32.2 \times .5}} = 6.23 > 1$

Hence, supercritical flow.

For down stream, $Fr = \frac{100/(4 \times 10)}{\sqrt{32.2 \times 4}} = 0.22 < 1$

Hence, subcritical flow.

It indicates that the flow is from supercritical to subcritical.

Therefore, a hydraulic jump can be expected to occur to dissipate the energy difference.

The low stage y_1 for various section can be computed by,

$$10.207 = y_1 + \frac{100^2}{2 \times 32.2 \times b^2 \times y_1^2}$$

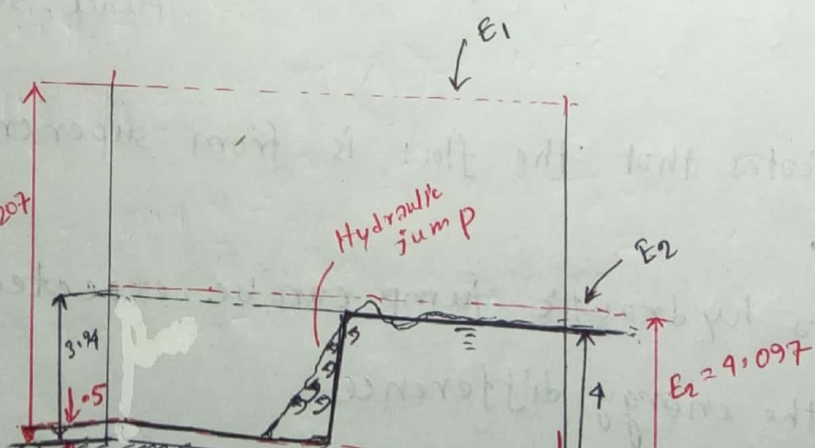
Similarly,

For high stage y_2 can be computed from,

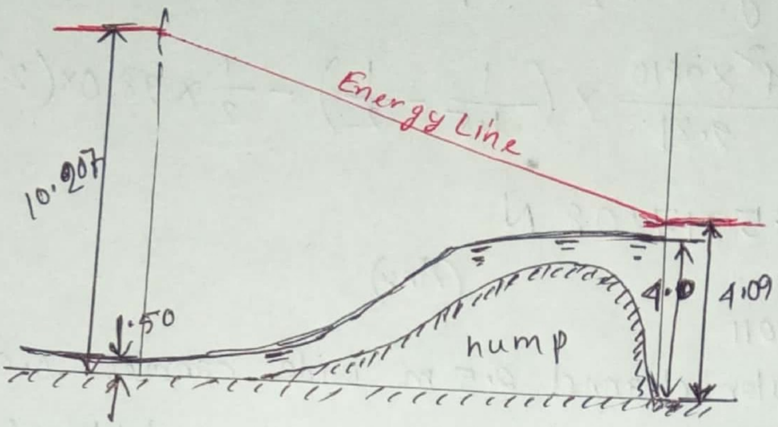
$$4.097 = y_2 + \frac{100^2}{2 \times 32.2 \times b^2 \times y_2^2}$$

Section width b, ft	Low stage y_1	F_1	High stage y_2	F_2
8.0	0.5	78.6	3.940	71.9
8.5	.47	78.7	3.960	75.9
9.0	.443	78.8	3.979	79.9
9.5	.419	78.8	3.987	83.6
10.0	.398	78.8	4.00	87.8

Flow profile:

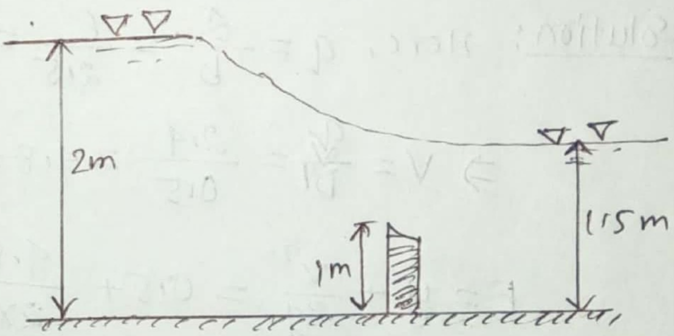


The jump canbe eliminated by raising the bottom of the transition, or humped.



2010

Figure shows a sharp crested weir in a rectangular channel. If The discharge per unit width of the weir is 4 m²/sec. estimate the energy loss due to weir and force on the weir plate for the submerged flow condition as shown in below:



Solution: Here, $Q = 4\ m^2/sec.$

$h_1 = 2m$

$h_2 = 1.5\ m$

Applying bernoulli's equation,

$$z_1 + h_1 + \frac{v_1^2}{2g} = z_2 + h_2 + \frac{v_2^2}{2g} + h_f \qquad [z_1 = z_2]$$

$$\Rightarrow h_1 - h_2 + \frac{v_1^2 - v_2^2}{2g} = h_f \qquad [b = 1]$$

$$\Rightarrow h_f = 2 - 1.5 + \frac{(\frac{4}{2})^2 - (\frac{4}{1.5})^2}{2 \times 9.81} = 0.34\ m$$

Now, Applying momentum equation,

$$\frac{Qw}{g}(v_2 - v_1) = F_1 - F_2 - F_f$$

$$\cancel{\Rightarrow F_f} = \frac{\cancel{4 \times 9810}}{\cancel{9.81}} \times \cancel{\left(\frac{1}{2}wh_2^2 - \frac{1}{2}wh_1^2\right)} -$$

$$\Rightarrow F_f = \frac{Qw}{g}\left(\frac{Q}{h_2} - \frac{Q}{h_1}\right) - \frac{1}{2}wh_1^2 + \frac{1}{2}wh_2^2$$

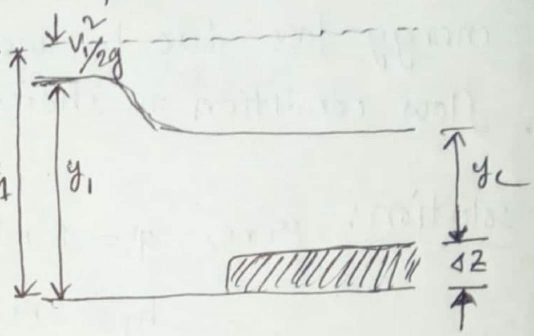
$$\Rightarrow F_f = \frac{4^2 \times 9810}{9.81} \times \left(\frac{1}{1.5} - \frac{1}{2}\right) - \frac{1}{2}\times 9810 \times (2^2 - 1.5^2)$$

$$\therefore F_f = -5917.08 \text{ N}$$

(Ans)

2007, 06, 2009, 2011

A rectangular channel 2.5 m wide carries 6.0 m3/s of flow at a depth of 0.50 m. Calculate the height of ~~to~~ a flat tapped hump required to be placed at a section to cause critical flow. The energy loss due to the obstruction by the hump can be taken as 0.1 times of the upstream velocity head.



Solution: Here, $q = \frac{Q}{b} = \frac{6}{2.5} = 2.4 \text{ m}^3/\text{s}$

$$V = \frac{q}{D} = \frac{2.4}{0.5} = 4.8 \text{ m/s}$$

$$E = y + \frac{v^2}{2g} = 0.5 + \frac{4.8^2}{2\times 9.8} = 1.674 \text{ m}$$

Now,

$$y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left[\frac{(2.4)^2}{9.8}\right]^{\frac{1}{3}} = 0.837$$

$$E_c = 1.5\, y_c = (1.5 \times .837) = 1.256$$

$$\text{Energy loss} = 0.1 \times \frac{v^2}{2g} = 0.1 \times \frac{4.8^2}{2\times 9.8} = 0.117$$

$$E = E_c + \Delta z + \text{Energy loss}$$

$$\Rightarrow \Delta z = (1.674 - 1.256 - 0.117) = 0.30 \text{ m}$$

If the hump hight is higher than 0.30 m, then critical flow will form. If lower than 0.30 m, critical flow will not form.

(Ans)

Computation of Uniform flow

2016, 2013

Compute the conveyance and velocity distribution co-efficient of a channel section 500 ft downstream from a natural stream channel consisting of a main section and an overflowed side section. The data obtained at the peak stage one as given below:

Sub section	A, ft^2	P, ft	n	α	β
Main section	5320	205	0.035	1.12	1.05
Sid section	5670	408	0.040	1.10	1.04

Solution: Here, The computations are given below:

Subsection	A, ft	P, ft	$R=\frac{A}{P}$	$R^{\frac{2}{3}}$	n	$K=\frac{1.49}{n}AR^{\frac{2}{3}}$	$\beta K^2/A$	$\alpha K^3/A^2$
Main section	5320	205	25.95	8.77	0.035	1.99×10^6	7.82×10^8	3.12×10^{11}
side section	5670	408	13.9	5.78	0.040	1.22×10^6	2.73×10^8	6.21×10^{10}
	$\Sigma=10990$					$\Sigma=3.21\times10^6$	$\Sigma=10.55\times10^8$	$\Sigma=3.74\times10^{11}$

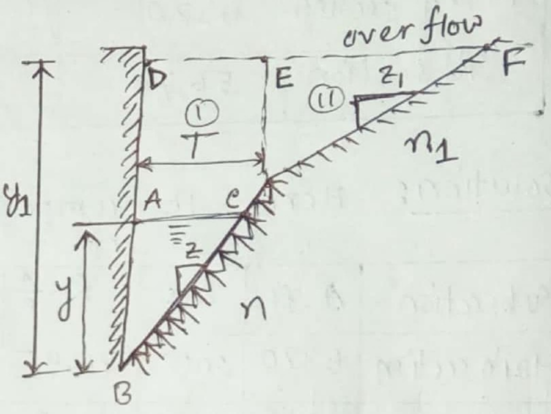
We know, $$\alpha = \frac{\Sigma(\alpha K^3/A^2)}{(\Sigma K)^3/(\Sigma A)^2} = \frac{3.74\times10^{11}}{(3.21\times10^6)^3/(10990)^2} = 1.366$$

$$\beta = \frac{\Sigma(\beta K^2/A)}{(\Sigma K)^2/(\Sigma A)} = \frac{10.55\times10^8}{(3.21\times10^6)^2\times 10990} = 1.125$$

(Ans.)

2013

\# compute the discharge in an over flowed highway gutter as shown in figure below, having a depth of flow of 3 in. and a longitudinal slope of 0.03. The gutter is made of concrete with $n = 0.017$ and has a triangle section with a verticle curb side, a sloped side of $z = 12$ and a top width of $T = 2$ ft. The overflowed soil aggregate pavement has a cross slope of $z_1 = 24$ and $n_1 = 0.02$



Solution: Here, $y_1 = 3$ inch.

$S = 0.03$

$n = 0.017$, $n_1 = 0.02$

$z = 12$, $z_1 = 24$

$T = 2$ ft $= 24$ in.

From given slope, $\frac{y}{T} = \frac{1}{12} \Rightarrow y = \frac{24}{12} = 2$ in.

For section ①, $A_1 = \frac{1}{2} \times AB \times AC + AC \times AD$

$\therefore A_1 = \frac{1}{2} \times 2 \times 24 + 24 \times (3-2) = 48 \text{ in}^2$

$\frac{EF}{EC} = \frac{z_1}{1} \Rightarrow EF = 24$ in.

Now, $A_2 = \frac{1}{2} \times EF \times EC = (\frac{1}{2} \times 24 \times 1) = 12 \text{ in}^2$

$\therefore A = (A_1 + A_2) = (48 + 12) = 60 \text{ in}^2 = 0.4167 \text{ ft}^2$

~~P = (AD + BC + CF) = 3 + √(2² + 24²) + √(1² + 24²) = 4.25~~

$P_1 = (AD + BC) = 3 + \sqrt{2^2 + 24^2} = 27.08 \text{ in} = 2.257$ ft

$P_2 = CF = \sqrt{1^2 + 24^2} = 24 \text{ in} = \mathbf{2}$ **ft**.

$\therefore P = P_1 + P_2 = (2.257 + 2) = 4.257$ ft

Equivalent roughness,

$$n = \left[\frac{P_1 n_1^{1.5} + P_2 n_2^{1.5}}{P}\right]^{\frac{2}{3}}$$

$$\therefore n = \left[\frac{2.257 \times (.017)^{1.5} + 2 \times (.02)^{1.5}}{4.257}\right]^{\frac{2}{3}} = 0.01844$$

$$\therefore R = \frac{A}{P} = \frac{0.4167}{4.257} = 0.0979 \text{ ft}$$

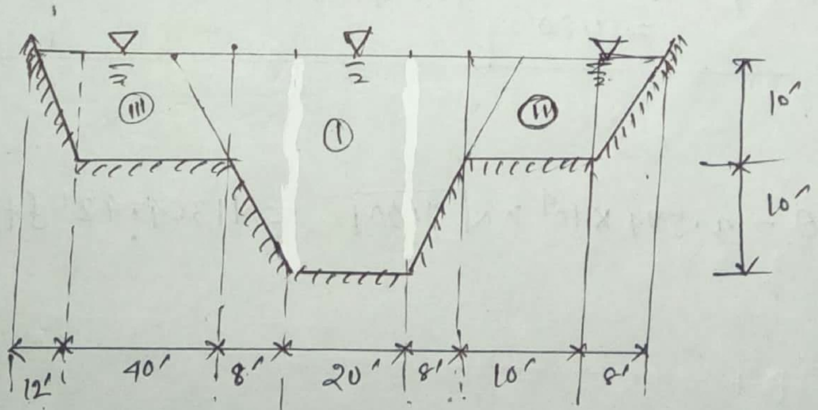
$$Q = \frac{1.49}{n} \times A R^{\frac{2}{3}} S^{\frac{1}{2}} = \frac{1.49}{.01844} \times .4167 \times (.0979)^{\frac{2}{3}} \times (0.03)^{\frac{1}{2}}$$

$$\therefore Q = 1.24 \text{ ft}^3/\text{s}$$

(Ans.)

2014

A channel consists of a main section and two side sections shown in figure below. Compute the total discharge assuming that the main section and the two side sections are seperated by (a) veptical division line and (b) extended sides of the main channel. Given, $n = 0.015$ for main channel, $n = 0.03$ for side channels and $S = 0.001$



Solution: (i) vertical division line: Here $K = \frac{1.49}{n} A R^{\frac{2}{3}}$

Subsection	A, ft^2	P, ft	$R=\frac{A}{P}$	n	K
Main section ①	$[\frac{1}{2}\times(20+36)\times 10]+(36\times 10) = 640$	$2\times\sqrt{10^2+8^2}+20 = 45.61$	14.03	0.025	2.22×10^5
side section ②	$(\frac{1}{2}\times 10\times 8)+(10\times 10) = 140$	$10+\sqrt{10^2+8^2} = 22.80$	6.13	0.03	2.33×10^4
side section ③	$40\times 10 + \frac{1}{2}\times 12\times 10 = 460$	$40+\sqrt{12^2+10^2} = 55.62$	8.27	0.03	9.34×10^4

$\Sigma K = 3.387\times10^5$

We know, $Q = \Sigma K S^{\frac{1}{2}} = 3.387\times10^5 \times \sqrt{.001}$

$\therefore Q = 10710.63\ ft^3/sec.$

(ii) extended side of the main channel:

Subsection	A, ft^2	P, ft	$R=\frac{A}{P}$	n	K
main section ①	$\frac{1}{2}\times(20+36+16)\times 20 = 720$	$2\times\sqrt{8^2+10^2}+20 = 45.61$	15.78	.025	2.70×10^5
side section ⑪	$8\times 10 = 80$	$10+\sqrt{10^2+8^2} = 22.80$	3.50	.03	9174.4
side section ⑪	$40\times10+\frac{1}{2}\times12\times10 = 420$	$40+\sqrt{12^2+10^2} = 55.62$	7.55	.03	8.02×10^4

$\Sigma K = 3.594\times10^5$

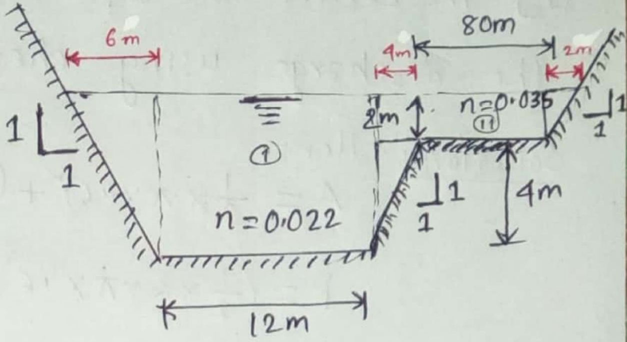
$\therefore Q = 3.594\times10^4 \times\sqrt{0.001} = 11364.42\ ft^3/sec.$

(Ans.)

2015

The cross section of the river may be idealized as shown in figure below. Find the discharge carried by the river if $S = 2\times10^{-4}$

Solution:



$$A = (\tfrac{1}{2}\times6\times6)+(12\times6)+(\tfrac{1}{2}\times4\times4)+(2\times84)+(\tfrac{1}{2}\times2\times2) = 268\ m^2$$

$$P_1 = (\sqrt{6^2+6^2}+12+\sqrt{4^2+4^2}) = 26.14\ m$$

$$P_2 = (80+\sqrt{2^2+2^2}) = 82.83\ m$$

$$P = (P_1+P_2) = (26.14+82.83) = 108.97\ m$$

Equivalent roughness, $n = \left[\dfrac{P_1 n_1^{1.5} + P_2 n_2^{1.5}}{P}\right]^{\frac{2}{3}}$

$$= \left[\frac{26.14\times(.022)^{1.5}+82.83\times(.035)^{1.5}}{108.97}\right]^{\frac{2}{3}}$$

$$= 0.032$$

$$R = \frac{A}{P} = \frac{268}{108.97} = 2.46\ m$$

Now, $Q = \dfrac{1}{n}\times A R^{\frac{2}{3}} S^{\frac{1}{2}} = \dfrac{1}{.032}\times268\times(2.46)^{\frac{2}{3}}\times(2\times10^{-4})^{\frac{1}{2}}$

$\therefore Q = 215.84\ m^3/s$

(Ans.)

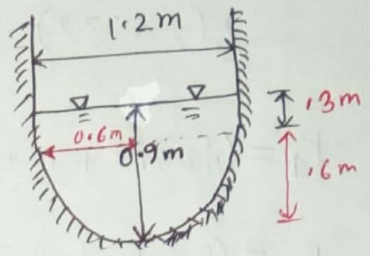
Development of uniform flow

2008

A channel has virtical walls 1.2 m apart and a semicircular invert. If the center line depth is 0.9 m and the bed slope is 1 in. 2000, Find the discharge using chezy's formula with $C = 54$.

Solution: Here,

$A = \frac{1}{2} \times \pi \times (.6)^2 + (.3 \times 1.2) = 0.9255 \text{ m}^2$



$P = \left(\frac{1}{2} \times 2 \times \pi \times .6\right) + (2 \times .3) = 2.485 \text{ m}$

$\therefore R = \frac{A}{P} = \frac{.9255}{2.485} = 0.37 \text{ m}$

We know, $V = C\sqrt{RS} = 54 \times \sqrt{0.37 \times \frac{1}{2000}} = 0.7345 \text{ m/s}$

$\therefore Q = AV = (0.9255 \times 0.7345) = 0.68 \text{ m}^3/\text{s}$

(Ans.)

2011

Design of channel

Design a trapizodal channel laid on a slope of 0.0016 and carrying a discharge of 400 cfs. The channel is to be excavated in earth containing non coloidal coarse gravels and pebbles, 25% of which is 1.25 in or over in diameter. Manning's $n = 0.025$

Solution: Here, $z = 2$ (Assume) $\therefore \tan\phi = \frac{1}{2} \Rightarrow \phi = 26.5°$

For very rounded material 1.25 in. dia. the angle of repose, $\theta = 33.5°$

$\therefore$ Tractive ratio, $K = \sqrt{1 - \frac{\sin^2\phi}{\sin^2\theta}} = \sqrt{1 - \frac{\sin^2 26.5}{\sin^2 33.5}} = 0.587$

For a size of 1.25 dia, $\tau_L = (0.4 \times 1.25) = 0.5 \text{ lb/ft}^2$

and, $\tau_s = K\tau_L = (0.587 \times 0.5) = 0.294 \text{ lb/ft}^2$

We know,

for trapezoidal channel maximum tractive force on sloping side is, $\tau_s = 0.750\, wyS$

Hence, $.750 \times 62.4 \times .0016\, y = 0.294$

$\Rightarrow y = 3.92$

Now, $A = \frac{(b+b+4y)\times y}{2} = (b+2y)y$

$P = b + 2y\sqrt{1+2^2} = b + 2\sqrt{5}\,y$

Let, base-depth ratio, $\frac{b}{y} = 4$ $\therefore b = 4y$

$\therefore A = 6y^2 = 92.2 \ ft^2$

$P = 8.47 \ y = 33.2 \ ft$

$\therefore R = \frac{A}{P} = \frac{92.2}{33.2} = 2.78 \ ft$

$$\therefore Q = \frac{1.49}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}} = \frac{1.49}{.025} \times 92.2 \times (2.78)^{\frac{2}{3}} \times (.0016)^{\frac{1}{2}}$$

$\therefore Q = 434.5$ cfs which is close to design discharge.

2013, 10

Design the section of channel of a canal to carry a discharge 200 ft^3/s through a land of erodible soils with $n = 0.02$ and $s = 0.002$. Assume other necessary data and use your own judgement.

Solution: Assume, $z = 2$ and permissible velocity, $V = 4.77$ f/s

using manning's formula,

$$V = \frac{1.49}{n} R^{\frac{2}{3}} \sqrt{S}$$

$$4.77 = \frac{1.49}{.020} \times R^{\frac{2}{3}} \times \sqrt{0.002}$$

$$\Rightarrow R^{\frac{2}{3}} = 1.43$$

$$\therefore R = 1.713 \ ft$$

Now, $A = \frac{Q}{V} = \frac{200}{4.77} = 41.93 \ ft^2$

$$P = \frac{A}{R} = \frac{41.93}{1.713} = 24.48 \ ft$$

considering trapezoidal section,

$A = (b + zy)y = (b+2y)y$

and, $P = b + 2y\sqrt{1+z^2} = b + 2\sqrt{5}\,y$

Hence, $(b+2y)\,y = 41.93$ —— (I)

and, $b + 2\sqrt{5}\,y = 24.52$ —— (II)

From (I) & (II), we obtain,

$(24.52 - 2\sqrt{5}y + 2y)\,y = 41.93$

$\Rightarrow 24.52y - 2.47y^2 = 41.93$

$\Rightarrow 2.47y^2 - 24.52y + 41.93 = 0$

$\therefore y = 2.196$ ft

and, $b = (24.52 - 2\sqrt{5} \times 2.196) = 14.7$ ft

(Ans)

THEORY

(MMR SIR)

CE 3121
Engineering Hydraulics

Prepared By
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Impact of Free Jets

Impact of Jets

The jet is a stream of liquid comes out from nozzle with a high velocity under constant pressure. When the jet impinges on plates or vanes, its momentum is changed and a hydrodynamic force is exerted. Vane is a flat or curved plate fixed to the rim of the wheel.

1. **Force exerted by the jet on a stationary plate**
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved
2. **Force exerted by the jet on a moving plate**
   a) Plate is vertical to the jet
   b) Plate is inclined to the jet
   c) Plate is curved

❑ Introduction about impact of jet

- The fluid stream coming from nozzle with high velocity & hence high k.E is known as fluid jet. So impact of jet means the force exerted by the fluid on a plate. The plate may be flat or curved which may be fixed or moving.

- Suppose the jet of liquid comes out of nozzle at velocity V, which is flowing under presssure when strikes to a plate exerts force on the plate

$$\text{impact of jet} = \text{rate of change of momentum}$$

$$= \frac{d(mV)}{dt} = \frac{m(V-0)}{t} = \frac{mV}{t}$$

❑ Introduction about impact of jet

▪

$F = m.V$

$= \rho AV*V$

$= \rho A\,V^2$

, where m = m/t , mass flow rate of fluid kg/sec.

$m = \rho AV.$

A = cross sectional area of jet.

Force exerted on stationary plate held normal to jet.

Consider a jet of water strike normally on the fixed plate held perpendicular to flow direction of jet as shown in fig.the jet after striking the plate will deflected through 90°.so final velocity of fluid in the direction of the jet after striking plate will be zero.

- Force exerted on stationary plate held normal to jet.

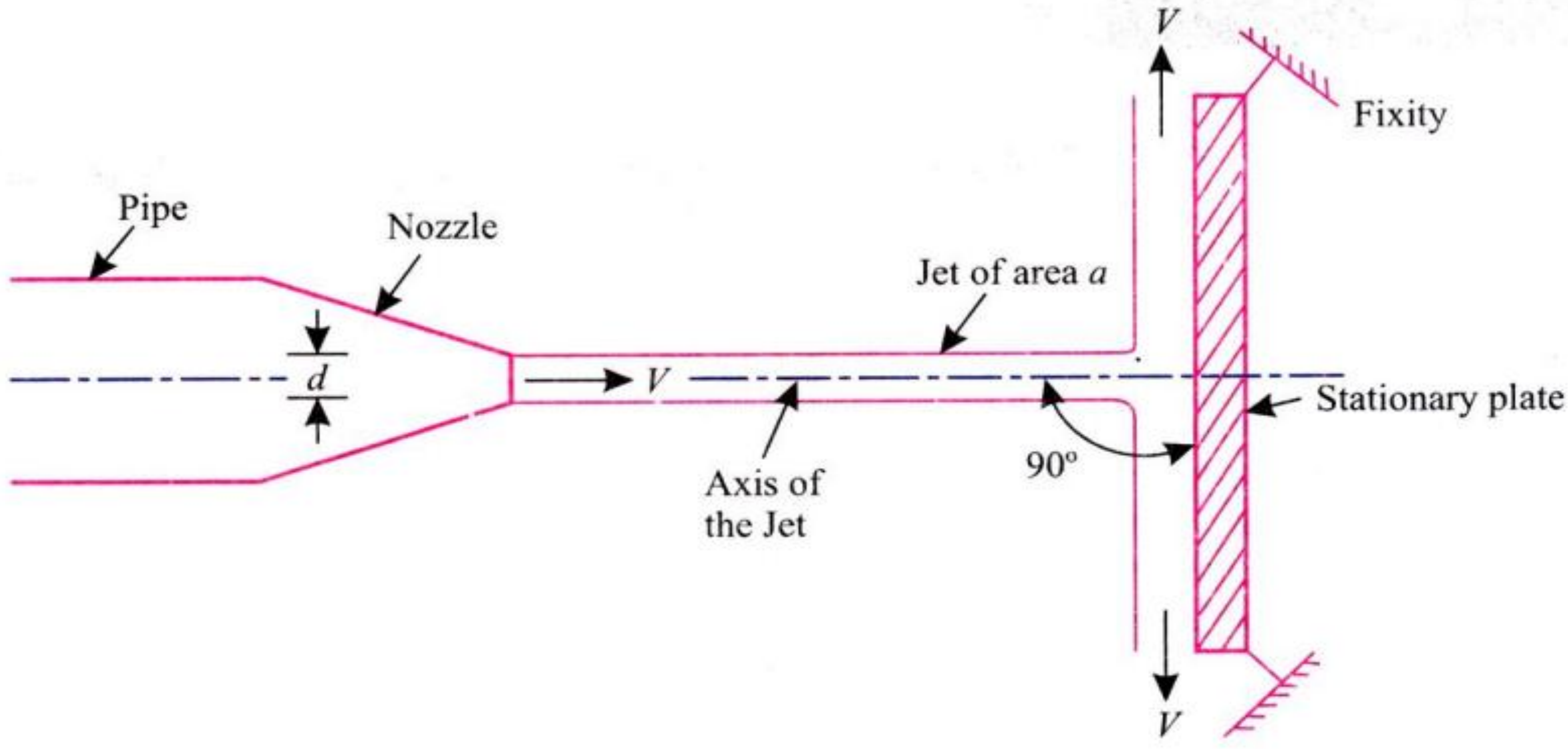


Fig. 2.1 Jet striking a fixed vertical plate

❑ Force exerted on stationary plate held normal to jet.

- Let, V = velocity of jet,

  d = diameter of jet,

  ρ = density of fluid,

  A = cross section area of jet,

  m = mass flow rate of fluid,

❑ Force exerted on stationary plate held normal to jet.

- Impact of jet = rate change of momentum in the direction of force

$$= \frac{initial\ momentum - final\ momentum}{time}$$

$$= \frac{[(mass * initial\ velocity) - (mass * final\ velocity)]}{time}$$

$$= \frac{mass}{time}\ [\text{velocity of jet before striking} - \text{velocity of jet after striking}]$$

$$= \rho AV(V\text{-}0)$$

$$= \rho A\, V^2$$

❑ Force exerted by the jet on moving flat plate.

– Fig. 2.6 shows a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

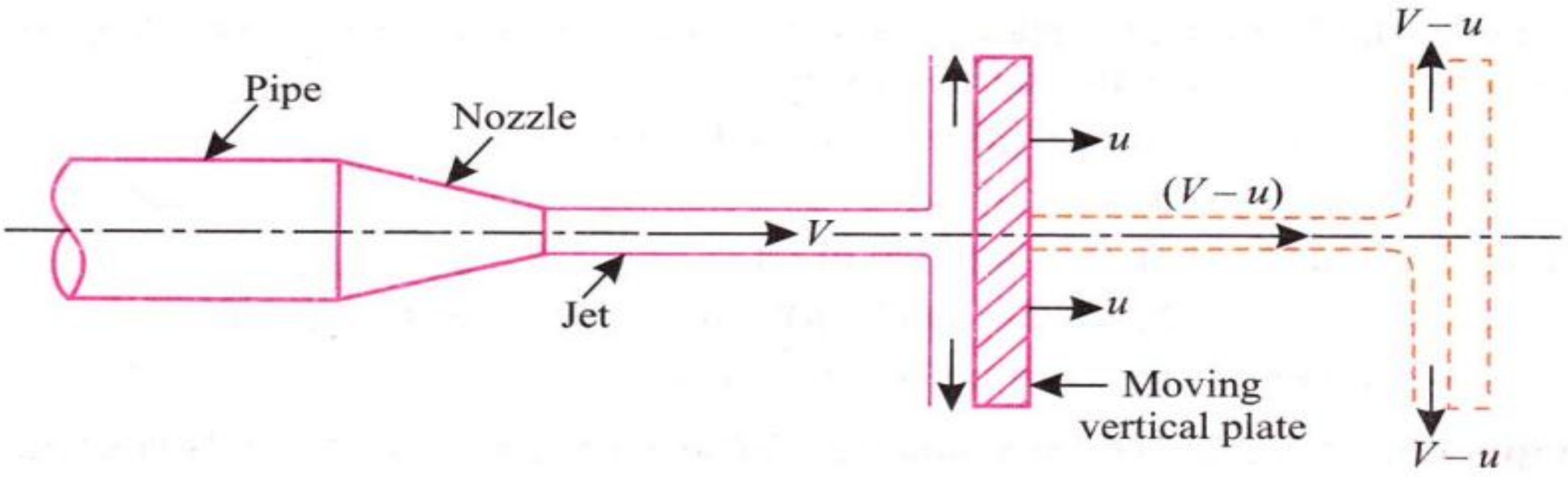


Fig. 2.6 Jet striking a flat vertical moving plate

❑ Force exerted by the jet on moving flat plate.

- Let,

  V = Velocity of the jet (absolute)

  u = Velocity of the flat plate

- In this case jet does not strike the plate with velocity V, but it strikes with a relative velocity (because plate is not stationary).
- Relative velocity of the jet with respect to plate = $(V-u)$

- **Mass of water striking the plate per sec,**

$$\dot{m} = density \times area\ of\ jet \times velocity\ with\ which\ jet\ strikes\ the\ plate$$

$$\dot{m} = \rho a(V-u) ---------(2.10)$$

- **Force exerted** by the jet on the moving plate in the direction of the jet,

$$F_x = \dot{m} \times [Initial\ velocity\ with\ which\ water\ strikes - Final\ velocity]$$

$$F_x = \rho a(V-u) \times [(V-u) - 0]$$

$$F_x = \rho a(V-u)^2 ---------(2.11)$$

❑ Force exerted by the jet on moving flat plate.

– In this case the work will be done by the jet on the plate, as plate is moving. (for the stationary plate, the work done is zero)

– **Work done per second** by the jet on the plate,

$$W = Force \times \left(\frac{distance\ travelled\ in\ the\ direction\ of\ force}{time}\right)$$

$$\therefore W = F_x \times Velocity\ with\ which\ plate\ moved\ in\ the\ direction\ of\ force$$

$$W = F_x \times u$$

$$W = \rho a (V - u)^2 \times u$$

$$W = \rho a u (V - u)^2 - - - - - - - - -(2.12)$$

(Here SI unit of W is Watt because it is work done per sec, i.e. Power)

❑ Force exerted on fixed inclined flat plate by jet.

Consider a jet striking on an incline fixed plate as shown in fig.

Let , V = velocity of jet.

A = cross section area of jet.

Θ = inclination of the plate with the jet.

❑ Force exerted on fixed inclined flat plate by jet.

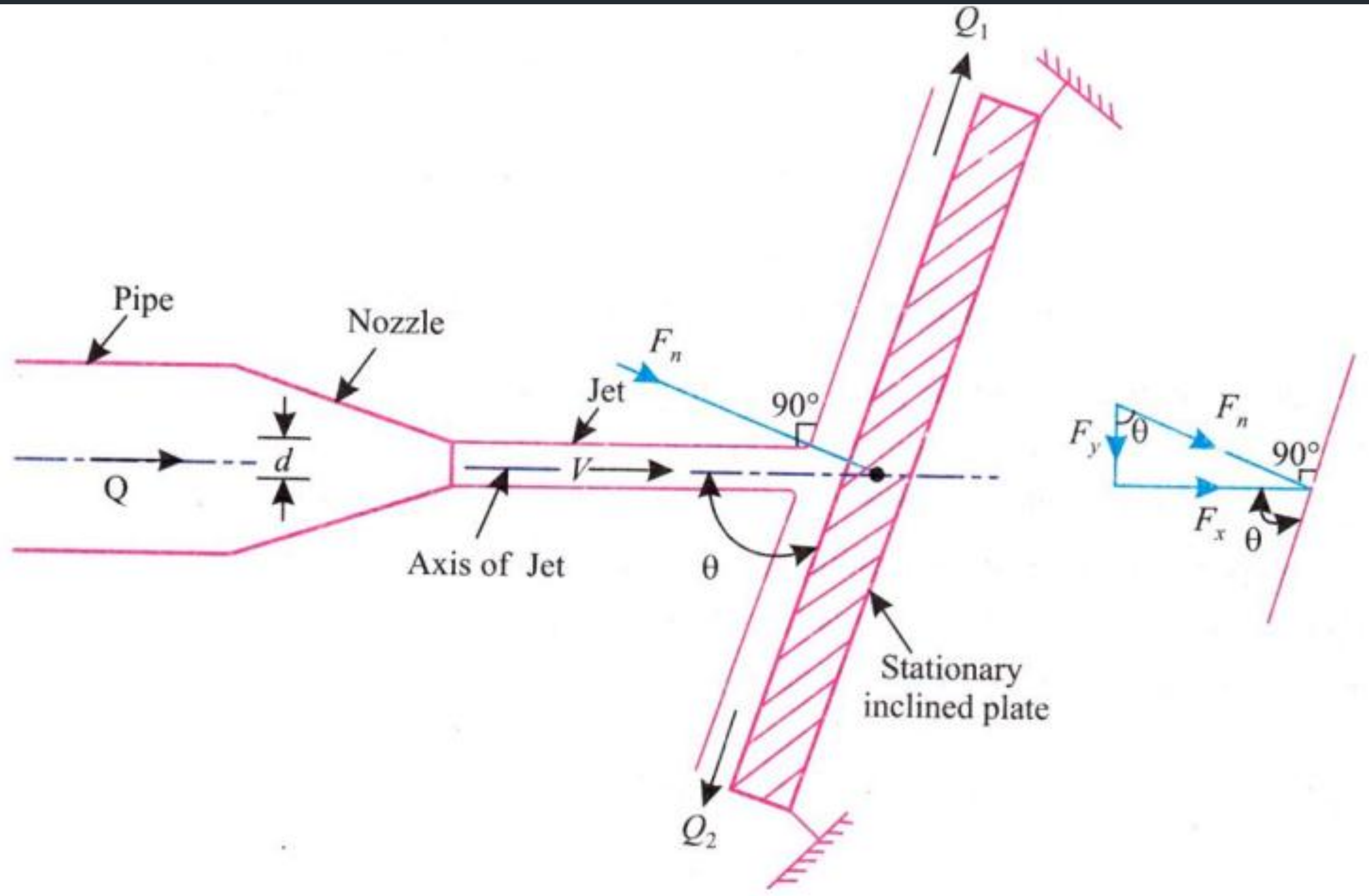


Fig. 2.2 Jet striking a fixed inclined plate

❑ Force exerted on fixed inclined flat plate by jet.

Assuming no loss of energy due to impact of jet then jet will move over the plate after striking with velocity equal to initial velocity of jet.

Let F_n = force exerted by the jet on the plate in direction normal to the plate.

F_x = force exerted by the jet on the plate in direction to the jet

F_y = force exerted by the jet on the plate in direction perpendicular to the jet.

❑ Force exerted on fixed inclined flat plate by jet.

$F_n = \frac{mass}{time}$ [velocity of jet before striking –velocity of jet after striking]

$= \rho AV(V\sin\Theta-0)$

$= \rho A V^2 \sin\Theta$

From fig. $F_x = F_n \cos(90-\Theta)$ & $F_y = F_n \cos\Theta$

$= (\rho A V^2 \sin\Theta) * \cos(90-\Theta)$

$= \rho A V^2 \sin^2\Theta$

$F_y = \rho A V^2 \sin\Theta * \cos\Theta$

If $\Theta = 90°$ then, . $F_x = \rho A V^2$, $F_y = 0$

❑ Force exerted by jet on the inclined plate moving in the direction of jet.

– Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet as shown in Fig. 2.7.

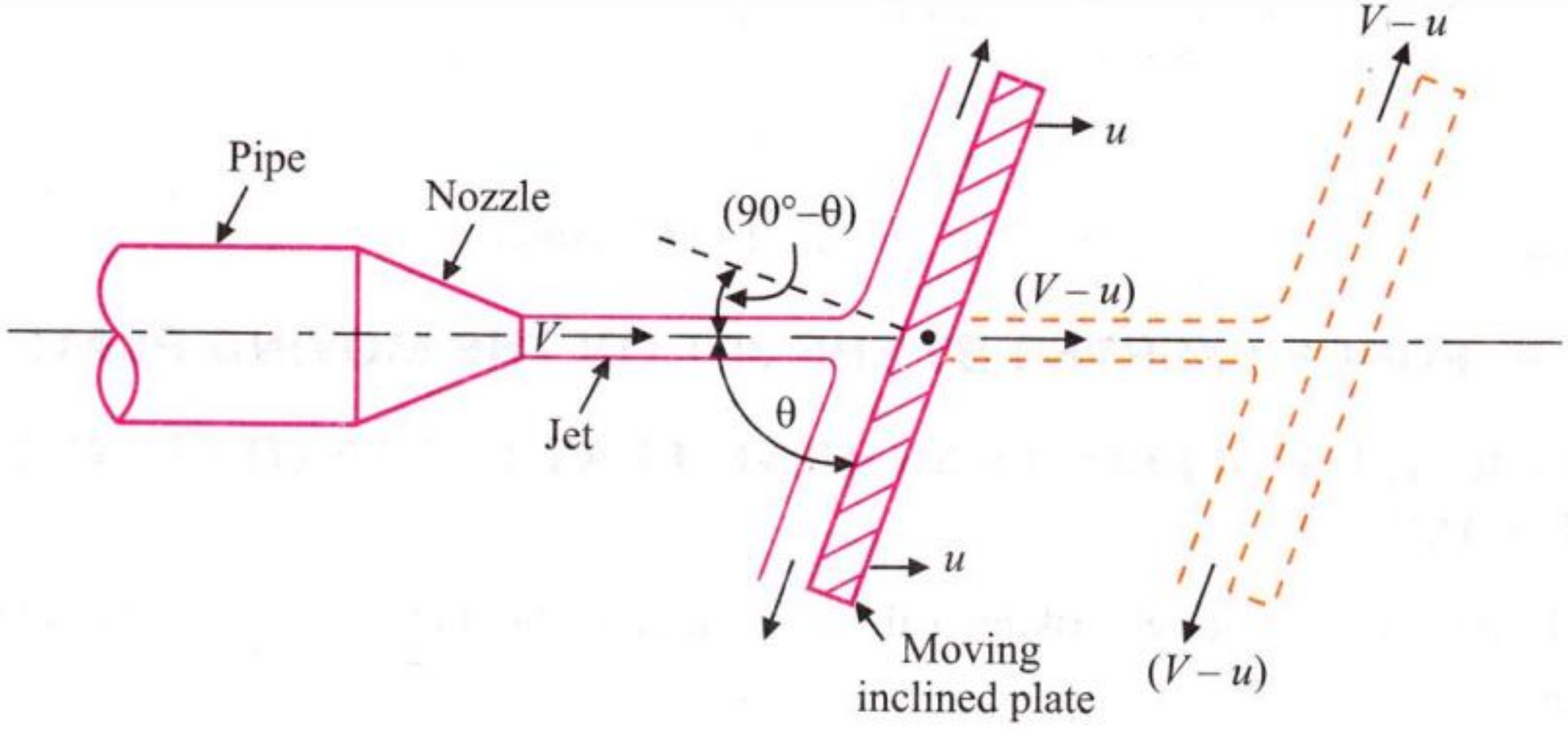


Fig. 2.7 *Jet striking an inclined moving plate*

Let

❑ Force exerted by jet on the inclined plate moving in the direction of jet.

– Let,

V = Absolute velocity of the jet of water

u = Velocity of the flat plate

a = Cross section area of jet

θ = Angle between jet and plate

– Relative velocity of jet of water $= (V - u)$

– Mass of water striking the plate per second,

$$\dot{m} = \rho a(V - u)$$

– If the plate is smooth and loss of energy due to impact of jet is assumed zero, the jet of water will leave the inclined plate with a velocity equals to (V – u).

– Force exerted by the jet of water on the plate in the direction normal to the plate,

$$F_n = \frac{mass}{time} \times [I.V.\,in\ the\ normal\ direction\ with\ which\ jet\ strikes - F.V.]$$

❑ Force exerted by jet on the inclined plate moving in the direction of jet.

$$F_n = \rho a(V-u) \times [(V-u)\, sin\,\theta - 0]$$

$$F_n = \rho a(V-u)^2 \sin\theta - - - - - - - - - -(2.13)$$

– This normal force F_n can be resolved into two components namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$$F_x = F_n \sin\theta = \rho a(V-u)^2 \sin^2\theta - - - - - - - - - -(2.14)$$

$$F_y = F_n \cos\theta = \rho a(V-u)^2 \sin\theta \cos\theta - - - - - - - - - -(2.15)$$

– Work done per second by the jet on the plate,

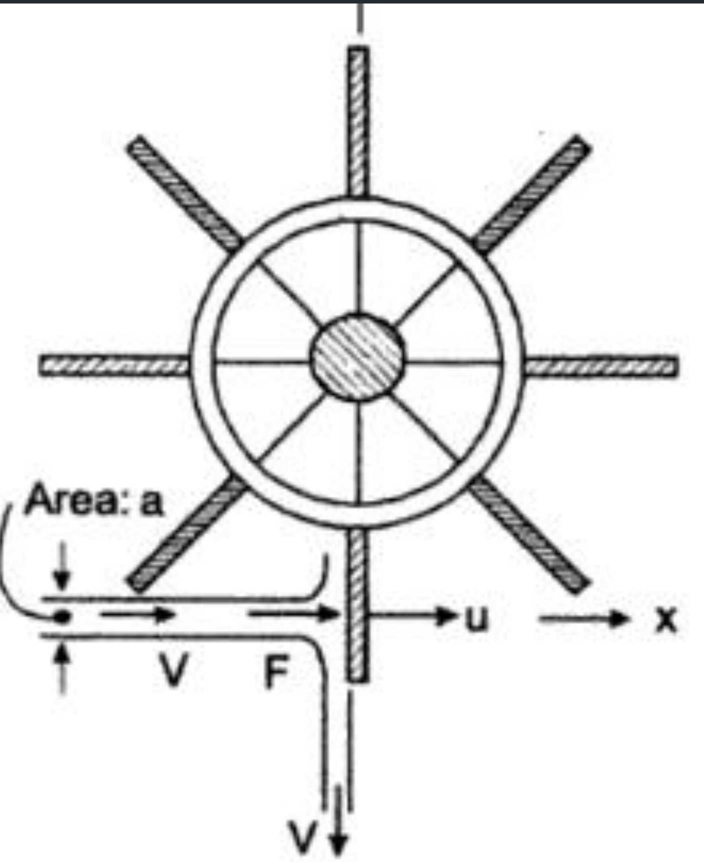
$$W = F_x \times Velocity\ with\ which\ plate\ moved\ in\ the\ X - direction$$

$$W = F_x \times u$$

$$W = \rho a(V-u)^2 \sin^2\theta \times u$$

$$W = \rho au(V-u)^2 \sin^2\theta - - - - - - - - - -(2.16)$$

- Force exerted by jet on series of moving flat vanes



(c) Series of Moving Vanes

Fig 13.2: Impact of Jet on Normal Flat Vanes

Force exerted by jet on series of moving flat vanes

13.3.3 Series of Moving Normal Flat Vanes

As shown in Fig 13.2(c), the vanes are so spaced around the periphery of the wheel that a vane is always in the normal position to receive the jet impact.

Volume rate of flow striking the vane $Q = aV$ and mass rate of flow striking the vane $Q_m = \rho aV$

V_{x_1} = initial fluid velocity at point of impact = $(V-u)$

V_{x_2} = final fluid velocity at exit = 0

$$\therefore \text{ Fluid force on vane: } F = F_x = Q_m(-\Delta V_x) = \rho\, av\,(V-u) \qquad (13.8)$$

Work done by jet per sec = $Fu = \rho\, aVu\,(V-u)$

Energy combined in jet per sec = weight rate of flow

$$\times \frac{V^2}{2g} = Q_m g \frac{V^2}{2g} = Q_m \frac{V^2}{2} = \rho a \frac{V^3}{2}$$

Force exerted by jet on series of moving flat vanes

Hence efficiency of system $\eta = \dfrac{\rho a\, vu\,(v-u) \times 2}{\rho a V^3} = \dfrac{2u\,(v-u)}{V^2}$ (13.9)

For maximum efficiency, dη/du may be equated to zero to obtain optimum wheel velocity.

$$\frac{d\eta}{du} = \frac{2}{V^2}\frac{d}{du}\left(V_u - u^2\right) = 0 \text{ or } V_\mu = u^2 \text{ or } u = V/2.$$

Hence, wheel velocity should be half the jet velocity to get best results.

$$\eta_{max} = \left.\frac{2u(V-u)}{V^2}\right|_{u=V/2} = 0.5 \text{ or } 50\% \qquad (13.10)$$

Problems

- **Problems - 1**

A jet of water 50 mm diameter strikes a flat plate held normal to the direction of jet. Estimate the force exerted and work done by the jet if.

a. The plate is stationary

b. The plate is moving with a velocity of 1 m/s away from the jet along the line of jet.

c. When the plate is moving with a velocity of 1 m/s towards the jet along the same line.

The discharge through the nozzle is 76 lps.

Problems

- **Problems - 3**

A jet of data 75 mm diameter has a velocity of 30 m/s. It strikes a flat plate inclined at 45° to the axis of jet. Find the force on the plate when.

a. The plate is stationary

b. The plate is moving with a velocity of 15 m/s along and away from the jet.

Also find power and efficiency in case (b)

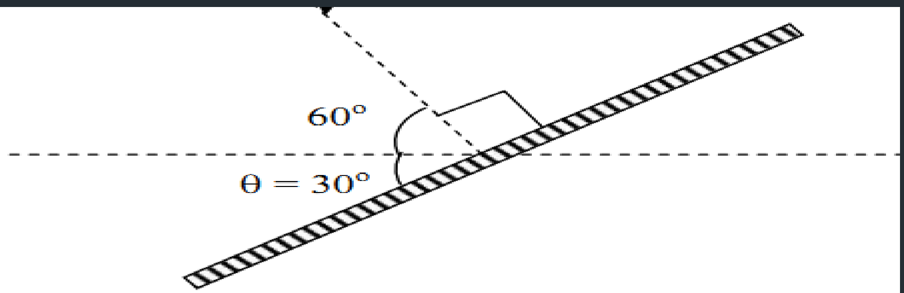
Problems

- **Problem – 4**

A 75 mm diameter jet having a velocity of 12 m/s impinges a smooth flat plate, the normal of which is inclined at 60^{o} to the axis of jet. Find the impact of jet on the plate at right angles to the plate when the plate is stationery.

What will be the impact if the plate moves with a velocity of 6 m/s in the direction of jet and away from it. What will be the force if the plate moves towards the plate.

Problems



When the plate is stationery

$F_n = \rho a V^2 \sin\theta$

$F_n = 1000 \text{ x } (4.418 \text{ x } 10^{-3})\ 12^2 \sin 30$

$\mathbf{F_n = 318.10\ N}$

When the plate is moving away from the jet

$F_n = \rho a\ (V - U)^2 \sin\theta$

$F_n = 1000 \text{ x } 4.418 \text{ x } 10^{-3}\ (12 - 6)^2 \sin 30$

$\mathbf{F_n = 79.52\ N}$

When the plate is moving towards the jet

$F_n = \rho a\ (V + U)^2 \sin\theta$

$F_n = 1000 \text{ x } 4.418 \text{ x } 10^{-3}\ (12 + 6)^2 \sin 30$

$\mathbf{F_n = 715.72\ N}$

Problems

Example 13.1: A jet of water of diameter 5 cm and velocity 20 m/s impinges on (a) a normal flat vane moving in the direction of jet at 7.6 m/s and (b) a series of normal flat vanes mounted on a wheel which has a tangential velocity of 7.5 m/s. Calculate force exerted, work done by water and efficiency of the system in both cases.

❑ Force exerted by jet on stationary curved vane

❑ When the jet strikes at the centre of the symmetrical blade :

consider jet of the water striking on the curved fixed blade at the centre of blade as shown in fig.

let V = velocity of liquid jet

A = area of cross section of jet

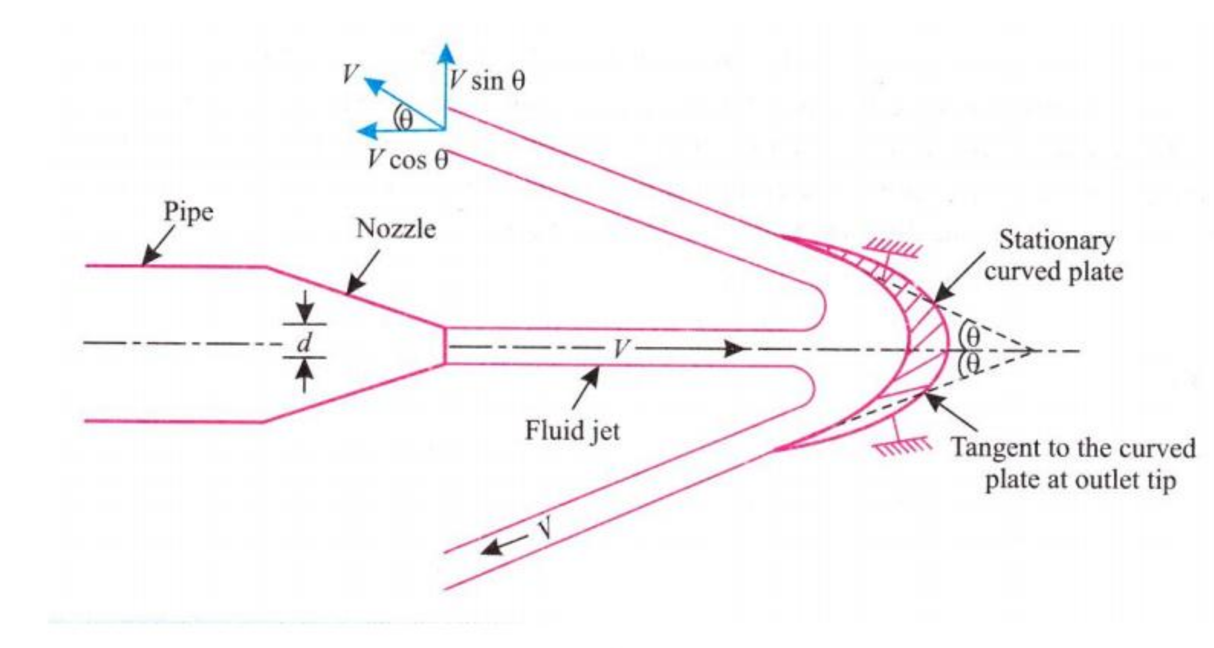


Fig. 2.3 Jet striking a fixed curved plate at the center

- Force exerted by jet on stationary curved vane
  - When the jet strikes at the centre of the symmetrical blade :

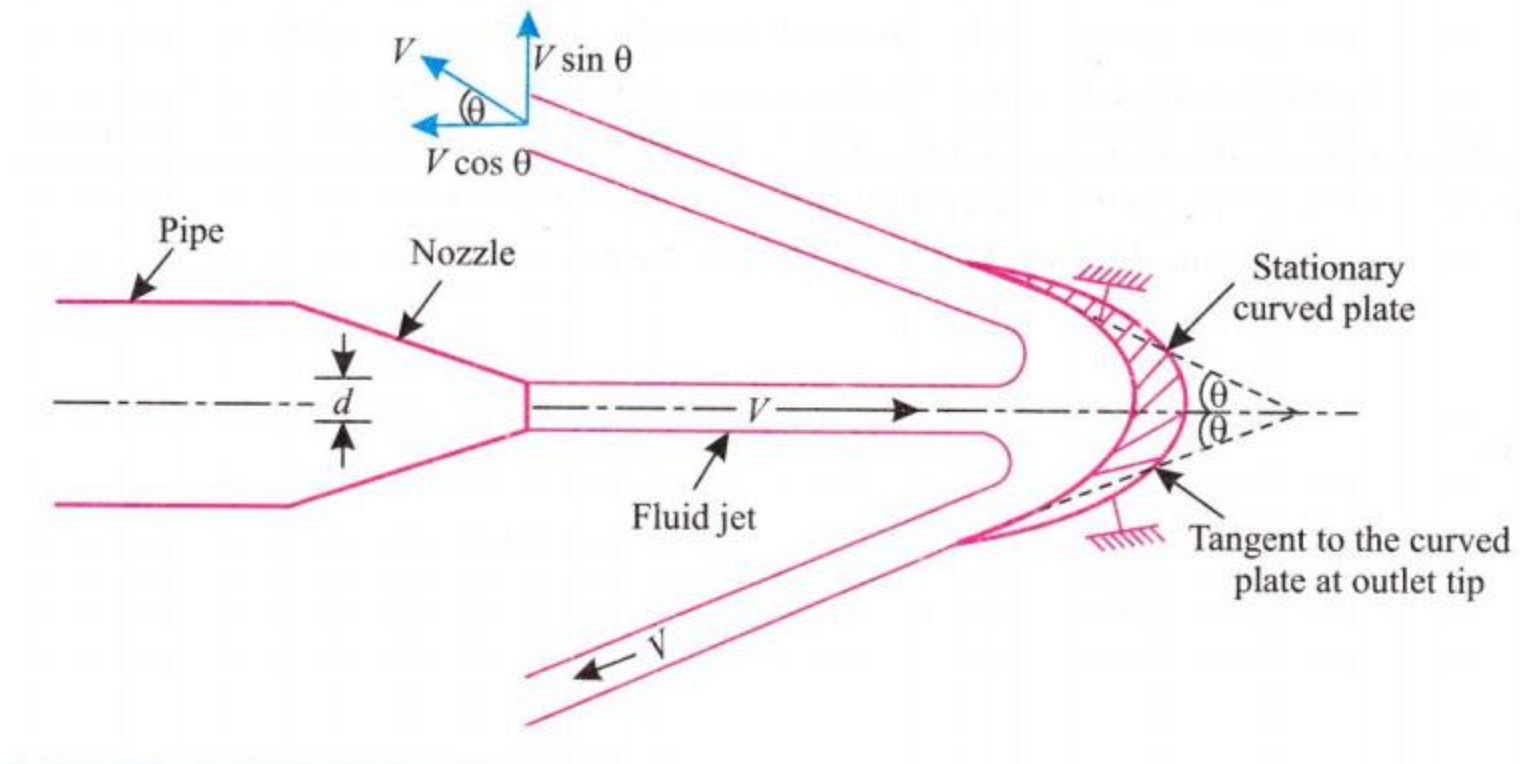


Fig. 2.3 Jet striking a fixed curved plate at the center

- Force exerted on single curved plate.
  - When the jet strikes at the centre of the symmetrical blade

The plate is smooth and there is no loss of energy due to impact of jet. Hence liquid leaving the plate with velocity V in the tangential direction of the curved plate.

Force exerted by jet in the direction of jet,

F_x = (mass of water/sec) * $(V_{1x} - V_{2x})$

where V_{1x} =initial velocity of water jet in direction of jet = V

V_{2x} =final velocity water in direction of jet = $-V\cos\Theta$

$$F_x = (\rho AV)*[V-(-V\cos\Theta)]$$

$$F_x = \rho AV^2[1+\cos\Theta]$$

- Force exerted by jet on stationary curved vane
- When the jet strikes at the centre of the symmetrical blade

Force exerted by jet on curved fixed plate in vertical direction

$$F_y = (\rho AV)*[V_{1y} - V_{2y}]$$

where V_{1y} =initial velocity of water jet in vertical direction=0

V_{2y} =final velocity water in vertical direction=$V\sin\Theta$

$$F_y = (\rho AV)*[0 - V\sin\Theta]$$

$$F_y = -\rho AV^2\sin\Theta$$

Angle of deflecton = 180°- Θ

❑ Force exerted by jet on stationary curved vane

❑ When jet strikes tangentially at one end of symmetrical plate :

consider a water jet striking on symmetrical curved plate tangentially at one end as shown in fig

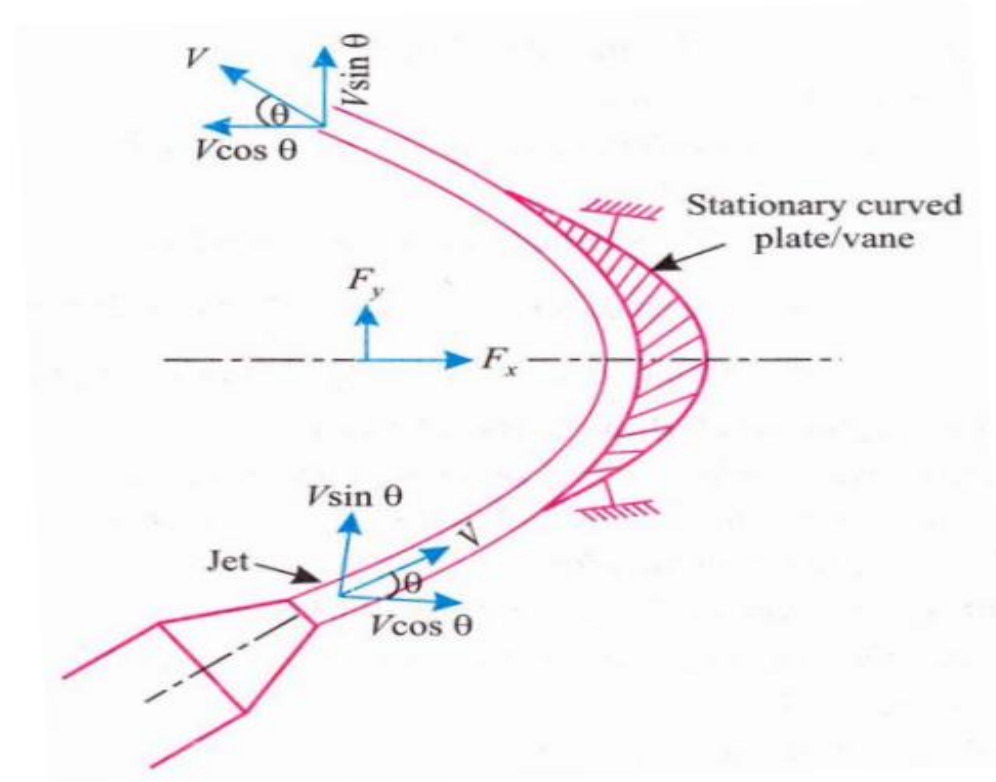


Fig. 2.4 Jet striking curved fixed plate at one end

- Force exerted by jet on stationary curved vane
  - When jet strikes tangentially at one end of symmetrical plate :

let V = velocity of water jet

Θ = angle between jet and x-axis at the tip of plate at inlet

$$F_x = (\rho AV)*(V_{1x} - V_{2x})$$
$$= (\rho AV)*[V\cos\Theta - (-V\cos\Theta)]$$
$$= 2\rho AV^2\cos\Theta$$
$$F_y = (\rho AV)*[V_{1y} - V_{2y}]$$
$$= (\rho AV)*[V\sin\Theta - V\sin\Theta] = 0$$

Angle of deflection = $180° - 2\Theta$

- Force exerted by jet on stationary curved vane

- When jet strikes tangentially at one end of unsymmetricall plate:

consider a water jet striking on unsymmetrical curved plate tangentially at one end as shown in fig.

let α = angle between water jet and x-axis at of inlet tip.

β = angle between water jet and x-axis at of outlet tip.

- Force exerted by jet on stationary curved vane
- When jet strikes tangentially at one end of unsymmetrically plate:

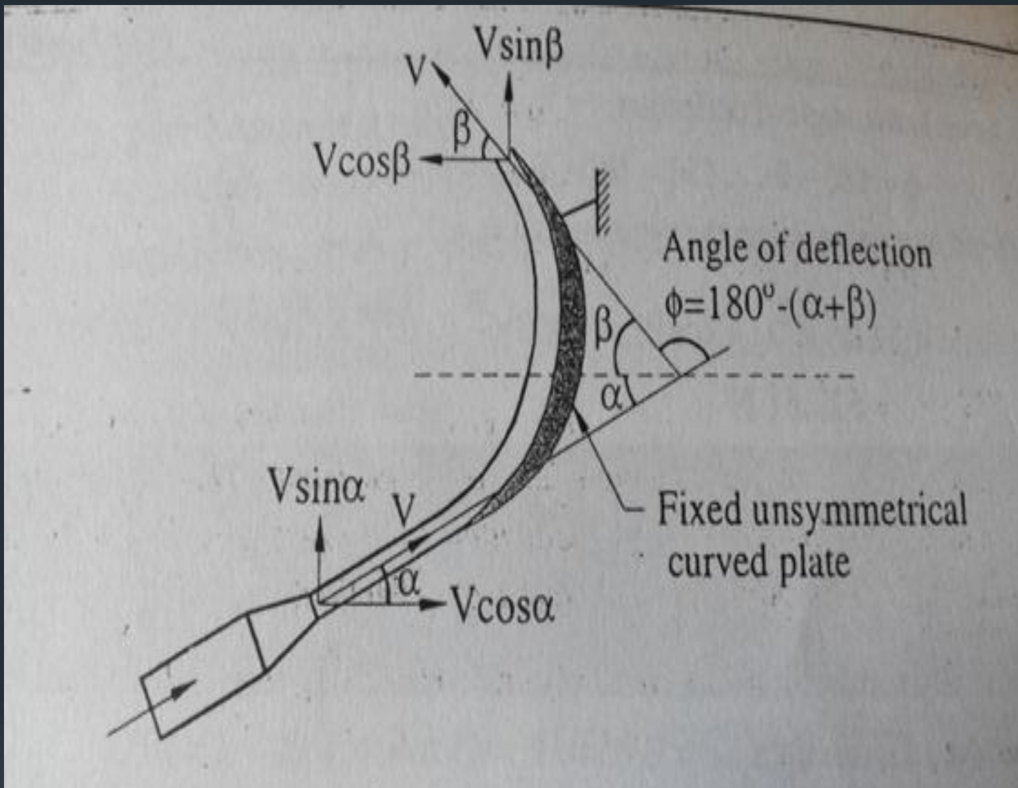


Fig. 2.13 Impact of jet on unsymmetrical curved plate (tangentially)

$F_x = (\rho AV)*(V_{1x} - V_{2x})$

$= \rho AV * [V\cos\alpha - V\cos\beta]$

$= \rho AV^2[\cos\alpha + \cos\beta]$

$F_y = (\rho AV)*[V_{1y} - V_{2y}]$

$= (\rho AV)*[V\sin\alpha - V\sin\beta]$

$= \rho AV^2[\sin\alpha - \sin\beta]$

Resultant force $F = \sqrt{F_x^2 + F_y^2}$

Resultant force inclination with Horizontal is ⦰ = tan^-1(F_y / F_x)

Angle of deflecton = 180°- (α +β)

A jet of water of diameter 50 mm strikes a stationary, symmetrical curved plate with a velocity of 40 m/s. Find the force extended by the jet at the centre of plate along its axis if the jet is deflected through 120° at the outlet of the curved plate.

A jet of water strikes a stationery curved plate tangentially at one end at an angle of 30°. The jet of 75 mm diameter has a velocity of 30 m/s. The jet leaves at the other end at angle of 20° to the horizontal. Determine the magnitude of force exerted along 'x' and 'y' directions.

C. Force exerted by a jet of water on the moving curved plate

I. Jet strikes the moving curved plate at the center

- Let a jet of water strikes a curved plate at the center of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig. 2.8.

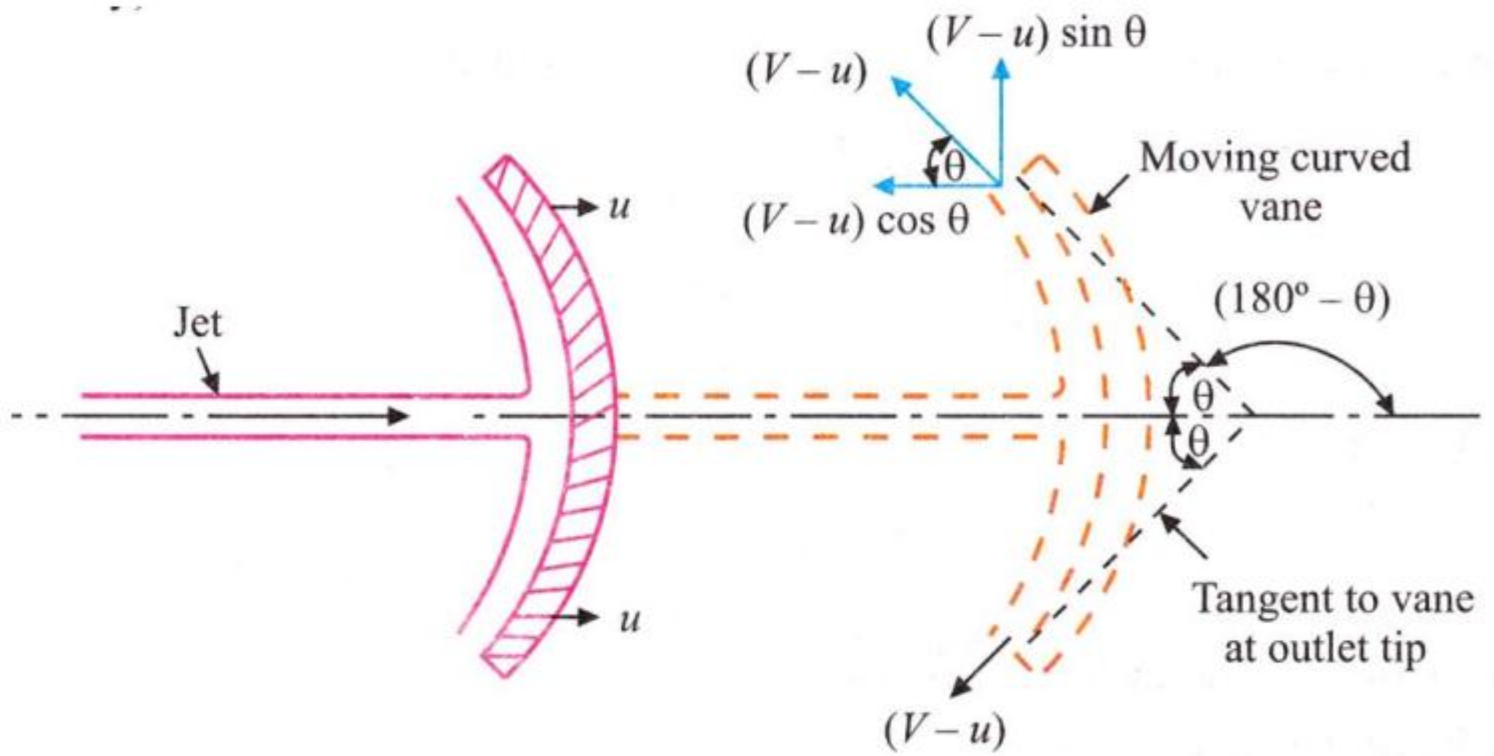


Fig. 2.8 Jet striking a moving curved plate at the center

C. Force exerted by a jet of water on the moving curved plate

I. Jet strikes the moving curved plate at the center

- Let,

  V = Absolute velocity of the jet of water

  u = Velocity of the flat plate in the direction of jet

  a = Cross section area of jet

- Relative velocity of the jet of water or the velocity with which jet strikes the curved plate $= (V - u)$

- If the plate is smooth and loss of energy due to impact of jet is assumed zero, then the velocity with which the jet will be leaving the curved vane equals to (V – u).

- Component of velocity in the direction of jet at outlet $= -(V - u)\cos\theta$.

  (−ve sign is taken as the velocity at outlet is in the opposite direction of the jet of water coming out at the nozzle)

- Mass of water striking the plate per second,

$$\dot{m} = \rho a(V - u)$$

C. Force exerted by a jet of water on the moving curved plate

I. Jet strikes the moving curved plate at the center

- **Force exerted** by the jet of water on the moving curved plate in the direction of the jet,

$$F_x = \dot{m} \times [Initial\ velocity\ with\ which\ jet\ strikes\ the\ plate - Final\ velocity]$$

$$F_x = \rho a(V-u) \times [(V-u) - (-(V-u)\cos\theta)]$$

$$F_x = \rho a(V-u) \times [(V-u) + (V-u)\cos\theta]$$

$$F_x = \rho a(V-u)^2 \times [1+\cos\theta] - - - - - - - - - - (2.17)$$

- Work done per second by the jet on the plate,

$$W = F_x \times Velocity\ with\ which\ plate\ moved\ in\ the\ X - direction$$

$$W = F_x \times u$$

$$W = \rho a(V-u)^2[1+\cos\theta] \times u$$

$$W = \rho a u(V-u)^2[1+\cos\theta] - - - - - - - - - - (2.18)$$

2.1 A jet of water of diameter 5cm moving with a velocity of 25 m/sec impinges on a fixed curved plate tangentially at one end at an angle of 30° with the horizontal. Determine force of the jet on the plate in the horizontal and the vertical direction if the jet is deflected through an angle of 130°. Also find direction and resultant force.

2.2 A jet of water impinges on a symmetrically curved vane at its center. The velocity of the jet is 60 m/s and the diameter 120 mm. The jet is deflected through an angle of 120°. Calculate the force on the vane if the vane is fixed. Also determine the force if the vane moves with a velocity of 25 m/s in the direction of the jet. What will be the power and efficiency? [GTU; DEC – 2010]

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

- Fig. 2.9 shows a jet of water striking a moving curved plate/vane/blade tangentially at one of its tips.
- As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero.
- In this case as plate is moving, the velocity with which jet of water strikes is equals to the relative velocity of the jet with respect to the plate.
- As the direction of jet velocity and vane velocity is not same, the relative velocity at inlet will be vector difference of the jet velocity and plate velocity at inlet.

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

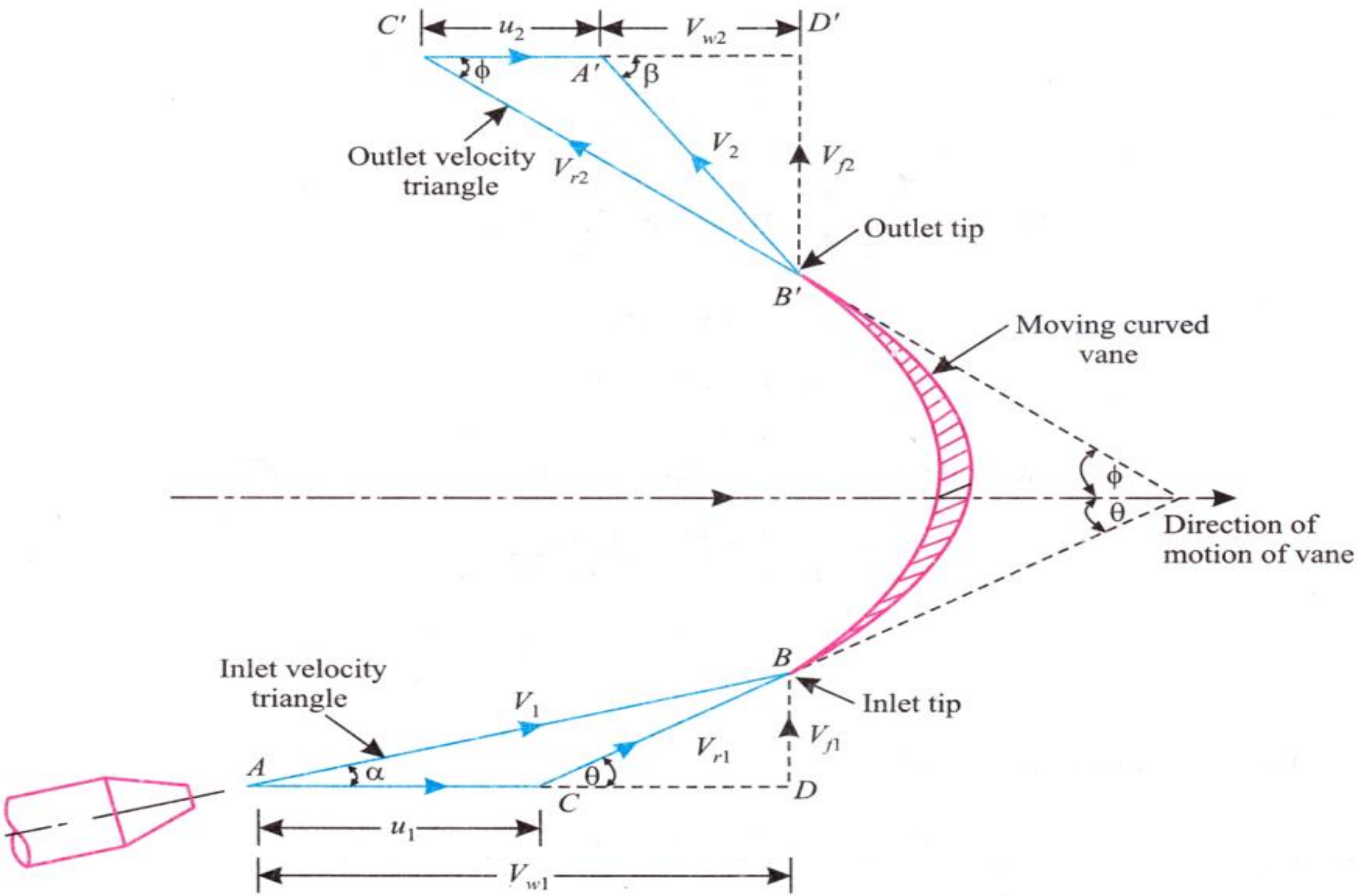


Fig. 2.9 *Jet striking unsymmetrical moving curved plate at one end*

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

– Let,

V_1 = Absolute velocity of the jet at inlet

V_2 = Absolute velocity of the jet at outlet

V_{r1} = Relative velocity of the jet and plate at inlet

V_{r2} = Relative velocity of the jet and plate at outlet

u_1 = Velocity of the vane at inlet

u_2 = Velocity of the vane at outlet

α = Angle between the direction of the jet and direction of motion of the plate at inlet

= Guide blade angle

θ = Angle made by the relative velocity V_{r1}, with the direction of motion of the vane at inlet

= Vane/blade angle at inlet

V_{w1} *and* V_{f1} = The components of the velocity of the jet V_1, in the direction of motion and perpendicular to the direction of motion of the vane respectively.

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

V_{w1} = Velocity of whirl at inlet

V_{f1} = Velocity of flow at inlet

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet

φ = Angle made by the relative velocity V_{r2}, with the direction of motion of the vane at outlet

= Vane/blade angle at outlet

V_{w2} *and* V_{f2} = The components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of the vane at outlet respectively.

V_{w2} = Velocity of whirl at outlet

V_{f2} = Velocity of flow at outlet

The triangles ABD and EGH are called the velocity triangles at inlet and outlet respectively.

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

- If the vane is smooth and having velocity in the direction of motion at inlet and outlet equal then we have,

$$u_1 = u_2 = u = velocity\ of\ vane\ in\ the\ direction\ of\ motion$$

And

$$V_{r1} = V_{r2}$$

- Mass of water striking the vane per second,

$$\dot{m} = \rho a V_{r1}$$

- **Force exerted** by the jet in the direction of motion,

F_x= mass of water striking per sec X [Initial velocity with which jet strikes in the direction of motion – Final velocity of jet in the direction of motion]

But,

Initial velocity with which jet strikes the vane = V_{r1} and,

The component of this velocity in the direction of motion = $V_{r1} \cos\theta = (V_{w1} - u_1)$

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

Similarly,

The component of the relative velocity at outlet in the direction of motion $= -V_{r2} \cos \varphi$

$= -[u_2 + V_{w2}]$

So,

$$\therefore F_x = \dot{m} \times [V_{r1} \cos \theta - (-V_{r2} \cos \varphi)]$$

$$F_x = \rho a V_{r1} \times [(V_{w1} - u_1) + (u_2 + V_{w2})]$$

As we know $u_1 = u_2$

$$F_x = \rho a V_{r1} \times [V_{w1} + V_{w2}] - - - - - - - - - -(2.19)$$

Equaion 2.19 is true only when angle β shown in Fig. 2.9 is acute angle $(< 90°)$.

- If $\beta = 90°$ then $V_{w2} = 0$ and equation 2.19 becomes,

$$F_x = \rho a V_{r1} V_{w1} - - - - - - - - - -(2.20)$$

- If β is an obtuse angle $(> 90°)$, the expression for F_x will become,

$$F_x = \rho a V_{r1} \times [V_{w1} - V_{w2}] - - - - - - - - - -(2.21)$$

II. Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips

- In general,

$$F_x = \rho a V_{r1} \times [V_{w1} \pm V_{w2}] - - - - - - - - -(2.22)$$

- **Work done per second** on the vane by the jet,

$$W = Force \times distance\ travelled\ per\ sec\ in\ the\ direction\ of\ force$$

$$W = F_x \times u$$

$$W = \rho a u V_{r1} \times [V_{w1} \pm V_{w2}] - - - - - - - - - -(2.23)$$

- Work done per second **<u>per unit weight</u>** of fluid striking per second ,

$$= \frac{\rho a u V_{r1} \times [V_{w1} \pm V_{w2}]}{(\rho a V_{r1}) \times g}$$

- Work done per second **<u>per unit mass</u>** of fluid striking per second ,

$$= \frac{\rho a u V_{r1} \times [V_{w1} \pm V_{w2}]}{(\rho a V_{r1})}$$

$$= u \times [V_{w1} \pm V_{w2}] \frac{N.m}{Kg} - - - - - - - - - -(2.25)$$

2.3 A jet of water having a velocity of 15 m/sec strikes a curved vane which is moving with a velocity of 5 m/sec. The vane is symmetrical and it so shaped that the jet is deflected through 120°. Find the angle of the jet at inlet of the vane so that there is no shock. What is the absolute velocity of the jet at outlet in magnitude and direction and the work done per unit weight of water? Assume the vane to be smooth. [17.21; R. K. Bansal]

Thank you

CE 3121
Engineering Hydraulics

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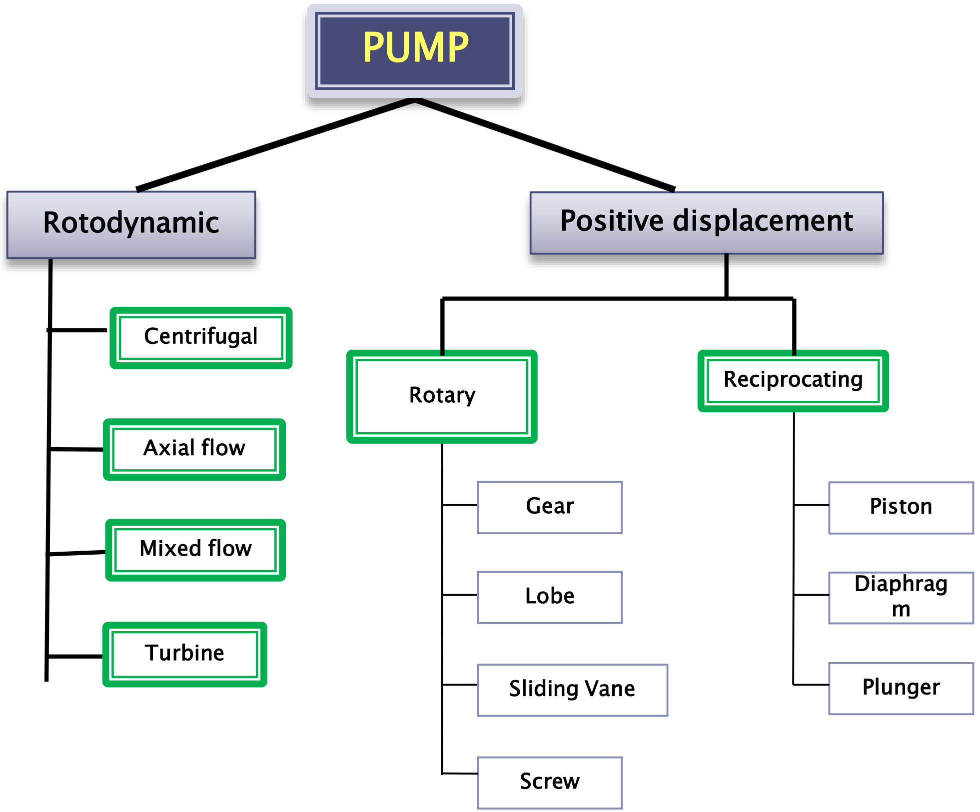
Pumps

Pumps

Pumps: A pump may be defined as a mechanical device which when imposed in a pipeline, converts the mechanical energy supplied to it from some external source into hydraulic energy and transfers the same to the liquid through the pipeline, thereby increasing the energy of flowing fluid.

There are two main categories of pump:

- Rotodynamic pumps
- Positive displacement pumps


PUMP
Rotodynamic
Positive displacement
Centrifugal
Axial flow
Mixed flow
Turbine
Rotary
Reciprocating
Gear
Lobe
Sliding Vane
Screw
Piston
Diaphrag
m
Plunger

Pumps

Positive displacement pumps: are defined as those which displace a fixed quantity of liquid with each motion of its pumping elements. They provide pressure by expanding and contracting space between the pumping elements.

A rotodynamic pump: is a pump that uses the rotation of an impeller or propeller to impart velocity to a liquid.

Difference between Centrifugal Pump and Reciprocating Pump:

S. No.	Centrifugal pump	Reciprocating pump
1.	It is one of the rotary pumps which used kinetic energy of impeller.	It is a positive displacement type pump which is forced by piston.
2.	It continuously discharges the fluid.	It does not discharge the fluid continuously.

Difference between Centrifugal Pump and Reciprocating Pump:

S. No.	Centrifugal pump	Reciprocating pump
3.	In centrifugal pump the flow rate decreases which increasing the pressure.	The pressure does not affect flow rate in reciprocating pumps.
4.	It is used for pumping high viscous fluid.	It is used for pump low viscous fluid.
5.	In this pumps discharge is inversely promotional to the viscosity of fluid.	In reciprocating pump viscosity of fluid does not affect the discharge rate.

Difference between Centrifugal Pump and Reciprocating Pump:

S. No.	Centrifugal pump	Reciprocating pump
6.	Efficiency of these pumps are low compare to reciprocating pump.	Efficiency is high.
7.	Centrifugal pump have problem of priming.	It does not have any problem of priming.
8.	It uses impellers to transfer energy to fluid.	It uses piston cylinder device to transfer energy to fluid.
9.	They are lighter than reciprocating pumps.	These are heavier compare to centrifugal pump.

Difference between Centrifugal Pump and Reciprocating Pump:

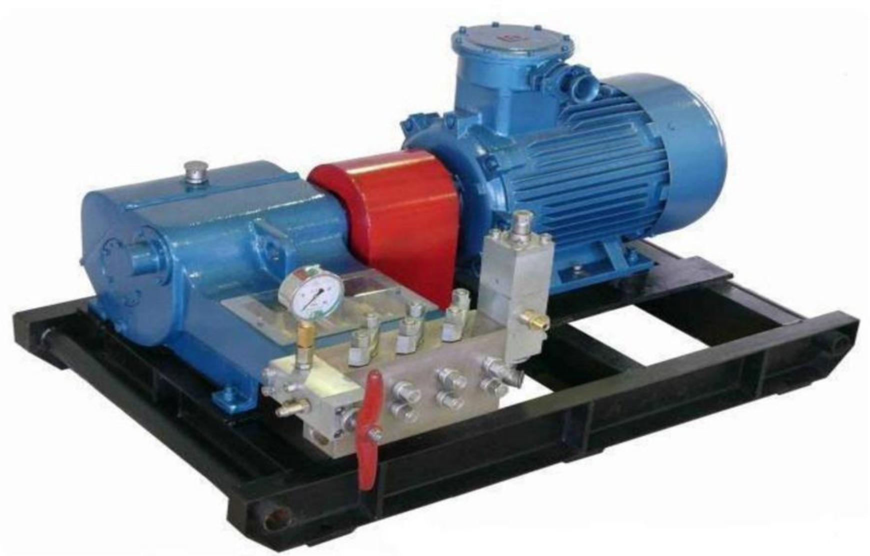
S. No.	Centrifugal pump	Reciprocating pump
10.	It gives higher discharge at low heads.	These gives higher heads at low discharge.
11.	It is less costly.	These are costly.
12.	These pumps required less maintenance.	These required higher maintenance.

Difference between Centrifugal Pump and Reciprocating Pump:

S. No.	Centrifugal pump	Reciprocating pump
13.	Centrifugal pumps are easy to install. These required less floor space.	These pumps are difficult to install. These required more floor area.
14.	It is mostly used for domestic purpose and where higher discharge at low head required.	These are mostly used in industries and high viscous fluid pumped at a high head.

Reciprocating pump

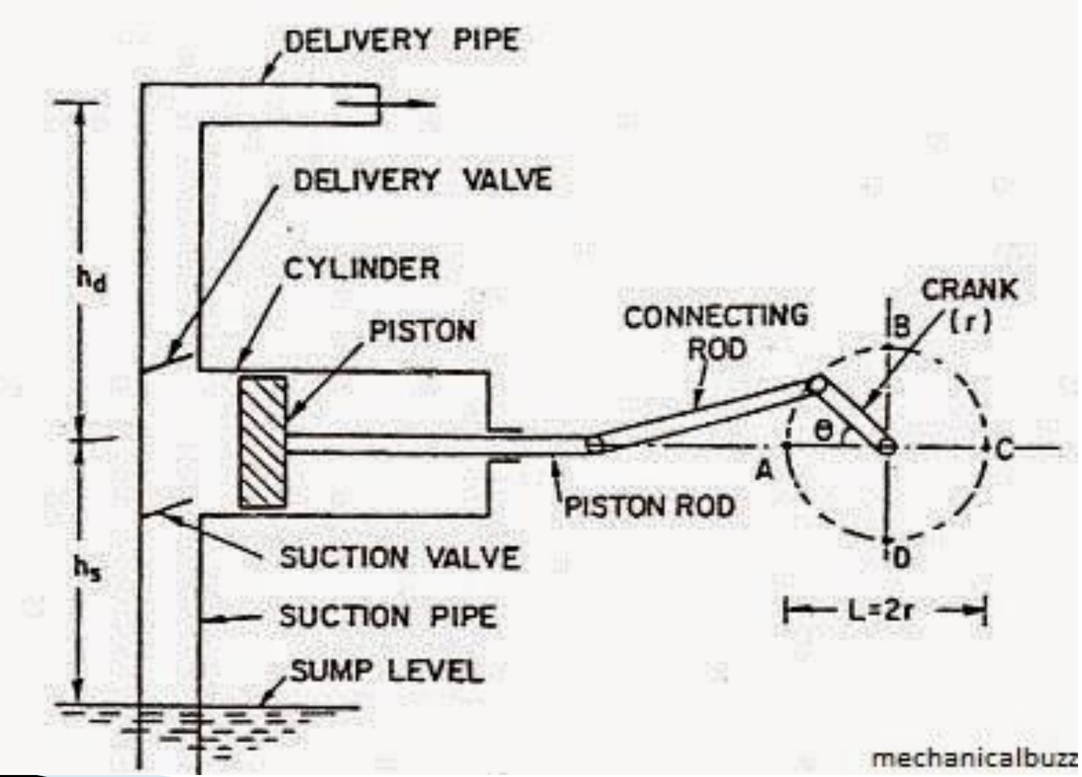
- Reciprocating pumps are positive displacement pump, i.e. initially, a small quantity of liquid is taken into a chamber and is physically displaced and forced out with pressure by a moving mechanical elements.
- The use of reciprocating pumps is being limited these days and being replaced by centrifugal pumps.



Reciprocating Pump

Reciprocating pump

Main components:



Working of Reciprocating Pump

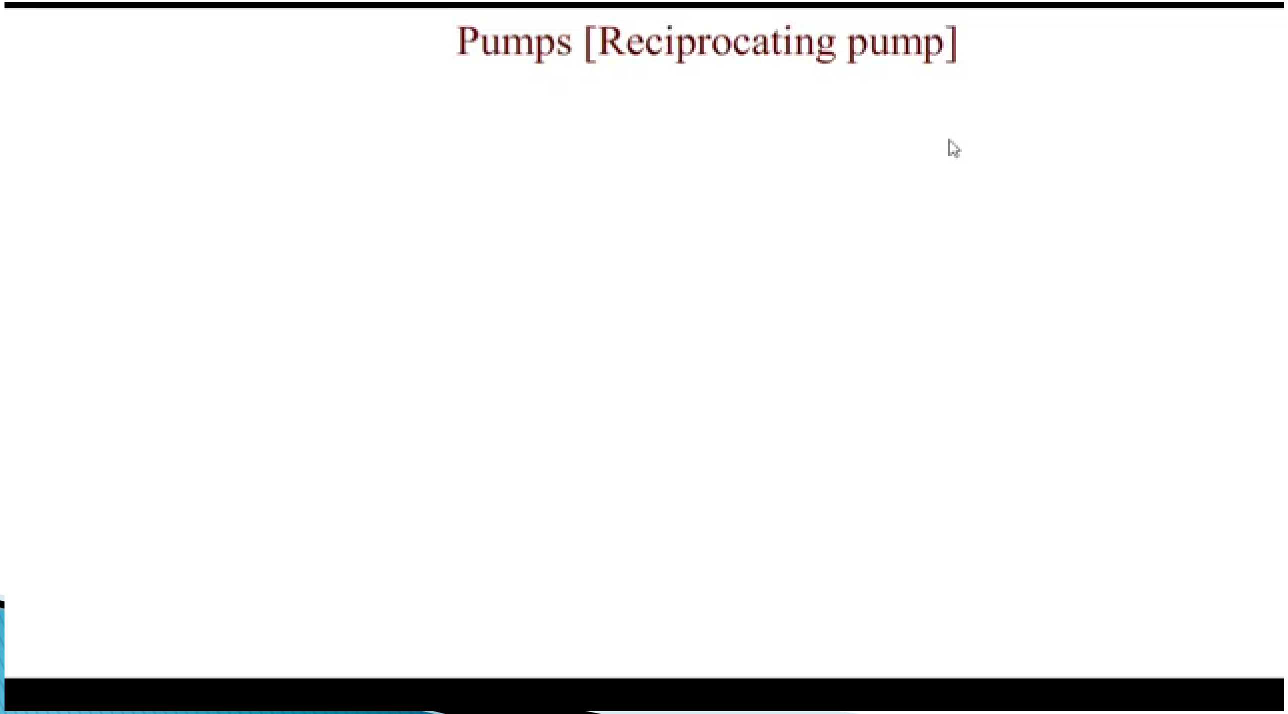
- When the piston moves from the left to the right, a suction pressure is produced in the cylinder. If the pump is started for the first time or after a long period, air from the suction pipe is sucked during the suction stroke, while the delivery valve is closed. Liquid rises into the suction pipe by a small height due to atmospheric pressure on the sump liquid.

Working of Reciprocating Pump

- During the delivery stroke, air in the cylinder is pushed out into the delivery pipe by the thrust of the piston, while the suction valve is closed. When all the air from the suction pipe has been exhausted, the liquid from the sump is able to rise and enter the cylinder.

Working of Reciprocating Pump

- During the delivery stroke it is displaced into the delivery pipe. Thus the liquid is delivered into the delivery tank intermittently, i.e. during the delivery stroke only.

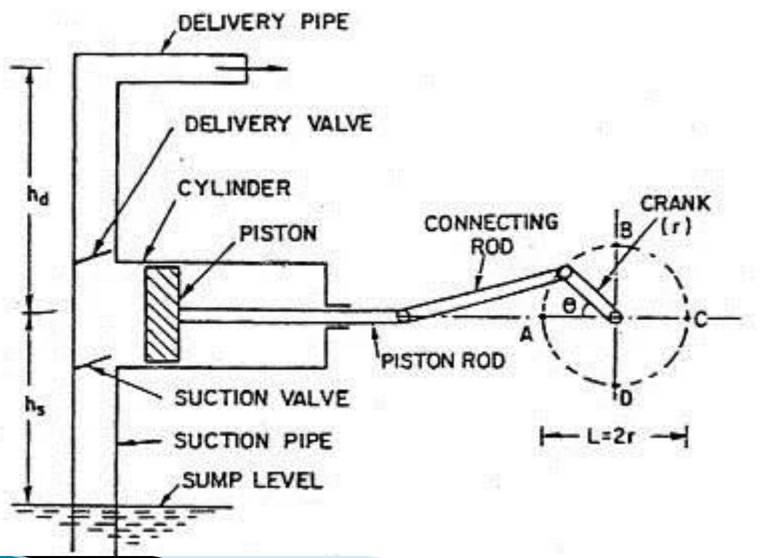


Classification of Reciprocating Pump

Following are the main types of reciprocating pumps:

- According to use of piston sides
  - Single acting Reciprocating Pump:

    If there is only one suction and one delivery pipe and the liquid is filled only on one side of the piston, it is called a single-acting reciprocating pump.



Classification of Reciprocating Pump

Following are the main types of reciprocating pumps:

- According to use of piston sides
  - Double acting Reciprocating Pump:

    A double-acting reciprocating pump has two suction and two delivery pipes, Liquid is receiving on both sides of the piston in the cylinder and is delivered into the respective delivery pipes.

Classification of Reciprocating pumps

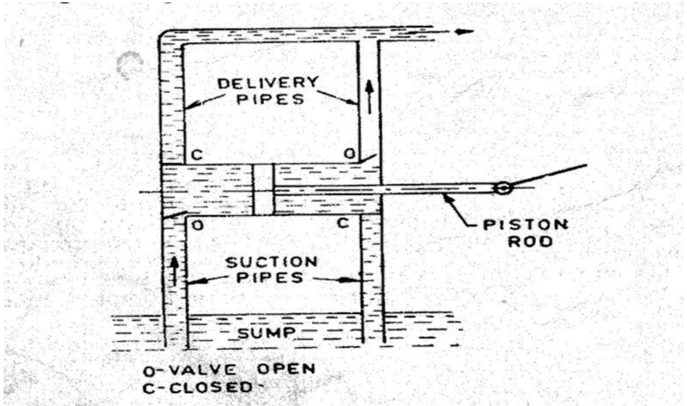


Fig. 11·2. Double-acting reciprocating pump.

Classification of Reciprocating pumps

- According to number of cylinder

  Reciprocating pumps having more than one cylinder are called multi-cylinder reciprocating pumps.

  - Single cylinder pump

    A single-cylinder pump can be either single or double acting

  - Double cylinder pump (or two throw pump)

    A double cylinder or two throw pump consist of two cylinders connected to the same shaft.

Classification of Reciprocating pumps

- According to number of cylinder
  - Triple cylinder pump (three throw pump)

    A triple-cylinder pump or three throw pump has three cylinders, the cranks of which are set at 120^0 to one another. Each cylinder is provided with its own suction pipe delivery pipe and piston.
  - There can be four-cylinder and five cylinder pumps also, the cranks of which are arranged accordingly.

Classification of Reciprocating pumps

- According to number of cylinder

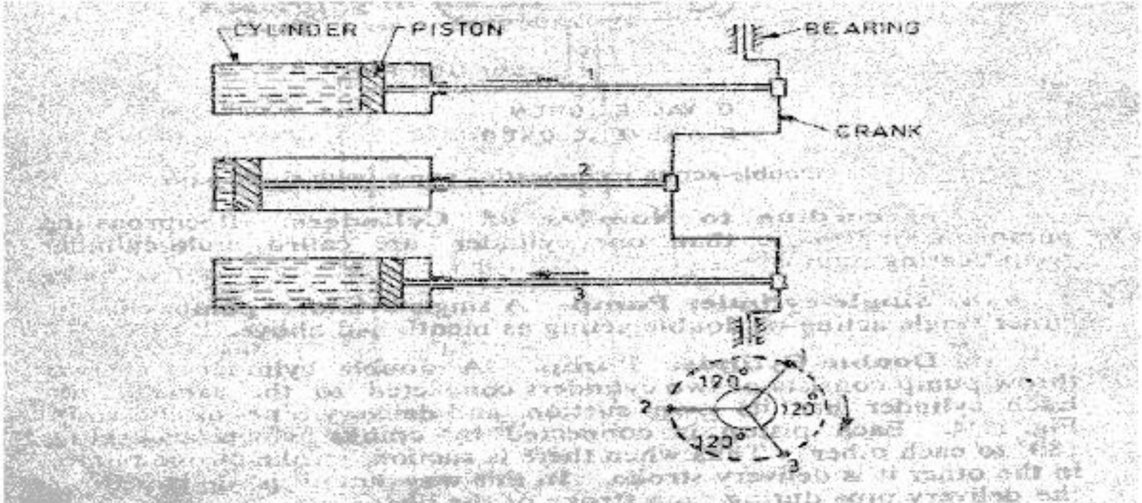


Fig. 11·5. Triple-cylinder pump.

Discharge through a Reciprocating Pump

Let

A = cross sectional area of cylinder

r = crank radius

N = rpm of the crank

L = stroke length (2r)

Discharge through pump per second=

Area x stroke length x rpm/60

$$Q_{th} = A \times L \times \frac{N}{60}$$

This will be the discharge when the pump is single acting.

Discharge through a Reciprocating Pump

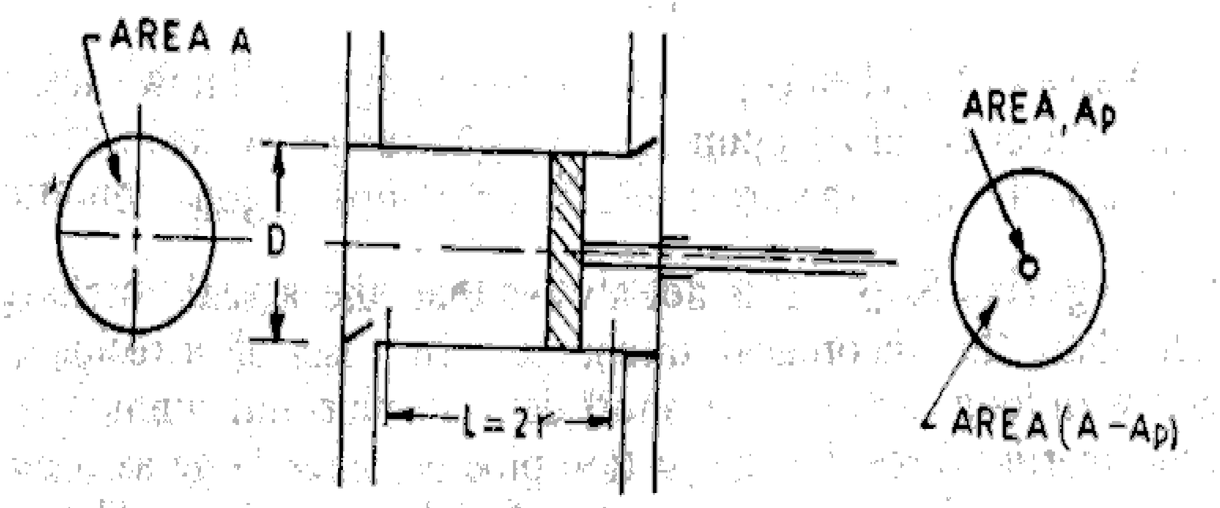


Fig. 11·6. Area of cylinder.

Discharge through a Reciprocating Pump

Discharge in case of double acting pump

Discharge/Second = $Q_{th} = \left[\frac{ALN}{60} + \frac{(A - A_P)LN}{60} \right]$

$$Q_{th} = \frac{(2A - A_P)LN}{60}$$

Where, A_p = Area of cross-section of piston rod

However, if area of the piston rod is neglected

Discharge/Second = $\frac{2ALN}{60}$

Discharge through a Reciprocating Pump

- Thus discharge of a double-acting reciprocating pump is twice than that of a single-acting pump.
- Owing to leakage losses and time delay in closing the valves, actual discharge Q_a usually lesser than the theoretical discharge Q_{th}.

Slip

Slip of a reciprocating pump is defined as the difference between the theoretical and the actual discharge.

i.e. Slip = Theoretical discharge – Actual discharge

= Q_{th}. – Q_a

Slip can also be expressed in terms of %age and given by

$$\%slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \left(1 - \frac{Q_{act}}{Q_{th}}\right)100 = (1 - C_d)100$$

Slip

Slip Where C_d is known as co-efficient of discharge and is defined as the ratio of the actual discharge to the theoretical discharge.

$C_d = Q_a / Q_{th}$.

Value of C_d when expressed in percentage is known as volumetric efficiency of the pump. Its value ranges between 95---98 %. Percentage slip is of the order of 2% for pumps in good conditions.

Negative slip

- It is not always that the actual discharge is lesser than the theoretical discharge. In case of a reciprocating pump with long suction pipe, short delivery pipe and running at high speed, inertia force in the suction pipe becomes large as compared to the pressure force on the outside of delivery valve. This opens the delivery valve even before the piston has completed its suction stroke. Thus some of the water is pushed into the delivery pipe before the delivery stroke is actually commenced. This way the actual discharge becomes more than the theoretical discharge.
- Thus co-efficient of discharge increases from one and the slip becomes negative.

Power Input

Consider a single acting reciprocating pump.

Let

h_s = Suction head or difference in level between centre line of cylinder and the sump.

h_d = Delivery head or difference in between centre line of cylinder and the outlet of delivery pipe.

H_{st} = Total static head

$= h_s + h_d$

Theoretical work done by the pump

$= \rho \, Q_{th} \, g \, H_{st}$

$$= \rho \left(\frac{ALN}{60} \right) g \left(h_s + h_d \right)$$

Power Input

Power input to the pump

$$= \rho\left(\frac{ALN}{60}\right)g\left(h_s + h_d\right)$$

However, due to the leakage and frictional losses, actual power input will be more than the theoretical power.

Let η = Efficiency of the pump.

Then actual power input to the pump

$$= \frac{1}{\eta}\rho\left(\frac{ALN}{60}\right)g\left(h_s + h_d\right)$$

Problem-1: A single-acting reciprocating pump discharge 0.018 m^3 /s of water per second when running at 60 rpm. Stroke length is 50 cm and the diameter of the piston is 22 cm. If the total lift is 15 m, determine:

a) Theoretical discharge of the pump

b) Slip and percentage slip of the pump

c) Co-efficient of discharge

d) Power required running the pump

Solution:

$L = 0.5$ m

$Q_a = 0.018 m^3 /s$

$D = 0.22$ m

$N = 60$ rpm

$H_{st} = 15$ m

Problem-1

Solution:

(a)

$$Q_{th} = A \times L \times \frac{N}{60} = \left(\frac{\pi}{4} D^2\right)\frac{LN}{60}$$

Q_{th} = (π/4)x(0.22)^{2}x(0.5x60/60)

Q_{th} = **0.019 m^3 /s**

(b) Slip = Q_{th} – Q_a

Slip = 0.019 – 0.018

= 0.001 m^3 /s

Percentage slip = (Q_{th} – Q_a)/ Q_{th}

= (0.019–0.018)/0.019

= 0.0526 or 5.26%

Problem-1

Solution:

(c) $C_d = Q_a / Q_{th}$

$= 0.018/0.019$

$= 0.947$

(d) Power Input

$= \rho Q_{th} g H_{st}$ (Neglecting Losses)

$= 1000 \times 0.019 \times 9.81 \times 15$

= 2796 w or 2.796 kW

.

Problem-2: A three-throw reciprocating pump delivering 0.1 **m^3 /s** of water against a head of 100 m. Diameter and stroke length of the cylinder are 25 cm and 50 cm respectively. Friction losses amount to 1 m in the suction pipe and 16 m in the delivery pipe. If the velocity of water in the delivery pipe is 1.4 m/s, pump efficiency 90% and slip 2%, determine the pump and the power required.

Solution:

H_{st} = 100 m

Qa = 0.1 **m^3 /s**

D = 0.25 m

L = 0.5 m

h_{fs} = 1 m

h_{fd} = 16 m

η_h = 0.9

s = 0.02

Vd = 1.4 m/s

$$Q_{th} = \frac{3ALN}{60}$$

Problem-2

Solution:

We know that, $s = (Q_{th} - Q_a)/\ Q_{th}$

$$0.02 = 1 - Q_a / Q_{th}$$

$$Q_a / Q_{th} = 0.98$$

$$Q_{th} = Q_a / 0.98$$

$$Q_a / 0.98 = 3/60 x \pi/4\ D^2 x L x N$$

$$0.1/ 0.98 = 3/60 x \pi/4\ (0.25)^2 x 0.5 x N$$

$$\mathbf{N = 83.15\ rpm}$$

Total head generated

$$H = H_{st} + h_{fs} + h_{fd} + Vd^2/(2g)$$

$$H = 100+1+16+ (1.4)^2/(2x9.81)$$

$$H = 117.1\ m$$

Problem-2

Solution:

Power required = $1/\ \eta_h\ (\ \rho\ Q_{th}\ g\ H)$

$= 1/0.9\ (1000 \times 0.1/0.98 \times 9.81 \times 117.1)$

$= 130.21 \times 10^3$ W

= 130.21 KW

24

Centrifugal Pumps

24.1 INTRODUCTION

Centrifugal pumps are classified as rotodynamic type of pumps in which a dynamic pressure is developed which enables the lifting of liquids from a lower to a higher level. The basic principle on which a centrifugal pump works is that when a certain mass of liquid is made to rotate by an external force, it is thrown away from the central axis of rotation and a centrifugal head is impressed which enables it to rise to a higher level. Now if more liquid is constantly made available at the centre of rotation, a continuous supply of liquid at a higher level may be ensured. Since in these pumps the lifting of the liquid is due to centrifugal action, these pumps are called 'centrifugal pumps'. In addition to the centrifugal action, as the liquid passes through the revolving wheel or impeller, its angular momentum changes, which also results in increasing the pressure of the liquid. As such centrifugal pumps behave quite differently from positive displacement pumps. A centrifugal pump does not push the liquid as in the case of a positive displacement pump, but it modifies the hydraulic gradient such that the liquid is lifted to a higher level.

According to the general direction of flow of liquid within the passage of the rotating wheel or impeller the rotodynamic pumps are classified as,

(*i*) Centrifugal pumps,

(*ii*) Half axial or screw or mixed flow pumps,

(*iii*) Axial flow or propeller pumps.

In the impeller of a centrifugal pump the liquid flows in the outward radial direction, while the flow of liquid in a propeller pump impeller is in the axial direction, parallel to the rotating shaft. The mixed flow pump impeller has an intermediate form so that the flow of liquid is in between the radial and axial directions. However, there are no rigid boundaries separating these three types of pumps, and often all the three types of pumps are called centrifugal pumps.

In general all the rotodynamic pumps closely resemble reaction type of hydraulic turbines and they may be regarded as reversed reaction turbines. Thus the action of a centrifugal pump is just the reverse of a radially inward flow reaction turbine. Similarly the axial flow pumps are reverse of propeller or Kaplan turbines and the mixed flow pumps are the reverse of mixed flow type turbines such as Francis turbine. In the present chapter only centrifugal pumps have been described.

24.2 ADVANTAGES OF CENTRIFUGAL PUMPS OVER RECIPROCATING PUMPS

The main advantage of a centrifugal pump is that its discharging capacity is very much greater than that of a reciprocating pump which can handle relatively small quantity of liquid only. A centrifugal pump can be used for lifting highly viscous liquids such as oils, muddy and sewage water, paper pulp, sugar molasses, chemicals etc. But a reciprocating pump can handle only pure water or less viscous liquids free from impurities as otherwise its valves may cause frequent trouble. A centrifugal pump can be operated at very high speeds without any danger of separation and cavitation. As such it can be coupled directly through flanged coupling to electric motor. On the other hand as indicated in Chapter 23 the maximum speed of a reciprocating pump is limited from the considerations of separation and cavitation. As such reciprocating pumps can be operated at low speeds only and for that these pumps are mostly belt driven. The maintenance cost of a centrifugal pump is low and only periodical check up is sufficient. But for a reciprocating pump the maintenance cost is high because the parts such as valves etc. may need frequent replacement. However, a reciprocating pump can build up very high pressures as high as 69×10^6 N/m^2 {700 kg(f)/cm^2} or even more and hence these pumps are used for lifting oil from very deep oil wells.

24.3 COMPONENT PARTS OF A CENTRIFUGAL PUMP

Fig. 24.1 shows the main component parts of a centrifugal pump which are described below:

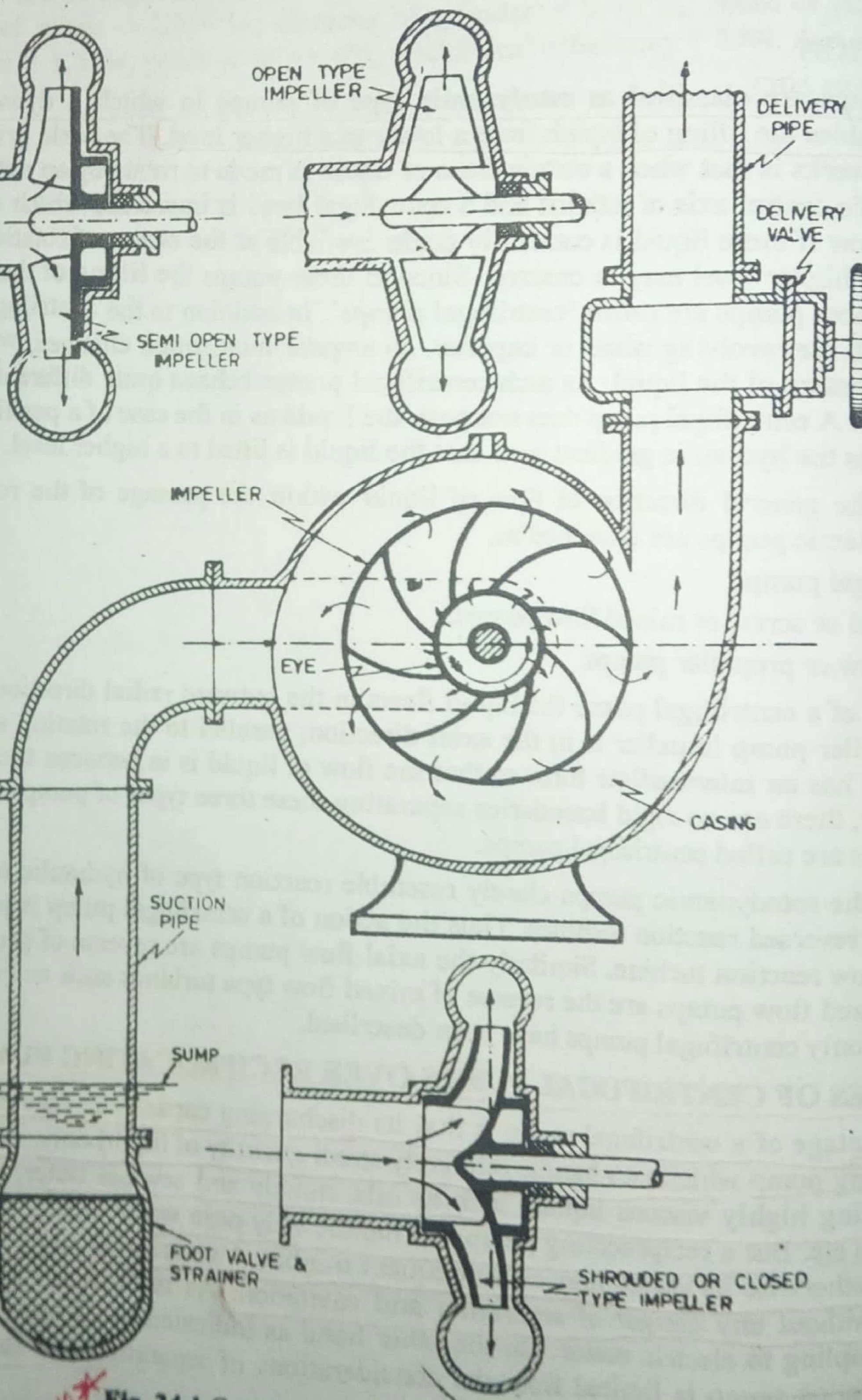


Fig. 24.1 Component parts of a centrifugal pump

(*i*) **Impeller.** It is a wheel or rotor which is provided with a series of backward curved blades or vanes. It is mounted on a shaft which is coupled to an external source of energy (usually an electric motor) which imparts the required energy to the impeller thereby making it to rotate.

The impellers may be classified as. (*a*) closed or shrouded impeller, (*b*) semi-open impeller; and (*c*) open impeller, which are all shown in Fig. 24.1. A 'closed or shrouded impeller' is that whose vanes are provided with metal cover plates or shrouds on both sides. These plates or shrouds are known as crown plate and lower or base plate as shown in Fig. 24.1. The closed impeller provides better guidance for the liquid and is more efficient. However, this type of impeller is most suited when the liquid to be pumped is pure and comparatively free from debris.

If the vanes have only the base plate and no crown plate, then the impeller is known as 'semi-open type impeller'. Such an impeller is suitable even if the liquids are charged with some debris.

An 'open impeller' is that whose vanes have neither the crown plate nor the base plate. Such impellers are useful in the pumping of liquids containing suspended solid matter, such as paper pulp, sewage and water containing sand or grit. These impellers are less liable to clog when handing liquids charged with a large quantity of debris.

(*ii*) **Casing.** It is an airtight chamber which surrounds the impeller. It is similar to the casing of a reaction turbine. The different types of casings that are commonly adopted are described later.

(*iii*) **Suction Pipe**. It is a pipe which is connected at its upper end to the inlet of the pump or to the centre of the impeller which is commonly known as eye. The lower end of the suction pipe dips into liquid in a suction tank or a sump from which the liquid is to be pumped or lifted up.

The lower end of the suction pipe is fitted with a *foot valve and strainer*. The liquid first enters the strainer which is provided in order to keep the debris (such as leaves, wooden pieces and other rubbish) away from the pump. It then passes through the foot valve to enter the suction pipe. A 'foot valve' is a non-return or one-way type of valve which opens only in the upward direction. As such the liquid will pass through the foot valve only upwards and it will not allow the liquid to move downwards back to the sump.

(*iv*) **Delivery Pipe.** It is a pipe which is connected at its lower end to the outlet of the pump and it delivers the liquid to the required height. Just near the outlet of the pump on the delivery pipe a delivery valve is invariably provided. A delivery valve is a regulating valve which is of sluice type and is required to be provided in order to control the flow from the pump into delivery pipe.

24.4 WORKING OF CENTRIFUGAL PUMP

The first-step in the operation of a centrifugal pump is priming. *Priming* is the operation in which the suction pipe, casing of the pump and the portion of the delivery pipe upto the delivery valve are completely filled with the liquid which is to be pumped, so that all the air (or gas or vapour) from this portion of the pump is driven out and no air pocket is left. It has been observed that even the presence of a small air pocket in any of the portion of pump may result in no delivery of liquid from the pump. The necessity of priming a centrifugal pump is due to the fact that the pressure generated in a centrifugal pump impeller is directly proportional to the density of the fluid that is in contact with it. Hence if an impeller is made to rotate in the presence of air, only a negligible pressure would be produced with the result that no liquid will be lifted up by the pump. As such it is essential to properly prime a centrifugal pump before it can be started. The various methods used for priming a centrifugal pump are discussed later.

After the pump is primed, the delivery valve is still kept closed and the electric motor is started to rotate the impeller. The delivery valve is kept closed in order to reduce the starting torque for the motor. The rotation of the impeller in the casing full of liquid produces a forced vortex which imparts a centrifugal head to the liquid and thus results in an increase of pressure throughout the liquid mass. The increase of pressure at any point is proportional to the square of the angular velocity and the distance of the point from the axis of rotation. Thus if the speed of rotation of the impeller of the pump is sufficiently high, the pressure in the liquid surrounding the impeller is considerably increased. Now as long the delivery valve is closed and the impeller is rotating, it just churns the liquid in the casing. When the delivery valve is opened the liquid is made to flow in an outward radial direction thereby leaving the vanes of the impeller at the outer circumference with high velocity and pressure. At the eye of the impeller due to the centrifugal action a partial vacuum is created. This causes the liquid from the sump, which is at atmospheric pressure, to rush through the suction pipe to the eye of the impeller thereby replacing the liquid which is being discharged from the entire circumference of the impeller. The high pressure of the liquid leaving the impeller is utilized in lifting the liquid to the required height through the delivery pipe.

As the liquid flows through the rotating impeller it receives energy from the vanes which results in an increase in both pressure and velocity energy. As such the liquid leaves the impeller with a high absolute velocity. In order that the kinetic energy corresponding to the high velocity of the leaving liquid is not wasted in eddies and efficiency of the pump thereby lowered, it is essential that this high velocity of the leaving liquid is gradually reduced to a lower velocity of the delivery pipe, so that the larger portion of the kinetic energy is converted into useful pressure energy. Usually this is achieved by shaping the casing such that the leaving liquid flows through a passage of gradually expanded area. The gradually increased cross-sectional area of the casing also helps in maintaining uniform velocity of flow throughout, because as the flow proceeds from the tongue T (Fig. 24.2) to the delivery pipe, more and more liquid is added from the impeller. There are different types of casings that are adopted for this purpose and on the basis of the type of casing used, the centrifugal pumps are classified into different types as described in the next section.

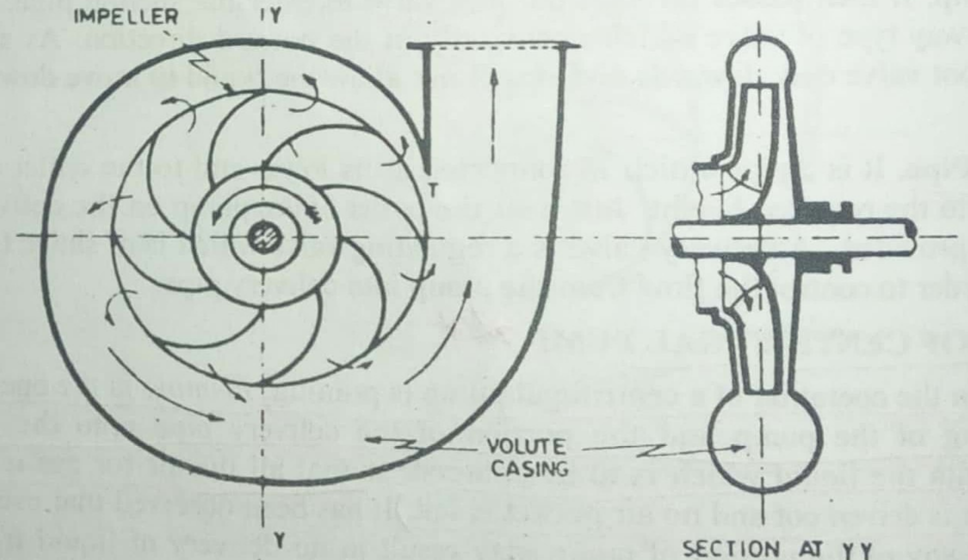


Fig. 24.2 Volute pump

24.5 TYPES OF CENTRIFUGAL PUMPS

According to the type of casing provided, centrifugal pumps are classified into the following two classes:

(1) Volute pump.

(2) Diffuser or turbine pump.

1. Volute Pump. In a volute pump the impeller is surrounded by a spiral shaped casing which is known as *volute chamber*. As shown in Fig. 24.2 the shape of the casing is such that the sectional area of flow around the periphery of the impeller gradually increases from the tongue T towards the

According to the number of impellers provided the pumps may be classified as single-stage and multi-stage. A *single stage centrifugal pump* has only one impeller mounted on the shaft. A *multi-stage centrifugal pump* has two or more impellers connected in series, which are mounted on the same shaft and are enclosed in the same casing.

On the basis of the direction of flow of the liquid through the impeller the pump may be classified as radial flow pump, mixed flow pump and axial flow pump.

A *radial flow pump* is that in which the liquid flows through the impeller in the radial direction only. Ordinarily all the centrifugal pumps are provided with radial flow impellers. In *mixed flow pumps* the liquid flows through the impeller axially as well as radially, that is there is a combination of radial and axial flows. A mixed flow impeller is just a modification of radial flow type in this respect that the former is capable of discharging a large quantity of liquid. As such mixed flow pumps are generally used where a large quantity of liquid is to be dicharged to low heights. In *axial flow pumps* the flow of liquid through the impeller is in the axial direction only. Axial flow pumps are usually designed to deliver very large quantities of liquid at relatively low heads. However, it is not justified to call axial flow pumps as centrifugal pumps, because there is hardly any centrifugal action in their operation.

Depending on the number of entrances to the impeller the centrifugal pumps may be classified as single suction pump and double suction pump. In a *single suction (or entry) pump* liquid is admitted from a suction pipe on one side of the impeller. In a *double suction (or entry) pump* liquid enters from both sides of the impeller. A double suction pump has an advantage that by this arrangement the axial thrust on the impeller is neutralised. Further it is suitable for pumping large quantities of liquid since it provides a large inlet area.

The centrifugal pumps may be designed with either horizontal or vertical disposition of shafts. Generally the pumps are provided with horizontal shafts. However, for deep wells and mines the pumps with vertical shafts are more suitable because the pumps with vertically disposed shafts occupy less space.

According to the head developed, the centrifugal pumps may be classified as *low head, medium head* and *high head* pumps. A *low head* pump is the one which is capable of working against a total head upto 15 m. A *medium head* pump is that which is capable of working against a total head more than 15 m but upto 40 m. A *high head* pump is the one which is capable of working against a total head above 40 m. Generally high head pumps are multi-stage pumps.

24.6 WORK DONE BY THE IMPELLER

The expression for the work done (or the energy supplied) by the impeller of a centrifugal pump on the liquid flowing through it may be derived in the same way as for a turbine. The liquid enters the impeller at its centre and leaves at its outer periphery. Fig. 24.5 shows a portion of the impeller of a Centrifugal pump with one vane and the velocity triangles at the inlet and the outlet tips of the vane. For the sake of convenience the same system of notation is employed as that for turbines. Thus, V is absolute velocity of liquid, u is peripheral (or tangential) velocity of the impeller V_r is relative velocity of liquid, V_f is velocity of flow of liquid, and V_w is velocity of whirl of the liquid at the entrance to the impeller. Similarly V_1, u_1 V_{r1}, V_{f1} and V_{w1} represent their counterparts at the exit point of the impeller. Further θ represents the impeller vane angle at the entrance and ϕ represents the impeller vane angle at the outlet. Similarly α is the angle between the direction of the absolute velocity of entering liquid and the peripheral velocity of the impeller at the entrance, and β is the angle between the absolute velocity of leaving liquid and the peripheral velocity of the impeller at the exit point.

At the entrance to the impeller since there are no guide vanes (as in the case of turbines) the direction of the absolute velocity of liquid at this point of the impeller is not directly known. However, for best efficiency of the pump it is commonly assumed that the liquid enters the impeller radially that is the absolute velocity of the liquid at the entrance to the impeller (or at the inlet tip of the impeller vane) is radial in direction. Thus, in this case $\alpha = 90°$ and the velocity of whirl V_w at inlet is equal to zero. Furthe it is desired that the liquid enters and leaves the vane without shock. This can be ensured if the inlet and oulet tips of the vane are parallel to the direction of the relative velocities at the two tips. As such it is assumed that the relative velocities V_r and V_{r1} are parallel to the tangents to the vane at the inlet and outle tips respectively as shown in Fig. 24.5.

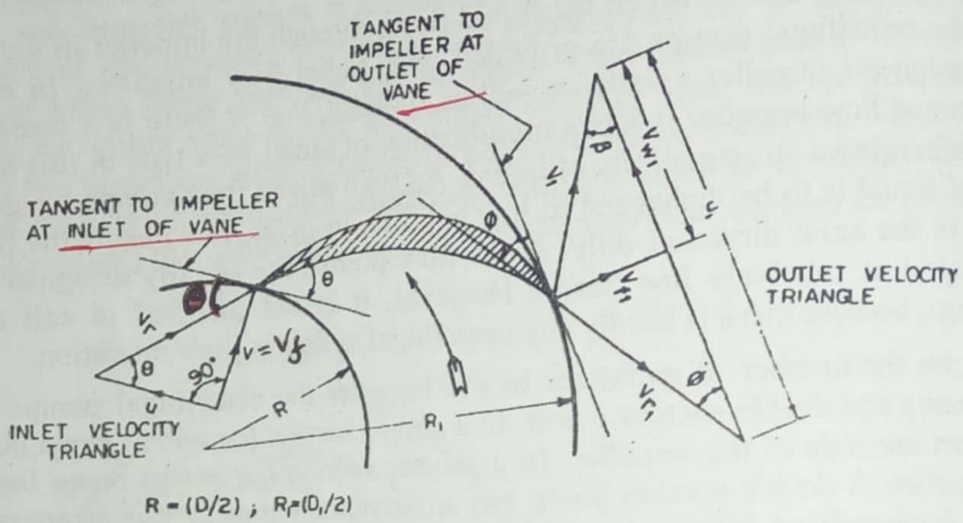


Fig. 24.5 Velocity triangles for an impeller vane

In the case of a radially inward flow reaction turbine the work done per second by the liquid on the runner = $(W/g)\,(V_w u \pm V_{w1}\, u_1)$; where W is weight of liquid striking the runner per second. Since a centrifugal pump is just the reverse of a radially inward flow reaction turbine, the same analysis as used for turbines may be applied to pumps. Thus, the work done per second by the impeller on the liquid may be written as

$$\boxed{\text{Work done } = \frac{W}{g}(V_{w1}u_1 - V_w u)} \qquad \ldots(24.1)$$

where W is weight of liquid per second that passes through the impeller. Since in this case as stated earlier the liquid enters the impeller radially, $\alpha = 90°$ and hence $V_w = 0$.

Thus equation 24.1 becomes

$$\text{Work done } = \frac{W}{g}(V_{w1}u_1) \qquad \ldots(24.2)$$

and work done per unit weight of liquid

$$= \frac{1}{g}(V_{w1}u_1) \qquad \ldots(24.3)$$

Thus equation 24.3 represents the head imparted by the impeller to the liquid.

Further from the outlet velocity triangle of Fig. 24.5, $V_{w1} = (u_1 - V_{f1} \cot \phi)$. As in the case of turbines u_1 and V_{f1} can be expressed in terms of the speed ratio K_u and the flow ratio ψ as follows:

$$u_1 = K_u\left(\sqrt{2gH_m}\right) \quad \text{and} \quad V_{f1} = \psi\left(\sqrt{2gH_m}\right)$$

where H_m is the total (or manometric) head as defined later. The usual range of values for the speed ratio and the flow ratio of impellers are given in Table 24.1.

TABLE 24.1

	Low N_s Impeller		High N_s Impeller
Speed ratio K_u	0.95	to	1.25
Flow ratio ψ	0.10	to	0.25

Moreover, equation 24.1 can be transformed in the following form:

Work done per kg per second

$$= \left(\frac{V_1^2 - V^2}{2g} + \frac{u_1^2 - u^2}{2g} + \frac{V_r^2 - V_{r_1}^2}{2g} \right) \quad \ldots(24.4)$$

Equation 24.4 shows that work done on the liquid consists of three parts. The first part represents the change in kinetic energy of the liquid, the second part represents effect of centrifugal head and the last part indicates the change in static pressure energy of the liquid if the losses in the impeller and the effect of difference in elevations of the inlet and the outlet points of the impeller are neglected. Usually equation 24.4 is known as the *fundamental equation* of centrifugal pump.

24.7 HEAD OF PUMP

The head of a centrifugal pump may be expressed in the following two ways:

(*a*) Static head,

(*b*) Manometric head (or total head or gross head or effective head).

(*a*) Static Head. As shown in Fig. 24.6 the static head is the vertical distance between the liquid surfaces in the sump and the tank to which the liquid is delivered by the pump. Thus if h_s is the vertical height of the centre line of the pump shaft above the liquid surface in the sump from which the liquid is being raised; and h_d is the vertical height of the liquid surface in the tank to which the liquid is delivered above the centre line of the pump shaft, then the static head (or lift) H_s may be expressed as

$$\boxed{H_s = h_s + h_d} \quad \ldots(24.5)$$

The term h_s is known as static suction lift, and h_d is known as static delivery lift. Thus static head (or lift) is the net total vertical height through which the liquid is lifted by the pump.

(*b*) Manometric head. It is the total head that must be produced by the pump to satisfy the external requirements. If there are no energy losses in the impeller and the casing of the pump, then the manometric head H_m will be equal to the energy given to the liquid by the impeller, which for radial entry to the impeller = $(V_{w1}u_1/g)$ N-m/N. But if the losses occur in the pump then

$$\boxed{H_m = \frac{V_{w1}u_1}{g} - \text{(losses of head in the pump)}} \quad \ldots(24.6)$$

The various losses of head that may occur in the pump have been described in the next section.

The manometric head may also be expressed in other forms which are as indicated below:

Applying Bernoulli's equation between the points, O at the liquid surface in the sump and 1 in the suction pipe just at the inlet to the pump (*i.e.*, at the centre line of the pump), the following expression is obtained if the liquid surface in the sump is taken as datum.

$$O = \frac{p_s}{w} + \frac{V_s^2}{2g} + h_s + h_{fs}$$

or

$$\frac{p_s}{w} = -\left(\frac{V_s^2}{2g} + h_s + h_{fs}\right) \quad \ldots(24.7)$$

The head $\left(h_d + h_{fd} + \frac{V_d^2}{2g}\right)$ is called the ***delivery head*** of the pump.

Now introducing equations 24.7 and 24.16 in equation 24.12 it becomes

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g} \quad \text{...(24.17)}$$

Eqation 24.17 represents another expression for the manometric head. It is observed from equation 24.17 that all the manometric head is not used to lift the liquid against the statc head, since some of it is used to overcome the various losses in the pipes. Hence

H_m = static head + friction and minor head loss in suction and delivery pipes + the velocity head in the delivery pipe.

If the velocity head in the delivery pipe ($V_d^2/2g$) is relatively small it may be neglected and then equation 24.17 becomes

$$H_m = h_s + h_d + h_{fs} + h_{fd} \quad \text{...(24.17a)}$$

24.8 LOSSES AND EFFICIENCIES

(A) **Losses.** The various losses occurring during the operation of a centrifugal pump may be classified as follows:

(1) Hydraulic losses.

(2) Mechanical losses.

(3) Leakage loss.

(1) **Hydraulic losses.** The *hydraulic losses* that may occur in a centrifugal pump installation may be grouped as

(*a*) Hydraulic losses in the pump.

(*b*) Other hydraulic losses.

The hydraulic losses that may occur within the pump consist of the following:

(*i*) Shock or eddy losses at the entrance to and the exit from the impeller.

(*ii*) Friction losses in the impeller.

(*iii*) Friction and eddy losses in the guide vanes (or diffuser) and casing.

It can be seen from Fig. 24.5 that for the given values of the blade angles θ and ϕ, and the speed of rotation, there can be only one rate of discharge that will ensure tangential entry to the impeller and tangential exit from the impeller. But often the pump is required to operate under varying conditions which results in the variation in the rate of discharge. As such at the entrance and the exit of the impeller the shock losses generally occur. Furthermore at the exit from the impeller there occurs a loss of energy due to the more or less abrupt change in the direction of the velocity of liquid as it enters the casing.

The other hydraulic losses consist of the following:

(*i*) Friction and other minor losses in the suction pipe.

(*ii*) Friction and other minor losses in the delivery pipe.

(2) **Mechanical losses**. The *mechanical losses* occur in the centrifugal pump on account of the following:

(*i*) Disc friction between the impeller and the liquid which fills the clearance spaces between the impeller and the casing.

(*ii*) Mechanical friction of the main bearings and glands.

(3) Leakage loss. In centrifugal pumps as ordinarily built, it is not possible to provide a complete water tight seal between the delivery and suction spaces. As such there is always a certain amount of liquid which slips or leaks from the high pressure to the low pressure points in the pump and it never passes through the delivery pipe. The liquid which escapes or leaks from a high pressure zone to a low pressure zone carries with it energy which is subsequently wasted in eddies. This loss of energy due to leakage of liquid represents the *leakage loss*.

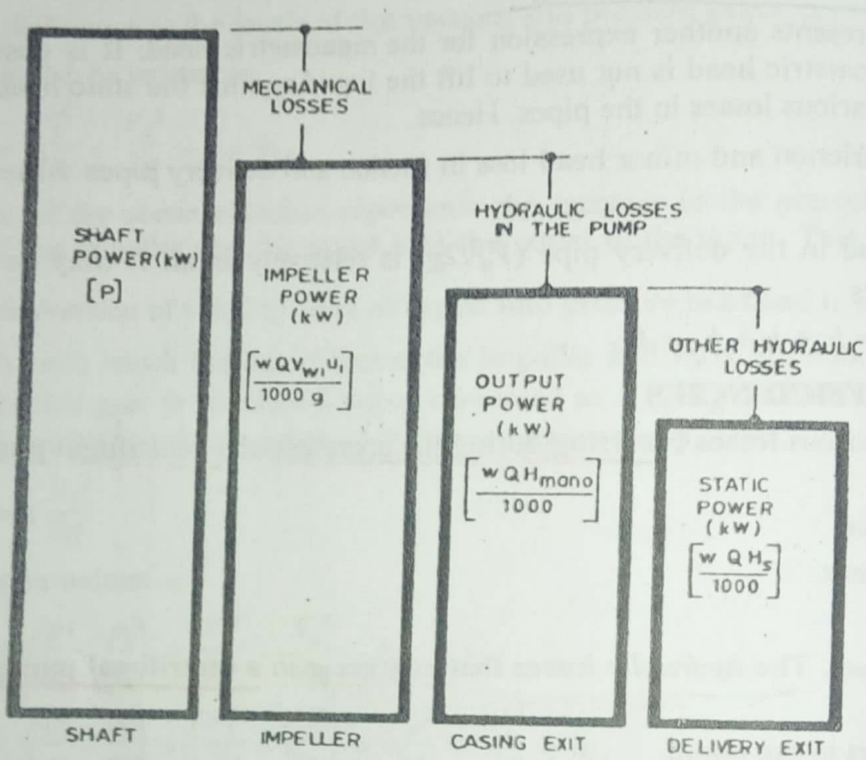


Fig. 24.7 Head losses in centrifugal pump

(B) Efficiencies. The efficiency of a centrifugal pump is expressed in the following form :

(*i*) Manometric efficiency,

(*ii*) Volumetric efficiency,

(*iii*) Mechanical efficiency,

(*iv*) Overall efficiency.

(*i*) **Manometric efficiency**. The manometric efficiency η_{mano} is defined as the ratio of the manometric head developed by the pump to the head imparted by the impeller to the liquid. Thus

$$\boxed{\eta_{mano} = \frac{H_m}{(V_{w1}u_1/g)} = \frac{gH_m}{V_{w1}u_1}} \quad \text{...(24.18)}$$

or

$$\eta_{mano} = \frac{H_m}{H_m + \text{Losses in the pump}}$$

If Q is the volume of liquid actually delivered per second by the pump and w is the specific weight of the liquid then

$$\eta_{mano} = \frac{wQH_m}{wQ(V_{w1}u_1/g)} \quad \text{...(24.19)}$$

or $$\eta_{mano} = \frac{\text{Power actually delivered by the pump}}{\text{Power imparted by the impeller}}$$

or $$\eta_{mano} = \frac{\text{Output of the pump}}{\text{Power imparted by the impeller}} \qquad ...(24.20)$$

[Power imparted by the impeller = Power delivered by the motor to the shaft (or shaft power) – Power lost in mechanical losses]

(*ii*) **Volumetric Efficiency**. The volumetric efficiency η_v is defined as the ratio of the quantity of liquid discharged per second from the pump to the quantity passing per second through the impeller. As stated earlier these two quantities differ by the rate ΔQ at which the liquid from the impeller leaks through the clearances between the impeller and the casing and finds its way back to the eye of the impeller. Thus

$$\eta_v = \frac{Q}{(Q+\Delta Q)} \qquad ...(24.21)$$

where Q is the quantity of liquid actually discharged per second from the pump.

(*iii*) **Mechanical Efficiency**. The mechanical efficiency η_{mech} is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft by the prime mover or motor. Thus

$$\eta_{mech} = \frac{w(q+\Delta Q)(V_{w_1} u_1 / g)}{\text{Power given to the shaft}} \qquad ...(24.22)$$

or $$\eta_{mech} = \frac{(V_{w1}u_1/g)}{\text{Energy head given to the shaft}}$$

or $$\eta_{mech} = \frac{(V_{w1}u_1/g)}{(V_{w1}u_1/g) + (\text{mechanical head losses in bearing})} \qquad ...(24.22a)$$

(*iv*) **Overall Efficiency**. The overall efficiency η_o of the pump is defined as the ratio of the power output from the pump to the power input from the prime mover driving the pump. Thus

$$\eta_o = \frac{wQH_m}{\text{Power given to the shaft}} \qquad ...(24.23)$$

The overall efficiency is also equal to the product of all the three efficiencies described above. That is

$$\eta_o = (\eta_{mano}) \times (\eta_v) \times (\eta_{mech})$$

$$= \frac{H_m}{(V_{w1}u_1/g)} \times \frac{Q}{(Q+\Delta Q)} \times \frac{w(Q+\Delta Q)(V_{w1}u_1/g)}{\text{Power given to the shaft}}$$

$$= \frac{wQH_m}{\text{Power given to the shaft}}$$

which is same as equation 24.23.

(C) **Effect of Vane Angle ϕ on Manometric Efficiency.** If the loss of head in the pump is neglected then the manometric head may be expressed as

$$H_m = \frac{V_{w1}u_1}{g} - \frac{V_1^2}{2g} \qquad ...(24.24)$$

From the outlet velocity triangle shown in Fig. 24.5,

$$V_1^2 = (V_{w1}^2 + V_{f1}^2) \text{ and } V_{w1} = (u_1 - V_{f1} \cot \phi).$$

Further the ratio of the manometric head H_m available from a pump and the head H_i actually imparted by the impeller to the liquid is termed as *hydraulic efficiency* η_h of the pump *i.e.*,

$$\eta_h = \frac{H_i - \text{losses of head in the pump}}{H_i}$$

or

$$\eta_h = \frac{H_m}{H_i}$$

However, the manometric efficiency η_{mano} of a pump has been defined earlier as (equation 24.18)

$$\eta_{mano} = \frac{H_m}{(V_{w1}u_1/g)} = \frac{H_m}{H_e}$$

Thus combining the above noted expressions the manometric efficiency becomes

$$\eta_{mano} = \frac{H_m}{H_i} \times \frac{H_i}{H_e} = (\eta_h \times \varepsilon)$$

Again for $\varepsilon = 1$; $H_i = H_e$ and $\eta_{mano} = \eta_h$

As stated earlier since the actual shape of the velocity triangles for impellers with finite numbers of vanes being not known, the impeller vanes are designed in accordance with the Euler's velocity triangles. As such in the various illustrative examples the value of ε has been assumed as 1 and accordingly H_i has been considered to be same as H_e.

24.9 MINIMUM STARTING SPEED

When the pump is started, there will be no flow of water until the pressure difference in the impeller is large enough to overcome the gross or manometric head. If the impeller is rotating, but there is no flow, then the water is rotating in a forced vortex. Therefore a centrifugal head or pressure head caused by the centrifugal force on the rotating water will be $(u_1^2 - u^2)/2g$. The flow will commence only if

$$[(u_1^2 - u^2)/2g] \geq H_m$$

or

$$\left[\frac{(\pi D_1 N/60)^2}{2g} - \frac{(\pi DN/60)^2}{2g}\right] \geq H_m$$

or

$$\left[\left(\frac{\pi N}{60}\right)^2 (D_1^2 - D^2)\right] \geq (2gH_m)$$

For determining the minimum speed required for the pump to commence the flow, the above expression may be written as

$$\left[\left(\frac{\pi N}{60}\right)^2 (D_1^2 - D^2)\right] = (2gH_m) \qquad \ldots(24.27)$$

from which the required value of N may be computed.

24.10 LOSS OF HEAD DUE TO REDUCED OR INCREASED FLOW

The efficiency of a pump will be maximum only when it is running and discharging at its designed speed. But if the discharge is either reduced or increased, then there will be a loss of head due to shock at the entrance to the impeller, which will result in lowering the efficiency of the pump.

As shown in Fig. 24.8, let *abd* be the inlet velocity triangle for the pump when running under normal conditions. The vanes at inlet tip will be parallel to *ab*. Now if the radial flow through the pump is

The specific speed of a centrifugal pump may be defined as *the speed in revolutions per minute of geometrically similar pump of such a size that under corresponding conditions it would deliver 1 litre of liquid per second against a head of 1 metre.* It is represented by N_s, and it may be expressed by a relation as derived below:

Discharge $\quad Q = (k\pi B_1 D_1)\, V_{f1}$

or $\quad Q \propto B_1 D_1 V_{f1}$

or $\quad Q \propto D_1^2 V_{f1}$; since $B_1 \propto D_1$

Also $\quad V_{f1} = \psi(\sqrt{2gH_m})$; or $V_{f1} \propto (\sqrt{H_m})$

Thus $\quad Q \propto D_1^2 (\sqrt{H_m})$

or
$$\frac{Q}{D_1^2(\sqrt{H_m})} = \text{constant} \qquad \text{...(24.32)}$$

Further $\quad u_1 = (\pi D_1 N/60)$

or $\quad D_1 \propto (u_1/N)$

Also $\quad u_1 = K_u(\sqrt{2gH_m})$; $u_1 \propto (\sqrt{H_m})$

Thus $\quad D_1 \propto (\sqrt{H_m}/N)$

or
$$\frac{\sqrt{H_m}}{D_1 N} = \text{constant} \qquad \text{...(24.33)}$$

Substituting the value of D_1 from equation 24.33 in equation 24.32 it becomes

$$Q \propto (H_m^{3/2}/N^2) \qquad \text{...(24.34)}$$

or $\quad N^2 \propto (H_m^{3/2}/Q)$

or $\quad N \propto (H_m^{3/4}/\sqrt{Q})$

or $\quad N = C(H_m^{3/4}/\sqrt{Q})$; where C is a constant

or
$$\frac{N\sqrt{Q}}{H_m^{3/4}} = C \qquad \text{...(24.35)}$$

Now according to the definition of the specific speed, putting $Q = 1$ litre/second and $H_m = 1$ m, then $C = N = N_s$. Thus

$$\boxed{N_s = \frac{N\sqrt{Q}}{H_m^{3/4}}} \qquad \text{...(24.36)}$$

Equation 24.36 represents the expression for the specific speed of a centrifugal pump, which is based on the unit discharge. It is the most commonly adopted expression for the specific speed of the pumps. The value of N_s usually varies from about 300 to 15,000 for a single impeller. Further the value of N_s is same in SI and metric system of units and it is equal to 0.67 times N_s in F.P.S system, where Q is in gallons per minute and H_m is in feet.

The values of Q and H_m to be substituted in equation 24.36 are those corresponding to the maximum efficiency of the pump at its normal working speed.

For a multi-stage pump the value of H_m to be used in equation 24.36 is obtained by dividing the total head developed by the number of stages. Similarly for a double suction pump half the actual discharge delivered by the pump is taken as Q.

Sometimes another definition of the specific speed of a centrifugal pump is used, which is based on unit power, though it is not very common. According to this basis the specific speed is defined as "

$$\left[\frac{N\sqrt{Q}\,\eta_{mano}^{3/4}}{gH^{3/4}}\right]_m = \left[\frac{N\sqrt{Q}\,\eta_{mano}^{3/4}}{gH^{3/4}}\right]_p \quad \text{...(24.42 a)}$$

Further the efficiency of the prototype pump being not equal to that of the model pump, by applying suitable correction factor, the efficiency of the prototype pump can be predicated from the efficiency obtained for its model. Based on the test efficiencies of single stage centrifugal pumps, G.F. Wislicenus has suggested the following expression relating the efficiencies of the model and the prototype pumps.

$$\frac{0.95-\eta_m}{0.95-\eta_p} = \left[\frac{0.658+\log_{10} Q_p}{0.658+\log_{10} Q_m}\right]^2 \quad \text{...(24.43)}$$

in which Q is the discharge in liters per minute.

24.14 PUMP IN SERIES—MULTI-STAGE PUMPS

The head produced by a centrifugal pump depends on the rim speed of the impeller. To increase the rim speed, either the rotative speed or the diameter of the impeller or both must be increased. Increasing either of these has the effect of increasing the stress in the impeller material. For this reason it is usually not possible to produce very high head with one impeller. Normally a pump with a single impeller can be used to deliver the required discharge against a maximum head of about 100 m. But if the liquid is required to be delivered against a still larger head then it can be done by using two or more *pumps in series*. The pumps in series may be used in boiler feeding system to satisfy high head demand, in multi-stations systems along pipelines such as long oil pipelines, for boosting the liquid like pumping installations interposed in the water mains, etc. The higher heads may also be produced by using multi-stage pumps.

A multi-stage pump consists of two or more identical impellers mounted on the same shaft, and enclosed in the same casing. All the impellers are connected in series, so that liquid discharged with increased pressure from one impeller passes through the connecting passages to the inlet of the next impeller and so on, till the discharge from the last impeller passes into the delivery pipe. The impellers are surrounded by guide vanes which are generally provided within the connecting passages, and are meant for the recuperation of the velocity energy of the liquid leaving the impeller into pressure energy. According to the number of impellers fitted in the casing a multi-stage pump is designated as two stage, three-stage, etc. Fig. 24.9 shows a three-stage centrifugal pump.

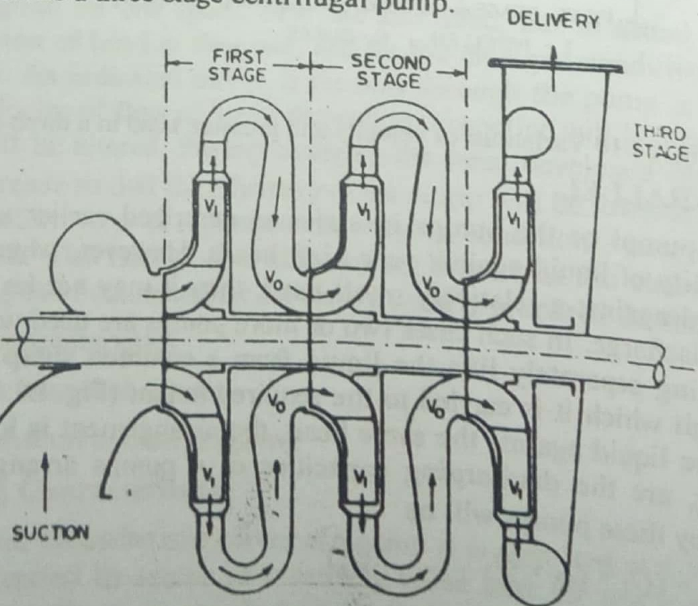


Fig. 24.9 Three-stage centrifugal pump

Large pumps are usually primed by evacuating the casing and the suction pipe with the aid of an pump or a stream ejector, the liquid is thus sucked into the suction pipe from the sump.

In some pumps, their internal construction is such that special arrangements containing a supply liquid are provided in the suction pipe, which facilitate automatic priming of the pump. Such pumps known as 'self priming pumps'.

24.22 CENTRIFUGAL PUMP-TROUBLES AND REMEDIES

Some of the troubles commonly experienced during the operation of the centrifugal pumps and remedial measures to be taken for the same are as listed below:

(1) *Pump fails to start pumping*:

(*i*) Pump is probably not properly primed. Reprime the pump, opening the air vent until a steady unbroken stream of liquid is obtained.

(*ii*) Total static head is probably much higher than that for which the pump is designed. Check the same with accurate vacuum and pressure gages, or may be determined by actual measurement the difference in elevation between pump and suction liquid level, and also between pump and point of discharge. Add to this static head the loss of head due to friction in pipes and that in the other fittings used in the installation.

(*iii*) Wrong direction of rotation of the impeller. Arrow on the pump case shows the proper direction of rotation.

(*iv*) Impeller may be clogged. Examine carefully for solids or foreign matter lodged in the impeller.

(*v*) Suction lift too high. Check with vacuum gage or by actual measurement where possible, adding to the static suction lift the loss of head in the pipe and fittings.

(*vi*) Strainer or suction line may be clogged. This would cause an excessive suction lift.

(*vii*) Speed may be too low. Check the speed with a techeometer and compare it with the one given on the name plate of the pump. When the pump is being driven by electric motor, check up to see whether the voltage may not be too low.

(2) *Pump working but not upto capacity and pressure*:

(*i*) Air may be leaking into pump through suction line or through stuffing boxes.

(*ii*) Speed may be too low.

(*iii*) Discharge head may be higher than anticipated.

(*iv*) Suction lift may be too high.

(*v*) Foot valve or end of suction pipe may not have sufficient submergence, or it may be partly clogged or entirely too small.

(*vi*) Impeller may be partly clogged or too small in diameter.

(*vii*) Rotation may be in the wrong direction.

(*viii*) Wearing rings may be worn, impeller may be damaged, shaft may be loose, stuffing box packing may be defective.

(*ix*) If hot or volatile liquids are being pumped, there may not be sufficient positive head on the suction.

(3) *Pump starts and then stops pumping*:

(*i*) Improperly primed or leaky suction line.

(*ii*) Air pockets in suction line.

(*iii*) Air may be entering suction pipe because of liquid being delivered in suction tank or sump too near the pump suction line.

(*iv*) Suction lift may be too high.

(4) *Pumps take too much power*:

(*i*) Speed may be too high.

(*ii*) Head may be too low and pump delivers too much liquid.

(*iii*) Liquid may have too high a specific gravity.

(*iv*) Pump may be operating in wrong direction.

(*v*) Shaft may be bent, impeller may be rubbing on casing, stuffing boxes may be too light, wearing rings may be worn.

Illustrative Example 24.1. *A centrifugal pump has an impeller 0.5 m outer diameter and when running at 600 r.p.m. discharges water at the rate of 8000 litres/minute against a head of 8.5 m. The water enters the impeller without whirl and shock. The inner diameter is 0.25 m, and the vanes are set back at outlet at an angle of 45° and the area of flow which is constant from inlet to outlet of the impeller is 0.06 m². Determine (a) the manometric efficiency of the pump, (b) the vane angle at inlet, and (c) the least speed at which the pump commences to work:*

Solution:

$$\boxed{V_f = V_{f_1} = \frac{Q}{A}}$$

$$= \frac{8000 \times 10^{-3}}{60 \times 0.06} = 2.22 \text{ m/s}$$

$$\boxed{u = \frac{\pi DN}{60}}$$

$$= \frac{\pi \times 0.25 \times 600}{60} = 7.85 \text{ m/s}$$

$$\boxed{u_1 = \frac{\pi D_1 N}{60}}$$

$$= \frac{\pi \times 0.50 \times 600}{60} = 15.71 \text{ m/s}$$

From the inlet velocity triangle of Fig. 24.5

$$\boxed{\tan\theta = \left(\frac{V_f}{u}\right)}$$

$$= \left(\frac{2.22}{7.85}\right) = 0.2828$$

$\therefore \quad \theta = 15° \, 47'.$

From the outlet velocity triangle

$$\boxed{V_{w_1} = \left[u_1 - \left(\frac{V_{f_1}}{\tan 45°}\right)\right]}$$

$$= (15.71 - 2.22)$$

$$= 13.49 \text{ m/s}$$

theoretical discharge Q_{th}. The ratio between the actual discharge Q_a and the theoretical discharge Q_{th} is defined as the *coefficient of discharge* C_d of the pump. That is

$$C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q_a}{Q_{th}} \qquad \ldots(23.13)$$

The value of C_d when expressed in terms of percentage is also known as *volumetric efficiency* of the pump.

(*b*) **Slip and Percentage Slip**. The difference between the theoretical discharge and the actual discharge is known as the *slip* of the pump. That is

$$\boxed{\text{slip} = (Q_{th} - Q_a)} \qquad \ldots(23.14)$$

Often the slip is expressed in terms of percentage which is then called *percentage slip* and it is given by

$$\text{Percentage slip} = \frac{Q_{th} - Q_a}{Q_{th}} \times 100 = (1 - C_d) \times 100 \qquad \ldots(23.15)$$

For the pumps maintained in good condition the percentage slip is of the order of 2 per cent or even less.

(*c*) **Negative Slip**. For most of the reciprocating pumps the actual discharge Q_a is less than the theoretical discharge Q_{th}, in which case C_d of the pump is less than one and the slip of the pump is positive, However, in some cases the actual discharge of the pump may be more than the theoretical discharge, in which case C_d will be more than one and the slip will be negative, which is then known as *negative slip*.

This may happen in the case of pumps having long suction pipe and low delivery head, especially when these are running at high speed. This is so because for such pumps the inertia pressure in the suction pipe will be large in comparison to the pressure on the outside of the delivery valve, which may cause the delivery valve to open before the suction stroke is completed. Some liquid is thus pushed directly into the delivery pipe even before the delivery stroke is commenced, which results in making the actual discharge more than the theoretical discharge.

23.6 EFFECT OF ACCELERATION OF PISTON ON VELOCITY AND PRESSURE IN THE SUCTION AND DELIVERY PIPES

During the reciprocating motion of the piston or plunger it does not move with uniform velocity for the entire stroke, but as indicated later it has an acceleration developed for the first half of each stroke and for the later half of the stroke there is retardation developed for it. Since the liquid flowing through the pump closely follows the piston or plunger, its acceleration and retarding effects will be correspondingly transmitted to the liquid flowing in the suction and delivery pipes. In other words the velocity of flow of liquid in the suction and delivery pipes will not be uniform but during the same stroke the velocity of flow in these pipes will vary with the crank position. These variations in the velocities of flow of liquid in the suction and delivery pipes give rise to inertia pressures which will cause a variation of pressure in the cylinder. The magnitudes of the inertia pressures so developed can be calculated as indicated below.

If the connecting rod is very long as compared with the length of the crank, then it can be assumed that the piston or plunger moves with simple harmonic motion. The assumption considerably simplifies the problem. Let the crank be rotating with an angular velocity ω radians per second and in time t seconds let it turn through an angle θ, from its inner dead centre. Then

$$\boxed{\theta = \omega t = \frac{2\pi N t}{60}}$$

If x is the linear displacement of the piston from the end of the stroke in t seconds then from Fig. 23.1, we have

$$x = r - r\cos\theta = r - r\cos(\omega t)$$

Further if v is the velocity of the piston and f is the acceleration of the piston then, we have

$$v = \frac{dx}{dt} = \omega r \sin(\omega t) = \omega r \sin\theta$$

and $$f = \frac{dv}{dt} = \omega^2 r\cos(\omega t) = \omega^2 r \cos\theta$$

Now if A represents the cross-sectional area of the piston or plunger and a is the area of the pipe (either suction or delivery pipe), then since the volume of liquid flowing from the pipe per second equals the volume of liquid flowing into the cylinder per second, then the velocity V of flow of liquid in the pipe is given by

$$V = \frac{A}{a}v = \frac{A}{a}\omega r\sin(\omega t) = \frac{A}{a}\omega r\sin\theta \qquad \ldots(23.16)$$

Similarly the acceleration of the liquid in the pipe is given by acceleration $= f\frac{A}{a} = \frac{A}{a}(\omega^2 r)\cos\theta.$

Now if l is the length of the pipe, then the weight of the liquid which is subjected to the above noted acceleration is (wal) and the corresponding mass of the liquid is $\left(\frac{wal}{g}\right)$.

Further if p_a is the intensity of pressure due to acceleration of liquid in the pipe then the accelerating force $= (p_a)a$.

But according to Newton's second law of motion

Accelerating force = mass × acceleration

Thus

$$(p_a)a = \frac{wal}{g} \times \frac{A}{a}(\omega^2 r)\cos\theta$$

$$\ldots(23.17)$$

or $$p_a = \frac{wl}{g} \times \frac{A}{a}(w^2 r)\cos\theta$$

From equation 23.17 the pressure head H_a due to acceleration may be obtained as

$$H_a = \frac{p_a}{w} = \frac{l}{g}\frac{A}{a}(\omega^2 r)\cos\theta \qquad \ldots(23.18)$$

It may thus be seen from equation 23.18 that the pressure head developed due to acceleration acting on the piston or plunger will vary with angle θ.

At the beginning of the stroke when $\theta = 0°$, $\cos\theta = 1$, *i.e.*, when piston or plunger is at its inner dead centre, then

$$H_a = \frac{l}{g}\frac{A}{a}(\omega^2 r) \qquad \ldots(23.19)$$

At the middle of the stroke when $\theta = 90°$, $\cos\theta = 0$, then

$$H_a = 0 \qquad \ldots(23.20)$$

At the end of the stroke when $\theta = 180°$, $\cos\theta = -1$, *i.e.*, when piston of plunger is at its outer dead centre, then

$$H_a = -\frac{l}{g}\frac{A}{a}(\omega^2 r) \qquad \ldots(23.21)$$

only the curves corresponding to the delivery of the liquid are shown. Thus the mean discharge $(Q_d)_{mean}$ for a double acting pump can also be obtained by integrating equation 23.31 as follows:

$$(Q_d)_{mean} = 2 \times \frac{1}{2\pi}\left[\int_0^{\pi} A\omega r \sin\theta d\theta\right]$$

$$= \frac{2A\omega r}{\pi} \quad \text{...(23.37)}$$

However the above expression for the mean discharge may also be obtained by multiplying $(V_d)_{mean}$ given by equation 23.35 by the area of the pipe $\left(\frac{\pi d_d^2}{4}\right)$.

Again by dividing equation 23.37 by equation 23.33 we obtain for double acting pump

$$\frac{(Q_d)_{mean}}{(Q_d)_{max}} = \frac{2}{\pi} \quad \text{...(23.38)}$$

However, if the area of piston rod is taken into account then it can be shown that the instantaneous discharges Q_{d_1} and Q_{d_2} for the two delivery pipes on either side of piston or plunger will be

$$\left.\begin{aligned} Q_{d_1} &= A\omega r \sin\theta \\ Q_{d_2} &= (A - A_p)\omega r[\sin(\pi - \theta)] \end{aligned}\right\} \quad \text{...(23.39)}$$

23.7 INDICATOR DIAGRAMS

(*a*) **Theoretical Indicator Diagram**. Fig. 23.4 shows the theoretical indicator diagram of a reciprocating pump which represents the work done by a single acting pump during one complete cycle as given by equation 23.3. In this diagram the pressure head on the piston or plunger is plotted along the vertical ordinate and the length of the stroke is represented by the abscissa. The horizontal line *fe* represents the atmospheric pressure. The line *dc* represents the pressure head in the cylinder during the suction stroke, which is below atmospheric pressure by an amount equal to suction head H_s. The line *ab* represents the pressure in the cylinder during the delivery stroke, which is above atmospheric pressure by an amount equal to delivery head H_d. The area *dcef* represents to some scale the work done by the piston or plunger during the suction stroke and the area *abef* represents to the same scale the work done by the piston or plunger during the delivery stroke. The total work done by the piston or plunger during the complete revolution of the crank is then represented to the same scale by area *abcd* which is equal to $(H_s + H_d)L$.

If the pump is double acting, this diagram will represent the pressure head on one side of the piston or plunger only. The work done per revolution will then be represented by twice the area of this diagram, if the area of the piston rod is neglected.

Such a diagram may be obtained automatically by means of an indicator placed on the cylinder and hence it is called an indicator diagram.

(*b*) **Effect of Acceleration in Suction Pipe on Indicator Diagram**. Due to the acceleration head developed in the suction pipe during the suction stroke, the theoretical indicator diagram will be modified as indicated below. If l_s, d_s, a_s are respectively the length, diameter and the cross-sectional area of the suction pipe, then the acceleration head H_{as} in the pipe is given by equation 23.18 as

$$H_{as} = \frac{l_s}{g}\frac{A}{a_s}\omega^2 r \cos\theta \quad \text{...(23.40)}$$

At the beginning of the suction stroke since $\theta = 0^\circ$, $\cos\theta = 1$, the acceleration head is

$$H_{as} = +\frac{l_s}{g} \times \frac{A}{a_s}\,\omega^2 r \qquad \ldots(23.41)$$

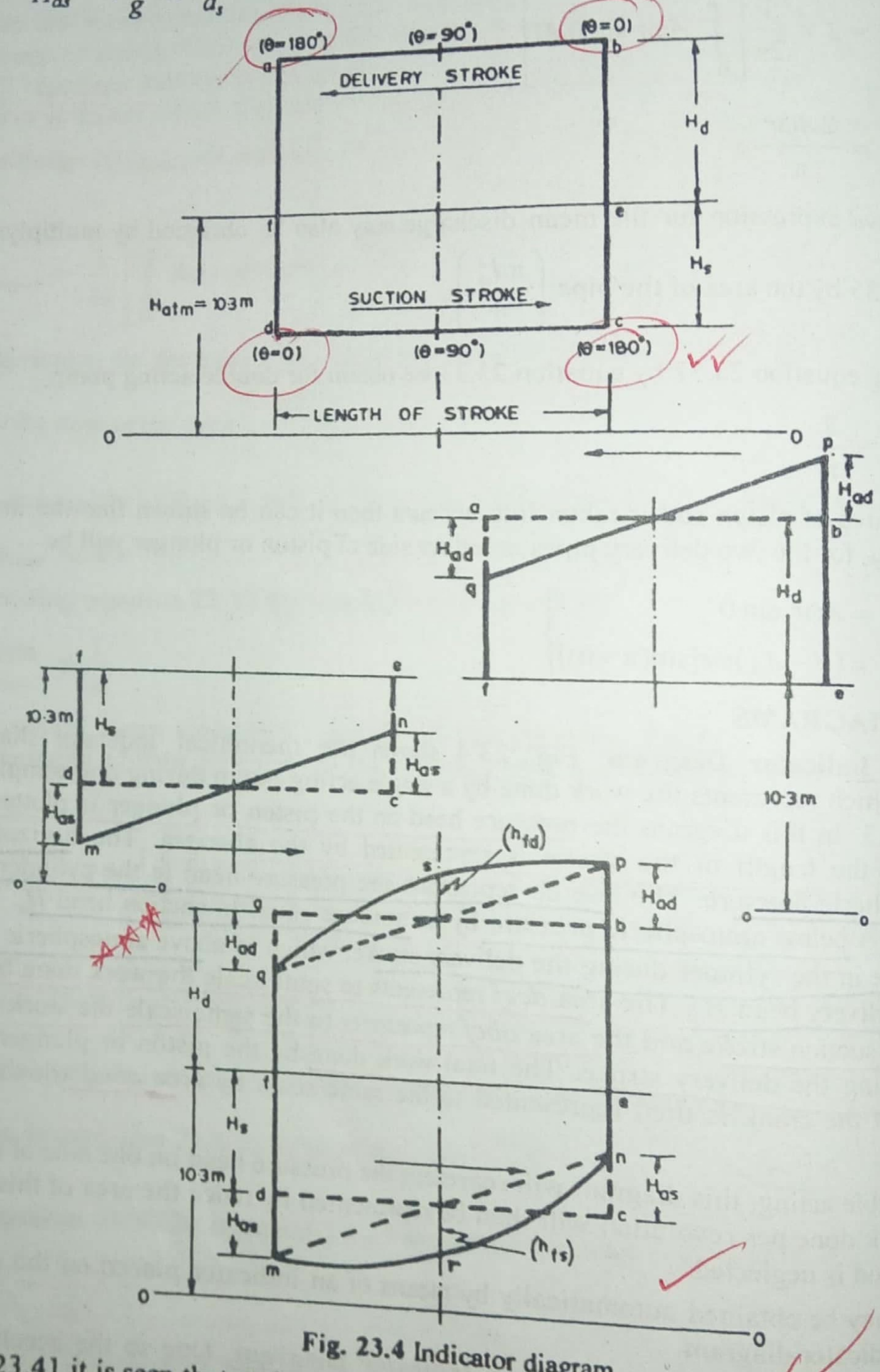


Fig. 23.4 Indicator diagram

From equation 23.41 it is seen that at the beginning of the suction stroke since H_{as} is positive the liquid in the suction pipe has to be accelerated, for which an additional drop in pressure in the cylinder is required. As a result the pressure head in the cylinder will further drop by an amount equal to H_{as}, so that at the beginning of the suction stroke the pressure head will be $(H_s + H_{as})$ below the atmospheric pressure head

At the middle of the suction stroke since $\theta = 90°$, $\cos\theta = 0$, the acceleration head $H_{as} = 0$. Thus at the middle of the suction stroke the pressure head will be only H_s below the atmospheric pressure head.

At the end of the suction stroke since $\theta = 180°$, $\cos\theta = -1$, the acceleration head is

$$H_{as} = -\frac{l_s}{g} \times \frac{A}{a_s} \omega^2 r \quad \text{...(23.42)}$$

From equation 23.42 it is seen that at the end of the suction stroke the liquid is to be retarded, corresponding to which a pressure rise in the cylinder is needed. As a result the pressure head in the cylinder will rise by an amount H_{as}, so that at the end of the suction stroke the pressure head will be $(H_a - H_{as})$ below the atmospheric pressure head.

The indicator diagram for the suction stroke will thus be modified as shown in Fig. 23.4. The base of the diagram is now changed from *dc* to *mn*. Also the work done during the suction stroke is now represented by the area *mnef*. However, area *mnef* being equal to the area *dcef*, the net work done during the suction stroke is not altered on account of the accelerating effects in the suction pipe. It is thus observed that the inertia pressure developed in the suction pipe does not affect the net work done, but merely causes a variation of pressure in the cylinder.

(c) **Maximum Speed of a Reciprocating Pump**. The speed of a reciprocating pump is usually restricted by the pressure corresponding to the point *m* of the indicator diagram. This is so because higher is the speed, the greater will be the acceleration head and lower will be the pressure in the cylinder at the beginning of the suction stroke. The pressure in the cylinder must not be allowed to fall to the value at which dissolved gases are liberated from the liquid, since under these conditions the cavitation may occur and the continuity of the flow may not exist. For water the value of this limiting pressure is about 2.6 m of water absolute or if atmospheric pressure is 10.3 m of water $(H_s + H_{as})$ at any point during the suction stroke must not be greater than (10.3–2.6) = 7.7 m of water. The maximum permissible speed of a reciprocating pump may thus be computed from the following expression

$$H_{sep} = H_s + H_{as}$$

or

$$H_{sep} = H_s + \frac{l_s}{g}\frac{A}{a_s}\omega^2 r$$

or

$$H_{sep} = H_s + \frac{l_s}{g}\frac{A}{a_s}\left(\frac{2\pi N}{60}\right)^2 r \quad \text{...(23.43)}$$

where H_{sep} represents the pressure head below atmospheric pressure head at which the separation of dissolved gases and cavitation may occur.

It may, however, be seen from equation 23.43 that the drop of pressure in the cylinder during the beginning of the suction stroke may also be limited by suitably adjusting H_s, l_s and (A/a_s) for the pump.

(d) **Effect of Acceleration in Delivery Pipe on Indicator Diagram**. As in the case of suction pipe the acceleration head will also be developed for the liquid flowing in the delivery pipe, due to which the indicator diagram will be modified. If l_d, d_d and a_d are respectively the length, diameter and the cross-sectional area of the delivery pipe then the acceleration head in the delivery pipe is given by equation 23.18 as

$$H_{ad} = \frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \cos\theta \quad \text{...(23.44)}$$

At the beginning of the delivery stroke $\theta = 0°$, $\cos\theta = 1$, the acceleration head is

$$H_{ad} = +\frac{l_d}{g} \times \frac{A}{a_d} \omega^2 r \quad \text{...(23.45)}$$

Again at the beginning of the delivery stroke since H_{ad} is positive, the liquid in the delivery pipe is to be accelerated, for which an additional pressure head is required to be developed in the cylinder. As a result at the beginning of the delivery stroke the pressure head in the cylinder will be $(H_d + H_{ad})$ above the atmospheric pressure head.

At the middle of the delivery stroke since $\theta = 90°$, $\cos\theta = 0$, the acceleration head $H_{ad} = 0$. Thus at this point of the delivery stroke the pressure head in the cylinder will be only H_d above the atmospheric pressure head.

At the end of the delivery stroke since $\theta = 180°$, $\cos\theta = -1$, the acceleration head is

$$H_{ad} = -\frac{l_d}{g} \times \frac{A}{a_d}\,\omega^2 r \qquad \ldots(23.4$$

Thus at the end of the delivery stroke the liquid is to be retarded, corresponding to which a drop in pressure in the cylinder is needed. As a result the pressure head in the cylinder will drop by an amount H_{ad}, so that at the end of the delivery stroke the pressure head will be $(H_d - H_{ad})$ above the atmospheric pressure head.

The indicator diagram for the delivery stroke will thus be modified as shown in Fig. 23.4. The top of the diagram is now changed from ab to pq. Again the work done during the delivery stroke is now represented by the area $pqfe$ which is however equal to the area $abef$. As such during the delivery stroke also the net work done is not altered on account of the accelerating effects in the delivery pipe, but they merely cause a variation of pressure in the cylinder.

During the delivery stroke the minimum pressure head in the cylinder is at the end of the stroke which is represented by the pressure head at point q of the indicator diagram and it is equal to $(H_{atm} + H_d - H_{ad})$ absolute. Again this pressure head must not be less than the absolute separation pressure head of the liquid, otherwise cavitation may occur at the end of the delivery stroke. For water the separation pressure head is about 2.6 m of water absolute, and hence in the limiting condition

$$10.3 + H_d - H_{ad} = 2.6$$

or

$$H_{ad} = 7.7 + H_d \qquad \ldots(23.4$$

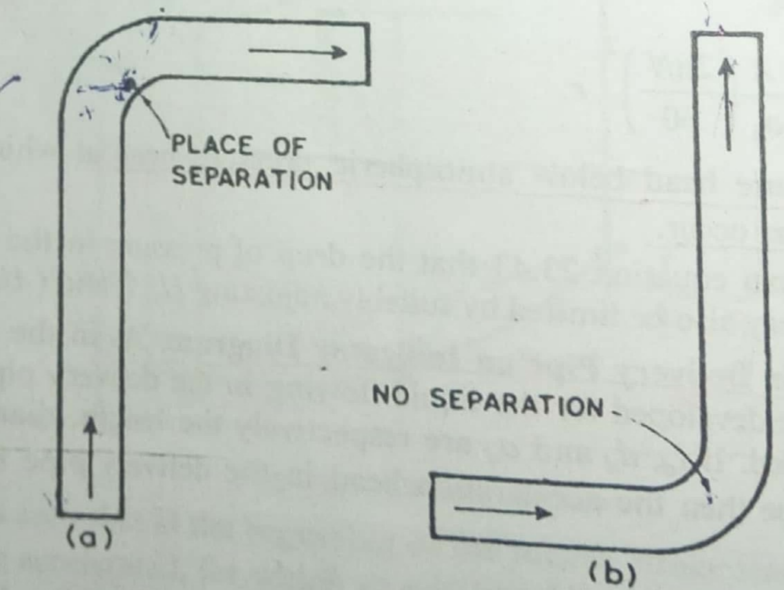


Fig. 23.5 Different arrangements of delivery pipe of a reciprocating pump

The delivery pipe of a reciprocating pump may have either of the two arrangements as shown in Fig. 23.5. In the arrangement shown in 23.5(a) the delivery pipe is first vertical and then horizontal this case the delivery head H_d will become zero at the bend, after which there is still a long horizontal pipe which will have a considerable value of accelerating head H_{ad}. Therefore according to equation 23.47 the separation may take place at the bend in this case. On the other hand in the arrangement shown in Fig. 23.5 (*b*) the pipe is horizontal first and then it is vertical. As such there is still considerable H_d available at the bend in this case. Therefore there is no possibility of separation to occur at the bend in this case.

(*e*) **Effect of Friction in Suction and Delivery Pipes on Indicator Diagram.** A further modification of the indicator diagram results from the effect of friction losses in the suction and delivery pipes. The loss of head due to friction h_f in these pipes is given by equation 23.25, which shows that the variation of h_f with θ is parabolic. At the beginning and the end of the suction as well as delivery strokes since $\theta = 0°$ and $\theta = 180°$, $\sin\theta = 0$, the head loss due to friction h_{fs} and h_{fd} in the suction and delivery pipes respectively will be equal to zero. At the middle of the stroke since $\theta = 90°$, $\sin\theta = 1$, the head loss due to friction h_{fs} and h_{fd} will be

$h_f = \frac{4fl\,V^2}{2gd}$

$$h_{fs} = \frac{fl_s}{2gd_s}\left(\frac{A}{a_s}\omega r\right)^2 \quad \ldots(23.48)$$

and

$$h_{fd} = \frac{fl_d}{2gd_d}\left(\frac{A}{a_d}\omega r\right)^2 \quad \ldots(23.49)$$

Now if the effect of the loss of head due to friction in the suction pipe is considered then as shown in Fig. 23.4 the base of the diagram will change from *mn* to *mrn*, the parabola *mrn* being the work done against friction in the suction pipe. Similarly by considering the effect of the loss of head due to friction in the delivery pipe the top of the diagram will change from *pq* to *psq* as shown in Fig. 23.4, the parabola *psq* being the work done against friction in the delivery pipe.

Thus the total work done during suction stroke is represented by area (*efmrn*) which is equal to [area (*efdc*) + area (*mrn*)]. Similarly the total work done during delivery stroke is represented by area (*efqsp*) which is equal to [area (*abef*) + area (*psq*)].

As the mean ordinate of a parabola equals two-thirds of its maximum ordinate the total area of the indicator diagram *mrnpsq* is equal to

$$L\left[H_s + H_d + \frac{2}{3}h_{f_s} + \frac{2}{3}h_{f_d}\right]$$

which to some scale represents the total work done during one complete revolution for a single acting pump. The total work done per second for a single acting pump is given as

$$\text{Work done} = \frac{wALN}{60}\left[H_s + H_d + \frac{2}{3}h_{f_s} + \frac{2}{3}h_{f_d}\right] \quad \ldots(23.50)$$

The absolute pressure head on the piston or plunger during suction stroke for any position of the crank angle θ may now be expressed as *i.e.*, for any crank angle θ may now be expressed as

$$H_{ts} = H_{atm} - H_s - H_{as} - h_{fs}$$

or

$$H_{ts} = H_{atm} - H_s - \left(\frac{l_s}{g} \times \frac{A}{a_s} \times \omega^2 r\cos\theta\right) - \frac{fl_s}{2gd_s}\left(\frac{A}{a_s}\omega r\sin\theta\right)^2 \quad \ldots(23.51)$$

From equation 23.51 it can be shown that the minimum value of the absolute pressure H_{ts} will occur for $\theta = 0°$ *i.e.*, at the beginning of the suction stroke. Therefore if cavitation occurs during the suction stroke, it will do so at the beginning of the stroke, at the junction of suction pipe with the cylinder.

Again the absolute pressure head on the piston or plunger during delivery stroke for any position of the crank *i.e.*, for any crank angle θ may be expressed as

$$H_{td} = H_{atm} + H_d + H_{ad} + h_{fd}$$

or

$$H_{td} = H_{atm} + H_d + \left(\frac{l_d}{g} \times \frac{A}{a_d} \times \omega^2 r \cos\theta\right) + \frac{fl_d}{2gd_d}\left(\frac{A}{a_d} \times \omega r \sin\theta\right)^2 \quad ...(23.52)$$

From equations 23.51 and 23.52 it is evident that the acceleration heads H_{as} and H_{ad} have maximum values at the ends of the stroke *i.e.*, when $\theta = 0°$ and $\theta = 180°$, and are equal to zero at the middle of the stroke *i.e.*, when $\theta = 90°$. On the other hand the frictional head losses h_{fs} and h_{fd} are equal to zero at the ends of the stroke and have a maximum value at the middle of the stroke.

23.8 AIR VESSELS

For a single acting pump, the flow rate in the delivery pipe at any instant varies considerably, having a value of zero during the whole of the suction stroke and rising to a maximum during the delivery stroke. In other words the supply of liquid by a single-acting pump will be quite intermittent. To some extent this difficulty may be overcome if the pump is double acting or if it is a multi-cylinder pump so that a continuous supply of liquid may be obtained. But in the case of almost all the pumps the velocity of flow in both suction and delivery pipes vary with the crank position, which results in developing a non-uniform rate of flow of liquid in these pipes. As such in order to obtain a continuous supply of liquid at a uniform rate from a single-acting pump and also to obtain a uniform rate of flow of liquid in both suction and delivery pipes of a reciprocating pump, a large air vessel is fitted to the suction as well as the delivery pipe at a point close to the cylinder. An *air vessel* is a close chamber having an opening at its base through which the liquid may flow into the vessel or it may flow out from the vessel. The top portion of the air-vessel contains compressed air, which will be further compressed when the liquid enters the vessel and it will expand when the liquid flows out from the vessel. Fig. 23.6 shows air-vessels fitted to suction and delivery pipes of a single-acting pump, the working of each of these is explained below.

Consider an air vessel fitted to the suction pipe of the pump. During the first half of the suction stroke the piston or plunger moves with acceleration and hence the liquid in the suction pipe has also to be accelerated. In other words the liquid in the suction pipe must have a velocity of flow more than the mean velocity, so that the corresponding flow rate of liquid entering the cylinder may be more than the mean discharge. This excess quantity of liquid required on account of accelerating effects will however be supplied from the air vessel, so that the velocity of flow of liquid in the portion of the suction pipe below the point at which the air vessel is connected to it will be equal to the mean velocity of flow only. During the second half of the suction stroke the piston or plunger moves with retardation and hence the liquid in the suction pipe is also required to be retarded. In other words the liquid in the suction pipe must have a velocity of flow less than the mean velocity, so that the corresponding flow rate of liquid entering the cylinder is less than the mean discharge. Since the velocity of flow of liquid in the portion of the suction pipe below the point at which the air vessel is connected to it is equal to the mean velocity of flow, the excess quantity of liquid flowing in the suction pipe will flow into the air-vessel thus compressing the air inside the vessel. In this way a relatively small quantity of liquid will only be entering the cylinder from the suction pipe. The liquid thus stored in the air-vessel will be supplied during the first half of the next suction stroke and the same cycle will thus be repeated.

By subtracting equation 23.77 from equation 23.73 the work saved per stroke by fitting an air vessel to the suction or delivery pipe of a double acting pump is obtained as

$$W_1 - W_2 = (2r)wA\left[\frac{fl}{2gd}\right]\left[\frac{A}{a}\omega r\right]^2\left[\frac{2}{3}-\frac{4}{\pi^2}\right] \qquad \cdots(23.78)$$

Again the percentage of the work saved during any stroke for a double acting pump

$$= \frac{W_1 - W_2}{W_1} \times 100 = \frac{\left(\frac{2}{3}\right)-\left(\frac{4}{\pi^2}\right)}{\left(\frac{2}{3}\right)} \times 100$$

$$= 39.2\%.$$

23.9 MULTI-CYLINDER PUMPS

As indicated earlier, pumps having more than one cylinder are known as 'multicylinder pumps'. Some of these pumps are shown in Fig. 23.2. The main advantage of multicylinder pumps are that these pumps even without air vessels deliver liquid more uniformly as compared with a single cylinder pump. This is made clear by the various discharge diagrams for different types of pumps shown in Fig. 23.7, in which the instantaneous discharge from each individual cylinder is compared with the mean discharge from the pump. It will be observed that by a suitable disposition of the discharge from the individual cylinder, the total discharge of a multicylinder pump may be made nearly equal to the mean discharge with almost negligible fluctuation. On account of much smaller fluctuations of velocity in both the suction and delivery pipes of a multicylinder pump, no air-vessel is required for the same.

23.10 OPERATING CHARACTERISTIC CURVES OF RECIPROCATING PUMPS

The operating characteristic curves indicating the performance of a reciprocating pump are shown in Fig. 23.8. These curves are obtained by plotting discharge, power input and overall efficiency against the head developed by the pump when it is operating at a constant speed. As shown in Fig. 23.8, under ideal conditions the discharge of a reciprocating pump operating at constant speed is independent of the head

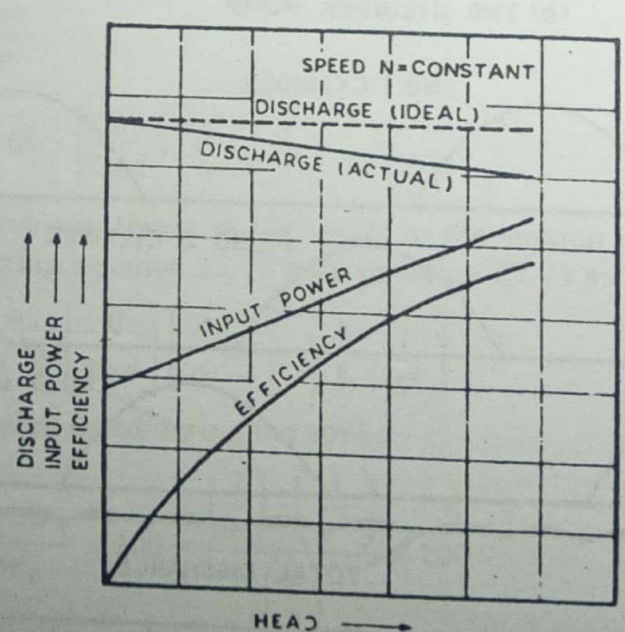


Fig. 23.8 Operating characteristic curves of a reciprocating pump

giving the numerical values of this function for different values of u. However, it can also be calculated from the following expression.

$$\int \frac{du}{1-u^3} = \left[\frac{1}{6}\log_e \frac{u^2+u+1}{(u-1)^2} - \frac{1}{\sqrt{3}}\cot^{-1}\left(\frac{2u+1}{\sqrt{3}}\right)\right] \quad ...(16.12)$$

Further the ratio $\left(\frac{y_c}{y_n}\right)$ in a wide rectangular channel can be expressed as

$$\left(\frac{y_c}{y_n}\right)^3 = \frac{C^2 S_0}{g}$$

where C is Chezy's coefficient.

Thus substituting the above expression in equation 16.11 and writing $\int \frac{du}{1-u^3}$ as $\phi(u)$, the Bresse's solution for the varied flow equation becomes

$$(x_2 - x_1) = \frac{y_n}{S_0}\left[(u_2 - u_1) - \left(1 - \frac{C^2 S_0}{g}\right) \times \{\phi(u_2) - \phi(u_1)\}\right] \quad ...(16.13)$$

in which u_2 and u_1 are the values of u at x_2 and x_1 respectively and $\phi(u_2)$ and $\phi(u_1)$ are the values of $\phi(u)$ for u equal to u_2 and u_1 respectively.

16.7 HYDRAULIC JUMP

The hydraulic jump is defined as the sudden and turbulent passage of water from a supercritical state to subcritical state. It has been classified as rapidly varied flow, since the change in depth of flow from rapid to tranquil state is in an abrupt manner over a relatively short distance. The flow in a hydraulic jump is accompanied by the formation of extremely turbulent rollers and there is a considerable dissipation of energy.

A hydraulic jump will form when water moving at a supercritical velocity in a relatively shallow stream strikes water having a relatively large depth and subcritical velocity. It occurs frequently in a canal below a regulating sluice, at the foot of a spillway, or at the place where a steep channel bottom slope suddenly changes to a flat slope.

In order to study the conditions of flow before and after the hydraulic jump the application of the energy equation does not provide an adequate means of analysis, because hydraulic jump is associated with an appreciable loss of energy which is initially unknown. As such in the analysis of hydraulic jump the momentum equation is used by considering the portion of the hydraulic jump as the control volume. The following assumptions are, however, made in this analysis:

(1) It is assumed that before and after jump formation the flow is uniform and the pressure distribution is hydrostatic.

(2) The length of the jump is small so that the losses due to friction on the channel floor are small and hence neglected.

(3) The channel floor is horizontal or the slope is so gentle that the weight component of the water mass comprising the jump is negligibly small.

Consider a hydraulic jump formed in a prismatic channel with horizontal floor carrying a discharge Q as shown in Fig. 16.8. Let the depth of flow before the jump at section 1 be y_1 and the depth of flow

However, as indicated in Illustrative Example 16.11, the energy loss ΔE in a hydraulic jump in a rectangular channel may also be expressed as

$$\Delta E = \frac{(V_1 - V_2)^3}{2g(V_1 + V_2)} \qquad \ldots(16.25\ a)$$

where V_1 and V_2 are the mean velocities of flow before and after the jump respectively.

The *height of the jump* h_j may be defined as the difference between the depths after and before the jump, *i.e.*, $h_j = (y_2 - y_1)$.

The *length of the jump* L_j may be defined as the distance measured from the front face of the jump to a point on the surface immediately downstream from the roller. However, the length of the jump cannot be determined analytically. In addition, practical complications arise from the general instability of the phenomenon and the difficulty of defining the beginning and the end sections of the jump. The length of the jump has been investigated experimentally by many hydraulicians and as a general statement it may be said that for a rectangular channel the length of the jump L_j varies between 5 and 7 times the height of the jump, that is,

$$L_j = (5 \text{ to } 7)\ h_j = (5 \text{ to } 7)\ (y_2 - y_1) \qquad \ldots(16.26)$$

(*b*) **Types of Hydraulic Jump.** Equation 16.23 emphasizes the importance of the Froude number Fr_1 of the incoming supercritical flow, as a parameter describing the phenomenon of hydraulic jump. As such according to the studies of U.S. Bureau of Reclamation, depending upon the value of Froude number Fr_1 of the incoming flow, there are five distinct types of the hydraulic jump which may occur on a horizontal floor. These different types of hydraulic jump are shown in Fig. 16.9 and are described below.

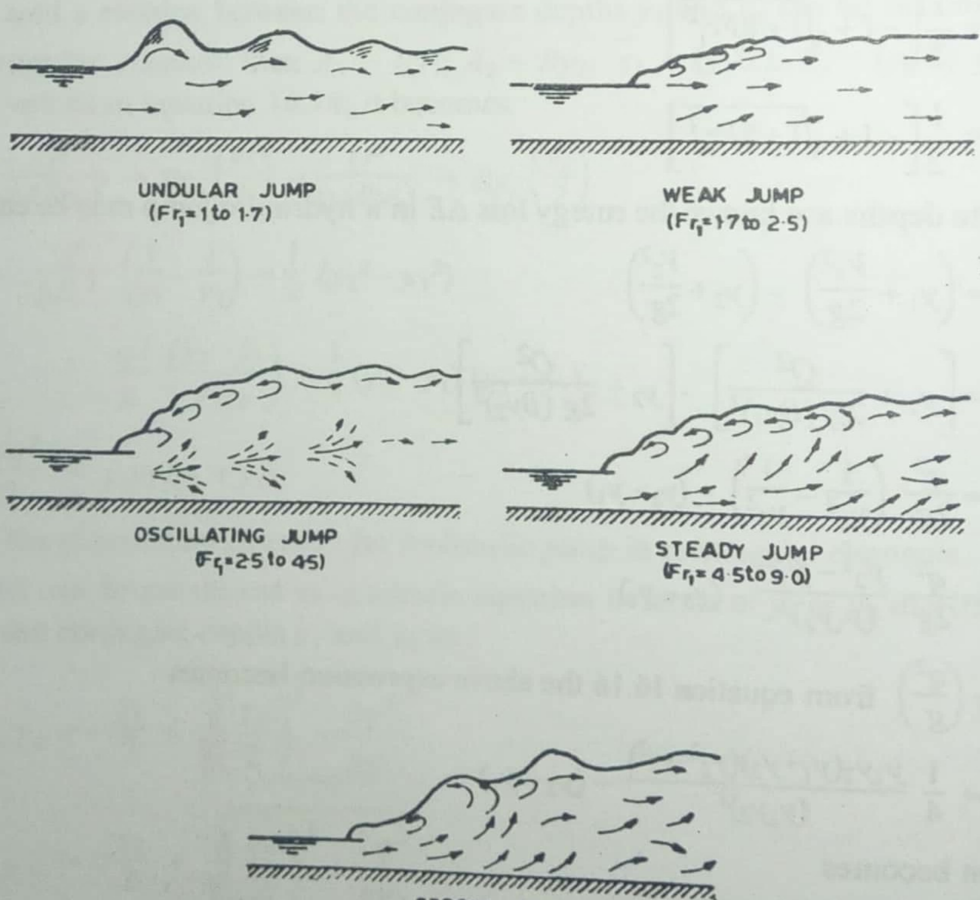


Fig. 16.9 Types of hydraulic jump

(1) For Fr_1 = 1.0 to 1.7, the water surface shows undulations and the jump is called an *undular jump*.

(2) For Fr_1 = 1.7 to 2.5, the jump formed is called *weak jump*, as the velocity throughout is fairly uniform and only a small amount of energy is dissipated. In this case a series of small rollers form on the jump surface, but the downstream water surface remains quite smooth.

(3) For Fr_1 = 2.5 to 4.5, jump formed is known as an *oscillating jump*. In this case the entering jet of water oscillates back and forth from the bottom to the surface and back again.

(4) For Fr_1 = 4.5 to 9.0 the jump formed is well stabilized and is called a *steady jump*. For this jump the energy dissipation ranges from 45 to 70 per cent.

(5) For Fr_1 = 9.0 and larger the jump formed is called a *strong jump*. In this case a rough surface prevails which continues downstream for a long distance. The jump action is quite rough but is effective since the energy dissipation may reach 85 per cent.

The above described different types of hydraulic jump refer only to channels of rectangular section. In other channel sections the shape of the jump is often complicated additionally by cross currents.

In the above paragraphs only the hydraulic jump in rectangular channel has been discussed, but it may however be mentioned that by using equation 16.14 the hydraulic jump in prismatic channel of any shape can be analysed.

(c) **Applications of Hydraulic Jump.** The phenomenon of hydraulic jump has many practical applications as listed below.

(1) It is a useful means of dissipating excess energy of water flowing over spillways and other hydraulic structures or through sluices and thus preventing possible erosion on the downstream side of these structures.

(2) It raises the water level in the channels for irrigation etc.

(3) It increases the weight on an apron of a hydraulic structure due to increased depth of flow and hence the uplift pressure acting on the apron is considerably counterbalanced.

(4) It increases the discharge through a sluice by holding back the tail water.

(5) It may be used for mixing chemicals in water and other liquids, since it facilitates thorough mixing due to turbulence created in it.

16.8 LOCATION OF HYDRAULIC JUMP

Often it is required to locate the exact position of the hydraulic jump in a channel under different conditions of flow. As such the following three typical cases for the location of the exact position of the hydraulic jump are described below.

Case (1)

In this case a jump forms below a regulating sluice in a mild sloped channel, see Fig. 16.10 (*a*). The jet of water issuing from the sluice will contract upto venacontracta section at a distance L_e from the sluice which is taken approximately equal to the sluice opening h. Thereafter it will follow M_3 profile as indicated by DE in Fig.16.10 (*a*). The location of the jump in this case will be considerably affected by the length of the channel reach on the downstream side of the sluice. Thus if there exists a long reach of channel with same slope on the downstream side of the sluice, then after the formation of the jump, uniform flow with depth of flow equal to the normal depth of flow will be developed. Hence the depth of flow after the jump or the sequent depth will be equal to the normal depth of flow. As such from the venacontracta section the depth of flow will gradually increase, following M_3 profile, upto a certain section on the downstream side, where the depth of flow will be equal to the initial depth required for the formation of the jump corresponding to the sequent depth equal to the normal depth of flow. At this

25

Miscellaneous Hydraulic Machines

25.1 INTRODUCTION

There are several hydraulic machines which are employed for either storing the hydraulic energy and then transmitting it when required or magnifying the hydraulic energy (mostly the pressure energy) several times and then transmitting the same. In general these machines are based on the principles of hydrostatics and hydrokinetics. In these machines the hydraulic energy is transmitted through the liquid medium for which water or oil is used. Some of these machines are described below.

25.2 HYDRAULIC ACCUMULATOR—SIMPLE AND DIFFERENTIAL TYPES

The hydraulic accumulator is a device which is used for temporarily storing or accumulating the liquid under pressure supplied by the pump when it is not required by the machines. Several hydraulic machines such as lifts, cranes, presses etc., are required to do a large amount of work during a small interval of time which is followed by an idle period. The demand of the liquid under pressure (or hydraulic energy) by these machines will however not be uniform throughout the period of their

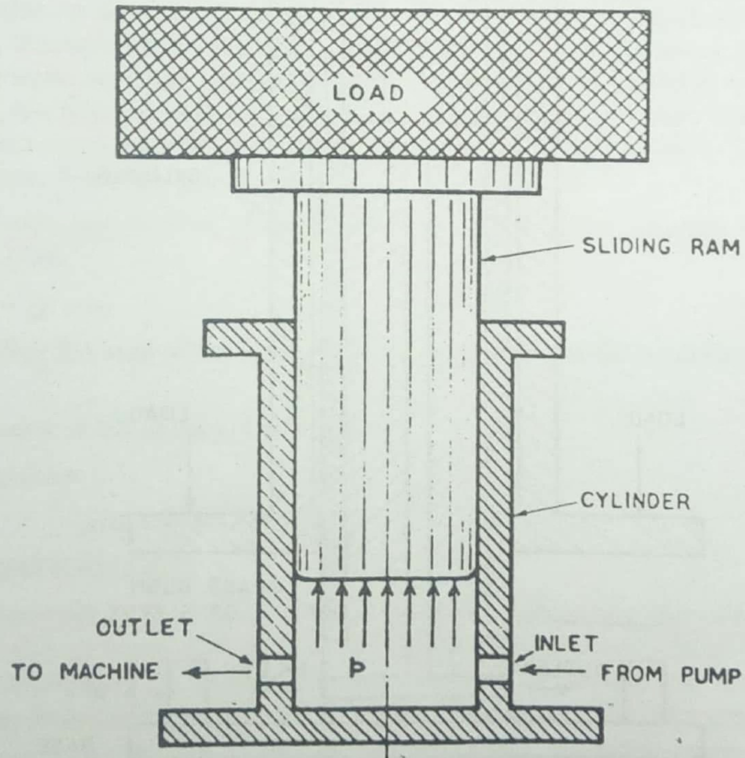


Fig. 25.1 Hydraulic accumulator

operation. For example, a hydraulic crane or lift requires the liquid under pressure to be supplied during the upward motion of the load only and practically no energy is used during the downward motion. But the pump normally supplies the liquid under pressure at a more or less uniform rate throughout the period. As such by introducing an accumulator in between the pump and the machines (*i.e.*, lift, press, etc.) the liquid under pressure can be stored in the accumulator during the idle period of the machine (*i.e.*, when the energy is not required by the machine), and this stored liquid will be supplied to the machine along with the uniform supply from the pump when during the working stroke of the machine it is in need of large quantity of liquid pressure. Thus by such an arrangement the capacity of the pump need not be as large as that required by the machine when it is doing maximum work, the machine will then receive a part of the required supply from the accumulator.

As shown in the Fig. 25.1 the hydraulic accumulator consists of a fixed vertical cylinder containing a sliding plunger or ram. The ram is loaded with weights. One side of the cylinder is connected to the pump and the other side to the machine. In the beginning the ram is at its lowermost position. Now when liquid is not required by the machine, then the pump will deliver the liquid under pressure to the cylinder which will raise the loaded ram till the cylinder is full. This constitutes the upward stroke of the ram during which the liquid under pressure (or hydraulic energy) is stored in the accumulator. Later when the machine requires liquid it will draw the same from the accumulator, and the ram will gradually move down. This is the downward stroke of the ram, during which the liquid under pressure (or the hydraulic energy) is delivered to the machine.

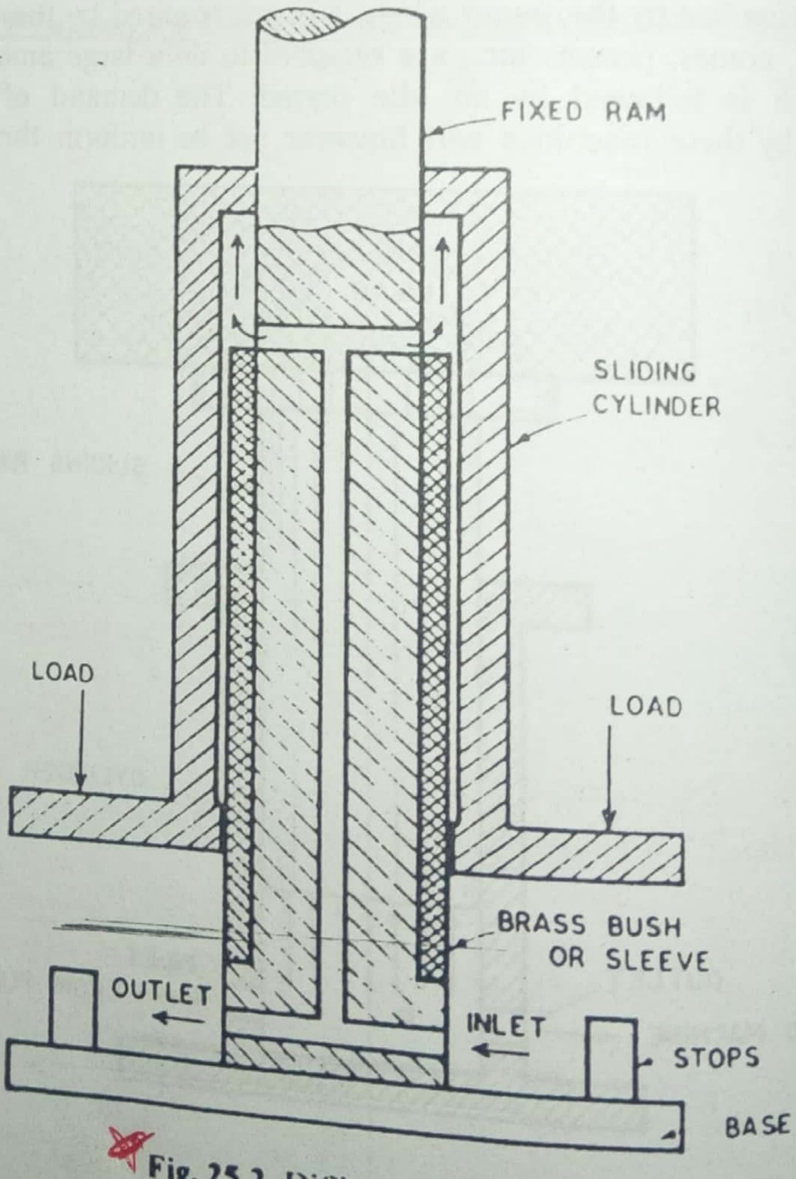


Fig. 25.2 Differential accumulator

The maximum amount of energy that the accumulator can store is known as the *capacity* of the accumulator.

Let D be the diameter of the sliding ram, L be the stroke or lift of the ram, p be the pressure intensity of the liquid supplied by the pump and W be the total weight of the ram (including the weight of the load on the ram), then

$$W = \frac{\pi}{4} D^2 \times p \qquad \text{...(25.1)}$$

Also the work done in lifting the ram or the capacity of the accumulator

$$= WL = \left(\frac{\pi}{4}\right) D^2 \times p \times L \qquad \text{...(25.2)}$$

But volume of the accumulator

$$= \left(\frac{\pi}{4}\right) D^2 L$$

$\therefore$ Capacity of accumulator

$$= (p \times \text{volume}) \qquad \text{...(25.3)}$$

Another form of accumulator, known as *Tweddell's differential accumulator* is as shown in Fig. 25.2. The advantage of this accumulator is that liquid can be stored at a high pressure by a comparatively small load on the ram. It consists of a fixed ram of which the lower portion is made larger than the upper portion by surrounding it with a closely fitting brass bush or sleeve. The fixed ram is surrounded by a sliding cylinder having a circular collar projecting outwards at the base, on which the required weights may be placed in order to load the cylinder. The fixed ram is provided with central vertical hole throughout its length, through which the liquid supplied from the pump enters the cylinder. This causes the loaded cylinder to move upwards, thus storing the hydraulic energy. When the machine draws liquid from the accumulator, the liquid leaves the cylinder through the same central hole. The liquid entering the cylinder exerts the pressure on the internal annular area of the cylinder which is equal to the horizontal cross-sectional area of the brass bush or sleeve.

Now if a is the cross-section area of the brass bush and p is the intensity of pressure of the liquid supplied by the pump, then

Load on cylinder = $(p \times a)$

Therefore by making the area of the bush small, it is possible to store liquid at a high pressure with a small load.

Again if L is the vertical lift of the cylinder, then

Capacity of accumulator

$$= paL = p \times \text{volume} \qquad \text{...(25.4)}$$

25.3 HYDRAULIC INTENSIFIER

The hydraulic intensifier is a device which is used for increasing the intensity of pressure of the liquid, by utilizing the energy of a larger quantity of liquid at low pressure. Often hydraulic machines such as press etc., require liquid at high pressure which may not be directly available from a pump. It can however be provided by introducing an intensifier between the pump and the machine.

As shown in Fig. 25.3 an intensifier consists of a fixed ram surrounded by a sliding cylinder or ram which contains high pressure liquid, which is supplied to the machine through the fixed ram. The sliding cylinder is encases by a fixed cylinder which contains the low pressure liquid from the main supply.

As shown in Fig. 25.3 valves *A* and *D* allow low pressure liquid from the supply, valve *C* is for exhaust and valve *B* allows high pressure liquid to be supplied to the machine. In the beginning when the sliding cylinder is at its bottom-most position, the fixed cylinder is full of low pressure liquid. The valves *B* and *D* are then closed, the valve *A* is opened thus admitting the low pressure liquid into the sliding cylinder, and the valve *C* is also opened which permits the low pressure liquid from the fixed cylinder to be discharged to exhaust and the sliding cylinder to move upwards. When the sliding cylinder reaches its topmost position, the inside of the sliding cylinder is full of low pressure liquid. Now the valves *A* and *C* are closed and the valves *D* and *B* are opened. The low pressure liquid from the supply then enters the fixed cylinder, which forces the sliding cylinder to move downwards, thereby producing high pressure liquid in the sliding cylinder, which is supplied to the machine. The same cycle of operation is repeated. The intensifier described here is thus single-acting since it supplies high pressure liquid during the downward stroke only. However, double acting intensifiers are also made which give a continuous supply of high pressure liquid. It is possible to raise the pressure intensity of liquid to as high as 157 MN/m^2 [1600 kg (f)/cm^2], by means of an intensifier.

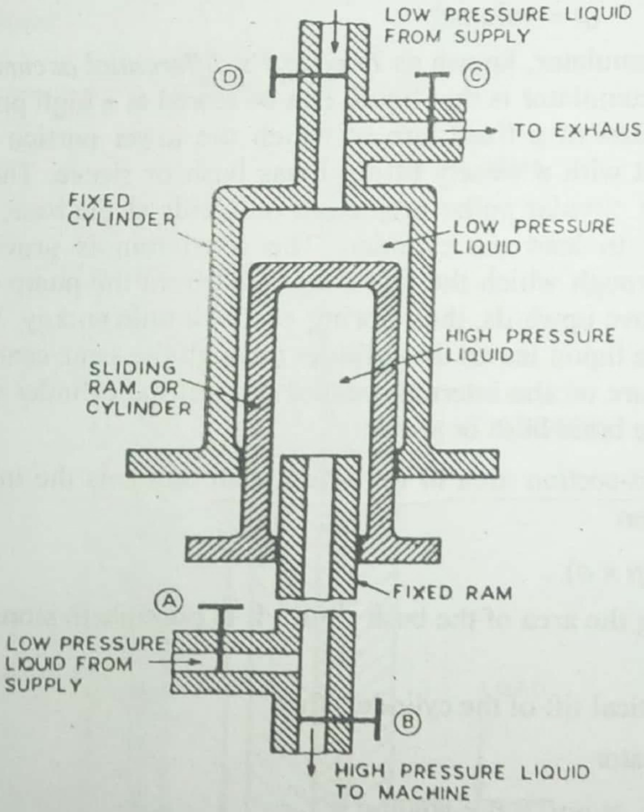


Fig. 25.3 Hydraulic intensifier

The magnitude of the pressure intensity developed by an intensifier and the quantity of high pressure liquid supplied by it may be computed as follows:

Let D_1 be the external diameter of the sliding cylinder and D_2 be the diameter of the fixed ram; and A_1 and A_2 be their respective cross-sectional areas. Also let p_1 be the intensity of pressure of low pressure liquid in the fixed cylinder and p_2 be the intensity of pressure of high pressure liquid inside the sliding cylinder. Then as the total upward force equals total downward force,

$$p_1 A_1 = p_2 A_2$$

or

$$p_2 = p_1 (A_1/A_2) = p_1 (D_1/D_2)^2 \quad ...(25.5)$$

Further if L is the stroke length or lift of the sliding cylinder then in one stroke of the sliding cylinder, the volume of low pressure liquid entering the fixed cylinder equals (A_1L) and the volume of the high pressure liquid supplied to the machine from the sliding cylinder equals (A_2L). Thus if Q_1 is the rate of discharge of low pressure liquid entering the fixed cylinder and Q_2 is the rate of discharge of high pressure liquid supplied to machine, then

$$\frac{Q_1}{Q_2} = \frac{A_1L}{A_2L} = \frac{A_1}{A_2}$$

or

$$Q_2 = Q_1(A_2/A_1) = Q_1(D_2/D_1)^2 \quad ...(25.6)$$

Sometimes compressed air is supplied to the fixed cylinder instead of low pressure liquid in which case it is known as *Hydro-Pneumatic Intensifier*. Steam under pressure may also be supplied to the fixed cylinder instead of low pressure liquid or compressed air. It is then called *Steam intensifier*.

25.4 HYDRAULIC PRESS

Hydraulic press was first built by Joseph Bramah in 1795, and is still in use since it was first developed. It is a machine which is based on Pascal's law of transmission of fluid pressure and in this machine by the application of a small force a large force may be developed. The working principle of a hydraulic press may be explained with the help of Fig. 25.4. Consider a ram and a plunger operating in two cylinders of different diameters which are inter-connected at the bottom through a chamber which is filled with some liquid. Let A be the area of the ram and a be the area of the plunger. Now if F is the force applied to the plunger then the corresponding pressure intensity developed is $p = (F/a)$. But according to Pascal's law, the same pressure intensity will be transmitted throughout the liquid, and therefore the ram will also be subjected to the same pressure intensity. Accordingly if W is the total weight lifted by the ram then $W = (pA)$ and hence

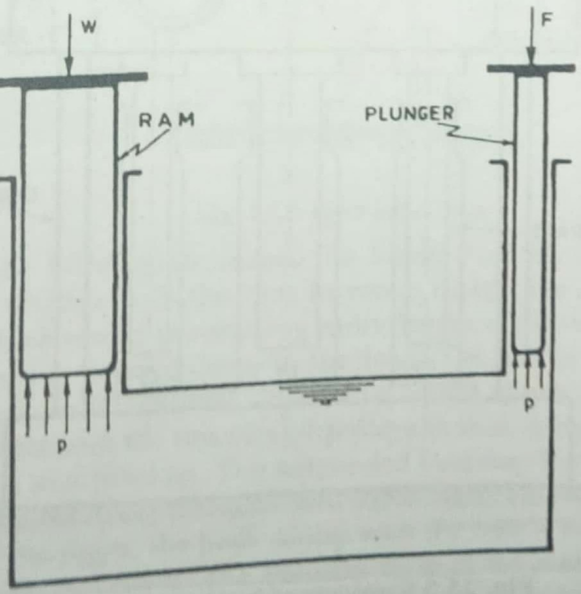


Fig. 25.4 Working principle of hydraulic press

$$p = \frac{F}{a} = \frac{W}{A} \quad ...(25.7)$$

or $$W = F\left(\frac{A}{a}\right) \qquad \text{...(25.8)}$$

From equation 25.8 it may be seen that by applying a small force F on the plunger a large force W may be developed at the ram. Also the mechanical advantage for this machine is equal to (A/a), the ratio of the areas of the ram and the plunger. As such by suitably adjusting the area of the plunger and the ram even a small force may be multiplied many times.

Fig. 25.5 shows the elements of a hydraulic press. In its simplest form it consists of a fixed table and a moving table mounted on sliding rams. It is usually preferred to have multiple rams *i.e.*, a number of rams of smaller size instead of one single ram of bigger size. The advantage of having multiple rams is that the total thrust on the moving table can be controlled to some extent by operating one or all the rams. The rams are operated by liquid under pressure which is supplied by pumps. Usually a hydraulic accumulator is provided in between the press and the pump, which permits the high pressure liquid to be stored in the accumulator cylinder while the press is at rest. In some large hydraulic presses a maximum total thrust ranging from about 50 MN to 100 MN can be produced.

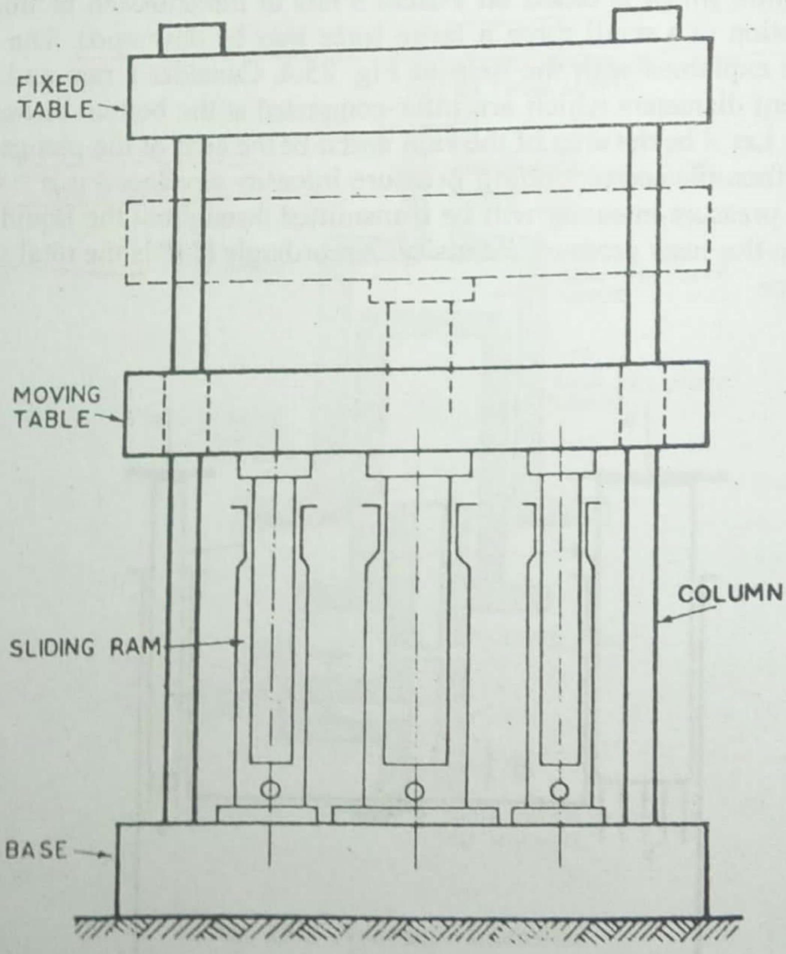


Fig. 25.5 Elements of hydraulic press

25.5 HYDRAULIC CRANE

The hydraulic crane is a device which is used for lifting heavy loads. It can lift loads upto about 25 MN, and it is widely used on docks, slidings, warehouses and workshops. It consists of a central pedestal supporting a mast from which is suspended a jib or arm. The jib can be raised or lowered in order to