Problem 17.1 *Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 20 \text{ m/s}$.

The force exerted by the jet of water on a stationary vertical plate is given by equation (17.1) as

$$F = \rho a V^2 \quad \text{where } \rho = 1000 \text{ kg/m}^3$$

$$\therefore \quad F = 1000 \times .004417 \times 20^2 \text{ N} = \mathbf{1766.8 \text{ N. Ans.}}$$

Problem 17.2 *Water is flowing through a pipe at the end of which a nozzle is fitted. The diameter of the nozzle is 100 mm and the head of water at the centre nozzle is 100 m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is given as 0.95.*

Solution. Given :

Diameter of nozzle, $d = 100 \text{ mm} = 0.1 \text{ m}$

Head of water, $H = 100 \text{ m}$

Co-efficient of velocity, $C_v = 0.95$

Area of nozzle, $a = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Theoretical velocity of jet of water is given as

$$V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 100} = 44.294 \text{ m/s}$$

But $$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$\therefore$ Actual velocity of jet of water, $V = C_v \times V_{th} = 0.95 \times 44.294 = 42.08 \text{ m/s}$.

Force on a fixed vertical plate is given by equation (17.1) as

$$F = \rho a V^2 = 1000 \times .007854 \times 42.08^2 \quad (\because \text{ In S.I. units } \rho \text{ for water} = 1000 \text{ kg/m}^3)$$

$$= 13907.2 \text{ N} = \mathbf{13.9 \text{ kN. Ans.}}$$

Problem 17.3 *A jet of water of diameter 75 mm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.*

Solution. Given :

Diameter of jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.075)^2 = 0.004417 \text{ m}^2$

Velocity of jet, $V = 25 \text{ m/s}$.

Angle between jet and plate $\theta = 60°$

(i) The force exerted by the jet of water in the direction normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin\theta$$

$$= 1000 \times .004417 \times 25^2 \times \sin 60° = \mathbf{2390.7 \text{ N. Ans.}}$$

(*ii*) The force in the direction of the jet is given by equation (17.3),

$$F_x = \rho a V^2 \sin^2 \theta$$
$$= 1000 \times .004417 \times 25^2 \times \sin^2 60° = \textbf{2070.4 N. Ans.}$$

Problem 17.4 *A jet of water of diameter 50 mm strikes a fixed plate in such a way that the angle between the plate and the jet is 30°. The force exerted in the direction of the jet is 1471.5 N. Determine the rate of flow of water.*

Solution. Given :

Diameter of jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

Angle, $\theta = 30°$

Force in the direction of jet, $F_x = 1471.5$ N

Force in the direction of jet is given by equation (17.3) as $F_x = \rho a V^2 \sin^2 \theta$

As the force is given in Newton, the value of ρ should be taken equal to 1000 kg/m³

$$\therefore \quad 1471.5 = 1000 \times .001963 \times V^2 \times \sin^2 30° = .05\ V^2$$

$$\therefore \quad V^2 = \frac{150}{.05} = 3000.0$$

$$V = 54.77 \text{ m/s}$$

∴ Discharge, $Q = \text{Area} \times \text{Velocity}$

$$= .001963 \times 54.77 = 0.1075 \text{ m}^3/\text{s} = \textbf{107.5 liters/s. Ans.}$$

Problem 17.5 *A jet of water of diameter 50 mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of the jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.*

Solution. Given :

Diameter of the jet, $d = 50 \text{ mm} = 0.05 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

Velocity of jet, $V = 40$ m/s

Angle of deflection $= 120°$

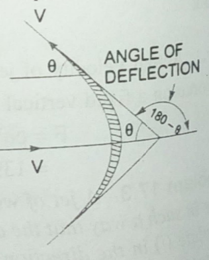

Fig. 17.5

From equation [17.6 (*A*)], the angle of deflection $= 180° - \theta$

$$\therefore \quad 180° - \theta = 120° \text{ or } \theta = 180° - 120° = 60°$$

Force exerted by the jet on the curved plate in the direction of the jet is given by equation (17.5) as

$$F_x = \rho a V^2 [1 + \cos \theta]$$
$$= 1000 \times .001963 \times 40^2 \times [1 + \cos 60°] = \textbf{4711.15 N. Ans.}$$

Problem 17.6 *A jet of water of diameter 75 mm moving with a velocity of 30 m/s, strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.*

Solution. Given :

Diameter of the jet, $d = 75 \text{ mm} = 0.075 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.075)^2 = .004417 \text{ m}^2$

Velocity of jet, $V = 30$ m/s

Angle made by the jet at inlet tip with horizontal, $\theta = 30°$

Angle made by the jet at outlet tip with horizontal, $\phi = 20°$

The force exerted by the jet of water in the direction of *x* is given by equation (17.8) and in the direction of *y* by equation (17.9),

$$\therefore \quad F_x = \rho a V^2 [\cos \theta + \cos \phi]$$
$$= 1000 \times .004417 [\cos 30° + \cos 20°] \times 30^2 = \textbf{7178.2 N. Ans.}$$

and
$$F_y = \rho a V^2 [\sin \theta - \sin \phi]$$
$$= 1000 \times .004417 [\sin 30° - \sin 20°] \times 30^2 = \textbf{628.13 N. Ans.}$$

▸ 17.3 FORCE EXERTED BY A JET ON A HINGED PLATE

Consider a jet of water striking a vertical plate at the centre which is hinged at *O*. Due to the force exerted by the jet on the plate, the plate will swing through some angle about the hinge as shown in Fig. 17.6

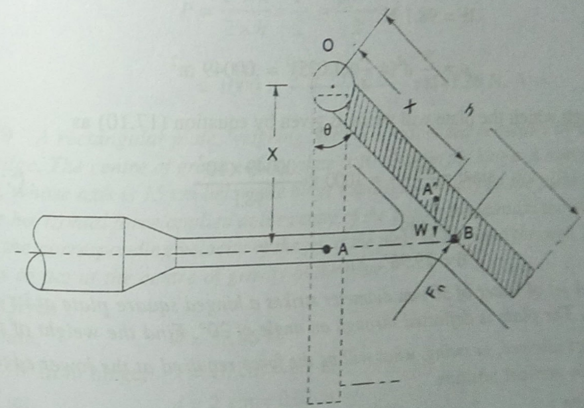

Fig. 17.6 *Force on a hinged plate.*

Let
x = distance of the centre of jet from hinge *O*,
θ = angle of swing about hinge,
W = weight of plate acting at C.G. of the plate.

The dotted lines show the position of the plate, before the jet strikes the plate. The point *A* on the plate will be at *A′* after the jet strikes the plate. The distance $OA = OA' = x$. Let the weight of the plate is acting at *A′*. When the plate is in equilibrium after the jet strikes the plate, the moment of all the forces about the hinge must be zero. Two forces are acting on the plate. They are :

1. Force due to jet of water, normal to the plate,

$$F_n = \rho a V^2 \sin \theta'$$

where θ' = Angle between jet and plate = $(90° - \theta)$

2. Weight of the plate, W

Moment of force F_n about hinge = $F_n \times OB = \rho a V^2 \sin(90° - \theta) \times OB = \rho a V^2 \cos\theta \times OB$

$$= \rho a V^2 \cos\theta \times \frac{OA}{\cos\theta} = \rho a V^2 \times OA = \rho a V^2 \times x$$

Moment of weight W about hinge $= W \times OA' \sin\theta = W \times x \times \sin\theta$

For equilibrium of the plate, $\rho a V^2 \times x = W \times x \times \sin\theta$

$$\therefore \quad \sin\theta = \frac{\rho a V^2}{W} \quad \ldots(17.10)$$

From equation (17.10), the angle of swing of the plate about hinge can be calculated.

Problem 17.7 *A jet of water of 2.5 cm diameter, moving with a velocity of 10 m/s, strikes a hinged square plate of weight 98.1 N at the centre of the plate. The plate is of uniform thickness. Find the angle through which the plate will swing.*

Solution. Given :

Diameter of jet, $d = 2.5$ cm $= 0.025$ m

Velocity of jet, $V = 10$ m/s

Weight of plate, $W = 98.1$ N

Area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (.025)^2 = .00049 \text{ m}^2$

The angle through which the plate will swing is given by equation (17.10) as

$$\sin\theta = \frac{\rho a V^2}{W} = 1000 \times \frac{.00049 \times 10^2}{98.1} \quad (\because \rho = 1000)$$

$$= .499$$

$$\therefore \quad \theta = \mathbf{29.96°.} \text{ Ans.}$$

Problem 17.8 *A jet of water of 30 mm diameter strikes a hinged square plate at its centre with a velocity of 20 m/s. The plate is deflected through an angle of 20°. Find the weight of the plate.*

If the plate is not allowed, to swing, what will be the force required at the lower edge of the plate to keep the plate in vertical position.

Solution. Given :

Diameter of the jet, $d = 30$ mm $= 3$ cm $= 0.03$ m

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(.03)^2 = .0007068 \text{ m}^2$

Velocity of jet, $V = 20$ m/s

Angle of swing, $\theta = 20°$

Using equation (17.10) for angle of swing,

$$\sin\theta = \frac{\rho a V^2}{W}$$

or

$$\sin 20° = 1000 \times \frac{.0007068 \times 20^2}{W} = \frac{282.72}{W}$$

$$\therefore \quad W = \frac{282.72}{\sin 20°} = 826.6 \text{ N}$$

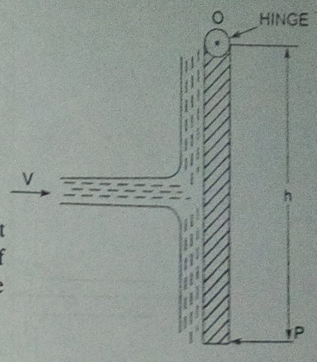

Fig. 17.7

If the plate is not allowed to swing, a force P will be applied at the lower edge of the plate as shown in Fig. 17.7. The weight of the plate is acting vertically downward through the C.G. of the plate.

Let F = Force exerted by jet of water

h = Height of plate

= Distance of P from the hinge.

The jet strikes at the centre of the plate and hence distance of the centre of the jet from hinge $= \frac{h}{2}$.

Taking moments* about the hinge, O, $P \times h = F \times \frac{h}{2}$.

$$\therefore \quad P = \frac{F \times h}{2 \times h} = \frac{F}{2} = \frac{\rho a V^2}{2} \quad (\because F = \rho a V^2)$$

$$= 1000 \times \frac{.0007068 \times 20^2}{2} = \mathbf{141.36 \text{ N.}} \text{ Ans.}$$

Problem 17.9 *A rectangular plate, weighing 58.86 N is suspended vertically by a hinge on the top of horizontal edge. The centre of gravity of the plate is 10 cm from the hinge. A horizontal jet of water 2 cm diameter, whose axis is 15 cm below the hinge impinges normally on the plate with a velocity of 5 m/s. Find the horizontal force applied at the centre of the gravity to maintain the plate in its vertical position. Find the corresponding velocity of the jet, if the plate is deflected through 30° and the same force continues to act at the centre of gravity of the plate.*

Solution. Given :

Weight of plate, $W = 58.86$ N

Distance of W from hinge, $x = 10$ cm $= 0.1$ m

Diameter of jet, $d = 2$ cm $= 0.02$ m

$\therefore$ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times .02^2 = .000314 \text{ m}^2$

Distance of the axis of the jet of water from hinge = 15 cm = 0.15 m

Velocity of jet, $V = 5$ m/s

(*i*) Let the force applied at the centre of gravity of the plate to keep the plate in vertical position = P as shown in Fig. 17.8 (*a*).

* The weight of the plate is passing through the hinge O. Hence moment of W about hinge is zero.

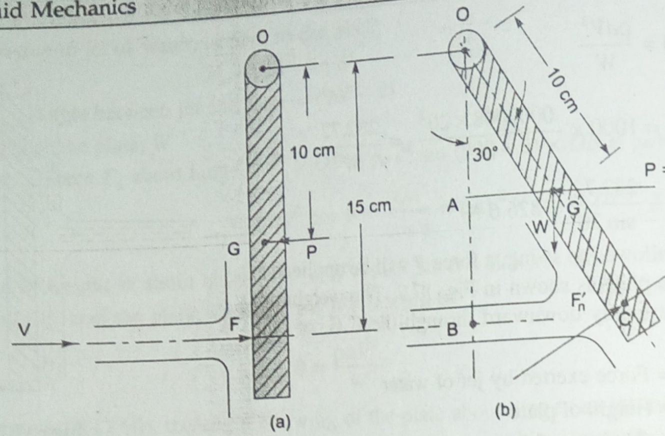

Fig. 17.8

The force exerted by a jet of water on the vertical plate,

$$F = \rho a V^2 = 1000 \times .000314 \times 5^2 = 7.85 \text{ N}$$

This force F is acting at a distance of 15 cm or 0.15 m from the hinge. Taking moments about hinge, we get

$$F \times 0.15 = P \times 0.10$$

$$\therefore \quad P = \frac{F \times 0.15}{0.10} = \frac{7.85 \times .15}{.10} = \textbf{11.775 N. Ans.}$$

(*ii*) The plate is deflected through an angle of 30° as shown in Fig. 17.8(*b*).

∴ Angle of swing $= 30°$

The force at the C.G. $= P = 11.775$ N

Let the velocity of the jet in this position = V m/s

For the deflected position of the plate as shown in Fig. 17.8 (*b*), the plate is in equilibrium under the action of three forces, which are :

(*i*) Weight of the plate, W acting at G at a distance 10 cm from O.

(*ii*) Horizontal force, P acting at G.

(*iii*) Normal force F_n' on the plate due to jet of water.

The angle between the jet and the plate, $\theta = 90° - 30° = 60°$

Hence, F_n' is given by equation (17.2) as

$$F_n' = \rho a V^2 \sin\theta = \rho a V^2 \sin 60°$$

$$= 1000 \times .000314 \times V^2 \times \sin 60° = 0.2717\, V^2$$

Taking moments of all forces about hinge O, we get ...(*i*)

$$F_n' \times OC = P \times OA + W \times AG$$

where $OB = OC \cos 30°$

$$\therefore \quad OC = \frac{OB}{\cos 30^\circ} = \frac{15}{\cos 30^\circ} = 17.32 \text{ cm} = 0.1732 \text{ m}$$

$$OA = OG \cos 30° = 10 \times .866 = 8.66 \text{ cm} = 0.0866 \text{ m}$$

$$AG = OG \sin 30° = 10 \times \frac{1}{2} = 5 \text{ cm} = .05 \text{ m}$$

Substituting these values in equation (*i*), we get

$$0.2717\, V^2 \times .1732 = 11.775 \times .0866 + 58.86 \times 0.05 = 3.962$$

$$\therefore \quad V = \sqrt{\frac{3.962}{0.2717 \times .1732}} = 9.175 \text{ m/s.}$$

Problem 17.10 *A jet of water of diameter 25 mm strikes a 20 cm × 20 cm square plate of uniform thickness with a velocity of 10 m/s at the centre of the plate which is suspended vertically by a hinge on its top horizontal edge. The weight of the plate is 98.1 N. The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that plate is kept vertical ? If the plate is allowed to deflect freely, what will be the inclination of the plate with vertical due to the force exerted by jet of water?*

Solution. Given :

Diameter of the jet, $d = 25 \text{ mm} = 25 \times 10^{-3} \text{ m} = .025 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(.025)^2 = .00049 \text{ m}^2$

Size of the plate, $= 20 \text{ cm} \times 20 \text{ cm}$

Weight of the plate, $W = 98.1$ N

Velocity of jet, $V = 10$ m/s

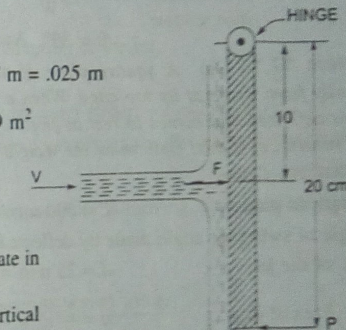

Fig. 17.9 (*a*)

(*i*) Let the force applied at the lower edge to keep the plate in vertical position is = P. See Fig. 17.9 (*a*).

Force exerted by the jet of water at the centre of the vertical plate,

$$F = \rho a V^2$$
$$= 1000 \times .00049 \times 10^2 = 49 \text{ N.}$$

This force is acting at a distance of $\frac{20}{2} = 10$ cm from the hinge. The force P is acting at a distance of 20 cm from the hinge.

Taking moments about hinge,

$$F \times 10 = P \times 20$$

$$\therefore \quad 49 \times 10 = P \times 20$$

$$\therefore \quad P = \frac{49 \times 10}{20} = \textbf{24.5 N. Ans.}$$

(*ii*) When the plate is allowed to deflect freely about hinge.

Let the inclination of the plate with vertical = θ

In this position, the angle between the plate and jet will be

$$= (90° - \theta).$$

∴ Force exerted by water normal to the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin(90° - \theta) = \rho a V^2 \cos\theta$$

The distance $$OB = \frac{OA}{\cos\theta} = \frac{10}{\cos\theta}$$

The weight W of the plate is acting at a distance 10 cm from hinge. Distance

$$DG = OG \sin\theta = 10 \times \sin\theta$$

Taking moments about hinge, we get

$$F_n \times OB = W \times GD$$

or $$\rho a V^2 \cos\theta \times \frac{10}{\cos\theta} = W \times 10 \times \sin\theta$$

$\therefore$ $$\rho a V^2 = W \times \sin\theta$$

$\therefore$ $$\sin\theta = \frac{\rho a V^2}{W} = 1000 \times \frac{.00049 \times 10^2}{98.1} = 0.5$$

$\therefore$ $$\theta = 30°.$$ **Ans.**

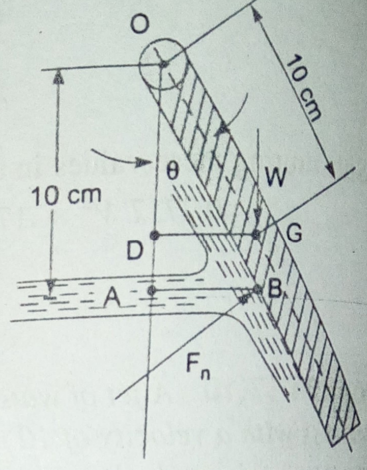

Fig. 17.9. (*b*)

Problem 17.10 (A) *A square plate of uniform thickness and length of side 300 mm hangs vertically from hinge at its top edge. When a horizontal water jet strikes the plate at its centre, the plate is deflected and comes to rest at angle of 30° to the vertical. The jet is 25 mm in diameter and has a velocity of 6 m/s. Determine the weight of the plate.*

Solution. Given :

Length of plate, $L = 300 \text{ mm} = 0.3 \text{ m}$

Angle of swing, or angle made by deflected plate with the vertical, $\theta = 30°$

Dia. of the jet, $d = 25 \text{ mm} = 0.025 \text{ m}$

$\therefore$ Area of jet, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.025^2) \text{ m}^2$

Velocity of jet, $V = 6$ m/s

Let W = Weight of plate

Using equation (17.10), we get $$\sin\theta = \frac{\rho \times a \times V^2}{W}$$

$\therefore$ $$W = \frac{\rho^* \times a \times V^2}{\sin\theta} = \frac{1000 \times \left(\frac{\pi}{4} \times 0.025^2\right) \times 6^2}{\sin 30°} = 35.33 \text{ N.}$$ **Ans.**

▸ 17.4 FORCE EXERTED BY A JET ON MOVING PLATES

The following cases of the moving plates will be considered :

1. Flat vertical plate moving in the direction of the jet and away from the jet,
2. Inclined plate moving in the direction of the jet, and
3. Curved plate moving in the direction of the jet or in the horizontal direction.

* If $\rho = 1000$ kg/m^3, then weight W will be in Newton.

If the plate is smooth and loss of energy due to impact of the jet is assumed zero, the jet of water will leave the inclined plate with a velocity equal to $(V-u)$.

The force exerted by the jet of water on the plate in the direction normal to the plate is given as

F_n = Mass striking per second × [Initial velocity in the normal direction with which jet strikes – Final velocity]

$$= \rho a (V-u) [(V-u) \sin\theta - 0] = \rho a (V-u)^2 \sin\theta \quad ...(17.13)$$

This normal force F_n is resolved into two components namely F_x and F_y in the direction of the jet and perpendicular to the direction of the jet respectively.

$$\therefore \quad F_x = F_n \sin\theta = \rho a (V-u)^2 \sin^2\theta \quad ...(17.14)$$

$$F_y = F_n \cos\theta = \rho a (V-u)^2 \sin\theta \cos\theta \quad ...(17.15)$$

∴ Work done per second by the jet on the plate

= F_x × Distance per second in the direction of x

$$= F_x \times u = \rho a (V-u)^2 \sin^2\theta \times u = \rho a (V-u)^2 u \sin^2\theta \text{ N m/s.} \quad ...(17.16)$$

Problem 17.11 *A jet of water of diameter 10 cm strikes a flat plate normally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find:*

(i) the force exerted by the jet on the plate

(ii) work done by the jet on the plate per second.

Solution. Given :

Diameter of the jet, $d = 10$ cm $= 0.1$ m

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$

Velocity of jet, $V = 15$ m/s

Velocity of the plate, $u = 6$ m/s.

(*i*) The force exerted by the jet on a moving flat vertical plate is given by equation (17.11),

$$F_x = \rho a (V-u)^2$$
$$= 1000 \times .007854 \times (15-6)^2 \text{ N} = \mathbf{636.17 \text{ N}}. \text{ Ans.}$$

(*ii*) Work done per second by the jet

$$= F_x \times u = 636.17 \times 6 = \mathbf{3817.02 \text{ Nm/s}}. \text{ Ans.}$$

Problem 17.12 *For Problem 17.11, find the power and efficiency of the jet.*

Solution. The given data from Problem 17.11 is

$a = .007854 \text{ m}^2$, $V = 15$ m/s, $u = 6$ m/s

Also work done per second by the jet = 3817.02 Nm/s

(*i*) Power of the jet in kW $= \frac{\text{Work done per second}}{1000} = \frac{3817.02}{1000} = \mathbf{3.817 \text{ kW}}$. Ans.

(*ii*) Efficiency of the jet (η) $= \frac{\text{Output of the jet per second}}{\text{Input of the jet per second}}$ ...(*i*)

where output of jet/sec = Work done by jet per second = 3817.02 Nm/s

And input per second = Kinetic energy of the jet/sec

$$= \frac{1}{2}\left(\frac{\text{mass}}{\text{sec}}\right) V^2 = \frac{1}{2}(\rho a V) \times V^2 = \frac{1}{2}\rho a V^3$$

$$= \frac{1}{2} \times 1000 \times .007854 \times 15^3 \text{ Nm/s} = 13253.6 \text{ Nm/s}$$

∴ η of the jet $= \frac{3817.02}{13253.6} = 0.288 = \mathbf{28.8\%}$. Ans.

Problem 17.12 (A) *A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find :*

(i) the force on the plate,

(ii) the work done, and

(iii) the efficiency of jet. (J.N.T.U., Hyderabad S 2002)

Solution. Given :

Dia. of jet = 50 mm = 0.05 m

∴ Area, $a = \frac{\pi}{4}(0.05^2) = 0.0019635 \text{ m}^2$

Velocity of jet, $V = 20$ m/s. Velocity of plate, $u = 5$ m/s

(*i*) The force on the plate is given by equation (17.11) as.

$$F_x = \rho a (V-u)^2$$
$$= 1000 \times 0.0019635 \times (20-5)^2 = \mathbf{441.78 \text{ N}}. \text{ Ans.}$$

(*ii*) The work done by the jet

$$= F_x \times u = 441.78 \times 5 = \mathbf{2208.9 \text{ Nm/s}}. \text{ Ans.}$$

(*iii*) The efficiency of the jet, $\eta = \frac{\text{Output of jet}}{\text{Input of jet}}$

$$= \frac{\text{Work done/s}}{\text{K.E. of jet/s}} = \frac{F_x \times u}{\frac{1}{2} m V^2}$$

$$= \frac{F_x \times u}{\frac{1}{2}(\rho A V) \times V^2}$$

$$= \frac{2208.9}{\frac{1}{2}(1000 \times 0.0019635 \times 20) \times 20^2} = \frac{2208.9}{6540}$$

$$= 0.3377 = \mathbf{33.77\%}. \text{ Ans.}$$

Problem 17.13 *A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate : (i) when the plate is stationary, and (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.*

Solution. Given :

Diameter of the jet, $d = 7.5$ cm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = .004417\ m^2$

Angle between the jet and plate $\theta = 90° - 45° = 45°$

Velocity of jet, $V = 30$ m/s.

(*i*) When the plate is stationary, the normal force on the plate is given by equation (17.2) as

$$F_n = \rho a V^2 \sin\theta = 1000 \times .004417 \times 30^2 \times \sin 45° = \mathbf{2810.96\ N.}\ \text{Ans.}$$

(*ii*) When the plate is moving with a velocity 15 m/s and away from the jet, the normal force on the plate is given by equation (17.13) as

$$F_n = \rho a (V - u)^2 \sin\theta \qquad \text{where } u = 15 \text{ m/s.}$$

$$= 1000 \times .004417 \times (30 - 15)^2 \times \sin 45° = \mathbf{702.74\ N.}\ \text{Ans.}$$

(*iii*) The power and efficiency of the jet when plate is moving is obtained as

Work done per second by the jet

= Force in the direction of jet × Distance moved by the plate in the direction of jet/sec

$= F_x \times u$, where $F_x = F_n \sin\theta = 702.74 \times \sin 45° = 496.9$ N

Work done/sec $= 496.9 \times 15 = 7453.5$ Nm/s

$$\therefore \quad \text{Power in kW} = \frac{\text{Work done per second}}{1000} = \frac{7453.5}{1000} = \mathbf{7.453\ kW.}\ \text{Ans.}$$

$$\text{Efficiency of the jet} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done per second}}{\text{Kinetic energy of the jet}}$$

$$= \frac{7453.5}{\frac{1}{2}(\rho a V) \times V^2} = \frac{7453.5}{\frac{1}{2}\rho a V^3} = \frac{7453.5}{\frac{1}{2} \times 1000 \times .004417 \times 30^3}$$

$$= 0.1249 = 0.125 = \mathbf{12.5\%.}\ \text{Ans.}$$

17.4.3 Force on the Curved Plate when the Plate is Moving in the Direction of Jet. Let a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet as shown in Fig. 17.12.

Let V = Absolute velocity of jet,

a = Area of jet,

u = Velocity of the plate in the direction of the jet.

Relative velocity of the jet of water or the velocity with which jet strikes the curved plate $= (V - u)$.

If plate is smooth and the loss of energy due to impact of jet is zero, then the velocity with which the jet will be leaving the curved vane $= (V - u)$.

This velocity can be resolved into two components, one in the direction of the jet and other perpendicular to the direction of the jet.

Component of the velocity in the direction of jet

$$= -(V - u)\cos\theta$$

(–ve sign is taken as at the outlet, the component is in the opposite direction of the jet).

Component of the velocity in the direction perpendicular to the direction of the jet $= (V - u)\sin\theta$.

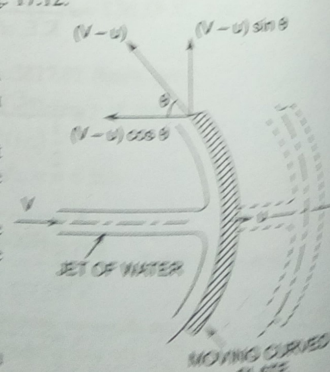

Fig. 17.12 Jet striking a curved moving plate.

Mass of the water striking the plate $= \rho \times a \times$ Velocity with which jet strikes the plate

$= \rho a (V - u)$

$\therefore$ Force exerted by the jet of water on the curved plate in the direction of the jet,

F_x = Mass striking per sec × [Initial velocity with which jet strikes the plate in the direction of jet – Final velocity]

$$= \rho a (V - u)\,[(V - u) - (-(V - u)\cos\theta)]$$

$$= \rho a (V - u)\,[(V - u) + (V - u)\cos\theta]$$

$$= \rho a (V - u)^2\,[1 + \cos\theta] \qquad \ldots(17.17)$$

Work done by the jet on the plate per second

= F_x × Distance travelled per second in the direction of x

$$= F_x \times u = \rho a (V - u)^2\,[1 + \cos\theta] \times u$$

$$= \rho a (V - u)^2 \times u\,[1 + \cos\theta] \qquad \ldots(17.18)$$

Problem 17.14 *A jet of water of diameter 7.5 cm strikes a curved plate at its centre with a velocity of 20 m/s. The curved plate is moving with a velocity of 8 m/s in the direction of the jet. The jet is deflected through an angle of 165°. Assuming the plate smooth find :*

(i) Force exerted on the plate in the direction of jet, (ii) Power of the jet, and (iii) Efficiency of the jet.

Solution. Given :

Diameter of the jet, $d = 7.5$ cm $= 0.075$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.075)^2 = 0.004417$

Velocity of the jet, $V = 20$ m/s

Velocity of the plate, $u = 8$ m/s

Angle of deflection of the jet, $= 165°$

$\therefore$ Angle made by the relative velocity at the outlet of the plate,

$$\theta = 180° - 165° = 15°.$$

(*i*) Force exerted by the jet on the plate in the direction of the jet is given by equation (17.17) as

$$= F_x = \rho a (V - u)^2 (1 + \cos\theta)$$

$$= 1000 \times .004417 \times (20 - 8)^2 [1 + \cos 15°] = \mathbf{1250.38\ N.}\ \text{Ans.}$$

(*ii*) Work done by the jet on the plate per second

$$= F_x \times u = 1250.38 \times 8 = 10003.04\ \text{N m/s}$$

$$\therefore \quad \text{Power of the jet} = \frac{10003.04}{1000} = \mathbf{10\ kW.}\ \text{Ans.}$$

$$\text{(iii) Efficiency of the jet} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Work done by jet/sec}}{\text{Kinetic energy of jet/sec}}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2}(\rho a V) \times V^2} = \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times V^3}$$

$$= \frac{1250.38 \times 8}{\frac{1}{2} \times 1000 \times .004417 \times 20^3} = 0.566 = \mathbf{56.6\%.}\ \text{Ans.}$$

Problem 17.15 *A jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tangentially without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s, calculate the magnitude and direction of the resultant force on the vane, if the vane is stationary.*

Solution. Given :

Velocity at inlet, $V_1 = 30$ m/s

Velocity at outlet, $V_2 = 25$ m/s

Mass per second $= 0.8$ kg/s

Force in the direction of jet,

$$F_x = \text{Mass per second} \times (V_{1x} - V_{2x})$$

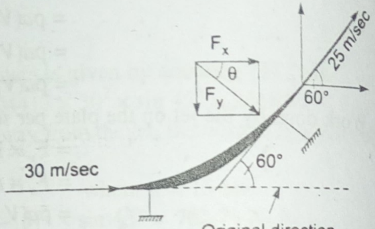

Fig. 17.13 (*a*)

where V_{1x} = Initial velocity in the direction of x

$= 30$ m/s

V_{2x} = Final velocity in the direction of x

$$= 25 \cos 60° = 25 \times \frac{1}{2} = 12.5 \text{ m/s}$$

$$\therefore \quad F_x = 0.8[30 - 12.5] = 0.8 \times 17.5 = 14.0 \text{ N}$$

Similarly, force normal to the jet,

$$F_y = \text{Mass per second} \times (V_{1y} - V_{2y})$$

$$= 0.8\ [0 - 25 \sin 60°] = -17.32 \text{ N}$$

–ve sign means the force, F_y, is acting in the vertically downward direction.

$\therefore$ Resultant force on the vane $= \sqrt{F_x^2 + F_y^2} = \sqrt{14^2 + (-17.32)^2} =$ **22.27 N. Ans.**

The angle made by the resultant with x-axis

$$\tan\theta = \frac{F_y}{F_x} = \frac{-17.32}{14.0} = -1.237$$

–ve sign means the angle θ is in the clockwise direction with x- axis as shown in Fig. 17.13 (*a*)

$$\therefore \quad \theta = \tan^{-1} 1.237 = \mathbf{51° \ 2.86'}. \text{ Ans.}$$

Problem 17.16 *(a) A stationary vane having an inlet angle of zero degree and an outlet angle of 25° as shown in Fig. 17.13(b), receives water at a velocity of 50 m/s. Determine the components of force acting on it in the direction of the jet velocity and normal to it. Also find the resultant force in magnitude and direction per unit weight of the flow.*

(b) If the vane stated above is moving with a velocity of 20 m/s in the direction of the jet, calculate the force components in the direction of the vane velocity and across it, also the resultant force in magnitude and direction. Calculate the work done and power developed per unit weight of the flow.

Solution. Given :

(*a*) Velocity of jet, $V = 50$ m/s

Angle at outlet, $= 25°$

For the stationary vane, the force in the direction of jet is given as

$$F_x = \text{Mass per sec} \times [V_{1x} - V_{2x}]$$

where $V_{1x} = 50$ m/s

$V_{2x} = -50 \cos 25° = -45.315$

$\therefore$ Force in the direction of jet per unit weight of water

$$= \frac{\text{Mass/sec}\,[50 - (-45.315)]}{\text{Weight of water/sec}}$$

or

$$F_x = \frac{(\text{Mass/sec})\,[50 + 45.315]}{(\text{Mass/sec}) \times g}$$

$$= \frac{1}{g}\,[50 + 45.315] \text{ N/N} = \frac{95.315}{9.81} = 9.716 \text{ N/N}$$

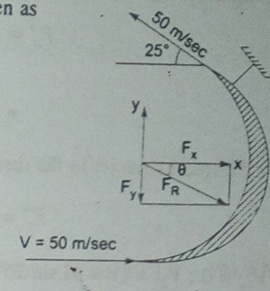

Fig. 17.13 (*b*)

Force exerted by jet in the direction perpendicular to the direction of the jet per unit weight of the flow,

$$F_y = \frac{(\text{Mass per sec})\,[V_{1y} - V_{2y}]}{g \times \text{Mass per sec}}$$

$$= \frac{1}{g}\,[V_{1y} - V_{2y}] = \frac{1}{g}\,[0 - 50 \sin 25°] \quad (\because V_{1y} = 0,\ V_{2y} = 50 \sin 25°)$$

$$= \frac{-50 \sin 25°}{9.81} = \mathbf{-2.154}. \text{ Ans.}$$

–ve sign means the force F_y is acting in the downward direction.

$\therefore$ Resultant force per unit weight of water $= \sqrt{F_x^2 + F_y^2}$

or

$$F_R = \sqrt{(9.716)^2 + (2.154)^2} = \mathbf{9.952\ N.}\ \text{Ans.}$$

The angle made by the resultant with the x-axis,

$$\tan\theta = \frac{F_y}{F_x} = \frac{2.154}{9.716} = 0.2217$$

$$\therefore \quad \theta = \tan^{-1} .2217 = \mathbf{12.50°}. \text{ Ans.}$$

(*b*) Velocity of the vane = 20 m/s.

When the vane is moving in the direction of the jet, the force exerted by the jet on the plate in the direction of jet,

$$F_x' = [\text{Mass of water striking/sec}] \times [V_{1x} - V_{2x}]$$

where V_{1x} = Initial velocity of the striking water

$= (V - u) = 50 - 20 = 30$ m/s

V_{2x} = Final velocity in the direction of x

$= -(V - u) \cos 25° = 30 \times \cos 25° = -27.189$ m/s.

$\therefore \quad F_x = \text{Mass per sec}\ [30 + 27.189]$

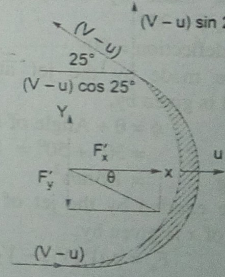

Fig. 17.14

Force in the direction of jet per unit weight,

$$F_x' = \frac{\text{Mass per sec } [30 + 27.189]}{\text{Mass per sec} \times g}$$

$$= \frac{(30 + 27.189)}{9.81} = 5.829 \text{ N.}$$

Force exerted by the jet in the direction perpendicular to direction of jet, per unit weight,

$$F_y' = \frac{1}{g} [V_{1y} - V_{2y}]$$

where $V_{1y} = 0$; $V_{2y} = (V - u) \sin 25^\circ = (50 - 20) \sin 25^\circ = 30 \sin 25^\circ$

$$F_y' = \frac{1}{9.81} [0 - 30 \sin 25^\circ] = -1.292 \text{ N}$$

$\therefore$ Resultant force $= \sqrt{(5.829)^2 + (1.292)^2} = 5.917$ N

The angle made by the resultant with x-axis, $\tan \theta = \frac{1.292}{5.829} = 0.2217$

$\therefore$ $\theta = \tan^{-1} .2217 = 12.30^\circ$

$\therefore$ Work done per second per unit weight of flow

$$= F_x' \times u = 5.829 \times 20 = 116.58 \text{ N m/s}$$

$\therefore$ Power developed $= \frac{\text{Work done per second}}{1000} = \frac{116.58}{1000} =$ **0.116 kW.** Ans.

Problem 17.17 *A jet of water of diameter 50 mm moving with a velocity of 25 m/s impinges on a fixed curved plate tangentially at one end at an angle of 30° to the horizontal. Calculate the resultant force of the jet on the plate if the jet is deflected through an angle of 50°. Take g = 10 m/s².*

Solution. Given :

Dia. of jet, $d = 50$ mm $= 0.05$ m²

$\therefore$ Area of jet, $a = \frac{\pi}{4} (0.05)^2 \text{m}^2$

Velocity of jet, $V = 25$ m/s

The angle made by the jet at inlet with horizontal, $\theta = 30^\circ$

Angle of deflection $= 50^\circ$

$\therefore$ Angle made by the jet at outlet with horizontal is given by,

$$\phi = \theta + \text{Angle of deflection}$$
$$= 30^\circ + 50^\circ = 80^\circ$$

Value of g $= 10$ m/s²

The force exerted by the jet of water in the direction of x is given by,

$$F_x = \rho a V [V_{1x} - V_{2x}] \quad \ldots(i)$$

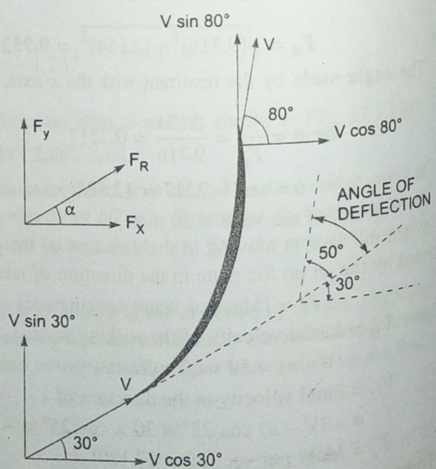

Fig. 17.14 (*a*)

where $\rho = 1000$ ($\because$ g is given as 10 m/s²)

$$a = \frac{\pi}{4} (0.05)^2; \; V = 25 \text{ m/s};$$
$$V_{1x} = V \cos 30^\circ = 25 \cos 30^\circ,$$
$$V_{2x} = V \cos 80^\circ = 25 \cos 80^\circ.$$

Substituting these values in equation (i), we get

$$F_x = 1000 \times \frac{\pi}{4} (0.05)^2 \times 25 \, [25 \cos 30^\circ - 25 \cos 80^\circ] = 849.7 \text{ N}$$

The force in the direction of y is given by,

$$F_y = \rho a V [V_{1y} - V_{2y}]$$
$$= 1000 \times \frac{\pi}{4} (0.05)^2 \times 25 [25 \sin 30^\circ - 25 \sin 80^\circ] = -594.9 \text{ N}$$

The –ve sign shows that force F_y is acting in the downward direction.

The resultant force is given by,

$$F_R = \sqrt{F_x^2 + F_y^2}$$
$$= \sqrt{849.7^2 + 594.9^2} = \textbf{1037 N.} \text{ Ans.}$$

And the angle made by the resultant with the horizontal is given by,

$$\tan \alpha = \frac{F_y}{F_x} = \frac{594.9}{849.7} = 0.7$$

$\therefore$ $\alpha = \tan^{-1} 0.7 =$ **35°.** Ans.

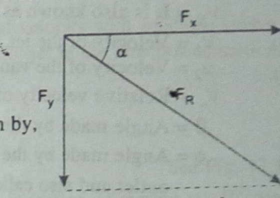

Fig. 17.14 (*b*)

17.4.4 Force Exerted by a Jet of Water on an Unsymmetrical Moving Curved Plate when Jet Strikes Tangentially at one of the Tips. Fig. 17.15 shows a jet of water striking a moving curved plate (also called vane) tangentially, at one of its tips. As the jet strikes tangentially, the loss of energy due to impact of the jet will be zero. In this case as plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to the plate. Also as the plate is moving in different direction of the jet, the relative velocity at inlet will be equal to the vector difference of the velocity of jet and velocity of the plate at inlet.

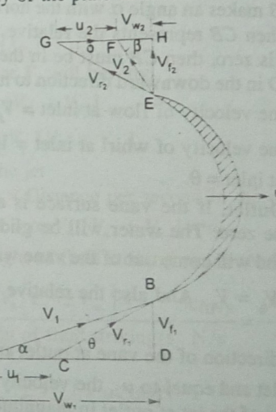

Fig. 17.15 *Jet striking a moving curved vane at one of the tips.*