(Pump) may be defined as a mechanical device which ব্যবহার করা হয়' expose in a pipe line, convert the mechanical energy supplies to it form some external sources into hydralic energy & transfer the same to the liquid through the pipe line, thereby increasing the energy of flowing liquid.

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36 Reciprocating Pumps

36.1 Introduction

A reciprocating pump, in its simplest form, consists of the following parts as shown in Fig. 36.1:

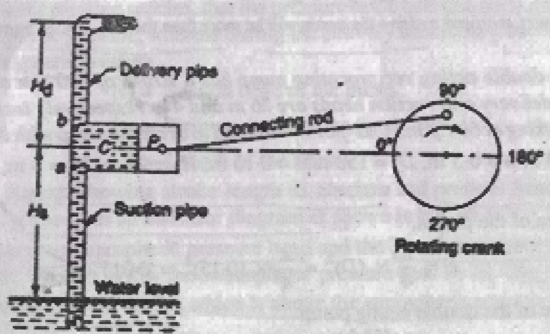


Fig. 36·1. Parts of a centrifugal pump.

1. A cylinder *C*, in which a piston *P* works. The movement of the piston is obtained by a connecting rod, which connects the piston and the rotating crank.
2. A suction pipe, connecting the source of water and the cylinder.
3. A delivery pipe, into which the water is discharged from the cylinder.
4. A suction valve *a*, which admits the flow from the suction pipe into the cylinder.
5. A delivery valve *b*, which admits the flow from the cylinder into the delivery pipe.

During the suction stroke, the piston *P* moves towards right (*i.e.*, from 0° to 180), thus creating vacuum in the cylinder. This vacuum causes the suction valve *a* to open and the water enters the cylinder. During the delivery stroke, the piston *P* move towards left (*i.e.*, from 180° to 360°) thus increasing pressure in the cylinder. This increase in pressure causes the suction valve *a* to close and delivery valve *b* to open, and the water is forced into the delivery pipe.

A reciprocating pump is also called a positive displacement pump, as it discharges a definite quantity of liquid during the displacement of its piston or plunger. This is why a reciprocating pump is ideally suitable for grouting operations in dam foundations.

36.2 Types of Reciprocating Pumps CL

The reciprocating pumps may be classified as discussed below :

1. *According to action of water*
   (*a*) single acting pump, and
   (*b*) double acting pump.
2. *According to number of cylinders*
   (*a*) single cylinder pump,
   (*b*) double cylinder pump, and
   (*c*) triple cylinder pump etc.
3. *According to the existence of air vessels*
   (*a*) with air vessel, and
   (*b*) without air vessel.

All the above mentioned pumps will discussed, in detail, at the appropriate places in the book.

36.3 Comparison of Centrifugal and Reciprocating Pumps CL

Following table gives the comparison of a centrifugal pump and a reciprocating pump :

S. No.	*Centrifugal pump*	*Reciprocating pump*
1.	Simple in construction, because of less number of parts.	Complicated in construction, because of more number of parts.
2.	Total weight of the pump is less for a given discharge.	Total weight of the pump is more for a given discharge.
3.	Suitable for large discharge and smaller heads.	Suitable for less discharge and higher heads.
4.	Requires less floor area and simple foundation.	Requires more floor area and comparatively heavy foundation.
5.	Less wear and tear.	More wear and tear.
6.	Maintenance cost is less.	Maintenance cost is high.
7.	Can handle dirty water.	Cannot handle dirty water.
8.	Can run at higher speeds.	Cannot run at higher speeds.
9.	Its delivery is continuous.	Its diivery is pulsating.
10.	No air vessels are required.	Air vessels are required.
11.	Thrust on the crankshaft is uniform.	Thrust on the crankshaft is not uniform
12.	Operation is quite simple.	Much care is required in operation.
13.	Needs priming.	Does not need priming.
14.	It has less efficiency.	It has more efficiency.

36.4 Discharge of a Reciprocating Pump

Consider a single acting reciprocating pump (*i.e.*, a pump, in which the water is acting on one side of the piston only) as shown in Fig. 36.1.

Let L = Length of the stroke or piston in metres,

A = Cross-sectional area of the piston in square metres, and

N = No. of revolutions, per minute of the crank.

$\therefore$ Discharge of water in one stroke

$= LA$

and discharge of the pump, $Q = \frac{LAN}{60}$ m³/s for single acting

If the pump is a double acting reciprocating pump (*i.e.*, a pump, in which the water is acting on both sides of the piston) the discharge is * taken to be double the discharge than that of a single acting pump. This is due to the reason that, in a double acting pump, the water is sucked on one side of the piston and delivered from the other side during the same stroke. These two processes (*i.e.*, suction on one side and delivery from the other) are reversed during the return stoke. Therefore the discharge of a double acting reciprocating pump,

$$Q = \frac{2LAN}{60}$$ for double acting

36·5 Slip of the Pump CL

In the last article, we have obtained the relation for the discharge of a single acting and double acting reciprocating pumps. But in practice, the actual discharge is less than the theoretical discharge. The difference between theoretical discharge and actual discharge is known as slip of the pump. This theory is similar to that which discussed for coefficient of discharge in Art. 11·8.

36·6 Negative Slip of the Pump CL slip = $Q_{th} - Q_{ac}$

Sometimes, the actual discharge of a reciprocating pump, is more than the theoretical discharge. In such cases, the coefficient of discharge will be more than unity, and the corresponding slip is known as negative slip of the pump.

This happens, when the suction pipe is long and delivery pipe is short and pump is running at high speeds. This causes the delivery valve to open before completion of the suction stroke and some water is pushed into the delivery pipe, before the piston commences its delivery stroke.

Example 36·1. *A single acting reciprocating pump has a plunger of diameter 300 mm and stroke of 200 mm. If the speed of the pump is 30 r.p.m. and it delivers 6·5 litres/sec of water, find the coefficient of discharge and the percentage slip of the pump.*

Solution. Given : D = 300 mm = 0·3 m; L = 200 mm = 0·2 m; N = 30 r.p.m. and Q_{ac} = 6·5 litres/s = 0·0065 m³/s.

Coefficient of discharge

We know that the area of the plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}3)^2 = 0{\cdot}0707 \text{ m}^2$$

and theoretical discharge, $Q_{th} = \frac{LAN}{60} = \frac{0{\cdot}2 \times 0{\cdot}0707 \times 30}{60} = 0{\cdot}0071$ m³/s

$\therefore$ Coefficient of discharge, $C_d = \frac{Q_{ac}}{Q_{th}} = \frac{0{\cdot}0065}{0{\cdot}0071} = 0{\cdot}92$ **Ans.**

*As a matter of fact, the discharge of a double acting reciprocating pump is not double of a single acting pump, because some area is blocked by the connecting rod. However, this area being small as compared to the discharge is neglected.

Percentage slip

We also know that percentage slip

$$= \frac{Q_{th} - Q_{ac}}{Q_{th}} = \frac{0{\cdot}0071 - 0{\cdot}0065}{0{\cdot}0071} = 0{\cdot}085 = 8{\cdot}5\% \textbf{ Ans.}$$

36·7 Power Required to Drive a Reciprocating Pump CL

Consider a reciprocating pump, first sucking a liquid (through the suction pipe) and then delivering the same (through the delivery pipe).

Let H_s = Suction head of the pump in metres,

H_d = Delivery head of the pump in metres,

A = Area of piston in m²,

w = Specific weight of the liquid, and

Q = Discharge of the liquid in m³/sec.

We know that force on the piston in forward stroke

$= w \cdot H_s A$ kN

and force on the piston in the backward stroke

$= w \cdot H_d A$ kN

$\therefore$ Work done by the pump $= wQ(H_s + H_d)$ kN-m

and theoretical power required to drive the pump

$= wQ(H_s + H_d)$ kW

Note. The actual power, required to drive the pump will be more than the theoretical, pump due to various losses.

Example 36·2. *A double acting reciprocating pump has a stroke of 300 mm and a piston of diameter 150 mm. The delivery and suction heads are 26 m and 4 m respectively including friction heads. If the pump is working at 60 r.p.m., find power required to drive the pump with 80% efficiency.*

Solution. L = 300 mm = 0·3 m; D = 150 mm = 0·15 m; H_d = 26 m; H_s = 4 m; N = 60 r.p.m. and η = 80% = 0·8.

We know that area of the piston,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}15)^2 = 0{\cdot}0177 \text{ m}^2$$

and theoretical discharge of the double acting pump

$$Q = \frac{2LAN}{60} = \frac{2 \times 0{\cdot}3 \times 0{\cdot}0177 \times 60}{60} = 0{\cdot}011 \text{ m}^3/\text{s}$$

$\therefore$ Theoretical power to drive the pump,

$$= wQ(H_s + H_d) = 9{\cdot}81 \times 0{\cdot}011 \times (4 + 26) = 3{\cdot}24 \text{ kW}$$

and actual power $= \frac{3{\cdot}24}{0{\cdot}8} = 4{\cdot}05$ kW **Ans.**

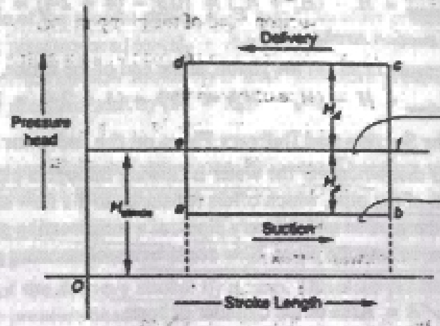
EXERCISE 36.1

1. A single acting reciprocating pump, having a bore of 150 mm diameter and a stroke of 300 mm length discharges 200 litres of water per minute. Neglecting losses, find (*i*) theoretical discharge in litres/min (*ii*) coefficient of discharge, and (*iii*) slip of the pump. [Ans. 210 litres/min; 0·95; 5·2%]

2. A single acting reciprocating pump having cylidner diameter of 150 mm and stroke 300 mm is used to raise water through a total height of 30 metres. Find the power required to drive the pump, if the crank rotates at 60 r.p.m. (Ans. 1·56 kW)
3. A double acting reciprocating pump of plunger diameter 100 mm and stroke of 250 mm length is discharging water into a tank fitted 20 m higher than the axis of the pump. If the pump is rotating at 45 r.p.m., find the power required to drive the pump. (Ans. 0·57 kW)

36·8 Indicator Diagram of a Reciprocating Pump

CL



atmospheric

suction

Fig. 36·2. Indicator diagram.

We have seen in the previous articles, that the pressure in the cylinder, during the suction stroke, is less than the atmospheric pressure; whereas during the delivery stroke, the pressure in the cylinder, is more than the atmospheric pressure.

Let H_s = Suction head of the pump.
H_d = Delivery head of the pump, and
L = Length of the stroke.

If we draw a diagram showing stroke length as abscissa and pressure head as ordinate, the diagram so obtained is known as an indicator diagram as shown in Fig. 36·2.

The line *ef* represents atmospheric pressure head and the line *ab* represents the pressure head during suction stroke; which is below the atmospheric pressure head by H_s. The line *cd* represents the pressure head during delivery stroke, which is above the atmospheric pressure head by H_d.

$$\therefore \text{ Work done by the pump} = wQ(H_s + H_d) = \frac{wLAN}{60}(H_s + H_d)$$

It is thus obvious, that the indicator diagram, of a reciprocating pump represents the work done to some scale.

36·9 Variation of Pressure in the Suction and Delivery Pipes due to Acceleration of the Piston

CL

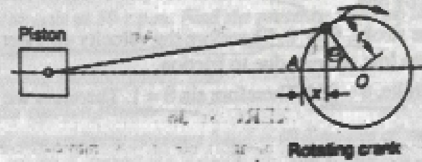


Fig. 36·3. Piston and crank.

We have seen in Art. 36·1 that the piston is connected to the rotating crank by a crank shaft. It is thus obvious that the piston will move, to and fro, with a simple harmonic motion. Therefore the

velocity of the piston will not be uniform at all points. It will be zero at its extreme ends, whereas it will be maximum at its centre as shown in Fig. 36·3.

Moreover, the piston will have an acceleration at the beginning and a retradation at end of every stroke. This acceleration and retardation of the piston causes a variation of pressure in the cylinder and consequently in the suction and delivery pipes. Now consider a reciprocating pump lifting water from a sump.

Prove that $H_a = -\frac{l}{g} \times \frac{A}{a}\omega^2 r$

Let A = Area of the cylinder,
a = Area of the pipe,
ω = Angular velocity of the rotating crank in rad/s,
r = Radius of the rotating crank, and
l = Length of the pipe.

Let the rotating crank start from A (known as inner dead centre). After t seconds, let the angle described by the rotating crank be θ radians. Such that

$$\theta = \omega t$$

and displacement of the piston in t seconds,

$$x = r - r\cos\theta = r - r\cos\omega t$$

We know that the velocity of the piston,

$$v = \frac{dx}{dt} = \omega r \sin \omega t$$

and acceleration of the piston,

$$a = \frac{dv}{dt} = \omega^2 r \cos \omega t$$

Since the flow of water in the pipe is equal to the flow of water in the cylinder, therefore velocity of water in the pipe

$$= \frac{A}{a} \times \text{Velocity of piston} = \frac{A}{a} \times \omega r \sin \omega t$$

and acceleration of water in the pipe $= \frac{A}{a} \times \omega^2 r \cos \omega t$

Now weight of water in the pipe $= wal$

$$\therefore \text{ Mass of water in the pipe} = \frac{wal}{g}$$

and acceleration force $= \boxed{\text{Mass} \times \text{Acceleration}}$

$$= \frac{wal}{g} \times \frac{A}{a} \times \omega^2 r \cos \omega t$$

We also know that intensity of pressure due to acceleration

$$= \frac{\text{Acceleration force}}{\text{Area}} = \frac{wl}{g} \times \frac{A}{a} \times \omega^2 r \cos \omega t$$

and acceleration pressure head,

$$H_a = \frac{\text{Intensity of pressure}}{\text{Sp. wt. of liquid}} = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r \cos \omega t$$

$$= \frac{l}{g} \times \frac{A}{a} \omega^2 r \cos \theta \qquad \ldots (\because \theta = \omega t)$$

Cor. 1. When $\theta = 0$ (*i.e.*, at the beginning of stroke),

$$\boxed{H_a = \frac{l}{g} \times \frac{A}{a}\,\omega^2 r} \qquad \ldots (\because \cos\theta = 1)$$

2. When $\theta = 90°$ (*i.e.*, at the mid of stroke)

$$H_a = 0 \qquad \ldots (\because \cos 90° = 0)$$

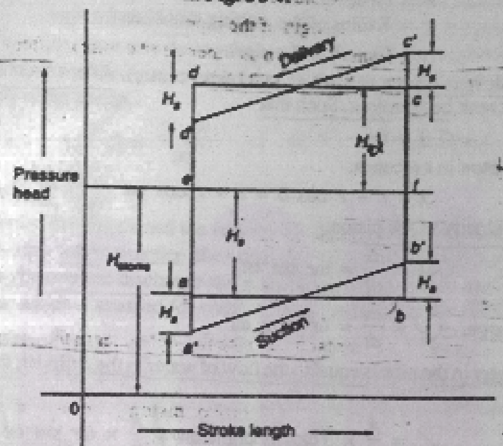
3. When $\theta = 180°$ (*i.e.*, at the end of stroke)

$$H_a = -\frac{l}{g} \times \frac{A}{a}\,\omega^2 r \qquad (\because \cos 180° = -1)$$

36.10. Effect of Acceleration of Piston on the Indicator Diagram

CL

We have seen in Art. 36.8 that some acceleration pressure head is caused due to the acceleration of the piston. We have also seen that at the beginning of the suction stroke, the pressure head is below the atmospheric pressure head by $(H_s + H_a)$, where H_a is the acceleration pressure head. In the middle of the suction stroke, the pressure head is below the atmospheric pressure head by H_s (as the acceleration pressure head, $(H_a = 0$, when $\theta = 90°)$. At the end of the suction stroke, the pressure head is below atmospheric pressure head by $(H_s - H_a)$. Therefore we can modify the indicator diagram, for the suction stroke, as shown in Fig. 36-4.



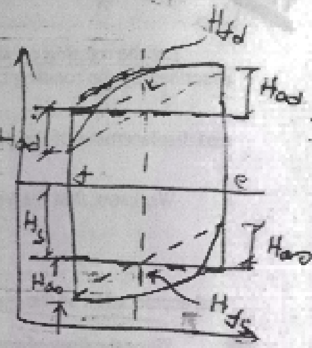


Fig. 36-4. Effect of acceleration of piston on the indicator diagram.

Similarly, at the beginning of the delivery stroke, the pressure head is above the atmospheric pressure by $(H_d + H_a)$. In the middle of the delivery stroke, the pressure head is above the atmospheric pressure head by H_d. At the end of the delivery stroke, the pressure head is above the atmospheric pressure head by $(H_d - H_a)$. Therefore we can modify the indicator diagram for delivery stroke also as shown in Fig. 36-4.

Example 36.3. CL *A single acting reciprocating pump, having plunger diameter 125 mm and stroke length 300 mm is drawing water from a depth of 4 metres from the axis the cylinder at 24 r.p.m. The length and diameter of suction pipe is 9 metres and 75 mm respectively. Find the pressure head on the piston at the beginning and end of the suction stroke, if the barometer reads 10·3 m of water.*

Solution. Given : D = 125 mm = 0·125 m; L = 300 mm = 0·3 m; or crank radius (r) = 0·3/2 = 0·15 m (because stroke length is equal to twice the crank radius); H_s = 4 m; N = 24 r.p.m.; l = 9 m; d = 75 mm; 0·075 m and barometer reading (H) = 10·3 m.

Pressure head at the beginning of the suction stroke

We know that area of the plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}125)^2 = 0{\cdot}0123 \text{ m}^2$$

Similarly area of the suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}075)^2 = 0{\cdot}0044 \text{ m}^2$$

and angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 24}{60} = 0{\cdot}8\pi \text{ rad/s}$$

$\therefore$ Acceleration pressure head,

$$H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{9}{9{\cdot}81} \times \frac{0{\cdot}0123}{0{\cdot}0044} \times (0{\cdot}8\pi)^2 \times 0{\cdot}15 \text{ m}$$
$$= 2{\cdot}4 \text{ m}$$

We also know that pressure head on the piston at the beginning of the stroke,

$$H_{piston} = H - (H_s + H_a) = 10{\cdot}3 - (4 + 2{\cdot}4) = 3{\cdot}9 \text{ m} \quad \textbf{Ans.}$$

Pressure head at the end of the suction stroke

We also know that the pressure head, on the piston, at the end of the stroke,

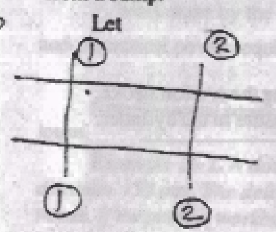
$$H_{piston} = H - (H_s - H_a) = 10{\cdot}3 - (4 - 2{\cdot}4) = 8{\cdot}7 \text{ m} \quad \textbf{Ans.}$$

36.11. Effect of Friction in the Suction and Delivery Pipes, on the Indicator Diagram

CL

We have seen in Art. 13.3 that whenever the water is flowing through a pipe, there is always some loss of head due to friction of the pipe, which offers resistance to the flow of water. Similarly, as the water is flowing through the suction and delivery pipes, of a reciprocating pump, there will be some loss of head, due to friction, in both the pipes. Now consider a reciprocating pump lifting water from a sump.

Let



A = Area of the cylinder or bore,
d = Diameter of the pipe,
a = Area of the pipe,
ω = Angular velocity of the rotating crank in rad/s,
r = Radius of the rotatory crank,
l = Length of the pipe,
f = Coefficient of friction, and
v = Velocity of water in the pipe

We know that velocity of the piston at any instant,

$$= \omega r \sin \omega t = \omega r \sin \theta$$

$\therefore$ Velocity of water in the pipe at that instant,

$$v = \frac{A}{a} \times \omega r \sin \theta$$

We know that the loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r \sin \theta\right)^2$$

Now we shall discuss the effect of this pipe friction on the indicator diagram at the beginning, middle and end of the stroke one by one.

1. At the beginning of the stroke, $\theta = 0$. Therefore the velocity of water in the pipe is zero, consequently there is no loss of head due to friction.
2. At the middle of the stroke, $\theta = 90°$. Therefore $\sin \theta = 1$. Therefore the loss of head due to friction,

$$H_f = \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2$$

3. At the end of the stroke, $\theta = 180°$. Therefore the velocity of water in the pipe is zero, consequently there is no loss of head due to friction.

Now we shall study the effect of friction alongwith the effect of acceleration of piston on the indicator diagram.

In the suction stroke

1. At the beginning of the suction stroke, H_f is zero. Therefore pressure head will be below the atmospheric pressure head by $(H_s + H_a)$.
2. In the middle of the suction stroke, H_a is zero. Therefore pressure head will be below the atmospheric pressure head by $(H_s + H_f)$.
3. At the end of the suction stroke, H_f is zero. Therefore pressure head will be below the atmospheric pressure head by $(H_s - H_a)$.

In the delivery stroke

1. At the beginning of the delivery stroke, H_f is zero. Therefore pressure head will be above the atmospheric pressure by $(H_d + H_a)$.
2. In the middle of the delivery stroke, H_a is zero. Therefore pressure head will be above the atmospheric pressure head by $(H_d + H_f)$.
3. At the end of the delivery stroke, H_f is zero. Therefore pressure head will be above the atmospheric pressure head by $(H_d - H_a)$.

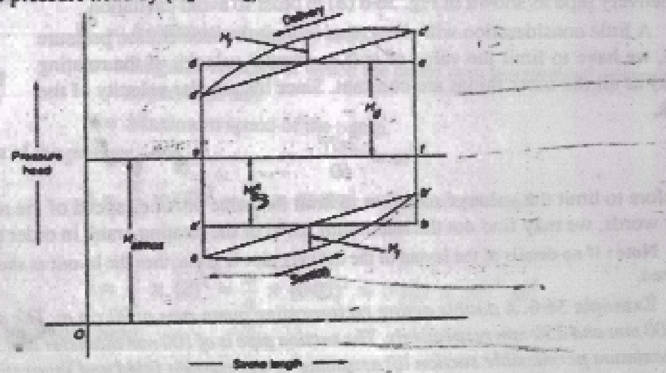


Fig. 36·5. Effect of friction in the suction and delivery pipes on the indicator diagram.

Now we can again modify the indicator diagram for suction stroke and delivery stroke as shown in Fig. 36·5.

Example 36·4. *A single acting reciprocating pump has plunger diameter of 200 mm and stroke length 300 mm. The suction pipe is 100 mm diameter and 8 metres long. The pump draws water 4 metres below the cylinder axis at 30 r.p.m. Find the pressure head on the piston :*

1. at the beginning of the suction stroke,

2. in the middle of the suction stroke, and

3. at the end of the suction stroke.

Take $f = 0{\cdot}01$ and atmospheric pressure head = 10·3 metres of water.

Solution. Given : D = 200 mm = 0·2 m; L = 300 mm = 0·3 m or crank radius $(r) = 0{\cdot}3/2 = 0{\cdot}15$ m (because stroke length is equal to twice the crank radius); d = 100 mm = 0·1 m; l_s = 8 m; H_s = 4 m; N = 30 r.p.m.; f = 0·01 and H = 10·3 m.

1. *Pressure head at the beginning of the suction stroke*

We know that area of the plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}2)^2 = 0{\cdot}03142 \text{ m}^2$$

Similarly area of suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}1)^2 = 0{\cdot}007\,85 \text{ m}^2$$

and angular velocity of crank, $\omega = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 30}{60} = \pi$ rad/s

∴ Acceleration pressure head,

$$H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{8}{9{\cdot}81} \times \frac{0{\cdot}031\,42}{0{\cdot}007\,85} \times (\pi)^2 \times 0{\cdot}15 \text{ m}$$
$$= 4{\cdot}8 \text{ m}$$

We know that the pressure head on the piston at the beginning of the stroke

$$H_{piston} = H - (H_s + H_a) = 10{\cdot}3 - (4 + 4{\cdot}8) = 1{\cdot}5 \text{ m} \quad \textbf{Ans.}$$

2. *Pressure head in the middle of the suction stroke*

We know that the velocity of water in the suction pipe in the* middle of the stroke

$$v = \frac{A}{a} \times \omega r \sin\theta = \frac{0{\cdot}031\,42}{0{\cdot}007\,85} \times \pi \times 0{\cdot}15 \times 1 = 1{\cdot}88 \text{ m/s}$$

and loss of head due to friction in the suction pipe,

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0{\cdot}01 \times 8 \times (1{\cdot}88)^2}{2 \times 9{\cdot}81 \times 0{\cdot}01} = 0{\cdot}6 \text{ m}$$

∴ Pressure head on the piston in the middle of the stroke,

$$H_{piston} = H - (H_s + H_f) \qquad ...(\because H_a = 0)$$
$$= 10{\cdot}3 - (4 + 0{\cdot}6) = 5{\cdot}7 \text{ m} \quad \textbf{Ans.}$$

3. *Pressure head at the end of the suction stroke*

We know that the pressure head on the piston at the end of the stroke,

$$H_{piston} = H - (H_s - H_a) \qquad ...(H_f = 0)$$
$$= 10{\cdot}3 - (4 - 4{\cdot}83) = 11{\cdot}1 \text{ m} \quad \textbf{Ans.}$$

Example 36·5. *A single acting reciprocating pump has a stroke of length 150 mm. The suction pipe is 7·5 metres long and the ratio of plunger diameter to the suction diameter is 4/3. The water level in the pump is 2·5 metres below the axis of the pump cylinder, and the pipe connecting the sump and pump cylinder is 75 mm diameter. If the crank is running at 75 r.p.m. find the pressure head on the piston (a) in the beginning of the suction stroke, (b) in the middle of the suction stroke, and (c) in the end of the suction stroke. Take coefficient of friction as 0·01.*

Solution. Given : L = 150 mm = 0·15 m or crank radius $(r) = 0{\cdot}15/2 = 0{\cdot}075$ m (because stroke length is equal to twice the crank radius); l_s = 7·5 m; $\dfrac{\text{Plunger diameter}}{\text{Suction diamete}} = \dfrac{4}{3}$ or $\dfrac{\text{Plunger area}}{\text{Suction area}} = \left(\dfrac{4}{3}\right)^2 = \dfrac{16}{9}$; H_s = 2·5 m; d = 75 mm = 0·075 m; N = 75 r.p.m. and f = 0·01.

* In the middle of the stroke, $\theta = 90°$ and $\sin 90° = 1$.

(a) Pressure head in the beginning of the suction stroke

We know that the angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 75}{60} = 2.5\,\pi \text{ rad/s}$$

and acceleration pressure head, $H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{7.5}{9.81} \times \frac{16}{9} \times (2.5\,\pi)^2 \times 0.075$ m

$= 6.3$ m

$\therefore$ Pressure head on the piston in the beginning of the stroke,

$$H_{piston} = H - (H_s + H_a) = H - (2.5 + 6.3) = H - 8.8 \text{ m}$$

$= 8.8$ m (vacuum) **Ans.**

(b) Pressure head in the middle of the suction stroke

We know that the velocity of water in the suction pipe in the middle of the stroke,

$$v = \frac{A}{a} \times \omega r \sin\theta = \frac{16}{9} \times 2.5\,\pi \times 0.075 \times 1 = 1.05 \text{ m/s}$$

and loss of head due to friction in the suction pipe,

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0.01 \times 7.5 \times (1.05)^2}{2 \times 9.81 \times 0.075} = 0.2 \text{ m}$$

$\therefore$ Pressure head on the piston in the middle of the stroke,

$$H_{piston} = H - (H_s + H_f) \qquad ...(\because H_a = 0)$$

$= H - (2.5 + 0.2) = H - 2.7$ m

$= 2.7$ m (vacuum) **Ans.**

(c) Pressure head in the end of the suction stroke

We also know that the pressure head on the piston in the end of the stroke,

$$H_{piston} = H - (H_s - H_a) = H - (2.5 - 6.3) = H + 3.8 \text{ m}$$

$= 3.8$ m (gauge) **Ans.**

36·12 Maximum Speed of the Rotating Crank

may hamper

It has been experienced that the continuous flow of water ceases to act (known as separation) whenever pressure in the pipe falls below the required pressure. In a reciprocating pump, the separation may take place in the suction pipe as well as delivery pipe. Now we shall discuss both the cases one by one.

(a) Separation in suction pipe C L

We have seen in Art. 36·11 that at the beginning of a suction stroke, the pressure head is below the atmospheric pressure head by $(H_s + H_a)$, where H_s is the suction head and H_a is the acceleration pressure head. It has been experimentally found that when this vacuum pressure head (*i.e.*, $H_s + H_a$) reaches 7·8 metres of water or 2·5 metres absolute [*i.e.*, $H - (H_s + H_a)$] the continuity of flow will stop, as the separation will take place; because the water will commence to vaporise. The head of water at which the separation takes place is known as separation head.

$$\therefore \qquad H_{sep} = H - (H_s + H_a) = H - H_s - H_a$$

Since the suction head is constant for a set reciprocating pump, therefore, in order to avoid separation at the beginning of the suction stroke, the acceleration pressure head should be limited. We know that the acceleration pressure head,

$$H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r$$

where l is the length of the suction pipe.

(b) Separation in delivery pipe C L

We have also seen, that the maximum pressure head in the delivery pipe is at the end of the delivery stroke is $(H + H_d - H_a)$. It has also been experimentally found that when this absolute pressure head is less than 2·5 m of water, the continuity of flow will stop and the separation will take place. The head at which the separation will take place is known as separation head and it denoted by H_{sep}

$$\therefore \qquad H_{sep} = H + H_d - H_a$$

In this case also, the relation for acceleration pressure head (as given above) is used. But the value of l is taken as the length of the delivery pipe.

It will be interesting to know that sometimes a reciprocating pump is used to deliver water through various types of delivery pipes. Two typical delivery pipes are shown in Fig. 36·6 (*a*) and (*b*). In Fig. 36·6 (*a*) the delivery pipe is first vertical, and then it is bent to be horizontal. In this case, the delivery head will be zero at B (*i.e.*, at the bend) and there will be still considerable acceleration pressure head. It is thus obvious, that there is always a possibility of separation taking place at B. In Fig. 36·6 (*b*) the pipe is first laid horizontal, and then it is bent to be vertical. In this case, there is a little possibility of separation taking place at B; as at this point there will be considerable delivery head. It is thus desirable to lay the delivery pipe as shown in Fig. 36·6 (*b*) in order to avoid cavitation.

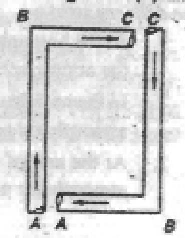


Fig. 36·6. Typical delivery pipes.

A little consideration will show, that to limit the acceleration pressure head, we have to limit the value of ω (*i.e.*, angular velocity of the rotating crank) as all the other things are constant. Since the angular velocity of the crank,

$$\omega = \frac{2\pi N}{60}$$

therefore to limit the value of ω means to limit the value of N *i.e.*, speed of the rotating crank. Or in other words, we may find out the maximum speed of the rotating crank in order to avoid separation.

Note : If no details of the layout of the delivery pipe is given, then the layout as shown in Fig. 36·6 (*b*) is assumed.

Example 36·6. *A double acting reciprocating pump runs at 90 r.p.m. The diameter and stroke are 100 mm and 250 mm respectively. The suction pipe is of 100 mm diameter and 5 m long. Calculate the maximum permissible suction lift assuming no air vessel is fitted and separation occurs at 2 m of water absolute.*

Solution. Given : $N = 90$ r.p.m.; $D = 100$ mm $= 0.1$ m; $L = 250$ mm $= 0.25$ m or crank radius $(r) = 0.25/2 = 0.125$ m (because stroke length is equal to twice the crank radius); $d = 100$ mm $= 0.1$ m; $l = 5$ m and $H_{sep} = 2$ m.

Let $\qquad H_s =$ Maximum permissible suction lift.

We know that area of the piston,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007\,85 \text{ m}^2$$

Similarly, area of suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.1)^2 = 0.007\,85 \text{ m}^2$$

and angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = 9.42 \text{ rad/s}$$

∴ Acceleration pressure head,

$$H_a = \frac{l_s}{g} \times \frac{A}{a} \times \omega^2 r = \frac{5}{9.81} \times \frac{0.007\,85}{0.007\,85} \times (9.42)^2 \times 0.125 \text{ m}$$
$$= 5.65 \text{ m}$$

We know that separation head (H_{sep}),

$$2 = H - H_s - H_a = 10.3 - H_s - 5.65 \quad \text{...(Assuming } H = 10.3 \text{ m)}$$
$$= 4.65 - H_s$$

∴ $$H_s = 4.65 - 2 = 2.65 \text{ m} \quad \textbf{Ans.}$$

07 CL (D)

Example 36-7. *A single acting reciprocating pump (with no air vessel) has a plunger of 80 mm diameter and a stroke of 150 mm. It draws water from a sump 3 m below the pump axis through a suction pipe 30 mm diameter and 4·5 m long.*

If separation occurs at a pressure of 80 kPa below atmospheric pressure, find the maximum speed at which the pump may be operated without separation. Assume that the plunger moves with simple harmonic motion.

Solution. Given : $D = 80$ mm $= 0.08$ m; $L = 150$ mm $= 0.15$ m; or crank radius $(r) = 0.15/2 = 0.075$ m; $H_s = 3$ m; $d = 30$ mm $= 0.03$ m; $l_s = 4.5$ and separation pressure below the atmospheric pressure = 80 kPa = 80 kN/m² or separation pressure head below the atmospheric head

$$(H - H_{sep}) = \frac{80 \times 10^3}{9.81 \times 10^3} = 8.2 \text{ m.}$$

Let H_a = Acceleration pressure head,

ω = Angular velocity at which the pump may be operated without separation, and

N = Maximum speed of the pump.

We know that area of the plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0.08)^2 = 5.03 \times 10^{-3} \text{ m}^2$$

and area of suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.03)^2 = 0.707 \times 10^{-3} \text{ m}^2$$

We know that the separation head,

$$H_{sep} = H - H_s - H_a$$

or

$$H_a = H - H_{sep} - H_s = (H - H_{sep}) - H_s = 8.2 - 3 = 5.2 \text{ m}$$

We also know that acceleration pressure head (H_a),

$$5.2 = \frac{l_s}{g} \times \frac{A}{a} \times \omega^2 r = \frac{4.5}{9.81} \times \frac{5.03 \times 10^{-3}}{0.707 \times 10^{-3}} \times \omega^2 \times 0.075$$
$$= 0.245\, \omega^2$$

∴ $$\omega^2 = 5.2/0.245 = 21.2 \quad \text{or} \quad \omega = 4.6 \text{ rad/s}$$

We know that angular velocity (ω),

$$4.6 = \frac{2\pi N}{60} = 0.105\, N \quad \text{or} \quad N = \frac{4.6}{0.105} = 43.8 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 36-8. *A single acting reciprocating pump of 250 mm diameter and 500 mm stroke delivers water through a 100 mm diameter vertical delivery pipe to a tank situated at 15 metres above it and 30 metres horizontally from it. Find the safe speed of the pump, if separation pressure corresponds to 23 kPa, when the delivery pipe is :*

(i) vertical from the pump and then horizontal upto the tank,

(ii) horizontal from the pump and then vertical upto the tank.

Assume atmosphere pressure head as 10·3 m.

Solution. Given : $D = 250$ mm $= 0.25$ m; $L = 500$ mm $= 0.5$ m or crank radius (r) $0.5/2 = 0.25$ m; $d_d = 100$ mm $= 0.1$ m; $H_d = 15$ m; Length of vertical delivery pipe = 15 m; Length of horizontal delivery pipe = 30 m or total length of delivery pipe $(l_d) = 15 + 30 = 45$ m; Separation pressure $(p_{sep}) = 23$ kPa $= 23$ kN/m² or separation pressure head $(H_{sep}) = \frac{p_{sep}}{w} = \frac{23 \times 10^3}{9.81 \times 10^3} = 2.3$ m and $H_a = 10.3$ m.

Safe speed for the pump, when the delivery is horizontal from the pump and then vertical upto the tank

Let H_a = Acceleration pressure head,

ω = Safe speed of the pump in rad/s,

N = Safe speed of the pump in r.p.m.

We know that area of piston

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0.25)^2 = 0.0491 \text{ m}^2$$

and area of delivery pipe,

$$a = \frac{\pi}{4} \times (d_d)^2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 \text{ m}^2$$

We know that the separation head (H_{sep}),

$$2.3 = H + H_d - H_a = 10.3 + 15 - H_a = 25.3 - H_a$$

∴ $$H_a = 25.3 - 2.3 = 23 \text{ m}$$

We also know that acceleration pressure head (H_a),

$$23 = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{45}{9.81} \times \frac{0.0491}{0.00785} \times \omega^2 \times 0.25$$
$$= 7.17\, \omega^2$$

∴ $$\omega^2 = 23/7.17 = 3.208 \quad \text{or} \quad \omega = 1.79 \text{ rad/s}$$

We know that angular velocity (ω),

$$1.79 = \frac{2\pi N}{60} = 0.105\, N \quad \text{or} \quad N = \frac{1.79}{0.105} = 17 \text{ r.p.m.} \quad \textbf{Ans.}$$

Safe speed for the pump, when the delivery pipe is vertical from the pump and then horizontal upto the tank.

In this case, the separation will take place at the bend after the vertical pipe. Therefore we shall take delivery head as zero in the relation for separation head. Thus separation head (H_{sep}),

$$2.3 = H - H_a = 10.3 - H_a$$

or

$$H_a = 10.3 - 2.3 = 8.0 \text{ m}$$

$H_{sep} = H + H_d - H_a$

We know that acceleration pressure head (H_a),

$$8 = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{45}{9{\cdot}81} \times \frac{0{\cdot}0491}{0{\cdot}00785} \times \omega^2 \times 0{\cdot}25$$

$$= 7{\cdot}17\ \omega^2$$

$\therefore \quad \omega^2 = 8/7{\cdot}13 = 1{\cdot}116 \quad$ or $\quad \omega = 1{\cdot}06$ rad/s

We know that angular velocity (ω),

$$1{\cdot}06 = \frac{2\pi N}{60} = 0{\cdot}105\ N \quad \text{or} \quad N = \frac{1{\cdot}06}{0{\cdot}105} = 10{\cdot}1 \text{ r.p.m. Ans.}$$

EXERCISE 36-2

1. A single acting reciprocating pump has plunger of diameter 150 mm and stroke of length 400 mm. It draws water from a depth of 4·5 m through a pipe 8 m long and 80 mm diameter at 25 r.p.m. If the atmospheric pressure head is 10·3 m, find the pressure head on the piston at the beginning and end of the suction stroke. **(Ans. 1·8 m; 9·8 m)**
2. A single acting reciprocating pump has plunger of diameter 150 mm and stroke of 300 mm. The lengths of suction and delivery pipe are 6·5 m and 39 m respectively and both the pipes are of the same diameter of 75 mm. The axis of the pump is 5 m above the level of water in the sump and 33 m below the delivery water level. If the atmospheric pressure head is 10·3 m of water and coefficient of friction for both the pipes is 0·01, find the pressure head on the piston at the beginning, middle and end of the suction strokes. Take speed of the crank as 30 r.p.m. **(Ans. 1·4 m; 4·67 m; 9·2 m)**
3. The bore and stroke of a single acting reciprocating pump are 100 mm and 200 mm respectively. The suction pipe is 80 mm in diameter and 4 m long and centre of the pump in 3·5 m above the water level of the sump. Find the maximum speed at which can be run without separation taking place. Assume separation to occur at 2·5 m of water and atmospheric pressure head = 10·3 m of water. **(Ans. 78·5 r.p.m.)**
4. A single acting reciprocating pump has stroke length of 300 mm and plunger diameter of 80 mm. The suction head is 3 metres and the suction pipe is 6 metres long. If the separation takes place at a pressure head of 2·5 metres, determine the maximum speed of the pump, so as to avoid separation. Take atmospheric pressure head as 10·3 metres of water. **(Ans. 58·6 r.p.m.)**

36·13 Air Vessels

An air vessel is a cast iron closed chamber, having an opening at its base, through which the water flows into the vessel or from the vessel. The vessel is filled up with compressed air.

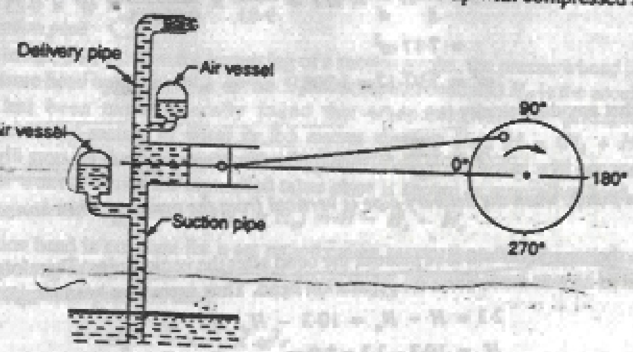


Fig. 36·7. Air vessels fitted to the suction and delivery pipes.

The air vessels are fitted to the suction pipe and delivery pipe close to the cylinder of the pump as shown in Fig. 36·7. The object of fitting the air vessels is to obtain a uniform discharge from a

reciprocating pump. Consider an air vessel fitted to the delivery pipe as shown in Fig. 34·7. During the first half of the delivery stroke, the piston moves with acceleration, thus forcing the water into the delivery pipe with a velocity more than the mean velocity. The excess flow of water, flows into the air vessel thus compressing the air inside the vessel. During the second half of the delivery stroke, the piston moves with retardation thus forcing the water into the delivery pipe, with a velocity less than the mean velocity. The water, stored into the air vessel, then starts flowing into the delivery pipe, thus making up the deficiency of the flow.

Thus the discharge in the delivery pipe, beyond the air vessel, is more or less uniform. But for all practical purposes, velocity of water in the delivery pipe, beyond air vessel is taken to be uniform. Similarly, on the suction side the water first flows from the suction pipe into the air vessel (during first half of the suction stroke) and then from the air vessel to the cylinder (during the second half of the suction stroke). Thus for all practical purpose, velocity of water in the suction pipe upto the air vessel is also taken to be uniform.

Example 36·9. *A double acting pump has a cylinder of 200 mm diameter and stroke of 400 mm. The pump delivers water to a height of 10 metres through a pipe 36 metres long and 150 mm diameter at 40 r.p.m. Find the pressure in the cylinder at the beginning of the delivery stroke, if a large air vessel is fitted in the delivery pipe at the same level of the pump, but 3 metres from the cylinder. Take f = 0·008.*

Solution. Given : D = 200 mm = 0·2 m; L = 400 mm = 0·4 m or crank radius (r) = 0·4/2 = 0·2 m; H_d = 10 m; l = 36 m; d_d = 150 mm = 0·15 m; N = 40 r.p.m.; Distance between the pump level and air vessel = 3 m and f = 0·008.

We know that area of the cylinder,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}2)^2 = 0{\cdot}03142 \text{ m}^2$$

and area of the delivery pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}15)^2 = 0{\cdot}01767 \text{ m}^2$$

Since the air vessel is fitted to the delivery pipe, at a distance of 3 metres from the cylinder, therefore :

1. there will be an acceleration pressure head in the delivery pipe for a length of 3 metres.
2. there will be a loss of head due to friction in the delivery pipe for a length of 36 − 3 = 33 metres.

We know that angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 40}{60} = 4{\cdot}19 \text{ rad/s}$$

and acceleration pressure head,

$$H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{3}{9{\cdot}81} \times \frac{0{\cdot}031\,42}{0{\cdot}017\,67} \times (4{\cdot}19)^2 \times 0{\cdot}2 \text{ m}$$

$$= 1{\cdot}91 \text{ m}$$

We know that the velocity of water in the delivery pipe,

$$v = \frac{Q}{a} = \frac{2LAN}{60 \times a} \quad \text{...(}\because \text{ of double acting pump)}$$

$$= \frac{2 \times 0{\cdot}4 \times 0{\cdot}0314 \times 40}{60 \times 0{\cdot}017\,67} = 0{\cdot}95 \text{ m/s}$$

80—

and loss of head due to friction in 33 m long pipe,

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0.008 \times 33 \times (0.95)^2}{2 \times 9.81 \times 0.15} = 0.32 \text{ m}$$

$\therefore$ Pressure head in the cylinder at the beginning of the delivery stroke,

$$= H_d + H_a + H_f = 10 + 1.91 + 0.32 = 12.23 \text{ m Ans.}$$

Example 36·10. *A single acting reciprocating pump is to raise a liquid of 11·8 kN/m³ through a vertical height of 11·5 metres, from 2·5 metre below pump axis to 9 metres above it. The plunger, which moves with S.H.M., has diameter 125 mm and stroke 225 mm. The suction and delivery pipes are 75 mm diameter and 3·5 metres and 13·5 metres long respectively. There is a large air vessel placed on the delivery pipe near the pump axis. But there is no air vessel on the suction pipe. If separation takes place at 90 kPa below atmospheric pressure, find*

(a) maximum speed, with which the pump can run without separation taking place, and

(b) power required to drive the pump, if f = 0·02.

Neglect slip for the pump.

Solution. Given : $w = 11.8$ kN/m³; $H = 11.5$ m; $H_s = 2.5$ m; $H_d = 9$ m; $D = 125$ mm $= 0.125$ m; $L = 225$ mm $= 0.225$ m; or crank radius $(r) = 0.225/2 = 0.1125$ m; $d = 75$ mm $= 0.075$ m; $l_s = 3.5$ m; $l_d = 13.5$ m; Separation pressure below the atmospheric pressure = 90 kPa or separation pressure head below the atmospheric pressure head $(H - H_{sep}) = \dfrac{90 \times 10^3}{11.8 \times 10^3} = 7.6$ m and and $f = 0.02$.

(a) Maximum speed, with which the pump can run without separation taking place

Let H_a = Acceleration pressure head,

ω = Angular velocity at which the pump can run without separation taking place, and

N = Maximum speed of the pump.

We know that area of plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0.125)^2 = 0.0123 \text{ m}^3$$

and area of suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0.075)^2 = 0.0044 \text{ m}^2$$

We know that the separation head,

$$H_{sep} = H - H_s - H_a$$

$$\therefore \quad H_a = (H - H_{sep}) - H_s = 7.6 - 2.5 = 5.1 \text{ m}$$

We also know acceleration pressure head (H_a),

$$5.1 = \frac{l_s}{g} \times \frac{A}{a} \times \omega^2 r = \frac{3.5}{9.81} \times \frac{0.0123}{0.0044} \times \omega^2 \times 0.1125$$

$$= 0.112\ \omega^2$$

$$\therefore \quad \omega^2 = 5.1/0.112 = 45.5 \quad \text{or} \quad \omega = 6.75 \text{ rad/s}$$

We know that angular velocity (ω),

$$6.75 = \frac{2\pi N}{60} = 0.105\ N \quad \text{or} \quad N = \frac{6.75}{0.105} = 64.3 \text{ r.p.m. Ans.}$$

62·19 rpm

(b) Power required to drive the pump

We know that the discharge of the pump,

$$Q = \frac{LAN}{60} = \frac{0.225 \times 0.0123 \times 64.3}{60} = 0.003 \text{ m}^3\text{/s}$$

$\therefore$ Velocity of water in the delivery pipe,

$$v = \frac{Q}{a} = \frac{0.003}{0.0044} = 0.68 \text{ m/s}$$

Since there is an air vessel on the delivery pipe near the pump axis, therefore there will be no acceleration head on the delivery side. Hence we shall consider the loss of head due to friction only.

$\therefore$ Loss of head due to friction,

$$H_{fd} = \frac{4fl_dv^2}{2gd} = \frac{4 \times 0.02 \times 13.5 \times (0.68)^2}{2 \times 9.81 \times 0.075} = 0.34 \text{ m}$$

Since there is no air vessel on the suction pipe, therefore there will be loss of head due to friction. But we shall consider the acceleration head only *i.e.*, $H_{as} = 5.1$ m

$\therefore$ Total head, against which the pump has to work,

$$H = (H_s + H_d) + (H_{as} + H_{ad}) + (H_{fs} + H_{fd})$$

$$= (2.5 + 9) + (5.1 + 0) + (0 + 0.34) = 16.94 \text{ m}$$

and power required to drive the pump,

$$P = wQH = 9.81 \times 0.003 \times 16.94 = 0.5 \text{ kW Ans.}$$

36·14 Maximum Speed of the Rotating Crank with Air Vessels

We have seen in Art. 36·13 that by fitting air vessels, the velocity of water in the suction pipe upto the air vessels (*i.e.*, for a length of l_s, in Fig. 36·8) is uniform. The acceleration and retardation to the velocity of water will take place in the suction pipe beyond the air vessel,(*i.e.*, for a length of l_s' in Fig. 36·8).

Similarly acceleration and retardation to the velocity of water will take place in the delivery pipe upto the air vessel (*i.e.*, for a length of l_d') in Fig. 36·8. The velocity of water in the delivery pipe beyond the air vessel (*i.e.*, for a length of l_d in Fig. 36·8) is constant.

Thus for finding out the maximum speed of the crank we have to limit the separation head. *i.e.*,

$$H_{sep} = H - [H_s + (H_a \text{ for } l_s') + (H_f \text{ for } l_s)]$$

The constant velocity of water, in the suction or delivery pipe, may be found out by dividing the discharge of the pump by the area of the respective pipe.

Now consider a reciprocating pump fitted with air vessels on suction and delivery pipes as shown in Fig. 36·8.

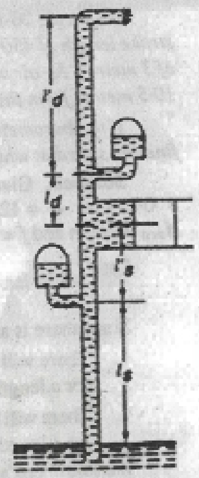


Fig. 36·8. Air vessels fitted to the pipes.

Let L = Length of the stroke,

A = Area of the piston,

N = Speed of the pump in r.p.m.,

ω = Angular velocity of the crank,

a = Area of the pipe,

r = Radius of the crank, and

v = Velocity of water in the pipe.

For single acting pump

We know that the discharge of a single acting pump,

$$Q = \frac{LAN}{60}$$

and velocity of water, $\quad v = \frac{Q}{a} = \frac{LAN}{60 \times a} = \frac{A}{a} \times \frac{LN}{60}$

Substituting $L = 2r$, $\omega = \frac{2\pi N}{60}$ or $N = \frac{60\omega}{2\pi}$ in the above equation,

$$v = \frac{A}{a} \times \frac{2r \times \frac{60\omega}{2\pi}}{60} = \frac{A}{a} \times \frac{\omega r}{\pi}$$

For double acting pump

We know that the discharge of a double acting pump,

$$Q = \frac{2LAN}{60}$$

and velocity of water, $\quad v = \frac{Q}{a} = \frac{2LAN}{60 \times a} = \frac{2A}{a} \times \frac{LN}{60}$

Substituting, $L = 2r$, $\omega = \frac{2\pi N}{60}$ or $N = \frac{60\omega}{2\pi}$ in the above equation,

$$v = \frac{2A}{a} \times \frac{2r \times \frac{60\omega}{2\pi}}{60} = \frac{2A}{a} \times \frac{\omega r}{\pi}$$

Example 36·11. *A single acting reciprocating pump has a plunger diameter of 250 mm and stroke length of 450 mm. The suction pipe is 125 mm diameter and 12 metres long with a suction lift of 3 metres. An air vessel is fitted to the suction pipe at a distance of 1·5 metre from the cylinder and 10·5 metres from the sump of water level.*

If the barometer reads 10·0 metres of water and separation takes place at 2·5 metres vacuum, find the speed at which the crank can operate without separation to occur. Take $f = 0{\cdot}01$.

Solution. Given : $D = 250$ mm $= 0{\cdot}25$ m; $L = 450$ mm $= 0{\cdot}45$ m or crank radius $(r) = 0{\cdot}45/2 = 0{\cdot}225$ m; $d = 125$ mm $= 0{\cdot}125$ m; $l = 12$ m; $H_s = 3$ m; $l_s' = 1{\cdot}5$ m; $l_s = 10{\cdot}5$ m; $H = 10$ m; $H_{sep} = 2{\cdot}5$ m and $f = 0{\cdot}01$.

Let $\quad \omega =$ Speed of the crank in rad/s, and

$N =$ Speed of the crank in r.p.m.

Since there is an air vessel fitted to the suction pipe, therefore

1. There will be a loss of head due to friction in the suction pipe for a length of 10·5 metres.
2. There will be an acceleration pressure head in the suction pipe for a length of 1·5 metre.

We know that area of the plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}25)^2 = 0{\cdot}0491 \text{ m}^2$$

and area of the suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}125)^2 = 0{\cdot}0123 \text{ m}^2$$

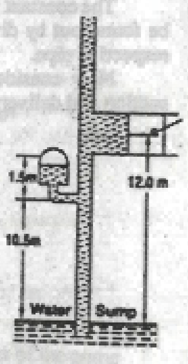


Fig. 36·9.

$\therefore$ Velocity of water in the suction pipe of single acting reciprocating pump,

$$v = \frac{A}{a} \times \frac{\omega r}{\pi} = \frac{0{\cdot}0491}{0{\cdot}0123} \times \frac{\omega \times 0{\cdot}225}{\pi} = 0{\cdot}286\ \omega \text{ m/s}$$

We know that the loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0{\cdot}01 \times 10{\cdot}5\ (0{\cdot}286\ \omega)^2}{2 \times 9{\cdot}81 \times 0{\cdot}125} = 0{\cdot}014\ \omega^2$$

and acceleration pressure head in the suction pipe for a length of 1·5 metre,

$$H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{1{\cdot}5}{9{\cdot}81} \times \frac{0{\cdot}0491}{0{\cdot}0123} \times \omega^2 \times 0{\cdot}225$$

$$= 0{\cdot}137\ \omega^2 \qquad \ldots(ii)$$

We know that the separation head (H_{sep})

$$2{\cdot}5 = H - (H_s + H_a + H_f) = 10 - (3 + 0{\cdot}137\,\omega^2 + 0{\cdot}014\ \omega^2)$$

$$= 7 - 0{\cdot}151\ \omega^2$$

$$\therefore \quad \omega^2 = \frac{7 - 2{\cdot}5}{0{\cdot}151} = 29{\cdot}8 \quad \text{or} \quad \omega = 5{\cdot}46 \text{ rad/s}$$

We know the angular velocity (ω),

$$5{\cdot}46 = \frac{2\pi N}{60} = 0{\cdot}105\ N \quad \text{or} \quad N = \frac{5{\cdot}46}{0{\cdot}105} = 52 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 36·12. *A single-acting single-cylinder reciprocating pump is installed 3·5 metres above the water of a sump. The suction pipe is 200 mm diameter and 10 metres long. The pump cylinder is of 300 mm diameter and 500 mm stroke. Find*

(a) *the speed, at which the separation may take place at the commencement of suction stroke.*

(b) *the change in speed of the pump, if an air vessel is fitted on the suction side 2·5 metres above the pump water level.*

Assume (i) barometric head = 10·3 metres of water, (ii) $f = 0{\cdot}01$ and, (iii) separation occurs at 2·5 metres of water absolute.

Solution. Given : $H_s = 3{\cdot}5$ m; $d = 200$ mm $= 0{\cdot}2$ m; $l = 10$ m; $D = 300$ mm $= 0{\cdot}3$ m; $L = 500$ mm $= 0{\cdot}5$ m or crank radius $(r) = 0{\cdot}5/2 = 0{\cdot}25$ m; $H = 10{\cdot}3$ m; $f = 0{\cdot}01$ and $H_{sep} = 2{\cdot}5$ m.

(a) Speed at which the separation may take place without air vessel

Let $\quad H_a =$ Acceleration pressure head,

$\omega =$ Angular velocity at which the separation may take place at the commencement of the suction stroke, and

$N =$ Speed of the crank in r.p.m.

We know that the area of pump cylinder,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}3)^2 = 0{\cdot}0707 \text{ m}^2$$

and area of suction pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}2)^2 = 0{\cdot}0314 \text{ m}^2$$

We know that separation head,

$$H_{sep} = H - H_s - H_a$$

or $$H_a = H - H_s - H_{sep} = 10{\cdot}3 - 3{\cdot}5 - 2{\cdot}5 = 4{\cdot}3 \text{ m}$$

We also know that acceleration pressure head (H_a),

$$4\cdot3 = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r = \frac{10}{9\cdot81} \times \frac{0\cdot0707}{0\cdot0314} \times \omega^2 \times 0\cdot25$$
$$= 0\cdot574\ \omega^2$$
$$\therefore \quad \omega^2 = 4\cdot3/0\cdot574 = 7\cdot49 \quad \text{or} \quad \omega = 2\cdot74 \text{ rad/s}$$

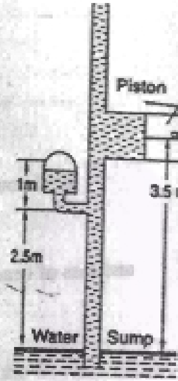
We know that angular velocity (ω),

$$2\cdot74 = \frac{2\pi N}{60} = 0\cdot105\ N \quad \text{or} \quad N = \frac{2\cdot74}{0\cdot105} = 26\cdot1 \text{ r.p.m.} \quad \text{A}$$

Change in the speed of the pump with air vessel

Since the air vessel is fitted in the suction pipe 2·5 metres above the sump water level, therefore

1. There will be loss of head due to friction in the suction pipe for a length $\frac{10 \times 2\cdot5}{3\cdot5} = 7\cdot14$ m.
2. There will be an acceleration pressure head in the suction pipe for a length of $10 - 7\cdot14 = 2\cdot86$ m.



We know that velocity of water,

$$v = \frac{A}{a} \times \frac{\omega r}{\pi} = \frac{0\cdot0707}{0\cdot0314} \times \frac{\omega \times 0\cdot25}{\pi}$$
$$= 0\cdot18\ \omega \text{ m/s}$$

and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0\cdot01 \times 7\cdot14\,(0\cdot18\ \omega)^2}{2 \times 9\cdot81 \times 0\cdot2} = 0\cdot002\ \omega^2$$

Fig. 36·10.

We also know that the acceleration pressure head in the suction pipe,

$$H_a = \frac{l}{g} \times \frac{A}{a}\ \omega^2 r = \frac{2\cdot86}{9\cdot81} \times \frac{0\cdot0707}{0\cdot0314} \times \omega^2 \times 0\cdot25 = 0\cdot164\ \omega^2$$

and the separation head, (H_{sep}),

$$2\cdot5 = H - (H_s + H_a + H_f)$$
$$= 10\cdot3 - (3\cdot5 + 0\cdot164\ \omega^2 + 0\cdot002\ \omega^2) = 6\cdot8 - 0\cdot166\ \omega^2$$
$$\therefore \quad \omega^2 = \frac{6\cdot8 - 2\cdot5}{0\cdot166} = 25\cdot9 \quad \text{or} \quad \omega = 5\cdot1 \text{ rad/s}$$

We know that angular velocity (ω)

$$5\cdot1 = \frac{2\pi N}{60} = 0\cdot105\ N \quad \text{or} \quad N = \frac{5\cdot1}{0\cdot105} = 48\cdot6 \text{ r.p.m.} \quad \textbf{Ans.}$$

Note. It will be interesting to know that the angular velocity of the rotating shaft has increased from 26·1 r.p.m. to 48·5 r.p.m. by fitting air vessel.

36·15. Work Done against Friction without Air Vessels

We have seen in Art. 36·11, that at the beginning and end of a stroke, there is no loss of head due to friction. The maximum loss of head is only in the middle of the stroke. We have also seen in Art. 36·12, that in the indicator diagram, the pressure head due to friction is a parabola. Now consider a reciprocating pump lifting water from a sump.

Let A = Area of the cylinder,

a = Area of the pipe,
ω = Angular velocity of the crank,
r = Radius of the crank,
l = Length of the pipe,
d = Diameter of the pipe, and
f = Coefficient of friction.

We know that in the middle of stroke speed of water in pipe,

$$v = \frac{A}{a} \times \omega r$$

and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2$$

We also know that area of a parabola is numerically equal to

$$= \frac{2}{3} \times \text{Base} \times \text{Height}$$

$\therefore$ Work done per stroke

$$\omega_n = \frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a}\ \omega r\right)^2$$

Note : The power required to overcome the friction of the delivery pipe will be given by the relation :

$$P = wQ\left(\frac{2}{3} \times H_f\right)$$

36·16. Work Done against Friction with Air Vessels

We have seen in Art. 36·13, that whenever an air vessel is fitted to a pipe, there is no acceleration of water, and hence there is no acceleration pressure head. Now consider a reciprocating pump lifting water from a sump.

Let A = Area of the cylinder,
ω = Angular velocity of the crank,
l = Length of the pipe,
a = Area of the pipe,
f = Coefficient of friction, and
d = Diameter of the suction pipe.

We have seen in Art. 36·14 that the velocity of water with air vessel in the pipe,

$$v = \frac{A}{a} \times \frac{\omega r}{\pi}$$

$\therefore$ Loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

and work done per stroke against friction

$$\omega_f = \frac{4fl}{2gd}\left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

Example 36·13. *A single acting reciprocating pump, running at 60 r.p.m., has a plunger diameter of 250 mm and a stroke of 500 mm. The delivery pipe is 100 mm diameter and 50 metres long. If the motion of the pump is simple harmonic, find the power required to overcome friction of the delivery pipe when :*

(a) no air vessel is fitted, and (b) a large air vessel is fitted at the centre line of the pump. Assume f = 0·01.

Solution. Given : $N = 60$ r.p.m.; $D = 250$ mm $= 0{\cdot}25$ m; $L = 500$ mm $= 0{\cdot}5$ m or crank radius $(r) = 0{\cdot}5/2 = 0{\cdot}25$ m; $d = 100$ mm $= 0{\cdot}1$ m; $l = 50$ m and $f = 0{\cdot}01$.

Power required to overcome friction of the delivery pipe, when no air vessel is fitted

We know that area of plunger,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}25)^2 = 0{\cdot}0491 \text{ m}^2$$

and area of delivery pipe,

$$a = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (0{\cdot}1)^2 = 0{\cdot}007\,85 \text{ m}^2$$

$\therefore$ Discharge of the pump,

$$Q = \frac{LAN}{60} = \frac{0{\cdot}5 \times 0{\cdot}0491 \times 60}{60} = 0{\cdot}025 \text{ m}^3/\text{s}$$

$\frac{L}{\pi}$ (ωr)

We also know that angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 6{\cdot}28 \text{ rad/s}$$

$\frac{4fl}{2gd}(v^2)$

and maximum velocity of water in the delivery pipe,

$$v = \frac{A}{a} \times \omega r = \frac{0{\cdot}0491}{0{\cdot}007\,85} \times 6{\cdot}28 \times 0{\cdot}25 = 9{\cdot}82 \text{ m/s}$$

$\therefore$ Loss of head due to friction,

$Q = \frac{LAN}{60}$ m³/sec

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0{\cdot}01 \times 50 \times (9{\cdot}82)^2}{2 \times 9{\cdot}81 \times 0{\cdot}1} = 98{\cdot}3 \text{ m}$$

and power required to overcome friction of the delivery pipe

$$P_1 = wQ\left(\frac{2}{3} \times H_f\right) = 9{\cdot}81 \times 0{\cdot}025 \times \left(\frac{2}{3} \times 98{\cdot}3\right) = 16{\cdot}1 \text{ kW} \quad \textbf{Ans.}$$

Power required to overcome friction of the delivery pipe, when a large air vessel is fitted at the centre line of the pump

We know that average velocity of water in the delivery pipe,

$$v = \frac{Q}{a} = \frac{0{\cdot}025}{0{\cdot}007\,85} = 3{\cdot}18 \text{ m/s}$$

$$V = \left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)$$

and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4 \times 0{\cdot}01 \times 50 \times (3{\cdot}18)^2}{2 \times 9{\cdot}81 \times 0{\cdot}1} = 10{\cdot}3 \text{ m}$$

$\therefore$ Power required to overcome friction of the delivery pipe,

$$P = wQH_f = 9{\cdot}81 \times 0{\cdot}025 \times 10{\cdot}3 = 2{\cdot}53 \text{ kW} \quad \textbf{Ans.}$$

36·17 Work Saved against Friction by Fitting Air Vessels

The work saved against friction by fitting air vessel, may be found out, first by finding out the work done against friction without air vessels, and then subtracting from it the work done against friction with air vessels.

Example 36·14. *Show from first principles, that the work saved against friction in the delivery pipe of a single acting reciprocating pump, by fitting air vessel, is 84·8 %.*

Solution. Given : Single acting reciprocating pump.

Let
A = Area of the cylinder,
L = Length of the stroke,
r = Radius of the crank,
l = Length of the delivery pipe,
d = Diameter of the delivery pipe,
ω = Angular velocity of the crank in rad/s, and
N = Speed of the crank in r.p.m.

We know that the velocity of water in the pipe, with air vessel,

$$v = \frac{A}{a} \times \frac{\omega r}{\pi}$$

and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

$\therefore$ Work done per stroke against friction

$$W_1 = \frac{4fl}{2gd}\left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2$$

We also know that the maximum velocity of water in the delivery pipe without air vessels,

$$v = \frac{A}{a} \times \omega r$$

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and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2$$

We also know that work done per stroke against friction,

$$W_2 = \frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2 \quad ...(ii)$$

$\therefore$ Saving in work done per stroke

$$= \frac{W_2 - W_1}{W_2}$$

$$= \frac{\frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2 - \frac{4fl}{2gd}\left(\frac{A}{a} \times \frac{\omega r}{\pi}\right)^2}{\frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2}$$

$$= \frac{\frac{2}{3} - \frac{1}{\pi^2}}{\frac{2}{3}} = 0{\cdot}848 = 84{\cdot}8\% \quad \textbf{Ans.}$$

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Example 36-15. *Find the saving, in work done, against friction in the delivery of a double acting reciprocating pump, by fitting air vessels.* ≈ 39.2%

Solution. Given : Double acting reciprocating pump.

Let A = Area of the cylinder,
L = Length of the crank,
r = Radius of the stroke,
l = Length of the delivery pipe,
d = Diameter of the delivery pipe,
a = Area of the delivery pipe,
ω = Angular velocity of the crank in rad/s, and
N = Speed of the crank in r.p.m.

We know that the velocity of water in the pipe, with air vessel,

$$v = \frac{2A}{a} \times \frac{\omega r}{\pi}$$

and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{2A}{a} \times \frac{\omega r}{\pi}\right)^2$$

$\therefore$ Work done per stroke against friction,

$$W_1 = \frac{4fl}{2gd}\left(\frac{2A}{a} \times \frac{\omega r}{\pi}\right)^2 \quad \text{...(i)}$$

We know that the maximum velocity of water in the delivery pipe, without air vessels,

$$v = \frac{A}{a} \times \omega r$$

and loss of head due to friction,

$$H_f = \frac{4flv^2}{2gd} = \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2$$

We also know that work done per stroke against friction,

$$W_2 = \frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2 \quad \text{...(ii)}$$

$\therefore$ Saving in work done per stroke

$$= \frac{W_2 - W_1}{W_2}$$

$$= \frac{\frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2 - \frac{4fl}{2gd}\left(\frac{2A}{a} \times \frac{\omega r}{\pi}\right)^2}{\frac{2}{3} \times \frac{4fl}{2gd}\left(\frac{A}{a} \times \omega r\right)^2}$$

$$= \frac{\frac{2}{3} - \frac{4}{\pi^2}}{\frac{2}{3}} = 0.392 = 39.2\% \text{ Ans.}$$

36-18 Flow of Water into and from the Air Vessel Fitted to the Delivery Pipe of a Single Acting Reciprocating Pump

We have seen in Art. 36-13 that the velocity of water, in the delivery pipe, beyond the air vessel is constant. But the velocity of water from the cylinder to the air vessel is subjected to acceleration and retardation. Now we shall discuss the rate of flow of water into and from the air vessel fitted to the delivery pipe of a single reciprocating pump.

Consider a single acting reciprocating pump, fitted with air vessels on both the suction and delivery pipes. Let us consider discharge from the cylinder to the delivery pipe. We know that the velocity of water in the delivery pipe from the cylinder upto the air vessel

$$= \frac{A}{a} \times \omega r \sin\theta$$

$\therefore$ Discharge from the cylinder,

$$= a \times \frac{A}{a} \times \omega r \sin\theta = A\omega r \sin\theta \quad \text{...(i)}$$

We also know that the velocity of water in a single acting reciprocating pump beyond the air vessel,

$$= \frac{A}{a} \times \frac{\omega r}{\pi}$$

$\therefore$ Discharge in the delivery pipe beyond the air vessel

$$= a \times \frac{A}{a} \times \frac{\omega r}{\pi} = \frac{A\omega r}{\pi} \quad \text{...(ii)}$$

A little consideration will show, that the difference of the above two discharges will be the discharge, into or form the air vessel.

$\therefore$ Discharge into the air vessel,

Q = Discharge from the cylinder – Discharge beyond air vessel

$$= A\omega r \sin\theta - \frac{A\omega r}{\pi}$$

$$= A\omega r\left(\sin\theta - \frac{1}{\pi}\right) \quad \text{...(iii)}$$

If the above equation works out to be +ve, it means that the discharge is taking place into the air vessel. But if this equation works out to be –ve, it means that the discharge is taking place from the air vessel.

Note : If we consider the flow, into or from the air vessel fitted to the suction pipe, then the above condition is reserved. *i.e.*, if the equation *(iii)* above works out to be +ve the discharge is taking place from the air vessel. But if this equation works out to be –ve, the discharge is taking place into the air vessel.

36-19 Flow of Water, into and from the Air Vessel Fitted to the Delivery Pipe of a Double Acting Reciprocating Pump

Consider a double acting reciprocating pump, fitted with air vessels on both the suction and delivery pipes. Let us consider the discharge from the cylinder to the delivery pipe.

We know that the velocity of water in the delivery pipe from the cylinder up to the air vessel

$$= \frac{A}{a}\,\omega r \sin\theta$$

$\therefore$ Discharge from the cylinder

$$= a \times \frac{A}{a} \times \omega r \sin\theta = A\omega r \sin\theta \quad \text{...(i)}$$

We know that the velocity of water in a double acting reciprocating pump beyond the air vessel

$$= \frac{2A}{a} \times \frac{\omega r}{\pi}$$

∴ Discharge in the delivery pipe beyond the air vessel

$$= a \times \frac{2A}{a} \times \frac{\omega r}{\pi} = \frac{2A\omega r}{\pi} \quad ...(ii)$$

A little consideration will show, that the difference of the above two discharges will be the discharge into or form the air vessel.

∴ Discharge into the air vessel,

Q = Discharge from the cylinder
– Discharge beyond the air vessel

$$= A\omega r \sin\theta - \frac{2A\omega r}{\pi}$$

$$= A\omega r\left(\sin\theta - \frac{2}{\pi}\right) \quad ...(iii)$$

If the above equation works out to be +ve, it means that the discharge is taking place into the air vessel. But if this equation works out to be –ve, it means that the discharge is taking place from the air vessel.

Note. If we consider the flow into or from the air vessel fitted to the suction pipe, then the above condition is reversed. *i.e.*, if the equation (*iii*) above works out to be +ve, the discharge is taking place from the air vessel. But if this equation work out to be –ve, the discharge is taking place into the air vessel.

Example 36·16. *A double acting reciprocating pump has a bore of 175 mm and a stroke of 350 mm. The suction pipe has a diameter of 150 mm and is fitted with an air vessel. Determine the crank angle, at which there is no flow of water to or from the vessel.*

Assume the crank speed as 150 r.p.m. and plunger has simple harmonic motion.

Solution. Given : D = 175 mm = 0·175 m; L = 350 mm = 0·35 m or crank radius (r) = 0·35/2 = 0·175 m and N = 150 r.p.m.

We know that area of bore,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}175)^2 = 0{\cdot}024 \text{ m}^2$$

and angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60} = 5\pi \text{ rad/s}$$

We also know that discharge from the sump upto the vessel

$$= \frac{2A\omega r}{\pi} = \frac{2 \times 0{\cdot}024 \times (5\pi) \times 0{\cdot}175}{\pi} = 0{\cdot}042 \text{ m}^3\text{/s} \quad ...(i)$$

and discharge beyond the air vessel

$$= A\omega r \sin\theta = 0{\cdot}024 \times (5\pi) \times 0{\cdot}175 \sin\theta$$

$$= 0{\cdot}066 \sin\theta \text{ m}^3\text{/s} \quad ...(ii)$$

*For no flow of water to or from the air vessel, the above two discharges should be equal. Therefore equating the above two discharges,

$$0{\cdot}666 \sin\theta = 0{\cdot}042$$

or $$\sin\theta = 0{\cdot}042/0{\cdot}066 = 0{\cdot}6364$$

∴ $$\theta = 39{\cdot}5^\circ \text{ or } 140{\cdot}5^\circ \quad \textbf{Ans.}$$

Example 36·17. *A 100 mm diameter suction pipe of a double acting reciprocating pump having 200 mm bore and 400 mm stroke is fitted with an air vessel. The pump runs at 120 r.p.m. and it may be assumed that the piston makes simple harmonic motion. Calculate the rate of flow into or from the air vessel in litres/s, when crank makes 30°, 90° and 120° with the inner dead centre.*

Solution. Given : d_s = 100 mm = 0·1 m; D = 200 mm = 0·2 m; L = 400 mm = 0·4 m or crank radius (r) = 0·4/2 = 0·2 m; and N = 120 r.p.m.

Rate of flow of water when the crank makes an angle of 30°

We know that area of bore,

$$A = \frac{\pi}{4} \times (D)^2 = \frac{\pi}{4} \times (0{\cdot}2)^2 = 0{\cdot}0314 \text{ m}^2$$

and angular velocity of the crank,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 4\pi \text{ rad/s}$$

∴ Rate of flow of water, when θ = 30°,

$$Q = A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$

$$= 0{\cdot}0314 \times 4\pi \times 0{\cdot}2\left(\sin 30^\circ - \frac{2}{\pi}\right) \text{ m}^3\text{/s}$$

$$= 0{\cdot}0789\ (0{\cdot}5 - 0{\cdot}637) \text{ m}^3\text{/s}$$

$$= -0{\cdot}0108 \text{ m}^3\text{/s} = -10{\cdot}8 \text{ litres/s} \quad \textbf{Ans.}$$

Since the result work out to be –ve and we are considering flow in the suction pipe, therefore the discharge is taking place into the air vessel. **Ans.**

Flow of water when the crank makes an angle of 90°

We know that the rate of flow of water, when θ = 90°

$$Q = A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$

$$= 0{\cdot}0314 \times 4\pi \times 0{\cdot}2\left(\sin 90^\circ - \frac{2}{\pi}\right) \text{ m}^3\text{/s}$$

*This may also be found from the relation of discharge to or from the air vessel. *i.e.*,

$$Q = A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$

For no flow of water to or from the air vessel, let us equate this equation to zero.

i.e., $$A\omega r\left(\sin\theta - \frac{2}{\pi}\right) = 0$$

$$\sin\theta - \frac{2}{\pi} = 0 \quad \text{...(Dividing both sides by } A\omega r)$$

$$\sin\theta = \frac{2}{\pi} = 0{\cdot}6364$$

∴ $$\theta = 39{\cdot}5^\circ \text{ or } 140{\cdot}5^\circ \quad \textbf{Ans.}$$

$$Q = 0{\cdot}0789\ (1{\cdot}0 - 0{\cdot}637)\ m^3/s$$
$$= +\ 0{\cdot}0286\ m^3/s = +\ 28{\cdot}6\ \text{litres/s}$$

Since the result works out to be +ve and we are considering flow in the suction pipe, therefore the discharge is taking place from the air vessel. **Ans.**

Rate of flow of water when the crank makes an angle of 120°

We also know that the rate of flow of water, when $\theta = 120°$,

$$Q = A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$
$$= 0{\cdot}0314 \times 4\pi \times 0{\cdot}2\left(\sin 120° - \frac{2}{\pi}\right) m^3/s$$
$$= 0{\cdot}0789\ (0{\cdot}866 - 0{\cdot}637)\ m^3/s$$
$$= +\ 0{\cdot}0181\ m^3/s = +\ 18{\cdot}1\ \text{litres/s}$$

Since the result works out to be +ve and we are considering flow in the suction pipe, therefore the discharge is taking place from the air vessel. **Ans.**

EXERCISE 36-3

1. A double acting reciprocating pump, having piston diameter 150 mm and stroke length 450 mm, draws water from a sump 3·6 metres below it through a 100 mm diameter and 6 metres long pipe. A large air vessel is fitted on the suction pipe close to the pump. Find the maximum speed of the pump, in order to avoid separation, which takes place at a pressure head of 2·5 metres, if the barometer reads 10·3 metres. Take $4f = 0{\cdot}025$. (Ans. 107·6 r.p.m.)
2. A single acting reciprocating pump has a plunger of 375 mm diameter and stroke of 600 mm. The delivery pipe is 90 metres long and 150 mm diameter. Find the power saved by installing an air vessel near the pump, if the pump runs at 50 r.p.m. Take $f = 0{\cdot}008$. (Ans. 31·5 kW)
3. A double acting reciprocating pump has an air vessel fitted on the suction pipe. The plunger is 150 mm diameter and 300 mm long. The suction pipe is 8 metres long and 100 mm diameter. Determine the rate of flow into or from the air vessel at crank positions of 30°, 90° and 120° from the inner dead centre. Take speed of the pump as 120 r.p.m. [Ans. 4·33 litres/s (into the air vessel); 12·54 litres/s (from air vessel); 8·04 litres/s (from air vessel)]

QUESTIONS

1. Why is a reciprocating pump called a positive displacement pump ?
2. Distinguish between coefficient of discharge and slip of a reciprocating pump.
3. Explain the working principle of reciprocating pump with sketches.
4. Show that the maximum acceleration head, in a reciprocating pump, without air vessel is given by the relation :

$$H_a = \frac{l}{g} \times \frac{A}{a} \times \omega^2 r$$

where l = Length of pipe,

$\frac{A}{a}$ = Ratio of cylinder area to pipe area, and

ω = Angular velocity of crank shaft.

5. Define 'separation' in a reciprocating pump, and explain how it can be avoided ?
6. Explain the function of air vessel in a reciprocating pump.

OBJECTIVE TYPE QUESTIONS

1. A reciprocating pump is suitable for
   (*a*) less discharge (*b*) more discharge
   (*c*) higher heads (*d*) both '*a*' and '*c*'
2. The slip of a reciprocating pump is
   (*a*) ratio of actual discharge to theoretical discharge
   (*b*) product of actual discharge and theoretical discharge
   (*c*) difference of theoretical discharge and actual discharge
   (*d*) sum of theoretical discharge and actual discharge
3. Air vessels are used in a reciprocating pump to
   (*a*) smoothen the flow (*b*) increase the flow
   (*c*) reduce acceleration head (*d*) all of these
4. By fitting air vessel in the reciprocating pump, there is always some saving in power. This saving in a single acting reciprocating pump is
   (*a*) 39·2% (*b*) 48·8% (*c*) 84·8% (*d*) 88·4%

ANSWER

1. (*d*) 2. (*c*) 3. (*c*) 4. (*c*)