



GEOTECHNICAL

ENGINEERING

HandNote On

GEOTECHNICAL ENGINEERING



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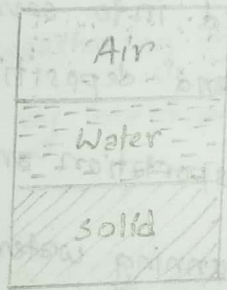
Introduction

☐ Soil: Soil is defined as uncemented aggregate of mineral grains and decayed organic matter with liquid and gas in the empty spaces between the solid particles.

Soil is used as a construction material in various civil engineering projects and it supports structural foundations.

☐ Types of soil according to phase:

Soil is a three phase system consisting of solid particles, water and air.



1. Unsaturated Soil = Solid + water + Air

2. Saturated soil = Solid + water (liquid)

3. Dry soil = solid + Air (Gas)

☐ Soil Mechanics: Soil Mechanics is a branch of science that deals with the study of physical properties of soil and behaviour of soil masses subjected to various types of loads.

☐ Soil Engineering: Soil Engineering is the application of the principles of soil mechanics to practical problems.

☐ Geotechnical Engineering: Geotechnical Engineering is a part of civil engineering that involves with natural materials found close to the surface of the earth. It includes the application of the principles of soil and rock mechanics to the design of foundation retaining earth structures.

☐ Residual Soil: The soil formed by weathered products, ^{deposits} at their place of origin are called residual soil.

An important characteristics of residual soil is the gradation of particle size. Grain size increases with depth. Fine grained soil is found at the surface. At greater depth, angular rock fragments are found.

☐ Transported soil: The soil formed by weathered products, deposits, away from the site of origin and which are transported by natural forces to a new site.

Transported soil may be classified into several groups depending on their mode of transportation and deposition:

1. Glacial soil: formed by transportation and deposition of glaciers.
2. Alluvial soil: transported by running water and deposited along stream.
3. Lacustrine soil: formed by deposition in the ~~sea~~ quiet lakes.
4. Marine soil: formed by deposition in the seas.
5. Aeolian soil: transported and deposited by wind.
6. colluvial soil: formed by movement of soil from its original place by gravity such as during landslides.

Soil Formation

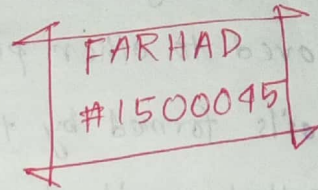
Soil Mechanics:

Soil mechanics is the branch of science that deals with the study of the physical property of soil and the behavior of soil masses subjected to various types of forces.

Pre classical period: (1700-1776)

Classical period: (1776-1856)

Modern period: (1910-1927)



Soil Formation:

Soils are formed by weathering of rocks. Weathering is the process of breaking down rocks by mechanical and chemical processes into smaller pieces.

Mechanical weathering: It is a physical process of weathering. It may be caused by -

1. unloading
2. Thermal expansion and contraction
3. Alternate wetting and drying
4. Crystal growth, including frost action
5. Organic activity, etc.

It is important to realize that, in mechanical weathering, large rocks are broken down into smaller pieces without any change in chemical composition.

Chemical weathering: In this process, the original rocks minerals are transformed into new minerals by chemical reaction. It may be caused by -

1. Hydrolysis
2. chelation
3. cation change

4. Oxidation and reduction

v. Carbonation.

Transportation of weathering product:

The products of weathering may stay in the same place or may be moved to other places by ice, water, wind and gravity.

The soils formed by the weathered products at their place of origin are called residual soils. An important characteristic of residual soil is the gradation of particle size.

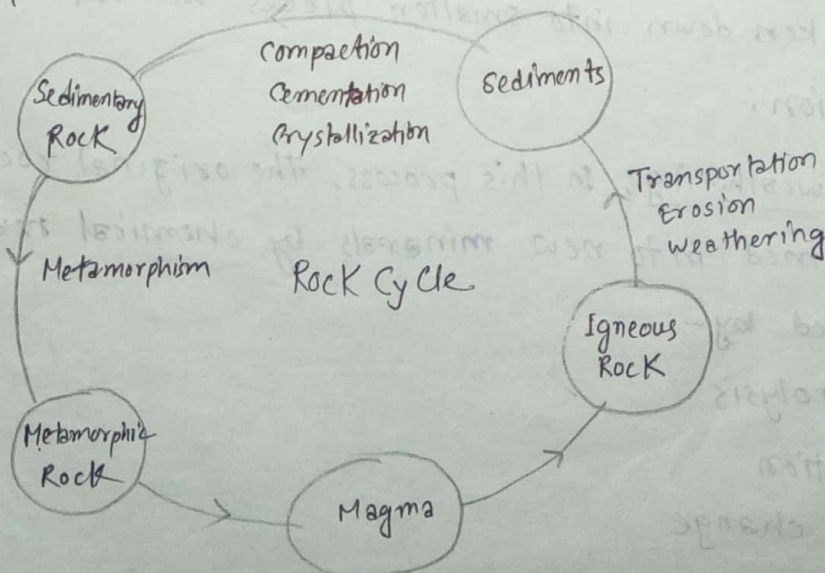
Transported soils may be classified into several groups:

1. Glacial soils.
2. Alluvial soils.
3. Lacustrine soils.
4. Marine soils.
5. Aeolian soils.
6. Colluvial soils.

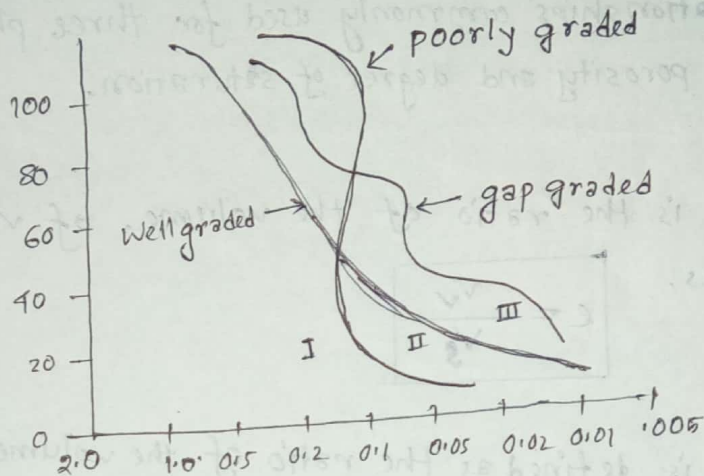
The deposits of gravel, sand, silt and clay formed by weathering may become compacted by overburden pressure and cement by different agents. Rocks formed in this way are called sedimentary rocks.

Metamorphism is the process of changing the composition and texture of rocks by heat and pressure.

This periodic sequence is called Rock cycle.



Particle size distribution curve:



* curve-I represents a type of soil in which most of soil grains are the same size. This is called poorly graded soil.

* curve-II represents a soil in which the particle sizes are distributed over a wide range, termed well graded.

* A soil might have a combination of two or more uniformly graded fractions, Curve III represents such a soil. This type of soil is termed gap graded.

Weight-Volume Relationship

Volume and weight relationship:

Total volume of a soil sample, $V = V_s + V_v = V_s + V_w + V_a$

where,

V_s = volume of soil solids

V_v = volume of voids

V_w = volume of water in the voids

V_a = volume of air in the voids.

Total weight of a soil sample, $w = w_s + w_w$; where, w_s = weight of soil solids
 w_w = weight of water

$$\frac{\gamma}{\omega + 1} = \rho$$

$$\frac{w_s}{V} = \rho_s$$

volume relationships:

The volume relationships commonly used for three phases in a soil element are void ratio, porosity and degree of saturation.

Void ratio (e):

Void ratio (e) is the ratio of the volume of voids to the volume of solids. Thus,

$$e = \frac{V_v}{V_s}$$

porosity (n):

porosity (n) is defined as the ratio of the volume of voids to the total volume.

$$n = \frac{V_v}{V_t}$$

degree of saturation (s):

The degree of saturation (s) is defined as the ratio of the volume of water to the volume of voids.

$$s = \frac{V_w}{V_v}$$

Density and unit weight (γ):

* Density is the mass of soil per unit volume. Thus, $\rho = \frac{M}{V}$

* unit weight is the weight of soil per unit volume. Thus,

$$\gamma = \frac{W}{V} = \frac{Mg}{V} = \rho g$$

* specific gravity is defined as the ratio of the unit weight of a given material to the unit weight of water.

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s}{\rho_w}$$

* Dry unit weight (γ_d) is defined as weight per unit volume of soil, excluding water.

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s (1 + \frac{W_w}{W_s})}{V} = \frac{W_s (1 + w)}{V}; \text{ where } w = \text{water content}$$

$$\text{but, } \gamma_d = \frac{W_s}{V} \quad \text{Now, } \gamma = \gamma_d (1 + w) \Rightarrow \gamma_d = \frac{\gamma}{1 + w}$$

Weight relationship:

Water content: It is defined as the ratio of the weight of water to the weight of solids in a given volume of soil. It is also referred as moisture content.

$$w = \frac{W_w}{W_s} = \frac{M_w}{M_s}$$

Density of soil:

Dry density: $\rho_d = \frac{M_s}{V}$

Moist density: $\rho = \frac{M}{V}$

Saturated density: $\rho_{sat} = \frac{M_s + M_w}{V}$ [$V_a = 0$ $S = 100\%$]

Submerged density: $\rho_{sub} = \rho_{sat} - \rho_w$

Submerged unit weight: $\gamma' = \gamma_{sat} - \gamma_w$

Density of water: $\rho_w = 1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$

Unit weight of water: $\gamma_w = 9.81 \text{ kN/m}^3 = 9810 \text{ N/m}^3 = 62.4 \text{ lb/ft}^3$

Relation among unit weight, void ratio, moisture content and specific gravity:

gravity:

Let, $V_s = 1$ $\therefore e = \frac{V_v}{V_s} = \frac{V_v}{1} \Rightarrow e = V_v$

$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\frac{W_s}{V_s}}{\gamma_w} = \frac{W_s}{\gamma_w} \Rightarrow W_s = \gamma_w G_s$

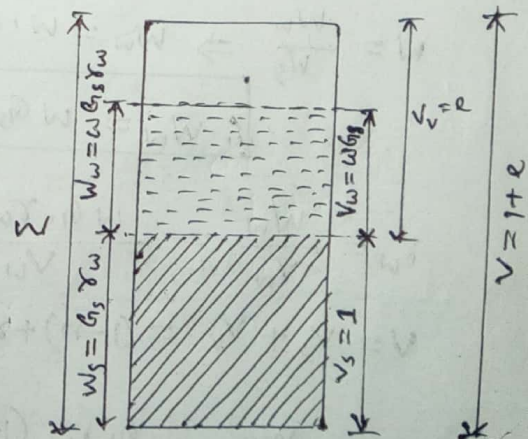
$w = \frac{W_w}{W_s} = \frac{W_w}{\gamma_w G_s} \Rightarrow W_w = w G_s \gamma_w$

$\gamma_w = \frac{W_w}{V_w} = \frac{w G_s \gamma_w}{V_w} \Rightarrow V_w = w G_s$

$V = V_s + V_v \Rightarrow V = 1 + e$

$S = \frac{V_w}{V_v} = \frac{w G_s}{e} \Rightarrow Se = w G_s$

$\gamma = \frac{W}{V} = \frac{W_s + W_w}{1 + e} = \frac{\gamma_w G_s + w G_s \gamma_w}{1 + e} \Rightarrow \gamma = \frac{(1 + w) G_s \gamma_w}{1 + e}$



$$\gamma_d = \frac{W_s}{V} \Rightarrow \boxed{\gamma_d = \frac{G_s \gamma_w}{1+e}}$$

For saturated soil, ($s=100\%$)

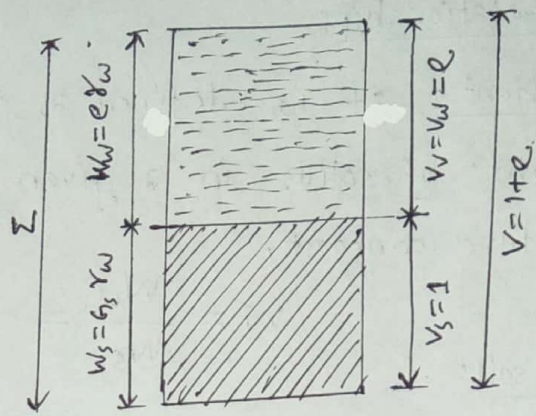
$$\boxed{s=1}$$

Hence, $se = w G_s$

$$\therefore e = w G_s$$

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{1+e}$$

$$\therefore \gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e}$$



Relationship among unit weight, porosity and moisture content:

$$\text{Let, } \boxed{v=1}, \quad n = \frac{V_v}{V} \quad \therefore n = V_v$$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\frac{W_s}{V_s}}{\gamma_w} = \frac{W_s}{V_s \gamma_w} \Rightarrow W_s = V_s \gamma_w G_s$$

$$\therefore W_s = G_s \gamma_w (1-n)$$

$$w = \frac{W_w}{W_s} \Rightarrow W_w = w \cdot W_s$$

$$\therefore W_w = w G_s \gamma_w (1-n)$$

$$\gamma_w = \frac{W_w}{V_w} = \frac{w G_s \gamma_w (1-n)}{V_w}$$

$$\therefore V_w = w G_s (1-n)$$

$$V = V_s + V_v = (1-n) + V_v \quad \therefore V_v = n$$

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w (1-n)}{1}$$

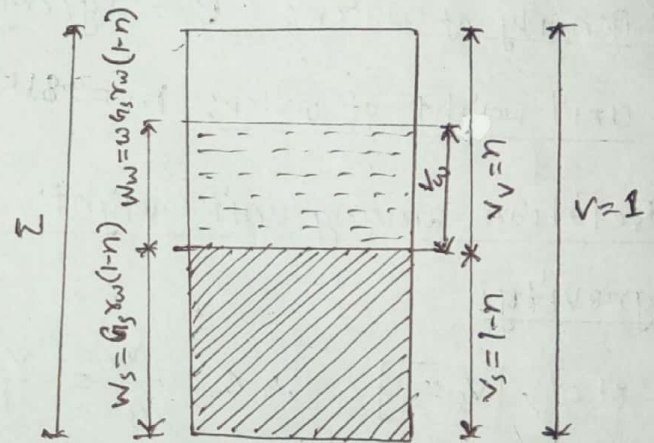
$$\therefore \gamma_d = G_s \gamma_w (1-n)$$

$$\gamma = \frac{W_s + W_w}{V} = G_s \gamma_w (1-n) + w G_s \gamma_w (1-n)$$

$$\therefore \gamma = G_s \gamma_w (1-n) (1+w)$$

$$s = \frac{V_w}{V_v} = \frac{w G_s (1-n)}{n}$$

$$\therefore sn = w G_s (1-n)$$



For saturated soil, ($s=100\%$)

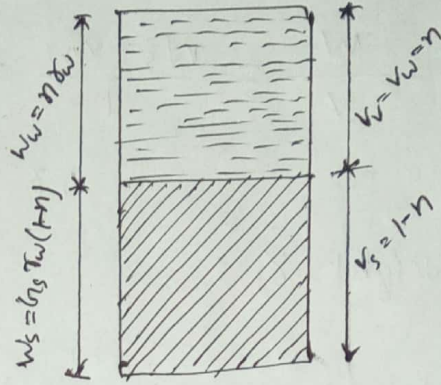
$$\therefore S=1$$

$$\gamma_{sat} = \frac{W_s + W_w}{V} = G_s \gamma_w (1-n) + n \gamma_w$$

$$\therefore \gamma_{sat} = [G_s (1-n) + n] \gamma_w$$

$$w_{sat} = \frac{W_w}{W_s} = \frac{n \gamma_w}{(1-n) \gamma_w G_s}$$

$$\therefore w_{sat} = \frac{n}{(1-n) G_s}$$



Various unit weight relationships: (Moist Unit Weight, γ):

$$(i) \gamma = \frac{(1+w) G_s \gamma_w}{1+e} = \frac{(1+w) G_s \gamma_w}{1 + \left(\frac{w G_s}{S}\right)}$$

proof: $\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + w G_s \gamma_w}{1+e} = \frac{(1+w) G_s \gamma_w}{1+e}$

And, $\gamma = \frac{(1+w) G_s \gamma_w}{1 + \left(\frac{w G_s}{S}\right)}$ [$\because Se = w G_s$]

$$(ii) \gamma = \frac{(G_s + se) \gamma_w}{1+e}$$

proof: $\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + w G_s \gamma_w}{1+e} = \frac{G_s \gamma_w + se \gamma_w}{1+e}$

$$\therefore \gamma = \frac{(G_s + se) \gamma_w}{1+e}$$

$$(iii) \gamma = G_s \gamma_w (1-n) (1+w)$$

proof: $\gamma = \frac{W}{V} = \frac{W_s + W_w}{1} = G_s \gamma_w (1-n) + w G_s \gamma_w (1-n)$

$$\therefore \gamma = G_s \gamma_w (1-n) (1+w)$$

$$(iv) \gamma = G_s \gamma_w (1-n) + n s \gamma_w$$

proof: $\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = G_s \gamma_w (1-n) + W G_s \gamma_w (1-n)$
 $\therefore \gamma = G_s \gamma_w (1-n) + s n \gamma_w$ [$\because s n = W G_s (1-n)$]

Dry unit weight, γ_d :

$$(i) \gamma_d = \frac{\gamma}{1+w}$$

proof: $\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s (1 + \frac{W_w}{W_s})}{V} = \gamma_d (1+w) \Rightarrow \gamma_d = \frac{\gamma}{1+w}$

$$(ii) \gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{G_s \gamma_w}{1 + (\frac{W_w}{W_s})}$$

proof: $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1+e}$ And, $\gamma_d = \frac{G_s \gamma_w}{1 + (\frac{W_w}{W_s})}$ [$\because s e = W G_s$]

$$(iii) \gamma_d = G_s \gamma_w (1-n)$$

proof: $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w (1-n)}{1} = G_s \gamma_w (1-n)$

$$(iv) \gamma_d = \frac{e s \gamma_w}{(1+e) W}$$

proof: $\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1+e} = \frac{\frac{s e}{W} \gamma_w}{1+e} = \frac{e s \gamma_w}{(1+e) W}$

$$(v) \gamma_d = \gamma_{sat} - n \gamma_w$$

proof: $\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s}{V} + \frac{W_w}{V} = \gamma_d + \frac{n \gamma_w}{1}$
 $\therefore \gamma_d = \gamma_{sat} - n \gamma_w$

$$(vi) \gamma_d = \gamma_{sat} - \frac{e \gamma_w}{1+e}$$

proof: $\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s}{V} + \frac{W_w}{V} = \gamma_d + \frac{e \gamma_w}{1+e}$
 $\therefore \gamma_d = \gamma_{sat} - \frac{e \gamma_w}{1+e}$

$$(vii) \gamma_s = \frac{(\gamma_{sat} - \gamma_w) G_s}{G_s - 1}$$

proof: $\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{W_s}{V} + \frac{W_w}{V} = \gamma_s + \gamma_w \dots \dots \dots \textcircled{1}$

$$\gamma_s = \frac{W_s}{V} = G_s \gamma_w (1-u) \Rightarrow 1-u = \frac{\gamma_s}{G_s \gamma_w} \therefore u = 1 - \frac{\gamma_s}{G_s \gamma_w}$$

Now, From eqⁿ ①, $\gamma_{sat} = \gamma_s + (1 - \frac{\gamma_s}{G_s \gamma_w}) \gamma_w$

$$\Rightarrow \gamma_{sat} = \gamma_s + \gamma_w - \frac{\gamma_s \gamma_w}{G_s}$$

$$\Rightarrow \gamma_{sat} - \gamma_w = \gamma_s (1 - \frac{1}{G_s})$$

$$\Rightarrow \gamma_s = \frac{(\gamma_{sat} - \gamma_w) G_s}{(G_s - 1)}$$

saturated unit weight:

$$(i) \gamma_{sat} = \frac{(G_s + e) \gamma_w}{1 + e}$$

proof: $\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{(1+e)} = \frac{(G_s + e) \gamma_w}{1+e}$

$$(ii) \gamma_{sat} = [(1-n) G_s + n] \gamma_w$$

proof: $\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w (1-n) + n \gamma_w}{1} = [(1-n) G_s + n] \gamma_w$

$$(iii) \gamma_{sat} = \left(\frac{1 + W_{sat}}{1 + W_{sat} G_s} \right) G_s \gamma_w$$

proof: $\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{W_s + W_{sat} \cdot W_s}{1 + e} = \frac{(W_{sat} + 1) W_s}{1 + e} = \frac{(W_{sat} + 1) G_s \gamma_w}{(1 + W_{sat} G_s)}$
 $[\because e = \frac{W_{sat} G_s}{1 + W_{sat} G_s}]$

$$(iv) \gamma_{sat} = n \left(\frac{1 + W_{sat}}{W_{sat}} \right) \gamma_w$$

proof: $\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{1 + e} = \frac{\frac{e \gamma_w}{W_{sat}} + e \gamma_w}{1 + e} = \frac{e \gamma_w (1 + \frac{1}{W_{sat}})}{1 + e}$
 $\therefore \gamma_{sat} = n \left(\frac{1 + W_{sat}}{W_{sat}} \right) \gamma_w \quad [\because n = \frac{e}{1 + e}]$

$$(v) \gamma_{sat} = \left(\frac{e}{w_{sat}} \right) \left(\frac{1 + w_{sat}}{1 + e} \right) \gamma_w$$

proof:
$$\gamma_{sat} = \frac{w_w + w_s}{V} = \frac{e \gamma_w + G_s \gamma_w}{1 + e} = \frac{e \gamma_w + \frac{e}{w_{sat}} \cdot \gamma_w}{1 + e}$$

$$= \frac{e \gamma_w w_{sat} + e \gamma_w}{w_{sat}(1 + e)}$$

$$\therefore \gamma_{sat} = \left(\frac{e}{w_{sat}} \right) \times \gamma_w \times \frac{(1 + w_{sat})}{(1 + e)}$$

$$(vi) \gamma_{sat} = \gamma_R + \left(\frac{e}{1 + e} \right) \gamma_w = m_R u + R \gamma_w$$

proof:
$$\gamma_{sat} = \frac{w_w + w_s}{V} = \frac{w_s}{V} + \frac{w_w}{V} = R \gamma + \frac{m_R e}{1 + e} \gamma_w$$

And,
$$\gamma_{sat} = \gamma_R + m_R u \gamma_w \quad [\because u = \frac{e}{1 + e}]$$

$$(vii) \gamma_{sat} = \left(1 - \frac{1}{G_s} \right) \gamma_R + \gamma_w$$

proof:
$$\gamma_{sat} = \frac{w_w + w_s}{V} = \frac{w_s}{V} + \frac{w_w}{V} = R \gamma + m_R u \gamma_w \quad \text{--- (1)}$$

$$\gamma_R = \frac{w_s}{V} = G_s \gamma_w (1 - u) \Leftrightarrow (1 - u) = \frac{R \gamma}{m_R \gamma_w} \Leftrightarrow 1 - u = 1 - \frac{R \gamma}{G_s m_R \gamma_w}$$

Now from eqⁿ (1),
$$\gamma_{sat} = R \gamma + m_R \gamma_w \left(1 - \frac{R \gamma}{G_s m_R \gamma_w} \right) = R \gamma + \gamma_w - \frac{R \gamma}{G_s}$$

$$\therefore \gamma_{sat} = \gamma_w + \left(1 - \frac{1}{G_s} \right) R \gamma$$

$$(viii) \gamma_{sat} = \gamma_R (1 + w_{sat})$$

proof:
$$\gamma_{sat} = \frac{w_s + w_w}{V} = \frac{w_s + w_s w_{sat}}{V} = \frac{w_s}{V} (1 + w_{sat})$$

$$\therefore \gamma_{sat} = \gamma_R (1 + w_{sat})$$

Relative density: The term Relative density is commonly used to indicate the in-situ denseness or looseness of granular soil. It is defined as,

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

The relationship for relative density can also be defined in terms of porosity:

$$D_r = \frac{(1 - n_{\min})(n_{\max} - n)}{(n_{\max} - n_{\min})(1 - n)}$$

where,

$$e_{\max} = \frac{n_{\max}}{1 - n_{\max}}$$

$$e_{\min} = \frac{n_{\min}}{1 - n_{\min}}$$

By using the definition of unit weight,

$$D_r = \frac{\left[\frac{1}{\gamma_d(\min)} \right] - \left[\frac{1}{\gamma_d} \right]}{\left[\frac{1}{\gamma_d(\min)} \right] - \left[\frac{1}{\gamma_d(\max)} \right]}$$

and, $e = \frac{n}{1 - n}$

$$\therefore D_r = \left[\frac{\gamma_d - \gamma_d(\min)}{\gamma_d(\max) - \gamma_d(\min)} \right] \times \left[\frac{\gamma_d(\max)}{\gamma_d} \right]$$

Weight-Volume Relationship

2006 Problem: 3.5 (B.M. Das - 8th edition)

The moist unit of a soil is 16.5 kN/m^3 . Given that $w = 15\%$ and $G_s = 2.7$ determine (i) Dry unit weight (ii) porosity and (iii) degree of saturation.

Solutions: (i) We know that,
$$\gamma_d = \frac{\gamma}{1+w} = \frac{16.5}{1+0.15} = 14.345 \text{ kN/m}^3 \quad (\text{Ans.})$$

(ii) We know that,
$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

$$\Rightarrow e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.7 \times 9.81}{14.345} - 1 = 0.8464$$

Now, Porosity,
$$n = \frac{e}{1+e} = \frac{0.8464}{1+0.8464} = 0.4584 \quad (\text{Ans.})$$

(iii) We know that,
$$se = w G_s$$

$$\Rightarrow s = \frac{w G_s}{e} = \frac{0.15 \times 2.7}{0.8464} = 0.4785$$

\therefore Degree of saturation, $s = 47.85\% \quad (\text{Ans.})$

2007, 01 Problem: 3.12

For a moist soil, the following: $V = 1.25 \text{ m}^3$, $W = 20 \text{ kN}$, $w = 9.5\%$ and $G_s = 2.72$

Determine (i) Bulk density (ii) void ratio (iii) Degree of saturation (iv) volume occupied by water

Solutions: (i)
$$\gamma = \frac{W}{V} = \frac{20}{1.25} = 16 \text{ kN/m}^3 \quad (\text{Ans.})$$

(ii)
$$\gamma_d = \frac{\gamma}{1+w} = \frac{16}{1+0.095} = 14.612 \text{ kN/m}^3$$

Again,
$$\gamma_d = \frac{G_s \gamma_w}{1+e} \Rightarrow e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.72 \times 9.81}{14.612} - 1 = 0.826 \quad (\text{Ans.})$$

(iii) we know, $se = wG_s$

$$\Rightarrow s = \frac{wG_s}{e} = \frac{0.095 \times 2.72}{0.826} = 0.313$$

\therefore Degree of saturation, $S = 31.3\%$

(iv) we know, volume occupied by water,

$$V_w = \frac{(\gamma - \gamma_d) V}{\gamma_w}$$

$$= \frac{(16 - 14.612) \times 1.25}{9.81}$$

$$V_w = 0.177 \text{ m}^3$$

(Ans)

$$V_w = wG_s V_s = wG_s (V - V_w)$$

$$\Rightarrow V_w = (0.095 \times 2.72) \times (1.25 - V_w)$$

$$\Rightarrow V_w + 1.2584 V_w = 0.323$$

$$\therefore V_w = 0.2567 \text{ m}^3$$

(Ans)

2008

The porosity of soil is 0.35. Given that $G_s = 2.65$. Calculate

(i) saturated unit weight (ii) Moisture content when the moist unit weight is 18 kN/m^3 .

Solution: (i) we know that, $\gamma_{\text{sat}} = \frac{(G_s + e) \gamma_w}{1 + e}$

$$\text{Here, } e = \frac{n}{1 - n} = \frac{0.35}{1 - 0.35} = 0.54$$

$$\therefore \gamma_{\text{sat}} = \frac{(2.65 + 0.54) \times 9.81}{1 + 0.54} = 20.33 \text{ kN/m}^3$$

(Ans)

(ii) We know that,

$$\gamma_{\text{moist}} = \frac{(G_s + se) \gamma_w}{1 + e}$$

$$\text{Or } \gamma_{\text{moist}} = \frac{G_s \gamma_w (1 + w)}{1 + e}$$

$$\Rightarrow 18 = \frac{(2.65 + s \times 0.54) \times 9.81}{1 + 0.54}$$

$$\Rightarrow s = 0.325$$

$$\text{Again, } se = wG_s \Rightarrow w = \frac{se}{G_s} = \frac{0.325 \times 0.54}{2.65} = 0.066$$

$= 6.6\%$ (Ans)

2007

An imaginary soil is contained in a container measuring 10 cm x 10 cm x 10 cm. The soil consists of spherical grains of size 10 cm in diameter. Determine the maximum possible void ratio, porosity and percent solids.

Solutions:

Total volume = $(10 \times 10 \times 10) = 1000 \text{ cm}^3$

volume of soil grains = $\frac{4}{3} \pi r^3 = \frac{4}{3} \times 3.1416 \times \left(\frac{10}{2}\right)^3 = 523.6 \text{ cm}^3$

\therefore Volume of void = $(1000 - 523.6) = 476.4$

void ratio, $e = \frac{V_v}{V_s} = \frac{476.4}{523.6} = 0.91$

porosity, $n = \frac{e}{1+e} = \frac{0.91}{1+0.91} = 0.4764$

% solid = $\frac{V_s}{V} \times 100 = \frac{523.6}{1000} \times 100 = 52.36\%$
(Ans)

2008, 2013

The unit weight of a soil is 95 lb/ft³. The moisture content of the soil is 15% and the degree of saturation is 60%. Determine

- (i) saturated unit weight (ii) void ratio and (iii) specific gravity of soil solids.

Solution:

we know that, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e}$

Given that, $\gamma = 95 \text{ lb/ft}^3$, $w = 15\%$, $S = 60\%$

$\gamma_d = \frac{\gamma}{1+w} = \frac{95}{1+0.15} = 82.62 \text{ lb/ft}^3$

Now, we know that, $S_e = W G_s$

$$\Rightarrow 0.60 \times e = 0.15 \times G_s$$

$$\therefore e = \frac{G_s}{4} \dots \textcircled{1}$$

Again, $\gamma_d = \frac{G_s \gamma_w}{1+e}$

$$\Rightarrow 82.61 = \frac{G_s \times 62.4}{1+e}$$

$$\Rightarrow G_s = \frac{82.61(1+e)}{62.4} = 1.324 + 1.324 e$$

$$\Rightarrow G_s = 1.324 + 1.324 \times \frac{G_s}{4} \quad \left[\text{From eq } \textcircled{1} \right]$$

$$\Rightarrow G_s - 0.331 G_s = 1.324$$

$$\therefore G_s = 1.98$$

Now, From eq $\textcircled{1} \Rightarrow e = \frac{1.98}{4} = 0.495$

and, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(1.98 + 0.495) \times 62.4}{1 + 0.495} = 103.30 \text{ lb/ft}^3$

2009 Ex-3.4

A 0.35 ft^3 moist soil weighs 35.6 lb . when dried in an oven, the soil weighs 28.0 lb . Assume $G_s = 2.67$. Determine (i) moist unit weight (ii) porosity (iii) moisture content (iv) Degree of saturation (v) Dry density. (Ans.)

Solutions: (i) $\gamma = \frac{W}{V} = \frac{35.6}{0.35} = 101.714 \text{ lb/ft}^3$

(v) $\gamma_d = \frac{W_d}{V} = \frac{28}{0.35} = 80 \text{ lb/ft}^3 \therefore e = \frac{80}{32.2} \text{ lb/ft}^3 = 2.48 \text{ lb/ft}^3$

(iii) we know, $\gamma_d = \frac{\gamma}{1+w} \Rightarrow 80 = \frac{101.714}{1+w} \Rightarrow w = 0.2714$
 \therefore moisture content, $w = 27.14\%$

(ii) we know that, $\gamma_d = \frac{G_s \gamma_w}{1+e}$
 $\Rightarrow 80 = \frac{2.67 \times 62.4}{1+e} \Rightarrow e = 1.0826$

(iv) we know, $se = w G_s \Rightarrow s = \frac{w G_s}{e}$
 $\Rightarrow s = \frac{0.2714 \times 2.67}{1.0826} = 0.66935$

\therefore Degree of saturation, $s = 66.935\%$

(Ans)

2010 **Problem: 3.15**

For a saturated soil, $w = 20\%$ and $G_s = 2.70$
 Determine (i) saturated unit weight (ii) dry unit weight (iii) Moist unit weight, when the degree of saturation becomes 75%.

Solutions: (i) we know that, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e}$ Here, given that $w = 20\%$
 $G_s = 2.7$
 and, $e = \frac{w G_s}{s} = \frac{0.20 \times 2.7}{0.75} = 0.72$

Now, $\gamma_{sat} = \frac{(2.7 + 0.72) \times 9.81}{1 + 0.72} = 19.506 \text{ KN/m}^3$

(iii) we know that, $\gamma_{moist} = \frac{(G_s + se) \gamma_w}{1+e} = \frac{(2.7 + 0.75 \times 0.72) \times 9.81}{1 + 0.72}$
 $\therefore \gamma_{moist} = 18.48 \text{ KN/m}^3$

(ii) We know that, $\gamma_d = \frac{\gamma}{1+w} = \frac{18.48}{1+0.2} = 15.4 \text{ KN/m}^3$

2011 **Problem: 3.17** 3.18

For a given sandy soil, $e_{max} = 0.70$, $e_{min} = 0.45$ and $G_s = 2.7$
 What are the moist and dry unit weight of compaction (KN/m³)
 in the field if relative density is 80% and $w = 10\%$.

Solution:

We know that,

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$\Rightarrow 0.8 = \frac{0.70 - e}{0.70 - 0.45}$$

$$\Rightarrow e = 0.50$$

Now, $\gamma_{moist} = \frac{G_s \gamma_w (1+w)}{1+e} = \frac{2.7 \times 9.81 \times (1+0.1)}{1+0.50} = 19.4238 \text{ KN/m}^3$

and, $\gamma_d = \frac{\gamma_{moist}}{1+w} = \frac{19.4238}{1+0.1} = 17.658 \text{ KN/m}^3$

2014, 2016

calculate the dry unit weight, the saturated unit weight and the effective (buoyant) unit weight of a soil having a void ratio of 0.7, and a specific gravity of 2.7. Also calculate the unit weight and the water content at a degree of saturation 70%.

Solution: (i) We know that, $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.7 \times 9.81}{1+0.7} = 15.58$

(ii) $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(2.7 + 0.7) \times 9.81}{1+0.7} = 19.62 \text{ KN/m}^3$

(iii) the effective unit weight, $\gamma' = \gamma_{sat} - \gamma_w$
 $= (19.62 - 9.81) \text{ KN/m}^3$
 $\therefore \gamma' = 9.81 \text{ KN/m}^3$

(iv) we know that, $\gamma = \frac{(G_s + se) \gamma_w}{1 + e} = \frac{(2.7 + 0.7 \times 0.7) \times 9.81}{1 + 0.7} = 18.41 \text{ KN/m}^3$

(v) we know that, $se = wG_s$
 $\Rightarrow w = \frac{se}{G_s} = \frac{0.7 \times 0.7}{2.7} = 0.1815$
 \therefore moisture content, $w = 18.15\%$ (Ans.)

2017
 # A soil sample has a unit weight of 105 pcf and a saturation of 50%. when its saturation is increased to 75%, its weight raises to 112.7 pcf. Determine the void ratio and specific gravity of this soil.

solution: we know that, $\gamma = \frac{(G_s + se) \gamma_w}{1 + e}$

For 50% saturation, $\gamma = 105 \text{ pcf}$

$$\therefore 105 = \frac{(G_s + 0.5e) \times 62.4}{1 + e}$$

$$\Rightarrow G_s + 0.5e = 1.6827 (1 + e)$$

$$\Rightarrow G_s - 1.1827e = 1.6827 \dots \dots \textcircled{1}$$

Again, For 75% saturation, $\gamma = 112.7 \text{ pcf}$

$$\therefore 112.7 = \frac{(G_s + 0.75e) \times 62.4}{1+e}$$

$$\Rightarrow G_s + 0.75e = 1.8061(1+e)$$

$$\Rightarrow G_s - 1.0561e = 1.8061 \quad \text{--- (11)}$$

From eqn (1) & (11) we obtain,

$$G_s = 2.8355$$

$$e = 0.975$$

(Ans)

Problem: 3.6

The moist unit weight of soil is 19.2 KN/m^3 . Given that $G_s = 2.69$, $e = 0.51$ and $w = 9.8\%$. Determine the weight of water, in KN , to be added per cubic meter of soil for 90% saturation.

Solution:

We know that,

$$\gamma = \frac{(G_s + se)\gamma_w}{1+e}$$

$$\therefore \gamma = \frac{(2.69 + 0.9 \times 0.51) \times 9.81}{1 + 0.51} = 20.45 \text{ KN/m}^3$$

$$\therefore \text{weight of water to be added} = (20.45 - 19.2) \text{ KN/m}^3$$

$$= 1.25 \text{ KN/m}^3$$

(Ans)

①

Physical Properties of soil

FARHAD

1500045

soil texture:

The texture of a soil is its appearance or "feel" and it depends on the relative sizes and shapes of the particles as well as the distribution of those sizes.

Coarse grained soils:

Fine Grained soils:

Gravel

sands

silts

clays

0.075 mm (uses)

0.06 mm (BS)

Sieve Analysis

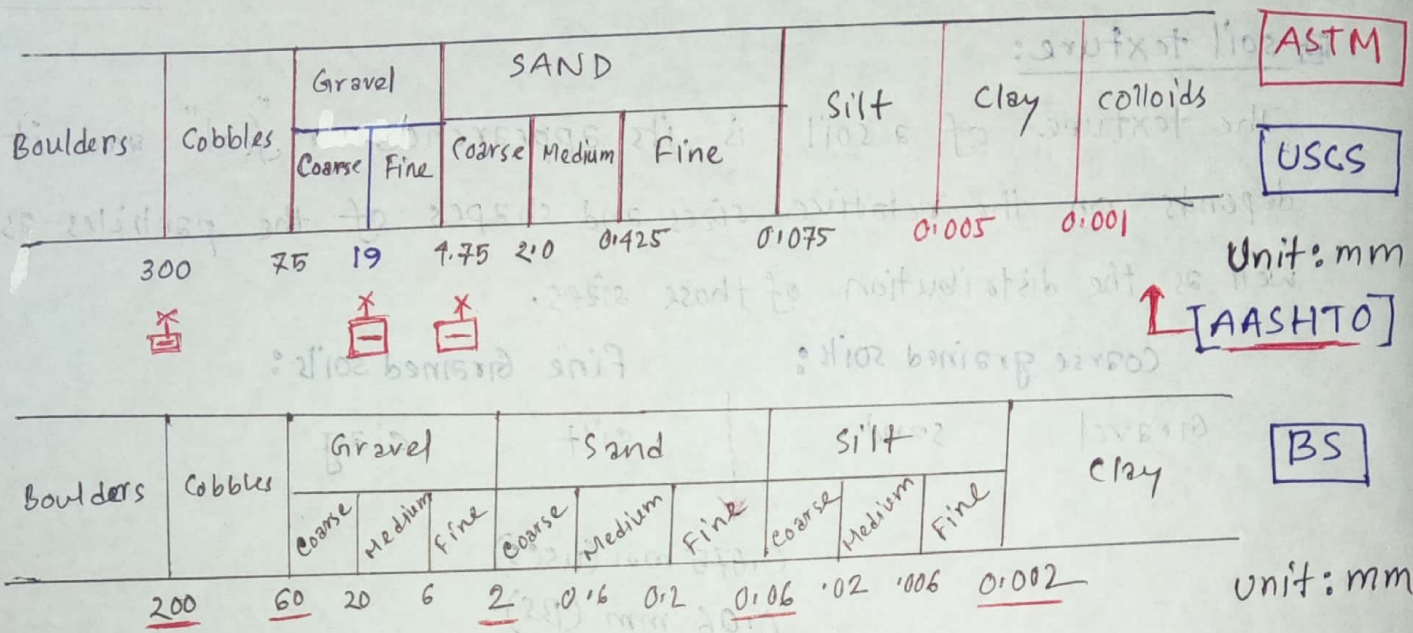
Hydrometer Analysis

Textural and other characteristics of soils:

Soil Name :	Gravel, sands	silts	clays
Grain size :	can see individual grains by eye	can not see	cannot see
Effect of Grain size on Engineering behavior :	Important	Relatively Unimportant	Relatively unimportant
Characteristics :	Non plastic Granular	Non plastic Granular	plastic -
Effect of water on engineering behavior	Relatively unimportant	Important	very Important

Physical Properties of Soil

Grain size:



ASTM : American Society of Testing and Materials.

USCS : Unified soil classification system.

BS : British Standard.

AASHTO : American Association for State Highway and Transportation Officials.

Grain size distribution:

Standard sieve size

sieve No.	open- ing (mm)	sieve No.	open- ing (mm)
4	4.75	30	0.600
8	2.36	40	0.425
10	2.00	50	0.355
16	1.18	100	0.150

200 No. sieve opening 0.075 mm

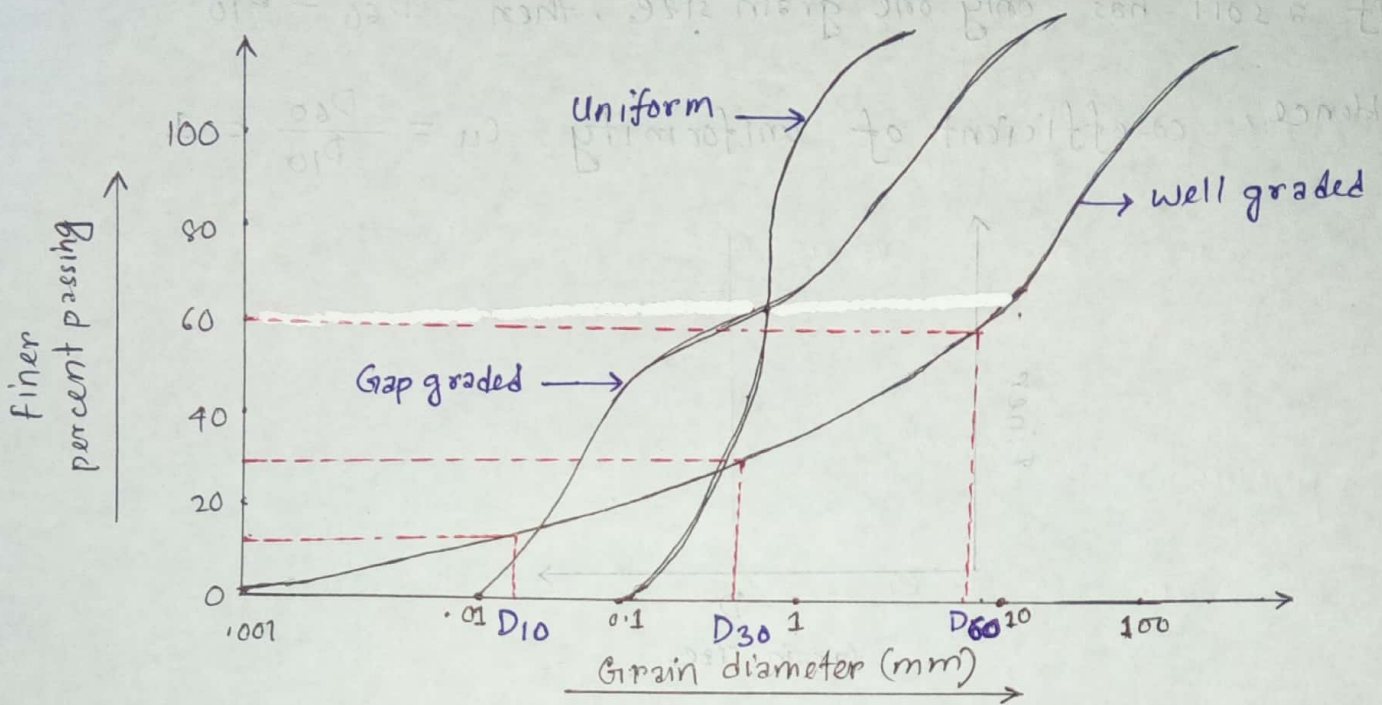


Fig. Typical grain size distribution

• Describe the shape

Example: Well graded

$D_{10} = 0.02 \text{ mm}$ (effective size)

$D_{30} = 0.6 \text{ mm}$

$D_{60} = 9 \text{ mm}$

co-efficient of uniformity,

$$C_u = \frac{D_{60}}{D_{10}} = \frac{9}{0.02} = 450$$

co-efficient of curvature,

$$C_c = \frac{(D_{30})^2}{(D_{10})(D_{60})} = \frac{(0.6)^2}{0.02 \times 9} = 2$$

• criteria

Well graded soil

For Gravels,

$$1 < C_c < 3 \text{ and } C_u \geq 4$$

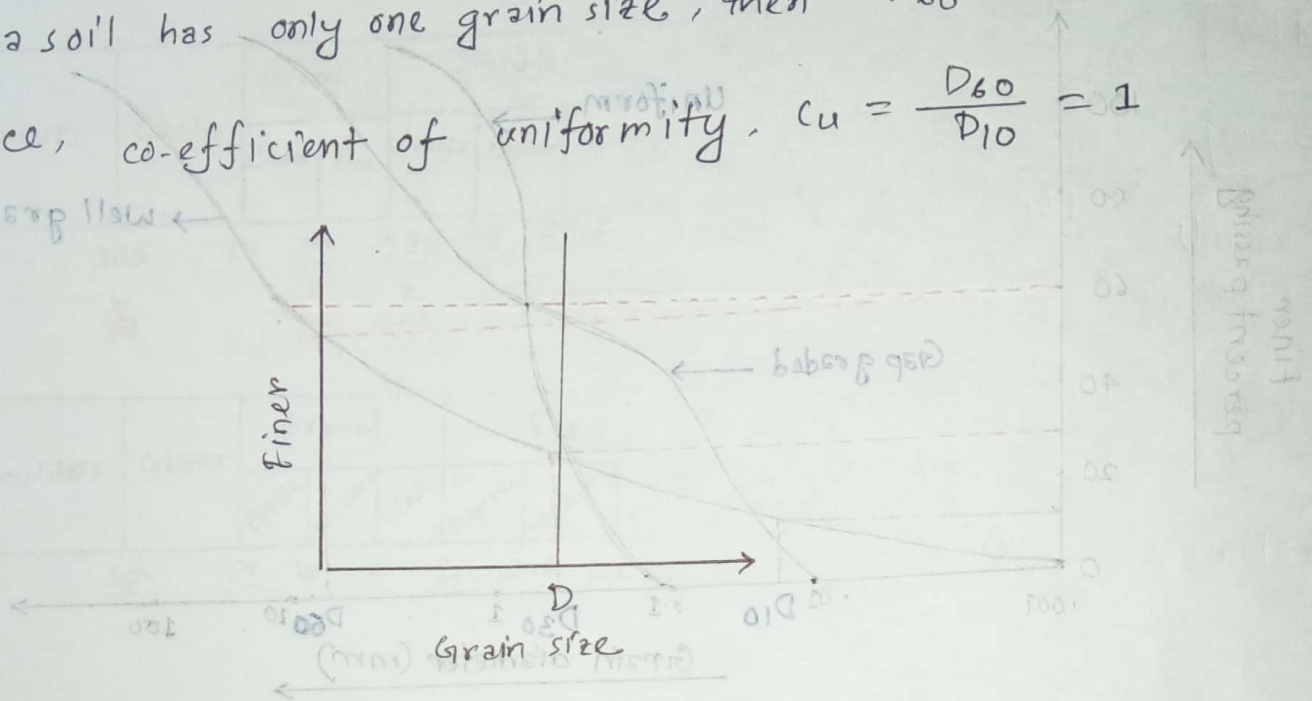
For Sands,

$$1 < C_c < 3 \text{ and } C_u \geq 6$$

What is the C_u for a soil only one grain size?

If a soil has only one grain size, then $D_{60} = D_{10}$

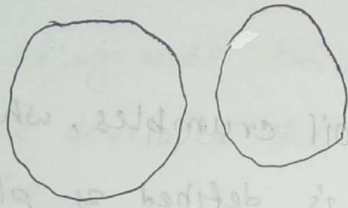
Hence, co-efficient of uniformity, $C_u = \frac{D_{60}}{D_{10}} = 1$



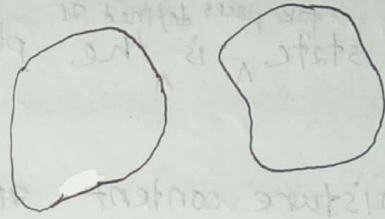
Engineering applications of Grain size distribution:

1. It will help us feel the soil texture and it will also be used for the soil classification. Example: Well graded
2. It can be used to define the grading specification of a drainage filter.
3. It can be a criterion for selecting fill materials of embankments and earth dams. Co-efficient of uniformity
4. It can be used to estimate the results of grouting and chemical injection and dynamic compaction. Co-efficient of uniformity
5. Effective size, D_{10} , can be co-related with the hydraulic conductivity.
6. It is more important to coarse-grained soils.

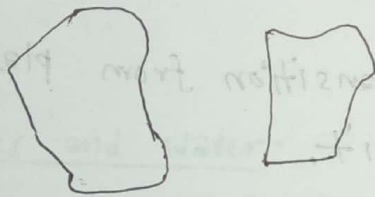
☐ Particle shape: It is important for granular soils.



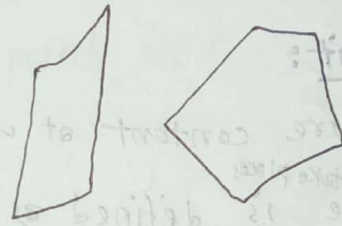
Rounded (low friction)



Sub-Rounded



Sub Angular



Angular (higher friction)

coarse grained soils

Clay particles are sheet-like.

☐ Atterberg Limits:

The atterberg limits are a basic measure of the critical water contents of a fine grained soil. The parameters of atterberg limit are shrinkage limit, plastic limit, Liquid limit.

shrinkage limit: The moisture content (in percent) at which the transition from solid to semi-solid state takes place is defined as shrinkage limit.

Or,

The moisture content at which the volume of soil mass ceases to change is defined as the shrinkage limit.

Plastic Limit:

The moisture content at which the transition from semi solid to plastic state ^{take place defined as} is the plastic limit.

or,

The moisture content at which the soil crumbles, when rolled into threads of 3.2 mm in diameter, is defined as plastic limit.

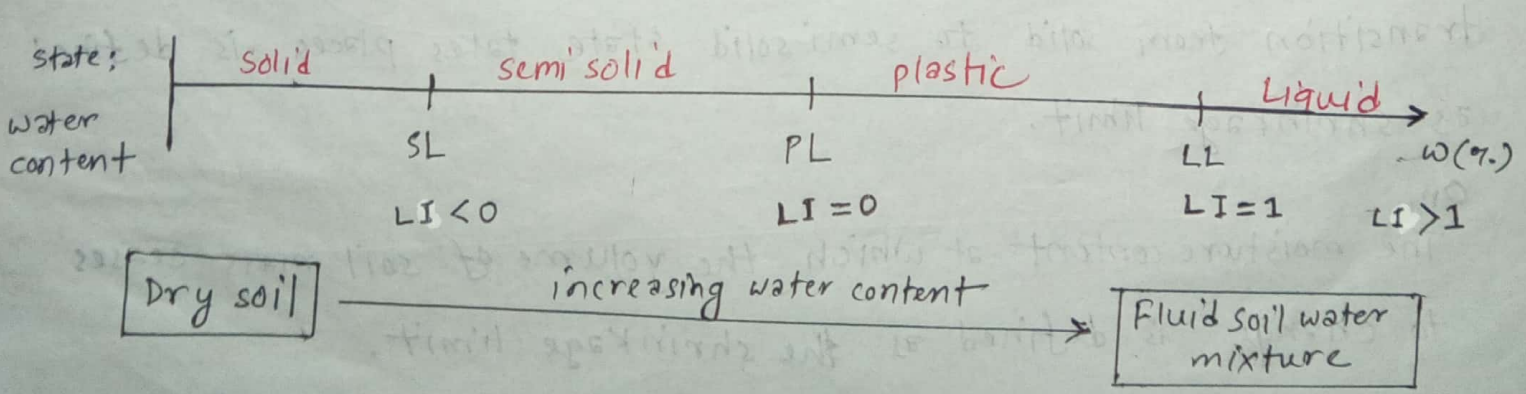
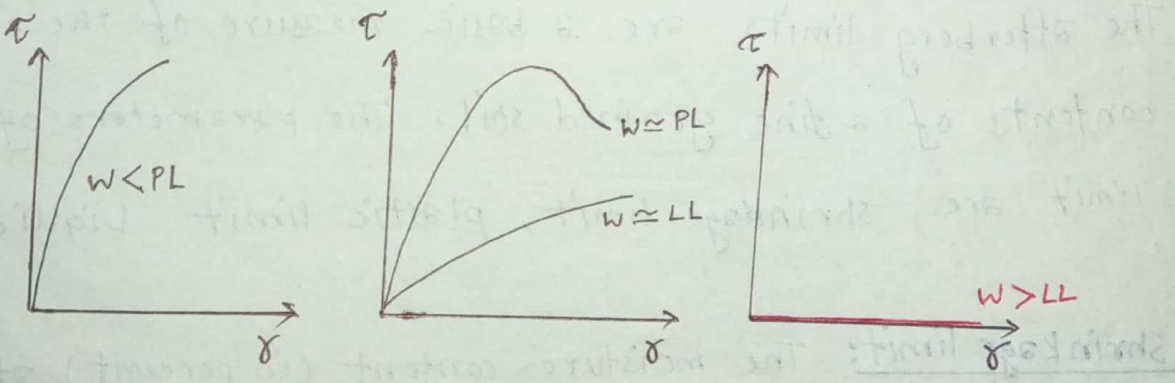
Liquid limit:

The moisture content at which the transition from plastic to liquid state ^{take place} is defined as liquid limit.

or,

The moisture content required to close a distance of 12.5 mm (0.5 inch) along the bottom of the groove after 25 blows is defined as the liquid limit.

Stress-strain curve:



Determination of Liquid Limit:

Casagrande Method

* Professor Casagrande standardized the test and developed the liquid limit device.

(i) Multipoint test

(ii) One-point test

Cone Penetrometer Method

* This method is developed by the Transport and Road Research Laboratory, UK.

(i) Multi point test

(ii) one point test

particle sizes and water:

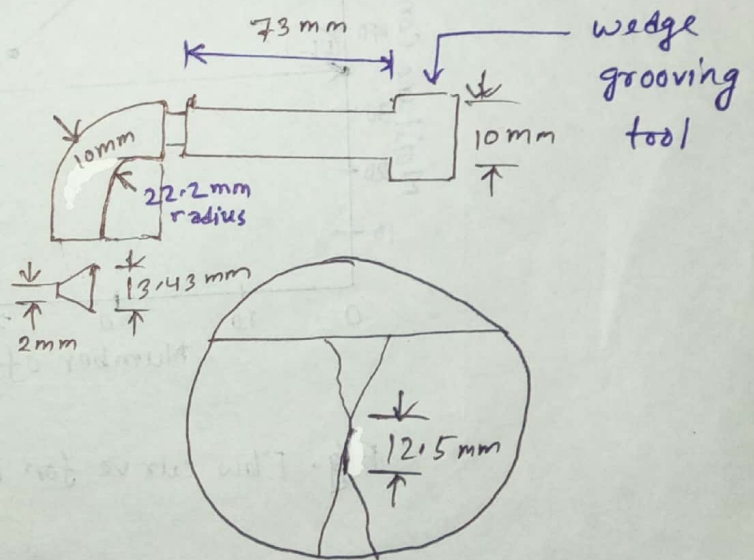
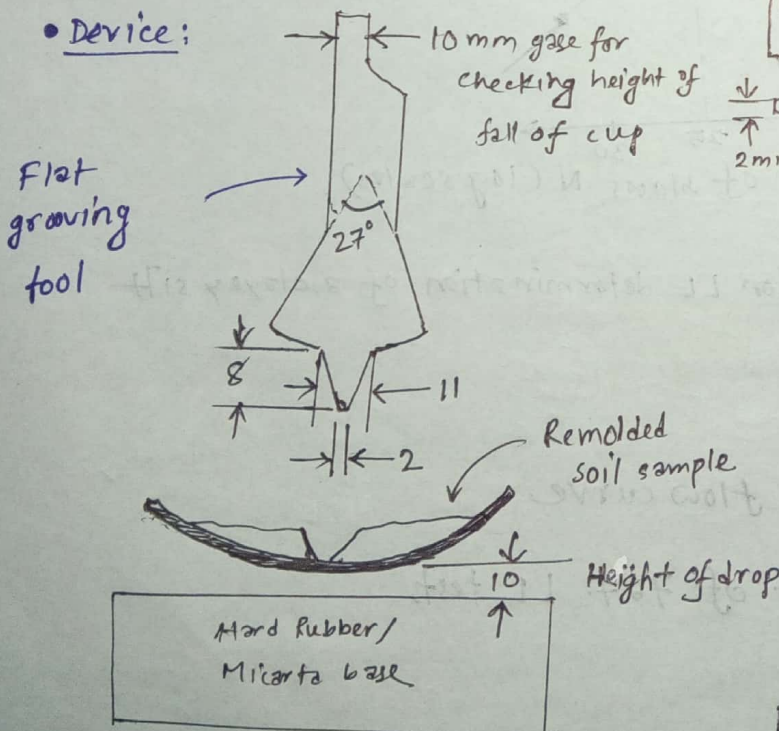
* passing through No. 40 sieve (0.425 mm)

* using deionized water

The type and amount of cations can significantly affect the measured results.

Casagrande method:

• Device:



Number of blows, $N \geq 25$

Closing distance = 12.5 mm (0.5 in)

Fig. Casagrande apparatus

* Multipoint method:

flow curve: The relation between moisture content and log N is approximated as a straight line. The line is referred to as flow curve.

flow index: The slope of flow line/curve is defined as flow index.

$$I_F = \frac{w_1 - w_2}{\log\left(\frac{N_2}{N_1}\right)}$$

where, $I_F = \text{flow index}$

equation of flow line, $w = -I_F \log N + C$ where $C = \text{constant}$

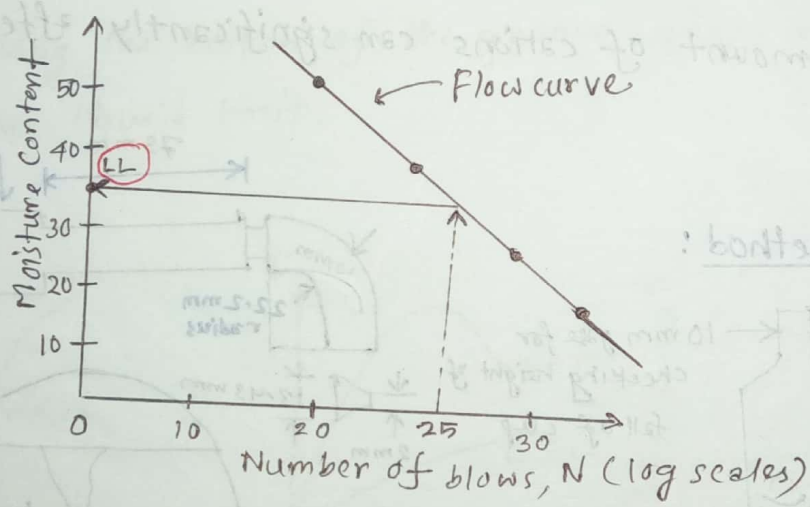


Fig. Flow curve for LL determination of a clayey soil

* One point Method:

- Assume a constant slope of the flow curve
- The slope is a statistical result of 76.7 LL tests

$$LL = W_n \left(\frac{N}{25} \right)^{\tan \beta}$$

where, N = number of blows
 W_n = corresponding moisture content

$$\tan \beta = 0.121$$

Limitations:

1. The β is an empirical coefficient, so it is not always 0.121
2. Good results can be obtained only for the blow number around 20 to 30.

Cone Penetrometer Method:

Device:

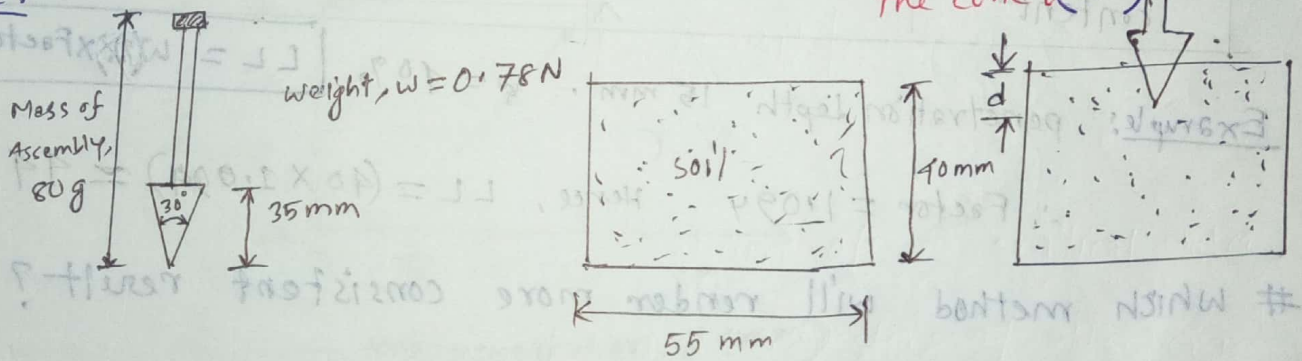
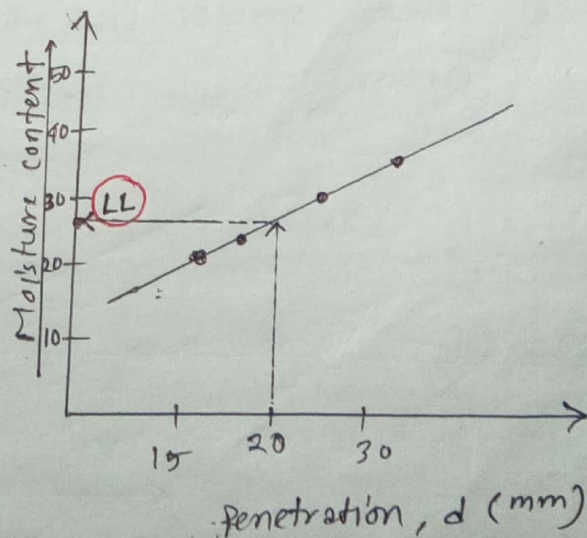


Fig. Details of cone penetrometer method

* Multipoint method:

between 15 mm and 25 mm
 and as close as possible
 to 20 mm



* one point method:

Table: suggested factors for cone penetration one point LL test
(From Clayton and Jukes, 1978)

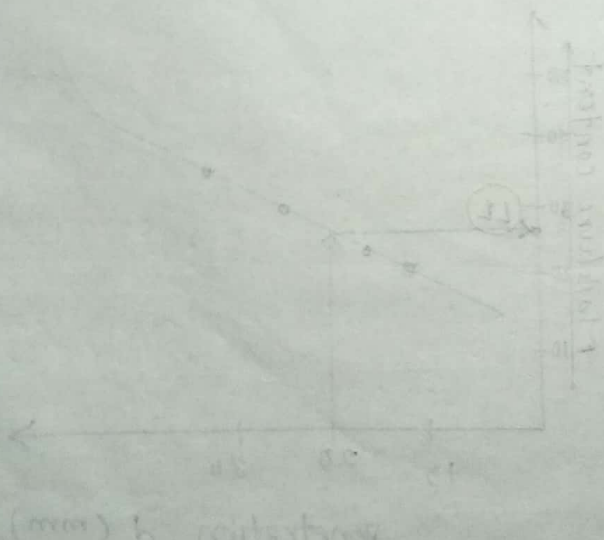
Penetration (mm)	Soil of high plasticity	soil of Intermediate plasticity	soil of low plasticity
15	1.098	1.094	1.057
16	1.075	1.076	1.052
18	1.055	1.058	1.042
⋮	⋮	⋮	⋮
Measured moisture content	above 50%	(35-50)%	below 35%

$$LL = W(\%) \times \text{Factor}$$

Example: penetration depth 15 mm, $w = 40\%$

\therefore Factor = 1.094 Hence, $LL = (40 \times 1.094) \approx 44$

Which method will render more consistent result?



* Multi-point method:
preferred if more than 5 points are available
to 50 mm

Plastic Limit:

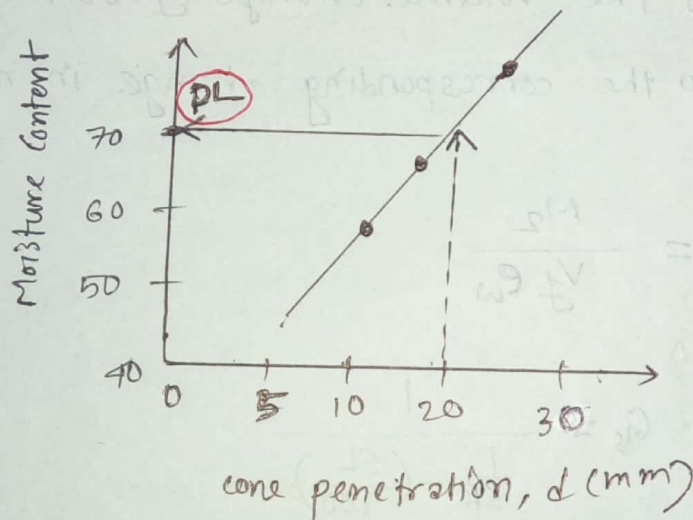
* Casagrade method:

The plastic limit is defined as the water content at which soil thread with 3/2 mm diameter just crumbles.

* Cone penetrometer test:

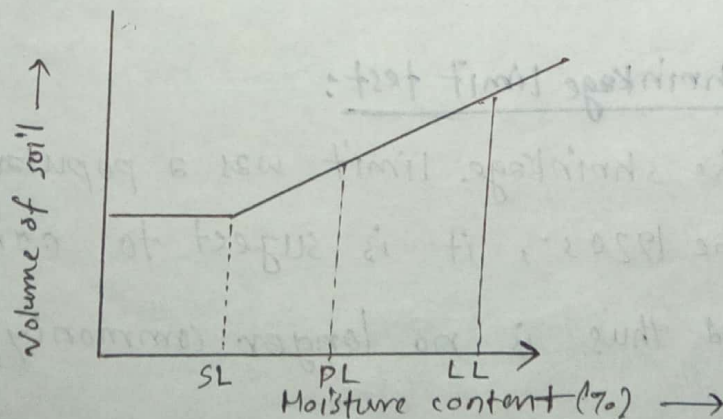
By using a cone of similar geometry but with a mass of 2.35 N (240g)

The moisture content corresponding to a cone penetration of $d = 20$ mm is defined as plastic limit.

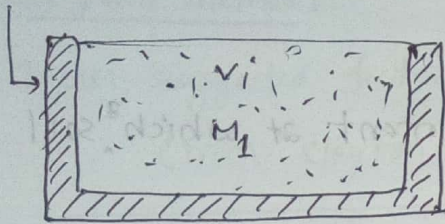


Shrinkage Limit:

The water content at which the soil volume ceases to change is defined as the shrinkage limit.

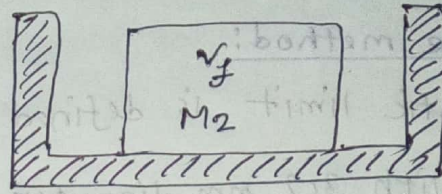


porcelain dish



Soil volume: V_i

Soil mass: M_1



Soil volume: V_f

Soil Mass: M_2

$$SL = W_i (\%) - \Delta W (\%)$$

$$= \left(\frac{M_1 - M_2}{M_2} \right) \times 100 - \left(\frac{V_i - V_f}{M_2} \right) \times e_w \times 100$$

Shrinkage ratio:

It is the ratio of the volume change of soil as a percentage of ^{the} dry volume to the corresponding change in moisture content.

$$SR = \frac{\left(\frac{\Delta V}{V_f} \right)}{\left(\frac{\Delta M}{M_2} \right)} = \frac{M_2}{V_f e_w}$$

specific gravity, $G_s = \frac{1}{\frac{1}{SR} - \left(\frac{SL}{100} \right)}$

At a given moisture content, $w (\%)$

$$\text{Volumetric shrinkage, } VS (\%) = SR [w (\%) - SL]$$

$$\text{Linear shrinkage, } LS (\%) = 100 \left[1 - \frac{100}{VS (\%) + 100} \right]$$

Limitations of shrinkage limit test:

Although the shrinkage limit was a popular classification test during the 1920s, it is subject to considerable uncertainty and thus is no longer commonly conducted.

One of the biggest problems with the SL test is that the amount of shrinkage depends not only on the grain size but also on the initial fabric of the soil.

The standard procedure is to start with the water content near the liquid limit

Especially with sandy and silty clays, this often results a $SL > PL$, which is meaningless.

Casagrande suggest that the initial water content be slightly greater than the PL , if possible. But admittedly, it is difficult to avoid entrapping air voids.

Indices:

Plasticity Index (PI): Plasticity index is the difference between the liquid limit and plastic limit. For describing it is the range of water content over which a soil is plastic.

$$PI = LL - PL$$

Burmister (1949) classified the plasticity index in a qualitative manner as follows:

PI	Description
0	Nonplastic
1-5	Slightly plastic
5-10	Low plasticity
10-20	Medium plasticity
20-40	High plasticity
> 40	very high plasticity

The plasticity Index is important in classifying fine grained soils. It is fundamental to the Casagrande plasticity chart, which is currently the basis for the USCS.

Liquid Index: The relative consistency of a cohesive soil in the nature state can be defined by a ratio called the Liquid Index.

$$LI = \frac{W - PL}{LL - PL} \quad \text{where, } W = \text{in situ moisture content of soil}$$

- $LL < 0$ - semi solid state, brittle fracture if sheared
- $0 < LL < 1$ - plastic state, plastic solid if sheared
- $LL > 1$ - Liquid state, viscous liquid if sheared

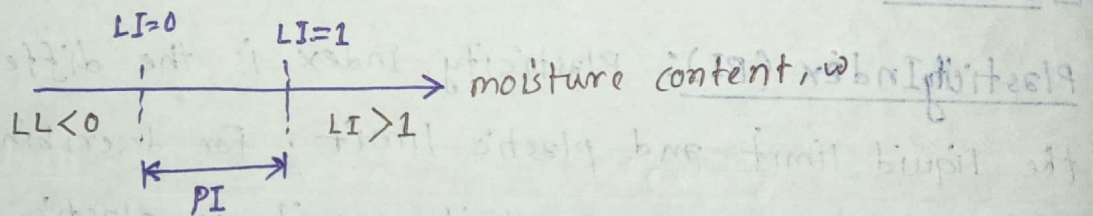


Figure. Liquid Index

consistency Index: Another index that is commonly used for engineering purposes is the consistency index (CI), which may be defined as,

$$CI = \frac{LL - W}{LL - PI}$$

If $W = LL$ then $CI = 0$

Again if $W = PI$ then $CI = 1$

Activity: Activity may be expressed as

$$A = \frac{PI}{(\% \text{ of clay size fraction, by weight})}$$

Activity is used as an index for identifying the swelling potential of clay soils.

Normal clays: $0.75 < A < 1.25$

In active clays: $A < 0.75$

Active clays: $A > 1.25$

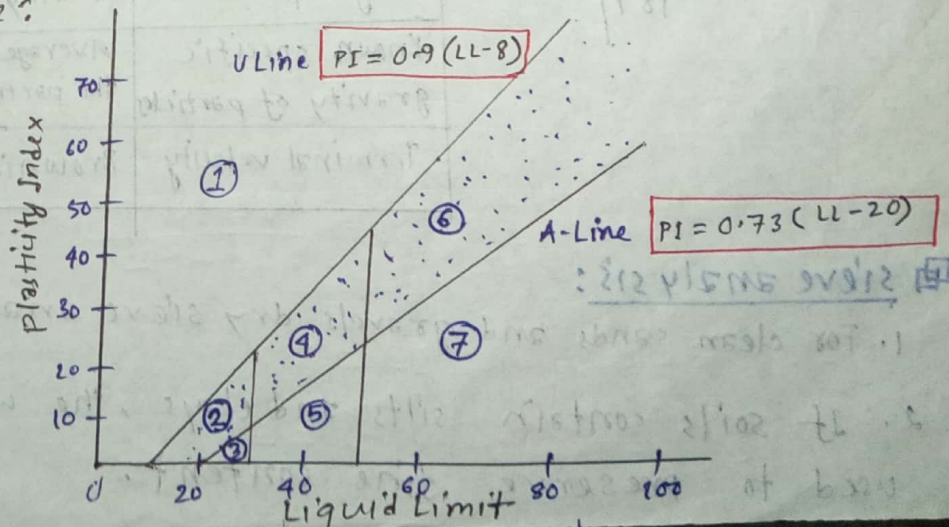
High activity: * Large volume change when wetted

* Large shrinkage when dried

* very reactive (chemically)

Plasticity chart:

Casagrande (1932) studied the relationship of the plasticity index to the liquid limit of a wide variety of natural soils. on the basis of test results, he proposed a plasticity chart as shown in figure:



- ① cohesionless soil
- ② Inorganic clays of low plasticity
- ③ silts of low compressibility
- ④ clays of medium plasticity
- ⑤ silts of high compressibility and organic silts
- ⑥ clays of high plasticity
- ⑦ silts of high compressibility and organic clays

Note:

Above U Line - cohesionless

Below U Line - Inorganic

Above A Line - clay & plasticity

Below A Line - silt & compressibility

LL < 30 - low plasticity & compressibility

30 < LL < 50 - Medium "

LL > 50 - High "

Engineering applications of Atterberg limit:

1. Atterberg limit enable clay soils to be classified.
2. Atterberg limit are usually correlated with some engineering properties such as permeability, compressibility, shear strength and others.
3. The values of SL can be used as a criterion to assess and prevent the excessive cracking of clay liners in the reservoir, embankment or canal.

Hydrometer Analysis:

Stokes's Law

$$V = \frac{(r_s - r_w) D^2}{18\eta}$$

Assumption	Reality
sphere particle	platy particle (clay particle) as $D \leq 0.005 \text{ mm}$
single particle	many particles in suspension
known specific gravity of particles	Average results of all the minerals in the particles, including the absorbed water films
Terminal velocity	Brownian motion as $D \leq 0.0002 \text{ mm}$

sieve analysis:

1. For clean sands and gravels dry sieve analysis can be used.
2. If soils contain silts and clays, the wet sieving is usually used to preserve fine content.

Physical Properties of Soil

2006

The Atterberg limits of a clay soil are liquid limit 52% and plastic limit 30% and shrinkage limit 18%. If the specimen of this soil shrinks from a volume of 39.5 cm³ at the liquid limit to a volume of 24.2 cm³ at the shrinkage limit, calculate the true specific gravity.

Solution: Given, $LL = 0.52$ $V_i = 39.5 \text{ cm}^3$
 $PL = 0.30$ $V_f = 24.2 \text{ cm}^3$
 $SL = 0.18$

$$\text{We know, } SL = \frac{M_1 - M_2}{M_2} \times 100 = \frac{V_i - V_f}{M_2} \times e_w \times 100$$

$$\Rightarrow SL = \frac{eV_i - eV_f}{eV_f} \times 100 = \frac{V_i - V_f}{M_2} \times e_w \times 100$$

$$\Rightarrow 18 = \frac{39.5 - 24.2}{M_2} \times 100 = \frac{39.5 - 24.2}{M_2} \times 1 \times 100$$

$$\Rightarrow M_2 = \frac{33.83 \times 100}{18} = 33.83$$

$$\text{Now, } G_s = \frac{M_2}{V_f \cdot e_w} = \frac{33.83}{24.2 \times 1} = 1.398$$

$$\therefore G_s = \left[\frac{1}{\frac{1}{SR} - \frac{SL}{100}} \right] \Rightarrow G_s = \left[\frac{1}{\frac{1}{1.398} - \frac{18}{100}} \right]$$

$$\therefore G_s = 1.87$$

(Ans.)

2013, 2017

The plastic limit of a soil is 25% and its plasticity index is 8%. When the soil is dried from its state at plastic limit, the volume change is 25% of its volume at plastic limit. Similarly, the corresponding volume change from the liquid limit to dry state is 34% of its volume at liquid limit. Determine the shrinkage limit and the shrinkage Ratio.

Solution:

We know that,

$$PI = LL - PL$$

$$\Rightarrow LL = PI + PL = (25 + 8) = 33\%$$

$$\text{Dry volume, } V_d = (1 - 0.25) V_p = 0.75 V_p \dots \textcircled{1}$$

$$\text{again, } V_d = (1 - 0.34) V_L = 0.66 V_L \dots \textcircled{II}$$

$$\text{From } \textcircled{1} \text{ \& } \textcircled{II} \text{ we obtain, } V_p = \frac{0.66}{0.75} V_L = 0.88 V_L$$

$$\text{Solid to Liquid} = \frac{8 \times 0.34 V_L}{(1 - 0.88) V_L} = 22.67$$

$$\therefore \text{Shrinkage Limit, } SL = LL - (\text{Solid to liquid}) = (33 - 22.67) = 10.33$$

$$\therefore SL = 10.33\%$$

$$\text{and Shrinkage Ratio} = \frac{(1 - 0.88) V_L \times 100}{0.66 V_L \times 8} = 2.273\%$$

(Ans.)

Example: 4.1, 4.2, 4.3

Following are the results of a shrinkage limit test:

Initial volume of soil in a saturated state = 24.6 cm³

Final " " " " dry " = 15.9 cm³

Initial mass of soil in saturated state = 44.0 g

Final " " " " dry " = 30.1 g

- (i) Determine the shrinkage limit of the soil
- (ii) Determine the shrinkage Ratio and also estimate the specific gravity of soil solids.
- (iii) If the soil is at a moisture content of 28%, estimate the maximum volumetric shrinkage and the linear shrinkage.

Solutions:

(i) We know that, $SL = \left(\frac{M_1 - M_2}{M_2} \right) \times 100 - \left(\frac{V_i - V_f}{M_2} \right) \times \rho_w \times 100$

$$= \frac{44 - 30.1}{30.1} \times 100 - \frac{24.6 - 15.9}{30.1} \times 1 \times 100$$

$\therefore SL = (46.8 - 28.9) = 17.28\%$ (Ans)

(ii) we know that, $SR = \frac{M_2}{V_f \rho_w} = \frac{30.1}{15.9 \times 1} = 1.89$

and, $G_s = \frac{1}{\frac{1}{SR} - \frac{SL}{100}} = \frac{1}{\frac{1}{1.89} - \frac{17.28}{100}} = 2.81$

(Ans)

(ii) we know that, $V_s = S R (W - SL)$ Here, $W = 28\%$
 $= 1.89 (28 - 17.28)$

$\therefore V_s = 20.26\%$

and,

$$LS = 100 \left[1 - \left(\frac{100}{V_s + 100} \right)^{\frac{1}{3}} \right]$$

$$LS = 100 \left[1 - \left(\frac{100}{20.26 + 100} \right)^{\frac{1}{3}} \right] = 5.96\%$$

(Ans)

Problem: 4.1, 4.2 (2012)

Results from Liquid and plastic limit test conducted on a soil are given below. Liquid limit tests:

Number of blows, N	Moisture content (%)
14	38.4
20	36.5
28	33.1
30	27.0

Plastic limit tests: $PL = 13.4\%$

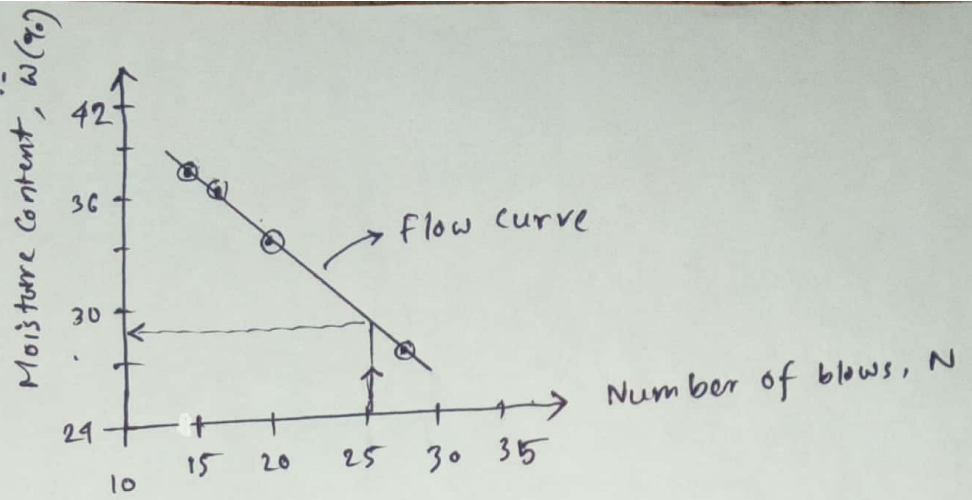
- (i) Draw flow curve and obtain Liquid Limit
- (ii) what is the plasticity index of the soil
- (iii) Determine the Liquid index of soil if $w_{insitu} = 32\%$

$$18.8 = \frac{1}{\frac{1}{32} - \frac{1}{13.4}} = \frac{1}{\frac{1}{32} - \frac{1}{27}}$$

(Ans)

Solutions: (i) Flow Curve:

From curve,
we obtain,
 $LL = 29$



(ii) We know that,

$$PI = LL - PL = (29 - 13.4) = 15.6\%$$

(iii) We know that,

$$LI = \frac{w - PL}{LL - PL} = \frac{32 - 13.4}{15.6} = 1.199.$$

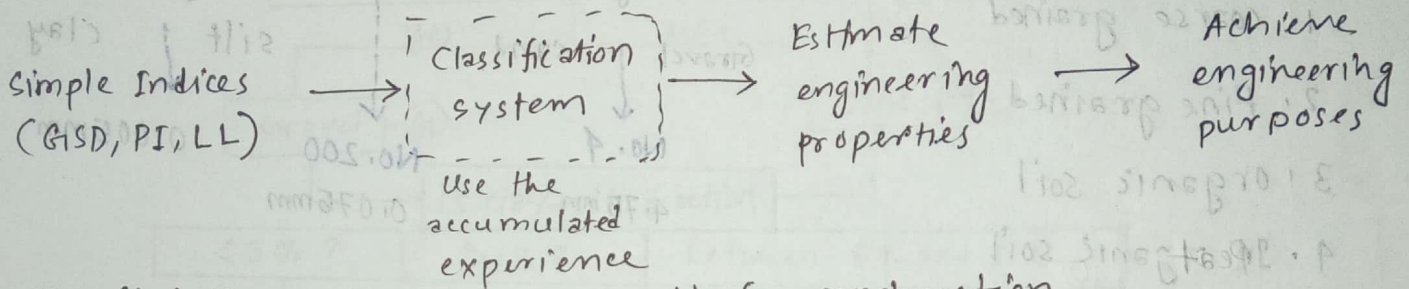
(Ans)

Soil Classification

FARHAD
1500045

Purpose of soil classification:

Classifying soil into groups with similar behaviour, in terms of simple indices, can provide geotechnical engineers a general guidance about engineering properties of soils through the accumulated experience.



* To find the suitability of soil for construction.

Classification system:

Two commonly used systems:

1. Unified soil classification system (USCS)
(based on the engineering behaviour of soil)

2. American Association of State Highway and Transportation officials (AASHTO)
(based on the particle size distribution of the percent of sand, silt and clay size fractions present in a given soil)

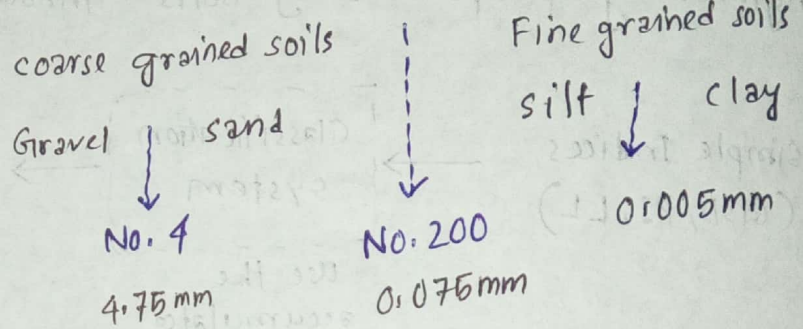
** Additional: Textural soil classification

Unified soil classification system:

The system was first developed by Casagrande (1942) for the purpose of air field construction during world war II.

Four major divisions:

1. coarse grained
2. fine grained
3. organic soil
4. inorganic soil



symbols: (Soil symbols)

G: Gravel

S: sand

M: silt

C: clay

O: organic

PT: Peat

LL symbols

H: High LL ($LL > 50$)

L: Low LL ($LL < 50$)

Gradation symbols

w: well graded

P: poorly graded

Example:

SW, well graded sand

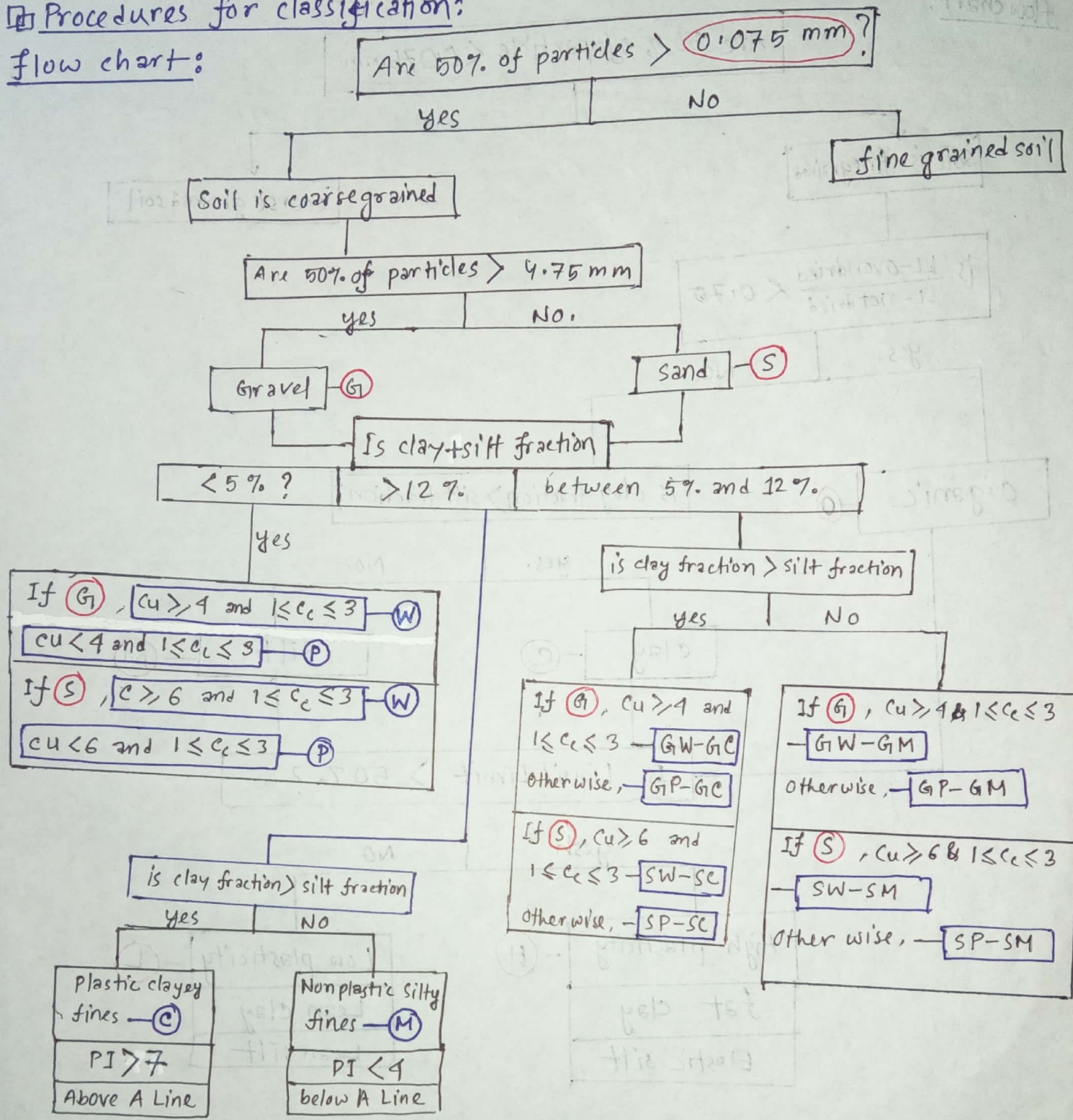
SC, clayey sand

SM, silty sand

Procedures for classification:

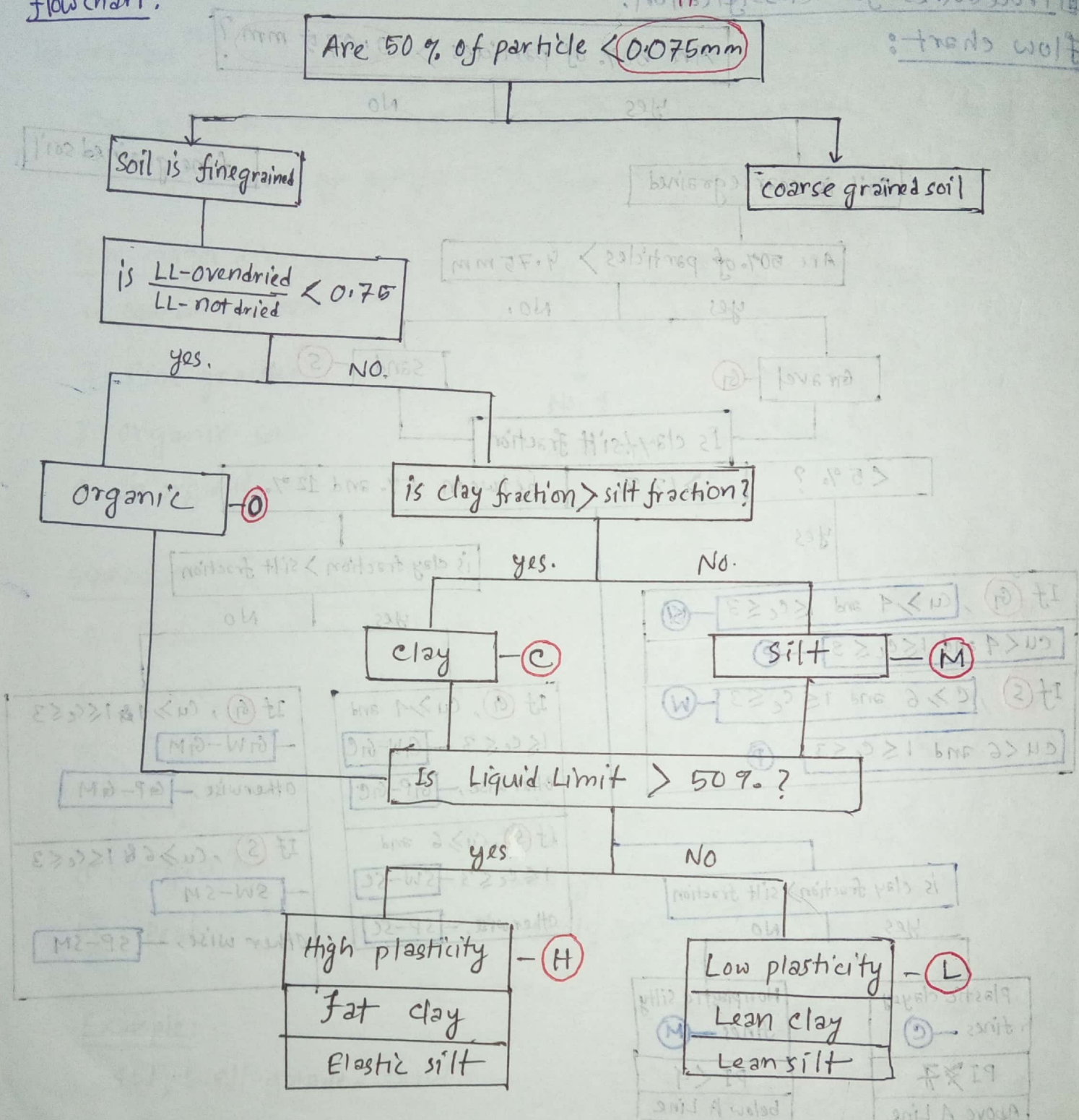
flow chart:

200 No. sieve



flow chart:

200 No. sieve
flow chart



Information that must be known for proper classification:

1. percent of gravel — that is, the fraction passing the 76.2 mm sieve and retained on the No. 4 sieve (4.75 mm)
2. Percent of sand — that is, the fraction passing the No. 4 sieve and retained on the No. 200 sieve (0.075 mm)
3. percent of silt and clay — that is, the fraction finer than the No. 200 sieve
4. Uniformity co-efficient (C_u) and the co-efficient of gradation (C_c)
5. LL and PL of the portion of soil passing the No. 40 sieve

Group symbols for coarse grained soils:

GW, GP, GM, GC, GC-GM, GW-GM, GW-GC, GP-GM and GP-GC

Group symbols for fine-grained soil:

CL, ML, OL, CH, MH, OH, CL-ML and Pt.

Fraction:

1. Fine fraction = percent passing No. 200 sieve
2. Coarse fraction = percent retained on No. 200 sieve
3. Gravel fraction = percent retained on No. 4 sieve
4. sand fraction = (percent retained on No. 200 sieve) - (percent retained on No. 4 sieve)

~~Soil Classification System~~

Border line cases (Dual system):

For the following three conditions, a dual symbol should be used.

1. coarse-grained soils with 5% - 12% fines:

The first symbol indicates whether the coarse fraction is well graded - or poorly graded. The second symbol describes the contained fines.

Example: SP-SM : poorly graded sand with silt

2. Fine grained soils with PI between 4 and 7 and LL between 12 and 25:

It is hard to distinguish between the silty and more clay like materials.

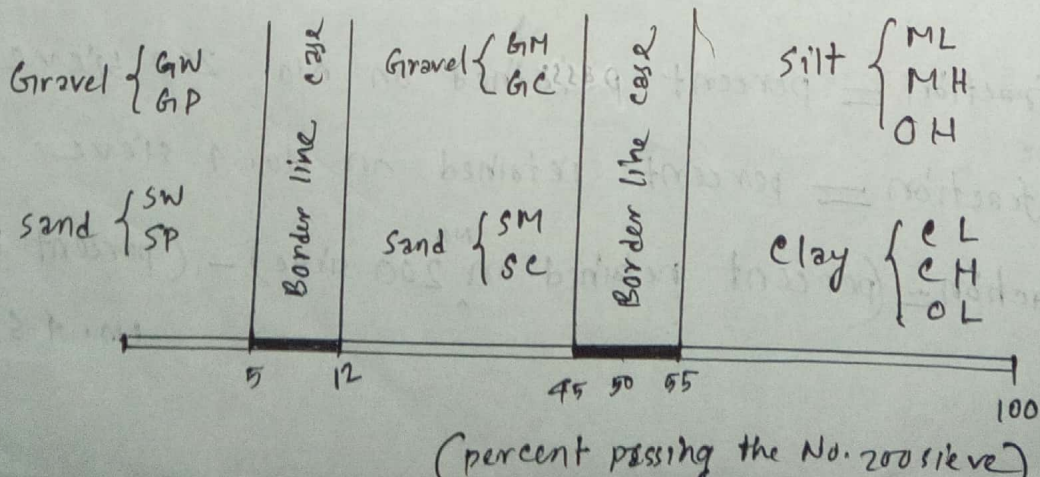
Example: CL-ML : silty clay

SC-SM : silty clayey sand

3. Soil contain similar fines and coarse grained fractions:

possible dual symbols GM-ML

summary



AASHTO Classification System:

The system was originally developed by Hogentogler and Terzaghi in 1929 as the "public Roads Classification system".

Major Groups: According to this system, soil is classified into 8 major groups. A1 - A7 and organic soils A8.

Required tests: Sieve Analysis and Atterberg limits.

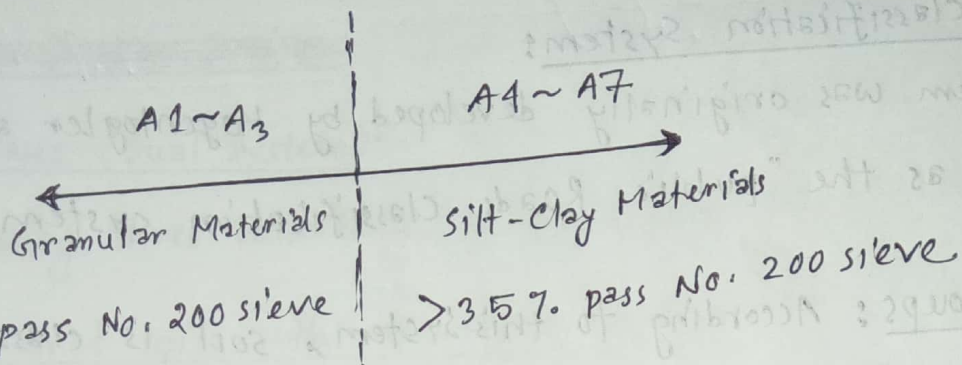
Group Index: The group index, an empirical formula, is used to further evaluate soils within a group (subgroups):

$$GI = (F_{200} - 35) [0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

Where, F_{200} = percent passing through the No. 200 sieve.

Rules for determining the group index:

1. If the equation yields a negative value for GI, it is taken as 0.
2. The group index is rounded off to the nearest whole number.
(Example: $GI = 3.4$ is rounded off 3; 3.5 is rounded off to 4)
3. There is no upper limit for the group index.
4. The group index of soils belonging to groups A-1-a, A-1-b, A-2-4, A-2-5 and A-3 is always 0.
5. For group A-2-6 and A-2-7 use the partial group index for PI,
 $GI = 0.01(F_{200} - 15)(PI - 10)$



criteria that the AASHTO classification system is based on:

1. Grain Size:

a. Gravel: fraction passing the 75 mm sieve and retained on the No. 10 (2mm) U.S. sieve.

b. sand: fraction passing the No. 10 (2mm) U.S. sieve and retained on the No. 200 (0.075 mm) U.S. sieve.

c. Silt and clay: fraction passing through ^{No.} 200 U.S. sieve.

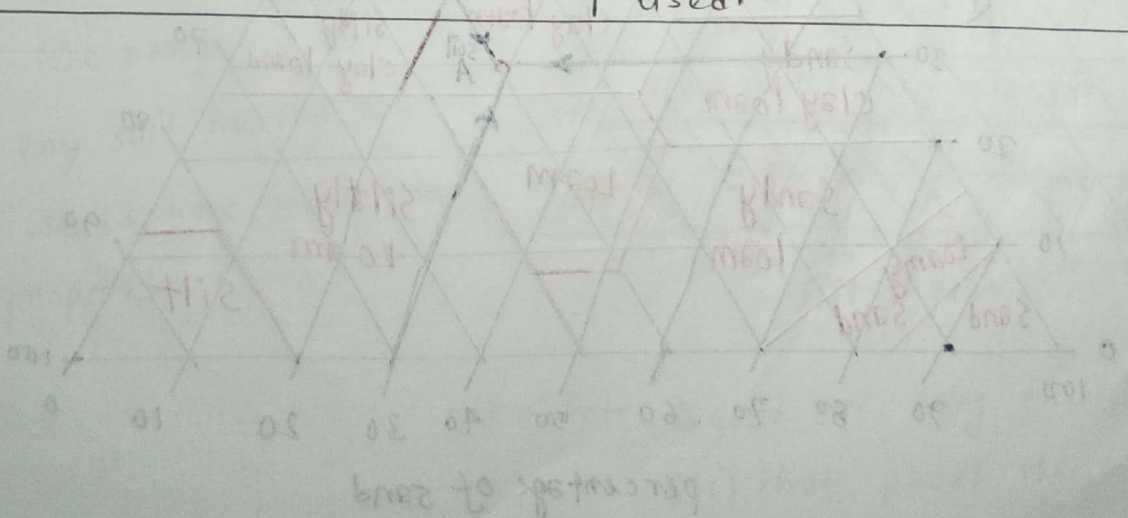
2. Plasticity: Silty — when fine fraction have a PI of 10 or less.
clays — when fine fraction have a PI of 11 or more.

3. If cobbles and boulders are encountered, they are excluded from the portion of the soil sample from which classification is made.

The rating for a pavement subgrade is inversely proportional to Group Index, GI

Comparison between AASHTO and USCS system:

AASHTO	USCS
1. Based on texture and plasticity of soil.	1. Based on texture and plasticity of soil.
2. Two major category of soil: coarse-grained and fine-grained soil is obtained by No. 200 sieve	2. Two major category of soil: coarse grained and fine-grained soil is obtained by No. 200 sieve
3. Soil is said to be fine grained when more than 35% passes No. 200 sieve.	3. Soil is said to be fine grained if more than 50% passes No. 200 sieve.
4. Appears to be more appropriate.	4. Appears to be less appropriate.
5. No. 10 sieve is used to separate gravel from sand.	5. No. 4 sieve is used to separate gravel from sand.
6. Classification of organic soil is absent.	6. Classification of organic soil is present.
7. Gravelly and sandy soils are not clearly separated	7. Gravelly and sandy soils are clearly separated
8. A symbol is used.	8. GW, SM, CH etc. symbols are used.

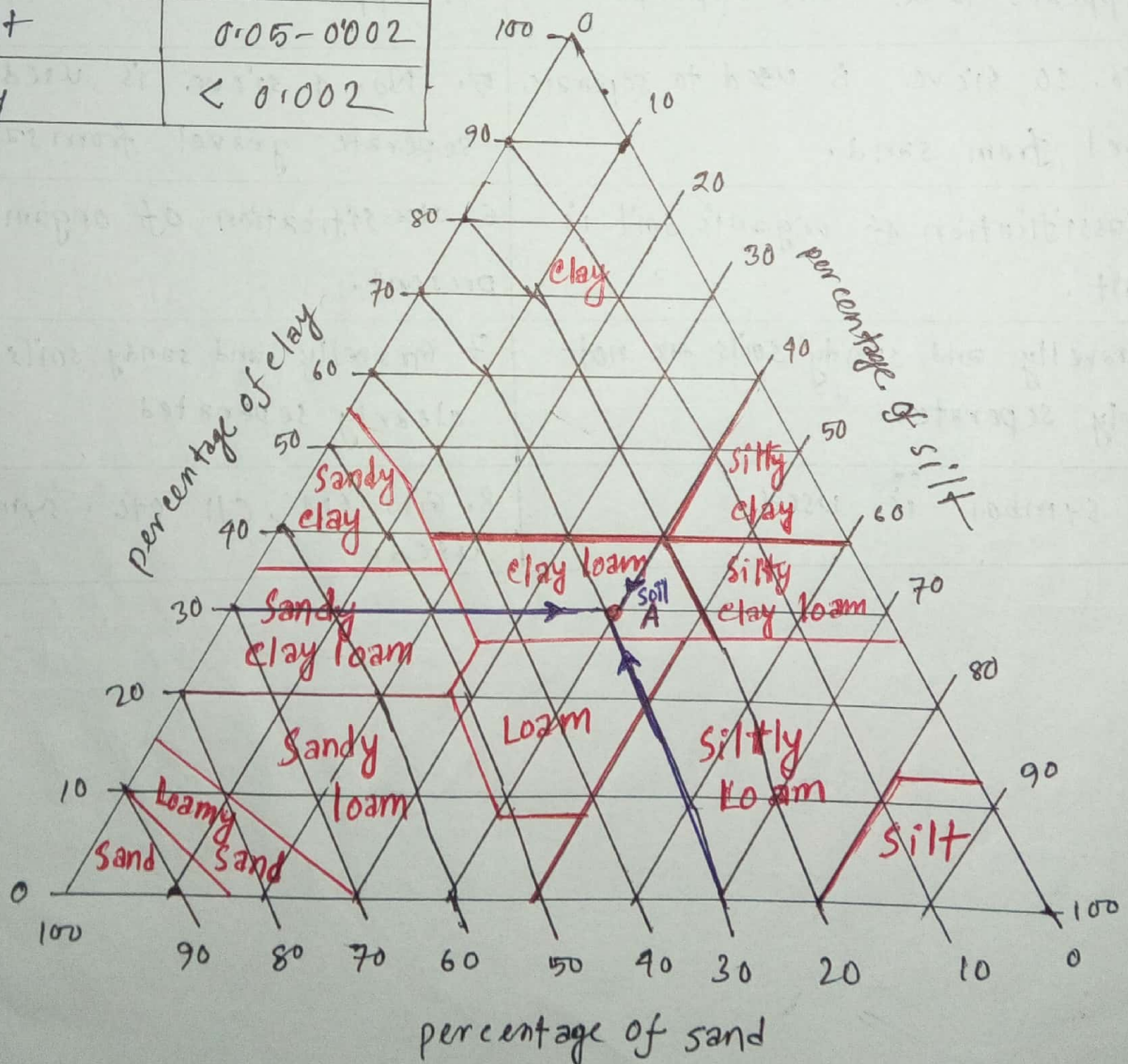


Textural classification:

In general sense, texture of soil refers to its surface appearance. Soil texture is influenced by the size of the individual particles present in it.

Textural classification system developed by the U.S. Department of Agriculture (USDA). This classification method is based on particle size limits as described under USDA system.

Soil fraction	Dia. in (mm)
Gravel	> 2.00
Sand	2 - 0.05
Silt	0.05 - 0.002
Clay	< 0.002



Example:

If the particle size distribution of soil A shows 30% sand, 40% silt and 30% clay size particle, its textural classification can be determined by proceeding indicated by the arrows. The soil sample falls in to the zone of clay loam.

Loam: In this system The term Loam used to describe a mixture of sand, silt and clay particles in various proportion. The term Loam is originated in Agricultural engineering where the suitability of a soil is judge for crops. This term is not used in soil engineering.

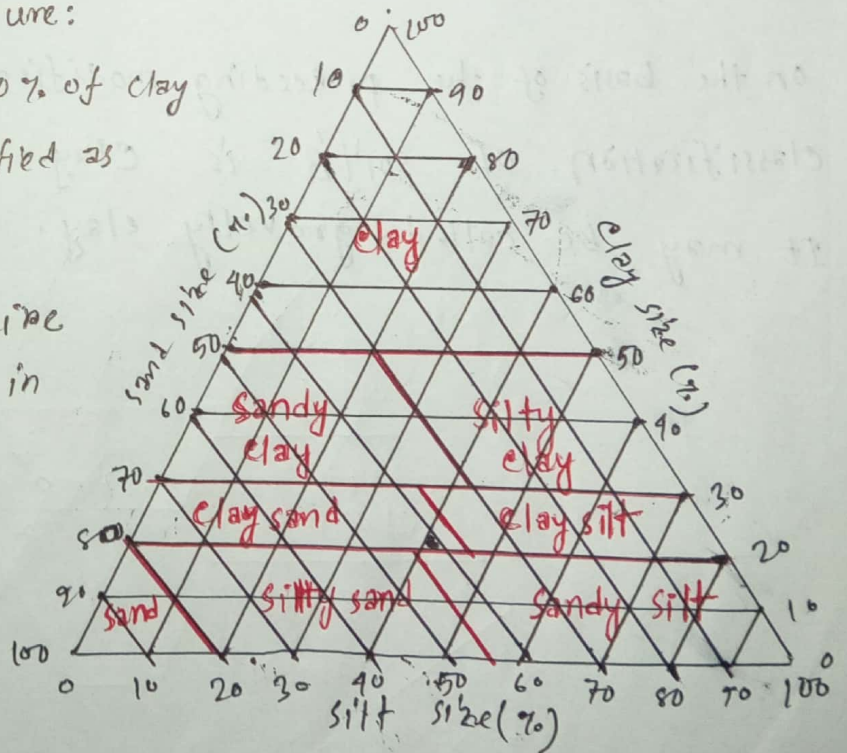
Mississippi River Commission (Modified triangular system)

Important observation from figure:

soil containing more than 50% of clay size particles would be classified as a clay.

where as sand and silt require 80% of the particles to be in that size range.

Also any soil having 20% clay would have some clay like properties.



the chart based on only the fraction of soil that passes the No. 10 sieve

The chart is based on only the fraction of soil

If the particle size distribution of a soil is such that a certain percentage of soil particles is larger than 2mm in diameter, a correction will be necessary.

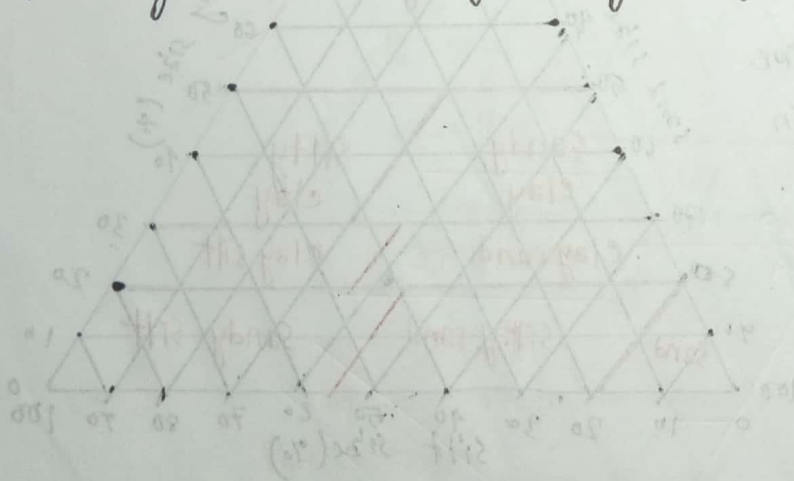
Example: If soil 'B' has a particle size distribution of 20% gravel, 10% sand, 30% silt and 40% clay, The modified textural composition are,

$$\text{sand size} = \frac{10 \times 100}{100 - 20} = 12.5\%$$

$$\text{silt size} = \frac{30 \times 100}{100 - 20} = 37.5\%$$

$$\text{clay size} = \frac{40 \times 100}{100 - 20} = 50\%$$

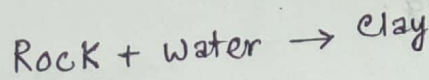
on the basis of the preceding modified percentages, USDA textural classification of soil 'B' is clay. For large percentage of gravel. It may be called gravelly clay.



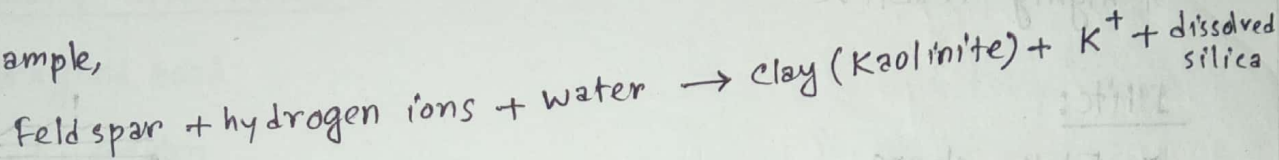
Clay Minerals

Origin of Clay Minerals:

The contact of rocks and water produces clays, either at or near the surface of the earth.



For example,



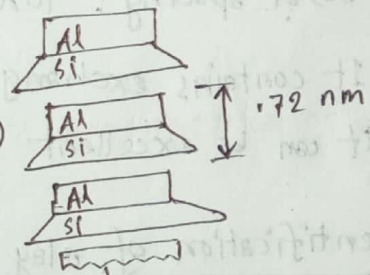
Basic Structural Units:

1. Tetrahedral sheet (silicon)
2. Octahedral sheet (Aluminium or Magnesium)
3. Gibbsite sheet (Aluminium)
4. Brucite sheet (Magnesium)

Different Clay Minerals:

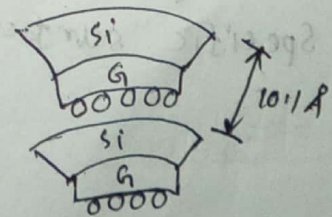
Kaolinite:

- * Platy shape.
- * ^{bonding:} Van der Waals forces and Hydrogen bonds. (strong)
- * Basal spacing: 7.2 Å
- * There is no interlayer swelling.



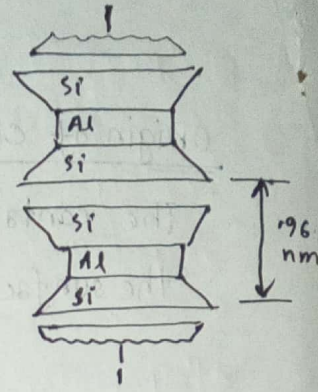
Halloysite:

- * Tubular shape while it is hydrated.
- * Basal spacing: 10.1 Å for hydrated halloysite and 7.2 Å for dehydrated.
- * There is no inter layer swelling.
- * A single layer^{of} water between unit layer.



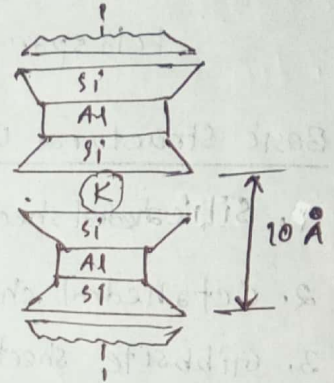
Montmorillonite:

- * Film like shape
- * bonding: Vander waals forces and cations (weak)
- * basal spacing: 9.6 \AA .
- * There is interlayer swelling.
- * highly reactive.



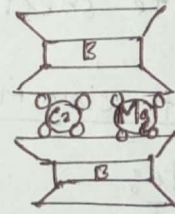
Illite:

- * Flaky shape
- * referred to as hydrous mica.
- * Basal spacing: 10 \AA
- * charge deficiency is balanced by K^+



Vermiculite:

- * The octahedral sheet is brucite
- * Basal spacing: 10 \AA to 14 \AA
- * It contains exchangeable cations Ca^{2+} and Mg^{2+}
- * It can be excellent insulation material after hydrated.



Identification of clay minerals:

1. X ray diffraction.
2. Differential Thermal Analysis (DTA)
3. plasticity chart,
4. Scanning Electron Microscope,
5. Specific surface etc.

charge Deficiencies;

The cations in the octahedral or tetrahedral sheet can be replaced by different kinds of cations without change in crystal structure. This is known as charge deficiencies.

Clay-water interrelation:

1. Hydrogen bond
2. Ion hydration
3. Osmotic pressure

Soil Structure

Soil structure:

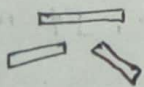
Soil structure describes the arrangement of the solid parts of the soil and the pore space located between them. It is determined by how individual soil grains bind together and aggregate, resulting in the arrangement of soil pores between them.

Soil fabric:

Soil fabric refers only to the geometric arrangement of soil particles.

Terminology:

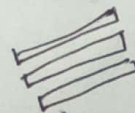
1. Dispersed fabric: No face to face association of clay mineral.
2. Flocculated fabric: Edge to Edge or Edge to face Association
3. Aggregated: Face to Face association



Dispersed



Flocculated



Aggregated

Fabric of Natural Clay soil:

1. Domain
2. cluster
3. ped
4. Silt grains
5. Micropore
6. Macropore

Fabric of Granular soil:

1. Loose packing.
2. Dense packing.
3. Honey combed fabric.

Soil Water

Soil water: Soil acts as a sponge to take up and retain water.

This water in soil is called soil water.

Soil water may be in the forms of ^{Gravitational} water and held water.

Gravitational water: This type of water is free to move through the pore space of soil mass under the influence of gravity.

Held water: This type of water is held in the proximity of the surface of the soil grains by certain forces of attraction.

Gravitational water may be subdivided into (i) Free water
(ii) capillary water

Free water: It has the usual properties of liquid water. It moves all time under the influence of gravity.

Capillary water: water which is in a suspended condition, held by the force of surface tension within the pores of soil, is called capillary water.

Held water can be classified as.

- (i) structural water.
- (ii) Absorbed water.

Structural water: ~~is~~ water that is chemically combined as a part of the crystal structure of the mineral of the soil grains is called structural water.

Note that, a moisture content $w=0$, the moist unit weight, $\gamma =$ the dry unit weight, γ_d , or,

$$\gamma = \gamma_d (w=0) = \gamma_d$$

When the moisture content is gradually increased and the same compactive effort is used for compaction, the weight of the soil solids in a unit volume gradually increases.

For example,

$$\text{at } w = w_1 \quad \gamma = \gamma_2$$

$$\therefore \gamma_d (w=w_1) = \gamma_d (w=0) + \Delta \gamma_d$$

Beyond a certain moisture content $w=w_2$, any increase in the moisture content tends to reduce the dry unit weight.

This phenomenon occurs because the water takes up the spaces that would have been occupied by the solid particles. The moisture content at which maximum dry unit weight is attained is generally referred to as the optimum moisture content.

Objectives of soil compaction: 2017

Compaction is applied to improve the properties of an existing soil or in the process of placing fill. The main objectives are to:

1. increase shear strength and therefore bearing capacity.
2. increase stiffness and therefore reduce future settlement.
3. decrease voids ratio and so permeability, thus reducing potential frost heave.

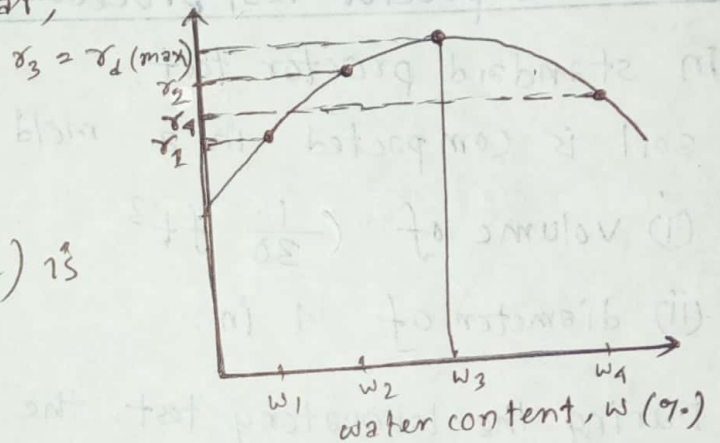
■ Optimum moisture content: 14, 07

The dry unit first increases as the moisture content increases, After a certain moisture content, any increase in moisture content tends to reduce the dry unit weight, So The moisture content at which maximum dry unit weight is attained is called optimum moisture content.

■ Explain compaction curve: 15

In the standard proctor test if we plotted (γ_d) vs. (w) in graph, then we can see that,

When percentage of water content is increased, the value of dry unit weight (γ_d) is also increased.



At one point, for a certain percentage of water content, the dry unit weight (γ_d) will be maximum.

After this water content, if we increase the percentage of water content, the value of dry unit weight (γ_d) will be decreased.

Field of compaction:

- (i) Foundation of many engineering structures.
- (ii) construction of highway, railway,
- (iii) construction of any earth structure (embankments, dams)
- (iv) For any filling work.
- (v) Airport runway.
- (vi) loose soils etc.

Standard proctor Test procedure:

In standard proctor test,

soil is compacted in a mold that has a

(i) volume of $\frac{1}{30} \text{ ft}^3$.

(ii) diameter of 4 in.

During the laboratory test, the mold is attached to a base plate at the bottom and to an extension at the top.

The dry soil of weight of 3 Kg is mixed with ~~with~~

varying amounts of water and then

The soil is compacted in three equal layers by a hammer, and each layer is compacted of 25 blows by the hammer.

The hammer has a

- (i) mass of 5.5 lb (ii) drop of 12 inch.

For each test, the moist unit weight of compaction, γ can be calculated as

$$\gamma = \frac{W}{V(m)}$$

Where,
 W = weight of compacted soil
 $V(m)$ = volume of mold

with known moisture content, dry unit weight can also be calculated as,

$$\gamma_d = \frac{\gamma}{1 + w}$$

The values of γ_d determined from the equation can be plotted against corresponding moisture contents to obtain the maximum dry unit weight and optimum moisture content for the soil.

■ Zero air void line: 13, 08, 07, 05

A line which shows the relation between dry unit weight and water content of compacted soil containing zero percentage of air, is known as zero air void line.

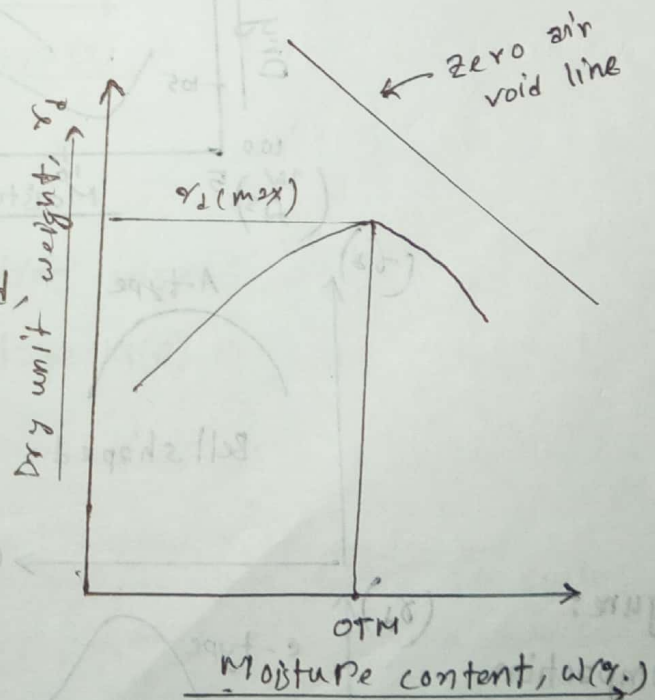
For a given moisture content w and degree of saturation S , γ_d is calculated as follows:

$$\gamma_d = \frac{G_s \gamma_w}{1 + \frac{G_s w}{S}}$$

When zero air void is obtained,

$S = 1$, the equation becomes,

$$\gamma_{zav} = \frac{G_s \gamma_w}{1 + G_s w}$$



■ Air void line: 14

A line which shows the relation between (γ_d) and (w) of compacted soil containing certain percentage of air, is known as Air void line.

Factors affecting the compaction: 2017, 01

(i) Moisture content: Moisture content has a strong influence on the degree of compaction achieved by a given soil.

Besides moisture content, other important factors that affect compaction are:

(ii) soil type: The soil type - that is grain size distribution, shape of the soil grains, specific gravity of soil solids and type of clay mineral present - has a great influence on the maximum dry unit weight and optimum moisture content.

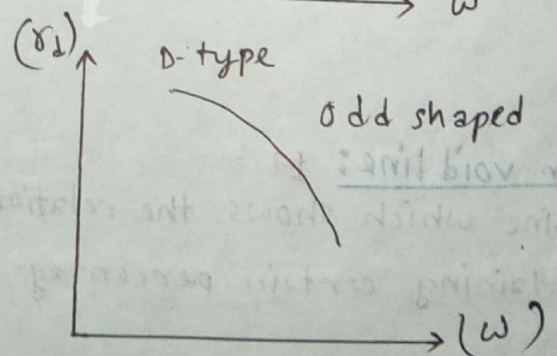
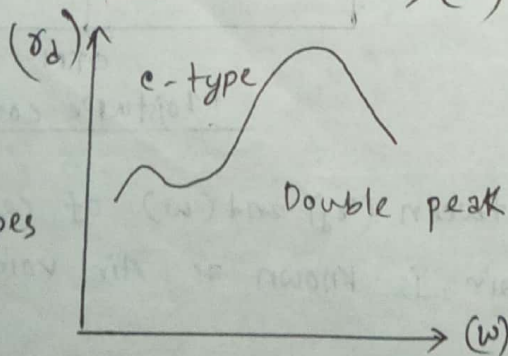
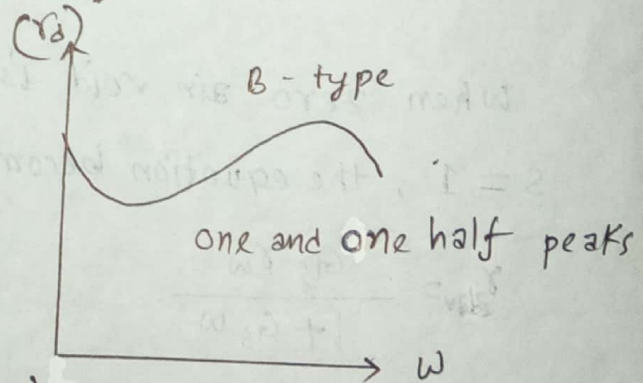
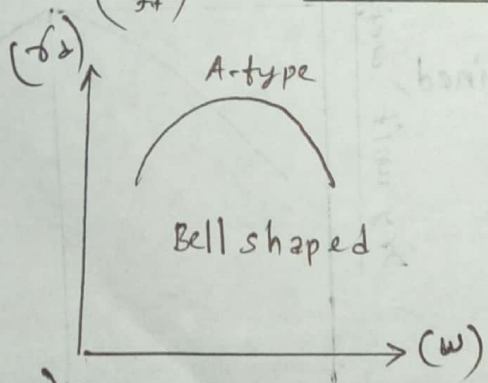
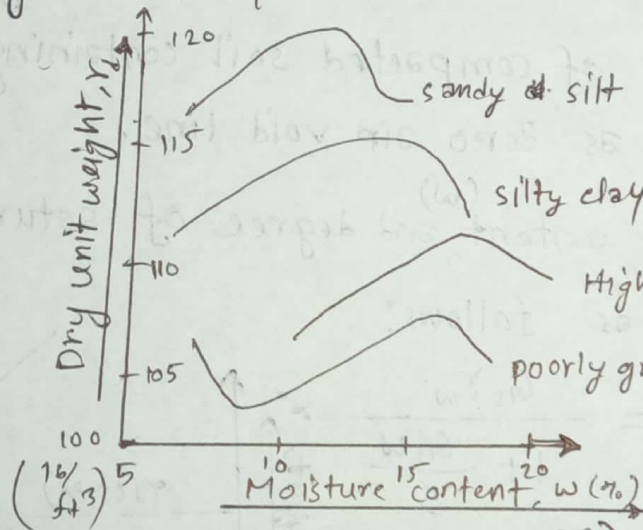


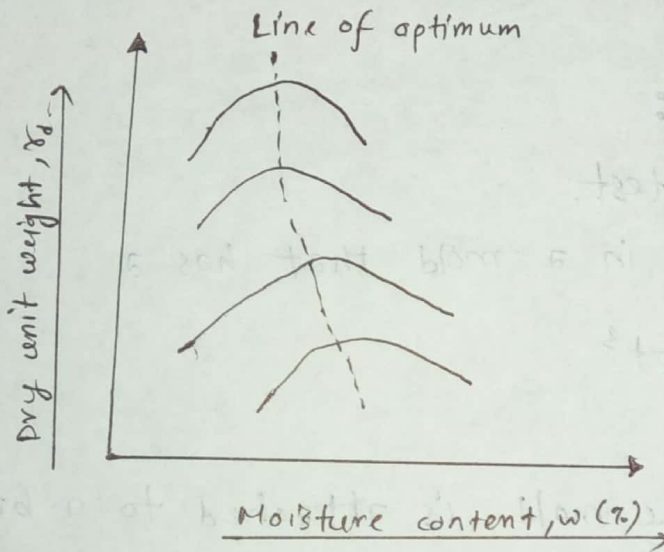
Figure:
Compaction curves of different types of soils

(ii) effect of compaction effort:

The compaction energy per unit volume used for the standard proctor test described as -

$$E = \frac{(\text{No. of blows/layer}) \times (\text{No. of layer}) \times (\text{weight of hammer}) \times (\text{Height of drop})}{\text{Volume of mold}}$$

If the compaction effort per unit volume of soil is changed, the moisture unit weight curve also changes.



Types of compaction curve:

There are different types of compaction curve:

- (i) Bell-shaped (A)
- (ii) one and one half peaks (B)
- (iii) Double peak (C)
- (iv) odd shaped (D)

Bell shaped: This type of curve is generally found for soils that have a Liquid Limit between 30 and 70

one and one-half peaks: This type of curve is found for soils that have a Liquid Limit less than 30

Double peak: This type of curve is found for soils that have a Liquid Limit greater than 70

Odd shaped: This type of curve do not have a definite peak and this curve is found for soils that have a liquid limit greater than 70.

Figure: (previous question)

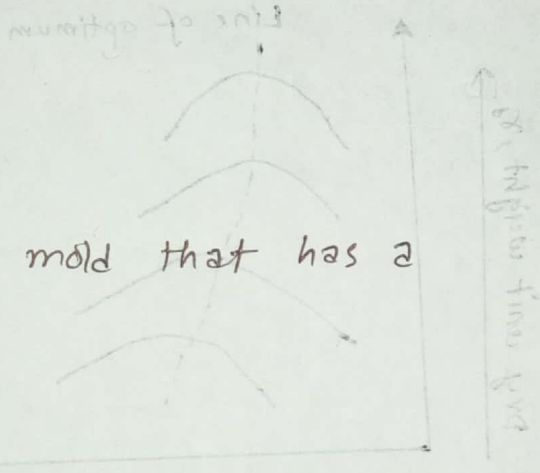
Modified proctor Test:

In modified proctor test,

soil is compacted in a mold that has a

(i) volume of, $\frac{1}{30} \text{ ft}^3$

(ii) diameter of 4 in



During the test, the mold is attached to a base plate at the bottom and to an extension at the top.

The dry soil of weight of 3 kg is mixed with varying amounts of water and then

the soil is compacted in five layers by a hammer and the number of blows for each layer is kept at 25 as in case of standard proctor test.

The hammer has a (i) mass of 10 lb (ii) drop of 18 in

Then the weight of the compacted soil is measured and

the moist unit weight of compaction, (γ) is calculated as follows:

$$\gamma = \frac{W}{V_{(m)}}$$

where,

W = weight of compacted soil

$V_{(m)}$ = volume of mold.

With known moisture content, dry unit weight (γ_d) can be calculated as,

$$\gamma_d = \frac{\gamma}{1 + w}$$

the values of γ_d determined from the equation can be plotted against corresponding moisture contents to obtain maximum γ_d and optimum moisture content.

Compaction Energy of proctor test: 08, 10

The compaction energy per unit volume can be given as -

$$E = \frac{N_b \times N_L \times W_h \times H_d}{V_{(m)}}$$

where,

N_b = No. of blows / layer

N_L = No. of Layers

W_h = Weight of hammer

H_d = Height of drop

For standard proctor test,

$$E_s = \frac{25 \times 3 \times 5.5 \times 1}{\left(\frac{1}{30}\right)}$$

$$= 12375 \text{ lb-ft/ft}^3$$

For modified proctor test,

$$E_m = \frac{25 \times 5 \times 10 \times 1.5}{\left(\frac{1}{30}\right)} = 56250 \text{ lb-ft/ft}^3$$

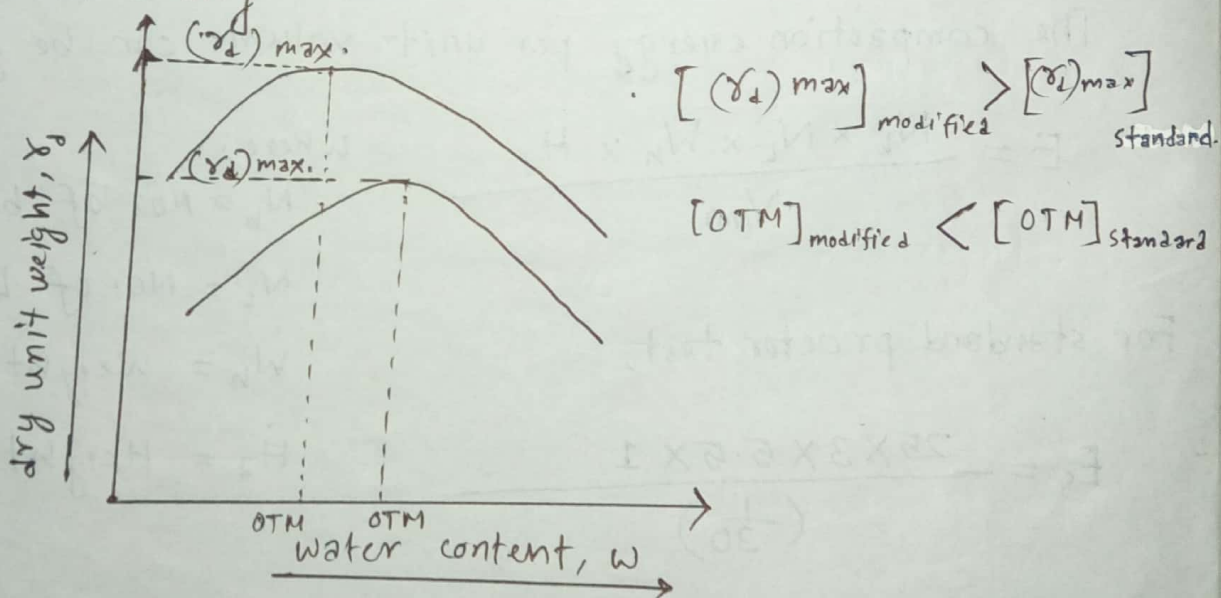
Now,

$$\frac{E_s}{E_m} = \frac{12375}{56250} = 0.22$$

Qualitative diagram of proctor test: 14, 11

Modified proctor test has a compaction energy of 56250 ft-lb/ft³. Which is 4.55 times of the compaction energy of 12375 ft-lb/ft³ in the standard proctor test. Likewise increased the compaction energy in the field will increase the maximum dry unit weight and decrease the associated optimum water content.

If the both results of proctor test plot in a single graph paper, it will easily be obtained:



■ Comparison between Standard and Modified proctor test: 14

Description	Standard	Modified
Weight of hammer	5.5 lb	10 lb
Height of drop	12 in	18 in
Number of soil layers	3	5
Energy	12375 ft-lb/ft ³	56250 ft-lb/ft ³

■ Compaction Equipment: 15, 01

1. Smooth wheel roller: * Drums are connected
* suitable for proof rolling subgrades and for finishing work of fills with sandy and clayey soils.

* provide 100% area coverage.

* ground contact pressure: (45 to 55) psig

* not suitable for producing high unit weights of compaction when used on thicker layers.

2. Pneumatic rubber-tired roller:

* The former are heavily loaded with several rows of tires.

* These tires are closely spaced - 4 to 6 in a row.

* can be used for sandy and clayey soil compaction.

* provide 70 to 80% Area coverage.

* contact pressure : (85 to 100) psi.

* compaction is achieved by a combination of pressure and kneading action.

3. Sheepsfoot roller: * connected with drums and the projections are set on drums.

* Projection Area : (4 to 13) in²

* suitable for clayey soil compaction.

* contact pressure : (200 to 1000) psi.

* provide 100% Area coverage.

4. Vibratory roller: * can be attached to any roller to provide vibratory effect to the soil.

* vibration is produced by rotating off-center weights.

5. Hand held vibrating plates:

* can be used for effective compaction of granular soil over a limited area.

* vibrating plate are gang-mounted on machines, that can be used in less restricted area.

Factors affecting Field Compaction:

- (i) Soil Type
- (ii) Moisture content
- (iii) Thickness of lift
- (iv) The intensity of pressure applied by compacting equipment.
- (v) The area over which the pressure is applied

These factors are important because the pressure applied at the surface decreases with depth, which results in a decrease in the degree of soil compaction.

During compaction, the dry unit weight of soil also is affected by the number of roller passes. The dry unit weight of soil at a given moisture content increases to a certain point with the number of roller passes. Beyond this point, it remains approximately constant. In most cases, about 10 to 15 roller passes yield the maximum dry unit weight economically attainable.

specification for field compaction: 2017

1. ⁰¹ Relative compaction; In most specifications for earthwork, the contractor is instructed to achieve a compacted field dry unit weight of 90 to 95% of the maximum dry unit weight determined in the laboratory by either standard or modified proctor test. This is a specification for relative compaction.

Relative compaction is the percentage ratio of the field density to the maximum density as determined in the laboratory. It is expressed as:

$$R(\%) = \frac{\gamma_d(\text{field})}{\gamma_d(\text{max-lab})} \times 100$$

2. Relative density: For the compaction of granular soils, specifications sometimes are written in terms of the relative density D_r . Relative density is expressed as

$$D_r = \left[\frac{\gamma_d(\text{field}) - \gamma_d(\text{min})}{\gamma_d(\text{max}) - \gamma_d(\text{min})} \right] \left[\frac{\gamma_d(\text{max})}{\gamma_d(\text{field})} \right]$$

Relative compaction in terms of relative density,

$$R = \frac{R_0}{1 - D_r(1 - R_0)} \quad \text{where, } R_0 = \frac{\gamma_d(\text{min})}{\gamma_d(\text{max})}$$

Determination of Field unit weight of compaction:

Sand cone method: 12,07

The sand cone device consists of a glass or plastic jar with a metal cone attached at its top.

1. The jar is filled with uniform dry Ottawa sand and the combined weight of jar, the cone and the sand filling the jar is determined (W_1)

2. In the field, a small hole is excavated in the area where soil has been compacted.

3. If the weight of the moist soil excavated from the hole (W_2) is determined, the dry unit weight of the soil (w_3) can be obtained as:

$$w_3 = \frac{W_2}{1+w}$$

4. After excavation of hole, the cone with sand filled jar attached to it, is inverted and placed over the hole. Sand is allowed to flow out of the jar to fill the hole and the cone. After that, combined weight of jar, cone and remaining sand in the jar is determined (W_4).

$$W_5 = W_1 - W_4$$

where, W_5 = weight of the sand to fill the hole and cone.

5. The volume of the excavated hole can be determined

$$V = \frac{W_5 - W_c}{\gamma_d(\text{sand})}$$

where, W_c = weight of sand fill the cone only.

$\gamma_d(\text{sand})$ = dry unit weight of

ottawa sand used

The values of W_c and $\gamma_d(\text{sand})$ determined from the calibration done in the laboratory.

6. The dry unit weight of compaction made in the field can be determined as follows:

$$\gamma_{(Field)} = \frac{W_3}{V} = \frac{\text{Dry weight of the soil excavated from the hole}}{\text{Volume of the hole}}$$

Special compaction technique:

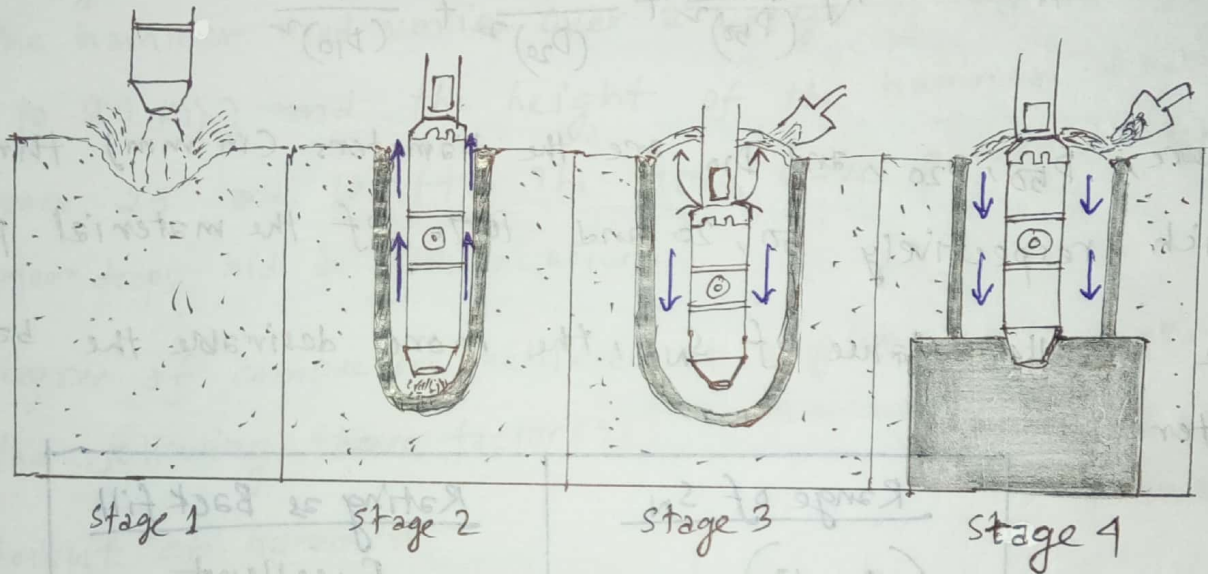
05 Vibroflotation: Vibroflotation is a technique for in situ densification of thick layers of loose granular soil deposits.

The process involves the use of a vibroflot unit (vibrating unit), which is about 7 ft long. The vibrating unit has an electric weight inside it and can develop a centrifugal force, which enables the vibrating unit to vibrate horizontally. There are openings at the bottom and top of the vibrating unit for water jets. The vibrating unit is attached to follow up pipe.

The entire vibroflotation compaction in the field can be divided into four stages:

Stage 1: The jet at the bottom of the vibrofloat is turned on and lowered into the ground.

Stage 2: The water jet creates a quick condition in the soil and it allows the vibrating unit to sink into ground.



Stage 3: Granular material is poured from the top of the hole. The water from the lower jet is transferred to the jet at the top of the vibrating unit. This water carries the granular material down the hole.

Stage 4: The vibrating unit is gradually raised in about 1ft lifts and held vibrating for about 30 seconds at each lift. The process compacts the soil to the desired unit weight.

■ Suitability Number: The grain-size distribution of the backfill material is an important factor that controls the rate of densification. Brown (1977) has defined a quantity called suitability number for rating backfills as.

$$S_N = 1.7 \sqrt{\frac{3}{(P_{50})^2} + \frac{1}{(D_{20})^2} + \frac{1}{(D_{10})^2}}$$

where, P_{50} , D_{20} , and D_{10} are the diameters (in mm) through which, respectively 50, 20 and 10% of the material passes.

The smaller value of S_N , the more desirable the backfill material.

<u>Range of S_N</u>	<u>Rating as Backfill</u>
(0-10)	Excellent
(10-20)	Good
(20-30)	Fair
(30-50)	Poor
> 50	Unsuitable

14, 08

■ Placement Water content: The water content used in the field compaction is called placement water content which is equal to, lower than or higher than optimum moisture content determined in the laboratory.

Dynamic Compaction:

Dynamic compaction is a technique that has gained popularity in United States for the densification of granular soil deposits.

The process consists primarily of dropping a heavy weight repeatedly on the ground at regular intervals. The weight of the hammer used varies over a range of 80 to 360 kN (18 to 80 kip) and the height of the hammer drop varies between 25 and 100 ft. The stress waves generated by the hammer drops aid in densification.

The degree of compaction achieved at a given site depends on the following three factors:

1. Weight of hammer
2. Height of hammer drop
3. Spacing of locations at which the hammer is dropped

The significant depth of influence for compaction,

$$D \approx \frac{1}{2} \sqrt{W_H h} \quad \text{where, } W_H = \text{dropping weight (metric ton)}$$

$h = \text{height of drop (m)}$

In English units,

$$D = 0.61 \sqrt{W_H h} \quad \text{where, } W_H \text{ in kip}$$

$D \text{ and } h \text{ in ft}$

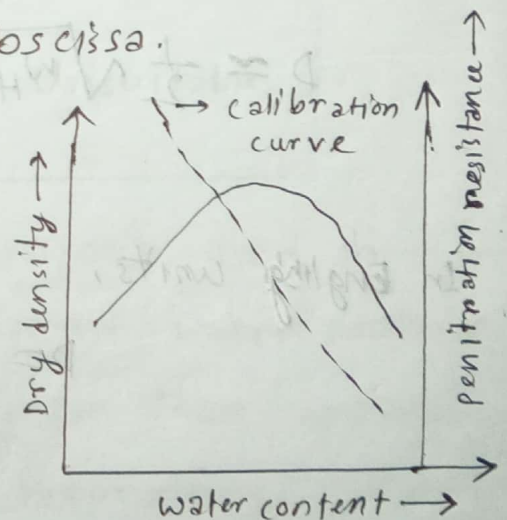
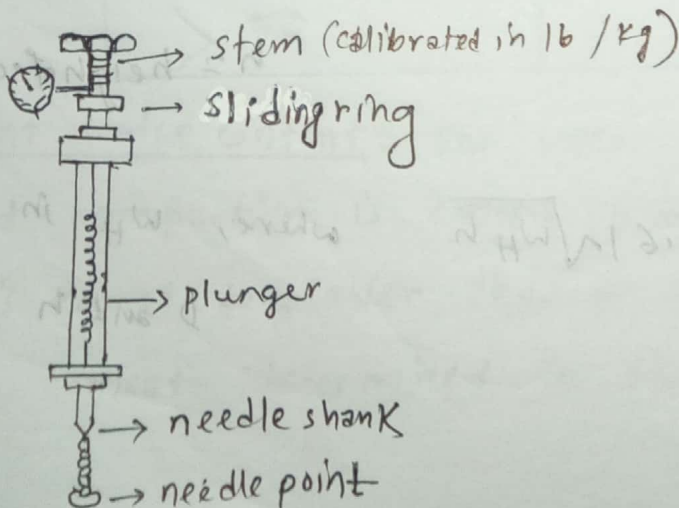
Field Compaction control: 05

Field compaction can be controlled by two methods:

1. calcium carbide method
2. Proctor needle method.

Proctor needle method:

1. It consists of a needle point attached to a graduated needle shank which in turn is attached to a spring loaded plunger.
2. The needle point is available in varying cross sectional area so that a wide range of penetration resistance can be measured.
3. The penetration force is read on a loaded gauge fixed over the handle.
4. To use the needle in the field, a calibration curve is plotted in the laboratory between penetration resistance as ordinate and water content as abscissa.



Soil Compaction

2009, 2015
2006, 2005,

The relative compaction of a sand in the field is 95%. The maximum and minimum dry unit weights of sand are $\gamma_d(\max) = 17 \text{ KN/m}^3$ and $\gamma_d(\min) = 15 \text{ KN/m}^3$. For the field conditions, determine (i) dry unit weight (ii) relative density of compaction (iii) moist unit weight at a moisture content of 10%.

Solutions:

(i) we know that,
$$R(\%) = \frac{\gamma_d(\text{field})}{\gamma_d(\text{max-lab})} \times 100$$

$$\Rightarrow 95 = \frac{\gamma_d(\text{field})}{17} \times 100$$

$$\therefore \gamma_d(\text{field}) = \frac{95 \times 17}{100} = 16.15 \text{ KN/m}^3 \quad (\text{Ans})$$

(ii) we know that,

$$D_r = \left[\frac{\gamma_d(\text{field}) - \gamma_d(\min)}{\gamma_d(\max) - \gamma_d(\min)} \right] \times \left[\frac{\gamma_d(\max)}{\gamma_d(\text{field})} \right]$$

$$\Rightarrow D_r = \frac{16.15 - 15}{17 - 15} \times \frac{17}{16.15}$$

$$\therefore D_r = 0.605 \quad (\text{Ans})$$

(iii) we know that,

$$\gamma_d = \frac{\gamma}{1+w}$$

$$\Rightarrow \gamma = \gamma_d (1+w)$$

$$\therefore \gamma = 16.15 \times (1 + 0.10) = 17.765 \text{ KN/m}^3$$

(Ans)

#Ex-6.1 (2010)

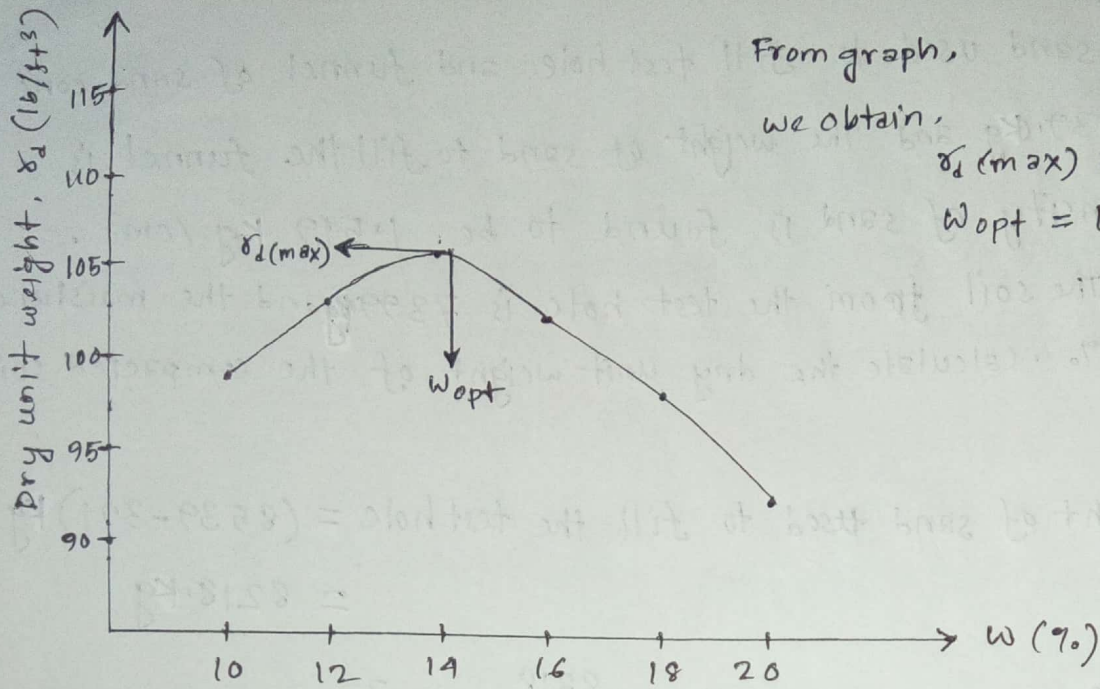
The laboratory test results of a standard proctor test are given in the following table:

Volume of mold (ft ³)	Weight of moist soil in mold (lb)	Moisture content, w (%)
$\frac{1}{30}$	3.63	10
$\frac{1}{30}$	3.86	12
$\frac{1}{30}$	4.02	14
$\frac{1}{30}$	3.98	16
$\frac{1}{30}$	3.88	18
$\frac{1}{30}$	3.73	20

Determine the maximum dry unit weight of compaction and optimum moisture content.

Solution:

Volume, V (ft ³)	Weight, W (lb)	Moist unit weight $\gamma = \frac{W}{V}$ (lb/ft ³)	Moisture content, w (%)	Dry unit weight $\gamma_d = \frac{\gamma}{1+w}$ (lb/ft ³)
$\frac{1}{30}$	3.63	108.9	10	99
$\frac{1}{30}$	3.86	115.8	12	103.4
$\frac{1}{30}$	4.02	120.6	14	105.8
$\frac{1}{30}$	3.98	119.4	16	102.9
$\frac{1}{30}$	3.88	116.4	18	98.6
$\frac{1}{30}$	3.73	111.9	20	93.3



From graph,
we obtain,

$$\gamma_d(\max) = 106 \text{ lb/ft}^3$$

$$w_{opt} = 14.2 \%$$

(Ans.)

Ex-6.7 (2011), 2013, 2014

The backfill material for a vibroflotation project has the grain sizes:

$D_{10} = 0.36 \text{ mm}$, $D_{20} = 0.52 \text{ mm}$ and $D_{50} = 1.42 \text{ mm}$. Determine the suitability

Number. Also determine the backfill rating.

Solution:

We know that,

$$S_N = 1.7 \sqrt{\frac{1}{(D_{10})^2} + \frac{1}{(D_{20})^2} + \frac{3}{(D_{50})^2}}$$

$$\Rightarrow S_N = 1.7 \sqrt{\frac{1}{(0.36)^2} + \frac{1}{(0.52)^2} + \frac{3}{(1.42)^2}}$$

$$\therefore S_N = 8.1$$

Suitability Number is between 0 to 10, Hence Rating as
backfill material is Excellent.

(Ans.)

2012, 2014

The weight of sand used to fill test hole and funnel of sand cone device is 8539.0g and The weight of sand to fill the funnel is 321.0g. The density of sand is found to be 1.549 g/cm³. The weight of the soil from the test hole is 7399g and the moisture content is 15%. Calculate the dry unit weight of the compacted soil.

Solution:

$$\text{The weight of sand used to fill the test hole} = (8539 - 321) \text{ g} \\ = 8218.0 \text{ g}$$

$$\text{Volume of the test hole, } V = \frac{8218}{1.549} \text{ cm}^3 \\ = 5305.36 \text{ cm}^3$$

$$\text{Moist density of compacted soil, } \rho = \frac{7399}{5305.36} \text{ g/cm}^3 \\ = 1.3946 \text{ g/cm}^3$$

$$\text{Moist unit weight of compacted soil, } \gamma = \rho g = \left(\frac{1.3946 \times 981}{1000} \right) \text{ N/cm}^3 \\ \Rightarrow \gamma = 1.3681 \times 10^{-2} \text{ N/cm}^3$$

$$\therefore \text{Dry unit weight of compacted soil, } \gamma_d = \frac{\gamma}{1+w}$$

$$\Rightarrow \gamma_d = \frac{1.3681 \times 10^{-2}}{1+0.15}$$

$$\Rightarrow \gamma_d = 1.19 \times 10^{-2} \text{ N/cm}^3$$

$$\therefore \gamma_d = 11.9 \text{ kN/m}^3 \text{ (Ans.)}$$

2017

A soil in the borrow pit has a water content of 11.7% and the dry density of 16.65 KN/m^3 . If 2070 m^3 of soil is excavated from it and compacted in an embankment at a porosity of 0.33. calculate the compacted volume of the embankment that can be constructed out of this volume of soil.

Solution:

Given that,

Volume of soil excavated from borrow pit, $V = 2070 \text{ m}^3$

and dry unit weight of soil at borrow pit, $\gamma_d = 16.65 \text{ KN/m}^3$

$$\therefore \text{Dry weight of soil required at the embankment, } W = V \gamma_d$$
$$\Rightarrow W = (2070 \times 16.65) \text{ KN}$$
$$\therefore W = 34465.5 \text{ KN}$$

Now,

compacted volume of the embankment that can be constructed

$$\text{out, } V = \frac{W}{\gamma_d} = \frac{W(1+e)}{G_s \gamma_w} \quad \left[\because \gamma_d = \frac{G_s \gamma_w}{1+e} \right]$$

Given that, porosity of soil at the embankment, $n = 0.33$

$$\therefore \text{void ratio, } e = \frac{n}{1-n} = \frac{0.33}{1-0.33} = 0.49$$

Let, specific gravity of soil at borrow pit, $G_s = 2.7$

Hence, compacted volume of the embankment that can be constructed

$$\text{out, } V = \frac{34465.5 \times (1+0.49)}{2.7 \times 9.81} = 1938.82 \text{ m}^3$$

(Ans.)

Problem-6.6: (8th edition - B.M DAS)

The in situ moist unit weight of a soil is 17.3 kN/m^3 and the moisture content is 16% . The specific gravity of soil solids is 2.72 . This is to be excavated and transported to a construction site for use in a compacted fill. If the specification calls for the soil to be compacted to a minimum dry density of 18.1 kN/m^3 at ^{the} same moisture content of 16% , how many cubic meters of soil from the excavation site are needed to produce 2000 m^3 of compacted fill? How many 20 ton truck loads are needed to transport the excavated soil?

Solution: Given that, $\gamma_{(\text{insitu})} = 17.3 \text{ kN/m}^3$, $w = 16\%$, $\gamma_d(\text{compacted}) = 18.1 \text{ kN/m}^3$

$$\therefore \gamma_d(\text{in situ}) = \frac{17.3}{1 + 0.16} = 14.914 \text{ kN/m}^3$$

Dry weight of soil required at the construction site, $W = V\gamma_d$
 $= (2000 \times 18.1) \text{ kN}$
 $= 36200 \text{ kN}$

\therefore volume of soil to be excavated from ^{embankment} a site, $V = \frac{W}{\gamma_d}$

$$= \frac{36200}{14.914} \text{ m}^3$$

$$= 2427.25 \text{ m}^3$$

(Ans.)

\therefore Number of truck loads = $\frac{\text{Weight of moist soil to be transported}}{20 \text{ ton}}$

$$= \frac{(2427.25 \times 17.3)}{20 \times 2 \times 0.4536 \times 9.81} = 235.00$$

(Ans.)

Problem-6.7

A Proposed embankment fill requires 5000 m^3 of compacted soil. The void ratio of compacted fill is specified as 0.75. Soil can be transported from one of the four borrow pits as described in the following table:

Borrow Pit	void ratio, e	G_s	cost (TK/m ³)
I	0.8	2.65	900
II	0.9	2.68	600
III	1.1	2.71	700
IV	0.85	2.74	1000

(a) Determine the volume of each borrow pit soil required to meet the specification of embankment site.

(b) Make necessary calculations to select the borrow pit which would be cost effective.

Solutions: (a) Dry weight of soil required at the embankment site:

$$W = V \gamma_d = 5000 \times \frac{G_s \gamma_w}{1+e} = 5000 \times \frac{9.81}{1+0.75} \times G_s = 28028.57 G_s$$

Borrow pit	W_s (KN)	γ_d at borrow pit $= \frac{G_s \gamma_w}{1+e}$ (KN/m ³)	Volume to excavated from borrow pit, $V = (W_s / \gamma_d)$	cost/m ³ (TK)	Total cost (TK)
I	74275.71	14.44	5143.75	900	46,29,375
II	75116.57	13.83	5431.42	600	32,58,852
III	75957.425	12.66	5999.797	700	41,99,857.9
IV	76798.28	14.53	5285.5	1000	52,85,500

(b) cost effective:

Compressibility of Soil

Farhad
#150045

¹¹
Compressibility of soil: When a compressive load is applied to a soil mass, a decrease in its volume takes place. The property of soil mass pertaining to its susceptibility to decrease in volume under pressure is known as compressibility of soil.

causes of compressibility: 11

The compression is caused by:

- (a) deformation of soil particles.
- (b) relocation of soil particles.
- (c) expulsion of water or air from the void spaces.

Types of settlement: 04, 08, 11, 13

1. Immediate settlement: This type of settlement is caused by elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Immediate settlement calculations are based on theory of elasticity.

¹⁴
2. Primary consolidation settlement: This type of settlement is the result of a volume change in saturated cohesive soils because of expulsion of water that occupies the void spaces.

3. Secondary consolidation settlement: This type of settlement is observed in saturated cohesive soils and is the result of plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

Fundamentals of consolidation:

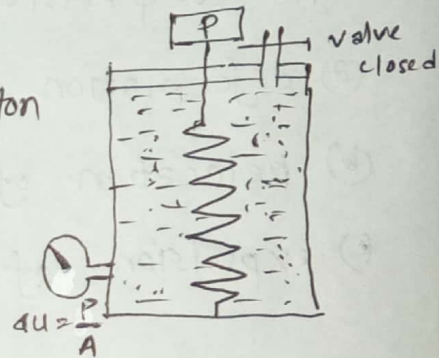
Time - dependent deformation of soil: (Spring cylinder model)

Time dependent deformation of saturated clayey soil best can be understood by considering a simple model that consist of a cylinder with a spring in its center.

Let the inside area of the cross section of cylinder = A

The cylinder is filled with water and has a frictionless water tight piston and valve.

At this time, if we place a load P on the piston and keep the valve closed, the entire load will be taken by the water in the cylinder, because water is incompressible. The



spring will not go through any deformation. The excess hydrostatic pressure at this time, $\Delta u = \frac{P}{A}$

The value observed in the pressure gauge attached to cylinder,

In general we can write, $P = P_s + P_w$ where,

so, when valve is closed, after placement of load P ,

$P_s =$ load carried by spring

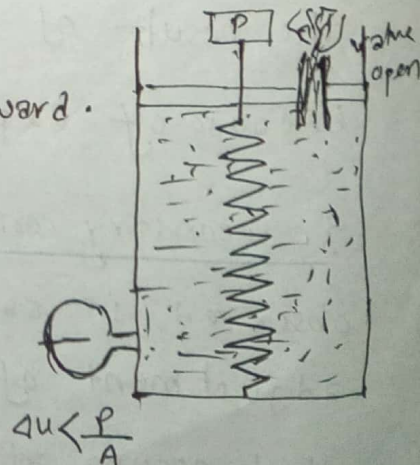
$P_w =$ (by water)

$$P_s = 0 \text{ and } P = P_w$$

Now, if valve is opened, the water will flow outward.

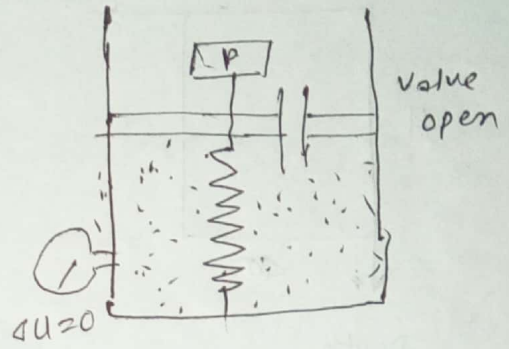
At this time, $P_s > 0$ and $P > P_w$

$$\text{Hence, } \Delta u < \frac{P}{A}$$



After some time, $\Delta u = 0$ and the system will reach a state of equilibrium.

$$P_s = P \quad \text{and} \quad P_w = 0$$



Stress diagram at different time condition:

consider a layer of saturated clay of thickness = H which is confined between two layers of sand.

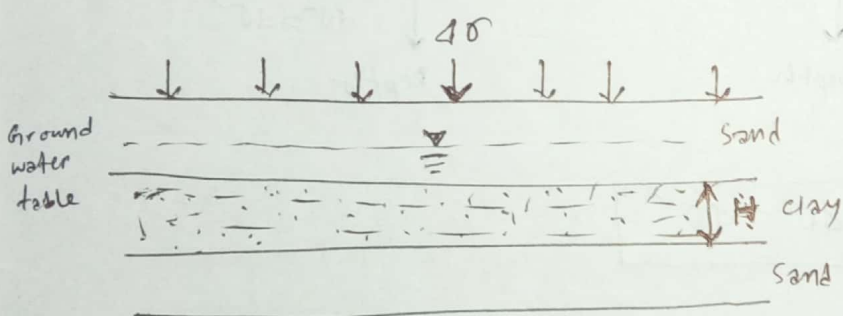
subjected an instantaneous increase of total stress = $\Delta \sigma$

$$\Delta \sigma = \Delta \sigma' + \Delta u$$

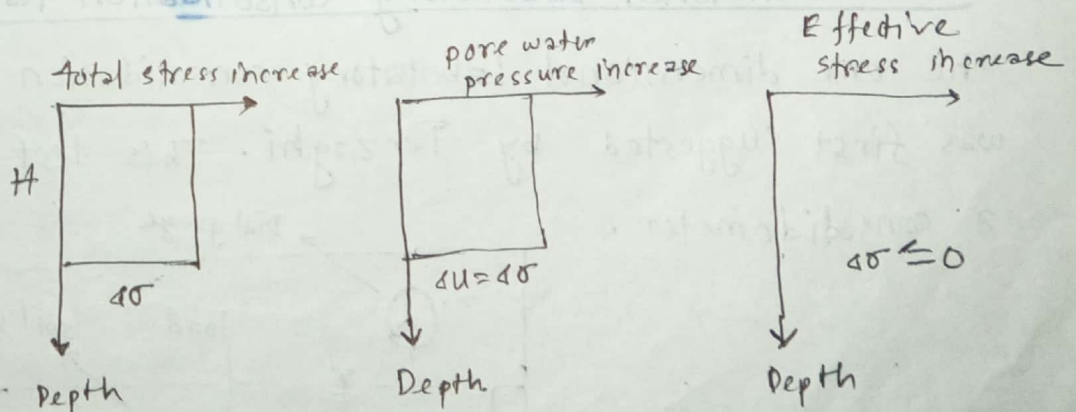
where,

$\Delta \sigma'$ = increase in effective stress

Δu = increase in the pore water pressure

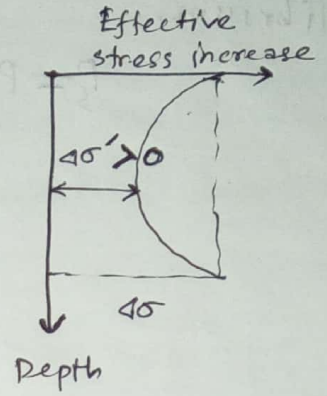
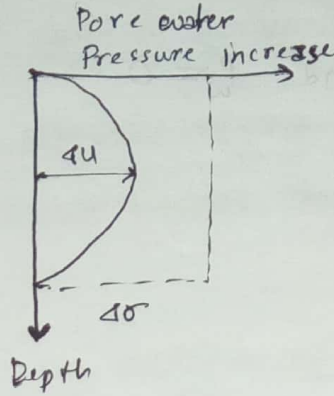
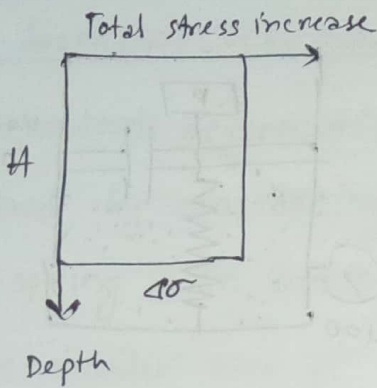


At time $t = 0$



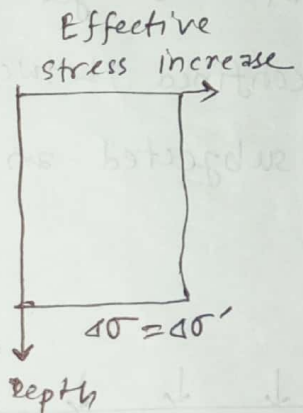
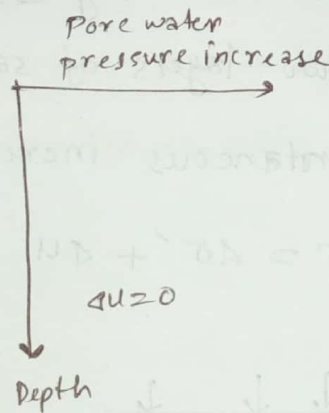
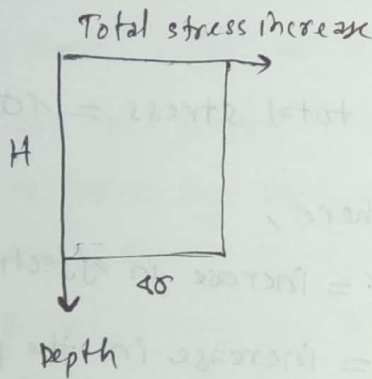
$$\Delta \sigma = \Delta u$$

At time $0 < t < \infty$



$$\Delta\sigma = \Delta\sigma' + \Delta u$$

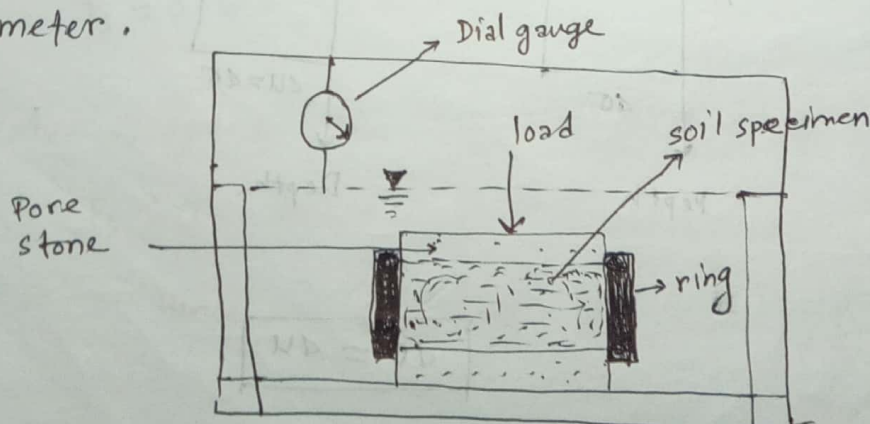
At time $t = \infty$



$$\Delta\sigma = \Delta\sigma'$$

One dimensional Laboratory Consolidation test:

The one dimensional Laboratory consolidation testing procedure was first suggested by Terzaghi. This test is performed in a consolidometer.

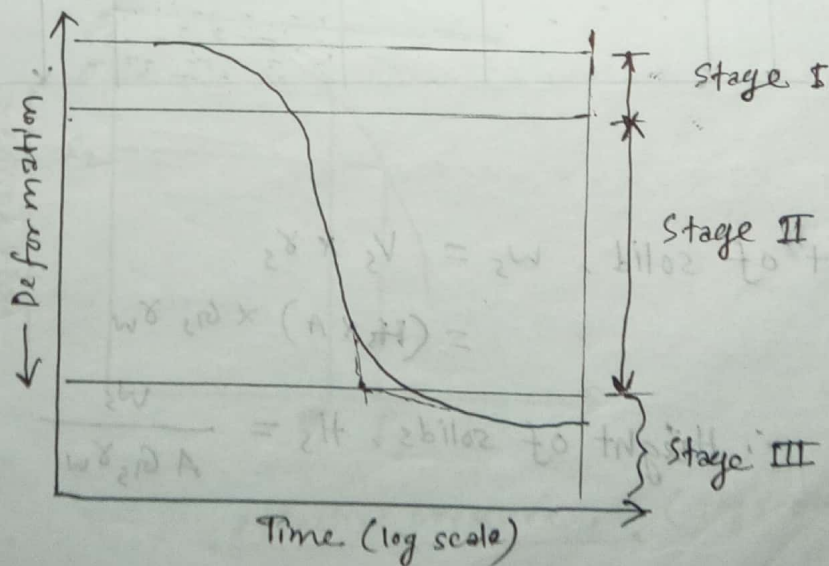


The specimens are usually 2.5 inch in diameter and 1 inch thick

Test procedure:

1. The soil specimen is placed inside a metal ring with two porous stones.
2. The load on the specimen is applied through a lever arm and compression is measured by a micrometer dial gauge.
3. The specimen is kept under water during the test, each load is kept for 24 hours.
4. After that, the load is usually doubled, which doubles the pressure on the specimen and the compression measurement is continued.
5. At the end of the test, the dry weight of test specimen is determined.

General shape of the plot of deformation of the specimen against time for a given load increment:



we can observe three distinct stage from the plot:

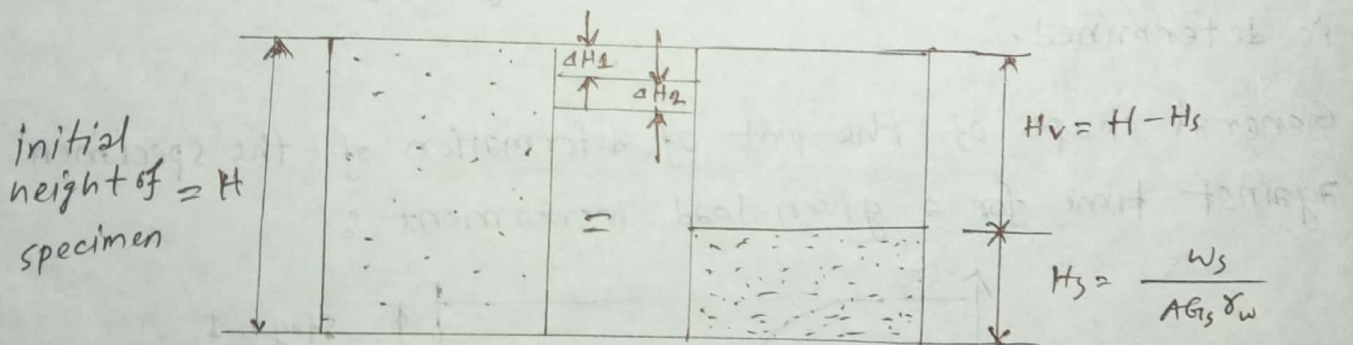
stage I: Initial compression, which is caused ^{mostly} by preloading.

stage II: Primary consolidation, during which excess pore water pressure gradually is transferred into effective stress because of the expulsion of pore water.

stage III: Secondary consolidation, which occurs after complete dissipation of the excess pore water pressure, when some deformation of the specimen takes place because of the plastic readjustment of soil fabric.

Plotting procedure of e - $\log P$ curve:

Let us, consider the following figure:



Calculation:

$$I. \text{ Weight of solid, } w_s = V_s \times \gamma_s$$

$$= (H_s \times A) \times G_s \gamma_w$$

$$\therefore \text{ Height of solids, } H_s = \frac{w_s}{A G_s \gamma_w}$$

2. Initial height of voids as, $H_v = H - H_s$

3. initial void ratio, $e_0 = \frac{V_v}{V_s} = \frac{H_v \cdot A}{H_s \cdot A} = \frac{H_v}{H_s}$

4. For first increment loading P_1 , which causes a deformation

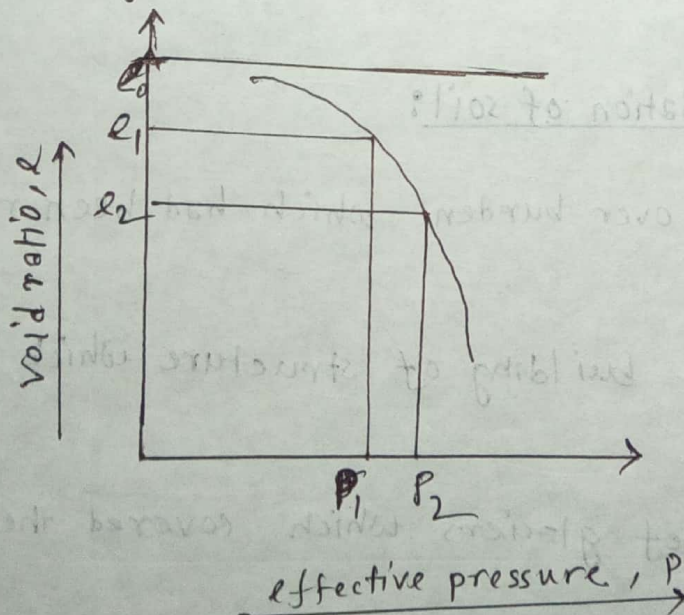
ΔH_1 change in void ratio, $\Delta e_1 = \frac{\Delta H_1}{H_s}$

5. New void ratio, $e_1 = e_0 - \Delta e_1$

6. For next loading P_2 , the void ratio at the end of consolidation,

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

The pressure p and corresponding void ratio e at the end of the consolidation are plotted on semilogarithmic graph paper where pressure p as abscissa and void ratio e as ordinate, e -log- p graph is obtained shown below:



2002

Q Difference between Normally consolidated and over consolidated soil.

Normally Consolidated	Over consolidated
1. If the present effective overburden pressure is the maximum minimum pressure that the soil was subjected to in the past, it is called normally consolidated soil.	1. If present effective overburden pressure is less than that which the soil experienced in the past, it is called over consolidated soil.
2. Liquidity Index lies between 0 to 1.	2. Liquidity Index varies between 0 to 0.6.
3. Change of void ratio is considerable.	3. Change of void ratio is negligible.
4. Over consolidation ratio is less than 1.	4. Over consolidation ratio is more than 1.

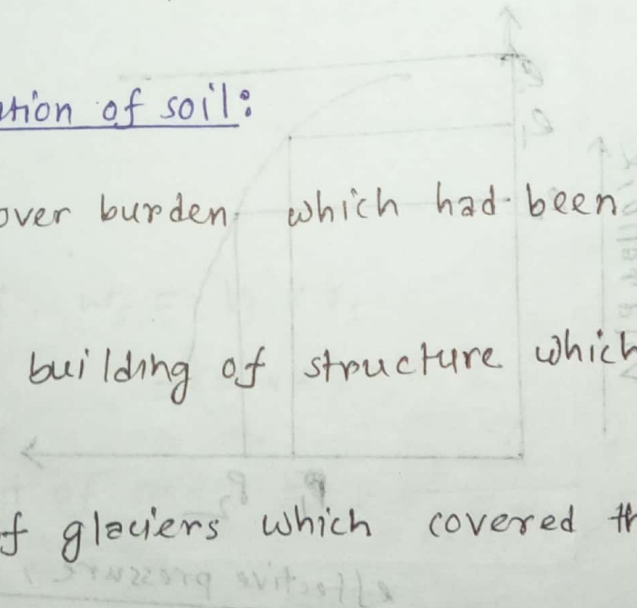
2010, 17

Q Pre consolidation pressure: The maximum effective past pressure is called pre consolidation pressure.

2002

Q causes of pre consolidation of soil:

1. Due to geologic over burden which had been removed later by erosion.
2. Due to loads of building of structure which had been destroyed.
3. Due to melting of glaciers which covered the soil deposit in the past.



4. Due to capillary pressure acting on the soil in past but removed later to rise in water table.

02, 04, 08, 09

Determination of preconsolidation pressure from e-log-P plot:
(Casagrande's simple graphic construction)

Procedure:

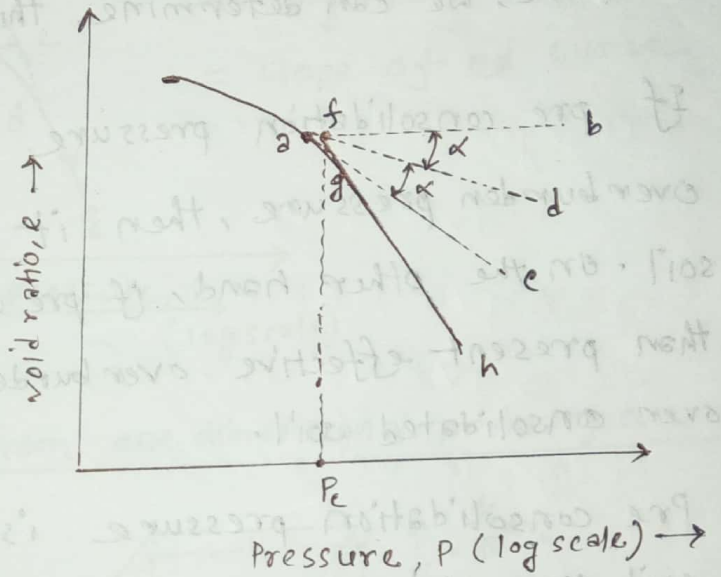
1. By visual observation, point a is established at which the e-log P has a minimum radius of curvature.

2. ~~The~~ The horizontal line ab is drawn.

3. A line ac is drawn which is tangent at a.

4. A line ad is drawn which is bisector of angle bac.

5. The straight portion gh of e-log-P plot is extended to intersect line ad at f. The abscissa of point f is the preconsolidation pressure.



Over consolidation ratio: The ratio of pre-consolidation pressure of a specimen to the present effective vertical pressure is defined as over consolidation ratio.

$$OCR = \frac{P_c}{P} \quad \text{where, } P_c = \text{pre consolidation pressure}$$

$P = \text{present effective vertical pressure}$

02, 03, 05, 06, 08, 17

☐ Importance of pre-consolidation pressure in settlement analysis:

In settlement analysis, it is important to know the soil type as well as its consolidation history. The soil may be normally consolidated, pre consolidated or over consolidated. By pre consolidation pressure, we can determine this classification of soil.

If pre consolidation pressure is less than present effective overburden pressure, then it is said to be normally consolidated soil. On the other hand, if pre consolidation pressure is greater than present effective overburden pressure, it is called over consolidated soil.

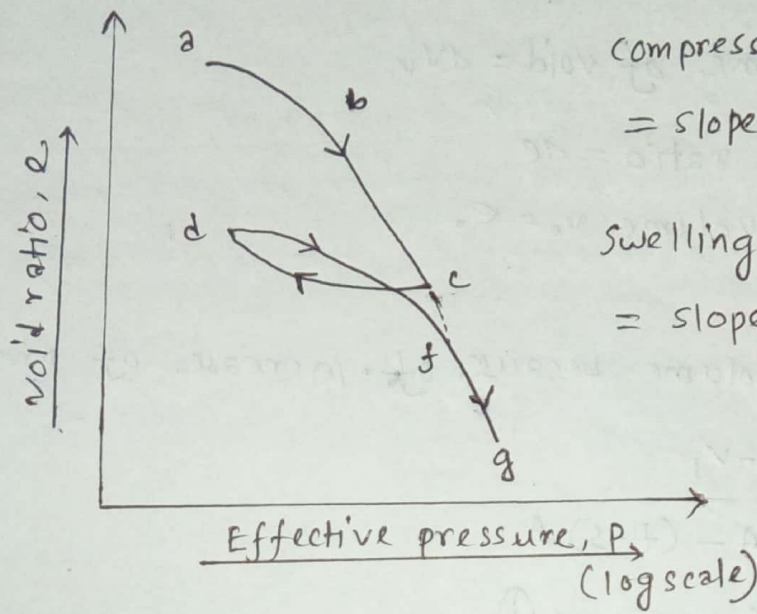
Pre consolidation pressure is used in many calculation of soil properties essential for structural analysis and soil mechanics. One of the primary uses of it is to predict settlement of a structure after loading.

It is important to know the pre consolidation pressure because it will help to determine the amount of loading that is appropriate for the site.

☐ e-log-P curve:

If the pressure P is plotted as abscissa in logarithmic scale and void ratio e is plotted as ordinate in normal scale. Then a curve is obtained, known as e-log-P curve.

☐ Loading, unloading and reloading showing in $e \log P$ curve:



Compression index, C_c
= slope of ac curve

Swelling index, C_s
= slope of cd curve

04, 05, 07, 08, 11

☐ Calculation of settlement from one-dimensional primary consolidation

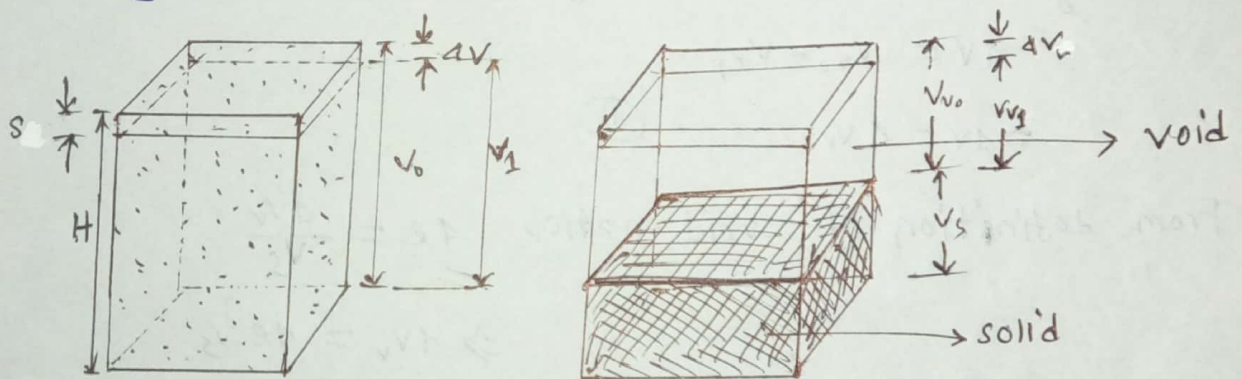


Fig. settlement caused by one dimensional consolidation

Let, thickness of clay layer = H

cross sectional area = A

Average effective overburden pressure = P_0

Initial volume = V_0

Final volume = V_1

Primary settlement = S

change in volume = ΔV

Initial void volume = V_{v0}

Final void volume = V_{v1}

change in volume of void = ΔV_v

change in void ratio = Δe

void ratio at volume $V_0 = e_0$

Now, change in volume because of increase of pressure, Δp

$$\Delta V = V_0 - V_1$$

$$\Rightarrow \Delta V = HA - (H-S)A$$

$$\Rightarrow \Delta V = AS \quad \text{--- (i)}$$

change in volume = change in volume of voids

$$\Delta V = V_{v0} - V_{v1}$$

$$\Rightarrow \Delta V = \Delta V_v \quad \text{--- (ii)}$$

From definition of void ratio,

$$e = \frac{\Delta V_v}{V_s}$$

$$\Rightarrow \Delta V_v = e \Delta V_s$$

$$\Rightarrow \Delta V_v = e \cdot \frac{V_0}{1+e}$$

$$\therefore \Delta V_v = \Delta e \cdot \frac{AH}{1+e} \quad \text{--- (iii)}$$

Equating (i) & (iii)

$$AS = \Delta V_v$$

$$\Rightarrow AS = \Delta e \cdot \frac{AH}{1+e} \quad \text{[from (iii)]}$$

$$\Rightarrow S = H \cdot \frac{\Delta e}{1+e} \quad \text{--- (iv)}$$

For normally consolidated clays that exhibit slope of e - $\log P$ plot

$$c_c = \frac{\Delta e}{\log\left(\frac{P_0 + \Delta P}{P_0}\right)} \Rightarrow \Delta e = c_c \log\left(\frac{P_0 + \Delta P}{P_0}\right)$$

Hence, Equation (IV) becomes,

$$S = \frac{H c_c}{1 + e_0} \log\left(\frac{P_0 + \Delta P}{P_0}\right)$$

But for over consolidated clay,

If $P_0 + \Delta P \leq P_c$ and then, slope of e - $\log P$ curve,

$$c_s = \frac{\Delta e}{\log\left(\frac{\Delta P + P_0}{P_0}\right)}$$

$$\Rightarrow \Delta e = c_s \log\left(\frac{\Delta P + P_0}{P_0}\right)$$

Hence, equation (IV) becomes,

$$S = \frac{H c_s}{1 + e_0} \log\left(\frac{P_0 + \Delta P}{P_0}\right)$$

If $P_0 + \Delta P > P_c$ then,

$$S = \frac{H c_s}{1 + e_0} \log\left(\frac{P_c}{P_0}\right) + \frac{H c_c}{1 + e_0} \log\left(\frac{P_0 + \Delta P}{P_c}\right)$$

compression index, e_c : 17

The modulus of the slope of the virgin compression curve is commonly called the compression index (e_c).

Skempton (1944) suggested the following empirical expression for the compression index for undisturbed clays:

$$e_c = 0.009 (LL - 10) \quad \text{where, } LL = \text{Liquid Limit}$$

swell index, e_s :

swell index is a measure of increase of volume due to removal of pressure.

It was expressed by Nagaraj and Murty (1985) as,

$$e_s = 0.0463 \left[\frac{LL(\%)}{100} \right] e_c$$

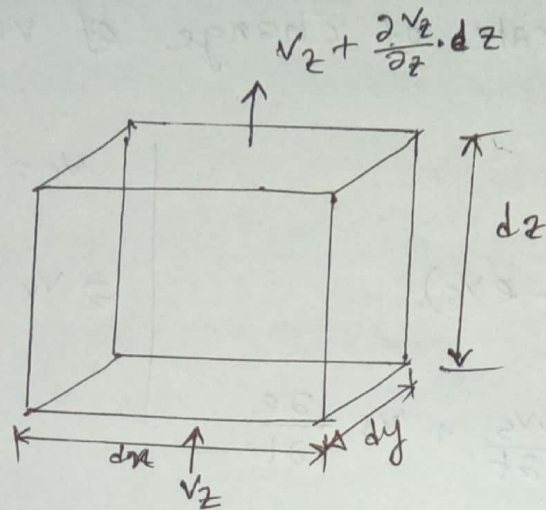
In most cases,

$$e_s \approx \left(\frac{1}{5} \text{ to } \frac{1}{10} \right) e_c$$

Assumption for one dimensional consolidation: (Terzaghi proposed)

1. soil sample system is homogenous.
2. soil sample is completely saturated.
3. compressibility of water is negligible.
4. compressibility of soil grains is negligible.
5. The flow of water is in one direction only.
6. Darcy's law is valid.

09
 Differential equation of Terzaghi's one dimensional consolidation theory



Let volume of soil element = V

velocity of flow in z direction = v_z

co-efficient of permeability = K

For the soil element shown,

Rate of outflow of water - Rate of inflow in water = Rate of volume change

$$\left(v_z + \frac{\partial v_z}{\partial z} \cdot dz \right) dx dy - v_z \cdot dx dy = \frac{\partial V}{\partial t}$$

$$\Rightarrow \frac{\partial v_z}{\partial z} dx dy dz = \frac{\partial V}{\partial t} \dots \dots \textcircled{1}$$

From darcy law,

$$v_z = Ki = -K \cdot \frac{\partial h}{\partial z} = -\frac{K}{\gamma_w} \cdot \frac{\partial u}{\partial z}$$

$$\therefore \frac{\partial v_z}{\partial z} = -\frac{K}{\gamma_w} \cdot \frac{\partial^2 u}{\partial z^2} \dots \dots \textcircled{11}$$

$$\begin{aligned} u &= \gamma_w h \\ \Rightarrow h &= \frac{u}{\gamma_w} \\ \Rightarrow \frac{\partial h}{\partial z} &= \frac{\partial u}{\partial z} \cdot \frac{1}{\gamma_w} \end{aligned}$$

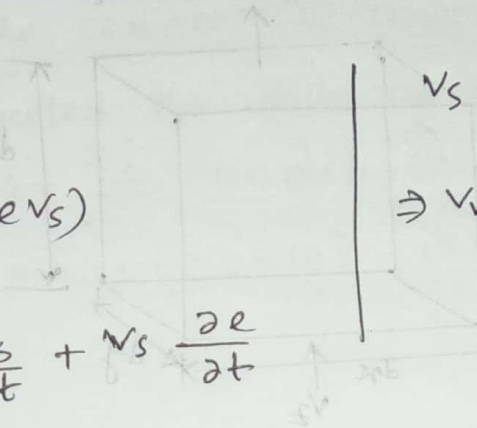
During consolidation, the rate of change in the volume of soil element = the rate of change of volume of voids

so,
$$\frac{\partial v}{\partial t} = \frac{\partial v_v}{\partial t}$$

$$\Rightarrow \frac{\partial v}{\partial t} = \frac{\partial}{\partial t} (v_s + e v_s)$$

$$\Rightarrow \frac{\partial v}{\partial t} = \frac{\partial v_s}{\partial t} + e \frac{\partial v_s}{\partial t} + v_s \frac{\partial e}{\partial t}$$

$$\Rightarrow \frac{\partial v}{\partial t} = v_s \cdot \frac{\partial e}{\partial t} \quad \left[\text{since, soil solids are incompressible, } \frac{\partial v_s}{\partial t} = 0 \right]$$



$$v_s = \frac{V_v}{1+e}$$

$$\Rightarrow v_v = v_s (1+e) = v_s + e v_s$$

The change in void ratio is linearly related with effective stress,

$$\partial e = a_v \cdot \partial (\Delta P') = -a_v \cdot \partial u$$

where, a_v = co-efficient of compressibility

$$\Rightarrow \frac{\partial e}{\partial t} = -a_v \cdot \frac{\partial u}{\partial t}$$

$$\therefore \frac{\partial v}{\partial t} = -a_v \cdot \frac{\partial u}{\partial t} \cdot v_s \quad \text{..... (11)}$$

Substituting $\frac{\partial v}{\partial t}$ and $\frac{\partial u}{\partial t}$ in equation (1) we get,

$$-\frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial z^2} \cdot dx dy dz = -a_v \cdot \frac{\partial u}{\partial t} \left[\frac{v}{1+e_0} \right] \quad \left(\because v_s = \frac{v}{1+e_0} \right)$$

$$\Rightarrow \frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial z^2} \cdot dx dy dz = a_v \cdot \frac{\partial u}{\partial t} \cdot \frac{dx dy dz}{1+e_0} \quad \left(\because v_0 = dx dy dz \right)$$

$$\Rightarrow \frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial z^2} = \left(\frac{a_v}{1+e_0} \right) \cdot \frac{\partial u}{\partial t}$$

$$\Rightarrow \frac{k}{\gamma_w} \cdot \frac{\partial^2 u}{\partial z^2} = m_v \cdot \frac{\partial u}{\partial t} \quad \left(\text{coefficient of volume compressibility, } m_v = \frac{\partial v}{\partial e} \right)$$

$$\Rightarrow \left(\frac{k}{m_v \cdot \gamma_w} \right) \cdot \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

$$\Rightarrow \frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \quad \left(\text{co-efficient of con-solidation, } c_v = \frac{k}{m_v \gamma_w} \right)$$

Applying boundary conditions,

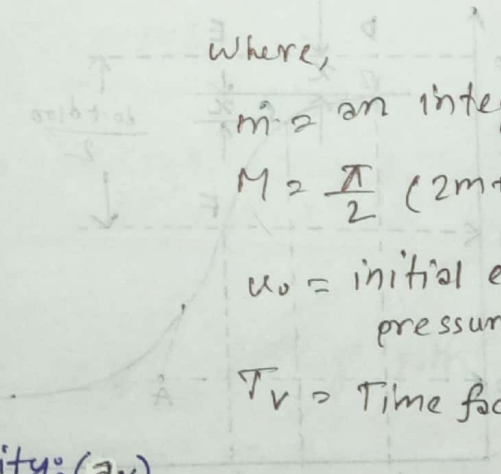
$$z=0, u=0$$

$$z=2H_{dr}, u=0$$

$$t=0, u=U_0$$

We get,

$$U = \sum_{m=0}^{\infty} \left[\frac{2U_0}{M} \sinh \left(\frac{Mz}{H_{dr}} \right) \right] \cdot e^{-M^2 T_v}$$



Where,

$m = \text{an integer}$

$$M = \frac{\pi}{2} (2m+1)$$

$U_0 = \text{initial excess pore water pressure}$

$$T_v = \text{Time factor} = \frac{c_v t}{H_{dr}^2}$$

17

Coefficient of compressibility: (a_v)

co-efficient of compressibility is a measure of relative volume change of a solid due to a pressure change.

Co-efficient of consolidation: 17

The co-efficient of consolidation, (c_v) generally decreases as the liquid limit of soil increases. The range of variation of c_v for a given liquid limit is wide.

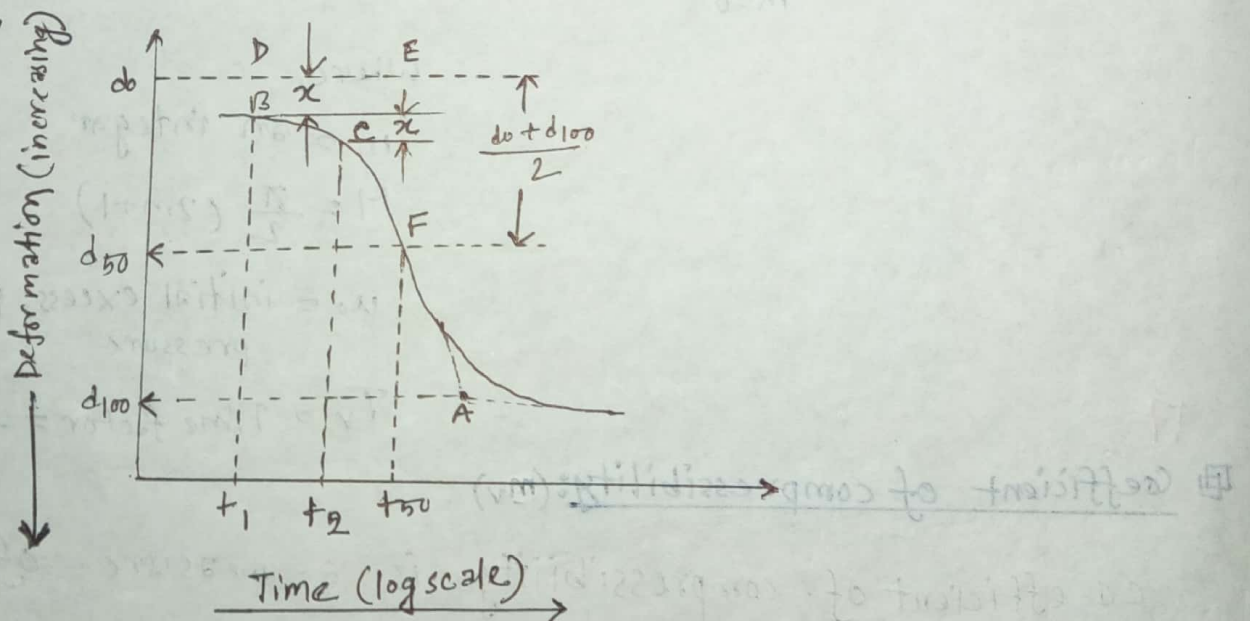
Determination of co-efficient of consolidation:

Two graphical methods commonly are used for determining c_v from laboratory one-dimensional tests.

- (i) Logarithm of time method.
- (ii) Square root of time method.

Logarithm of time method:

Logarithm of time method was proposed by Casagrande and Fadum (1940).



Procedure:

1. The straight line portions of primary and secondary consolidations are extended to intersect at A. The ordinate of A represents d_{100} .

2. Time t_1 and t_2 are selected on the curved portion such that $t_2 = 4t_1$. Let deformation during the time $(t_2 - t_1)$ being equal to α .

3. Horizontal line DE is drawn such that vertical distance $BD = \alpha$. The deformation corresponding to the line DE is d_0 .

4. The ordinate of point F represents d_{50} and abscissa represents corresponding time t_{50} .

5. For consolidation less than 50%.

$$\text{Time factor, } T_v = \frac{\pi}{9} \left(\frac{U\%}{100} \right)^2$$

$$\Rightarrow T_v = \frac{\pi}{9} \left(\frac{50}{100} \right)^2 \quad [U = 50\%]$$

$$T_v = 0.1963$$

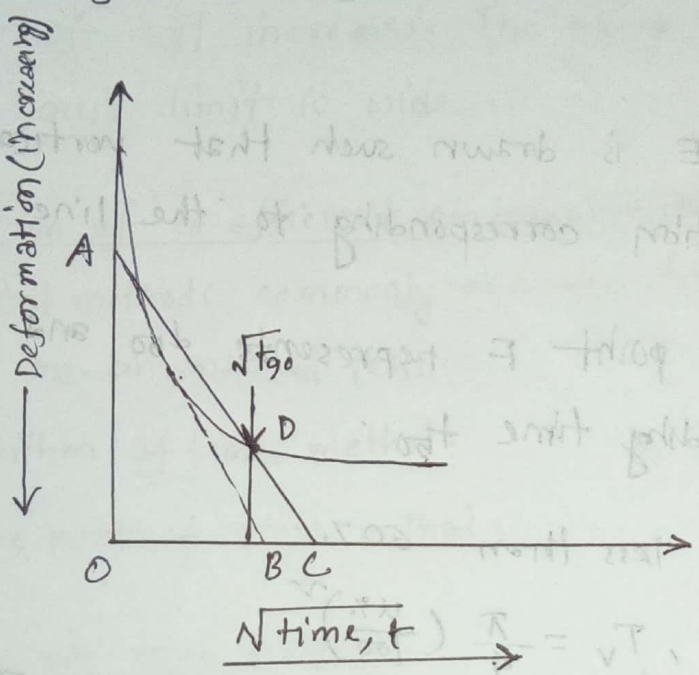
$$\text{Again, } T_v = \frac{c_v t_{50}}{H_{av}^2} \Rightarrow c_v = \frac{0.1963 H_{av}^2}{t_{50}}$$

where,

H_{av} = average longest drainage path during consolidation.

Square-root of time method:

Square-root of time method is proposed by Taylor (1942).



Procedure:

1. A line AB is drawn through the early portion of this curve.
2. A line AC is drawn such that $OC = 1.15 \cdot OB$
3. Line AC intersects the curve at D point. The ordinate of D point represents 90% consolidation (u_{90}) and abscissa represents $\sqrt{t_{90}}$
4. For consolidation greater than 60%.

Time factor, $T_v = 1.781 - 0.933 \log(100 - u\%)$

$\Rightarrow T_v = 1.781 - 0.933 \log(100 - 90) \quad [u = 90\%]$

$\Rightarrow T_{90} = 0.848$

Again,

$$T_v = \frac{C_v t_{90}}{H_{dr}^2}$$

$$\Rightarrow C_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

Here, C_v can be calculated.

Consolidation in sandy soil under stress:

1. When sandy soil layer is subjected to a stress increase, the pore water pressure is suddenly increased.
2. The drainage caused by the increase in pore water pressure is completed immediately.
3. Pore water drainage is accompanied by a reduction in the volume of soil mass, which results in settlement.
4. Because of rapid drainage of pore water in sandy soil, immediate settlement and consolidation occur simultaneously.

Consolidation in clay soil under stress:

1. When a saturated compressible clay layer is subjected to stress increase, elastic settlement occurs immediately.
2. The hydraulic conductivity of ^{clay} soil is significantly smaller than that of sand, so excess pore water pressure generated by loading gradually dissipates over a long period.
3. Associated volume change in clay layer may continue long after the immediate settlement.
4. The settle^{ment} caused by consolidation in clay layer may be several times greater than immediate settlement.

☐ Difference between consolidation and compaction:

Consolidation	Compaction
1. It is a gradual process	1. It is a fast process
2. Volume is reduced by static loading.	2. Volume is reduced by mechanical means.
3. Both water and air voids are removed.	3. Mainly air voids are removed
4. It is a natural process in which saturated soil deposits.	4. It is an artificial process which is applied to increase density of soil.
5. It is done by applying long time loading.	5. It is done by applying short time loading.

☐ Secondary consolidation settlement:

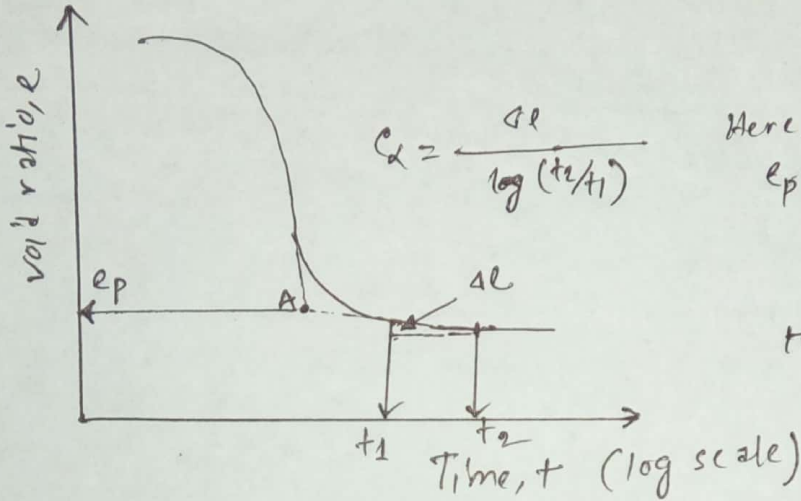
At the end of primary consolidation, that is, after complete dissipation of excess pore water pressure, some settlement is observed because of plastic adjustment of soil fabrics. This stage of consolidation is called secondary settlement.

During secondary consolidation, the plot of deformation against the log of time is practically linear.

(115) The secondary compression index can be defined as.

$$C_{\alpha} = \frac{\Delta e}{\log(t_2/t_1)}$$

where, Δe = change of void ratio
 t_1, t_2 = time
 C_{α} = secondary compression index.



Here,
 e_p = void ratio at the end of primary consolidation
 H = thickness of layer

Total magnitude of secondary consolidation, ^{settlement} can be calculated as,

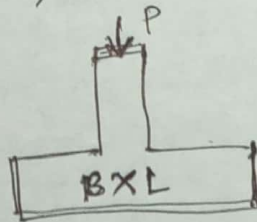
$$s_s = C'_{\alpha} H \log\left(\frac{t_2}{t_1}\right)$$

and where, $C'_{\alpha} = \frac{C_{\alpha}}{1+e_p}$

and $e_p = e_0 - \Delta e$

7] Elastic settlement: (immediate settlement)

$$s_e = PB \left(\frac{1 - \mu^2}{E_s} \right) \times I_p$$



where, μ = poisson's ratio.

P = pressure
 B = least dimension of foundation

E_s = modulus of elasticity

and, I_p = Non dimensional influence factor

$$I_p = \frac{1}{\pi} \left[m_1 \ln \left(\frac{1 + \sqrt{m_1^2 + 1}}{m_1} \right) + \ln \left(m_1 + \sqrt{m_1^2 + 1} \right) \right]$$

where, $m_1 = \frac{L}{B}$

Compressibility of soil

2005, 2007 Ex-11.6

For a normally consolidated clay layer in the field, the following are given: Thickness of clay layer = 8.5 ft; void ratio, $e_0 = 0.8$; compression index, $c_c = 0.28$; Average effective pressure on the clay layer, $p_0 = 2650 \text{ lb/ft}^2$; $4P = 970 \text{ lb/ft}^2$; secondary compression index, $c_\alpha = 0.02$
 what is the total settlement of the clay layer five years after the completion of primary settlement? Time for completion of primary settlement = 1.5 years

Solution:

We know that,
 for primary ^{consolidation} settlement, $S_p = \frac{4e H}{1+e_0}$

Here, $H = 8.5 \text{ ft}$, $e_0 = 0.8$ and

$$4e = c_c \log \left(\frac{p_0 + 4P}{p_0} \right) = 0.28 \times \log \left(\frac{2650 + 970}{2650} \right)$$

$$\therefore 4e = 0.038$$

$$\therefore S_p = \frac{0.038 \times 8.5}{1 + 0.8} = 0.179 \text{ ft}$$

Now, for secondary consolidation settlement,

$$S_s = c'_\alpha H \log \left(\frac{t_2}{t_1} \right)$$

Here, $c'_\alpha = \frac{c_c}{1+e_p}$ and $e_p = e_0 - 4e = (0.8 - 0.038) = 0.762$

Hence, $S_s = \frac{0.02}{1 + 0.762} \times 8.5 \times \log \left(\frac{5}{1.5} \right) = 0.05045 \text{ ft}$

\therefore Total settlement = $(S_p + S_s) = (0.179 + 0.05045) = 0.22945 \text{ ft} = 2.7534 \text{ in}$
 (Ans.)

2006
 # A 10 m thick clay layer beneath the foundation of a building is overlain by a permeable stratum and is underlain by an impervious rock. The coefficient of consolidation of the clay was found to be $0.035 \text{ cm}^2/\text{min}$. The final expected settlement for the layer is 90 mm. Determine (i) How much time will it take for 90% of the total settlement to take place. (ii) compute the settlement that would occur in one year.

Solution: (i) Due to impervious rock, Drainage is one way.

Hence, $H_{dr} = 10 \text{ m}$

given that, $c_v = 0.035 \text{ cm}^2/\text{min} = 0.035 \times 10^{-4} \text{ m}^2/\text{min}$

$U = 90\%$

For $U > 60\%$, we know that, $T_v = 1.781 - 0.933 \log(100 - U)$

$$\Rightarrow T_{90} = 1.781 - 0.933 \log(100 - 90)$$

$$\therefore T_{90} = 0.848$$

Now, $T_v = \frac{c_v t_v}{H_{dr}^2} \Rightarrow t_{90} = \frac{T_{90} H_{dr}^2}{c_v}$

$$\therefore t_{90} = \frac{0.848 \times (10)^2}{0.035 \times 10^{-4}} = 24228571.43 \text{ min}$$

$$= 46.097 \text{ years}$$

(ii) ~~Not sure~~ $t_v \propto U^2$?? $U = h \gamma_w$ $U = \frac{S_t}{S_p} \Rightarrow S_t = (0.9 \times 90) = 81 \text{ mm (Ans)}$

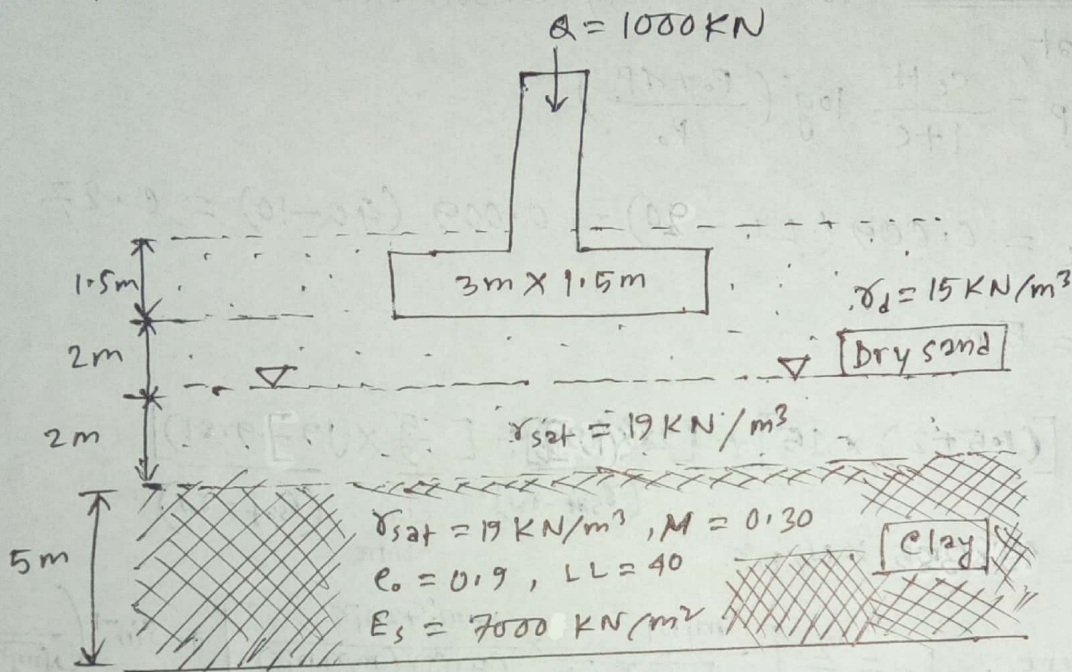
Not sure $\Rightarrow \frac{t_1}{t_2} = \frac{U_1^2}{U_2^2} = \frac{h_1^2}{h_2^2}$ $\Rightarrow \frac{46.097}{1} = \frac{81^2}{h_2^2}$

$\therefore h_2 = 11.93 \text{ mm (Ans)}$

Labels: Degree of consolidation, settlement at that time, Total settlement

2008 **Ex-11.14**

calculate the elastic and consolidation settlement of the 5 m thick clay layer that will result from the load carried by a footing measuring 3m x 1.5 m in plan. Assume the clay to be normally consolidated.



Solution:

Elastic settlement: We know that,

$$S_e = PB \frac{1 - \nu^2}{E_s} \times I_p$$

Here, $P = \frac{Q}{A} = \frac{1000}{3 \times 1.5} = 222.22 \text{ kN/m}^2$

$B = 1.5 \text{ m}$ (least dimension)

$\nu = 0.30$, $m_1 = \frac{L}{B} = \frac{3}{1.5} = 2$

$$I_p = \frac{1}{\pi} \left[m_1 \ln \left(\frac{1 + \sqrt{m_1^2 + 1}}{m_1} \right) + \ln \left(m_1 + \sqrt{m_1^2 + 1} \right) \right]$$

$$= \frac{1}{\pi} \left[2 \times \ln \left(\frac{1 + \sqrt{5}}{2} \right) + \ln \left(2 + \sqrt{5} \right) \right]$$

$\therefore I_p = 0.766$

Hence, $S_e = 222.22 \times 1.5 \times \frac{1 - 0.3^2}{7000} \times 0.766$

$\therefore S_e = 0.0332 \text{ m}$

consolidation settlement: for normally consolidated,

We know that,

$$S_p = \frac{c_c H}{1+e} \log \left(\frac{P_o + 4P}{P_o} \right)$$

Here, $c_c = 0.009 (LL - 10) = 0.009 (40 - 10) = 0.27$

$H = 5 \text{ m}$

$$P_o = [(1.5 + 2) \times 15] + [2 \times (19 - 9.8)] + \left[\frac{5}{2} \times (19 - 9.81) \right]$$

$(\gamma_{sat} - \gamma_w)$ clay centre $(\gamma_{sat} - \gamma_w)$

$\therefore P_o = 93.855 \text{ kN/m}^2$

Now, ΔP_{av} : Here, $I_q = \frac{2}{\pi} \left[\frac{m_1 n_1}{\sqrt{1+m_1^2+n_1^2}} \times \frac{1+m_1^2+2n_1^2}{(1+n_1^2)(m_1^2+n_1^2)} \right] + \sin^{-1} \left(\frac{m_1}{\sqrt{m_1^2+n_1^2} \sqrt{1+n_1^2}} \right)$
 radian mode

$m_1 = \frac{L}{B}$	$b = \frac{B}{2}$	z_1	$n_1 = \frac{z_1}{b}$	$P = \frac{Q}{A}$	I_q	$\Delta P = PI_q$
2	0.75	4.0	5.33	222.22	0.141	$\Delta P_1 = 31.33$
2	0.75	6.5	8.0	222.22	0.057	$\Delta P_2 = 14.89$
2	0.75	9.0	10.67	222.22	0.06	$\Delta P_3 = 13.33$

$\therefore \Delta P_{av} = \frac{\Delta P_1 + 4\Delta P_2 + \Delta P_3}{6} = \frac{31.33 + 4 \times 14.89 + 13.33}{6} = 17.37$

So, $S_p = \frac{0.27 \times 5}{1 + 0.9} \times \log \left(\frac{93.855 + 17.37}{93.855} \right) = 0.052 \text{ m}$
 (Ans.)

2008

For a normally consolidated clay, the following are given:

$$P_0 = 100 \text{ kN/m}^2, (P_0 + \Delta P) = 200 \text{ kN/m}^2, e_0 = 0.85, e = 0.74$$

The hydraulic conductivity of the clay for preceding loading range is $3.5 \times 10^{-5} \text{ cm/sec}$. (i) How long (in days) will it take for a 5 m thick clay layer (drained on top only) in the field to reach 90% consolidation? (ii) What is the settlement at that time.

Solution: (i) We know that,

$$\begin{aligned} \text{The co-efficient of compressibility, } m_v &= \frac{a_v}{1 + e_{av}} \\ &= \frac{\left(\frac{\Delta e}{\Delta P}\right)}{1 + \left(\frac{e_0 + e}{2}\right)} \\ &= \frac{\frac{(0.85 - 0.74)}{(200 - 100)}}{1 + \frac{0.85 + 0.74}{2}} \end{aligned}$$

$$m_v = 6.13 \times 10^{-4} \text{ m}^2/\text{kN}$$

Now, The co-efficient of consolidation,

$$C_v = \frac{T_v \cdot H_{dr}^2}{t_v} \quad \text{and also,}$$

$$C_v = \frac{K}{m_v \gamma_w} = \frac{3.5 \times 10^{-5} \times 10^{-2}}{6.13 \times 10^{-4} \times 9.81}$$

$$\therefore C_v = 5.822 \times 10^{-5} \text{ m}^2/\text{sec.}$$

Hence,

$$t_{90} = \frac{T_{90} H_{dr}^2}{C_v}$$

$$\text{Here, } U = 90\% \therefore T_{90} = 1.781 - 0.933 (100 - 90)$$

$$= 0.848$$

$$= \frac{0.848 \times 5^2}{5.822 \times 10^{-5}}$$

$$\therefore t_{90} = 3.64 \times 10^5 \text{ sec} = 4.215 \text{ days}$$

(ii) we know that,

$$S_p = \frac{4eH}{1+e_0}$$

$$= \frac{(0.85-0.74) \times 5}{1+0.85}$$

$$= 0.2973 \text{ m}$$

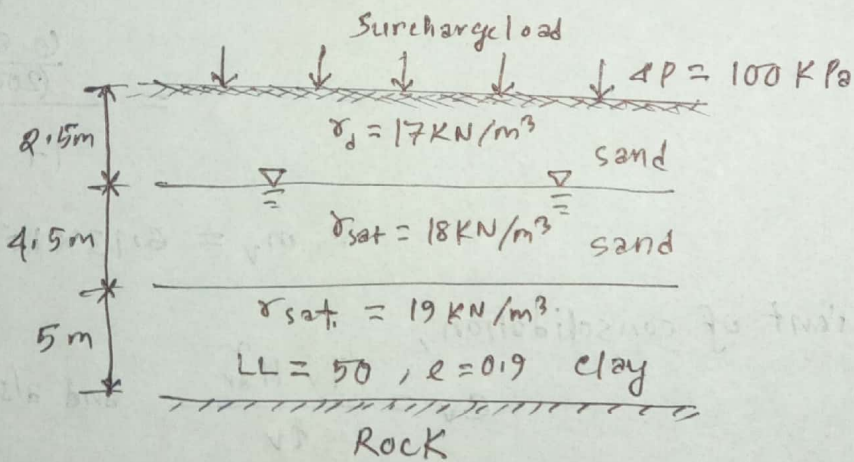
And, $U = \frac{S_t}{S_p} \Rightarrow S_t = U \times S_p = (0.9 \times 0.2973) = 0.27 \text{ m}$

(Ans.)

2013,
2011,

2009 **Ex-11.4** 2014,

A soil profile is shown in figure. The uniformly distributed load on the ground surface is $4P$. Estimate primary consolidation settlement if (a) the clay is normally consolidated (b) the pre consolidation pressure is 150 kPa ($e_s = \frac{1}{6} c_c$)



Solution:

(i) Here, $P_0 = 2.5 \times 17 + 4.5 (18 - 9.81) + \frac{5}{2} (19 - 9.81)$

$$\therefore P_0 = 102.33 \text{ kN/m}^2$$

Now, for normally consolidated clay,

$$S_p = \frac{c_c H}{1+e_0} \log \left(\frac{P_0 + 4P}{P_0} \right) \quad \text{Here, } c_c = 0.009 (LL - 10)$$

$$= \frac{0.36 \times 5}{1+0.19} \log \left(\frac{102.33 + 100}{102.33} \right)$$

$$= 0.36$$

$$(u=100) \therefore s_p = 0.28 \text{ m}$$

(Ans.)

(ii) Given that, preconsolidation pressure $p_c = 150 \text{ kPa}$

$$\text{Hence, } (p + \Delta p) = 202.33 > p_c = 150 \text{ kPa}$$

Now,

$$s_p = \frac{c_s H}{1+e_0} \log \frac{p_c}{p_0} + \frac{c_c H}{1+e_0} \log \frac{p+\Delta p}{p_c} \quad \left. \begin{array}{l} \text{Here, } c_s = \frac{1}{6} c_c \\ = \frac{0.36}{6} \\ = 0.06 \end{array} \right\}$$

$$= \frac{0.06 \times 5}{1+0.9} \log \left(\frac{150}{702.33} \right) + \frac{0.36 \times 5}{1+0.9} \log \left(\frac{202.33}{150} \right)$$

$$\therefore s_p = 0.15 \text{ m}$$

(Ans.)

2009 Ex-11.10

In laboratory test, on a clay specimen (drained on both sides),

the following results were obtained:

Thickness of clay layer = 50 mm.

$$p_1 = 100 \text{ kPa}, e_1 = 0.75$$

$$p_2 = 200 \text{ kPa}, e_2 = 0.71$$

Time for 90% consolidation $t_{90} = 6.0 \text{ min}$. Determine the hydraulic conductivity of the clay for the loading range.

Solution: We know that,

$$\text{Hydraulic conductivity, } k = c_v m_v \gamma_w$$

$$\text{Here, } m_v = \frac{\alpha_v}{1+e_{av}} = \frac{\left(\frac{\Delta e}{\Delta p}\right)}{1+\left(\frac{e_1+e_2}{2}\right)} = \frac{\frac{(0.75-0.71)}{(200-100)}}{1+\left(\frac{0.75+0.71}{2}\right)} = 2.31 \times 10^{-4} \text{ m}^2/\text{kN}$$

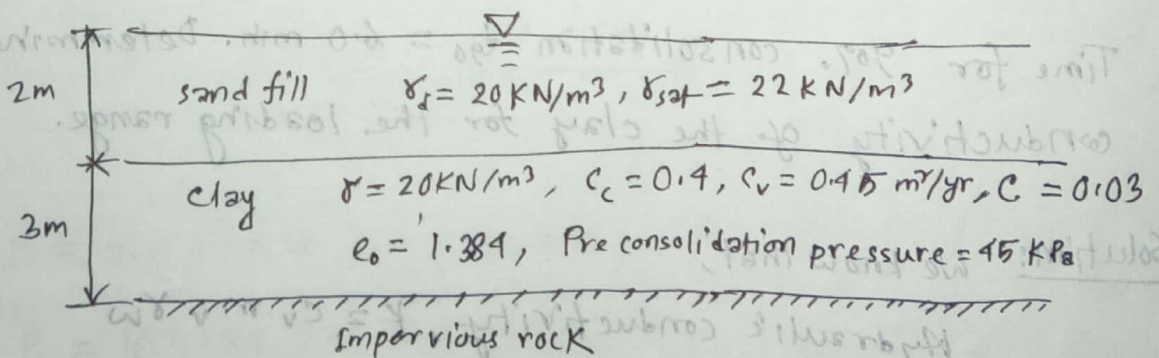
and, $C_v = \frac{T_v H_{dr}^2}{t_v}$ Here, for $U=90\%$, $T_v = 1.781 - 0.933 \log(100-U)$
 $= \frac{0.848 \times \left(\frac{50}{2 \times 1000}\right)^2}{6} = 0.848$

$\therefore C_v = 8.83 \times 10^{-5} \text{ m}^2/\text{min}$

Hence, $K = C_v m_v \gamma_w = (8.83 \times 10^{-5} \times 2.31 \times 10^{-4} \times 9.81) \text{ m/min}$
 $\therefore K = 2.002 \times 10^{-7} \text{ m/min}$
 (Ans.)

2012

Calculate the consolidation settlement of the soil profile shown in figure below, 2 years after the water table is lowered from the ground surface to the bottom of the clay layer. Assume that the clay layers remains full saturated after the lowering of the water table and account for capillary tension in calculations.



Solution:

Handwritten calculations for consolidation settlement, including terms like $\frac{(15.0 - 25.0)}{(200 - 100)}$ and $\frac{(9.8)}{(9.8)}$.

$$U = \frac{2}{9} = 0.222$$

$$U = \frac{30}{80} = 0.375$$

(i) We know that $U = \frac{2}{9}$ given that $U = 0.222$

(ii) We know that $U = \frac{30}{80} = 0.375$

(Ans)

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.375}{1-0.375} \right) = 151.50 \text{ days}$$

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.222}{1-0.222} \right) = 80.8 \text{ days}$$

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.222}{1-0.222} \right) = 80.8 \text{ days}$$

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.222}{1-0.222} \right) = 80.8 \text{ days}$$

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.222}{1-0.222} \right) = 80.8 \text{ days}$$

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.222}{1-0.222} \right) = 80.8 \text{ days}$$

$$t_{50} = \frac{H^2}{c_v} \left(\frac{U}{1-U} \right) = \frac{4^2}{0.003 \times 10^{-4}} \left(\frac{0.222}{1-0.222} \right) = 80.8 \text{ days}$$

2015, 2017
 # A normally consolidated clay layer is 4m thick (one-way drainage):

From the application of a given pressure, the total anticipated primary consolidation settlement will be 80 mm.

- (i) What is the average degree of consolidation for the clay when the settlement is 30 mm?
- (ii) If the average value of c_v for the pressure range is $0.003 \text{ cm}^2/\text{sec}$, how long will it take for 50% settlement to occur?
- (iii) How long will take for 50% consolidation if the clay is drained at both top and bottom?

Solution: (i) We know that, $U = \frac{S_t}{S_p}$ given that, $S_p = 80 \text{ mm}$
and $S_t = 30 \text{ mm}$

$$\Rightarrow U = \frac{30}{80} = 0.375$$

$$\therefore U = 37.5 \%$$

(Ans)

(ii) We know that, $t_v = \frac{T_v H_{dr}^2}{c_v}$ Here, $c_v = 0.003 \text{ cm}^2/\text{sec}$
 $= 0.003 \times 10^{-4} \text{ m}^2/\text{sec}$

Again,

$$U = 50\% \text{ Hence, } T_v = \frac{\pi}{4} \left(\frac{U}{100} \right)^2 \quad H_{dr} = 4 \text{ m}$$

$$\therefore T_{50} = \frac{\pi}{4} \left(\frac{50}{100} \right)^2 = 0.19635$$

$$\text{Now, } t_{50} = \frac{T_{50} H_{dr}^2}{c_v} = \frac{0.19635 \times (4)^2}{0.003 \times 10^{-4}} = 1,04,72,000 \text{ sec.}$$

$$= 121.20 \text{ days.}$$

(Ans)

(iii) If clay layer is drained at both top and bottom, $H_{dr} = \frac{4}{2} \text{ m} = 2 \text{ m}$

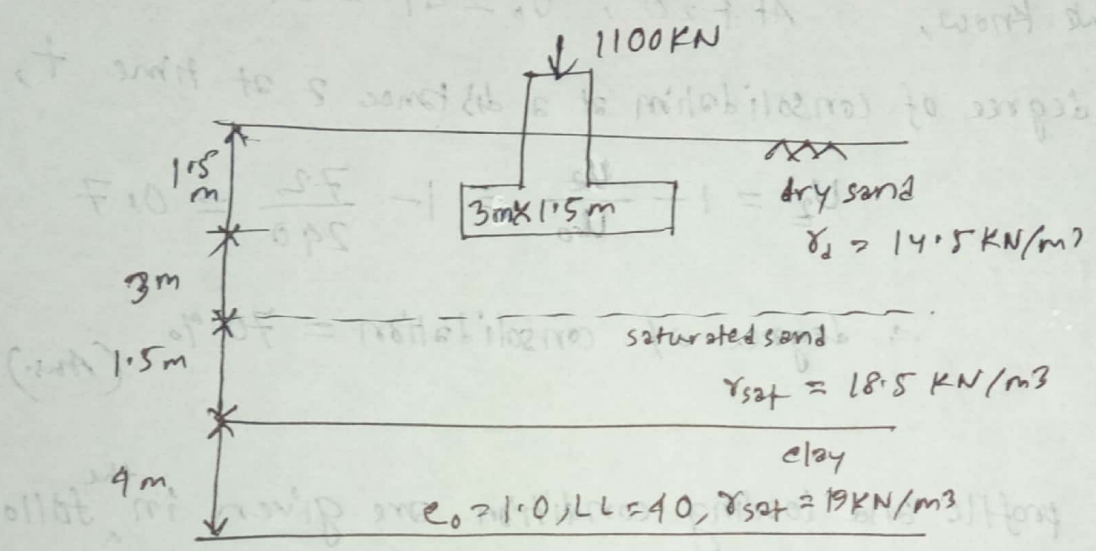
$$\therefore t_{50} = \frac{0.19635 \times 2^2}{0.003 \times 10^{-4}} = 26,18,000 \text{ sec.}$$

$$= 30.30 \text{ days.}$$

(Ans)

2016

calculate the consolidation settlement of 4 m thick layer (shown in figure below) that will result from the load carried by a footing measuring 3m x 1.5 m in plan. Assume the clay to be normally consolidated and $\Delta P_{ave} = 13.00 \text{ KN/m}^2$



Solution: We know, for normally consolidation,

$$S_p = \frac{c_c H}{1+e_0} \log \left(\frac{P_0 + \Delta P}{P_0} \right)$$

Here, $c_c = 0.009 (LL - 10) = 0.009 (40 - 10) = 0.27$

$H = 4 \text{ m}$, $e_0 = 1.0$

$$P_0 = (3 + 1.5) \times 14.5 + 1.5 \times (18.5 - 9.81) + \frac{4}{2} \times (19 - 9.81)$$

$\therefore P_0 = 96.665 \text{ KN/m}^2$

$\Delta P_{ave} = 13.00 \text{ KN/m}^2$

$$\therefore S_p = \frac{0.27 \times 4}{1+1} \times \log \left(\frac{96.665 + 13.00}{96.665} \right) = 0.03 \text{ m}$$

(Ans.)

2016
 # A saturated clay specimen is subjected to a pressure of 240 KN/m^2 . After the lapse of a time, it is determined that the pore pressure in the specimen is 72 KN/m^2 . What is the degree of consolidation?

Solution:

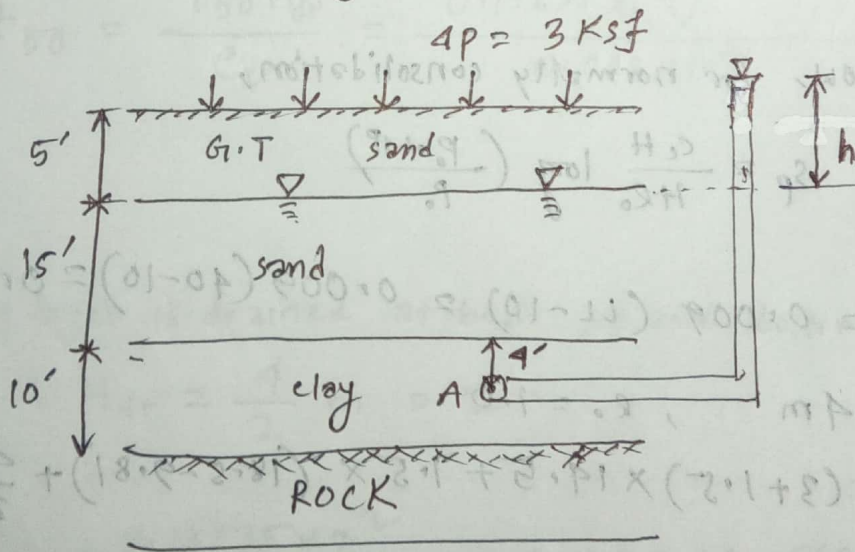
We know, At $t=0$, $U_0 = q = 240 \text{ KN/m}^2$

The degree of consolidation at a distance z at time t ,

$$U_z = 1 - \frac{u_z}{u_0} = 1 - \frac{72}{240} = 0.7$$

\therefore degree of consolidation = 70% (Ans.)

2017
 # The soil profile and loading condition are given in the following figure:



(i) $h = ?$ when $t=0$.

After the lapse of a time

(ii) if $h = 15 \text{ ft}$, Degree of consolidation = ?

(cont)

Solution: (i) At time $t=0$, $U_0 = \Delta P = 3 \text{ Ksf} = 3000 \text{ lb/ft}^2$

we know,

$$U_0 = \gamma_w H$$

$$\Rightarrow h = \frac{U_0}{\gamma_w} = \frac{3000}{62.4} = 48.077 \text{ ft}$$

(Ans)

After a time,

(ii) If, $h = 15 \text{ ft}$

$$U_A = \gamma_w h = [62.4 \times (15)] = 936 \text{ lb/ft}^2$$

Now,

we know that,

$$\text{Degree of consolidation, } U_A = 1 - \frac{U_A}{U_0} \quad \text{Here } U_0 = 3000 \text{ lb/ft}^2$$

$$= 1 - \frac{936}{3000}$$

$$= 0.688$$

\therefore Degree of consolidation, $U = 68.8 \%$

(Ans.)

2017

A clay layer in the field is 15 ft thick and drained the top only. Under a given surcharge, the estimate consolidation settlement is 10 inch.

(i) What is the average degree of consolidation for the clay when the settlement is 3 inch.

(ii) If the average value of C_v for the pressure range is 0.0005 in²/sec. how long will it take place for 50% consolidation to occur.

(iii) If the 15 ft clay layer is drained ^{on} both sides, how long will it take place for 50% consolidation to occur, Assume time factor is 0.197 for 50% consolidation.

Same as 2015

Permeability

15000415
FARHAD

Permeability:

Permeability is the measure of the soil's ability to permit water to flow through its pores and voids.

Soils are permeable due to the existence of inter connected voids through which water can flow from points of high energy to points of low energy.

Factors affecting permeability: 07,06,04

1. Grain size.
2. Properties of the pore fluid.
3. void ratio of the soil.
4. structural arrangement of soil particles.
5. Entrapped air and foreign matter.
6. Absorbed water in soils.

Factors affecting hydraulic conductivity:

1. Degree of saturation $\uparrow \rightarrow K(\uparrow)$
2. Grain size distribution
3. pore size distribution
4. void ratio $\uparrow \rightarrow K(\uparrow)$
5. fluid viscosity
6. Roughness of mineral particles.
7. composition of soil particles
8. density and concentration of permeant

Importance of permeability:

1. permeability influences the rate of settlement of a saturated soil under load.
2. The design of earth dams is very much based on the permeability of the soils used.
3. Filters made of soils are designed based upon their permeability.
4. The stability of ~~slopes~~ slopes and retaining structures can be greatly affected by the permeability of the soils involved.

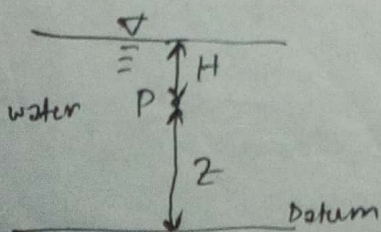
The study of permeability is important for:

1. estimating the quantity of underground seepage.
2. investigating problems involving pumping seepage of water for underground constructions.
3. Analysing the stability of earth dams and earth retaining walls subjected to seepage forces.

Bernoulli's Equation:

According to Bernoulli's equation, the total head at a point in water under motion can be expressed as:

$$h = \frac{P}{\rho_m} + \frac{v^2}{2g} + z \quad \text{where, } h = \text{total head}$$



↓
Pressure head

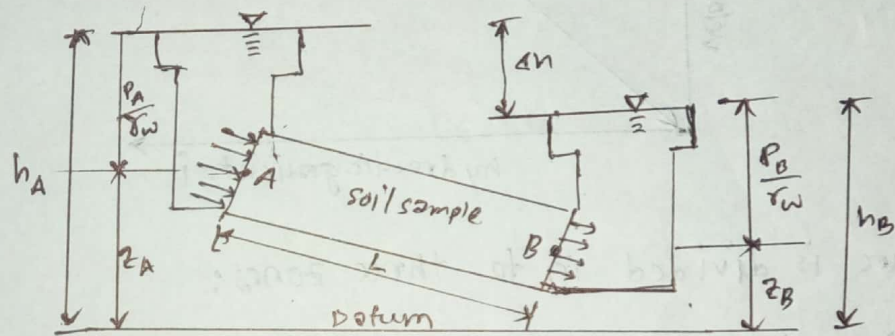
↓
velocity head

P = water pressure
 v = velocity of water
 z = elevation head

When water flows through soils, the seepage velocity is often very small. It is even smaller when squared and velocity head in Bernoulli's equation becomes negligible. Therefore

$$h = \frac{P}{\gamma_w} + z$$

The heads of water at points A and B as the water flows from A to B are given as follows: (with respect to a datum)



Total head at A: $h_A = \frac{P_A}{\gamma_w} + z_A$

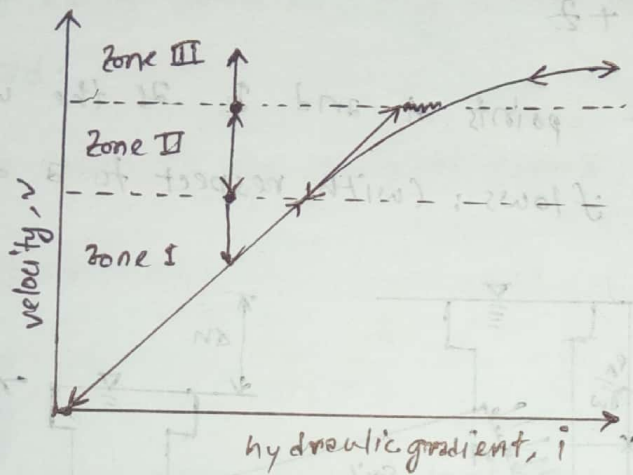
Total head at B: $h_B = \frac{P_B}{\gamma_w} + z_B$

The loss of head between A and B: $\Delta h = h_A - h_B = \left(\frac{P_A}{\gamma_w} + z_A \right) - \left(\frac{P_B}{\gamma_w} + z_B \right)$

The head loss may be expressed as, $i = \frac{\Delta h}{L}$ where, $i =$ hydraulic gradient

$L =$ distance between A and B

▣ Variation of flow velocity with hydraulic gradient:
 The variation of flow velocity, v with hydraulic gradient, i is as shown in figure:



The figure is divided into three zones:

1. Laminar flow zone (zone I)
2. Transition zone (zone II)
3. Turbulent zone (zone III)

When hydraulic gradient is increased gradually, the flow remains laminar in zone I and velocity, v bears a linear relationship to hydraulic gradient. At a higher hydraulic gradient, the flow becomes turbulent (zone III)

In most soils, the flow of water through void spaces can be considered laminar, thus, $v \propto i$

In fractured rock, stones, gravels and very coarse sands, turbulent flow condition exist.

Darcy's law and Hydraulic conductivity: 17, 11, 10, 05, 05, 03
 Henri Darcy in 1856 derived an empirical formula for the behavior of flow through saturated soils, which may be expressed as:

$$v = Ki \dots \textcircled{1} \text{ where, } v = \text{discharge velocity} = \frac{q}{A}$$

$$\Rightarrow \frac{q}{A} = Ki$$

$$\Rightarrow q = KiA = \frac{Q}{t}$$

Q = volume of collected water

K = co-efficient of permeability
 or hydraulic conductivity

i = hydraulic gradient

A = gross cross sectional area.

Equation $\textcircled{1}$ is valid for laminar flow condition. ~~and a relation between discharge~~

when, A & i becomes unity, then,

$$KA = K = q$$

Hence, co-efficient of permeability or hydraulic conductivity

is defined as the discharge per unit time that will occur through unit cross sectional area of soil under unit hydraulic gradient.

Relation between Discharge velocity and seepage velocity: 17, 11

Discharge velocity: It is defined as the quantity of water flowing per unit time ~~per~~ through unit gross cross sectional area of soil perpendicular to the direction of flow.

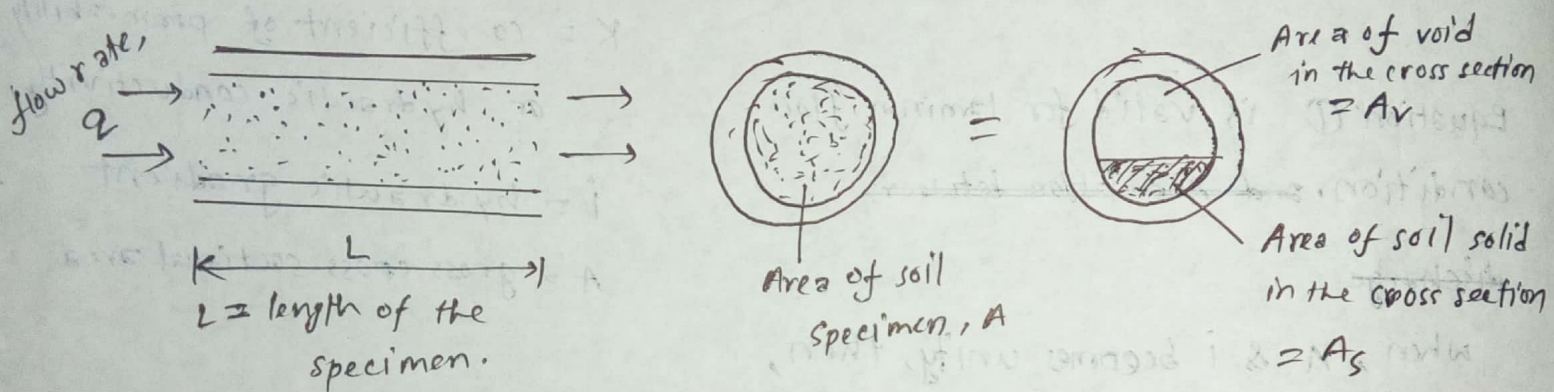
$$\text{discharge velocity, } v = \frac{Q}{A}$$

seepage velocity: It is defined as the quantity of water flowing per unit time through ~~the soil~~ unit cross sectional area of void space of soil perpendicular to the direction of flow.

$$\text{seepage velocity, } v_s = \frac{q}{A_v}$$

seepage velocity is the actual velocity of water through the void spaces.

Let us consider the figure:



If the quantity of water flowing through the soil

in unit time is q , then

$$q = AV = A_v v_s \quad \text{where, } v_s = \text{seepage velocity}$$

$A_v = \text{Area of void in the cross section}$

$$\Rightarrow \frac{v}{v_s} = \frac{A_v}{A}$$

$$\Rightarrow \frac{v}{v_s} = \frac{A_v}{A_v + A_s}$$

$$\Rightarrow \frac{v}{v_s} = \frac{A_v L}{(A_v + A_s) L}$$

$$\Rightarrow \frac{v}{v_s} = \frac{v_v}{v_v + v_s}$$

$$\Rightarrow \frac{v}{v_s} = \frac{\frac{v_v}{v_s}}{\frac{v_v}{v_s} + 1} \Rightarrow \frac{v}{v_s} = \frac{e}{1+e} \Rightarrow \frac{v}{v_s} = \eta \quad \therefore v = \eta v_s$$

Laboratory determination of Hydraulic conductivity:

Two standard laboratory test are used to determine the hydraulic conductivity of soil-

1. constant head test and
2. Falling head test

03 Constant head test: (for sandy soil) [coarse grained soil]

A typical arrangement of the constant head permeability test is shown in figure:

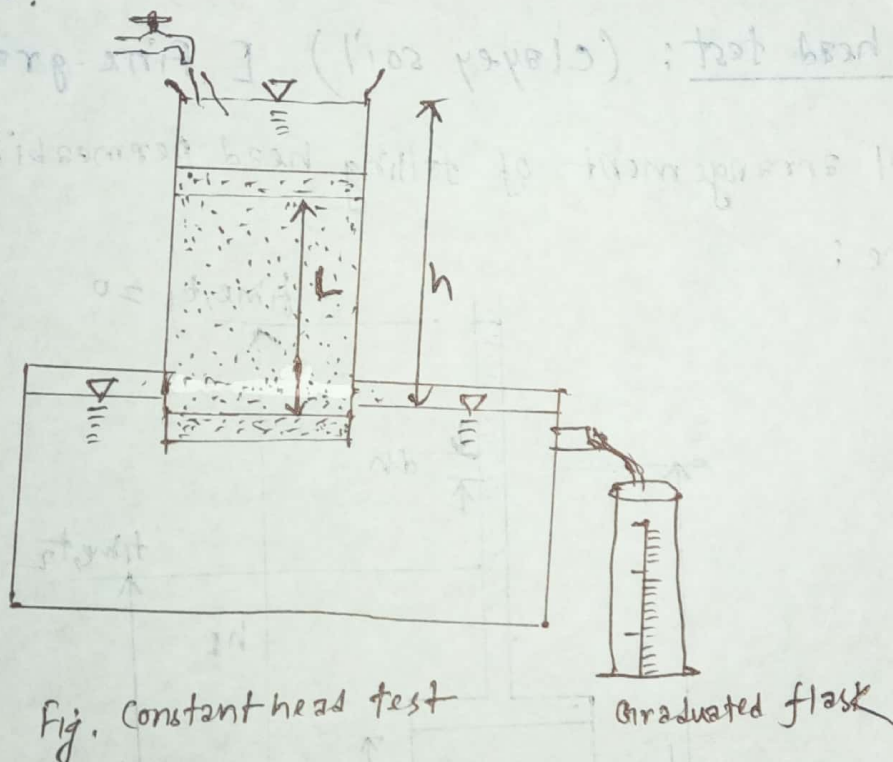


Fig. Constant head test

Graduated flask

In this type of laboratory set up,

The water supply is adjusted in such a way that head difference h between inlet and outlet remains constant—
water is collected in a graduated flask for a definite time duration.

The total volume of water collected may be expressed as:

$$Q = Avt = A(Ki)t$$

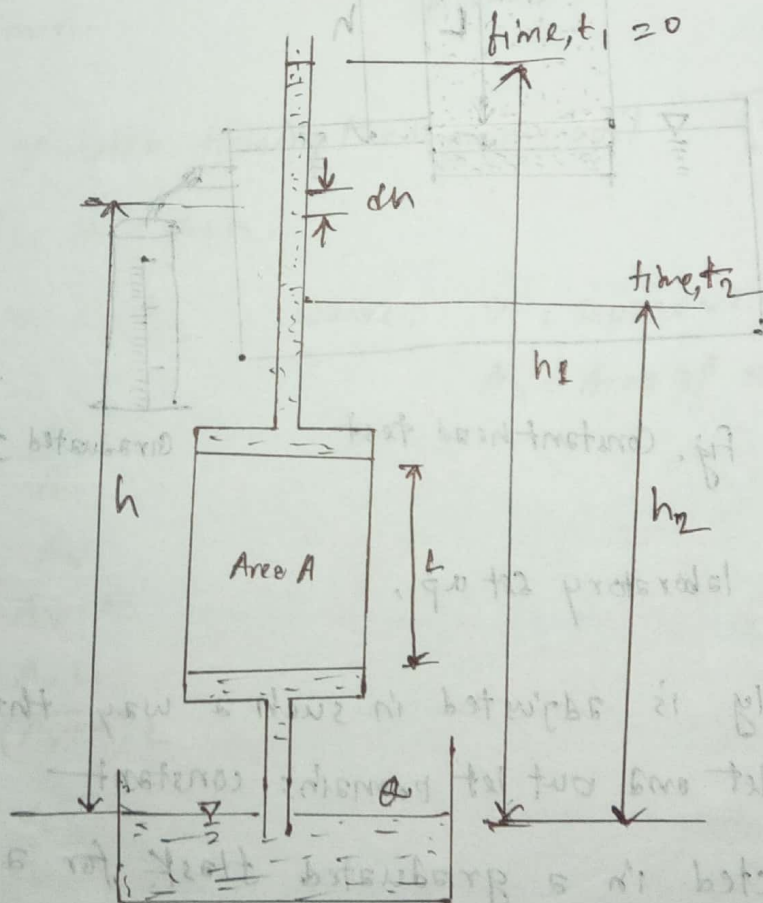
$$\Rightarrow Q = AKi \frac{h}{L} \cdot t$$

$$\Rightarrow K = \frac{QL}{hAt}$$

From this equation hydraulic conductivity can be determined.

02,04
 Falling head test: (clayey soil) [fine-grained soil]

A typical arrangement of falling head permeability test is shown in figure:



The initial head difference, h_1 at time $t=0$ is recorded, and, water is allowed to flow through specimen such that the final head difference, at time $t=t_2$ is h_2 .

The rate of flow of the water through specimen at any time t can be given by:

$$q = K \frac{h}{L} A = -a \frac{dh}{dt} \quad \text{where, } q = \text{flow rate}$$

$a = \text{cross sectional area of stand pipe}$

$$\Rightarrow dt = \frac{aL}{AK} \left(-\frac{dh}{h} \right)$$

$A = \text{cross sectional area of soil specimen}$

Integrating between two time limits, we obtain,

$$\int_0^{t_2} dt = \frac{aL}{AK} \int_{h_1}^{h_2} -\frac{dh}{h}$$

$$\Rightarrow [t]_0^{t_2} = \frac{-aL}{AK} [\ln h]_{h_1}^{h_2}$$

$$\Rightarrow t = \frac{-aL}{AK} (\ln h_2 - \ln h_1)$$

$$\Rightarrow t = \frac{aL}{AK} (\ln h_1 - \ln h_2)$$

$$\Rightarrow t = \frac{aL}{AK} \ln \frac{h_1}{h_2}$$

$$\Rightarrow \boxed{K = \frac{aL}{At} \ln \frac{h_1}{h_2}} \quad \text{--- (1)}$$

From this equation, hydraulic conductivity can be determined.

Q If the time intervals for drop in levels from h_1 to h_2 and h_2 to h_3 are equal, prove that $h_2 = \sqrt{h_1 \times h_3}$ 2008

(After equation-1)

For the same soil sample if h_2 and h_3 be the head for the same time interval we can write,

$$K = \frac{2L}{At} \ln \frac{h_2}{h_3} \dots \textcircled{11}$$

From ① & ⑪ we obtain,

$$\ln \frac{h_1}{h_2} = \ln \frac{h_2}{h_3}$$

$$\Rightarrow \frac{h_1}{h_2} = \frac{h_2}{h_3} \Rightarrow h_2^2 = h_1 \times h_3$$

$$\therefore h_2 = \sqrt{h_1 \times h_3}$$

(Proved)

Q Difference between constant head and falling head test:

Constant head Test	Falling head Test
1. Applicable for coarse grained soil.	1. Applicable for clayey soil.
2. Head difference between inlet and outlet is constant.	2. Head difference between inlet and outlet varies time to time.
3. Three tubes are connected with permeameter	3. One stand pipe is connected with permeameter.
4. Hydraulic conductivity is determined by $K = \frac{QL}{hAt}$	4. Hydraulic conductivity is determined by, $K = \frac{2L}{At} \ln \frac{h_1}{h_2}$

Relationships for hydraulic conductivity: (Granular soil)

* For fairly uniform sand, Hazen (1930) proposed an empirical relationship for hydraulic conductivity.

$$K \text{ (cm/sec)} = c D_{10}^2 \quad \text{when, } c = \text{a constant that varies from 1 to 1.5}$$

this equation is based primarily on

Hazen's observations of loose, clean and filter sand. It is not very reliable

D_{10} = effective size, in mm

* Based on Kozeny-Carman equation, $K = 1.99 \times 10^4 \left(\frac{1}{S_s}\right)^2 \frac{e^3}{1+e}$

Again, $S_s = \frac{SF}{D_{eff}} \left(\frac{1}{cm}\right)$ with $D_{eff} = \frac{100\%}{\sum \left(\frac{f_i}{D_{(av)_i}}\right)}$

where,

f_i = fraction of particles between two sieve sizes, in percent

$$D_{(av)_i} \text{ (cm)} = [D_{L_i} \text{ (cm)}]^{0.5} \times [D_{S_i} \text{ (cm)}]^{0.5}$$

SF = shape factor: (varies from 6 to 8) (note: L = large sieve, S = small sieve)

* Chapuis (2004) proposed an empirical relationship for K

$$K \text{ (cm/s)} = 2.4622 \left[D_{10}^2 \frac{e^3}{1+e} \right]^{0.7825}$$

(cohesive soil)

* Taylor (1948) proposed a linear relationship between the logarithm of K and the void ratio, e as:

$$\log K = \log K_0 - \frac{e_0 - e}{C_K} \quad \text{where,}$$

$K_0 =$ in situ hydraulic conductivity at e_0

* Samarasinghe et al. (1982) conducted laboratory test on New Liskeard clay and proposed that, for normally consolidated clays,

$K =$ hydraulic conductivity at e

$C_K =$ hydraulic conductivity change index.

$$K = C \left(\frac{e^n}{1 + e} \right)$$

where, C and n are constants to be determined experimentally.

□ Typical values of co-efficient of permeability for various soil:

Material	co-efficient of permeability [K (mm/s)]
coarse	10^0 to 10^3
Fine gravel, coarse and medium sand	10^{-2} to 10^0
Fine sand, clayey silt	10^{-4} to 10^{-4}
Dense silt, clayey silt	10^{-4} to 10^{-4}
silty clay, clay	10^{-5} to 10^{-5}

Validity of Darcy's law:

Darcy's law is valid for laminar flow of water through void spaces. A criterion for investigating the range can be furnished by Reynold's number.

For flow through soils, Reynold's number can be given

by the relation:

$$Re = \frac{v D \rho}{\mu}$$

where, v = discharge velocity, cm/s (super critical)

For laminar flow conditions in soil,

D = average dia. of soil particle, cm

experimental results show

ρ = density of the fluid, g/cm^3

that,

$$Re = \frac{v D \rho}{\mu} \leq 1$$

μ = co-efficient of viscosity, $g/(cm \cdot s)$

Hydraulic conductivity from consolidation test:

The co-efficient of permeability of clay soil is often determined by the consolidation test,

$$T_v = \frac{C_v t}{H^2}$$

where, T_v = Time factor

C_v = co-efficient of consolidation

and, the co-efficient of consolidation is,

$$C_v = \frac{k}{\gamma_w m_v}$$

H = length of average drainage path

t = time

Here, $m_v = \frac{\Delta e}{\Delta p (1+e)}$

where,

Δe = change of void ratio for incremental loading

$\Delta p =$ incremental pressure applied

$e =$ initial void ratio.

Hence, combining these equations, we obtain,

$$k = \frac{T_v \gamma_w \Delta e H^2}{4\sigma(1+e)t}$$

For 50% consolidation, $T_v = 0.198$ and the corresponding time $= t_{50}$

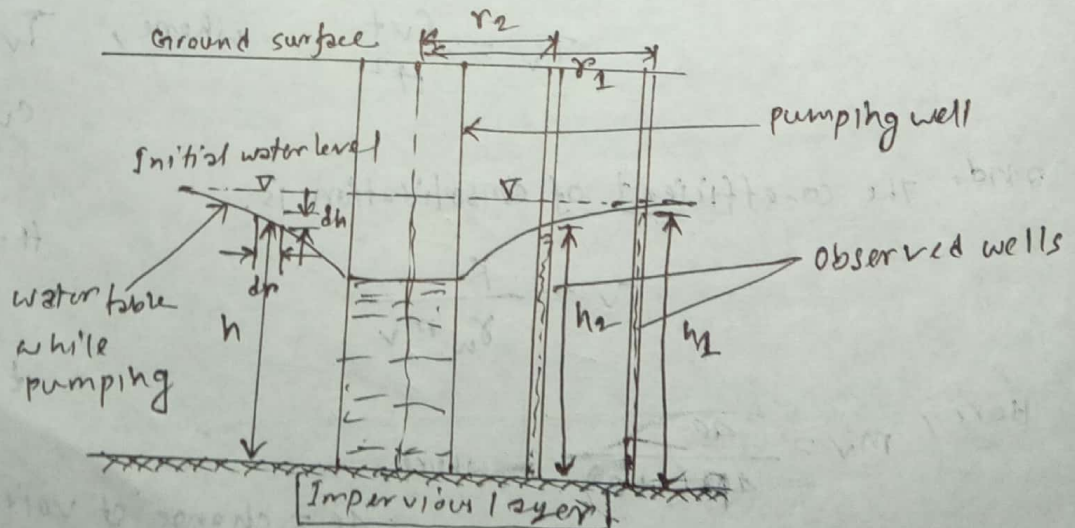
Hence,

$$k = \frac{0.198 \gamma_w \Delta e H^2}{4\sigma(1+e)t_{50}}$$

Permeability test in the field : pumping well

The average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from well.

During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various distances are made around the test well.



Steady state: The equilibrium state when no further drawdown develops as pumping continues.

We know,

$$q = KiA \Rightarrow q = K \left(\frac{dh}{dr} \right) 2\pi r h \quad \left[i = \frac{dh}{dr} \text{ \& } A = 2\pi r h \right]$$

$$\Rightarrow \frac{dr}{r} = \frac{2\pi K}{q} h dh$$

Integrating this equation,

$$\int_{r_2}^{r_1} \frac{dr}{r} = \frac{2\pi K}{q} \int_{h_2}^{h_1} h dh$$

$$\Rightarrow [\ln r]_{r_2}^{r_1} = \frac{2\pi K}{2q} [h^2]_{h_2}^{h_1}$$

$$\Rightarrow \ln r_1 - \ln r_2 = \frac{\pi K}{q} (h_1^2 - h_2^2)$$

$$\Rightarrow \ln \frac{r_1}{r_2} = \frac{\pi K}{q} (h_1^2 - h_2^2)$$

Now,
$$K = \frac{q \ln \frac{r_1}{r_2}}{\pi (h_1^2 - h_2^2)}$$

From this equation, hydraulic conductivity can be determined in the field.

Average hydraulic conductivity: (if thickness of aquifer = H)

The steady state of discharge,

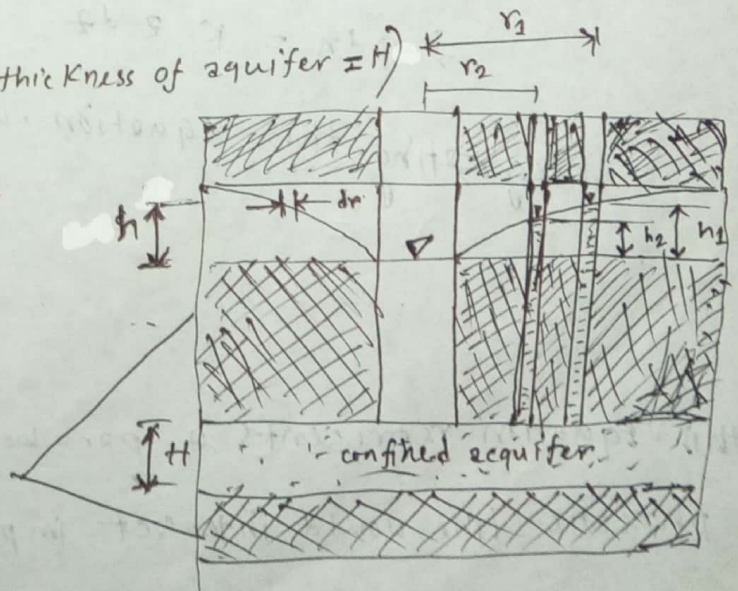
$$q = K \left(\frac{dh}{dr} \right) 2\pi r H$$

$$\Rightarrow \int_{r_2}^{r_1} \frac{dr}{r} = \int_{h_2}^{h_1} \frac{2\pi K H}{q} dh$$

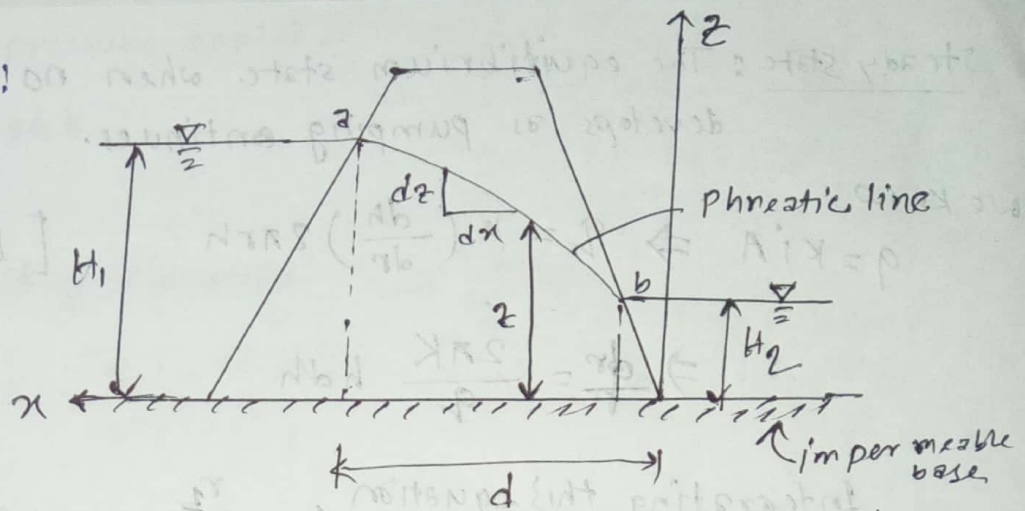
$$\Rightarrow K = \frac{q \ln \left(\frac{r_1}{r_2} \right)}{2\pi H (h_1 - h_2)}$$

confined aquifer

impermeable layer



Dupuit's solution:



This figure shows the section of an earth dam in which is the phreatic surface i.e. the upper most line of seepage. The quantity of seepage through a unit length at right angles to the cross section can be given from Darcy's law as

$$q = K i A$$

Dupuit (1863) assumed that the hydraulic gradient $i = \text{slope of the free surface} = \frac{dz}{dx}$ and it is constant with depth.

$$q = K \frac{dz}{dx} [z] \quad (1)$$

$$\Rightarrow q = K \frac{dz}{dx} z$$

$$\Rightarrow q dx = K z dz$$

Integrating this equation, we get, $q \int_0^d dx = K \int_{H_2}^{H_1} z dz$

$$\Rightarrow q d = \frac{K}{2} (H_1^2 - H_2^2)$$

$$\Rightarrow q = \frac{K}{2d} (H_1^2 - H_2^2)$$

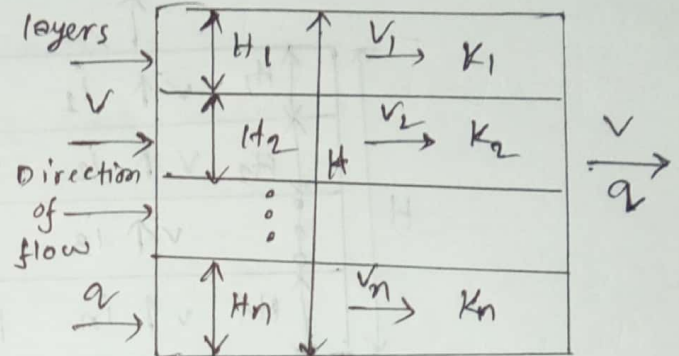
This equation represents a parabolic surface. If $H_2 = 0$ the phreatic line would intersect impervious surface.

Equivalent permeability: (stratified soil)

* when flow takes place in horizontal direction to the bedding plane:

Let, H_1, H_2, \dots, H_n = thickness of layers

and, K_1, K_2, \dots, K_n = hydraulic conductivity of the layers.



Here, hydraulic gradient, i

will be same for all the layers. However,

Fig. Flow parallel to bedding plane

$V = Ki$ since K is different, the velocity of flow will be different in different layer.

Let, $K_H =$ ~~average~~ Equivalent hydraulic conductivity of soil in horizontal direction

$$Q = K_H (eq) i H = K_1 i H_1 + K_2 i H_2 + \dots + K_n i H_n$$

$$\Rightarrow K_H (eq) = \frac{K_1 H_1 + K_2 H_2 + \dots + K_n H_n}{H}$$

This is the equivalent hydraulic conductivity of soil in the horizontal direction to the bedding plane.

* When flow takes place in vertical direction to the bedding planes

05, 06, 07, 10

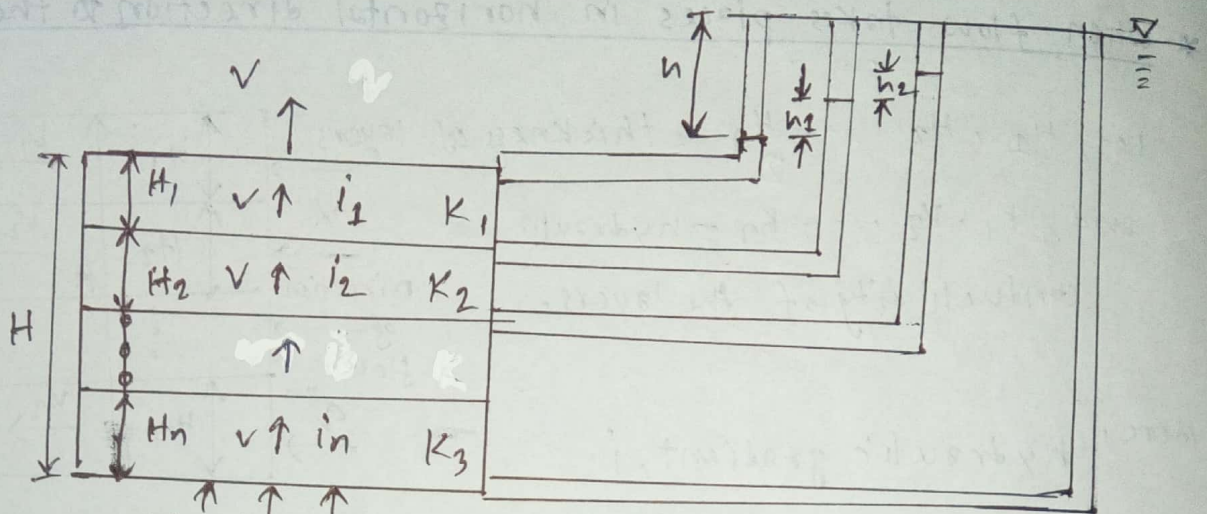


Fig. Flow verticle to the bedding plane

In this case, the velocity of flow through all the layers is the same. Hence unit discharge will be same through each layers.

However, the total head loss, $H = h_1 + h_2 + \dots + h_n$

and, $v = v_1 = v_2 = \dots = v_n$

But, hydraulic gradient, $i = \frac{h}{H} \Rightarrow h = Hi$

So, $h = H_1 i_1 + H_2 i_2 + \dots + H_n i_n$ ①

Using Darcy's Law, $v = Ki$

$v = K_v (eq) \frac{h}{H} = K_1 i_1 = K_2 i_2 = \dots = K_n i_n$ ②

From ① & ② we obtain, [substituting these value in ①]

$$K_v (eq) = \frac{H}{\left(\frac{H_1}{K_1}\right) + \left(\frac{H_2}{K_2}\right) + \dots + \left(\frac{H_n}{K_n}\right)}$$

where, K_v is the equivalent hydraulic conductivity in vertical direction

□ Prove that, "Average permeability parallel to the bedding plane is higher than that of perpendicular to the bedding plane."

considering a three layers system having,

$$k_1 = 2, k_2 = 1, k_3 = 4 \text{ units}$$

$$\text{and } h_1 = 3, h_2 = 1.5, h_3 = 2 \text{ units}$$

$$\therefore K_H (\text{eq}) = \frac{(2 \times 3) + (1 \times 1.5) + (4 \times 2)}{(3 + 1.5 + 2)} = 2.38$$

and,

$$K_V (\text{eq}) = \frac{(3 + 1.5 + 2)}{\left(\frac{3}{2}\right) + \left(\frac{1.5}{1}\right) + \left(\frac{2}{4}\right)} = 1.86$$

$$\therefore K_H > K_V \quad (\text{Proved})$$

Permeability

2002 **Problem: 7.3** (BM Das - 8th edition)

For a constant head laboratory test, the following data are:

Length of specimen = 10 inch, Diameter of specimen = 2.5 in, Head difference = 18 in. water collected in 2 minutes = 0.031 in³. Determine (i) Hydraulic conductivity of soil. (ii) Discharge velocity (iii) seepage velocity if the void ratio of the soil specimen is 0.46.

Solutions: (i) we know that,

$$\text{Hydraulic conductivity, } K = \frac{QL}{Aht} = \frac{0.031 \times 10}{\frac{\pi}{4} \times (2.5)^2 \times 18 \times 2}$$

$$\therefore K = 1.754 \times 10^{-3} \text{ in/min}$$

(ii) Discharge velocity, $v = Ki$

$$\Rightarrow v = K \times \frac{h}{L} = 1.754 \times 10^{-3} \times \frac{18}{10} = 3.16 \times 10^{-3} \text{ in/min}$$

(iii) seepage velocity, $v_s = \frac{v}{n}$ Here, $n = \frac{e}{1+e} = \frac{0.46}{1+0.46} = 0.315$

$$\therefore v_s = \frac{3.16 \times 10^{-3}}{0.315} = 0.01 \text{ in/min}$$

(Ans)

2005, 2003

For a falling head test the following data are:

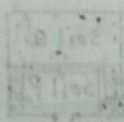
length of ^{soil} specimen = 15 in, Area of soil specimen = 3 in², area of standpipe = 0.15 in²

Head difference at $t=0$ is 25 in.

Head difference at $t=8$ min. is 10 in.

Determine (i) Hydraulic conductivity of soil.

(ii) What was the head difference at time $t=4$ min?



Solutions: (i) We know that,

$$\text{Hydraulic conductivity, } K = \frac{aL}{At} \ln\left(\frac{h_1}{h_2}\right)$$

$$\Rightarrow K = \frac{0.15 \times 15}{3 \times 8} \ln\left(\frac{25}{h_2}\right)$$

(ii) At time $t = 4$ min,

$$K = \frac{aL}{At} \ln\left(\frac{h_1}{h_2}\right)$$

$$\Rightarrow 0.086 = \frac{0.15 \times 15}{3 \times 4} \ln\left(\frac{25}{h_2}\right)$$

$$\Rightarrow \ln\left(\frac{25}{h_2}\right) = 0.4587$$

$$\Rightarrow \frac{25}{h_2} = e^{0.4587} = 1.582$$

$$\therefore h_2 = 15.80 \text{ in}$$

2006

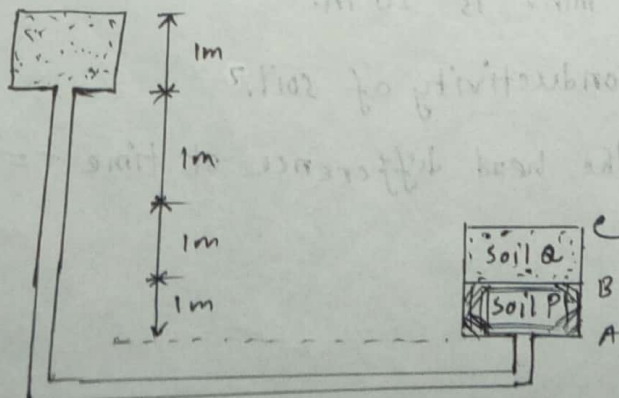
(Any)

The experimental permeability test set up shown is figure below, flow takes place under a constant head through the soil p and q.

Determine (i) the piezometric head at point A

(ii) If 45% of the excess hydro-static pressure is lost in flowing through soil p, what are the hydraulic and piezometric head at point B.

(iii) What is the discharge per unit area?



Solution: (i) At A, piezometric head, $h_w = (1+1+1+1) = 4 \text{ m}$

(ii) Total head at A, $Z_i = z + h_w$ Here, hydraulic head, $z = -2$

$$\therefore Z_i = -2 + 4 = 2 \text{ m}$$

head loss from A to B = 45% of Total head at A

$$= (0.45 \times 2) = 0.9 \text{ m}$$

$$\therefore \text{Total head at B} = (2 - 0.9) = 1.1 \text{ m}$$

we know that, $Z_i = z + h_w$

$$\therefore \text{piezometric head at B, } h_w = Z_i - z$$

$$\Rightarrow h_w = 1.1 - (-2) = 3.1 \text{ m}$$

(iii) we know that,

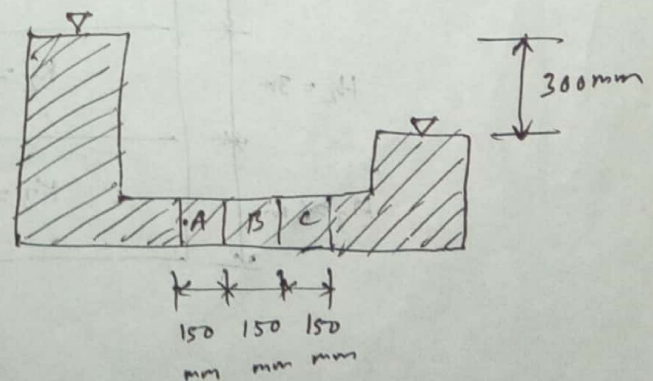
$$Q = K i A = K \times \frac{h}{L} \times A = K \times \frac{0.9}{1} \times 1 = 0.9 K$$

(Ans.)

2007 **Ex-7.13**

Fig. below shows the three layers of soil in a tube that is 100mm x 100mm in cross section. water is supplied to maintain a constant head difference of 300mm, across the sample. The hydraulic conductivity of the soil in direction of flow through them are as follows:

Soil	K (cm/s)
A	10^{-2}
B	3×10^{-3}
C	9.9×10^{-4}



Find the rate of water supply in cm^3/hr

Solutions: Here, the flow direction is vertical.

For vertical direction of flow equivalent permeability,

$$K_v(eq) = \frac{H}{\left(\frac{H_1}{K_1}\right) + \left(\frac{H_2}{K_2}\right) + \left(\frac{H_3}{K_3}\right)}$$

$$= \frac{15 + 15 + 15}{\frac{15}{10^{-2}} + \frac{15}{3 \times 10^{-3}} + \frac{15}{4.9 \times 10^{-4}}}$$

$$K_v(eq) = 1.21 \times 10^{-3} \text{ cm/sec.}$$

\therefore rate of water supply, $q = K_v A$

$$= 1.21 \times 10^{-3} \times \frac{30}{45} \times \left(\frac{100}{10} \times \frac{100}{10}\right)$$

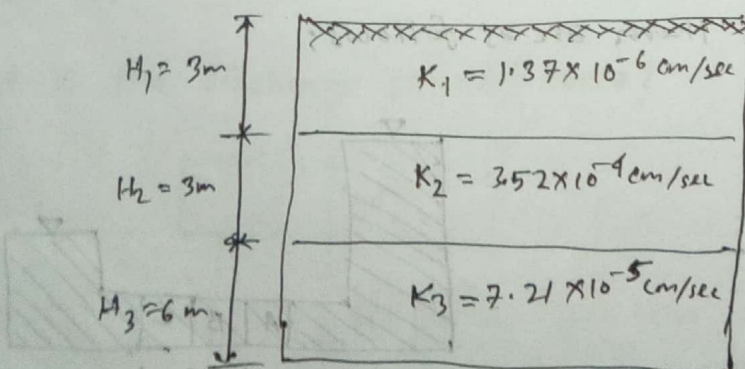
$$= 0.0808 \text{ cm}^3/\text{sec.}$$

$$= 290.9 \text{ cm}^3/\text{hr.}$$

(Ans)

2009

A layered soil profile is shown in figure below. Determine the horizontal and vertical permeability of the soil layer.



Solution: Horizontal permeability, $K_H = \frac{K_1 H_1 + K_2 H_2 + K_3 H_3}{H_1 + H_2 + H_3}$

$$= \frac{1.37 \times 10^{-6} \times 300 + 3.52 \times 10^{-4} \times 300 + 7.21 \times 10^{-5} \times 600}{300 + 300 + 600}$$

$\therefore K_H = 1.249 \times 10^{-4} \text{ cm/sec.}$

and vertical permeability, $K_V = \frac{H_1 + H_2 + H_3}{\frac{H_1}{K_1} + \frac{H_2}{K_2} + \frac{H_3}{K_3}}$

$$= \frac{300 + 300 + 600}{\frac{300}{1.37 \times 10^{-6}} + \frac{300}{3.52 \times 10^{-4}} + \frac{600}{7.21 \times 10^{-5}}}$$

$$= 5.26 \times 10^{-6} \text{ cm/sec.}$$

2010 (Ans.)

For a variable head permeability test, the followings are given:
 length of specimen = 15 in. area of specimen = 3 in² and $k = 0.0688 \text{ in/min}$
 what should be the area of the standpipe for the head to drop from 25 to 12 in in 8 min?

Solution: We know that,

$$k = \frac{2L}{At} \ln \left(\frac{h_1}{h_2} \right)$$

$$\Rightarrow 0.0688 = \frac{2 \times 15}{3 \times 8} \ln \left(\frac{25}{12} \right)$$

$$\therefore 2 = 0.115 \text{ in}^2$$

\therefore Area of stand pipe = 0.115 in² (Ans.)

2009 **Problem: 7.2**

Find the flow rate in $\text{m}^3/\text{sec}/\text{m}$ length (at right angle to the cross section shown) through the permeable soil layer in figure below. Given, $H = 4\text{m}$, $H_1 = 2\text{m}$, $h = 3.1\text{m}$, $L = 30\text{m}$, $\alpha = 14^\circ$, $K = 0.05\text{ cm/sec}$.

Solution: We know,

$$q = K i A$$

Here,

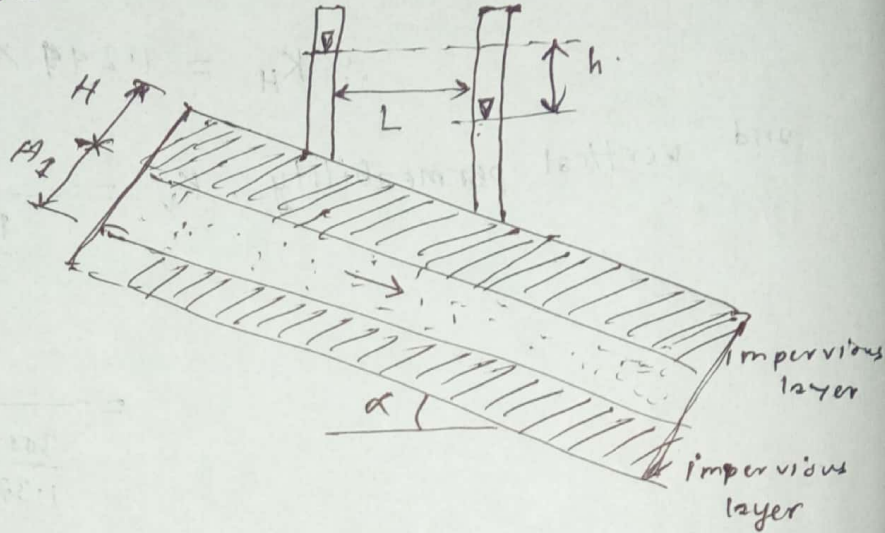
$$i = \frac{h}{\left(\frac{L}{\cos \alpha}\right)}$$

$$\Rightarrow i = \frac{3.1}{\left(\frac{30}{\cos 14}\right)} = 0.100$$

$$A = (H_1 \times 1) = (2 \times 1) = 2\text{ m}^2$$

$$\therefore \text{flow rate, } q = K i A = \frac{0.05}{10^2} \times 0.1 \times 2 = 1.00264 \times 10^{-4}\text{ m}^3/\text{sec}/\text{m}$$

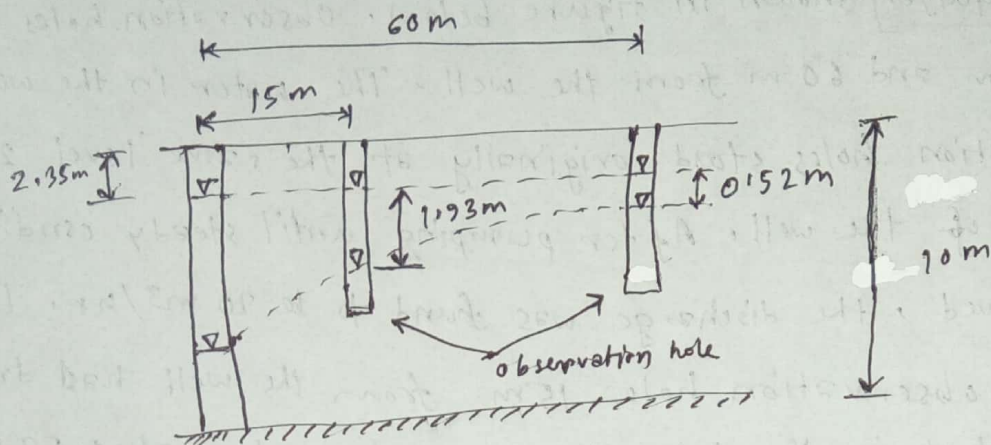
(Ans)



2011

During preparation for a pumping test, a well was sunk through a stratum of dense sand 10 m deep and into a clay of very low permeability beneath. Observation holes were drilled at 15 m and at 60 m from the well. The water in the well and the observation holes stood originally at the same level, 2.5 m below the top of the well. After pumping until steady condition had achieved, the discharge was found to be $19.7\text{ m}^3/\text{hour}$. The water level in the observation hole 15 m from the well had dropped 1.93 m and that is the hole 60 m away had

dropped 0.52 m. Determine the field co-efficient of permeability of dense sand in m/s.



Solution:

We know that,

co-efficient of permeability in field,

$$K = \frac{q \ln\left(\frac{r_1}{r_2}\right)}{\pi (h_1^2 - h_2^2)}$$

Here, $q = 19.7 \text{ m}^3/\text{hr}$
 $= \frac{19.7}{3600} \text{ m}^3/\text{sec}$
 $= 5.472 \times 10^{-3} \text{ m}^3/\text{s}$

and, $r_1 = 60 \text{ m}$

$h_1 = (10 - 2.35 - 0.52) = 7.13$

$r_2 = 15 \text{ m}$

$h_2 = (10 - 2.35 - 1.93) = 5.72$

$\therefore K = \frac{5.472 \times 10^{-3} \times \ln\left(\frac{60}{15}\right)}{\pi \times (7.13^2 - 5.72^2)} = 1.3327 \times 10^{-9} \text{ m/sec.}$ (Ans)

2014

The effective size of a soil is found 0.55 mm from grain size analysis. What will be the co-efficient of permeability of that soil? use Hazen's equation.

Solution: Hazen's equation is, $K = c D_{10}^2$ Here, $c = (1 - 1.5)$

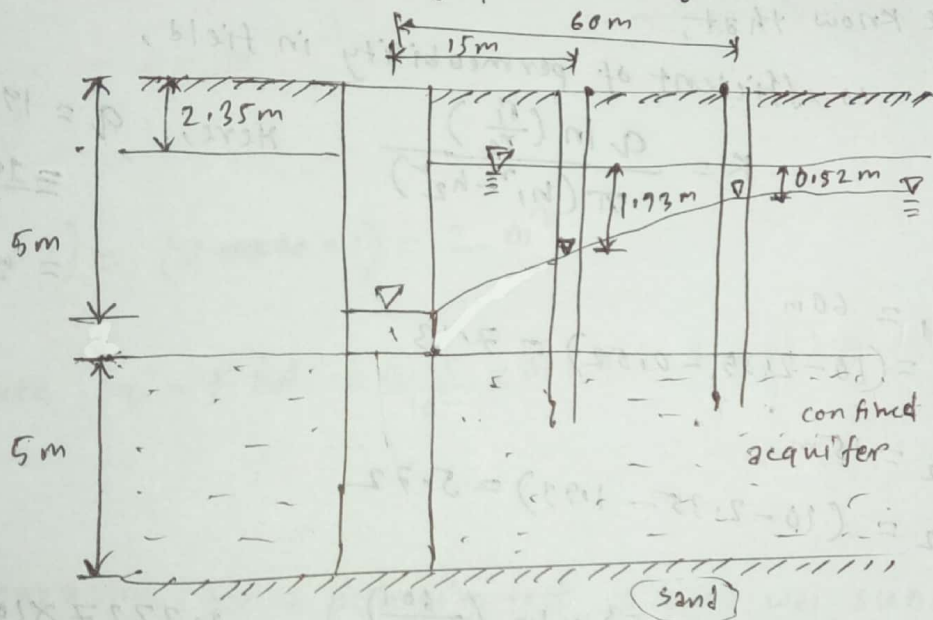
$D_{10} = 0.55 \text{ mm}$

$= 1 \times (0.55)^2$

$\therefore K = 0.3025 \text{ cm/sec.}$ (Ans)

2013

During preparations for a pumping test a well was sunk through a confined aquifer shown in figure below. Observation holes were drilled at 15 m and 60 m from the well. The water in the well and in the observation holes stood originally at the same level 2.35 m below the top of the well. After pumping until steady conditions had been achieved, the discharge was found to be $20 \text{ m}^3/\text{hr}$. The water level in the observation hole 15 m from the well had dropped 1.93 m, and that in the hole 60 m away had dropped 0.52 m. Determine the field coefficient of permeability of sand in m/sec.



Solution: For confined aquifer,

we know that,
$$K = \frac{q \ln \left(\frac{r_1}{r_2} \right)}{2\pi H (h_1 - h_2)}$$

Here, $r_1 = 60 \text{ m}$, $r_2 = 15 \text{ m}$, $H = 5 \text{ m}$, $q = 20 \text{ m}^3/\text{hr}$

$h_1 = (5 - 2.35 - 0.52) = 2.18 \text{ m}$, $h_2 = (5 - 2.35 - 1.93) = 0.72 \text{ m}$

$$\therefore K = \frac{20 \times \ln \left(\frac{60}{15} \right)}{2\pi \times 5 \times (2.18 - 0.72)} = 0.6045 \text{ m/hr}$$

(Ans.)

2016

- # The initial head is 300 mm in a falling head permeability test. It drops by 10 mm in 3 minutes. How much longer should this test continue, if the head is to drop to 120 mm?

Solution:

We know that, for falling head test,

$$K = \frac{2L}{At} \ln \left(\frac{h_1}{h_2} \right)$$

Hence,
$$\frac{t_1}{t_2} = \frac{\ln \left(\frac{h_1}{h_2} \right)}{\ln \left(\frac{h_1}{h_3} \right)}$$

$$\Rightarrow \frac{3}{t_2} = \frac{\ln \left(\frac{300}{290} \right)}{\ln \left(\frac{300}{180} \right)}$$

$$\Rightarrow t_2 = 45.209 \text{ min.} \quad (\text{Ans})$$

given that,

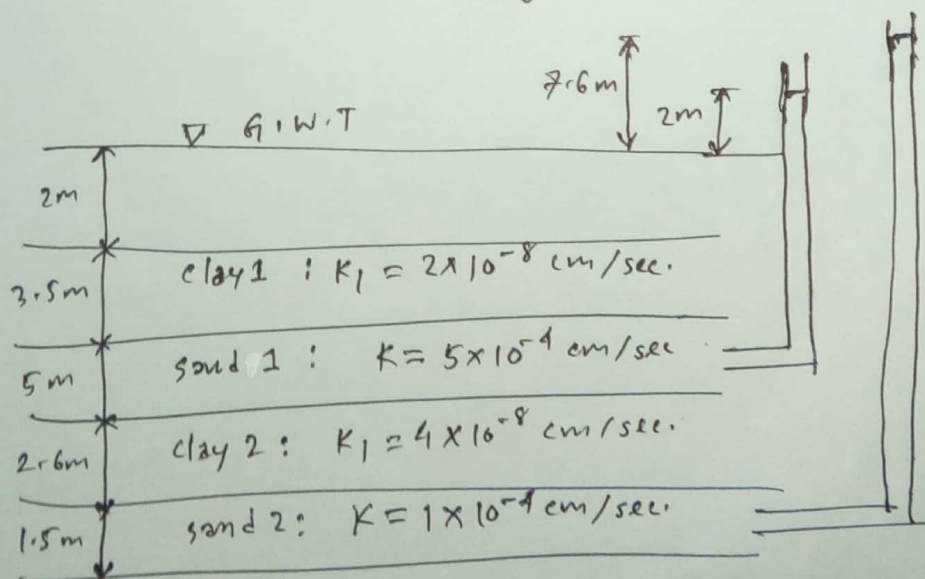
At $t=0$, $h_1 = 300 \text{ mm}$

$t = 3 \text{ min}$, $h_2 = (300 - 10) = 290 \text{ mm}$

$t = ?$, $h_3 = (300 - 120) = 180 \text{ mm}$

2012

- # with reference to the figure, (i) plot the variation of elevated head, pressure head and total head for the soil profile. (ii) Determine the average Darcy seepage velocity across clay 1 & 2.





Seepage

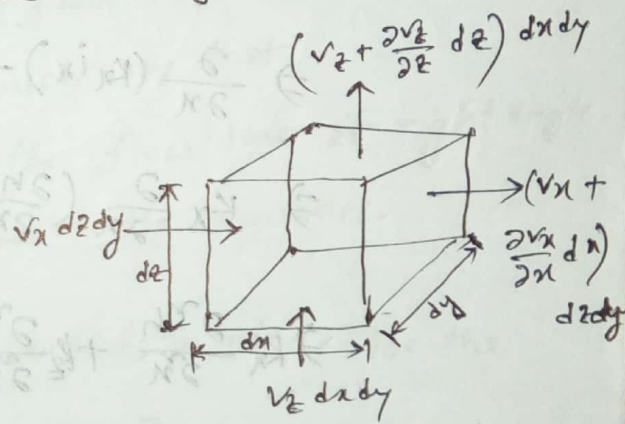
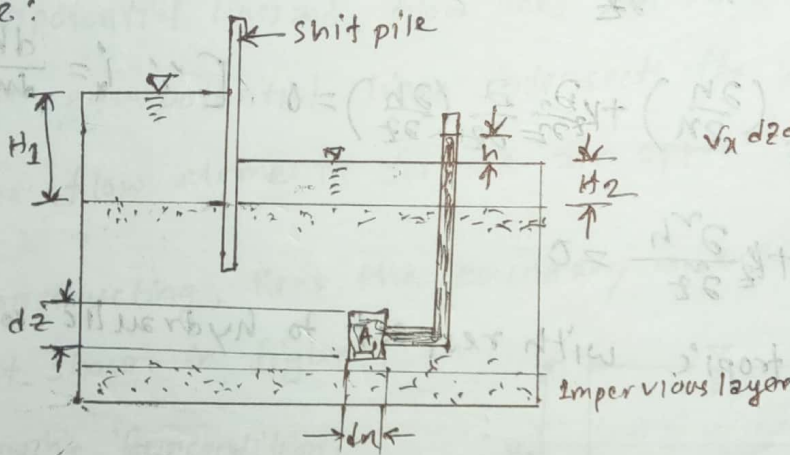
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Seepage: Seepage, in soil engineering, movement of water in soils, often a critical problem in building foundations. Seepage depends on several factors including permeability of the soil and the pressure gradient essentially the combination of forces acting on water through gravity and other factors.

Laplace equation of continuity:

Let us, consider a single row of sheet piles is assumed to be impervious, that have been driven into a permeable soil layer as shown in figure:

figure:



The steady state flow of water from the upstream to downstream side through the permeable layer is two-dimensional flow.

For flow at point A, we consider an element block

The rate of flow into block : $v_x dz dy$ in horizontal direction

and, $v_z dx dy$ in vertical direction

The rate of out flow from block : $(v_x + \frac{\partial v_x}{\partial x} dx) dz dy$ in horizontal direction

and, $(v_z + \frac{\partial v_z}{\partial z} dz) dx dy$ in vertical direction

Assuming, water is incompressible and no volume change in the soil mass occurs.

We know that, rate of inflow = rate of out flow

$$v_x dz dy + v_z dx dy = (v_x + \frac{\partial v_x}{\partial x} dx) dz dy + (v_z + \frac{\partial v_z}{\partial z} dz) dx dy$$

$$\Rightarrow \frac{\partial v_x}{\partial x} dx dy dz + \frac{\partial v_z}{\partial z} dx dy dz = 0$$

$$\Rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0$$

$$\Rightarrow \frac{\partial}{\partial x} (K_x i_x) + \frac{\partial}{\partial z} (K_z i_z) = 0 \quad \left[\text{From Darcy's law, } v = Ki \right]$$

$$\Rightarrow K_x \frac{\partial}{\partial x} \left(\frac{\partial h}{\partial x} \right) + K_z \frac{\partial}{\partial z} \left(\frac{\partial h}{\partial z} \right) = 0 \quad \left[i_x = \frac{dh}{dx} \text{ \& } i_z = \frac{dh}{dz} \right]$$

$$\Rightarrow K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0$$

if the soil is isotropic with respect to hydraulic conductivity

then, $K_x = K_z$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

This is the continuity equation for two dimensional flow.

☐ Flow nets: A flow net is a grid obtained by drawing a series of streamlines and equipotential lines.

☐ Flow line: Flow line is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium.

☐ Equipotential Line: Equipotential line is a line along which the potential head at all points are equal.

⇒ To complete the graphic construction of a flow net, one must draw the equipotential lines and flow lines in such way that

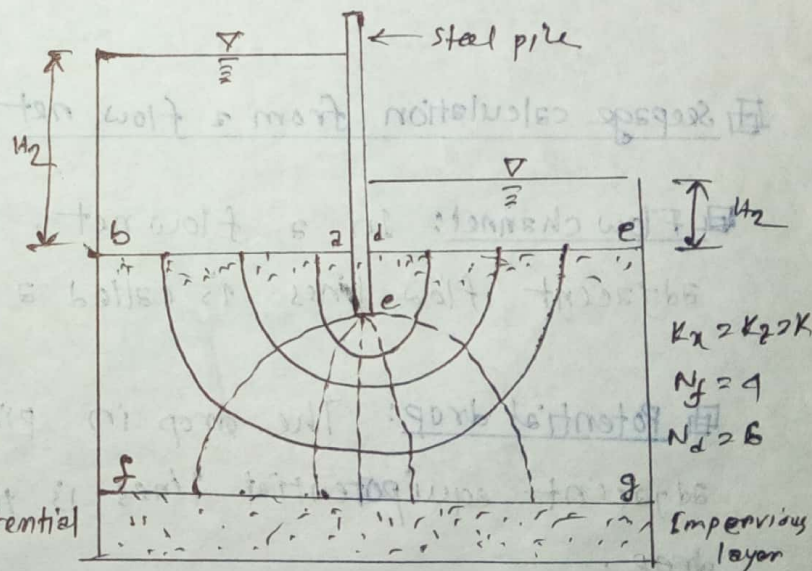
1. The equipotential lines intersect the flow lines at right angle.
2. The flow elements formed are approximate squares.

⇒ While constructing, keep the boundary conditions in mind. For the flow net shown in figure:

The following four conditions apply:

1. The upstream and downstream surfaces of the permeable layer (lines ab and de) are equipotential lines.

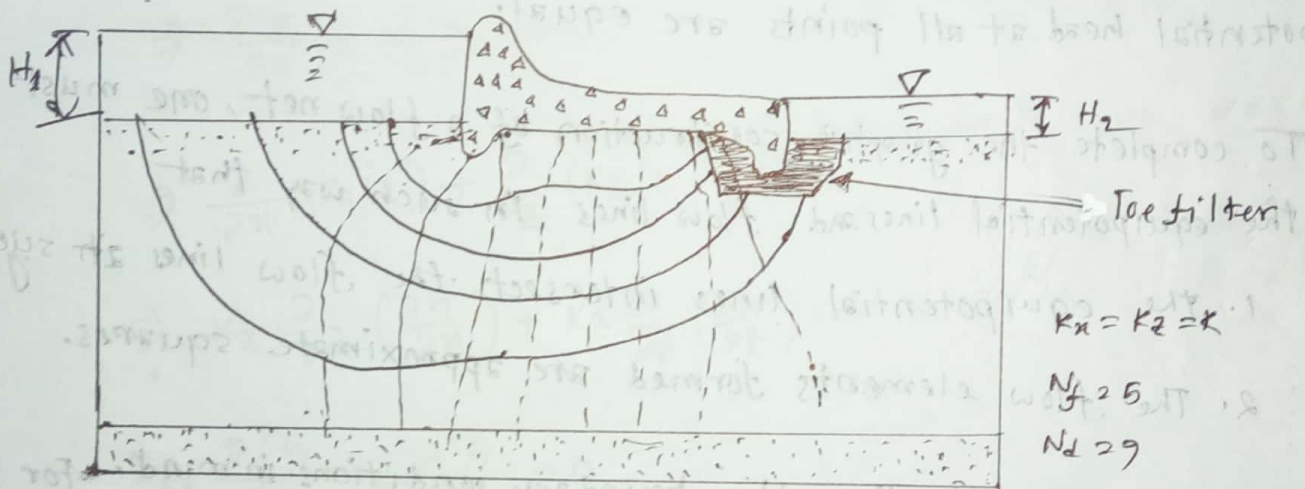
2. Because ab and de are equipotential lines, all the flow lines intersect them at right angle.



3. the boundary of impervious layer - that is line fg - is a flow line and ac is the surface of the impervious ~~layer~~ sheet pile, line acd .

4. The equipotential lines intersect acd and fg at right angles.

Flow net under a dam with toe filter:



Seepage calculation from a flow net:

Flow channel: In a flow net, the strip between any two adjacent flow lines is called a flow channel.

Potential drop: The drop in piezometric level between any two adjacent equipotential lines is the same. This is called potential drop.

Let us, a flow channel with the equipotential lines forming square elements as shown in figure:

The flow rate in a element:

$$\Delta q_1 = \Delta q_2 = \dots = \Delta q$$

From Darcy's law, $q = KiA$. Thus

$$\Delta q = K \left(\frac{h_1 - h_2}{l_1} \right) l_1 = K \left(\frac{h_2 - h_3}{l_2} \right) l_2 = \dots$$

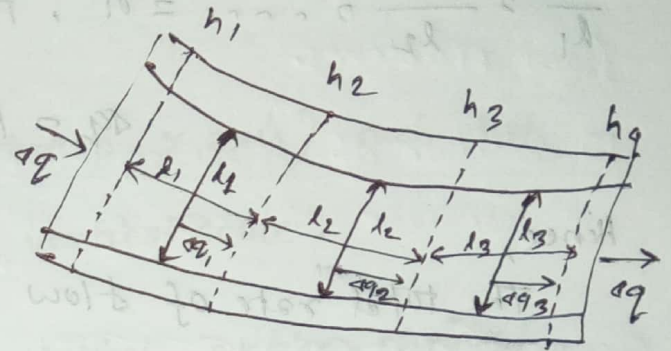


Fig. seepage through a flow channel with square element.

Since,

the flow elements are drawn as approximate squares. Thus,

$$h_1 - h_2 = h_2 - h_3 = \dots = \frac{H}{N_d}$$

and $\Delta q = K \frac{H}{N_d}$ where, H = head difference between up stream and down stream side.

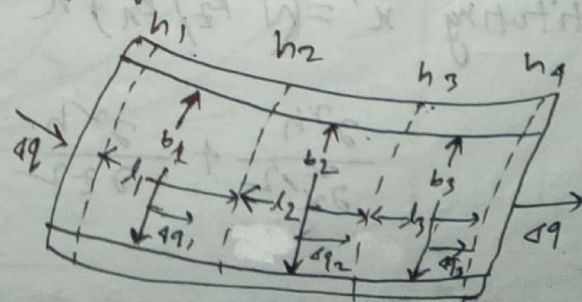
N_d = Number of potential drop

if the number of flow channels in a flow net = N_f

∴ The total rate of flow through all the channel per unit length:

$$Q = K \frac{HN_f}{N_d}$$

Alternatively, one can draw a rectangular channel mesh for a flow channel as shown in figure:



In this case, the flow rate, $q_1 = K \left(\frac{h_1 - h_2}{l_1} \right) b_1 = K \left(\frac{h_2 - h_3}{l_2} \right) b_2 = \dots$

If $\frac{b_1}{l_1} = \frac{b_2}{l_2} = \dots = n$, then,

$$q_1 = KH \left(\frac{n}{N_d} \right)$$

Hence,

$$\text{The total rate of flow, } q = KH \left(\frac{N_f}{N_d} \right) n$$

Flow nets in Anisotropic Soil:

In nature, most soils exhibit some degree of anisotropy. To account for soil anisotropy with respect to hydraulic conductivity, we must modify the flow net construction.

The differential equation of continuity for a two dimensional flow,

$$K_x \frac{\partial^2 h}{\partial x^2} + K_z \frac{\partial^2 h}{\partial z^2} = 0$$

For anisotropic soils, $K_x \neq K_z$. In this case the equation represents two families of curves do not meet at 90° .

$$\frac{\partial^2 h}{\left(\frac{K_z}{K_x} \right) \partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

substituting $x' = \left(\sqrt{K_z / K_x} \right) x$, we can obtain

$$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

To construct the flow net, use the following procedure:

step 1: Adopt a vertical scale (that is z axis) for drawing the cross section.

step 2: Adopt a horizontal scale (that is x axis) such that
horizontal axis $= \sqrt{\frac{K_z}{K_x}}$ x vertical scale.

step 3: with adopted scales, ^{plot} the vertical section through permeable layer parallel to the direction of flow.

step 4: Draw the flow net for the permeable layer on the section obtained from step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.

The rate of seepage per unit length can be calculated by:

$$q = \sqrt{K_x K_z} \frac{HN_f}{N_d}$$

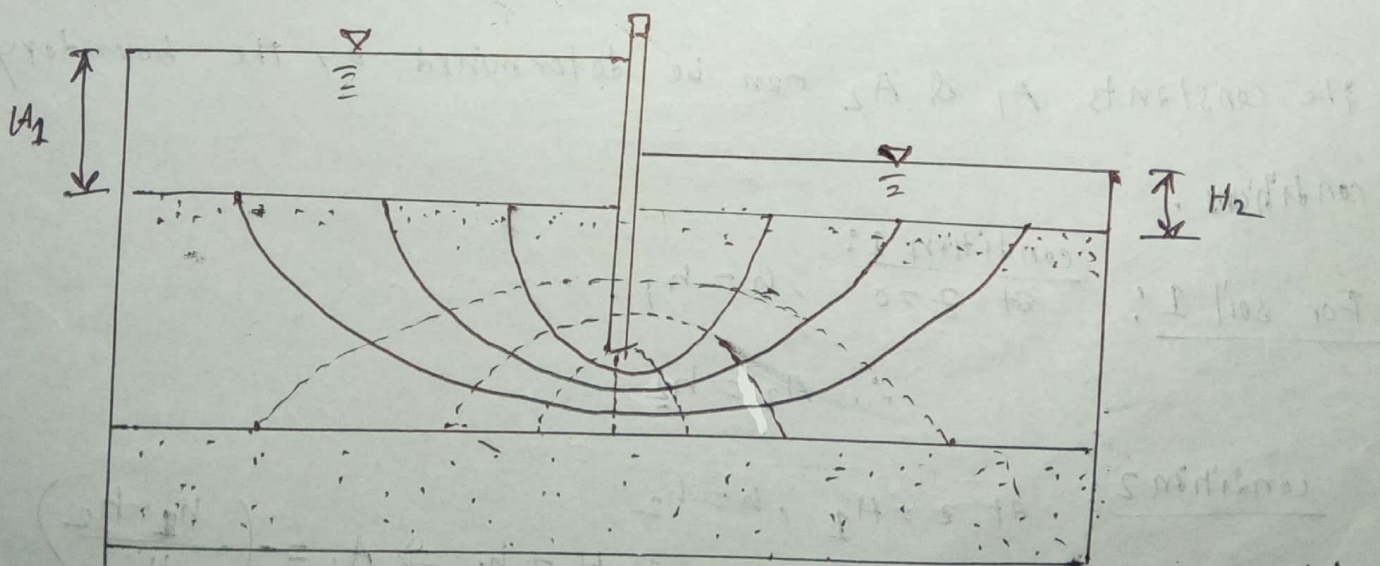


Fig. A flow element in anisotropic (in true section)

Continuity Equation for solution of simple flow problems:

The continuity equation can be used in solving some simple flow problems.

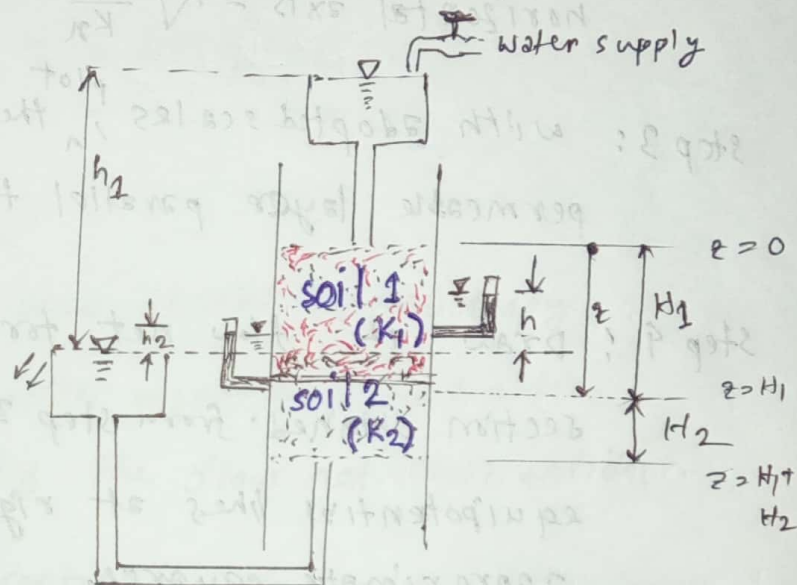
1D solution of Laplace equation:

We consider the case of only vertical flow as shown in figure:

~~Because of this equation it is easy to get by hand.~~

Because of the flow is in only the z direction, the continuity equation is simplified to the form:

$$\frac{\partial^2 h}{\partial z^2} = 0$$



The solution of the equation is easy to get by having a integration of h with respect to z twice:

$$h = A_1 z + A_2 \dots \text{where } A_1 \text{ \& } A_2 \text{ are constants}$$

The constants A_1 & A_2 can be determined by the boundary conditions.

condition 1:
 For soil 1: at $z = 0$, $h = h_1$
 $\therefore A_2 = h_1$

condition 2:
 at $z = H_1$, $h = h_2$
 Then, $h_2 = A_1 H_1 + h_1 \Rightarrow A_1 = -\left(\frac{h_2 - h_1}{H_1}\right)$

Now,

From eqn ① we get, $h = -\left(\frac{h_1 - h_2}{H_1}\right)z + h_1$ for $0 \leq z \leq H_1$

For soil 2:

condition 1: at $z = H_1$, $h = h_2$

$$A_2 = h_2 - A_1 H_1$$

condition 2: at $z = H_1 + H_2$, $h = 0$

$$0 = A_1 (H_1 + H_2) + A_2$$

$$\Rightarrow A_1 H_1 + A_1 H_2 + h_2 - A_1 H_1 = 0$$

$$\Rightarrow A_1 = -\frac{h_2}{H_2}$$

Now, From eqn ① we obtain, $h = -\left(\frac{h_2}{H_2}\right)z + h_2 \left(1 + \frac{H_1}{H_2}\right)$... ③

for $H_1 \leq z \leq (H_1 + H_2)$

At any given time,

flow through soil 1 = flow through soil 2

$$q_1 = q_2 = q$$

$$\text{then, } q = K_1 \left(\frac{h_1 - h_2}{H_1}\right) A = K_2 \left(\frac{h_2 - 0}{H_2}\right) A$$

$$\Rightarrow h_2 = \frac{h_1 K_1}{H_1 \left(\frac{K_1}{H_1} + \frac{K_2}{H_2}\right)}$$

substituting the value of h_2 in eqⁿ (iii) we get,

$$h = h_1 \left[\left(\frac{K_1}{K_1 H_2 + K_2 H_1} \right) (H_1 + H_2 - z) \right] \text{ for } H_1 \leq z \leq H_1 + H_2$$

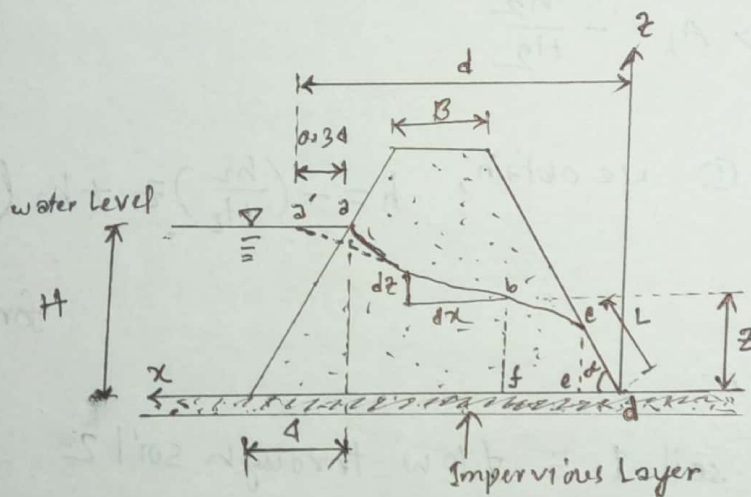
similarly,

substituting the value of h_2 in eqⁿ (ii) we get,

$$h = h_1 \left(1 - \frac{K_2 z}{K_1 H_2 + K_2 H_1} \right) \text{ for } 0 \leq z \leq H_1$$

Seepage through an earth dam on an impervious Base:

The following figure shows a homogeneous earth dam resting



Let,

the hydraulic conductivity of the compacted material of which the earth dam is made be equal to K . The free surface of water passing through earth dam is given by $abcd$

considering the triangle cde ,

$$\text{rate of seepage per unit length of the dam, } q = KiA \dots \textcircled{1}$$

where,

$$A = ce \times l = L \sin \alpha$$

$$i = \frac{dz}{dx} = \tan \alpha$$

Hence, $q = KL \sin \alpha \cdot \tan \alpha$ (ii)

again,

rate of seepage through section of is, $q = KiA$
 $= K \left(\frac{dz}{dx} \right) \times (z \times l)$

$$\therefore q = Kz \frac{dz}{dx}$$
 (iii)

For continuous flow,

$$Kz \left(\frac{dz}{dx} \right) = KL \sin \alpha \cdot \tan \alpha$$

$$\Rightarrow \int z dz = \int L \sin \alpha \tan \alpha dx$$

when, $x = L \cos \alpha$ then, $z = L \sin \alpha$

when, $x = d$ then, $z = H$

Integrating within these limits we obtain,

$$\left[\frac{z^2}{2} \right]_{L \sin \alpha}^H = L \sin \alpha \tan \alpha \left[x \right]_{L \cos \alpha}^d$$

$$\Rightarrow \frac{1}{2} (H^2 - L^2 \sin^2 \alpha) = L \sin \alpha \tan \alpha (d - L \cos \alpha)$$

$$\Rightarrow \frac{H^2}{2} - \frac{L^2 \sin^2 \alpha}{2} = \frac{Ld \sin^2 \alpha}{\cos \alpha} - \frac{L^2 \sin^2 \alpha}{1}$$

$$\Rightarrow \frac{L^2 \sin^2 \alpha}{2} - Ld \frac{\sin^2 \alpha}{\cos \alpha} + \frac{H^2}{2} = 0$$

$$\Rightarrow L^2 \cos \alpha - 2Ld + H^2 \frac{\cos \alpha}{\sin^2 \alpha} = 0 \left[\text{Multiplying by } 2 \frac{\cos \alpha}{\sin^2 \alpha} \right]$$

$$\Rightarrow L = \frac{-(-2d) \pm \sqrt{(-2d)^2 - 4 \cdot \cos \alpha \cdot \frac{H^2 \cos \alpha}{\sin^2 \alpha}}}{2 \cos \alpha}$$

$$\therefore L = \frac{d}{\cos \alpha} - \sqrt{\frac{d^2}{\cos^2 \alpha} - \frac{H^2}{\sin^2 \alpha}}$$

Properties of flow net:

1. The flow lines and equipotential lines meet at right angle to one another.
2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.
3. Smaller the dimension of the field, greater will be hydraulic gradient and velocity of flow through it.
4. The quantity of water through each flow channel is the same. Similarly the same potential drop occurs between two successive equipotential lines.

Applications of flow net:

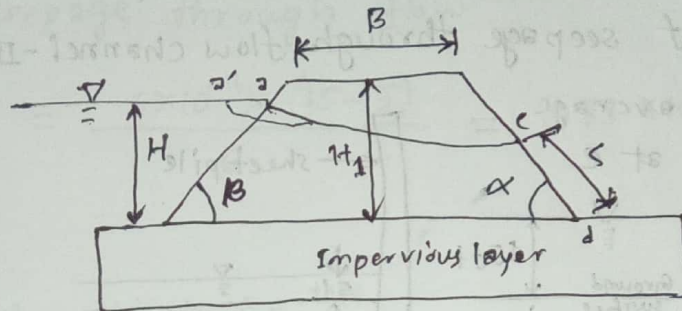
A flow net can be utilized for the following purposes:

- (i) Determination of seepage.
- (ii) Determination of hydraulic pressure.
- (iii) Determination of seepage pressure.
- (iv) Determination of exit gradient, etc.

Seepage

2006, 2010 Example-8.5

An earth dam is shown in figure below. Determine the seepage rate, q in $\text{ft}^3/\text{day}/\text{ft}$ length. $\alpha = 45^\circ$, $\beta = 45^\circ$, $B = 15 \text{ ft}$, $H = 30 \text{ ft}$, $H_1 = 40 \text{ ft}$ and $K = 4.2 \times 10^{-4} \text{ ft}/\text{min}$.



Solution:

We know that,

$$A = \frac{H}{\tan \beta} = \frac{30}{\tan 45^\circ} = 30 \text{ ft}$$

then,

$$d = 0.3A + \frac{H_1 - H}{\tan \beta} + B + \frac{H_1}{\tan \alpha}$$

$$\Rightarrow d = (0.3 \times 30) + \frac{40 - 30}{\tan 45^\circ} + 15 + \frac{40}{\tan 45^\circ}$$

$$\Rightarrow d = 74 \text{ ft}$$

and,

$$L = \frac{d}{\cos \alpha} - \sqrt{\left(\frac{d}{\cos \alpha}\right)^2 - \left(\frac{H}{\sin \alpha}\right)^2}$$

$$\Rightarrow L = \frac{74}{\cos 45^\circ} - \sqrt{\left(\frac{74}{\cos 45^\circ}\right)^2 - \left(\frac{30}{\sin 45^\circ}\right)^2}$$

$$\Rightarrow L = 8.986 \text{ ft}$$

Now, $q = KL \tan \alpha \sin \alpha$

$$\Rightarrow q = 4.2 \times 10^{-4} \times 8.986 \times \tan 45^\circ \times \sin 45^\circ$$

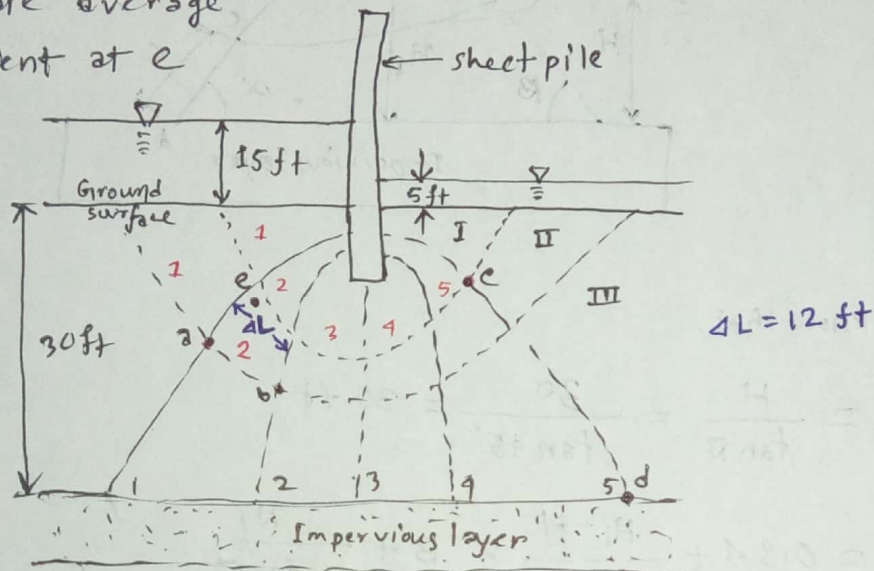
$$\therefore q = 2.67 \times 10^{-3} \text{ ft}^3/\text{min}/\text{ft} = 3.843 \text{ ft}^3/\text{day}/\text{ft}$$

(Ans.)

2012 Example-8.2

A flow net for flow around a single row of sheet in a permeable soil layer is shown in figure below. Given hydraulic conductivity is 5×10^{-3} cm/sec, determine (i) How high (Above the ground surface) the water will rise if the piezometers are placed at points a, b, c and d. (ii) The rate of seepage through flow channel-II per unit length.

(ii) The approximate average hydraulic gradient at e



Solutions: (i) From figure, we have $N_d = 6$, $H_1 = 15$ ft and $H_2 = 5$ ft.

So head loss of each potential drop is,

$$\Delta H = \frac{H_1 - H_2}{N_d} = \frac{15 - 5}{6} = 1.67 \text{ ft}$$

At point a, we have gone through one potential drop. So the water in the piezometer will rise to an elevation of

$$(15 - 1.67) \text{ ft} = 13.33 \text{ ft above the ground surface.}$$

At point b, we have two potential drops. so the water in the piezometer will rise to an elevation of

$$(15 - 2 \times 1.67) = 11.66 \text{ ft above the ground surface}$$

At point c & d, we have 5 potential drops. so, elevation of
($15 - 5 \times 1.67$) = 4.98 ft above the ground surface

~~At point~~

(i) The rate of seepage through flow channel II per unit length

$$dq_2 = \frac{KH}{N_d} = \frac{5 \times 10^{-3} \times (15 - 5)}{6} = 8.33 \times 10^{-3} \text{ ft}^3 / \text{sec} / \text{ft}$$

(Ans.)

(ii) Average hydraulic gradient, $i = \frac{\Delta H}{\Delta L}$

$$\text{At e, } i = \frac{1.67}{12} = 0.139$$

(Ans.)

In Situ Stresses

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Effective pressure: The pressure transmitted through grain to grain at the contact points through a soil mass is termed as effective pressure (σ') or inter granular pressure. This pressure is responsible for ^{the} decrease in the void ratio or increase in the frictional resistance of a soil mass.

pore water pressure: If the pores of a soil mass are filled with water and if a pressure induced into the pore water, tries to separate the grains, this pressure is termed as pore water pressure (u) or neutral pressure.

The effect of this pressure is to increase volume or decrease the frictional resistance of the soil mass.

Total pressure: The summation of effective pressure and pore water pressure is defined as Total pressure (σ).

$$\sigma = \sigma' + u$$

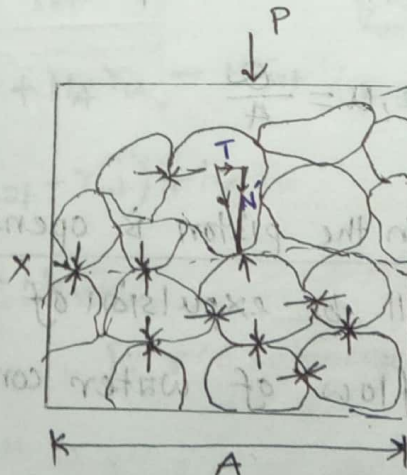
Total pressure
effective pressure
pore water pressure

By Terzaghi (1923),

$$P = \sum N' + uA$$

$$\Rightarrow \frac{P}{A} = \frac{\sum N'}{A} + u$$

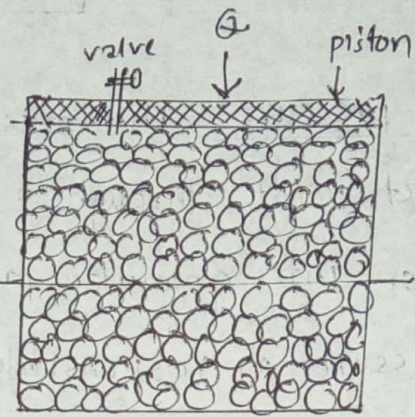
$$\Rightarrow \sigma = \sigma' + u$$



Effective stress & Pore water pressure:

Case 1: Dry sand under load (Q)

The load Q applied at the surface of the soil is transferred to the soil grains in the mold through their points of contact.



If the cross sectional area of the cylinder is A , the average stress at any level $x-y$ may be written as,

$$\sigma_a = \frac{Q}{A}$$

Since, this stress is responsible for the deformation of soil, it is termed as effective stress. Hence,

$$\sigma_a = \sigma'$$

Case 2: Fully saturated sand under load (Q)

If the valve provide in the piston is closed,

As we assume that water is incompressible, the applied load Q will be transmitted to the water in the pores. The pressure developed in the water is called pore water pressure.

This pressure prevents the compression of the soil. Hence,

$$\text{pore water pressure, } u = \frac{Q}{A}$$

If the valve provide in the piston is opened,

immediately there will be expulsion of water through the hole in the piston. The flow of water continues for some time and then stops.

The expulsion of water from the pores decreases the pore water pressure and correspondingly increases effective pressure.

At any stage, the total pressure $\frac{Q}{A}$ is divided between water and the points of contact of grains.

\therefore Total pressure = effective pressure + pore water pressure

$$\sigma = \sigma' + u$$

Final equilibrium will be reached when there is no expulsion of water. At this stage, pore water pressure, $u = 0$

Hence, $\sigma = \sigma'$

Stresses in saturated soil without seepage:

The following figure shows a column of saturated soil mass with no seepage of water in any direction.

The total stress at the elevation of point A,

$$\sigma = H_A \gamma_w + (H_A - H) \gamma_{sat}$$

$$= H_A \gamma_w + H_A \gamma_{sat} - H \gamma_{sat}$$

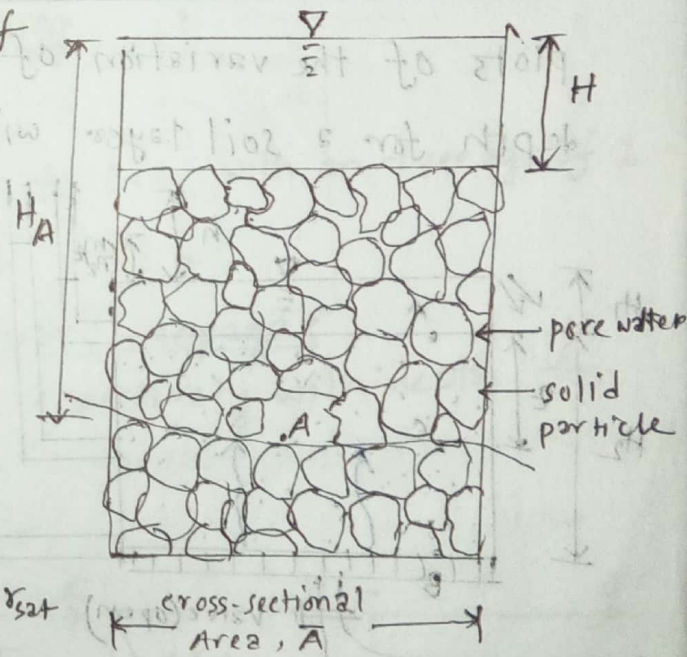
$$= H_A \gamma_w + H_A (\gamma_{sat} - \gamma_w) + H_A \gamma_w - H \gamma_{sat}$$

$$= H_A (\gamma_{sat} - \gamma_w) - H (\gamma_{sat} - \gamma_w) + H_A \gamma_w$$

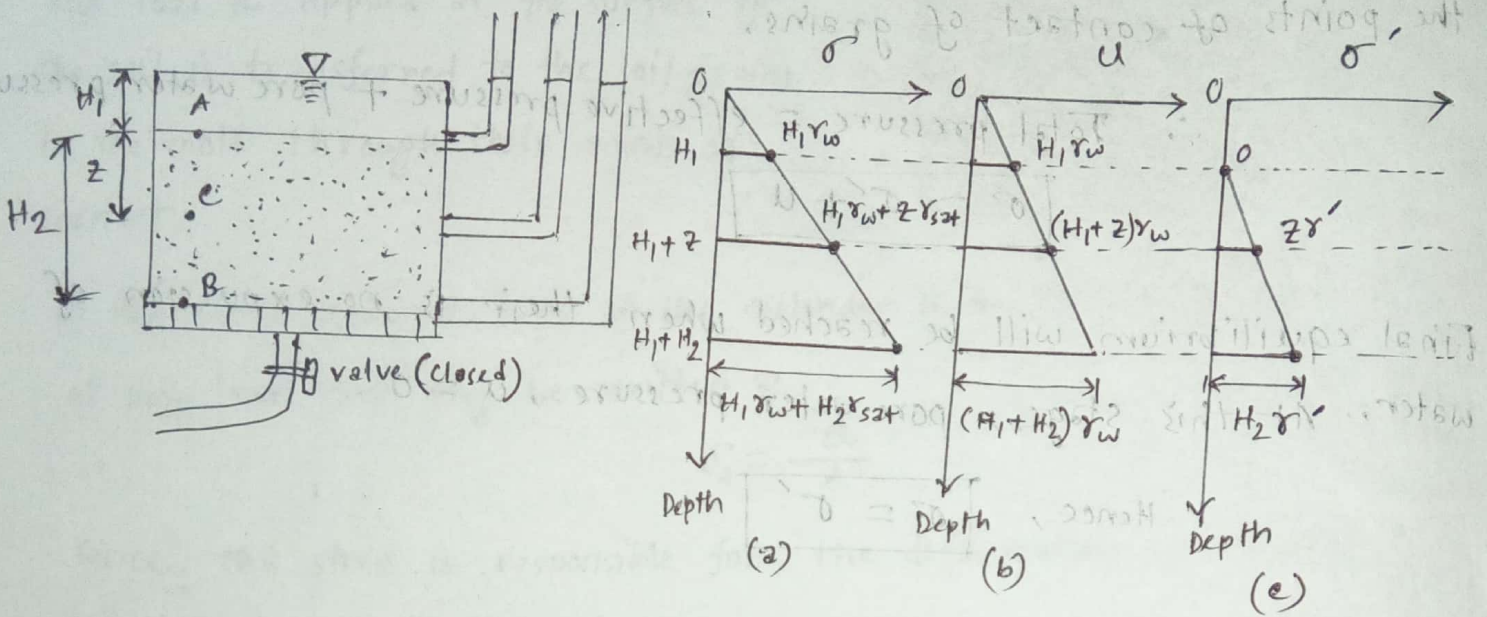
$$= (H_A - H) (\gamma_{sat} - \gamma_w) + H_A \gamma_w$$

$$= (H_A - H) \gamma' + H_A \gamma_w \quad [\because \gamma' = \text{submerged unit weight of soil} = \gamma_{sat} - \gamma_w]$$

$$\therefore \sigma = \sigma' + u$$

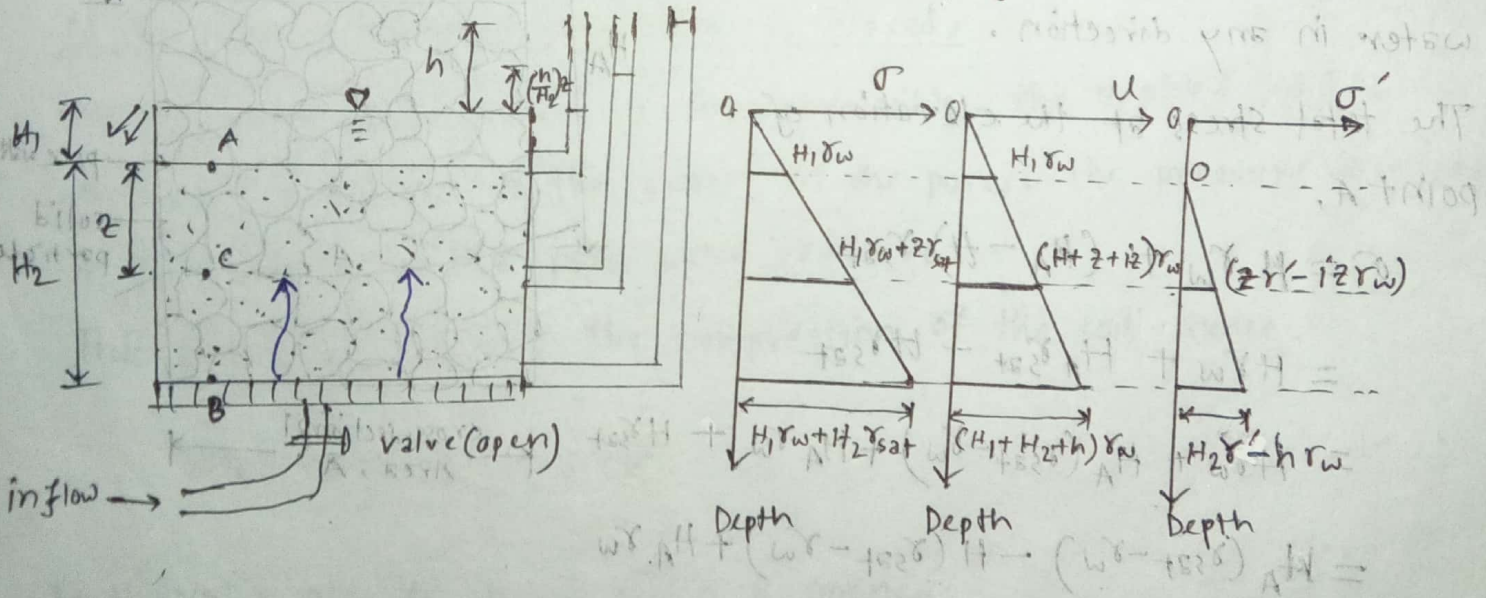


plots of the variation of σ , u and σ' respectively, with depth for a submerged layer of soil placed in a tank with no seepage.



Stresses in saturated soil with upward seepage.

plots of the variation of σ , u and σ' respectively with depth for a soil layer with upward seepage.



A + A,

$$\sigma_A = H_1 \gamma_w$$

$$u_A = H_1 \gamma_w$$

$$\sigma'_A = \sigma_A - u_A = 0$$

A + B,

$$\sigma_B = H_1 \gamma_w + H_2 \gamma_{sat}$$

$$u_B = (H_1 + H_2 + h) \gamma_w$$

$$\sigma'_B = \sigma_B - u_B = H_2 (\gamma_{sat} - \gamma_w) - h \gamma_w = H_2 \gamma' - h \gamma_w$$

At C,

$$\sigma_c = H_1 \gamma_w + z \gamma_{sat}$$

$$u_c = \left(H_1 + z + \frac{h}{H_2} z \right) \gamma_w = \left(H_1 + z + i z \right) \gamma_w$$

$$\sigma'_c = \sigma_c - u_c = z (\gamma_{sat} - \gamma_w) - \left(\frac{h}{H_2} \right) z \gamma_w = z \gamma' - i z \gamma_w$$

Note that $\frac{h}{H_2} = \text{hydraulic gradient } (i)$

If the rate of seepage and thereby hydraulic gradient gradually are increased, a limiting condition will be reached, at which point:

$$\sigma'_c = z \gamma' - i_{cr} z \gamma_w \quad \text{Where, } i_{cr} = \text{critical hydraulic gradient} = \frac{\gamma'}{\gamma_w}$$

(for zero effective stress)

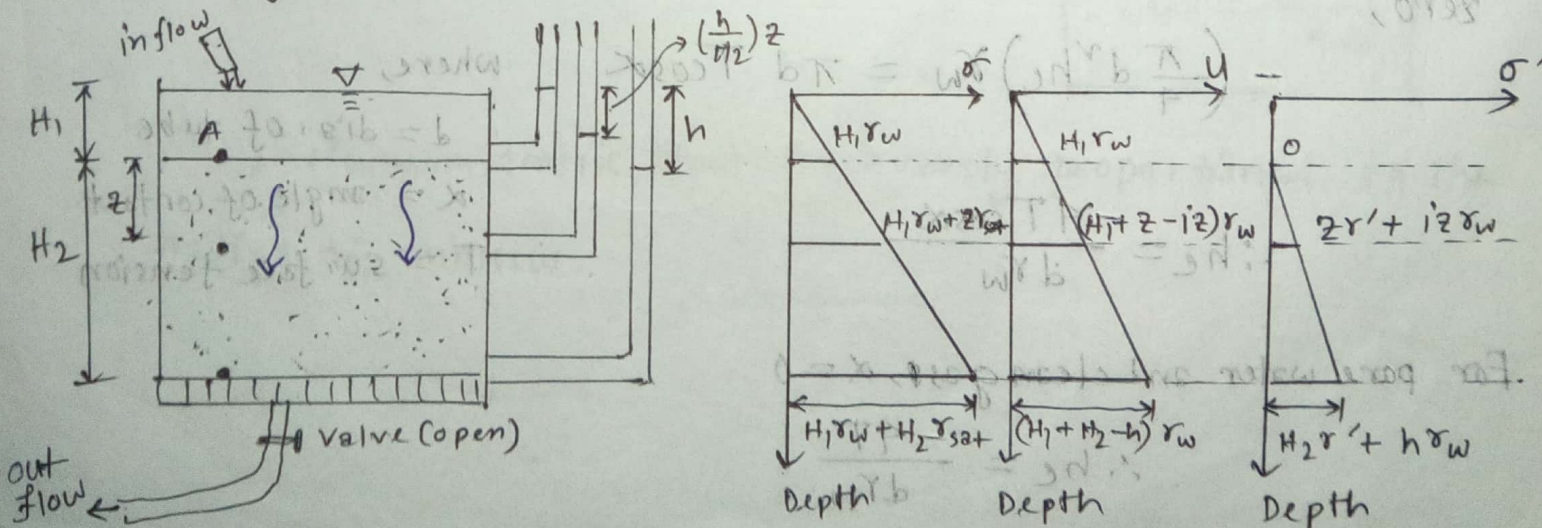
* under such condition, soil stability is lost.

The situation is referred to as boiling or quick condition.

For most soils, i_{cr} varies from 0.9 to 1.1 with an average of 1.

Stresses in saturated soil with Downward seepage:

plots of the variation of σ , u and σ' respectively with depth for a soil layer with downward seepage:

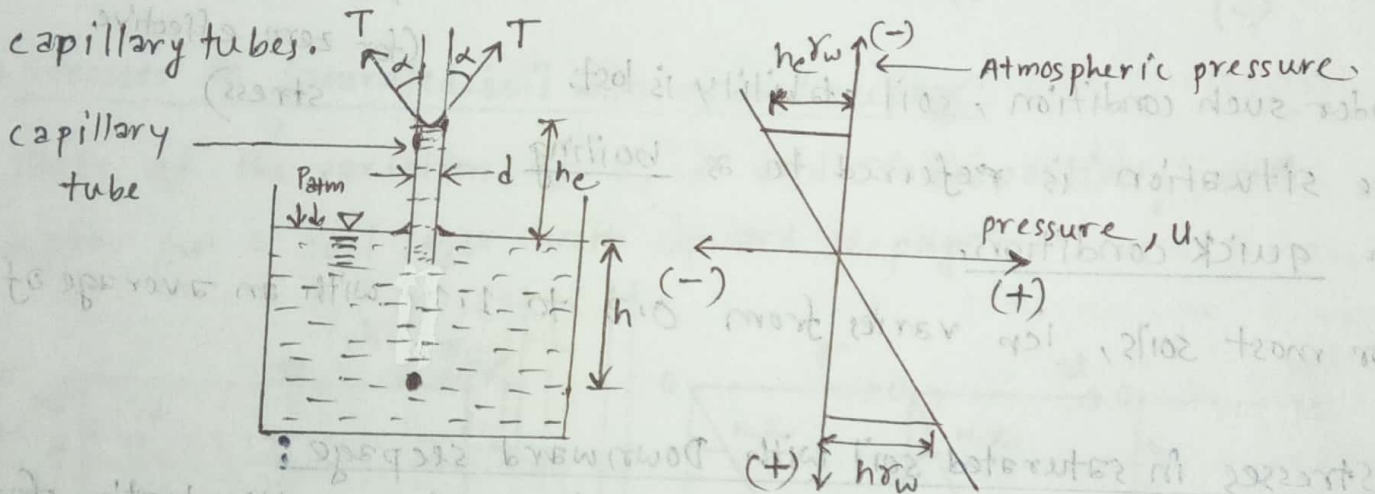


capillary rise in soils:

capillary rise: The rise of water level in a capillary tube or in a fine pores of the soil is due to the existence of surface tension which pulls the water up against the gravitational force is called capillary rise.

The height of capillary rise above the ground water surface depends upon the diameter of the capillary tube and the value of the surface tension.

The continuous void spaces in soil can behave as bundles of capillary tubes.



For equilibrium, summation of force along vertical direction will be zero,

$$\left(\frac{\pi}{4} d^2 h_c \right) \gamma_w = \pi d T \cos \alpha$$

$$\therefore h_c = \frac{4T \cos \alpha}{d \gamma_w}$$

For pure water and clean glass, $\alpha = 0$

$$\therefore h_c = \frac{4T}{d \gamma_w}$$

where,

d = dia. of tube

α = angle of contact

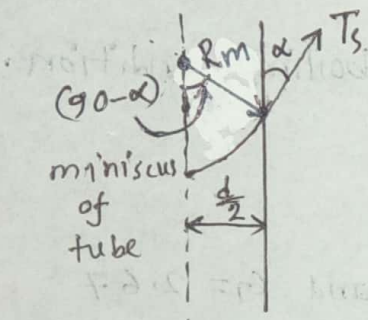
T = surface tension

At 20°C temperature, for water, $T = 72 \text{ mN/m}$

Hence, height of capillary rise, $h_c \propto \frac{1}{d}$

Thus, the smaller the capillary tube diameter, the larger the capillary rise.

□ Prove that, $U_c = \frac{2T_s}{R_m}$ where $T_s = \text{surface tension}$ & $R_m = \text{radius of meniscus}$



Here,

$$\frac{d}{2} = R_m \sin(90 - \alpha) = R_m \cos \alpha$$

$$\Rightarrow R_m = \frac{d}{2 \cos \alpha}$$

We know,

$$h_c = \frac{4 T_s \cos \alpha}{d \gamma_w}$$

$$h_c = \frac{4 T_s \cos \alpha}{2 R_m \cos \alpha \cdot \gamma_w} \quad [\because d = 2 R_m \cos \alpha]$$

$$\therefore h_c = \frac{2 T_s}{R_m \gamma_w}$$

Now,

maximum tension at the level of meniscus, $U_c = \gamma_w h_c$

$$\Rightarrow U_c = \gamma_w \cdot \frac{2 T_s}{R_m \gamma_w}$$

$$\therefore U_c = \frac{2 T_s}{R_m}$$

Thus, Maximum tensile stress is inversely proportional to the radius of meniscus.

Seepage force:

By the virtue of the viscous friction exerted on water flowing through soil pores, an energy is effected between the water and the soil. The force corresponding to this energy transfer is called seepage force or seepage pressure.

hydraulic critical gradient:

The hydraulic gradient at quick condition or boiling condition is called critical hydraulic gradient.

For loose deposits of sand or silt, $e = 0.67$ and $G_s = 2.67$

$$\text{Hence, } i = i_{cp} = \frac{G_s - 1}{1 + e} = \frac{2.67 - 1}{1 + 0.67} = 1$$

Quick Sand:

When flow takes place in upward direction, the seepage pressure also acts in the upward direction and the effective pressure is reduced. If the seepage pressure becomes equal to the pressure due to submerged weight of soil. The effective pressure is reduced to zero. In such a case, a cohesionless soil loses all its shear strength and the soil particles have a tendency to move up in the direction of flow. This phenomenon of lifting of soil particles is called quick sand or quick condition.

During quick condition, $\sigma' = z\gamma' - iz\gamma_w = 0 \Rightarrow \gamma' = i\gamma_w$

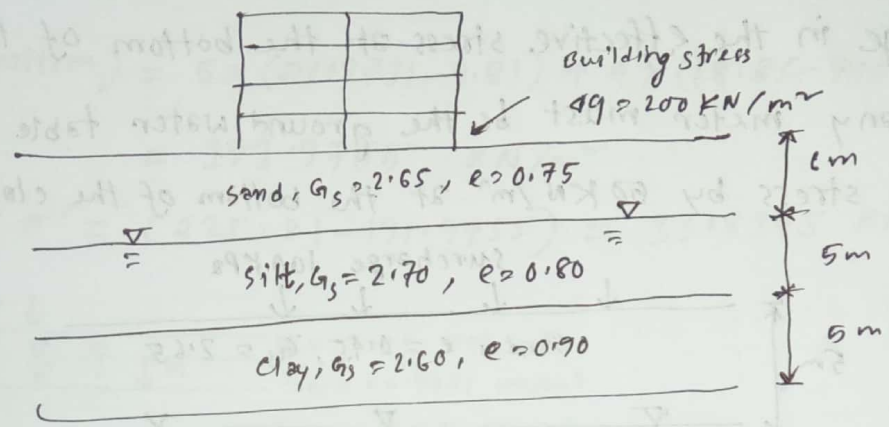
$$\text{Hence, } i = i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e}$$

From this equation, hydraulic gradient will be calculated.

In situ stresses

2006 Example - 9.1

A soil profile is shown in figure. Calculate the total stress, effective stress and pore water pressure.



Solution: (i) For sand, $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.75} = 14.855 \text{ kN/m}^3$

(ii) For silt, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(2.7 + 0.8) \times 9.81}{1+0.8} = 19.075 \text{ kN/m}^3$

(iii) For clay, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(2.6 + 0.9) \times 9.81}{1+0.9} = 18.071 \text{ kN/m}^3$

Depth, D (m)	Total Stress, σ (kN/m ²)	pore water pressure, u (kN/m ²)	effective stress, $\sigma' = \sigma - u$ (kN/m ²)
0	200	0	200
6 m	$200 + (6 \times 14.855)$ = 289.13	0	289.13
(6+5) = 11 m	$289.13 + (5 \times 19.075)$ = 384.505	(5 × 9.81) = 49.05	335.455
(11+5) = 16 m	$384.505 + (5 \times 18.071)$ = 474.86	$49.05 + (5 \times 9.81)$ = 98.1	376.76

(Ans.)

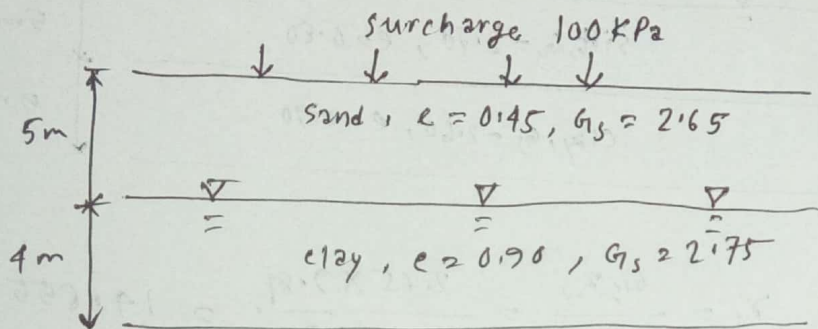
2008 ~~2008~~

The soil profile is shown in figure below:

(i) calculate the variation of σ , u and σ' with depth.

(ii) If the water table rises to the top of the ground surface, what is the change in the effective stress at the bottom of the clay layer?

(iii) How many meter must be the ground water table rise to decrease the effective stress by 50 kN/m^2 at the bottom of the clay layer?



Solution: (i) For sand, $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.45} = 17.93 \text{ kN/m}^3$

For clay, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(2.75 + 0.9) \times 9.81}{1+0.9} = 18.85 \text{ kN/m}^3$

Depth, D (m)	Total stress, σ (kN/m ²)	Pore water pressure, u (kN/m ²)	effective stress, $\sigma' = \sigma - u$ (kN/m ²)
0	100	0	100
5	$100 + (5 \times 17.93)$ $= 189.65$	0	189.65
$(5+4)$ $= 9$	$189.65 + (4 \times 18.85)$ $= 265.05$	(4×9.81) $= 39.24$	225.81

(Ans.)

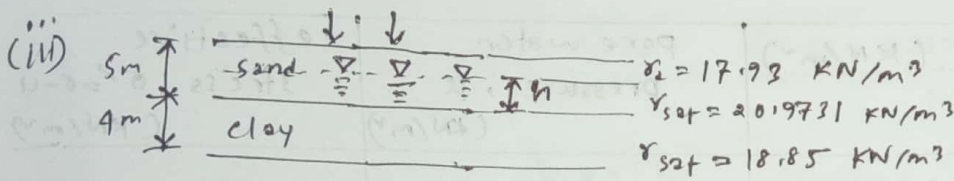
(ii) water table rises the ground surface, sand layer is saturated

For sand, $\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.65 + 0.45) \times 9.81}{1 + 0.45} = 20.9731 \text{ KN/m}^3$

$\therefore \sigma' \text{ (at bottom)} = 5 \times (20.9731 - 9.81) + 4 \times (18.85 - 9.81) + 100$
 $= 191.9755 \text{ KN/m}^2$

decrease in $\sigma' = (225.81 - 191.9755) = 33.8345 \text{ KN/m}^2$

(Ans)



$\therefore \sigma' \text{ (at bottom)} = (5-h) \times 17.93 + h \times (20.9731 - 9.81) + 4 \times (18.85 - 9.81) + 100$

$\Rightarrow (225.81 - 50) = 89.65 - 17.93h + 11.1631h + 36.16 + 100$

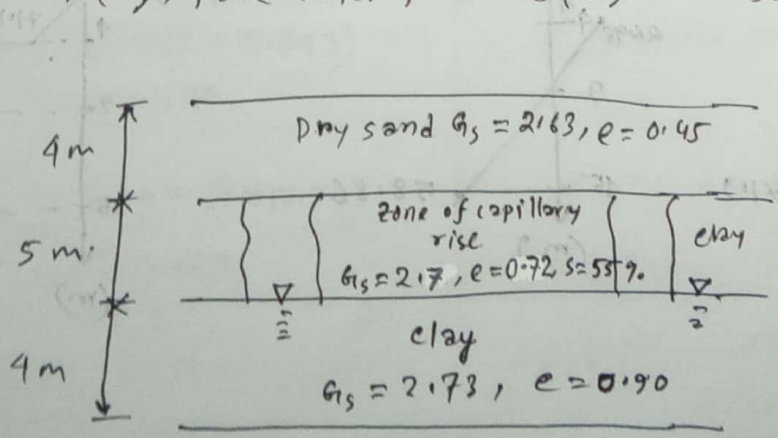
$\Rightarrow 175.81 = 225.81 - 6.7669h$

$\Rightarrow h = \frac{50}{6.7669} = 7.39 \text{ m}$

(Ans)

2009, 2010

A layered soil profile is shown in figure below. Determine and plot total stress (σ), pore water pressure (u) and effective stress (σ')

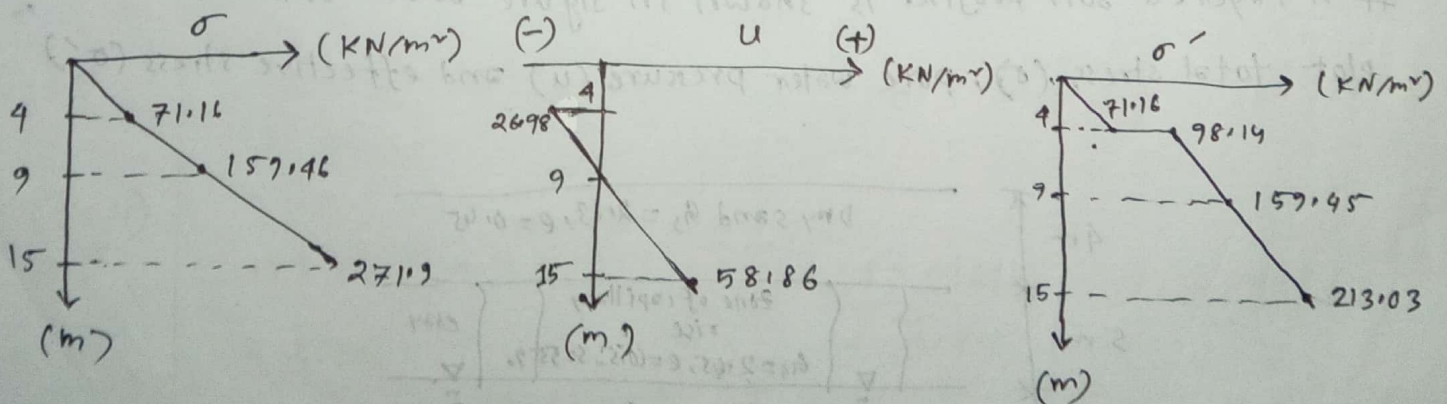


Solution: For dry sand, $\gamma_d = \frac{G_s \gamma_w}{1+e} = \frac{2.63 \times 9.81}{1+0.45} = 17.79 \text{ KN/m}^3$

For clay, $\gamma = \frac{(G_s + Se) \gamma_w}{1+e} = \frac{(2.7 + 0.55 \times 72) \times 9.81}{1+0.72} = 17.658 \text{ KN/m}^3$

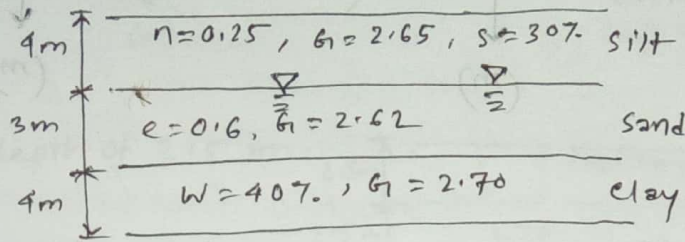
For saturated clay, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1+e} = \frac{(2.73 + 0.9) \times 9.81}{1+0.9} = 18.74 \text{ KN/m}^3$

Depth, D (m)	Total stress, σ (KN/m ²)	pore water pressure, u (KN/m ²)	effective stress, $\sigma' = \sigma - u$ (KN/m ²)
0	0	0	0
4	$(4 \times 17.79) = 71.16$	immediately above: 0 immediately below: $-s \gamma_w h = -(0.55 \times 9.81 \times 4) = -26.98$	71.16 98.14
(A + 5) = 9	$71.16 + (17.658 \times 5) = 159.45$	0	159.45
(9 + 6) = 15	$159.45 + (6 \times 18.74) = 271.89$	$(6 \times 9.81) = 58.86$	213.03



2013

For a given soil profile, determine the total pressure, effective pressure and pore water pressure, and draw the pressure diagram.



Solution:

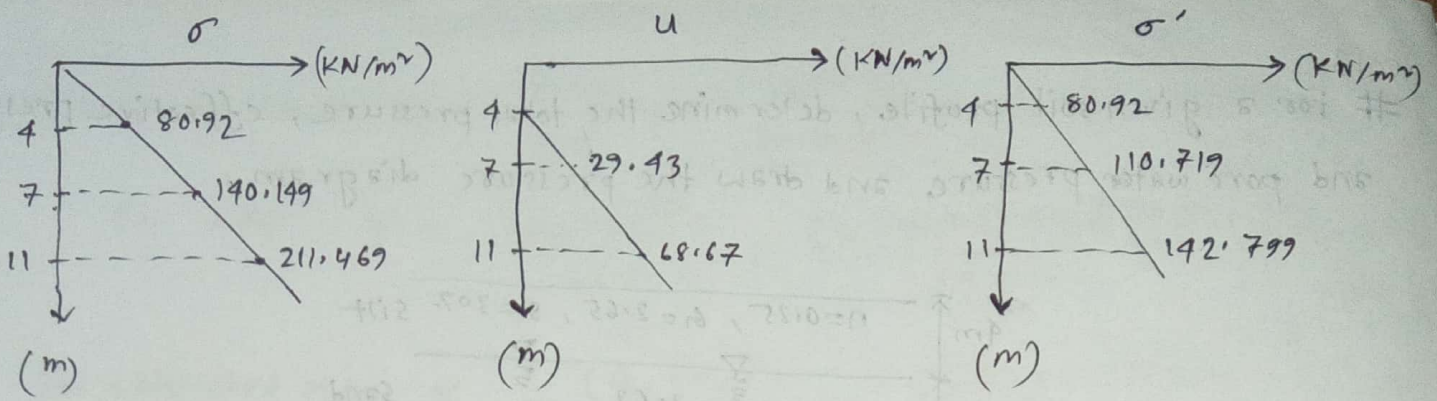
For silt, $\gamma = \frac{(G_s + S e) \gamma_w}{1 + e} = \frac{(2.65 + 0.30 \times \frac{0.25}{1-0.25}) \times 9.81}{1 + (\frac{0.25}{1-0.25})} = 20.23 \text{ KN/m}^3$

For sand, $\gamma_{sat} = \frac{(G_s + e) \gamma_w}{1 + e} = \frac{(2.62 + 0.6) \times 9.81}{1 + 0.6} = 19.743 \text{ KN/m}^3$

For clay, $e = \frac{G_s W}{S} = \frac{(2.70 \times 0.4)}{1} = 1.08$

$\therefore \gamma_{sat} = \frac{(G_s + e) \gamma_w}{1 + e} = \frac{(2.7 + 1.08) \times 9.81}{1 + 1.08} = 17.83 \text{ KN/m}^3$

Depth, D (m)	Total Stress, σ (KN/m ²)	pore water pressure, u (KN/m ²)	effective stress, $\sigma' = \sigma - u$ (KN/m ²)
0	0	0	0
4	$(4 \times 20.23) = 80.92$	0	80.92
$(4+3)=7$	$80.92 + (3 \times 19.743) = 140.149$	$(3 \times 9.81) = 29.43$	110.719
$(7+4)=11$	$140.149 + (4 \times 17.83) = 211.467$	$29.43 + (4 \times 9.81) = 68.67$	142.799



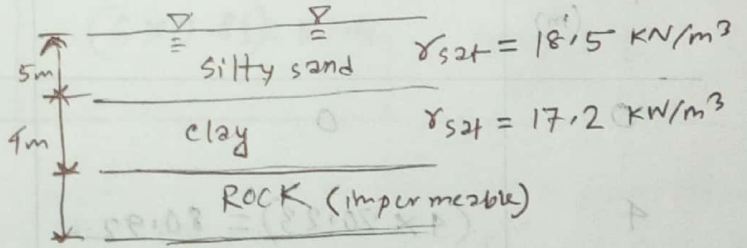
2014

For the site profile shown in following table, draw total stress, pore water pressures and effective pressure vertical stresses against depth for the following conditions: (i) water table at the ground surface and (ii) water table at a depth of 2.5 m assuming the silty sand stratum above the water table to remain saturated with capillary water (i.e. negative pressure are present). Saturated unit weight of silty sand = 18.5 KN/m^3 and saturated unit weight of clay = 17.2 KN/m^3

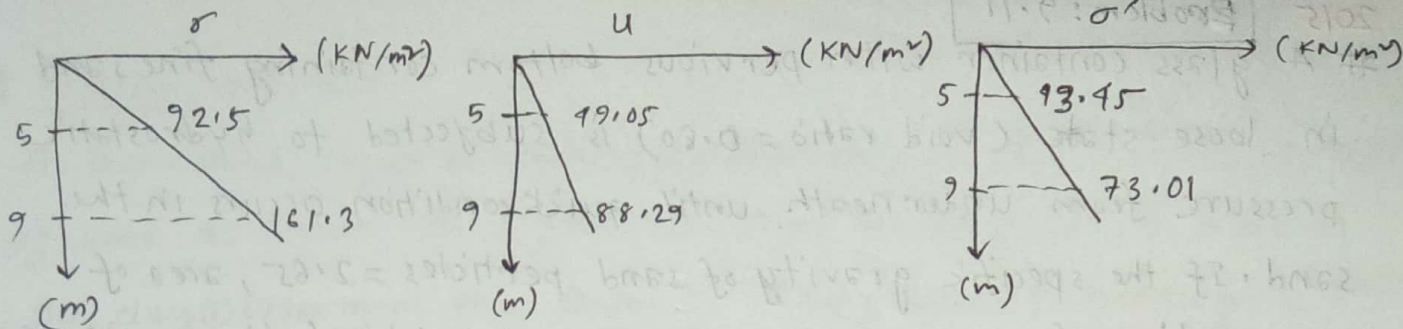
Depth (m)	Stratum
0-5	Silty sand
5-9	Peaty clay
>9	Rock (impermeable)

Solution:

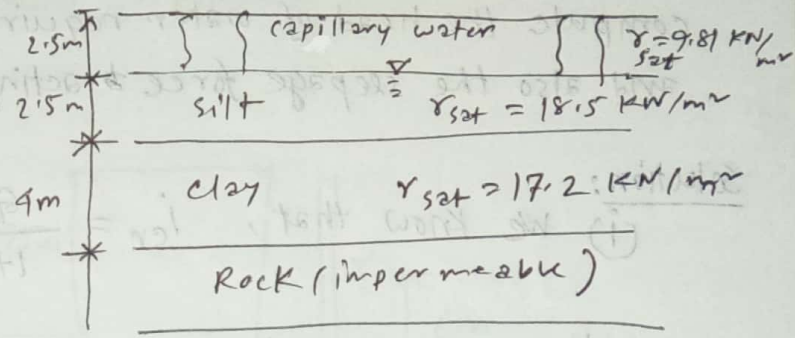
(i) water table at ground surface,



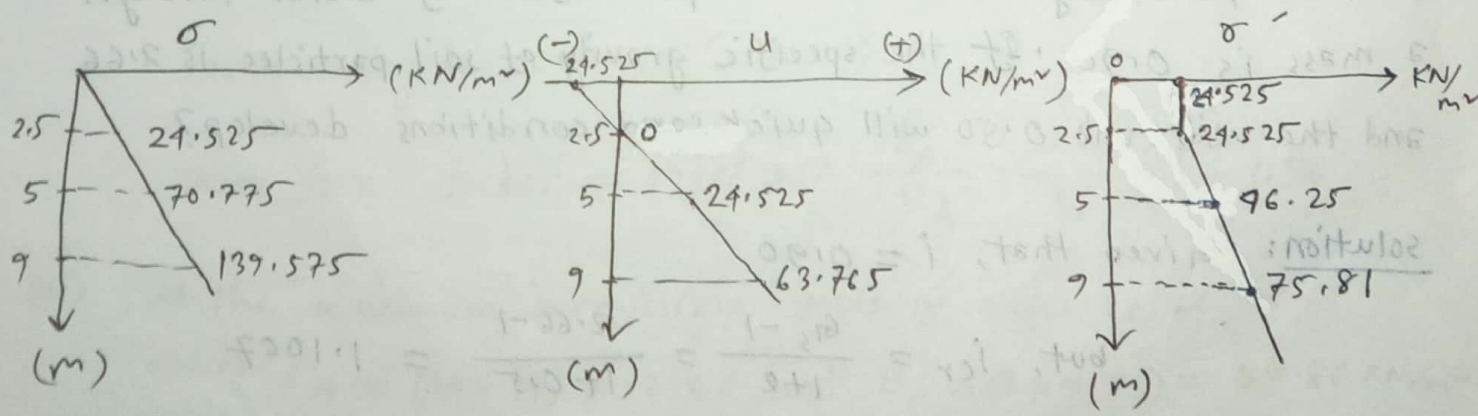
Depth, D (m)	Total stress, σ (KN/m ²)	pore water pressure u (KN/m ²)	effective stress, $\sigma' = \sigma - u$
0	0	0	0
5	$(5 \times 18.5) = 92.5$	$(5 \times 9.81) = 49.05$	43.45
$(5+4) = 9$	$92.5 + (4 \times 17.2) = 161.3$	$49.05 + (4 \times 9.81) = 88.29$	73.01



(ii) water table at depth of 2.5 m



Depth, D (m)	Total stress, σ (KN/m ²)	pore water pressure, u (KN/m ²)	effective stress, $\sigma' = \sigma - u$ (KN/m ²)
0	0	just above: 0 just below: $-s\gamma_w h = -(1 \times 9.81 \times 2.5) = -24.525$	0
2.5	$(2.5 \times 9.81) = 24.525$	0	24.525
5	$24.525 + (2.5 \times 18.5) = 70.775$	$(2.5 \times 9.81) = 24.525$	46.25
9	$70.775 + (4 \times 17.2) = 139.575$	$24.525 + (4 \times 9.81) = 63.765$	75.81



2015

Problem: 9.11

A glass container with pervious bottom containing fine sand in loose state (void ratio = 0.80) is subjected to hydrostatic pressure from underneath until quick condition occurs in the sand. If the specific gravity of sand particles = 2.65, area of cross section of sand sample = 10 cm² and height of the sample = 10 cm compute the head of water required to cause quick sand condition and also the seepage force acting from below.

Solution:

(i) We know that, $i_{cr} = \frac{G_s - 1}{1 + e} = \frac{2.65 - 1}{1 + 0.8} = 0.9167$

Here, hydraulic gradient, $i = \frac{h}{H_2} = \frac{h}{10}$

For quick condition, $\frac{h}{10} = 0.9167$

$\Rightarrow h = (0.9167 \times 10)$

$\therefore h = 9.167 \text{ cm}$

(ii) seepage force per unit volume:

S.F = $i \gamma_w = (0.9167 \times 9.81) = 9.0925 \text{ KN/m}^3$

(Ans.)

2016

The hydraulic gradient for an upward flow of water through a mass is 0.90. If the specific gravity of soil particles is 2.66 and the void ratio 0.50 will quick sand conditions develop?

Solution: Given that, $i = 0.90$

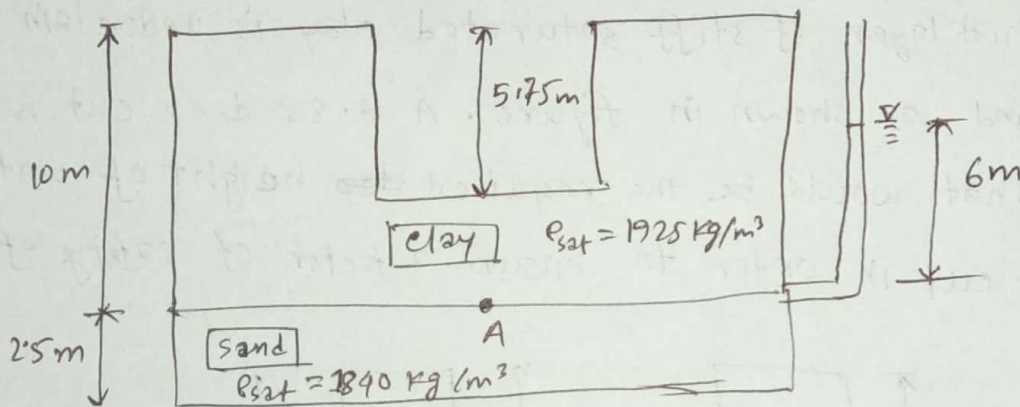
but, $i_{cr} = \frac{G_s - 1}{1 + e} = \frac{2.66 - 1}{1 + 0.5} = 1.1067$

$\therefore i < i_{cr}$ quick condition will not develop.

(Ans.)

Problem: 9.8, 9.9

A 10 m-thick layer of stiff saturated clay is underlain by a layer of sand as shown in figure. A 5.75 m-deep cut is made in the clay. (i) Determine the factor of safety against heaving at point A. (ii) What would be the maximum permissible depth of cut before heaving would occur?



Solution:

(i) Consider the stability of point A in terms of heaving

$$\gamma_{sat}(\text{clay}) = \frac{1925 \times 9.81}{1000} = 18.88 \text{ kN/m}^3$$

$$\text{Now, } \sigma_A = (10 - 5.75) \times 18.88 = 80.24 \text{ kN/m}^2$$

$$\text{and } u_A = (6 \times 9.81) = 58.86 \text{ kN/m}^2$$

For heaving to occur, $\sigma' = 0 \Rightarrow \sigma = u$

$$\text{Therefore factor of safety} = \frac{\sigma_A}{u_A} = \frac{80.24}{58.86} = 1.36$$

(ii) Let the maximum permissible depth of cut be H

$$\text{Now, } \sigma_A = (10 - H) \times 18.88 \text{ and } u_A = (6 \times 9.81) = 58.86 \text{ kN/m}^2$$

For heaving to occur, $\sigma_A = U_A$

$$\Rightarrow (10-H) \times 18.88 = 58.86$$

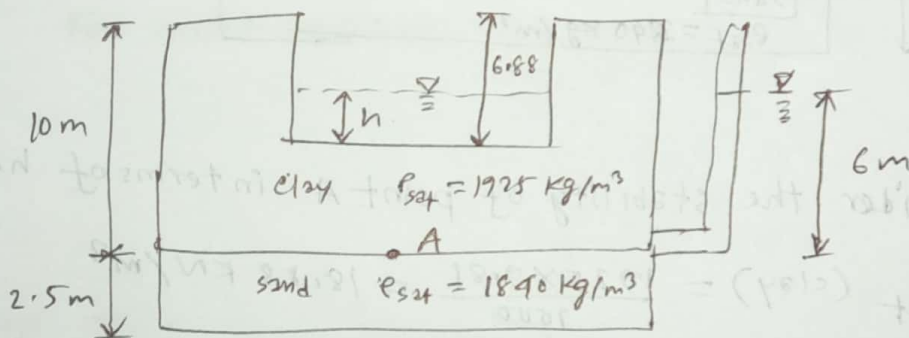
$$\Rightarrow 10-H = 3.1176$$

$$\therefore H = 6.8824 \text{ m}$$

(Ans.)

Problem: 9.10

A 10 m thick layer of stiff saturated clay is underlain by a layer of sand as shown in figure. A 6.88 deep cut is made in clay. What would be the required ~~depth~~ height of water inside the cut in order to ensure a factor of safety of 1.5



Solution:

Here,

$$\gamma_{sat}(\text{clay}) = \frac{1925 \times 9.81}{1000} = 18.88 \text{ kN/m}^3$$

$$\sigma_A = (10 - 6.88) \times 18.88 + h \times 9.81 = 58.86 + 9.81h$$

$$U_A = (6 \times 9.81) = 58.86 \text{ kN/m}^2$$

Factor of safety, $\frac{\sigma_A}{U_A} = 1.5$

$$\Rightarrow \frac{58.86 + 9.81h}{58.86} = 1.5 \Rightarrow h = 3.0 \text{ m}$$

(Ans.)

Shear Strength of soil

Shear strength of soil: Shear strength of a soil is the internal resistance per unit area that the soil mass can offer to resist failure and sliding along any plane inside it.

04, 07, 17

Mohr-Coulomb Failure Criterion: Mohr (1900) presented a theory for rupture in materials that contended ^{that} a material fails because of a critical combination of normal stress and shearing stress and not from either maximum normal or shear stress alone.

Thus, the functional relationship between normal stress and shear stress on a failure plane can be expressed as:

$$\tau_f = f(\sigma)$$

The failure envelope is a curved line. For most soil mechanics problems, it is sufficient to approximate the shear stress on the failure plane as a linear function of the normal stress.

The linear function can be written as:

$$\tau_f = c + \sigma \tan \phi \quad \text{where, } \begin{array}{l} c = \text{cohesion} \\ \phi = \text{angle of} \\ \text{internal friction} \end{array}$$

This preceding equation is called Mohr-Coulomb failure criterion.

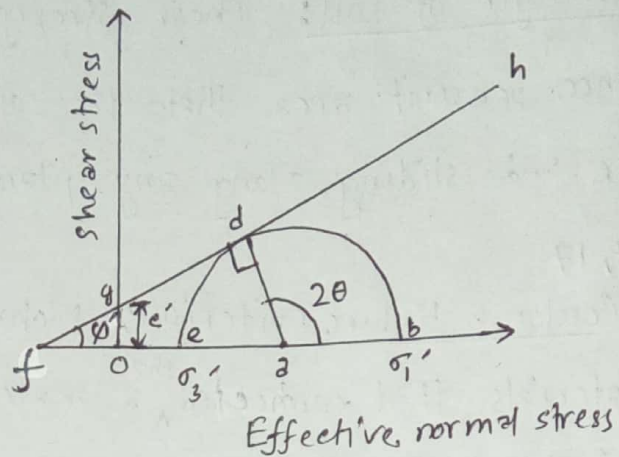
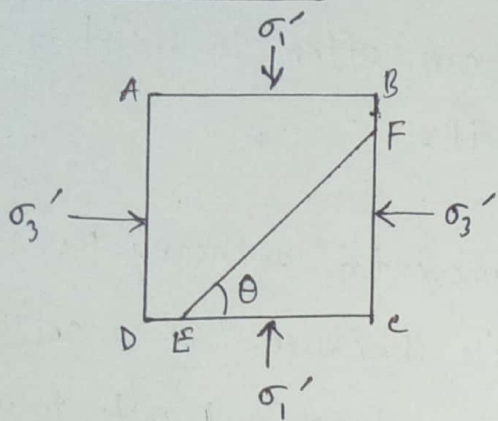
The Mohr-Coulomb failure criterion, expressed in terms of effective stress (σ'), will be of the form:

$$\tau_f = c' + \sigma' \tan \phi'$$

σ = normal stress on failure plane

τ_f = shear strength

Deduce the relationship between principle stresses at failure using mohr-coulomb criteria: 03, 05, 06, 08, 10



Let us consider the figure shows an elementary soil mass with failure plane.

In figure, fgh is the failure envelope defined by the relationship $\tau_f = c + \sigma \tan \phi$. The radial line ab defines major principal plane and the radial line ad defines the failure plane.

Here, $\angle dae = 90^\circ - \phi'$

$$\therefore 90^\circ - \phi' + 2\theta = 180^\circ \Rightarrow 2\theta = 90^\circ + \phi' \Rightarrow \theta = \frac{90}{2} + \frac{\phi'}{2}$$

$$\therefore \theta = 45^\circ + \frac{\phi'}{2}$$

From triangle aocg,

$$\sin \phi' = \frac{ad}{af}$$

Here, $ad = \frac{\sigma_1' - \sigma_3'}{2}$

$$of = c' \cot \phi'$$

$$\Rightarrow \sin \phi' = \frac{ad}{of + oa}$$

$$oa = \frac{\sigma_1' + \sigma_3'}{2}$$

$$\Rightarrow \sin \phi' = \frac{(\sigma_1' - \sigma_3')/2}{c' \cot \phi' + (\sigma_1' + \sigma_3')/2}$$

$$\Rightarrow \sin \phi' = \frac{\sigma_1' - \sigma_3'}{2c' \cot \phi' + (\sigma_1' + \sigma_3')}$$

$$\Rightarrow \sigma_1' \sin \phi' + \sigma_3' \sin \phi' + 2c' \cos \phi' = \sigma_1' - \sigma_3'$$

$$\Rightarrow \sigma_1' (1 - \sin \phi') = \sigma_3' (1 + \sin \phi') + 2c' \cos \phi'$$

$$\Rightarrow \sigma_1' = \sigma_3' \frac{1 + \sin \phi'}{1 - \sin \phi'} + 2c' \frac{\cos \phi'}{1 - \sin \phi'}$$

$$\Rightarrow \sigma_1' = \sigma_3' \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) + 2c' \tan \left(45^\circ + \frac{\phi'}{2} \right)$$

$$\therefore \sigma_1' = \sigma_3' \tan^2 \theta + 2c' \tan \theta$$

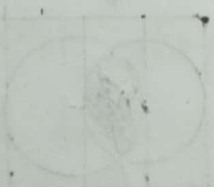
The expression in terms of total stress,

$$\sigma_1 = \sigma_3 \tan^2 \theta + 2c \tan \theta$$

Laboratory test for determination of shear strength parameters:

There are various methods available to determine the shear strength parameters (i.e., c, ϕ, c', ϕ') of various soil specimen in the laboratory. They are as follows:

- (i) Direct shear test.
- (ii) Triaxial shear test.
- (iii) Unconfined compression test.
- (iv) vane shear test.

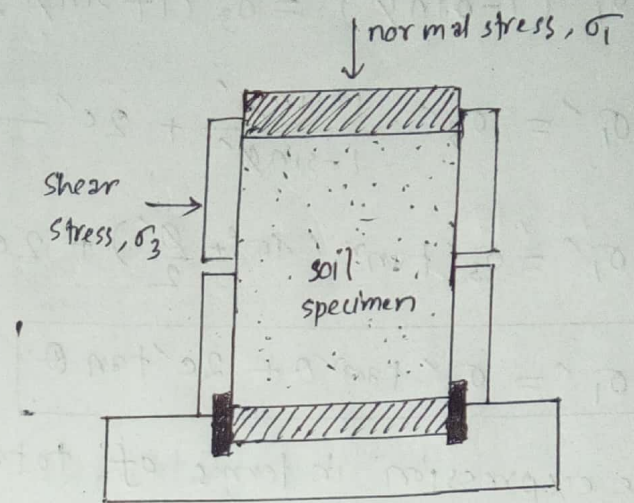


Direct shear Test:

The direct shear test is the oldest and simple form of shear test arrangement.

The test equipment consists of a metal box in which the soil specimen placed.

soil specimen may be square or circular. (size: 51 mm x 51 mm or 102 mm x 102 mm and ^{about} 25 mm high)



The box is split horizontally into two halves.

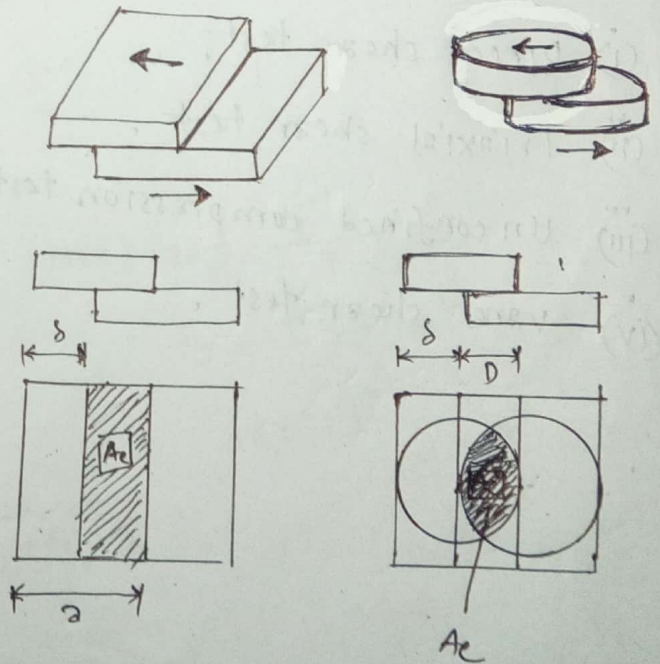
Normal force is applied through a metal platen. (as great as 1050 kN/m²) shear force is applied by moving one half of the box relative to the other to cause failure in the soil specimen.

Depending on equipment, the shear test can be

1. stress controlled
2. strain controlled.

In stress controlled tests, shear force is applied in equal increments until specimen fails.

In strain-controlled tests, a constant rate of shear displacement is applied to one half of the box by a motor that acts through gears.



The advantages of the strain controlled test is that in case of dense sand, peak shear resistance as well as ultimate strength can be observed and plotted.

In stress-controlled tests, only the peak shear resistance can be observed and plotted.

Note that, peak shear resistance in **stress** controlled tests can be only approximated because failure occurs at a stress level ^{where} some between the pre failure ~~and~~ load increment and the failure load increment.

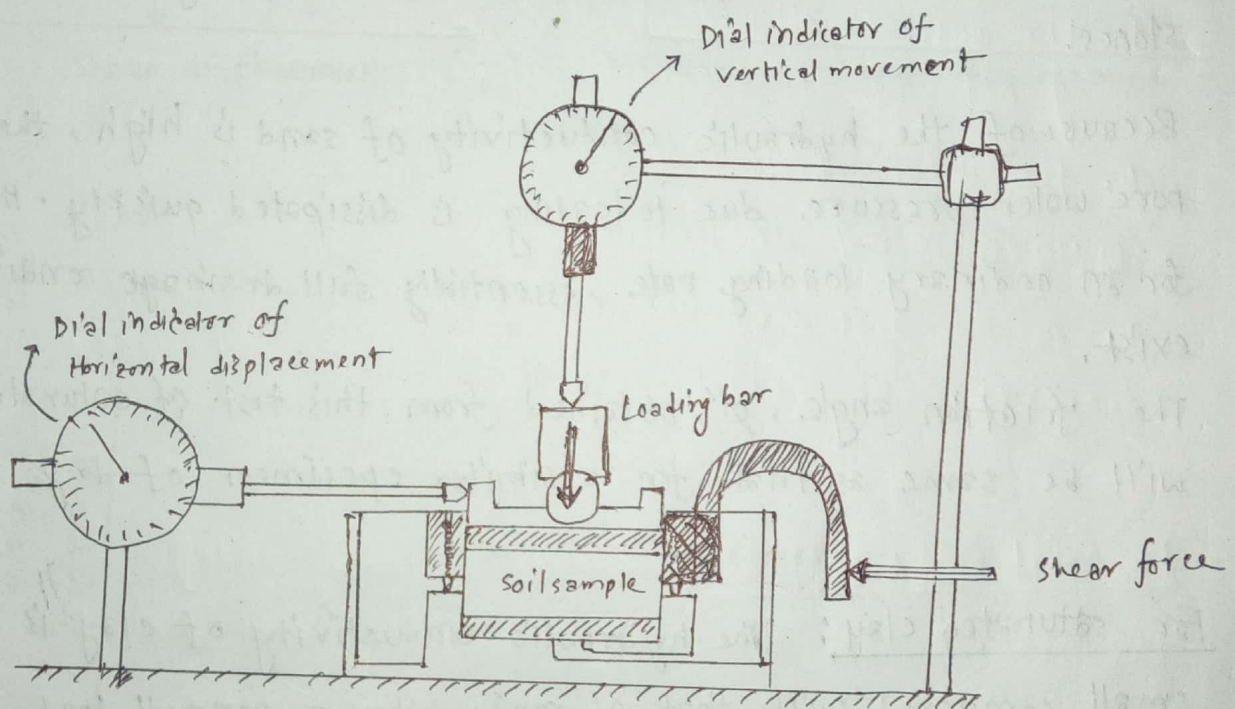


Fig. Direct shear test equipment

Direct shear test is used to determine the shear strength of both cohesive as well as non-cohesive soils.

Drained Direct Shear Test:

For saturated sand: In drained direct shear test, the shear box kept inside a container that can be filled with water to saturate the specimen.

A drained test is made on a saturated soil by keeping the rate of loading slow enough^{so} that the excess pore water pressure generated in the soil is dissipated completely by drainage. Pore water from the specimen is drained through two porous stones.

Because of the hydraulic conductivity of sand is high, the excess pore water pressure due to loading is dissipated quickly. Hence for an ordinary loading rate, essentially full drainage conditions exist.

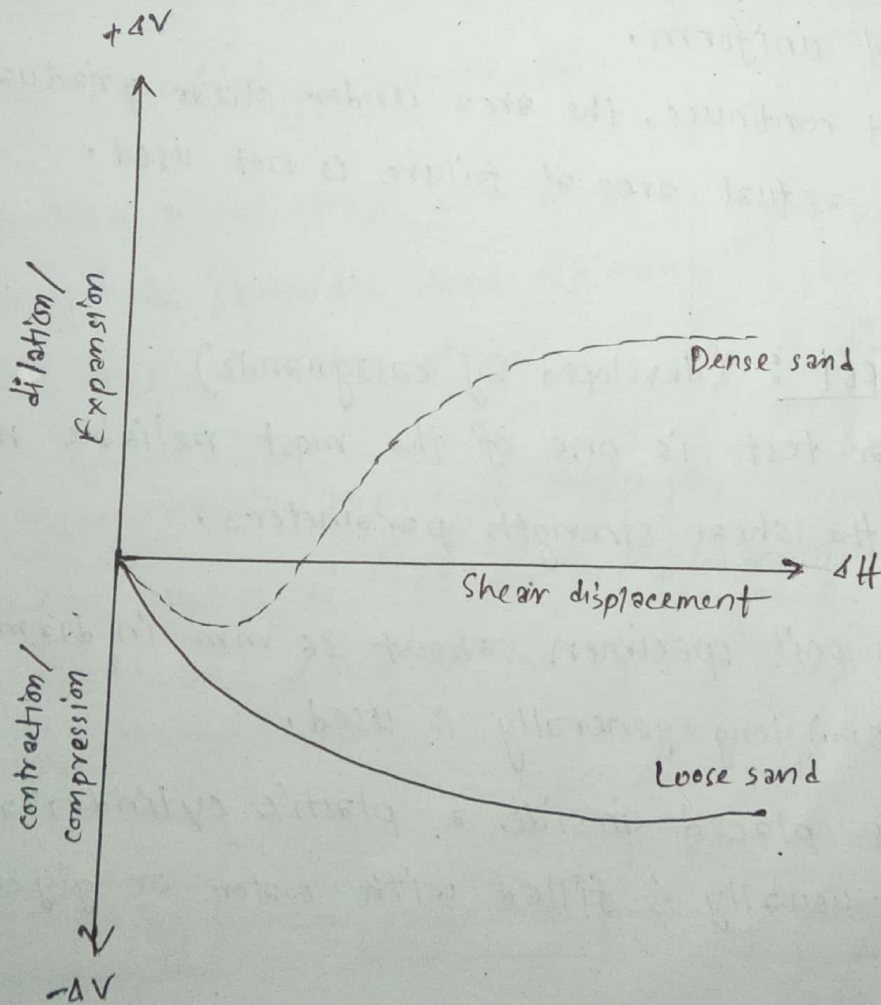
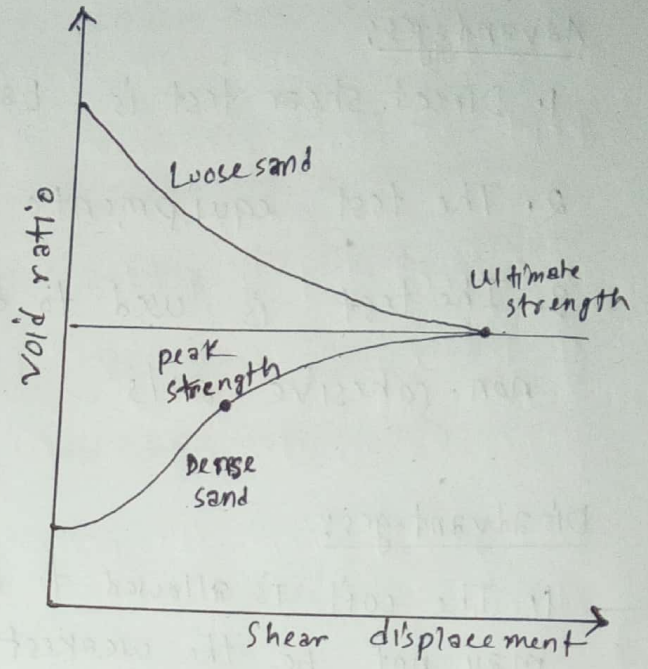
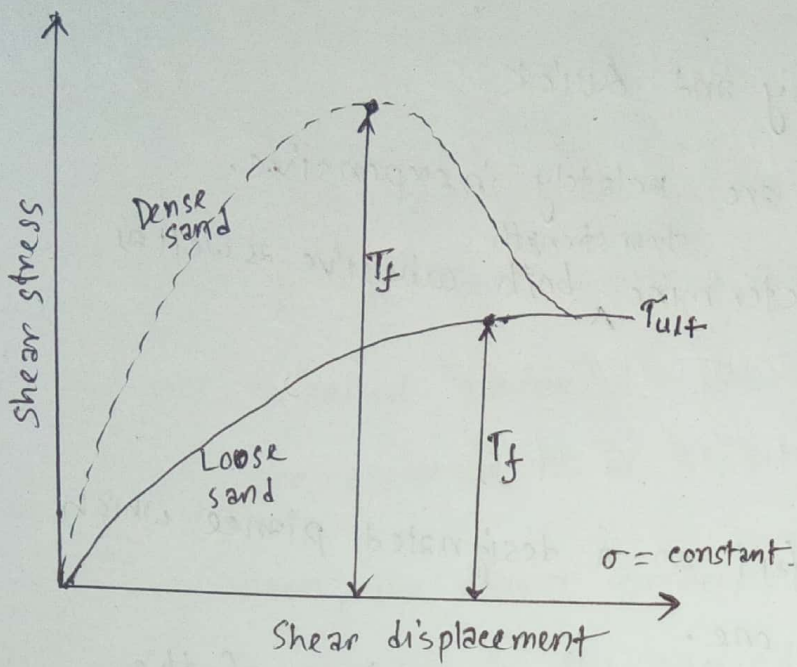
The friction angle, ϕ' , obtained from this test of saturated sand will be same as that for a similar specimen of dry sand.

For saturated clay: The hydraulic conductivity of clay is very small compared with that of sand. When a normal load is applied to a clay soil specimen, a sufficient length of time must elapse for full consolidation. For this reason, the shear load must be applied very slowly.

Note that, the value of $c' \approx 0$ for normally consolidated clay.

At large shearing displacement, we obtain the residual strength of clay.

Figures: (Direct shear test)



Advantages and Disadvantages of Direct shear test; 04, 07

Advantages:

1. Direct shear test is Easy and Quick
2. The test equipments are relatively inexpensive.
3. The test is used to determine ^{shear strength} both cohesive as well as non-cohesive soils

Disadvantages:

1. The soil is allowed to fail on a designated plane which may not be the weakest one.
2. The shear stress distribution over shear surface of the specimen is not uniform.
3. As the test continues, the area under shear gradually decreases. The actual area at failure is not used.

Triaxial Shear Test : (developed by Casagrande)

The triaxial shear test is one of the most reliable method for determining the shear strength parameters.

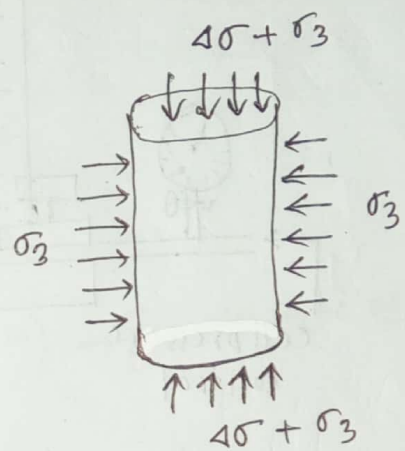
In this test, a soil specimen about 36 mm in diameter and 76 mm (3 in.) long generally is used.

The specimen is placed inside a plastic cylindrical chamber that usually is filled with water or glycerine.

principles of the Triaxial compression test:

- * The triaxial compression test is used under controlled drainage conditions.
- * A cylindrical specimen of soil is subjected to a confining fluid / air pressure and then loaded axially to cause failure.
- * The test is called 'triaxial' because the three principal stresses are assumed to be known and are controlled.

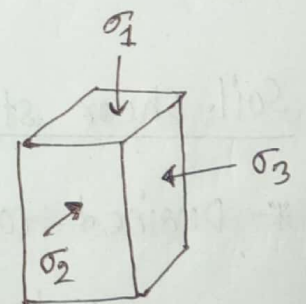
* During shear, the major principle stress, σ_1 is equal to the applied axial stress ($\Delta\sigma = P/A$) plus the chamber (confining) pressure, σ_3



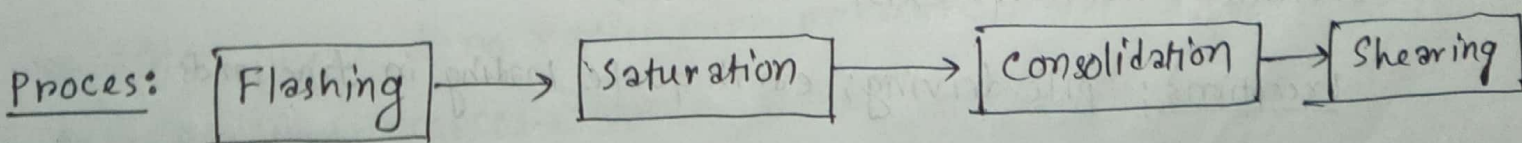
* The applied axial stress, $\Delta\sigma = \sigma_1 - \sigma_3$ is termed the "principle stress difference" or sometimes "deviator stress"

$$\sigma_1 = \Delta\sigma + \sigma_3$$

* The intermediate principle stress, σ_2 and minor principal stress, σ_3 are identical in the test and equal to the confining pressure.



$$\sigma_2 = \sigma_3$$



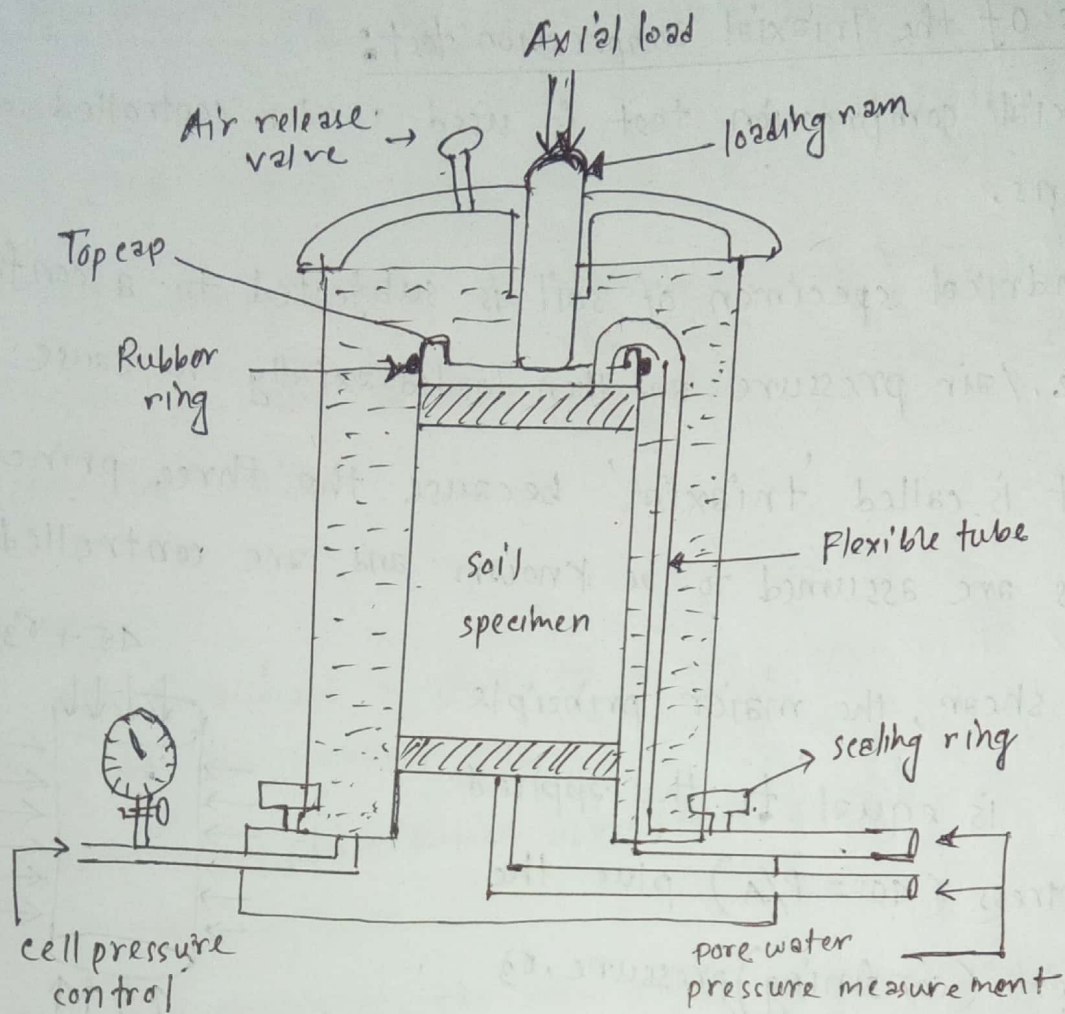


Fig. Triaxial Test Equipment

Soil Shear strength under Drained and undrained conditions:

* Drained condition occurs when,

rate at which loads are applied $<$ rate at which soil material can drain

* sand drain fast. Therefore under most loading conditions

Drained conditions exist in sands.

Exceptions: pile driving; earthquake loading in fine sands.

* In clays, drainage does not occur quickly. therefore in clays, the short term shear strength may correspond to undrained conditions.

But in long term shear strength is assumed drained condition in clays.

Types of Triaxial Test:

There are ^{Mainly} three types of triaxial test:

- (i) Consolidated Drained test (CD Test)
- (ii) Consolidated Undrained test (CU Test)
- (iii) Unconsolidated undrained test (UU Test)
- (iv) Un-confined compression test (UC Test) → [special type of UU]

Consolidated Drained Test:

* Also called slow test

* Drainage valves open during consolidation as well as shearing phases.

* Complete sample drainage is achieved prior to the application of the vertical load.

* The load is applied at such a slow strain rate that particle readjustments in the specimen do not induce any excess ^{pore} pressure.

* Since there is no excess pore pressure, total stress will equal to effective stresses. Hence $\sigma = \sigma'$ and $c = c'$

* The test can take up to 2 weeks.

* The test simulates long term shear strength for cohesive soils.

ed Test results:

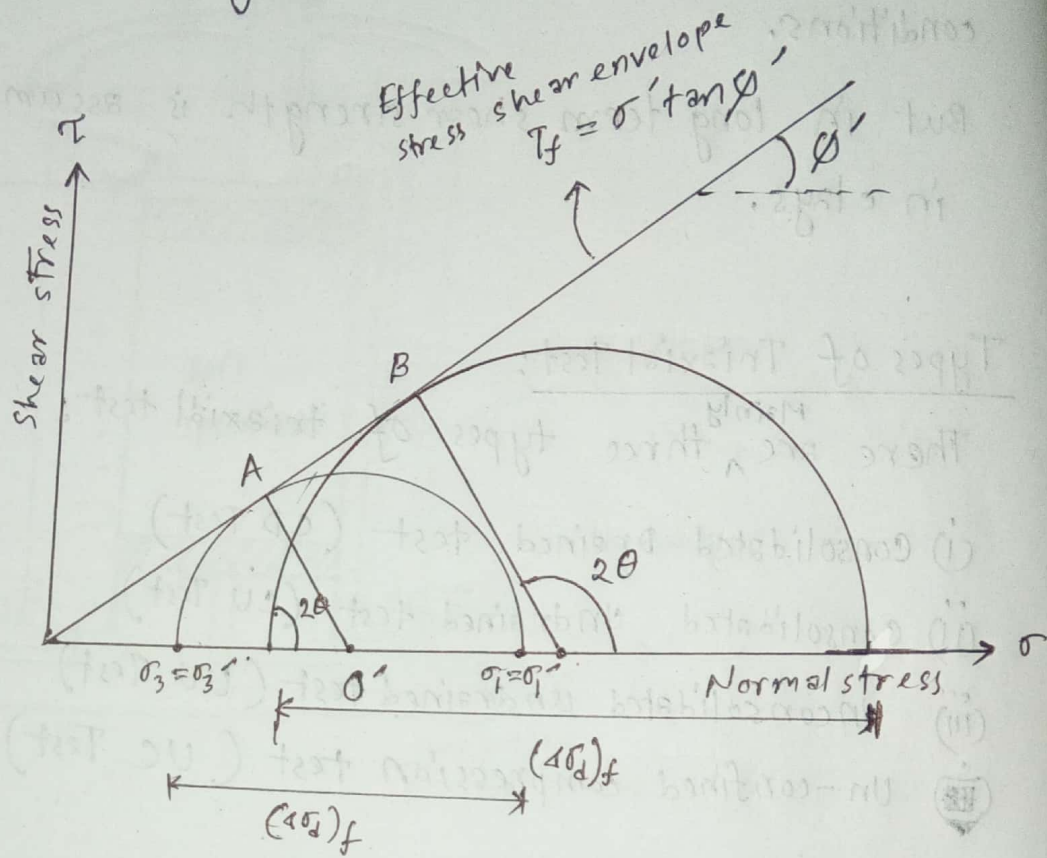
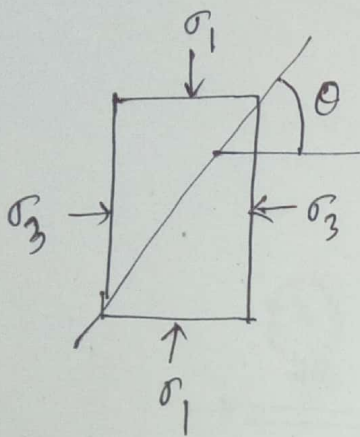


Fig. Drained test on sand and normally consolidated clay.

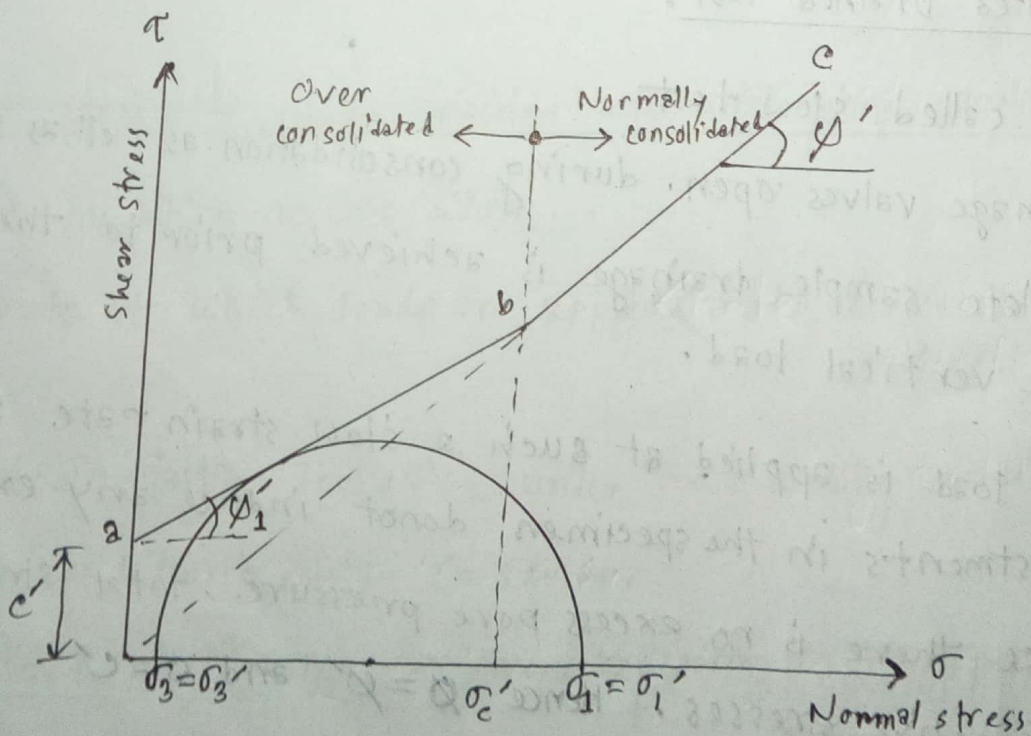
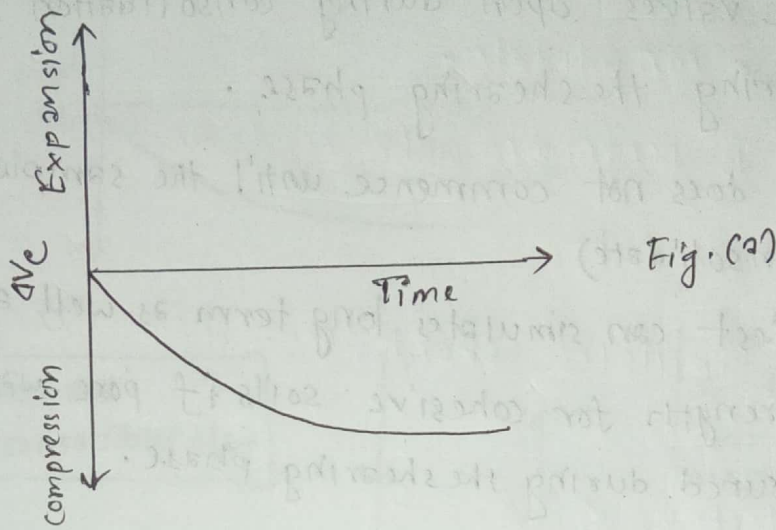


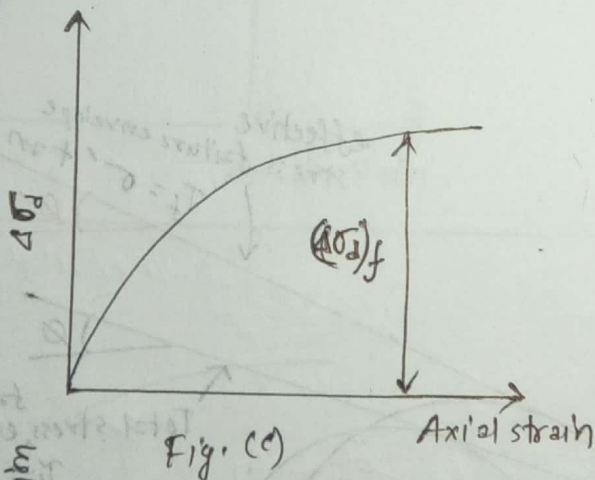
Fig. Drained test on overconsolidated clay

(a) Volume change of specimen caused by confining pressure:

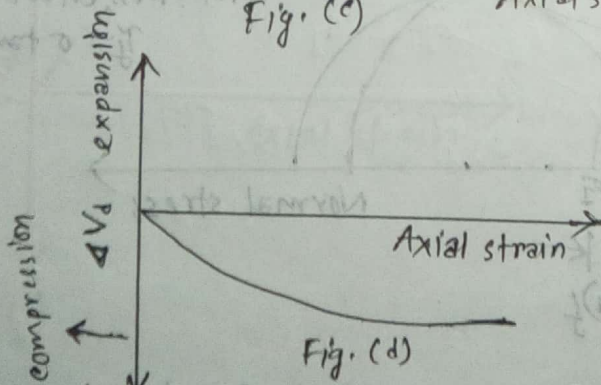
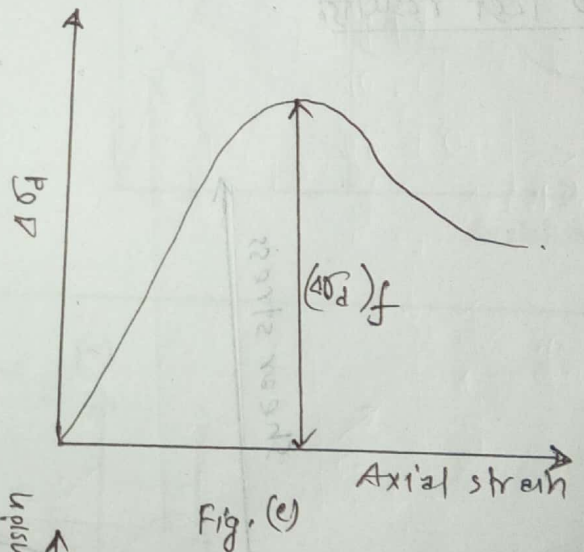


(b) $\Delta\sigma_d$ vs. ΔH

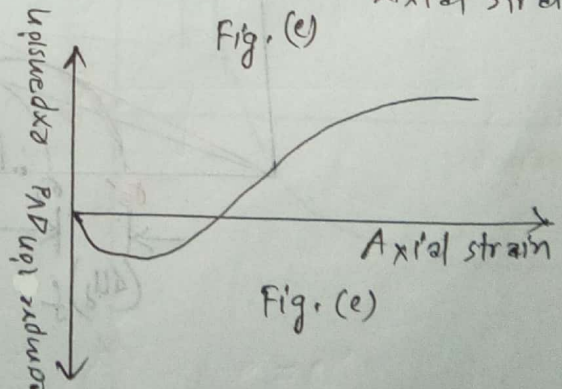
for loose sand and normally consolidated clay.



(c) $\Delta\sigma_d$ vs. ΔH for dense sand and over consolidated clay



(d) Volume change by deviator stress for dense sand and normally consolidated clay



(e) Volume change by deviator stress for dense sand and normally consolidated clay

Unconsolidated undrained:

- * This test is also called quick test
- * σ_3 and σ_1 are applied fast so the soil does not have time to consolidate.
- * The test is performed with the drain valve closed for all phases of the test
- * UU test simulates short term shear strength for cohesive soils.
- * For this test, $\phi = \phi' = 0$ and $c_u = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1' - \sigma_3'}{2}$

UU Test Results:

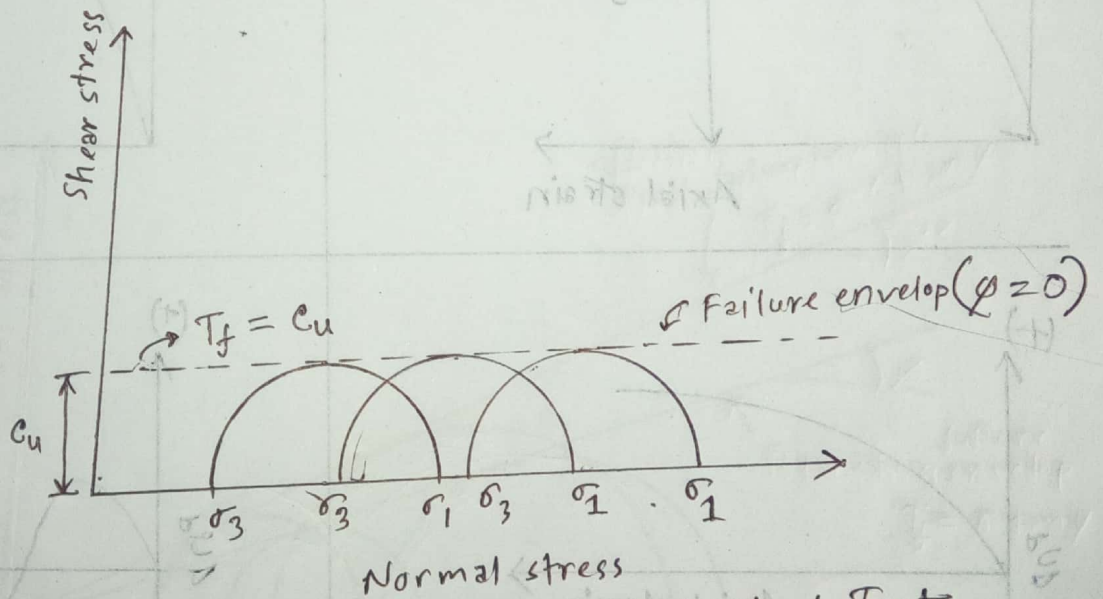


Fig. Unconsolidated Undrained Test

unconfined Compression Test :

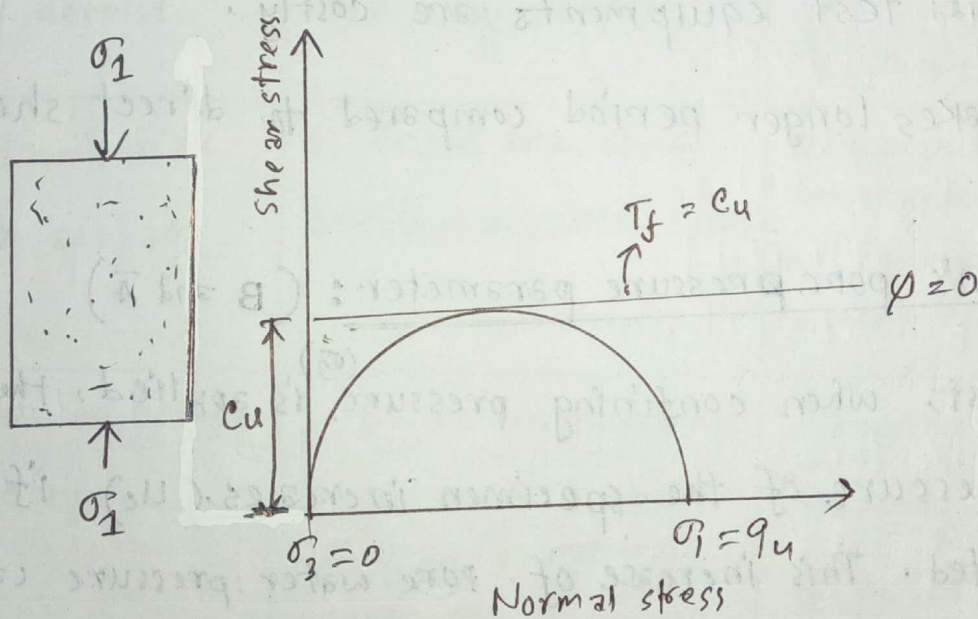
* The unconfined compression test is special type of unconsolidated undrained test that is commonly used for clay specimens.

* In this test, confining pressure is 0.

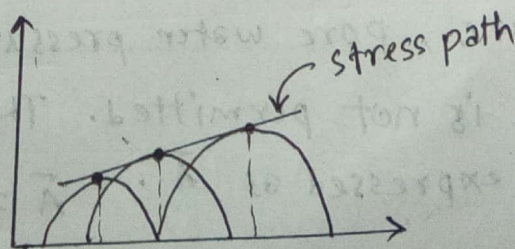
* An axial load is rapidly applied to the specimen to cause failure.

* In this test, $T_f = \frac{\sigma_1}{2} = \frac{q_u}{2} = c_u$ where, $q_u =$ unconfined compression strength.
and $\phi = 0$.

Unconfined compression test result:



Stress path : A stress path is a line that connects a series of points, each of which represents a successive state experienced by a soil specimen during the progress of test.



Advantages of Triaxial test over direct shear test: 03, 08

1. More versatile
2. Drainage can be well controlled
3. There is no rotation of the principle stress like direct shear test.
4. The failure plane is allowed to occur anywhere.
5. Stress distribution in failure plane is uniform.

Disadvantage of Triaxial Test:

1. Triaxial test equipments are costly.
2. It takes longer period compared to direct shear test

Skempton's pore pressure parameter: (B and \bar{A})

In CD test, when confining pressure (σ_3) is applied, the pore water pressure of the specimen increases (u_e) , if drainage is prevented. This increase of pore water pressure can be expressed as B .

$$B = \frac{u_e}{\sigma_3}$$

In CU test, when the deviator stress (σ_d) is applied to cause shear failure, pore water pressure (u_d) will increase because drainage is not permitted. The increase in the pore water pressure is expressed as \bar{A} .

$$\bar{A} = \frac{\Delta u_d}{\Delta \sigma_d}$$

In UU test, $U_c = B\sigma_3$ and $\Delta U_d = \bar{A} \Delta\sigma_d$

$$\therefore U = U_c + \Delta U_d = B\sigma_3 + \bar{A} \Delta\sigma_d = B\sigma_3 + \bar{A} (\sigma_1 - \sigma_3)$$

where, B and \bar{A} = Skempton's pore pressure parameter.

Practical application of CD, CU and UU test:

CD	CU	UU
1. Embankment constructed very slowly, in layers over a soft clay deposit	1. Embankment constructed rapidly over a soft clay deposit.	1. Embankment constructed rapidly over a soft clay deposit.
2. Earth dam with steady state seepage.	2. Rapid drawdown behind an earth dam.	2. Large earth dam constructed rapidly with no change in water content of soft clay.
3. Excavation on natural slope in clay.	3. Rapid construction of an earth embankment on a natural slope.	3. Footing placed rapidly on clay deposit.
—	—	—

Vane Shear Test:

Fairly reliable result for the undrained shear strength, c_u ($\phi = 0$ concept), of very soft to medium cohesive soils may be obtained directly from vane tests.

The shear vane usually consists of four thin, equal sized steel plates welded to steel torque rod.

First,

The vane is pushed in to the soil.

Then, torque is applied at the top of the torque rod to rotate the vane at a uniform speed.

A cylinder of soil of height h and diameter d will resist the torque until the soil fails.

At failure torque T can be expressed as,

$$T = \pi c_u \left[\frac{d^2 h}{2} + \beta \frac{d^3}{h} \right]$$

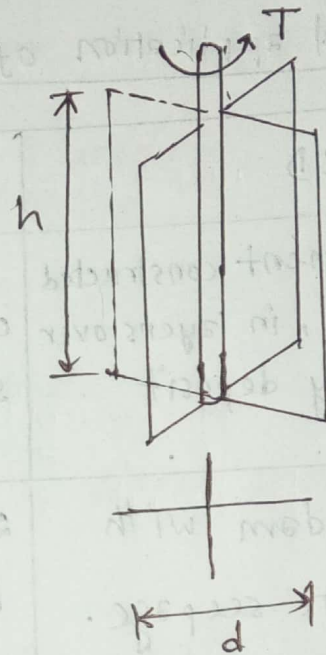
where,

c_u = undrained shear strength

$\beta = \frac{1}{2}$ for triangle,

$= \frac{2}{3}$ for uniform

$= \frac{3}{5}$ for parabolic mobilization.



Shear strength of soil

99, 01, 02

A series of triaxial shear test gives the result:

<u>Test No</u>	<u>Cell Pressure</u> (KPa)	<u>Deviator Shear</u> (KPa)
1	200	675
2	300	860
3	400	1060

determine shear strength parameter.

Solution:

For test 1: $\sigma_3 = 200 \text{ KPa}$, $\sigma_d = 675 \text{ KPa}$

$$\therefore \sigma_1 = \sigma_3 + \sigma_d = (200 + 675) \text{ KPa} = 875 \text{ KPa}$$

For test 2: $\sigma_1 = 300 + 860 = 1160 \text{ KPa}$

For test 3: $\sigma_1 = 400 + 1060 = 1460 \text{ KPa}$

We know, $\sigma_1 = \sigma_3 \tan^2 \theta + 2c \tan \theta$

$$\text{Hence, } 875 = 200 \tan^2 \theta + 2c \tan \theta \quad \dots \text{ (I)}$$

$$1160 = 300 \tan^2 \theta + 2c \tan \theta \quad \dots \text{ (II)}$$

$$1460 = 400 \tan^2 \theta + 2c \tan \theta \quad \dots \text{ (III)}$$

$$\text{From eq}^n \text{ (III), } 2c \tan \theta = 1460 - 400 \tan^2 \theta \quad \dots \text{ (IV)}$$

using the value of $2c \tan \theta$ in eqⁿ (II) =

$$1160 = 300 \tan^2 \theta + 1460 - 400 \tan^2 \theta$$

$$\Rightarrow \tan^2 \theta = 300/100 = 3$$

$$\Rightarrow \tan \theta = \sqrt{3} \quad \therefore \theta = 60^\circ$$

$$\text{From eq}^n \text{ (I), } 2c \tan 60 = 1460 - 400 \tan^2 60$$

$$\Rightarrow c = 75.056$$

$$\text{we know, } \theta = 45^\circ + \frac{\phi}{2} \Rightarrow \frac{\phi}{2} = \theta - 45^\circ = (60 - 45) = 15$$

$$\therefore \phi = 30^\circ \text{ (Ans.)}$$

2002, 05, 07

A cylinder of soil fails under an axial vertical stress of 160 kN/m^2 when it is laterally unconfined. The failure plane makes 50° angle with horizontal. Find cohesion and angle of internal friction.

Solution:

$$\sigma_1 = 160 \text{ kN/m}^2, \theta = 50^\circ$$

$$\theta = 45^\circ + \frac{\phi}{2} \Rightarrow \frac{\phi}{2} = (50^\circ - 45^\circ) = 5^\circ \therefore \phi = 10^\circ$$

Now,

$$\sigma_1 = \sigma_3 \tan^2 \theta + 2c \tan \theta$$

$$\Rightarrow 160 = 0 + 2c \tan 50^\circ \quad [\because \text{For confined sample, } \sigma_3 = 0]$$

$$\Rightarrow c = 67.13 \text{ kN/m}^2$$

(Ans.)

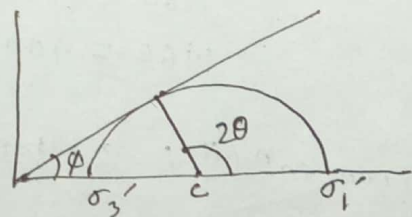
2009, 05

A consolidated drained triaxial test was conducted on a normally consolidated clay. Results are: $\sigma_3 = 200 \text{ kN/m}^2$ (chamber confining pressure), Deviator pressure ($\Delta \sigma_2$) = 350 kN/m^2 . Find (i) ϕ (ii) θ (iii) σ' (iv) τ_f

Solution: For normally consolidated soil, $c = 0$

$$\sigma_3' = \sigma_3 = 200 \text{ kN/m}^2$$

$$\begin{aligned} \sigma_1' = \sigma_2 &= \sigma_3 + (\Delta \sigma_2) \\ &= 200 + 350 \\ &= 550 \text{ kN/m}^2 \end{aligned}$$



$$\sin \phi = \frac{AC}{OC} = \frac{\frac{\sigma_1' - \sigma_3'}{2}}{\frac{\sigma_1' + \sigma_3'}{2}} = \frac{550 - 200}{550 + 200} = \frac{350}{750} = 0.467$$

$$\therefore \phi = 27.82^\circ$$

$$\theta = 45^\circ + \frac{\phi}{2} = 45^\circ + \frac{27.82}{2} = 58.91^\circ$$

$$\sigma' = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta = \frac{550 + 200}{2} + \frac{550 - 200}{2} \cos (2 \times 58.91^\circ)$$

$$\therefore \sigma' = 293.33 \text{ kN/m}^2$$

$$\tau_f = \sigma' \tan \phi = (293.33 \times \tan 27.82^\circ) = 154.79 \text{ kN/m}^2 \quad (\text{Ans.})$$

2003

A sample of dry sand was tested in a triaxial machine. If the angle of shearing resistance was 30° and confining pressure 150 kPa. Find deviator stress at which the sample fails.

Solution: $\phi = 30^\circ$ $\sigma_3 = 150 \text{ kPa}$.

$$\begin{aligned} \sigma_1 &= \sigma_3 \tan^2 \theta + 2c \tan \theta \\ &= 150 \tan^2 \left(45 + \frac{30}{2}\right) \quad [\because \text{For dry sand, } c = 0] \\ &= 450 \text{ kPa.} \end{aligned}$$

$$\sigma_1 = \sigma_3 + (\Delta \sigma_d)_f$$

$$\Rightarrow (\Delta \sigma_d)_f = \sigma_1 - \sigma_3 = (450 - 150) = 300 \text{ kPa.} \quad (\text{Ans.})$$

2008

A cylinder sample, having a cohesion of 80 kPa. and on an angle of internal friction of 20° is subjected to a cell pressure of 100 kPa. Find (i) Maximum deviator stress at which the sample fails and (ii) Angle made by failure plane with the axis of sample.

solve: \rightarrow

2010

A triaxial sample of soil having $c = 60 \text{ kPa}$ and $\phi = 25^\circ$ is subjected to cell pressure of 225 kPa . Determine (i) the deviator stress at failure, and (ii) the angle made by the failure plane with the axis of deviator stress.

Solution: $c = 60 \text{ kPa}$, $\phi = 25^\circ$ $\therefore \theta = (45 + \frac{25}{2}) = 57.5^\circ$

$$\sigma_3 = 225 \text{ kPa.}$$

$$\sigma_1 = \sigma_3 \tan^2 \theta + 2c \tan \theta$$

$$= 225 \tan^2 57.5 + 2 \times 60 \times \tan 57.5$$

$$= 742.74 \text{ kPa.}$$

Now, $\sigma_1 = \sigma_3 + (\Delta \sigma_d)_f$

$$\Rightarrow (\Delta \sigma_d)_f = \sigma_1 - \sigma_3$$

$$\Rightarrow (\Delta \sigma_d)_f = (742.74 - 225)$$

$$\therefore (\Delta \sigma_d)_f = 517.74 \text{ kPa.}$$

(Ans.)

08
A cylinder sample - ...

Solution: $c = 80 \text{ kPa}$, $\phi = 20^\circ$ $\therefore \theta = 45^\circ + \frac{20^\circ}{2} = 55^\circ$

$$\sigma_3 = 100 \text{ kPa.}$$

$$\sigma_1 = \sigma_3 \tan^2 \theta + 2c \tan \theta$$

$$= 100 \times \tan^2 55^\circ + 2 \times 80 \times \tan 55^\circ$$

$$\therefore \sigma_1 = 432.66 \text{ kPa.}$$

$$\sigma_1 = \sigma_3 + (\Delta \sigma_d)_f \Rightarrow (\Delta \sigma_d)_f = (432.66 - 100) = 332.66 \text{ kPa.}$$

(Ans.)

2008

The results of two drained triaxial tests on a saturated soil follows:

specimen-I : $\sigma_3 = 100 \text{ KPa}$ $4\sigma_d = 220 \text{ KPa}$

specimen-II : $\sigma_3 = 200 \text{ KPa}$ $4\sigma_d = 430 \text{ KPa}$

Determine the shear strength parameters.

Solution: Here, For specimen-I, $\sigma_1 = (\sigma_3 + 4\sigma_d) = (100 + 220) = 320 \text{ KPa}$

Hence, $320 = 100 \tan^2 \theta + 2c \tan \theta$ (I)

For specimen-II, $\sigma_1 = (200 + 430) = 630 \text{ KPa}$.

Hence, $630 = 200 \tan^2 \theta + 2c \tan \theta$ (II)

From (I) & (II) we obtain, $\tan^2 \theta = \frac{630 - 320}{200 - 100} = 3.1$

$\Rightarrow \tan \theta = 1.761$ $\therefore \theta = 60.4^\circ$

and, $\theta = 45^\circ + \frac{\phi}{2}$ $\therefore \phi = 30.8^\circ$

Now, From (I),

$$320 = 100 \times 3.1 + 2c \times 1.761$$

$$\Rightarrow c = 2.84$$

(Ans.)

2009

The result of two drained triaxial tests on a saturated clay are given below:

Test 1 : cell pressure $\sigma_3 = 100 \text{ KPa}$ and Deviator stress at failure, $\sigma_d = 215 \text{ KPa}$

Test 2 : cell pressure $\sigma_3 = 200 \text{ KPa}$, and Deviator stress at failure, $\sigma_d = 258 \text{ KPa}$.

calculate the shear strength parameters of soil.

Solution: For test ①, $\sigma_1 = (100 + 215) = 315$

$$\therefore 315 = 100 \tan^2 \theta + 2c \tan \theta \dots \text{①}$$

For test ②, $\sigma_1 = (200 + 258) = 458$

$$458 = 200 \tan^2 \theta + 2c \tan \theta \dots \text{②}$$

From ① & ②, $\tan^2 \theta = 1.43 \Rightarrow \tan \theta = 1.196 \therefore \theta = 50.096^\circ$

$$\therefore \phi = (50.096 - 45) \times 2 = 10.19^\circ$$

From eqⁿ ①, $c = 71.9$

(Ans.)

2011

For a clay soil $\phi' = 32^\circ$, $\phi_{cu} = 20^\circ$. A consolidated-undrained triaxial test was conducted on the clay soil with a confining pressure of 13 lb/in^2 . Determine the deviator stress and the pore water pressure.

solution: we know, $\sin \phi_{cu} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{\sigma_3 + 4\sigma_d - \sigma_3}{\sigma_3 + 4\sigma_d + \sigma_3}$

$$\Rightarrow \sin 20^\circ = \frac{4\sigma_d}{4\sigma_d + 2\sigma_3} = \frac{4\sigma_d}{4\sigma_d + 2 \times 13}$$

$$\Rightarrow 4\sigma_d = 13.515 \text{ lb/in}^2$$

$$\therefore \sigma_1 = (13 + 13.515) = 26.515 \text{ lb/in}^2$$

Now, we know that,

$$\sin \phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{\sigma_1 - 4u_d - \sigma_3 + 4u_d}{\sigma_1 - 4u_d + \sigma_3 - 4u_d} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(4u_d)}$$

$$\Rightarrow \sin 32 = \frac{26.515 - 13}{26.515 + 13 - 2(4u_d)} \Rightarrow 4u_d = 7.0056 \text{ lb/in}^2$$

(Ans.)

2012, 14

The results of an undrained shear box test are in the following table:

Normal stress (KPa)	200	300	400
Shear stress (KPa)	113	141	167

- (i) Find the apparent cohesion (c_u) and angle of resistance (ϕ_u)
- (ii) What value of c_u would be expected from an unconfined compression test on the same soil?
- (iii) If another specimen of the soil is subjected to an undrained triaxial test with lateral pressure 275 KPa. Find the total axial pressure at which failure would be expected.

Solution: (i) We know, $T_f = c_u + \sigma \tan \phi_u$

Here,

$$113 = c_u + 200 \tan \phi_u \quad \text{--- (i)}$$

$$141 = c_u + 300 \tan \phi_u \quad \text{--- (ii)}$$

$$167 = c_u + 400 \tan \phi_u \quad \text{--- (iii)}$$

From (i) & (ii), $\tan \phi_u = 0.28$ $\therefore \phi_u = 15.64^\circ$ Then, from (i), $c_u = 57 \text{ KPa}$

From (ii) & (iii), $\tan \phi_u = 0.26$ $\therefore \phi_u = 14.57^\circ$ Then, from (ii), $c_u = 63 \text{ KPa}$

From (iii) & (i), $\tan \phi_u = 0.27$ $\therefore \phi_u = 15.11^\circ$ Then, from (iii), $c_u = 59 \text{ KPa}$

$$\therefore \text{Apparent cohesion, } c_u = \frac{57 + 63 + 59}{3} = 59.67$$

$$\text{and angle of friction, } \phi = \frac{15.64 + 14.57 + 15.11}{3} = 15.107 \quad (\text{Ans})$$

(ii) For an unconfined compression test, $\sigma_3 = 0$ & $c_u = \frac{\sigma_1}{2}$

Hence,

We know, $\sigma_1 = 2c_u \tan \theta$ Here, $c_u = 59.67$ and $\phi = 15.107^\circ$

$$\sigma_1 = 2 \times 59.67 \times \tan \left(45 + \frac{15.107}{2} \right)$$

$$\sigma_1 = 155.828$$

$$\therefore c_u = \frac{\sigma_1}{2} = \frac{155.828}{2} = 77.914$$

(Ans.)

(iii) Here, $\sigma_3 = 275 \text{ kPa}$, $\theta = \left(45^\circ + \frac{\phi}{2} \right) = \left(45 + \frac{15.107}{2} \right) = 52.5535^\circ$

We know,

$$\sigma_1 = \sigma_3 \tan^2 \theta + 2c_u \tan \theta$$

$$\sigma_1 = 275 \times \tan^2 52.5535 + 2 \times 59.67 \times \tan 52.5535$$

$$\therefore \sigma_1 = 624.697 \text{ kPa}$$

\therefore Total axial pressure, $\sigma_1 = 624.697 \text{ kPa}$.

(Ans.)

2013 Example: 12.7

A consolidated-undrained test on a normally consolidated soil yielded the following results:

$$\sigma_3 = 10 \text{ lb/in}^2$$

$$(\Delta\sigma_d)_f = 9.4 \text{ lb/in}^2$$

$$(\Delta\sigma_u)_f = 6.8 \text{ lb/in}^2 \text{ . calculate the undrained and drained friction angle of soil}$$

↳ pore water pressure (Δu_d)

Solution:

$$\sin \phi_{cu} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{(\Delta\sigma_d)_f}{2\sigma_3 + (\Delta\sigma_d)_f} = \frac{9.4}{2 \times 10 + 9.4} = 0.32$$

Undrained friction angle, $\phi_{cu} = 18.646^\circ$

$$\text{Now, } \sin \phi' = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)} = \frac{9.4}{10 + 9.4 + 10 - 2 \times 6.8}$$

$$\Rightarrow \sin \phi' = 0.595$$

∴ Drained friction angle, $\phi' = 36.51^\circ$ (Ans.)

2015, 2016

For a normally consolidated insensitive clay $\phi_{cd} = 30^\circ$, deviator stress at failure of the same soil is 250 kN/m^2 in CU test. If Skempton's A-parameter at failure is 0.62. Find out ϕ_{cu} for this soil. Also find out the confining pressure during the consolidation state.

Solution: For consolidated drained test,

$$\sin \phi_{cd} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \frac{(\sigma_3' + \Delta\sigma_d) - \sigma_3'}{\sigma_3' + \Delta\sigma_d + \sigma_3'} = \frac{\Delta\sigma_d}{2\sigma_3' + \Delta\sigma_d}$$

$$\sin 30^\circ = \frac{250}{2 \times \sigma_3' + 250} \Rightarrow \sigma_3' = 125 \text{ kN/m}^2$$

For ~~direct~~ ^{CD} test

$$\Rightarrow \sigma_3' = 125 \text{ KN/m}^2$$

$$\therefore \sigma_1' = (125 + 250) = 375 \text{ KN/m}^2$$

Now,

For ~~direct~~ CU test,

$$\text{Given, } \bar{A} = 0.62$$

$$\text{we know, } \bar{A} = \frac{4U_d}{4\sigma_3'}$$

$$\therefore 4U_d = (0.62 \times 250) = 155 \text{ KN/m}^2$$

Hence,

$$\sigma_1 = (\sigma_1' + 4U_d) = (375 + 155) = 530 \text{ KN/m}^2$$

$$\text{Confining pressure, } \sigma_3 = (\sigma_3' + 4U_d) = (125 + 155) = 280 \text{ KN/m}^2 \quad (\text{Ans})$$

Now,

$$\sin \phi_{cu} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} = \frac{530 - 280}{530 + 280} = 0.30864$$

$$\therefore \phi_{cu} = 17.98^\circ \quad (\text{Ans})$$

Example: 12.4

A consolidated drained triaxial test was conducted on a normally consolidated clay. The results are as follows:

$$\sigma_3 = 276 \text{ kN/m}^2 \quad \text{and} \quad (\sigma_3)_f = 276 \text{ kN/m}^2, \quad \phi = 19.45^\circ$$

(i) Find the normal stress σ' and shear stress T_f on the failure plane.

(ii) Determine the effective normal stress on the plane of maximum shear stress.

Here, $\sigma_3 = \sigma_3' = 276 \text{ kN/m}^2$, $\phi = \phi' = 19.45^\circ$

Solution: (i) $\sigma_1 = \sigma_3 + (\sigma_3)_f = (276 + 276) = 552 \text{ kN/m}^2 = \sigma_1'$

Now, we know,

$$\sigma' \text{ (on the failure plane)} = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$

$$\therefore \sigma' = \frac{552 + 276}{2} + \frac{552 - 276}{2} \times \cos \left[2 \times \left(45 + \frac{19.45}{2} \right) \right]$$

$$= 368.03 \text{ kN/m}^2$$

And,

$$T_f = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\theta$$

$$= \frac{552 - 276}{2} \times \sin \left[2 \times \left(45 + \frac{19.45}{2} \right) \right]$$

$$\therefore T_f = 130.12 \text{ kN/m}^2$$

(Ans.)

(ii) The maximum σ occur on the plane with $\theta = 45^\circ$

$$\therefore \sigma' = \frac{552 + 276}{2} + \frac{552 - 276}{2} \times \cos(2 \times 45^\circ)$$

$$= 414 \text{ KN/m}^2 \quad (\text{Ans})$$

(ii) Determine the effective normal stress on the plane of maximum shear stress.

Solution: $\sigma_1 = \sigma_3 + (\sigma_3 - \sigma_1) \cos^2 \theta$
 Here, $\sigma_3 = 0$, $\sigma_1 = 552 \text{ KN/m}^2$, $\theta = 45^\circ$

Now, we know $\sigma' = \sigma_3 \cos^2 \theta + \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$

$$\therefore \sigma' = \frac{225 + 552}{2} + \frac{225 - 552}{2} \times \cos(2 \times 45^\circ)$$

$$= 368.03 \text{ KN/m}^2$$

$$\tau_f = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

$$= \frac{552 - 225}{2} \times \sin(2 \times 45^\circ)$$

$$= 100.15 \text{ KN/m}^2$$

(Ans)