REINFORCED CONCRETE

HandNote On

REINFORED CONCRETE

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Contents

THEORY

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concrete: Concrete is a stonelike material obtained by permitting a carefully proportioned mixture of cement, sand and gravel or other aggregate and water to harden in forms of the shape and dimensions of the desired structure.

Major components of concrete:

1. Binding material: cement or Lime
2. Aggregate: Fine and coarse aggregate
3. water

The bulk of the material consists of fine and coarse aggregate. Cement and water interact chemically to bind the aggregate particles into a solid mass.

Why is concrete widely used as an engineering materials:

concrete is used as most common construction materials for having the following properties:

1. The raw materials of concrete is country wide.
2. Any desirable shape can be made of it.
3. It has a high compressive strength.
4. It is free from corrosion and there is no appreciable effect of atmospheric agents on it.
5. It is not burnt in normal temperature.
6. It is low cost comperatively.
7. It is durable and sustainable.

disadvantages of concrete:

1. It has less tensile strength.
2. It is difficult to quality control of it.
3. Because of having less tensile strength, steel is sopreferable than concrete.
4. Due to initial setting time, it can not be used until hardening.

What are properties of concrete:

1. Strength 2. Elastic properties 3. Fatigue 4. Durability
5. Impermeability 6. workability.

Why is strength considered as the principle property?

Strength of concrete is commonly considered its most valuable property. Although in many practical cases other properties like durability and impermeability may be considered more important, Never the less strength usually gives the overall picture of quality of concrete.

Strength of concrete are following types:

i. compressive strength ii. Tensile strength
iii. Flexural strength iv. shear strength

If any concrete structure aequire a desirable strength ~~than~~ then in parallel other properties will be fulfilled.

This is why strength is considered as the principle property.

Factors affecting the strength of concrete:

1. Water cement ratio.
2. Size of aggregate.
3. Proportion of aggregate.
4. curing.
5. concrete porosity.
6. Degree of compaction.

Reinforcement concrete: 16, 14, 10, 09

The reinforcement, usually round steel rods is placed in the forms in advance of the concrete. When it is completely sorrounded by hardened concrete mass, it forms an integral part of the member. The resulting combination of two materials, is known as reinfored concrete.

Advantages of Reinforcement concrete over plain concrete:

1. RCC is more economical than plain concrete for same strength
2. RCC has high tensile strength as compared to plain concrete
3. RCC is more durable than plain concrete.
4. RCC has greater ductility and toughness.
5. RCC has good compressive strength and excellent formability of concrete.

08,13,15

Why is mild steel used as reinforcement in RCC structure:

The following are the required characterestics of material which can be used as good reinforcement:

(i) It should be able to develop perfect bond with concrete.

(ii) The co-efficient of thermal expansion should be nearly same as that of concrete.

(iii) It should be easily available.

(iv) It should have high tensile strength.

(v) It should be easily cut and bend.

(vi) It should not produce any harmful effect combining with concrete.

It is found that all these requirements are fulfilled by mild steel. That is why mild steel is used as reinforcement in RCC structure.

Loads:

Loads that act on structures can be divided into three broad categories:

(i) dead loads (ii) Live loads (iii) environmental loads.

Dead loads: Dead loads are those that are constant in magnitude and fixed in location throughout the life time of the structure. Usually major part of the dead load is the weight of the structure itself.

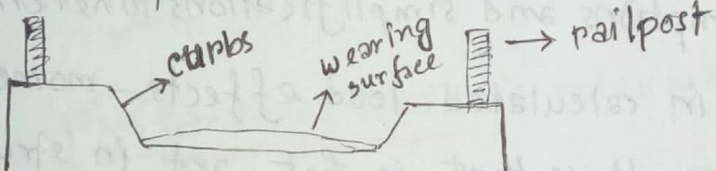
This can be calculated with good accuracy from the design configuration, dimensions of structure, and density of the material.

For buildings: dead loads are

* floor fill * finish floors * plastered ceilings
* allowance is made for suspended loads such as pipings and lighting fixtures.

For bridges: dead loads are

* wearing surface * side walks * curbs * an allowance is made for piping and other suspended loads.



Live loads: If the magnitude and position of a load is change with respect to time change, then this type of loads is known as live loads.

Live loads for ~~bridge~~ highway bridges are specified by the American Association of state Highway and Transportation Officials (AASHTO)

Environmental loads: Environmental loads consist mainly of snow loads, wind pressure and suction, earthquake loads, soil pressures on subsurface portion of structures. Like live loads, environmental loads at any given time are uncertain both in magnitude and distribution.

⊞ sources of uncertainty & margin of safety: 17

There are a number of sources of uncertainty in the analysis, design, and construction of reinforced concrete structure. These sources of uncertainty, which require a definite margin of safety, may be listed as follows:

1. Actual loads may differ from those assumed.
2. Actual loads may be distributed in a manner different from that assumed.
3. The assumptions and simplifications inherent in any analysis may result in calculated load effects - moment, shears etc - different from those that, in fact, act in structure.
4. The actual structural behaviour may differ from that assumed, owing to imperfect knowledge.
5. Actual member dimensions may differ from those specified.
6. Reinforcement may not be in its proper position.
7. Actual material strength may be different from that specified.

⊞ safety provisions of ACI code:

The design strength ϕS_n of a structure or member must be at least equal to the required strength U calculated from the factored loads i.e.

Design strength $\geq$ Required strength

$\phi S_n \geq U$ where, S_n = nominal strength

In specific terms for a member subjected, say, to moment, shear and axial load:

$$\phi M_n \geq M_u$$

$$\phi V_n \geq V_u$$

$$\phi P_n \geq P_u$$

~~where~~ Basically, 'U' (factored load) is calculated from,

$U = 1.2D + 1.6L$ where, D = dead loads
L = Live loads

Strength reduction factor: 01, 05, 13, 14

To provide safety, the theoretical ultimate strength is reduced by a co-efficient, which is known as strength reduction factor and it is denoted by ϕ.

The value for bending is higher than that for shear or ~~bending~~ bearing. Because the flexure failure of beam depends on the quality of steel which can be controlled more precisely than shear failure as shear failure depends on the quality of concrete which varies from time to time. thus, More safety is required for shear strength.

ϕ for shear — 0.75

ϕ for bending — 0.90

00, 03, 04, 06, 07, 08

Fundamental assumptions for reinforced concrete behaviour:

The fundamental propositions on which the mechanics of reinforced concrete is based are as follows:

1. The internal forces, at any section of a member are in equilibrium with the effect of the external loads at that sections.

2. The strain in an embedded reinforcing bar, is same as that of surrounding concrete.

3. Cross sections that were plane prior to loading continue to be plane in the member under load.

4. Concrete is not capable of resisting any tension stress Because the tensile strength of concrete is only a fraction of its compressive strength.

5. The theory is based on the actual stress-strain relationships and strength properties of two constituent materials.

Elastic behaviour of concrete:

* At low stresses, up to about $\frac{f_c'}{2}$, the concrete is seen to behave nearly elastically, i.e. stresses and strains are quite closely proportional. $\sigma \propto E$

* The compression strain in the concrete, at any given load, is equal to the compression in the steel.

$$\epsilon_c = \frac{f_c}{E_c} \quad \text{and} \quad \epsilon_s = \frac{f_s}{E_s}$$

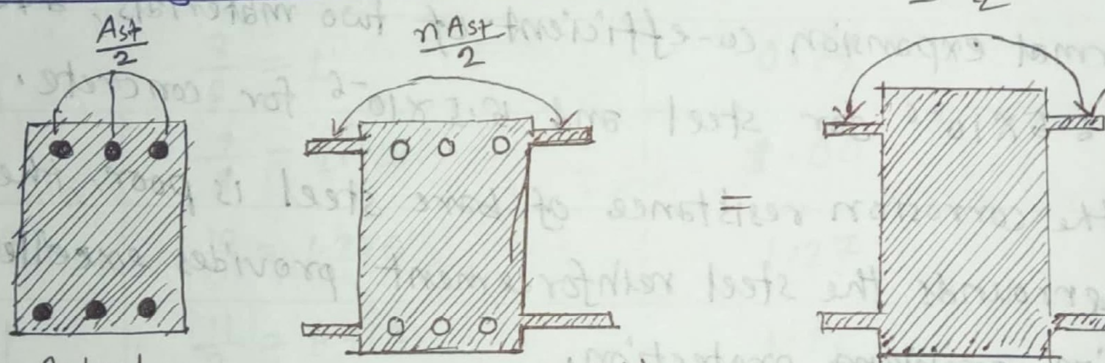
When, $\epsilon_c = \epsilon_s$

$$\Rightarrow \frac{f_c}{E_c} = \frac{f_s}{E_s}$$

$$\Rightarrow f_s = \frac{E_s}{E_c} \times f_c$$

$\Rightarrow f_s = n f_c$ where, $n = \frac{E_s}{E_c}$ is known as modular ratio

Axial Load of transformed section: 2017



Actual section

Transformed section $A_t = A_c + nA_{st}$

Transformed section $A_t = A_g + (n-1)A_{st}$

Let, A_c = net area of concrete i.e. gross area minus area occupied by reinforcing bars.

A_g = gross area

A_{st} = total area of reinforcing bar

P = axial load.

Then, $P = P_c + P_s = f_c A_c + f_s A_{st} = f_c A_c + n f_c A_{st}$

$\therefore P = f_c(A_c + nA_{st})$ where, $(A_c + nA_{st}$ is transformed area)

$$\Rightarrow P = f_c(A_g - A_{st} + nA_{st})$$

$$\Rightarrow P = f_c[A_g + (n-1)A_{st}]$$

Reinforcing steels for concrete:

- The useful strength of ordinary reinforcing steels in tension as well as compression.
- Steel is ~~a~~ high cost material compared with concrete. It follows that the two materials are the best used in combination if the concrete is made to resist the compressive stresses and the steel the tensile stresses.
- For most effective reinforcing action, it is essential that steel and concrete deform together.
- The thermal expansion co-efficient of two materials, a ~~be~~ sufficiently close. 6.5×10^{-6} for steel and 5.5×10^{-6} for concrete.
- While the corrosion resistance of bare steel is poor, the concrete that surrounds the steel reinforcement provides excellent corrosion ~~problem~~ protection.
- The thermal conductivity of steel is high and conversely the thermal conductivity of concrete is ~~lo~~ relatively low. A moderate amount of concrete cover ~~prodti~~ provides sufficient ~~strength~~ thermal insulation for embedded reinforcement

Reinforcing bar:

According to ACI code:

Bar No.	Diameter, d (in) = $\frac{\text{Bar No.}}{8}$	Area $(in^2) = \frac{\pi}{4} \times (d)^2$
3	$\frac{3}{8} = 0.375$	0.11
4	$\frac{4}{8} = 0.5$	0.20
5	$\frac{5}{8} = 0.625$	0.31
6	$\frac{6}{8} = 0.75$	0.44
7	$\frac{7}{8} = 0.875$	0.60
8	$\frac{8}{8} = 1$	0.79
9	$\frac{9}{8} = 1.128$	1.00
10	$\frac{10}{8} = 1.270$	1.27
11	$\frac{11}{8} = 1.410$	1.56
14	$\frac{14}{8} = 1.693$	2.25
18	$\frac{18}{8} = 2.257$	4

Fundamental assumptions relating to flexure and flexural shear:

1. A cross section that was plane before loading remain plane under load.

2. The bending stress 'f' at any point depends on the strain at that point in a manner given by the stress-strain diagram of the material.

3. The distribution of the shear stresses 'v' over the depth of the section depends on the shape of the cross section and of the stress-strain diagram.

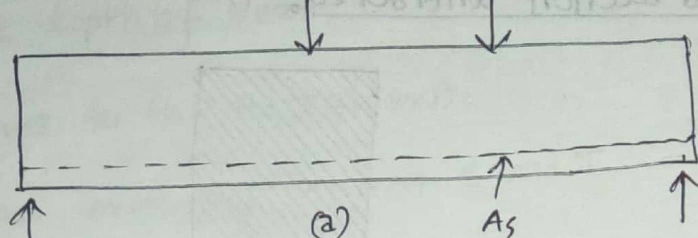
4. Owing to the combined action of shear stresses and flexural stresses, at any point in a beam, there are inclined stress of tension and compression, the largest of which form an angle of 90° with each other.

5. Since the horizontal and vertical shearing stresses are equal ~~such that~~ and the flexural stresses are zero at the neutral plane, the inclined tensile and compressive stresses at any point in that plane form an ~~45°~~ angle of 45° with horizontal, the intensity of each being equal to the unit shear at the point.

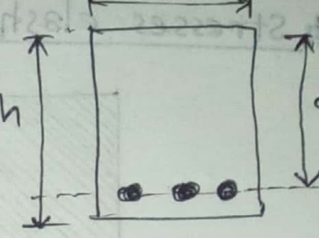
6. When the stresses in the outer fibers are smaller than the proportional limit 'fp', the beam behaves elastically.

Reinforced concrete beam behaviour: 16, 14, 10, 09

When the load on such a beam is gradually increased from zero to the magnitude that will cause the beam to fail, several different stages of behaviour can be clearly distinguished.



(a)



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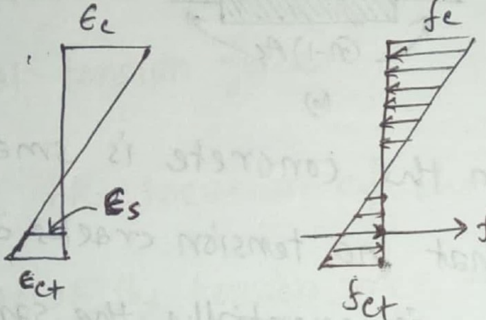
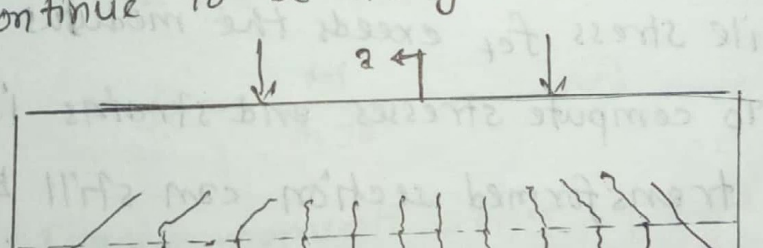


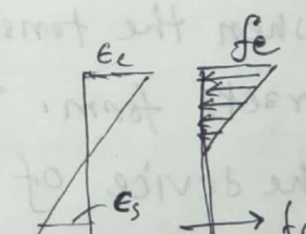
(c)

At low loads, The entire concrete is effective in resisting stress, in compression on one side and in tension on other side of the neutral axis. At this stage, all stress in the concrete are of small magnitude and are proportional to strains. [Fig.(c)]

When the load is further increased, concrete does not resist any tensile stress and at this stage, tension cracks develop. stresses and strains continue to be closely proportional. [Fig. (d)]

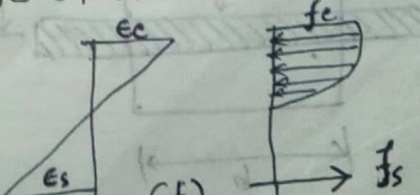


(d)



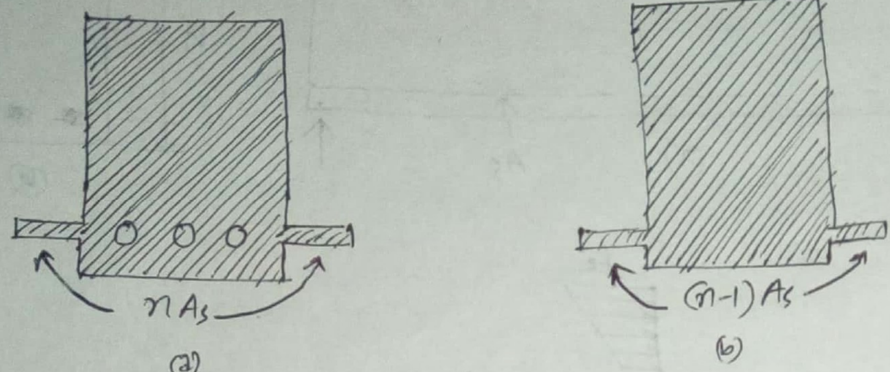
(e)

when the load is still further increased, stresses and strains rise correspondingly and are no longer proportional. The ensuing non linear relation between stresses and strains is that given by the concrete stress-strain curve. [Fig (f)]



(f)

Stresses elastic and section uncracked: 17

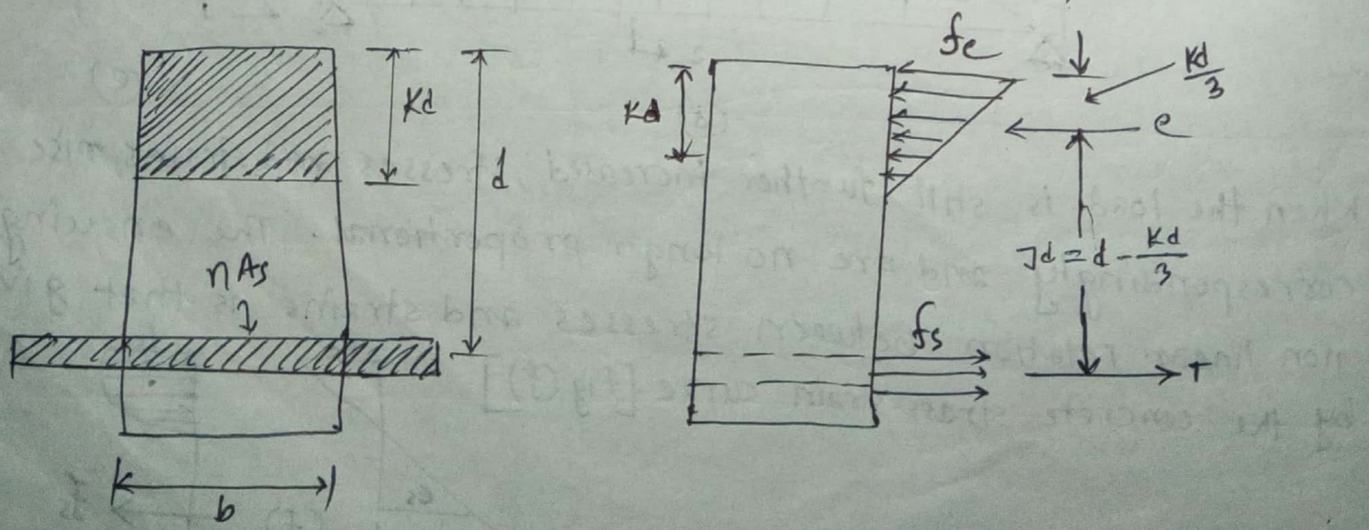


As long as the tensile stress in the concrete is smaller than the modulus of rupture, so that no tension cracks develop. The stress and strain distribution is essentially the same in an elastic, homogeneous beam.

The only difference is the presence of another material, the steel reinforcement. In the elastic range, for any given value of strain, the stress in the steel is n times that of concrete.

Stresses elastic and section cracked: 17

When the tensile stress f_{ct} exeeds the modulus of rupture, cracks form. To compute stresses and strains if desired, the device of transformed section can still be used.



Here,

d = effective depth of beam.

Kd = distance to the netral axis.

jd = internal lever arm between C and T

C = Total compression force.

T = Total tension force.

To determine the location of the neutral axis

The moment of the tension area about the axis = the moment of the compression area.

which gives,

$$b\frac{(Kd)^2}{2} = nA_s(d-Kd) \quad \text{---- (I)}$$

Then,

$$C = \frac{f_c}{2}bKd \text{ --- (II) and } \quad T = A_s f_s \text{ ---- (III)}$$

The couple constitued by two forces C and T be equal numerically to the external bending moment M. Taking moments about C gives

$$M = Tjd = A_s f_s jd \quad \text{---- (IV)}$$

$$\Rightarrow f_s = \frac{M}{A_s Jd} \quad \text{----- (V)}$$

conversely, Taking moments about T. gives,

$$M = Cjd = \frac{f_c}{2}bKdjd = \frac{f_c}{2}Kjbd^2 \quad \text{---- (VI)}$$

$$\Rightarrow f_c = \frac{2M}{Kjbd^2} \quad \text{------ (VII)}$$

Reinforcement ratio,

$$\rho = \frac{A_s}{bd} \Rightarrow A_s = \rho bd$$

Now, from eqn ①

$$b\frac{(kd)^2}{2} = n\rho bd(d-kd)$$

$$\Rightarrow k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \quad \text{.......} \quad (\text{VIII})$$

From Figure,

$$jd = d - \frac{kd}{3}$$

$$\Rightarrow j = 1 - \frac{k}{3} \quad \text{.....} \quad (\text{IX})$$

Difference between WSD and USD method: 09,10,04

WSD	USD
1. WSD means Working stress Design.	1. USD means Ultimate strength design.
2. It is the old method and used for patiular cases.	2. It is the modern method and universally used.
3. It counts the stress upon proportional limit.	3. It counts the stress upon yield point.
4. Stress is increased by multiplying factor of safety.	4. Stress is decreased by multiplying factor of safety.
5. Design load is computed as $W = $ Dead loads + Live loads	5. Design load is computed as $W = 1.2D + 1.6L$
6. Modular ratio is used for determining the moment capacity.	6. Steel ratio is used for determining moment capacity

Advantages and Disadvantages of WSD and USD method:

Advantages of WSD:

(i) The critical deflection is normally under service load for which WSD method is appropriate.

(ii) With high stress steel reinforcing crack width at working load can be a consideration.

Disadvantages of WSD:

(i) Different types of loads acting simultaneously have different degrees of uncertainties. This can not be counted in WSD method.

(ii) The actual factor of safety is not known in this method.

Advantages of USD:

(i) USD method ensures proper factor of safety.

(ii) USD method is necessary for pre-stressed concrete

Disadvantages of USD:

(i) Limitations of this method is the collapse of flexure, compression, shear and Tension.

(ii) It is high cost design method.

Transformed RC section:

If The cross section of a composite beam is transformed in to an equivalent cross section, it is called transforced RC section.

Doubly reinforced Beam:

The R.C.C beams in which the steel reinforcement is placed in the tension as well as compression zone are called doubly reinforced beams.

Necessity of doubly reinforced beam: 08, 10, 11, 15

1. When the dimension of the beam are restricted due to any constraints like availability of head room, or architectural considerations and the moment of resistance of singly reinforced beam is less than the external moment.
2. When external loads may occur either face of the member and may cause tension on both faces of member.
3. When the loads are eccentric
4. When the beam is subjected to sudden lateral loads.
5. In case of continuous beams or slab, the sections at support are generally designed as doubly reinforced sections.

Example: Limited head room ~~for~~ in multistoried buildings.

Difference between singly and doubly reinforced beam.

Singly reinforced beam	Doubly reinforced beam
(i) No steel is provided in compression zone.	(i) Steel is provided in compression zone as well as tension zone.
(ii) suitable for unrestricted dimension of beam	(ii) suitable for limited dimension of beam
(iii) All the compressive stress is carried by concrete.	(iii) compressive stress is carried by concrete as well as steel.
(iv) It offers less resistance to earthquake forces	(iv) It offers more resistance to earthquake forces ~~the~~ rather than singly reinforced beam.

why reinforcement are provided in compression zone: 15,16

(i) when a section is limited to specific dimension due to architectural reasons such as a need for limited head room in multy storied buildings.

(ii) Long term deflection is reduced with the inclussion of reinforcement in compression zone.

(iii) To support the stirrups.

(iv) Ductility of the beam is increased with the inclussion of reinforcement in compression zone.

Behavior of diagonally cracked beam: 2014

Two types of behavior have been observed in the diagonally cracked beam.

(i) The diagonal crack, once formed, spreads either immediately or at only slightly higher load, traversing the entire beam from tension reinforcement to the compression face, splitting it in two and failing the beam.
This process is sudden and without warning.

(ii) Alternatively, the diagonal crack, once formed, spreads toward and partially into the compression zone, but stops short of penetrating to the compression face.
In this case, No sudden collapse occurs.

Behavior of web reinforced concrete beam:

After diagonal cracks have developed, web reinforcement augments the shear resistance of a beam in four seperate ways:

(i) part of the shear force is resisted by bars that traverse a particular crack.

(ii) The presence of these same bars restricts the growth of diagonal cracks and reduces their penetration into the compression zone.

(iii) The stirrups also counteract the widening of the cracks, so that two crack faces stay in close contact.

(iv) The stirrup are arranged so that they tie the longitudinal reinforcement into the main bulk of the concrete. This provides some measure of restraint against the splitting of concrete.

Types of failure in RC Beam:

1. Shear failure 2. Flexural failure 3. Tension failure 4. compression failure.

Shear failure: It will typically occur in the connections between member to column connection or member to girder connection etc. connections typically have high shearing forces that an engineer must ~~have~~ consider when designing the connections.

Flexural failure: flexural failure occur in flexural member such as members, girders and in some cases, compression members such as column that are subjected to bending stresses. Flexural members typically fail when flexural loadings cause the element to buckle.

Tension failure: Tension failure generally occur in brace member or hangers. This type of failure occurs when the steel member is stretched to a level that exceeds the material strength of the member.

compression failure: Compression failures typically occur in compression member, such as column and braces, when the compressive axial force applied to the element caused the element to either buckle or become over stressed.

T Beam

T Beam: 15, 14, 10, 06, 04, 01, 03, 11

A T-beam is a beam which consists of a rectangular flange cast integrally with the beam subjected to tension and a rectangular web below the flange.

Advantages of T-beam over rectangular beam: 17, 11, 07, 06, 01

(i) In T-beam flange can resist more compressive stress than rectangular beam.

(ii) In T-beam web thickness is less and so that material use is less than a rectangular beam.

(iii) T-beam is economical compared to a rectangular beam.

(iv) For larger span, T-beams are usually preferred than rectangular beam as the deflection is reduced to a good extent.

Disadvantages of T-beam:

(i) There is a considerable increase in the shear stress at the junction of the flange and the web of the beam due to the change in cross section. So casting should be done very carefully to ensure both are bonded well.

(ii) Since the beam slab is rigid, it becomes very weak in resisting lateral shear forces.

Differences between T-beam and Rectangular beam: 17

T-beam	Rectangular beam
(i) T beam offers more moment of inertia.	(i) Rectangular beam offers less moment of inertia than T-beam.
(ii) The flexural capacity of T beam varies based on the sign of moment (positive or negative)	(ii) Rectangular beam only depends on the location of reinforcement to yield the flexural capacity.
(iii) T beam is economical compared to a rectangular beam.	(iii) Rectangular beam is high cost than T beam.
(iv) The neutral axis of T beam lies in the web.	(iv) The neutral axis of rectangular beam lies in the flange.

How T-beam is formed: 16

Concrete beams are often casted integrally with the slab and formed T-shapped beam.

Here slab portion carries the compressive load and web portion carries the tension

These beams are very efficient.

recommendation

☐ criteria/ for effective flange width: 14

① For symmetrical T-beam / having slab both sides,

a) $16 h_f + b_w$

b) $b \leq span/4$

c) c/c beam distance

② beams having slabs on one side only,

a) $b_w + span/12$

b) $b_w + 6 h_f$

c) $b_w + \frac{1}{2} \times$ beam clear distance

③ Isolated T beam,

a) $b_{eff} \leq 4 b_w$

b) $h_f \geq b_w/2$

☐ Tension controlled failure in T beam: 16, 12

T beam is formed by the combination of flange and web. Flange has compressive reinforcement. compressive load is resisted by flange, both reinforcement and concrete. And web has tension reinforcement. But Tension is resisted by only steel or reinforcement. Hence if failur occurs in T beam, it will be expected tension failure.

ACI specifications for T-beam:

Effective flange width

① Effective flange width should not exeed $\frac{1}{4}$ th of span

② The overhanging width on either side of the beam web should not exeed 8 times the thickness of the slab nor $\frac{1}{2}$ of the clear distance to next beam.

③ Steel ratio should not exist the following limit

$$\rho_{max} = 0.75(\rho_b + \rho_f)$$

where, ρ_b = balanced steel ratio for rectangular portion

ρ_f = balanced steel ratio for flange

④ Tensile steel ratio should not be less than

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \text{ and } \geq \frac{200}{f_y}$$

Slab

Slab:

Slab is a common flat structural element, usually horizontal with top and bottom surface parallel or nearby so. It is supported by reinforced contrete beams, masonry or reinfored concrete walls, structural steel members or directly by columns.

Temperature or shrinkage reinforcement / Distribution steel:

In reinforced concrete, steel reinforced which is provided to resist shrinkage stress is called shrinkage reinforcement. Shrinkage reinforcement is required at right angles to the main reinforcement to minimize cracking and to tie the structure together to ensure it is acting as assumed in the design.

Necessity of shrinkage reinforcement in one-way slab: 16, 15, 14, 12, 11

(i) To reduce the shrinkage of steel of main reinforcement

(ii) It distributes cracks, so deflection of main reinforcement will be controlled.

(iii) Due to temperature change, steel contraction or expansion will happen which can be controlled by shrinkage reinforcement.

(iv) To resist bending moment.

ACI code specifications for distribution steel reinforcement:

(i) If $f_y = 60$ KSI $\quad A_s\,(min) = 0.0018\,bt$

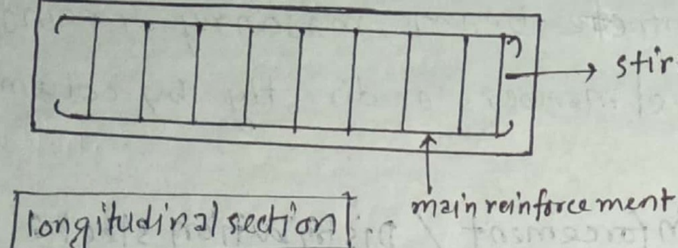
(ii) If $f_y < 60$ KSI $\quad A_s\,(min) = 0.0020\,bt$

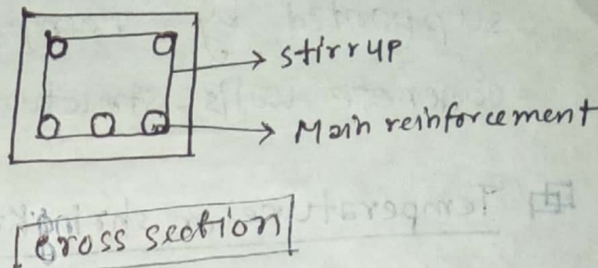
(iii) If $f_y > 60$ KSI $\quad A_s\,(min) = \dfrac{0.0018 \times 60000}{f_y}\,bt$

Where, $b = 12$ inch and t = thickness of slab

Stirrup

longitudinal & cross section of beam:





Diagonal Tension: 15,08

In a simple beam subjected to bending, the shear stress is produced maximum at neutral axis and zero at the outer fibers. And Tensile and compressive flexural stresses develops maximum at the outer fibers. If the beam is reinforced with longitudinal steels only, cracks may form diagonally. The stresses producing these cracks are known as diagonal stress/tension.

cracks in RC beam: 17

(i) Flexural cracks:

If A concrete beam reinforced with longitudinal steels only, At low loads the entire concrete is effective in resisting stress. When load is increased, tension cracks develop. These crack are verticle at the centre of the span. This type of cracks are called flexural cracks.

(ii) Diagonal tension cracks:

These cracks become more inclined as they approaches the support where they slope at 45° angle. These are called diagonal tension cracks. Diagonal tension cracks are of two types.

(a) Web shear cracks: This type of cracks form when the diagonal tension stresses in vicinity of the neutral axis becomes equal to the tensile strength of the concrete.

(b) Flexural shear cracks: when both the shear force and bending moment have large values, the diagonal tension at the upper end of cracks exeeds tensile strength of the concrete and the cracks bends in a diagonal direction and continue to grow in length and width. These are called flexural shear cracks.

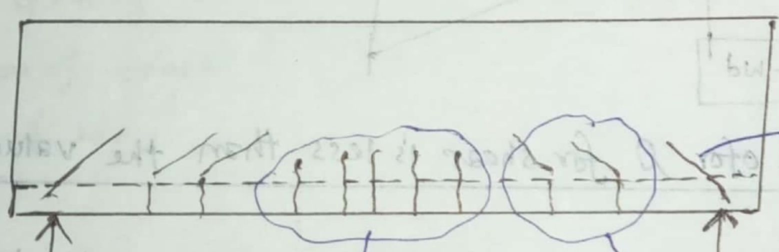
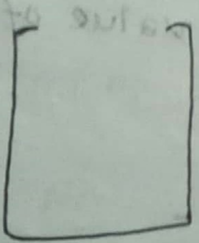


Fig. cracks in RC beam.

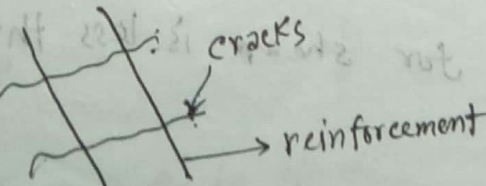
Forms of web reinforcement:

The forms of web reinforcement are:

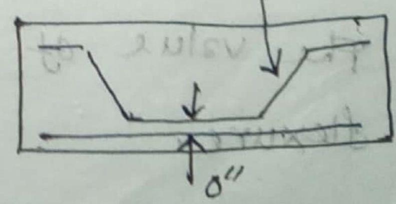
1. vertical stirrup
2. Inclined stirrup
3. bent up bars



U-stirrup



inclined stirrup



Function of stirrup:

(i) Resist shear force

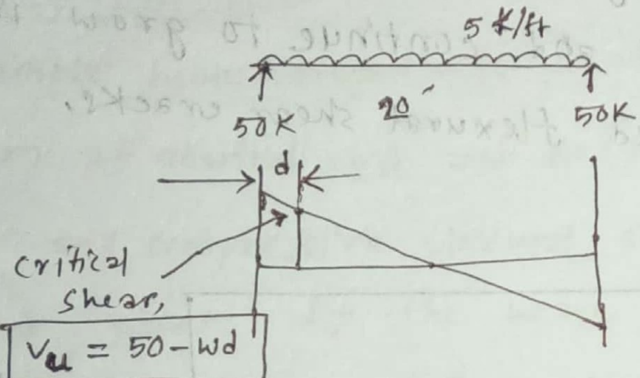
(ii) Increase strength and ductility by providing confinement

(iii) Hold the main reinforcement or longitudinal reinforcement in position.

critical section for shear:

critical section for shear is at a distance 'd' from the support

Example:



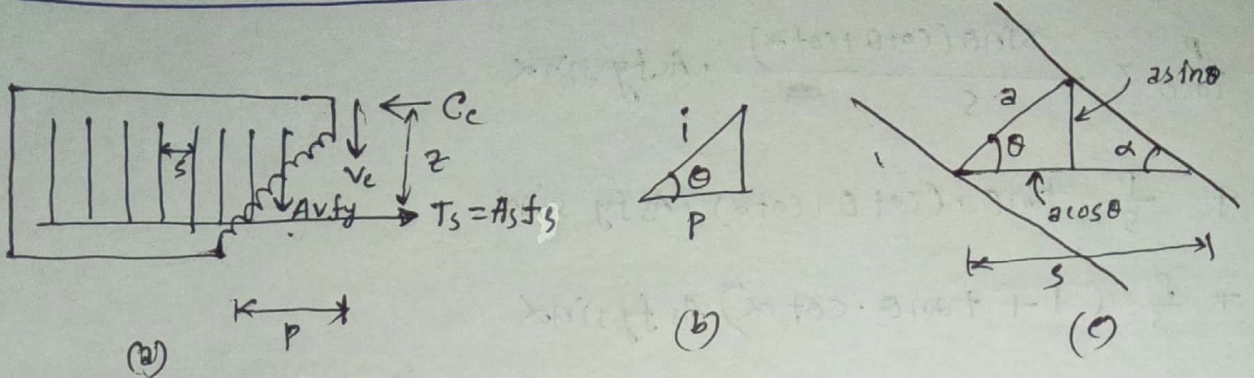
Why the value of ϕ for shear is less than the value of ϕ for flexure:

Shear failure of reinforced concrete is difficult to predict. shear collapse is likely to occur suddenly, with no advance warnig of distress. This is in strong contrast with the nature of flexure failure.

For typically under reinforced beam, flexure failure accompanied by obvious cracking of the concrete and large diflection, giving ample warning and providing the opportunity to take corrective measures. Because of these differences,

The value of ϕ for shear is less than the value of ϕ for flexure.

Derive an expression for spacing of inclined / vertical stirrup:



Let us consider diagonal crack as shown in figure,

p = Horizontal projection of cracks.

i = inclined length of crack

a = Inclined spacing of bar

s = Horizontal spacing of bar

θ = Inclination of crack

α = Inclination of bar,

For total crack, $\frac{p}{i} = \cos\theta$ [figure (b)]

$$\Rightarrow i = \frac{p}{\cos\theta}$$

For the crack in between two bar from figure (c)

$$s = a\cos\theta + \frac{a\sin\theta}{\tan\alpha}$$

$$\Rightarrow a = \frac{s}{\cos\theta + \frac{\sin\theta}{\tan\alpha}}$$

$$\Rightarrow a = \frac{s}{\sin\theta\,(\cot\theta + \cot\alpha)}$$

if n = no. of stirrup then, $n = \frac{i}{a}$

Now, $V_n = V_c + n A_s f_y \sin\alpha$

$\Rightarrow V_n = V_c + \frac{i}{a} A_s f_y \sin\alpha$

$\Rightarrow V_n = V_c + \frac{P}{\cos\theta} \times \frac{\sin\theta(\cot\theta + \cot\alpha)}{s} \cdot A_s f_y \sin\alpha$

$\Rightarrow V_n = V_c + \frac{P}{s} \tan\theta \cdot (\cot\theta + \cot\alpha) \cdot A_s f_y \sin\alpha$

$\Rightarrow V_n = V_c + \frac{P}{s} (1 + \tan\theta \cdot \cot\alpha) A_s f_y \sin\alpha$

$\therefore V_u = \phi V_n = \phi \left[\frac{P}{s} A_s f_y \sin\alpha (1 + \tan\theta \cdot \cot\alpha) \right] + \phi V_c$

Now, $P = d$ = effective depth, $\theta = 45°$ for ~~verticle~~ diagonal cracks,

$\alpha = 90°$ & $\theta = 90°$ for verticle stirrup.

Now,

$\therefore$ When $\theta = 90°$ & $\alpha = 90°$

$$V_u = \phi \frac{A_s f_y d}{s} + \phi V_c$$

$$\Rightarrow \boxed{s = \frac{\phi A_s f_y d}{V_u - \phi V_c}}$$

when $\theta = 45°$

$$V_u = \phi \cdot \frac{d}{s} A_v f_y (\sin\alpha + \cos\alpha) + \phi V_c$$

$$\Rightarrow \boxed{s = \frac{\phi A_v f_y (\sin\alpha + \cos\alpha) d}{V_u - \phi V_c}}$$

Bond stress

FARHAD
1500045

Bond:
It is defined as the cohesion or adhesion between steel and concrete over entire length of steel bar.

Type of Bond failure:
For reinforcing bar in tension, two types of bond failure may occur.

1. Direct pull out of the bar, which occurs when ample confinement is provided by the surrounding concrete.

For example, when relatively small diameter bars are used with sufficiently large concrete cover distances and bar spacing.

2. Splitting of concrete along the bars, occurs when the concrete cover, confinement or bar spacing is insufficient to resist tension resulting from the wedging effect of the bar deformations.

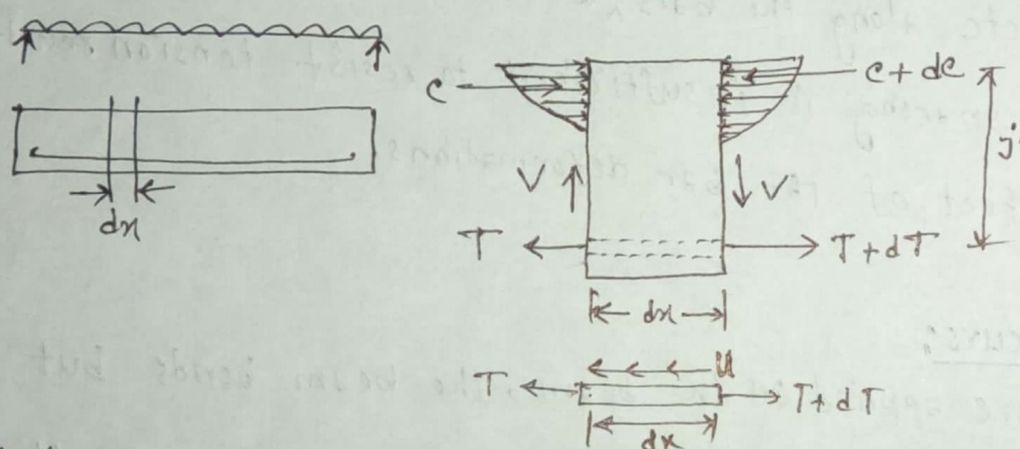
How bond failure occurs:
When heavy loads are applied on the beam, the beam bends but the steel bars try to maintain their original length. As a result the bar may slip longitudinally with respect to the adjucent concrete the bar is pulled through the concrete. Such a beam will collapse as the bar is pulled through the concrete. This is how a beam fails in bond. This type of failure is bond failure.

Development Length:
Development length of the reinforcing bar is defined as that length of embedment necessary to develop the full tensile strength of the bar, controlled by either pull out or splitting.

Factors affecting Development Length:

1. Tensile strength of concrete.
2. cover distance and bar spacing.
3. Tranverse reinforcement.
4. vertical bar location.
5. Epoxy coated reinforcing bar.
6. over design at reinforcing bar.
7. bar diameter.

Derive expression for flexural / bending bond stress:



Let us consider a beam as shown in figure.

An elementary portion of the beam of length $= dx$

For equilibrium, $M = A_s f_y \, jd = T \, jd$

Similarly, $M + dM = (T + dT) \, jd$

Change in moment, $M + dM - M = (T + dT) jd - T jd$

$\Rightarrow dM = dT \, jd$

$\Rightarrow dT = \dfrac{dM}{jd}$ .....①

For flexural bond stress, u

$$dT = u\,(\Sigma_0\, dx) \quad \text{.....(ii)} \quad \text{where, } \Sigma_0 = \text{perimeter}$$

From (i) & (ii) we obtain,

$$\frac{dM}{jd} = u\,(\Sigma_0\, dx)$$

$$\Rightarrow \frac{dM}{dx} = u\,\Sigma_0\, jd$$

$$\Rightarrow V = u\,\Sigma_0\, jd$$

$$\boxed{\therefore u = \frac{V}{\Sigma_0\, jd}}$$

This is the expression for flexural bond stress.

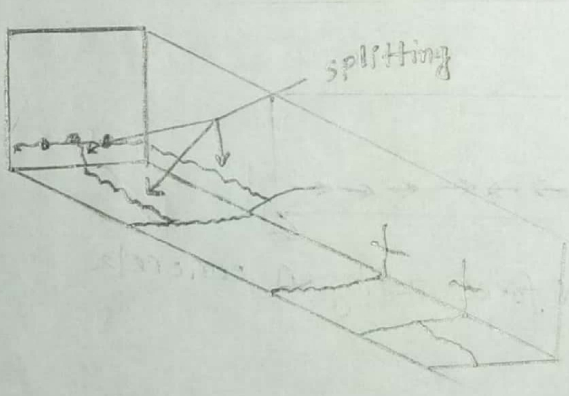


Fig. splitting of concrete along reinforcement

Anchorage bond: It is the average bond stress over entire length of the bar, with in which the bar tension being changed from T to zero (anchorage length) or zero to T (development length).

Bond force due to flexure:

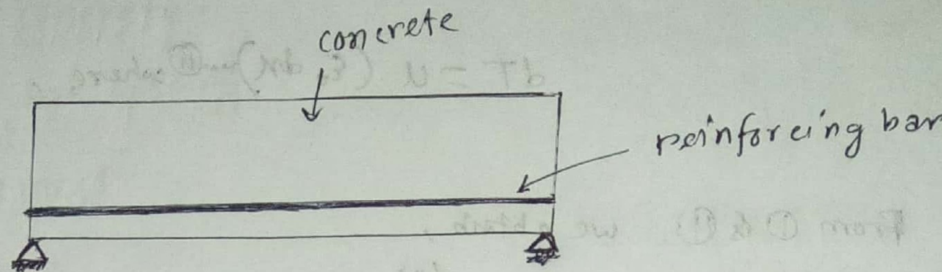


Fig. beam before loading

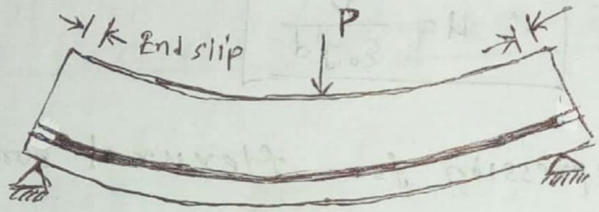


Fig. Unrestrained slip between steel and concrete

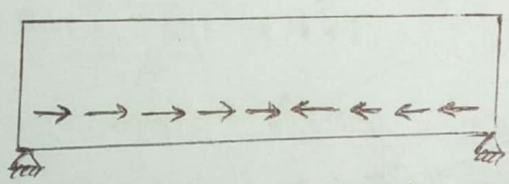


Fig. bond force acting on concrete

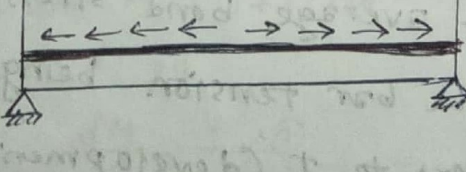


Fig. bond forces acting on steel

FORMULA

Singly (WSD)

Design step:

1. Assume,

depth, h = $\boxed{}$

width, b = $\boxed{}$

2. **Total load calculation:**

$$T.L = D.L + L.L$$

[N.B: including to its own weight থাকলে self weight যোগ করতে হবে না। in addition / super imposed থাকলে

$$\text{self weight} = \left(\frac{bh}{144} \times 150\right) plf$$

যোগ করতে হবে]

3. **Maximimum Moment calculion:**

[From BMD Diagram / Moment capacity দেওয়া থাকবে]

4. **depth check:**

$$n = \frac{E_s}{E_c} = \frac{27 \times 10^6}{57000\sqrt{f_c'}}$$

$$r = \frac{f_s}{f_c} = \frac{.4 f_y}{.45 f_c'}$$

$$K = \frac{n}{n+r}, \quad j = 1 - \frac{K}{3}$$

$$R = \frac{1}{2} f_c j K$$

$$\boxed{d_{req} = \sqrt{\frac{M}{Rb}}}$$

4. Total depth required, $h_{req} = (d_{req} + 2.5)$

if h_{req} < Assumed depth, h = $\boxed{}$

It is singly reinforce beam.

$\boxed{\text{Design OK}}$

5. **Steel Area calculation:**

$$A_{st} = \frac{M}{f_s j d}$$ Assumed effective depth.

[N.B: BMD তে যদি positive and Negative Moment দুইটাই থাকে, তাহলে A_{st} দুইটা বের করতে হবে.

For tension, $A_{st} = \dfrac{M_{max}(+ve)}{f_s j d}$ → Assumed effective depth

For compression, $A_{st} = \dfrac{M_{max}(-ve)}{f_s j d}$]

Analysis step:

1. $\rho = \dfrac{A_s}{bd}$

2. $n = \dfrac{E_s}{E_c}$

3. $K = \sqrt{(\rho n)^2 + 2(\rho n)} - \rho n$

4. $j = 1 - \dfrac{K}{3}$

5. $M_c = \dfrac{1}{2} f_c j K b d^2$

$M_s = A_s f_s j d$

[N.B: M_c এবং M_s এর মধ্যে যেইটা Minimum হবে সেইটাই Allowable moment capacity]

#3 → 0.11 in²

#4 → 0.20 in²

#5 → 0.31 in²

#6 → 0.44 in²

#7 → 0.60 in²

#8 → 0.79 in²

#9 → 1.00 in²

#10 → 1.27 in²

#11 → 1.56 in²

singly (USD)

Design step: $\phi = 0.9$

1. Load calculation: $W_u = 1.2\,D.L + 1.6\,L.L$

2. Moment calculation: $M_u = \frac{W_u L^2}{8}$ (for simply supported)

$M_u = \frac{W_u L^2}{2}$ (for cantilever beam)

3. Depth:

$M_u = \phi \rho b d^2 \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

$\Rightarrow d_{req} = \boxed{}$

effective depth

Here,

$\phi = 0.9$ (for design)

$\rho = .85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$

$\frac{\beta}{\alpha} = 0.59$ [$\epsilon_u = 0.003$, $\epsilon_y = 0.005$]

$\beta_1 = 0.85$ (when $f_c' = 4000$ psi)

$\beta_1 = 0.80$ (when $f_c' = 3000$ psi)

Total depth required, $h_{req} = (d_{req} + 2.5)$

4. Steel Area calculation:

$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$

d_{req}

Here,

$a = \frac{A_s f_y}{0.85 f_c' b}$

$\Rightarrow a = \boxed{} \times A_s$

$\Rightarrow A_s = \boxed{}$

Analysis's step:

1. $A_s = \boxed{}$ ← ($\square$ # $\square$ bars)

2. $\rho = \frac{A_s}{bd}$

3. $\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$ ($\epsilon_u = 0.003$)

$\beta_1 = 0.85$ ($f_c' \le 4000$ psi)

$\beta_1 = 0.80$ ($f_c' = 5000$ psi)

4. Moment calculation:

If, $\rho < \rho_b$ it is under reinforced beam. (tension steel yields at failure)

$\boxed{M_n = A_s f_y \left(d - \frac{a}{2}\right)}$

Here,

$a = \frac{A_s f_y}{.85 f_c' b}$

$\boxed{M_u = \phi M_n}$

$\phi = 0.483 + 83.3\,\epsilon_t$

Here, $\epsilon_t = \epsilon_u \times \left(\frac{d - c}{c}\right)$

$c = \frac{a}{\beta_1}$

[N.B: ϵ_t এর মান 0.005 এর বেশি আসলে 0.005 নিতে হবে]

if, $\rho > \rho_b$, it is over reinforced beam. (concrete yields at failure)

$\boxed{M_n = A_s f_s \left(d - \frac{a}{2}\right)}$

Here, $f_s = E_s \epsilon_u \left(\frac{d - c}{c}\right)$

Here,

$E_s = 29 \times 10^6$

$\epsilon_u = 0.003$

$\boxed{c = \frac{a}{\beta_1}}$

At equilibrium,

$\boxed{C = T}$

$\Rightarrow 0.85 f_c' \beta_1 c b = A_s f_s$

$\Rightarrow c = \boxed{}$

$a = \frac{a}{\beta_1} \Rightarrow a = \boxed{}$

$f_s = \boxed{} < f_y$

যদি, $f_s > f_y$

then, $M_n = A_s f_y \left(d - \frac{a}{2}\right)$

$M_n = \boxed{}$

$\boxed{M_u = \phi M_n}$

Doubly (WSD)

Design steps:

1. depth check: $n = \frac{E_s}{E_c}$, $r = \frac{f_s}{f_c}$,

$K = \frac{n}{n+r}$, $j = 1 - \frac{K}{3}$

$R = \frac{1}{2} f_c j K$, $d_{req} = \sqrt{\frac{M}{Rb}}$

if, $d_{req} >$ Effective depth (given)

It is doubly reinforced beam.

2. $M_1 = Kbd^2$ → given effective depth

3. $M_2 = M - M_1$

4. $A_{s_1} = \frac{M_1}{f_s j d}$

5. $A_{s_2} = \frac{M_2}{f_s (d - d')}$

6. $A_s = A_{s_1} + A_{s_2}$

7. $A_s' = \frac{M_2}{f_s' (d - d')}$ if, $f_s' < f_s$

Here, $f_s' = 2 f_s \times \frac{K - \frac{d'}{d}}{1 - K}$

[N.B: If $f_s' > f_s$, then, $A_s' = \frac{M_2}{f_s (d - d')}$]

A_s → Tension reinforcement

A_s' → Compression reinforcement

Doubly (USD)

Design steps:

1. load calculation: $W_u = 1.2\, D.L + 1.6\, L.L$

2. Moment calculation:

$M_u = \frac{W_u L^2}{8}$ (for simply supported beam)

3. Depth check: $\phi = 0.9$ (for design)

$M_u = \phi \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

Here, $\frac{\beta}{\alpha} = 0.59$

$d_{req} = \boxed{}$

$\rho = 0.85 \beta_1 \times \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$

ϵ_y → 0.005

if $d_{req} >$ given effective depth.

It is doubly reinforced beam.

4. $M_n = A_s f_y \left(d - \frac{a}{2}\right)$ Here, $A_s = \rho b d$

and, $a = \frac{A_s f_y}{.85 f_c' b}$

5. $M_1 = \frac{M_u}{\phi} - M_n$

6. $A_s' = \frac{M_1}{f_s' (d - d')}$ Here,

$f_s' = E_s \epsilon_s'$

$\epsilon_s' = \epsilon_u \frac{c - d'}{c}$

$c = \frac{a}{\beta_1}$

7. Tension reinforcement

$= A_s + \frac{A_s' f_s'}{f_y}$

Doubly (USD)

Analysis:

1. $A_s' = \boxed{}$, $\rho' = \frac{A_s'}{bd}$

2. $A_s = \boxed{}$, $\rho = \frac{A_s}{bd}$

3. Check:

$$\rho_{max} = 0.85\,\beta_1 \times \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} \quad \epsilon_y \rightarrow 0.004$$

if $\rho_{max} < \rho$, It should be doubly reinforced beam analysis.

4. $$\bar{\rho}_{cy} = 0.85 \times \beta_1 \times \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \times \frac{d'}{d} + \rho' \quad \epsilon_y \rightarrow 0.002$$

if $\boxed{\bar{\rho}_{cy} < \rho}$, compression bar will yield when beam fails.

5. $M_{n_1} = A_s' f_y (d-d')$

$M_{n_2} = (A_s - A_s') f_y (d - \frac{a}{2})$ Here, $a = \frac{(A_s - A_s') f_y}{.85 f_c' b}$

$M_n = M_{n_1} + M_{n_2}$

6. $M_u = \phi M_n$ Here, $\phi = 0.483 + 83.3\,\epsilon_t$

and, $\epsilon_t = \epsilon_u \left(\frac{d_t - c}{c}\right)$

or,

if $\boxed{\bar{\rho}_{cy} > \rho}$, compression bar will not yield when beam fails.

5. $M_{n_1} = A_s' f_s' (d-d')$

$M_{n_2} = 0.85 f_c' ab (d - \frac{a}{2})$

$M_n = M_{n_1} + M_{n_2}$

6. $M_u = \phi M_n$

At equilibrium, $C = T$

$$A_s' f_s' + 0.85 f_c' ab = A_s f_y$$

$f_s' = E_s \epsilon_s'$

$= E_s \epsilon_u \left(\frac{c-d'}{c}\right)$

$a = c\beta_1$

$\Rightarrow c = \boxed{}$ $a = c\beta_1 = \boxed{}$

$f_s' = \boxed{}$

Doubly (WSD)

Analysis:

1. $A_s' = \boxed{}$

2. $A_s = \boxed{}$

3. $n = \frac{E_s}{E_c}$, $r = \frac{f_s}{f_c}$, $K = \frac{n}{n+r}$

$j = 1 - \frac{K}{3}$

4. $M_1 = \frac{1}{2} f_c\, j K b d^2$

5. $A_{s_1} = \frac{M_1}{f_s\, j d}$

6. $A_{s_2} = A_s - A_{s_1}$

7. $M_2 = A_{s_2} f_s (d-d')$

$M_2 = A_{s_2} f_s' (d-d')$

8. $M = M_1 + M_2$ [N.B: M_2এর মানের ছোট মানটা নিতে হবে]

$$f_s' = 2 f_s \times \frac{K - \frac{d'}{d}}{1-K}$$

T beam (WSD)

Analysis steps:

$16h_f + b_w = \square$

$\frac{L}{4} = \square$

center line spacing $= \square$

1. $b = \square$ (less) [effective flange width]

2. $\rho = \frac{A_s}{bd}$

3. **check:** $a = \frac{A_s f_y}{.85 f_c' b}$

   if $\boxed{a > h_f}$, T beam analysis is required.

4. $M_{n_1} = A_{sf} f_y \left(d - \frac{h_f}{2}\right)$

   Here, $A_{sf} = \frac{.85 f_c' (b - b_w) h_f}{f_y}$

   $M_{n_2} = (A_s - A_{sf}) f_y \left(d - \frac{a}{2}\right)$

   Here, $a = \frac{(A_s - A_{sf}) f_y}{.85 f_c' b_w}$

5. $M_n = M_{n_1} + M_{n_2}$

6. $M_u = \phi M_n$

   Here, $\phi = 0.483 + 83.3 \epsilon_t$

   and, $\epsilon_t = \epsilon_u \left(\frac{d-c}{c}\right)$

Design step:

1. Effective flange width, $b = \square$

   $16h_f + b_w = \square$

   $\frac{L}{4} = \square$

   center line spacing $= \square$

2. **Load calculation:**

   যদি Moment capacity না দেওয়া থাকে,

   $D.L = \left[\frac{c/c \times h_f}{144} + \frac{(h - h_f) \times b_w}{144}\right] \times 150$ plf

   $L.L = \square \times c/c$ plf

   $W_u = 1.2\, D.L + 1.6\, L.L$

3. Moment calculation:

   For simply supported beam, $M_u = \frac{W_u L^2}{8}$

4. $A_s = \frac{M_u}{\phi f_y \left(d - \frac{h_f}{2}\right)}$

5. **check:** $\rho = \frac{A_s}{bd}$

   $a = \frac{A_s f_y}{.85 f_c' b}$ if $\boxed{a > h_f}$. T beam design is required.

6. $\phi M_{n_1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$

   Here, $A_{sf} = \frac{.85 f_c' (b - b_w) h_f}{f_y}$

7. $\phi M_{n_2} = M_u - \phi M_{n_1}$

8. $A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y \left(d - \frac{a}{2}\right)}$

   Here, $a = \frac{(A_s - A_{sf}) f_y}{.85 f_c' b_w}$

9. $A_s = A_{sf} + (A_s - A_{sf})$

T beam (WSD)

Analysis steps:

1. $e = \dfrac{A_s}{bd}$

2. $n = \dfrac{E_s}{E_c}$

3. $en = \square$

4. $\dfrac{t}{d} = \square$ Here, $t = h_f$

5. $K = \dfrac{en + \frac{1}{2}\left(\frac{t}{d}\right)^2}{en + \left(\frac{t}{d}\right)}$

6. $j = \dfrac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3 \times \left(\frac{1}{2en}\right)}{6 - 3\left(\frac{t}{d}\right)}$

7. <u>check:</u> if $\boxed{Kd > h_f}$, T beam analysis is required.

8. $M_s = A_s f_s j d$

   $M_c = f_c \left(1 - \dfrac{t}{2Kd}\right) j t b d$

9. Allowable moment capacity হবে এই দুইটার minimum টা।

Design steps:

1. $A_s = \dfrac{M}{f_s \left(d - \frac{t}{2}\right)}$ Here, $t = h_f$

2. $e = \dfrac{A_s}{bd}$

3. $n = \dfrac{E_s}{E_c}$

4. $en = \square$

5. $\dfrac{t}{d} = \square$

6. $K = \dfrac{en + \frac{1}{2}\left(\frac{t}{d}\right)^2}{en + \left(\frac{t}{d}\right)}$

7. <u>check:</u> if $\boxed{Kd > h_f}$

   T beam design is required.

8. $j = \dfrac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3 \times \left(\frac{1}{2en}\right)}{6 - 3\left(\frac{t}{d}\right)}$

9. $A_s = \dfrac{M}{f_s j d}$

$$* \; z = \frac{3Kd - 2t}{2Kd - t} \times \left(\frac{t}{3}\right)$$

Stirrup Design (USD)

1. **Load calculation:**

$W_u = 1.2\,D.L + 1.6\,L.L$

2. V_u at support, $V_{max} = \frac{W_u L}{2}$

(for simply supported)

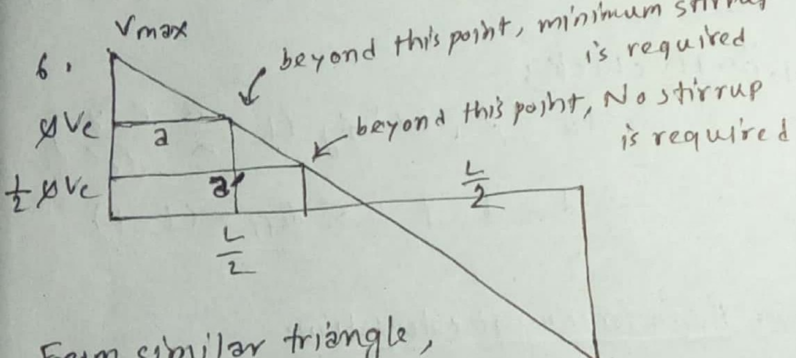
3. V_u at a distance 'd' from support,

$V_{ud} = \frac{W_u L}{2} - \frac{wd}{12}$ ('d' in inch)

4. $\phi V_c = \phi\, 2\sqrt{f_c'}\, b_w d$ $(\phi = 0.75)$

5. **Check:** if $\phi V_c < V_{ud}$ stirrup is required.

6.



From similar triangle,

$\frac{V_{max}}{\frac{L}{2}} = \frac{V_{max} - \phi V_c}{a} \Rightarrow a = \boxed{}$

$\frac{V_{max}}{\frac{L}{2}} = \frac{V_{max} - \frac{1}{2}\phi V_c}{a'} \Rightarrow a' = \boxed{}$

7. Actual spacing, $s = \frac{\phi A_v f_y d}{V_{ud} - \phi V_c}$ (f_y in ksi)

$S_{max} = \frac{A_v f_y}{.75\sqrt{f_c'}\, b_w}$ (f_y in psi)

$S_{max} = \frac{A_v f_y}{50\, b_w}$ (f_y in psi)

$S_{max} = \frac{d}{2}$

$S_{max} = 24$ in.

* minimum টা হবে design spacing.

(WSD)

1. **Load calculation:** $W = D.L + L.L$

2. V_u at support, $V_{max} = \frac{WL}{2}$

(for simply supported beam)

3. V_u at a distance 'd' from support

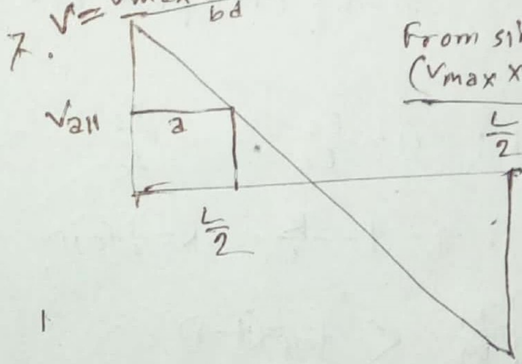
$V_{ud} = \frac{WL}{2} - \frac{wd}{12}$ (d in inch)

4. $V_{dev} = \frac{V_{ud}}{bd} \times 1000$ (psi)

5. $V_{all} = 1.1\sqrt{f_c'}$ (psi)

6. **Check:** $V_{all} < V_{dev}$, stirrup is required.

7. $V = \frac{V_{max} \times 1000}{bd}$



From similar triangle;

$\frac{(V_{max} \times 1000)/bd}{\frac{L}{2}} = \frac{V - V_{all}}{a}$

$\Rightarrow a = \boxed{}$

8. Actual spacing, $s = \frac{A_v f_v}{(V_{ud} - V_{all})\, b_w}$

$S_{max} = \frac{A_v f_v}{50\, b_w}$

$S_{max} = \frac{d}{2}$

$S_{max} = 24$ in;

* minimum টা হবে design spacing.

Stair Design (WSD)

1. Let, thickness of slab = 8"

2. Load calculation:

Dead load for landing; $W_1 = \frac{t}{12} \times 150$ psf

Dead load for inclined portion, $W = W_1 \times \frac{\sqrt{T^2+R^2}}{T}$ psf

Dead Load due to step on inclined portion $= \frac{R}{2} \times 150$ psf

Total load on landing = (D.L + L.L)

Total load on inclined portion = (D.L + L.L)

T.L (inclined)

3. Moment calculation: $M_{max} = \boxed{}$ from $\boxed{BMD}$

4. depth check: $n = \frac{E_s}{E_c}$, $r = \frac{f_s}{f_c}$

$k = \frac{n}{n+r}$, $j = 1 - \frac{k}{3}$, $R = \frac{1}{2} f_c j k$

$d_{req} = \sqrt{\frac{M}{Rb}} < d_{eff} = (t-1)$

5. Reinforce calculation;

$A_s = \frac{M}{f_s j \underbrace{d}_{\rightarrow d_{eff}}}$

provide # □ bar @ $= \frac{\square \times 12}{A_s} = \square$" c/c

for landing; T.L (landing)

$M = \frac{WL^2}{8}$

$A_s = \frac{M}{f_s j d}$

provide # □ bar @ $= \frac{\square \times 12}{A_s} = \square$" c/c

6. Distribution reinforcement: $A_s = 0.0018\, bt$ ($f_y = 60$)

$A_s = 0.002\, bt$ ($f_y < 60$)

$A_s = \frac{0.0018 \times 60000}{f_y} bt$ ($f_y > 60$)

(USD)

1. Let, thickness of slab = 8"

2. Load calculation;

D.L for landing, $W_1 = \frac{t}{12} \times 150$

D.L for inclined portion, $W = W_1 \times \frac{\sqrt{T^2+R^2}}{T}$

D.L due to step on " $= \frac{R}{2} \times 150$

Total load on landing = (1.2 D.L + 1.6 L.L)

" " on inclined portion = (1.2 D.L + 1.6 L.L)

T.L (landing) T.L (inclined) T.L (landing)

3. Moment calculation: $M_{max} = \boxed{}$ (from BMD)

4. depth check:

$M_u = \phi \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

$\Rightarrow d = \boxed{} < d_{eff} = (t-1)$

5. Reinforcement calculation;

$A_s = \frac{M_u}{\phi f_y \left(\underbrace{d}_{d_{eff}} - \frac{a}{2}\right)}$ Here, $a = \frac{A_s f_y}{.85 f_c' b}$

provide # □ bar @ $= \frac{\square \times 12}{A_s} = \square$" c/c

for landing: T.L (landing)

$M_u = \frac{WL^2}{8}$ $A_s = \frac{M_u}{\phi f_y \left(\underbrace{d}_{d_{eff}} - \frac{a}{2}\right)}$

provide # □ bar @ $= \frac{\square \times 12}{A_s} = \square$" c/c

6. Distribution reinforcement: (same as WSD)

ANALYSIS

&

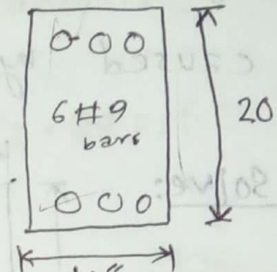
DESIGN

FARHAD
1500045

Transformed Section

Ex - 1.1

Problem: A RC beam which section 16×20 inch^2 is reinforced with 6 #9 bar. Determine the axial load that will stress the concrete to 1200. The concrete cylindrical strength $f_c' = 3500$ psi. How much load carried by steel.



Solve: Here,

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3500}} = 8.6 \simeq 8$$

$$A_g = (16\times20)\ in^2 = 320\ in^2$$

$$A_{st} = (6\times1)\ in^2 = 6\ in^2$$

∴ The load on the beam, $P = f_c\,[A_g + (n-1)A_{st}]$

$$= 1200\,[320 + (8-1)\,6]$$

$$= 434400\ lb$$

Of this total load,

The concrete is seen to carry, $P_c = f_c\, A_c = f_c\,(A_g - A_{st})$

$$= 1200\,(320-6)$$

$$= 376800\ lb$$

and the steel, $P_s = f_s\, A_{st} = n\, f_c\, A_{st}$

$$= (8\times1200\times6)$$

$$= 57600\ lb$$

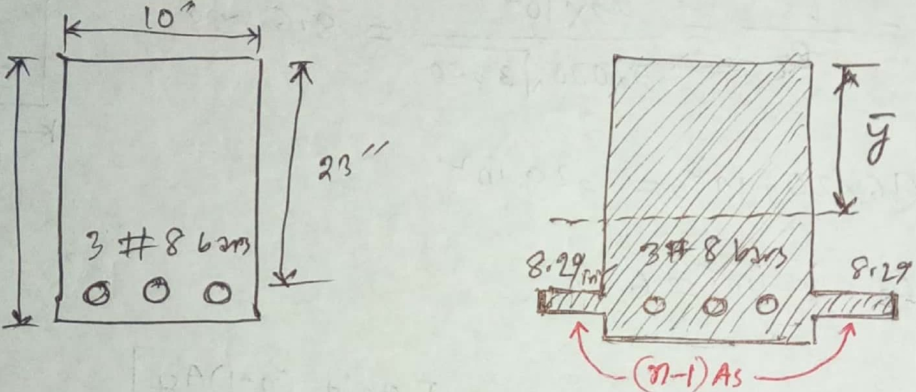
(Ans.)

Ex-3.1

Problem:

A rectangular beam has the dimensions $b = 10$ in. $h = 25$ in. and $d = 23$ in. and is reinforced with three No. 8 bars. the concrete cylinder strength f_c' is 4000 psi and the tensile strength in bending (modulus of rupture) is 475 psi. The yield point of the steel f_y is 60000 psi. Determine the stresses caused by a bending moment $M = 45$ ft-kips.

Solve:



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$$

$$A_s = (3 \times .79) = 2.37$$

retangular outline area, $(n-1)\,A_s = 16.59$ in²

The location of the neutral axis from top,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(25 \times 10) \times 12.5 + 16.59 \times 23}{(25 \times 10) + 16.59}$$

$$\therefore \bar{y} = 13.15$$

The moment of inertia about the axis,

$$I = \left(\frac{bh^3}{12} + A_1 d^2\right) + \left(\underbrace{\frac{bh^3}{12}}_{\text{negligible for steel}} + A_2 d^2\right)$$

$$= \frac{10\times 25^3}{12} + (10\times 25)\times(13.15-12.5)^2 + 16.59\times(23-13.15)^2$$

$$= 14736 \text{ in}^4$$

For $M = 45$ K-ft

The concrete compression stress at the top fibre is,

$$f_c = \frac{Mc}{I} = \frac{(45\times 12000)\times 13.15}{14736} = 481.88 \text{ psi}$$

and similarly, the concrete tension stress at the bottom fibre is, $f_{ct} = \frac{(45\times12000)\times(25-13.15)}{14736} = 434.24$ psi

Since $f_{ct} < f_r = 475$ psi. Hence no tension cracks will form.

The stress in the steel is, $f_s = n f_{ct}$

for effective depth, $f_{ct} = \frac{(45\times12000)\times(23-13.15)}{14736} = 360.953$ psi

$$\therefore f_s = (8\times 360.953) = 2887.62 \text{ psi}$$

(Ans.)

Ex-3.2 Previous problem

If the beam is subject to a bending moment $M = 90$ K ft calculate the relevant properties and stresses.

Solve: Then,

For $M = 90$ K-ft

$$f_c = \frac{(90 \times 12000) \times 13.15}{14736} = 963.76 \text{ psi}$$

$$f_{ct} = \frac{(90 \times 12000) \times (25 - 13.15)}{14736} = 868.5 \text{ psi}$$

since, $f_{ct} > f_r = 475$ psi, cracks will have formed.

Hence, The analysis must be adapted appropriately

Locating neutral axis:

Here, $\rho = \frac{A_s}{bd} = \frac{(3 \times .79)}{10 \times 23} = 0.0103$

$\rho n = (0.0103 \times 8) = 0.0824$

$$\therefore K = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$= \sqrt{(0.0824)^2 + 2 \times 0.0824} - 0.0824$$

$\therefore K = 0.332$ $\quad \therefore J = 1 - \frac{K}{3} = \left(1 - \frac{.332}{3}\right) = 0.889$

$\therefore Kd = (.332 \times 23) = 7.63$ inch

$\therefore$ The steel stress is, $f_s = \frac{M}{A_s\, j d} = \frac{(90 \times 12000)}{2.37 \times .889 \times 23}$

$= 22286.70$ psi

And,
The concrete stress is, $f_c = \frac{2M}{kj\, bd^2} = \frac{2 \times (90 \times 12000)}{.332 \times .889 \times 10 \times 23^2}$

$= 1383.43$ psi

Ex-3.3

~~In~~ Previous problem:
Determine the nominal moment M_n at which the beam will fail. (Ans.)

Solve: Here,
$\rho = \frac{A_s}{bd} = \frac{2.37}{10 \times 23} = 0.0103$

$\rho_b = 0.85\, \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$

$= .85 \times .85 \times \frac{4000}{60000} \times \frac{.003}{.003 + .002} = 0.0289$

$\therefore \rho < \rho_b$. Hence the beam will cause failure by yielding of steel. , ~~...~~

$\rho_{max} = 0.85 \times .85 \times \frac{4000}{60000} \times \frac{.003}{.003 + .004} = 0.0206$

$\therefore \rho < \rho_{max}$. Hence it is under reinforced beam.

$\therefore$ Nominal moment, $M_n = A_s f_y \left(d - \frac{a}{2}\right)$

Here, $a = \frac{A_s f_y}{.85 f_c' b}$

$= \frac{2.37 \times 60000}{.85 \times 4000 \times 12}$

$= 3.4853$

$\therefore M_n = 2.37 \times 60000 \times \left(23 - \frac{3.4853}{2}\right)$

$= 3022795.588$ lb in.

$= 251.9$ K-ft

(Ans.)

Or, $M_n = \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \times \frac{\rho f_y}{f_c'}\right)$

$= .0103 \times 10 \times 23^2 \times 60000 \times \left(1 - .59 \times \frac{.0103 \times 60000}{4000}\right)$

$= 2971214.251$ lb in.

$= 247.6$ K-ft

When, the beam reaches M_n, the distance to it's neutral axis is,

$c = \frac{\rho f_y d}{\alpha f_c'} = \frac{.0103 \times 60000 \times 23}{.72 \times 4000} = 4.94$ in

Cracked and Uncracked section

2017

Problem: A rectangular beam has following properties: b = 12 inch, h = 20 inch, d = 17.5 inch. The beam is reinforced with 3 # 8 bar, If $f_c' = 4000$ psi, $f_y = 60{,}000$ psi, modulus of rupture = 475 psi. calculate

(i) cracking moment of the section and moment on whether the beam has cracked or not for a 30 K-ft imposed moment.

(ii) the stresses in steel and concrete when the section is subjected to a moment of 40-K-ft.

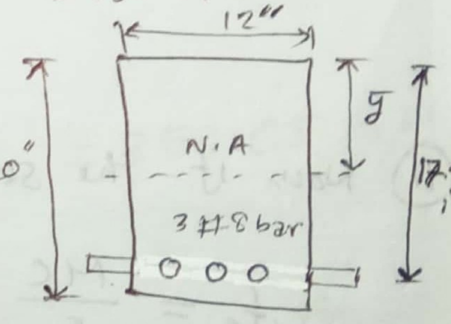
(iii) the nominal moment capacity.

Solution: Here $f_c' = 4000$ psi $\therefore f_c = .45 f_c' = (.45 \times 4000)$

$= 1800$ psi

We Know,

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$$



Now,

$A_s = (3 \times .79) = 2.37 \text{ in}^2$

$(n-1)A_s = (8-1) \times 2.37 = 16.59 \text{ in}^2$

$A_1 = bh = (12 \times 20) = 240 \text{ in}^2$; $A_2 = 16.59 \text{ in}^2$

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(240 \times \frac{20}{2}) + (16.59 \times 17.5)}{240 + 16.59} = 10.48 \text{ inch}$$

$\therefore$ Location of the neutral axis from top, $\bar{y} = 10.48$ in

Moment of Inertial about the neutral axis,

$$I = \left(\frac{bh^3}{12} + A_1 d^2\right) + \left(\underbrace{\frac{bh^3}{12}}_{\text{(negligible for steel)}} + A_2 d^2\right) = \frac{12 \times 20^3}{12} + 240 \times (10.48 - 10)^2 + 16.59 \times (17.5 - 10.48)^2$$

$\therefore I = 8872.86 \text{ in}^4$

modulus of rupture

① $\therefore$ cracking moment of the section, $M = \frac{f_r I}{c}$

$$= \frac{475 \times 8872.86}{(20 - 10.48)}$$

$$= 442711 \text{ lb-in}$$

$$= 36.89 \text{ K ft}$$

Now, if the section is subtected to a moment of 30 K-ft section will be uncracked. Since cracking moment of the section is 36.89 K-ft.

[Or, $f_{ct} = \frac{MC}{I} = \frac{(30 \times 12000) \times (20 - 10.48)}{8872.86}$

$\therefore f_{ct} = 386.26 < 475 \text{ psi} (= f_r)$]

Ans.)

⑪ Now, if the section is subjected to a moment of 40 K-ft

$$f_c = \frac{MC}{I} = \frac{(40 \times 12000) \times 10.48}{8872.86} = 566.94 \text{ psi}$$

$$f_{ct} = \frac{MC}{I} = \frac{(40 \times 12000) \times (20.0 - 10.48)}{8872.86} = 515.00 \text{ psi}$$

which is greater than modulus of rupture. Hence it is a cracked section. Analysis should be based on cracked section.

~~But for effective depth,~~

$\cancel{f_c} = \cancel{\frac{MC}{I}} = \cancel{\frac{(40 \times 12000) \times (17.5}{}}$

Now, $\rho = \frac{A_s}{bd} = \frac{2.37}{12 \times 17.5} = 0.0113$

$\rho n = 8 \times 0.0113 = 0.0903$

$\therefore K = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$

$= \sqrt{(.0903)^2 + 2 \times .0903} - 0.0903$

$\therefore K = 0.344$

$Kd = 0.344 \times 17.5 = 6.02$

$J = 1 - \frac{K}{3} = 0.885$

$\therefore f_s = \frac{M}{A_s jd} = \frac{(40 \times 12000)}{2.37 \times .885 \times 17.5} = 13077.104$ psi

$\therefore f_c = \frac{2M}{Kjbd^2} = \frac{2 \times 40 \times 12000}{0.344 \times .885 \times 12 \times 17.5^2} = 858.05$ psi

Ans.

(iii) Nominal moment capacity:

$\rho = 0.0113$

$\rho_b = 0.85 \, \beta_1 \, \frac{f_c'}{f_y} \, \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$

$= 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + .002} = 0.0289$

$\therefore \rho < \rho_b$ Hence under reinforced beam.

Now,

$M_n = A_s f_y \left(d - \frac{a}{2}\right)$ Here, $a = \frac{A_s f_y}{.85 f_c' b}$

$= 2.37 \times 60000 \times \left(17.5 - \frac{3.485}{2}\right)$ $= \frac{2.37 \times 60000}{.85 \times 4000 \times 12}$

$= 2240695.588$ lb in $= 3.485$

$= 186.72$ K-ft

(Ans:)

Design of Singly Reinforce Beam (WSD)

FARHAD
1500045

Design steps:

1. Write the given data:
   * span length
   * loads (live load)
   * Grade of concrete and steel [f_c' & f_y] f_c' = compressive strength
     f_y = yeild strength

2. Write the design constant:
   * modular ratio, $n = \frac{E_s}{E_c}$ where, $E_s = 29 \times 10^6$ psi
     $E_c = 57000\sqrt{f_c'}$
   * permissible stresses,
     - $f_s = 0.4 f_y$
     - $f_c = 0.45 f_c'$
   * Stress ratio, $r = \frac{f_s}{f_c}$
   * Critical axis co-efficient, $K = \frac{n}{n+r}$
   * Lever arm co-efficient, $j = 1 - \frac{K}{3}$
   * Moment of Resistance co-efficient, $R = \frac{1}{2} f_c jK$

3. Find out the cross section dimensions:
   * Assume the depth, h = span length or less
   * Assume the width, $b = \frac{h}{2}$ [$\nless$ 10 inch] or less
   * Dead load = self weight of beam $= \frac{\gamma . b . h}{144}$ plf
     (γ = unitweight of concrete) where, $\gamma = 150\ lb/ft^3$

* Total load, W = (Dead load + Live load)

* Use W as distributed load on the beam and calculate the maximum bending Moment with the help of SFD and BMD.

(For simply supported beam, Max. B.M = $\frac{WL^2}{8}$)

* From formulae, $D = \sqrt{\frac{M}{Rb}}$, calculate D (effective depth)

* Hence, $D_{required}$ = (D + clear cover) where (c.c = 2.5 inch)

if $D_{required} < h$, Design is OK.

if $D_{required} > h$, Resign the depth.

4. <u>Find out the area of steel:</u>

$$A_{st} = \frac{M}{f_s\, Jd}$$

* No. of bars = $\frac{\text{Area of steel } (A_{st})}{\text{Area of any one bar}}$

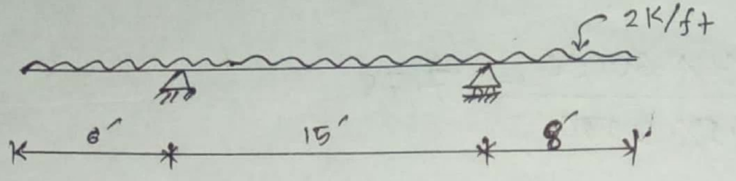
For Analysis:

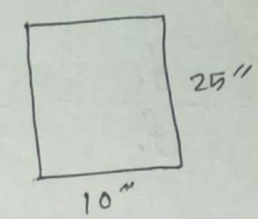
$$K = \sqrt{[(\rho n)^2 + 2\rho n]} - \rho n$$ where, reinforcement ratio, $\rho = \frac{A_{st}}{bd}$

Design

Problem: Design the beam when $f_c' = 3000$ psi ; $f_y = 60000$ psi

(2015) → same type





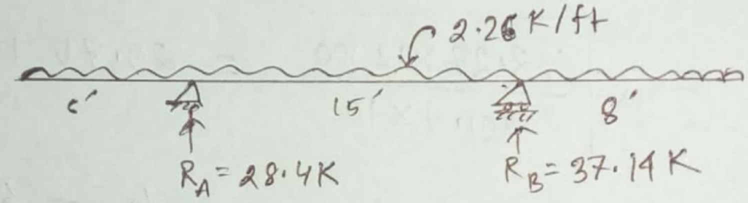
Solution: Assume the depth = 25 inch

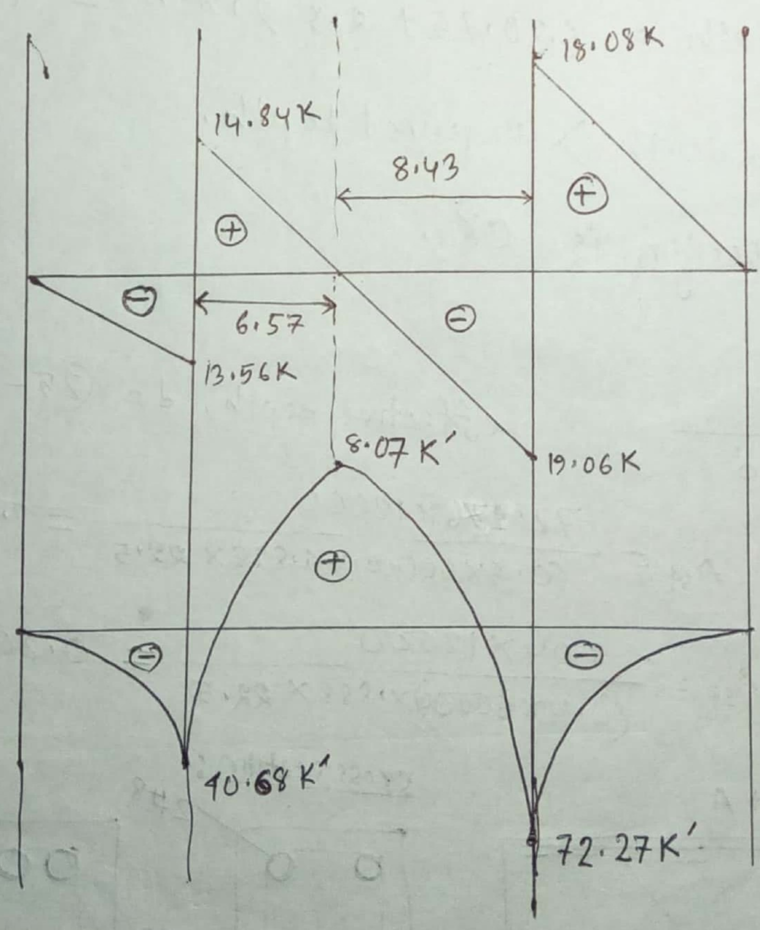
Assume the width = 10 inch

$$\text{self weight} = \frac{bd}{144} \times 150 = D.L$$

$$\text{Dead load} = \left(\frac{25 \times 10}{144} \times 150\right) \text{ plf} = 260.42 \text{ plf}$$

$$\therefore \text{Total load} = (260.42 + 2000) = 2260.42 \text{ plf} = 2.26 \text{ Klf}$$





$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 \times 60000}{0.45 \times 3000} = 17.78$$

$$\therefore K = \frac{n}{n+r} = \frac{9}{9+17.78} = 0.336$$

$$\therefore J = 1 - \frac{K}{3} = \left(1 - \frac{0.336}{3}\right) = 0.888$$

Now, $R = \frac{1}{2} f_c jK = \frac{1}{2} \times (.45 \times 3000) \times 0.888 \times 0.336 = 201.4$

$$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{72.27 \times 12000}{201.4 \times 10}} = 20.75 \text{ inch}$$

$\therefore$ Required depth $= (20.75 + 2.5)$ inch $= 23.25$ inch

Hence, Assumed depth > Required depth.

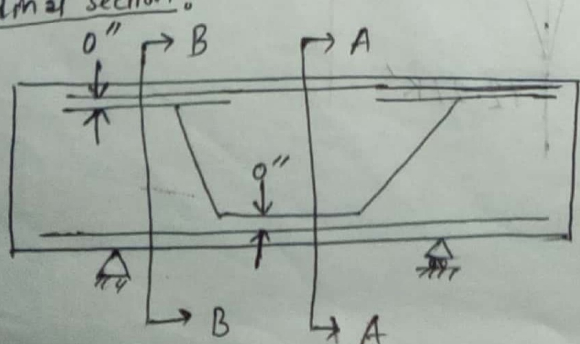
$\therefore$ Design is OK.

Now, $A_{st} = \frac{M}{f_s jd}$ $\quad$ effective depth, $d = (25 - 2.5) = 22.5$ inch

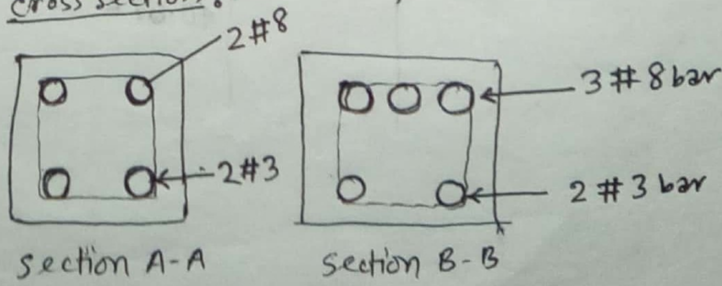
For compression, $A_{st} = \frac{72.27 \times 12000}{(0.4 \times 60000) \times 0.888 \times 22.5} = 7.81 \text{ inch}^2 = 3\#8$ bar

For, tension, $A_{st} = \frac{8.04 \times 12000}{(0.4 \times 60000) \times .888 \times 22.5} = 0.20 \text{ inch}^2 = 2\#3$ bar

longitudinal section:



cross section:



Design

Problem: Determine the dimension and reinforcement of a rectangular beam of moment capacity 100k-ft. ($f_s = 20000$ psi, $f_c' = 3000$ psi)

Solution:

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$$

$$r = \frac{f_s}{f_c} = \frac{20000}{.45\times 3000} = 14.815$$

$$K = \frac{n}{n+r} = \frac{9}{9+14.815} = 0.378$$

$$J = 1 - \frac{K}{3} = 0.874$$

$$R = \frac{1}{2} f_c jK = \frac{1}{2}\times(.45\times3000)\times .874\times 0.378 = 222.94$$

$$\therefore d = \sqrt{\frac{100\times12000}{10\times 222.94}} = 23.2 \text{ inch}$$

Now, $$A_{st} = \frac{M}{f_s j d} = \frac{100\times12000}{20000\times0.874\times23.2} = 2.96 \text{ inch}^2$$

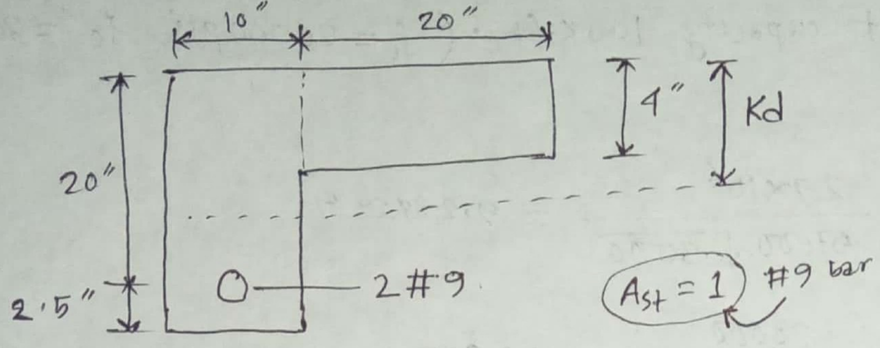
$$= 3\#9 \text{ bar}$$

(Ans)

Analysis

Problem: Determine the moment capacity of the beam. ($f_c' = 3000$ psi)

$f_y = 60000$ psi



$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.289 \approx 9$$

Solution:

$$Q_c = 10 \times Kd \times \frac{Kd}{2} + 20\times 4 \times \left(Kd - \frac{4}{2}\right) = 5(Kd)^2 + 80Kd - 160$$

$$Q_T = n\, A_{st}\,(d - Kd) = \frac{E_s}{E_c} \times (2\times 1) \times (d - Kd) = 9\times 2 \times (d - Kd) = 18d - 18Kd$$

Now, $Q_c = Q_T$

$$5(Kd)^2 + 80Kd - 160 = 18d - 18Kd$$

$$\Rightarrow 5(Kd)^2 + 98Kd - 520 = 0$$

$$\Rightarrow Kd = 4.34$$

$$\therefore K = \frac{4.34}{20} = 0.217$$

$$J = 1 - \frac{K}{3} \; 2\left(1 - \frac{0.217}{3}\right) = 0.928$$

we know,

$$M_c = \frac{1}{2} f_c J K b d^2 = \frac{1}{2} \times .45 \times 3000 \times .928 \times .217 \times 10 \times 20^2$$

$$= 543715.2 \text{ lb inch}$$

$$M_c = 543.72 \text{ K-inch} \quad \text{(less)}$$

Again,

$$M_s = A_s f_s j d = 2\times(.4\times 60000)\times 0.928 \times 20 = 890880 \text{ lb in} = 890.88 \text{ kin.}$$

Hence, moment capacity = 543.72 kin. (Ans.)

Design

Problem: A rectangular beam is to carry a uniformly distributed live load of 680 plf and support the dead load of a wall weighing 380 plf in addition to its own weight on a simple span of 24 ft. Design the beam for flexure using intermediate-grade steel at a working stress of 20,000 psi and 3000 psi concrete at a working stress of 1350 psi.

Solution: Given that,

$L.L = 680$ plf

$D.L = 380$ plf

$f_s = 20000$ psi

$f_c = 1350$ psi

$f'_c = 3000$ psi

$L = 24$ ft

Assume, depth, $h = 24$ inch

width, $b = 12$ inch

[N.B: * including থাকলে লাগবে না
* in addition/ super impose থাকলে লাগবে]

$$\text{Total load} = \left(680 + 380 + \boxed{\frac{24 \times 12}{144} \times 150}\right) = 1360 \text{ plf} = 1.36 \text{ klf}$$

For simply supported beam, $M_{max} = \frac{wL^2}{8} = \frac{1360 \times (24)^2}{8} = 97920$ lb-ft

Now,

$$r = \frac{f_s}{f_c} = \frac{20000}{1350} = 14.81$$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$$

$$\therefore K = \frac{n}{n+r} = \frac{9}{9 + 14.81} = 0.378$$

$$J = 1 - \frac{K}{3} = 1 - \frac{.378}{3} = 0.874$$

$$R = \frac{1}{2} f_c j K = \left(\frac{1}{2} \times 1350 \times .874 \times .378\right) = 223$$

$$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{(97920 \times 12)}{223 \times 12}} = 20.95 \text{ inch}$$

$\therefore$ required depth $= (20.95 + 2.5) = 23.45$ inch

Assume depth > required depth

$\therefore$ design is OK.

Now,

$A_s = \dfrac{M}{f_s j d}$ effective depth, $d = (24 - 2.5) = 21.5$

$$A_s = \frac{97920 \times 12}{20000 \times .874 \times 21.5} = 3.13 \text{ in}^2 = 3 \# 10$$

Analysis

Problem: A simply supported rectangular beam has a total cross-section of 10×16 inch² and a length of 20 ft. It is reinforced with 4 #5 bars in one row. The distance from the centre of the bars to the lower surface of the beam is 2.5 inch with 2500 psi concrete and an allowable stress of 20000 psi in the steel. What is the resisting Moment of the beam. WSD

Solution: Given that, $b = 10''$ $f'_c = 2500$ psi

$h = 16''$ $f_s = 20000$ psi

$d = (16 - 2.5) = 13.5''$

$L = 20$ ft

$$\text{Steel ratio, } \rho = \frac{A_s}{bd} = \frac{4 \times .31}{10 \times 13.5} = 9.185 \times 10^{-3}$$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{2500}} = 10.175 \simeq 10$$

$\therefore \rho n = 0.092$

$$\therefore K = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = \sqrt{(.092)^2 + 2 \times .092} - 0.092 = 0.347$$

$\therefore j = 1 - \frac{k}{3} = \left(1 - \frac{.347}{3}\right) = 0.884$

$\therefore R = \frac{1}{2} f_c j K = \frac{1}{2} \times (.45 \times 2500) \times .884 \times 0.347 = 172.546$

Now,

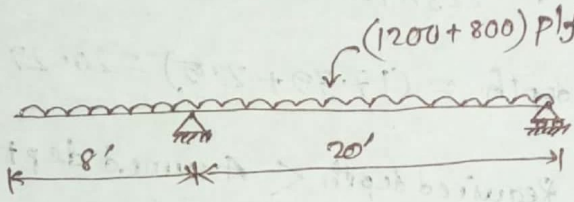
$M_c = R b d^2 = 172.546 \times 10 \times (13.5)^2 = 314464.63$ lb-in $= 314.465$ K-in

$M_s = A_s f_s j d = (4 \times .31) \times 20000 \times .884 \times 13.5 = 295.96$ K-in.

(Ans.)

2017, 2010

Problem: Design a over hanging beam in figure to support a live load of 1200 plf and a dead load of 800 plf including a its self weight using $f_c' = 3000$ psi, $f_y = 5000$ psi.

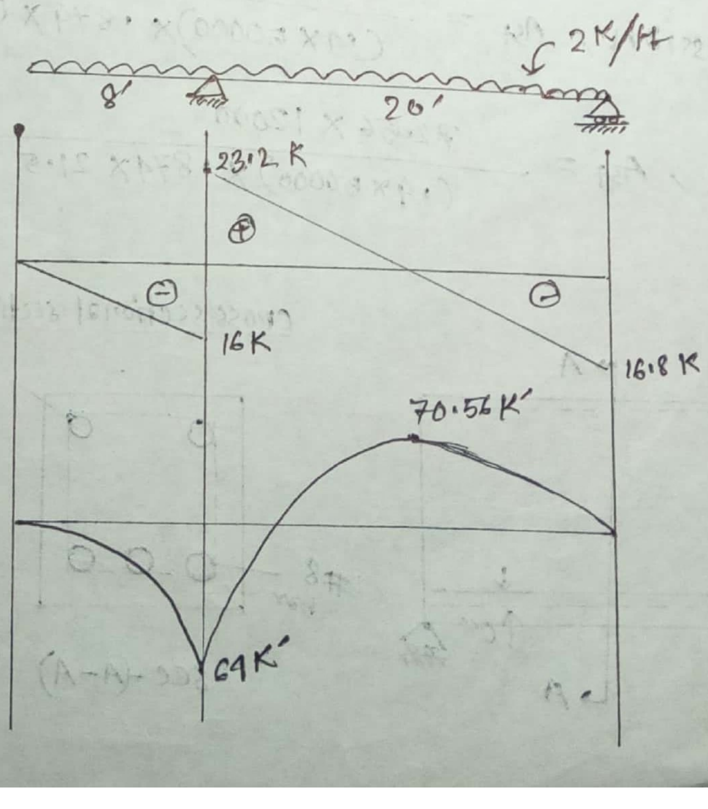


Solution:

Assuming, depth, $h = 24$ in.

width, $b = 12$ in.

Total load = (1200+800) plf = 2000 plf = 2 Klf



Now,

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4\times 50000}{0.45\times 3000} = 14.815$$

$$K = \frac{n}{n+r} = \frac{9}{9+14.815} = 0.378$$

$$j = 1-\frac{K}{3} = 1-\frac{0.378}{3} = 0.874$$

$$R = \frac{1}{2} f_c\, jK = \frac{1}{2}\times(0.45\times3000)\times 0.874\times 0.378 = 223$$

$$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{70.56\times12000}{223\times12}} = 17.79 \text{ in.}$$

Hence, required depth $= (17.79+2.5) = 20.29$ in.

∴ Required depth < Assumed depth.

∴ Design is OK

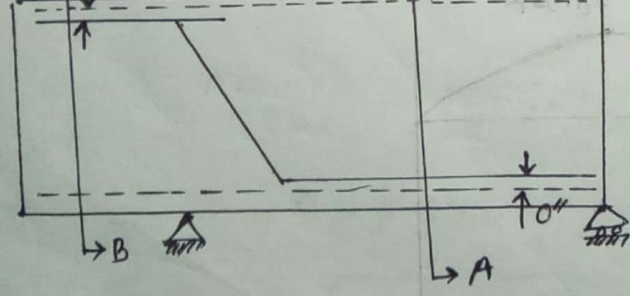
Now,

$$A_{st} = \frac{M}{f_s\, jd}$$

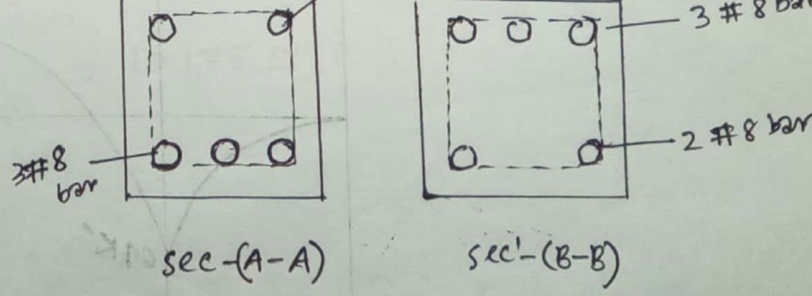
For compression, $A_{st} = \frac{64\times12000}{(.4\times50000)\times .874\times(24-2.5)} = 2.04 \text{ in}^2$ (3 # 8 bar)

For tension, $A_{st} = \frac{72.56\times12000}{(.4\times50000)\times .874\times 21.5} = 2.31 \text{ in}^2$ (3 # 8 bar)

longitudinal section:

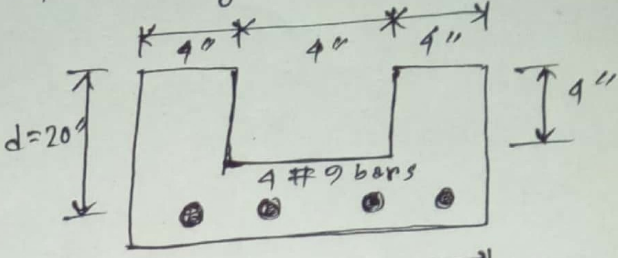


cross sectional section:



Analysis

problem: calculate the working moment capacity of the section given below: Assume, $f_c' = 3000$ psi & $f_y = 60$ ksi



Solve:

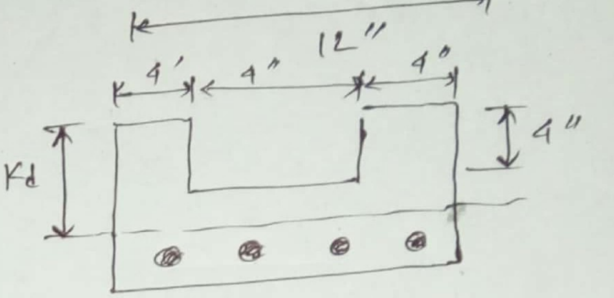
At equilibrium,

$$Q_c = Q_T$$

$$Q_c = 12 \times Kd \times \frac{Kd}{2} - (4\times4) \times \left(Kd - \frac{4}{2}\right)$$

$$= 6(Kd)^2 - 16(Kd-2)$$

$$= 6(Kd)^2 - 16Kd + 32$$



And,

$$Q_T = n A_{st} (d - Kd) \qquad \text{Here } n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$$

$$= 9 \times (4\times1) \times (20 - Kd)$$

$$= 36(20 - Kd)$$

$$= 720 - 36Kd$$

$$\therefore Q_c = Q_T$$

$$\Rightarrow 6(Kd)^2 - 16Kd + 32 = 720 - 36Kd$$

$$\Rightarrow 6(Kd)^2 + 20Kd - 688 = 0$$

$$\Rightarrow Kd = 9.17$$

$$\therefore K = 0.4585 \text{ and, } j = \left(1 - \frac{K}{3}\right) = \left(1 - \frac{0.4585}{3}\right) = 0.847$$

Now, $M_c = \frac{1}{2} f_c j K b d^2 = \frac{1}{2} \times (.45 \times 3000) \times 0.4585 \times 0.847 \times 12 \times (20)^2$

$\therefore M_c = 1258252.3816$ in $= 1258.25$ K in. (less)

$M_s = A_s f_s j d = 4 \times (60000 \times .4) \times .847 \times 20 = 1626240$ lb in

$\therefore M_s = 1626.24$ K in.

Working Moment capacity $= 1258.25$ K in.

(Ans.)

Singly reinforced Beam (USD)

Farhad
#1500045

Design (Ex-3.6)

Problem: Find the cross section of the concrete and area of steel required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.27 Kips/ft and a service live load of 2.15 Kips/ft. Material strength are $f_c' = 4000$ psi and $f_y = 60000$ psi.

Solution: $W_u = 1.2\,D.L + 1.6\,L.L = (1.2 \times 1.27 + 1.6 \times 2.15) = 4.96$ K/ft

$$M_u = \frac{W_u L^2}{8} = \frac{4.96 \times (15)^2}{8} = 139.5 \text{ Kft}$$

Now, $M_u = \phi \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

Assume

$\Rightarrow 139.5 \times 12000 = 0.9 \times .018 \times 10 \times d^2 \times 60000 \left(1 - 0.59 \times \frac{.018 \times 60000}{4000}\right)$

$\Rightarrow d^2 = 204.85$

$\therefore d = 14.31$ inch

Here, $\beta_1 = 0.85$ ($\because f_c' = 4000$)

$\beta_1 = 0.85 - 0.05 \times \frac{f_c' - 4000}{1000}$

$$\rho = 0.85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

$$= 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{.003}{.003 + .005}$$

$$= 0.018$$

Hence, required total depth, $h = (14.31 + 2.5) = 16.81$ inch $\simeq 17$ inch.

Now, $A_s = \dfrac{M_u}{\phi f_y (d - \frac{a}{2})}$

Here, $a = \dfrac{A_s f_y}{.85 f_c' b}$

$\Rightarrow a = \dfrac{60000 A_s}{.85 \times 4000 \times 10}$

$\therefore a = 1.7647 A_s$

$$\Rightarrow A_s = \frac{139.5 \times 12000}{0.9 \times 60000 \times \left(14.31 - \frac{1.7647 A_s}{2}\right)}$$

$\Rightarrow A_s = 2.58$ inch (3 #9 bars)

(Ans.)

Analysis (Ex-3.5)

Problem: A rectangular beam has width 12 in. and effective depth 17.5 inch. It is reinforce with 4 # 9 bar in one one row. If $f_y = 60000$ psi and $f_c' = 4000$ psi, what is the normal flexural strength and what is the maximum moment that can be utilized in design. according to ACI code.

Solution: $A_s = (4\times1) = 4 \text{ in}^2$

$$\rho = \frac{A_s}{bd} = \frac{4}{12\times17.5} = 0.019$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u+\epsilon_y}$$ Here, $\beta_1 = 0.85$ ($\because f_c' = 4000$)

$$\Rightarrow \rho_b = 0.85\times0.85\times\frac{4000}{60000}\times\frac{.003}{.003+.002}$$

$$\therefore \rho_b = 0.0289$$

Hence, $\rho < \rho_b$. It is under reinforced beam.

Now,

$$M_n = A_s f_y \left(d-\frac{a}{2}\right)$$ Here, $a = \frac{A_s f_y}{.85 f_c' b}$

$$\Rightarrow M_n = 4\times60000\times\left(17.5-\frac{5.88}{2}\right)$$ $$\Rightarrow a = \frac{4\times60000}{.85\times4000\times12} = 5.88$$

$\therefore M_n = 3494400$ lb in.

So, $M_u = \phi M_n$

$\Rightarrow M_u = 0.87\times3494400$

$\therefore M_u = 3040128$ lb-in.

(Ans)

Here, $\phi = 0.483 + 83.3\,\epsilon_t$

Again, $\epsilon_t = \epsilon_u\left(\frac{d-c}{c}\right)$

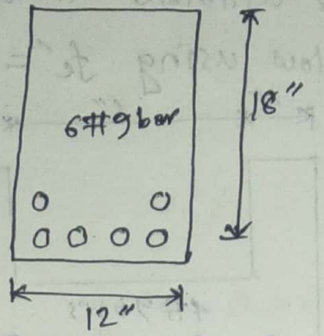
$$= .003\times\frac{17.5-\frac{5.88}{.85}}{\frac{5.88}{.85}}$$ $[\because c = \frac{a}{\beta_1}]$

$\therefore \epsilon_t = 0.0046$

$\therefore \phi = 0.483 + 83.3\times0.0046 = 0.87$

Analysis

Problem: A beam section is shown in figure below. Calculate nominal and ultimate moment.



$f_c' = 3$ ksi

$f_y = 60$ ksi

Solution:

$A_s = (6 \times 1) = 6$ in^2

$\rho = \frac{A_s}{bd} = \frac{6}{12 \times 18} = 0.0278$ $\beta_1 = 0.85$ [$\because f_c' = 3000$ psi]

$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.002} = 0.0217$

$\therefore \rho > \rho_b$ Hence, it is over reinforced beam.

Now, $C = T$

$\Rightarrow 0.85 f_c' \beta_1 b c = A_s f_s$

$\Rightarrow 0.85 \times 3000 \times .85 \times 12 \times c = 6 \times E_u \cdot \epsilon_u \times \frac{d-c}{c}$

$\Rightarrow 24480\, c = 6 \times 29 \times 10^6 \times 0.003 \times \frac{18-c}{c}$

$\Rightarrow c^2 = 21.3253\,(18-c)$

$\Rightarrow c = 11.46$

$\Rightarrow \frac{a}{\beta_1} = 11.46 \Rightarrow a = (11.46 \times .85) = 9.74$

$\therefore f_s = E_s \epsilon_u \frac{d-c}{c} = 29 \times 10^6 \times .003 \times \frac{18 - 11.643}{11.643} = 49649.21$ psi

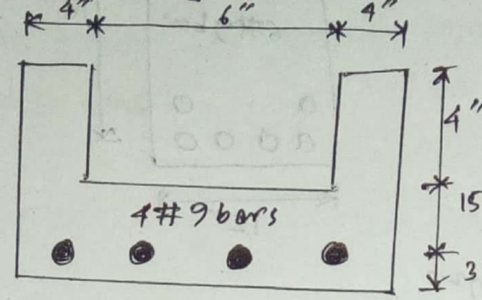
Hence, $M_n = A_s f_s \left(d - \frac{a}{2}\right) = 6 \times 49649.21 \times \left(18 - \frac{9.74}{2}\right)$

$\therefore M_n = 3911364.76$ lb in $= 391.14$ Kft

(Ans.)

<u>Analysis problem</u>

Problem: Determine the ultimate moment capacity of the beam section in the figure below using $f_c' = 4000$ psi, $f_y = 60000$ psi



Solve: Here,

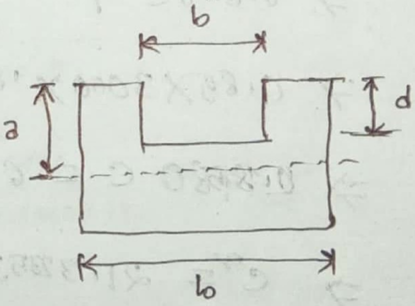
$A_s = (4 \times 1) = 4 \text{ in}^2$

$$\rho = \frac{A_s}{bd - (6 \times 4)} = \frac{4}{(14 \times 19) - 24} = 0.01653$$

$$\rho_b = 0.85 \times \beta_1 \times \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

$$\Rightarrow \rho_b = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.002} = 0.0289$$

$\therefore \rho < \rho_b$. Hence it is under reinforced section.



At equilibrium, $C = T$

$$0.85 f_c' (ab - 6 \times 4) = A_s f_y$$

$$\Rightarrow 0.85 \times 4000 \times (a \times 14 - 24) = 4 \times 60000$$

$$\Rightarrow a = 6.76''$$

$\therefore a > (d' = 4'')$

$$\text{Hence, } \quad \bar{y} = \frac{b\frac{a^2}{2} - b'\frac{d'^2}{2}}{ab - b'd'} = \frac{14 \times \frac{6.76^2}{2} - \frac{6 \times 4^2}{2}}{14 \times 6.76 - 6 \times 4} = 3.85''$$

$$\therefore M_n = A_s f_y (d - \bar{y})$$

$$= 4 \times 60000 \times (19 - 3.85)$$

$$\therefore M_n = 3636000 \text{ lb.in.}$$

Now. $M_u = \phi M_n$

$\Rightarrow M_u = 0.833 \times 3636000$

$\Rightarrow M_u = 3028788$ lb in.

$\therefore M_u = 3028.788$ K in.

(Ans.)

Here, $\phi = 0.483 + 83.3\,\epsilon_t$ where, $c = \frac{a}{\beta_1}$

$\epsilon_t = \epsilon_u \times \frac{d-c}{c}$

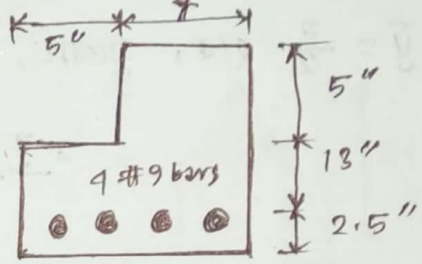
$= 0.003 \times \frac{19 - \frac{6.76}{.85}}{\frac{6.76}{.85}}$

$\therefore \epsilon_t = 0.0042$

Hence, $\phi = 0.483 + 83.3 \times 0.0042$

$\therefore \phi = 0.833$

Problem: Neglecting lack of symmetry. Determine the ultimate moment capacity of the section shown in figure below: using $f_c' = 4000$ psi and $f_y = 60000$ psi



Solve: $A_s = (4 \times 1) = 4\ in^2$

$\rho = \frac{A_s}{bd - (5\times5)} = \frac{4}{(12\times18)-25} = 0.02094$

$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.002}$

$\Rightarrow \rho_b = 0.0289 \therefore \rho < \rho_b$ Hence, it is under reinforced beam.

At equilibrium,

$C = T$

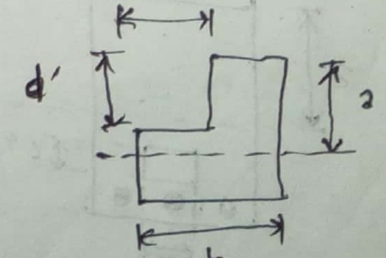
$0.85 f_c' (ab - 5\times5) = A_s f_y$

$\Rightarrow 0.85 \times 4000 \times (12\times a - 25) = 4 \times 60000$

$\Rightarrow a = 7.96''$ $\therefore a > 5''$

$\therefore \bar{y} = \frac{b\frac{a^2}{2} - b'\frac{d'^2}{2}}{ab - d'b'} = \frac{12\times\frac{7.96^2}{2} - 5\times\frac{5^2}{2}}{12\times7.96 - 5\times5}$

$\Rightarrow \bar{y} = 4.5''$



$\therefore M_n = A_s f_y (d - \bar{y}) = 4 \times 60000 \times (18 - 4.5) = 3240000$ lb in

Now, $M_u = \phi M_n$

$\Rightarrow M_u = 0.713 \times 3240000$

$\Rightarrow M_u = 2310120$ lb in

$\therefore M_u = 2310.12$ K in.

(Ans.)

Here, $\phi = 0.483 + 83.3\,\epsilon_t$

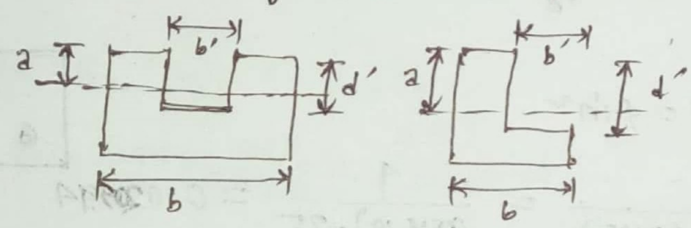
$\epsilon_t = \epsilon_u \times \dfrac{d-c}{c}$ where $c = \dfrac{a}{\beta_1}$

$\Rightarrow \epsilon_t = 0.003 \times \dfrac{18 - \frac{7.96}{.85}}{\frac{7.96}{.85}}$

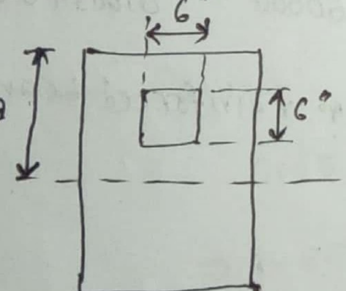
$\therefore \epsilon_t = 0.00276$

$\therefore \phi = 0.483 + 83.3 \times 0.00276 = 0.713$

[N.B: যদি a এর মান unsymmetry depth, d′ এর চেয়ে কম হয় তাহলে $\bar{y} = \frac{a}{2}$ হবে, সুতরাং, $M_n = A_s f_y (d - \bar{y}) = A_s f_y \left(d - \frac{a}{2}\right)$]



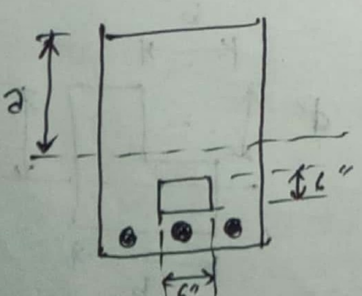
Hollow in compression zone: (Need to substract hollow section area)



Here, $C = 0.85 f_c' (ab - 6 \times 6)$

$T = A_s f_y$

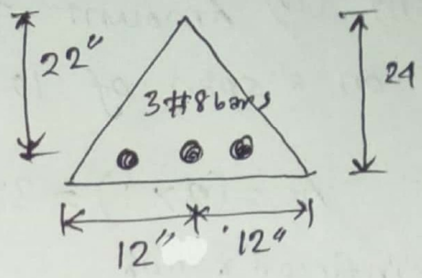
Hollow in tension zone: (Need not to substract)



Here, $C = 0.85 f_c' ab$

$T = A_s f_y$

Problem: calculate the moment capacity of the beam section in figure below: Assume $f_y = 60000$ psi, $f_c' = 3000$ psi



Solution: $A_s = (3 \times .79) = 2.37\ in^2$

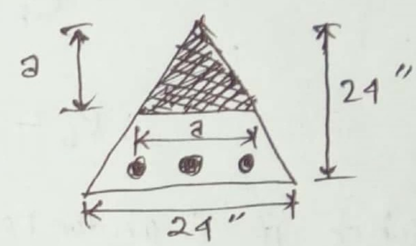
$$\therefore \rho = \frac{A_s}{.5 \times 22 \times 22} = \frac{2.37}{242} = 0.011$$

$$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85 \times 0.85 \times \frac{3000}{60000} \times \frac{0.003}{.003 + .002}$$

$\therefore \rho_b = 0.0217$

$\therefore \rho < \rho_b$ Hence it is under reinforced beam.

At equilibrium,



$$C = T$$

$$.85 f_c' \times (\tfrac{1}{2} \times a \times a) = A_s f_y$$

$$\Rightarrow .85 \times 3000 \times \tfrac{1}{2} \times a^2 = 2.37 \times 60000$$

$$\Rightarrow a = 10.56"$$

$$\bar{y} = \frac{2a}{3} = \frac{2 \times 10.56}{3} = 7.04$$

$$\therefore M_n = A_s f_y (d - \bar{y}) = 2.37 \times 60000 \times (22 - 7.04) = 2127312 \text{ lb in.}$$

$\therefore M_U = \phi M_n$

Here, $\phi = 0.483 + 83.3\ \epsilon_t$

$$\epsilon_t = \epsilon_u \frac{d-c}{c} = 0.003 \times \frac{22 - \frac{10.56}{.85}}{\frac{10.56}{.85}}$$

$\therefore \epsilon_t = 0.0023$

$\therefore \phi = (0.483 + 83.3 \times 0.0023) = 0.6746$

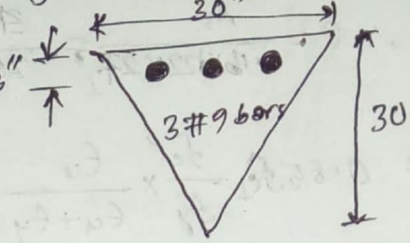
$\Rightarrow M_u = 0.6746 \times 2127312$

$\Rightarrow M_u = 1435084.675$ lb in

$\therefore M_u = 1435.085$ k in.

(Ans.)

Problem: A reinforced concrete beam section is shown in figure: compute (i) reinforced ratio (ii) mode of failure (iii) ultimate moment (iv) Amount of live load that can be applied on the beam on a span of 10 ft. Assume $f_c' = 4000$ psi & $f_y = 60000$ psi



Solve: $A_s = (3 \times 1) = 3 \text{ in}^2$

(i) reinforced ratio:

$$\rho = \frac{A_s}{.5 \times 27 \times 27}$$

$$\rho = \frac{3}{364.5} = 0.00823$$

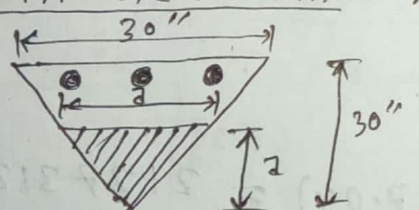
(ii) mode of failure:

$$\rho_b = 0.85 \beta_1 (f_c'/f_y) \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.002}$$

$\therefore \rho_b = 0.0289$ $\quad \therefore \rho < \rho_b$ Hence it is under reinforce section.

since it is under reinforced section, Failure will occur by yielding of steel.

(iii) ultimate moment: At equilibrium, $C = T$



$$0.85 f_c' (\tfrac{1}{2} \times a \times a) = A_s f_y$$

$$\Rightarrow .85 \times 4000 \times \tfrac{1}{2} a^2 = 60000 \times 3$$

$$\Rightarrow a = 10.29$$

$$\therefore \bar{y} = \frac{2a}{3} = \frac{2 \times 10.29}{3} = 6.86$$

$$\therefore M_n = A_s f_y (d - \bar{y}) = 3 \times 60000 \times (27 - 6.86) = 3625200 \text{ lb in.}$$

$$\therefore M_u = \phi M_n = (0.79 \times 3625200)$$

$= 2863.908$ K in.

$\therefore M_u = 2863908$ lb in $= 2863.908$ K in.

Here $\phi = .483 + 83.3 \epsilon_t$
$= .483 + 83.3 \times .0037$
$= 0.79$

(iv) Amount of live load: For simply supported beam, $M_u = \frac{W_u L^2}{8}$ Here, $L = 10$ ft

$$\therefore W_u = \frac{8 \times 2863.908}{(10)^2 \times 12} = 19.093 \text{ K/ft}, \quad W_u = 1.2 D.L + 1.6 L.L$$

$$\therefore 1.6 L.L = 19.093 - 1.2 \times \frac{.5 \times 30 \times 30}{144} \times .15 = 18.5305$$

$$\therefore L.L = \frac{18.5305}{1.6} = 11.58 \text{ K/ft}$$

(Ans)

Doubly Reinforced Beam (WSD)

Farhad
1500045

Design

Problem: Design a rectangular beam of width 9.5 inch and effective depth 22 inch. Given, $f_c' = 3000$ psi, $f_s = 20000$ psi and working moment capacity, $M = 100$ Kft.

Solution: $n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{f_c'}} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$

$r = \frac{f_s}{f_c} = \frac{20000}{.45 f_c'} = \frac{20000}{.45 \times 3000} = 14.185$

$K = \frac{n}{n+r} = \frac{9}{9+14.185} = 0.378$

$R = \frac{1}{2} f_c j K = \frac{1}{2} \times (.45 \times 3000) \times 0.874 \times 0.378 = 223$

$d_{req} = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{100 \times 12000}{223 \times 9.5}} = 23.8 \simeq 24$ inch

$\therefore d_{req} > 22$ inch. Hence, it is doubly reinforced beam.

Moment carried by concrete, $M_1 = Rbd^2 = 223 \times 9.5 \times 22^2$

$= 1025354$ lb in

$= 85.446$ K ft

Excessive moment, $M_2 = M - M_1 = (100 - 85.446)$ Kft

$= 14.554$ ft

$A_{s1} = \frac{M_1}{f_s j d} = \frac{85.446 \times 12000}{20000 \times 0.874 \times 22} = 2.67$ in^2

$A_{s2} = \frac{M_2}{f_s(d-d')} = \frac{14.554 \times 12000}{20000 \times (22 - 2.5)} = 0.448$ in^2

Now, $A_s' = \dfrac{M_2}{f_s'(d-d')}$ Here, $f_s' = 2f_s \times \dfrac{K - \frac{d'}{d}}{1-K}$

$= 2 \times 20000 \times \dfrac{0.378 - \frac{2.5}{22}}{1 - 0.378}$

$= \dfrac{14.554 \times 12000}{17000.877 \times (22 - 2.5)}$ $= 17000.877 < f_s$

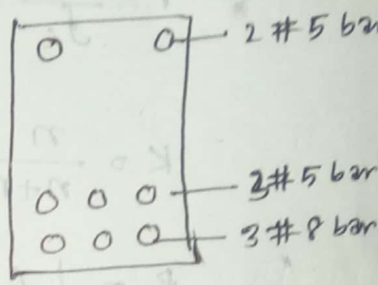
$\therefore A_s' = 0.527 \text{ in}^2$ (2 #5 bar)   [N.B: যদি $f_s' > f_s$ হয়, তাহলে $A_s' = \dfrac{M_2}{f_s(d-d')} = A_{s2}$]

$A_s = A_{s1} + A_{s2} = (2.67 + 0.448) \text{ in}^2$

$\therefore A_s = 3.118 \text{ in}^2$ (3 #5 bar and 3 # 8 bar)

(Ans.)

Cross section:



Design

Problem: A rectangular beam limitted by architectural condition to a width 13 in. and a total depth 24 inch. It must resist a total working moment 2530 K in. If $f_s = 20000$ psi, $f_c' = 4000$ psi What reinforcement is required for flexural design.

Solution: Assume, Tensile reinforcement is provided in two layer.

Hence, effective depth = (24 − 4) = 20 inch.

$n = \dfrac{E_s}{E_c} = \dfrac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$

$r = \dfrac{f_s}{f_c} = \dfrac{20000}{0.45 \times 4000} = 11.11$

$K = \dfrac{n}{n+r} = \dfrac{8}{8 + 11.11} = 0.42$

$\therefore j = 1 - \dfrac{K}{3} = 1 - \dfrac{0.42}{3} = 0.86$

Scanned by CamScanner

$R = \frac{1}{2} f_c j K = \frac{1}{2} \times (.45 \times 4000) \times .42 \times .86 = 325.08$

$\therefore d_{req.} = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{2530 \times 1000}{325.08 \times 13}} = 24.47$ inch

$\because d_{req} >$ effective depth, d. Hence it is doubly reinforced beam.

Now,

$M_1 = Rbd^2 = 325.08 \times 13 \times 20^2 = 1690416$ lb in. $= 1690.416$ Kin

$M_2 = (2530 - 1690.416) = 839.584$ K in

$$As_1 = \frac{M_1}{f_s j d} = \frac{1690416}{20000 \times 0.86 \times 20} = 4.914 \text{ in}^2$$

$$As_2 = \frac{M_2}{f_s (d-d')} = \frac{839.584 \times 1000}{20000 \times (20-2.5)} = 2.40 \text{ in}^2$$

$$As' = \frac{M_2}{f_s' (d-d')} = \frac{839.584 \times 1000}{20000 \times (20-2.5)}$$

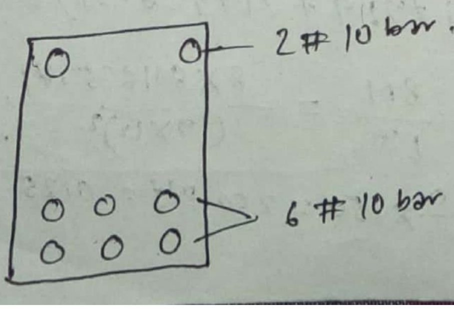
Here, $f_s' = 2 f_s \frac{K - \frac{d'}{d}}{1-K} = 2 \times 20000 \times \frac{0.42 - \frac{2.5}{20}}{1-0.42} = 20344.82$

$As' = 2.4 \text{ in}^2$ (2# 10 bar)

$\because f_s' > f_s \quad \therefore f_s' = 20000$

$As = As_1 + As_2 = (4.914 + 2.4) = 7.314 \text{ in}^2$ (6 # 10 bar)

Cross section:



Analysis

Problem: A simply reinforced beam with effective width b = 12 in. And effective depth d = 16.5 in. is reinforced with 6 # 8 bars no tension side and 2 # 7 bar on compression side. If $f_c' = 2500$ psi, $f_s = 20000$ psi, $d' = 2.5$ inch, what uniform load can be sustained on the beam of span of about 19 feet. [What Live load?]

Solution: $A_s' = 2 \times 0.60 = 1.2 \text{ in}^2$

$A_s = 6 \times 0.79 = 4.74 \text{ in}^2$

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{2500}} = 10.17 \simeq 10$, $r = \frac{f_s}{f_c} = \frac{20000}{.45 \times 2500} = 17.78$

$\therefore K = \frac{n}{n+r} = \frac{10}{10+17.78} = 0.38$, $j = 1 - \frac{K}{3} = 1 - \frac{0.38}{3} = 0.88$

$M_1 = \frac{1}{2} f_c jKbd^2 = \frac{1}{2} \times (.45 \times 2500) \times 0.36 \times 0.88 \times 12 \times 16.5^2 = 582179.4$ lb in.

$A_{s1} = \frac{M_1}{f_s j d} = \frac{582179.4}{20000 \times .88 \times 16.5} = 2.005 \text{ in}^2$

$A_{s2} = A_s - A_{s1} = (4.74 - 2.005) = 2.735 \text{ in}^2$

Now,

$M_2 = A_{s2} f_s (d - d') = 2.735 \times 20000 \times (16.5 - 2.5) = 765800$ lb inch.

and, $M_2 = A_s' f_s' (d - d')$ Here, $f_s' = 2 f_s \frac{K - \frac{d'}{d}}{1-K} = 2 \times 20000 \times \frac{0.36 - \frac{2.5}{16.5}}{1 - 0.36}$

$\therefore f_s' = 13030.30$ psi

$= 1.2 \times 13030.30 \times (16.5 - 2.5)$ $\therefore f_s' < f_s$

$= 219456.31$ lb inch (less)

$\therefore M = M_1 + M_2 = (582179.4 + 219456.31) = 801635.71$ lb-in.

Now, $M = \frac{WL^2}{8} \Rightarrow W = \frac{8M}{L^2} = \frac{8 \times 801635.71}{(19 \times 12)^2} = 123.37$ lb/in $= 1.48$ K/ft (Ans.)

* $D.L = \frac{bh}{144} \times 150 = \frac{12 \times 20}{144} \times 150 = 250$ plf $= 0.25$ Klf $\therefore L.L = (T.L - D.L)$
$= (1.48 - 0.25)$
$= 1.23$ K/ft (Ans.)

Doubly Reinforced Beam (USD)

Design [Ex-3.13]

Problem: A rectangular beam that must carry a service live load of 2.47 Kips/ft and a calculated dead load that of 1.05 Kips/ft on an 18 ft simple span is limited in cross-section for architectural reasons to 10 inch width and 20 inch in total depth. If $f_y = 60000$ psi, $f_c' = 4000$ psi, what steel area must be provided?

Solution: $W_u = 1.2\,D.L + 1.6\,L.L = (1.2 \times 1.05 + 1.6 \times 2.47) = 5.212$ K/ft

$\therefore M_u = \frac{w_u L^2}{8} = \frac{5.212 \times 18^2}{8} = 211.086$ K-ft $= 2533032$ lb in.

$\beta = 0.425$, $\alpha = 0.72$, $\frac{\beta}{\alpha} = 0.59$

Now,

$M_u = \phi \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

Here, $\rho = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

$\Rightarrow \rho = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.005}$

$\therefore \rho = 0.018$

$\Rightarrow 2533032 = 0.9 \times 0.018 \times 10 \times d^2 \times 60000 \times \left(1 - 0.59 \times \frac{.018 \times 60000}{4000}\right)$

$\Rightarrow d_{req}^2 = 310.001$

$\therefore d_{req} = 17.607$ inch

$\therefore d_{req.} >$ effective depth $= (20-4) = 16$ inch

Hence, it is doubly reinforce beam.

$M_n = A_s f_y \left(d - \frac{a}{2}\right)$

Here, $A_s = \rho b d = (0.018 \times 10 \times 16) = 2.88\ in^2$

and, $a = \frac{A_s f_y}{.85 f_c' b} = \frac{2.88 \times 60000}{.85 \times 4000 \times 10} = 5.08$

$\Rightarrow M_n = 2.88 \times 60000 \times \left(16 - \frac{5.88}{2}\right)$

$\therefore M_n = 2325888$ lb in.

Excessive moment, $M_1 = \frac{M_u}{\phi} - M_n = \frac{2533032}{0.9} - 2325888$

$\therefore M_1 = 488592$ lb-in.

$$A_s' = \frac{M_1}{f_s'(d-d')}$$ Here, $f_s' = E_s \epsilon_s' = 29\times10^6 \times \epsilon_u \left(\frac{c-d'}{c}\right)$

$$\Rightarrow f_s' = 29\times10^6 \times .003 \times \frac{\frac{5.08}{0.85} - 2.5}{\frac{5.08}{0.85}}$$

$$\therefore f_s' = 50750 \text{ psi}$$

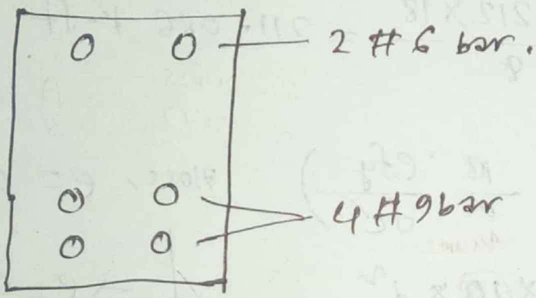
$$\Rightarrow A_s' = \frac{488592}{50750 \times (16-2.5)}$$

$\therefore A_s' = 0.713 \text{ in}^2$ (2#6 bars)

$\therefore$ Tension reinforcement $= A_s + A_s' \times \frac{f_s'}{f_y} = \left(2.88 + 0.713 \times \frac{50750}{60000}\right)$

$= 3.483 \text{ in}^2$ (4#9 bars)

Cross section:



Analysis

Ex-3.12

Problem: A rectangular beam has a width of 12 inch and effective depth to the centroid of the tension reinforcement of 24 inch. The tension reinforcement consists of 6 # 10 bars in two rows. compression reinforcement consisting of 2 #8 bars is placed 2.5 inch from the compression face. If $f_y = 60000$ psi and $f_c' = 5000$ psi what is the design moment capacity of the beam.

Solution: $A_s' = (2\times .79) = 1.58 \text{ in}^2$, $\rho' = \frac{A_s'}{bd} = \frac{1.58}{12\times24} = 0.0055$

$A_s = (6\times1.27) = 7.62 \text{ in}^2$, $\rho = \frac{A_s}{bd} = \frac{7.62}{12\times24} = 0.0265$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85\times0.80\times\frac{5000}{60000}\times\frac{.003}{.003+0.004}$$

$\therefore \rho_{max} = 0.024 < \rho$ Hence, it is doubly reinforced beam analysis.

Now,

$$\bar{\rho}_{cy} = 0.85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \times \frac{d'}{d} + \rho'$$

$$= 0.85 \times 0.80 \times \frac{5000}{60000} \times \frac{0.003}{0.003 - 0.002} \times \frac{2.5}{24} + 0.0055$$

$\therefore \bar{\rho}_{cy} = 0.0232 < \rho$ Hence, compression bar will yield when the beam fails.

$\therefore M_{n_1} = A_s' f_y (d - d') = 1.58 \times 60000 \times (24 - 2.5) = 2038200$ lb-in.

$M_{n_2} = (A_s - A_s') f_y \left(d - \frac{a}{2}\right)$ | Here, $a = \frac{A_s f_y}{.85 f_c' b} = \frac{(7.62 - 1.58) \times 60000}{0.85 \times 5000 \times 12}$

$\Rightarrow M_{n_2} = (7.62 - 1.58) \times 60000 \times \left(24 - \frac{7.106}{2}\right)$ | $\therefore a = 7.106$

$\therefore M_{n_2} = 7409992.8$ lb-in.

$\therefore M_n = (M_{n_1} + M_{n_2}) = (2038200 + 740992.8) = 9448192.8$ lb-in

Now,

$M_u = \phi M_n$

$\Rightarrow M_u = (0.8995 \times 9448192.8)$

$\Rightarrow M_u = 8498649.424$ lb-in

$\therefore M_u = 8498.65$ K-in

Here $\phi = 0.483 + 83.3\,\epsilon_t$

$\epsilon_t = \epsilon_u \left(\frac{d_t - c}{c}\right) = 0.003 \times \frac{24 - \frac{7.108}{.80}}{\frac{7.108}{.80}}$

$\therefore \epsilon_t = 0.005$

$\therefore \phi = (0.483 + 83.3 \times 0.005) = 0.8995$

(Ans.)

Analysis

Problem: A rectangular beam with a dimension of 10"x 20" in reinforced with 4 #9 bars on the tension side. and 2 #6 bar on the compression sides, compression steel is placed 2.5" below compression face. The effective depth of the beam may be considered as 16". compute the design moment capacity. considering $f_c' = 4000$ psi and $f_y = 60000$ psi.

Solution: $A_s' = (2 \times 0.44) = 0.88\ in^2$, $\rho' = \frac{A_s'}{bd} = \frac{.88}{10 \times 16} = 0.0055$

$A_s = (4 \times 1.0) = 4\ in^2$, $\rho = \frac{A_s}{bd} = \frac{4}{10 \times 16} = 0.025$

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{.003 + 0.004}$$

$\therefore \rho_{max} = 0.0206 < \rho$. Hence it is doubly reinforced beam analysis.

Now,

$$\bar{\rho}_{cy} = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u - \epsilon_y} \times \frac{d'}{d} + \rho'$$

$$\Rightarrow \bar{\rho}_{cy} = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{.003 - 0.002} \times \frac{2.5}{16} + 0.0055$$

$\therefore \bar{\rho}_{cy} = 0.028 > \rho$. Hence compression bar will <u>not</u> yield when the beam fails.

Now,

$C = T$

$\Rightarrow A_s' f_s' + 0.85 f_c' a b = A_s f_y$

$\Rightarrow 0.88 \times 87000 \times \frac{c - 2.5}{c} + 0.85 \times 4000 \times c \times 0.85 \times 10 = 4 \times 60000$

$\Rightarrow c = 6.65"$

Here, $f_s' = E_s \epsilon_s'$

$= 29 \times 10^6 \times \epsilon_u \left(\frac{c - d'}{c}\right)$

$= 29 \times 10^6 \times .003 \times \frac{c - 2.5}{c}$

$\therefore f_s' = 87000 \times \frac{c - 2.5}{c}$

and, $a = c \beta_1$

$\therefore a = (6.65 \times .85) = 5.6525$

$f_s' = 87000 \times \frac{6.65 - 2.5}{6.65} = 54293.23 < f_y$

Now, $M_{n_1} = A_s' f_s' (d - d') = .88 \times 54293.23 \times (16 - 2.5) = 645003.57$ lb in.

$M_{n_2} = 0.85 f_c' a b (d - \frac{a}{2}) = 0.85 \times 4000 \times 5.6525 \times 10 \times (16 - \frac{5.6525}{2})$

$\therefore M_{n_2} = 2531797.14$ lb in.

$\therefore M_n = M_{n_1} + M_{n_2} = (645003.57 + 2531797.14)$ lb in.

$\therefore M_n = 3176800.71$ lb in $= 3178.8$ K in

Now,

$M_u = \phi M_n$

Here, $\phi = 0.483 + 83.3\, \epsilon_t$

$\epsilon_t = \epsilon_u \times \left(\frac{d_t - c}{c}\right)$

Here, $d_t = (20 - 2.5)$

$\Rightarrow d_t = 17.50$

$\epsilon_t = 0.003 \times \frac{17.5 - 6.65}{6.65}$

$\therefore \epsilon_t = 0.0049$

$\therefore \phi = 0.483 + 83.3 \times 0.0049 = 0.89$

$\Rightarrow M_u = 0.89 \times 3178.8$

$\therefore M_u = 2829.132$ K in.

(Ans.)

Analysis (2016) → same type

T- Beam (USD)

FARHAD 1500045

E-3.14

Problem: (2006)

An isolated T beam is composed of a flange 28 in. (b) wide and 6 in. (h_f) deep cast monolithically with a web of 10 in. (b_w) width that extends 24 in. below the bottom surface of the flange to produce a beam of 30 in. (h) total depth. Tensile reinforcement consists of six No.10 bars (A_s) placed in two horizontal rows. The centroid of the bar group is 26 in. (d) from the top of the beam. It has been determined that the concrete has a strength of 3000 psi and that the yeild stress of the steel is 60000 psi. What is the design moment capacity of the beam.

Solution:

$$\rho = \frac{A_s}{bd} = \frac{6\times 1.27}{28\times 26} = 0.0105$$

Now,

$$a = \frac{A_s f_y}{.85 f_c' b} = \frac{(1.27\times 6)\times 60000}{.85\times 3000\times 28} = 6.40 > h_f (=6)$$

Hence T-beam analysis is required.

$$A_{sf} = \frac{0.85 f_c' (b-b_w) h_f}{f_y} = \frac{0.85\times 3000\times (28-10)\times 6}{60000}$$

$$= 4.59 \text{ in}^2$$

$$M_{n1} = A_{sf}\times f_y \times (d - \frac{h_f}{2})$$

$$= 4.59\times 60000\times \left((30-4) - \frac{6}{2}\right) = 6334200 \text{ lb in}$$

$M_{n_2} = (A_s - A_{sf}) f_y (d - \frac{a}{2})$ where, $a = \frac{(A_s - A_{sf}) f_y}{.85 f_c' b_w}$

$= (7.62 - 4.59) \times (26 - \frac{7.13}{2}) \times 60000$

$= 4078683$ lb in.

$= \frac{(7.62 - 4.59) \times 60000}{.85 \times 3000 \times 10}$

$= 7.13$

$\therefore M_n = M_{n_1} + M_{n_2}$

$= (6334200 + 4078683)$ lb in

$= 10412883$ lb in.

$= 867.74$ K-ft

Now,

$M_u = \phi M_n$

But $\phi = 0.483 + 83.3\, \epsilon_t$ where $\epsilon_t = \epsilon_u (\frac{d-c}{c})$

$c = \frac{a}{\beta_1} = \frac{7.13}{0.85} = 8.39$ $[\because f_c' = 3000 \therefore \beta_1 = .85]$

$\therefore \epsilon_t = \epsilon_u (\frac{d-c}{c}) = 0.003 \times (\frac{26 - 8.39}{8.39}) = 0.0063 > 0.005$

$\therefore \epsilon_t = 0.005$

$\therefore \phi = 0.483 + (83.3 \times 0.005) = 0.90$

$\therefore M_u = \phi M_n = (0.90 \times 867.74) = 780.97$ K-ft

(Ans.)

Design E-3.15

Problem: A floor system consists of a 3 in. concrete slab (h_f) supported by continuous T beam with a 24 ft span (L), 47 in. on centers. Web dimension as determined by negative moment requirements at the supports, are $b_w = 11$ in. and $d = 20$ in. What tensile steel area is required at mid span to resist a factored moment of 6400 in kips. if $f_y = 60000$ psi and $f_c' = 3000$ psi

Solution: Effective flange width (b):

$$16h_f + b_w = (16 \times 3) + 11 = 59 \text{ in.}$$

$$\frac{L}{4} = \frac{24 \times 12}{4} = 72 \text{ in.}$$

centerline spacing = 47 in.

The center line T beam spacing controls in this case. Hence

$b = 47$ in (less)

Now,

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)} = \frac{6400 \times 1000}{0.9 \times 60000 \times \left(20 - \frac{3}{2}\right)} \quad [a = h_f]$$

$$= 6.41 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{6.41}{47 \times 20} = 0.00682$$

$$a = \frac{\rho f_y d}{0.85 f_c'} = \frac{0.00682 \times 60000 \times 20}{0.85 \times 3000} = 3.21 > h_f (= 3)$$

Hence, T beam design is required.

$\phi M_{n1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$ where, $A_{sf} = \frac{0.85 f_c' (b - b_w) h_f}{f_y}$

$= 0.9 \times 4.59 \times 60000 \left(20 - \frac{3}{2}\right)$ $\quad = \frac{0.85 \times 3000 \times (47-11) \times 3}{60000}$

$= 4585410$ lb in. $\quad = 4.59$ in^2

$\therefore \phi M_{n2} = M_u - \phi M_{n1}$

$= (6400 \times 1000 - 4585410)$

$= 1814590$ lb-in

Now,

$$A_s - A_{sf} = \frac{\phi M_{n2}}{\phi f_y \left(d - \frac{a}{2}\right)}$$

where, $a = \frac{(A_s - A_{sf}) \times f_y}{0.85 f_c' b_w}$

$a = \frac{(A_s - A_{sf}) \times 60000}{0.85 \times 3000 \times 11}$

$= 2.139 (A_s - A_f)$

$$\Rightarrow (A_s - A_{sf}) = \frac{1814590}{.9 \times 60000 \times \left(20 - \frac{2.139(A_s - A_{sf})}{2}\right)}$$

$\therefore A_s - A_{sf} = 1.8665$ in^2

$\therefore A_s = A_{sf} + (A_s - A_{sf})$

$= (4.59 + 1.87)$

$= 6.46$ in^2 (6 # 10 No. bars)

(Ans.)

2015, 2017 - same type, 2007, 2008, 2011, 2013, 2006, 2014 → M_u given

Problem: Design

A concrete floor system is supported by T-beams spaced at 9 ft on centers. The beam has the dimension $b_w = 11$ inch, $h_f = 4.5$ inch, $d = 22$ inch. If the beam is to support a live load of 200 psf on a simple span of 22 ft, what tensil steel area is required? Assume $f_c' = 3000$ psi, $f_y = 60000$ psi

Solve: Effective flange with (b)

$$16 h_f + b_w = 16 \times 4.5 + 11 = 83''$$

$$\frac{L}{4} = \frac{22 \times 12}{4} = 66''$$

center line spacing $= (9 \times 12) = 108''$

$\therefore b = 66''$ (less)

Now,
$$D.L = \left[\frac{c/c \times h_f}{144} + \frac{(h - h_f) \times b_w}{144}\right] \times 150 \text{ plf}$$

$$= \left[\frac{108 \times 4.5}{144} + \frac{[(22+4) - 4.5] \times 11}{144}\right] \times 150$$

$$= 752.6 \text{ plf}$$

$$L.L = 200 \text{ psf} = (\underbrace{200 \times 9}_{c/c}) = 1800 \text{ plf}$$

$$\therefore T.L = 1.2\, D.L + 1.6\, L.L = (1.2 \times 752.6 + 1.6 \times 1800)$$

$$= 3783.12 \text{ plf} = 3.8 \text{ klf}$$

$$M_u = \frac{WL^2}{8} = \frac{3.8 \times (22)^2}{8} = 229.9 \text{ Kft}$$

Now,

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{229.9 \times 12}{.9 \times 60 \times (22 - \frac{4.5}{2})} \quad [\text{Here, } a = h_f]$$

$$= 2.587 \text{ in}^2$$

$$\therefore \rho = \frac{A_s}{bd} = \frac{2.587}{66 \times 22} = 0.00178$$

$$a = \frac{\rho f_y d}{0.85 f_c'} = \frac{.00178 \times 60 \times 22}{.85 \times 3} = 0.92 < h_f = 4.5$$

Hence, it is required to design as rectangular beam.

$$\therefore A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})} = \frac{229.9 \times 12}{.9 \times 60 \times (22 - \frac{.92}{2})} = 2.372 \text{ in}^2$$

$$= 3 \# 8 \text{ bar}$$

(Ans.)

2015, 2005

Analysis

A floor system is supported by RC beams spaced at 10 ft on centers on a simply supported span of 15 ft. The beam is reinforced with three No. 9 bars. If $h_f = 4$ in., $b_w = 10$ in and $d = 17.5$ in. determine the moment capacity of a typical interior beam by USD method. Use $f_c' = 4000$ psi and $f_y = 40,000$ psi

Solve: Effective flange width (b)

$$16h_f + b_w = 16 \times 4 + 10 = 74 \text{ in}$$

$$\frac{L}{4} = \frac{15 \times 12}{4} = 45 \text{ in.}$$

$$\text{center line beam spacing} = 10 \times 12 = 120 \text{ in.}$$

$$\therefore b = 45 \text{ in (less)}$$

Now,

$$a = \frac{A_s f_y}{.85 f_c' b} = \frac{(3 \times 1) \times 40000}{.85 \times 4000 \times 45} = 0.784 < h_f = 4 \text{ in.}$$

Hence, it is required to as rectangular beam.

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = 3 \times 40000 \times \left(17.5 - \frac{.784}{2}\right) = 2052941.18 \text{ lb in.}$$

$$M_u = \phi M_n$$

Here, $\phi = 0.483 + 83.3\,\epsilon_t$

But, $\epsilon_t = \epsilon_u \left(\frac{d-c}{c}\right) = 0.003 \times \dfrac{17.5 - \frac{a}{\beta_1}}{\frac{a}{\beta_1}}$

$$= 0.003 \times \frac{17.5 - \frac{.784}{.85}}{\frac{.784}{.85}} \qquad [\because f_y = 4000,\ \beta_1 = .85]$$

$$= 0.054 > 0.005$$

$$\therefore \phi = 0.483 + 83.3 \times .005$$

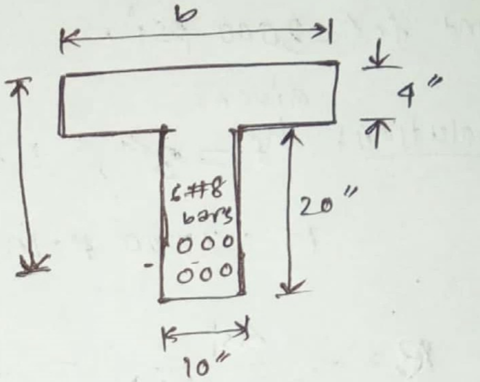
$$= 0.90$$

$$\therefore M_u = (.9 \times 2052941.18) \text{ lb in.}$$

$$= 1847647.06 \text{ lb in.} \text{ (Ans)}$$

Analysis

Problem: A floor slab 4" thick is supported by R.C beams 9' on centers, The beams are simple supported of 19' span. web dimension = 10" × 20". As = 6 # 8 bars in two rows. 2 inch center to center vertically. The center of the lower row being $2\frac{1}{2}$" above the lower surface of the beam. $f_c' = 2500$ psi, $f_s = 20000$ psi. Find Allowable working moment of the beam.



Solution: effective flange width (b)

$$16h_f + b_w = 16\times4 + 10 = 74$$

$$\frac{L}{4} = \frac{19\times12}{4} = 57''$$

$$c/c = (9\times12) = 108''$$

Hence, $b = 57''$ Here $d = (24-4) = 20$ in. $h_f = t = 4''$

$$\rho = \frac{A_s}{bd} = \frac{6\times.79}{57\times20} = 0.00416$$

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{2500}} = 10.17 \simeq 10 \qquad \therefore \rho n = 0.0416$$

$$\frac{t}{d} = \frac{4}{20} = 0.2 \qquad \therefore K = \frac{\rho n + \frac{1}{2}\left(\frac{t}{d}\right)^2}{\rho n + \left(\frac{t}{d}\right)} = \frac{0.0416 + .5\times(.2)^2}{.0416 + .2} = 0.255$$

$kd = 5.1 > h_f$ Hence T beam analysis is required.

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3\times\left(\frac{1}{2\rho n}\right)}{6 - 3\left(\frac{t}{d}\right)} = \frac{6 - 6\times.2 + 2\times(.2)^2 + (.2)^3 + \frac{1}{2\times.0416}}{6 - 3\times(.2)}$$

$\therefore j = 0.904$.   Now, $M_s = A_s f_s jd = (6\times.79)\times20000\times.904\times20$

$\therefore M_s = 1713984$ lb in. (Allowable)

~~check for concrete stress~~ $M_c = f_c\left(1 - \frac{t}{2Kd}\right)btjd = 1125\times\left(1 - \frac{4}{2\times5.1}\right)\times57\times4\times.904\times20$

$= 2818884.706$ lb in

$$\cancel{f_c = \frac{M}{\left(1 - \frac{t}{2Kd}\right)btjd} = \frac{1713984}{\left(1 - \frac{4}{2\times5.1}\right)\times57\times4\times.904\times20}}$$

(Ans.)

Design 2004, 2007

Problem: A floor system consist of a 3" concrete slab supported by continuous T beam of 24' span, 47" on center web dimensions as determined by negative flexural reinforcement at the supports are b=11" and d = 20". What tensile steel area is required at midspan to resist a working moment of 2500 K-in. if $f_s = 20000$ psi and $f_c' = 3000$ psi.

Given,

Solution: $h_f = 3''$, $L = 24'$, $c/c = 47''$, $b_w = 11''$, $d = 20''$

$M = 2500$ k-in.

effective flange width, (b):

$16h_f + b_w = 59''$

$\frac{L}{4} = \frac{24\times12}{4} = 72''$

$c/c = 47''$

Hence, $b = 47''$

$$A_s = \frac{M}{f_s(d-\frac{h_f}{2})} = \frac{2500\times1000}{20000\times(20-\frac{3}{2})} = 6.76 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{6.76}{47\times20} = 0.0072$$

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$$

$\therefore \rho n = 0.0648$, $\frac{t}{d} = \frac{3}{20} = 0.15$, $K = \frac{\rho n + \frac{1}{2}(\frac{t}{d})^2}{\rho n + (\frac{t}{d})} = \frac{.0648 + .5\times(.15)^2}{.0648 + .15}$

$\therefore K = 0.354$

$Kd = 7.04 > h_f$. Hence T beam design is required.

$$j = \frac{6-6(\frac{t}{d})+2(\frac{t}{d})^2+(\frac{t}{d})^3\times(\frac{1}{2\rho n})}{6-3(\frac{t}{d})} = \frac{6-6\times.15+2\times(.15)^2+(.15)^3\times(\frac{1}{2\times.0648})}{6-3\times.15}$$

$\therefore j = 0.927$

$$A_s = \frac{M}{f_s jd} = \frac{2500\times1000}{20000\times.927\times20} = 6.74 \text{ in}^2 \quad (6 \# 10 \text{ bars})$$

Check for concrete stress: $f_c = \frac{M}{(1-\frac{t}{2Kd})\,bt\,jd} = \frac{2500\times1000}{(1-\frac{3}{2\times7.04})\times47\times3\times.927\times20}$

$\therefore f_c = 1215.27$ psi < 1350 psi

(OK) (Ans.)

2012

A T-beam with $b = 32$ in, $b_w = 8$ in. $d = 12$ in. $A_s = 3.00$ in^2,

$f_c' = 3000$ psi and $f_y = 60000$ psi and $h_f = 2$ in.

(i) Determine the stresses in concrete and steel if the applied moment is 1.5×10^6 lb-in.

(ii) The ultimate strength of the beam.

Solve: (i)

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.289 \simeq 9$$

$$\rho = \frac{A_s}{bd} = \frac{3}{32 \times 12} = 0.0078$$

$$\rho n = (9 \times .0078) = 0.0702$$

$$t/d = \frac{2}{12} = 0.167$$

$$\therefore K = \frac{\rho n + \frac{1}{2}\left(\frac{t}{d}\right)^2}{\rho n + \left(\frac{t}{d}\right)} = \frac{0.0702 + .5 \times (0.167)^2}{0.0702 + 0.167}$$

$$\Rightarrow K = 0.35$$

$$\therefore Kd = (0.35 \times 12) = 4.25 > h_f = 2 \text{ in}$$

Hence, T-beam analysis is required.

$$\therefore j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \frac{(t/d)^3}{2\rho n}}{6 - 3 \times \left(\frac{t}{d}\right)}$$

$$= \frac{6 - 6 \times .167 + 2 \times (.167)^2 + \frac{(.167)^3}{2 \times .0702}}{6 - (3 \times .167)} = 0.925$$

We know,

$\therefore M = A_s f_s jd$

$$\Rightarrow f_s = \frac{M}{A_s jd} = \frac{1.5\times10^6}{3\times0.925\times12} = 45051 \text{ psi}$$

Again,

$$M = f_c \left(1-\frac{t}{2Kd}\right) bt\, jd$$

$$\Rightarrow f_c = \frac{M}{\left(1-\frac{t}{2Kd}\right) bt\, jd} = \frac{1.5\times10^6}{\left(1-\frac{2}{2\times4.25}\right)\times32\times2\times.925\times12}$$

$\therefore f_c = 2761 \text{ psi}$

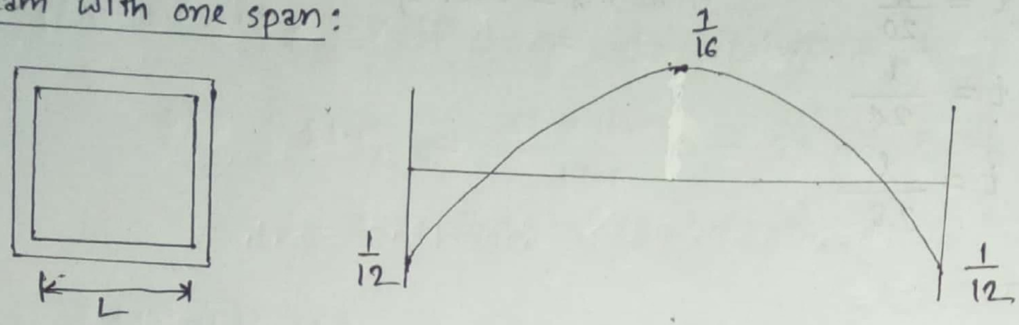
(Ans.)

(ii) same as (Ex - 3.14) Analysis

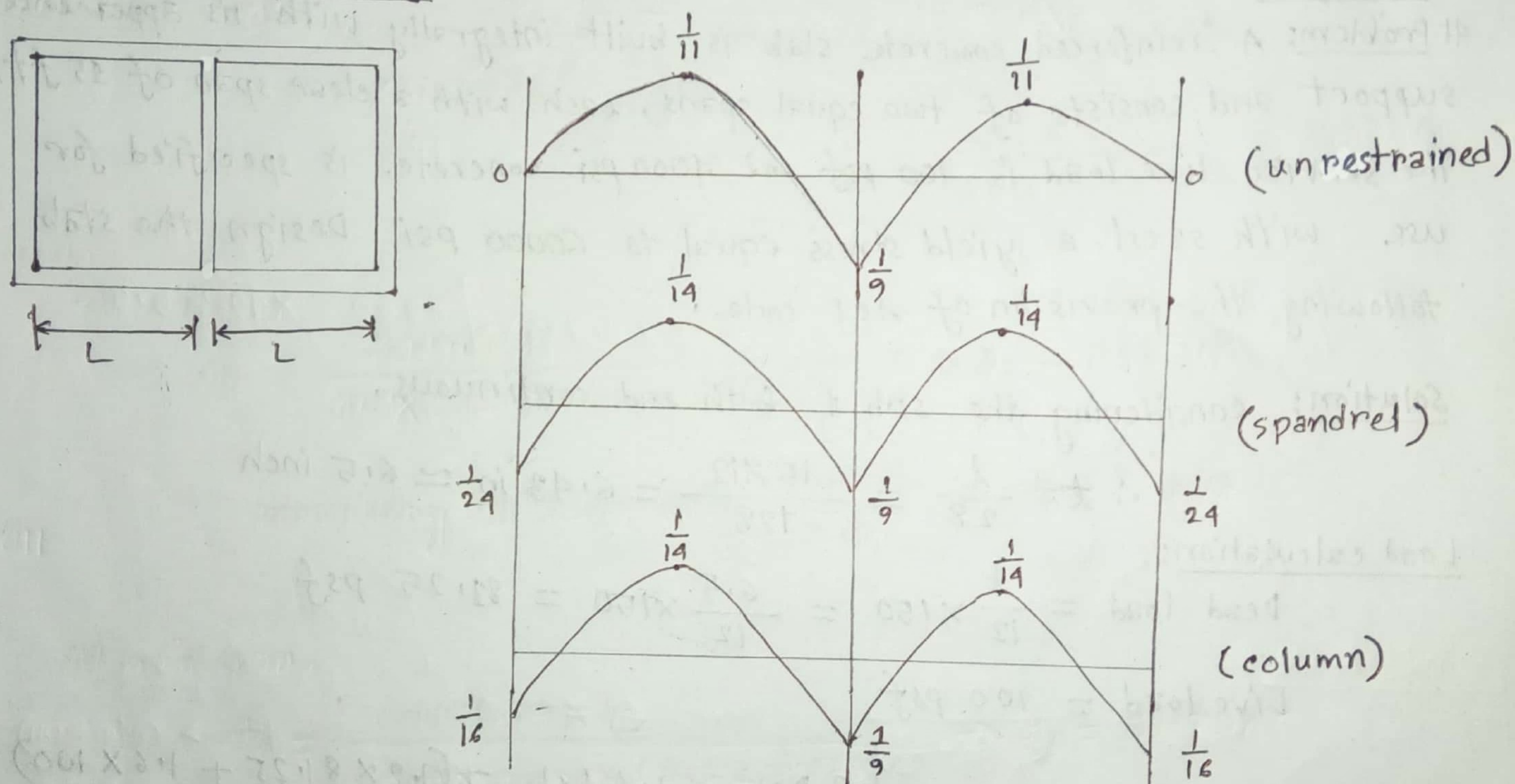
Design of Slab

Farhad Hossain
#1500045

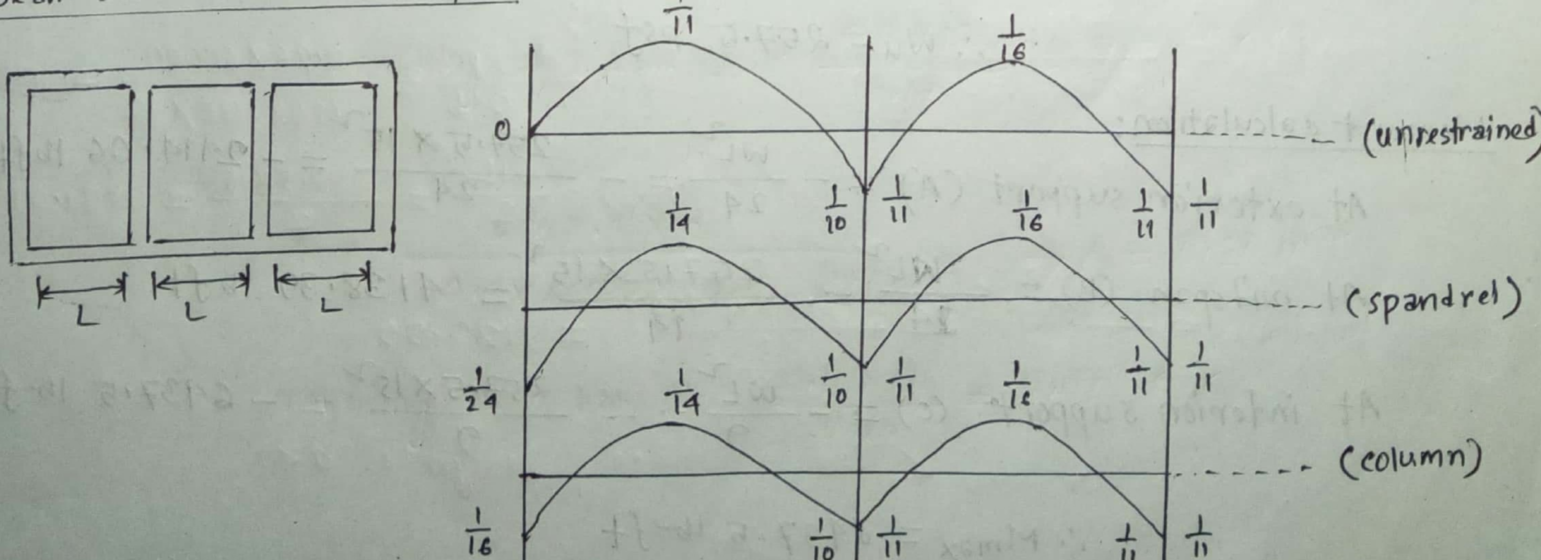
Beam with one span:



Beam with two spans only:



Beam with more than two spans:



Minimum thickness of one-way slab:

Simply supported $\Rightarrow t = \frac{l}{20}$ Here, l = clear span

One end continuous $\Rightarrow t = \frac{l}{24}$

Both end continuous $\Rightarrow t = \frac{l}{28}$

Cantilever $\Rightarrow t = \frac{l}{10}$

Ex 13.1 USD

Problem: A reinforced concrete slab is built integrally with its appearence support and consists of two equal spans, each with a clear span of 15 ft. The service live load is 100 psf and 4000 psi concrete is specified for use with steel a yield stress equal to 60000 psi. Design the slab following the provision of ACI code.

Solution: considering the slab is both end continuous,

$$\therefore t = \frac{l}{28} = \frac{15 \times 12}{28} = 6.43 \text{ in} \simeq 6.5 \text{ inch}$$

Load calculation:

Dead load $= \frac{t}{12} \times 150 = \frac{6.5}{12} \times 150 = 81.25$ psf

Live load $= 100$ psf

$\therefore$ Total load, $W_u = 1.2 D.L + 1.6 L.L = (1.2 \times 81.25 + 1.6 \times 100)$

$\therefore W_u = 257.5$ psf

Moment calculation:

At exterior support (A) $= -\frac{WL^2}{24} = -\frac{257.5 \times 15^2}{24} = -2414.06$ lb ft

At mid span (B) $= \frac{WL^2}{14} = \frac{257.5 \times 15^2}{14} = 4138.39$ lb ft

At interior support (c) $= -\frac{WL^2}{9} = -\frac{257.5 \times 15^2}{9} = -6437.5$ lb-ft

$\therefore M_{max} = 6437.5$ lb-ft

Depth check: $M_u = \phi e b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{e f_y}{f_c'}\right)$

Assume

$\Rightarrow (6437.5 \times 12) = 0.9 \times 0.018 \times 12 \times d^2 \times 60000 \left(1 - 0.59 \times \frac{0.018 \times 60}{4}\right)$

$\Rightarrow d^2 = 7.88$

$\therefore d = 2.81$ inch $<$ effective depth $= (6.5 - 1.0)$

$= 5.5$ inch

Here, $e = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$

$\Rightarrow e = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{(0.003 + 0.005)}$

$\therefore e = 0.018$

$\therefore d_{eff} = 5.5$ inch.

Steel calculation: $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})}$ Here, $a = \frac{A_s f_y}{.85 f_c' b}$

$a = \frac{A_s \times 60000}{.85 \times 4000 \times 12} = 1.471 A_s$

Now,

At exterior,

$A_s = \frac{2414.06 \times 12}{0.9 \times 60000 \times (5.5 - \frac{1.471 A_s}{2})} \Rightarrow A_s = 0.1 \text{ in}^2$

providing # 3 bar @ $= \frac{0.11 \times 12}{0.1} = 13.2''$ c/c

At mid span,

$A_s = \frac{4138.39 \times 12}{0.9 \times 60000 \times (5.5 - \frac{1.471 A_s}{2})} \Rightarrow A_s = 0.17 \text{ in}^2$

providing # 3 bar @ $= \frac{0.11 \times 12}{0.17} = 7.76''$ c/c

At interior,

$A_s = \frac{6437.5 \times 12}{0.9 \times 60000 \times (5.5 - \frac{1.471 A_s}{2})} \Rightarrow A_s = 0.26 \text{ in}^2$

providing # 3 bar @ $= \frac{0.11 \times 12}{0.26} = 5.07''$ c/c

Distribution reinforcement: $f_y = 60000$ psi

Hence, $A_s = 0.0018\,bt = (0.0018 \times 12 \times 6.5) = 0.14 \text{ in}^2$ Assume

providing # 3 bar @ $= \dfrac{0.11 \times 12}{0.14} = 9.43''$ c/c

Shear check:

The factored shear force at a distance d from the face of interior support is,

$$V_u = 1.15 \times \frac{WL}{2} - \frac{Wd}{12}$$ (inch)

$$= 1.15 \times \frac{257.5 \times 15}{2} - \frac{257.5 \times 5.5}{12}$$

$$\therefore V_u = 2102.92 \text{ lb}$$

design strength of slab,

$$V_{allowable} = 2\phi\sqrt{f_c'}\,bd = 2 \times 0.75 \times \sqrt{4000} \times 12 \times 5.5$$ Assume

$$\therefore V_{all} = 6261.31 \text{ lb}$$

$\therefore V_u < V_{all}$ Hence, Design is OK

Bond check:

$$U_{dev.} = \frac{V_{max}}{\Sigma_o (d - \frac{a}{2})}$$

Here, $\Sigma_o = n\pi D = \dfrac{b}{\text{spacing}} \times \pi \times D$ dia of bar (minimum)

$$\Rightarrow \Sigma_o = \frac{12}{5.07} \times 3.1416 \times \frac{3}{8} = 2.79$$

and, $a = \dfrac{A_s f_y}{.85 f_c' b} = \dfrac{0.26 \times 60000}{.85 \times 4000 \times 12} = 0.38$ max

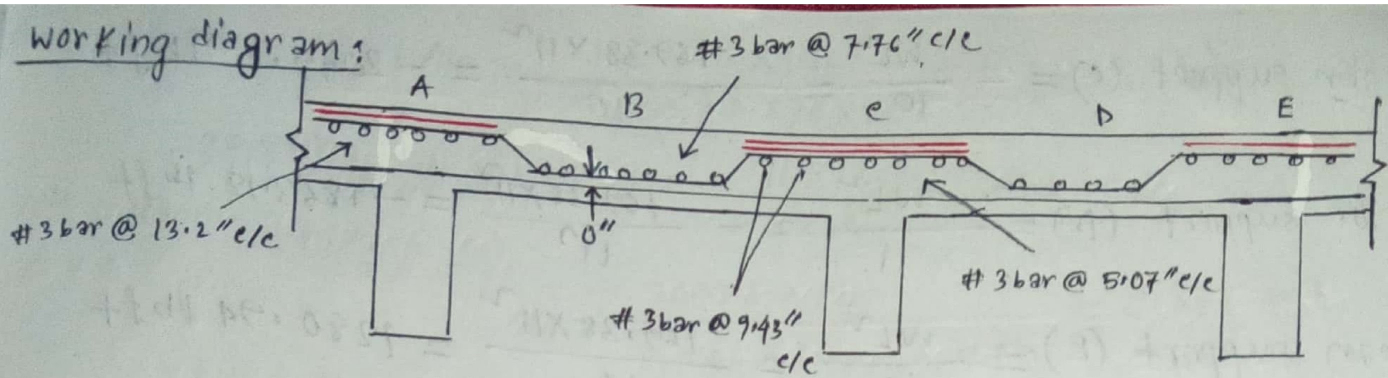
$V_{max} = \dfrac{WL}{2} = \dfrac{257.5 \times 15}{2} = 1931.25$

$$\Rightarrow U_{dev} = \frac{1931.25}{2.79 \times (5.5 - \frac{0.38}{2})}$$

$$\therefore U_{dev} = 130.36 \text{ lb/in}^2$$

$$U_{allowable} = \frac{6.7\sqrt{f_c'}}{D} = \frac{6.7 \times \sqrt{4000}}{\frac{3}{8}} = 1129.99$$

$\therefore U_{dev} < U_{all}$ Hence, Design is OK working diagram:



At support A and E, No of extra top $= \dfrac{(0.1 - \frac{0.11}{2})}{0.11} = 0.4 \simeq 1$ (providing #3 bar)

At support C, No of extra top $= \left(\dfrac{0.26 - \frac{0.11}{2}}{0.11}\right) = 1.86 \simeq 2$ (providing #3 bar)

WSD

Problem: A reinforced concrete slab is built integrally with its appearence support and consists of three equal spans, each span of 11 ft and The working live load is 110 psf. If $f_c' = 3000$ psi, $f_y = 50000$ psi. Design the slab using WSD.

Solution: considering the slab is both end continuous.

$\therefore t = \dfrac{l}{28} = \dfrac{11 \times 12}{28} = 4.71 \simeq 4.75$ ft

Load calculation; Dead load $= \dfrac{t}{12} \times 150 = \left(\dfrac{4.75}{12} \times 150\right) = 59.38$ psf

Live load = 110 psf

$\therefore$ Total load, $W = (D.L + L.L) = (59.38 + 110) = 169.38$ psf

Moment calculation;

At exterior support (A) $= -\dfrac{WL^2}{24} = -\dfrac{169.38 \times 11^2}{24} = -853.96$ lbft

At mid span (B) $= \dfrac{WL^2}{14} = \dfrac{169.38 \times 11^2}{24} = 1463.93$ lbft

At interior support (C) $= -\frac{WL^2}{10} = -\frac{169.38 \times 11^2}{10} = -2049.5$ lb ft

At interior support (D) $= -\frac{WL^2}{11} = -\frac{169.38 \times 11^2}{11} = -1863.18$ lb ft

At mid span support (E) $= \frac{WL^2}{16} = \frac{169.38 \times 11^2}{16} = 1280.94$ lb ft

$\therefore M_{max} = 2049.5$ lb ft

Depth check: $n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \simeq 9$

$r = \frac{f_s}{f_c} = \frac{.4 \times 50000}{.45 \times 3000} = 14.81$

$K = \frac{n}{n+r} = \frac{9}{9+14.81} = 0.378$

$j = 1 - \frac{K}{3} = 1 - \frac{0.378}{3} = 0.874$

$R = \frac{1}{2} f_c j K = \frac{1}{2} \times (.45 \times 3000) \times 0.874 \times 0.378 = 223$

$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{2049.5 \times 12}{223 \times 12}} = 3.03''$ Assume

$\therefore t_{req} = 3.03 + c.c + \frac{\phi}{2} = 3.03 + (0.75 + 0.25) = 4.03'' < t = 4.75''$

effective depth, $d_{eff} = (4.75 - 1) = 3.75$ in.

Steel Area calculation: $A_s = \frac{M}{f_s j d}$ Here, $f_s = (.4 \times 50000) = 20000$

At exterior support (A), $A_s = \frac{853.96 \times 12}{(.4 \times 50000) \times 0.874 \times 3.75} = 0.156$ in^2

providing # 3 bar @ $= \frac{0.11 \times 12}{.156} = 8.46''$ c/c

At mid span (B), $A_s = \dfrac{1463.93 \times 12}{20000 \times 0.874 \times 3.75} = 0.268 \ in^2$

providing # 3 bar @ $= \dfrac{.11 \times 12}{.268} = 4.93'' \ c/c$

At interior support (C),

$$A_s = \frac{2049.5 \times 12}{20000 \times 0.874 \times 3.75} = 0.375 \ in^2$$

providing # 3 bar @ $= \dfrac{0.11 \times 12}{0.375} = 3.52'' \ c/c$

At interior support (D), $A_s = \dfrac{1863.18 \times 12}{20000 \times 0.874 \times 3.75} = 0.341 \ in^2$

providing # 3 bar @ $= \dfrac{0.11 \times 12}{0.341} = 3.87'' \ c/c$

At mid span (E), $A_s = \dfrac{1280.94 \times 12}{20000 \times 0.874 \times 3.75} = 0.234 \ in^2$

providing # 3 bar @ $= \dfrac{0.11 \times 12}{0.234} = 5.64'' \ c/c$

Distribution reinforcement: $f_y = 50000$ psi

Assume

Hence, $A_s = 0.0020\,bt = 0.0020 \times 12 \times 4.75 = 0.114 \ in^2$

providing # 3 bar @ $= \dfrac{0.11 \times 12}{0.114} = 11.58'' \ c/c$

Shear check:

$$V_w = 1.15 \frac{wL}{2} - \frac{wd}{12} = 1.15 \times \frac{169.38 \times 11}{2} - \frac{169.38 \times 3.75}{12}$$

$\therefore V_w = 1018.4$ lb

$V_{all} = 1.1\sqrt{f_c'}\ bd = 1.1 \times \sqrt{3000} \times 12 \times 3.75 = 2711.22$

$\therefore V_w < V_{all}$.

Hence Design is OK

Bond check:

$$U_{dev} = \frac{V_{max}}{\Sigma_0 \, jd}$$

Here, $V_{max} = \frac{WL}{2} = \frac{169.38 \times 11}{2}$

$\therefore V_{max} = 931.59$

$$\Rightarrow U_{dev} = \frac{931.59}{4.01625 \times 0.874 \times 3.75}$$

$\Sigma_0 = n \pi D = \frac{b}{\text{spacing}} \times \pi \times D$

$\therefore U_{dev} = 70.77$

$$\Rightarrow \Sigma_0 = \frac{12}{3.52} \times 3.1416 \times \frac{3}{8}$$

$\therefore \Sigma_0 = 4.01625$

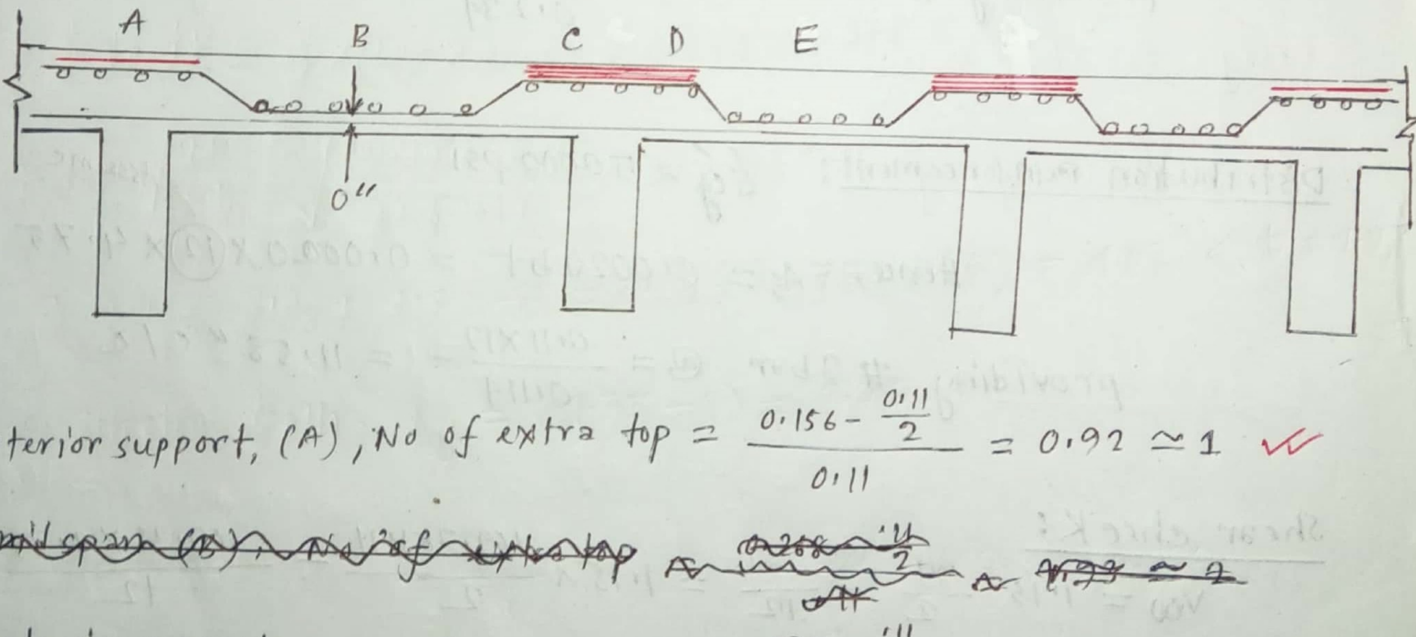
$$U_{all} = \frac{3.4\sqrt{f_c'}}{D}$$

$$\Rightarrow U_{all} = \frac{3.4\sqrt{3000}}{\frac{3}{8}}$$

$\therefore U_{all} = 496.60$

$\therefore U_{dev} < U_{all}$ Hence design is OK.

working diagram:



At exterior support, (A), No of extra top $= \frac{0.156 - \frac{0.11}{2}}{0.11} = 0.92 \simeq 1$

~~At mid span (B), No of extra top = ... = ... ≃ 2~~

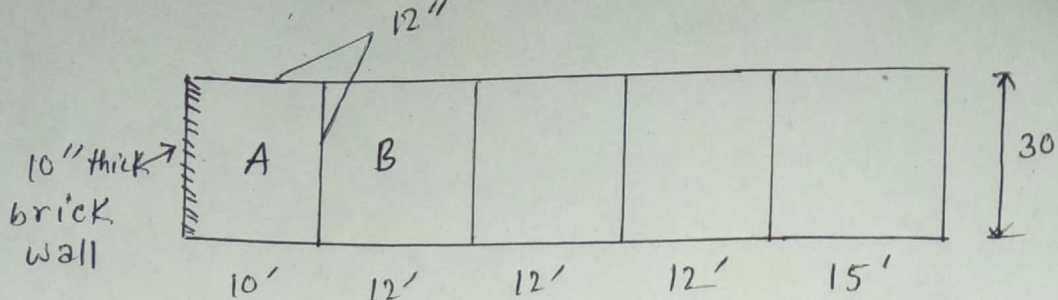
At interior support, No of extra top $= \frac{.375 - \frac{.11}{2}}{.11} = 2.91 \simeq 3$

~~At mid span (E), No of extra top = ... = ... ≃ 2~~

2013

Problem:

Design slab panel A and B

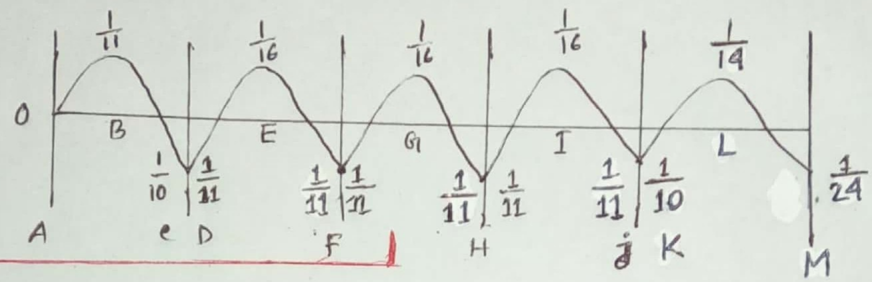


Hints: considering, the slab is one end continuous. $\therefore \boxed{t = \frac{l}{24}}$ $[l = 14']$

max

$\boxed{15' - 1'}$ beam width বাদ দিতে হবে

Moment calculation:



$M_c = -\frac{1}{10} \times wL^2$ $[L = 10' - 1' = 9']$

$M_D = -\frac{1}{11} \times wL^2$ $[L = 12' - 1' = 11']$

beam width বাদ দিতে হবে

$M_j = -\frac{1}{11} \times wL^2$ $[L = 12' - 1' = 11']$

$M_K = -\frac{1}{10} \times wL^2$ $[L = 15' - 1' = 14']$

beam width বাদ দিতে হবে

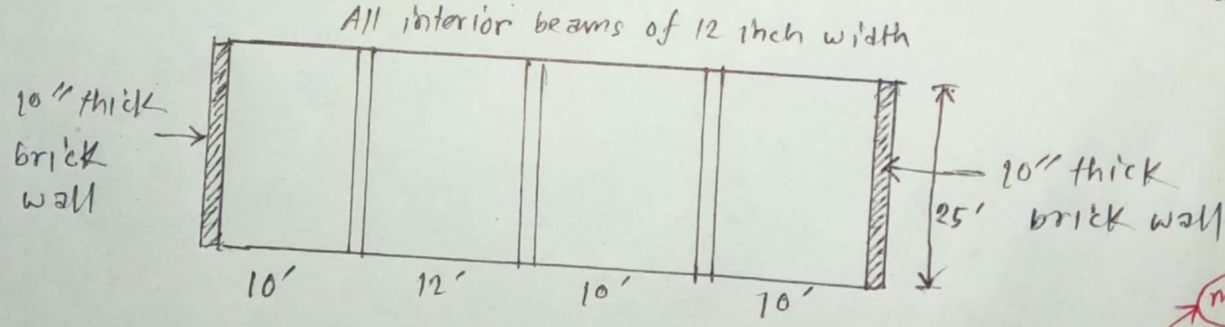
2011

Problem:

$LL = 100$ psf

$f_c' = 4000$ psi

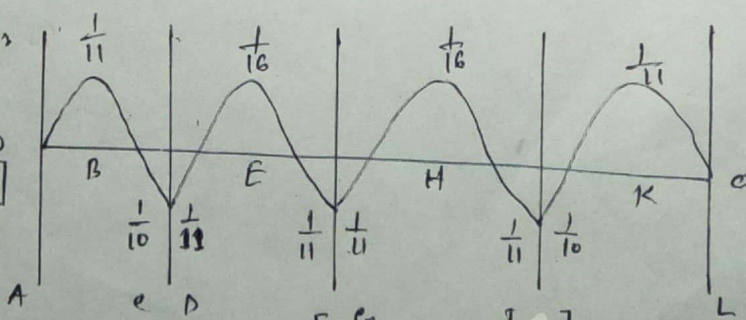
$f_y = 60000$ psi



Hints: considering, the slab is simply supported, $\therefore \boxed{t = \frac{l}{20}}$ $[l = 11']$

max

Moment calculation:



$M_c = -\frac{1}{10} \times wL^2$ $[L = 9']$

$M_D = -\frac{1}{11} \times wL^2$ $[L = 11']$

$M_F = -\frac{1}{11} wL^2$ $[L = 11']$

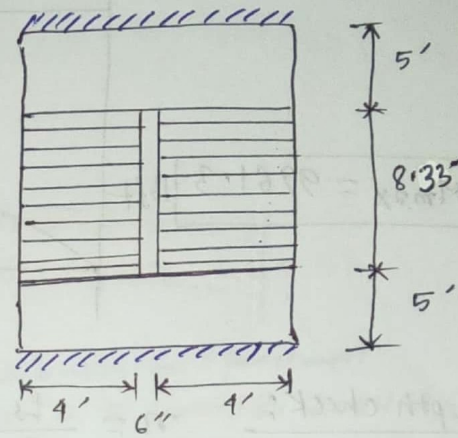
$M_G = -\frac{1}{11} wL^2$ $[L = 9']$

Stair

FARHAD
1500045

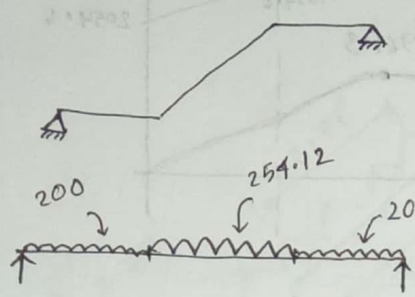
(WSD)

Problem: Design a stair case which carries Live load = 100 psf, T = 10", R = 6", $f_c' = 4000$ psi and $f_y = 60000$ psi



Solution:

Equivalent structure:



Let, ~~the~~ thickness of slab = 8"

Load calculation: Dead load for landing, $w_1 = \frac{t}{12} \times 150 = \left(\frac{8}{12} \times 150\right) = 100$ psf

Dead load for inclined portion, $w = w_1 \times \frac{\sqrt{T^2+R^2}}{T}$

$= 100 \times \frac{\sqrt{10^2+6^2}}{10}$

$= 116.62$ psf

load due to step on inclined portion $= \frac{R}{2} \times 150$

$= \frac{6}{12 \times 2} \times 150$ psf

$= 37.5$ psf

Now,

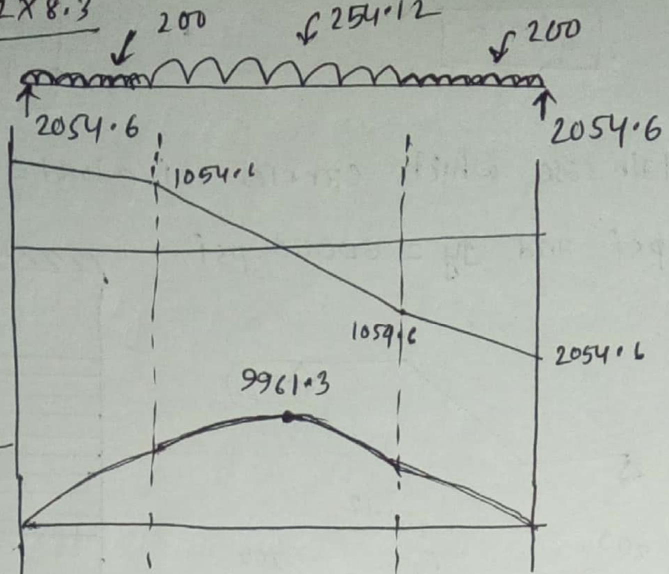
Total load on landing = (D.L + L.L) = (100+100) = 200 psf

Total load on inclined portion = (D.L + L.L) = (116.62 + 37.5) + 100

$= 254.12$ psf

$$\frac{WL}{2} = \frac{200 \times 5 \times 2 + 254.12 \times 8.3}{2}$$

$$\boxed{\therefore R = 2054.6}$$



$$\boxed{\therefore M_{max} = 9961.3} \text{ lbft}$$

depth check:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$$

$$r = \frac{f_s}{f_c} = \frac{.4 \times 60000}{.45 \times 4000} = 13.33$$

$$\therefore K = \frac{n}{n+r} = \frac{9}{9+13.33} = 0.375$$

$$\therefore J = 1 - \frac{K}{3} = 0.875$$

$$\therefore R = \frac{1}{2} f_c\, j K = \frac{1}{2} \times .45 \times 3000 \times .875 \times .375 = 295.3125$$

$$\therefore d_{req} = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{9961.3 \times 12}{295.3125 \times 12}} = 5.81''$$

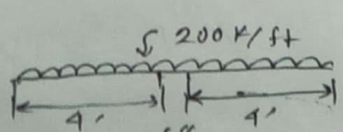
$$d_{act} = t - c.c - \frac{\phi}{2} = (8-1) = 7'' > d_{req} \quad \text{(OK)}$$

Reinforcement calculation:

$$A_s = \frac{M}{f_s j d} = \frac{9961.3 \times 12}{(.4 \times 60000) \times .875 \times 7} = 0.813 \text{ in}^2$$

$$\text{provide \# 5 bars @ } \frac{0.31 \times 12}{.813} = 4.57'' \text{ c/c}$$

For landing:



$$\therefore M = \frac{WL^2}{8} = \frac{200 \times (8.5)^2}{8} = 1806.25 \text{ K-ft}$$

$$A_s = \frac{M}{f_s j d} = \frac{1806.25 \times 12}{24000 \times .875 \times 7} = 0.147 \text{ in}^2$$

$$\text{provide \# 3 bar @ } = \frac{0.11 \times 12}{.147} = 8.98'' \text{ c/c}$$

Distribution reinforcement: $A_{sf} = 0.0018\,bt = 0.0018 \times 12 \times 8 = 0.1728\ in^2$

provide #3 bars @ $\frac{.11 \times 12}{.1728} = 7.64$ c/c

working diagram:

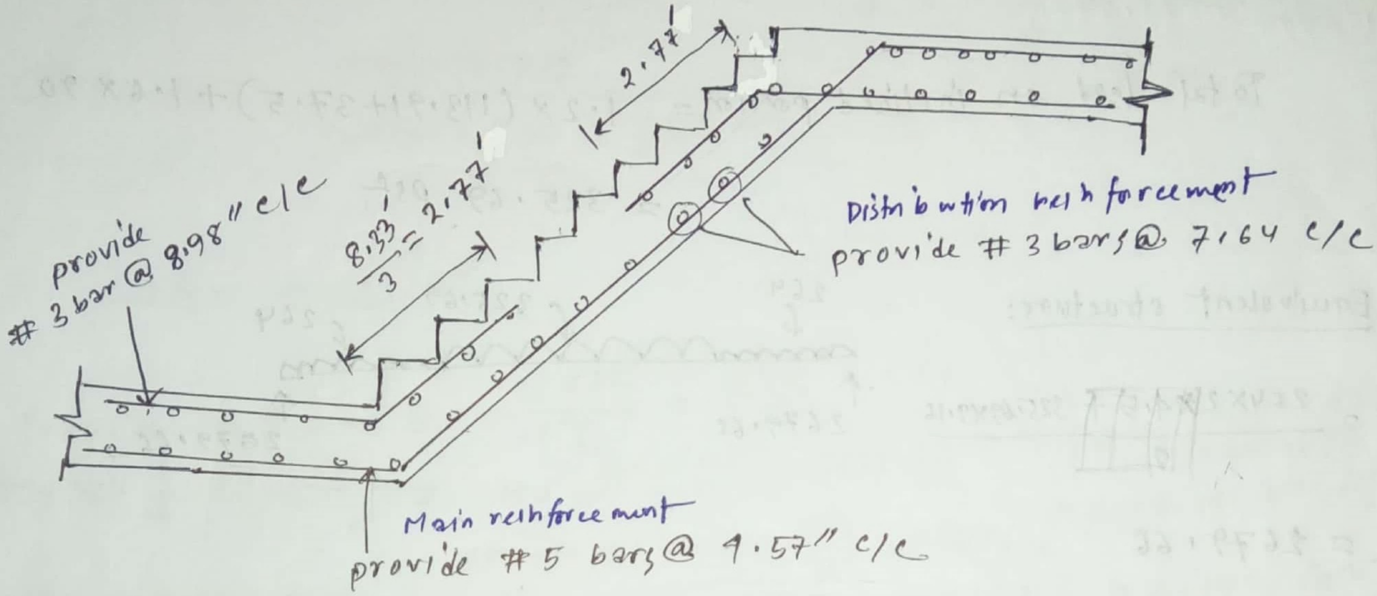


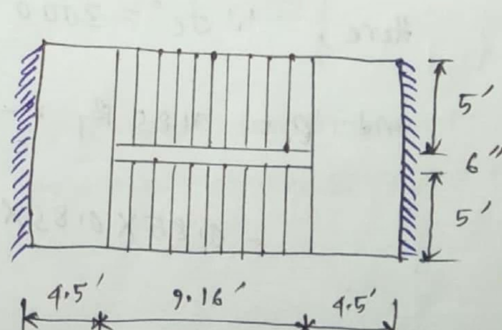
Fig. Reinforcement details of stair case

(USD)

Problem: Design the stair shown in the figure for a live load of 90 psf. Use $f_c' = 3000$ psi and $f_y = 50000$ psi

Solution:

Let, thickness of slab = 8"



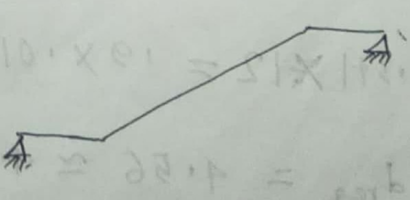
$N_T = 10$

$T = \frac{9.16}{10} \times 12$

Load calculation:

Dead load for landing, $w_1 = \frac{t}{12} \times 150$

$w_1 = \frac{8}{12} \times 150 = 100$ psf



Dead load for inclined portion, $w = w_1 \times \frac{\sqrt{T^2 + R^2}}{T}$

$= 100 \times \frac{\sqrt{11^2 + 6^2}}{11}$

$\therefore W = 113.91$ psf

load due to step on inclined portion $= \frac{R}{2} \times 150 = \frac{6}{2 \times 12} \times 150 = 37.5$ psf

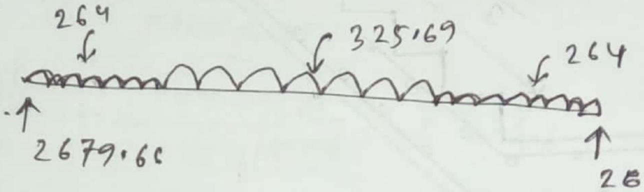
$\therefore$ Total load for landing $= 1.2\,D.L + 1.6\,L.L = (1.2 \times 100 + 1.6 \times 90)$ psf

$= 264$ psf

Total load on inclined portion $= 1.2 \times (113.91 + 37.5) + 1.6 \times 90$

$= 325.69$ psf

Equivalent structure:



$$\frac{WL}{2} = \frac{264 \times 2 \times 4.5 + 325.69 \times 9.16}{2}$$

$\therefore R = 2679.66$

$$\therefore M_{max} = 2679.66 \times \left(4.5 + \frac{9.16}{2}\right) - 264 \times 4.5 \times \left(\frac{4.5}{2} + \frac{9.16}{2}\right) - 325.69 \times \frac{9.16}{2} \times \frac{9.16}{4}$$

$= 12801.371$ lb ft

depth check: $M_u = \phi \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

Here, $\because f_c' = 3000 \quad \therefore \beta_1 = .85$

and $\rho_{\cdot} = 0.85\, \beta_1 \times \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$

$= 0.85 \times 0.85 \times \frac{3000}{50000} \times \frac{.003}{.003 + .005} = 0.0163$

Now, $M = \phi \rho b d^2 f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right)$

$$12801.371 \times 12 = .9 \times .0163 \times 12 \times d^2 \times 50000 \times \left(1 - 0.59 \times \frac{.0163 \times 50000}{3000}\right)$$

$\Rightarrow d_{req} = 4.56 \simeq 4.75''$

$d_{act} = t - c/c - \frac{\phi}{2} = (8-1) = 7'' > d_{req}$ (OK)

Reinforcement calculation:

$$A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})}$$

Here, $a = \frac{A_s f_y}{.85 f_c' b}$

$\Rightarrow a = \frac{A_s \times 50000}{.85 \times 3000 \times 12}$

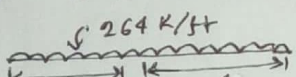
$\Rightarrow a = 1.634 A_s$

Now, $A_s = \frac{12801.371 \times 12}{.9 \times 50000 \times (7 - \frac{1.634 A_s}{2})}$

$\therefore A_s = 0.52\ in^2$

Provide # 5 bars @ $\frac{.31 \times 12}{.52} = 7.15''$ c/c

For landing:



$M = \frac{264 \times (10.5)^2}{8} = 3638.25$ K-ft, $A_s = \frac{3638.25 \times 12}{.9 \times 50000 \times (7 - \frac{1.634 A_s}{2})}$

$\Rightarrow A_s = 0.141\ in^2$

provide # 3 bar @ $= \frac{.11 \times 12}{.141} = 9.36''$ c/c

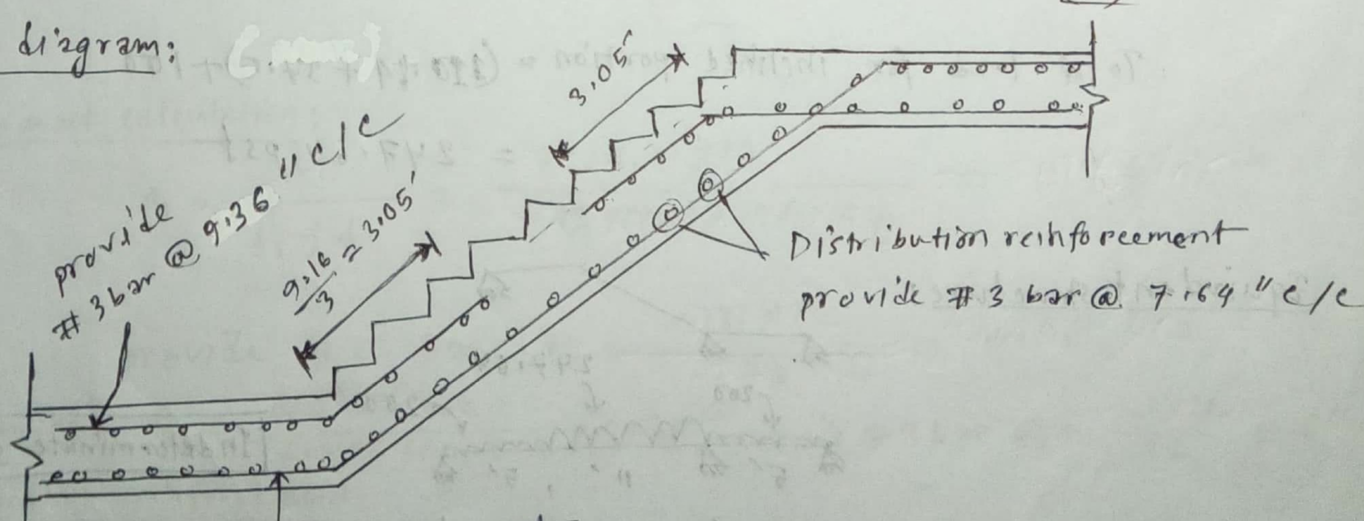
Distribution reinforcement:

$A_{sf} = 0.0020\ bt = 0.0020 \times 12 \times 8 = 0.192\ in^2$

provide # 3 bars @ $\frac{.11 \times 12}{.192} = 6.875''$ c/c

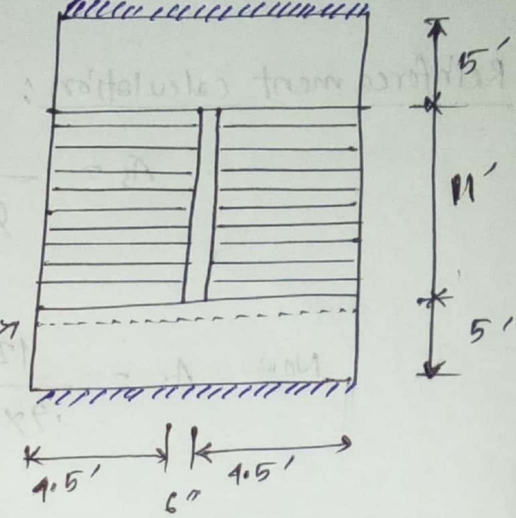
(Ans)

Working diagram:



providing # 5 bar @ $\frac{.31 \times 12}{.52} = 7.15''$ c/c

Problem: Design a stair shown in figure for a live load 100 psf. $f_c' = 3000$ psi & $f_y = 50{,}000$ psi



Solution:

Let thickness of slab = 8"

Dead load for landing, $w_1 = \frac{t}{12} \times 150$

$= \frac{8}{12} \times 150$

$= 100$ psf

$N_T = 10$

$\therefore T = \frac{11}{10} \times 12$

$= 13.2"$

Dead load for inclined portion, $w = w_1 \times \frac{\sqrt{T^2 + R^2}}{T}$

$= 100 \times \frac{\sqrt{13^2 + 6^2}}{13}$

≈ 110.14 psf

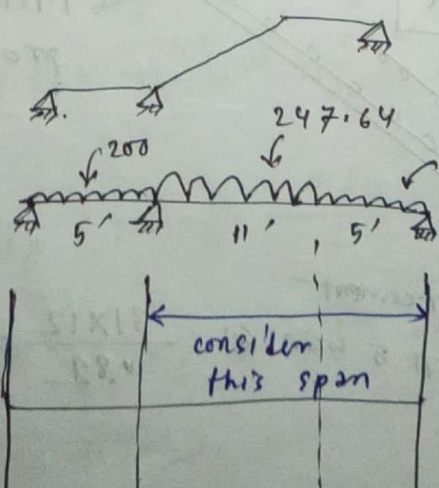
Load due to step on inclined portion $= \frac{R}{2} \times 150 = \frac{6}{2 \times 12} \times 150 = 37.5$ psf

$\therefore$ Total load for landing $= (100 + 100) = 200$ psf

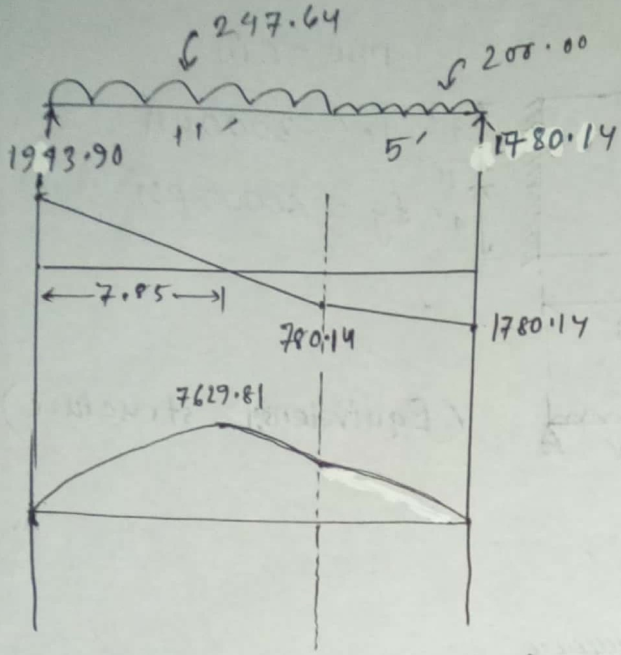
Total load for inclined portion $= (110.14 + 37.5) + 100$

$= 247.64$ psf

Equivalent structure:



Indeterminate structure



∴ $M_{max} = 7629.81$ lb ft

depth check : $n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}}$

$= 9.29 \simeq 9$

$r = \frac{f_s}{f_c} = \frac{.4\times 50000}{.45\times 3000}$

$= 14.81$

∴ $d_{req} = \sqrt{\frac{M}{Rb}}$

$K = \frac{n}{n+r} = \frac{9}{9+14.81} = 0.378$

$= \sqrt{\frac{7629.81\times12}{223\times12}}$

∴ $j = 1 - \frac{K}{3} = 0.874$

$R = \frac{1}{2} f_c jK = \frac{1}{2}\times .45\times3000\times.874\times .378$

$= 5.85$

$= 223$

$d_{act} = \cancel{8} - c/c - \frac{\phi}{2} = (8-1) = 7'' > d_{req}$

(OK)

Reinforcement calculation:

$A_s = \frac{M}{f_s j d} = \frac{7629.81\times12}{(.4\times50000)\times .874\times 7} = 0.75 \text{ in}^2$

provide #5 bars @ $= \frac{.31\times12}{.75} = 4.96''$ c/c

for landing: $A_s = \frac{\frac{200\times(5.5)^2}{8}\times12}{20000\times .874\times 7} = 0.22 \text{ in}^2$ (provide #3 bar @ $= \frac{.11\times12}{.22} = 6''$ c/c)

Distribution reinforcement:

$A_{sf} = 0.002\,bt = .002\times12\times8 = 0.192 \text{ in}^2$

∴ provide #3 bars @ $= \frac{.11\times12}{.192} = 6.875''$ c/c

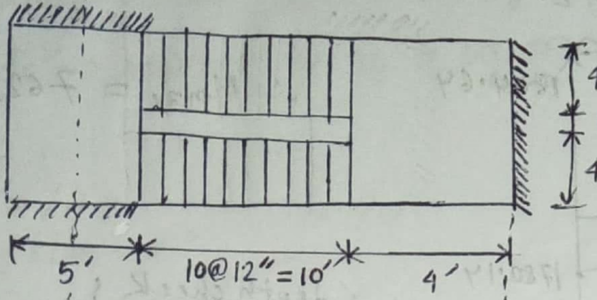
Working diagram: (same)

(Ans)

2015

Problem:

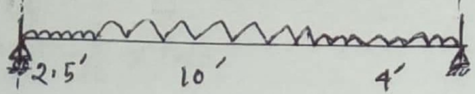
rise = 6 in



$f_c' = 3000\,psi$

$f_y = 60000\,psi$

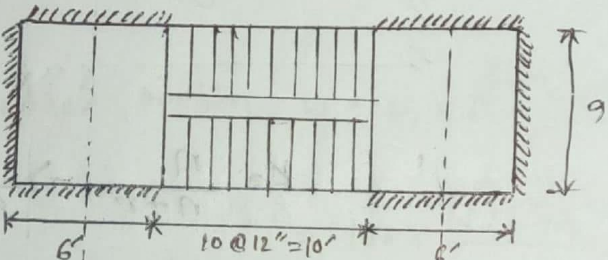
Hints:



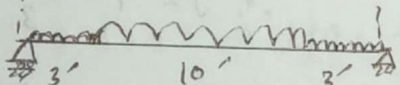
(Equivalent structure)

2012

Problem:



Hints:



(Equivalent structure)

Stirrup

FARHAD
1500045

Problem: Design a beam only for shear to carry ultimate shear force of 27 Kips. consider that no webreinforcement will be provided. Given, $f_c' = 4000$ psi

Solution:

Here, $V_u = 27$ Kips

$\therefore$ Shear capacity, $V_c = 2\lambda\sqrt{f_c'}\, b_w d$

$= 2 \times 1 \times \sqrt{4000} \times b_w d = 126.49\, b_w d$

But, if web reinforcement is not used, $V_c = \frac{1}{2} \times (2\lambda\sqrt{f_c'}\, b_w d)$

$= \frac{1}{2} \times 126.49$

$= 63.25\, b_w d$

Now, $V_u = \phi V_n$

$= \phi (V_c + V_s)$

$= \phi (V_c + 0)$ [$V_s = 0$, because, no web reinforcement will be provided]

$V_u = \phi V_c$

$\Rightarrow (27 \times 1000) = .75 \times 63.25\, b_w d$

$\therefore b_w d = 569.21\ in^2$

Let, $b_w = 16$ in. $\therefore d = \frac{569.21}{16} = 35.58$ in $\simeq 36$ in

(Ans.)

(USD)

Problem: A simple supported beam on a 20 feet span is carrying a factored load of 9.4 K/ft. width of the beam is 16 inch and effective depth of 22 inch. The beam is reinforced with 4 #9 bar.
If $f_c' = 4000$ psi

(i) Up to what part web reinforcement should be provided.

(ii) What will be the spacing of vertical stirrup. if $\boxed{f_y = 60000 \text{ psi}}$ → WSD / USD

Solution:

$$W_u = 9.4 \text{ K/ft}$$

$$\therefore V_{max} = \frac{WL}{2} = \frac{9.4 \times 20}{2} = 94 \text{ K} \quad (V_u \text{ at support})$$

V_u at a distance 'd' from support $= (94 - 9.4d) = 94 - 9.4 \times \frac{22}{12}$

$$\therefore V_u = 76.77 \text{ K}$$

Now,

$$\phi V_c = 0.75 \times 2\sqrt{f_c'}\, b_w d$$

condition: $\boxed{\phi V_c \le V_u}$

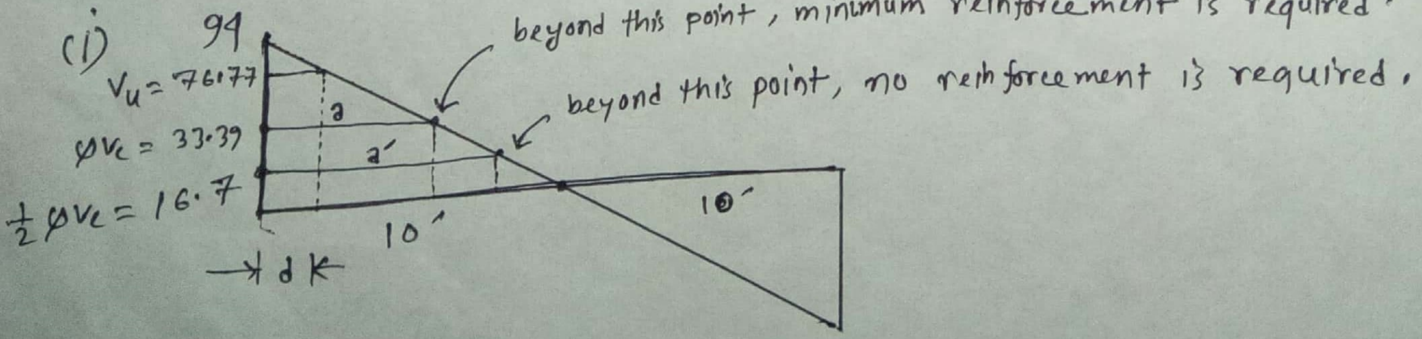
$$= .75 \times 2 \times \sqrt{4000} \times 22 \times 16$$

$$= 33393.65 \text{ lb}$$

$$\phi V_c = 33.39 \text{ K} < V_u.$$

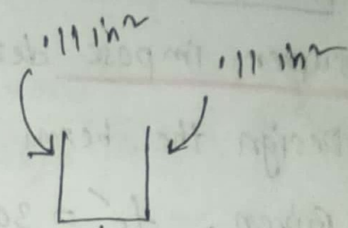
Hence, stirrup are required.

(i)



Now, from similar triangle,

$$\frac{94}{10} = \frac{94-33.39}{a} \Rightarrow a = 6.45'$$



again,

$$\frac{94}{10} = \frac{94-16.7}{a'} \Rightarrow a' = 8.22'$$

(ii) spacing, $S = \dfrac{\phi A_v f_y d}{V_u - \phi V_c} = \dfrac{0.75 \times (2 \times .11) \times 60 \times 22}{76.77 - 33.39} = 5.02''$

[Let, provide #3 bars and U stirrup]

$$S_{max} = \frac{A_v f_y}{.75\sqrt{f_c'}\ b_w} = \frac{.22 \times 60000}{.75\sqrt{4000} \times 16} = 17.39 \text{ in.}$$

$$S_{max} = \frac{A_v f_y}{50\ b_w} = \frac{.22 \times 60000}{50 \times 16} = 15.5 \text{ in}$$

$$S_{max} = \frac{d}{2} = \frac{22}{2} = 11 \text{ in.}$$

$$S_{max} = 24 \text{ in.}$$

Hence, provide #3 U stirrup @ 5" c/c (minimum spacing)

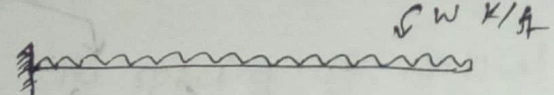
(Ans)

(WSD)

Problem: A cantilever beam of 10 ft span is supporting super impose dead load of 1 K/ft and a live load of 1.1 K/ft. Design the beam for shear. assuming section of the beam 10 in × 18 in

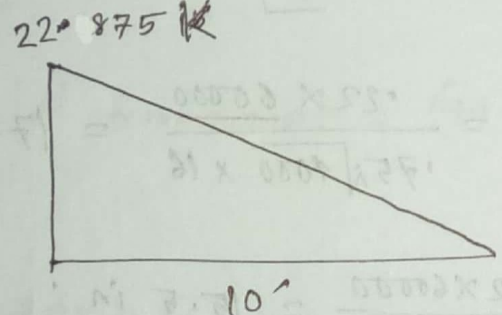
Given, $f_c' = 3000$ psi and $\boxed{f_s = 20000 \text{ psi}}$ → WSD method

Solution:



$$\text{self weight of the beam} = \frac{bh}{144} \times 150 = \frac{10 \times 18}{144} \times 150 = 187.5 \text{ lb/ft}$$

∴ Total working load = $(1 + 1.1 + 0.1875) = 2.2875$ K/ft



∴ V_u at support $= 22.875$ K

V_u at a distance 'd' from support $= 22.875 - 2.2875\,d$

$$= 22.875 - 2.2875 \times \frac{(18 - 2.5)}{12}$$

$$\therefore V_u = 19.92 \text{ K}$$

Now,

$$V_{dev} = \frac{V}{bd} = \frac{19.92}{10 \times 15.5} = 0.13 \text{ Ksi} = 130 \text{ psi}$$

$$V_{all} = 1.1\sqrt{f_c'} = 1.1\sqrt{3000} = 60.25 \text{ psi} < V_{dev}$$

Hence, stirrup are required.

Let, #3 U stirrup be used, $\therefore A_v = (0.11 \times 2) = 0.22\ in^2$

$$\text{Spacing, } s = \frac{A_v f_v}{v' b} = \frac{A_v f_v}{(v_{dev} - v_{all})\, b} = \frac{.22 \times 20000}{(130 - 60.25) \times 10} = 6.31\ in$$

$$\simeq 6.25\ in.$$

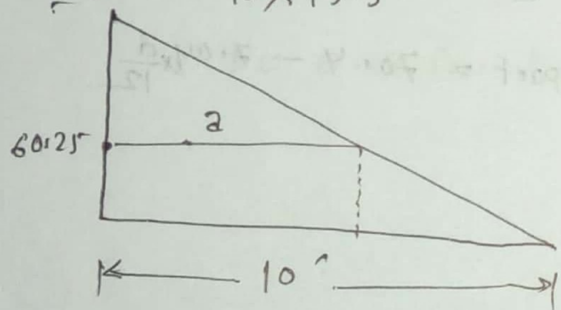
$$S_{max} = \frac{A_v f_v}{50 b_w} = \frac{.22 \times 20000}{50 \times 10} = 8.8\ in.$$

$$S_{max} = \frac{d}{2} = \frac{15.5}{2} = 7.75\ in.$$

$S_{max} = 24$ in.

Hence. #3 U stirrup @ 6.25" c/c

$$v_{max} = \frac{22.875 \times 1000}{10 \times 15.5} = 147.58\ psi$$



From similar triangle,

$$\frac{147.58}{10} = \frac{147.58 - 60.25}{a} \Rightarrow a = 5.92'$$

First stirrup should be placed at a distance $\frac{s}{2}$ from support.

(Ans.)

2006, 2015

Problem: A rectangular beam is to carry a service dead load of 1.6 K/ft including its own weight and service live load of 3.2 K/ft on a simple span of 20 ft. Select the width and effective depth of the beam in which web reinforcement provides shear strength $V_s = 2V_c$. Use $f_c' = 4000$ psi. Find the spacing of verticle stirrup at a distance 4 ft from support if $f_y = 40000$ psi

Solution: (i) Here, D.L = 1.6 K/ft

L.L = 3.2 K/ft

$$\therefore w_u = (1.6 \times 1.2 + 1.6 \times 3.2) = 7.04 \text{ K/ft}$$

Now, V_u at support $= \frac{wL}{2} = \frac{7.04 \times 20}{2} = 70.4$ K

V_u at a distance 'd' from support $= 70.4 - 7.04 \times \frac{d}{12}$

$$V_u = \phi(V_c + V_s)$$

$$\Rightarrow 70.4 - 7.04 \times \frac{d}{12} = \phi(V_c + 2V_c)$$

$$\Rightarrow 70.4 - 7.04 \times \frac{d}{12} = 3\phi V_c = \frac{3 \times 0.75 \times 2\sqrt{4000}\, b_w d}{1000}$$

$$\Rightarrow d = \frac{70.4}{0.2846\, b_w + 0.587}$$

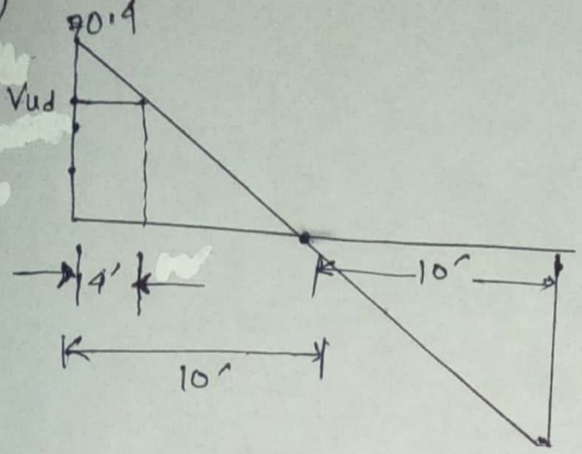
Let, $b_w = 12''$ $\quad \therefore d = 17.5$ in.

$$\therefore V_u = 70.4 - 7.04 \times \frac{17.5}{12} = 60.13 \text{ K}$$

$$\phi V_c = \phi 2\sqrt{4000}\, b_w d = .75 \times 2 \times \sqrt{4000} \times 12 \times 17.5 = 19922.35 \text{ lb}$$

$$\therefore \phi V_c = 19.92 \text{ K}$$

(ii)



From similar triangle,

$$\frac{70.4}{10} = \frac{V_{ud}}{10-4} \Rightarrow V_{ud} = 42.24\ K$$

Let, #3 U stirrup be used.

Now, spacing, $S = \frac{\phi A_v f_y d}{V_{ud} - \phi V_c} = \frac{0.75 \times .22 \times 40 \times 17.5}{42.24 - 19.92}$

$\therefore S = 5.17$ in. $\simeq 5''$

(Ans.)