



**STRUCTURAL**

**ANALYSIS**

**&**

**DESIGN**

HandNote On

# STRUCTURAL ANALYSIS & DESIGN



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Influence Line
Wheel Load
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Influence Line

Equations

2D

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M = 0$$

3D

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

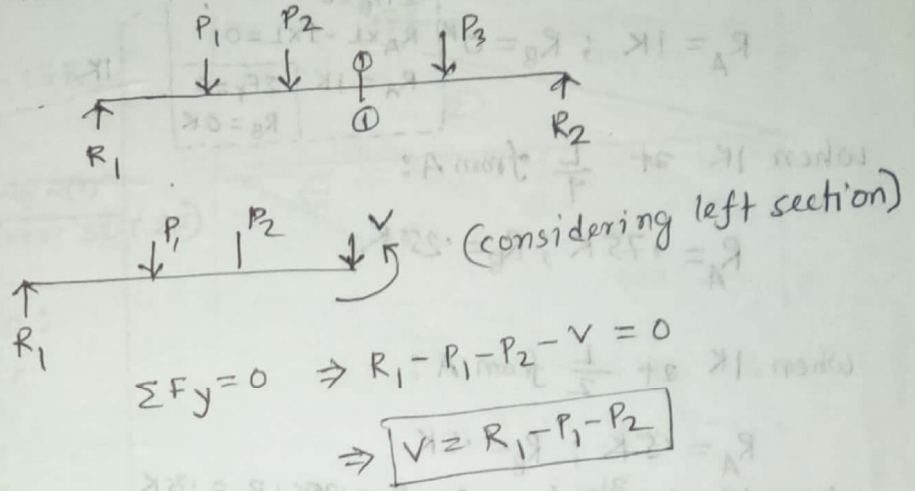
$$\Sigma F_z = 0$$

$$\Sigma M_x = 0$$

$$\Sigma M_y = 0$$

$$\Sigma M_z = 0$$

Shear force: Algebraic sum of all transverse forces either considering left or right section of a beam is called shear force.



Kinds of Loads:

1. Dead Loads.
2. Live Loads.

Dead loads: Dead loads are loads which are always fixed in position, always acting and of unchanging magnitude.

Live loads: Live loads are loads which are more or less temporary and which vary in magnitude.

Influence Lines: An influence line is a diagram showing the variation in shear, moment, stress in a member, reaction or other direct function due to a unit load moving across the structure.

construction of influence line: An influence line is constructed by plotting directly under the point where unit load is placed an ordinate, the height of which represents to some scale the value of the particular function being studied when the load is in that position.

Influence line for Reaction:

# Draw IL for  $R_A$  and  $R_B$

When 1K at A:

$R_A = 1K ; R_B = 0K$

$$\begin{aligned} \sum M_B = 0 & \Rightarrow R_A \times L - 1 \times L = 0 \\ R_A = 1K & \quad \sum F_y = 0 \\ R_B = 0K & \end{aligned}$$

When 1K at  $\frac{L}{4}$  from A:

$R_A = .75K ; R_B = .25K$

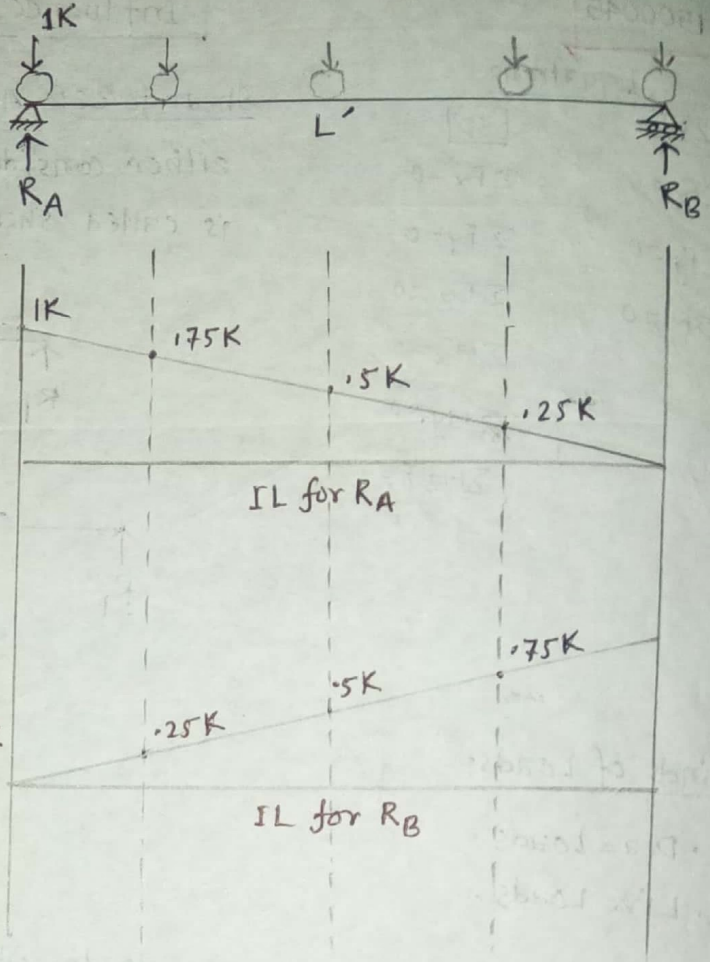
When 1K at  $\frac{L}{2}$  from A:

$R_A = .5K ; R_B = .5K$

When 1K at  $\frac{3L}{4}$  from A:  $R_A = .25K ; R_B = .75K$

When 1K at B:

$R_A = 0 ; R_B = 1K$



In case of distributed load:

$\frac{1}{L} = \frac{i}{x}$  [From similar Triangle]

$\Rightarrow i = \frac{x}{L}$

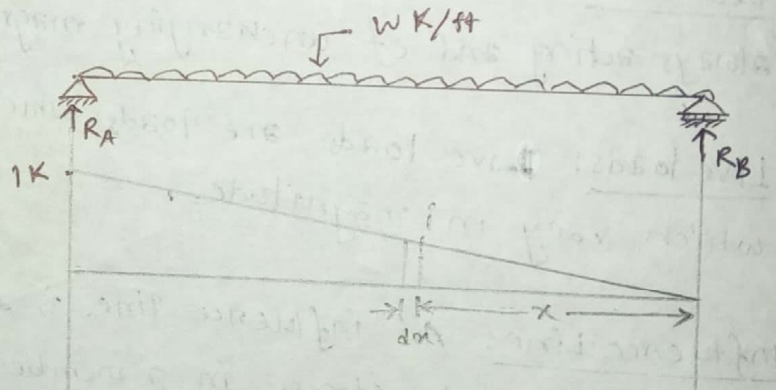
Now,  $\int dR_A = \int_0^L i \times w \times dx$

$\Rightarrow R_A = \int_0^L \frac{x}{L} \times w \times dx$

$\therefore R_A = \left[ \frac{w x^2}{2L} \right]_0^L = \frac{wL}{2}$

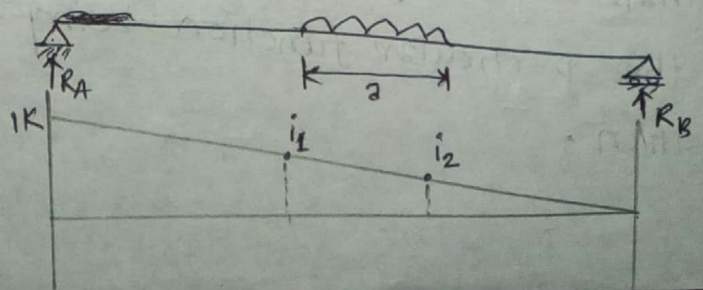
$= \frac{1}{2} \times (1 \times L) \times w$

Reaction = Area of influence line  $\times$  distributed load.



$R_A = \frac{i_1 + i_2}{2} \times a \times w$

Area of influence line



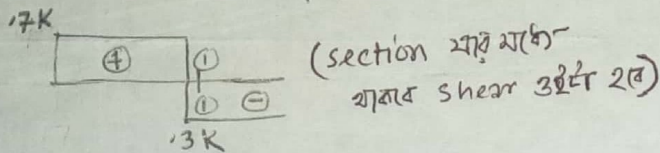
Influence Line for shear force:  
(# Draw IL for  $V_{1-1}$ )

When 1K at A:

$$R_A = 1K, R_B = 0; V_{1-1} = 0K$$

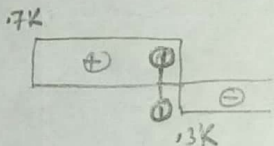
When 1K at just left of section ①-①:

$$R_A = .7K; R_B = .3K; V_{1-1} = -.3K$$



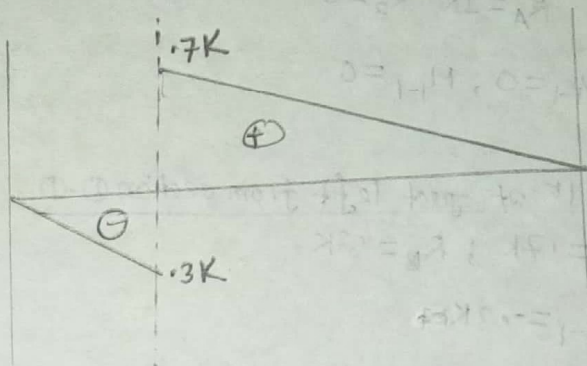
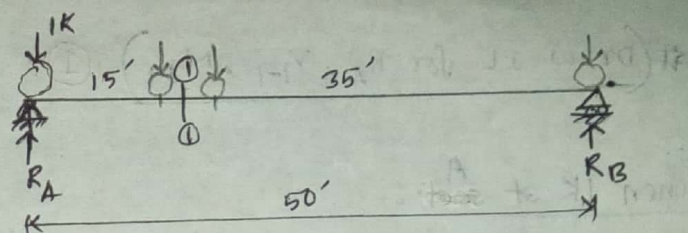
When 1K at just right of section ①-①:

$$R_A = .7K, R_B = .3K; V_{1-1} = .7K$$



When 1K at B:

$$R_A = 0; R_B = 1K; V_{1-1} = 0K$$



Influence Line for Moment:  
(# Draw IL for  $M_{1-1}$ )

When 1K at A:

$$R_A = 1K, R_B = 0K; M_{1-1} = 0$$

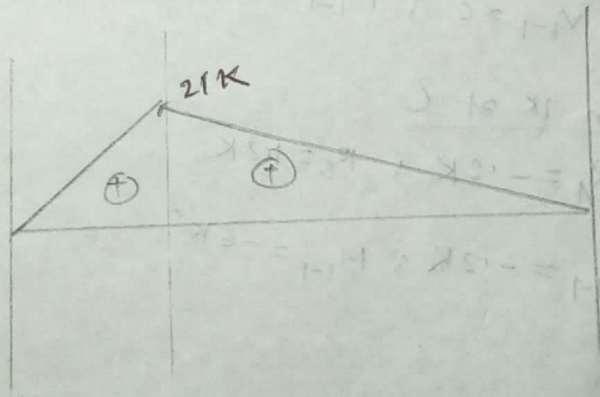
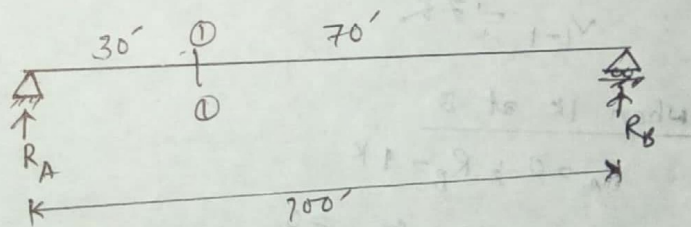
When 1K at section ①-①:

$$R_A = .7K, R_B = .3K; M_{1-1} = 21K$$

$$[M_{1-1} = .7 \times 30]$$

When 1K at B:

$$R_A = 0K, R_B = 1K; M_{1-1} = 0$$



# (Draw IL for  $R_A$ ,  $V_{1-1}$ ,  $M_{1-1}$ ) (1)

When 1K at A:

$$R_A = 1K; R_B = 0$$

$$V_{1-1} = 0; M_{1-1} = 0$$

When 1K at just left from section ①-①

$$R_A = 1.7K; R_B = 1.3K$$

$$V_{1-1} = -0.3K$$

When 1K at ~~left~~ section ①-①

$$R_A = 1.7K; R_B = 1.3K$$

$$M_{1-1} = 2.1K'$$

When 1K at just right from section ①-①

$$R_A = 1.7K; R_B = 1.3K$$

$$V_{1-1} = 0.7K$$

When 1K at B

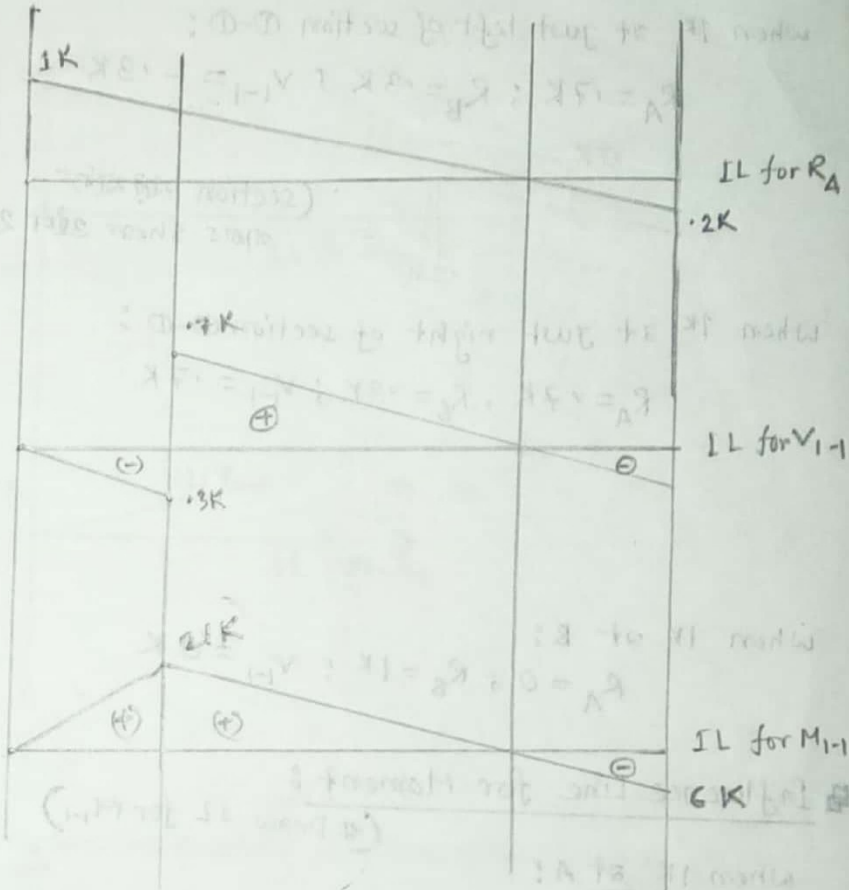
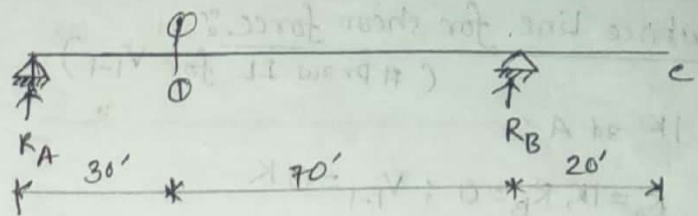
$$R_A = 0; R_B = 1K$$

$$V_{1-1} = 0; M_{1-1} = 0$$

When 1K at C

$$R_A = -0.2K; R_B = 1.2K$$

$$V_{1-1} = -0.2K; M_{1-1} = -6K'$$

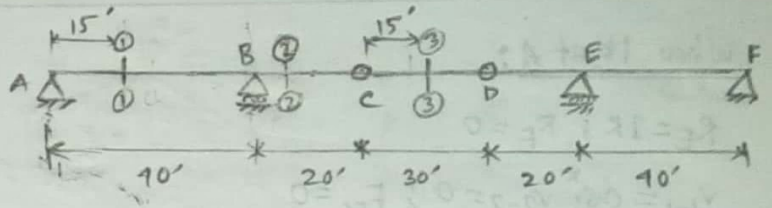


# Draw IL for  $V_{1-1}, V_{2-2}, V_{3-3}, M_{1-1}, M_{2-2}, M_{3-3}$  (2)

When 1K at A:  $R_A = 1K; R_B = 0K;$

$$V_{1-1} = 0; V_{2-2} = 0; V_{3-3} = 0$$

$$M_{1-1} = 0; M_{2-2} = 0; M_{3-3} = 0$$



When 1K at just left of section ①-①

$$R_A = 0.625K; R_B = 0.375K$$

$$V_{1-1} = -0.375K; V_{2-2} = 0; V_{3-3} = 0$$

When 1K at section ①-①

$$R_A = 0.625K; R_B = 0.375K$$

$$M_{1-1} = 0.375K; M_{2-2} = 0; M_{3-3} = 0K'$$

When 1K at just right of section ①-①

$$R_A = 0.625K; R_B = 0.375K$$

$$V_{1-1} = 0.625K; V_{2-2} = 0; V_{3-3} = 0$$

When 1K at B:  $R_A = 0; R_B = 1K$

$$V_{1-1} = 0; V_{2-2} = 0; V_{3-3} = 0$$

$$M_{1-1} = 0; M_{2-2} = 0; M_{3-3} = 0$$

When 1K at just Right of section ②-②

$$V_{1-1} = 0; V_{2-2} = 1K; V_{3-3} = 0$$

$$M_{1-1} = 0; M_{2-2} = 0; M_{3-3} = 0$$

When 1K at C:

$$R_A = -0.5K; R_B = 1.5K$$

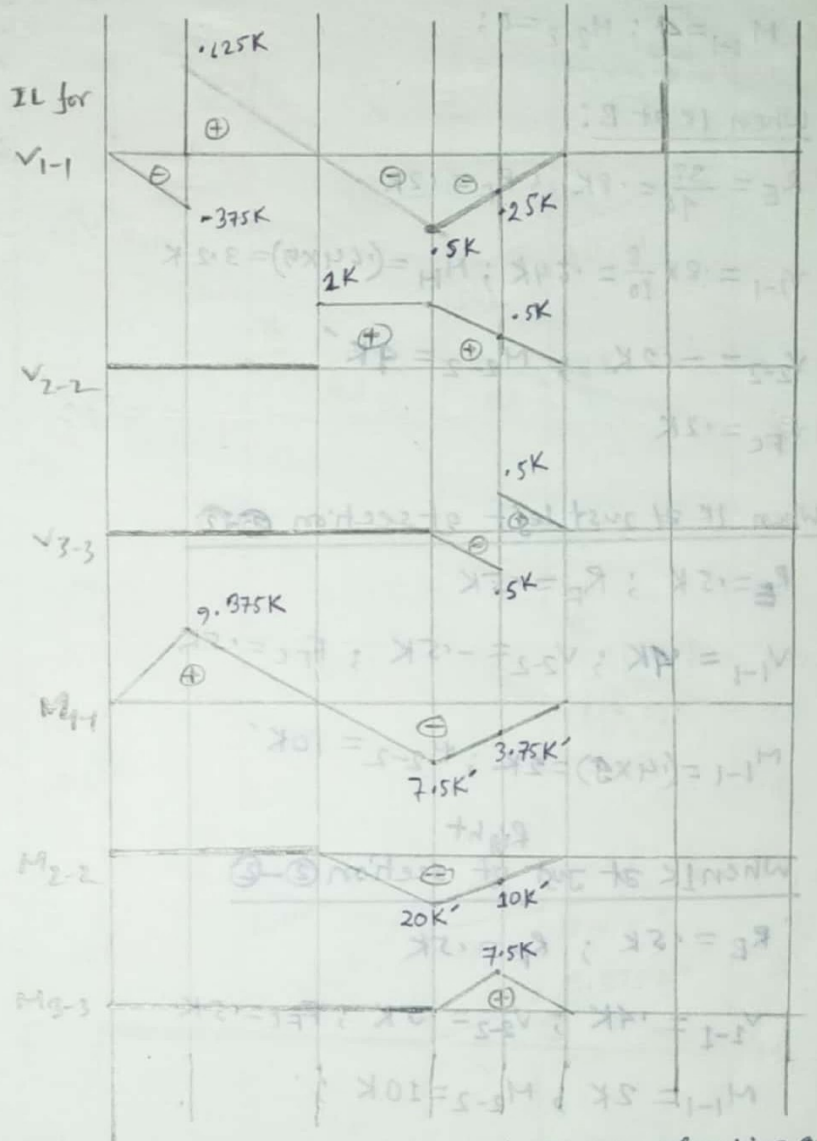
$$V_{1-1} = -0.5K; V_{2-2} = 1K; V_{3-3} = 0$$

When 1K at just left of section ③-③

$$R_A = -0.25K; R_B = 0.75K$$

$$\therefore V_{1-1} = -0.25K, V_{2-2} = 0.5K, V_{3-3} = -0.5K$$

$$M_{1-1} = -3.75K', M_{2-2} = -10K'; M_{3-3} = 7.5K'$$



When 1K at just Right of section ③-③

$$R_A = -0.25K; R_B = 0.75K$$

$$V_{1-1} = -0.25K; V_{2-2} = 0.5K; V_{3-3} = 0.5K$$

$$M_{1-1} = -3.75K'; M_{2-2} = -10K'; M_{3-3} = 7.5K'$$

When 1K at D:

$$V_{1-1} = 0; V_{2-2} = 0; V_{3-3} = 0$$

$$M_{1-1} = 0; M_{2-2} = 0; M_{3-3} = 0$$



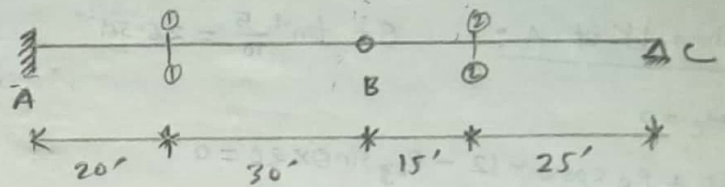
# Draw IL for  $V_{1-1}, V_{2-2}, M_{1-1}, M_{2-2}, M_A$ : (5)

When 1K at A

$$R_A = 1; R_C = 0$$

$$V_{1-1} = 0; V_{2-2} = 0; M_A = 0$$

$$M_{1-1} = 0; M_{2-2} = 0$$



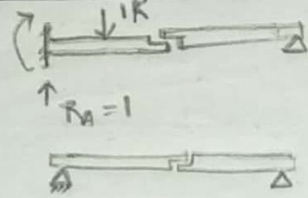
When 1K at just left of section 1-1

$$R_A = 1K; R_C = 0K$$

$$V_{1-1} = 0; V_{2-2} = 0$$

$$M_{1-1} = 0; M_{2-2} = 0$$

$$M_A = -20K'$$



When 1K at just right of section 1-1

$$R_A = 1K; R_C = 20K$$

$$V_{1-1} = 1K; V_{2-2} = 0$$

$$M_{1-1} = -20K'; M_{2-2} = 0$$

When 1K at B

$$R_A = 1K; R_C = 0K$$

$$V_{1-1} = 1K; V_{2-2} = 0$$

$$M_{1-1} = -30K'; M_{2-2} = 0$$

$$M_A = -50K'$$

When 1K at just left of section 2-2

$$R_C = \frac{15}{40} = 0.375K; R_B = 0.625K$$

$$V_{1-1} = 0.625K; V_{2-2} = -0.375K$$

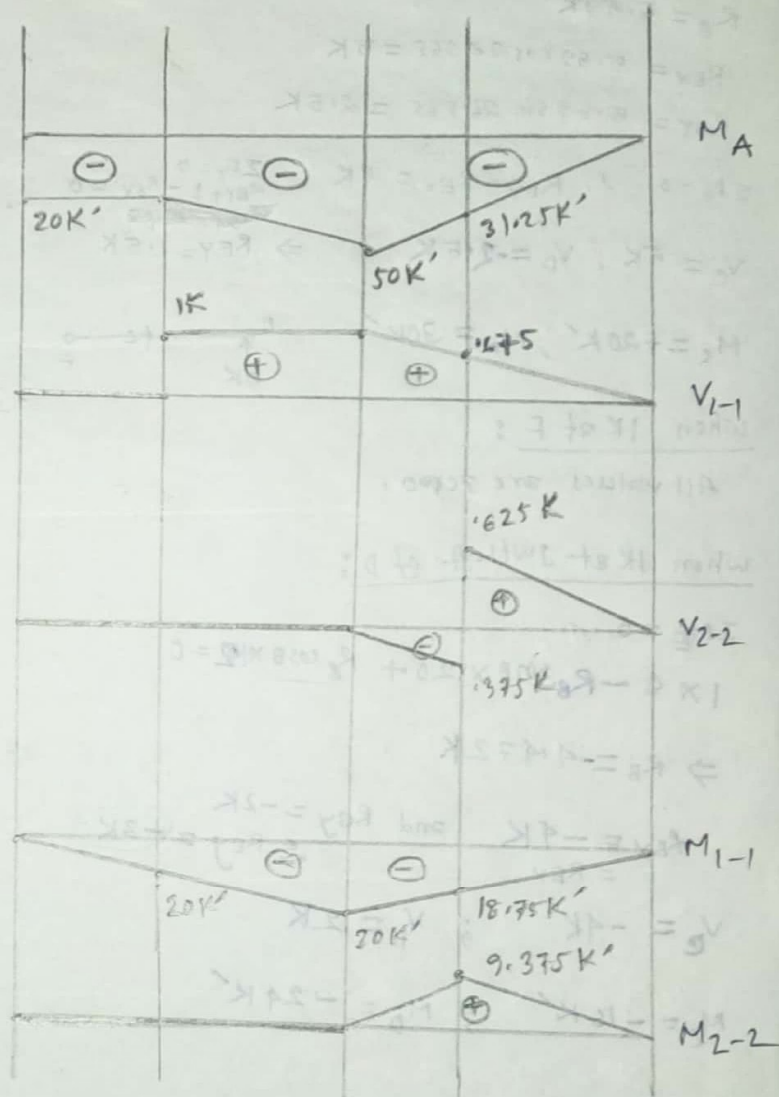
$$M_{1-1} = -18.75K'; M_{2-2} = 9.375K'; M_A = -31.25$$

When 1K at just right of section 2-2

$$R_C = 0.375K; R_B = 0.625K$$

$$V_{1-1} = 0.625K; V_{2-2} = 0.625K$$

$$M_{1-1} = -18.75K'; M_{2-2} = 9.375K'; M_A = -31.25$$



When 1K at C:

$$V_{1-1} = 0; V_{2-2} = 0$$

$$M_{1-1} = 0; M_{2-2} = 0$$

$$M_A = 0$$

# Draw IL for  $V_c, V_D, M_c, M_D$  and  $R_B$  (5)

When 1K at A:

$$\theta = \tan^{-1} \frac{5}{10} = 26.565^\circ$$

$$\sum M_E = 0$$

$$-1 \times 10 + R_B \cos \theta \times 12 - R_{By} \sin \theta \times 20 = 0$$

$$R_B = 5.59 \text{ K}$$

$$R_{Bx} = 5.59 \cos 26.565 = 5 \text{ K}$$

$$R_{By} = 5.59 \sin 26.565 = 2.5 \text{ K}$$

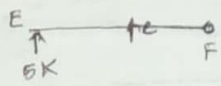
$$\sum F_x = 0 \quad \therefore R_{Bx} = R_{Ex} = 5 \text{ K}$$

$$\sum F_y = 0 \quad R_{Ey} + 1 - R_{By} = 0$$

$$\Rightarrow R_{Ey} = 1.5 \text{ K}$$

$$V_c = 5 \text{ K}, \quad V_D = -2.5 \text{ K}$$

$$M_c = +20 \text{ K}', \quad M_D = 30 \text{ K}'$$



When 1K at F:

All values are zero.

When 1K at just left of D:

$$\sum M_E = 0$$

$$1 \times 8 - R_B \sin \theta \times 20 + R_B \cos \theta \times 12 = 0$$

$$\Rightarrow R_B = -4.472 \text{ K}$$

$$R_{Bx} = -4 \text{ K} \quad \text{and} \quad R_{By} = -2 \text{ K}$$

$$\therefore R_{Ey} = -3 \text{ K}$$

$$V_c = -4 \text{ K} \quad ; \quad V_D = 2 \text{ K}$$

$$M_c = -16 \text{ K}' \quad ; \quad M_D = -24 \text{ K}'$$

When 1K at just right of D:

$$V_D = +3 \text{ K}$$

When 1K at B:

$$\sum M_E = 0$$

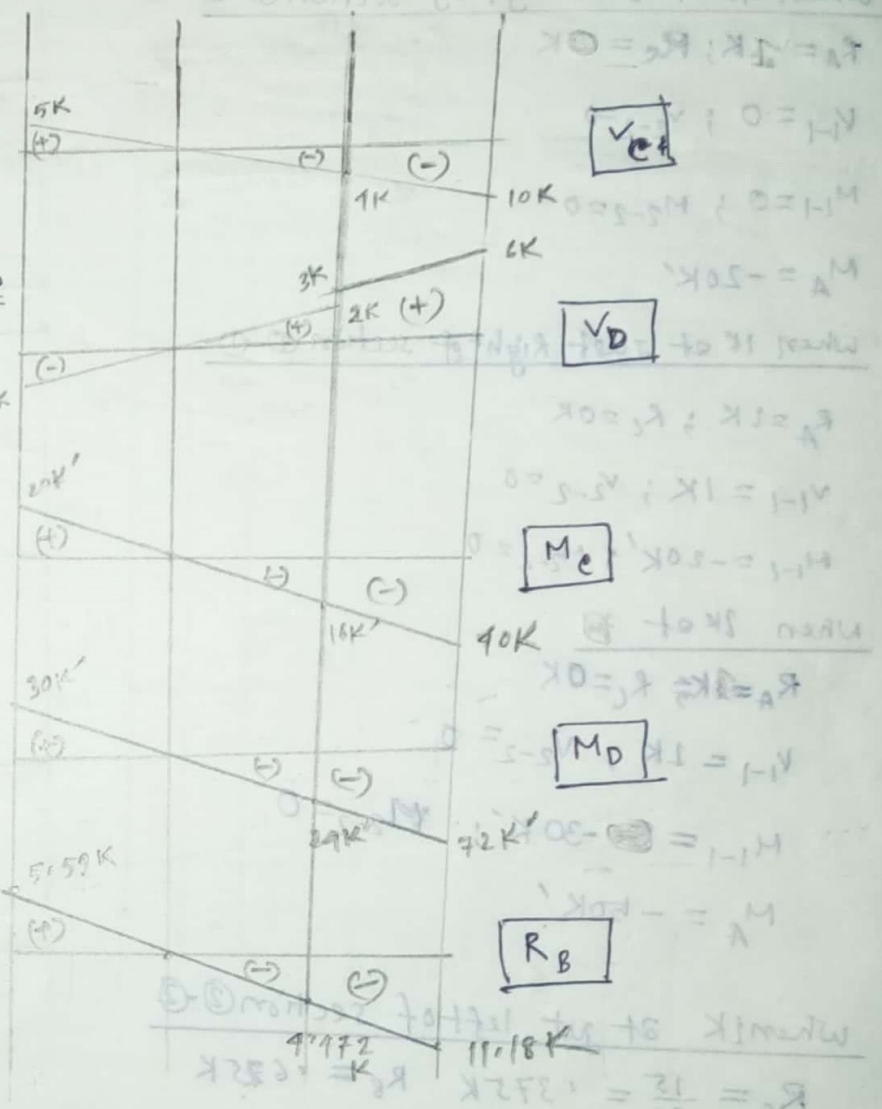
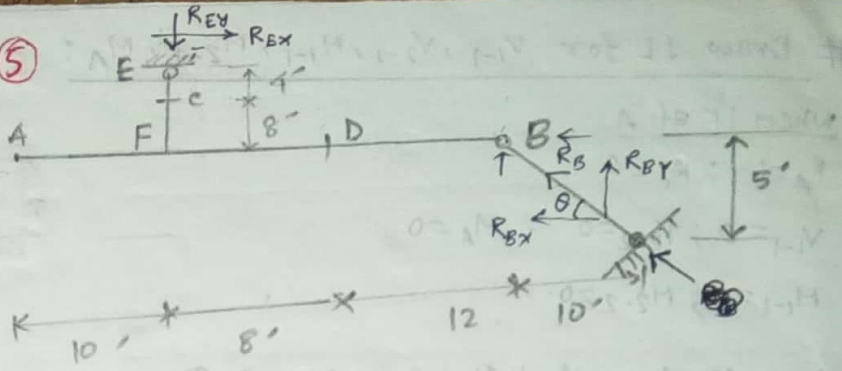
$$1 \times 20 + R_B \cos \theta \times 12 - R_B \sin \theta \times 20 = 0$$

$$\Rightarrow R_B = 11.18 \text{ K}$$

$$R_{Bx} = -10 \text{ K} \quad R_{By} = -5 \text{ K}$$

$$\therefore R_{Ex} = -10 \text{ K}, \quad R_{Ey} = -5 \text{ K}$$

$$V_c = -10 \text{ K}, \quad V_D = 5 \text{ K}, \quad M_c = -40 \text{ K}', \quad M_D = -72 \text{ K}'$$



When 1K at just right of D:

When 1K at E

$$R_A = 0; R_B = 0; R_D = 1.25K, R_C = -1.25K$$

$$V_a = 0; V_b = -0.25K; V_c = -0.25K$$

$$M_a = 0; M_b = 0; M_c = -7.5K'$$

When 1K at F

$$R_F = 1K; \text{ all values are zero.}$$

# Draw SL for  $R_A, R_B, V_b, V_c, M_b, M_c$ . MC: 5

When 1K at A:

$$R_A = 1K; R_B = 0$$

$$V_b = 0, V_c = 0$$

$$M_b = 0, M_c = 0, M_e = 0$$

When 1K at just left of section (b)

$$R_A = 0.6K; R_B = 0.4K$$

$$V_b = -0.4K; V_c = -0.4K$$

$$M_b = 12K'; M_c = 6K'$$

$$M_c = -12K'$$

When 1K at just right of section (b)

$$V_b = +0.6K \text{ and same as before}$$

When 1K at B

$$R_A = 0; R_B = 1K$$

$$V_b = 0; V_c = -1K$$

$$M_b = 0; M_c = -15K'$$

$$M_c = -30K'$$

When 1K at just <sup>left</sup> of section (c)

$$R_A = 0, R_B = 0, R_C = 1K$$

$$V_b = 0; V_c = +1K$$

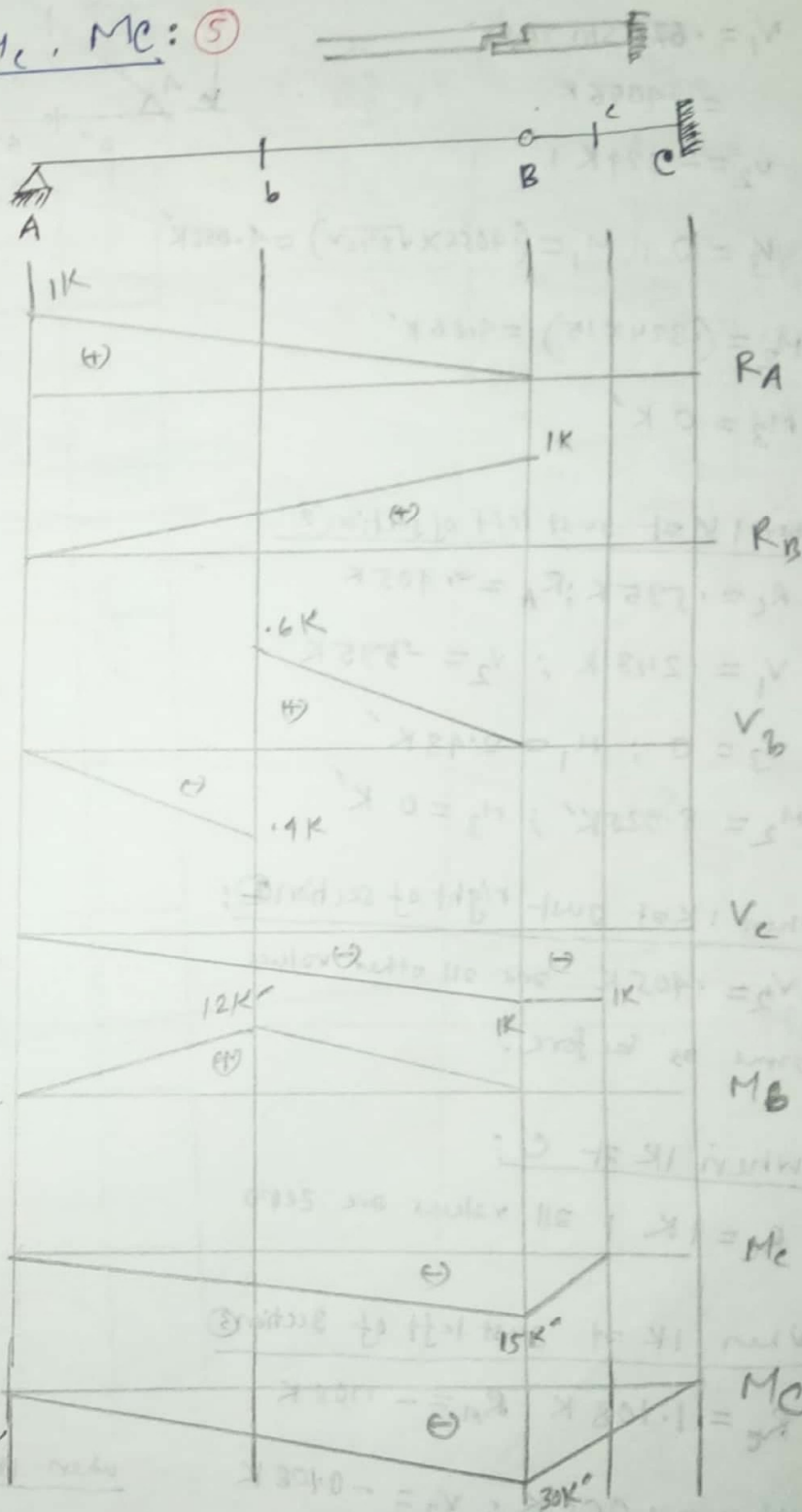
$$M_b = 0; M_c = 0K'; M_e = -15K'$$

When 1K at just right of section (c)

$$V_c = 0; \text{ and same as before.}$$

When 1K at C

$$R_C = 1K, \text{ and all other values are zero.}$$



# Draw IL for  $R_c$ ,  $V_1, V_2, V_3, M_1, M_2, M_3$ : (load moves from B to D) (6)

When 1K at B:

$R_c = .324 K, R_A = .676 K$

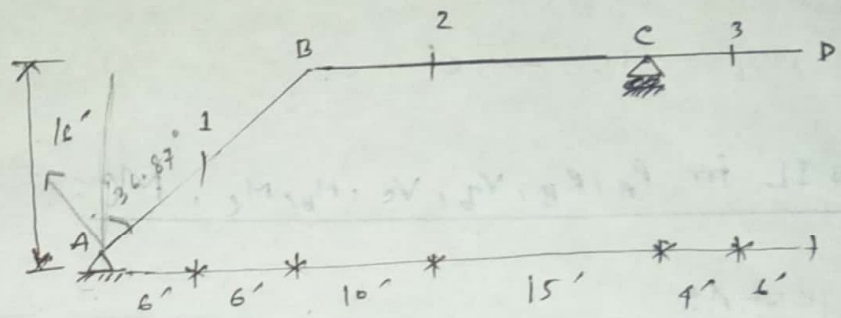
$V_1 = .876 \sin 36.87^\circ = .4056 K$

$V_2 = -.324 K$

$V_3 = 0; M_1 = (.4056 \times \sqrt{84} \times 2) = 4.056 K'$

$M_2 = (.324 \times 15) = 4.86 K'$

$M_3 = 0 K'$



When 1K at just left of section 2:

$R_c = .595 K; R_A = .405 K$

$V_1 = .243 K; V_2 = -.595 K$

$V_3 = 0; M_1 = 2.43 K'$

$M_2 = 8.925 K'; M_3 = 0 K'$

When 1K at just right of section 2:

$V_2 = .405 K$  and all other values same as before.

When 1K at C:

$R_c = 1 K$ ; all values are zero

When 1K at just left of section 3:

$R_c = 1.108 K, R_A = -.108 K$

$V_1 = -.065 K, V_2 = -.108 K$

$M_1 = -.065 K'; M_2 = -2.38 K'$

$M_3 = 6 K'; V_3 = 0 K'$

When 1K at just right of section 3:

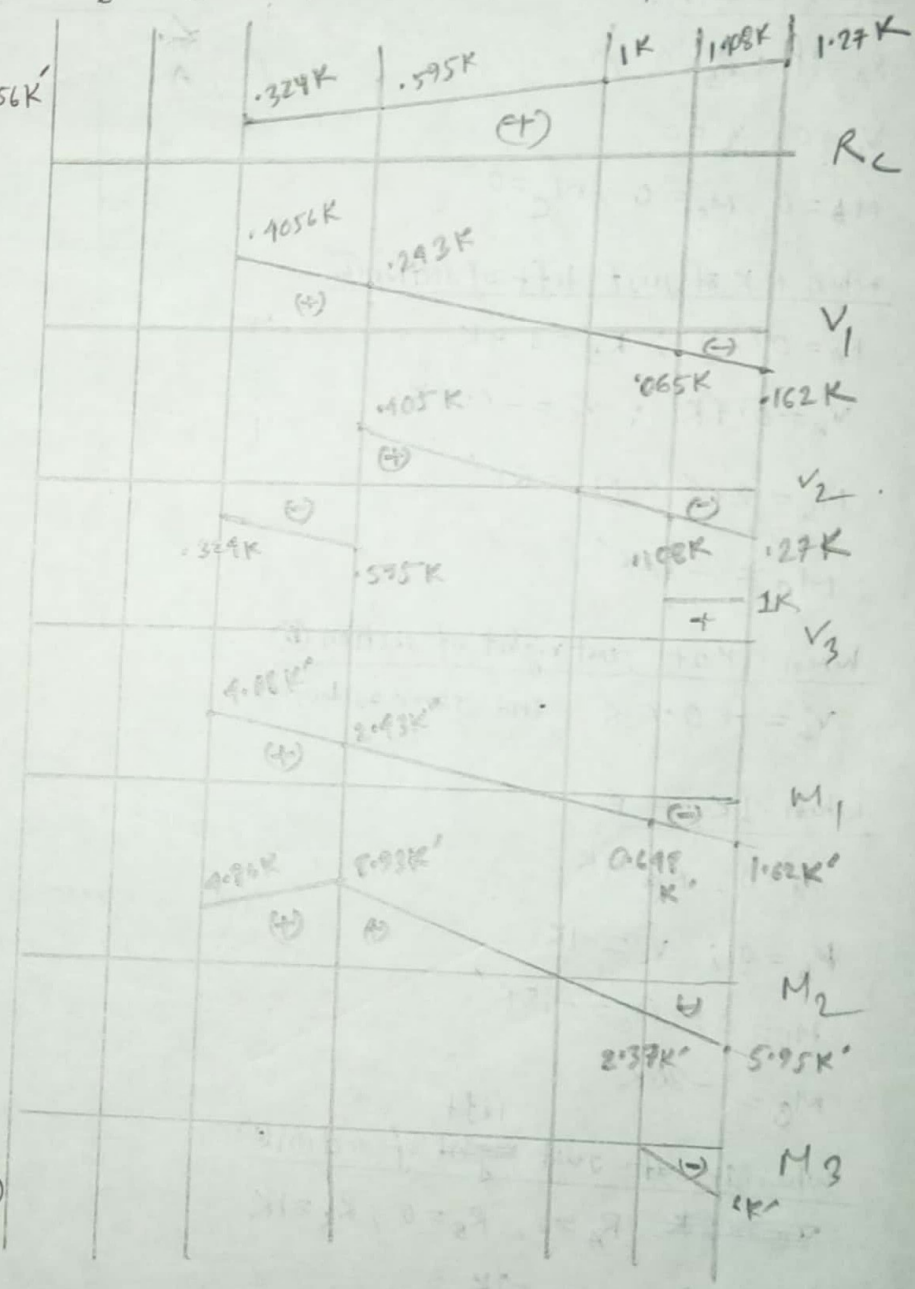
$V_3 = 1 K$

When 1K at D:

$R_c = 1.27 K, R_A = -.27 K$

$V_1 = -.162 K; V_2 = -.27 K; M_3 = 1 K$

$M_1 = -.162 K'; M_2 = -5.95 K'; M_3 = -6 K'$



# Draw IL for  $V_1, V_2, M_1, M_2$ : (7)

When 1K at A

$$R_C = -0.33K (\downarrow)$$

$$R_B = 1.33K (\uparrow)$$

$$V_1 = 0.33K$$

$$V_2 = -0.33K$$

$$M_1 = 0K'; M_2 = -2K'$$

When 1K at B

$R_B = 1K$  and all values are zero

When 1K at just left of section ①

$$R_B = 0; R_C = 1K (\uparrow)$$

$$V_1 = -1K; V_2 = +1K$$

$$M_1 = 0K'; M_2 = 6K'$$

When 1K just right of section ②

$$V_1 = 0K; \text{all are same as}$$

before.

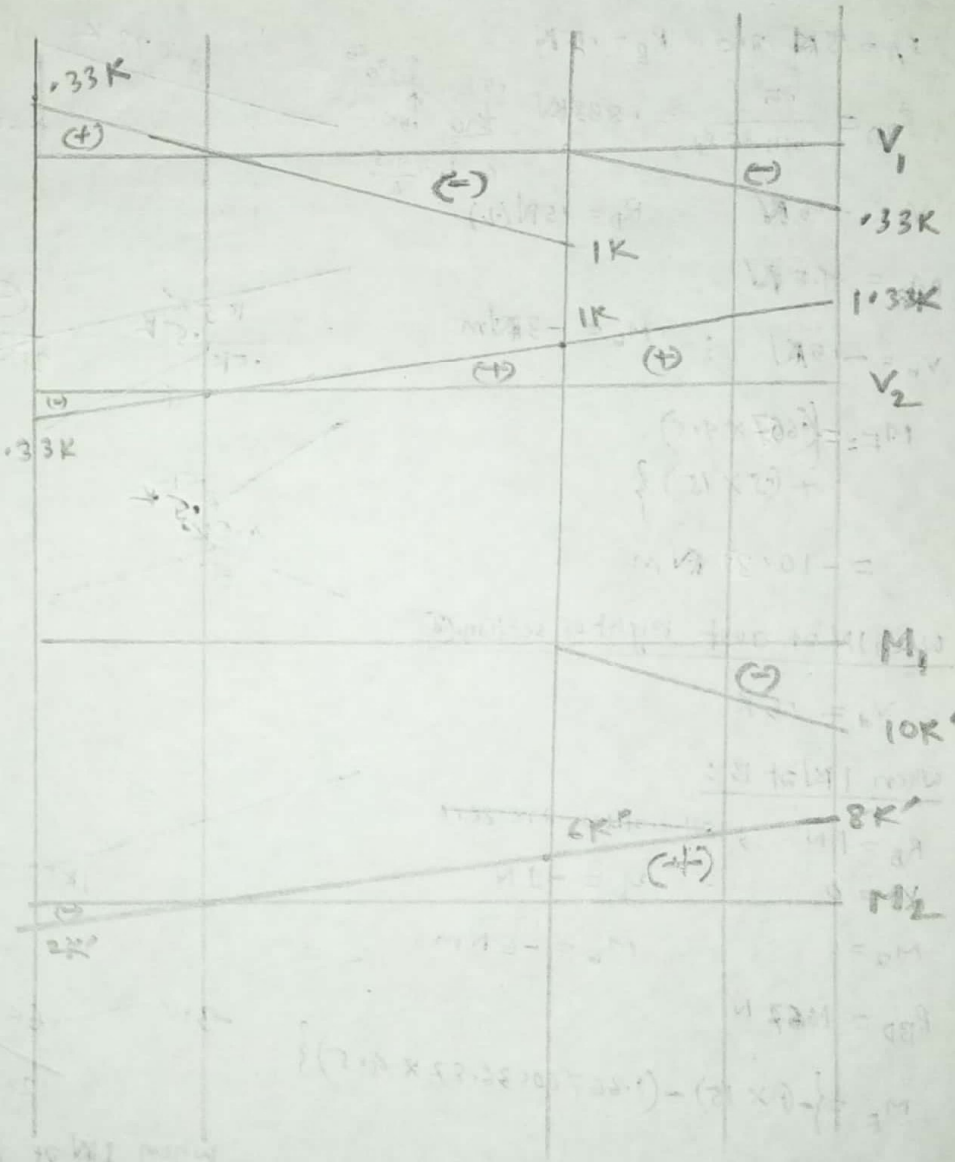
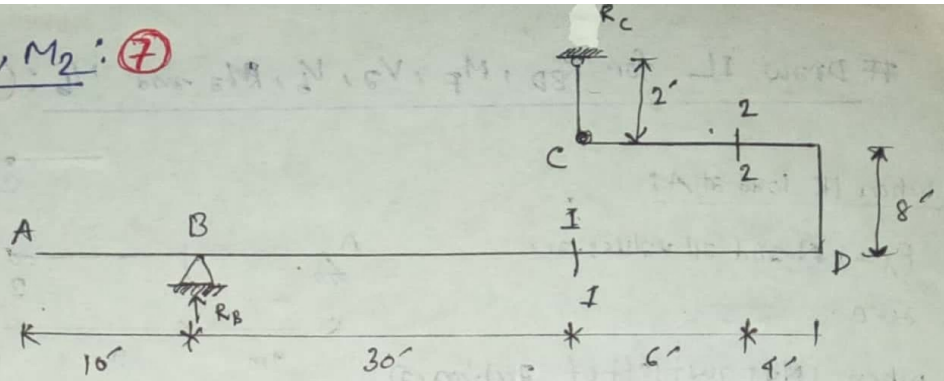
When 1K at ~~the~~ D:

$$R_C = 1.33K (\uparrow)$$

$$R_B = -0.33K (\downarrow)$$

$$V_1 = -0.33K; V_2 = 1.33K$$

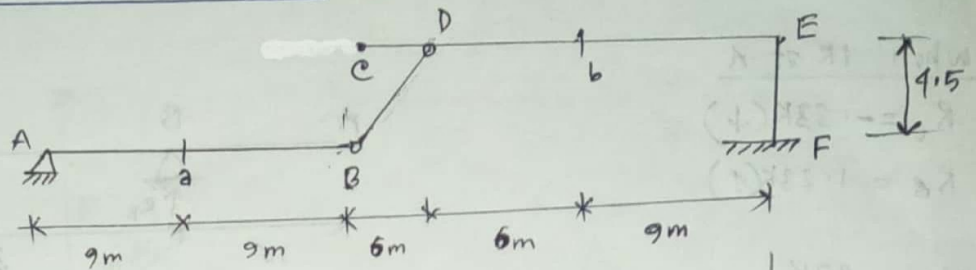
$$M_1 = -10K'; M_2 = 8K'$$



# Draw IL for  $R_{BD}$ ,  $M_F$ ,  $V_a$ ,  $V_b$ ,  $M_a$  and  $M_b$ : (12)

When 1K load at A:

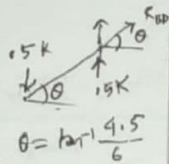
$R_A = 1\text{N}$  and all values are zero.



When 1K at just left of section (a)

$R_A = 0.5\text{N}$  and  $R_B = 0.5\text{N}$

$$R_{BD} = \frac{0.5}{\sin 36.87} = 0.833\text{N}$$

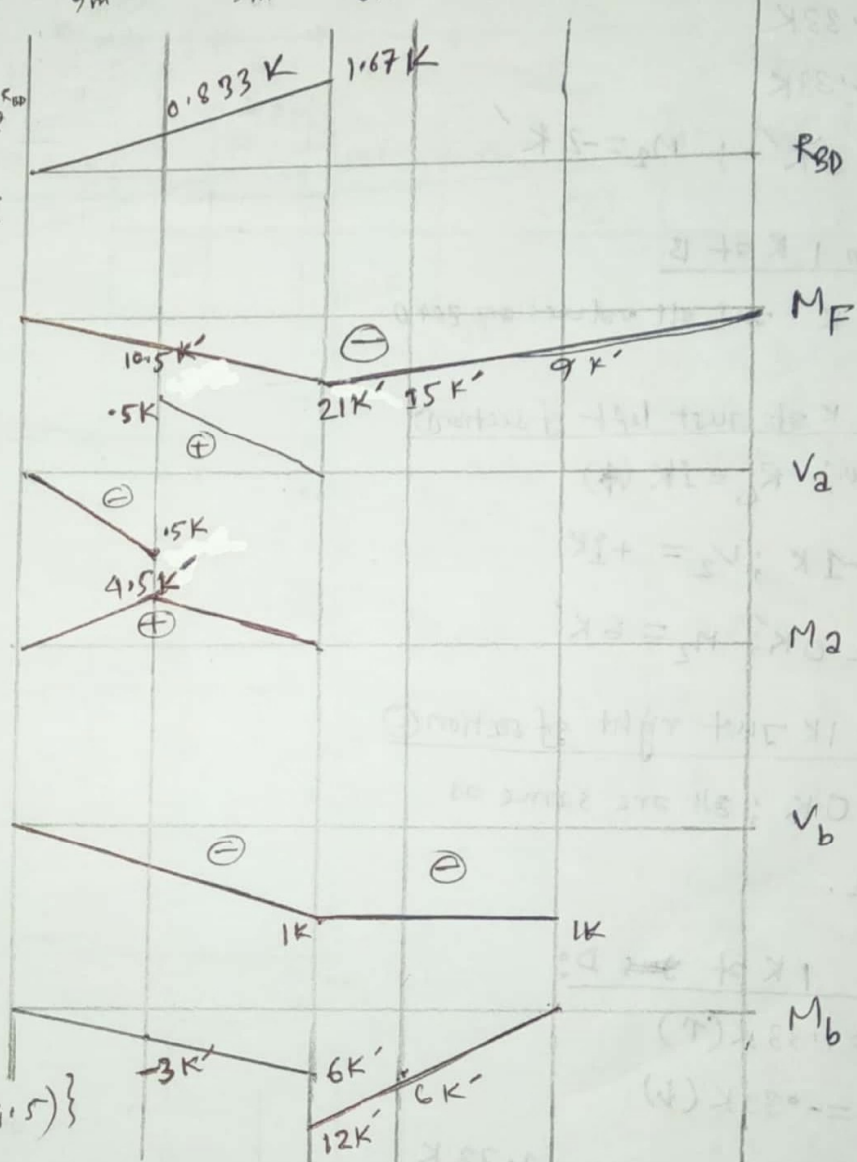
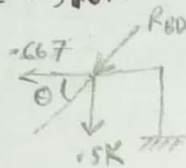


$V_a = -0.5\text{N}$  ;  $R_D = 15\text{N}(\downarrow)$

$M_a = 4.5\text{Nm}$

$V_b = -0.5\text{N}$  ;  $M_b = -3\text{Nm}$

$$M_F = \left\{ \begin{array}{l} -(0.667 \times 9.5) \\ + (0.5 \times 15) \end{array} \right\} = -10.5\text{Nm}$$



When 1N at just right of section (a)

$V_a = 0.5\text{K}$

When 1K at B:

$R_B = 1\text{N}$  ; all values are zero

$V_a = 0$  ;  $V_b = -1\text{N}$

$M_a = 0$  ;  $M_b = -6\text{Nm}$

$R_{BD} = 1.667\text{N}$

$$M_F = \left\{ \begin{array}{l} -(1 \times 15) \\ -(1.667 \cos 36.87 \times 4.5) \end{array} \right\} = -21\text{Nm}$$

When 1K at C:  $R_D = 0$

$R_{BD} = 0$  ;  $V_a = 0$  ;  $M_a = 0$

$R_F = 1\text{N}$  ;  $V_b = -1\text{K}$  ;  $M_b = -12\text{Nm}$

$M_F = -21\text{Nm}$

When 1K at D:

$R_{BD} = 0$  ;  $R_F = 1\text{N}$  ;  $M_F = -15\text{Nm}$

$V_a = 0$  ;  $M_a = 0$  ;  $V_b = -1\text{N}$  ;  $M_b = -6\text{Nm}$

When 1K at just left of section (b)

$M_a = 0$  ;  $V_b = -1\text{K}$  ;  $R_{BD} = 0$

$M_b = 0$  ;  $M_b = 0\text{K}$

$M_F = -9\text{Nm}$

When 1K at just right of section (b)

$V_b = 0$

When 1K at E

all values are zero.

# Draw IL for  $V_{1-1}$ ,  $V_{2-2}$ ,  $M_{1-1}$  and  $M_{2-2}$ .

When 1K at B:

$R_A = 1K$   
 $V_{1-1} = 0$ ;  $V_{2-2} = 0$ ;  $M_{1-1} = 0$ ;  $M_{2-2} = 0$

When 1K at just left of section ①-①

$R_A = 1K$   
 $V_{1-1} = 0$ ;  $V_{2-2} = 0$ ;  $M_{1-1} = 0$ ;  $M_{2-2} = 0$

When 1K at just right of section ①-①

$R_A = 1K$   
 $V_{1-1} = 1K$ ;  $V_{2-2} = 0$ ;  $M_{1-1} = 0$ ;  $M_{2-2} = 0$

When 1K at G:

$R_A = 1K$   
 $V_{1-1} = 2K$ ;  $V_{2-2} = 0$ ;  $M_{1-1} = -12K'$   
 $M_{2-2} = 0$

When 1K at D

$R_C = 1K$ ;  $R_A = 1K$   
 $V_{1-1} = 1K$ ;  $V_{2-2} = 0$   
 $M_{1-1} = -20K'$ ;  $M_{2-2} = 0$

When 1K at E

$R_F = 1K$ ,  $R_A = 1K$   
 $V_{1-1} = 1K$ ;  $V_{2-2} = 0$   
 $M_{1-1} = -12K'$ ;  $M_{2-2} = 0$

When 1K at just left of section ②-②

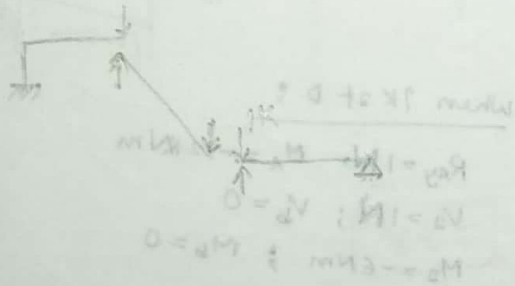
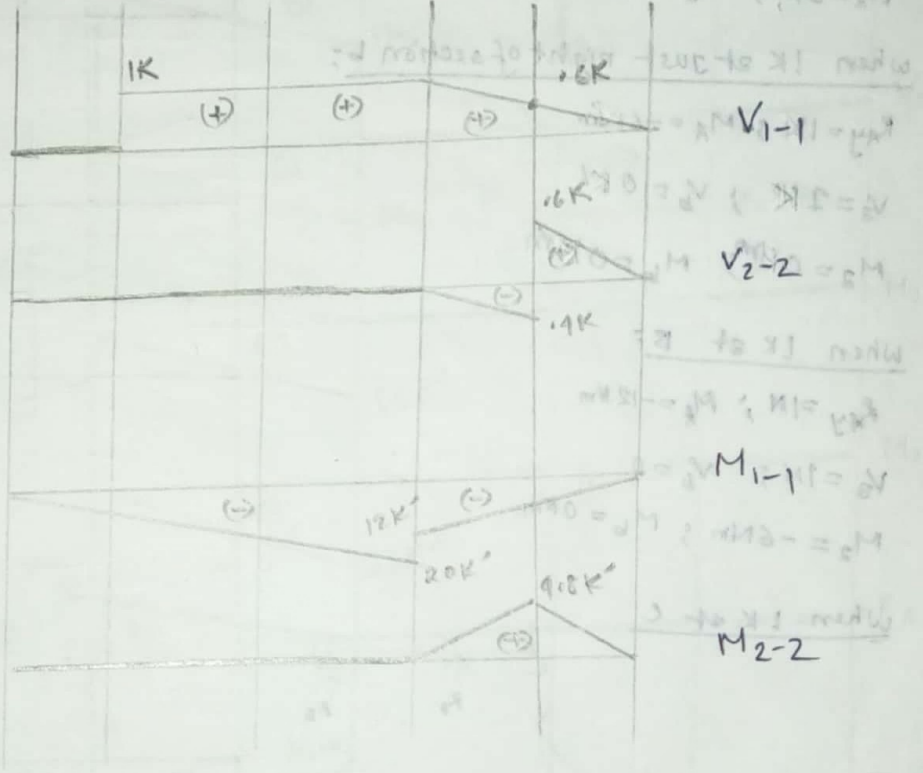
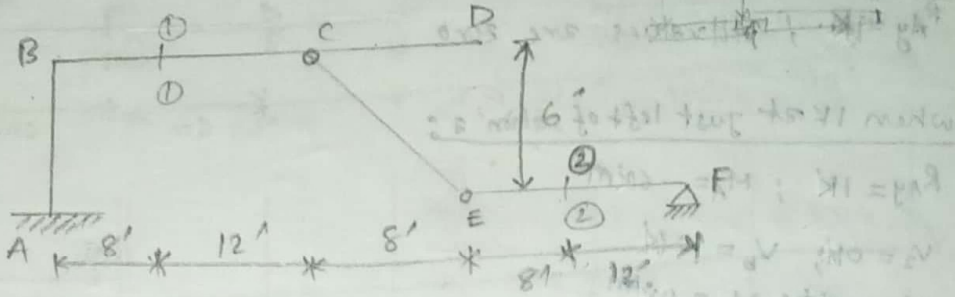
$R_E = 1.6K$ ,  $R_F = 0.4K$ ,  $R_A = 1.6K$   
 $V_{1-1} = 0.6K$ ;  $V_{2-2} = -0.4K$   
 $M_{1-1} = -7.2K'$ ;  $M_{2-2} = 4.8K'$

When 1K at just right of section ②-②

$V_{2-2} = 0.6K$

When 1K at F

All values are zero.



# DRAW IL for  $R_{BD}$ ,  $M_A$ ,  $V_a$ ,  $V_b$ ,  $M_a$  and  $M_b$ : (11) 2011, 16

When IK at A:

$R_{Ay} = 1N$ ; All values are zero

when IK at just left of section a:

$R_{Ay} = 1N$ ;  $M_A = -6Nm$

$V_a = 0N$ ;  $V_b = 0N$

$M_a = 0Nm$ ;  $M_b = 0Nm$

when IK at just right of section b:

$R_{Ay} = 1N$ ;  $M_A = -6Nm$

$V_a = 1N$ ;  $V_b = 0N$

$M_a = 0Nm$ ;  $M_b = 0Nm$

When IK at B:

$R_{Ay} = 1N$ ;  $M_A = -12Nm$

$V_a = 1N$ ;  $V_b = 0$

$M_a = -6Nm$ ;  $M_b = 0Nm$

When IK at C:

$R_B = 1.4N$ ;  $R_E = -1.4N$

$R_B = 1.4N$ ;  $M_A = -16.8Nm$

$M_a = 1.4N$ ;  $V_b = -1.4N$

$M_b = -8.4$ ;  $M_b = -2.4Nm$

when IK at just left of section b:

$R_{Ay} = 0.6N$ ;  $M_A = -7.2Nm$

$V_a = 1.6N$ ;  $M_a = -3.6Nm$

$D_{Ay} = 0.6N$ ;  $V_b = -0.4N$

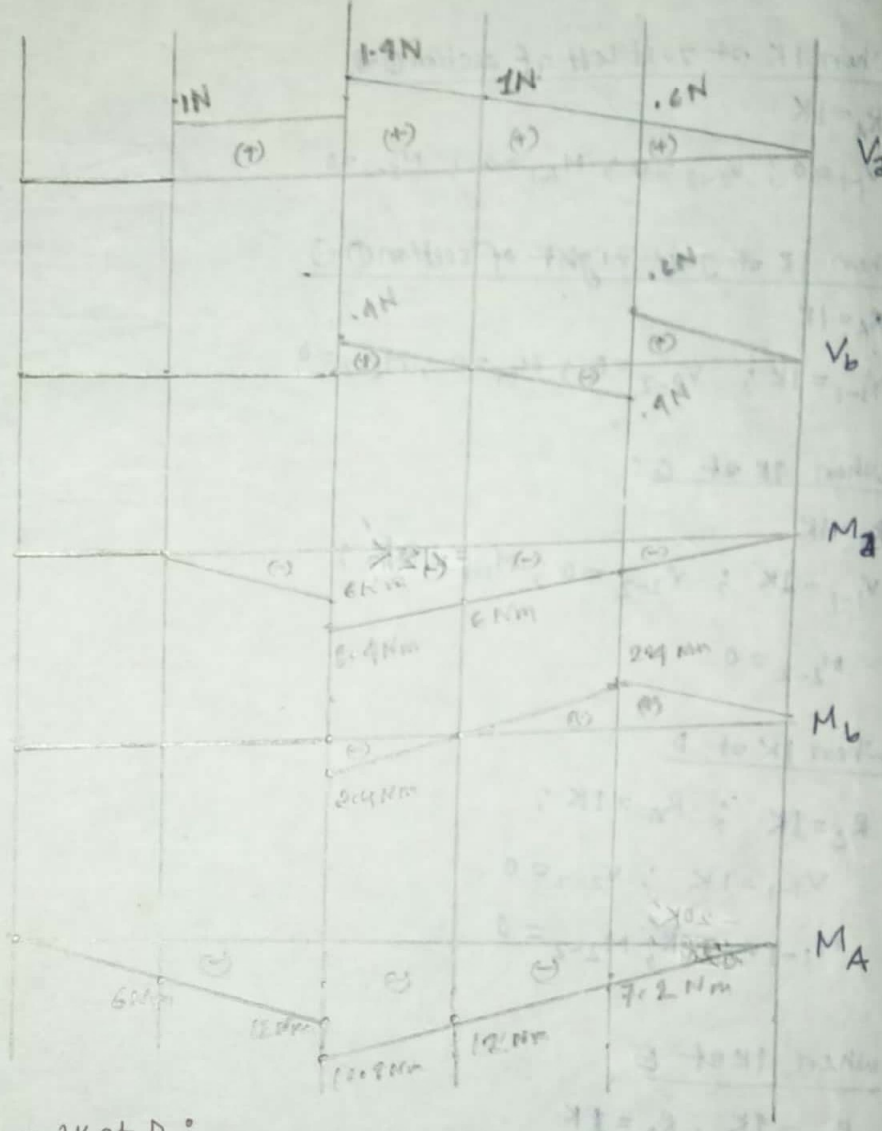
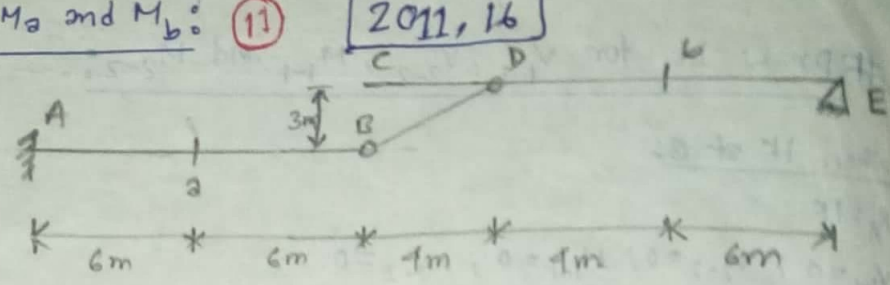
$E_y = 0.4N$ ;  $M_b = 2.4Nm$

When IK at just right of section b:

$V_b = 0.6N$

when IK at E:

All values are zero



When IK at D:

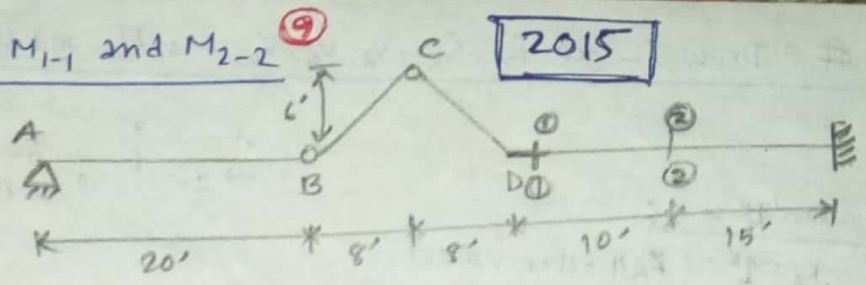
$R_{Ay} = 1N$ ;  $M_A = -12Nm$

$V_a = 1N$ ;  $V_b = 0$

$M_a = -6Nm$ ;  $M_b = 0$

# Draw IL for  $R_A$ ,  $R_{BC}$ ,  $V_{1-1}$ ,  $V_{2-2}$ ,  $M_{1-1}$  and  $M_{2-2}$

2015



When 1K at A

$R_A = 1K$ ,  $R_{BC} = 0$

$V_{1-1} = 0$ ,  $V_{2-2} = 0$ ,  $M_{1-1} = 0$ ,  $M_{2-2} = 0$

When 1K at B

$R_A = 0K$ ;  $R_{BC} = 1.66K$

$V_{1-1} = -1K$ ;  $V_{2-2} = -1K$

$M_{1-1} = -16K'$ ;  $M_{2-2} = -26K'$

When 1K at D

$R_A = 0K$ ;  $R_{BC} = 0$ ;  $V_{1-1} = -1K$

$V_{2-2} = -1K$ ;  $M_{1-1} = 0K'$ ;  $M_{2-2} = -10K'$

When 1K at just right of section 1-1

$R_A = 0K$ ;  $R_{BC} = 0.4K$

$V_{1-1} = 0$ ;  $V_{2-2} = -1K$

$M_{1-1} = 0K'$ ;  $M_{2-2} = 0K'$

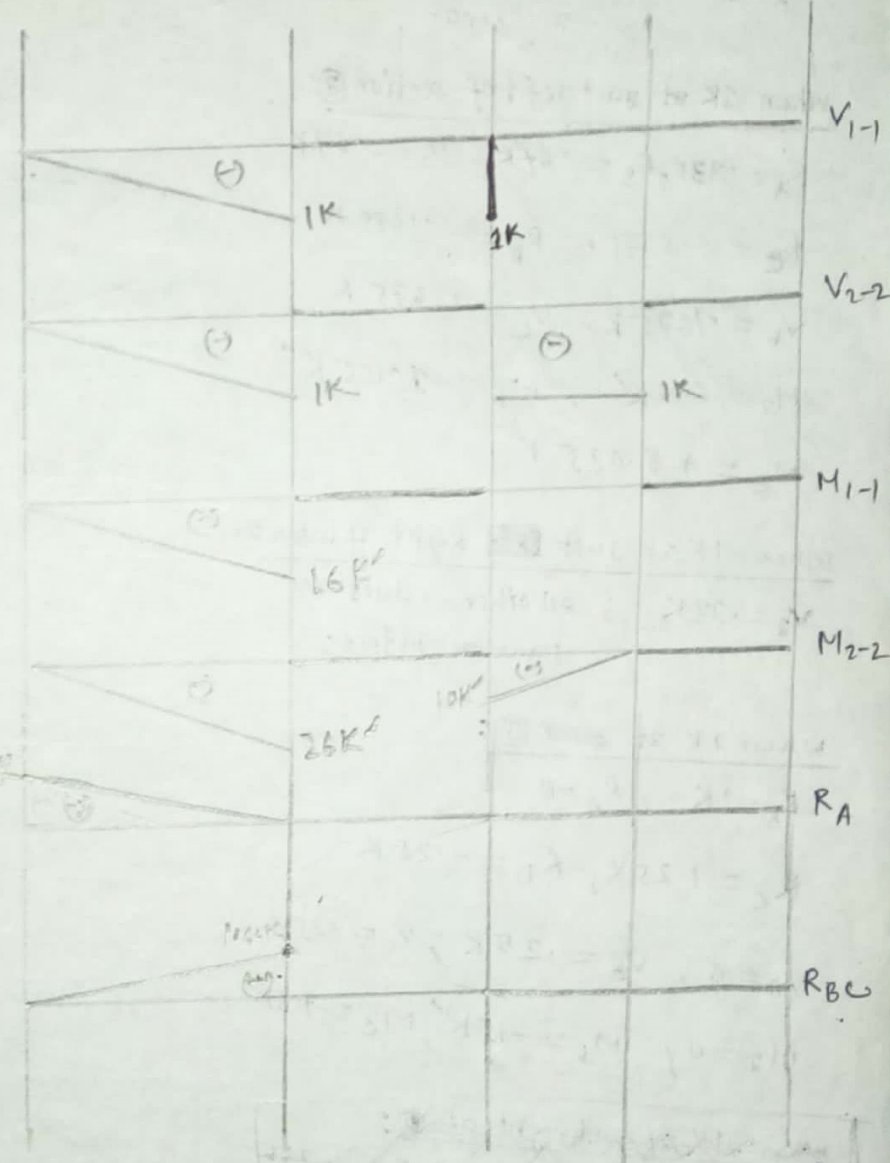
When 1K at just left of section 2-2

$R_A = 0K$ ;  $R_{BC} = 0K$ ;  $V_{1-1} = 0K$

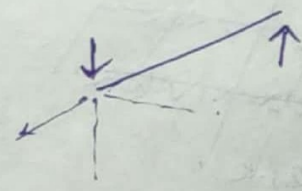
$V_{2-2} = -1K$ ;  $M_{1-1} = 0K'$ ;  $M_{2-2} = 0K'$

When 1K at right of 2-2:

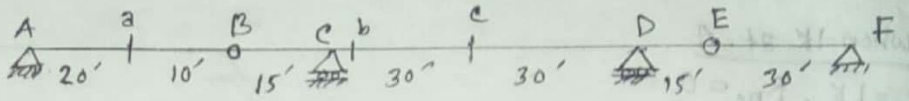
All values are zero.



$R_{BC} = 2$



#3 Draw IL for  $R_A, R_B, V_a, V_b, V_c, M_a, M_b$  and  $M_c$



When 1K at A:

$R_A = 1K$ ; All other values are zero.

When 1K at just left of section (a):

$R_A = .33K, R_B = .67K, V_a = -.67K$

$R_D = 0.8375K, R_D = -.1675K$

$V_b = .1675K, V_c = .1675K$

$M_a = 6.6K', M_b = -10.05K'$

$M_c = +5.025K'$

When 1K at just right of section (a):

$V_a = .33K$ ; all other values are same as before.

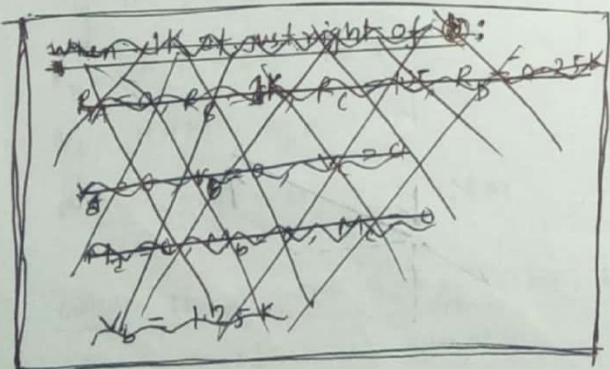
When 1K at (b):

$R_B = 1K, R_A = 0$

$R_C = 1.25K, R_D = -.25K$

$V_a = 0, V_b = .25K, V_c = .25K$

$M_a = 0, M_b = -15K', M_c = -7.5K'$



When 1K at c:

$R_C = 1K$  and all other values are zero.

When 1K at just right of section (b):

$V_b = 1K$  and all other values are zero.

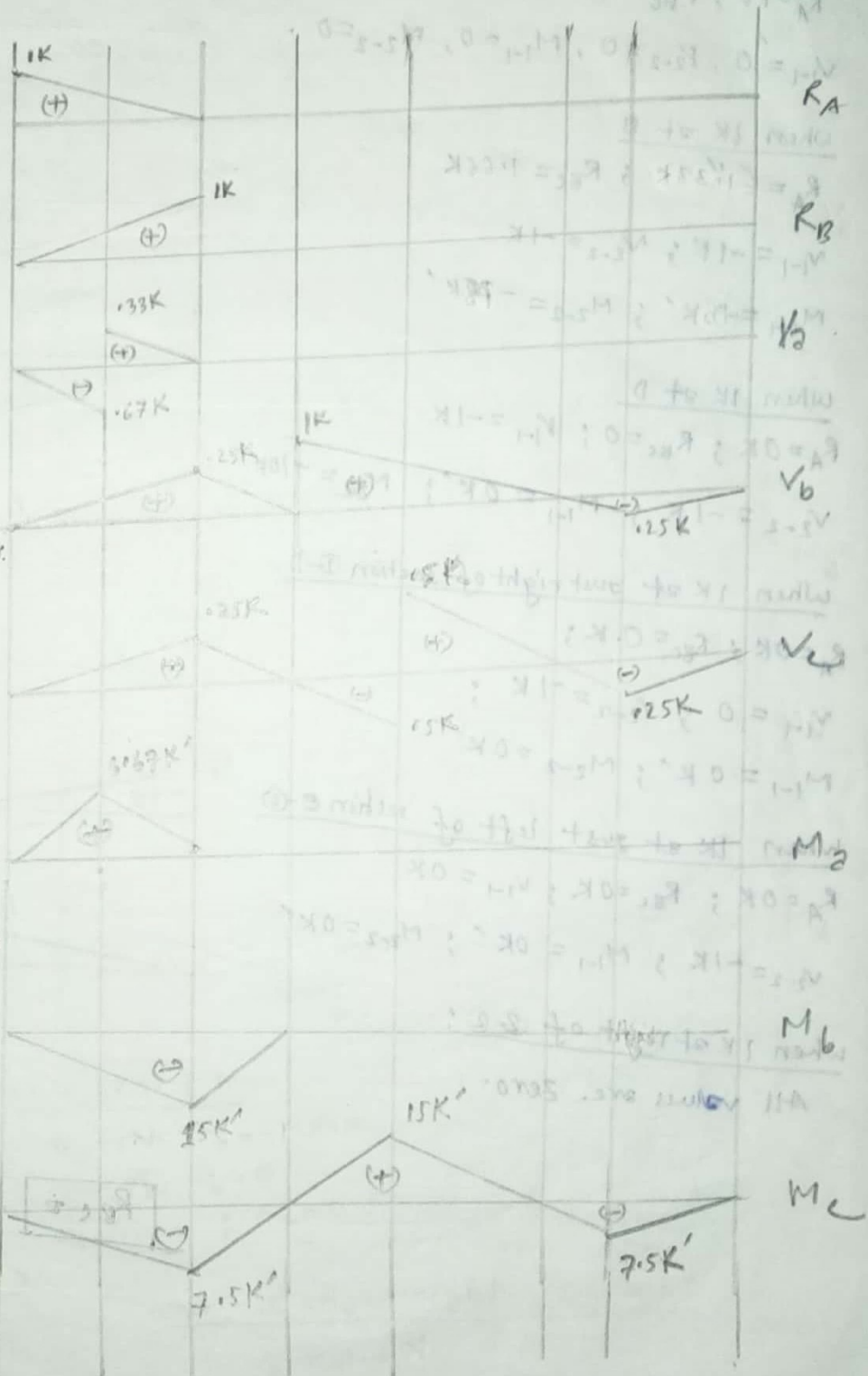
When 1K at left of section (c):

$R_E = R_D = .5K, V_a = 0, V_b = .5K$

$V_c = -.5K, M_a = 0, M_b = 0, M_c = 15K'$

When 1K at right of section (c):

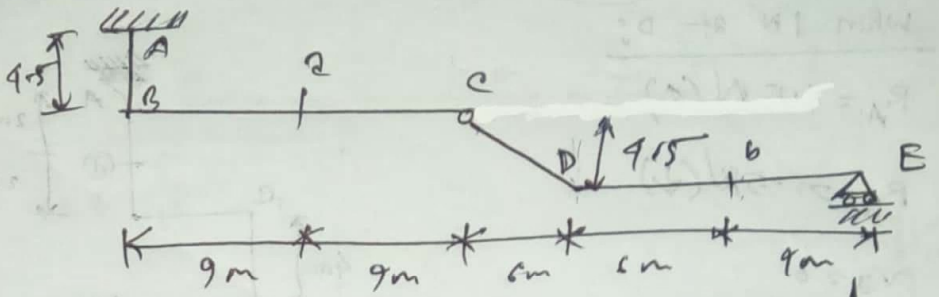
$V_c = .5K$  and same as before.



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B-C D-E

# Draw IL for  $M_A, V_2, V_6, M_2$  &  $M_6$  (13)



1k at B

$R_A = 1, R_E = 0$   
all values are zero

1k at section a:

$R_A = 1, R_E = 0, M_A = 9$   
 $V_2 = 0$  (left)  $M_2 = 0$   
 $V_2 = 1$  (right)  
 $V_6 = 0, M_6 = 0$

1k at c:

$R_A = 1, R_E = 0, M_A = 15$   
 $V_2 = 1, M_2 = 9$   
 $V_6 = 0, M_6 = 0$

1k at D:

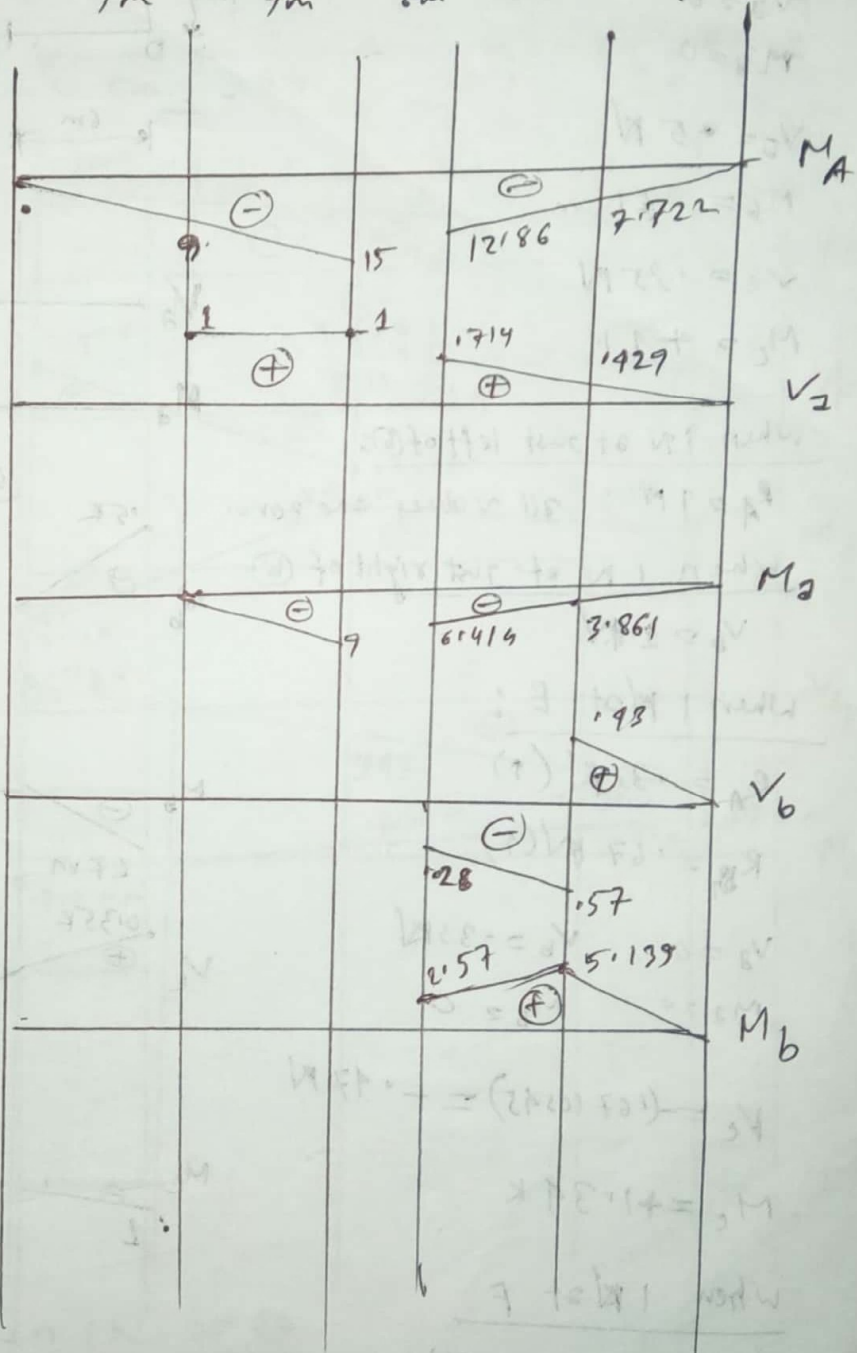
$R_C = 1.714, R_E = 0.286$   
 $R_A = 1.714, M_A = -12.86$   
 $V_2 = 1.714, M_2 = -6.424$   
 $V_6 = -1.286, M_6 = 2.57$

1k at section b:

$R_C = 0.429, R_E = 1.571$   
 $R_A = 0.429, M_A = -7.222$   
 $V_2 = 1.429, M_2 = 3.861$

1k at E

$R_E = 1k, R_C = 0, R_A = 0$   
all values are zero.



# Draw IL for  $V_2$ ,  $V_b$ ,  $M_2$ ,  $M_b$  &  $M_c$ : (14)

When 1N at D:

$$R_A = 1.15 \text{ kN} (\uparrow)$$

$$R_G = 1.5 \text{ kN} (\downarrow)$$

$$V_2 = 0$$

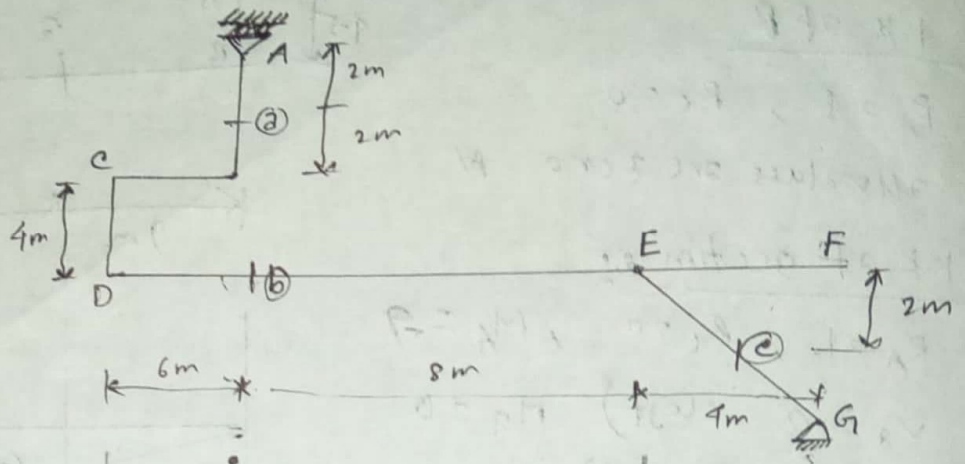
$$M_2 = 0$$

$$V_b = +5 \text{ kN}$$

$$M_b = -6 \text{ kNm}$$

$$V_c = 1.35 \text{ kN}$$

$$M_c = -1 \text{ kNm}$$



When 1N at just left of (b):

$$R_A = 1 \text{ kN}; \text{ all values are zero.}$$

When 1N at just right of (b):

$$V_b = 1 \text{ kN}$$

When 1 kN at E:

$$R_A = 0.33 \text{ kN} (\uparrow)$$

$$R_G = 1.67 \text{ kN} (\uparrow)$$

$$V_2 = 0 \quad V_b = +0.33 \text{ kN}$$

$$M_2 = 0 \quad M_b = 0$$

$$V_c = -(1.67 \cos 45^\circ) = -1.17 \text{ kN}$$

$$M_c = +1.34 \text{ kNm}$$

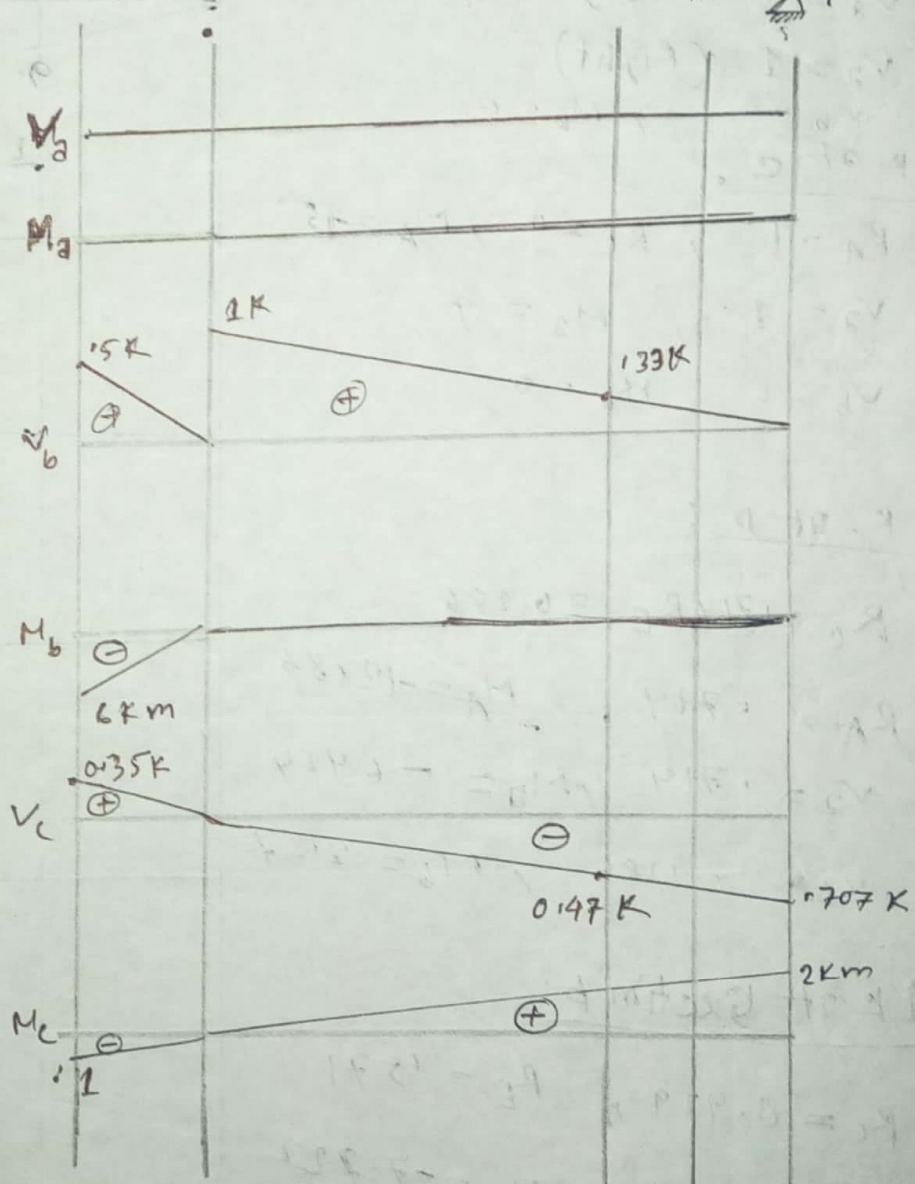
When 1 kN at F:

$$R_G = 1 \text{ kN}$$

$$V_2 = 0 \quad V_b = 0$$

$$M_2 = 0 \quad M_b = 0$$

$$V_c = -1.707 \text{ kN} \quad M_c = +2 \text{ kNm}$$



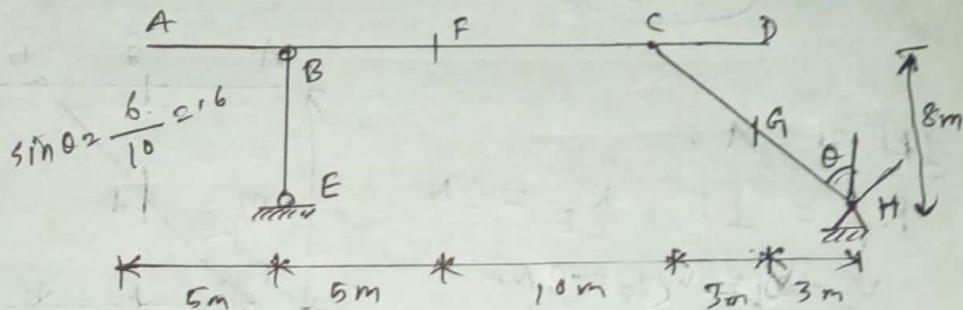
# Draw IL for  $R_{BE}$ ,  $V_F$ ,  $M_F$ ,  $V_G$  &  $M_G$ : (10) (A-D)

When 1N at A

$$R_H = -0.238 \text{ N}, R_B = 1.238 \text{ N}$$

$$V_F = 0.238 \text{ N}, M_F = 3.81 \text{ Nm}$$

$$V_G = 1.143 \text{ N}, M_G = -1.714$$



When 1N at B

$$R_B = 1 \text{ N}, R_H = 0 \text{ N}$$

$$V_F = 0, M_F = 0 \text{ Nm}$$

$$V_G = 0, M_G = 0$$

When 1N at F

$$R_H = 0.238 \text{ N}, R_B = 0.762 \text{ N}$$

$$V_F = -0.238 \text{ N (left)}$$

$$V_F = 0.762 \text{ N}, M_F = 3.81 \text{ Nm}$$

$$V_G = 0.143 \text{ N}, M_G = 2.1714$$

When 1N at C

$$R_H = 0.714 \text{ N}, R_B = 0.286 \text{ N}$$

$$V_F = 0.714, M_F = 1.43 \text{ Nm}$$

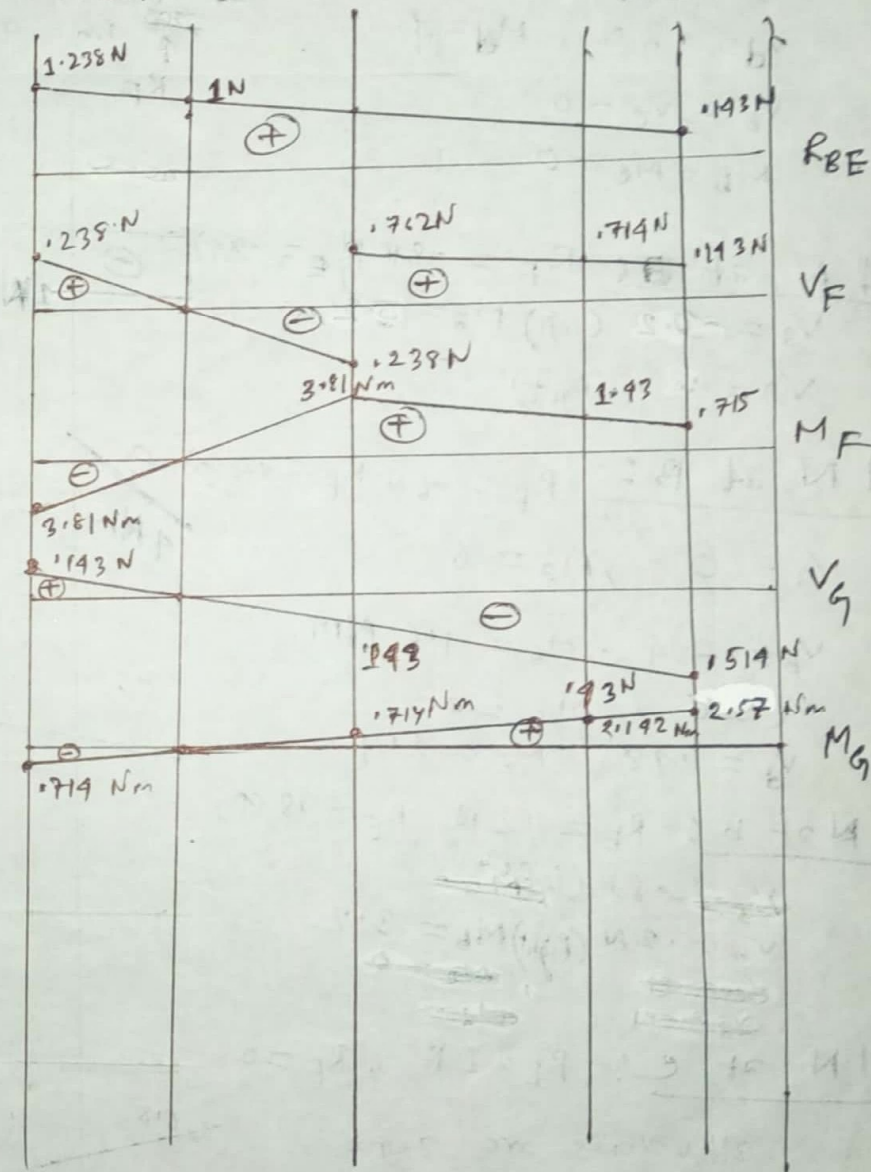
$$V_G = -0.143 \text{ N}, M_G = 2.142 \text{ Nm}$$

When 1N at D

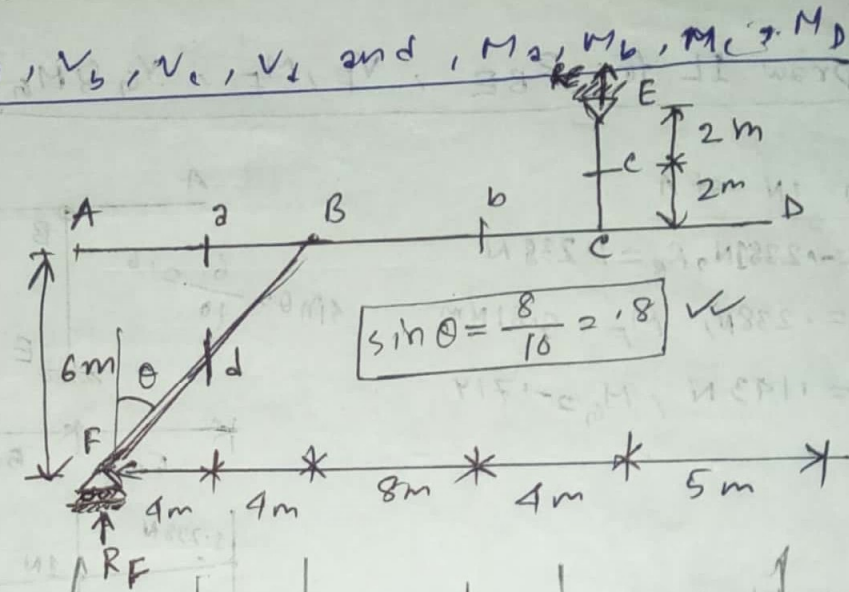
$$R_H = 0.857 \text{ N}, R_B = 0.143 \text{ N}$$

$$V_F = 0.143, M_F = 0.715 \text{ Nm}$$

$$V_G = -0.514 \text{ N}, M_G = 2.57 \text{ Nm}$$



17 Draw IL for  $V_2, V_3, V_c, V_d$  and  $M_a, M_b, M_c, M_d$   
(A-D)



1N at A:  
 $R_F = 1N, R_E = 0N$   
 $V_a = -1N, M_a = -4Nm$   
 $V_d = -8N, M_d = 4$   
 $V_b = V_c = 0$   
 $M_b = M_c = 0$

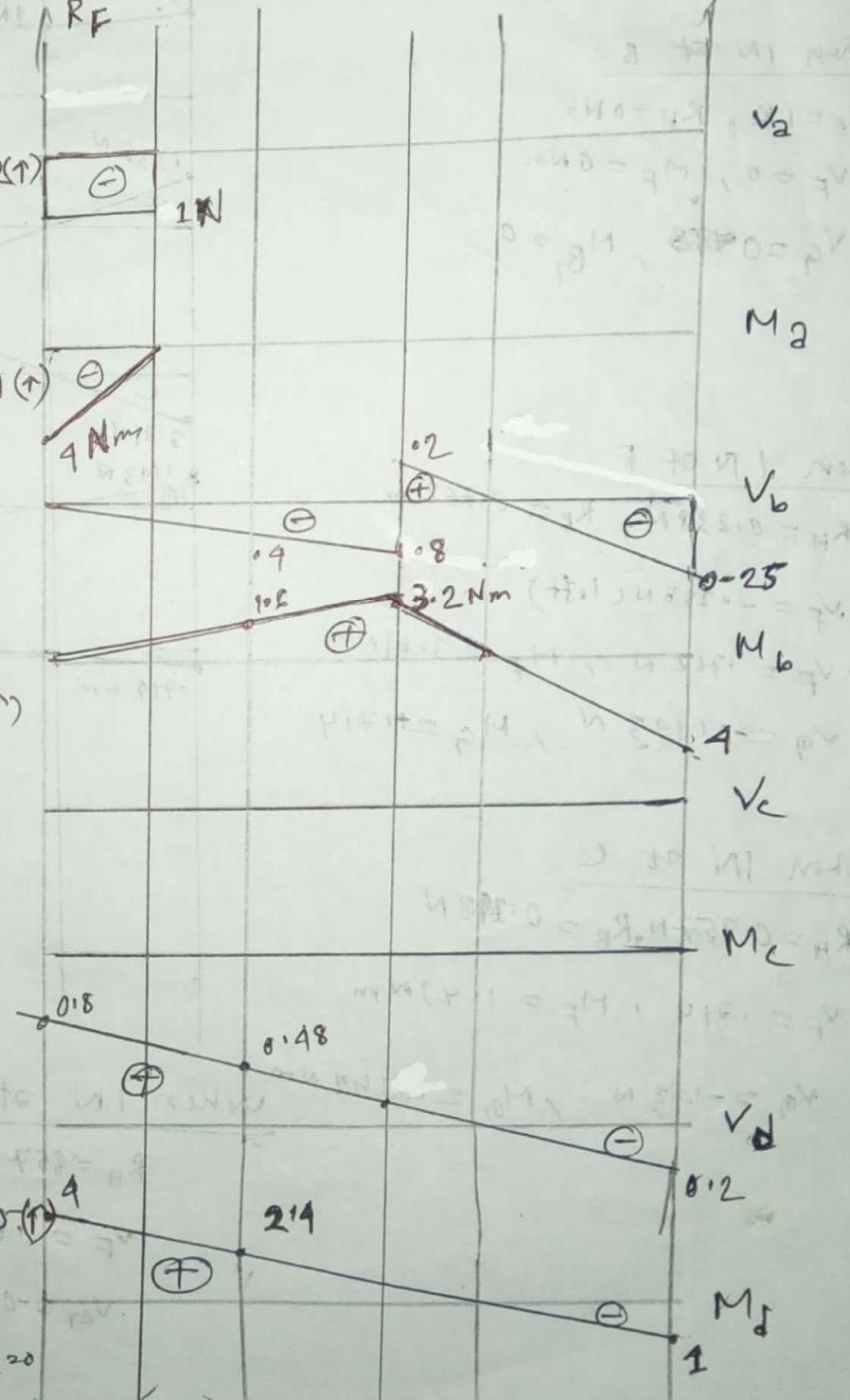
1N at a:  $R_F = -0.8N, R_E = 0.2(\uparrow)$   
 $V_a = -1$  (left)  $M_a = 0Nm$   
 $V_a = 0$  (right)

1N at B:  $R_F = -0.6N, R_E = 0.4(\uparrow)$   
 $V_b = 0, M_b = 0$   
 $V_b = -1.4, M_b = 1.6Nm$   
 $V_c = 0, M_c = 0$   
 $V_d = 0.48, M_d = 2.14$

1N at b:  $R_F = -0.2N, R_E = 0.8(\uparrow)$   
 $V_b = -0.8N$  (left)  
 $V_b = 0.2N$  (right)  $M_b = 3.2$

1N at c:  $R_E = 1N, R_F = 0$   
 all values are zero

1N at D:  $R_F = -0.25N, R_E = 0.25(\uparrow)$   
 $V_d = 0, M_d = 0$   
 $V_b = -0.25, M_b = 4Nm$   
 $V_d = -2, M_d = 10, V_c = M_c = 0$



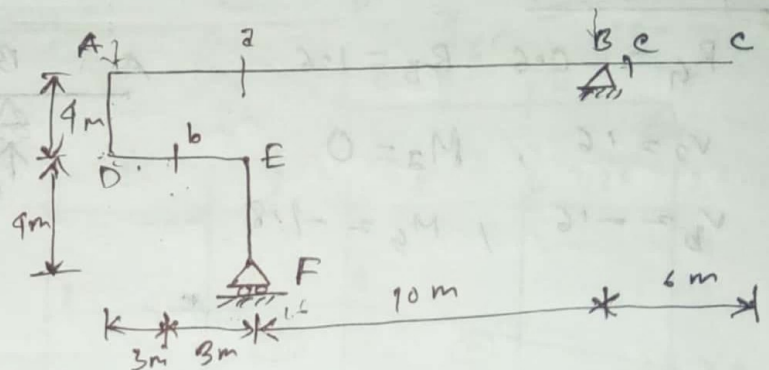
18 Draw IL for  $V_2, M_2, V_b, M_b, V_c, M_c$ : A-E

1K at A:

$$R_B = -0.6 \text{ K}, R_F = 1.6$$

$$V_2 = 0.6, M_2 = -6$$

$$V_b = -1, M_b = 4.8$$



1K at Section-2:

$$R_F = 1 \text{ K}, R_B = 0$$

$$V_2 = 0 \quad (\text{left}) \quad M_2 = 0$$

$$= 1 \quad (\text{right}) \quad \times$$

1K at B:

$$R_B = 1, R_F = 0$$

all values are zero,

$$V_c = 0 \text{ N (left)} \quad M_c = 0$$

$$V_c = +1 \text{ N (right)}$$

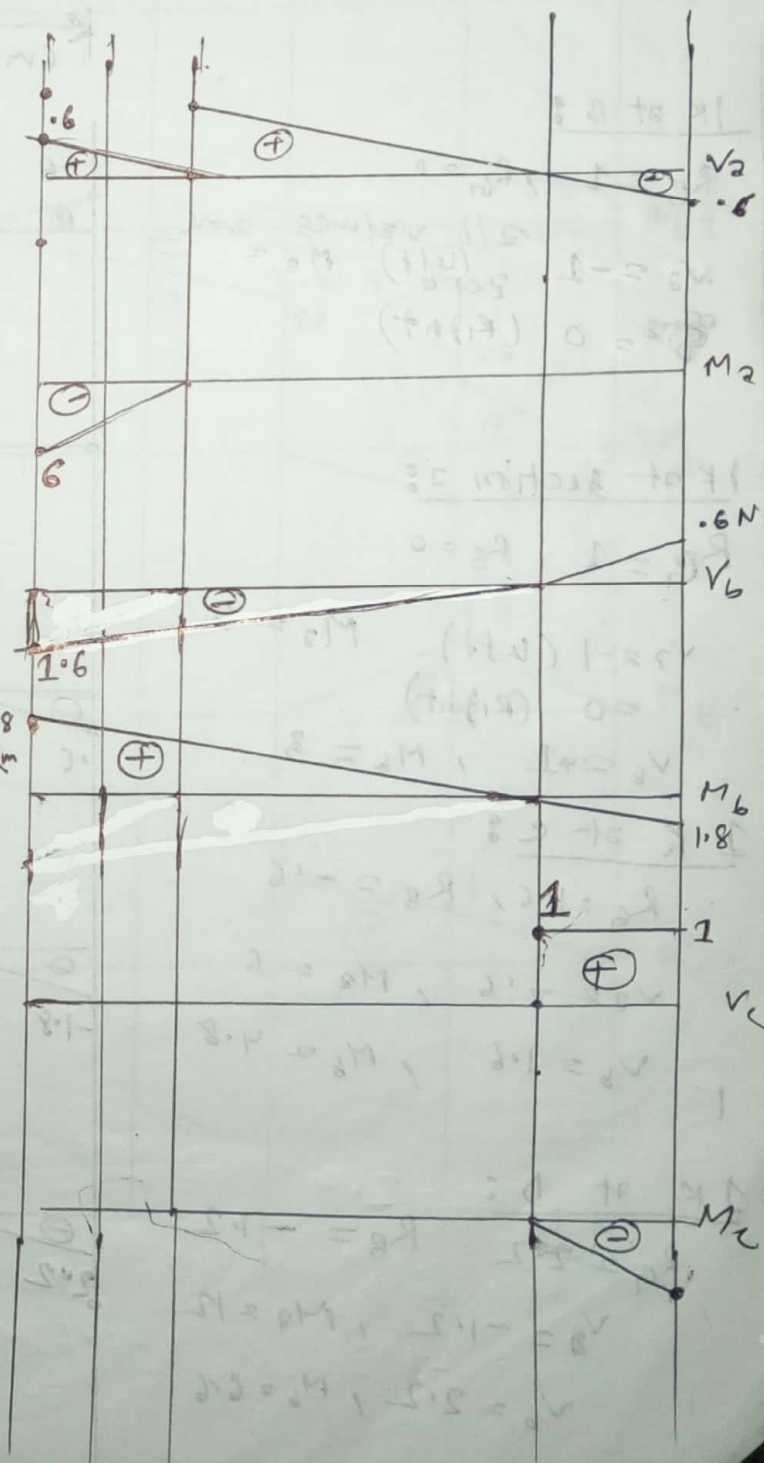
1K at C:

$$R_B = 1.6, R_F = -0.6$$

$$V_2 = -0.6, M_2 = 0$$

$$V_b = 0.6, M_b = -1.8$$

$$V_c = 1, M_c = -6$$



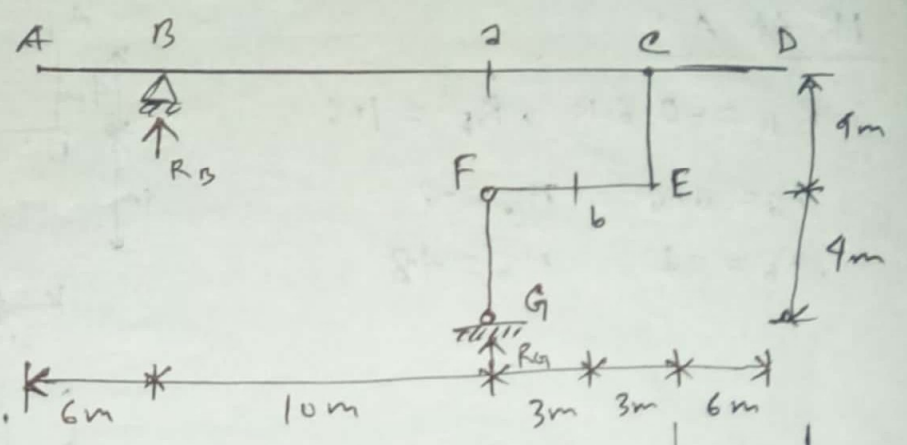
19) Draw SL for  $v_2, v_b, M_2, M_b, R_{FG}$ : A-D

1K at A:

$R_G = -0.6$     $R_B = 1.6$

$v_2 = 1.6$  ,    $M_2 = 0$

$v_b = -1.6$  ,    $M_b = -1.8$



1K at B:

$R_B = 1$  ,  $R_G = 0$   
 all values are zero.

1K at section 2:

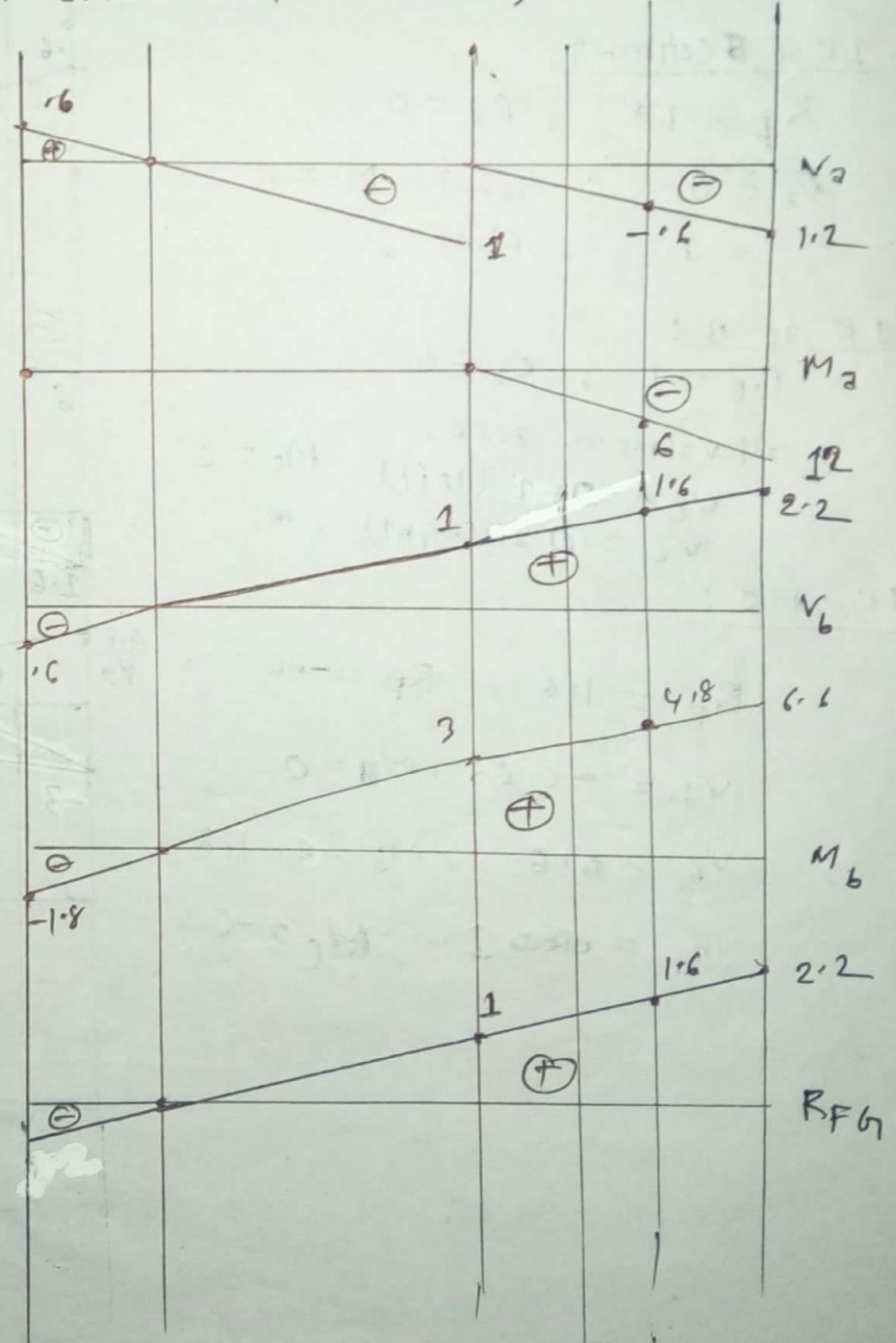
$R_G = 1$  ,  $R_B = 0$   
 $v_2 = -1$  (left)    $M_2 = 0$   
 $= 0$  (right)  
 $v_b = +1$  ,    $M_b = 3$

1K at c:

$R_G = 2.16$  ,  $R_B = -1.6$   
 $v_2 = -1.6$  ,    $M_2 = 6$   
 $v_b = 1.6$  ,    $M_b = 4.8$

1K at D:

$R_G = 2.12$     $R_B = -1.2$   
 $v_2 = -1.2$  ,    $M_2 = 12$   
 $v_b = 2.12$  ,    $M_b = 6.6$



21) Draw IL for  $V_2, V_6, M_2, M_6$ ;  $R_{CE}$ : B-D & E-F (2017)

1 K at B:  
 $R_A = 1, R_C = 0$   
 All values are zero.

1 K at section 2:  
 $R_A = 0.1K, R_C = 0$   
 $V_2 = 0K$  (left)  
 $= 1K$  (right)  
 $M_2 = 0$

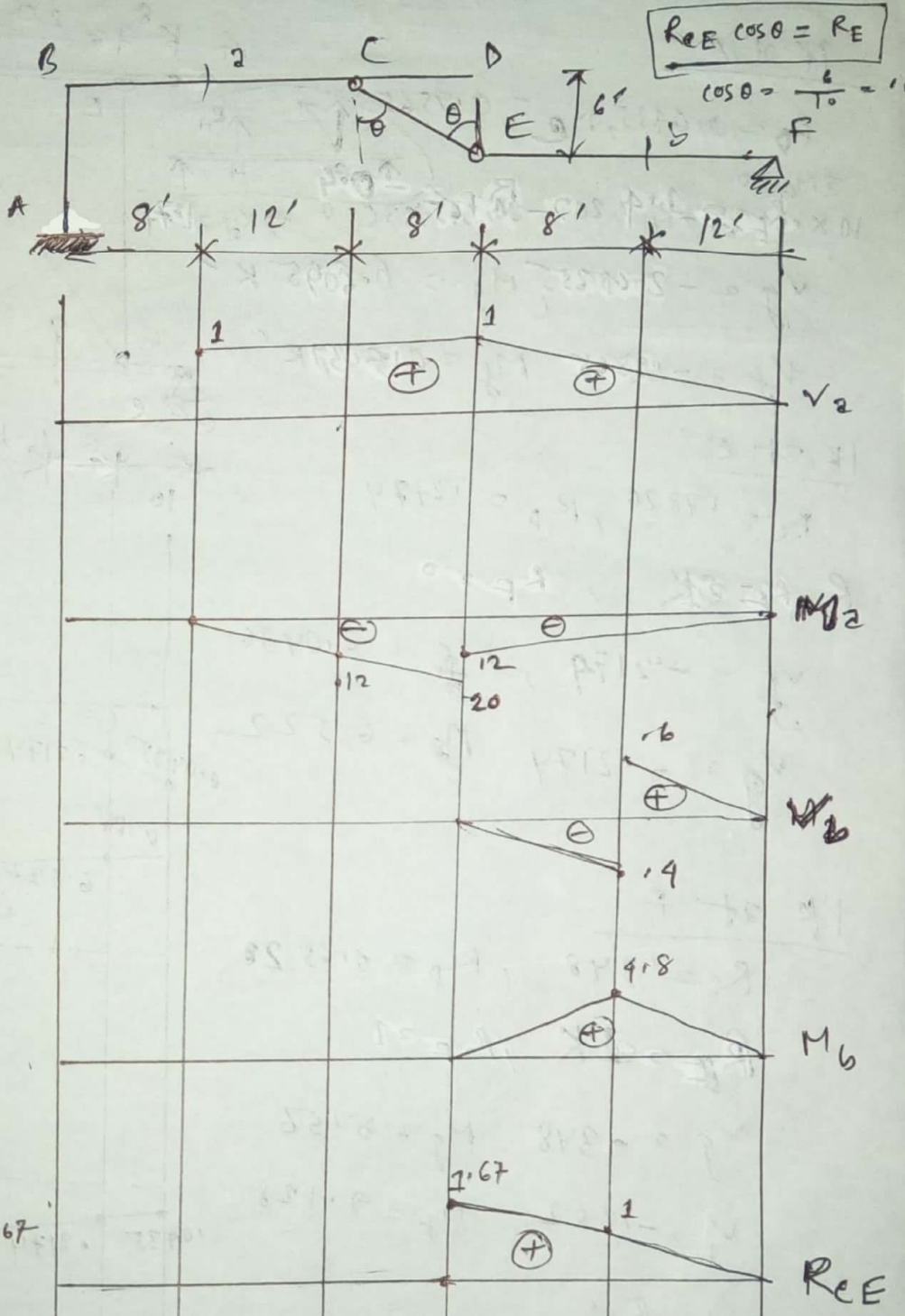
1 K at C:  
 $R_A = 1K, R_C = 0$   
 $V_2 = 1, M_2 = -12$   
 $V_6 = 0, M_6 = 0$

1 K at D:  
 $R_A = 1, R_C = 0$   
 $V_2 = 1, M_2 = -20$

1 K at E:  
 $R_E = 1, R_F = 0, R_A = 1$   
 $V_2 = 1, M_2 = -12$   
 $V_6 = 0, M_6 = 0, R_{CE} = 1.67$

1 K at section 6:  
 $R_E = 0.6, R_F = 0.4, R_A = 1$   
 $V_2 = -0.4$  (left)  $M_2 = 4.8$   
 $= 0.6$  (right)  $R_{CE} = 1$

1 K at F:  
 $R_F = 1, R_E = 0, R_A = 0$   
 $V_2 = 0, M_2 = 0, R_{CE} = 0$



$R_{CE} \cos \theta = R_E$   
 $\cos \theta = \frac{6}{10} = 0.6$

8. Draw IL for  $V_f, M_f, M_g, V_g$ : A-B

IK at A

$R_D = 0.0435, R_E = 0.9565$   
 $\sum M_G = 0$   
 $10 \times 0.9565 - R_H \times 20 - 0.435 \times 35 = 0 \Rightarrow R_H = 0.4$   
 $V_g = -0.4435, M_g = 6.905 \text{ K}$   
 $V_f = -0.0435, M_f = 0.609 \text{ K}$

IK at E

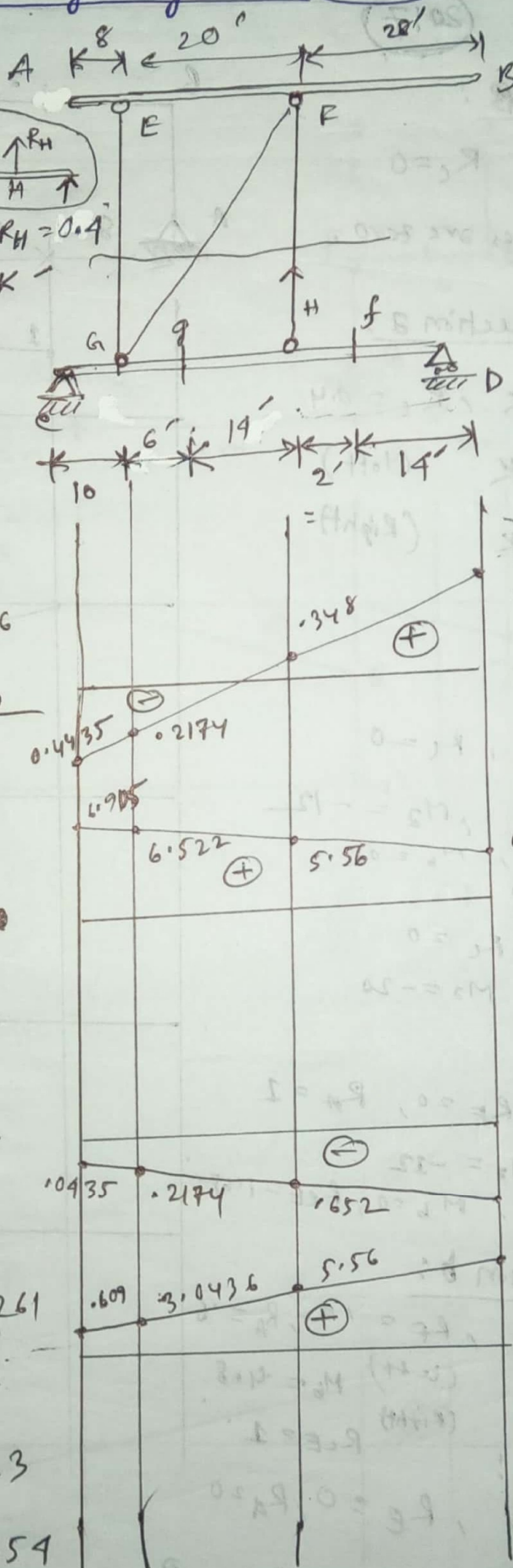
$R_E = 1.7826, R_D = 0.2174$   
 $R_H = 0 \text{ K}$   
 $V_g = -0.2174, M_g = 3.0436$   
 $V_f = -0.2174, M_f = 6.522$

IK at F

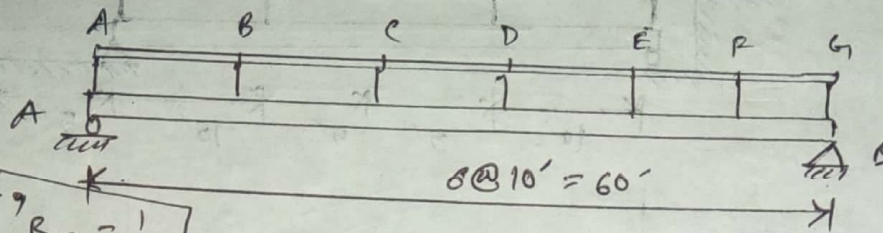
$R_E = 0.348, R_D = 0.652$   
 $R_H = 1 \text{ K}$   
 $V_g = -0.348, M_g = 5.56$   
 $V_f = -0.652, M_f = 9.128$

IK at B

$R_E = -0.261, R_D = 1.261$   
 $R_H = -2.4$   
 $V_g = 1.139, M_g = 4.23$   
 $V_f = -1.261, M_f = 17.654$



16 Draw SL for (i) shear in point BC (ii) Moment at B



1 k at B:

$$R_A = \frac{5}{6}, R_B = \frac{1}{6}$$

$$V_{B-C} = -\frac{1}{6}$$

1 k at C:

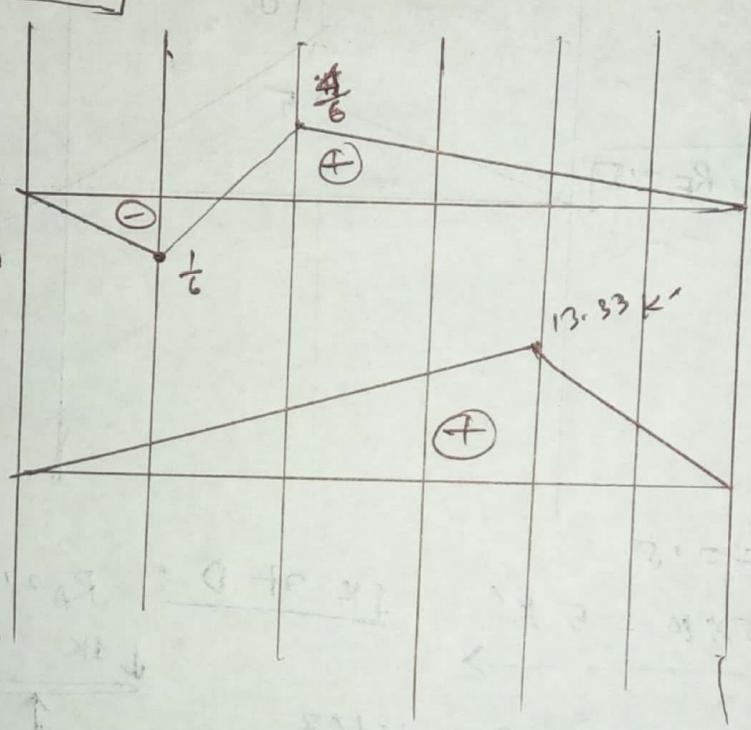
$$R_A = \frac{4}{6}, R_B = \frac{2}{6}$$

$$V_{B-C} = \frac{2}{3}$$

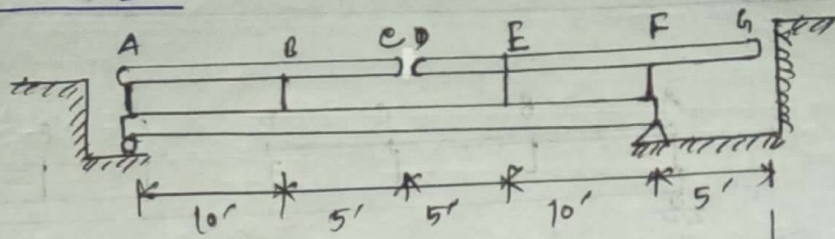
1 k at E:

$$R_B = \frac{4}{6}$$

$$M_B = \frac{4}{6} \times 20 = 13.33 \text{ k'}$$



17 Draw IL for  $M_E$

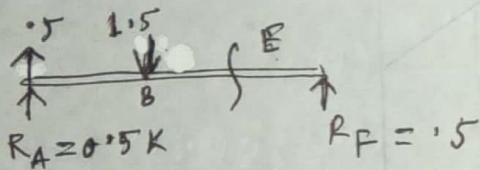
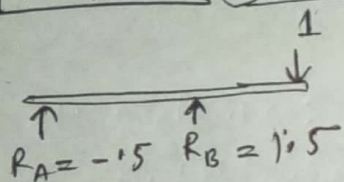


1K at A

$R_A = 1$

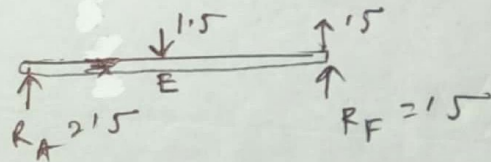
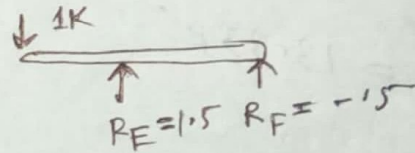
$M_E = 0$

1K at C:  $R_A = 1.5, R_F = 1.5$



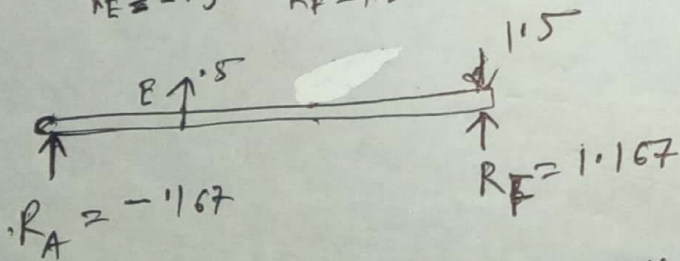
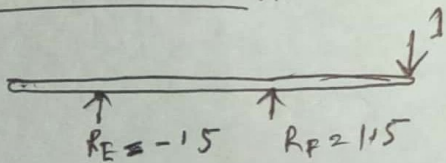
$M_E = 1 \times 20 + 1.5 \times 10 = 5 \text{ K'}$

1K at D:  $R_A = 2.5, R_F = 1.5$

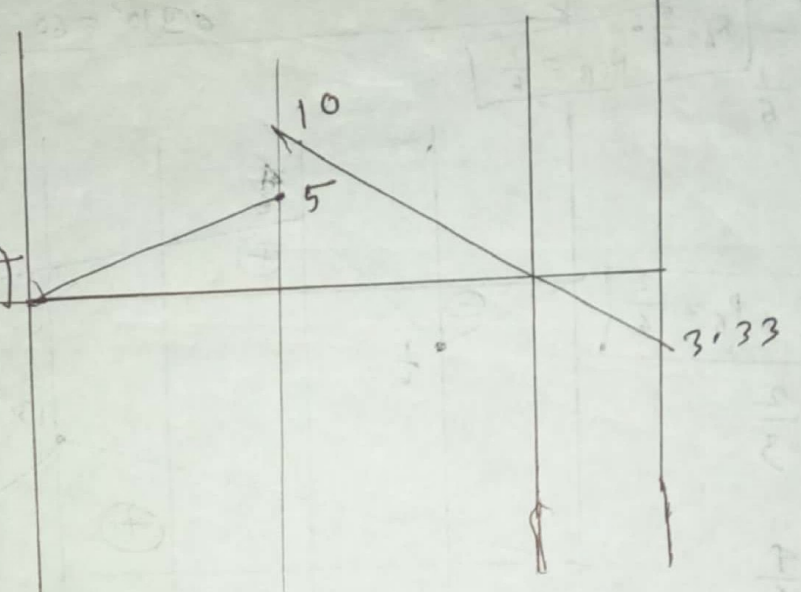


$M_E = 0.5 \times 10 + 1.5 \times 10 = 10$

1K at G:  $R_A = -1.67, R_B = 1.67$



$M_E = 1.67 \times 10 - 1.5 \times 10 = -3.33$



2019

# Draw IL for  $M_B, M_G, V_G, V_2, H_E$  (A-D)

$\sin \theta = \frac{12}{16.97} = 0.707$

1 K at A:

$R_F = 2.11, R_E = -1.11$

$V_2 = 1.11$

$M_B = -8$

$M_G = -1.49, M_G = 7.45$

1 K at B:

$R_F = 1.67, R_E = -1.67$

$V_2 = 1.67, V_G = -1.18$

$M_G = 5.19$

1 K at E:

$R_E = 1 K, R_F = 0$

$V_2 = -1$  (left)

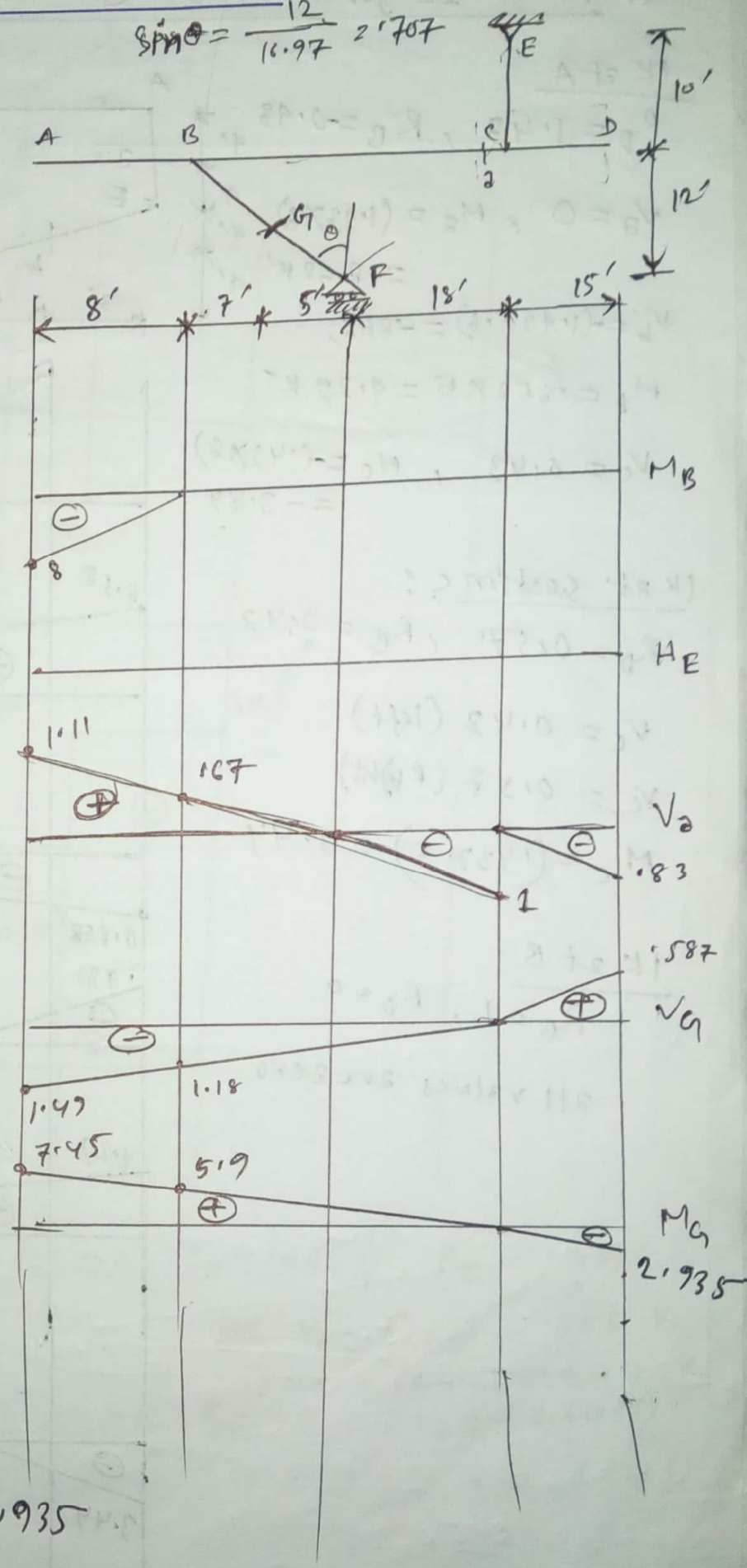
$V_2 = 0$  (right)

1 K at D:

$R_E = -0.83, R_F = 1.83$

$V_2 = -0.83$

$V_G = 1.587, M_G = -2.935$



20/2

# Draw IL for  $V_a, V_b, V_c, M_a, M_b, M_c$

A-B

1K at A

$R_D = 1.43, R_B = -0.43$

$V_a = 0, M_a = (1.43 \times 6) = 8.58 \text{ K'}$

$V_b = -(1.43 \times 0.6) = -0.858,$

$M_b = 0.858 \times 5 = 4.29 \text{ K'}$

$V_c = 0.43, M_c = (1.43 \times 8) = -3.44$

1K at section c:

$R_D = 0.57, R_B = 0.43$

$V_c = 0.43$  (left)

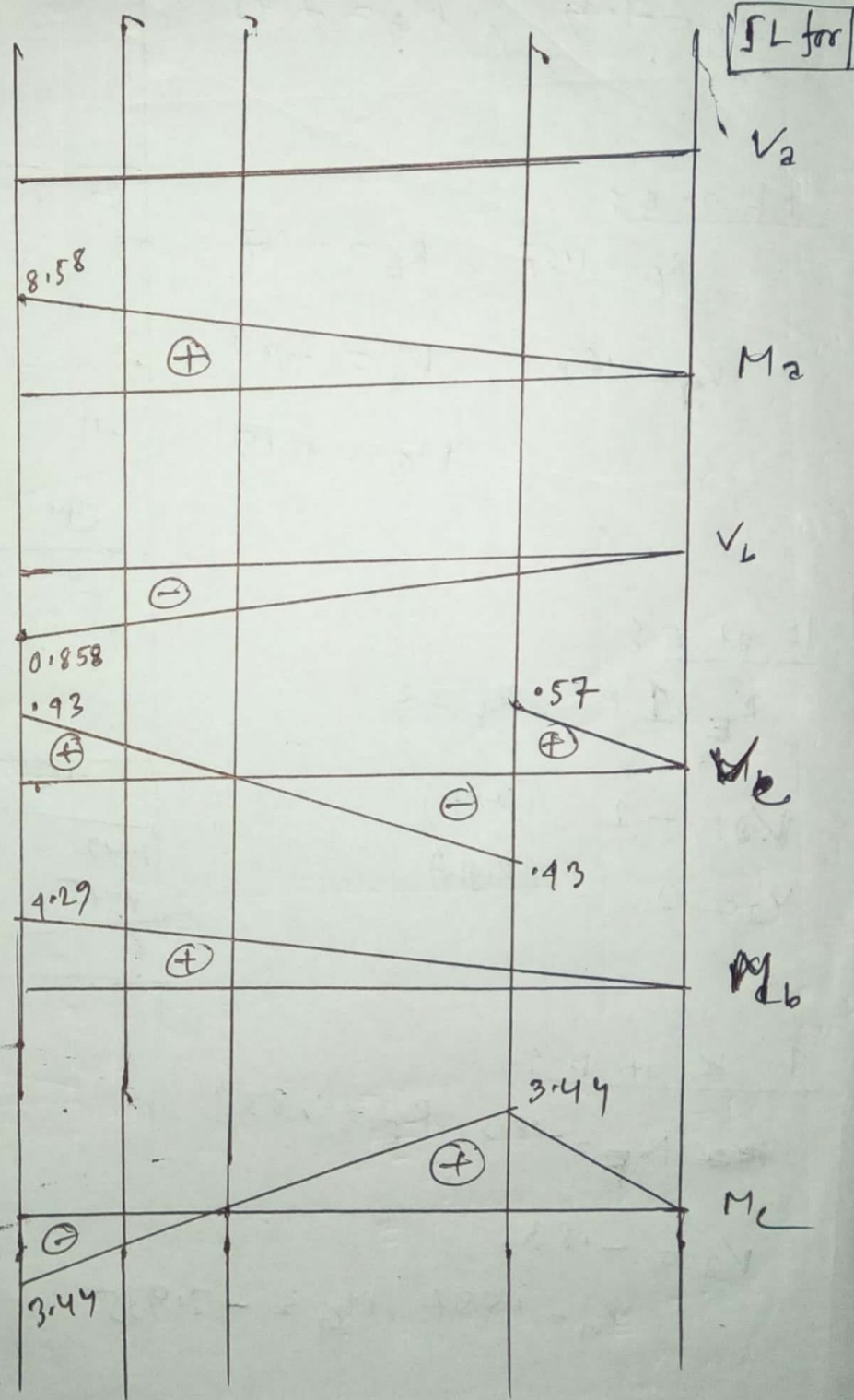
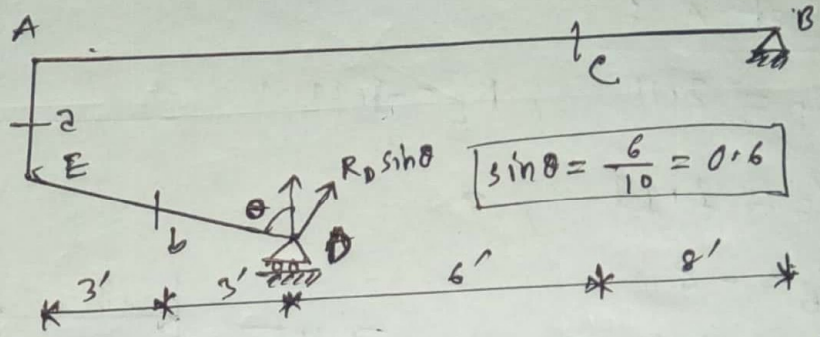
$V_c = 0.57$  (right)

$M_c = (0.43 \times 5) = 2.15$

1K at B

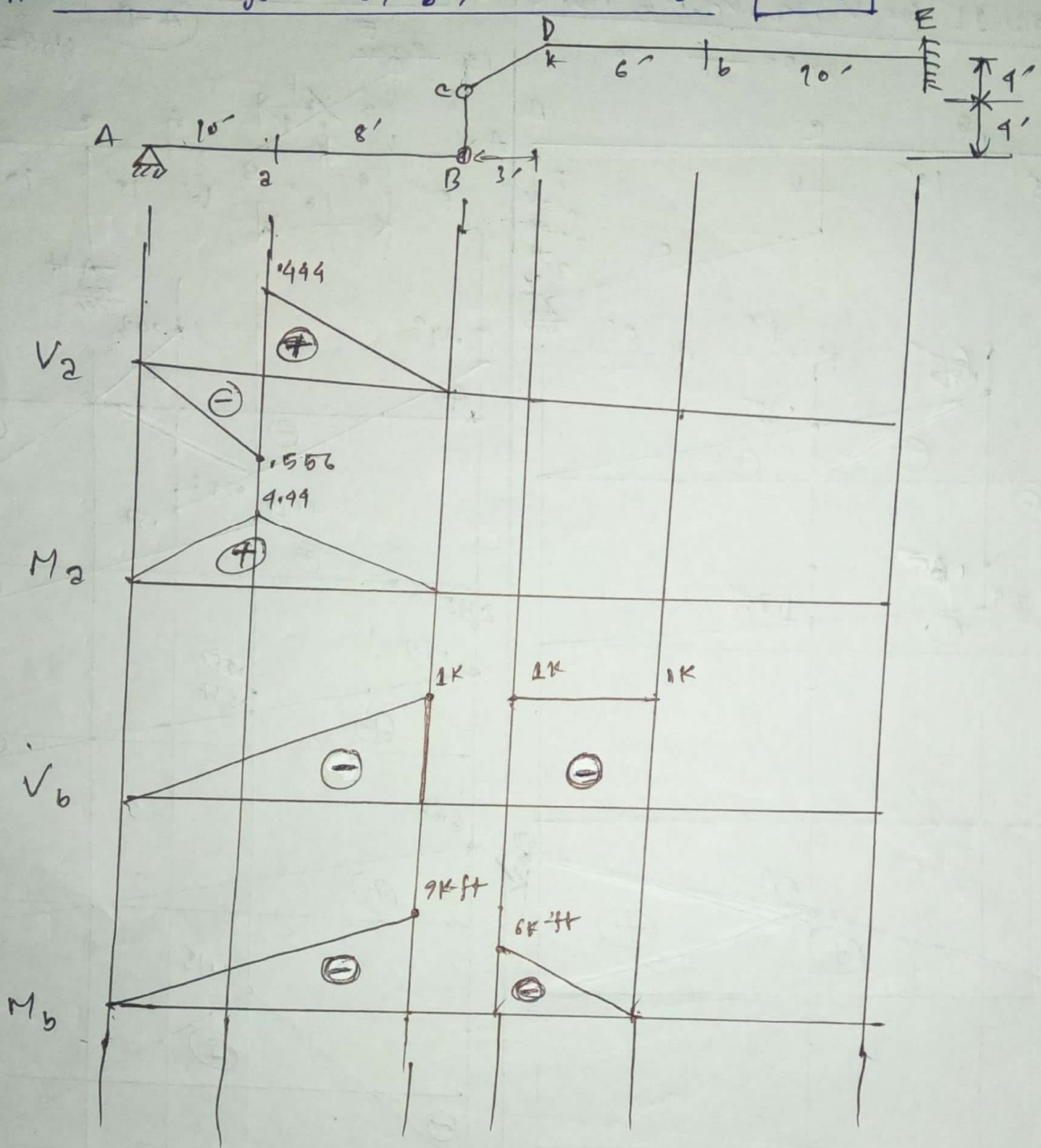
$R_B = 1, R_D = 0$

all values are zero



# Draw SL for  $v_2, v_b, M_2$  and  $M_b$ !

2010



1K at A:  $R_A = 1K$ ,

at section 2:  $R_A = 1.444$ ,  $R_B = 1.556$

at B:  $R_B = 1K$ ,

$V_2 = (\text{left}) - 0.556K$

$V_2 = (\text{right}) 0.444K$

at D:  $R_E = 1K$ ,

at section b:  $R_E = 1$ ,  $V_b = -1K$  (left)

at E:  $R_E = 1K$ ,

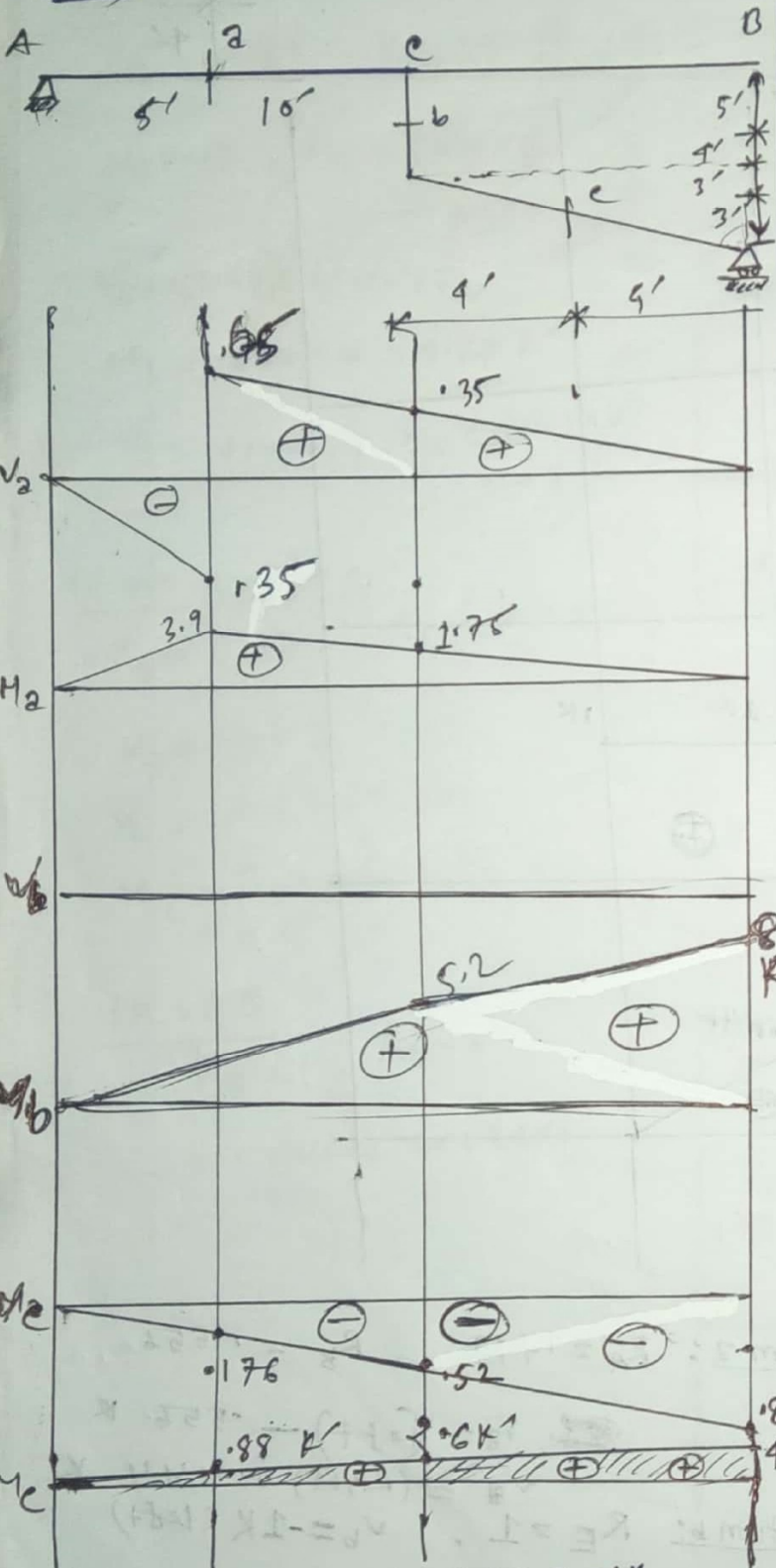
$= 0$  (right)

# 2009

Draw IL for  $V_2, V_b, V_c, M_2, M_b, M_c$

(A-B)

$$\sin \theta = \frac{8}{10} = 0.8$$



1K at A:  $R_A = 1$ , 1K at B:  $R_B = 1K$

1K at c:  $R_A = 0.35K$ ,  $R_B = 0.65K$ ,

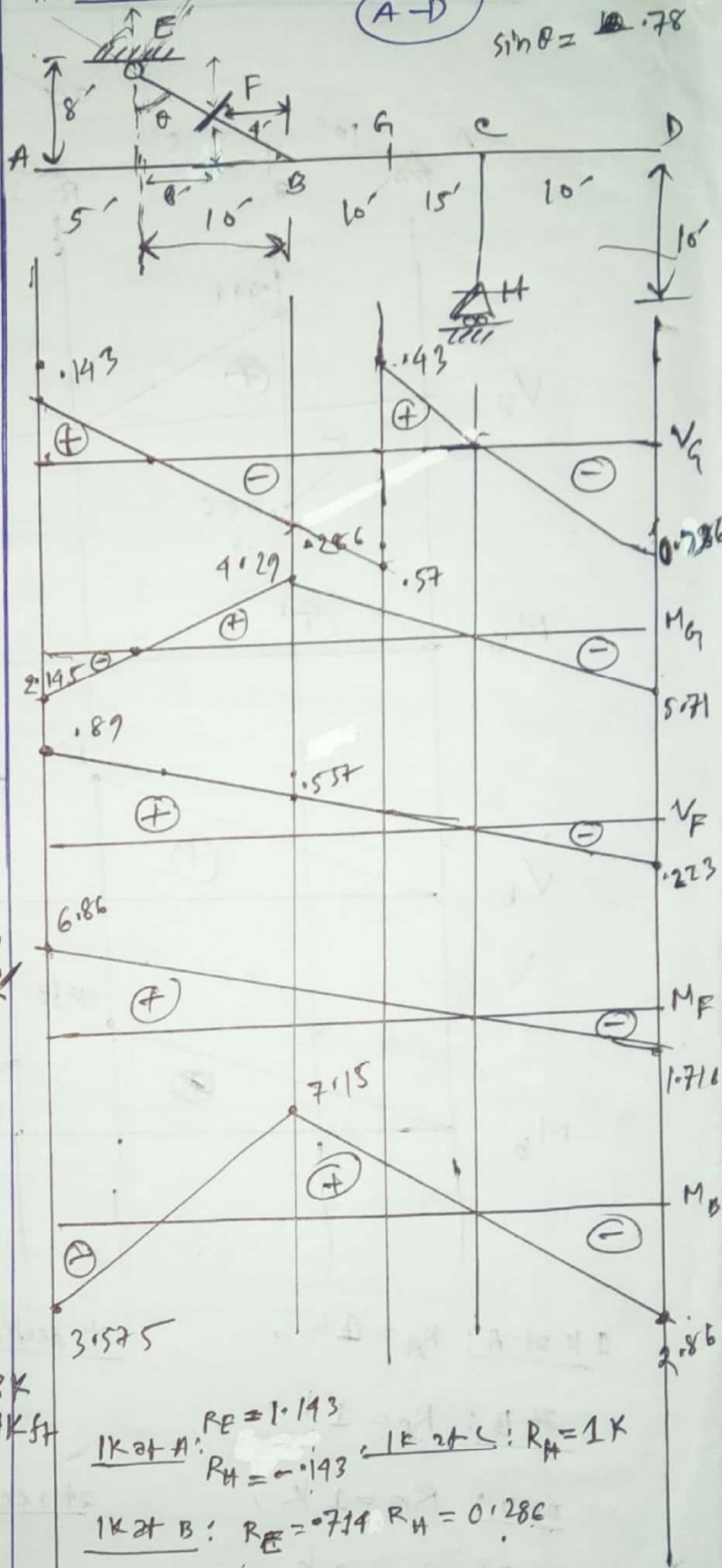
1K at Section 2:  $R_A = 0.78$ ,  $R_B = 0.22$

2008

# Draw IL for  $V_G, V_F, M_G, M_F, M_B$

(A-D)

$$\sin \theta = 0.78$$



1K at A:  $R_E = 1.143$ ,  $R_H = -0.143$  1K at C:  $R_H = 1K$

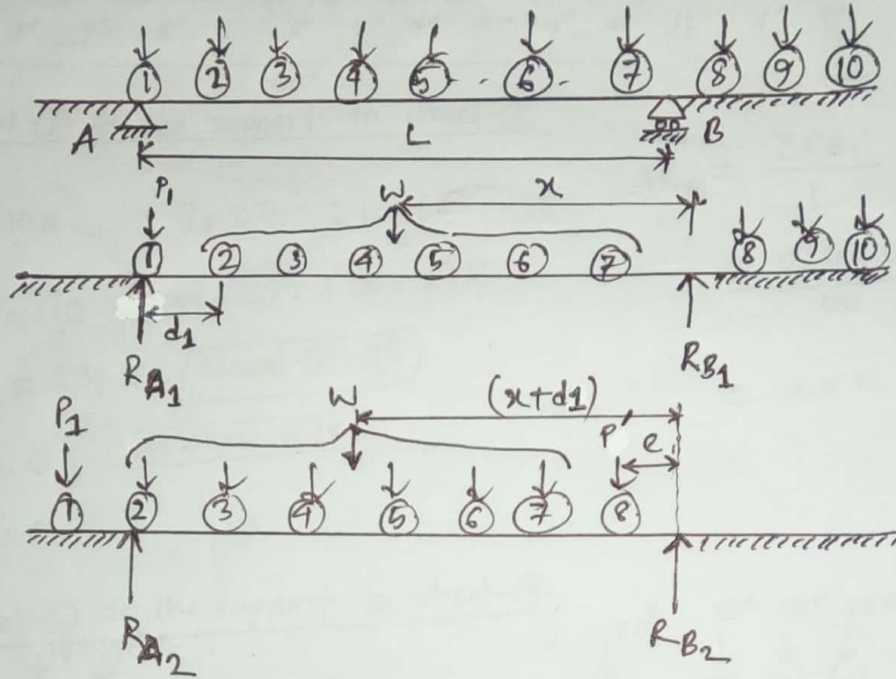
1K at B:  $R_E = 0.714$ ,  $R_H = 0.286$

1K at D:  $R_H = 1.286$ ,  $R_E = 0.286$

1K at G:  $R_E = 0.43$ ,  $R_H = 0.57$

## Wheel Load

# criteria for the maximum reaction of a simple beam subjected to series of concentrated loads move from right to left.



Before movement,  $\Sigma M_B = 0$

$$\Rightarrow R_{A1} L - P_1 L - Wx = 0$$

$$\Rightarrow R_{A1} = \frac{Wx}{L} + P_1 \dots \dots \dots \textcircled{1}$$

After movement,  $\Sigma M_B = 0$

$$\Rightarrow R_{A2} \times L - W(x+d_1) - P'e = 0$$

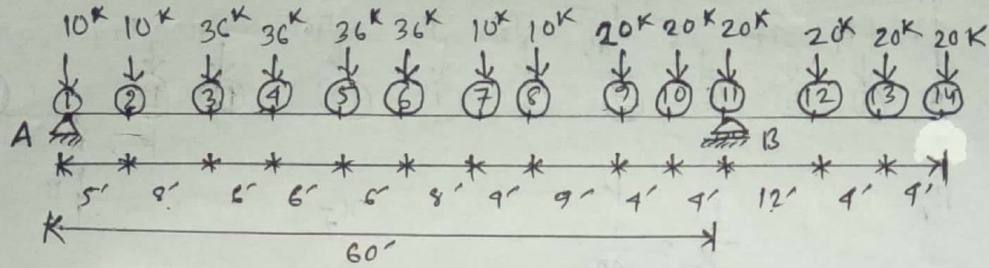
$$\Rightarrow R_{A2} = \frac{W(x+d_1)}{L} + \frac{P'e}{L} \dots \dots \dots \textcircled{11}$$

From  $\textcircled{1}$  &  $\textcircled{11}$ ,

change of reaction  $\Delta R = R_{A2} - R_{A1} = \frac{Wd_1}{L} + \frac{P'e}{L} - P_1$

Hence,  $\Delta R = \frac{\Sigma P d_1}{L} + \frac{P'e}{L} - P_1$

# Problem: Determine maximum reaction at A of a simple beam due to the loading shown in figure:



Solution:

When wheel (1) at the support to wheel (2)

Here,  $P_1 = 10K$  ;  $d_1 = 5'$  ;  $L = 60'$

$$\Sigma P = (10 + 36 \times 4 + 10 \times 2 + 20 \times 3) K$$

$$= 234 K \quad \text{Wheel-2 - (11)}$$

$$P' = 0 K \quad \text{No wheel enter}$$

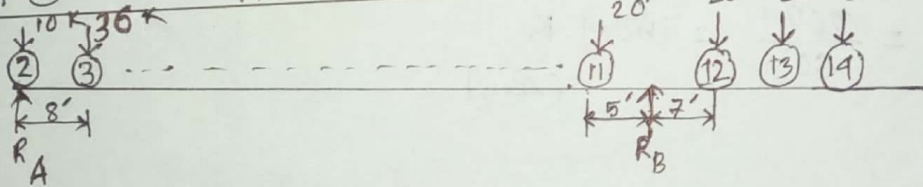
$$e = 0$$

$$\Delta R_A = \frac{\Sigma P d_1}{L} + \frac{P'e}{L} - P_1$$

$$= \frac{234 \times 5}{60} + \frac{0}{60} - 10$$

$$= 9.5 K \quad \text{(increasing)}$$

When wheel (2) at the support to wheel (3)



Here,

$$P_1 = 10K ; d_1 = 8' ; L = 60'$$

$$\Sigma P = (234 - 10) K = 224 K \quad \text{W-3 - (11)}$$

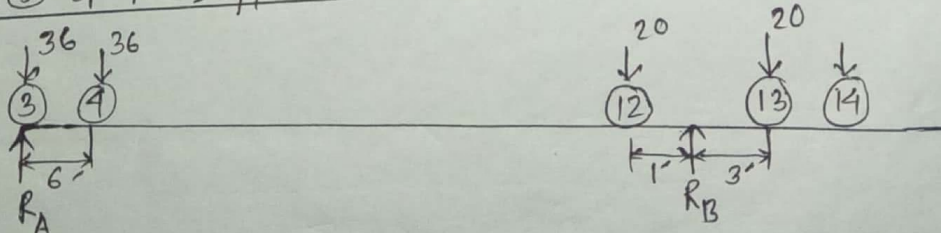
$$P' = 20 K \quad \text{Wheel-12}$$

$$e = 1'$$

$$\Delta R_A = \frac{224 \times 8}{60} + \frac{20 \times 1}{60} - 10$$

$$= 20.2 K \quad \text{(increasing)}$$

When wheel (3) at the support to wheel (4)



Here,  $P_1 = 36K$  ;  $d_1 = 6'$  ;  $L = 60'$

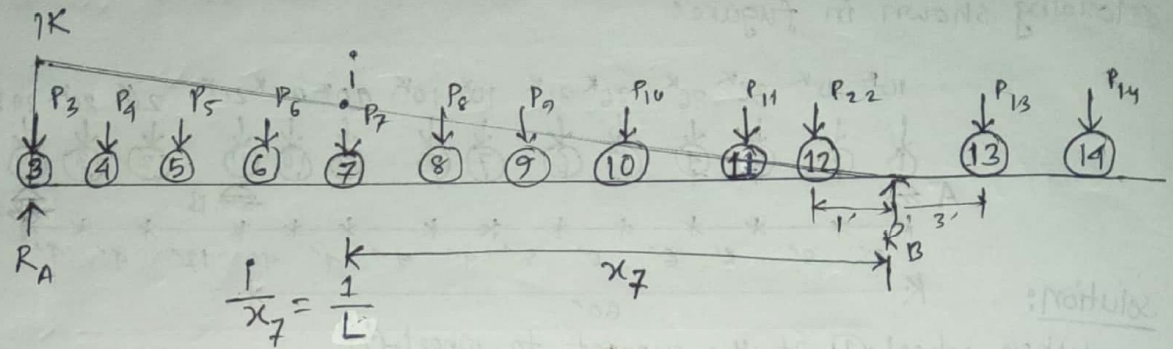
$$\Sigma P = (224 - 36 + 20) K = 208 K \quad \text{W-4 - (13)}$$

$$\text{W-12} \leftarrow P' = 20 K ; e = 3'$$

$$\Delta R_A = \frac{208 \times 6}{60} + \frac{20 \times 3}{60} - 36$$

$$= -14.2 K \quad \text{(decreasing)}$$

Hence, when wheel (3) at support A, Reaction at A will be maximum.



$$\Rightarrow i = \frac{x_7}{L} \quad \therefore R_{A(7)} = \frac{x_7}{L} \times P_7$$

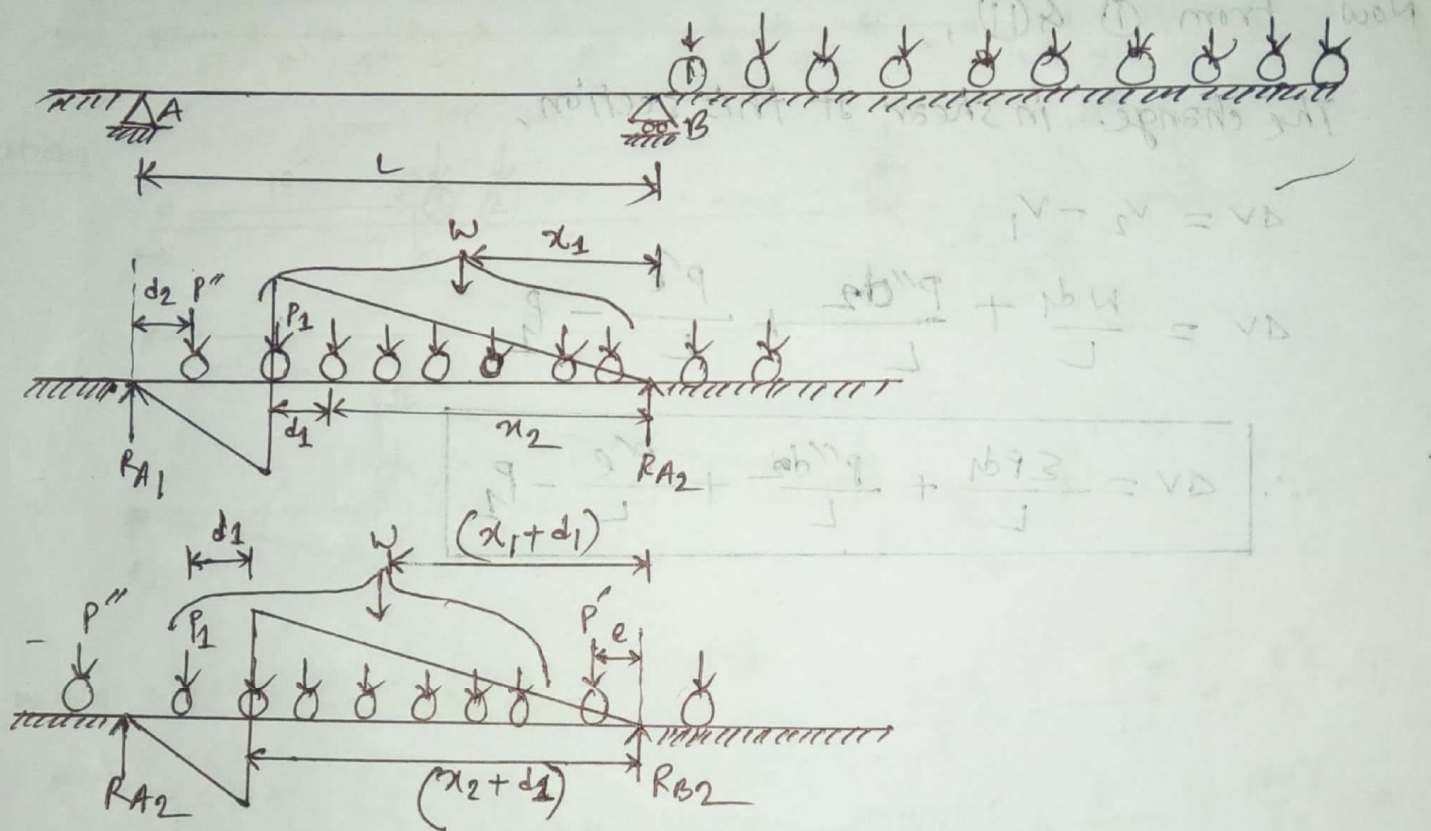
# Maximum Reaction,  $R_A = \frac{1}{L} [P_{12}x_{12} + P_{11}x_{11} + \dots + P_3x_3]$

$$\text{Max. } R_A = \frac{1}{60} [(20 \times 1) + (20 \times 13) + (20 \times 17) + (20 \times 21) + (10 \times 30) + (10 \times 34) + (36 \times 42) + (36 \times 48) + (36 \times 54) + (36 \times 60)]$$

$$\text{Max. } R_A = \frac{9024}{60} = 150.4 \text{ K (Ans)}$$

09, 11, 14, 16, 17

# Criteria for the maximum shear of a simple beam subjected to series of concentrated loads move from right to left.



Before movement,

$$\sum M_B = 0 \Rightarrow R_{A1} \times L - w x_1 - P''(L - d_2)$$

$$\Rightarrow R_{A1} = \frac{w x_1}{L} + P'' \frac{(L - d_2)}{L}$$

Now, shear force of the section,

$$V_1 = R_{A1} - P''$$

$$\Rightarrow V_1 = \frac{w x_1}{L} + P'' \frac{(L - d_2)}{L} - P'' = \frac{w x_1}{L} + \frac{P'' d_2}{L} \quad \text{--- (1)}$$

After movement,

$$\sum M_B = 0 \Rightarrow R_{A2} \times L - w(x_1 + d_1) - P' e = 0$$

$$\Rightarrow R_{A2} = \frac{w(x_1 + d_1)}{L} + \frac{P' e}{L}$$

Now, shear of the section, maximum at of section

$$V_2 = RA_2 - P_1$$

$$\Rightarrow V_2 = \frac{w(x_1 + dx)}{L} + \frac{p'e}{L} - P_1 \quad \text{--- (11)}$$

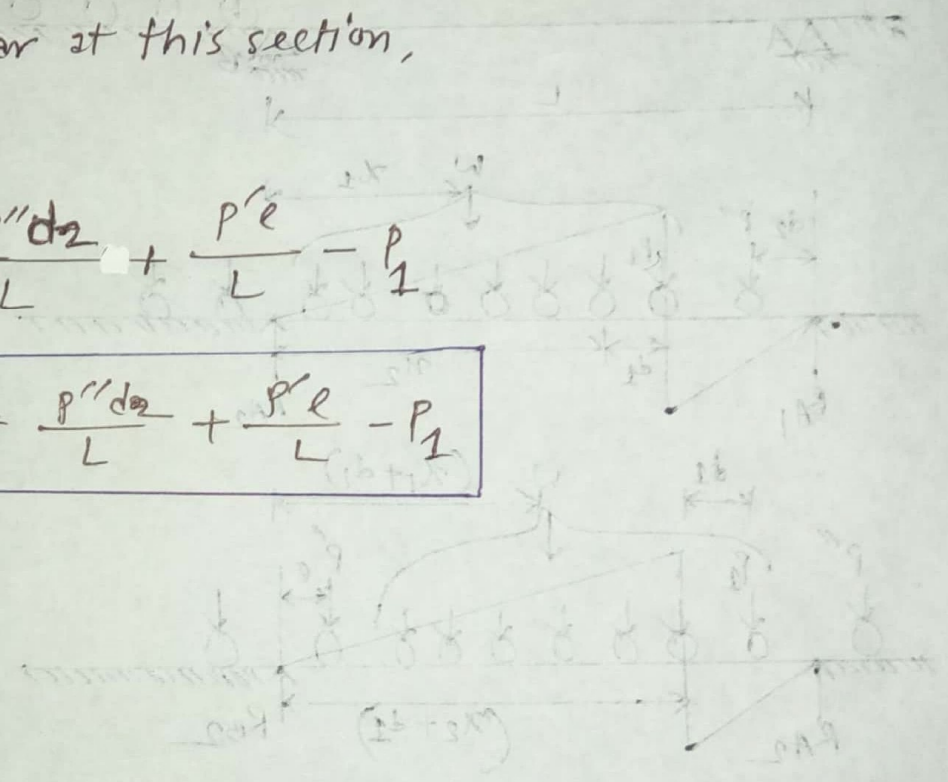
Now From (1) & (11),

The change in shear at this section,

$$\Delta V = V_2 - V_1$$

$$\Delta V = \frac{w dx}{L} + \frac{p'' dx}{L} + \frac{p'e}{L} - P_1$$

$$\therefore \Delta V = \frac{\sum P dx}{L} + \frac{p'' dx}{L} + \frac{p'e}{L} - P_1$$



Before movement,

$$\sum M = 0 \Rightarrow RA_1 \times L - w \times L \times \frac{L}{2} - P_1 \times (L - e) = 0$$

$$\Rightarrow RA_1 = \frac{wL}{2} + \frac{P_1(L - e)}{L}$$

Now shear force at the section,

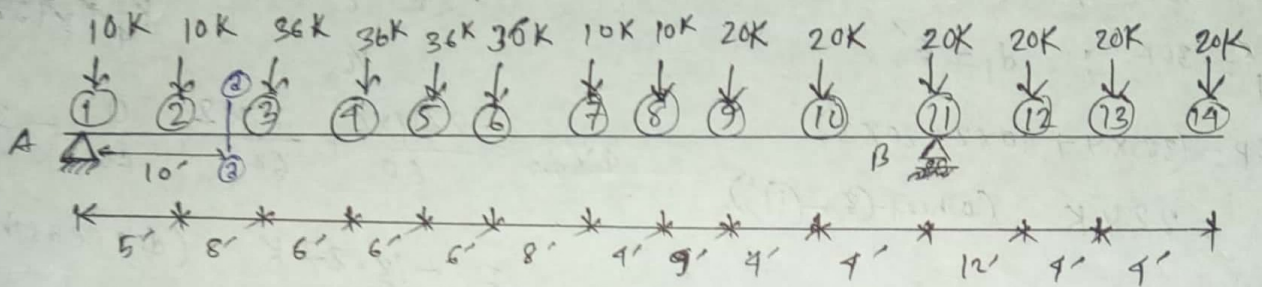
$$\Rightarrow V_1 = \frac{w x_1}{L} + \frac{P_1(L - e)}{L} - P_1$$

After movement,

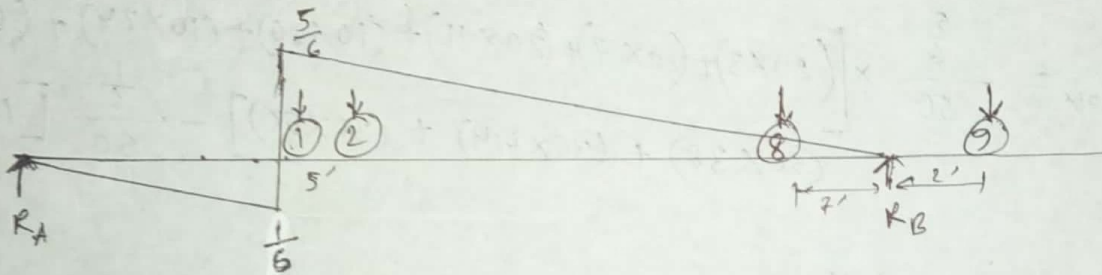
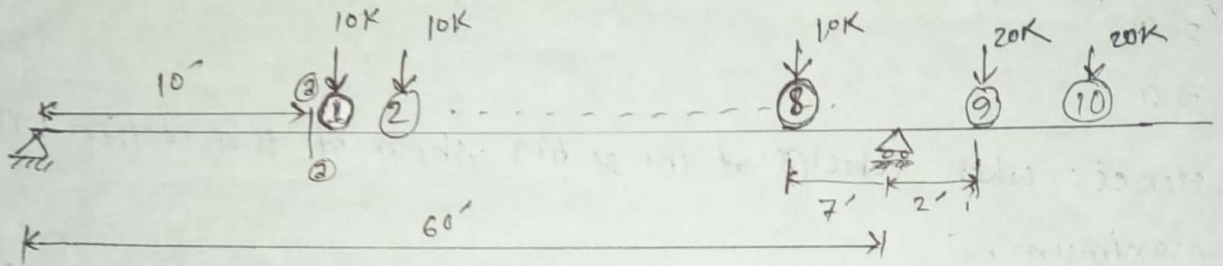
$$\sum M = 0 \Rightarrow RA_2 \times L - w(L + dx) \times \frac{L + dx}{2} - P_1(L - e) = 0$$

$$\Rightarrow RA_2 = \frac{w(L + dx)}{2} + \frac{P_1(L - e)}{L}$$

# Problem: Find the maximum shear at section 2-2 due to moving load as shown in figure:



Solution:



When wheel-1 at the section to wheel-2

Here,  $P_1 = 10K$

$$\sum P = (10 \times 2 + 36 \times 4 + 10 \times 2) K = 184 K \quad (\text{wheel-1} - \text{8})$$

$$d_1 = 5' ; d_2 = 0$$

$$P'' = 0 ; P_1 = 10K \quad (\text{wheel-1})$$

$$\boxed{W-9} \leftarrow P' = 20K ; e' = 3$$

When wheel-2 at the section to wheel-3

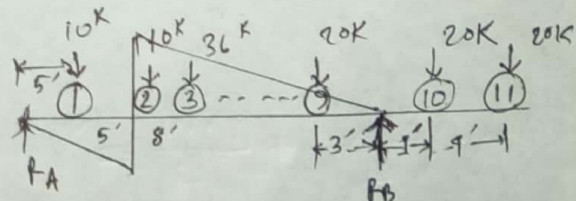
$$P_1 = 10K ; d_1 = 8' ; d_2 = 5'$$

$$P'' = 10K ; \sum P = (10 + 36 \times 4 + 10 \times 2 + 20) = 194 \quad (\text{wheel-2} - \text{9})$$

$$P' = (20 + 20) K = 40K \quad (\text{wheel-10} \& \text{11})$$

$$e = \left( \frac{7+3}{2} \right) = \frac{10}{2} = 5'$$

$$\begin{aligned} \Delta V_{2-2} &= \frac{\sum P d_1}{L} + \frac{P'' d_2}{L} + \frac{P' e}{L} - P_1 \\ &= \frac{184 \times 5}{60} + \frac{0}{60} + \frac{20 \times 3}{60} - 10 \\ &= 6.83 K \quad (\text{increasing}) \end{aligned}$$



$$\begin{aligned} \Delta V_{2-2} &= \frac{194 \times 8}{60} + \frac{10 \times 5}{60} + \frac{40 \times 5}{60} - 10 \\ &= 20.03 K \quad (\text{increasing}) \end{aligned}$$

When wheel (3) at the section to wheel (4)

$P'' = 10K$  ;  $d_2 = 2'$

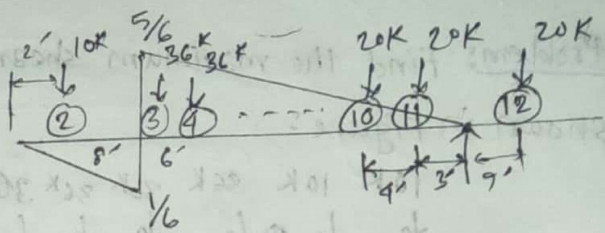
$P_1 = 36K$  ;  $d_1 = 6'$

$\Sigma P = (36 \times 4 + 10 \times 2 + 20 \times 3)$

$= 224K$  (wheel (3) - (11))

$P' = 0$

$e = 0$



$4V_{2-2} = \frac{224 \times 6}{60} + \frac{10 \times 2}{60} + \frac{0}{60} - 36$

$= -13.27K$  (decreasing)

Hence, when wheel (3) at the section, shear at that section will be maximum.

$\therefore V_{max} = \frac{5}{6} \times \left[ \frac{(20 \times 3) + (20 \times 7) + (20 \times 11) + (10 \times 20) + (10 \times 24) + (36 \times 32) + (36 \times 38) + (36 \times 44) + (36 \times 50)}{50} \right] - \frac{1}{60} [10 \times 2]$

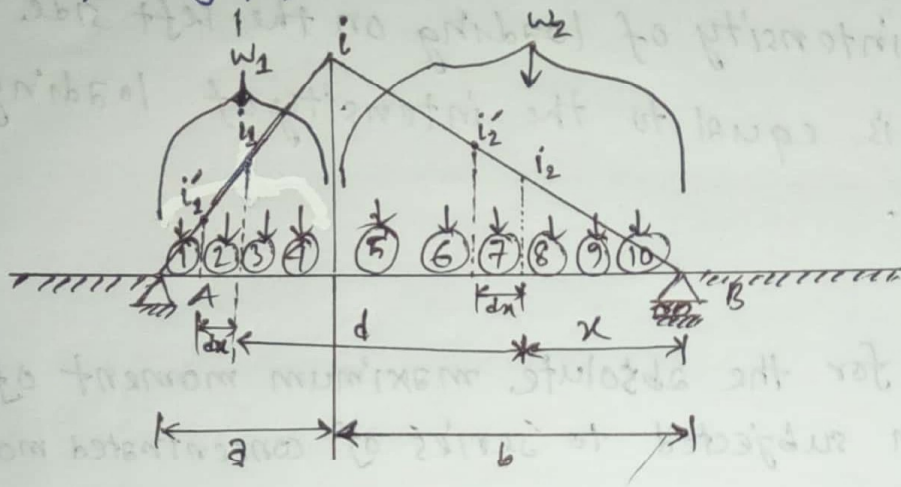
$= 112.4K$

(Ans)

*(Faint handwritten notes and diagrams, including a diagram of a beam with a triangular load and point loads, and various calculations for shear and moment.)*

12, 13, 15

# Criteria for the maximum moment at section of a simple beam subjected to series of concentrated loads move from right to left.



considering the right side of section,

$$i_2 = \frac{x}{b} i, \quad i_2' = \frac{(x+dx)}{b} i$$

$$\therefore M_2 = i \frac{x}{b} w_2, \quad M_2' = i' \frac{(x+dx)}{b} w_2$$

$$\therefore \text{increase of moment, } \Delta M_2 = M_2' - M_2 = i \frac{w_2}{b} dx$$

considering the left side of the section,

$$i_1 = \frac{(L-x-d)}{a} i, \quad i_1' = \frac{(L-x-d-dx)}{a} i$$

$$\therefore M_1 = i \frac{(L-x-d)}{a} w_1, \quad M_1' = i' \frac{(L-x-d-dx)}{a} w_1$$

$$\therefore \text{decrease of moment, } \Delta M_1 = M_1 - M_1' = i \frac{w_1 dx}{a}$$

Net increase in moment,  $dM = \Delta M_1 + \Delta M_2$

$$\therefore dM = i \frac{w_2}{b} dx - i \frac{w_1}{a} dx$$

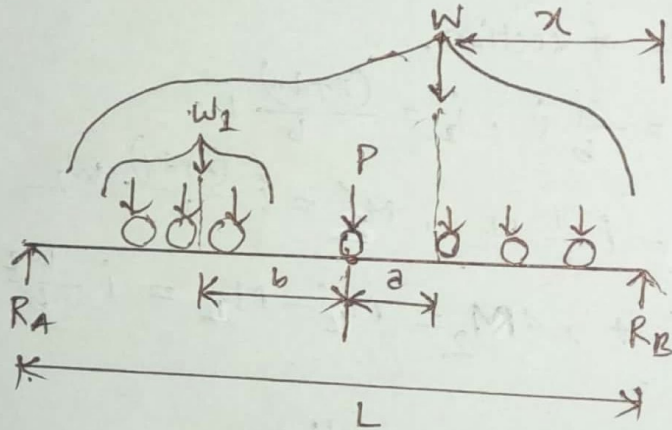
$$\Rightarrow \frac{dM}{dx} = -i \frac{w_1}{a} + i \frac{w_2}{b} = 0$$

$$\Rightarrow \boxed{\frac{w_1}{a} = \frac{w_2}{b} = \frac{w_1 + w_2}{a + b} = \frac{w}{L}}$$

So, The maximum moment at a given section occurs when the intensity of loading on the left side of the section is equal to the intensity of loading on the span.

10

# Criteria for the absolute maximum moment of a simple beam subjected to series of concentrated moving loads.



$$M = R_A (L - a - x) - W_1 b \quad \text{and} \quad R_A = \frac{Wx}{L}$$

$$\Rightarrow M = \frac{Wx}{L} (L - a - x) - W_1 b$$

$$\Rightarrow M = \frac{W}{L} (Lx - ax - x^2) - W_1 b$$

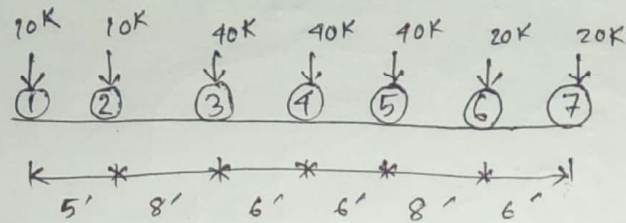
$$\Rightarrow \frac{dM}{dx} = \frac{W}{L} (L - a - 2x) = 0$$

$$\Rightarrow \frac{dM}{dx} = \frac{W}{L} (L - a - 2x) = 0$$

$$\Rightarrow \therefore 2x = L - a$$

$$\Rightarrow \boxed{x = \frac{L}{2} - \frac{a}{2}}$$

# For a single span of 100 ft. calculate (i) maximum moment at mid span and (ii) the absolute maximum moment due to the loading shown in figure below:



Solution: criteria for max. moment at center is:  $\frac{w_1}{a_1} = \frac{w}{L}$

For wheel 1 at center:

$$\text{Right: } \frac{0}{50} < \frac{180}{100}$$

$$\text{Left: } \frac{10}{50} < \frac{180}{100}$$

Wheel 2:

$$\text{Right: } \frac{10}{50} < \frac{180}{100}$$

$$\text{Left: } \frac{20}{50} < \frac{180}{100}$$

Wheel 3:

$$\text{Right: } \frac{20}{50} < \frac{180}{100}$$

$$\text{Left: } \frac{60}{50} < \frac{180}{100}$$

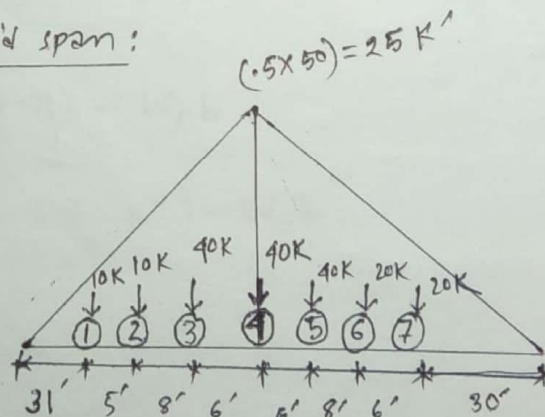
Wheel 4:

$$\text{Right: } \frac{60}{50} < \frac{180}{100}$$

$$\text{Left: } \frac{100}{50} > \frac{180}{100}$$

which satisfies the criteria.

Now, wheel 4 at mid span:



$$\begin{aligned} \text{Max. moment at center} &= \frac{25}{50} [(30 \times 20) + (36 \times 20) + (44 \times 40) + (50 \times 40)] + \\ &\quad \frac{25}{50} [(31 \times 10) + (36 \times 10) + (44 \times 40)] \\ &= 3755 \text{ K}' \end{aligned}$$

Absolute max. moment:

position of centroid of total load from right end of loading

$$\bar{x} = \frac{1}{180} (20 \times 0 + 20 \times 6 + 40 \times 14 + 40 \times 20 + 40 \times 26 + 10 \times 34 + 10 \times 39)$$

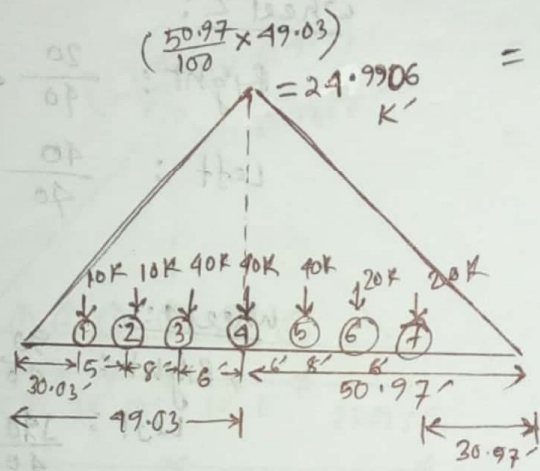
$$= 18.06'$$

$\bar{x}$  (Wheel 2) distance

$\therefore$  position of wheel 1 from left end,  $x = \frac{L}{2} - \frac{a}{2}$  where,  $a = (20 - 18.06)$

$$= \left( \frac{100}{2} - \frac{1.94}{2} \right) \text{ ft}$$

$$= 49.03 \text{ ft}$$

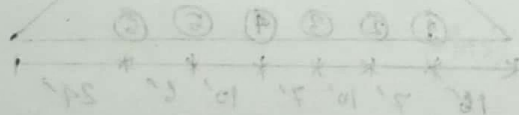


$\therefore$  Absolute <sup>max.</sup> moment =  $\frac{24.9906}{49.03} \times (30.03 \times 20 + 35.03 \times 10 + 43.03 \times 40 + 49.03 \times 40)$

$$+ \frac{24.9906}{50.97} (30.97 \times 20 + 36.97 \times 20 + 44.97 \times 40)$$

$$= 3756.703 \text{ K'}$$

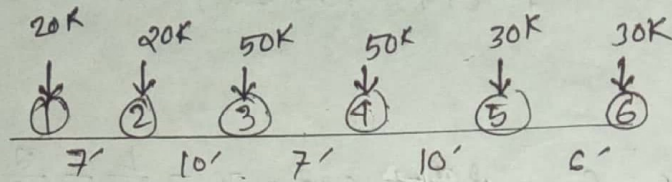
(Ans.)



$$\text{Max. moment at center} = \frac{10}{180} [(30 \times 10) + (50 \times 20) + (50 \times 30) + (50 \times 40)] + \frac{20}{180} [(30 \times 30) + (30 \times 40)]$$

$$= 3056 \text{ K'}$$

# For a simple span of 80ft, compute ① the maximum moment at mid-span ② the absolute <sup>max.</sup> moment due to the loading shown in figure below:



Solution: criteria for max. moment at center is:  $\frac{W_1}{2} = \frac{W}{L}$

For wheel 1 at center:

$$\text{Right: } \frac{0}{40} < \frac{170}{80}$$

$$\text{Left: } \frac{20}{40} < \frac{200}{80}$$

wheel 2:

$$\text{Right: } \frac{20}{40} < \frac{200}{80}$$

$$\text{Left: } \frac{40}{40} < \frac{200}{80}$$

wheel 3:

$$\text{Right: } \frac{40}{40} < \frac{200}{80}$$

$$\text{Left: } \frac{90}{40} < \frac{200}{80}$$

wheel 4:

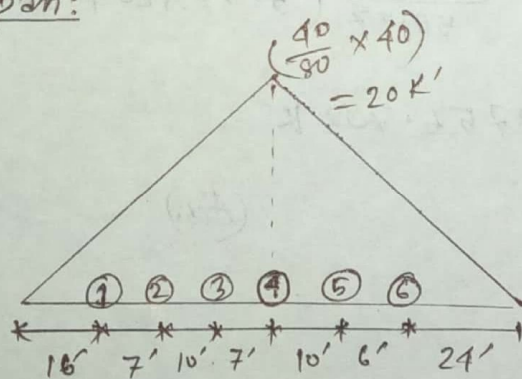
$$\text{Right: } \frac{90}{40} < \frac{200}{80}$$

$$\text{Left: } \frac{140}{40} > \frac{200}{80}$$

which satisfies the criteria

~~Maximum moment~~

Now, wheel 4 at mid span:



$$\begin{aligned} \text{Max. moment at center} &= \frac{20}{40} [(20 \times 16) + (20 \times 23) + (50 \times 33) + (50 \times 40)] \\ &+ \frac{20}{40} [(30 \times 24) + (30 \times 30)] \\ &= 3025 K' \end{aligned}$$

Absolute max. moment:

position of centroid of total load from right end of loading,

$$\bar{x} = \frac{(20 \times 0) + (30 \times 6) + (50 \times 16) + (50 \times 23) + (20 \times 33) + (20 \times 40)}{200}$$

$$= 17.95'$$

position of wheel - 4 from left end,  $x = \frac{L}{2} - \frac{a}{2}$  where,  $\bar{x}$

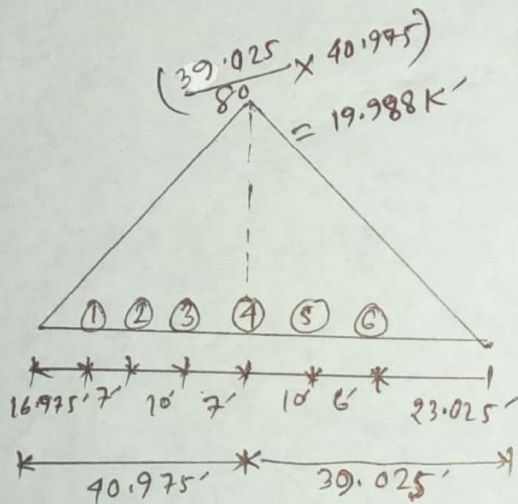
$$a = (16 - 17.95)$$

$$= -1.95$$

$$= \frac{80}{2} - \left(-\frac{1.95}{2}\right)$$

$$= \frac{80}{2} + \frac{1.95}{2}$$

$$= 40.975$$

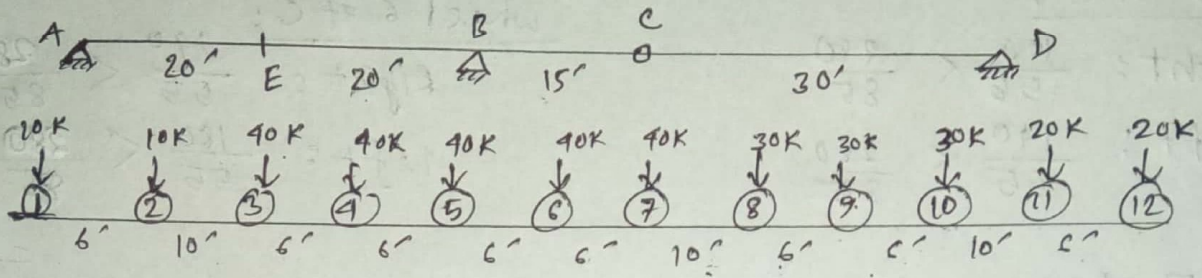


$$\begin{aligned} \therefore \text{Absolute max. moment} &= \frac{19.988}{40.975} \left( 20 \times 16.975 + 20 \times 23.975 + 50 \times 33.975 \right. \\ &\quad \left. + 50 \times 40.975 \right) + \frac{19.988}{39.025} \left( 30 \times 23.025 + 30 \times 29.025 \right) \\ &= 3027.36 \text{ k}' \end{aligned}$$

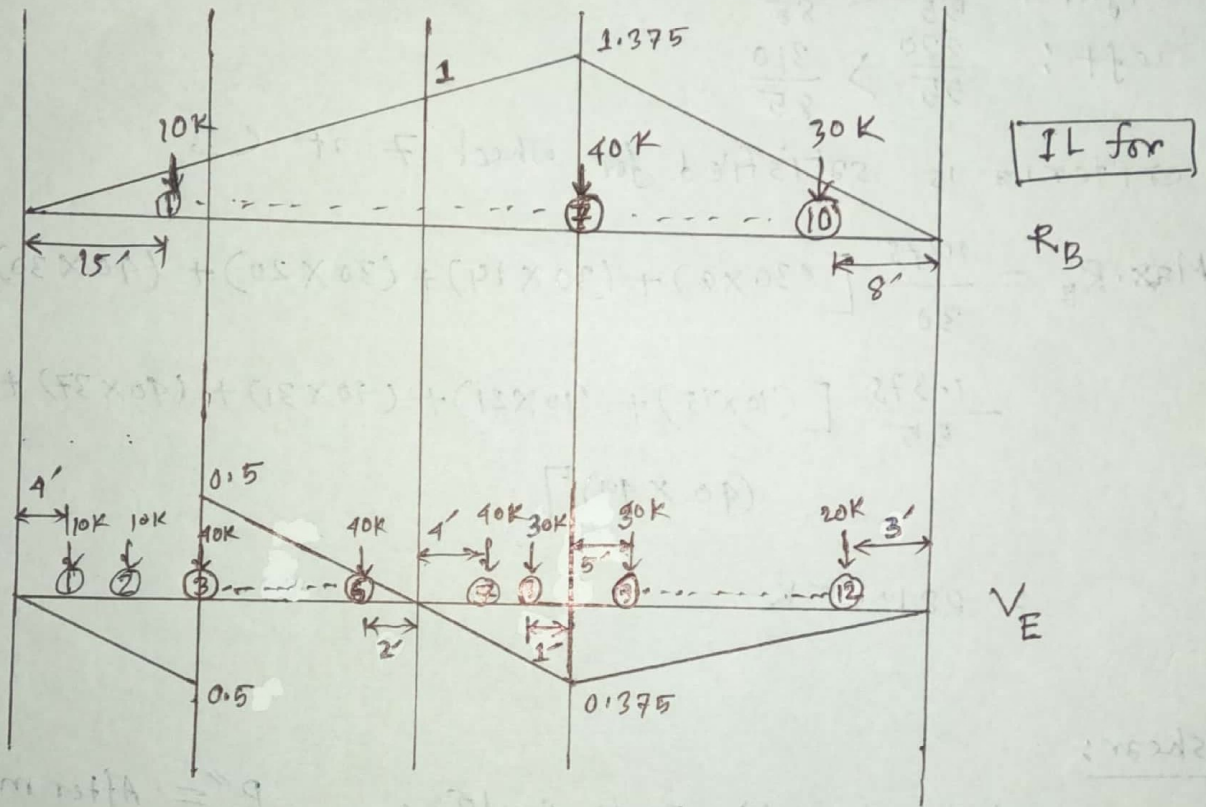
(Ans.)

2005, 03

# Apply proper criteria to obtain the position of loads and hence compute the maximum values of reactions at B and shear at E. structure subjected to moving loads as shown below.



Solution:



For Reaction: IL of  $R_B$  is same as Moment diagram. Hence

criteria for max. reaction at B is  $\frac{w_1}{x_1} = \frac{w}{L}$

Wheel 1 at C: Right:  $\frac{0}{55} < \frac{140}{85}$

Left:  $\frac{10}{55} < \frac{140}{85}$

Wheel 2 at C:

Right:  $\frac{10}{55} < \frac{180}{85}$

Left:  $\frac{20}{55} < \frac{180}{85}$

wheel 3 at c: Right:  $\frac{20}{55} < \frac{220}{85}$   
 Left:  $\frac{60}{55} < \frac{220}{85}$

wheel 4 at c:  
 Right:  $\frac{60}{55} < \frac{250}{85}$   
 Left:  $\frac{100}{55} < \frac{250}{85}$

wheel 5 at c:

Right:  $\frac{100}{55} < \frac{280}{85}$   
 Left:  $\frac{140}{55} < \frac{280}{85}$

wheel 6 at c:

Right:  $\frac{140}{55} < \frac{310}{85}$   
 Left:  $\frac{180}{55} < \frac{310}{85}$

wheel 7 at c:

Right:  $\frac{180}{55} < \frac{310}{85}$   
 Left:  $\frac{220}{55} > \frac{310}{85}$

∴ criteria is satisfied for wheel 7 at c

$$\text{Max. } R_B = \frac{1.375}{30} [(30 \times 8) + (30 \times 14) + (30 \times 20) + (40 \times 30)] +$$

$$\frac{1.375}{55} [(10 \times 15) + (10 \times 21) + (40 \times 31) + (40 \times 37) + (40 \times 43) +$$

$$(40 \times 49)]$$

$$= 281.75 \text{ K}$$

For shear:

when, wheel ① at section E to wheel ②,

$P'' =$  After movement,  
 span (अंतर अंतर)  
 अंतर अंतर.

$$\Delta V_E = \frac{\sum P d_1}{L} + \frac{P'' d_2}{L} + \frac{P' e}{L} - P_1$$

Here,  $P_1 = 10 \text{ K}, L = 85'$   
 $d_1 = 6'$

$$\therefore \Delta V_E = \frac{310 \times 6}{85} + 0 + 0 - 10$$

$P'' = 0$

$d_2 = 0$

$P' = 0$

$e = 0$

$\sum P = 310 \text{ K}$  (Wheel ①-⑩)

∴  $\Delta V_E = 11.88$  (increasing)

wheel ① at section E to wheel ②

Here  $P_1 = 10 \text{ K}$        $\Sigma P = 310 \text{ K}$  [wheel ①-10]

$d_1 = 10'$        $P' = (20+20) = 40 \text{ K}$

$P'' = 0$        $e' = \left(\frac{9+3}{2}\right) = 6$

$d_2 = 0$

$$\therefore \Delta V_E = \frac{310 \times 10}{85} + 0 + \frac{40 \times 6}{85} - 10 = 29.294 \text{ K (increasing)}$$

wheel ③ at section E to wheel ④

Here,

$P_1 = 40 \text{ K}$        $\Sigma P = 340 \text{ K}$  [wheel ③-12]

$d_1 = 6'$        $P' = 0$

$P'' = 10 \text{ K}$        $e = 0$

$d_2 = 4'$        $\therefore \Delta E = \frac{340 \times 6}{85} + \frac{10 \times 4}{85} + 0 - 40 = -15.53 \text{ K}$   
(decreasing)

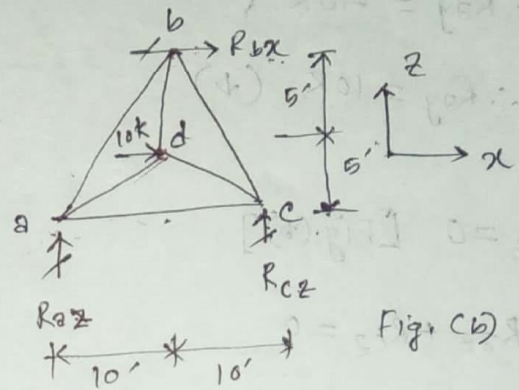
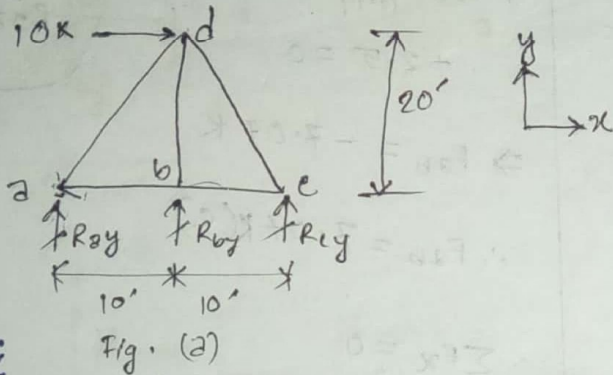
Hence, criteria is satisfied for wheel ③ at section E,

$$\begin{aligned} \therefore \text{Max. shear at E, } V_E &= \frac{0.5}{20} [(40 \times 2) + (40 \times 8) + (40 \times 14) + (40 \times 20)] \\ &\quad - \frac{0.5}{20} [(10 \times 4) + (10 \times 10)] - \frac{0.375}{15} [(40 \times 4) + \\ &\quad (30 \times 14)] - \frac{0.375}{30} [(20 \times 3) + (20 \times 9) + (30 \times 15) \\ &\quad + (30 \times 25)] \\ &= 44 - 3.5 - 14.5 - 19.5 \\ &= 6.5 \text{ K} \end{aligned}$$

(Ans)

## 3D Space Truss

**Problem:** Compute the reactions and bar forces of the following truss as shown in figure below:



Solution:

Member	Projection			Length = $\sqrt{x^2+y^2+z^2}$
	x	y	z	
ab	10'	0'	10'	14.14'
ac	20'	0'	10'	22.36'
ad	10'	20'	5'	22.91'
bc	10'	0'	10'	14.14'
bd	0'	20'	5'	20.62'
cd	10'	20'	5'	22.91'

$$\sum F_x = 0 \quad [\text{Fig. (b)}]$$

$$R_{bx} + 10 = 0$$

$$\Rightarrow R_{bx} = -10 \text{ K}$$

$$\therefore R_{bx} = 10 \text{ K} (\leftarrow)$$

$$\sum M_x = 0 \quad [\text{Fig. (a) \& Fig. (b)}]$$

$$R_{by} \times 10 = 0$$

$$\therefore R_{by} = 0 \text{ K}$$

$$\sum M_z = 0 \quad [\text{Fig. (a)}]$$

$$R_{by} \times 10 + R_{cy} \times 20 - 10 \times 20 = 0$$

$$\therefore R_{cy} = 10 \text{ K}$$

$$\sum M_y = 0 \quad [\text{Fig. (b)}]$$

$$R_{cz} \times 20 - 10 \times 5 - R_{bx} \times 10 = 0$$

$$\Rightarrow R_{cz} = \frac{50 - 100}{20} = -2.5 \text{ K}$$

$$\therefore R_{cz} = 2.5 \text{ K} (\downarrow)$$

$$\Sigma F_y = 0 \text{ [Fig (a)]}$$

$$R_{ay} + R_{by} + R_{cy} = 0$$

$$\Rightarrow R_{ay} + 0 + 10 = 0$$

$$\Rightarrow R_{ay} = -10 \text{ K}$$

$$\therefore R_{ay} = 10 \text{ K (}\downarrow\text{)}$$

$$\Sigma F_z = 0 \text{ [Fig (b)]}$$

$$R_{az} + R_{cz} = 0$$

$$\Rightarrow R_{az} - 2.5 = 0$$

$$\therefore R_{az} = 2.5 \text{ K}$$

Now, joint a:  $\Sigma F_y = 0$

$$F_{ady} + R_{ay} = 0$$

$$\Rightarrow F_{ad} \times \frac{20}{22.91} = 10$$

$$\therefore F_{ad} = 11.455 \text{ K (T)}$$

joint b:  $\Sigma F_y = 0$

$$F_{bdy} + R_{by} = 0$$

$$F_{bd} = 0$$

joint c:  $\Sigma F_y = 0$

$$F_{cdy} + R_{cy} = 0$$

$$\Rightarrow F_{cd} \times \frac{20}{22.91} = -10$$

$$\Rightarrow F_{cd} = -11.455 \text{ K}$$

$$F_{cd} = 11.455 \text{ K (C)}$$

At joint a:

$$\Sigma F_z = 0$$

$$F_{abz} + F_{adz} + R_{az} = 0$$

$$\Rightarrow F_{ab} \times \frac{10}{14.14} + 11.455 \times \frac{5}{22.91}$$

$$+ 2.5 = 0$$

$$\Rightarrow F_{ab} = -7.07 \text{ K}$$

$$\therefore F_{ab} = 7.07 \text{ K (C)}$$

$$\Sigma F_x = 0$$

$$F_{abx} + F_{adx} + F_{ac} = 0$$

$$7.07 \times \frac{10}{14.14} + 11.455 \times \frac{10}{22.91} + F_{ac} = 0$$

$$\Rightarrow F_{ac} = -10 \text{ K}$$

$$\therefore F_{ac} = 10 \text{ K (C)}$$

At joint e:

$$\Sigma F_z = 0$$

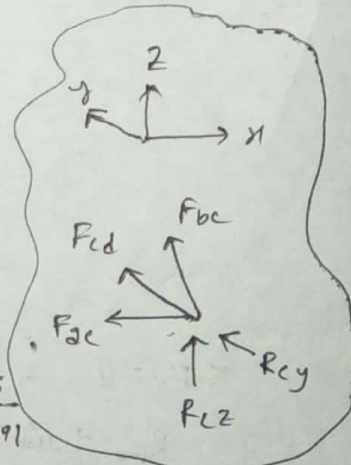
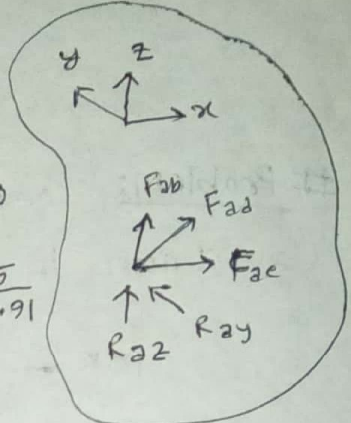
$$R_{ez} + F_{ecz} + F_{edz} = 0$$

$$-2.5 + F_{ec} \times \frac{10}{14.14} + 11.455 \times \frac{5}{22.91}$$

$$= 0$$

$$\Rightarrow F_{ec} = 7.07 \text{ K (T)}$$

(Ans)



# compute the reactions and bar forces of the following space truss shown in figure below:- [With out considering moment equation] → Another way to solve

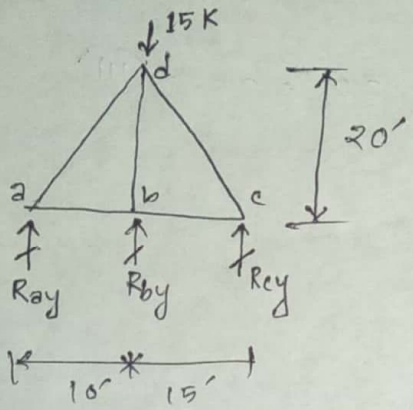


Fig. (a)

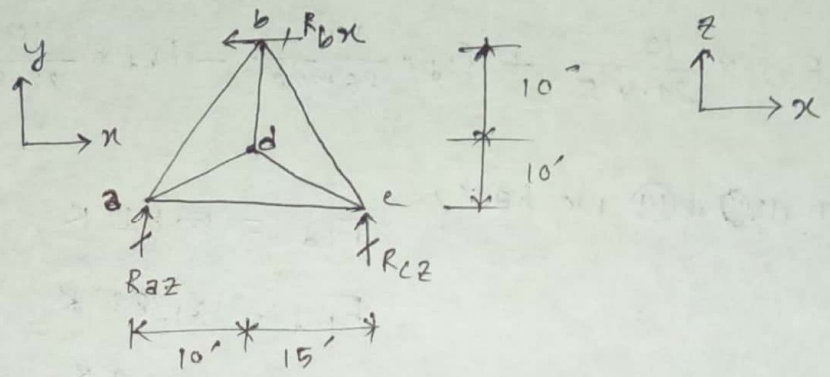


Fig. (b)

Solution:

Member	Projection			Length = $\sqrt{x^2+y^2+z^2}$
	x	y	z	
ab	10'	0'	20'	22.36'
ac	25'	0'	0'	25'
ad	10'	20'	10'	24.495'
bd	0'	20'	10'	22.36'
cd	15'	20'	10'	26.926'

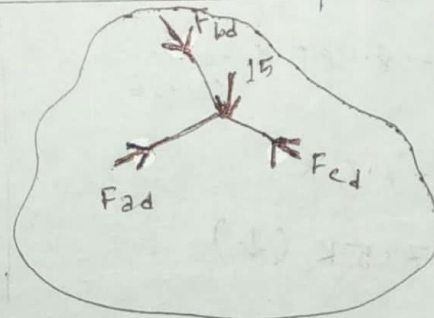
considering joint d:

$$\sum F_x = 0$$

$$F_{ad}x - F_{cd}x = 0$$

$$\Rightarrow F_{ad} \times \frac{10}{24.495} = F_{cd} \times \frac{15}{26.926}$$

$$\therefore F_{ad} = 1.3646 F_{cd} \dots \dots \textcircled{1}$$



$$\sum F_y = 0$$

$$F_{ad}y + F_{bd}y + F_{cd}y - 15 = 0$$

$$\Rightarrow F_{ad} \times \frac{20}{24.495} + F_{bd} \times \frac{20}{22.36} + F_{cd} \times \frac{20}{26.926} = 15 \dots \dots \textcircled{11}$$

$$\Sigma F_z = 0$$

$$F_{adz} + F_{cdz} - F_{bdz} = 0$$

$$\Rightarrow F_{ad} \times \frac{10}{24.495} + F_{cd} \times \frac{10}{26.926} - F_{bd} \times \frac{10}{22.36} = 0 \quad \text{--- (11)}$$

from (1), (11) & (11) we have,

$$F_{ad} = 5.51 \text{ K}$$

$$F_{bd} = 8.385 \text{ K}$$

$$F_{cd} = 4.04 \text{ K}$$

$$\Sigma F_x = 0 \text{ [Fig (b)] } R_{bx} = 0$$

considering joint b:

$$\Sigma F_x = 0$$

$$F_{abx} - F_{bcx} = 0$$

$$\Rightarrow F_{ab} \times \frac{10}{22.36} = F_{bc} \times \frac{15}{25}$$

$$\Rightarrow F_{ab} = 1.3416 F_{bc} \quad \text{--- (1)}$$

$$\Sigma F_y = 0$$

$$F_{bdy} + R_{by} = 0$$

$$\Rightarrow R_{by} = -8.385 \times \frac{20}{22.36}$$

$$\Rightarrow R_{by} = -7.5 \text{ K}$$

$$R_{by} = 7.5 \text{ K } (\downarrow)$$

$$\Sigma F_z = 0$$

$$F_{abz} - F_{bdz} + F_{bcz} = 0$$

$$\Rightarrow F_{ab} \times \frac{20}{22.36} - 8.385 \times \frac{10}{22.36} + F_{bc} \times \frac{20}{25} = 0 \quad \text{--- (2)}$$



From (1) & (2) we obtain,  $F_{ab} = +2.6155 \text{ K}$

$$F_{bc} = +1.875 \text{ K}$$

considering joint a:

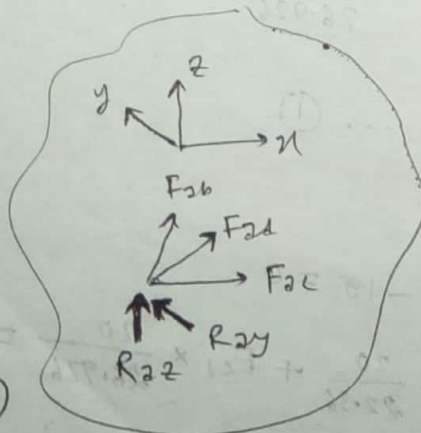
$$\Sigma F_z = 0$$

$$F_{abz} + F_{adz} + R_{az} = 0$$

$$2.6155 \times \frac{20}{22.36} + 5.51 \times \frac{10}{24.495} + R_{az} = 0$$

$$\Rightarrow R_{az} = -4.5 \text{ K}$$

$$\therefore R_{az} = 4.5 \text{ K } (\downarrow)$$



$$\sum R_y = 0$$

$$F_{ady} + R_{ay} = 0$$

$$\Rightarrow R_{ay} = -5.51 \times \frac{20}{24.495}$$

$$\Rightarrow R_{ay} = -4.5 \text{ K}$$

$$R_{ay} = 4.5 \text{ K} (\downarrow)$$

$$\sum F_x = 0$$

$$F_{abx} + F_{adx} + F_{acx} = 0$$

$$2.151 \times \frac{10}{22.36} + 5.51 \times \frac{10}{24.495} + F_{ac} \times \frac{25}{25} = 0$$

$$\therefore F_{ac} = -3.372 \text{ K}$$

$$\therefore F_{ac} = 3.372 \text{ K} (\text{C})$$

considering point e

$$\sum F_z = 0$$

$$R_{ez} + F_{bez} + F_{dez} = 0$$

$$R_{ez} + 1.875 \times \frac{20}{25} + 4.04 \times \frac{10}{26.926} = 0$$

$$\Rightarrow R_{ez} = -3 \text{ K}$$

$$\therefore R_{ez} = 3 \text{ K} (\downarrow)$$

$$\sum F_y = 0$$

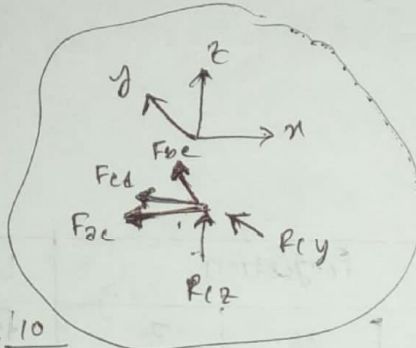
$$F_{edy} + R_{ey} = 0$$

$$\Rightarrow R_{ey} = -4.04 \times \frac{20}{26.926}$$

$$\Rightarrow R_{ey} = -3 \text{ K}$$

$$\therefore R_{ey} = 3 \text{ K} (\downarrow)$$

(Ans.)



# problem: compute the reaction and bar forces of the following space truss shown in figures:

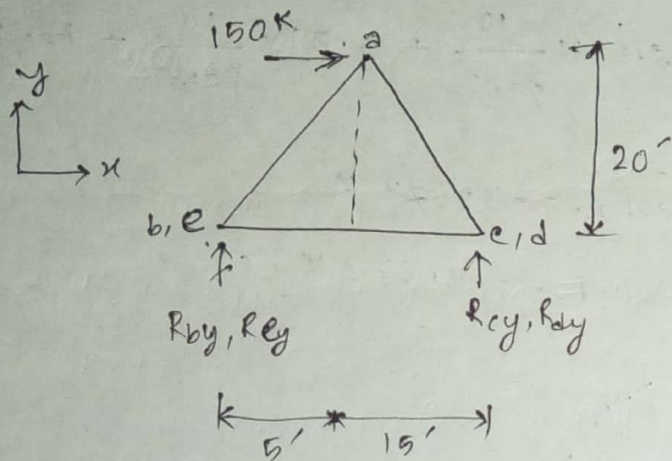


Fig. (a)

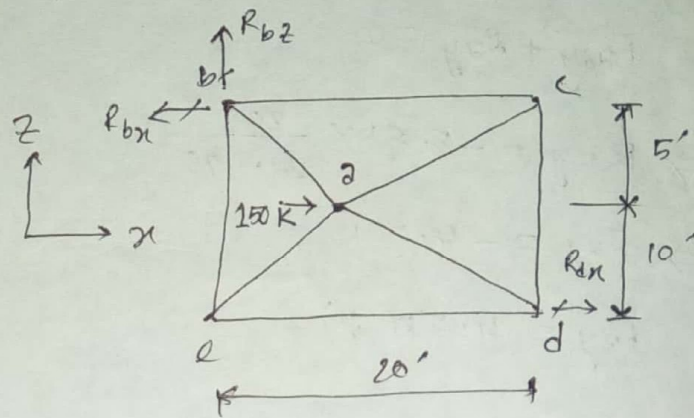


Fig. (b)

Solution:

Member	Projection			Length = $\sqrt{x^2 + y^2 + z^2}$
	x	y	z	
ab	5'	20'	5'	21.21
ac	15'	20'	5'	25.495
ad	15'	20'	10'	26.926
ae	5'	20'	10'	22.913
bc	20'	0'	0'	20
bd	0'	0'	15'	15
cd	0'	0'	15'	15
de	20'	0'	0'	20

$$\sum M_{yb} = 0 \quad [\text{Fig. (b)}]$$

$$R_{dx} \times 15 + 150 \times 5 = 0$$

$$R_{dx} = -50 \text{ K} \quad \leftarrow$$

$$\therefore R_{dx} = 50 \text{ K} \quad (\leftarrow)$$

$$\sum F_x = 0 \quad [\text{Fig. (b)}]$$

$$R_{bx} - 150 - R_{dx} = 0$$

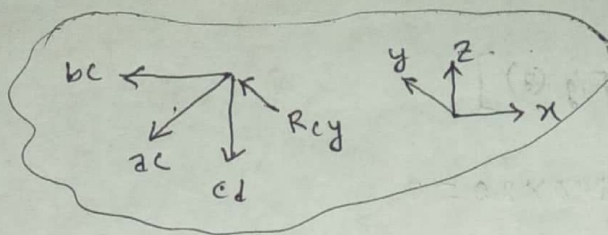
$$\Rightarrow R_{bx} = 150 - 50$$

$$\therefore R_{bx} = 100 \text{ K} \quad (\leftarrow)$$

$$\sum F_z = 0 \quad [\text{Fig. (b)}]$$

$$R_{bz} = 0$$

At joint C



$$\Sigma F_z = 0$$

$$F_{ac}z + F_{cd} = 0$$

$$\Rightarrow F_{ac} \times \frac{5}{25.495} + F_{cd} = 0$$

$$\Rightarrow F_{ac} = -5.099 F_{cd} \dots \textcircled{i}$$

$$\Sigma F_y = 0$$

$$F_{acy} + R_{cy} = 0$$

$$\Rightarrow R_{cy} = -\frac{20}{25.495} F_{ac}$$

$$\Rightarrow R_{cy} = -0.784 F_{ac} \dots \textcircled{ii}$$

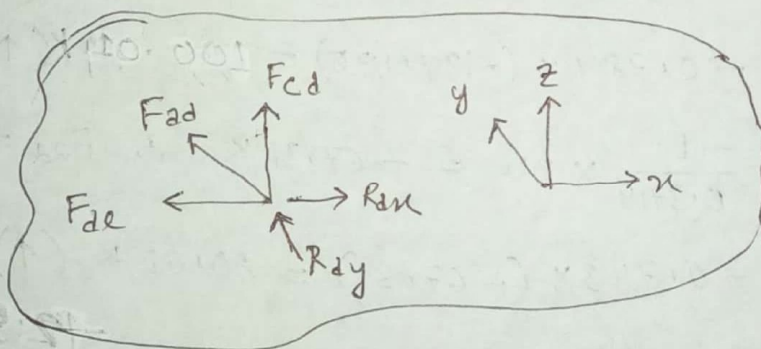
$$\Sigma F_x = 0$$

$$F_{bc} + F_{ac}x = 0$$

$$\Rightarrow F_{bc} = -\frac{15}{25.495} F_{ac}$$

$$\Rightarrow F_{bc} = -0.588 F_{ac} \dots \textcircled{iii}$$

At joint d



$$\Sigma F_z = 0$$

$$F_{cd} + F_{ad}z = 0$$

$$\Rightarrow F_{cd} = -\frac{10}{26.926} F_{ad}$$

$$\Rightarrow F_{cd} = -0.3714 F_{ad} \dots \textcircled{iv}$$

$$\Sigma F_y = 0$$

$$R_{dy} + F_{ady} = 0$$

$$\Rightarrow R_{dy} = -\frac{20}{26.926} F_{ad}$$

$$\Rightarrow R_{dy} = -0.743 F_{ad} \dots \textcircled{v}$$

$$\Sigma F_x = 0$$

$$F_{de} + F_{ad}x - R_{dx} = 0$$

$$\Rightarrow F_{de} = -\frac{15}{26.926} F_{ad} - 50$$

$$\Rightarrow F_{de} = -0.5571 F_{ad} - 50 \dots \textcircled{vi}$$

Now,

$$\sum M_{z-be} = 0 \quad [\text{Fig. (a)}]$$

$$20R_{dy} + 20R_{ey} - 150 \times 20 = 0$$

~~$$\Rightarrow 15 \times (-0.743 F_{ad}) + 15 \times (-0.6573 F_{ac} - 50)$$~~

$$\Rightarrow -0.743 F_{ad} - 0.784 F_{ac} = 150 \quad [\text{From eq}^n \textcircled{V} \text{ \& } \textcircled{II}]$$

$$\Rightarrow -0.743 \left( \frac{-1}{0.3714} F_{ad} \right) - 0.784 (-5.099 F_{ad}) = 150 \quad [\text{From eq}^n \textcircled{I} \text{ \& } \textcircled{IV}]$$

$$\Rightarrow F_{ad} = 25 \text{ K (T)}$$

From,

$$\textcircled{I} \Rightarrow F_{ac} = -127.475 \text{ K} \quad \therefore F_{ac} = 127.475 \text{ K (C)}$$

$$\textcircled{II} \Rightarrow F_{bc} = -0.588 \times (-127.475) = 74.955 \text{ K (T)}$$

$$\textcircled{III} \Rightarrow R_{ey} = -0.784 \times (-127.475) = 99.94 \text{ K (}\uparrow\text{)}$$

$$\textcircled{IV} \Rightarrow F_{ad} = \frac{-1}{0.3714} \times 25 = -67.31 \text{ K} \quad \therefore F_{ad} = 67.31 \text{ K (C)}$$

$$\textcircled{V} \Rightarrow R_{dy} = -0.743 \times (-67.31) = 50.01 \text{ K (}\uparrow\text{)}$$

$$\textcircled{VI} \Rightarrow F_{de} = -0.5571 \times (-67.31) - 50 = -12.50 \text{ K} \quad \therefore F_{de} = 12.50 \text{ K (C)}$$

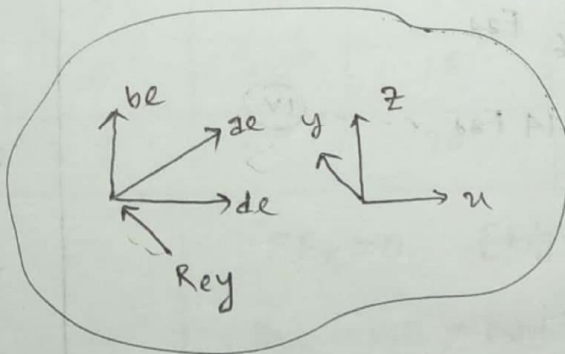
At joint e

$$\sum F_z = 0$$

$$F_{be} + F_{ae} z = 0$$

$$\Rightarrow F_{be} = -F_{ae} \times \frac{10}{22.913}$$

$$\Rightarrow F_{be} = -0.4364 F_{ae} \quad \textcircled{1}$$



$$\Sigma F_y = 0$$

$$F_{ae_y} + R_{ey} = 0$$

$$\Rightarrow R_{ey} = -F_{ae} \times \frac{20}{22.913}$$

$$\Rightarrow R_{ey} = -0.873 \cdot F_{ae} \quad \text{--- (1)}$$

$$\Sigma F_x = 0$$

$$F_{ae_x} + F_{de} = 0$$

$$\Rightarrow F_{ae} \times \frac{5}{22.913} - 12.50 = 0$$

$$\Rightarrow F_{ae} = 57.28 \text{ K (T)}$$

Now,

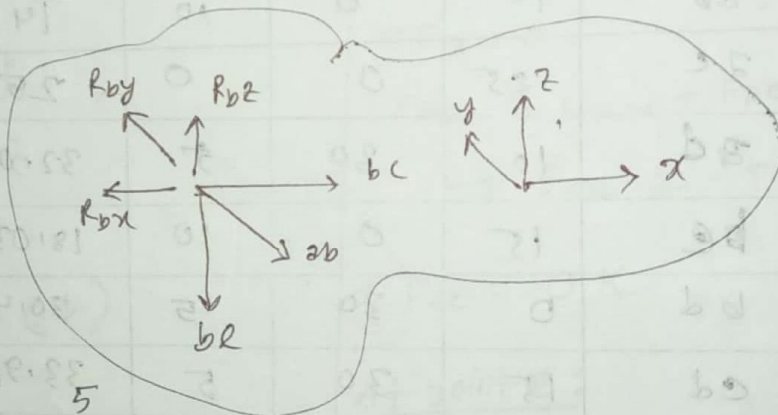
$$\text{From (1)} \Rightarrow F_{be} = -25 \text{ K}$$

$$\therefore F_{be} = 25 \text{ K (C)}$$

$$\text{(1)} \Rightarrow R_{ey} = -50 \text{ K}$$

$$\therefore R_{ey} = 50 \text{ K (T)}$$

At joint b



$$\Sigma F_z = 0$$

$$R_{bz} - F_{be} - F_{abz} = 0$$

$$\Rightarrow R_{bz} = F_{be} + F_{ab} \times \frac{5}{21.21}$$

$$\Rightarrow -25 + F_{ab} \times \frac{5}{21.21} = 0$$

$$\Rightarrow F_{ab} = 106.05 \text{ K (T)}$$

$$\Sigma F_y = 0$$

$$R_{by} + F_{aby} = 0$$

$$\therefore R_{by} = -106.05 \times \frac{20}{21.21} = -100.0 \text{ K}$$

$$\therefore R_{by} = 100.0 \text{ K (D)}$$

# compute the reaction bar forces and reactions of the structures shown in figure below:

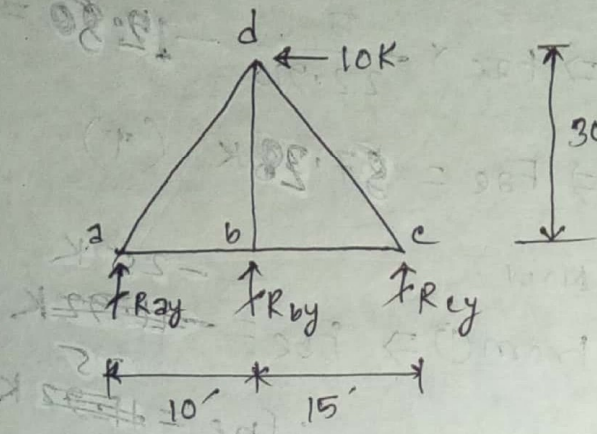


Fig. (a)

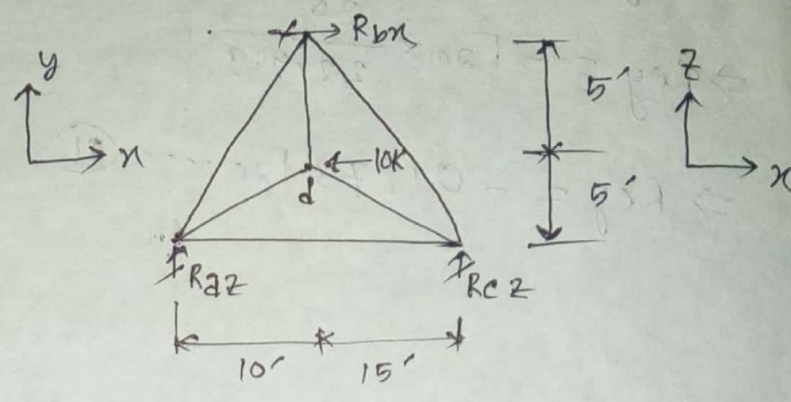


Fig. (b)

Solution:

Member	Projection			Length $\sqrt{x^2+y^2+z^2}$
	x	y	z	
ab	10	0	10	14.14
ac	25	0	0	25
ad	10	30	5	32.02
bc	15	0	10	18.03
bd	0	30	5	30.41
cd	15	30	5	33.91

$$\Sigma F_x = 0$$

$$R_{bx} = 10$$

$$\Sigma M_x = 0$$

$$R_{by} = 0$$

$$\Sigma M_z = 0 \quad [\text{Fig. (a)}]$$

$$R_{cy} \times 25 + 10 \times 30 + R_{bx} \times 10 = 0$$

$$\Rightarrow R_{cy} \times 25 = -300 \text{ K}$$

$$\Rightarrow R_{cy} = -12 \text{ K}$$

$$R_{cy} = 12 \text{ K } (\downarrow)$$

$$\Sigma M_{y2} = 0 \quad [\text{Fig (b)}]$$

$$R_{bx} \times 10 - 10 \times 5 - R_{c2} \times 25 = 0$$

$$\Rightarrow R_{c2} \times 25 = 10 \times 10 - 10 \times 5$$

$$\Rightarrow R_{c2} = 2 \text{ K } (\uparrow)$$

$$\Sigma F_y = 0 \quad [\text{Fig (a)}]$$

$$R_{ay} + R_{by} + R_{cy} = 0$$

$$\Rightarrow R_{ay} + 0 + 12 = 0$$

$$R_{ay} = -12 \text{ K } (\uparrow)$$

$$\Sigma F_z = 0 \quad [\text{Fig (b)}]$$

$$R_{az} + R_{cz} = 0$$

$$\Rightarrow R_{az} = -2 \text{ K}$$

$$\therefore R_{az} = 2 \text{ K } (\downarrow)$$

Now,

$$\text{Joint a: } \Sigma F_y = 0$$

$$\Sigma F_{ady} + R_{ay} = 0$$

$$\Rightarrow F_{ad} \times \frac{30}{32.02} = -12$$

$$\Rightarrow F_{ad} = -12.81 \text{ K}$$

$$\therefore F_{ad} = 12.81 \text{ K } (e)$$

$$\text{Joint b: } \Sigma F_y = 0$$

$$F_{bdy} + R_{by} = 0$$

$$F_{bd} = 0$$

$$\text{Joint c: } \Sigma F_y = 0$$

$$F_{cdy} + R_{cy} = 0$$

$$\Rightarrow F_{cd} \times \frac{30}{33.91} = 12$$

$$\Rightarrow F_{cd} = 13.56 \text{ K } (T)$$

$$\text{At joint a:}$$

$$\Sigma F_z = 0$$

$$F_{az2} + F_{bz2} + R_{az} = 0$$

$$\Rightarrow F_{ab} \times \frac{10}{14.14} = \left( 12.81 \times \frac{5}{32.02} \right) + 2$$

$$\Rightarrow F_{ab} = 5.656 \text{ K } (T)$$

$$\Sigma F_x = 0$$

$$F_{abx} + F_{adx} + F_{ac} = 0$$

$$\Rightarrow F_{ac} = -5.656 \times \frac{10}{14.14} + 12.81 \times \frac{10}{32.02}$$

$$\Rightarrow F_{ac} = 0 \text{ K}$$

$$\text{At joint c:}$$

$$\Sigma F_z = 0$$

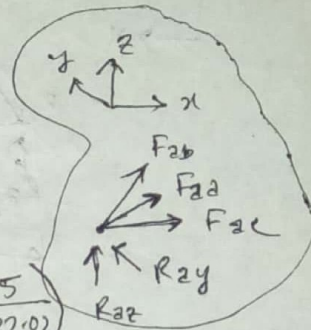
$$R_{cz} + F_{bcz} + F_{cdz} = 0$$

$$2 + f_{bc} \times \frac{10}{18.03} + 13.56 \times \frac{5}{33.91} = 0$$

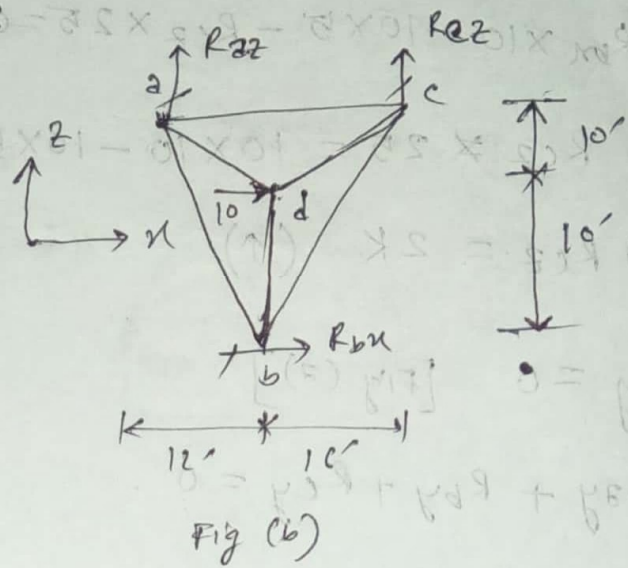
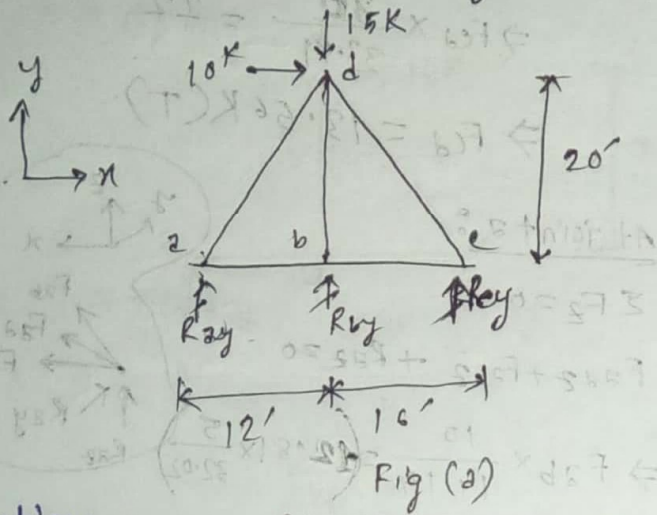
$$\Rightarrow F_{bc} = -7.21 \text{ K}$$

$$\therefore F_{bc} = 7.21 \text{ K } (e)$$

(Ans)



# compute the reaction and bar forces of the following space truss as shown in figure below:



Solution:

Member	Projection			Length $\sqrt{x^2+y^2+z^2}$
	x	y	z	
ab	12	0	20	23.32
ac	28	0	0	28.00
ad	12	20	10	25.37
bc	16	0	20	25.6
bd	0	20	10	22.36
cd	16	20	10	27.15

$$\sum F_x = 0 \quad [\text{Fig. (b)}]$$

$$R_{bx} + 10 = 0$$

$$R_{bx} = -10 \text{ K}$$

$$\therefore R_{bx} = 10 \text{ K } (\leftarrow)$$

$$\sum M_{x2} = 0 \quad [\text{Fig (b)}]$$

$$R_{by} \times 20 - 15 \times 10 = 0$$

$$\therefore R_{by} = 7.5 \text{ K } (\uparrow)$$

$$\sum M_{z2} = 0 \quad [\text{Fig (b)}]$$

$$R_{by} \times 12 + R_{cy} \times 28 - 10 \times 20 - 15 \times 12 = 0$$

$$\Rightarrow R_{cy} \times 28 = 380 - 7.5 \times 12$$

$$\Rightarrow R_{cy} = 10.36 \text{ K } (\uparrow)$$

$$\sum M_{y2} = 0 \quad [\text{Fig (b)}]$$

$$R_{cz} \times 28 + R_{bx} \times 20 + 10 \times 10 = 0$$

$$\Rightarrow R_{cz} \times 28 = -100 + 200$$

$$\therefore R_{cz} = 3.57 \text{ K } (\uparrow)$$

$$\sum F_y = 0 \quad [\text{Fig (z)}]$$

$$R_{ay} + R_{by} + R_{cy} = 15$$

$$\Rightarrow R_{ay} = 15 - 10.36 - 7.5$$

$$\Rightarrow R_{ay} = -2.86 \text{ K}$$

$$\therefore R_{ay} = 2.86 \text{ K} (\downarrow)$$

$$\sum F_z = 0 \quad [\text{Fig (b)}]$$

$$R_{az} + R_{cz} = 0$$

$$\Rightarrow R_{az} = -3.57 \text{ K}$$

$$\therefore R_{az} = 3.57 \text{ K} (\downarrow)$$

Now,

joint a:  $\sum F_y = 0$

$$F_{ady} + R_{ay} = 0$$

$$\Rightarrow F_{ad} \times \frac{20}{25.37} = +2.86$$

$$\Rightarrow F_{ad} = 3.63 \text{ K (T)}$$

joint b:  $\sum F_y = 0$

$$F_{bdy} + R_{by} = 0$$

$$\Rightarrow F_{bd} \times \frac{20}{22.36} = -7.5$$

$$\Rightarrow F_{bd} = -8.39 \text{ K}$$

$$\therefore F_{bd} = 8.39 \text{ K (C)}$$

joint c:  $\sum F_y = 0$

$$F_{cdy} + R_{cy} = 0$$

$$F_{cd} \times \frac{20}{27.5} = -10.36$$

$$\Rightarrow F_{cd} = -14.245 \text{ K}$$

$$\therefore F_{cd} = 14.245 \text{ K (C)}$$

At joint a:

$$\sum F_z = 0$$

$$F_{abz} + F_{adz} -$$

$$R_{az} = 0$$

$$\Rightarrow F_{ab} \times \frac{20}{23.32} = -3.57 - 3.63 \times \frac{10}{25.37}$$

$$\Rightarrow F_{ab} = -5.83 \text{ K}$$

$$\therefore F_{ab} = 5.83 \text{ K (C)}$$

$$\sum F_x = 0$$

$$F_{adx} + F_{ac} + F_{abx} = 0$$

$$\Rightarrow F_{ac} = 5.83 \times \frac{12}{23.32} - 3.63 \times \frac{12}{25.37}$$

$$\Rightarrow F_{ac} = 1.283 \text{ (T)}$$

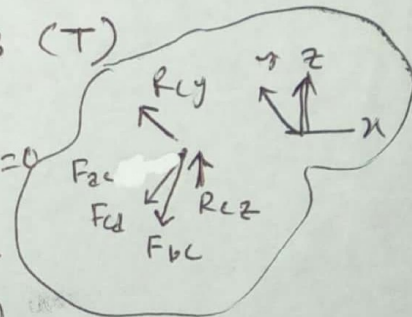
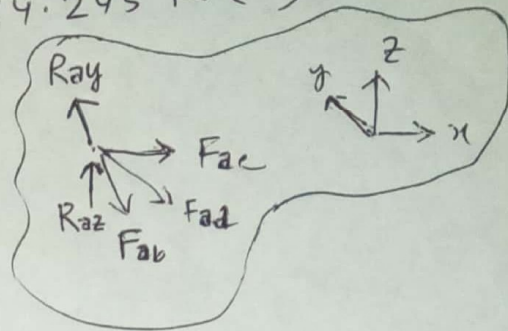
At joint c:  $\sum F_z = 0$

$$R_{cz} - F_{cdz} - F_{bcz} = 0$$

$$\Rightarrow F_{bc} \times \frac{20}{25.61} = 3.57 + 14.245 \times \frac{10}{27.5}$$

$$\Rightarrow F_{bc} = +11.20 \text{ K}$$

$$\therefore F_{bc} = 11.20 \text{ K (T)}$$

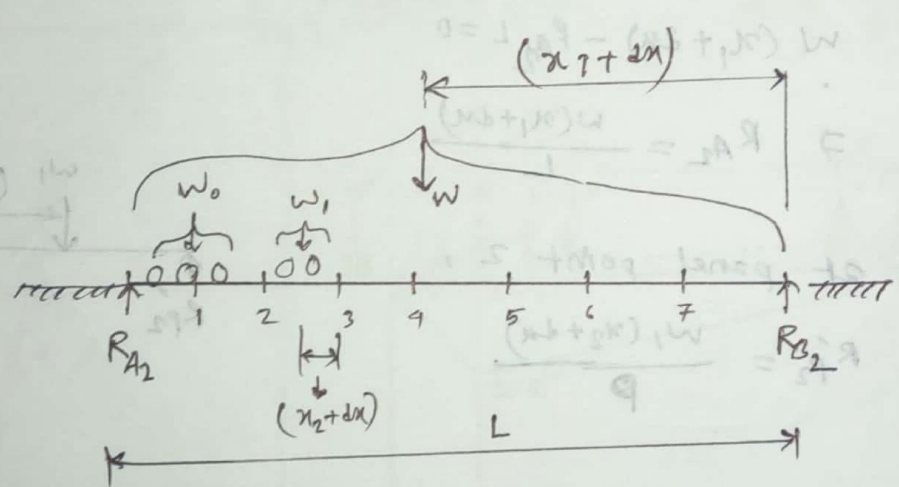
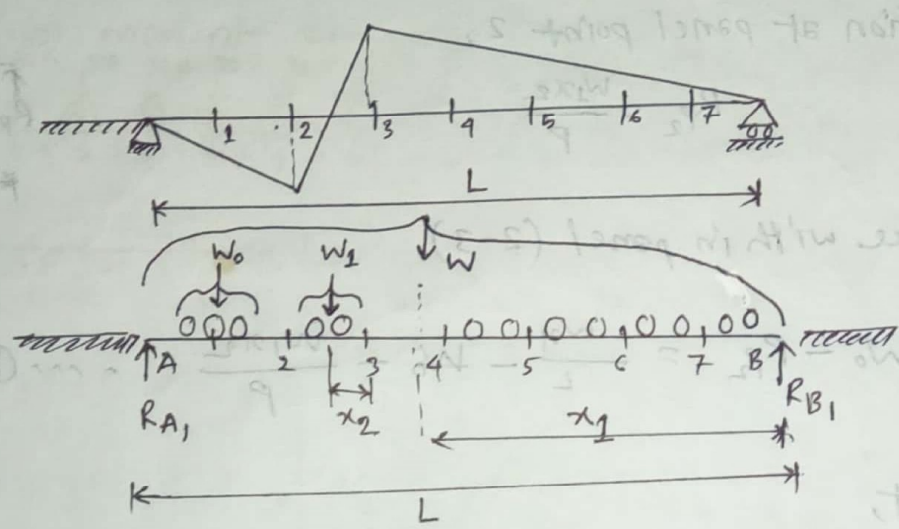


(Ans)

16, 15, 08

**Panel Load**

Criteria for the maximum shear of a floor beam subjected to series of concentrated loads move right to left.



- Here,  $W$  = the sum of all of loads on the span
- $W_0$  = The sum of all of loads left of the panel.
- $W_1$  = The sum of all of loads in the panel.
- $x_1$  = The distance of center of gravity all the loads ( $W$ ) to right support (B)
- $x_2$  = The distance of center of gravity all the loads with in panel to right point of panel.
- $L$  = span length of beam.

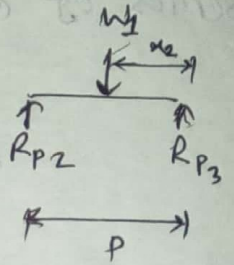
$$\frac{W}{L} = \frac{W}{L} \therefore$$

Before movement,

$$\sum M_B = 0 \quad \cdot Wx_1 - R_{A1}L = 0 \quad \Rightarrow R_{A1} = \frac{Wx_1}{L}$$

Also reaction at panel point 2,

$$R_{P2} = \frac{W_1x_2}{P}$$



Now, shear force within panel (2-3)

$$V_1 = R_{A1} - W_0 - R_{P2} = \frac{Wx_1}{L} - W_0 - \frac{W_1x_2}{P} \quad \dots \textcircled{1}$$

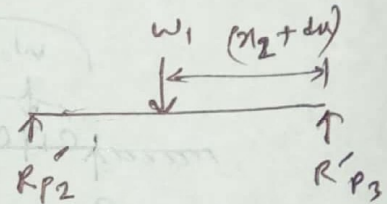
After movement,

$$\sum M_B = 0 \quad W(x_1 + dx) - R_{A2}L = 0$$

$$\Rightarrow R_{A2} = \frac{W(x_1 + dx)}{L}$$

Also, reaction at panel point 2,

$$R'_{P2} = \frac{W_1(x_2 + dx)}{P}$$



Now, shear force within panel (2-3)

$$V_2 = R_{A2} - W_0 - R'_{P2} = \frac{W(x_1 + dx)}{L} - W_0 - \frac{W_1(x_2 + dx)}{P} \quad \dots \textcircled{2}$$

Now, From ① - ② we obtain.

$$dV = V_2 - V_1 = \frac{W(x_1 + dx)}{L} - W_0 - \frac{W_1(x_2 + dx)}{P} - \left( \frac{Wx_1}{L} - W_0 - \frac{W_1x_2}{P} \right)$$

$$\Rightarrow dV = \frac{Wdx}{L} - \frac{W_1dx}{P} = 0$$

$$\therefore \frac{dV}{dx} = \frac{W}{L} - \frac{W_1}{P}$$

Now, For maximum,  $\frac{dV}{dx} = 0$

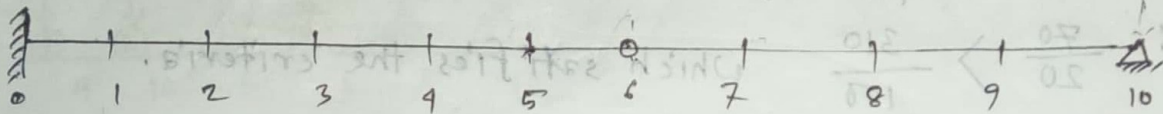
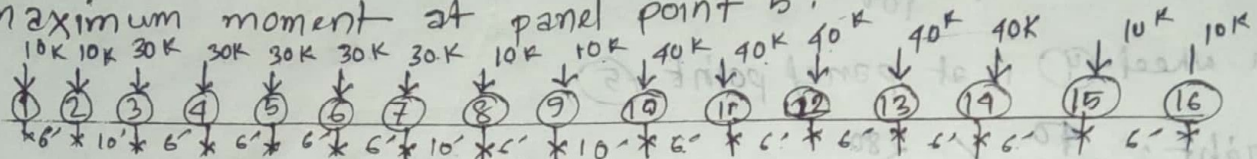
$$\frac{dV}{dx} = \frac{W}{L} - \frac{W_1}{P} = 0$$

$$\therefore \frac{W}{L} = \frac{W_1}{P}$$

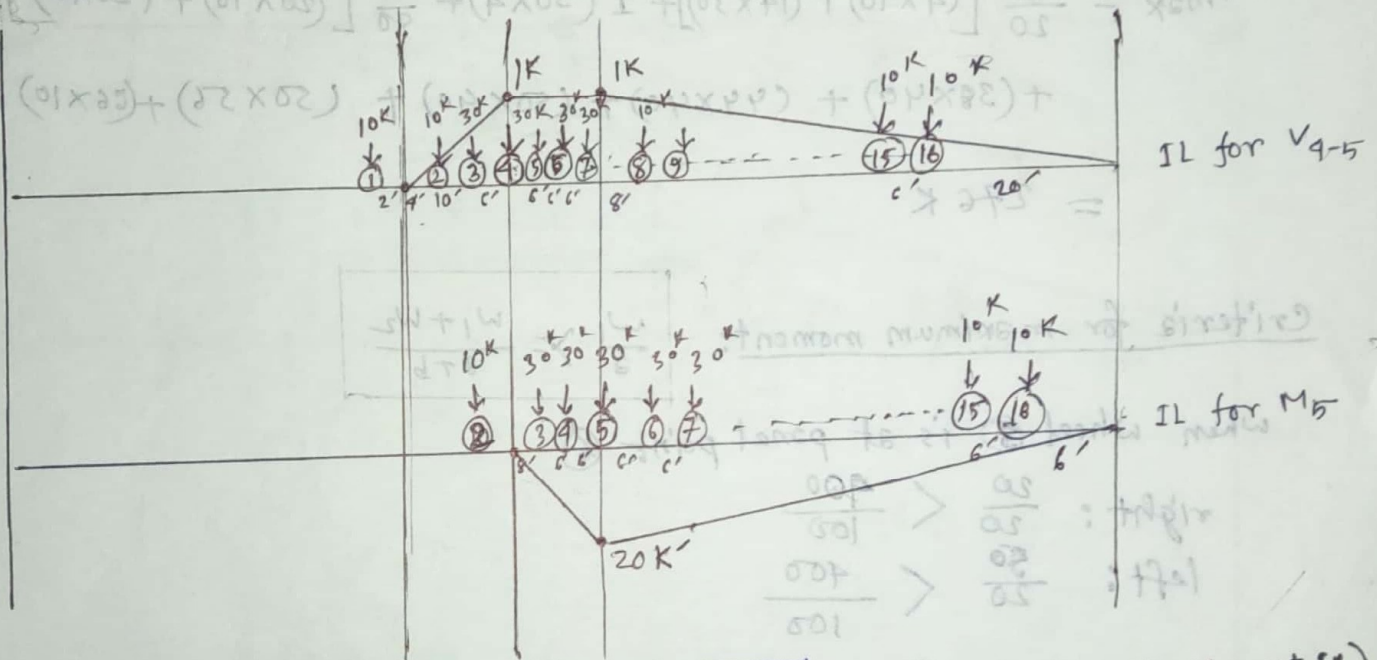
2006

# using proper criteria to obtain the position of moving loads and hence calculate the maximum shear in the panel 4-5 and

the maximum moment at panel point 5.



$10 @ 20' = 200'$



Here,

Criteria for maximum shear:

$\frac{W_1}{a} = \frac{W_1 + W_2}{a + b}$  (IL is similar to Moment IL)

When wheel ① is at panel point ⑤

Right:  $\frac{0}{20} < \frac{350}{100}$

Left:  $\frac{10}{20} < \frac{350}{100}$

When wheel ② is at panel point ⑤

Right:  $\frac{10}{20} < \frac{340}{100}$

Left:  $\frac{20}{20} < \frac{350}{100}$

When wheel ③ is at panel point ⑤,

$$\text{right: } \frac{20}{20} < \frac{290}{100}$$

$$\text{left: } \frac{50}{20} < \frac{320}{100}$$

when wheel ④ is at panel point ⑤,

$$\text{right: } \frac{40}{20} < \frac{280}{100}$$

$$\text{left: } \frac{70}{20} > \frac{310}{100}$$

which satisfies the criteria.

$$V_{\max} = \frac{1}{20} [(4 \times 10) + (14 \times 30)] + 1(30 \times 4) + \frac{1}{80} [(20 \times 10) + (26 \times 10) + (32 \times 40) + (38 \times 40) + (44 \times 40) + (50 \times 40) + (50 \times 56) + (66 \times 10) + (72 \times 10)]$$

$$= 276 \text{ K}$$

Criteria for maximum moment:

$$\frac{w_1}{a} = \frac{w_1 + w_2}{a + b}$$

when wheel ③ is at panel point ⑥

$$\text{right: } \frac{20}{20} < \frac{390}{100}$$

$$\text{left: } \frac{50}{20} < \frac{400}{100}$$

when wheel ④ is at panel point ⑥

$$\text{right: } \frac{40}{20} < \frac{390}{100}$$

$$\text{left: } \frac{70}{20} < \frac{400}{100}$$

when wheel ⑤ is at panel point ②

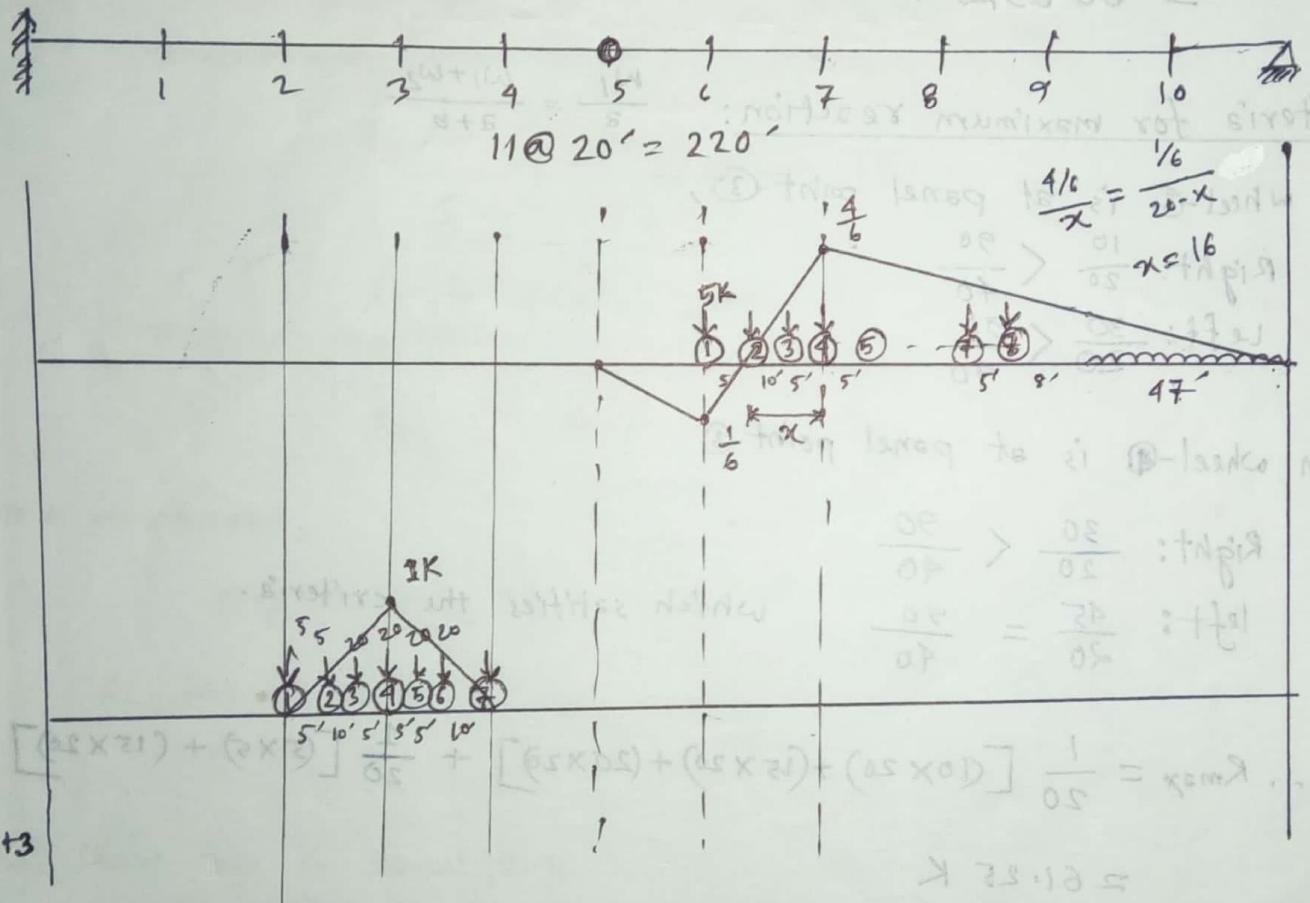
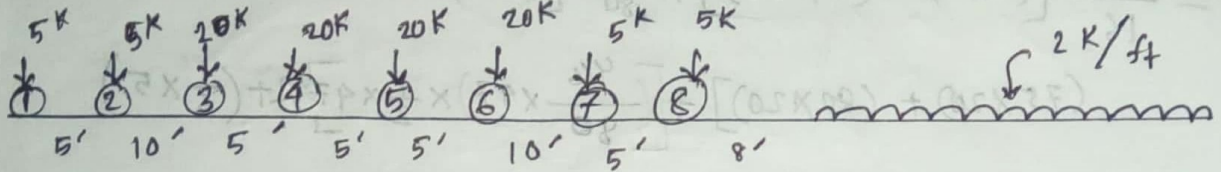
$$\text{right: } \frac{60}{20} < \frac{390}{100}$$

$$\text{left: } \frac{90}{20} > \frac{390}{100}$$

which satisfies the criteria.

$$M_{\max} = \frac{20}{20} [(8 \times 30) + (14 \times 30) + (20 \times 30)] + \frac{20}{80} [(6 \times 10) + (12 \times 10) + (18 \times 40) + (24 \times 40) + (30 \times 40) + (36 \times 40) + (42 \times 40) + (52 \times 10) + (58 \times 10) + (68 \times 30) + (74 \times 30)] = 4145 \text{ K}$$

# Problem: using proper criteria to determine the maximum shear in panel 6-7 and maximum floor beam reaction at panel point 3 due to the series of moving concentrated loads shown in figure below:



criteria for maximum shear:

$$\frac{W_l}{P} = \frac{W}{L}$$

Now,

when wheel ③ at panel point ⑦,

right:  $\frac{10}{20} < \frac{184}{120} \rightarrow (5 \times 2 + 20 \times 4 + 5 \times 2 + 2 \times 42)$

left:  $\frac{30}{20} < \frac{184}{120}$

When wheel-④ is at panel point ⑦

right:  $\frac{30}{20} < \frac{194}{120}$

left:  $\frac{45}{20} > \frac{189}{120}$

which satisfies the criteria.

$$V_{max} = \frac{4/6}{16} [(1 \times 5) + (11 \times 20)] + \frac{4/6}{80} [(55 \times 5) + (60 \times 5) + (70 \times 20) + (75 \times 20) + (80 \times 20)] + \left[ \left( \frac{4/6}{80} \times 47 \right) \times \frac{1}{2} \times 47 \times 2 - \left( \frac{1}{6} \times 5 \right) \right]$$

= 69.242 K

criteria for maximum reaction:

$$\frac{W_1}{a} = \frac{W_1 + W_2}{a + b}$$

when wheel-③ is at panel point-③,

Right:  $\frac{10}{20} < \frac{90}{40}$

Left:  $\frac{30}{20} < \frac{90}{40}$

when wheel-④ is at panel point-③

Right:  $\frac{30}{20} < \frac{90}{40}$

left:  $\frac{45}{20} = \frac{90}{40}$

which satisfies the criteria.

$$\therefore R_{max} = \frac{1}{20} [(10 \times 20) + (15 \times 20) + (20 \times 20)] + \frac{1}{20} [(5 \times 5) + (15 \times 20)]$$

= 61.25 K

(Ans.)  $\frac{W_1}{a} = \frac{W_1 + W_2}{a + b}$

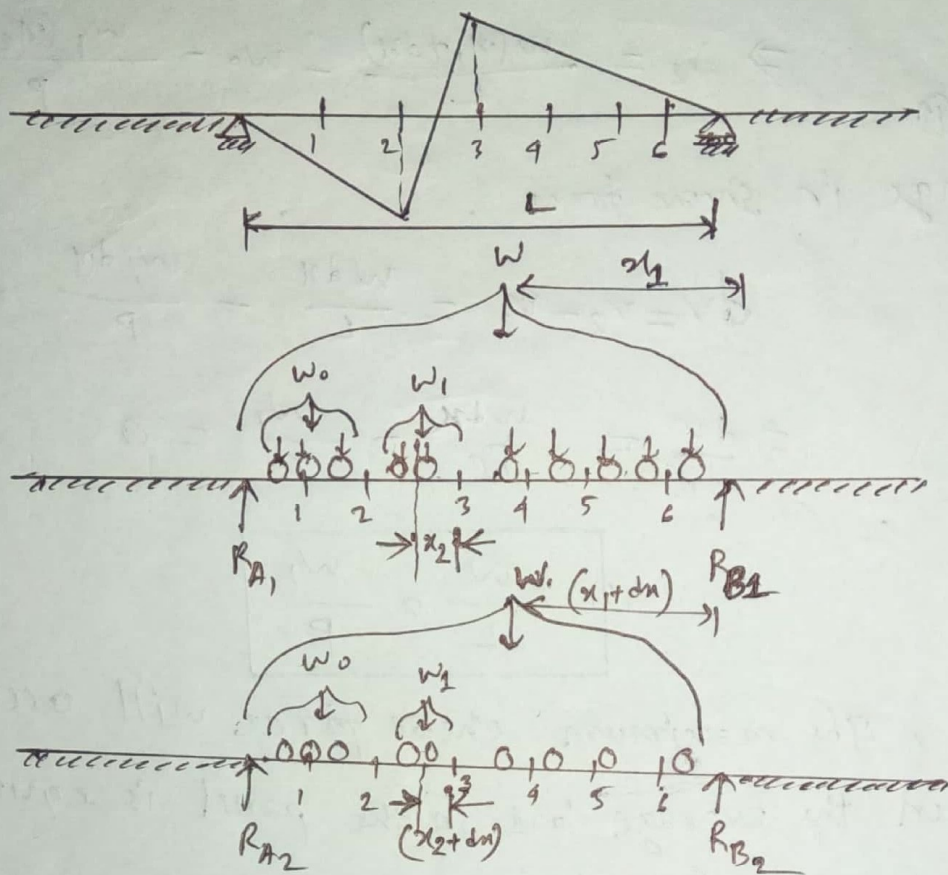
criteria for maximum reaction

When wheel ③ is at panel point ③

right:  $\frac{10}{20} < \frac{90}{40}$   
 left:  $\frac{30}{20} < \frac{90}{40}$

16.15.08

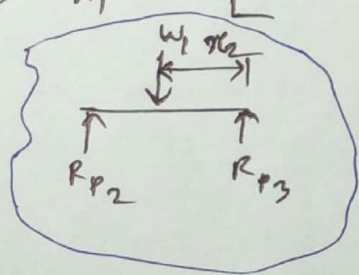
# Criteria for the maximum shear of a floor beam subjected to series of concentrated loads move from right to left.



Before movement,

$$\sum M_B = 0 \Rightarrow R_{A1} \times L - W x_1 = 0 \Rightarrow R_{A1} = \frac{W x_1}{L}$$

Reaction at panel 2,  $R_{P2} = \frac{w_1 x_2}{P}$



Now, Shear force in panel (2-3),

$$V_1 = R_{A1} - W_0 - R_{P2} = \frac{W x}{L} - W_0 - \frac{w_1 x_2}{P} \dots \text{--- (1)}$$

After movement,

$$\sum M_B = 0 \Rightarrow R_{A2} = \frac{W (x_1 + dx)}{L}$$

2nd,  $R_{P2}' = \frac{w_1 (x_2 + dx)}{L}$

Now, shear force in panel (2-3),

$$V_2 = R_{A2} - W_0 - R'_{P2}$$

$$\Rightarrow V_2 = \frac{w(x_1 + dx)}{L} - w_0 - \frac{w_1(x_2 + dx)}{P}$$

From (i) & (ii)

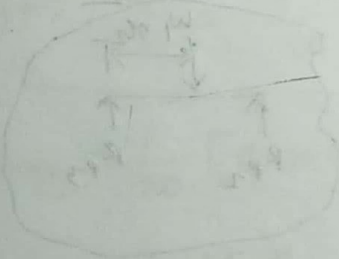
\(\therefore\) The change in shear force,

$$dV = V_2 - V_1 = \frac{w dx}{L} - \frac{w_1 dx}{P}$$

$$\Rightarrow \frac{dV}{dx} = \frac{w}{L} - \frac{w_1}{P} = 0$$

$$\boxed{\frac{w}{L} = \frac{w_1}{P}}$$

Therefore, The maximum shear force will occur within panel when the average load in the panel is equal to average load in the span of the beam.



$$\text{① } \frac{w(x_1 + dx)}{L} - w_0 - \frac{w_1(x_2 + dx)}{P} = \frac{w(x_1)}{L} - w_0 - \frac{w_1(x_2)}{P}$$

$$\frac{w(x_1 + dx)}{L} - \frac{w_1(x_2 + dx)}{P} = \frac{w(x_1)}{L} - \frac{w_1(x_2)}{P}$$

$$\frac{w dx}{L} - \frac{w_1 dx}{P} = 0$$

# Three Hinged Arch

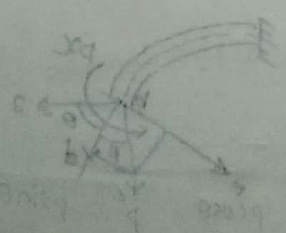
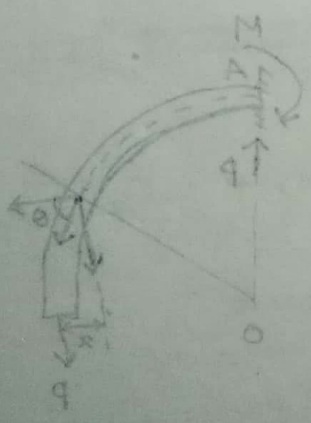
Arch: An arch may be defined as mechanical arrangement of wedge shaped blocks of stones or bricks mutually supporting each other and supported at the end by piers, or abutments. In general, Arch is defined as a curved beam.

Three hinged arch: An arch hinged at both supports and having third hinge anywhere in the rib is called a three hinged arch. A three hinge arch is a determinant structure.

Beam: Beam is a flexural member; which is long in transverse direction.

## Advantages of Arch:

1. Architecturally looks good.
2. Arch can take load 10 times more than straight beam.



$$\frac{b}{r} = \theta \text{ rad, } b \text{ is arc length}$$

Shear force and normal force on arch

Consider section at section point, Bending moment,  $M_c = -P \cdot x$

Let, shear force =  $V_x$  and normal force =  $N_x$

$$V_x = P \cos \theta$$

$$N_x = -P \sin \theta$$

Reaction of the arch in supports:

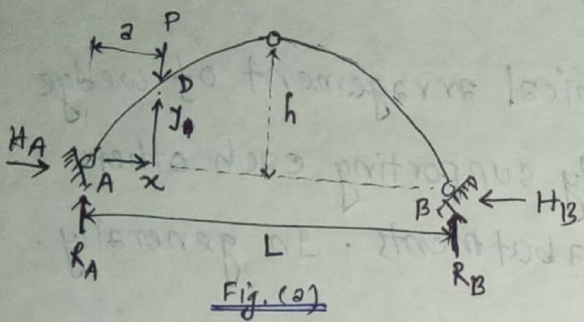


Fig. (a)

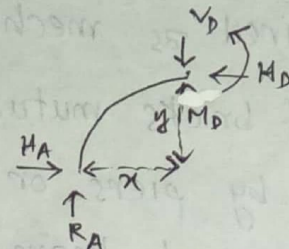


Fig. (b)

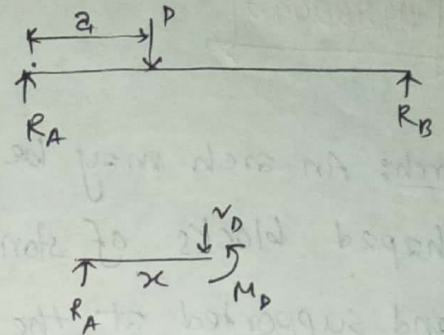


Fig. (c)

$$\sum M_A = 0$$

$$\sum F_y = 0$$

$$P \cdot a - R_B \cdot L = 0$$

$$R_A - P + R_B = 0$$

$$\Rightarrow R_B = \frac{Pa}{L}$$

$$\Rightarrow R_A = P - R_B = P - \frac{Pa}{L}$$

$$R_A = \frac{P(L-a)}{L}$$

From fig. (b),

$$\sum M_D = R_A x - H_A y \quad \text{--- (1)}$$

$$\sum F_x = 0$$

$$\Rightarrow H_A - H_B = 0$$

$$\therefore H_A = H_B = H$$

But,

For simply supported beam -

From fig. (c)

$$\sum M_D = R_A x = M_s$$

So, From (1)

$$\sum M_D = M_s - H y$$

Here, Reactions of arch and Reactions of a beam are equal.

Parabolic equation of the arch:  $y = \frac{4hx}{L^2} (L-x)$

Now,  $M_H = Hy$  [Let]

Hence,  $M_H = H \left[ \frac{4hx}{L^2} (L-x) \right]$

and,  $\tan \theta = \frac{dy}{dx}$

Shear force and Normal force on Arch:

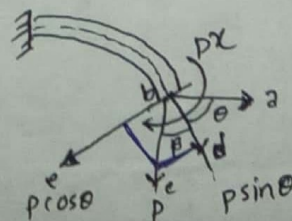
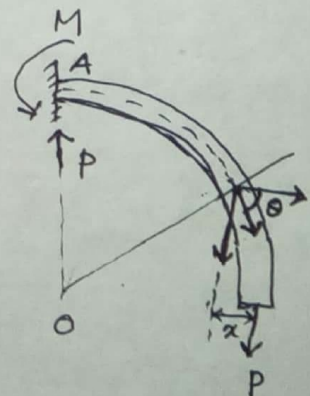
Consider,

at section point, Bending moment,  $M_x = -Px$

Let, shear force =  $F_x$  and Normal force =  $N_x$

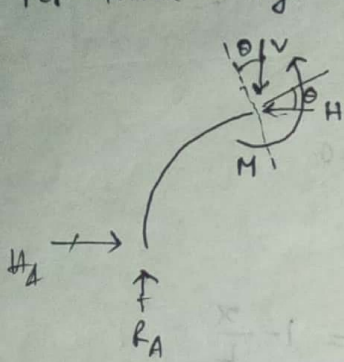
$$\therefore F_x = P \cos \theta$$

$$\text{And, } N_x = -P \sin \theta$$



$$\begin{aligned} \angle abc &= 90^\circ = \theta + \beta \\ \angle abd &= \theta; \angle ebd = 90^\circ \\ \angle bdc &= \beta \therefore \angle ebc = \theta \end{aligned}$$

For three hinge arch,



$$\tan \theta = \frac{y}{x}$$

$$\therefore \text{Shear force, } F_x = V \cos \theta - H \sin \theta$$

$$\text{and, Normal force, } N = V \sin \theta + H \cos \theta$$

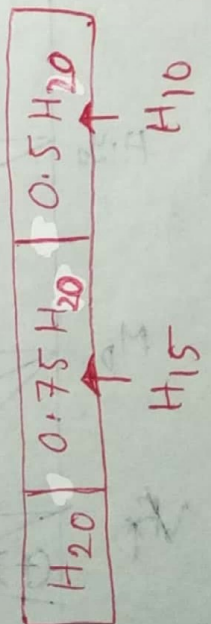
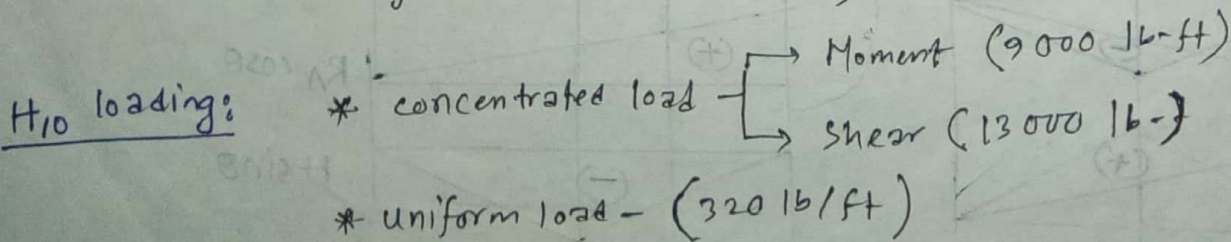
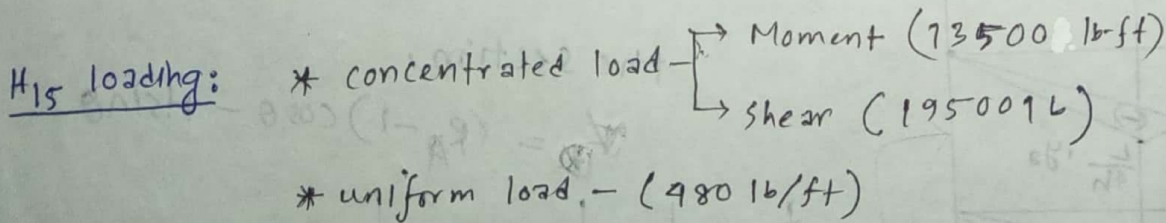
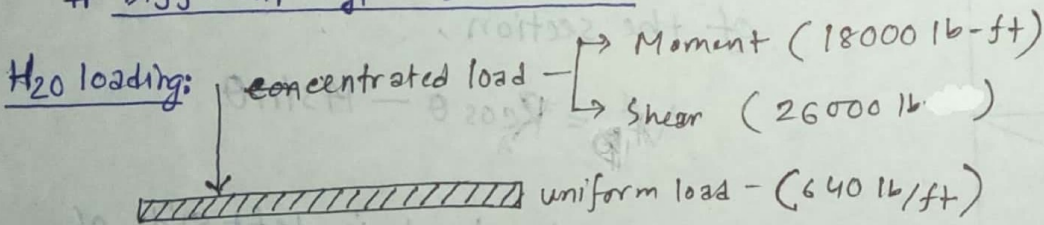
### Influence Line:

An influence line is a diagram showing the variance of shear, stress, moment in a member, reaction or other direct function due to a unit load moving across the structure.

### construction of Influence line:

An influence line is constructed by plotting directly under the point where unit load is placed and ordinate, the height of which represents to some scale the value of the particular function being studied, when the load is in that position.

### Different types of loading:



# # IL diagram of three hinged arch:

When 1K at A;

$$R_A = 1 \text{ K}; R_B = 0$$

When 1K at D;

$$R_A = \frac{1(L-x)}{L} = 1 - \frac{x}{L}$$

$$\therefore R_B = \frac{x}{L}$$

$$\sum M_c = 0$$

$$R_B \times \frac{L}{2} - H \cdot h = 0$$

$$H = \frac{R_B L}{2h} = \frac{x}{2h}$$

At maximum,  $x = \frac{L}{2}$

$$\therefore H = \frac{L}{4h}$$

$\therefore$  Maximum Bending moment

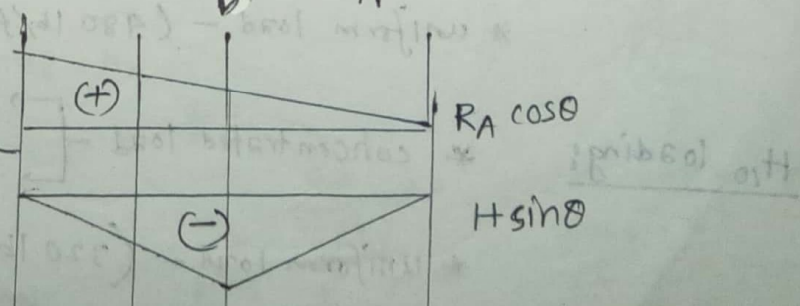
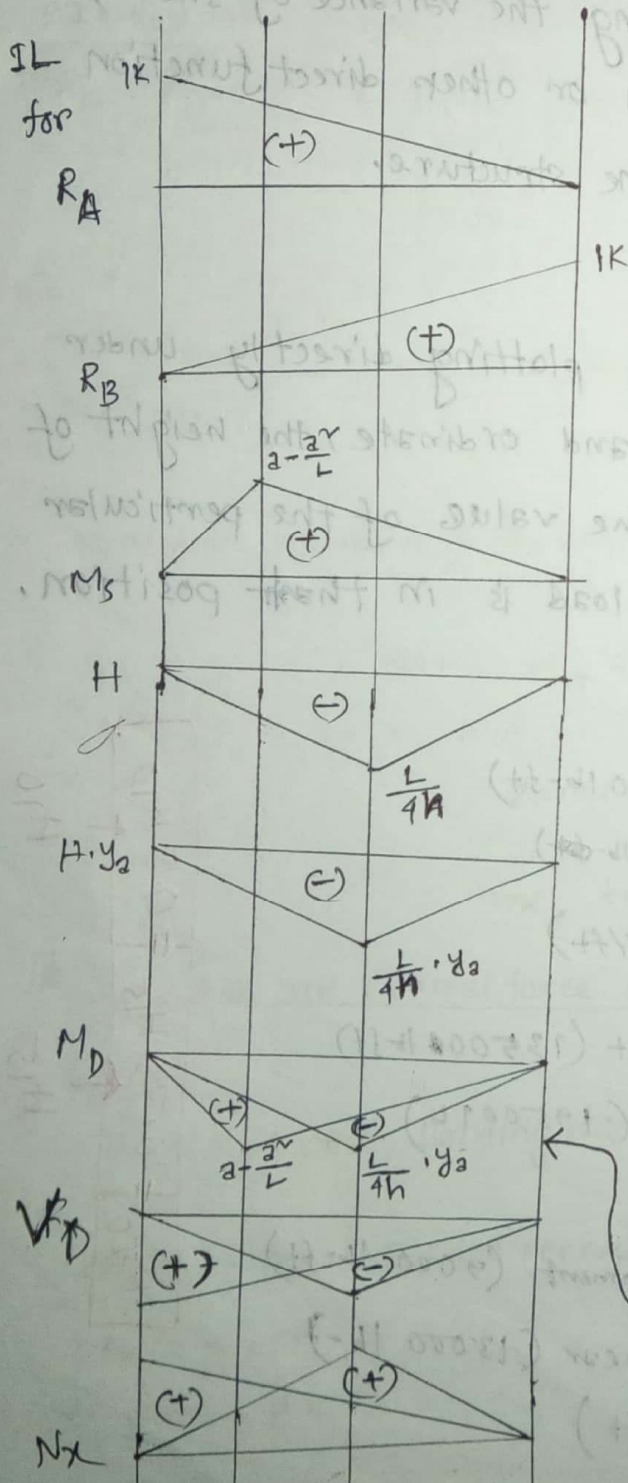
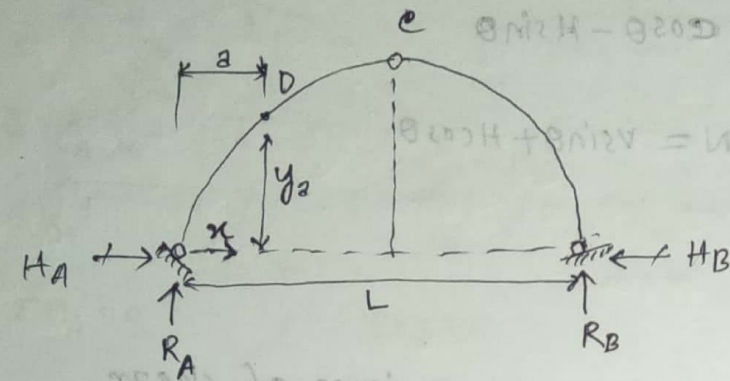
$$M = M_s - H y_2 = R_A x - H y_2$$

When the load is at right side of the section,

$$V_D = R_A \cos \theta - H \sin \theta$$

When the load is left side of the section,

$$V_D = (R_A - 1) \cos \theta - H \sin \theta$$





Now,

$$(+)\ M_{max} = 40PR \times 0.64 + (6.67 - 4.43) \times 18 \quad [\text{Due to } H_2O \text{ loading}]$$

$$= (40PT - 40RT) \times 0.64 + 40.32$$

$$= \left\{ \left( \frac{1}{2} \times 30 \times 6.67 \right) - \left( \frac{1}{2} \times 30 \times 5.72 \right) \right\} \times 0.64 + 40.32$$

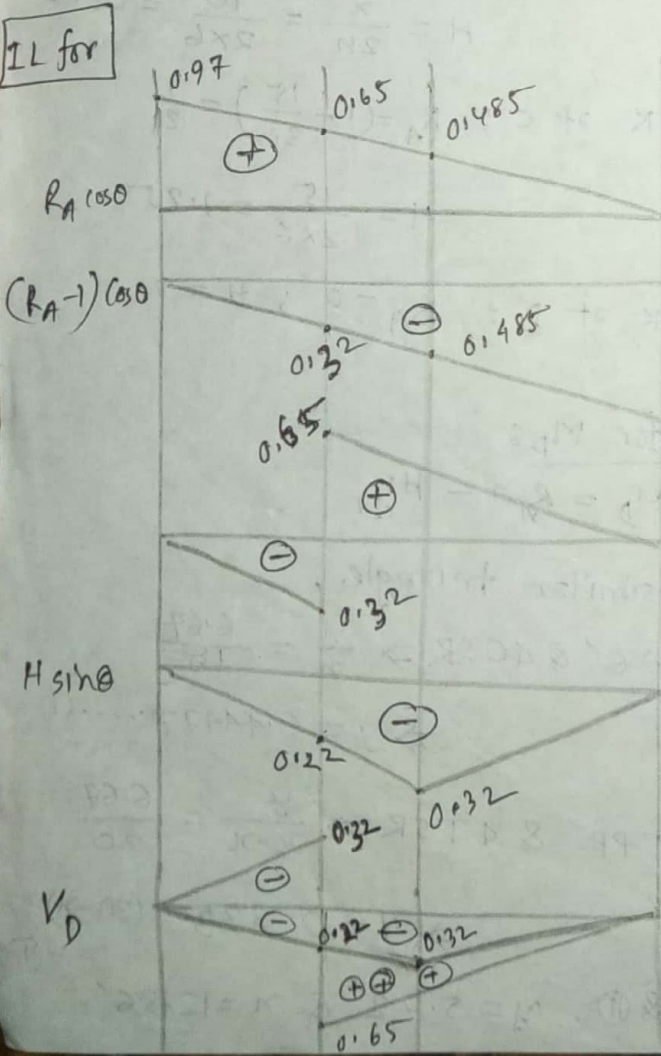
$$= 49.44 \text{ K}$$

$$(-)\ M_{max} = 4TQR \times 0.64 + (6.67 - 5) \times 18$$

$$= (40QT - 40RT) \times 0.64 + 30.06$$

$$= \left\{ \left( \frac{1}{2} \times 30 \times 6.67 \right) - \left( \frac{1}{2} \times 30 \times 5.72 \right) \right\} \times 0.64 + 30.06$$

$$= 39.18 \text{ K}$$



IL for  $V_D$ :

$$V_D = (R_A - 1) \cos \theta - H \sin \theta \quad (\text{IK at just left of D})$$

$$V_D = R_A \cos \theta - H \sin \theta \quad (\text{IK at just right of D})$$

$$\tan \theta = \frac{4h}{L} - \frac{8h^2}{L^2}$$

$$\Rightarrow \tan \theta = \frac{4 \times 6}{30} - \frac{8 \times 6 \times 10}{30^2}$$

$$\Rightarrow \tan \theta = 0.267$$

$$\therefore \theta = 14.93^\circ$$

$$\therefore \sin \theta = 0.26$$

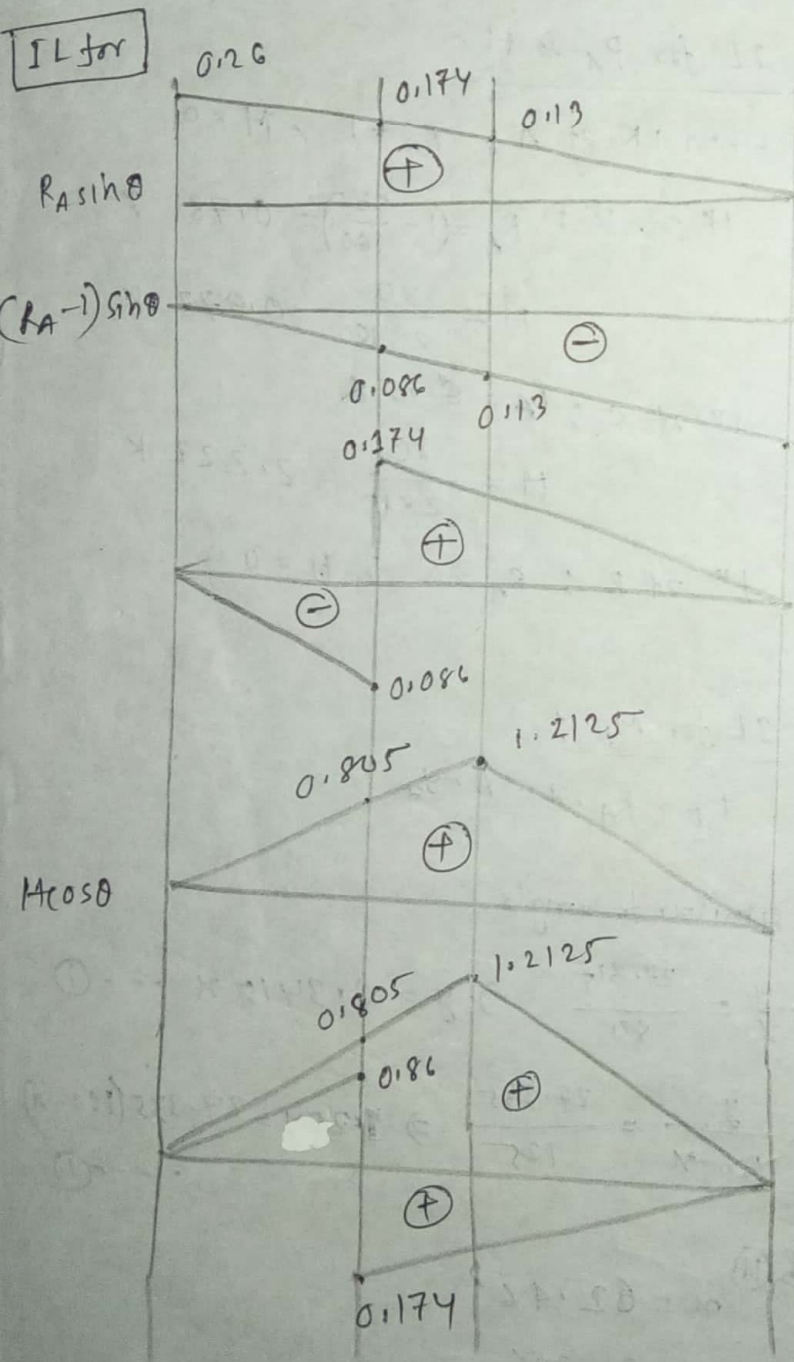
$$\cos \theta = 0.97$$

$$(+V_{D(max)}) = \left[ \left( \frac{1}{2} \times 20 \times 0.65 \right) - \left( \frac{1}{2} \times 30 \times 0.32 \right) + \left( \frac{1}{2} \times 10 \times 0.22 \right) \right] \times 0.64 + (0.65 - 0.22) \times 26$$

$$= (1.792 + 11.18) = 12.972 \text{ K}$$

$$(-V_{D(max)}) = \left[ \left( \frac{1}{2} \times 10 \times 0.32 \right) + \left( \frac{1}{2} \times 10 \times 0.22 \right) \right] \times 0.64 + (26 \times 0.32)$$

$$= (1.728 + 8.32) = 10.048 \text{ K}$$



IL for  $N_D$ :

$$N_D = (R_A - I) \sin \theta + H \cos \theta \text{ (Left)}$$

$$N_D = R_A \sin \theta + H \cos \theta \text{ (Right)}$$

$$(+N_{D(max)}) = \left[ \left( \frac{1}{2} \times 30 \times 1.2125 \right) + \left( \frac{1}{2} \times 20 \times 0.174 \right) - \left( \frac{1}{2} \times 0.086 \times 10 \right) \right] \times 0.64 + 26 \times 1.2125$$

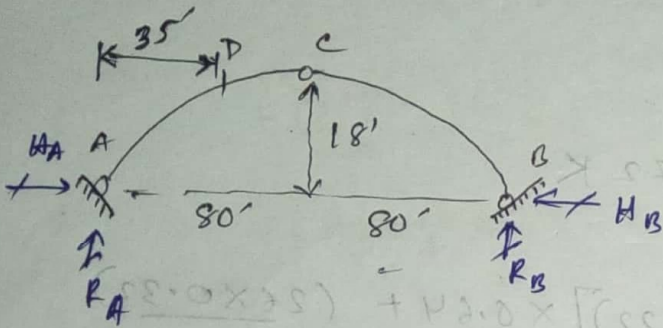
$$= 12.4784 + 31.525$$

$$= 44 \text{ K}$$

# # Problem: (Due to $H_2O$ loading)

Given that,

$$L = 160', \quad h = 18', \quad a = 35'$$



$$y_2 = \frac{4ha}{L^2} (L-a)$$

$$= \frac{4 \times 18 \times 35}{160^2} (160 - 35)$$

$$= 0.15 \text{ K}$$

IL for  $R_A$  &  $H$ :

when 1K at A:  $R_A = 1, H = 0$

$$1K \text{ at D: } R_A = \left(1 - \frac{35}{160}\right) = 0.781 \text{ K}$$

$$H = \frac{35}{2 \times 18} = 0.972 \text{ K}$$

$$1K \text{ at c: } R_A = 0.5 \text{ K}$$

$$H = \frac{80}{2 \times 18} = 2.222 \text{ K}$$

$$1K \text{ at B: } R_A = 0, H = 0$$

IL for  $M_D$ :

$$M_D = R_A \cdot a - H \cdot y_2$$

From similar triangle,

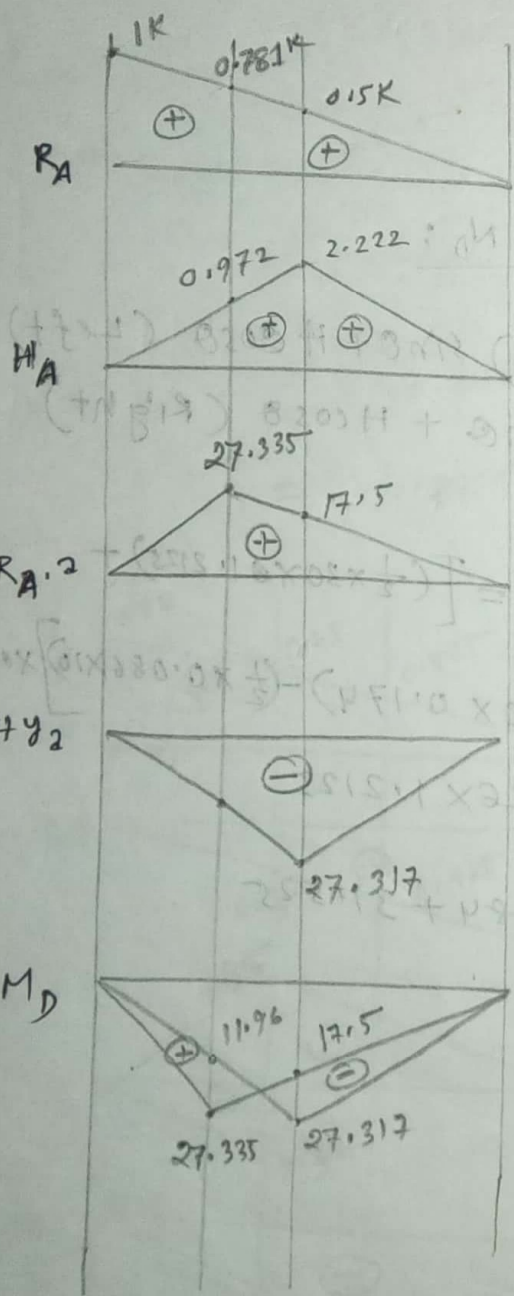
$$\frac{y}{x} = \frac{27.317}{80} \Rightarrow y = 0.3415x \quad \text{--- (I)}$$

$$\text{And, } \frac{y}{160-x} = \frac{27.335}{125} \Rightarrow 1.25y = 27.335(160-x) \quad \text{--- (II)}$$

From (I) & (II)

$$x = 62.46'$$

$$y = 21.33'$$



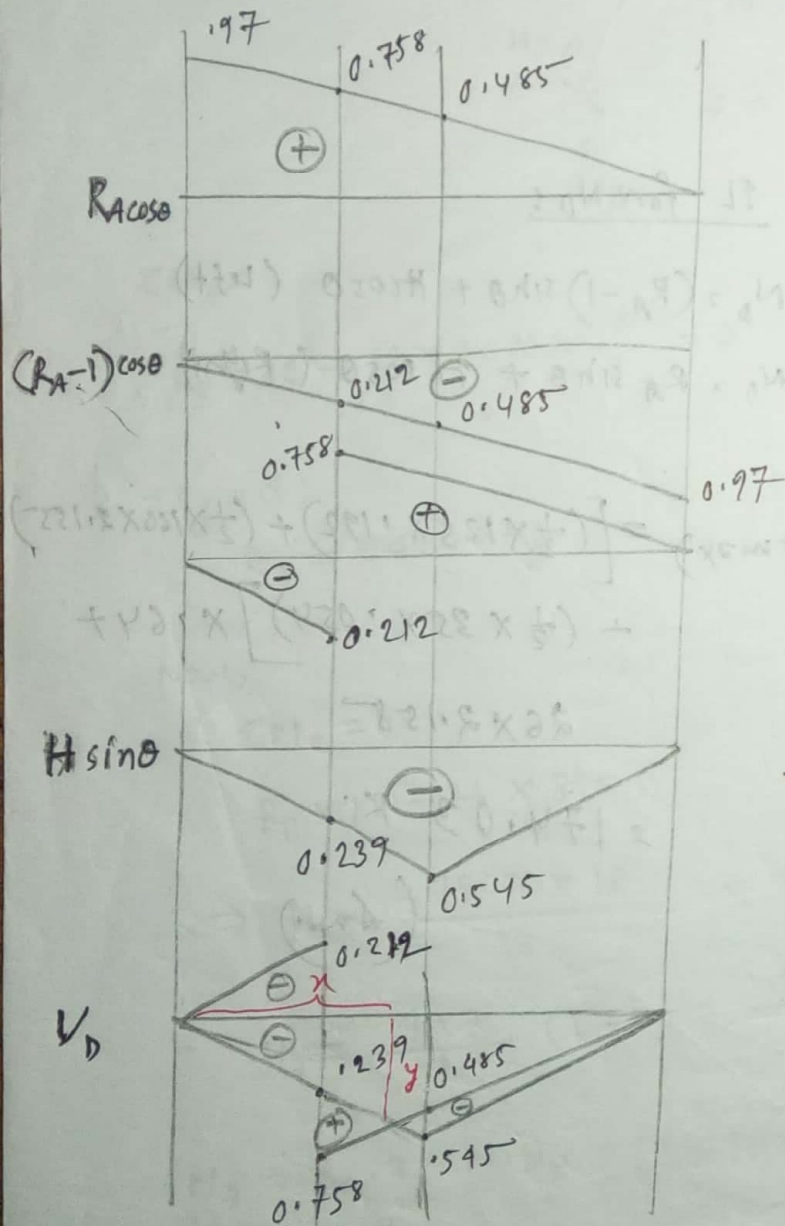
Now,

$$(+)\ M_{max} = \left[ \left( \frac{1}{2} \times 160 \times 27.335 \right) - \left( \frac{1}{2} \times 160 \times 21.33 \right) \right] \times 0.64 + 18 \left( \frac{27.335 - 11.96}{11.96} \right)$$

$$= 584.206 \text{ k'}$$

$$(-)\ M_{max} = \left[ \left( \frac{1}{2} \times 160 \times 27.317 \right) - \left( \frac{1}{2} \times 160 \times 21.33 \right) \right] \times 0.64 + 18 \left( \frac{27.317 - 17.5}{17.5} \right)$$

$$= 483.24 \text{ k'}$$



IL for  $V_D$ :

$$V_D = (R_A - 1) \cos \theta - H \sin \theta \quad (\text{Left})$$

$$V_D = R_A \cos \theta - H \sin \theta \quad (\text{Right})$$

$$\tan \theta = \frac{4 \times 18}{160} - \frac{8 \times 18 \times 35}{160^2}$$

$$\theta = 14.20$$

$$\sin \theta = 0.2454$$

$$\cos \theta = 0.97$$

~~(+)  $V_D$  (max)~~

From similar triangle,

$$\frac{y}{160-x} = \frac{0.758}{125}$$

$$\Rightarrow 125y = (160-x) \cdot 0.758 \quad \dots \textcircled{1}$$

And,

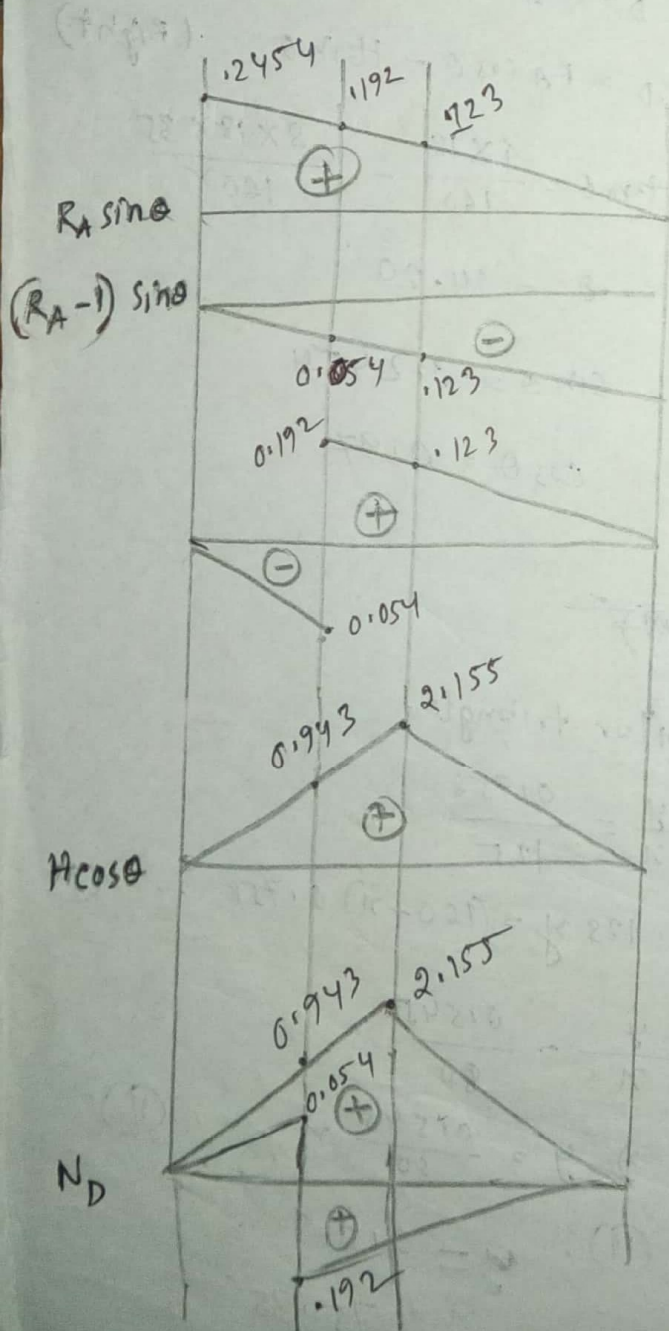
$$\frac{y}{x} = \frac{0.545}{80}$$

$$\Rightarrow y = \frac{0.545}{80} x \quad \dots \textcircled{2}$$

From  $\textcircled{1}$  &  $\textcircled{2}$ ,  $y = 0.51$   
 $x = 75.35$

$$\begin{aligned}
 (+) V_D (\text{max}) &= \left[ \left( \frac{1}{2} \times 125 \times 0.758 \right) - \left( \frac{1}{2} \times 160 \times 0.51 \right) + \left( \frac{1}{2} \times 35 \times 0.239 \right) \right] \times 0.64 \\
 &\quad + 26 (0.76 - 0.239) \\
 &= 20.4308 \text{ K}
 \end{aligned}$$

$$\begin{aligned}
 (-) V_D (\text{max}) &= \left[ \left( \frac{1}{2} \times 35 \times 0.212 \right) + \left( \frac{1}{2} \times 35 \times 0.239 \right) + \left( \frac{1}{2} \times 160 \times 0.595 \right) - \left( \frac{1}{2} \times 160 \times 0.51 \right) \right] \times 0.64 + \underline{26 \times 0.239} \\
 &= 13.0572 \text{ K}
 \end{aligned}$$



IL for  $N_D$ :

$$N_D = (R_A - 1) \sin \theta + H \cos \theta \quad (\text{Left})$$

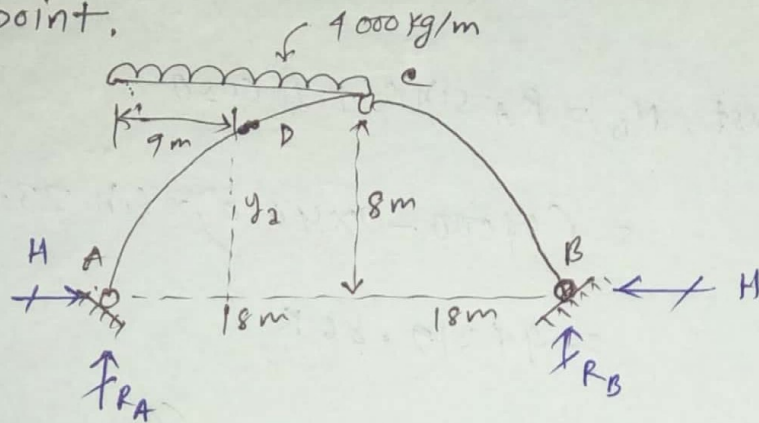
$$N_D = R_A \sin \theta + H \cos \theta \quad (\text{Right})$$

$$\begin{aligned}
 (+) N_D (\text{max}) &= \left[ \left( \frac{1}{2} \times 125 \times 0.192 \right) + \left( \frac{1}{2} \times 160 \times 2.155 \right) \right. \\
 &\quad \left. + \left( \frac{1}{2} \times 35 \times 0.054 \right) \right] \times 0.64 + \\
 &\quad 26 \times 2.155 \\
 &= 174.03 \text{ K}
 \end{aligned}$$

(Ans)

# Problem: A uniformly distributed load of  $4000 \text{ kg/m}$  covers left hand half of the span of a three hinged parabolic arch span  $36 \text{ m}$  and center rise  $8 \text{ m}$ . Determine the horizontal thrust, Also find Bending moment and shear force and normal thrust at the loaded quarter point.

Solution:



$$\sum M_A = 0$$

$$R_B \times 36 - 18 \times 4000 \times \frac{18}{2} = 0 \Rightarrow R_B = 18000 \text{ kg}$$

$$\sum F_y = 0,$$

$$R_A = 54000 \text{ kg}$$

Now,

$$\sum M_C = 0$$

$$R_B \times 18 - H \times 8 = 0$$

$$\Rightarrow H = \frac{18000 \times 18}{8} = 40,500 \text{ kg}$$

$$y_2 = \frac{4ha}{L^2} (L-a) = \frac{4 \times 8 \times 9}{36^2} (36-9) = 6 \text{ m}$$

$$M_D = R_A \cdot a - H \cdot y_2 = (54,000 \times 9 - 40,500 \times 6) = 81000 \text{ kg}$$

$$\tan \theta = \frac{4h}{L} - \frac{8ha}{L^2} = \frac{4 \times 8}{36} - \frac{8 \times 8 \times 9}{36^2} = 0.44$$

$$\therefore \theta = 23.96^\circ$$

$$\therefore \text{Shear force } V_D = R_A \cos \theta - H \sin \theta$$

$$= (54000 - 9 \times 4000) \times \cos 23.96 - 40500 \sin 23.96$$

$$= 1.93 \text{ K}$$

$$\text{Normal thrust, } N_D = R_A \sin \theta + H \cos \theta$$

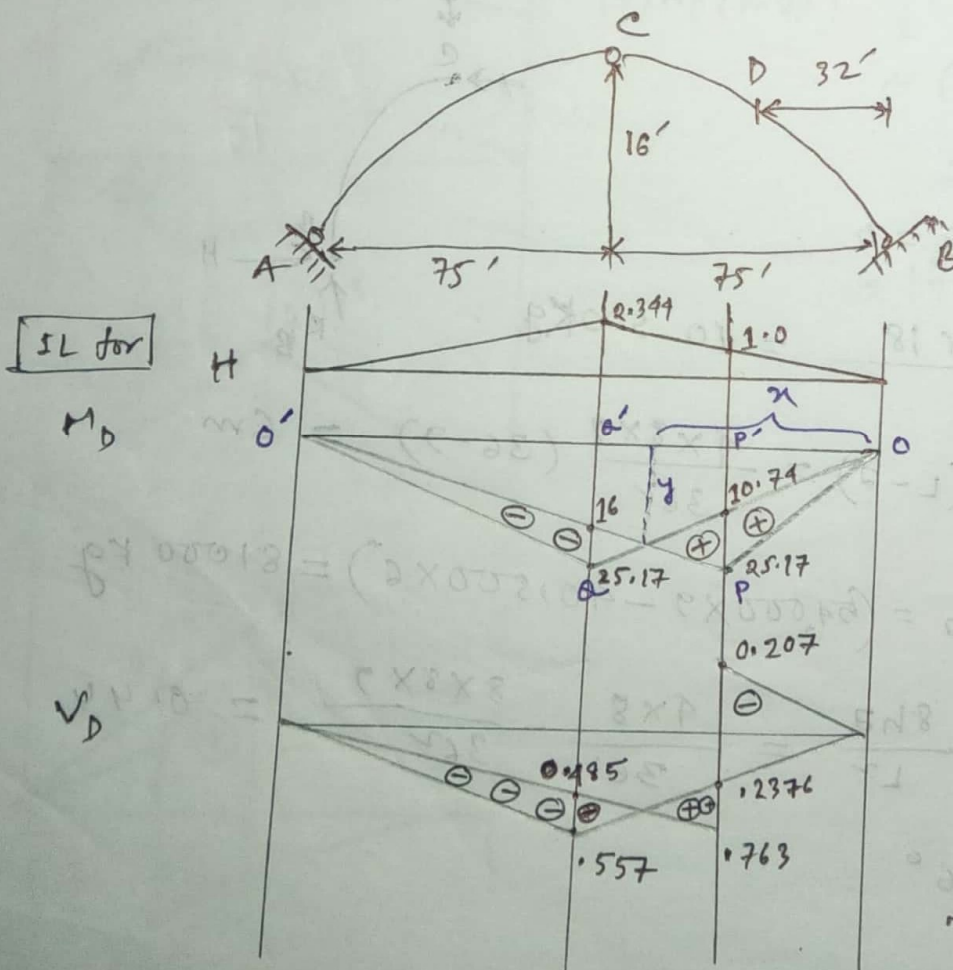
$$= (54000 - 9 \times 4000) \times \sin 23.96 + 40500 \times \cos 23.96$$

$$= 44319.86 \text{ K}$$

(Ans.)

class test + 15 series

# Draw influence line for bending moment, shear force, and Normal thrust at a section D of the following three hinged parabolic arch. Also obtain the maximum moment at section D for H15 loading.



Given that,

$$L = 150$$

$$h = 16$$

$$a = 32'$$

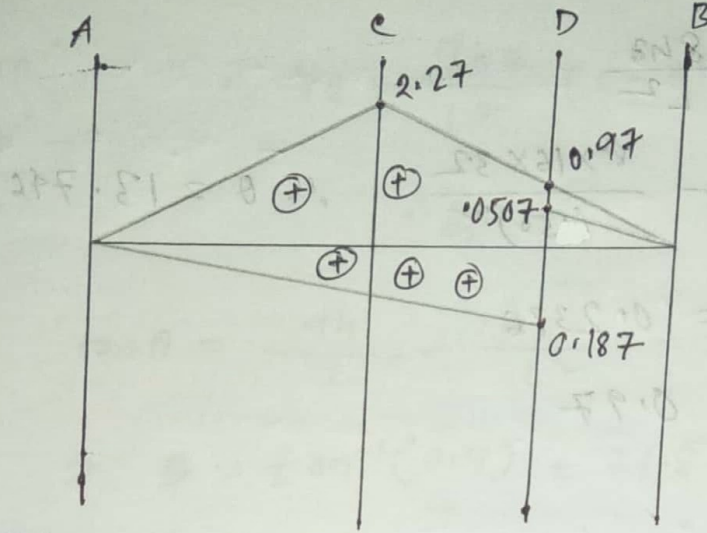
$$\therefore y_2 = \frac{4ha^2}{L^2} (L-a)$$

$$= \frac{4 \times 16 \times 32^2}{150^2} \times (150 - 32)$$

$$\therefore y_2 = 10.74'$$

IL for

$N_D$



IL for  $M_D$ :

$$M_D = R_B \cdot a - H \cdot y_2$$

From  $\triangle OPP'$ ,  $\frac{y}{150-x} = \frac{25.17}{118}$  --- ①

From  $\triangle OQQ'$ ,  $\frac{y}{x} = \frac{25.17}{75}$  --- ②

From ① & ②,  $x = 58.29'$  &  $y = 19.56'$

$$\begin{aligned} (+) M_{max} &= \left[ \frac{1}{2} \times 150 \times (25.17 - 19.56) \right] \times 48 + (25.17 - 10.74) \times 13.5 \\ &= 396.765 \text{ K ft} \end{aligned}$$

$$\begin{aligned} (-) M_{max} &= \left[ \frac{1}{2} \times 150 \times (25.17 - 19.56) \right] \times 48 + (25.17 - 16) \times 13.5 \\ &= 325.755 \text{ K ft} \end{aligned}$$

IL for  $V_D$ :

For Right,  $V_D = (R_B - 1) \cos \theta - H \sin \theta$

For left  $V_D = R_B \cos \theta - H \sin \theta$

$$\tan \theta = \frac{4h}{L} - \frac{8ha}{L^2}$$

$$\tan \theta = \frac{4 \times 16}{150} - \frac{8 \times 16 \times 32}{(150)^2} \quad \therefore \theta = 13.746^\circ$$

$$\therefore \sin \theta = 0.2376$$

$$\text{and } \cos \theta = 0.97$$

IL for  $N_D$ :

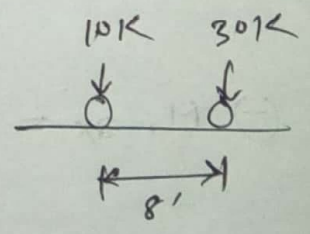
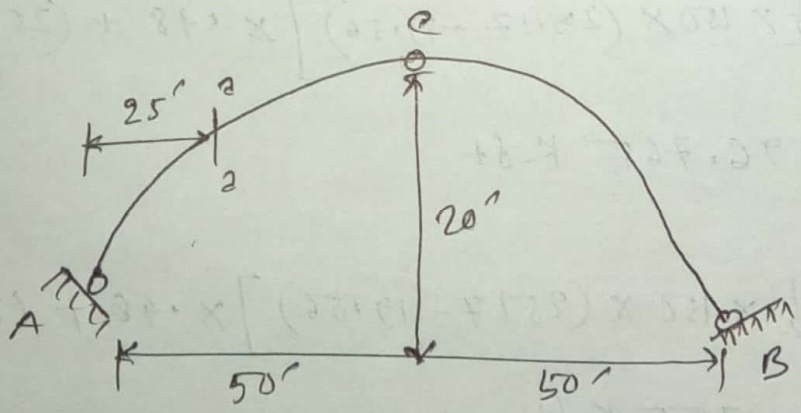
$$N_D = (R_B - 1) \sin \theta + H \cos \theta \quad (\text{for Right})$$

$$N_D = R_B \sin \theta + H \cos \theta \quad (\text{for left})$$

2010, 08

# A three hinged parabolic arch shown in figure, below a span of 100' and a central rise of 20'. Draw influence line for Shear force, normal thrust and bending moment at quarter span 2-2 and hence compute the maximum shear, normal thrust and bending moment due to following wheel loads.

Solution:



Hence,  $L = 100'$   
 $h = 20'$   
 $a = 25'$

$$\therefore y_2 = \frac{4ha}{L^2} (L-a) = \frac{4 \times 20 \times 25}{100^2} \times (100-25)$$

$$\therefore y_2 = 15$$

$$\tan \theta = \frac{4h}{L} - \frac{8ha}{L^2} = \frac{4 \times 20}{100} - \frac{8 \times 20 \times 25}{100^2}$$

$$\Rightarrow \theta = \tan^{-1}(0.4) = 21.8^\circ$$

$$\therefore \sin \theta = 0.37$$

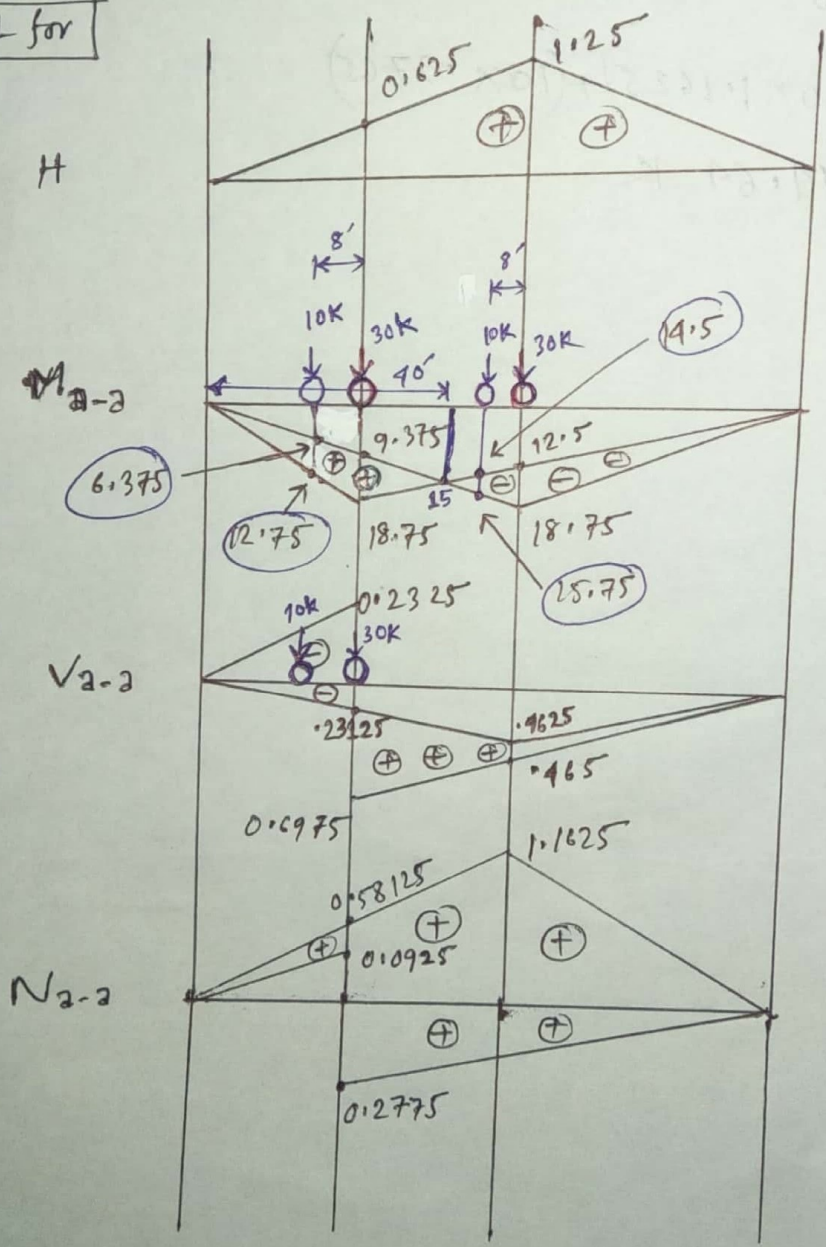
$$\therefore \cos \theta = 0.93$$

$$\frac{x}{y} = \frac{50}{18.75} \dots \textcircled{1}$$

$$\frac{100-x}{y} = \frac{75}{18.75} \dots \textcircled{2}$$

$$\textcircled{1} \& \textcircled{2} \Rightarrow x = 40, y = 15$$

IL for



Max. stress for  $M_{a-a}$ :  $\uparrow$

$$\frac{y}{25-8} = \frac{9.375}{25} \Rightarrow y = \underline{\underline{6.375}}$$

again,  $\frac{y}{25-8} = \frac{18.75}{25} \Rightarrow y = \underline{\underline{12.75}}$

$$\textcircled{+} \text{ Max. } M_{a-a} = 30(18.75 - 9.375) + 10(12.75 - 6.375)$$

$$= 345 \text{ Kft}$$

$$\frac{y}{50+8} = \frac{12.5}{50} \Rightarrow y = \underline{\underline{14.5}}$$

$$\frac{y}{50-8} = \frac{18.75}{50} \Rightarrow y = \underline{\underline{15.75}}$$

$$\textcircled{-} \text{ Max } M_{a-a} = 30(18.75 - 12.5) + 10(15.75 - 14.5)$$

$$= 200 \text{ Kft}$$

Max. stress for  $V_{2-2}$ : (+)  $\text{Max. } V_{2-2} = 30 \times (16975 - 127125) + 10 \times 0$   
 $= 13.9875 \text{ K}$

similarly,  $\frac{y}{25-8} = \frac{.2325}{25}$

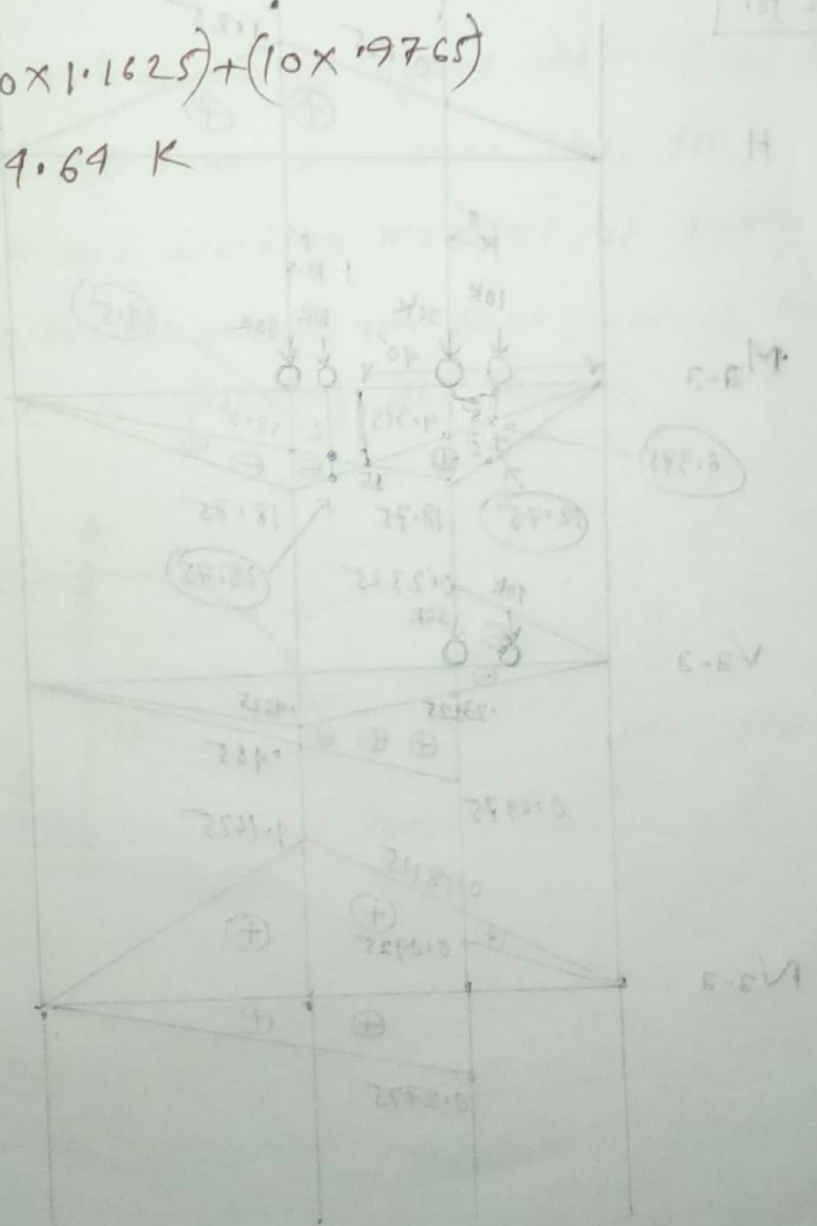
$\Rightarrow y = 0.11581$

(-)  $\text{Max. } V_{2-2} = 30 \times (.2325) + 10 \times 11581$   
 $= 8.556 \text{ K}$

Max. stress for  $N_{2-2}$ :

$\frac{y}{50-8} = \frac{1.1625}{50} \Rightarrow y = 0.9765$

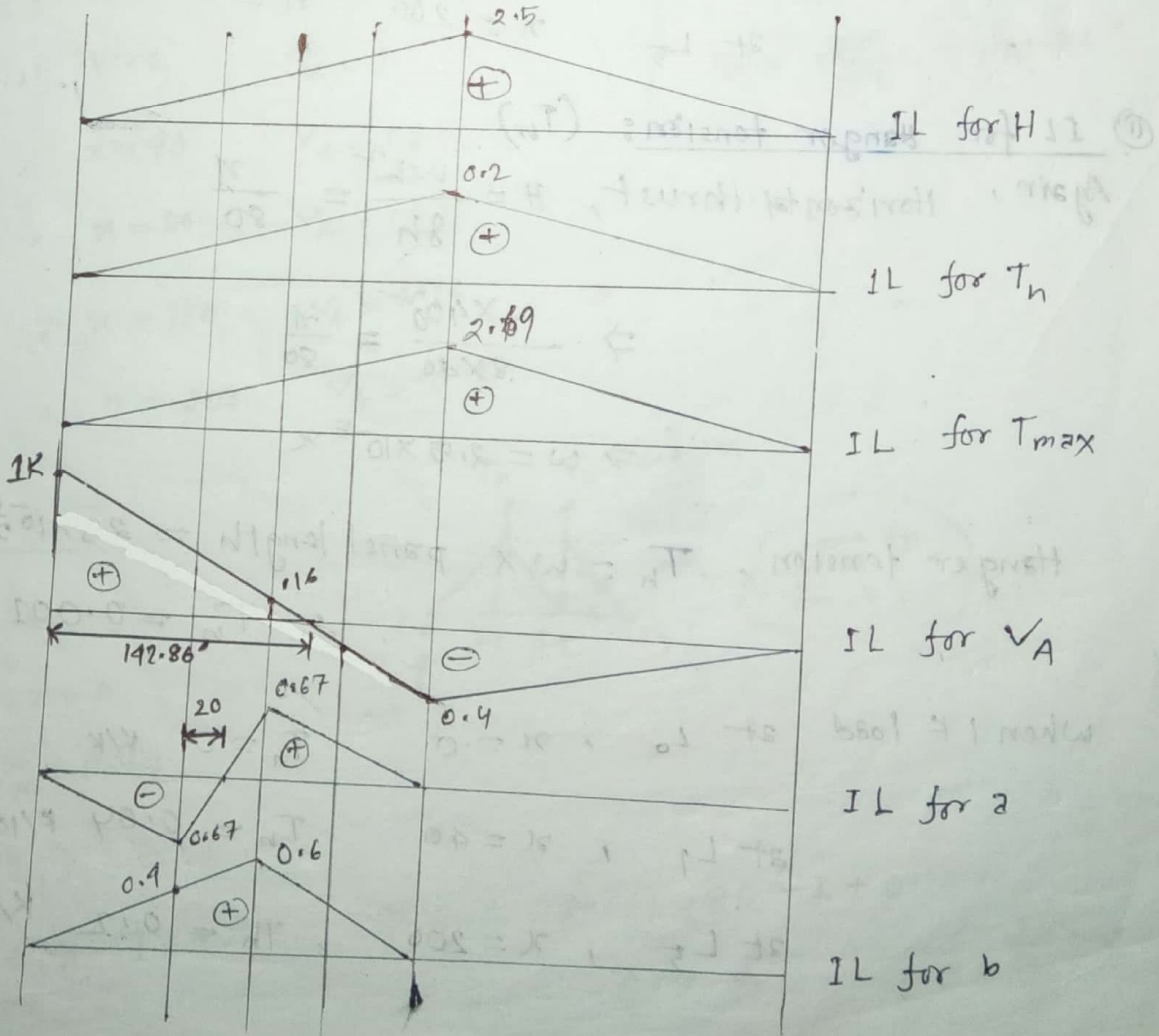
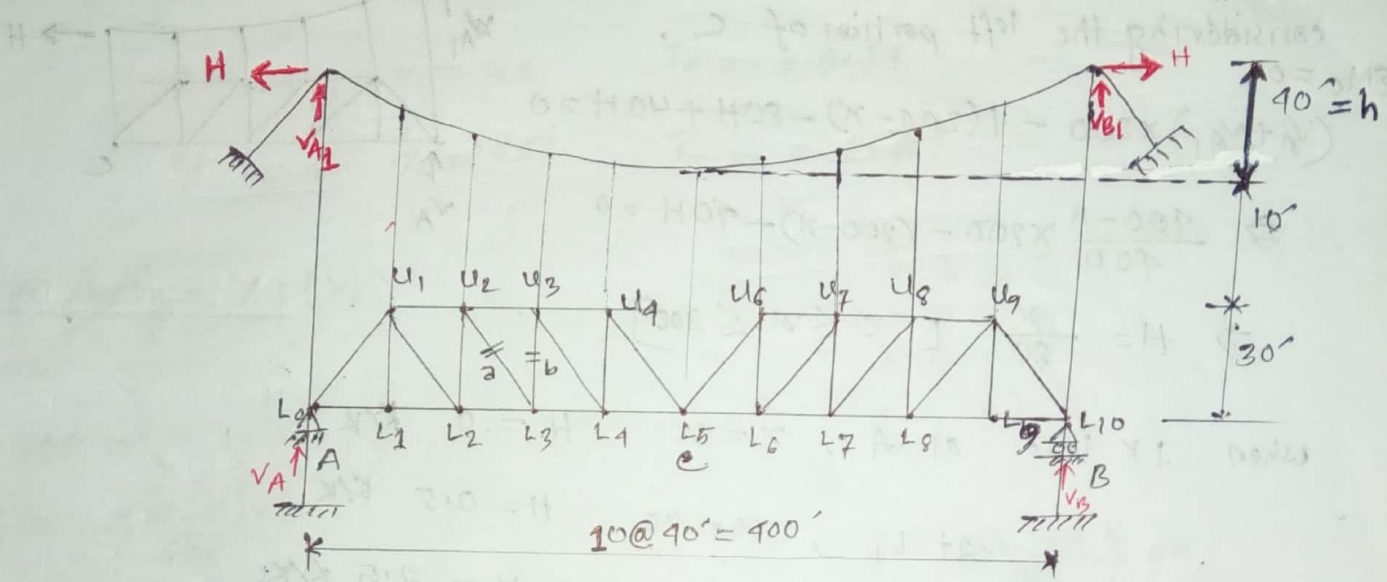
(+)  $\text{Max. } N_{2-2} = (30 \times 1.1625) + (10 \times 9765)$   
 $= 49.69 \text{ K}$



Handwritten calculations on the left side of the page, including:  
 $(24512 - 27.91) \times 101 = 24512 - 27.91 \times 101$   
 $(24512 - 27.91) \times 101 = 24512 - 27.91 \times 101$   
 $(24512 - 27.91) \times 101 = 24512 - 27.91 \times 101$   
 $(24512 - 27.91) \times 101 = 24512 - 27.91 \times 101$

# Suspension Cable Bridge

**# Problem:** Deduce IL for hanger tension, maximum cable tension and stress in member 'a' and 'b' of the following suspension bridge shown in figure below. Also calculate the maximum force or stress of this member due to uniform load of  $5 \text{ K/ft}$  with a concentrated load of  $10 \text{ K}$ .



① IL for horizontal thrust  $H$ : (H)  
considering whole free body diagram,

$$\Sigma M_B = 0$$

$$(V_A + V_{A1}) \times 400 - 1(400 - x) + H \times 80 - H \times 80 = 0$$

$$\Rightarrow V_A + V_{A1} = \frac{400 - x}{400} \dots \dots \text{--- (1)}$$

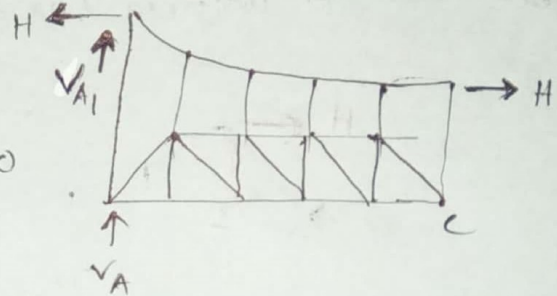
considering the left portion of C,

$$\Sigma M_C = 0$$

$$(V_A + V_{A1}) \times 200 - 1(200 - x) - 80H + 40H = 0$$

$$\Rightarrow \frac{400 - x}{400} \times 200 - (200 - x) - 40H = 0$$

$$\Rightarrow H = \frac{x}{80} \quad [0 \leq x \leq 200]$$



When 1 K load at A,  $x = 0$        $H = 0$  K/K  
 at  $L_1$ ,  $x = 40$        $H = 0.5$  K/K  
 at  $L_5$ ,  $x = 200$        $H = 2.5$  K/K

② IL for hanger tension: ( $T_h$ )

Again, Horizontal thrust,  $H = \frac{wL^2}{8h} = \frac{x}{80}$

$$\Rightarrow \frac{w \times 400^2}{8 \times 40} = \frac{x}{80}$$

$$\Rightarrow w = 2.5 \times 10^{-5} x$$

Hanger tension,  $T_h = w \times \text{panel length} = 2.5 \times 10^{-5} x \times 40$

$$\therefore T_h = 0.001 x$$

When 1 K load at  $L_0$ ,  $x = 0$        $T_h = 0$  K/K  
 at  $L_1$ ,  $x = 40$        $T_h = 0.04$  K/K  
 at  $L_5$ ,  $x = 200$        $T_h = 0.2$  K/K

(iii) IL for maximum cable tension: ( $T_{max}$ )

We know,  $T_{max} = H (1 + 16\theta^2)^{\frac{1}{2}} = \frac{x}{80} \left[ 1 + 16 \times \left( \frac{40}{100} \right)^2 \right]^{\frac{1}{2}} \quad (\because \theta = \frac{h}{L})$

$\therefore T_{max} = 0.01346 x$

When 1K load at  $L_0$ ,  $x = 0$   $T_{max} = 0$

at  $L_1$ ,  $x = 40$   $T_{max} = 0.54$

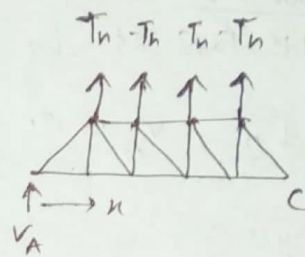
at  $L_5$ ,  $x = 200$   $T_{max} = 2.69$

(iv) IL for shear at A: ( $V_A$ )

$\Sigma M_c = 0$

$V_A \times 200 + T_h \times (1+2+3+4) \times 40 - 1(200-x) = 0$

$\Rightarrow V_A = \frac{200 - 1.4x}{200}$



$\frac{200 - 1.4x}{200} = 0$

$x = \frac{200}{1.4} = 142.86'$

When 1K at  $L_0$ ,  $x = 0$   $V_A = 1$

at  $L_1$ ,  $x = 40$   $V_A = 0.72$

at  $L_2$ ,  $x = 80$   $V_A = 0.44$

at  $L_3$ ,  $x = 120$   $V_A = 0.16$

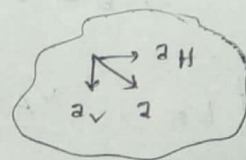
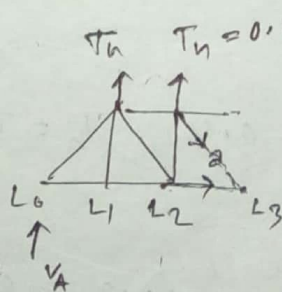
at  $L_5$ ,  $x = 200$   $V_A = -0.4$

(v) IL for member 'a':

When 1K at A,  $V_A = 1$

$a = 0$

at  $L_2$ ,  $V_A = 0.44$



$\Sigma F_y = 0$

$2 \times 0.001x + V_A - 1 - 2v = 0 \Rightarrow 2v = 2 \times 0.001 \times 80 - 1 + 0.44$

$\therefore 2v = -0.4 \Rightarrow v = -0.4 \times \frac{\sqrt{20^2 + 40^2}}{30}$

$\therefore v = -0.67$

at  $L_3$ ,  $\Sigma F_y = 0$

$V_A = 0.16$

$2 \times 0.001x + V_A - 2v = 0$

$\Rightarrow 2v = 2 \times 0.001 \times 120 + 0.16$

$\therefore 2v = 0.4$

$\Rightarrow a = 0.4 \times \frac{\sqrt{30^2 + 40^2}}{30} = 0.67$

at  $L_5$ ,  $V_A = -0.14$ ,  $2v = 0$   $\therefore a = 0$

⑤ IL for member b:

when 1k load at  $L_0$ ,  $V_A = 1$ ,  $b = 0$

at  $L_2$ ,  $V_A = 0.44$ ,  $T_h = 0.08$

$\Sigma F_y = 0$

$b + 0.44 + 2 \times 0.08 - 1 = 0$

$\Rightarrow b = 0.40$

at  $L_3$ ,  $V_A = 0.16$ ,  $T_h = 0.12$

$\Sigma F_y = 0$

$b + 0.16 + 2 \times 0.12 - 1 = 0$

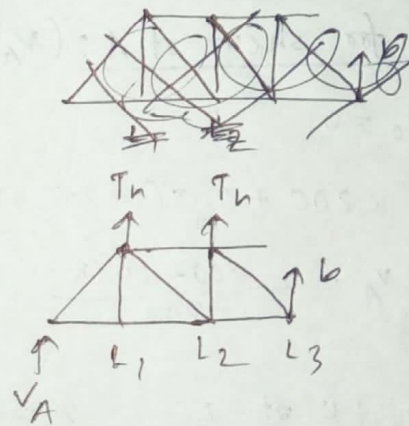
$\Rightarrow b = 0.16$

at  $L_5$ ,  $V_A = -0.14$ ,  $T_h = 0.2$

$\Sigma F_y = 0$

$b - 0.14 + 2 \times 0.2 = 0$

$b = 0$



Due to 5 K/ft uniform load and 10 Kip concentrated load.

$$\text{Max. stress for hanger tension} = \left(2 \times \frac{1}{2} \times 200 \times 0.2\right) \times 5 + 10 \times 0.2$$

$$= 202 \text{ Kips.}$$

$$\text{Max. stress for cable tension} = \left(2 \times \frac{1}{2} \times 200 \times 2.69\right) \times 5 + 10 \times 2.69$$

$$= 2716.9 \text{ Kips.}$$

Max. stress for a,

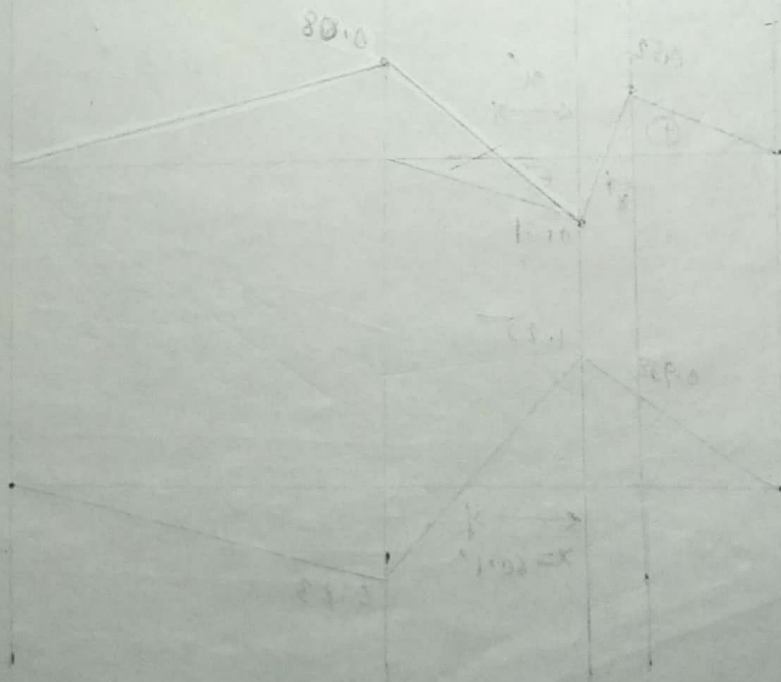
$$\text{Max. } (+)_{\Delta} \text{ stress} = \left(\frac{1}{2} \times 100 \times 0.67\right) \times 5 + 10 \times 0.67 = 174.2 \text{ Kips}$$

$$\text{Max. } (-)_{\Delta} \text{ stress} = \left(\frac{1}{2} \times 100 \times 0.67\right) \times 5 + 10 \times 0.67 = 174.2 \text{ Kips.}$$

$$\text{Max. stress for b} = \left(\frac{1}{2} \times 200 \times 0.6\right) \times 5 + 10 \times 0.6$$

$$= 306 \text{ Kips.}$$

(Ans)



$$\frac{31}{x-100} = \frac{13}{56}$$

$$\frac{13}{x-100} = \frac{570}{x}$$

$$13x = 570(x-100)$$

$$13x = 570x - 57000$$

$$557x = 57000$$

$$x = \frac{57000}{557}$$

$$x = 102.33$$

$$\frac{13}{x-100} = \frac{13}{56}$$

$$\frac{13}{x-100} = \frac{570}{x}$$

$$13x = 570(x-100)$$

$$13x = 570x - 57000$$

$$557x = 57000$$

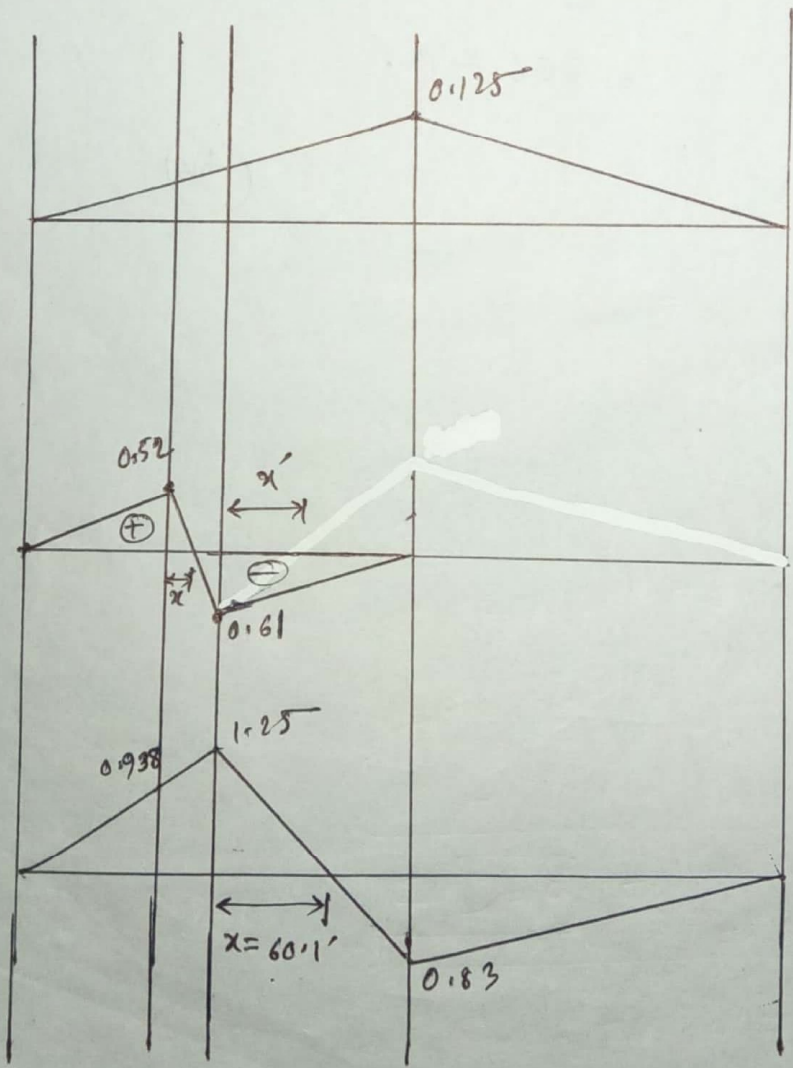
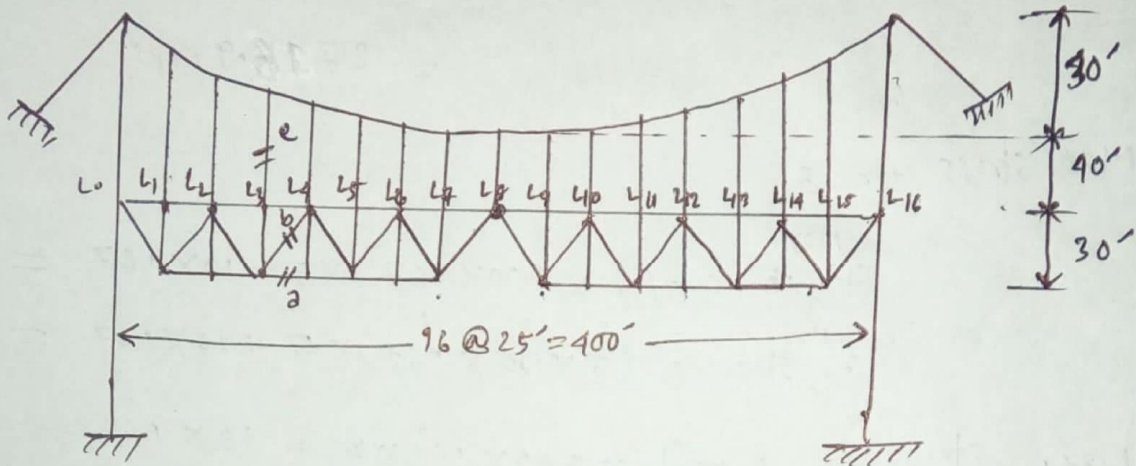
$$x = \frac{57000}{557}$$

$$x = 102.33$$

2014

# Draw influence lines for stress in member a, b and c of the following suspension bridge shown in figure below and hence determine the maximum stress of the same members due to uniform load of 5 kips/ft and concentrated load of 20 kips.

Solution:



IL for

c

b

$$\frac{0.52}{x} = \frac{1.61}{25-x}$$

$$\Rightarrow x = 11.5$$

a

$$\frac{1.25}{x} = \frac{0.83}{100-x}$$

$$\Rightarrow x = 60.1$$

Here,  $H = \frac{x}{2h} = \frac{x}{2 \times 30} = \frac{x}{60}$

again,  $H = \frac{wL^2}{8h} = \frac{wL^2}{80}$

$\Rightarrow w = \frac{x \times 8h}{60L^2} = \frac{x \times 8 \times 30}{60 \times (900)} = 2.5 \times 10^{-5} x$

IL for c:

$C = T_h = w \times \text{panel length}$

$\therefore T_h = 2.5 \times 10^{-5} x \times 25 = 6.25 \times 10^{-4} x$

When 1K at  $L_0 \rightarrow T_h = 0$

$\sum M_{L_0} = 0$  1K at  $L_8 \rightarrow T_h = 1.25$

$V_A \times 200 + T_h (1+2+3+4+5+6+7) \times 25$

$-1(200-x) = 0$

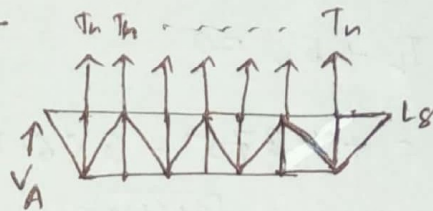
$\therefore V_A = \frac{200 - 1.4375x}{200}$

when 1K at  $L_0 \rightarrow V_A = 1$

1K at  $L_3 \rightarrow V_A = 0.461$

1K at  $L_4 \rightarrow V_A = 0.281$

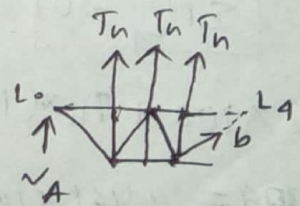
1K at  $L_8 \rightarrow V_A = -0.14375$



IL for b:

1K at  $L_0 \rightarrow V_A = 1$   $\therefore \sum = 0$

1K at  $L_3 \rightarrow V_A = 0.461$



$\sum F_y = 0$

$V_A + 3 \times T_h - 1 + bV = 0$

$\Rightarrow 0.461 + 3 \times 6.25 \times 10^{-4} \times 75 - 1 + bV = 0$

$\Rightarrow bV = 0.398375$   $\therefore b = 0.398375 \times \frac{\sqrt{30^2 + 25^2}}{30} = 0.52 \text{ K}$

$$1K \text{ at } L_4 \rightarrow V_A = 0.1281$$

$$\Sigma F_y = 0$$

$$\therefore V_A + 3T_h + b_v = 0 \Rightarrow b_v = -V_A - 3T_h = 0.1281 - 3 \times 6.25 \times 10^{-4} \times 1000$$

$$\Rightarrow b_v = -0.4685$$

$$\therefore b = -0.4685 \times \frac{\sqrt{30^2 + 25^2}}{30} = -0.61K$$

$$1K \text{ at } L_8 \rightarrow V_A = -0.4375$$

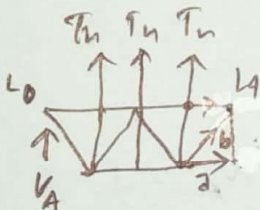
$$\Sigma F_y = 0$$

$$V_A + 3T_h + b_v = 0$$

$$\Rightarrow b_v = +0.4375 - 3 \times 6.25 \times 10^{-4} \times 200 = 0.0625$$

$$\Rightarrow b = 0.0625 \times \frac{\sqrt{30^2 + 25^2}}{30} = 0.08 \approx 0K$$

IL for a:



~~IL for a:~~

$$1K \text{ at } L_0 \rightarrow V_A = 1$$

$$\therefore b = 0$$

$$1K \text{ at } L_3 \rightarrow V_A = 0.461$$

$$\boxed{\Sigma M_{L_4} = 0}$$

$$V_A \times 100 + T_h \times (1+2+3) \times 25 - 1 \times 25 - 2 \times 30 = 0$$

$$\Rightarrow 30a = 0.461 \times 100 + 6.25 \times 10^{-4} \times 75 \times (1+2+3) \times 25 - 25 = 28.73125$$

$$\therefore a = 0.938K$$

$$1K \text{ at } L_4 \rightarrow V_A = 0.1281 \text{ or}$$

$$\boxed{\Sigma M_{L_4} = 0} \Rightarrow V_A \times 100 + T_h \times (1+2+3) \times 25 - 2 \times 30 = 0$$

$$\Rightarrow a = 1.25K$$

$$\boxed{\Sigma M_{L_3} = 0}$$

$$V_A \times 75 + T_h \times (1+2) \times 25 + b_h \times 30 - 2 \times 30 = 0$$

Easy

$$\Rightarrow 0.281 \times 75 + 6.25 \times 10^{-4} \times 100 \times (1+2) \times 25 + 0.61 \times \frac{25}{\sqrt{25^2 + 30^2}} \times 30 = 30a$$

$$\Rightarrow a = 1.25 \text{ K}$$

$$1 \text{ K at } L_8 \rightarrow V_A = -0.4375 \text{ or}$$

$$\boxed{\Sigma M_{L_1} = 0}$$

$$\Rightarrow V_A \times 100 + T_h \times (1+2+3) \times 25 - a \times 30 = 0$$

$$\Rightarrow a = -0.83 \text{ K}$$

$$\boxed{\Sigma M_{L_3} = 0}$$

$$V_A \times 75 + T_h \times (1+2) \times 25 + b_h \times 30 - a \times 30 = 0$$

↑ Easy

$$\Rightarrow -0.4375 \times 75 + 6.25 \times 10^{-4} \times 200 \times 25 \times 3 - 0.108 \times \frac{25}{\sqrt{25^2 + 30^2}} = 30a$$

$$\therefore a = -0.83 \text{ K}$$

Now,

For c,

$$\text{Maximum stress, (+)} = \left( \frac{1}{2} \times 400 \times 1125 \right) \times 5 + 1125 \times 20$$

$$= 127.5 \text{ Kips/ft}^2$$

Maximum stress for b:

$$\text{Max. (+) stress} = \left[ \frac{1}{2} \times (75 + 11.5) \times 152 \right] \times 5 + 0.152 \times 20$$

$$= 122.85 \text{ K/ft}^2$$

$$\text{(-) Max. stress} = \left[ \frac{1}{2} \times (25 - 11.5 + 100) \times 161 \right] \times 5 + 0.61 \times 20$$

$$= 185.29 \text{ K/ft}^2$$

Maximum stress for a:

$$\text{(+) Max. stress} = \left[ \frac{1}{2} \times (100 + 601) \times 1.25 \right] \times 5 + 1.25 \times 20$$

$$= 525.3125 \text{ K/ft}^2$$

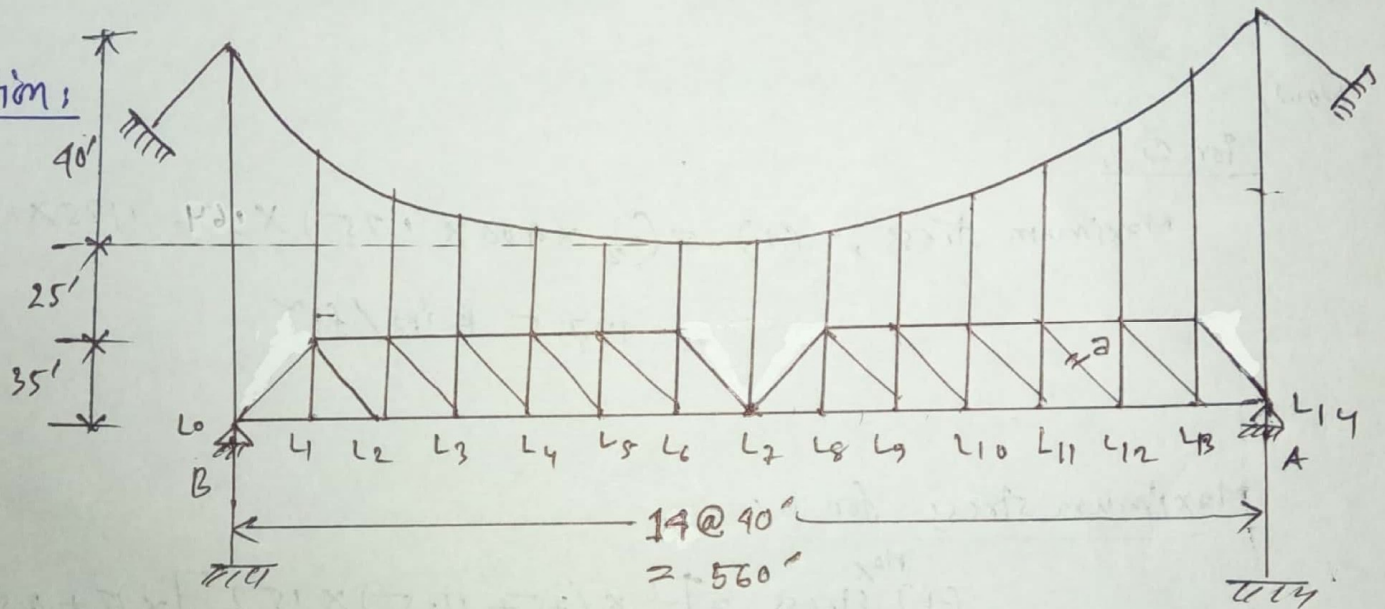
$$\begin{aligned} \Rightarrow \text{Max. stress} &= \left[ \frac{1}{2} \times (200 + 100 - 60.1) \times 183 \right] \times 5 + 183 \times 20 \\ &= 514.3925 \text{ K/ft}^2 \end{aligned}$$

(Ans.)

**Class Test** -15 series

# Draw influence lines for hanger tension, maximum cable tension, and stress in the member 'a' of the following suspension bridge shown in figure. Find the maximum stress in the member 'a' if the bridge is subjected to a uniform load of 5 K/ft and moving concentrated load of 15 kips.

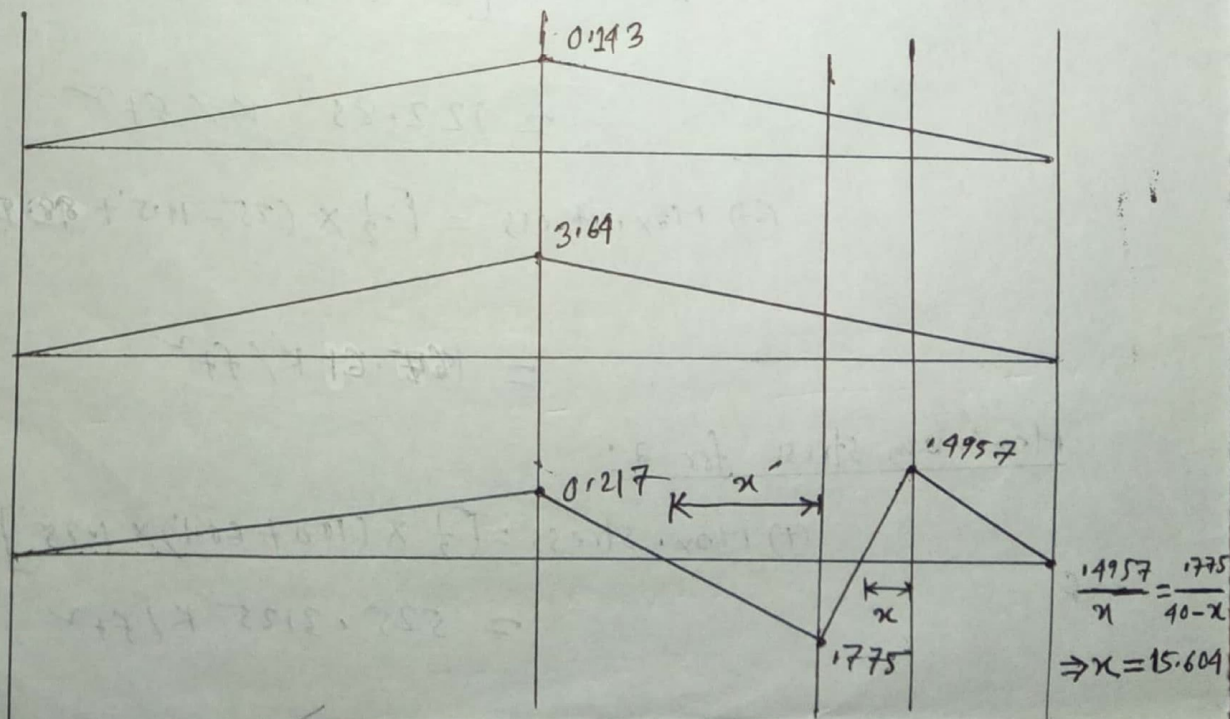
Solution:



IL for

$T_h$

$T_{max}$



$$\begin{aligned} \frac{.775}{x'} &= \frac{.217}{160-x'} \\ \Rightarrow x' &= 125' \end{aligned}$$

$$\begin{aligned} \frac{1.9957}{x} &= \frac{1.775}{40-x} \\ \Rightarrow x &= 15.604 \end{aligned}$$

Here,

$$H = \frac{\alpha}{2h} = \frac{\alpha}{2 \times 40} = \frac{\alpha}{80}$$

again,

$$H = \frac{wL^2}{8h} \quad \therefore \frac{wL^2}{8h} = \frac{\alpha}{80}$$

$$\Rightarrow w = \frac{\alpha \times 8 \times 40}{80 \times (560)^2} = 1.2755 \times 10^{-5} \alpha$$

IL for hanger tension:

$$T_n = w \times \text{panel length} = 1.2755 \times 10^{-5} \alpha \times 40 \\ = 5.1 \times 10^{-4} \alpha$$

when IK at  $L_4 \rightarrow T_n = 0$

IK at  $L_7 \rightarrow T_n = 0.143$

IL for maximum cable tension:

$$T_{\max} = H \sqrt{1 + 16h^2} = \frac{\alpha}{80} \times \sqrt{1 + 16 \times \left(\frac{40}{560}\right)^2} = 0.013 \alpha$$

$\therefore$  when IK at  $L_4 \rightarrow T_{\max} = 0$

IK at  $L_7 \rightarrow T_{\max} = 3.64$

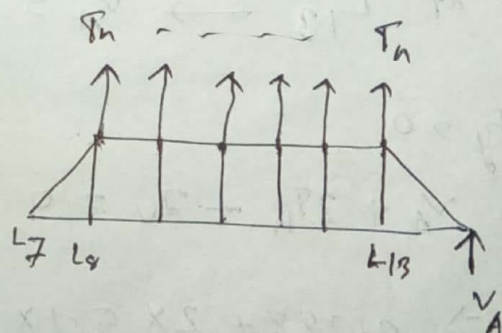
Now,

$$\sum M_{L_7} = 0$$

$$V_A \times 280 + T_n \times (1+2+3+4+5+6) \times 40$$

$$- 1(280 - \alpha) = 0$$

$$\Rightarrow V_A = \frac{280 - 1.4289 \alpha}{280}$$



When 1K at  $L_{14} \rightarrow V_A = 1 \text{ K}$

1K at  $L_{12} \rightarrow V_A = 0.592 \text{ K}$

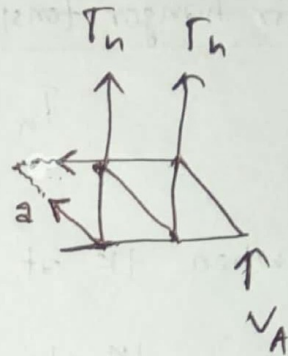
1K at  $L_{11} \rightarrow V_A = 0.388 \text{ K}$

1K at  $L_7 \rightarrow V_A = -0.4284 \text{ K}$

IL for stress in member a:

When 1K at  $L_{14} \rightarrow V_A = 1 \text{ K} \therefore a = 0$

1K at  $L_{12} \rightarrow V_A = 0.592 \text{ K}$



$\Sigma F_y = 0$

$V_A + 2T_h - 1 + 2v = 0$

$\Rightarrow 0.592 + 2 \times 5.1 \times 10^{-4} \times 80 - 1 + 2v = 0$

$\Rightarrow 2v = 0.3264 \therefore v = 0.3264 \times \frac{\sqrt{35^2 + 40^2}}{35} = 0.4957 \text{ K}$

1K at  $L_{11} \rightarrow V_A = 0.388 \text{ K}$

$\Sigma F_y = 0$

$V_A + 2T_h + 2v = 0$

$\Rightarrow 0.388 + 2 \times 5.1 \times 10^{-4} \times 20 + 2v = 0$

$\Rightarrow 2v = -0.5104 \therefore v = -0.5104 \times \frac{\sqrt{35^2 + 40^2}}{35} = -0.775 \text{ K}$

1K at  $L_7 \rightarrow V_A = -0.4284$

$\Sigma F_y = 0 \quad V_A + 2T_h + 2v = 0 \Rightarrow -0.4284 + 2 \times 5.1 \times 10^{-4} \times 280 + 2v = 0$

$\Rightarrow 2v = 0.1428 \therefore v = 0.217 \text{ K}$

Maximum stress for a: 1

$$\begin{aligned} (+) \text{ Max. Stress} &= \left[ \frac{1}{2} \times (280 + 160 - 125) \times 217 \right] \times 5 + (217 \times 15) \\ &= 174.1425 \text{ K/ft}^2 \quad (\text{max}) \end{aligned}$$

$$\begin{aligned} (+) \text{ Max. stress} &= \left[ \frac{1}{2} \times (80 + 15.604) \times 4957 \right] \times 5 + (4957 \times 15) \\ &= 125.913 \text{ K/ft}^2 \end{aligned}$$

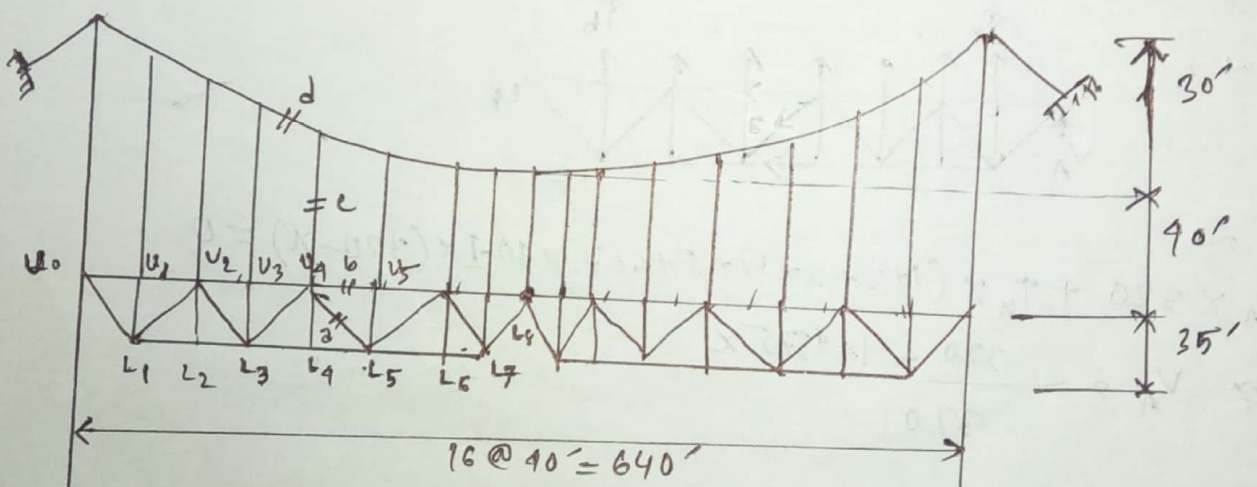
$$\begin{aligned} (-) \text{ Max stress} &= \left[ \frac{1}{2} \times (125 + 40 - 15.604) \times 775 \right] \times 5 + \\ &\quad (775 \times 15) \\ &= 301.08 \text{ K/ft}^2 \end{aligned}$$

2015

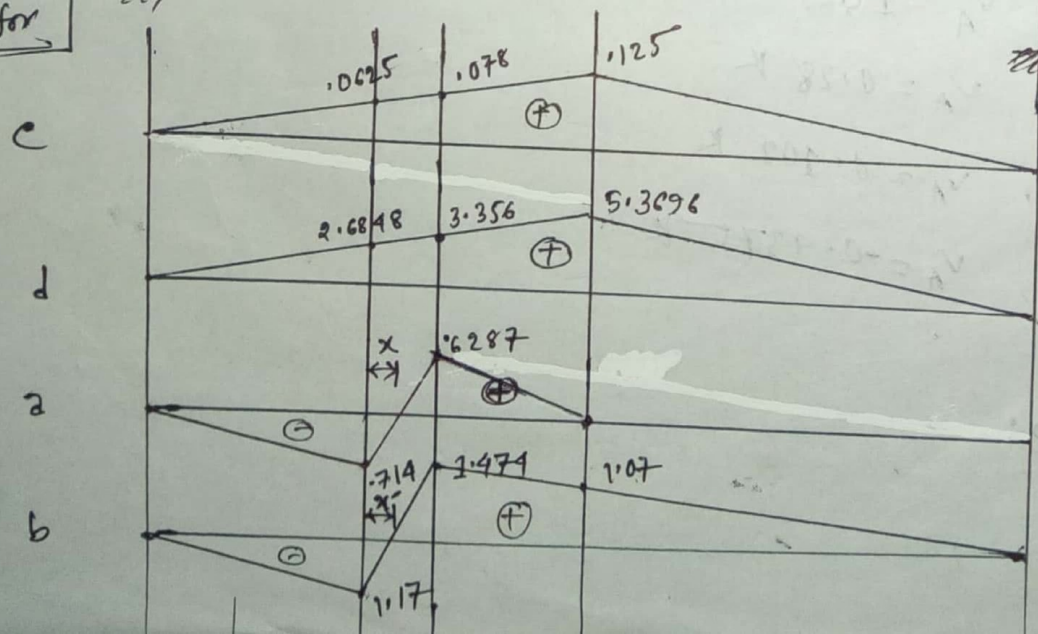
(Ans.)

# Compute the maximum stress developed in member 'a', 'b', hanger 'c' and cable tension 'd' due to uniform <sup>dead</sup> load of 5 K/ft and the live load of 4 K/ft of the following suspension bridge shown in figure below:

Solution:



IL for



$$\frac{x}{0.714} = \frac{40-x}{1.474} \Rightarrow x = 21.27'$$

$$\frac{x'}{1.17} = \frac{40-x'}{1.474} \Rightarrow x' = 17.7'$$

$$\text{Here, } H = \frac{\gamma}{2h} = \frac{\gamma}{2 \times 30} = \frac{\gamma}{60}$$

$$\text{again, } H = \frac{WL^2}{8h} \Rightarrow \frac{WL^2}{8h} = \frac{\gamma}{60} \Rightarrow W = \frac{\gamma \times 8 \times 30}{60 \times 60} = 9.766 \times 10^{-6} \gamma$$

IL for hanger tension:

$$T_h = C = W \times \text{panel length}$$

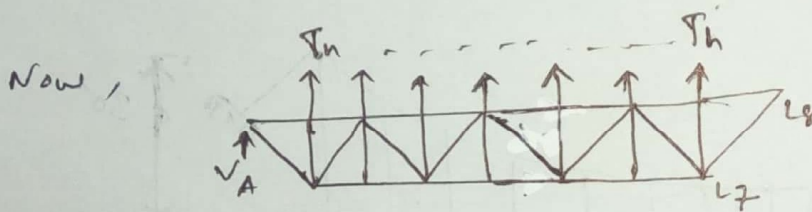
$$\Rightarrow C = (9.766 \times 10^{-6} \gamma \times 40) = 3.9064 \times 10^{-4} \gamma$$

When 1 K at  $u_0$ ,  $C = 0$  K

$$\text{at } L_4, C = 0.0625 \text{ K}$$

$$\text{at } L_5, C = 0.078 \text{ K}$$

$$\text{at } L_8, C = 0.125 \text{ K}$$



$$\Sigma M_{L_8} = 0$$

$$V_A \times 320 + T_h \times (1+2+3+4+5+6+7) \times 40 - 1 \times (320 - x) = 0$$

$$\Rightarrow V_A = \frac{320 - 1.9375x}{320}$$

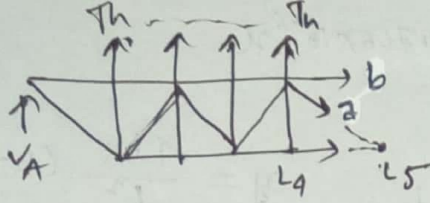
When 1 K at  $u_0$ ,  $V_A = 1$  K

$$\text{at } L_1, V_A = 0.28 \text{ K}$$

$$\text{at } L_4, V_A = 0.102 \text{ K}$$

$$\text{at } L_5, V_A = -0.4375 \text{ K}$$

IL for a & b:



1K at  $L_0$ :  $V_A = 1$ , Hence,  $a = 0$ ,  $b = 0$

1K at  $L_4$ :  $V_A = 0.28$ ,  $T_h = 0.0625$

$$\Sigma F_y = 0 \Rightarrow 0.28 + 4 \times 0.0625 - 1 - 2v = 0 \Rightarrow 2v = -0.47$$

$$\Rightarrow a = -0.47 \times \frac{\sqrt{35^2 + 40^2}}{35} = -0.714$$

~~$\Sigma F_x = 0 \Rightarrow a_x + b_x = 0 \Rightarrow a_x = -b_x = 0.714 \times \frac{40}{\sqrt{35^2 + 40^2}}$~~

~~$a_x = 0.54 \text{ K}$~~

$\Sigma M_{L_5} = 0$

$$0.28 \times 200 + 0.0625 \times (1+2+3+4) \times 40 - 1 \times 40 + b \times 35 = 0$$

$$\Rightarrow b = -1.17 \text{ K}$$

1K at  $L_5$ :  $V_A = 0.102 \text{ K}$ ,  $T_h = 0.078$

$$\Sigma F_y = 0 \Rightarrow 0.102 + 4 \times 0.078 - 2v = 0 \Rightarrow 2v = 0.414$$

$$\therefore a = 0.6287$$

$\Sigma M_{L_5} = 0$

$$0.102 \times 200 + 0.078 \times (1+2+3+4) \times 40 + b \times 35 = 0$$

$$\Rightarrow b = 1.474$$

1K at  $L_8$ :  $V_A = -0.4375$ ,  $T_h = 0.125$

$$\Sigma F_y = 0 \Rightarrow -0.4375 + 4 \times 0.125 - 2v = 0 \Rightarrow 2v = 0.0625$$

$$\therefore a = 0.095 = 0 \text{ K}$$

$\Sigma M_{L_5} = 0$

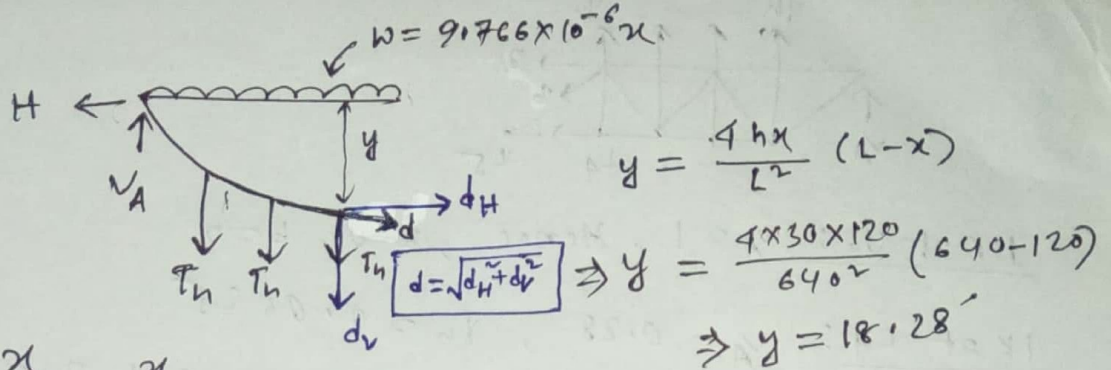
$$-0.4375 \times 200 + 0.125 \times (1+2+3+4) \times 40 + b \times 35 = 0$$

$$\Rightarrow b = 1.07 \text{ K}$$

1K at  $L_{10}$ :  $V_A = 0$ , Hence all values are zero.

Now,

IL for d:



$$\sum F_x = 0$$

$$\Rightarrow H = d_H = \frac{x}{2h} = \frac{x}{60}$$

$$\sum F_y = 0$$

$$\Rightarrow V_A - 3T_n + d_v = 0 \Rightarrow d_v = -3T_n + V_A$$

$$= -3 \times 3.9064 \times 10^{-4} x + \frac{wL}{2}$$

$$= -1.1718 \times 10^{-3} x + 9.766 \times 10^{-6} \times \frac{640}{2} x$$

$$\Rightarrow d_v = +1.9532 \times 10^{-3} x$$

$$\text{Now, } d = \sqrt{\left(\frac{x}{60}\right)^2 + (1.9532 \times 10^{-3} x)^2}$$

$$\therefore d = 0.01678 x$$

1K at  $V_A$ :  $d = 0$

1K at  $L_4$ :  $d = 2.6848$  K

1K at  $L_5$ :  $d = 3.356$  K

1K at  $L_8$ :  $d = 5.3696$  K

Due to uniform dead load of 5K/ft and live load of 1K/ft:

$$\text{Max. (-) a} = \left[ \frac{1}{2} \times (160 + 21.27) \times 0.714 \right] \times 9 = 582.42 \text{ K/ft}^2$$

$$\text{Max. (+) a} = \left[ \frac{1}{2} \times (160 - 21.27) \times 0.6287 \right] \times 9 = 392.99 \text{ K/ft}^2$$

$$\text{Max. (-) b} = \left[ \frac{1}{2} \times (160 + 17.7) \times 1.17 \right] \times 9 = 935.5905 \text{ K/ft}^2$$

$$\text{Max. (+) b} = \left[ \frac{1}{2} \times (160 - 17.7) \times 1.474 \right] \times 9 = 3066.436 \text{ K/ft}^2$$

$$\text{Max (+) c} = \left( \frac{1}{2} \times 640 \times 0.125 \right) \times 9 = 300 \text{ K/ft}^2$$

$$\text{Max (+) d} = \left( \frac{1}{2} \times 640 \times 5.3696 \right) \times 9 = 15464.448 \text{ K/ft}^2$$

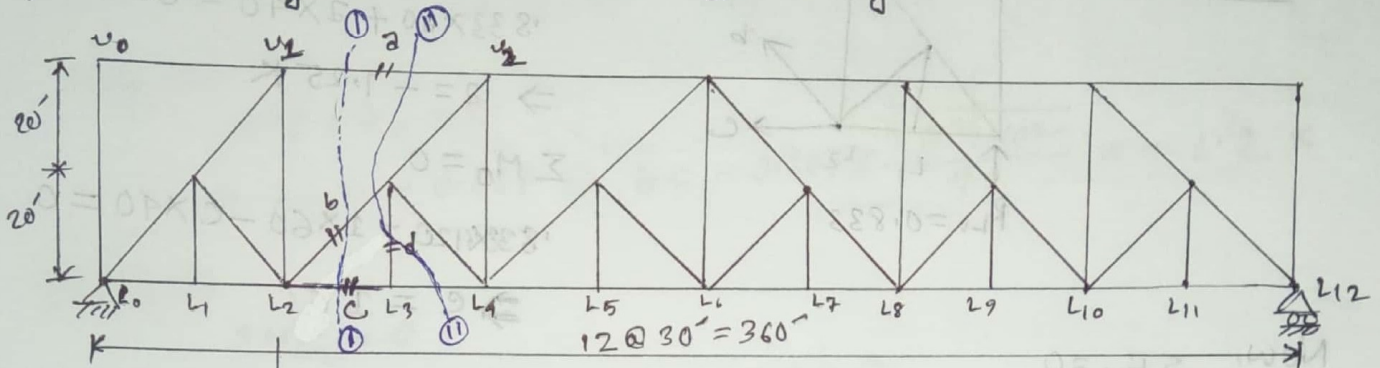
(Ans)

# Sub-divided Truss

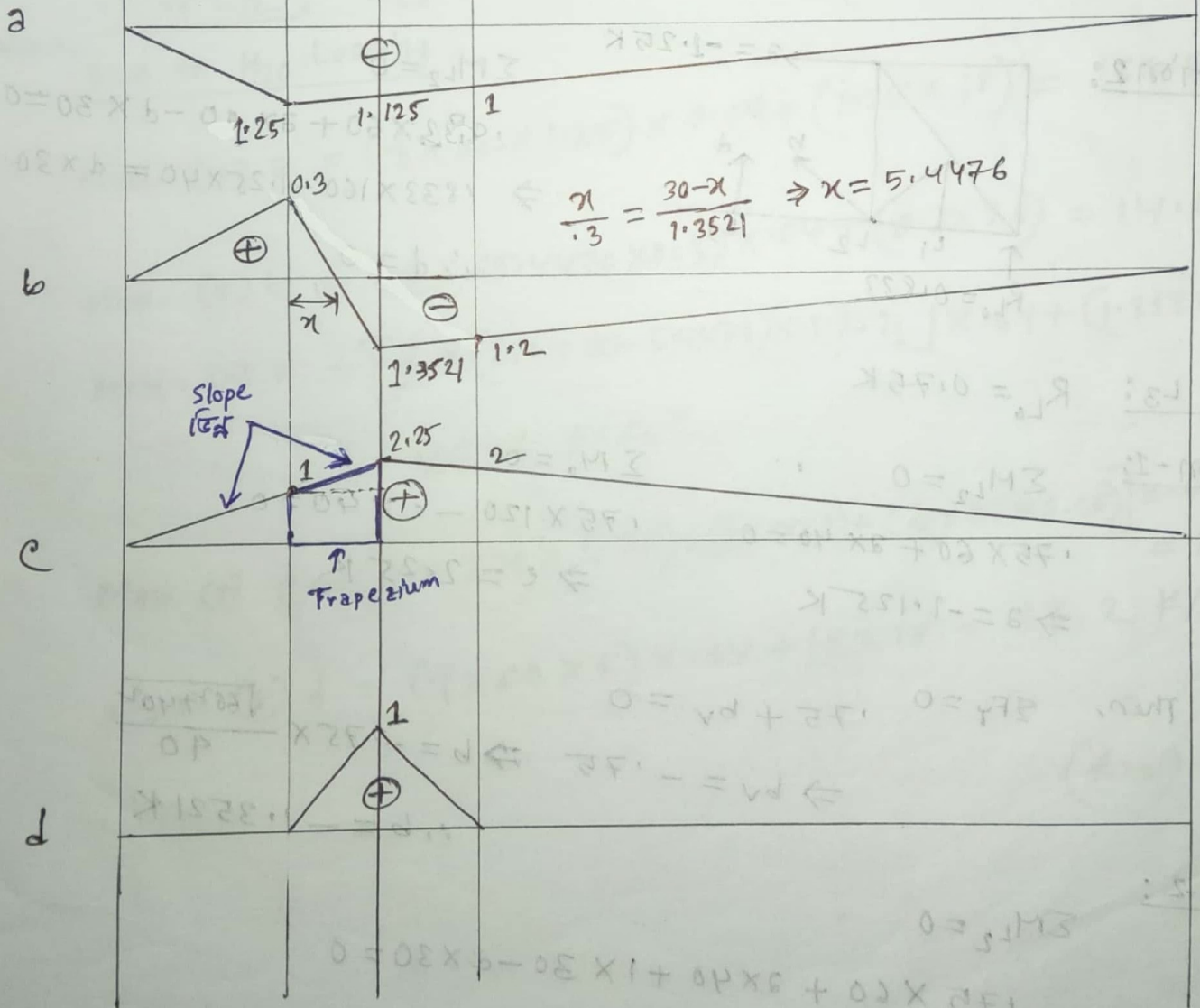
## Truss analysis due to lane loading

2012, 10

# Calculate the maximum live load stresses in member a, b, c and d of the following structure due to  $H_{20}$  loading.



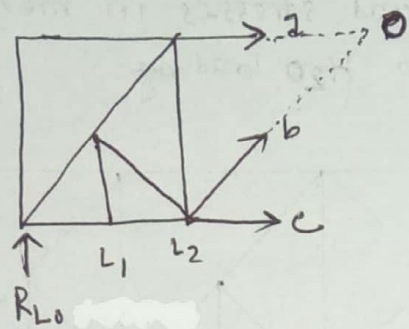
IL for



1 K at L<sub>0</sub>:  $R_{L_0} = 1 K$ , Hence,  $a=0$ ,  $b=0$ ,  $c=0$ ,  $d=0$

1 K at L<sub>2</sub>:  $R_{L_0} = 0.833 K$ ,

section-1:



$$\Sigma M_{L_2} = 0$$

$$0.833 \times 60 + a \times 40 = 0$$

$$\Rightarrow a = -1.25 K$$

$$\Sigma M_0 = 0$$

$$0.833 \times 120 - 1 \times 60 - c \times 40 = 0$$

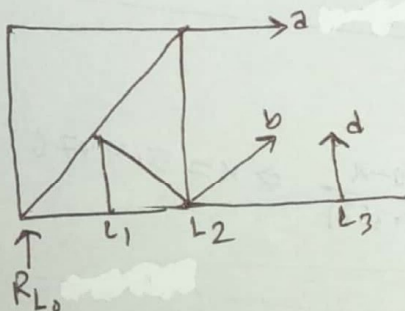
$$\Rightarrow c = 1 K$$

Now,  $\Sigma F_y = 0$

$$0.833 - 1 + b_v = 0$$

$$\Rightarrow b_v = 0.167 \quad \therefore b = 0.167 \times \frac{\sqrt{60^2 + 40^2}}{40} = 0.30 K$$

section 2:



$$\Sigma M_{L_2} = 0$$

$$R_{L_0} \times 60 + a \times 40 - d \times 30 = 0$$

$$\Rightarrow 0.833 \times 60 - 1.25 \times 40 = d \times 30$$

$$\therefore d = 0$$

1 K at L<sub>3</sub>:  $R_{L_0} = 0.75 K$

section-1:

$$\Sigma M_{L_2} = 0$$

$$0.75 \times 60 + a \times 40 = 0$$

$$\Rightarrow a = -1.125 K$$

$$\Sigma M_0 = 0$$

$$0.75 \times 120 - c \times 40 = 0$$

$$\Rightarrow c = 2.25 K$$

Then,  $\Sigma F_y = 0$   $0.75 + b_v = 0$

$$\Rightarrow b_v = -0.75 \Rightarrow b = -0.75 \times \frac{\sqrt{60^2 + 40^2}}{40}$$

$$\therefore b = -1.3521 K$$

section-2:

$$\Sigma M_{L_2} = 0$$

$$0.75 \times 60 + a \times 40 + 1 \times 30 - d \times 30 = 0$$

$$\Rightarrow d = 1 K$$

1K at  $L_1$ :  $R_L = 0.667$

section-1:  $\Sigma M_2 = 0$

$$0.667 \times 60 + a \times 40 = 0$$

$$\Rightarrow a = -1 \text{ K}$$

$$\Sigma M_0 = 0$$

$$0.667 \times 120 - c \times 40 = 0$$

$$\Rightarrow c = 2 \text{ K}$$

$$\Sigma F_y = 0$$

$$0.667 + b_v = 0$$

$$\Rightarrow b_v = -0.667 \Rightarrow b = -0.667 \times \frac{\sqrt{60^2 + 40^2}}{40} = -1.2 \text{ K}$$

section-2:

$$\Sigma M_{L_2} = 0$$

$$0.667 \times 60 + a \times 40 - d \times 30 = 0$$

$$\Rightarrow d = 0 \text{ K}$$

1K at  $L_{12}$ :  $R_L = 0$  Hence, all values are zero.

Now, due to  $H_2O$  Loading,

$$\text{Max. (+) } a = \left(\frac{1}{2} \times 360 \times 1.25\right) \times 0.64 + (1.25 \times 18) = 166.5 \text{ K/ft}^2$$

$$\text{Max. (+) } b = \left(\frac{1}{2} \times 65.4476 \times 0.3\right) \times 0.64 + (0.3 \times 26) = 14.083 \text{ K/ft}^2$$

$$\text{Max. (-) } b = \left[\frac{1}{2} \times (270 + 30 - 5.4476) \times 1.3521\right] \times 0.64 + (1.3521 \times 26) = 162.6 \text{ K/ft}^2$$

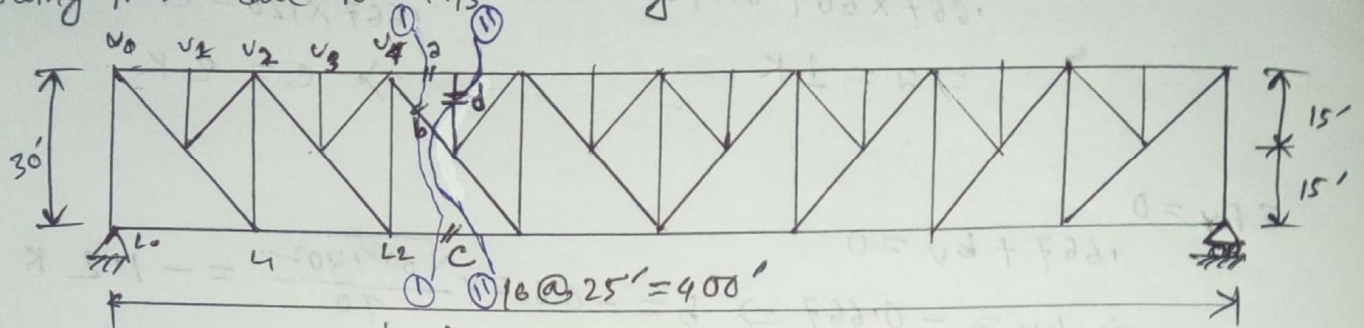
$$\text{Max (+) } c = \left[\left(\frac{1}{2} \times 270 \times 2.25\right) + \left(\frac{1}{2} \times 60 \times 1\right) + \left(\frac{1}{2} \times 30 \times 3.25\right)\right] \times 0.64 + (2.25 \times 18) = 285.3 \text{ K/ft}^2$$

$$\text{Max. (+) } d = \left(\frac{1}{2} \times 60 \times 1\right) \times 0.64 + (1 \times 18) = 37.2 \text{ K/ft}^2$$

(Ans.)

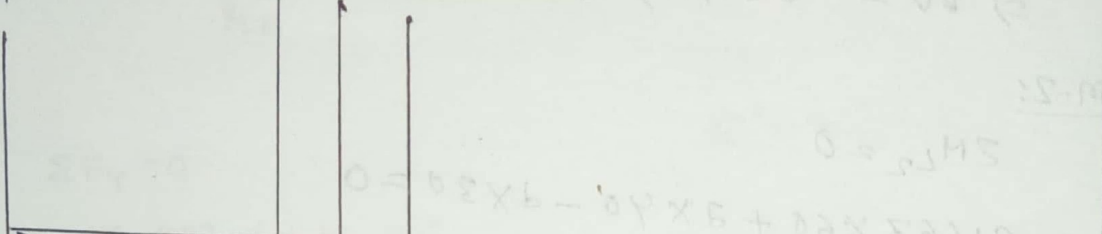
2013

# calculate the maximum stresses in the member a, b, c, and d of the following truss due to H15 loading.

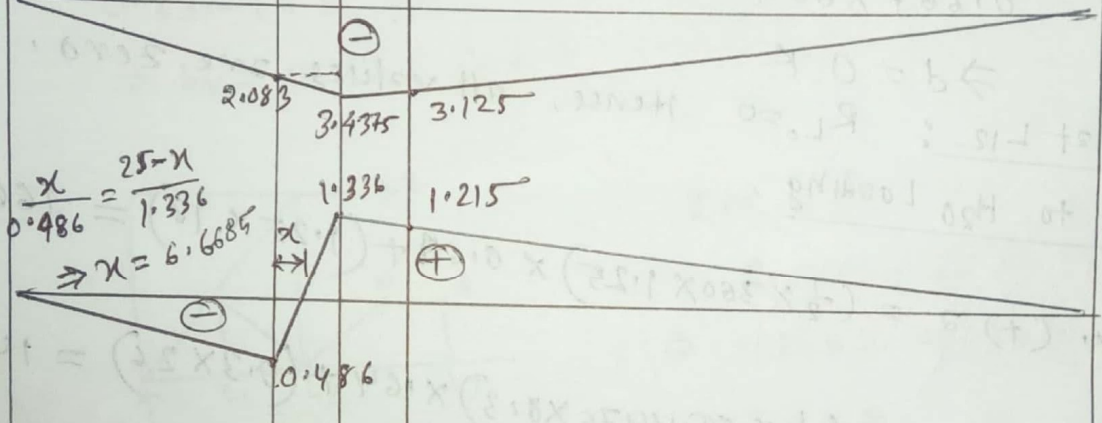


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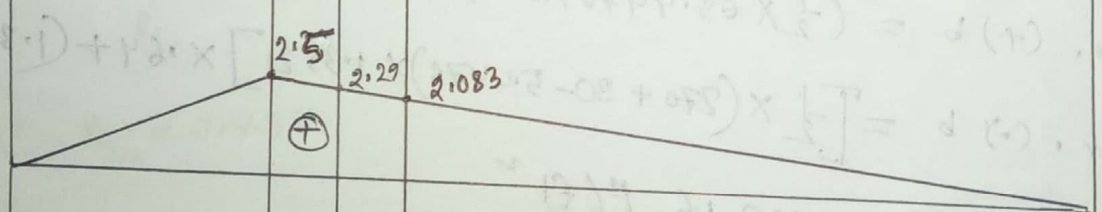
a



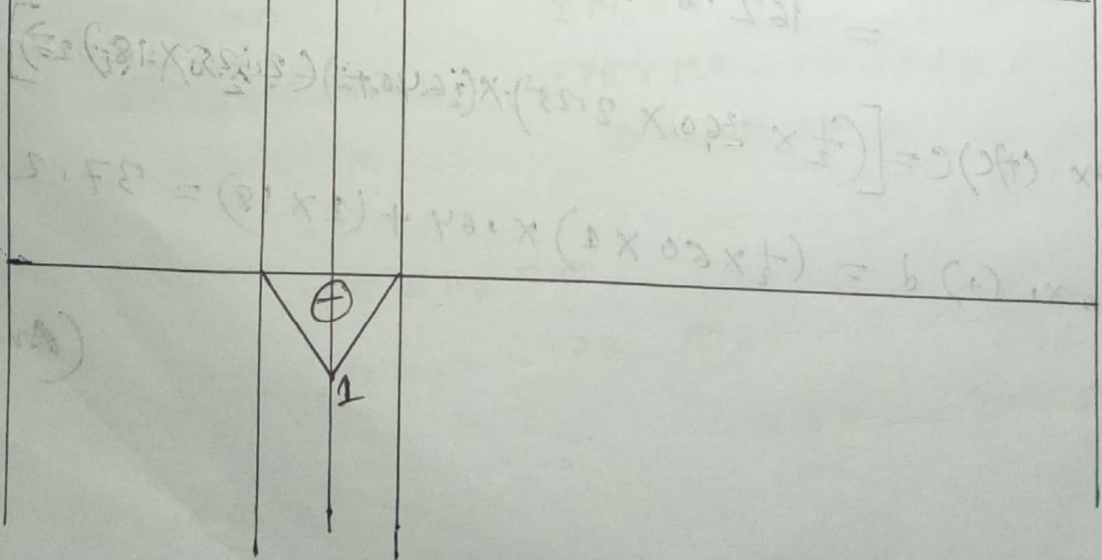
b



c



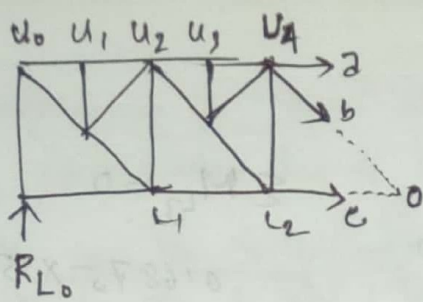
d



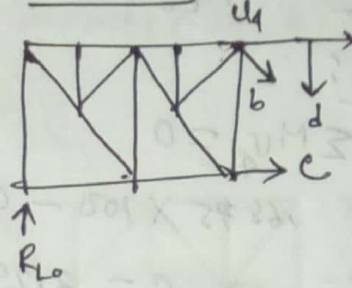
$$\frac{x}{0.486} = \frac{25-x}{1.336}$$

$$\Rightarrow x = 6.6685$$

section 1:



section-2:



1K at L0:  $R_{L0} = 1$ , Hence,  $a = 0, b = 0$   
 $c = 0, d = 0$

1K at L2:  $R_{L0} = 0.75K$

section 1:  $\sum M_{u4} = 0$

$\sum M_o = 0$

$0.75 \times 100 - c \times 30 = 0$   
 $\Rightarrow c = 2.5K$

$0.75 \times 150 - 1 \times 50 + a \times 30 = 0$   
 $\Rightarrow a = -2.083K$

$\sum F_y = 0$

$0.75 - 1 - b_v = 0$

$\Rightarrow b_v = -0.25$

$\therefore b = -0.25 \times \frac{\sqrt{36^2 + 58^2}}{30}$   
 $= -0.486K$

section 2:

$\sum M_{u4} = 0 \Rightarrow 0.75 \times 100 - 2.5 \times 30 + d \times 25 = 0$   
 $\Rightarrow d = 0K$

1K at L3:  $R_{L0} = 0.625$

section 1:

$\sum M_{u4} = 0$   
 $0.625 \times 100 - c \times 30 = 0$   
 $\therefore c = 2.083K$

$\sum M_o = 0$

$0.625 \times 150 + a \times 30 = 0$   
 $a = -3.125K$

$\sum F_y = 0$

$0.625 - b_v = 0 \Rightarrow b_v = 0.625$

$\therefore b = 1.215K$

section 2:

$\sum M_{u4} = 0$   
 $0.625 \times 100 - 2.083 \times 30 + d \times 25 = 0$   
 $\Rightarrow d = 0K$

1K at  $U_5$ :  $R_L = 0.6875 K$

Section 1:

$$\sum M_{U_4} = 0$$

$$0.6875 \times 100 - c \times 30 = 0$$

$$\Rightarrow c = 2.2917 K$$

$$\sum M_{L_3} = 0$$

$$0.6875 \times 150 + 2 \times 30 = 0$$

$$\Rightarrow 2 = -3.4375 K$$

$$\sum F_y = 0$$

$$0.6875 - b_v = 0 \Rightarrow b_v = 0.6875 \therefore b = 1.336 K$$

Section 2:

$$\sum M_{U_4} = 0$$

$$0.6875 \times 100 + 1 \times 25 - 2.2917 \times 30 + d \times 25 = 0$$

$$\Rightarrow d = -1 K$$

1K at  $L_8$ :  $R_L = 0$  Hence all values are zero.

Due to  $H_{15}$  loading:

$$\text{Max } (-) a = \left[ \left( \frac{1}{2} \times 3.4375 \times 275 \right) + \left( \frac{1}{2} \times 100 \times 2.883 \right) + \left( \frac{1}{2} \times 25 \times 5.5205 \right) \right] \times 0.64 + (3.4375 \times 13.5)$$
$$= 459.726 K/ft^2$$

$$\text{Max } (+) b = \left[ \frac{1}{2} \times (275 + 25 - 6.6685) \times 1.336 \right] \times 0.48 + (1.336 \times 19.5)$$
$$= 120.106 K/ft^2$$

$$\text{Max } (-) b = \left[ \frac{1}{2} \times (100 + 6.6685) \times 1.486 \right] \times 0.48 + (1.486 \times 19.5)$$
$$= 21.92 K/ft^2$$

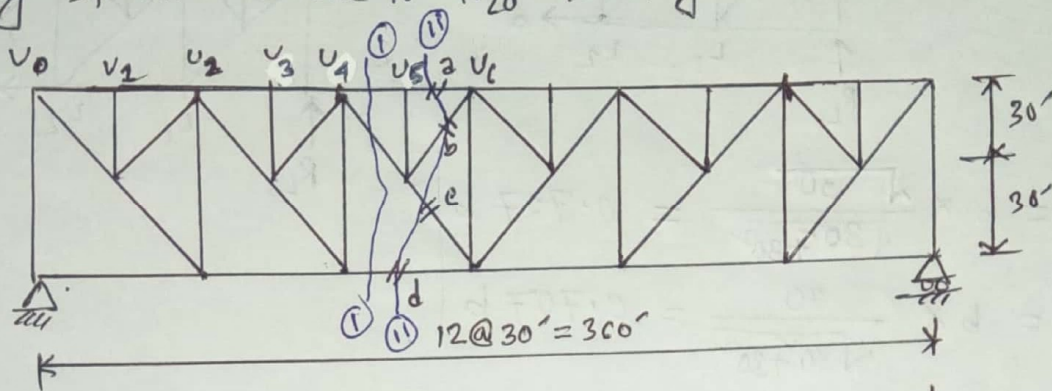
$$\text{Max } (+) c = \left( \frac{1}{2} \times 400 \times 2.5 \right) \times 0.48 + (2.5 \times 13.5)$$
$$= 273.75 K/ft^2$$

$$\text{Max } (+) d = \left( \frac{1}{2} \times 50 \times 1 \right) \times 0.48 + (1 \times 13.5)$$
$$= 25.5 K/ft^2$$

(Ans)

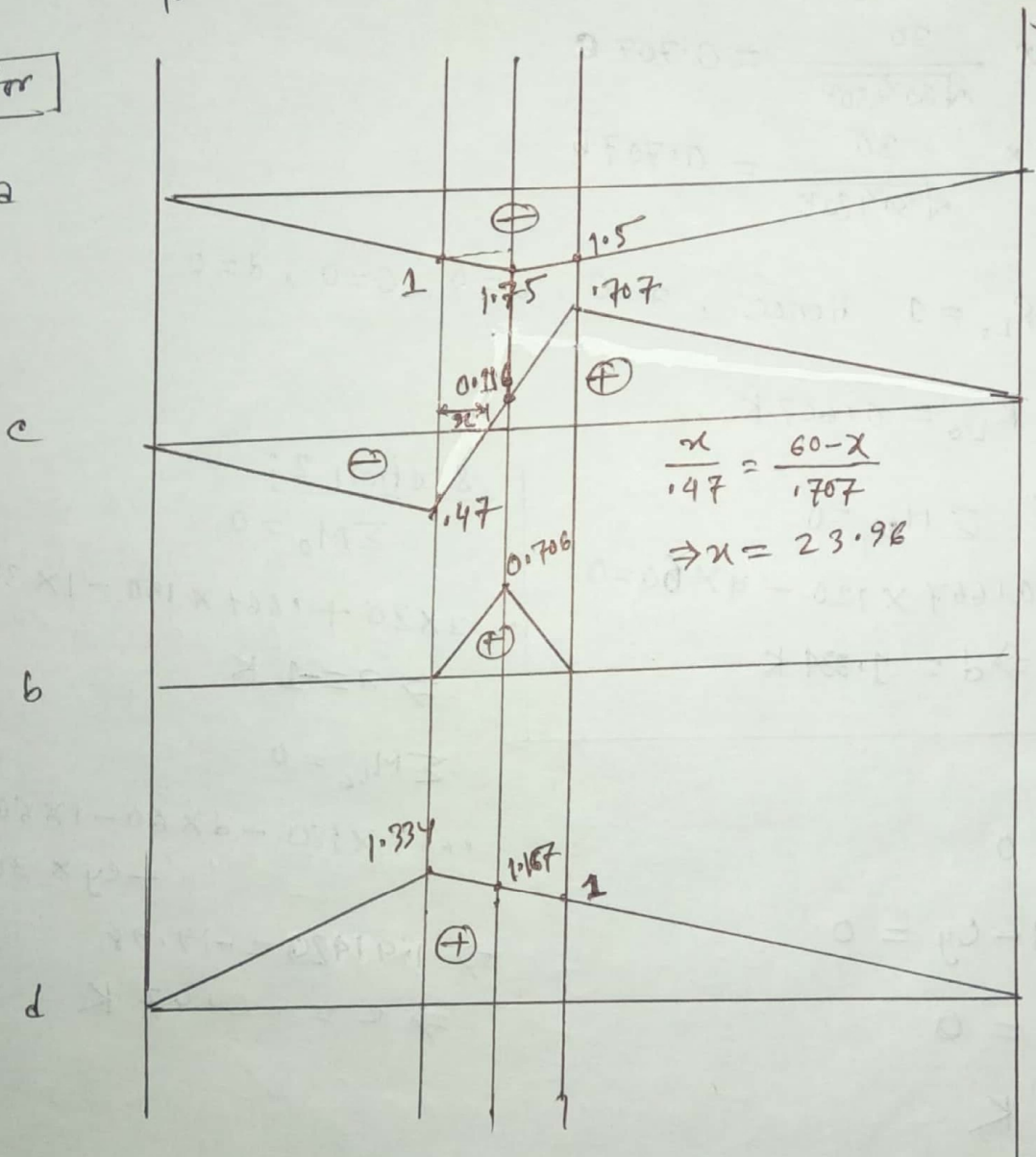
2016

# Calculate the maximum live load stress in member a, b, c and d of the following structure due to H<sub>20</sub> loading.



IL for

a



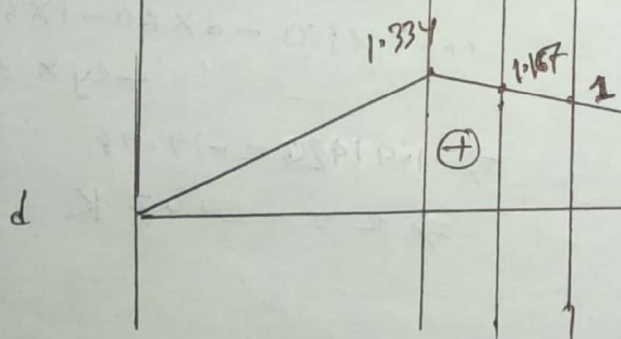
c

$$\frac{x}{1.47} = \frac{60-x}{1.707}$$

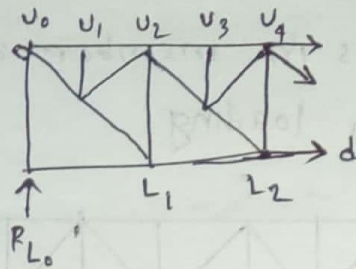
$$\Rightarrow x = 23.92$$

b

d



section 1:



$$b_x = b \times \frac{30}{\sqrt{30^2 + 30^2}} = 0.707b$$

$$b_y = b \times \frac{30}{\sqrt{30^2 + 30^2}} = 0.707b$$

$$c_x = c \times \frac{30}{\sqrt{30^2 + 30^2}} = 0.707c$$

$$c_y = c \times \frac{30}{\sqrt{30^2 + 30^2}} = 0.707c$$

1K at L0:  $R_{L0} = 1$  Hence,  $a=0, b=0, c=0, d=0$

1K at L2:  $R_{L0} = 0.667K$ ,

section 1:  $\sum M_{u4} = 0$

$$0.667 \times 120 - d \times 60 = 0$$

$$\Rightarrow d = 1.334 K$$

section 2:

$$\sum M_0 = 0$$

$$2 \times 30 + 0.667 \times 150 - 1 \times 30 - d \times 30 = 0$$

$$\Rightarrow a = -1 K$$

$$\sum M_{u6} = 0$$

$$0.667 \times 180 - d \times 60 - 1 \times 60 - c_x \times 30 - c_y \times 30 = 0$$

$$\Rightarrow 42.42c = -19.98$$

$$\Rightarrow c = -0.47 K$$

Now,  $\sum F_y = 0$

$$0.667 + b_y - 1 - c_y = 0$$

$$\Rightarrow 0.707b = 0$$

$$\therefore b = 0 K$$

1 K at L3:  $R_{L0} = 0.5$

section 1:  $\sum M_{u4} = 0$

$$0.5 \times 120 - d \times 60 = 0 \Rightarrow d = 1 K$$

section 2:

$$\sum M_o = 0$$

$$2 \times 30 + 1.5 \times 150 - d \times 30 = 0$$

$$\Rightarrow d = -1.5 \text{ K}$$

$$\sum F_y = 0$$

$$1.5 + b_y - c_y = 0$$

$$\Rightarrow b_y = 0 \quad \therefore b = 0 \text{ K}$$

$$\sum M_{u_6} = 0$$

$$1.5 \times 180 - d \times 60 - c_x \times 30 - c_y \times 30 = 0$$

$$\Rightarrow 42.42c = 30$$

$$\Rightarrow c = 0.707 \text{ K}$$

1 K at  $U_5$ :  $R_{L_0} = 0.583$

section 1:

$$\sum M_{u_9} = 0$$

$$0.583 \times 120 - d \times 60$$

$$\Rightarrow d = 1.167 \text{ K}$$

section 2:

$$\sum M_o = 0$$

$$2 \times 30 + 0.583 \times 150 - d \times 30 = 0$$

$$\Rightarrow d = -1.75 \text{ K}$$

$$\sum M_{u_6} = 0$$

$$1.583 \times 180 - 1 \times 30 - d \times 60 - c_x \times 30 - c_y \times 30 = 0$$

$$\Rightarrow 42.42c = 4.92$$

$$\Rightarrow c = 0.116 \text{ K}$$

Now,

$$\sum F_y = 0$$

$$0.583 + b_y - 1 - c_y = 0$$

$$\Rightarrow b_y = 0.5$$

$$\therefore b = 0.706 \text{ K}$$

1 K at  $L_6$ :  $R_{L_0} = 0$  Hence, all values are zero.

Due to  $H_2O$  loading:

$$\text{Max } (-) a = \left[ \left( \frac{1}{2} \times 210 \times 1.75 \right) + \left( \frac{1}{2} \times 120 \times 1 \right) + \left( \frac{1}{2} \times 30 \times 2.75 \right) \right] \times 0.64 + (2.75 \times 18) = 231.9 \text{ K/ft}^2$$

$$\text{Max } (+) b = \left( \frac{1}{2} \times 0.706 \times 60 \right) + (0.706 \times 18) = 33.89 \text{ K/ft}^2$$

$$\text{Max } (+) d = \left( \frac{1}{2} \times 1.334 \times 360 \right) \times 0.64 + (1.334 \times 18) = 177.69 \text{ K/ft}^2$$

$$\text{Max. } \ominus C = \left[ \frac{1}{2} \times (120 + 23.96) \times 0.47 \right] \times 0.64 + (0.47 \times 26)$$

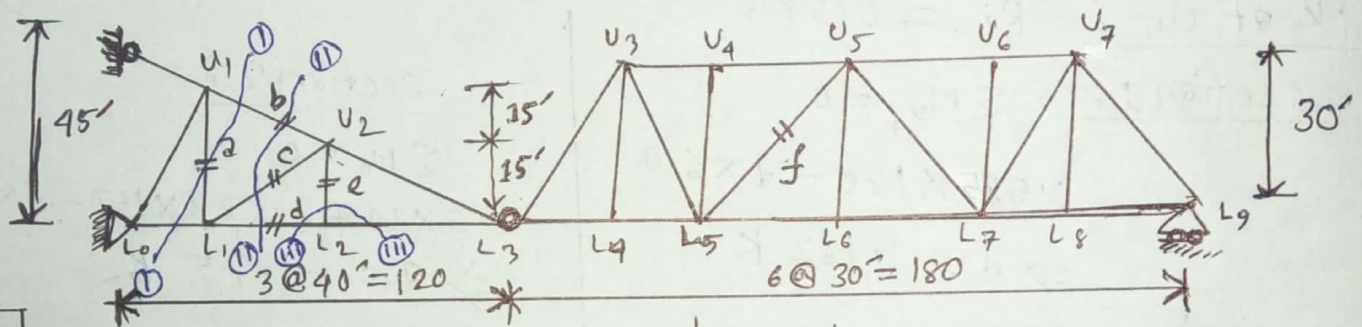
$$= 33.87 \text{ K/ft}^2$$

$$\text{Max } (+) C = \left[ \frac{1}{2} \times (180 + 60 - 23.96) \times 0.707 \right] \times 0.64 + (0.707 \times 26)$$

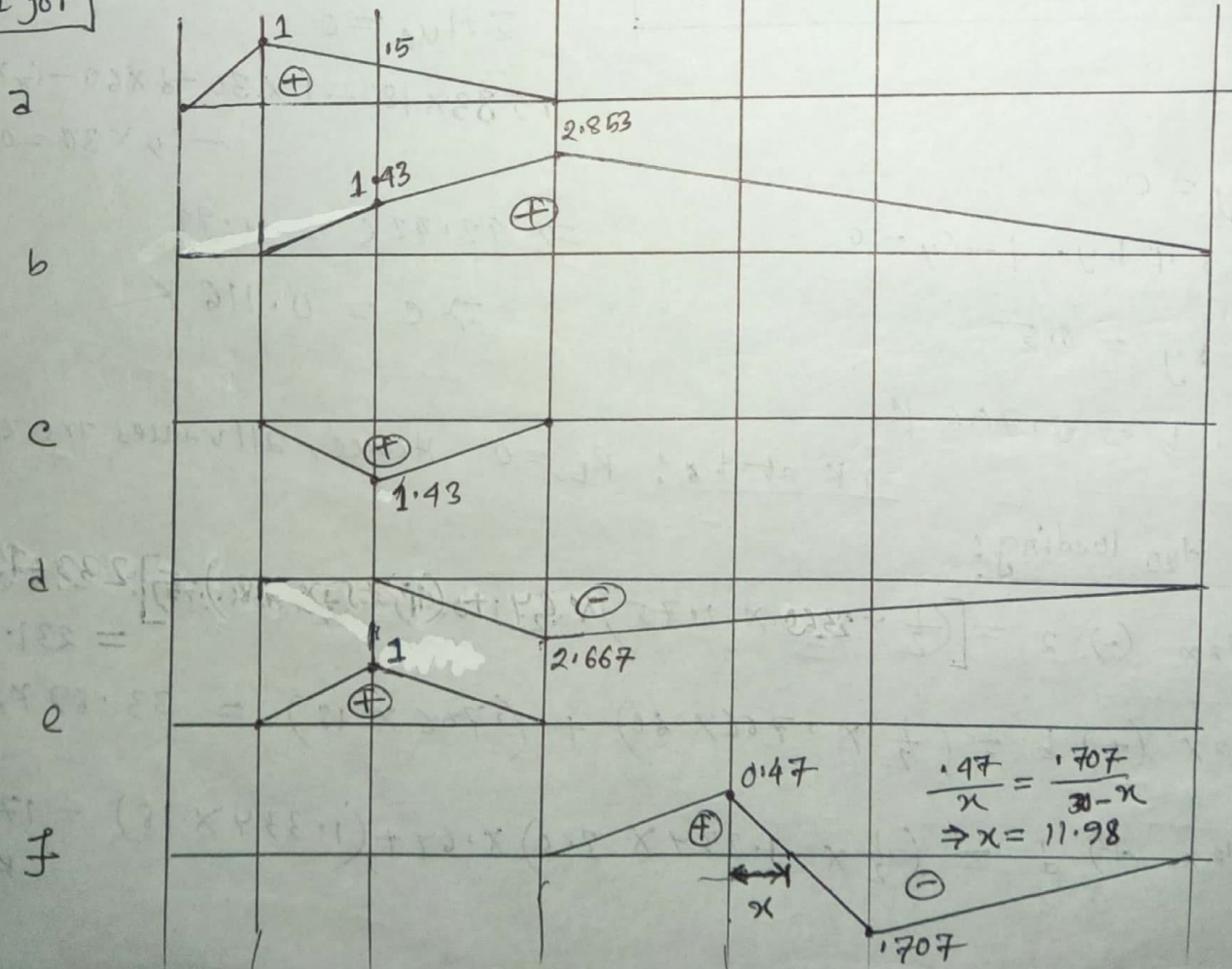
$$= 67.26 \text{ K/ft}^2$$

(Ans.)

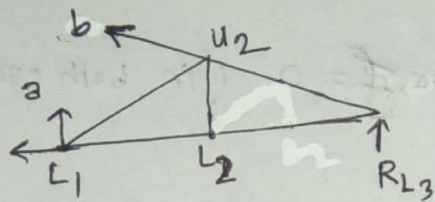
# calculate the maximum stresses in member a, b, c, d, e and f of the following compound truss due to H20 loading.



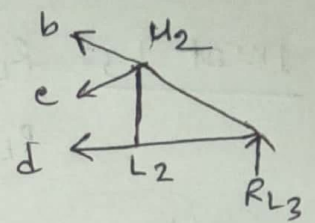
IL for



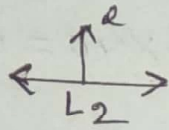
section 1:



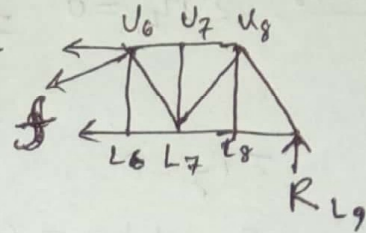
section 2:



section 3:



section 4:



$$f_x = f \times \frac{30}{\sqrt{30^2 + 30^2}} = 0.707f$$

$$f_y = 0.707f$$

$$b_x = b \times \frac{40}{\sqrt{40^2 + 15^2}} = 0.936b$$

$$b_y = b \times \frac{15}{\sqrt{40^2 + 15^2}} = 0.35b$$

$$c_x = c \times \frac{40}{\sqrt{40^2 + 15^2}} = 0.936c$$

$$c_y = c \times \frac{15}{\sqrt{40^2 + 15^2}} = 0.35c$$

1K at L0:  $R_{L3} = 0$  Hence,  $a = 0$ ,  $b = 0$ ,  $c = 0$ ,  $d = 0$ ,  $e = 0$ ,  $f = 0$

1K at L1:  $R_{L3} = 0$ ,  $a = 1$ ,  $b = 0$ ,  $c = 0$ ,  $d = 0$ ,  $e = 0$ ,  $f = 0$

1K at L2:  $R_{L3} = 0$ ,  $f = 0$

section 1:  $\sum M_{L1} = 0 \Rightarrow b_x \times 15 + b_y \times 40 - 1 \times 40 = 0$

$$\Rightarrow 28.04b = 40$$

$$\therefore b = 1.43K$$

$$\sum F_y = 0$$

$$a - 1 + b_y = 0$$

$$\Rightarrow a = 0.15K$$

section-2:  $\sum M_{U2} = 0 \Rightarrow d \times 15 - R_{L3} \times 40 = 0 \therefore d = 0$

$$\sum F_y = 0 \Rightarrow b_y - c_y - 1 = 0 \Rightarrow c_y = -0.15 \therefore c = -1.43K$$

section 3:  $\sum F_y = 0 \Rightarrow e - 1 = 0 \therefore e = 1$

~~section 4~~

1K at L3:  $R_{L3} = 0K$  (just left)  
 $R_{L3} = 1K$  (just Right)

$f = 0$  (in both case)

section 1:  $\sum M_{L1} = 0 \Rightarrow 1 \times 80 - b_x \times 15 - b_y \times 40 = 0$   
 $\Rightarrow b = 2.853 K$   
 $\sum F_y = 0$   
 $a - 1 + b_y = 0 \Rightarrow a = 0 K$

section 2:  $\sum M_{u2} = 0 \Rightarrow d \times 15 + 1 \times 40 = 0 \Rightarrow d = -2.667 K$

$\sum F_y = 0$   
 $b_y - c_y - 1 = 0 \Rightarrow c = 0 K$

section 3:  $e = 0$

1K at L5:  $R_{L9} = \frac{2}{6} = 0.333 K$

section 4:  $\sum F_y = 0 \Rightarrow f_y - R_{L9} = 0 \Rightarrow f_y = 0.333 K$   
 $\therefore f = 0.47 K$

1K at L6:  $R_{L9} = 0.5 K$

section 4:  $\sum F_y = 0 \Rightarrow f_y - 0.5 + 1 \Rightarrow f_y = -0.5$   
 $\therefore f = -0.707$

1K at L9:  $R_{L9} = 1K$  all values are zero.

~~max~~ Due to  $H_2O$  loading,

$$\text{Max. (+) a} = \left(\frac{1}{2} \times 120 \times 1\right) \times 0.64 + (1 \times 18) = 56.4 \text{ K/ft}^2$$

$$\text{Max. (+) b} = \left(\frac{1}{2} \times 2.853 \times 260\right) \times 0.64 + (2.853 \times 18) = 288.7236 \text{ K/ft}^2$$

$$\text{Max. (-) c} = \left(\frac{1}{2} \times 80 \times 1.43\right) \times 0.64 + (1.43 \times 18) = 62.348 \text{ K/ft}^2$$

$$\text{Max. (-) d} = \left(\frac{1}{2} \times 220 \times 2.667\right) \times 0.64 + (2.667 \times 18) = 235.7628 \text{ K/ft}^2$$

$$\text{Max. (+) e} = \left(\frac{1}{2} \times 1 \times 80\right) \times 0.64 + (1 \times 18) = 43.6$$

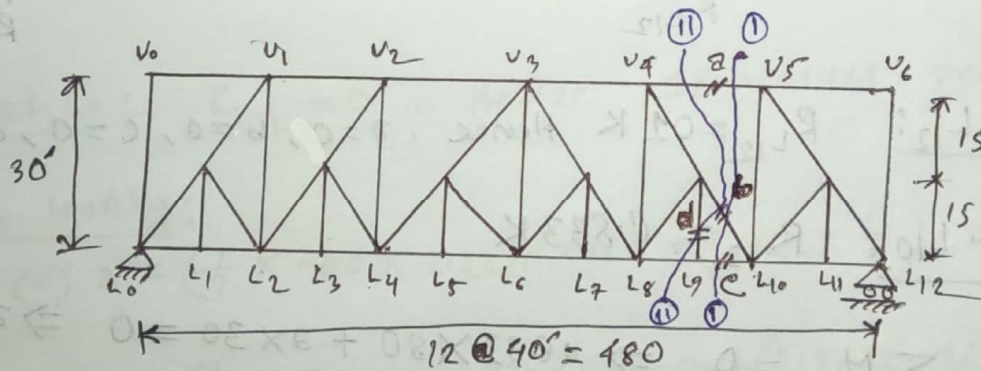
$$\begin{aligned} \text{Max. (+) f} &= \left[\frac{1}{2} \times (80 + 71.98) \times 0.47\right] \times 0.64 + (0.47 \times 26) \\ &= 18.534 \text{ K/ft}^2 \end{aligned}$$

$$\begin{aligned} \text{Max. (-) f} &= \left[\frac{1}{2} \times (90 + 30 - 11.98) \times 0.707\right] \times 0.64 + (0.707 \times 26) \\ &= 42.82 \text{ K/ft}^2 \end{aligned}$$

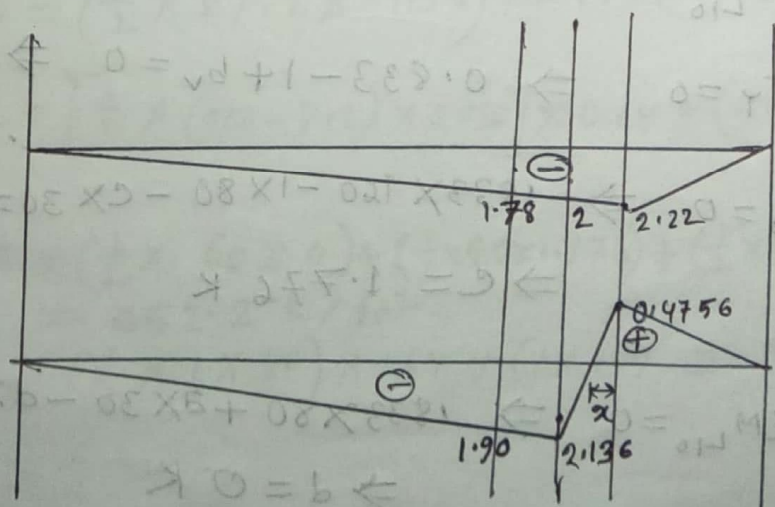
(Ans.)

2015

# calculate the maximum stresses in member a, b, c and d of the following truss due to  $H_2O$  loading.

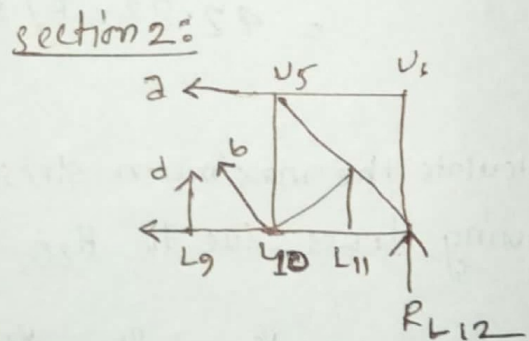
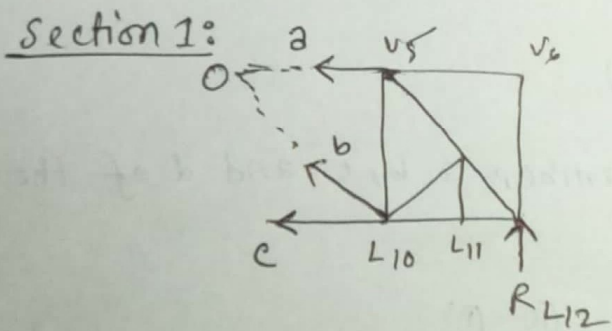
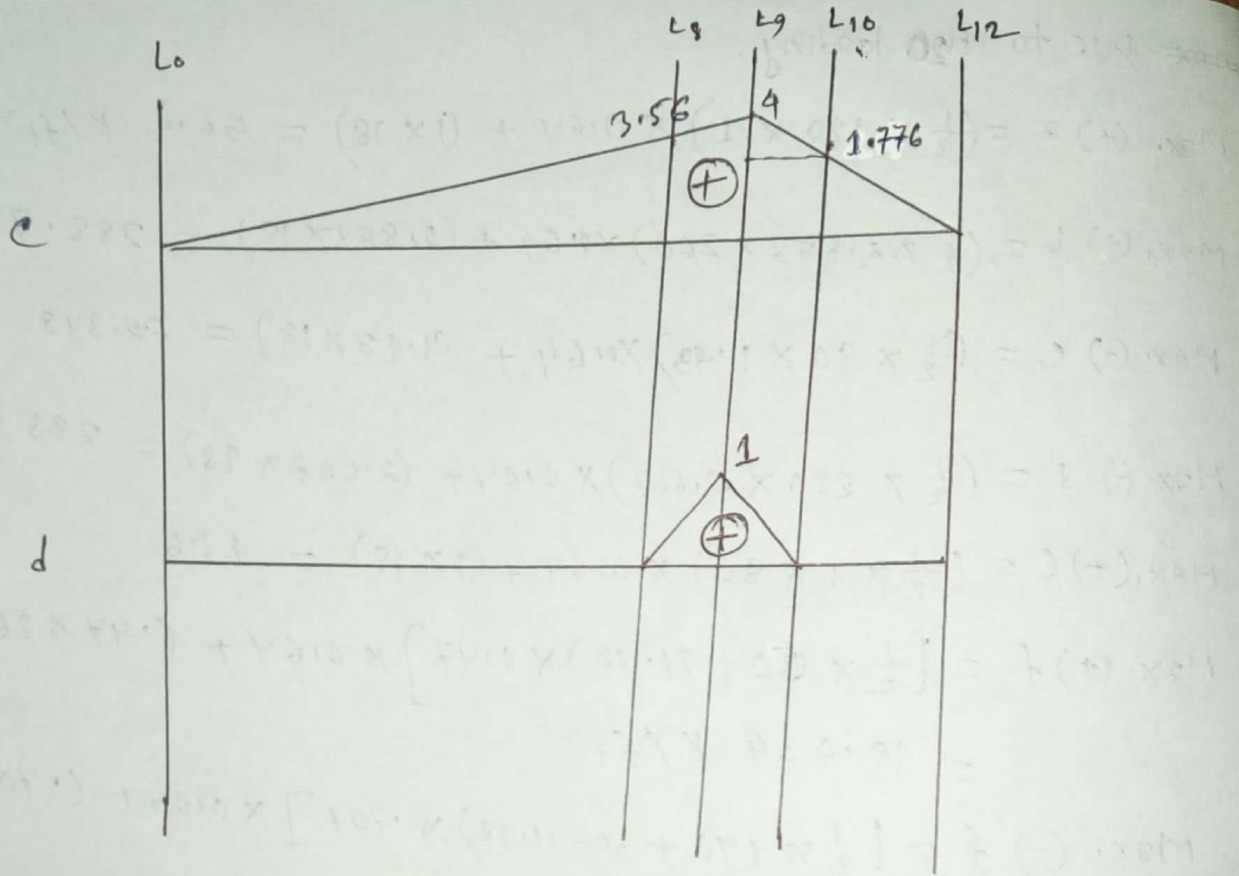


IL for



$$\frac{1.4756}{x} = \frac{2.136}{40-x}$$

$$\Rightarrow x = 7.2'$$



When 1 K at L12:  $R_{L12} = 1 \text{ K}$  Hence,  $a = 0, b = 0, c = 0, d = 0$

When 1 K at L10:  $R_{L12} = 0.833 \text{ K}$

Section 1:  $\sum M_{L10} = 0 \Rightarrow 0.833 \times 80 + a \times 30 = 0 \Rightarrow a = -2.22 \text{ K}$

$$\sum F_y = 0 \Rightarrow 0.833 - 1 + b_v = 0 \Rightarrow b = 0.167 \times \frac{\sqrt{30^2 + 80^2}}{30}$$

$$\therefore b = 0.4756 \text{ K}$$

$$\sum M_o = 0 \Rightarrow 0.833 \times 160 - 1 \times 80 - c \times 30 = 0$$

$$\Rightarrow c = 1.776 \text{ K}$$

Section 2:  $\sum M_{L10} = 0 \Rightarrow 0.833 \times 80 + a \times 30 - d \times 30 = 0$

$$\Rightarrow d = 0 \text{ K}$$

When 1K at  $L_9$ :  $R_{L12} = 0.75 \text{ K}$

section 1:  $\sum M_{L10} = 0 \Rightarrow 0.75 \times 80 + 2 \times 30 = 0 \Rightarrow a = -2 \text{ K}$

$$\sum F_y = 0 \Rightarrow 0.75 + b_v = 0 \Rightarrow b = -2.136 \text{ K}$$

$$\sum M_0 = 0 \Rightarrow 0.75 \times 160 - c \times 30 = 0 \Rightarrow c = 4 \text{ K}$$

section 2:  $\sum M_{L0} = 0 \Rightarrow 0.75 \times 80 + 2 \times 30 - d \times 30 = 0$   
 $\Rightarrow d = 1 \text{ K}$

When 1K at  $L_8$ :  $R_{12} = 0.667 \text{ K}$

section 1:  $\sum M_{L10} = 0 \Rightarrow 0.667 \times 80 + 2 \times 30 = 0 \Rightarrow a = -1.78 \text{ K}$

$$\sum F_y = 0 \Rightarrow 0.667 + b_v = 0 \Rightarrow b = -1.90 \text{ K}$$

$$\sum M_0 = 0 \Rightarrow 0.667 \times 160 - c \times 30 = 0 \Rightarrow c = 3.56 \text{ K}$$

section 2:  $\sum M_{L10} = 0 \Rightarrow 0.667 \times 80 + 2 \times 30 - d \times 30 = 0$   
 $\Rightarrow d = 0 \text{ K}$

When 1K at  $L_0$ :  $R_{L12} = 0$ , Hence all values are zero.

Due to Heo loading:

$$\text{Max. (+) } a = \left(\frac{1}{2} \times 480 \times 2.22\right) \times 0.64 + (2.22 \times 18) = 380.952 \text{ K/ft}^2$$

$$\text{Max. (+) } b = \left(\frac{1}{2} \times 87.2 \times 0.4756\right) \times 0.64 + (0.4756 \times 26) = 25.64 \text{ K/ft}^2$$

$$\text{Max. (-) } b = \left[\frac{1}{2} \times (400 - 71.2) \times 2.136\right] \times 0.64 + (2.136 \times 26) = 324.02 \text{ K/ft}^2$$

$$\text{Max (+) } c = \left(\frac{1}{2} \times 60 \times 4\right) + \left(\frac{1}{2} \times 80 \times 1.776\right) + \left(\frac{1}{2} \times 40 \times 5.776\right) \times 0.64 + (4 \times 18) = 652.2 \text{ K/ft}^2$$

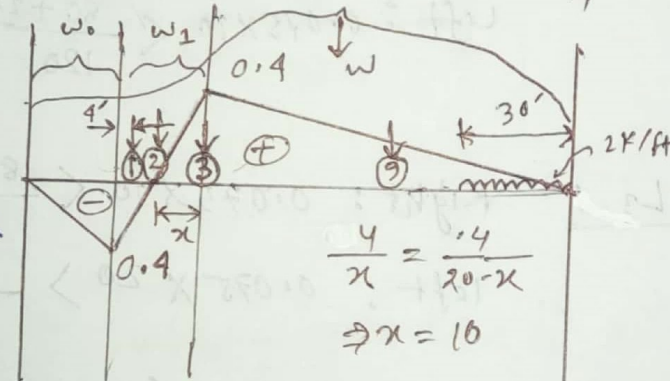
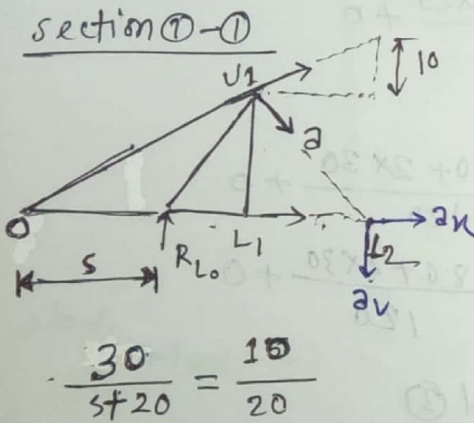
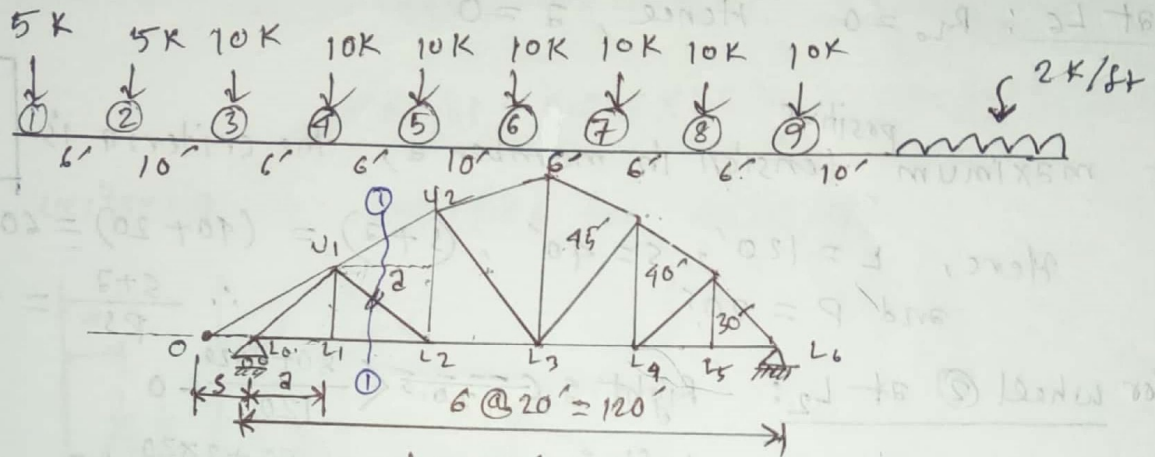
$$\text{Max (+) } d = \left(\frac{1}{2} \times 1 \times 80\right) \times 0.64 + (1 \times 18) = 43.6 \text{ K/ft}^2$$

(Ans)

# Non-Parallel Chord Truss

2014

# Calculate the maximum positive and negative stresses in the member 'a' of the truss due to series of moving loads, shown in figure below:



$$a_x = \frac{20 \cdot a}{\sqrt{30^2 + 20^2}} = 0.555 \cdot a$$

$$a_v = \frac{30 \cdot a}{\sqrt{30^2 + 20^2}} = 0.832 \cdot a$$

$$\frac{30}{5+20} = \frac{10}{20}$$

$$\Rightarrow s = 40$$

When 1K at  $L_0$ :  $R_{L_0} = 1K$  Hence,  $a = 0$

$$1K \text{ at } L_1: R_{L_0} = \frac{5}{6} = 0.833 K$$

Now,  $\sum M_0 = 0$

$$a_v \times (40 + 20) - 0.833 \times 40 + 1 \times 60 + 2x \times 30$$

$$\Rightarrow (0.832 \times 60) a + (0.555 \times 30) a = -26.68$$

$$\Rightarrow a = -0.40 K$$

or,  $\sum M_0 = 0$

$$0.833 \times 40 - 1 \times 60 - a_v \times 80 = 0$$

$$\Rightarrow a = -0.40 K$$

$$\underline{1K \text{ at } L_2: R_L = \frac{4}{6} = 0.667 K}$$

$$\Sigma M_o = 0$$

$$0.667 \times 40 - a_v \times 80 = 0 \Rightarrow a = 0.40 K$$

$$\underline{1K \text{ at } L_6: R_L = 0 \quad \text{Hence, } a = 0}$$

For maximum <sup>positive</sup> tension in member a, the criteria is

$$\frac{W}{L} + \frac{W_o}{S} = \frac{W_1}{P} \times \frac{S+a}{S}$$

$$\text{Here, } L = 120', S = 40', (S+a) = (40+20) = 60$$

$$\text{and } P = 20'$$

$$\therefore \frac{S+a}{PS} = \frac{60}{20 \times 40} = 0.075$$

$$\underline{\text{Wheel (2) at } L_2: \text{ Right: } 0.075 \times 5 < \frac{80+2 \times 20}{120} + 0}$$

$$\text{Left: } 0.075 \times 10 < \frac{80+2 \times 20}{120} + 0$$

$$\underline{\text{Wheel (3) at } L_3: \text{ Right: } 0.075 \times 10 < \frac{80+2 \times 30}{120} + 0}$$

$$\text{Left: } 0.075 \times 20 > \frac{80+2 \times 30}{120} + 0$$

Hence, criteria is satisfied for wheel (3)

$$\text{Max (+) } a = \frac{-0.4}{10} \times (5 \times 0 + 5 \times 6) + \frac{.4}{10} \times (10 \times 10) + \frac{.4}{80} \times \left[ (30 \times \frac{1}{2} \times 30) \times 2 \right.$$

$$\left. + 10 \times (40 + 46 + 52 + 58 + 68 + 74) \right]$$

$$= (-1.2 + 4 + 21.4) = 24.2 \text{ K/ft}$$

Now, for maximum negative tension, the criteria is,

$$\frac{W}{L} - \frac{W_o}{S+L} = \frac{W_1}{P} \times \frac{(S+a+P)}{(S+L)}$$

$$\text{Here, } S = 40', L = 120', a = 20', P = 20' \therefore (S+L) = 160'$$

$$\therefore \frac{S+a+P}{(S+L)P} = \frac{80}{160 \times 20} = \frac{0.5}{20} = 0.025$$

When wheel ⑧ is at  $L_1$ :

$$\text{Left: } 0.025 \times 10 + 1 < \frac{40 + 84 \times 2}{120} - \frac{80 \times 2}{160}$$

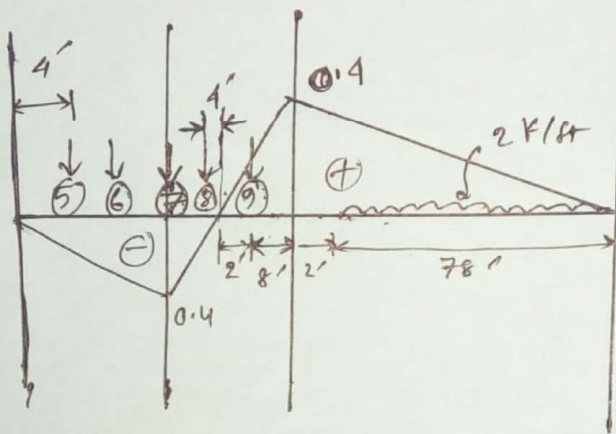
$$\text{Right: } 0.025 \times 28 < \frac{40 + 84 \times 2}{120} - \frac{80 \times 2}{160}$$

When wheel ⑦ is at  $L_1$ :

$$\text{Left: } 0.025 \times 20 < \frac{50 + 78 \times 2}{120} - \frac{78 \times 2}{160}$$

$$\text{Right: } 0.025 \times 30 > \frac{50 + 78 \times 2}{120} - \frac{78 \times 2}{160}$$

Hence, criteria is satisfied for wheel-⑦



Now,

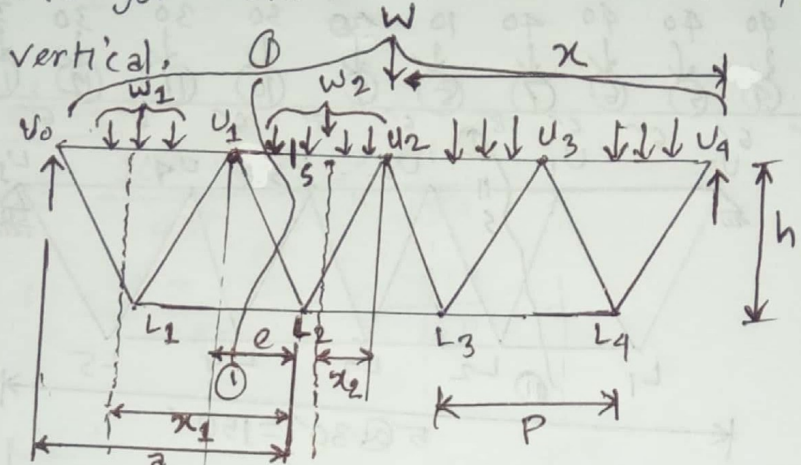
$$\begin{aligned} \text{Max. } (-) \text{ } z &= -\left(\frac{0.4}{80} \times 78\right) \times \frac{1}{2} \times 78 \times 2 - \frac{0.4}{10} \times (9 \times 2) \\ &+ \frac{0.4}{10} \times (10 \times 4 + 10 \times 10) + \frac{0.4}{20} \times (10 \times 4 + 10 \times 14) \\ &= -30.42 - 0.72 + 5.6 + 3.6 \\ &= -21.94 \text{ K} \end{aligned}$$

(Ans.)

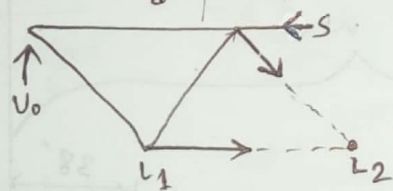
17, 16, 15, 14, 12, 11, 10

Truss without Vertical Chord

# Deduce the criteria for maximum stress in the upper chord of truss without vertical.



section - 1:



Before movement,

$$R_{U01} = \frac{Wx}{L}$$

$$\boxed{\sum M_{L2} = 0}$$

$$\Rightarrow R_{U01} \times a - w_1 \times x_1 - \frac{w_2 x_2}{P} \times e - S_1 \times h = 0$$

$$\Rightarrow S_1 = \frac{Wx}{L} \times \frac{a}{h} - \frac{w_1 x_1}{h} - \frac{w_2 x_2}{2h} \quad [\text{if } e = \frac{P}{2}]$$

..... (i)

After movement,

$$R_{U02} = \frac{W(x+dx)}{L}$$

$$\boxed{\sum M_{L2} = 0} \Rightarrow \frac{W(x+dx)}{L} \times a - w_1(x_1+dx) - \frac{w_2(x_2+dx)}{P} \times e - S_2 \times h = 0$$

$$\therefore S_2 = \frac{W(x+dx)}{L} \times \frac{a}{h} - \frac{w_1(x_1+dx)}{h} - \frac{w_2(x_2+dx)}{2h} \quad \dots \dots (ii)$$

From (i) & (ii),

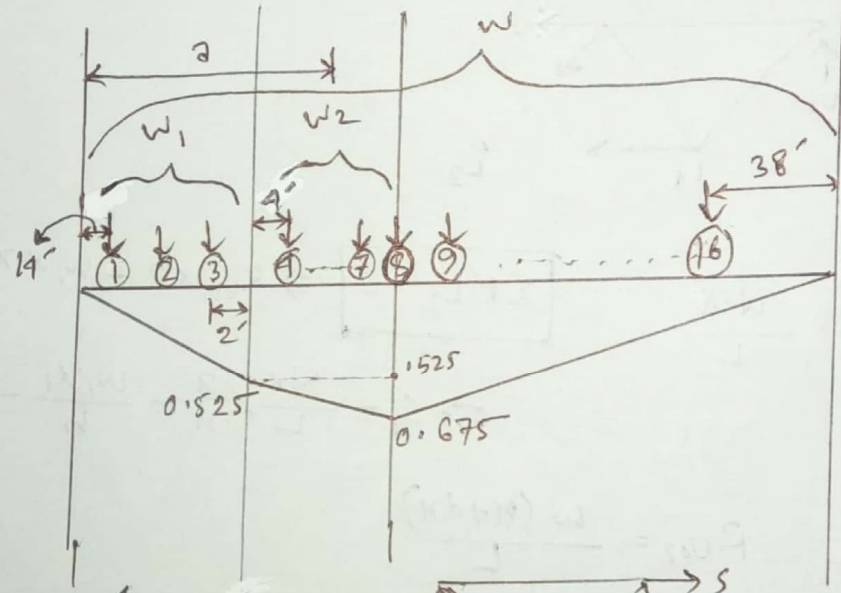
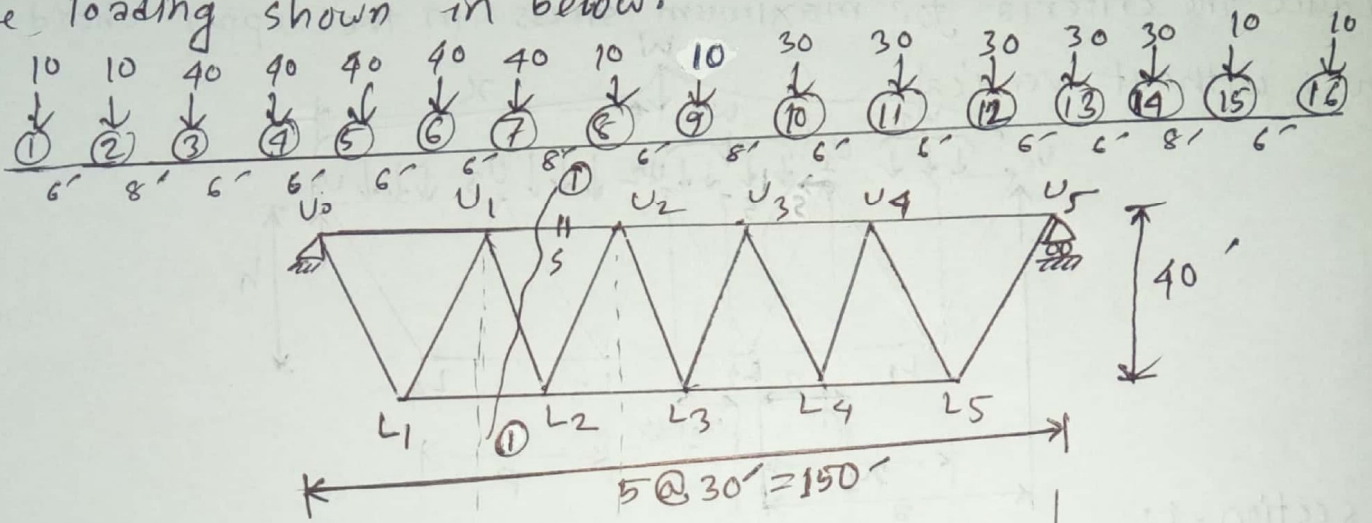
$$ds = S_2 - S_1 = \frac{w_2 dx}{Lh} - \frac{w_1 dx}{h} - \frac{w_2 dx}{2h}$$

$$\therefore \frac{ds}{dx} = \frac{w_2}{Lh} - \frac{w_1}{h} - \frac{w_2}{2h} = 0$$

$$\boxed{\frac{W}{L} = \frac{w_1 + \frac{w_2}{2}}{2}}$$

2014, 11

# Find out the maximum stress of the same member due to the loading shown in below.



IL for

1 K at  $U_0$   
 $R_{U_0} = 1 \text{ K}$   
 $S = 0$

1 K at  $U_4$   $R_{U_0} = \frac{4}{5} = 0.8 \text{ K}$

Section 1-1:

$$\sum M_{L_2} = 0 \Rightarrow 0.8 \times (30 + 15) - 1 \times 15 + S \times 40 = 0$$

$$\Rightarrow S = -0.525 \text{ K}$$

1 K at  $U_2$ :

$$\sum M_{L_2} = 0 \Rightarrow 1.8 \times (30 + 15) + S \times 40 = 0$$

$$\Rightarrow S = 0.675 \text{ K}$$

1 K at  $U_5$ ;  $R_{U_0} = 0$  Hence  $S = 0$

we know the criteria's for with out vertical chord,

$$\frac{w}{L} \leq \frac{w_1 + \frac{w_2}{2}}{a} \quad \text{Here, } L=150, a=45$$

When wheel ⑥ at  $U_2$ :

$$\text{Right: } \frac{10 + \frac{130}{2}}{45} < \frac{410}{150}$$

$$\text{Left: } \frac{10 + \frac{170}{2}}{45} < \frac{410}{150}$$

Wheel ⑦ at  $U_2$ :

$$\text{Right: } \frac{20 + \frac{160}{2}}{45} < \frac{410}{150}$$

$$\text{Left: } \frac{20 + \frac{200}{2}}{45} < \frac{410}{150}$$

wheel ⑧ at  $U_2$ :

$$\text{Right: } \frac{60 + \frac{180}{2}}{45} > \frac{410}{150}$$

$$\text{Left: } \frac{60 + \frac{170}{2}}{45} > \frac{410}{150}$$

Hence, criteria is satisfied for wheel ⑧

Now,

$$S_{max} = \frac{0.525}{30} \times [(10 \times 14) + (10 \times 20) + (40 \times 28)] + 0.525 \times (40 + 40 + 40 + 40)$$

$$+ \frac{(0.675 - 0.525)}{30} \times [(40 \times 4) + (40 \times 10) + (40 \times 16) + (40 \times 22)]$$

$$+ \frac{0.675}{90} [(10 \times 38) + (10 \times 44) + (30 \times 52) + (30 \times 58) + (30 \times 64) + (30 \times 70) + (30 \times 76) + (10 \times 84) + (10 \times 90)]$$

$$= (25.55 + 84 + 10.4 + 91.2) = 211.15$$

(Ans)