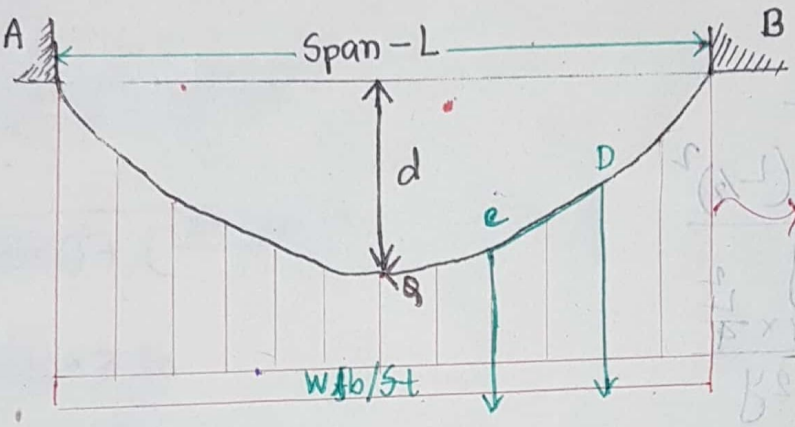
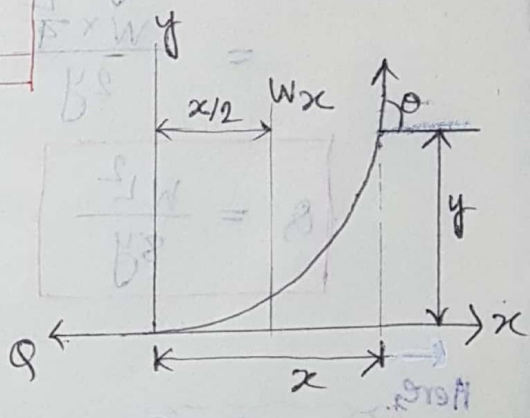
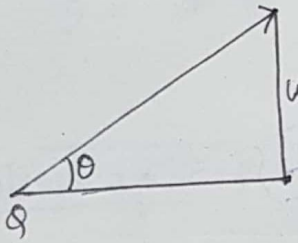
Cable

Cable take only tension.



W lb/ft → weight per feet.





$\tan\theta = \frac{Wx}{Q}$

∴ Cable slope, $\theta = \tan^{-1}\frac{Wx}{Q}$

$\theta = \tan^{-1}\frac{WL}{2Q}$ $(\because L = 2x)$

$\Sigma F_x = 0$

$\Rightarrow Q = F\cos\theta$ ..... (i)

$\Sigma F_y = 0$

$\Rightarrow Wx = F\sin\theta$ ..... (ii)

$\Sigma M_c = 0$

$\Rightarrow -Q \times y + (Wx) \times \left(\frac{x}{2}\right) = 0$

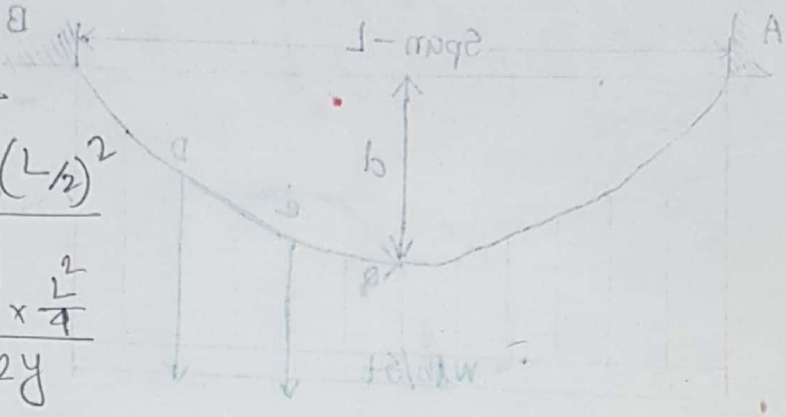
$\Rightarrow \boxed{y = \frac{Wx^2}{2Q}}$ ..... (iii)

Tension

$$F = \sqrt{Q^2 + (Wx)^2}$$

$$F_{max} = W y_{(max)}$$

$$F_{max} = \sqrt{Q^2 + \left(\frac{WL}{2}\right)^2} \quad \cdots \quad (iv)$$



$$Q = \frac{Wx^2}{2y}$$

$$= \frac{W \times (L/2)^2}{2y}$$

$$= \frac{W \times \frac{L^2}{4}}{2y}$$

$$\boxed{Q = \frac{WL^2}{8y}}$$

Here,

Span / L

Length of the cable

$$\boxed{S_{(max)} = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}}$$

(*) $L = 500$ ft, $d = 55$ ft, $w = 2$ lb/ft, $Q = ?$ $\theta = ?$

$$Q = \frac{2 \times (500)^2}{8 \times 55}$$

$$= 1250$$

$w = 2$ lb/ft
$L = 500$ ft
$d = 55$

$$F_{max} = \sqrt{Q^2 + \left(\frac{wL}{2}\right)^2}$$

$$= \sqrt{(1250)^2 + \left(\frac{2 \times 500}{2}\right)^2}$$

$$= 1346.3 \text{ lb}$$

$$\tan\theta = \frac{2 \times 500}{2 \times 1250}$$

$$\therefore \theta = 21.8^\circ$$

Problem - 1337

$S = 600$ ft, $w = 3$ lb/ft, $d = 150$ ft

we know,

$$S_{max} = L + \frac{8d^2}{3L} - \frac{3d^4}{5L^3}$$

$$\Rightarrow 600 = L + \frac{8 \times (150)^2}{3 \times L} - \frac{3 \times (150)^4}{5 \times L^3}$$

$$\Rightarrow 600L = L^2 + 64066.67 - 60000$$

$$\Rightarrow L^2 - 600L + 64066.67 - 60000 = 0$$

$$\therefore L = \frac{600 \pm \sqrt{(-600)^2 + 4 \times 600 \times 6106.67}}{2}$$

$\therefore L = 473.2$ ft or, 126.79 ft

$\therefore L = 473.2$ ft [$L \neq 126.79$ ft (Answer) sag, $d > 126.79$]

Tension of the lowest point,

$$Q = \frac{WL^2}{8d}$$

$$= \frac{3 \times (473.2)^2}{8 \times 150}$$

$$= 559.796 \text{ \#}$$

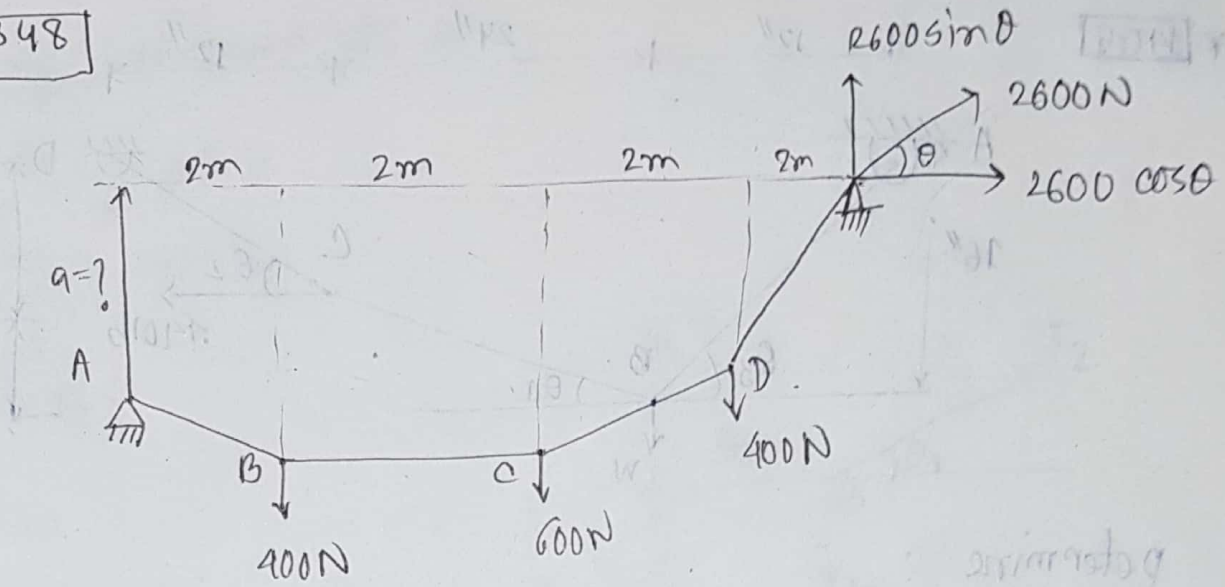
$$F_{max} = \sqrt{Q^2 + \left(\frac{WL}{2}\right)^2}$$

$$= \sqrt{(559.796)^2 + \left(\frac{3 \times 473.2}{2}\right)^2}$$

$$= 963.98 \text{ lb}$$

Ans.

Problem-1348



At point C

$F_y = 0$

$400 + 600 = 2600 \sin\theta$

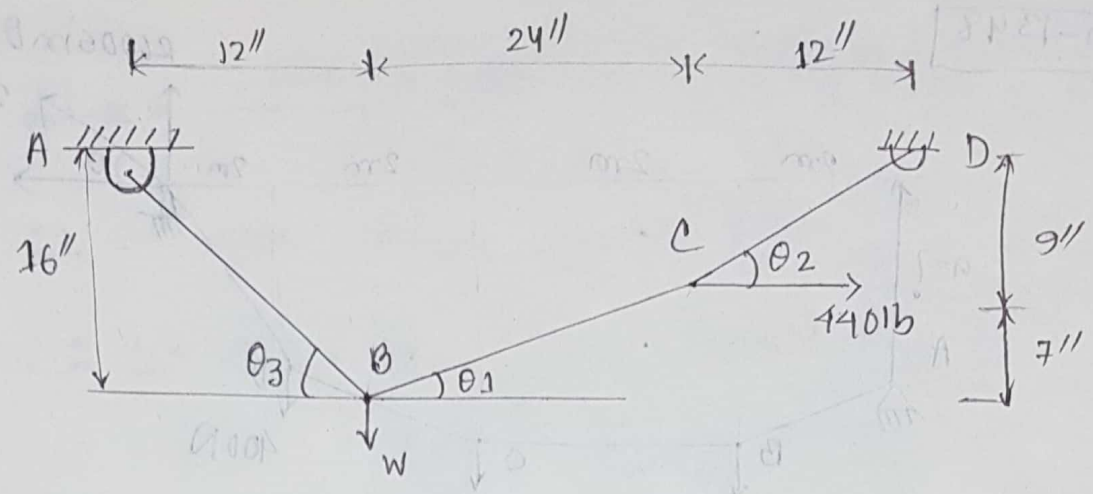
$\Rightarrow \theta = 22.62^\circ$

$\Sigma M_A = 0$

$\Rightarrow a \times 2600 \cos 22.6^\circ - 8 \times 2600 \sin 22.6^\circ + 400 \times 2 + 600 \times 4 + 6 \times 400 = 0$

$\therefore a = 1\ m$ (Answer)

*1405



Determine :

(a) Tension at D.

(b) W = ?

(c) Tension in each portion of cable.

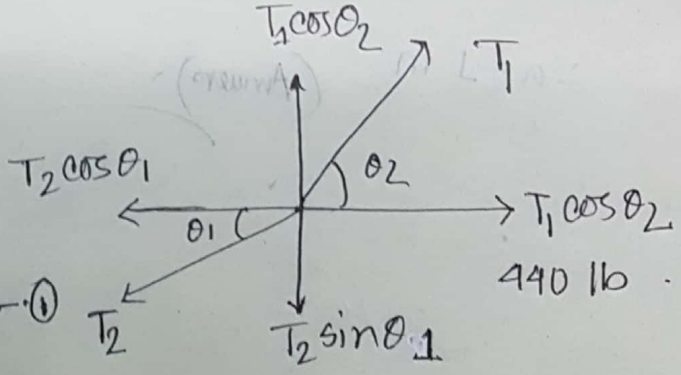
Solution :

$\theta_1 = \tan^{-1}\frac{7}{24} = 16.26^\circ$

$\theta_2 = \tan^{-1}\frac{9}{12} = 36.87^\circ$

$\theta_3 = \tan^{-1}\frac{16}{12} = 53.13^\circ$

At point C;



$\Sigma F_x = 0$

$\Rightarrow T_1 \cos\theta_2 + 440 - T_2 \cos\theta_1 = 0 \quad \cdots (i)$

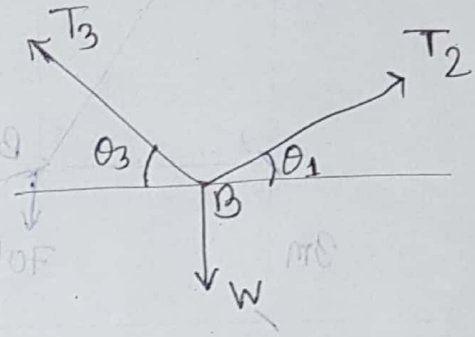
$\Sigma F_y = 0$

$\Rightarrow T_1 \cos\theta_2 - T_2 \sin\theta_1 = 0 \quad \cdots (ii)$

from eqn (i) and (ii) ⇒

$T_1 = 350$ lb

$T_2 = 750$ lb.



At point B,

$\Sigma F_x = 0$

$\Rightarrow T_2 \cos\theta_1 - T_3 \cos\theta_3 = 0$ ——— (iii)

$\Sigma F_y = 0$

$\Rightarrow T_2 \sin\theta_1 + T_3 \sin\theta_3 - W = 0$ ——— (iv)

Solving (iii) & (iv) ⇒

$T_3 = 1200$ lb

$W = 1170$ lb

At point D:

$D_x = T_1 \cos\theta_2$

$= 350 \cos 36.87$.

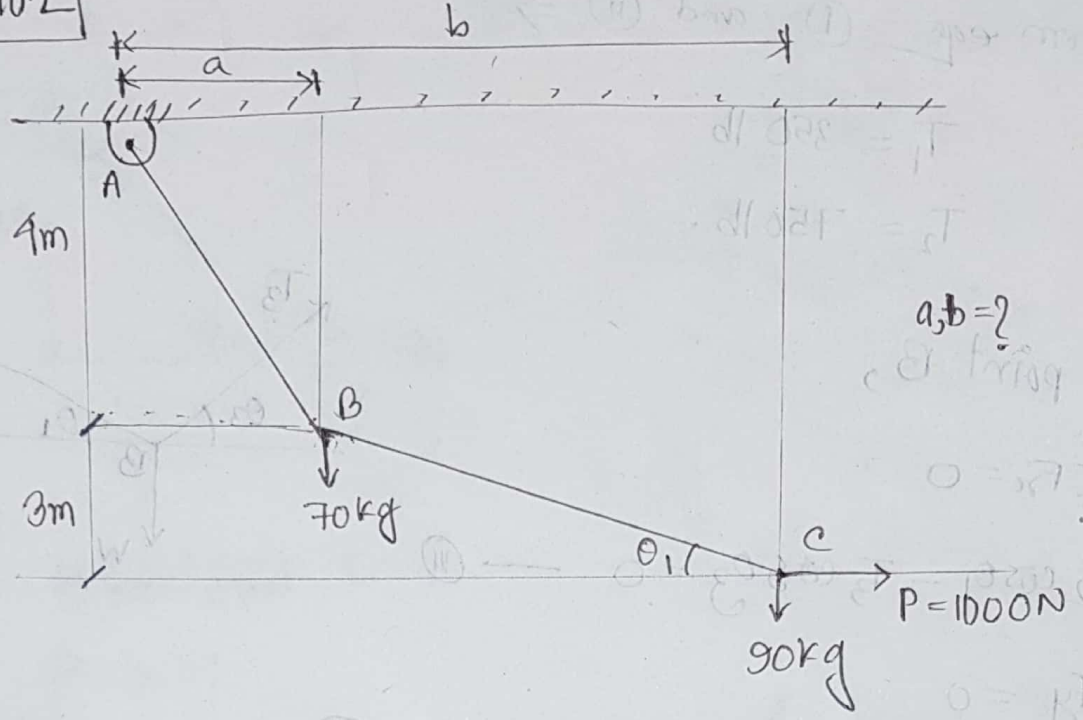
$= 280$ lb.

$D_y = T_1 \sin 36.87$.

$= 210$ lb

Answer

Problem-1402



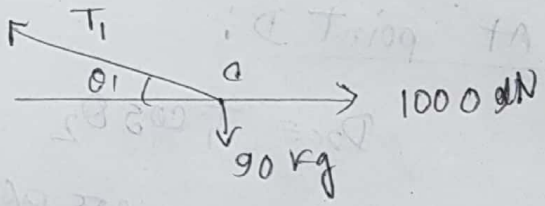
$\Sigma M_A = 0$

$\Rightarrow -70 \times a - 90 \times b + 1000 \times 7 = 0$

$\therefore 70a + 90b = 7000$ ——— (I)

Free c point,

$F_x = 0$



$\Rightarrow T_1 \cos\theta_1 = 1000$ ··· (II)

$\Sigma F_y = 0$

$\Rightarrow 90 - T_1 \sin\theta_1 = 0$

$\Rightarrow T_1 \sin\theta_1 = 90$ ··· (III)

$\therefore \tan\theta_1 = \frac{90}{1000}$

$\therefore \theta_1 = 5.14°$

$$\tan 5.14^\circ = \frac{3}{b-a}$$

$\Rightarrow b - a = 33.33$

$\Rightarrow b = a + 33.33$

From, eqn (1),

$$70a + 90a + 3000 = 7000$$

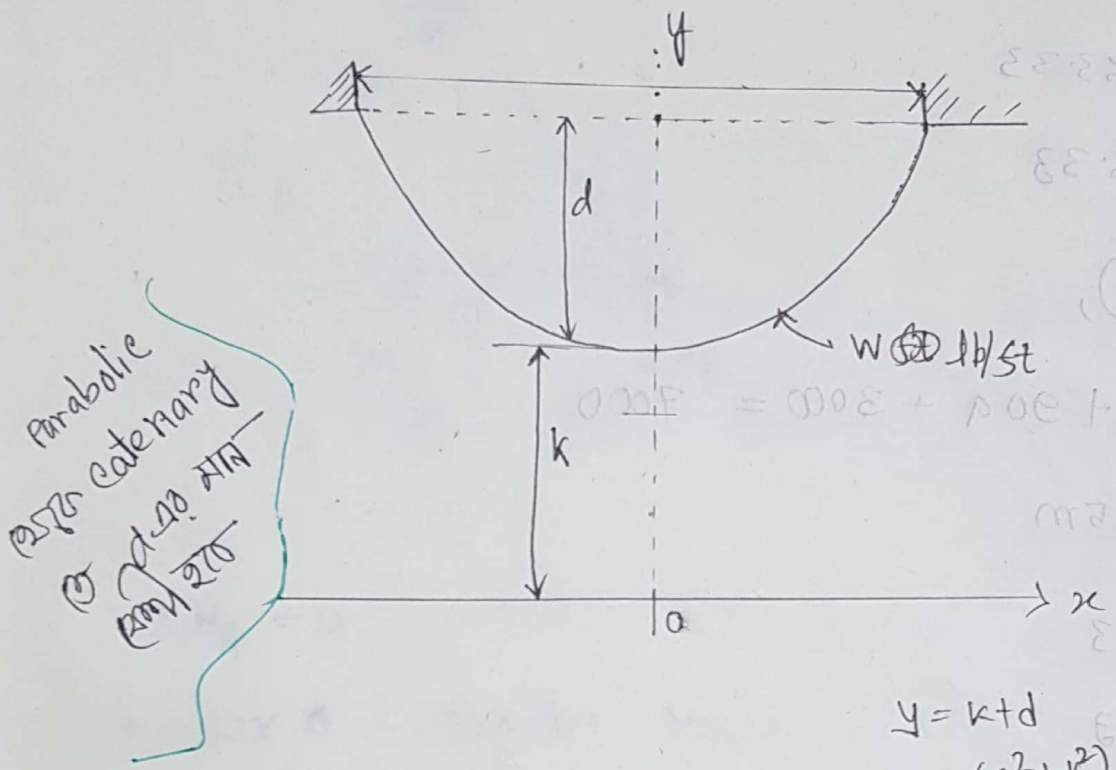
$\therefore a = 25\,m$

$b = a + 33.33$

$= 25 + 33.33$

$= 58.33\,m$

catenary



$y = k + d$

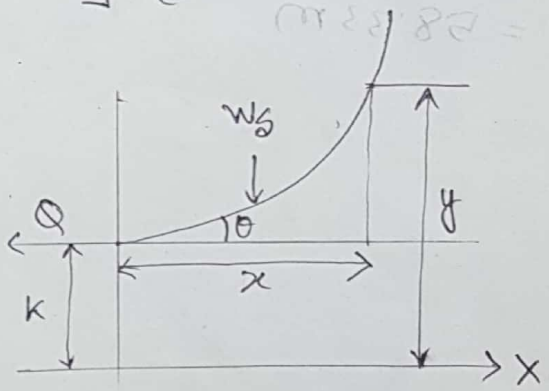
$y = (s^2 + k^2)^{1/2}$

$\Sigma F_x = 0$

$\Rightarrow F\cos\theta = Q \quad \cdots (i)$

$\Sigma F_y = 0$

$\Rightarrow F\sin\theta = ws \quad \cdots (ii)$



$(ii) \div (i) \Rightarrow$

$\tan\theta = \frac{ws}{Q} \quad \left(\frac{Q}{w} = k, \text{Let}\right)$

$\therefore \tan\theta = \frac{s}{k} = \frac{dy}{dx}$

$$ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} dx$$

$$= \left[1 + (s/k)^2\right]^{1/2} dx$$

$$ds = \frac{(k^2+s^2)^{1/2}}{k} dx$$

$$\Rightarrow \int \frac{dx}{k} = \int \frac{ds}{\sqrt{k^2+s^2}}$$

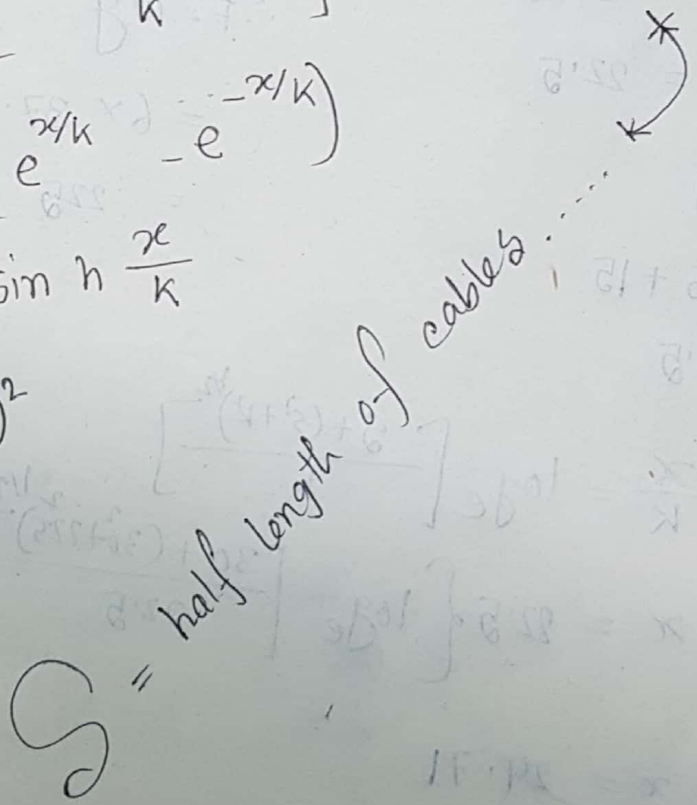
$$\Rightarrow \frac{x}{k} = \log_e \left[\frac{s+(k^2+s^2)^{1/2}}{k}\right]$$

$$\Rightarrow s = \frac{k}{2}\left(e^{x/k} - e^{-x/k}\right)$$

$$= k \sin h \frac{x}{k}$$

$$F^2 = Q^2 + (ws)^2$$

$$\Rightarrow F = wy$$



1302

* A 60ft cable which weight - 360 lb and d = 15 ft. Find the tension at support and span.

$S = L/2$ Here, $L = 60$ ft

$\therefore S = 30$ ft.

$d = 15$ ft

$w = \frac{360}{60}$

$= 6$ lb/ft

we know,

$y = k + d$

and $y = \sqrt{S^2 + k^2}$

$\Rightarrow k^2 + 2kd + d^2 = S^2 + k^2$

$\Rightarrow 2kd + d^2 = S^2$

$\Rightarrow k = \frac{S^2 - d^2}{2d}$

$= \frac{30^2 - 15^2}{2 \times 15}$

$= 22.5$

$\therefore F = wy$

$= 6 \times 37.5$

$= 225$

$y = k + d$

$= 22.5 + 15$

$= 37.5$

Again, $\frac{x}{k} = \log_e \left[\frac{S + (S^2 + k^2)^{1/2}}{k} \right]$

$\Rightarrow x = 22.5 \left\{ \log_e \left[\frac{30 + (30^2 + 22.5^2)^{1/2}}{22.5} \right] \right\}$

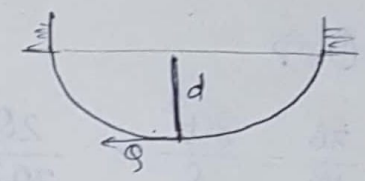
$\therefore x = 24.71$

Span, $L = 2x$

$= 24.71 \times 2$

$= 49.43$ (Answer)

1339



$w = 3.4$ lb/ft.

$L = 600$ ft.

Sag, $d = 80$ ft

length of cable, $S = ?$

$$H = \frac{wL^2}{8d}$$

$$= \frac{3.4 \times (600)^2}{8 \times 80}$$

$= 1912.5$ lb/ft

$\therefore F = \sqrt{H^2 + (wL/2)^2}$

$= 2167.5$ lb.

$$S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}$$

$= 627.23$ ft (Answer)

1364

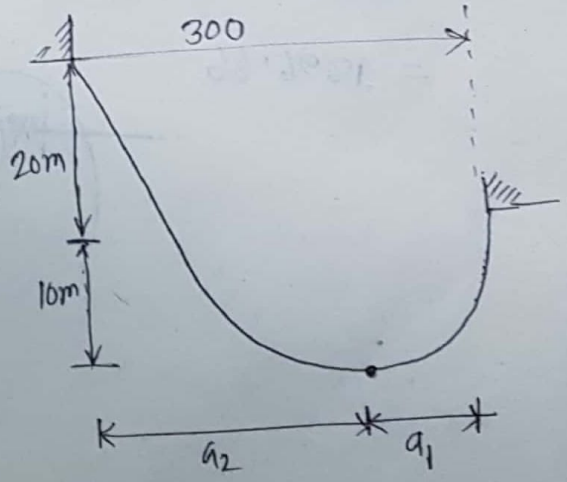
weight of the cable = 3kg/m

maximum tension = ?

$$\text{Sag, } y = \frac{wx^2}{2H}$$

[Right] $10 = \frac{wa_1^2}{2H}$ ——— (I)

[Left] $30 = \frac{wa_2^2}{2H}$ ——— (II)



Lowest point horizontal.

Lowest Sag horizontal.

- tension minimum.

(II) ÷ (I) ⇒

$$\frac{30}{10} = \frac{3q_2^2}{2Q} \times \frac{2Q}{3q_1^2}$$

$$\Rightarrow \quad 3 = \frac{q_2^2}{q_1^2}$$

$$\therefore q_2 = \sqrt{3}\, q_1 \quad \text{— (III)}$$

Again, $q_1 + q_2 = 300$

$$\Rightarrow q_1 + \sqrt{3} q_1 = 300$$

$$\Rightarrow q_1 = 109.80$$

(III) ⇒

$$\therefore q_2 = 190.20$$

দ্বিতীয়ত,

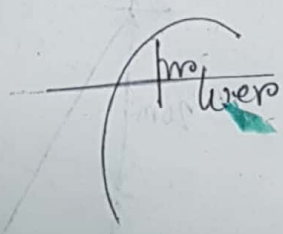
$$Q = \frac{wq_1^2}{2 \times 10} = 452.1015 \text{ d}$$

$$Q = \frac{wq_2^2}{2 \times 30} = \frac{3 \times (190.20)^2}{2 \times 30} = 1808.802$$

$$\therefore F_{max} = \sqrt{Q^2 + wq_2^2}$$

$$= \sqrt{(1808.802)^2 + (3 \times 190.20)^2}$$

$$= 1896.66$$



1302 Here, the total length, $2S = 60$

$\Rightarrow S = 30$

weight of the cable $= 360^{\#}$

$\therefore w = \frac{360}{60} = 6$ lb/ft

Here, $d = 15$ ft

we know, $k = \frac{S^2 - d^2}{2d}$

$= \frac{30^2 - 15^2}{2 \times 15}$

$= 22.5'$

$y_{max} = k + d = 22.5 + 15 = 37.5'$

$F_{max} = wy_{(max)}$

$= 6 \times 37.5'$

$= 225$ ft

$F_{max} = \sqrt{H^2 + \left(\frac{WL}{2}\right)^2}$ এটাও হতে পারে।