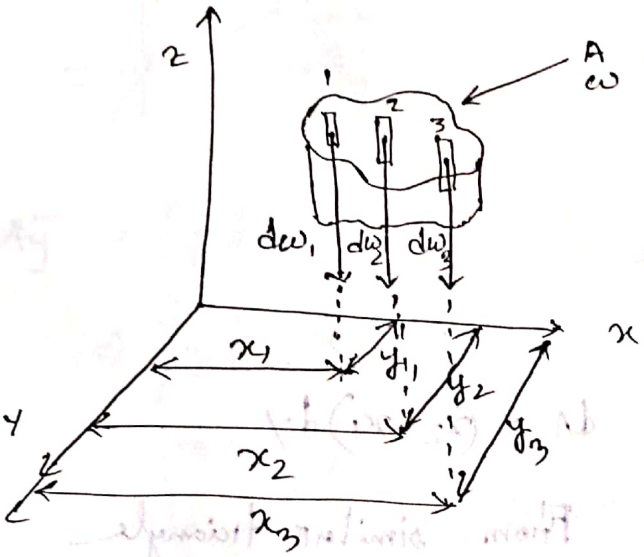
MRA
1st class

Centroids



Moment

$$\omega\bar{x} = d\omega_1 x_1 + d\omega_2 x_2 + d\omega_3 x_3 + \cdots\cdots d\omega_n x_n$$

$$= \sum x\, d\omega$$

$$= \int x\, d\omega$$

$$\therefore \boxed{\bar{x} = \frac{\int x\, d\omega}{\omega}} \quad \text{or,} \quad = \frac{\int x\, dm}{m} = \frac{\int x\, dv}{v} = \frac{\int x\, dA}{A} = \frac{\int x\, dL}{L}$$

$$\boxed{\bar{y} = \frac{\int y\, d\omega}{\omega}} = \frac{\int y\, dm}{m} = \frac{\int y\, dv}{v} = \int \frac{y\, dA}{A} = \frac{\int y\, dL}{L}$$

$$\boxed{\bar{z} = \frac{\int z\, d\omega}{\omega}} = \frac{\int z\, dm}{m} = \frac{\int y\, dv}{v} = \int \frac{z\, dA}{A} = \int \frac{z\, dL}{L}$$

Analytic mechanics by faires

MRA
2nd Class

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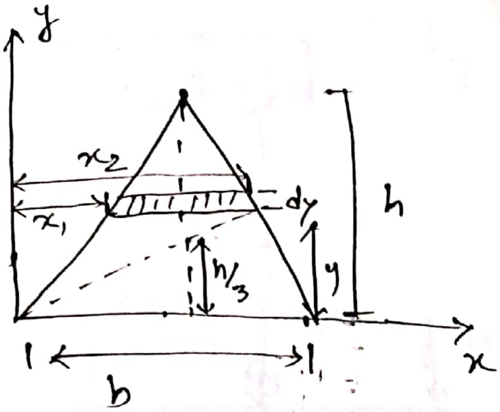


$$A\bar{y} = \int y \, dA$$

$$dA = (x_2 - x_1)\, dy$$

$$= \int_0^h y (x_2 - x_1)\, dy$$

From similar triangle

$$\Rightarrow \int_0^h y \frac{b(h-y)}{h} dy$$

$$\frac{x_2 - x_1}{h - y} = \frac{b}{h}$$

$$= \frac{b}{h} \int_0^h (hy - y^2)\, dy$$

$$(x_2 - x_1) = \frac{b(h-y)}{h}$$

$$= \frac{b}{h} \left[\frac{h \cdot y^2}{2} - \frac{y^3}{3} \right]_0^h$$

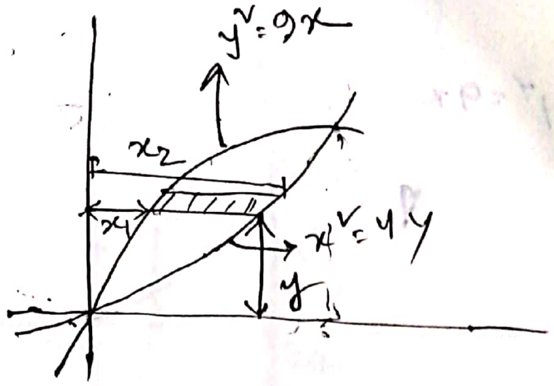
$$\Rightarrow \frac{b}{h} \left(\frac{h^3}{2} - \frac{h^3}{3} \right)$$

$$\Rightarrow \frac{bh^2}{6}$$

$$A\bar{y} = \frac{bh^2}{6}$$

$$\bar{y} = \frac{bh^2}{6} \times \frac{1}{A}$$

$$= \frac{\cancel{b}h^{\cancel{2}}}{\cancel{6}\,3} \times \frac{\cancel{2}}{\cancel{b}\cancel{h}} = \frac{h}{3}$$



$\int_0 x\,dy$

$y^2 = 9x$

$y^2 = 9x$

$A\bar{y} = \int y\,dA$

$dA = (x_2 - x_1)\,dy$

$= \left(2y^{1/2} - \frac{y^2}{9}\right) dy$

$\Rightarrow \int y\left(2y^{1/2} - \frac{y^2}{9}\right) dy$

$= \int_0^{6.86} y\left(2y^{1/2} - \frac{y^2}{9}\right) dy$

$= \int_0^{6.86} \left(2y^{3/2} - \frac{y^3}{9}\right) dy$

$y^2 = 9x$

$x^2 = 4y$

$\frac{x^4}{16} = y^2$

$A\bar{y} = \left[2 \cdot \frac{y^{5/2}}{5/2} - \frac{y^4}{36}\right]_0^{6.86}$

$\frac{x^4}{16} = 9x$

or, $x^3 = 144$

$x = 5.24$

$y = 6.86$

$\Rightarrow 37.08$

$\bar{y} = \frac{37.08}{12}$

$\Rightarrow 3.09$

$\int dA = \int_0^{6.86} \left(2y^{1/2} - \frac{y^2}{9}\right) dy$

$\Rightarrow 12$

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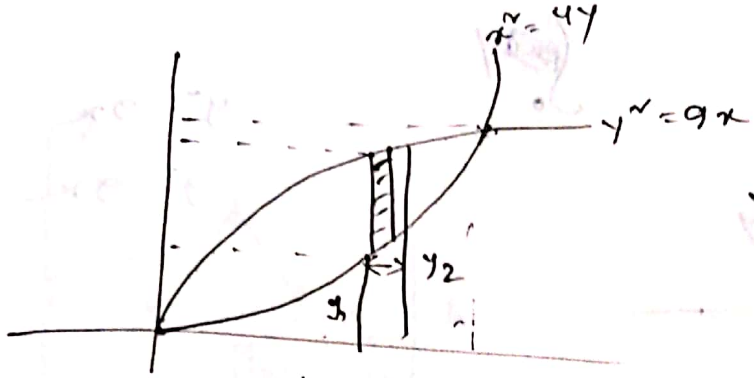

$$\int dA = \int_0^{5.24} (y_2 - y_1)\, dx$$

$$= \int_0^{5.24} \left(3x^{1/2} - \frac{x^2}{4}\right) dx$$

$A = 12$

$$A\bar{x} = \int x\, dA \qquad dA\ (y_2 - y_1)\, dx$$

$$\Rightarrow \int_0^{5.24} x \left(3x^{1/2} - \frac{x^2}{4}\right) dx$$

$\bar{x} \Rightarrow 2.36''$

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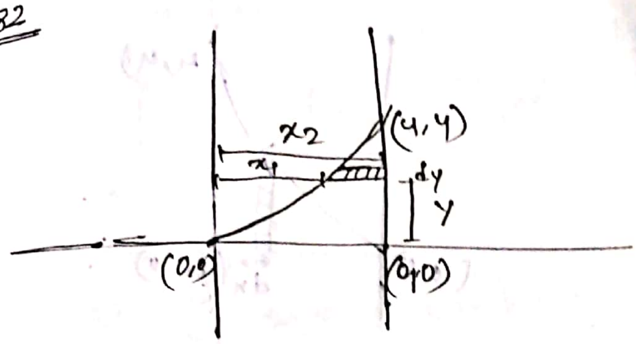
682



$x^2 = 4y$
$16 = 4y$
$y = 4$

$x = 4$

$dA = (x_2 - x_1)\, dy$

$A\bar{y} = \int_0^4 y\, dA$

$= \int_0^4 y\,(x_2 - x_1)\, dy$

$= \int_0^4 y\,(4 - 2y^{1/2})\, dy$

$= \int_0^4 (4y - 2y^{3/2})\, dy$

$\frac{3}{2}+1$

$= \frac{3+2}{2}$

$= \left[\frac{4y^2}{2} - \frac{2y^{5/2}}{5/2}\right]_0^4$

$= \left[2y^2 - \frac{4y^{5/2}}{5}\right]_0^4$

$= \left[32 - \frac{4 \times 4^{5/2}}{5}\right]$

$A\bar{y} = \cancel{6.4}\ \frac{32}{5}$

$\int dA = \int (x_2 - x_1)\, dy$

$= \int_0^4 (4 - 2y^{1/2})\, dy$

$A = \left[4y - \frac{2 \times y^{3/2}}{3}\right]_0^4$

$= \frac{16}{3}$

$\bar{y} = \frac{32}{5} \div \frac{16}{3}$

$= \frac{6}{5}$ unit

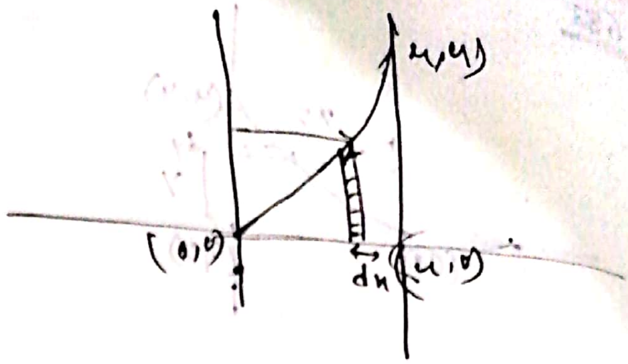
$A\bar{x} = \int x \, dA$

$= \int x \, y \, dx$

$\Rightarrow \int \frac{x^3}{4} \, dx$

$\Rightarrow \left[\frac{x^4}{4 \times 4} \right]_0^4$

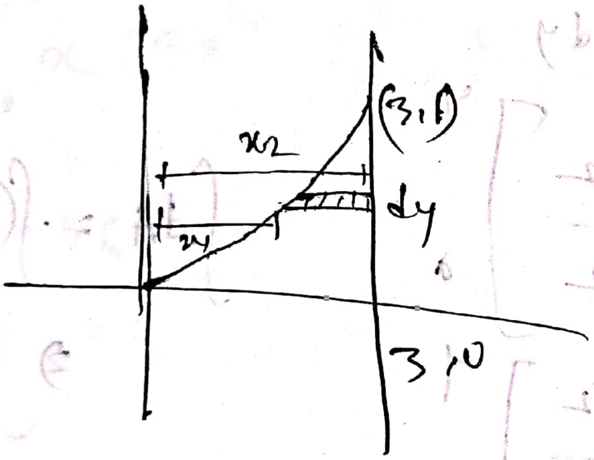
$\Rightarrow 16$



$\bar{x} = 16 \div \frac{16}{3}$

$\Rightarrow 3$ unit

634



$A\bar{y} = \int y \, dA$

$= \int y (x_2 - x_1) \, dy$

$= \int y \, (3 - 3y^{1/2}) \, dy$

$\Rightarrow \int_0^1 3y - 3y^{3/2} \, dy$

$= \frac{3}{10}$ squnit

$\frac{3}{2} - \frac{6}{5}$

$= \frac{15 - 12}{10} = \frac{3}{10}$

$x^2 = 9y$

$\int dA = \int_0^1 (3 - 3y^{1/2}) \, dy$

$A = 1$

$\bar{y} = \frac{3}{10}$ unit

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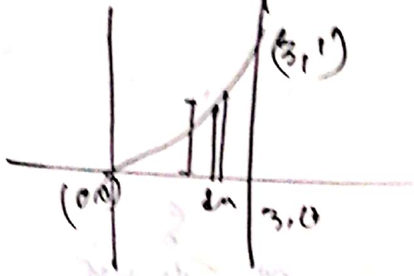
$A\bar{x} = \int x \, dA$

$= \int x \, y \, dx$

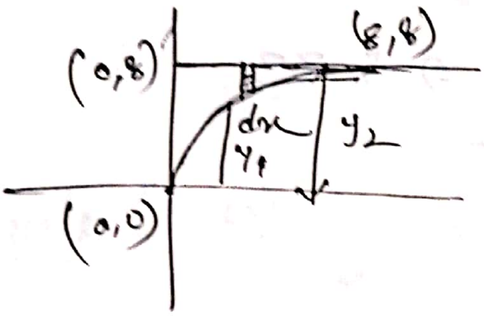
$\Rightarrow \int \frac{x^3}{9} \, dx$

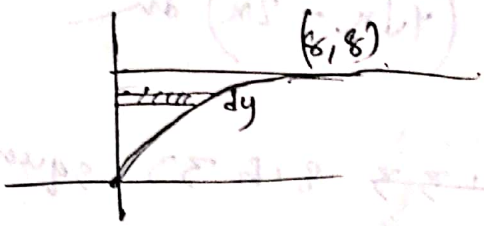
$\Rightarrow \frac{9}{4}$ square unit



$\bar{x} = \frac{9}{4}$ unit

35





$= \int x \, dA$

$= \int_0^8 x (y_2 - y_1) \, dx$

$\Rightarrow \int_0^8 x (8 - \sqrt{8x}) \, dx$

$= 51.20$

$\bar{x} = 2.4$ unit

$\int dA = \int_0^8 (y_2 - y_1) \, dx$

$A = \int (8 - \sqrt{8} \, x^{1/2}) \, dx$

$= 21.33$

$A\bar{y} = \int y \, dA$

$= \int_0^8 y \, x \, dy$

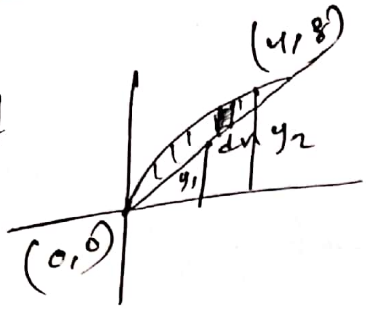
$\Rightarrow \int_0^8 y \times \frac{y^2}{8} \, dy$

$\Rightarrow \int_0^8 \frac{y^3}{8} \, dy$

$= 128$ unit

$\bar{y} = 6$ unit Ans.

6861



$y^2 = \cancel{16}^{8}\ \frac{x \cdot 4}{\cancel{x}}$

$A = \int_0^4 (y_2 - y_1)\,dx$

$\Rightarrow \int_0^4 (4\sqrt{x} - 2x)\,dx$

$\Rightarrow 5.33$

$A\bar{x} = \int_0^4 x\, dA$

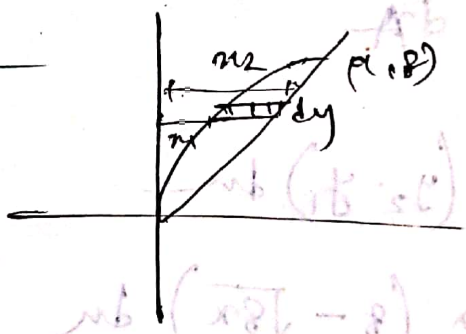
$= \int_0^4 x(y_2 - y_1)\,dx$

$\Rightarrow \int_0^4 x(4\sqrt{x} - 2x)\,dx$

$= \cancel{5.33}\ 8.533$ square unit

$\bar{x} = \frac{8.533}{5.33}$

$= 1.60$ unit Ans



$y = 2x$

$A\bar{y} = \int_0^8 y\, dA$

$= \int_0^8 y(x_2 - x_1)\,dy$

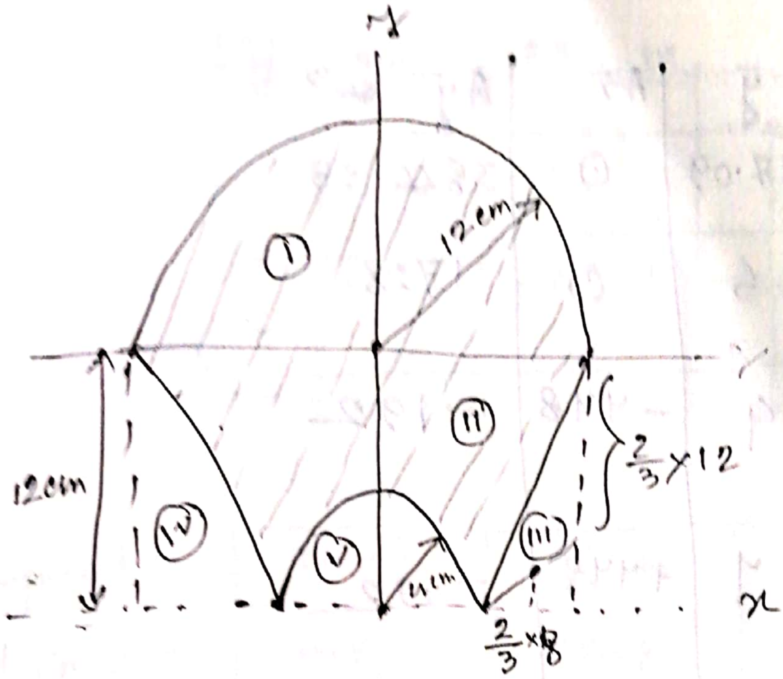
$= \int_0^8 y\left(\frac{y}{2} - \frac{y^2}{16}\right)dy$

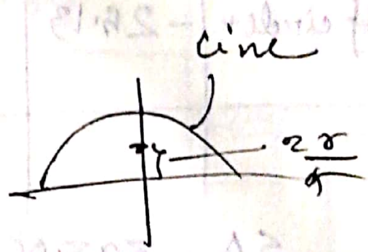
$\Rightarrow \int_0^8 y\left(\frac{8y - y^2}{16}\right)dy$

$\Rightarrow 21.33$

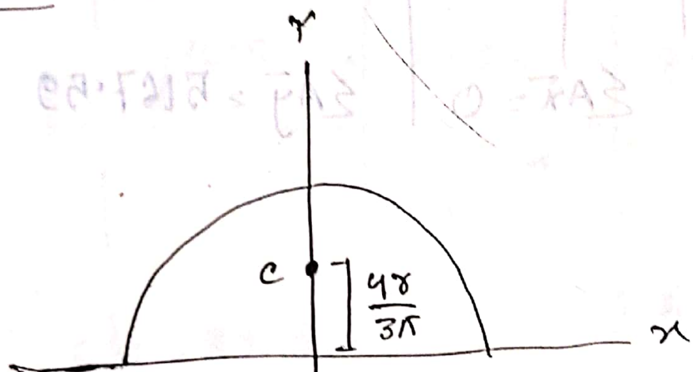
$\bar{y} = \frac{21.33}{5.33}$

$\Rightarrow 4$ unit Ans

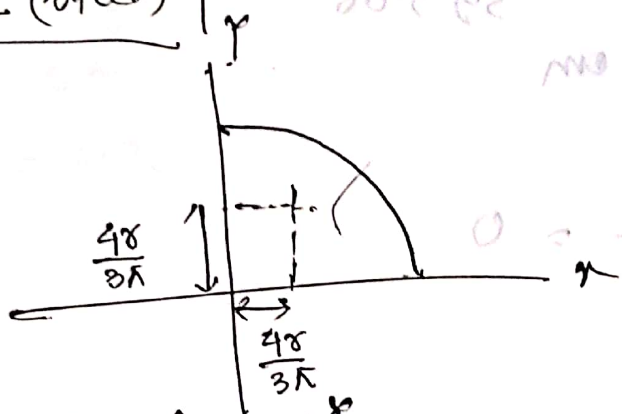




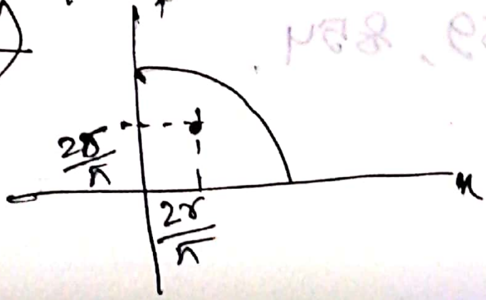
Half circle (area)



Quartare circle (area)



" " (line)



808

Component	A (cm)	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$ cm³
Half circle - I	226.19	0	17.09	0	3866.25
Rectangular (II)	288	0	6	0	1728
Triangle - (III)	-48	$(4+\frac{2}{3}\times8)$ 9.33	4	-448	-192
Triangle - (IV)	-48	-9.33	4	+448	-192
Half circle V	-25.13	0	1.69	0	-42.66

$\Sigma A = 393.06$ $\Sigma A\bar{x} = 0$ $\Sigma A\bar{y} = 5167.59$

$$A\bar{y} = \int y \, dA$$

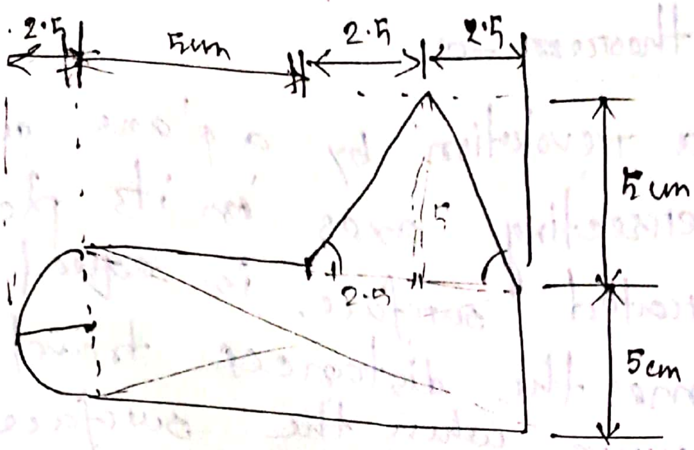
$$\bar{y} = \frac{\Sigma A\bar{y}}{A} = \frac{5167.59}{393.06}$$

$= 13.15$ cm

$$\bar{x} = \frac{\Sigma Ax}{A} = 0$$

Assainment

808, 836, 839, 854



Component	A (cm²)	$\bar{y}$ cm	$A\bar{y}$ cm³	$\bar{x}$	$A\bar{x}$
Half circle-1	9·817	~~1·061~~ 2·5	~~10·41~~ 24·54	~~5~~	~~1·061~~ ~~49·085~~
Rectangular	50	2·5	125	~~11·061~~	~~553·05~~
triangle	12·5	6·66	83·25		

$\Sigma A = 72·317$

$\Sigma Ay = 218·66$

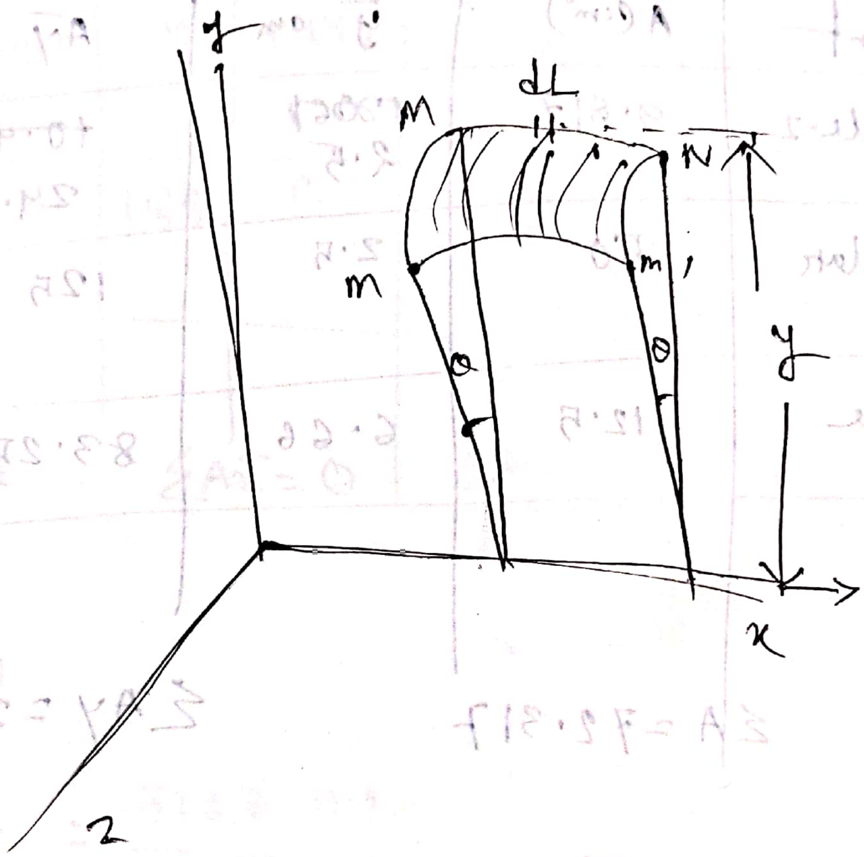
5·39

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$= 3·02$

Pappus and Guldinus theorem -01.

If a surface of a revolution by a plane curve revolving about any non-intersecting axes in its plane the area of the generated surface is equal to the length of the curve, times the distance travel by the centroid of the curve when the surface being generated.



$$\int dA = \int \theta y \, dL$$

$$\Rightarrow A = \theta \int y \, dL$$

$$\boxed{A = \theta \bar{y} L}$$

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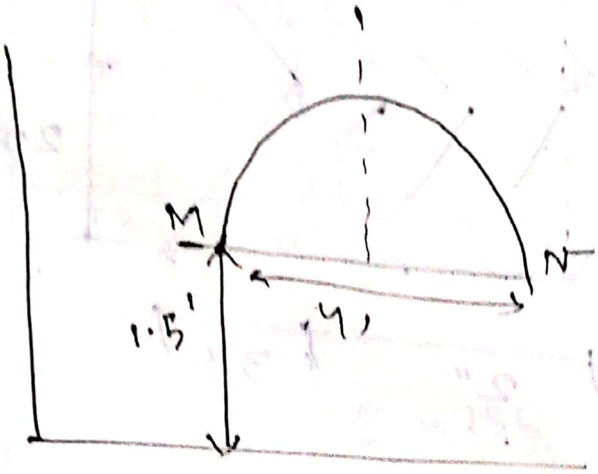
Theorem :

$\boxed{V = \theta \bar{y} A}$



$\theta = 270°$

$A = \theta \bar{y} L$

$= \frac{3\pi}{2} \times \left(1.5 + \frac{4}{\pi}\right) \times 2\pi$

$= 82.11$

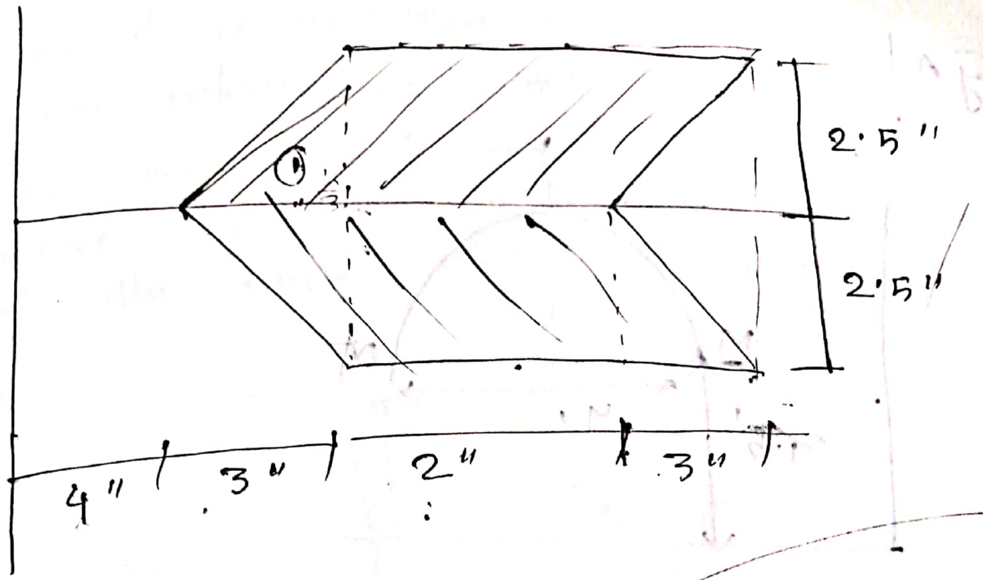
$\theta = 270°$

$= \frac{270 \times \pi}{180}$

$L = 2\pi$

$\bar{y} = 1.5 + \frac{2r}{\pi}$

$= 1.5 + \frac{4}{\pi}$



$A = \theta \times \bar{x}$

$\bar{V} = \theta \bar{x} A$

$= 203.3 \times 2\pi$

$= 1277.33$

$\theta = 2\pi$

$A = \frac{1}{2} \times 2.5 \times 5 \times$

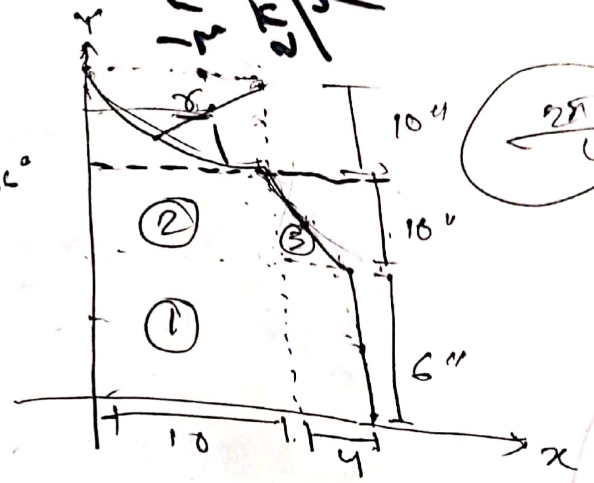
$= 25$

Components	A	$\bar{x}$	$A\bar{x}$
Triangle	$\frac{1}{2} \times 5 \times 3 = 7.5$	6	45
Rectangular	$5 \times 5 = 25$	9.5	237.5
Triangle (M)	-7.5	~~10~~ 11	-79.2

$\Sigma A = 25$

$\Sigma A\bar{x} = 203.3$

$\theta = 136°$
A = ?
V = ?



$\frac{2\pi r}{4}$

$$\theta = \frac{136° \times \pi}{180}$$

$$\vec{A} = \cancel{\theta \bar{y} l}$$
$$= \theta \bar{x} l$$
$$= 2.373 \times 181.29$$
$$= 430.21$$

$\theta = 136°$

$l = 6 + 10.77 + 15.70$

Component	L	A	$\bar{x}$	$A\bar{x}$	$L\bar{x}$
Rectangular ①	6	14×6 =	7	588	42
Rectangular (ii)	10	10×10	5	500	50
Triangle	10.77	20	11.33	226.6	122.0241
Rectangular (iii)	10	10×10	5	500	50
Circle	15.7	$\cancel{\frac{25 \times 10}{4}}$ = 78.53	5.75	~~-90.275~~ -451.60	-82.73 =

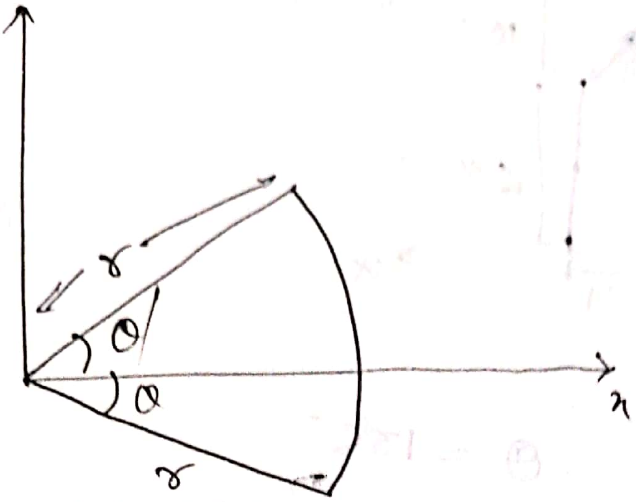
$\Sigma A\bar{x} = 1363$

$\Sigma L\bar{x} = 181.23$

$$V = \theta \bar{x} A$$
$$= 2.373 \times 1363$$
$$= 3234.4$$

Q =

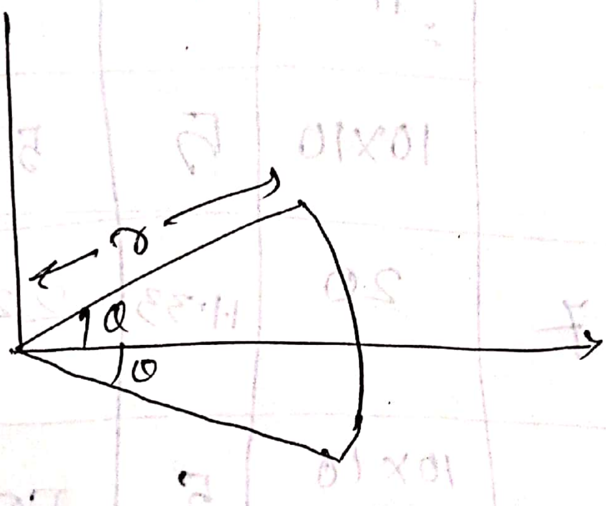
Area



$$\bar{x} = \frac{2r\sin\theta}{3\theta^c} \qquad \bar{y} = 0$$

$$A = \theta^c r^2$$

Line



$$\bar{x} = \frac{r\sin\theta}{\theta}, \quad \bar{y} = 0$$

$$L = 2\theta r$$

$$\bar{x}_1 = \frac{2r\sin\alpha}{3\theta}$$

$$= \frac{2 \times 4.8 \times \sin 30^\circ}{3 \times \frac{30 \times \pi}{180}}$$

$$= 30.55$$

$$x_2 = \frac{2 \times 72 \times \sin 30^\circ}{3 \times \frac{30 \times \pi}{180}}$$

$$= 45.83$$

$$\vec{x} = (x_2 - x_1)$$

$$= \underline{15.28}$$