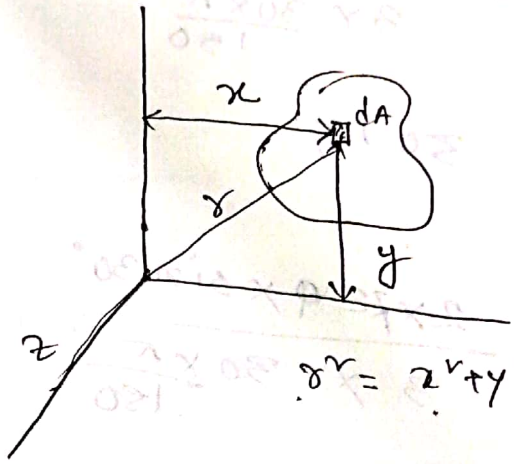
Moment of Inertia

The inertia is the property which ....

$$I = \int a^2 \, dA$$



second moment of area.

$$I_x = \int y^2 \, dA$$

$$I_y = \int x^2 \, dA$$

$$r^2 = x^2 + y^2$$

Polar moment of Inertia

$$J = \int r^2 \, dA$$

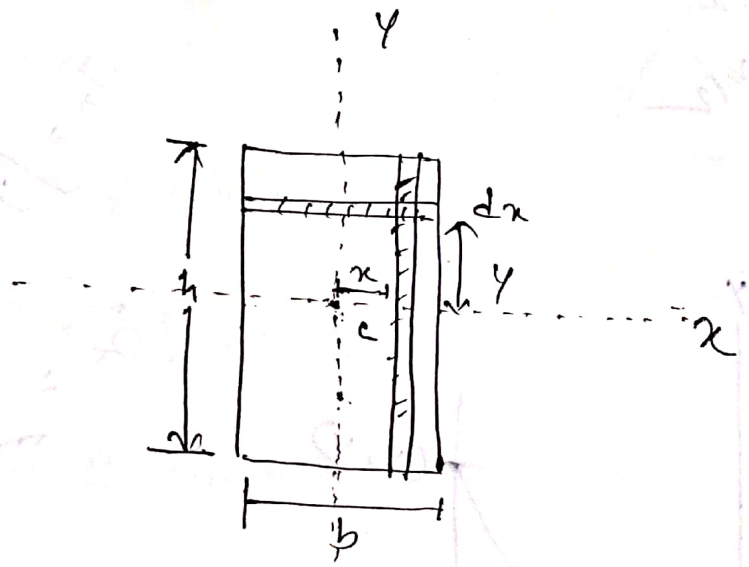
$$= \int (x^2 + y^2) \, dA$$

$$J = I_x + I_y$$

Radius of Gyration

$$K_x = \sqrt{\frac{I_x}{A}}$$

$$K_y = \sqrt{\frac{I_y}{A}}$$



$$\cancel{I_x = \left(\frac{h}{2}\right)^2 \times h \times b}$$

$$\cancel{= \frac{h^3 b}{4}}$$

$$\cancel{I_y = \frac{b^3 h}{4}}$$

$$I_x = \int y^2 \, dA$$

$$= \int_{-h/2}^{h/2} y^2 b \, dy$$

$$= b \left[\frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$\Rightarrow b \frac{h^3}{24} + \frac{h^3}{24}$$

$$\Rightarrow \frac{b h^3}{12}$$

$$I_y = \int x^2 dA$$

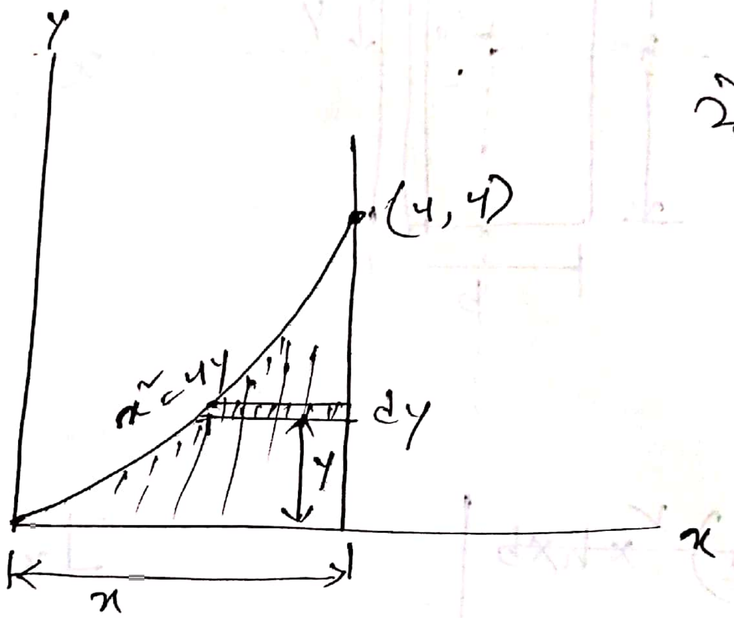
$$= \int_{-b/2}^{b/2} x^2 h \, dx$$

$$= h\left[\frac{x^3}{3}\right]_{-b/2}^{b/2}$$

$$I_y = \frac{hb^3}{12}$$

$$Q_x = \int y_c \, dA$$

$$\hat{x} = \frac{\int x \, dA}{A}$$



$$I_x = \int y^2 dA$$

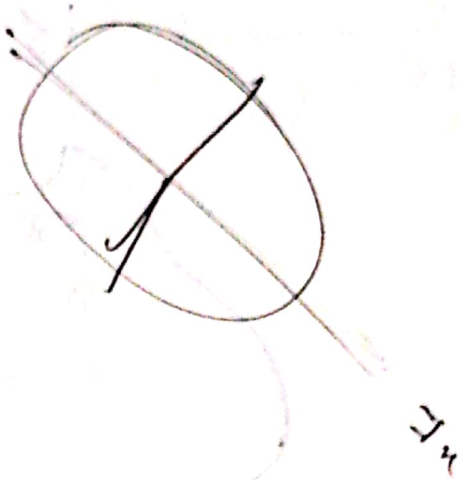
$$= \int y^2 (x_2 - x_1) dy$$

$$= \int_0^4 y^2 (4 - 2y^{1/2}) dy$$

$$\Rightarrow \left[\frac{4y^3}{3} + \frac{2y^{+5/2}}{5/2}\right]_0^4$$

$$\Rightarrow \frac{4\times4^3}{3} + \frac{2\times 4^{5/2}}{5/2}$$

$= 12.19$ unit^4



$$I_y = \int x^2 \, dA$$

$$= \int x^2 (y_2 - y_1)\, dx$$

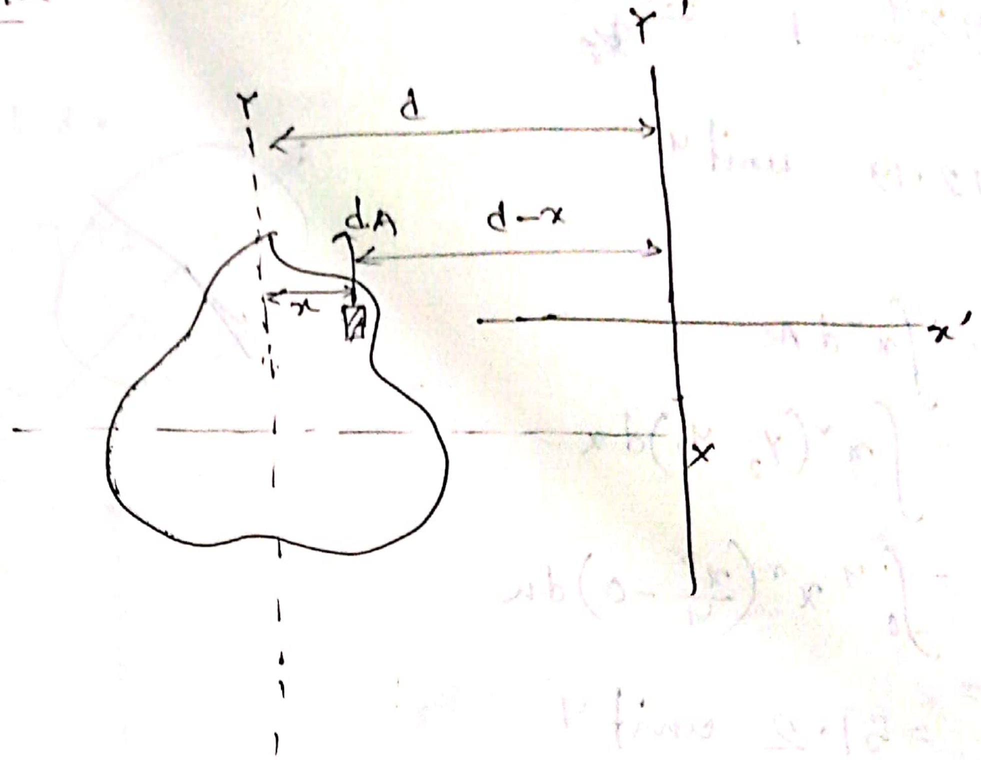
$$= \int_0^4 x^2 \left(\frac{x^2}{4} - 0\right) dx$$

$= 51.2$ unit^4

* Example – 136

* Example – 138

Transfer formula



$$I_y = \int x^2 \, dA$$

$$I_{y'} = \int (d-x)^2 \, dA$$

$$= \int (d^2 - 2dx + x^2) \, dA$$

centroid (0,0)

$$= \int x^2 \, dA - 2d \int \cancel{x} \, dA + d^2 \int dA$$

$$\to 0$$

$$= \bar{I}_y + Ad^2$$

$$\boxed{I_{y'} = \bar{I}_y + Ad^2}$$

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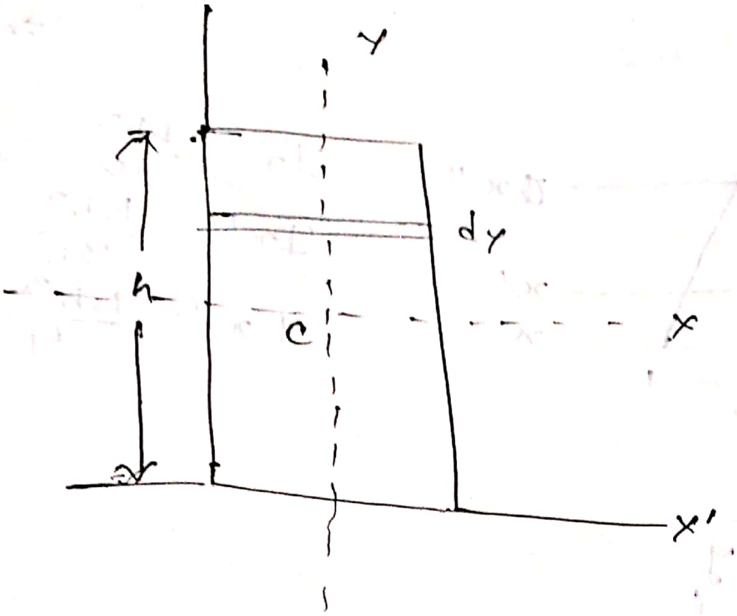


$$\boxed{I_x = \frac{bh^3}{12}}$$

$$I_y = \frac{hb^3}{12}$$

$$I_{x'} = \int y^2 \, dA$$

$$= \int_0^h y^2 b \, dy$$

$$\boxed{I_{x'} \Rightarrow \frac{bh^3}{3}}$$

$$I_{x'} = \bar{I}_x + Ad^2$$

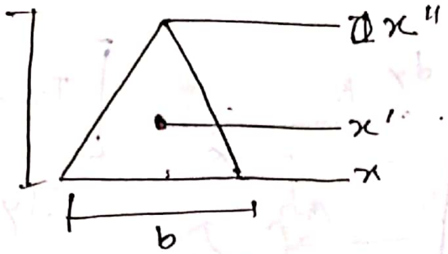
$$= \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2$$

$$\Rightarrow \frac{bh^3}{12} + \frac{bh^3}{4}$$

$$= \frac{bh^3}{3}$$

$$I_y' = \frac{hb^3}{3}$$

02 Triangle

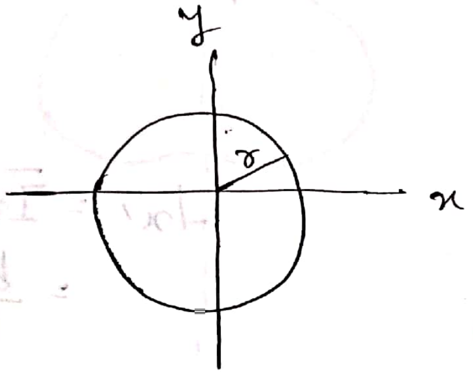


$I_x = \frac{bh^3}{12}$

$\bar{I}_{x'} = \frac{bh^3}{36}$

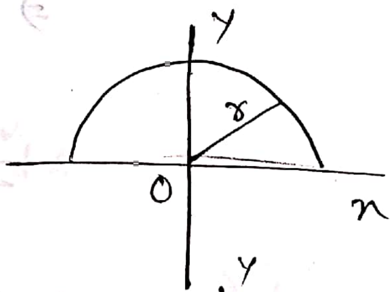
$I_{x''} = \frac{bh^3}{4}$

03 Circle



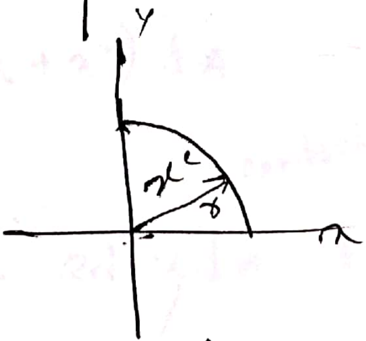
$\bar{I}_x = \bar{I}_y = \frac{\pi r^4}{4}$

04) Half circle



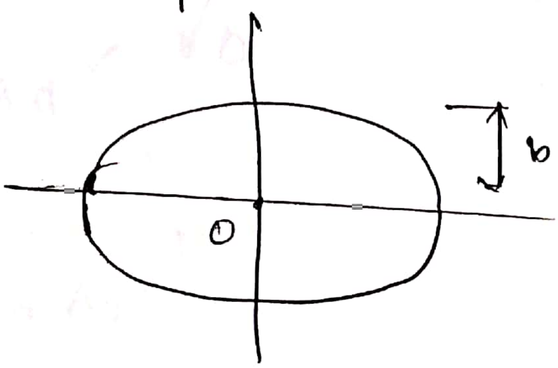
$I_x = I_y = \frac{\pi r^4}{8}$

05) Quarter circle



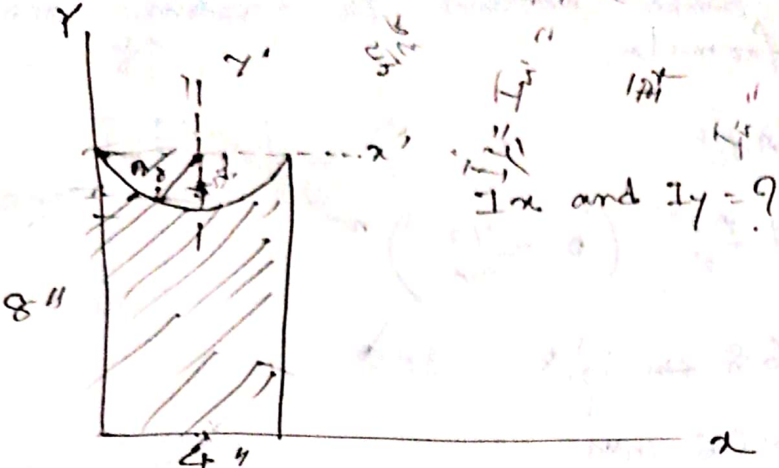
$I_x = I_y = \frac{\pi r^4}{16}$

06) Ellipse



$\bar{I}_x = \frac{\pi a b^3}{4}$

$\bar{I}_y = \frac{\pi a^3 b}{4}$



I_x and $I_y = ?$

Rectangle section

$$I_x = \frac{bh^3}{3} = \frac{4 \times 8^3}{3} = 682.67 \text{ in}^4$$

$$I_y = \frac{8(4)^3}{3} = 170.67 \text{ in}^4$$

$I_x' = I_y' = \frac{\pi r^4}{4}$

$I_x' = \bar{I}_x + Ah^2$

$\bar{I}_x$

$I'_y = \bar{I}_y + Ah^2$

$I_x = \bar{I}_x + Ah^2$

$I_y = \bar{I}_y + Ah^2$

Half circle

$$I_x' = I_y' = 6.28 \text{ in}^4$$

$$I_x' = \bar{I}_x + Ad^2$$

$$6.28 = \bar{I}_x + \frac{\pi 2^2}{2}$$

$$\bar{I}_x = 1.75 \text{ cm}^3$$

Her eyes just like two drops of coffee in the cloud of milk

Her [redacted]

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Again transfer formula

$$I_x = I_x + Ad^2$$

$$= 1.75 + \frac{\pi 2^2}{2}\left(8 - \frac{4r}{3\pi}\right)^2$$

$$\Rightarrow \cancel{46.68\ cm}\ in^8$$

$$= 323.06\ in^4$$

$$I_y = I_y' + Ad^2$$

$$= 6.28 + \frac{\pi(2)^2}{2}(2)^2$$

$$= 31.41\ in^4$$

Moment of inertia of composite area

$$I_x = 682.67 - 323.06$$

$$I_y = 170.67 - 31.41$$

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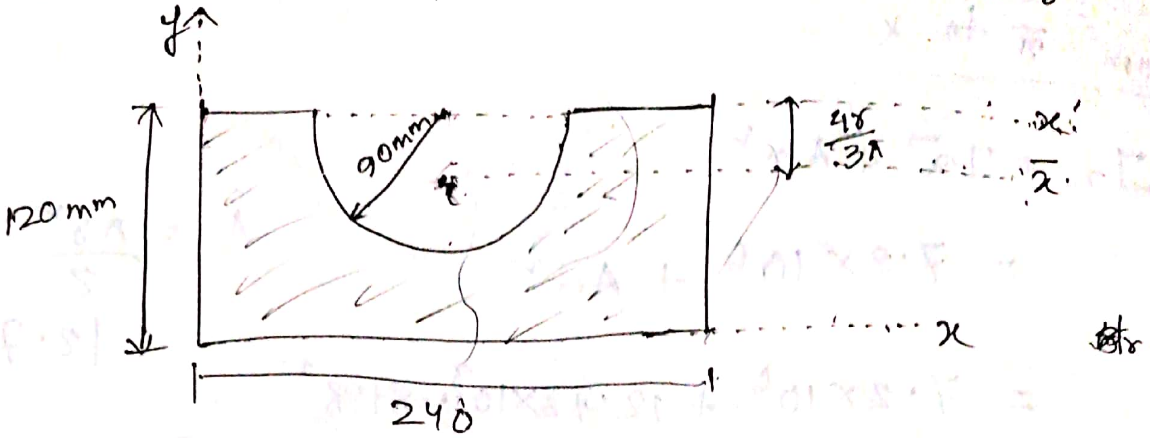
* Calculate moment of inertia about x-axis



<u>Soln</u> <u>Rectangular</u>

$$I_x = \frac{bh^3}{3} \qquad b = 240 \text{ mm}$$

$$= 138.2\times10^6 \text{ mm}^4 \qquad h = 120 \text{ mm}$$

<u>Semicircle</u>

$$I_{x'} = \frac{\pi r^4}{8} \qquad r = 90\text{mm}$$

$$= 25.76\times10^6 \text{ mm}^4$$

From x' to $\bar{x}$

$$I_{x'} = I_{\bar{x}} + Aa^2 \qquad a = \frac{4r}{3\pi} = 38.2$$

$$\Rightarrow I_{\bar{x}} = I_{x'} - Aa^2 \qquad A = \frac{\pi r^2}{2} = 12.72\times10^3$$

$$= \left[25.76\times10^6 - 12.72\times10^3 \times (38.2)^2\right.$$

$$= 7.2\times10^6 \text{ mm}^2$$

from $\bar{x}$ to x

$I_x = I_{\bar{x}} + Aa^2$

$= 7.2 \times 10^6 + Aa^2$

$A = \frac{\pi r^2}{2}$

$= 12.72 \times 10^3$

$= 7.2 \times 10^6 + 12.72 \times 10^3 \times 81.8^2$

$a = 81.8$ mm

$\Rightarrow 92.3 \times 10^6$ mm^2

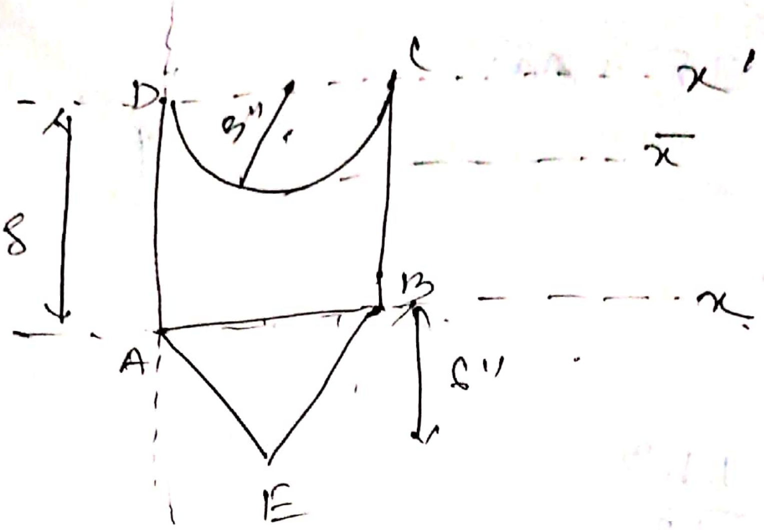
Total moment of inerchia

$= 138.2 \times 10^6 - 92.3 \times 10^6$

$\Rightarrow 45.9 \times 10^6$ mm^4

Ans.

* Calculate moment of inertia about x-axis



Rectangular

$$I_x = \frac{bh^3}{3}$$ $b = 6''$, $h = 8''$

$$= \frac{6 \times 8^3}{3}$$

$$= 1024 \text{ Inc}^4$$

Semi circular

$$I_{x'} = \frac{\pi r^4}{8} = \frac{\pi \times 3^4}{8}$$

$$= 31.8 \text{ Inc}^4$$

$I_{x'} = I_{\bar{x}} + Aa^2$

$a = \frac{4r}{3\pi}$

$I_{\bar{x}} = 31.8 - 14.13 \times 1.273$

$= \frac{4 \times 3}{3\pi}$

$= 13.8\ldots \text{ Inc}^4$

From $\bar{x}$ to x

$I_x = I_{\bar{x}} + Aa^2$

Triangle

$I_x = \frac{bh^3}{12}$

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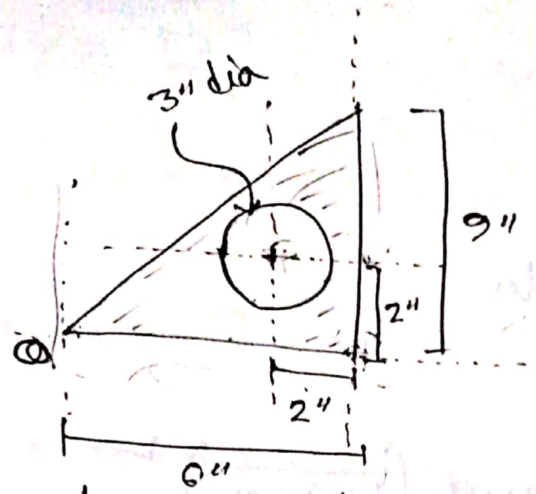
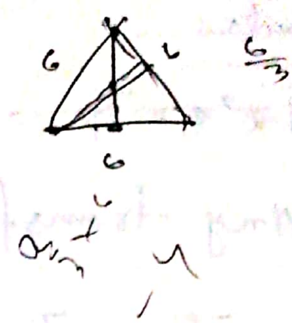
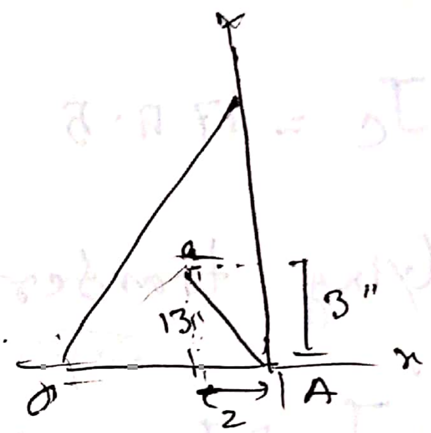




Find the polar moment of Inertia at Point O.



~~For circle~~

$\bar{I}_x = \frac{\pi r^4}{4} = \frac{\pi 3^4}{4}$

$= 63.61$

$\bar{I}_y$

For triangle

$I_x = \frac{bh^3}{12} = \frac{6\times 9^3}{12} = \cancel{3197\ in^4}\ 364.5\ in^4$

$I_y = \frac{hb^3}{12} = \frac{9\times 6^3}{12} = 162\ in^4$

$\cancel{I_x' = I_x + Aa^2}$

$J_A' = I_x + I_y = 526.5\ in^4$

~~For circle~~

~~$I_x = I_y = \frac{\pi r^4}{}$~~

Applying transfer formula

$$J_A = J_c + Ad^2$$

$$526.5 = J_c + \frac{1}{2}\times6\times9\left(\sqrt{3^2+2^2}\right)^2$$

$$J_C = 175.5$$

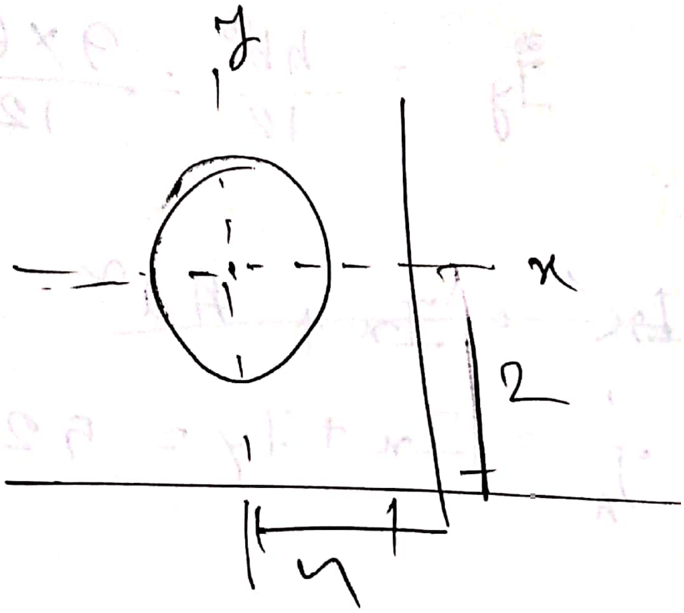
Again Applying transfer formula

$$J_o = J_c + Ad^2$$

$$= 175.5 + 27\times\left(\sqrt{3^2+4^2}\right)^2$$

$$= 850.5$$

For circle



$I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi (1.5)^4}{4}$

$= 3.97 \text{ in}^4$

$J_c = I_x + I_y$

$= 3.97 + 3.97$

$= 7.94 \text{ in}^4$

$J_o = J_c + Ad^2$

$\Rightarrow 7.94 \times \pi (1.5)^2 \times \sqrt{2^2 + 4^2}$

$\Rightarrow 149.31 \text{ in}^4$

$\therefore$ Polar moment of Inertia of composite figure

$\Rightarrow 850.5 - 149.31$

$\Rightarrow 701.19$

Polare gairation

$K_o = \sqrt{\frac{J_o}{A}}$

Assignment 1157, 1163, 1164, 1165, 1244