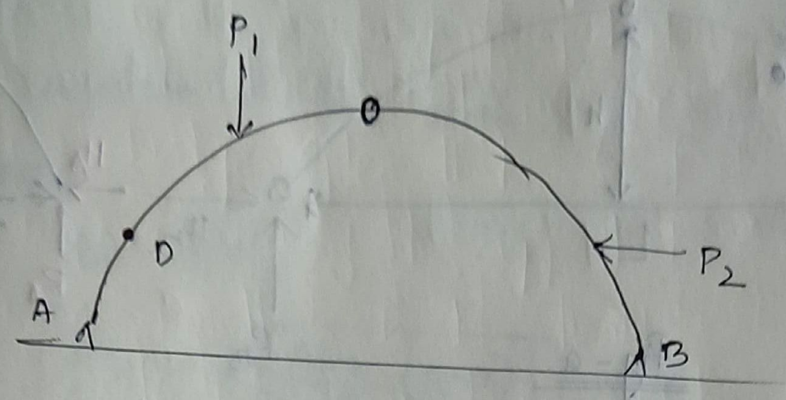
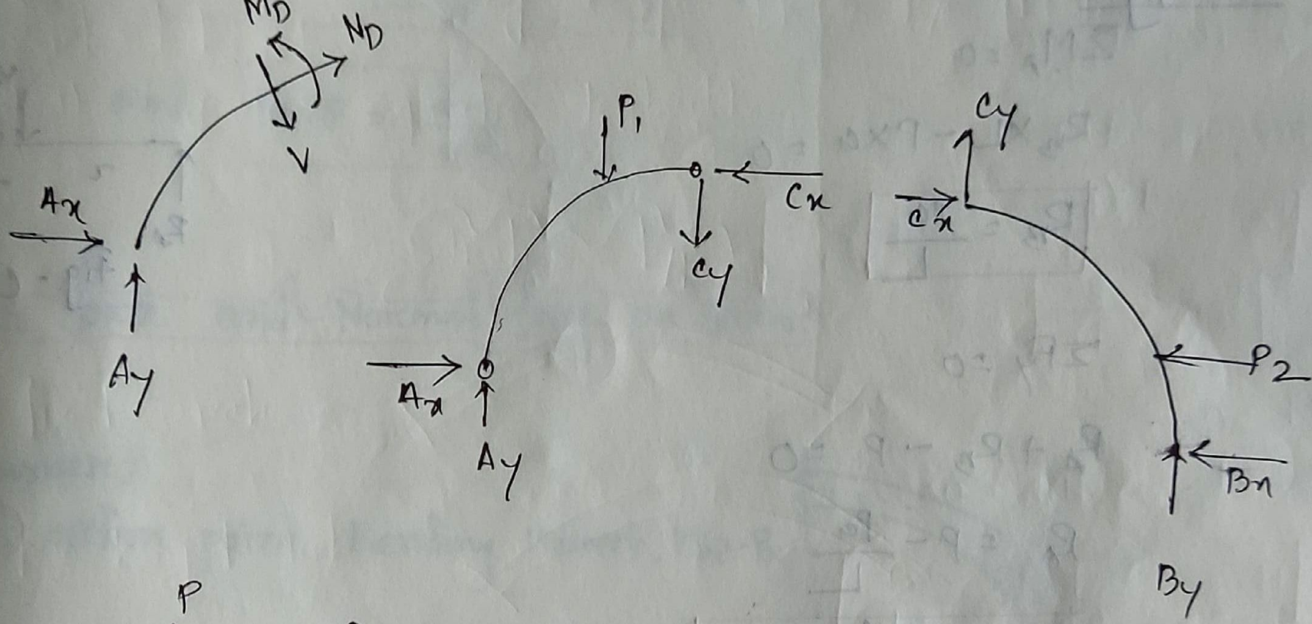
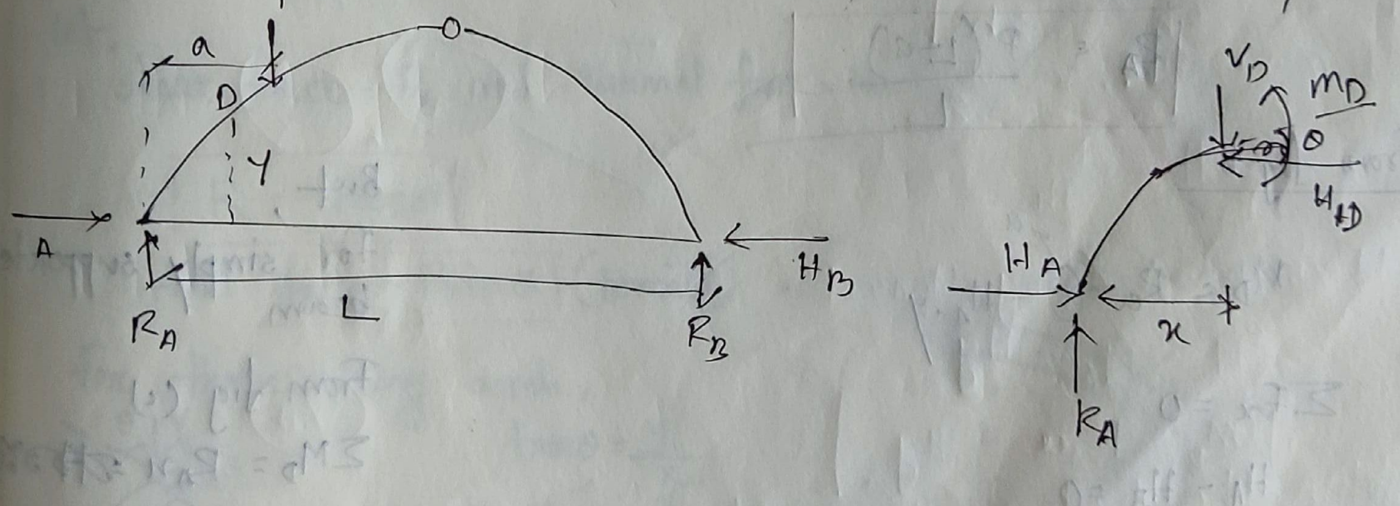
Three Hinge Arches







Formation of Determination Internal Force of Three Hinged Arches:

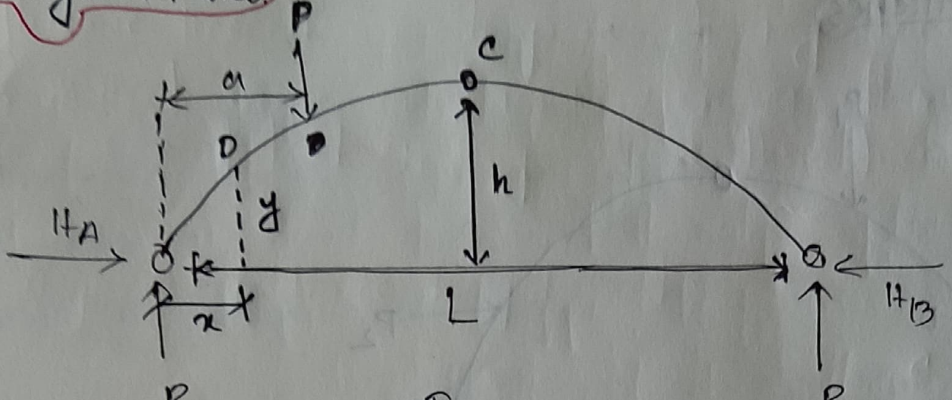


Fig - a

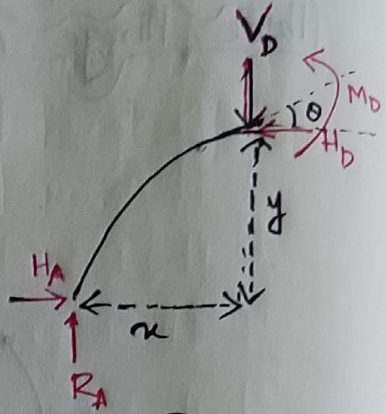


Fig - b

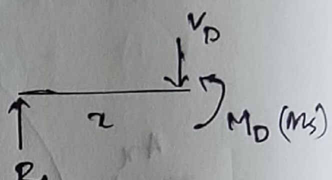


Fig - c

From fig a

$$\Sigma M_A = 0$$

$$R_B \times L - P \times a = 0$$

$$\boxed{R_B = \frac{Pa}{L}}$$

$$\Sigma F_y = 0$$

$$R_A + R_B - P = 0$$

$$R_A = P - \frac{Pa}{L}$$

$$\boxed{R_A = \frac{P(L-a)}{L}}$$

From fig b

$$M_D = R_a \times \overset{a'}{x} - H_A \cdot y$$

$$\Sigma F_x = 0$$

$$H_A - H_B = 0$$

$$H_A = H_B = H$$

But,
For simply supported beam
From fig (c)

$$\Sigma M_D = R_A x = M_s$$

So, From ①

$$\Sigma M_D = R_A x - Hy$$

$$\boxed{\Sigma M_D = M_s - Hy}$$

Here,

Reactions of an arch and Reactions of a beam are equal

Parabolic equation of the arch, $y = \frac{4hx}{L^2}(L-x)$

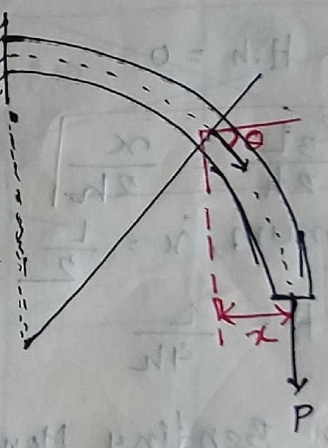
Now,

$$M_H = H * y$$

$$\boxed{M_H = H\left[\frac{4hx}{L^2}(L-x)\right]}$$

and $\boxed{\tan\theta = \frac{dy}{dx}}$

Shear force and Normal force on Arch:



Consider,

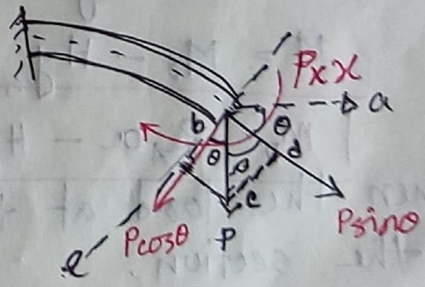
at section point, Bending Moment, $M_x = -Px$

let,

shear force $= F_x$ and Normal force $= N_x$

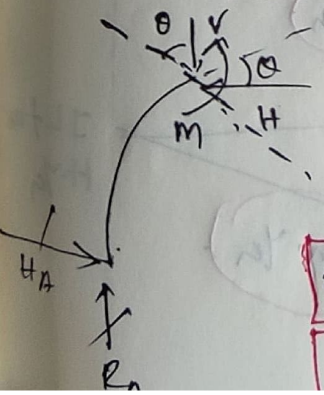
$\therefore \boxed{F_x = P\cos\theta}$

And $\boxed{N_x = -P\sin\theta}$ (tensile)



For three hinge arch,

$\tan\theta = \frac{y}{x}$



$\boxed{\text{shear force} = V\cos\theta - H\sin\theta}$

$\boxed{\text{Normal force} = V\sin\theta + H\cos\theta}$

$\angle abc = 90° = \theta + \beta$

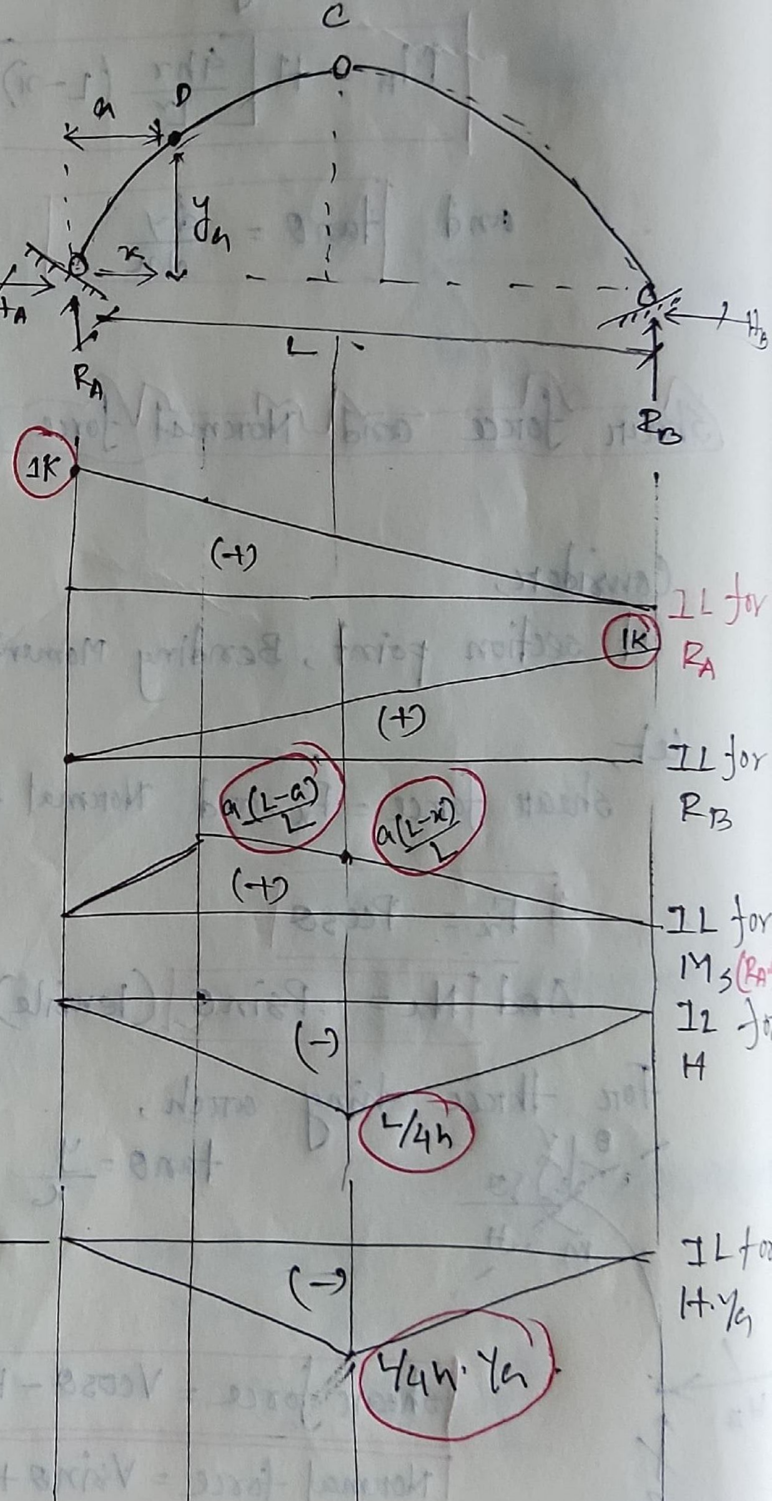
$\angle abd = \theta$; $\angle ebd = 90 - \theta$

$\angle bdc = \beta$; $\angle ebc = \theta$

Influence line:

An influence line is a diagram showing the variance of shear, stress, moment in a member, reaction or other direct function due to a unit load moving across the structure.

IL diagram of three hinged arch:



When 1K at A:

$R_A = 1K; \; R_B = 0$

When 1K at D:

$$\boxed{R_A = \frac{1(L-x)}{L} = 1 - \frac{x}{L}}$$

$$\therefore \boxed{R_B = \frac{x}{L}}$$

$\Sigma M_c = 0$

$R_B \times \frac{L}{2} - H.h = 0$

$$\boxed{H = \frac{R_B L}{2h} = \frac{x}{2h}}$$

At maximum, $x = \frac{L}{2}$

$\therefore H = \frac{L}{4h}$

Maximum Bending Moment

$M = M_s - Hy_a$

$$\boxed{M_D = R_A a - Hy_a}$$

when the load at the right of the section,

$$\boxed{V_D = R_A \cos\theta - H\sin\theta}$$

when 1k load at left side of the section

$$\boxed{V_D = (R_A - 1)\cos\theta - H\sin\theta}$$

IL for M_D

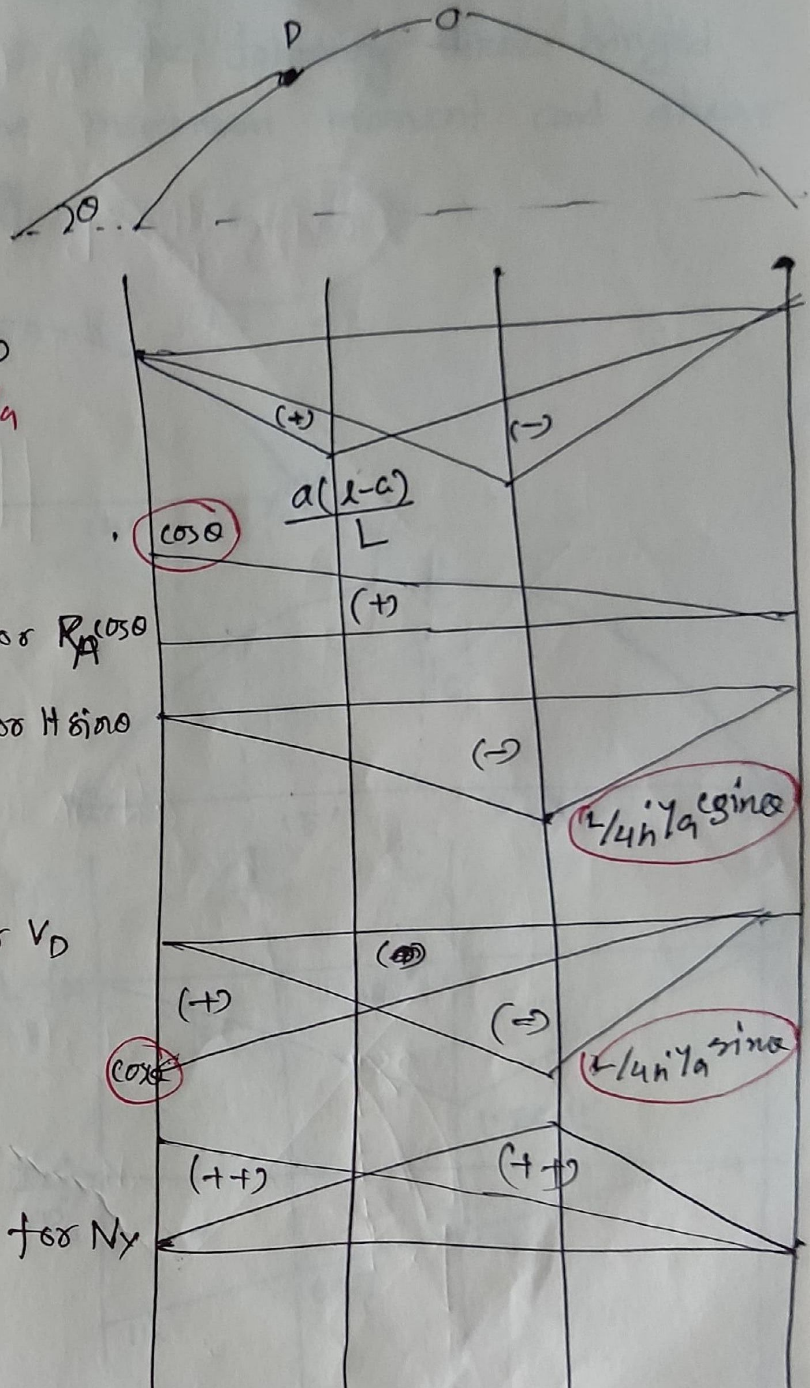
$M_s - H \cdot y_a$

IL for $R_A \cos\theta$

IL for $H\sin\theta$

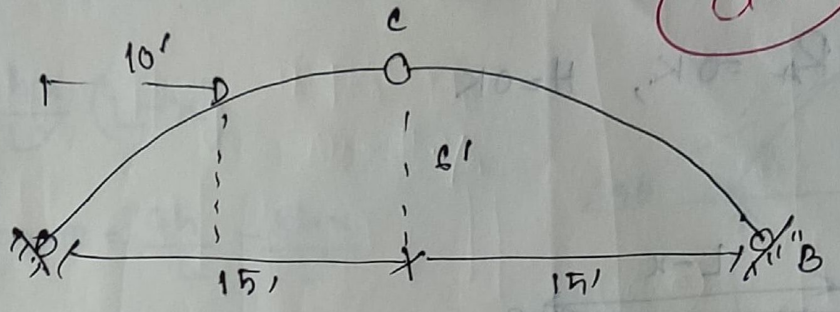
IL for V_D

IL for N_Y

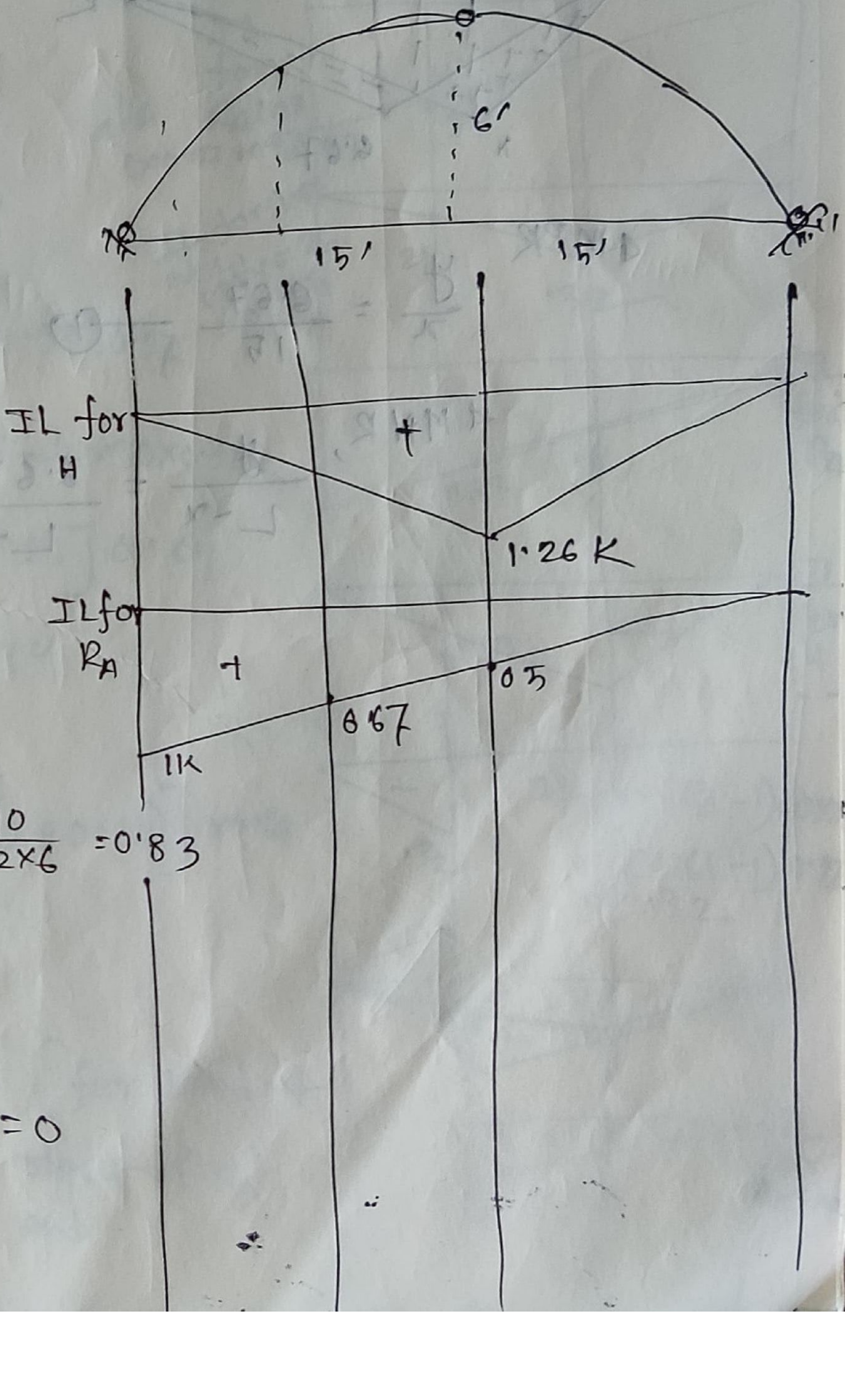


Prob 1. Draw influence lines for bending moment, shear force, and normal thrust at a section D of the following three hinged parabolic arch. Also obtain the maximum moment and shear at section D due H_{20} loading

Class



Solution:



Here, $L = 30'$

$h = 6'$

$a = 10'$

$$Y_a = \frac{4ha}{L^2}(L-a)$$

$$= \frac{4\times6\times10}{30^2}(30-10)$$

$$= 5.33 \text{ ft}$$

IL for horizontal thrust, H

Unit load at A, $H = 0$

Unit load at D, $H = \frac{a}{2h} = \frac{10}{2\times6} = 0.83$

Or,

$$R_A = \frac{L-a}{L} = \frac{2}{3}K$$

$\Sigma M_C = 0$

$R_A \times 15 - H\times6 - 1(15-10) = 0$

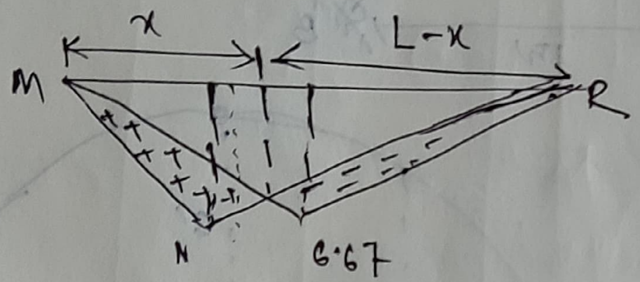
$H = 0.83K$

unit load at C, $H = \frac{L}{4h} = \frac{30}{4\times 6}$

$= 1.25K$

unit load at B,

$R_A = 0K, \quad H = 0K$



ΔMPR $\quad \frac{y}{x} = \frac{6.67}{15}$ — (i)

ΔMNR, $\quad \frac{y}{L-x} = \frac{6.67}{L-a}$ — (ii)

Sheare forcce,

$V_D = R_A \cos\theta - H \sin\theta$ [for load at right of D]

$V_D = (R_A - 1)\cos\theta - H\sin\theta$ [for load at left of D],

$$y = \frac{4hx}{L^2}(L-x)$$

$$\therefore \frac{dy}{dx} = \tan\theta = \frac{4h}{L} - \frac{8hx}{L^2}$$

$$= \frac{4\times 6}{30} - \frac{8\times 6\times 10}{30^2}$$

$$= 0.267$$

$$\theta = \tan^{-1}(0.267) = 14.93^\circ$$

$\sin 14.93^\circ = 0.26$ and $\cos 14.93^\circ = 0.97$

$$V_D \max(+ve) = \left[\frac{1}{2}\times 20\times 0.65 - \frac{1}{2}\times 30\times 0.32 + \frac{1}{2}\times 10\times 0.21\right] 0.64$$

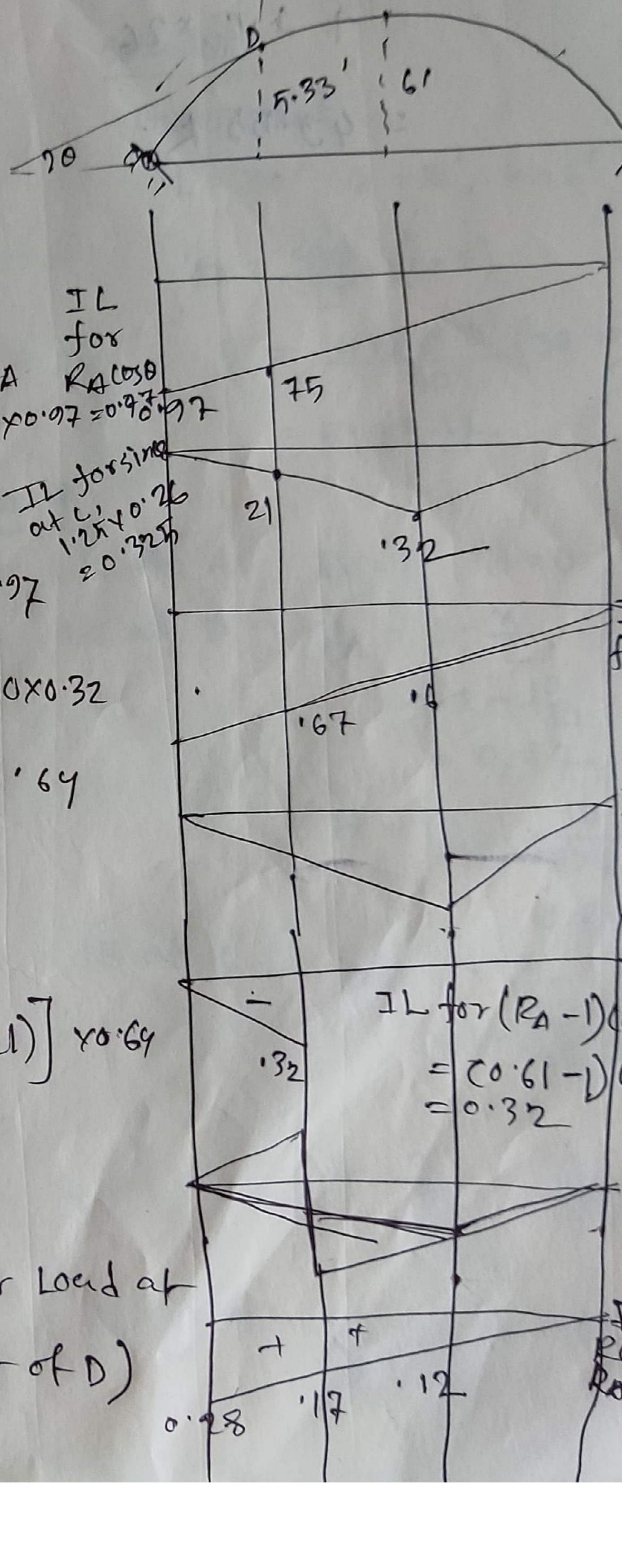
$$= 1.76 + 11.44$$

$$= 13.2\,K$$

$$V_D(\max)(-ve) = \left[\frac{1}{2}\times 10\times(0.32+0.21)\right]\times 0.64 + 0.32\times 26$$

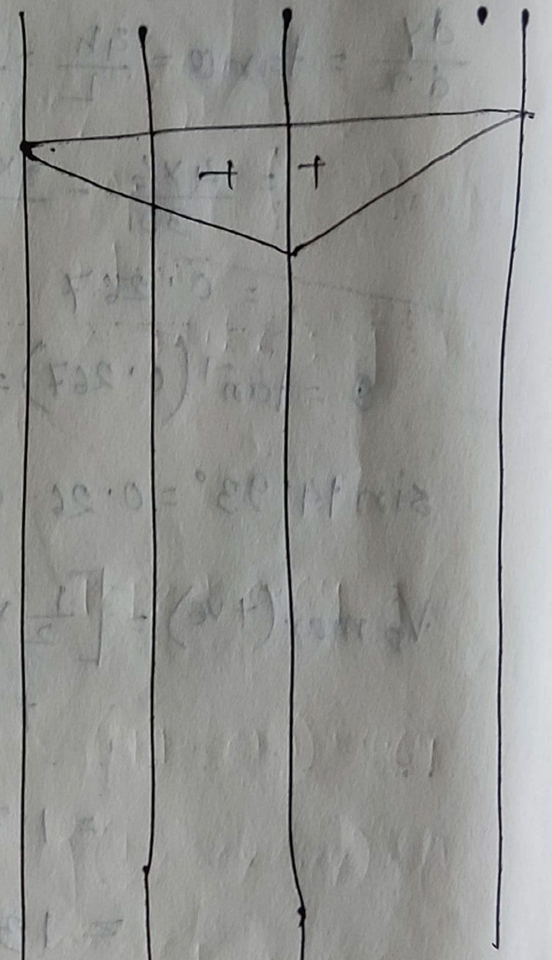
$$= 10.02\,K$$

$N.T_D = R_A \sin\theta + H\cos\theta$ (for Load at right of D) and



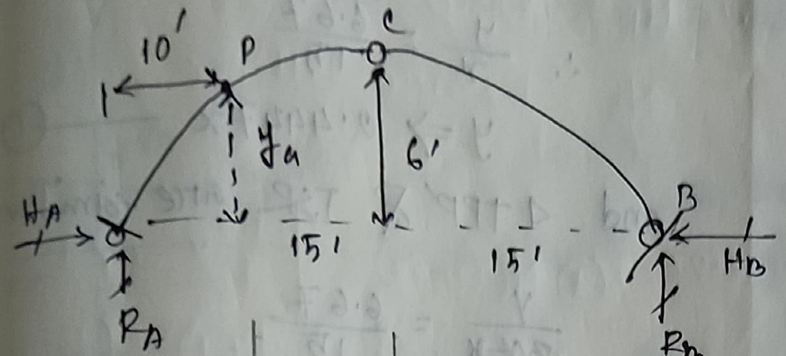
$N T_D = (R_A - 1)\sin\theta + H\cos\theta$ [for load at right of D]

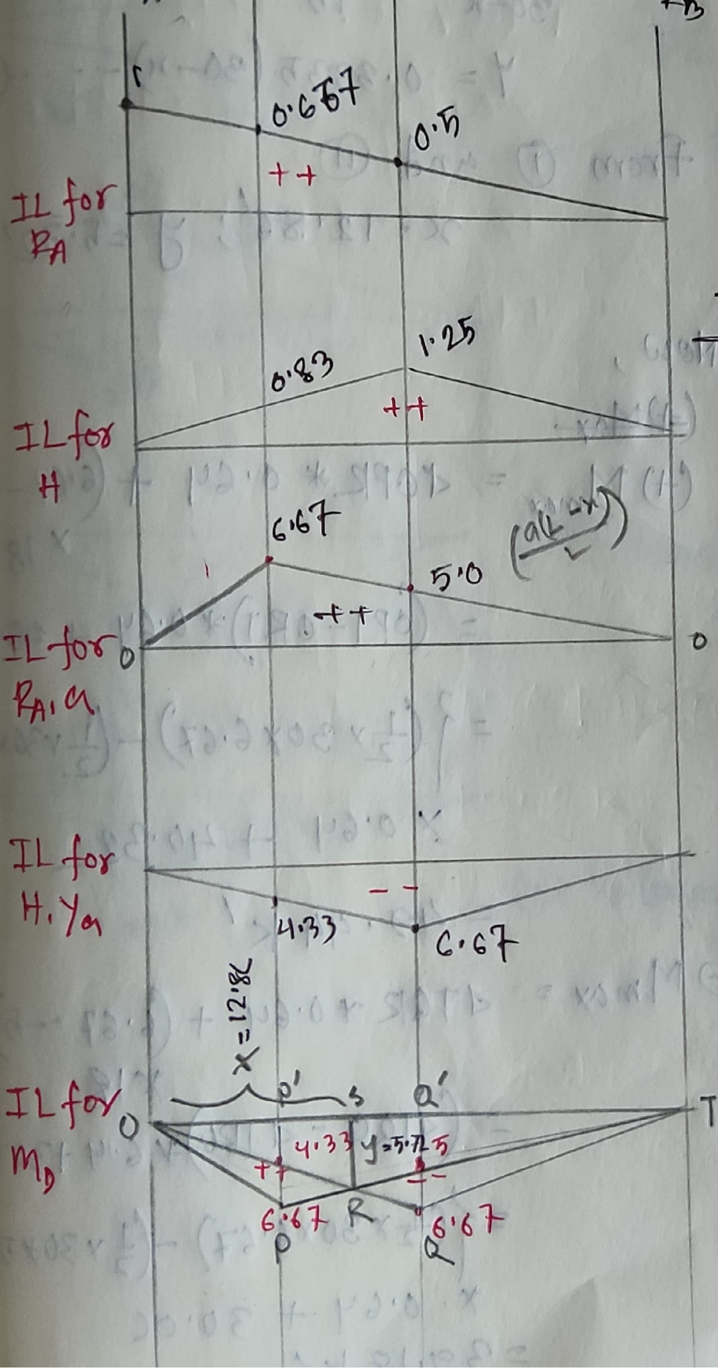
$N.T = \frac{1}{2} \times 30 \times 1.2 - \frac{1}{2} \times 10 \times 0.08 + \frac{1}{2} \times 0.17 \times 20] \times 0.64$
$+ 1.2 \times 26$
$= 43.55\ k$



Problem : 01

Draw IL for three hinged arch - (i) Horizontal thrust, H (ii) Moment at D, M_D (iii) Shear at D, V_D (iv) Normal thrust, N_D. Also compute shear force, bending moment, normal thrust due to H20 loading.





Solution:

Given that,

$L = 30'$ $a = 10'$, $h = 6'$

$$y_a = \frac{4ha}{L^2}(L-a)$$

$$= \frac{4 \times 6 \times 10'}{30^2}(30-10)$$

$$= 5.33\ ft$$

IL for H

When, 1 k at A ; $R_A = 1$ $H = 0$

1K at D ; $R_A = 1 - \frac{x}{L}$

$= 1 - \frac{10}{30}$

$= 0.67$

$H = \frac{x}{2h} = \frac{10}{2 \times 6}$

$= 0.83$

1K at C ; $R_A = 1 - \frac{x}{L}$

$= 1 - \frac{15}{30}$

$= 0.5$

$H = \frac{x}{2h} = \frac{15}{2 \times 6}$

$= 1.25$

1K at B ; $R_A = 0$ $H = 0$

$R_B = 1$

IL for M_D

$M_D = R_A a - H y_A$

From similar triangle,

$\triangle OQQ'$ & $\triangle OSR$ are simillar

$\therefore \frac{y}{x} = \frac{6.67}{15}$

$y \Rightarrow 0.4447x$ ——— (i)

and $\triangle TPP'$ & TSR are simillar

$\frac{y}{30-x} = \frac{6.67}{15}$

$y = 0.3335(30-x)$ ...... (ii)

from (i) and (ii)

$x = 12.86'$; $y = 5.72'$

Now,

~~(+) Max~~

$(+) M_{max} = \triangle OPR * 0.64 + (6.67 - 4.4) \times 18$

$= (OPT - ORT) * 0.64 + 40.32$

$= \left\{\left(\frac{1}{2} \times 30 \times 6.67\right) - \left(\frac{1}{2} \times 30 \times 5.72\right)\right\} \times 0.64 + 40.32$

$= 49.44 \; k'$

$(-) M_{max} = \triangle TQR * 0.64 + (6.67 - 5) \times 18$

$= (\triangle TQO - \triangle TRO) * 6.4 + 30.06$

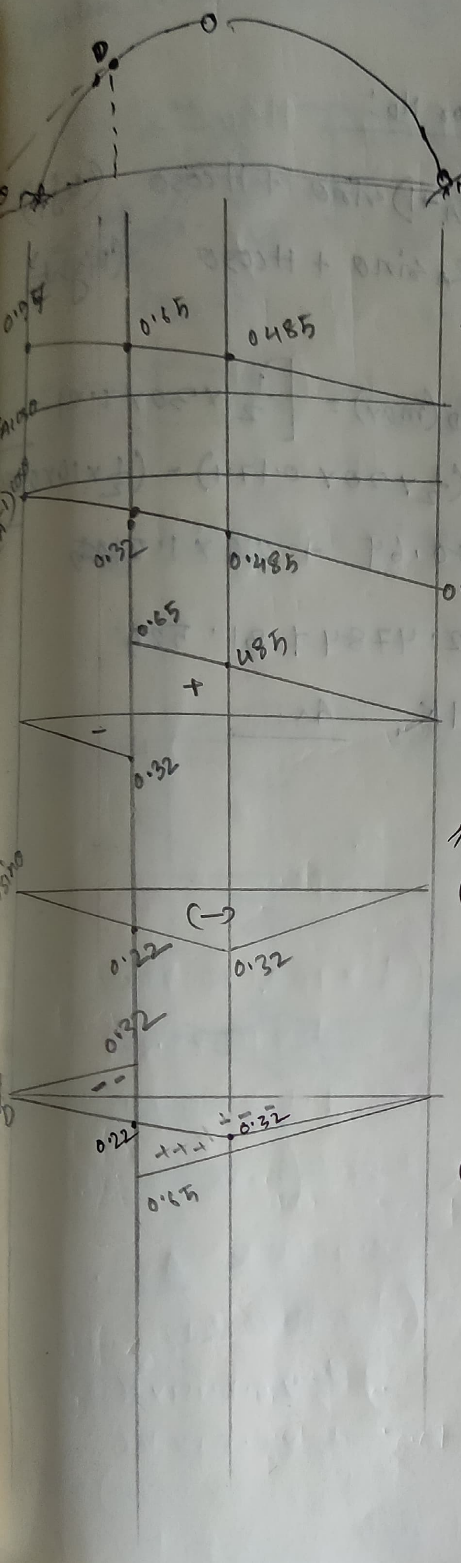
$= \left\{\left(\frac{1}{2} \times 30 \times 6.67\right) - \left(\frac{1}{2} \times 30 \times 5.72\right)\right\} \times 0.64 + 30.06$

$= 39.18 \; k'$

IL for V_D:

$V_D = (R_A - 1)\cos\theta - H\sin\theta$ [1k at just left of D]

$V_D = R\cos\theta - H\sin\theta$ [1k at just right of D]



IL for,

we know,

$$y = \frac{4hx}{L^2}(L-x)$$

$$\frac{dy}{dx} = \tan\theta = \frac{4h}{L} - \frac{8hx}{L^2}$$

$$\therefore \tan\theta = \frac{4h}{L} - \frac{8ha}{L^2}$$

$$= \frac{4\times6}{30} - \frac{8\times6\times10}{30^2}$$

$$\tan\theta = 0.267$$

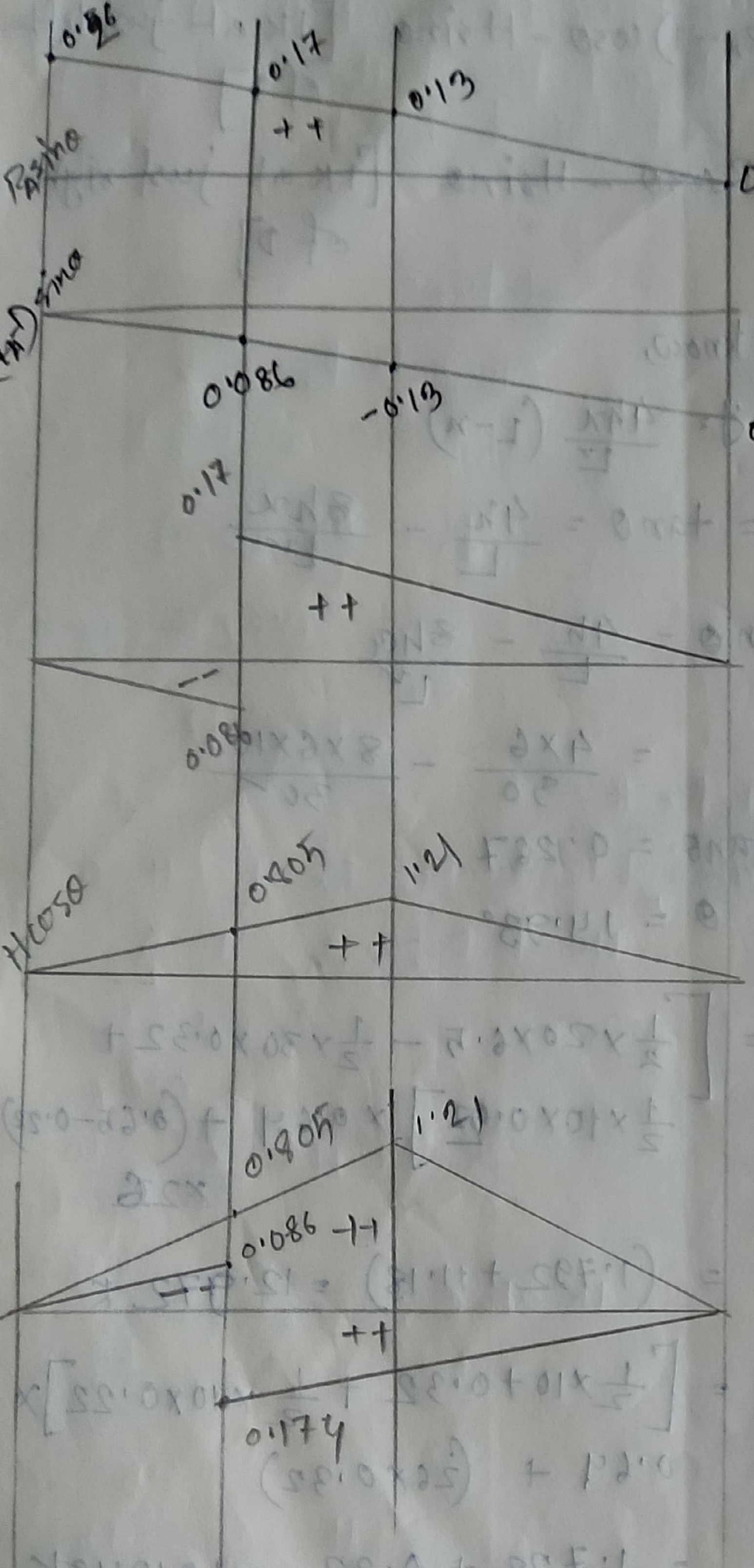
$$\theta = 14.93^\circ$$

$$(+)\,V_{D(max)} = \left[\frac{1}{2}\times20\times6.5 - \frac{1}{2}\times30\times0.32 + \frac{1}{2}\times10\times0.92\right]\times0.64 + (0.65-0.22) \times 26$$

$$= (1.792 + 11.18) = 12.972\text{ k}$$

$$(-)\,V_{D(max)} = \left[\frac{1}{2}\times10+0.32 + \frac{1}{2}\times10\times0.22\right]\times 0.64 + (26\times0.32)$$

$$= 1.728 + 8.32 = 10.048\text{K}$$



IL for N_D:

$N_D = (R_A - 1)\sin\theta + H\cos\theta$ (Left)

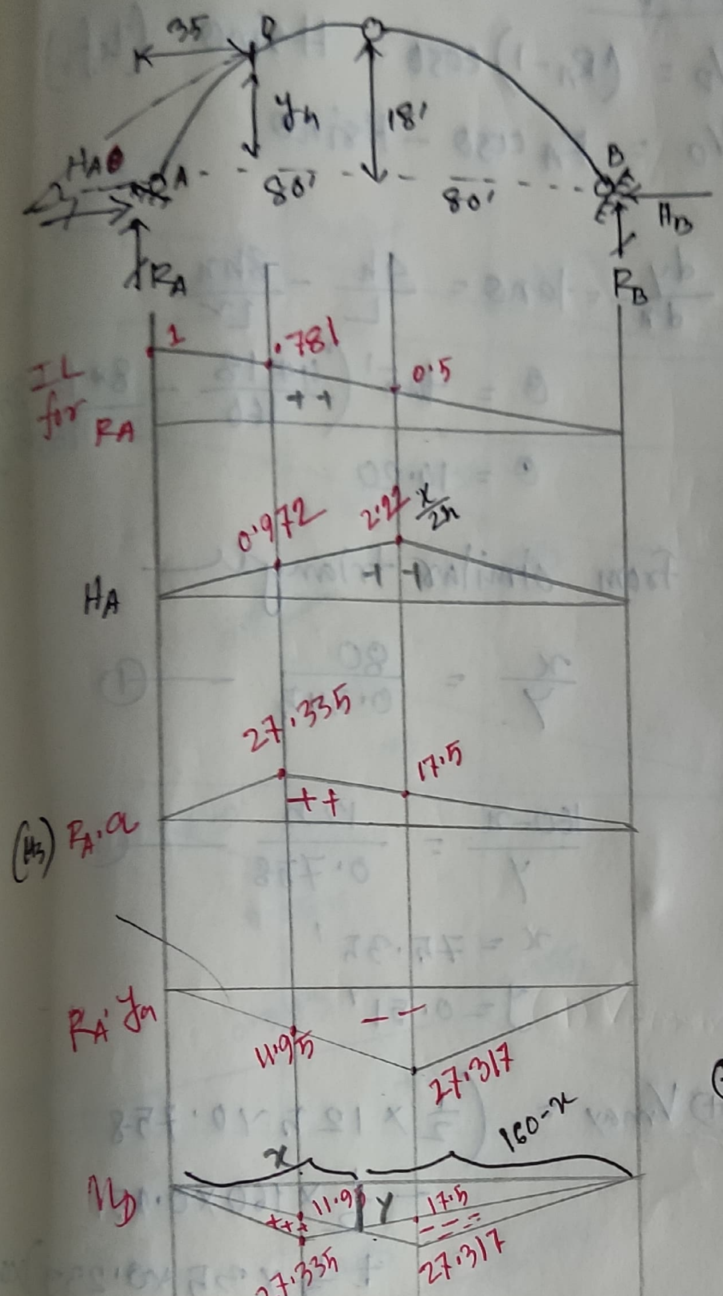
$N_D = R_A \sin\theta + H\cos\theta$ (Right)

$(+)\ N_D(max) = \Big[\frac{1}{2} \times 30 \times 1.21$
$+ (\frac{1}{2} \times 20 \times 0.174) - (\frac{1}{2} \times 10 \times 0.0$
$\times 0.64 + 26 \times 1.21\cancel{85}$

$= 12.4784 + 31.525$

$= 44K$ Ans

Problem: (Due to H_{20} loading)



IL for M_D

Given that $L = 160'$; $h = 18'$; $a = 35$

$$y_a = \frac{4ha}{L^2}(L-a)$$

$$= \frac{4 * 18 * 35}{160^2}(160-35)$$

$$= 12.305'$$

From similar triangle.

$$\frac{x}{y} = \frac{80}{27.317} \quad \text{--- (I)}$$

$$\frac{160-x}{y} = \frac{125}{27.335} \quad \text{--- (II)}$$

From (I) and (II)

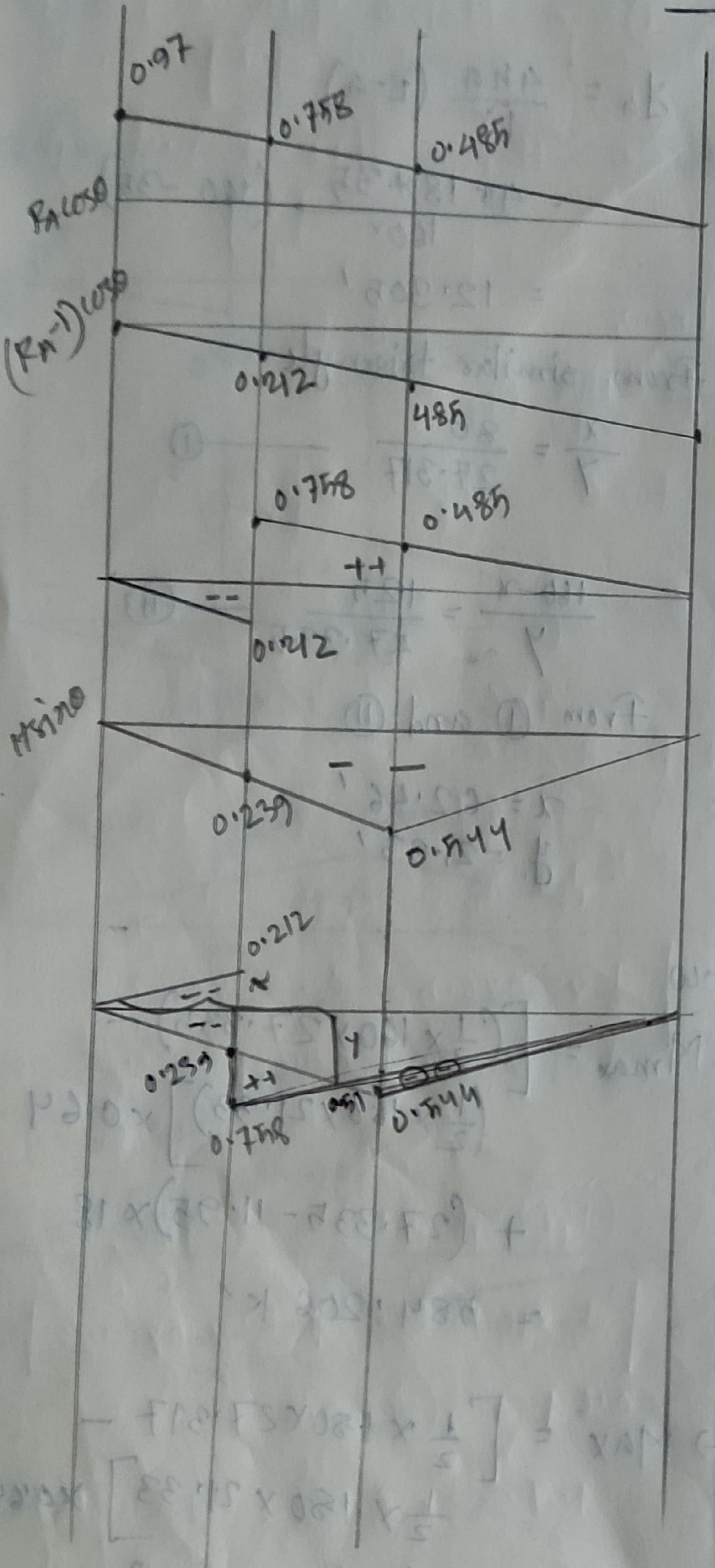
$$x = 62.46'$$
$$y = 21.33'$$

Now

$$(+)\ M_{max} = \left[\left(\frac{1}{2}\times 160\times 27.335\right) - \left(\frac{1}{2}\times 160\times 21.33\right)\right]\times 0.64$$
$$+ (27.335 - 11.95)\times 18$$
$$= 584.206\ K'$$

$$(-)\ Max = \left[\frac{1}{2}\times 1.80\times 27.317 - \frac{1}{2}\times 180\times 21.33\right]\times 0.64$$
$$+ (27.317 - 17.5)\times 18$$
$$= 483.24\ K'$$

IL for V_D



$V_D = (R_A - 1)\cos\theta - H\sin\theta$ (left)

$V_D = R_A\cos\theta - H\sin\theta$

$\frac{dy}{dx} = \tan\theta = \frac{4h}{L} - \frac{8hx}{L^2}$

$\theta = \tan^{-1}\left(\frac{4*18}{160} - \frac{8*18*35}{160^2}\right)$

$\theta = 14.20$

from similar triangle

$\frac{x}{y} = \frac{80}{0.545}$ — (i)

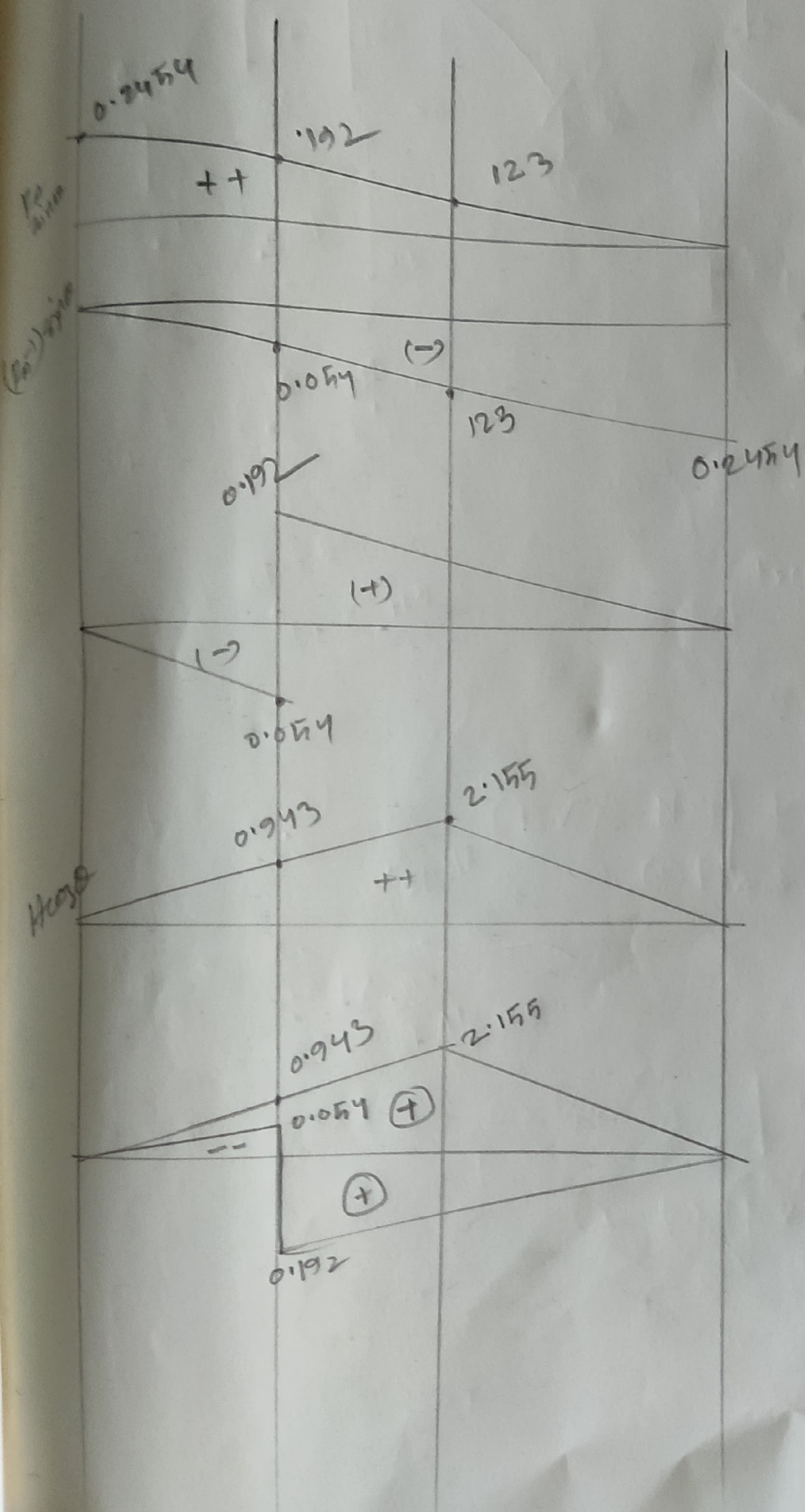
$\frac{160-x}{y} = \frac{125}{0.758}$ — (ii)

$x = 75.35'$

$y = 0.51'$

$(+) V_{max} = \left(\frac{1}{2} \times 125 \times 0.758 - \frac{1}{2} \times 160 \times 0.051 + \frac{1}{2} \times 35 \times 0.239\right) \times 0.64$
$+ 26 \times (0.758 - 0.239)$
$= 20.4308\ K$ Ans.

$(-) V_{max} = \left(\frac{1}{2} \times 35 \times 0.212 + \frac{1}{2} \times 0.544 \times 160 - \frac{1}{2} \times 0.51 \times 160 + \frac{1}{2} \times 35 \times 0.239\right) \times 0.64$
$+ 26 \times 0.239 = 13.0572\ K$



IL for N_D

$N_D = (R_A - 1)\sin\theta + H\cos\theta$

$N_D = R_A \sin\theta - H\cos\theta$

$$(+)\ N_D(max) = \left[\frac{1}{2}\times 125 \times 0.192 + \left(\frac{1}{2}\times 160 \times 2.155\right) - \frac{1}{2}\times 35 \times 0.054\right] \times 0.64 + 26 \times 2.155$$

$$= 174.03\ K$$

Ans