Structural Analysis & Design - I

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A Hand-note On

STRUCTURAL ANALYSIS & DESIGN - I

CE 312

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Topics

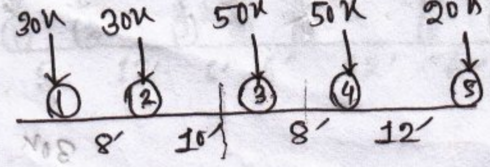
Wheel load
Truss
3D truss
Three hinged arch
Suspension bridge

Wheel load

10

AHSAN
090063

1. For a span of 70 ft, calculate the maximum moment at the center and absolute maximum moment due to the following loads as shown below.

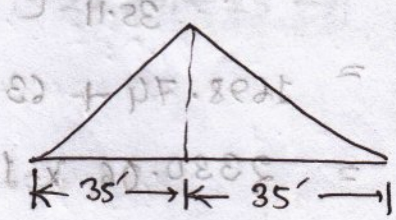


$$\bar{x} = \frac{50\times(12+20)+30(30+38)}{20+2\times50+2\times30} = 20.22$$

when wheel ① at center,

Right: $\frac{0}{35} < \frac{160}{70}$

Left: $\frac{30}{35} < \frac{160}{70}$



wheel ② at center,

Right: $\frac{30}{35} < \frac{180}{70}$

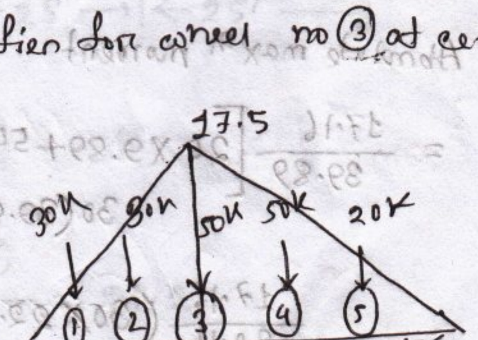
Left: $\frac{60}{35} < \frac{180}{70}$

wheel ③ at center,

Right: $\frac{60}{35} < \frac{180}{70}$

Left: $\frac{110}{35} > \frac{180}{70}$

Criteria satisfies for wheel no ③ at center



$$M_{max} = \frac{17.5}{35}\left[20\times15+50(27+35)\right] + \frac{17.5}{35}\left[30(17+25)\right]$$

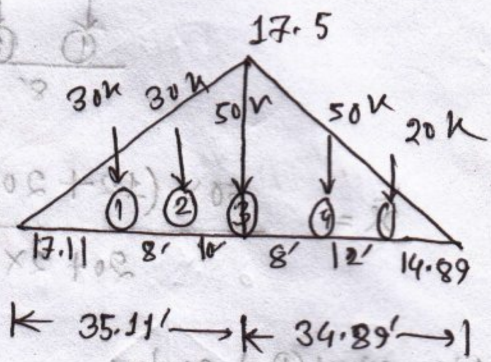
$= 1700 + 630$

$= 2330$ k-ft.

$a = 20.22 - 20 = 0.22'$

$\frac{a}{2} = 0.11'$

$x = \frac{L}{2} - \frac{a}{2} = \frac{70}{2} - 0.11 = 34.89$



Absolute max^m moment

$= \frac{17.5}{34.89}\left[20 \times 14.89 + 50(26.89 + 34.89)\right]$

$+ \frac{17.5}{35.11}\left[30(17.11 + 25.11)\right]$

$= 1698.74 + 631.32$

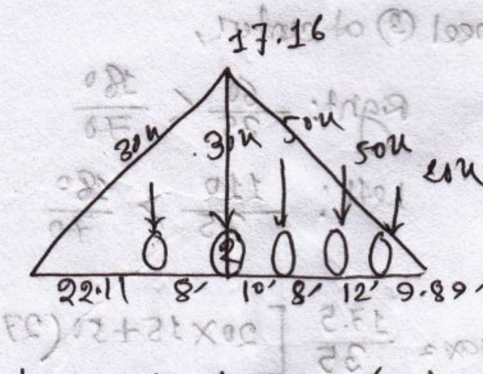
$= 2330.06$ k-ft (Ans).

Let, max^m absolute moment will occur by wheel 2

$a = 30 - 20.22 = 9.78'$

$\frac{a}{2} = 4.89$

$x = \frac{L}{2} + \frac{a}{2} = 35 + 4.89 = 39.89'$



Absolute max^m moment

$= \frac{17.16}{39.89}\left[20 \times 9.89 + 50(21.89 + 29.89) + 30(39.89)\right] +$

$\frac{17.16}{30.11}(30 \times 22.11)$

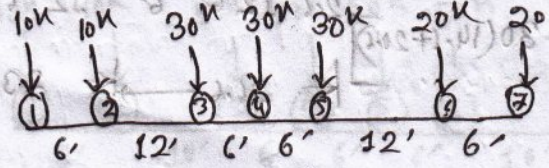
$= 1713.63 + 378.02$

$= 2091.65$ k-ft

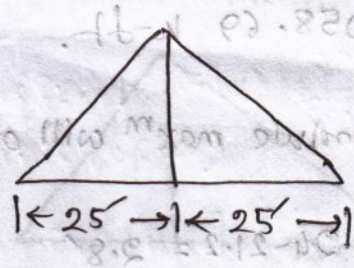
$M_{max} = 2330.06$ k-ft. (Ans).

06

2. For a simple span of 50 ft. calculate (i) the maximum moment at mid span and (ii) the absolute maximum moment due to the loading shown in figure below.



Solution:



wheel 3 at mid span,

Right : $\frac{20}{25} < \frac{110+20}{50}$

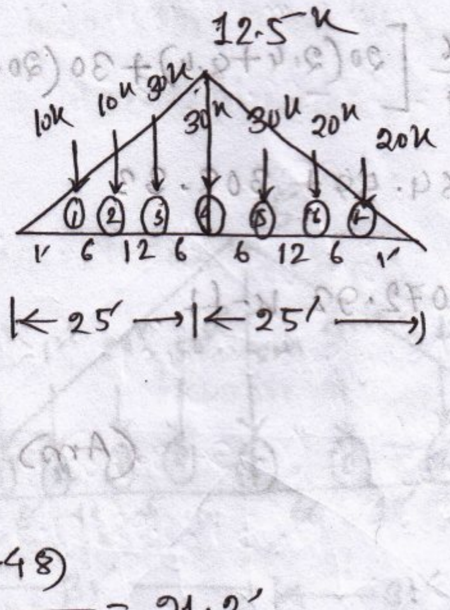
Left : $\frac{50}{25} < \frac{110+20}{50}$

wheel 4 at mid span

Right : $\frac{50}{25} < \frac{150}{50}$

Left : $\frac{80}{25} > \frac{150}{50}$

For wheel 4 maxm moment will occur.



Maxm moment

$= \frac{12.5}{25}\left[20(1+7)+30(19+25)\right] + \frac{12.5}{25}\left[10(1+7)+30\times 19\right]$

$= 740 + 325 = 1065$ k-ft.

$$\bar{x} = \frac{20\times 6 + 30(18+24+30) + 10(42+48)}{2\times 10 + 3\times 30 + 2\times 20} = 21.2'$$

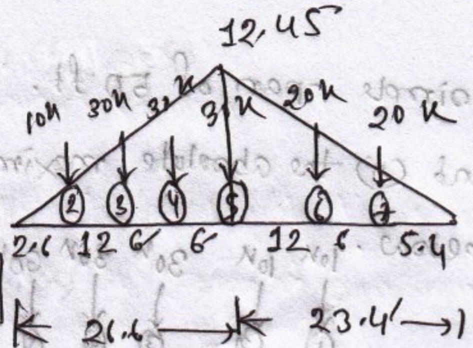
Let, absolute maxm will occur at wheel 5

$a = 21.2 - 18 = 3.2$

$\frac{a}{2} = 1.6$

$x = \frac{L}{2} - \frac{a}{2} = 25 - 1.6 = 23.4$

'Absolute' max^m moment]—



$$= \frac{12.45}{23.4}\left[20(5.4+11.4)+30\times 23.4\right]+$$

$$\frac{12.45}{21.6}\left[10\times 2.6+30(14.6+20.6)\right]$$

$= 552.27 + 506.42$

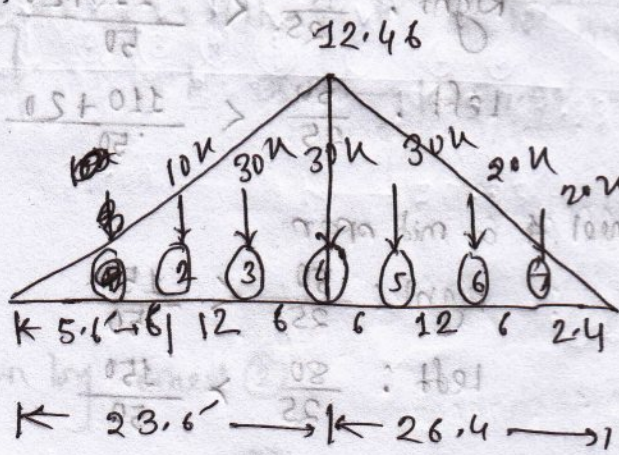
$= 1058.69$ k-ft.

Let, absolute max^m will occur at wheel 4.

$a = 24 - 21.2 = 2.8'$

$\frac{a}{2} = 1.4$

$x = \frac{L}{2} + \frac{a}{2} = 25 + 1.4 = 26.4$



Absolute max^m moment

$$= \frac{12.46}{26.4}\left[20(2.4+8.4)+30(20.4+26.4)\right]+\frac{12.46}{23.6}\left[10\times 5.6+30\times 17.6\right]$$

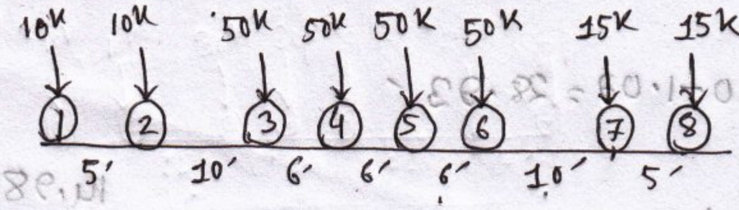
$= 764.59 + 308.33$

$= 1072.92$ k-ft.

(Ans).

$\boxed{07}$

3. For a span of 60 ft. calculate due to loading shown (i) the maximum moment at the center and (ii) the absolute maximum moment.

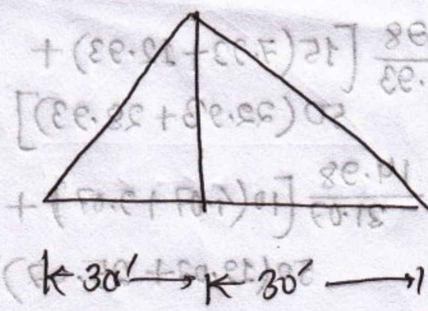


Solution:

wheel 3 at mid span,

Right: $\frac{20}{30} < \frac{235}{60}$

Left: $\frac{70}{30} < \frac{235}{60}$



wheel 4 at mid span,

Right: $\frac{70}{30} < \frac{250}{60}$

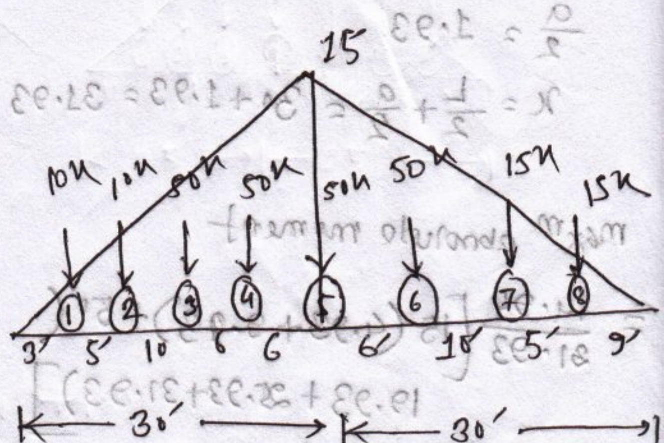
Left: $\frac{120}{30} < \frac{250}{60}$

wheel 5 at mid span,

Right: $\frac{120}{30} < \frac{250}{60}$

Left: $\frac{170}{30} > \frac{250}{60}$

max^m moment will occur for wheel 5



$$\text{Max}^m \text{ moment} = \frac{15}{30}\left[15(9+14)+50(24+30)\right] + \frac{15}{30}\left[10(3+8)+50(18+24)\right]$$

$= 1522.5 + 1105$

$= 2627.5$ k-ft.

(Ans).

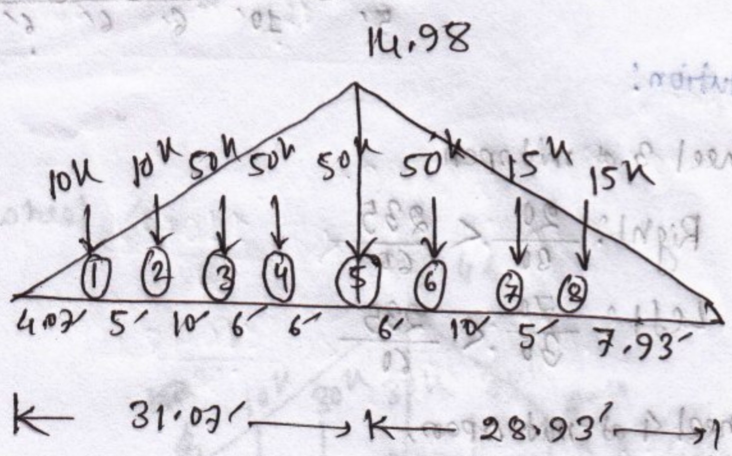
$$\bar{x} = \frac{15\times5+50(15+21+27+33)+10(43+48)}{2\times10+4\times50+2\times15} = 23.14'$$

Let, absolute max^m will occur at wheel 5

$a = 23.14 - 21 = 2.14$

$\frac{a}{2} = 1.07$

$x = \frac{L}{2} - \frac{a}{2} = 30 - 1.07 = 28.93$



max^m absolute moment =

$$\frac{14.98}{28.93}\left[15(7.93+12.93) + 50(22.93+28.93)\right] + \frac{14.98}{31.07}\left[10(4.07+9.07) + 50(19.07+25.07)\right]$$

$= 1504.68 + 1127.43$

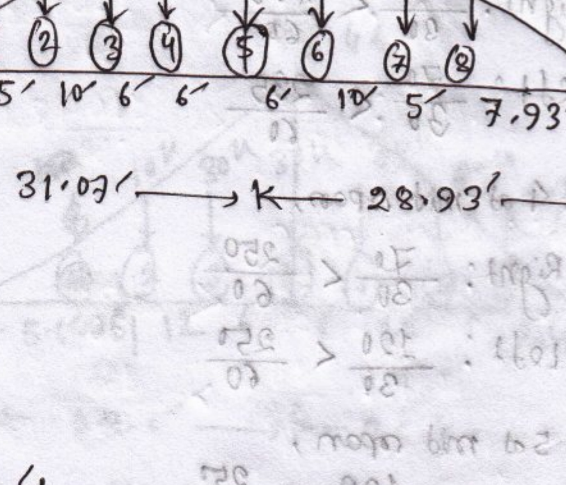
$= 2632.11$ k-ft

Let absolute max^m will occur at wheel 4

$a = 27 - 23.14 = 3.86$

$\frac{a}{2} = 1.93$

$x = \frac{L}{2} + \frac{a}{2} = 30 + 1.93 = 31.93$



max^m absolute moment

$$= \frac{14.94}{31.93}\left[15(4.93+9.93) + 50(19.93+25.93+31.93)\right] + \frac{14.94}{28.07}\left[10(7.07+12.07) + 50 \times 22.07\right]$$

$= 1924.19 + 689.2$

$= 2613.39$ k-ft

$\therefore$ max^m absolute moment $= 2632.11$ k-ft (Ans).

Non-Parallel Chord

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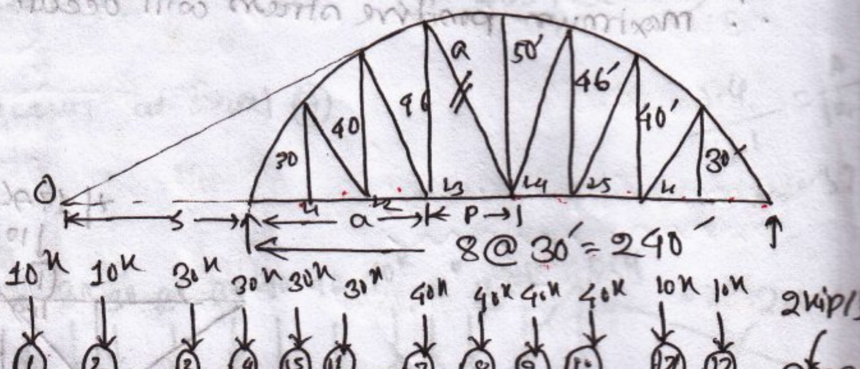
08

Q. Calculate the maximum positive and negative stresses in member 'a' of a truss due to moving load as shown in figure below:

Solution:



From similar triangle,

$$\frac{46}{s+90} = \frac{50}{s+120}$$

$\therefore s = 255'$

when unit load at L3:

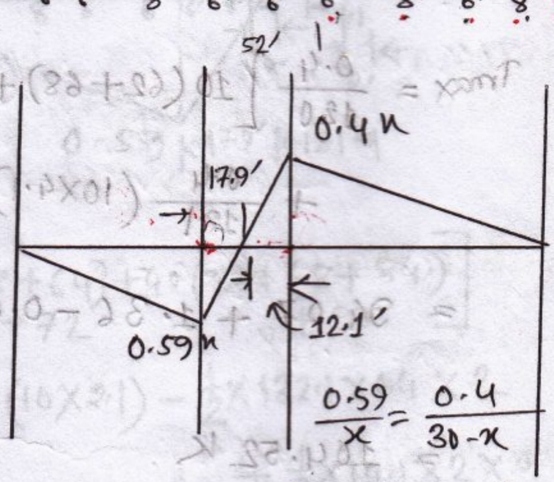
$$RL_0 = \frac{L-x}{L} = \frac{240-90}{240} = 0.625^k$$

$\Sigma M_0 = 0,$

$$1 \times (255+90) + a \times \frac{46}{\sqrt{46^2+30^2}} \times (255+120) - 0.625 \times 255 = 0$$

$$\Rightarrow \quad 185.625 + 314.104\, a = 0$$

$$\therefore a = -0.59^k$$



$$\frac{0.59}{x} = \frac{0.4}{30-x}$$

$\therefore x = 17.88'$

$\simeq 17.9'$

when unit Load at L4: $R_{L0} = \dfrac{240-120}{240} = 0.5^k$

$\Sigma M_0 = 0,$

~~1x (255+120) − ax~~

$$a \times \frac{46}{\sqrt{46^2+30^2}} \times (255+120) = 0.5 \times 255$$

$$\therefore a = 0.4^k$$

10783

Maximum positive:

$$\frac{W}{L} + \frac{W_0}{S} = \frac{W_1}{P}\left(\frac{S+a}{S}\right)$$

wheel ③ at L_4:

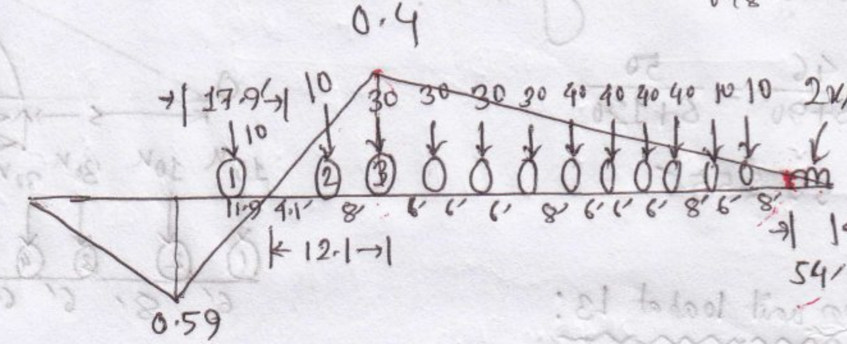
Right: $\dfrac{320+2\times54}{240} + 0 > \dfrac{20}{30}\left(\dfrac{255+90}{255}\right)$

Left: $\dfrac{320+2\times54}{240} + 0 < \dfrac{50}{30}\left(\dfrac{255+90}{255}\right)$

$\therefore$ maximum positive stress will occur at wheel ③.

$\dfrac{0.4}{120} = \dfrac{A}{54}$

0.18

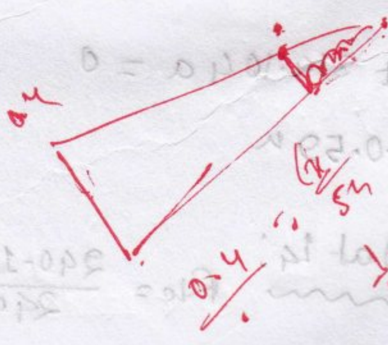


$$T_{max} = \frac{0.4}{120}\Big[10(62+68) + 40(76+82+88+94) + 30(102+108+114+120)\Big]$$

$$+ \frac{0.4}{12.1}(10\times4.1) - \frac{0.59}{17.9}(10\times1.9) + \frac{1}{2}\times54\times0.18\times2$$

$$= 94.07 + 1.36 - 0.63 + 9.72$$

$$= 104.52\text{ K}$$



2.35 1.87 1.62

07

Maximum negative:

$$\frac{W}{L} - \frac{W_0}{S+L} = \frac{W_1}{P}\left(\frac{S+a+P}{S+L}\right)$$

Wheel ⑨ at L_3:

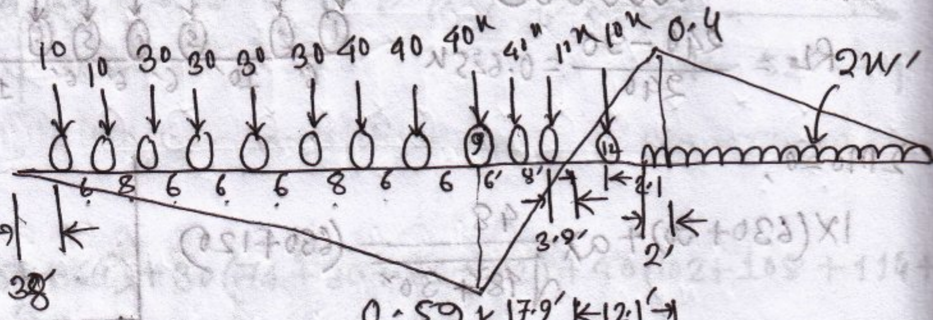
Left: $\frac{320+2\times122}{240} - \frac{2\times120}{255+240} > \frac{60+2\times2}{30}\left(\frac{255+90+30}{255+240}\right)$

Right: $\frac{320+2\times122}{240} - \frac{2\times120}{255+240} < \frac{100+2\times2}{30}\left(\frac{255+90+30}{255+240}\right)$

Maximum negative stress will occur at wheel ⑨.

$\frac{0.4}{12.1} = \frac{A}{10.1}$

$A = 0.33$



$$C_{max} = \frac{0.59}{90}\left[10(38+44) + 30(46+52+58+64) + 40(78+84+90) + 72\right]$$

$$+ \frac{0.59}{17.9}(10\times3.9 + 40\times11.9) - \frac{0.4}{12.1}(10\times2.1) - \frac{1}{2}\times132.1\times0.4\times2$$

$$+ \frac{1}{2}\times10.1\times2\times0.33$$

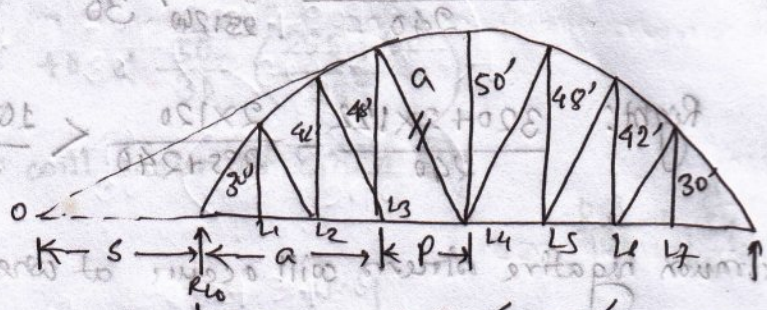
$= 119.44 + 16.97 - 0.69 - 52.84 + 3.33$

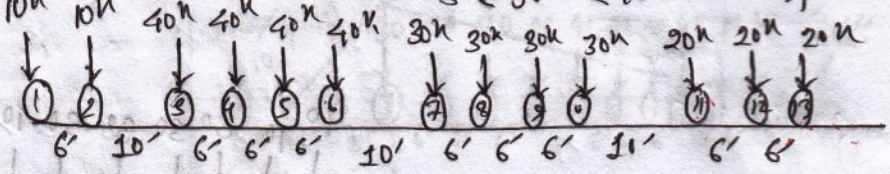
$= 86.21^k$

(Ans)

07

6. Find out the maximum stress in the member 'a' of the non-parallel chord truss due to a series of moving concentrated load shown below:





From similar triangle,

$$\frac{48}{s+90} = \frac{50}{s+120}$$

$\therefore s = 630'$

when unit load at L_3:

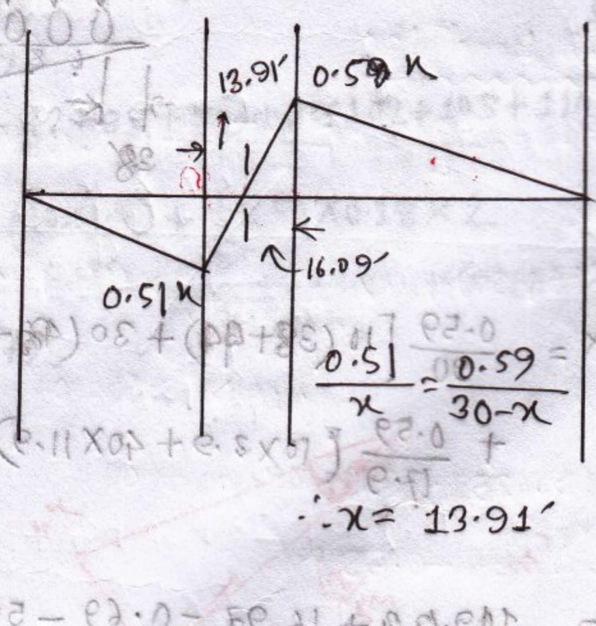
$$R_{L_0} = \frac{240-90}{240} = 0.625^k$$

$\Sigma M_0 = 0,$

$$1\times(630+90) + a\times\frac{48}{\sqrt{48^2+30^2}}(630+120) - 0.625\times 630 = 0$$

$\Rightarrow 326.25 + 6\frac{6}{3}6\,a = 0$

$\therefore a = -0.51^k$



$$\frac{0.51}{x} = \frac{0.59}{30-x}$$

$\therefore x = 13.91'$

when unit load at L_4:

$$R_{L_0} = \frac{240-120}{240} = 0.5^k$$

$\Sigma M_0 = 0,$

$$a\times\frac{48}{\sqrt{48^2+30^2}}(630+120) = \cancel{0.625}\times 630$$
$$0.5$$

$\therefore a = 0.59^k$

1.5

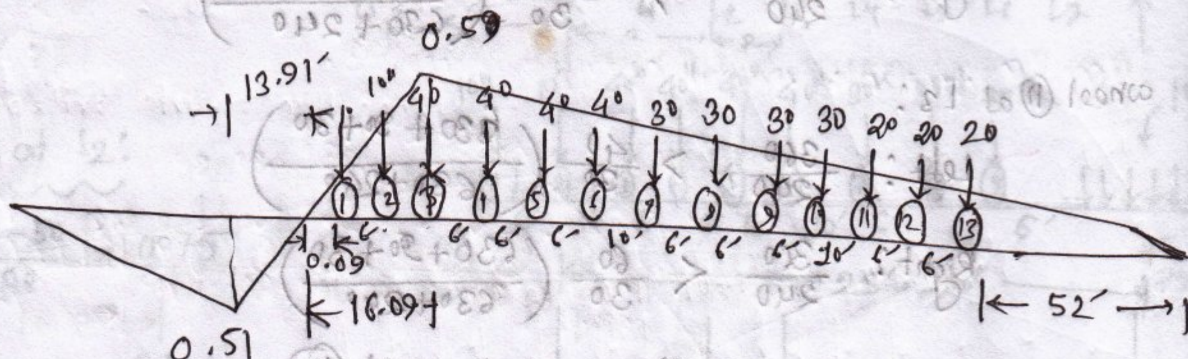
Maximum positive: $\frac{W}{L} + \frac{W_0}{S} = \frac{W_1}{P}\left(\frac{S+a}{S}\right)$

wheel ③ at L_4:

Right: $\frac{360}{240} + 0 > \frac{20}{30}\left(\frac{630+90}{630}\right)$

Left: $\frac{360}{240} + 0 < \frac{60}{30}\left(\frac{630+90}{630}\right)$

maxm positive stress will occur at wheel ③



$$T_{max} = \frac{0.59}{120}\left[20(52+58+64) + 30(74+80+86+92) + 40(102+108+114+120)\right] + \frac{0.591}{16.09}\left[10(0.09+6.09)\right]$$

$= 153.4 + 2.27$

$= 155.67\ k$

(Calculation got ONE)

131.24 k

1.5 1 0.86

Maximum negative:

$$\frac{W}{L} - \frac{W_0}{S+L} = \frac{W_1}{P}\left(\frac{S+a+P}{S+L}\right)$$

Wheel (12) at L_3:

Left: $\frac{360}{240} - 0 > \frac{20}{30}\left(\frac{630+90+30}{630+240}\right)$

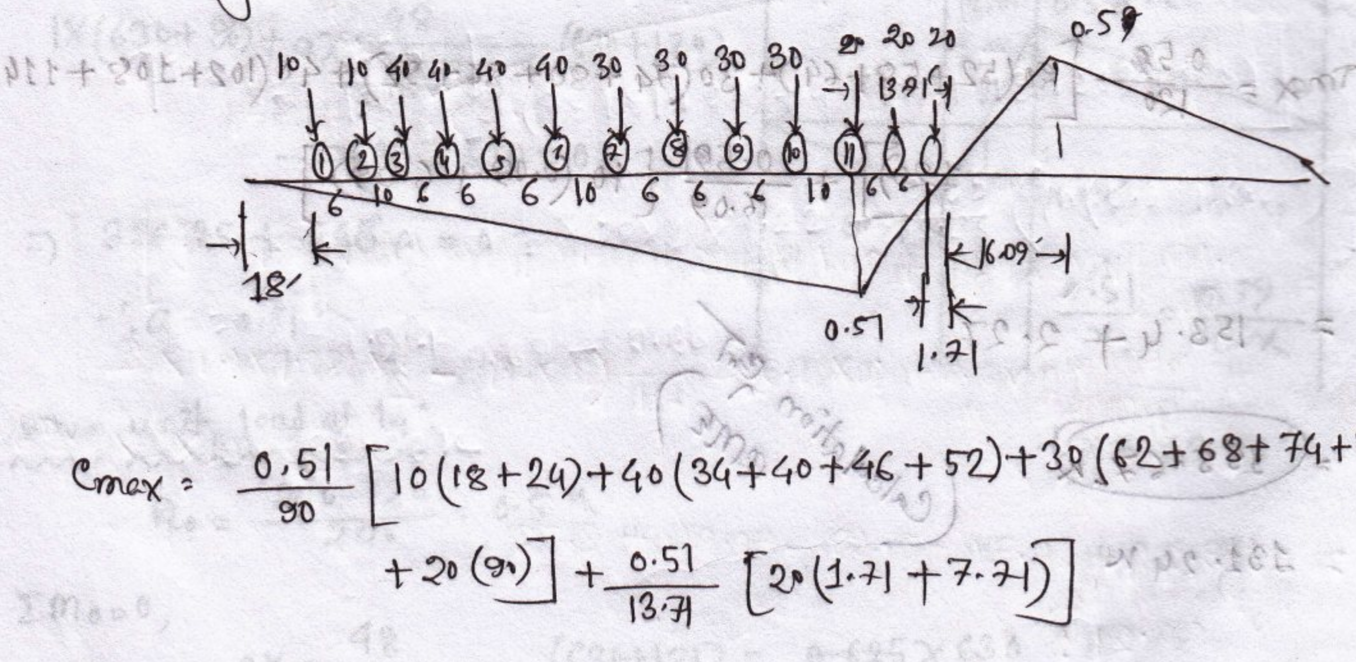
Right: $\frac{360}{240} > \frac{40}{30}\left(\frac{630+90+30}{630+240}\right)$

Wheel (11) at L_3:

Left: $\frac{360}{240} > \frac{40}{30}\left(\frac{630+90+30}{630+240}\right)$

Right: $\frac{360}{240} < \frac{60}{30}\left(\frac{630+90+30}{630+240}\right)$

Max^m negative stress will occur at wheel (11)



$$C_{max} = \frac{0.51}{90}\left[10(18+24)+40(34+40+46+52)+30(62+68+74+80)+20(90)\right]+\frac{0.51}{13.71}\left[20(1.71+7.71)\right]$$

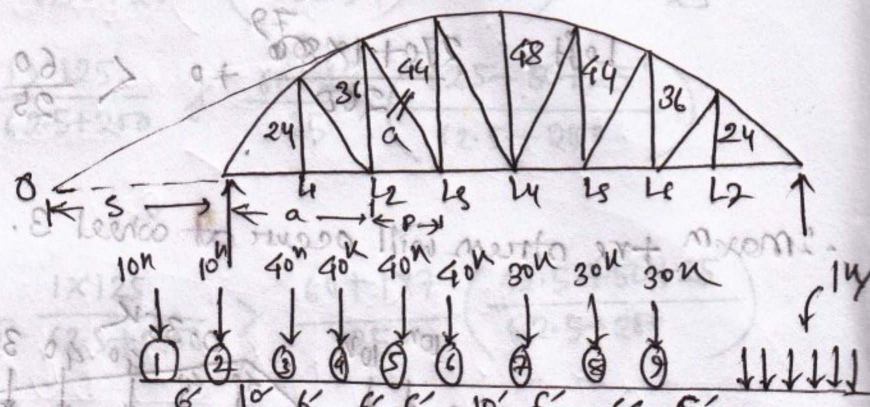
$= 99.85 + 7$

$= \boxed{106.86^k}$. (Ans)

114.47^k

09

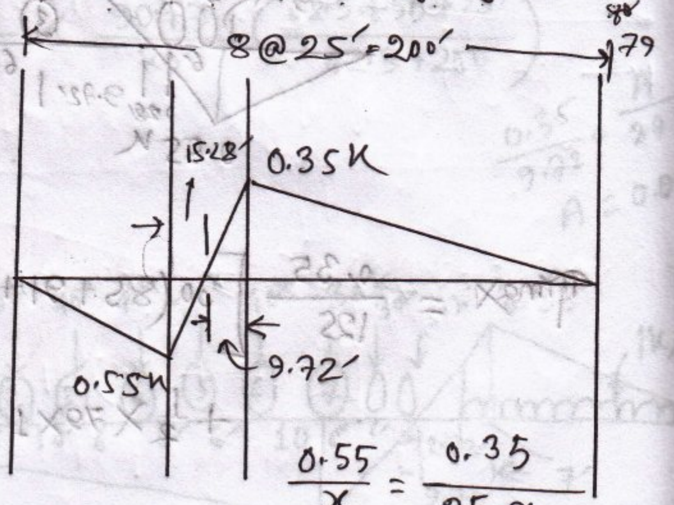
6. Calculate maximum +ve and −ve stresses in the members 'a' of the non-parallel chord truss due to the moving concentrated load as shown in figure below:



From similar triangle,

$$\frac{36}{S+50} = \frac{44}{S+75}$$

$\therefore S = 62.5'$



When unit load at L_2:

$$R_{L_0} = \frac{200-50}{200} = 0.75$$

$\Sigma M_O = 0$,

$$1\times(62.5+50) + a \times \frac{36}{\sqrt{36^2+25^2}} \times (62.5+75) - 0.75 \times 62.5 = 0$$

$\Rightarrow 65.625 + 112.94a = 0$

$\therefore a = -0.55k$

$$\frac{0.55}{x} = \frac{0.35}{25-x}$$

$\therefore x = 15.28$

When unit load at L_3:

$$R_{L_0} = \frac{200-75}{200} = 0.625k$$

$\Sigma M_O = 0$,

$$a \times \frac{36}{\sqrt{36^2+25^2}}(62.5+75) = 0.625 \times 62.5$$

$\therefore a = 0.35k$

1.25 1.8

Max^m +ve stress:

$$\frac{W}{L} + \frac{W_a}{s} = \frac{W_1}{p}\left(\frac{s+a}{s}\right)$$

wheel 3 at L_3:

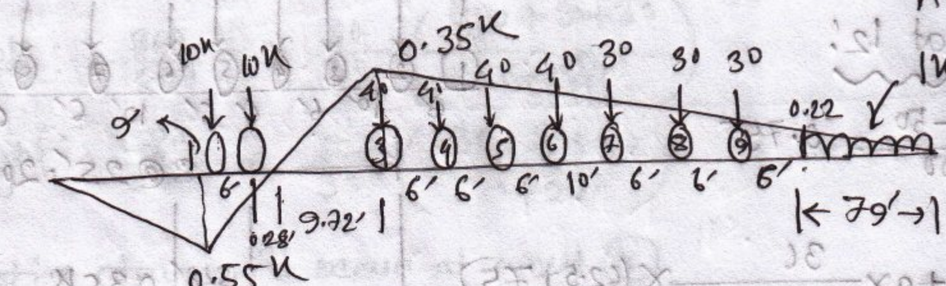
Right: $$\frac{270+1\times 79}{200} + 0 > \frac{20}{25}\left(\frac{62.5+50}{62.5}\right)$$

Left: $$\frac{270+1\times 79}{200} + 0 < \frac{60}{25}\left(\frac{62.5+50}{62.5}\right)$$

$$\frac{0.35}{125} = \frac{A}{79}$$

$A = 0.22$

∴ Max^m +ve stress will occur at wheel 3.



$$M_{max} = \frac{0.35}{125}\Big[30(85+91+97)+40(107+113+119+125)\Big] + \frac{1}{2}\times 79\times 1\times 0.22 - \frac{0.55}{15.28}\Big(10(0.28+6.28)\Big)$$

$= 74.9 + 8.69 - 2.36$

$= 81.23\text{K}$

2.04 0.48 1.50

2.07 1.53 0.52

Max^m −ve stress:

$$\frac{W}{L} - \frac{W_1}{S+L} = \frac{W_1}{P}\left(\frac{S+a+P}{S+L}\right)$$

wheel 8 at L_2:

Left: $\frac{270+138}{200} - \frac{1\times125}{62.5+200} > \left(\frac{62.5+50+25}{62.5+200}\right) \frac{30+13}{25}$

Right: $\frac{270+138}{200} - \frac{1\times125}{62.5+200} > \frac{60+13}{25}\left(\frac{62.5+50+25}{62.5+200}\right)$

wheel 7 at L_2:

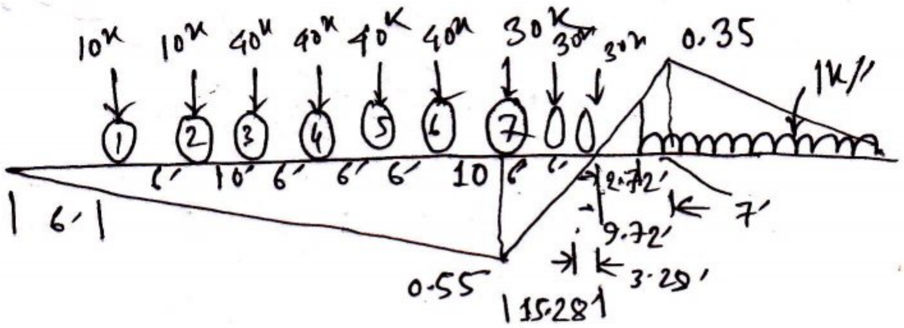
Left: $\frac{270+132}{200} - \frac{1\times125}{62.5+200} > \frac{60+1\times7}{25}\left(\frac{62.5+50+25}{62.5+200}\right)$

Right: $\frac{270+132}{200} - \frac{1\times125}{62.5+200} < \frac{90+7}{25}\left(\frac{62.5+50+25}{62.5+200}\right)$

$\frac{0.35}{9.72} = \frac{A}{2.72}$

$A = 0.098$

∴ Max^m −ve stress will occur at wheel 7.



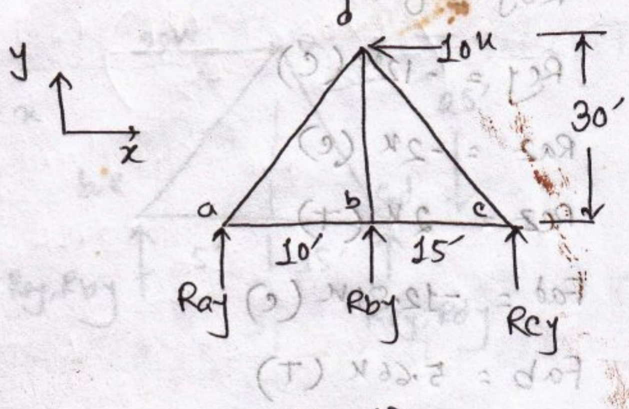
$$C_{max} = \frac{0.55}{50}\left[10(6+12)+40(22+28+34+40)+30\times50\right] + \frac{0.55}{15.28}\left[30(3.28+9.28)\right] - \frac{1}{2}\times134.72\times0.35 + \frac{1}{2}\times2.72\times0.098$$

$= 73.04 + 13.56 - 23.576 + 0.13$

$= 63.154k$ (Ans).

3D

11|10|07



Elevation

$\Sigma M_{x\text{-}ac} = 0,$

$R_{by} \times 10 = 0$

$\therefore R_{by} = 0$

$\Sigma M_{y\text{-}a} = 0,$

$R_{bx} \times 10 - 10 \times 5 - R_{cz} \times 25 = 0 \quad \text{(1)}$

$\Sigma F_x = 0, \quad R_{bx} = 10\,K$

$(1) \Rightarrow \quad 100 - 50 - 25 R_{cz} = 0$

$\therefore R_{cz} = 2\,k$

$\Sigma F_z = 0, \quad R_{az} + R_{cz} = 0$

$\therefore R_{az} = -2^k$

$\Sigma M_{z\text{-}a} = 0,$

$R_{by} \times 10 + R_{cy} \times 25 + 10 \times 30 = 0$

$\therefore R_{cy} = -12^k$

$\Sigma F_y = 0,$

$R_{ay} + R_{by} + R_{cy} = 0$

$\Rightarrow R_{ay} + 0 - 12 = 0$

$\therefore R_{ay} = 12\,k.$

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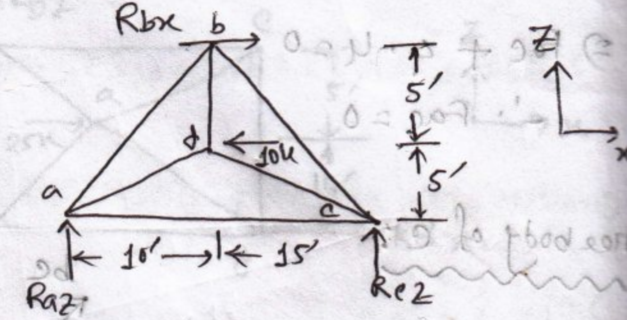
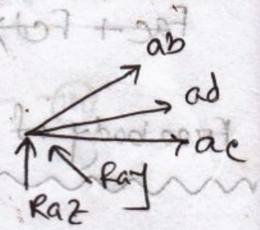


Plan

Member	Position			Length
	x	y	z	$\sqrt{x^2+y^2+z^2}$
ab	10	0	10	14.14
ad	10	30	5	32.02
ac	25	0	0	25
bd	0	30	5	30.41
bc	15	0	10	18.03
cd	15	30	5	33.91

Free body of a:



$\Sigma F_y = 0,$

$R_{ay} + F_{ad} \times \frac{30}{32.02} = 0$

$\Rightarrow 12 + 0.94\, F_{ad} = 0$

$\therefore F_{ad} = -12.81^k = 12.81^k \; (C)$

$\Sigma F_z = 0,$

$R_{az} + F_{ab} \times \frac{10}{14.14} + F_{ad} \times \frac{5}{32.02} = 0$

$\Rightarrow -2 + 0.707\, F_{ab} - 2 = 0$

$\therefore F_{ab} = 5.66^k \; (T)$

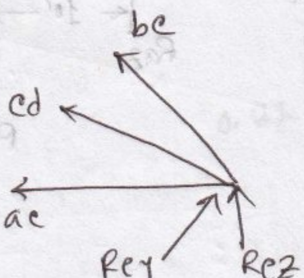
$\Sigma Fx = 0,$

$$Fac \times \frac{25}{25} + Fab \times \frac{10}{14.14} + Fad \times \frac{10}{32.02} = 0$$

$\Rightarrow Fac + 4 - 4 = 0$

$\therefore Fac = 0$

Free body of c:



$\Sigma Fy = 0,$

$$Rcy + Fcd \times \frac{30}{33.91} = 0$$

$\Rightarrow -12 + 0.88\, Fcd = 0$

$\therefore Fcd = 13.564^k$ (T)

$\Sigma Fz = 0,$

$$Rcz + Fbc \times \frac{10}{18.03} + Fcd \times \frac{5}{33.91} = 0$$

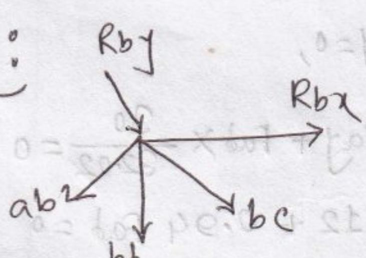
$\Rightarrow 2 + 0.55\, Fbc + 2 = 0$

$\therefore Fbc = -7.273^k = 7.273^k$ (C)

$\cancel{\Sigma Fx = 0},$

$\cancel{Fac + Fcd \times}$

Free body of b:



$\Sigma Fy = 0,$

$Rby = 0$

$\Sigma Fz = 0,$

$$Fbd + Fbc \times \frac{10}{18.03} + Fab \times \frac{10}{14.14} = 0$$

$\Rightarrow Fbd - 4.034 + 4.003 = 0$

$\therefore Fbd = 0^k$ (T)

$Ray = 12^k$ (T)

$Rby = 0^k$

$Rcy = -12^k$ (C)

$Raz = -2^k$ (C)

$Rcz = 2^k$ (T)

$Fad = -12.81^k$ (C)

$Fab = 5.66^k$ (T)

$Fac = 0^k$

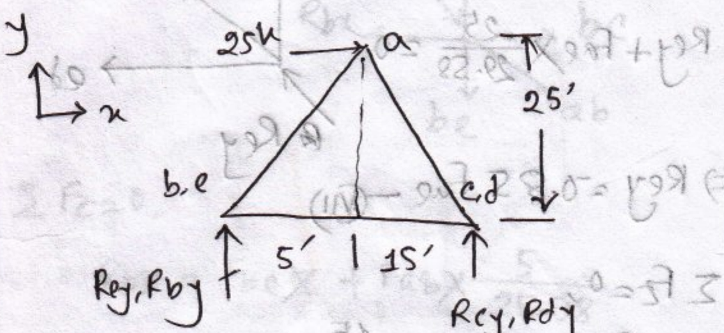
$Fcd = 13.564^k$ (T)

$Fbe = -7.273^k$ (C)

$Fbd = 0^k$.

(Ans).

$\boxed{05,09}$



$\Sigma M_{y-b} = 0,$

$25 \times 5 + R_{dx} \times 20 = 0$

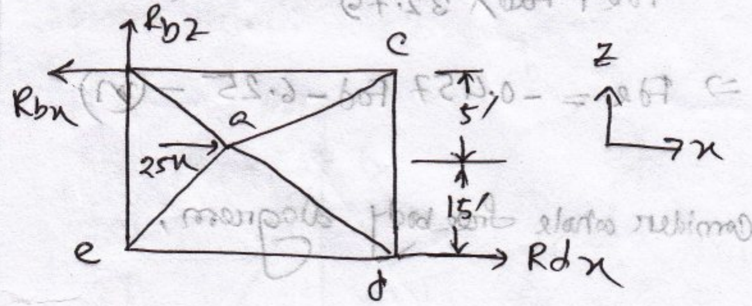
$\therefore R_{dx} = -6.25^K$

$\Sigma F_x = 0,$

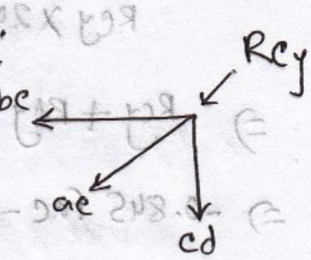
$R_{bx} = 25 + R_{dx}$

$= 25 - 6.25 = 18.75^K$

Member	Position			Length
	x	y	z	$\sqrt{x^2+y^2+z^2}$
ab	5	25	5	25.98
ac	15	25	5	29.58
ad	15	25	15	32.79
ae	5	25	15	29.58
bc	20	0	0	20
cd	0	0	20	20
de	20	0	0	20
be	0	0	20	20



Consider free body of c:



$\Sigma F_y = 0,$

$R_{cy} + F_{ac} \times \frac{25}{29.58} = 0$

$\Rightarrow R_{cy} = -0.845\ F_{ac}$ — (i)

$\Sigma F_z = 0,$

$F_{cd} + F_{ac} \times \frac{5}{29.58} = 0$

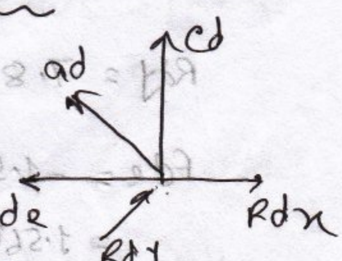
$\Rightarrow \cancel{F_{cd}}\ F_{ac} = -5.916\ F_{cd}$ — (ii)

$\Sigma F_x = 0,$

$F_{bc} + F_{ac} \times \frac{15}{29.58} = 0$

$\Rightarrow F_{bc} = -0.507\ F_{ac}$ — (iii)

Consider free body of d:



$\Sigma F_y = 0,$

$R_{dy} + F_{ad} \times \frac{25}{32.79} = 0$

$\Rightarrow R_{dy} = -0.76\ F_{ad}$ — (iv)

$\Sigma F_z = 0,\ F_{cd} + F_{ad} \times \frac{15}{32.79} = 0$

$\Rightarrow F_{cd} = -0.457\ F_{ad}\ \cancel{=0}$ — (v)

$\Sigma F_y = 0$, $\Sigma F_x = 0$,

$$F_{de} + F_{ad} \times \frac{15}{32.79} = R_{dx} = -6.25$$

$$\Rightarrow F_{de} = -0.457\, F_{ad} - 6.25 \quad \text{— (vi)}$$

Consider whole free body diagram,

$$\Sigma M_{z-b,e} = 0$$

$$R_{ey} \times 20 + R_{dy} \times 20 = 25 \times 25$$

$$\Rightarrow R_{ey} + R_{dy} = 31.25$$

$$\Rightarrow -0.845\, F_{ac} - 0.76\, F_{ad} = 31.25$$

$$\Rightarrow -0.845(-5.916\, F_{cd}) - 0.76(-2.19\, F_{cd}) = 31.25$$

$$\Rightarrow 5\, F_{cd} + 1.66\, F_{cd} = 31.25$$

$$\therefore F_{cd} = 4.69^k \text{ (T)}$$

\# (ii) $F_{ac} = -27.75^k = 27.75^k$ (C)

$R_{ey} = 23.45^k$ (T)

$F_{be} = 14.07^k$ (T)

~~Rdy~~ =

$F_{ad} = -10.26^k$ (T)

$= 10.26^k$ (C)

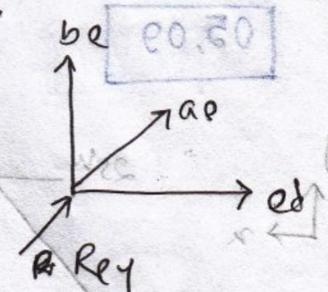
$R_{dy} = 7.8^k$ (T)

$F_{de} = -1.56^k$

$= 1.56^k$ (C)

consider point e:

05.09



$\Sigma F_y = 0$,

$$R_{ey} + F_{ae} \times \frac{25}{29.58} = 0$$

$$\Rightarrow R_{ey} = -0.85\, F_{ae} \quad \text{— (vii)}$$

$\Sigma F_z = 0$,

$$F_{be} + F_{ae} \times \frac{15}{29.58} = 0$$

$$\Rightarrow F_{ae} = -1.972\, F_{be} \quad \text{— (viii)}$$

$\Sigma F_x = 0$,

$$F_{ed} + F_{ae} \times \frac{5}{29.58} = 0$$

$\Rightarrow$ ~~$4.69 + 0.169\, F_{ae} = 0$~~

$\therefore$ ~~$F_{ae} = -27.75^k = 27.75^k$ (C)~~

~~$F_{be} = 14.07^k$ (T)~~

~~$R_{ey} = 23.588^k$ (T)~~

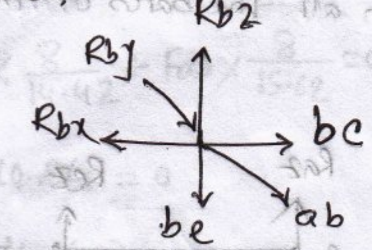
$$\Rightarrow -1.56 + 0.169\, F_{ae} = 0$$

$$\therefore F_{ae} = 9.23^k \text{ (T)}$$

$$F_{be} = -4.68^k = 4.68^k \text{ (C)}$$

$$R_{ey} = -7.85^k = 7.85^k \text{ (C)}$$

consider point b:



$\Sigma F_z = 0,$

$R_{bz} = F_{be} \cancel{x} + F_{ab} \times \frac{5}{25.98}$

$\Rightarrow R_{bz} = -4.68 + 0.192\, F_{ab}$

$\Rightarrow 0 = -4.68 + 0.192\, F_{ab}$

$\therefore F_{ab} = 24.375^k \, (T)$

$\Sigma F_y = 0,$

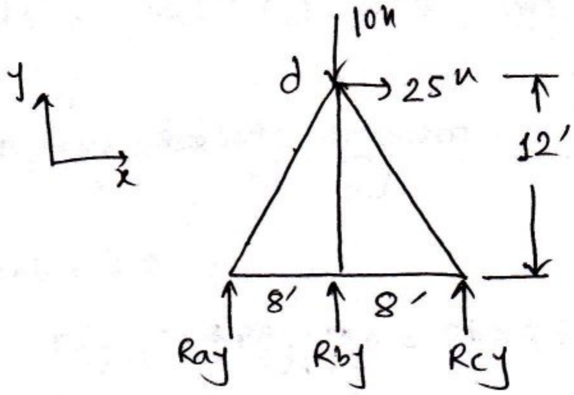
$R_{by} + F_{ab} \times \frac{25}{25.98} = 0$

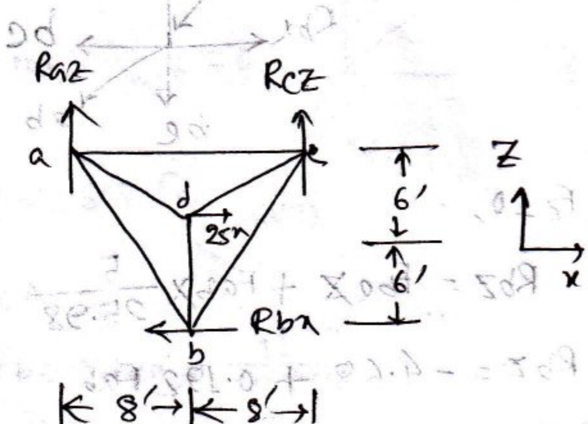
$\Rightarrow R_{by} = -23.46^k$

$= 23.46^k \, (T).$

08,06

Q. Find the reaction and bar forces in all the bars of the following space frame shown below:





$\Sigma M_{x\text{-}ac} = 0,$

$R_{by} \times 12 = 10 \times 6$

$\therefore R_{by} = 5^k$

$\Sigma f_x = 0, \quad R_{bx} = 25^k$

$\Sigma M_{z,a} = 0,$

$10 \times 8 + 25 \times 12 = R_{by} \times 8 + R_{cy} \times 16$

$\Rightarrow 380 = 40 + R_{cy} \times 16$

$\therefore R_{cy} = 21.25^k$

$\Sigma f_y = 0,$

$R_{ay} + R_{by} + R_{cy} = 10$

$\Rightarrow R_{ay} = 10 - 5 - 21.25$

$= -16.25^k$

$\Sigma M_{y\text{-}a} = 0,$

$R_{bx} \times 12 = R_{cz} \times 16 + 25 \times 6$

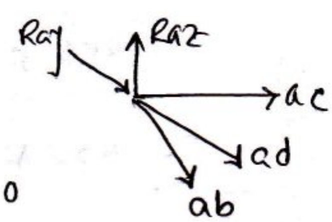
$\Rightarrow 25 \times 12 = 16 R_{cz} + 150$

$\therefore R_{cz} = 9.375^k$

$\Sigma f_z = 0, \quad R_{az} = -9.375^k$

Member	Position			Length
	x	y	z	$\sqrt{x^2+y^2+z^2}$
ac	16	0	0	16
ab	8	0	12	14.42
ad	8	12	6	15.62
cd	8	12	6	15.62
bc	8	0	12	14.42
bd	0	12	6	13.42

consider point a:



$\Sigma f_y = 0,$

$R_{ay} + F_{ad} \times \frac{12}{15.62} = 0$

$\Rightarrow -16.25 + 0.768 F_{ad} = 0$

$\therefore F_{ad} = 21.16^k \text{ (T)}$

$\Sigma f_z = 0,$

$R_{az} = F_{ab} \times \frac{12}{14.42} + F_{ad} \times \frac{6}{15.62}$

$\Rightarrow -9.375 = 0.83 F_{ab} + 8.13$

$\therefore F_{ab} = -21.09^k = 21.09^k \text{ (C)}$

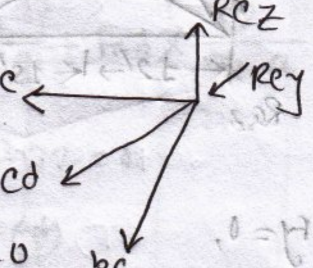
$\Sigma F_x = 0,$

$$F_{ac} + F_{ab} \times \frac{8}{14.42} + F_{ad} \times \frac{8}{15.62} = 0$$

$$\Rightarrow F_{ac} - 11.70 + 10.84 = 0$$

$$\therefore F_{ac} = 0.86\,k\ (T)$$

consider point c:



$\Sigma F_y = 0,$

$$R_{cy} + F_{cd} \times \frac{12}{15.62} + \cancel{F_{bc}\times} = 0$$

$$\Rightarrow 21.25 + 0.768\,F_{cd} = 0$$

$$\therefore F_{cd} = -27.66^k = 27.66^k\ (C)$$

$\Sigma F_z = 0,$

$$R_{cz} = F_{cd} \times \frac{6}{15.62} + F_{bc} \times \frac{12}{14.42}$$

$$\Rightarrow 9.375 = -10.62 + 0.83\,F_{bc}$$

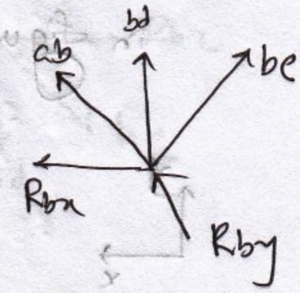
$$\therefore F_{bc} = 24.09\,k\ (T)$$

$\Sigma F_x = 0,$

$$\cancel{F_{ac} + F_{cd} \times \frac{8}{15.62} + F_{bc} \times \frac{8}{14.42} = 0}$$

$\Rightarrow$ ~~0.86..~~

Consider point b:



$\Sigma F_z = 0,$

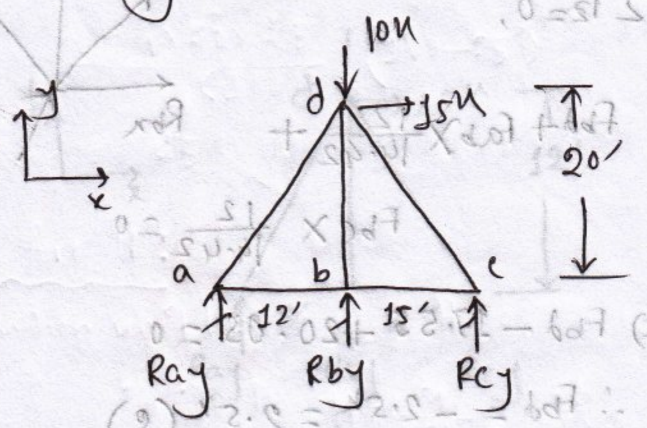
$$F_{bd} + F_{ab} \times \frac{12}{14.42} + F_{be} \times \frac{12}{14.42} = 0$$

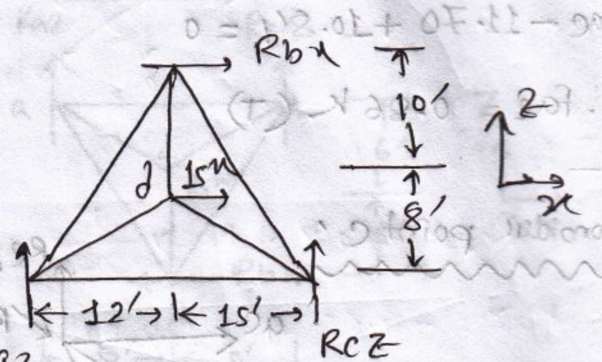
$$\Rightarrow F_{bd} - 17.55 + 20.05 = 0$$

$$\therefore F_{bd} = -2.5^k = 2.5^k\ (C)$$

03

Q. Find the reactions and bar forces of the space truss shown in figure below:





$\Sigma M_{x\text{-}ac} = 0,$

$R_{by} \times 18 = 10 \times 8$

$\therefore R_{by} = 4.44^{k}$

$\Sigma F_x = 0, \quad R_{bx} = -15^{k}$

Member	Position			Length $\sqrt{x^2+y^2+z^2}$
	x	y	z	
ab	12	0	18	21.63
ad	12	20	8	24.66
ac	27	0	0	20
bd	0	20	10	22.36
bc	15	0	18	23.43
cd	15	20	8	26.25

$\Sigma M_{z\text{-}a} = 0,$

$R_{by} \times 12 + R_{cy} \times 27 = 10 \times 12 + 15 \times 20$

$\Rightarrow 4.44 \times 12 + 27 R_{cy} = 420$

$\therefore R_{cy} = 13.58^{k}$

$\Sigma F_y = 0,$

$R_{ay} + R_{by} + R_{cy} = 10$

$\Rightarrow R_{ay} = 10 - 4.44 - 13.58$

$= -8.02^{k}$

$\Sigma M_{y\text{-}a} = 0,$

$R_{bx} \times 18 + 15 \times 8 = R_{cz} \times 27$

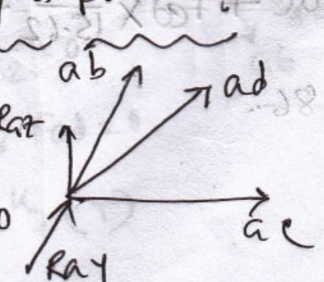
$\Rightarrow -15 \times 18 + 15 \times 8 = 27 R_{cz}$

$\therefore R_{cz} = -5.55^{k}$

$\Sigma F_z = 0,$

$R_{az} = 5.55^{k}$

consider free body of point 'a':



$\Sigma F_y = 0,$

$R_{ay} + F_{ad} \times \frac{20}{24.66} = 0$

$\Rightarrow -8.02 + 0.81 F_{ad} = 0$

$\therefore F_{ad} = 9.9^{k}$ (T)

$\Sigma F_z = 0,$

$R_{az} + F_{ab} \times \frac{18}{21.63} + F_{ad} \times \frac{8}{24.66} = 0$

$\Rightarrow 5.55 + 0.83 F_{ab} + 3.21 = 0$

$\therefore F_{ab} = -10.56^k = 10.56^k (C)$

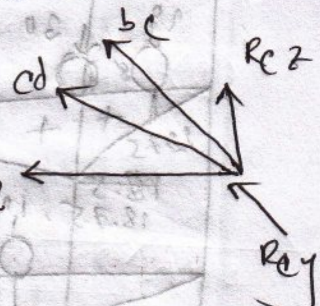
$\Sigma F_x = 0,$

$F_{ac} + F_{ab} \times \frac{12}{21.63} + F_{ad} \times \frac{12}{24.66} = 0$

$\Rightarrow F_{ac} - 5.86 + 4.82 = 0$

$\therefore F_{ac} = 1.04^k (T)$

Consider free body of point c:



$\Sigma F_y = 0,$

$R_{cy} + F_{cd} \times \frac{20}{26.25} = 0$

$\Rightarrow 13.58 + 0.76 F_{cd} = 0$

$\therefore F_{cd} = -17.87^k = 17.87^k (C)$

$\Sigma F_Z = 0,$

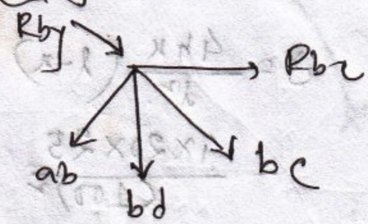
$R_{cz} + F_{bc} \times \frac{18}{23.43} + F_{cd} \times \frac{8}{26.25} = 0$

$\Rightarrow -5.55 + 0.768 F_{bc} - 5.45 = 0$

$\therefore F_{bc} = 14.32^k (T)$

(4)

Consider free body of point b:



$\Sigma F_z = 0,$

$F_{bd} + F_{bc} \times \frac{18}{23.43} + F_{ab} \times \frac{18}{21.63} = 0$

$\Rightarrow F_{bd} + 11 - 8.79 = 0$

$\therefore F_{bd} = -2.21^k$

$= 2.21^k (C)$

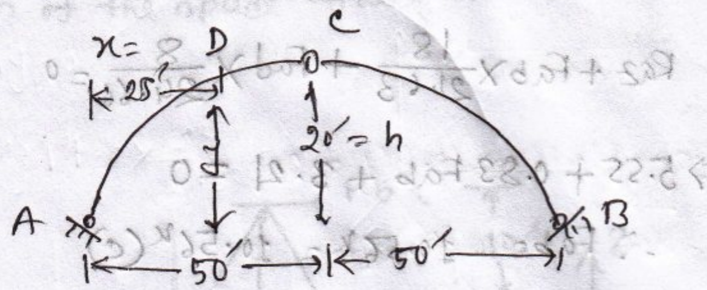
CE-10

(4.)

$y = \frac{4hx}{l^2}(l-x)$

$= \frac{4\times 20\times 25}{(100)^2}(100-25)$

$= 15'$



IL for H:

when unit load at A, H = 0

" " " D, $H = \frac{x}{2h} = \frac{25}{2\times 20}$

$= 0.625$

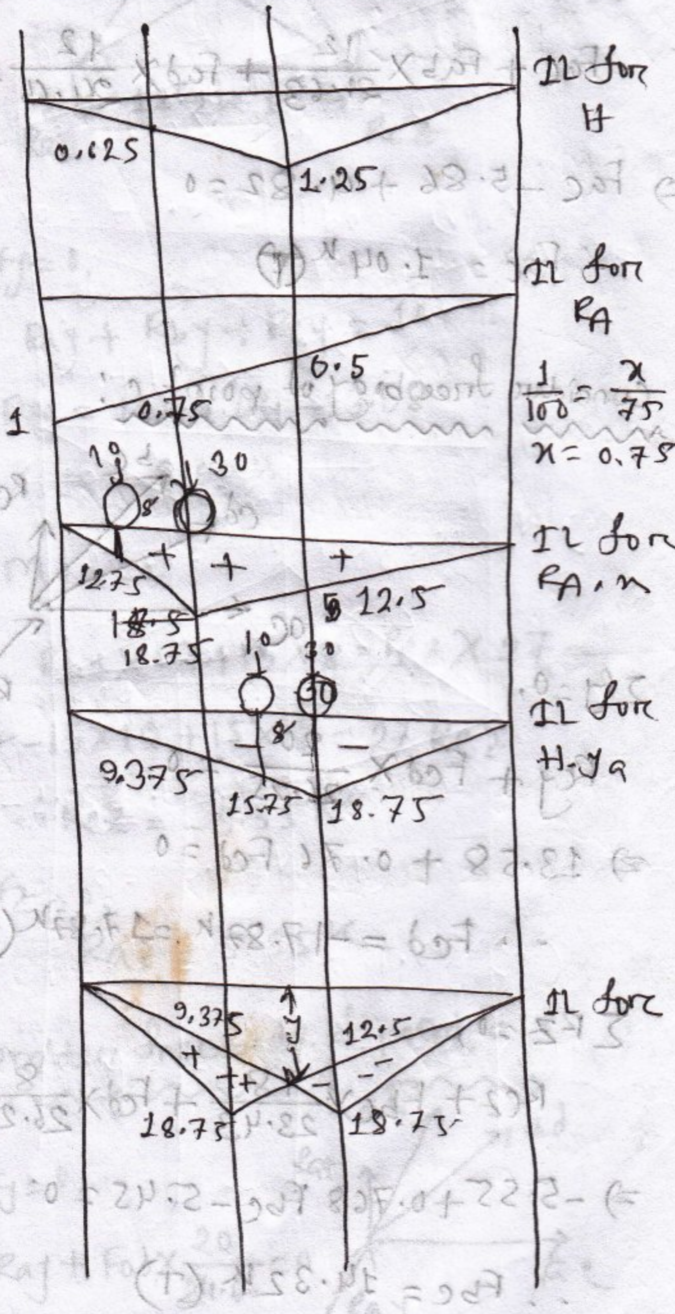
" " " C, $H = \frac{l}{4h} = \frac{100}{4\times 20} = 1.25$

$\frac{18.75}{25} = \frac{x}{17} \quad \therefore x = 12.75$

$\frac{18.75}{50} = \frac{x}{42} \quad \therefore x = 15.75$

$M_{max} = 30\times 18.75 + 10\times 12.75 -$

$30\times 18.75 - 10\times 15.75$



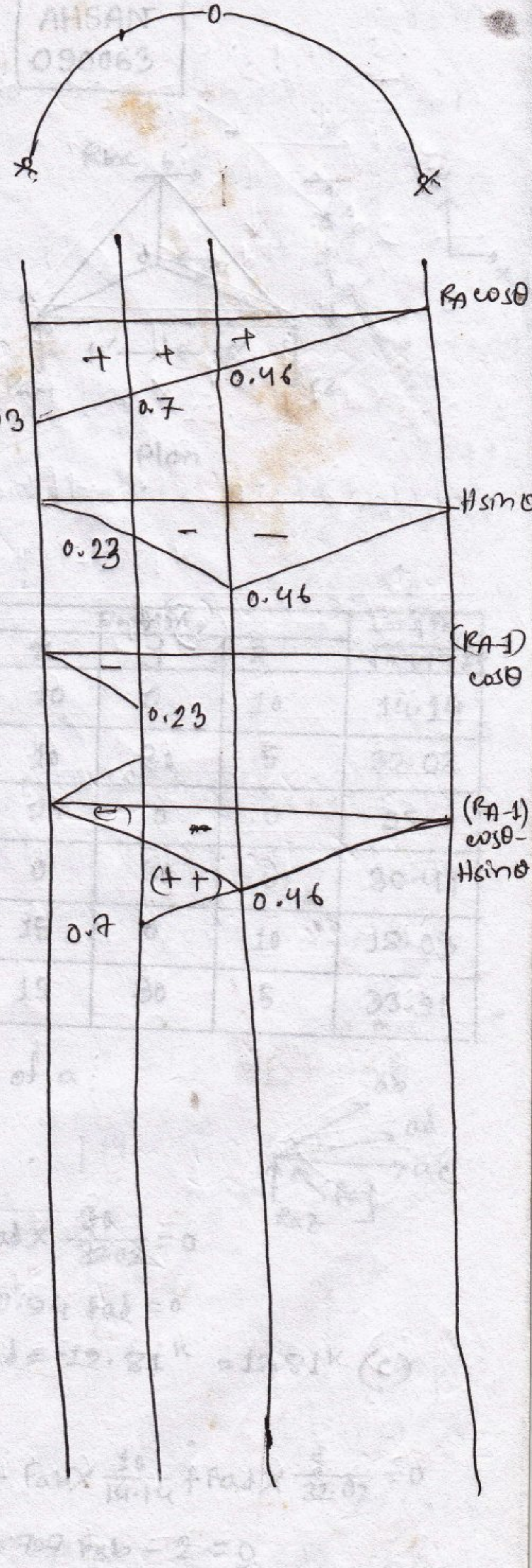
$$y = \frac{4hx}{l^2}(l-x)$$

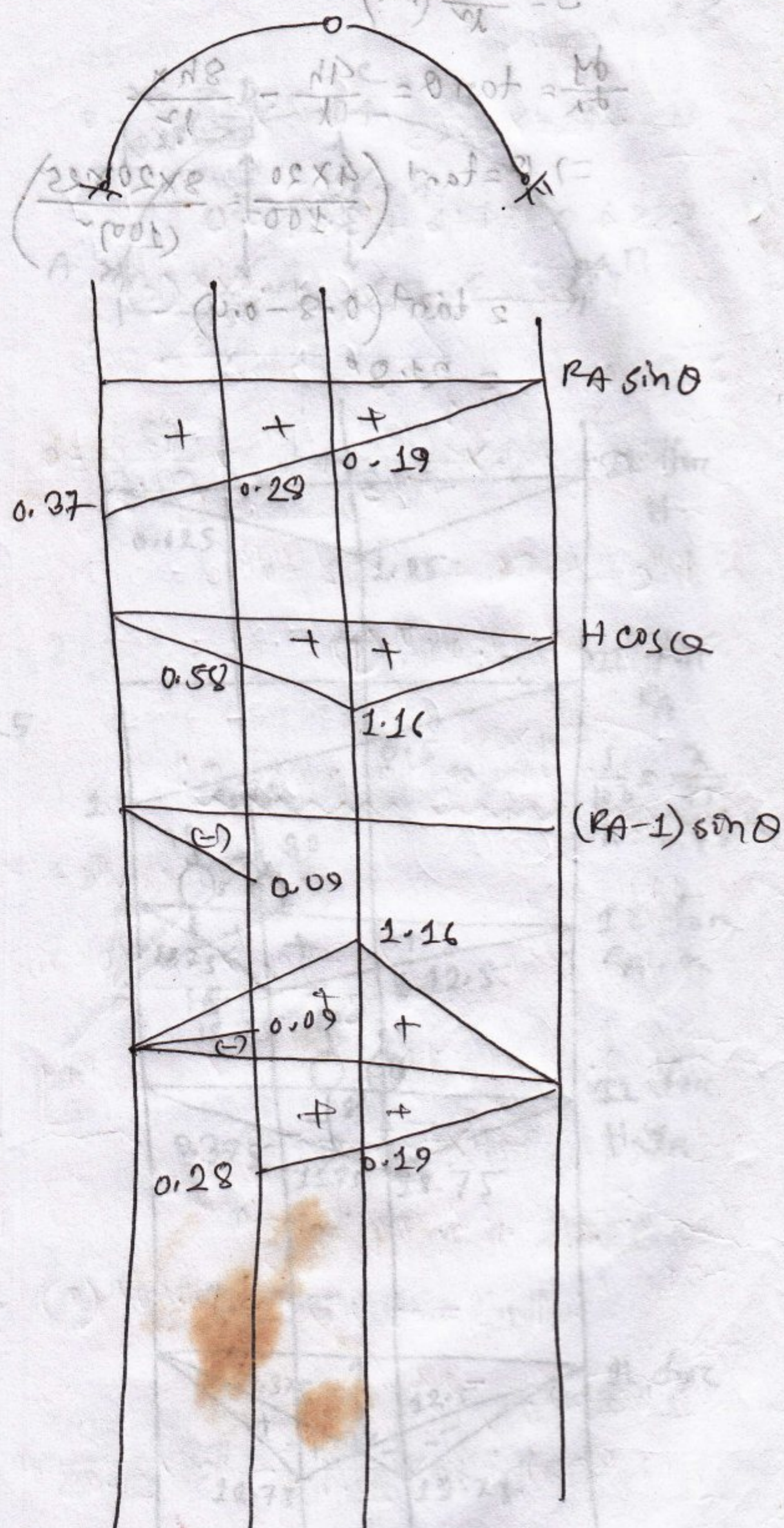
$$\frac{dy}{dx} = \tan\theta = \frac{4h}{l} - \frac{8hx}{l^2}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4\times20}{100} - \frac{8\times20\times25}{(100)^2}\right)$$

$$= \tan^{-1}(0.8 - 0.4)$$

$$= 21.8^\circ$$





RA sinθ
0.37
0.29
0.19
H cosθ
0.58
1.16
(RA-1) sinθ
0.09
1.16
0.07
0.28
0.19

Suspension Bridge

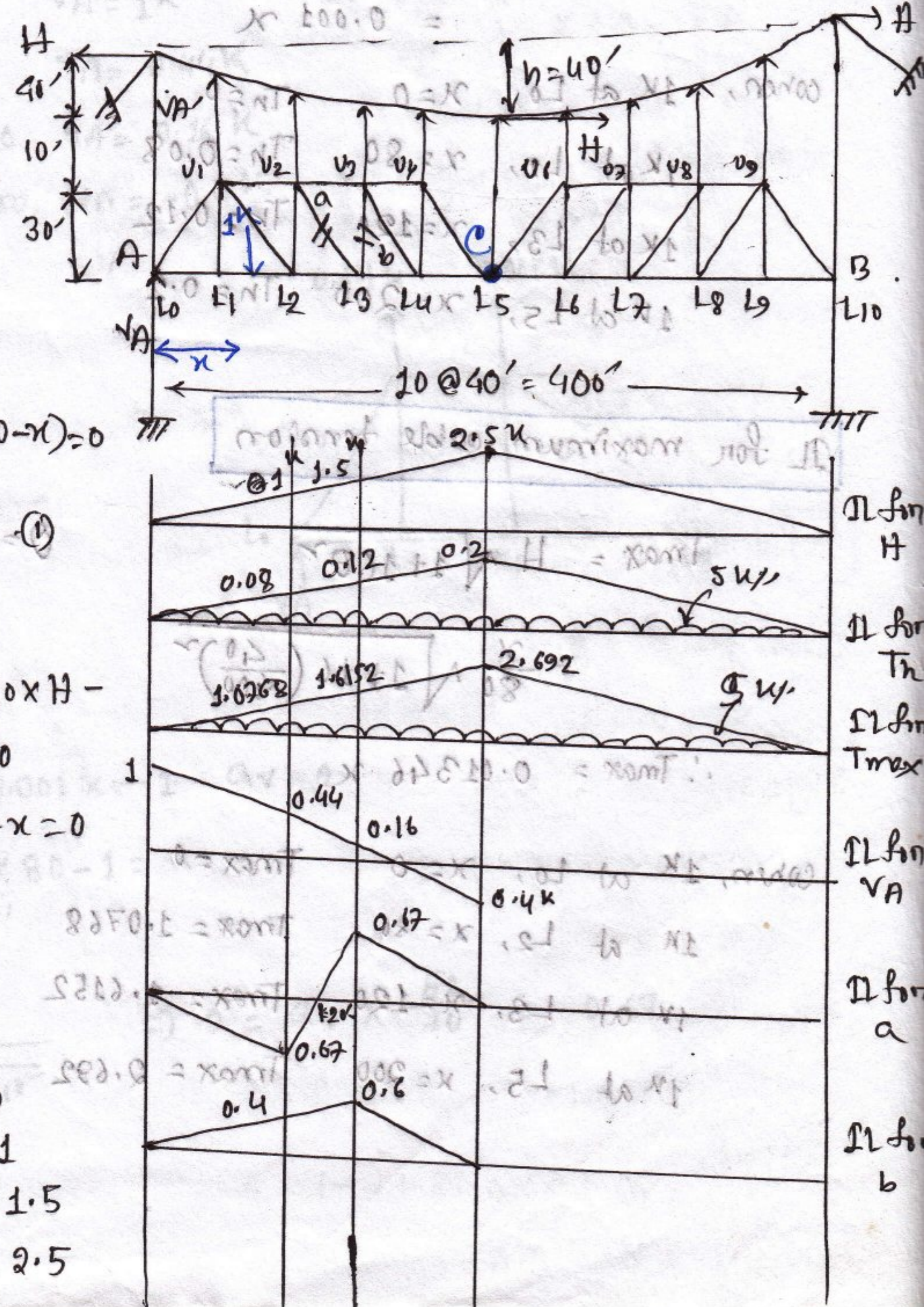
090063
AHSAN

10

Q. Deduce influence lines for hanger tension, maximum cable tension and stress in member a and b of the following suspension bridge as shown in fig below. Also compute the maximum force/stress of this members due to uniform load of 5 k/ft.

Solution:

Considering whole free body diagram.



$\Sigma M_B = 0,$

$(V_A + V_A') \times 400 - 1 \times (400 - x) = 0$

$\therefore V_A + V_A' = \dfrac{400-x}{400}$ ---- (1)

$\Sigma M_C = 0,$

$(V_A + V_A') \times 200 + 40 \times H - 80 \times H - 1 \times (200 - x) = 0$

$\Rightarrow \dfrac{400-x}{400} \times 200 - 40H - 200 - x = 0$

$\Rightarrow 40H = \dfrac{400-x}{2} - 200 - x$

$\therefore H = \dfrac{x}{80}$

when 1^k at L_0, $x = 0$ $H = 0$

1^k at L_2, $x = 80$ $H = 1$

1^k at L_3, $x = 120$ $H = 1.5$

1^k at L_5, $x = 200$ $H = 2.5$

IL for hanger tension

$$H = \frac{wl^2}{8h} \Rightarrow \frac{x}{80} = \frac{w \times (400)^2}{8 \times 40}$$

$$\Rightarrow w = 2.5 \times 10^{-5} x$$

Hanger tension, $T_h = w \times$ Panel length

$$= 2.5 \times 10^{-5} \times 40\, x$$

$$= 0.001\, x$$

when, 1K at L_0, $x = 0$ $T_h = 0$

1K at L_2, $x = 80$ $T_h = 0.08$

1K at L_3, $x = 120$ $T_h = 0.12$

1K at L_5, $x = 200$ $T_h = 0.2$

IL for maximum cable tension

$$T_{max} = H \sqrt{1 + 16\theta^2}$$

$$= \frac{x}{80} \sqrt{1 + 16 \left(\frac{40}{400}\right)^2}$$

$$\therefore T_{max} = 0.01346\, x$$

when, 1K at L_0, $x = 0$ $T_{max} = 0$

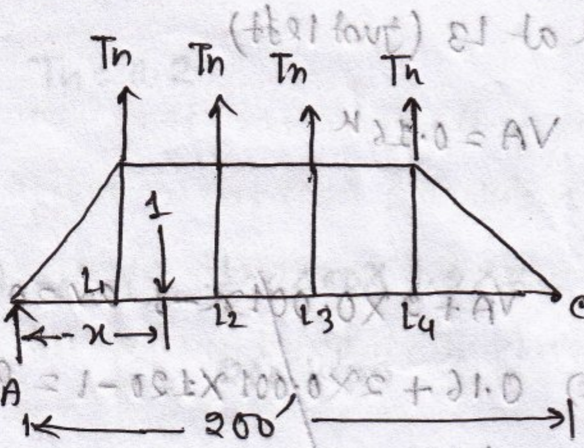
1K at L_2, $x = 80$ $T_{max} = 1.0768$

1K at L_3, $x = 120$ $T_{max} = 1.6152$

1K at L_5, $x = 200$ $T_{max} = 2.692$

$$\theta = \frac{h}{l}$$

IL for VA



$\Sigma M_c = 0,$

$VA \times 200 + Tn \times (1+2+3+4) \times 40$

$\quad - 1 \times (200 - x) = 0$

$\Rightarrow VA = \dfrac{200 - 1.4x}{200}$

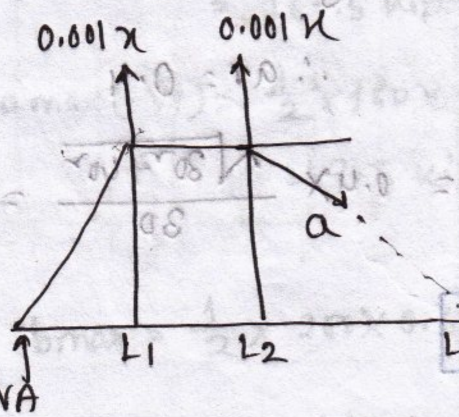
when, 1^k at L_0, $x = 0$ $\quad VA = 1^k$

1^k at L_2, $x = 80$ $\quad VA = 0.44^k$

1^k at L_3, $x = 120$ $\quad VA = 0.16^k$

1^k at L_5, $x = 200$, $VA = -0.4^k$

IL for a & b

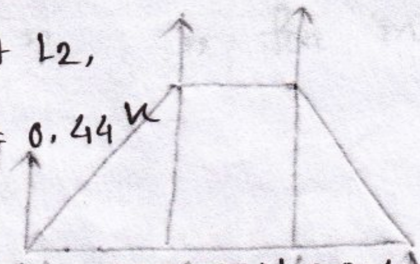


when 1^k at L_0,

$VA = 1^k \quad a = 0$

when 1^k at L_2,

$VA = 0.44^k$



$\Sigma F_y = 0,$

$VA + 2 \times \cancel{0.0001}\, 0.001x - 1 - a_v = 0$

$\Rightarrow 0.44 + 2 \times 0.001 \times 80 - 1 = a_v$

$\therefore a_v = -0.4$

$a_v = a \times \dfrac{30}{\sqrt{30^2 + 40^2}} \quad \Rightarrow a = a_v \times \dfrac{50}{30} = -0.67$

when 1k at L3 (just left)

$VA = 0.16^{k}$

$\Sigma Fy = 0,$

$\Rightarrow VA + 2 \times 0.001x - 1 - ar = 0$

$\Rightarrow 0.16 + 2 \times 0.001 \times 120 - 1 = ar$

$\therefore ar = -0.6$

$$a = -0.6 \times \frac{\sqrt{30^2 + 40^2}}{30} =$$

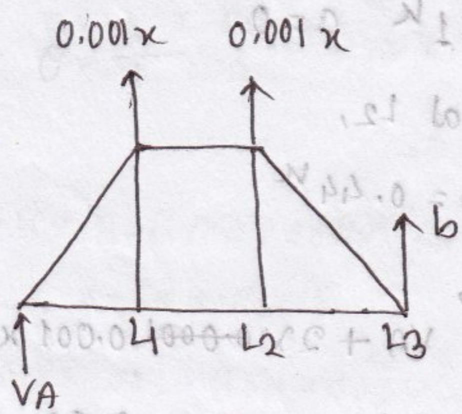
just right:

$\Sigma Fy = 0, \quad VA + 2 \times 0.001x - ar = 0$

$\therefore ar = 0.4$

$$\therefore a = 0.4 \times \frac{\sqrt{30^2 + 40^2}}{30} = 0.67^{k}$$

$\boxed{\text{IL for b}}$



when 1k at L2,

$VA = 0.44 \qquad Tn = 0.08$

$\Sigma Fy = 0,$

$VA + 2 \times 0.08 + b - 1 = 0$

$\therefore b = 0.4$

when 1k at L3,

$VA = 0.16^{k} \qquad Tn = 0.12$

$\Sigma Fy = 0, \quad VA + 2 \times 0.12 + b - 1 = 0$

$\therefore b = 0.6$

When 1k at L5, $V_A = -0.4$ $T_h = 0.2$

$b = 0$

Maximum stress for hanger tension $= \frac{1}{2} \times 400 \times 0.2 \times 5$

$= 200$ kips

" " for cable tension $T_{max} = \frac{1}{2} \times 400 \times 2.692 \times 5$

$= 2692$ kips

" " for member a, $a_{max}(+ve) = \frac{1}{2} \times 100 \times 0.67 \times 5$

$= 167.5$ kips

$a_{max}(-ve) = \frac{1}{2} \times 100 \times 0.67 \times 5$

$= 167.5$ kips

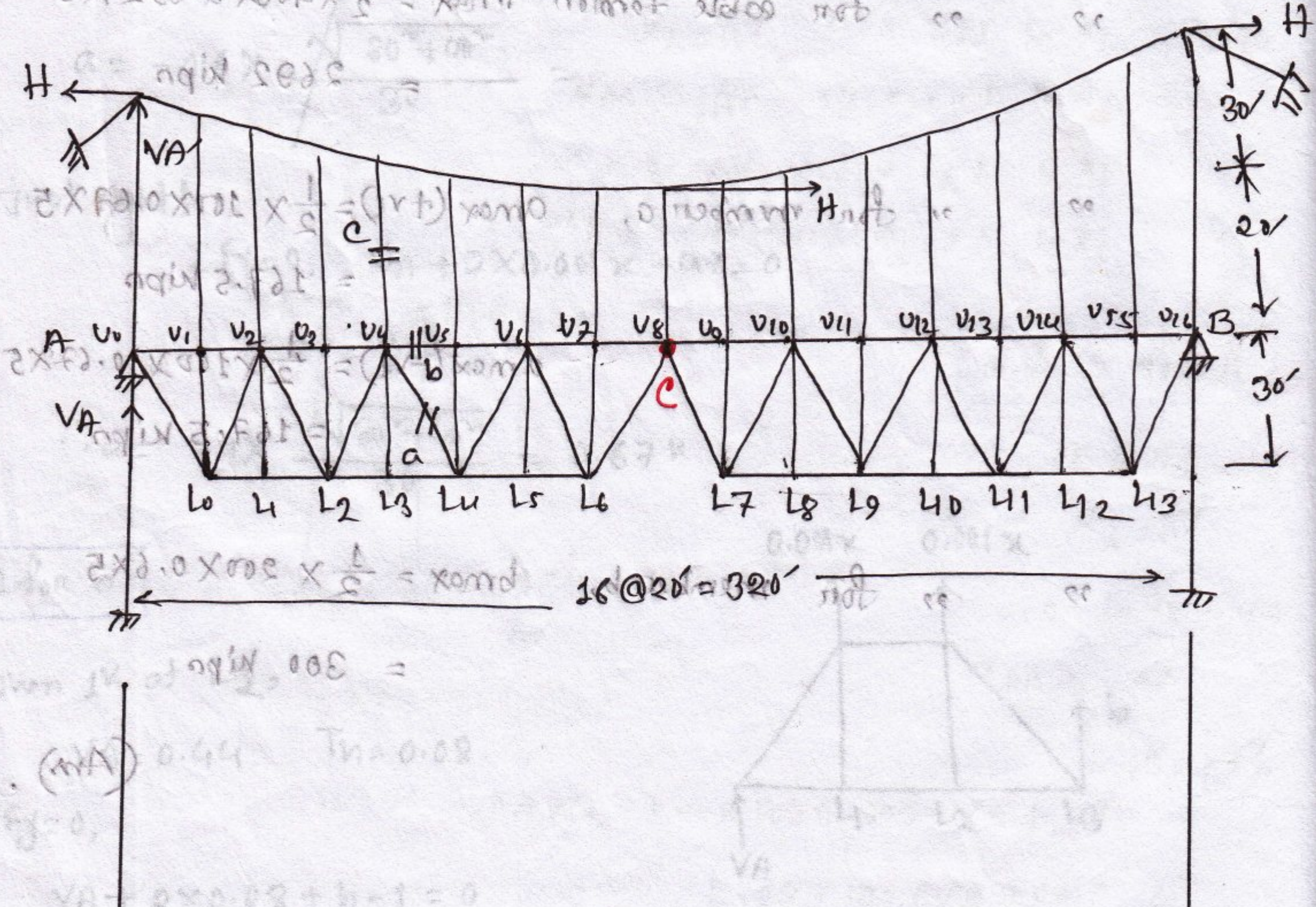
" " for member b, $b_{max} = \frac{1}{2} \times 200 \times 0.6 \times 5$

$= 300$ kips.

(Ans).

3

Q. Draw the influence lines for stress in members a, b and c of the following suspension bridge shown in figure below and hence determine the maximum stress of the same members due to a uniformly distributed moving load of 3 k/ft.



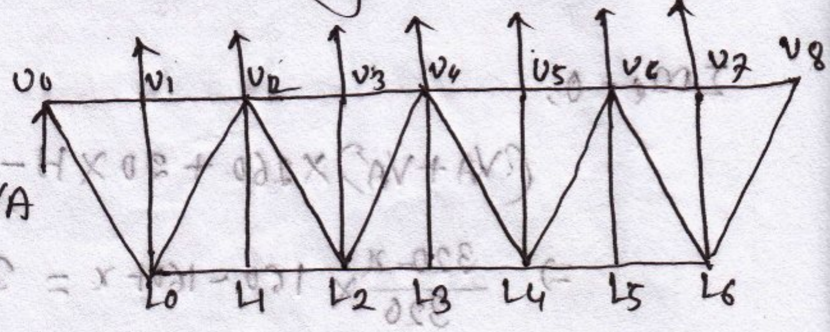
1^k at U_5, $x = 100$, $c = 0.078$

1^k at U_6, $x = 120$, $c = 0.094$

1^k at U_7, $x = 140$, $c = 0.109$

1^k at U_8, $x = 160$, $c = 0.125$

Consider the free body of left part of the bridge:



$\Sigma M_{U8} = 0$,

$VA \times 160 + 7.81 \times 10^{-4} \times x$

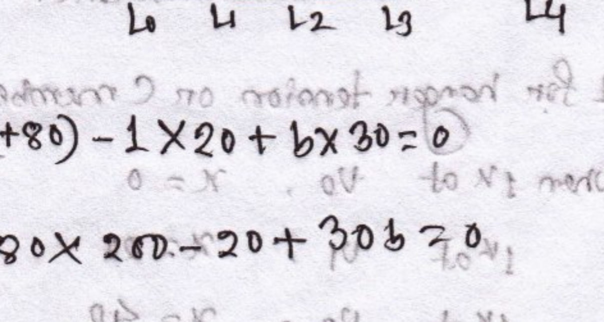
$(1+2+3+4+5+6+7+8) \times 20$

$- 1 \times (160 - x) = 0$

$\Rightarrow 160\,VA + 0.44x - 160 + x = 0$

$\Rightarrow VA = \frac{\cancel{160 - 1.56x}}{\cancel{160}} \quad \frac{160 - 1.44x}{160}$

$\boxed{\text{IL for } a \ \& \ b}$



When 1^k at U_0:

$a = b = c = 0$

When 1^k at U_4: $x = 80$

$\Sigma M_{L4} = 0$,

$VA \times 160 + 7.81 \times 10^{-4} x \,(20+40+60+80) - 1 \times 20 + b \times 30 = 0$

$\Rightarrow \frac{160 - 1.44 \times 180}{160} \times 160 + 7.81 \times 10^{-4} \times 80 \times 200 - 20 + 30b = 0$

$\therefore b = -0.168^k$

Consider the whole free body diagram,

$\Sigma M_B = 0,$

$$(V_A + V_A') \times 320 - 1 \times (320 - x) = 0$$

$$\therefore V_A + V_A' = \frac{320 - x}{320} \quad \ldots \text{(i)}$$

$\Sigma M_e = 0,$

$$(V_A + V_A') \times 160 + 20 \times H - 50 \times H - 1 \times (160 - x) = 0$$

$$\Rightarrow \frac{320 - x}{320} \times 160 - 160 + x = 30H$$

$$\therefore H = \frac{x}{60} \quad \ldots \text{(ii)}$$

Now,

$$H = \frac{\omega l^2}{8h} \Rightarrow \frac{x}{60} = \frac{\omega \times (320)^2}{8 \times 30}$$

$$\therefore \omega = 3.91 \times 10^{-5} x$$

Hanger tension, $T_h = \omega \times$ Panel length

$$= 3.91 \times 10^{-5} \times 20x$$

$$= 7.81 \times 10^{-4} x$$

IL for hanger tension on C member:

When 1^k at v_0, $x = 0$ $C = 0$

1^k at v_1, $x = 20$ $C = 0.016^k$

1^k at v_2, $x = 40$ $C = 0.031^k$

1^k at v_3, $x = 60$ $C = 0.047^k$

1^k at v_4, $x = 80$ $C = 0.062^k$