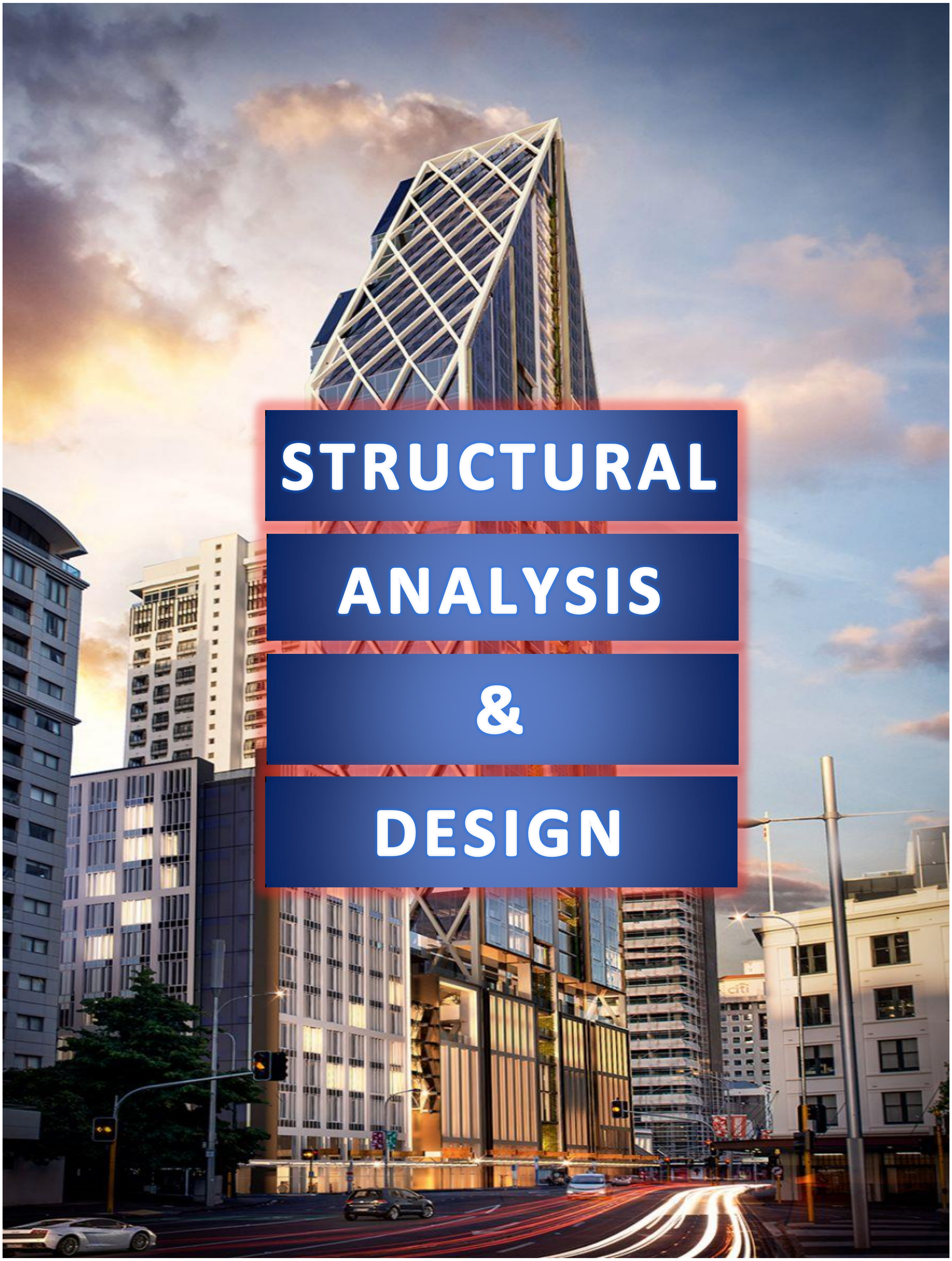
STRUCTURAL ANALYSIS & DESIGN



HandNote On

STRUCTURAL ANALYSIS & DESIGN

RUET

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Contents

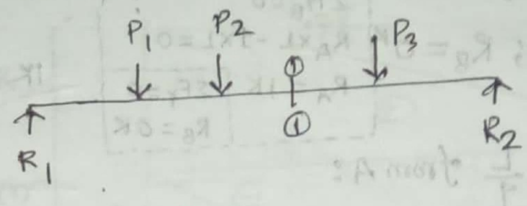
FARHAD
#1500045

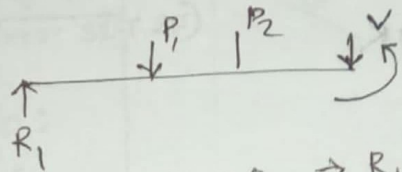
Influence Line

Equations

2D	3D
$\Sigma F_x = 0$	$\Sigma F_x = 0$
$\Sigma F_y = 0$	$\Sigma F_y = 0$
$\Sigma M = 0$	$\Sigma F_z = 0$
	$\Sigma M_x = 0$
	$\Sigma M_y = 0$
	$\Sigma M_z = 0$

Shear force: Algebraic sum of all transverse forces either considering left or right section of a beam is called shear force.





(considering left section)

$$\Sigma F_y = 0 \Rightarrow R_1 - P_1 - P_2 - V = 0$$

$$\Rightarrow \boxed{V = R_1 - P_1 - P_2}$$

Kinds of Loads:

1. Dead Loads.
2. Live Loads.

Dead loads: Deads loads are loads which are always fixed in position, always acting and of unchanging magnitude.

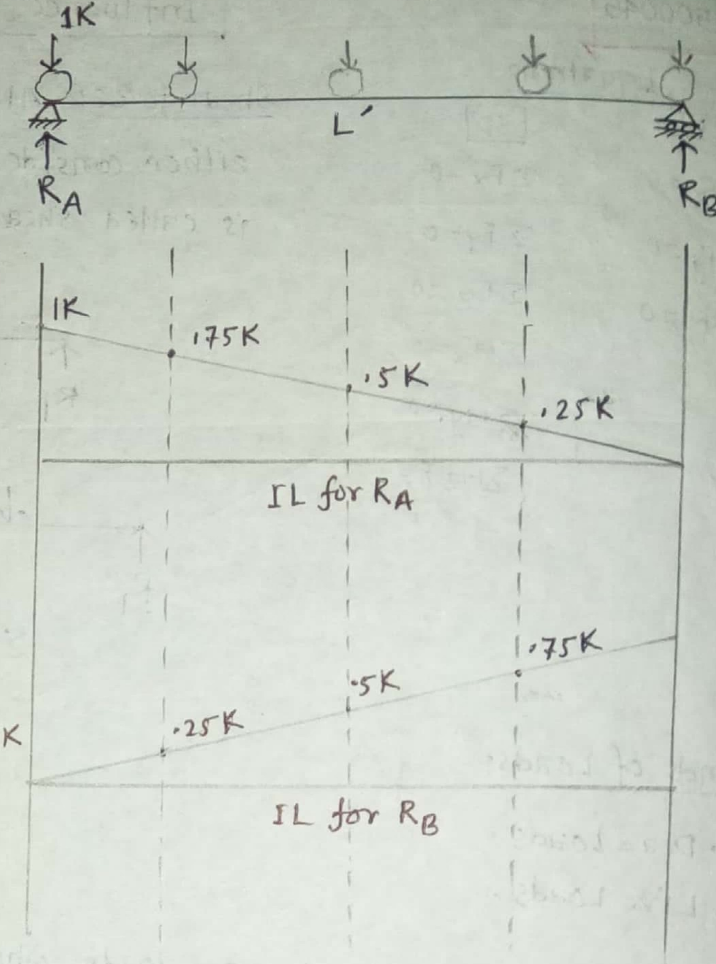
Live loads: Live loads are loads which are more or less temporary and which vary in magnitude.

Influence Line: An influence line is a diagram showing the variation in shear, moment, stress in a member, reaction or other direct function due to a unit load moving across the structure.

construction of influence Line: An influence line is constructed by plotting directly under the point where unit load is placed an ordinate, the height of which represents to some scale the value of the particular function being studied when the load is in that position.

Influence line for Reaction:

Draw IL for R_A and R_B



when 1^K at A:

$R_A = 1K$; $R_B = 0^K$

$\Sigma M_B = 0$
$R_A \times L - 1 \times L = 0$
$R_A = 1K$

$\Sigma F_Y = 0$
$R_B = 0K$

when 1K at $\frac{L}{4}$ from A:

$R_A = .75K$; $R_B = .25K$

when 1K at $\frac{L}{2}$ from A:

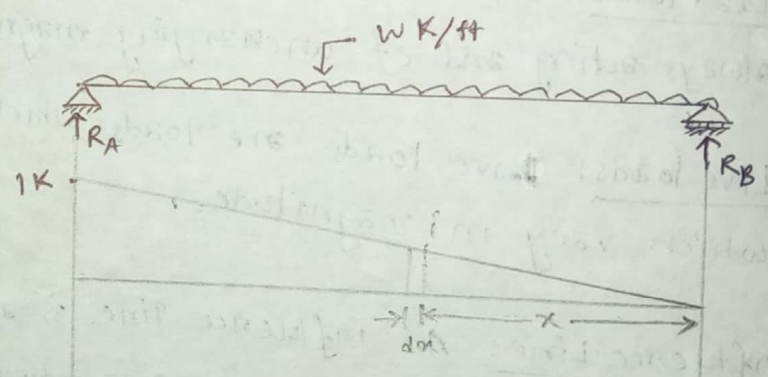
$R_A = .5K$; $R_B = .5K$

when 1K at $\frac{3L}{4}$ from A: $R_A = .25K$; $R_B = .75K$

when 1K at B:

$R_A = 0$; $R_B = 1K$

In case of distributed load:



$\frac{1}{L} = \frac{i}{x}$ [From similar Triangle]

$\Rightarrow i = \frac{x}{L}$

Now,

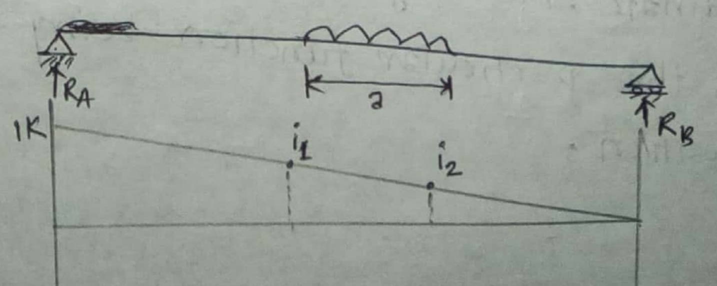
$\int_0^L dR_A = \int_0^L i \times w \times dx$

$\Rightarrow R_A = \int_0^L \frac{x}{L} \times w\, dx$

$\therefore R_A = \left[\frac{w x^2}{2L}\right]_0^L = \frac{wL}{2}$

$= \frac{1}{2} \times (1 \times L) \times w$

Reaction = Area of influence line × distributed load.

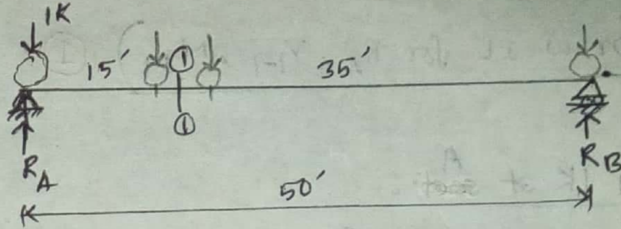


$$R_A = \boxed{\frac{i_1 + i_2}{2} \times a} \times w$$

Area of influence Line

Influence Line for shear force:

(# Draw IL for V_{1-1})

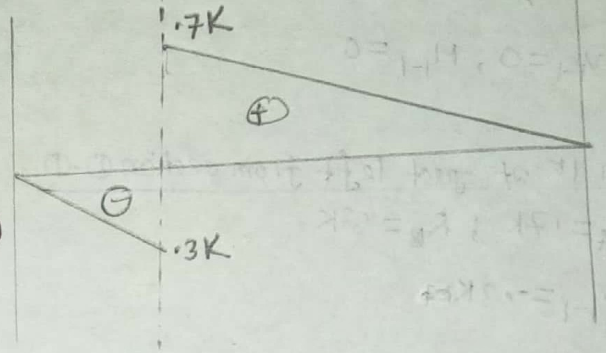


when 1K at A:

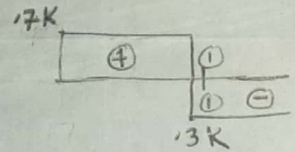
$R_A = 1K,\ R_B = 0\ ;\ V_{1-1} = 0K$

when 1K at just left of section ①-①:

$R_A = .7K\ ;\ R_B = .3K\ ;\ V_{1-1} = -.3K$

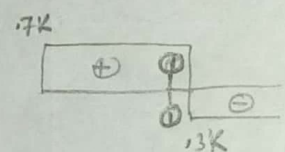


(section যার মধ্যে থাকবে shear উল্টা হবে)



when 1K at just right of section ①-①:

$R_A = .7K,\ R_B = .3K\ ;\ V_{1-1} = .7K$

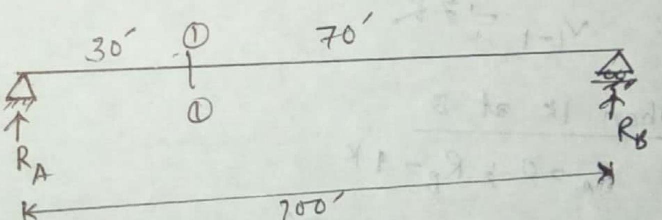


when 1K at B:

$R_A = 0\ ;\ R_B = 1K\ ;\ V_{1-1} = 0K$

Influence Line for Moment:

(# Draw IL for M_{1-1})



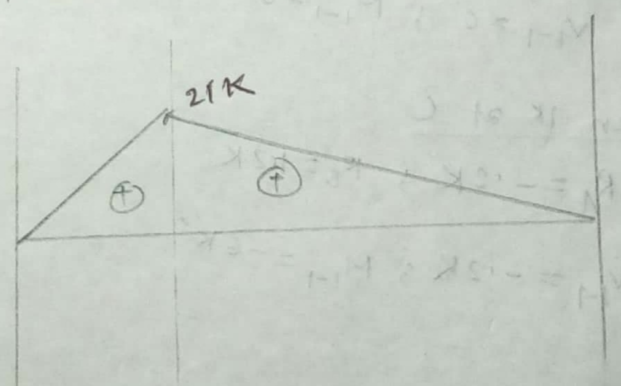
when 1K at A:

$R_A = 1K,\ R_B = 0K\ ;\ M_{1-1} = 0$

when 1K at section ①-①:

$R_A = .7K,\ R_B = .3K\ ;\ M_{1-1} = 21K$

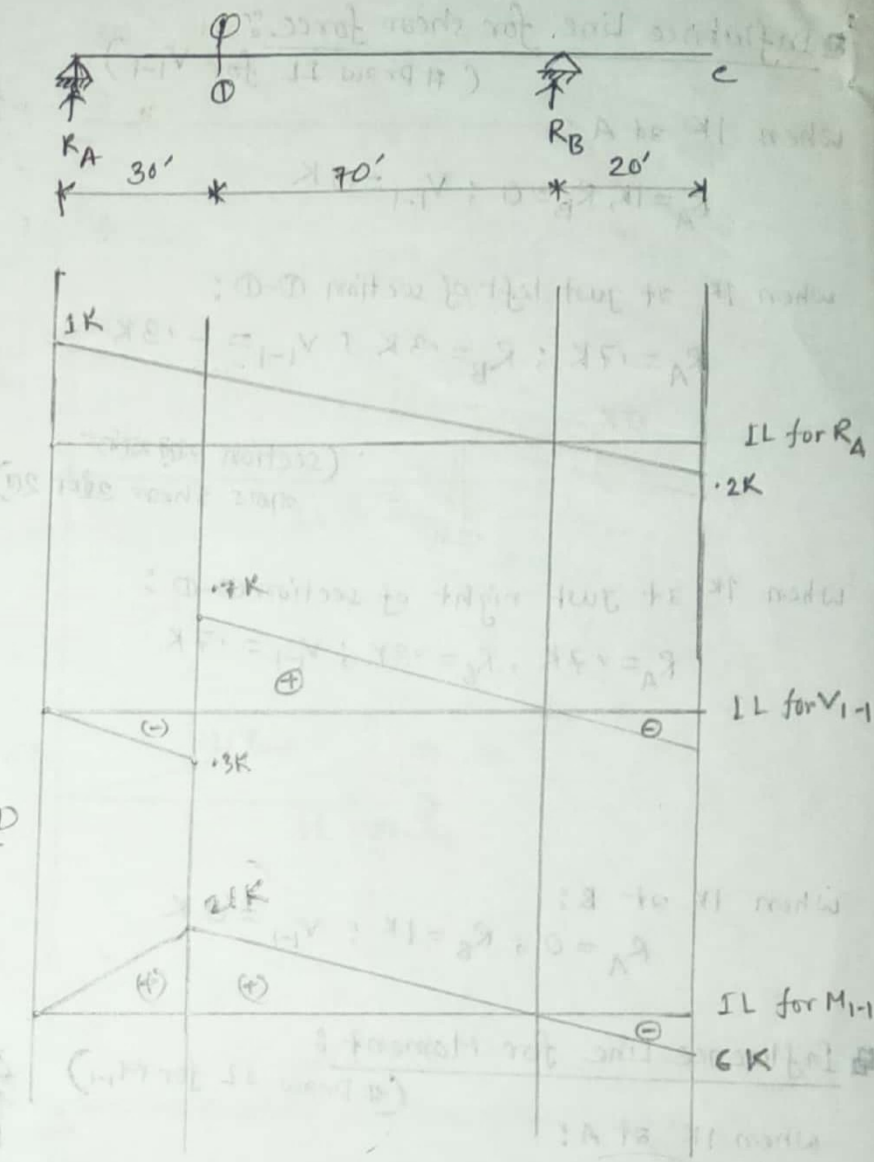
$[M_{1-1} = .7 \times 30]$



when 1K at B:

$R_A = 0K,\ R_B = 1K\ ;\ M_{1-1} = 0$

(Draw IL for R_A, V_{1-1}, M_{1-1}) 1



when 1K at A:

$R_A = 1K;\ R_B = 0$

$V_{1-1} = 0,\ M_{1-1} = 0$

when 1K at just left from section ①-①

$R_A = .7K;\ R_B = .3K$

$V_{1-1} = -.3K$

when 1K at ~~left~~ section ①-①

$R_A = .7K;\ R_B = .3K$

$M_{1-1} = 21K'$

when 1K at just right from section ①-①

$R_A = .7K;\ R_B = .3K$

$V_{1-1} = .7K$

when 1K at B

$R_A = 0;\ R_B = 1K$

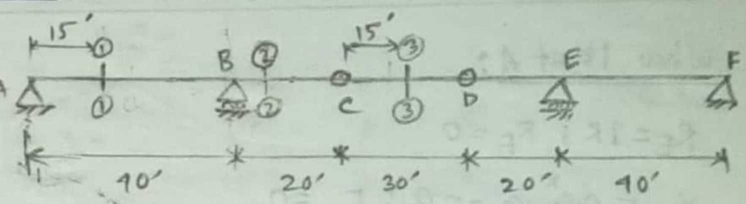
$V_{1-1} = 0;\ M_{1-1} = 0$

when 1K at C

$R_A = -.2K;\ R_B = 1.2K$

$V_{1-1} = -.2K;\ M_{1-1} = -6K'$

Draw IL for $V_{1-1}, V_{2-2}, V_{3-3}, M_{1-1}, M_{2-2}, M_{3-3}$ ②



When 1K at A: $R_A = 1K$; $R_B = 0K$;

$V_{1-1} = 0$; $V_{2-2} = 0$; $V_{3-3} = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$; $M_{3-3} = 0$

When 1K at just left of section ①-①

$R_A = \cdot625K$; $R_B = \cdot375K$

$V_{1-1} = -\cdot375K$; $V_{2-2} = 0$; $V_{3-3} = 0$

When 1K at section ①-①

$R_A = \cdot625K$; $R_B = \cdot375K$

$M_{1-1} = 9\cdot375K$; $M_{2-2} = 0$; $M_{3-3} = 0K'$

When 1K at just right of section ①-①

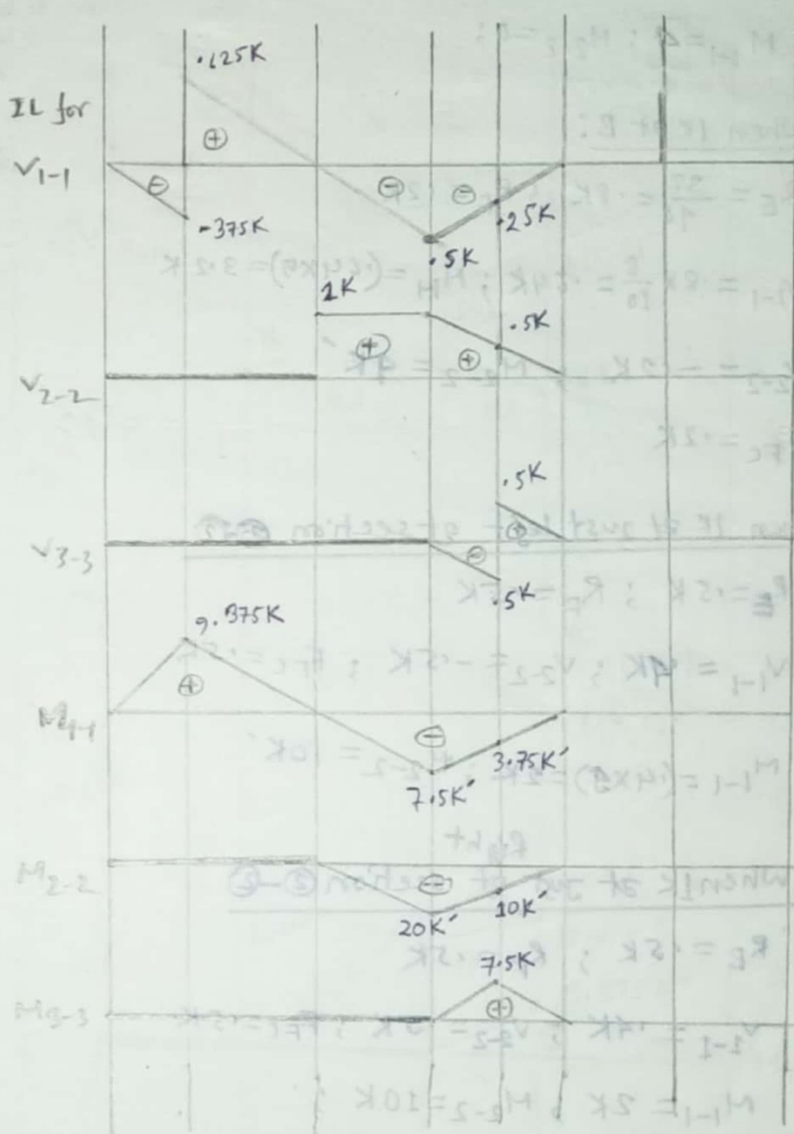
$R_A = \cdot625K$; $R_B = \cdot375K$

$V_{1-1} = \cdot625K$; $V_{2-2} = 0$; $V_{3-3} = 0$

When 1K at B: $R_A = 0$; $R_B = 1K$

$V_{1-1} = 0$; $V_{2-2} = 0$; $V_{3-3} = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$; $M_{3-3} = 0$



When 1K at just Right of section ②-②

$V_{1-1} = 0$; $V_{2-2} = 1K$; $V_{3-3} = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$; $M_{3-3} = 0$

When 1K at C:

$R_A = -\cdot5K$, $R_B = 1\cdot5K$

$V_{1-1} = -\cdot5K$; $V_{2-2} = 1K$; $V_{3-3} = 0$

When 1K at just left of section ③-③

$R_A = -\cdot25K$; $R_B = \cdot75K$

$\therefore V_{1-1} = -\cdot25K$, $V_{2-2} = \cdot5K$, $V_{3-3} = -\cdot5K$

$M_{1-1} = -3\cdot75K'$, $M_{2-2} = -10K'$; $M_{3-3} = 7\cdot5K'$

When 1K at just Right of section ③-③

$R_A = -\cdot25K$; $R_B = \cdot75K$

$V_{1-1} = -\cdot25K$; $V_{2-2} = \cdot5K$; $V_{3-3} = \cdot5K$

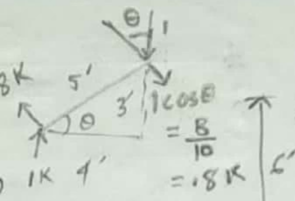
$M_{1-1} = -3\cdot75K'$; $M_{2-2} = -10K'$; $M_{3-3} = 7\cdot5K$

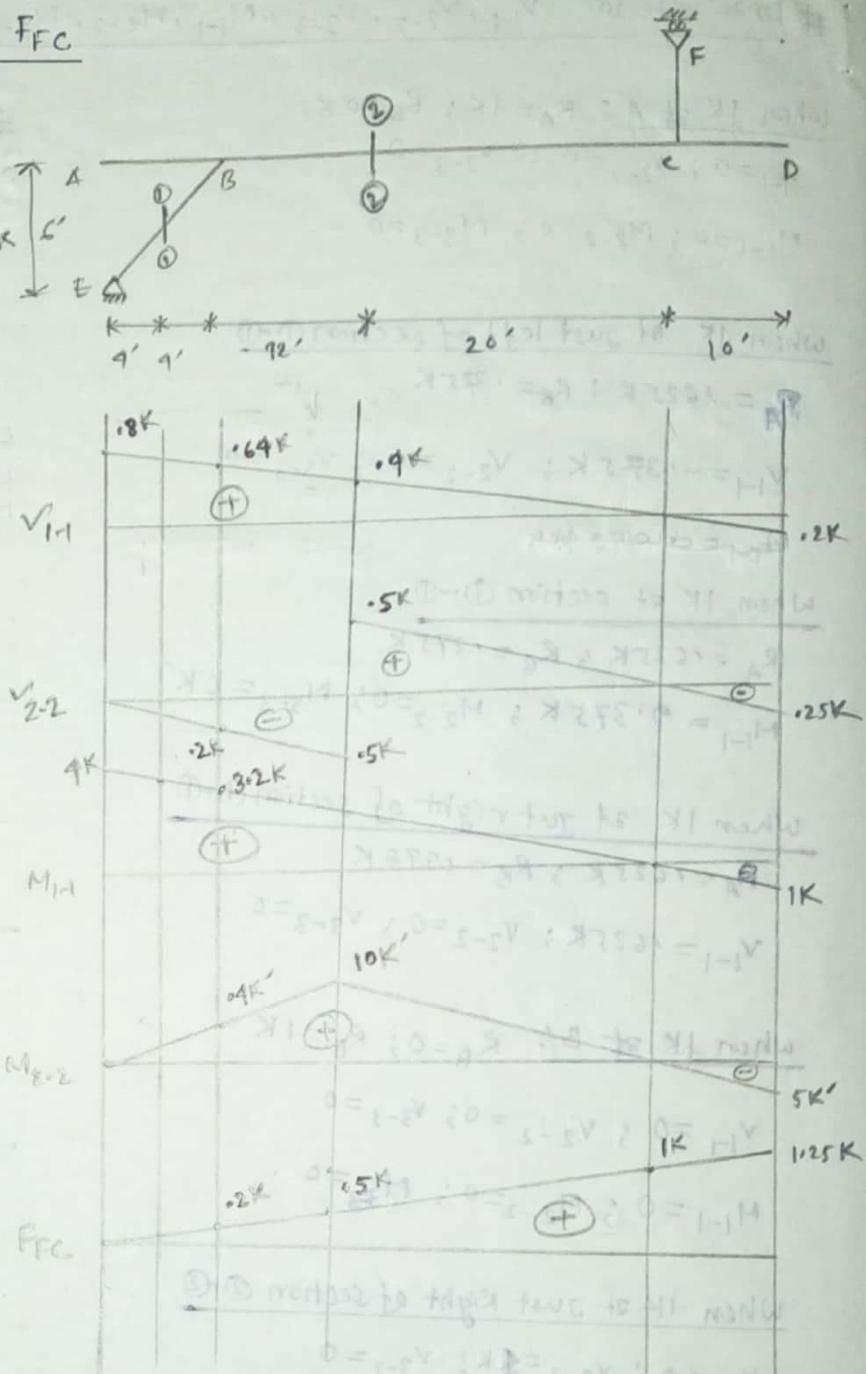
When 1K at D:

$V_{1-1} = 0$; $V_{2-2} = 0$; $V_{3-3} = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$; $M_{3-3} = 0$

Draw IL for $V_{1-1}, V_{2-2}, V_{3-3}, M_{1-1}, M_{2-2}, F_{FC}$





When 1K at A:

$R_E = 1K$; $R_F = 0$

$V_{1-1} = 0.8^K$; $V_{2-2} = 0$; $F_{FC} = 0$

$M_{1-1} = \boxed{4}$; $M_{2-2} = 0$;
$(.8\times5)$

When 1K at B:

$R_E = \frac{32}{40} = .8K$; $R_F = .2K$

$V_{1-1} = .8 \times \frac{8}{10} = .64K$; $M_{1-1} = (.64\times5) = 3.2K$

$V_{2-2} = -.2K$; $M_{2-2} = 4K'$

$F_{FC} = .2K$

When 1K at just left of section 2-2

$R_E = .5K$; $R_F = .5K$

$V_{1-1} = \boxed{.4K}$; $V_{2-2} = -.5K$; $F_{FC} = .5K$
$(.5\times\frac{8}{10})$

$M_{1-1} = (.4\times5) = 2K$; $M_{2-2} = 10K'$

When 1K at just Right of section 2-2

$R_E = .5K$; $R_F = .5K$

$V_{1-1} = .4K$; $V_{2-2} = .5K$; $F_{FC} = .5K$

$M_{1-1} = 2K$; $M_{2-2} = 10K$;

When 1K at C:

$R_E = 0$; $R_F = 1K$

$V_{1-1} = 0$; $M_{1-1} = 0$; $F_{FC} = 1K$

$V_{2-2} = 0$; $M_{2-2} = 0$

When 1K at D:

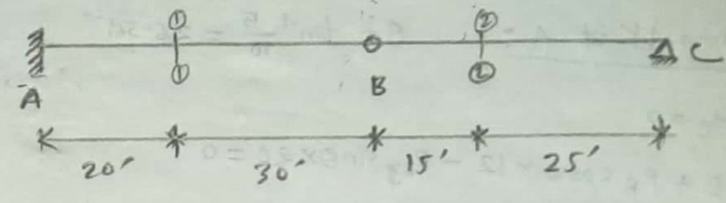
$R_E = -0.25K$; $R_F = 1.25K$

$V_{1-1} = (-.25\times\frac{8}{10}) = -.2K$; $M_{1-1} = (-.2\times5) = -1K$

$V_{2-2} = -.25K$; $M_{2-2} = (-.25\times20) = -5K'$

$F_{FC} = 1.25K$

Draw IL for V_{1-1}, V_{2-2}, M_{1-1}, M_{2-2}, M_A : 5



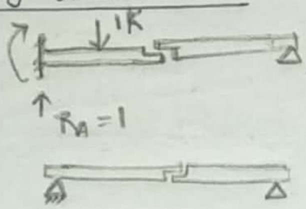
when 1K at A

$R_A = 1$; $R_C = 0$

$V_{1-1} = 0$; $V_{2-2} = 0$; $M_A = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$

when 1K at just left of section 1-1



$R_A = 1K$; $R_C = 0K$

$V_{1-1} = 0$; $V_{2-2} = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$

$M_A = -20K'$

when 1K at just Right of section 1-1

$R_A = 1K$; $R_C = 0K$

$V_{1-1} = 1K$; $V_{2-2} = 0$

$M_{1-1} = -20K'$; $M_{2-2} = 0$

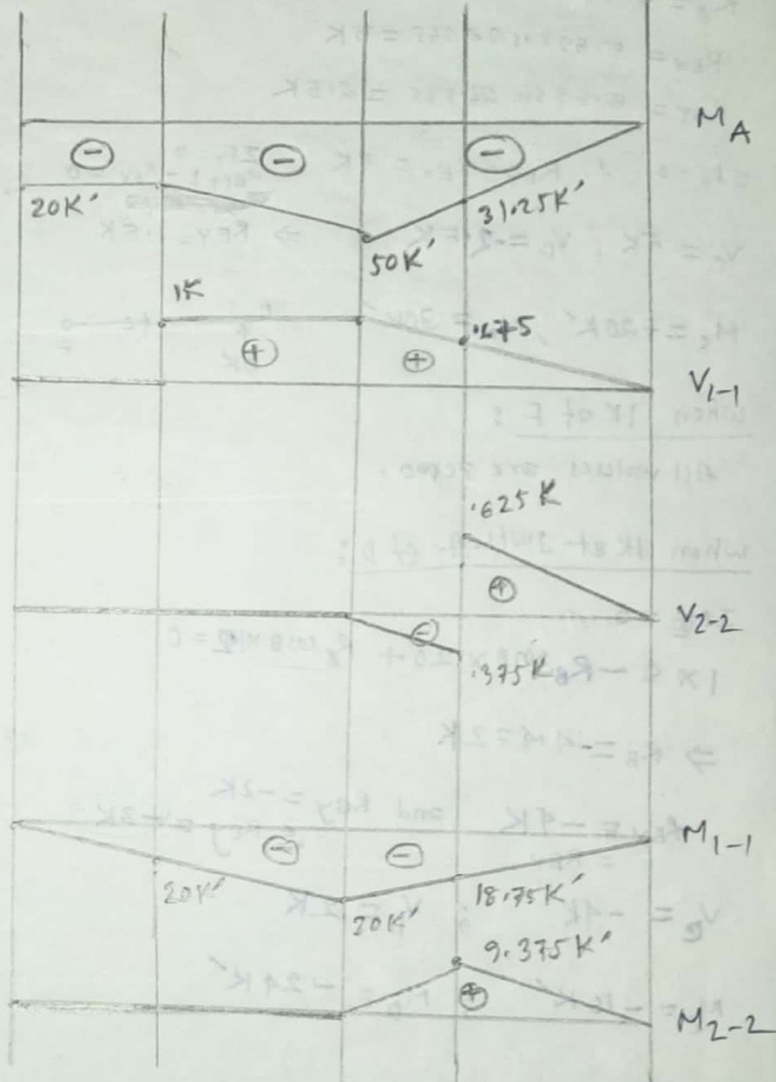
when 1K at B

$R_A = 1K$; $R_C = 0K$

$V_{1-1} = 1K$; $V_{2-2} = 0$

$M_{1-1} = -30K'$; $M_{2-2} = 0$

$M_A = -50K'$



when 1K at just left of section 2-2

$R_C = \frac{15}{40} = .375K$ $R_B = .625K$

$V_{1-1} = .625K$; $V_{2-2} = -.375$

$M_{1-1} = -18.75K'$; $M_{2-2} = 9.375K'$; $M_A = -31.25$

when 1K at just Right of section 2-2

$R_C = .375K$ $R_B = .625K$

$V_{1-1} = .625K$; $V_{2-2} = .625K$

$M_{1-1} = -18.75K'$; $M_{2-2} = 9.375K'$; $M_A = -31.25$

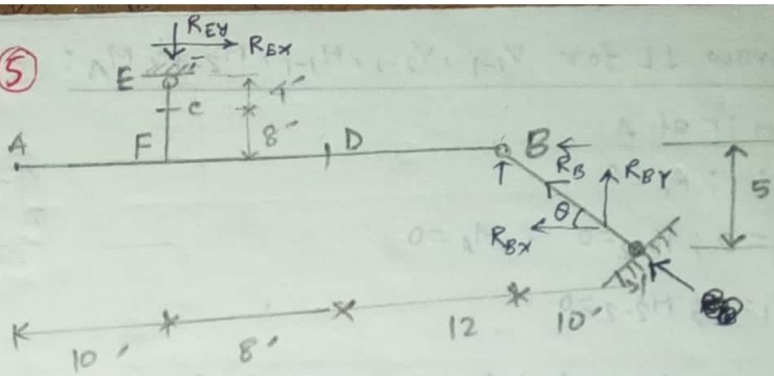
when 1K at C:

$V_{1-1} = 0$; $V_{2-2} = 0$

$M_{1-1} = 0$; $M_{2-2} = 0$

$M_A = 0$

Draw IL for V_C, V_D, M_C, M_D and R_B ⑤



When 1K at A : $\theta = \tan^{-1}\frac{5}{10} = 26.565°$

$\Sigma M_E = 0$

$-1 \times 10 + R_B \cos\theta \times 12 - R_{B} \sin\theta \times 20 = 0$

$R_B = 5.59K$

$R_{BX} = 5.59 \cos 26.565 = 5K$

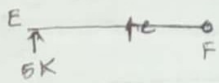
$R_{BY} = 5.59 \sin 26.565 = 2.5K$

$\Sigma F_x = 0 \quad \therefore R_{BX} = R_{EX} = 5K$

$\Sigma F_Y = 0$
$R_{EY} + 1 - R_{BY} = 0$
$\Rightarrow R_{EY} = 1.5K$

$V_C = 5K$, $V_D = -2.5K$

$M_C = +20K'$, $M_D = 30K'$



when 1K at F :

All values are zero.

when 1K at justleft of D:

$\Sigma M_E = 0$

$1 \times 8 - R_B \sin\theta \times 20 + R_B \cos\theta \times 12 = 0$

$\Rightarrow R_B = -4.472K$

$R_{BX} = -4K = R_{EX}$ and $R_{By} = -2K$ $\therefore R_{Ey} = -3K$

$V_C = -4K$; $V_D = 2K$

$M_C = -16K'$; $M_D = -24K'$

when 1K at Just Right of D:

$V_D = +3K$

when 1K at B:

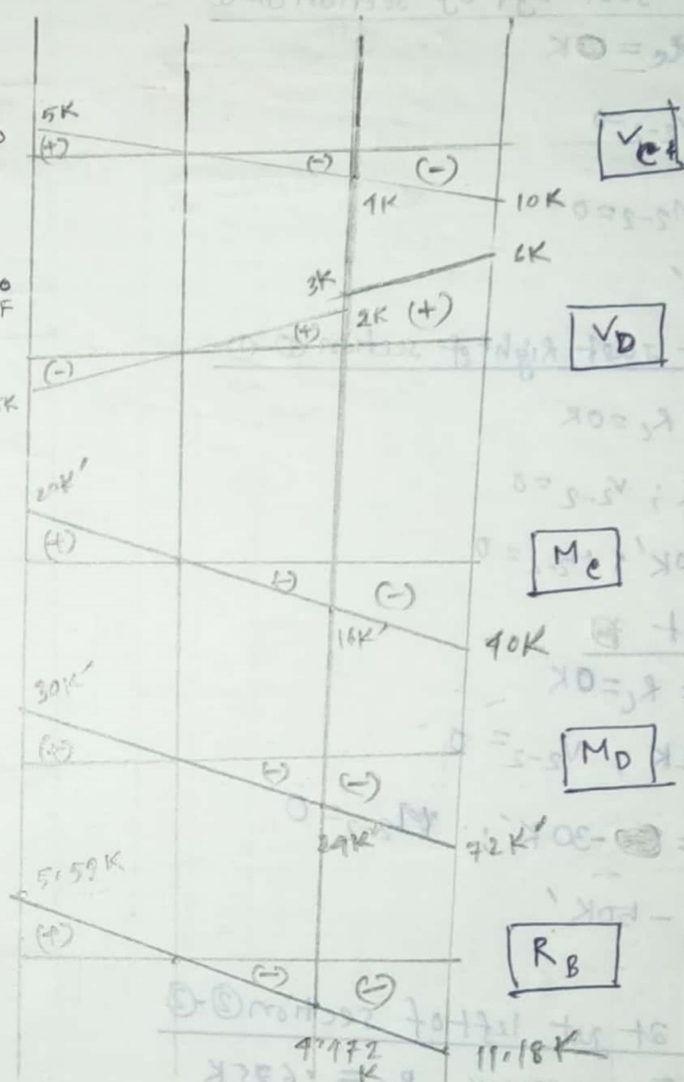
$\Sigma M_E = 0$

$1 \times 20 + R_B \cos\theta \times 12 - R_B \sin\theta \times 20 = 0$

$\Rightarrow R_B = -11.18K$

$R_{Bx} = -10K$ $R_{By} = -5K$ $\therefore R_{EX} = -10K$, $R_{EY} = -6$

$V_C = -10K$, $V_D = 6K$, $M_C = -40K'$, $M_D = -72K'$



When 1K at E

$R_A = 0$; $R_B = 0$; $R_D = 1.25K$, $R_c = -.125K$

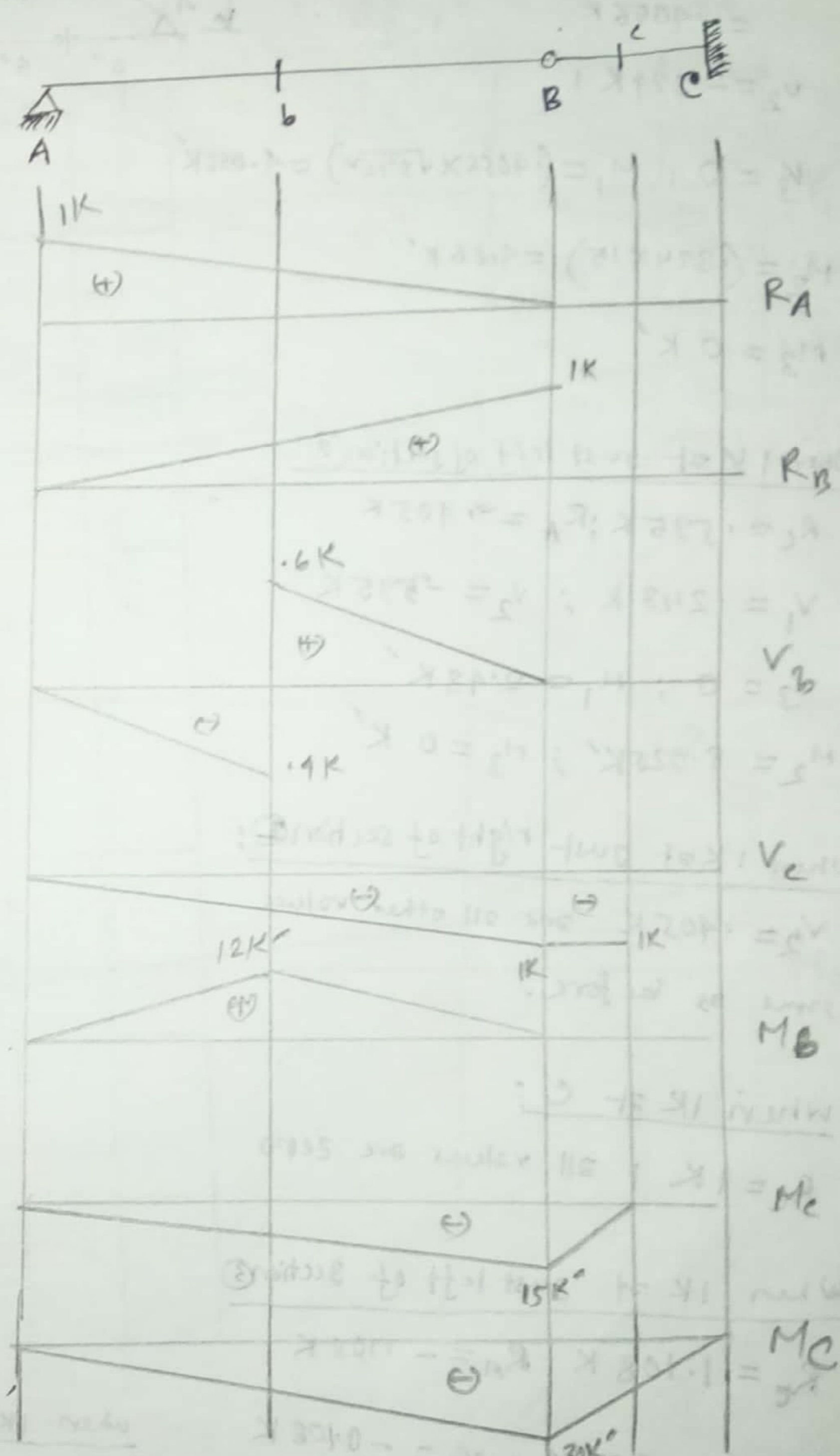
$V_a = 0$; $V_b = -0.25K$; $V_c = -0.25K$

$M_a = 0$; $M_b = 0$; $M_c = -7.5K'$

When 1K at F

$R_F = 1K$; all values are zero.

Draw IL for $R_A, R_B, V_b, V_c, M_b, M_c, M_C$: 5



When 1K at A:

$R_A = 1K$; $R_B = 0$

$V_b = 0$, $V_c = 0$

$M_b = 0$, $M_c = 0$, $M_C = 0$

when 1 K at just left of section (b)

$R_A = 0.6K$; $R_B = 0.4K$

$V_b = -0.4K$; $V_c = -0.4K$

$M_b = 12K'$; $M_c = 6K'$

$M_C = -12K'$

When 1K at just right of section (b)

$V_b = +0.6K$ and same as before

When 1K at B

$R_A = 0$; $R_B = 1K$

$V_b = 0$; $V_c = -1K$

$M_b = 0$; $M_c = -15K'$

$M_C = -30K'$

When 1K at just ~~right~~ left of section (c)

~~$V_c = 0K$~~ $R_A = 0$, $R_B = 0$, $R_c = 1K$

$V_b = 0$; $V_c = +1K$

$M_b = 0$; $M_c = 0K'$; $M_C = -15K'$

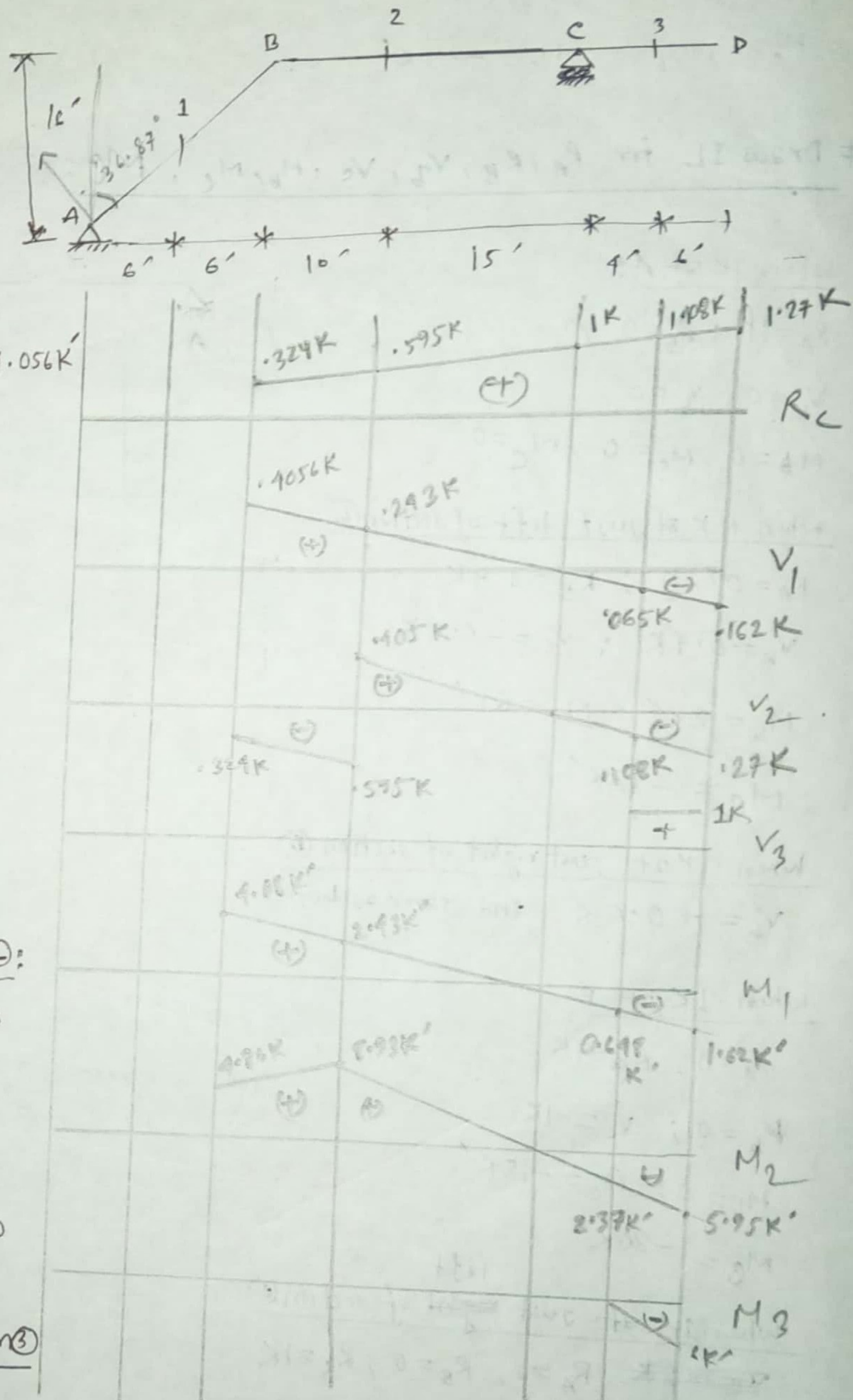
When 1K at just Right of section (c)

$V_c = 0$; and same as before.

When 1K at C

$R_c = 1K$, and all other values are zero.

Draw IL for R_C, $V_1, V_2, V_3, M_1, M_2, M_3$: (load moves from B to D) 6



When 1K at B:

$R_C = .324\text{ K}$, $R_A = .676\text{ K}$

$V_1 = .676 \sin 36.87°$

$= .4056\text{ K}$

$V_2 = -.324\text{ K}$;

$V_3 = 0$; $M_1 = (.4056 \times \sqrt{8^2+6^2}) = 4.056\text{ K}'$

$M_2 = (.324 \times 15) = 4.86\text{ K}'$

$M_3 = 0\text{ K}'$

When 1 K at just left of section ②:

$R_C = .595\text{ K}$; $R_A = .405\text{ K}$

$V_1 = .243\text{ K}$; $V_2 = -.595\text{ K}$

$V_3 = 0$; $M_1 = 2.43\text{ K}'$

$M_2 = 8.925\text{ K}'$; $M_3 = 0\text{ K}'$

When 1K at just right of section ②:

$V_2 = .405\text{ K}$ and all other values same as before.

When 1K at C:

$R_C = 1\text{K}$; all values are zero

When 1K at just left of section ③

$R_C = 1.108\text{ K}$ $R_A = -.108\text{ K}$

$V_1 = -.065\text{ K}$; $V_2 = -0.108\text{ K}$

$M_1 = -0.65\text{ K}'$; $M_2 = -2.38\text{ K}'$

$M_3 = 6\text{ K}'$; $V_3 = 0\text{ K}$

when 1K at just right of section ③

$V_3 = 1\text{K}$

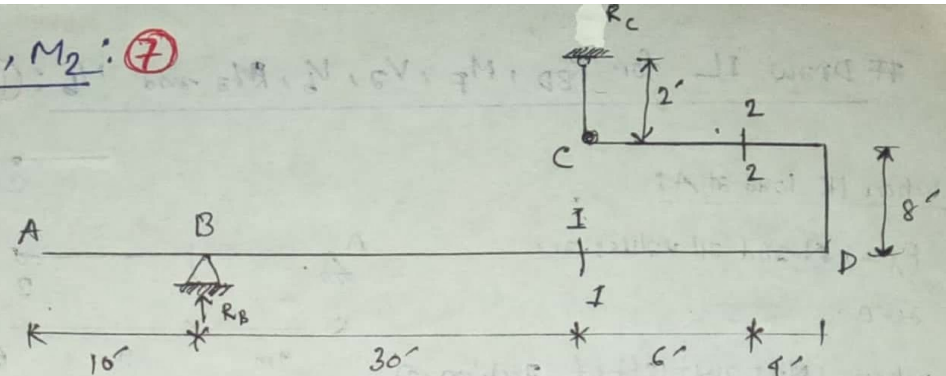
When 1K at D:

$R_C = 1.27\text{ K}$ $R_A = -.27\text{ K}$

$V_1 = -.162\text{ K}$; $V_2 = -.27\text{K}$; $V_3 = 1\text{K}$

$M_1 = -1.62\text{K}'$; $M_2 = -5.95\text{K}'$; $M_3 = -6\text{K}'$

Draw IL for V_1, V_2, M_1, M_2: 7



When 1K at A

$R_C = -.33K (\downarrow)$

$R_B = 1.33K (\uparrow)$

$V_1 = .33K$

$V_2 = -.33K$

$M_1 = 0K'$; $M_2 = -2K'$

When 1 K at B

$R_B = 1K$ and all values are zero

When 1 K at just left of section ①

$R_B = 0$; $R_C = 1K (\uparrow)$

$V_1 = -1K$; $V_2 = +1K$

$M_1 = 0K'$; $M_2 = 6K'$

when 1K just right of section ②

$V_1 = 0K$; all are same as before.

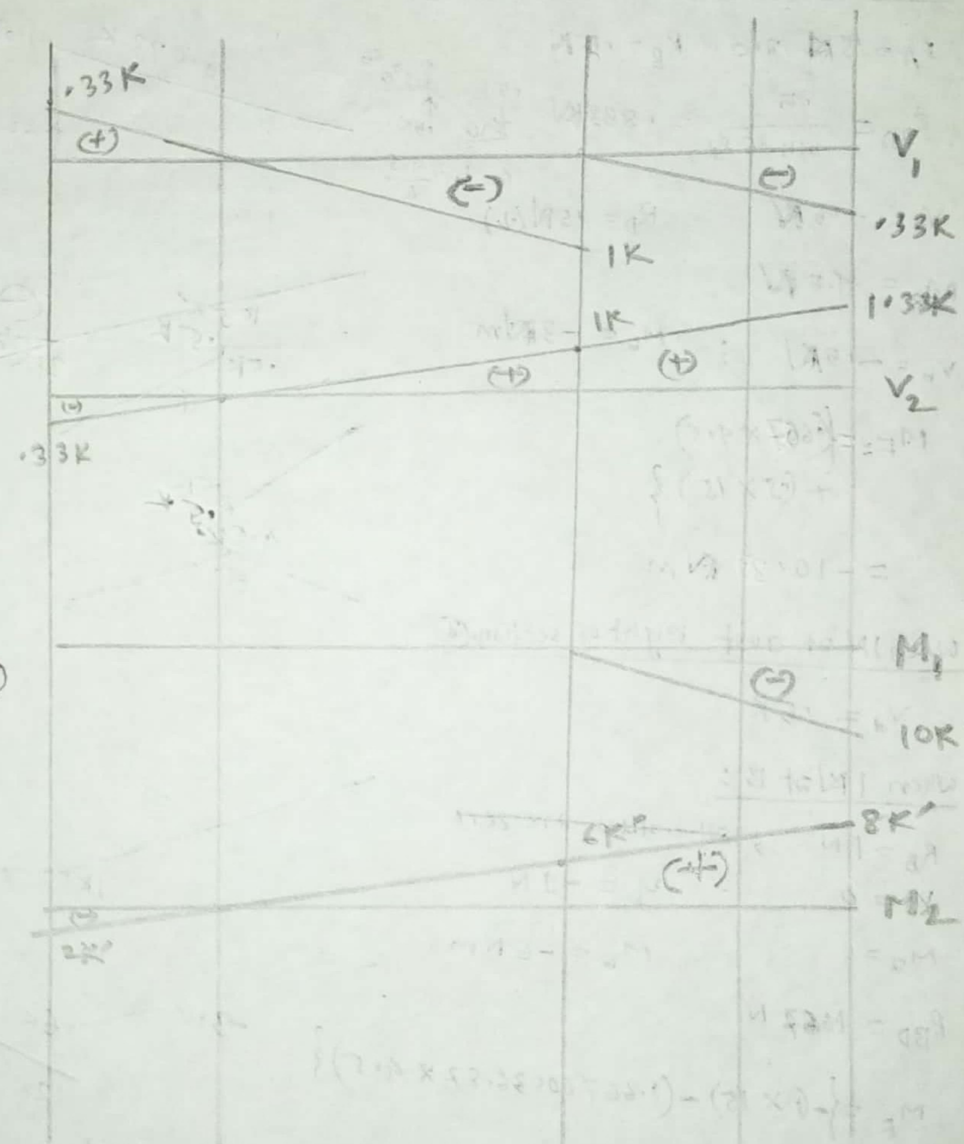
when 1 K at D:

$R_C = 1.33 K (\uparrow)$

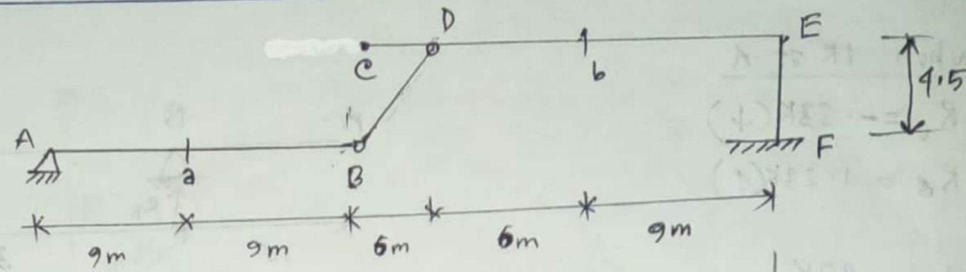
$R_B = -.33K (\downarrow)$

$V_1 = -0.33 K$; $V_2 = 1.33 K$

$M_1 = -10K'$; $M_2 = 8K'$



Draw IL for R_{BD}, M_F, V_a, V_b, M_a and M_b: 12



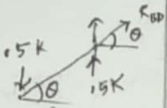
when 1K load at A:

$R_A = 1$ N and all values are zero.

when 1N at just left of section (a)

$R_A = .5$ N and $R_B = .5$ N

$R_{BD} = \frac{.5}{\sin 36.87} = .833$ N



$\theta = \tan^{-1}\frac{4.5}{6}$

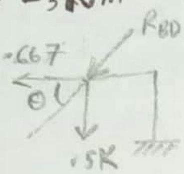
$V_a = -.5$ N $\qquad R_D = .5$ N (↓)

$M_a = 4.5$ N

$V_b = -.5$ N ; $\qquad M_b = -3$ Nm

$M_F = -\{(.667 \times 4.5) + (.5 \times 15)\}$

$= -10.5$ Nm



when 1N at just right of section (a)

$V_a = .5$ K

when 1N at B:

$R_B = 1$ N ; ~~all values are zero~~

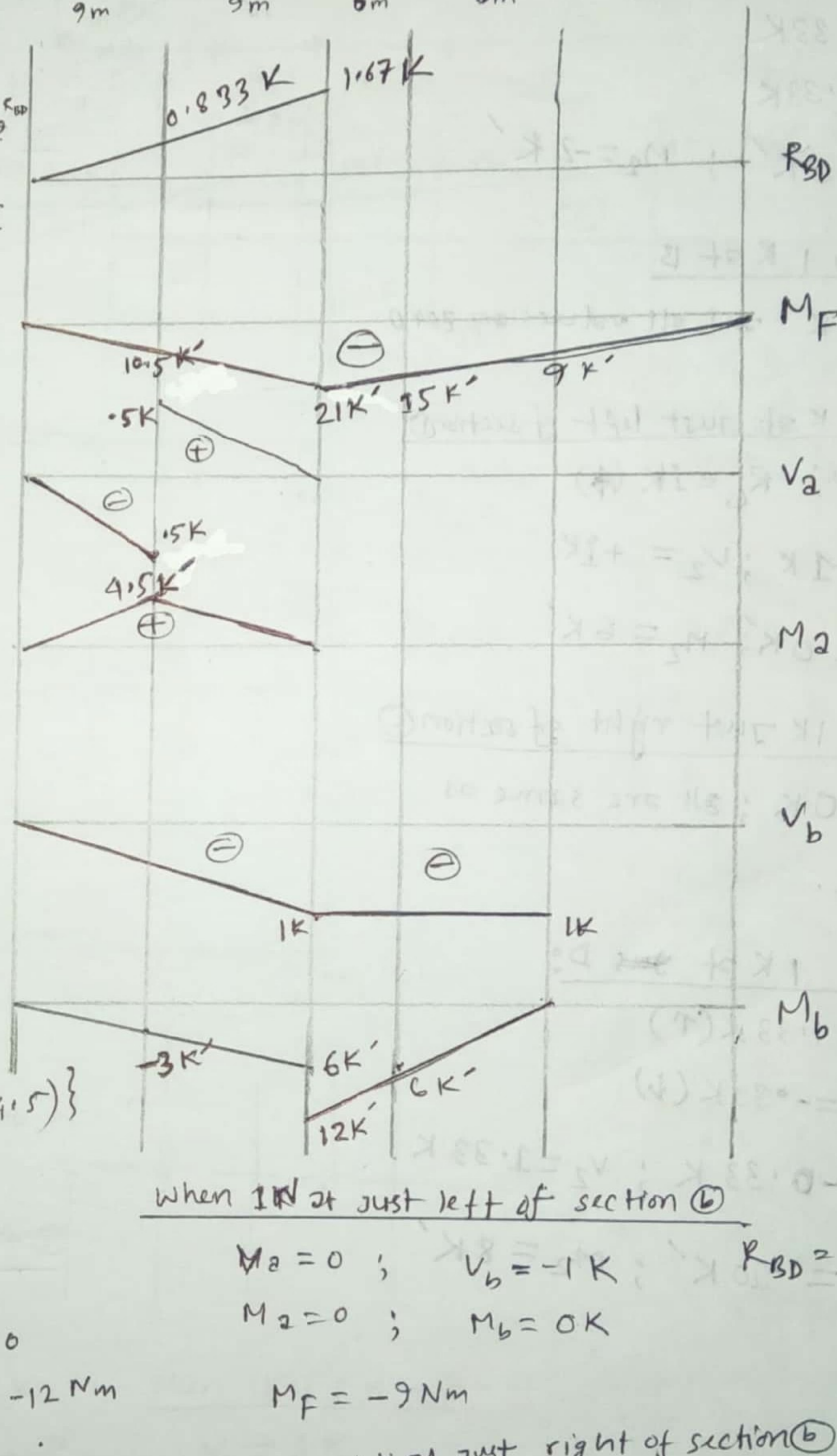
$V_a = 0$ $\qquad V_b = -1$ N

$M_a = 0$ $\qquad M_b = -6$ Nm

$R_{BD} = 1.667$ N

$M_F = \{-(1 \times 15) - (1.667 \cos 36.87 \times 4.5)\}$

$= -21$ N m



when 1N at just left of section (b)

$M_a = 0$; $\quad V_b = -1$ K $\qquad R_{BD} = 0$

$M_a = 0$; $\quad M_b = 0$ K

$M_F = -9$ Nm

when 1K at just right of section (b)

$V_b = 0$

when 1 K at E

all values are zero.

when 1 K at C: $R_D = 0$

$R_{BD} = 0$; $\quad V_a = 0$ $\quad ; M_a = 0$

$R_F = 1$ N; $\quad V_b = -1$ K $\quad ; M_b = -12$ Nm

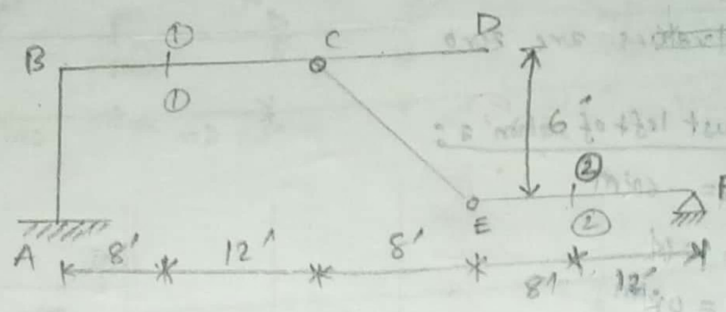
$M_F = -21$ N

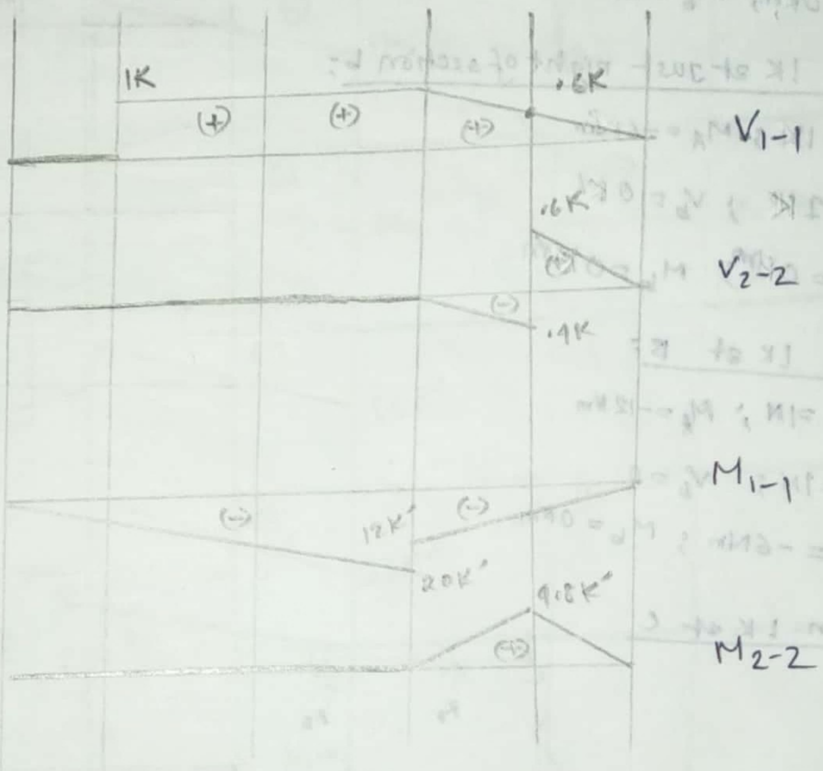
when 1K at D:

$R_{BD} = 0$; $R_F = 1$ N ; $\quad M_F = -15$ N

$V_a = 0$; $\quad M_a = 0$; $\quad V_b = -1$ N ; $M_b = -6$ Nm

Draw IL for V_{1-1}, V_{2-2}, M_{1-1} and M_{2-2}:





When 1K at B:

$R_A = 1K$

$V_{1-1}=0$; $V_{2-2}=0$; $M_{1-1}=0$; $M_{2-2}=0$

When 1K at just left of section ①-①

$R_A = 1K$

$V_{1-1}=0$; $V_{2-2}=0$; $M_{1-1}=0$; $M_{2-2}=0$

When 1K at just right of section ①-①

$R_A = 1K$

$V_{1-1}=1K$; $V_{2-2}=0$; $M_{1-1}=0$; $M_{2-2}=0$

When 1K at C:

$R_A = 1K$

$V_{1-1}=1K$; $V_{2-2}=0$; $M_{1-1}=-12K'$;

$M_{2-2}=0$

When 1K at D

$R_C = 1K$; $R_A = 1K$;

$V_{1-1}=1K$; $V_{2-2}=0$

$M_{1-1}=-20K'$; $M_{2-2}=0$

When 1K at E

$R_F = 1K$, $R_A = 1K$

$V_{1-1}=1K$; $V_{2-2}=0$

$M_{1-1}=-12K'$; $M_{2-2}=0$

When 1K at just left of section ②-②

$R_E = .6K$ $R_F = .4K$, $R_A = .6K$

$V_{1-1}=.6K$; $V_{2-2}=-.4K$

$M_{1-1}=-7.2K'$; $M_{2-2}=4.8K'$

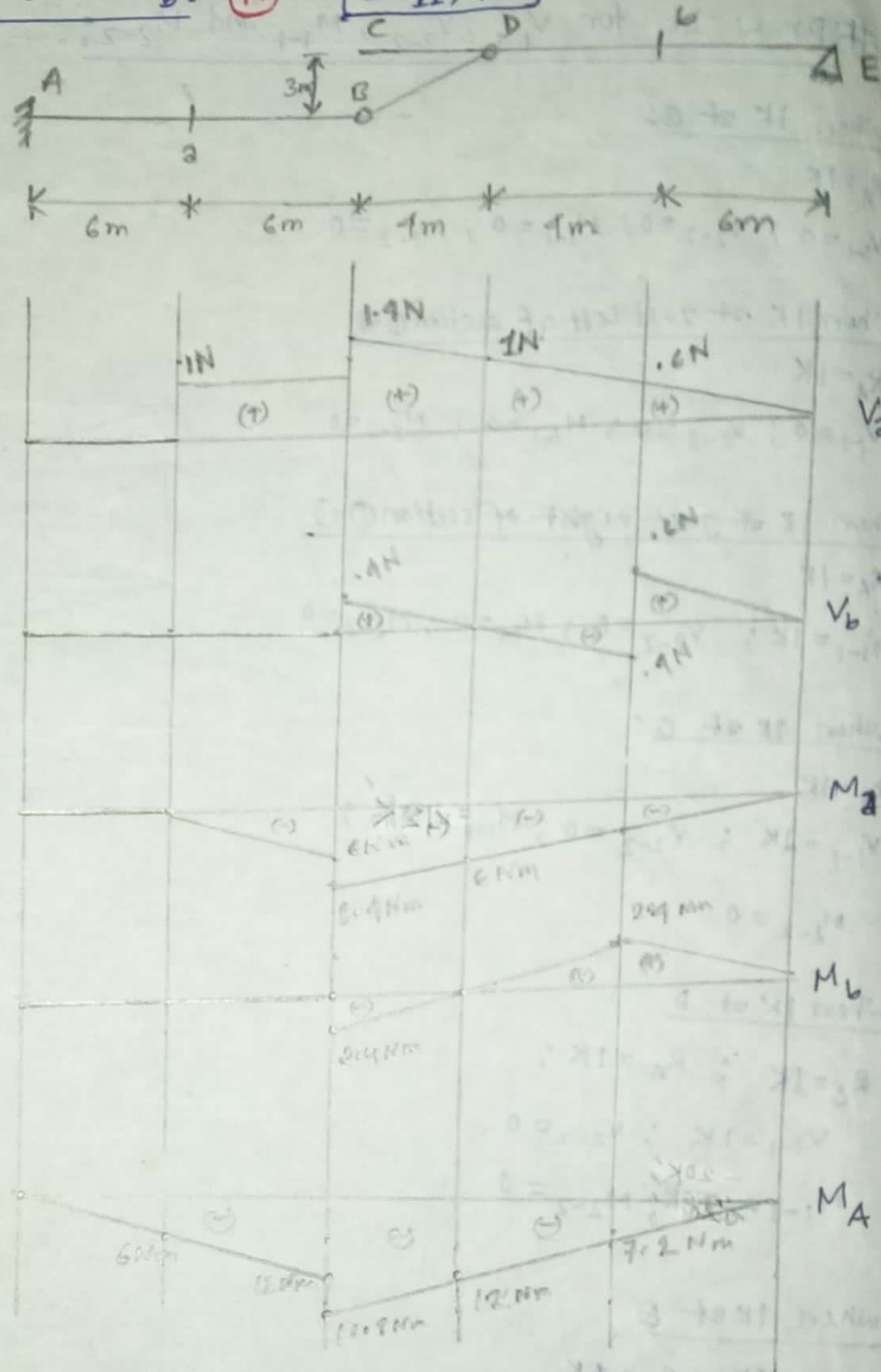
When 1K at just right of section ②-②

$V_{2-2}=.6K$

When 1K at F

All values are zero.

DRAW IL for R_{BD}, M_A, V_a, V_b, M_a and M_b: (11) $\boxed{2011, 16}$



When 1K at A:

$R_{Ay} = 1N$; All values are zero

when 1K at just left of section a:

$R_{Ay} = 1N$; $M_A = -6Nm$

$V_a = 0N$; $V_b = 0N$

$M_a = 0Nm$; $M_b = 0Nm$

when 1K at just right of section b:

$R_{Ay} = 1N$; $M_A = -6Nm$

$V_a = 1N$; $V_b = 0N$

$M_a = 0Nm$; $M_b = 0Nm$

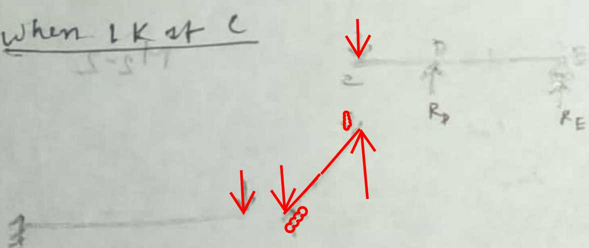
when 1K at B:

$R_{Ay} = 1N$; $M_A = -12Nm$

$V_a = 1N$; $V_b = 0$

$M_a = -6Nm$; $M_b = 0Nm$

when 1K at C



$R_D = 1.4N$; $R_E = -.4N$

$R_B = 1.4N$; $M_A = -16.8Nm$

$V_a = 1.4N$; $V_b = .4N$

$M_a = -8.4$; $M_b = -2.4Nm$

When 1K at D:

$R_{Ay} = 1N$, $M_A = -12KNm$

$V_a = 1N$; $V_b = 0$

$M_a = -6Nm$; $M_b = 0$

when 1K at just left of section b

$R_{Ay} = 0.6N$, $M_A = -7.2Nm$

$V_a = .6N$; $M_a = -3.6Nm$

$D_y = 0.6N$ $\quad$ $V_b = -0.4N$

$E_y = 0.4N$ $\quad$ $M_b = 2.4Nm$

when 1K at just right of section b:

$V_b = 0.6N$

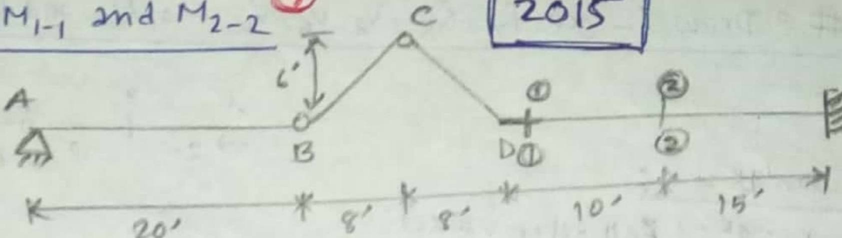
when 1K at E:

All values are zero

Draw IL for R_A, R_{BC}, V_{1-1}, V_{2-2}, M_{1-1} and M_{2-2}

9

2015



When 1K at A

$R_A = 1K$, $R_{BC} = 0$

$V_{1-1} = 0$, $V_{2-2} = 0$, $M_{1-1} = 0$, $M_{2-2} = 0$

When 1K at B

$R_A = 0K$; $R_{BC} = 1.66K$

$V_{1-1} = -1K$; $V_{2-2} = -1K$

$M_{1-1} = -16K'$; $M_{2-2} = -26K'$

When 1K at D

$R_A = 0K$; $R_{BC} = 0$; $V_{1-1} = -1K$

$V_{2-2} = -1K$; $M_{1-1} = 0K'$; $M_{22} = -10K'$

When 1K at just right of section ①-①

$R_A = 0K$; $R_{BC} = 0.K$;

$V_{1-1} = 0$; $V_{2-2} = -1K$;

$M_{1-1} = 0K'$; $M_{2-2} = 0K'$

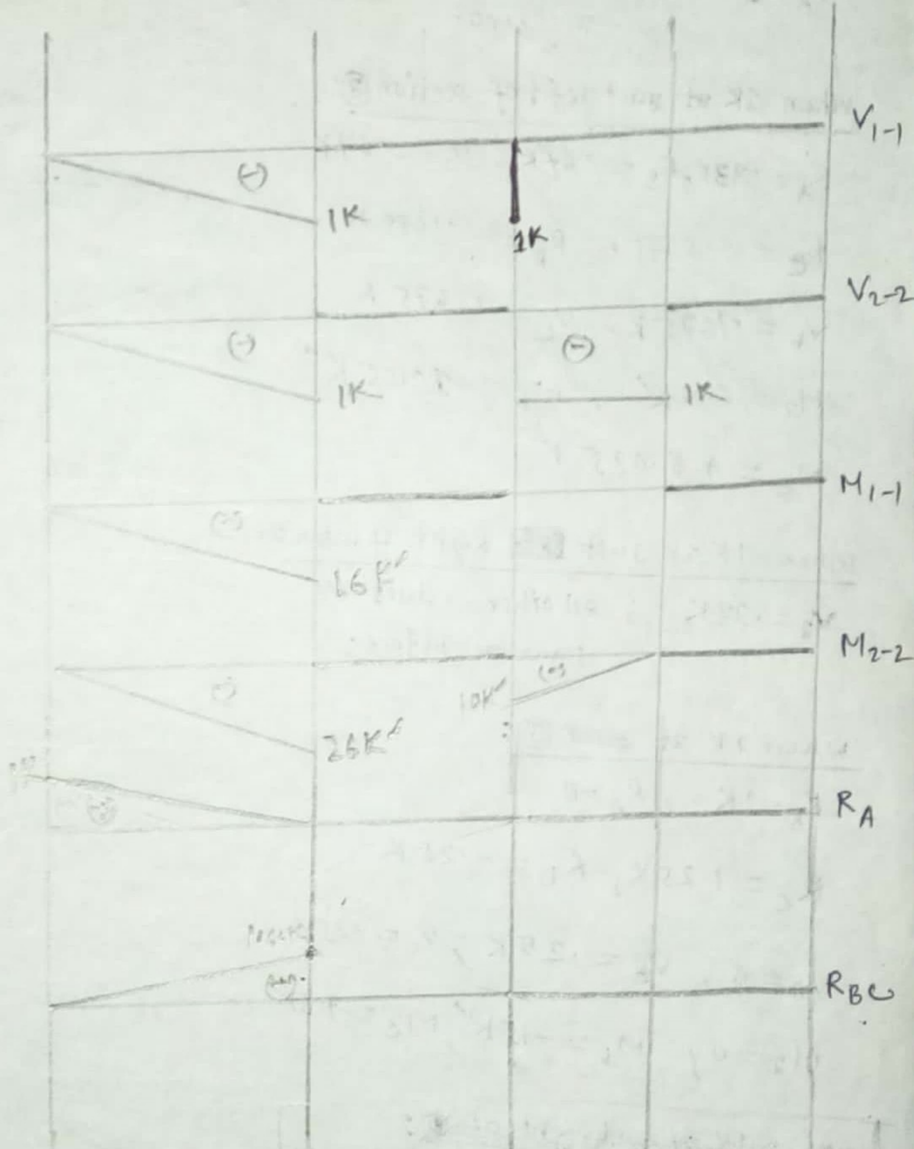
When 1K at just left of section ②-②

$R_A = 0K$; $R_{BC} = 0K$; $V_{1-1} = 0K$

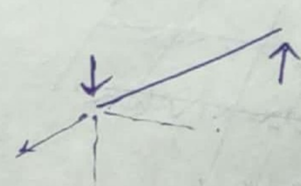
$V_{2-2} = -1K$; $M_{1-1} = 0K'$; $M_{2-2} = 0K'$

When 1K at right of ②②:

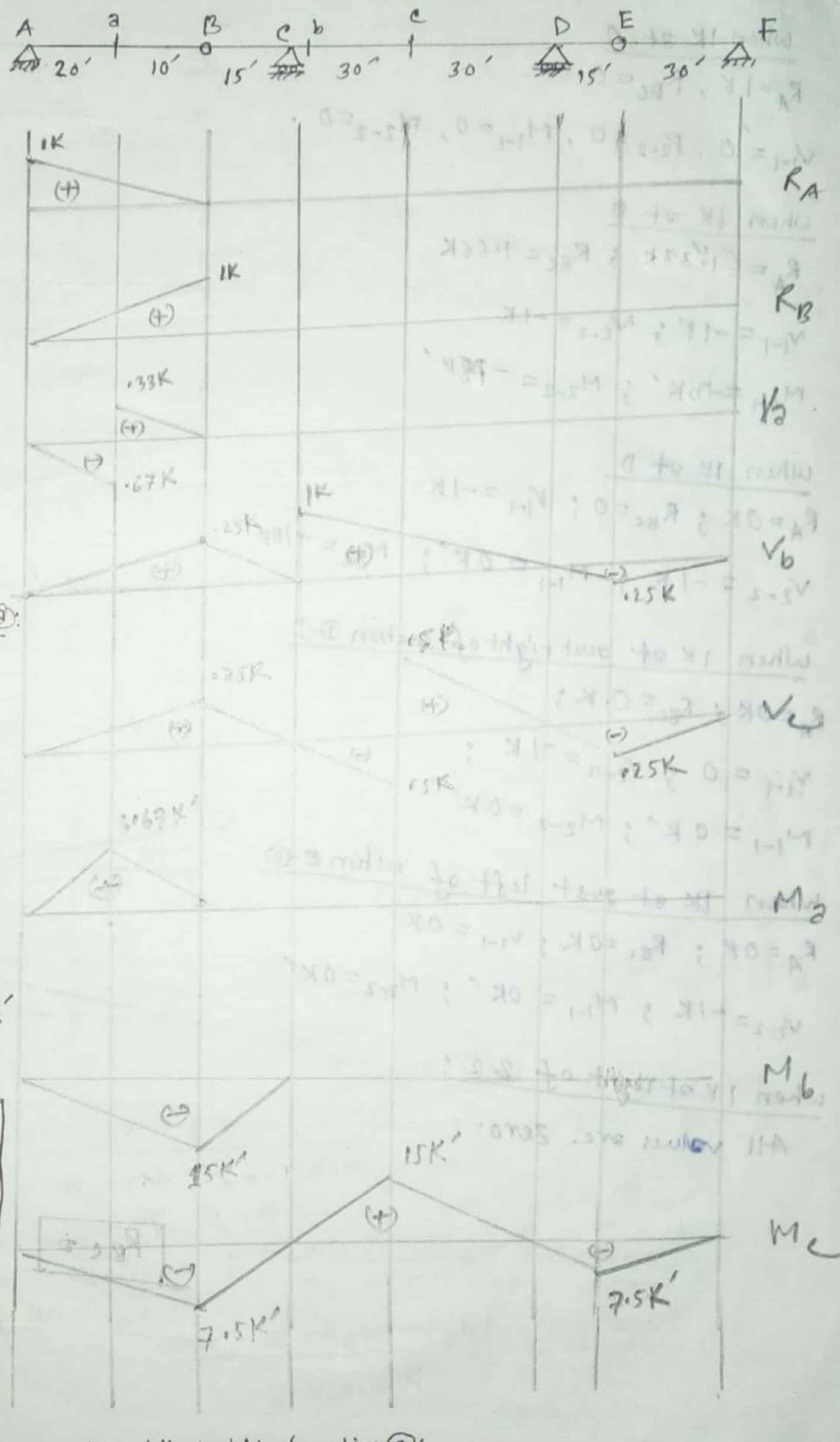
All values are zero.



R_{BC}:



3 Draw IL for R_A, R_B, V_a, V_b, V_c, M_a, M_b and M_c



When 1K at A:

$R_A = 1^K$; All other values are zero.

When 1K at just left of section ⓐ:

$R_A = .33K$, $R_B = .67K$, $V_a = -.67K$

$R_C = 0.8375K$, $R_D = -.1675K$

$V_b = .1675K$, $V_c = .1675K$

$M_a = 6.6K'$, $M_b = -10.05K'$

$M_c = +5.025K'$

When 1K at just Right of section ⓐ:

$V_a = .33K$; all other values are same as before.

When 1K at Ⓑ:

$R_B = 1K$, $R_A = 0$

$R_C = 1.25K$, $R_D = -.25K$

$V_a = 0$, $V_b = .25K$, $V_c = .25K$

$M_a = 0$; $M_b = -15K'$, $M_c = -7.5K'$

~~When 1K at just right of B:~~

When 1K at C:

$R_C = 1K$ and all other values are zero.

When 1K at just right of section ⓑ:

$V_b = 1K$ and all other values are zero.

When 1K at left of section ⓒ:

$R_C = R_D = .5K$; $V_a = 0$, $V_b = .5K$

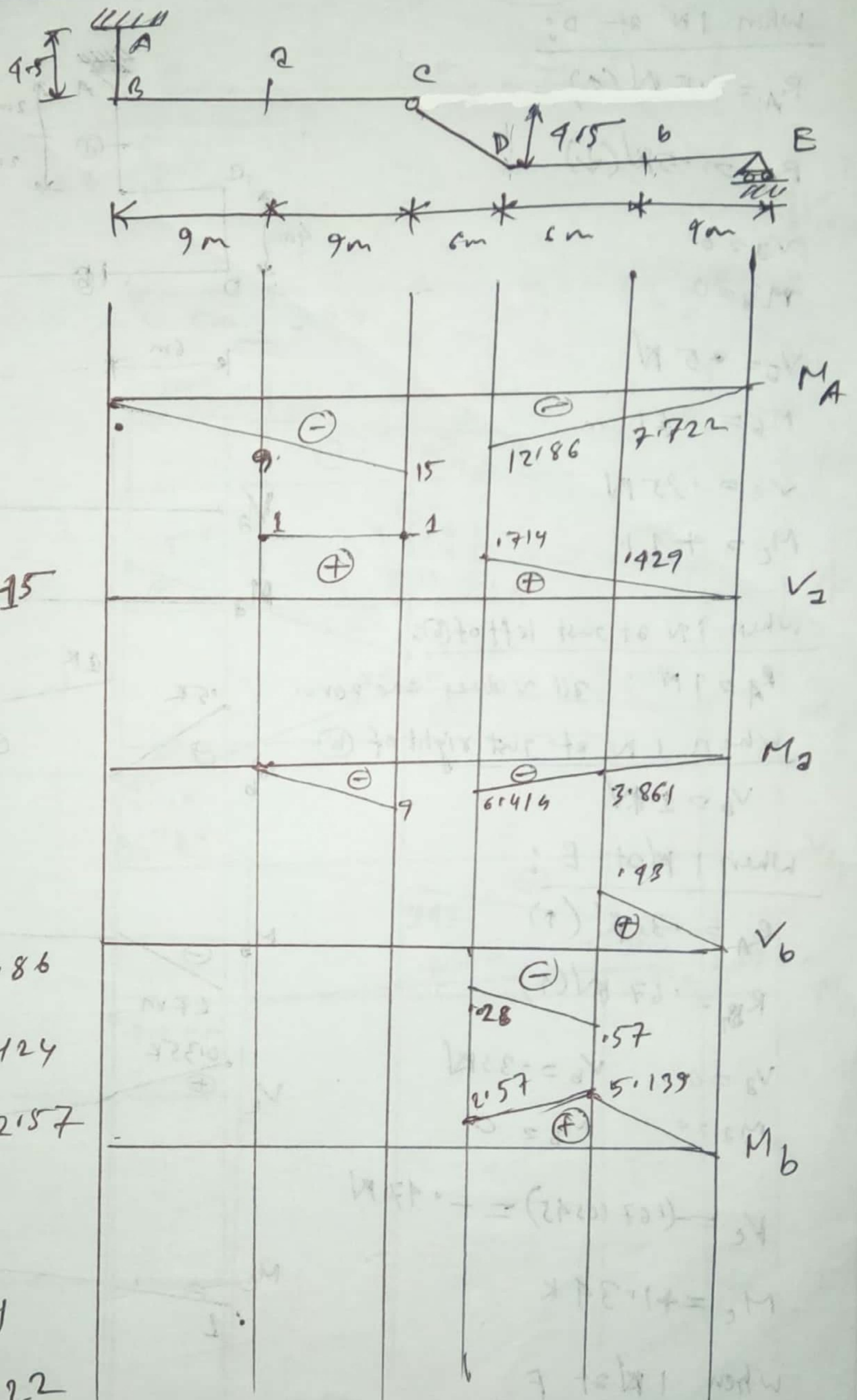
$V_c = -.5K$, $M_a = 0$, $M_b = 0$, $M_c = 15$ K ft

When 1K at right of section ⓒ:

$V_c = .5K$ and same as before.

5. $\boxed{2013, 17}$

Draw IL for M_A, V_a, V_b, M_a & M_b: (13) (B-C) (D-E)



1 K at B

$R_A = 1$, $R_C = 0$

all values are zero

1 K at section a:

$R_A = 1$, $R_C = 0$, $M_A = -9$

$V_a = 0$ (Left) $M_a = 0$

$V_a = 1$ (Right)

$V_b = 0$, $M_b = 0$

1K at C:

$R_A = 1$, $R_C = 0$, $M_A = -15$

$V_a = 1$, $M_a = 9$

$V_b = 0$ $M_b = 0$

1 K at D:

$R_C = .714, R_E = 0.286$

$R_A = .714$, $M_A = -12.86$

$V_a = .714$, $M_a = -6.424$

$V_b = -.286$, $M_b = 2.57$

1 K at Section b:

$R_C = 0.429$, $R_E = .571$

$R_A = 0.429$, $M_A = -7.722$

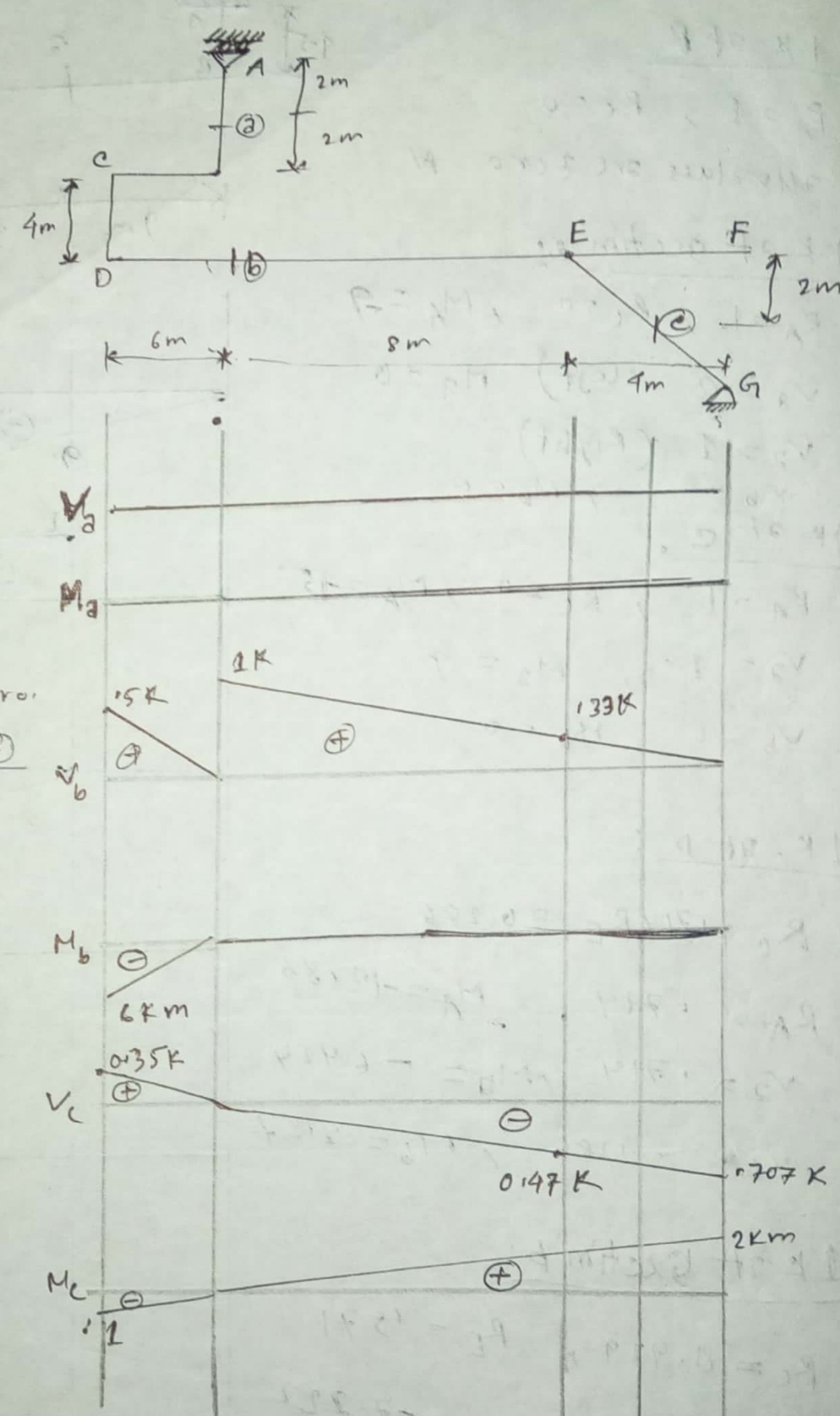
$V_a = .429$, $M_a = 3.861$

1 K at E

$R_E = 1K$, $R_C = 0$, $R_A = 0$

all values are zero.

Draw IL for V_a, V_b, V_c, M_a, M_b & M_c: (14)



When 1N at D:

$R_A = 1.5\ N\ (\uparrow)$

$R_G = .5\ N\ (\downarrow)$

$V_a = 0$

$M_a = 0$

$V_b = .5\ N$

$M_b = -6\ N\,m$

$V_c = .35\ N$

$M_c = -1\ N$

$\theta = 45°$, $.35\ N$, $.5\ N$

When 1N at just left of (b):

$R_A = 1\ N$; all values are zero.

When 1N at just right of (b)

$V_b = 1\ N$

When 1 N at E:

$R_A = .33\ N\ (\uparrow)$

$R_G = .67\ N\ (\uparrow)$

$V_a = 0$ $\quad V_b = .33\ N$

$M_a = 0$ $\quad M_b = 0$

$V_c = -(.67 \cos 45) = -.47\ N$

$M_c = +1.34\ K$

When 1 N at F

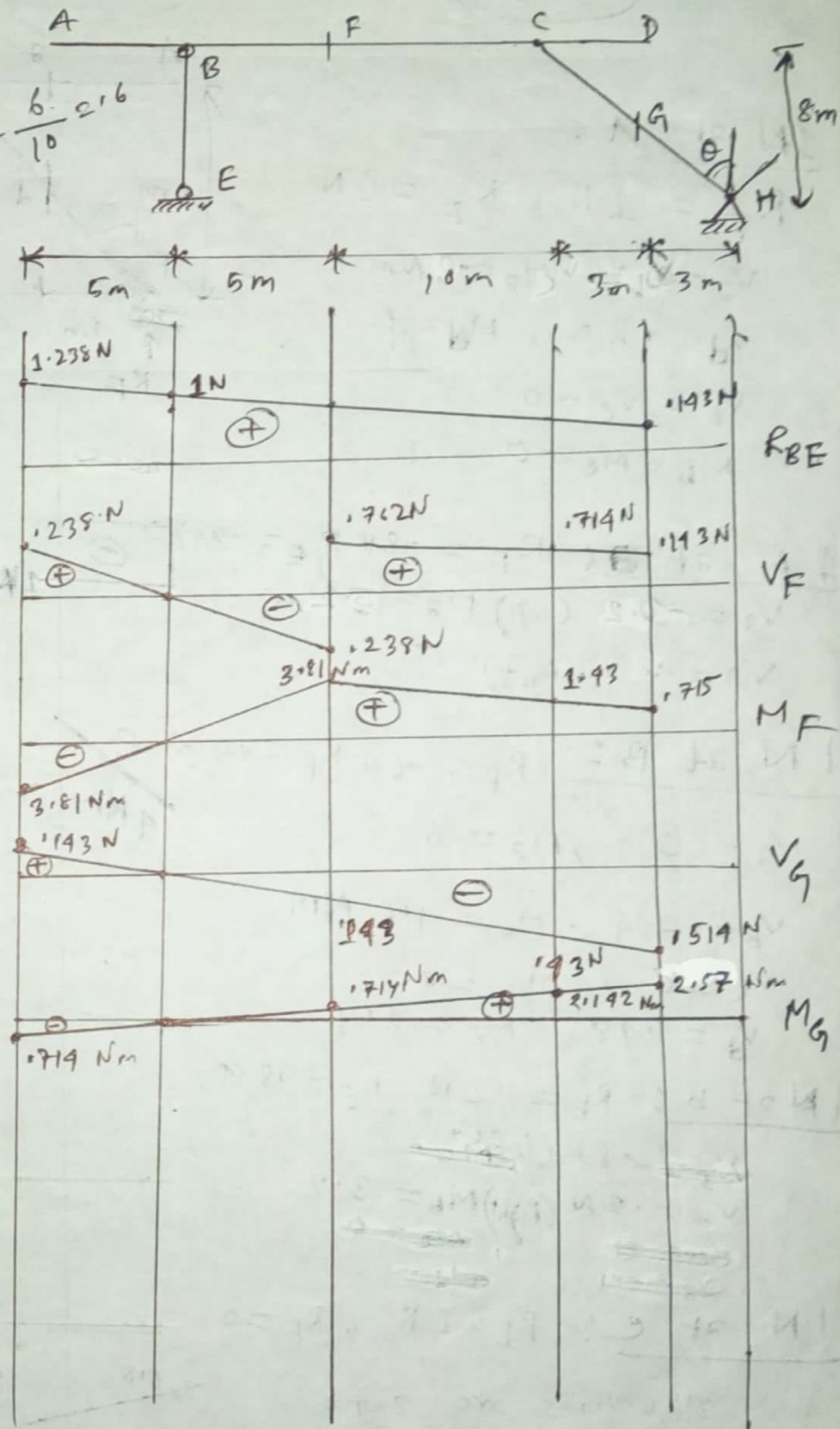
$R_G = 1\ N$ and ...

$V_a = 0$ $\quad V_b = 0$

$M_a = 0$ $\quad M_b = 0$

$V_c = -.707\ N$ $\quad M_c = +2\ N$

Draw IL for R_{BE}, V_F, M_F, V_G & M_G: (10) (A–D)



When 1N at A

$R_H = -.238\ N,\ R_B = 1.238\ N$

$V_F = .238N,\quad M_F = 3.81Nm$

$V_G = .143\ N,\ M_G = -.714$

When 1N at B

$R_B = 1K,\ R_H = 0N,$

$V_F = 0,\ M_F = 0\ Nm$

$V_G = 0,\ M_G = 0$

When 1N at F

$R_H = 0.238N,\ R_B = 0.762\ N$

$V_F = -.238N$ (left)

$V_F = .762\ N,\ M_F = 3.81\ Nm$

$V_G = -0.143\ N,\ M_G = +.714$

When 1N at C

$R_H = 0.714\ N, R_B = 0.286\ N$

$V_F = .714,\ M_F = 1.43Nm$

$V_G = -.143\ N,\ M_G = 2.142\ Nm$

When 1N at D

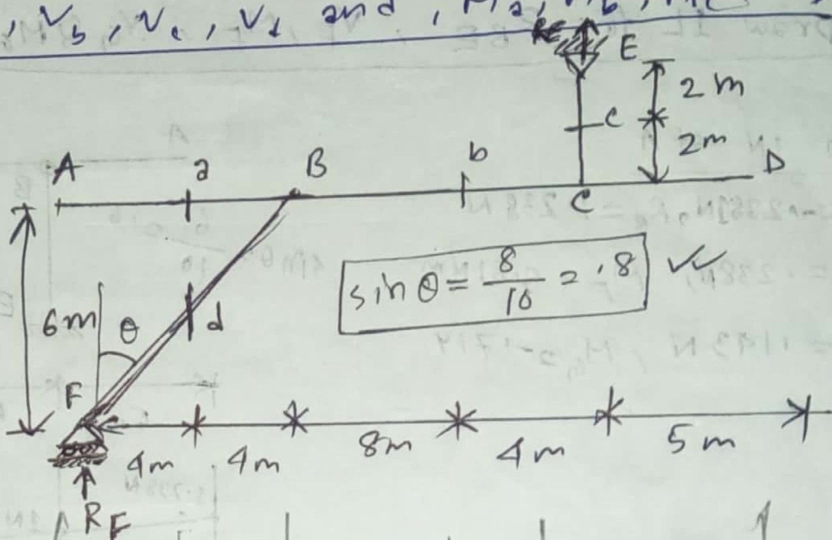
$R_H = .857N,\ R_B = 0.143N$

$V_F = .143,\ M_F = 0.715\ Nm$

$V_G = -0.514\ N,\ M_G = 2.57\ Nm$

(17) Draw IL for V_a, V_b, V_c, V_d and, M_a, M_b, M_c, M_D

(A – D)



1N at A:

$R_F = 1N$ $\quad R_E = 0N$

$V_a = -1N$, $M_a = -4Nm$

$V_d = .8N$, $M_d = 4$

$V_b = V_c = 0$

$M_b = M_c = 0$

1N at a: $R_F = .8N$ $R_E = .2(\uparrow)$

$V_a = -1$ (left) $M_a = 0Nm$

$V_a = 0$ (Right)

1N at B: $R_F = .6N$ $R_E = .4(\uparrow)$

$V_a = 0$, $M_a = 0$

$V_b = -.4$, $M_b = 1.6Nm$

$V_c = 0$, $M_c = 0$

$V_d = .48$, $M_d = 2.4$

1N at b: $R_F = .2N$ $R_E = .8(\uparrow)$

$V_b = -.8N$ (left)

$V_b = .2N$ (Right) $M_b = 3.2$

1N at c: $R_E = 1N$, $R_F = 0$

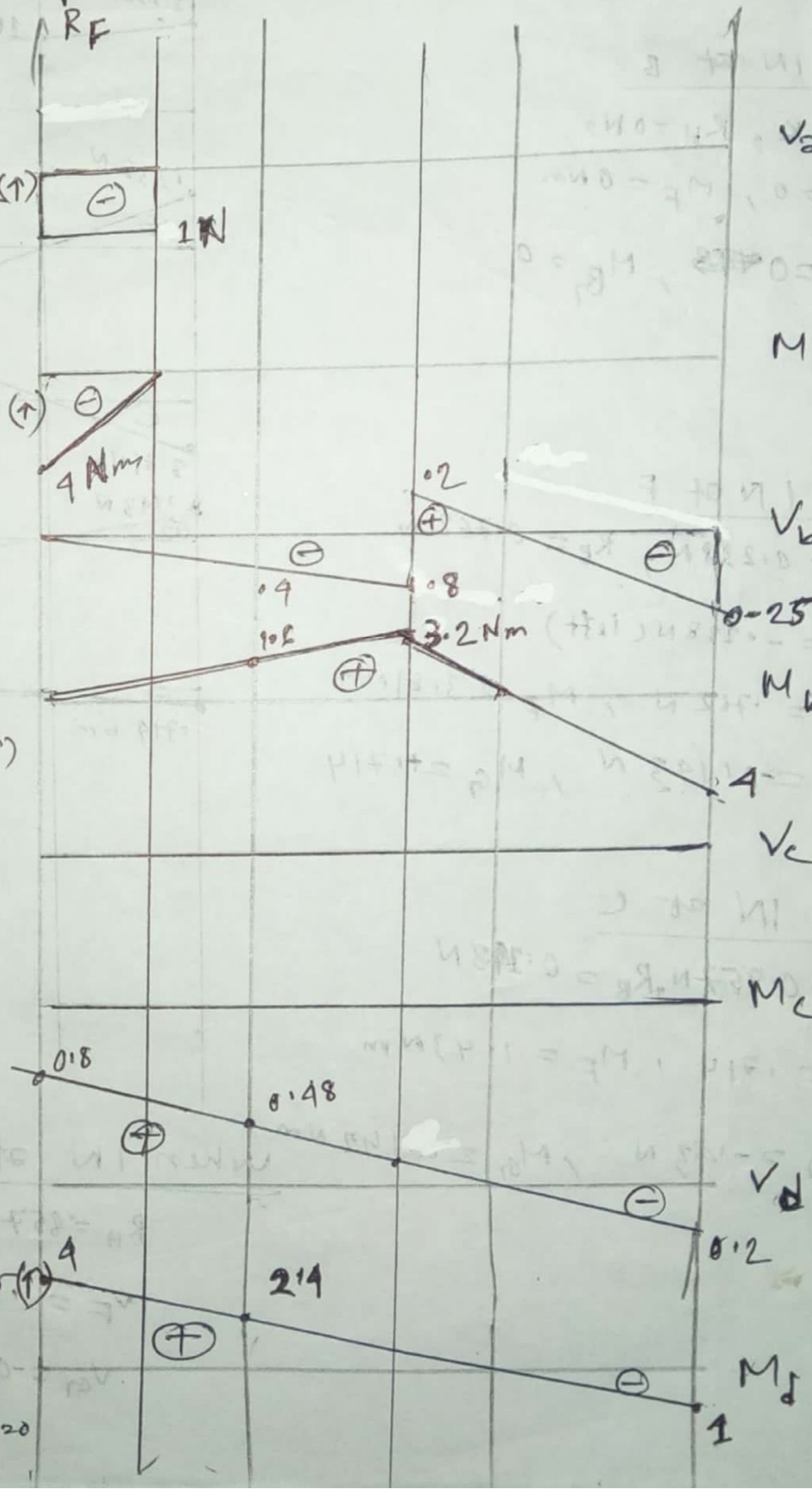
all values are zero

1N at D: $R_F = -.25N$, $R_E = 1.25(\uparrow)$

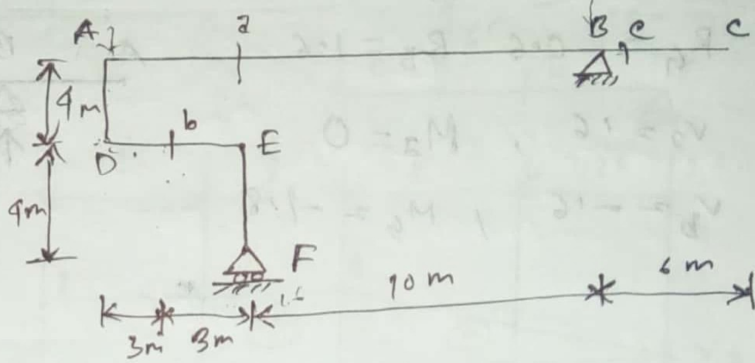
$V_a = 0$ $\quad M_a = 0$

$V_b = -.25$ $\quad M_b = 4Nm$

$V_d = -.2$ $\quad M_d = -1.0$ $\quad V_c = M_c = 0$



(18) Draw IL for $V_a, M_a, V_b, M_b, V_c, M_c$: (A-B)



1K At A :

$R_B = -0.6\,K$, $R_F = 1.6$

$V_a = 0.6$, $M_a = -6$

$V_b = -1$, $M_b = 4.8$

1K at section-a :

$R_F = 1\,K$, $R_B = 0$

$V_a = 0$ (left) $M_a = 0$

$= 1$ (Right)

1K at B :

$R_B = 1$, $R_F = 0$

all values are zero,

$V_c =$ 0 N (left) $M_c = 0$

$V_c = +1\,N$ (Right) ,

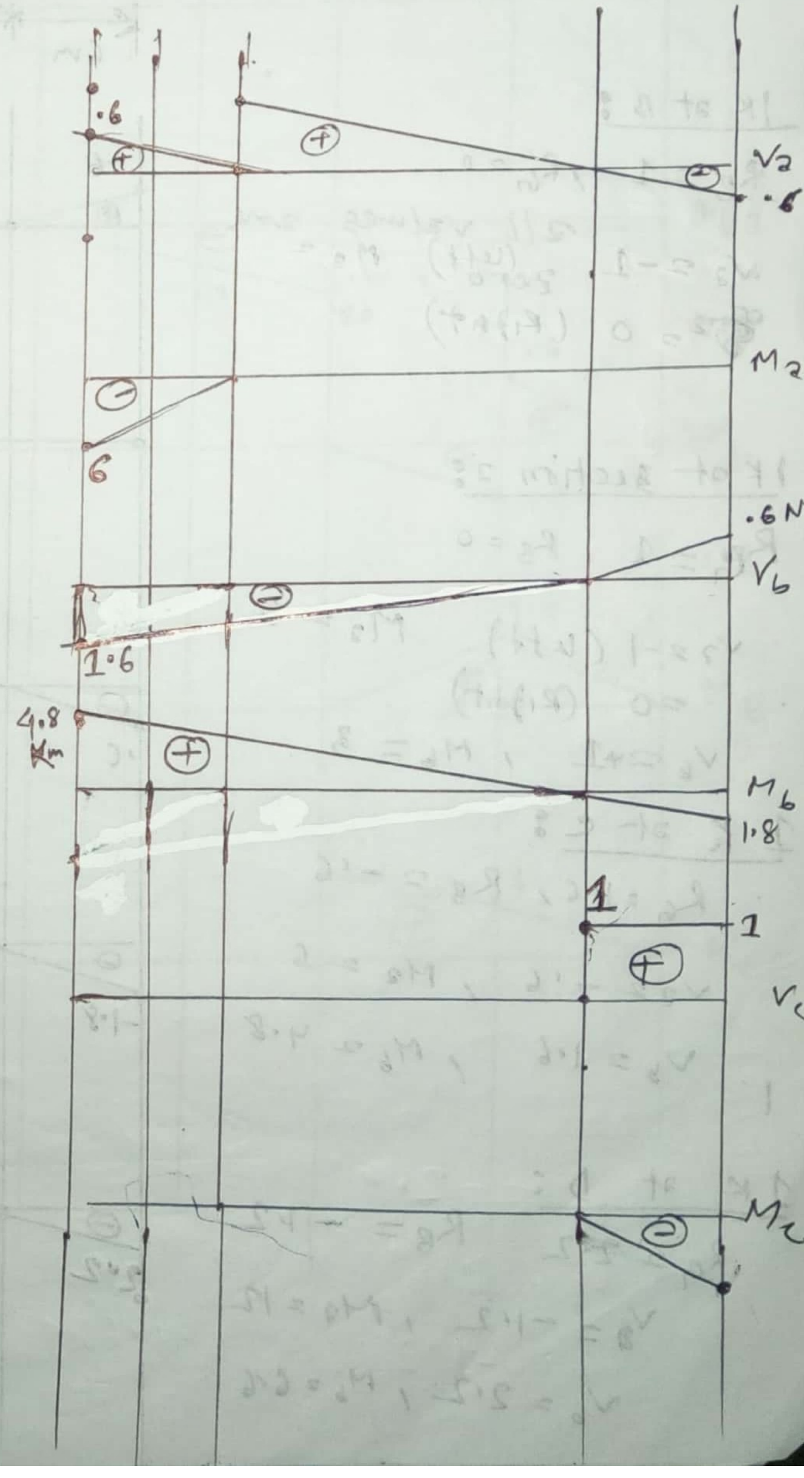
1K at C :

$R_B = 1.6$, $R_F = -.6$

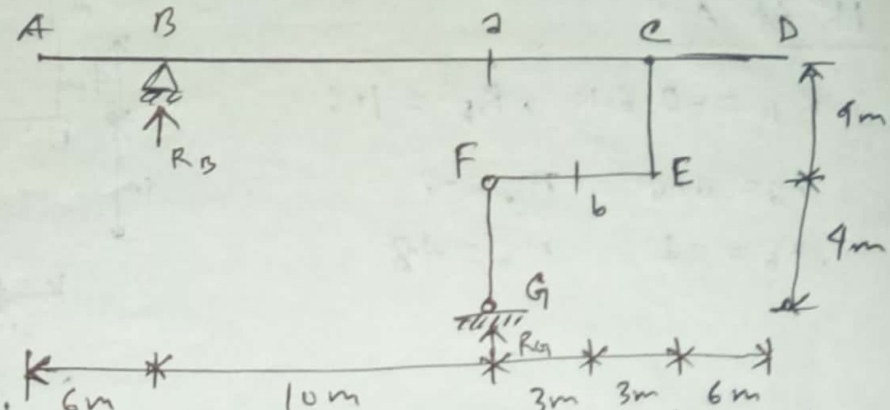
$V_a = -0.6$, $M_a = 0$

$V_b = 0.6$, $M_b = -1.8$

$V_c = 1$, $M_c = -6$



(19) Draw ILD for $V_a, V_b, M_a, M_b, R_{FG}$: (A – D)



1K at A:

$R_G = -0.6 \quad R_B = 1.6$

$V_a = .6 \;,\; M_a = 0$

$V_b = -.6 \;,\; M_b = -1.8$

1K at B:

$R_B = 1 \;,\; R_G = 0$

all values are zero.

1K at section a:

$R_G = 1 \;,\; R_B = 0$

$V_a = -1$ (left) $\quad M_a = 0$

$= 0$ (Right)

$V_b = +1 \;,\; M_b = 3$

1 K at c:

$R_G = 1.6 \;,\; R_B = -.6$

$V_a = -.6 \;,\; M_a = 6$

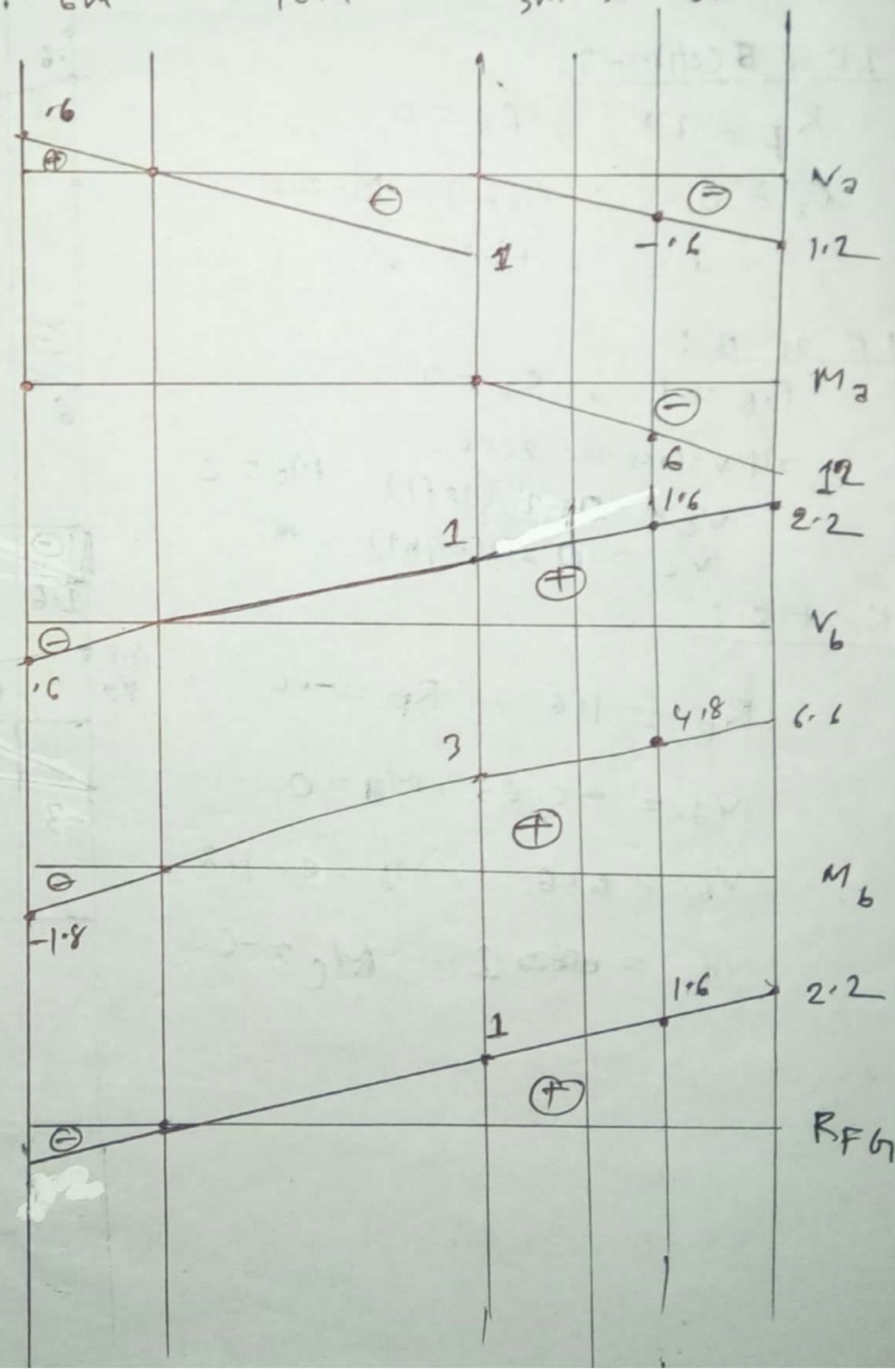
$V_b = 1.6 \;,\; M_b = 4.8$

1K at D:

$R_G = 2.2 \quad R_B = -1.2$

$V_a = -1.2 \;,\; M_a = 12$

$V_b = 2.2 \;,\; M_b = 6.6$

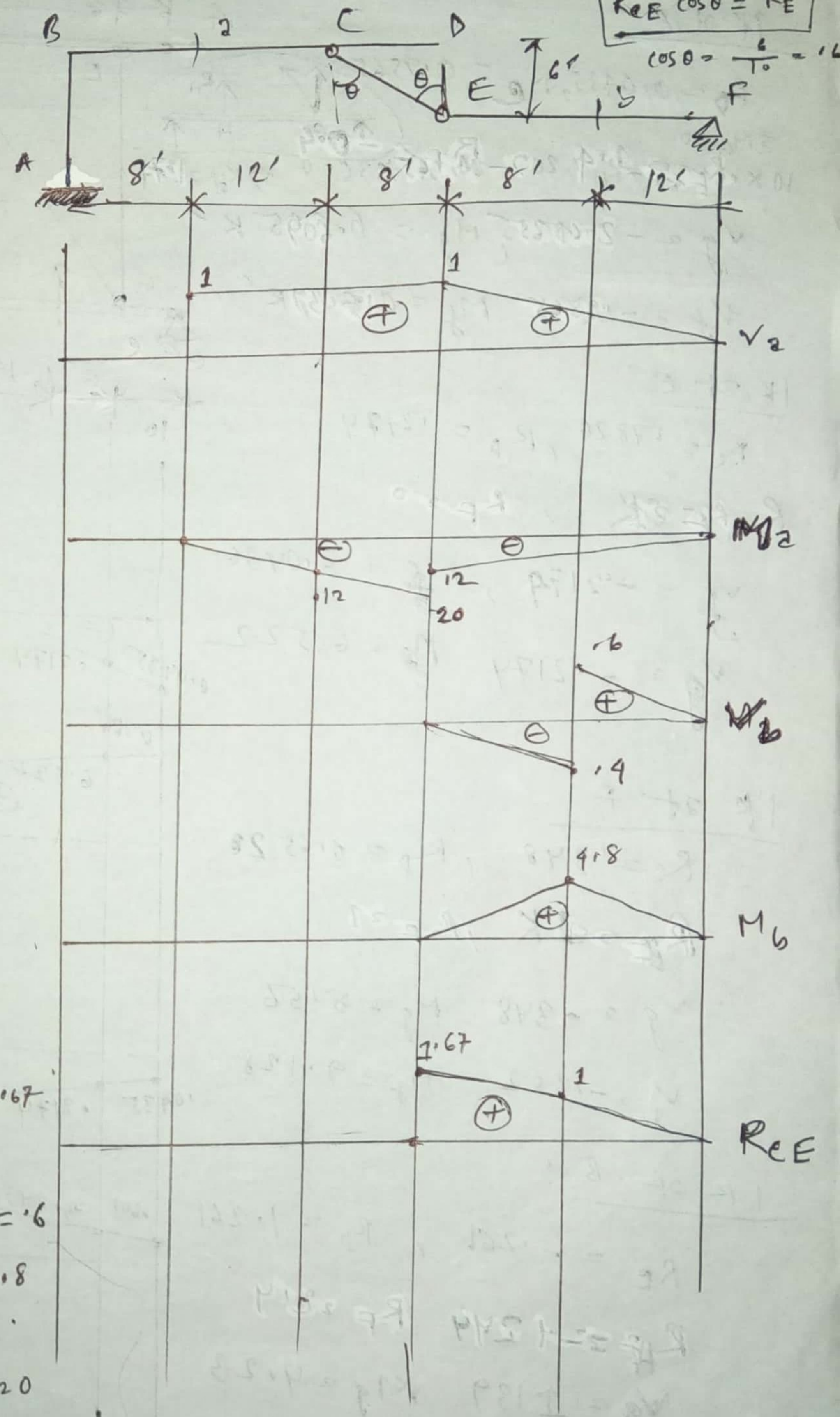


(21) Draw IL for V_a, V_b, M_a, M_b; R_{CE}: (B-D) & (E-F)

(2017)

$R_{CE}\cos\theta = R_E$

$\cos\theta = \frac{6}{10} = .6$



1 K at B:

$R_A = 1$, $R_C = 0$

all values are zero.

1 K at section a:

$R_A = 0$ K, $R_C = 0$

$V_a = 0$ K (left)

$= 1$ K (Right)

$M_a = 0$

1K at C:

$R_A = 1$ K, $R_C = 0$

$V_a = 1$, $M_a = -12$

$V_b = 0$, $M_b = 0$

1K at D:

$R_A = 1$, $R_C = 0$

$V_a = 1$, $M_a = -20$

1K at E:

$R_E = 1$; $R_F = 0$, $R_A = 1$

$V_a = 1$, $M_a = -12$

$V_b = 0$, $M_b = 0$, $R_{CE} = 1.67$

1K at section b:

$R_E = .6$, $R_F = .4$, $R_A = .6$

$V_b = -.4$ (left) $M_b = 4.8$

$= .6$ (Right)

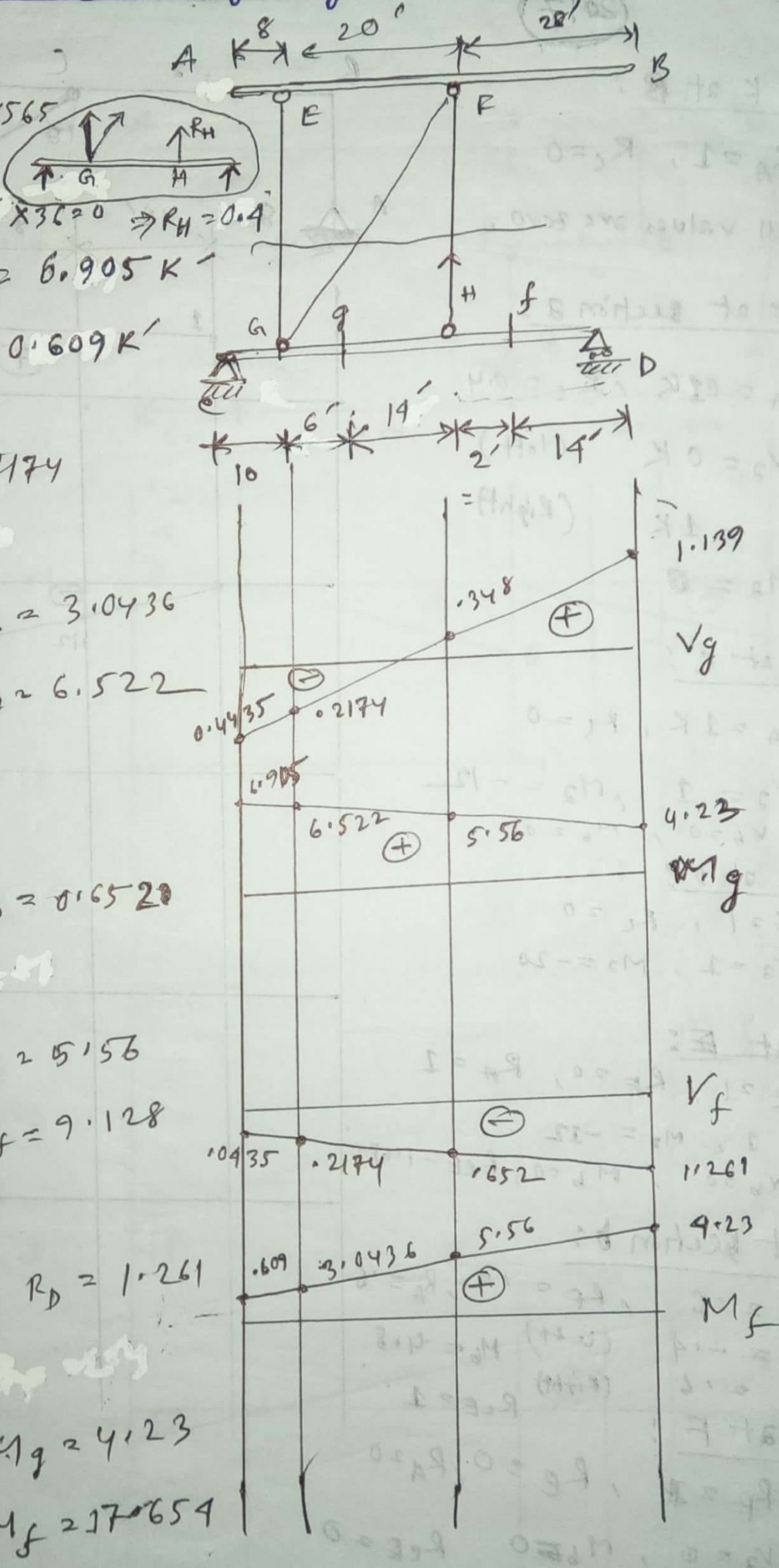
$R_{CE} = 1$.

1K at F:

$R_F = 1$, $R_E = 0$, $R_A = 0$

$V_a = 0$, $M_b = 0$ $R_{CE} = 0$

8. Draw IL for V_f, M_f, V_g, M_g : (A – B)



1K at A

$R_D = 0.0435$, $R_E = 0.9565$

$\Sigma M_G = 0$

$10 \times .9565 - R_H \times 20 - .0435 \times 36 = 0 \Rightarrow R_H = 0.4$

$V_g = -0.4435$, $M_g = 6.905\ K'$

$V_f = -.0435$ $M_f = 0.609\ K'$

1K at E

$R_E = .7826$, $R_D = .2174$

$R_H = 0K$,

$V_f = -.2174$, $M_f = 3.0436$

$V_g = -.2174$ $M_g = 6.522$

1K at F

$R_E = .348$, $R_D = 0.6520$

$R_H = 1K$

$V_g = .348$ $M_g = 5.56$

$V_f = -.652$ $M_f = 9.128$

1K at B !

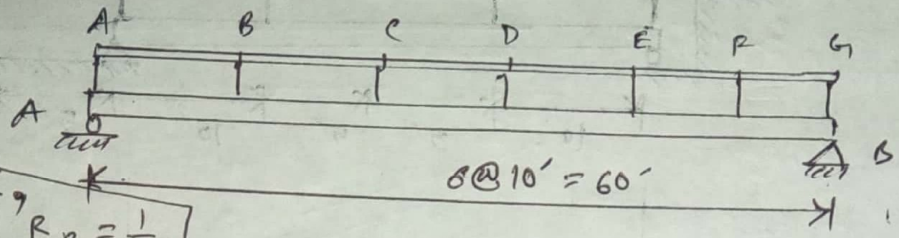
$R_E = -.261$, $R_D = 1.261$

$R_H = -2.4$

$V_g = 1.139$ $M_g = 4.23$

$V_f = -1.261$, $M_f = 17.654$

(16) Draw IL for (i) shear in panel BC (ii) Moment at E



1K at B: $\boxed{R_A = \frac{5}{6},\ R_B = \frac{1}{6}}$

$V_{B-C} = -\frac{1}{6}$

1K at C:

$\boxed{R_A = \frac{4}{6},\ R_B = \frac{2}{6}}$

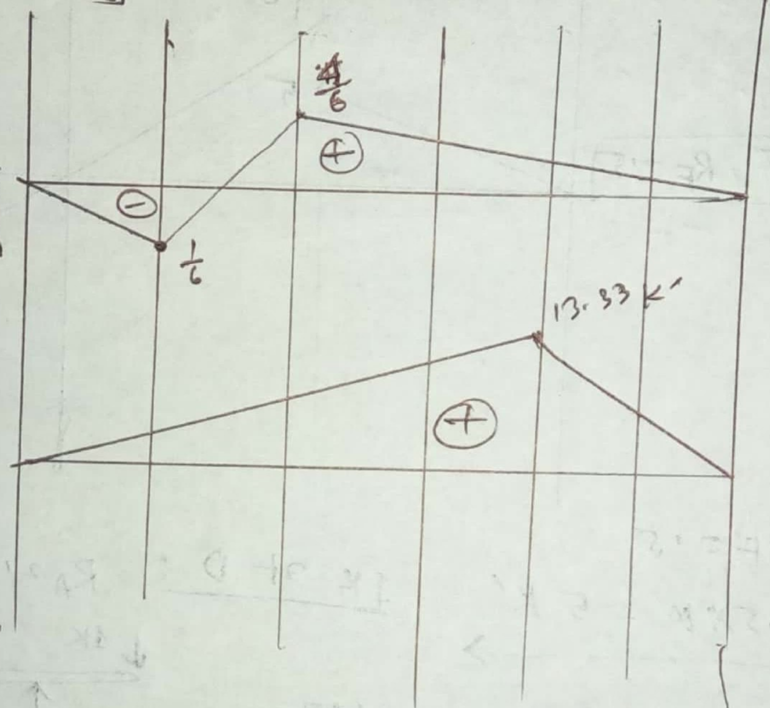
$V_{B-C} = \frac{2}{3}$

1K at E

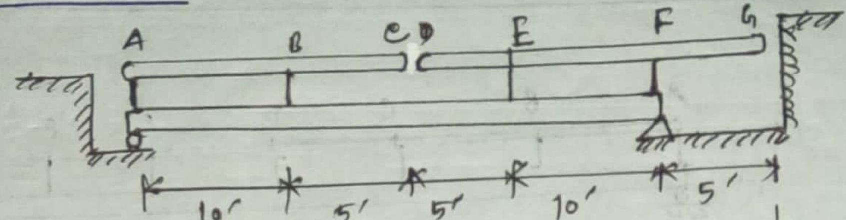
$R_B = \frac{4}{6}$

$M_E = \frac{4}{6} \times 20$

$= 13.33K'$



(17) Draw IL for M_E:

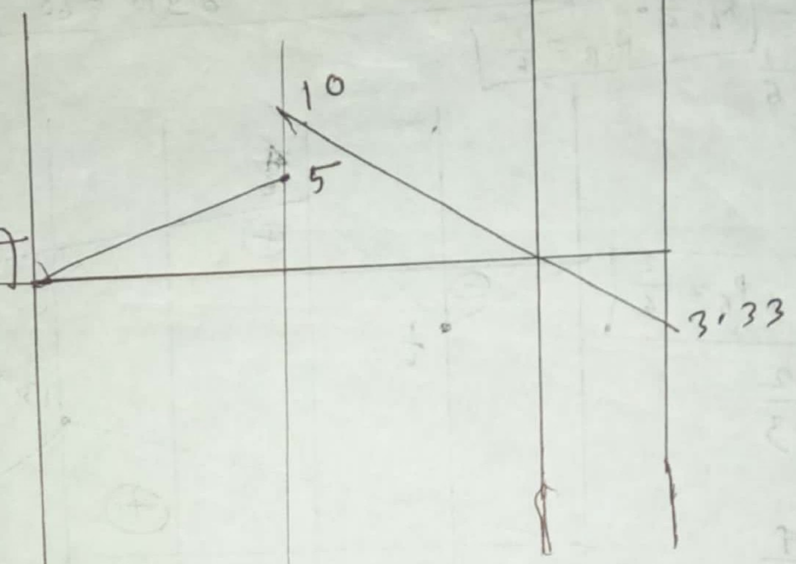


1K at A

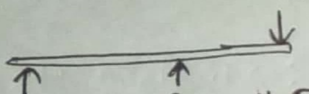
$\boxed{R_A = 1}$

~~all values~~

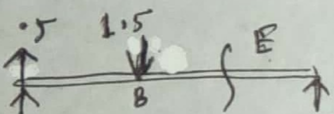
$M_E = 0$



1K at C: $\boxed{R_A = .5, \; R_F = .5}$



$R_A = -.5$ $R_B = 1.5$

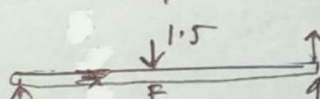


$R_A = 0.5K$ $R_F = .5$

$M_E = 1 \times 20 * 1.5 \times 10 = 5K'$

1K at D: $R_A = .5$, $R_F = .5$

$R_E = 1.5$ $R_F = -.5$

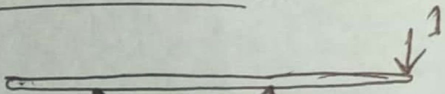


$R_A = .5$ $R_F = .5$

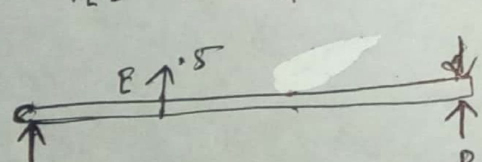
$M_E = 0.5 \times 10 + .5 \times 10$

$= 10$

1K at G: $R_A = -.167$, $R_B = 1.167$



$R_E = -.5$ $R_F = 1.5$



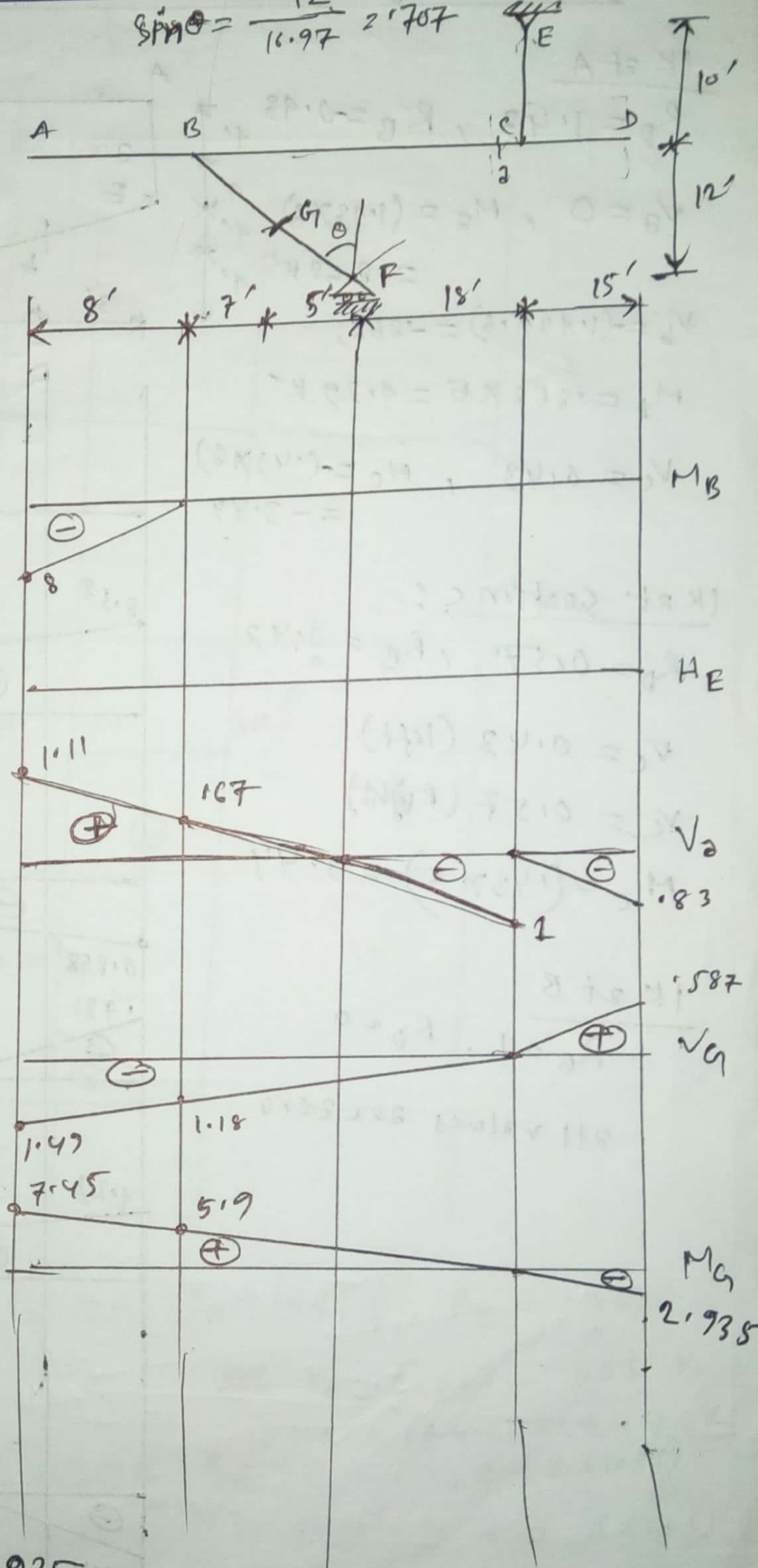
$R_A = -.167$ $R_F = 1.167$

$M_E = 1.167 \times 10 - 1.5 \times 10 = -3.33$

2014

Draw IL for M_B, M_G, V_G, V_a, H_E: (A – D)

$\sin\theta = \frac{12}{16.97} = .707$



1K at A:

$R_F = 2.11$, $R_E = -1.11$

$V_a = 1.11$,

$M_B = -8$

$V_G = -1.49$, $M_G = 7.45$

1 K at B:

$R_F = 1.67$, $R_E = -.67$

$V_a = .67$, $V_G = -1.18$

$M_G = 5.9$

1K at E:

$R_E = 1$ K, $R_F = 0$

$V_a = -1$ (left)

$V_a = 0$ (Right)

1 K at D:

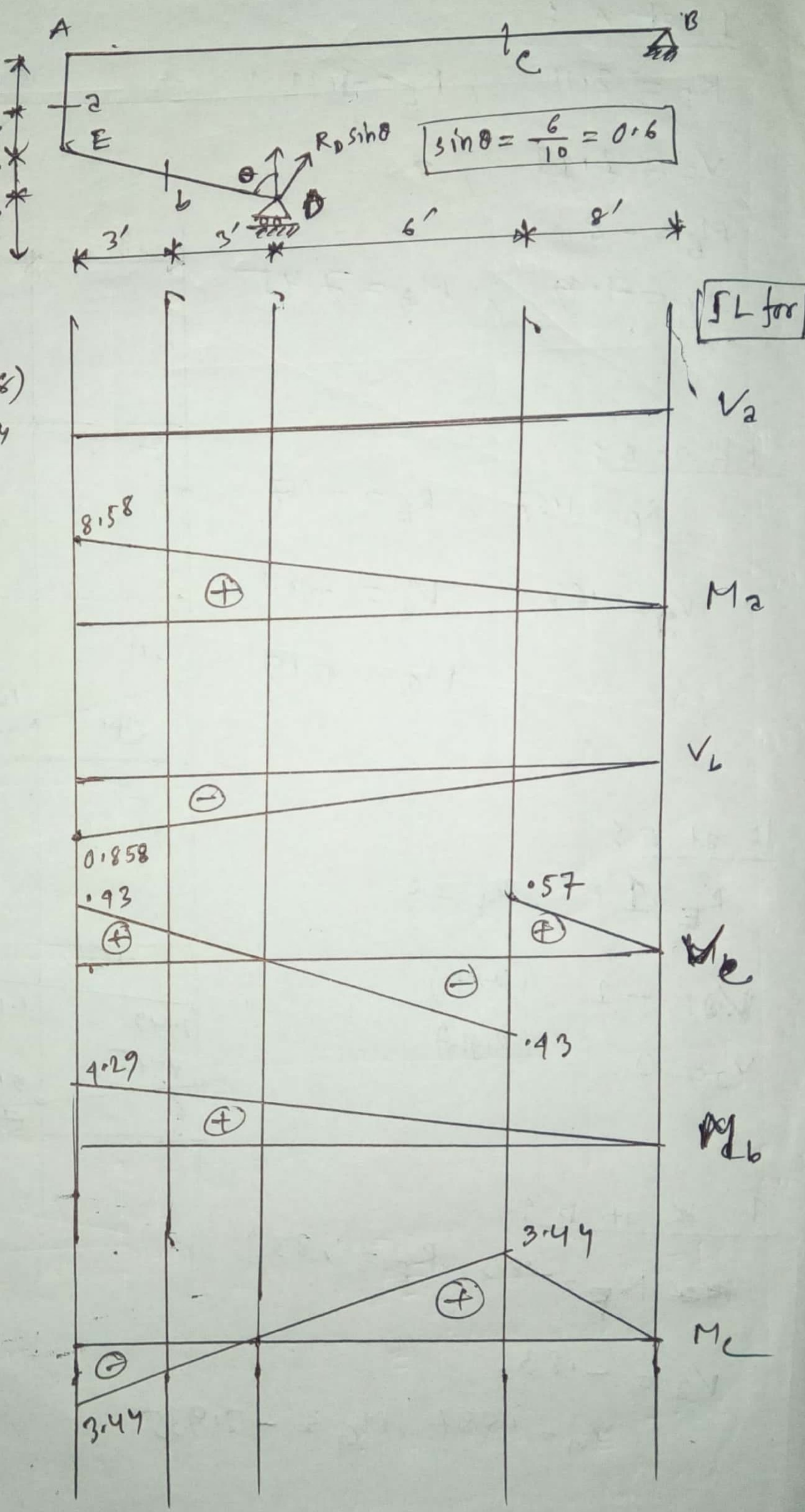
$R_F = -.83$ $R_E = 1.83$

$V_a = -.83$,

$V_G = .587$, $M_G = -2.935$

2012

Draw IL for $V_a, V_b, V_c, M_a, M_b, M_c$: $\boxed{A-B}$



$\boxed{\sin\theta = \frac{6}{10} = 0.6}$

1K at A

$R_D = 1.43$, $R_B = -0.43$

$V_a = 0$, $M_a = (1.43 \times 6)$
$= 8.58K'$

$V_b = -(1.43 \times .6) = -.858$,

$M_b = .858 \times 5 = 4.29K'$

$V_c = 0.43$, $M_c = -(.43 \times 8)$
$= -3.44$

1K at section c:

$R_D = 0.57$, $R_B = 0.43$

$V_c = 0.43$ (left)

$V_c = 0.57$ (Right)

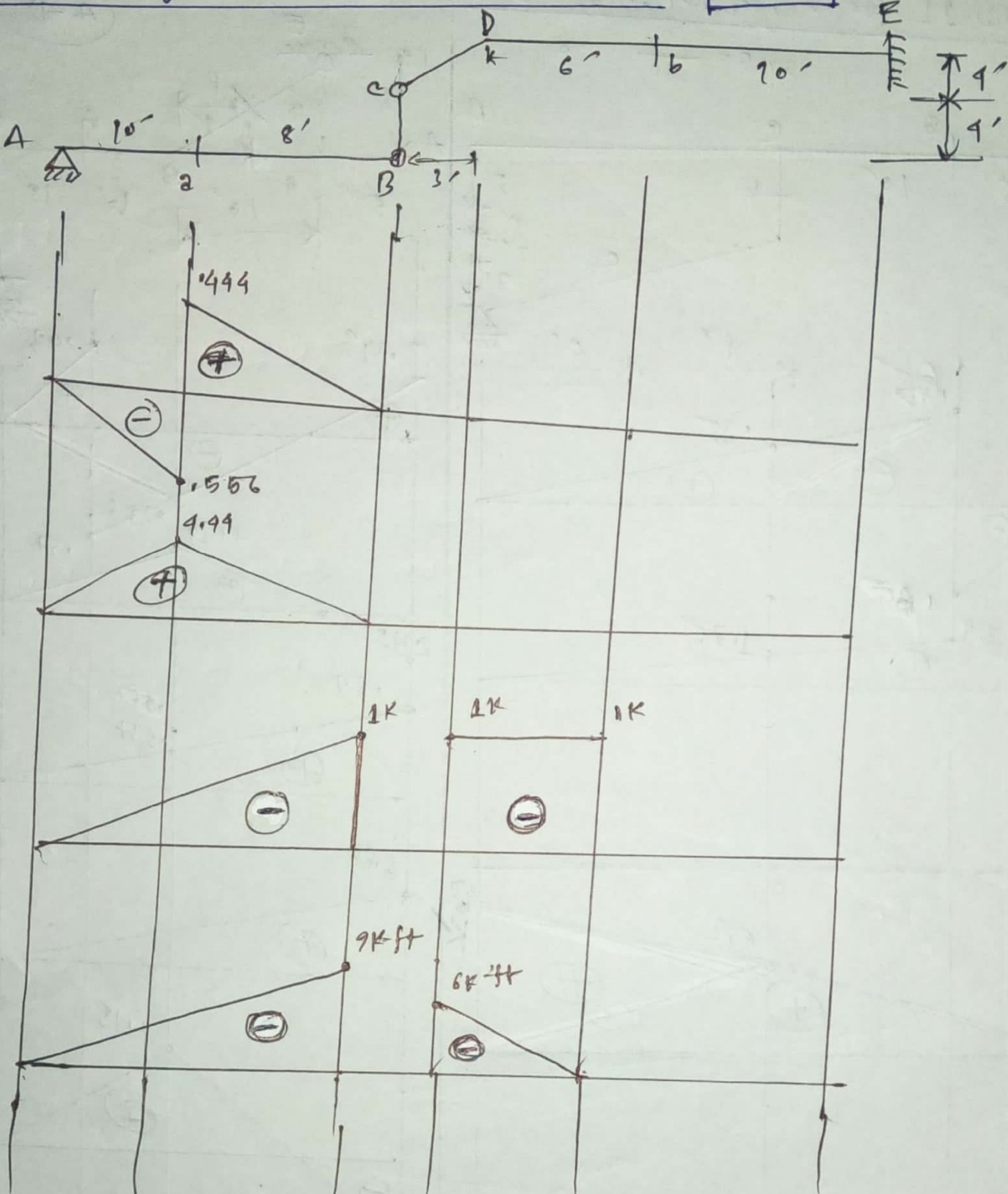
$M_c = (.43 \times 5) = 3.44$

1K at B

$R_B = 1$, $R_D = 0$

all values are zero

Draw IL for V_a, V_b, M_a and M_b: $\boxed{2010}$



1 K at A: $R_A = 1K$,

at B: $R_B = 1K$,

at D: $R_E = 1K$,

at E: $R_E = 1K$,

at section a: $R_A = .444$, $R_B = .556$

$V_a =$ (Left) $- .556$ K

$V_a =$ (Right) 0.444 K

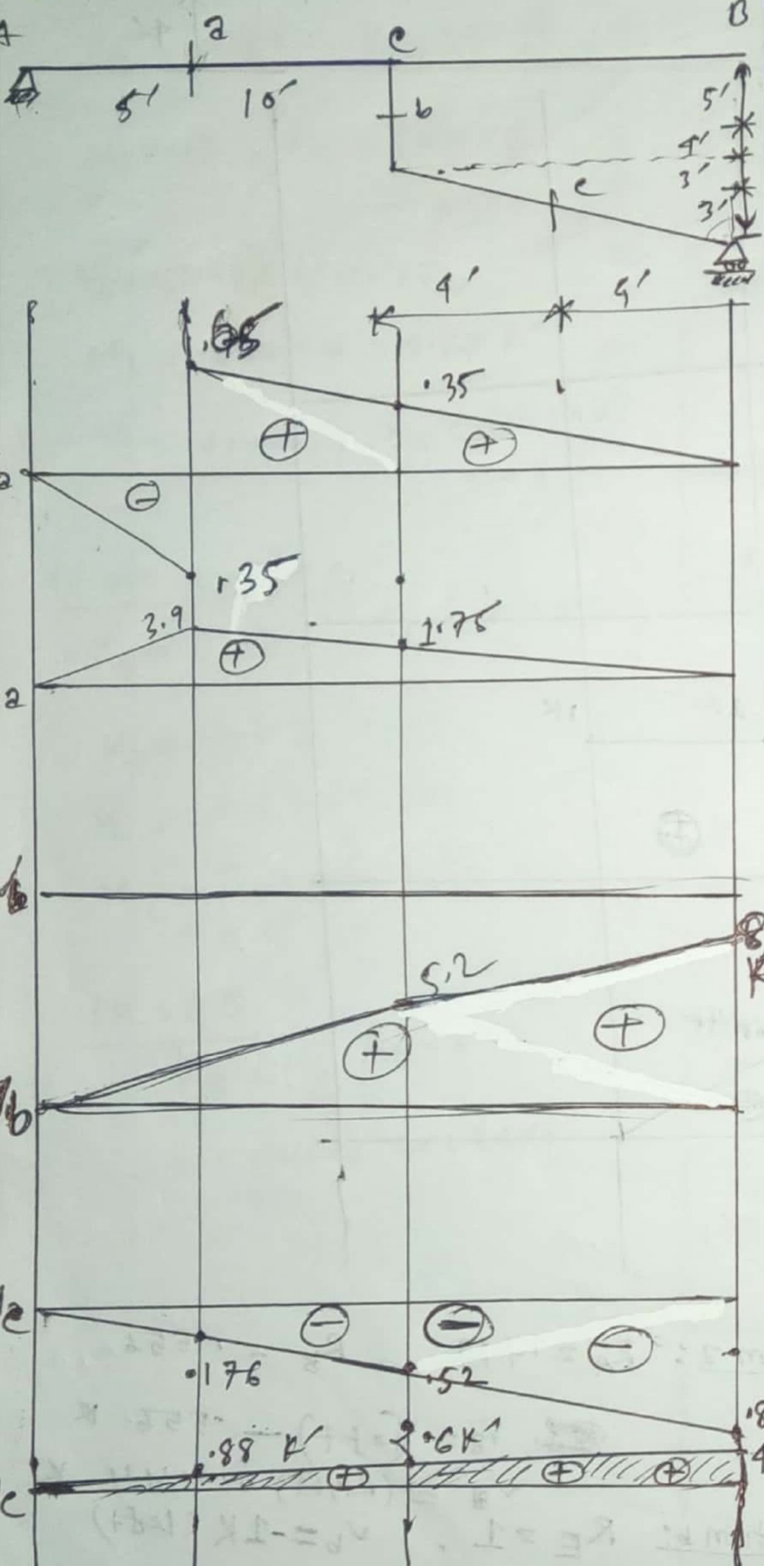
at section b: $R_E = 1$, $V_b = -1K$ (left)

$= 0$ (Right)

2009

Draw IL for $V_a, V_b, V_c, M_a, M_b, M_c$

(A-B) $\sin\theta = \frac{8}{10} = .8$



1K at A: $R_A = 1$, 1K at B: $R_B = 1K$

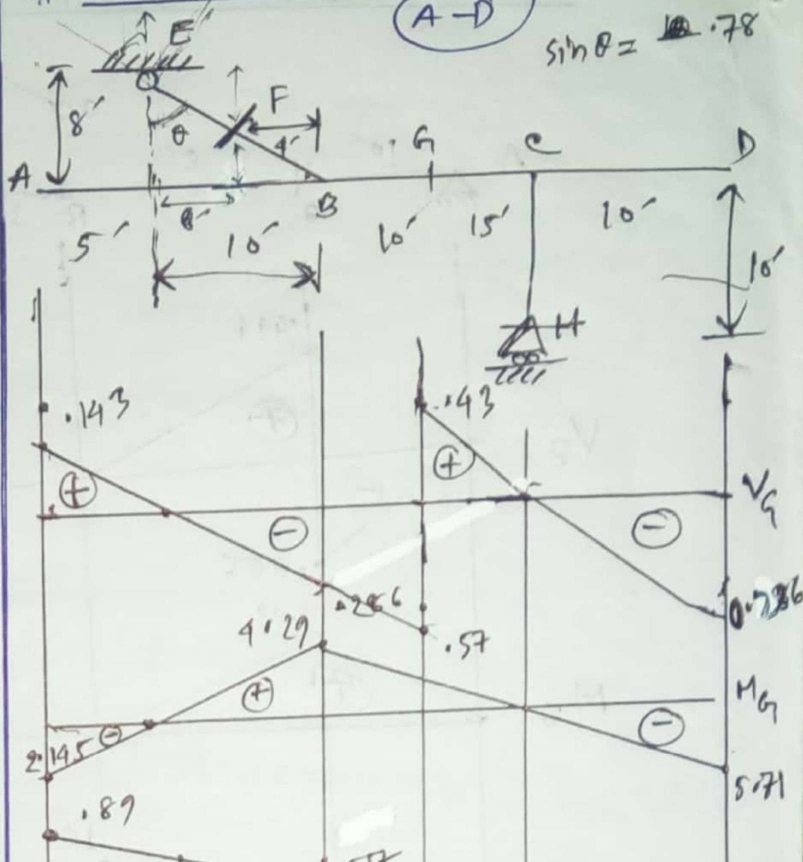
1K at c: $R_A = 0.35\,K$, $R_B = 0.65\,K$,

1K at section a: $R_A = 0.78$, $R_B = 0.22$

2008

Draw IL for V_G, V_F, M_G, M_F, M_B

(A-D) $\sin\theta = .78$



1K at A: $R_E = 1.143$, $R_H = -.143$; 1K at C: $R_H = 1K$

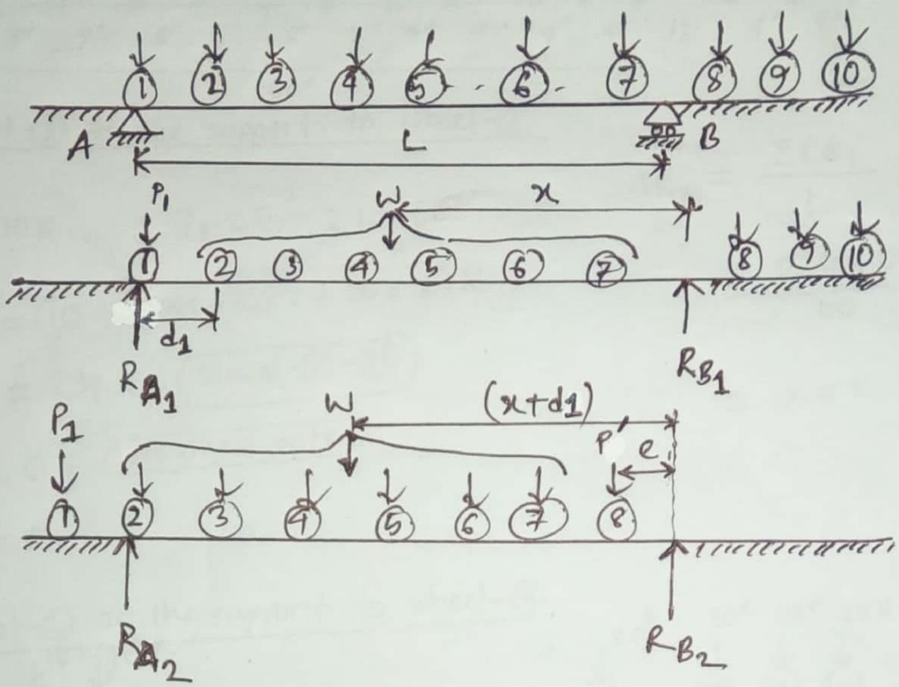
1K at B: $R_E = .714$ $R_H = 0.286$

1K at D: $R_H = 1.286$, $R_E = -.286$

1K at G: $R_E = .43$, $R_H = .57$

Wheel Load

Criteria for the maximum reaction of a simple beam subjected to series of concentrated loads move from right to left.



Before movement, $\Sigma M_B = 0$

$$\Rightarrow R_{A_1} L - P_1 L - Wx = 0$$

$$\Rightarrow R_{A_1} = \frac{Wx}{L} + P_1 \quad \ldots\ldots (I)$$

After movement, $\Sigma M_B = 0$

$$\Rightarrow R_{A_2} \times L - W(x+d_1) - P'e = 0$$

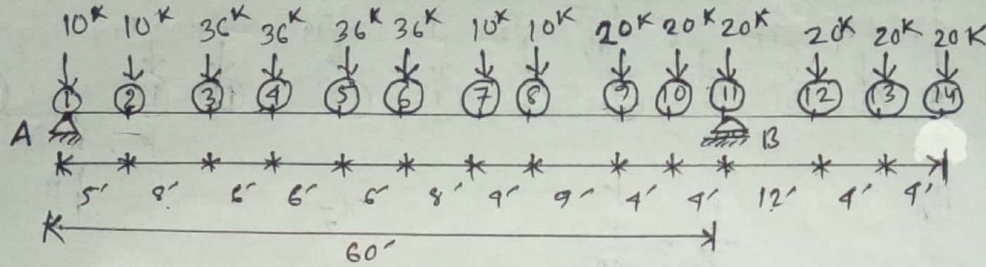
$$\Rightarrow R_{A_2} = \frac{W(x+d_1)}{L} + \frac{P'e}{L} \quad \ldots\ldots (II)$$

From (I) & (II),

change of reaction $\Delta R = R_{A_2} - R_{A_1} = \frac{Wd_1}{L} + \frac{P'e}{L} - P_1$

Hence, $\boxed{\Delta R = \frac{\Sigma P d_1}{L} + \frac{P'e}{L} - P_1}$

Problem: Determine maximum reaction at A of a simple beam due to the loading shown in fugure:



Solution:

When wheel ① at the support to wheel-②

Here, $P_1 = 10K$; $d_1 = 5'$; $L = 60'$

$\Sigma P = (10 + 36\times4 + 10\times2 + 20\times3)K$

$= 234K$ (Wheel-② - ⑪)

$P' = 0K$ (No wheel enter)

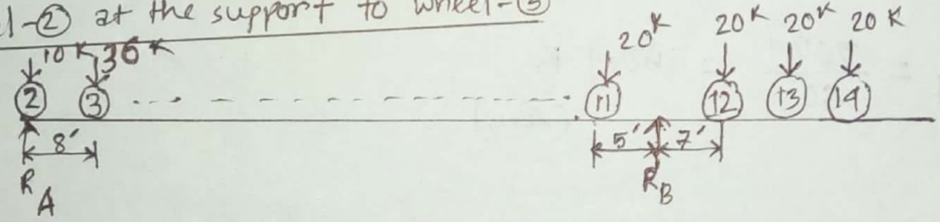
$e = 0$

$$\Delta R_A = \frac{\Sigma P d_1}{L} + \frac{P'e}{L} - P_1$$

$$= \frac{234\times5}{60} + \frac{0}{60} - 10$$

$= 9.5K$ (increasing)

When wheel-② at the support to wheel-③



Here, $P_1 = 10K$; $d_1 = 8'$; $L = 60'$

$\Sigma P = (234 - 10)K = 224K$ $\boxed{W-③-⑪}$

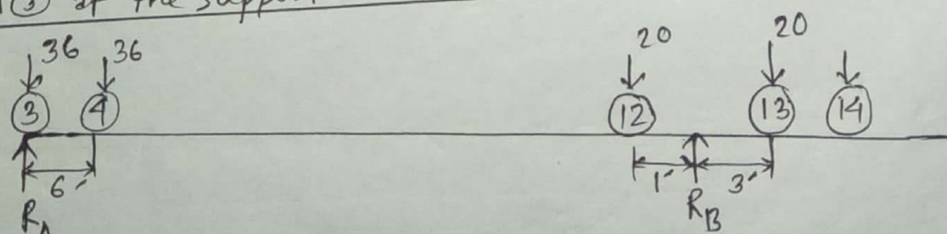
$P' = 20K$ (Wheel-12)

$e = 1'$

$$\Delta R_A = \frac{224\times8}{60} + \frac{20\times1}{60} - 10$$

$= 20.2K$ (increasing)

When wheel ③ at the support to wheel ④



Here, $P_1 = 36K$; $d_1 = 6'$; $L = 60'$

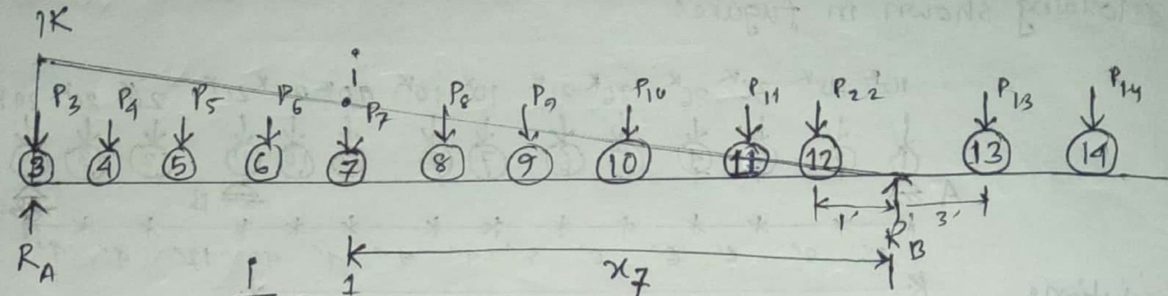
$\Sigma P = (224 - 36 + 20)K = 208K$ $\boxed{W-④-⑬}$

(W-12) ← $P' = 20K$; $e = 3'$

$$\Delta R_A = \frac{208\times6}{60} + \frac{20\times3}{60} - 36$$

$= -14.2K$ (decreasing)

Hence, when wheel ③ at support A, Reaction at A will be maximum.



$$\frac{i}{x_7} = \frac{1}{L}$$

$$\Rightarrow i = \frac{x_7}{L} \qquad \therefore R_{A(7)} = \frac{x_7}{L} \times P_7$$

Maximum Reaction, $R_A = \frac{1}{L}\left[P_{12}x_{12} + P_{11}x_{11} + \cdots + P_3 x_3\right]$

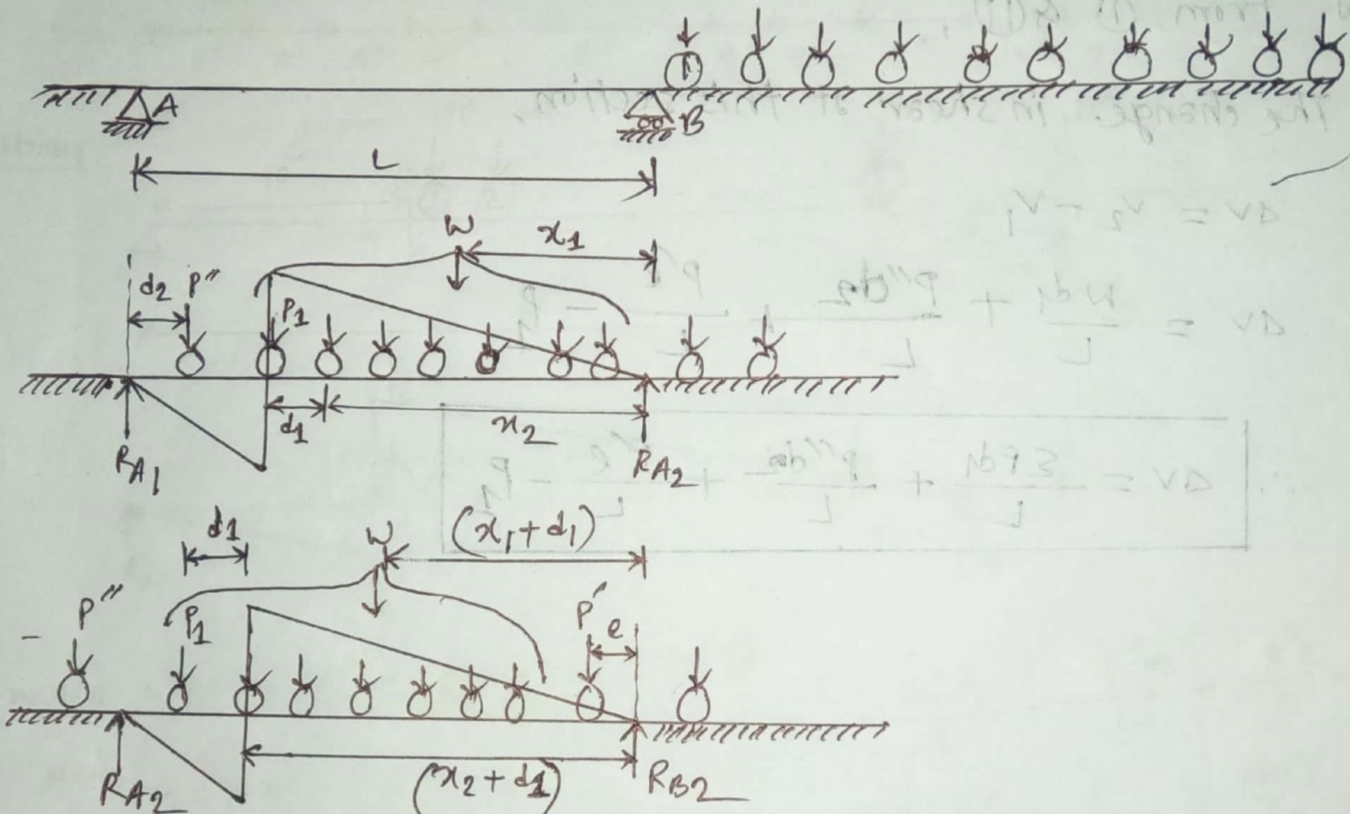
$$\text{Max. } R_A = \frac{1}{60}\left[(20\times 1)+(20\times 13)+(20\times 17)+(20\times 21)+(10\times 30)+(10\times 34)+(36\times 42)\right.$$
$$\left.+(36\times 48)+(36\times 54)+(36\times 60)\right]$$

$$\text{Max. } R_A = \frac{9024}{60} = 150.4 \text{ K}$$

(Ans)

09, 11, 14, 16, 17

Criteria for the $\boxed{\text{maximum shear}}$ of a simple beam subjected to series of concentrated loads move from right to left.



Before movement,

$$\Sigma M_B = 0 \Rightarrow R_{A_1} \times L - W x_1 - P''(L-d_2)$$

$$\Rightarrow R_{A_1} = \frac{W x_1}{L} + P'' \frac{(L-d_2)}{L}$$

Now, shear force of the section,

$$V_1 = R_{A_1} - P''$$

$$\Rightarrow V_1 = \frac{W x_1}{L} + P''\frac{(L-d_2)}{L} - P'' = \frac{W x_1}{L} + \frac{P'' d_2}{L} \quad \text{--- (1)}$$

After movement,

$$\Sigma M_B = 0 \Rightarrow R_{A_2} \times L - W(x_1+d_1) - P'e = 0$$

$$\Rightarrow R_{A_2} = \frac{W(x_1+d_1)}{L} + \frac{P'e}{L}$$

Now, Shear of the section,

$$V_2 = R_{A_2} - P_1$$

$$\Rightarrow V_2 = \frac{w(x_1+d_1)}{L} + \frac{P'e}{L} - P_1 \quad \text{--- (ii)}$$

Now From (i) & (ii),

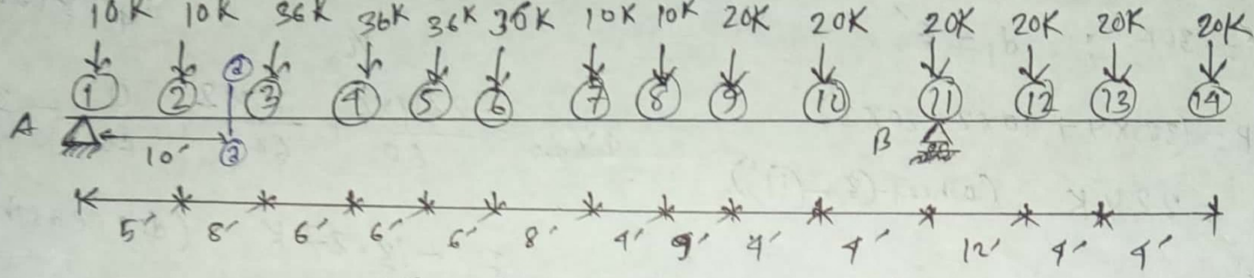
The change in shear at this section,

$$\Delta V = V_2 - V_1$$

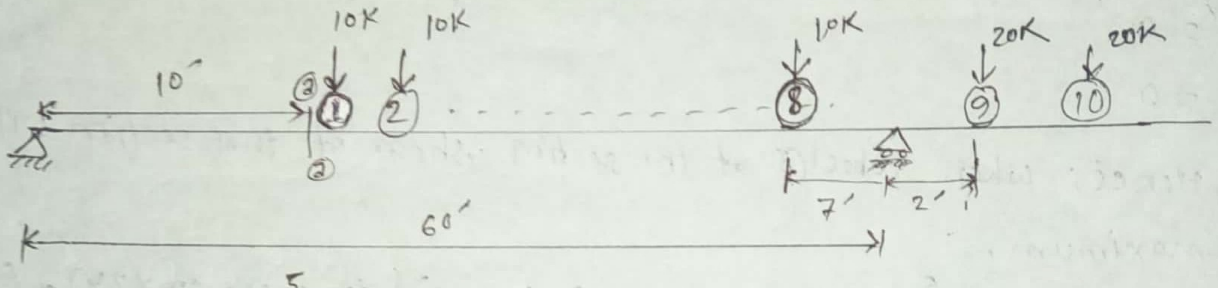
$$\Delta V = \frac{w d_1}{L} + \frac{P'' d_2}{L} + \frac{P'e}{L} - P_1$$

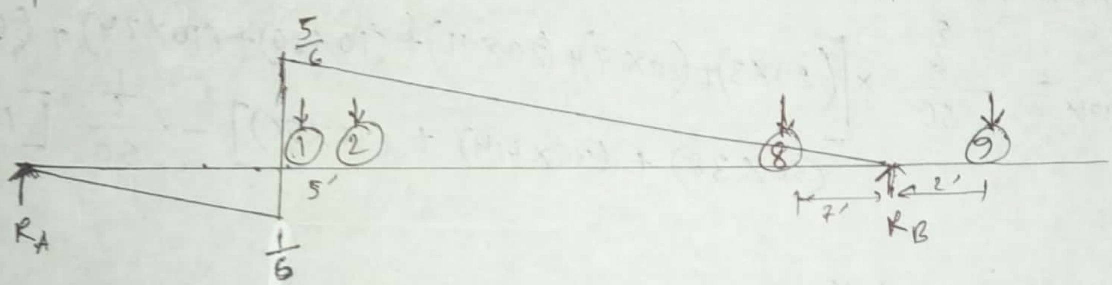
$$\therefore \boxed{\Delta V = \frac{\Sigma P d_1}{L} + \frac{P'' d_2}{L} + \frac{P'e}{L} - P_1}$$

Problem: Find the maximum shear at section a-a due to moving load as shown in figure:



Solution:





When wheel-1 at the section to wheel-2

$$\Delta V_{a\text{-}a} = \frac{\Sigma P d_1}{L} + \frac{P'' d_2}{L} + \frac{P' e}{L} - P_1$$

Here, $P_1 = 10K$

$\Sigma P = (10\times2 + 36\times4 + 10\times2)\ K$

$= 184\ K$ (wheel-1 – 8)

$d_1 = 5'$; $d_2 = 0$

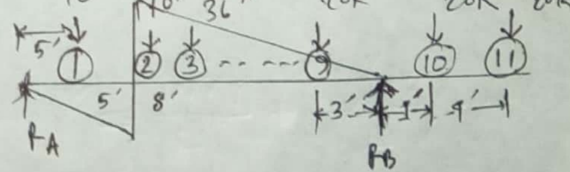
$P'' = 0$; $P_1 = 10K$ (wheel-1)

$\boxed{W-9} \leftarrow P' = 20K$; $e' = 3$

$$= \frac{184\times5}{60} + \frac{0}{60} + \frac{20\times3}{60} - 10$$

$= 6.33\ K$ (increasing)

When wheel-2 at the section to wheel 3



$P_1 = 10K$; $d_1 = 8'$; $d_2 = 5'$

$P'' = 10K$; $\Sigma P = (10 + 36\times4 + 10\times2 + 20) = 194$ $\boxed{\text{Wheel-2 – 9}}$

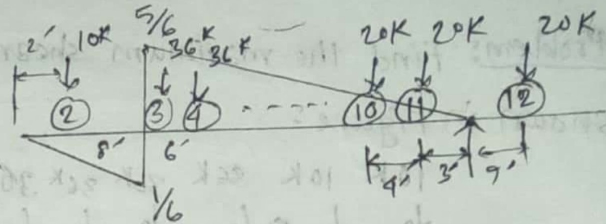
$P' = (20+20)K = 40K$ $\boxed{\text{wheel-10 \& 11}}$

$e = \left(\frac{7+3}{2}\right) = \frac{10}{2} = 5'$

$$\Delta V_{a\text{-}a} = \frac{194\times8}{60} + \frac{10\times5}{60} + \frac{40\times5}{60} - 10$$

$= 20.03\ K$ (increasing)

When wheel(3) at the section to wheel (4)



$P'' = 10K$; $d_2 = 2'$

$P_1 = 36K$; $d_1 = 6'$

$\Sigma P = (36 \times 4 + 10 \times 2 + 20 \times 3)$

$= 224 K$ (wheel-(3)-(11))

$P' = 0$

$e = 0$

$$\Delta V_{2-2} = \frac{224 \times 6}{60} + \frac{10 \times 2}{60} + \frac{0}{60} - 36$$

$= -13.27 K$ (decreasing)

Hence, when wheel(3) at the section, shear at that section will be maximum.

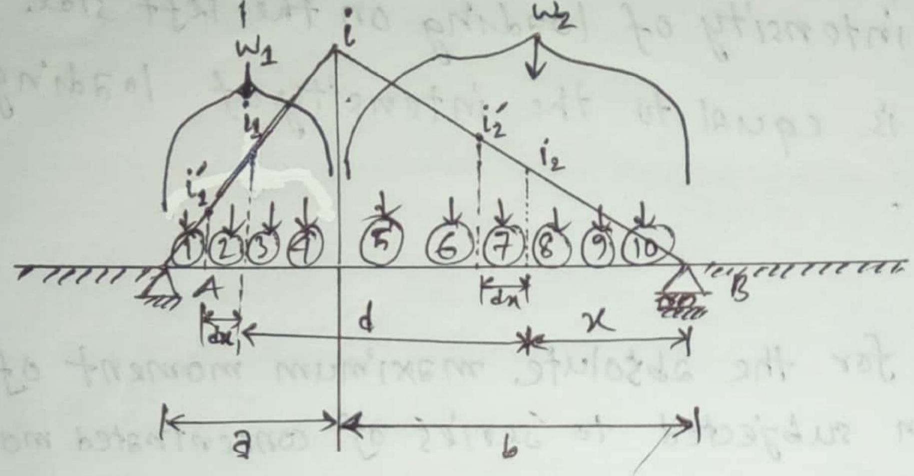
$$\therefore V_{max} = \frac{\frac{5}{6}}{50} \times \Big[(20 \times 3) + (20 \times 7) + (20 \times 11) + (10 \times 20) + (10 \times 24) + (36 \times 32) + (36 \times 38) + (36 \times 44) + (36 \times 50) \Big] - \frac{\frac{1}{6}}{60} \left[10 \times 2 \right]$$

$= 112.4 K$

(Ans.)

12, 13, 15

Criteria for the $\boxed{\text{maximum moment}}$ at section of a simple beam subjected to series of concentrated loads move from right to left.



considering the right side of section,

$$i_2 = \frac{x}{b} i \ , \ i_2' = \frac{(x+dx)}{b} i$$

$$\therefore M_2 = i\frac{x}{b} w_2 \ , \ M_2' = i\frac{(x+dx)}{b} w_2$$

$\therefore$ increase of moment, $\Delta M_2 = M_2' - M_2 = i\frac{w_2}{b}dx$

considering the left side of the section,

$$i_1 = \frac{(L-x-d)}{a} i \ , \quad i_1' = \frac{(L-x-d-dx)}{a} i$$

$$\therefore M_1 = i\frac{(L-x-d)}{a} w_1 \ , \ M_1' = i\frac{(L-x-d-dx)}{a} w$$

$\therefore$ decrease of moment, $\Delta M_1 = M_1' - M_1 = -i\frac{w_1 dx}{a}$

Net increase in moment, $dM = \Delta M_1 + \Delta M_2$

$$\therefore dM = i\frac{w_2}{b}dx - i\frac{w_1}{a}dx$$

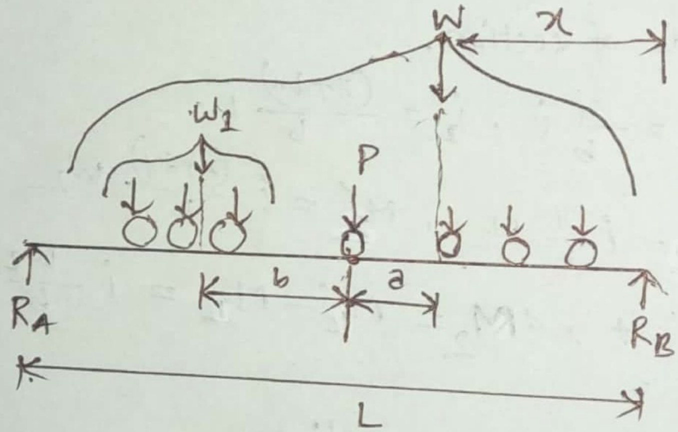
$$\Rightarrow \frac{dM}{dx} = -i\frac{w_1}{a} + i\frac{w_2}{b} = 0$$

$$\Rightarrow \boxed{\frac{W_1}{a} = \frac{W_2}{b} = \frac{W_1+W_2}{a+b} = \frac{W}{L}}$$

So, The maximum moment at a given section occurs when the intensity of loading on the left side of the section is equal to the intensity of loading on the span.

10

Criteria for the absolute maximum moment of a simple beam subjected to series of concentrated moving loads.



$$M = R_A(L-a-x) - W_1 b \qquad \text{and } R_A = \frac{Wx}{L}$$

$$\Rightarrow M = \frac{Wx}{L}(L-a-x) - W_1 b$$

$$\Rightarrow M = \frac{W}{L}(Lx-ax-x^2) - W_1 b$$

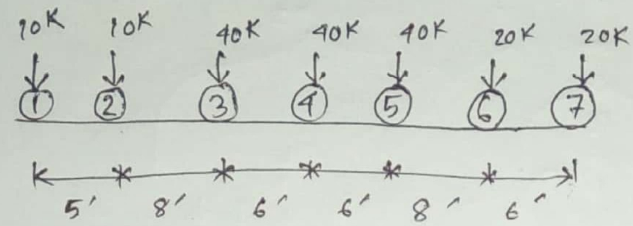
$$\Rightarrow \frac{dM}{dx} = \frac{W}{L}(L-a-2x) - 0$$

$$\Rightarrow \frac{dM}{dx} = \frac{W}{L}(L-a-2x) = 0$$

$$\therefore 2x = L-a$$

$$\Rightarrow \boxed{x = \frac{L}{2} - \frac{a}{2}}$$

For a single span of 100 ft. calculate (i) maximum moment at mid span and (ii) the absolute maximum moment due to the loading shown in figure below:



Solution: criteria for max. moment at center is : $\frac{W_1}{a_1} = \frac{W}{L}$

For wheel 1 at center:

Right: $\frac{0}{50} < \frac{180}{100}$

Left: $\frac{10}{50} < \frac{180}{100}$

wheel 2:

Right: $\frac{10}{50} < \frac{180}{100}$

Left: $\frac{20}{50} < \frac{180}{100}$

wheel 3:

Right: $\frac{20}{50} < \frac{180}{100}$

Left: $\frac{60}{50} < \frac{180}{100}$

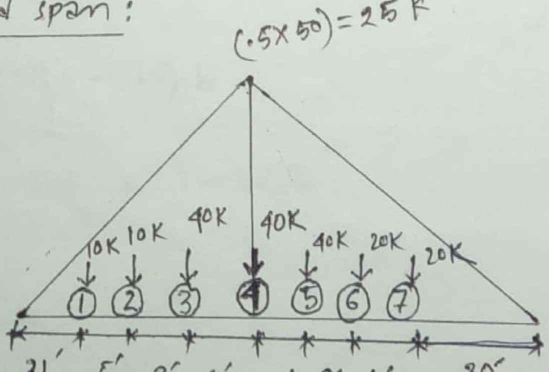
wheel 4:

Right: $\frac{60}{50} < \frac{180}{100}$

Left: $\frac{100}{50} > \frac{180}{100}$

which satisfies the criteria.

Now, wheel 4 at mid span:



Max. moment at center $= \frac{25}{50}\left[(30\times20)+(36\times20)+(44\times40)+(50\times40)\right] +$

$\frac{25}{50}\left[(31\times10)+(36\times10)+(44\times40)\right]$

$= 3755\ K'$

Absolute max. moment:

position of centroid of total load from right end of loading

$$\bar{x} = \frac{1}{180}(20\times 0 + 20\times 6 + 40\times 14 + 40\times 20 + 40\times 26 + 10\times 34 + 10\times 39)$$

$= 18.06'$

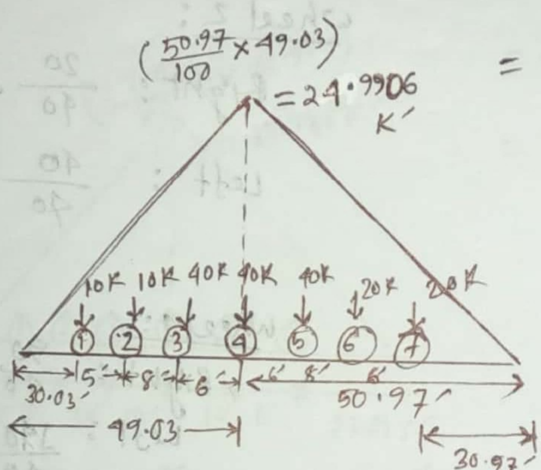
$\bar{x}$ যে wheel এর কাছে তার distance

$\therefore$ position of wheel 4 from left end, $x = \frac{L}{2} - \frac{a}{2}$ where, $a = (20 - 18.06)$

$= 1.94'$

$= \left(\frac{100}{2} - \frac{1.94}{2}\right)$ ft

$= 49.03$ ft



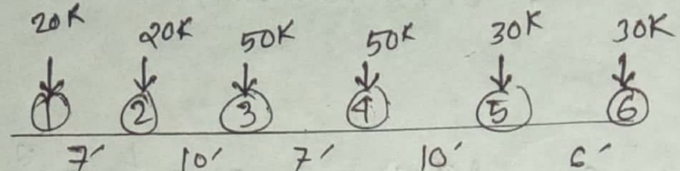
$$\therefore \text{Absolute max. moment} = \frac{24.9906}{49.03}\times(30.03\times 10 + 35.03\times 10 + 43.03\times 40 + 49.03\times 40)$$

$$+ \frac{24.9906}{50.97}(30.97\times 20 + 36.97\times 20 + 44.97\times 40)$$

$= 3756.703$ K'

(Ans.)

For a simple span of 80ft, compute (i) the maximum moment at mid-span (ii) the absolute max. moment due to the loading shown in figure below:



Solution: criteria for max. moment at center is : $\frac{W_1}{a_1} = \frac{W}{L}$

For wheel 1 at center:

Right: $\frac{0}{40} < \frac{170}{80}$

Left: $\frac{20}{40} < \frac{200}{80}$

wheel 2:

Right: $\frac{20}{40} < \frac{200}{80}$

Left : $\frac{40}{40} < \frac{200}{80}$

wheel 3:

Right: $\frac{40}{40} < \frac{200}{80}$

Left: $\frac{90}{40} < \frac{200}{80}$

wheel 4:

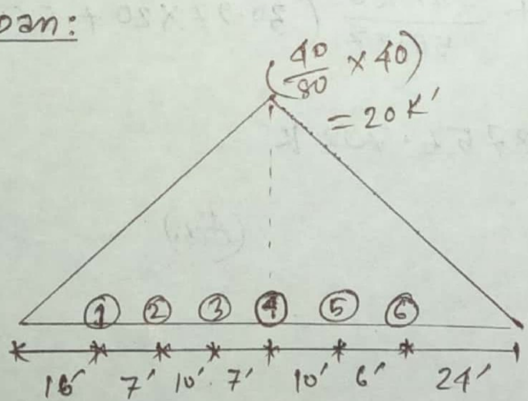
Right: $\frac{90}{40} < \frac{200}{80}$

Left: $\frac{140}{40} > \frac{200}{80}$

which satisfies the criteria

~~Maximum moment~~ .

Now, wheel 4 at mid span:



$$\text{Max. moment at center} = \frac{20}{40}\left[(20\times16) + (20\times23) + (50\times33) + (50\times40)\right] + \frac{20}{40}\left[(30\times24) + (30\times30)\right]$$

$$= 3025\ K'$$

Absolute max. moment:

position of centroid of total load from right end of loading,

$$\bar{x} = \frac{(30\times0)+(30\times6)+(50\times16)+(50\times23)+(20\times33)+(20\times40)}{200}$$

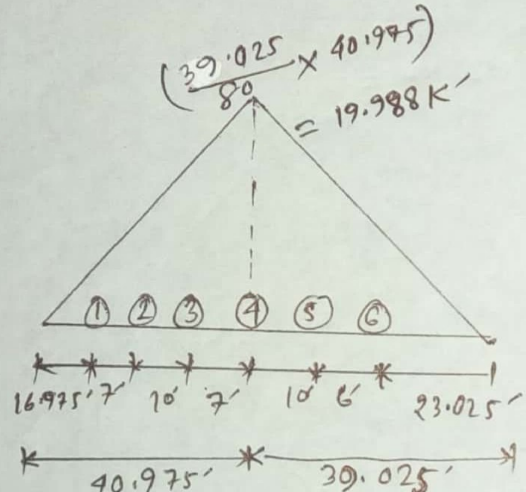
$$= 17.95'$$

position of wheel-4 from left end, $x = \frac{L}{2} - \frac{a}{2}$ where, $a = (16 - 17.95) = -1.95$

$$= \frac{80}{2} - \left(\frac{-1.95}{2}\right)$$

$$= \frac{80}{2} + \frac{1.95}{2}$$

$$= 40.975$$



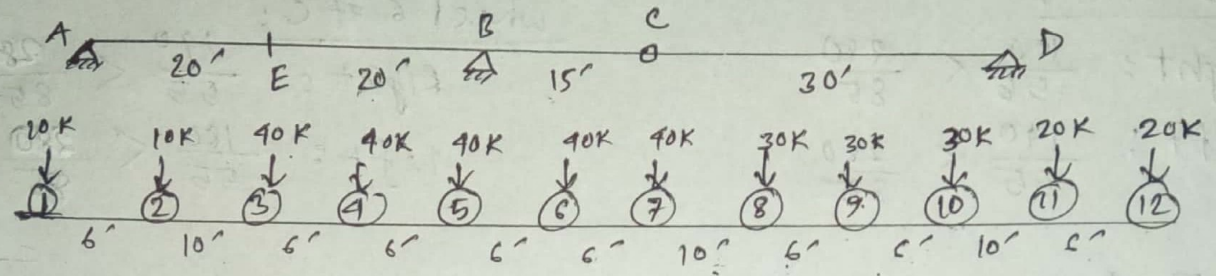
$\therefore$ Absolute max. moment $= \frac{19.988}{40.975}(20\times16.975 + 20\times23.975 + 50\times33.975 + 50\times40.975) + \frac{19.988}{39.025}(30\times23.025 + 30\times29.025)$

$$= 3027.36\ K'$$

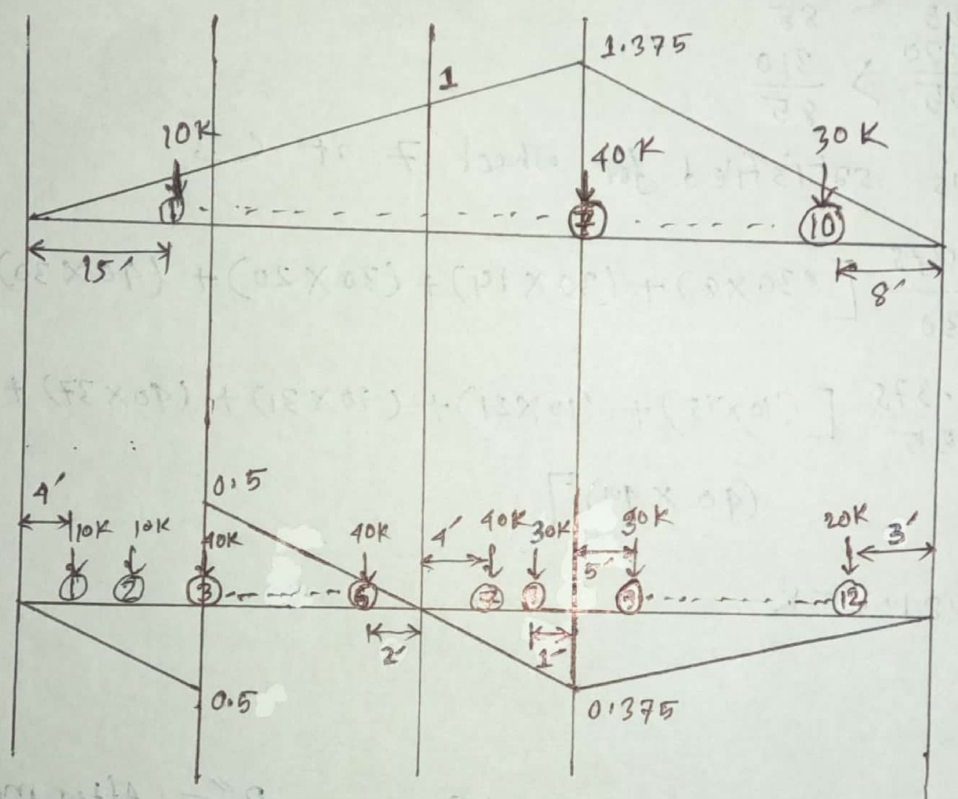
(Ans.)

2005, 03

Apply proper criteria to obtain the position of loads and hence compute the maximum values of reactions at B and shear at E. structure subjected to moving loads as shown below.



Solution:



IL for R_B

V_E

For Reaction: IL of R_B is same as Moment diagram. Hence criteria for max. reaction at B is $\frac{W_1}{L_1} = \frac{W}{L}$

Wheel 1 at C: Right: $\frac{0}{55} < \frac{140}{85}$

Left: $\frac{10}{55} < \frac{140}{85}$

Wheel 2 at C:

Right: $\frac{10}{55} < \frac{180}{85}$

Left: $\frac{20}{55} < \frac{180}{85}$

Wheel 3 at C: Right: $\frac{20}{55} < \frac{220}{85}$

Left: $\frac{60}{55} < \frac{220}{85}$

Wheel 4 at C:

Right: $\frac{60}{55} < \frac{250}{85}$

Left: $\frac{100}{55} < \frac{250}{85}$

Wheel 5 at C:

Right: $\frac{100}{55} < \frac{280}{85}$

left: $\frac{140}{55} < \frac{280}{85}$

Wheel 6 at C:

Right: $\frac{140}{55} < \frac{310}{85}$

left: $\frac{180}{55} < \frac{310}{85}$

Wheel 7 at C:

Right: $\frac{180}{55} < \frac{310}{85}$

left: $\frac{220}{55} > \frac{310}{85}$

$\therefore$ criteria is satisfied for wheel 7 at C

$$\text{Max. } R_B = \frac{1.375}{30}\left[(30\times 8)+(30\times 14)+(30\times 20)+(40\times 30)\right] + \frac{1.375}{55}\left[(10\times 15)+(10\times 21)+(40\times 31)+(40\times 37)+(40\times 43)+(40\times 49)\right]$$

$= 281.75\text{ K}$

For shear:

when, wheel 1 at section E to wheel 2,

$$\Delta V_E = \frac{\Sigma P d_1}{L} + \frac{P'' d_2}{L} + \frac{P' e}{L} - P_1$$

P'' = After movement, span থেকে চলে যাবে হবে।

Here, $P_1 = 10\text{K}$, $L = 85'$

$d_1 = 6'$

$P'' = 0$

$d_2 = 0$

$P' = 0$

$e = 0$

$\Sigma P = 310\text{ K}$ (wheel 1 - 10)

$\therefore \Delta V_E = \frac{310\times 6}{85} + 0 + 0 - 10$

$\therefore \Delta V_E = 11.88$ (increasing)

Wheel ② at section E to wheel ③

Here $P_1 = 10\,K$ $\Sigma P = 310\,K$ $\boxed{\text{wheel-①-10}}$

$d_1 = 10'$ $P' = (20+20) = 40K$

$P'' = 0$ $e' = \left(\frac{9+3}{2}\right) = 6$

$d_2 = 0$

$$\therefore \Delta V_E = \frac{310 \times 10}{85} + 0 + \frac{40 \times 6}{85} - 10 = 29.294\ K \text{ (increasing)}$$

Wheel ③ at section E to wheel ④

Here,

$P_1 = 40K$ $\Sigma P = 340K$ $\boxed{\text{wheel ③ – ⑫}}$

$d_1 = 6'$ $P' = 0$

$P'' = 10K$ $e = 0$

$d_2 = 4'$ $\therefore \Delta E = \frac{340 \times 6}{85} + \frac{10 \times 4}{85} + 0 - 40 = -15.53K$

(decreasing)

Hence, criteria is satisfied for wheel ③ at section E,

$$\therefore \text{Max. shear at E}, V_E = \frac{0.5}{20}\left[(40\times 2) + (40\times 8) + (40\times 14) + (40\times 20)\right] - \frac{0.5}{20}\left[(10\times 4) + (10\times 10)\right] - \frac{0.375}{15}\left[(40\times 4) + (30\times 14)\right] - \frac{0.375}{30}\left[(20\times 3) + (20\times 9) + (30\times 19) + (30\times 25)\right]$$

$$= 44 - 3.5 - 14.5 - 19.5$$

$$= 6.5\ K$$

(Ans.)

3D Space Truss

 # Problem: compute the reactions and bar forces of the following truss as shown in figure below:

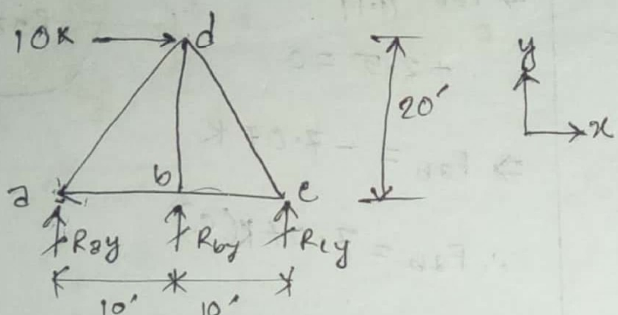


Fig. (a)

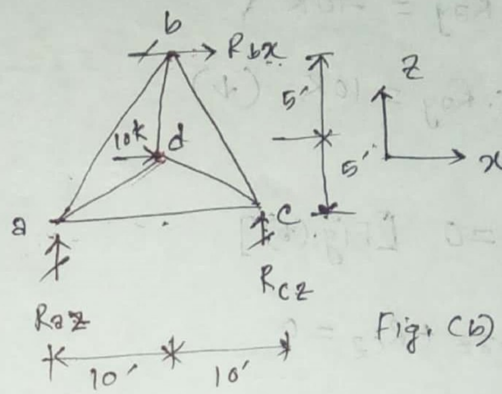


Fig. (b)

Solution:

Member	Projection x	Projection y	Projection z	Length $=\sqrt{x^2+y^2+z^2}$
ab	10′	0′	10′	14.14′
ac	20′	0′	0′	20′
ad	10′	20′	5′	22.91′
bc	10′	0′	10′	14.14′
bd	0′	20′	5′	20.62′
cd	10′	20′	5′	22.91′

$\Sigma F_x = 0$ [Fig. (b)]

$R_{bx} + 10 = 0$

$\Rightarrow R_{bx} = -10\text{ K}$

$\therefore R_{bx} = 10\text{ K } (\leftarrow)$

$\Sigma M_{xa} = 0$ [Fig (a) & Fig. (b)]

$R_{by} \times 10 = 0$

$\therefore R_{by} = 0\text{ K}$

$\Sigma M_{za} = 0$ [Fig. (a)]

$R_{by} \times 10 + R_{cy} \times 20 - 10 \times 20 = 0$

$\therefore R_{cy} = 10\text{ K}$

$\Sigma M_{ya} = 0$ [Fig. (b)]

$R_{cz} \times 20 - 10 \times 5 - R_{bx} \times 10 = 0$

$\Rightarrow R_{cz} = \dfrac{50 - 100}{20} = -2.5\text{ K}$

$\therefore R_{cz} = 2.5\text{ K } (\downarrow)$

$\Sigma F_y = 0$ [Fig(a)]

$R_{ay} + R_{by} + R_{cy} = 0$

$\Rightarrow R_{ay} + 0 + 10 = 0$

$\Rightarrow R_{ay} = -10K$

$\therefore R_{ay} = 10K \ (\downarrow)$

$\Sigma F_z = 0$ [Fig(b)]

$R_{az} + R_{cz} = 0$

$\Rightarrow R_{az} - 2.5 = 0$

$\therefore R_{az} = 2.5K$

Now, <u>joint a</u>: $\Sigma F_y = 0$

$F_{ady} + R_{ay} = 0$

$\Rightarrow F_{ad} \times \frac{20}{22.91} = 10$

$\therefore F_{ad} = 11.455 \ K \ (T)$

<u>joint b</u>: $\Sigma F_y = 0$

$F_{bdy} + R_{by} = 0$

$F_{bd} = 0$

<u>joint c</u>: $\Sigma F_y = 0$

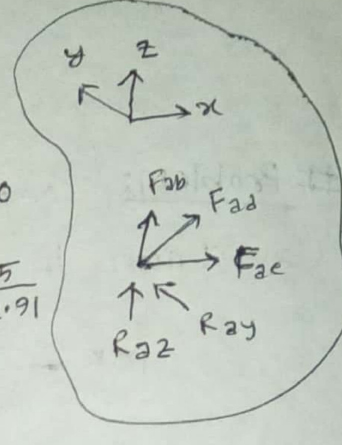
$F_{cdy} + R_{cy} = 0$

$\Rightarrow F_{cd} \times \frac{20}{22.91} = -10$

$\Rightarrow F_{cd} = -11.455 \ K$

$F_{cd} = 11.455 \ K \ (C)$

<u>At joint a</u>:



$\Sigma F_z = 0$

$F_{abz} + F_{adz} + R_{az} = 0$

$\Rightarrow F_{ab} \times \frac{10}{14.14} + 11.455 \times \frac{5}{22.91} + 2.5 = 0$

$\Rightarrow F_{ab} = -7.07 \ K$

$\therefore F_{ab} = 7.07 K (C)$

$\Sigma F_x = 0$

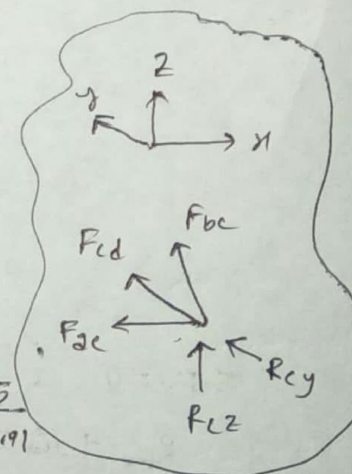
$F_{abx} + F_{adx} + F_{ac} = 0$

$7.07 \times \frac{10}{14.14} + 11.455 \times \frac{10}{22.91} + F_{ac} = 0$

$\Rightarrow F_{ac} = -10K$

$\therefore F_{ac} = 10K \ (C)$

<u>At joint c</u>:



$\Sigma F_z = 0$

$R_{cz} + F_{bcz} + F_{cdz} = 0$

$-2.5 + F_{bc} \times \frac{10}{14.14} + 11.455 \times \frac{5}{22.91} = 0$

$\Rightarrow F_{bc} = 7.07K \ (T)$

(Ans)

compute the reactions and barforces of the following space truss shown in figure below:- [With out considering moment equation] → Another way to solve

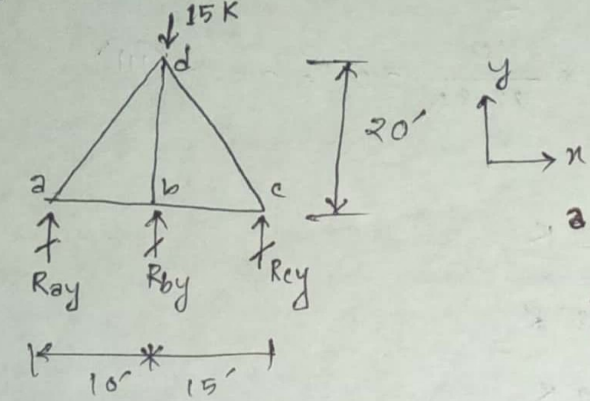


Fig. (a)

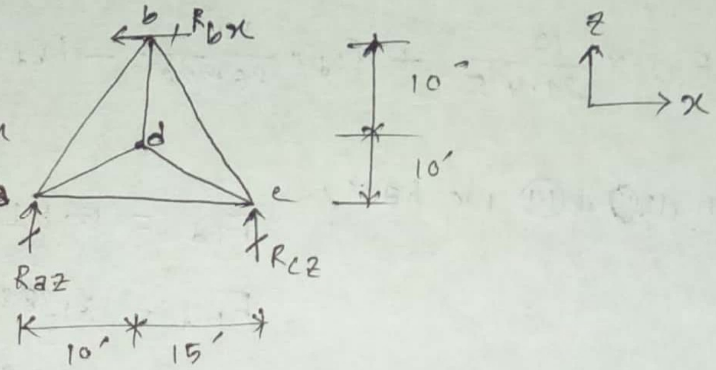
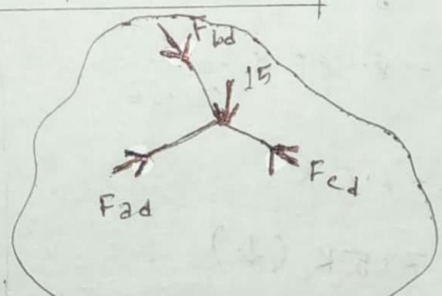


Fig. (b)

Solution:

Member	Projection			Length =
	x	y	z	$\sqrt{x^2+y^2+z^2}$
ab	10'	0'	20'	22.36'
ac	25'	0'	0'	25'
ad	10'	20'	10'	24.495'
bc	15'	0'	20'	25'
bd	0'	20'	10'	22.36'
cd	15'	20'	10	26.926'

considering joint d:



$\Sigma F_x = 0$

$F_{adx} - F_{cdx} = 0$

$\Rightarrow F_{ad} \times \dfrac{10}{24.495} = F_{cd} \times \dfrac{15}{26.926}$

$\therefore F_{ad} = 1.3646\, F_{cd}$ ...... (i)

$\Sigma F_y = 0$

$F_{ady} + F_{bdy} + F_{cdy} - 15 = 0$

$\Rightarrow F_{ad} \times \dfrac{20}{24.495} + F_{bd} \times \dfrac{20}{22.36} + F_{cd} \times \dfrac{20}{26.926} = 15$ ...... (ii)

$\Sigma F_z = 0$

$F_{adz} + F_{cdz} - F_{bdz} = 0$

$\Rightarrow F_{ad} \times \dfrac{10}{24.495} + F_{cd} \times \dfrac{10}{26.926} - F_{bd} \times \dfrac{10}{22.36} = 0 \quad \cdots\cdots (iii)$

From (i), (ii) & (iii) we have,

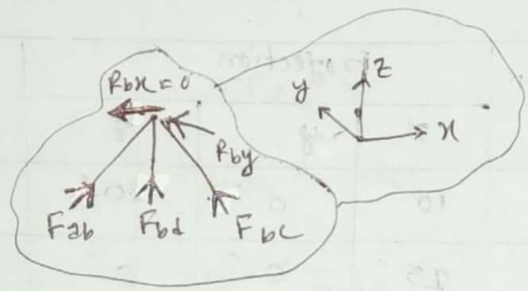
$F_{ad} = 5.51\ K$

$F_{bd} = 8.385\ K$

$F_{cd} = 4.04\ K$

$\Sigma F_x = 0$ [Fig (b)] $\quad R_{bx} = 0$

<u>considering joint b:</u>



$\Sigma F_x = 0$

$F_{abx} - F_{bcx} = 0$

$\Rightarrow F_{ab} \times \dfrac{10}{22.36} = F_{bc} \times \dfrac{15}{25}$

$\Rightarrow F_{ab} = 1.3416\ F_{bc} \quad \cdots\cdots (1)$

$\Sigma F_y = 0$

$F_{bdy} + R_{by} = 0$

$\Rightarrow R_{by} = -8.385 \times \dfrac{20}{22.36}$

$\Rightarrow R_{by} = -7.5\ K$

$R_{by} = 7.5\ K\ (\downarrow)$

$\Sigma F_z = 0$

$F_{abz} - F_{bdz} + F_{bcz} = 0$

$\Rightarrow F_{ab} \times \dfrac{20}{22.36} - 8.385 \times \dfrac{10}{22.36} + F_{bc} \times \dfrac{20}{25} = 0 \quad \cdots\cdots (ii)$

From (i) & (ii) we obtain, $F_{ab} = +2.5155\ K$

$F_{bc} = +1.875\ K$

<u>considering joint a:</u>

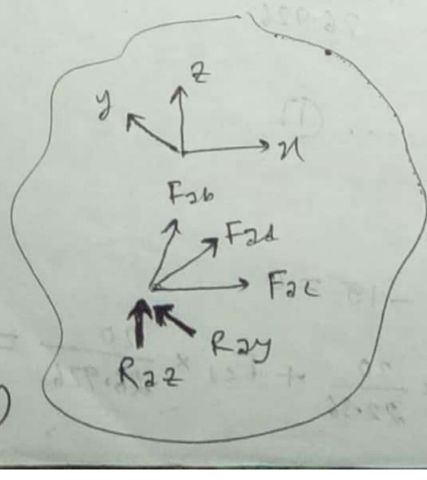
$\Sigma F_z = 0$

$F_{abz} + F_{adz} + R_{az} = 0$

$2.51 \times \dfrac{20}{22.36} + 5.51 \times \dfrac{10}{24.495} + R_{az} = 0$

$\Rightarrow R_{az} = -4.5\ K$

$\therefore R_{az} = 4.5\ (\downarrow)$



$\Sigma F_y = 0$

$F_{ady} + R_{ay} = 0$

$\Rightarrow R_{ay} = -5.51 \times \frac{20}{24.495}$

$\Rightarrow R_{ay} = -4.5 \text{ K}$

$R_{ay} = 4.5 \text{K} (\downarrow)$

$\Sigma F_x = 0$

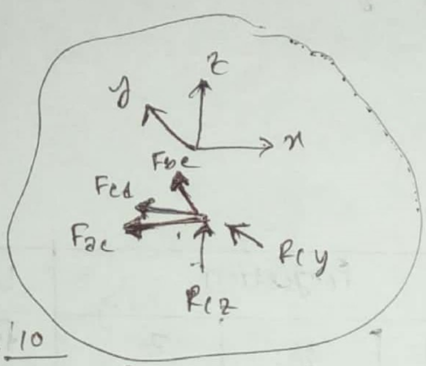
[redacted] $F_{abx} + F_{adx} + F_{acx} = 0$

$2.51 \times \frac{10}{22.36} + 5.51 \times \frac{10}{24.495} + F_{ac} \times \frac{25}{25} = 0$

$\therefore F_{ac} = -3.372 \text{ K}$

$\therefore F_{ac} = 3.372 \text{ K (C)}$

considering point c



$\Sigma F_z = 0$

$R_{cz} + F_{bcz} + F_{cdz} = 0$

$R_{cz} + 1.875 \times \frac{20}{25} + 4.04 \times \frac{10}{26.926} = 0$

$\Rightarrow R_{cz} = -3 \text{ K}$

$\therefore R_{cz} = 3 \text{K} (\downarrow)$

$\Sigma F_y = 0$

$F_{cdy} + R_{cy} = 0$

$\Rightarrow R_{cy} = -4.04 \times \frac{20}{26.926}$

$\Rightarrow R_{cy} = -3 \text{ K}$

$\therefore R_{cy} = 3 \text{K} (\downarrow)$

(Ans.)

problem: compute the reaction and bar forces of the following space truss shown in figure:

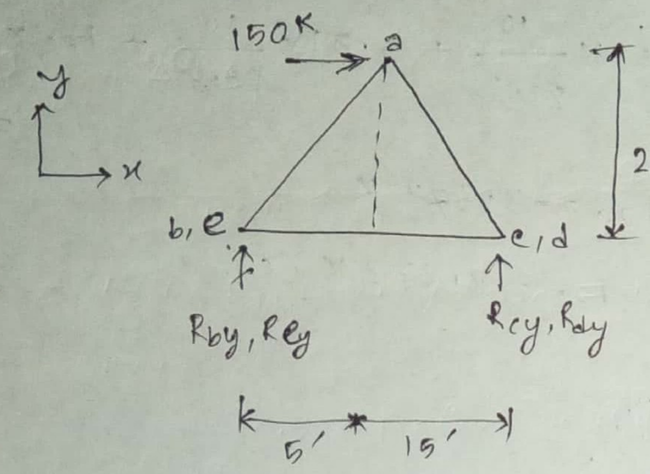


Fig. (a)

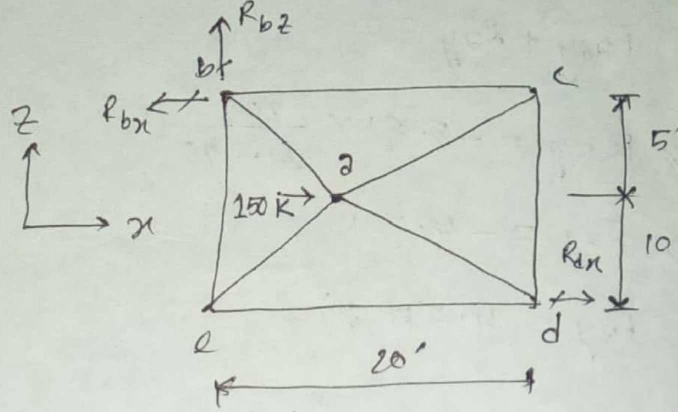


Fig.(b)

Solution:

Member	Projection			Length = $\sqrt{x^2+y^2+z^2}$
	x	y	z	
ab	5′	20′	5′	21.21
ac	15′	20′	5′	25.495
ad	15′	20′	10′	26.926
ae	5′	20′	10′	22.913
bc	20′	0′	0′	20
be	0′	0′	15′	15
cd	0′	0′	15′	15
de	20′	0′	0′	20

$\Sigma M_{yb} = 0$ [Fig. (b)]

$R_{dx} \times 15 + 150 \times 5 = 0$

$R_{dx} = -50K$

$\therefore R_{dx} = 50K\ (\leftarrow)$

$\Sigma F_x = 0$ [Fig. (b)]

$R_{bx} - 150 - R_{dx} = 0$

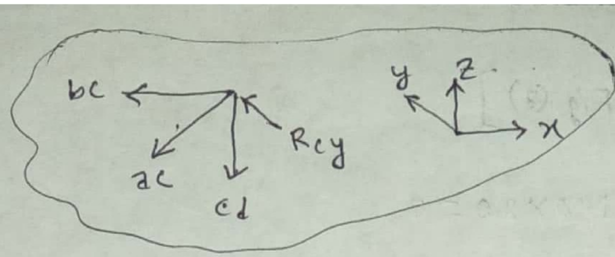
$\Rightarrow R_{bx} = 150 - 50$

$\therefore R_{bx} = 100K\ (\leftarrow)$

$\Sigma F_z = 0$ [Fig. (b)]

$R_{bz} = 0$

At joint c



$\Sigma F_z = 0$

$F_{acz} + F_{cd} = 0$

$\Rightarrow F_{ac} \times \frac{5}{25.495} + F_{cd} = 0$

$\Rightarrow F_{ac} = -5.099\, F_{cd}$ ...... (i)

$\Sigma F_y = 0$

$F_{acy} + R_{cy} = 0$

$\Rightarrow R_{cy} = -\frac{20}{25.495} F_{ac}$

$\Rightarrow R_{cy} = -0.784\, F_{ac}$ ...... (ii)

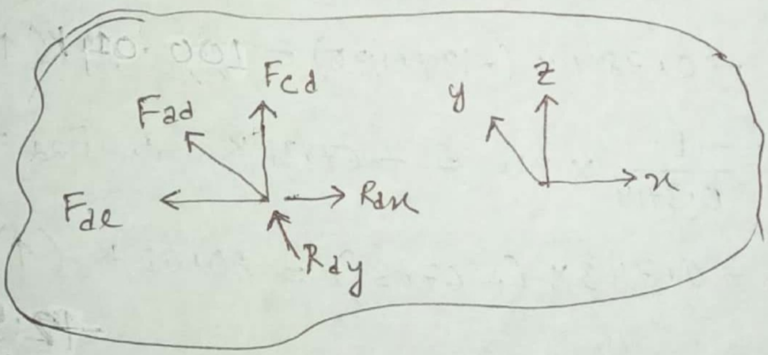
$\Sigma F_x = 0$

$F_{bc} + F_{acx} = 0$

$\Rightarrow F_{bc} = -\frac{15}{25.495} F_{ac}$

$\Rightarrow F_{bc} = -0.588\, F_{ac}$ ...... (iii)

At joint d



$\Sigma F_z = 0$

$F_{cd} + F_{adz} = 0$

$\Rightarrow F_{cd} = -\frac{10}{26.926} F_{ad}$

$\Rightarrow F_{cd} = -0.3714\, F_{ad}$ ...... (iv)

$\Sigma F_y = 0$

$R_{dy} + F_{ady} = 0$

$\Rightarrow R_{dy} = -\frac{20}{26.926} F_{ad}$

$\Rightarrow R_{dy} = -0.743\, F_{ad}$ ...... (v)

$\Sigma F_x = 0$

$F_{de} + F_{adx} - R_{dx} = 0$

$\Rightarrow F_{de} = -\frac{15}{26.926} F_{ad} - 50$

$\Rightarrow F_{de} = -0.5571\, F_{ad} - 50$ ...... (vi)

Now,

$\Sigma M_{z\text{-}be} = 0$ [Fig.(2)]

$20 R_{dy} + 20 R_{ey} - 150 \times 20 = 0$

$\Rightarrow \cancel{15 \times (-0.743 F_{ad}) + 15 \times (-0.6543 F_{ad} - 50) =}$

$\Rightarrow -0.743 F_{ad} - 0.784 F_{ac} = 150$ [From eq^n (V) & (III)]

$\Rightarrow -0.743 \left(\frac{-1}{0.3714} F_{ed}\right) - 0.784 (-5.099 F_{ed}) = 150$ [From eq^n (I) & (IV)]

$\Rightarrow F_{ed} = 25 \text{ K} \quad (T)$

From,

(I) $\Rightarrow F_{ac} = -127.475 \text{ K} \quad \therefore F_{ac} = 127.475 \text{ K } (C)$

(II) $\Rightarrow F_{bc} = -0.588 \times (-127.475) = 74.955 \text{ K } (T)$

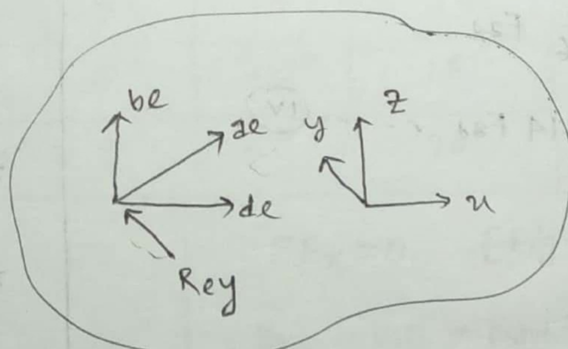
(III) $\Rightarrow R_{ey} = -0.784 \times (-127.475) = 99.94 \text{ K} (\uparrow)$

(IV) $\Rightarrow F_{ad} = \frac{-1}{0.3714} \times 25 = -67.31 \text{ K} \quad \therefore F_{ad} = 67.31 \text{ K } (C)$

(V) $\Rightarrow R_{dy} = -0.743 \times (-67.31) = 50.01 \text{ K } (\uparrow)$

(VI) $\Rightarrow F_{de} = -0.5571 \times (-67.31) - 50 = -12.50 \text{ K} \quad \therefore F_{de} = 12.50 \text{ K} (C)$

At jointe



$\Sigma F_z = 0$

$F_{be} + F_{aez} = 0$

$\Rightarrow F_{be} = -F_{ae} \times \frac{10}{22.913}$

$\Rightarrow F_{be} = -0.4364 F_{ae}$ ---- (1)

$\Sigma F_y = 0$

$\Sigma F_{aey} + R_{ey} = 0$

$\Rightarrow R_{ey} = -F_{ae} \times \frac{20}{22.913}$

$\Rightarrow R_{ey} = -0.873 F_{ae}$ ...... (II)

$\Sigma F_x = 0$

$F_{aex} + F_{de} = 0$

$\Rightarrow F_{ae} \times \frac{5}{22.913} - 12.50 = 0$

$\Rightarrow F_{ae} = 57.28\ K$ (T)

Now,

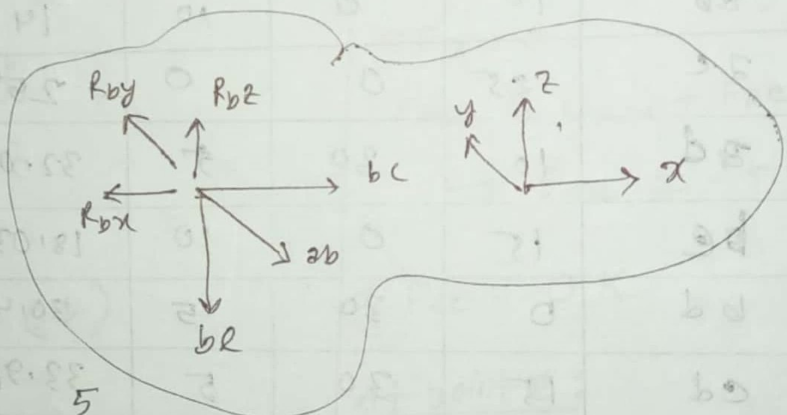
From (I) $\Rightarrow F_{be} = -25\ K$

$\therefore F_{be} = 25\ K$ (C)

(II) $\Rightarrow R_{ey} = -50\ K$

$\therefore R_{ey} = 50\ K$ (↓)

At joint b



$\Sigma F_z = 0$

$R_{bz} - F_{be} - F_{abz} = 0$

$\Rightarrow R_{bz} = F_{be} + F_{ab} \times \frac{5}{21.21}$

$\Rightarrow -25 + F_{ab} \times \frac{5}{21.21} = 0$

$\Rightarrow F_{ab} = 106.05\ K$ (T)

$\Sigma F_y = 0$

$R_{by} + F_{aby} = 0$

$\therefore R_{by} = -106.05 \times \frac{20}{21.21} = -100.0\ K$

$\therefore R_{by} = 100.0\ K$ (↓)

compute the reaction bar force's and reactions of the structures shown in figure below:

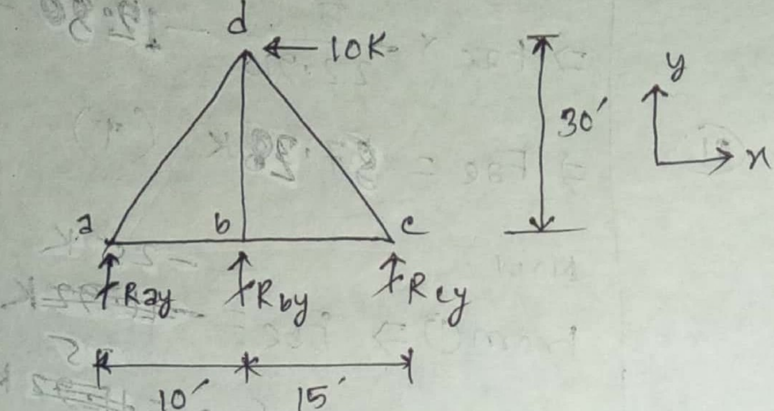


Fig. (a)

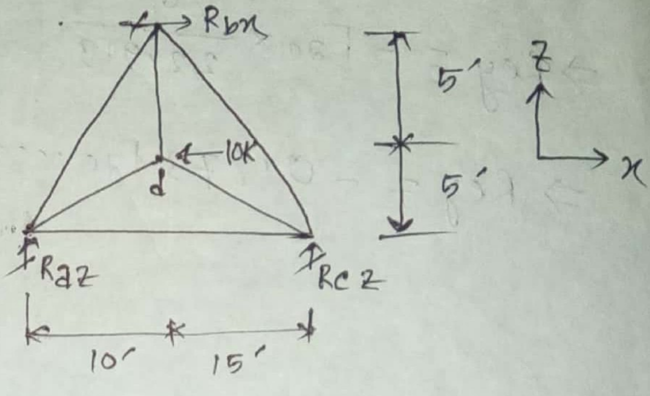


Fig. (b)

Solution:

Member	Projection x	y	z	Length $\sqrt{x^2+y^2+z^2}$
ab	10	0	10	14.14
ac	25	0	0	25
ad	10	30	5	32.02
bc	15	0	10	18.03
bd	0	30	5	30.41
cd	15	30	5	33.91

$\Sigma F_x = 0$

$R_{bx} = 10$

$\Sigma M_{xa} = 0$

$R_{by} = 0$

$\Sigma M_{za} = 0$ [Fig (a)]

$R_{cy} \times 25 + 10 \times 30 + R_{by} \times 10 = 0$

$\Rightarrow R_{cy} \times 25 = -300K$

$\Rightarrow R_{cy} = -12K$

$R_{cy} = 12K \ (\downarrow)$

$\Sigma M_{ya} = 0$ [Fig (b)]

$R_{bx} \times 10 - 10 \times 5 - R_{cz} \times 25 = 0$

$\Rightarrow R_{cz} \times 25 = 10 \times 10 - 10 \times 5$

$\Rightarrow R_{cz} = 2K$ (↑)

$\Sigma F_y = 0$ [Fig (a)]

$R_{ay} + R_{by} + R_{cy} = 0$

$\Rightarrow R_{ay} + 0 - 12 = 0$

$R_{ay} = 12K$ (↑)

$\Sigma F_z = 0$ [Fig (b)]

$R_{az} + R_{cz} = 0$

$\Rightarrow R_{az} = -2K$

$\therefore R_{az} = 2K$ (↓)

Now,

Joint a: $\Sigma F_y = 0$

$\Sigma F_{ady} + R_{ay} = 0$

$\Rightarrow F_{ad} \times \frac{30}{32.02} = -12$

$\Rightarrow F_{ad} = -12.81K$

$\therefore F_{ad} = 12.81K$ (C)

Joint b: $\Sigma F_y = 0$

$F_{bdy} + R_{by} = 0$

$\therefore F_{bd} = 0$

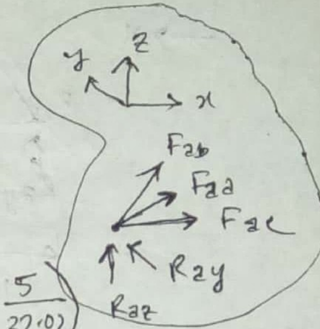
joint c: $\Sigma F_y = 0$

$F_{cdy} + R_{cy} = 0$

$\Rightarrow F_{cd} \times \frac{30}{33.91} = 12$

$\Rightarrow F_{cd} = 13.56K$ (T)

At joint a:



$\Sigma F_z = 0$

$F_{adz} + F_{abz} + R_{az} = 0$

$\Rightarrow F_{ab} \times \frac{10}{14.14} = \left(12.81 \times \frac{5}{32.02}\right) + 2$

$\Rightarrow F_{ab} = 5.656K$ (T)

$\Sigma F_x = 0$

$F_{abx} + F_{adx} + F_{ac} = 0$

$\Rightarrow F_{ac} = -5.656 \times \frac{10}{14.14} + 12.81 \times \frac{10}{32.02}$

$\Rightarrow F_{ac} = 0K$

At joint c:

$\Sigma F_z = 0$

$R_{cz} + F_{bcz} + F_{cdz} = 0$

$2 + F_{bc} \times \frac{10}{18.03} + 13.56 \times \frac{5}{33.91} = 0$

$\Rightarrow F_{bc} = -7.21K$

$\therefore F_{bc} = 7.21K$ (C)

(Ans.)

compute the reaction and bar forces of the following space truss as shown in figure below:

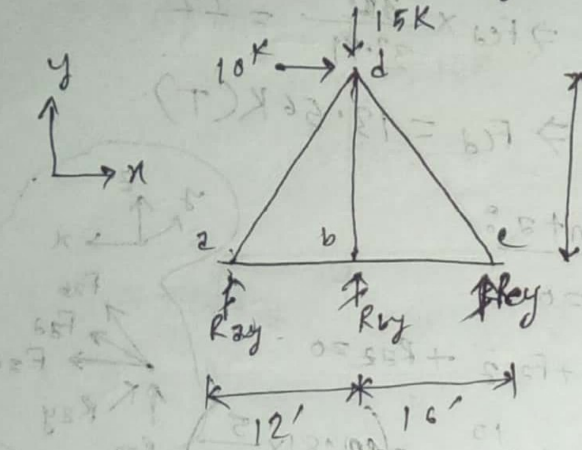


Fig (a)

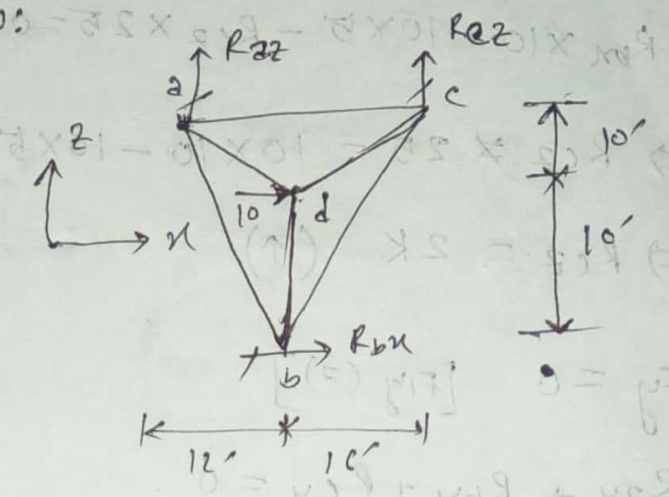


Fig (b)

Solution:

Member	Projection x	y	z	Length $\sqrt{x^2+y^2+z^2}$
ab	12	0	20	23.32
ac	28	0	0	28.00
ad	12	20	10	25.37
bc	16	0	20	25.6
bd	0	20	10	22.36
cd	16	20	10	27.50

$\Sigma F_x = 0$ [Fig. (b)]

$R_{bx} + 10 = 0$

$R_{bx} = -10\,K$

$\therefore R_{bx} = 10\,K\ (\leftarrow)$

$\Sigma M_{xa} = 0$ [Fig (b)]

$R_{by} \times 20 - 15 \times 10 = 0$

$\therefore R_{by} = 7.5\,K\ (\uparrow)$

$\Sigma M_{za} = 0$ [Fig (a)]

$R_{by} \times 12 + R_{cy} \times 28 - 10 \times 20 - 15 \times 12 = 0$

$\Rightarrow R_{cy} \times 28 = 380 - 7.5 \times 12$

$\Rightarrow R_{cy} = 10.36\,K\ (\uparrow)$

$\Sigma M_{ya} = 0$ [Fig (b)]

$R_{cz} \times 28 + R_{bx} \times 20 + 10 \times 10 = 0$

$\Rightarrow R_{cz} \times 28 = -100 + 200$

$\therefore R_{cz} = 3.57\,K\ (\uparrow)$

$\therefore \Sigma F_y = 0$ [Fig (a)]

$R_{ay} + R_{by} + R_{cy} = 15$

$\Rightarrow R_{ay} = 15 - 10.36 - 7.5$

$\Rightarrow R_{ay} = -2.86$ K

$\therefore R_{ay} = 2.86$ K (↓)

$\Sigma F_z = 0$ [Fig (b)]

$R_{az} + R_{cz} = 0$

$\Rightarrow R_{az} = -3.57$ K

$\therefore R_{az} = 3.57$ K (↓)

Now,

joint a: $\Sigma F_y = 0$

$F_{ady} + R_{ay} = 0$

$\Rightarrow F_{ad} \times \frac{20}{25.37} = +2.86$

$\Rightarrow F_{ad} = 3.63$ K (T)

joint b: $\Sigma F_y = 0$

$F_{bdy} + R_{by} = 0$

$\Rightarrow F_{bd} \times \frac{20}{22.36} = -7.5$

$\Rightarrow F_{bd} = -8.39$ K

$\therefore F_{bd} = 8.39$ K (C)

Joint C: $\Sigma F_y = 0$

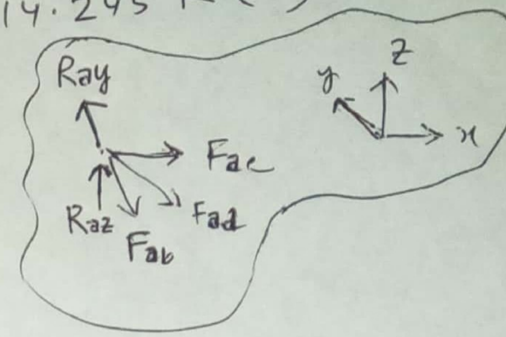
$F_{cdy} + R_{cy} = 0$

$F_{cd} \times \frac{20}{27.5} = -10.36$

$\Rightarrow F_{cd} = -14.245$ K

$\therefore F_{cd} = 14.245$ K (C)

At joint a:



$\Sigma F_z = 0$

$F_{abz} + F_{adz} - R_{az} = 0$

$\Rightarrow F_{ab} \times \frac{20}{23.32} = -3.57 - 3.63 \times \frac{10}{25.37}$

$\Rightarrow F_{ab} = -5.83$ K

$\therefore F_{ab} = 5.83$ K (C)

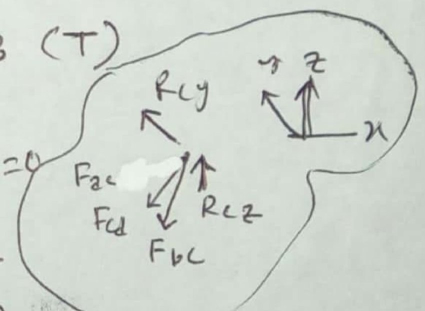
$\Sigma F_x = 0$

$F_{adx} + F_{ac} + F_{abx} = 0$

$\Rightarrow F_{ac} = 5.83 \times \frac{12}{23.32} - 3.63 \times \frac{12}{25.37}$

$\Rightarrow F_{ac} = 1.283$ (T)

At joint C: $\Sigma F_z = 0$



$R_{cz} - F_{cdz} - F_{bcz} = 0$

$\Rightarrow F_{bc} \times \frac{20}{25.61} = 3.57 + 14.245 \times \frac{10}{27.5}$

$\Rightarrow F_{bc} = +11.20$ K

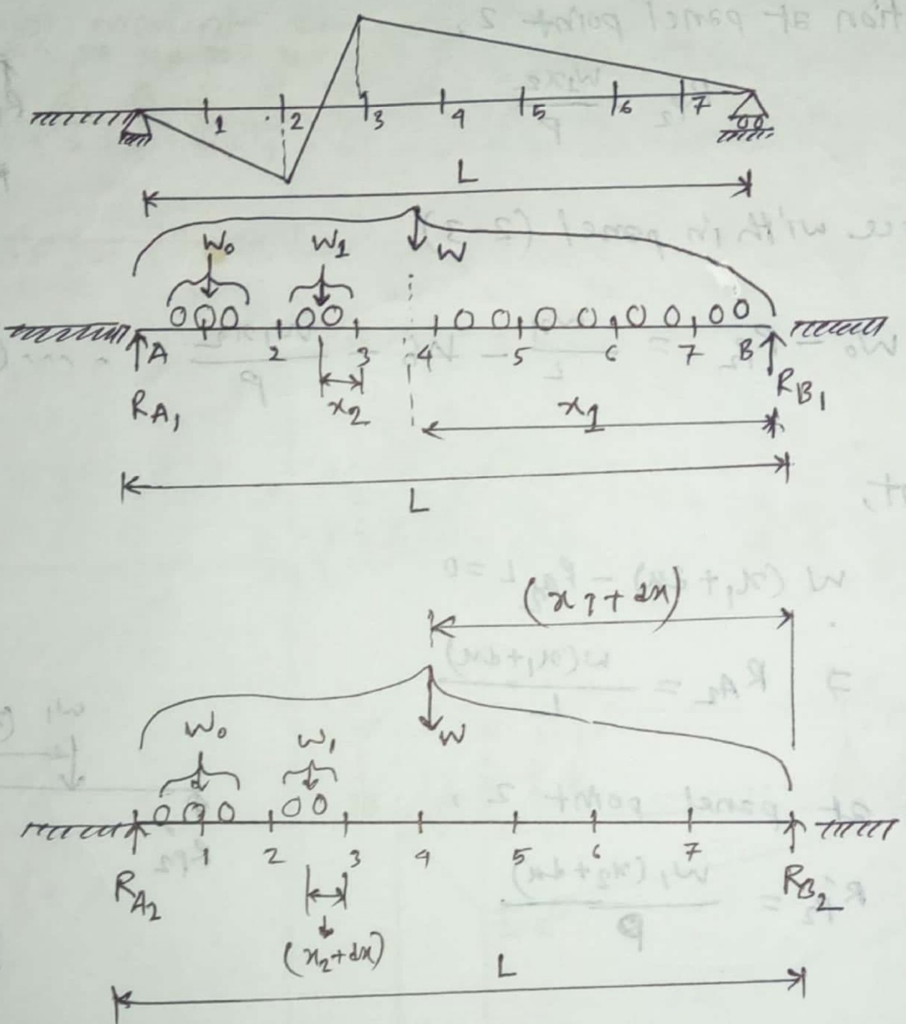
$\therefore F_{bc} = 11.20$ K (T)

(Ans)

16, 15, 08

Panel Load

Criteria for the maximum shear of a floor beam subjected to series of concentrated loads move right to left.



Here, W = the sum of all of loads on the span

W_0 = The sum of all of loads left of the panel.

W_1 = The sum of all of loads in the panel.

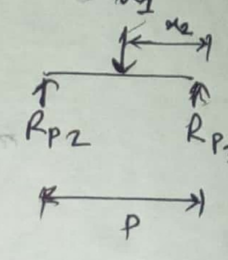
x_1 = The distance of center of gravity all the loads (W) to right support (B)

x_2 = The distance of center of gravity all the loads with in panel to right point of panel.

L = span length of beam.

Before movement,

$$\Sigma M_B = 0 \quad \therefore W x_1 - R_{A_1} L = 0 \Rightarrow R_{A_1} = \frac{W x_1}{L}$$



Also reaction at panel point 2,

$$R_{P_2} = \frac{W_1 x_2}{P}$$

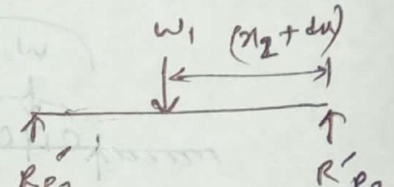
Now, shear force with in panel (2-3)

$$V_1 = R_{A_1} - W_0 - R_{P_2} = \frac{W x_1}{L} - W_0 - \frac{W_1 x_2}{P} \quad \text{---- (1)}$$

After movement,

$$\Sigma M_B = 0 \quad W(x_1 + dx) - R_{A_2} L = 0$$

$$\Rightarrow R_{A_2} = \frac{W(x_1 + dx)}{L}$$



Also, reaction at panel point 2,

$$R'_{P_2} = \frac{W_1 (x_2 + dx)}{P}$$

Now, shear force with in panel (2-3)

$$V_2 = R_{A_2} - W_0 - R'_{P_2} = \frac{W(x_1 + dx)}{L} - W_0 - \frac{W_1 (x_2 + dx)}{P} \quad \text{---- (2)}$$

Now, From (1) - (2) we obtain.

$$dV = V_2 - V_1 = \frac{W(x_1 + dx)}{L} - W_0 - \frac{W_1 (x_2 + dx)}{P} - \frac{W x}{L} + W_0 + \frac{W_1 x_2}{P}$$

$$\Rightarrow dV = \frac{W dx}{L} - \frac{W_1 dx}{P} = 0$$

$$\therefore \frac{dV}{dx} = \frac{W}{L} - \frac{W_1}{P}$$

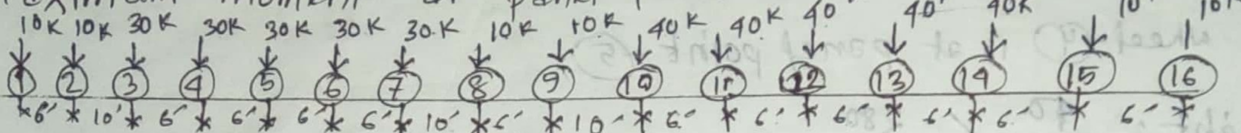
Now, For maximum, $\frac{dV}{dx} = 0$

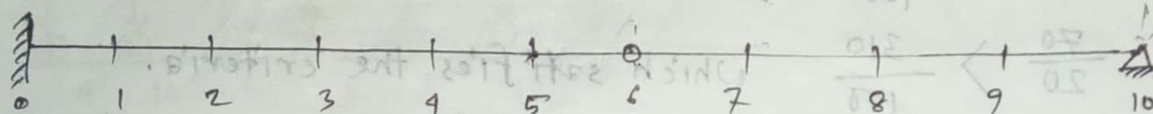
$$\frac{dV}{dx} = \frac{W}{L} - \frac{W_1}{P} = 0$$

$$\boxed{\therefore \frac{W}{L} = \frac{W_1}{P}}$$

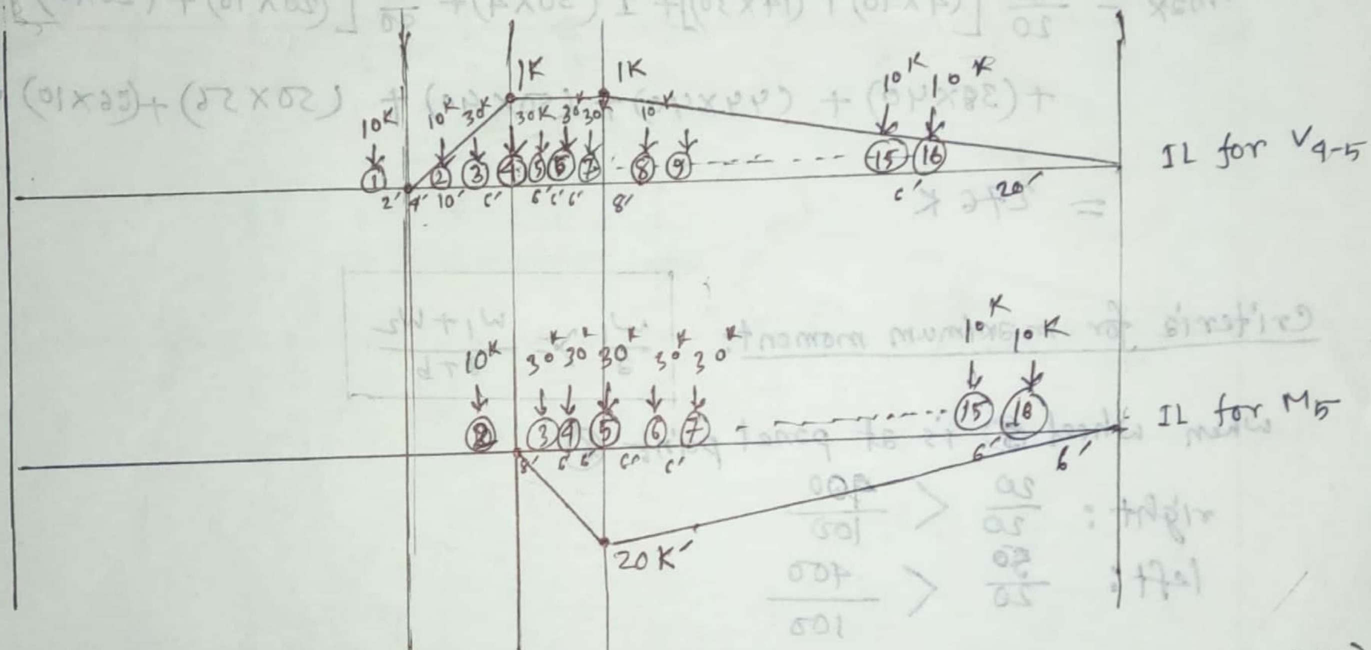
2006

using proper criteria to obtain the position of moving loads and hence calculate the maximum shear in the panel 4-5 and the maximum moment at panel point 5.





10 @ 20' = 200'



Here,

Criteria for maximum shear: $\boxed{\dfrac{W_1}{a} = \dfrac{W_1 + W_2}{a+b}}$ (IL is similiar to Moment IL)

When wheel ① is at panel point ⑤

Right: $\dfrac{0}{20} < \dfrac{350}{100}$

left: $\dfrac{10}{20} < \dfrac{350}{100}$

when wheel ② is at panel point ⑤

Right: $\dfrac{10}{20} < \dfrac{340}{100}$

Left: $\dfrac{20}{20} < \dfrac{350}{100}$

when wheel-③ is at panel point-⑤,

right: $\frac{20}{20} < \frac{290}{100}$

left: $\frac{50}{20} < \frac{320}{100}$

when wheel-④ is at panel point-⑤,

right: $\frac{40}{20} < \frac{280}{100}$

left: $\frac{70}{20} > \frac{310}{100}$ which satifies the criteria.

$$V_{max} = \frac{1}{20}\left[(4\times10)+(14\times30)\right] + 1(30\times4) + \frac{1}{80}\left[(20\times10)+(26\times10)+(32\times40)\right.$$
$$\left. +(38\times40)+(44\times40)+(50\times40)+(50\times56)+(66\times10)+(72\times10)\right]$$
$$= 276\text{ K}$$

Criteria for maximum moment: $\boxed{\frac{W_1}{a} = \frac{W_1+W_2}{a+b}}$

when wheel ③ is at panel point ⑥.

right: $\frac{20}{20} < \frac{390}{100}$

left: $\frac{50}{20} < \frac{400}{100}$

when wheel ④ is at panel point ⑥

right: $\frac{40}{20} < \frac{390}{100}$

left: $\frac{70}{20} < \frac{400}{100}$

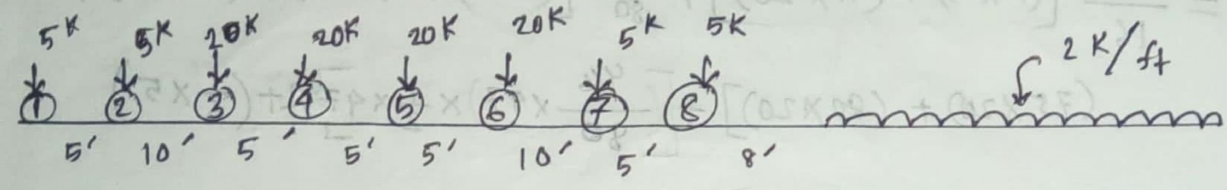
when wheel ⑤ is at panel point ⑥

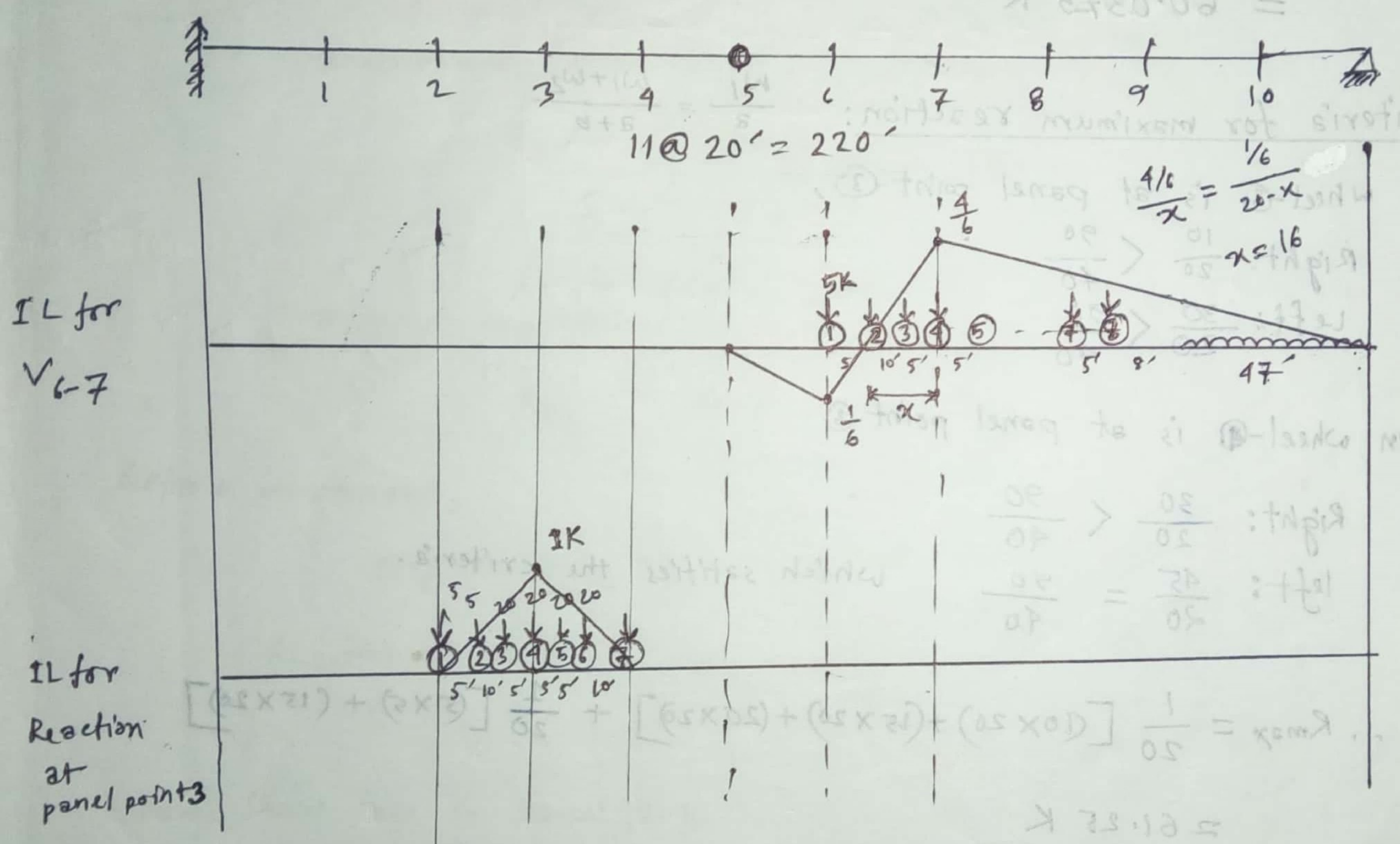
right: $\frac{60}{20} < \frac{390}{100}$

left: $\frac{90}{20} > \frac{390}{100}$ which satifies the criteria.

$$M_{max} = \frac{20}{20}\left[(8\times30)+(14\times30)+(20\times30)\right] + \frac{20}{80}\left[(6\times10)+(12\times10)+\right.$$
$$(18\times40)+(24\times40)+(30\times40)+(36\times40)+(42\times40)+$$
$$\left.(52\times10)+(58\times10)+(68\times30)+(74\times30)\right] = 4145\text{ K}'$$

Problem: using proper criteria to determine the maximum shear in panel 6–7 and maximum floor beam reaction at panel point 3 due to the series of moving concentrated loads shown in figure below:





criteria for maximum shear: $\boxed{\dfrac{W_1}{P} = \dfrac{W}{L}}$

Now,

when wheel 3 at panel point 7,

right: $\frac{10}{20} < \frac{184}{120}$ → $(5\times2+20\times4+5\times2+2\times42)$

Left: $\frac{30}{20} < \frac{184}{120}$

when wheel-4 is at panel point 7

right: $\frac{30}{20} < \frac{194}{120}$

Left: $\frac{45}{20} > \frac{189}{120}$ which satisfies the criteria.

$$V_{max} = \frac{4/6}{16}\left[(1\times5)+(11\times20)\right] + \frac{4/6}{80}\left[(55\times5)+(60\times5)+(70\times20)+(75\times20)+(80\times20)\right] + \left[\left(\frac{4/6}{80}\times47\right)\times\frac{1}{2}\times47\right]\times2 - \left(\frac{1}{6}\times5\right)$$

$= 69.242$ K

criteria for maximum reaction: $\frac{W_1}{a} = \frac{W_1+W_2}{a+b}$

when wheel-3 is at panel point-3,

Right: $\frac{10}{20} < \frac{90}{40}$

Left: $\frac{30}{20} < \frac{90}{40}$

when wheel-4 is at panel point-3

Right: $\frac{30}{20} < \frac{90}{40}$

left: $\frac{45}{20} = \frac{90}{40}$ which satisfies the criteria.

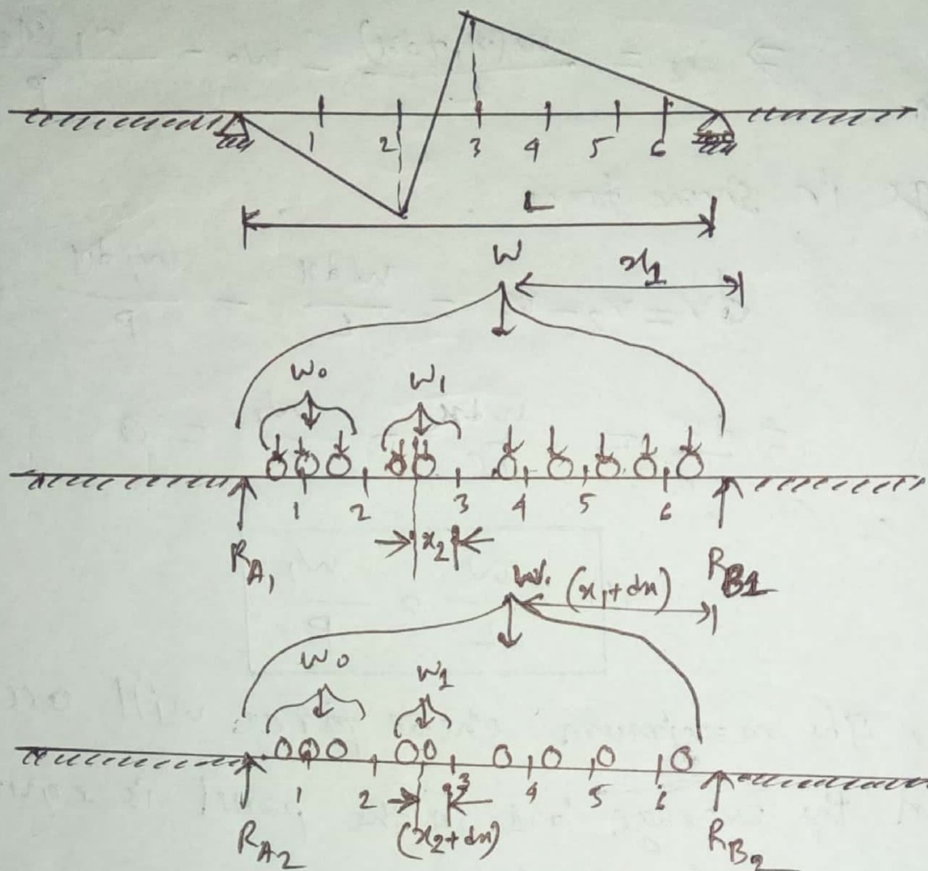
$$\therefore R_{max} = \frac{1}{20}\left[(10\times20)+(15\times20)+(20\times20)\right] + \frac{1}{20}\left[(5\times5)+(15\times20)\right]$$

$= 61.25$ K

(Ans.)

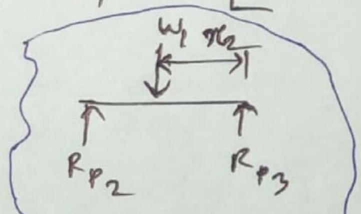
16.15.08

Criteria for the $\boxed{\text{maximum shear}}$ of a floor beam subjected to series of concentrated loads move from right to left.



Before movement,

$$\Sigma M_B = 0 \Rightarrow R_{A_1} \times L - W x_1 = 0 \Rightarrow R_{A_1} = \frac{W x_1}{L}$$



Reaction at pand 2, $R_{P_2} = \dfrac{W_1 x_2}{P}$

Now, Shear force in panel (2-3),

$$V_1 = R_{A_1} - W_0 - R_{P_2} = \frac{Wx}{L} - W_0 - \frac{W_1 x_2}{P} \quad \cdots\cdots (1)$$

After movement,

$$\Sigma M_B = 0 \Rightarrow R_{A_2} = \frac{W(x_1+dx)}{L}$$

and, $R_{P_2}' = \dfrac{W_1(x_2+dx)}{L}$

Now, shear force in panel (2-3),

$$V_2 = R_{A_2} - W_0 - R'_{P_2}$$

$$\Rightarrow V_2 = \frac{W(x_1+dx)}{L} - W_0 - \frac{W_1(x_2+dx)}{P} \quad \text{.......} (\text{ii})$$

From (i) & (ii)

$\therefore$ The change in shear force,

$$dV = V_2 - V_1 = \frac{W\,dx}{L} - \frac{W_1\,dx}{P}$$

$$\Rightarrow \frac{dV}{dx} = \frac{W}{L} - \frac{W_1}{P} = 0$$

$$\therefore \boxed{\frac{W}{L} = \frac{W_1}{P}}$$

There fore, The maximum shear force will occur within panel when the average load in the panel is equal to average load in the span of the beam.

FARHAD
#1500045

Three Hinged Arch

Arch: An arch may be defined as mechanical arragement of wedge shaped blocks of stones of bricks mutually supporting each other and supported at the end by piers, or abutments. In generally, Arch is defined as a curved beam.

Three hinged arch: An arch hinged at both supports and having third hinge any where in the rib is called a three hinged Arch. A three hinge arce is a determinant structure.

Beam: Beam is a flexural member; which is long in transverse direction.

Advantages of Arch:

1. Architecturally looks good.
2. Arch can take load 10 times more than straight beam.

Reaction of the arch in supports:

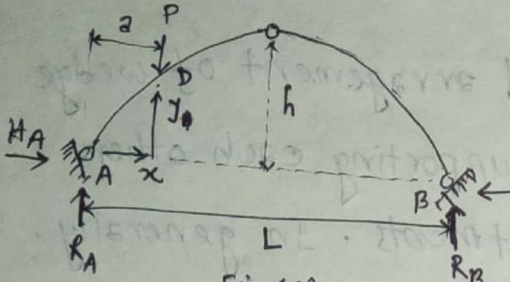


Fig. (a)

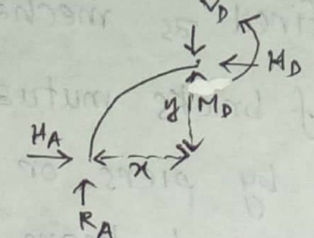


Fig. (b)

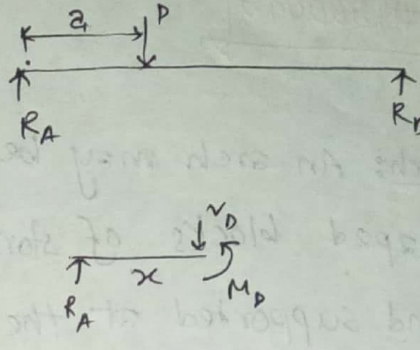


Fig. (c)

$\Sigma M_A = 0$

$P.a - R_B \cdot L = 0$

$\Rightarrow \boxed{R_B = \frac{Pa}{L}}$

$\Sigma F_y = 0$

$R_A - P + R_B = 0$

$\Rightarrow R_A = P - R_B = P - \frac{Pa}{L}$

$\therefore \boxed{R_A = \frac{P(L-a)}{L}}$

From fig. (b),

$\Sigma M_D = R_A x - H_A y_D$ ---①

$\Sigma F_x = 0$

$\Rightarrow H_A - H_B = 0$

$\therefore H_A = H_B = H$

But, For simply supported beam.

From fig. (c)

$\Sigma M_D = R_A x = M_s$

So, From ①

$\boxed{\Sigma M_D = M_s - H y_D}$

Here,

Reactions of an arch and Reactions of a beam are equal.

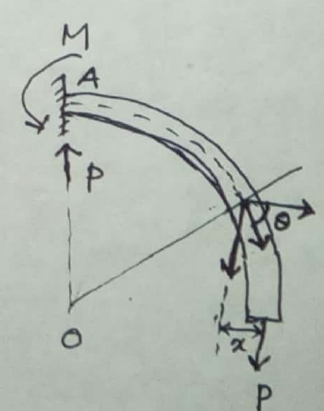
Parabolic equation of the arch, $y = \frac{4hx}{L^2}(L-x)$

Now, $M_H = Hy$ [Let)

Hence, $M_H = H\left[\frac{4hx}{L^2}(L-x)\right]$

and, $\tan\theta = \frac{dy}{dx}$

Shear force and Normal force on Arch:



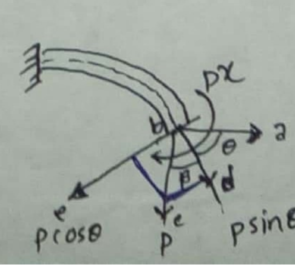
Consider,

at section point, Bending moment, $M_x = -Px$

Let, shear force $= F_x$ and Normal force $= N_x$

$\therefore F_x = P\cos\theta$

And, $N_x = -P\sin\theta$

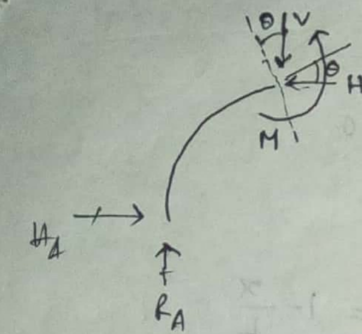


$\angle abc = 90^\circ = \theta + \beta$

$\angle abd = \theta$; $\angle ebd = 90^\circ$

$\angle bdc = \beta$ $\therefore \angle ebc = \theta$

For three hinge arch,



$\tan\theta = \frac{y}{x}$

$\therefore$ Shear force, $F_x = V\cos\theta - H\sin\theta$

and, Normal force, $N = V\sin\theta + H\cos\theta$

Influence Line:

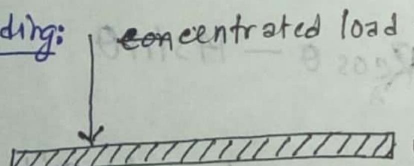
An influence line is a diagram showing the variance of shear, stress, moment in a member, reaction or other direct function due to a unit load moving across the structure.

construction of Influence line:

An influence line is constructed by plotting directly under the point where unit load is placed and ordinate, the height of which represents to some scale the value of the perticular function being studied, when the load is in that position.

Different types of loading:

H_{20} loading: concentrated load – Moment (18000 lb-ft), Shear (26000 lb)



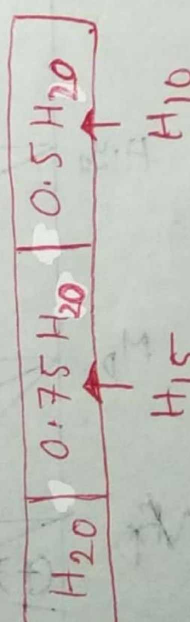
uniform load – (640 lb/ft)

H_{15} loading: * concentrated load – Moment (13500 lb-ft), Shear (19500 lb)

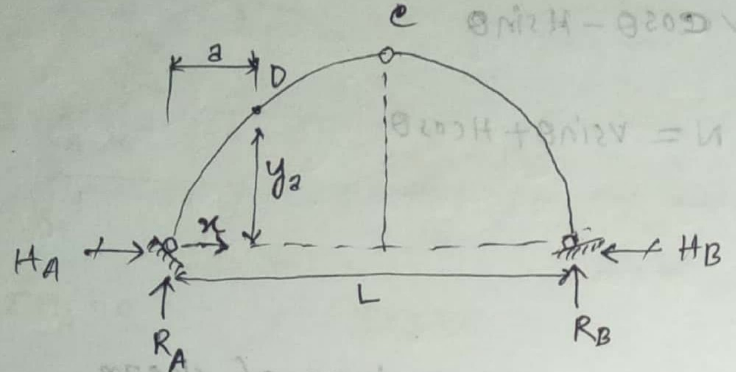
* uniform load – (480 lb/ft)

H_{10} loading: * concentrated load – Moment (9000 lb-ft), Shear (13000 lb)

* uniform load – (320 lb/ft)



IL diagram of three hinged arch:



When 1K at A:

$R_A = 1K$; $R_B = 0$

When 1K at D:

$$R_A = \frac{1(L-x)}{L} = 1 - \frac{x}{L}$$

$$\therefore R_B = \frac{x}{L}$$

$$\Sigma M_C = 0$$

$$R_B \times \frac{L}{2} - H \cdot h = 0$$

$$H = \frac{R_B L}{2h} = \frac{x}{2h}$$

At maximum, $x = \frac{L}{2}$

$$\therefore H = \frac{L}{4h}$$

∴ Maximum Bending moment

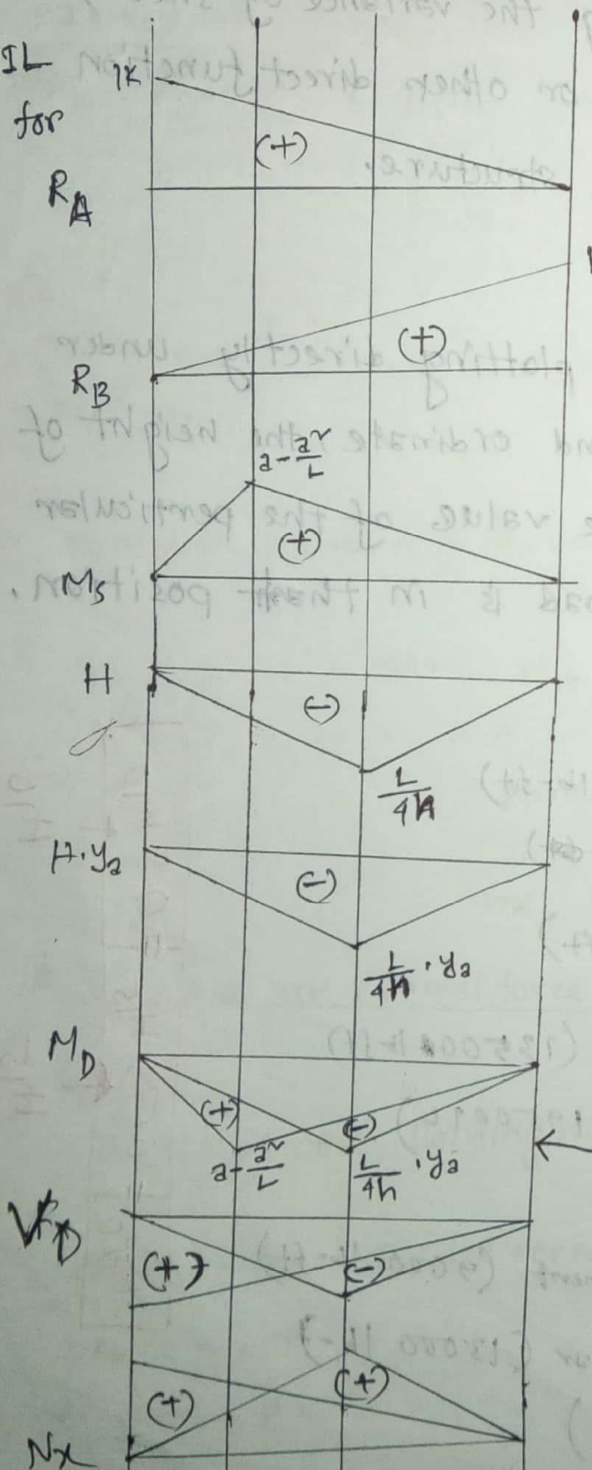
$$M = M_s - Hy_a = R_A x - Hy_a$$

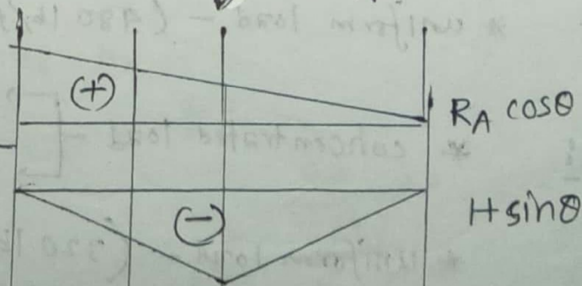
When the load is at right side of the section,

$$V_D = R_A \cos\theta - H\sin\theta$$

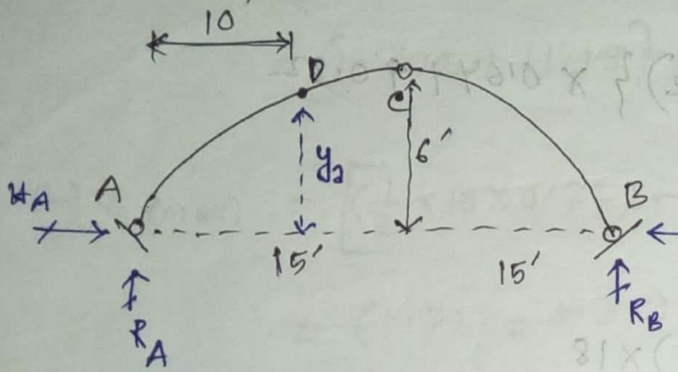
When the load is left side of the section,

$$V_D = (R_A - 1)\cos\theta - H\sin\theta$$





Problem: Draw IL for three hinged arch – (i) Horizontal thrust, H (ii) Moment at D, M_D (iii) Shear at D, V_D (iv) Normal thrust, N_D. Also compute shear force, bending moment, normal thrust due to H_{20} loading

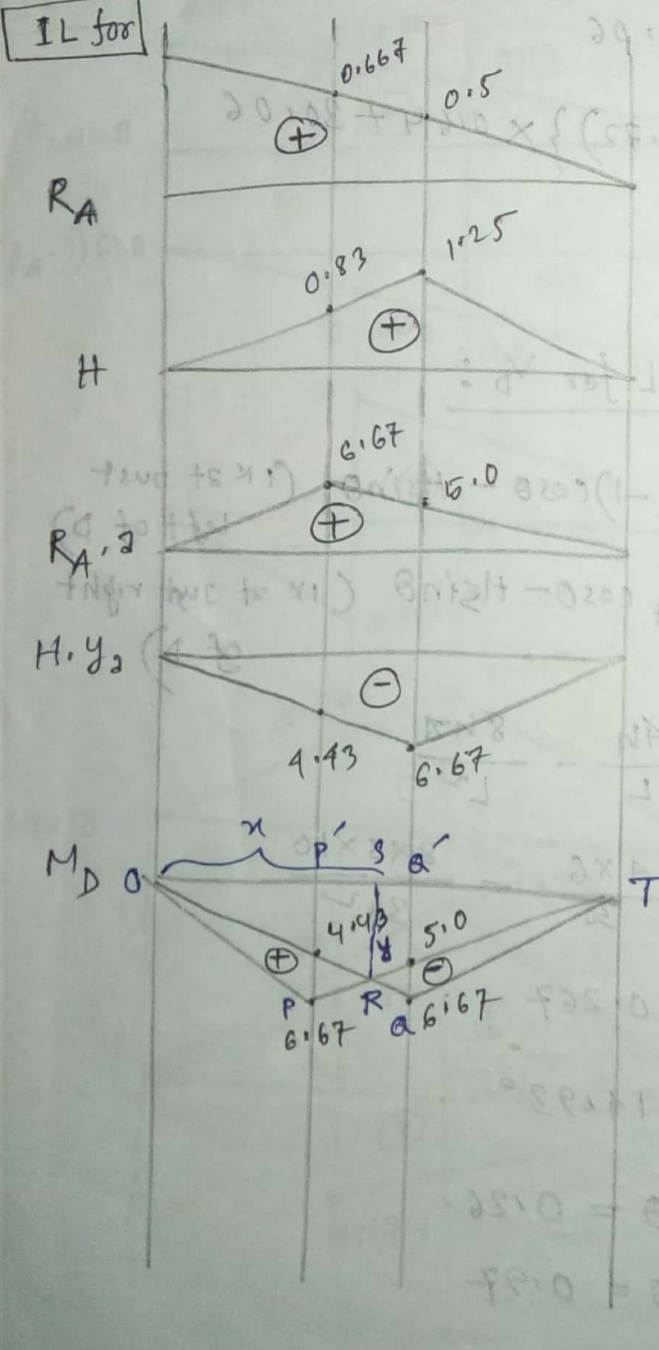


Solution: Given that,

$L = 30'$, $a = 10'$, $h = 6'$

$$y_a = \frac{4ha}{L^2}(L-a)$$

$$= \frac{4\times 6\times 10}{30^2}(30-10) = 5.33\ ft$$



IL for H:

When,

1K at A; $R_A = 1$; $H = 0$

1K at D; $R_A = 1 - \frac{x}{L} = \left(1 - \frac{10}{30}\right) = 0.67$

$$H = \frac{x}{2h} = \frac{10}{2\times 6} = 0.83$$

1K at C; $R_A = \left(1 - \frac{15}{30}\right) = \frac{1}{2}$

$$H = \frac{15}{2\times 6} = 1.25$$

1K at B: $R_A = 0$; $H = 0$

IL for M_D:

$$M_D = R_A a - H y_a$$

From similar triangle,

$\triangle OQQ'$ & $\triangle OSR \Rightarrow \frac{y}{x} = \frac{6.67}{15}$

$\Rightarrow y = 0.4447x$ ---- (i)

And, $\triangle TPP'$ & $\triangle TSR \Rightarrow \frac{y}{30-x} = \frac{6.67}{20}$

$\Rightarrow y = 0.3335(30-x)$ ---- (ii)

From (i) & (ii), $y = 5.72'$ & $x = 12.86'$

Now,

$(+) M_{max} = \triangle OPR \times 0.64 + (6.67 - 4.43) \times 18$ [Due to H20 loading]

$= (\triangle OPT - \triangle ORT) \times 0.64 + 40.32$

$= \{(\frac{1}{2} \times 30 \times 6.67) - (\frac{1}{2} \times 30 \times 5.72)\} \times 0.64 + 40.32$

$= 49.44\ K'$

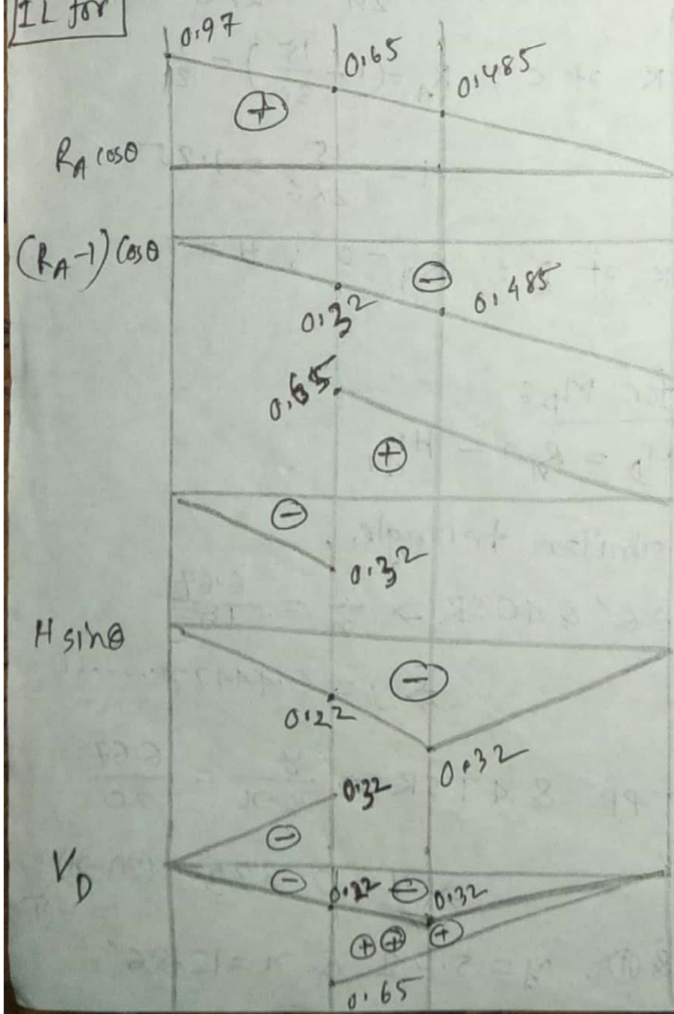
$(-) M_{max} = \triangle TQR \times 0.64 + (6.67 - 5) \times 18$

$= (\triangle OQT - \triangle ORT) \times 0.64 + 30.06$

$= \{(\frac{1}{2} \times 30 \times 6.67) - (\frac{1}{2} \times 30 \times 5.72)\} \times 0.64 + 30.06$

$= 39.18\ K'$

IL for



IL for V_D:

$V_D = (R_A - 1)\cos\theta - H\sin\theta$ (1K at just left of D)

$V_D = R_A \cos\theta - H\sin\theta$ (1K at just right of D)

$\tan\theta = \frac{4h}{L} - \frac{8hx}{L^2}$

$\Rightarrow \tan\theta = \frac{4 \times 6}{30} - \frac{8 \times 6 \times 10}{30^2}$

$\Rightarrow \tan\theta = 0.267$

$\therefore \theta = 14.93°$

$\therefore \sin\theta = 0.26$

$\cos\theta = 0.97$

$$(+)\,V_{D(max)} = \left[\left(\tfrac{1}{2}\times 20\times 0.65\right) - \left(\tfrac{1}{2}\times 30\times 0.32\right) + \left(\tfrac{1}{2}\times 10\times 0.22\right)\right]\times 0.64$$

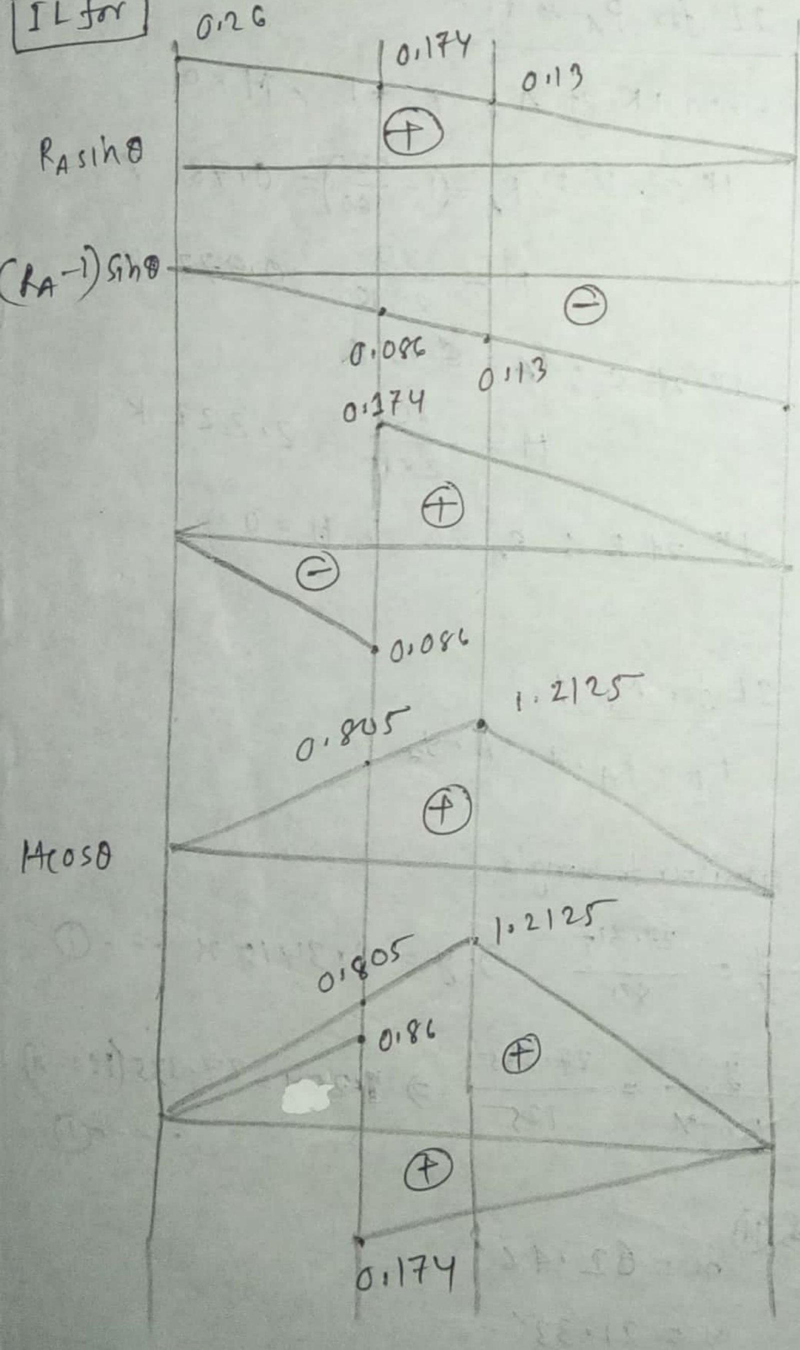
$$+\ (0.65 - 0.22)\times 26$$

$$= (1.792 + 11.18) = 12.972\ K$$

$$(-)\,V_{D\,(max)} = \left[\left(\tfrac{1}{2}\times 10\times 0.32\right) + \left(\tfrac{1}{2}\times 10\times 0.22\right)\right]\times 0.64 + (26\times 0.32)$$

$$= (1.728 + 8.32) = 10.048\ K$$

$\boxed{\text{IL for}}$



IL for N_D:

$N_D = (R_A - 1)\sin\theta + H\cos\theta$ (Left)

$N_D = R_A \sin\theta + H\cos\theta$ (Right)

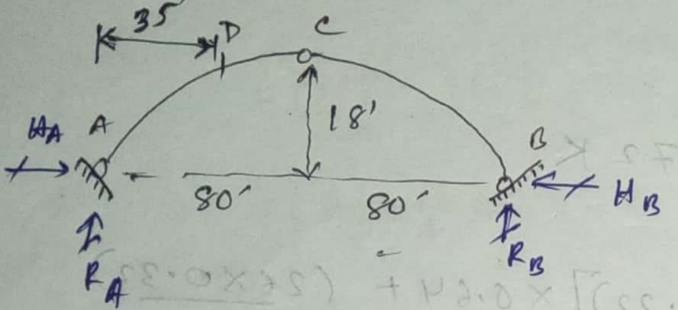
$$(+)\,N_{D\,(max)} = \left[\left(\tfrac{1}{2}\times 30\times 1.2125\right) + \left(\tfrac{1}{2}\times 20\times 0.174\right) - \left(\tfrac{1}{2}\times 0.086\times 10\right)\right]\times 0.64$$

$$+\ 26\times 1.2125$$

$$= 12.4784 + 31.525$$

$$= 44K$$

Problem: (Due to H_{20} loading)



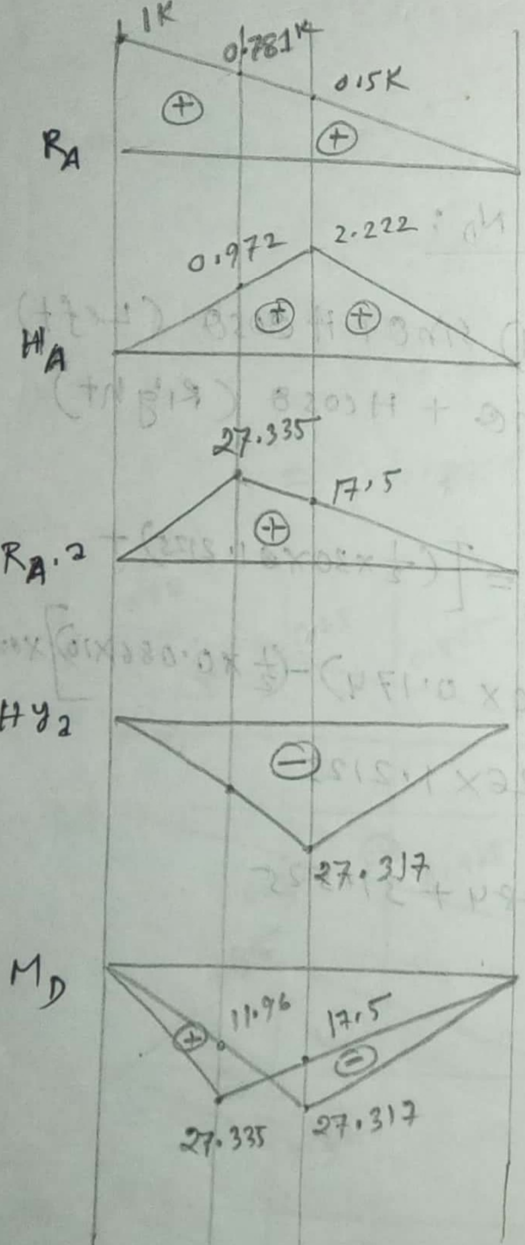
Given that,

$L = 160'$, $h = 18'$, $a = 35'$

$$\therefore y_a = \frac{4ha}{L^2}(L-a)$$

$$= \left(\frac{4\times 18\times 35}{160^2}\right)(160-35)$$

$$= 12.305'$$



IL for R_A & H:

when 1K at A: $R_A = 1$, $H = 0$

1K at D: $R_A = \left(1-\frac{35}{160}\right) = 0.781\ K$

$H = \frac{35}{2\times 18} = 0.972\ K$

1K at C: $R_A = 0.5\ K$

$H = \frac{80}{2\times 18} = 2.222\ K$

1K at B: $R_A = 0$, $H = 0$

IL for M_D:

$M_D = R_A . a - H . y_a$

From similar triangle,

$$\frac{y}{x} = \frac{27.317}{80} \Rightarrow y = 0.3415x \quad \text{--- (i)}$$

And, $$\frac{y}{160-x} = \frac{27.335}{125} \Rightarrow 125y = 27.335(160-x) \quad \text{--- (ii)}$$

From (i) & (ii) $x = 62.46'$

$y = 21.33'$

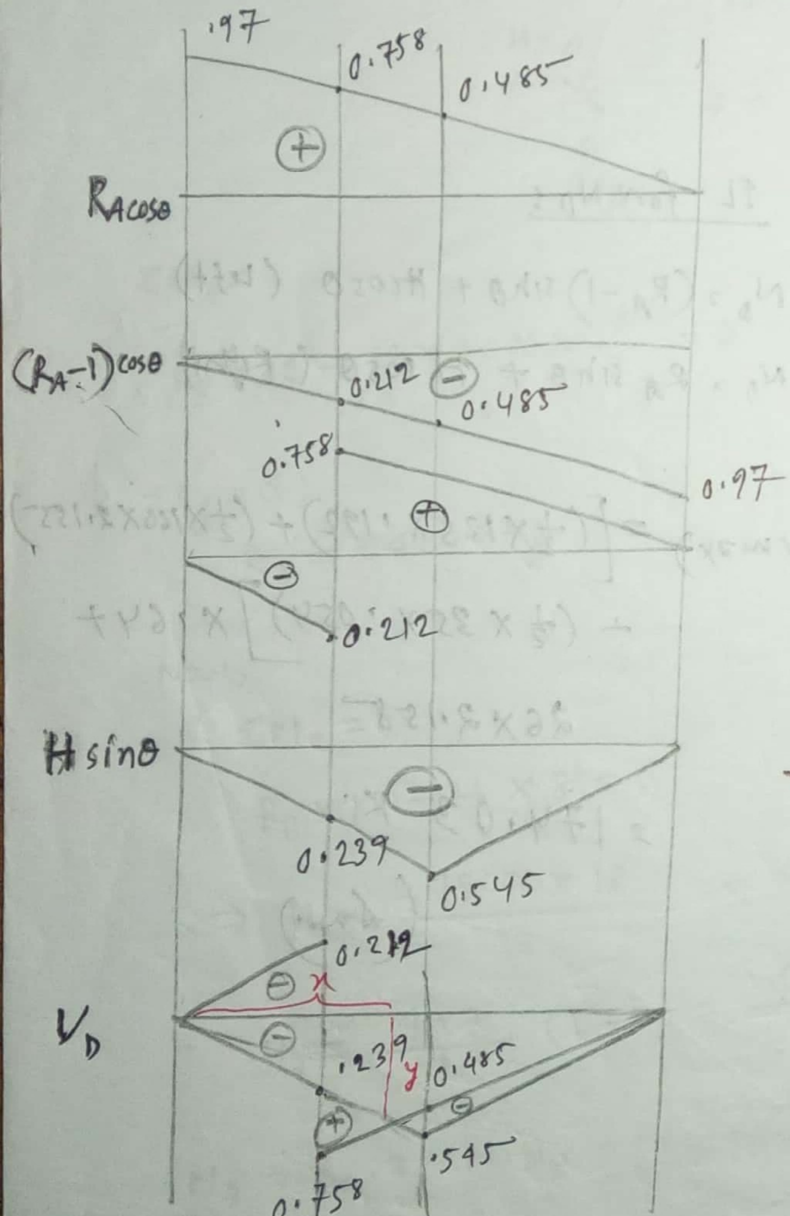
Now,

$$(+)\ M_{max} = \left[\left(\tfrac{1}{2}\times 160\times 27.335\right) - \left(\tfrac{1}{2}\times 160\times 21.33\right)\right]\times 0.64 + 18\,(27.335 - 11.96)$$

$$= 584.206\ K'$$

$$(-)\ M_{max} = \left[\left(\tfrac{1}{2}\times 160\times 27.317\right) - \left(\tfrac{1}{2}\times 160\times 21.33\right)\right]\times 0.64 + 18\,(27.317 - 17.5)$$

$$= 483.24\ K'$$



IL for V_D :

$V_D = (R_A - 1)\cos\theta - H\sin\theta$ (Left)

$V_D = R_A\cos\theta - H\sin\theta$ (Right)

$$\tan\theta = \frac{4\times 18}{160} - \frac{8\times 18\times 35}{160^2}$$

$$\theta = 14.20$$

$$\sin\theta = 0.2454$$

$$\cos\theta = 0.97$$

~~(+) $V_{D(max)}$ =~~

From similar triangle,

$$\frac{y}{160-x} = \frac{0.758}{125}$$

$$\Rightarrow 125y = (160-x)\,0.758 \quad \ldots (i)$$

And, $$\frac{y}{x} = \frac{0.545}{80}$$

$$\Rightarrow y = \frac{0.545}{80}x \quad \ldots (ii)$$

From (i) & (ii), $y = 0.51'$

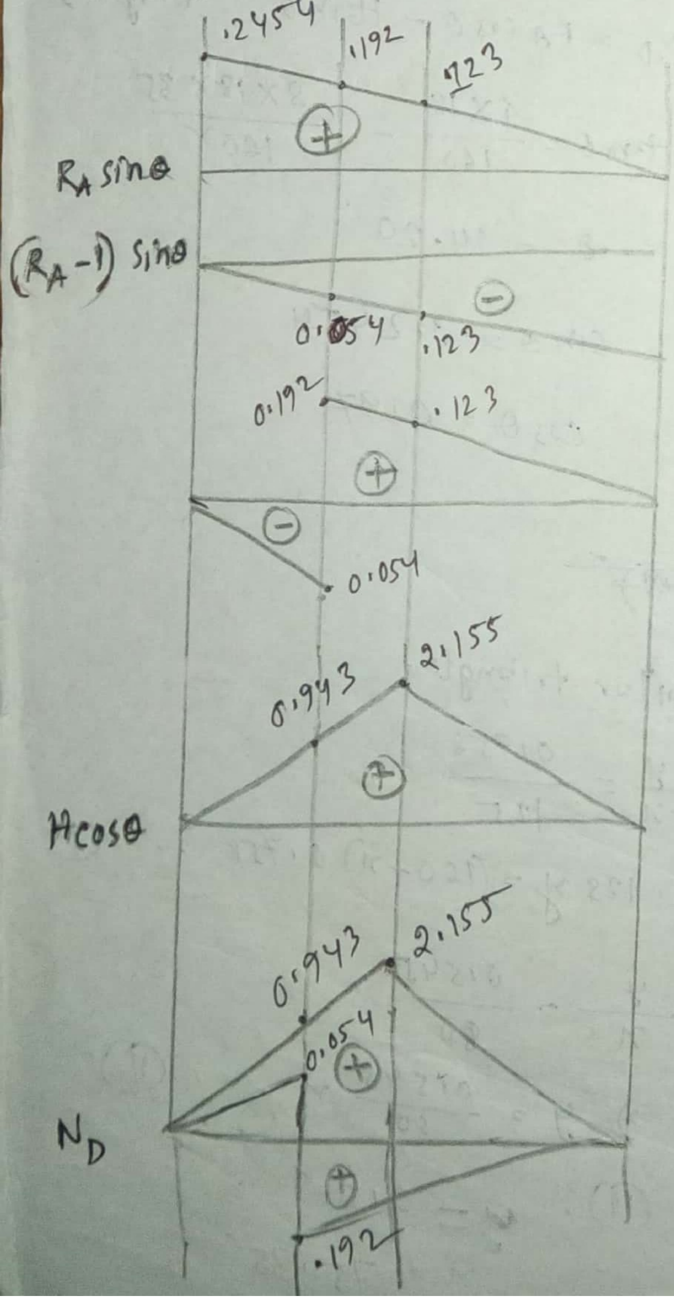
$x = 75.35'$

$$(+)\,V_D\,(max) = \left[\left(\tfrac{1}{2}\times 145\times .758\right) - \left(\tfrac{1}{2}\times 160\times 0.51\right) + \left(\tfrac{1}{2}\times 35\times .239\right)\right]\times 0.64 + 26\,(.76 - 0.239)$$

$$= 20.4308\ K$$

$$(-)\,V_D\,(max) = \left[\left(\tfrac{1}{2}\times 35\times .212\right) + \left(\tfrac{1}{2}\times 35\times .239\right) + \left(\tfrac{1}{2}\times 160\times .595\right) - \left(\tfrac{1}{2}\times 160\times 0.51\right)\right]\times 0.64 + 26\times .239$$

$$= 13.0572\ K$$



IL for N_D:

$N_D = (R_A - 1)\sin\theta + H\cos\theta$ (Left)

$N_D = R_A \sin\theta + H\cos\theta$ (Right)

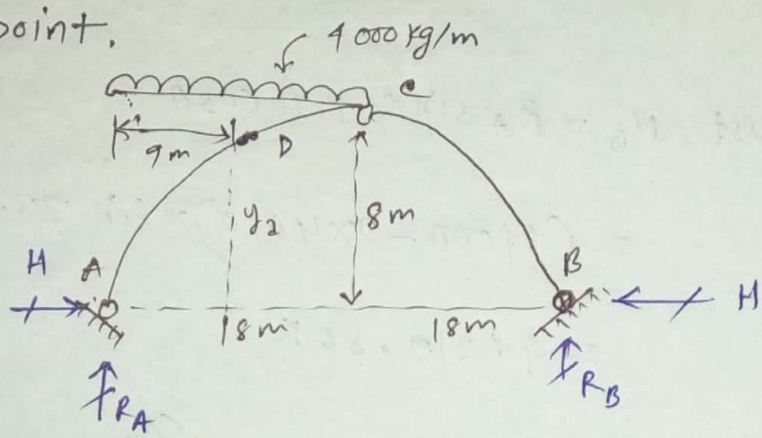
$$(+)\,N_D\,(max) = \left[\left(\tfrac{1}{2}\times 125\times .192\right) + \left(\tfrac{1}{2}\times 160\times 2.155\right) - \left(\tfrac{1}{2}\times 35\times .054\right)\right]\times .64 + 26\times 2.155$$

$$= 174.03\ K$$

(Ans.)

Problem: A uniformly distributed load of 4000 kg/m covers left hand half of the span of a three hinged parabolic arch span 36m and center rise 8m. Determine the horizontal thrust, Also find Bending moment and shear force and normal thrust at the loaded quarter point.

Solution:

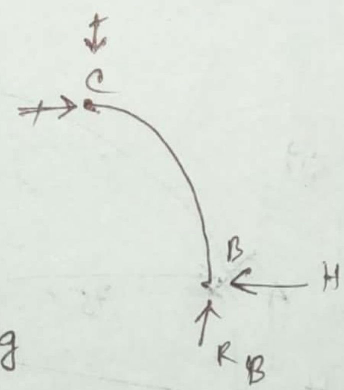


$\Sigma M_A = 0$

$R_B \times 36 - 18 \times 4000 \times \frac{18}{2} = 0 \Rightarrow R_B = 18000$ kg

$\Sigma F_y = 0,$

$R_A = 54000$ kg



Now,

$\Sigma M_C = 0$

$R_B \times 18 - H \times 8 = 0$

$\Rightarrow H = \frac{18000 \times 18}{8} = 40,500$ kg

$y_a = \frac{4ha}{L^2}(L-a) = \frac{4 \times 8 \times 9}{36^2}(36-9) = 6$ m

$M_D = R_A \cdot a - H \cdot y_a = (54,000 \times 9 - 40,500 \times 6) = 81000$ kg

$\tan\theta = \frac{4h}{L} - \frac{8ha}{L^2} = \frac{4 \times 8}{36} - \frac{8 \times 8 \times 9}{36^2} = 0.44$

$\therefore \theta = 23.96°$

$\therefore$ Shear force $V_D = R_A \cos\theta - H\sin\theta$

$$= (54000 - 9\times4000)\times\cos 23.96 - 40500 \sin 23.96$$

$$= 1.93\ K.$$

Normal thrust, $N_D = R_A \sin\theta + H\cos\theta$

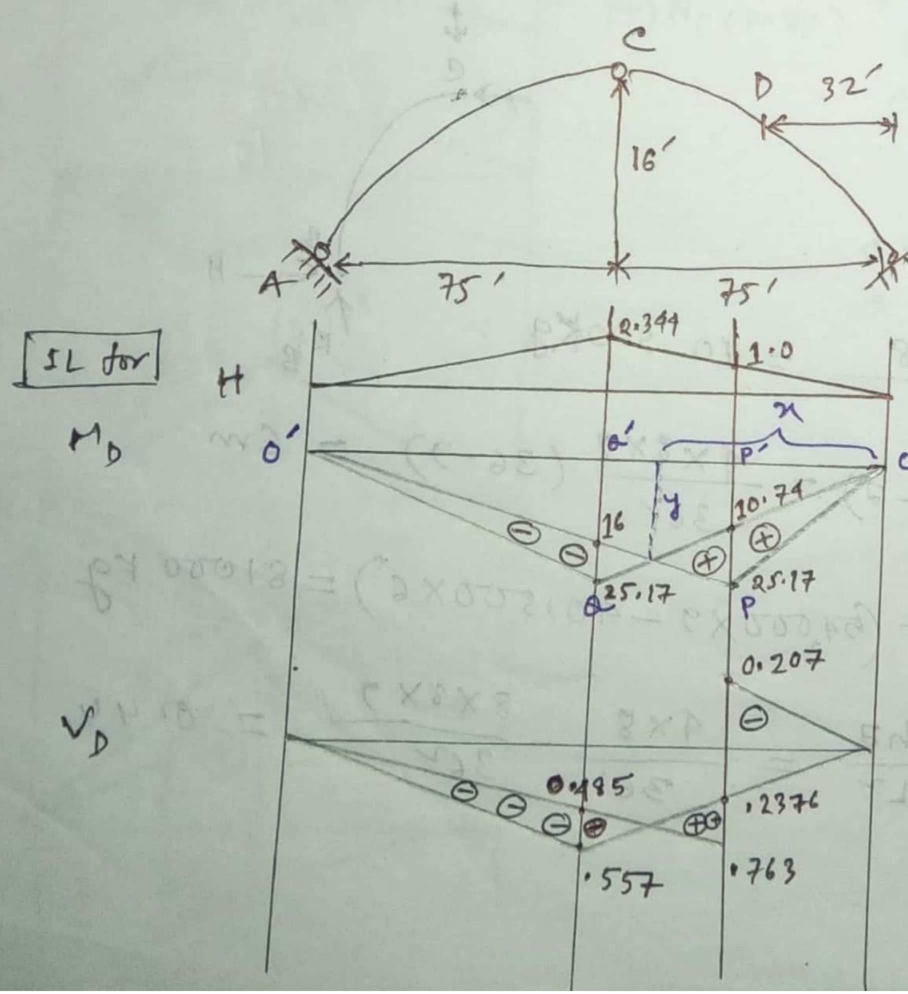
$$= (54000 - 9\times4000)\times \sin 23.96 + 40500 \times \cos 23.96$$

$$= 44319.86\ K.$$

(Ans.)

$\boxed{\text{class test - 15 series}}$

Draw influence line for bending moment, shear force, and Normal thrust at a section D of the following three hinged parabolic arch. Also obtain the maximum moment at section D for H15 loading.



Given that,

$L = 150$

$h = 16$

$a = 32'$

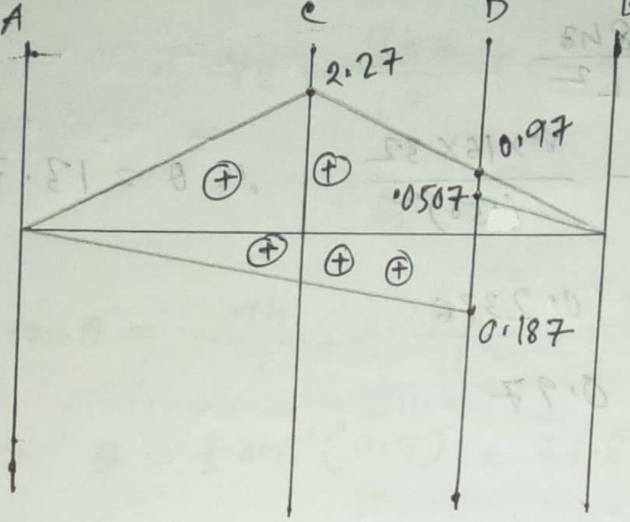
$$\therefore y_a = \frac{4ha}{L^2}(L-a)$$

$$= \frac{4\times16\times32}{150^2}\times(150-32)$$

$$\therefore y_a = 10.74'$$

IL for

N_D



IL for M_D:

$M_D = R_B \cdot a - H \cdot y_a$

From, $\triangle OPP'$, $\frac{y}{150-x} = \frac{25.17}{118}$ --- ①

From $\triangle OQQ'$, $\frac{y}{x} = \frac{25.17}{75}$ --- ⑪

From ① & ⑪, $x = 58.29'$ & $y = 19.56'$

$(+) M_{max} = \left[\frac{1}{2} \times 150 \times (25.17 - 19.56)\right] \times .48 + (25.17 - 10.74) \times 13.5$

$= 396.765$ K ft

$(-) M_{max} = \left[\frac{1}{2} \times 150 \times (2517 - 19.56)\right] \times .48 + (25.17 - 16) \times 13.5$

$= 325.755$ K ft

IL for V_D: For Right, $V_D = (R_B - 1)\cos\theta - H\sin\theta$

For left $V_D = R_B \cos\theta - H\sin\theta$

$$\tan\theta = \frac{4h}{L} - \frac{8ha}{L^2}$$

$$\tan\theta = \frac{4\times 16}{150} - \frac{8\times 16\times 32}{(150)^2} \qquad \therefore \theta = 13.746^\circ$$

$$\therefore \sin\theta = 0.2376$$

and $\cos\theta = 0.97$

IL for N_D:

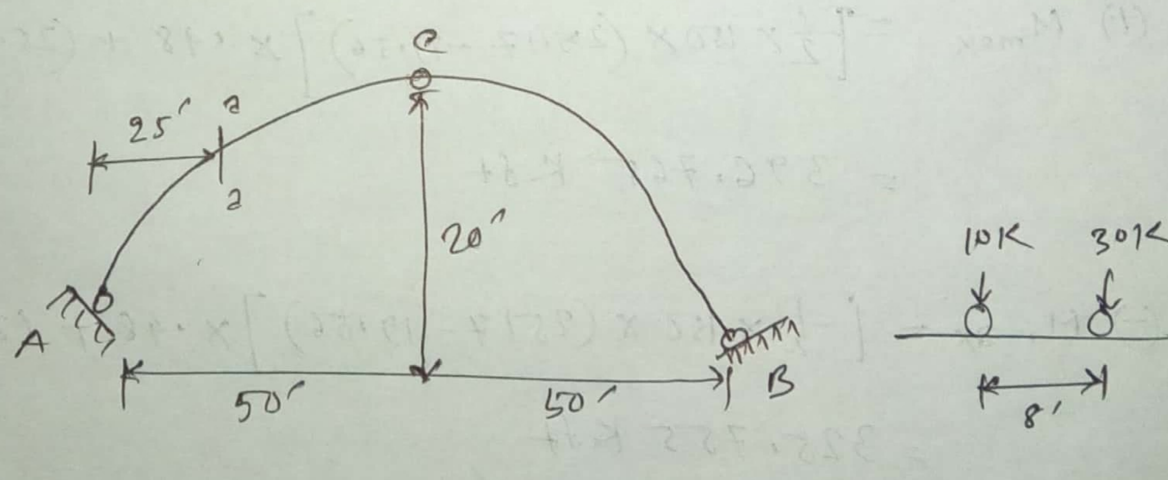
$$N_D = (R_B - 1)\sin\theta + H\cos\theta \quad \text{(for Right)}$$

$$N_D = R_B \sin\theta + H\cos\theta \quad \text{(for left)}$$

2010, 08

A three hinged parabolic arch shown in figure below a span of 100' and a central rise of 20'. Draw influence line for Shear force, normal thrust and bending moment at quarter span a-a and hence compute the maximum shear, normal thrust and bending moment due to following wheel loads.

Solution:



Here, $L = 100'$

$h = 20'$

$a = 25'$

$\therefore y_a = \frac{4ha}{L^2}(L-a) = \frac{4\times20\times25}{100^2}\times(100-25)$

$\therefore y_a = 15$

$\tan\theta = \frac{4h}{L} - \frac{8ha}{L^2} = \frac{4\times20}{100} - \frac{8\times20\times25}{100^2}$

$\Rightarrow \theta = \tan^{-1}(0.4) = 21.8^\circ$

$\therefore \sin\theta = 0.37$

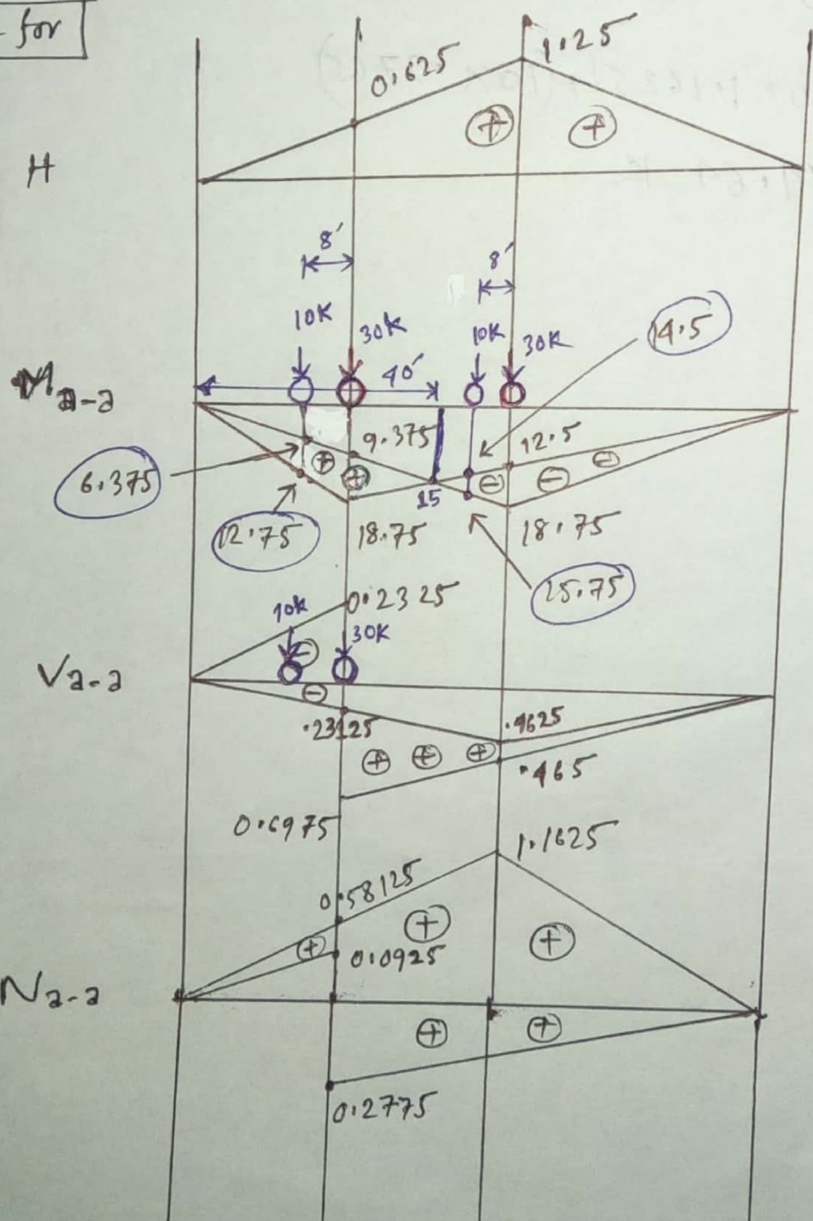
$\therefore \cos\theta = 0.93$

$\frac{x}{y} = \frac{50}{18.75}$ ...(i)

$\frac{100-x}{y} = \frac{75}{18.75}$ ...(ii)

(i) & (ii) $\Rightarrow x = 40, y = 15$

IL for



Max. stress for M_{a-a}:

$\frac{y}{25-8} = \frac{9.375}{25} \Rightarrow y = 6.375$

again, $\frac{y}{25-8} = \frac{18.75}{25} \Rightarrow y = 12.75$

(+) Max. $M_{a-a} = 30\times(18.75 - 9.375) + 10\times(12.75 - 6.375)$

$= 345$ Kft

$\frac{y}{50+8} = \frac{12.5}{50} \Rightarrow y = 14.5$

$\frac{y}{50-8} = \frac{18.75}{50} \Rightarrow y = 15.75$

(−) Max $M_{a-a} = 30\times(18.75 - 12.5) + 10\times(15.75 - 14.5)$

$= 200$ Kft

Max. stress for V_{2-2}: $(+)\ \text{Max. } V_{2-2} = 30 \times (.6975 - .23125) + 10 \times 0$

$= 13.9875\ K$

similarly, $\dfrac{y}{25-8} = \dfrac{.2325}{25}$

$\Rightarrow y = 0.1581$

$(-)\ \text{Max. } V_{2-2} = 30 \times (.2325) + 10 \times .1581$

$= 8.556\ K$

Max. stress for N_{2-2}:

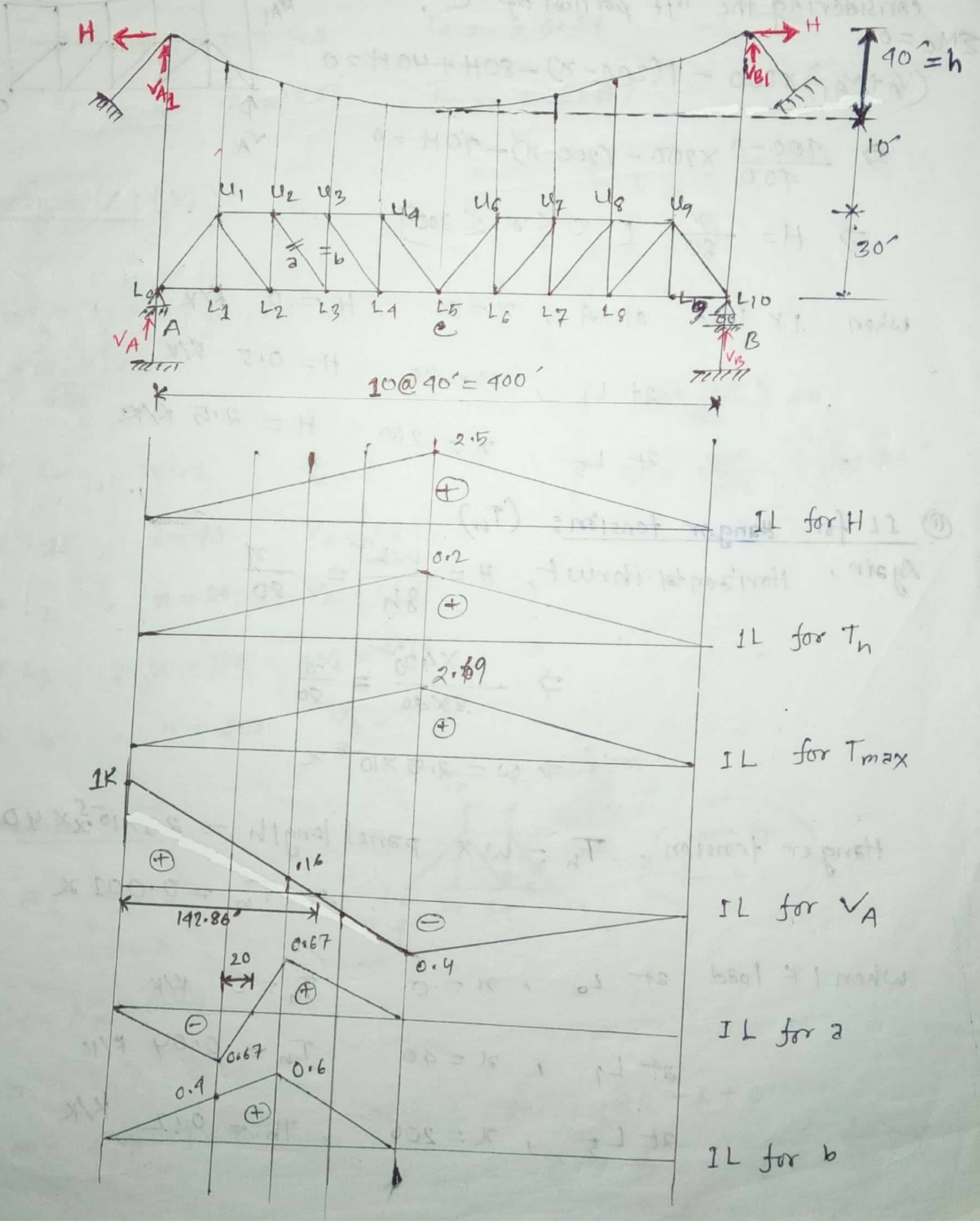
$\dfrac{y}{50-8} = \dfrac{1.1625}{50} \Rightarrow y = 0.9765$

$\therefore (+)\ \text{Max. } N_{2-2} = (30 \times 1.1625) + (10 \times .9765)$

$= 44.64\ K$

Suspension Cable Bridge

 # Problem: Deduce IL for hanger tension, maximum cable tension and stress in member 'a' and 'b' of the following suspension bridge shown in figure below. Also calculate the maximum force or stress of this member due to uniform load of 5 K/ft with a concentrated load of 10K.



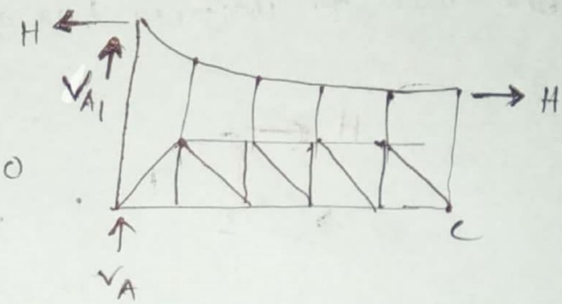
① **IL for horizontal thrust : (H)**

considering whole free body diagram,

$\Sigma M_B = 0$

$(V_A + V_{A_1}) \times 400 - 1(400 - x) + H \times 80 - H \times 80 = 0$

$\Rightarrow V_A + V_{A_1} = \dfrac{400 - x}{400}$ ......①



considdering the left portion of C,

$\Sigma M_C = 0$

$(V_A + V_{A_1}) \times 200 - 1(200 - x) - 80H + 40H = 0$

$\Rightarrow \dfrac{400 - x}{400} \times 200 - (200 - x) - 40H = 0$

$\Rightarrow H = \dfrac{x}{80}$ $[\,0 \leq x \leq 200\,]$

when 1K load at A, $x = 0$ $H = 0$ K/K

at L_1, $x = 40$ $H = 0.5$ K/K

at L_5, $x = 200$ $H = 2.5$ K/K

② **IL for Hanger tension: (T_h)**

Again, Horizontal thrust, $H = \dfrac{wL^2}{8h} = \dfrac{x}{80}$

$\Rightarrow \dfrac{w \times 400^2}{8 \times 40} = \dfrac{x}{80}$

$\Rightarrow w = 2.5 \times 10^{-5} x$

Hanger tension, $T_h = w \times$ panel length $= 2.5 \times 10^{-5} x \times 40$

$\therefore T_h = 0.001 x$

when 1K load at L_0, $x = 0$ $T_h = 0$ K/K

at L_1, $x = 40$ $T_h = 0.04$ K/K

at L_5, $x = 200$, $T_h = 0.2$ K/K

(iii) IL for maximum cable tension: (T_{max})

we know, $T_{max} = H\left(1+16\theta^2\right)^{\frac{1}{2}} = \frac{x}{80}\left[1+16\times\left(\frac{40}{400}\right)^2\right]^{\frac{1}{2}}$ $\left(\because \theta = \frac{h}{L}\right)$

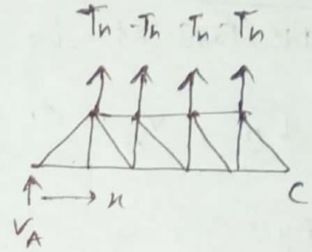
$\therefore T_{max} = 0.01346\,x$

when 1K load at L_0, $x = 0$ $T_{max} = 0$

at L_1, $x = 40$ $T_{max} = 0.54$

at L_5, $x = 200$ $T_{max} = 2.69$

(iv) IL for shear at A: (V_A)



$\Sigma M_c = 0$

$V_A \times 200 + T_h \times (1+2+3+4)\times 40 - 1(200-x) = 0$

$\Rightarrow V_A = \frac{200 - 1.4x}{200}$

$\frac{200-1.4x}{200} = 0$

$x = \frac{200}{1.4} = 142.86$

when 1K at L_0, $x = 0$ $V_A = 1$

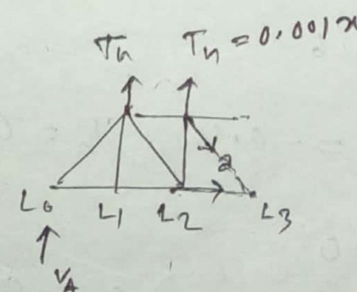
at L_1, $x = 40$ $V_A = .72$

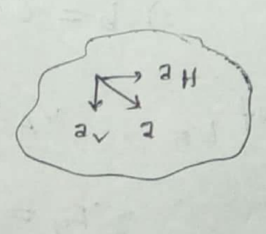
at L_2, $x = 80$ $V_A = .44$

at L_3, $x = 120$ $V_A = 0.16$

at L_5, $x = 200$ $V_A = -0.4$

(v) IL for member 'a':





when 1K at A, $V_A = 1$

$a = 0$

at L_2, $V_A = 0.44$

$\Sigma F_y = 0$

$2\times 0.001x + V_A - 1 - a_v = 0$ $\Rightarrow a_v = 2\times .001\times 80 - 1 + 0.44$

$\therefore a_v = -0.4 \Rightarrow a = -0.4\times\frac{\sqrt{30^2+40^2}}{30}$

$\therefore a = -0.67$

at L_3, $\quad \Sigma F_y = 0$

$V_A = 0.16 \quad 2 \times 0.001x + V_A - a_v = 0$

$\Rightarrow a_v = 2 \times 0.001 \times 120 + 0.16$

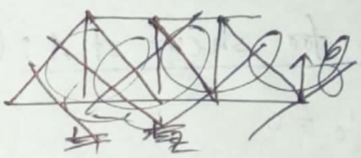
$\therefore a_v = 0.4$

$\Rightarrow a = 0.4 \times \dfrac{\sqrt{30^2 + 40^2}}{30} = 0.67$

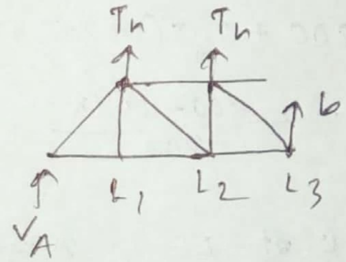
at L_5, $\quad V_A = -0.4, \quad a_v = 0 \quad \therefore a = 0$

(v) IL for member b:

when 1K load at L_0, $V_A = 1$, $b = 0$



at L_2; $V_A = 0.44$, $T_h = 0.08$



$\Sigma F_y = 0$

$b + 0.44 + 2 \times .08 - 1 = 0$

$\Rightarrow b = 0.40$

at L_3, $\quad V_A = 0.16$, $T_h = 0.12$

$\Sigma F_y = 0$

$b + 0.16 + 2 \times .12 - 1 = 0$

$\Rightarrow b = 0.6$

at L_5, $\quad V_A = -0.4$, $T_h = 0.2$

$\Sigma F_y = 0$

$b - 0.4 + 2 \times 0.2 = 0$

$b = 0$

Due to 5 K/ft uniform load and 10 Kip concentrated load.

Max. stress for hanger tension $= \left(2 \times \frac{1}{2} \times 200 \times 0.2\right) \times 5 + 10 \times .2$

$= 202$ Kips.

Max. stress for cable tension $= \left(2 \times \frac{1}{2} \times 200 \times 2.69\right) \times 5 + 10 \times 2.69$

$= 2716.9$ Kips.

Max. stress for a,

(+) Max. stress $= \left(\frac{1}{2} \times 100 \times .67\right) \times 5 + 10 \times .67 = 174.2$ Kips

(−) Max. stress $= \left(\frac{1}{2} \times 100 \times .67\right) \times 5 + 10 \times .67 = 174.2$ Kips.

Max. stress for b $= \left(\frac{1}{2} \times 200 \times .6\right) \times 5 + 10 \times .6$

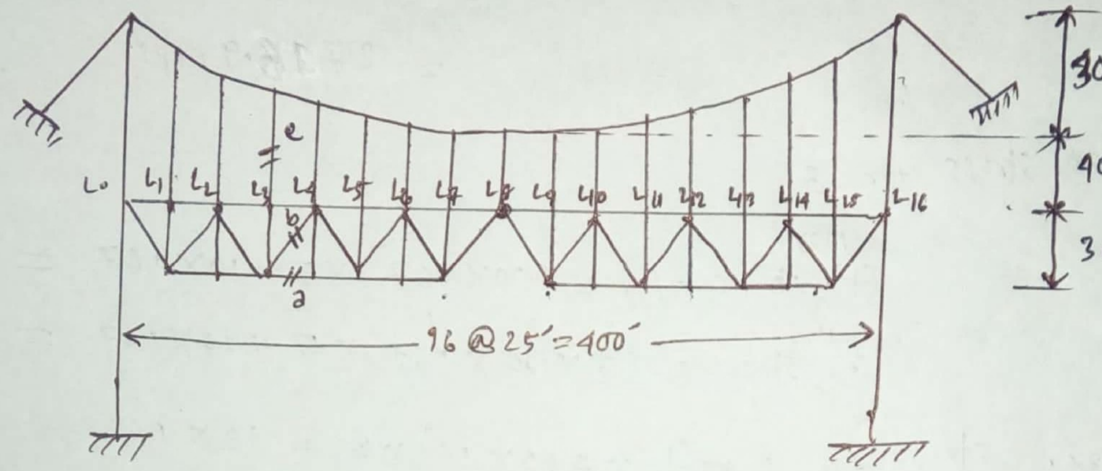
$= 306$ Kips.

(Ans)

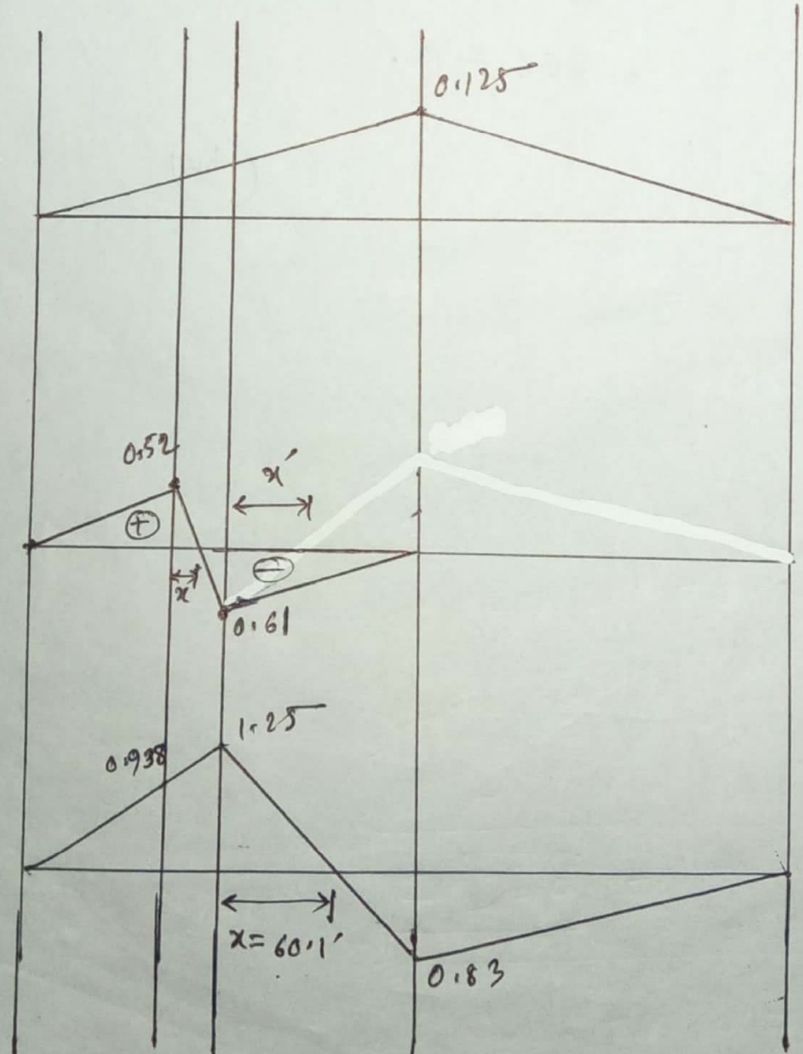
2014

Draw influence lines for stress in member a, b and c of the following suspension bridge shown in figure below and hence determine the maximum stress of the same members due to uniform load of 5 Kips/ft and concentrated load of 20 Kips.

Solution:



IL for



c

b

$$\frac{.52}{x} = \frac{.61}{25-x}$$

$$\Rightarrow x = 11.5$$

a

$$\frac{1.25}{x} = \frac{0.83}{100-x}$$

$$\Rightarrow x = 60.1'$$

Here, $H = \frac{x}{2h} = \frac{x}{2\times30} = \frac{x}{60}$

again, $H = \frac{wL^2}{8h}$ ∴ $\frac{wL^2}{8h} = \frac{x}{60}$

$\Rightarrow w = \frac{x\times8h}{60L^2} = \frac{x\times8\times30}{60\times(400)^2} = 2.5\times10^{-5}x$

IL for c:

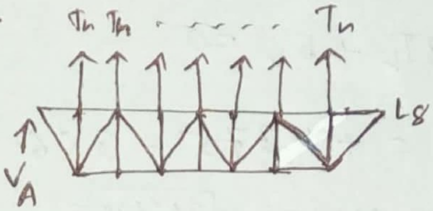
$c \rightleftharpoons T_h = w \times$ panel length

∴ $T_h = 2.5\times10^{-5}x\times25 = 6.25\times10^{-4}x$

When 1K at $L_0 \rightarrow T_h = 0$

1K at $L_8 \rightarrow T_h = .125$

$\Sigma M_{L_8} = 0$



$V_A\times200 + T_h(1+2+3+4+5+6+7)\times25' - 1(200-x) = 0$

∴ $V_A = \frac{200-1.4375x}{200}$

when 1K at $L_0 \rightarrow V_A = 1$

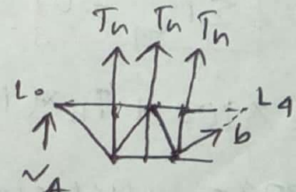
1K at $L_3 \rightarrow V_A = 0.461$

1K at $L_4 \rightarrow V_A = 0.281$

1K at $L_8 \rightarrow V_A = -0.14375$

IL for b: 1K at $L_0 \rightarrow V_A = 1$ ∴ $a = 0$

1K at $L_3 \rightarrow V_A = 0.461$



$\Sigma F_y = 0$

$V_A + 3\times T_h - 1 + b_v = 0$

$\Rightarrow 0.461 + 3\times6.25\times10^{-4}\times75 - 1 + b_v = 0$

$\Rightarrow b_v = 0.398375$ ∴ $b = .398375\times\frac{\sqrt{30^2+25^2}}{30} = 0.52$ K

1K at $L_4 \rightarrow V_A = 0.281$

$\Sigma F_y = 0$

$\therefore V_A + 3T_h + b_v = 0 \quad \Rightarrow b_v = -V_A - 3T_h = 0.281 - 3 \times 6.25 \times 10^{-4} \times 100$

$\Rightarrow b_v = -0.4685$

$\therefore b = -0.4685 \times \frac{\sqrt{30^2+25^2}}{30} = -0.61K$

1K at $L_8 \rightarrow V_A = -0.4375$

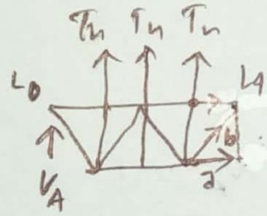
$\Sigma F_y = 0$

$V_A + 3T_h + b_v = 0$

$\Rightarrow b_v = +.4375 - 3 \times 6.25 \times 10^{-4} \times 200 = 0.0625$

$\Rightarrow b = 0.0625 \times \frac{\sqrt{30^2+25^2}}{30} = 0.08 \simeq 0K$

IL for a:



~~ΣM~~

1K at $L_0 \rightarrow V_A = 1$

$\therefore b = 0$

1K at $L_3 \rightarrow V_A = 0.461$

$\boxed{\Sigma M_{L_4} = 0}$

$V_A \times 100 + T_h \times (1+2+3) \times 25 - 1 \times 25 - a \times 30 = 0$

$\Rightarrow 30a = .461 \times 100 + 6.25 \times 10^{-4} \times 75 \times (1+2+3) \times 25 - 25 = 28.73125$

$\therefore a = 0.938K$

1K at $L_4 \rightarrow V_A = 0.281$ Or $\boxed{\Sigma M_{L_4} = 0} \Rightarrow V_A \times 100 + T_h \times (1+2+3) \times 25 - a \times 30 = 0$

$\Rightarrow a = 1.25K$

Easy

$\boxed{\Sigma M_{L_3} = 0}$

$V_A \times 75 + T_h \times (1+2) \times 25 + b_h \times 30 - a \times 30 = 0$

$\Rightarrow 0.281 \times 75 + 6.25 \times 10^{-4} \times 100 \times (1+2) \times 25 + 0.61 \times \frac{25}{\sqrt{25^2+30^2}} \times 30 = 30a$

$\Rightarrow a = 1.25$ K

1K at $L_8 \rightarrow V_A = -0.4375$ or $\boxed{\Sigma M_{L_4} = 0}$

$\Rightarrow V_A \times 100 + T_h \times (1+2+3) \times 25 - a \times 30 = 0$

$\Rightarrow a = -0.83$ K

Easy

$\boxed{\Sigma M_{L_3} = 0}$

$V_A \times 75 + T_h \times (1+2) \times 25 + b_h \times 30 - a \times 30 = 0$

$\Rightarrow -.4375 \times 75 + 6.25 \times 10^{-4} \times 200 \times 25 \times 3 - 0.108 \times \frac{25}{\sqrt{25^2+30^2}} = 30a$

$\therefore a = -0.83$ K

Now,

For c,

Maximum stress, $(+) = \left(\frac{1}{2} \times 400 \times .125\right) \times 5 + .125 \times 20$

$= 127.5$ Kips/ft^2

Maximum stress for b:

$(+)$ Max. stress $= \left[\frac{1}{2} \times (75 + 11.5) \times .52\right] \times 5 + 0.52 \times 20$

$= 122.85$ K/ft^2

$(-)$ Max. stress $= \left[\frac{1}{2} \times (25 - 11.5 + 100) \times .61\right] \times 5 + 0.61 \times 20$

$= 185.29$ K/ft^2

Maximum stress for a:

$(+)$ Max. stress $= \left[\frac{1}{2} \times (100 + 60.1) \times 1.25\right] \times 5 + 1.25 \times 20$

$= 525.3125$ K/ft^2

(−) Max. stress $= \left[\frac{1}{2} \times (200 + 100 - 60.1) \times .83\right] \times 5 + .83 \times 20$

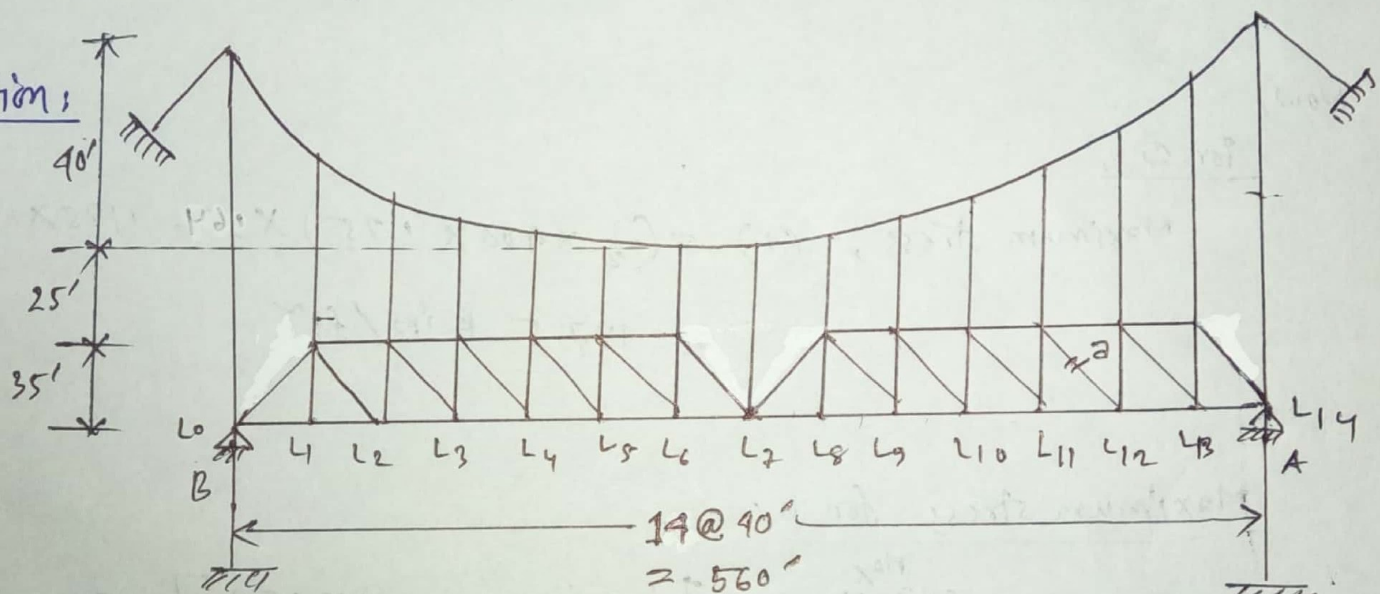
$= 514.3925$ K/ft²

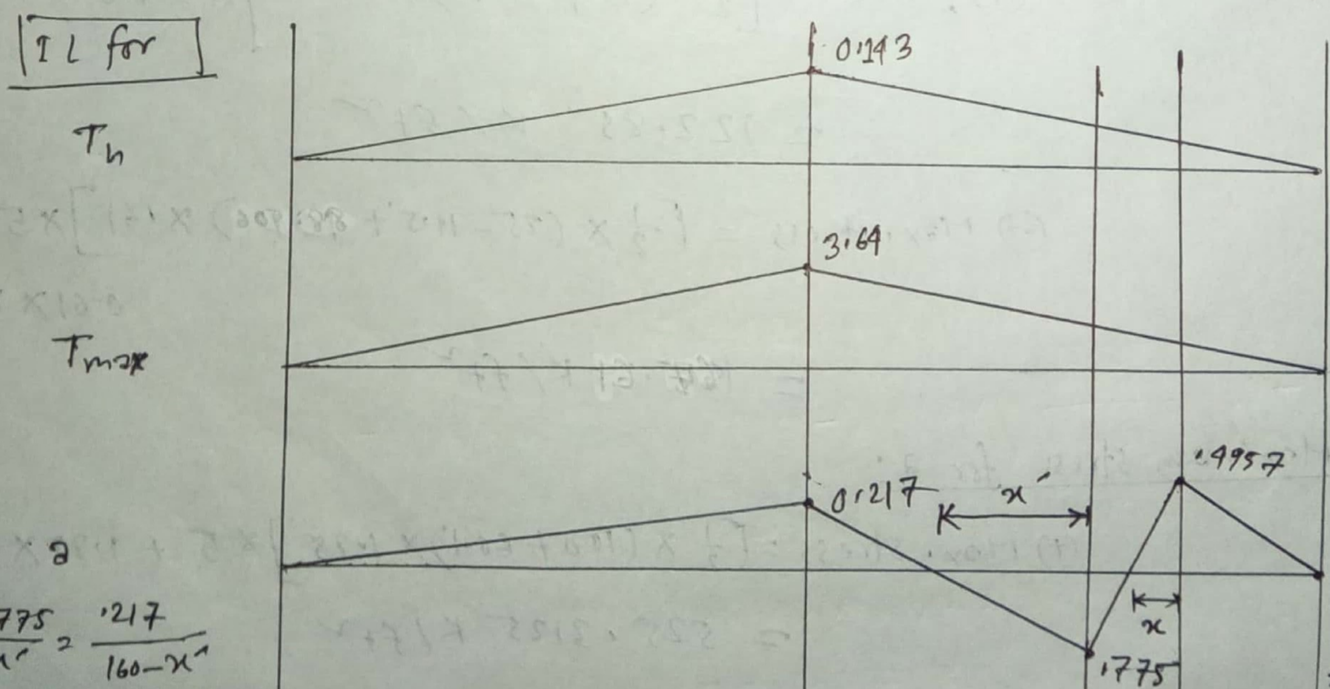
(Ans)

Class Test −15 series

Draw influence lines for hanger tension, maximum cable tension, and stress in the member a of the following suspension bridge shown in figure. Find the maximum stress in the member 'a' if the bridge is subjected to a uniform load of 5 K/ft and moving concentrated load of 15 kips.

Solution:





$\frac{.775}{x'} = \frac{.217}{160 - x'}$

$\Rightarrow x' = 125'$

$\frac{.4957}{x} = \frac{.775}{40 - x}$

$\Rightarrow x = 15.604$

Here,

$$H = \frac{x}{2h} = \frac{x}{2\times 40} = \frac{x}{80}$$

again,

$$H = \frac{wL^2}{8h} \quad \therefore \frac{wL^2}{8h} = \frac{x}{80}$$

$$\Rightarrow w = \frac{x \times 8 \times 40}{80 \times (560)^2} = 1.2755\times10^{-5}\, x$$

<u>IL for hanger tension:</u>

$$T_h = w \times \text{panel length} = 1.2755\times10^{-5}\, x \times 40$$
$$= 5.1\times10^{-4}\, x$$

when 1K at L_{14} $\rightarrow$ $T_h = 0$

1K at L_7 $\longrightarrow$ $T_h = 0.143$

<u>IL for maximum cable tension:</u>

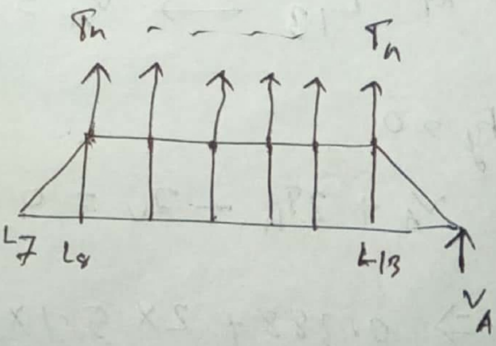
$$T_{max} = H\sqrt{1+16\theta^2} = \frac{x}{80}\times\sqrt{1+16\times\left(\frac{40}{560}\right)^2} = 0.013\,x$$

$\therefore$ when 1K at L_{14} $\longrightarrow$ $T_{max} = 0$

1K at L_7 $\longrightarrow$ $T_{max} = 3.164$

Now,

$\Sigma M_{L_7} = 0$



$$V_A \times 280 + T_h \times (1+2+3+4+5+6)\times 40$$
$$- 1(280 - x) = 0$$

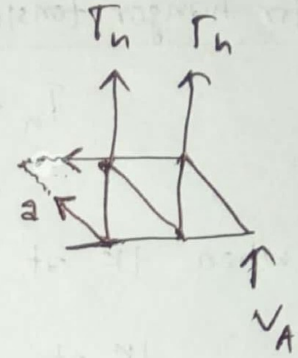
$$\Rightarrow V_A = \frac{280 - 1.4284\,x}{280}$$

when 1K at L_{14} $\longrightarrow$ $V_A = 1$ K

1K at L_{12} $\longrightarrow$ $V_A = 0.592$ K

1K at L_{11} $\longrightarrow$ $V_A = 0.388$ K

1K at L_7 $\longrightarrow$ $V_A = -0.4284$ K



IL for stress in member a :

when 1K at L_{14} $\rightarrow V_A = 1K$ $\therefore a = 0$

1K at $L_{12} \rightarrow V_A = 0.592$ K

$\Sigma F_y = 0$

$V_A + 2T_h - 1 + a_v = 0$

$\Rightarrow 0.592 + 2 \times 5.1 \times 10^{-4} \times 80 - 1 + a_v = 0$

$\Rightarrow a_v = .3264$ $\therefore a = .3264 \times \frac{\sqrt{35^2+40^2}}{35} = 0.4957$ K

1K at $L_{11} \longrightarrow V_A = 0.388$ K

$\Sigma F_y = 0$

$V_A + 2T_h + a_v = 0$

$\Rightarrow 0.388 + 2 \times 5.1 \times 10^{-4} \times 120 + a_v = 0$

$\Rightarrow a_v = -0.5104$ $\therefore a = -0.5104 \times \frac{\sqrt{35^2+40^2}}{35} = -0.775$ K

1K at $L_7 \rightarrow V_A = -0.4284$

$\Sigma F_y = 0$ $V_A + 2T_h + a_v = 0$ $\Rightarrow -0.4284 + 2 \times 5.1 \times 10^{-4} \times 280 + a_v = 0$

$\Rightarrow a_v = 0.1428$ $\therefore a = 0.217$ K

Maximum stress for a:

$(+)_1$ Max. Stress $= \left[\frac{1}{2} \times (280+160-125) \times .217\right] \times 5 + (217 \times 15)$
$= 174.1425$ K/ft^2 (max)

$(+)_2$ Max. Stress $= \left[\frac{1}{2} \times (80+15.604) \times .4957\right] \times 5 + (.4957 \times 15)$
$= 125.913$ K/ft^2

$(-)$ Max stress $= \left[\frac{1}{2} \times (125+40-15.604) \times .775\right] \times 5 + (.775 \times 15)$

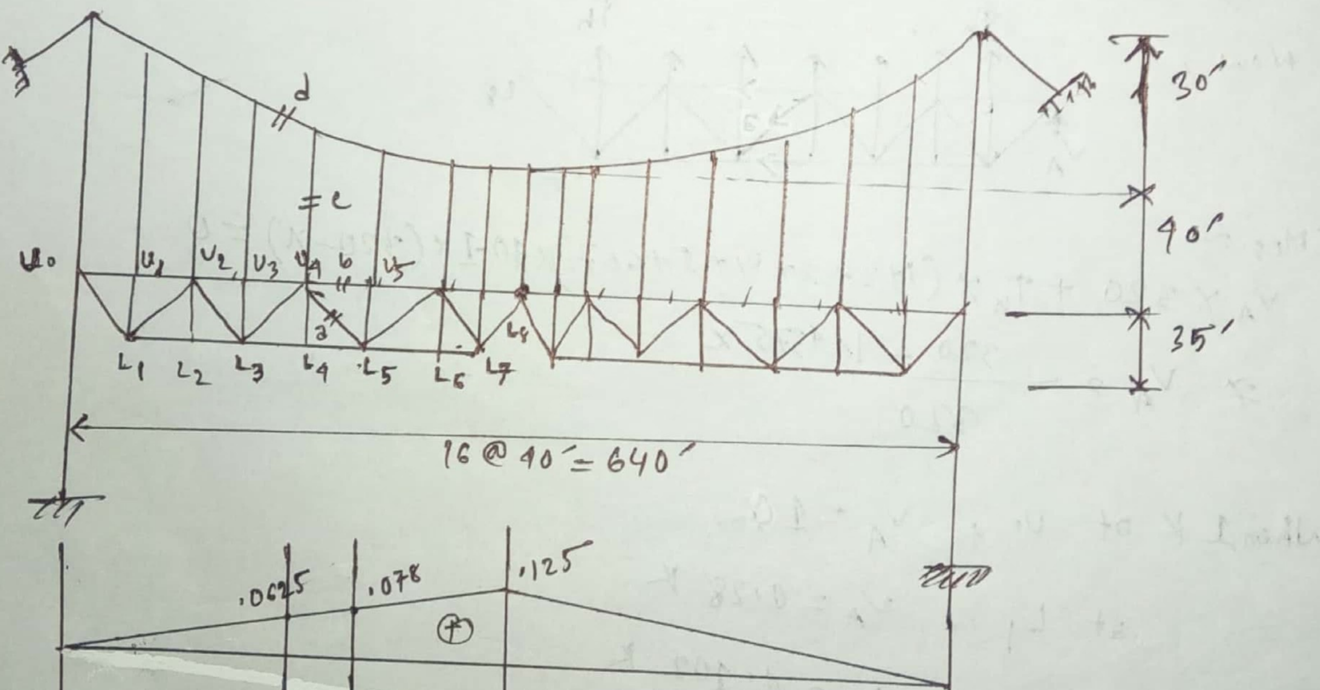
$= 301.08$ K/ft^2

(Ans.)

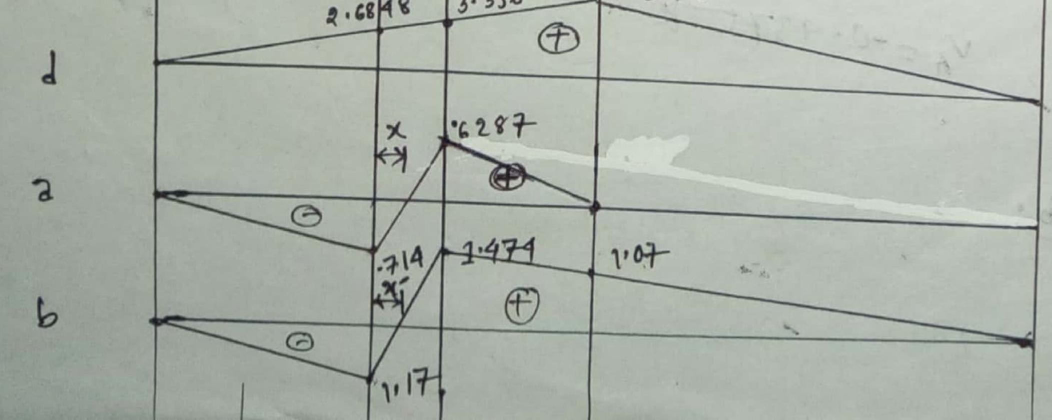
2015

\# Compute the maximum stress developed in member 'a', 'b', hanger 'c' and cable tension 'd' due to uniform dead load of 5 K/ft and the live load of 4 K/ft of the following suspension bridge shown in figure below:

Solution:



IL for



$\frac{x}{.714} = \frac{40-x}{.6287} \Rightarrow x = 21.27'$

$\frac{x'}{1.17} = \frac{40-x'}{1.474} \Rightarrow x' = 17.7'$

Here, $H = \frac{x}{2h} = \frac{x}{2\times 30} = \frac{x}{60}$

again, $H = \frac{wL^2}{8h} \Rightarrow \frac{wL^2}{8h} = \frac{x}{60} \Rightarrow w = \frac{x\times 8\times 30}{60\times 640^2} = 9.766\times10^{-6}x$

<u>IL for hanger tension:</u>

$T_h = C = w \times \text{panel length}$

$\Rightarrow C = (9.766\times10^{-6}x\times 40) = 3.9064\times10^{-4}x$

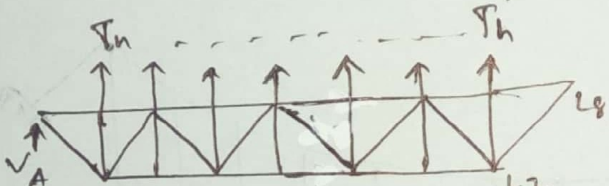
When 1 K at U_0, $C = 0$ K

at L_4, $C = 0.0625$ K

at L_5, $C = 0.078$ K

at L_8, $C = 0.125$ K

Now,



$\Sigma M_{L_8} = 0$

$V_A\times 320 + T_h\times(1+2+3+4+5+6+7)\times 40 - 1\times(320-x) = 0$

$\Rightarrow V_A = \frac{320 - 1.4375x}{320}$

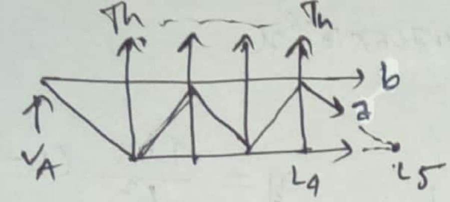
When, 1 K at U_0, $V_A = 1$ K

at L_1, $V_A = 0.28$ K

at L_4, $V_A = 0.102$ K

at L_5, $V_A = -0.4375$ K

<u>IL for a & b:</u>



<u>1 K at U_0:</u> $V_A = 1$, Hence, $a = 0$, $b = 0$

<u>1 K at L_4:</u> $V_A = 0.28$, $T_h = 0.0625$

$\Sigma F_y = 0 \Rightarrow 0.28 + 4 \times .0625 - 1 - a_v = 0 \Rightarrow a_v = -0.47$

$\Rightarrow a = -0.47 \times \frac{\sqrt{35^2 + 40^2}}{35} = -0.714$

$\cancel{\Sigma F_x = 0 \Rightarrow a_x + b = 0 \Rightarrow b = -0.714 \times \frac{40}{\sqrt{35^2+40^2}}}$

$\cancel{= -0.54\,K}$

$\Sigma M_{L_5} = 0$

$.28 \times 200 + 0.0625 \times (1+2+3+4) \times 40 - 1 \times 40 + b \times 35 = 0$

$\Rightarrow b = -1.17\ K$

<u>1K at L_5:</u> $V_A = 0.102\ K$, $T_h = 0.078$

$\Sigma F_y = 0 \Rightarrow 0.102 + 4 \times 0.078 - a_v = 0 \Rightarrow a_v = 0.414$

$\therefore a = 0.6287$

$\Sigma M_{L_5} = 0$

$.102 \times 200 + 0.078 \times (1+2+3+4) \times 40 + b \times 35 = 0$

$\Rightarrow b = 1.474$

<u>1K at L_8:</u> $V_A = -0.4375$, $T_h = 0.125$

$\Sigma F_y = 0 \Rightarrow -0.4375 + 4 \times 0.125 - a_v = 0 \Rightarrow a_v = 0.0625$

$\therefore a = 0.095 = OK$

$\Sigma M_{L_5} = 0$

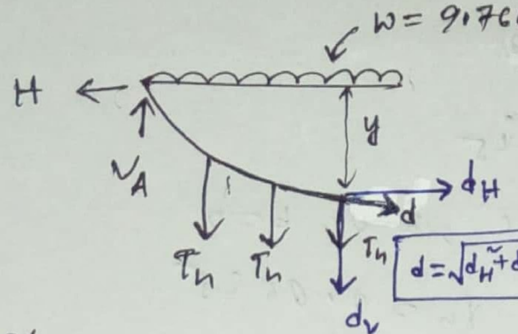
$-0.4375 \times 200 + 0.125 \times (1+2+3+4) \times 40 + b \times 35 = 0$

$\Rightarrow b = 1.07\ K$

<u>1K at L_{16}:</u> $V_A = 0$. Hence all values are zero.

Now,

IL for d :



$\boxed{d=\sqrt{d_H^2+d_v^2}}$

$$y = \frac{4hx}{L^2}(L-x)$$

$$\Rightarrow y = \frac{4\times30\times120}{640^2}(640-120)$$

$$\Rightarrow y = 18.28'$$

$\Sigma F_x = 0$

$$\Rightarrow H = d_H = \frac{x}{2h} = \frac{x}{60}$$

$\Sigma F_y = 0$

$$\Rightarrow V_A - 3T_h + d_v = 0 \quad \Rightarrow d_v = -3T_h + V_A$$

$$= -3\times3.9064\times10^{-4}x + \frac{wL}{2}$$

$$= -1.1718\times10^{-3}x + 9.766\times10^{-6}\times\frac{640}{2}x$$

$$\Rightarrow d_v = +1.9532\times10^{-3}x$$

$$\text{Now, } d = \sqrt{\left(\frac{x}{60}\right)^2 + (1.9532\times10^{-3}x)^2}$$

$$\therefore d = 0.01678x$$

1K at U_0 : $d = 0$

1K at L_4 : $d = 2.6848$ K

1K at L_5 : $d = 3.356$ K

1K at L_8 : $d = 5.3696$ K

Due to uniform dead load of 5K/ft and live load of 4K/ft :

$$Max.(-)a = \left[\frac{1}{2}\times(160+21.27)\times.714\right]\times(5+4) = 582.42 \text{ K/ft}^2$$

$$Max.(+)a = \left[\frac{1}{2}\times(160-21.27)\times.6287\right]\times9 = 392.49 \text{ K/ft}^2$$

$$Max.(-)b = \left[\frac{1}{2}\times(160+17.7)\times1.17\right]\times9 = 935.5905 \text{ K/ft}^2$$

$$Max.(+)b = \left[\frac{1}{2}\times(480-17.7)\times1.474\right]\times9 = 3066.436 \text{ K/ft}^2$$

$$Max\ (+)c = \left(\frac{1}{2}\times640\times.125\right)\times9 = 360 \text{ K/ft}^2$$

$$Max\ (+)d = \left(\frac{1}{2}\times640\times5.3696\right)\times9 = 15464.448 \text{ K/ft}^2$$

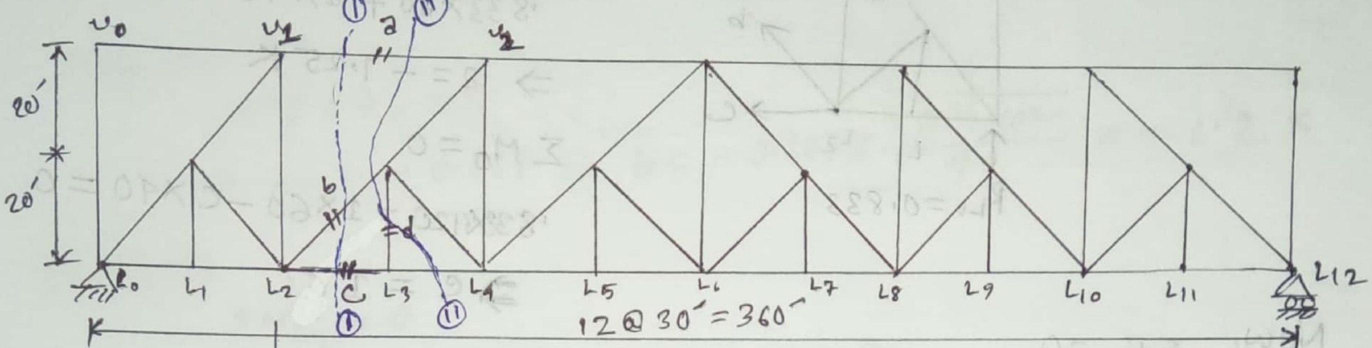
(Ans)

Sub-divided Truss

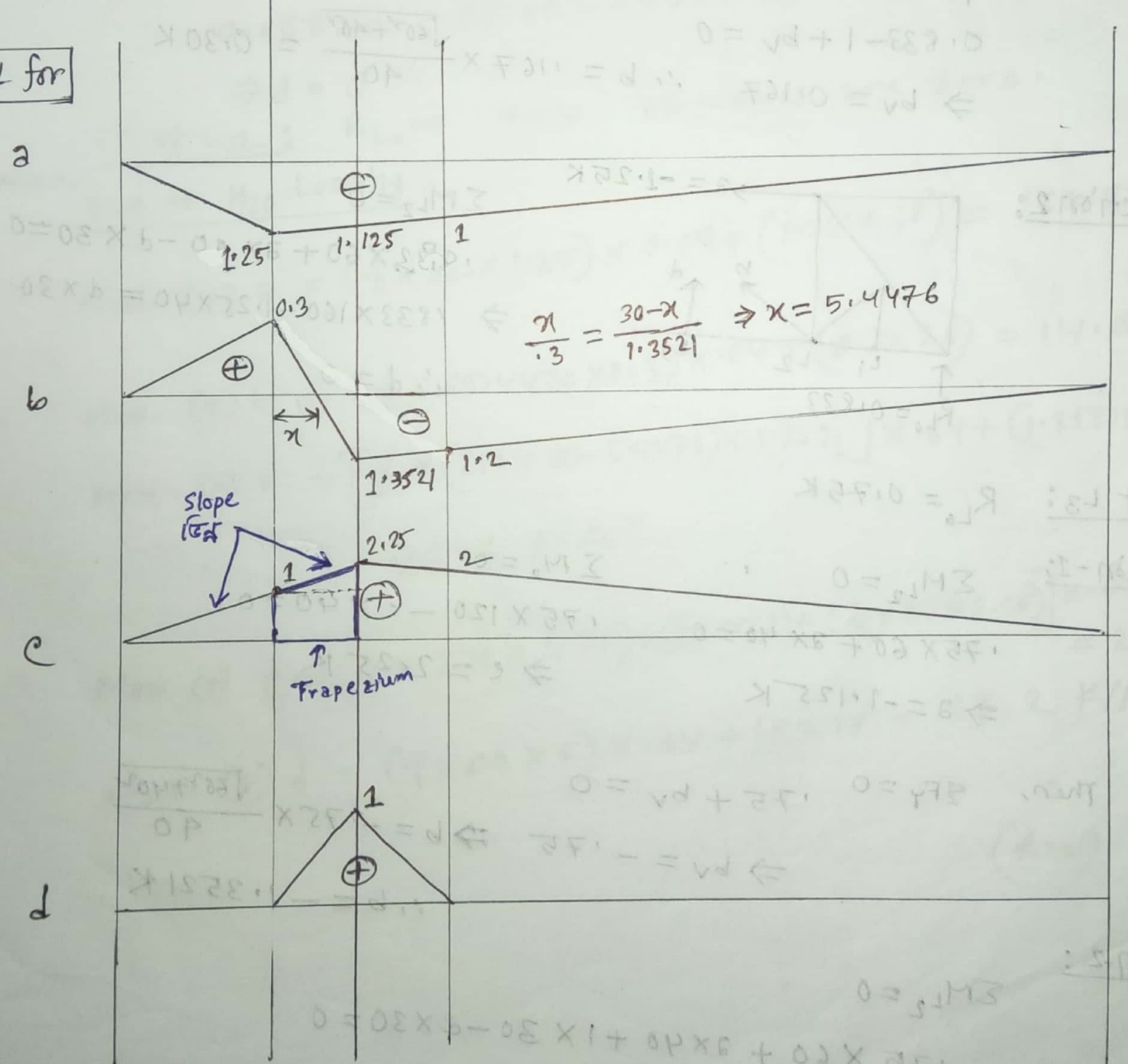
Truss analysis due to lane loading

2012, 10

Calculate the maximum live load stresses in member a, b, c and d of the following structure due to H20 loading.



IL for

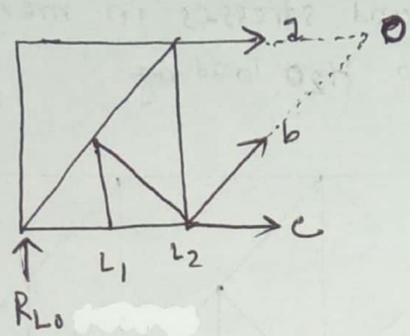


$$\frac{x}{.3} = \frac{30-x}{1.3521} \Rightarrow x = 5.4476$$

$1K$ at L_0: $R_{L_0} = 1K$, Hence, $a=0$, $b=0$, $c=0$, $d=0$

$1K$ at L_2: $R_{L_0} = 0.833K$,

section-1:



$\Sigma M_{L_2} = 0$

$.833 \times 60 + a \times 40 = 0$

$\Rightarrow a = -1.25K$

$\Sigma M_0 = 0$

$.833 \times 120 - 1 \times 60 - c \times 40 = 0$

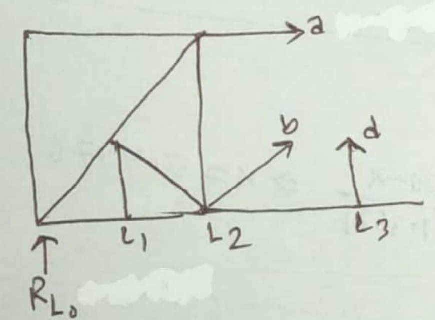
$\Rightarrow c = 1K$

Now, $\Sigma F_Y = 0$

$0.833 - 1 + b_v = 0$

$\Rightarrow b_v = 0.167$ $\therefore b = .167 \times \frac{\sqrt{60^2+40^2}}{40.} = 0.30K$

section 2:



$\Sigma M_{L_2} = 0$

$R_{L_0} \times 60 + a \times 40 - d \times 30 = 0$

$\Rightarrow .833 \times 60 - 1.25 \times 40 = d \times 30$

$\therefore d = 0$

$1K$ at L_3: $R_{L_0} = 0.75K$

section-1:

$\Sigma M_{L_2} = 0$

$.75 \times 60 + a \times 40 = 0$

$\Rightarrow a = -1.125K$

$\Sigma M_0 = 0$

$.75 \times 120 - c \times 40 = 0$

$\Rightarrow c = 2.25K$

Then, $\Sigma F_Y = 0$ $.75 + b_v = 0$

$\Rightarrow b_v = -.75$ $\Rightarrow b = -.75 \times \frac{\sqrt{60^2+40^2}}{40}$

$\therefore b = -1.3521K$

section-2:

$\Sigma M_{L_2} = 0$

$.75 \times 60 + a \times 40 + 1 \times 30 - d \times 30 = 0$

$\Rightarrow d = 1K$

1K at L_4: $R_{L_0} = 0.667$

section-1: $\Sigma M_2 = 0$

$.667 \times 60 + a \times 40 = 0$

$\Rightarrow a = -1$ K

$\Sigma M_0 = 0$

$.667 \times 120 - c \times 40 = 0$

$\Rightarrow c = 2$ K

$\Sigma F_Y = 0$

$.667 + b_V = 0$

$\Rightarrow b_V = -0.667 \Rightarrow b = -0.667 \times \frac{\sqrt{60^2+40^2}}{40} = -1.2$ K

section-2:

$\Sigma M_{L_2} = 0$

$0.667 \times 60 + a \times 40 - d \times 30 = 0$

$\Rightarrow d = 0$ K

1K at L_{12}: $R_{L_0} = 0$ Hence, all values are zero.

Now, Due to H_{20} Loading,

Max. (+) a $= (\frac{1}{2} \times 360 \times 1.25) \times 0.64 + (1.25 \times 18) = 166.5$ K/ft^2

Max. (+) b $= (\frac{1}{2} \times 65.4476 \times 0.3) \times .64 + (0.3 \times 26) = 14.083$ K/ft^2

Max. (-) b $= \left[\frac{1}{2} \times (270 + 30 - 5.4476) \times 1.3521\right] \times .64 + (1.3521 \times 26)$

$= 162.6$ K/ft^2

Max (+) c $= \left[(\frac{1}{2} \times 270 \times 2.25) + (\frac{1}{2} \times 60 \times 1) + (\frac{1}{2} \times 30 \times 3.25)\right] \times .64 + (2.25 \times 18)$

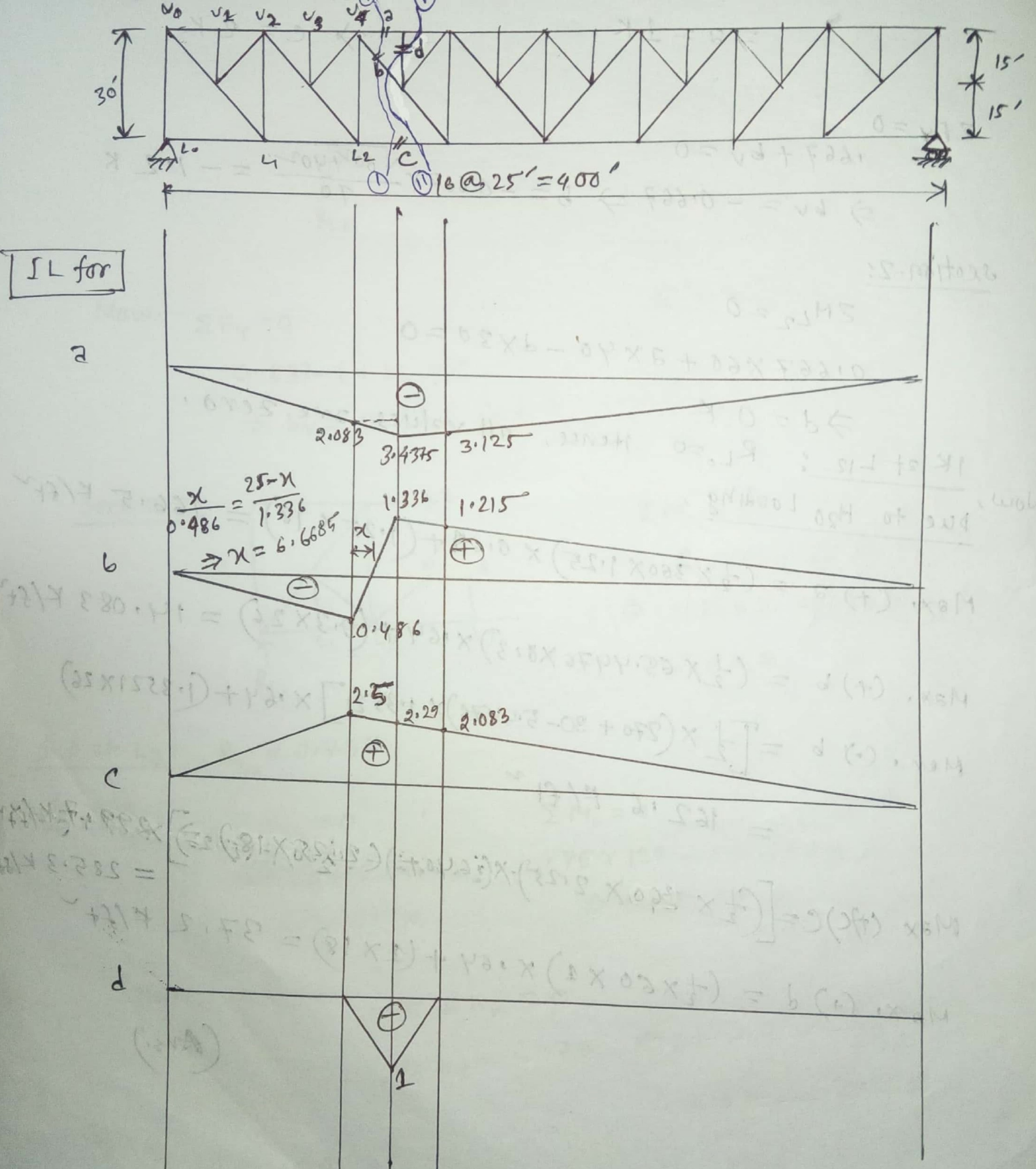
$= 285.3$ K/ft^2

Max. (+) d $= (\frac{1}{2} \times 60 \times 1) \times .64 + (1 \times 18) = 37.2$ K/ft^2

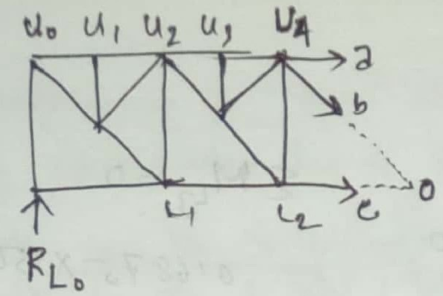
(Ans.)

2013

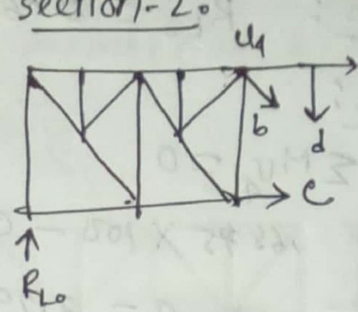
calculate the maximum stresses in the member a, b, c, and d of the following truss due to H_{15} loading.



section1:



section-2:



1K at L_0: $R_{L_0} = 1$, Hence, $a = 0, b = 0$
$c = 0, d = 0$

1 K at L_2: $R_{L_0} = 0.75K$

section1: $\Sigma M_{U_4} = 0$.

$.75 \times 100 - C \times 30 = 0$

$\Rightarrow C = 2.5K$

$\Sigma M_o = 0$

$.75 \times 150 - 1 \times 50 + a \times 30 = 0$

$\Rightarrow a = -2.083\ K$

$\Sigma F_y = 0$

$.75 - 1 - b_v = 0 \quad \Rightarrow b_v = -.125 \quad \therefore b = -.125 \times \frac{\sqrt{36^2 + 58^2}}{30}$

$= -0.486\ K$

section 2: $\Sigma M_{U_4} = 0 \quad \Rightarrow 0.75 \times 100 - 2.5 \times 30 + d \times 25 = 0$

$\Rightarrow d = 0\ K$

1K at L_3: $R_{L_0} = 0.625$

section1: $\Sigma M_{U_4} = 0$

$0.625 \times 100 - C \times 30 = 0$

$\therefore C = 2.083\ K$

$\Sigma M_o = 0$

$.625 \times 150 + a \times 30 = 0$

$a = -3.125\ K$

$\Sigma F_y = 0$

$0.625 - b_v = 0 \quad \Rightarrow b_v = 0.625 \quad \therefore b = 1.215\ K$

section2: $\Sigma M_{U_4} = 0$

$0.625 \times 100 - 2.083 \times 30 + d \times 25 = 0$

$\Rightarrow d = 0\ K$

1K at U_5: $R_{L_0} = 0.6875$ K

section 1:

$\Sigma M_{U_4} = 0$

$.6875 \times 100 - c \times 30 = 0$

$\Rightarrow c = 2.2917$ K

$\Sigma M_{L_3} = 0$

$0.6875 \times 150 + a \times 30 = 0$

$\Rightarrow a = -3.4375$ K

$\Sigma F_Y = 0$

$0.6875 - b_v = 0 \Rightarrow b_v = 0.6875 \quad \therefore b = 1.336$ K

Section: 2

$\Sigma M_{U_4} = 0$

$0.6875 \times 100 + 1 \times 25 - 2.2917 \times 30 + d \times 25 = 0$

$\Rightarrow d = -1$ K

1K at L_8: $R_{L_0} = 0$ Hence all values are zero.

Due to H_{15} loading:

Max (−) a $= \left[\left(\frac{1}{2} \times 3.4375 \times 275\right) + \left(\frac{1}{2} \times 100 \times 2.083\right) + \left(\frac{1}{2} \times 25 \times 5.5205\right)\right] \times .64 + (3.4375 \times 13.5)$

$= 459.726$ K/ft²

Max (+) b $= \left[\frac{1}{2} \times (275 + 25 - 6.6685) \times 1.336\right] \times 0.48 + (1.336 \times 19.5)$

$= 120.106$ K/ft²

Max. (−)b $= \left[\frac{1}{2} \times (100 + 6.6685) \times .486\right] \times .48 + (.486 \times 19.5)$

$= 21.92$ K/ft²

Max (+) c $= \left(\frac{1}{2} \times 400 \times 2.5\right) \times .48 + (2.5 \times 13.5)$

$= 273.75$ K/ft²

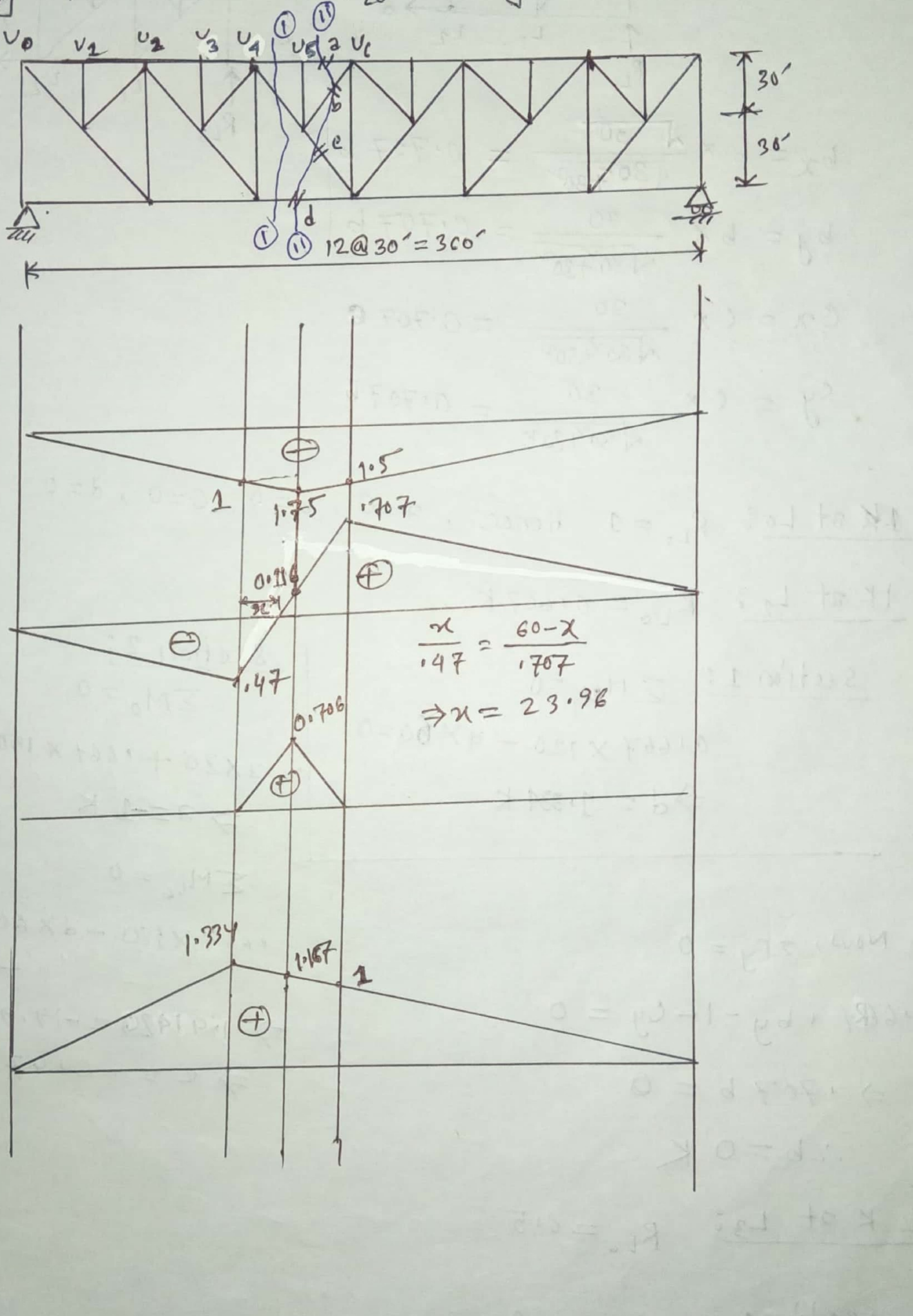
Max (+)d $= \left(\frac{1}{2} \times 50 \times 1\right) \times .48 + (1 \times 13.5)$

$= 25.5$ K/ft²

(Ans)

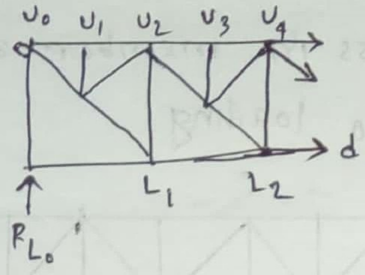
2016

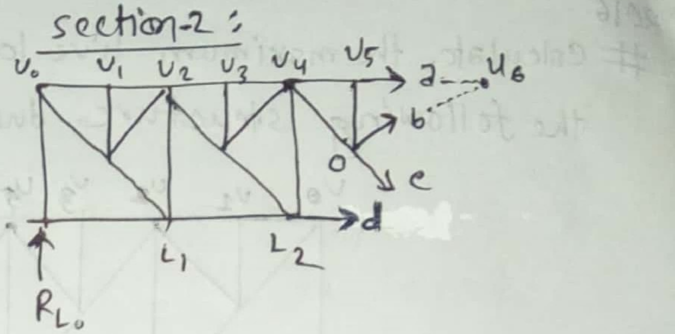
Calculate the maximum live load stress in member a, b, c and d of the following structure due to H_{20} loading.



$$\frac{x}{.47} = \frac{60-x}{.707}$$

$$\Rightarrow x = 23.96$$





$$b_x = b \times \frac{30}{\sqrt{30^2+30^2}} = 0.707\,b$$

$$b_y = b \times \frac{30}{\sqrt{30^2+30^2}} = 0.707\,b$$

$$c_x = c \times \frac{30}{\sqrt{30^2+30^2}} = 0.707\,c$$

$$c_y = c \times \frac{30}{\sqrt{30^2+30^2}} = 0.707\,c$$

1K at L_0: $R_{L_0} = 1$ Hence, $a=0$, $b=0$, $c=0$, $d=0$

1K at L_2: $R_{L_0} = 0.667K$,

section 1: $\Sigma M_{U_4} = 0$

$0.667 \times 120 - d \times 60 = 0$

$\Rightarrow d = 1.334$ K

section 2:

$\Sigma M_0 = 0$

$a \times 30 + .667 \times 150 - 1 \times 30 - d \times 30 = 0$

$\Rightarrow a = -1$ K

$\Sigma M_{U_6} = 0$

$.667 \times 180 - d \times 60 - 1 \times 60 - c_x \times 30 - c_y \times 30 = 0$

$\Rightarrow 42.42c = -19.98$

$\Rightarrow c = -0.47$ K

Now, $\Sigma F_y = 0$

$.667 + b_y - 1 - c_y = 0$

$\Rightarrow .707\,b = 0$

$\therefore b = 0$ K

1 K at L_3: $R_{L_0} = 0.5$

section 1: $\Sigma M_{U_4} = 0$

$0.5 \times 120 - d \times 60 = 0 \quad \Rightarrow d = 1$ K section 2:

$\Sigma M_o = 0$

$a \times 30 + .5 \times 150 - d \times 30 = 0$

$\Rightarrow a = -1.5$ K

$\Sigma M_{U_6} = 0$

$.5 \times 180 - d \times 60 - C_x \times 30 - C_y \times 30 = 0$

$\Rightarrow 42.42c = 30$

$\Rightarrow c = 0.707$ K

$\Sigma F_y = 0$

$.5 + b_y - C_y = 0$

$\Rightarrow b_y = 0 \quad \therefore b = 0$ K

1K at U_5: $R_{L_0} = 0.583$

section 1: $\Sigma M_{U_4} = 0$

$0.583 \times 120 - d \times 60$

$\Rightarrow d = 1.167$ K

section 2:

$\Sigma M_o = 0$

$a \times 30 + 0.583 \times 150 - d \times 30 = 0$

$\Rightarrow a = -1.75$ K

$\Sigma M_{U_6} = 0$

$.583 \times 180 - 1 \times 30 - d \times 60 - C_x \times 30 - C_y \times 30 = 0$

$\Rightarrow 42.42C = 4.92$

$\Rightarrow c = 0.116$ K

Now,

$\Sigma F_y = 0$

$0.583 + b_y - 1 - C_y = 0$

$\Rightarrow b_y = 0.5$

$\therefore b = 0.706$ K

1K at L_6: $R_{L_0} = 0$ Hence, all values are zero.

Due to H_{20} loading:

$Max\ (-)\ a = \left[\left(\frac{1}{2} \times 210 \times 1.75\right) + \left(\frac{1}{2} \times 120 \times 1\right) + \left(\frac{1}{2} \times 30 \times 2.75\right)\right] \times .64 + (2.75 \times 18) = 231.9\ K/ft^2$

$Max\ (+)\ b = \left(\frac{1}{2} \times .706 \times 60\right) + (.706 \times 18) = 33.89\ K/ft^2$

$Max\ (+)\ d = \left(\frac{1}{2} \times 1.334 \times 360\right) \times .67 + (1.334 \times 18) = 177.69\ K/ft^2$

$$\text{Max.} \ (-) \ C = \left[\tfrac{1}{2} \times (120 + 23.96) \times 0.47\right] \times 0.64 + (0.47 \times 26)$$

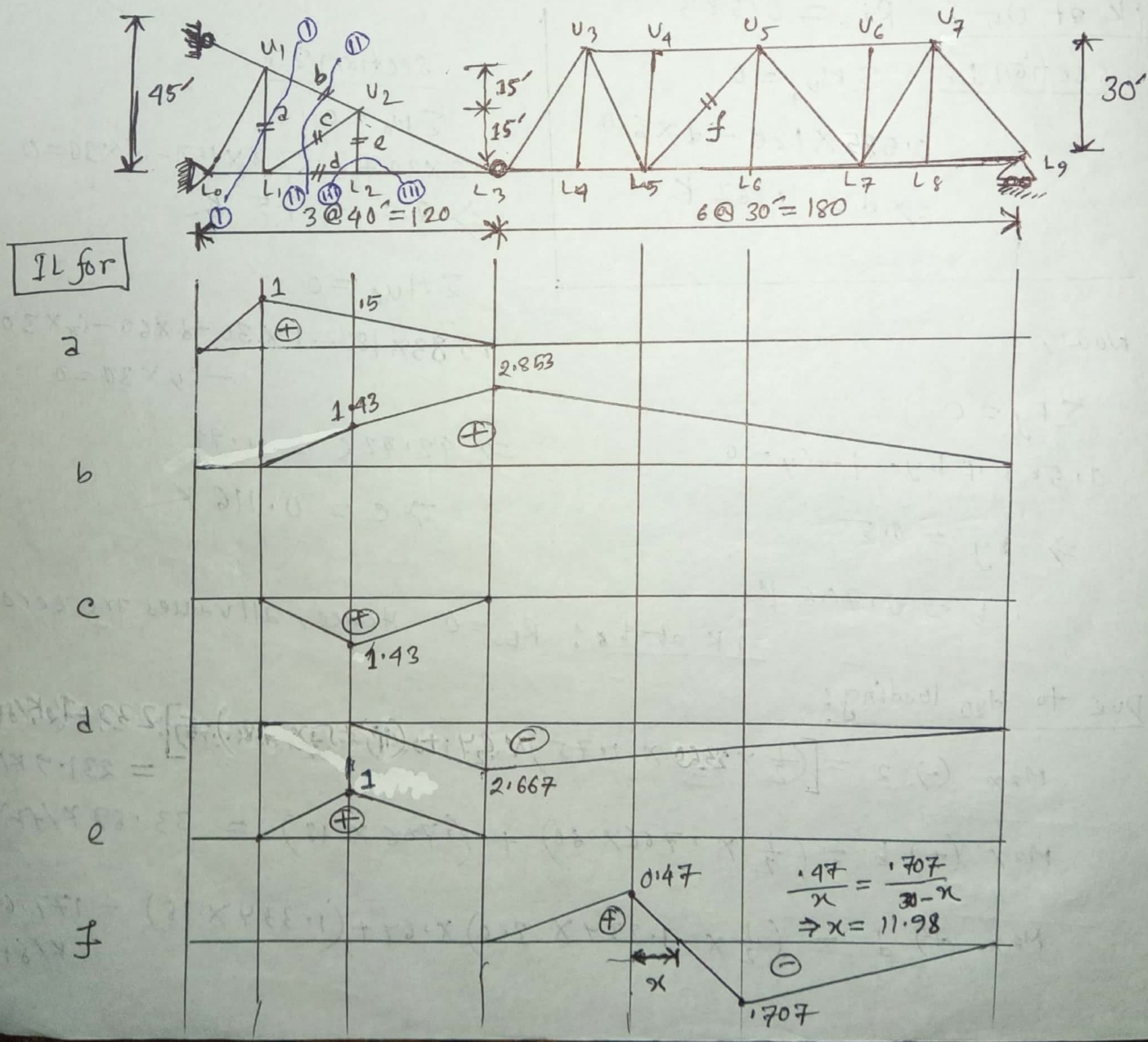
$$= 33.87 \ \text{K/ft}^2$$

$$\text{Max} \ (+) \ C = \left[\tfrac{1}{2} \times (180 + 60 - 23.96) \times .707\right] \times 0.64 + (.707 \times 26)$$

$$= 67.26 \ \text{K/ft}^2$$

(Ans.)

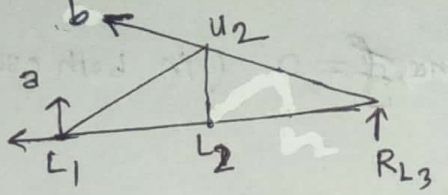
Calculate the maximum stresses in member a, b, c, d, e and f of the following compound truss due to H20 loading.



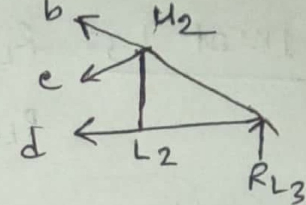
$$\frac{.47}{x} = \frac{.707}{30 - x}$$

$$\Rightarrow x = 11.98$$

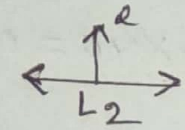
section 1:



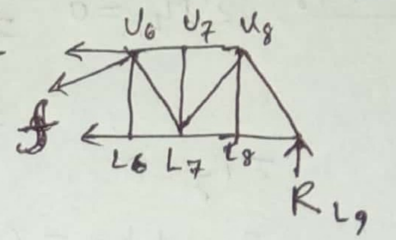
section 2:



section 3:



section 4:



$b_x = b \times \frac{40}{\sqrt{40^2+15^2}} = 0.936b$

$b_y = b \times \frac{15}{\sqrt{40^2+15^2}} = 0.35b$

$c_x = c \times \frac{40}{\sqrt{40^2+15^2}} = 0.936c$

$c_y = c \times \frac{15}{\sqrt{40^2+15^2}} = 0.35b$

$f_x = f \times \frac{30}{\sqrt{30^2+30^2}} = 0.707f$

$f_y = 0.707f$

1K at L_0: $R_{L_3} = 0$ Hence, $a=0$, $b=0$, $c=0$, $d=0$, $e=0$, $f=0$

1 K at L_1: $R_{L_3} = 0$, $a=1$, $b=0$, $c=0$, $d=0$, $e=0$, $f=0$

1K at L_2: $R_{L_3} = 0$, $f=0$

section 1: $\Sigma M_{L_1} = 0 \Rightarrow b_x \times 15 + b_y \times 40 - 1 \times 40 = 0$

$\Rightarrow 28.04b = 40$

$\therefore b = 1.43K$

$\Sigma F_y = 0$

$a - 1 + b_y = 0$

$\Rightarrow a = 0.5K$

section-2: $\Sigma M_{U_2} = 0 \Rightarrow d \times 15 - R_{L_3} \times 40 = 0 \quad \therefore d = 0$

$\Sigma F_y = 0 \Rightarrow b_y - c_y - 1 = 0 \Rightarrow c_y = -0.5 \quad \therefore c = 1.43K$

section 3: $\Sigma F_y = 0 \Rightarrow e - 1 = 0 \quad \therefore e = 1$

~~section 4~~

$1K$ at L_3: $R_{L_3} = 0K$ (just left) $f = 0$ (in both case)
$R_{L_3} = 1K$ (Just Right)

section 1: $\Sigma M_{L_1} = 0 \Rightarrow 1\times 80 - b_x\times 15 - b_y\times 40 = 0$
$\Rightarrow b = 2.853 K$

$\Sigma F_y = 0$,
$a - 1 + b_y = 0 \Rightarrow a = 0K$

section 2: $\Sigma M_{u_2} = 0 \quad d\times 15 + 1\times 40 = 0 \Rightarrow d = -2.667K$

$\Sigma F_y = 0$
$b_y - c_y - 1 = 0 \Rightarrow c = 0K$

section 3: $e = 0$

$1K$ at L_5: $R_{L_9} = \frac{2}{6} = 0.333K$
section 4: $\Sigma F_y = 0 \Rightarrow f_y - R_{L_9} = 0 \Rightarrow f_y = 0.333K$
$\therefore f = 0.47K$

$1K$ at L_6: $R_{L_9} = 0.5K$
section 4: $\Sigma F_y = 0$, $\Rightarrow f_y - 0.5 + 1 \Rightarrow f_y = -0.5$
$\therefore f = -0.707$

$1K$ at L_9: $R_{L_9} = 1K$ all values are zero.

~~max~~ Due to H_{20} loading,

$$Max. (+) a = \left(\frac{1}{2} \times 120 \times 1\right) \times 0.64 + (1 \times 18) = 56.4 \ K/ft^2$$

$$Max. (+) b = \left(\frac{1}{2} \times 2.853 \times 260\right) \times 0.64 + (2.853 \times 18) = 288.7236 \ K/ft^2$$

$$Max. (-) c = \left(\frac{1}{2} \times 80 \times 1.43\right) \times 0.64 + (1.43 \times 18) = 62.348 \ K/ft^2$$

$$Max. (-) d = \left(\frac{1}{2} \times 220 \times 2.667\right) \times 0.64 + (2.667 \times 18) = 235.7628 \ K/ft^2$$

$$Max. (+) e = \left(\frac{1}{2} \times 1 \times 80\right) \times 0.64 + (1 \times 18) = 43.6$$

$$Max. (+) f = \left[\frac{1}{2} \times (30 + 11.98) \times 0.47\right] \times 0.64 + (.47 \times 26)$$

$$= 18.534 \ K/ft^2$$

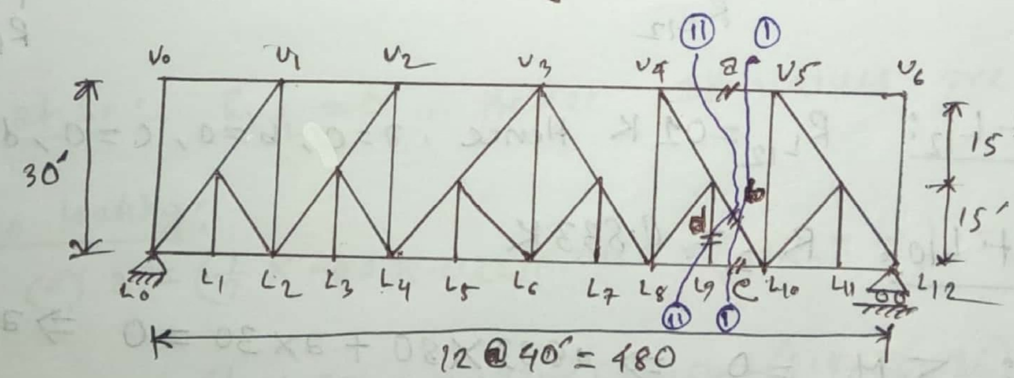
$$Max. (-) f = \left[\frac{1}{2} \times (90 + 30 - 11.98) \times .707\right] \times 0.64 + (.707 \times 26)$$

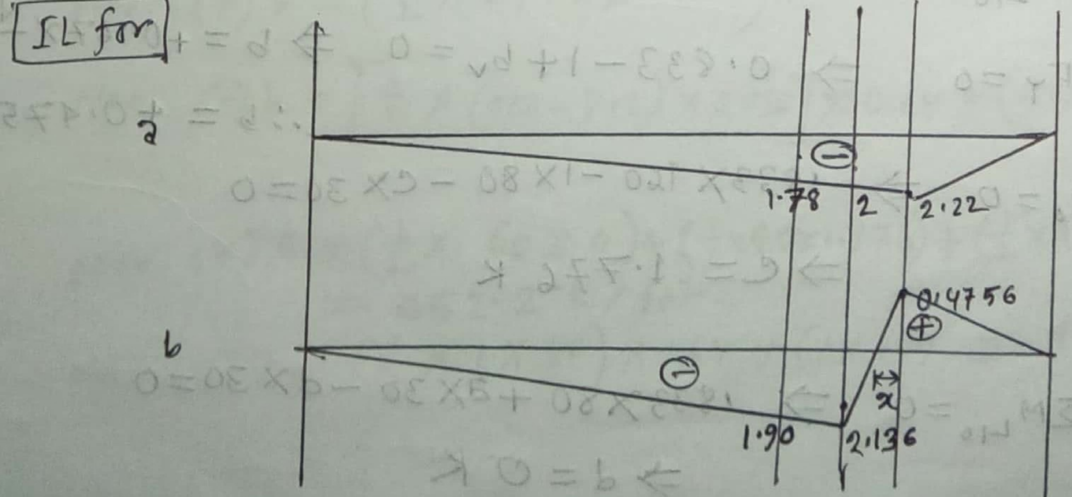
$$= 42.82 \ K/ft^2$$

(Ans)

2015

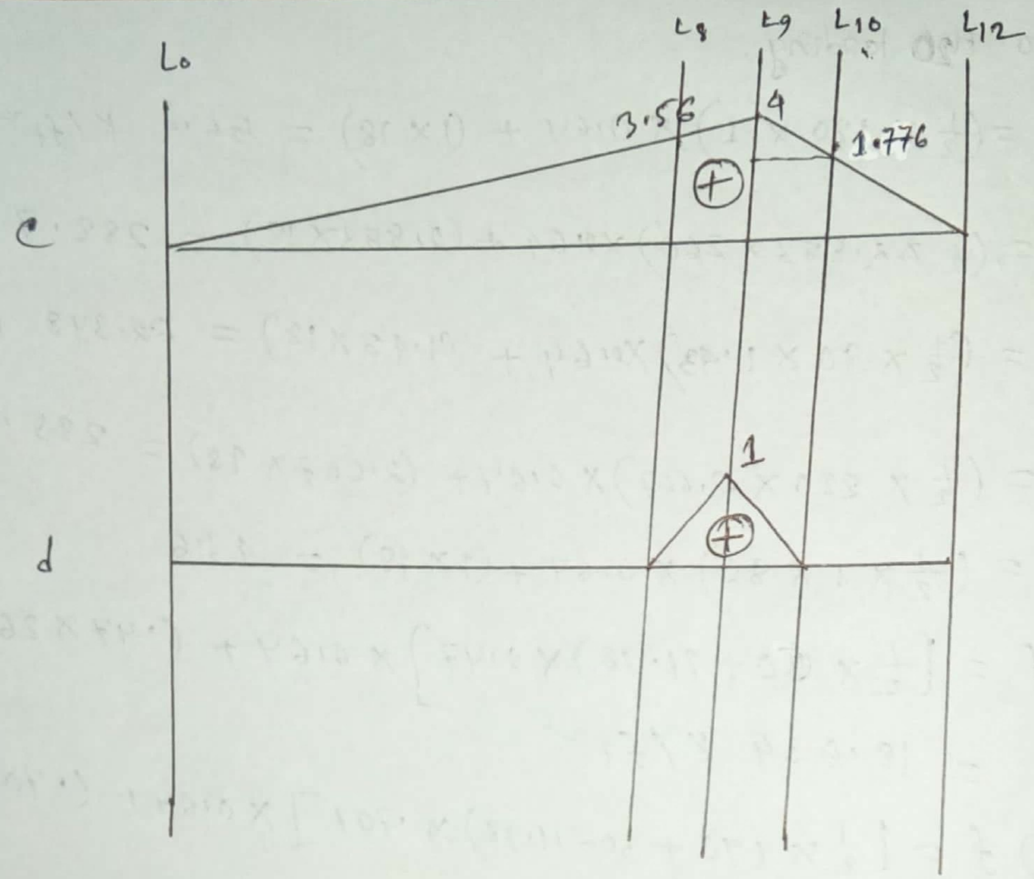
Calculate the maximum stresses in Member a, b, c and d of the following truss due to H_{20} loading.



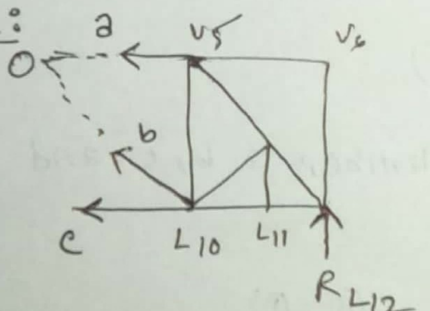


$$\frac{.4756}{x} = \frac{2.136}{40 - x}$$

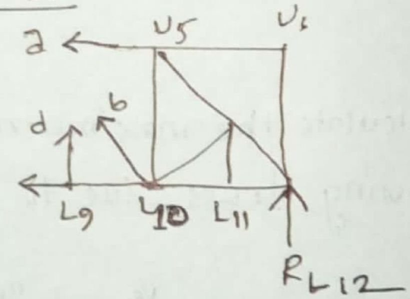
$$\Rightarrow x = 7.2'$$



Section 1:



section 2:



When 1 K at L_{12}: $R_{L_{12}} = 1$ K Hence, $a=0,\ b=0,\ c=0,\ d=0$

When 1K at L_{10}: $R_{12} = 0.833$ K

section 1: $\Sigma M_{L_{10}} = 0 \Rightarrow .833 \times 80 + a \times 30 = 0 \Rightarrow a = -2.22$ K

$\Sigma F_y = 0 \Rightarrow 0.833 - 1 + b_v = 0 \Rightarrow b = 0.167 \times \frac{\sqrt{30^2+80^2}}{30}$

$\therefore b = 0.4756$ K

$\Sigma M_o = 0 \Rightarrow .833 \times 160 - 1 \times 80 - c \times 30 = 0$

$\Rightarrow c = 1.776$ K

Section 2: $\Sigma M_{L_{10}} = 0 \Rightarrow .833 \times 80 + a \times 30 - d \times 30 = 0$

$\Rightarrow d = 0$ K

When 1K at L_9: $R_{L_{12}} = 0.75$ K

Section 1: $\Sigma M_{L_{10}} = 0 \Rightarrow 0.75 \times 80 + a \times 30 = 0 \Rightarrow a = -2$ K

$\Sigma F_y = 0 \Rightarrow 0.75 + b_v = 0 \Rightarrow b = -2.136$ K

$\Sigma M_0 = 0 \Rightarrow .75 \times 160 - c \times 30 = 0 \Rightarrow c = 4$ K

Section 2: $\Sigma M_{L_{10}} = 0 \Rightarrow .75 \times 80 + a \times 30 - d \times 30 = 0$

$\Rightarrow d = 1$ K

When 1K at L_8: $R_{12} = 0.667$ K

Section 1: $\Sigma M_{L_{10}} = 0 \Rightarrow 0.667 \times 80 + a \times 30 = 0 \Rightarrow a = -1.78$ K

$\Sigma F_y = 0 \Rightarrow 0.667 + b_v = 0 \Rightarrow b = -1.90$ K

$\Sigma M_0 = 0 \Rightarrow 0.667 \times 160 - c \times 30 = 0 \Rightarrow c = 3.56$ K

Section 2: $\Sigma M_{L_{10}} = 0 \Rightarrow 0.667 \times 80 + a \times 30 - d \times 30 = 0$

$\Rightarrow d = 0$ K

When 1K at L_0: $R_{L_{12}} = 0$, Hence all values are zero.

Due to H_{20} loading:

Max. (−) $a = (\frac{1}{2} \times 480 \times 2.22) \times 0.64 + (2.22 \times 18) = 380.952$ K/ft²

Max. (+) $b = (\frac{1}{2} \times 87.2 \times .4756) \times 0.64 + (0.4756 \times 26) = 25.64$ K/ft²

Max. (−) $b = [\frac{1}{2} \times (400 - 71.2) \times 2.136] \times 0.64 + (2.136 \times 26) = 324.02$ K/ft²

Max (+) $c = [(\frac{1}{2} \times 60 \times 4) + (\frac{1}{2} \times 80 \times 1.776) + (\frac{1}{2} \times 40 \times 5.776)] \times .64 + (4 \times 18)$

$= 652.2$ K/ft²

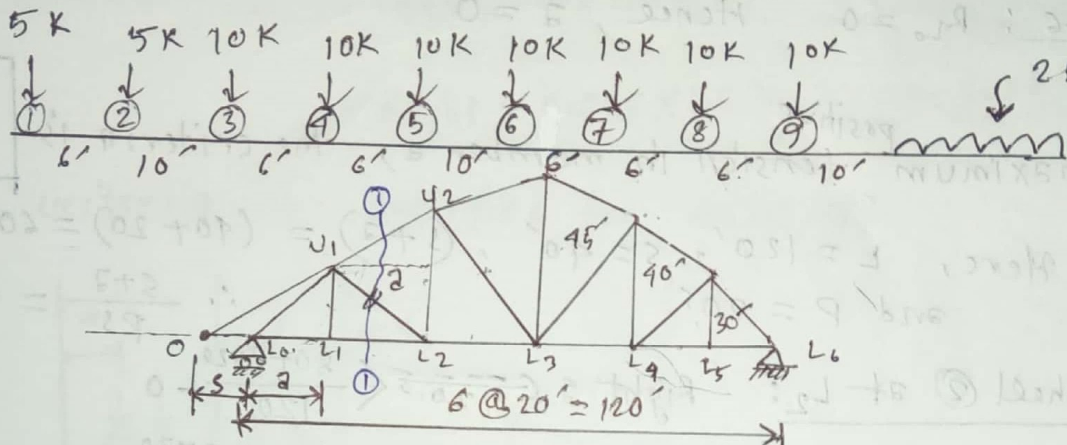
Max (+) $d = (\frac{1}{2} \times 1 \times 80) \times .64 + (1 \times 18) = 43.6$ K/ft²

(Ans.)

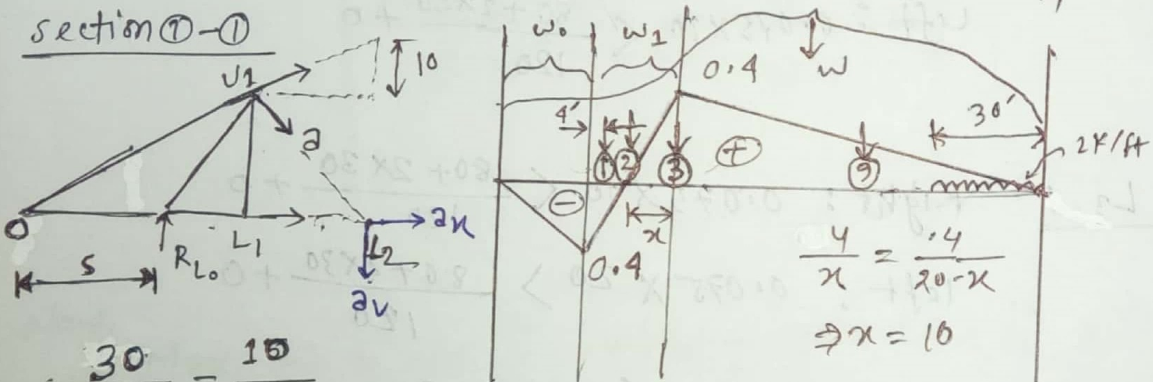
Non-Parallel Chord Truss

2014

Calculate the maximum positive and negative stresses in the member 'a' of the truss due to series of moving loads shown in figure below:



section ①-①



$$\frac{y}{x} = \frac{.4}{20-x}$$

$$\Rightarrow x = 10$$

$$a_x = \frac{20\,a}{\sqrt{30^2+20^2}} = 0.555\,a$$

$$a_v = \frac{30\,a}{\sqrt{30^2+20^2}} = 0.832\,a$$

$$\frac{30}{s+20} = \frac{10}{20}$$

$$\Rightarrow s = 40$$

When 1K at L_0: $R_{L_0} = 1K$ Hence, $a = 0$

1K at L_1: $R_{L_0} = \frac{5}{6} = 0.833$ K

Now, $\Sigma M_0 = 0$

$$a_v \times (40+20) - 0.833 \times 40 + 1 \times 60 + a_x \times 30$$

$$\Rightarrow (.832 \times 60)a + (.555 \times 30)a = -26.68$$

$$\Rightarrow \boxed{a = -0.40 K}$$

or, $\Sigma M_0 = 0$

$$.833 \times 40 - 1 \times 60 - a_v \times 80 = 0$$

$$\Rightarrow a = -0.40 K$$

1K at L_2: $R_{L_0} = \frac{4}{6} = 0.667$ K

$\Sigma M_o = 0$

$.667 \times 40 - a_v \times 80 = 0 \Rightarrow a = 0.40$ K

1K at L_6: $R_{L_0} = 0$ Hence, $a = 0$

For maximum positive tension in member a, the criteria is $\boxed{\frac{W}{L} + \frac{W_o}{S} = \frac{W_1}{P} \times \frac{S+a}{S}}$

Here, $L = 120'$, $S = 40'$, $(S+a) = (40+20) = 60$
and $P = 20'$
$\therefore \frac{S+a}{PS} = \frac{60}{20 \times 40} = 0.075$

wheel ② at L_2: Right: $0.075 \times 5 < \frac{80+2\times20}{120} + 0$
Left: $0.075 \times 10 < \frac{80+2\times20}{120} + 0$

wheel ③ at L_3: Right: $0.075 \times 10 < \frac{80+2\times30}{120} + 0$
left: $0.075 \times 20 > \frac{80+2\times30}{120} + 0$

Hence, criteria is satisfied for wheel ③

$$\text{Max} (+)\ a = \frac{-0.4}{10} \times (5\times0 + 5\times6) + \frac{.4}{10} \times (10\times10) + \frac{.4}{80} \times \Big[\Big(30 \times \tfrac{1}{2} \times 30\Big) \times 2$$
$$+ 10 \times (40+46+52+58+68+74)\Big]$$
$$= (-1.2 + 4 + 21.4) = 24.2 \text{ K/ft}^2$$

Now, for maximum negative tension, the criteria is,

$$\boxed{\frac{W}{L} - \frac{W_o}{S+L} = \frac{W_1}{P} \times \frac{(S+a+P)}{(S+L)}}$$

Here, $S = 40'$, $L = 120'$, $a = 20'$, $P = 20'$ $\therefore (S+L) = 160'$

$\therefore \frac{S+a+P}{(S+L)P} = \frac{80}{160 \times 20} = \frac{0.5}{20} = 0.025$

When wheel (8) is at L_1:

Left: $0.025 \times 10 + 4\times2 < \frac{40+84\times2}{120} - \frac{80\times2}{160}$

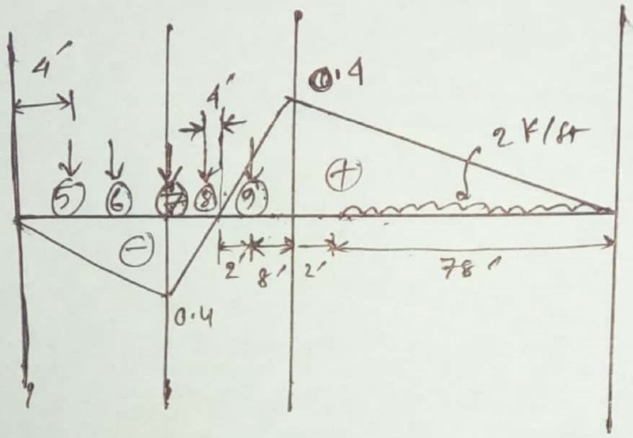
Right: $0.025 \times 28 < \frac{40+84\times2}{120} - \frac{80\times2}{160}$

wheel (7) is at L_1:

Left: $0.025 \times 20 < \frac{50+78\times2}{120} - \frac{78\times2}{160}$

Right: $0.025 \times 30 > \frac{50+78\times2}{120} - \frac{78\times2}{160}$

Hence, criteria is satisfied for wheel-(7)



Now,

$$\text{Max.} \ominus \, a = -\left(\frac{0.4}{80} \times 78\right) \times \frac{1}{2} \times 78 \times 2 - \frac{.4}{10} \times (9 \times 2) + \frac{0.4}{10} \times (10 \times 4 + 10 \times 10) + \frac{0.4}{20} \times (10 \times 4 + 10 \times 14)$$

$$= -30.42 - .72 + 5.6 + 3.6$$

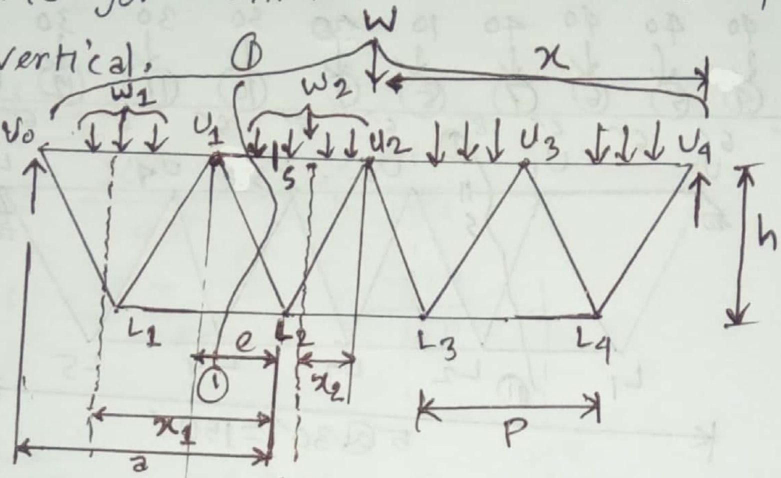
$$= -21.94 \text{ K}$$

(Ans.)

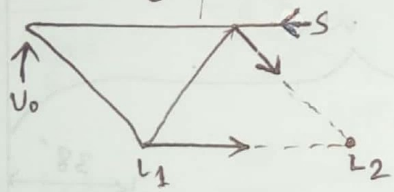
17, 16, 15, 14, 12, 11, 10

Truss without Vertical Chord

Deduce the criteria for maximum stress in the upper chord of truss without vertical.



section-1:



Before movement,

$$R_{U_{01}} = \frac{Wx}{L}$$

$$\boxed{\sum M_{L_2} = 0} \Rightarrow R_{U_{01}} \times a - W_1 \times x_1 - \frac{W_2 x_2}{P} \times e - S_1 \times h = 0$$

$$\Rightarrow S_1 = \frac{Wx}{L} \times \frac{a}{h} - \frac{W_1 x_1}{h} - \frac{W_2 x_2}{2h} \quad [\text{if } e = \tfrac{P}{2}] \quad \text{.....(I)}$$

After movement,

$$R_{U_{02}} = \frac{W(x+dx)}{L}$$

$$\boxed{\sum M_{L_2} = 0} \quad \frac{W(x+dx)}{L} \times a - W_1(x_1+dx) - \frac{W_2(x_2+dx)}{P} \times e - S_2 \times h = 0$$

$$\therefore S_2 = \frac{W(x+dx)}{L} \times \frac{a}{h} - \frac{W_1(x_1+dx)}{h} - \frac{W_2(x_2+dx)}{2h} \quad \text{....(II)}$$

From (I) & (II),

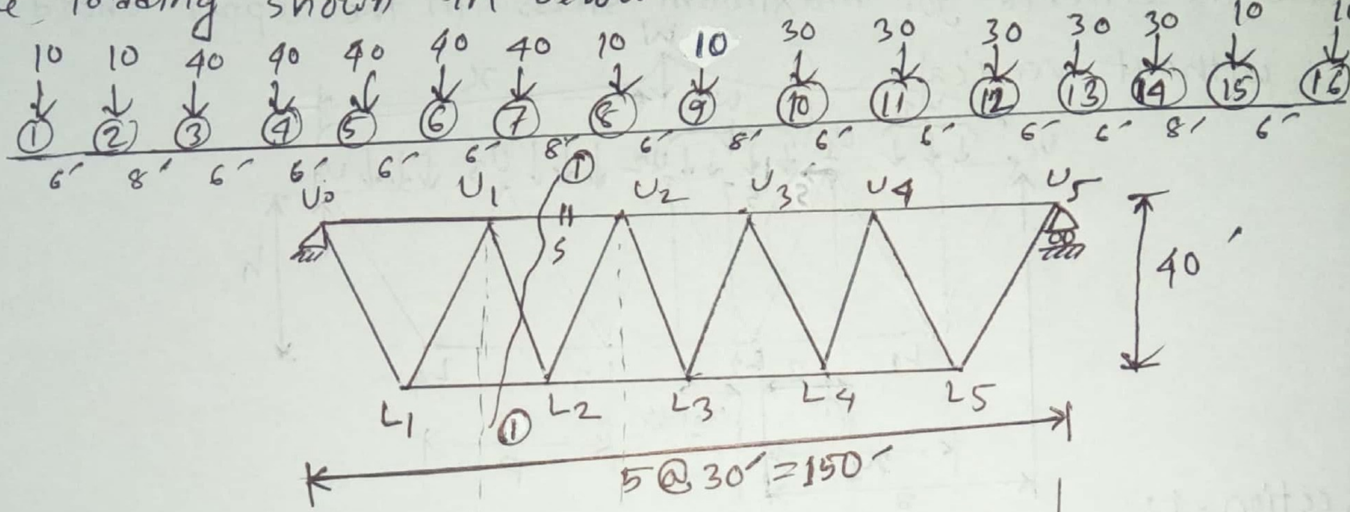
$$ds = S_2 - S_1 = \frac{Wa\,dx}{Lh} - \frac{W_1\,dx}{h} - \frac{W_2\,dx}{2h}$$

$$\therefore \frac{ds}{dx} = \frac{Wa}{Lh} - \frac{W_1}{h} - \frac{W_2}{2h} = 0$$

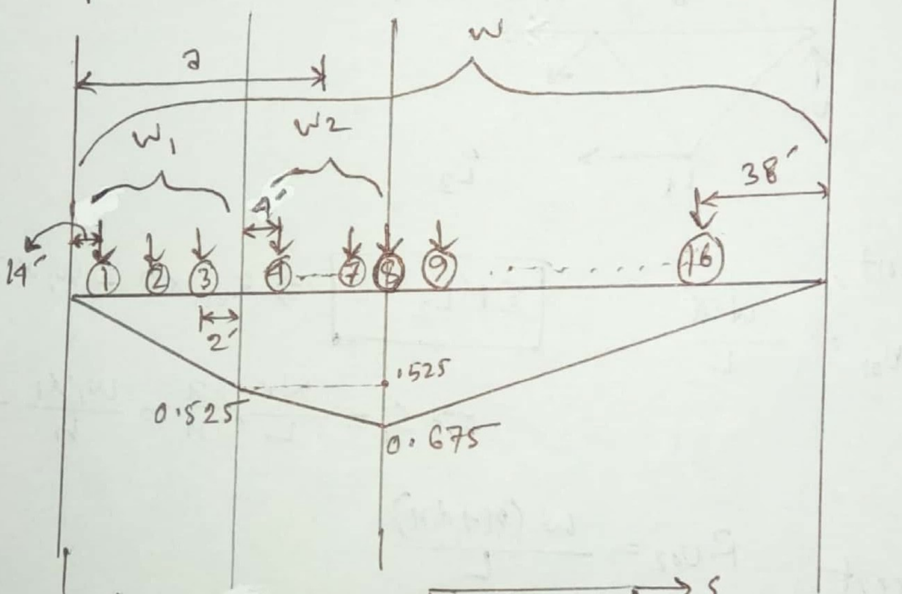
$$\therefore \boxed{\frac{W}{L} = \frac{W_1 + \frac{W_2}{2}}{a}}$$

2014, 11

Find out the maximum stress of the same member due to the loading shown in below.



IL for

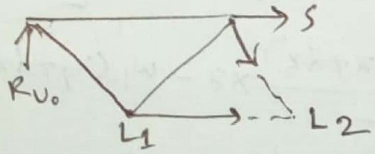


1 K at U_0

$R_{U_0} = 1\,K$

$S = 0$

1 K at U_1 $\quad R_{U_0} = \frac{4}{5} = 0.8\,K$



Section 1-1:

$\Sigma M_{L_2} = 0 \quad \Rightarrow 0.8 \times (30 + 15) - 1 \times 15 + S \times 40 = 0$

$\Rightarrow S = -0.525\,K$

1K at U_2:

$\Sigma M_{L_2} = 0 \quad \Rightarrow .8 \times (30 + 15) + S \times 40 = 0$

$\Rightarrow S = 0.675\,K$

1K at U_5: $\quad R_{U_0} = 0 \quad$ Hence $\quad S = 0$

we know the criteria for without vertical chord,

$$\frac{W}{L} = \frac{W_1 + \frac{W_2}{2}}{a} \qquad \text{Here, } L = 150,\ a = 45$$

When wheel ⑥ at U_2:

Right: $\frac{10 + \frac{130}{2}}{45} < \frac{410}{150}$

Left: $\frac{10 + \frac{170}{2}}{45} < \frac{410}{150}$

Wheel ⑦ at U_2:

Right: $\frac{20 + \frac{160}{2}}{45} < \frac{410}{150}$

Left: $\frac{20 + \frac{200}{2}}{45} < \frac{410}{150}$

wheel ⑧ at U_2:

Right: $\frac{60 + \frac{160}{2}}{45} > \frac{410}{150}$

Left: $\frac{60 + \frac{170}{2}}{45} > \frac{410}{150}$

Hence, criteria is satisfied for wheel-⑧

Now,

$$S_{max} = \frac{0.525}{30} \times [(10\times 14) + (10 \times 20) + (40 \times 28)] + .525 \times (40+40+40+40)$$

$$+ \frac{(0.675 - 0.525)}{30} \times [(40\times 4) + (40 \times 10) + (40 \times 16) + (40 \times 22)]$$

$$+ \frac{0.675}{90} [(10\times 38) + (10\times 44) + (30\times 52) + (30 \times 58) + (30\times 64) + (30\times 70) + (30\times 76) + (10\times 84) + (10\times 90)]$$

$$= (25.55 + 84 + 10.4 + 91.2) = 211.15$$

(Ans)