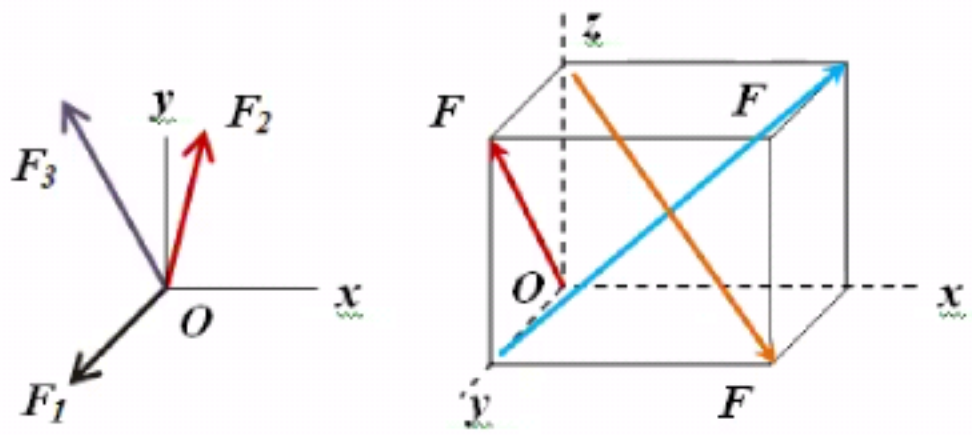


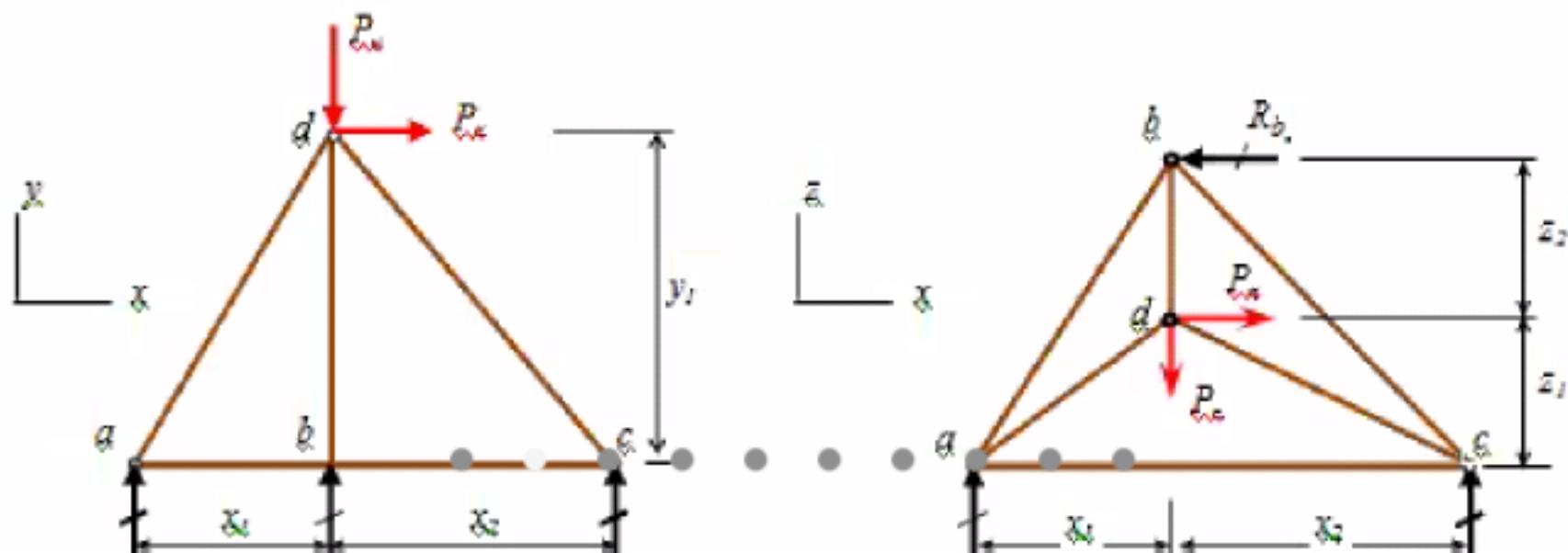
Non-coplanar force systems, in which the lines of action of the forces do not lie in the same plan. A non-concurrent system may be either coplanar or non-coplanar. It may be either concurrent or non-concurrent.

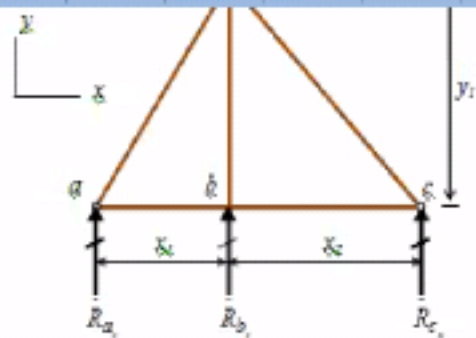


Resolution of non-coplanar force

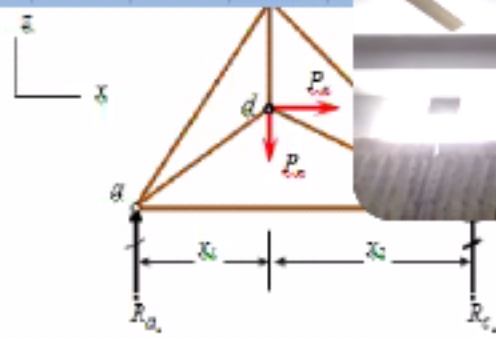


Space Trusses: A space truss consists of members joined together at their ends to form a stable three-dimensional structure. Tetrahedron is the simplest form of space truss as shown below. A simple space truss can be built from the basic tetrahedral element by three additional members and a joint, and continuing in this manner to form a system of multi-connected tetrahedrons.

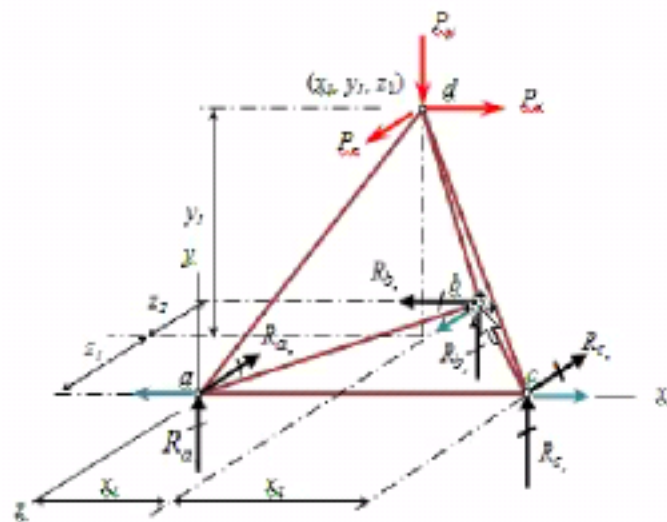




Elevation



Top View

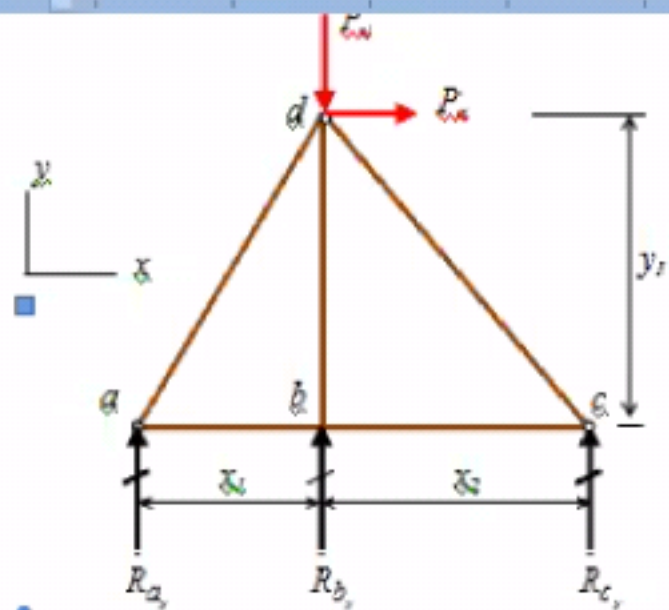


Typical diagram of space truss.

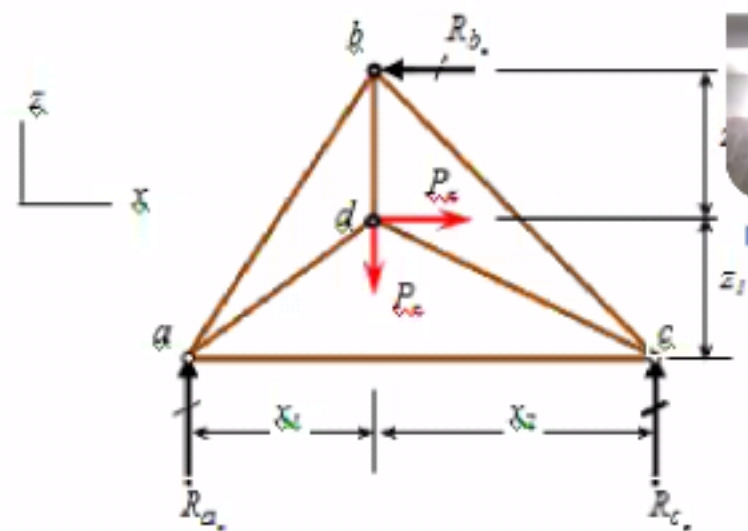
Assumptions of Analysis: ● ● ● ● ● ●

The members of a space truss may be treated as axial force member (as two force

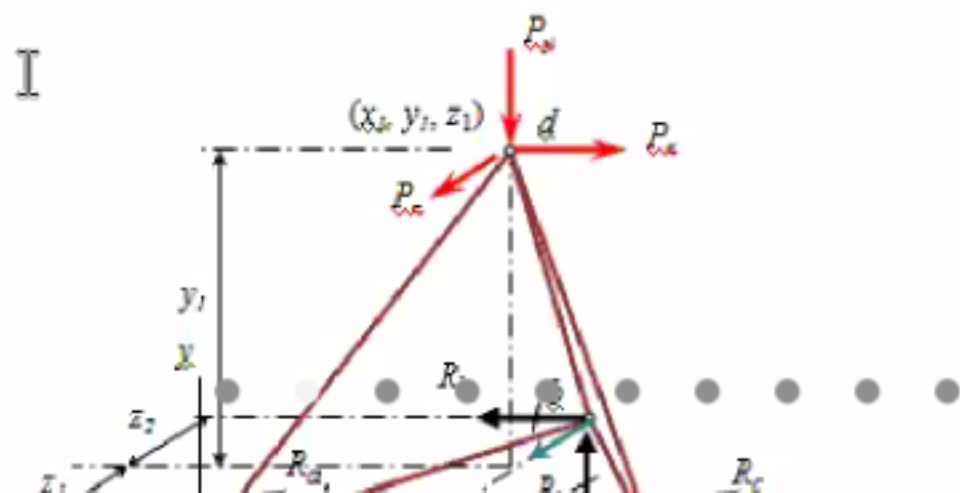


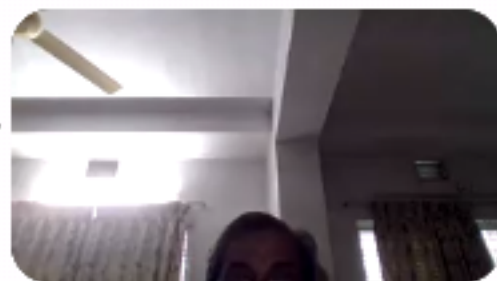
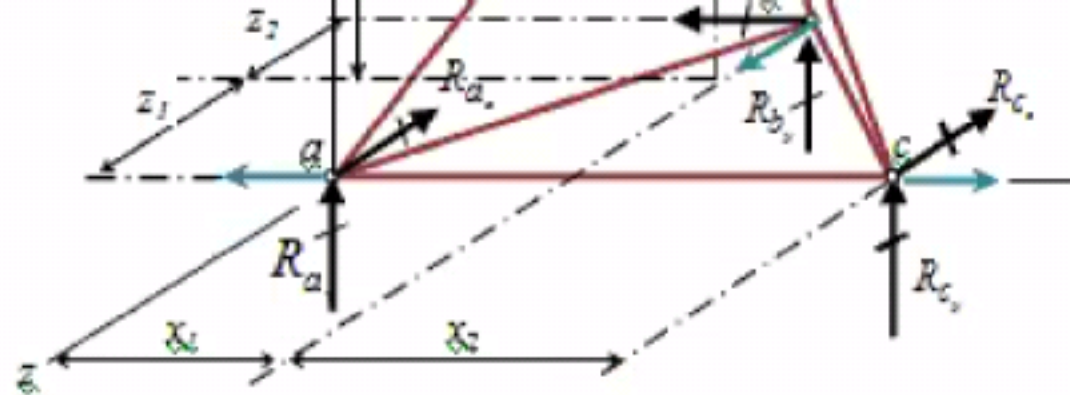


Elevation



Top View





Typical diagram of space truss.

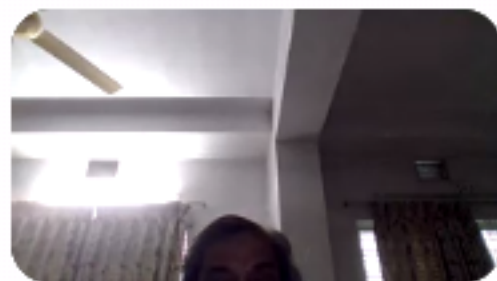
Assumptions of Analysis:

The members of a space truss may be treated as axial force member (as two force member in plan truss) and connected by ball and socket joints.

The force applied at joint only.

The weight of members can be neglected (or transfer to the joint).

Welded or bolted connections of the members intersect at a common point (Joint is assumed to be fixed end moment free condition).



Methods

Method of Joints

Three equilibrium equations are applied in each joint to determine bar forces.

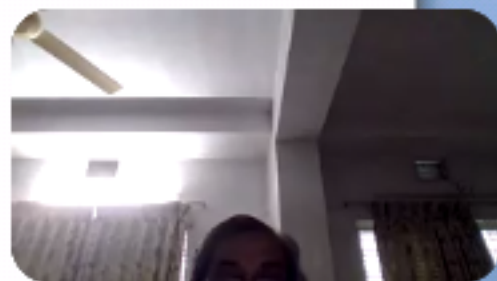
$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0,$$

Method of Sections

To method of sections, a rigid segment of the truss can be isolate by pass a surface through not more than six members. Such isolated portion is a free body in equilibrium under the action of bar forces, applied forces and support reactions. In case of method of section of three dimensional space trusses, six independent equations of static can be written as follows:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0, \quad \sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$$

A necessary (although not sufficient) condition for statical determination of three dimensional truss with respect to both inner and outer forces is that the total number



Methods

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A necessary (although not sufficient) condition for statical determination of three dimensional truss with respect to both inner and outer forces is that the total number of bars plus the total number of independent reaction components shall equal three times the number of joints.

In general

Methods

Method of Joints

Three equilibrium equations are applied in each joint to determine bar forces.

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0,$$

Method of Sections

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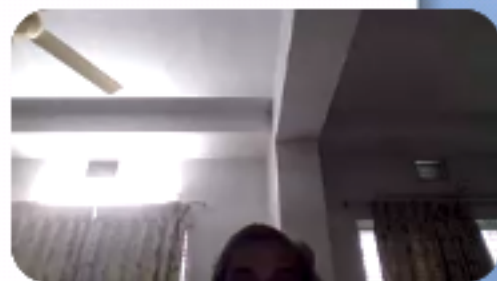
≡

A necessary (although not sufficient) condition for statical determination of three dimensional truss with respect to both inner and outer forces is that the total number of bars plus the total number of independent reaction components shall equal three times the number of joints.

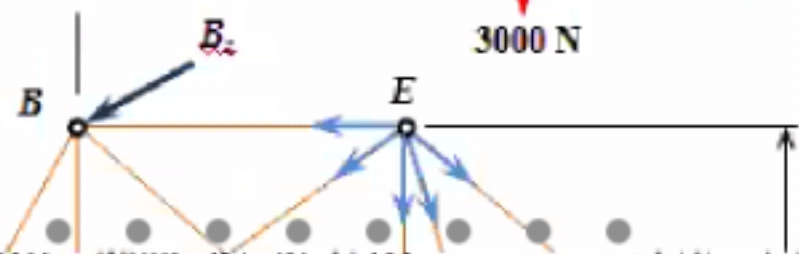
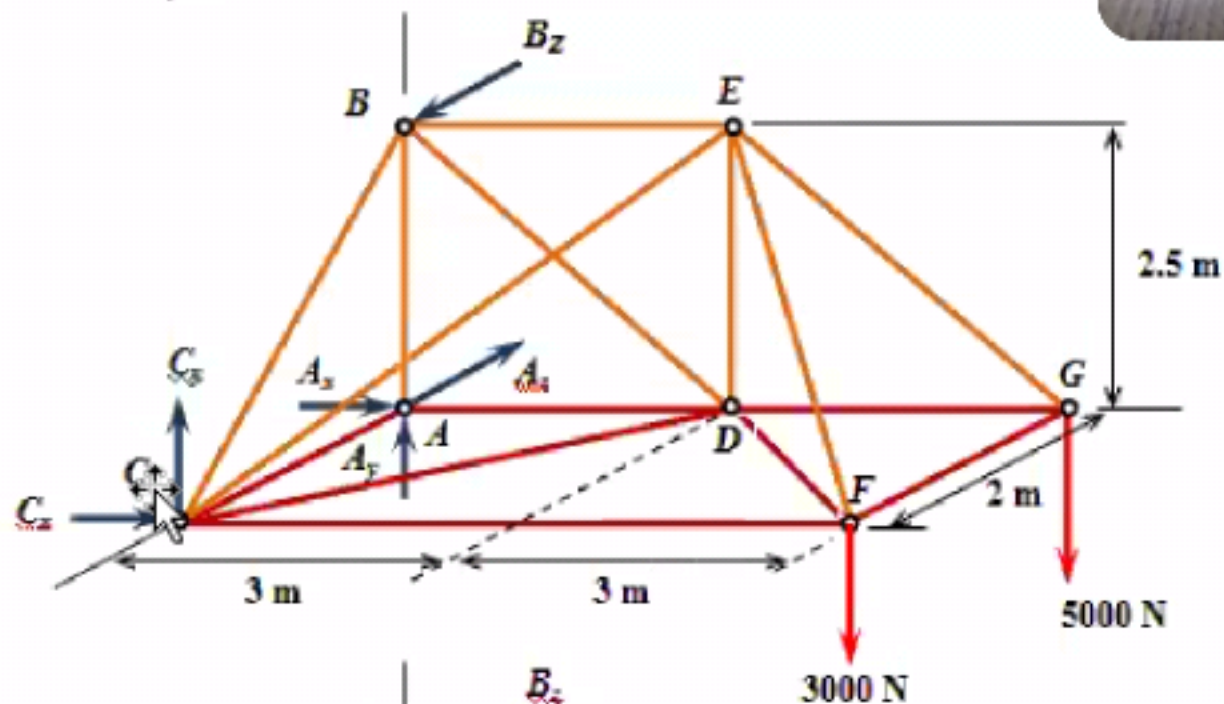
In general

$b + r < 3n$, the structure is unstable.

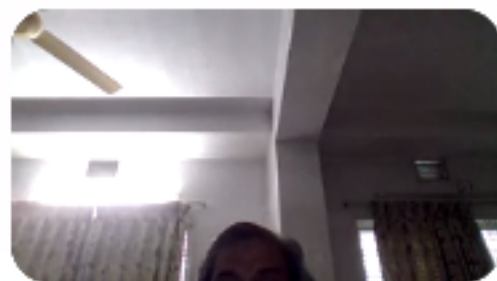
$b + r = 3n$, the structure is statically determinate.



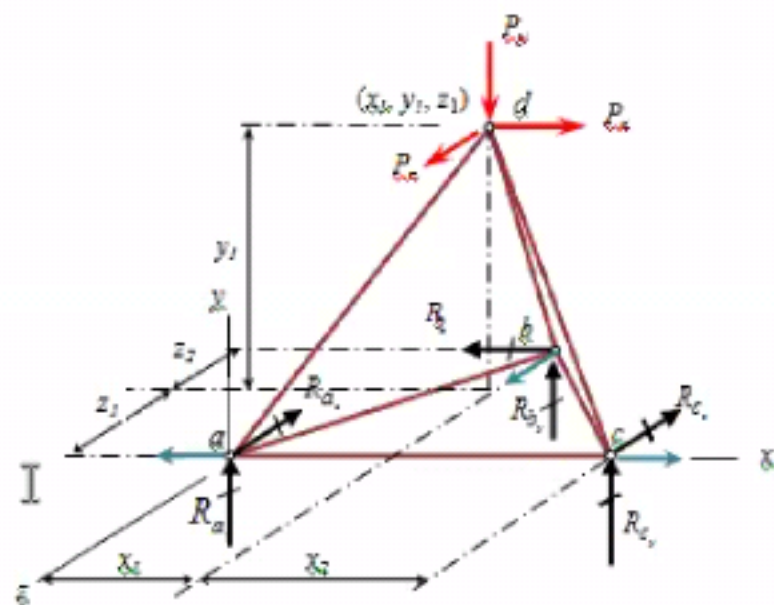
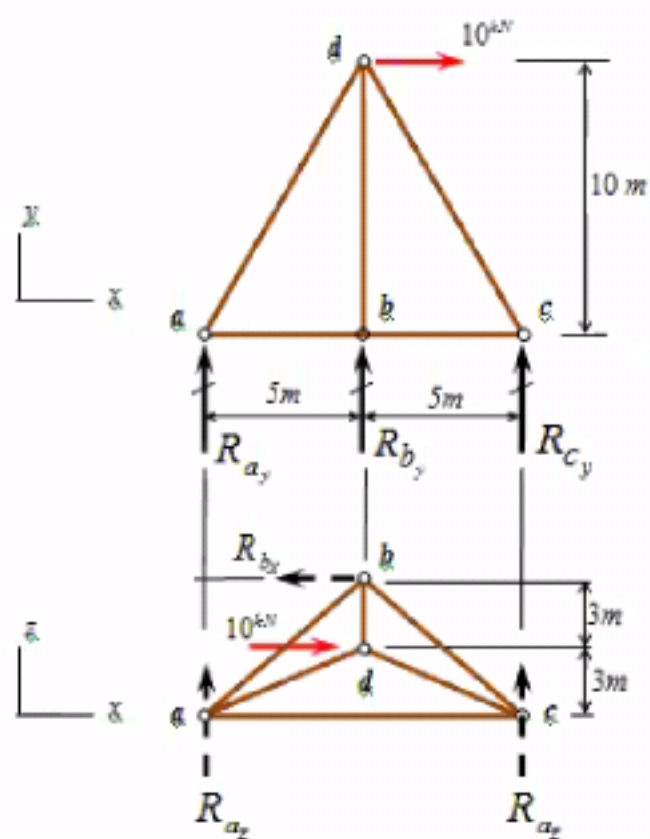
3. Find force in bar EC, CF and BE.



$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}} DE + \frac{3}{\sqrt{3^2 + 2^2}} DF + FG = 0; \quad DE = -1200 \text{ N}$$

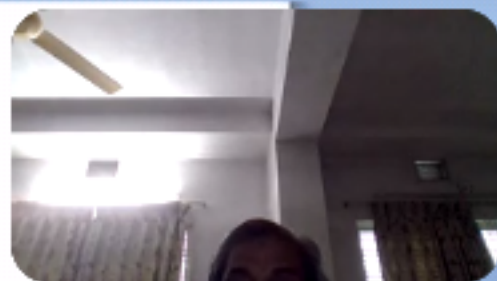
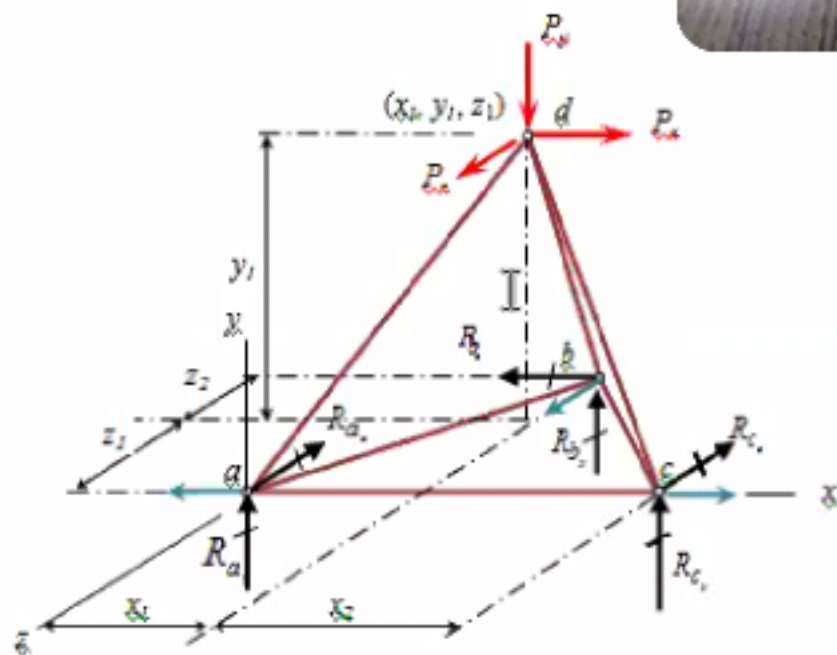
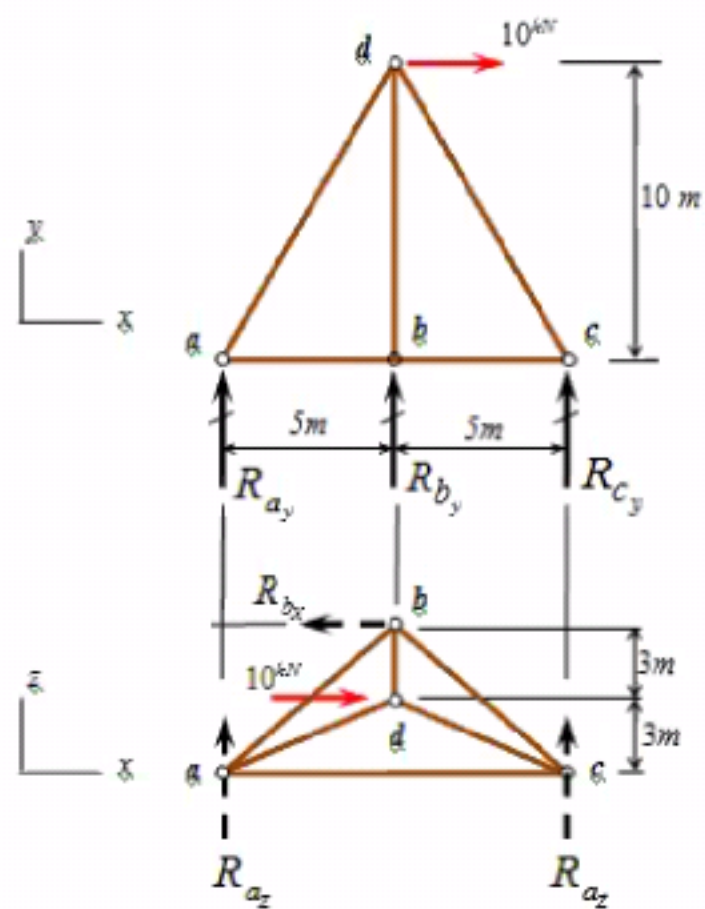


Example 1 Determine reactions and bar forces of the following space truss.



Member	Projection			length	x-slope	y-slope	z-slope
	x	y	z				
1	5	10	0	11.180	0.4472	0.8944	0

Example 1 Determine reactions and bar forces of the following space truss.



Member

Projection

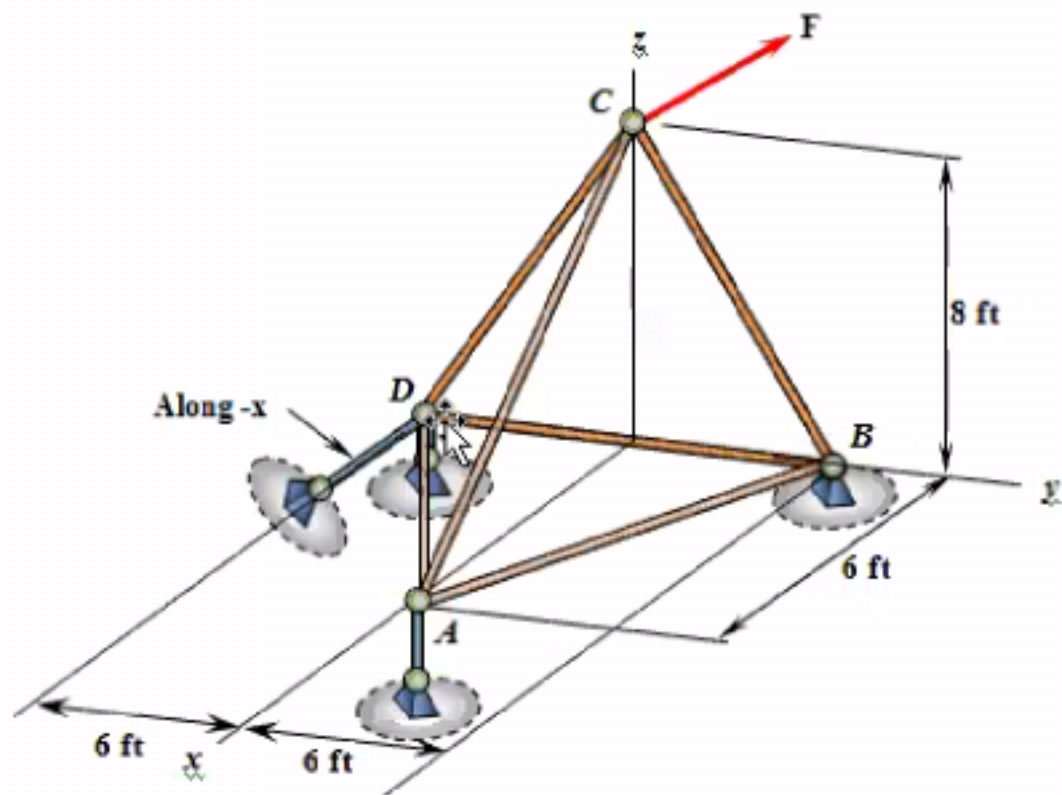
length

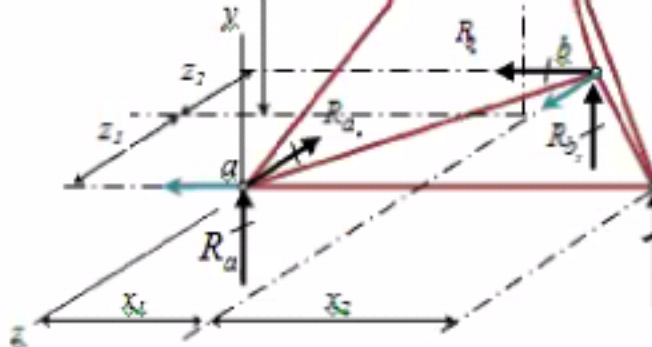
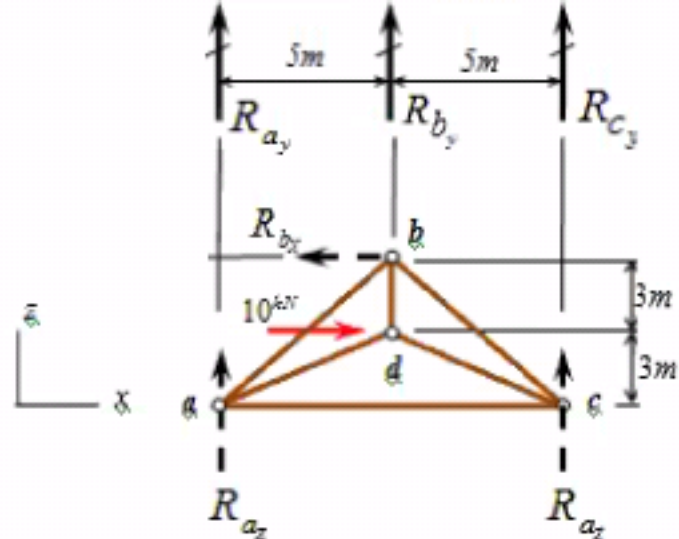
x-slope

y-slope

z-slope

8. The space truss is shown below. Determine nature and magnitude of force in each member. (i) $\mathbf{F} = [-500\mathbf{i} + 600\mathbf{j} + 400\mathbf{k}]$ and (ii) $\mathbf{F} = [600\mathbf{i} + 450\mathbf{j} - 750\mathbf{k}]$





Member	Projection			length	x-slope	y-slope	z-slope
	x	y	z				
<u>ad</u>	5	10	3	11.58	0.4318	0.8636	0.2591
<u>bd</u>	0	10	3	10.44	0	0.9579	0.2874
<u>cd</u>	5	10	3	11.58	0.4318	0.8636	0.2591
<u>ab</u>	5	0	6	7.81	0.6402	0	0.7682
<u>bc</u>	5	0	6	7.81	0.6402	0	0.7682
<u>ac</u>	10	0	0	10.00	1.00	0	0

For entire structure

$$\sum M_x = 0;$$

$$\rightarrow R_{b_x} = 0$$

For entire structure



R_{az} R_{az} 

Member	Projection			length	x-slope	y-slope	z-slope
	x	y	z				
<i>ad</i>	5	10	3	11.58	0.4318	0.8636	0.2874
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<i>bc</i>	5	0	6	7.81	0.6402	0	0.7682
<i>ac</i>	10	0	0	10.00	1.00	0	0

**For entire structure**

$$\sum M_x^{ac} = 0; \quad \text{I}$$

$$\rightarrow R_{by} = 0$$

$$\sum M_z^{ao} = 0;$$

$$\rightarrow 10 \cdot 10 - (R_{cy})(10) = 0;$$

$$\rightarrow R_{cy} = +10 \text{ kN } (\uparrow)$$

$$\sum F_y = 0;$$

$$\rightarrow R_{ay} + R_{by} + R_{cy} = 0;$$

$$\rightarrow R_{ay} = -10 \text{ kN } (\downarrow)$$

$$\sum F_x = 0;$$

$$\rightarrow 10 - R_{bx} = 0;$$

$$\rightarrow R_{bx} = +10 \text{ kN } (\leftarrow)$$

$$\sum M_y^o = 0;$$

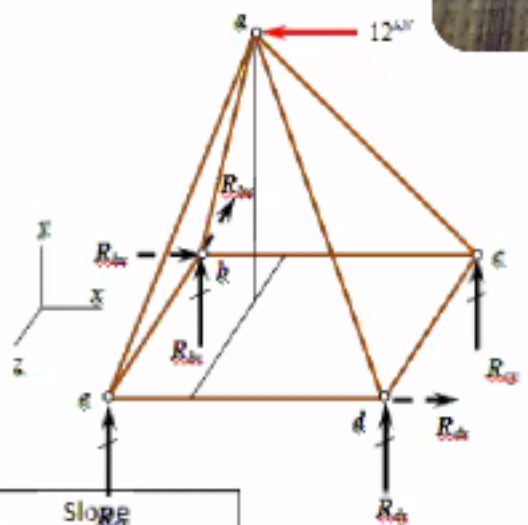
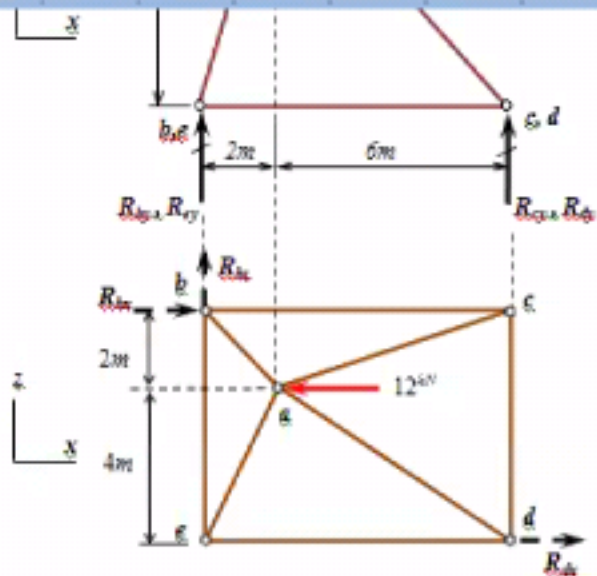
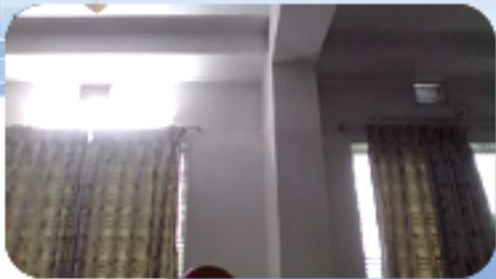
$$\rightarrow -10(3) - R_{cz}(10) = 0$$

$$\rightarrow R_{cz} = -3 \text{ kN } (\downarrow)$$

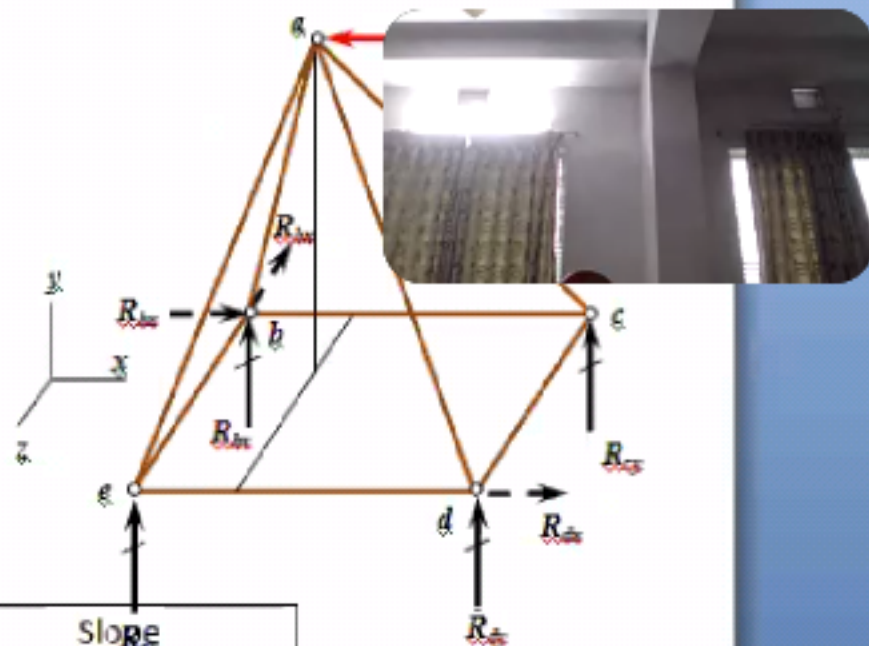
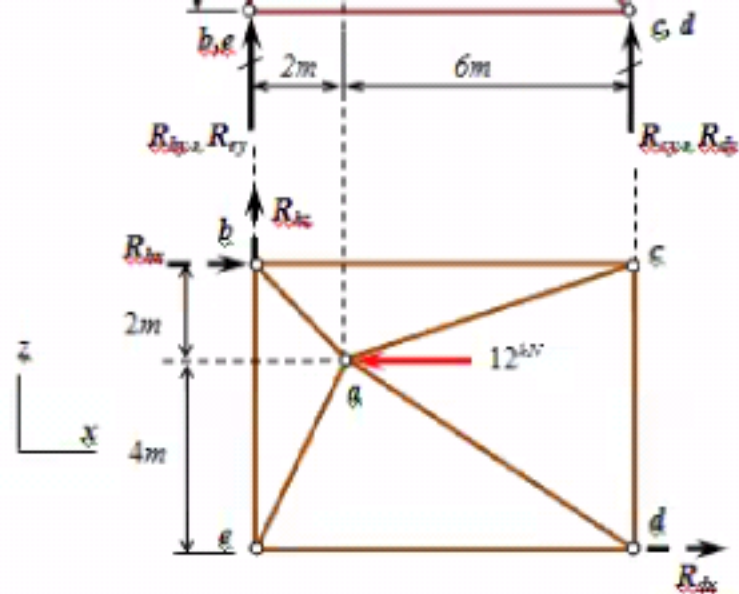
$$\sum F_z = 0;$$

$$\rightarrow R_{az} + R_{cz} = 0;$$

$$\rightarrow R_{az} = +3 \text{ kN } (\uparrow)$$



Member	Projection			Sum(Sq)	Length	Slope		
	x	y	z			x	y	z
<u>ab</u>	2	8	2	72	8.485	0.236	0.943	0.236
<u>ac</u>	6	8	2	104	10.198	0.588	0.784	0.196
<u>ad</u>	6	8	4	116	10.770	0.557	0.743	0.371
<u>ae</u>	2	8	4	84	9.165	0.218	0.873	0.436
<u>bc</u>	8	0	0	64	8.000	1.000	0.000	0.000
<u>cd</u>	0	0	6	36	6.000	0.000	0.000	1.000
<u>de</u>	8	0	0	64	8.000	1.000	0.000	0.000
<u>eb</u>	0	0	5	36	6.000	0.000	0.000	1.000



+

Member	Projection			Sum(Sq)	Length	Slope		
	x	y	z			x	y	z
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<u>eb</u>	0	0	6	36	6.000	0.000	0.000	1.000

obtain the solution

At Joint c,

$$\sum F_x = F_{acx} + F_{bc} = 0; F_{bc} = -(0.588) * (10.20) = -6.0 \text{ kN}(c)$$

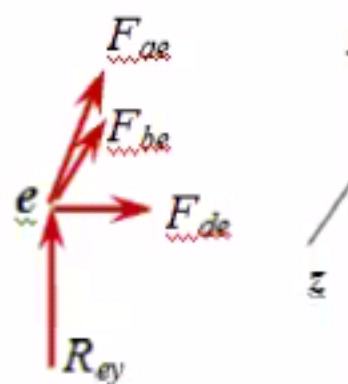
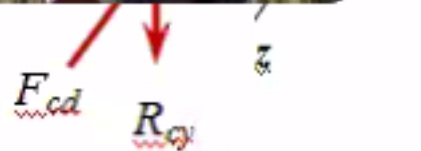
At Joint e,

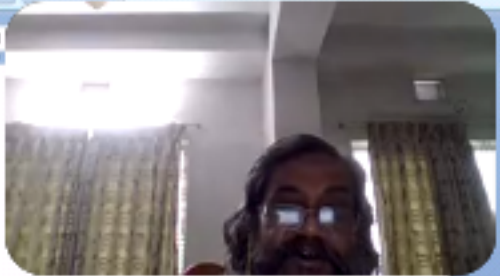
$$\sum F_y = R_{ey} + F_{aey} = +4 + 0.873F_{ae} = 0; F_{ae} = -4.58 \text{ kN}(c)$$

$$\sum F_z = F_{be} + F_{aesz} = F_{be} + 0.436(-4.58) = 0; F_{be} = +2.0 \text{ kN}(t)$$

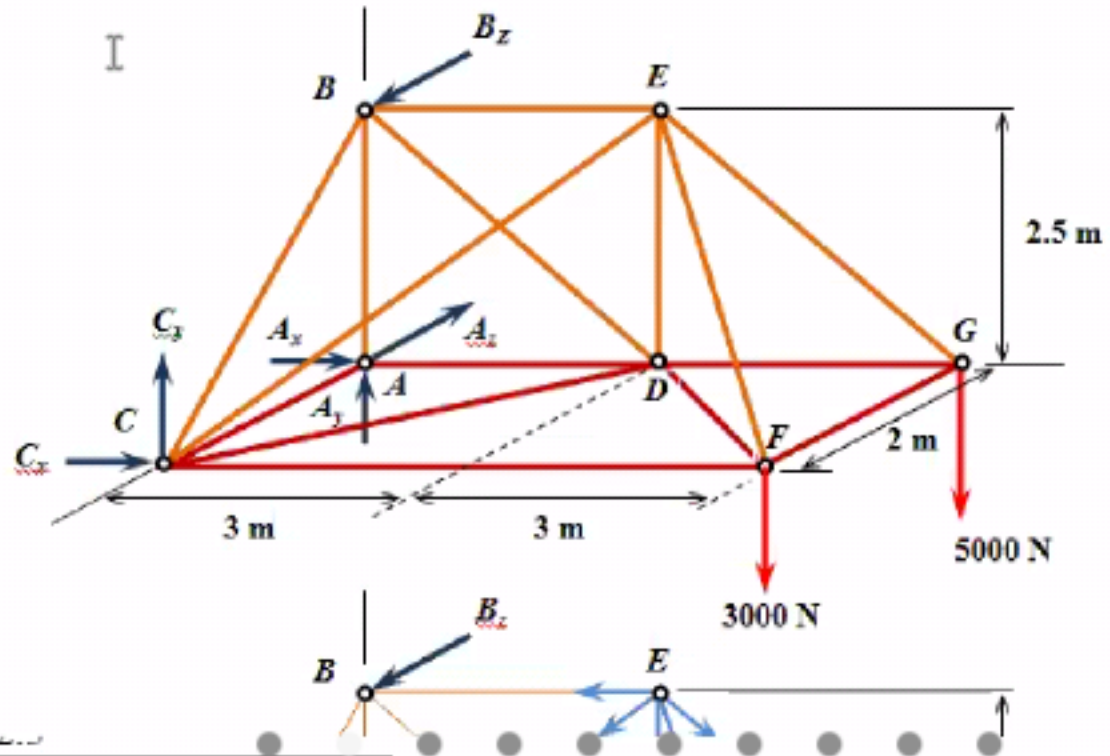
$$\sum F_x = F_{de} + F_{aex} = F_{de} + 0.218(-4.58) = 0; F_{de} = +1.0 \text{ kN}(t)$$

Free the joints b and d to obtain other non forces

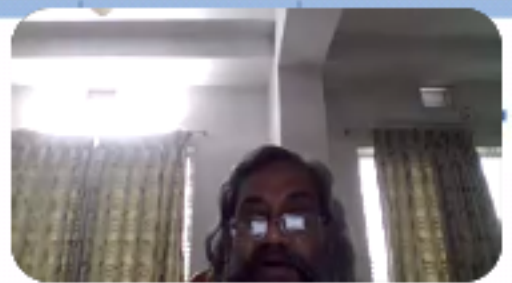
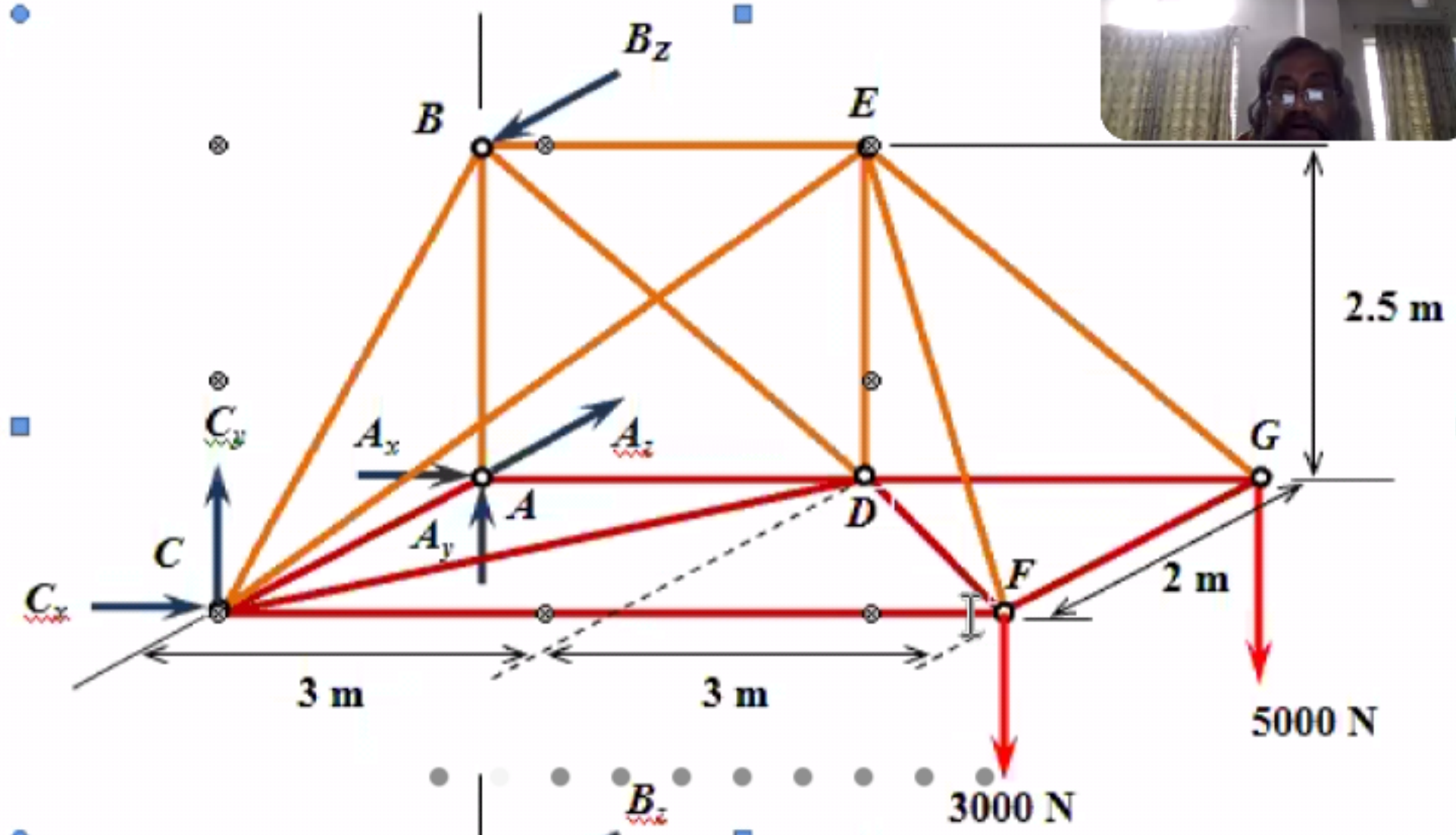


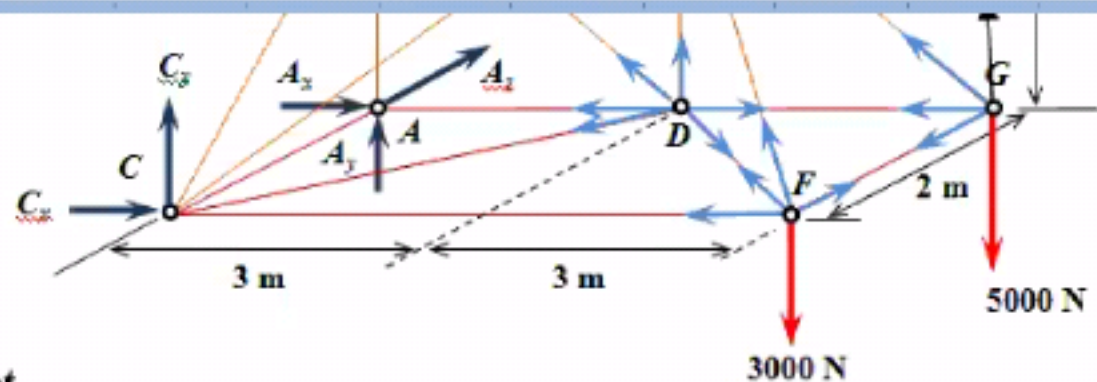


3. Find force in bar EC, CF and BE.



Force in bar EC, CF and BE.





Method of joint

At Joint G

$$\sum F_x = -\frac{3}{\sqrt{3^2 + 2.5^2}}GE - GD = 0; \quad GD = -0.604GE; \quad GD = -4717.38 \text{ N (c)}$$

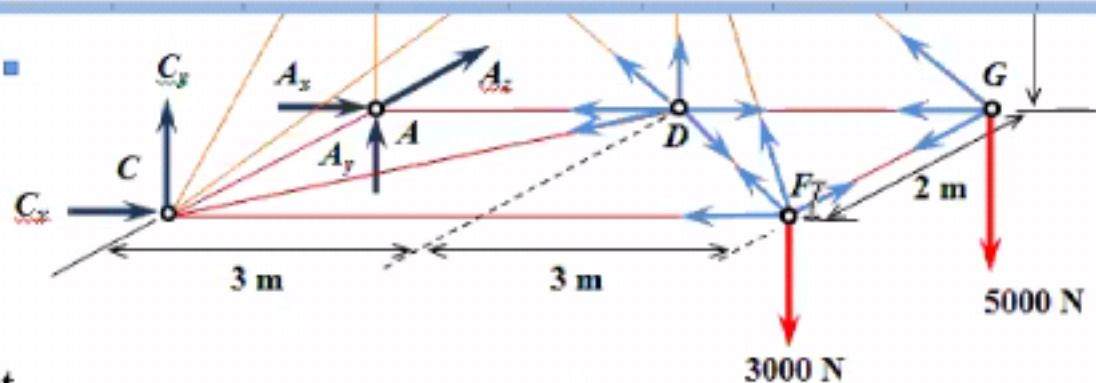
$$\sum F_y = -\frac{2.5}{3.905}GE + 5000 = 0; \quad GE = 2000\sqrt{15.25} \text{ N (t)}; \quad GE = +7810.24 \text{ N (t)}$$

$$\sum F_z = GF = 0; \quad GF = 0 \quad GF = 0$$

At Joint F

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + CF + \frac{3}{\sqrt{3^2 + 2^2}}DF = 0; \quad CF = 0$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}}EF - 3000 = 0; \quad EF = +1200\sqrt{19.25} \text{ N (t)}$$



Method of joint

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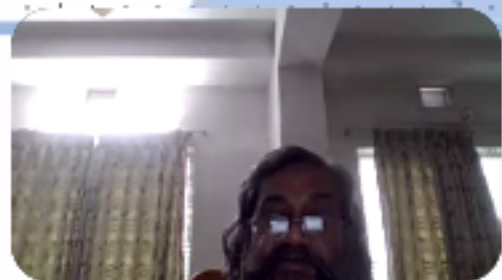
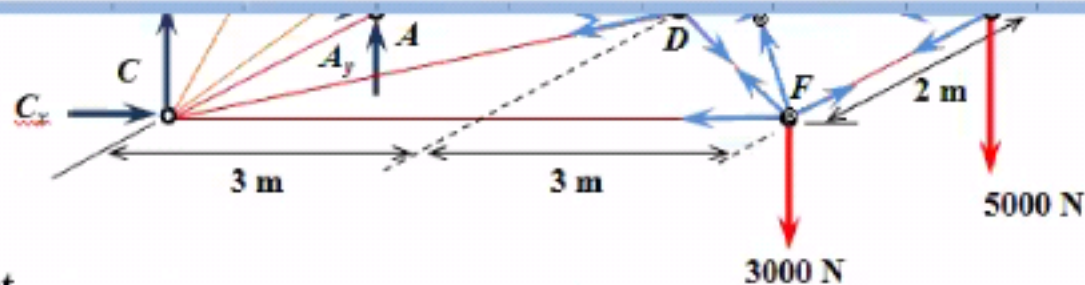
$$\sum F_z = GF = 0; \quad GF = 0 \quad GF = 0$$

At Joint F

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + CF + \frac{3}{\sqrt{3^2 + 2^2}}DF = 0; \quad CF = 0$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}}EF - 3000 = 0; \quad EF = +1200\sqrt{19.25} \text{ N (t)}$$

$$\sum F_z = \dots$$



Method of joint

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$$\sum F_x = -\frac{3}{\sqrt{3^2 + 2.5^2}}GE - GD = 0; \quad GD = -0.604GE; \quad GD = -4717.38 \text{ N (c)}$$

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$$\sum F_z = GF = 0; \quad GF = 0 \quad GF = 0$$

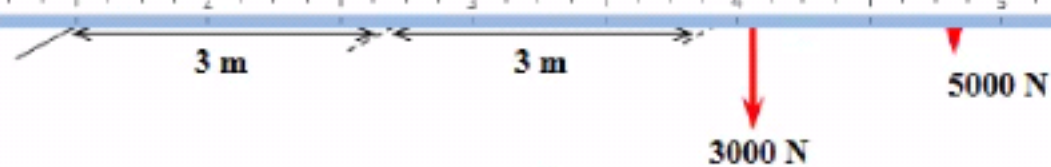
I

At Joint F

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + CF + \frac{3}{\sqrt{3^2 + 2^2}}DF = 0; \quad CF = 0$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}}EF - 3000 = 0; \quad EF = +1200\sqrt{19.25} \text{ N (t)}$$

$$\sum F_z = \frac{2}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + \frac{2}{\sqrt{3^2 + 2^2}}DF + FG = 0; \quad DF = -1200\sqrt{13} \text{ N}$$



Method of joint

At Joint G

$$\sum F_x = -\frac{3}{\sqrt{3^2 + 2.5^2}}GE - GD = 0; \quad GD = -0.604GE; \quad GD = -4717.38 \text{ N (c)}$$

$$\sum F_y = -\frac{2.5}{3.905}GE + 5000 = 0; \quad GE = 2000\sqrt{15.25} \text{ N (t)}; \quad GE = +7810.24 \text{ N (t)}$$

$$\sum F_z = GF = 0; \quad GF = 0 \quad GF = 0$$

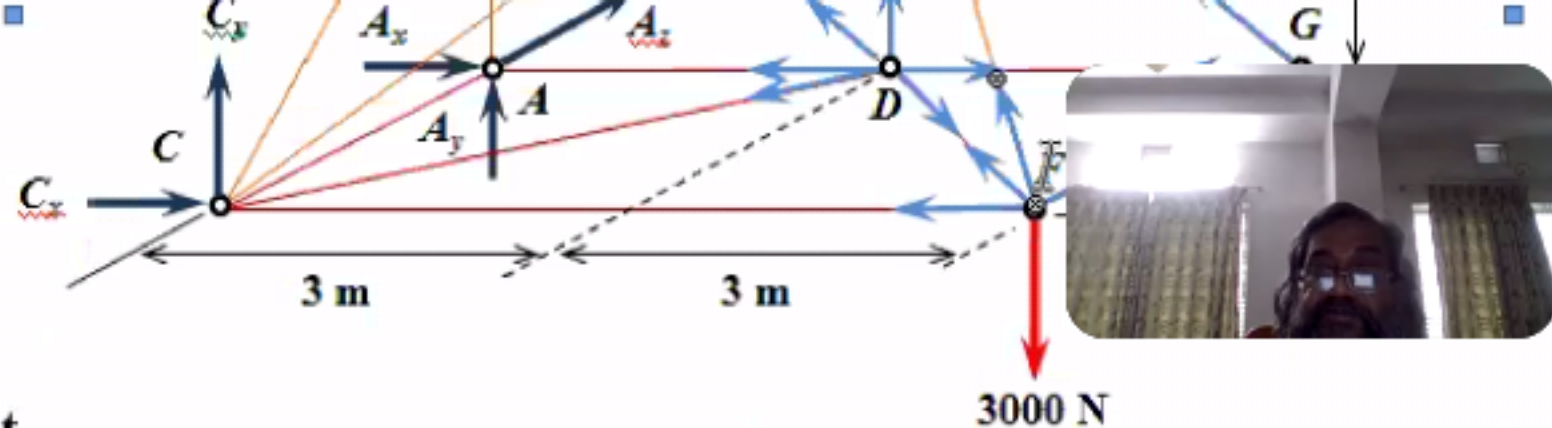
At Joint F

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + CF + \frac{3}{\sqrt{3^2 + 2^2}}DF = 0; \quad CF = 0$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}}EF - 3000 = 0; \quad EF = +1200\sqrt{19.25} \text{ N (t)}$$

$$\sum F_z = \frac{2}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + \frac{2}{\sqrt{3^2 + 2^2}}DF + FG = 0; \quad DF = -1200\sqrt{13} \text{ N}$$





Method of joint

At Joint G

$$\sum F_x = -\frac{3}{\sqrt{3^2 + 2.5^2}}GE - GD = 0; \quad GD = -0.604GE; \quad GD = -4717.38 \text{ N (c)}$$

$$\sum F_y = -\frac{2.5}{3.905}GE + 5000 = 0; \quad GE = 2000\sqrt{15.25} \text{ N (t)}; \quad GE = +7810.24 \text{ N (t)}$$

$$\sum F_z = GF = 0; \quad GF = 0 \quad GF = 0$$

At Joint F

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}}EF + CF + \frac{3}{\sqrt{3^2 + 2^2}}DF = 0; \quad CF = 0$$

$$\sum F_y = 3.905$$

$$\sum F_z = GF = 0; \quad GF = 0 \quad GF = 0$$

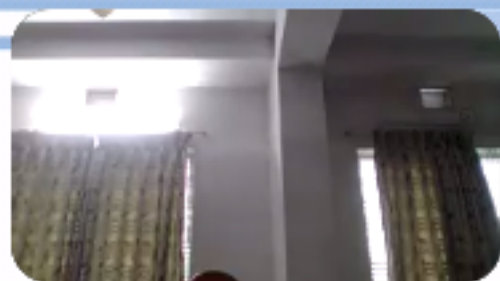
At Joint F

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}} EF + CF + \frac{3}{\sqrt{3^2 + 2^2}} DF = 0; \quad CF = 0$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}} EF - 3000 = 0; \quad EF = +1200\sqrt{19.25} N (\text{t})$$

$$\sum F_z = \frac{2}{\sqrt{3^2 + 2.5^2 + 2^2}} EF + \frac{2}{\sqrt{3^2 + 2^2}} DF + FG = 0; \quad DF = -1200\sqrt{13} N$$





At joint E

$$\sum F_x = \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}} (EF = 1200\sqrt{19.25}) + \frac{2.5}{\sqrt{2.5^2 + 3^2}} (EG = -2000\sqrt{15.25})$$

$$-BE - \frac{3}{\sqrt{3^2 + 2.5^2 + 2^2}} CE = 0; \quad BE = 2200 \text{ N}(t)$$

$$\sum F_z = \frac{2}{\sqrt{3^2 + 2.5^2 + 2^2}} CE + \frac{2}{\sqrt{3^2 + 2.5^2 + 2^2}} (EF = +1200\sqrt{19.25}) = 0; \quad CE = -1200\sqrt{19.25} \text{ N}$$

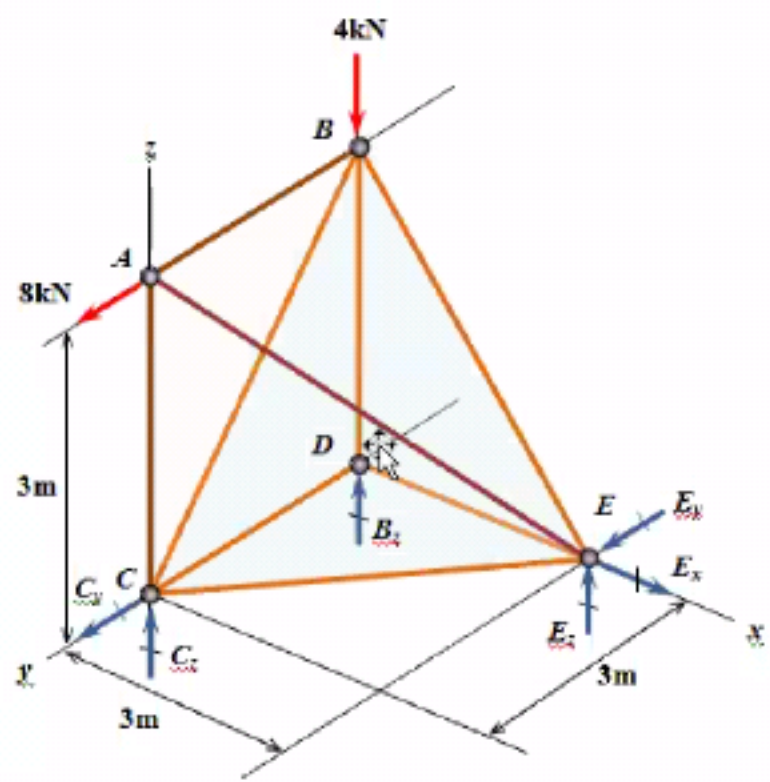
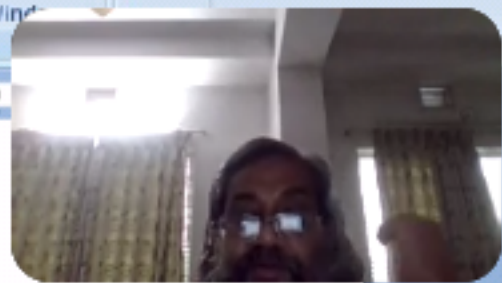
$$\sum F_y = \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}} (EF = 1200\sqrt{19.25}) + \frac{2.5}{\sqrt{3^2 + 2.5^2 + 2^2}} (CE = -1200\sqrt{19.25}) +$$

$$\frac{2.5}{\sqrt{3^2 + 2.5^2}} (EG = 2000\sqrt{15.25}) + ED = 0; \quad ED = -5000 \text{ N}(c)$$

Method of Sections



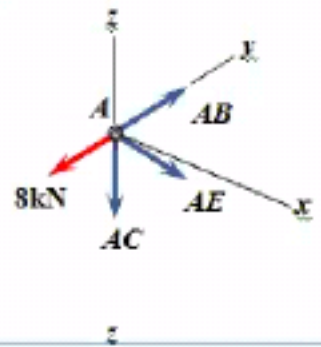
below: Hibbeler, p. 502



At joint A

$$\sum F_x = \frac{3}{\sqrt{3^2+3^2+3^2}} AE - 0 = 0; \quad AE = 0;$$

$$\sum F_y = 8 + \frac{3}{\sqrt{3^2+3^2+3^2}} (AE = 0) - B = 0 \quad AD = +8 \text{ kN (t)}$$



$$\sum F_z = \frac{3}{\sqrt{3^2 + 3^2 + 3^2}} AE - 0 = 0; \quad AE = 0;$$

$$\sum F_y = 8 + \frac{3}{\sqrt{3^2 + 3^2 + 3^2}} (AE = 0) - AB = 0; \quad AB = +8 \text{ kN}(t);$$

$$\sum F_z = \frac{3}{\sqrt{3^2 + 3^2 + 3^2}} (AE = 0) - AC = 0; \quad AC = 0;$$

At joint B

$$\sum F_x = \frac{3}{\sqrt{3^2 + 3^2}} BE - 0 = 0; \quad BE = 0;$$

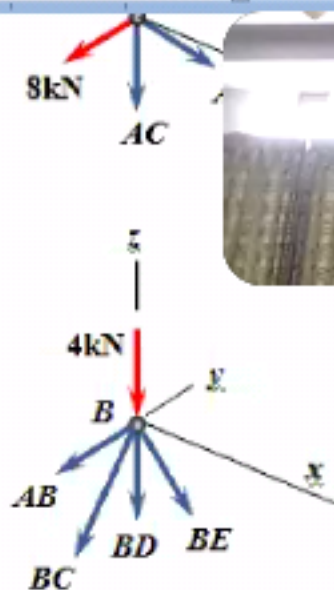
$$\sum F_y = -\frac{3}{\sqrt{3^2 + 3^2}} BC - (AB = 8) = 0; \quad BC = AB\sqrt{2} = -8\sqrt{2} \text{ kN}(c);$$

$$\sum F_z = -\frac{3}{\sqrt{3^2 + 3^2}} (BC = -8\sqrt{2}) - BD - \frac{3}{\sqrt{3^2 + 3^2}} (BE = 0) - 4 = 0; \quad BD = +4 \text{ kN}(t)$$

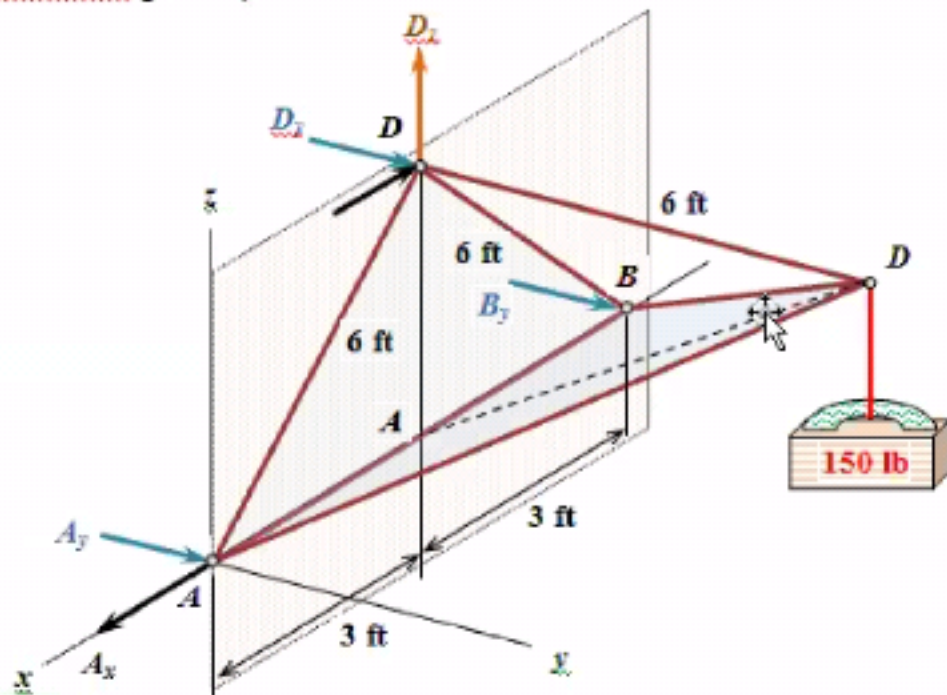
From equilibrium equations at joint D,

$$DC = 0, \quad BE = 0,$$

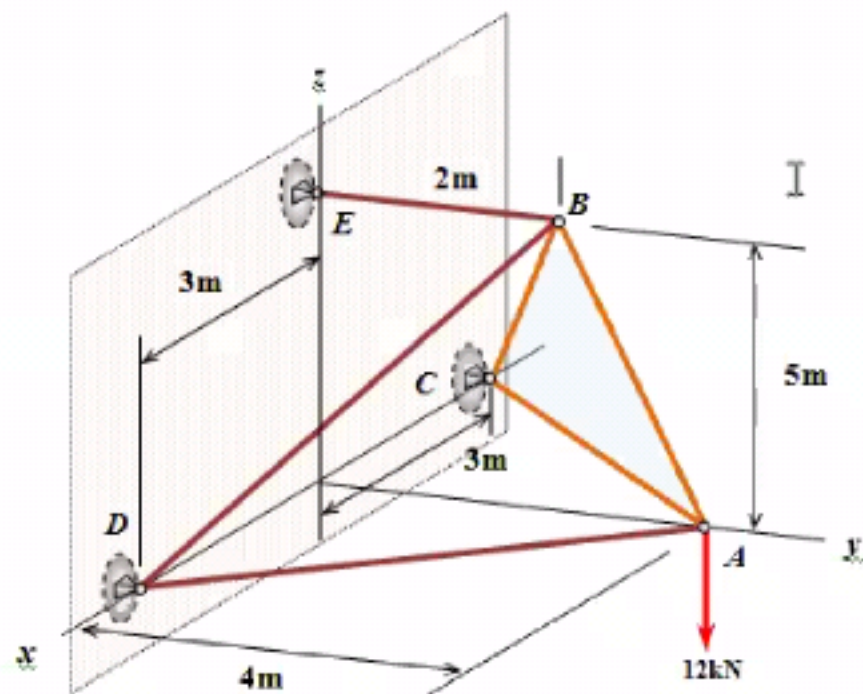
From equilibrium equations at joint C, $CE = 0,$



5. Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb (Hibbeler p. 303)

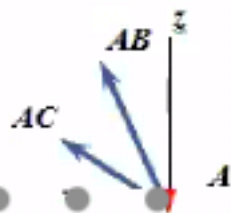


6. Find forces in all members. Hibbeler, p. 303

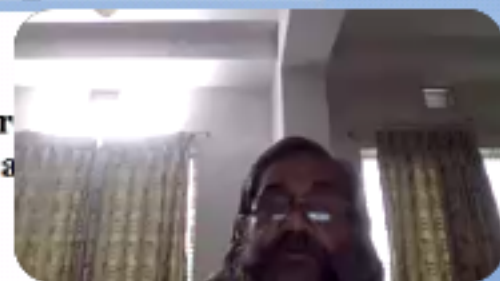
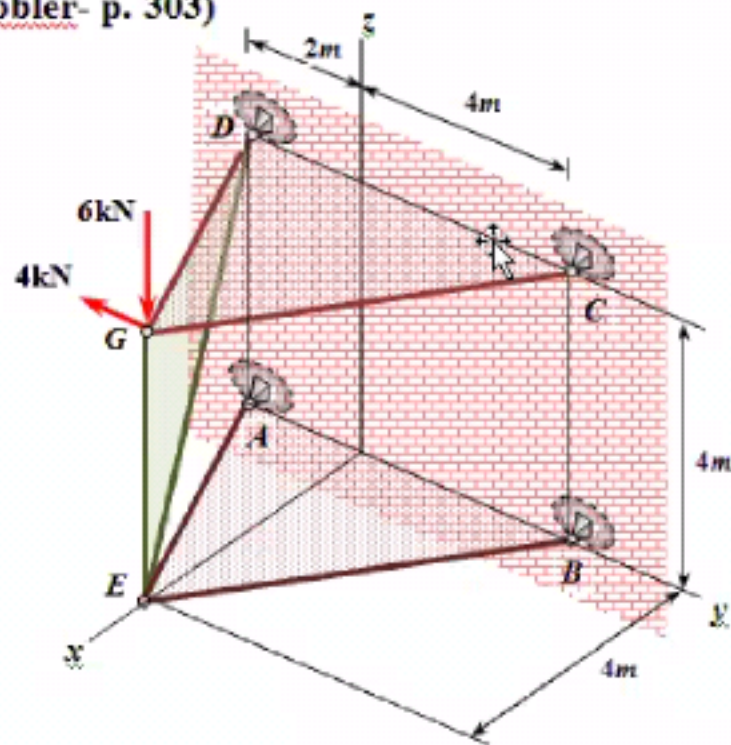


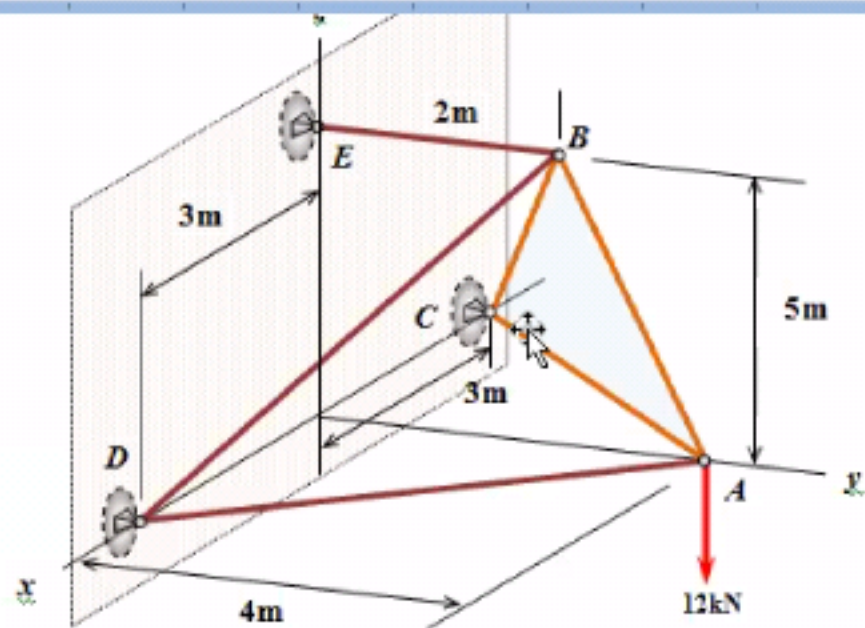
At Joint A

$$\sum F_x = \frac{3}{\sqrt{3^2 + 4^2}} AD - \frac{3}{\sqrt{3^2 + 4^2}} AC = 0; \quad AD = AC$$



7. Determine the force in member of the space truss and state if the member are in tension or compression. The truss is supported by ball and socket joint at A , B , C , and D . (Hibbler- p. 303)



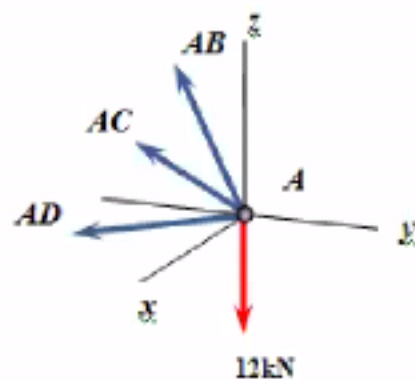


At Joint A

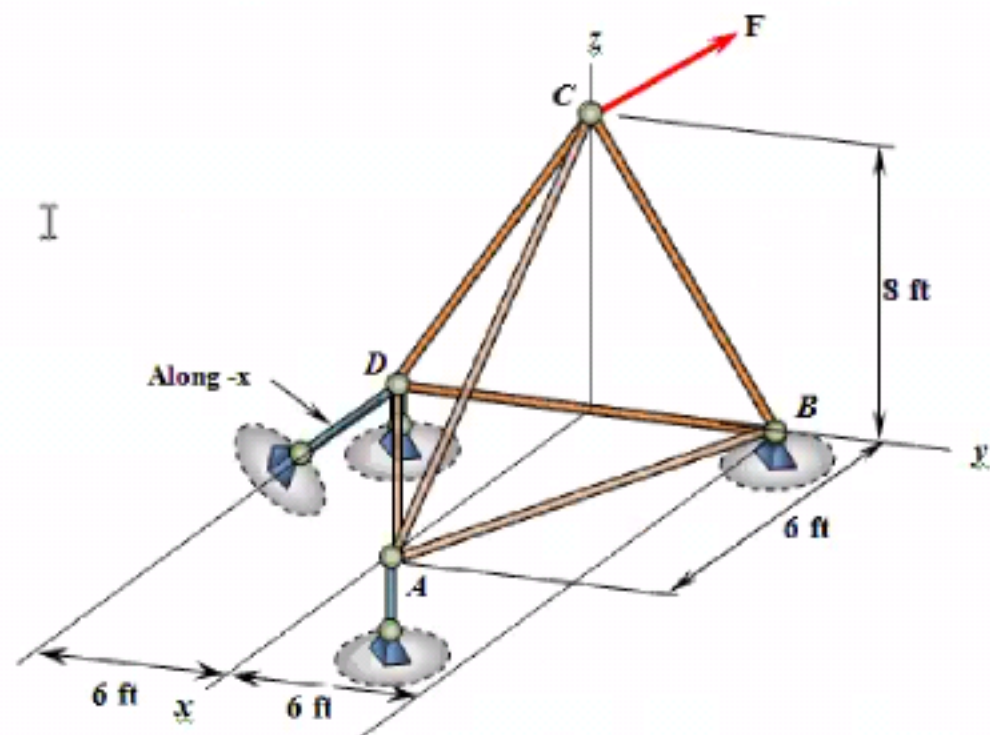
$$\sum F_x = \frac{3}{\sqrt{3^2+4^2}} AD - \frac{3}{\sqrt{3^2+4^2}} AC = 0; \quad AD = AC$$

$$\sum F_y = \frac{4}{\sqrt{3^2+4^2}} AD + \frac{4}{\sqrt{3^2+4^2}} AC + AB \frac{2}{\sqrt{5^2+2^2}} = 0;$$

$$\sum F_z = AB \frac{5}{\sqrt{5^2+2^2}} - 12 = 0; \quad AB = 2.4\sqrt{29} = 12.92 \text{ kN}(t)$$



8. The space truss is shown below. Determine nature and magnitude of force in each member. (i) $F = [-500i + 600j + 400k]$ and (ii) $F = [600i + 450j - 750k]$



At Joint c,

$$\sum F_x = F_{ac} + F_{bc} = 0; F_{bc} = -(0.588) * (10.20) = -6.0 \text{ kN}(c)$$

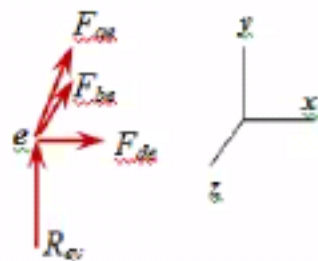


At Joint e,

$$\sum F_y = R_{ey} + F_{ae} = +4 + 0.873F_{ae} = 0; F_{ae} = -4.58 \text{ kN}(c)$$

$$\sum F_z = F_{be} + F_{aez} = F_{be} + 0.436(-4.58) = 0; F_{be} = +2.0 \text{ kN}(t)$$

$$\sum F_x = F_{de} + F_{aex} = F_{de} + 0.218(-4.58) = 0; F_{de} = +1.0 \text{ kN}(t)$$



Free the joints b and d to obtain other non forces

