Criteria for the maximum reaction of a simple beam subjected to series of concentrated loads move from right to left.

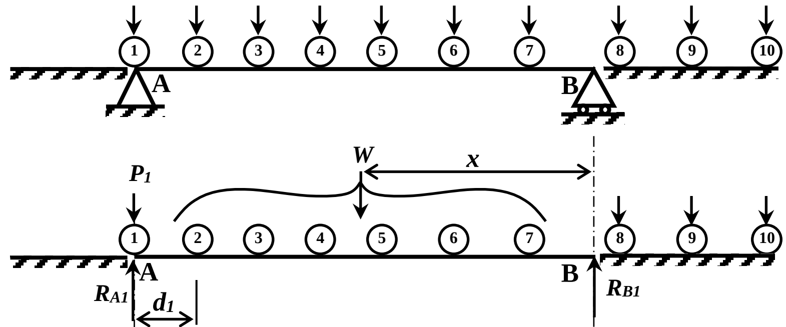


Fig. 1(a)

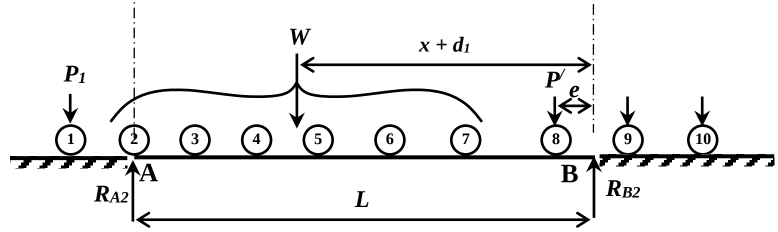


Fig. 1(a)

Let, P_1 = the load which was over the left support and is moved off the span after movement.

d_1 = the distance between P_1 and the following wheel.

L = span length of the beam.

$W = \sum P$ = the sum of all the loads which are on the span before movement and stay on during movement.

$P^/$= the load which enters in the span after the movement.

e= the distance of load $P^/$ from right support at B.

From Fig. 1(a), before the movement of wheels,

$$\sum M_B = R_{A1}L - Wx - P_1 L = 0$$

or, $$R_{A1} = \frac{Wx}{L} + P_1 \qquad (1)$$

From Fig. 1(b), after the movement of wheels,

$$\sum M_B = R_{A2}L - W(d_1 + x) - P^/ e = 0$$

or, $$R_{A2} = \frac{W(d_1 + x)}{L} + \frac{P^/ e}{L} \qquad (2)$$

The change in reaction at support A due to the movement of the wheel can be obtained by subtracting Eq. (1) from Eq. (2), then equation is given by

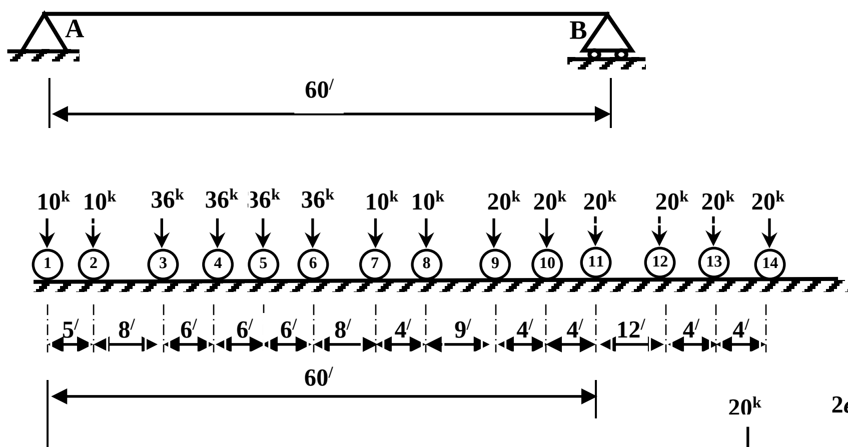
$$\Delta R = R_{A2} - R_{A1} = \frac{W(d_1 + x)}{L} + \frac{P^/ e}{L} - \frac{Wx}{L} - P_1$$

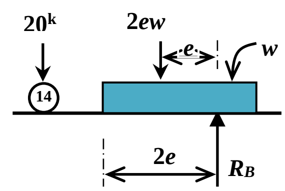
or, $$\Delta R = \frac{Wd_1}{L} + \frac{P^/ e}{L} - P_1 \qquad (3)$$

or, $$\Delta R = \frac{\sum P d_1}{L} + \frac{P^{/} e}{L} - P_1 \qquad (4)$$

N.B. In the above expression, P_1 is the first wheel as shown in Fig. 1. However, for the subsequent action wheel 2 must be replaced by P_1. Similarly, P_8 is $P^{/}$ in the expression which is outside of span before movement and enters in the span after movement. The wheel 8 must be replaced by wheel 9. This procedure continues till the ΔR changes its sign of character.

Example1





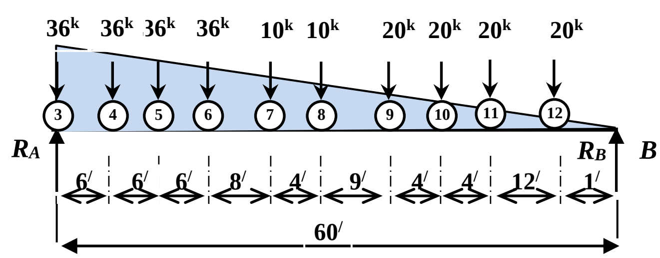
Criteria for maximum reaction

$$\Delta R = \frac{\sum P d_1}{L} + \frac{P^{/} e}{L} - P_1$$

Wheel 1-2 $\quad \Delta R = \frac{234 \times 5}{60} + 0 - 10 = an\ increase$

Wheel 2-3 $\quad \Delta R = \frac{224 \times 8}{60} + 20 \times \frac{1}{60} - 10 = an\ increase$

Wheel 3-4 $\quad \Delta R = \frac{208 \times 6}{60} + 20 \times \frac{3}{60} - 36 = a\ decrease$



$$R_A = \frac{1}{60}\left[1 \times 20 + 13 \times 20 + 17 \times 20 + 21 \times 20 + 30 \times 10 + 34 \times 10 + 42 \times 36 + 48 \times 36 + 54 \times 36 + 60 \times 36\right]$$

= 150.4 kips

Criteria for the maximum shear of a simple beam subjected to series of concentrated loads move from right to left.

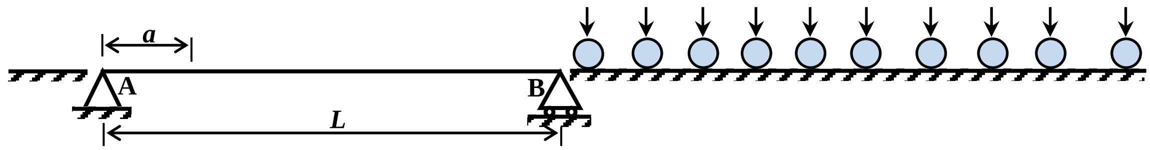


Fig. 2(a)

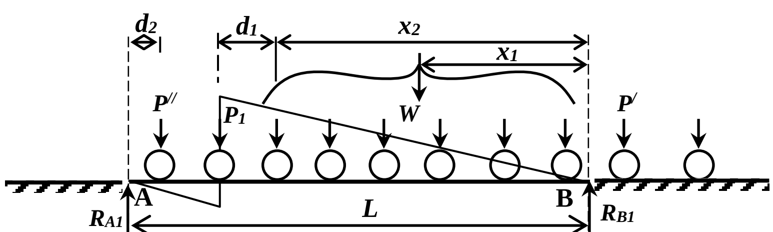


Fig. 2(b)

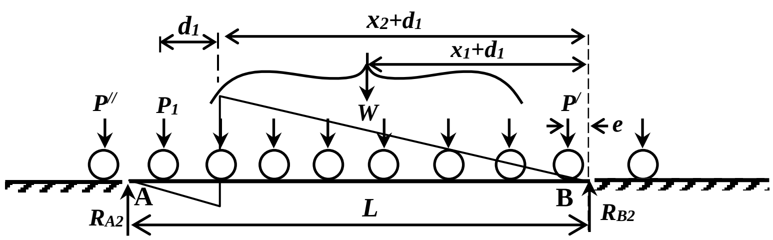


Fig. 2(c)

A simply supported beam of span length *L* is shown in Fig. 2(a). The beam is subjected to series moving loads. Loads move from right to left and pass through a section at a distance '*a*' from the left support. It is required to determine the criteria to obtain the position of loads for maximum shear. Fig. 2(b) and Fig. 2(c) show the position wheel loads before and after movement.

Let, P_1 = the load which was over section and is moved off the section after movement.
$P^{/}$ = the load which was off the span before movement and enter in the span after movement.
$P^{//}$ = the load which was in negative shear zone before movement and is moved off the span after movement.
d_1 = the distance between P_1 and the following wheel.
d_2 = the distance of wheel load $P^{//}$ from left support.
L = span length of the beam.
$W = \sum P$ = the sum of all the loads which are on the span before movement and stay on during movement.

e = the distance of load $P^{/}$ from right support at B.

From Fig. 2(b), the reaction at left support before the movement of wheels R_{A1} can be obtained as follows:

$$\sum M_B = R_{A1}L - P^{//}(L-d_2) - Wx_1 - P_1(d_1+x_2) = 0$$

or, $$R_{A1} = \frac{Wx_1}{L} + P_1\frac{(d_1+x_2)}{L} + P^{//}\frac{(L-d_2)}{L}$$

Now, shear force of the section,

$$V_1 = R_{A1} - P^{//}$$

$$V_1 = \frac{Wx_1}{L} + P\frac{(d_1+x_2)}{L} + P^{//}\frac{(L-d_2)}{L} - P^{//}$$

$$V_1 = \frac{Wx_1}{L} + P\frac{(d_1+x_2)}{L} - \frac{P^{//}d_2}{L} \qquad (1)$$

From Fig. 2(c), the reaction at left support after the movement of wheels R_{A2} can be obtained as follows:

$$\sum M_B = R_{A2}L - W(d_1+x_1) - P_1(2d_1+x_2) - P^{/}e = 0$$

or, $$R_{A2} = \frac{W(d_1+x_1)}{L} + P_1\frac{(2d_1+x_2)}{L} + P^{/}\frac{e}{L}$$

Now, shear force of the section,

$$V_2 = R_{A2} - P_1$$

$$V_2 = \frac{W(d_1+x_1)}{L} + P_1\frac{(2d_1+x_2)}{L} + P^{/}\frac{e}{L} - P_1 \qquad (2)$$

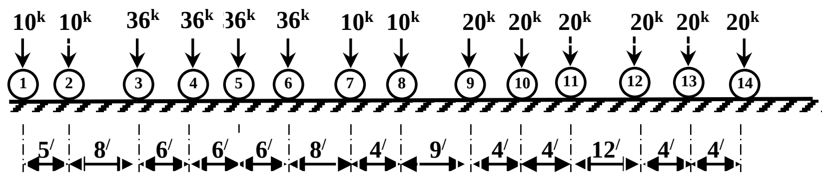
The change in shear at this section due to the movement of the wheel can be obtained by subtracting Eq. (1) from Eq. (2),

$$\Delta V = V_2 - V_1 = \frac{W(d_1+x_1)}{L} + P_1\frac{(2d_1+x_2)}{L} + P^{/}\frac{e}{L} - P_1 - \frac{Wx_1}{L} - P\frac{(d_1+x_2)}{L} + \frac{P^{//}d_2}{L}$$

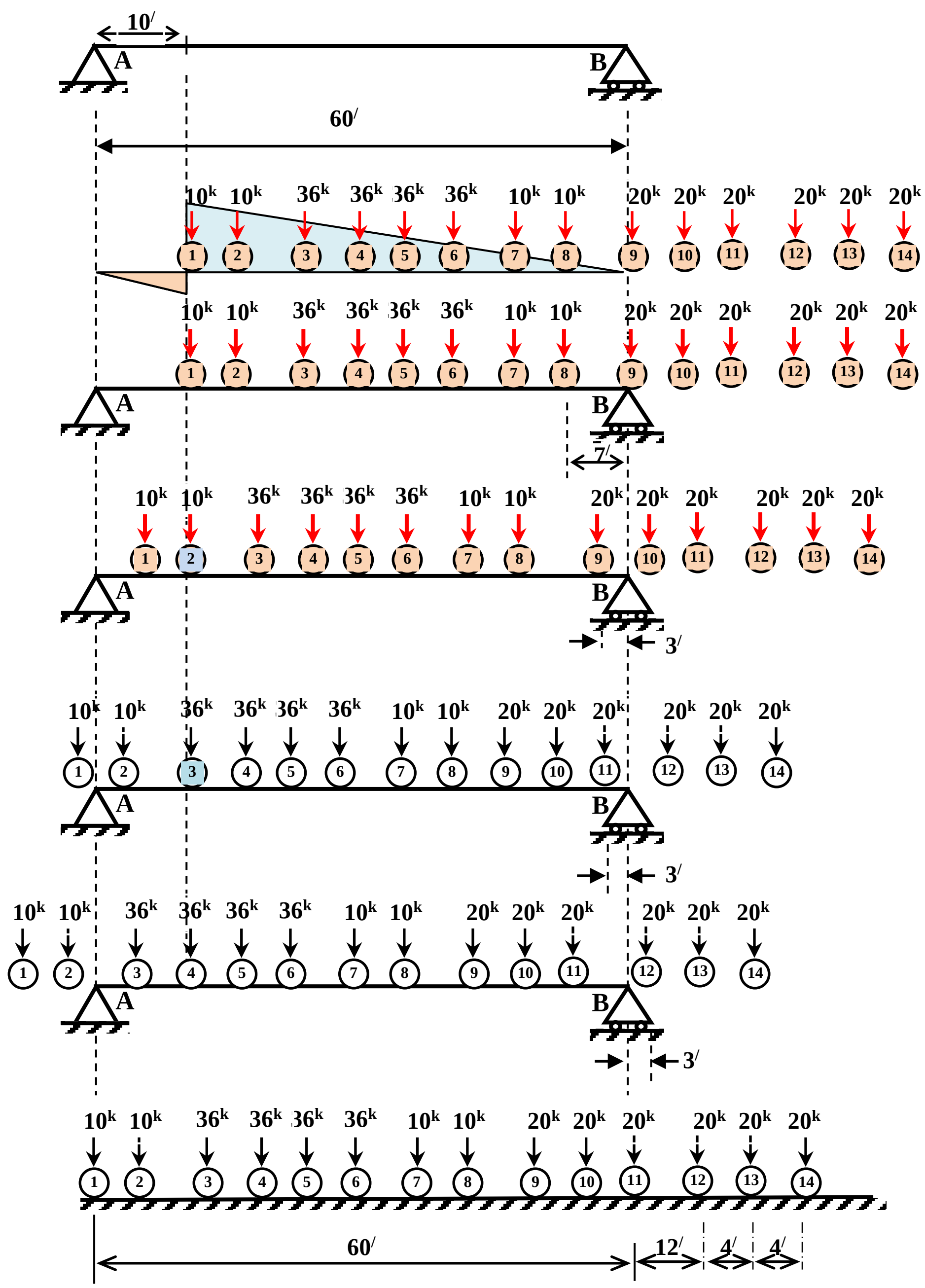
or, $$\Delta V = \frac{(W+P_1)d_1}{L} + \frac{P^{//}d_2}{L} + \frac{P^{/}e}{L} - P_1 \qquad (3)$$

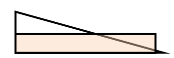
or, $$\Delta V = \frac{\sum Pd_1}{L} + \frac{P^{//}d_2}{L} + \frac{P^{/}e}{L} - P_1 \qquad (4)$$

The maximum shear force at this section will occur when ΔV tends to zero. This procedure continues till the ΔV changes its sign of character.


10k 10k 36k 36k 36k 36k 10k 10k 20k 20k 20k 20k 20k 20k
1 2 3 4 5 6 7 8 9 10 11 12 13 14
5/ 8/ 6/ 6/ 6/ 8/ 4/ 9/ 4/ 4/ 12/ 4/ 4/

Example 2


10/
A
B
60/
7/
3/
3/
3/
60/
12/
4/
4/



Wheel 1 at section to wheel 2 at section

$$\Delta V = \frac{\sum P d_1}{L} + \frac{P^{//} d_2}{L} + \frac{P^{/} e}{L} - P_1$$

$\sum P = 184k$, d_1 = 5$^{/}$, $P^{//}$= 0.0, d_2 = 0.0, $P^{/}$= 20k, e = 3, P_1=10k

$$\Delta V = \frac{184\times5}{60} + \frac{0\times0}{60} + \frac{20\times3}{60} - 10 = 6.33 = \textit{an increase.}$$

Wheel 2 at section to wheel 3 at section

$\sum P = 194k$, d_1 = 8$^{/}$, $P^{//}$= 10k, d_2 = 5$^{/}$, $P^{/}$= 20k+20k=40k, e = (3+7)/2 =5$^{/}$, P_1=10k

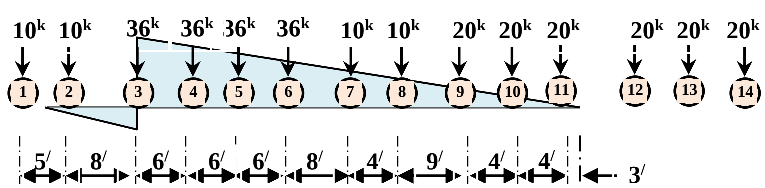
$$\Delta V = \frac{194\times8}{60} + \frac{10\times5}{60} + \frac{40\times5}{60} - 10 = 20.03 = \textit{an increase.}$$

Wheel 3 at section to wheel 4 at section

$\sum P = 224k$, d_1 = 6$^{/}$, $P^{//}$= 10k, d_2 = 2, $P^{/}$= 0, e = 0, P_1=36k

$$\Delta V = \frac{224\times6}{60} + \frac{10\times2}{60} + \frac{0\times0}{60} - 36 = -13.27 = \textit{a decrease.}$$

Therefore, **wheel 3** will produce maximum shear at the section.



$$V_{\max} = \frac{0.83}{50}\begin{bmatrix} 3\times20 + 7\times20 + 11\times20 + 20\times10 + 24\times10 + 32\times36 + 38\times36 \\ +44\times36 + 50\times36 \end{bmatrix} - \frac{0.17}{10}[2\times10]$$

= 111.94k

Criteria for the maximum moment at section of a simple beam subjected to series of concentrated loads move from right to left.

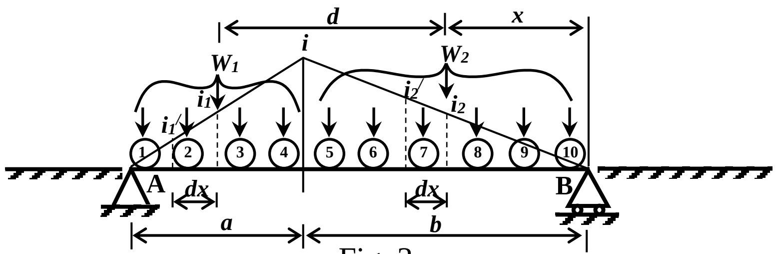


Fig. 3

Let, W_1 = load to the left of the section

W_2 = load to the right of the section

W = total load in the span

i = ordinate of influence line for bending moment at the section

i_1, i_2 = are the ordinates of influence line for bending moment under load W_1 and W_2 before movement.

$i_1^{/}$, $i_2^{/}$ = are the ordinates of influence line for bending moment under load W_1 and W_2 after movement.

Considering the right side of the section as shown in Fig. 3

$$i_2 = \frac{x}{b} i\,,\ \ i_2^{/} = \frac{(x+dx)}{b} i\,,$$

Corresponding moment, $M_2 = i\frac{x}{b}W_2\,,\ \ M_2^{/} = i\frac{(x+dx)}{b}W_2$

Increase of moment in the right hand side, $\Delta M_2 = M_2^{/} - M_2 = i\frac{W_2}{b}dx$

Considering the left side of the section

$$i_1 = \frac{(L-x-d)}{a} i\,,\ \ i_1^{/} = \frac{(L-x-d-dx)}{a} i\,,$$

Corresponding moment, $M_1 = i\frac{(L-x-d)}{a}W_1\,,\ \ M_1^{/} = i\frac{(L-x-d-dx)}{a}W_1$

Decrease of moment in the left hand side, $\Delta M_1 = M_1^{/} - M_1 = -i\frac{W_1}{a}dx$

Net increase in moment, $dM = \Delta M_1 + \Delta M_2 = -i\frac{W_1}{a}dx + i\frac{W_2}{b}dx$

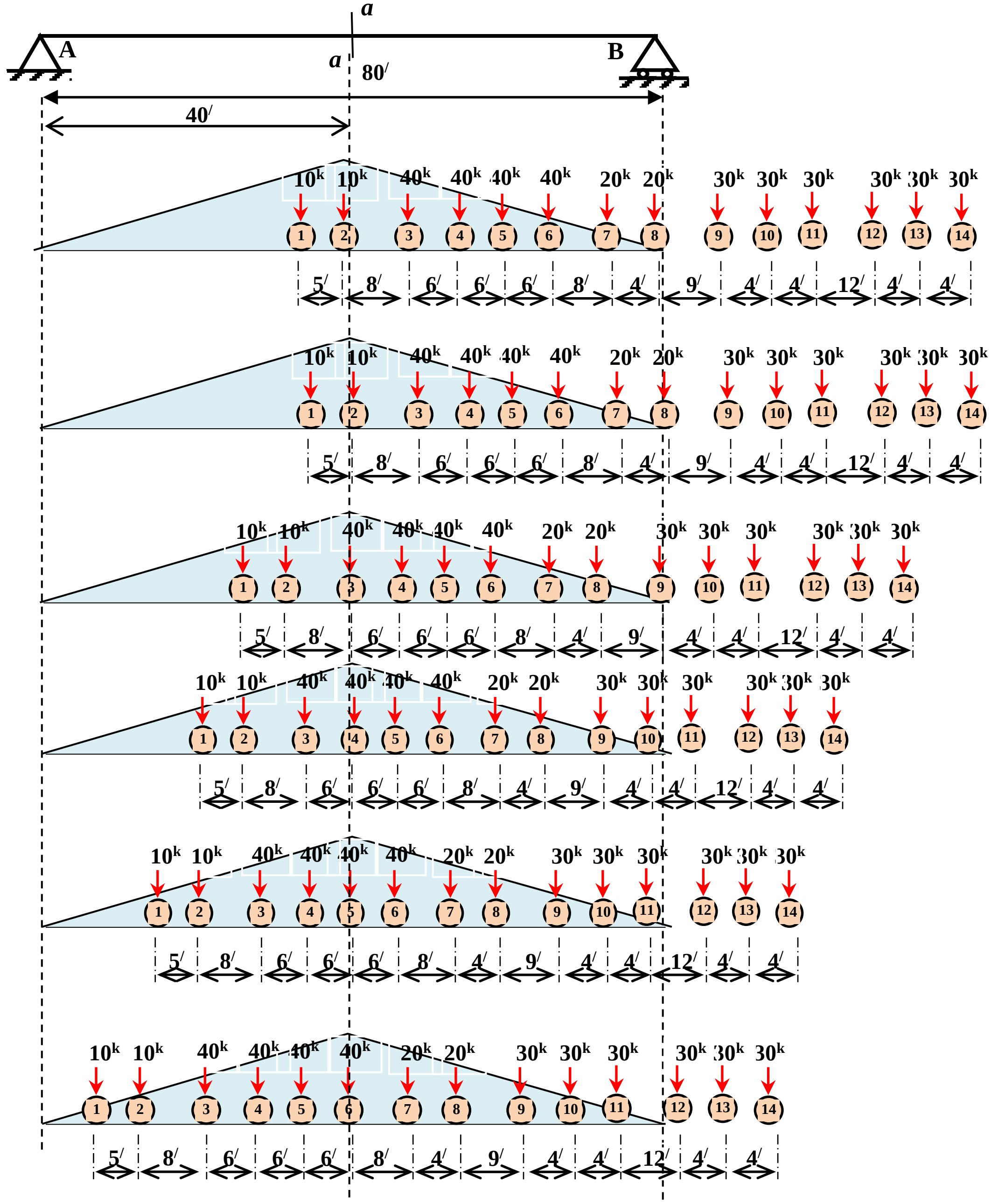
Therefore, $\frac{dM}{dx} = -i\frac{W_1}{a} + i\frac{W_2}{b}$

For maximum derivative of moment with respect x must be zero

or, $\frac{dM}{dx} = -i\frac{W_1}{a} + i\frac{W_2}{b} = 0\,,$ or, $\frac{W_1}{a} = \frac{W_2}{b} = \frac{W_1+W_2}{a+b} = \frac{W}{L}$

The maximum moment at a given section occurs when the intensity of loading on the left side of the section is equal to the intensity of loading on the span.

3. **Determine maximum moment at *a-a* due to a series of moving loads pass over a simple beam of 80 ft span.**



Wheel 1

Right

$\frac{0}{40}\langle\frac{200}{80}$

Left

$\frac{10}{40}\langle\frac{200}{80}$

Wheel 2

Right

$\frac{10}{40}\langle\frac{220}{80}$

Left

$\frac{20}{40}\langle\frac{220}{80}$

Wheel 3

Right

$\frac{20}{40}\langle\frac{250}{80}$

Left

$\frac{60}{40}\langle\frac{250}{80}$

Wheel 4

Right

$\frac{60}{40}\langle\frac{280}{80}$

Left

$\frac{100}{40}\langle\frac{280}{80}$

Wheel 5

Right

$\frac{100}{40}\langle\frac{310}{80}$

Left

$\frac{140}{40}\langle\frac{310}{80}$

Wheel 6

Right

$\frac{140}{40}\langle\frac{310}{80}$

Left

$\frac{180}{40}\rangle\frac{310}{80}$; Satisfy the criteria, so Wheel 6 must be on the section.

$$M_{\max,at\,a-a} = \frac{20}{40}(11*30+15*30+19*30+28*20+32*20+40*40)+$$

$$\frac{20}{40}(9*10+14*10+22*40+28*40+34*40) = 3870\,kft$$

Criteria for the absolute maximum moment of a simple beam subjected to series of concentrated moving loads.

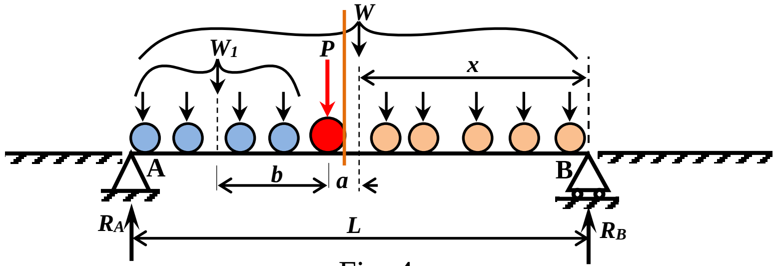


Fig. 4

Fig. 4 represent a simply supported beam subjected to move loads, where

- P = represent one of the loads under which the maximum moment will occur.
- W_1 = represent the sum of all the loads to the left of P.
- W = the sum of all the loads on the span.
- a = distance between the center of gravity of all the loads and P.
- b = distance between the center of gravity of the loads W_1 and P.
- L = span length of beam.
- R_A = reaction at A due to applied loading.
- R_B = reaction at B due to applied loading.
- x = distance of the center of gravity all the loads to the support B.

The expression of bending moment at the load P

$$M = R_A (L - a - x) - W_1 b$$

in which

$$\mathrm{R_A} = \frac{Wx}{L}$$

Then

$$M = \frac{W}{L}(Lx - ax - x^2) - W_1 b$$

Differentiating with respect to x and equating to zero, then the equation takes the form,

$$\frac{dM}{dx} = \frac{W}{L}(L - a - 2x) = 0$$

or, $$x = \frac{L}{2} - \frac{a}{2}$$

Therefore, the maximum moment under any load will occur when that load and the center of gravity of all the loads on the beam are equidistant from the center of the beam.

4. Determine the absolute maximum moment for following moving loads on a simply supported beam.

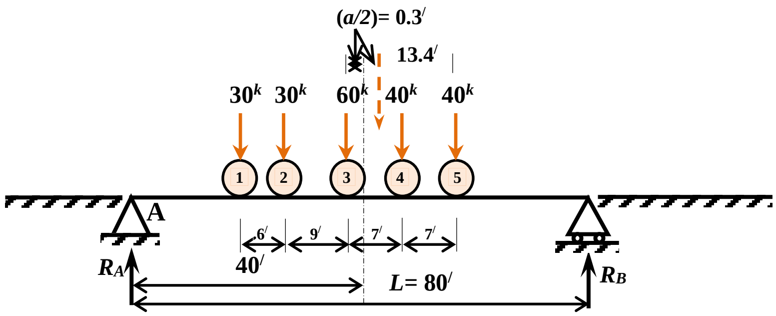
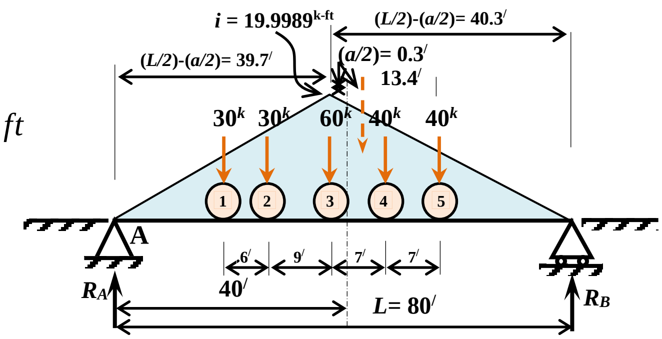


Fig. 4

Fig. 4 represent a simply supported beam subjected to move loads, where

C.G. from wheel 5



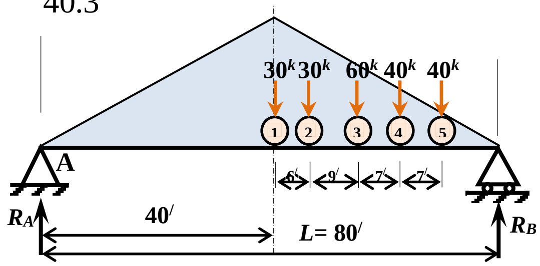
$$\bar{x} = \frac{40*7+60*14+30*23+30*29}{2*30+60+2*40} = 13.4\ ft$$

a = 14-13.4 = 0.6 ft

$$\frac{a}{2} = \frac{0.6}{2} = 0.3\,ft$$

$$M_{abs-\max} = \frac{19.9989}{39.7}(24.7*30+30.7*30+39.7*60) + \frac{19.9989}{40.3}(26.3*40+33.3*40) = 3220.29\,kft$$

Maximum moment at mid-span



Wheel 1 at section

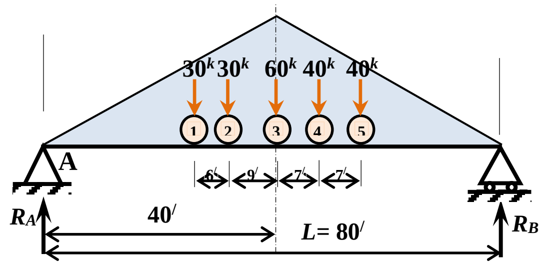
Right **Left**

$$\frac{0}{40} \langle \frac{200}{80} \qquad \frac{30}{40} \langle \frac{200}{80}$$

Wheel 2 at section

Right **Left**

$$\frac{30}{40} \langle \frac{200}{80} \qquad \frac{60}{40} \langle \frac{200}{80}$$



Wheel 3 at section

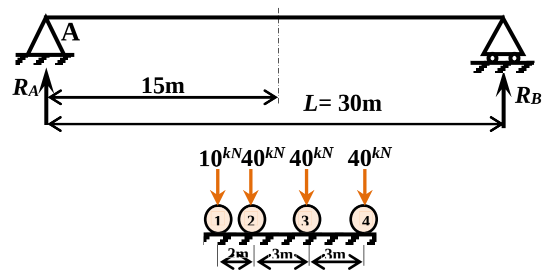
Right **Left**

$$\frac{60}{40} \langle \frac{200}{80} \qquad \frac{120}{40} \rangle \frac{200}{80}$$

Satisfy the criteria

$$M_{\max} = \frac{20}{40}[25*30+31*30+40*60] + \frac{20}{40}[26*40+33*40] = 3220\,kft$$

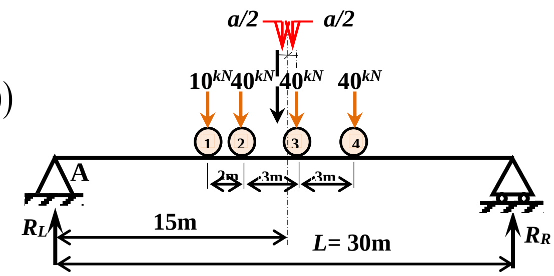
5. **A system of moving loads traverses on a simply beam of span length of 30m. Determine the absolute maximum moment and the maximum moment at mid-span.**



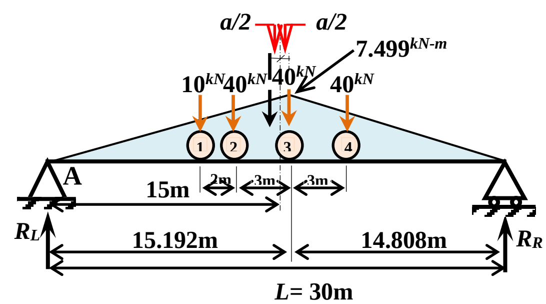
Distance CG from P_4

$$\bar{x} = \frac{40*3+40*6+10*8}{10+40+40+40} = 3.38m$$

$$\frac{a}{2} = \left(\frac{3.38-3}{2}\right) = 0.192\,m$$



$$\sum M_{abs} = \frac{7.499}{15.192}(10.192*10+12.192*40+15.192*40) + \frac{7.499}{14.808}*11.808*40 = 830.19\,kN-m$$



Wheel 3

Right   **Left**

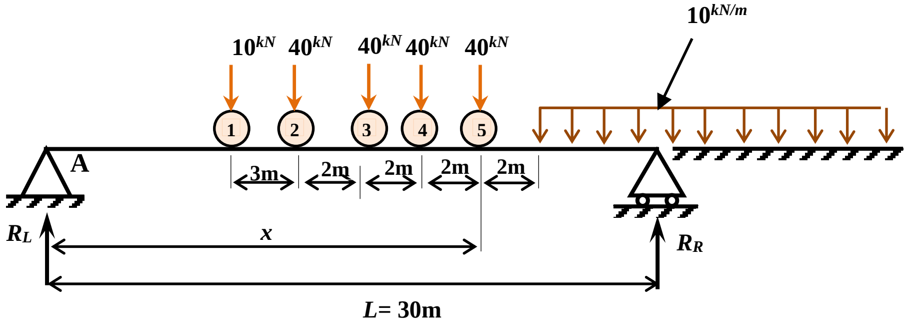
$\frac{50}{15}\langle\frac{130}{30}$ $\frac{90}{15}\rangle\frac{130}{30}$

Satisfy the criteria

$$\sum M_{\max} = \frac{15}{30}(10*10+40*12+40*15+40*12) = 830kN-m$$

Find the absolute maximum w.r.t. wheel 2

6. Find the absolute maximum moment and maximum moment at mid-span.



$$R_L = \frac{10*\frac{(L-x-2)^2}{2} + 40*(L-x) + 40*(L-x+2) + 40*(L-x+4) + 40*(L-x+6) + 10*(L-x+9)}{30}$$

$$R_L = \frac{10*\frac{(L-x-2)^2}{2} + 10*(4L-4x+4L-4x+8+4L-4x+16+4L-4x+24+L-x+9)}{30}$$

$$R_L = \frac{10*\frac{(L-x-2)^2}{2} + 10*(17L-17x+57)}{30}$$

Moment about the wheel 5

$$M_5 = \left(\frac{10*\frac{(L-x-2)^2}{2} + 10*(17L-17x+57)}{30}\right)x + 80 + 160 + 240 + 90$$

$$M_5 = \frac{(30-x-2)^2}{6}x + \frac{(17*30-17x+57)x}{3} + 570$$

$$M_5 = \frac{(28-x)^2}{6}x + \frac{(567-17x)x}{3} + 570 = \frac{1}{6}\left[784x - 56x^2 + x^3\right] + \frac{1}{3}\left[567x - 17x^2\right] + 570$$

$$\frac{dM_5}{dx} = \frac{1}{6}\left[784 - 112x + 3x^2\right] + \frac{1}{3}\left[567 - 34x\right] = 0$$

$$784 - 112x + 3x^2 + 1134 - 68x = 0$$

$$x^2 - 60x + 639.33 = 0$$

$$(x-30)^2 - 900 + 639.33 = 0$$

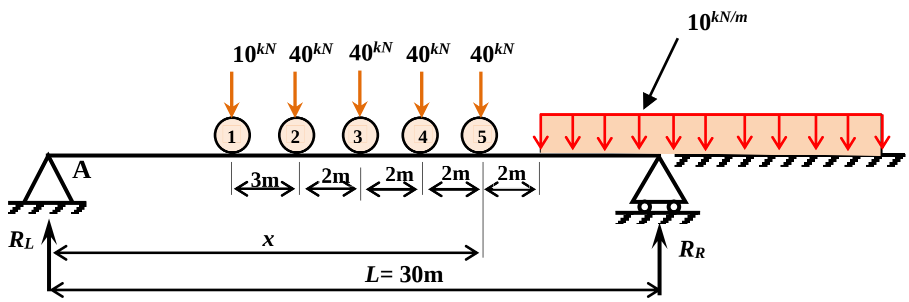
$$(x-30)^2 = 260.67$$

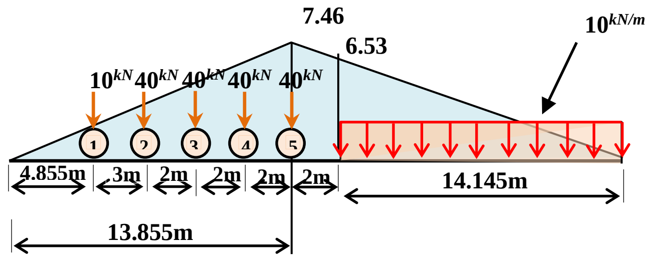
$$(x-30) = \pm 16.15, \qquad x = 13.855m$$

Wheel 5

Right | **Left**

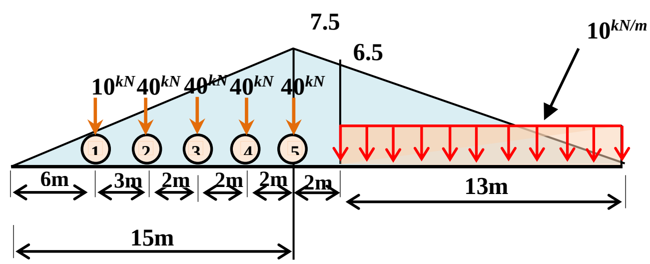
$$\frac{130}{15} \langle \frac{300}{30} \qquad \frac{170}{15} \rangle \frac{300}{30}$$





$$M_{abs} = \frac{7.46}{13.855}[10*4.855+40*7.855+40*9.855+40*11.855+40*13.855]+\frac{1}{2}*6.53*14.145*10$$

$$M_{abs} = 1423.13\,kN-m$$



$$M_{\max-mid} = \frac{7.5}{15}[6*10+9*40+11*40+13*40+15*40]+\frac{1}{2}*6.5*13*10 = 1412.5 \quad kN-m$$

Wheel 2

Right

(10/15) less than (240/30)

Left

(50/15) less than (240/30)

Criteria for the maximum shear of a floor beam subjected to series of concentrated loads move from right to left.

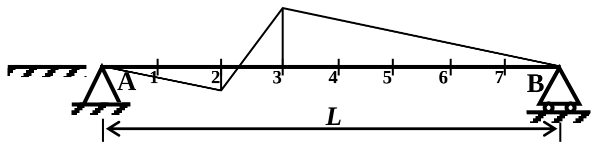


Fig. 5(a)

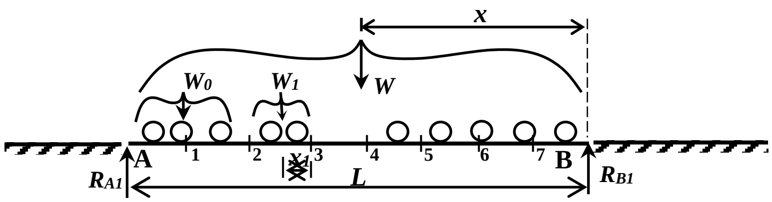


Fig. 5(b)

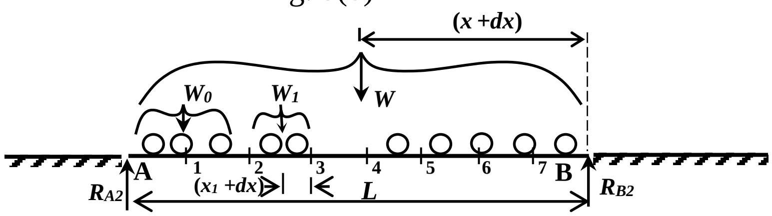


Fig. 5(c)

A simply supported floor beam subjected to moving concentrated loads is shown in Fig. 5. It is required to obtain the criteria for allocating the position of loads to find the maximum shear in a panel. Following notations are used:

W = the sum of all loads on the span.

W_o = the sum of all the loads to the left of the panel.

W_1 = the sum of all the loads in the panel.

x = the distance of the center of gravity all the loads (W) to right support B.

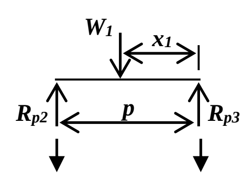
x_1 = the distance of the center of gravity all the loads within panel to right panel point.

L = span length of beam.

Before movement,

$$\sum M_B = Wx - R_{A1}L = 0\ ;$$

or, $R_{A1} = \dfrac{Wx}{L}$



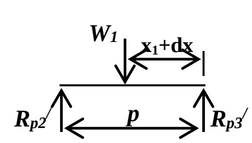
Also, reaction at panel point 2,

$$R_{p2} = \frac{W_1 x_1}{p}$$

Now, shear force within panel (2-3)

$$V_1 = R_{A1} - W_0 - R_{p2} = \frac{Wx}{L} - W_0 - \frac{W_1 x_1}{p} \qquad (1)$$

After movement,



$$\sum M_B = W(x + dx) - R_{A2}L = 0\ ;$$

or, $R_{A2} = \frac{W(x+dx)}{L}$

Also, reaction at panel point 2,

$$R'_{p2} = \frac{W_1(x_1+dx)}{p}$$

Now, shear force within panel(2-3)

$$V_2 = R_{A2} - W_0 - R'_{p2} = \frac{W(x+dx)}{L} - W_0 - \frac{W_1(x_1+dx)}{p} \qquad (2)$$

The change in shear force within panel(2-3) can be obtained by using Eq. (1) and (2)

$$dV = V_2 - V_1 = \frac{W(x+dx)}{L} - W_0 - \frac{W_1(x_1+dx)}{p} - \frac{Wx}{L} + W_0 + \frac{W_1x_1}{p}$$

or, $dV = V_2 - V_1 = \frac{Wdx}{L} - \frac{W_1dx}{p} = 0$

or, $\frac{dV}{dx} = \frac{W}{L} - \frac{W_1}{p}$

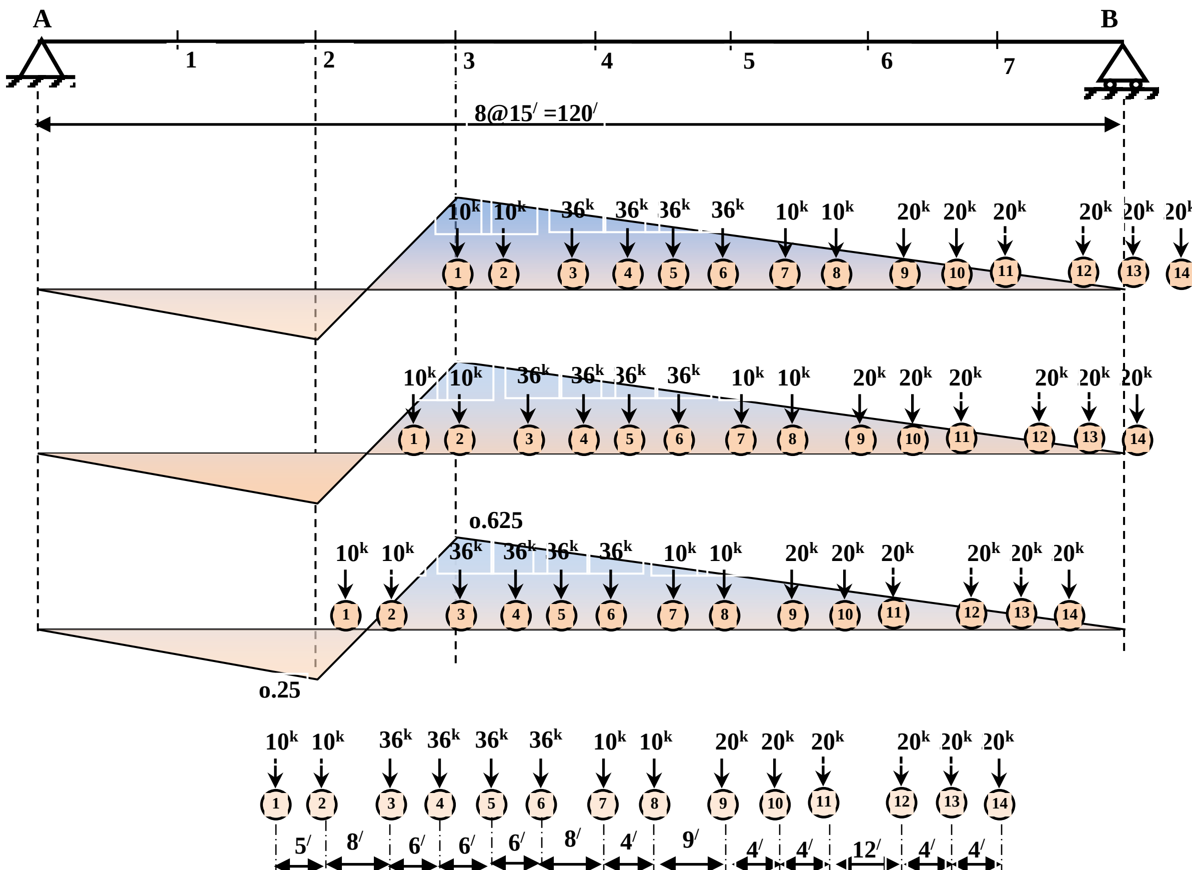
For the maximum value of shear force, the derivative of shear force with respect to x must be equal to zero. Therefore,

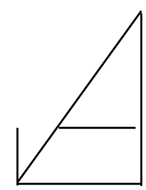
$$\frac{W}{L} - \frac{W_1}{p} = 0$$

or, $\frac{W}{L} = \frac{W_1}{p}$

Therefore, the maximum shear force will occur within panel when the average load in the panel is equal to average load in the span of the beam.

7. Determine the maximum shear within panel 2-3 due to following moving wheel loads.





Wheel 1

To right of p.p. 3 $\frac{264}{120} \rangle \frac{0}{15}$

To left of p.p. 3 $\frac{264}{120} \rangle \frac{10}{15}$ Wheel 1 does not satisfy criterion.

Wheel 2

To right of p.p. 3 $\frac{284}{120} \rangle \frac{10}{15}$

To left of p.p. 3 $\frac{304}{120} \rangle \frac{20}{15}$ Wheel 2 does not satisfy criterion.

Wheel 3

To right of p.p. 3 $\frac{304}{120} \rangle \frac{20}{15}$

To left of p.p. 3 $\frac{304}{120} \langle \frac{56}{15}$ Wheel 3 does satisfy criterion.

Alternately

Wheel 1

To right of p.p. 3 0

To left of p.p. 3 10 $\frac{264}{8} = 33$ *Does not satisfy.*

Wheel 2

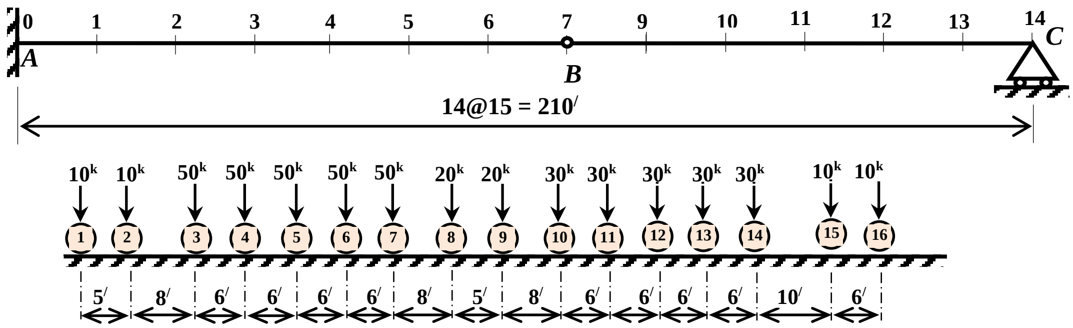
To right of p.p. 3 10

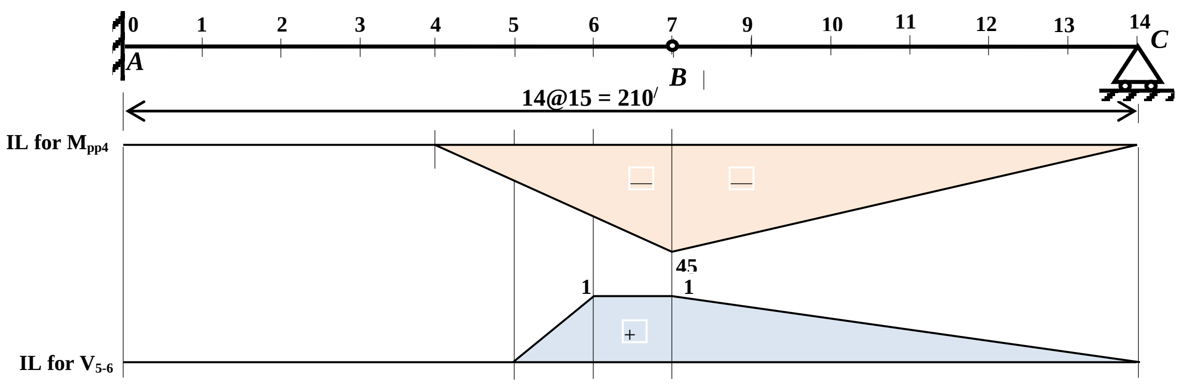
To left of p.p. 3 20 $\frac{284}{8} = 35.5$ *Does not satisfy.*

Wheel 3

To right of p.p. 3 20
To left of p.p. 3 56 $\Big\} \frac{304}{8} = 38$ *Satisfy.*

8. Determine the maximum moment at pp 4 and maximum shear in the panel 5-6 for the following beam due to the moving loads as shown below.





Wheel 1

To right of p.p. 6 $\frac{0}{1} = 0 \langle \frac{480}{8} = 60$

To left of p.p. 6 $\frac{10}{1} = 10 \langle \frac{480}{8} = 60$ does not satisfy.

Wheel 2

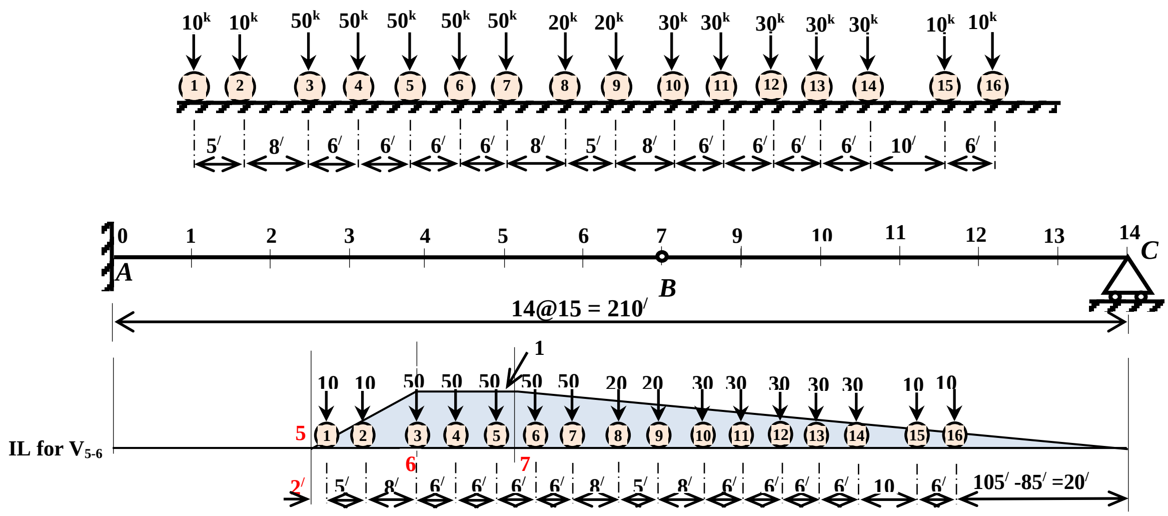
To right of p.p. 6 $\frac{10}{1} = 0 \langle \frac{480}{8} = 60$

To left of p.p. 6 $\frac{20}{1} = 20 \langle \frac{480}{7} = 60$ does not satisfy. 7

Wheel 3

To right of p.p. 6 $\frac{20}{1} = 20 \langle \frac{480}{8} = 60$

To left of p.p. 6 $\frac{70}{1} = 70 \rangle \frac{480}{8} = 60$ Satisfy.



$$V_{5-6} = \frac{1}{15}\left(2*10+7*10+15*50\right)+50+50+\frac{1}{90}\begin{pmatrix}20*10+26*10+36*30+42*30+48*30+54*30\\+60*30+68*20+73*20+81*50+87*50\end{pmatrix}$$

= 56 + 50 + 50 +188.44 = 344.44 kips

Wheel 3 at pp. 7 ***d_{3-16}* = 85 ft** So all wheel on the span

To right of p.p. 7 $\frac{20}{3} = 6.67 \langle \frac{480}{9} = 53.33$

To left of p.p. 7 $\frac{70}{3} = 23.33 \langle \frac{480}{9} = 53.33$ Does not satisfy.

Wheel 4 at pp. 7

To right of p.p. 7 $\frac{70}{3} = 23.33 \langle \frac{480}{9} = 53.33$

To left of p.p. 7 $\frac{120}{3} = 40 \langle \frac{480}{9} = 53.33$ Does not satisfy.

Wheel 5 at pp. 7

To right of p.p. 7 $\frac{120}{3} = 40 \ \langle \frac{480}{9} = 53.33$

To left of p.p. 7 $\frac{170}{3} = 56.67 \rangle \frac{480}{9} = 53.33$ Satisfy.

***D_{5-14}* = 73 ft; *d_{1-5}* = 25 ft *d_{pp4-w1}* = 20 ft; *$d_{w14-pp14}$* = 17 ft**

$$M_{pp4} = \frac{45}{45}\left(20*10+25*10+33*50+39*50+45*50\right)+$$

$$\frac{45}{90}\begin{pmatrix}17*10+23*10+33*30+39*30+45*30+51*30+57*30+\\65*20++70*20+78*50+84*50\end{pmatrix}$$

= 6300 + 8975 = 15275 k-ft

Criteria for the maximum moment at any section of a compound beam subjected to series of concentrated loads move from right to left.

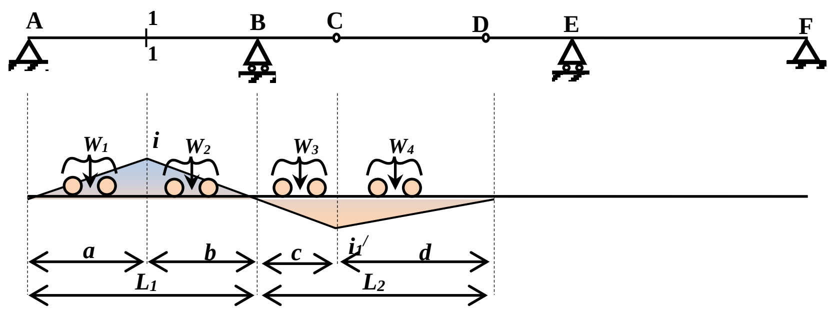


Fig. 6

A line diagram of balance cantilever bridge is shown in the above figure. It is required to obtain the criteria to determine the position moving concentrated loads and hence to find the maximum moment at a section 1-1. As the loads move from the right to left, the moment at section 1-1 will change. The change in moment may be calculated as:

The change in moment due to movement of loads in between A and B,

$$\Delta M_1 = -\frac{i}{a}W_1\Delta x + \frac{i}{b}W_2\Delta x$$

The change in moment due to movement of loads in between B and D,

$$\Delta M_2 = -\frac{i^/}{d}W_4\Delta x + \frac{i^/}{c}W_3\Delta x$$

The net change in moment,

$$\Delta M = \Delta M_1 + \Delta M_2 = -\frac{i}{a}W_1\Delta x + \frac{i}{b}W_2\Delta x - \frac{i^/}{d}W_4\Delta x + \frac{i^/}{c}W_3\Delta x \qquad (1)$$

From influence line diagram,

$$\frac{i}{b} = \frac{i^/}{c}; \qquad i^/ = \frac{c}{b}i$$

Substitute this value in Eq. (1), we have

$$\Delta M = i\left(\frac{W_2}{b} - \frac{W_1}{a}\right)\Delta x + \frac{c}{b}i\left(\frac{W_3}{c} - \frac{W_4}{d}\right)\Delta x \qquad (2)$$

Finally the Eq. (2) can be expressed in differential form as:

$$\frac{dM}{dx} = i\left(\frac{W_2}{b} - \frac{W_1}{a}\right) + \frac{c}{b}i\left(\frac{W_3}{c} - \frac{W_4}{d}\right) \qquad (3)$$

To obtain the maximum value of moment, Eq. (3) must be equal to zero. Therefore

$$i\left(\frac{W_2}{b} - \frac{W_1}{a}\right) + \frac{c}{b}i\left(\frac{W_3}{c} - \frac{W_4}{d}\right) = 0$$

or, $$\left(\frac{W_2}{b} - \frac{W_1}{a}\right) + \frac{c}{b}\left(\frac{W_3}{c} - \frac{W_4}{d}\right) = 0 \qquad (4)$$

If there is no load in right side i.e. in between B and C then Eq. (4) takes the form as:

$$\frac{W_2}{b} - \frac{W_1}{a} = 0$$

or, $$\frac{W_2}{b} = \frac{W_1}{a} = \frac{W_1 + W_2}{a+b} = \frac{W}{L_1}$$

If there is no load in left side i.e. in between A and B then Eq. (4) takes the form as:

$$\frac{W_3}{c} - \frac{W_4}{d} = 0$$

or, $$\frac{W_3}{c} = \frac{W_4}{d} = \frac{W_3 + W_4}{c+d} = \frac{W^{/}}{L_2}$$