Influence Lines: An influence line is a diagram showing the variation in the shear, moment, stress in a member, reaction, or other direct function at a particular section or point or member, due to a unit load moving across the structure.

Construction of Influence Line: An influence line is constructed by plotting directly under the point where the unit load is placed an ordinate the height of which represents to some scale the value of the particular function being studied when the load is in that point.

Purpose of Influence Lines:

Influence lines can be used for two very important purposes:

1. To determine what position of live loads will lead to a maximum value of the particular function for which an influence line has been constructed.
2. To compute the value of that function with the loads so placed or, in fact, for any loading condition.

Theorem 1. To obtain the maximum value of a function due to a single concentrated live load, the load should be placed at the point where the ordinate to the influence line for that function is a maximum.

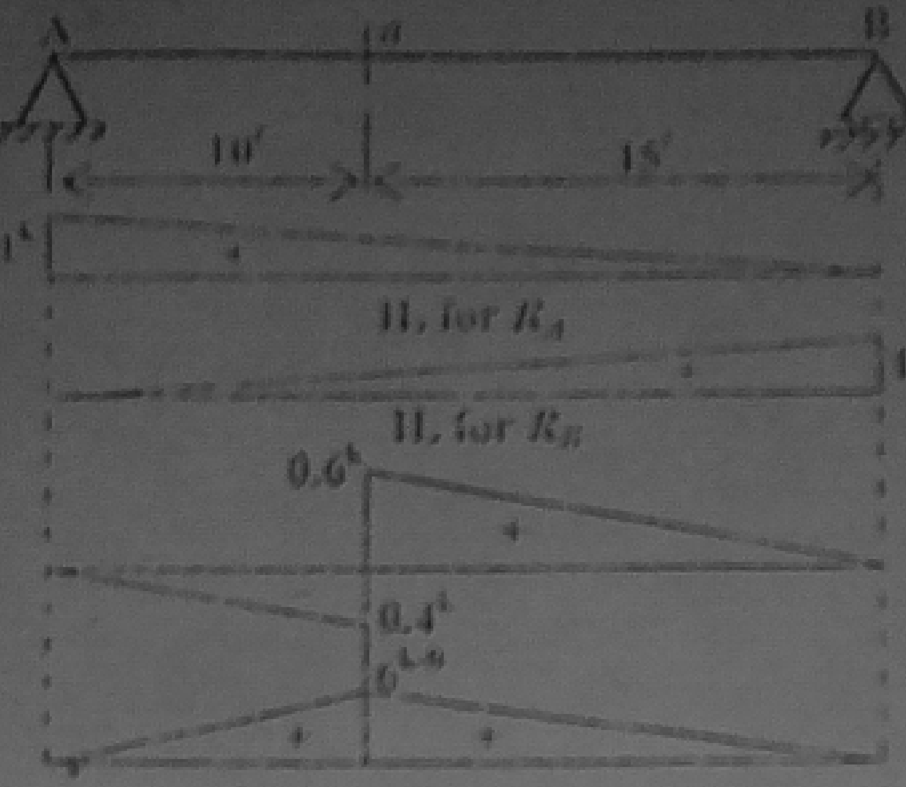
Theorem 2. The value of a function due to the action of a single concentrated live load equals the product of the magnitude of the load and the ordinate to the influence line for that function, measured at the point of application of the load. value = load x ordinate

- **Theorem 3.** To obtain the maximum value of a function due to a uniformly distributed live load, the load should be placed over all those portions of the structure for which the ordinates to the influence line for that function have the sign of the character of the function desired.

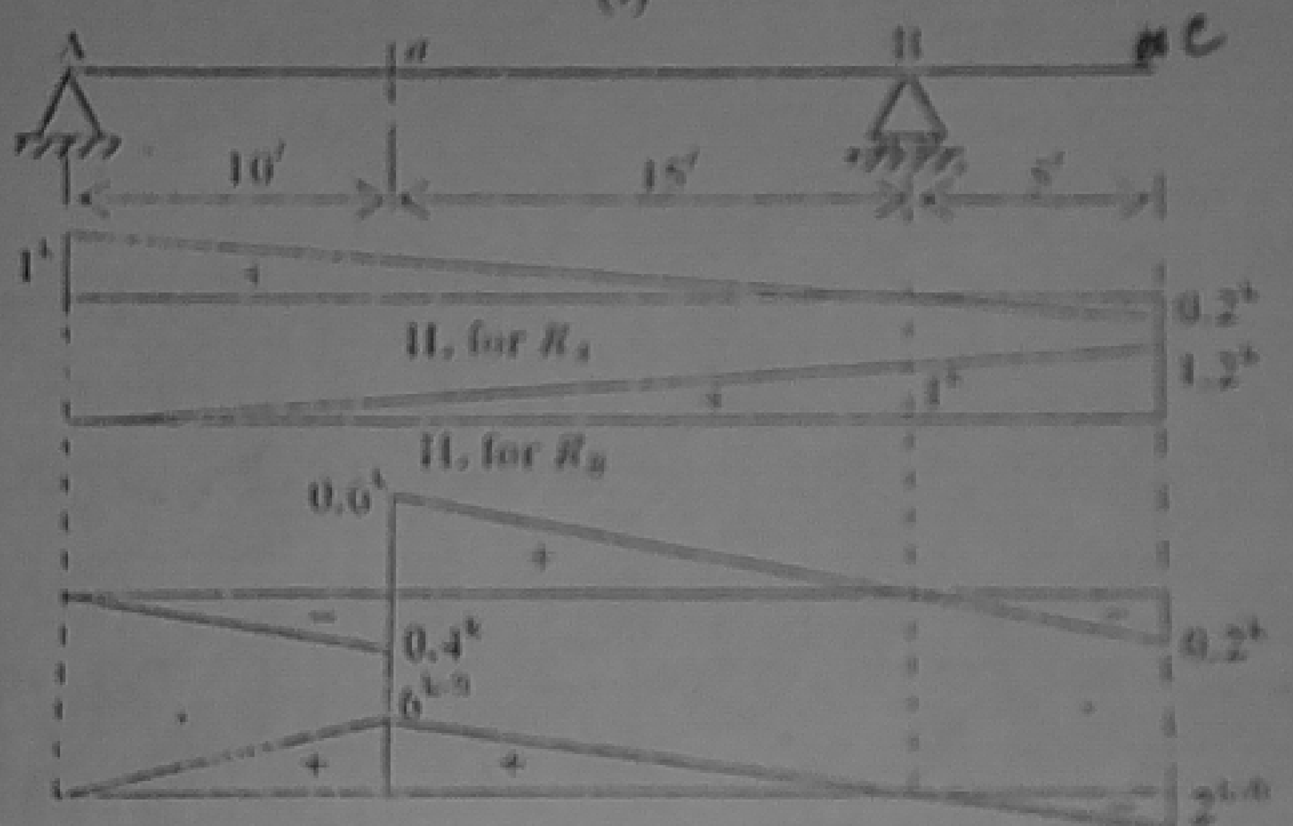
Theorem 4. The value of a function due to a uniformly distributed live load is equal to the product of the intensity of the loading and the net area under that portion of the influence line, for that function under consideration, which corresponds to the portion of the structure loaded.

load value = load intensity x area under IL

Q.1 Draw IL diagrams for R_A, R_B, V_a and M_a of the following structures as a unit load moves from A to B.

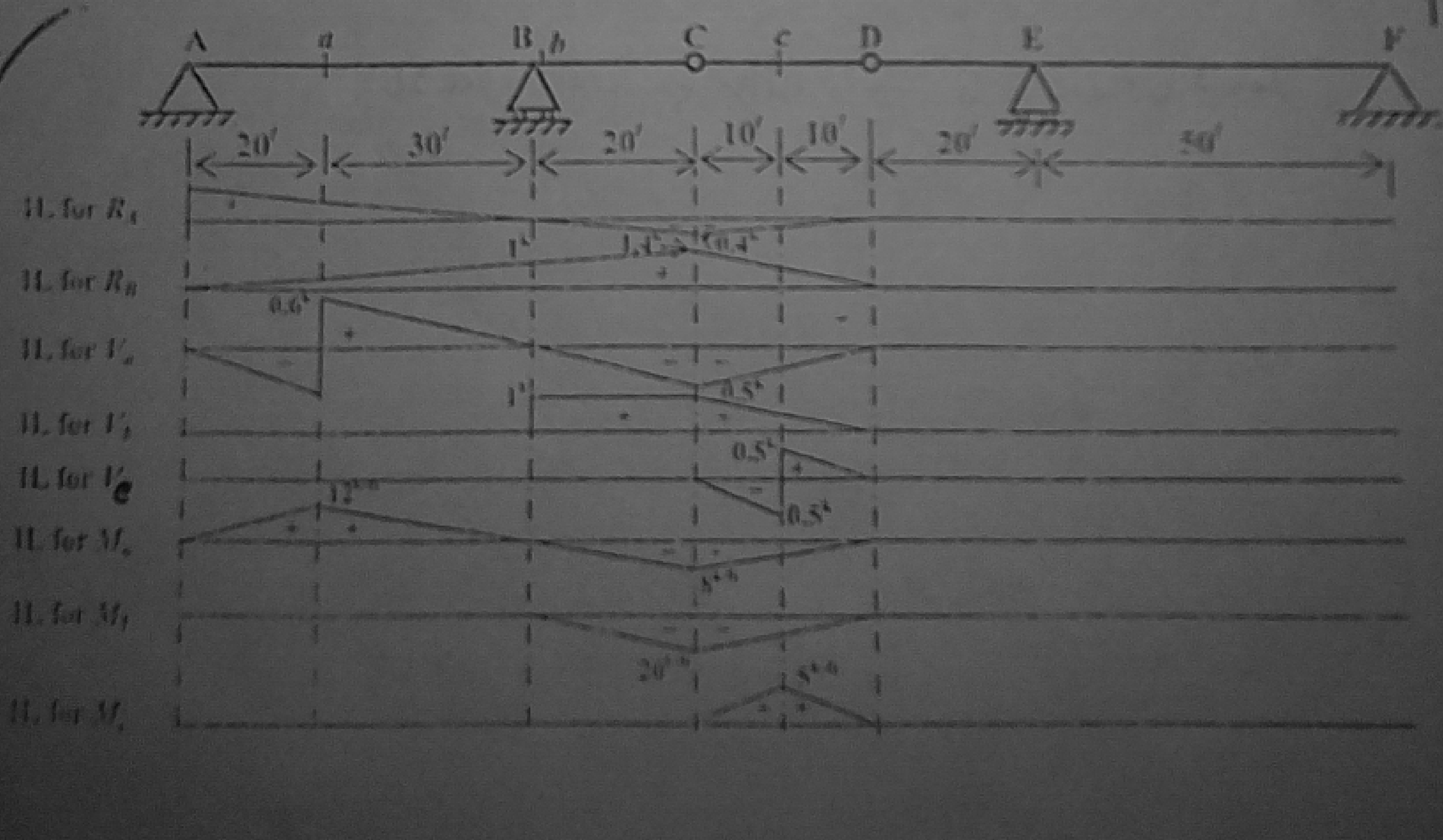


(i)

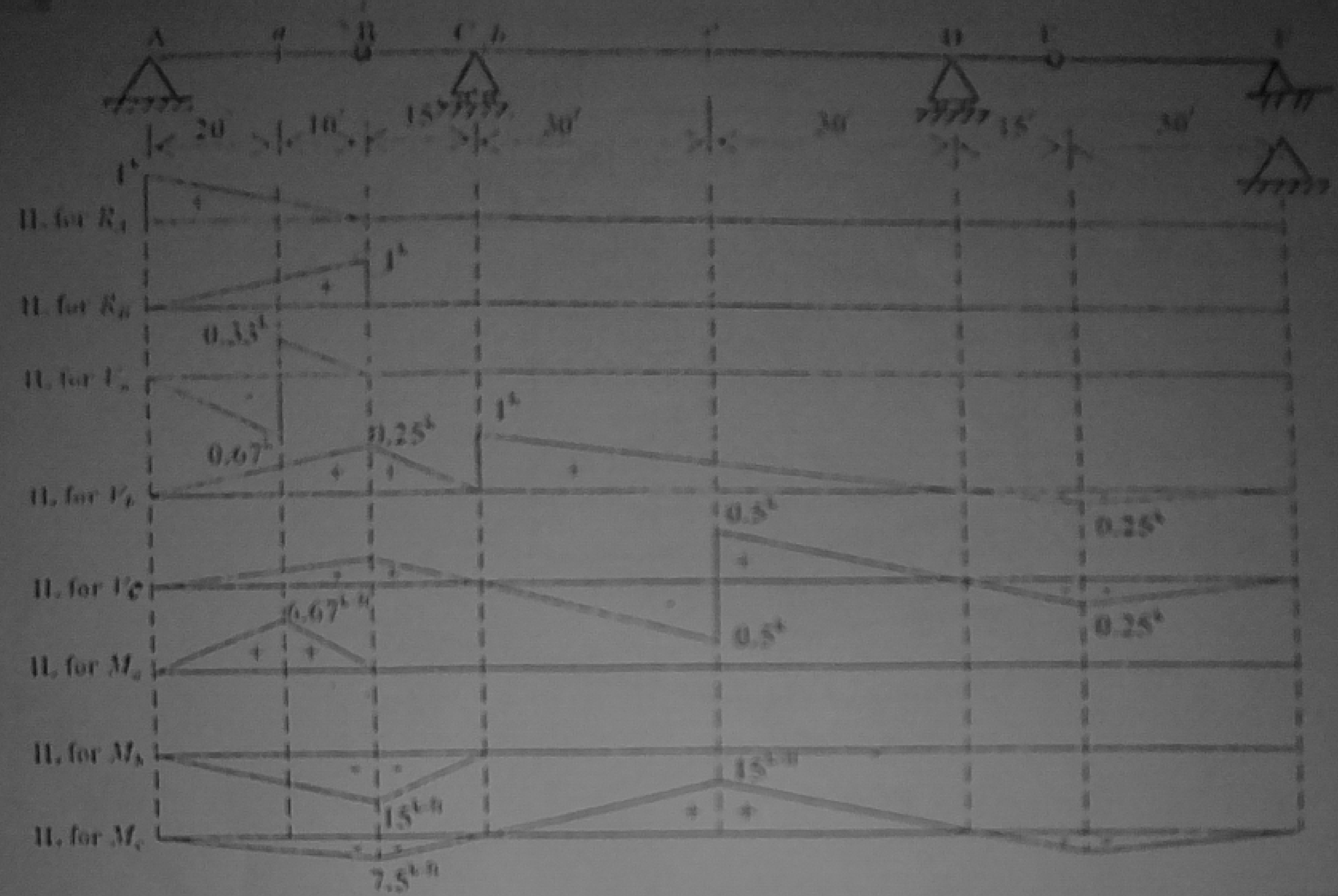


(ii)

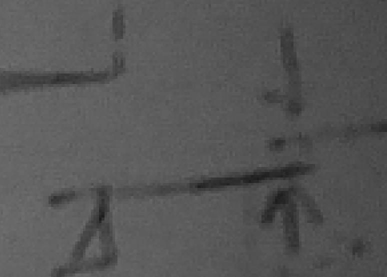
Q.2 Draw IL diagrams for R_A, R_B, V_a, V_b, V_c, M_a, M_b and M_c of the following balanced cantilever bridge as a unit load moves from A to F.



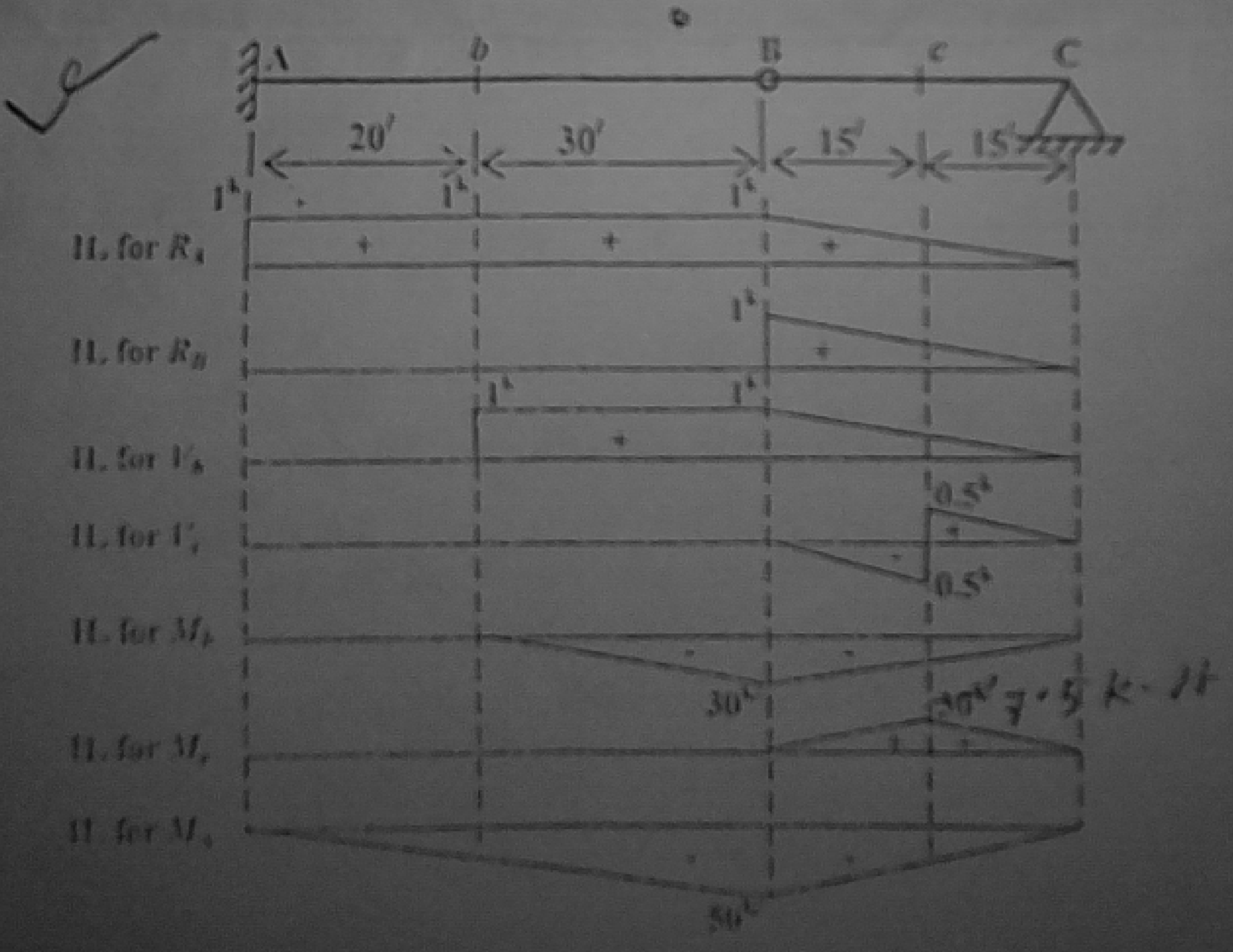
Q.3. Draw IL diagrams for $R_A, R_B, V_a, V_b, V_c, M_a, M_b$ and M_c of the following compound bridge as shown below in a line diagram as a unit load moves from A to F.



Prob. 3

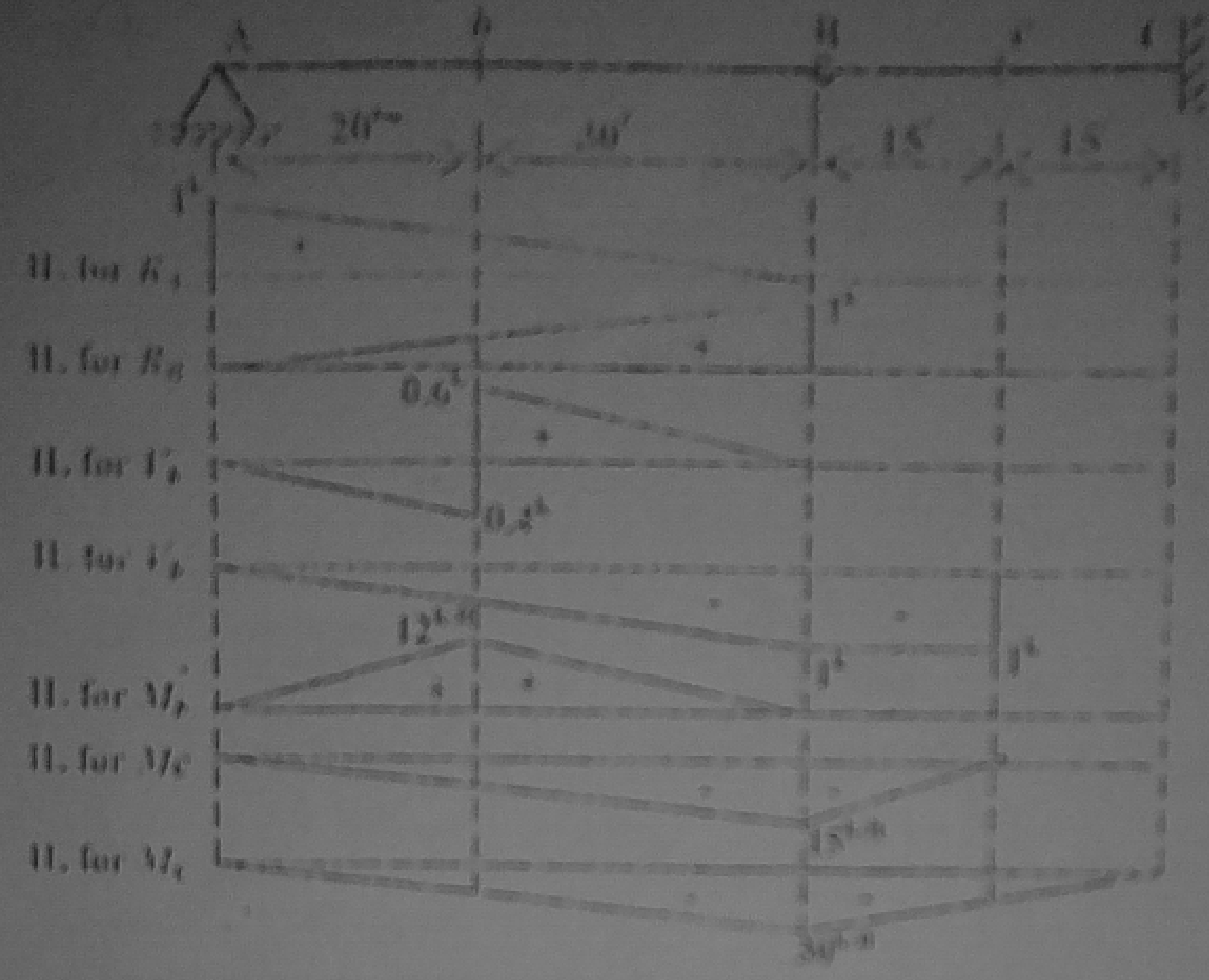


Q.4. Draw IL diagrams for R_A, R_B, V_b, V_c, M_b, M_c and reactive moment at support M_A of the following compound beam as a unit load moves from A to C.



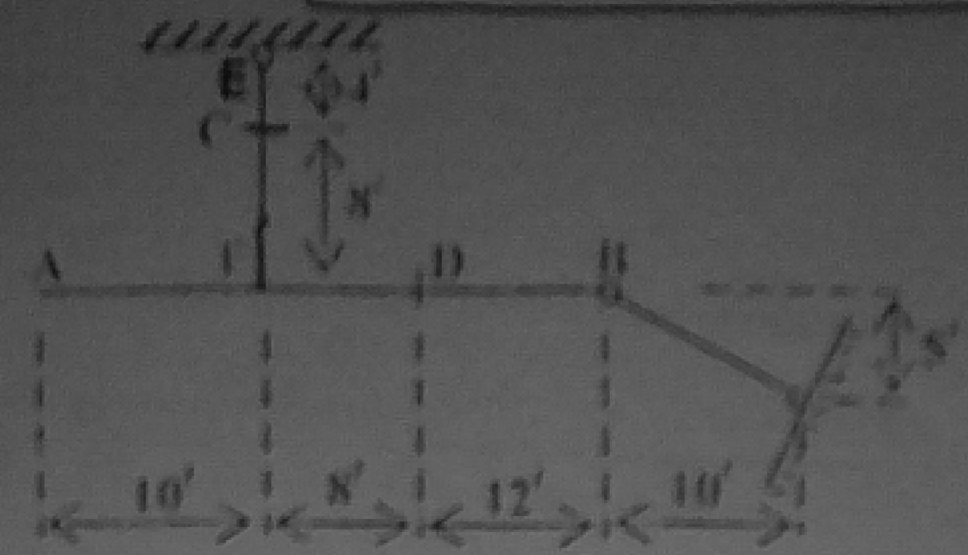
Prob. 4

Q.5. Draw IL diagrams for R_A, R_B, V_b, V_c, M_b, M_c and M_C (reactive moment) of the balance cantilever bridge as shown below in a line diagram as a unit load moves from A to F.

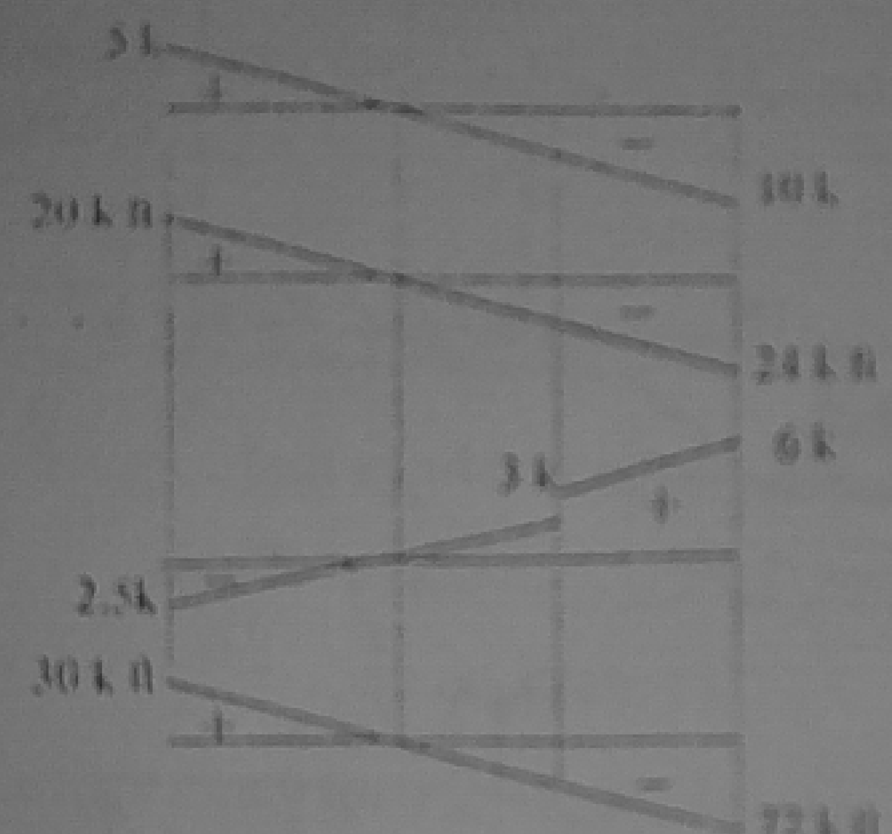


Prob. 5

5. Draw the influence lines for shear and moment at C and D in the beam shown as a unit load moves from A to B. (Shedd & Vawter, p-140, Prob - 92)



Prob. 5



When 1^k at A

$R_B = 5.59k$, $R_{Bx} = 5.0k$, $R_{By} = 2.5k$; $V_C = 5.0k$, $V_D = -2.5k$, $M_C = 20k\ ft$, $M_D = 30k\ ft$.

When 1^k at F

$R_B = 0.0$, $R_{Bx} = 0.0k$, $R_{By} = 0.0k$; $V_C = 0.0k$, $V_D = 0.0k$, $M_C = 0k\ ft$, $M_D = 0k\ ft$.

When 1^k at left of BD:

$R_B = 4.472k$, $R_{Bx} = -4k$, $R_{By} = -2k$; $V_D = 2k$, $M_D = -24k\ ft$.

When 1^k at ~~left~~ right of B D:

$R_B = 4.472k$, $R_{Bx} = -4k$, $R_{By} = -2k$; $V_D = 3k$, $M_D = -24k\ ft$.

When 1^k at B

$R_B = 11.18k$, $R_{Bx} = -10k$, $R_{By} = -5.0k$; $V_C = -10k$, $V_D = 6k$, $M_C = -40k\ ft$, $M_D = -72k\ ft$.

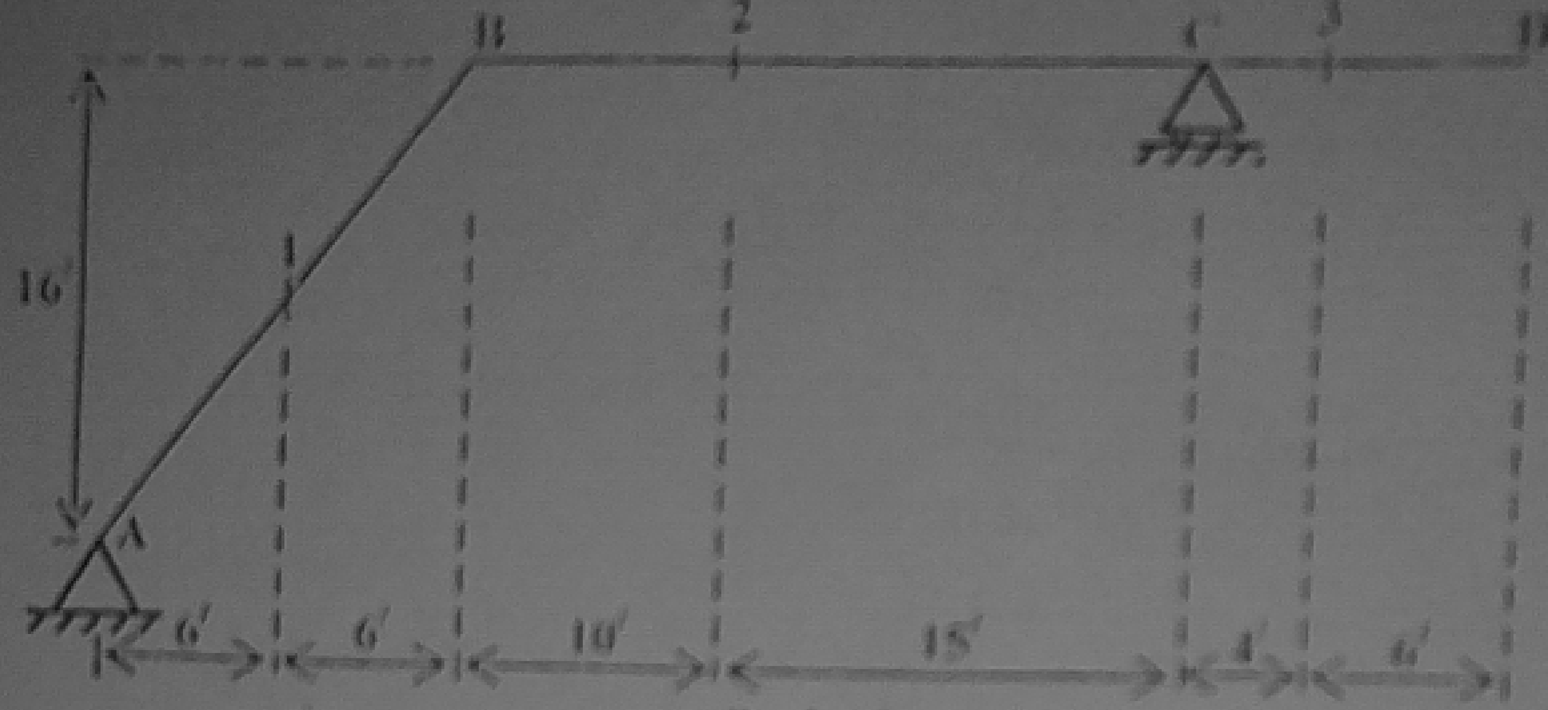
6. Draw the following influence lines for the structure shown. In all cases the load moves from B to D. (Shedd & Vawter, p-140, Prob.- 93)

(i) Reaction at C.

(ii) Shear and moment at 1.

(v) Shear and moment at 2.

(vi) Shear and moment at 3.

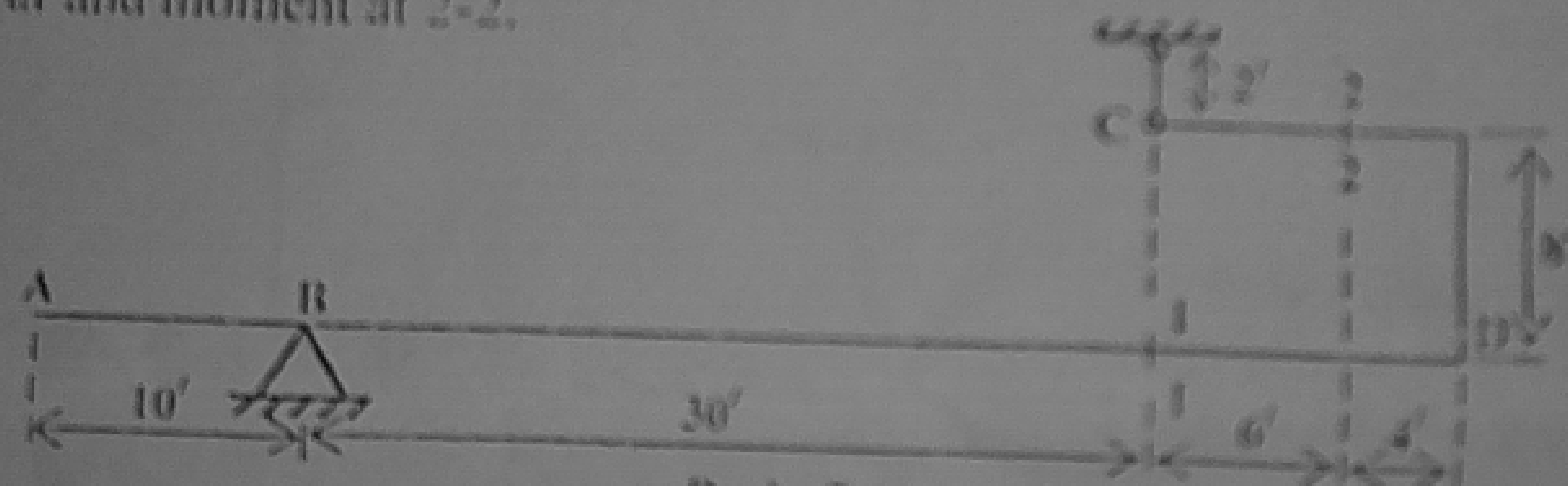


Prob. 6

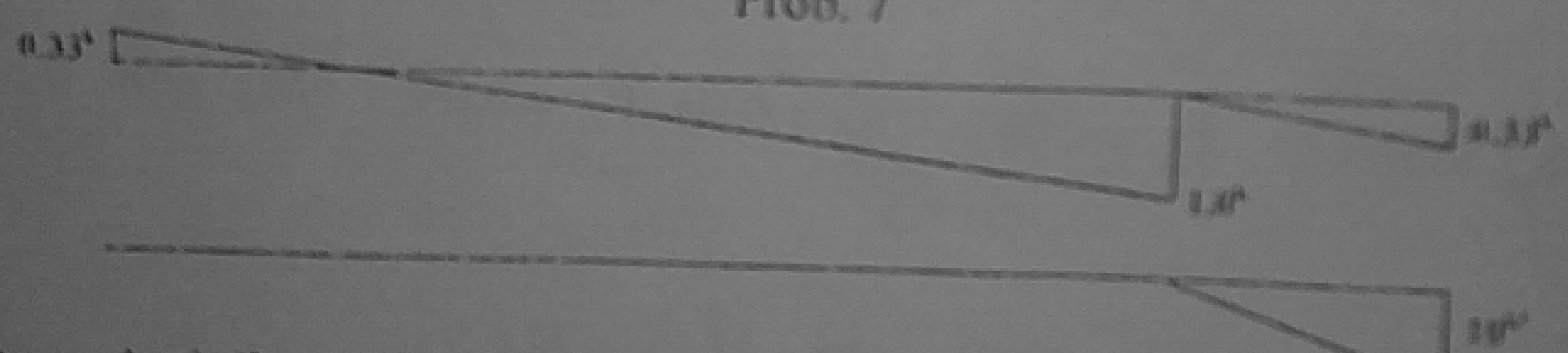
7. As a unit load moves from A to D, draw the influence lines for: (Shedd & Vawter, p-141, Prob.- 96)

(i) Shear and moment at 1-1.

(ii) Shear and moment at 2-2.

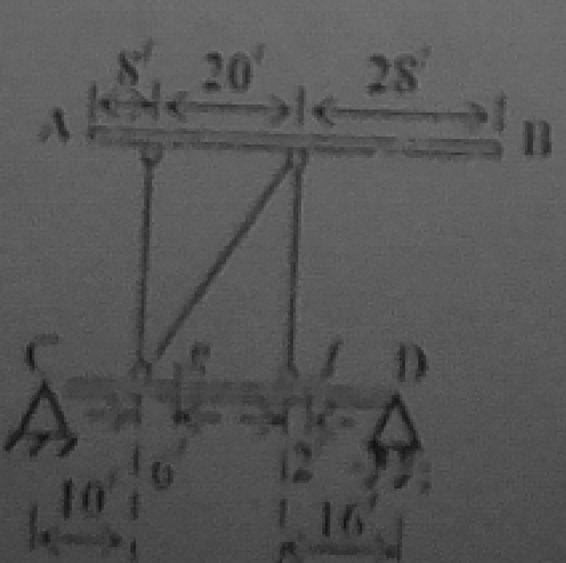


Prob. 7



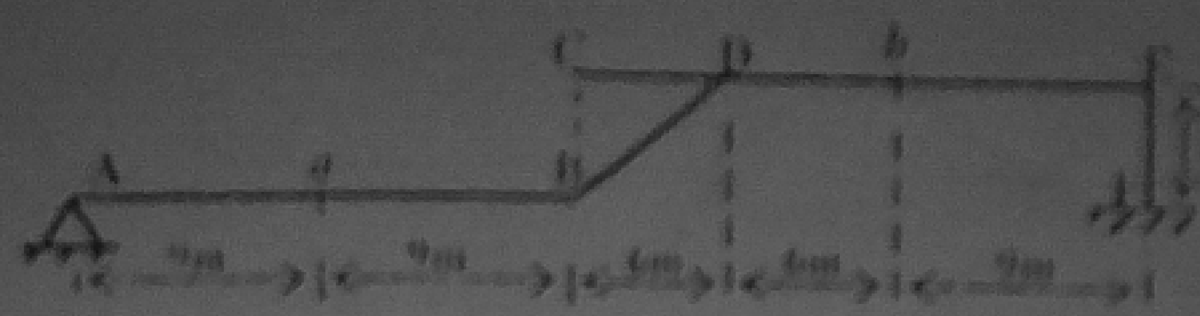
8.(i) Draw the influence lines for shear and moment at *f*, in the beam CD of the frame shown, as a unit load moves from A to B.

(ii) Draw the influence lines for shear and moment at *g* in the same beam for the same movement of the unit load. (Shedd & Vawter, p-141, Prob.- 97)



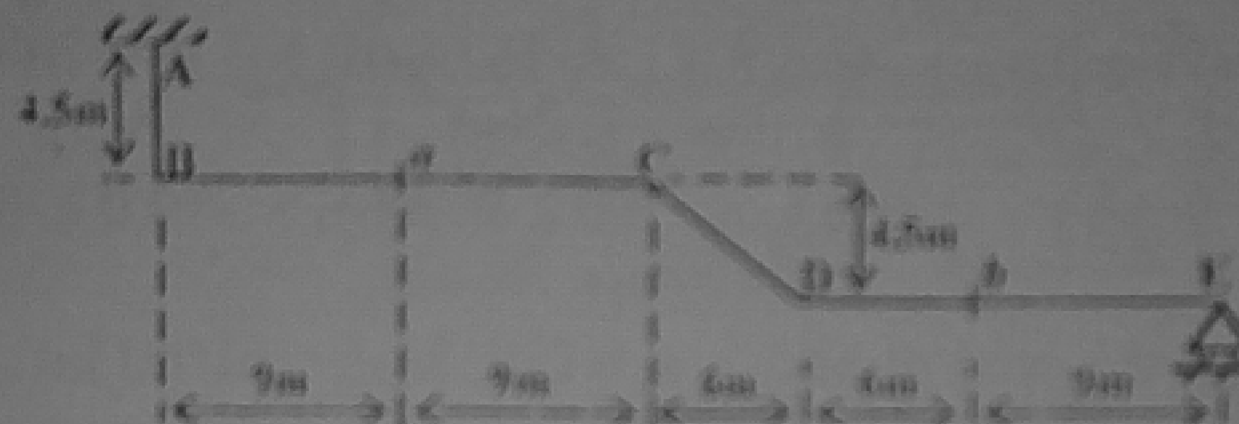
Prob. 8

12. Draw the influence lines for R_{EF}, M_F, V_a, V_b, M_a and M_b as a unit load moves from A to B and from C to E.



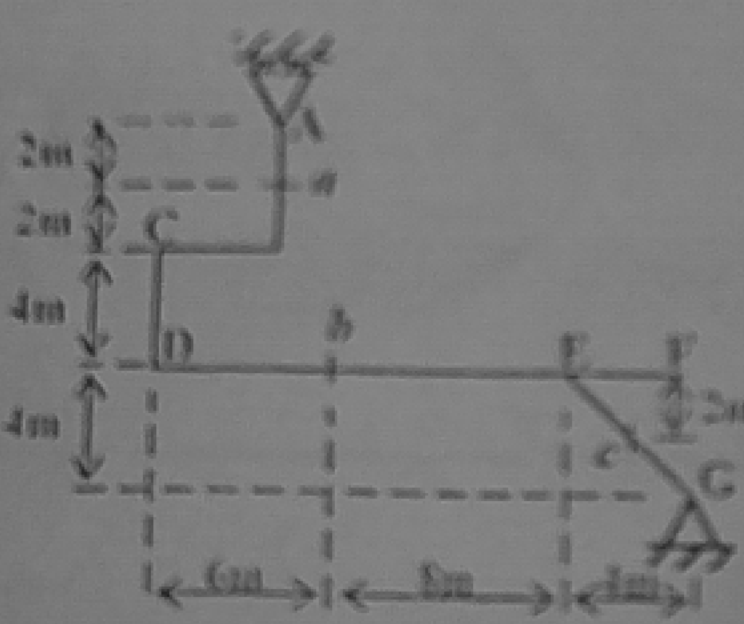
Prob. 12

13. Draw the influence lines for M_A, V_a, V_b, M_a and M_b as a unit load moves from B to C and from D to E.



Prob. 13

14. Draw the influence lines for V_a, V_b, V_c, M_a, M_b and M_c as a unit load moves from D to F.



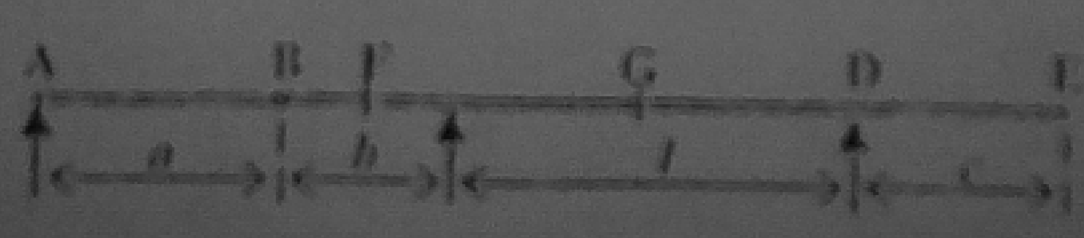
Prob. 14

15. As a unit load moves from A to E on the following structure, draw the influence lines for: (Vazirani & Ratwani, Vol-I, p-502)

(i) Reactions R_A, R_B, R_C, and R_D.

(ii) Shear and moment at F.

(iii) Shear and moment at G.



Prob. 15

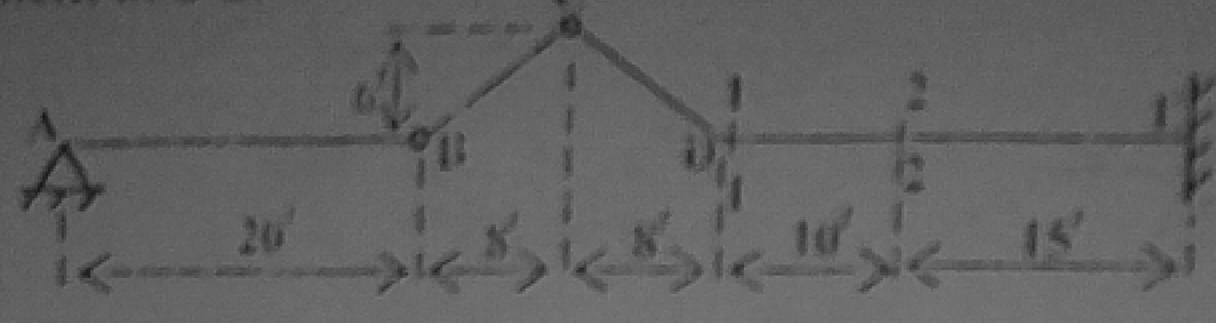
9. As a unit load moves from A to B and from D to E on the following structure, draw the influence lines for:

(i) Reaction at support A.

(ii) Reaction in the member BC.

(iii) Shear and moment at 1-1 (Just to the right of D).

(iv) Shear and moment at 2-2.



Prob. 9

When 1k at A

$R_A = 1k; F_{BC} = 0.0, V_{1-1} = 0.0, V_{2-2} = 0.0; M_{1-1} = 0.0; M_{2-2} = 0.0$

When 1k at B

$R_A = -1.33k; F_{BC} = 1.66k; V_{1-1} = -1.0; V_{2-2} = -1.0, M_{1-1} = -16.0$ k-ft; $M_{2-2} = -26.0$ k-ft

When 1k at D

$R_A = 0.0k; F_{BC} = 0.0k; V_{1-1} = -1.0; V_{2-2} = -1.0, M_{1-1} = 0.0$ k-ft; $M_{2-2} = -10.0$ k-ft

When 1k at right of 1-1

$R_A = 0.0k; F_{BC} = 0.0k; V_{1-1} = 0.0; V_{2-2} = -1.0; M_{1-1} = 0.0$ k-ft; $M_{2-2} = -10.0$ k-ft

When 1k at left of 2-2

$R_A = 0.0k; F_{BC} = 0.0k; V_{1-1} = 0.0; V_{2-2} = -1.0, M_{1-1} = 0.0$ k-ft; $M_{2-2} = 0.0$ k-ft

When 1k at left of 2-2

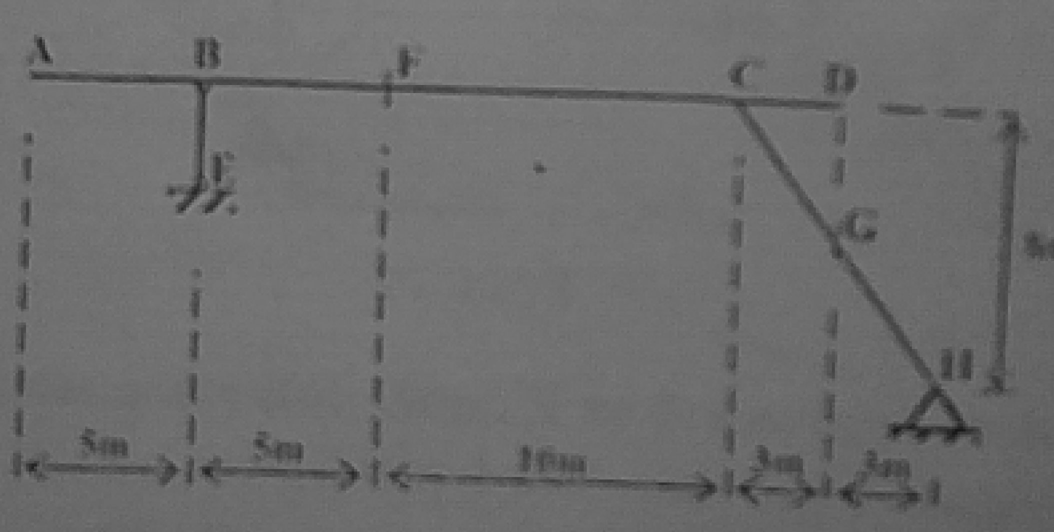
$R_A = 0.0k; F_{BC} = 0.0k; V_{1-1} = 0.0; V_{2-2} = 0.0, M_{1-1} = 0.0$ k-ft; $M_{2-2} = 0.0$ k-ft

10. As a unit load moves from A to D on the following structure, draw the influence lines for:

(i) Reaction in the member BE.

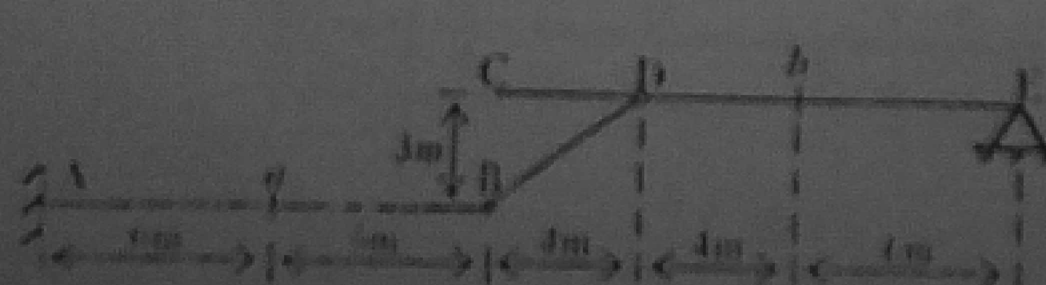
(ii) Shear and moment at F.

(iii) Shear and moment at G.



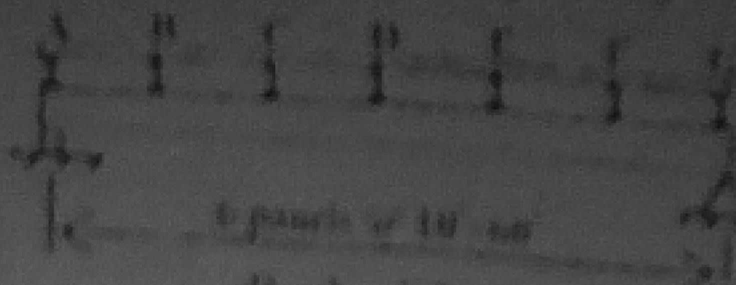
Prob. 10

11. Draw the influence lines for R_{ED}, M_a, V_a, V_b, M_a and M_b as a unit load moves from A to B and from C to E.



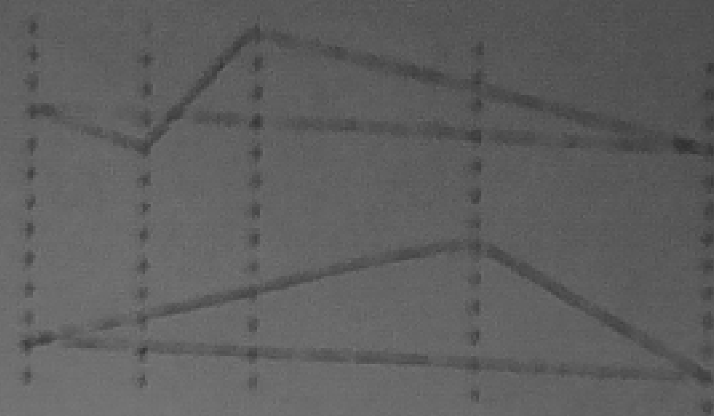
Prob. 11

For the girder shown, construct the influence lines for (i) shear in panel BC and (ii) moment at E. (Norris & Wilbur, 4th Ed, p-172)



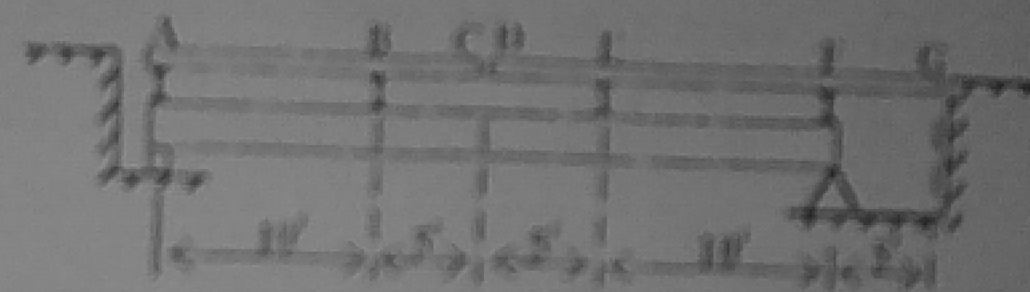
Prob. 16

V_B = - 1/6 k, V_c = +2/3 k

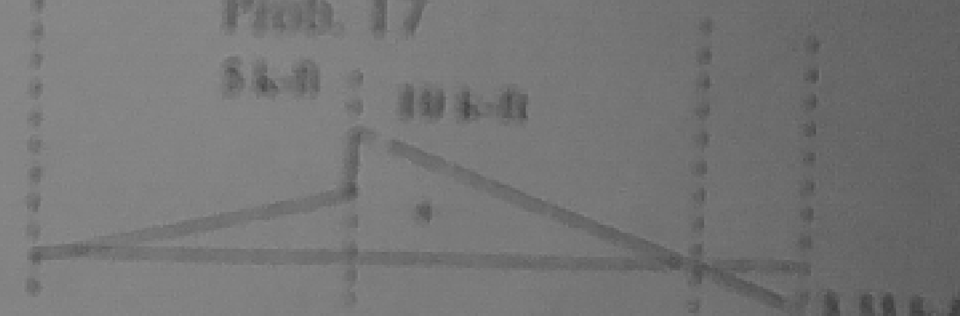


M_E = +13.33 k-ft

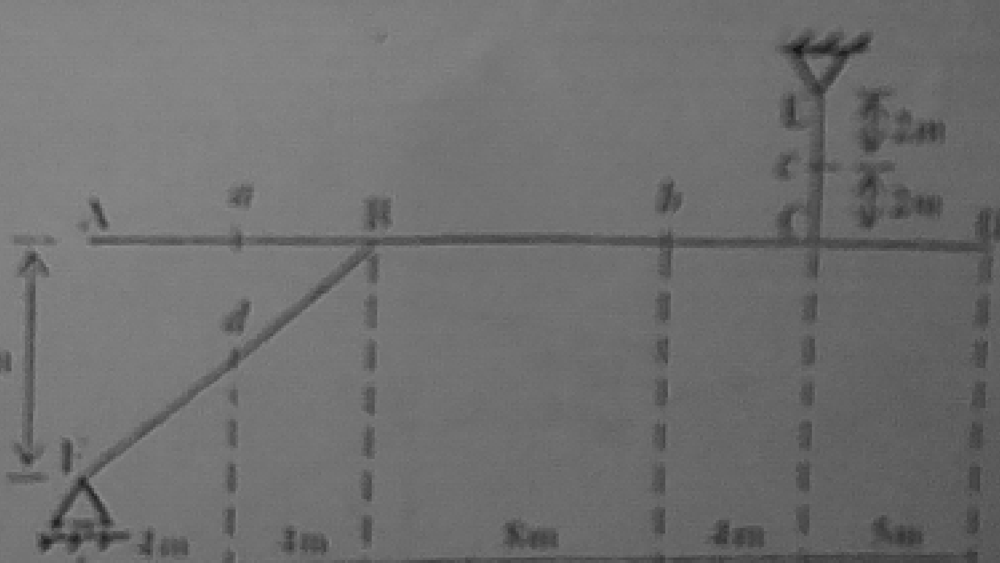
17. Note the unusual stringer arrangement for this girder. Construct the influence lines for the bending moment in the girder at E. (Norris & Wilbur, 4th Ed, p-173)



Prob. 17

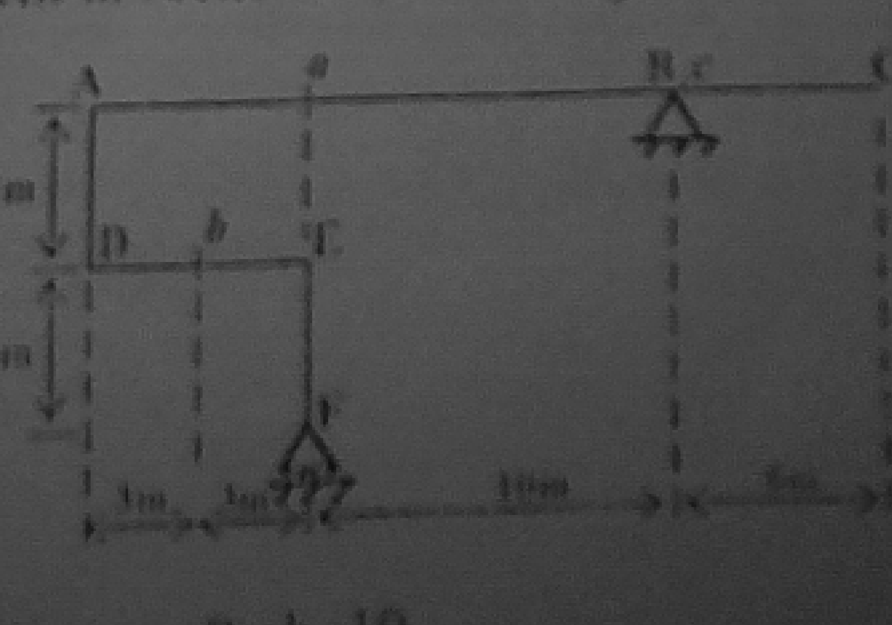


17.a As a unit load moves from A to D on the following structure, draw the influence lines for shear and moment at sections *a*, *b*, *c*, and *d*.



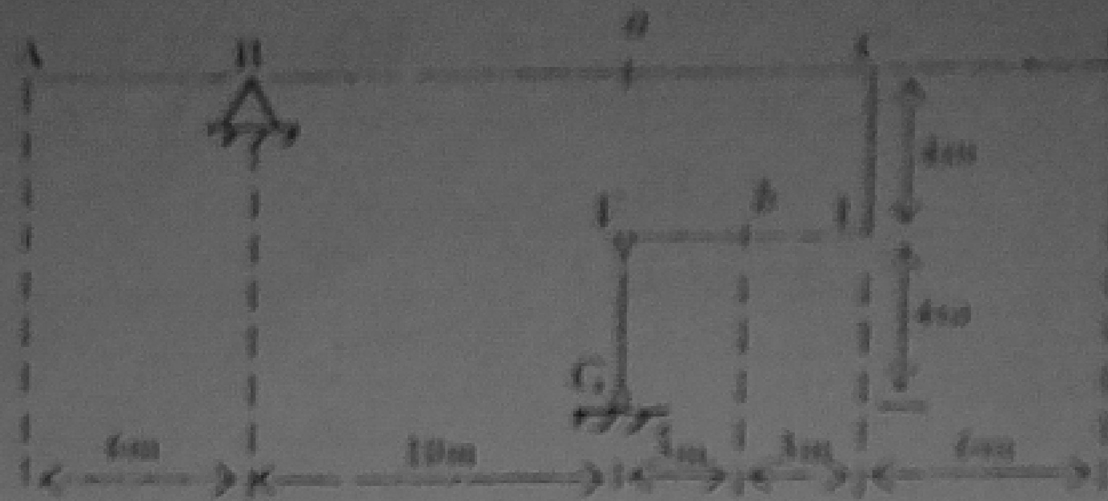
Prob. 18

18. As a unit load moves from A to D on the following structure, draw the influence lines for shear and moment at sections *a*, *b*, and *c*(just to the right of B).



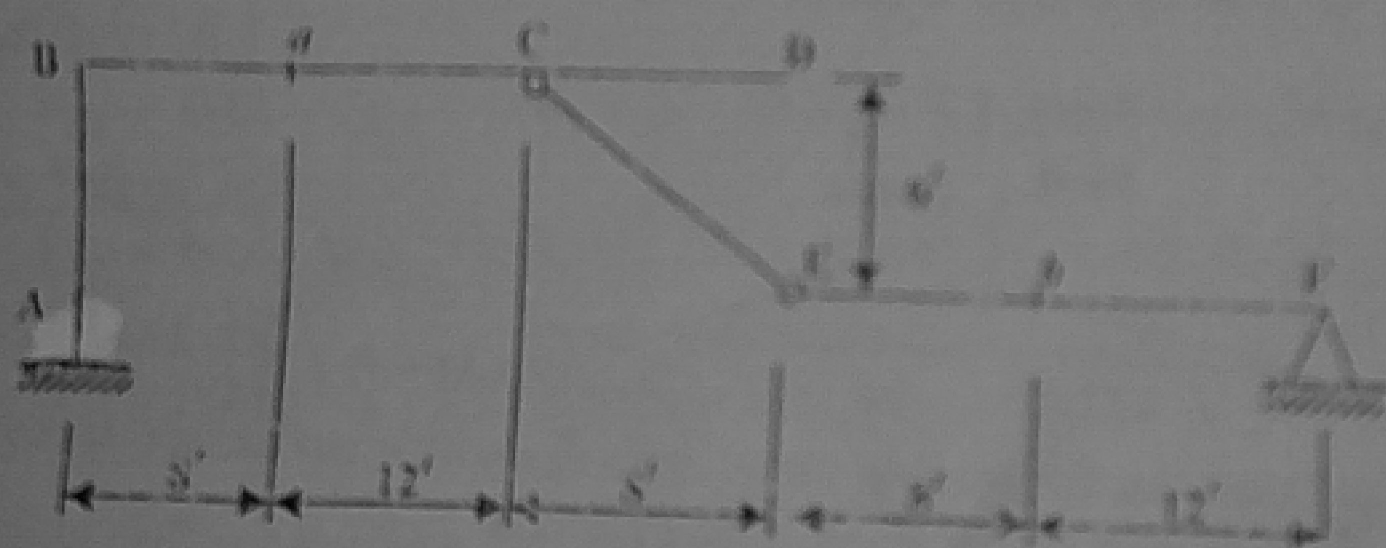
Prob. 19

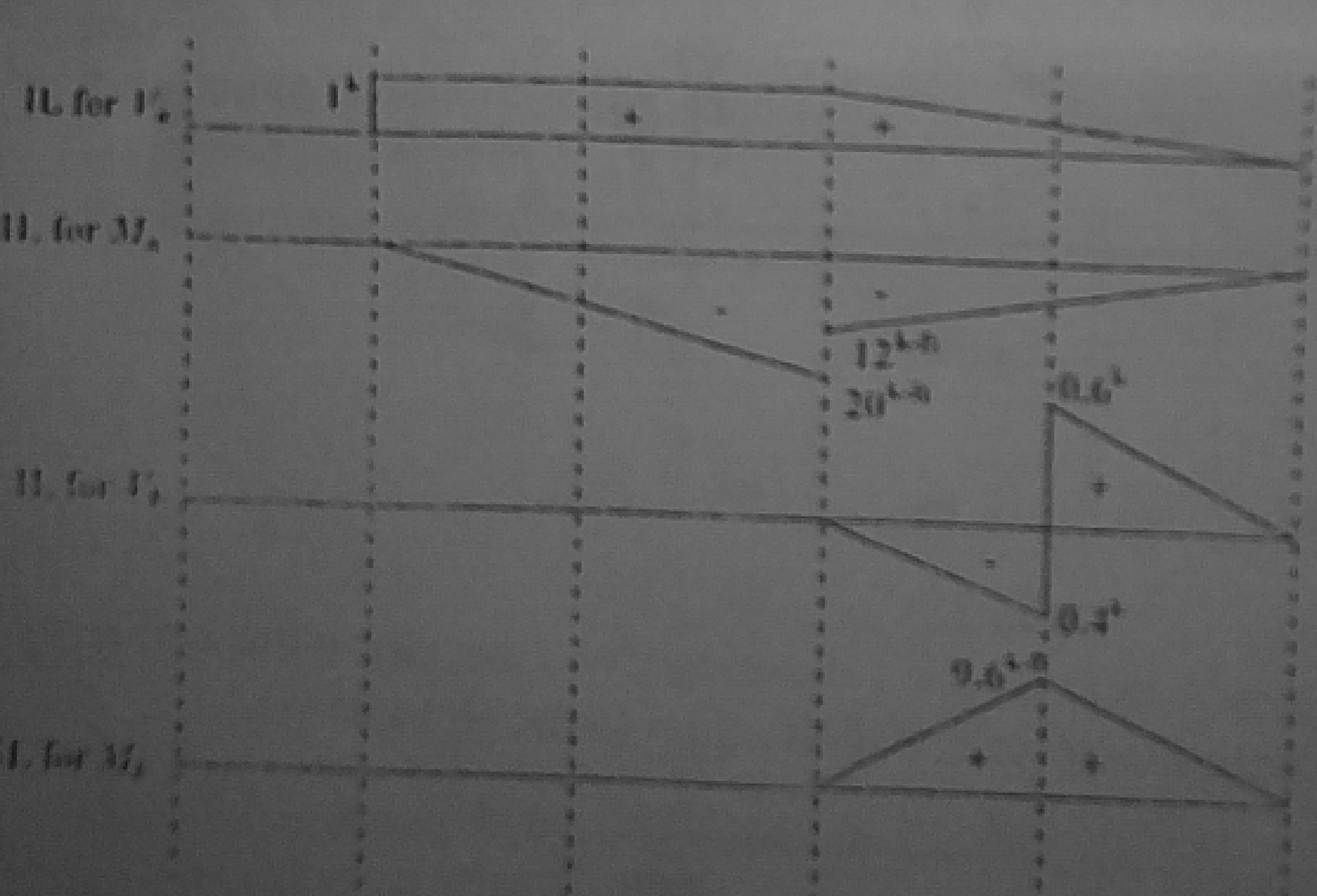
19. As a unit load moves from A to D on the following structure, draw the influence lines for shear and moment at sections a, and b. Also, draw the influence lines for reaction in member FG.



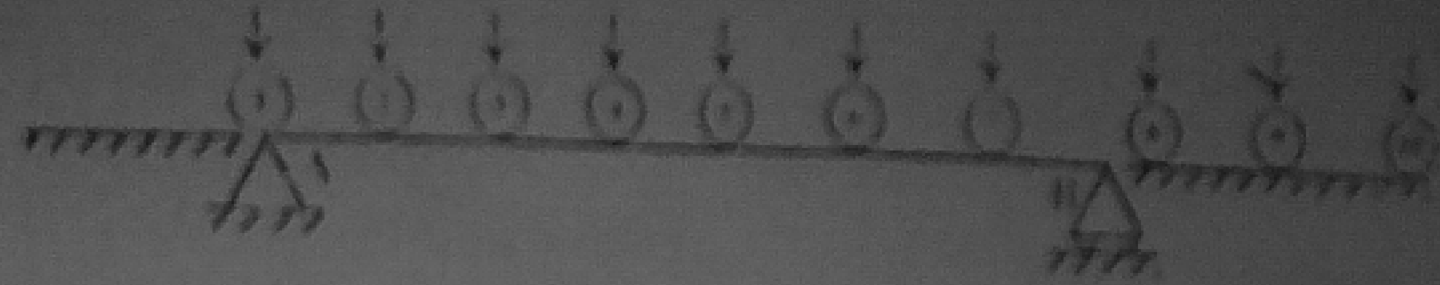
Prob. 20

21. As a unit load moves from A to D and E to F on the following structure, draw the influence lines for shear and moment at sections a, and b. Also, draw the influence lines for reaction in member FG.





[illegible] for the maximum reaction of a simple beam subjected to series of concentrated loads move from right to left.



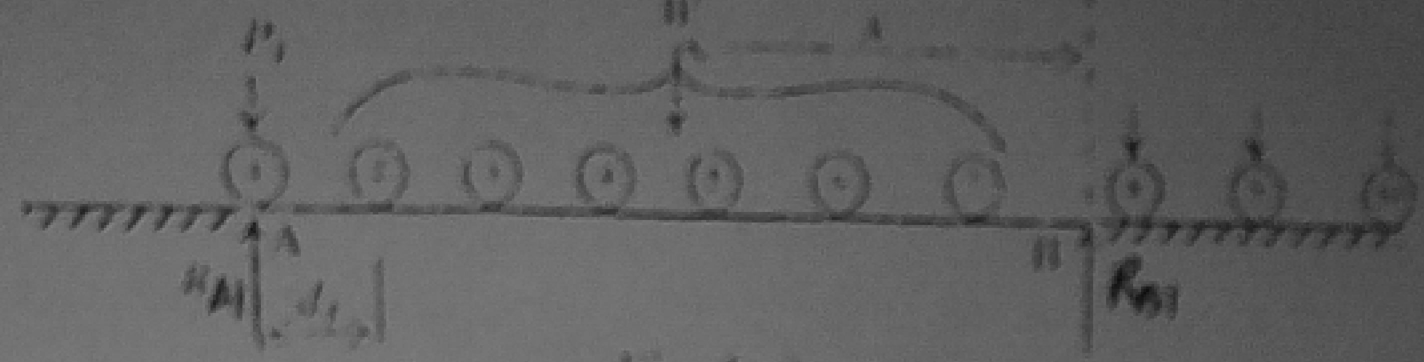


Fig. 1(a)

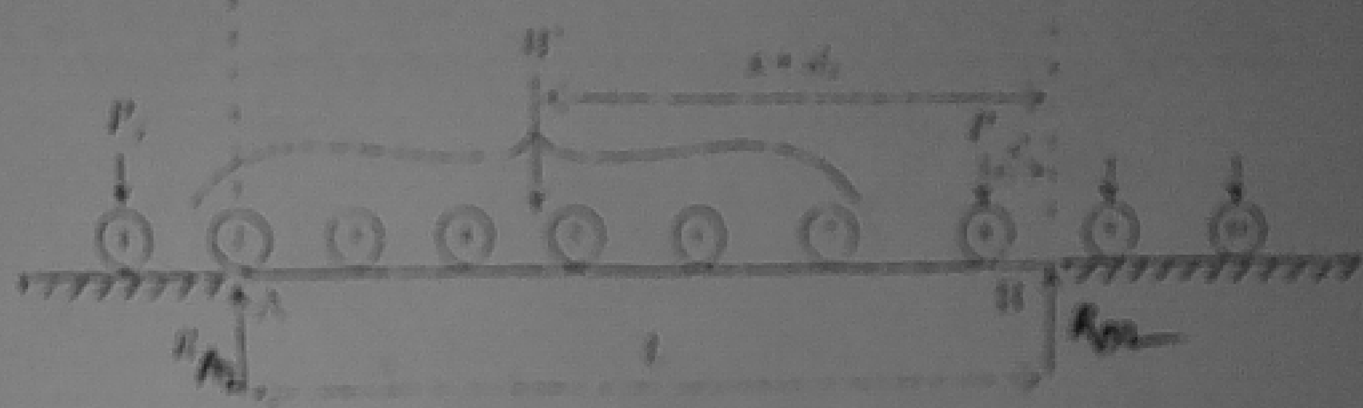


Fig. 1(a)

Let P_1 = the load which was over the left support and is moved off the span after movement.

d_1 = the distance between P_1 and the following wheel.

L = span length of the beam.

$W = \sum P$ = the sum of all the loads which are on the span before movement and stay on during movement.

P' = the load which enters in the span after the movement.

e = the distance of load P' from right support at B.

From Fig. 1(a), before the movement of wheels,

$$\sum M_B = R_{A1}L - Wx - P_1L = 0$$

or, $$R_{A1} = \frac{Wx}{L} + P_1 \quad (1)$$

From Fig. 1(b), after the movement of wheels,

$$\sum M_B = R_{A2}L - W(d_1 + x) - P'e = 0$$

or, $$R_{A2} = \frac{W(d_1 + x)}{L} + \frac{P'e}{L} \quad (2)$$

The change in reaction at support A due to the movement of the wheel can be derived by subtracting Eq (1) from Eq (2) then equation is given by

$$\Delta R = R_{A2} - R_{A1} = \frac{W(d_1 + x)}{L} + \frac{P'e}{L} - \frac{Wx}{L} - P_1$$

or, $$\Delta R = \frac{Wd_1}{L} - \frac{P'e}{L} - P_1 \quad (3)$$

or, $$\Delta R = \frac{\sum P d_1}{L} + \frac{P' e}{L} - P_1 \qquad (4)$$

N.B. In the above expression, P_1 is the first wheel as shown in Fig. 1. However, for the subsequent action wheel 2 must be replaced by P_2. Similarly, P_8 is P' in the expression which is outside of span before movement and enters in the span after movement. The wheel 8 must be replaced by wheel 9. This procedure continues uil the ΔR changes its sign of character.

Criteria for the maximum shear of a simple beam subjected to series of concentrated loads move from right to left.

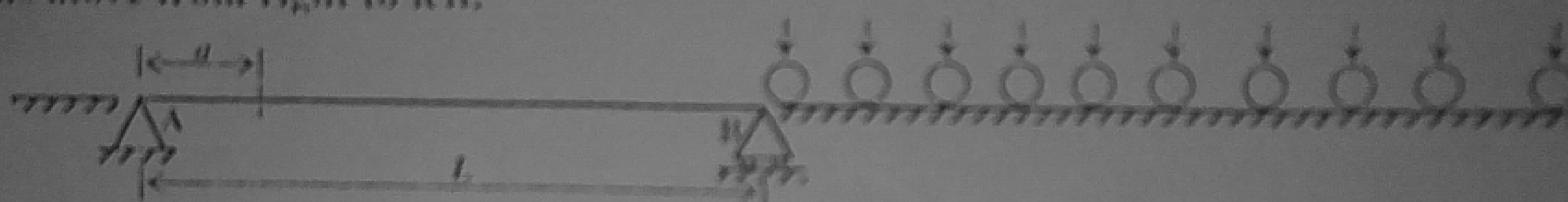


Fig. 2(a)

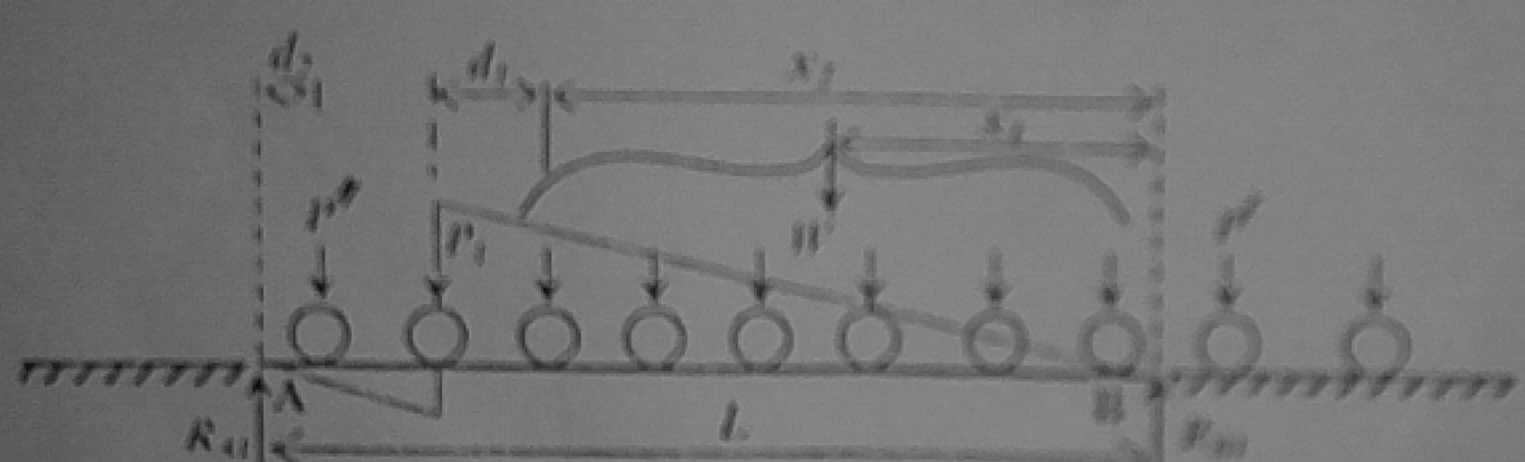


Fig. 2(b)

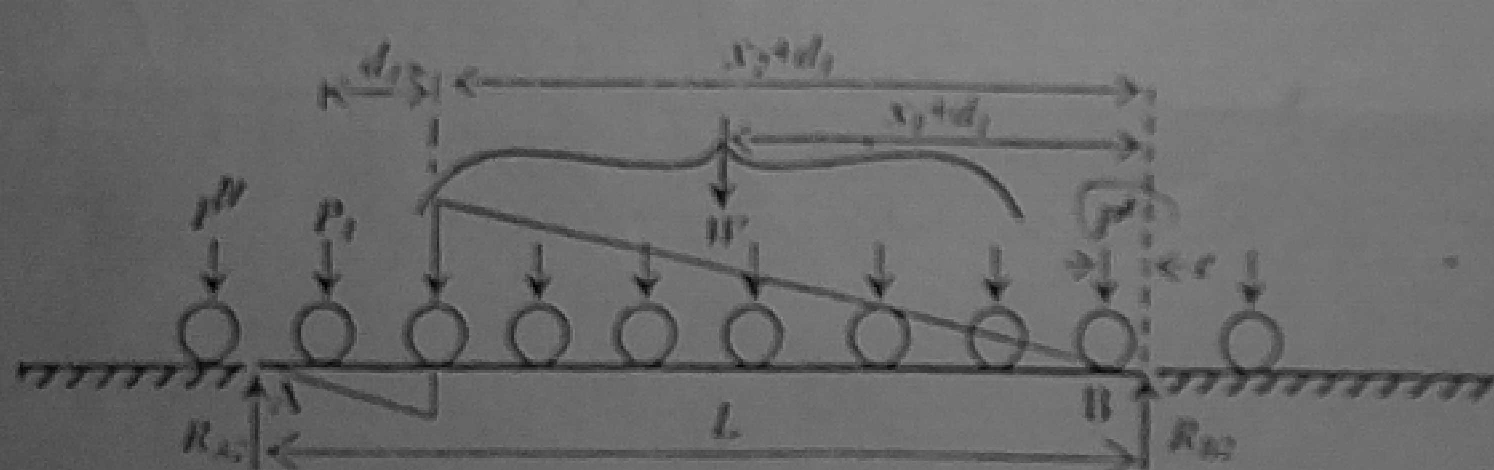


Fig. 2(c)

A simply supported beam of span length L is shown in Fig. 2(a). The beam is subjected to series moving loads. Loads move from right to left and pass through a section at a distance 'a' from the left support. It is required to determine the criteria to obtain the position of loads for maximum shear. Fig. 2(b) and Fig. 2(c) show the position wheel loads before and after movement.

Let. P_1 = the load which was over section and is moved off the section after movement.

P' = the load which was off the span before movement and enter in the span after movement.

P'' = the load which was in negative shear zone before movement and is moved off the span after movement.

d_1 = the distance between P_1 and the following wheel.

d_2 = the distance of wheel load P'' from left support.

l = span length of the beam.

$W = \sum P$ = the sum of all the loads which are on the span before movement and stay on during movement.

P' = the load which enters in the span after the movement.

e = the distance of load P' from right support at B.

From Fig. 2(b), the reaction at left support before the movement of wheels R_{A1} can be obtained as follows:

$$\sum M_{B0} = R_{A1}L - P''(L - d_2) - Wx_1 - P_1(d_1 + x_2) = 0$$

or,
$$R_{A1} = \frac{Wx_1}{L} + P_1\frac{(d_1 + x_2)}{L} + P''\frac{(L - d_2)}{L}$$

Now, shear force of the section,

$$V_1 = R_{A1} - P''$$

$$V_1 = \frac{Wx_1}{L} + P\frac{(d_1 + x_2)}{L} + P''\frac{(L - d_2)}{L} - P''$$

$$V_1 = \frac{Wx_1}{L} + P\frac{(d_1 + x_2)}{L} - \frac{P''d_2}{L} \qquad (1)$$

From Fig. 2(c), the reaction at left support after the movement of wheels R_{A2} can be obtained as follows:

$$\sum M_B = R_{A2}L - W(d_1 + x_1) - P_1(2d_1 + x_2) - P'e = 0$$

or,
$$R_{A2} = \frac{W(d_1 + x_1)}{L} + P_1\frac{(2d_1 + x_2)}{L} + P'\frac{e}{L}$$

Now, shear force of the section,

$$V_2 = R_{A2} - P_1$$

$$V_2 = \frac{W(d_1 + x_1)}{L} + P_1\frac{(2d_1 + x_2)}{L} + P'\frac{e}{L} - P_1 \qquad (2)$$

The change in shear at this section due to the movement of the wheel can be obtained by subtracting Eq. (1) from Eq. (2).

$$\Delta V = V_2 - V_1 = \frac{W(d_1 + x_1)}{L} + P_1\frac{(2d_1 + x_2)}{L} + P'\frac{e}{L} - P_1 - \frac{Wx_1}{L} - P\frac{(d_1 + x_2)}{L} + \frac{P''d_2}{L}$$

or,
$$\Delta V = \frac{(W + P_1)d_1}{L} + \frac{P''d_2}{L} + \frac{P'e}{L} - P_1 \qquad (3)$$

or,
$$\Delta V = \frac{\sum Pd_1}{L} + \frac{P''d_2}{L} + \frac{P'e}{L} - P_1 \qquad (4)$$

The maximum shear force at this section will occur when ΔV tends to zero. This procedure continues till the ΔV changes its sign of character.

Criteria for the maximum moment at section of a simple beam subjected to series of concentrated loads move from right to left.

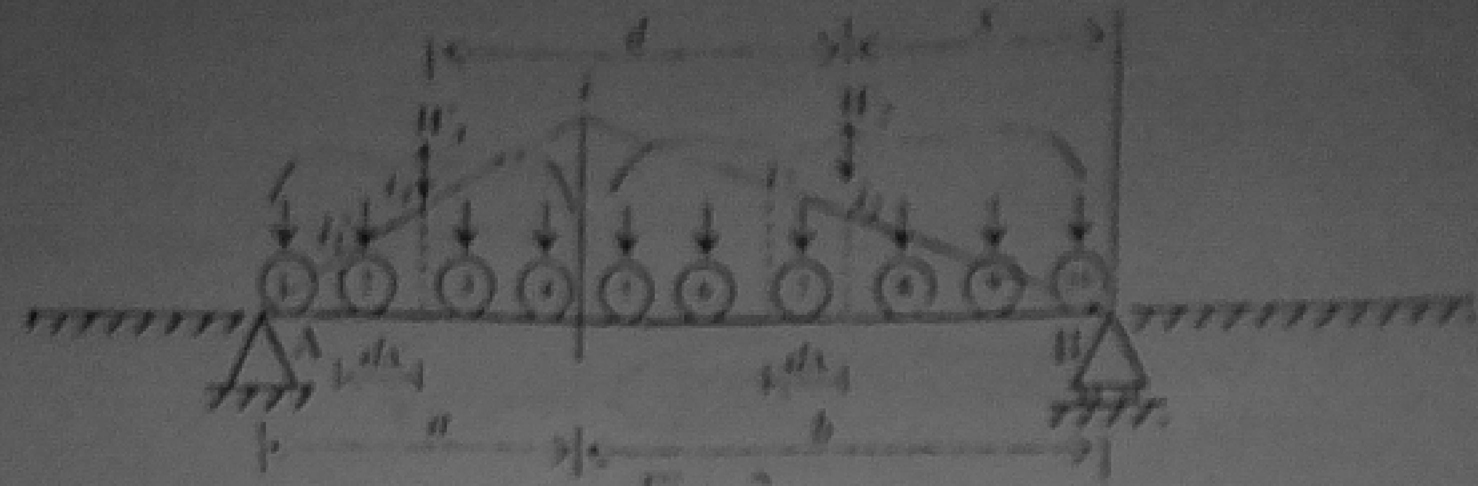


Fig. 3

Let, W_1 = load to the left of the section

W_2 = load to the right of the section

W = total load to the left of the section

i = ordinate of influence line for bending moment at the section

i_1, i_2 = are the ordinates of influence line for bending moment under load W_1 and W_2 before movement.

i_1', i_2' = are the ordinates of influence line for bending moment under load W_1 and W_2 after movement.

Considering the right side of the section as shown in Fig. 3

$$i_2 = \frac{x}{b} i, \quad i_2' = \frac{(x + dx)}{b} i,$$

Corresponding moment, $M_2 = i\frac{x}{b}W_2, \quad M_2' = i\frac{(x+dx)}{b}W_2$

Increase of moment in the right hand side, $\Delta M_2 = M_2' - M_2 = i\frac{W_2}{b}dx$

Considering the left side of the section

$$i_1 = \frac{(L - x - d)}{a} i, \quad i_1' = \frac{(L - x - d - dx)}{a} i,$$

Corresponding moment, $M_1 = i\frac{(L-x-d)}{a}W_1, \quad M_1' = i\frac{(L-x-d-dx)}{a}W_1$

Decrease of moment in the left hand side, $\Delta M_1 = M_1' - M_1 = -i\frac{W_1}{a}dx$

Net increase in moment, $dM = \Delta M_1 + \Delta M_2 = -i\frac{W_1}{a}dx + i\frac{W_2}{b}dx$

Therefore, $\frac{dM}{dx} = -i\frac{W_1}{a} + i\frac{W_2}{b}$

For maximum derivative of moment with respect x must be zero

or, $\frac{dM}{dx} = -i\frac{W_1}{a} + i\frac{W_2}{b} = 0$, or, $\frac{W_1}{a} = \frac{W_2}{b} = \frac{W_1 + W_2}{a + b} = \frac{W}{L}$

$\frac{W_1}{a} = \frac{W_2}{b} = \frac{W_1+W_2}{a+b} = \frac{W}{L}$

The maximum moment at a given section occurs when the intensity of loading on the left side of the section is equal to the intensity of loading on the span.

Criteria for the absolute maximum moment of a simple beam subjected to series of concentrated moving loads.