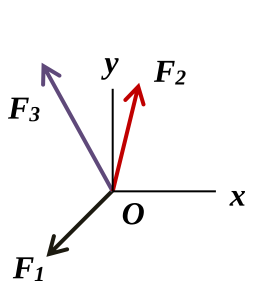
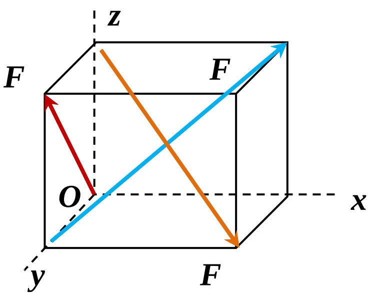
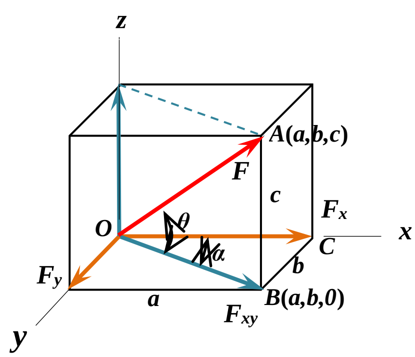
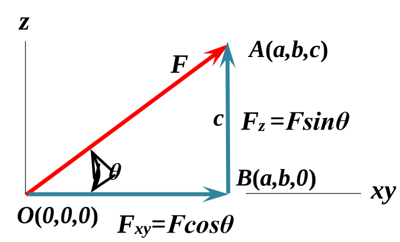
**Non-coplanar force systems,** in which the lines of action of the forces do not lie in the same plan. A non-concurrent system may be either coplanar or non-coplanar. It may be **either concurrent or non-concurrent.**




**Resolution of non-coplanar force**



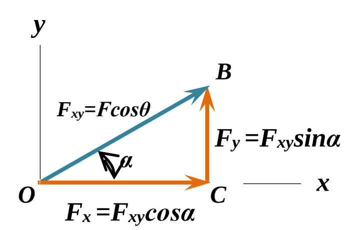




$$F_z = F\sin\theta = F\frac{c}{OA} = F\frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$F_{xy} = F\cos\theta = F\frac{OB}{OA} = F\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+c^2}}$$

$$F_x = F_{xy}\cos\alpha = F\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+c^2}}\times\frac{a}{\sqrt{a^2+b^2}} = F\frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$F_y = F_{xy}\cos\alpha = F\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+c^2}}\times\frac{b}{\sqrt{a^2+b^2}} = F\frac{b}{\sqrt{a^2+b^2+c^2}}$$



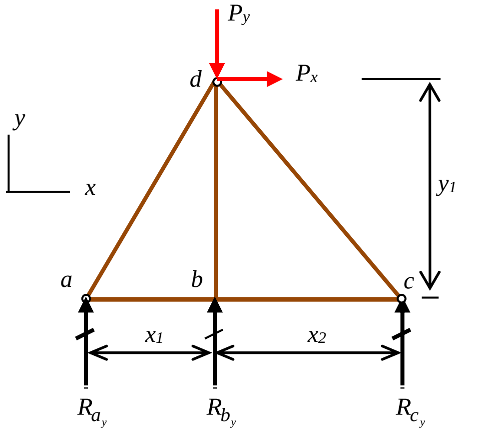

Therefore, components are

$$F_x = F\frac{a}{\sqrt{a^2+b^2+c^2}}$$

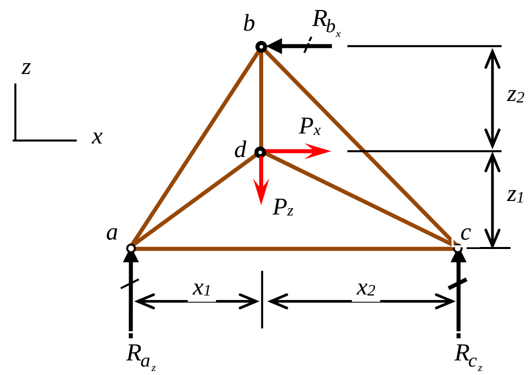
$$F_y = F\frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$F_z = F\frac{c}{\sqrt{a^2+b^2+c^2}}$$

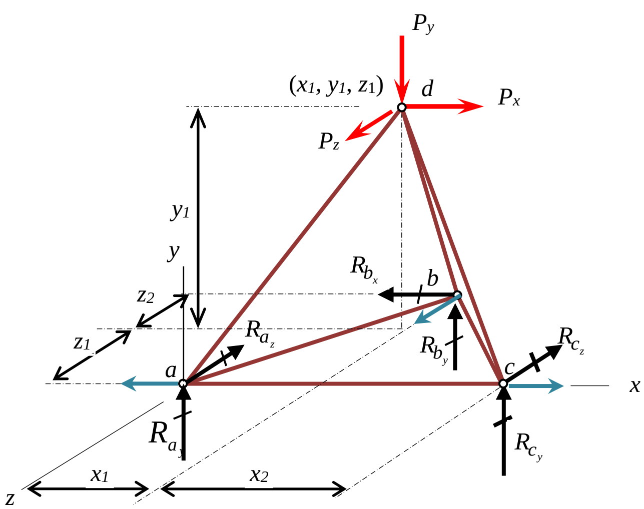
**Space Trusses:** A space truss consist of members joint together at their ends to form a stable three dimensional structure. Tetrahedron is the simplest form of space truss as shown below. A simple space truss can be built from the basic tetrahedral element by three additional members and a joint, and continuing in this manner to form a system of multi-connected tetrahedrons.




**Elevation**




**Top View**




**Typical diagram of space truss.**

**Assumptions of Analysis:**

The members of a space truss may be treated as axial force member (as two force member in plan truss) and connected by ball and socket joints.

The force applied at joint only.

The weight of members can be neglected (or transfer to the joint).

Welded or bolted connections of the members intersect at a common point(Joint is assumed to be fixed end moment free condition).

# Methods

## Method of Joints

Three equilibrium equations are applied in each joint to determine bar forces.

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0,$$

## Method of Sections

To method of sections, a rigid segment of the truss can be isolate by passing a surface through not more than six members. Such isolated portion is a free body in equilibrium under the action of bar forces, applied forces and support reactions. In case of method of section of three dimensional space trusses, six independent equations of static can be written as follows:

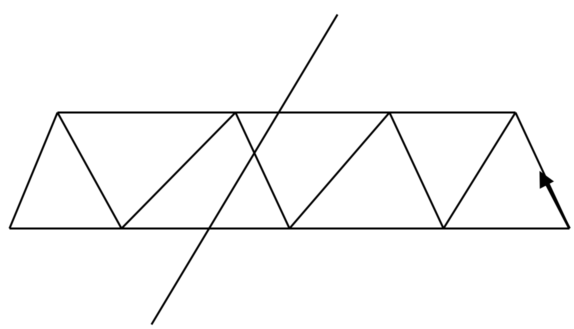
$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0, \quad \sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0$$

A necessary (although not sufficient) condition for statical determination of three dimensional truss with respect to both inner and outer forces is that the total number of bars plus the total number of independent reaction components shall equal three times the number of joints.

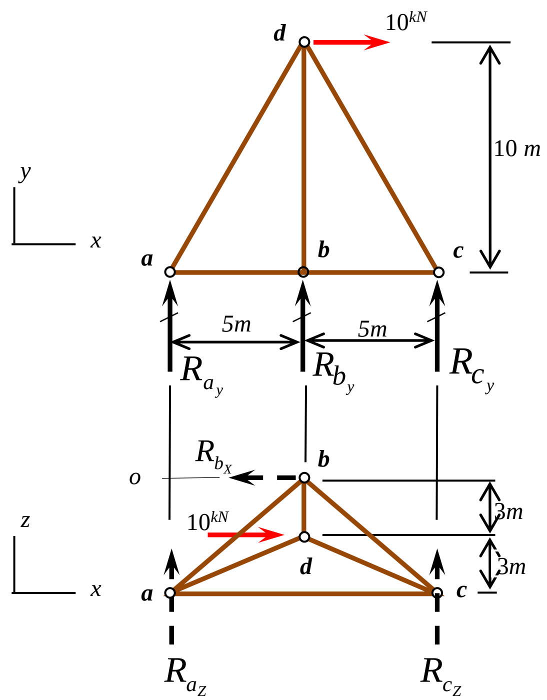
**In general**

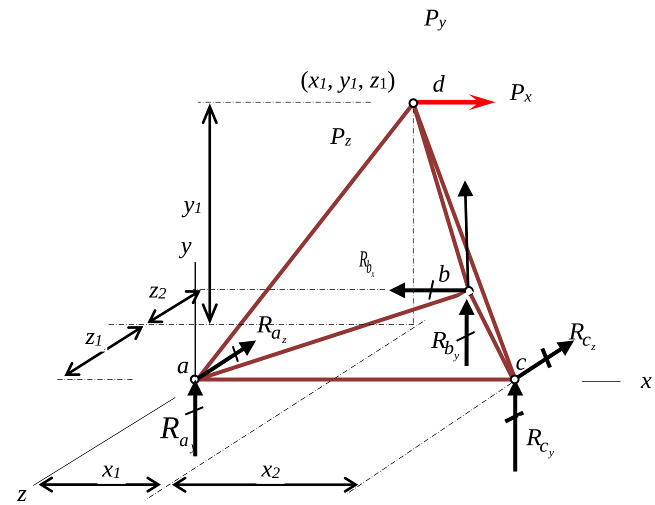
b + r < 3n, the structure is unstable.
b + r = 3n, the structure is statically determinate.
b + r > 3n, the structure is indeterminate.

**v**



**Example 1 Determine reactions and bar forces of the following space truss.**




| Member | Projection | | | length | x-slope | y-slope | z-slope |
|---|---|---|---|---|---|---|---|
| | *x* | *y* | *z* | | | | |
| *ad* | 5 | 10 | 3 | 11.58 | 0.4318 | 0.8636 | 0.2591 |
| *bd* | 0 | 10 | 3 | 10.44 | 0 | 0.9579 | 0.2874 |
| *cd* | 5 | 10 | 3 | 11.58 | 0.4318 | 0.8636 | 0.2591 |
| *ab* | 5 | 0 | 6 | 7.81 | 0.6402 | 0 | 0.7682 |
| *bc* | 5 | 0 | 6 | 7.81 | 0.6402 | 0 | 0.7682 |
| *ac* | 10 | 0 | 0 | 10.00 | 1.00 | 0 | 0 |

**For entire structure**

$\sum M_x^{ac} = 0$; $\rightarrow R_{by} = 0$

$\sum M_z^{ao} = 0$; $\rightarrow 10*10 - (R_{cy})(10) = 0$; $\rightarrow R_{cy} = +10$ kN (↑)

$\sum F_y = 0$; $\rightarrow R_{ay} + R_{by} + R_{cy} = 0$; $\rightarrow R_{ay} = -10$ kN (↓)

$\sum F_x = 0$; $\rightarrow 10 - R_{bx} = 0$; $\rightarrow R_{bx} = +10$ kN (←)

$\sum M_y^{o} = 0$; $\rightarrow -10(3) - R_{cz}(10) = 0$ $\rightarrow R_{cz} = -3$ kN (↓)

$\sum F_Z = 0$; $\rightarrow R_{az} + R_{cz} = 0$; $\rightarrow R_{az} = +10$ kN (↑)

**All members are assumed to be in tension.**

At joint *d*,

$\sum F_x = 0$; $10 - F_{adx} + F_{dcx} = 0$; $10 - 0.4318F_{ad} + 0.4318F_{dc} = 0$

$\sum F_y = 0$; $F_{ady} + F_{bdy} + F_{cdy} = 0$; $-0.8636F_{ad} - 0.8636F_{cd} - 0.9579F_{bd} = 0$

$\sum F_z = 0$; $F_{adz} - F_{bdz} + F_{cdz} = 0$; $0.2591F_{ad} + 0.2591F_{cd} - 0.2874F_{bd} = 0$

$F_{bd} = 0.0$; $F_{ad} = +11.58\ kN(t)$, $F_{cd} = -11.58\ kN(c)$

At joint *b*,

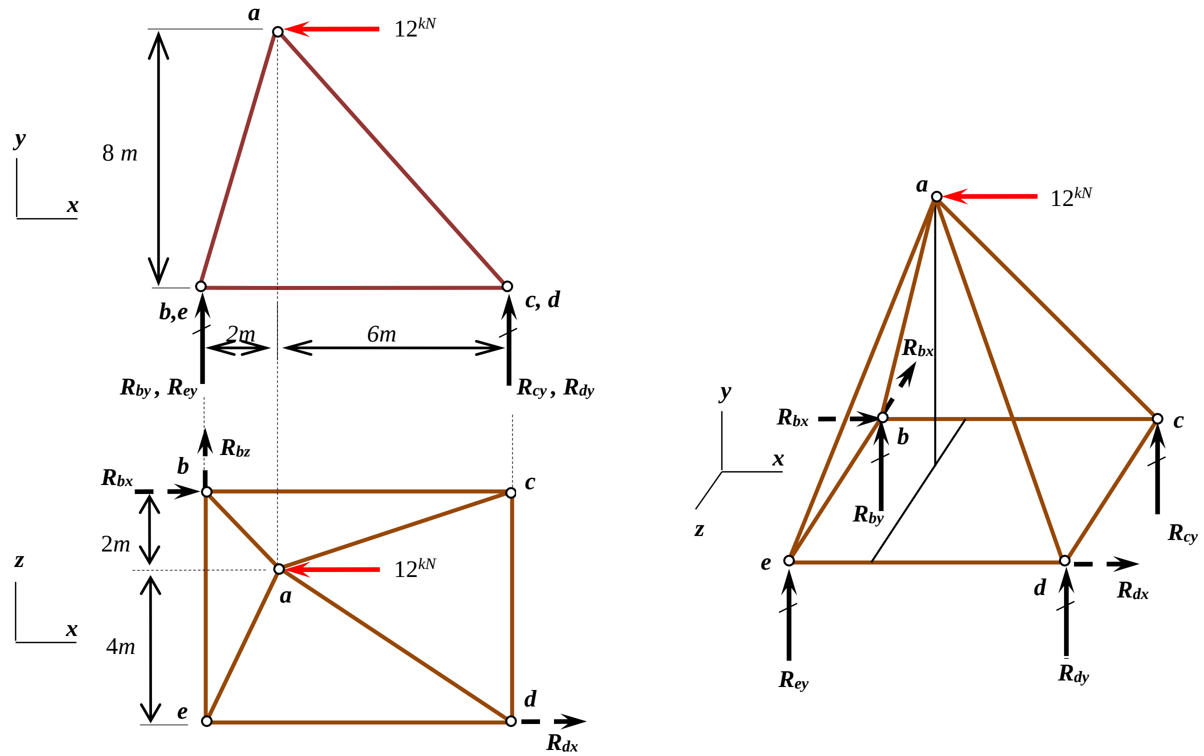
$\sum F_y = 0$; $F_{bdy} + R_{by} = 0$; $0.9579F_{bd} + R_{by} = 0$; $F_{bd} = R_{by} = 0.0$;

$\sum F_x = 0$; $-R_{bx} - F_{abx} + F_{bcx} = 0$; $-10 - 0.6402F_{ab} + 0.6402F_{bc} = 0$

$\sum F_z = 0$; $F_{abz} + F_{bcz} + \cancel{F_{bdz}} = 0$; $0.7682F_{ab} + 0.7682F_{bc} = 0$

$F_{ab} = -7.81$ kN(*c*), $F_{cd} = +7.81$ kN(*t*)

**2. Determine the bar forces and reactions of the following space truss.**




| Member | Projection | | | Sum(Sq) | Slope | | | |
|---|---|---|---|---|---|---|---|---|
| | x | y | z | | Length | x | y | z |
| *ab* | 2 | 8 | 2 | 72 | 8.485 | 0.236 | 0.943 | 0.236 |
| *ac* | 6 | 8 | 2 | 104 | 10.198 | 0.588 | 0.784 | 0.196 |
| *ad* | 6 | 8 | 4 | 116 | 10.770 | 0.557 | 0.743 | 0.371 |
| *ae* | 2 | 8 | 4 | 84 | 9.165 | 0.218 | 0.873 | 0.436 |
| *bc* | 8 | 0 | 0 | 64 | 8.000 | 1.000 | 0.000 | 0.000 |
| *cd* | 0 | 0 | 6 | 36 | 6.000 | 0.000 | 0.000 | 1.000 |
| *de* | 8 | 0 | 0 | 64 | 8.000 | 1.000 | 0.000 | 0.000 |
| *eb* | 0 | 0 | 6 | 36 | 6.000 | 0.000 | 0.000 | 1.000 |

**$b + r = 3n$, b+r = 8 + 7 = 15, 3n=3*5 = 15, satisfy the determinacy and stability criteria.**
**Eq. of Equilibrium = 6, No. of reactions at support = 7,**

**Five joints can provide 15 simultaneous equations those can be solved to the bar forces and reaction. Alternately, consider one of bar forces as unknown and use moment equation to obtain the solution**

**At Joint** ***c*****,** $\sum F_z = F_{cdz} + F_{acz} = F_{cd} + 0.196F_{ac} = 0$ **;** $F_{ac} = -5.102F_{cd}$

**At Joint** ***d*****,** $\sum F_z = F_{cdz} + F_{adz} = F_{cd} + 0.371F_{ad} = 0$ **;** $F_{ad} = -2.695F_{cd}$

**At Joint** ***c*****,** $\sum F_y = R_{cy} + F_{acy} = R_{cy} + 0.784F_{ac} = 0$ **;** $R_{cy} = -0.784F_{ac} = 4.0F_{cd}$

**At Joint** ***d*****,** $\sum F_y = R_{dy} + F_{ady} = R_{dy} + 0.743F_{ad} = 0$ **;** $R_{dy} = -0.743F_{ad} = +2.0F_{cd}$

$$\sum M_z^{be} = -10*8 - (R_{cy})*8 - (R_{dy})*8 = 0$$

Putting the value of $R_{cy}$ and $R_{dy}$, we have

$-96 - (4.0)F_{cd}*8 - (2.0)F_{cd}*8 = 0$ $F_{cd} = -2.0\ kN$ (c)

$F_{ac} = -5.102F_{cd} = -5.102*(-2.0) = +10.20\ kN\ (t)$

$F_{ad} = -2.695F_{cd} = -2.695*(-2.0) = +5.39\ kN(t)$

$R_{cy} = 4.0F_{cd} = +4.0*(-2) = -8.0\ kN\ (\downarrow)$

$R_{dy} = +2.0F_{cd} = +2.0(-2.0) = -4.0\ kN\ (\downarrow)$

For entire structure,

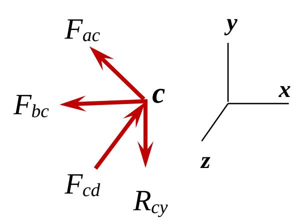
$\sum M_x^{bc} = +R_{ey}*6 + (-4.0)*6 = 0;$ $R_{ey} = +4.0\ kN\ (\uparrow)$

$\sum F_x = R_{by} + R_{cy} + R_{dy} + R_{ey} = 0;$ $R_{by} = 8 + 4 - 4 = 8\ kN\ (\uparrow)$

Also, $\sum M_z^{cd} = 0;$ can provide $R_{by}$ = ?

At Joint c,

$\sum F_x = F_{acx} + F_{bc} = 0; F_{bc} = -(0.588)*(10.20) = -6.0\ kN(c)$

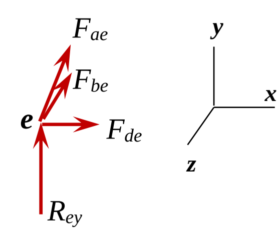



**At Joint e,**

$\sum F_y = R_{ey} + F_{aey} = +4 + 0.873F_{ae} = 0;$ $F_{ae} = -4.58\ kN(c)$

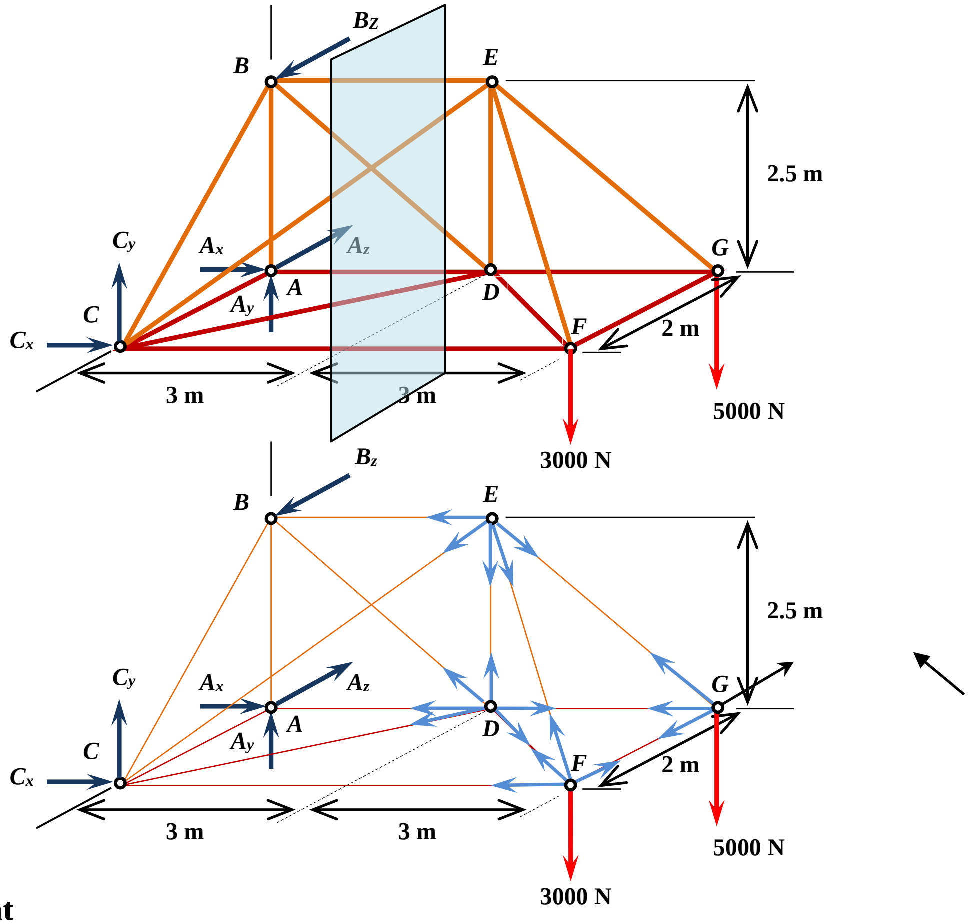
$\sum F_z = F_{be} + F_{aez} = F_{be} + 0.436(-4.58) = 0;$ $F_{be} = +2.0\ kN(t)$

$\sum F_x = F_{de} + F_{aex} = F_{de} + 0.218(-4.58) = 0;$ $F_{be} = +1.0\ kN(t)$




**Free the joints b and d to obtain other non forces**

**3. Find force in bar EC, CF and BE.**




**Method of joint**

**At Joint G**

$$\sum F_x = -\frac{3}{\sqrt{3^2+2.5^2}}GE - GD = 0; \quad GD = -0.604GE\,; \qquad GD = -4717.38\text{ N (c)}$$

$$\sum F_y = -\frac{2.5}{3.905}GE + 5000 = 0; \quad GE = 2000\sqrt{15.25}N\ \textbf{(t);} \qquad GE = +7810.24\text{ N (t)}$$

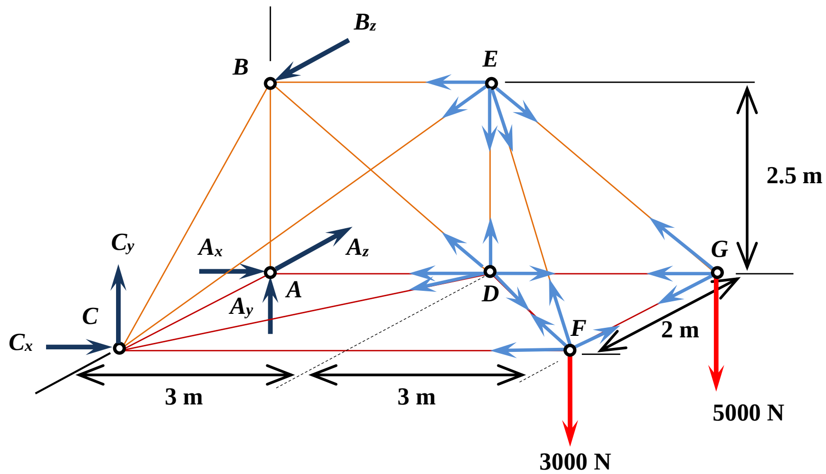
$$\sum F_z = GF = 0; \quad GF = 0 \qquad GF = 0$$

**At Joint F**

$$\sum F_x = \frac{3}{\sqrt{3^2+2.5^2+2^2}}EF + CF + \frac{3}{\sqrt{3^2+2^2}}DF = 0; \qquad CF = 0$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2+2.5^2+2^2}}EF - 3000 = 0; \quad EF = +1200\sqrt{19.25}N\ \textbf{(t)}$$

$$\sum F_z = \frac{2}{\sqrt{3^2+2.5^2+2^2}}EF + \frac{2}{\sqrt{3^2+2^2}}DF + FG = 0; \quad DF = -1200\sqrt{13}N$$




At joint E

$$\sum F_x = \frac{3}{\sqrt{3^2+2.5^2+2^2}}\left(EF = 1200\sqrt{19.25}\right) + \frac{2.5}{\sqrt{2.5^2+3^2}}\left(EG = ?-2000\sqrt{15.25}\right)$$

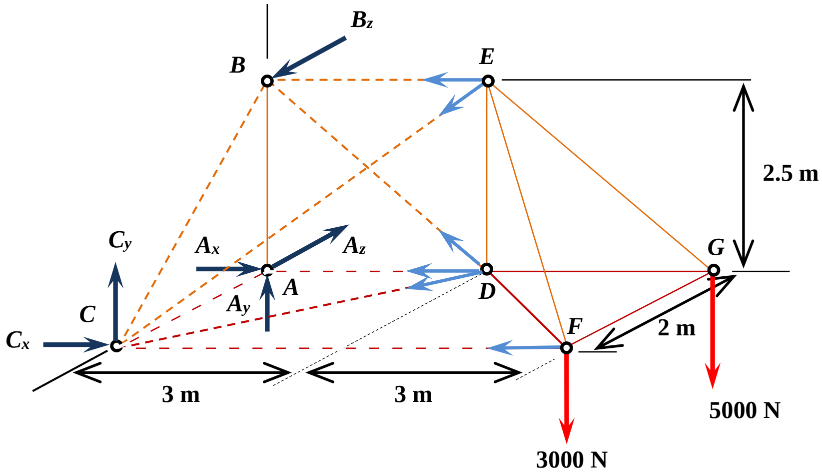
$$-BE - \frac{3}{\sqrt{3^2+2.5^2+2^2}} CE = 0; \quad BE = 2200\ N(t)$$

$$\sum F_z = \frac{2}{\sqrt{3^2+2.5^2+2^2}} CE + \frac{2}{\sqrt{3^2+2.5^2+2^2}} (EF = +1200\sqrt{19.25}) = 0; \; CE = -1200\sqrt{19.25} N$$

$$\sum F_y = \frac{2.5}{\sqrt{3^2+2.5^2+2^2}} (EF = 1200\sqrt{19.25}) + \frac{2.5}{\sqrt{3^2+2.5^2+2^2}} (CE = -1200\sqrt{19.25}) +$$

$$\frac{2.5}{\sqrt{3^2+2.5^2}} (EG = 2000\sqrt{15.25}) + ED = 0; \quad ED = -5000\ N(c)$$

## Method of Sections



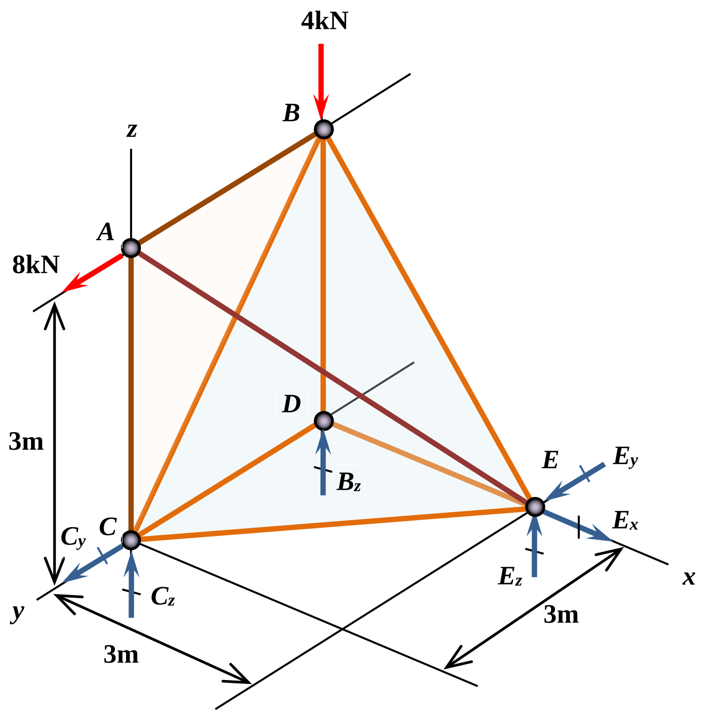

Free body diagram of portion *DEFG*.

$$\sum M_D^{x\to DG} = 3000*2 + (CE_z = \frac{2}{\sqrt{19.25}} CE)*2.5 = 0; \quad CE = -1200\sqrt{19.25}\ N(c)$$

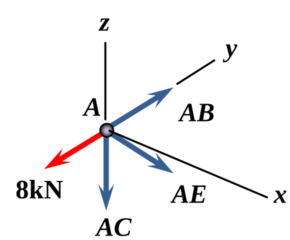
$$\sum M_D^{y\to DE} = 0; \quad CF = 0;$$

$$\sum M_D^{z} = 0; \quad CE_x * 2.5 + BE * 2.5 - 3000*3 - 5000*3 = 0$$

4. **Determine the force acting in the members of space truss shown in figure below. Hibbeler, p. 302**




**At joint A**

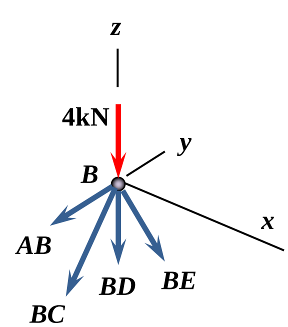



$$\sum F_x = \frac{3}{\sqrt{3^2+3^2+3^2}} AE - 0 = 0; \quad AE = 0;$$

$$\sum F_y = 8 + \frac{3}{\sqrt{3^2+3^2+3^2}} (AE = 0) - AB = 0; \quad AB = +8\,kN(t);$$

$$\sum F_z = \frac{3}{\sqrt{3^2+3^2+3^2}} (AE = 0) - AC = 0; \quad AB = 0;$$

**At joint B**




$$\sum F_x = \frac{3}{\sqrt{3^2+3^2}} BE - 0 = 0; \quad BE = 0;$$

$$\sum F_y = -\frac{3}{\sqrt{3^2+3^2}} BC - (AB = 8) = 0; \quad BC = AB\sqrt{2} = -8\sqrt{2}\ kN(c);$$

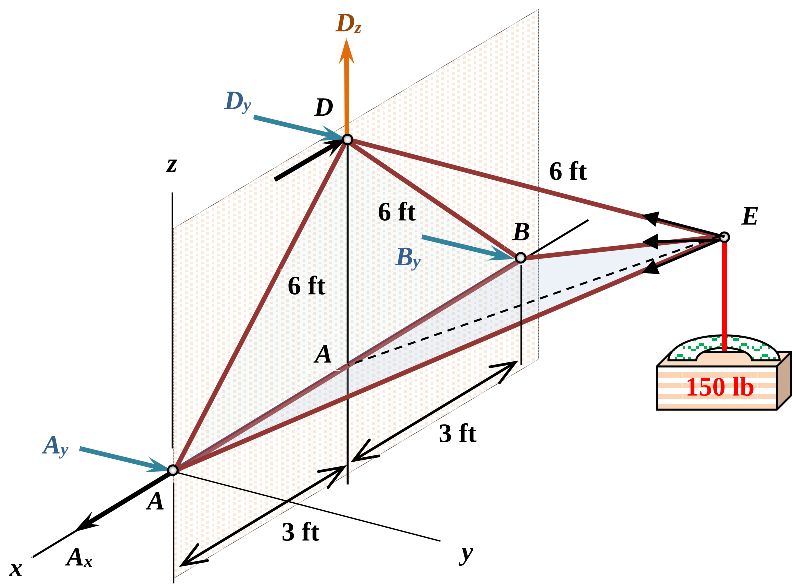
$$\sum F_z = -\frac{3}{\sqrt{3^2+3^2}} (BC = -8\sqrt{2}) - BD - \frac{3}{\sqrt{3^2+3^2}} (BE = 0) - 4 = 0; \quad BD = +4kN(t)$$

**From equilibrium equations at joint D,**

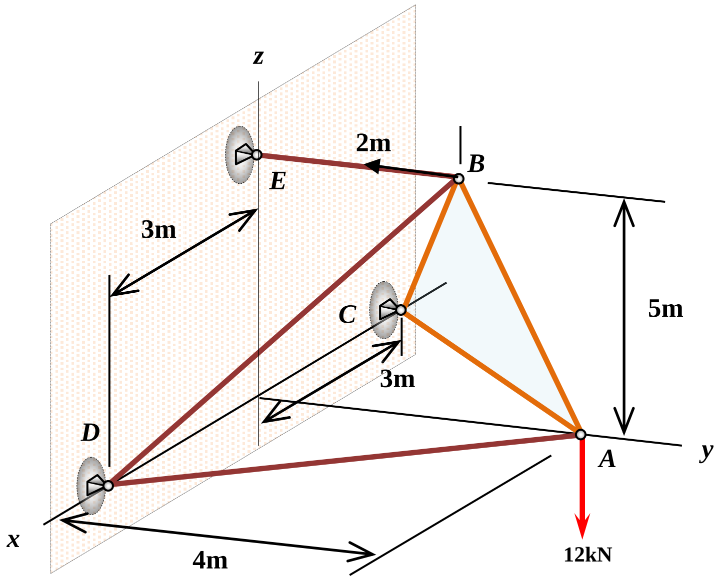
***DC = 0,  BE = 0,***

**From equilibrium equations at joint C,  *CE=0,***

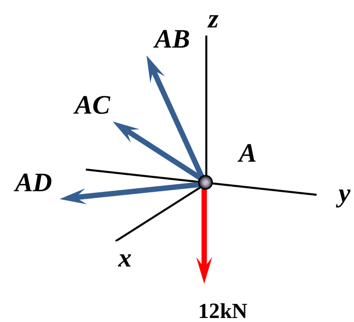
5. **Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb. (Hibbeler p. 303)**



**6.** **Find forces in all members. Hibbeler, p. 303**




**At Joint *A***




$$\sum F_x = \frac{3}{\sqrt{3^2+4^2}} AD - \frac{3}{\sqrt{3^2+4^2}} AC = 0; \quad AD = AC$$

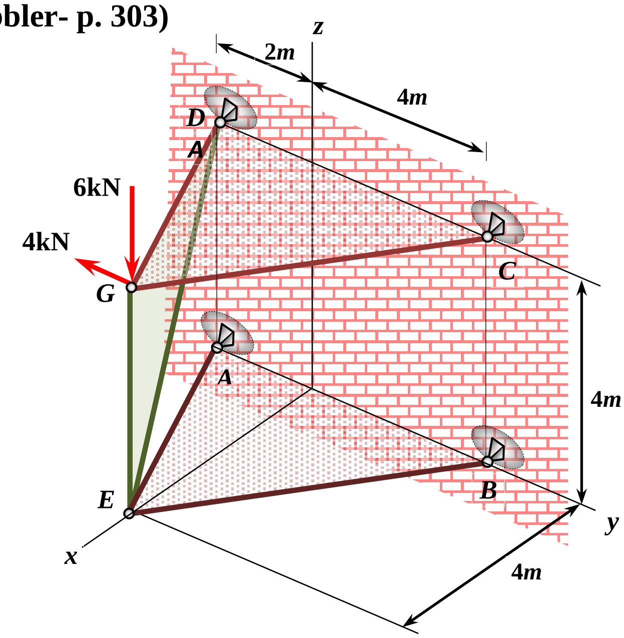
$$\sum F_y = \frac{4}{\sqrt{3^2+4^2}} AD + \frac{4}{\sqrt{3^2+4^2}} AC + AB \frac{2}{\sqrt{5^2+2^2}} = 0;$$

$$\sum F_z = AB \frac{5}{\sqrt{2^2+5^2}} - 12 = 0; \quad AB = 2.4\sqrt{29} = 12.92\ kN(t)$$

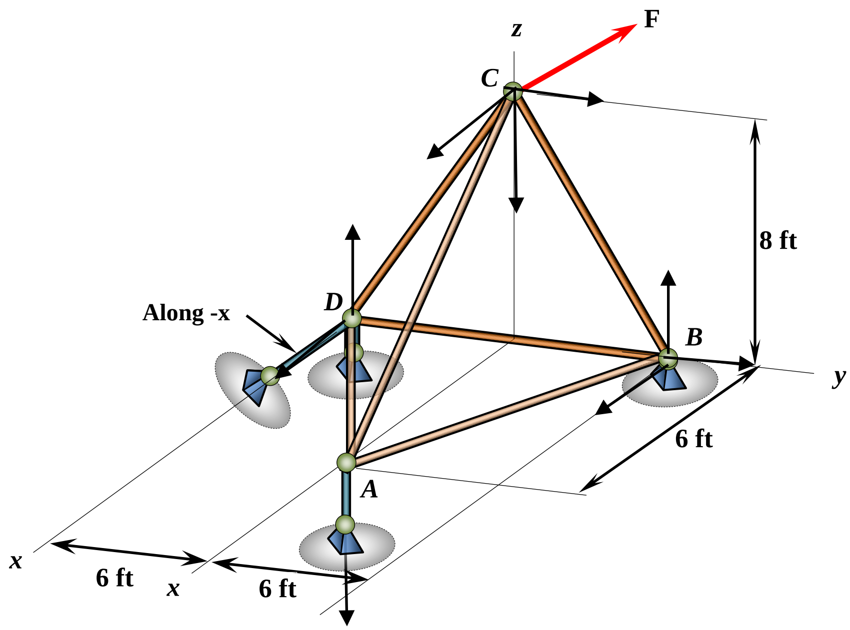
$$AD = AC = -3\ kN(c)$$

**Joint *B* shall fetch the stresses in bars EB, BD, and CB**

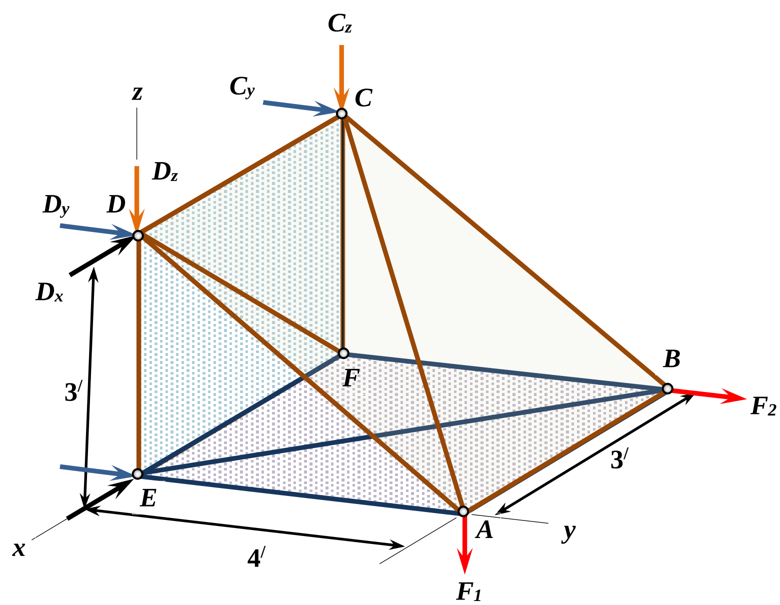
**7.** **Determine the force in member of the space truss and state if the member are in tension or compression. The truss is supported by ball and socket joint at *A*, *B*, C, and *D*. (Hibbler- p. 303)**



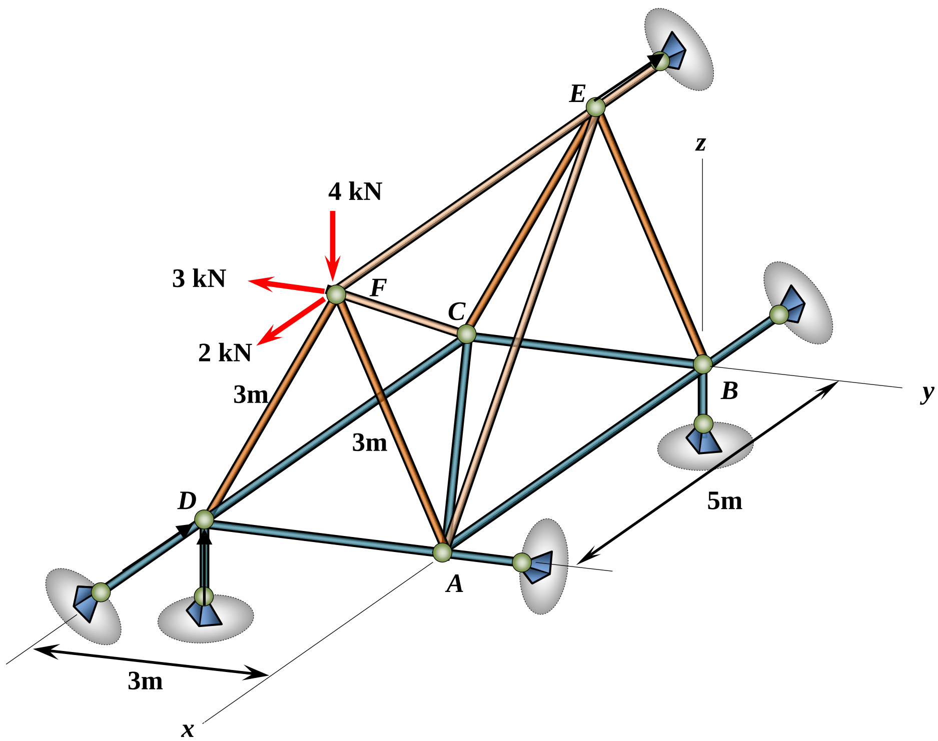
**8.** The space truss is shown below. Determine nature and magnitude of force in each member. (i) **F = [** –500**i** + 600**j** + 400**k] and** (ii) **F = [** 600**i** + 450**j** - 750**k]**



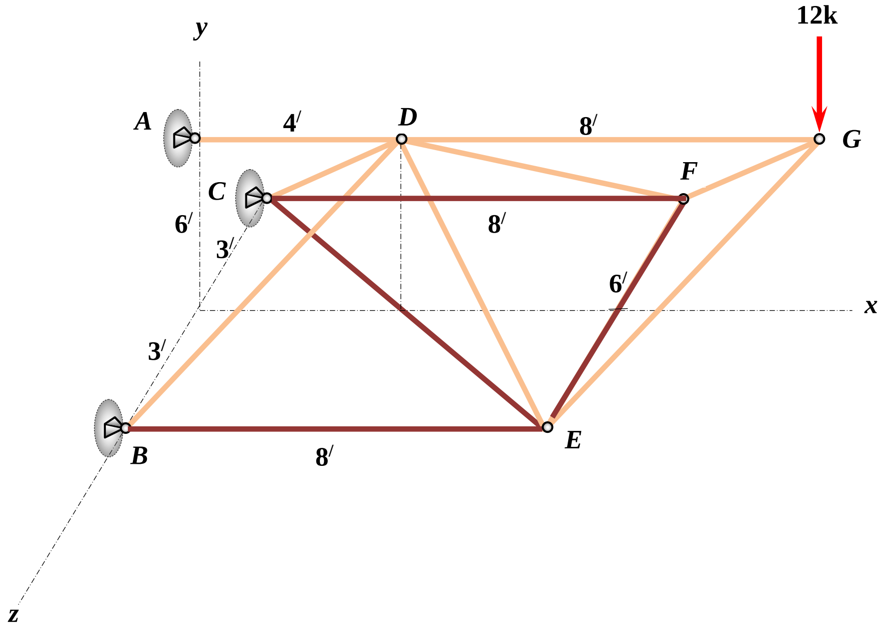
9. **The space truss is shown in figure below. Determine nature and magnitude of each member. (i) $F_1$ = – 500k lb and $F_2$ = + 400j and (ii) $F_1$ = 200i + 300j – 500k lb and $F_2$ = + 400j.**



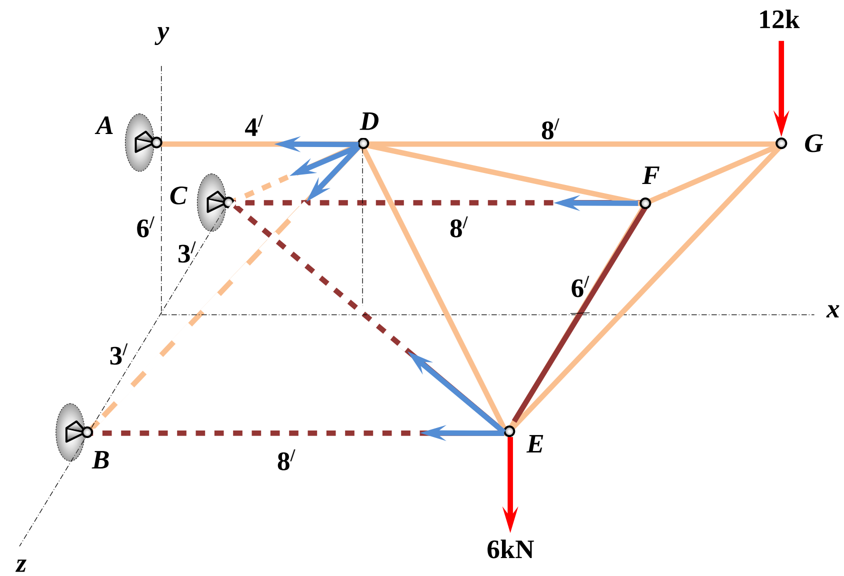
10. Determine the force in members *EF*, *AF*, and *DF* of the space truss and state if the members are in tension or compression. The truss is supported by short links at *A*, *B*, *D*, and *E*.



**11. Determine the force in bars BE, CE, and CF of the space truss shown below.**




Free DEFG portion, Take moment about D to obtain the bar forces.

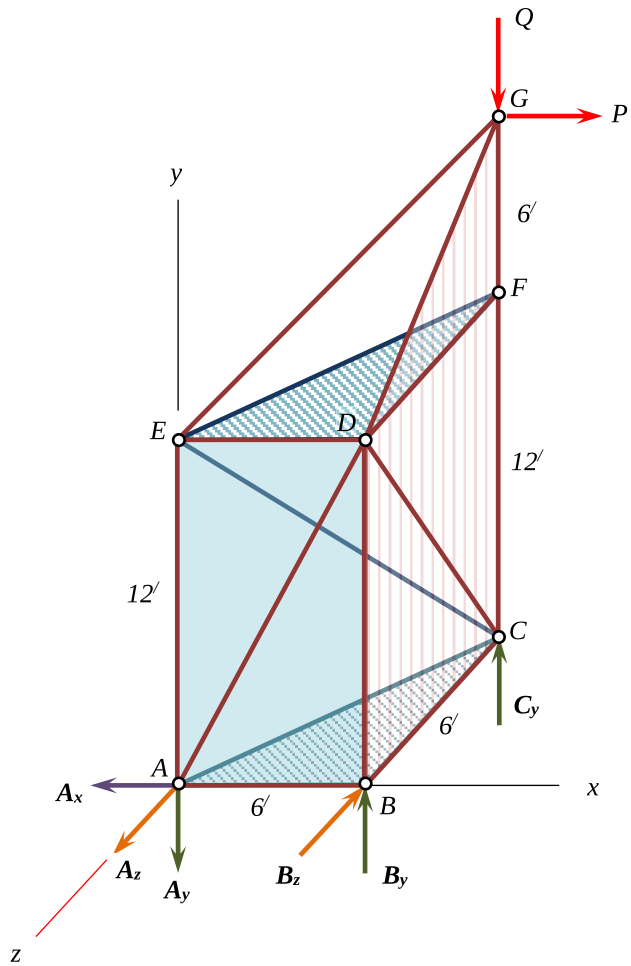


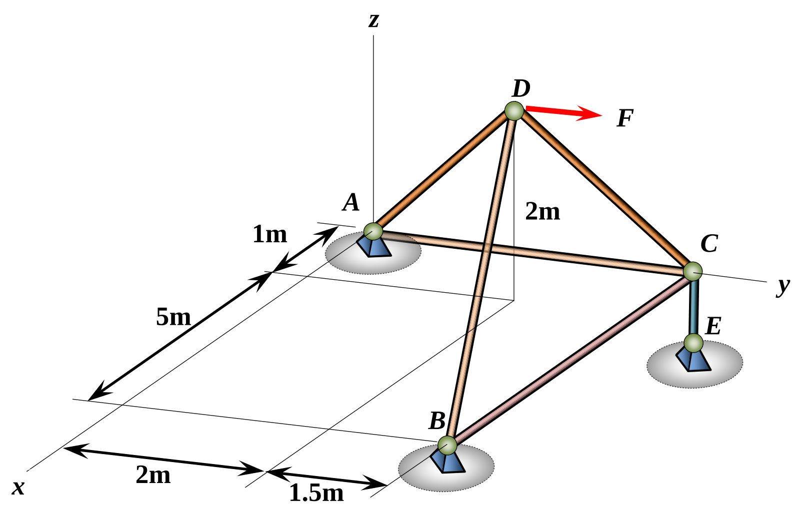

$$\sum M_x^D = \frac{3}{\sqrt{3^2+4^2}} CE*6 + 6*3 = 0; \quad CE = -5\,kN(c)$$

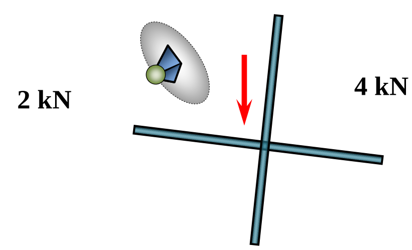
$$\sum M_y^D = CF*3 - BE*3 = 0; \quad CF = BE$$

$$\sum M_z^D = \frac{4}{\sqrt{3^2+4^2}} CE*6 + 6*4 + 12*8 + BE*6 + CF*6 = 0; \quad CE = -8\,kN(c)$$

**12.** The reaction components of the space truss shown in figure below. The applied loads in kips are $P = 2i$ and $Q = 3j$. Determine the forces in each bar. ($AC = 2.83$ T; $CD = 4.48$ T; $EG = 3.46$ T)

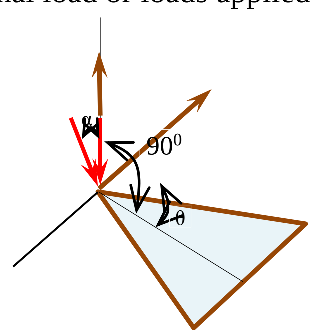





## Special Theorems (Norris)

**Theorem-1.** If all the bars meeting at a joint, with the exception of one bar n, lie in a plane, the component normal to that plane of the force in bar *n* is equal to the component normal to that plane of any external load or loads applied at that joint.




On the basis of Theorem 1, two corollary theorems can be stated:

**Theorem-2.** If all the bars meeting at a joint, with the exception of one bar n, lie in a plane, and if no external load is applied at this joint, the force in bar n is equal to zero.

**Theorem-3.** If all but two bars at a joint having no bar force and these two are not collinear, and if no external loads acts at a joint, the bar force in each of these two bars is zero.