



*Heaven 's Light is Our Guide*

DEPARTMENT OF CIVIL ENGINEERING

RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY

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Prepared by

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**Course Title :** Structural Analysis and Design-I

**Course Code:** CE 3111

**Lecture:** 3 hrs/ week ,

**Credit:** 3.00

**Prereq.** CE 2111

- **Content of Course:**

- Stability and determinacy of structures, analysis of statically determinate arches. Influence lines for statically determinate structure: moving loads on beams, frames and trusses. Cable supported structures and space trusses..

## Reference Books

- 1. Theory of Simple Structures by T.C. Shedd and J.Vawter (2<sup>nd</sup> Edition)
- 2. Elementary Structural Analysis by Utku, Norris & Wilber (4<sup>th</sup> Edition)
- 3. Analysis of structures by V N Vazirani ( Vol. 1) , M M Ratwani
- 4. Indeterminate Structural Analysis by J. Sterling Kinney

**Topics:**

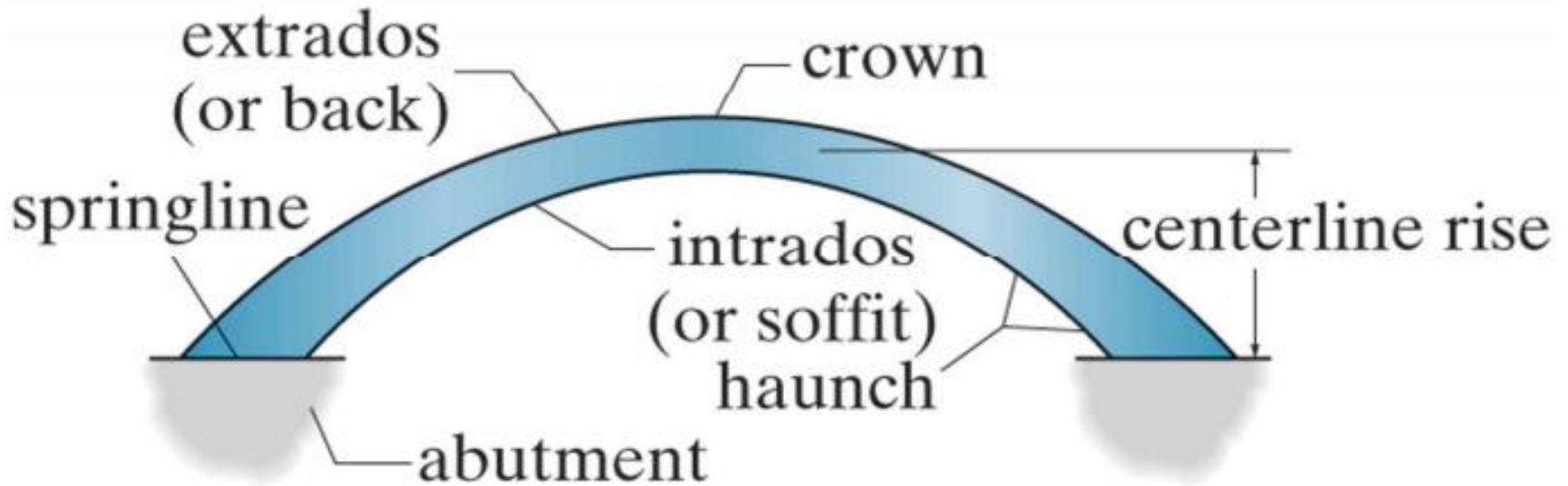
**Three Hinges Arches**

# Topics to be Covered

- Introduction
- Define Arches
- Example of Arches
- Advantages and disadvantages of Arches
- Derive Formula for internal reactions
- Derives Formula shear force and normal forces of Arches
- Sample Example to Analysis and Design of Three Hinge Arches

# Introduction

## Arches



# Sydney Harbour Arch Bridges



# China Curved Truss Arch Steel Bridge .



# New River Gorge Bridge, in Fayetteville, West Virginia



An Arch is defined as a curved beam/girder having convexity upwards and supported at its ends. The supports must effectively arrest displacement in the vertical and horizontal only then there will be arch action.

#### Advantages of Arch:

- **It offers higher levels of resistance compared to other designs**
- **Architecture look nice.**
- **It offers the option to span a greater distance**
- **It can be construction from almost any material.**
- **It provides an advantage when carrying loads.**
- **It continues to provide support without distortion over time.**
- **It can become stronger over time.**
- **It adapts to local environmental conditions better.**

## Disadvantages of Arch:

- **It offers a finite span length to use.**
- **It is a time-consuming project to complete.**
- **It is a structure which requires careful maintenance.**
- **It is a bridge option that cannot be built in some locations.**
- **It requires more side support to complete a successful span.**
- **It can sometimes be too flexible.**
- **It cannot have any design flaws for it to work as intended.**

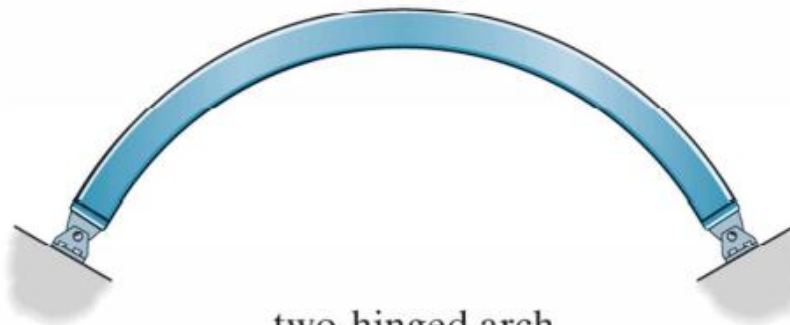
**It requires expertise to build**

# Arches Types

## Arches Types

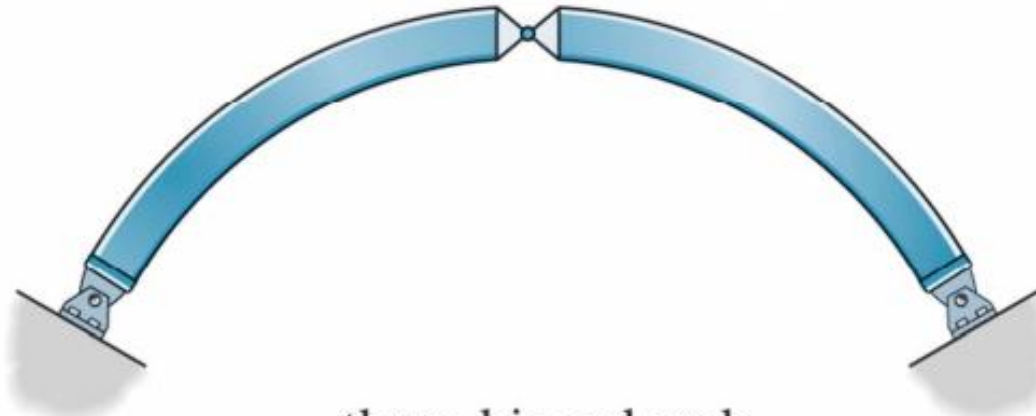


fixed arch

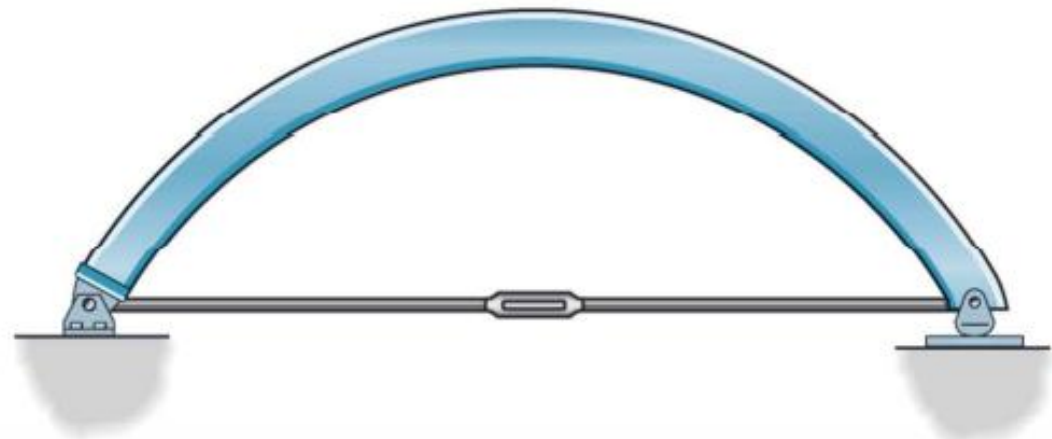


two-hinged arch

# Arches Types



three-hinged arch



tied arch

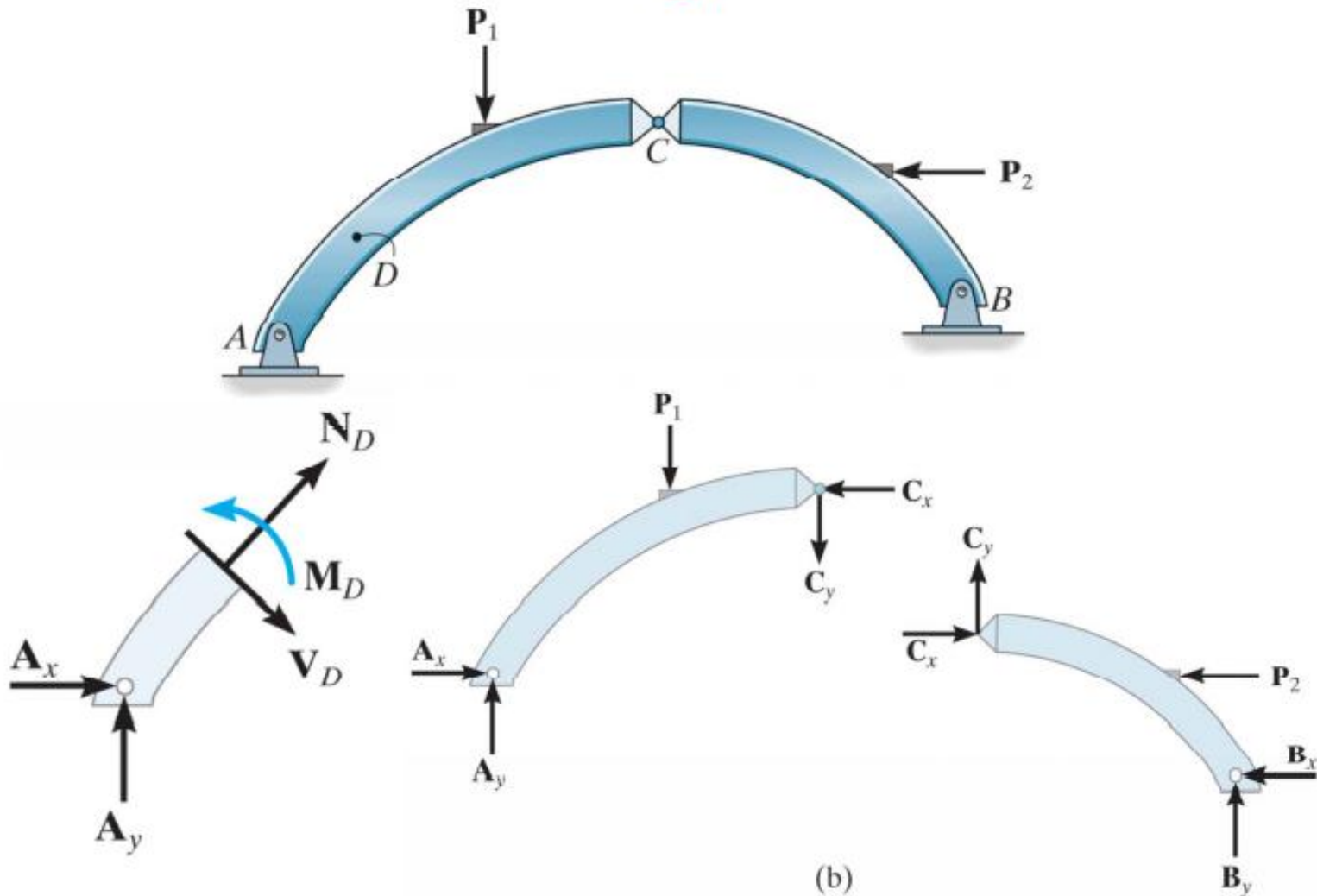
# Arches Types

## Example of Fixed Arch

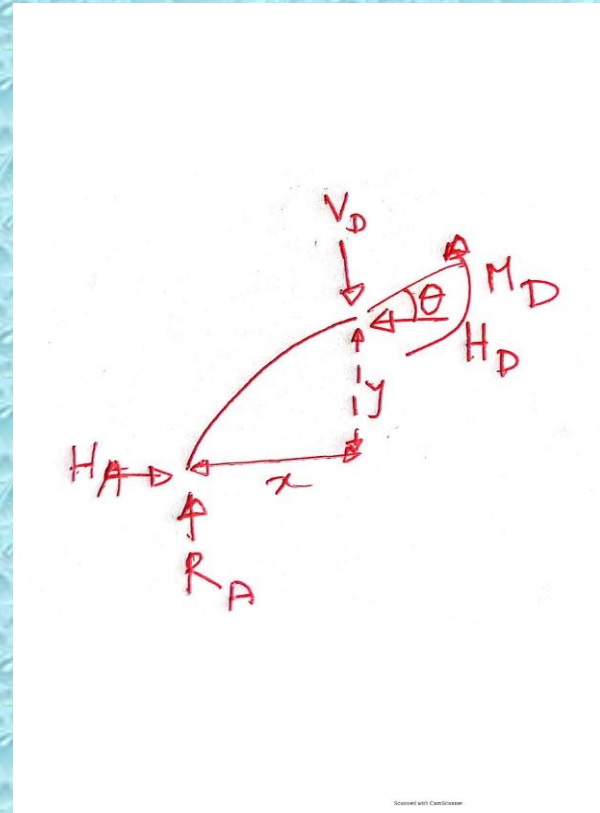
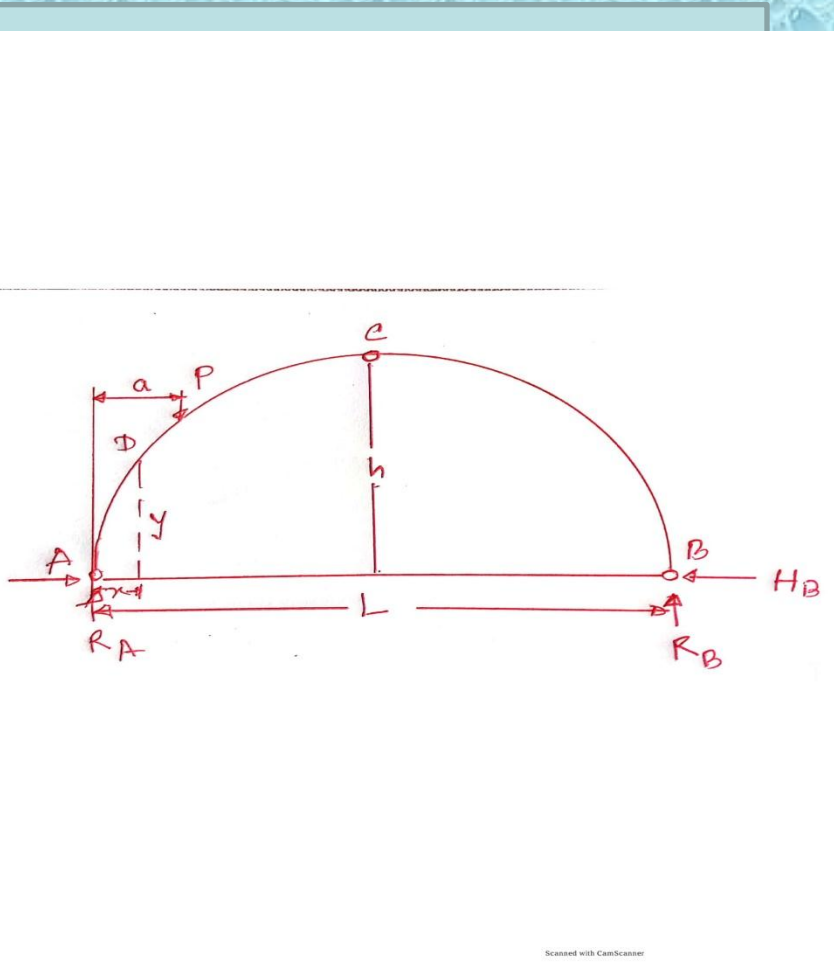


# Example of Three Hinge Arches

## Three Hinge Arches



# Formulation of Determination Internal Force of Three Hinged Arches



$$\sum M_A = 0$$

$$Pa - R_B L = 0$$

$$R_B = \frac{Pa}{L}$$

$$\sum F_y = R_A - P + R_B = 0$$

$$R_A = P - \frac{Pa}{L} = \frac{P(L-a)}{L} = R_A$$

$$\sum M_A = R_A X - H_A Y \quad [M_D = M_s - H_y]$$

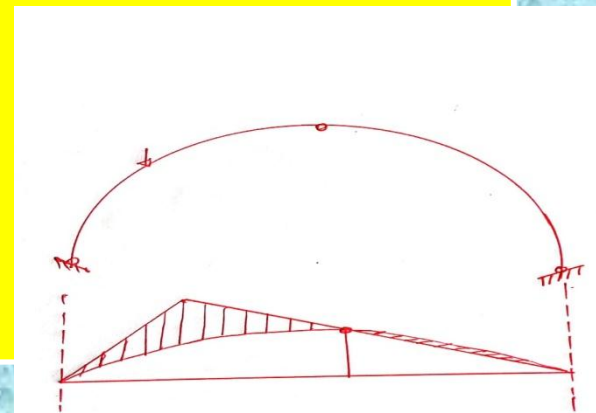
$$\sum F_x = H_A - H_B$$

$$H_A = H_B = H \quad [M_s = R_A \cdot X \text{ top simply supported beam}]$$

$$Y = \frac{4hx}{L^2}(L-a)$$

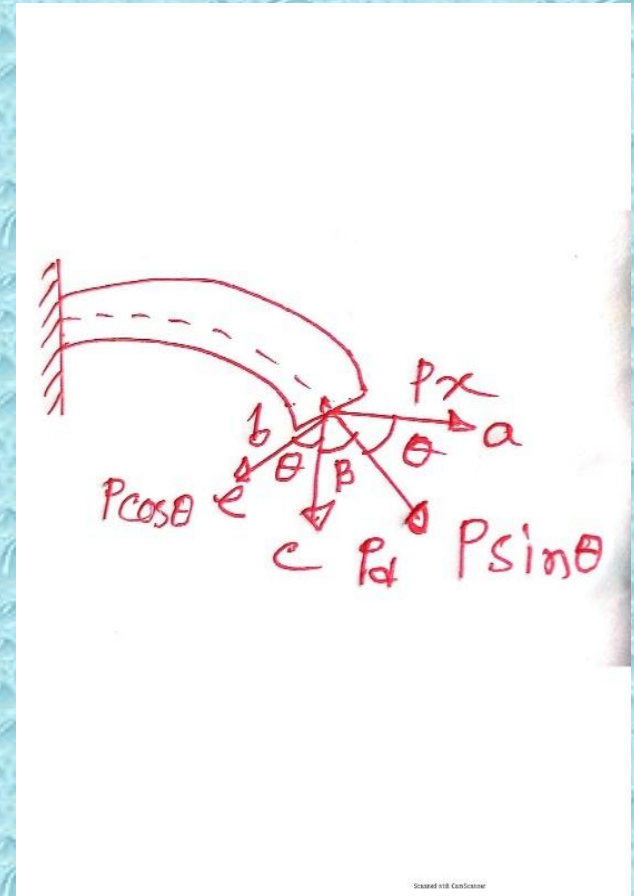
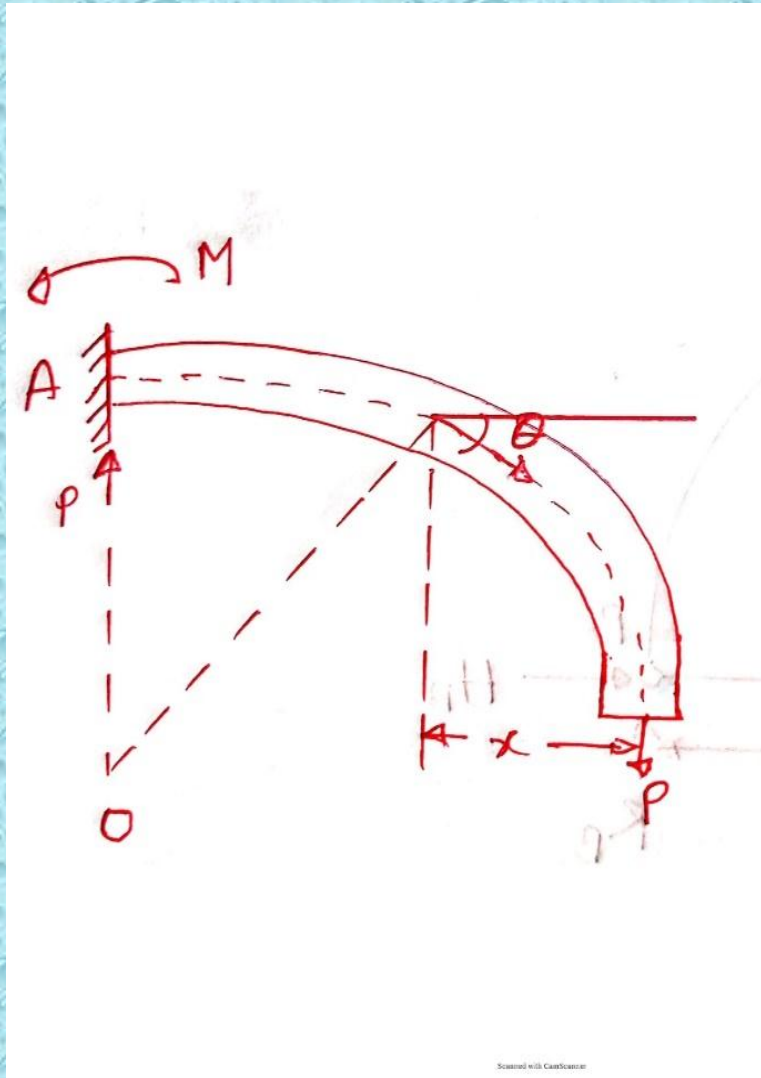
$$MH = Hy = H \left[ \frac{4hx}{L^2}(L-x) \right]$$

$$\tan \theta = \frac{dy}{dx}$$



B.M. diagram

# Formulation of Determination Shear force and Normal force of Three Hinged Arches



At section point considered

Bending moment ,  $M_x = - p$

$\angle abc = 90 = \theta + \beta$

$\angle abd = \theta$

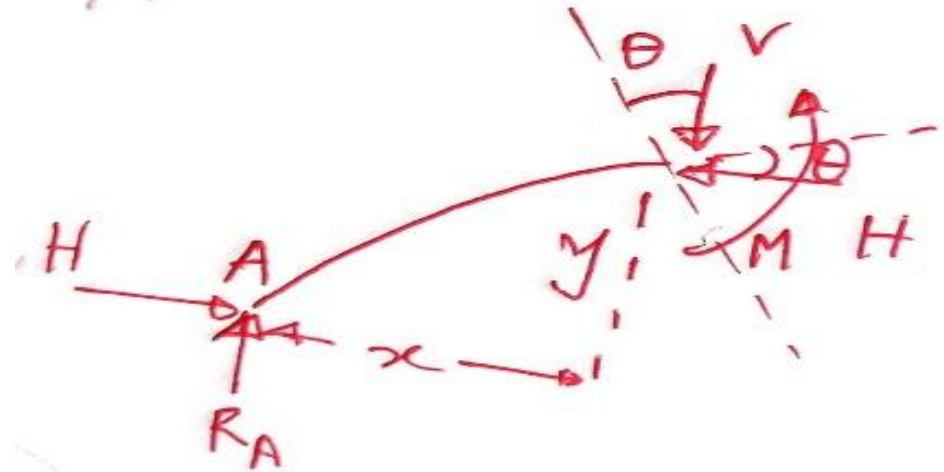
$\theta + \beta = 90, \beta + \theta = 90 = dbc,$

Shear force =  $F_x = p \cos \theta$

Normal force,  $N_x = -p \sin \theta$  (tensile)

For three hinge arch:

$$\tan \theta = \frac{y}{x}$$

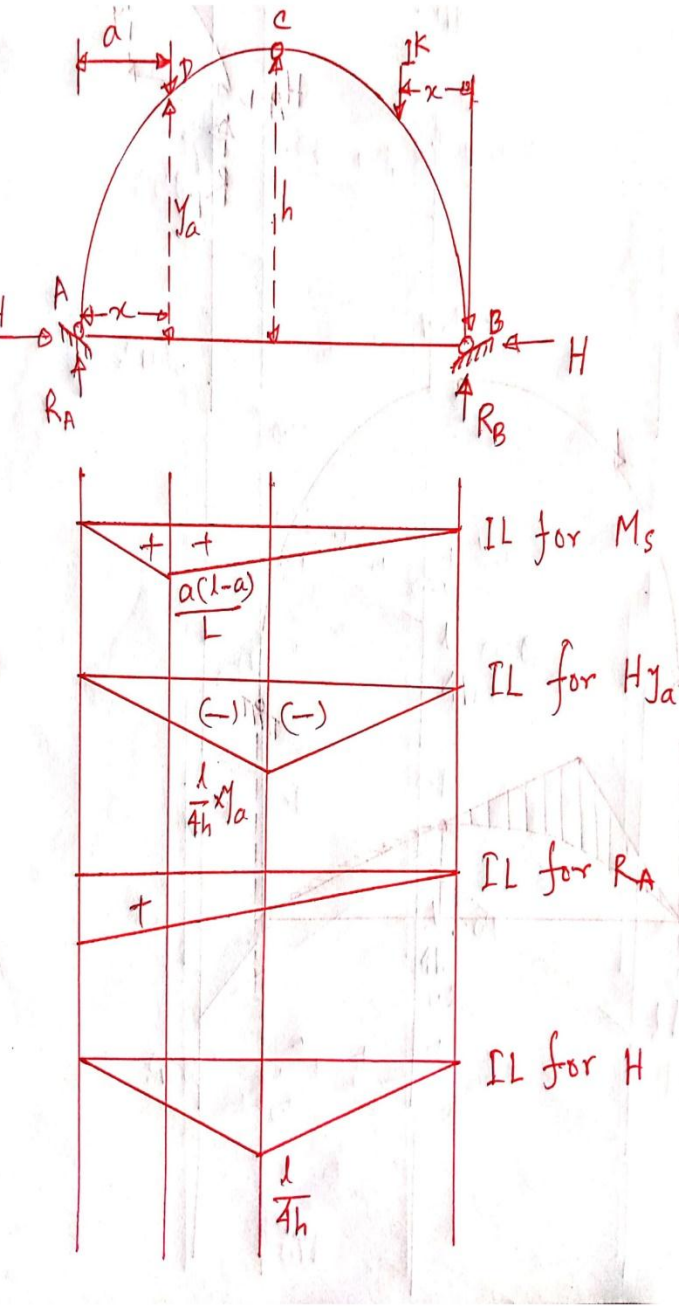


Shear force =  $V \cos \theta - H \sin \theta$

Normal force =  $V \sin \theta + H \cos \theta$

**Influence Lines:** An influence line is a diagram showing the variation in the shear, moment, stress in a member reaction, or other direct function, due to unit load moving across the structures.

An influence line is constructed by plotting directly under the point where the unit load is placed an ordinate the height of which represents to some scale the value of particular function being studied when the load is in that position.



When load at A,

$$R_A = 1$$

$$R_A = \frac{1(L-x)}{L} = 1 - \frac{x}{L}$$

$$R_B = \frac{x}{L}$$

Horizontal Thrust,

$$R_B \cdot \frac{L}{2} - H \cdot h = 0$$

$$H = \frac{R_B \cdot \frac{L}{2}}{h}$$

$$H = \frac{\frac{x}{L} \cdot \frac{L}{2}}{h}$$

$$H = \frac{x}{2h}$$

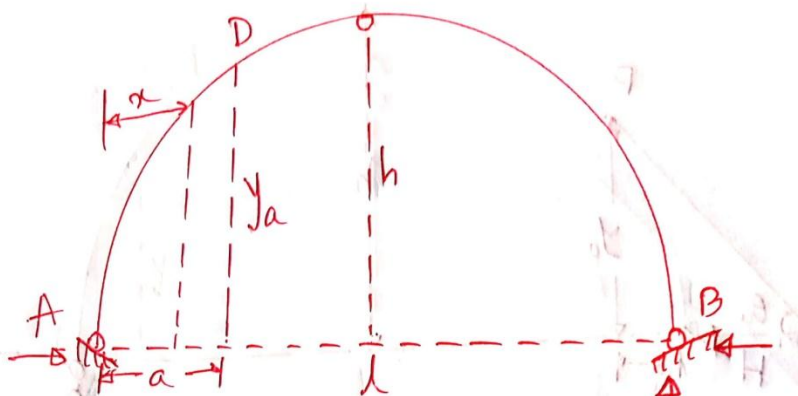
$$R_A = \frac{X'}{L}$$

When the unit load is in the right crown,

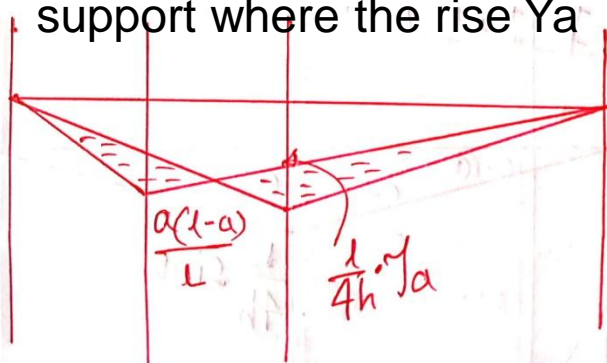
$$R_A \times \frac{l}{2} = H \times h$$

$$H = \frac{l}{2h} \cdot R_A$$

$$M_D = R_A \cdot a - H \cdot Y_a$$



Influence line for B.M. at any point a distance from left support where the rise Ya



IL for BM at D

When the unit load is to the right of the section,

$$M_D = R_L \cdot a - H \cdot Y_a$$

When the unit load is to the left of the section at X distance from the left support,

$$M_D = R_L \cdot a - 1(a-x) - H \cdot Y_a$$

At  $x=0$ ,  $R_L \cdot a - 1(a-x) = 0$ ,  $H \cdot Y_a = 0$

At  $x=a$ , when load at cross section,

$$R_L \cdot a - 1(a-x) = R_L \cdot a = \frac{l-a}{l} \cdot a = \frac{a(l-a)}{l}$$

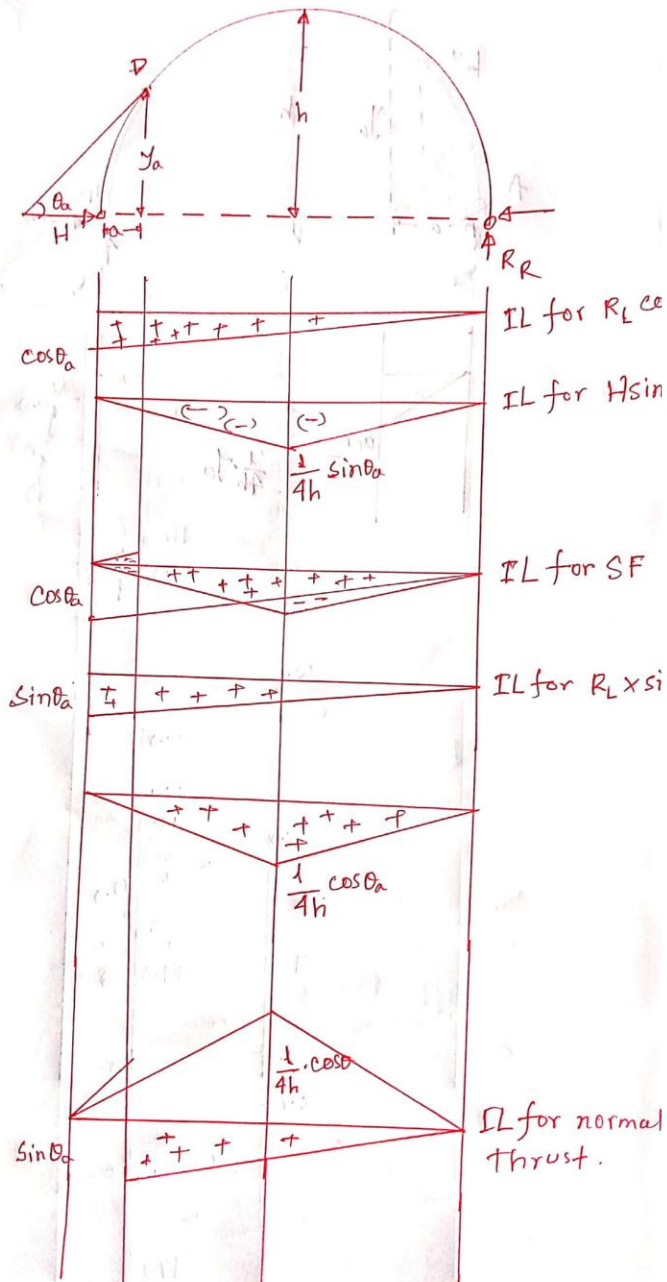
Simply supported beam.

When the load at the crown,

$$M_D = -\frac{l}{4h} \cdot Y_a, \text{ for horizontal thrust,}$$

$$M_x = \frac{x}{2l} \cdot Y_x$$

# Influence line for S.F and Normal thrust



When the load is to the right of the section ,  $F_a$

$$F_a = R_L \cos \theta_a - H \sin \theta_a$$

When the load is to the left of the section,

$$F_a = (R_L - 1) \cos \theta_a - H \sin \theta_a$$

When the load is to the right of the section,

$$N_a = R_L \sin \theta_a + H \cos \theta_a$$

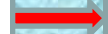
When the load is to the left of the section,

$$N_a = (R_L - 1) \sin \theta_a + H \cos \theta_a$$

$$N_a = R_L \sin \theta_a + H \cos \theta_a$$

## Lane Loading:

Concentrated load



1. 18000 for moment
2. 26000 for shear

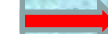
Uniform load 640 lbs. per linear foot of lane



H<sub>20</sub> Loading

## Lane Loading:

Concentrated load



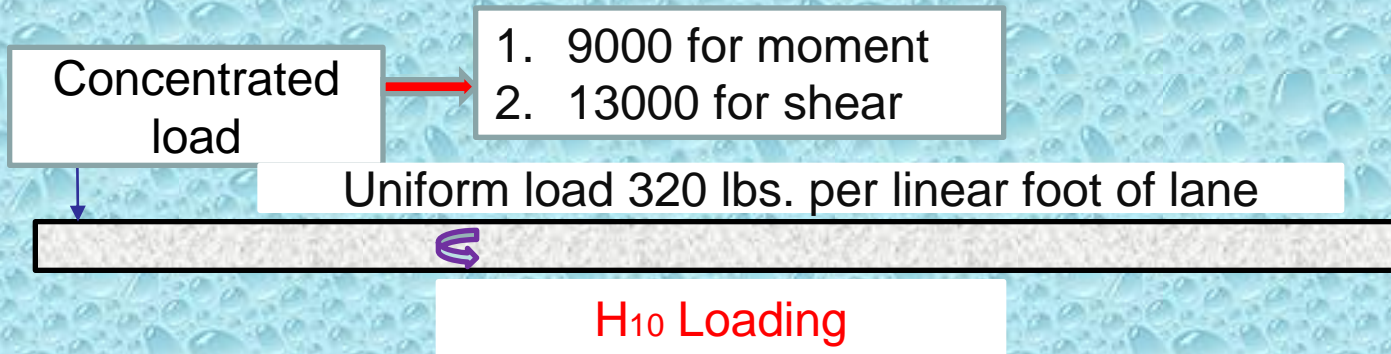
1. 13500 for moment
2. 19500 for shear

Uniform load 480 lbs. per linear foot of lane



H<sub>15</sub> Loading

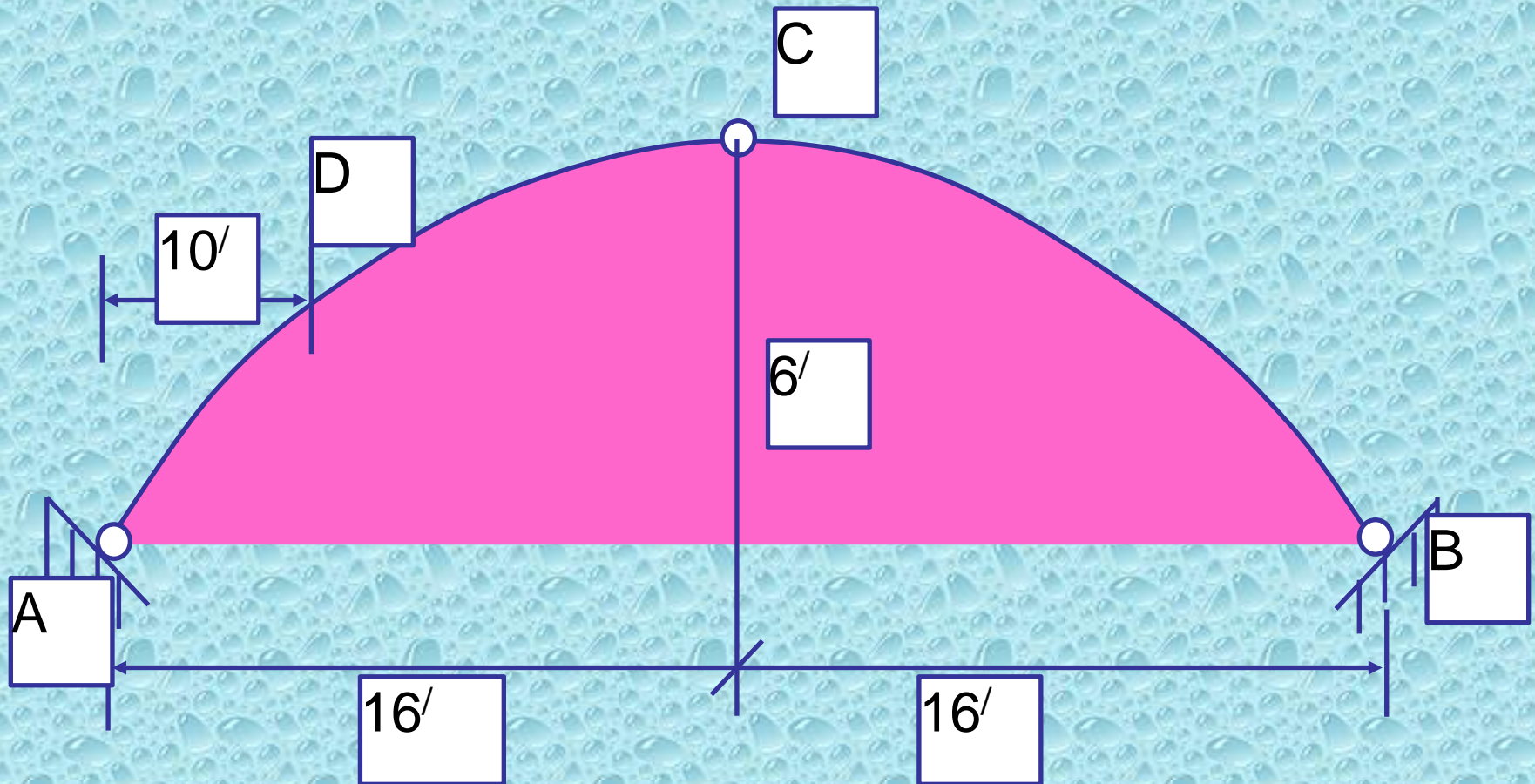
# Lane Loading:



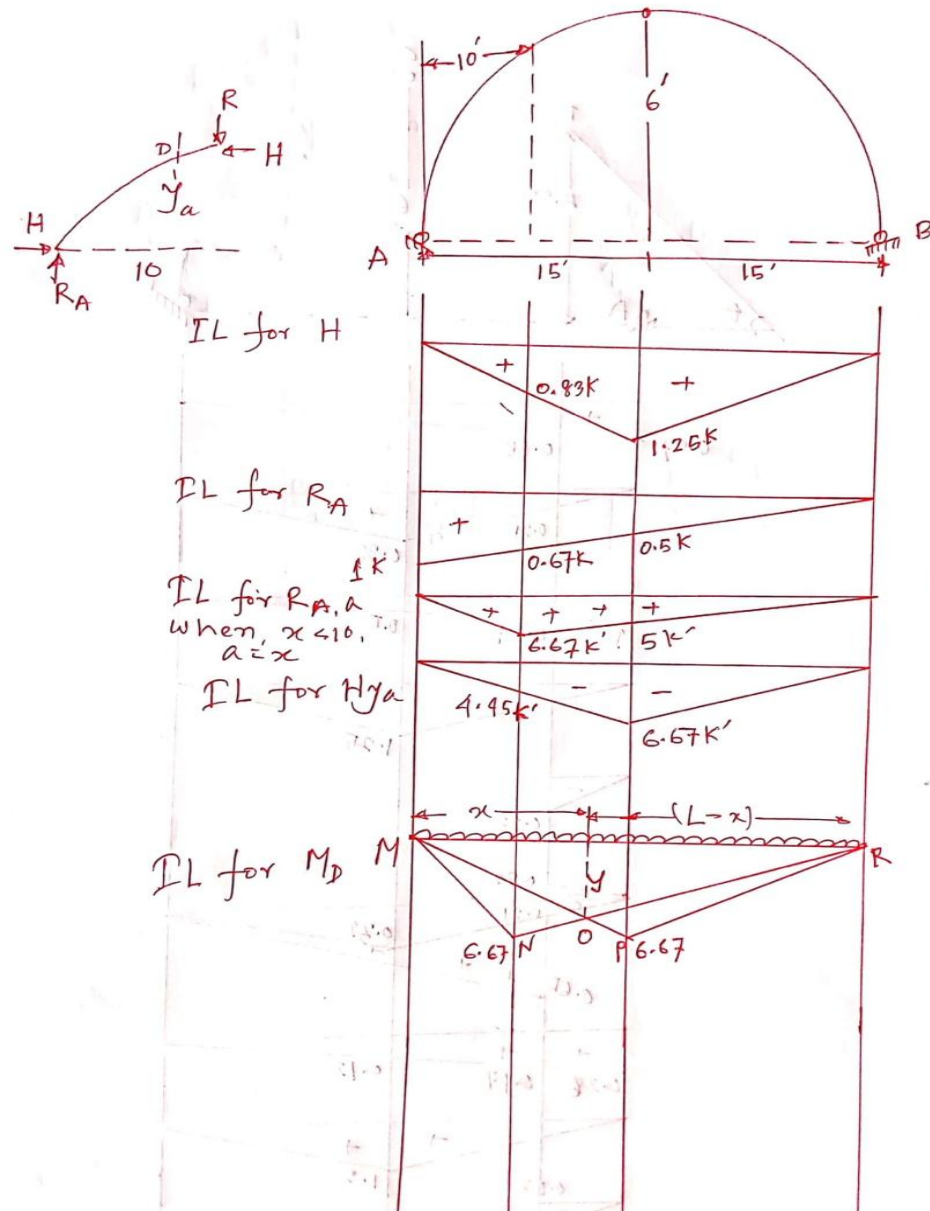
Equivalent loading for loaded lengths exceeding 60 ft.

1. Subdivided Trusses
2. Non-parallel Truss
3. Truss without vertical Truss
4. K-Truss

Prob 1. Draw influence lines for bending moment, shear force, and normal thrust at a section D of the following three hinged parabolic arch. Also obtain the maximum moment and shear at section D due to  $H_{20}$  loading.



Draw for (i) Horizontal thrust (ii) MD (iii) VD (iv) Normal thrust, also compute SF, BM, NT, and HT due to H<sub>2</sub>O loading



Here,  $L = 30'$

$h = 6'$

$a = 10'$

$$Y_a = \frac{4ha}{L^2} (L - a)$$

$$= \frac{4 \times 6 \times 10}{30^2} (30 - 10)$$

$$= 5.33 \text{ ft.}$$

IL for horizontal thrust, H

Unit load at A,  $H = 0$

$$\text{Unit load at D, } H = \frac{a}{2h} = \frac{10}{2 \times 6} = 0.83$$

Or,

$$R_A = \frac{L - a}{L} = \frac{2}{3} \text{ k}$$

$$\sum M_c = 0$$

$$R_A \times 15 - H \times 6 - 1(15 - 10) = 0$$

$$H = 0.83 \text{ k}$$

$$\text{Unit load at C, } H = \frac{L}{4h} = \frac{30}{4 \times 6} = 1.25 \text{ k}$$

$$\text{Unit load at B, } R_A = 0 \text{ k, } H = 0 \text{ k}$$

Now,  $M_D = R_A \cdot a - H \cdot Y_a$

$$\frac{Y}{X} = \frac{6.67}{15}, Y = \frac{6.67}{15} \cdot X$$
$$\frac{Y}{L-x} = \frac{6.67}{L-a} = \frac{6.67}{20}$$
$$Y = \frac{6.67}{20}(30-x)$$

$$X = 12.86', y = 5.72'$$

$$\begin{aligned} M_{\max(+ve)} &= \text{are of } \triangle OMN \times .64 + 18(6.67 - 4.45) \\ &= (\triangle MNR - \triangle MOR) \times .64 + 39.96 \\ &= (.5 \times 30 \times 6.67 - .5 \times 30 \times 5.72) \times .64 + 39.96 \\ &= 49.08 \text{ ft.} \end{aligned}$$

$$\begin{aligned} M_{\max(-ve)} &= \text{are of } \triangle OPR \times .64 + 18(6.67 - 5) \\ &= (\triangle MPR - \triangle MOR) \times .64 + 30.06 \\ &= (.5 \times 30 \times 6.67 - .5 \times 30 \times 5.72) \times .64 + 30.06 \\ &= 39.18 \text{ ft.} \end{aligned}$$

Shear force,

$$V_D = R_L \cos \theta - H \sin \theta \text{ (for load at right of D)}$$
$$V_D = (R_L - 1) \cos \theta - H \sin \theta \text{ (for load at left of D)}$$

## Conclusons:

1. Load carrying capacity of Arches is higher than flat beam structures
2. Arches is structurally efficient than horizontal beam
3. Arches are used in longer span
4. Arches are used in Bridges, Building, Airport, gate, Stadium etc.
5. Arch have construction difficulty and costly.



**Thanks All**