Lecture 1 Notes

Serker Sir
26-11-2020

Concrete:

- Is a stone like material obtained by mixing cement, sand, gravel or other agg and water.
- compressive strength : very high &
  Tensile strength is small compared to compre. strength
- Not suitable to use in members subjected to tension fully (e.g. tie rods) or partially (such as beam).

RCC vs Conc

→ RCC তে rod থাকে, concrete এ RCC নাই
→ RCC তে ductility (নমনীয়তা) থাকে।
→ RCC ↑ tensile strength. Concrete ↓ tensile strength

* Steel এর modulus of elasticity $\boxed{200 \text{ GPa} = E_{st}}$
  = 29000 ksi

Steel এর strength > concrete এর strength. তাহলে কেন construction এ concrete use করি?

→ Co. Binding Property ভালো → to increase durability to weather
→ Con. Cost কম → Shape দেওয়া easy → creating process easy

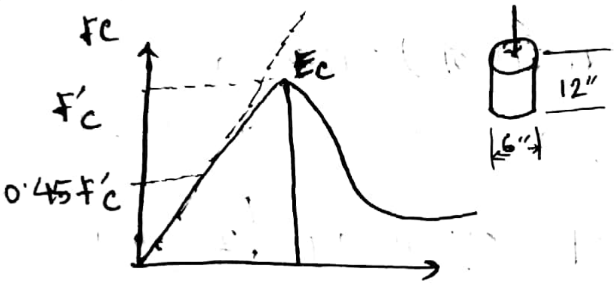
Concrete এর strength কে কেন সবচেয়ে গুরুত্বপূর্ণ property হিসেবে ধরি?

→ কতটুকু load নিতে পারে তা calculate করতে useful
→ conc এর strength থেকে এর অন্য অনেক property সম্পর্কে জেনে যেতে পারি। Exp: ↑Strength থাকলে ↓permeability জানব কেননা pore ↓. pore কম concrete এর জন্য ভালো, durability ↑ তখন

→ Strength এর সাথে Elastic Modulus এর formula আছে।

☐ W/C ↓ strength বেশি।
কিন্তু বেশি W/C কম মানে workability কমে যাবে।
তাই W/C ratio অনেক important.

☐ ~~Rcc~~ Concrete against এ
① Low tensile strength
② Forms & shoring
③ Relative low strength per unit volume or wt.

☐ 

f_c' = compressive strength of concrete at 28 days

f_y = yeild strength of steel.

ε_u = ultimate strength of conc.

☐ 600 Grade Steel = yeild strength 60,000 psi

☐ Concrete Strain at max compressive stress.
- ε_u varies btwn 0.0015 – 0.003
- for normal strength concrete, $\varepsilon_u \approx 0.002$

☐ Max useable strain, ε_u
- ACI Code $\boxed{\varepsilon_u = 0.003}$
- Used for flexural & axial compression.

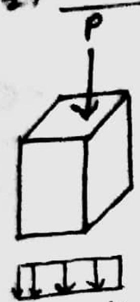
☐ $\sigma = E.\epsilon$

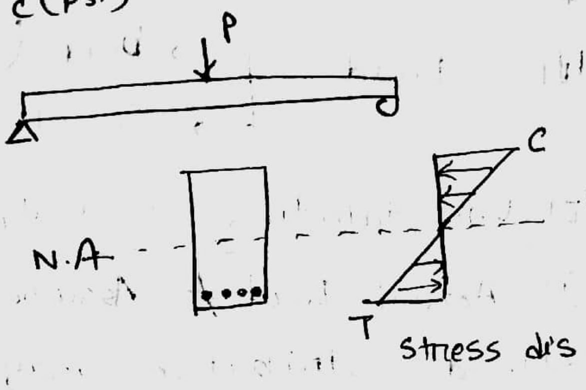
∴ Steel $\varepsilon_y = \frac{\sigma}{200\,GPa}$.

* unit wt of concrete $\boxed{W_c \cong 150 \text{ pcf}}$

* $E_c \text{ (psi)} = 57{,}000 \sqrt{f'_c \text{ (psi)}}$

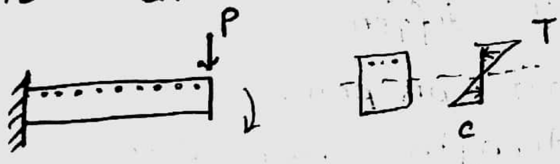
Why RCC



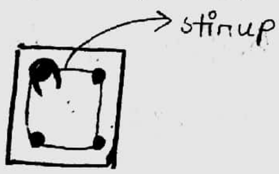


Stress distribution

* যদি max T. stress capacity x psi আর concrete টার capability y psi. $x > y$ হলে fail. RCC না use করলে beam size ↑ হতে হবে। $\sigma = \frac{Mc}{I}$ Area বাড়াতে হবে। তাই Tension zone এ Reinforcement বসানো হত।



* Deformed bar conc আর steel এর bonding বেশি। thermal expansion co-eff ও good. steel এর, তাই steel use করা হয়।



* Deformed bar এর ঘাঁজ গুলো rib/rip. $(3/8'' - \frac{9}{8})''$ dia

* Bar no 3–9 এর Area memorize must.

Safety Provisions

three reasons why safety factor are necessary

① Variability in resistance

② " " Loading

③ consequences of Failure

Dead Load :

① Wt of all permanent construction

② Constant magnitude & fixed location

live load :

① User এর load.

Live load	minimum
Stair ~~(L.L)~~ & exitways :	100 psf

* Storage ware house ~~(L.L)~~ : 125 psf (light)
  250 psf (heavy)

* floor : 40 psf

Slide 2 Behavior of RC Member under Loading

* Concrete 10% of it's compressive ~~stress~~ strength resist করতে পারে।

* Concrete tension stress resist করতে পারে না,

ϵ_c = strain in con , ϵ_s = steel
f_c = stress in con , f_s = steel
E_s = mod of elas con , E_s = steel

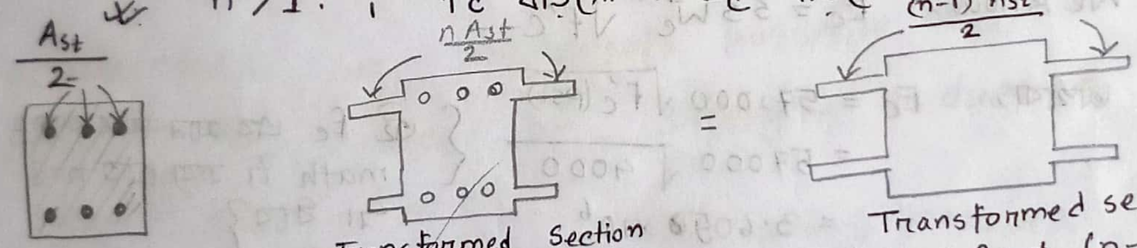
* At low stress $f'_c/2$ con nearly elastic

* strain $\epsilon_c = \frac{f_c}{E_c}$ (stress) $= \epsilon_s = \frac{f_s}{E_s}$

or, $f_s = \frac{E_s}{E_c} f_c = n f_c$ [n = modular ratio]

where, modular ration, $\boxed{n = \frac{E_s}{E_c}}$ (বেশি / কম)

* $n > 1$. , f'_c বাড়লে E_c বাড়বে, n কমবে,



Actual section — Transformed Section — Transformed section

net $A_t = A_c + n \cdot A_{st}$ — gross $A_t = A_g + (n-1) A_{st}$

Here, A_c = ~~gro~~ net area of conc. i.e. (gross area – area occupied by reinforcing bars)

A_g = gross area

A_{st} = total area of reinforcing bars

Note: no. 9 bar এর Area = 1 sq inch
$\therefore A_{st}$ এখানে $= 6 \times 1$ in^2 $= 6$ in^2

P = axial Load applied
P_c = load carried by conc
P_s = " " " steel

$\frac{f_s}{E_s} = \frac{f_c}{E_c}$
$\therefore f_s = \frac{E_s}{E_c} f_c$
$f_s = n f_c$

Therefore, P Load apply করলে conc & st both নিবে,

$P = P_c + P_s = f_c A_c + f_s A_{st}$

$\Rightarrow P = f_c A_c + (n \cdot f_c) A_{st}$

$\Rightarrow P = \underset{\text{stress of conc}}{f_c} \underbrace{(A_c + n A_{st})}_{\text{transformed area}} \longrightarrow P = f_c [A_g + (n-1) A_{st}]$

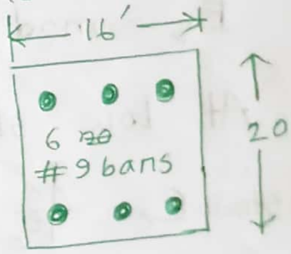
$A_c = A_g - A_{st}$ = ~~trans~~ core Area.

$A_t = A_c + nA_{st}$ = transformed area

$= A_g - A_{st} + nA_{st} = A_g + (n-1)A_{st}$ = transformed area

$f'_{c_{psi}}$ এর value দেয়া থাকলে $E_c = 57000\sqrt{f'_c(psi)}$ then $n = E_s/E_c$ দিয়ে বের করব

8. A column has a cross section of 16x20 in and is reinforced by six No. 9 bars. Determine the axial Load that will stress concrete to 1200 psi



given, $f_c = 1200$ psi

We know, #9 bar area = 1 in^2

∴ 6 ~~no~~ #9 bar area, $A_{st} = 6 in^2$

Gross area $= A_g = 16 \times 20 = 320 \ in^2$

modular ratio, $n = \frac{E_s}{E_c} = 8$ (ধরি)

or, ধরি, f'_c এর value 4,000 psi

We know, $E_c = 33 w_c^{1.5} \sqrt{f'_c}$

অথবা, $E_c = 57,000\sqrt{f'_c(psi)}$
$= 57000\sqrt{4000}$
$= 3.6050 \times 10^6$

{ এই f_c এর মান use করেও math টা করা যেত n=8 না ধরে }

$\therefore P = f_c (A_c + nA_{st})$
$= f_c \{ A_g + (n-1)A_{st} \}$
$= 1200 \{ 320 + (8-1)6 \}$ lb
$= 434400$ lb

$P_c = A_c f_c = f_c [A_g - A_{st}] =$ lb
$= 1200 [320 - 6] = 376800$ lb লোড carry করে.

$P_s = A_s f_s = A_{st} \cdot n \cdot f_c = 6 \times 8 \times 1200$
$= 57600$ lb load carry করে

Short Note on Strength reduction factor

Strength reduction factor (ϕ) is used in Reinforced concrete design & analysis and its magnitude is different for various reinforced concrete members for safety purposes. The strength reduction factor is used to decrease the estimated strength of structural members i.e, to compute the design strength of concrete elements. in order to account for uncertainties in material, possible design. & construction errors. The value of strength reduction factor is less than 1.

Based on Strength reduction factor for various member ~~&co~~ ACI 318-19

Actions on Structural members	Strength reduction factor
Tension controlled beams & slabs	0·90
Shears & torsions in beams	0·75
Columns	0·65 (tie), 0·75 (Sprirally RCC)
Bearing on concrete	0·65
Plain concrete elements	0·60

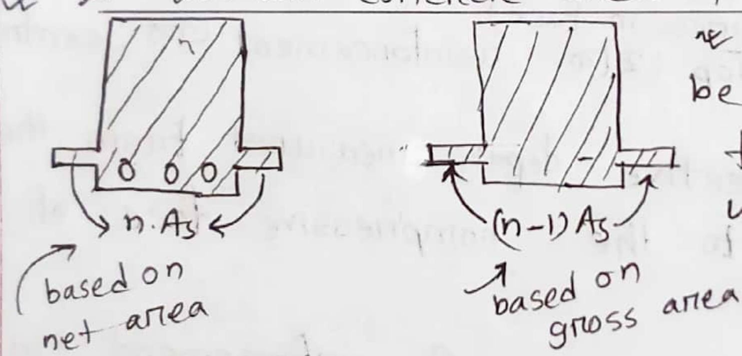
As the failure of column is brittle, they are more critical than failure of the beam which is ductile. Columns fail suddenly without showing any sign. That is why for more safty the capacity reduction factor is higher than of beams.

Senken Sin
Lecture 3 02-01-21

আগের দিন continue.

Stress elastic & Section Uncracked

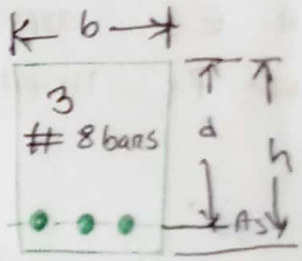
- Along As tensile stress in conc is smaller than the modulus of rupture, the strain & stress distribution is as c.
- No tension cracks developed
- Strain & stress distribution is smaller to that of a homogeneous beam.
- equivalent concrete area of steel find out



- the beam section can be analyzed using the transformed section & usual method of analysis

Problem : For a rectangular beam section assume $b = 10in$, $h = 25in$, $d = 23in$. Reinforcement consists of 3No. 8 bars. Concrete Cylinder strength at 28 days, $f_c' = 4000$ psi. Concrete tensile strength in bending = 475psi; $f_y = 60{,}000$psi. Determine the stress caused by a bending moment 45kip

- effective length, $d = 23''$



$$E_c = 57{,}000\sqrt{f_c'\ (psi)}$$

$$= 57000 \times \sqrt{4000}$$

$$= 3604996.533\ psi$$

$$\approx 3600000\ psi = 3.6 \times 10^6\ psi$$

#5→0.31in² #6→0.44 #7→0.6in² #8→0.79in² #9→1in² #10→1.27

#4→0.20in² #3→0.11 #2→0.05 #

We know, $E_{st} = 29{,}000{,}000$ psi (29×10^6)

∴ modular ratio, $n = \frac{E_{st}}{E_c} = 8.055 \approx 8$

∴ A_s = total area for reinforcement bars $= \{A\#8\}\times 3$

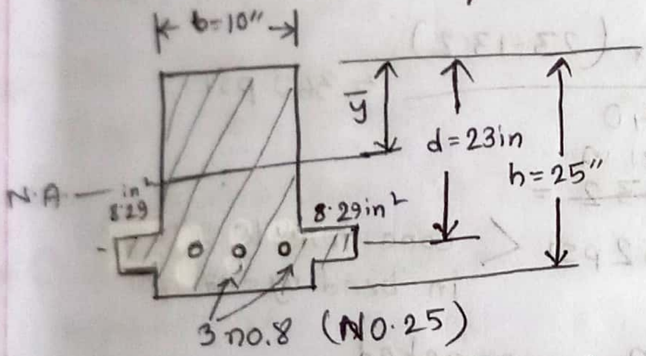
$= \{0.79\times3\}\ in^2$

$= 2.37\ in^2$

∴ Transformed area / equivalent area of st to con $= (n-1)A_{st}$

$= (8-1)\ 2.37$

$= 16.59\ in^2$



{diagram এর centroid বের করতে হবে $\bar{y}$. এখানে symmetric না তাই $\bar{y} \neq h/2$}

$A_1 = (10\times25)$ □ , $A_2 = (n-1)A_{st}$

$= 250\ in^2$ $= 16.59\ in^2$

From top

$$\bar{y} = \frac{250\times\frac{25}{2} + 16.59\times23}{250+16.59} = 13.153\ in$$

Moment of area about N.A., I = বাদ

$$\left\{I = \frac{bh^3}{12} + Ad^2\right\} = \left\{\frac{10\times25^3}{12} + (10\times25)(12.5-13.15)^2\right\} + \left\{\left(\frac{bh^3}{12}\right) + 16.59\times(23-13.15)^2\right\}$$

$= 13126.45833 + 1609.603275 \approx 14{,}740\ in^4$

Now, $\sigma = \frac{MC}{I}$ Here C = N.A থেকে top / bottom এর distance

∴ tensile stress এ $c = (25 - 13.15)$

compressive " এ $c = (13.15)$

steel এ stress এ $c = (23-13.15) \longrightarrow f_c$ (পেলাম)

$f_s = n f_c$.

concrete এর ~~tensile~~ compressive stress, [top এ comp এখানে]

$$f_c = \frac{MC}{I} = \frac{45000 \times \overset{ft\to in}{12} \times 13.2}{14740} = 484 psi$$

[bottom এ tension]

$$f_{ct} = \frac{MC}{I} = \frac{45000 \times 12 \times (25-13.2)}{14740} psi = 432 psi$$

∴ section is uncracked $\xrightarrow{\text{যেহেতু}}$ mod of rup $> f_{ct}$

$$f_{st} = \frac{MC}{I} = \frac{45000 \times 12 \times (23-13.2)}{14740} = 360 psi$$

or, $f_{st} = n \times \overset{\text{ঐ level এ}}{f_c} = \cancel{8 \times 432} =$

The value of $f_{ct} = 432 psi$ < Conc tensile strength in bending $= 475 psi$

∴ the section remain uncracked

Stress Elastic & Section Cracked

✓ Crack ~~করার~~ সাথে সাথে conc এর tensile stress capacity zero ধরে নিব, $f_{ct} = 0$

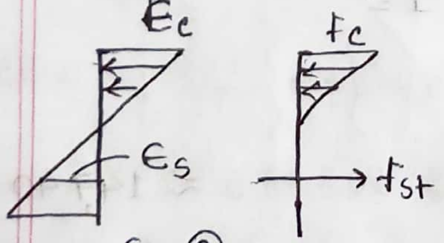


fig (e)

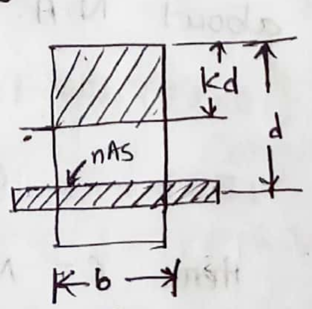


fig (a)

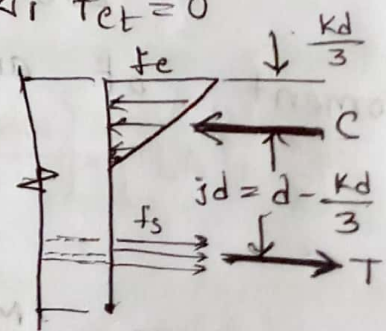


fig (b)

C = Total compressive force

T = Total Tensile force = ~~$A_{st} \cdot f_{st}$ for ste~~

kd = N.A. depth from Top

d = effective depth

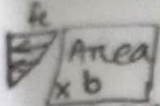
n As = equivalent area of conc from st

j.d = distance btwn C & T = internal lever arm btw C & T

 n/s → st কে equivalent conc এ transform.

For steel, $T = A_{st} \cdot f_{st}$

Stress block


Area × b = volume

$C = \left(\frac{1}{2} f_c \times kd\right) \times b$ কেননা 3D এখানে

The location of neutral axis can be obtained by equating the moment of compression area to the tension area:

$C_{(area)} \times arm = T_{(area)} \times arm$

$(kd \times b) \times \frac{kd}{2} = (nA_s) \times (d - kd)$ [NA বরাবর moment নিয়ে] —— ①

$kd = \cancel{A}$

Total comp. force $C = \left(\frac{1}{2} f_c \times kd\right) \times b$ —— ②

Total Ten. force $T = A_s f_s$ —— ③

For equilibrium $C = T$ & couple constituted by the forces C & T be numerically equal to the external bending moment M.

Internal resisting moment বের করতে,

Taking moment about C, $M = T \cdot jd = A_s f_s jd$ —— ④

stress in steel, $\boxed{f_s = \frac{M}{A_s \cdot jd}}$ —— ⑤

Taking moment about T, $M = C \cdot jd$
$= \frac{1}{2} \cdot f_c \cdot kd \cdot b \cdot jd$

k, j আলাদা আলাদা factor

d = effective depth

$\therefore \boxed{M = \frac{f_c \cdot kd \cdot jd \cdot b}{2}}$ —— ⑥

$\therefore \boxed{f_c = \frac{2M}{k \cdot j \cdot d^2 b}}$ —— ⑦

দুইটা M থেকে যার value lower হবে সেটা max moment capacity.

reinforcement ratio, rho $\boxed{\rho = \frac{A_s}{bd}}$ —— ⑧ [1–5% ~ 0.01 ~ 0.05]

Substituting eqn ⑧ & ① & solving for k,

$k = \sqrt{\{(\rho n)^2 + 2\rho n\}} - \rho n$ —— ⑨

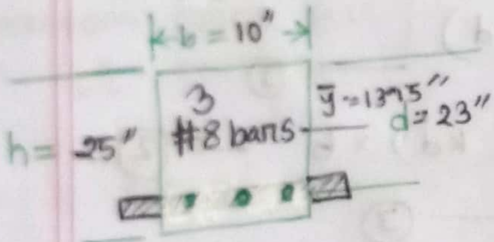
from figure ⓑ, $jd = d - kd/3$

$\therefore j = 1 - \frac{k}{3}$ —— ⑩

k = 0.3 ~ 0.4 পর্যন্ত হয়

j = 0.85 ~ 0.9

Problem: For a rectangular beam section assume, $b = 10''$, $h = 25''$, $d = 23''$. Reinforcement consist of 3 No 8 bars. Conc. Cylinder strength at 28 days $f'_c = 4000$ psi. Conc tensile strength in bending $= 475$ psi; $F_y = 60{,}000$ psi. Determine the stress caused by a bending moment 90kipft



$E_c = 57000\sqrt{f'_c} = 57000\sqrt{4000}$ psi
$\approx 3.6 \times 10^6$ psi

modular ratio, $n = E_s/E_c$

$= \dfrac{29 \times 10^6}{3.6 \times 10^6} \approx 8$

Considering the section is uncraked:

A_s = Area of reinforcement bars $= 3 \times 0.79$ in^2 $= 2.37$ in^2

transformed area/equi. st area of conc $= (n-1)A_s$
$= 7 \times 2.37 = 16.59$ in^2

$A_1 = 10 \times 25 = 250$ in^2
$A_2 = 16.59$ in^2

from top $\bar{y} = \dfrac{250 \times \frac{25}{2} + 16.59 \times 23}{250 + 16.59}$

$= 13.15$ in

$I = \cancel{(250)} + \left\{ \dfrac{10 \times 25^3}{12} + (10 \times 25)\left(\tfrac{25}{2} - 13.15\right)^2 \right\}$

$+ \left\{ 16.59 \times (23 - 13.15)^2 \right\} \approx 14740$ in^4

Now $\sigma = \dfrac{Mc}{I} = f$

distance N.A. to bottom

$f_{ct} = \dfrac{90 \times 10^3 \times 12 \times (25 - 13.15)}{14740} = 868.25$ psi

$f_{ct} = \dfrac{90 \times 10^3 \times (13.15) \times 12}{14740} = 963.50$ psi

The value of f_{ct} is larger than conc tensile strength in bending $= 475$ psi. therefore the section is cracked. And the beam should be analyzed considering cracked section.

reinforcement ratio, $\rho = \frac{A_s}{b \cdot d} = \frac{2.37}{10 \times 23} = 0.010304$

modular ratio, $n = 8$.

$\therefore k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n = \sqrt{(0.0103 \times 8)^2 - 2 \times 0.0103 \times 8} - 0.0103 \times 8$.

$\therefore k = 0.331$. $\therefore kd = 0.331 \times 23 = 7.63$ → সুতরাং N.A. উপরে উঠে গিয়েছে।

$j = 1 - k/3 = 1 - \frac{0.331}{3} = 0.89$.

$\therefore$ Maximum concrete stress, $f_c = \frac{2M}{kjd^2b}$

$= \frac{2 \times 90000 \times 12}{0.331 \times 0.89 \times (23)^2 \times 10}$

$= 1386.05 \approx 1390 psi$

* Steel stress, $f_s = \frac{M}{j \cdot d \cdot A_s} = \frac{90000 \times 12}{0.89 \times 23 \times 2.37}$

$= 22261.66 psi$

$\approx 22,300 psi$.

Flexural Strength - General Analysis

* near ultimate load stress ~~&~~ strain
* parabolic stress block করে analysis করছি।

value of c & βc জানা নেই।

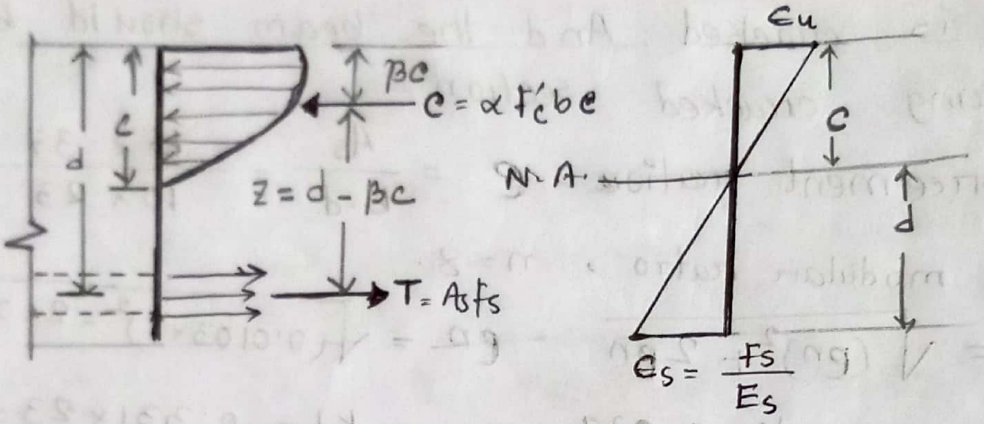


fig: Internal stress & strain distribution when the beam is about to fail

* failure 3 way তে হতে পারে : beam এ load gradually increase
→ conc & steel both fail.
→ conc fail , → st fail

* mode of failure ① fs = fyeild , conc crash
② exact criterion is not yet known

* comp failure এ fs < fs কিন্তু conc crashed. no sign shown before failure. brittle type fail. over reinforced tension এ এরকম হয়।

③ balanced failure theoretically possible : st & con both fail. Normally তা হয় না।

Senken Sir
07-01-2021
Lecture 4

* f'c এর average একটা value ধরে নিব।

Considering tension failure : under reinforced

- considering a rectangular beam, the comp. area is therefore ~~bxc)~~ bc.
- Total comp. on this area, $C = f_{av} bc$ —— (1)
  Here f_{av} = average comp stress in the area bc.
- f_{av} which can be developed before failure occurs > cylinder strength f'_c on the particular conc.

Let. $\alpha = \frac{f_{av}}{f_c'}$ ——— (2)

$C = f_{av}\, bc \Rightarrow C = \alpha f_c' bc$ ——— (3)

[• for a given distance c to the N.A., the Location of C can be defined as some fraction β of this distance.

• There fore it is only necessary to know α & β to define completely the effect of the conc. comp. stresses.]

Values of α & β : (f_c' বাড়লে α কমবে। Stronger conc α reduce করব)

□ $\alpha = 0.72 \rightarrow$ for $f_c' \leq 4000$ psi and decrease by 0.04 for every 1000 psi above 4000 psi

Example: $f_c' = 3000, 4000 \ldots$ psi হলে $\alpha = 0.72$

$f_c' = 5000$ psi হলে $\alpha = 0.74 - 0.04 = 0.68$

$f_c' = 6000$ psi " $\alpha = 0.74 - (2 \times 0.04) = 0.64$

□ $\beta = 0.425 \rightarrow$ for $f_c' \leq 4000$ psi. and decrease by 0.025 for every 1000 psi above 4000 psi

✓ The decrease in for high-strength conc. is related to the fact that such conc are more brittle

→ • The ultimate strength can be calculated from the Laws of equilibrium & the assumptions that the plane cross section remain plane.

• for equilibrium, $C = T$ ——— (4)

$\Rightarrow \alpha f_c' bc = A_s f_s$

• Bending Moment, $M = Tz \Rightarrow M = A_s f_s (d - \beta c)$ ——— (5)

or, $M = Cz \Rightarrow M = \alpha f_c' bc\,(d - \beta c)$ ——— (6)

• For tension failure by yeilding of steel, $f_s = f_y$.

∴ from eqtn (4) $\alpha f_c' bc = A_s f_y$

distance of N.A from top, $c = \frac{A_s f_y}{\alpha f_c' b}$ ——— (7)

$\varepsilon_u = 0.003$

$\varepsilon_y = F_y / E_s$

Note
[Ultimate Moment Capacity = Nonimal Moment capacity × Capacity reduction factor]

- The steel area also expressed nondimentionally as a fraction of the effective area of the section & is called reinforcement ratio, ρ

$$\rho = \frac{A_s}{bd} \quad \text{——— (8)}$$

- So for tension failure the distance to the N.A. is $\rightarrow$ $c = \dfrac{A_s f_y}{\alpha f_c' \, b}$

$$\Rightarrow c = \frac{\rho f_y}{\alpha f_c'} d \quad \text{——— (9)}$$

- In tension failure the ultimate moment is given by (combine Eqtn 5 & 9)

$$M_u' = A_s f_s d \left(1 - \frac{\beta f_y}{\alpha f_c'} \rho\right) \quad \text{——— (10)}$$

Moment capacity of the beam

- Putting the extreme values of $\alpha = 0.72$ & $\beta = 0.425$

$$\boxed{M_u' = A_s f_s d \left(1 - 0.59 \frac{f_y}{f_c'} \rho\right)} \quad \text{——— (4)}$$

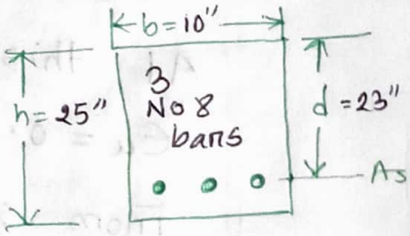
ρ = Actual
ρ_b = theoretical reinforcement ratio

Note: কোনো section কি tension / comp / balance failure হবে তা নির্ণয়ের জন্য reinforcement ratio ρ use করা হয়

- ✓ tension failure (beam under reinforced) $\rightarrow \rho < \rho_b$
- ✓ Comp failure (over reinforced) $\rightarrow \rho > \rho_b$
- ✓ balanced failure $\rightarrow \rho = \rho_b$

we always (1–4%) এর মধ্যে
0.1 / 0.2 আসলে ভুল।

Problem: For a rectangular beam section. $b = 10''$, $h = 25''$, $d = 23''$. Reinforcement consists of 3 No. 8 bars. Conc. Cylinder strength at 28 days, $f_c' = 4000$ psi. Conc tensile strength in bending $= 475$ psi. $f_y = 60{,}000$ psi $\Rightarrow$ Determine M_n at which the beam fails.



Actual reinforcement ratio, $\rho = \dfrac{A_s}{bd}$

$$= \frac{3 \times 0.79}{10 \times 23} = 0.0103$$

theoretical reinforcement ration, $\rho_b = \dfrac{\alpha f_c'}{f_y} \dfrac{\epsilon_u}{\epsilon_u + \epsilon_y}$

we know, for, $f_c' \le 4000$ psi, $\alpha = 0.72$ & $\beta = 0.425$

$\epsilon_u = 0.003$ & $\epsilon_y = f_y / E_s = \dfrac{60{,}000}{29 \times 10^6} = 2.069 \times 10^{-3}$

$$\therefore \rho_b = \frac{\overset{0.72}{\cancel{0.003}} \times 4000}{60000} \times \frac{0.003}{0.003 + 2.069 \times 10^{-3}} = 0.028$$

$\because \rho < \rho_b$ $\therefore$ Tension failure occurs. Hence $f_s = f_y$.

$\therefore$ Nominal Moment Capacity, $M_n = \rho f_y b d^2 \left(1 - \dfrac{\beta \rho f_y}{\alpha f_c'}\right)$

$$\Rightarrow M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$M_n = 0.0103 \times 60000 \times 10 \times (23)^2 \left(1 - 0.59 \times \frac{0.0103 \times 60000}{4000}\right)$$

$= 2971214.251$ lb-in $= 247.6$ ft-kip ≈ 248 ft-kips

N.A. from top,

$$c = \frac{\rho f_y d}{\alpha f_c'} = \frac{0.0103 \times 60000 \times 23}{0.72 \times 4000} = 4.935 \text{ inch}$$

[∵ N.A উপরে উঠে গেছে]

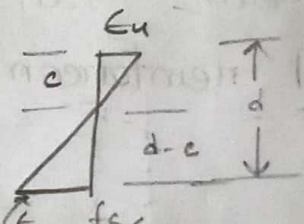
$\boxed{4}$ In oven reinforcement beams the steel does not yeild at failure and steel is proportional to the steel strain.

i.e $\rightarrow$ $f_s = \epsilon_s \cdot E_s$ ——— (12)

At this condition strain in the concrete beams

$\epsilon_u = 0.003$ $\longrightarrow$ 1st Scritarion.

From strain distribution :



$$f_s = \left(\epsilon_u \frac{d-c}{c}\right) \cdot E_s \text{ — (13)}$$

$(\epsilon_s = f_s / E_s$

$\therefore \frac{\epsilon_u}{\epsilon_s} = \frac{c}{d-c}$

$\therefore \epsilon_s = \frac{d-c}{c} \times \epsilon_u$

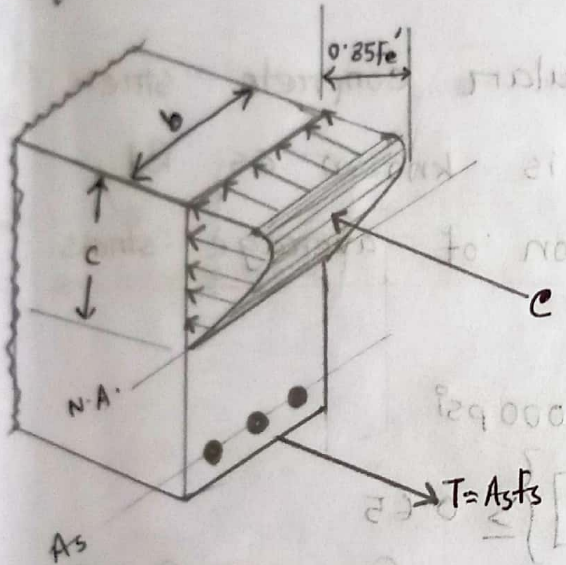
Total force $C = T \rightarrow C = A_s f_s$

$\Rightarrow \alpha f_c' bc = \epsilon_u A_s E_s \frac{d-c}{c}$ ——— (14)

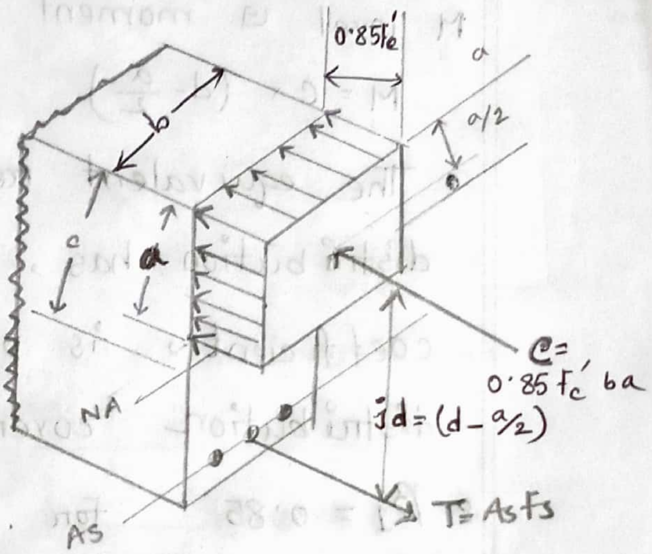
By solving equadratic eqtn c can be obtained and the moment capacity of the beam can be computed.

Flexural Stress - Equivalent Stress Block

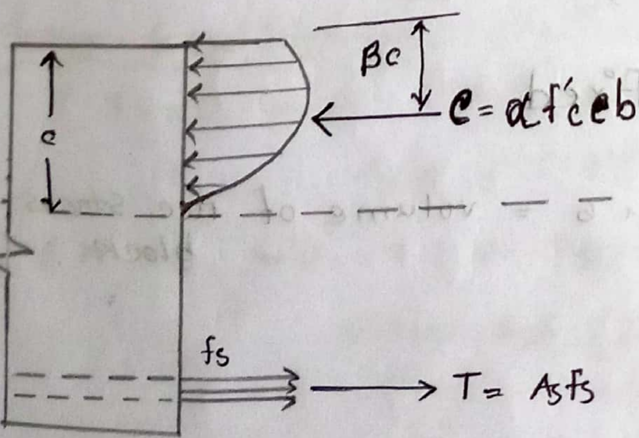
The compressive zone is ~~on~~ modeled with a equivalent stress block.



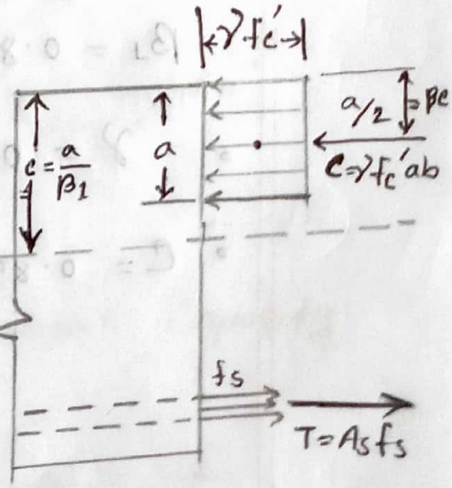
Actual stress block ডা২



Equilivalent rectangular stress block ডা২



Actual
ⓐ



Equivalent
ⓑ

lever arm (C—T distance)= $d - a/2$ effective length

Force C এর Location & মান change হওয়া সম্ভাবনা

[Note

C level এ moment নিলে,

$M = T \times (d - a/2) = A_s f_s \left(d - \frac{a}{2}\right)$

M level এ moment নিলে,

$M = C \times \left(d - \frac{a}{2}\right)$]

- The equivalent rectangular concrete stress distribution has, what is known as β_1, coefficient, is proportion of average stress distribution covers. $= \beta_1$

- $\beta_1 = 0.85$ for $f_c' \leq 4000$ psi

- $\beta_1 = 0.85 - \left\{0.05\left[\frac{f_c' - 4000}{1000}\right]\right\} \geq 0.65$

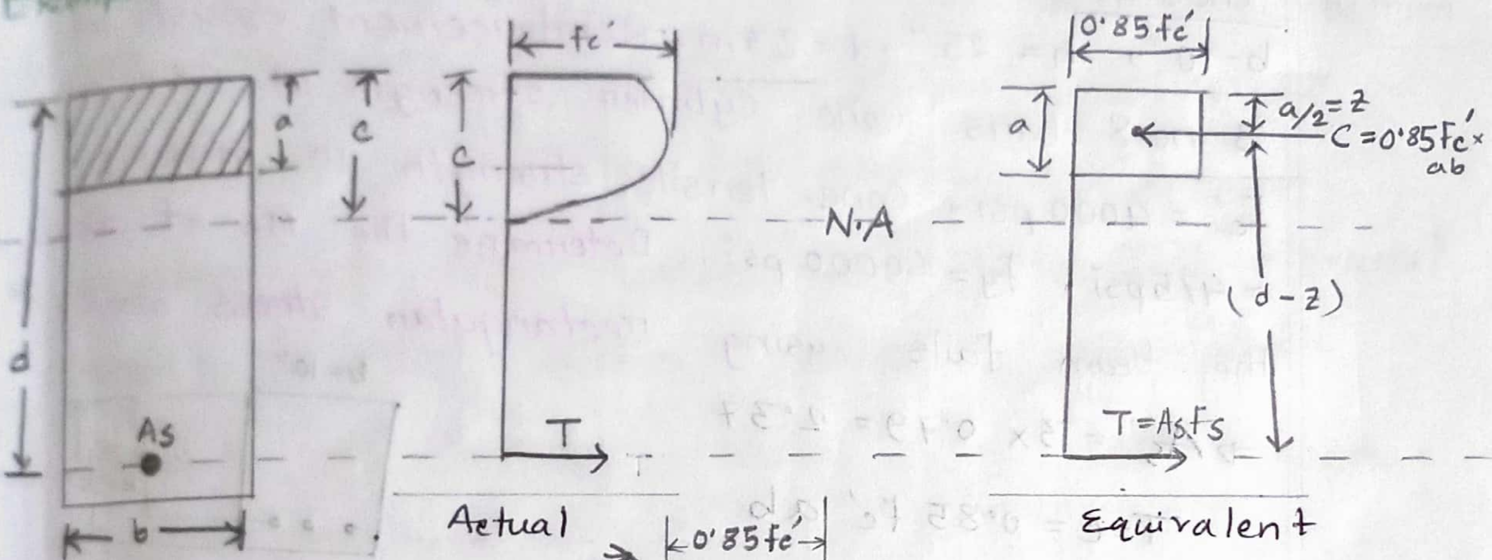
↳ এটার মানে f_c' প্রত্যেক 1000 বৃদ্ধির জন্য β_1 0.05 করে ~~স~~ হ্রাস পাবে, যেমন: $f_c' = 6000$ হলে,

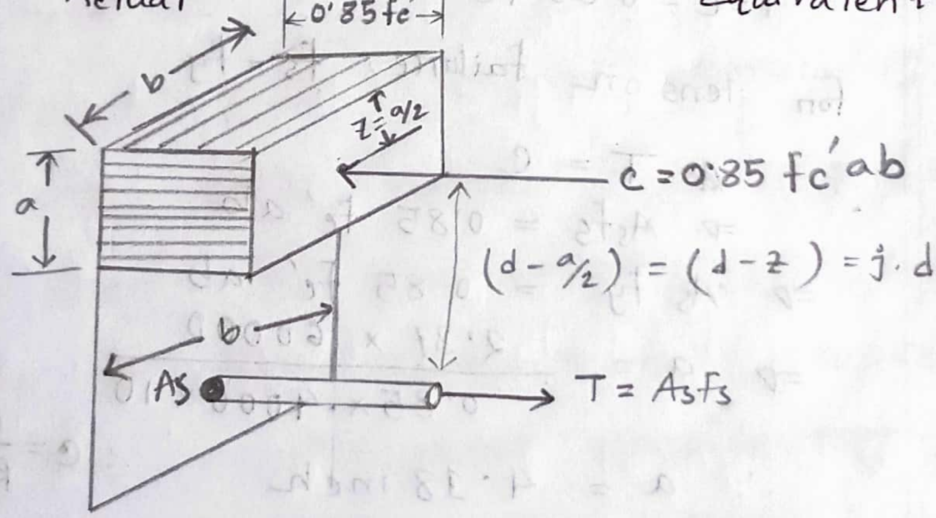
$\beta_1 = 0.85 - (\overset{\text{2k বাড়ছে}}{2} \times 0.05) = 0.75$

- $\gamma = 0.85$ fixed,

- $C = 0.85 f_c \times a \times b$ = volume of rec. stress block

Example of Rectangular reinforced concrete beam:





① Setup Equilibrium: (failure for tension consider) $f_s = f_y$

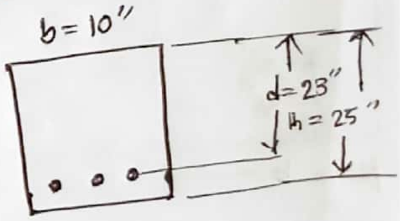
$\Sigma F_x = 0 \Rightarrow T = C$

$A_s f_s = 0 \cdot 85 f_c' ab$

(Considerin force level) $\Sigma M = 0 \Rightarrow T(d - a/2) = M_n$ = Moment Capacity

$\Rightarrow M_n = A_s f_s (d - a/2)$

Problem: For a rectangular beam section assume b = 10", h = 25", d = 23 in. Reinforcement consists of 3 no. 8 bars. Conc. Cylinder strength at 28 days. $f_c' = 4000$ psi. Conc. tensile strength in bending = 475 psi. $f_y = 60{,}000$ psi. Determine the M_n at which the beam fails using rectangular stress block.



$\Rightarrow A_s = 3 \times 0.79 = 2.37$

$T = C = 0.85 f_c' ab$

for tension failure, $f_s = f_y$

$\therefore T = C$

$\Rightarrow A_s f_s = 0.85 f_c' ab$

$\Rightarrow A_s f_y = 0.85 f_c' ab$

$\Rightarrow a = \dfrac{2.37 \times 60000}{0.85 \times 4000 \times 10}$

$\therefore a = 4.18$ inch

$\therefore c = \dfrac{a}{\beta_1} = \dfrac{4.18}{0.85} = 4.91''$

Now, $\Sigma M = 0$

$M_n = T(d - a/2) = A_s f_s (d - a/2)$

$= A_s f_y (d - a/2)$

$= 2.37 \times 60000 \left(23 - \dfrac{4.18}{2}\right)$

$= 2973402.$

= max Moment capacity

"Design of Singly Reinforced Beams"

Lecture 5
16-01-21

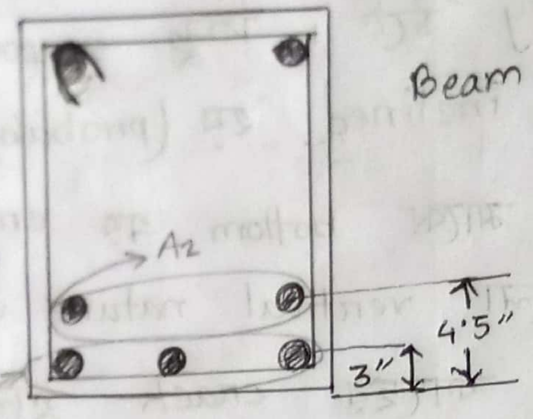
Notes

* Beam / Girder / member এ মাঝের crack গুলো bottom হতে vertically উঠে। কিন্তু support এর কাছের crack গুলো inclined হয় (probably 45°)
* member এর মাঝে bottom এর crack গুলো flexural crack. এগুলো vertical nature এ spreaded.
* support এর ~~কাছের~~ crack গুলো shear crack. এগুলো inclined.
* ~~me~~ মাঝের crack গুলো (flexural) repair এর জন্য সময় পাওয়া যায়। কিন্তু support এর কাছের (shear) crack গুলো সময় নাও পেতে পারি। এগুলো sudden failure হয়। এর জন্য reinforcement capacity থেকে একটু কম reinforcement provide করে member under reinforced রাখা হয় যাতে failure হলেও তার আগে repairing এর জন্য time পাই।

Effective depth Calculation

Beam size = $\overset{b}{12} \times \overset{h}{24}$ in

2#8 bars &
3#9 bars



∴ সব reinforcement এর centroid,

$$y = \frac{A_1y_1 + A_2y_2}{A_1 + A_2} = \frac{(3\times 1)\times 3 + (2\times 0.79)\times 4.5}{(3\times 1) + (2\times 0.79)}$$

$$= 3.52 \text{ inch}$$

∴ effective depth, $d = \overset{h}{t} - y = 24 - 3.52 = 20.48$ ''

Design of Tension-Reinforcement Rectangular Beam

- The nominal strength of a proposed member is calculated based on the best current knowledge of member and material's behavior.

$\boxed{\phi < 1}$

- The nominal strength is modified by a strength reduction factor (ϕ), less than unity, to obtain strength.

- Ultimate Moment, $M_u \leq \phi M_n$
- Ultimate Axial force, $P_u \leq \phi P_n$
- Ultimate Shear force, $V_u \leq \phi V_n$

Scanned with CamScanner

Note: আগের formula হতে, • M_u = applied load এর জন্য যে formula moment এটা দিয়ে হিসাব

যেমন simple supported beam এ distributed load এ থাকলে

max moment $M_u = \frac{wL^2}{8}$.

• ϕM_n = member nominal moment capacity $= \phi \cdot A_s f_y (d - a/2)$

Equivalent Rectangular Stress Distribution

(আগের diagram)

• Actual stress distribution can be replaced by equivalent rectangular stress distribution.

• ~~Two~~ Two Conditions:

↳ The total comp. force (C) and its location must be the same in the equivalent rectangular stress distribution.

• $C = \alpha f_c' c b = \gamma f_c' a b$, $\gamma = \alpha \frac{c}{a}$, $\alpha = \beta_1 c$

Concrete Stress block Parameters: f_c' psi

f_c' (psi)	≤4000	5000	6000	7000	≥8000
α	0·72	0·68	0·64	0·60	0·56
β	0·425	0·400	0·375	0·350	0·325
$\beta_1 = 2\beta$	0·85	0·80	0·75	0·70	0·65
$\gamma = \alpha/\beta_1$	0·85	0·85	0·85	0·86	0·86

Derived parameters

Experimentally Derived parameters

- $C = 0.85 f_c' ab$

- f_c' 4000 থেকে বৃদ্ধির ক্ষেত্রে,

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000}{1000} \quad \& \quad 0.65 \le \beta_1 \le 0.85$$

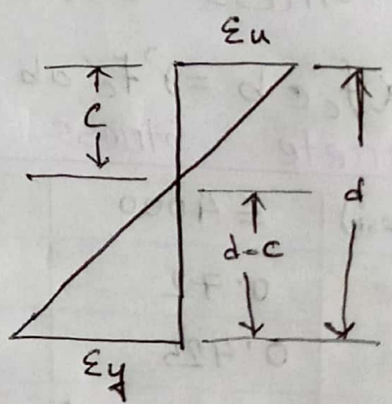
for $f_c' \le 4000$ psi $\longrightarrow a = 0.85c$

3 possibilities in inelastic behaviour

- Compression failure (over-reinforced beam)
  → brittle fail, sudden
- Tension failure (under-reinforced beam)
  actual ratio → $e < e_b$ ← theo. ratio
- Balanced Failure (balanced reinforcedment)
  → $e = e_b$ [theoretically possible, Not practically]

Balanced Strain Condition

From strain diagram,



$$\frac{c}{d-c} = \frac{\varepsilon_u}{\varepsilon_y} \Rightarrow \varepsilon_y \cdot c = \varepsilon_u d - \varepsilon_u \cdot c$$

$$\Rightarrow c = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} d$$

Balance condition এ $f_s = f_y$

for equilibrium $\Rightarrow C = T$

$$\Rightarrow 0.85 f_c' ab = A_s f_y = e_b bd \cdot f_y$$

$$\therefore e_b = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \qquad [a = \beta_1 c]$$

"Under Reinforced Beam"

ACI Provisions for Under Reinforced beams:

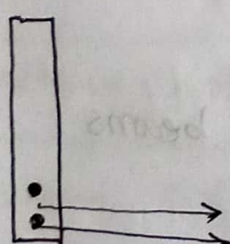
- An ACI (American Concrete Institute) code limitation on max. reinforcement ratio takes two forms:

(1) Minimum tensile reinforcement strain allowed at nominal strength in the design of beams.

(2) Strength reduction factor may depend on net tensile strength at nominal strength.

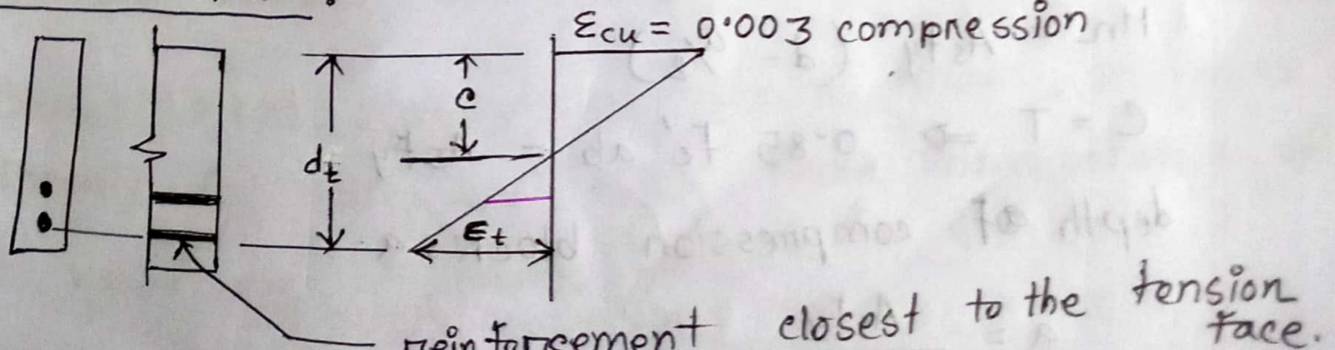
- Net tensile ~~strength~~ strain is the tensile strain of the reinforcement farthest from the compression face of the concrete at depth d_t

- For single layer reinforcement $d_t = d$
  For multiple " " $d_t > d$



দুই layer reinforcement এর
→ centroid বরাবর tensile.
→ last layer (farthest) বরাবর net tensile.

Net tensile strain:



From strain diagram, $\varepsilon_s = \varepsilon_u \frac{d-c}{c}$

Substituting d_t for d and ε_t for ε_s

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c}$$

∴ Reinforcement ratio for any value of net tensile strain,

$$\rho = 0.85 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_t}$$ general formula of ρ

* এখানে, balanced reinforcement ratio পেতে হলে ε_t এর স্থানে ε_y বসাবো [$f_s = f_y$ in balance]

তখন, $\rho_b = 0.85 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

* $\varepsilon_t = 0.005$ বসালে ρ_{max} পেয়ে যাবো।

* At the nominal member strength minimum value of $\varepsilon_t = 0.004$

Maximum reinforcement ratio for beams

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.004}$$

$$M_n = A_s f_y (d - a/2)$$

$C = T \Rightarrow 0.85 f_c' ab = A_s f_y$

depth of compression block, a

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Substituting $A_s = e\, bd$

$$a = \frac{e f_y d}{0.85 f_c'}$$

$$\therefore M_n = A_s f_y d\left(1 - 0.59 \frac{f_y}{f_c'} e\right)$$

$$= e b d^2 f_y \left(1 - 0.59 \frac{e f_y}{f_c'}\right)$$

It can be written as,

$$M_n = R\, bd^2$$

Here $R = e f_y \left(1 - 0.59 \frac{e f_y}{f_c'}\right)$

R is the flexural resistance factor depends on the reinforcement ratio & the strength of the materials.
reinforcement → e
strength of the materials → (f_y, f_c')

⊞ Strength Reduction factor, ϕ

Types of member	net tensile strain	Strength reduction factor
Tension controlled	greater than or equal to 0.005 [≥ 0.005]	0.90
Compression controlled	less than 0.002 [< 0.002]	0.65

* Column ϕ < beam ϕ → কারণ column failure বেশি harmful
* Tension member (beam) → under rein. ϕ > over rein ϕ of comp. member

• Btwn 0.002 & 0.005 net tensile strain, ϕ varies linearly & ACI code allows linear interpolation of ϕ based on [graph টা]

V.J. Why a lower ϕ-factor is used for comp. controlled sections?

⟹ A lower value of ϕ-factor is used for comp controlled sections than is used for tension controlled sections cause:

(1) comp controlled sections have less ductility

(2) are more sensitive to variations in concrete structures

(3) & generally occur in members that support larger loaded areas than members with tension controlled sections.

(4) Members with spiral reinforcement are assigned a higher ϕ than tied columns since they have greater ductility or toughness.

Limitation on e:

(1) $e \leq 0.75\ e_b$

This will ensure yeilds; $\varepsilon_s (1.8 \text{ to } 2.0)\ \varepsilon_y$ at failure

* মানে e_{max}, e_b এর 75% max হওয়া যাবে,

* best হয় $e = (40\% - 50\%) e_b$ রাখা যাতে space থাকে। crack & deflection কম হয়.

$f_y = f_s = E_s \varepsilon_s$

$\therefore \varepsilon_s = \varepsilon_y = \frac{f_s}{E_s}$

$= \frac{60000}{29 \times 10^6}$

② Lower limit on e

$$A_{s(min)} = \boxed{\frac{3\sqrt{f_c'}}{f_y}} * b_w d \geq \boxed{\frac{200}{f_y}} * b_w d$$

অথবা

Here, f_c, f_y in psi

মানে A_s min এত থাকা লাগবে। design এ A_s কম আসলেও এই minimum মানটা দিয়ে count করব। দিলে excess deflection হবে।

Problem A rectangular beam has width 12" & d = 17.5". If it is reinforced with 4#9 bars in one row &. If f_y = 60,000 psi, f_c' = 4000 psi, What is the nominal flexural strength and what is the max moment that can be utilized in design?

→ f_y = 60,000 psi, f_c' = 4000 psi

$A_s = 4 \times 1 = 4 \text{ in}^2$

Actual → $\therefore e = \frac{A_s}{bd} = \frac{4}{12 \times 17.5} = 0.019$

theore. → $\therefore e_b = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

$= 0.85 \times 0.85 \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.0021}$

$= 0.0285$

$\varepsilon_u = 0.003$

$f_s = f_y = $

$\varepsilon_y = \varepsilon_s = \frac{f_y}{E_s}$

$= \frac{60 \times 10^3}{29 \times 10^6}$

[এটা ACI code মানে নাকি তার জন্য e_{min} count করব, $e > e_{min}$ হওয়া লাগবে। $e_{min} = \frac{200}{f_y} = 0.003 < e$ ∴ হবে]

[অথবা $e_{max} \leq 0.75 e_b$ হয় নাকি দেখব]

$\therefore e < e_b$, tension failure will occur.

$\therefore f_s = f_y$

$$\therefore a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4 \times 60000}{0.85 \times 4000 \times 12} = 5.88''$$

$$\therefore M_n = A_s f_y \left(d - a/2\right) = 4 \times 60000 \left(17.5 - \frac{5.88}{2}\right)$$

$$= 3494.4 \text{ k-in}$$

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92''$$

$$\therefore \varepsilon_t = \varepsilon_u \frac{d_t - c}{c} \Rightarrow \varepsilon_t = 0.003 \, \frac{17.5 - \cancel{5.88}\ 6.92}{6.92}$$

$$= 0.0046 \approx 0.005$$

$\therefore$ strength reduction factor,

$\phi = 0.90$ [As $\varepsilon_t \geq 0.005$ হলে $\phi = 0.9$]

$$\because M_u = \phi M_n \Rightarrow M_u = 0.90 \times 3494.4$$

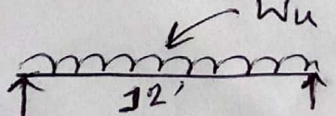
$$= 3144.96 \text{ k-in}$$

$$= 3145 \text{ k-in}$$

[$\because$ member এর internal resistance max 3144.96 k-in allowed]

Problem Extension: How much Live load can be applied on the beam, assume simple span of 12'.

$\rightarrow M_u = 3145/12 = 262.08$ k-ft



$$\rightarrow M_u = \frac{w_u L^2}{8} \Rightarrow w_u = \frac{8 M_u}{L^2}$$

$$= \frac{8 \times 262.08}{(12)^2}$$

$$= 14.56 \text{ k/ft}$$

$W_u = 1.2\ \underbrace{D.L}_{\text{beam self wt}} + 1.6\ \underbrace{L.L}_{\text{যা চেয়েছে}}$

17.5+2.5

ধরি, $h = d + 2.5'' = 20''$

$W_u = 1.2\ D.L + 1.6\ L.L.$

$\Rightarrow 14.56 \times 10^3 = 1.2 \times \left\{ \frac{b}{12} \times \frac{h}{12} \times 150 \right\} + 1.6\ L.L$

$\Rightarrow 14.56 \times 10^3 = 1.2 \times \frac{12}{12} \times \frac{20}{12} \times 150 + 1.6\ LL$

$\therefore L.L = 8.9125$ ~~lb~~.

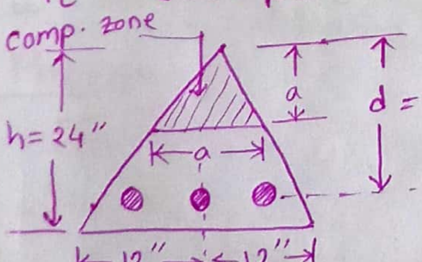
Load Combination

- Basic load combination :

  factor Load, $\omega_u = \underbrace{1.2\ D.L.}_{\text{Load factor}} + 1.6\ L.L$

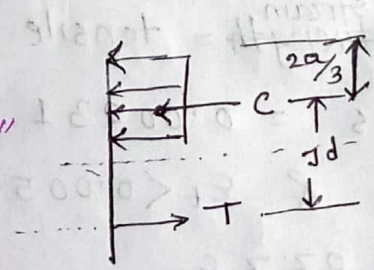
- self wt of beam (used in D.L) $= \omega = \frac{b}{12} \times \frac{h}{12} \times 150 = ?$ plf

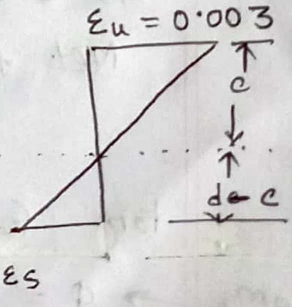
Problem : Calculate the moment capacity of the beam shown in figure below. Assume $f_y = 60,000$ psi & $f_c' = 3000$ psi



3#8

cross section





$A_s = 3 \times 0.79 = 2.37\ in^2$

$C = T \Rightarrow 0.85\ f_c' \times a^2/2 = A_s f_y$

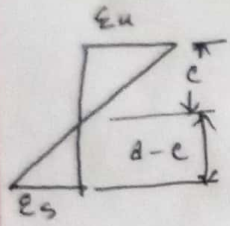
$\Rightarrow a^2 = \frac{2.37 \times 60000 \times 2}{0.85 \times 3000}$

$\therefore a = 10.56''$

assume $f_s = f_y$, that is tension steel yeilds

compression area $= \frac{1}{2} \cdot a \cdot a = \frac{a^2}{2}$

The distance to N.A. =' $c = \frac{a}{\beta_1} = \frac{10.56}{0.85} = 12.42''$



Now from similar triangle,

$$\frac{\varepsilon_s}{\varepsilon_u} = \frac{d-c}{c}$$

$$\Rightarrow \varepsilon_s = \frac{22 - 12.42}{12.42} \times 0.003$$

$$\therefore \varepsilon_s = 0.00234$$

$f_y = E_y \varepsilon_y$ $\quad \varepsilon_y = \frac{f_y}{E_s} = \frac{60,000}{29 \times 10^6} = 0.00207$

$\therefore$ $\boxed{\varepsilon_s > \varepsilon_y}$ therefore the member will fail by yeilding of steel. (Tension)

As one layer reinforcement,
Net tensile ~~strength~~ strain = tensile ~~strength~~ strain

$$\varepsilon_t = \varepsilon_s = 0.00231$$

for $\varepsilon_t \geq 0.002$ & $\varepsilon_t < 0.005$

from graph → $\phi = 0.483 + 83.3\,\varepsilon_t$

$= 0.483 + 83.3 \times 0.00231 = 0.675$

$\therefore jd = d - \frac{2a}{3} = 22 - \frac{2 \times 10.56}{3} = 14.96''$

$M_n = A_s f_y \left(d - \frac{2a}{3}\right) = 2.37 \times 60 \times 10^3 \left(22 - \frac{2 \times 10.56}{3}\right)$

$= 2.13 \times 10^6$ lb-in $= 178$ ft-kips.

$M_u = \phi M_n = 0.675 \times 178 = 120.15$ ft-kips (Ans)

Over-Reinforced Beam

- Flexural strength of an over-reinforced member for which f_s is less than f_y at flexural failure. (Comp failure). $\boxed{f_s < f_y}$
- In this case the steel strain will be less than yeild strain of steel. $\boxed{\varepsilon_s < \varepsilon_y}$
- From the equilibrium, $C = T$, {N.A. এর depth / comp block depth আসে}

$0.85 f'_c ab = \rho bd \cdot f_s$, $[a = \beta_1 c]$ $[f_s = E_s \varepsilon_s]$

or, $0.85 f'_c \beta_1 bc = \rho bd \cdot E_s \varepsilon_s$

$\therefore c = ?$ পাবো

$\varepsilon_s = \varepsilon_u \frac{d-c}{c}$

$0.85 \beta_1 f'_c bc = \rho \varepsilon_u \frac{d-c}{c} E_s bd$ ∴ $c = ?$ পাবো

অথবা a এর term এ $a = \beta_1 c$ দিয়ে c পাবো।

$\Rightarrow 0.85 f'_c a^2 = \rho \varepsilon_u E_s \beta_1 d^2 - \rho E_s \varepsilon_u ad$

Problem: A rectangular beam has width 12" & effective depth 18.1" & $d_t = 19.1$ in. It is reinforced with, $A_s = 6.32 in^2$ in two rows. $F_y = 60,000$ psi and $f'_c = 4000$ psi. What is the nominal flexural strength and what is the maximum moment that can be utilized in design according to the ACI code?

$F_y = E_y \varepsilon_y$

$\varepsilon_y = \frac{60\times10^3}{29\times10^6} = 0.00207$

→ Actual reinforcement ratio, $\rho = \frac{A_s}{bd}$

$= \frac{6.32}{12 \times 18.1}$

$= 0.0291$

$\rho_b = 0.85\, \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

$= 0.85 \times 0.85 \times \frac{4000}{60000} \frac{0.003}{0.003 + 0.00207}$

$= 0.0285$

[For ρ_{max} $\varepsilon_t = 0.004 / 0.005$ ধরতে বলা হয়েছিল]

$\rho_{max} = 0.85 \times 0.85 \times \frac{4}{60} \frac{0.003}{0.003 + 0.004}$

$= 0.0206$

∵ ρ is greater than both ρ_b and ρ_{max}.

∴ The beam is over-reinforced beam. And during failure $f_s < f_y$.

$C_1 = T$

or, $\frac{0.85 f_c' a^2}{\rho E_s \varepsilon_u} = \frac{\rho \varepsilon_u E_s \beta_1 d^2}{\rho \varepsilon_u E_s} - \frac{\rho E_s \varepsilon_u a d}{\rho E_s \varepsilon_u}$

or, $\left(\frac{0.85 f_c'}{\rho E_s \varepsilon_u}\right) a^2 + a \cdot d - \beta_1 d^2 = 0$

or, $1.3430 a^2 + \cancel{22a}\ 18.1 a - 0.85 \times (18.1)^2 = 0$

∴ $a = 9.16$

$a = \beta_1 c$ ∴ $c = \frac{9.16}{0.85} = 10.78''$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60000}{29\times10^6} = 0.0021$$

$$\varepsilon_s = \varepsilon_u \frac{d-c}{c} = 0.003 \frac{18.1 - 10.78}{10.78}$$
$$= 0.00204 < \varepsilon_y$$

$\therefore F_s = E_s \varepsilon_s = 29\times10^6 \times 0.00204 = 59076$ psi

$\therefore f_s < F_y$ $\therefore$ the section is over reinforced.

$$M_n = A_s F_s \left(d - \frac{a}{2}\right) = 6.32 \times 59076 \left(18.1 - \frac{9.16}{2}\right)$$
$$= 5047837.233 \text{ lb-in}$$
$$= 420.65 \approx 421 \text{ k-ft}$$

$$\varepsilon_t = \varepsilon_u \frac{d_t - c}{c} = 0.003 \frac{19.1 - 10.78}{10.78}.$$
$$= 0.00231$$

$\therefore$ for $0.002 \le \varepsilon t \le 0.005$, $\varphi = 0.483 + 83.3\ \varepsilon t$

$$\approx 0.68$$

$\therefore M_u = \phi M_n = 0.68 \times 421 = 284.35$ k-ft (Ans)

Scanned with CamScanner