

# Singly (WSD)

## Design step:

✓ 1. Assume,

depth,  $h = \square$

width,  $b = \square$

✓ 2. Total load calculation:

$$TL = D.L + L.L$$

N.B: including to its own weight  
ଅନୁରାମ self weight (ଆମେ ବାବଦ  
ରାଏ ଆମ

in addition/super imposed ଅନୁରାମ

$$\text{self weight} = \left(\frac{bh}{144} \times 150\right) \text{ p/f}$$

(ଆମେ ବାବଦ ରାଏ 1 meter ଦୂରାପନ)

$$\text{ଅନୁରାମ} = (bh \times 24) = \text{KN/m}$$

✓ 3. Maximum moment calculation:

[From BMD Diagram / Moment  
capacity (ନିର୍ଦ୍ଧାରିତ ଅନୁରାମ)]

✓ 4. depth check

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^9}{57000 \sqrt{f_c'}}$$

$$r = \frac{f_s}{f_c} = \frac{0.4 f_y}{0.45 f_c'}$$

$$k = \frac{n}{n+r} ; j = 1 - \frac{k}{3}$$

$$R = \frac{1}{2} f_c' j k$$

$$d_{req} = \sqrt{\frac{M}{R b}}$$

total depth required,  $h_{req}$

$$= (d_{req} + 2.5)$$

→ clear cover

if  $h_{req} < \text{Assumed depth, } h = \square$

It is singly reinforce beam

Design OK

✓ 1) steel area calculation:

$$A_{st} = \frac{M}{f_s j d}$$

→ Assume effective depth

[N.B: BMD -ve ଅର୍ଥାନ୍ତେ positive and negative Moment ପୂର୍ଣ୍ଣ ଅନୁରାମ, ଗଠନ  
A<sub>st</sub> ପୂର୍ଣ୍ଣ (ସବୁ ବାବଦ ରାଏ)]

for tension,

$$A_{st} = \frac{M_{max}(+ve)}{f_s j d}$$

for compression

$$A_{st} = \frac{M_{max}(-ve)}{f_s j d}$$

## Analysis step:

✓ 1.  $\rho = \frac{A_s}{bd}$

✓ 2.  $n = \frac{E_s}{E_c}$

✓ 3.  $k = \sqrt{(n\rho)^2 + 2\rho n} - \rho n$

✓ 4.  $j = 1 - \frac{k}{3}$

✓ 5.  $M_c = \frac{1}{2} f_c' k j b d^2$

$$M_s = A_s f_s j d$$

[N.B:  $M_c$  ଓ  $M_s$  ଗଠନ ସର୍ବନିମ୍ନ (ଅନୁରାମ) Allowable moment capacity]

Singly (USD)

Design step:

✓ 1. Load calculation:

$$W_u = 1.2 D.L + 1.6 L.L$$

✓ 2. Moment calculation:  $M_u = \frac{W_u L^2}{8}$

$$M_u = \frac{W_u L^2}{2}$$

for simply supported  
for cantilever beam

✓ 3. Depth:

$$M_u = \phi \rho b d^2 f_y \left(1 - \frac{\rho}{\alpha} \frac{f_y}{f_c'}\right)$$

$$\Rightarrow d_{req} = \square$$

Here,

$$\phi = 0.9 \text{ (for design)}$$

$$\rho = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{E_u}{E_u + E_y}$$

$$\frac{\beta_1}{\alpha} = 0.59$$

$$\beta_1 = 0.85 \text{ (} f_c' = 4000 \text{ psi)}$$

$$\beta_1 = 0.80 \text{ (} f_c' = 3000 \text{ psi)}$$

$$E_u = 0.003$$

$$E_y = 0.005$$

Total depth required,  $h_{req} = (d_{req} + 2.5)$

Analysis step

✓ 1.  $A_s = \square \leftarrow (\square \# \square \text{ bar})$

✓ 2.  $\rho = \frac{A_s}{bd}$

✓ 3.  $\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{E_u}{E_u + E_y}$   
( $E_u = 0.002$ )

$$\beta_1 = 0.85 \text{ (} f_c' \leq 4000 \text{)}$$

$$\beta_1 = 0.80 \text{ (} f_c' = 5000 \text{)}$$

✓ 4. Moment calculation

If,  $\rho < \rho_b$  it is under reinforced beam.  
(tension steel yield at failure)

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

Here,

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$M_u = \phi M_n$$

$$\phi = 0.483 + 83.3 \epsilon_f$$

Here,

$$\epsilon_f = \epsilon_u \times \left(\frac{d-c}{c}\right)$$

$$c = \frac{a}{\beta_1}$$

[N.B =  $\epsilon_f$  को सब मान 0.005 को सब मान 0.005 लिखो 2/10]

Here

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$a = \square \times A_s$$

If  $\rho > \rho_b$  it is over reinforced beam (concrete yield at failure)

$$M_n = A_s f_s \left(d - \frac{a}{2}\right)$$

condition  $f_s \neq f_y$

Here,

$$f_s = E_s \epsilon_c \left(\frac{d-c}{c}\right)$$

Here,

$$E_s = 29 \times 10^9$$

$$\epsilon_u = 0.003$$

$$c = \frac{a}{\beta_1}$$

At equilibrium

$$C = T$$

$$0.85 f_c' \beta_1 c b = A_s f_s$$

$$\Rightarrow c = \square$$

$$c = \frac{a}{\beta_1} \Rightarrow a = \square$$

$$f_s = \square$$

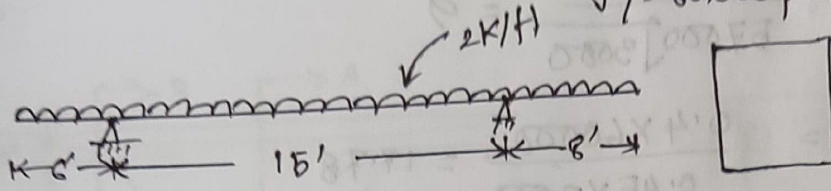
$$M_n = \square$$

$$M_u = \phi M_n$$

# Singly Reinforce Beam (WSD)

Design

# Problem: Design the beam when  $f'_c = 3000 \text{ psi}$ ,  
 $f_y = 60,000 \text{ psi}$



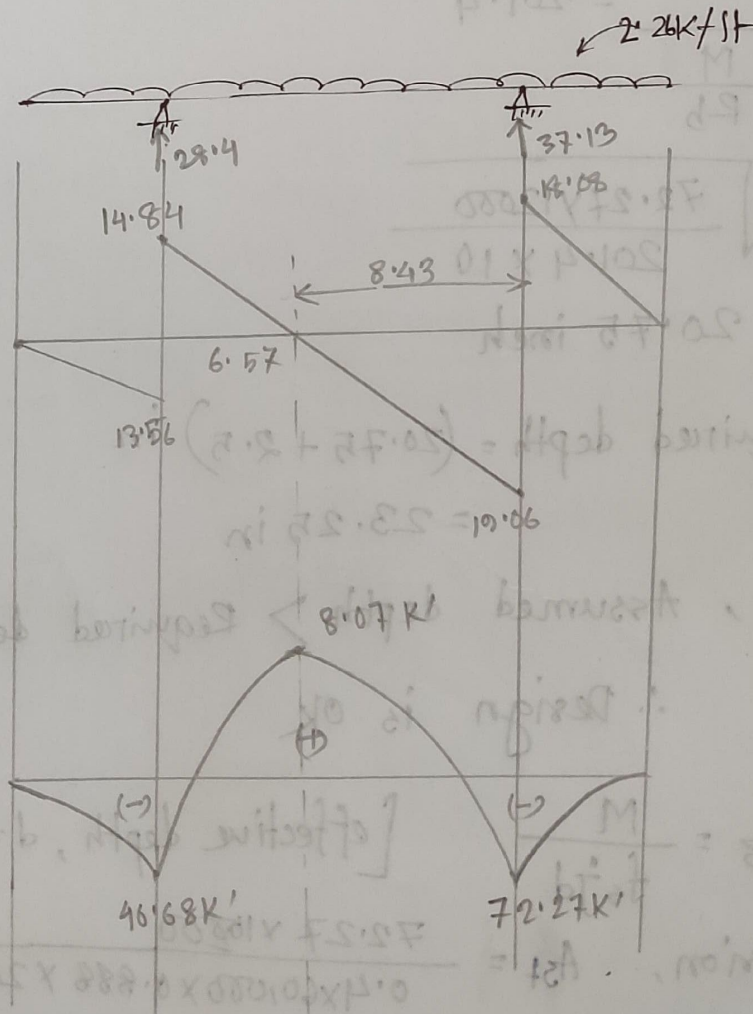
\*  $f_s / f_y$  (method 2) is not used  
method 1 is used  
\* But  $f_s$  (method 2) is used  
WSD (method 1) is used  
method 2 is used (of  $f_y$ )

Solution:

Step 1 Assume the depth = 25 inch  
Assume the width = 10 inch

Step 2 Dead load =  $\left( \frac{25 \times 10}{144} \times 150 \right) \text{ plf} = 260.41 \text{ plf}$

$\therefore$  Total load =  $(260.41 + 2000) \text{ plf} = 2260.41 \text{ plf} = 2.26 \text{ klf}$



Step-03

depth check

$$\therefore m = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \sqrt{3080}} = 9$$

$$\therefore r = \frac{f_s}{f_c} = \frac{0.4 \times 60000}{0.45 \times 3080} = 17.78$$

$$\therefore k = \frac{m}{m+r} = \frac{9}{9+17.78} = 0.336$$

$$j = 1 - \frac{k}{3} = \left(1 - \frac{0.336}{3}\right) = 0.888$$

$$R = \frac{1}{2} f_c j k = \frac{1}{2} (0.45 \times 3080) \times 0.888 \times 0.336 = 201.4$$

$$\begin{aligned} \therefore d &= \sqrt{\frac{M}{R_b}} \\ &= \sqrt{\frac{72.27 \times 12000}{201.4 \times 10}} \\ &= 20.75 \text{ inch} \end{aligned}$$

$$\begin{aligned} \therefore \text{Required depth} &= (20.75 + 2.5) \text{ in} \\ &= 23.25 \text{ in} \end{aligned}$$

Hence, Assumed depth > Required depth

$\therefore$  Design is OK

Step-04

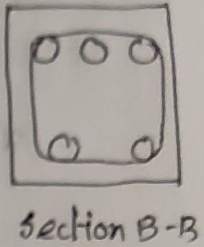
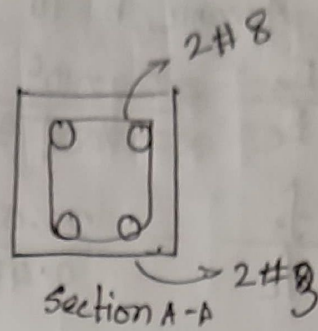
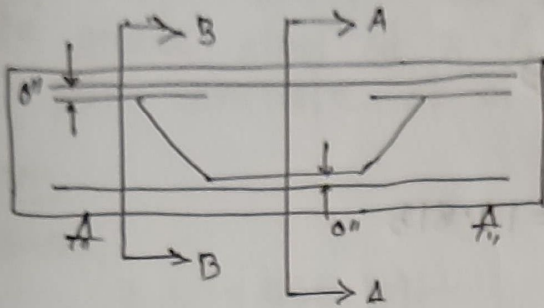
$$\text{Now, } A_s = \frac{M}{f_s j d}$$

$$\left[ \text{effective depth, } d = (25 - 2.5) = 22.5 \text{ in} \right]$$

$$\begin{aligned} \text{for compression, } A_{st} &= \frac{72.27 \times 12000}{0.4 \times 60000 \times 0.888 \times 20.75} \\ &= 1.81 \text{ in}^2 = 3\#8 \end{aligned}$$

for tension,

$$A_s = \frac{8.04 \times 12080}{0.24 \times 60,000 \times 0.88 \times 22.5} = 0.20 \text{ in}^2 \quad 2\#3$$



## Design

Problem 02: Determine the dimension and reinforcement of a tangular beam of moment capacity 100 K-ft ( $f_s = 20,000$  psi,  $f_c = 3000$  psi)

Solution: Assume  $b = 10$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \sqrt{3000}} = 9$$

$$r = \frac{f_s}{f_c} = \frac{20,000}{0.45 \times 3000} = 14.815$$

$$k = \frac{n}{n+r} = \frac{9}{9+14.815} = 0.378$$

$$j = 1 - \frac{k}{3} = 0.874$$

$$R = \frac{1}{2} f_c j k = \frac{1}{2} \times (0.45 \times 3000) \times 0.874 \times 0.378 = 222.94$$

$$d_{req} = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{100 \times 12000}{222.94 \times 10}} = 23.2 \text{ in}$$

Now,

$$A_{st} = \frac{M}{f_s j d} = \frac{100 \times 12000}{20,000 \times 0.874 \times 23.2} = 2.96 \text{ in}^2$$

= 3 # 9 bars Ans.

## Design

Problem-03: A rectangular beam is to carry a uniformly distributed live load of 680 plf and support the dead load of a wall weighing 380 plf in addition to its own weight on a simple span of 24 ft. Design the beam for flexure using intermediate grade steel at a working stress of 20,000 psi and 3000 psi concrete at a working stress of 1350 psi

Solution:

Given,

$$L.L = 680 \text{ ; plf}$$

$$D.L = 380 \text{ ; plf}$$

$$f_s = 20,000 \text{ psi}$$

$$f_c' = 3000 \text{ psi}$$

$$f_c = 1350 \text{ psi}$$

$$L = 24 \text{ ft}$$

(i)

Assume depth,  $h = 24 \text{ in}$

width,  $b = 12 \text{ in}$

$$\text{Total load} = (680 + 380 + \frac{24 \times 12}{144} \times 150) = 1360 \text{ plf} = 1.36 \text{ klf}$$

(ii)

For simply supported beam,  $M_{max} = \frac{WL^2}{8} = \frac{1.36 \times 24^2}{8} = 97.92 \text{ K-ft}$

(iii)

Now,

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \times 3000} = 9$$

$$r = \frac{f_s}{f_c} = \frac{20,000}{1350} = 14.81$$

$$k = \frac{n}{n+r} = \frac{9}{9+14.81} = 0.378$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.378}{3} = 0.874$$

$$R = \frac{1}{2} f_c j k$$

$$= \frac{1}{2} (1350 \times 0.874 \times 0.378)$$

$$= 223$$

$$\therefore d = \sqrt{\frac{M}{Rb}}$$

$$= \sqrt{\frac{97.92 \times 1000 \times 12}{223 \times 12}}$$

$$= 20.95 \text{ in}$$

$$\therefore \text{Require depth} = (20.95 + 2.5) = 23.45 \text{ in}$$

$\therefore$  Assume depth  $>$  require depth

$\therefore$  design OK

(iv)

Now,

$$A_s = \frac{M}{f_s j d}$$

effective,  $d = (24 - 2.5)$   
 $= 21.5$

$$= \frac{97.92 \times 1000 \times 12}{20,000 \times 0.874 \times 21.5}$$

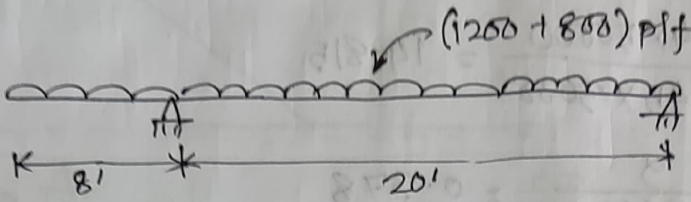
$$= 3.13 \text{ in}^2$$

$$= 4\#9$$

Design

check depth

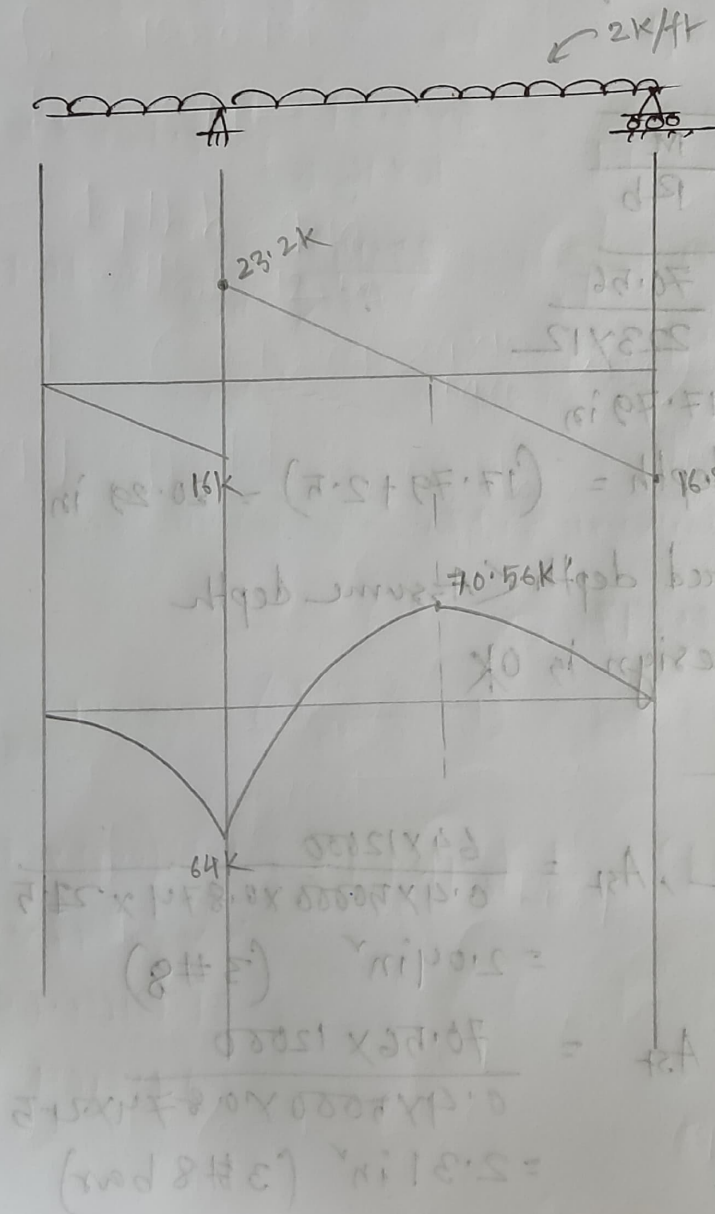
Problem: Design a over hanging beam in figure to support a live load of 1200 plf and a dead load of 800 plf including a its self weight using  $f_c' = 3000 \text{ psi}$ ,  $f_y = 5000 \text{ psi}$ .



Solution:

Assuming. depth,  $h = 24 \text{ in}$   
width,  $b = 12 \text{ in}$

Total load =  $(1200 + 800) \text{ plf} = 2000 \text{ plf} = 2 \text{ K/ft}$



for compression force  $A_s = \frac{M}{f_y d}$   
for tension force  $A_s = \frac{M}{f_y d}$

Now, depth check

$$m = \frac{E_s}{E_c} = \frac{29 \times 10^9}{57000 \sqrt{3000}} = 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4 \times 50000}{0.45 \times 3000} = 14.815$$

$$k = \frac{m}{m+r} = \frac{9}{9+14.815} = 0.378$$

$$j = 1 - k/3 = 1 - 0.378/3 = 0.874$$

$$R = \frac{1}{2} f_c j k$$
$$= \frac{1}{2} \times (0.45 \times 3000) \times 0.874 \times 0.378$$
$$= 223$$

$$A_3 \quad d_{req} = \sqrt{\frac{M}{R_b}}$$
$$= \sqrt{\frac{70.56}{223 \times 12}}$$
$$= 17.79 \text{ in}$$

$$\therefore \text{required depth} = (17.79 + 2.5) = 20.29 \text{ in}$$

$\therefore$  Required depth < Assume depth

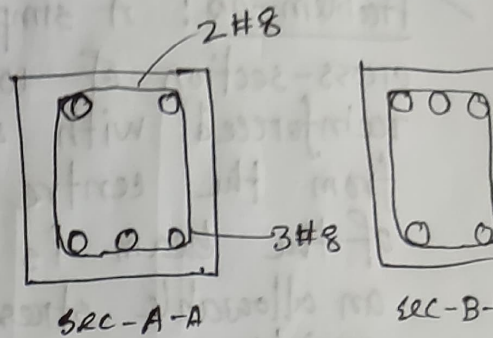
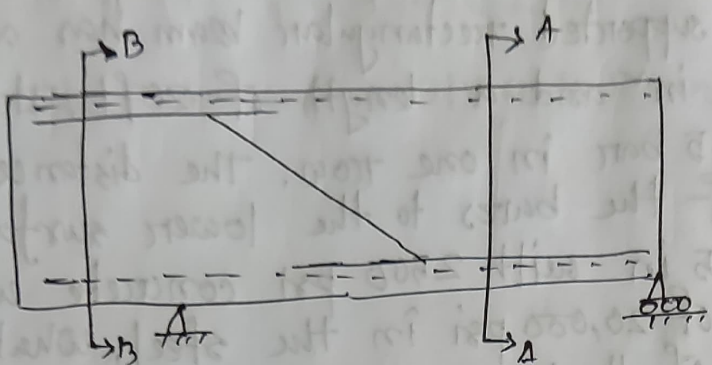
$\therefore$  Design is OK

Now,

$$A_s = \frac{M}{f_s j d}$$

For compression zone,  $A_{st} = \frac{64 \times 12000}{0.4 \times 50000 \times 0.874 \times 24.5}$ 
$$= 2.04 \text{ in}^2 \quad (3 \# 8)$$

For tension zone,  $A_{st} = \frac{70.56 \times 12000}{0.4 \times 50000 \times 0.874 \times 24.5}$ 
$$= 2.31 \text{ in}^2 \quad (3 \# 8 \text{ bar})$$



$$L = 50 \text{ ft}$$

$$q = (16 \cdot 5) = 80 \text{ psf}$$

$$N = 16$$

$$b = 10$$

$$f_c = 5000$$

$$f_s = 60000$$

$$m = \frac{E_s}{E_c} = \frac{29000000}{5000} = 5800$$

$$k = \sqrt{\frac{12EI}{L^3}}$$

$$I = 1 - \frac{1}{3} k^2 = 1 - \frac{1}{3} (0.314)^2 = 0.881$$

$$M_o = R_p q L = \frac{1}{2} (16 \cdot 5) (50) = 2000 \text{ ft-lb}$$

## Analysis

Problem-05: A simply supported rectangular beam has a cross-section of  $10 \times 16$  in<sup>2</sup> and a length of 20 ft. It is reinforced with 4 #5 bars in one row. The distance from the centre of the bars to the lower surface of the beam is 2.5 in with 2500 psi concrete and an allowable stress of 20,000 psi in the steel, what is resisting moment of the beam.

Solution:

$$b = 10''$$

$$h = 16''$$

$$d = (16 - 2 \cdot 5) = 13.5''$$

$$L = 20 \text{ ft}$$

$$f_c' = 2500$$

$$f_s = 20,000$$

$$\text{Steel ratio, } \rho = \frac{A_s}{bd} = \frac{4 \times 0.31}{10 \times 13.5} = 9.185 \times 10^{-3}$$

$$n = \frac{f_s}{E_c} = \frac{29 \times 10^9}{57500 \sqrt{2500}} = 10.175 \approx 10$$

$$\therefore k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$= \sqrt{(9.185 \times 10^{-3} \times 10)^2 + 2 \times (9.185 \times 10^{-3} \times 10)} - (9.185 \times 10^{-3})$$

$$= 0.347$$

$$j = 1 - \frac{k}{3} = \left(1 - \frac{0.347}{3}\right) = 0.884$$

Now,

$$M_c = Rbd^2 = \frac{1}{2} f_c' j k b d^2 = \frac{1}{2} \times (0.45 \times 2500) \times 0.884$$

$$\times 0.347 \times 10 \times 13.5^2$$

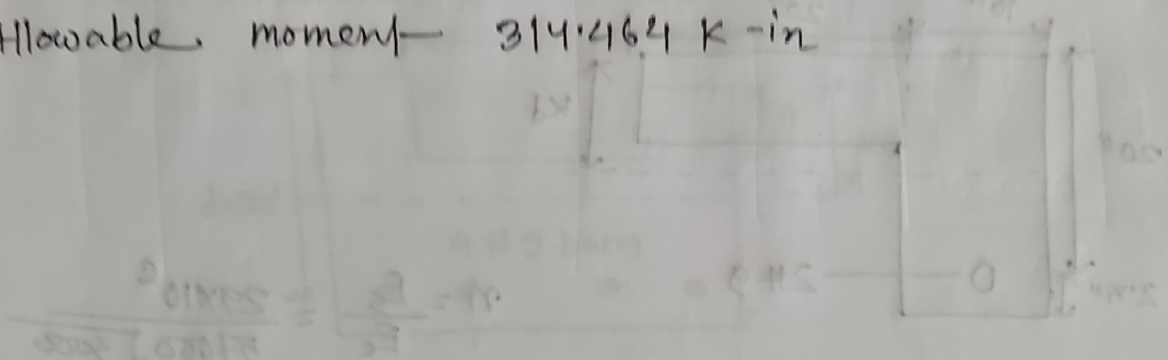
$$= 314464.63 \text{ lb-in}$$

$$= 314.464 \text{ k-in}$$

$$M_s = A_s f_s j d = (4 \times 0.31) \times 20,000 \times 0.884 \times 13.5$$

$$= 295.96 \text{ k-in}$$

Allowable moment — 314.464 K-in



$$Q_1 = \frac{10 \times k_9 \times \frac{k_9}{2} + 20 \times k_9 \times \left(k_9 + \frac{k_9}{2}\right)}{2} = 10 \times k_9 \times \frac{k_9}{2} + 20 \times k_9 \times \left(k_9 + \frac{k_9}{2}\right)$$

$$Q_2 = \frac{2 \times 5 \times (9 - k_9) \times (1 + 5) \times \frac{k_9}{2}}{2} = (9 - k_9) \times 15 \times k_9$$

$$Q_1 = Q_2$$

$$10 \times k_9 \times \frac{k_9}{2} + 20 \times k_9 \times \left(k_9 + \frac{k_9}{2}\right) = (9 - k_9) \times 15 \times k_9$$

$$5k_9^2 + 40k_9^2 + 20k_9^2 = 135k_9 - 15k_9^2$$

$$65k_9^2 - 135k_9 = 0$$

$$k_9(65k_9 - 135) = 0$$

$$k_9 = \frac{135}{65} = 2.077$$

$$M_s = \frac{1}{2} \times 10 \times k_9 \times \frac{k_9}{2} \times (1 + 5) \times \frac{k_9}{2} + 20 \times k_9 \times \left(k_9 + \frac{k_9}{2}\right) \times \frac{k_9}{2}$$

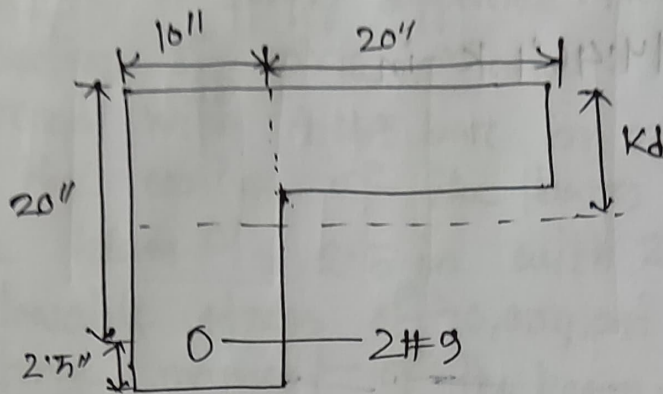
$$= \frac{1}{2} \times 10 \times 2.077 \times \frac{2.077}{2} \times (1 + 5) \times \frac{2.077}{2} + 20 \times 2.077 \times \left(2.077 + \frac{2.077}{2}\right) \times \frac{2.077}{2}$$

$$= 820.88 \text{ k-in}$$

Moment capacity = 1278.25 k-in

# Analysis

Problem-06: Determine the moment capacity of the beam



$$\eta = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \sqrt{3000}} = 9$$

Solution:

$$Q_c = 10 \times kd \times \frac{kd}{2} + 20 \times 4 \times \left(kd - \frac{4}{2}\right) = 5(kd)^2 + 80kd - 160$$

$$Q_T = n A_s (d - kd) = \frac{E_s}{E_c} \times (2 \times 1) \times (d - kd)$$

$$= 9 \times 2 \times (d - kd) = 18d - 18kd$$

Now,  $Q_c = Q_T$

$$5(kd)^2 + 80kd - 160 = 18d - 18kd$$

$$\Rightarrow 5(kd)^2 + 98kd - 160 = 0$$

$$\Rightarrow kd = 4.34$$

$$k = \frac{4.34}{20} = 0.217$$

*Handwritten notes:  $J = 1 - k/3 = (1 - \frac{0.217}{3}) = 0.928$*

$$J = 1 - \frac{k}{3} = \left(1 - \frac{0.217}{3}\right) = 0.928$$

*Handwritten notes:  $M_c = \frac{1}{2} f_c J k b d^2 = \frac{1}{2} \times (0.45 \times 3000) \times 0.928 \times 0.217 \times 10^3$*

$$M_c = \frac{1}{2} f_c J k b d^2 = \frac{1}{2} \times (0.45 \times 3000) \times 0.928 \times 0.217 \times 10^3$$
$$= 543715.2 \text{ lb-in.}$$

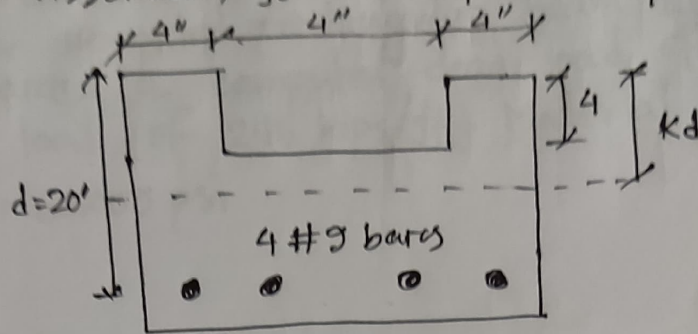
$$= 543.72 \text{ K-in} \quad \underline{\text{Ans.}}$$

$$M_s = A_s f_s j d = 2 \times (0.4 \times 60,000) \times 0.928 \times 20$$
$$= 890.88 \text{ K-in}$$

Neutral section  
↑  
10"

Analysis

Problem-07: Calculate the working moment capacity of the section given below. Assume,  $f_c' = 3000 \text{ psi}$  &  $f_y = 60 \text{ ksi}$



$$Q_c = 12 \times kd \times \frac{kd}{2} - (4 \times 4) \times (kd - 4/2)$$

$$= 6(kd)^2 - 16(kd - 2)$$

$$Q_T = n A_{st} (d - kd)$$

$$= 9 \times (4 \times 1) \times (20 - kd)$$

At equilibrium

$$Q_c = Q_T$$

$$6(kd)^2 - 16kd + 32 = 720 - 36kd$$

$$\Rightarrow 6(kd)^2 + 20kd - 688 = 0$$

$$\Rightarrow kd = 9.17$$

$$k = \frac{9.17}{20} = 0.4585$$

$$j = (1 - k/3) = 0.847$$

$$M_c = \frac{1}{2} f_c' j k b d^2 = \frac{1}{2} \times (0.4585 \times 3000) \times 0.847 \times 0.4585 \times 20^2$$

$$= 1258.25 \text{ K-in}$$

$$M_s = A_s f_y j d = 4 \times (60,000 \times 0.4) \times 0.847 \times 20$$

$$= 1626.24 \text{ Ki}$$

$\therefore$  Moment capacity = 1258.25 K-in Ans.

$$n = \frac{E_s}{E_c}$$

$$= \frac{29 \times 10^9}{57000 \sqrt{3000}}$$

$$= 9$$

## Singly Reinforced Beam (USD)

### Problem-01

Find the cross section of the concrete and area of steel required for a simply supported rectangular beam with a span of 15 ft that is to carry a computed dead load of 1.27 kips/ft and a service live load of 2.15 kips/ft. Material strength are  $f'_c = 4000$  psi and  $f_y = 60,000$  psi

### Solution:

$$W_u = 1.2 \text{ D.L} + 1.6 \text{ C.L.L.}$$
$$= (1.2 * 1.27 + 1.6 * 2.15) = 4.96 \text{ K/ft}$$

$$M_u = \frac{W_u L^2}{8} = \frac{4.96 * (15)^2}{8} = 139.5 \text{ Kft}$$

Now,

$$M_u = \phi \rho b d^2 f_y \left(1 - \frac{\rho}{\alpha} \frac{f_y}{f'_c}\right)$$

$$139.5 * 12000 = 0.9 * 0.018 * 10 * d^2$$
$$\times \left(1 - 0.59 \frac{0.018 * 60,000}{4000}\right)$$

$$d^2 = 204.85$$

$$d = 14.31 \text{ in}$$

required total depth,  $h = (14.31 + 2.5)$   
 $\approx 17 \text{ in}$

$$\rho = 0.85 \beta_1 \frac{f'_c}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

$$\rho = 0.85 * 0.85 * \frac{4000}{60,000} * \frac{0.003}{0.003 + 0.002}$$
$$= 0.018$$

$$\frac{\rho}{\alpha} = 0.59$$

Now,

$$A_s = \frac{M_u}{\phi f_y (d - a/2)}$$

$$A_s = \frac{139.5 * 12000}{0.9 * 60,000 \left(14.31 - \frac{1.7647 A_s}{2}\right)}$$

$$A_s = 2.16 \text{ inch} \quad (3 \# 9)$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$= \frac{60,000 A_s}{0.85 * 4000 * 10}$$

$$= 1.7647 A_s$$

Ans.

## Analysis

Problem-02: A rectangular beam has width 12 in and effective depth 17.5 in. It is reinforced with 4#9 bar in one row.  $f_y = 60,000$  psi and  $f_c' = 4000$  psi, what is the normal strength and what is the maximum moment that can be utilized in design according to ACI code.

Solution:

$$\rho = \frac{A_s}{bd} = \frac{4 \times 1}{12 \times 17.5} = 0.019$$

$$\begin{aligned} \rho_b &= 0.85 \rho_1 \frac{f_c'}{f_y} \times \frac{E_u}{E_u + E_s} \\ &= 0.85 \times 0.85 \times \frac{4000}{60,000} \times \frac{0.003}{0.003 + 0.002} \\ &= 0.0289 \end{aligned}$$

$\rho < \rho_b$ , it is under-reinforced beam

Now,

$$\begin{aligned} M_n &= A_s f_y \left( d - \frac{a}{2} \right) & a &= \frac{A_s f_y}{0.85 f_c' b} \\ &= 4 \times 60,000 \left( 17.5 - \frac{5.88}{2} \right) & &= \frac{4 \times 60,000}{0.85 \times 4000 \times 12} \\ &= 3494400 \text{ lb-in} & &= 5.88 \end{aligned}$$

$$\text{So, } M_u = \phi M_n$$

$$= 0.87 \times 3494400$$

$$= 3040280 \text{ lb-in}$$

Ans,

$$\phi = 0.483 + 83.3 \epsilon_t$$

$$\epsilon_t = \epsilon_u \left( \frac{d-c}{c} \right)$$

$$= 0.003 \times \frac{17.5 - \frac{a}{\rho_1}}{\frac{a}{\rho_1}}$$

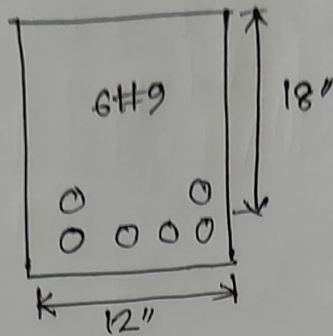
$$= 0.003 \times \frac{17.5 - \frac{5.88}{0.85}}{5.88}$$

$$\epsilon_t = 0.0046$$

$$\begin{aligned} \phi &= 0.483 + 83.3 \times 0.0046 \\ &= 0.87 \end{aligned}$$

Analysis

Problem: 03 A beam section is shown in figure below. Calculate nominal and ultimate moment.



$f_c' = 3 \text{ ksi}$   
 $f_y = 60 \text{ ksi}$

Solution:

$$\rho = \frac{A_s}{bd} = \frac{(6 \times 1)}{12 \times 18} = 0.0278$$

$$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{E_u}{E_u + E_s}$$

$$= 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.002}$$

$$= 0.0217$$

$\rho > \rho_b$ . So it is over reinforced beam.

We know,

$$M_n = A_s f_s \left( d - \frac{a}{2} \right)$$

$$= 6 \times 49649.21 \times \left( 18 - \frac{9.74}{2} \right)$$

$$= 3911364.761 \text{ lb-in}$$

Ans.

$$M_u = \phi M_n$$

$$\phi = 0.483 + 83.3 \epsilon_t$$

$$\epsilon_t = \epsilon_u \times \left( \frac{d-c}{c} \right)$$

$$= 0.003 \times \left( \frac{18 - 11.46}{11.46} \right)$$

$$= 0.0017$$

$$\therefore \phi = 0.483 + 83.3 \times 0.0017$$

$$= 0.625$$

$$M_u = 2444602.97 \text{ lb-in}$$

Ans.

$$f_s = E_s \epsilon_u \left( \frac{d-c}{c} \right)$$

$$c = \frac{a}{\beta_1}$$

At equilibrium

$$0.85 \times f_c' \beta_1 c b = A_s f_s$$

$$0.85 \times 3000 \times 0.85 \times c \times 12 = 6 \times f_s$$

$$24480c = 6 \times 29 \times 10^6 \times 0.003 \times \frac{18-c}{c}$$

$$c^2 = 21323.53 (18-c)$$

$$c = 11.46$$

$$a = c \beta_1 = 11.46 \times 0.85 = 9.74$$

$$\therefore f_s = 29 \times 10^6 \times 0.003 \times \left( \frac{18 - 11.46}{11.46} \right)$$

$$= 49649.21 \text{ psi}$$



# WSD Method

✓ 1. Thickness of slab:  
same as USD

✓ 2. Load calculation

$$\text{T.L.} = \text{D.L.} + \text{L.L.}$$

$$\bullet \left[ \text{self load} = \frac{t}{12} \times 150 \text{ psf} \right]$$

✓ 3. Moment calculation:

[from BMD Diagram]

✓ 4. Depth check

$$= n = \frac{E_s}{E_c}$$

$$= r = \frac{f_s}{f_c}$$

$$= k = \frac{n}{n+r}; \quad j = 1 - k/3$$

$$= R = 1/2 f_c j k$$

$$= d = \sqrt{\frac{M}{Rb}}$$

$$\approx \text{tr req} = d + c + \frac{d_{\text{sig}}}{2}$$

→ distribution rod

effective depth (A<sub>s</sub> dep = 1)

∴ tr req < Assume depth

design OK

✓ 5. steel area calculation:

$$A_s = \frac{M}{f_s j d} \quad f_s = 0.8 f_y$$

✓ 6. Distribution reinforcement  
same as USD

✓ 7. Shear check

$$V_w = 1.15 \frac{WL}{2} - \frac{Wd}{12}$$

$$V_{all} = 1.1 \sqrt{f_c'} b d$$

∴  $V_w < V_{all}$  design

✓ 8. Bond check

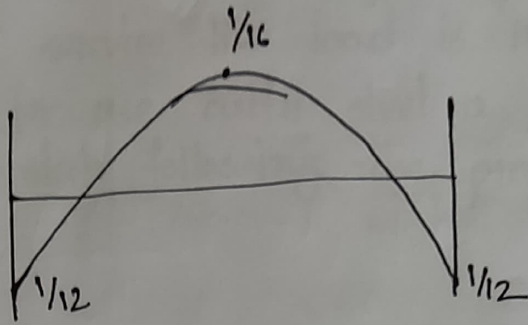
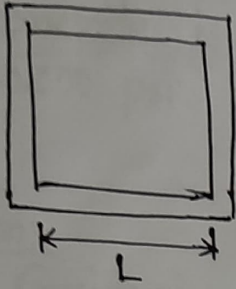
$$U_{dev} = \frac{V_{max}}{E_o j d}$$

$$U_{all} = \frac{3.4 \sqrt{f_c'}}{D}$$

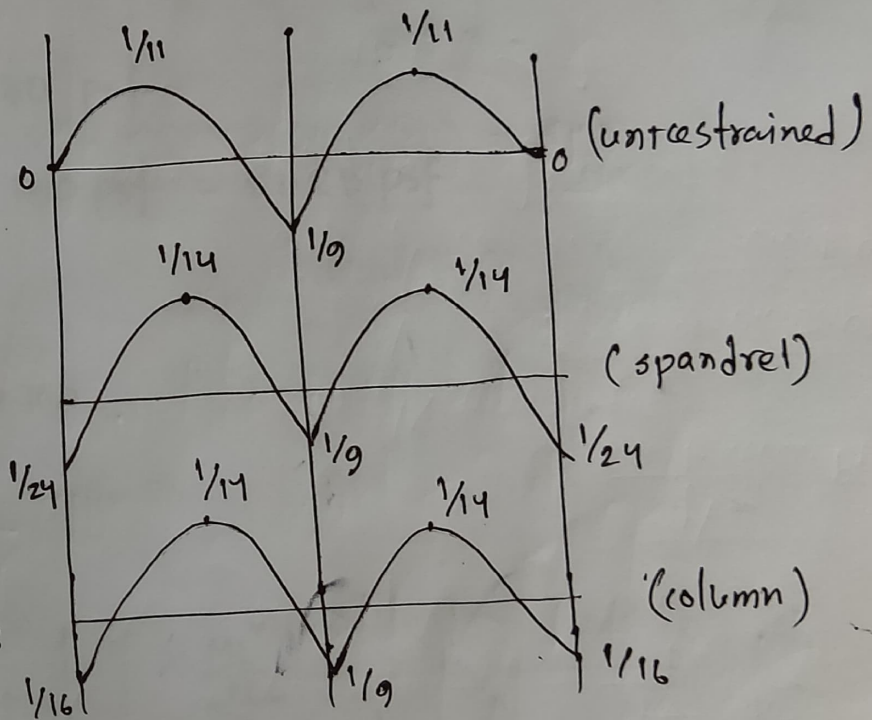
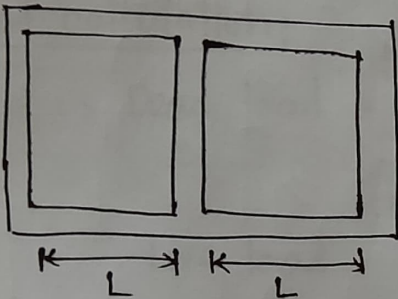
$U_{dev} < U_{all}$

design OK

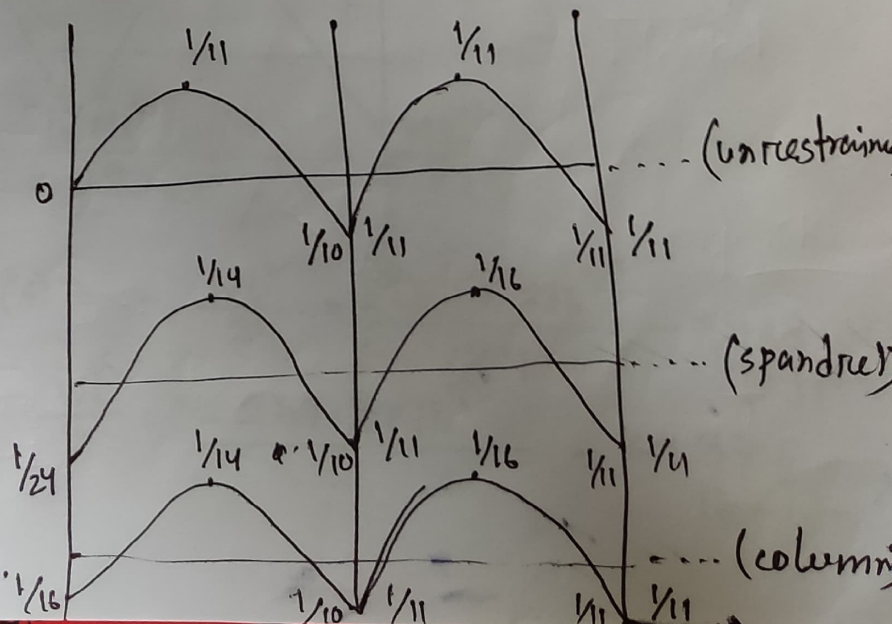
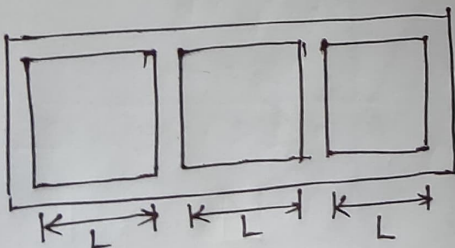
Beam with one span:



Beam with two spans only:



Beam with more than two span:



WSD  
Problem-01: A reinforced concrete slab is built integrally with its appearance support and consists of two equal spans, each with a clear span of 15 ft. The service live load is 100 psf and 4000 psi concrete is specified for use with steel a yield stress equal to 60,000 psi. Design the slab following the provision of ACI code.

Solution:

Considering the slab is both end continuous,

$$\therefore t = \frac{l}{28} = \frac{15 \times 12}{28} = 6.43 \text{ in} \approx 6.5 \text{ in}$$

Load calculation:

$$\text{Dead load} = \frac{t}{12} \times 150 \text{ psf}$$

$$= \frac{6.5}{12} \times 150 \text{ psf} = 81.25 \text{ psf}$$

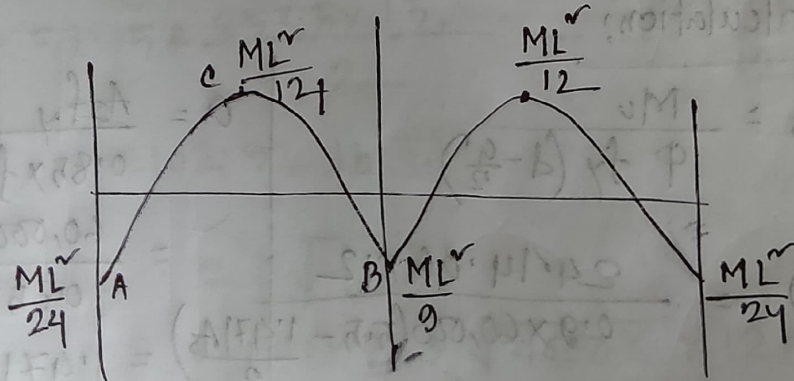
$$\text{Live load} = 100 \text{ psf}$$

$$L.L = 1.2 D.L + 1.6 L.L$$

$$= 1.2 * 81.25 + 1.6 * 100$$

$$\therefore W_u = 257.5 \text{ psf}$$

Moment calculation:



$$\therefore \text{At exterior support (A)} = - \frac{W L^2}{24} = - \frac{257.5 \times 15^2}{24} = - 2414.06 \text{ lb-ft}$$

$$\therefore \text{At mid span (B) interior support} = - \frac{W L^2}{9} = - \frac{257.5 \times 15^2}{9} = - 6937.5 \text{ lb-ft}$$

At interior support (0) =  $-\frac{wL^2}{12} = -\frac{257.5 \times 15^2}{12}$   
 mid span  
 $= -4828.125$   
 $= 4138.39 \text{ lb}$

4. Depth check:

$$M_u = \phi \rho b d^2 f_y \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$\rho = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

$$= 0.85 \times 0.85 \times \frac{4000}{60,000} \times \frac{0.003}{0.003 + 0.005}$$

$$= 0.018$$

$$\frac{6437.5}{12} = 0.9 \times 0.018 \times 12 \times d^2 \times 60,000 \left(1 - 0.5 \frac{0.018 \times 60,000}{4000}\right)$$

$$d_{req} = 2.76 \text{ inch}$$

effective thickness,  $t_{eff} = (6.5 - 1) = 5.5$ .

$d_{req} < t_{eff}$ . Design OK

5. Steel calculation:

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

$$a = \frac{A_s f_y}{0.85 \times f_c' \times b}$$

$$= \frac{60,000 \times A_s}{0.85 \times 4000}$$

$$= 1.47 A_s$$

$$A_s \text{ (at exterior)} = \frac{2414.06 \times 12}{0.9 \times 60,000 \left(5.5 - \frac{1.47 A_s}{2}\right)}$$

$$A_s = 0.1 \text{ in}^2$$

providing #3 bar @  $= \frac{0.11 \times 12}{0.1} = 13.2 \text{ " c/c}$

$$A_s(\text{mid span}) = \frac{4138.39}{0.9 \times 60,000 \times \left(5.5 - \frac{1.471 A_s}{2}\right)}$$

$$A_s = 0.17 \text{ in}^2$$

$$\text{providing } \#3 \text{ bar } @ = \frac{0.11 \times 12}{0.17} = 7.76'' \text{ c/c}$$

$$A_s(\text{interior span}) = \frac{6437.5 \times 12}{0.9 \times 60,000 \times \left(5.5 - \frac{1.471 A_s}{2}\right)}$$

$$A_s = 0.26 \text{ in}^2$$

$$\text{providing } \#3 \text{ bar } @ = \frac{0.11 \times 12}{0.26} = 5.07'' \text{ c/c}$$

### 8. Distribution reinforcement:

$$A_s(\text{min}) = 0.0018 b t$$

$$= 0.0018 * 12 * 6.5$$

$$= 0.14$$

$$\text{providing } \#3 \text{ bar } @ = \frac{0.11 \times 12}{0.14} = 9.42'' \text{ c/c}$$

### 7.1 shear check:

$$V_u = 1.15 * \frac{wL}{2} - \frac{w d}{12}$$

$$= 1.15 * \frac{257.5 \times 15}{2} - \frac{257.5 \times 5.5}{12}$$

$$= 2102.92 \text{ lb}$$

$$V_{\text{allowable}} = 2 \phi \sqrt{f_c'} b d$$

$$= 2 \times 0.75 \times \sqrt{4000} \times 12 \times 5.5$$

$$= 6261.31 \text{ lb}$$

$$\therefore V_u < V_{\text{all}}$$

design is OK

8) Bond check:

$$U_{dev} = \frac{V_{max}}{E_o (d - \alpha/2)}$$

$$= \frac{1931.25}{2.79 \times (5.5 - \frac{0.38}{2})}$$

$$= 130.36 \text{ lb/in}$$

$$U_{au} = \frac{6.7 \sqrt{f_c'}}{D}$$

$$= \frac{6.7 \times \sqrt{4080}}{0.374}$$

$$= 1129.99$$

$$V_{max} = \frac{WL}{2} = \frac{257.5 \times 2}{2}$$

$$= 1931.25$$

$$E_o = n \pi D$$

$$= \frac{b}{\text{spacing}} \times \pi \times D$$

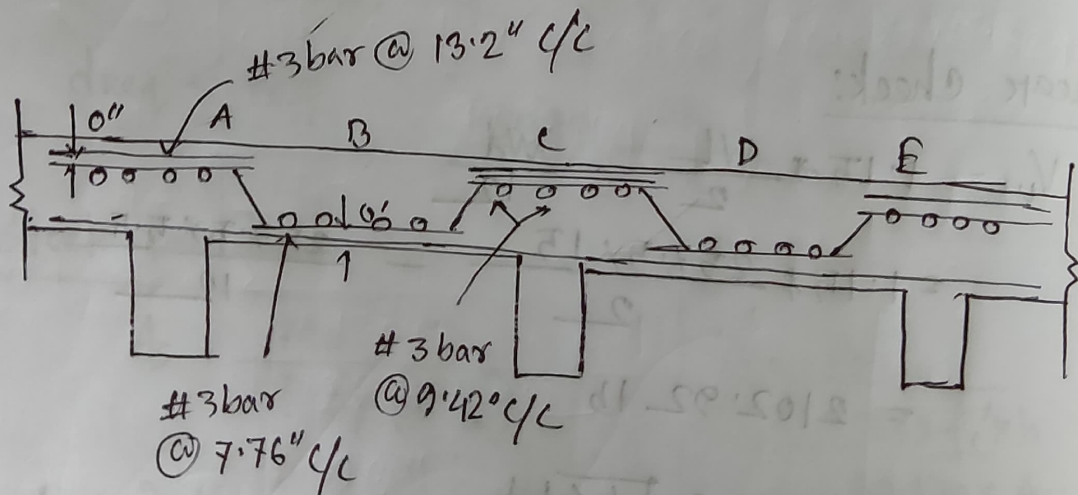
$$= \frac{12}{5.07} \times \pi \times 0.3$$

$$= 2.79$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.26 \times 60,000}{0.85 \times 4}$$

$$= 0.38$$

$U_{dev} < U_{au}$  Hence, Design is OK Am.



At support - A and E,

At support C,

WSD  
Problem-02: A reinforced concrete slab is built integrally with its appearance support and consists of three equal spans, each span of 11 ft and the working line load is 110 psf. If  $f'_c = 3000$  psi,  $f_y = 50,000$  psi. Design the slab using WSD.

Solution:

Considering the slab is both end continuous

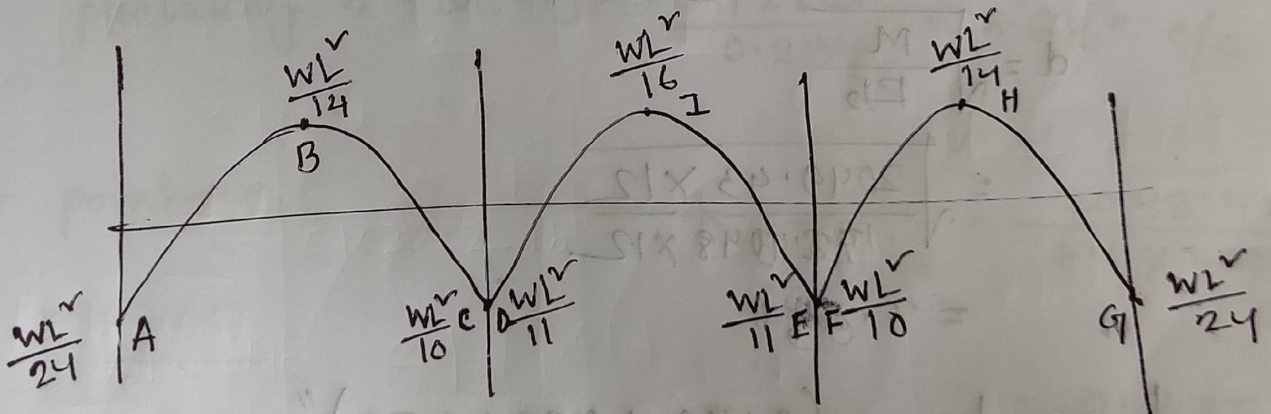
$$\therefore t = l/28 = \frac{11 \times 12}{28} = 4.71 \text{ ft} \approx 4.75 \text{ ft}$$

Load calculation:

$$\begin{aligned} \text{Dead load} &= \frac{t}{12} \times 150 \text{ psf} \\ &= 59.375 \text{ psf} \end{aligned}$$

$$\begin{aligned} \text{T.L} &= 59.375 + 110 \\ &= 169.375 \text{ psf} \end{aligned}$$

Moment calculation:



$$\begin{aligned} \text{Moment at point A, G} &= -\frac{WL^2}{24} = \frac{169.375 \times 11^2}{24} \\ &= 853.93 \text{ lb-ft} \end{aligned}$$

$$\begin{aligned} \text{Moment at point B, H} &= +\frac{WL^2}{14} = \frac{169.375 \times 11^2}{14} \\ &= 1463.88 \text{ lb-ft} \end{aligned}$$

$$\text{Moment at point J} = + \frac{wL^2}{16} = 1280.89 \text{ lb-ft}$$

$$\text{Moment at point C, F} = - \frac{wL^2}{10} = 2049.43 \text{ lb-ft}$$

$$\text{Moment at point D, E} = - \frac{wL^2}{11} = 1863.125 \text{ lb-ft}$$

Depth check

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000 \sqrt{3000}} = 9$$

$$r = \frac{f_s}{f_c} = \frac{0.47 \times 50000}{0.47 \times 3000} = 14.81$$

$$k = \frac{n}{n+r} = \frac{9}{9+14.81} = 0.378$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.378}{3} = 0.874$$

$$R = \frac{1}{2} f_c j k = \frac{1}{2} \times (0.47 \times 3000) \times 0.874 \times 0.378 \\ = 223.1045$$

$$d = \sqrt{\frac{M}{Rb}}$$

$$= \sqrt{\frac{2049.43 \times 12}{223.1045 \times 12}}$$

$$= 3.03''$$

$$d_{req} = t_{req} = (3.03 + 0.25 + 0.75)'' = 4.03''$$

$$t_{req} < t_{assume}$$

Design OK

$$d_{effective} = (4.75 - 1) = 3.75''$$

## steel calculation

$$\text{At point A, G} \longrightarrow A_s = \frac{M}{f_s j d} = \frac{853.93 \times 12}{(0.4 \times 50,000) \times 0.875 \times 3.75}$$
$$= 0.156 \text{ in}^2$$

$$\text{providing \# 3 bar @} = \frac{0.11 \times 12}{0.156} = 8.46 \text{ " c/c}$$

$$\text{At point B, H} \longrightarrow A_s = \frac{M}{f_s j d} = \frac{1463.88 \times 12}{(0.4 \times 50,000) \times 0.875 \times 3.75}$$

$$= 0.268 \text{ in}^2$$
$$\text{providing \# 3 bar @} = \frac{0.11 \times 12}{0.268} = 4.92 \text{ " c/c}$$

$$\text{At point I} \longrightarrow A_s = \frac{M}{f_s j d} = \frac{1289.89 \times 12}{0.4 \times 50,000 \times 0.875 \times 3.75}$$

$$= 0.234 \text{ in}^2$$
$$\text{providing \# 3 bar @} = \frac{0.11 \times 12}{0.234} = 5.64 \text{ " c/c}$$

$$\text{At point C, F} \longrightarrow A_s = \frac{M}{f_s j d} = \frac{2049.43 \times 12}{0.4 \times 50,000 \times 0.875 \times 3.75}$$

$$= 0.375 \text{ in}^2$$
$$\text{providing \# 3 bar @} = \frac{0.11 \times 12}{0.375} = 3.52 \text{ " c/c}$$

$$\text{At point D, E} \longrightarrow A_s = \frac{M}{f_s j d} = \frac{1863.125 \times 12}{0.4 \times 50,000 \times 0.875 \times 3.75}$$

$$= 0.34 \text{ in}^2$$
$$\text{providing \# 3 bar @} = \frac{0.11 \times 12}{0.34} = 3.88 \text{ " c/c}$$

## Distribution reinforcement

$$f_y = 50,000 \text{ psi}$$

$$\text{So, } A_s = 0.0020bt$$

$$= 0.0020 \times 12 \times 4.75$$

$$= 0.114 \text{ in}^2$$

## shear check

$$V_u = 1.15 \frac{WL}{2} - \frac{Wd}{12}$$

$$= 1.15 \frac{169.375 \times 11}{2} - \frac{169.375 \times 3.75}{12}$$

$$= 1018.36 \text{ lb}$$

$$V_{all} = 1.1 \sqrt{f_c'} bd = 1.1 \times \sqrt{3000} \times 12 \times 3.75$$

$$= 2711.22 \text{ lb}$$

$$V_u < V_{all}$$

[Design OK]

## Bond check

$$U_{dev} = \frac{V_{max}}{C_o j d}$$

$$= \frac{931.56}{7.3899 \times 0.874 \times 3.75}$$

$$= 38.46$$

$$U_{all} = \frac{3.4 \sqrt{f_c'}}{D} = \frac{3.4 \sqrt{3000}}{0.69}$$

$$= 269.89$$

$$U_{all} > U_{dev}$$

[Design OK]

$$V_{max} = \frac{WL}{2}$$

$$= \frac{169.375 \times 11}{2}$$

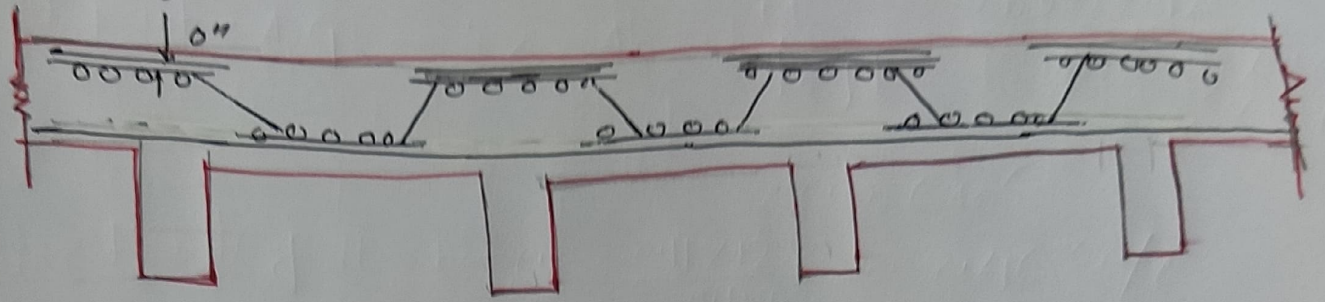
$$= 931.56$$

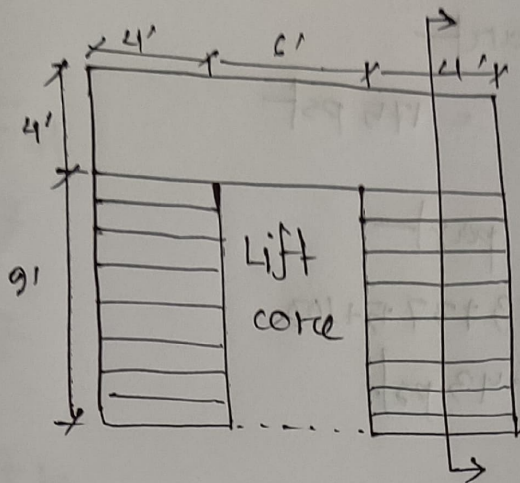
$$C_o = n \pi D$$

$$= \frac{12}{3.14} \times \pi \times 0.69$$

$$= 7.3899$$

Working diagram

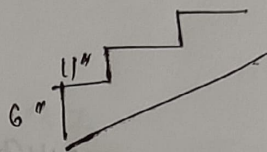




$$LL = 100 \text{ psf}$$

$$f_c' = 3000 \text{ psi}$$

$$f_y = 50,000 \text{ psi}$$



Design the staircase by using WSD method

1. Thickness:

Assume thickness of slab = 6"

2. Load calculation:

$$\text{Dead load} = \frac{t}{12} \times 150$$

$$= \frac{6}{12} \times 150$$

$$= 75 \text{ psf}$$

Live load

$$\text{for top} = \frac{R}{2} = \frac{6/2}{12} = 0.75 \text{ psf}$$

$$\text{for inclined portion} = \frac{75 \times \sqrt{6^2 + 11^2}}{11}$$

$$= 85.43$$

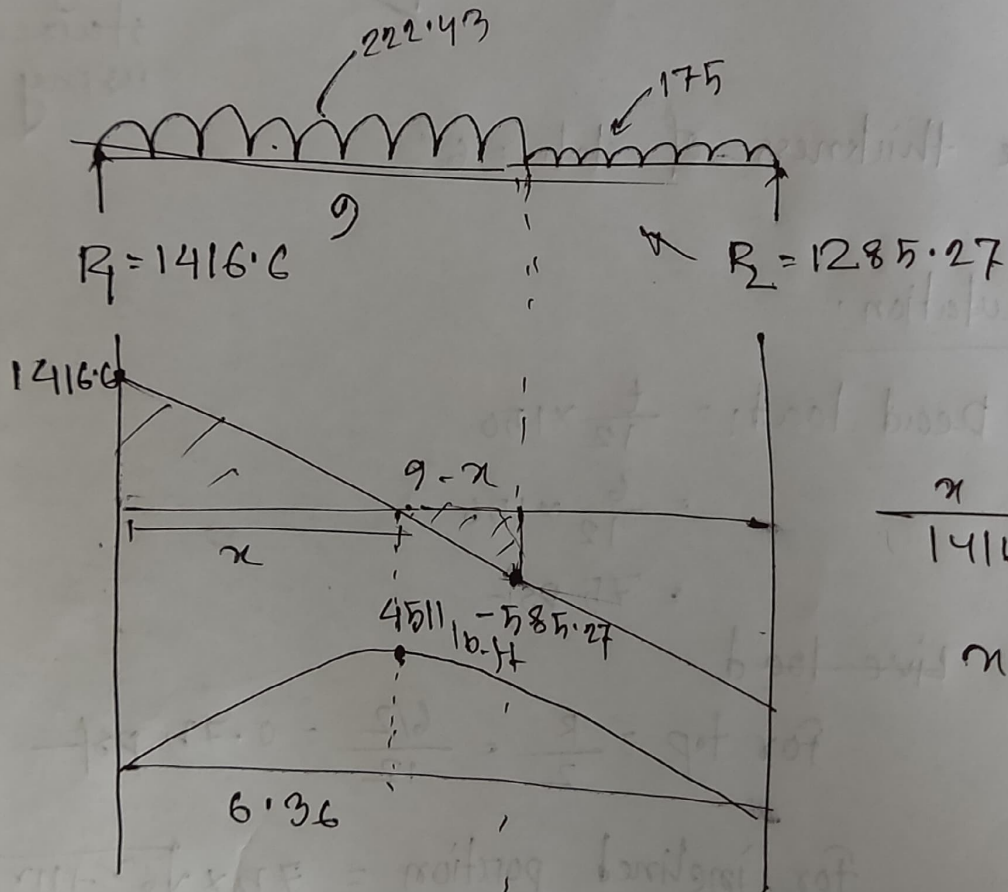
$$\text{load for step} = \frac{6}{2 \times 12} \times 150 = 37.5$$

$$T.L = D.L + L.L$$

=

Total load of straight part  
 $= 75 + 100 = 175 \text{ psf}$

Total load of inclined part  
 $= 85 \cdot 43 + 37 \cdot 5 + 100$   
 $= 222 \cdot 43 \text{ psf}$



$$\frac{x}{1416.6} = \frac{9 - x}{1285.27}$$

$$x = 6.37'$$

$$n = 9 \quad n = \frac{E_s}{E_c} = 9$$

$$r = \frac{f_s}{f_c} = \frac{0.8 \times 50,000}{0.4 \times 3,000}$$

$$= 14.81$$

$$k = \frac{n}{n+y}$$

$$= \frac{9}{9+14.81}$$

$$= 0.37$$

$$j = 0.87$$

$$R = \frac{1}{2} f_c j k$$

$$= \frac{1}{2} \times (0.47 \times 3000) \times 0.87 \times 0.37$$

$$= 223$$

$$d = \sqrt{\frac{M}{R b}}$$

$$= \sqrt{\frac{4511 \times 12}{223 \times 12}}$$

$$= 4.5''$$

$$t = 4.5 + 1 = 5.5'' \quad \text{OK}$$

$$A_s = \frac{M}{f_s j d}$$

$$= \frac{4511 \times 12}{20,080 \times 0.875 \times 5.6}$$

$$= 0.69$$

Distribution:

$$A_s = 0.00206t$$

$$= 0.002 \times 12 \times 6$$

$$= 0.144 \text{ in}^2$$