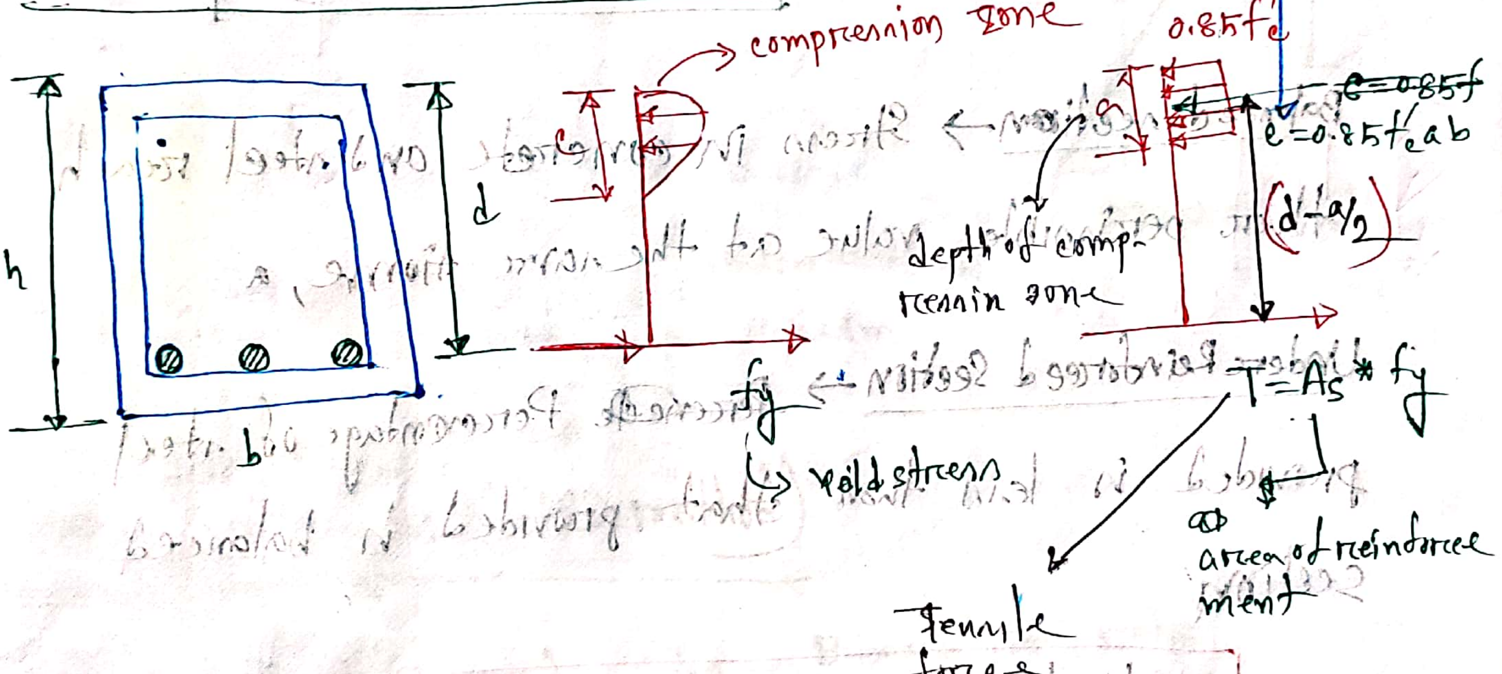
Class lecture · Ahsan habib vai

Lecture-1 Part-1

Beam Analysis (USD method)

Determination of Nominal / Theoretical Moment Strength of Beam.



Force equilibrium condition

$$T = C$$

$$A_s f_y = 0.85 f'_c ab$$

$$\Rightarrow \boxed{a = \frac{A_s f_y}{0.85 f'_c b}}$$

Nominal moment capacity, $\boxed{M_n = T * (d - a/2)}$

$$\boxed{M_n = A_s f_y (d - a/2)}$$

Ultimate moment capacity,

$$Mu = \phi Mn$$

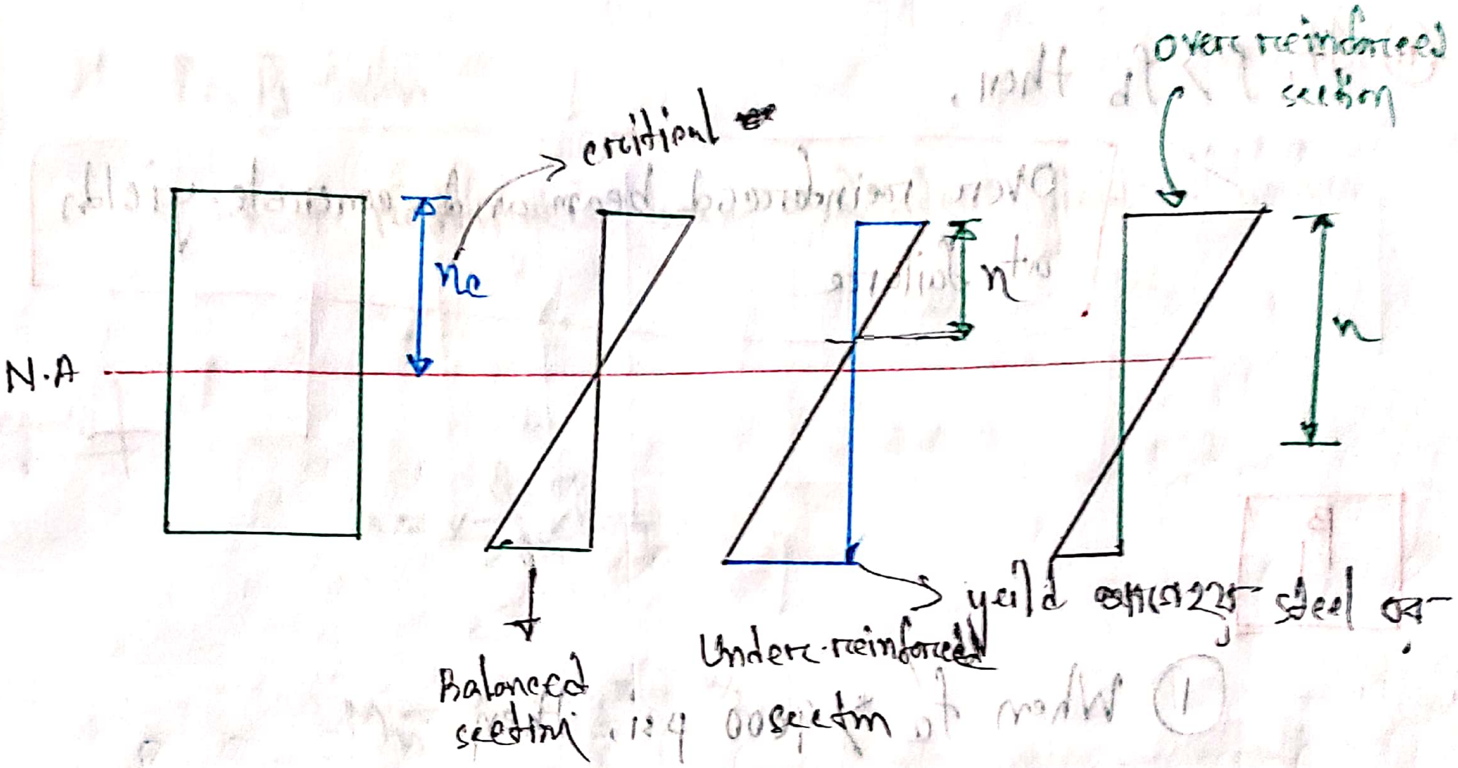
$$\boxed{Mu = \phi * A_s * f_y * (d - a/2)}$$

Balanced section → Stress in concrete and steel reach there perissible value at the same time,

Under Reinforced Section → ~~Percenetde~~ Percentage of steel provided is less than that provided in balanced section

practical design এর সময় যে steel টা ইউজ করা হয় সেটা under reinforced থাকে। এর কারণ হচ্ছে, যেন beam fail করে, তখন ~~আগে~~ concret আগে fail করে, সেজন্যে আগে একটা Alarm পাওয়া যায়।

over Reinforced section — greater than that ~~prodiv~~ provided in balanced section.



Beam ~~Aon~~ Analysis (USD method)

step-1 (calculate ρ and ρ_b)

steel ratio, $\boxed{\rho = \frac{A_s}{bd}}$

Balanced steel ratio

$$\boxed{\rho_b = 0.85\beta_1 \times \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}}$$

0.003

0.002

Depends on f_c'

β_1

① If $\rho \le \rho_b$ then, Under reinforced beam and steel yields at failure

(ii) If, $\rho > \rho_b$ then,

Over reinforced beam and concrete yields at failure

$\boxed{\beta_1}$

① When $f'_c > 4000$ psi, then

$$\boxed{\beta_1 = 0.85 - \left(\frac{f'_c - 4000}{1000}\right) \times 0.05 \geq 0.65}$$

(II) When $f'_c \leq 4000$ psi

$$\boxed{\beta_1 = 0.85}$$

For each 1000psi increase in f'_c over 4000 psi β_1 decrease at a rate of 0.05

For example, when $f'_c = 5000$ psi, then
$\beta_1 = 0.80$

* If $\rho < \rho_b$ then

Under reinforced beam and steel yield at failure

Step-2]

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s * f_y * (d - a/2)$$

$$M_u = \underbrace{\phi}_{} M_n$$

⟶ strength reduction factor

$$\boxed{\varphi}$$

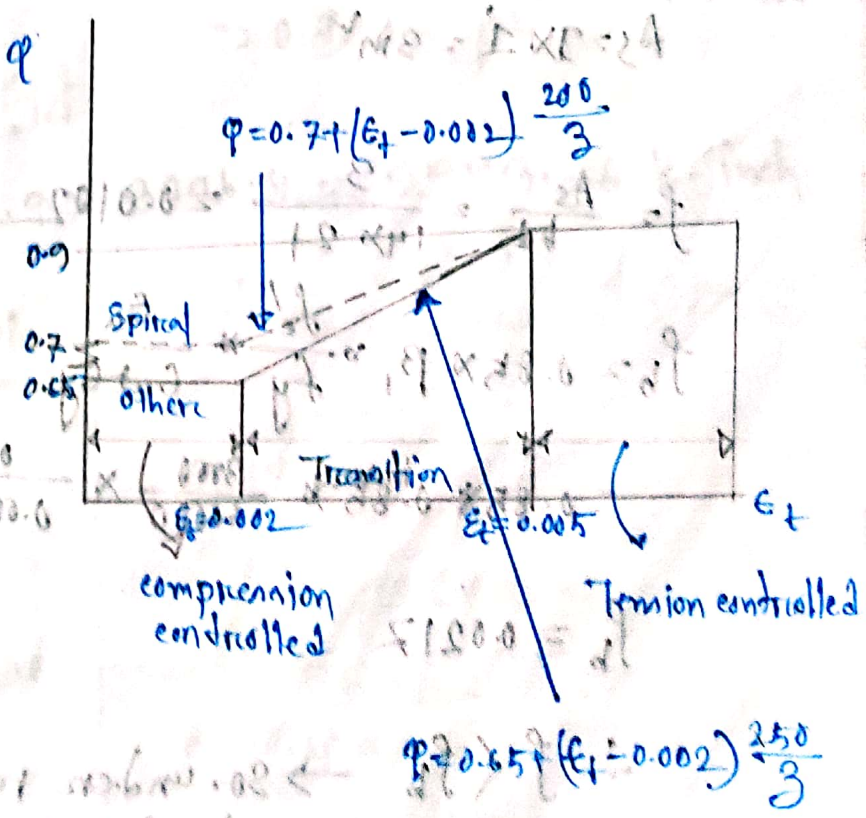
* when $\epsilon_t = 0.002$, then

$\varphi = 0.65$

* When $\epsilon_t = 0.005$, then

$\varphi = 0.9$

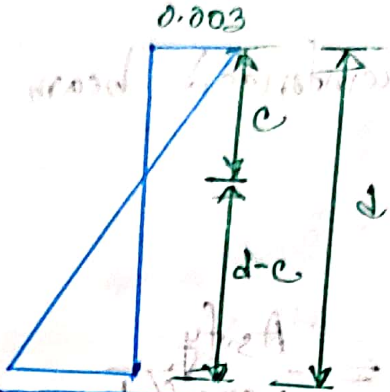
$$\boxed{\varphi = 0.483 + 83.3 \times \epsilon_t}$$



$\boxed{\text{Determination } \varepsilon_t}$

→ max. strain of concrete

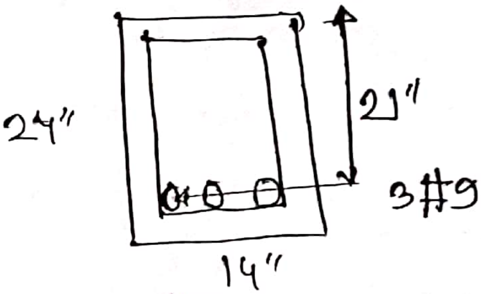
$$\boxed{\varepsilon_t = \varepsilon_u * \frac{d-c}{c} \le 0.005}$$



$$\boxed{c = \frac{a}{\beta_1}}$$

$$\boxed{\varepsilon_t = \frac{d-c}{c} * 0.003}$$

Math 1 Determination the nominal moment capacity of the beam shown in figure. $f'_c = 3000$ psi and $f_y = 60000$ psi



Soln,

$A_s = 3 \times 1 = 3 \text{ in}^2$

$$\rho = \frac{A_s}{bd} = \frac{3}{14 \times 21} = 0.0102$$

$$\rho_b = 0.85 \times \beta_1 \times \frac{f'_c}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= 0.85 * 0.85 * \frac{3000}{60000} \times \frac{0.003}{0.003 + .002}$$

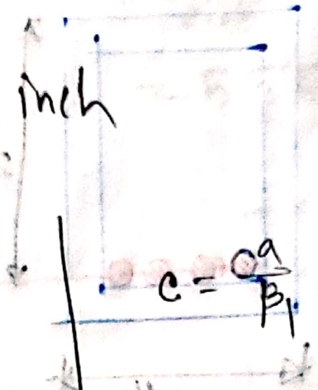
$\left[f'_c < 4000 \text{ psi}, \; \beta_1 = 0.85 \right]$

$$\rho_b = 0.0217$$

$\rho < \rho_b$ → so under reinforced beam. Steel failed at yield at failure

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$= \frac{3\times60}{0.85\times4\times14} = 5.04 \text{ inch}$$



$$c = \frac{a}{\beta_1} = \frac{5.04}{0.85} = 5.93$$

$$M_n = A_s * f_y * (d - a/2)$$

$$= 3\times60 * \left(21 - \frac{5.04}{2}\right)$$

$$= 3326.4 \text{ k-inch}$$

$$\epsilon_t = 0.003 \times \frac{21-5.93}{5.93}$$

$$= \cancel{0.0037}$$

$$= 0.007625$$

$$\phi = 0.483 + 83.3 \times 0.007625$$

$$\cancel{= 0.9} \qquad \cancel{1.1187}$$

$$= 1.118 > 0.9$$

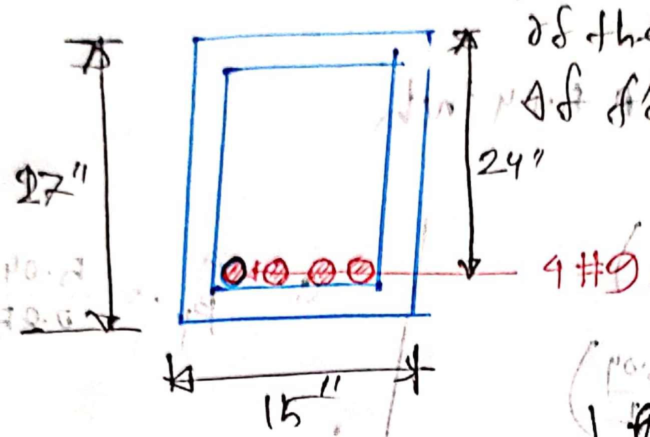
$$= 0.9$$

$$M_u = \phi M = 0.9 \times 3326.4 = 2993.76 \text{ k-inch}$$

Beam Analysis [Part-02]

Math-2

Determine design moment capacity of the beam shown in fig. If $f'_c = 4000$ psi and $f_y = 60000$ psi



Soln:

Steel ratio, $\rho = \frac{A_s}{b\,d}$

$$= \frac{4\times 1^2}{15\times 24} = 0.0111$$

$f'_c = 4000$ psi

$\therefore \beta_1 = 0.85$

Balanced steel ratio

$$\rho_b = 0.85\,\beta_1 \frac{f'_c}{f_s} \times \frac{\varepsilon_u}{\varepsilon_u+\varepsilon_y}$$

$$\rho_b = 0.85\times 0.85 \frac{4000}{60000} \times \frac{0.003}{0.003+0.002}$$

$$= 0.0289$$

$\therefore \rho_b > \rho \rightarrow$ So the beam is under-reinforced beam.

$$a = \frac{A_s f_y}{0.85 f'_c \times b}$$

$$= \frac{4\times 4000}{0.85\times 60000\times 15}$$

~~= 0.02091~~

$$= \frac{4\times 60000}{0.85\times 4000\times 15}$$

$$= 4.7058$$

$$M_n = A_s f_y (d - a/2)$$

$$= 4\times 60 \times \left(24 - \frac{4.70}{2}\right)$$

$= 5195.294$ k-inch

$M_u = \varphi M_n$

=

$$\varepsilon_t = \varepsilon_u \times \frac{d-c}{c}$$

$$= 0.003 \times \frac{24-5.54}{5.54}$$

$$c = \frac{a}{\beta_1} = \frac{4.70}{0.85} = 5.54$$

~~$\varepsilon_t = 0.01002$~~

$\varepsilon_t = 0.01 > 0.005$

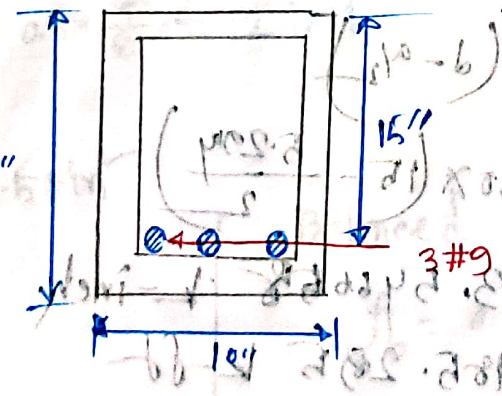
$\therefore \varphi = 0.9$

$M_u = 0.9 \times 5195.294$

$= 4675.7646$ kinch

$M_u = 389.6470$ k-ft

Math-03:



Determine design moment capacity of the beam shown in figure. If $f'_c = 4000$ psi and $f_y = 60000$ psi

Soln:

Steel ratio, $\rho = \frac{A_s}{bd} = \frac{3 \times 1}{10 \times 15} = 0.02$

Balanced steel ratio, $\rho_b = 0.85 \beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

If $f'_c = 4000$ psi

$\therefore \beta_1 = 0.85$

$$= 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003+0.002}$$

$= 0.0289$

∵ $f_b > f$ → so, undere reeinforced beam

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$= \frac{3\times1\times60}{0.85\times4\times10}$$

$$= 5.294$$

$$c = \frac{a}{\beta_1}$$

$$= \frac{5.294}{0.85}$$

$$= 6.228$$

$$\varepsilon_t = \varepsilon_u \times \frac{d-c}{c}$$

$$= 0.003\times\frac{15-6.228}{6.228}$$

~~= 1.4083~~

$$= 0.0042265$$

$$0.002 < \varepsilon_t = 0.004 < 0.005$$

$$\therefore \phi = 0.483 + 83.3\,\varepsilon_t$$

$$= 0.483 + 83.3\times0.0042265$$

$$= 0.83497$$

$$M_n = A_s\times f_y\left(d - a/2\right)$$

$$= (3\times1)\times60\times\left(15 - \frac{5.294}{2}\right)$$

$$= 2223.54668 \text{ k-inch}$$

$$= 185.295 \text{ k-ft}$$

$$M_u = \phi M_n$$

$$= 1856.5280 \text{ k-inch}$$

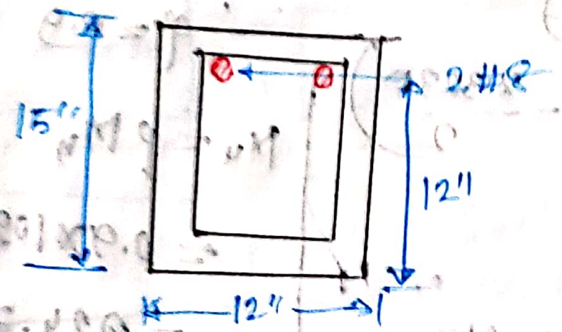
$$= 154.7106 \text{ k-ft}$$

Math-11 Determine the flexural moment capacity of the cantilever beam having concrete clear cover 2.5 inch Given that $f_y = 60$ ksi, $f_c' = 4$ ksi (Assume, ~~sigl~~-single reinforced beam)

Soln:

numbered of bar

dia of bar = $\frac{8}{8}$



#8, $A_s = 0.79$ in²

#9; $A_s = 1$ in²

d = h − clear cover of concrete − 0.5 * Bar dia

$= 15 - 2.5 - 0.5 \times \frac{8}{8} = 12$ inch

$A_s = 2 \times 0.79 = 1.58$ in²

steel ratio,

$\rho = \frac{A_s}{bd}$

$= \frac{1.58}{12 \times 12} = 0.0109$

$f_c' = 4$ ksi

$\beta_1 = 0.85$

Balanced steel ratio,

$\rho_b = 0.85 \beta_1 \frac{f_c'}{f_y} * \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

$= 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003 + 0.002}$

$= 0.0289$

$\rho < \rho_b \rightarrow$ so, under reinforced beam

$$a = \frac{A_s f_y}{0.85 \times f_c' b}$$

$$= \frac{1.58 \times 60}{0.85 \times 4 \times 12}$$

$$= 2.3235$$

$$c = \frac{a}{\beta_1}$$

$$= 2.73356$$

$$\varepsilon_t = \varepsilon_u \times \frac{d-c}{c}$$

$$= 0.003 \times \frac{12-2.73336}{2.73336}$$

$$= 0.01069 > 0.005$$

$$\therefore \varphi = 0.9$$

$$M_n = A_s f_s \left(d - \frac{a}{2}\right)$$

$$= 1.58 \times 60 \times \left(12 - \frac{2.3235}{2}\right)$$

$$= 1027.4661$$

$$= 85.6224 \text{ k-ft}$$

$$M_u = \varphi M_n$$

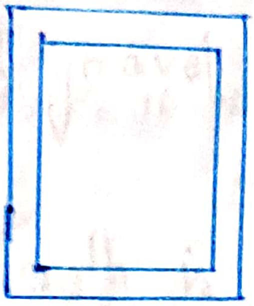
$$= 0.9 \times 1027.4661$$

$$= 924.72$$

$$277.86 \text{ k-ft}$$

Part -03

Beam Design (USD method)



assumption

$h = \frac{1}{10} * \text{Beam Span}$

$b = h/2$

$\text{weight of Beam} = \frac{b * h}{144} * 150$

$W_u = 1.2 D.L + 1.6^{*} L.L$

~~facture load~~

factored load

Fore simply supported load, $M_u = \frac{wL^2}{8}$

~~fore~~ Max moment

fore cantilivere beam,

$M_u = \frac{wL^2}{2}$

Design Site

Concrete — Concrete is a stone like material (artificial) obtained by mixing cement, and sand and gravel or other aggregate and water

compressive strength is high but tensile strength is very small compared to compressive strength

advantages of concrete over steel —

① Longevity

② Well resistance against water

③ Easy to produce any desired size

④ cheaper and available

> The strength of concrete will have to be good. If the strength is good, then porosity will have to be low

Rcc. vs concrete

(i) ~~RCC~~ There are reinforcement in RCC, where there is no reinforcement in concrete.

(ii) There is ductility in RCC

(iii) tensile strength in RCC is higher than concrete

(iv) Though ~~seesteel~~ strength of steel is greater than concrete, so why we use concrete for construction?

Ans — (i) Concrete has good bending property

(ii) To increase durability of construction to weather

(iii) construction cost is law.

(iv) Easy to give desire shape

(v) concreting process is easy

⇒ Why the strength of concrete is considered as the most important property?

Ans → The load bearing capacity can be ~~a~~ calculated ~~so~~ easily.

~~About Many other property~~

→ from ~~of~~ concrete strength, many other property can ~~a~~ be known

→ For example, from strength of the concrete, we can know about the perci~~b~~meability of concrete, which gives the information about porusity.

→ If w/c ratio ~~a~~ decrease, streangth of concrete increase. But if w/c ratio decrease more, then workibility decrease, ~~more~~.

⊞ 60 grade steel = yeild strength 60000 psi

⊞ concrete strain at max compressive stress

ε_u varies between 0.0015 - 0.003

⊞ For normal strength, concrete $\varepsilon_u \approx 0.002$.

⊞ ACI code $\boxed{\varepsilon_u = 0.003}$ used for flexural and axial compression

Factore affecting choise of Reinforced concrete for structure

(i) R.C.C is economical than concrete

(ii) Suitable then steel

(iii) fire resistance

(iv) Rigidity

(v) Low maintainance

(vi) ~~to~~ Availability of materials

disadvantage

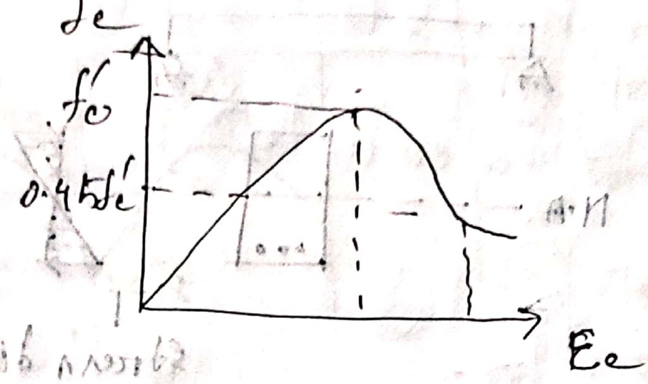
(i) Low tensile strength

(ii) forms and shoring cost

(iii) Relatively low strength per unit volume

(iv) Slow formation

Concrete properties



It is assumed that a concrete behaves as elastic half of It's elastic limit

Poision Ratio, υ

$V \simeq 0.15$ to 0.20 ; usually use $\upsilon = 0.17$

Modulus of elasticity for normal weight concrete,

$$E_c(psi) = 57000\sqrt{f'_c(psi)}$$

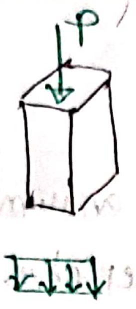
$$E_c(psi) = 33\, w^{1.5}\sqrt{f'_c(psi)}$$

→ compressive strength of concrete at 28 days

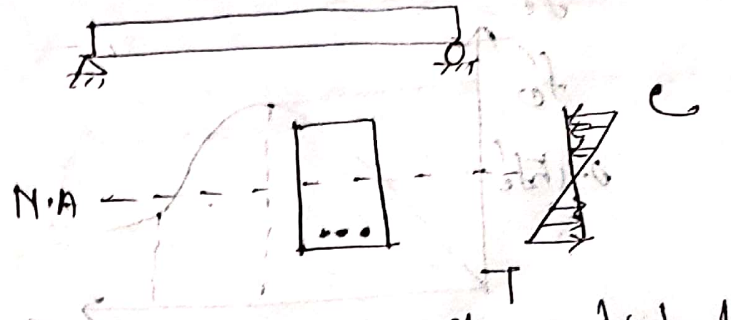
unit weight of concrete, $w_c \cong 150$ lb/ft3

Modulus of Rupture, $f_r = 7.5\sqrt{f'_c(psi)}$

Why Rcc

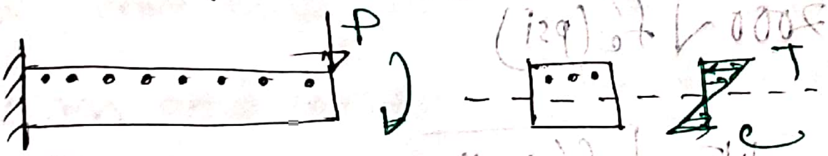


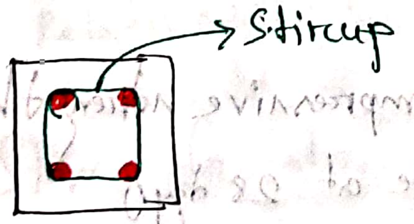
Stress distribution



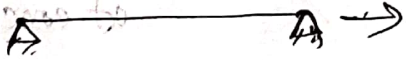
stress distribution

* If max. T stress capacity x psi and max. C stress y psi, and it x > y, then it will fail. If we donot use rcc, then then ~~the~~ of beam size ~~whi~~ will have to be heigh. For this reason, reinforcement should here given in tension zone.

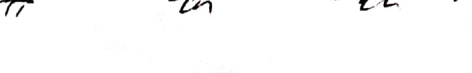


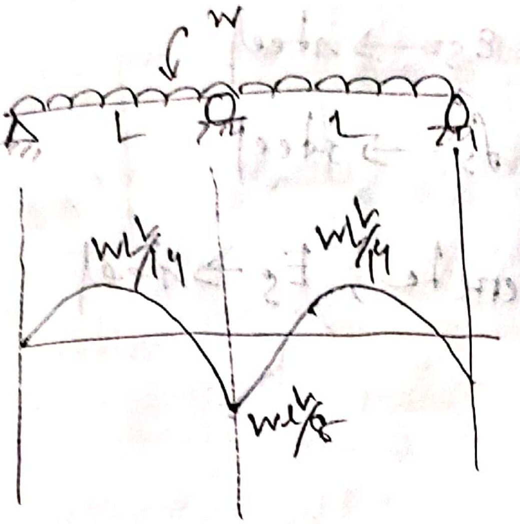



→ Simply supported beam


→ both end hinged beam


→ both end fixed beam


→ continuous beam



why mild steel is used as Reinforced bar

(i) Well chemical & mechanical bonding

(ii) Thermall expansion co-efficient of steel and concrete are ~~equal~~ almost equal.

Safety ~~previo~~ Provisions

Three reasons why safety factors are necessary -

(i) variability in resistance

(ii) " in loading

(iii) consequences of failure

Dead load

(i) weight of all permanent construction

(ii) constant magnitude and fixed location

live load

(i) User

ε_e = strain in concrete , $\varepsilon_s \rightarrow$ steel

f_e' = stress in concrete $f_s \rightarrow$ steel

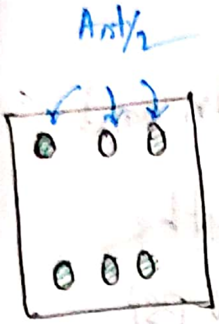
E_e = modulus of elasticity of concrete, $E_s \rightarrow$ steel

Strain $\boxed{\varepsilon_c = \frac{fe}{E_c}}$

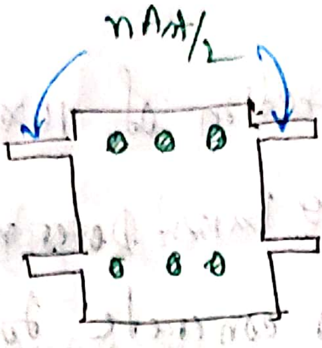
$$\varepsilon_c = \varepsilon_s$$

$$\Rightarrow f_s = \frac{E_s}{E_e} fe$$

$\boxed{f_s = n fe}$ [n = modular ratio]

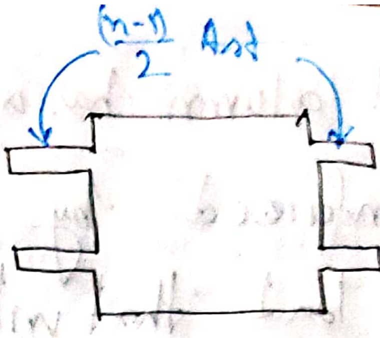


Actual section



Transformed section

$=$



Transformed section

$A_t = A_c + n A_{st}$

↳ total area

↳ Area of concrete

↳ Area of steel

$A_t = A_g + (n-1) A_{st}$

↳ Gross area

Therefore P load is applied, ~~An con~~ then concrete and steel will carry total load

$P = P_c + P_s$

$= f_c A_c + (n f_c) A_{st}$

$$\boxed{P = f_c (A_c + n A_{st})}$$

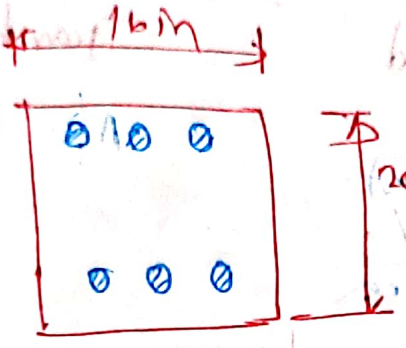
↓ stress of concrete

↳ transformed area

$A_c = A_g - A_{st}$ = concrete area

$A_t = A_c + n A_{st}$ = transformed area

Math 1 A column has a cross-section of 16x 20 in and is reinforced by six no. 9 bars. Determine the axial load that will stress concrete to 12000 psi



6 #9 bars

Soln: given $f_c = 12000$ psi

we know, area of #9 bar $= 1$ in^2

$\therefore$ 6 #9 bars area $= 6$ in^2 $= A_{st}$

Gross area, $A_g = 16 \times 20 = 320$ in^2

modular ratio, $n = \frac{E_s}{E_c} = 8$ (Let)

$E_c = 33\, w^{1.5} \sqrt{f_c'}$

$E_c = 5700 \sqrt{f_c'}$

$A_t = A_g + (n-1) A_{st}$

$= 320 + (8-1) \times 6$

$= 362$ in^2

$P = f_c A_t$

$= 1200 \times 362$

$= 434400 \text{ lb}$

$P_c = A_c f_c = f_c (A_g - A_{st})$

$= 1200 \times (320 - 6)$

$= 376800 \text{ lb}$

$P_s = A_s f_s = A_s n f_c$

$= 6 \times 8 \times 1200$

$= 57600 \text{ lb}$

Cross-sectional area of MS bar

① #5 bar, $A = 0.31 \text{ in}^2$

⑪ #6 bar, $A = 0.37 \text{ in}^2$

(111) #7 bar, $A = 0.6 \text{ in}^2$

(1V) #8 bar, $A = 0.79 \text{ in}^2$

(V) #9 bar, $A = 1 \text{ (in}^2)$

1 Short Note on strength reduction Factor

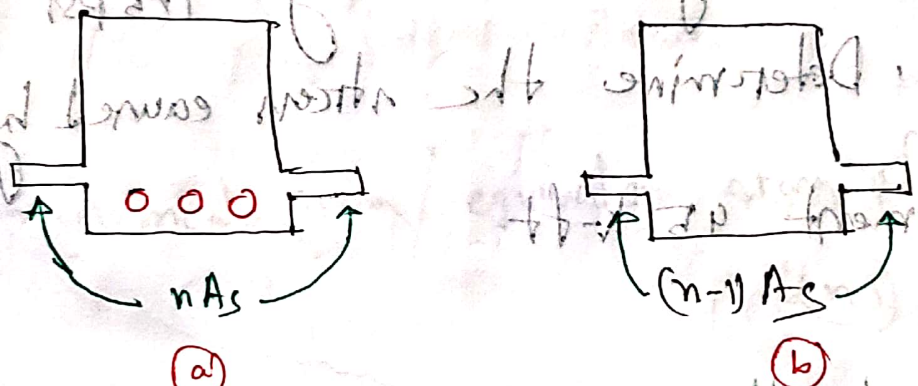
Strength reduction factor (φ) is used in Reinforced concrete design and analysis and its magnitude is different for various reinforced concrete member for ~~ss~~ safety purposes. The strength reduction factor is used to decrease the estimated strength of structural members. i.e to ~~mo~~ compute the design strength of concrete elements, in order to account for uncertainties in material, possible design and construction errors. The value of strength reduction factor is less than 1.

Strength reduction factors are for various members based on ACI 318-19

Actions on Structural members	Strength reduction factor
Tension controlled beam and slabs	0.90
shear and torsion in beams	0.75
columns	0.65 (tie), 0.75 (spirally Rec)
Bearing on concrete	0.65
Plain concrete elements	0.60

As the failure of column is brittle, they are more crutical than failure od the beam which is ductile. column fail suddenly without showing any sign. That is why for more safety the capacity reduction factor is higher than of beam.

⇒ Stress elastic and section uncracked



As long as the tensile stress in the concrete is smaller than the modulus of rupture, so that no tension cracks develop. The stress and strain distribution is essentially the same as in an elastic, homogeneous beam. The only difference is the presence of onother material, the steel reinforcement. In the elastic range for any given value of strain, the stress in the steel is n times that of concrete

Math) For a rectangular beam section assume $b = 10$ in, $h = 25$ in, $d = 23$ in. Reinforcement consists of 3 No. 8 bars Concrete cylinder strength at 28 days $f_c' = 4000$ psi concrete tensile strength in bending $= 475$ psi, $f_y = 60000$ psi. Determine the stress caused by a bending moment 45 k-ft

Soln:

effective length, $d = 23''$

$$E_c = 57000\sqrt{f_c'}\ (psi)$$

$$= 57000\sqrt{4000}$$

$$= 3604996.533\ psi$$

$$= 3600,000\ psi$$

$$= 3.6 \times 10^6\ psi$$

we know $E_s = 29 \times 10^6$ psi

modular ratio, $n = \frac{E_s}{E_c}$

$= \frac{29 \times 10^6}{3.6 \times 10^6}$

$= 8.055 \approx 8$

A_s = total area of reinforced bars $= (0.79 \times 3)$ in²

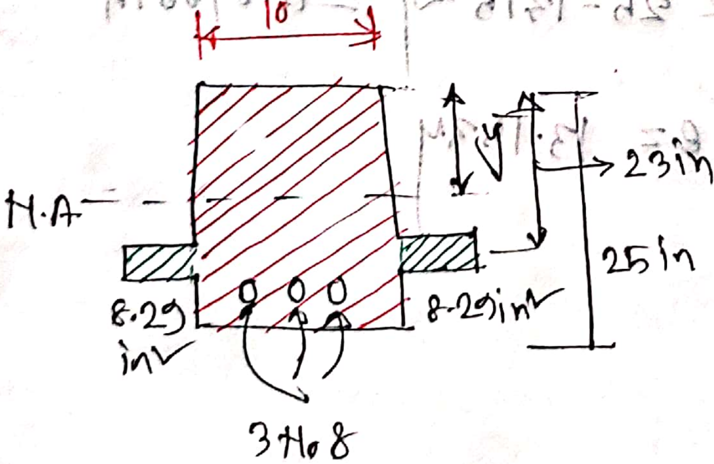
$= 2.37$ in²

Transformed area / equivalent area of concrete

~~$A_2 = (n-1) \cdot A_s$~~

~~$= (8-1) \times 2.37$ in²~~

$= 16.59$ in²



$A_2 = (n-1) A_s$

$= (8-1) \times 2.37$

$= 16.59$ in²

$A_1 = (10 \times 25) = 250$ in²

From top

$$\bar{y} = \frac{250 \times \frac{25}{2} + 16.59 \times 23}{250 + 16.59} = 13.1534 \text{ in}$$

Moment of area about N.A, I

$$I = \left\{ \frac{10 \times 25^3}{12} + 250 \times \left(13.1534 - \frac{25}{2}\right)^2 \right\}$$

$$+ \left\{ \frac{bh^3}{12} + 16.59 \times (23 - 13.15)^2 \right\}$$

$$= 14736.058 \text{ in}^4$$

$$\approx 14740 \text{ in}^4$$

~~tensile stress In~~

For tensile stress $c = 25 - 13.1534 = 11.8466$ in

For compressive stress $c = 13.1534$

compressive stress,

$$f_c = \frac{Mc}{I} = \frac{45000 \times 12 \times 13.1534}{14790}$$

$$= 481.8792 \text{ psi}$$

$$f_{ct} = \frac{Mc}{I} = \frac{45000 \times 12 \times (25 - 13.1534)}{14798}$$

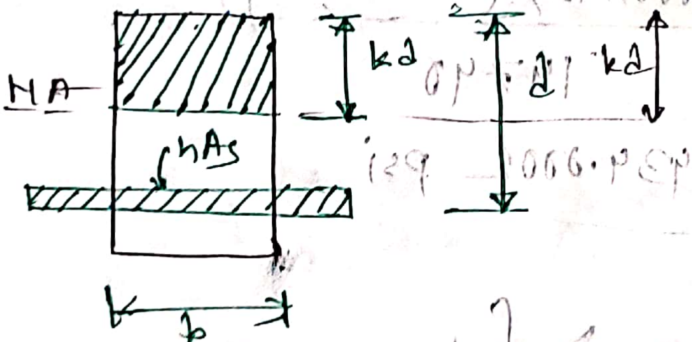
$$= 434.0002 \text{ psi}$$

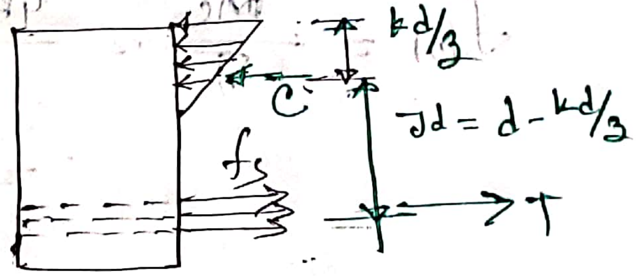
hence, modulus of rupture > f_{ct}

∴ section is uncracked.

Stress elastic and section cracked

When the tensile stress f_{ct} exeeds the modulus of rupture, cracks form. To compute stresses and strains if desired, the device of transformed section can still be used.





Here, d = effective depth of beam

kd = distance to the neutral axis

jd = internal lever arm between c and T

c = Total compression force

T = Total tension force

To determine the location of the neutral axis.

The moment of the tension area about the axis = the moment of the compression area

Which gives $b \cdot kd \cdot \frac{kd}{2} = nA_s(d-kd)$

$$\Rightarrow b\frac{(kd)^2}{2} = nA_s(d-kd) \quad \text{--- (i)}$$

Then, $C = \frac{f_c}{2} bkd$ --- (ii)

$T = A_s f_s$ --- (iii)

The couple constituted by two forces C and T be equal numerically to the external bending moment M. Taking moment about C gives

$$M = Tjd = A_s f_s jd \quad \text{--- (iv)}$$

$$f_s = \frac{M}{A_s jd} \quad \text{--- (v)}$$

Conversely taking moment about T, gives

$$M = C\,jd$$

~~$C = M$~~

$$M = \frac{1}{2} f_c \, b \, kd \, jd$$

$$= \frac{f_c}{2} jk \, b \, d^2 \quad \text{--- (vii)}$$

Reinforced ratio, $\rho = \frac{A_s}{bd}$

$$A_s = \rho \, bd$$

From (v) ⇒

$$b \frac{(kd)^2}{2} = n\rho \, bd \, (d - kd)$$

$$\Rightarrow k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n \quad \text{--- (viii)}$$

From figure $jd = d - kd/3$

$$j = 1 - k/3 \quad \text{--- (ix)}$$

$$M = C \times jd$$

$$= \frac{1}{2} f_c b \times kd \times jd$$

$$= \frac{1}{2} f_c jk\, bd^2$$

$$M = Kbd^2$$

$$\therefore K = \frac{1}{2} f_c jk$$

→ ~~flexrau~~ flexural resistance factor

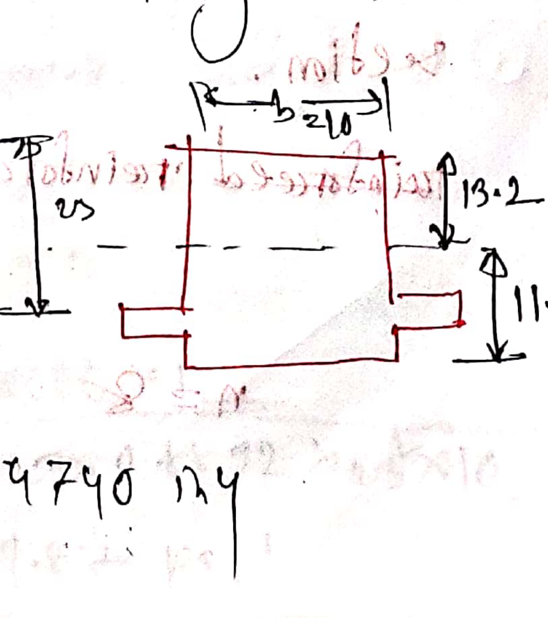
Math 1 For a rectangular beam section assume $b = 10''$, $h = 25''$, $d = 23''$. Reinforcement consist of 3 No. 8 bars. concrete ~~considered~~ cylinder strength at 28 days $f_c' = 4000$ psi. tensile strength in bending $= 475$ psi. $f_y = 60000$ psi. Determine the stress caused by a bending moment 90 kip-ft.

~~Forming from previous math~~

Soln: from previous math

$n = 8$

$\bar{y} = 13.2$



$I = 14740$ in^4

Now, $f_c = \frac{Mc}{I}$

$$= \frac{90\times1000\times12\times13.2}{14740}$$

$$= 967.1641 \text{ psi}$$

$$f_{ct} = \frac{90\times1000\times12\times11.8}{14740}$$

$$= 864.5861 \text{ psi}$$

The value of f_{ct} in larger than concrete tensile strength in bending = 475 psi. ~~therefore the~~

Therefore the section in cracked. And the beam should be analyzed considering cracked section.

~~reinforced~~ reinforcement ratio

$$\rho = \frac{A_s}{bd} = \frac{2.37}{10\times23} = 0.010309$$

$n = 8$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$= \sqrt{(0.01034 \times 8)^2 + (2 \times 0.01034 \times 8)} - (0.01034 \times 8)$$

$$= 0.3323$$

$kd = 0.3323 \times 23 = 7.6440$

$$j = 1 - k/3$$

$$= 1 - \frac{0.3323}{3}$$

$$= 0.8892$$

Steel stress, $f_s = \frac{M}{j d A_s}$

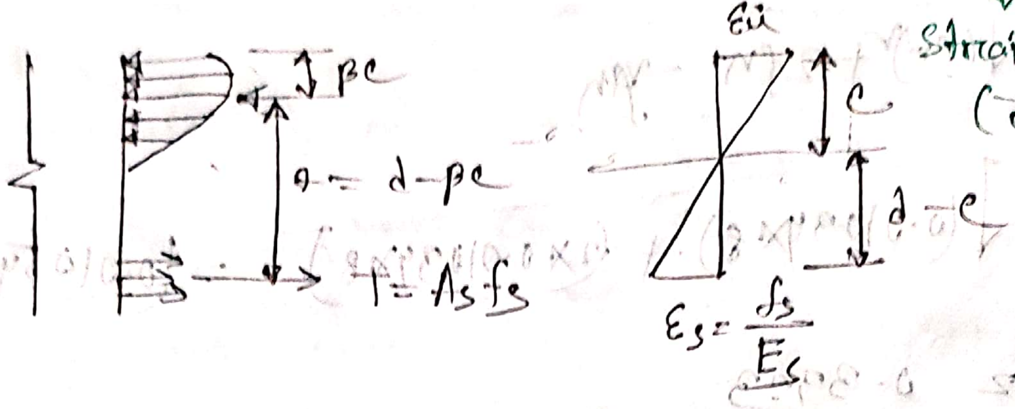
$$= \frac{90 \times 1000 \times 12}{0.8892 \times 23 \times 2.37}$$

$$= 22281.6895$$

Max. concrete stress, $f_c = \frac{2M}{kj d^2 b}$

$$= \frac{2 \times 90000 \times 12}{0.3323 \times 0.8892 \times 23^2 \times 10}$$

$$= 1381.87 \text{ psi}$$



$\varepsilon_u = 0.003$
↓ Ultimate Strain of concrete (Just before failure)

Fig: Internal stress and strain distribution when the beam is about to fail.

Failure may occure in three ways

① concrete and steel both failure
(ii) concrete failure
(iii) steel failure

Mode of failure

① $F_s = f_{yeild}$, concrete crush

Three types of failure

① Balanced failure— If the steel reaches yeild stress and conercete fails by pressing simultanously, the failure is called balanced failure, and the section that produce balance failure is called balanced failure section.

Balanced reinforcement ratio, ~~ρ_b~~ ρ_b

failure condition, $f_s = f_y$, depth of ~~neutr~~ N.A $= c_b$

⑪ Under reinforced section— If the concrete reaches ~~yeill~~ yeild stress first and concrete crushes secondly, the mood of failure is called U.R.S. The section that produce tension failure, is know as U.R.S

Under reinforced section gives warning ~~@~~ before failure.

$\rho = \frac{A_s}{bd}$ $\rho < \rho_b$

$c < c_b \rightarrow$ neutral axis moves down

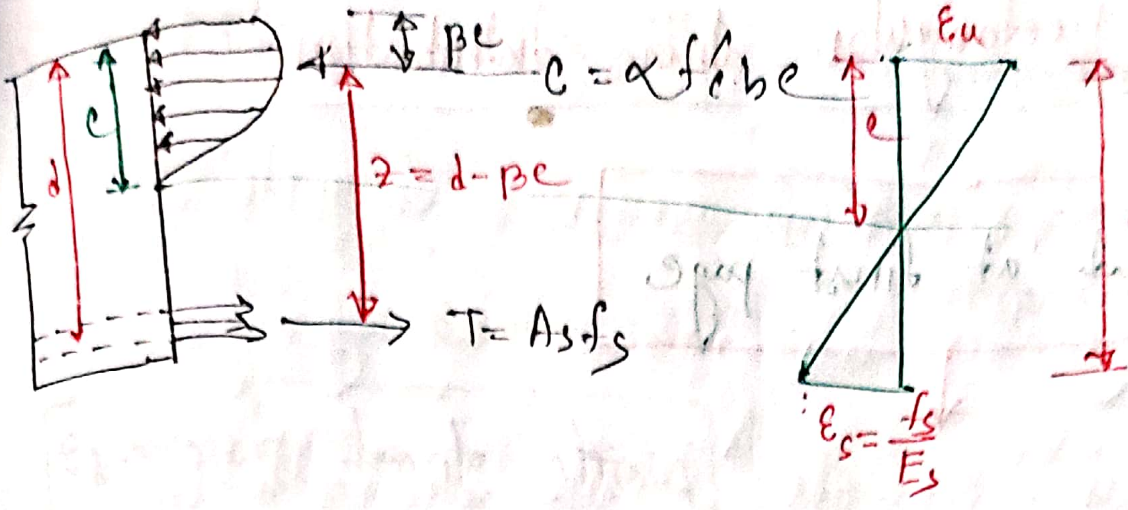
Over reinforced section— Beam fails by crushing of concrete before steel reaches yeild stress, the mode is called over reinforced section. This failure is called compression failure. The section produce compression failure is known as Over reinforced section

It does not gives warning

$\rho > \rho_d$

$c > c_b$

~~PAa~~



f_{av} = average compression stress

$$\alpha = \frac{f_{av}}{f'_c} \quad \Big| \quad \gamma = \frac{\alpha}{\beta_1} \quad \Big| \quad a = \beta_1 c$$

$$C = \alpha f'_c b c$$

$\gamma = 0.85$ for $f'_c \le 6000$ psi

Analysis of under reinforced section

$f_s = f_y$

$C = T$

$0.85 f'_c a b = A_s f_y$

$$\Rightarrow \boxed{a = \frac{A_s f_y}{0.85 a b f'_c}}$$

$$\boxed{a = \frac{\rho f_y d}{0.85 f'_c}} \leftarrow \boxed{\rho = \frac{A_s}{bd}}$$

$$\boxed{\begin{aligned} M_n &= Tz = A_s f_y (d - a/2) \\ M_n &= Cz \\ &= 0.85 f'_c a b (d - a/2) \end{aligned}}$$

$$\boxed{M_n = \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f'_c}\right)}$$

Equivalent rectangular stress distribution

Derived at first page

↓

nominal moment capacity,

$$M_n = A_s f_s \left(d - a/_2\right)$$
$$= 0.85 f_c' ab \left(d - a/_2\right)$$

Ultimate moment / Design moment.

$$M_u = \phi M_n$$

Value of φ depends on

① Mode of failure

⑪ consequence of failure

① $\varphi = 0.9$ for $\varepsilon_t \geqslant 0.005$ ⟶ tension failure

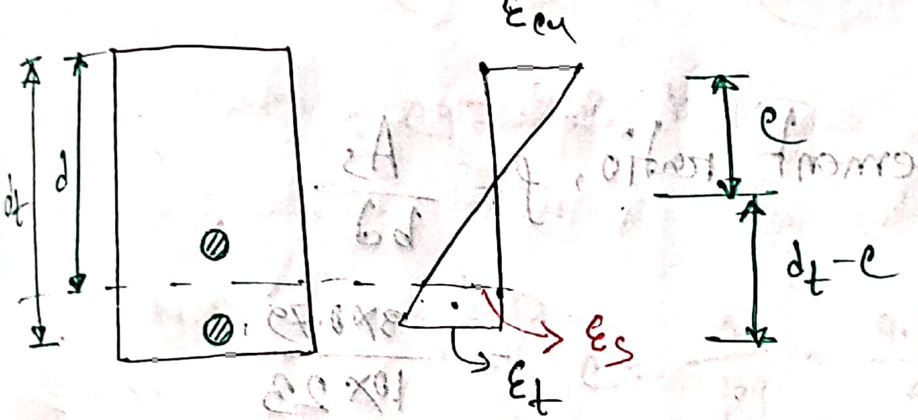
⑪ $\varphi = 0.65$ for $\varepsilon_t \leq 0.002$ ⟶ compression failure

(iii) $0.65 + (\varepsilon_t - 0.002) * \frac{250}{3}$ → transition mode

[between tension and compression]

ε_t = net tensile strain

d = effective depth



$$\varepsilon_t = \varepsilon_{cu} * \frac{d_t - c}{c}$$

↳ সবচেয়ে নিচের layer এর strain

d = distance between compression phase and ~~centriod~~ centreoid of tension reinforcement

d_t = distance between compression phase and centroid of farthest layer of reinforcement

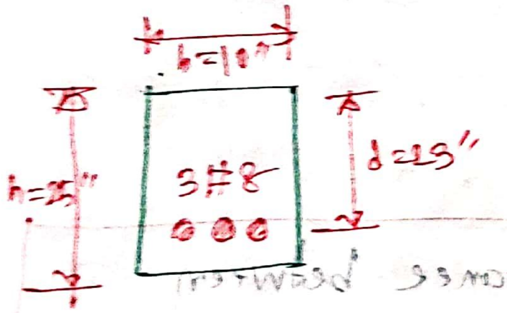
$d_t > d$ for multiple layer of reinforcement	$d_t = d$; for single layer reinforcement

Math 1

For a rectangulare beam section $b = 10''$, $h = 25''$, $d = 23''$. Reinforcement consists of 3 No. 8 bars. Concrete cylinder strength at 28 ~~dy~~ days, $f_c' = 4000$ psi. Concrete tensile strength in bending = 975 psi. $f_y = 60{,}000$ psi. Determine M_n at which the beam fails.

Soln:

Actual reinforcement ratio, $\rho = \frac{A_s}{bd}$



$$= \frac{3 \times 0.79}{10 \times 23}$$

$$= 0.0103$$

Balanced reinforcement ratio,

$$\rho_b = 0.85\, \beta_1 \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$\because f_c' \leq 4000$ psi

$\therefore \beta_1 = 0.85$

$$= 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.002}$$

$$= 0.0289$$

$\rho < \rho_b \rightarrow$ underreinforced condition

~~Mn = As~~ $a = \frac{A_s f_s}{0.85 f_c' b}$

$= \frac{3 \times 0.79 \times 60}{0.85 \times 4 \times 18} = 4.1823''$

$M_n = A_s f_y (d - a/2)$

$= (0.79 \times 3) \times 60 \times (23 - \frac{4.1823}{2})$

$= 2972.769$ k-ft

N.A from top $c = \frac{a}{\beta_1} = \frac{4.1823}{0.85}$

= ~~fro~~ 4.920 inch

Math 1

A reectangulare beam section $b = 10''$, $h = 25''$, $d = 23''$, Reinforcement consist of 3#8 bars. Concrete cylinders strength at 28 days, $f'_c = 4000$ psi. concrete tensile strength in bending = 475 psi. $f_y = 60,000$ psi.

① Compute ~~ult~~ ultimate moment for the beam section

(ii) What live load can be applied on the ~~bem~~ beam if the span is 10 ft simply supported.

Solⁿ:

① actual reinforcement ratio, $\rho = \frac{A_s}{bd}$

$$= \frac{3 \times 0.79}{10 \times 23} = 0.0103$$

Balanced reinforcement ratio,

for $f'_c \leq 4000$ psi

$\beta_1 = 0.85$

$$\rho_b = 0.85 \beta_1 \frac{f'_c}{f_s} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$\rho_b = 0.85 \times 0.85 \times \frac{4000}{60000} \times \frac{0.003}{0.003 + 0.002}$$

$$= 0.0289$$

$\rho_b > \rho$ → under reinforced section

$$a = \frac{A_s f_s}{0.85 f'_c b}$$

$$= \frac{3 \times 0.79 \times 60}{0.85 \times 6 \times 10}$$

$$= 4.18$$

$$c = \frac{a}{\beta_1}$$

$$= \frac{4.18}{0.85}$$

$$c = 4.92$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$= (3 \times 0.79) \times 60 \times \left(23 - \frac{4.18}{2}\right)$$

$$= 2973.902 \text{ k-inch}$$

$$= 247.78 \text{ k-ft}$$

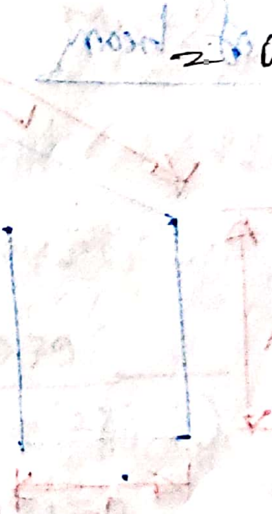
$$\approx 248 \text{ k-ft}$$

$$\varepsilon_u = 0.003$$

$$\varepsilon_t = \varepsilon_u \times \frac{d-c}{c}$$

$$= 0.003 \times \frac{23-4.92}{4.92}$$

$$= 0.01102$$



~~$\phi = 0.65 + 0.01102$~~

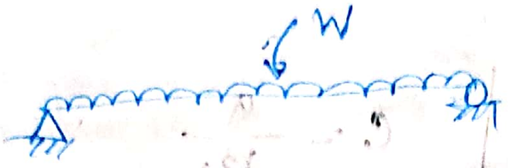
$$\phi = 0.65 + (\varepsilon_t - 0.002)\frac{250}{3}$$

$$= 0.65 + (0.01102 - 0.002) \times \frac{250}{3} \approx 0.9$$

~~$M_u = 0.9 \times$~~ $M_u = \phi M_n$

$$= 248 \times 0.9$$

$$= 223.2 \text{ k-ft}$$



Factored load ; $W_u = 1.2\,DL + 1.6\,LL$

$$M_u = \frac{W_u L^2}{8}$$

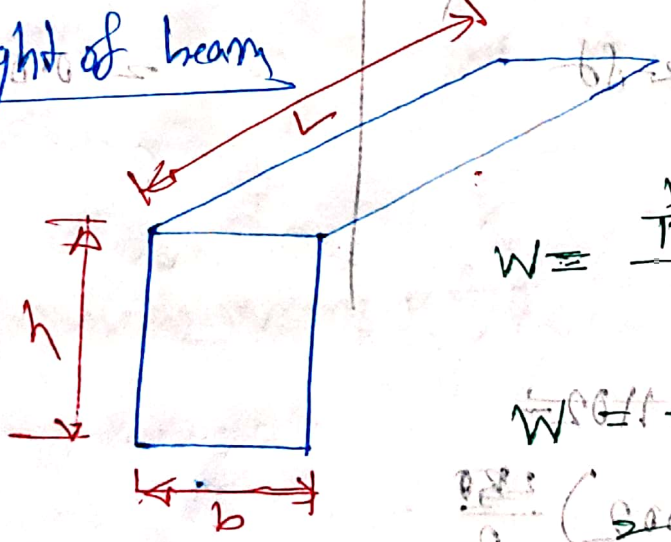
$$M_u = \frac{W_u \times 16^2}{8}$$

$$M_u = 32\,W_u$$

$$223.2 = 32\,W_u$$

$$\Rightarrow W_u = 6.975 \text{ k/ft}$$

Self weight of beam



$$W = \frac{\frac{b}{12} \times \frac{h}{12} \times L \times 150}{L}$$

$$W = \frac{10}{12} \times \frac{25}{12} \times 150$$

$= 260.41667$ ~~k/ft~~ lb/ft

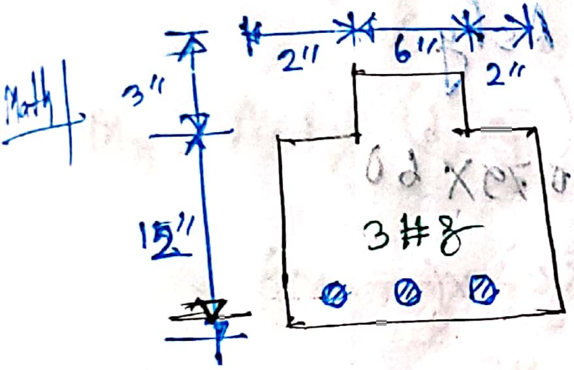
$W_u = 1.2\, D.L + 1.6\, LL$

$\Rightarrow 6.97 \times 1000 = 1.2 \times 260.5 + 1.6\, LL$

$\Rightarrow L.L = \dfrac{(6.97 \times 1000) - (1.2 \times 260.5)}{1.6}$

$\Rightarrow L.L = 4160.93$ lb/ft

$L.L = 4.16$ k/ft



$f'_c = 5000$ psi

$f_y = 60{,}000$ psi

$M_n = ?$ $M_u = ?$

reinforced ratio, $\rho = \dfrac{A_s}{bd}$

$= \dfrac{3 \times 0.79}{10 \times 15 - (2 \times 3 \times 2)}$

$= 0.017173$

Balanced reinforced ratio, $\rho_b = 0.85\, \beta_1 \dfrac{f'_c}{f_y} \times \dfrac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

For $f'_c = 5000$ psi

$\beta_1 = 0.85 - 0.05$

$= 0.80$

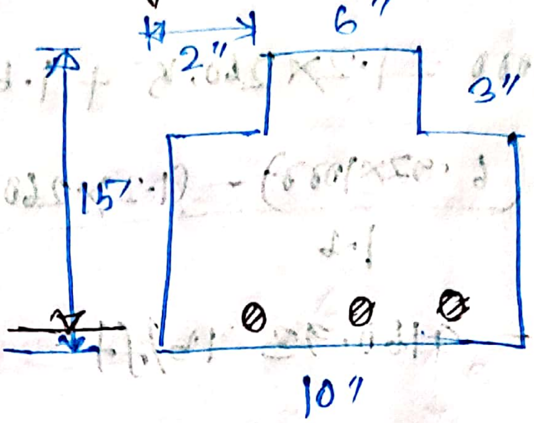
$= 0.85 \times 0.80 \times \dfrac{4}{60} \times \dfrac{0.003}{0.003 + 0.002}$

$= 0.0272$

$\rho_b > \rho \longrightarrow$ Under reinforced section

So, steel will yield.

$\therefore f_s = f_y$



a

C = T

~~$\Rightarrow 0.85 f_c'$~~

$\Rightarrow 0.85 f_c' a b - 0.85 f_c' (2\times3\times2) = A_s f_y$

$\Rightarrow 0.85 f_c' a\times10 - 0.85\times f_c'\times12 = 3\times0.79\times60$

~~$\Rightarrow a = \frac{3\times0.79\times60}{0.85\times5\times10}$~~

$$a = \frac{(3\times0.79\times60) + (0.85\times5\times12)}{0.85\times5\times10}$$

$a = 4.5458''$

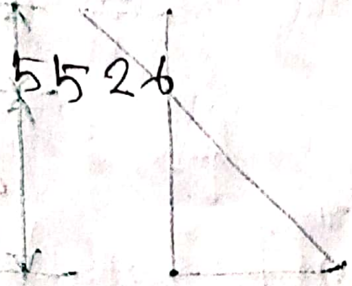
$c = \frac{a}{\beta_1} = \frac{4.5458}{0.8}$

$c = 5.6823''$

~~$\bar{y} = \dfrac{\left(10\times15\times\frac{15}{2}\right) - \left(2\times3\times\frac{3}{2}\times2\right)}{(10\times15) - (2\times2\times3)}$~~

$$\bar{y} = \frac{\left(10\times4.55\times\frac{4.55}{2}\right) - \left(2\times3\times\frac{3}{2}\times2\right)}{(10\times4.55) - (2\times3\times2)}$$

$\bar{y} = 2.5526$



$M_n = A_s f_y (d - \bar{y})$

$= (3\times0.79)\times60\times(15 - 2.5526)$

$= 1770.018$ k-inch

$= 147.5015$ k-ft

$\varepsilon_t = \dfrac{d-c}{c}\times\varepsilon_u$

~~$= \dfrac{15-5.5458}{5.5458}\times$~~

$= \dfrac{15-5.6823}{5.6823}\times0.003$

~~$= 1.63977$~~

$= 0.004919$

≈ 0.005

$\therefore \varphi = 0.9$

$M_u = \varphi M_n$

$= 0.9\times147.5015$

$= 132.75135$ k-ft

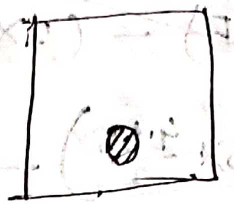
Over reinforced section

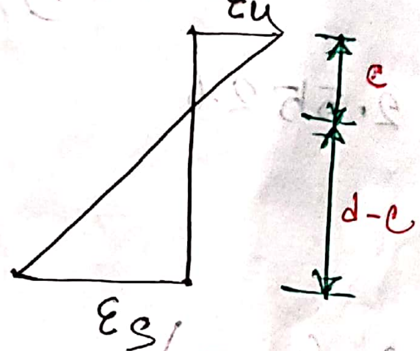
→ $\rho > \rho_b$

→ $c > c_b$

steel will not yield

→ $f_s < f_y$





$$\frac{\varepsilon_u}{\varepsilon_s} = \frac{c}{d-c}$$

$$\Rightarrow \boxed{\varepsilon_s = \frac{d-c}{c}\varepsilon_u}$$

$$f_s = \varepsilon_s E_s$$

$$\boxed{f_s = \left(\frac{d-c}{c}\varepsilon_u\right)E_s}$$

$$C = T$$

$$\Rightarrow 0.85 f_c' ab = A_s f_s$$

$$\Rightarrow \boxed{0.85 f_c' \beta_1 cb = A_s \times \left(\frac{d-c}{c}\varepsilon_u\right)E_s}$$

$$\boxed{c = \frac{a}{\beta_1}}$$

Why the height of beam (h) is greater than width of beam (b) ?

Ans. We know deflection of beam $y \propto \frac{1}{I}$

Where $I = \frac{bh^3}{12}$

If $h > b$, then I increases and consequently deflection decreases. To keep less deflection of beam. $h > b$ is kept.

Q. Determine the amount of reinforcement to assure

(i) tension failure

(ii) Balanced failure

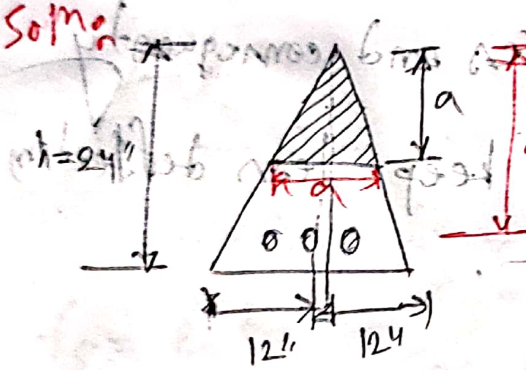
(iii) compression failure

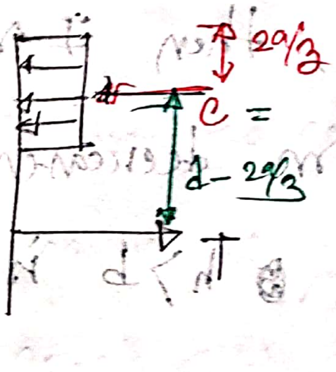
Problem on under reinforced beam section

Math] Calculate the moment capacity of the beam shown in figure below. Assume $f_y = 60{,}000$ psi, $f_c' = 3000$ psi

3 #8 bars

Sol^n





f_m

$f_c \le 4000$ psi

$\beta_1 = 0.85$

$A_s = 3 \times 0.79$

$= 2.37$

$$\rho = \frac{A_s}{bd}$$

$$= \frac{2.37}{\frac{1}{2} \times 24 \times 24}$$

$$= 0.008229$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

$$= 0.85 \times 0.85 \times \frac{3000}{60{,}000} \times \frac{0.003}{0.003 + 0.002}$$

$$= 0.021675$$

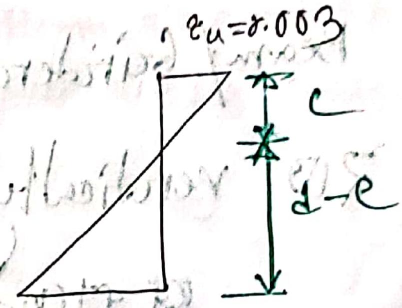
$\rho < \rho_b \rightarrow$ Under reinforced condition

$\therefore f_y = f_s$

$C = T$

$\Rightarrow$ ~~$\frac{1}{2}\times a\times a\times f'_c = A_s f_y$~~ $\Rightarrow \frac{1}{2} a\times a\times 0.85 f'_c = A_s f_y$

$\Rightarrow a^2 = \frac{2 A_s f_y}{0.85 f'_c}$



$a^2 = \frac{2\times 2.37\times 60}{3\times 0.85}$

~~$a = 11.4547$~~

$a = 10.5607''$

$c = \frac{a}{\beta_1} = \frac{10.5607}{0.85}$

$= 12.4244$

$\varepsilon_t = \frac{d-c}{c}\times\varepsilon_u$

$= \frac{22-12.4244}{12.4244}\times 0.003$

$= 0.00231$

$\varphi = 0.65 + (\varepsilon_t - 0.002)\frac{250}{3}$

$= 0.676$

$M_n = A_s f_y\left(d - \frac{2a}{3}\right)$

$= 2.37\times 60\times\left(22 - \frac{2\times 10.5607}{3}\right)$

$= 2127.2456$ k-inch

$= 177.27047$ k-ft

$M_u = \varphi M_n$

$=$ ~~$0.9\times$~~

$= 0.675\times 177.2704$

$= 119.8348$

$\simeq 120$ k-ft

Design of Singly Reinforced Beam

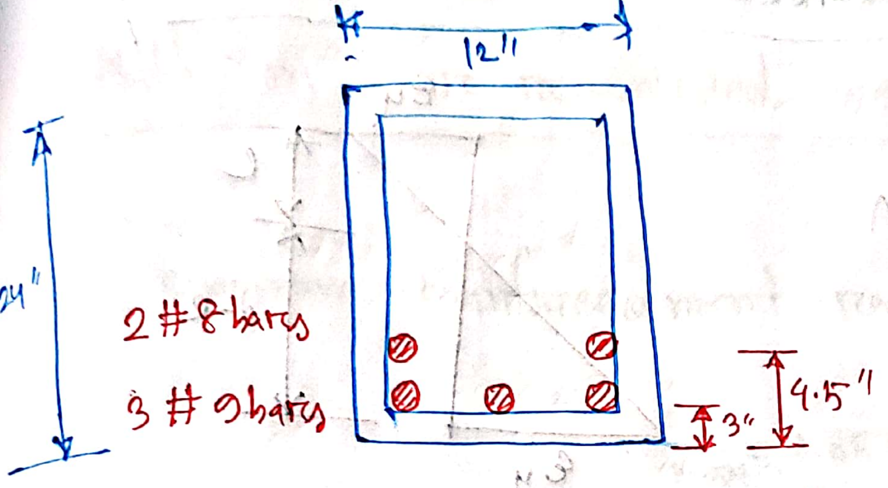
- Beam/Girder/member এ মাঝের crack গুলো bottom হতে vertically উঠে। কিন্তু support এর ~~কাছাকাছি~~ এর কাছের crack গুলো inclined হয় (probably 45°)

- member এর মাঝে Bottom এর crack গুলো flexural crack. গুলো vertical nature এ spreaded.

- Support এর কাছের crack গুলো shear crack গুলো inclined

- মাঝের crack গুলো (flexural) repair এর জন্য সময় পাওয়া যায়। কিন্তু support এর কাছের (shear) crack গুলো সময় নাও দিতে পারি। হঠাৎ এ গুলোর sudden failure হয়। এর জন্য reinforcement capacity থেকে একটু কম reinforcement provide করে member under reinforced রাখা হয়, যাতে failure হলেও তার আগে repairing এর জন্য time পাওয়া যায়।

Effective depth calculation (two layers of reinforcement)



Beam size = b×h

= 12×24 in

centroid of reinforcement,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(3\times1)\times 3 + (2\times0.79)\times 4.5}{\underbrace{(3\times1)}_{3\#9\text{ bars}} + \underbrace{(2\times0.79)}_{2\#8\text{ bars}}}$$

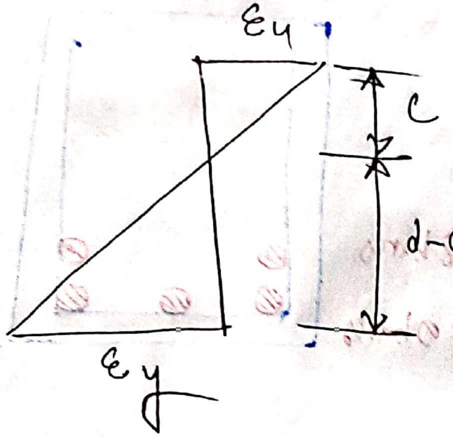
= 3.52 inch

Effective depth, $d = t - \bar{y}$

$= 24 - 3.52$

$= 20.482$

Derivation of balanced reinforced ratio



from strain diagram

$$\frac{c}{d-c} = \frac{\varepsilon_u}{\varepsilon_y}$$

$$\Rightarrow c\varepsilon_y = \varepsilon_u d - \varepsilon_u c$$

$$\Rightarrow (\varepsilon_y + \varepsilon_u)c = \varepsilon_u d$$

$$\Rightarrow \therefore c = \frac{\varepsilon_u}{\varepsilon_y + \varepsilon_u} d$$

In Balanced condition, $f_s = f_y$

$[\rho = \frac{A_s}{bd}]$

For equilibrium, $C = T$

$$\Rightarrow 0.85 f_c' ab = A_s f_y = \rho_b bd f_y$$

$$\Rightarrow \rho_b = \frac{0.85 f_c' ab}{bd f_y}$$

$$\rho_b = 0.85 \frac{f_c'}{f_y} \times a \times \frac{1}{d}$$

$$= 0.85 \times \frac{f_c'}{f_y} \times \beta_1 c \times \frac{1}{d} \quad [a = \beta_1 c]$$

$$\rho_d = 0.85 \beta_1 \times \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$$

Design of singly Reinforced Beam

Math on over reinforced section

Maximum reinforcement ratio,

$$\rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + 0.005}$$

Math| A rectangular beam has width 12" and effective depth d = 18.1" and $d_t = 19.1$ in. It is reinforced with $A_s = 6.32\ in^2$ in two rows. $f_y = 60,000$ psi and $f_c' = 4000$ psi. What is the nominal flexural strength and what is the maximum moment that can be utilized in design according to ACI code?

Design of singly reinforced Beam

Minimum thickness	tmin
(i) Simply supported ——	L/16
(ii) One end continous ——	L/18.5
(iii) Both end continuous ——	L/24
(iv) cantilivere ——	L/8

Minimum Reinforcement,

$$A_{s\,min} = \frac{3\sqrt{f_c'}}{f_y} bwd \geq \frac{200\, bwd}{f_y}$$

Maximum Reinforcement,

$$\rho_{max} = 0.85\, \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + 0.005}$$

Math | Design a simply reinforced beam to support a supercimposed dead load = 1.5 k/ft and live load = 2 k/ft. over a simply supported span of 18'. Follow USD method and $f'_c = 4$ ksi

$f_y = 60$ ksi

Soln:

minimum thickness $t_{min} = \frac{L}{16} = \frac{18 \times 12}{16}$

$= 12''$

Let, 12"×18" section will be used

Load calculation

Dead load (DL)

① Self weight $= \frac{12}{12} \times \frac{18}{12} \times 150' = 225$ lb/ft

$= 0.225$ k/ft

(ii) Super imposed dead load = 1.5 k/ft

Dead load = 1.725 k/ft

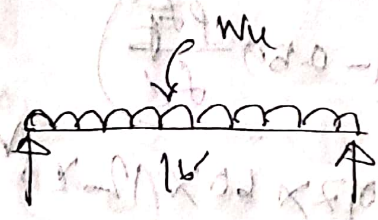
Live load = 1.9 k/ft

factored load, $W_u = 1.2DL + 1.6LL$

$= (1.2 \times 1.725) + (1.6 \times 1.9)$

$= 5.11$ k/ft

Moment calculation



$$M_u = \frac{w_u l^2}{8} = \frac{5.11 \times 16^2}{8} = 163.52 \text{ k-ft}$$

Depth check

$$\underset{\downarrow}{M_u} = \varphi \underline{M_n} \rightarrow \text{provided by beam section}$$

Due to external load

$$M_u = \varphi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \quad — ①$$

$$\rho_{max} = 0.85\ \beta_1 \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + 0.005}$$

$$= 0.85 \times 0.85 \times \frac{4'}{60} \times \frac{0.003}{0.003+0.005}$$

$$= 0.01806$$

Let $\rho = 0.017$

~~Q~~. Now, from ① ⇒

$$M_u = \phi \rho f_y b d^2 \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right)$$

$$163.527 \times 12 = 0.9 \times 0.017 \times 60 \times 12 \times d^2 \left(1 - 0.59 \frac{0.017 \times 60}{4}\right)$$

$$163.527 \times 12 = 9.358 \times d^2$$

⇒ $d^2 = 209.6714$

$d = 14.48''$

∴ $d_{require} = 14.48''$

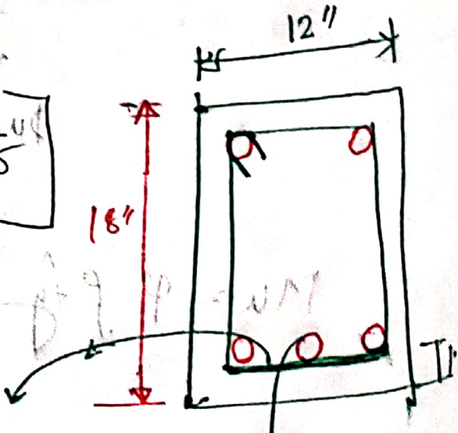
$d_{actual} = t - [\text{clear cover} + \phi_{stirrup} + \phi/2]$

$$d_{actual} = 18 - \left[1.5 + 3/4 + \frac{1}{2} \times \frac{4}{8}\right]$$

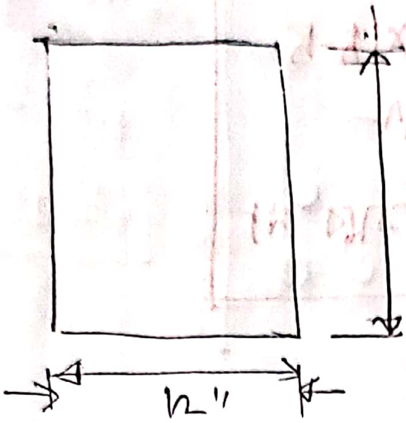
$$\cancel{= 12 \quad 2.5} = 18 - 2.5$$

$= 15.5'' > d_{req.}$

design is ok



Reinforcaement calculation



$M_u = \varphi M_n$

$\Rightarrow M_u = \varphi A_s f_y (d - a/2)$

$\Rightarrow A_s = \frac{M_u}{\varphi f_y (d - a/2)}$ —①

$a = \frac{A_s f_y}{0.85 f_c' b}$

$a = \frac{A_s 60}{0.85 \times 4 \times 12}$

$a = 1.4705 A_s$

① $\Rightarrow A_s = \frac{163.5 \times 12}{0.9 \times 60 \times (15.5 - \frac{A_s}{2})}$

$\Rightarrow A_s = \frac{163.5 \times 12}{0.9 \times 60 \times (15.5 - \frac{1.4205 A_s}{2})}$

$\Rightarrow A_s = \frac{163.5 \times 12}{54 (15.5 - 0.7353 A_s)}$

$\Rightarrow A_s = \frac{163.5 \times 12}{837 - 39.7062 A_s}$

$\Rightarrow -39.7062 A_s^2 + 837 A_s = 163.5 \times 12$

$\Rightarrow 39.7062 A_s^2 - 837 A_s + 1962 = 0$

$A_s = 2.6864$ in^2

$$A_s = \rho b d$$

$$= 0.017 \times 12 \times 15.5$$

$$= 3.162 \text{ in}^2$$

এভাবে করা যাবে না

Trial and error method

$$A_s = \frac{M_u}{\phi f_y (d - a/2)}$$

Let, $a = \frac{18}{4} = 4.5$

$$A_s = \frac{163.5 \times 12}{0.9 \times 60 \times \left(15.5 - \frac{4.5}{2}\right)} = 2.742 \text{ in}^2$$

check, $a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.742 \times 60}{0.85 \times 4 \times 12} = 4.03288$

Not ok

Let, $a = 4$, $A_s = 2.6913 \text{ in}^2$ $a = 3.96$ in

ok

A_s ~~in~~ $A_{s(min)} = \frac{3\sqrt{f_c'}}{f_y} b_w d$

$$= \frac{3\sqrt{4000}}{60000} \times 12 \times 15.5 = 0.58 \text{ in}^2$$

$$A_{s(min)} = \frac{200 \times b_w d}{f_y} = \frac{200 \times 12 \times 15.5}{60,000} = 0.62 \text{ in}^2$$

$$\boxed{A_{s(min)} \cdot 0.62 \text{ in}^2}$$

No. of bar calculation

4 # 8 bar ; Area = 0.79 in² ⟶ $A_s = 3.16$ in²

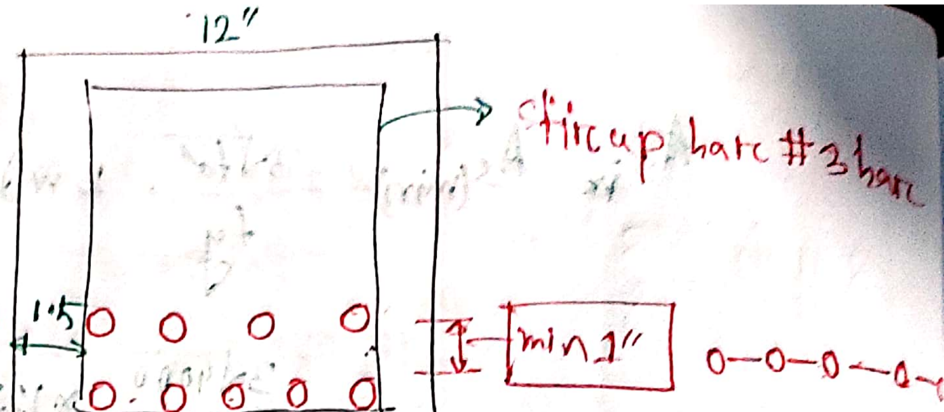
5 # 7 bar, Area = 0.6 in² ⟶ $A_s = 3$ in²

7 # 6 bar Area = 0.37 in² ⟶ $A_s = 2.6$ in²

9 # 5 bar Area = 0.31 in² ⟶ $A_s = 2.79$ in²

$$\text{number of bar} = \frac{2.69}{0.31} = 8.67 \approx 9$$

দুটি বার bar এর gap
minimum 1" হতে হবে



$d = 15.5"$

$d_t = 14.5"$

$\text{Speceing forc bare} = 12 - \{1.5 \times 2 + 2 \times \frac{3}{8}\}$

$= 8.25$

x + 1 + x + 1 + x
1 + x + 1 + x
4 + 5x
7.125

max. aggregate size $1.33 \times \frac{3}{4}$

Fromblem on WSD method

Math] Design a singly reinforced rectangular beam using the following information

① Slab thickness 6"

(II) Live load on slab 120 psf

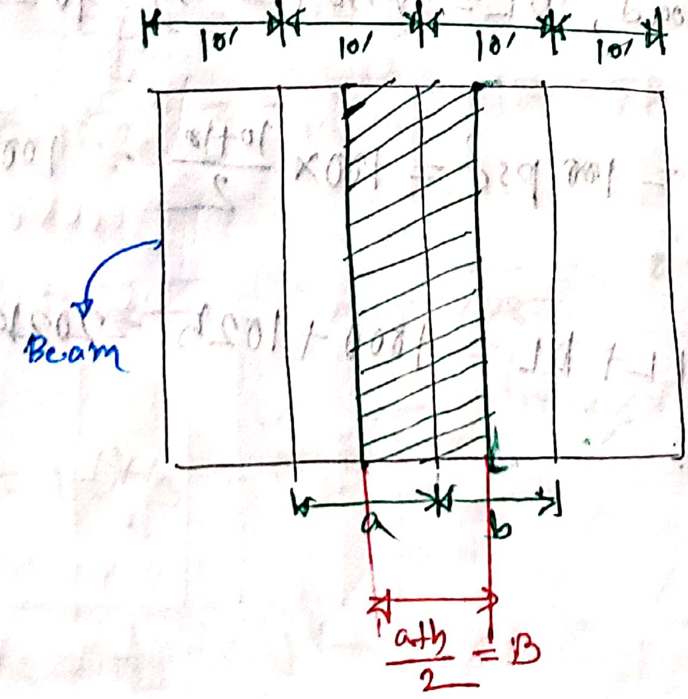
(III) beam are fixed at both end

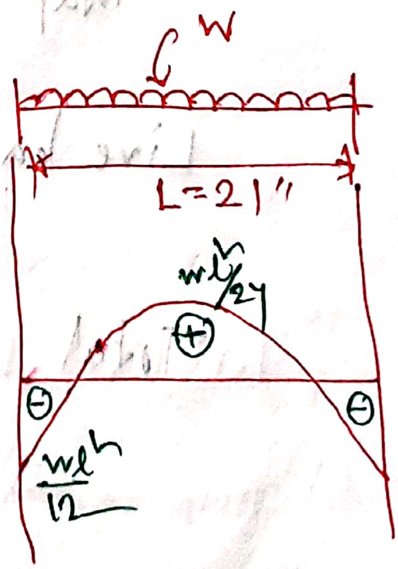
(IV) $f_c' = 3000$ psi

(V) $f_y = 6000$ psi

(VI) follow wsd method

(VII) Floor plan is shown ~~in fig~~ in figure





Solⁿ:

Thickness of the beam, $t = L/24$

$= \frac{21 \times 12}{24} = 10.5"$

Let size of beam $= 12" \times 22"$

load calculation

weight of slab $= \frac{t}{12} \times \left(\frac{10+10}{2}\right) \times 150$

$= \frac{6}{12} \times 10 \times 150 = 750 \text{ lb/ft}$

self weight of beam $= \frac{12}{12} \times \frac{22}{12} \times 150 = 275 \text{ lb/ft}$

Total dead load, $DL = (750 + 275) = 1025 \text{ lb/ft}$

Live load, $LL = 100 \text{ psf} = 100 \times \frac{10+10}{2} = 1000 \text{ lb/ft}$

Total load $= LL + DL = 1000 + 1025 = 2025 \text{ lb/ft}$

moment calculation

Max (−ve) moment $= \frac{wl^2}{12} = \frac{2.025 \times 21^2}{12} = 74.418$ k-ft

Max (+ve) moment $= \frac{wl^2}{24} = \frac{2.025 \times 21^2}{24} = 37.293$ k-ft

Relevant Properties

$f_c = 0.45 f'_c = 0.45 \times 3000 = 1350$ psi

$f_s = 0.4 f_y = 0.4 \times 60000 = 24000$ psi

$n = \frac{E_s}{E_c} = 9$

$r = \frac{f_s}{f_c} = \frac{24000}{1350} = 17.778$

$k = \frac{n}{n+r} = \frac{9}{9+17.778} = 0.34$

$j = 1 - k/3 = 0.89$

$R = \frac{1}{2} f_c j k$

$= \frac{1}{2} \times 1350 \times 0.89 \times 0.34$

$R = 203.788$ psi

Depth check

$$d = \sqrt{\frac{M}{Rb}}$$

$$= \sqrt{\frac{74.42\times12\times1000}{203.78\times12}}$$

$d_{req} = 19.1''$

da. $d_{actual} = 22-2.5 = 19.5'' > d_{req}$

Design is ok

depth check should be based on max. moment

Reinforcement calculation

for (+ve) bending moment $+A_s = \frac{M}{f_s j d}$

$$= \frac{37.21\times12\times1000}{24000\times0.89\times19.5}$$

$= 1.072\ in^2$

for (-ve) bending moment $-A_s = \frac{M}{f_s j d}$

$$= \frac{74.42\times12\times1000}{24000\times0.89\times19.5}$$

$= 2.144\ in^2$

$A_s (min) = \frac{200}{f_y} bd$

$= \frac{200}{6000} \times12\times 19.5$

$= 0.78\ in^2$

No of calculation of bars

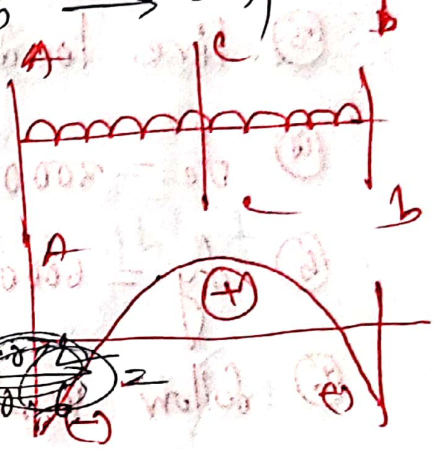
4 #7 ⟶ $A_s = 2.4\ in^2$

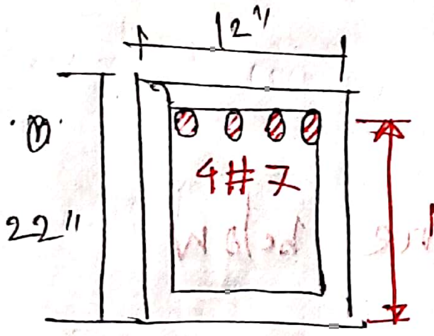
7 #5 ⟶ $A_s = 2.17\ in^2$

5 #6 ⟶ $A_s = 2.2\ in^2$

2 #8 ⟶ $A_s = 1.58\ in^2$

#9 —— 1
#8 —— 0.79
#7 ⟶ 0.6
#6 ⟶ 0.44
#5 ⟶ 0.31

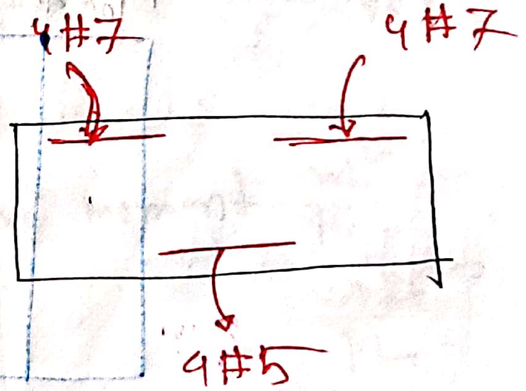




For #7 bars

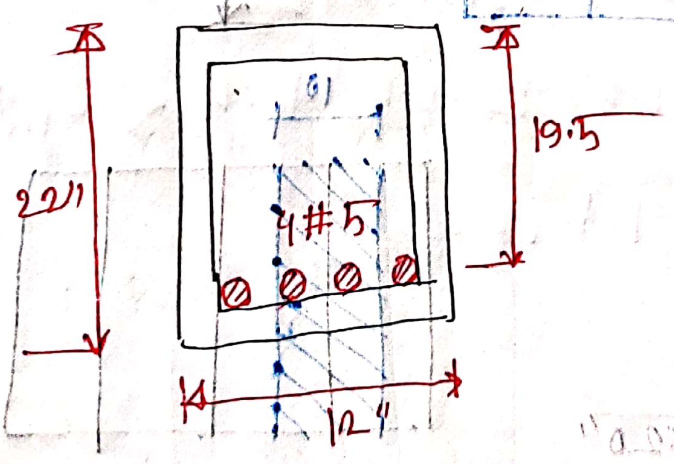
$$\text{No of bars} = \frac{2.144}{0.6} = 3.57 \simeq 4$$

section A :



For #5 bars

$$\text{No of bars} = \frac{1.072}{0.31} = 3.45 \simeq 4$$



section C

7 Design is singly reinforced rectangular beam using following instruction

① Beam are fixed at end

② Slab thickness 6"

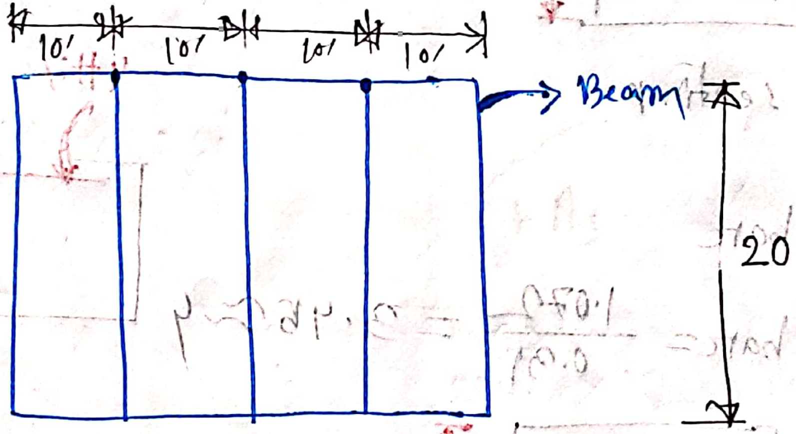
③ live load on the slab 120 psf

④ $f'_c = 3000$ psi

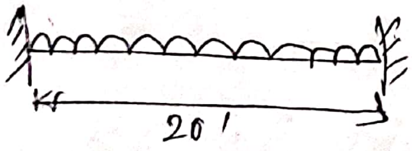
⑤ $f_y = 6000$ psi

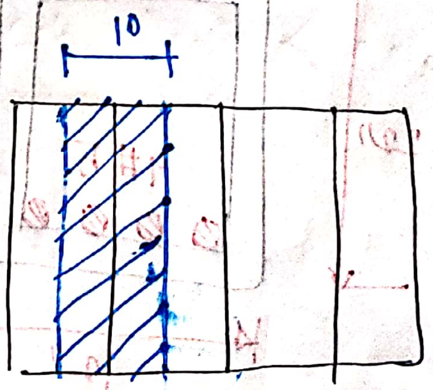
⑥ follow ~~usd~~ WSD method

⑦ Floor plan is shown in figure below



Sol^n:





Let, size of beam 12"x20"

weight of slab = $\frac{t}{12}\times 10\times 150$

$= \frac{6}{12}\times 10\times 150$

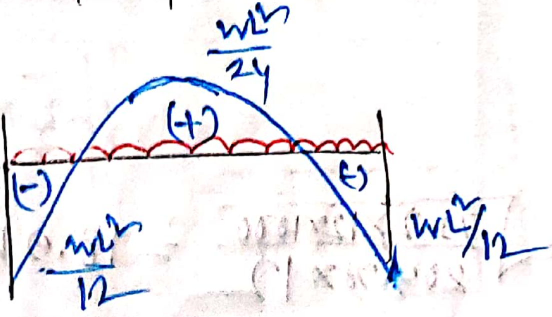
$= 750$ lb/ft

self weight of beam = $\frac{12\times 20}{12\times 12}\times 150 = 250$ lb/ft

Total dead load = 750 + 250 = 1000 lb/ft

total live load load = 120 × 10 = 1200 lb/ft

Total load = D.L + L.L = 1 + 1.2 = 2.2 k/ft



max (-ve) moment = $\frac{wL^2}{12}$

$= \frac{2.2\times 20^2}{12}$

$= 73.33$ k-ft

max (+ve) moment = $\frac{wL^2}{24}$

$= \frac{2.2\times 20^2}{24}$

$= 36.667$ k-ft

Relavent properties

$f_c = 0.45 f_c' = 0.45 \times 3000 = 1350$ psi

$f_s = 0.4 f_y = 24000$ psi

$n = \frac{E_s}{E_c} = 9 \quad r = \frac{24000}{1350} = 17.778$

$k = \frac{n}{n+r} = 0.34$

$j = 1 - k/3 = 1 - \frac{0.34}{3} = 0.89$

$R = \frac{1}{2} f_c j k$

$= \frac{1}{2} \times 1350 \times 0.89 \times 0.34$

$= 203.78$ psi

Depth check

$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{73.2 \times 12 \times 1000}{203.78 \times 12}} = 18.952$ (required)

$d_{actual} = t - 2.5 = 20 - 2.5 = 17.5 < d_{required}$

Not ok

Section should be revised

Let, section is 12" x 24"

self weight $= \frac{12 \times 24}{12 \times 12} \times 150 = 300$ lb/ft

Total load

$= 750 + 300 + 1200$

$= 2250$ lb/ft

~~depth~~

Max (-ve) moment, $M = \frac{wl^2}{12} = \frac{2.25 \times 20^2}{12} = 75$ k-ft

Max (+ve) moment, $M = \frac{wl^2}{24} = \frac{2.25 \times 20^2}{24} = 37.5$ k-ft

depth check

$$d = \sqrt{\frac{M}{R b}}$$

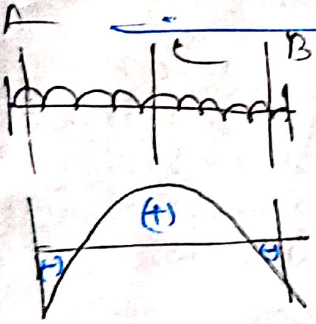
$$= \sqrt{\frac{75 \times 12 \times 1000}{203.78 \times 12}} = 19.184'' \text{ (req.)}$$

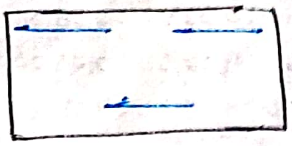
$d_{actual} = t - 2.5$

$= 24 - 2.5 = 21.5'' > d_{req.}$

ok

Reinforcement calculation





for (+ve) bending $A_s = \frac{M}{f_s j d}$

$= \frac{37.5 \times 12000}{24000 \times 0.89 \times 21.5}$

$= 0.98$ in²

for (-ve) bending $A_s = \frac{M}{f_s j d}$

$= 1.96$ in²

$A_{min} = \frac{200}{f_y} \times bd = 0.86$ in²

Flexural Aalysis and Design of beam

* Fundamental assumption relating to flexure and flexural shear are follow—

1. A cross section that was plane before loading remains plane under loading

2. The bending stress 'f' at any point depends on the strain at that point in a manner given by the stress-strain diagram of the ~~d~~ material.

3. The distribution of the shear stress "v" over the depth of the section depends on the shape of the cross-section and of the stress-strain diagram.

4. Owing to the combined action of shear-stress and flexural stresses, at any point in a beam, there are inclined stresses of tension and compression, the largest of which form an angle of 90° with each other. The intensity of max. or principal stress at any point is given by

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2}$$

f = intensity of normal fiber stress

v = intensity of tangential shearing stress

5. Since the horizontal and vertical shear stress are equal and the flexural stresses are zero at neutral. The inclined and tensile and compressive stresses in that plane form angle 45° with the horizontal, the intensity of each being equal to the unit shear at the point.

6. Bending stress at a distance y from neutral axis $= f$
external bending moment at section $= M$
Moment of inertia $= I$

$$f_{max} = \frac{Mc}{I} = \frac{M}{S}$$

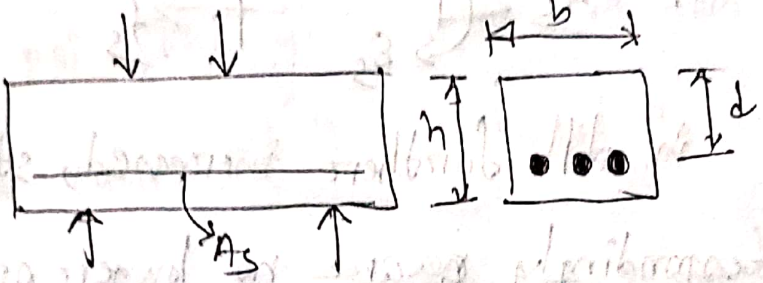
$$\hookrightarrow I/c$$

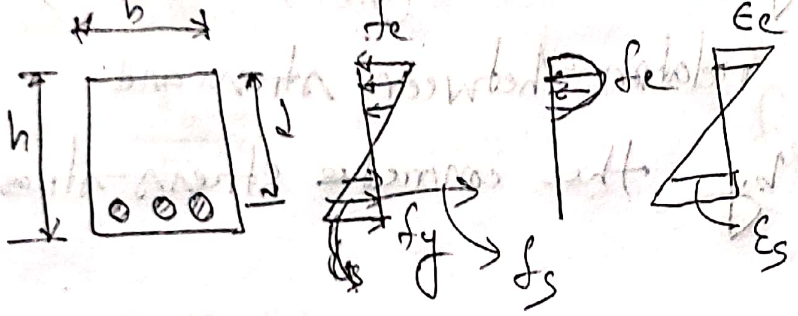
Shear stress v, at any point in the cross-section is given by $v = \frac{VQ}{Ib}$

$$v_{max} = \frac{3}{2}\frac{V}{bh}$$

Reinforced concrete Beam Behaviour [exam এ আসবে]

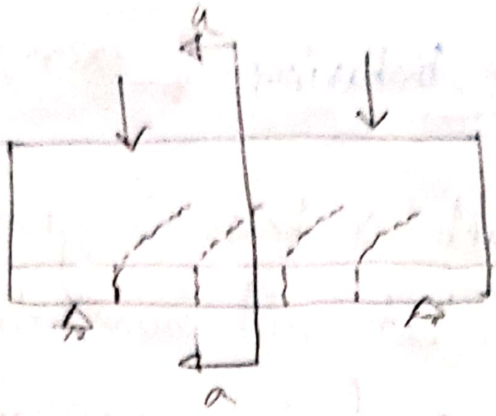
When the load on such a beam is gradually increased from to the magnidude, that will cause the beam to fail, several different stages of behaviour can be clearly distinguished

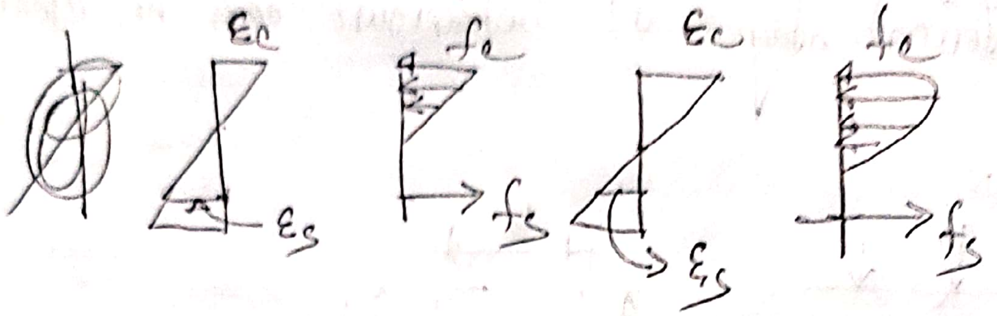




At low loads the entire concrete is effective in resisting the stress, in compression on one side and on tension on other sides of the neutral axis. At this stage, ~~As at this~~ all stress in the concrete are of small magnidude and are proportional to strain.

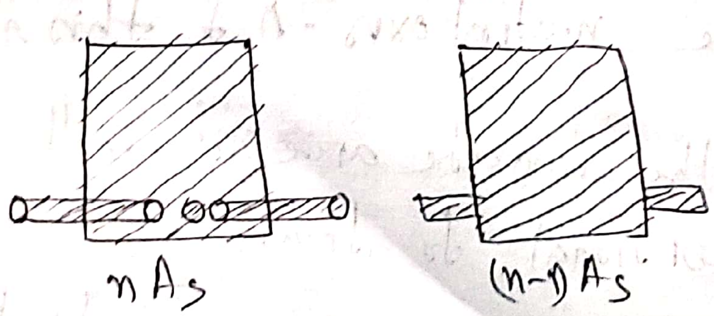
When the load is further increased, concrete does not resist any tensile stress and at this stage, tension cracks develops, stress and strain continue to be a closely proportional





When the load is still furthere increased, stress and strain rise correspondingly ~~ar~~ are no longer proportional) The ensuing non linear relation between stress and strain is that given by the concrete stress-strain curve

Stresses elastic in crack ~~a~~ & uncrack section



n= number of reinforcement bars

Problem-03:1 A rectangular beam has the dimension b = 10'' h = 25'' & d = 23''. It's reinforce #8 bars. So, the bar area, $A_s = 3.27$ in². The concrete cylinder strength = 4000 psi. The tensile strength in ~~bond~~ bending (ε_s) = 475 psi. The yield point of steel $f_y = 60,000$ psi. The stress-strain curve of the material is follow. Determine the stress caused by bending moment 45 kip-ft

Soln:

$$\varepsilon_c = \frac{f_c}{E_s} \qquad \varepsilon_s = \frac{f_s}{E_s} \qquad f_s = \frac{E_s}{E_c} * f_c = n f_c$$

$f_c = 4000$ psi $\qquad f_c' =$ crushing strength in 28 days

↳ design strength

$n = \frac{E_s}{E_c}$ (from s-s diagram)

$= \frac{29 \times 10^6}{3.6 \times 10^6} = 8$

$(n-1) A_s = (8-1) \times 3.27$

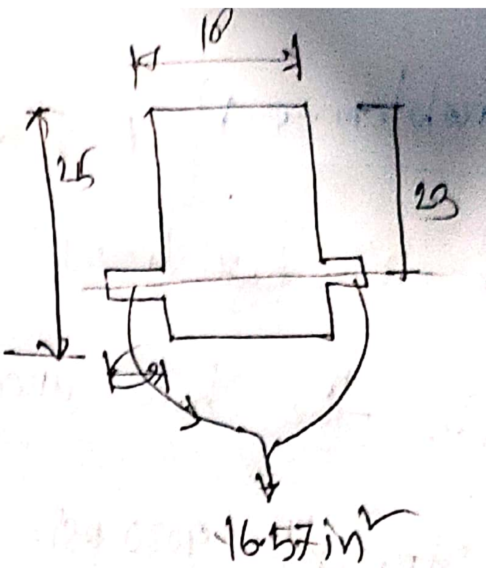
$= 22.89$

$M = 45$ kip-ft

$= 45 \times 12000$

$= 540000$ lb-in

$$\bar{y} = \frac{(25\times10)\times\frac{25}{2}+(16.57\times23.)}{(25\times10)+16.57}$$



$$\bar{y} = 13.148$$

$$\bar{y} = 13.2$$

$$I = \frac{10\times25^3}{12}+\left(\frac{25}{2}-13.2\right)^2\times(25\times10)$$

$$+\left\{\frac{bh^3}{12}+16.57\times(23-13.2)^2\right\}$$

$$= 14736.054 \text{ in}^4$$

$$= 14740 \text{ in}^4$$

For tensile stress, $e = 25-13.1534 = 11.846$ in

for compressive stress, $e =$ 13.1534 in

compressive stress, $f_c = \frac{Mc}{I} = \frac{45000\times12\times13.1534}{14740}$

$$= 481.8748 \text{ psi}$$

$$f_{ct} = \frac{Mc}{I} = \frac{45000\times12\times11.846}{14740}$$

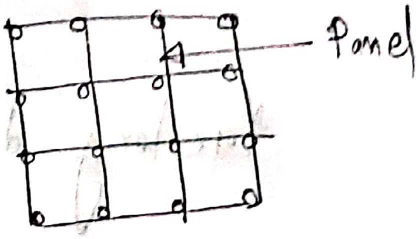
$$= 433.9782 \text{ psi}$$

here, modulus of ruptarre > f_{ct}

∴ Seetlm in uneracked

Design of one-way slab

if $L/B \geq 2$ - one way slab



Support condition -

support at 2 side — one way slab

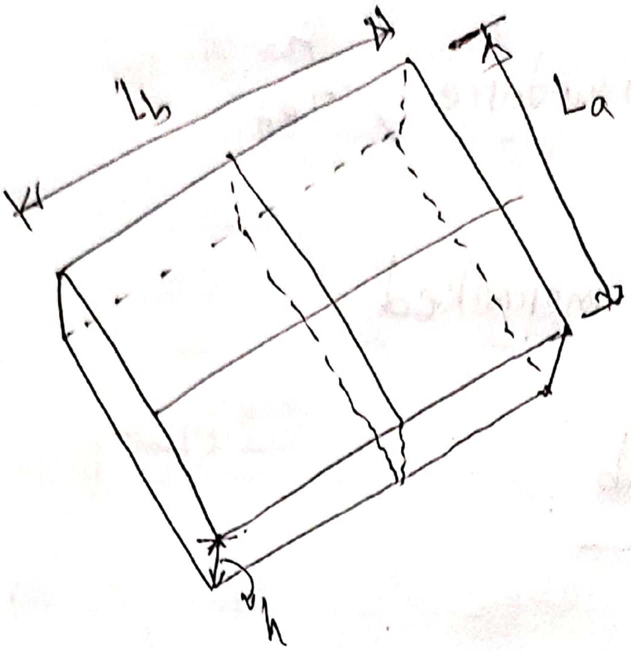
support at 4 side — Two way slab ($L/b \leq 2$)

Main Reinfrocement shorter distance/ direction ↳ থাকবে ।

for simply supported Beam,

$$\text{Deflection} = \frac{WL^4}{384 EI}$$

~~for long~~ direction



for long direction, = $\frac{W_a L_a^4}{384EI}$

for short direction = $\frac{W_b L_b^4}{384EI}$

$\Rightarrow \frac{W_a}{W_b} = \frac{L_b^4}{L_a^4}$

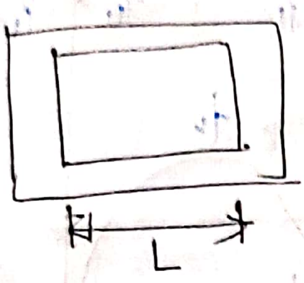
Q. prove that max. loads are transfered to short way direction

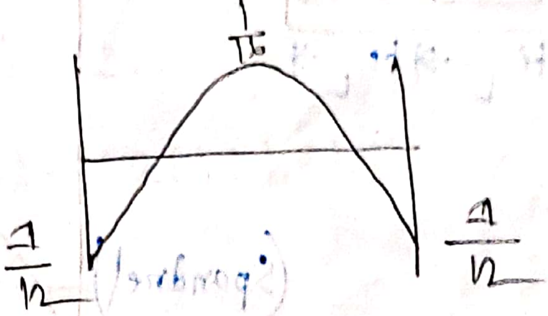
Q. Why use temperatu shrinkage in RCC?

⇒ To reduce expansion and contraction due to the change of heat, we provide temperature shrinkage RCC to protect from producing crack.

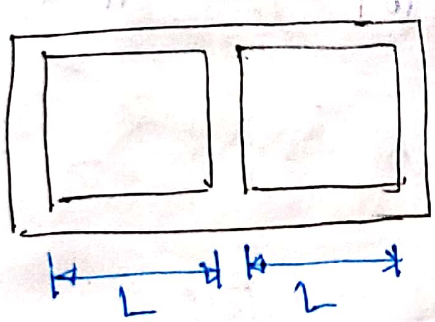
Q. fundamental diff. between two way and one way slab.

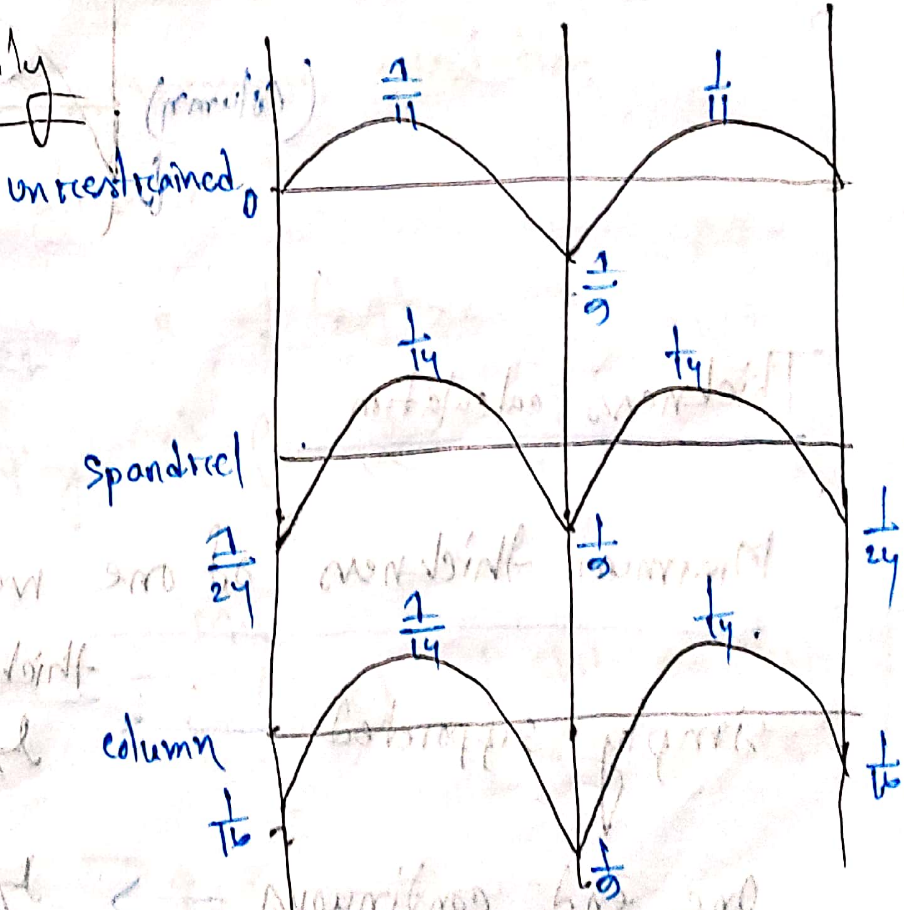
Beam with one span



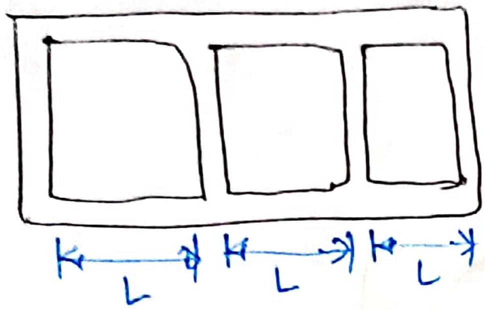


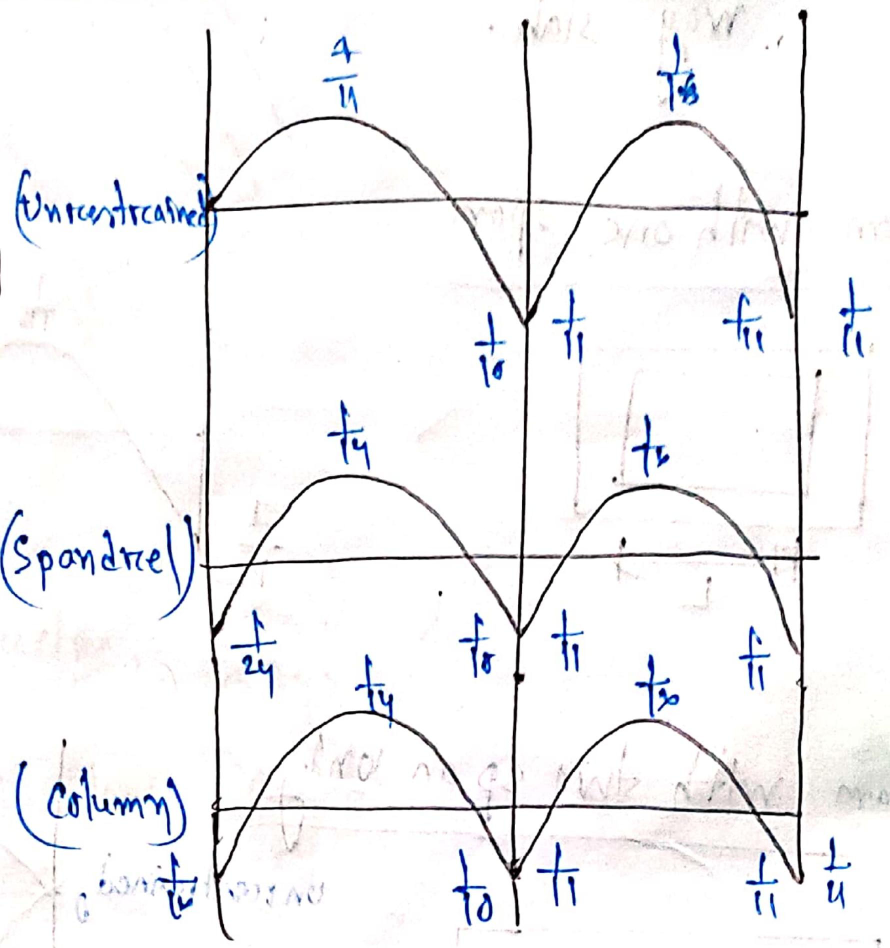
Beam with two span only





Beam with more than two Spans





Thickness calculation

Minimum thickness of one way slab

		thickness
simply supported	→	L/20
One end continuous	→	L/24
Both end continuous	→	L/24
cantilever	→	L/10

L = Clear span length

Design Step

(i) Thickness calculation

(ii) load calculation

WSD method, load = DL+LL

USD method, Load = 1.2DL+1.6LL

(iii) Moment calculation

2 span - 3 moment calculation

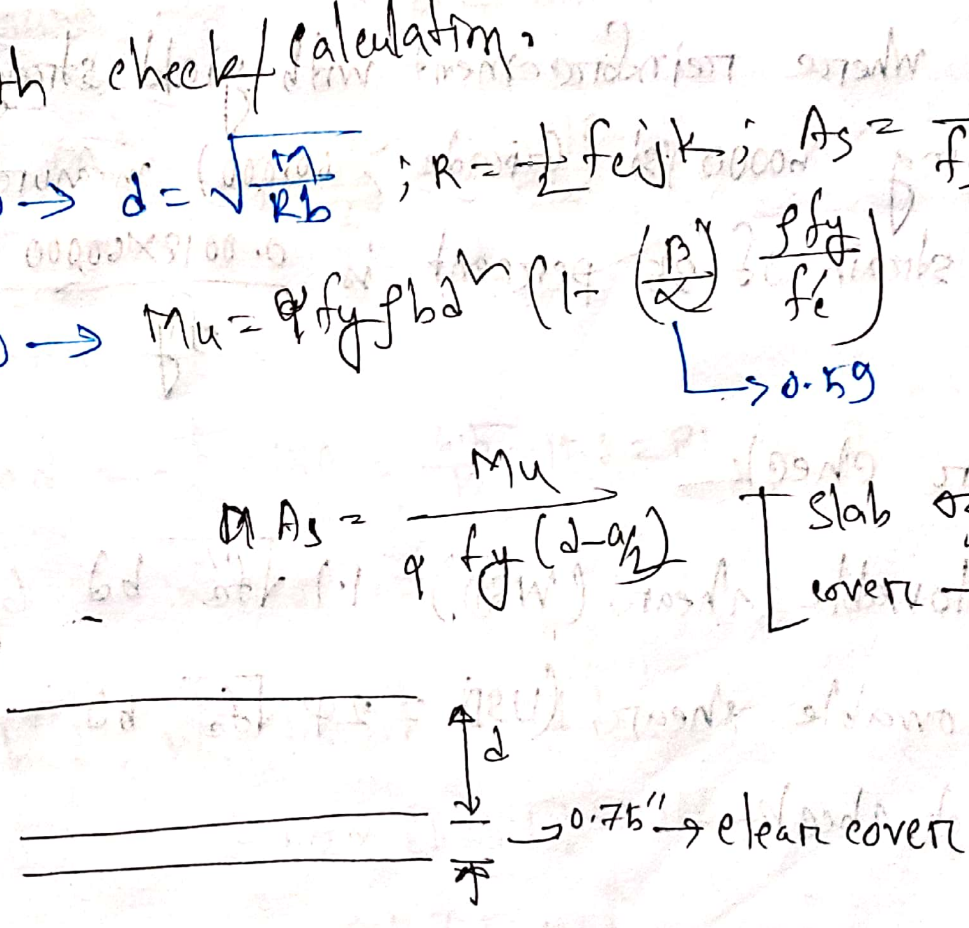
more of 2 span - 5 moment calculation

(iv) Depth check/calculation:

WSD → $d = \sqrt{\frac{M}{Rb}}$; $R = \frac{1}{2} f_c j k$; $A_s = \frac{M}{f_s j d}$

USD → $M_u = \phi f_y \rho b d^2 \left(1 - \underbrace{\left(\frac{\beta}{\alpha}\right)}_{\rightarrow 0.59} \frac{\rho f_y}{f_c'}\right)$

$A_s = \frac{M_u}{\phi f_y (d - a/2)}$ [Slab এর clear cover = 0.75'']



(v) Reinforcement calculation

WSD → $A_s = \frac{M}{f_s j d}$

USD → $A_s = \frac{M_u}{\varphi f_y (d - a/2)}$

* Slabs where Grade 40 or 50 deformed bars are used ' 0.0020

\# Slabs where Grade 60 bars or weeded wire fabric (smooth/deformed) are used 0.0018

* Slabs, where reinforcement with yield strength exceeding 60000 psi (Grade > 60,000) measured at yield strain of 35 percent is $\frac{0.0018 \times 60,000}{f_y}$

(vi) Shear check

Allowable shear (WSD) = $1.1 \sqrt{f_c'}\ bd$

Allowable shear, (USD) = $2\varphi \sqrt{f_c'}\ bd$ $\varphi = 0.75$

(vii) Bond check

Example- 13·1

One way slab design - A reinforced concrete slab is built integrally with its support and consists of two equal spans, each with a clear span of 15 ft. The service live load is 100 psf. and 4000 psi concrete is specified for use with steel with a ~~yeild~~ yield stress equal to 60,000 psi. Design the slabs following the provision of the ACI code.

@ USD

Soln: considering the slab ~~in bot~~ is continuous in both ~~en~~ end, $\therefore t = \frac{l}{28} = \frac{15 \times 12}{28} = 6.428 \approx 6.5$ in

load calculation

Dead load $= \frac{t}{12} \times 150 = \frac{6.5}{12} \times 150 = 81.25$ psf

live load $= 100$ psf

Total load, $W_u = 1.2 DL + 1.6 LL$

$= 1.2 \times 81.25 + 1.6 \times 100$

$= 257.5$ psf

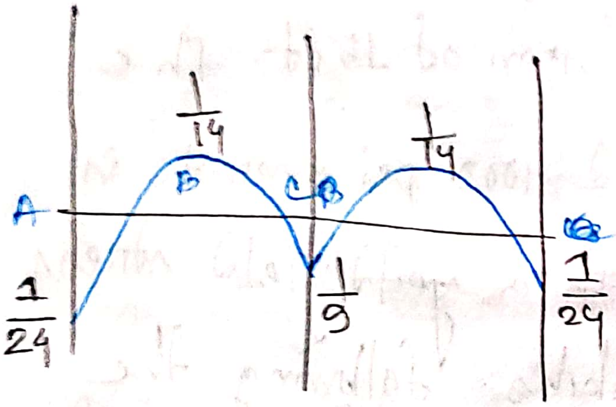
Moment Calculation

At exterior support (A) $= -\frac{WL^2}{24}$

$= -\frac{257.5\times15^2}{24}$

$= -2414.06$ lb-ft

$=$



At midspan, (B) $= \frac{WL^2}{14} = \frac{257.5\times15^2}{14}$

$= 4138.3928$ lb-ft

At interior support (C) $= -\frac{WL^2}{9} = -\frac{257.5\times15^2}{9}$

$= -6437.5$ lb-ft

(Max)

Depth check

$$Mu = \phi f_y \rho b d^2 \left(1 - \frac{\beta}{\alpha}\cdot\frac{\rho f_y}{f_c'}\right) \quad --- ①$$

The maximum ~~practical~~ practical reinforcement ratio by ACI code

$$\rho = (0.85)^2 \times \frac{4}{60} \times \frac{0.003}{0.003+0.004} = 0.021$$

$=$

① ⇒ $6437.5 \times 12 = 0.9 \times 60000 \times 0.0214 \times 12 \times d^2 \left(1 - 0.59 \times \frac{0.0214 \times 60}{4}\right)$

$d^2 = 6.9706$

$d = 2.6405$ in

$t = d + c.c + \phi/2$

$= 2.64 + 0.75 + \frac{0.5}{2}$

$= 3.63'' < 6.5$

$\therefore d_{effective} = 6.5 - cc - \phi/2$

$= 6.5 - 0.75 - 0.25$

$= 5.5$ inch

Steel calculation / Reinforcement calculation

At point C

$A_s = \frac{M_u}{\phi f_y (d - a/2)}$

$= \frac{6437.5 \times 12}{0.9 \times 60000 \times (5.5 - \frac{1.47 A_s}{2})}$

$a = \frac{A_s \times f_y}{0.85 f_c' \times b}$

$a = \frac{A_s \times 60000}{0.85 \times 4000 \times 12}$

$a = 1.47 A_s$

$A_s (5.5 - 0.735 A_s) = \frac{6437.5 \times 12}{0.9 \times 60000}$

$5.5 A_s - 0.735 A_s^2 = 1.4305$

$0.735 A_s^2 - 5.5 A_s + 1.4305 = 0$

$A_s = 0.27 \text{ in}^2$

providing #3 bars @ $\frac{0.11 \times 12}{0.27} = 4.88''$ c/c

At point B,

$$A_s = \frac{4138.39 \times 12}{0.9 \times 60,000 \times (5.5 - 0.735 A_s)}$$

$$\Rightarrow 5.5 A_s - 0.735 A_s^2 = 0.91964$$

$$\Rightarrow 0.735 A_s^2 - 5.5 A_s + 0.91964 = 0$$

$$A_s = 0.17112 \text{ in}^2$$

providing #3 bar @ $= \frac{0.11 \times 12}{0.17} = 7.76$" c/c

At point A,

$$A_s = \frac{2414.06 \times 12}{0.9 \times 60000 \times (5.5 - 0.735 A_s)}$$

$$0.735 A_s^2 - 5.5 A_s + 0.53645 = 0$$

$$A_s = 0.10 \text{ in}^2$$

providing #3 bar @ $= \frac{0.11 \times 12}{0.10} = 13.2$" c/c

Themperature shrinkage of reinforcement distribution $= 0.0018 \times b \times t$ [for $f_y = 60,000$ psi ratio is 0.0018]

$$= 0.0018 \times 12 \times 6.5$$

$$= 0.1404 \text{ in}^2$$

providing #3 bar @ $= \frac{0.11 \times 12}{0.14} = 9.43$" c/c

Sheare. check

Developed sheare,

$$V_{dev} = \frac{WL}{2} = \frac{257.5 \times 15}{2} = 1931.25 \text{ lb}$$

$$\text{Sheare stress}, (V_{stress}) = \frac{1931.25}{12 \times 5.5} = 29.261310 \text{ psi}$$

$$\text{Allowable shear} = 2\phi\sqrt{fc'}\, bd$$

$$= 2 \times 0.75 \times \sqrt{4000} \times 12 \times 5.5$$

$$= 6261.3097 \text{ lb}$$

Vallowable > Vdeveloped

So, design is ok

~~$V_{devo} = 1.15$~~ (A).

$$V_{dev} = 1.15 \times \frac{wL}{2} - \frac{wd}{12} \xrightarrow{} \text{Inch এ আসবে}$$

$$= 1.15 \times \frac{257.5 \times 15}{2} - \frac{257.5 \times 5.5}{12}$$

= ~~2102.~~ 2102.916 ~~16~~ lb

Vdev < Vallowable

Design is ok

Bond check

$$U_{dev} = \frac{V_{max}}{\Sigma_o \left(d - a/2\right)}$$

$$= \frac{1931.25}{2.79 \times \left(5.5 - \frac{0.39}{2}\right)}$$

$U_{dev} = 130.36$ lb/in²

[illegible] =

Here

dia of bars

$\Sigma_o = n \pi D$

$= \frac{b}{spacing} \times \pi D$

$\Sigma_o = \frac{12}{5.07} \times 3.1416 \times \frac{3}{8}$

$= 2.79$

$$a = \frac{A_s f_y}{0.85 \beta_1 b f'_c}$$

$$= \frac{0.27 \times 60000}{0.85 \times 4000 \times 12}$$

$= 0.397$

$V_{max} = \frac{wL}{2}$

$= \frac{257.5 \times 15}{2} = 1931.25$

$$U_{allwable} = \frac{6.7\sqrt{f'_c}}{D}$$

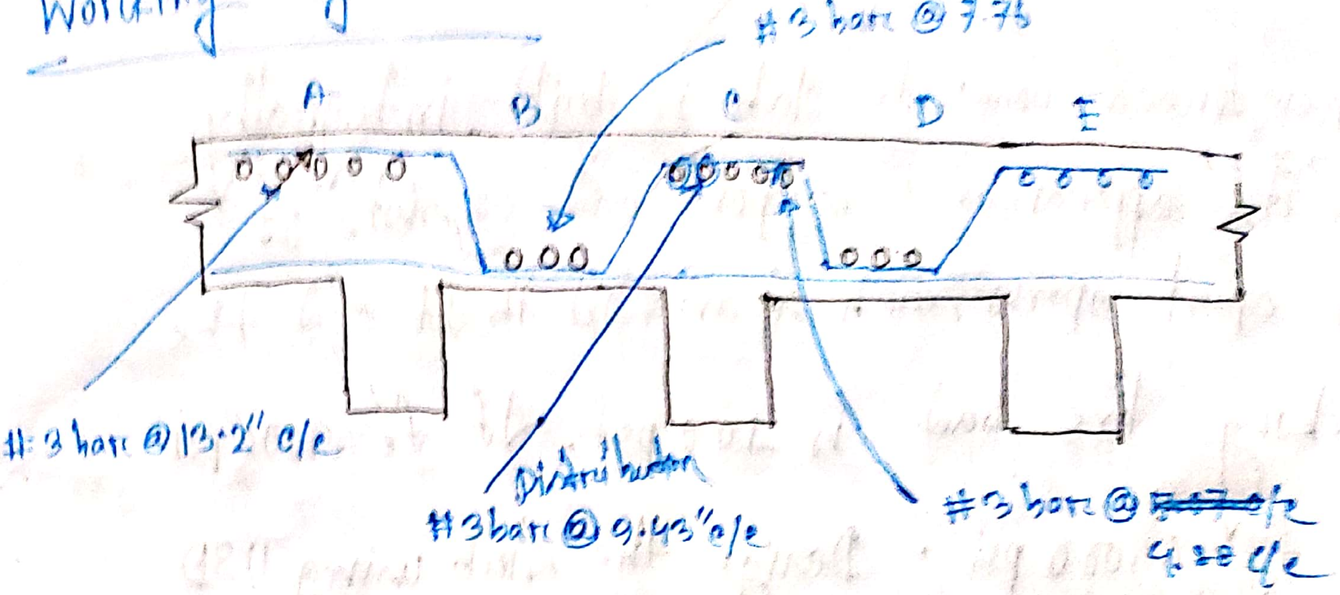
$$= \frac{6.7 \times \sqrt{4000}}{3/8}$$

$= 1129.9872$ lb

$U_{dev} < U_{all}$

Hence, design is ok.

Working diagram



At support A & E, No. of extra top $= \dfrac{0.1 - \frac{0.11}{2}}{0.11}$

$= 0.4 \simeq 1$ (providing #3 bar)

At support C, No. of extra top $= \left(\dfrac{0.26 - \frac{0.11}{2}}{0.11}\right)$

$= 1.86 \simeq 2$

(providing #3 bar)

Problem-2

A reinforced concrete slab is built integrally with its appearance support and consists of two equal spans, each span of 15 ft and the working live load is 100 psf. If $f_c' = 4000$ psi and $f_y = 60{,}000$ psi. Design the slab using USD

[WSD method]

Sol^n:

Thickness calculation

Considering both end continuous,

$$t = \frac{l}{28} = \frac{15 \times 12}{28} = 6.43 \text{ in} \simeq 6.5 \text{ inch}$$

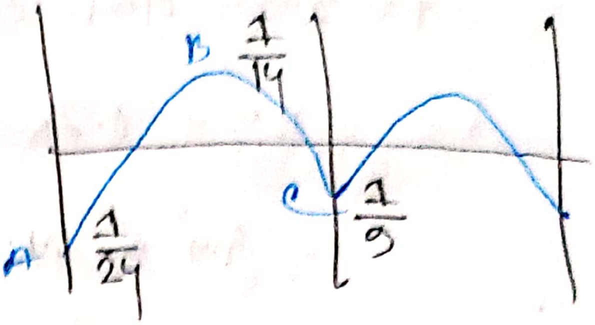
load calculation

$$DD = \frac{t}{12} \times 150 = \frac{6.5}{12} \times 150 = 81.25 \text{ psf}$$

$$LL = 100 \text{ psf}$$

Total load, $w = DD + LL = 81.25 + 100 = 181.25$ psf

Moment calculation



$$M_A = -\frac{wl^2}{24} = -\frac{181.25 \times 15^2}{24}$$

$$= -1699.2187 \text{ lb-ft}$$

$$M_B = \frac{wl^2}{14} = \frac{181.25 \times 15^2}{14} = 2912.9464 \text{ lb-ft}$$

$$M_C = -\frac{wl^2}{9} = -\frac{181.25 \times 15^2}{9} = -4531.25 \text{ lb-ft}$$

Depth check

$$d = \sqrt{\frac{M}{Rb}} \quad \text{--- ①}$$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.044 \simeq 8$$

$$r = \frac{f_s}{f_c} = \frac{0.4 f_y}{0.45 f_c'} = \frac{0.4 \times 60000}{0.45 \times 4000} = 13.33$$

$$k = \frac{n}{n+r} = \frac{8}{8+13.33} = 0.3750$$

$$j = 1 - k/3 = 1 - \frac{0.3750}{3} = 0.875$$

(V) $R = \frac{1}{2} f_c j k = \frac{1}{2} \times (0.45 \times 4000) \times 0.875 \times 0.375$

~~d =~~ $= 295.31$

① ⇒ $d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{\cancel{295.31}\ 4531.25 \times 12}{295.31 \times \boxed{12}}}$ ⟶ one way slab এর ক্ষেত্রে always use 12

$= 3.917$

$t = 3.9 + \text{clear cover} + \varphi/2$

$= 3.9 + 0.75 + 0.45$

$= 4.9' \leq 6.5''$

ok

~~Aes~~ effective depth,

$d_{eff} = 6.5 - \overset{cc}{0.75} - \overset{\varphi/2}{0.25}$

$= 5.5''$

Reinforcement calculation

For A,

$A_s = \frac{M_A}{f_s j d} = \frac{1699.2 \times 12}{0.4 \times 60,000 \times 0.875 \times 5.5}$

$= 0.18 \text{ in}^2$

(providing #3 bar @ $\frac{0.11 \times 12}{0.18} = 7.33''$ c/c)

For B, $A_s = \frac{M_B}{f_s j d} = \frac{2912.9464 \times 12}{0.4 \times 60,000 \times 0.875 \times 5.5}$

$= 0.30 \text{ in}^2$

(providing #3 bar @ $\frac{0.11 \times 12}{0.30} = 4.4''$ c/c)

For C, $A_s = \frac{M_C}{f_s j d} = \frac{9531.25 \times 12}{0.4 \times 60000 \times 0.875 \times 5.5} = 0.47 \text{ in}^2$

(providing #3 bars @ $= \frac{0.11 \times 12}{0.4} = 2.81''$ c/c)

Reinforcement Distribution

$A_s = 0.0018 \times bt$ → assume (one way slab এর ক্ষেত্রে always same)

$= 0.0018 \times 12 \times 0.5$

$= 0.14 \text{ in}^2$

providing #3 bar @ $= \frac{0.11 \times 12}{0.14} = 9.43''$ c/c

Shear check

$V_{developed} = 1.15 \times \frac{WL}{2} - \frac{Wd}{12}$ → always in inch

$= 1.15 \times \frac{181.25 \times 15}{2} - \frac{181.25 \times 5.5}{12}$

$= 1480.21$ lb

$V_{allowable} = 1.1 \sqrt{f'_c}\, bd = 1.1 \times \sqrt{4000} \times 12 \times 5.5$

$= 4591.63$ lb

$V_{all} > V_{dev}$ ∴ so design is ok

Bond check

$$U_{developed} = \frac{V_{max}}{\Sigma_o j d}$$

$$= \frac{1359.38}{5.03 \times 0.875 \times 5.5}$$

$$U_{dev} = 56.16 \text{ psf}$$

$$U_{all} = \frac{3.4\sqrt{f_c'}}{D} \leq 350$$

$$= \frac{3.4 \times \sqrt{4000}}{3/8}$$

$$= 573.43$$

$$U_{dev} < U_{all}$$

So, design is ok.

$$V_{max} = \frac{wl}{2}$$

$$= \frac{181.25 \times 15}{2}$$

$$= 1359.38 \text{ lb}$$

$$\Sigma_o = n \pi D$$

$$= \frac{b}{spacing} \times \pi \times \frac{3}{8}$$

$$= \frac{12}{2.81} \times \pi \times \frac{3}{8}$$

$$= 5.03$$

$$j = 0.875$$

$$d = 5.5''$$

Working diagram

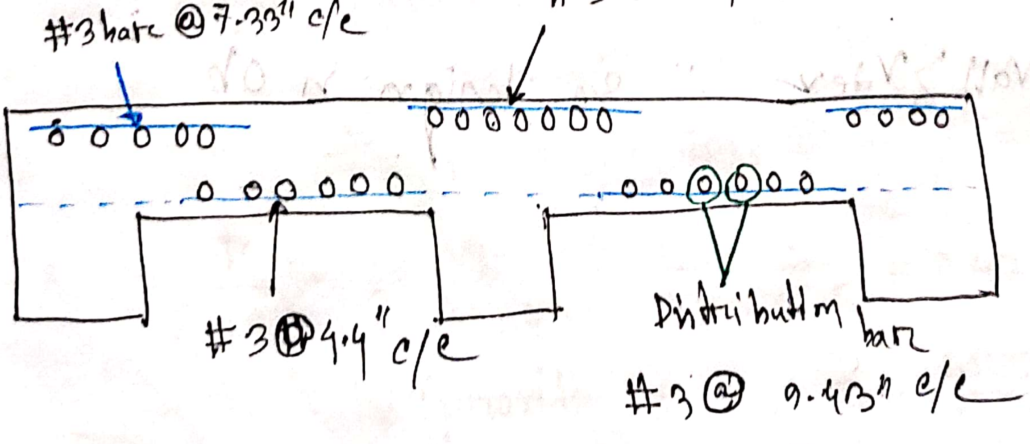


Fig - Reinfocement detal of slab

NB:

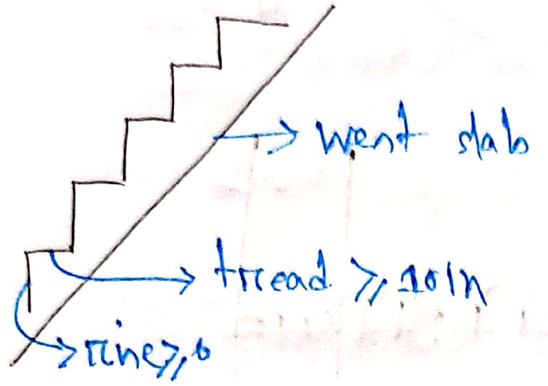
Main reinforcement → Shorter direction

Distribution " → long direction

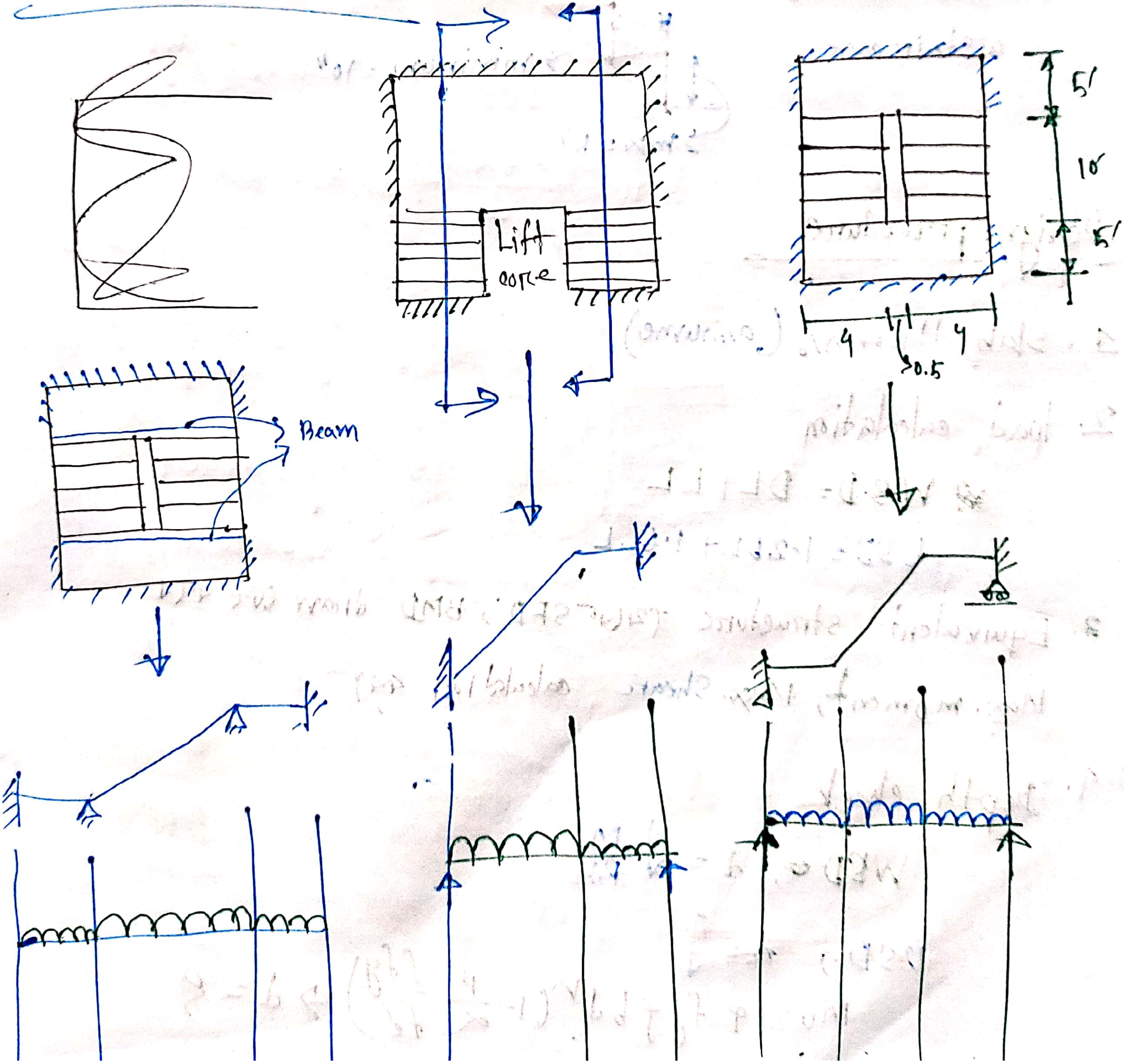
Horizontally " → Top bars

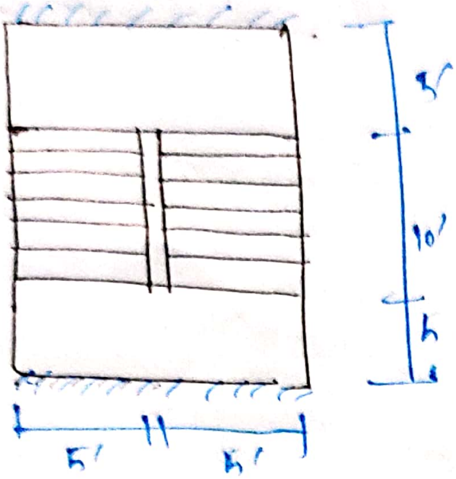
Vertically → Distribution bars.

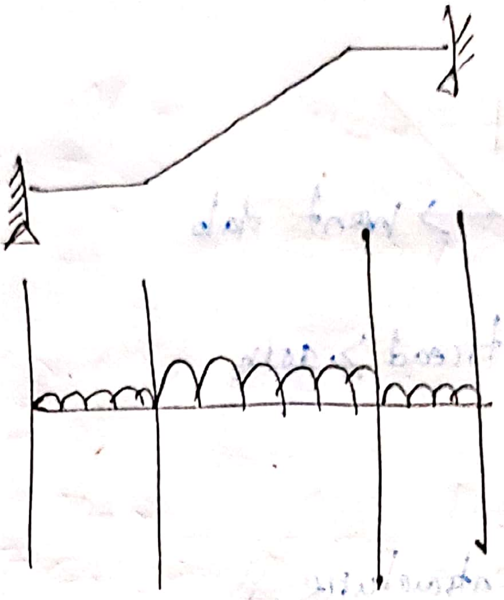
Staire Case.



equivalent structure

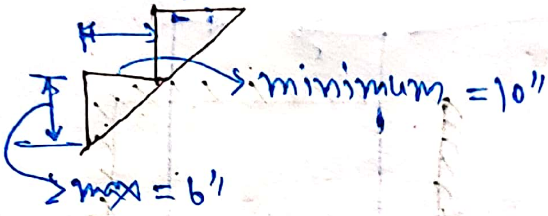






7'x15.4'

minimum area of a staircase



Design procedure

1. Slab thickness (assume)

2. load calculation

   W.S.D = DL+LL

   USD = 1.2DL + 1.6LL

3. Equivalent structure থেকে SFD, BMD draw ~~করে~~ করে– Max. moment, Max. Sheare calculation করব

4. Depth check

   WSD, $d = \sqrt{\frac{M}{Rb}}$

   ~~USD, $d = \sqrt{\quad}$~~

   $Mu = \varphi f_y \rho b d^2 (1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}) \Rightarrow d = ?$

$$\rho = \rho_{max}$$

$$\rho_{max} = 0.85\,\beta_1 \frac{f_c'}{f_y}\,\frac{\overset{\to 0.003}{\varepsilon_u}}{\varepsilon_u + \underset{\to 0.004}{\varepsilon_y}}$$

5. Reinforcement calaculation

WSD → $A_s = \dfrac{M}{f_s j d}$

USD → $A_s = \dfrac{M_u}{\varphi f_y (d - a/2)}$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Distrubution reindorcement bars

$A_s = 0.002\,bt$ → when $f_y \leq 60000$

$A_s = 0.0018\,bt$ → when $f_y = 60,000$

$A_s = \dfrac{0.0018 \times 60000}{f_y}\,bt$ → when $f_y > 60,000$

6. Shear check

WSD → $V_{dev} = 1.15\dfrac{wL}{2} - \dfrac{wd}{2}$

$V_{all} = 1.1\sqrt{f_c'}\,bd$

USD → V

7. Bond check

WSD →

$$U_{dev} = \frac{V_{max}}{\Sigma_o jd} \; ; \; V_{max} = \frac{wL}{2}$$

$$\Sigma_o = n\pi D$$

$$U_{all} = \frac{3.4\sqrt{f_c'}}{D} \le 350 \text{ psi}$$

USD →

$$U_{dev} = \frac{V_{max}}{\Sigma_o(d-a/2)} \; ; \; a = \frac{A_s f_y}{0.85 f_c' b}$$

$$V_{max} = \frac{wL}{2}$$

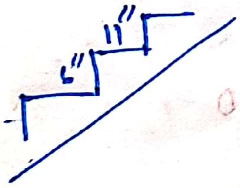
$$U_{all} = \frac{5.6\sqrt{f_c'}}{D} \le 560 \text{ psi}$$

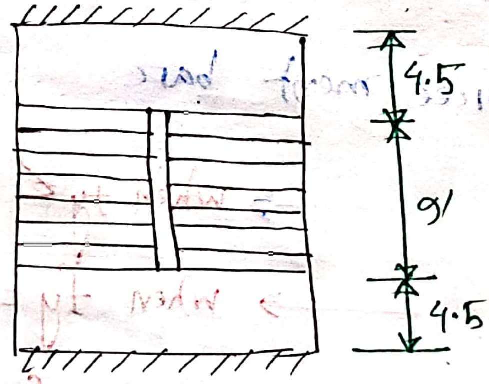
Math }

USD

T = 11"

R = 6"





$f_c' = 3000$ psi

$f_y = 50000$ psi

LL = 90 psf

Sol^n:

1. Slab thickness, t = 8" (assumed)

2. load calculation

weight od straight part

$$= \frac{8}{12} \times 150 = 100 \text{ psf}$$

wt. of inclined part

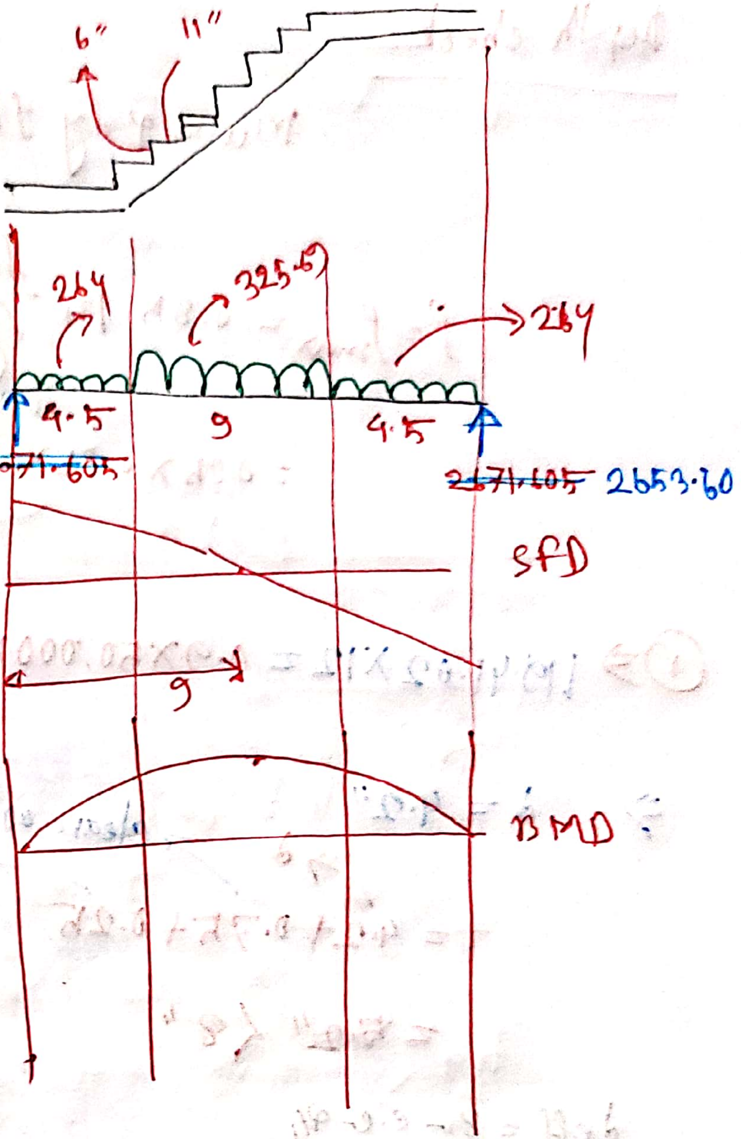
$$= 100 \times \frac{\sqrt{11^2+6^2}}{11} = 1139 \text{ psf}$$

Dead load for steps

$$= \frac{R}{12 \times 2} \times \frac{11}{11} \times 150$$

$$= \frac{6}{24} \times \frac{11}{11} \times 150 = 37.5 \text{ psf}$$

total dead load for inclined part $= 1139 + 37.5 = 151.41$ psf



Factor load for straight part $= 1.2DL + 1.6LL$

$$W_u = 1.2 \times 100 + 1.6 \times 90 = 264 \text{ psf}$$

factor load for inclined part

$$= 1.2DL + 1.6LL$$

$$W_u = 1.2 \times 151.41 + 1.6 \times 90$$

$$= 325.69 \text{ psf}$$

$$\text{Shear} = \frac{(264 \times 4.5 \times 2) + (325.69 \times 9)}{2}$$

$= ~~2671.605~~$ lb $= 2653.60$

From BMD

$$M_{max} = 2653.60 \times 9 - 264 \times 4.5 \times (4.5 + 2.25) - 325.69 \times 4.5 \times \frac{4.5}{2}$$

$$= 11941.02 \text{ lb-ft}$$

Depth check

$$M_u = \varphi f_y \rho b d^2 \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right) \quad \text{— (r)}$$

$$\rho = \rho_{max} = 0.85 \beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_y + \varepsilon_y}$$

$$= 0.85 \times 0.85 \times \frac{3000}{60000} \times \frac{0.003}{0.003 + 0.004} = 0.019$$

$$(r) \Rightarrow 11941.02 \times 12 = 0.9 \times 50{,}000 \times 0.019 \times 12 \times d^2 \left(1 - 0.59 \frac{0.019 \times 50000}{3000}\right)$$

$$\Rightarrow d = 4.2''$$

$$t = \overset{d}{4.2} + \overset{\text{clear cover}}{0.75} + \overset{a/2}{0.25}$$

$$= 5.2'' < 8'' \qquad \text{OK} \qquad [\text{or first trial needed}]$$

$$d_{eff} = 8 - c.c - a/2$$

$$= 8 - 0.75 - 0.25 = 7''$$

reinforcement calculation

$$A_s = \frac{M_u}{\varphi f_y (d - a/2)} \quad \text{— (1')} \qquad [d_{eff} = 7'']$$

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

$$a = \frac{A_s \times 50000}{0.85 \times 3000 \times 12}$$

$$a = 1.63 A_s$$

$$a/2 = 0.815 A_s$$

$$A_s = \frac{11941.02 \times 12}{0.9 \times 50{,}000 \times (7 - 0.815 A_s)}$$

$$A_s = 0.48 \text{ in}^2$$

providing #4 bars @ $\frac{\overset{\text{area of \#4 bar}}{0.2} \times 12}{0.48} = 5''$ c/c

Distribution reinforcement

$A_s = 0.002\,bt$

$= 0.002 \times 12 \times 8$

$= 0.192\ \text{in}^2$

providing #3 bars @ $\frac{0.11 \times 12}{0.19} = 6.95''$ c/c

$= 7''$ c/c

Shear check

$V_{dev} = \frac{V_{max}}{bd} = \frac{2653.36}{12 \times 7} = 31.59$ psi

$V_{all} = 2\varphi\sqrt{f_c'} = 2 \times 0.75 \sqrt{3000} = 81.15$ psi

$V_{all} > V_d$ ok

Bond check

$U_{dev} = \frac{V_{max}}{\Sigma_0 j d}$

$= \frac{2653.36}{3.14 \times 0.875 \times 7}$

$= 137.97$ psi

$\Sigma_0 = n \pi d$

$= \frac{b}{\text{spacing}} \times \pi \times d$

$= 2 \times \pi \times 0.5$

$= 3.1416$

$j = \frac{7}{8} = 0.875$

$U_{all} = \frac{5.6\sqrt{f_c'}}{D} \leq 560$ psi

$= \frac{5.6 \times \sqrt{3000}}{0.5} \leq 560$ psi

$= 613.45 \simeq 560$

Working diagram of staircase

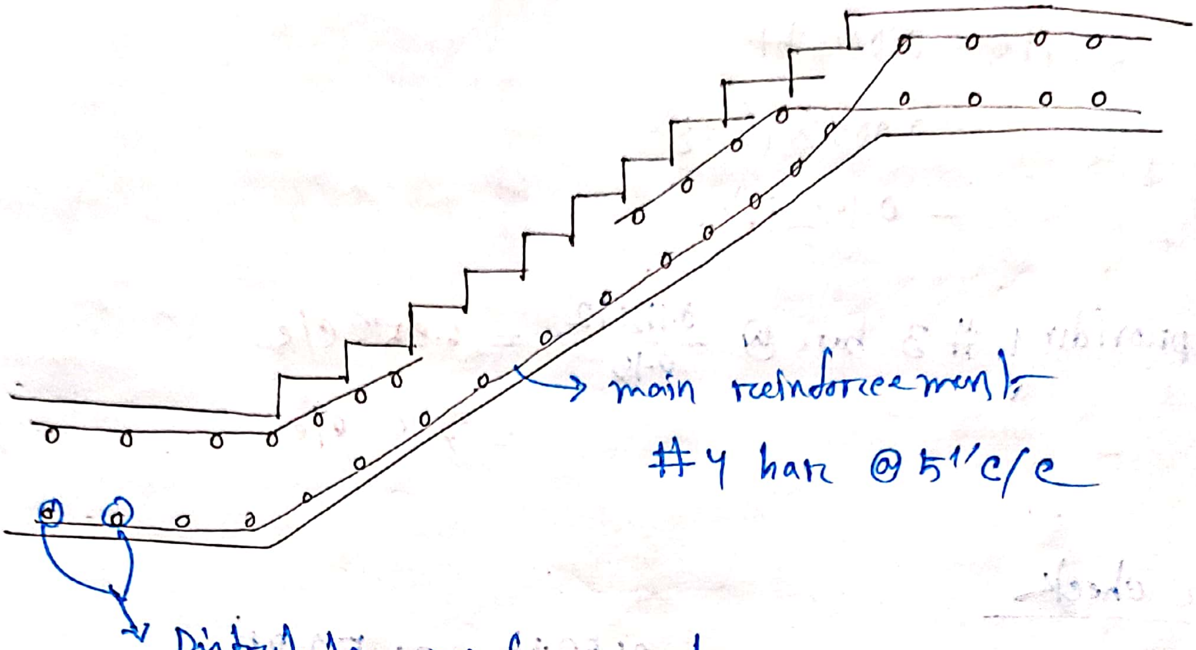
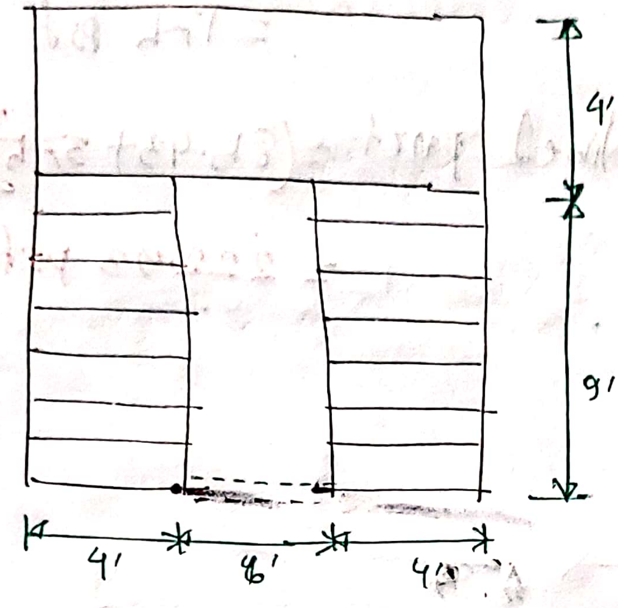


Fig: Reinforcement detail of staircase.

Staire case math on WSD method

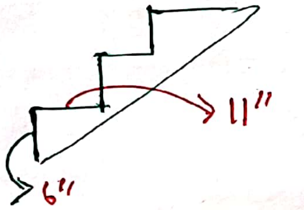


$LL = 100$ psf

$f_c' = 3000$ psi

$f_y = 50{,}000$ psi

Design the staircase using $\boxed{\text{WSD}}$ method



Solⁿ:

① assume, Slab thickness = 6"

⑪ Dead load for straight part $= \frac{6}{12} \times 150 = 75$ psf

Dead load for inclined part $= 75 \times \frac{\sqrt{11^2+6^2}}{11}$

$= 85.43$ psf

load due to step $= \cancel{\frac{R}{2} \times 150}$ $\cancel{\frac{\text{Riskex trade}}{2}}$

$$= \frac{\frac{\text{Rise}}{2} \times \text{trade} \times 150}{11}$$

$$= \frac{6}{12 \times 2} \times \frac{11}{11} \times 150 = 37.5 \text{ psf}$$

Total load for straight part = DL+LL

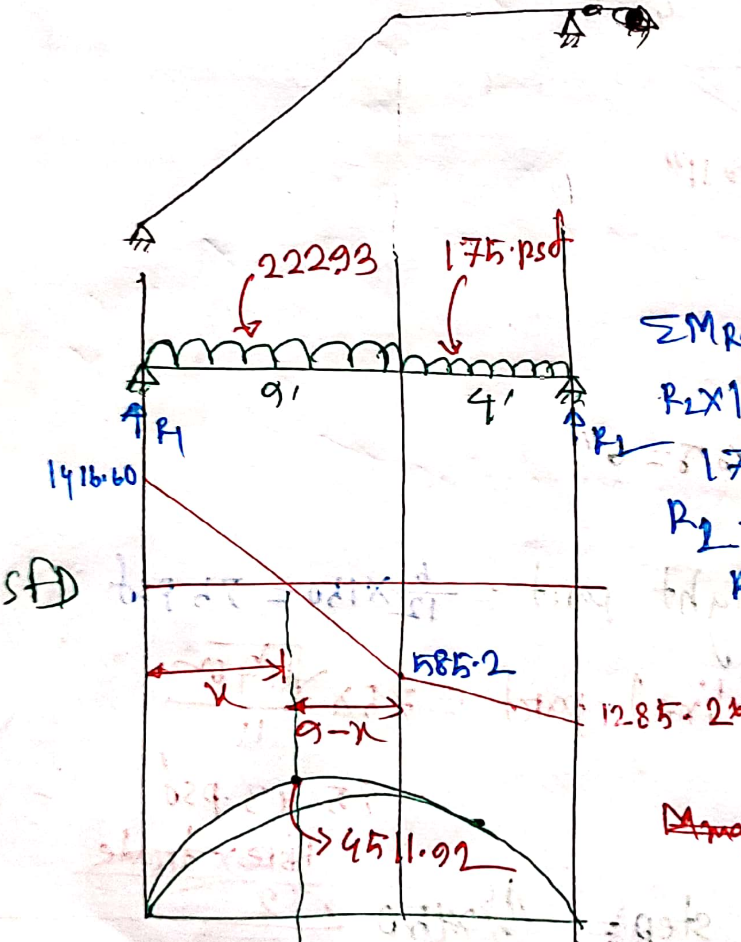
= 75+100

= 175 psf

Total load for inclined part = (85.43+37.5)+100

= 222.93 psf

Equivalent Structure



$\Sigma M_{R_1} = 0$

$R_2 \times 13 - 222.93 \times 9 \times \frac{9}{2} - 175 \times 4 \times (9+2) = 0$

$R_2 = 1285.26$

$R_1 = 1416.60$

$\frac{1416.60}{585.2} = \frac{x}{9-x}$

~~Mmax~~ $\Rightarrow 6x = 6.37$

$R = \frac{1}{2} f_c j k$

$M_{max} = 1416.60 \times 6.37 - 222.93 \times \frac{6.37^2}{2}$

$= 4511.92$ lb-ft

Depth check

$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{3000}} = 9.29 \simeq 9$$

$$r = \frac{f_s}{f_c} = \frac{0.4\times f_y}{0.45 f_c'} = \frac{0.4\times50000}{0.45\times3000} = 14.8$$

$$k = \frac{n}{n+r} = \frac{9}{9+14.8} = 0.38$$

$$j = 1 - k/3 = 1 - \frac{0.38}{3} = 0.87$$

$$R = \frac{1}{2} j k f_c = \frac{1}{2}\times0.87\times0.38\times0.45\times3000 = 223.16$$

$$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{4511.92\times12}{223.16\times12}} = 4.50 \text{ inch}$$

$t = 4.5 + 0.75 + 0.25 = 5.50" < 6"$ OK

$d_{eff} = 6 - c.c - \phi/2$

$= 6 - 0.75 - 0.25 = 5"$

Reinforcement calculation

$$A_s = \frac{M}{f_s j d} = \frac{4511.92\times12}{0.4\times50{,}000\times5} = 0.62\ in^2$$

providing #4 bars @ $\frac{0.20\times12}{0.62} = 3.87"$ c/c

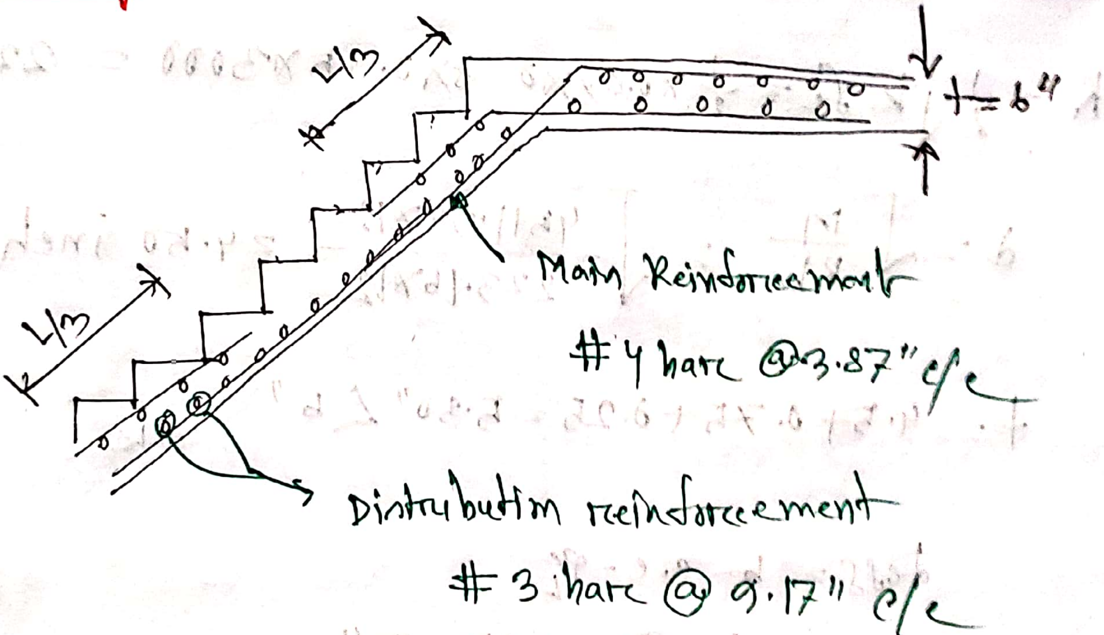
Distribution Reinforcement

$A_s = 0.002\, bt$

$= 0.002 \times 12 \times 6 = 0.144\, in^2$

providing #3 bars @ $\frac{0.11 \times 12}{0.144} = 9.17''$ c/c

Working diagram



Main Reinforcement
#4 bars @ 3.87" c/c

Distribution reinforcement
#3 bars @ 9.17" c/c

fig: Reinforcement detail of staircase