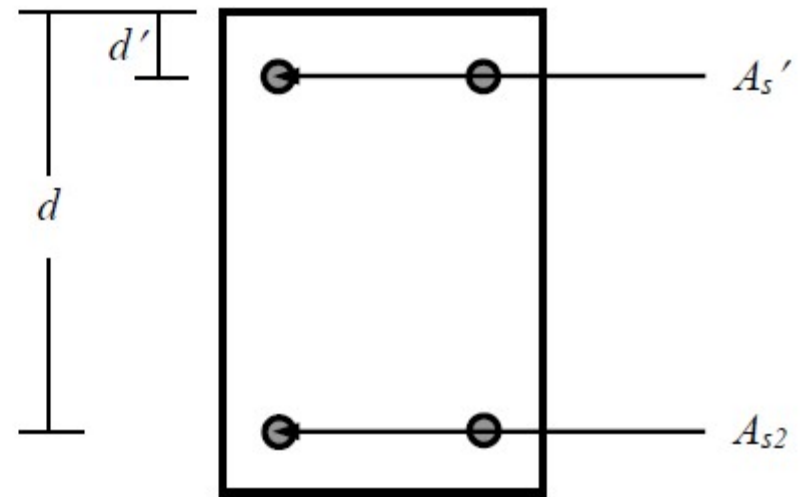
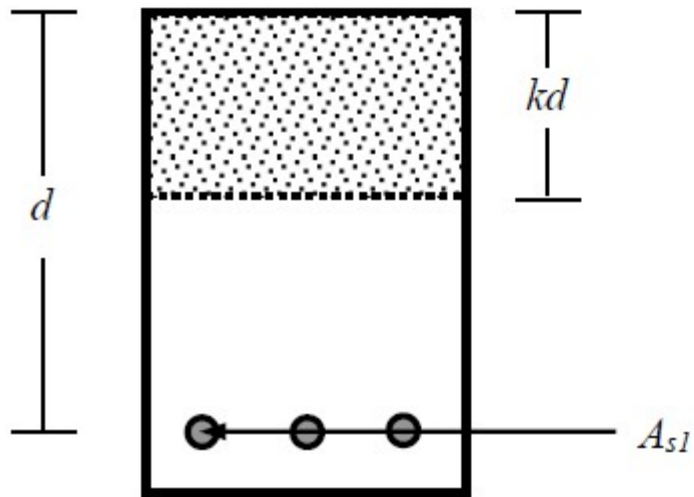
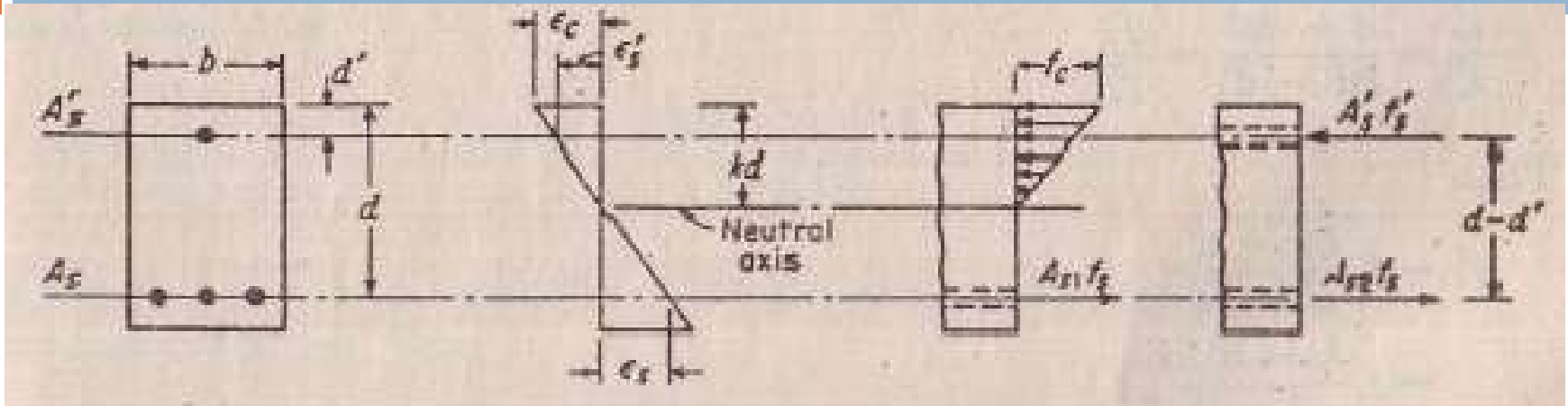


ANALYSIS AND DESIGN OF DOUBLY REINFORCED BEAMS BY WSD METHOD



The moment M_1 is given by

$$M_1 = Rbd^2 = A_{s1} (f_s j d)$$

The required steel area (A_{s1}) resisting the moment M_1 is given by

$$A_{s1} = M_1 / (f_s j d)$$


The remaining moment M_2 is resisted by additional tensile steel (area A_{s2}) and compression steel (area A_s')

$$M_2 = A_{s2} f_s (d - d') = A_s' f_s' (d - d')$$

The additional tensile reinforcement $A_{s2} = M_2 / [f_s (d - d')]$

The total tensile reinforcement $A_s = A_{s1} + A_{s2} = M_1 / (f_s j d) + M_2 / [f_s (d - d')]$

the compression reinforcement $A_s' = M_2 / [f_s' (d - d')]$



The compressive stress f_s' can be evaluated based on the strain compatibility:

$$\varepsilon_s/(d-kd) = \varepsilon_s'/(kd-d') \Rightarrow f_s' = f_s (k-d'/d)/(1-k)$$

To approximate the effects of nonlinear stress-strain behavior and plastic flow of concrete, ACI code provisions specify that effective modular ratio of $2E_s/E_c$ be used to transform the steel area to concrete,

$$f_s' = 2f_s(k-d'/d)/(1-k) \leq f_s$$

A beam with $b = 12$ in; $d = 18$ in. Tension reinforcement 4#10 bars
 Compression reinforcement 3#8 bars placed 2.5 in below compression
 face. $f_s = 24$ ksi $f_c = 1800$ psi. Compute allowable working moment

$$k = 0.375 \quad j = 0.875$$

$$M_1 = 0.5 f_c j k b d^2 = 1150 \text{ in-kips}$$

$$A_{s1} = M_1 / (f_s j d) = 3.05 \text{ in}^2$$

$$A_{s2} = A_s - A_{s1} = 5.06 - 3.05 = 2.01 \text{ in}^2$$

$$f_s' = 2 f_s (k - d'/d) / (1 - k) \leq f_s$$

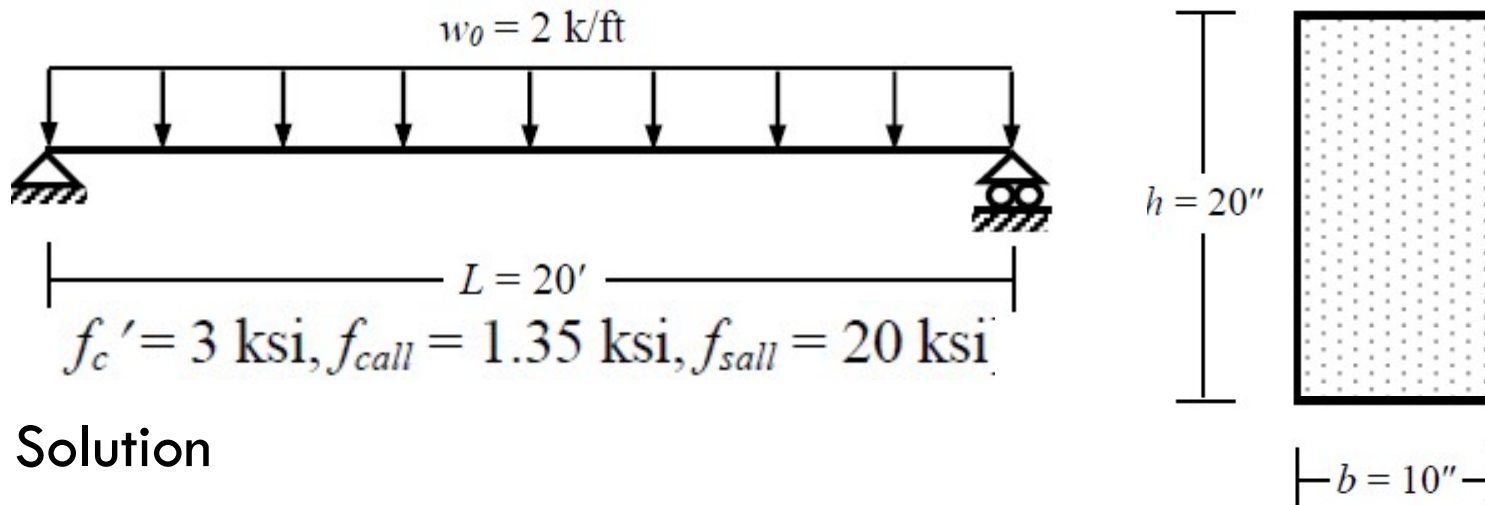
$$f_s' = 18.1 \text{ ksi}$$

$$A_{s2} f_s = 2.01 * 24 = 48.2 \text{ k}$$

$$A_s' / f_s' = 2.37 * 18.1 = 42.5 \text{ k}$$

$$M_2 = A_s' / f_s' (d - d') = 42.5 * (18 - 2.5) = 600 \text{ k-in}$$

Use the WSD Method to design the simply supported RC beam loaded as shown below, in addition to its self-weight



Solution

$$f_c' = 3 \text{ ksi}; f_c = 0.45f_c' = 1.35 \text{ ksi} \quad f_s = 20 \text{ ksi}$$

$$n = 9 \quad r = 14.81 \quad k = 0.378; \quad j = 1 - k/3 = 0.874$$

$$R = 0.5f_c k j = 0.223 \text{ ksi}$$

Self-weight of beam = $0.15 \times 10 \times 20 / (12 \times 12) = 0.208$ k/ft

Total load $w = 2 + 0.208 = 2.208$ k/ft

Maximum bending moment $M_{max} = wL^2/8 = 2.208 \times 20^2/8 = 110.4$ k-ft

Assuming two layers of bottom steel, $d = 20 - 4 = 16''$

$$M_1 = Rbd^2 = 0.223 \times 10 \times 16^2/12 = 47.56 \text{ k-ft}$$

$M_1 < M$ Doubly reinforced design is required

$$M_2 = M - M_1 = 62.85 \text{ k-ft}$$

$$A_s = A_{s1} + A_{s2} = M_1 / (f_s j d) + M_2 / [f_s (d - d')]$$

$$= 47.56 \times 12 / (20 \times 0.874 \times 16) + 62.85 \times 12 / [20 \times (16 - 2.5)]$$

$$= 2.04 + 2.79 = 4.83 \text{ in}^2; \text{ i.e., 6 \#8 Bars in two layers}$$