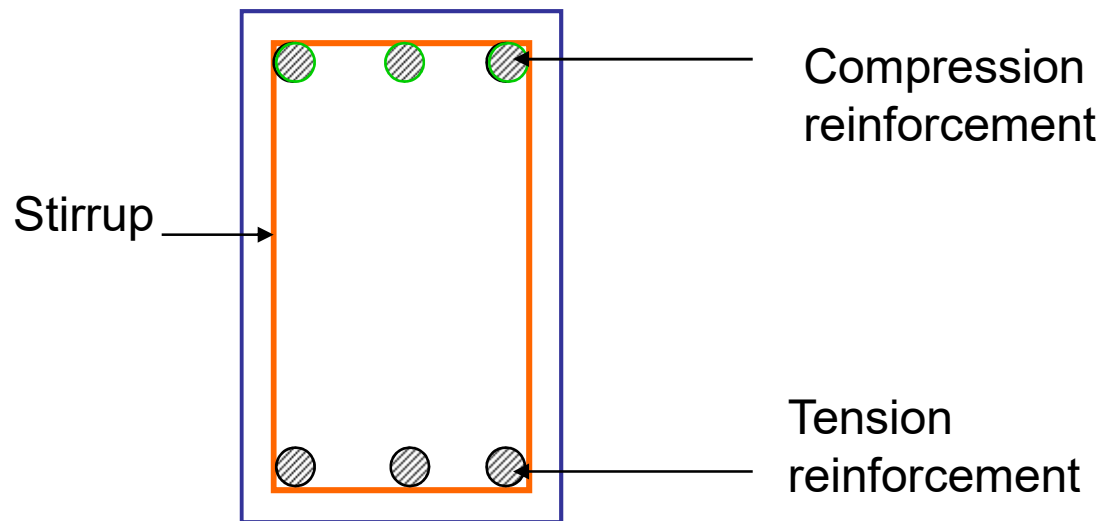


CE 3115 REINFORCED CONCRETE-I
BEAMS WITH COMPRESSION REINFORCEMENT

What is a doubly reinforced beam?

- **When a beam section is limited to specific dimension, concrete can not develop the compression force to resist the given bending moment and reinforcement is added in the compression zone, such a beam is known as doubly reinforced beam**



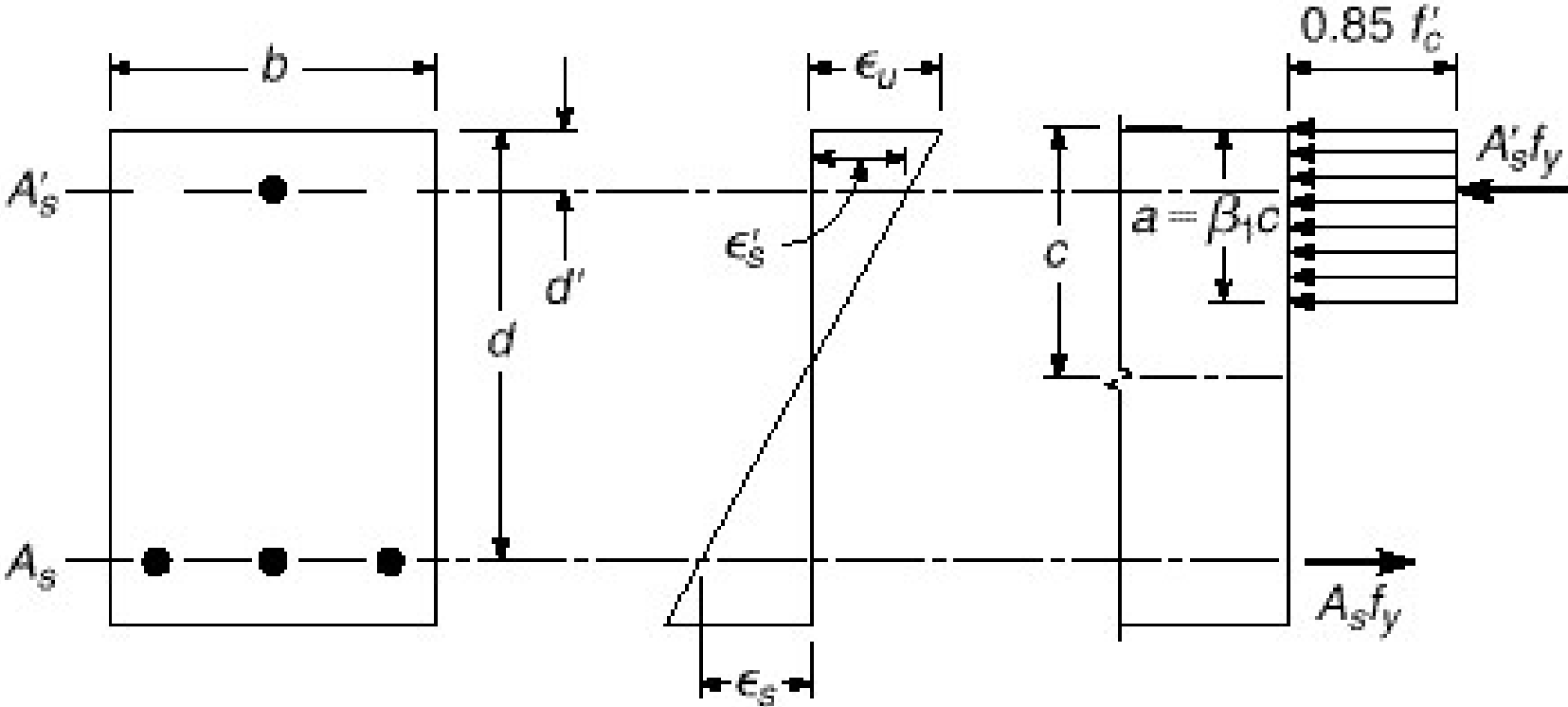
Why reinforcements are provided in compression zone?

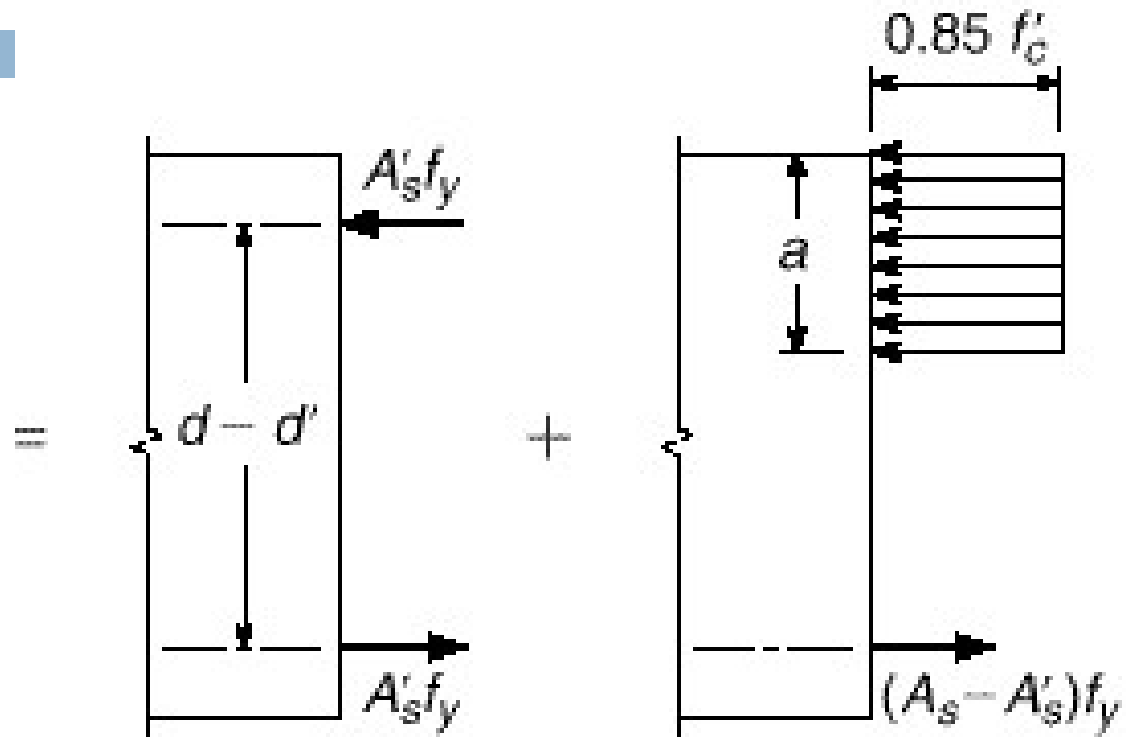
- **When a section is limited to specific dimension due to architectural reasons such as a need for limited headroom in multistory buildings**
- **Long-term deflection is reduced with the inclusion of reinforcement in compression zone**
- **Increases ductility.**
- **Change of mode of failure from compression to tension.**
- **Fabrication ease.**

Analysis of Doubly Reinforced Beams

-
- Two cases may arise while analyze a beam with reinforcement in compression zone
- Both of the tension and compression steel are at yield stress
- Compression steel below the yield stress

Both of the tension and compression steel are at yield stress





- Let,
- $A_s =$ Area of tension steel
- $A_s' =$ Area of compression steel
- $d' =$ Distance of the compression steel from the compression face
- $d =$ effective depth
- Let the total resisting moment $M_n = M_{n1} + M_{n2}$
- Moment M_{n1} is provided by the couple consisting of the force provided by A_s' and equal area of tension steel. Therefore,

$$M_{n1} = A_s' f_y (d - d')$$

- Moment M_{n2} is provided by the couple consisting of the remaining tension steel ($A_s - A'_s$) and the concrete compression.

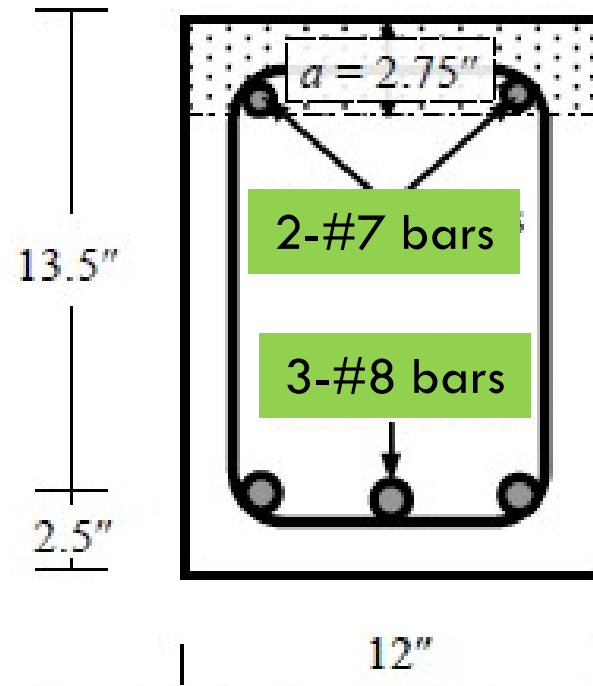
- Therefore,
$$M_{n2} = (A_s - A'_s) f_y \left(d - \frac{a}{2} \right)$$

$$a = \frac{(A_s - A'_s) f_y}{0.85 f'_c b}$$

- Total resisting moment,

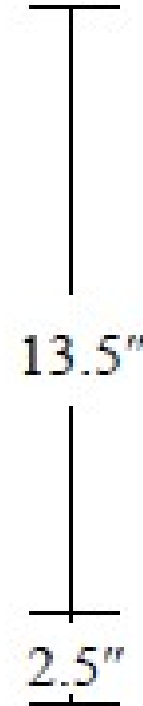
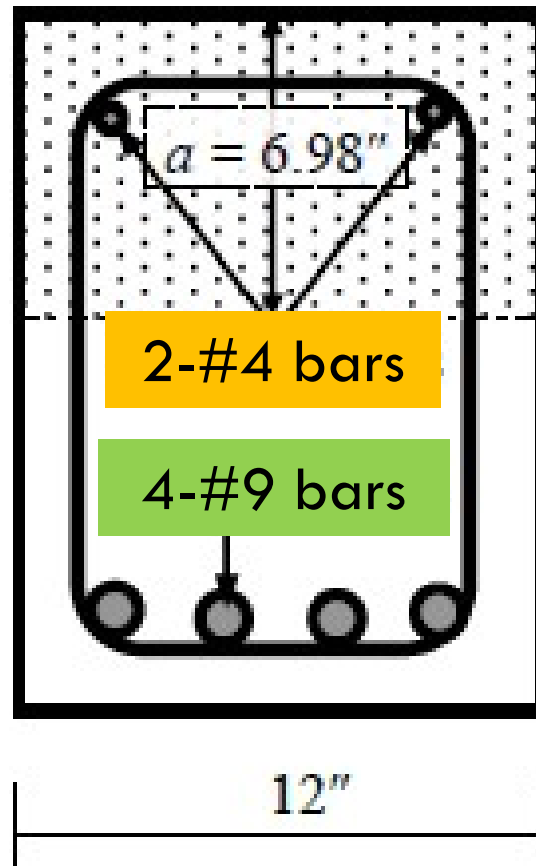
$$M_n = M_{n1} + M_{n2} = A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2} \right)$$

Calculate the ultimate moment capacities of the RC sections shown below



$$f_c' = 3 \text{ ksi}, f_y = 60 \text{ ksi}$$

Calculate the ultimate moment capacities of the RC sections shown below



12"

$$f_c' = 3 \text{ ksi}, f_y = 60 \text{ ksi}$$

Solution:

$$A_s = 4 * 1.0 = 4.0 \text{ in}^2$$

$$A_s' = 2 * 0.2 = 0.40 \text{ in}^2$$

$$\rho = \frac{A_s}{bd} = \frac{4}{12 * 13.5} = 0.0247$$

$$\rho' = \frac{A_s'}{bd} = \frac{0.40}{12 * 13.5} = 0.0024$$

$$\rho_{\max} = 0.85\beta_1 \frac{f_c'}{f_y} \frac{\epsilon_u}{\epsilon_u + 0.004} = 0.0154$$

- **If the reinforcement ratio is less than this limiting value, the compression steel stress at failure is less than the yield stress.**

- The balanced reinforcement ratio is $\bar{\rho}_b = \rho_b + \rho' \frac{f'_s}{f_y}$

- where, $\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

$$f'_s = E_s \varepsilon'_s = E_s \left[\varepsilon_u - \frac{d'}{d} (\varepsilon_u + \varepsilon_y) \right] \leq f_y$$

- 
- The maximum reinforcement ratio permitted by the ACI Code is

$$\bar{\rho}_{\max} = \rho_{\max} + \rho' \frac{f'_s}{f_y}$$

$$\text{where } f'_s = E_s \left[\varepsilon_u - \frac{d'}{d} (\varepsilon_u + 0.004) \right]$$

Minimum tensile reinforcement ratio that to ensure the yielding of compression steel at failure


$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$$

$$\bar{\rho}_{cy} = 0.85 * 0.85 \frac{3}{60} \frac{2.5}{13.5} \frac{0.003}{0.003 - 0.0021} + 0.0014 = 0.024$$

$\bar{\rho}_{cy} > \rho$ Compression steel yields at failure

$$a = \frac{(A_s - A'_s)f_y}{0.85f'_c b} \quad a = \frac{(4 - 0.40) * 60}{0.85 * 3 * 12} = 7.05in \quad c = a/\beta_1 = 8.3 in$$

$$f'_s = E_s \epsilon_s = E_s * \epsilon_u * \frac{c - d'}{c} = 29000 * 0.003 * \frac{8.3 - 2.5}{8.3} = 60.8ksi$$


$$\begin{aligned}M_n &= A'_s f_y (d - d') + (A_s - A'_s) f_y \left(d - \frac{a}{2} \right) \\ &= 0.4 * 60 * (13.5 - 2.5) + (4 - 0.4) * 60 * \left(13.5 - \frac{7.05}{2} \right) =\end{aligned}$$

COMPRESSION STEEL BELOW THE YIELD STRESS

□ Whether or not the compression steel will have yielded at failure can be determined as follows

□ From geometry, $\frac{c}{d'} = \frac{\epsilon_u}{\epsilon_u - \epsilon_y}$

□ Or $c = \frac{\epsilon_u}{\epsilon_u - \epsilon_y} d'$

□ Summing

□ Minimum tensile reinforcement ratio that to ensure the yielding of compression steel at failure:

$$\bar{\rho}_{cy} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \frac{\epsilon_u}{\epsilon_u - \epsilon_y} + \rho'$$

Compression Steel below Yield Stress

- If the tensile steel reinforcement is less than $\bar{\rho}_b$ and less than $\bar{\rho}_{cy}$ then the tensile steel at the yield stress at failure but the compression steel is not.
- The compression steel stress can also be expressed in terms of the still-unknown neutral axis depth as

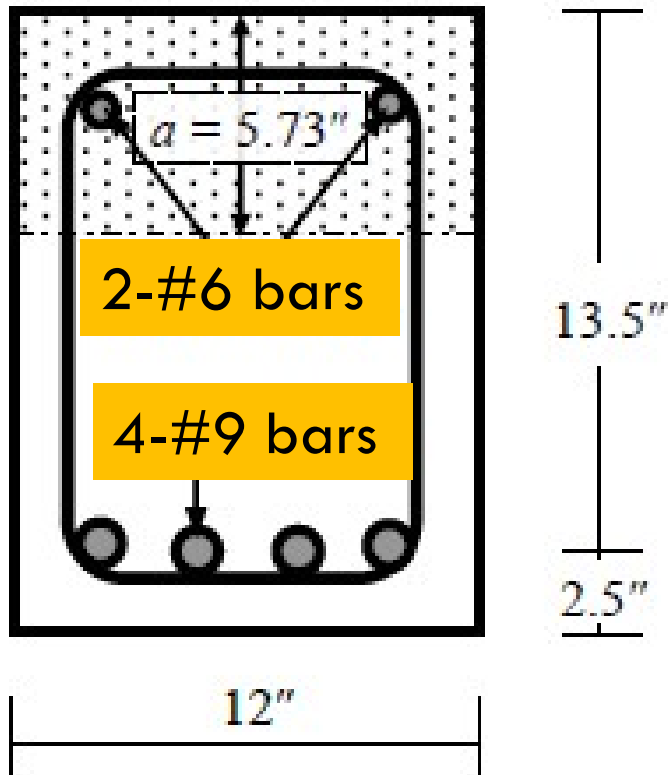
$$f'_s = E_s \varepsilon_u \frac{c - d'}{c}$$

- 
- Considering horizontal force equilibrium

- $T = C \Rightarrow T = A_s f_y = C_c + C_s$

$$A_s f_y = 0.85 f'_c \beta_1 b c + A'_s \varepsilon_u E_s \frac{c - d'}{c}$$

- Solving this equation the only unknown c is easily obtained.



$$A_s = 3.8 \text{ in}^2 \quad A_s' = 0.88 \text{ in}^2$$

$$d = 13.5 \text{ in} \quad d' = 2.5 \text{ in}$$

$$\rho_s = A_s/bd = 3.80/162 = 0.0235$$

$$\rho_s' = A_s'/bd = 0.88/162 = 0.0054$$

$$\bar{\rho}_{cy} = 0.85 * 0.85 \frac{3}{60} \frac{2.5}{13.5} \frac{0.003}{0.003 - 0.0021} + 0.0054 = 0.0277$$

$$C_c = 0.85 f'_c ab = 0.85 f'_c (\beta_1 c) b$$

$$\Rightarrow C_c = 0.85 * 3 * 0.85 c * 12 = 26.01 c$$

$$f'_s = E_s \varepsilon_u \frac{c - d'}{c} = \frac{87(c - d')}{c}$$

$$C_s = A'_s f'_s = 0.88 \left[\frac{87(c - d')}{c} \right] = \left[\frac{76.56(c - 2.5)}{c} \right]$$

$$T = A_s f_y = 3.8 * 60 = 228 k$$

From equilibrium of horizontal forces $T = C = C_c + C_s$