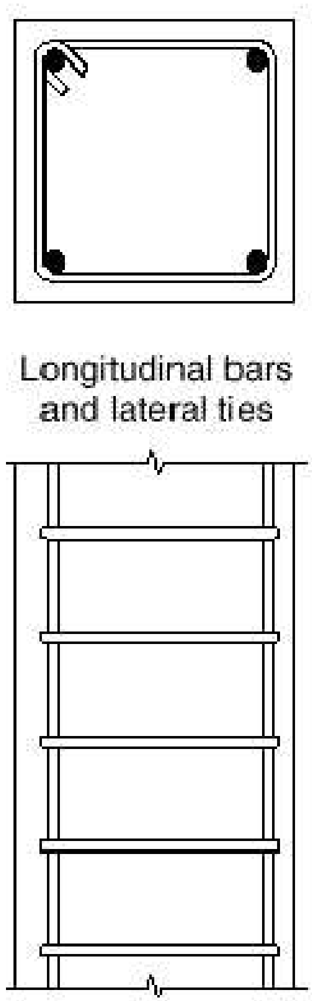
Behavior of a RC Member Under Loading

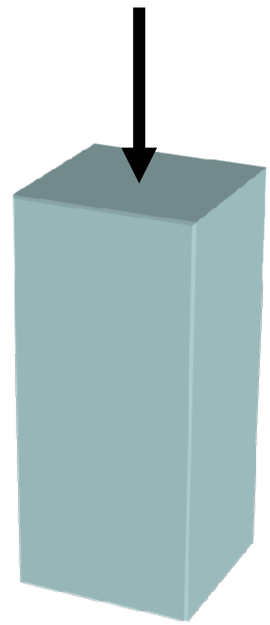
N.H.M. Kamrujjaman Serker *PhD*
Professor
Department of Civil Engineering
RUET

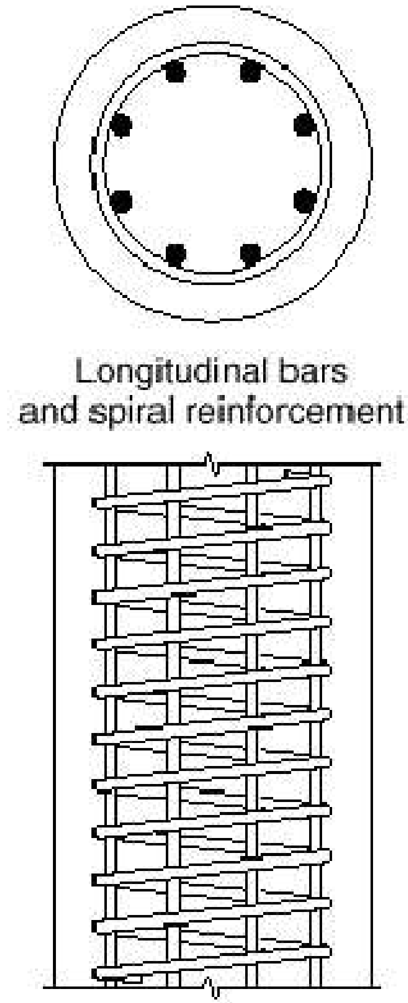
FUNDAMENTAL ASSUMPTIONS FOR REINFORCED CONCRETE BEHAVIOR

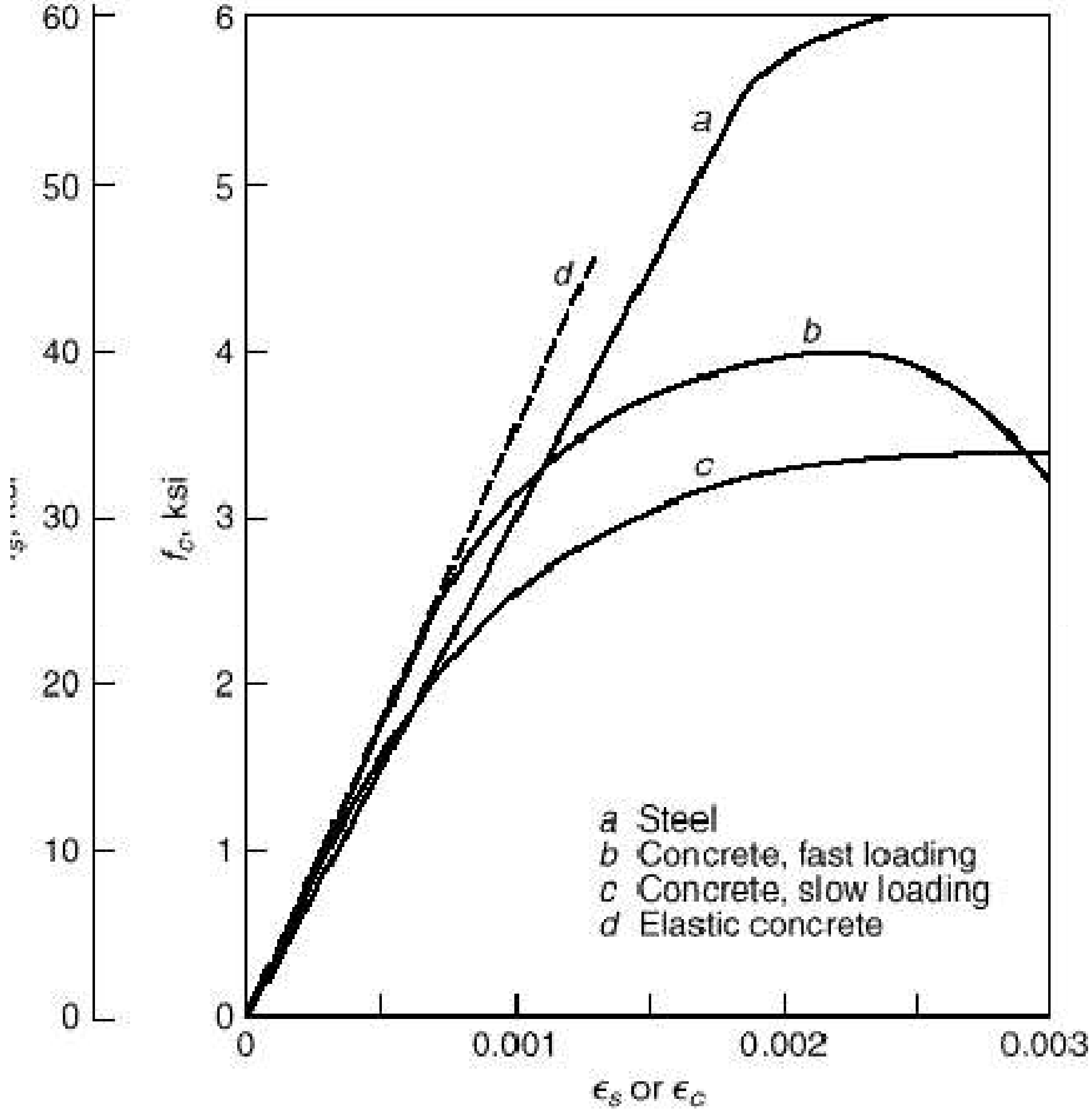
- The internal forces, such as bending moments, shear forces and normal and shear stresses at any section of a member are in equilibrium with the effect of the external loads at that section
- The strain in an embedded reinforcing bar is the same as that of the surrounding concrete.
- A cross section plane before loading remains plane under load
- Concrete is not capable of resisting any tension stress.
- The theory is based on the actual stress-strain relationship and strength properties of the two constituents materials or some reasonable and equivalent simplification thereof.

Behavior of Members Subject to Axial Load








60
50
40
30
20
10
0
6
5
4
3
2
1
0
f_c, ksi
a
b
c
d
0
0.001
0.002
0.003
ϵ_s or ϵ_c
a Steel
b Concrete, fast loading
c Concrete, slow loading
d Elastic concrete

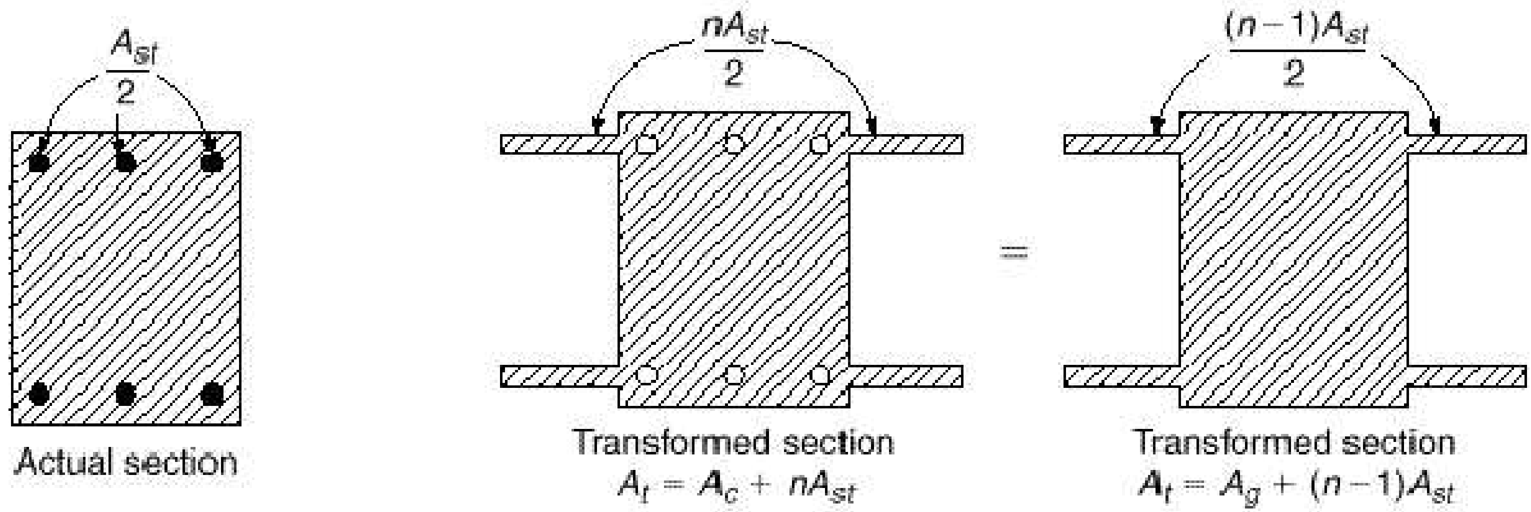
Elastic Behavior

- **At low stress upto about $f_c^{/}/2$ concrete behaves nearly elastically. i.e. stress and strain are closely proportional.**
- **Because the compression strain in the concrete, at any load, is equal to the compression strain in the steel**

$$Therefore, \quad \varepsilon_c = \frac{f_c}{E_c} = \varepsilon_s = \frac{f_s}{E_s}$$

$$Therefore, \quad f_s = \frac{E_s}{E_c} f_c = n f_c \qquad (1)$$

$$where, n = \frac{E_s}{E_c}$$ **Modular Ratio**



Let Ac = net area of concrete i.e. gross area minus area occupied by reinforcing bars

Ag = gross area Ast = total area of reinforcing bars

P = axial load

Therefore $P = f_c A_c + f_s A_{st}$

$$\Rightarrow P = f_c A_c + nf_c A_{st}$$

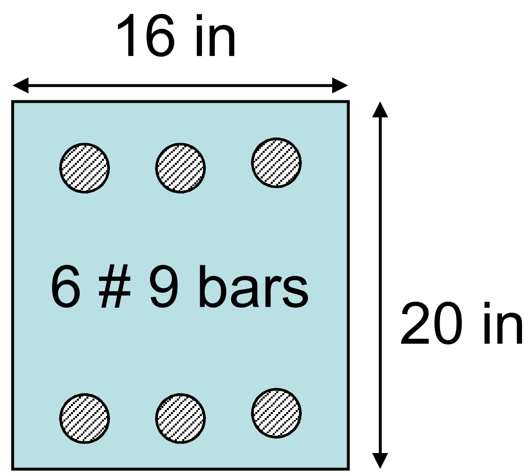
$$\Rightarrow P = f_c (A_c + nA_{st}) \qquad (2)$$

- The term $A_c + nA_{st}$ is the fictitious concrete cross section called the transformed area..
- The transformed area consists of concrete area plus ***n*** times the area of the reinforcement.
- The transformed area can also be obtained by replacing the steel area with concrete. Consists of concrete area plus n times the area of the reinforcement.

$$i.e. \qquad P = f_c \left\lfloor A_g + (n-1) A_{st} \right\rfloor \qquad (3)$$

- If load and cross-sectional dimensions are known, the concrete stress can be found by solving equation (2) or (3)
- Steel stress can be calculated from equation (1)

A column has a cross section of 16 x 20 in and is reinforced by six No. 9 bars. Determine the axial load that will stress concrete to 1200 psi.



A_g =16x 20 =320 in^2

A_{st} =6.00 in^2

n = Es / Ec = 8

$$E_c = 33 w_c^{1.5} \sqrt{f_c^{/}}$$

$$E_c = 57{,}000 \sqrt{f_c^{/}}$$

$$P = f_c \left[A_g + (n-1) A_{st} \right] = 434{,}000 lb$$

$$P_c = A_c f_c = f_c \left[A_g - A_{st} \right] = 377{,}000 lb$$

$$P_s = A_{st} f_s = n f_c A_{st} = 57{,}600 lb$$

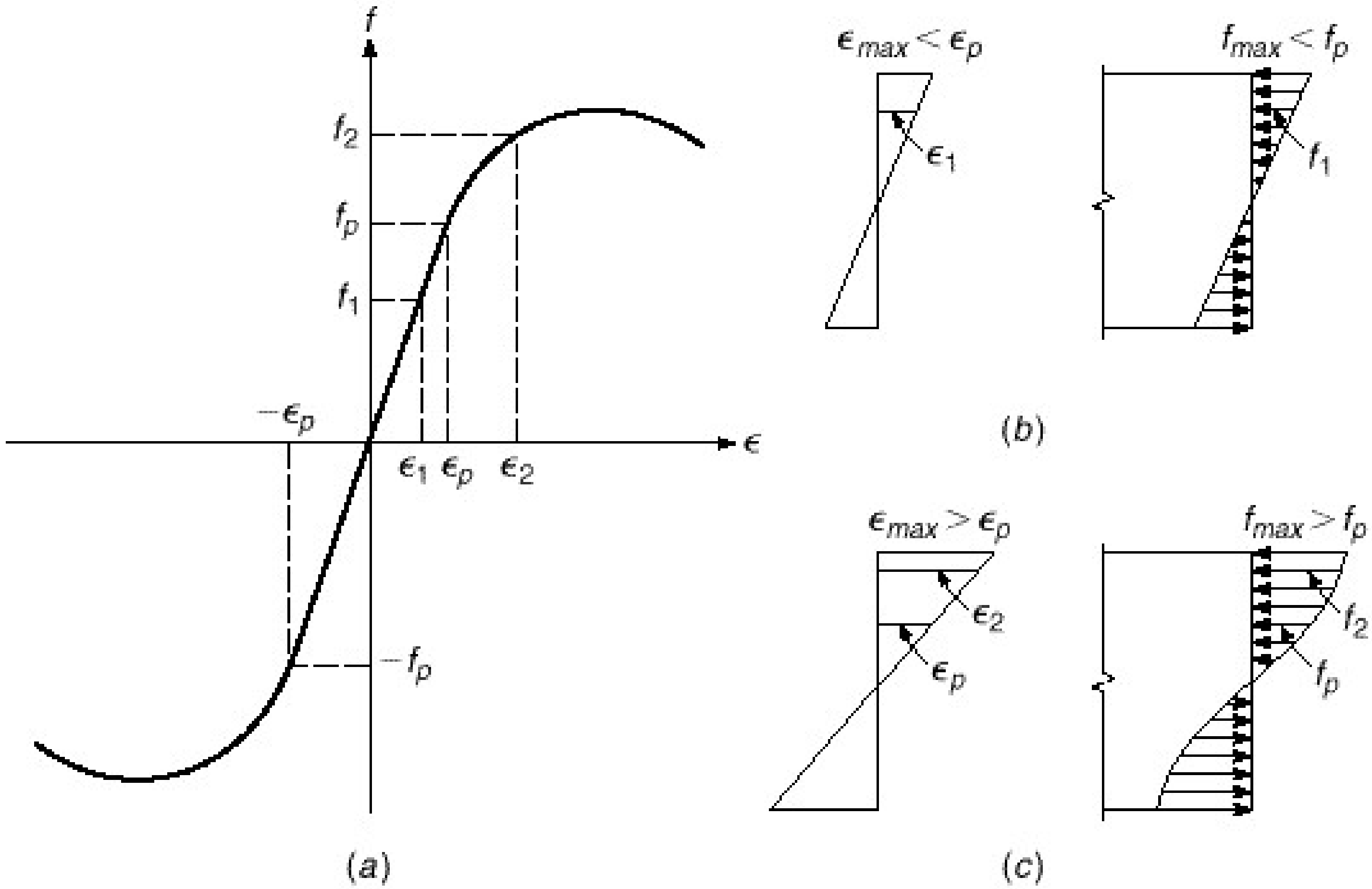
TABLE A.1

Designations, diameters, areas, and weights of standard bars

Bar No.				
Inch-Pound	SI	Diameter, in.	Cross-Sectional Area, in^2	Nominal Weight, lb/ft
3	10	$\frac{3}{8}$ = 0.375	0.11	0.376
4	13	$\frac{1}{2}$ = 0.500	0.20	0.668
5	16	$\frac{5}{8}$ = 0.625	0.31	1.043
6	19	$\frac{3}{4}$ = 0.750	0.44	1.502
7	22	$\frac{7}{8}$ = 0.875	0.60	2.044
8	25	1 = 1.000	0.79	2.670
9	29	$1\frac{1}{8}$ = 1.128[c]	1.00	3.400
10	32	$1\frac{1}{4}$ = 1.270[c]	1.27	4.303
11	36	$1\frac{3}{8}$ = 1.410[c]	1.56	5.313
14	43	$1\frac{3}{4}$ = 1.693[c]	2.25	7.650
18	57	$2\frac{1}{4}$ = 2.257[c]	4.00	13.600

Basic Assumptions

- A cross section plane before loading remains plane under load
- The bending stress at any point depends on the strain at that point in a manner given by the stress-strain diagram of the material.
- The distribution of the shear stresses over the depth of the section depends on the shape of the cross section and of the stress-strain diagram.
- Owing to the combined action of shear stresses and flexural stresses, at any point in a beam there are inclined stresses of tension and compression.
- Since the horizontal and vertical shearing stresses are equal and the flexural stresses are zero at the neutral plane, the inclined tensile and compressive stresses at any point in that plane form an angle of 45 degree with the horizontal, the intensity of each being equal to the unit shear at the point


f
f_2
f_p
f_1
$-\epsilon_p$
ϵ
ϵ_1 ϵ_p ϵ_2
$-f_p$
(a)
$\epsilon_{max} < \epsilon_p$
ϵ_1
$f_{max} < f_p$
f_1
(b)
$\epsilon_{max} > \epsilon_p$
ϵ_2
ϵ_p
$f_{max} > f_p$
f_2
f_p
(c)

- **When the stresses in the outer fibers are smaller than the proportional limit, the beam behaves elastically (Fig. b)**

The N.A.passes through the c.g. of the cross section

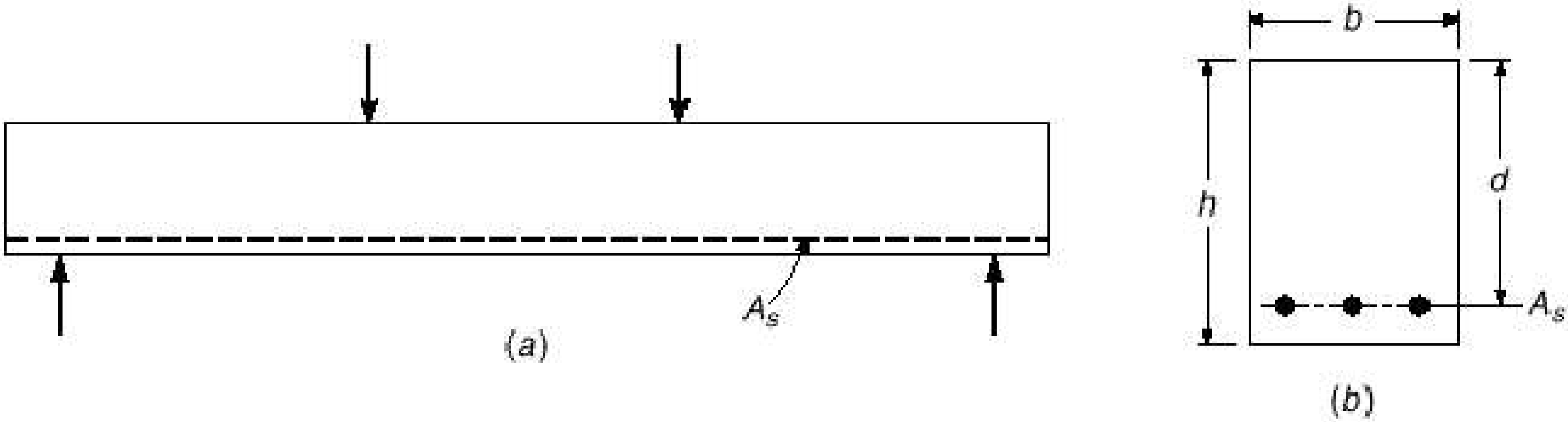
The stress at any given point in the cross section is computed by the equation

$$f = \frac{My}{I}$$

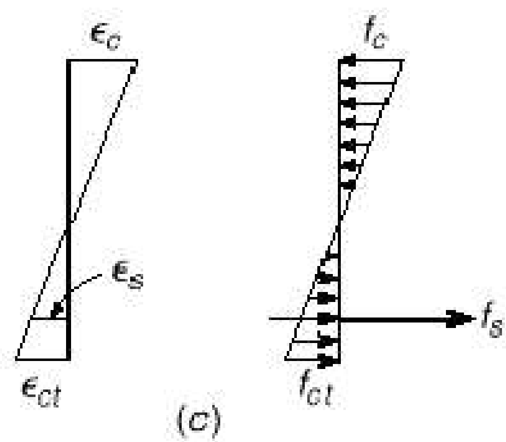
The shear stress at any given point in the cross section is given by

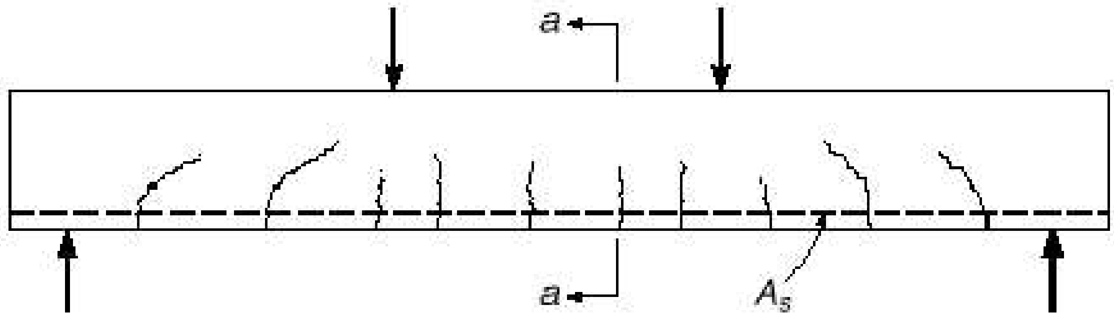
$$= \frac{VQ}{Ib}$$

Reinforced Concrete Beam Behavior

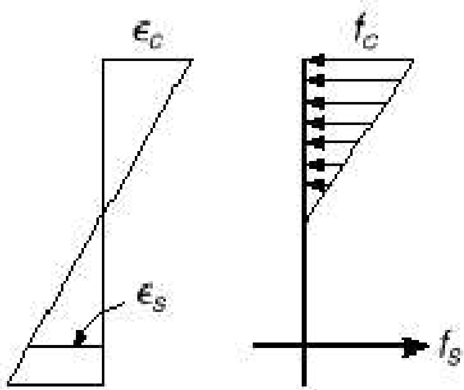


No Cracking



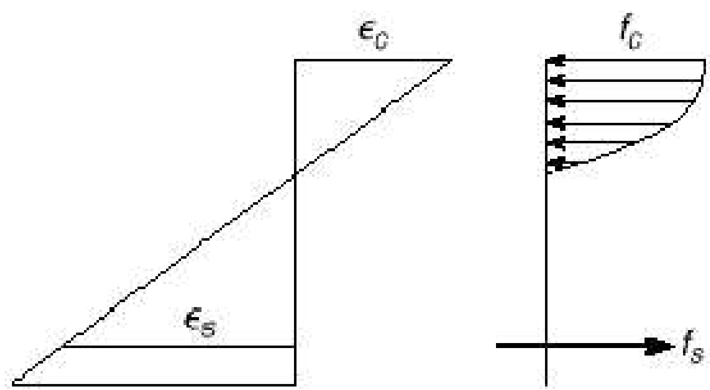


(d)



(e)

Up to elastic limit



Just before failure

Reinforced Concrete Beam Behavior

- ***Stage 1:*** No external loads self weight.
- ***Stage 2:*** the external load P cause the bottom fibers to equal to modulus of rupture of the concrete. Entire concrete section was effective, steel bar at tension side has same strain as surrounding concrete.
- ***Stage 3:*** The tensile strength of the concrete exceeds the rupture f_r and cracks develop. The neutral axis shifts upward and cracks extend to neutral axis. Concrete loses tensile strength and steel starts working effectively and resists the entire tensile load

- ***Stage 4:*** The reinforcement yields.
- ***Stage 5:*** Failure of the beam.

- The behavior of a RC beam under progressive loading can be summarized under the following categories
- Stresses elastic and section uncracked (Figure (c))
- Stresses elastic and section cracked (Fig (e))
- Ultimate flexural strength (Fig. (f))

Stresses Elastic and Section Uncracked

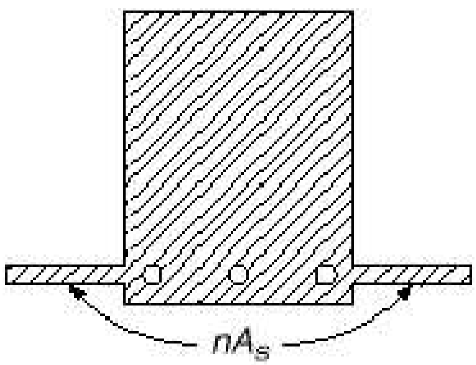
As long as the tensile stress in the concrete is smaller than the modulus of rupture, the strain and stress distribution is as shown in figure (c).

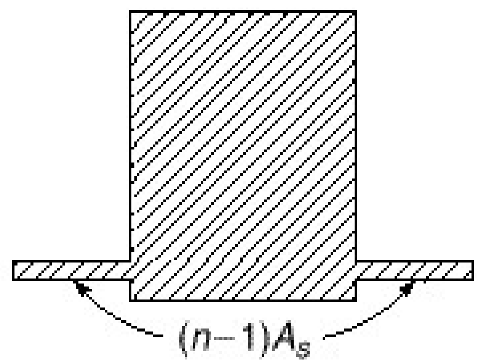
No tension cracks develop.

the strain and stress distribution is similar to that of a homogeneous beam

What about the presence of STEEL REINFORCEMENT??

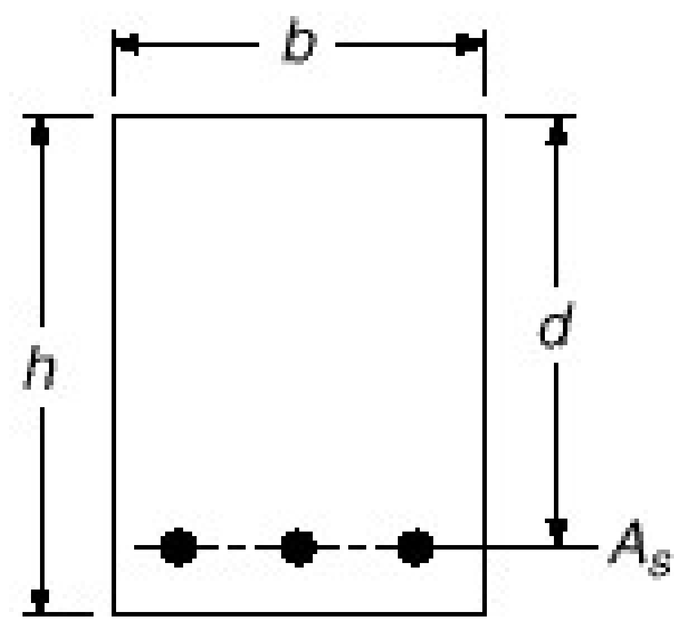
Stresses Elastic and Section Uncracked





- The beam section can be analyzed using the Transformed section and usual method of analysis

For a rectangular beam section assume $b = 10in.$ $h = 25in.$ $d = 23in.$ Reinforcement consists of 3 No. 8 bars. Concrete Cylinder strength at 28 days, $f_c^{/} = 4000$ *psi* Concrete tensile strength in bending = 475 psi; f_y = 60,000psi. Determine the stress caused by a bending moment 45 kips-ft



Solution:

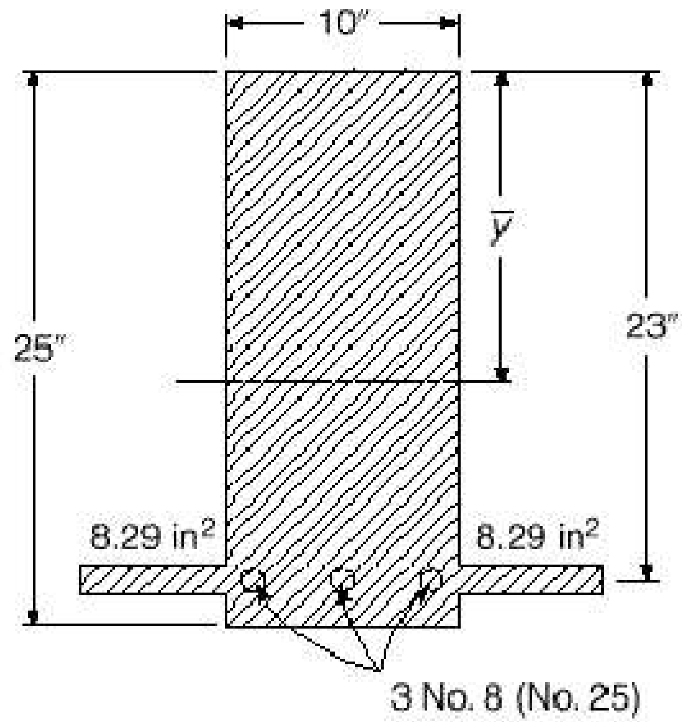
$$E_c = 57{,}000\sqrt{f_c^{/}} = 3{,}600{,}000 \text{ psi}$$

$$E_s = 29{,}000{,}000 \text{ psi}$$

$$\text{n} = E_s / E_c = 8$$

$$A_s = 3 \text{ x } 0.79 = 2.37 \text{ in}^2$$

$$(\text{n-1})\, A_s = 7 \text{ x } 2.37 = 16.59 \text{ in}^2$$

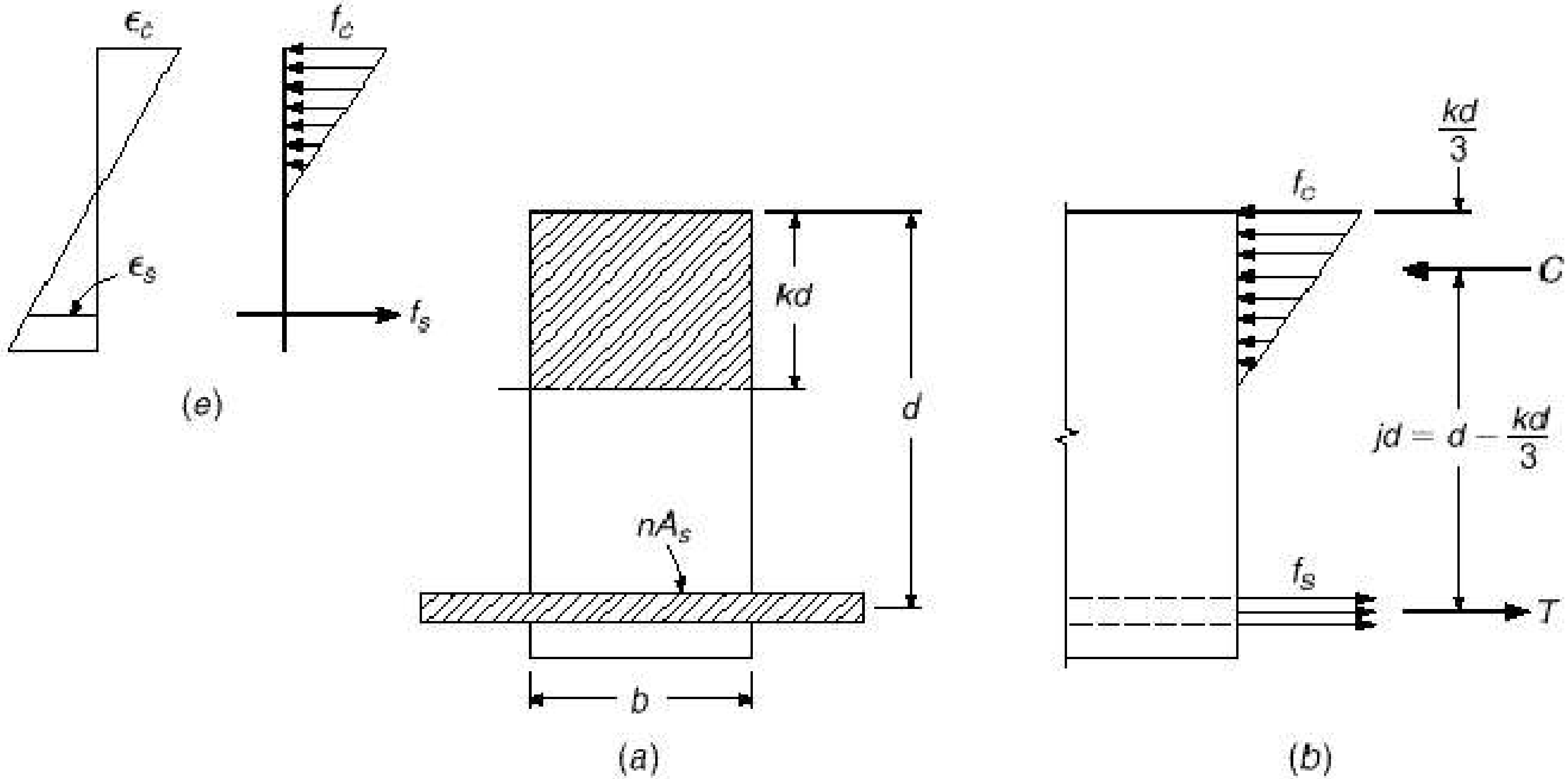


Location of neutral axis (from top) y = 13.2 in.

Moment of inertia about the N.A. I = 14,740 in4

The value of f_{ct} = 432 psi is smaller than Concrete tensile strength in bending = 475 psi, therefore, the section remain uncracked

Stresses Elastic and Section Cracked



- *d* = effective depth of the beam = the distance between the compression face and centroid of the tension steel provided
- *k*d = distance of neutral axis from the compression face
- *j*d = internal lever arm between C and T

The location of neutral axis can be obtained by equating the moment of the compression area to tension area

$$b\times(kd)\times\frac{kd}{2}=nA_s(d-kd) \qquad \text{……………….(1)}$$

Total compression force C = $\frac{1}{2}\times(kd)\times f_c\times b$ ……..(2)

Total tension force T $=A_s f_s$ ……………..(3)

For equilibrium C = T and couple constituted by the forces C and T be numerically equal to the external bending moment M.

Taking moment about C, we get, M = Tjd = $A_s f_s jd$ …………..(4)

$$f_s = \frac{M}{A_s jd} \qquad \text{………………..(5)}$$

Taking moments about T gives

$$M = Cjd = \frac{f_c}{2} b(kd) \times jd = \frac{f_c}{2} kjbd^2 \qquad \text{……………(6)}$$

and $$f_c = \frac{2M}{kjbd^2} \qquad \text{……………(7)}$$

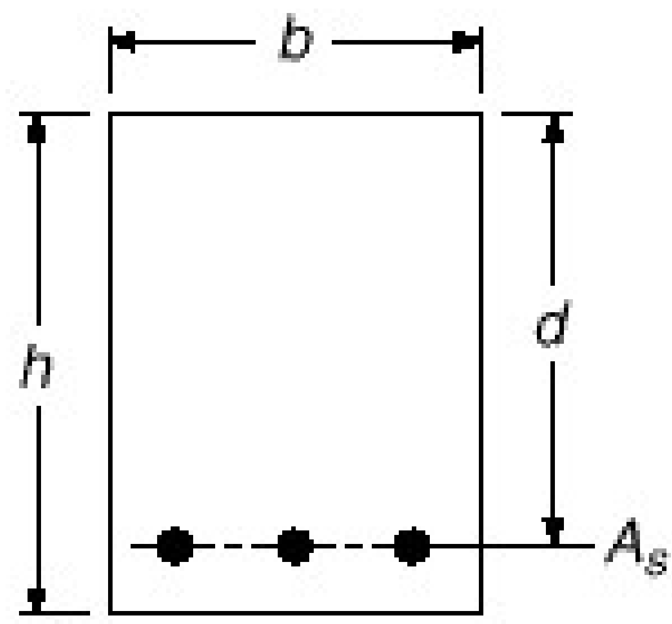
Reinforcement ratio $p = \frac{A_s}{bd}$ ………………….(8)

Substituting equation (8) into equation (1) and solving for k

$$k = \sqrt{(pn)^2 + 2pn} - pn \quad \text{……………………(9)}$$

From figure (b) jd = d – kd/3 i.e. $j = 1 - \frac{k}{3}$ …..(10)

For a rectangular beam section assume $b = 10in.$ $h = 25in.$ $d = 23in.$ Reinforcement consists of 3 No. 8 bars. Concrete Cylinder strength at 28 days, $f_c' = 4000\ psi$ Concrete tensile strength in bending = 475 psi; f_y = 60,000psi. Determine the stress caused by a bending moment 90 kips-ft .



Consider the section is Uncracked :
Calculate Concrete tensile stress

$$f_{ct} = \frac{My}{I} = \frac{90{,}000 \times 12 \times 11.8}{14{,}740} = 864\,psi$$

The value of f_{ct} is larger than **Concrete tensile strength in bending = 475 psi**, therefore, the section is cracked. And the beam should be analyzed considering cracked section.

Calculate relevant properties

p = As / bd = 2.37 / (10 x 23) = 0 .0103
n= 8

$k = \sqrt{(pn)^2 + 2pn} - pn$ => k = 0.33 and kd = 0.33 x 23 = 7.60 in

$j = 1 - \frac{k}{3}$ = 1- 0.33/3 = 0.89

Maximum concrete stress $f_c = \frac{2M}{kjbd^2} = \frac{2 \times 90{,}000 \times 12}{0.33 \times 0.89 \times 10 \times (23)^2}$

= 1390 psi

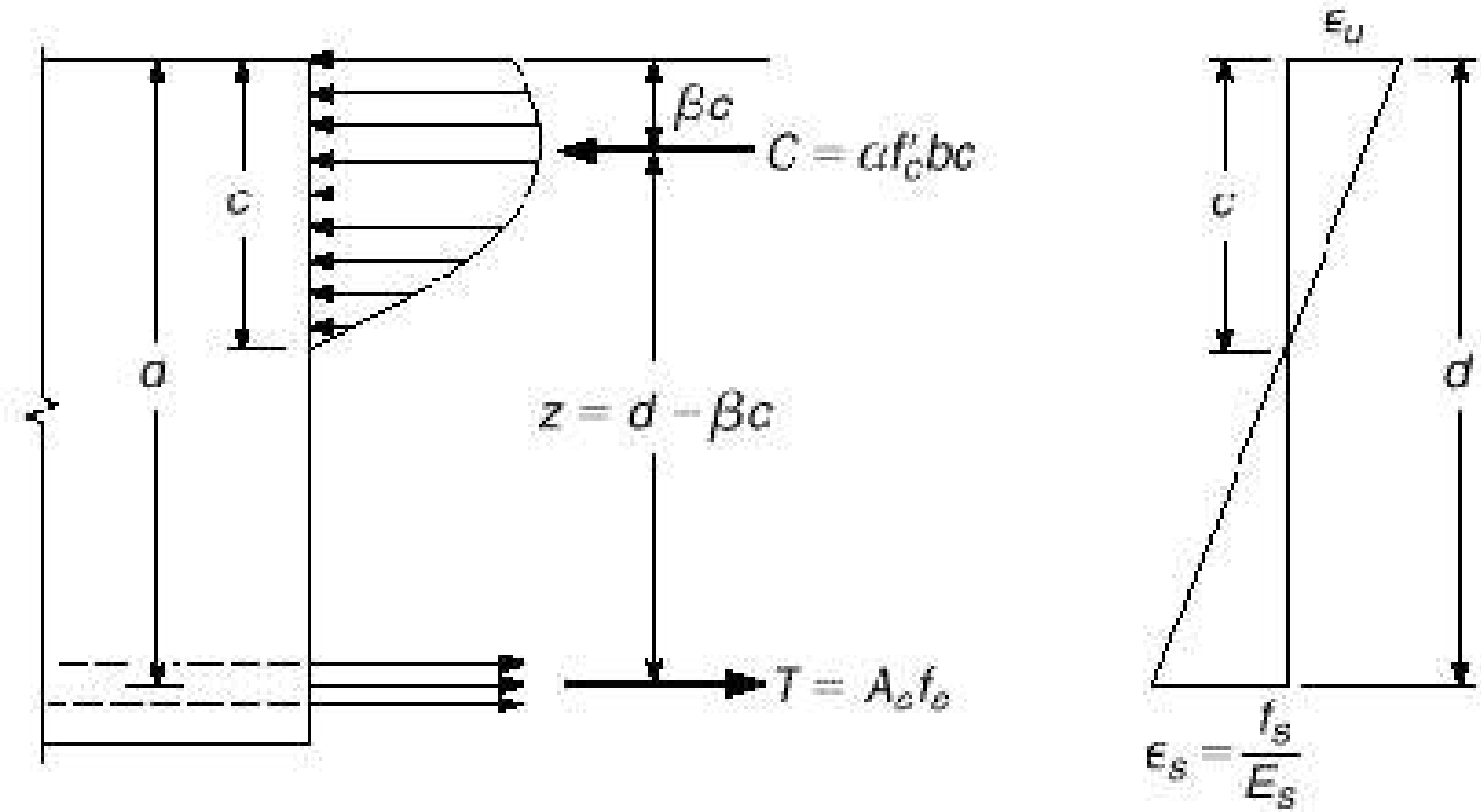
Steel stress $f_s = \frac{M}{A_s jd} = \frac{90{,}000 \times 12}{2.37 \times 0.89 \times 23}$ = 22,300 psi

- Did you notice that the position of N.A. has been changed
- What happened to location of N.A.?
- Shifted up or down?

Flexural Strength – General Analysis

- At or near the ultimate load, stresses are no longer proportional to strain
- More realistic methods of analysis rather than assumed elastic behavior of the materials is necessary to predict the member strength.
- At or near the ultimate load, geometrical shape of distribution of concrete compressive stresses is invariable (parabolic, trapezoidal etc)
- The geometrical shape of distribution of concrete compressive stresses depends on a number of factors such as compressive strength, duration and rate of loading
- Therefore the present analysis is based on known laws of mechanics and experimental data

Flexural Strength – General Analysis



Internal stress and strain distribution when the beam is about to fail

Flexural Strength – General Analysis

- Our target is to calculate the nominal moment capacity of the beam at which it will fail by tension yielding of steel or crushing of concrete in the outer compression fiber
- For the first mode of failure $f_s = f_y$
- For the second mode of failure exact criterion is not yet known

Flexural Strength – General Analysis

- But it is considered that at failure ε_u =0.003 to 0.004
- For safety, it is considered that ε_u =0.003
- However, it is not really necessary to know the exact shape of the compressive stress distribution of concrete
- What we should know??
  - The total resultant compressive force C in the concrete
  - Its vertical location, i.e. its distance from the outer compression fiber

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Flexural Strength – General Analysis

- Consider a rectangular beam, the compression area is therefore bc.
- Total compression on this area is
  - $$C = f_{av} bc \qquad (1)$$
- where f_{av} is the average compression stress in the area bc
- The average compression stress which can be developed before failure occurs becomes larger than the cylinder strength $f_c^{/}$ of the particular concrete
- Let $$\alpha = f_{av} / f_c^{/} \qquad (2)$$
- Therefore $$C = \alpha f_c^{/} bc \qquad (3)$$

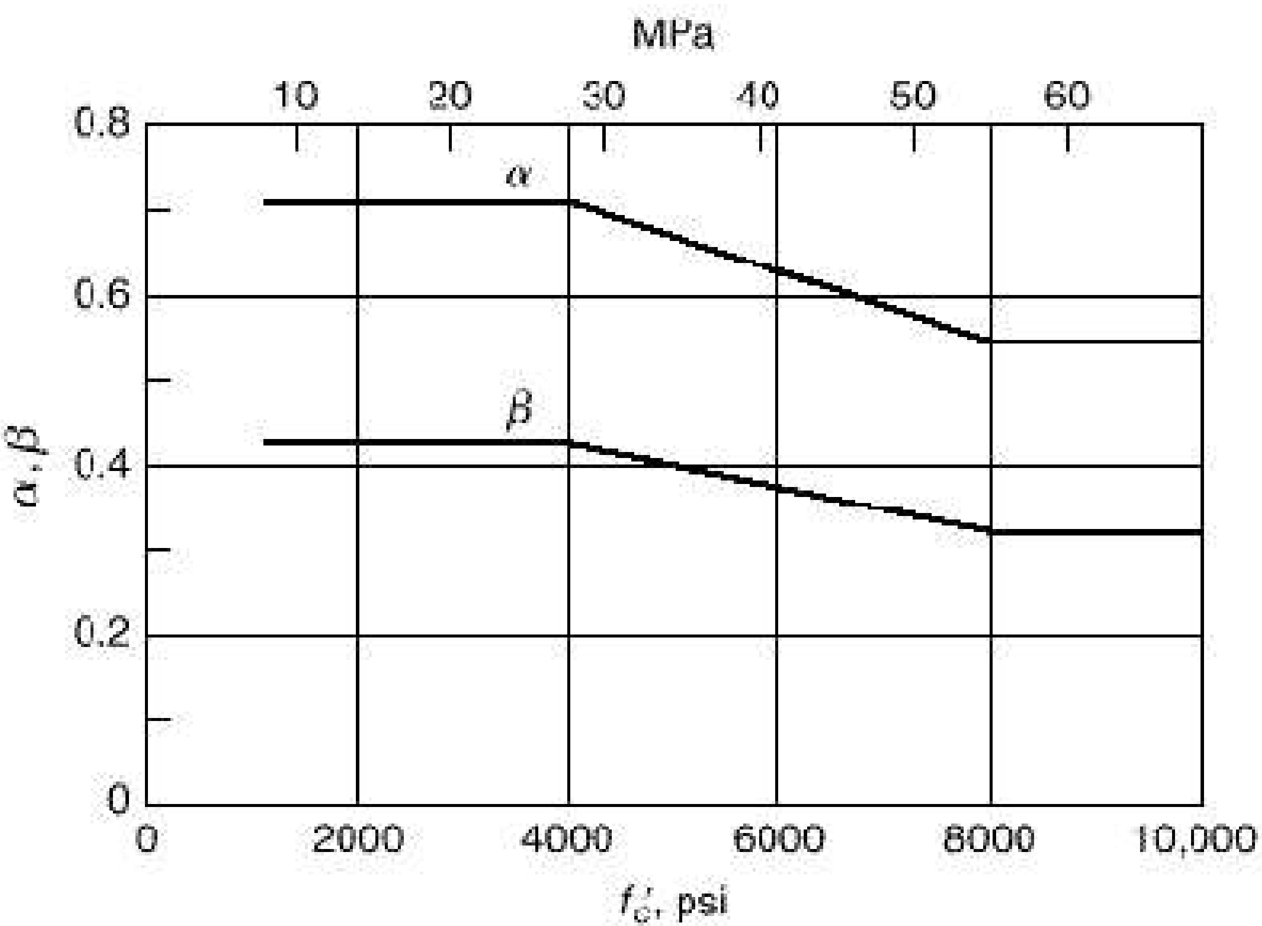
Flexural Strength – General Analysis

- For a given distance ***c*** to the neutral axis, the location of **C** can be defined as some fraction β of this distance
- Therefore, it is only necessary to know *α and* β to define completely the effect of the concrete compression stresses

Flexural Strength – General Analysis

- ***Values of α and β***
- ***α equals 0.72 for $f_c^{/}$*** **≤ 4000 psi and decrease by 0.04 for every 1000psi above 4000**
- ***β equals 0.425 for $f_c^{/}$*** **≤ 4000 psi and decrease by 0.025 for every 1000psi above 4000**
- **The decrease in for high-strength concrete is related to the fact that such concrete are more brittle**

Flexural Strength – General Analysis



Flexural Strength – General Analysis

- The ultimate strength can be calculated from the laws of equilibrium and the assumption that the plane cross section remain plane
- For Equilibrium C = T
- Or $\alpha f_c^{/} bc = A_s f_s$ (4)

- Bending moment $M = Tz = A_s f_s (d - \beta c)$ (5)
- Or $M = Cz = \alpha f_c^{/} bc\,(d - \beta c)$ (6)
- For tension failure by yielding of steel, $f_s = f_y$. Therefore from Eqn. (4)

$$c = \frac{A_s f_y}{\alpha f_c^{/} b} \qquad (5)$$

Flexural Strength – General Analysis

- The steel area also expressed nondimensionally as a fraction of the effective area of the section and is called reinforcement ratio, *p*
- $$p = \frac{A_s}{bd} \qquad (6)$$

- So for tension failure the distance to the neutral axis is

$$c = \frac{pf_y}{\alpha f_c^{/}} d \qquad (7)$$

- In tension failure the ultimate moment is given by (combine Eqn 5 and 7)

$$M_u' = A_s f_y d\left(1 - \frac{\beta f_y}{\alpha f_c'} p\right) \qquad (8)$$

Flexural Strength – General Analysis

Putting the extreme values of α= 0.72 and β= 0.425

Eqn (8) becomes

$$M'_u = A_s f_y d\left(1 - 0.59\frac{f_y}{f'_c}p\right) \qquad (9)$$

Flexural Strength – General Analysis

In over reinforced beams the steel does not yield at failure and steel stress is proportional to the steel strain.

i.e. fs = Ɛs*Es (10)

At this condition strain in the concrete becomes Ɛu = 0.003

From strain distribution

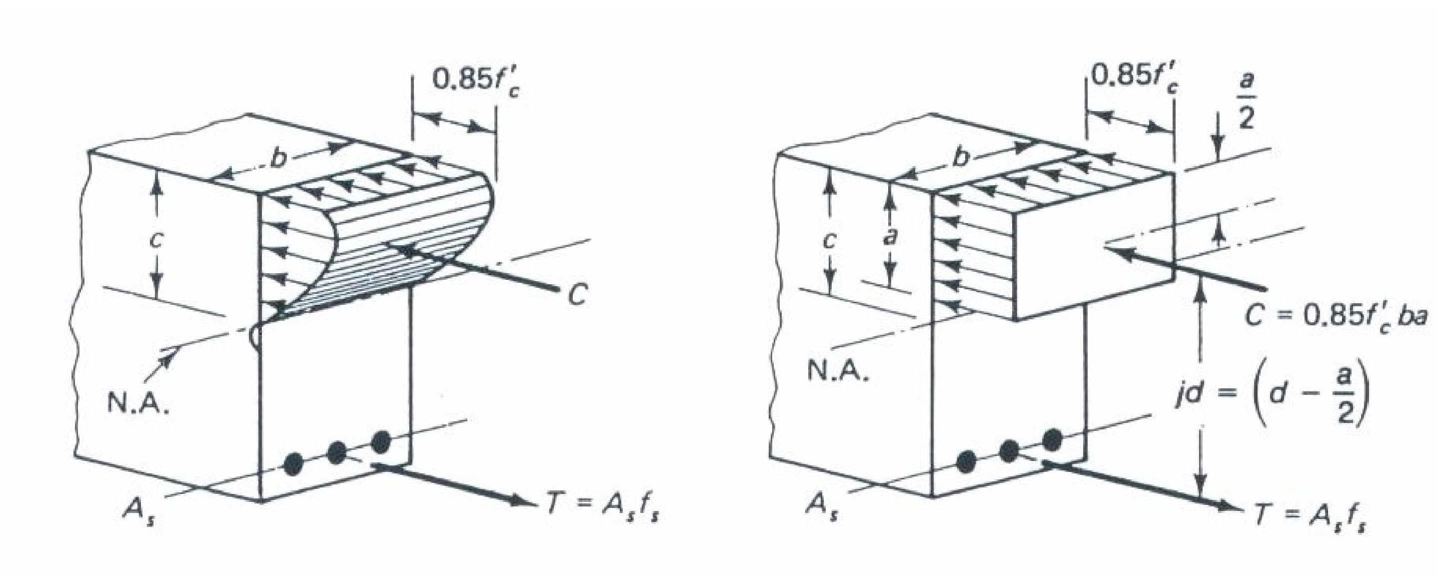
$$f_s = \varepsilon_u E_s \frac{d-c}{c} \tag{11}$$

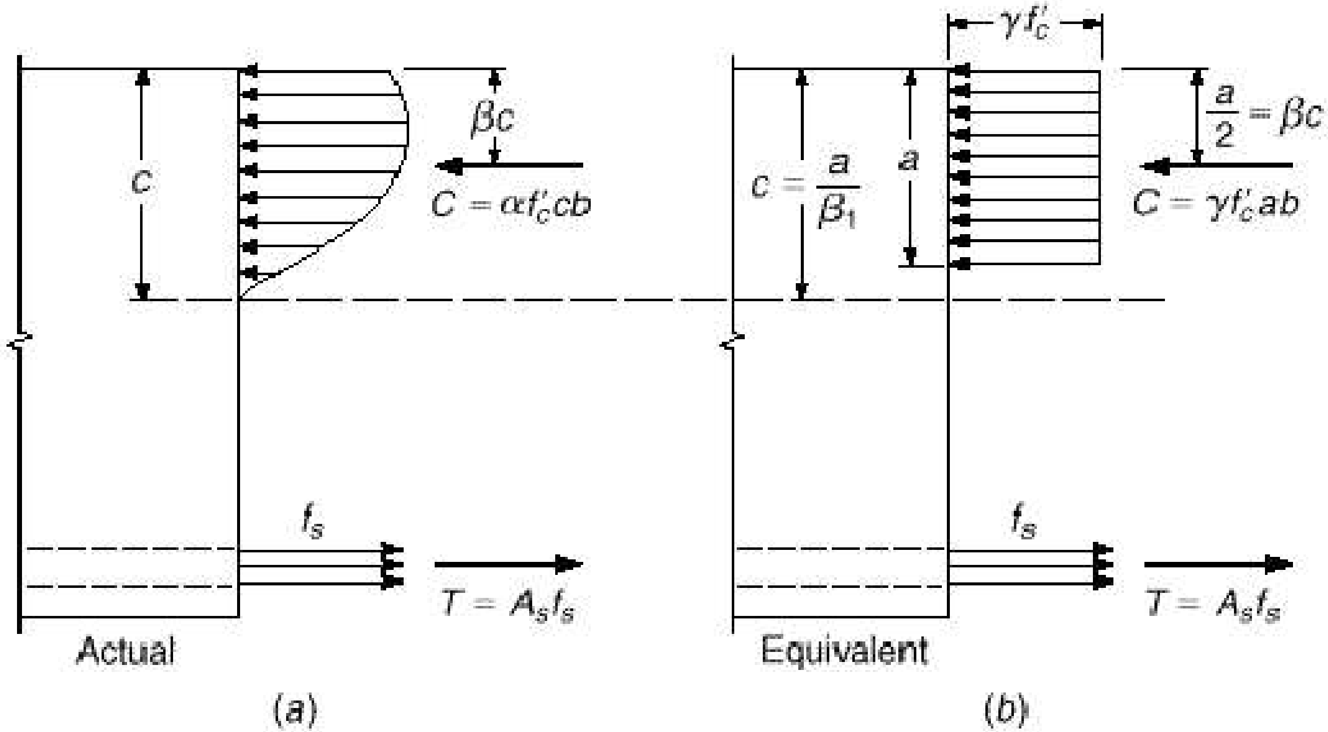
$$\alpha f_c' bc = \varepsilon_u A_s E_s \frac{d-c}{c} \tag{12}$$

By solving the quadratic equation c can be obtained and the moment capacity of the beam can be computed

Flexural Stress- Equivalent Stress Block

The compressive zone is modeled with a equivalent stress block.




γf'c
βc
c
C = αf'c cb
c = a/β1
a
a/2 = βc
C = γf'c ab
fs
T = As fs
fs
T = As fs
Actual
Equivalent
(a)
(b)

Flexural Stress

The equivalent rectangular concrete stress distribution has what is known as a β_1 coefficient is proportion of average stress distribution covers.

$$\beta_1 = 0.85 \text{ for } f_c^{/} \leq 4000 \text{ psi}$$

$$\beta_1 = 0.85 - 0.05 * \left[\frac{f_c^{/} - 4000}{1000} \right] \geq 0.65$$

Flexural Stress

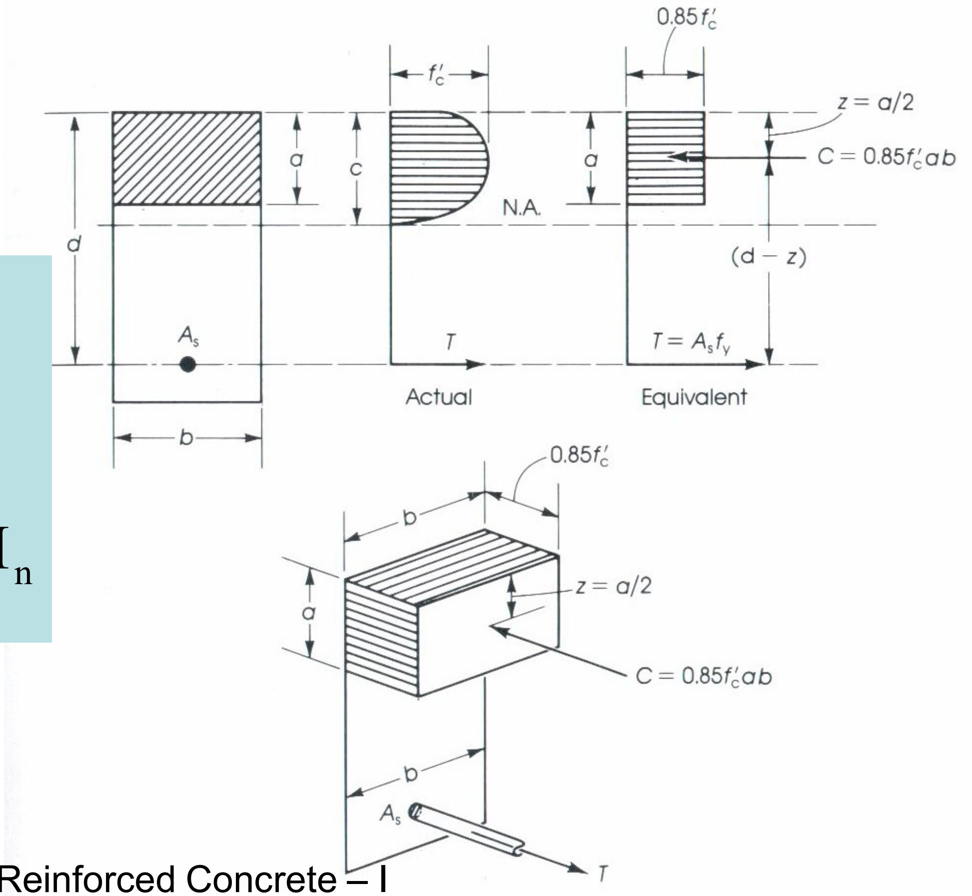
Example of rectangular reinforced concrete beam.

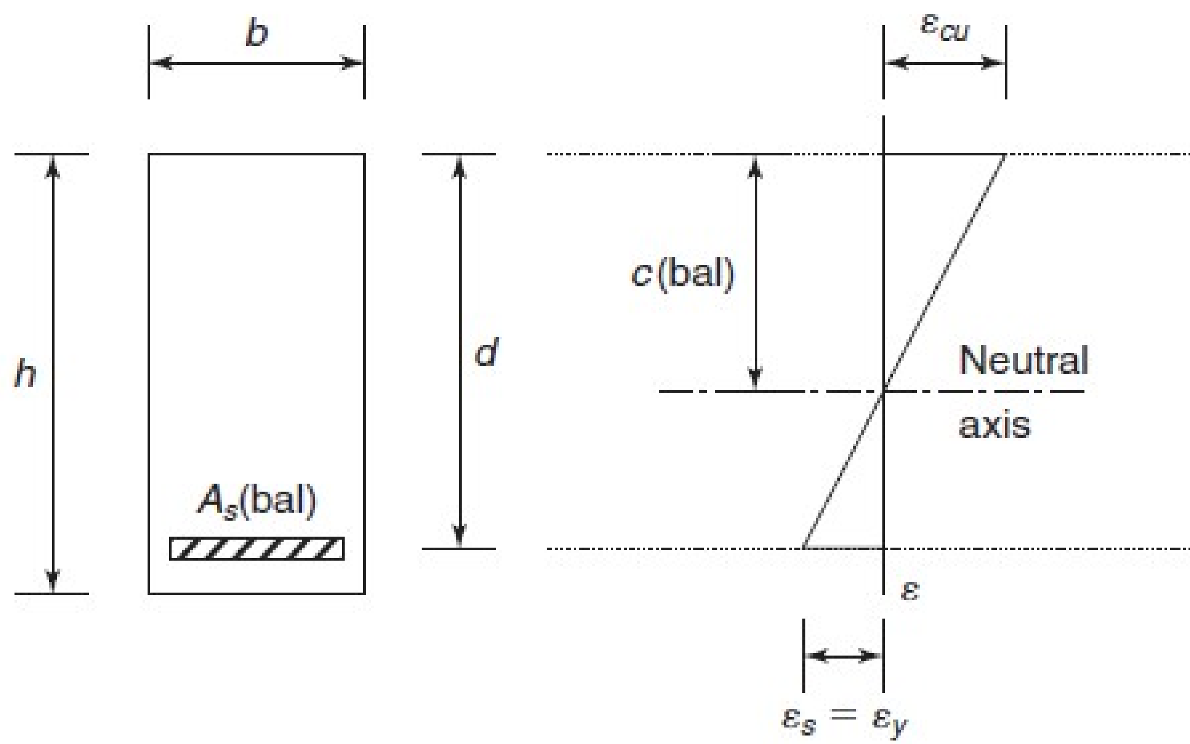
(1) Setup equilibrium.

$$\sum F_x = 0 \quad \Rightarrow \quad \mathrm{T} = \mathrm{C}$$

$$A_s f_s = 0.85 f_c ab$$

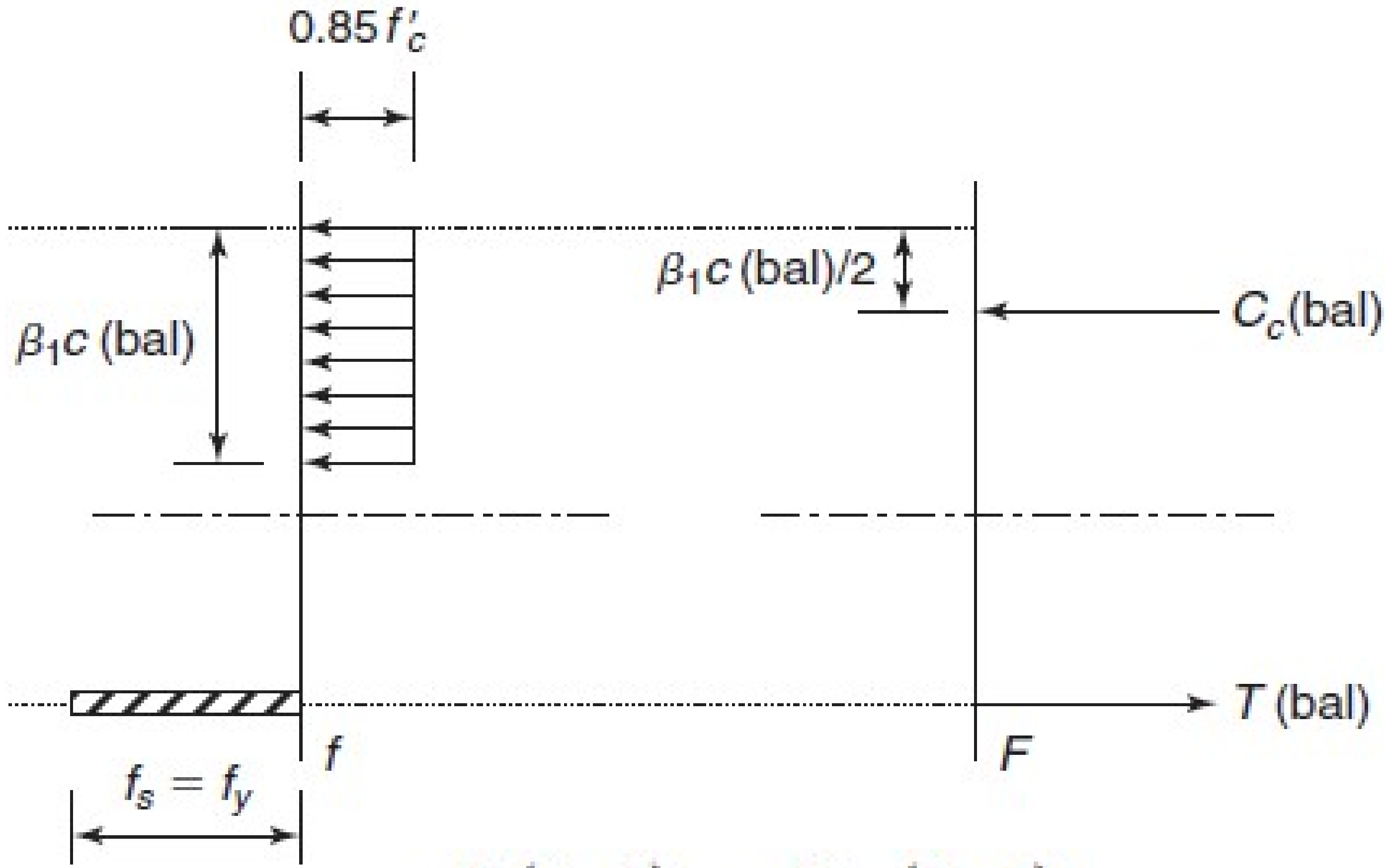
$$\sum M = 0 \quad \Rightarrow \quad \mathrm{T}\left(d - \frac{a}{2}\right) = \mathrm{M_n}$$





(a) Beam section. (b) Balanced strain distribution.

$$\frac{c(\text{bal})}{\varepsilon_{cu}} = \frac{d}{\varepsilon_{cu} + \varepsilon_y} \qquad c(\text{bal}) = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right)d$$



$$T(\text{bal}) = C_c(\text{bal})$$

$$A_s(\text{bal})f_y = 0.85\, f'_c\, b\beta_1 c(\text{bal})$$

$$A_s(\text{bal}) = \frac{1}{f_y}[0.85\, f'_c\, b\beta_1 c(\text{bal})]$$

$$\rho_b = \frac{A_s(\text{bal})}{bd} = \frac{0.85\,\beta_1 f'_c}{f_y} \times \frac{b}{bd} \times \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right) d$$

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right)$$

Substitute ε_{cu} = 0.003 and then multiplying both the numerator and denominator by Es = 29,000,000 psi

$$\rho_b = \frac{0.85\,\beta_1 f'_c}{f_y}\left(\frac{87{,}000}{87{,}000 + f_y}\right)$$

Flexural Stress

Example of rectangular reinforced concrete beam.

(2) Find flexural capacity.

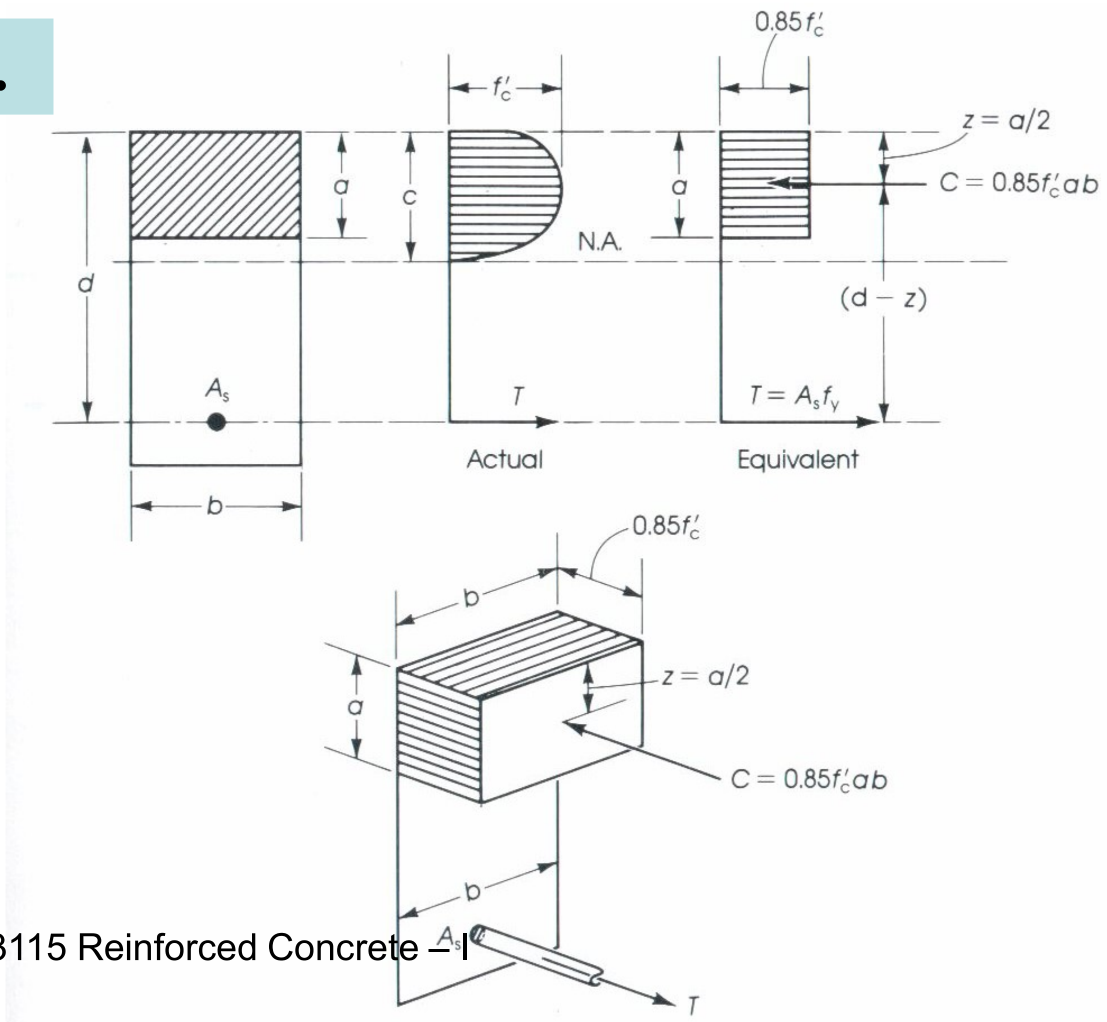
$$T = A_s f_s$$

$$C = 0.85 f_c ab$$

$$a = \frac{A_s f_y}{0.85 f_c b}$$

$$M_n = Tjd$$

$$= A_s f_y \left(d - \frac{a}{2} \right)$$



Flexural Stress

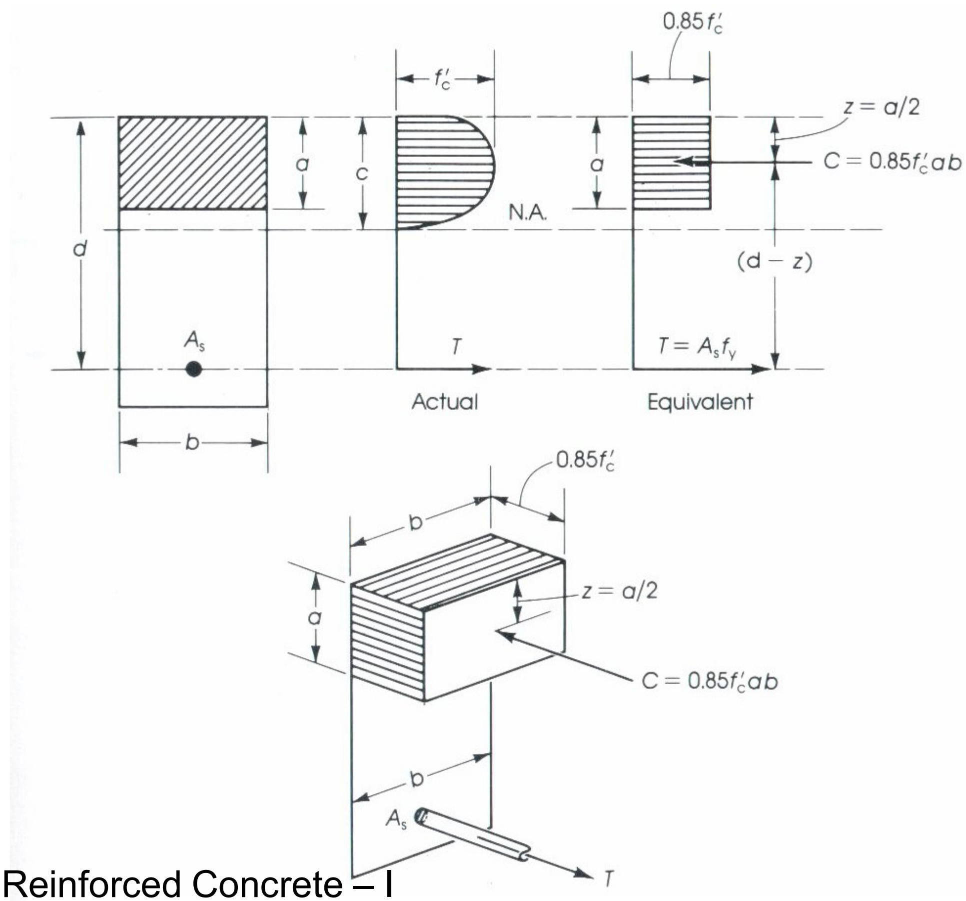
Example of rectangular reinforced concrete beam.

(3) Need to confirm $\varepsilon_s > \varepsilon_y$

$$\varepsilon_y = \frac{\sigma_y}{E_s}$$

$$c = \frac{a}{\beta_1}$$

$$\varepsilon_s = \frac{(d-c)}{c}\varepsilon_c > \varepsilon_y$$



Three possibilities in Inelastic Behavior

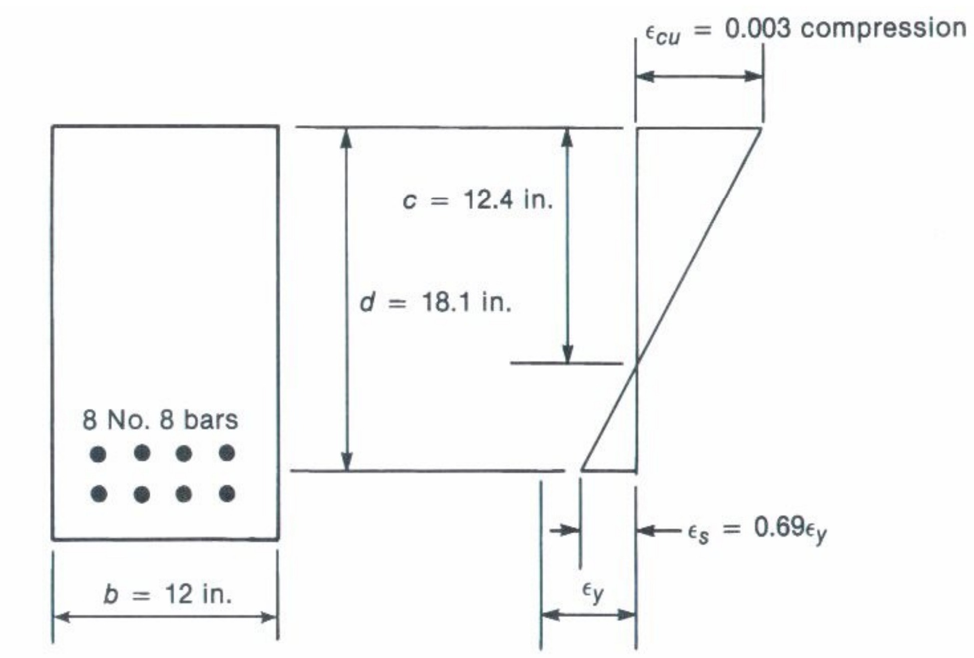
- Compression Failure - (over-reinforced beam)
- Tension Failure - (under-reinforced beam)
- Balanced Failure - (balanced reinforcement)

Inelastic Behavior

Compression Failure

The concrete will crush before the steel yields. This is a sudden failure.

The beam is known as an ***over-reinforced beam***.

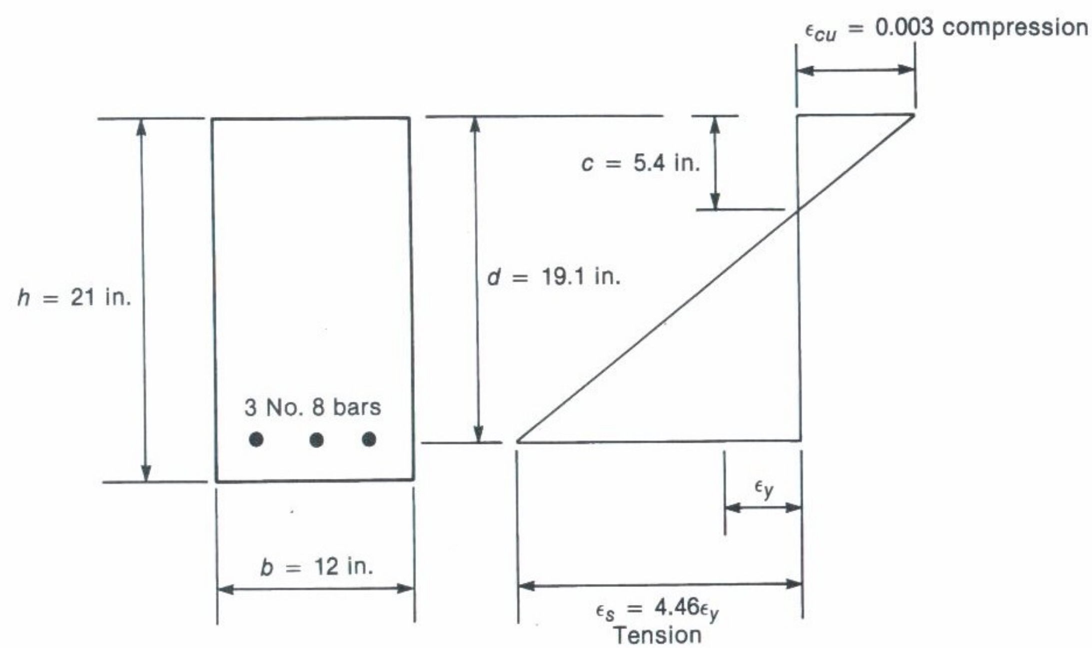


Inelastic Behavior

Tension Failure

The reinforcement yields before the concrete crushes The concrete crushes is a secondary compression failure.

The beam is known as an ***under-reinforced beam***.

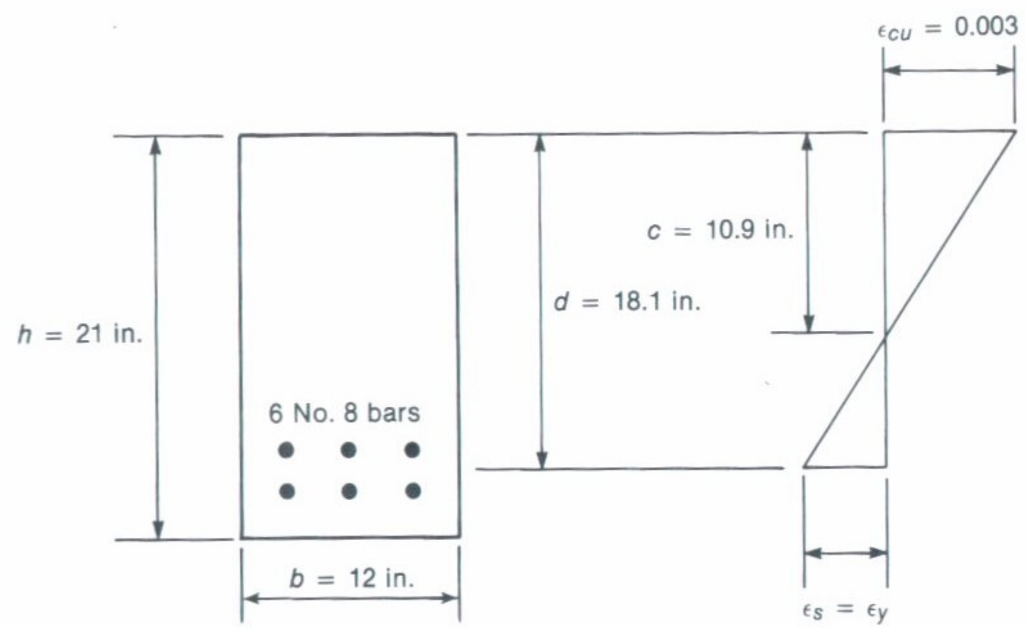


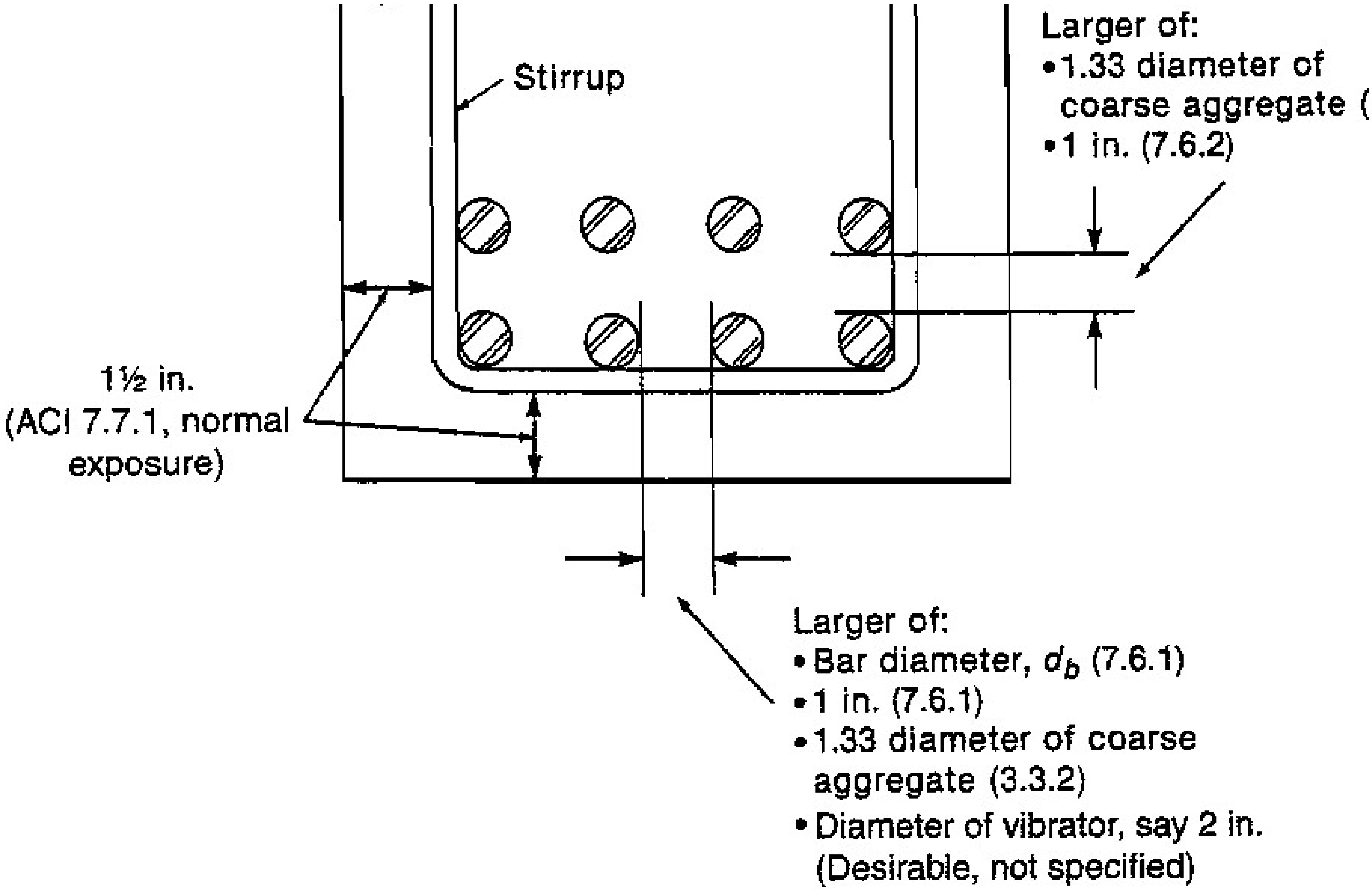
Inelastic Behavior

Balanced Failure

The concrete crushes and the steel yields simultaneously.

The beam is known as an *balanced-reinforced beam*.





(b) Minimum bar spacing and cover limits in ACI Code.

Flexural Stress

Example of rectangular reinforced concrete beam.

Given a rectangular beam

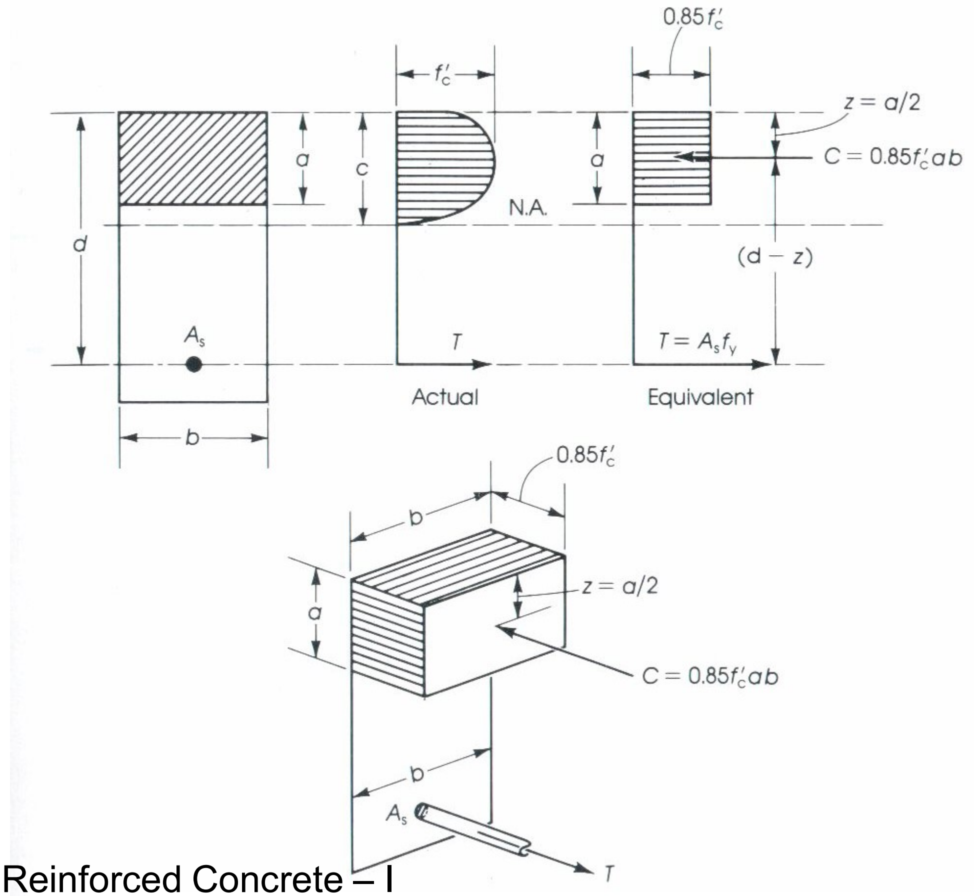
f_c = 4000 psi β_1 = 0.85

f_y = 60 ksi (4 #7 bars)

b = 12 in. d = 15.5 in. h= 18 in.

Find the neutral axis.

Find the moment capacity of the beam.



Flexural Stress - Example

For a non-rectangular beam

Given that the beam with concrete rated at f_c =6000 psi and the steel is rated at f_s =60,000 psi. d= 12.5 in.

(a) Determine the area of the steel for a balanced system.

(b) Determine the moment capacity of the beam. M_n

(c) Determine the NA.