OPEN- CHANNEL HYDRAULICS

CHOW

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CONTENTS

PART II. UNIFORM FLOW

Afroin 1700082

Nisha

CHAPTER 1

OPEN-CHANNEL FLOW AND ITS CLASSIFICATIONS

1-1. Description. The flow of water in a conduit may be either *open-channel flow* or *pipe flow*. The two kinds of flow are similar in many ways but differ in one important respect. Open-channel flow must have a *free surface*, whereas pipe flow has none, since the water must fill the whole conduit. A free surface is subject to atmospheric pressure. Pipe flow, being confined in a closed conduit, exerts no direct atmospheric pressure but hydraulic pressure only.

The two kinds of flow are compared in Fig. 1-1. Shown on the left side is pipe flow. Two piezometer tubes are installed on the pipe at sections 1 and 2. The water levels in the tubes are maintained by the pressure in the pipe at elevations represented by the so-called *hydraulic grade line*. The pressure exerted by the water in each section of the pipe is indicated in the corresponding tube by the height y of the water column above the center line of the pipe. The total energy in the flow of the section with reference to a datum line is the sum of the elevation z of the pipe-center line, the piezometric height y, and the velocity head $V^2/2g$, where V is the mean velocity of flow.[1] The energy is represented in the figure by what is called the *energy grade line* or simply the *energy line*. The loss of energy that results when water flows from section 1 to section 2 is represented by h_f. A similar diagram for open-channel flow is shown on the right side of Fig. 1-1. For simplicity, it is assumed that the flow is parallel and has a uniform velocity distribution and that the slope of the channel is small. In this case, the water surface is the hydraulic grade line, and the depth of the water corresponds to the piezometric height.[2]

Despite the similarity between the two kinds of flow, it is much more difficult to solve problems of flow in open channels than in pressure pipes. Flow conditions in open channels are complicated by the fact that the

[1] It is here assumed that the velocity is uniformly distributed across the conduit section; otherwise a correction would have to be made, such as is described in Art. 2-7 for open channels.

[2] If the flow were curvilinear or if the slope of the channel were large, the piezometric height would be appreciably different from the depth of flow (Arts. 2-9 and 2-10). As a result, the hydraulic grade line would not coincide exactly with the water surface.

position of the free surface is likely to change with respect to time and space and also by the fact that the depth of flow, the discharge, and the slopes of the channel bottom and of the free surface are interdependent. Reliable experimental data on flow in open channels are usually difficult to obtain. Furthermore, the physical condition of open channels varies much more widely than that of pipes. In pipes the cross section of flow is fixed, since it is completely defined by the geometry of the conduit. The cross section of a pipe is generally round, but that of an open channel may be of any shape—from the circular to the irregular forms of natural streams. In pipes, the interior surface ordinarily ranges in roughness

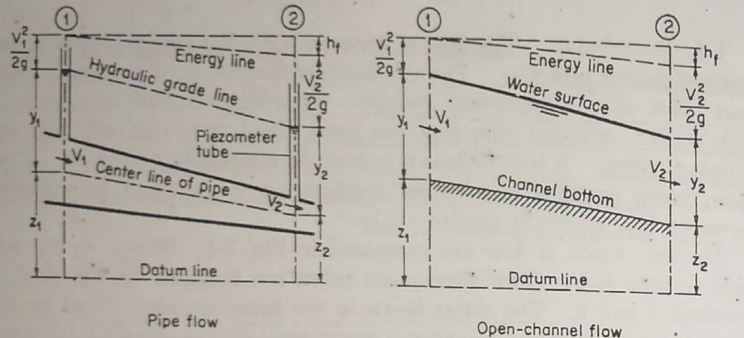


FIG. 1-1. Comparison between pipe flow and open-channel flow.

from that of new smooth brass or wooden-stave pipes, on the one hand, to that of old corroded iron or steel pipes, on the other. In open channels the surface varies from that of the polished metal used in testing flumes to that of rough irregular river beds. Moreover, the roughness in an open channel varies with the position of the free surface. Therefore, the selection of friction coefficients is attended by greater uncertainty for open channels than for pipes. In general, the treatment of open-channel flow is somewhat more empirical than that of pipe flow. The empirical method is the best available at present and, if cautiously applied, can yield results of practical value.

The flow in a closed conduit is not necessarily pipe flow. It must be classified as open-channel flow if it has a free surface. The storm sewer, for example, which is a closed conduit, is generally designed for open-channel flow because the flow in the sewer is expected to maintain a free surface most of the time.

1-2. Types of Flow. Open-channel flow can be classified into many types and described in various ways. The following classification is made according to the change in flow depth with respect to time and space.

Steady Flow and Unsteady Flow: Time as the Criterion. Flow in an open channel is said to be *steady* if the depth of flow does not change or if it can be assumed to be constant during the time interval under consideration. The flow is *unsteady* if the depth changes with time. In most open-channel problems it is necessary to study flow behavior only under steady conditions. If, however, the change in flow condition with respect to time is of major concern, the flow should be treated as unsteady. In floods and surges, for instance, which are typical examples of unsteady flow, the stage of flow changes instantaneously as the waves pass by, and the time element becomes vitally important in the design of control structures.

For any flow, the discharge Q at a channel section is expressed by

$$Q = VA \tag{1-1}$$

where V is the mean velocity and A is the flow cross-sectional area normal to the direction of the flow, since the mean velocity is defined as the discharge divided by the cross-sectional area.

In most problems of steady flow the discharge is constant throughout the reach of the channel under consideration; in other words, the flow is *continuous*. Thus, using Eq. (1-1),

$$Q = V_1A_1 = V_2A_2 = \cdots \tag{1-2}$$

where the subscripts designate different channel sections. This is the *continuity equation* for a continuous steady flow.

Equation (1-2) is obviously invalid, however, where the discharge of a steady flow is *nonuniform* along the channel, that is, where water runs in or out along the course of flow. This type of flow, known as *spatially varied or discontinuous flow*, is found in roadside gutters, side-channel spillways, the washwater troughs in filters, the effluent channels around sewage-treatment tanks, and the main drainage channels and feeding channels in irrigation systems.

The law of continuity of unsteady flow requires consideration of the time effect. Hence, the continuity equation for continuous unsteady flow should include the time element as a variable (Art. 18-1).

Uniform Flow and Varied Flow: Space as the Criterion. Open-channel flow is said to be *uniform* if the depth of flow is the same at every section of the channel. A uniform flow may be steady or unsteady, depending on whether or not the depth changes with time.

Steady uniform flow is the fundamental type of flow treated in open-channel hydraulics. The depth of the flow does not change during the time interval under consideration. The establishment of *unsteady uniform flow* would require that the water surface fluctuate from time to time while remaining parallel to the channel bottom. Obviously, this is a practically impossible condition. The term "uniform flow" is, therefore, used hereafter to refer only to steady uniform flow.

Flow is *varied* if the depth of flow changes along the length of the channel. Varied flow may be either steady or unsteady. Since unsteady uniform flow is rare, the term "unsteady flow" is used hereafter to designate *unsteady varied flow* exclusively.

Varied flow may be further classified as either *rapidly* or *gradually varied*. The flow is rapidly varied if the depth changes abruptly over a comparatively short distance; otherwise, it is gradually varied. A rapidly varied flow is also known as a *local phenomenon;* examples are the hydraulic jump and the hydraulic drop.

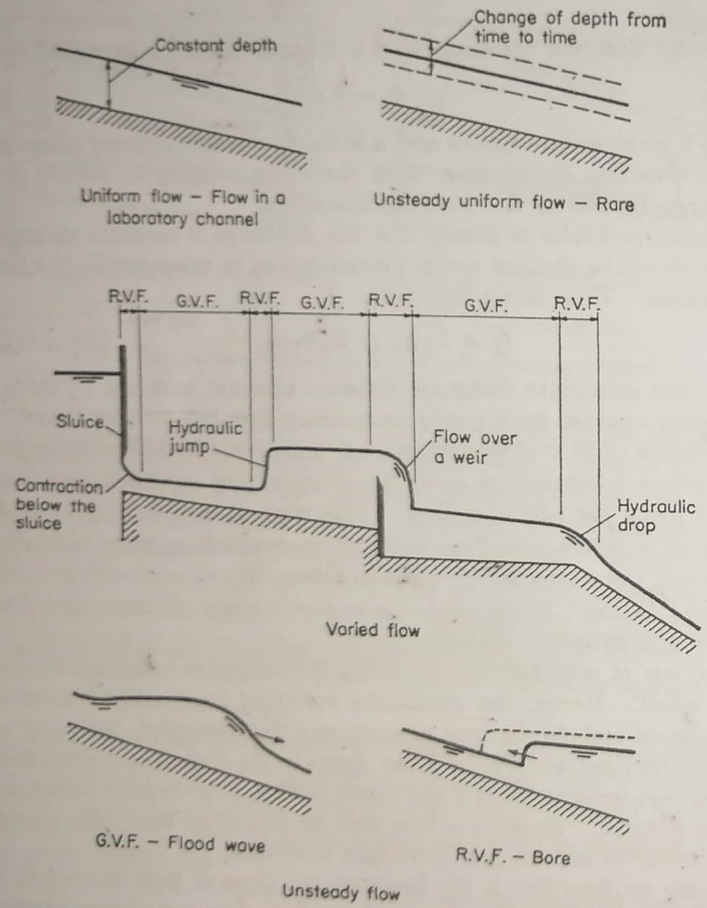


FIG. 1-2. Various types of open-channel flow. G.V.F. = gradually varied flow; R.V.F. = rapidly varied flow.

For clarity, the classification of open-channel flow is summarized as follows:

A. Steady flow
   1. Uniform flow
   2. Varied flow
      a. Gradually varied flow
      b. Rapidly varied flow
B. Unsteady flow
   1. Unsteady uniform flow (rare)
   2. Unsteady flow (i.e., unsteady varied flow)
      a. Gradually varied unsteady flow
      b. Rapidly varied unsteady flow

Various types of flow are sketched in Fig. 1-2. For illustrative purposes, these diagrams, as well as other similar sketches of open channels in this book, have been drawn to a greatly exaggerated vertical scale, since ordinary channels have small bottom slopes.

1-3. State of Flow. The state or behavior of open-channel flow is governed basically by the effects of viscosity and gravity relative to the inertial forces of the flow. The surface tension of water may affect the behavior of flow under certain circumstances, but it does not play a significant role in most open-channel problems encountered in engineering.

Effect of Viscosity. Depending on the effect of viscosity relative to inertia, the flow may be laminar, turbulent, or transitional.

The flow is *laminar* if the viscous forces are so strong relative to the inertial forces that viscosity plays a significant part in determining flow behavior. In laminar flow, the water particles appear to move in definite smooth paths, or streamlines, and infinitesimally thin layers of fluid seem to slide over adjacent layers.

The flow is *turbulent* if the viscous forces are weak relative to the inertial forces. In turbulent flow, the water particles move in irregular paths which are neither smooth nor fixed but which in the aggregate still represent the forward motion of the entire stream.

Between the laminar and turbulent states there is a mixed, or *transitional*, state.

The effect of viscosity relative to inertia can be represented by the *Reynolds number*, defined as

$$R = \frac{VL}{\nu} \tag{1-3}$$

where V is the velocity of flow in fps; L is a characteristic length in ft, here considered equal to the hydraulic radius R of a conduit; and ν (nu) is the kinematic viscosity of water in ft²/sec. The kinematic viscosity in ft²/sec is equal to the dynamic viscosity μ (mu) in slug/ft-sec divided by the mass density ρ (rho) in slug/ft³. For water at 68°F (20°C), $\mu = 2.09 \times 10^{-5}$ and $\rho = 1.937$; hence, $\nu = 1.08 \times 10^{-5}$.

An open-channel flow is laminar if the Reynolds number R is small and turbulent if R is large. Numerous experiments have shown that the flow in a pipe changes from laminar to turbulent in the range of R between the critical value 2,000 and a value that may be as high as 50,000.* In these experiments the diameter of the pipe was taken as the characteristic length in defining the Reynolds number. When the hydraulic radius is taken as the characteristic length, the corresponding range is from 500 to 12,500,* since the diameter of a pipe is four times its hydraulic radius.

The laminar, turbulent, and transitional states of open-channel flow can be expressed by a diagram that shows a relation between the Reynolds number and the friction factor of the Darcy-Weisbach formula. Such a diagram, generally known as the *Stanton diagram* [1], has been developed for flow in pipes. The *Darcy-Weisbach formula*,[1] also developed primarily for flow in pipes, is

$$h_f = f \frac{L}{d_0} \frac{V^2}{2g} \tag{1-4}$$

where h_f is the frictional loss in ft for flow in the pipe, f is the friction factor, L is the length of the pipe in ft, d_0 is the diameter of the pipe in ft, V is the velocity of flow in fps, and g is the acceleration due to gravity in ft/sec².

Since $d_0 = 4R$ and the energy gradient $S = h_f/L$, the above equation may be rewritten for the friction factor

$$f = \frac{8gRS}{V^2} \tag{1-5}$$

This equation may also be applied to uniform and nearly uniform flows in open channels.

The f-R relationship for smooth pipes can be expressed by the *Blasius equation* [5]

$$f = \frac{0.223}{\mathrm{R}^{0.25}} \qquad (1\text{-}6)\dagger$$

* It should be noted that there is actually no definite upper limit.

[1] As a result of Darcy's study [2] on flow in pipes, his name is commonly associated with that of Weisbach [3] in designating this equation which Weisbach first formulated. Actually, d'Aubuisson [4] presented, prior to Darcy, a formula that can be reduced to the form of Eq. (1-4).

† In this equation, the hydraulic radius is used as the characteristic length in defining the Reynolds number. If the diameter of pipe were used as the characteristic length, the numerical constant of the numerator in this equation would be 0.316.

which is believed to be valid only where the value of R is between 750 and 25,000. For higher values of R, von Kármán [6] developed a general expression, which was later modified by Prandtl [7] to agree more closely with the data obtained by Nikuradse [8]. The resulting *Prandtl-von Kármán* equation is

$$\frac{1}{\sqrt{f}} = 2 \log (\mathrm{R}\sqrt{f}) + 0.4 \tag{1-7}$$

Equations (1-6) and (1-7) will be used in the following discussion as a basis for comparing flow conditions in open channels. It may be noted that corresponding equations for flow in open channels have been derived by Keulegan [9] and appear to be very similar to the pipe-flow equations given above. It must be remembered, however, that, owing to the free surface and to the interdependence of the hydraulic radius, discharge, and slope, the f-R relationship in open-channel flow does not follow *exactly* the simple concepts that hold for pipe flow. Some specific features of the f-R relationship in open-channel flow are described below.

Experimental data available for the determination of the f-R relationship in open-channel flow can be found in various publications on hydraulics.[1] Figure 1-3, which plots the relationship for flow in *smooth channels*, is based on data developed at the University of Illinois[2] [21] and the University of Minnesota [20]. In this plot the following features may be noted:

1. The plot shows clearly how the state of flow changes from laminar to turbulent as the Reynolds number increases. The discontinuity of the plot and the spread of data characterize the transitional region, as they do in the Stanton diagram for flow in pipes. The transitional range, however, is not so well defined as it is for pipe flow. The lower critical Reynolds number depends to some extent on channel shape. The value varies from 500 to 600, being generally larger than the value for pipe flow. For practical purposes, the transitional range of R for open-channel flow may be assumed to be 500 to 2,000. It should be noted, however, that the upper value is arbitrary, since there is no definite upper limit for all flow conditions.

2. The data in the laminar region can be defined by a general equation

$$f = \frac{K}{\mathrm{R}} \tag{1-8}$$

From Eqs. (1-3) and (1-5) it can be shown that

$$K = \frac{8gR^2S}{\nu V} \tag{1-9}$$

[1] See [10] to [23].

[2] The data for the rectangular channel were furnished through the courtesy of Professor W. M. Lansford and processed for the present purpose by the author.

Since V and R have specific values for any given channel shape, K is a purely numerical factor dependent only on channel shape. For laminar flow in smooth channels, the value of K can be determined theoretically [20]. The plot in Fig. 1-3 indicates that K is approximately 24 for the rectangular channels and 14 for the triangular channel under consideration.

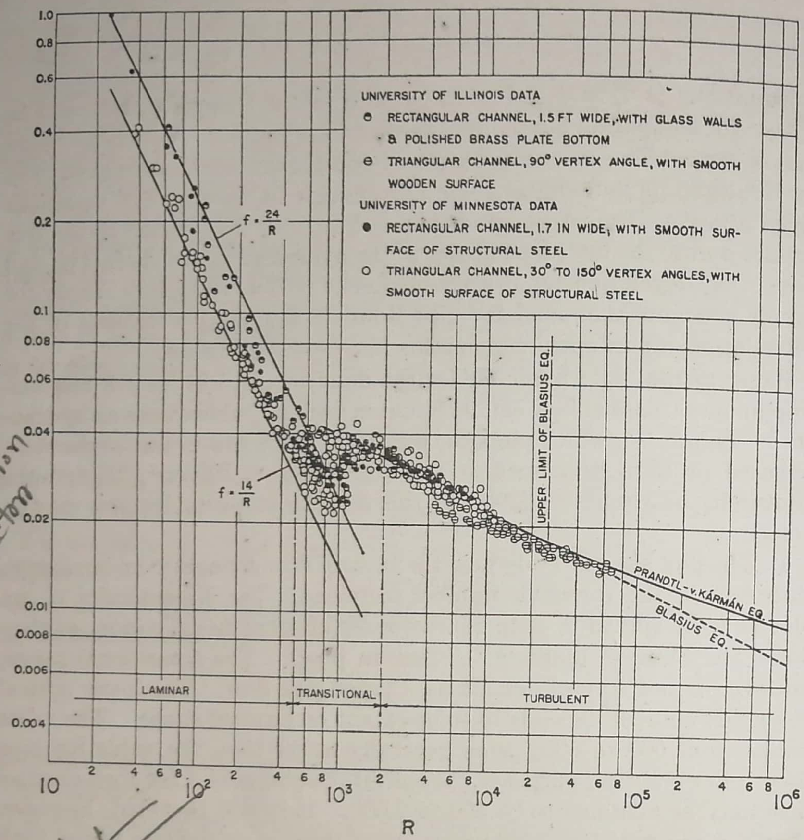


FIG. 1-3. The f-**R** relationship for flow in smooth channels.

3. The data in the turbulent region correspond closely to the Blasius-Prandtl-von Kármán curve. This indicates that the law for turbulent flow in smooth pipes may be approximately representative of all smooth channels. The plot also shows that the shape of the channel does not have an important influence on friction in turbulent flow, as it does in laminar flow.

The data for laminar flow obtained at the University of Minnesota [20] and the data for turbulent flow collected individually by Kirschmer

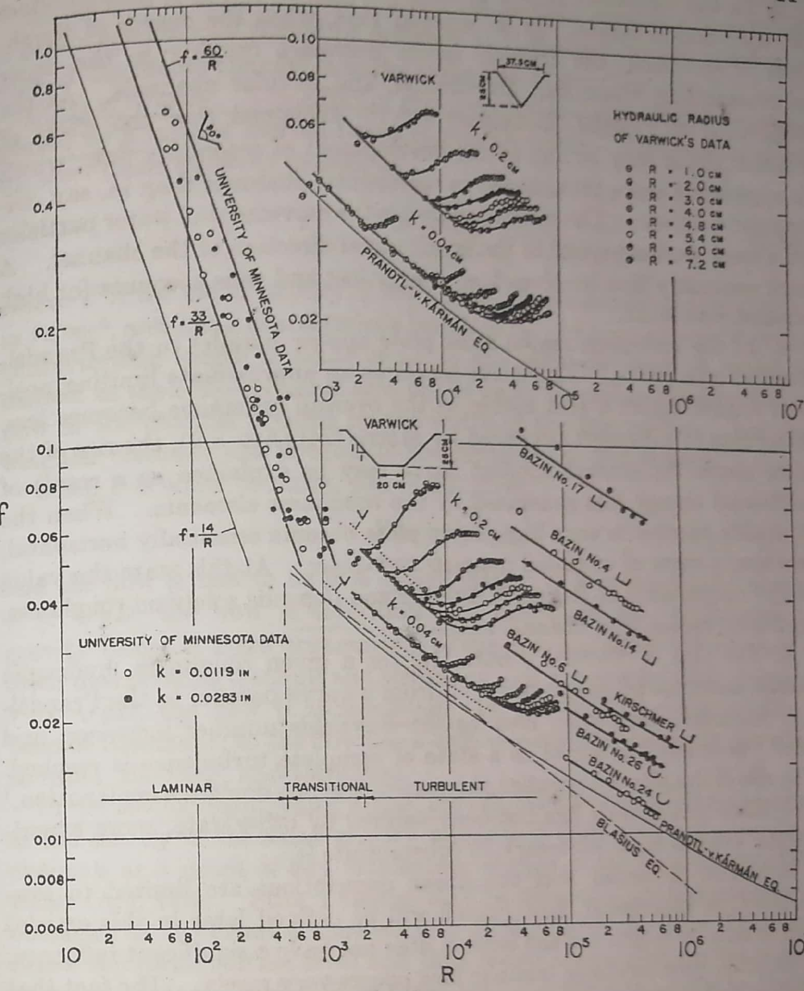


FIG. 1-4. The f-**R** relationship for flow in rough channels. Bazin's channels: No. 4, gravel embedded in cement; No. 6, unpolished wood; No. 14, unpolished wood roughened by transverse wooden strips 27 mm long, 10 mm high, and 10 mm in spacing; No. 17, same as No. 14 except with a spacing of 50 mm; No. 24, cement lining; and No. 26, unpolished wood. Kirschmer's channel: smooth concrete.

[15,16], Eisner [22], and Koženy [23] are shown in the diagram for flow in *rough channels* (Fig. 1-4). In some of the data channel roughness is represented by k, which is a size measure of the roughness particles forming the channel surface. The diagram illustrates the following features:

1. In the laminar region the data can be defined by Eq. (1-8). In this region, the value of K is generally higher than it is for smooth channels

As the flow in most channels is turbulent, a model employed to simulate a prototype channel should be designed so that the Reynolds number of flow of the model channel is in the turbulent range.

Effect of Gravity. The effect of gravity upon the state of flow is represented by a ratio of inertial forces to gravity forces. This ratio is given by the *Froude number*,[1] defined as

$$\mathbf{F} = \frac{V}{\sqrt{gL}} \tag{1-10}$$

where V is the mean velocity of flow in fps, g is the acceleration of gravity in ft/sec^2, and L is a characteristic length in ft. In open-channel flow the characteristic length is made equal to the *hydraulic depth* D, which is defined as the cross-sectional area of the water normal to the direction of flow in the channel divided by the width of the free surface. For rectangular channels this is equal to the depth of the flow section.

When $\mathbf{F}$ is equal to unity, Eq. (1-10) gives

$$V = \sqrt{gD} \tag{1-11}$$

and the flow is said to be in a *critical* state. If $\mathbf{F}$ is less than unity, or $V < \sqrt{gD}$, the flow is *subcritical.* In this state the role played by gravity forces is more pronounced; so the flow has a low velocity and is often described as tranquil and streaming. If $\mathbf{F}$ is greater than unity, or $V > \sqrt{gD}$, the flow is *supercritical.* In this state the inertial forces become dominant; so the flow has a high velocity and is usually described as rapid, shooting, and torrential.

In the mechanics of water waves, the critical velocity $\sqrt{gD}$ is identified as the *celerity* of the small gravity waves that occur in shallow water in channels as a result of any momentary change in the local depth of the water (Art. 18-6). Such a change may be developed by disturbances or obstacles in the channel that cause a displacement of water above and below the mean surface level and thus create waves that exert a weight or gravity force. It should be noted that a gravity wave can be propagated upstream in water of subcritical flow but not in water of supercritical flow, since the celerity is greater than the velocity of flow in the former case and less in the latter. Therefore, the possibility or impossibility of propagating a gravity wave upstream can be used as a criterion for distinguishing between subcritical and supercritical flow.

Since the flow in most channels is controlled by the gravity effect, a model used to simulate a prototype channel for testing purposes must be

[1] Other dimensionless ratios used for the same purpose include (1) the *kinetic-flow factor* $\lambda = V^2/gL = \mathbf{F}^2$, first used by Rehbock [25] and then by Bakhmeteff [26]; (2) the *Boussinesq number* $\mathbf{B} = V/\sqrt{2gR}$, first used by Engel [27]; and (3) the *kineticity* or *velocity-head ratio* $k = V^2/2gL$, proposed by Stevens [28] and Posey [29], respectively.

designed for this effect; that is, the Froude number of the flow in the model channel must be made equal to that of the flow in the prototype channel.

1-4. Regimes of Flow. A combined effect of viscosity and gravity may produce any one of four *regimes of flow* in an open channel, namely, (1) *subcritical-laminar*, when **F** is less than unity and **R** is in the laminar range; (2) *supercritical-laminar*, when **F** is greater than unity and **R** is in the laminar range; (3) *supercritical-turbulent*, when **F** is greater than unity

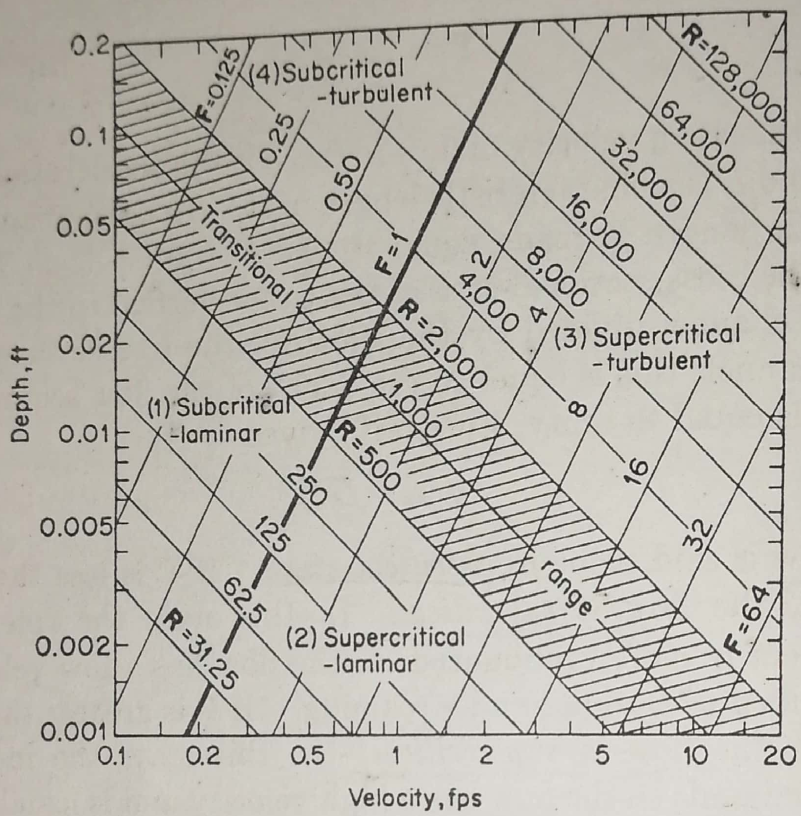


FIG. 1-5. Depth-velocity relationships for four regimes of open-channel flow. (*After Robertson and Rouse* [30].)

and **R** is in the turbulent range; and (4) *subcritical-turbulent*, when **F** is less than unity and **R** is in the turbulent range. The depth-velocity relationships for the four flow regimes in a wide open channel can be shown by a logarithmic plot (Fig. 1-5) [30]. The heavy line for **F** = 1 and the shaded band for the laminar-turbulent transitional range intersect on the graph and divide the whole area into four portions, each of which represents a flow regime. The first two regimes, subcritical-laminar and supercritical-laminar, are not commonly encountered in applied open-channel hydraulics, since the flow is generally turbulent in the channels considered in engineering problems. However, these regimes occur frequently where there is very thin depth—this is known as *sheet flow*—and they become significant in such problems as the testing of hydraulic models, the study of overland flow, and erosion control for such flow.

Photographs of the four regimes of flow are shown in Fig. 1-6. In each

CHAPTER 2

OPEN CHANNELS AND THEIR PROPERTIES

2-1. Kinds of Open Channel. An open channel is a conduit in which water flows with a free surface. Classified according to its origin a channel may be either *natural* or *artificial.*

Natural channels include all watercourses that exist naturally on the earth, varying in size from tiny hillside rivulets, through brooks, streams, small and large rivers, to tidal estuaries. Underground streams carrying water with a free surface are also considered natural open channels.

The hydraulic properties of natural channels are generally very irregular. In some cases empirical assumptions reasonably consistent with actual observations and experience may be made such that the conditions of flow in these channels become amenable to the analytical treatment of theoretical hydraulics. A comprehensive study of the behavior of flow in natural channels requires knowledge of other fields, such as hydrology, geomorphology, sediment transportation, etc. It constitutes, in fact, a subject of its own, known as *river hydraulics.*

Artificial channels are those constructed or developed by human effort: navigation channels, power canals, irrigation canals and flumes, drainage ditches, trough spillways, floodways, log chutes, roadside gutters, etc., as well as model channels that are built in the laboratory for testing purposes. The hydraulic properties of such channels can be either controlled to the extent desired or designed to meet given requirements. The application of hydraulic theories to artificial channels will, therefore, produce results fairly close to actual conditions and, hence, are reasonably accurate for practical design purposes.

Under various circumstances in engineering practice the artificial open channel is given different names, such as "canal," "flume," "chute," "drop," "culvert," "open-flow tunnel," etc. These names, however, are used rather loosely and can be defined only in a very general way. The *canal* is usually a long and mild-sloped channel built in the ground, which may be unlined or lined with stone masonry, concrete, cement, wood, or bituminous materials. The *flume* is a channel of wood, metal, concrete, or masonry, usually supported on or above the surface of the ground to carry water across a depression. The *chute* is a channel having steep slopes. The *drop* is similar to a chute, but the change in elevation is effected in a short distance. The *culvert* flowing partly full is a covered channel of comparatively short length installed to drain water through highway and railroad embankments. The *open-flow tunnel* is a comparatively long covered channel used to carry water through a hill or any obstruction on the ground.

2-2. Channel Geometry. A channel built with unvarying cross section and constant bottom slope is called a *prismatic channel*. Otherwise, the channel is *nonprismatic;* an example is a trough spillway having variable width and curved alignment. Unless specifically indicated, the channels described in this book are prismatic.

The term *channel section* used in this book refers to the cross section of a channel taken normal to the direction of the flow. A *vertical channel section*, however, is the vertical section passing through the lowest or bottom point of the channel section. For horizontal channels, therefore, the channel section is always a vertical channel section.

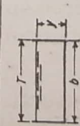
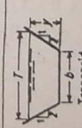
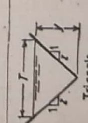
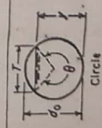
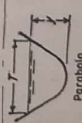
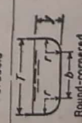
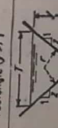
Natural channel sections are in general very irregular, usually varying from an approximate parabola to an approximate trapezoid. For streams subject to frequent floods, the channel may consist of a main channel section carrying normal discharges and one or more side channel sections for accommodating overflows.

Artificial channels are usually designed with sections of regular geometric shapes. Table 2-1 lists seven geometric shapes that are in common use. The trapezoid is the commonest shape for channels with unlined earth banks, for it provides side slopes for stability. The rectangle and triangle are special cases of the trapezoid. Since the rectangle has vertical sides, it is commonly used for channels built of stable materials, such as lined masonry, rocks, metal, or timber. The triangular section is used only for small ditches, roadside gutters, and laboratory works. The circle is the popular section for sewers and culverts of small and medium sizes. The parabola[1] is used as an approximation of sections of small and medium-size natural channels. The round-cornered rectangle is a modification of the rectangle. The round-bottom triangle is an approximation of the parabola; it is a form usually created by excavation with shovels.

Closed geometric sections other than the circle are frequently used in sewerage, particularly for sewers large enough for a man to enter. These sections are given various names according to their form; they may be

[1] The side slope $z:1$ of a parabolic section at the intersection of the sides with the free surface can be computed easily by the simple formula $z = T/4y$. Russian engineers [1] also use semielliptical and parabolic sections of higher order: $y = ax^p$ with $p = 3$ or 4. The constant a is computed from the side slope assumed at the free surface.

TABLE 2-1. GEOMETRIC ELEMENTS OF CHANNEL SECTIONS

Section	Area A	Wetted perimeter P	Hydraulic radius R	Top width T	Hydraulic depth D	Section factor Z
 Rectangle	by	$b + 2y$	$\frac{by}{b + 2y}$	b	y	$by^{1.5}$
 Trapezoid	$(b + zy)y$	$b + 2y\sqrt{1 + z^2}$	$\frac{(b + zy)y}{b + 2y\sqrt{1 + z^2}}$	$b + 2zy$	$\frac{(b + zy)y}{b + 2zy}$	$\frac{[(b + zy)y]^{1.5}}{\sqrt{b + 2zy}}$
 Triangle	zy^2	$2y\sqrt{1 + z^2}$	$\frac{zy}{2\sqrt{1 + z^2}}$	$2zy$	$\frac{1}{2}y$	$\frac{\sqrt{2}}{2}zy^{2.5}$
 Circle	$\frac{1}{8}(\theta - \sin\theta)d_0^2$	$\frac{1}{2}\theta d_0$	$\frac{1}{4}\left(1 - \frac{\sin\theta}{\theta}\right)d_0$	$(\sin\frac{1}{2}\theta)d_0$ or $2\sqrt{y(d_0 - y)}$	$\frac{1}{8}\left(\frac{\theta - \sin\theta}{\sin\frac{1}{2}\theta}\right)d_0$	$\frac{\sqrt{2}}{32}\frac{(\theta - \sin\theta)^{1.5}}{(\sin\frac{1}{2}\theta)^{0.5}}d_0^{2.5}$
 Parabola	$\frac{2}{3}Ty$	$T + \frac{8}{3}\frac{y^2}{T}$ *	$\frac{2T^2y}{3T^2 + 8y^2}$ *	$\frac{3}{2}\frac{A}{y}$	$\frac{2}{3}y$	$\frac{2}{9}\sqrt{6}\,Ty^{1.5}$
 Round-cornered rectangle ($y > r$)	$\left(\frac{\pi}{2} - 2\right)r^2 + (b + 2r)y$	$(\pi - 2)r + b + 2y$	$\frac{(\pi/2 - 2)r^2 + (b + 2r)y}{(\pi - 2)r + b + 2y}$	$b + 2r$	$\frac{(\pi/2 - 2)r^2}{b + 2r} + y$	$\frac{[(\pi/2 - 2)r^2 + (b + 2r)y]^{1.5}}{\sqrt{b + 2r}}$
 Round-bottomed triangle	$\frac{T^2}{4z} - \frac{r^2}{z}(1 - z\cot^{-1}z)$	$\frac{T}{z}\sqrt{1 + z^2} - \frac{2r}{z}(1 - z\cot^{-1}z)$	$\frac{A}{P}$	$2[z(y - r) + r\sqrt{1 + z^2}]$	$\frac{A}{T}$	$A\sqrt{\frac{A}{T}}$

* Satisfactory approximation for the interval $0 < x \leq 1$, where $x = 4y/T$. When $x > 1$, use the exact expression $P = (T/2)[\sqrt{1 + x^2} + 1/x \ln(x + \sqrt{1 + x^2})]$.

the width is greater than 5 to 10 times the depth of flow, depending on the condition of surface roughness. Thus, a *wide open channel* can safely be defined as a rectangular channel whose width is greater than 10 times the depth of flow. For either experimental or analytical purposes, the flow in the central region of a wide open channel may be considered to be the same as the flow in a rectangular channel of infinite width.

2-6. Measurement of Velocity. According to the stream-gaging procedure of the U.S. Geological Survey,[1] the channel cross section is divided into vertical strips by a number of successive verticals, and mean velocities in verticals are determined by measuring the velocity at 0.6 of the depth in each vertical, or, where more reliable results are required, by taking the average of the velocities at 0.2 and 0.8 of the depth. When the stream is covered with ice, the mean velocity is no longer close to 0.6 of the water depth, but the average at 0.2 and 0.8 of the water depth still gives reliable results. The average of the mean velocities in any two adjacent verticals multiplied by the area between the verticals gives the discharge through this vertical strip of the cross section. The sum of discharges through all strips is the total discharge. The mean velocity of the whole section is, therefore, equal to the total discharge divided by the whole area.

It should be noted that the above methods are simple and approximate. For precise measurements more elaborate methods must be used, which are beyond the scope of this book.

2-7. Velocity-distribution Coefficients. As a result of nonuniform distribution of velocities over a channel section, the velocity head of an open-channel flow is generally greater than the value computed according to the expression $V^2/2g$, where V is the mean velocity. When the energy principle is used in computation, the true velocity head may be expressed as $\alpha V^2/2g$, where α is known as the *energy coefficient* or *Coriolis coefficient*, in honor of G. Coriolis [12] who first proposed it. Experimental data indicate that the value of α varies from about 1.03 to 1.36 for fairly straight prismatic channels. The value is generally higher for small channels and lower for large streams of considerable depth.

The nonuniform distribution of velocities also affects the computation of momentum in open-channel flow. From the principle of mechanics, the momentum of the fluid passing through a channel section per unit time is expressed by $\beta wQV/g$, where β is known as the *momentum coefficient* or *Boussinesq coefficient*, after J. Boussinesq [13] who first proposed it; w is the unit weight of water; Q is the discharge; and V is the mean velocity. It is generally found that the value of β for fairly straight prismatic channels varies approximately from 1.01 to 1.12.

The two velocity-distribution coefficients are always slightly larger than the limiting value of unity, at which the velocity distribution is

[1] For details see [9] to [11].

strictly uniform across the channel section. For channels of regular cross section and fairly straight alignment, the effect of nonuniform velocity distribution on the computed velocity head and momentum is small, especially in comparison with other uncertainties involved in the computation. Therefore, the coefficients are often assumed to be unity. In channels of complex cross section, the coefficients for energy and momentum can easily be as great as 1.6 and 1.2, respectively, and can vary quite rapidly from section to section in case of irregular alignment. Upstream from weirs, in the vicinity of obstructions, or near pronounced irregularities in alignment, values of α greater than 2.0 have been observed.[1] Precise studies or analyses of flow in such channels will require measurement of the actual velocity and accurate determination of the coefficients. In regard to the effect of channel slope, the coefficients are usually higher in steep channels than in flat channels.

For practical purposes, Kolupaila [16] proposed the values shown below for the velocity-distribution coefficients. Actual values of the coefficients for a number of channels may be found in [17] and [18].

Channels	Value of α			Value of β		
	Min	Av	Max	Min	Av	Max
Regular channels, flumes, spillways	1.10	1.15	1.20	1.03	1.05	1.07
Natural streams and torrents	1.15	1.30	1.50	1.05	1.10	1.17
Rivers under ice cover	1.20	1.50	2.00	1.07	1.17	1.33
River valleys, overflooded	1.50	1.75	2.00	1.17	1.25	1.33

2-8. Determination of Velocity-distribution Coefficients. Let ΔA be an elementary area in the whole water area A, and w the unit weight of water; then the weight of water passing ΔA per unit time with a velocity v is $wv\,\Delta A$. The kinetic energy of water passing ΔA per unit time is $wv^3\,\Delta A/2g$. This is equivalent to the product of the weight $wv\,\Delta A$ and the velocity head $v^2/2g$. The total kinetic energy for the whole water area is equal to $\Sigma wv^3\,\Delta A/2g$.

Now, taking the whole area as A, the mean velocity as V, and the

[1] A value of $\alpha = 2.08$ was computed by Lindquist [14] using data from weir measurements made by Ernest W. Schoder and Kenneth B. Turner.

In the case of closed conduits, much larger values of α have been observed [15]. A value of $\alpha = 3.87$, observed at the outlet section of a draft tube in the Rublevo power plant, is probably the largest known value obtained from actual measurements; the real value there must have been still larger—10.2% more, if the effect of a 15° curvature of the streamlines is taken into account. The largest known value from laboratory measurements is believed to be $\alpha = 7.4$, which was derived by V. S. Kviatkovskii in 1940 in the VIGM (All-Union Institute for Hydraulic Machinery, U.S.S.R.) for the spiral flow under a model turbine wheel.

corrected velocity head for the whole area as $\alpha V^2/2g$, the total kinetic energy is $\alpha wV^3A/2g$. Equating this quantity with $\Sigma wv^3\,\Delta A/2g$ and reducing,

$$\alpha = \frac{\int v^3\,dA}{V^3A} \approx \frac{\Sigma v^3\,\Delta A}{V^3A} \tag{2-4}$$

The momentum of water passing ΔA per unit time is the product of the mass $wv\,\Delta A/g$ and the velocity v, or $wv^2\,\Delta A/g$. The total momentum is $\Sigma wv^2\,\Delta A/g$. Equating this quantity with the corrected momentum for the whole area, or $\beta wAV^2/g$, and reducing,

$$\beta = \frac{\int v^2\,dA}{V^2A} \approx \frac{\Sigma v^2\,\Delta A}{V^2A} \tag{2-5}$$

O'Brien and Johnson [19] used a graphical solution of the above formulas as follows:

From the measured velocity-distribution curves, the area within each curve of equal velocity is planimetered. Taking the velocity indicated by each equal-velocity curve as v, a curve of v^3 against the corresponding planimetered area is constructed. It is evident that the area beneath this v^3 curve is the integral $\Sigma v^3\,\Delta A$, which can be obtained by planimetering again. Similarly, integrals $\Sigma v^2\,\Delta A$ and $\Sigma v\,\Delta A$ can also be obtained. The integral $\Sigma v\,\Delta A$ divided by A gives V. With these quantities determined, the above equations can be solved for the coefficients α and β.

For approximate values, the energy and momentum coefficients can be computed by the following formulas:[1]

$$\alpha = 1 + 3\epsilon^2 - 2\epsilon^3 \tag{2-6}$$
$$\beta = 1 + \epsilon^2 \tag{2-7}$$

where $\epsilon = v_M/V - 1$, v_M being the maximum velocity and V being the mean velocity.

Computation of the velocity-distribution coefficients for irregular natural channels will be discussed later (Art. 6-5). In most practical problems dealing with regular channels it is not necessary to consider the variation of velocity throughout the cross section, since use of the average velocity will give the accuracy required. The expressions $V^2/2g$ and wQV/g are used extensively in this book with the understanding either that these items have been corrected for the effect of the nonuniform velocity distribution, or that a value of unity is assumed.[2]

[1] These formulas are obtained by assuming a logarithmic distribution of velocity (Art. 8-5, Prob. 8-9). Assuming a linear velocity distribution, Rehbock [20] obtained $\alpha = 1 + \epsilon^2$ and $\beta = 1 + \epsilon^2/3$.

[2] For discussions on this subject, the reader may look into [21] and [22]. However, he should use judgment in reading these references because they contain erroneous

2-9. Pressure Distribution in a Channel Section. The pressure at any point on the cross section of the flow in a channel of small slope can be measured by the height of the water column in a piezometer tube installed at the point (Fig. 2-7). Ignoring minor disturbances due to turbulence, etc., it is apparent that this water column should rise from the point of measurement up to the hydraulic grade line or the water surface. Therefore, the pressure at any point on the section is directly proportional to the depth of the point below the free surface and equal to the hydrostatic pressure corresponding to this depth. In other words, the distribution of pressure over the cross section of the channel is the same as the distribution of hydrostatic pressure; that is, the distribution is linear and can be represented by a straight line *AB* (Fig. 2-7*a*). This is known as the *hydrostatic law of pressure distribution.*

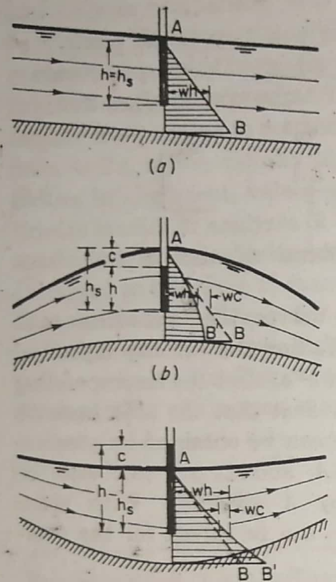


FIG. 2-7. Pressure distribution in straight and curved channels of small or horizontal slope at the section under consideration. h = piezometric head; h_s = hydrostatic head; and c = pressure-head correction for curvature. (*a*) Parallel flow; (*b*) convex flow; (*c*) concave flow.

Strictly speaking, the application of the hydrostatic law to the pressure distribution in the cross section of a flowing channel is valid only if the flow filaments have no acceleration components in the plane of cross section. This type of flow is theoretically known as *parallel flow*,[1] that is, such that the streamlines have neither substantial curvature nor divergence. Consequently, there are no appreciable acceleration components normal to the direction of flow that would disturb the hydrostatic-pressure distribution in the cross section of a parallel flow.

statements. Some authors have proposed the use of the momentum coefficient to replace the energy coefficient even in computations based on the energy principle. This is not correct. Whether the energy coefficient or the momentum coefficient is to be used depends on whether the energy or the momentum principle is involved. The two coefficients are derived independently from basically different principles (Art. 3-6). Neither of them is wrong and neither can be replaced by the other; both should be used in the correct sense.

[1] Specific qualifications for parallel flow were clearly stated for the first time by Bélanger [23].

In actual problems uniform flow is practically parallel flow. Gradually varied flow may also be regarded as parallel flow, since the change in depth of flow is so mild that the streamlines have neither appreciable curvature nor divergence; that is, the curvature and divergence are so small that the effect of the acceleration components in the cross-sectional plane is negligible. *For practical purposes, therefore, the hydrostatic law of pressure distribution is applicable to gradually varied flow as well as to uniform flow.*

If the curvature of streamlines is substantial, the flow is theoretically known as *curvilinear flow*. The effect of the curvature is to produce appreciable acceleration components or centrifugal forces normal to the direction of flow. Thus, the pressure distribution over the section deviates from the hydrostatic if curvilinear flow occurs in the vertical plane. Such curvilinear flow may be either convex or concave (Fig. 2-7*b* and *c*). In both cases the nonlinear pressure distribution is represented by AB' instead of the straight distribution AB that would occur if the flow were parallel. It is assumed that all streamlines are horizontal at the section under consideration. In concave flow the centrifugal forces are pointing downward to reinforce the gravity action; so the resulting pressure is greater than the otherwise hydrostatic pressure of a parallel flow. In convex flow the centrifugal forces are acting upward against the gravity action; consequently, the resulting pressure is less than the otherwise hydrostatic pressure of a parallel flow. Similarly, when divergence of streamlines is great enough to develop appreciable acceleration components normal to the flow, the hydrostatic pressure distribution will be disturbed accordingly.

Let the deviation from an otherwise hydrostatic pressure h_s in a curvilinear flow be designated by c (Fig. 2-7*b* and *c*). Then the true pressure or the piezometric height $h = h_s + c$.

If the channel has a curved longitudinal profile, the approximate centrifugal pressure may be computed, by Newton's law of acceleration, as the product of the mass of water having height d and a cross section of 1 sq ft, that is, wd/g, and the centrifugal acceleration v^2/r; or

$$P = \frac{wd}{g}\frac{v^2}{r} \tag{2-8}$$

where w is the unit weight of water, g is the gravitational acceleration, v is the velocity of flow, and r is the radius of curvature. The pressure-head correction is, therefore,

$$c = \frac{d}{g}\frac{v^2}{r} \tag{2-9}$$

For computing the value of c at the channel bottom, r is the radius of curvature of the bottom, d is the depth of flow, and for practical purposes

CHAPTER 3

ENERGY AND MOMENTUM PRINCIPLES

3-1. Energy in Open-channel Flow. (It is known in elementary hydraulics that the total energy in foot-pounds per pound of water in any streamline passing through a channel section may be expressed as the total head in feet of water, which is equal to the sum of the elevation

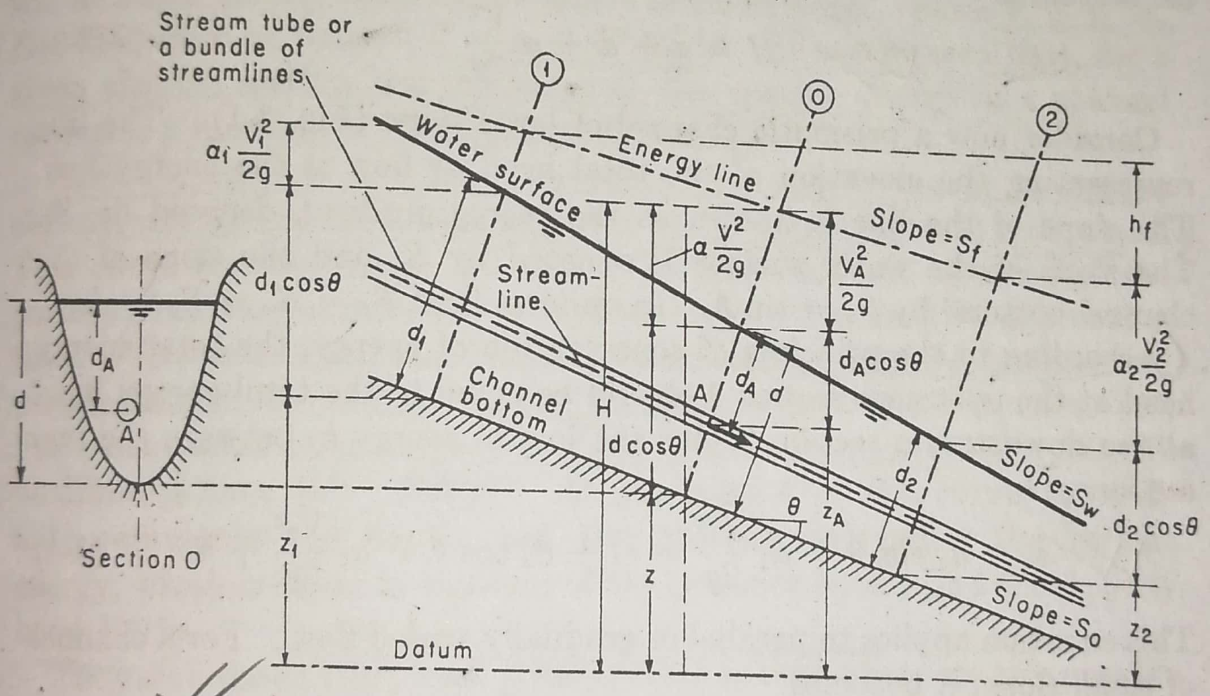


FIG. 3-1. Energy in gradually varied open-channel flow.

above a datum, the pressure head, and the velocity head.). For example, with respect to the datum plane, the total head H at a section O containing point A on a streamline of flow in a channel of large slope (Fig. 3-1) may be written

$$H = z_A + d_A \cos\theta + \frac{V_A^2}{2g} \tag{3-1}$$

where z_A is the elevation of point A above the datum plane, d_A is the depth of point A below the water surface measured along the channel section, θ is the slope angle of the channel bottom, and $V_A^2/2g$ is the velocity head of the flow in the streamline passing through A.

In general, every streamline passing through a channel section will have a different velocity head, owing to the nonuniform velocity distribution in actual flow. Only in an ideal parallel flow of uniform velocity distribution can the velocity head be truly identical for all points on the cross section. In the case of gradually varied flow, however, it may be assumed, for practical purposes, that the velocity heads for all points on the channel section are equal, and the energy coefficient may be used to correct for the over-all effect of the nonuniform velocity distribution. Thus, the total energy at the channel section is

$$H = z + d \cos \theta + \alpha \frac{V^2}{2g} \tag{3-2}$$

For channels of small slope, $\theta \approx 0$. Thus, the total energy at the channel section is

$$H = z + d + \alpha \frac{V^2}{2g} \tag{3-3}$$

Consider now a prismatic channel of large slope (Fig. 3-1). The line representing the elevation of the total head of flow is the energy line. The slope of the line is known as the *energy gradient*, denoted by S_f. The slope of the water surface is denoted by S_w and the slope of the channel bottom[1] by $S_0 = \sin \theta$. In uniform flow, $S_f = S_w = S_0 = \sin \theta$. According to the principle of conservation of energy, the total energy head at the upstream section 1 should be equal to the total energy head at the downstream section 2 plus the loss of energy h_f between the two sections; or

$$z_1 + d_1 \cos \theta + \alpha_1 \frac{V_1^2}{2g} = z_2 + d_2 \cos \theta + \alpha_2 \frac{V_2^2}{2g} + h_f \tag{3-4}$$

This equation applies to parallel or gradually varied flow. For a channel of small slope, it becomes

$$z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g} + h_f \tag{3-5}$$

Either of these two equations is known as the *energy equation*. When $\alpha_1 = \alpha_2 = 1$ and $h_f = 0$, Eq. (3-5) becomes

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} = \text{const} \tag{3-6}$$

This is the well-known *Bernoulli energy equation*.[2]

[1] The slope is generally defined as tan θ. For the present purpose, however, it is defined as sin θ.

[2] It is believed that this equation is ascribed to the Swiss mathematician Daniel Bernoulli only by inference, to give recognition to his pioneer achievement in hydrodynamics, in particular the introduction of the concept of "head." Actually, this equation was first formulated by Leonhard Euler and later popularized by Julius Weisbach [1].

3-2. Specific Energy. *Specific energy*[1] in a channel section is defined as the energy per pound of water at any section of a channel measured with respect to the channel bottom. Thus, according to Eq. (3-2) with $z = 0$, the specific energy becomes

$$E = d \cos \theta + \alpha \frac{V^2}{2g} \tag{3-7}$$

or, for a channel of small slope and $\alpha = 1$,

$$E = y + \frac{V^2}{2g} \tag{3-8}$$

which indicates that the specific energy is equal to the sum of the depth of water and the velocity head. For simplicity, the following discussion will be based on Eq. (3-8) for a channel of small slope. Since $V = Q/A$, Eq. (3-8) may be written $E = y + Q^2/2gA^2$. It can be seen that, for a given channel section and discharge Q, the specific energy in a channel section is a function of the depth of flow only.

When the depth of flow is plotted against the specific energy for a given channel section and discharge, a *specific-energy curve* (Fig. 3-2) is obtained. This curve has two limbs AC and BC. The limb AC approaches the horizontal axis asymptotically toward the right. The limb BC approaches the line OD as it extends upward and to the right. Line OD is a line that passes through the origin and has an angle of inclination equal to 45°. For a channel of large slope, the angle of inclination of the line OD will be different from 45°. (Why?) At any point P on this curve, the ordinate represents the depth, and the abscissa represents the specific energy, which is equal to the sum of the pressure head y and the velocity head $V^2/2g$.

The curve shows that, for a given specific energy, there are two possible depths, for instance, the *low stage* y_1 and the *high stage* y_2. The low stage is called the *alternate depth* of the high stage, and vice versa. At point C, the specific energy is a minimum. It will be proved later that this condition of minimum specific energy corresponds to the critical state of flow. Thus, at the critical state the two alternate depths apparently become one, which is known as the *critical depth* y_c. When the depth of flow is greater than the critical depth, the velocity of flow is less than the critical velocity for the given discharge, and, hence, the flow is subcritical. When the depth of flow is less than the critical depth, the flow is supercritical. Hence, y_1 is the depth of a supercritical flow, and y_2 is the depth of a subcritical flow.

If the discharge changes, the specific energy will be changed accordingly. The two curves $A'B'$ and $A''B''$ (Fig. 3-2) represent positions of

[1] The concept of specific energy was first introduced by Bakhmeteff [2] in 1912.

the specific-energy curve when the discharge is less and greater, respectively, than the discharge used for the construction of the curve AB.

3-3. Criterion for a Critical State of Flow. The critical state of flow has been defined (Art. 1-3) as the condition for which the Froude number is equal to unity. A more common definition is that it is the state of flow at which the specific energy is a minimum for a given discharge.[1] A

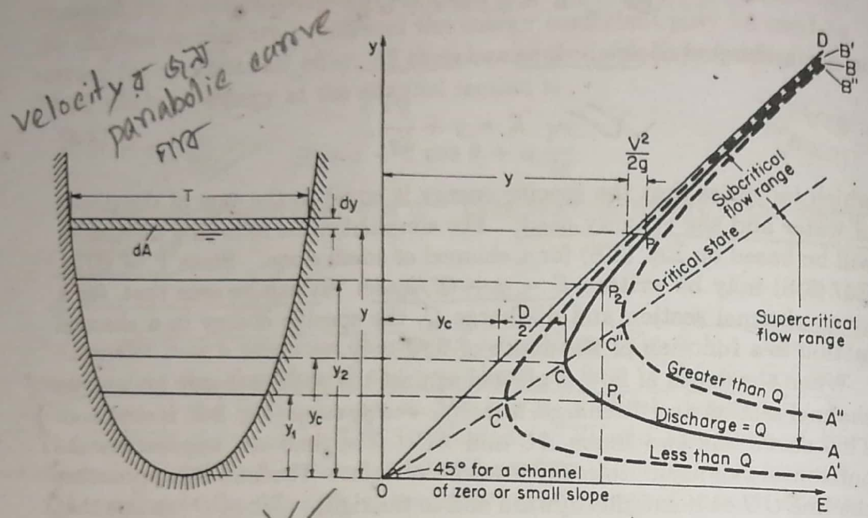


FIG. 3-2. Specific-energy curve.

theoretical criterion for critical flow may be developed from this definition as follows:

Since $V = Q/A$, Eq. (3-8), the equation for specific energy in a channel of small slope with $\alpha = 1$, may be written

$$E = y + \frac{Q^2}{2gA^2} \tag{3-9}$$

Differentiating with respect to y and noting that Q is a constant,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 1 - \frac{V^2}{gA}\frac{dA}{dy}$$

The differential water area dA near the free surface (Fig. 3-2) is equal to $T\,dy$. Now $dA/dy = T$, and the hydraulic depth $D = A/T$; so the above equation becomes

$$\frac{dE}{dy} = 1 - \frac{V^2T}{gA} = 1 - \frac{V^2}{gD}$$

[1] The concept of critical depth based on the theorem of minimum energy was first introduced by Böss [3].

At the critical state of flow the specific energy is a minimum, or $dE/dy = 0$. The above equation, therefore, gives

$$\frac{V^2}{2g} = \frac{D}{2} \tag{3-10}$$

This is the criterion for critical flow, which states that *at the critical state of flow, the velocity head is equal to half the hydraulic depth.* The above equation may also be written $V/\sqrt{gD} = 1$, which means $\mathbf{F} = 1$; this is the definition of critical flow given previously (Art. 1-3).

If the above criterion is to be used in any problem, the following conditions must be satisfied: (1) flow parallel or gradually varied, (2) channel of small slope, and (3) energy coefficient assumed to be unity. If the energy coefficient is not assumed to be unity, the critical-flow criterion is

$$\alpha\frac{V^2}{2g} = \frac{D}{2} \tag{3-11}$$

For a channel of large slope angle θ and energy coefficient α, the criterion for critical flow can easily be proved to be

$$\alpha\frac{V^2}{2g} = \frac{D\cos\theta}{2} \tag{3-12}$$

where D is the hydraulic depth of the water area normal to the channel bottom. In this case, the Froude number may be defined as

$$\mathbf{F} = \frac{V}{\sqrt{gD\cos\theta/\alpha}} \tag{3-13}$$

It should be noted that the coefficient α of a channel section actually varies with depth. In the above derivation, however, the coefficient is assumed to be constant; therefore, the resulting equation is not absolutely exact.

3-4. Interpretation of Local Phenomena. Change of the state of flow from subcritical to supercritical or vice versa occurs frequently in open channels. Such change is manifested in a corresponding change in the depth of flow from a high stage to a low stage or vice versa. If the change takes place rapidly over a relatively short distance, the flow is rapidly varied, and is known as a *local phenomenon*. The hydraulic drop and hydraulic jump are the two types of local phenomena, and may be described as follows:

Hydraulic Drop. A rapid change in the depth of flow from a high stage to a low stage will result in a steep depression in the water surface. Such a phenomenon is generally caused by an abrupt change in the channel slope or cross section and is known as a *hydraulic drop* (Fig. 1-2). At the transitory region of the hydraulic drop a reverse curve usually appears, connecting the water surfaces before and after the drop. The point of inflection on the reverse curve marks the approximate position of the critical depth at which the specific energy is a minimum and the flow passes from a subcritical state to a supercritical state.

The *free overfall* (Fig. 3-3) is a special case of the hydraulic drop. It occurs where the bottom of a flat channel is discontinued. As the free overfall enters the air in the form of a nappe, there will be no reverse curve in the water surface until it strikes some object at a lower elevation. It is the law of nature that, if no energy were added from the outside, the

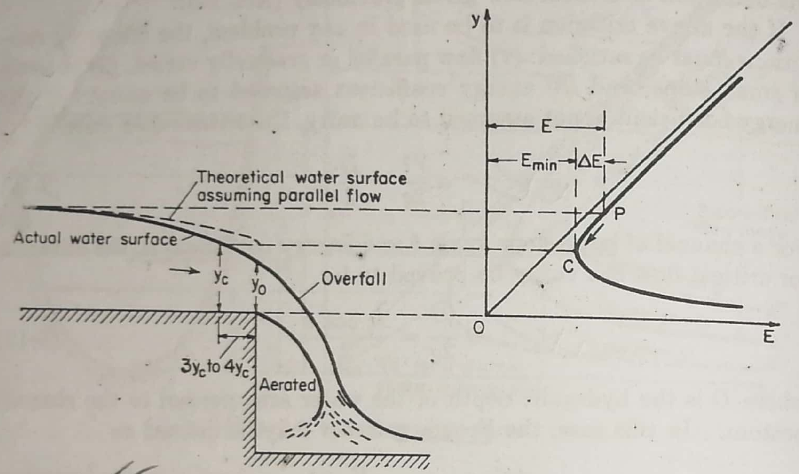


FIG. 3-3. Free overfall interpreted by specific-energy curve.

water surface would seek its lowest possible position corresponding to the least possible content of energy dissipation. If the specific energy at an upstream section is E, as shown on the specific-energy curve, it will continue to be dissipated on the way downstream and will finally reach a minimum energy content E_{min}. The specific-energy curve shows that the section of minimum energy or the critical section should occur at the brink. The brink depth cannot be less than the critical depth because further decrease in depth would require an increase in specific energy, which is impossible unless compensating external energy is supplied. The theoretical water-surface curve of an overfall is shown with a dashed line in Fig. 3-3.

It should be remembered that the determination of critical depth by Eq. (3-10) or (3-11) is based on the assumption of parallel flow and is applicable only approximately to gradually varied flow. The flow at the brink is actually curvilinear, for the curvature of flow is pronounced; hence, the method is invalid for determining the critical depth as the

depth at the brink. The actual situation is that the brink section is the *true* section of minimum energy, but it is not the critical section as computed by the principle based on the parallel-flow assumption. Rouse [4] found that for small slopes the computed critical depth is about 1.4 times the brink depth, or $y_c = 1.4y_0$, and that it is located about $3y_c$ to $4y_c$ behind the brink in the channel. The actual water surface of the overfall is shown by the full line (Fig. 3-3).

It should be noted that, if the change in the depth of flow from a high stage to a low stage is gradual, the flow becomes a gradually varied flow

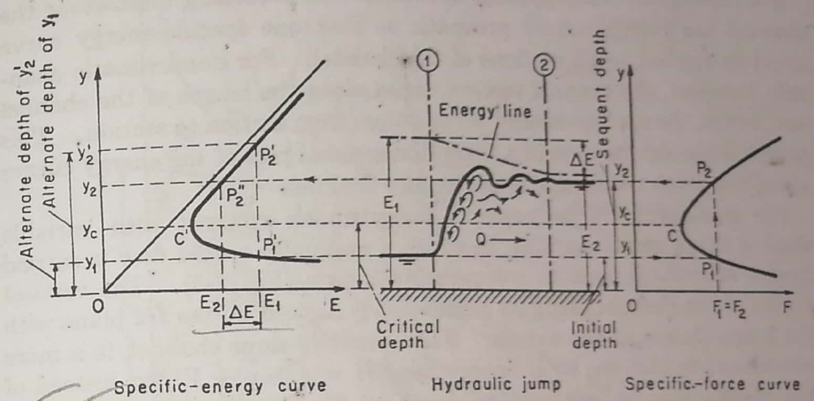


FIG. 3-4. Hydraulic jump interpreted by specific-energy and specific-force curves.

having a prolonged reversed curve of water surface; this phenomenon may be called a *gradual hydraulic drop* and is no longer a local phenomenon.

Hydraulic Jump. When the rapid change in the depth of flow is from a low stage to a high stage, the result is usually an abrupt rise of water surface (Fig. 3-4, in which the vertical scale is exaggerated). This local phenomenon is known as the *hydraulic jump*. It occurs frequently in a canal below a regulating sluice, at the foot of a spillway, or at the place where a steep channel slope suddenly turns flat.

If the jump is low, that is, if the change in depth is small, the water will not rise obviously and abruptly but will pass from the low to the high stage through a series of undulations gradually diminishing in size. Such a low jump is called an *undular jump*.

When the jump is high, that is, when the change in depth is great, the jump is called a *direct jump*. The direct jump involves a relatively large amount of energy loss through dissipation in the turbulent body of water in the jump. Consequently, the energy content in the flow after the jump is appreciably less than that before the jump.

It may be noted that the depth before the jump is always less than the alternate-depth line. At the critical section, it is noted that the three lines, namely, the water surface, the critical-depth line, and the alternate-depth line, intersect at a single point. It is seen that, in passing through the critical section, the water surface enters the supercritical-flow region smoothly.

The three-dimensional plot of energy curves is complicated. The description given here is used only for helping the reader to visualize the problem. In actual applications, the energy curves may be constructed

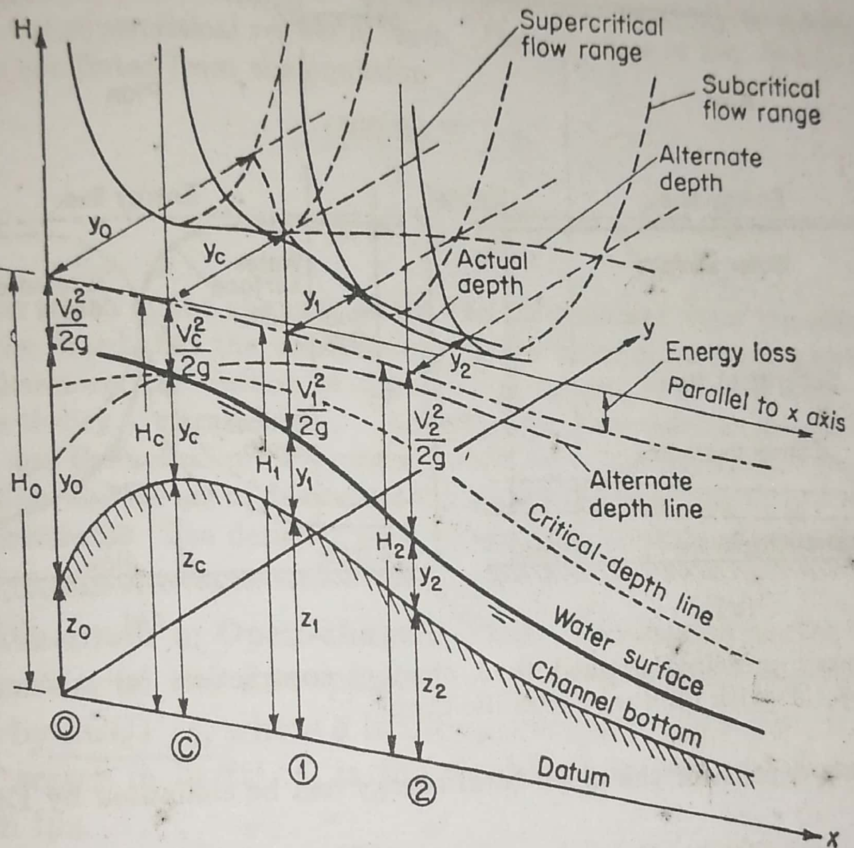


FIG. 3-5. Energy in a nonprismatic channel of variable-slope, carrying gradually varied flow from subcritical to supercritical state.

separately on a number of two-dimensional Hy planes for the chosen sections. The data obtained from these curves are then used to plot the water surface, critical-depth line, and alternate-depth line on a two-dimensional Hx plane. For simple channels, the energy curves are not necessary because the critical depth and alternate depths can easily be computed directly.

Example 3-1: A rectangular channel 10 ft wide is narrowed down to 8 ft by a contraction 50 ft long, built of straight walls and a horizontal floor. If the discharge is 100 cfs and the depth of flow is 5 ft on the upstream side of the transition section, determine the flow-surface profile in the contraction (*a*) allowing no gradual hydraulic drop in the contraction, and (*b*) allowing a gradual hydraulic drop having its point of inflection at the mid-section of the contraction. The frictional loss through the contraction is negligible.

critical depth at this section is equal to the total head divided by 1.5 (Prob. 3-3), or $5.062/1.5 = 3.375$ ft. By Eq. (3-10), the critical velocity is equal to $V_c = \sqrt{3.375g} = 10.45$ fps. Hence, the width of this critical section should be $100/(10.45 \times 3.38) = 2.83$ ft.

With the size of the mid-section determined, the side walls of the contraction can be drawn in with straight lines. The low and high stages at each section are then computed by the equation previously given. As the flow upstream from the critical section is subcritical, its water surface should follow the high stage. Downstream from the critical section, the flow is supercritical and its surface profile follows the low-stage line.

The critical-depth line is shown to separate the high from the low stage or the subcritical from the supercritical region of flow. On the basis of Eq. (3-10), the critical depth can be computed from the equation

$$\frac{(100/by_c)^2}{2g} = \frac{y_c}{2}$$

or

$$y_c = \sqrt[3]{\frac{10,000}{gb^2}}$$

where b is the width of the channel, which can be measured from the plan.

It should be noted that the vertical scale of the channel profile is greatly exaggerated. Furthermore, the outline of the gradual hydraulic drop is only theoretical, based on the theory of parallel flow. In reality, the flow near the drop is more or less curvilinear, and the actual profile would deviate from the theoretical one.

This example also serves to demonstrate a method of designing a channel transition (Arts. 11-5 to 11-7). The designer may fit any type of contraction walls he desires to suit a given flow profile, or vice versa.

3-6. Momentum in Open-channel Flow. As stated earlier (Art. 2-7), the momentum of the flow passing a channel section per unit time is expressed by $\beta wQV/g$, where β is the momentum coefficient, w is the unit weight of water in lb/ft³, Q is the discharge in cfs, and V is the mean velocity in fps.

According to Newton's second law of motion, the change of momentum per unit of time in the body of water in a flowing channel is equal to the resultant of all the external forces that are acting on the body. Applying this principle to a channel of large slope (Fig. 3-7), the following expression for the momentum change per unit time in the body of water enclosed between sections 1 and 2 may be written:

$$\frac{Qw}{g}(\beta_2 V_2 - \beta_1 V_1) = P_1 - P_2 + W \sin\theta - F_f \qquad (3\text{-}14)$$

where Q, w, and V are as previously defined, with subscripts referring to sections 1 and 2; P_1 and P_2 are the resultants of pressures acting on the two sections; W is the weight of water enclosed between the sections; and F_f is the total external force of friction and resistance acting along the surface of contact between the water and the channel. The above equation is known as the *momentum equation*.[1]

[1] The application of the momentum principle was first suggested by Bélanger [5].

For a parallel or gradually varied flow, the values of P_1 and P_2 in the momentum equation may be computed by assuming a hydrostatic distribution of pressure. For a curvilinear or rapidly varied flow, however, the pressure distribution is no longer hydrostatic; hence the values of P_1 and P_2 cannot be so computed but must be corrected for the curvature effect of the streamlines of the flow. For simplicity, P_1 and P_2 may be replaced, respectively, by $\beta_1'P_1$ and $\beta_2'P_2$, where β_1' and β_2' are the correction coefficients at the two sections. The coefficients are referred

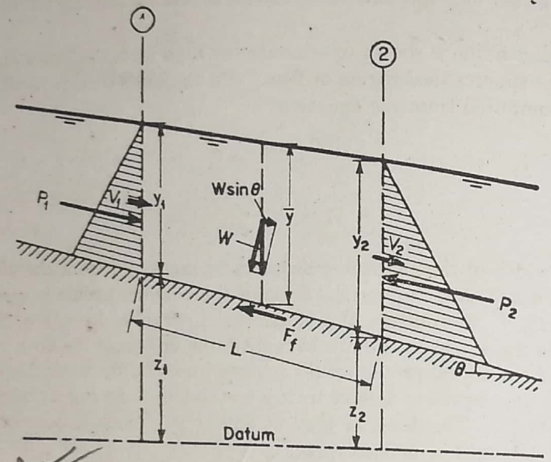


FIG. 3-7. Application of the momentum principle.

to as *pressure-distribution coefficients.* Since P_1 and P_2 are forces, the coefficients may be specifically called *force coefficients.* It can be shown that the force coefficient is expressed by

$$\beta' = \frac{1}{A\bar{z}} \int_0^A h\, dA = 1 + \frac{1}{A\bar{z}} \int_0^A c\, dA \tag{3-15}$$

where $\bar{z}$ is the depth of the centroid of the water area A below the free surface, h is the pressure head on the elementary area dA, and c is the pressure-head correction [Eq. (2-9)]. It can easily be seen that β' is greater than 1.0 for concave flow, less than 1.0 for convex flow, and equal to 1.0 for parallel flow.

It can be shown that the momentum equation is similar to the energy equation when applied to certain flow problems. In this case, a gradually varied flow is considered; accordingly, the pressure distribution in the sections may be assumed hydrostatic, and $\beta' = 1$. Also, the slope of the channel is assumed relatively small.[1] Thus, in the short reach of a

[1] If the slope angle θ is large, then $P_1 = \frac{1}{2}wd_1^2 \cos\theta$ and $P_2 = \frac{1}{2}wd_2^2 \cos\theta$, where d_1 and d_2 are the depths of flow section and $\cos\theta$ is a correction factor (Art. 2-10).

rectangular channel of small slope and width b (Fig. 3-7),

$$P_1 = \tfrac{1}{2}wby_1^2$$

and

$$P_2 = \tfrac{1}{2}wby_2^2$$

Assume

$$F_f = wh_f'b\bar{y}$$

where h_f' is the friction head and $\bar{y}$ is the average depth, or $(y_1 + y_2)/2$. The discharge through the reach may be taken as the product of the average velocity and the average area, or

$$Q = \tfrac{1}{2}(V_1 + V_2)b\bar{y}$$

Also, it is evident (Fig. 3-7) that the weight of the body of water is

$$W = wb\bar{y}L$$

and

$$\sin\theta = \frac{z_1 - z_2}{L}$$

Substituting all the above expressions for the corresponding items in Eq. (3-14) and simplifying,

$$z_1 + y_1 + \beta_1 \frac{V_1^2}{2g} = z_2 + y_2 + \beta_2 \frac{V_2^2}{2g} + h_f' \tag{3-16}$$

This equation appears to be practically the same as the energy equation (3-5).

Theoretically speaking, however, the two equations not only use different velocity-distribution coefficients, although these are nearly equal, but also involve different meanings of the frictional losses. In the energy equation, the item h_f measures the *internal* energy dissipated in the whole mass of the water in the reach, whereas the item h_f' in the momentum equation measures the losses due to *external* forces exerted on the water by the walls of the channel. Ignoring the small difference between the coefficients α and β, it seems that, in gradually varied flow, the internal-energy losses are practically identical with the losses due to external forces. In uniform flow, the rate with which surface forces are doing work is equal to the rate of energy dissipation. In that case, therefore, a distinction between h_f and h_f' does not exist except in definition.

The similarity between the applications of the energy and momentum principles may be confusing. A clear understanding of the basic differences in their constitution is important, despite the fact that in many instances the two principles will produce practically identical results. The inherent distinction between the two principles lies in the fact that energy is a scalar quantity whereas momentum is a vector quantity; also, the energy equation contains a term for internal losses, whereas the momentum equation contains a term for external resistance.

Generally speaking, the energy principle offers a simpler and clearer explanation than does the momentum principle. But the momentum principle has certain advantages in application to problems involving high internal-energy changes, such as the problem of the hydraulic jump. If the energy equation is applied to such problems, the unknown internal-energy loss represented by h_f is indeterminate, and the omission of this term would result in considerable errors. If instead the momentum equation is applied to these problems, since it deals only with external forces, the effects of the internal forces will be entirely out of consideration and need not be evaluated. The term for frictional losses due to external forces, on the other hand, is unimportant in such problems and can safely be omitted, because the phenomenon takes place in a short reach of the channel and the effect due to external forces is negligible compared with the internal losses. Further discussions on the solution of the hydraulic-jump problem by both principles will be given later (Example 3-3).

An example showing the application of the momentum principle to the problem of a broad-crested weir is given below.

Example 3-2. Derive the discharge per unit width of a broad-crested weir across a rectangular channel.

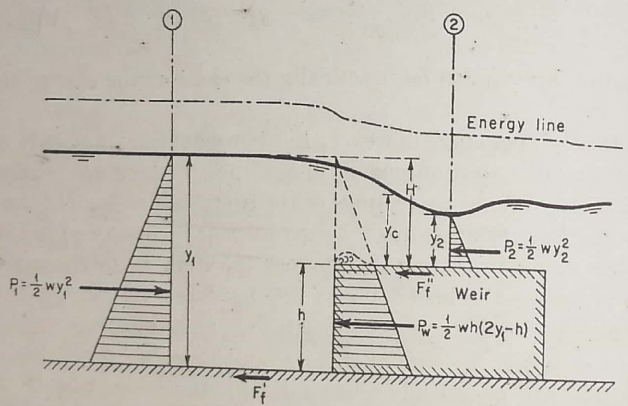


FIG. 3-8. Momentum principle applied to flow over a broad-crested weir.

Solution. The assumptions to be made in this solution (Fig. 3-8) are (1) the frictional forces F_f' and F_f'' are negligible; (2) the depth y_2 is the minimum depth on the weir; (3) at the channel sections under consideration there is parallel flow; and (4) the water pressure P_w on the weir surface is equal to the total hydrostatic pressure measured below the upstream water surface, or

$$P_w = \tfrac{1}{2}wh[y_1 + (y_1 - h)] = \tfrac{1}{2}wh(2y_1 - h)$$

The accuracy of the last assumption has been checked experimentally [6]. If the momentum equation (3-14) is applied to the body of water between the upstream approach section 1 and the downstream section 2 at the minimum depth on the top of the weir, the following equation may be written:

$$\frac{qw}{g}\left(\frac{q}{y_2} - \frac{q}{y_1}\right) = \tfrac{1}{2}wy_1^2 - \tfrac{1}{2}wy_2^2 - \tfrac{1}{2}wh(2y_1 - h)$$

where q is the discharge per unit width of the weir.

Experiments by Doeringsfeld and Barker [6] have shown that, on the average, $y_1 - h = 2y_2$. In that case the above equation can be simplified and solved for q,

$$q = 0.433\sqrt{2g}\left(\frac{y_1}{y_1 + h}\right)^{1/2} H^{3/2} \tag{3-17}$$

Considering the limit of h from zero to infinity, this equation varies from $q = 3.47H^{3/2}$ to $q = 2.46H^{3/2}$. It is interesting to note that the practical range of the coefficient to $H^{3/2}$ obtained by actual observations[1] is from 3.05 to 2.67. In applying the momentum principle to this problem, it can be seen that knowledge of the internal-energy losses due to separation of flow at the entrance and to other causes is not needed in the analysis.

3-7. Specific Force. In applying the momentum principle to a short horizontal reach of a prismatic channel, the external force of friction and the weight effect of water can be ignored. Thus, with $\theta = 0$ and $F_f = 0$ and assuming also $\beta_1 = \beta_2 = 1$, Eq. (3-14) becomes

$$\frac{Qw}{g}(V_2 - V_1) = P_1 - P_2$$

The hydrostatic forces P_1 and P_2 may be expressed as

$$P_1 = w\bar{z}_1 A_1 \quad \text{and} \quad P_2 = w\bar{z}_2 A_2$$

where $\bar{z}_1$ and $\bar{z}_2$ are the distances of the centroids of the respective water areas A_1 and A_2 below the surface of flow. Also, $V_1 = Q/A_1$ and $V_2 = Q/A_2$. Then, the above momentum equation may be written

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \tag{3-18}$$

[1] The value of the coefficient actually depends on many factors: mainly, the rounding of the upstream corner, the length and slope of the weir crest, and the height of the weir. Many experiments on *broad-crested weirs* have been performed. From several of the well-known experiments King [7] has interpolated the data and prepared tables for the coefficient under various conditions. A comprehensive analysis including more recent data and a presentation of the results for practical applications were made by Tracy [8]. The well-known experiments on broad-crested weirs are (1) *Bazin tests* performed in Dijon, France, in 1886 [9]; (2) *U.S.D.W.B. Cornell tests* performed at Cornell University in 1899 by the U.S. Deep Waterways Board under the direction of G. W. Rafter, and *U.S.G.S. Cornell tests* performed by the U.S. Geological Survey under the direction of Robert E. Horton in 1903 [10]; (3) *Michigan tests* performed at the University of Michigan during 1928–1929 [11]; and (4) *Minnesota* and *Washington tests* performed, respectively, at the university of Minnesota and Washington State University [6]. For some formulas and coefficients of discharge developed in the U.S.S.R., see [12]. For an analytical treatment of the problem, see [13].

The two sides of Eq. (3-18) are analogous and, hence, may be expressed for any channel section by a general function

$$F = \frac{Q^2}{gA} + \bar{z}A \tag{3-19}$$

This function consists of two terms. The first term is the momentum of the flow passing through the channel section per unit time per unit weight of water, and the second is the force per unit weight of water. Since both terms are essentially force per unit weight of water, their sum may be called the *specific force*.[1] Accordingly, Eq. (3-18) may be expressed

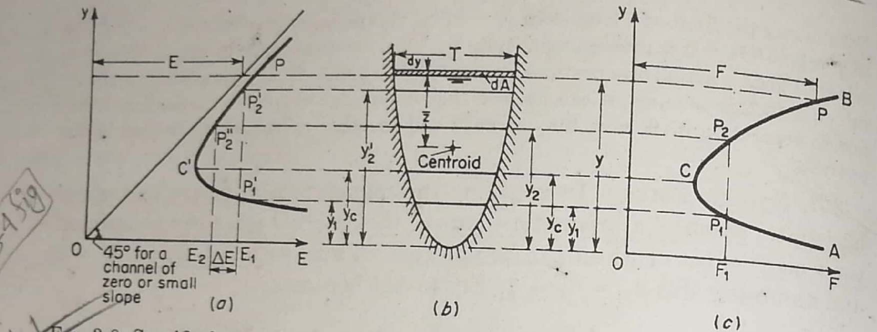


FIG. 3-9. Specific-force curve supplemented with specific-energy curve. (a) Specific-energy curve; (b) channel section; (c) specific-force curve.

as $F_1 = F_2$. This means that the specific forces of sections 1 and 2 are equal, provided that the external forces and the weight effect of water in the reach between the two sections can be ignored.

By plotting the depth against the specific force for a given channel section and discharge, a *specific-force curve* is obtained (Fig. 3-9). This curve has two limbs AC and BC. The limb AC approaches the horizontal axis asymptotically toward the right. The limb BC rises upward and extends indefinitely to the right. For a given value of the specific force, the curve has two possible depths y_1 and y_2. As will be shown later, the two depths constitute the initial and sequent depths of a hydraulic jump. At point C on the curve the two depths become one, and the specific force is a minimum. The following argument shows that the depth at the minimum value of the specific force is equal to the critical depth.[2]

[1] This has been variously called the "force plus momentum," the "momentum flux," the "total force," or, briefly, the "force" of a stream (see pp. 81 and 82 of [14]). The function represented by Eq. (3-19) was formulated by Bresse [15] for the study of the hydraulic jump to be described in Example 3-3.

[2] The concept of critical depth based on the theorem of momentum is believed to have been developed by Boussinesq [16].

For a minimum value of the specific force, the first derivative of F with respect to y should be zero, or, from Eq. (3-19),

$$\frac{dF}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{d(\bar{z}A)}{dy} = 0$$

For a change dy in the depth, the corresponding change $d(\bar{z}A)$ in the static moment of the water area about the free surface is equal to $[A(\bar{z} + dy) + T(dy)^2/2] - \bar{z}A$ (Fig. 3-9). Ignoring the differential of higher degree, that is, assuming $(dy)^2 = 0$, the change in static moment becomes $d(\bar{z}A) = A\,dy$. Then the preceding equation may be written

$$\frac{dF}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + A = 0$$

Since $dA/dy = T$, $Q/A = V$, and $A/T = D$, the above equation may be reduced to

$$\frac{V^2}{2g} = \frac{D}{2} \tag{3-10}$$

This is the criterion for the critical state of flow, derived earlier (Art. 3-3). Therefore, it is proved that the depth at the minimum value of the specific force is the critical depth.[1] It may also be stated that *at the critical state of flow the specific force is a minimum for the given discharge*.

Now, compare the specific-force curve with the specific-energy curve (Fig. 3-9). For a given specific energy E_1, the specific-energy curve indicates two possible depths, namely, a low stage y_1 in the supercritical flow region and a high stage y_2' in the subcritical flow region.[2] For a given value of F_1, the specific-force curve also indicates two possible depths, namely, an initial depth y_1 in the supercritical region and a sequent depth y_2 in the subcritical flow region. It is assumed that the low stage and the initial depth are both equal to y_1. Thus, the two curves indicate jointly that the sequent depth y_2 is always less than the high stage y_2'. Furthermore, the specific-energy curve shows that the energy content E_2 for the depth y_2 is less than the energy content E_1 for the depth y_2'. Therefore, in order to maintain a constant value of F_1, the depth of flow may be changed from y_1 to y_2 at the price of losing a certain amount of energy, which is equal to $E_1 - E_2 = \Delta E$. One example of this is the

[1] It should be noted that the above proof is based on the assumptions of parallel flow and uniform velocity distribution. However, the concept of critical depth is a general concept that is valid for all flows, whether derived from energy or from momentum considerations. This validity has been proved by Jaeger [14,17,18], and the proof is known as the *Jaeger theorem* [19].

[2] In order to make a clear distinction between the sequent depth and the high stage of the alternate depths, the sequent depth is designated by y_2 and the high stage by y_2'. In some other places in this book, however, both are designated by y_2.

hydraulic jump on a horizontal floor, in which the specific forces before and after the jump are equal and the loss of energy is a consequence of the phenomenon. This will be explained further in the following example. It may be noted at this point, however, that the depths y_1 and y_2' shown by the specific-energy curve are the alternate depths; whereas the depths y_1 and y_2 shown by the specific-force curve are, respectively, the initial depth and the sequent depth of a hydraulic jump.

Example 3-3. Derive a relationship between the initial depth and the sequent depth of a hydraulic jump on a horizontal floor in a rectangular channel.

Solution. The external forces of friction and the weight effect of water in the hydraulic jump on a horizontal floor are negligible, because the jump takes place in a relatively short distance and the slope angle of the horizontal floor is zero. The specific forces of sections 1 and 2 (Fig. 3-4), respectively, before and after the jump, can therefore be considered equal; that is,

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \tag{3-18}$$

For a rectangular channel of width b, $Q = V_1 A_1 = V_2 A_2$, $A_1 = by_1$, $A_2 = by_2$, $\bar{z}_1 = y_1/2$, and $\bar{z}_2 = y_2/2$. Substituting these relations and $\mathbf{F}_1 = V_1/\sqrt{gy_1}$ in the above equation and simplifying,

$$\left(\frac{y_2}{y_1}\right)^3 - (2\mathbf{F}_1^2 + 1)\left(\frac{y_2}{y_1}\right) + 2\mathbf{F}_1^2 = 0 \tag{3-20}$$

Factoring,

$$\left[\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2\right]\left(\frac{y_2}{y_1} - 1\right) = 0$$

Then, let

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2 = 0$$

The solution of this quadratic equation is

$$\frac{y_2}{y_1} = \tfrac{1}{2}\left(\sqrt{1 + 8\mathbf{F}_1^2} - 1\right) \tag{3-21}$$

For a given Froude number $\mathbf{F}_1$ of the approaching flow, the ratio of the sequent depth to the initial depth is given by the above equation.

It should be understood that the momentum principle is used in this solution because the hydraulic jump involves a high amount of internal-energy losses which cannot be evaluated in the energy equation.

The joint use of the specific-energy curve and the specific-force curve helps to determine graphically the energy loss involved in the hydraulic jump for a given approaching flow. For the given approaching depth y_1, points P_1 and P_1' are located on the specific-force curve and the specific-energy curve, respectively (Fig. 3-4). The point P_1' gives the initial energy content E_1. Draw a vertical line, passing through the point P_1 and intercepting the upper limb of the specific-force curve at point P_2, which gives the sequent depth y_2. Then, draw a horizontal line passing through the point P_2 and intercepting the specific-energy curve at point P_2'', which gives the energy content E_2 after the jump. The energy loss in the jump is then equal to $E_1 - E_2$, represented by ΔE.

3-8. Momentum Principle Applied to Nonprismatic Channels.

The specific force, like the specific energy, varies with the shape of the channel

section. In applying the momentum principle to nonprismatic channels, therefore, a three-dimensional plot similar to that shown for the application of the energy principle (Fig. 3-5) can be constructed. For practical purposes, however, this is rarely necessary.

Where there is no intervention of external forces or where these forces are either negligible or given, the momentum principle can be applied to its best advantage to problems, such as the hydraulic jump, that deal with high internal-energy losses that cannot be evaluated if the energy principle alone is used. The following example shows how the momentum principle is applied to the design of a channel transition in which a hydraulic jump is involved.

Example 3-4. A rectangular channel 8 ft wide, carrying 100 cfs at a depth of 0.5 ft, is connected by a straight-wall transition to a channel 10 ft wide, flowing at a depth

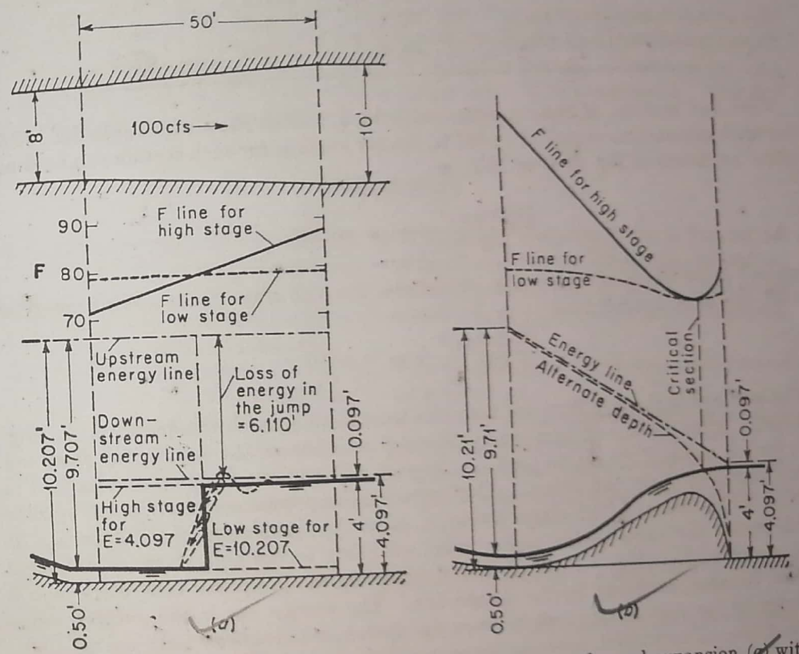


FIG. 3-10. Energy and momentum principles applied to a channel expansion (a) with hydraulic jump; (b) without hydraulic jump.

of 4 ft (Fig. 3-10). Determine the flow profile in the transition if the frictional loss through the transition is negligible. If a hydraulic jump occurs in the transition, how can it be eliminated?

Solution. From the given data, the total energy with respect to the channel bottom in the approaching flow is $E = 0.5 + [100/(0.5 \times 8)]^2/2g = 10.207$ ft, and in the downstream, $E = 4.0 + [100/(4 \times 10)]^2/2g = 4.097$ ft. It is apparent that this for instance, by bolting cross timbers to the bottom of the transition. It can be assumed in this example that the energy difference of 6.110 ft is dissipated uniformly in the transition by artificial roughness. Thus, the energy line in the transition is simply a straight line joining the total heads of the two end sections (Fig. 3-10b). For design purposes, it is convenient first to assume the flow profile and then to proportion the dimensions of the transition so that the jump can be eliminated. In proportioning the transition, the jump is eliminated either by varying the width or by raising the bottom of the transition. In this example, it is assumed that the bottom is to be raised, or "humped" (Fig. 3-10b). The subsequent procedure of the computation is to (1) assume the flow profile; (2) compute the velocity head, which is equal to the difference between the total head and the water-surface elevation, at a number of selected sections; (3) compute the velocity and then the water area and depth of flow for each section; (4) determine the elevation of the bottom of the transition, which is equal to the elevation of the water surface minus the depth of flow; (5) compute the alternate depth, since the bottom of the transition is fixed; and (6) compute F_1 and F_2 lines for the low and high stages, and plot them on a convenient scale. It can be seen that the two F lines intersect and become tangent to each other at a critical section, where the flow changes from low to high stage, that is, from supercritical to subcritical state. If the critical-depth line is plotted, it will intersect the alternate-depth line and the water surface simultaneously at the critical section. Based on the critical-depth line, a line of minimum specific energy can also be constructed. This line should be tangent to the total-energy line at the critical section.

PROBLEMS

3-1. With reference to a channel of small slope and a section shown in Fig. 2-2, (*a*) construct a family of specific-energy curves for Q = 0, 50, 100, 200, 300, and 400 cfs, (*b*) draw the locus of the critical-depth point on these curves, (*c*) plot a curve of the critical depth against the discharge, and (*d*) plot a family of curves of alternate depths, y_1 vs. y_2, for the given discharges.

3-2. Construct the specific-energy curve for a 36-in. pipe carrying an open-channel flow of 20 cfs (*a*) on a flat slope, and (*b*) on a 30° slope.

3-3. Show that at the critical state of flow the specific-energy head in a rectangular channel is equal to 1.5 times the depth of flow, assuming zero slope and $\alpha = 1$.

3-4. Derive the equations for the locus of the critical-depth point on the specific-energy curve and for the curve of critical depth vs. discharge, as obtained in Prob. 3-1.

3-5. Prove Eq. (3-12).

3-6. Prove Eq. (3-13).

3-7. Prove that at the critical state of flow the discharge is a maximum for a given specific energy.[1]

3-8. Show that the relation between the alternate depths y_1 and y_2 in a rectangular channel can be expressed by

$$\frac{2y_1^2y_2^2}{y_1 + y_2} = y_c^3 \qquad (3\text{-}22)$$

where y_c is the critical depth. Using values of y_1/y_c as ordinates and of y_2/y_c as abscissas, construct a dimensionless graph for the above equation and study its characteristics.

[1] The concept of critical depth based on the theorem of maximum discharge was first introduced by Bélanger [20].

3-9. Solve the problem given in Example 3-1 (*a*) if there is a total energy loss of 0.60 ft uniformly distributed throughout the length of the contraction, and (*b*) if a gradual hydraulic drop is desired with its point of inflection at a distance 20 ft upstream from the exit section.

3-10. Applying the momentum principle and the continuity equation to the analysis of a submerged hydraulic jump which occurs at the sluice outlet in a rectangular channel (Fig. 3-11), prove that

$$\frac{y_s}{y_2} = \sqrt{1 + 2F_2^2\left(1 - \frac{y_2}{y_1}\right)} \tag{3-23}$$

where y_s is the submerged depth; y_1 is the height of sluice-gate opening; y_2 is the tail-water depth; and $F_2^2 = q^2/gy_2^3$, q being the discharge per unit width of the channel. Neglect the channel-bed friction F_f.

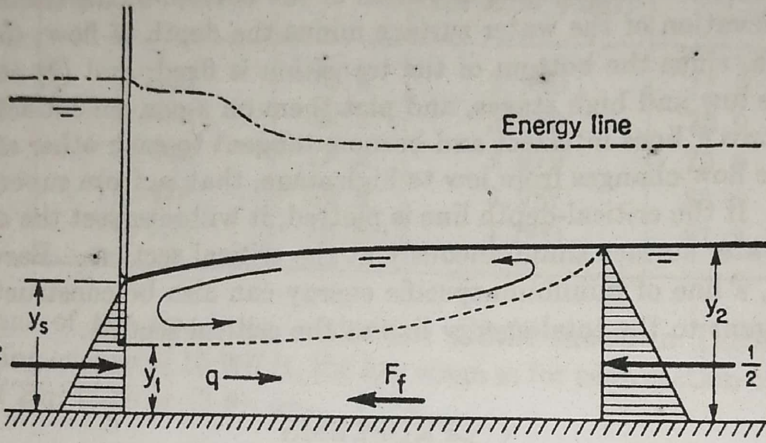


FIG. 3-11. A submerged hydraulic jump at sluice outlet.

3-11. Prove that the energy loss in a horizontal hydraulic jump is

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} \tag{3-24}*$$

3-12. If a hydraulic jump is formed on the horizontal floor at the toe of the spillway described in Prob. 2-11, determine the sequent depth and the energy loss involved in the jump.

3-13. With reference to a channel of small slope and a section shown in Fig. 2-2, (*a*) construct a family of specific-force curves for Q = 0, 50, 100, 200, 300, and 400 cfs, and (*b*) plot a family of curves of initial depth against sequent depth for the given discharges.

3-14. Construct the specific-force curve for a 36-in. pipe carrying an open-channel flow of 20 cfs on a small slope.

3-15. Prove Eq. (3-15).

3-16. Using the momentum principle, show that the Froude number of a parallel or gradually varied flow in a channel of slope angle θ may be defined by

$$F = \frac{V}{\sqrt{gD\cos\theta/\beta}} \tag{3-25}$$

* This formula was shown by Bresse early in 1860 [15]. At the same time Bresse introduced the concept of critical depth, as a depth at which the subcritical flow changes to supercritical, or vice versa.

CHAPTER 4

CRITICAL FLOW: ITS COMPUTATION AND APPLICATIONS

4-1. Critical Flow. As described in the previous chapter, the critical state of flow through a channel section is characterized by several important conditions.[1] Recapitulating, they are (1) the specific energy is a minimum for a given discharge; (2) the discharge is a maximum for a given specific energy (Prob. 3-7); (3) the specific force is a minimum for a given discharge; (4) the velocity head is equal to half the hydraulic depth in a channel of small slope; (5) the Froude number is equal to unity; and (6) the velocity of flow in a channel of small slope with uniform velocity distribution is equal to the celerity of small gravity waves in shallow water caused by local disturbances.

Discussions on critical state of flow have referred mainly to a particular section of a channel, known as the *critical section*. If the critical state of flow exists throughout the entire length of the channel or over a reach of the channel, the flow in the channel is a *critical flow*. Since, as indicated by the critical-flow criterion Eq. (3-10), the depth of critical flow depends on the geometric elements A and D of the channel section when the discharge is constant, the critical depth in a prismatic channel of uniform slope will be the same in all sections, and critical flow in a prismatic channel should, therefore, be uniform flow. At this condition, the slope of the channel that sustains a given discharge at a uniform and critical depth is called the *critical slope* S_c. A slope of the channel less than the critical slope will cause a slower flow of subcritical state for the given discharge, as will be shown later, and, hence, is called a *mild* or *subcritical slope*. A slope greater than the critical slope will result in a faster flow of supercritical state, and is called a *steep* or *supercritical slope*.

A flow at or near the critical state is unstable. This is because a minor change in specific energy at or close to critical state will cause a major change in depth. This fact can also be recognized in the specific-energy curve (Fig. 3-2). As the curve is almost vertical near the critical depth, a slight change in energy would change the depth to a much smaller or much greater alternate depth corresponding to the specific energy after

[1] For a historical account of the theory of critical flow, see [1].

the change. It can be observed also that, when the flow is near the critical state, the water surface appears unstable and wavy. Such phenomena are generally caused by the minor changes in energy due to variations in channel roughness, cross section, slope, or deposits of sediment or debris. In the design of a channel, if the depth is found at or near the critical depth for a great length of the channel, the shape or slope of the channel should be altered, if practicable, in order to secure greater stability.

The criterion for a critical state of flow (Art. 3-3) is the basis for the computation of critical flow, which will be explained in subsequent articles. Two major applications of critical-flow theory are flow control and flow measurement, which will also be discussed in this chapter.

4-2. The Section Factor for Critical-flow Computation. Substituting $V = Q/A$ in Eq. (3-10) and simplifying,

$$Z = \frac{Q}{\sqrt{g}} \qquad (4\text{-}1)$$

When the energy coefficient is not assumed to be unity,

$$Z = \frac{Q}{\sqrt{g/\alpha}} \qquad (4\text{-}2)$$

In the above equations, $Z = A\sqrt{D}$, which is the *section factor for critical-flow computation* [Eq. (2-3)]. Equation (4-2) states that the section factor Z for a channel section at the critical state of flow is equal to the discharge divided by the square root of g/α. Since the section factor Z is a function of the depth, the equation indicates that there is *only one* possible critical depth for maintaining the given discharge in a channel and similarly that, when the depth is fixed, there can be *only one* discharge that maintains a critical flow and makes the depth critical in the given channel section.

Equation (4-1) or (4-2) is a very useful tool for the computation and analysis of critical flow in an open channel. When the discharge is given, the equation gives the critical section factor Z_c and, hence, the critical depth y_c. On the other hand, when the depth and, hence, the section factor are given, the critical disharge can be computed by Eq. (4-1) in the following form:

$$Q = Z\sqrt{g} \qquad (4\text{-}3)$$

or by Eq. (4-2) in the following form:

$$Q = Z\sqrt{\frac{g}{\alpha}} \qquad (4\text{-}4)$$

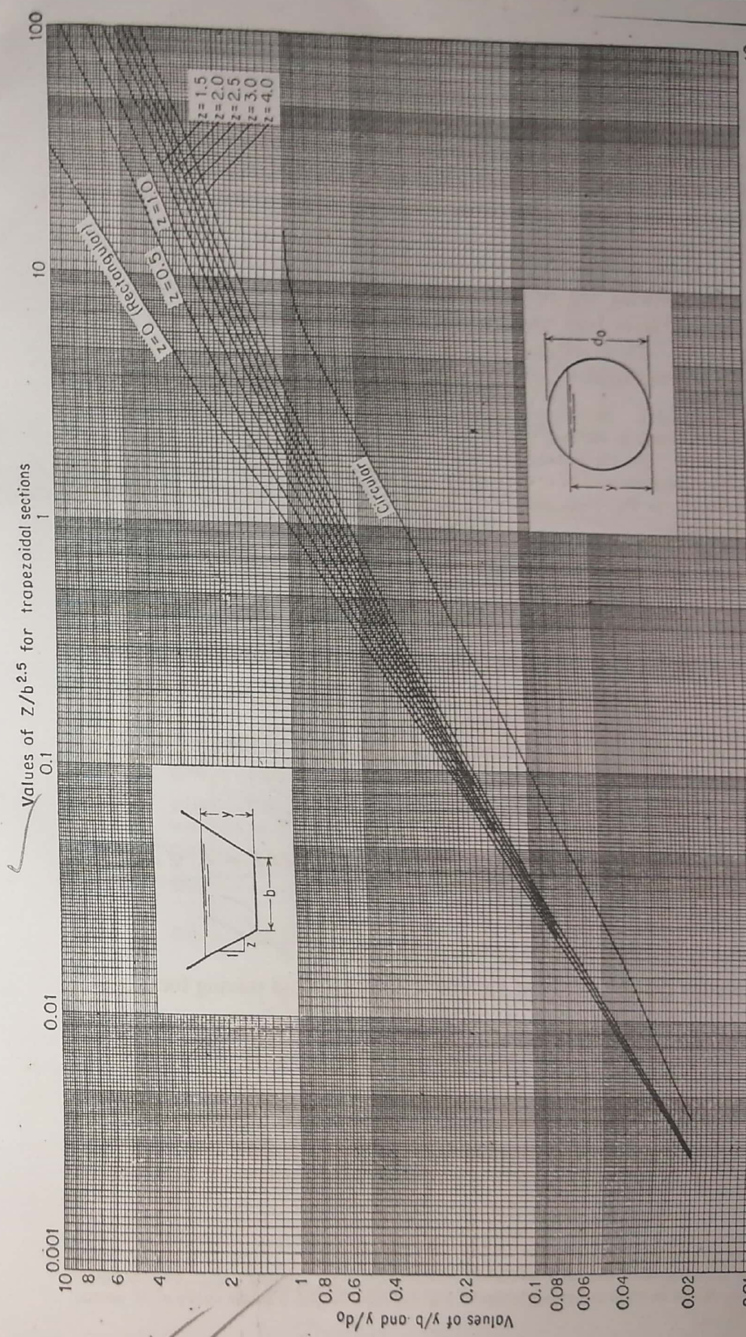


FIG. 4-1. Curves for determining the critical depth.

A subscript c is sometimes used to specify the condition of critical flow. Formulas for the section factor Z of seven common channel sections are given in Table 2-1. The Z values for a circular section can be found either from the curve in Fig. 2-1 or from the table in Appendix A.

In order to simplify the computation of critical flow, dimensionless curves showing the relation between the depth and section factor Z (Fig. 4-1) have been prepared for rectangular, trapezoidal, and circular channels. These self-explanatory curves will help to determine the depth y for a given section factor Z, and vice versa.

Example 4-1. Derive an equation showing critical discharge through a rectangular channel section in terms of the channel width and the total head.

Solution. For the rectangular section, Table 2-1 gives the section factor $Z = by^{1.5}$. At the critical state of flow, the depth $y = H/1.5$ (see Prob. 3-3). Substituting these expressions in Eq. (4-3), using $g = 32.16$, and simplifying, we find that the critical discharge is

$$Q_c = 3.087bH^{1.5} \tag{4-5}$$

4-3. The Hydraulic Exponent for Critical-flow Computation. Since the section factor Z is a function of the depth of flow y, it may be assumed that

$$Z^2 = Cy^M \tag{4-6}$$

where C is a coefficient and M is a parameter called the *hydraulic exponent for critical-flow computation.*

Taking logarithms on both sides of Eq. (4-6) and then differentiating with respect to y,

$$\frac{d(\ln Z)}{dy} = \frac{M}{2y} \tag{4-7}$$

Now, taking logarithms on both sides of Eq. (2-3), or $Z = A\sqrt{A/T}$, and then differentiating with respect to y,

$$\frac{d(\ln Z)}{dy} = \frac{3}{2}\frac{T}{A} - \frac{1}{2T}\frac{dT}{dy} \tag{4-8}$$

Equating the right sides of Eqs. (4-7) and (4-8) and solving for M,

$$M = \frac{y}{A}\left(3T - \frac{A}{T}\frac{dT}{dy}\right) \tag{4-9}$$

This is a general equation for the hydraulic exponent M, which is a function of the channel section and the depth of flow. For a trapezoidal section, the expressions for A and T obtained from Table 2-1 are substituted in Eq. (4-9); the resulting equation [2] is simplified and becomes

$$M = \frac{3[1 + 2z(y/b)]^2 - 2z(y/b)[1 + z(y/b)]}{[1 + 2z(y/b)][1 + z(y/b)]} \tag{4-10}^*$$

* This equation was also developed independently by Chugaev [3]. In this equation, M can be regarded as a function of $z(y/b)$; accordingly, a single curve of M versus $z(y/b)$ may be constructed. It is obvious that this curve would be identical with the curve for $z = 1$ in Fig. 4-2. For convenience in application, however, a family of curves of M versus y/b are shown, using z as a parameter.

This equation indicates that the value of M for the trapezoidal section is a function of z and y/b. For values of z = 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0, a family of curves for M versus y/b are constructed (Fig. 4-2). These curves indicate that the value of M varies in a range from 3.0 to 5.0.

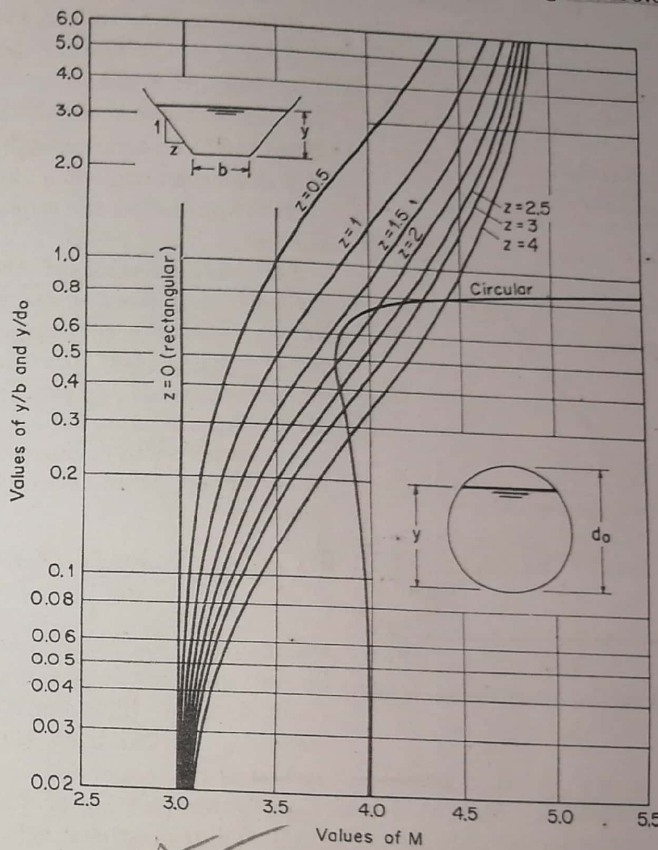


FIG. 4-2. Curves of M values.

A curve for a circular section with M plotted against y/d_0, where d_0 is the diameter, is also shown (Fig. 4-2). This curve was developed by a similar procedure but constructed from a much more complicated formula. The curve shows that the value of M varies within a rather narrow range for values of y/d_0 less than 0.7 or so, but increases rapidly as the value of y/d_0 becomes greater than 0.7. The significance of this characteristic is that, when the depth of flow in a circular section approaches the top of the circle, the section factor and with it the critical discharge, as shown by Eq. (4-3), become indefinitely large. In other words, it is practically impossible to maintain a critical flow in a circular conduit at a depth approaching the top of the section. In fact, the wavy surface of the critical flow will touch the top of the conduit before it actually comes so near as to approach the top. A similar characteristic and phenomenon occur also in other types of closed conduit with gradually closing crown, when the water surface approaches the crown of the conduit.

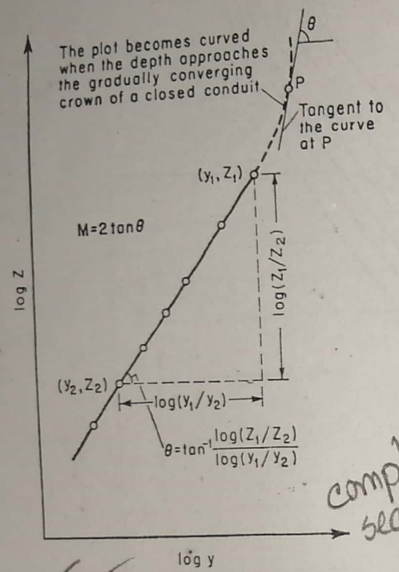


FIG. 4-3. Graphical determination of the M value.

For channel sections of other than trapezoidal or circular shape, exact values of M can be computed directly by Eq. (4-9), provided that the derivative dT/dy can be evaluated. Approximate values of M for any channel section, however, may be obtained from the following equation

$$M = 2\frac{\log(Z_1/Z_2)}{\log(y_1/y_2)} \tag{4-11}$$

where Z_1 and Z_2 are section factors for any two depths y_1 and y_2 of the given section. This equation can easily be derived from Eq. (4-6). In applying Eq. (4-11), a graphical method is recommended instead of direct computation. This involves a logarithmic plotting of Z as ordinate against the depth as abscissa (Fig. 4-3). For most channels, except for closed conduits with depth approaching a gradually closing crown and some channels of peculiar shapes, the plot takes a more or less straight-line form. The hydraulic exponent is equal to twice the slope of the plotted straight line. For a depth approaching the gradually closing crown of a closed conduit, the plot becomes a curve, and the hydraulic exponent of a given depth is equal to twice the slope of the tangent to the curve at that depth.

The hydraulic exponent M is described here only as a characteristic value of a channel section under the condition of critical flow. The application of this exponent will be further described in the computation of gradually varied flow (Art. 10-2).

4-4. Computation of Critical Flow. Computation of critical flow involves the determination of critical depth and velocity when the discharge and the channel section are known. Three methods illustrated by simple examples will be given below. On the other hand, if the critical depth and channel section are known, the critical discharge can be determined by the method described in Art. 4-2.

A. Algebraic Method. For a simple geometric channel section, the critical flow can be determined by an algebraic computation using the basic equations. The method has already been used (Example 3-1), but the following example is given for further illustration:

Example 4-2. Compute the critical depth and velocity of the trapezoidal channel (Fig. 2-2) carrying a discharge of 400 cfs.

Solution. The hydraulic depth and water area of the trapezoidal section are expressed in terms of the depth y as

$$D = \frac{y(10 + y)}{10 + 2y} \quad \text{and} \quad A = y(20 + 2y)$$

The velocity is

$$V = \frac{Q}{A} = \frac{400}{y(20 + 2y)}$$

Substituting the above expressions for D and V in Eq. (3-10) and simplifying,

$$2{,}484(5 + y) = [y(10 + y)]^3$$

Solving this equation for y by a trial-and-error procedure, $y_c = 2.15$ ft. This is the critical depth. The corresponding area is $A_c = 52.2$ ft², and the critical velocity is $V_c = 400/52.2 = 7.66$ fps.

B. Graphical Method. For a complicated or natural channel section, a graphical procedure for critical-flow computation is generally employed. By this procedure a curve of y versus Z is constructed. The value of $Q/\sqrt{g}$ is then computed. Using Eq. (4-1), the critical depth may be obtained directly from the curve, where $Z = Q/\sqrt{g}$.

Example 4-3. A 36-in. concrete circular culvert carries a discharge of 20 cfs. Determine the critical depth.

Solution. Construct a curve of y vs. Z (Fig. 4-4). Then compute $Z = Q/\sqrt{g} = 20/\sqrt{g} = 3.53$. From the curve the critical depth for this value of Z is found to be $y_c = 1.44$ ft.

The dimensionless curve (Fig. 2-1) or the table in Appendix A for the geometric elements of a circular section might also be used to solve this problem. Since $d_0 = 3.0$ ft and $d_0^{2.5} = 15.6$, $Z/d_0^{2.5} = 3.53/15.6 = 0.226$. From the dimensionless curve or from the table, $y/d_0 = 0.48$, and so $y_c = 0.48 \times 3 = 1.44$ ft.

C. Method of Design Chart. The design chart for determining the critical depth (Fig. 4-1) can be used with great expediency.

In Example 4-2, $Z = 400/\sqrt{g} = 70.5$ [Eq. (4-1)]. The value of $Z/b^{2.5}$ is 0.0394. For this value, the chart gives $y/b = 0.108$ or $y_c = 2.16$ ft.

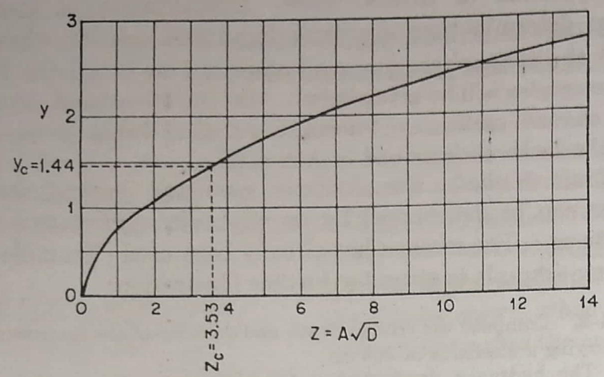


FIG. 4-4. Curve of y versus Z for a circular section.

In Example 4-3, $Z/d_0^{2.5} = 0.226$. For this value the chart gives $y/d = 0.48$ or $y_c = 1.44$ ft.

4-5. Control of Flow. The control of flow in an open channel is defined loosely in many ways. As used here the term means the establishment of a definitive flow condition in the channel or, more specifically, a definitive relationship between the stage and the discharge of the flow. When the control of flow is achieved at a certain section of the channel, this section is a *control section.* It will be shown later that the control section controls the flow in such a way that it restricts the transmission of the effect of changes in flow condition either in an upstream direction or in a downstream direction depending on the state of flow in the channel. Since the control section holds a definitive stage-discharge relationship, it is always a suitable site for a gaging station and for developing the *discharge rating curve,* a curve representing the depth-discharge relationship at the gaging station.

At the critical state of flow a definitive stage-discharge relationship can be established and represented by Eq. (4-1). This equation shows that the stage-discharge relationship is theoretically independent of the channel roughness and other uncontrolled circumstances. Therefore, a critical-flow section is a control section.

The location of the control section in a prismatic channel is generally governed by the state of flow, which in turn is determined by the slope of the channel. Take for an example a long straight prismatic channel in which a pool is created by a dam across the channel and the water flows over the dam through an overflow spillway (Fig. 4-5). Three flow conditions in the channel are shown, representing the subcritical, critical, and supercritical flows, respectively. The slopes of the channel in the three cases are, correspondingly, *mild* or subcritical, critical, and *steep* or supercritical.

If the channel has a critical slope (middle sketch in Fig. 4-5), then the flow is initially uniform and critical throughout the channel. In the presence of the dam, however, the flow through the pool will be subcritical and the pool surface will approach the horizontal. At the downstream end a so-called *drawdown curve* will be developed, extending upstream

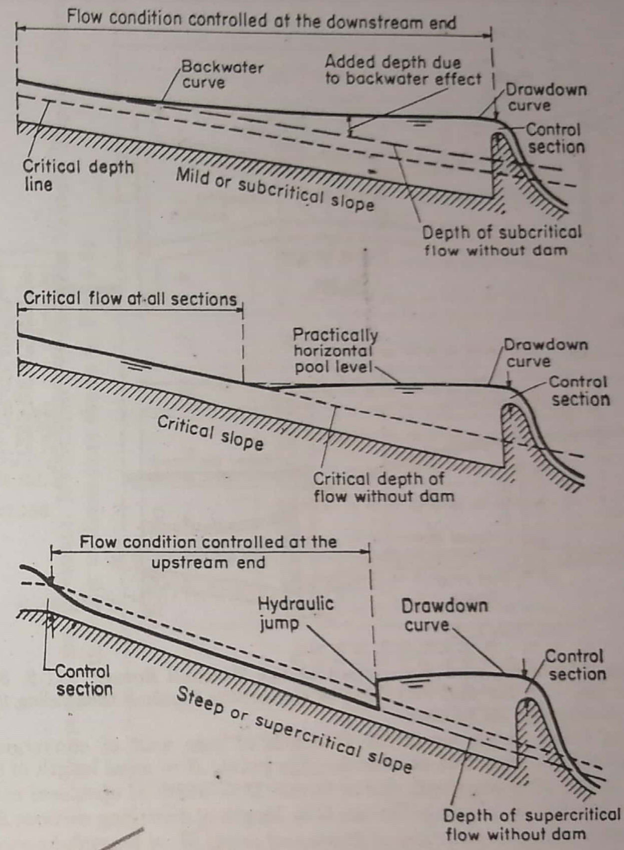


FIG. 4-5. Flow conditions in a long prismatic channel.

from a section near the spillway crest and becoming asymptotic to the pool level.

If the channel has a subcritical slope (top sketch in Fig. 4-5), the flow is initially subcritical. In the presence of the dam, the pool surface will be further raised for a long distance upstream from the pool in a so-called *backwater curve.* The additional depth of water is required to build up enough head to give the increased velocity necessary to pass water over the spillway. This effect of backing up the water behind the dam is known as the *backwater effect.* At the downstream end the backwater curve is connected with a smooth drawdown curve which leads the water over the spillway.

If the channel has a supercritical slope (bottom sketch in Fig. 4-5), the flow is initially supercritical. In the presence of the dam, the backwater

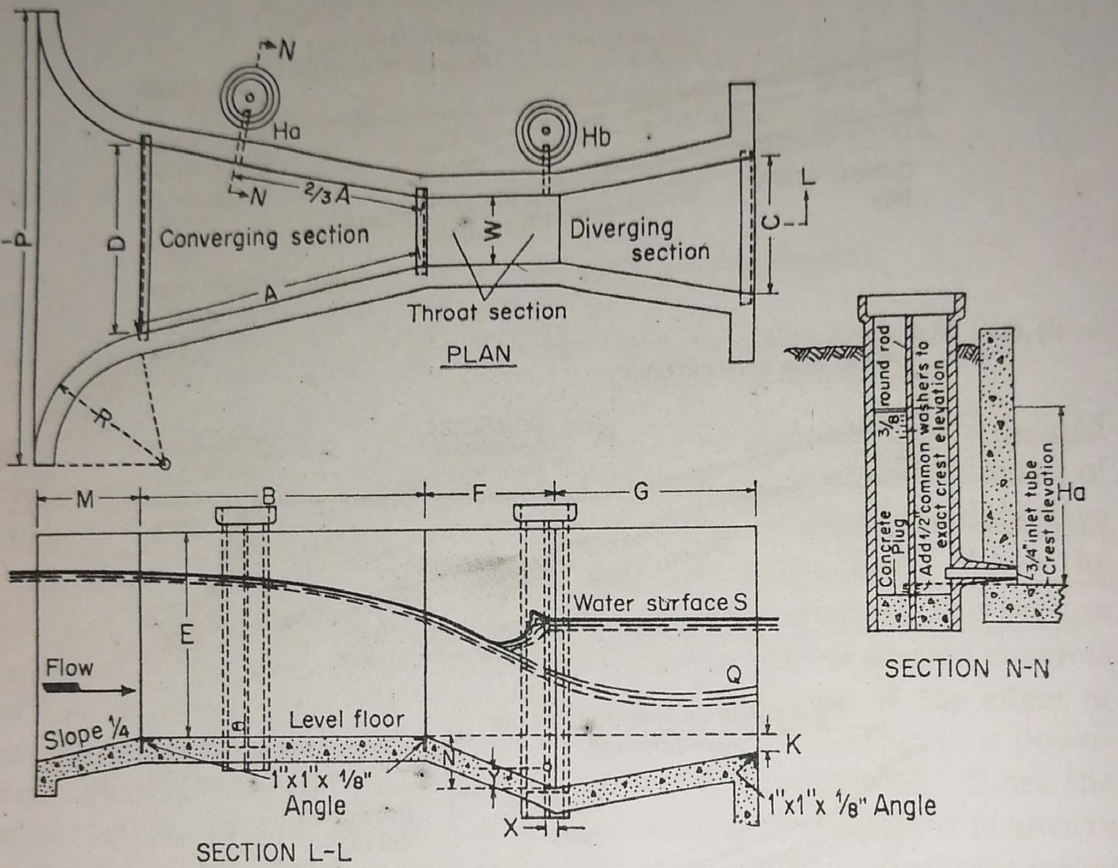


FIG. 4-6. Plan, elevation, and dimensions of the Parshall flume. (*U.S. Soil Conservation Service* [26].) Plan and elevation of a concrete Parshall measuring flume showing lettered dimensions as follows:

W = size of flume in in. or ft; A = length of side wall of converging section; $\frac{2}{3}A$ = distance back from end of crest to gage point; B = axial length of converging section; C = width of downstream end of flume; D = width of upstream end of flume; E = depth of flume; F = length of throat; G = length of diverging section; K = difference in elevation between lower end of flume and crest; M = length of approach floor; N = depth of depression in throat below crest; P = width between ends of curved wing walls; R = radius of curved wing wall; X = horizontal distance to H_b gage point from low point in throat; Y = vertical distance to H_b gage point from low point in throat. See the table on the next page for actual dimensions for various sizes of flume.

effect originating from the pool will not extend far upstream. Instead, the flow in the upstream channel will continue in the downstream direction at a supercritical state until the flow-surface profile is actually below the pool level;[1] then it will rise abruptly to the pool elevation in a hydrau-

[1] It should be noted that the pool level in this case is not horizontal but curved. The curved water surface has an S1 profile, which will be described later (Art. 9-4).

lic jump. The backwater effect will not extend upstream through the hydraulic jump. The flow upstream from the jump is governed entirely by the upstream conditions.

The above example explains the important fact that on subcritical slopes the effect of change in water-surface elevation downstream is transmitted upstream by a backwater curve, whereas on supercritical slopes the effect cannot be transmitted far upstream. The flow condition in a subcritical channel is affected by downstream conditions; but, in a supercritical channel, the flow condition is dependent entirely upon the condition upstream or at the place where water enters the channel. Accordingly, the control of flow is said to be at the downstream end for channels with subcritical slope and at the upstream end for channels with supercritical slope.

When the channel is on a subcritical slope a control section at the downstream end may be a critical section, such as that created on the top of an overflow spillway. On a supercritical slope, the control section at the upstream end may also be a critical section, as shown in the figure. A sluice gate or an orifice or other control structure may also be used to create a control section. It should be noted that whether the channel slope is critical, subcritical, or supercritical will depend not only on the measure of the actual slope but also on the discharge or the depth of flow.

4-6. Flow Measurement. It was mentioned in the preceding article that, at a critical control section, the relationship between the depth and the discharge is definitive, independent of the channel roughness and other uncontrollable circumstances. Such a definitive stage-discharge relationship offers a theoretical basis for the measurement of discharge in open channels.

Based on the principle of critical flow, various devices for flow measurement have been developed. In such devices the critical depth is usually created either by the construction of a low hump on the channel bottom, such as a weir, or by a contraction in the cross section, such as a *critical-flow flume*. The use of a weir is a simple method, but it causes relatively high head loss. If water contains suspended particles, some will be deposited in the upstream pool formed by the weir, resulting in a gradual change in the discharge coefficient. These difficulties, however, can be overcome at least partially by the use of a critical-flow flume.

The critical-flow flume, also known as the *Venturi flume*, has been designed in various forms.[1] It is usually operated with an unsubmerged

[1] The critical-flow flumes mentioned in the text are those developed and studied in the United States. Outstanding designs of critical-flow flumes were also developed and tested by Jameson [4,5], Engel [6,7], and Linford [8] in England; by Crump [9] and Inglis [10] in India; by De Marchi [11,12], Contessini [11], Nebbia [13–15], and Citrini [16,17] in Italy; by Khafagi [18] in Switzerland; and by Balloffet [19] in Argentina.

the tailwater depth $D = 2.5$ ft, and the elevation of the crest above the channel bottom is $X = 2.5 - 0.81 = 1.69$ ft.

From Fig. 4-8, the head loss corresponding to $H_b/H_a = 0.7$, $Q = 20$ cfs, and $W = 4$ ft is 0.43 ft. Therefore, the depth of water upstream from the flume will be $2.50 + 0.43 = 2.93$ ft.

Similarly, try 2- and 3-ft flumes. It is found that the respective crest elevations are 1.53 and 1.23 ft and that the respective upstream water depths are 2.98 and 3.12 ft.

In deciding the most practical size of flume to use, it will be necessary to examine the freeboard of the channel and the effect of rise of the water surface upon the flow through the headgate. If these conditions are satisfactory, the 2-ft flume will be the

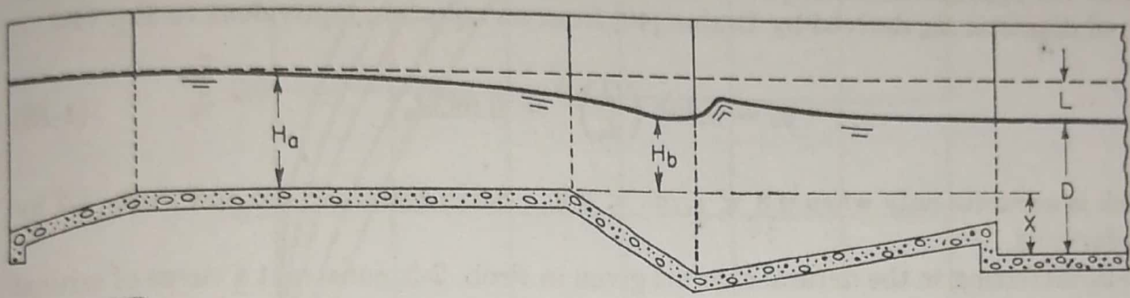


Fig. 4-9. Section of a Parshall flume illustrating the determination of the proper crest elevation [26].

most economical because of its small dimensions. However, when the width of the channel is considered, the final selection may be in favor of the 3- or 4-ft flume because moderate or long wing walls may be required for a small structure. Usually, the throat width of the flume will be from one-third to one-half of the channel width.

PROBLEMS

4-1. Prove the following critical-discharge equations for the triangular, trapezoidal, and circular sections:

Channel Section	*Equation*	
Triangular	$Q_c = 2.295zH_e^{2.5}$	(4-19)
Trapezoidal	$Q_c = \dfrac{5.671[(b + zy)y]^{1.5}}{(b + 2zy)^{0.5}}$	(4-20)
Circular	$Q_c = \dfrac{0.251(\theta - \sin\theta)^{1.5}}{(\sin \frac{1}{2}\theta)^{0.5}} d_0^{2.5}$	(4-21)
Parabolic	$Q_c = 2.005TH_e^{1.5}$	(4-22)

In the above equations, $\alpha = 1$ and H_e is the specific-energy head; other notation follows that of Table 2-1.

4-2. Compute the hydraulic exponent M of the trapezoidal channel section (Fig. 2-2) having a flow depth of 6 ft, using (*a*) Eq. (4-10), (*b*) Fig. 4-2, and (*c*) the graphical method based on Eq. (4-11).

4-3. Compute the hydraulic exponent M of a 36-in. circular conduit having a flow depth of 24 in. above the invert, using (*a*) Fig. 4-2 and (*b*) the graphical method based on Eq. (4-11).

4-4. Prove that the critical depth and velocity for a rectangular channel are

CHAPTER 5

DEVELOPMENT OF UNIFORM FLOW AND ITS FORMULAS

5-1. Qualifications for Uniform Flow. The uniform flow to be considered has the following main features: (1) the depth, water area, velocity, and discharge at every section of the channel reach are constant; and (2) the energy line, water surface, and channel bottom are all parallel; that is, their slopes are all equal, or $S_f = S_w = S_0 = S$. For practical purposes, the requirement of constant velocity may be liberally interpreted as the requirement that the flow possess a constant mean velocity. Strictly speaking, however, this should mean that the flow possesses a constant velocity at every point on the channel section within the uniform-flow reach. In other words, the velocity distribution across the channel section is unaltered in the reach. Such a stable pattern of velocity distribution can be attained when the so-called "boundary layer" is fully developed (Art. 8-1).

Uniform flow is considered to be steady only, since unsteady uniform flow is practically nonexistent. In natural streams, even steady uniform flow is rare, for rivers and streams in natural states scarcely ever experience a strict uniform-flow condition. Despite this deviation from the truth, the uniform-flow condition is frequently assumed in the computation of flow in natural streams. The results obtained from this assumption are understood to be approximate and general, but they offer a relatively simple and satisfactory solution to many practical problems.

As turbulent uniform flow is most commonly encountered in engineering problems, it will be discussed extensively in the following chapters. Laminar uniform flow has limited engineering applications, and will be described only in Art. 6-10.

It should be noted that uniform flow cannot occur at very high velocities, usually described as *ultrarapid*. This is because, when uniform flow reaches a certain high velocity, it becomes very unstable. At higher velocities the flow will eventually entrain air and become unsteady. The criterion for instability of uniform flow will be discussed in Art. 8-8.

5-2. Establishment of Uniform Flow. When flow occurs in an open channel, resistance is encountered by the water as it flows downstream.

This resistance is generally counteracted by the components of gravity forces acting on the body of the water in the direction of motion (Fig. 5-2). A uniform flow will be developed if the resistance is balanced by the gravity forces. The magnitude of the resistance, when other physical factors of the channel are kept unchanged, depends on the velocity of flow.

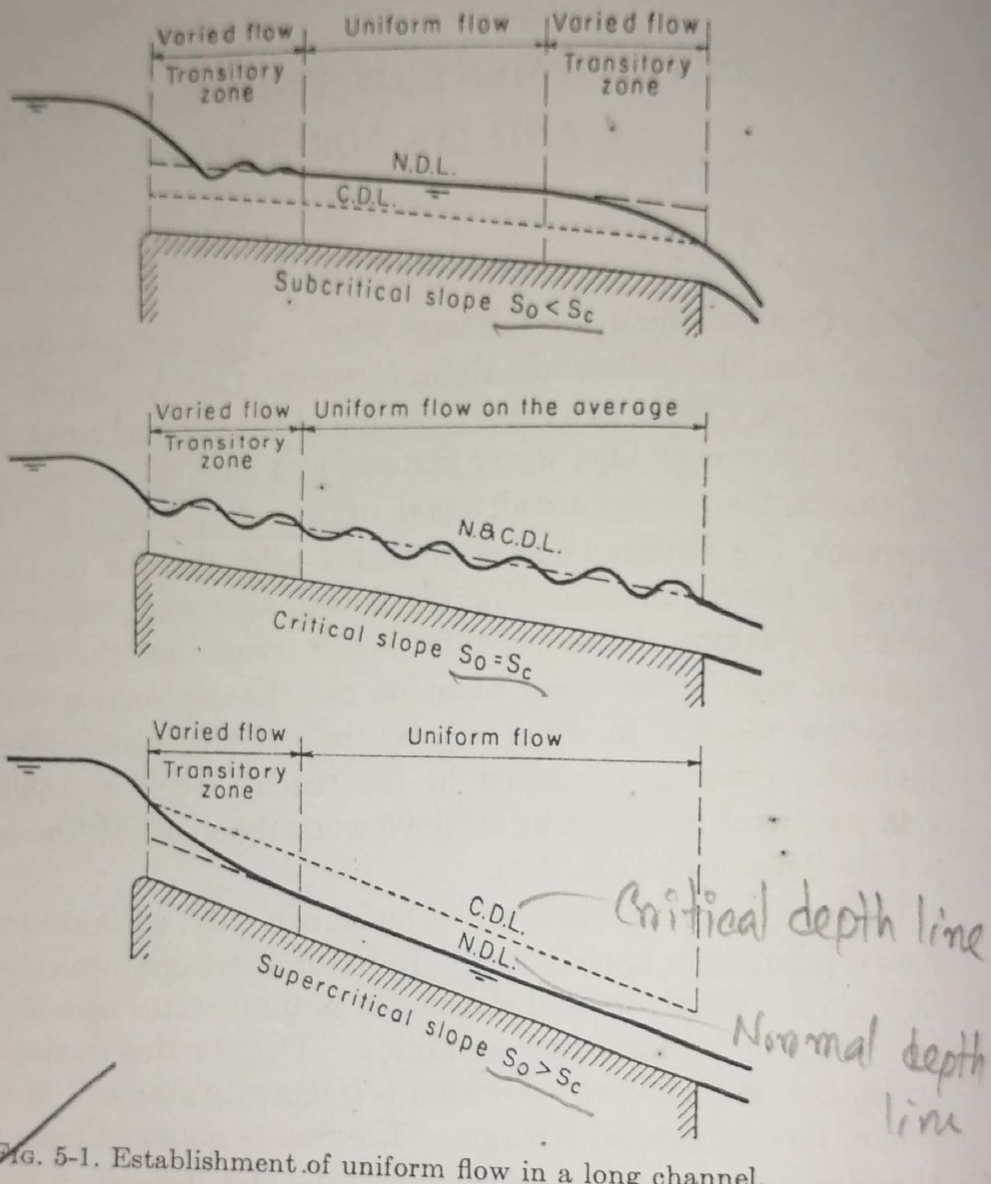


FIG. 5-1. Establishment of uniform flow in a long channel.

If the water enters the channel slowly, the velocity and hence the resistance are small, and the resistance is outbalanced by the gravity forces, resulting in an accelerating flow in the upstream reach. The velocity and the resistance will gradually increase until a balance between resistance and gravity forces is reached. At this moment and afterward the flow becomes uniform. The upstream reach that is required for the establishment of uniform flow is known as the *transitory zone*. In this zone the flow is accelerating and varied. If the channel is shorter than the transitory length required by the given conditions, uniform flow cannot be attained. Toward the downstream end of the channel the resistance

5-4. The Chézy Formula. As early as 1769 the French engineer Antoine Chézy was developing probably the first uniform-flow formula, the famous *Chézy formula*[1] which is usually expressed as follows:

$$V = C\sqrt{RS} \tag{5-2}$$

where V is the mean velocity in fps, R is the hydraulic radius in ft, S is the slope of the energy line, and C is a factor of flow resistance, called *Chézy's C*.

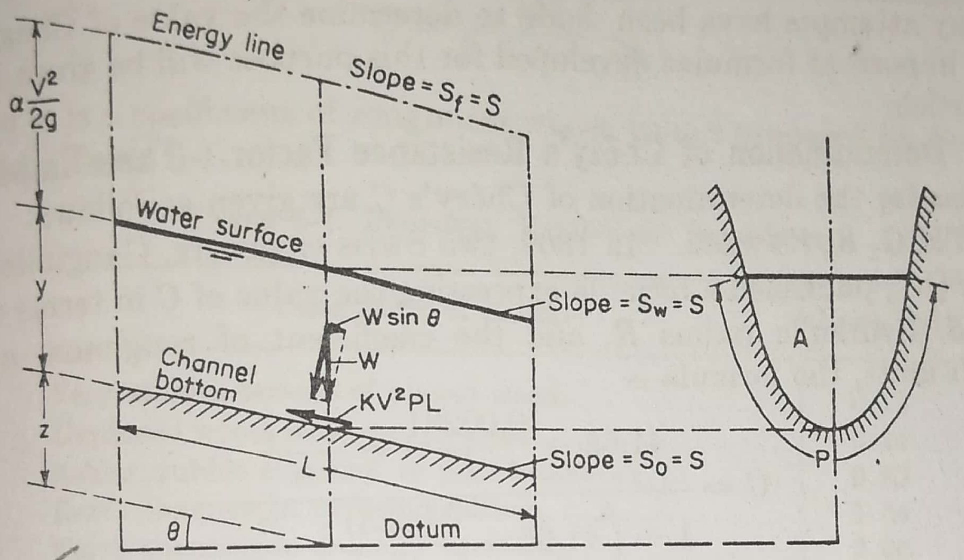


FIG. 5-2. Derivation of the Chézy formula for uniform flow in open channel.

The Chézy formula can be derived mathematically from two assumptions. The first assumption was made by Chézy. It states that the force resisting the flow per unit area of the stream bed is proportional to the square of the velocity; that is, this force is equal to KV^2 where K is a constant of proportionality. The surface of contact of the flow with the stream bed is equal to the product of the wetted perimeter and the length of the channel reach, or PL (Fig. 5-2). The total force resisting the flow[2] is then equal to KV^2PL.

[1] The source of this famous formula is not mentioned in most hydraulics textbooks. In fact, this knowledge has long been sought for. In 1876, the German engineer Gotthilf Heinrich Ludwig Hagen mentioned in his work [7] that Gaspard de Prony had stated that Chézy set up this formula in 1775, on the occasion of a report that Chézy made on the Canal de l'Yvette in conjunction with Jean-Rodolphe Perronet. "But," says Hagen, "I have sought in vain for further information on the subject." Then, in 1897, the American engineer Clemens Herschel through the assistance of a friend in Paris traced the original Canal de l'Yvette report to its hiding place, then translated the portion relating to the formula, and published it in [8]. Chézy's report revealed that the formula was developed and verified by experiments made on an earthen canal, the Courpalet Canal, and on the Seine River in late 1769.

[2] This channel resisting force may also be explained by the principles of fluid dynamics. The open channel can be conceived as a flat plate warped into a cylinder

The second assumption is the basic principle of uniform flow, which is believed to have been claimed first by Brahms [9] in 1754. It states that, in uniform flow, the effective component of the gravity force causing the flow must be equal to the total force of resistance. The effective gravity-force component (Fig. 5-2) is parallel to the channel bottom and equal to $wAL \sin\theta = wALS$, where w is the unit weight of water, A is the water area, θ is the slope angle, and S is the channel slope.[1] Thence, $wALS = KV^2PL$. Let $A/P = R$ and let $\sqrt{w/K}$ be replaced by a factor C; then the previous equation is reduced to the Chézy formula, or $V = \sqrt{(w/K)(A/P)S} = C\sqrt{RS}$.

Many attempts have been made to determine the value of Chézy's C. Three important formulas developed for this purpose will be given in the next article

5-5. Determination of Chézy's Resistance Factor. Three important formulas for the determination of Chézy's C are given as follows:

A. The G. K. Formula. In 1869, two Swiss engineers, Ganguillet and Kutter [10], published a formula expressing the value of C in terms of the slope S, hydraulic radius R, and the coefficient of roughness n. In English units, the formula is

$$C = \frac{41.65 + \dfrac{0.00281}{S} + \dfrac{1.811}{n}}{1 + \left(41.65 + \dfrac{0.00281}{S}\right)\dfrac{n}{\sqrt{R}}} \tag{5-3}$$

The coefficient n in this formula is specifically known as *Kutter's n*. The value of n will be discussed in Arts. 5-7 and 5-8.

The G. K. formula was derived elaborately from flow-measurement data in channels of various types, including Bazin's gagings and the gagings of many European rivers and of the Mississippi River.[2] Although

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but unclosed on one side which corresponds to the free surface of the open-channel flow. A fluid flowing in the unclosed cylinder will create a drag or resisting force on the inside surface. This force is equal to the drag created by the flow of fluid along a flat plate whose two surfaces offer resistance to the flow. The latter is equal to $C_d\rho V^2PL/2$, where C_d is the coefficient of drag and ρ is the mass density of the fluid. Thus, the factor $C_d\rho/2$ is equivalent to the constant of proportionality K.

[1] The slope under consideration is defined as the sine of the angle of inclination, or $S = \sin\theta$.

[2] The Mississippi River gagings were made by Humphreys and Abbot on the lower Mississippi River between 1850 and 1860, and the data thus obtained were published in a report submitted to the U.S. Army Corps of Topographical Engineers in 1861 [11]. The term containing S was introduced into the G. K. formula simply in order to make the formula agree with the Humphreys and Abbot data. This seems somewhat ridiculous now, because these data are known to have been quite inaccurate (see pp. 133–136 of [2]). Some authors have suggested that the slope term $0.00281/S$ of

bution studied by Keulegan (Art. 8-4). The practical application of this formula is limited, since further investigation is needed for determination of the proper values of ϵ.

TABLE 5-3. TENTATIVE VALUES OF POWELL'S ϵ

Description of channel	Powell's ϵ	
	New	Old
Neat cement surface	0.0002	0.0004
Unplaned-plank flumes	0.0010	0.0017
Concrete-lined channels	0.004	0.006
Earth, straight and uniform	0.04	
Dredged earth channels	0.10	

Example 5-1. Compute the velocity and discharge in the trapezoidal channel described in Example 2-1, having a bottom width of 20 ft, side slopes 2:1, and a depth of water 6 ft. Given: Kutter's $n = 0.015$, and $S = 0.005$.

Solution. From Example 2-1, $A = 192.0$ ft² and $R = 4.10$ ft. Using the G. K. formula, the value of Chézy's C is

$$C = \frac{41.65 + \dfrac{0.00281}{0.005} + \dfrac{1.811}{0.015}}{1 + \left(41.65 + \dfrac{0.00281}{0.005}\right)\dfrac{0.015}{\sqrt{4.10}}} = 124.2$$

Then, by the Chézy formula,

$$V = 124.2\sqrt{4.10 \times 0.005} = 17.8 \text{ fps}$$

Therefore,

$$Q = 192.0 \times 17.8 = 3{,}420 \text{ cfs}$$

5-6. The Manning Formula. In 1889 the Irish engineer Robert Manning[1] presented a formula, which was later modified to its present

[1] Manning first presented the formula in a paper read on December 4, 1889, at a meeting of the Institution of Civil Engineers of Ireland. The paper was later published in the Transactions of the Institution [15]. The formula was first given in a complicated form and then simplified to $V = CR^{2/3}S^{1/2}$, where V is the mean velocity, C is a factor of flow resistance, R is the hydraulic radius, and S is the slope. This was further modified by others and expressed in metric units as $V = (1/n)R^{2/3}S^{1/2}$. Later, it was converted back again to English units, resulting in $V = (1.486/n)R^{2/3}S^{1/2}$. In this conversion, as in the conversion of the Ganguillet and Kutter formula, the numerical value of n is kept unaffected. Consequently, the same value of n is widely used in both systems of units.

In the view of modern fluid mechanics, which pays much attention to dimensions, the dimensions of n become a matter of consideration. Directly from the Manning formula, the dimensions of n are seen to be $TL^{-1/3}$. Since it is unreasonable to suppose that the roughness coefficient would contain the dimension T, some authors assume that the numerator contains $\sqrt{g}$, thus yielding the dimensions of $L^{1/6}$ for n. Also, for physical reasons, it will be seen that $n = [\phi(R/k)]k^{1/6}$ [Eq. (8-26)], where k is

well-known form

$$V = \frac{1.49}{n}R^{2/3}S^{1/2} \tag{5-6}$$

where V is the mean velocity in fps, R is the hydraulic radius in ft, S is the slope of energy line, and n is the coefficient of roughness, specifically known as *Manning's n*. This formula was developed from seven different formulas, based on Bazin's experimental data, and further verified by 170 observations.[1] Owing to its simplicity of form and to the satis-

a linear measure of roughness and $\phi(R/k)$ is a function of R/k. If $\phi(R/k)$ is considered dimensionless, n will have the same dimensions as those of $k^{1/6}$, that is, $L^{1/6}$.

On the other hand, of course, it is equally possible to assume that the numerator of $1.486/n$ can absorb the dimensions of $L^{1/2}T^{-1}$, or that $\phi(R/k)$ involves a dimensional factor, thus leaving no dimensions for n. Some authors, therefore, preferring the simpler choice, consider n a dimensionless coefficient.

It is interesting to note that the conversion of the units for the Manning formula is independent of the dimensions of n, as long as the same value of n is used in both systems of units. If n is assumed dimensionless, then the formula in English units gives the numerical constant $3.2808^{1/3} = 1.486$ since 1 meter = 3.2808 ft. Now, if n is assumed to have the dimensions of $L^{1/6}$, its numerical value in English units must be different from its value in metric units, unless a numerical correction factor is introduced for compensation. Let n be the value in metric units and n' the value in English units. Then, $n' = (3.2808^{1/6})n = 1.2190n$. When the formula is converted from metric to English units, the resulting form takes the numerical constant $3.2808^{1/3+1/6} = 3.2808^{1/2} = 1.811$, since n has the dimensions of $L^{1/6}$. Thus, the resulting equation should be written $V = 1.811R^{2/3}S^{1/2}/n'$. Since the same value of n is used in both systems, the practical form of the formula in the English system is $V = 1.811R^{2/3}S^{1/2}/1.2190n = 1.486R^{2/3}S^{1/2}/n$, which is identical with the formula derived on the assumption that n has no dimensions.

In a search of early literature on hydraulics, the author has failed to find any significant discussion regarding the dimensions of n. It seems that this was not a problem of concern to the forefathers of hydraulics. It is most likely, however, that n was unconsciously taken as dimensionless in the conversion of the Manning formula, because such a conversion, as shown above, is more direct and simpler.

Now, considering the approximations involved in the derivation of the formula and the uncertainty in the value of n, it seems unjustifiable to carry the numerical constant to more than three significant figures. For practical purposes, a value of 1.49 is believed to be sufficiently accurate [16].

Manning mentioned that the simplified form of the formula had been suggested independently by G. H. L. Hagen prior to Manning's own work, according to a statement by Major Cunningham [17]. Hagen's formula was believed to have appeared first in 1876 [7]. It is also known that Philippe-Gaspard Gauckler [18] had an early proposal of the simplified form of Manning's formula in 1868 and that Strickler [19] presented independently the same form of the formula in 1923.

[1] For the derivation of the exponent of R, use was made of Bazin's experimental data on artificial channels [12]. For different shapes and roughnesses, the average value of the exponent was found to vary from 0.6499 to 0.8395. Considering these variations, Manning adopted an approximate value of 2/3 for the exponent. On the

for n between 0.011 and 0.040. For practical purposes, the following approximate forms of Eq. (5-9) are generally suggested for use:

$$y = 1.5\sqrt{n} \quad \text{for } R < 1.0 \text{ m} \tag{5-10}$$
$$y = 1.3\sqrt{n} \quad \text{for } R > 1.0 \text{ m} \tag{5-11}$$

5-7. Determination of Manning's Roughness Coefficient. In applying the Manning formula or the G. K. formula, the greatest difficulty lies in the determination of the roughness coefficient n; for there is no exact method of selecting the n value. At the present stage of knowledge, to select a value of n actually means to estimate the resistance to flow in a given channel, which is really a matter of intangibles. To veteran engineers, this means the exercise of sound engineering judgment and experience; for beginners, it can be no more than a guess, and different individuals will obtain different results.

In order to give guidance in the proper determination of the roughness coefficient, four general approaches will be discussed; namely, (1) to understand the factors that affect the value of n and thus to acquire a basic knowledge of the problem and narrow the wide range of guesswork, (2) to consult a table of typical n values for channels of various types, (3) to examine and become acquainted with the appearance of some typical channels whose roughness coefficients are known, and (4) to determine the value of n by an analytical procedure based on the theoretical velocity distribution in the channel cross section and on the data of either velocity or roughness measurement. The first three approaches will be given in the next three articles, and the fourth approach will be taken up in Art. 8-7.

5-8. Factors Affecting Manning's Roughness Coefficient. It is not uncommon for engineers to think of a channel as having a single value of n for all occasions. In reality, the value of n is highly variable and depends on a number of factors. In selecting a proper value of n for various design conditions, a basic knowledge of these factors should be found very useful. The factors that exert the greatest influence upon the coefficient of roughness in both artificial and natural channels are therefore described below. It should be noted that these factors are to a certain extent interdependent; hence discussion about one factor may be repeated in connection with another.

A. Surface Roughness. The surface roughness is represented by the size and shape of the grains of the material forming the wetted perimeter and producing a retarding effect on the flow. This is often considered the only factor in selecting a roughness coefficient, but it is actually just one of several major factors. Generally speaking, fine grains result in a relatively low value of n and coarse grains, in a high value of n.

In alluvial streams where the material is fine in grain, such as sand, clay, loam, or silt, the retarding effect is much less than where the material is coarse, such as gravels or boulders. When the material is fine, the value of n is low and relatively unaffected by change in flow stage. When the material consists of gravels and boulders, the value of n is generally high, particularly at low or high stage. Larger boulders usually collect at the bottom of the stream, making the channel bottom rougher than the banks and increasing the value of n at low stages. At high stages, a portion of the energy of flow is used in rolling the boulders downstream, thus increasing the value of n. A theoretical discussion of surface roughness will be given in Art. 8-2.

B. Vegetation. Vegetation may be regarded as a kind of surface roughness, but it also markedly reduces the capacity of the channel and retards the flow. This effect depends mainly on height, density, distribution, and type of vegetation, and it is very important in designing small drainage channels.

At the University of Illinois an investigation has been made to determine the effect of vegetation on the coefficient of roughness [22]. On one of the drainage ditches in central Illinois under investigation, an average n value of 0.033 was measured in March, 1925, when the channel was in good condition. In April, 1926, there were bushy willows and dry weeds on the side slopes, and n was found to be 0.055. This increase in n represents the result of one year's growth of vegetation. During the summers of 1925 and 1926 there was a thick growth of cattails on the bottom of the channel. The n value at medium summer stages was about 0.115, and at a nearly bankfull stage it was 0.099. The cattails in the channel were washed out by the high water in September, 1926; the average value of n found after this occurrence was 0.072. The conclusions drawn from this investigation were, in part, as follows:

1. The minimum value of n that should be used for designing drainage ditches in central Illinois is 0.040. This value is obtainable at high stages during the summer months in the most carefully maintained channels, where the bottom of the channel is clear of vegetation and the side slopes are covered with grass or low weeds, but no bushes. This low value of n should not be used unless the channel is to be cleared annually of all weeds and bushes.

2. A value of $n = 0.050$ should be used if the channel is to be cleared in alternate years only. Large weeds and bushy willows from 3 to 4 ft high on the side slopes will produce this value of n.

3. In channels that are not cleared for a number of years, the growth may become so abundant that values of $n > 0.100$ may be found.

4. Trees from 6 to 8 in. in diameter growing on the side slopes do not impede the flow so much as do small bushy growths, provided overhanging branches are cut off.

The U.S. Soil Conservation Service has made studies on flow of water in small shallow channels protected by vegetative linings (Chap. 7, Sec. C). It was found that n values for these channels varied with the shape and cross section of the channel, the slope of the channel bed, and the depth of flow. Comparing two channels, all other factors being equal, the lesser average depth gives the higher n value, owing to a larger proportion of affected vegetation. Thus, a triangular channel has a higher n value than a trapezoidal channel, and a wide channel has a lower n value than a narrow channel. A flow of sufficient depth tends to bend over and submerge the vegetation and to produce low n values. A steep slope causes greater velocity, greater flattening of the vegetation, and low n values.

The effect of vegetation on flood plains will be discussed later in item *H*.

C. Channel Irregularity. Channel irregularity comprises irregularities in wetted perimeter and variations in cross section, size, and shape along the channel length. In natural channels, such irregularities are usually introduced by the presence of sand bars, sand waves, ridges and depressions, and holes and humps on the channel bed. These irregularities definitely introduce roughness in addition to that caused by surface roughness and other factors. Generally speaking, a gradual and uniform change in cross section, size, and shape will not appreciably affect the value of n, but abrupt changes or alternation of small and large sections necessitates the use of a large value of n. In this case, the increase in n may be 0.005 or more. Changes that cause sinuous flow from side to side of the channel will produce the same effect.

D. Channel Alignment. Smooth curvature with large radius will give a relatively low value of n, whereas sharp curvature with severe meandering will increase n. On the basis of flume tests, Scobey [23] suggested that the value of n be increased 0.001 for each 20 degrees of curvature in 100 ft of channel. Although it is doubtful whether curvature ever increases n more than 0.002 or 0.003, its effect should not be ignored, for curvature may induce the accumulation of drift and thus indirectly increase the value of n. Generally speaking, the increase of roughness in unlined channels carrying water at low velocities is negligible. An increase of 0.002 in n value would constitute an adequate allowance for curve losses in most flumes containing pronounced curvatures, whether built of concrete or other materials. The meandering of natural streams, however, may increase the n value as high as 30%.

E. Silting and Scouring. Generally speaking, silting may change a very irregular channel into a comparatively uniform one and decrease n, whereas scouring may do the reverse and increase n. However, the dominant effect of silting will depend on the nature of the material deposited. Uneven deposits such as sand bars and sand waves are channel irregularities and will increase the roughness. The amount and uniformity of scouring will depend on the material forming the wetted perimeter. Thus, a sandy or gravelly bed will be eroded more uniformly than a clay bed. The deposition of silt eroded from the uplands will tend to even out the irregularities in a channel dredged through clay. The energy used in eroding and carrying the material in suspension or rolling it along the bed will also increase the n value. The effect of scouring is not significant as long as the erosion on channel bed caused by high velocities is progressing evenly and uniformly.

F. *Obstruction.* The presence of log jams, bridge piers, and the like tends to increase n. The amount of increase depends on the nature of the obstructions, their size, shape, number, and distribution.

G. *Size and Shape of Channel.* There is no definite evidence about the size and shape of a channel as an important factor affecting the value of n. An increase in hydraulic radius may either increase or decrease n, depending on the condition of the channel (Fig. 5-4).

H. *Stage and Discharge.* The n value in most streams decreases with increase in stage and in discharge. When the water is shallow, the irregularities of the channel bottom are exposed and their effects become pronounced. However, the n value may be large at high stages if the banks are rough and grassy.

When the discharge is too high, the stream may overflow its banks and a portion of the flow will be along the flood plain. The n value of the flood plains is generally larger than that of the channel proper, and its magnitude depends on the surface condition or vegetation. If the bed and banks of a channel are equally smooth and regular and the bottom slope is uniform, the value of n may remain almost the same at all stages; so a constant n is usually assumed in the flow computation. This happens mostly in artificial channels. On flood plains the value of n usually varies with the stage of submergence of the vegetation at low stages. This can be seen, for example, from Table 5-4, which shows the n values for various flood stages according to the type of cover and depth

TABLE 5-4. VALUES OF n FOR VARIOUS STAGES IN THE NISHNABOTNA RIVER, IOWA, FOR THE AVERAGE GROWING SEASON

Depth of water, ft	Channel section	Flood-plain cover				
		Corn	Pasture	Meadow	Small grains	Brush and waste
Under 1	0.03	0.06	0.05	0.10	0.10	0.12
1 to 2	0.03	0.06	0.05	0.08	0.09	0.11
2 to 3	0.03	0.07	0.04	0.07	0.08	0.10
3 to 4	0.03	0.07	0.04	0.06	0.07	0.09
Over 4	0.03	0.06	0.04	0.05	0.06	0.08

CHAPTER 6

COMPUTATION OF UNIFORM FLOW

6-1. The Conveyance of a Channel Section. The discharge of uniform flow in a channel may be expressed as the product of the velocity, represented by Eq. (5-1), and the water area, or

$$Q = VA = CAR^xS^y = KS^y \tag{6-1}$$

where

$$K = CAR^x \tag{6-2}$$

The term K is known as the *conveyance* of the channel section; it is a measure of the carrying capacity of the channel section, since it is directly proportional to Q.

When either the Chézy formula or the Manning formula is used as the uniform-flow formula, i.e., when $y = \frac{1}{2}$, the discharge by Eq. (6-1) becomes

$$Q = K\sqrt{S} \tag{6-3}$$

and the conveyance is

$$K = \frac{Q}{\sqrt{S}} \tag{6-4}$$

This equation can be used to compute the conveyance when the discharge and slope of the channel are given.

When the Chézy formula is used, Eq. (6-2) becomes

$$K = CAR^{1/2} \tag{6-5}$$

where C is Chézy's resistance factor. Similarly, when the Manning formula is used,

$$K = \frac{1.49}{n} AR^{2/3} \tag{6-6}$$

The above two equations are used to compute the conveyance when the geometry of the water area and the resistance factor or roughness coefficient are given. Since the Manning formula is used extensively, most of the following discussions and computations will be based on Eq. (6-6).

6-2. The Section Factor for Uniform-flow Computation. The expression $AR^{2/3}$ is called the *section factor for uniform-flow computation;* it is an important element in the computation of uniform flow. From Eq.

(6-6), this factor may be expressed as

$$AR^{2/3} = \frac{nK}{1.49} \tag{6-7}$$

and, from Eq. (6-4),

$$AR^{2/3} = \frac{nQ}{1.49\sqrt{S}} \tag{6-8}$$

Primarily, Eq. (6-8) applies to a channel section when the flow is uniform. The right side of the equation contains the values of n, Q, and S; but the left side depends only on the geometry of the water area. Therefore, it shows that, for a given condition of n, Q, and S, there is *only one* possible depth for maintaining a uniform flow, provided that the value of $AR^{2/3}$ always increases with increase in depth, which is true in most cases. This depth is the *normal depth.* When n and S are known at a channel section, it can be seen from Eq. (6-8) that there can be *only one* discharge for maintaining a uniform flow through the section, provided that $AR^{2/3}$ always increases with increase of depth.[1] This discharge is the *normal discharge.*

Equation (6-8) is a very useful tool for the computation and analysis of uniform flow. When the discharge, slope, and roughness are known, this equation gives the section factor $A_nR_n^{2/3}$ and hence the normal depth y_n. On the other hand, when n, S, and the depth, hence the section factor, are given, the normal discharge Q_n can be computed from this equation in the following form:

$$Q = \frac{1.49}{n} AR^{2/3}\sqrt{S} \tag{6-9}$$

This is essentially the product of the water area and the velocity defined by the Manning formula. The subscript n is sometimes used to specify the condition of uniform flow.

In order to simplify the computation, dimensionless curves showing the relation between depth and section factor $AR^{2/3}$ (Fig. 6-1) have been prepared for rectangular, trapezoidal, and circular channel sections. These self-explanatory curves will help to determine the depth for a given section factor $AR^{2/3}$, and vice versa. The $AR^{2/3}$ values for a circular section can also be found from the table in Appendix A.

[1] This is true for channels in which the value of $AR^{2/3}$ always increases with increase of depth, since Eq. (6-8) will give one value of $AR^{2/3}$, which in turn gives only one depth. In the case of a closed conduit having a gradually closing top, the value of $AR^{2/3}$ will first increase with depth and then decrease with depth when the full depth is approached, because a maximum value of $AR^{2/3}$ usually occurs in such a conduit at a depth slightly less than the full depth. Consequently, it is possible to have two depths for the same value of $AR^{2/3}$, one greater and the other less than the depth for the maximum value of $AR^{2/3}$. For further discussion on this subject see Art. 6-4.

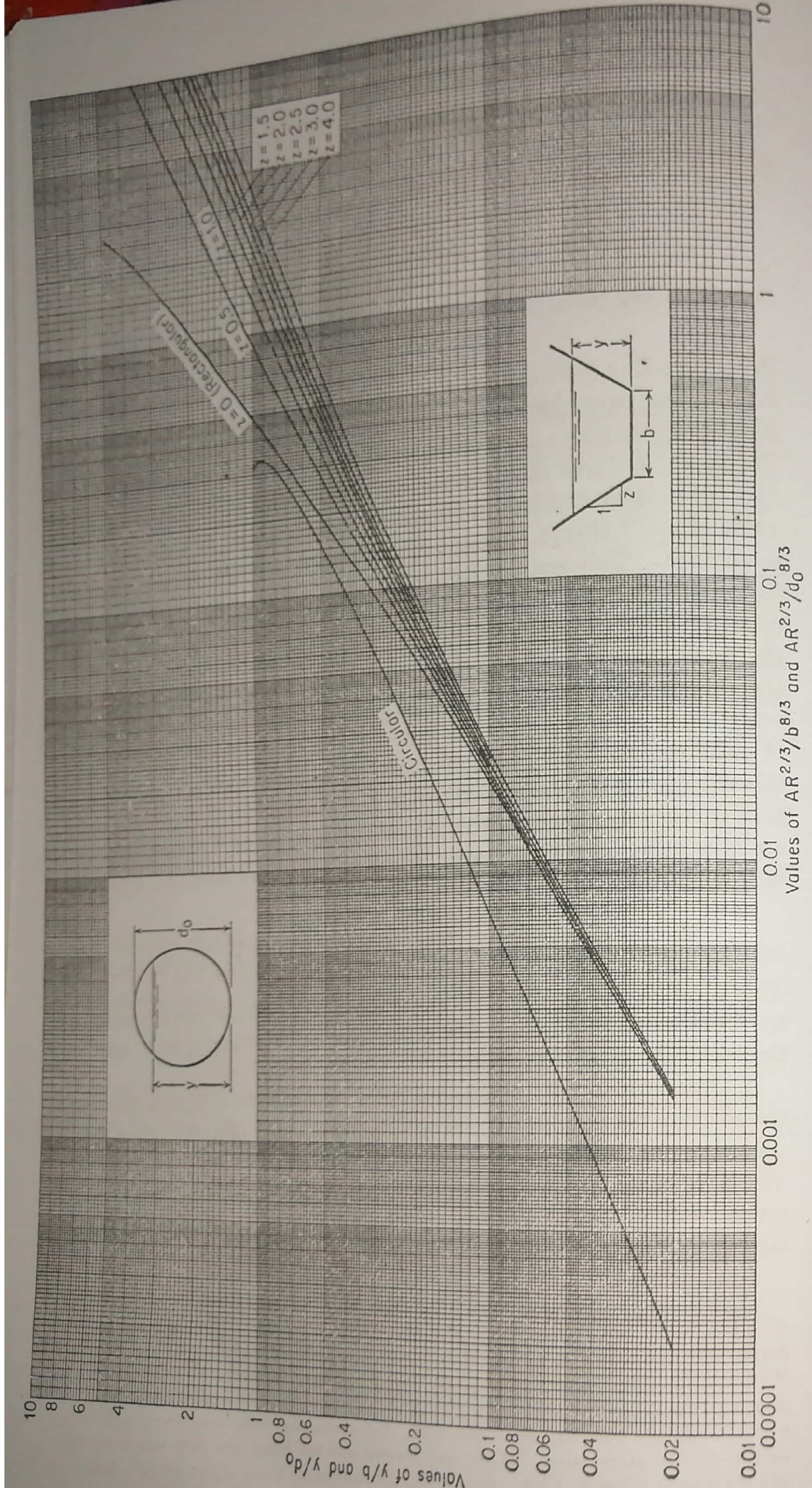

Fig. 6-1. Curves for determining the normal depth.

6-3. The Hydraulic Exponent for Uniform-flow Computation. Since the conveyance K is a function of the depth of flow y, it may be assumed that

$$K^2 = Cy^N \tag{6-10}$$

where C is a coefficient and N is a parameter called the *hydraulic exponent for uniform-flow computation.*

Taking logarithms on both sides of Eq. (6-10) and then differentiating with respect to y,

$$\frac{d(\ln K)}{dy} = \frac{N}{2y} \tag{6-11}$$

Now, taking logarithms on both sides of Eq. (6-6), $K = 1.49AR^{2/3}/n$, and then differentiating this equation with respect to y under the assumption that n is independent of y,

$$\frac{d(\ln K)}{dy} = \frac{1}{A}\frac{dA}{dy} + \frac{2}{3}\frac{1}{R}\frac{dR}{dy} \tag{6-12}$$

Since $dA/dy = T$ and $R = A/P$, the above equation becomes

$$\frac{d(\ln K)}{dy} = \frac{1}{3A}\left(5T - 2R\frac{dP}{dy}\right) \tag{6-13}$$

Equating the right sides of Eqs. (6-11) and (6-13) and solving for N,

$$N = \frac{2y}{3A}\left(5T - 2R\frac{dP}{dy}\right) \tag{6-14}$$

This is the general equation for the hydraulic exponent N. For a trapezoidal channel section having a bottom width b and side slopes 1 on z, the expressions for A, T, P, and R may be obtained from Table 2-1. Substituting them in Eq. (6-14) and simplifying, the resulting equation[1] is

$$N = \frac{10}{3}\,\frac{1 + 2z(y/b)}{1 + z(y/b)} - \frac{8}{3}\,\frac{\sqrt{1+z^2}\,(y/b)}{1 + 2\sqrt{1+z^2}\,(y/b)} \tag{6-15}$$

This equation indicates that the value of N for the trapezoidal section is a function of z and y/b. For values of $z = 0$, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0, a family of curves for N versus y/b can be constructed (Fig. 6-2).[2] These curves indicate that the value of N varies within a range of 2.0 to 5.3.

The curve for a circular section with N plotted against y/d_0, where d_0 is the diameter, is also shown in Fig. 6-2. This curve shows that the

[1] This equation [1] was also developed independently by Chugaev [2] through the use of the Chézy formula.

[2] Similar curves to those in Fig. 6-2 for trapezoidal channels were constructed by Kirpich [3] and also prepared independently by Pavlovskiĭ [4] and Rakhmanoff [5].

and abscissa are interchanged. If a constant n value is assumed, Eq. (6-6) indicates that $K \propto AR^{2/3}$; hence, these curves for $AR^{2/3}$ should show the same characteristics as if the curves were plotted for K. From Eq. (6-10), it can be seen that the hydraulic exponent for the straight-line range of the plot is equal to twice the slope of the plotted straight line. Thus, if any two points with coordinates (K_1, y_1) and (K_2, y_2) are taken from the straight line, the approximate value of N may be computed by the following equation:

$$N = 2\frac{\log(K_1/K_2)}{\log(y_1/y_2)} \tag{6-16}$$

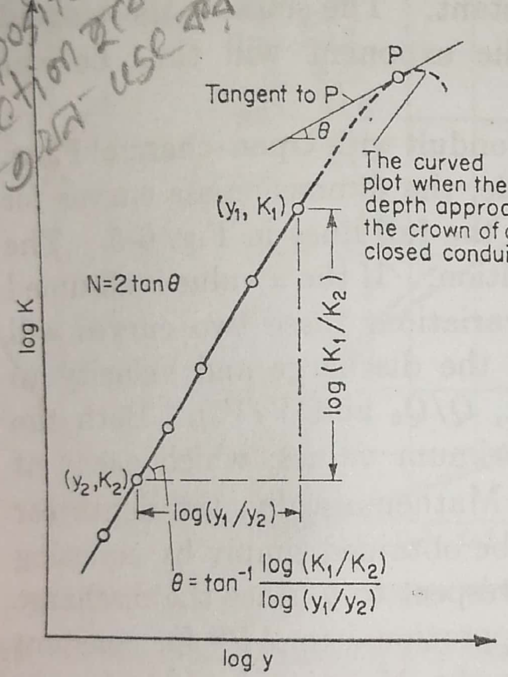

FIG. 6-3. Graphical determination of N by logarithmic plotting.

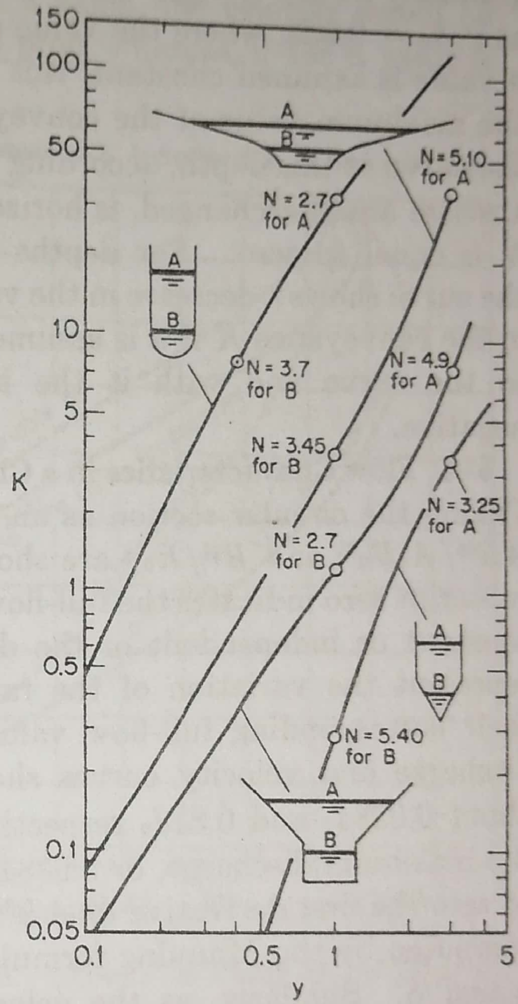

FIG. 6-4. Typical channel sections having appreciable variation in N value with respect to depth. (*After R. R. Chugaev* [2].)

When the cross section of a channel changes abruptly with respect to depth, the hydraulic exponent will change accordingly. Several typical sections are shown in Fig. 6-4. In such cases the logarithmic plot of N against y may appear as a broken line or an evident curve. For the nearly straight portions of the broken line or curve, the hydraulic exponents may be assumed constant.

When the depth of flow approaches the gradually closing crown of a closed conduit, the logarithmic plot will appear as a curve. The hydraulic exponent in the range of the curved plot is equal to twice the slope of the tangent to the curve at the given depth (Fig. 6-3). For practical purposes, depth, because the depth for maximum discharge is so close to the top that there is always a possibility of slight backwater to increase this depth closer to and eventually equal to the full depth.

6-5. Flow in a Channel Section with Composite Roughness. In simple channels, the roughness along the wetted perimeter may be distinctly different from part to part of the perimeter, but the mean velocity can still be computed by a uniform-flow formula without actually subdividing the section. For example, a rectangular channel built with a wooden bottom and glass walls must have different n values for the bottom and the walls. In applying the Manning formula to such channels, it is sometimes necessary to compute an *equivalent n value* for the entire perimeter and use this equivalent value for the computation of the flow in the whole section.

For the determination of the equivalent roughness, the water area is divided imaginatively into N parts of which the wetted perimeters P_1, P_2, . . . , P_N and the coefficients of roughness n_1, n_2, . . . , n_N are known. Horton [6] and Einstein [7,8] assumed that each part of the area has the same mean velocity, which at the same time is equal to the mean velocity of the whole section; that is, $V_1 = V_2 = \cdots = V_N = V$. On the basis of this assumption, the equivalent coefficient of roughness may be obtained by the following equation:

$$n = \left[\frac{\sum_1^N (P_N n_N^{1.5})}{P}\right]^{2/3} = \frac{(P_1 n_1^{1.5} + P_2 n_2^{1.5} + \cdots + P_N n_N^{1.5})^{2/3}}{P^{2/3}} \quad (6\text{-}17)$$

There are many other assumptions for the determination of an equivalent roughness. Pavlovskiĭ [9] and also Mühlhofer [10] and Einstein and Banks [11] assumed that the total force resisting the flow (that is, KV^2PL; see Art. 5-4) is equal to the sum of the forces resisting the flow developed in the subdivided areas. By this assumption, the equivalent roughness coefficient is

$$n = \frac{\left[\sum_1^N (P_N n_N^2)\right]^{1/2}}{P^{1/2}} = \frac{(P_1 n_1^2 + P_2 n_2^2 + \cdots + P_N n_N^2)^{1/2}}{P^{1/2}} \quad (6\text{-}18)$$

Lotter [12] assumed that the total discharge of the flow is equal to the sum of the discharges of the subdivided areas. Thus, the equivalent roughness coefficient is

$$n = \frac{PR^{5/3}}{\sum_1^N \left(\frac{P_N R_N^{5/3}}{n_N}\right)} = \frac{PR^{5/3}}{\frac{P_1 R_1^{5/3}}{n_1} + \frac{P_2 R_2^{5/3}}{n_2} + \cdots + \frac{P_N R_N^{5/3}}{n_N}} \quad (6\text{-}19)$$

channel section is equal to the total discharge divided by the total water area.

Owing to the differences that exist among the velocities of the subsections, the velocity-distribution coefficients of the whole section are different from those of the subsections. The values of these coefficients may be computed as follows:

Let $v_1, v_2, \ldots, v_N$ be the mean velocities in the subsections; let $\alpha_1, \alpha_2, \ldots, \alpha_N$ and $\beta_1, \beta_2, \ldots, \beta_N$ be the velocity-distribution coefficients for the corresponding subsections; let $\Delta A_1, \Delta A_2, \ldots, \Delta A_N$ be

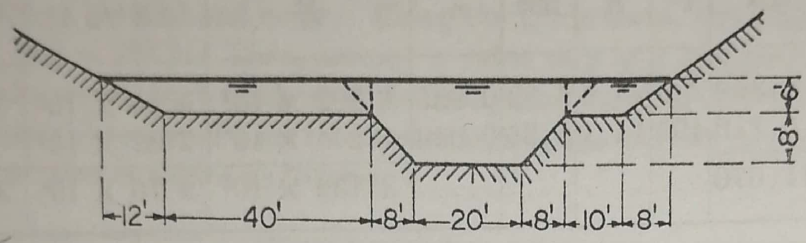

FIG. 6-6. A channel consisting of one main section and two side sections.

the water areas of the corresponding subsections; let $K_1, K_2, \ldots, K_N$ be the conveyances of the corresponding subsections; let V be the mean velocity of the total section; and let A be the total water area. From the continuity equation and Eq. (6-3), the following can be written:

$$v_1 = \frac{K_1}{\Delta A_1} S^{1/2} \qquad v_2 = \frac{K_2}{\Delta A_2} S^{1/2} \qquad \cdots \qquad v_N = \frac{K_N}{\Delta A_N} S^{1/2}$$

$$Q = VA = v_1 \Delta A_1 + v_2 \Delta A_2 + \cdots + v_N \Delta A_N$$

$$= (K_1 + K_2 + \cdots + K_N) S^{1/2} = \left(\sum_1^N K_N\right) S^{1/2}$$

and

$$V = \frac{\left(\sum_1^N K_N\right) S^{1/2}}{A}$$

Incorporating the above expressions with Eqs. (2-4) and (2-5) and simplifying, the velocity-distribution coefficients of the entire section are

$$\alpha = \frac{\sum_1^N (\alpha_N K_N^3 / \Delta A_N^2)}{\left(\sum_1^N K_N\right)^3 / A^2} \tag{6-30}$$

and

$$\beta = \frac{\sum_1^N (\beta_N K_N^2 / \Delta A_N)}{\left(\sum_1^N K_N\right)^2 / A} \tag{6-31}$$

Example 6-1. Compute the velocity-distribution coefficients at a peak flow in a natural stream channel consisting of a main section and an overflow side section. The data obtained at the peak flow stage are:

Subsection	A, ft²	P, ft	n value	α	β
Main section........	5,360	225	0.035	1.10	1.04
Side section.........	5,710	405	0.040	1.11	1.04

Solution. The computations are given below.

Subsection	ΔA	P	R	$R^{2/3}$	n	K	$\beta K^2/\Delta A$	$\alpha K^3/\Delta A^2$
Main section.	5,360	225	23.8	8.29	0.035	1.892×10^6	6.94×10^8	25.93×10^{10}
Side section...	5,710	405	14.1	5.85	0.040	1.244×10^6	2.82×10^8	6.56×10^{10}
Total......	11,070	...	....	....	.....	3.136×10^6	9.76×10^8	32.49×10^{10}

By Eqs. (6-30) and (6-31), the coefficients are

$$\alpha = \frac{32.49 \times 10^{10}}{(3.136 \times 10^6)^3/11{,}070^2} = 1.29$$

and

$$\beta = \frac{9.76 \times 10^8}{(3.136 \times 10^6)^2/11{,}070} = 1.10$$

6-6. Determination of the Normal Depth and Velocity. The normal depth and velocity may be computed by a uniform-flow formula. In the following computations, the Manning formula is used with three different methods of solution.[1]

A. Algebraic Method. For geometrically simple channel sections, the uniform-flow condition may be determined by an algebraic solution, as illustrated by the following example:

Example 6-2. A trapezoidal channel (Fig. 2-2), with $b = 20$ ft, $z = 2$, $S_0 = 0.0016$, and $n = 0.025$, carries a discharge of 400 cfs. Compute the normal depth and velocity.

Solution 1: The Analytical Approach. The hydraulic radius and water area of the given section are expressed in terms of the depth y as

$$R = \frac{y(10 + y)}{10 + y\sqrt{5}} \quad \text{and} \quad A = y(20 + 2y)$$

The velocity is

$$V = \frac{Q}{A} = \frac{400}{y(20 + 2y)}$$

Substituting the given quantities and the above expressions in the Manning formula

[1] Besides the methods described here, there are other methods for the computation of uniform flow, such as the use of hydraulic tables. Popular tables for this purpose can be found in [16] to [20].

and simplifying,

$$\frac{200}{y(10 + y)} = \frac{1.49}{0.025}\left[\frac{y(10 + y)}{10 + y\sqrt{5}}\right]^{2/3} 0.0016^{1/2}$$

or

$$7{,}680 + 1{,}720y = [y(10 + y)]^{2.5}$$

Solving this equation for y by trial and error, $y_n = 3.36$ ft. This is the normal depth. The corresponding area is $A_n = 89.8$ ft² and the normal velocity is $V_n = 400/89.8 = 4.46$ fps. From Example 4-2, it is known that the critical depth for the same discharge in the channel is 2.15 ft. Since the normal depth is greater than the critical depth, the flow is subcritical.

Solution 2: The Trial-and-error Approach. Some engineers prefer to solve this type of problem by trial and error. Using the given data, the right side of Eq. (6-8) is $nQ/1.49\sqrt{S} = 167.7$. Then, assume a value of y and compute the section factor $AR^{2/3}$. Make several such trials until the computed value of $AR^{2/3}$ is very closely equal to 167.7; then the assumed y for the closest trial is the normal depth. This trial-and-error computation is shown as follows:

y	A	R	$R^{2/3}$	$AR^{2/3}$	Remarks
3.00	78.0	2.34	1.762	137.4	y too small
3.50	94.5	2.65	1.915	181.0	y too large
3.30	87.7	2.53	1.852	162.6	
3.35	89.5	2.56	1.870	167.2	
3.36	89.8	2.56	1.870	168.0	The closest

The normal depth is, therefore, $y_n = 3.36$ ft.

B. Graphical Method. For channels of complicated cross section and variable flow conditions, a graphical solution of the problem is found to be convenient. By this procedure, a curve of y against the section factor $AR^{2/3}$ is first constructed and the value of $nQ/1.49\sqrt{S}$ is computed. According to Eq. (6-8), it is evident that the normal depth may be found from the y-$AR^{2/3}$ curve where the coordinate of $AR^{2/3}$ equals the computed value of $nQ/1.49\sqrt{S}$. When the discharge changes, new values of $nQ/1.49\sqrt{S}$ are then computed and the corresponding new normal depths can be found from the same curve.

Example 6-3. Determine the normal depth of flow in a 36-in. culvert (Example 4-3) laid on a slope of 0.0016, having $n = 0.015$, and carrying a discharge of 20 cfs.

Solution. Construct a curve of y vs. $AR^{2/3}$ for the given culvert (Fig. 6-7). Compute $nQ/1.49\sqrt{S} = 0.015 \times 20/1.49\sqrt{0.0016} = 5.04$. From the y-$AR^{2/3}$ curve, find the depth corresponding to the value of 5.04 for $AR^{2/3}$. This depth is the required normal depth, or $y_n = 2.16$ ft. Since this depth is greater than the critical depth determined in Example 4-3 under the same condition, the flow is subcritical.

The table in Appendix A for the geometric elements of a circular section may also be used for the solution of this problem. Since $d_0 = 3.0$ ft and $d_0^{8/3} = 18.75$, $AR^{2/3}/d_0^{8/3} = 5.04/18.75 = 0.269$. From the table, $y/d_0 = 0.72$, or $y = 0.72 \times 3 = 2.16$ ft.

C. Method of Design Chart. The design chart for determining the normal depth (Fig. 6-1) can be used with great expediency.

In Example 6-2, $AR^{2/3} = 167.7$. The value of $AR^{2/3}/b^{8/3}$ is 0.0569. For this value, the chart gives $y/b = 0.168$, or $y_n = 3.36$ ft.

In Example 6-3, $AR^{2/3}/d_0^{8/3} = 0.269$. For this value, the chart gives $y/d_0 = 0.72$, or $y = 0.72 \times 3 = 2.16$ ft.

6-7. Determination of the Normal and Critical Slopes. When the discharge and roughness are given, the Manning formula can be used to determine the slope of a prismatic channel in which the flow is uniform at a given normal depth y_n. The slope thus determined is sometimes called specifically the *normal slope* S_n.

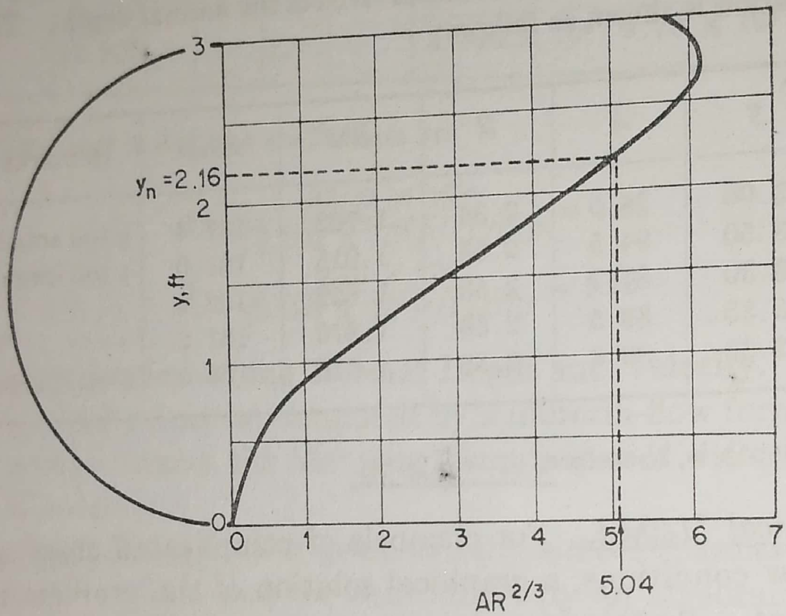

FIG. 6-7. A curve of y vs. $AR^{2/3}$ for a circular section.

By varying the slope of the channel to a certain value, it is possible to change the normal depth and make the uniform flow occur in a critical state for the given discharge and roughness. The slope thus obtained is the *critical slope* S_c, and the corresponding normal depth is equal to the critical depth. The smallest critical slope for a channel of given shape and roughness is called the *limit slope* S_L.

Furthermore, by adjusting the slope and the discharge, a critical uniform flow may be obtained at the given normal depth. The slope thus obtained is known as the *critical slope at the given normal depth* S_{cn}.

The following examples will illustrate the above discussion.

Example 6-4. A trapezoidal channel has a bottom width of 20 ft, side slopes of 2:1, and $n = 0.025$.

a. Determine the normal slope at a normal depth of 3.36 ft when the discharge is 400 cfs.

PROBLEMS

6-1. Determine the normal discharges in channels having the following sections for $y = 6$ ft, $n = 0.015$, and $S = 0.0020$:

a. A rectangular section 20 ft wide
b. A triangular section with a bottom angle equal to 60°
c. A trapezoidal section with a bottom width of 20 ft and side slopes of 1 on 2
d. A circular section 15 ft in diameter
e. A parabolic section having a width of 16 ft at the depth of 4 ft

6-2. Prove the following equation for the discharge in a triangular highway gutter (Fig. 6-10) having one side vertical, one side sloped at 1 on z, Manning's n, depth of flow y, and longitudinal slope S:

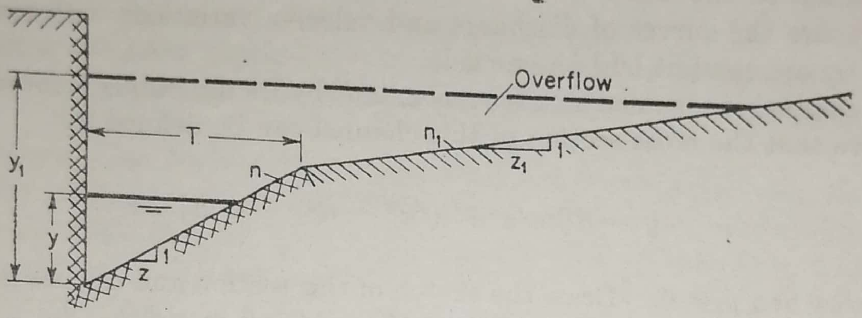

FIG. 6-10. A highway gutter section.

$$Q = \frac{0.47}{n} f(z) y^{8/3} S^{1/2} \tag{6-48}$$

where

$$f(z) = \frac{z^{5/3}}{[1 + \sqrt{1 + z^2}]^{2/3}}$$

6-3. Compute the discharge in the triangular highway gutter described in the preceding problem when $z = 24$, $n = 0.017$, $y = 0.22$ ft, and $S = 0.03$.

6-4. Using the Manning formula, determine the hydraulic exponent N for the following channel sections: (*a*) a very narrow rectangle, (*b*) a very wide rectangle, (*c*) a very wide parabola for which the wetted perimeter is practically equal to the top width, and (*d*) an equilateral triangle with a vertex at the bottom.

6-5. Using the Chézy formula,[1] show that the general equation for the hydraulic exponent N is

$$N = \frac{y}{A}\left(3T - R\frac{dP}{dy}\right) \tag{6-49}$$

6-6. Solve Prob. 6-4 if the determination of the hydraulic exponent is based on the Chézy formula. Compare the results with those obtained in Prob. 6-4.

[1] The G. K. formula shows that Chézy's C is a function of the hydraulic radius and hence of the depth y. Thus, the Chézy formula has not been found very convenient for determination of the N value. For canals in earth and gravelly soil, the N value is generally found to have an increase of 0.30 to 0.50 due to the variation in Chézy's C with respect to the depth. This increase, however, brings the N value closer to that based on the Manning formula.

6-7. Compute the hydraulic exponent N of the trapezoidal channel section (Fig. 2-2) having a normal depth of 6 ft, using (*a*) Eq. (6-15), (*b*) Fig. 6-2, and (*c*) the graphical method based on Eq. (6-16).

6-8. Compute the hydraulic exponent N of a 36-in. circular conduit having a normal depth of 24 in. above the invert, using (*a*) Fig. 6-2 and (*b*) the graphical method based on Eq. (6-16).

6-9. Using the Manning formula, show that the depths for a maximum discharge and velocity in a circular conduit are, respectively, $0.938d_0$ and $0.81d_0$.

6-10. On the basis of the Chézy formula, determine the respective depths for maximum discharge and maximum velocity in a circular conduit.

6-11. At what depths will the maximum discharge and velocity occur in a square conduit laid flat on one side?

6-12. Prepare the curves of discharge and velocity variations with respect to the depth in a square conduit laid on one side.

6-13. A channel is assumed to have a constant hydraulic radius R for any depth of flow. Prove that the cross section of this channel can be defined by

$$y = R[\ln (x + \sqrt{x^2 - R^2}) - \ln R] \tag{6-50}$$

where $x = R$ when $y = 0$. Draw the sketch of this section and discuss its properties. (HINT: From the given condition, $R = A/P = dA/dP = x\,dy/\sqrt{dx^2 + dy^2}$. Solve this differential equation, and evaluate the integration constant by the condition that $x = R$ when $y = 0$. Mathematically, the section is formed by two catenaries as sides. For practical purposes, an artificial bottom should be provided since the theoretical section is bottomless. A uniform-flow formula, such as the Manning formula, indicates that the hydraulic radius is the sole shape parameter for the velocity. The adequacy of this indication can be verified experimentally by testing a channel built of the section of constant hydraulic radius. If the indication is true, then, once this channel is designed for a safe velocity, it should be nonscouring and nonsilting over a wide range of stages. In earthen canals, however, the large variation in water surface during the change of stage would erode the sides very easily.)

6-14. Verify Eqs. (6-17) to (6-19).

6-15. A rectangular testing channel is 2 ft wide and laid on a slope of 0.1035%. When the channel bed and walls were made smooth by neat cement, the measured normal depth of flow was 1.36 ft for a discharge of 8.9 cfs. The same channel was then roughened by cemented sand grains, and thus the measured normal depth became 1.31 ft for a discharge of 5.2 cfs.

a. Determine the discharge for a normal depth of 1.31 ft if the bed were roughened and the walls were kept smooth.

b. Determine the discharge for a normal depth of 1.31 ft if the walls were roughened and the bed were smooth.

c. The discharges for the conditions described in *a* and *b* were actually measured and found to be 6.60 and 6.20 cfs, respectively. Determine the corresponding n values, and compare these values with those computed by Eqs. (6-17) to (6-19).

6-16. A channel consists of a main section and two side sections (Fig. 6-6). Compute the total discharge, assuming that the main section and the two side sections are separated (*a*) by vertical division lines and (*b*) by extended sides of the main channel. Given: $n = 0.025$ for the main channel, $n = 0.030$ for the side channels, and $S = 0.001$.

6-17. The hydrographic survey of a stream indicates that the hydraulic properties of the stream are relatively uniform for a length of over 2 miles. The data obtained by the survey are:

a. The cross section of the stream at a typical upstream station in the uniform reach is given by the following coordinates:

	Station	*Elev. m.s.l.*	*Station*	*Elev. m.s.l.*
Left bank:	0 + 00	590.0	6 + 00	543.7
	1 + 00	580.7	8 + 00	540.0
	1 + 50	578.2	10 + 00	572.2
	3 + 00	582.0	11 + 00	573.2
	4 + 00	581.0	12 + 00	568.5
	5 + 00	580.0	14 + 00	590.0

b. The value of n for the main channel is estimated as 0.035, for the side channels as 0.050.

c. The natural slope of the stream is about 1 ft/mile.

Construct a synthetic rating curve. It is suggested that the water areas of the main channel and the side channels be separated by the extended sides of the main channel.

6-18. Compute the discharge in an overflowed highway gutter (Fig. 6-10) having a depth of flow of 3 in. and a longitudinal slope of 0.03. The gutter is made of concrete with $n = 0.017$ and has a triangular section with a vertical curb side, a sloped side of $z = 12$, and a top width of $T = 2$ ft. The overflowed soil-aggregate pavement has a cross slope of $z_1 = 24$ and $n_1 = 0.020$.

6-19. For an equal amount of discharge, an ice-covered channel should have greater depth of flow than an uncovered channel, for two reasons: (1) the wetted perimeter is greater in an ice-covered channel and thus results in greater resistance or less velocity, and (2) the thickness of the ice cover is greater than a depth of water of equal weight, since the specific gravity of ice is about 0.917. Show that the increase in depth due to resistance in an ice-covered wide open channel may be expressed by

$$\Delta y = \left[1.32\left(\frac{n_1}{n}\right)^{3/5} - 1\right] y \tag{6-51}$$

where n_1 is the roughness coefficient of the channel with ice cover, n is the roughness coefficient of the channel without ice cover, and y is the depth of flow in the channel carrying the same discharge but without ice cover.

6-20. Compute the conveyance and velocity-distribution coefficients of a channel section 500 ft downstream from the section described in Example 6-1. The survey data at the section for the same flood are:

Subsection	A, ft²	P, ft	n	α	β
Main section........	5,320	205	0.035	1.12	1.05
Side section.........	5,670	408	0.040	1.10	1.04

6-21. Solve Example 6-2 by the G. K. Formula.

6-22. A rectangular channel with 20 ft width, $S = 0.006$, and $n = 0.015$, carries a discharge of 200 cfs. Compute the normal depth and velocity.

6-23. Using the Manning formula, determine the normal depths in channels having the following sections when $Q = 100$ cfs, $n = 0.015$, and $S = 0.0020$:

CHAPTER 7

DESIGN OF CHANNELS FOR UNIFORM FLOW

Channels to be discussed in this chapter include nonerodible channels erodible channels, and grassed channels. For erodible channels, the discussion will be limited mostly to those which scour but do not silt (see Preface).

A. NONERODIBLE CHANNELS

7-1. The Nonerodible Channel. Most lined channels and built-up channels can withstand erosion satisfactorily and are therefore considered *nonerodible*. Unlined channels are generally erodible, except those excavated in firm foundations, such as rock bed. In designing nonerodible channels, such factors as the maximum permissible velocity (Art. 7-9) and the permissible tractive force (Art. 7-13) are not the criteria to be considered. The designer simply computes the dimensions of the channel by a uniform-flow formula and then decides the final dimensions on the basis of hydraulic efficiency, or empirical rule of best section, practicability, and economy [1,2]. The factors to be considered in the design are: the kind of material forming the channel body, which determines the roughness coefficient; the minimum permissible velocity, to avoid deposition if the water carries silt or debris; the channel bottom slope and side slopes; the freeboard; and the most efficient section, either hydraulically or empirically determined.

7-2. Nonerodible Material and Lining.[1] The nonerodible materials used to form the lining of a channel and the body of a built-up channel include concrete, stone masonry, steel, cast iron, timber, glass, plastic, etc. The selection of the material depends mainly on the availability and cost of the material, the method of construction, and the purpose for which the channel is to be used.

The purpose of lining a channel is in most cases to prevent erosion, but occasionally it may be to check seepage losses. In lined channels, the *maximum permissible velocity*, i.e., the maximum that will not cause erosion, can be ignored, provided that the water does not carry sand, gravel, or stones. If there are to be very high velocities over a lining, however, it should be remembered that there is a tendency for the rapidly

[1] For detailed information on channel lining, see [3].

moving water to pick up lining blocks and push them out of position. Accordingly, the lining should be designed against such possibilities.

7-3. The Minimum Permissible Velocity. The *minimum permissible velocity*, or the *nonsilting velocity*, is the lowest velocity that will not start sedimentation and induce the growth of aquatic plant and moss. This velocity is very uncertain and its exact value cannot be easily determined. For water carrying no silt load or for desilted flow, this factor has little significance except for its effect on plant growth. Generally speaking, a mean velocity of 2 to 3 fps may be used safely when the percentage of silt present in the channel is small, and a mean velocity of not less than 2.5 fps will prevent a growth of vegetation that would seriously decrease the carrying capacity of the channel.

7-4. Channel Slopes. The longitudinal bottom slope of a channel is generally governed by the topography and the energy head required for the flow of water. In many cases, the slope may depend also on the purpose of the channel. For example, channels used for water-distribution purposes, such as those used in irrigation, water supply, hydraulic mining, and hydropower projects, require a high level at the point of delivery; therefore, a small slope is desirable in order to keep the loss in elevation to a minimum.

The side slopes of a channel depend mainly on the kind of material. Table 7-1 gives a general idea of the slopes suitable for use with various

TABLE 7-1. SUITABLE SIDE SLOPES FOR CHANNELS BUILT IN VARIOUS KINDS OF MATERIALS

Material	*Side slope*
Rock	Nearly vertical
Muck and peat soils	¼:1
Stiff clay or earth with concrete lining	½:1 to 1:1
Earth with stone lining, or earth for large channels	1:1
Firm clay or earth for small ditches	1½:1
Loose sandy earth	2:1
Sandy loam or porous clay	3:1

kinds of material. For erodible material, however, a more accurate determination of the slopes should be checked against the criterion of maximum permissible velocity (Art. 7-10) or by the principle of tractive force (Art. 7-14). Other factors to be considered in determining slopes are method of construction, condition of seepage loss, climatic change, channel size, etc. Generally speaking, side slopes should be made as steep as practicable and should be designed for high hydraulic efficiency and stability. For lined canals, the U.S. Bureau of Reclamation [4] has been considering standardizing on a 1.5:1 slope for the usual sizes of canals. One advantage of this slope is that it is sufficiently flat to

allow the practicable use of just about any type of lining or lining treatment now or in the future anticipated by the Bureau.

7-5. Freeboard. The *freeboard* of a channel is the vertical distance from the top of the channel to the water surface at the design condition. This distance should be sufficient to prevent waves or fluctuations in water surface from overflowing the sides. This factor becomes important particularly in the design of elevated flumes, for the flume substructure may be endangered by any overflow.

There is no universally accepted rule for the determination of freeboard, since wave action or water-surface fluctuation in a channel may be created by many uncontrollable causes. Pronounced waves and fluctuation of water surface are generally expected in channels where the velocity is so high and the slope so steep that the flow becomes very unstable, or on curves where high velocity and large deflection angle may cause appreciable superelevated water surface on the convex side of a curve, or in channels where the velocity of flow approaches the critical state at which the water may flow at alternate depths and thus jump from the low stage to the high stage at the least obstruction. Other natural causes such as wind movement and tidal action may also induce high waves and require special consideration in design.

Freeboards varying from less than 5% to greater than 30% of the depth of flow are commonly used in design. For smooth, interior, semicircular metal flumes on tangents, carrying water at velocities not greater than 80% of the critical velocity with a maximum of 8 fps, experience has indicated that a freeboard of 6% of the flume diameter should be used. For flumes on curves with high velocity or deflections, wave action will be produced; so freeboard must be increased to prevent water from slopping over.

Freeboard in an unlined canal or lateral will normally be governed by considerations of canal size and location, storm-water inflow, and water-table fluctuations caused by checks, wind action, soil characteristics, percolation gradients, operating road requirements, and availability of excavated material. According to the U.S. Bureau of Reclamation [4], the approximate range of freeboard frequently used extends from 1 ft for small laterals with shallow depths to 4 ft in canals of 3,000 cfs or more capacity with relatively large water depths. The Bureau recommends that preliminary estimates of the freeboard required under ordinary conditions be made according to the following formula:

$$F = \sqrt{Cy} \qquad (7\text{-}1)$$

where F is the freeboard in ft, y is the depth of water in the canal in ft, and C is a coefficient varying from 1.5 for a canal capacity of 20 cfs to 2.5 for a canal capacity of 3,000 cfs or more. This approximation is based upon average Bureau practice; it will not, however, serve for all conditions.

For lined canals or laterals, the height of lining above the water surface will depend upon a number of factors: size of canal, velocity of water, curvature of alignment, condition of storm- and drain-water inflow, fluctuations in water level due to operation of flow-regulating structures, and wind action. In a somewhat similar manner, the height of bank above the water surface will vary with size and location of canal, type of soil, amount of intercepted storm or drain water, etc. As a guide for lined-canal design, the U.S. Bureau of Reclamation [3] has prepared curves (Fig. 7-1) for average freeboard and bank heights in relation to capacities.

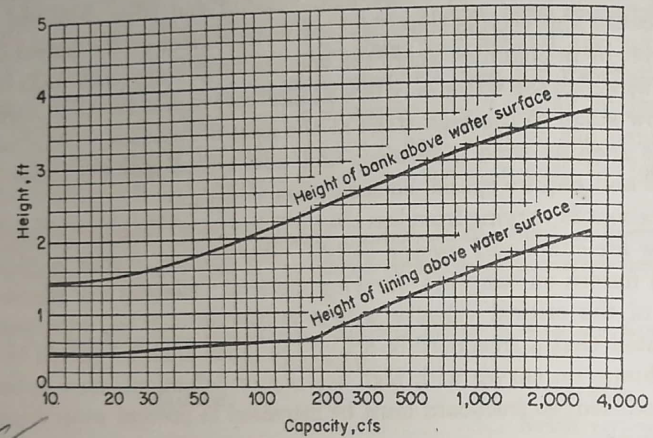

Fig. 7-1. Recommended freeboard and height of bank of lined channels. (*U.S. Bureau of Reclamation.*)

7-6. The Best Hydraulic Section. It is known that the conveyance of a channel section increases with increase in the hydraulic radius or with decrease in the wetted perimeter. From a hydraulic viewpoint, therefore, the channel section having the least wetted perimeter for a given area has the maximum conveyance; such a section is known as the *best hydraulic section*. The semicircle has the least perimeter among all sections with the same area; hence it is the most hydraulically efficient of all sections.

The geometric elements of six best hydraulic sections are listed in Table 7-2, but these sections may not always be practical owing to difficulties in construction and in use of material. In general, a channel section should be designed for the best hydraulic efficiency but should be modified for practicability. From a practical point of view, it should

be noted that a best hydraulic section is the section that gives the minimum area for a given discharge but not necessarily the minimum excavation. The section of minimum excavation occurs only if the water surface is at the level of the bank tops. Where the water surface is below the bank tops, as frequently occurs, channels narrower than those of the best hydraulic section will give minimum excavation. If the water surface overtops the banks and these are even with the ground level, wider channels will provide minimum excavation.

Table 7-2. Best Hydraulic Sections

Cross section	Area A	Wetted perimeter P	Hydraulic radius R	Top width T	Hydraulic depth D	Section factor Z
Trapezoid, half of a hexagon	$\sqrt{3}\,y^2$	$2\sqrt{3}\,y$	$\frac{1}{2}y$	$\frac{4}{3}\sqrt{3}\,y$	$\frac{3}{4}y$	$\frac{3}{2}y^{2.5}$
Rectangle, half of a square	$2y^2$	$4y$	$\frac{1}{2}y$	$2y$	y	$2y^{2.5}$
Triangle, half of a square	y^2	$2\sqrt{2}\,y$	$\frac{1}{4}\sqrt{2}\,y$	$2y$	$\frac{1}{2}y$	$\frac{\sqrt{2}}{2}y^{2.5}$
Semicircle	$\frac{\pi}{2}y^2$	πy	$\frac{1}{2}y$	$2y$	$\frac{\pi}{4}y$	$\frac{\pi}{4}y^{2.5}$
Parabola, $T = 2\sqrt{2}\,y$	$\frac{4}{3}\sqrt{2}\,y^2$	$\frac{8}{3}\sqrt{2}\,y$	$\frac{1}{2}y$	$2\sqrt{2}\,y$	$\frac{2}{3}y$	$\frac{8}{9}\sqrt{3}\,y^{2.5}$
Hydrostatic catenary	$1.39586y^2$	$2.9836y$	$0.46784y$	$1.917532y$	$0.72795y$	$1.19093y^{2.5}$

The principle of the best hydraulic section applies only to the design of nonerodible channels. For erodible channels, the principle of tractive force must be used to determine an efficient section (Art. 7-15).

Example 7-1. Show that the best hydraulic trapezoidal section is one-half of a hexagon.

Solution. Table 2-1 gives the water area and wetted perimeter of a trapezoid as

$$A = (b + zy)y \qquad \text{and} \qquad P = b + 2\sqrt{1 + z^2}\,y$$

where y is the depth, b is the bottom width, and z:1 is the side slope.

First, consider A and z to be constant. Differentiating the above two equations with respect to y and solving simultaneously for dP/dy,

$$\frac{dP}{dy} = 2(\sqrt{1 + z^2} - z) - \frac{b}{y}$$

For a minimum wetted perimeter, $dP/dy = 0$, or

$$b = 2y(\sqrt{1 + z^2} - z)$$

Substituting this equation for b in the previous two equations for A and P and solving simultaneously for P,

$$P = 2\sqrt{A(2\sqrt{1+z^2} - z)}$$

Now, find the value of z that makes P the least. Differentiating P with respect to z, equating dP/dz to zero, and solving for z,

$$z = \frac{\sqrt{3}}{3} = \tan 30°$$

This means that the section is a half hexagon.

7-7. Determination of Section Dimensions. The determination of section dimensions for nonerodible channels includes the following steps:

1. Collect all necessary information, estimate n, and select S.
2. Compute the section factor $AR^{2/3}$ by Eq. (6-8), or

$$AR^{2/3} = \frac{nQ}{1.49\sqrt{S}} \tag{6-8}$$

3. Substitute in Eq. (6-8) the expressions for A and R obtained from Table 2-1, and solve for the depth. If there are other unknowns, such as b and z of a trapezoidal section, then assume the values for these unknowns and solve Eq. (6-8) for the depth. By assuming several values of the unknowns, a number of combinations of section dimensions can be obtained. The final dimensions are decided on the basis of hydraulic efficiency and practicability. For lined canals, the trapezoidal section is commonly adopted, and the U.S. Bureau of Reclamation [3] has developed experience curves (Fig. 7-2) showing the average relation of bottom widths and water depths to canal capacities. These curves can be used as a guide in selecting proper section dimensions.

The determination of the depth for the computed value of $AR^{2/3}$ can be simplified by use of the design chart (Fig. 6-1). Some engineers prefer a solution by trial and error, similar to Solution 2 for Example 6-2 of Art. 6-6.

4. If the best hydraulic section is required directly, substitute in Eq. (6-8) the expressions for A and R obtained from Table 7-2 and solve for the depth. This best hydraulic section may be modified for practicability.
5. For the design of irrigation channels, the channel section is sometimes proportioned by empirical rules such as the simple rule given by the early U.S. Reclamation Service [5] for the full supply depth of water in feet.

$$y = 0.5\sqrt{A} \tag{7-2}$$

where A is the water area in ft². For a trapezoidal section it can be shown that this rule may also be expressed by a simple formula

$$x = 4 - z \tag{7-3}$$

where x is the width-depth ratio b/y and z is the horizontal projection of the side slope corresponding to 1 ft vertical. Similarly, engineers in India [6] have used an empirical formula $y = \sqrt{A/3} = 0.577\sqrt{A}$, which is equivalent to $x = 3 - z$ for trapezoidal sections; and Philippine engineers [7] use Eq. (7-3) with $z = 1.5$, or $x = 2.5$, for earth canals.

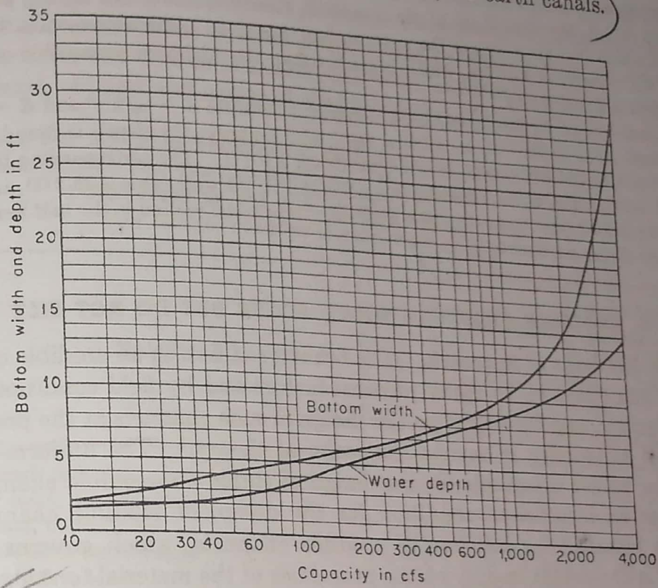

FIG. 7-2. Experience curves showing bottom width and depth of lined channels. (*U.S. Bureau of Reclamation.*)

6. Check the minimum permissible velocity if the water carries silt.
7. Add a proper freeboard to the depth of the channel section.

Example 7-2. A trapezoidal channel carrying 400 cfs is built with nonerodible bed having a slope of 0.0016 and $n = 0.025$. Proportion the section dimensions.

Solution. By Eq. (6-8),

$$AR^{2/3} = \frac{0.025 \times 400}{1.49\sqrt{0.0016}} = 167.7$$

Substituting $A = (b + zy)y$ and $R = (b + zy)y/(b + 2\sqrt{1+z^2}\,y)$ in the above expression,

$$\frac{[(b + zy)y]^{5/3}}{(b + 2\sqrt{1+z^2}\,y)^{2/3}} = 167.7$$

Assuming $b = 20$ ft and $z = 2$ and simplifying,

$$7{,}680 + 1{,}720y = [y(10 + y)]^{2.5}$$
$$y = 3.36 \text{ ft}$$

It should be noted that this solution is exactly the same as the computation of the normal depth given in Solution 1 of Example 6-2. Accordingly, the solutions by trial and error and by the graphical method described in Example 6-2 can also be applied to the present problem.

Similarly, assume other suitable values of b and z, and compute the corresponding depths. The final decision on dimensions will depend on practical considerations. If the values of b and z are decided at the beginning of the computation, the depth will be computed only once.

Suppose that $b = 20$ ft, $z = 2$, and $y = 3.36$ ft are the final values. Assign a free-board of 2 ft; the total depth of the channel is, therefore, 5.36 ft and the top width of the channel (not the width of the water surface) is 41.4 ft. The water area is 89.8 ft^2, and the velocity is 4.46 fps, which is greater than the minimum permissible velocity for inducing silt, if any.

When the best hydraulic section is required, substitute $A = \sqrt{3}\, y^2$ and $R = 0.5y$, obtained from Table 7-2, in $AR^{2/3} = 167.7$ and simplify; the depth is found to be $y = 6.6$ ft. Add 3 ft freeboard; the total depth is 9.6 ft. The corresponding bottom width is 7.6 ft, the top width of the channel is 18.7 ft, the water area is 75.2 ft^2, and the velocity is 5.32 fps. Since the best hydraulic trapezoidal section is the half hexagon, the side slopes are 1 on $\sqrt{3}/3$.

B. ERODIBLE CHANNELS WHICH SCOUR BUT DO NOT SILT

7-8. Methods of Approach. The behavior of flow in an erodible channel is influenced by so many physical factors and by field conditions so complex and uncertain that precise design of such channels at the present stage of knowledge is beyond the realm of theory.[1] The uniform-flow formula, which is suitable for the design of stable nonerodible channels, provides an insufficient condition for the design of erodible channels. This is because the stability of erodible channels, which governs the design, is dependent mainly on the properties of the material forming the channel body, rather than only on the hydraulics of the flow in the channel. Only after a stable section of the erodible channel is obtained can the uniform-flow formula be used for computing the velocity of flow and discharge.

Two methods of approach to the proper design of erodible channels are described here: the *method of permissible velocity* and the *method of tractive force*. The method of permissible velocity has been used extensively for the design of earth canals in the United States to ensure freedom from scour. The method of tractive force has sometimes been used in Europe; it is now under comprehensive investigation by the U.S. Bureau

[1] It has been noticed that certain channels are erodible whereas others very similar in channel geometry, hydraulics, and soil physical properties are not. As a further step in investigation, the chemical properties of the material forming the channel body should be explored. It may be that an ion exchange between water and soil or hydration of the material is providing a binder in some places and thus affecting the erosion. For a general discussion of the complexity of this problem, see [8] and [9].

1. For the given kind of material forming the channel body, estimate the roughness coefficient n (Art. 5-7), side slope z (Table 7-1), and the maximum permissible velocity V (Table 7-3 and Figs. 7-3 to 7-5).
2. Compute the hydraulic radius R by the Manning formula.
3. Compute the water area required by the given discharge and permissible velocity, or $A = Q/V$.
4. Compute the wetted perimeter, or $P = A/R$.
5. Using the expressions for A and P from Table 2-1, solve simultaneously for b and y. The solution may be expedited by using the charts given in Appendix B.
6. Add a proper freeboard, and modify the section for practicability.

Example 7-3. Compute the bottom width and the depth of flow of a trapezoidal channel laid on a slope of 0.0016 and carrying a design discharge of 400 cfs. The channel is to be excavated in earth containing noncolloidal coarse gravels and pebbles.

Solution. For the given conditions, the following are estimated: $n = 0.025$, $z = 2$, and maximum permissible velocity $= 4.5$ fps.

Using the Manning formula, solve for R.

$$4.5 = \frac{1.49}{0.025} R^{2/3} \sqrt{0.0016}$$

or

$$R = 2.60 \text{ ft}$$

Then $A = 400/4.5 = 88.8$ ft², and $P = A/R = 88.8/2.60 = 34.2$ ft. Now

$$A = (b + zy)y = (b + 2y)y = 88.8 \text{ ft}^2$$

and

$$P = b + 2\sqrt{1 + z^2}\, y = (b + 2\sqrt{5}\, y) = 34.2 \text{ ft}$$

Solving the above two equations simultaneously, $b = 18.7$ ft and $y = 3.46$ ft.

Next class

movable channel bed

7-11. The Tractive Force. When water flows in a channel, a force is developed that acts in the direction of flow on the channel bed. This force, which is simply the pull of water on the wetted area, is known as the *tractive force*.[1] In a uniform flow the tractive force is apparently equal to the effective component of the gravity force acting on the body of water, parallel to the channel bottom and equal to $wALS$, where w is the unit weight of water, A is the wetted area, L is the length of the channel reach, and S is the slope (Art. 5-4). Thus, the average value of the tractive force per unit wetted area, or the so-called *unit tractive force* τ_0, is equal to $wALS/PL = wRS$, where P is the wetted perimeter and R is the hydraulic radius; that is

$$\tau_0 = wRS \tag{7-5}$$

In a wide open channel, the hydraulic radius is equal to the depth of flow y; hence $\tau_0 = wyS$.

[1] This is also known as the *shear force* or the *drag force*. The idea of tractive force is generally believed to have been first introduced into hydraulic literature by du Boys in 1879 [p. 149 of 30]. However, the principle of balancing this force with the channel resistance in a uniform flow was stated by Brahms early in 1754 (see Art. 5-4).

It should be noted that the unit tractive force in channels, except for wide open channels, is not uniformly distributed along the wetted perimeter. Many attempts have been made to determine the distribution of the tractive force in a channel. Leighly [31] attempted to determine this distribution in many trapezoidal and several rectangular and triangular channels from the published data on the velocity distribution in the

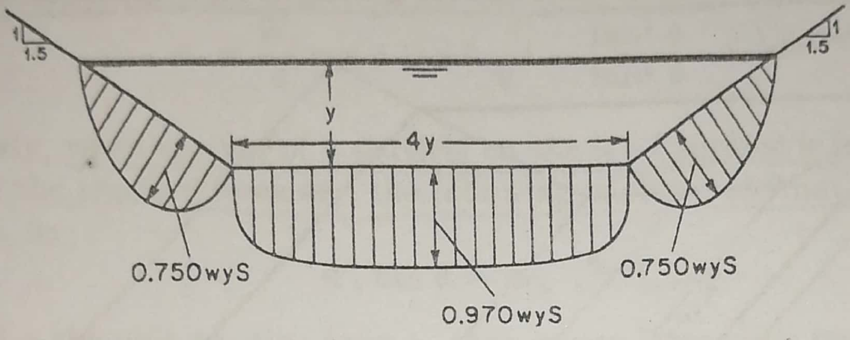

FIG. 7-6. Distribution of tractive force in a trapezoidal channel section.

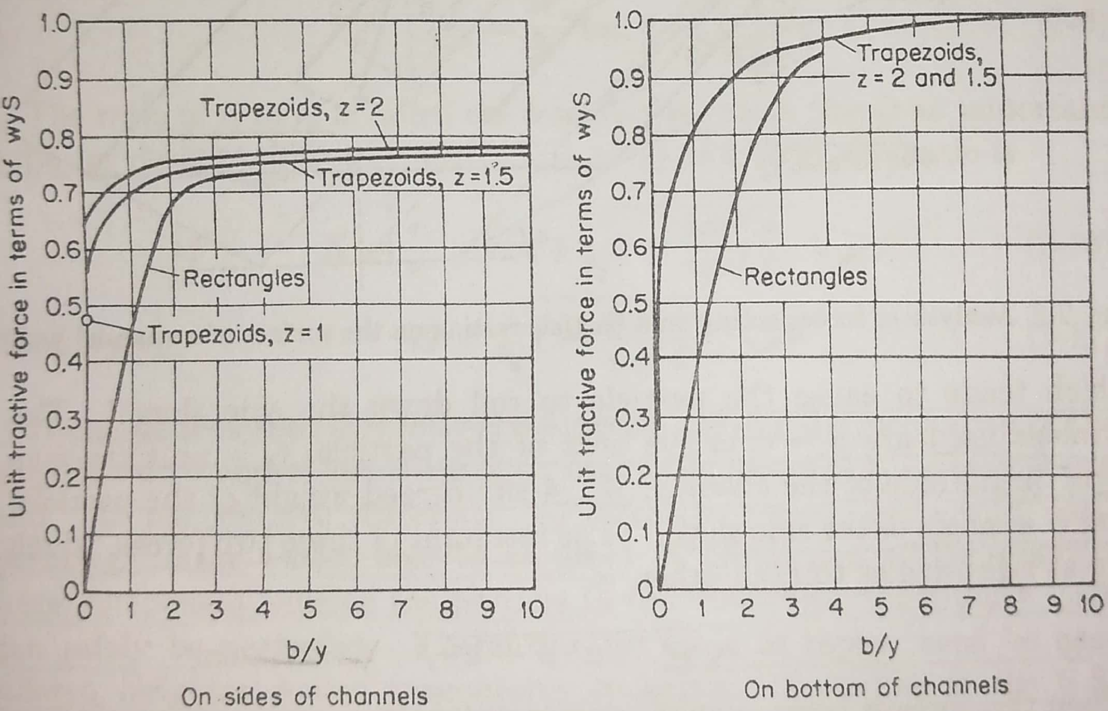

FIG. 7-7. Maximum unit tractive forces in terms of wyS.

channels. Unfortunately, owing to deficiency of data, the results of his study were not very conclusive. In the U.S. Bureau of Reclamation, Olsen and Florey [32] and other engineers have used the membrane analogy and analytical and finite-difference methods for determining the distribution of tractive force in trapezoidal, rectangular, and triangular channels. A typical distribution of tractive force in a trapezoidal channel resulting from the membrane-analogy study is shown in Fig. 7-6. The pattern of distribution varies with the shape of the section but is practically unaffected by the size of the section. Based on such studies, curves (Fig. 7-7) showing the maximum unit tractive forces on the sides and bottom of various channel sections have been prepared for use in canal design. Generally speaking, for trapezoidal channels of the shapes ordinarily used in canals, the maximum tractive force on the bottom is close to the value wyS, and on the sides close to $0.76\ wyS$.

7-12. Tractive-force Ratio. On a soil particle resting on the sloping side of a channel section (Fig. 7-8) in which water is flowing, two forces are acting: the tractive force $a\tau_s$ and the gravity-force component $W_s \sin \phi$,

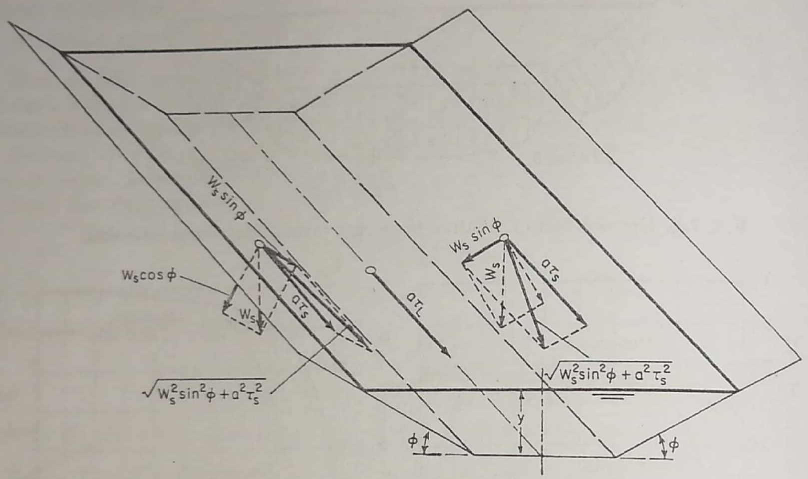

FIG. 7-8. Analysis of forces acting on a particle resting on the surface of a channel bed.

which tends to cause the particle to roll down the side slope.[1] The symbols used are a = effective area of the particle, τ_s = unit tractive force on the side of the channel, W_s = submerged weight of the particle, and ϕ = angle of the side slope. The resultant of these two forces, which are at right angles to each other, is

$$\sqrt{W_s^2 \sin^2 \phi + a^2 \tau_s^2}$$

When this force is large enough, the particle will move.

By the principle of frictional motion in mechanics, it may be assumed that, when motion is impending, the resistance to motion of the particle

[1] The concept of the three-dimensional analysis of the gravity and tractive forces acting on a particle resting on a slope at the state of impending motion was first given by Forchheimer [33]. A complete analysis of a channel section using this concept was first developed by Chia-Hwa Fan [34]. The analysis was also developed independently by the U.S. Bureau of Reclamation under the direction of E. W. Lane [29,35].

is equal to the force tending to cause the motion. The resistance to motion of the particle is equal to the normal force $W_s \cos \phi$ multiplied by the coefficient of friction, or $\tan \theta$, where θ is the angle of repose. Hence,

$$W_s \cos \phi \tan \theta = \sqrt{W_s^2 \sin^2 \phi + a^2 \tau_s^2} \qquad (7\text{-}6)$$

Solving for the unit tractive force τ_s that causes impending motion on a sloping surface,

$$\tau_s = \frac{W_s}{a} \cos \phi \tan \theta \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \theta}} \qquad (7\text{-}7)$$

Similarly, when motion of a particle on the level surface is impending owing to the tractive force $a\tau_L$, the following is obtained from Eq. (7-6) with $\phi = 0$:

$$W_s \tan \theta = a\tau_L \qquad (7\text{-}8)$$

Solving for the unit tractive force τ_L that causes impending motion on a level surface,

$$\tau_L = \frac{W_s}{a} \tan \theta \qquad (7\text{-}9)$$

The ratio of τ_s to τ_L is called the *tractive-force ratio;* this is an important ratio for design purposes. From Eqs. (7-7) and (7-9), the ratio is

$$K = \frac{\tau_s}{\tau_L} = \cos \phi \sqrt{1 - \frac{\tan^2 \phi}{\tan^2 \theta}} \qquad (7\text{-}10)$$

Simplifying,[1]

$$K = \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \theta}} \qquad (7\text{-}11)$$

It can be seen that this ratio is a function only of the inclination of the sloping side ϕ and of the angle of repose of the material θ. For cohesive and fine noncohesive materials, the cohesive forces, even with comparatively clear water, become so great in proportion to the gravity-force component causing the particle to roll down that the gravity force can safely be neglected. Therefore, the angle of repose need be considered only for coarse noncohesive materials. According to the U.S. Bureau of Reclamation's investigation, it was found in general that the angle of repose increases with both size and angularity of the material. For use in design, curves (Fig. 7-9) were prepared by the Bureau, showing values of the angle of repose for noncohesive material above 0.2 in. in diameter for various degrees of roughness. The diameter referred to is the diameter of a particle than which 25% (by weight) of the material is larger.

[1] Equation (7-10) was presented by the U.S. Bureau of Reclamation [35,36] and Eq. (7-11) by Fan [34]. The two equations are mathematically identical.

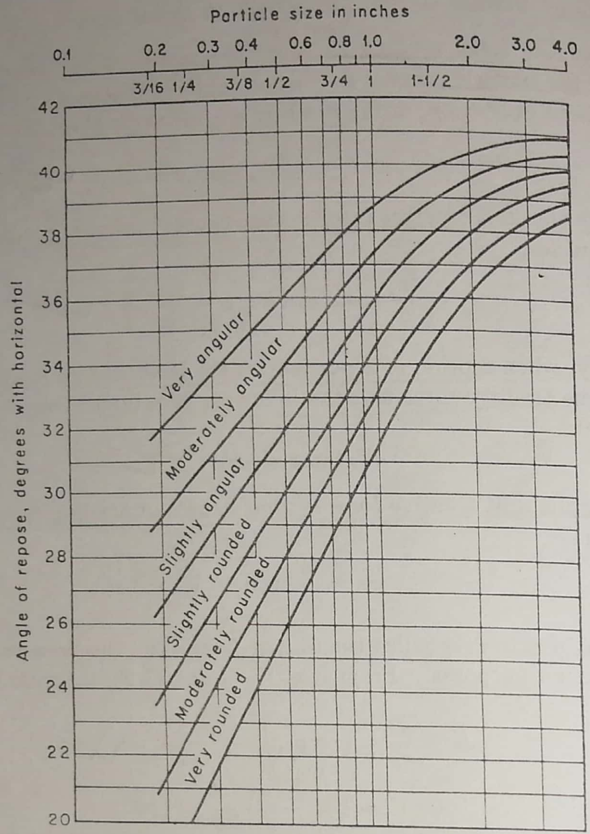

FIG. 7-9. Angles of repose of noncohesive material. (*U.S. Bureau of Reclamation.*)

7-13. Permissible Tractive Force. The *permissible tractive force* is the maximum unit tractive force that will not cause serious erosion of the material forming the channel bed on a level surface. This unit tractive force can be determined by laboratory experiments, and the value thus obtained is known as the *critical tractive force*. However, experience has shown that actual canals in coarse noncohesive material can stand substantially higher values than the critical tractive forces measured in the laboratory. This is probably because the water and soil in actual canals contain slight amounts of colloidal and organic matter which provide a binding power and also because slight movement of soil particles can be tolerated in practical designs without endangering channel stability. Since the permissible tractive force is the design criterion for field conditions, the permissible value may be taken less than the critical value.

The determination of permissible tractive force is now based upon

particle size for noncohesive material and upon compactness or voids ratio for cohesive material. Other soil properties such as the plasticity index[1] or the chemical action may probably also be taken as indexes for defining permissible tractive force more precisely. However, sufficient data and information on these indexes are lacking. The U.S. Bureau of Reclamation has made a comprehensive study of the problem, using data for coarse noncohesive material obtained from the San Luis Valley

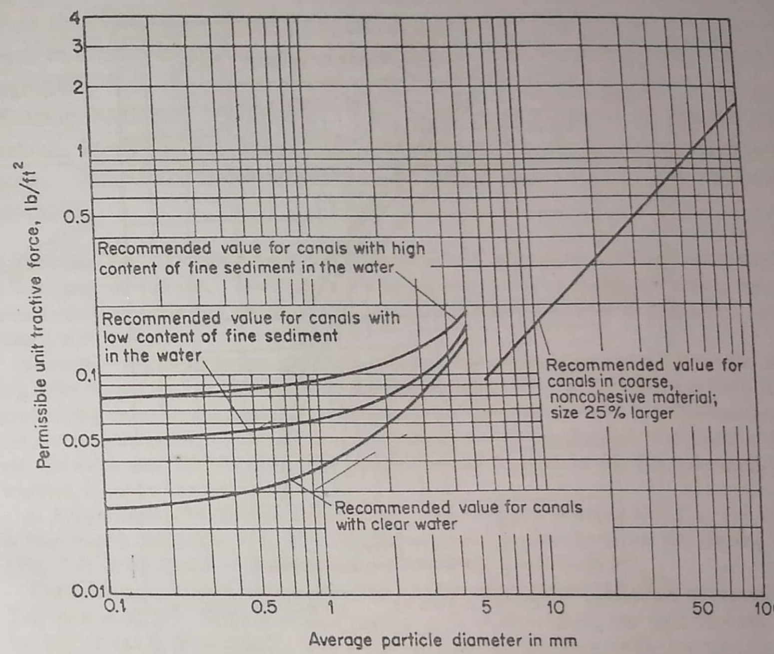

FIG. 7-10. Recommended permissible unit tractive forces for canals in noncohesive material. (*U.S. Bureau of Reclamation.*)

canals [37], values converted from permissible velocities, given by Etcheverry and by Fortier and Scobey, the U.S.S.R. values, etc. (Art. 7-9). As a result, values of permissible tractive force recommended for canal design were developed as follows:

[1] The plasticity index is the difference in per cent of moisture between plastic limit and liquid limit in Atterberg soil tests. This index has been investigated by the U.S. Bureau of Reclamation as a soil characteristic that can be used to indicate resistance to scour for cohesive materials. For canal design, a plasticity index of 7 may be taken tentatively as the critical value, with scour occurring for moderate tractive forces below this value. However, scours are still observed in many cases where the index is above 7. Research shows that determination of the plasticity index in conjunction with consolidated-shear tests may possibly be necessary.

For *coarse* noncohesive material, with sufficient factor of safety, the Bureau recommends tentatively a value of permissible tractive force in pounds per square foot equal to 0.4 times the diameter in inches of a particle than which 25% (by weight) of the material is larger. This recommendation is shown by the straight line in the design chart (Fig. 7-10).

For *fine* noncohesive material, the size specified is the median size, or size smaller than 50% of the weight. Three design curves (Fig. 7-10) are tentatively recommended (1) for canals with high content of fine sediment in the water, (2) for canals with low content of fine sediment in the water, and (3) for canals with clear water.

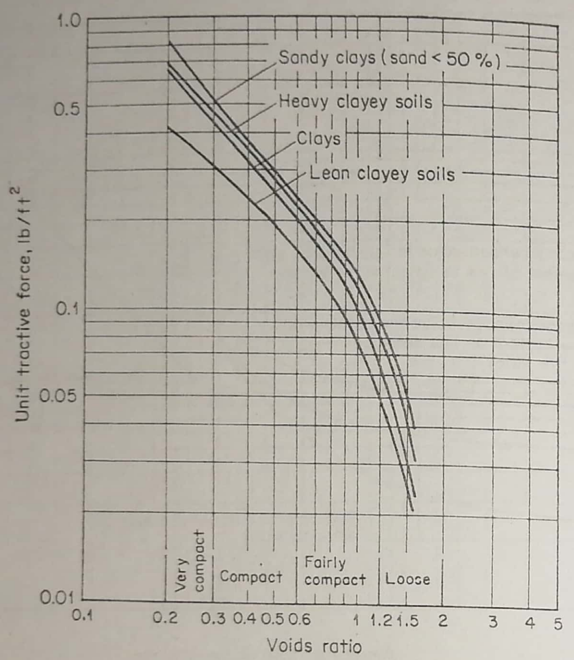

FIG. 7-11. Permissible unit tractive forces for canals in cohesive material as converted from the U.S.S.R. data on permissible velocities.

For cohesive materials, the data based on conversion of permissible velocities to unit tractive forces and given in Table 7-3 and Fig. 7-11 are recommended as design references.

The permissible tractive forces mentioned above refer to straight channels. For sinuous channels, the values should be lowered in order to reduce scour. Approximate percentages of reduction, suggested by Lane [29], are 10% for slightly sinuous canals, 25% for moderately sinuous canals, and 40% for very sinuous canals.

7-14. Method of Tractive Force. The first step in the design of erodible channels by the method of tractive force consists in selecting an approximate channel section by experience or from design tables,[1] collecting samples of the material forming the channel bed, and determining the required properties of the samples. With these data, the designer investigates the section by applying tractive-force analysis to ascertain probable stability by reaches and to determine the minimum section that appears stable. For channels in noncohesive materials the rolling-down effect should be considered in addition to the effect of the distribution of tractive forces; for channels in cohesive material the rolling-down effect is negligible, and the effect of the distribution of tractive force alone is a criterion sufficient for design. The final proportioning of the channel section, however, will depend on other nonhydraulic practical considerations. The analysis for tractive force is best described by the following example:

Example 7-4. Design a trapezoidal channel laid on a slope of 0.0016 and carrying a discharge of 400 cfs. The channel is to be excavated in earth containing noncolloidal coarse gravels and pebbles, 25% of which is 1.25 in. or over in diameter. Manning's $n = 0.025$.

Solution. For trapezoidal channels, the maximum unit tractive force on the sloping sides is usually less than that on the bottom (Fig. 7-7); hence, the side force is the controlling value in the analysis. The design of the channel should therefore include (*a*) the proportioning of the section dimensions for the maximum unit tractive force on the sides and (*b*) checking the proportioned dimensions for the maximum unit tractive force on the bottom.

a. Proportioning the Section Dimensions. Assuming side slopes of 2:1, or $z = 2$, and a base-depth ratio $b/y = 5$, the maximum unit tractive force on the sloping sides (Fig. 7-7) is $0.775wyS = 0.775 \times 62.4 \times 0.0016y = 0.078y$ lb/ft².

Considering a very rounded material 1.25 in. in diameter, the angle of repose (Fig. 7-9) is $\theta = 33.5°$. With $\theta = 33.5°$ and $z = 2$, or $\phi = 26.5°$, the tractive-force ratio by Eq. (7-11) is $K = 0.587$. For a size of 1.25 in., the permissible tractive force on a level bottom is $\tau_L = 0.4 \times 1.25 = 0.5$ lb/ft² (same from Fig. 7-10), and the permissible tractive force on the sides is $\tau_s = 0.587 \times 0.5 = 0.294$ lb/ft².

For a state of impending motion of the particles on side slopes, $0.078y = 0.294$, or $y = 3.77$ ft. Accordingly, the bottom width is $b = 3.77 \times 5 = 18.85$ ft. For this trapezoidal section, $A = 99.5$ ft² and $R = 2.79$ ft. With $n = 0.025$ and $S = 0.0016$, the discharge by the Manning formula is 470 cfs. Further computation will show that, for $z = 2$ and $b/y = 4.1$, the section dimensions are $y = 3.82$ ft and $b = 15.66$ ft and that the discharge is 414 cfs, which is close to the design discharge.

Alternative section dimensions may be obtained by assuming other values of z or side slopes.

b. Checking the Proportioned Dimensions. With $z = 2$ and $b/y = 4.1$, the maximum unit tractive force on the channel bottom (Fig. 7-7) is $0.97wyS = 0.97 \times 62.4 \times 3.82 \times 0.0016 = 0.370$ lb/ft², less than 0.5 lb/ft², which is the permissible tractive force on the level bottom.

[1] Typical average earth sections of irrigation canals and laterals, constructed or proposed by the U.S. Bureau of Reclamation and selected for the flows required on the basis of economy and stability, are given in Fig. 5, paragraph 1.12C, of [4].

FLOW IN OPEN CHANNELS

Flow in Open Channels

Second Edition

K Subramanya
Formerly, Professor
Department of Civil Engineering
Indian Institute of Technology
Kanpur

Nisha
1700082

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(artificial) channels are prismatic channels over long stretches. The rectangle, trapezoid, triangle and circle are some of the commonly-used shapes in man-made channels. All natural channels generally have varying cross-sections and consequently are non-prismatic.

Rigid and Mobile Boundary Channels

On the basis of the nature of the boundary open channels can be broadly classified into two types: (i) rigid channels and (ii) mobile boundary channels.

Rigid channels are those in which the boundary is not deformable in the sense that the shape, planiform and roughness magnitudes are not functions of the flow parameters. Typical examples include lined canals, sewers and non-erodible unlined canals. The flow velocity and shear-stress distribution will be such that no major scour, erosion or deposition takes place in the channel and the channel geometry and roughness are essentially constant with respect to time. The rigid channels can be considered to have only one degree of freedom; for a given channel geometry the only change that may take place is the depth of flow which may vary with space and time depending upon the nature of the flow. This book is concerned essentially with the study of rigid boundary channels.

In contrast to the above, we have many unlined channels in alluvium—both man-made channels and natural rivers—in which the boundaries undergo deformation due to the continuous process of erosion and deposition due to the flow. The boundary of the channel is mobile in such cases and the flow carries considerable amounts of sediment through suspension and in contact with the bed. Such channels are classified as mobile-boundary channels. The resistance to flow, quantity of sediment transported, channel geometry and planiform, all depend on the interaction of the flow with the channel boundaries. A general mobile-boundary channel can be considered to have four degrees of freedom. For a given channel not only the depth of flow but also the bed width, longitudinal slope and planiform (or layout) of the channel may undergo changes with space and time depending on the type of flow. Mobile-boundary channels, usually treated under the topic of *sediment transport* or *sediment engineering*[1,2] attract considerable attention of the hydraulic engineer and their study constitutes a major area of multi-disciplinary interest.

Mobile-boundary channels are dealt with briefly in Chapter 11. The discussion in rest of the book is confined to rigid-boundary open channels only. Unless specifically stated, the term *channel* is used in this book to mean the rigid—boundary channels.

1.3 CLASSIFICATION OF FLOWS

Steady and Unsteady Flows

A steady flow occurs when the flow properties, such as the depth or discharge

at a section do not change with time. As a corollary, if the depth or discharge changes with time the flow is termed *unsteady*.

In practical applications due to the turbulent nature of the flow and also due to the interaction of various forces, such as wind, surface tension, etc. at the surface there will always be some fluctuations of the flow properties with respect to time. To account for these the definition of steady flow is somewhat generalised and the classification is done on the basis of gross characteristics of the flow. Thus, for example, if there are ripples resulting in small fluctuations of depth in a canal due to wind blowing over the free surface and if the nature of the water-surface profile due to the action of an obstruction is to be studied, the flow is not termed unsteady. In this case, a time average of depth taken over a sufficiently long time interval would indicate a constant depth at a section and as such for the study of gross characteristics the flow would be taken as steady. However, if the characteristics of the ripples were to be studied, certainly an unsteady wave movement at the surface is warranted. Similarly, a depth or discharge slowly varying with respect to time may be approximated for certain calculations to be steady over short time intervals.

Flood flows in rivers and rapidly-varying surges in canals are some examples of unsteady flows. Unsteady flows are considerably more difficult to analyse than steady flows. Fortunately, a large number of open channel problems encountered in practice can be treated as steady-state situations to obtain meaningful results. A substantial portion of this book deals with steady-state flows and only a few relatively simple cases of unsteady-flow problems are presented in Chapter 10.

Uniform and Non-uniform Flows

If the flow properties, say the depth of flow, in an open channel remain constant along the length of the channel, the flow is said to be *uniform*. As a corollary of this, a flow in which the flow properties vary along the channel is termed as *non-uniform flow* or *varied flow*.

A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow (Fig. 1.1(a)). In this case the depth of flow will be constant along the channel length and hence the free surface will be parallel to the bed. It is easy to see that an unsteady uniform flow is practically an impossibility and hence the term uniform flow is used to mean *steady uniform flow*.

Flow in a non-prismatic channel and flow with varying velocities in a prismatic channel are examples of *varied flow*. Varied flow can be either steady or unsteady.

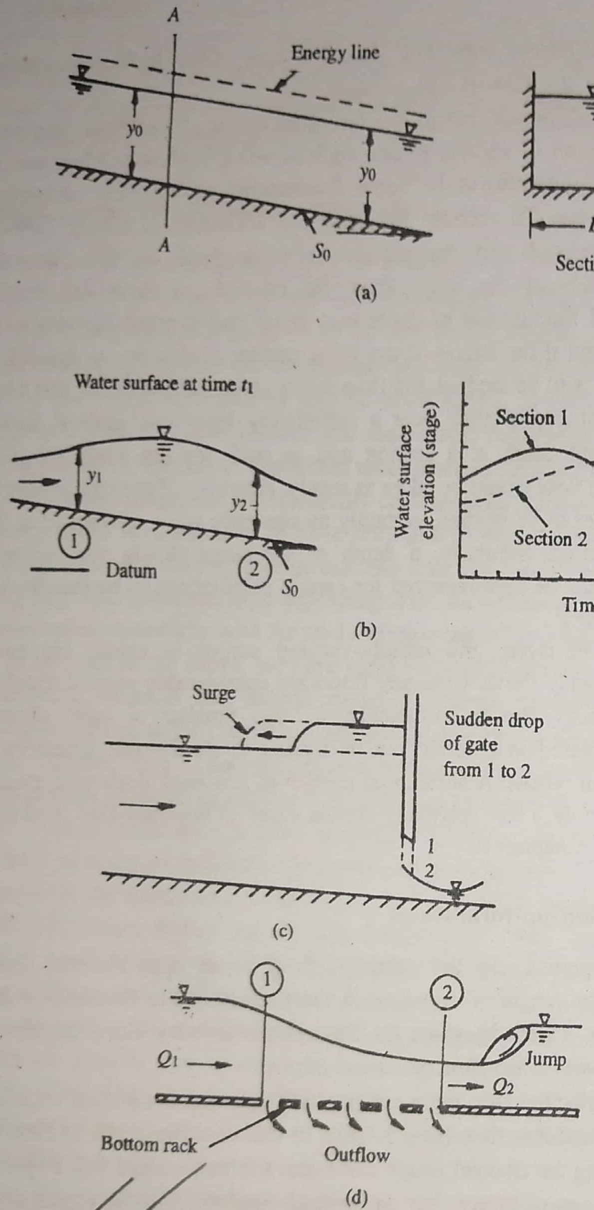


Fig. 1.1 Various types of open channel flows

(a) Uniform flow

(b) Gradually-varied unsteady flow

(c) Rapidly-varied unsteady flow

(d) Spatially-varied flow

Gradually-varied and Rapidly-varied Flows

If the change of depth in a varied flow is gradual so that the curvature of streamlines is not excessive, such a flow is said to be a *gradually-varied flow* (GVF). Frictional resistance plays an important role in these flows. The backing up of water in a stream due to a dam or drooping of the water surface due to a sudden drop in a canal bed are examples of steady GVF. The passage of a flood wave in a river is a case of unsteady GVF (Fig. 1.1(b)).

If the curvature in a varied flow is large and the depth changes appreciably over short lengths, such a phenomenon is termed as *rapidly-varied flow* (RVF). The frictional resistance is relatively insignificant in such cases and it is usual to regard RVF as a local phenomenon. A hydraulic jump occurring below a spillway or a sluice gate is an example of steady RVF. A surge moving up a canal (Fig. 1.1(c)) and a bore travelling up a river are examples of unsteady RVF.

Spatially-varied Flow

Varied flow classified as GVF and RVF assumes that no flow is externally added to or taken out of the canal system. The volume of water in a known time interval is conserved in the channel system. In steady-varied flow the discharge is constant at all sections. However, if some flow is added to or abstracted from the system the resulting varied flow is known as a *spatially varied flow* (SVF).

SVF can be steady or unsteady. In the steady SVF the discharge while being steady–varies along the channel length. The flow over a bottom rack is an example of steady SVF (Fig. 1.1(d)). The production of surface runoff due to rainfall, known as overland flow, is a typical example of unsteady SVF.

Classification Thus open channel flows are classified for purposes of identification and analysis as follows.

Fig. 1.1 (a) through (d) shows some typical examples of the above types of flows

- Open channel-flow
  - Steady
    - Uniform
    - Gradually-varied (GVF)
    - Rapidly-varied (RVF)
    - Spatially-varied (SVF)
  - Unsteady
    - Gradually-varied (GVUF)
    - Rapidly-varied (RVUF)
    - Spatially-varied (SVUF)

1.4 VELOCITY DISTRIBUTION

The presence of corners and boundaries in an open channel causes the velocity vectors of the flow to have components not only in the longitudinal direction but also in the lateral as well as normal direction to the flow. In a macro-analysis, one is concerned only with the major component, viz. the longitudinal component, v_x. The other two components being small are ignored and v_x is designated as v. The distribution of v in a channel is dependent on the geometry of the channel. Figure 1.2(a) and (b) show isovels (contours of equal velocity) of v for a natural and rectangular channel respectively. The influence of the channel geometry is apparent. The velocity v is zero at the solid boundaries and gradually increases with distance from the boundary. The maximum velocity of the cross-section occurs at a certain distance below the free surface. This dip of the maximum velocity point, giving surface velocities which are less than the maximum velocity, is due to secondary currents and is a function of the aspect

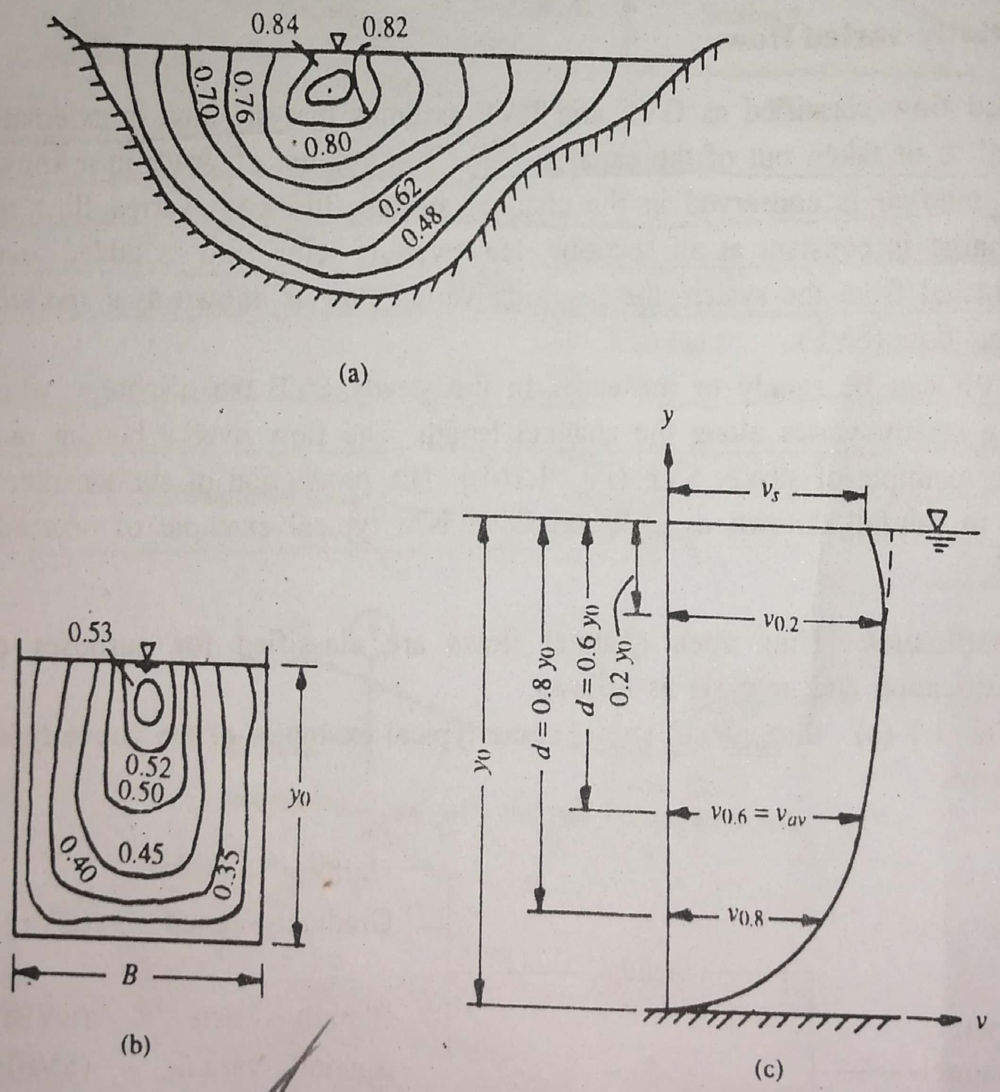


Fig. 1.2 Velocity distribution in open channels:
(a) Natural channel
(b) Rectangular channel
(c) Typical velocity profile

...critical depth for a specific energy head of 1.5 m in the following channels:

(a) Rectangular channel, $B = 2.0$ m
(b) Triangular channel, $m = 1.5$
(c) Trapezoidal channel, $B = 2.0$ m and $m = 1.0$
(d) Circular channel, $D = 1.50$ m

Solution

(a) *Rectangular Channel*

By Eq. (2.10)

$$E_c = \frac{3}{2} y_c = 1.50\text{m}$$

$$y_c = \frac{1.50 \times 2}{3} = 1.00\text{m}$$

(b) *Triangular Channel*

By Eq. (2.15)

$$E_c = 1.25\, y_c = 1.50 \text{ m}$$

$$y_c = \frac{1.50}{1.25} = 1.20\text{m}$$

(c) *Trapezoidal Channel*

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{Q^2}{2g A_c^2}$$

Since by Eq. (2.4a) $\frac{Q^2}{g} = A_c^3 / T_c$, $E_c = y_c + \frac{A_c}{2T_c}$

$$1.5 = y_c + \frac{(2.0 + y_c) y_c}{2(2.0 + 2y_c)}$$

Solving by trial and error, $y_c = 1.095$ m.

(d) *Circular Channel*

$$E_c = y_c + \frac{A_c}{2T_c}$$

By non-dimensionalising with respect to the diameter D.

$$\frac{y_c}{D} + \frac{(A_c / D^2)}{2(T_c / D)} = \frac{E_c}{D} = \frac{1.5}{1.5} = 1.0$$

From Table 2A.1, values of (A_c/D^2) and (T_c/D) for a chosen (y_c/D) are read and a trial and error procedure is adopted to solve for y_c/D. It is found that

$$\frac{y_c}{D} = 0.69 \quad \text{and} \quad y_c = 0.69 \times 1.50 = 1.035\text{m}$$

3-8-21

2.7 TRANSITIONS

The concepts of specific energy and critical depth are extremely useful in the analysis of problems connected with transitions. To illustrate the various aspects, a few simple transitions in rectangular channels are presented here. The principles are nevertheless equally applicable to channels of any shape and other types of transitions.

2.7.1 Channel with a Hump

(a) *Subcritical Flow*

Consider a horizontal, frictionless rectangular channel of width B carrying Q at a depth y_1.

Let the flow be subcritical. At a section 2 (Fig. 2.9) a smooth hump of height ΔZ is built on the floor. Since there are no energy losses between sections 1 and 2, and construction of a hump causes the specific energy at section 2 to decrease by ΔZ. Thus the specific energies at sections 1 and 2 are given by

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

and

$$E_2 = E_1 - \Delta Z \qquad (2.28)$$

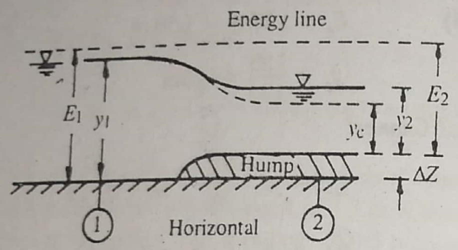


Fig. 2.9 Channel transition with a hump

Since the flow is subcritical, the water surface will drop due to a decrease in the specific energy. In Fig. 2.10, the water surface which was at P at section 1 will come down to point R at section 2. The depth y_2 will be given by

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2gB^2y_2^2} \qquad (2.29)$$

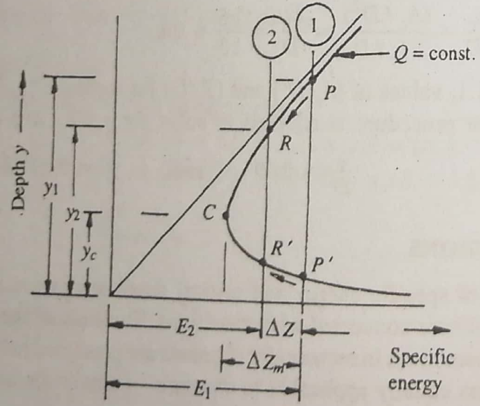


Fig. 2.10 Specific energy diagram for Fig. 2.9

It is easy to see from Fig. 2.10 that as the value of ΔZ is increased, the depth at section 2, i.e. y_2, will decrease. The minimum depth is reached when the point R coincides with C, the critical-depth point. At this point the hump height will be maximum, say $= \Delta Z_m$, $y_2 = y_c$ = critical depth and $E_2 = E_c$. The condition at ΔZ_m is given by the relation

$$E_1 - \Delta Z_m = E_2 = E_c = y_c + \frac{Q^2}{2gB^2y_c^2} \qquad (2.30)$$

The question naturally arises as to what happens when $\Delta Z > \Delta Z_m$. From Fig. 2.10 it is seen that the flow is not possible with the given specific energy. The upstream depth has to increase to cause an increase in the specific energy at section 1. If this modified depth is represented by y'_1, then

$$E'_1 = y'_1 + \frac{Q^2}{2gB^2y'^2_1} \qquad \{\text{with } E'_1 > E_1 \text{ and } y'_1 > y_1\} \qquad (2.31)$$

At section 2 the flow will continue at the minimum specific-energy level, i.e. at the critical condition. At this condition, $y_2 = y_c$ and

$$E'_1 - \Delta Z = E_2 = E_c = y_c + \frac{Q^2}{2gB^2y_c^2} \qquad (2.32)$$

Recollecting the various sequences, when $0 < \Delta Z < \Delta Z_m$ the upstream water level remains stationary at y_1 while the depth of flow at section 2 decreases with ΔZ reaching a minimum value of y_c at $\Delta Z = \Delta Z_m$ (Fig. 2.11). With further increase in the value of ΔZ, i.e. for $\Delta Z > \Delta Z_m$, y_1 will change to y'_1 while y_2 will continue to remain at y_c.

The variation of y_1 and y_2 with ΔZ in the subcritical regime can be clearly noticed in Fig. 2.11.

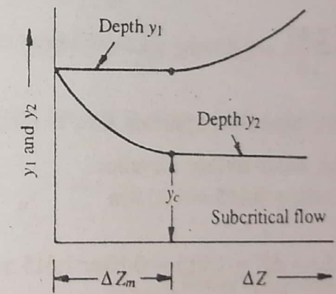


Fig. 2.11 Variation of y_1 and y_2 in subcritical flow over a hump

(b) *Supercritical Flow*

If y_1 is in the supercritical flow regime, Fig. 2.10 shows that the depth of flow increases due to the reduction of specific energy. In Fig. 2.10 point P' corresponds to y_1 and point R' to depth at the section 2. Up to the critical depth, y_2 increases to reach y_c at $\Delta Z = \Delta Z_m$. For $\Delta Z > \Delta Z_m$, the depth over the hump $y_2 = y_c$ will remain constant and the upstream depth y_1 will change. It will decrease to have a higher specific energy E'_1. The variation of the depths y_1 and y_2 with ΔZ in the supercritical flow is shown in Fig. 2.12.

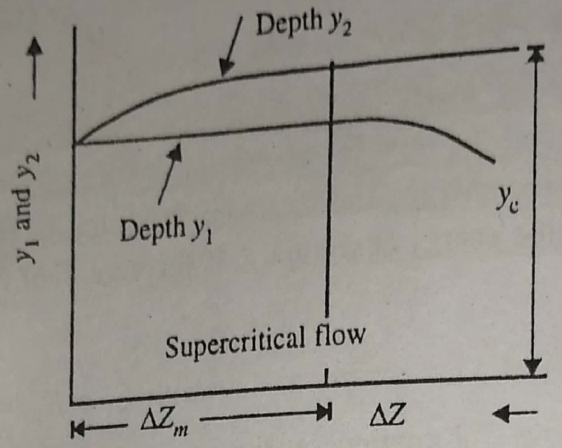


Fig. 2.12 Variation of y_1 and y_2 in supercritical flow over a hump

EXAMPLE 2.7 A rectangular channel has a width of 2.0 m and carries a discharge of 4.80 m³/s with a depth of 1.60 m. At a certain section a small, smooth hump with a flat top and of height 0.10 m is proposed to be built. Calculate the likely change in the water surface. Neglect the energy loss.

Solution

Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively as in Fig. 2.9.

$$q = \frac{4.80}{2.0} = 2.40 \text{ m}^3/\text{s/m}$$

$$V_1 = \frac{2.40}{1.6} = 1.50 \text{ m/s}, \quad \frac{V_1^2}{2g} = 0.115\text{m}$$

$F_1 = V_1/\sqrt{gy_1} = 0.391$, hence the upstream flow is subcritical and the hump will cause a drop in the water-surface elevation.

$$E_1 = 1.60 + 0.115 = 1.715 \text{ m}$$

At section 2,

$$E_2 = E_1 - \Delta Z = 1.715 - 0.10 = 1.615 \text{ m}$$

$$y_c = \left(\frac{(2.4)^2}{9.81}\right)^{1/3} = 0.837 \text{ m}$$

$$E_c = 1.5\, y_c = 1.256 \text{ m}$$

The minimum specific energy at section 2, E_{c2} is less than E_2, the available specific energy at that section. Hence $y_2 > y_c$ and the upstream depth y_1 will remain unchanged. The depth y_2 is calculated by solving the specific-energy relation

$$y_2 + \frac{V_2^2}{2g} = E_2$$

i.e.
$$y_2 + \frac{(2.4)^2}{2 \times 9.81 \times y_2^2} = 1.615$$

Solving by trial and error, $y_2 = 1.481$ m.

EXAMPLE 2.8 In Example 2.7, if the height of the hump is 0.5 m, estimate the water surface elevation on the hump and at a section upstream of the hump.

Solution

From Example 2.7 : $F_1 = 0.391$, $E_1 = 1.715$ m and $y_c = y_{c2} = 0.837$ m.

Available specific energy at section 2 = $E_2 = E_1 - \Delta Z$

$E_1 = 1.715 - 0.500 = 1.215$ m

$E_{c2} = 1.5\, y_{c2} = 1.256$ m.

The minimum specific energy at section 2 is greater than E_2, the available specific energy at that section. Hence, the depth at section 2 will be at the critical depth. Thus $y_2 = y_{c2} = 1.256$ m. The upstream depth y_1 will increase to a depth y'_1 such that the new specific energy at the upstream section 1 is

$$E'_1 = E_{c2} + \Delta Z$$

Thus
$$E'_1 = y'_1 + \frac{V_1'^2}{2g} = E_{c2} + \Delta Z$$

$$y'_1 + \frac{q^2}{2gy_1'^2} = 1.256 + 0.500 = 1.756$$

$$y'_1 + \frac{(2.4)^2}{2 \times 9.81 \times y_1'^2} = 1.756$$

$$y'_1 + \frac{0.2936}{y_1'^2} = 1.756$$

Solving by trial and error and selecting the positive root which gives $y'_1 > y_2$, $y'_1 = 1.648$ m.

The nature of the water surface is shown in Fig. 2.13.

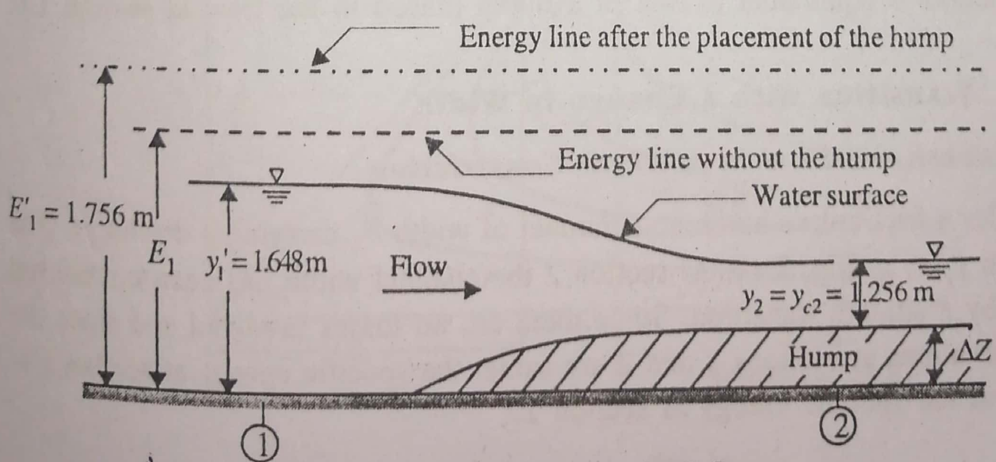


Fig. 2.13 Example 2.8

$$= -\frac{Q^2}{gA^3}\frac{dA}{dy}\frac{dy}{dx}$$

Since $dA/dy = T$,

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = -\frac{Q^2T}{gA^3}\frac{dy}{dx} \tag{4.7}$$

Equation (4.4) can now be rewritten as

$$-S_f = -S_0 + \frac{dy}{dx} - \left(\frac{Q^2T}{gA^3}\right)\frac{dy}{dx}$$

Re-arranging

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}} \tag{4.8}$$

This forms the basic differential equation of GVF and is also known as the dynamic equation of GVF. If a value of the kinetic-energy correction factor α greater than unity is to be used, Eq. (4.8) would then read as

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2T}{gA^3}}} \tag{4.8a}$$

Other Forms of Eq. (4.8)

(a) If K = conveyance at any depth y and K_0 = conveyance corresponding to the normal depth y_0, then

$$K = Q/\sqrt{S_f} \quad \text{(By assumption 2 of GVF)} \tag{4.9}$$

and

$$K_0 = Q/\sqrt{S_0} \quad \text{(Uniform flow)}$$

$$S_f/S_0 = K_0^2/K^2 \tag{4.10}$$

Similarly, if Z = section factor at depth y and Z_c = section factor at the critical depth y_c,

$$Z^2 = A^3/T$$

and

$$Z_c^2 = \frac{A_c^3}{T_c} = \frac{Q^2}{g}$$

Hence,

$$\frac{Q^2T}{gA^3} = \frac{Z_c^2}{Z^2} \tag{4.11}$$

Using Eqs (4.10) and (4.11), Eq. (4.8) can now be written as

Channel	Region	Condition	Type
Mild slope	1	$y > y_0 > y_c$	M_1
	2	$y_0 > y > y_c$	M_2
	3	$y_0 > y_c > y$	M_3
Steep slope	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_c > y_0 > y$	S_3
Critical slope	1	$y > y_0 = y_c$	C_1
	3	$y < y_0 = y_c$	C_3
Horizontal bed	2	$y > y_c$	H_2
	3	$y < y_c$	H_3
Adverse slope	2	$y > y_c$	A_2
	3	$y < y_c$	A_3

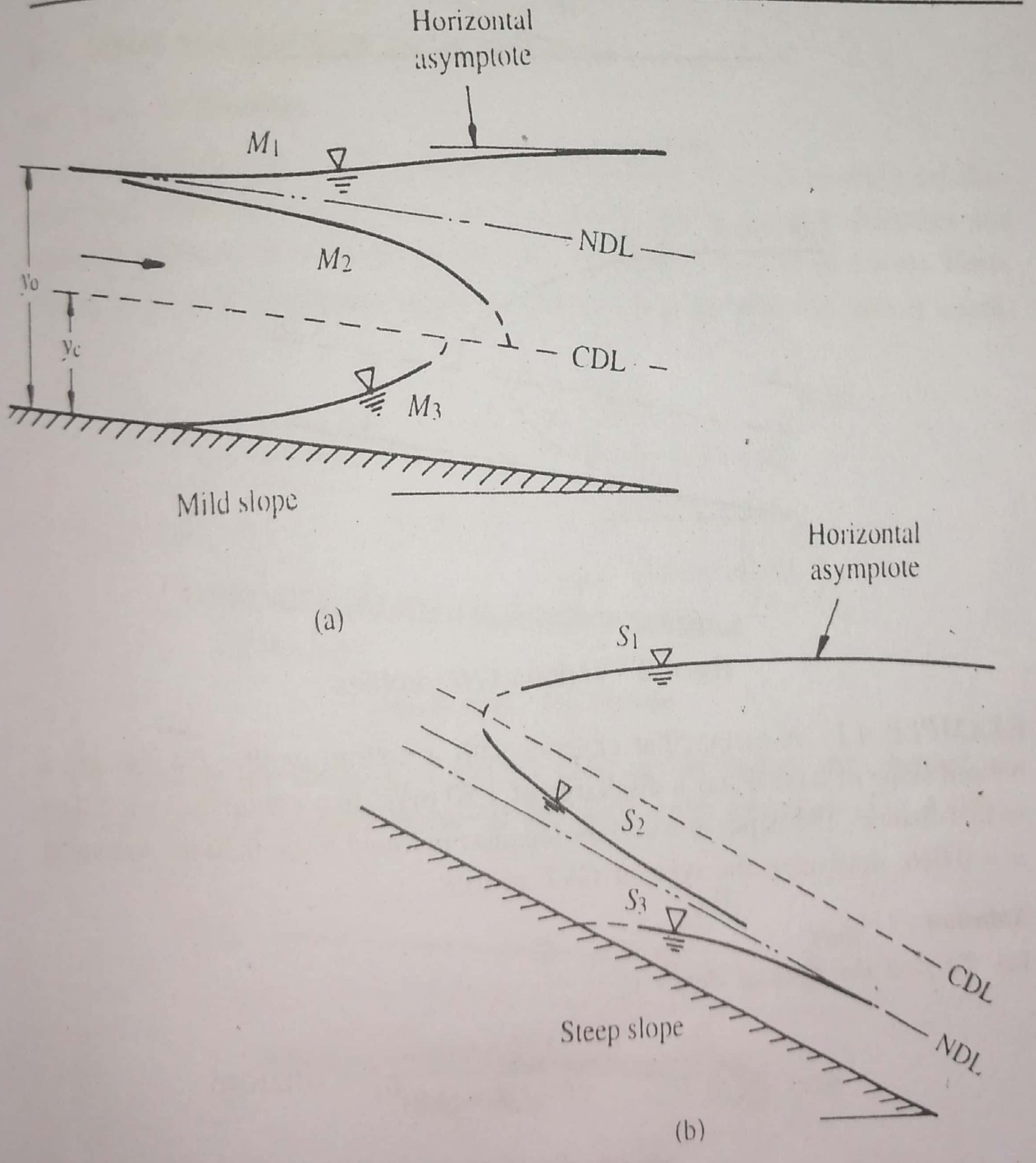


Fig. 4.3 (Contd)

FLOW THROUGH OPEN CHANNELS

K G RANGA RAJU

Second Edition

or
$$(h_1 + h_2) = \frac{2U_1^2}{g}\frac{h_1}{h_2}$$

i.e.
$$\frac{h_2}{h_1}\left(1 + \frac{h_2}{h_1}\right) = 2F_1^2$$

where
$$F_1 = \frac{U_1}{\sqrt{gh_1}}$$

Simplifying
$$\frac{h_2}{h_1} = \frac{1}{2}\left[\sqrt{1 + 8F_1^2} - 1\right] \tag{1.25}$$

The depths h_1 and h_2 are known as *sequent depths* or *conjugate depths.*

Yet another case, namely that of flow downstream of a sharp-crested weir, is chosen for the illustration of the use of the momentum equation (see Fig. 1.8). The space underneath the falling sheet of water is invariably

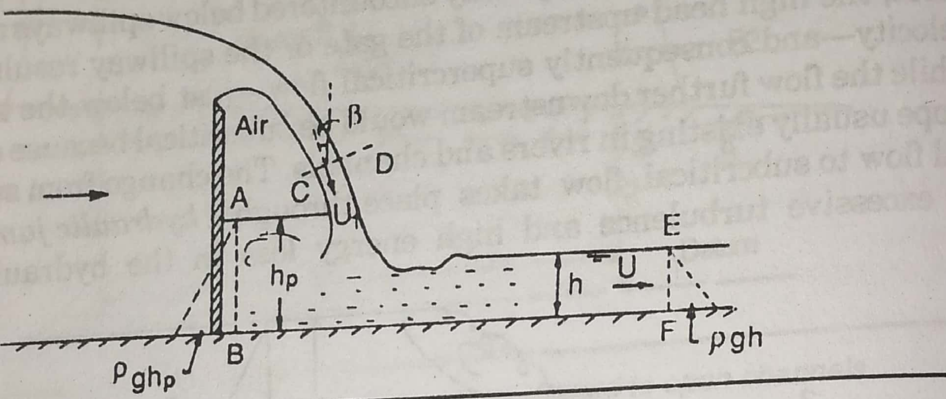


Fig. 1.8 *Momentum equation applied to the flow downstream of a sharp-crested weir*

ventilated to keep the pressure of air in this area practically equal to the atmospheric pressure. The depth of water in this pocket, h_p, can be calculated from the momentum equation by considering the control volume enclosed by the lines AB, CD and EF. Neglecting F_f and F_a and treating the channel to be horizontal, the momentum equation for this control volume becomes

$$\frac{\rho g h_p^2}{2} - \frac{\rho g h^2}{2} = \rho U h (U - U_1 \sin\beta) \tag{1.26}$$

Since the value of β is usually very small, sin β = 0. Thus Eq. (1.26) becomes

$$h_p^2 - h^2 = \frac{2U^2 h}{g}$$

or
$$\frac{h_p}{h} = \sqrt{1 + 2F^2} \tag{1.27}$$

1.5 Energy and Momentum Coefficients

The velocity of flow remains constant over a cross-section only in the case of noncurvilinear flow of an ideal fluid. The velocity varies over the cross-section in the case of real-fluid flow; the velocity at the boundary is equal to zero and it increases with an increase in distance from the boundary. Such variations in velocity need to be taken into account in the calculation of the kinetic energy and momentum flux in an open channel. In other words, the energy and momentum equations discussed in Sec. 1.4 need to be suitably modified.

The kinetic energy of a mass m having a velocity U is $mU^2/2$. Considering a channel of area A in which u is the velocity over an elementary area dA, one can write the total kinetic energy as:

$$\text{K.E.} = \int_A \rho u \, dA \, u^2/2 = \frac{1}{2}\int_A \rho u^3 \, dA \tag{1.28}$$

The ratio between the kinetic energy calculated using Eq. (1.28) and the kinetic energy calculated by assuming an average velocity U over the cross-section is denoted by α called the *energy correction factor*. Since the kinetic energy on the basis of an average velocity over the cross-section is

$$\text{K.E.} = \frac{1}{2}\rho U^3 A \tag{1.29}$$

Eqs. (1.28) and (1.29) give

$$\alpha = \frac{1}{A}\int_A \left(\frac{u}{U}\right)^3 dA \tag{1.30}$$

If the flow is two-dimensional as in a wide rectangular channel, $A = Bh$ and $dA = Bdy$, y being the distance from the bed. Accordingly, Eq. (1.30) reduces to

$$\alpha = \frac{1}{h}\int_0^h \left(\frac{u}{U}\right)^3 dy \tag{1.31}$$

If α and U are known, the true kinetic energy may thus be computed as

$$\alpha \frac{1}{2}\rho U^3 A$$

One can integrate Eq. (1.30) or Eq. (1.31) and find α if the velocity u is known as an algebraic function of y. When such a simple law does not exist (or is not known), the results of actual measurement may be used to evaluate α graphically as shown in Example 1.1. Once the value of α is known for a particular flow the kinetic energy may be evaluated from Eq. (1.29) after inserting α in this equation. Thus the energy equation becomes

$$h_1 + \alpha_1 \frac{U_1^2}{2g} + z_1 = h_2 + \alpha_2 \frac{U_2^2}{2g} + z_2 + E_L \tag{1.32}$$

and

$$E = h + \alpha \frac{U^2}{2g} \tag{1.33}$$

The momentum flux of a mass rate of flow m at a constant velocity U is mU. In case the velocity varies over the cross-section one can write the momentum flux as

$$\text{Momentum flux} = \int_A (\rho u dA)u = \int_A \rho u^2 dA \tag{1.34}$$

When the momentum flux is expressed in terms of the average velocity ignoring the velocity variation over the cross-section, it may be written as

$$\text{Momentum flux} = \rho U^2 A \tag{1.35}$$

The ratio of momentum flux calculated using Eqs. (1.34) and (1.35) is denoted by β which is called the *momentum correction factor*, i.e.

$$\beta = \frac{1}{A}\int_A \left(\frac{u}{U}\right)^2 dA \tag{1.36}$$

and for two-dimensional flow

$$\beta = \frac{1}{h}\int_0^h \left(\frac{u}{U}\right)^2 dy$$

Equations (1.36) and (1.37) can be integrated (graphically or otherwise) for a known velocity distribution and β evaluated; the momentum flux can then be computed from a known average velocity as $\beta\rho U^2 A$. As such, the momentum equation can be written in the modified form

$$\Sigma F_x = \beta_2 \rho Q U_2 - \beta_1 \rho Q U_1 \tag{1.38}$$

It can be seen that invariably $\alpha > \beta > 1.0$. It is obvious that when there is marked variation of velocity over a cross-section, α and β are considerably larger than unity. For that reason the values of α and β in laminar flow are generally larger than in turbulent flow. Even in the case of turbulent flow, high values of α and β may be obtained in odd-shaped channels or when the flow is concentrated in one part of the cross-section. But generally speaking, α and β in turbulent flow are of the order of 1.10 and 1.05 (or even less) respectively and it is customary to assume these to be unity in a majority of problems. Interestingly, measurements[5] in curved irrigation canals of trapezoidal shape have revealed values of α and β in the foregoing range only. Unless otherwise mentioned, the assumption that $\alpha = \beta = 1.0$ has been made in the chapters to follow.

Example 1.1

The following velocities were measured at the centre line of a very wide rectangular channel of depth equal to 120 mm. Find U, α and β.

y mm	0	3.0	10.0	15.0	20.0	40.0	60.0	80.0	100.0	120.0
u m/s	0	1.25	1.75	2.05	2.20	2.55	2.75	2.85	2.90	3.00

Solution

Using the above data plots of u vs. y, u^2 vs. y and u^3 vs. y may be prepared as shown in Fig. 1.9. The areas A_1, A_2 and A_3 on these three plots may be obtained by graphical integration and expressed in the proper units, viz. m^2/s, m^3/s^2 and m^4/s^3 respectively.

Now $$U = \frac{1}{h}\int_0^h u\,dy = \frac{A_1}{h}$$

From Fig. 1.9 $A_1 = 0.3032\ m^2/s$ and hence $U = 2.53$ m/s. From Eqs (1.31) and (1.37), $\alpha = \frac{A_3}{U^3 h}$ = and $\beta = \frac{A_2}{U^2 h}$. The values of A_2 and A_3 may be obtained from Fig. 1.9 as 0.811 m^3/s^2 and 2.20 m^4/s^3 respectively. Thus $\alpha = 1.13$ and $\beta = 1.05$.

Example 1.2

The velocity distribution in a wide rectangular channel may be approximated by the equation $u = 0.4 + 0.6\,y/h$ m/s. Find U, α and β if $h = 1.0$ m.

Solution

$$U = \frac{1}{h}\int_0^h u\,dy = \frac{1}{1}\int_0^1 (0.4 + 0.6y)dy$$

$$= [0.4y + 0.6y^2/2]_0^1 = 0.7$$

$$U = 0.7 \text{ m/s}$$

$$\alpha = \frac{1}{U^3 h}\int_0^h u^3 dy = \frac{1}{0.7^3 \times 1}\int_0^1 (0.4 + 0.6y)^3 \times dy$$

$$\alpha = \frac{1}{0.343}\int_0^1 (0.4^3 + 0.6^3 y^3 + 3 \times 0.4 \times 0.6^2 y^2 + 3 \times 0.4^2 \times 0.6y)\,dy$$

$$= \frac{1}{0.343}\int_0^1 (0.064 + 0.216y^3 + 0.432y^2 + 0.288y)dy$$

$$= \frac{1}{0.343}\left[0.064 + \frac{0.216}{4} + \frac{0.432}{3} + \frac{0.288}{2}\right]$$

$$= 1.18$$

$$\therefore \quad \alpha = 1.18$$

$$\beta = \frac{1}{U^2 h}\int_0^h u^2 dy$$

$$= \frac{1}{0.7^2 \times 1}\int_0^1 (0.4 + 0.6)^2 dy = \frac{1}{0.49}\int_0^1 (0.16 + 0.36y^2 + 0.48y)dy$$

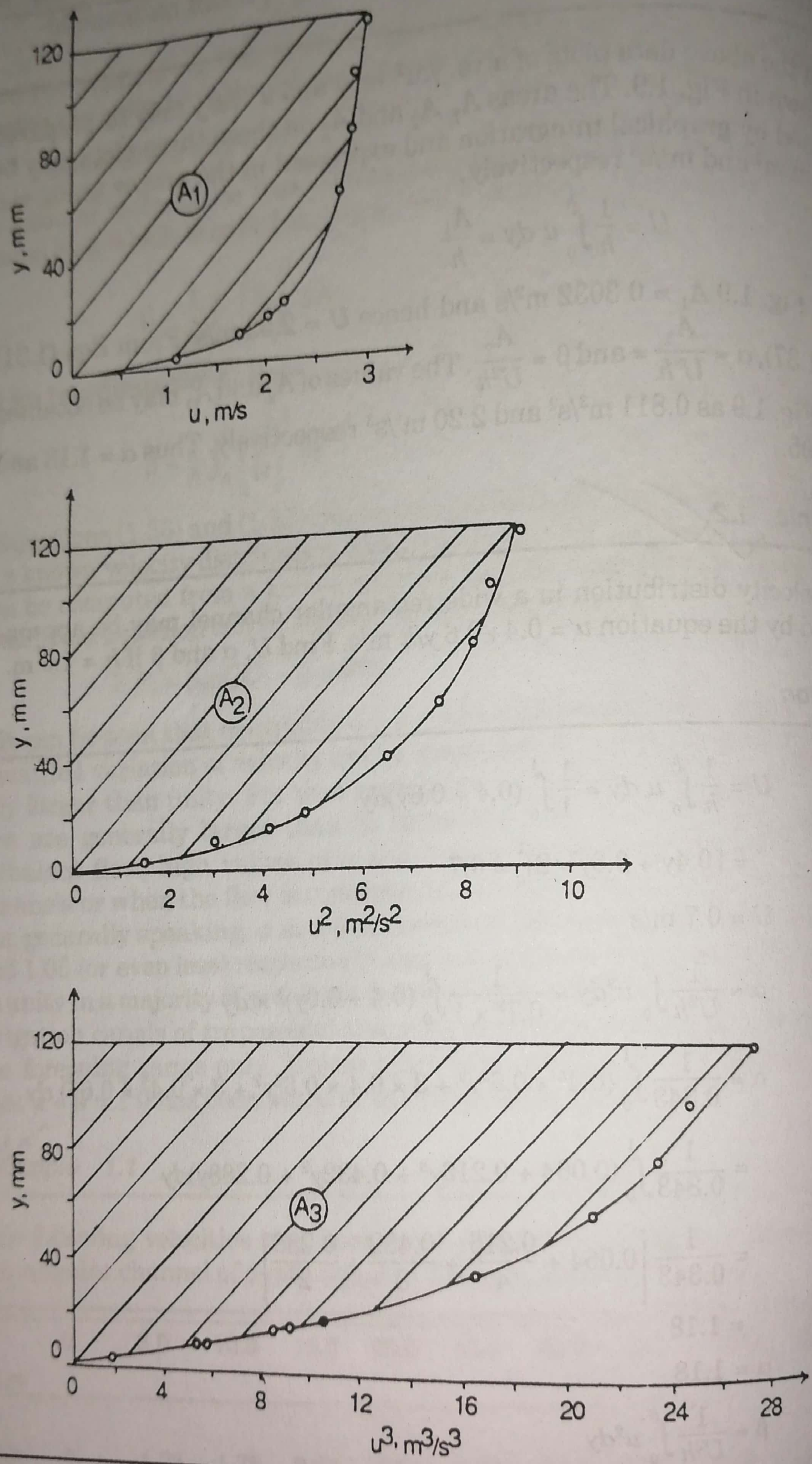


Fig. 1.9 *Graphical computation of U, α and β*

$$= \frac{1}{0.49}\left[0.16 + \frac{0.36}{3} + \frac{0.48}{2}\right] = 1.06$$

$\therefore \quad \beta = 1.06$

A computer programme for the calculation of α and β from a measured velocity profile in a two-dimensional channel is given in Table AP-1 of the Appendix.

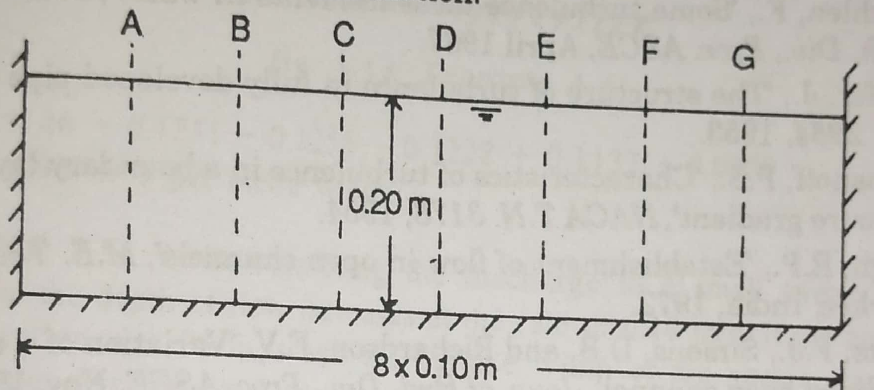
PROBLEMS

1.1 The velocity distribution in a very wide river 3.0 m deep is found to be approximately in accordance with the equation $u = 1+2(y/h)^{1/2}$. Calculate α and β.

1.2 The velocity distribution in a semi-circular channel of diameter $2R_0$ follows the law $u/U_0 = (y/R_0)^{1/7}$, in which y is the distance normal to the surface at which the velocity is u and U_0 is the velocity at the centre of the semi-circle. If $R_0 = 2.0$ m and $U_0 = 2.0$ m/s, find U, α and β.

1.3 Water flows in a rectangular channel 1.0 m wide at a depth of 0.10 m and a velocity of 1.5 m/s. Find the state of flow. $\nu = 10^{-6}$ m²/s.

1.4 The following velocities were measured downstream of an open channel expansion. Find α and β at this section.



Distance from the bed, mm	2.0	5.0	8.0	10.0	20.0	30.0	50.0	100.0	195.0
	Velocity, m/s								
Section A	0.40	0.43	0.48	0.51	0.53	0.56	0.59	0.62	0.65
Section B	0.53	0.60	0.65	0.68	0.70	0.74	0.78	0.81	0.85
Section C	0.61	0.67	0.71	0.75	0.79	0.81	0.84	0.90	0.93
Section D	0.68	0.73	0.80	0.84	0.87	0.89	0.90	0.97	1.00
Section E	0.61	0.67	0.71	0.75	0.79	0.81	0.84	0.90	0.93
Section F	0.53	0.60	0.65	0.68	0.70	0.74	0.78	0.81	0.85
Section G	0.40	0.43	0.48	0.51	0.53	0.56	0.59	0.62	0.65

1.5 State whether the following flows are steady or unsteady and uniform or nonuniform:

(a) River flow around a bridge pier.

(b) Flow in a long, prismatic irrigation canal.

(c) Movement of water around a boat in a lake.

1.6 Uniform flow of an ideal fluid on a sloping channel is impossible. Why?

1.7 Why is the flow necessarily nonuniform in a channel of zero bed slope?

1.8 The following pressures were measured on a wall. Find the force per unit length of the wall, pressure coefficient at the base of the wall and force in excess of the hydrostatic value.

3.2.1 Three approaches to the problem

A completely theoretical solution for the incipient motion condition is not available and recourse is invariably made to experimentation. The experimental data are analysed on the premise that there is a particular velocity—often called the *competent velocity*—which causes motion or that there is a certain force which leads to the movement of the particles. Depending on whether lift or drag on the particle is considered important, the latter approach may again be subdivided into the *lift approach* and the *critical tractive force approach*. Over the years the critical tractive force approach has found favour with hydraulic engineers and is used quite often today; as such, only this approach is discussed.

3.2.2 Critical tractive force approach

The average shear stress on the bed of an open channel at which the sediment particles just begin to move is called the *critical tractive stress*. Shields[1] was the first to give a semi-theoretical analysis for the determination of the critical tractive stress and his results are most commonly used today. The details of his analysis are presented below.

Considering particles of size d and of mass density ρ_s in a fluid of mass density ρ, the force required to move the particle F_1 may be written as

$$F_1 = A_1 g(\rho_s - \rho)\, d^3 = A_1 g \Delta\rho_s\, d^3 \qquad (3.1)$$

in which $\Delta\rho_s = \rho_s - \rho$ and A_1 is a coefficient depending on the particle shape and angle of internal friction. The fluid drag on the particle F_2 may be written as

$$F_2 = C_D A_2 d^2 \rho \frac{u_d^2}{2} \qquad (3.2)$$

in which u_d is the velocity at the top of the particles, C_D is the drag coefficient at the Reynolds number corresponding to u_d and A_2 is a coefficient depending on the particle shape. From the laws of velocity distribution,

$$\frac{u_d}{u_*} = f_1\left(\frac{u_* d}{\nu}\right)$$

in which u_* is the shear velocity $= \sqrt{\tau_0/\rho}$ and ν is the kinematic viscosity of the fluid. Since C_D is a function of $u_d d/\nu$, it follows from the above equation that C_D is also a function of $u_* d/\nu$. Thus Eq. (3.2) may be written as

$$F_2 = f_2\left(\frac{u_* d}{\nu}\right)\left(\frac{\rho}{2} u_*^2\right) f_1^2\left(\frac{u_* d}{\nu}\right) A_2 d^2$$

Equating the above value of F_2 to F_1 in Eq. (3.1) under the critical condition and using the subscript c to designate the critical condition

$$A_1 g \Delta\rho_s d^3 = f_2\left(\frac{u_{*c}\,d}{\nu}\right)\left(\frac{\rho}{2}u_*^2\right)f_1^2\left(\frac{u_{*c}\,d}{\nu}\right)A_2 d^2$$

Designating $\frac{\tau_c}{g(\Delta\rho_s)d}$ as τ_c^* and $u_{*c}\,d/\nu$ as R_c^* and simplifying the above equation

$$\tau_c^* = \frac{2A_1}{A_2}f(R_c^*)$$

For particles of not very dissimilar shapes and for a constant value of the angle of internal friction, A_1 and A_2 are practically constant and the foregoing equation reduces to

$$\tau_c^* = f(R_c^*) \tag{3.3}$$

Data collected by various investigators using sediment of different sizes and mass densities are plotted in Fig. 3.1. The data justify Eq. (3.3) in the sense that τ_c^* is seen to be uniquely related to R_c^*. Since R_c^* is proportional to the ratio of the sediment size to the laminar sublayer thickness, the value of R_c^* gives an indication of the type of boundary. The boundary is rough at large values of R_c^* and, understandably therefore, τ_c^* becomes independent of R_c^* at high values of R_c^*; at $R_c^* \geq 70$, τ_c^* remains constant at 0.045. When the boundary is smooth or in transition, τ_c^* is a function of R_c^* as shown in Fig. 3.1. The bold line shown in Fig. 3.1 is the relationship

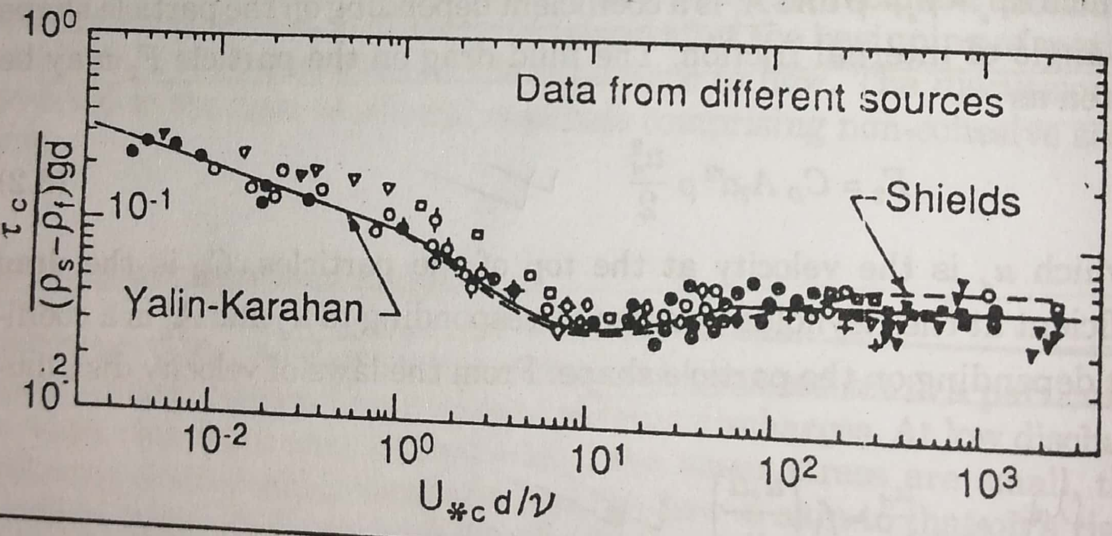


Fig. 3.1 *Yalin-Karahan curve for incipient motion condition*

proposed by Yalin and Karahan[2]. The relationship proposed earlier by Shields on the basis of a relatively smaller amount of data is shown by the dotted line in Fig. 3.1. It is recommended that the relationship of Yalin and Karahan be used for the determination of the critical tractive stress.

The computation of τ_c may be carried out using Fig. 3.1 only by trial and error. A value of τ_c is assumed and thus u_{*c} and R_c^* calculated; τ_c^* may then be read from Fig. 3.1 and τ_c calculated. This value may then be compared with the assumed value; if the two do not match, the trial is continued with the newly obtained value of τ_c. The correct value of τ_c^* is that which comes out to be the same as that assumed in the preceding trial.

The trial and error may be eliminated by a simple modification of Fig. 3.1 as follows:

$$(R_c^*)^{2/3}(\tau_c^*)^{-1/3} = \left(\frac{u_{*c}d}{\nu}\right)^{2/3}\left(\frac{g\Delta\rho_s d}{\tau_c}\right)^{1/3}$$

$$= \left(\frac{u_{*c}d}{\nu}\right)^{2/3}\left(\frac{g\Delta\rho_s}{\rho}\frac{d}{u_{*c}^2}\right)^{1/3}$$

$$= \left(\frac{\Delta\rho_s}{\rho}\right)^{1/3}\frac{g^{1/3}d}{\nu^{2/3}}$$

Designating $R_c^{*2/3}\,\tau_c^{*-1/3}$ as R_1^*,

$$R_1^* = \left(\frac{\Delta\rho_s}{\rho}\right)^{1/3}\left(\frac{g^{1/3}d}{\nu^{2/3}}\right) = R_c^{*2/3}\,\tau_c^{*-1/3} \tag{3.4}$$

Choosing different values of R_c^*, τ_c^* may be read from Yalin-Karahan relation shown in Fig. 3.1 and Eq. (3.4) used to calculate R_1^*. Figure 3.2 shows a plot of τ_c^* against R_1^* based on such calculations. Since R_1^* is a known quantity τ_c^* can easily be read from Fig. 3.2 and hence τ_c calculated as shown in the following example; obviously the boundary will be moving when the shear stress on the bed is equal to or greater than τ_c.

Example 3.1

Find the depth of flow at which sediment of 1 mm diameter starts moving in a wide rectangular channel set at a slope of 10^{-4}. (Relative density of sediment = 2.65 and $\nu = 10^{-6}$ m²/s.)

Solution

$$R_1^* = \left(\frac{\Delta\rho_s}{\rho}\right)^{1/3}\frac{g^{1/3}d}{\nu^{2/3}}$$

$$= \left(\frac{2650 - 1000}{1000}\right)^{1/3} \times \frac{(9.8)^{1/3} \times 1 \times 10^{-3}}{(10^{-6})^{2/3}}$$

$$= 25.3$$

The corresponding value of τ_c^* read from Fig. 3.2 is 0.035. Hence

$$\tau_c = 0.035g(\Delta\rho_s)\,d$$

$$= 0.035 \times 9.81 \times 1650 \times 1 \times 10^{-3}$$

$$= 0.565 \text{ N/m}^2$$

The material starts moving when $\tau_0 = \tau_c$. Since this a wide rectangular channel, $\tau_0 = \rho g h S$, i.e.

$$\rho g h S = 0.565$$

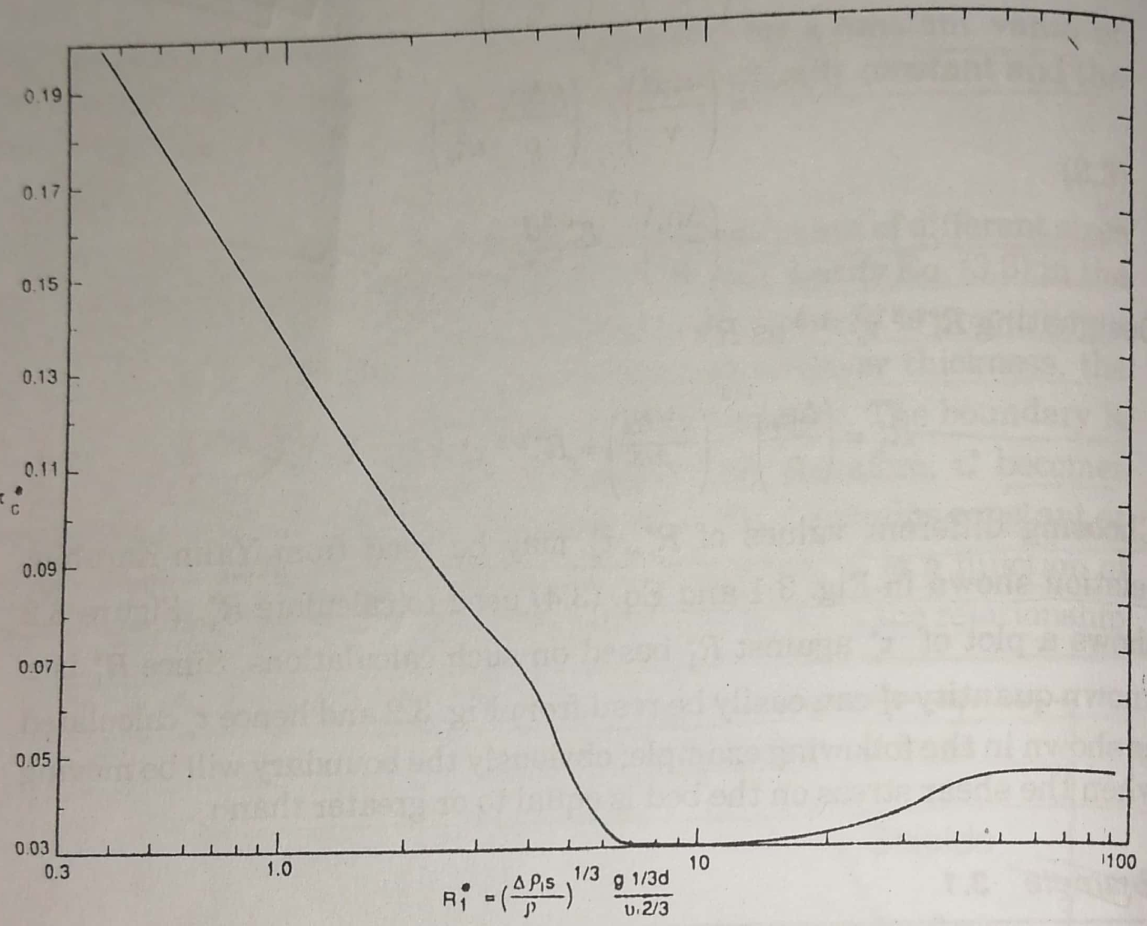


Fig. 3.2 *Modified form of Yalin-Karahan curve*

$$9810 \times h \times 10^{-4} = 0.565$$

$$h = 0.575 \text{ m}$$

Hence the material moves at a depth of 0.575 m.

3.3 Regimes of Flow

At shear stresses greater than critical, the initially plane sediment bed is disturbed on account of the sediment motion. Depending on the sediment, fluid and flow conditions, the bed and water surface assume various forms; these types of bed and water surfaces are classified according to their characteristics and are called[3] *regimes of flow*.

$$\tau_{bm} \le \tau_{b1}$$

and

$$\tau_{sm} \le \tau_{s1} \qquad (4.9)$$

The discharge capacity of such a channel can be obtained by using Manning's equation in which the rugosity coefficient is defined by Strickler's equation, viz. Eq. (2.31). The design is usually carried out by using a trial-and-error process as shown in Example 4.5.

Example 4.5

Design a canal to carry 41.50m³/s of clear water through 3.0 mm gravel (angle of repose = 31°) on a slope of 10^{-4}. The canal is to be trapezoidal in shape having side slopes of 2 H : 1 V. The average temperature = 20°C ($\nu = 10^{-6} m^2/s$).

Solution

$$n_s = \frac{d^{1/6}}{25.6} = \frac{(3.0 \times 10^{-3})^{1/6}}{25.6} = 0.0148$$

$$R_1^* = \left(\frac{\Delta\rho_s}{\rho}\right)^{1/3} \frac{g^{1/3} d}{\nu^{2/3}} = \frac{(1.65)^{1/3} \times (9.8)^{1/3} \times 3 \times 10^{-3}}{(10^{-6})^{2/3}}$$

$$= 75.7$$

The corresponding value of τ_c^* from Fig. 3.2 is 0.045

$$\therefore \quad \tau_c = 0.045\, g\Delta\rho_s d = 0.045 \times 9.81 \times 1650 \times 3 \times 10^{-3} = 2.19 \text{ N/m}^2$$

Taking τ_{b1} as 0.90 τ_c,

$$\tau_{b1} = 1.97 \text{ N/m}^2$$

From Eq. (4.8)

$$K = \cos\theta \sqrt{1 - \frac{\tan^2\theta}{\tan^2\varphi}}$$

$$\tan\varphi = \tan 31° = 0.600, \ \cos\theta = \frac{2}{\sqrt{5}}, \ \tan\theta = \frac{1}{2}$$

$$\therefore \quad K = \frac{2}{\sqrt{5}} \sqrt{1 - \frac{0.25}{0.36}} = 0.494$$

$$\therefore \quad \tau_{sl} = 0.494 \times 1.97 = 0.973 \text{ N/m}^2$$

The rest of the computations are done in tabular form as shown below by assuming different values of B/h. The values of h in column 4 and U and Q in columns 8 and 9 respectively are computed as shown at the foot of the table for one trial.

1	2	3	4	5	6	7	8	9
B/h	$\frac{\tau_{sm}}{\rho g h S}$	$\frac{\tau_{bm}}{\rho g h S}$	h, m	B, m	A, m²	R, m	U, m/s	Q, m³/s
7.0	0.78	0.985	1.27	8.89	14.52	0.997	0.676	9.67

9.0	0.78	0.985	1.27	11.43	17.74	1.037	0.692	12.28
25.0	0.78	0.985	1.27	31.75	43.55	1.164	0.748	32.57
32.0	0.78	0.985	1.27	40.64	54.83	1.183	0.755	41.40

Columns 2 and 3 : Read from Fig. 2.9

Column 4 : Putting $\tau_{bm} = \tau_{bl} = 1.97\ \text{N/m}^2$, $h = \dfrac{1.97}{9810 \times 0.985 \times 10^{-4}}$

$= 2.04$ m

Putting $\tau_{sm} = \tau_{sl} = 0.973\ \text{N/m}^2$, $h = \dfrac{0.973}{0.78 \times 9810 \times 10^{-4}} = 1.27$ m

Obviously the lesser of the two values will be chosen.

Column 8 : $U = \dfrac{1}{n_s} R^{2/3} S^{1/2} = \dfrac{1}{0.0148} (0.997)^{2/3} (10^{-4})^{1/2}$

$= 0.666$ m/s

Column 9 : $Q = AU = 14.52 \times 0.666 = 9.67\ \text{m}^3/\text{s}$

Different values of B/h are assumed and the computations carried out till the discharge obtained in column 9 matches with the design discharge. In the present case

$$h = 1.27 \text{ m and } B = 40.64 \text{ m}$$

4.3.2 Most efficient section

The sediment particles at some locations of the trapezoidal section discussed above are practically under the incipient motion condition; at other locations, the shear stresses are smaller than those required to cause motion to occur. Obviously the most efficient cross-sectional shape in the present case is one in which the particles are at the incipient motion condition all over the periphery rather than over a part of the periphery. Assuming that the tractive stress at a section is proportional to the local depth and using an analysis similar to the one discussed in Sec. 4.3.1, Glover and Florey[6] found that the shape of the most efficient section is given by (see Fig. 4.8)

$$h_1 = h \cos\left(\frac{x}{h} \tan \varphi\right) \tag{4.10}$$

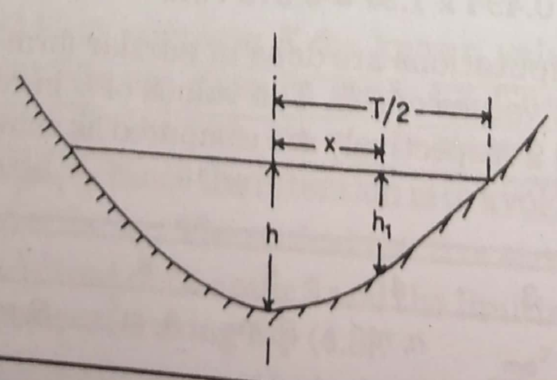


Fig. 4.8 *Shape of most efficient section in erodible boundary*