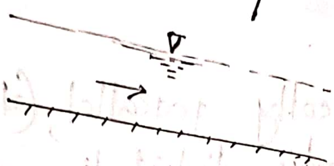
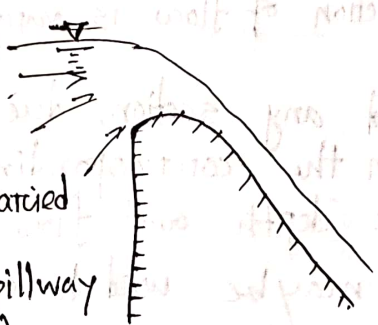
Class lecture of 1st week (Home work)

a) Uniform flow are mostly steady



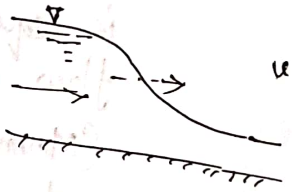
b) Unsteady uniform flows are very rare in nature

c)



steady varied flow (over a spillway crest)

d)



unsteady varied flow (flood wave)

e)

unsteady varied flow (tidal surge)

Gradually - Varied flow in Open Channels

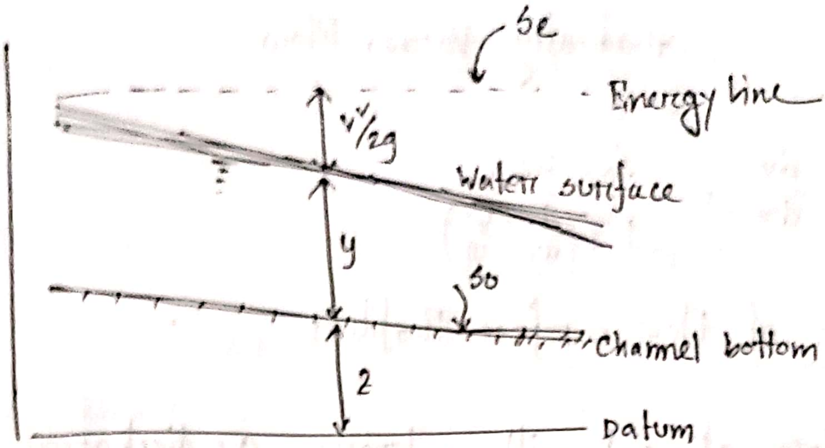
◫ Gradually varied flow - The flow in which depth of flow and mean velocity change gradually along the length of the channel, is called gradually varied flow.

◫ Dynamic Equation of Gradually Varied Flow

objective: To get the relationship between the water surface slop and other characteristics of flow

The following assumptions are made in the derivation of the equ

1. The flow is steady;
2. The streamlines are practically parallel, ~~(true when~~ that means the hydrostatic distribution of pressure is assumed over the section (true when the variation in depth along the direction of flow is very gradual)
3. The loss of head at any section, due to friction, is equal to that in the corresponding uniform flow with the same depth and flow characteristics (Manning's formula may be used to calculate the slope of the energy line)
4. The slope of the channel is small
5. The channel is prismatic
6. The velocity distribution across the section is fixed.
7. The roughness co-efficient is constant in the reach.



$$H = 2 + y + \frac{v^2}{2g}$$

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) \quad \text{——— ①}$$

$\frac{dH}{dx}$ = the slope of energy line ~~(-ve)~~ $(-S_e)$

$\frac{dz}{dx}$ = the bed slope $(-S_0)$

$$-S_e = -S_0 + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{v^2}{2g}\right) = -S_0 + \frac{dy}{dx} + \frac{dy}{dx}\cdot\frac{d}{dy}\left(\frac{v^2}{2g}\right)$$

$$S_0 - S_e = \frac{dy}{dx}\left[1 + \frac{d}{dy}\left(\frac{v^2}{2g}\right)\right]$$

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_e}{1 + \frac{d}{dy}\left(\frac{v^2}{2g}\right)}} \quad \text{- - - - ①}$$

This equation is known as the dynamic equation of Gradually varied flow

$$\frac{d}{dy}\left(\frac{v^2}{2g}\right) = \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right) = -\frac{Q^2}{gA^3}\left(\frac{dA}{dy}\right) = -\frac{Q^2 T}{gA^3} = -F_r^2$$

$T = \frac{dA}{dy}$ → Top width

From equation ① we obtain

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - \frac{Q^2 T}{gA^3}}$$

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2}}$$

Gradually Varied Flow

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 + \frac{d}{dy}\left(\frac{V^2}{2g}\right)}$$

Classification of Flow surface Profiles:

For a given channel with a known Q = discharge, n = Manning co-efficient,

S_0 = channel bed slope

y_c = critical water depth

y_0 = uniform flow depth can be computed.

There are three posible relation between y_0 and y_c

1) $y_0 > y_c$

2) $y_0 < y_c$,

3) $y_0 = y_c$.

For horizontal ($S_0 = 0$); and adverse slope ($S_0 < 0$) channels

$$\boxed{Q = A \frac{1}{n} R^{2/3} S_0^{1/2}}$$

Horizontal channel, $\boxed{S_0 = 0 \longrightarrow Q = 0}$

Adverse channel, $\boxed{S_0 < 0 \longrightarrow Q \text{ cannot be computed}}$

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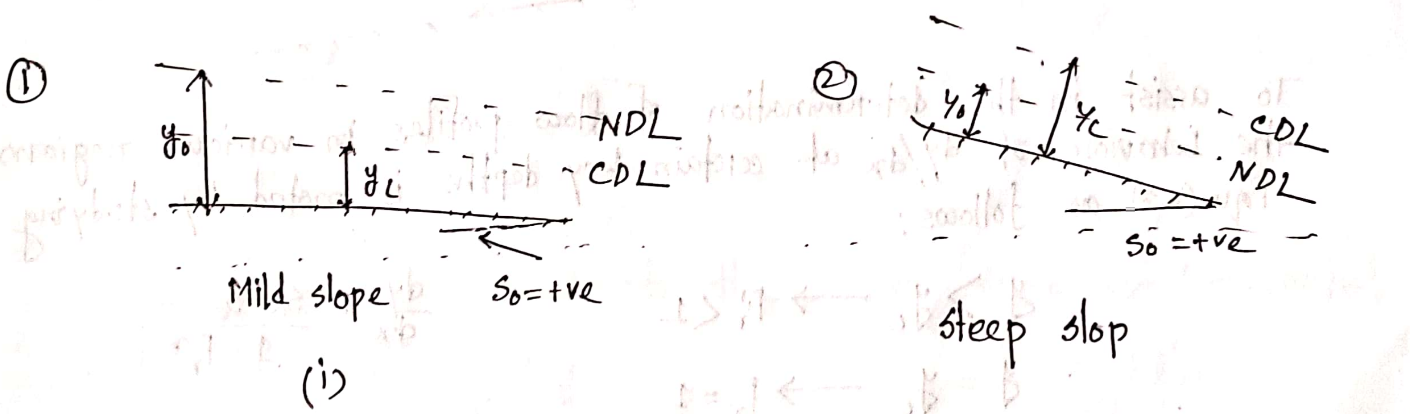
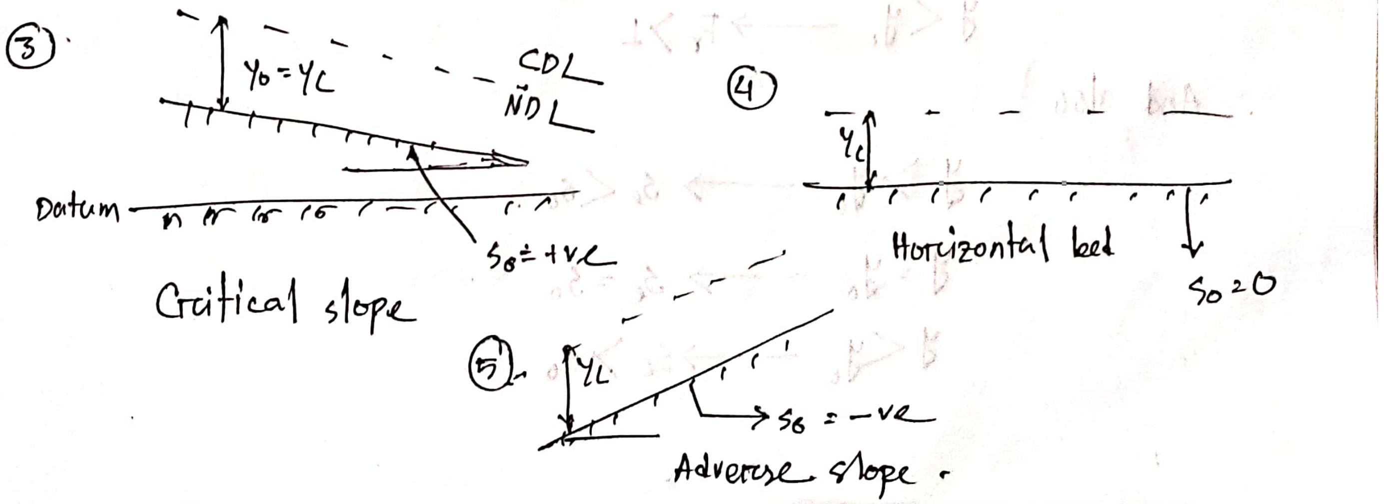
Number	Channel category	symble	Characteristic condition	Remark
1.	Mild slop	M	$y_0 > y_c$	Subcritical flow at normal depth
2.	Steep slop	S	$y_c > y_0$	Supercritical flow at normal depth
3.	Critical slop	C	$y_c = y_0$	Critical flow at normal depth
4.	Horizontal bed	H	$S_0 = 0$	Cannot sustain uniform flow
5.	Adverse slop	A	$S_0 < 0$	Can not sustain uniform flow





Gradually - Varied Flow

For a given Q, n and S_0 at a channel,

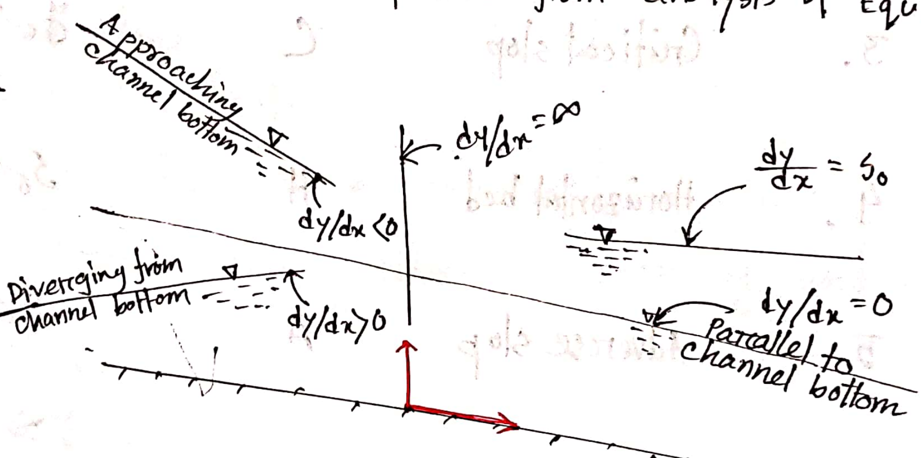
y_0 = Uniform flow depth

y_c = critical flow depth

y = Non-uniform flow depth (Gradually-varied flow)

The depth y is measured vertically from the channel bottom, the slope of the water surface dy/dx is relative to this channel bottom, is basic to the prediction of surface profiles from analysis of Equ. (6.3). Fig. (6.3)

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2}$$



To assist in the determination of flow profiles in various region, the behavior of dy/dx at certain key depths is noted by studying Equ. (6.3) as follows:

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2}$$

$y > y_c \longrightarrow F_r < 1$

$y = y_c \longrightarrow F_r = 1$

$y < y_c \longrightarrow F_r > 1$

And also

$y > y_0 \longrightarrow S_e < S_0$

$y = y_0 \longrightarrow S_e = S_0$

$y < y_0 \longrightarrow S_e > S_0$

Explain the behavior of flow profile when (i) $y \to y_0$ (ii) $y \to y_c$ (iii) $y \to \alpha$

1. As $y \to y_0$, $V \to V_0$, $S_e = S_0$

$$\lim_{y \to y_0} \frac{dy}{dx} = \frac{S_0 - S_0}{1 - F_r^2} = \frac{0}{cons} = 0$$

The water surface approaches the normal depth asymptotically.

2. As $y \to y_c$ $F_r^2 = 1$, $1 - F_r^2 = 0$,

$$\lim_{y \to y_c} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0 - S_e}{0} = \infty$$

The water surface meets the critical depth line vertically

3. As $y \to \infty$, $V = 0$ $F_r = 0 \longrightarrow S_e \to 0$

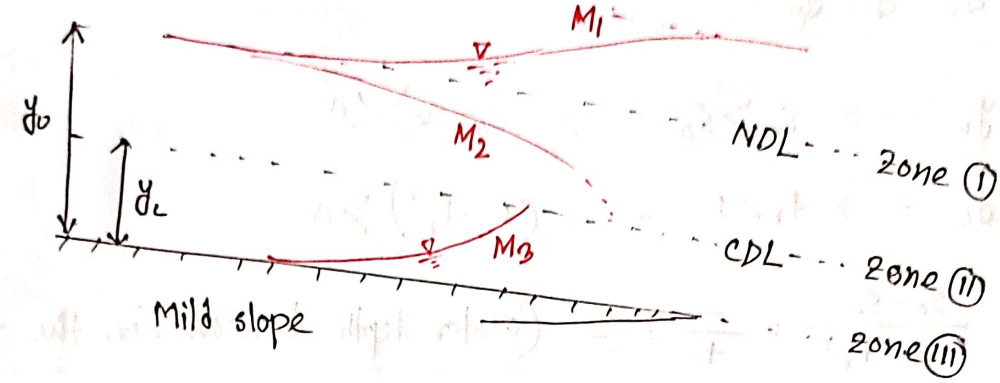
$$\lim_{y \to \alpha} \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{S_0}{1} = S_0$$

The water surface meets the large depth as horizontal asymptotically.

Based on this information, the various possible gradually varied flow profiles are grouped into twelve types

Channel	Region	Condition	Type
Mild slope	1	$y > y_0 > y_c$	M_1
	2	$y_0 > y > y_c$	M_2
	3	$y_0 > y_c > y$	M_3
Steep slope	1	$y > y_c > y_0$	S_1
	2	$y_c > y > y_0$	S_2
	3	$y_c > y_0 > y$	S_3
Critical slope	1	$y > y_0 = y_c$	C_1
	3	$y < y_0 = y_c$	C_3
Horizontal bed	2	$y > y_c$	H_2
	3	$y < y_c$	H_3
Adverse slope	2	$y > y_c$	A_2
	3	$y < y_c$	A_3

M - curves



M_1 - Curve

$y > y_c \rightarrow F_r < 1$
$y = y_c \rightarrow F_r = 1$
$y < y_c \rightarrow F_r > 1$

$y > y_0 \rightarrow S_e < S_0$
$y = y_0 \rightarrow S_e = S_0$
$y < y_0 \rightarrow S_e > S_0$

$y > y_0 > y_c$

$S_e < S_0 \longrightarrow$ Mild slope channel

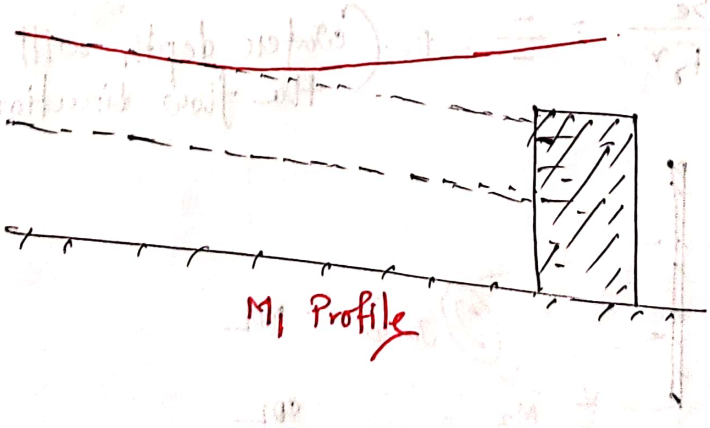
$y_0 > y_c \longrightarrow$ subcritical flow

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2}$$

$F_r < 1 \longrightarrow$ subcritical flow $\longrightarrow (1 - F_r^2) > 0$

$y > y_0 \longrightarrow S_e < S_0$

$\frac{dy}{dx} = \frac{+}{+} > 0$ (Water depth will increase in the flow direction)



M_1 Profile

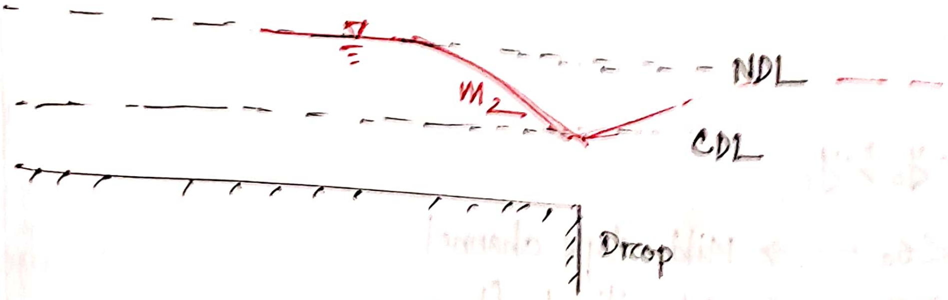
M_2-Curve

$$y_0 > y > y_c$$

$y < y_0 \longrightarrow S_e > S_0 \longrightarrow (S_0 - S_e) < 0$

$y > y_c \longrightarrow F_r < 1 \longrightarrow (1 - F_r^2) > 0$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{+} = -$$ (Water depth decrease in the flow direct



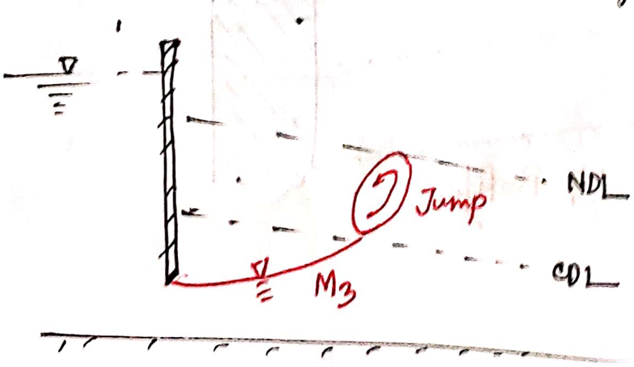
M_3-Curve

$$y_0 > y_c > y$$

$y < y_0 \longrightarrow V > V_0 \longrightarrow S_e > S_0 \longrightarrow (S_0 - S_e) < 0$

$y < y_c \longrightarrow$ Supercritical flow $\longrightarrow F_r > 1 \longrightarrow (1 - F_r^2) < 0$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = +$$ (Water depth will increase in the flow direction)



Home Task:

A rectangular channel with a bottom width of 4.0 m and a bottom slope of 0.0008 has a discharge of 1.50 m³/sec. In a gradually varied flow in this channel, the ~~ty~~ depth at a certain location is found to be 0.30m. assuming $n = 0.016$, determine the type of GVF profile

Answer

$$A = By_0 = 4y_0$$

$$P = B + 2y_0 = 4 + 2y_0$$

$$R = \frac{A}{P} = \frac{4y_0}{4+2y_0}$$

$$Q = A\frac{1}{n}R^{2/3}S_0^{1/2}$$

$$1.50 = \frac{1}{0.016} \times \frac{(4y_0)^{5/3}}{(4+2y_0)^{2/3}} \times 0.0008^{1/2}$$

$$0.0842 = \frac{y_0^{5/3}}{(4+2y_0)^{2/3}}$$

$$y_0 = 0.43 \text{ m} \quad \text{—— (1)}$$

$$y_c' = \left(\frac{Q^2}{gB^2}\right)^{1/3}$$

$$= \left(\frac{1.50^2}{9 \times 4^2}\right)^{1/3}$$

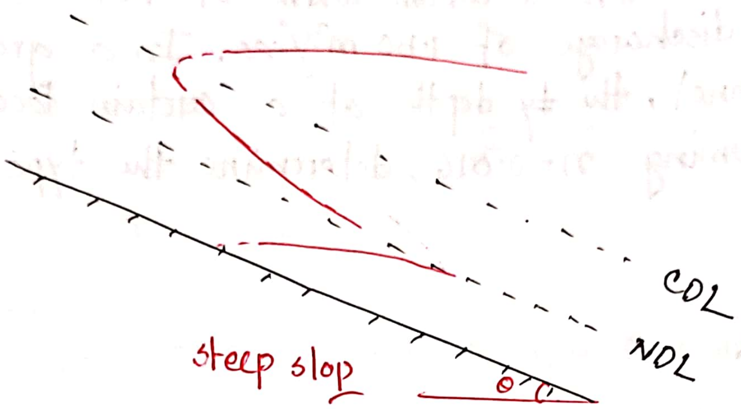
$$= 0.24 \text{ m}$$

Given that $y = 0.30$

$$y_0 > y > y_c \quad \text{(Region 2)}$$

water surface profile is of the M_2 type.

S - curves



$y > y_c > y_0$

$y_c > y > y_0$

$y_c > y_0 > y$

S_1 - Curve

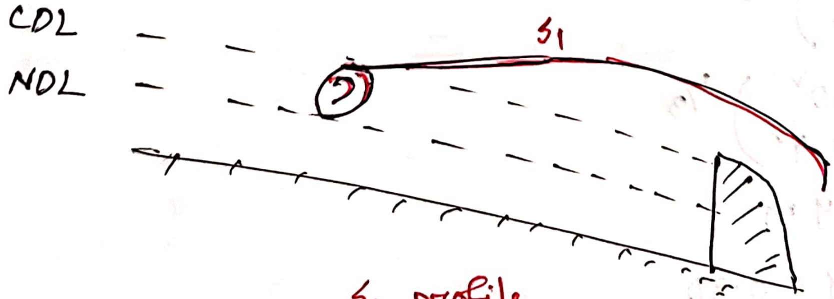
$y > y_c > y_0$

$y > y_0 \longrightarrow V < V_0 \longrightarrow S_e < S_0 \longrightarrow S_0 - S_e > 0$

$y > y_c \longrightarrow F_r < 1 \longrightarrow (1 - F_r^2) > 0$

$$\therefore \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{+} = +$$

(Water depth is increase in the flow direction)



S_1 profile

S_2 - Curve

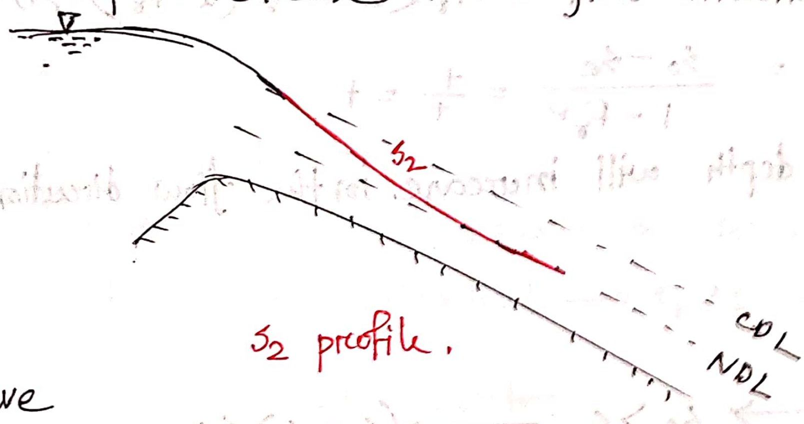
$y_c > y > y_0$

$y > y_0 \longrightarrow V < V_0 \longrightarrow S_e < S_0 \longrightarrow S_0 - S_e > 0$

$y < y_c \longrightarrow F_r > 1 \longrightarrow (1 - F_r^2) < 0$

$\therefore \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{-} = -$

(water depth decrease in the flow direction



S_2 profile.

S_3 - Curve

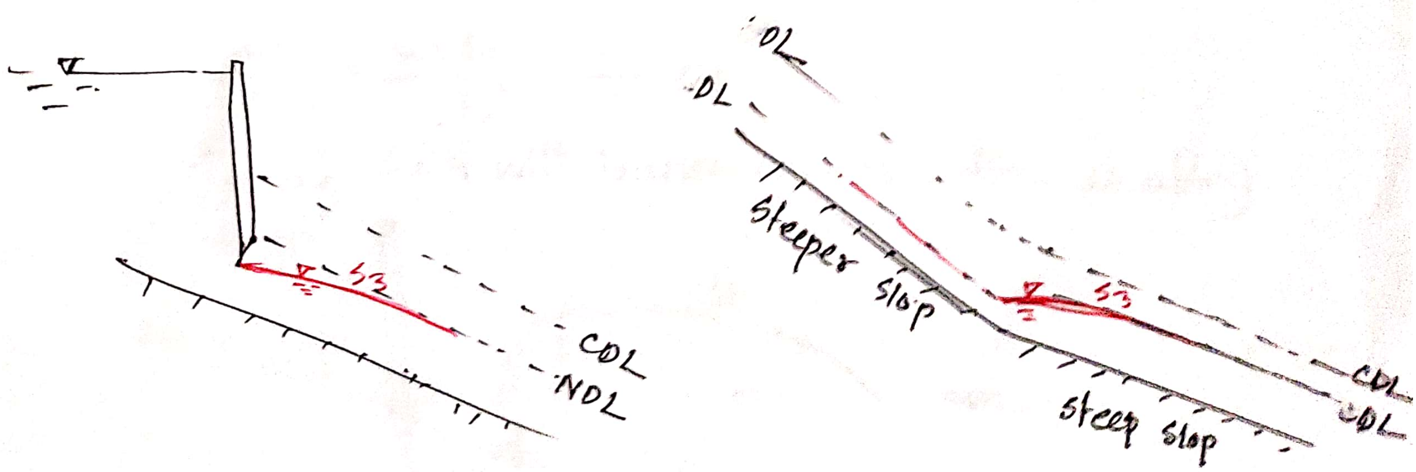
$y_c > y_0 > y$

$y < y_0 \longrightarrow V > V_0 \longrightarrow S_e > S_0 \longrightarrow S_0 - S_e < 0$

$y < y_c \longrightarrow F_r > 1 \longrightarrow (1 - F_r^2) < 0$

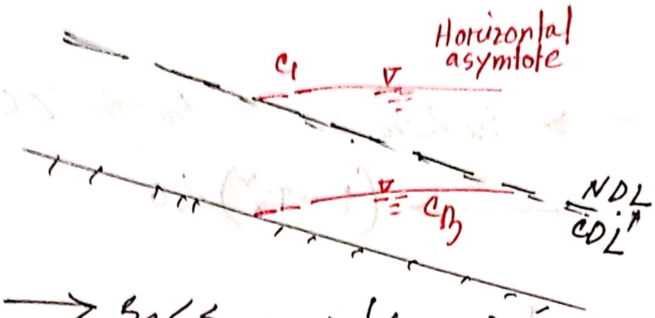
$\therefore \frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = +$

(water depth will increase in the flow direction)



S_3 profile.

C-Curve



$y > y_0 = y_c$

$y < y_0 = y_c$

C_1 curve

$y > y_0 \longrightarrow S_e < S_0 \longrightarrow (S_0 - S_e) > 0$

$y > y_c \longrightarrow$ subcritical $\longrightarrow F_r < 1 \longrightarrow (1 - F_r^2) > 0$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{+}{+} = +$$

(water depth will increase in the flow direction)

C_3 curve:

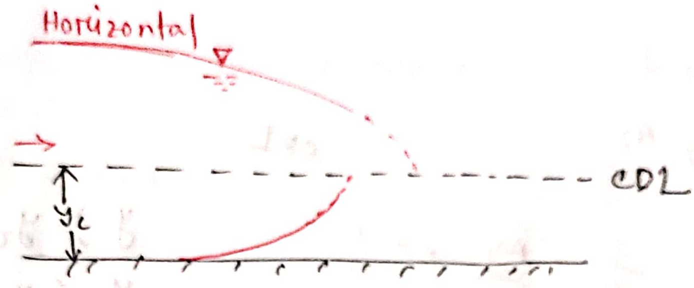
$y < y_0 \longrightarrow S_e > S_0 \longrightarrow (S_0 - S_e) < 0$

$y < y_c \longrightarrow$ supercritical $\longrightarrow F_r > 1 \longrightarrow (1 - F_r^2) < 0$

$$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = +$$

(water depth will increase in the flow direction)

H-Curve



$y > y_c$
$y < y_c$

H_2 - curve

$\infty > y > y_c$

$y > y_c \longrightarrow S_e < S_0 = 0 \longrightarrow (S_0 - S_e) > 0$

$y > y_c \longrightarrow$ subcritical $\longrightarrow F_r < 1 \longrightarrow (1 - F_r^2) > 0$

$\frac{dy}{dx} = \frac{+}{+} = +$ (water

??

H_3 - curve

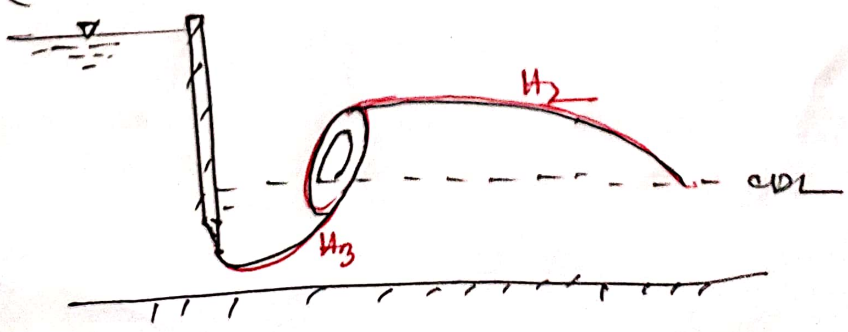
$y < y_c$

$y < y_c \longrightarrow S_e > S_0 = 0 \longrightarrow (S_0 - S_e) = < 0$

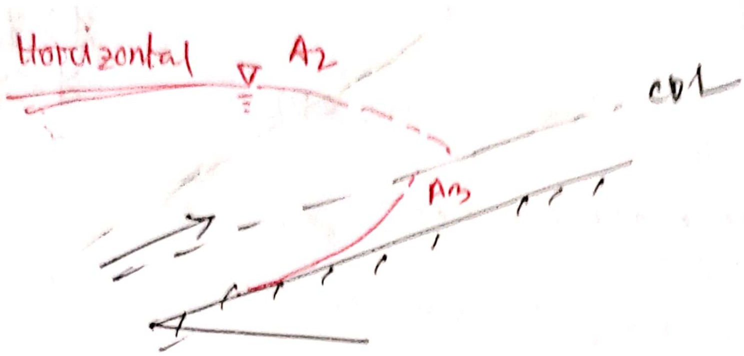
$y < y_c \longrightarrow$ supercritical flow $\longrightarrow F_r < 1 \longrightarrow (1 - F_r^2) < 0$

$\frac{dy}{dx} = \frac{S_0 - S_e}{1 - F_r^2} = \frac{-}{-} = +$

(water depth will increase in the flow direction)

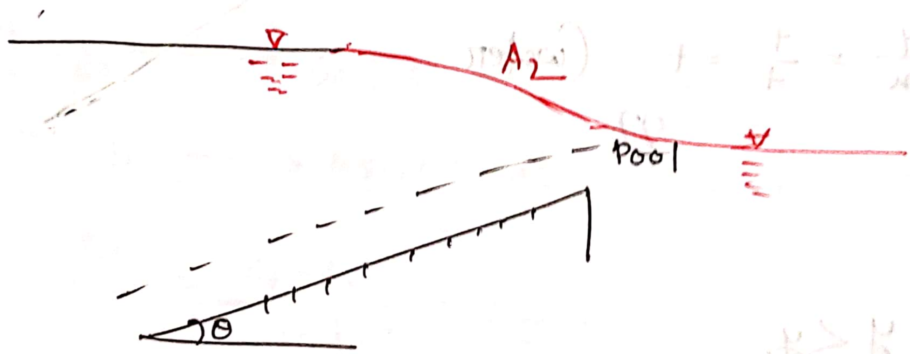


A curve



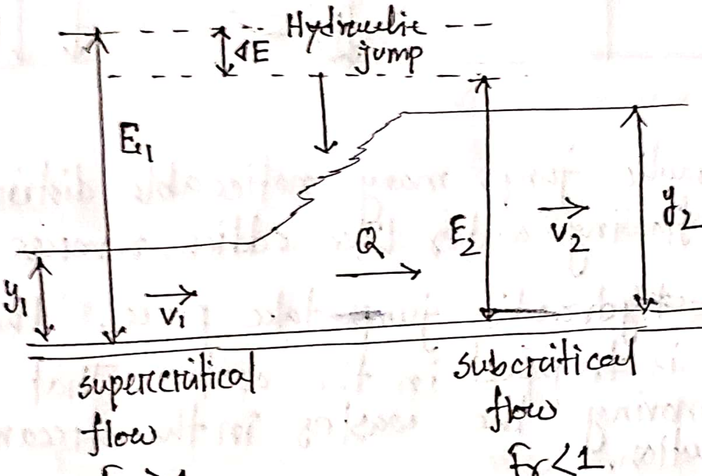
$y > y_c$

$y < y_c$

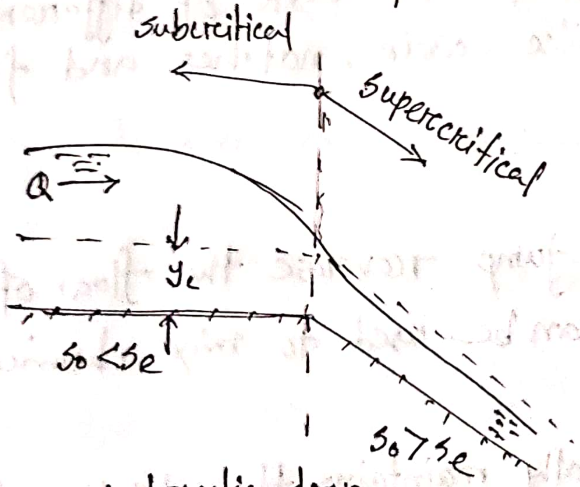


Hydraulic Jump

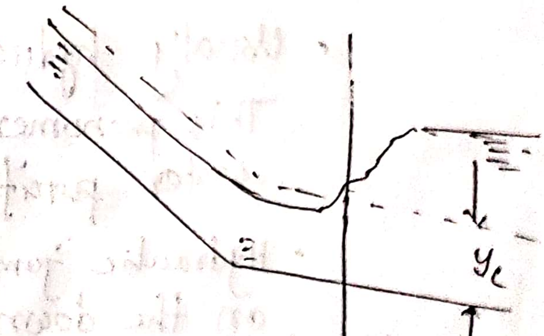
Definition:
A hydraulic jump is a phenomenon in the science of hydraulics which is frequently observed in open channel flow such as river and spillways. When liquid at high velocity discharges into a zone of lower velocity, a rather abrupt rise occurs in the liquid surface.



Hydraulic Jump



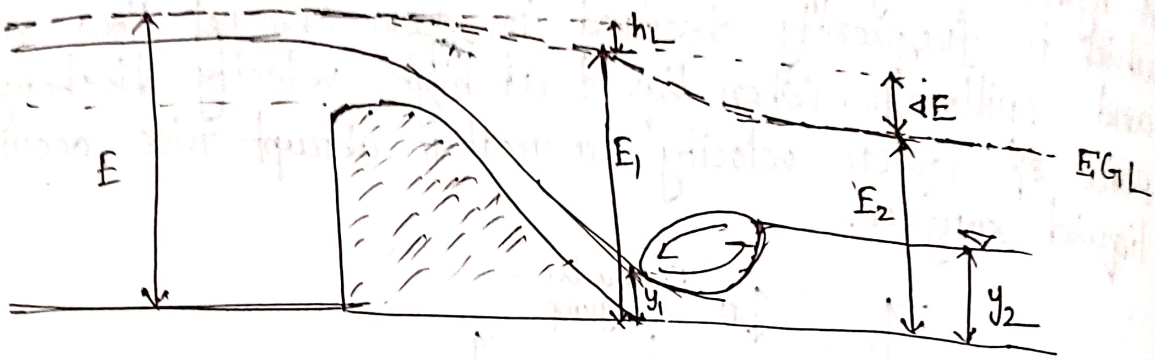
Hydraulic drop



Hydraulic jump

Effect of Hydraulic Jump

- Actually the ~~dic d~~ hydraulic jump usually acts as the energy dissipater. It distributes the surplus energy of water.



- Due to the hydraulic jump, many noticeable disturbance are created in the flowing water like eddies, reverse flow.
- Usually when the hydraulic jump take place, the considerable amount of air is trapped in the water. That air can be helpful in removing the wastes in the streams that are causing pollution.
- Hydraulic jump also make the work of different hydraulic structures, effective like weirs, notches and flumes etc.

Application of Hydraulic Jump:

- Usually hydraulic jump reverse the flow of water. This phenomenon can be used to mix chemicals for water purification
- Hydraulic jump usually maintains the high water level on the down stream side. This high water level can be used for irrigation purposes.

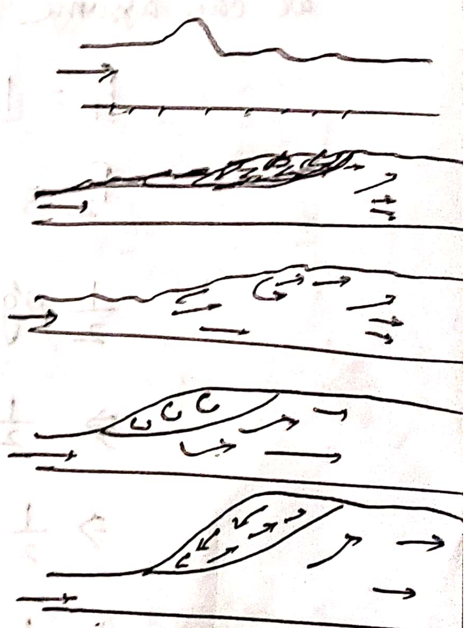
- Hydraulic jump can be used to remove the debris from water supply and sewage line to prevent the blocking.
- It prevents the scouring action on the downstream side of the dam structure.

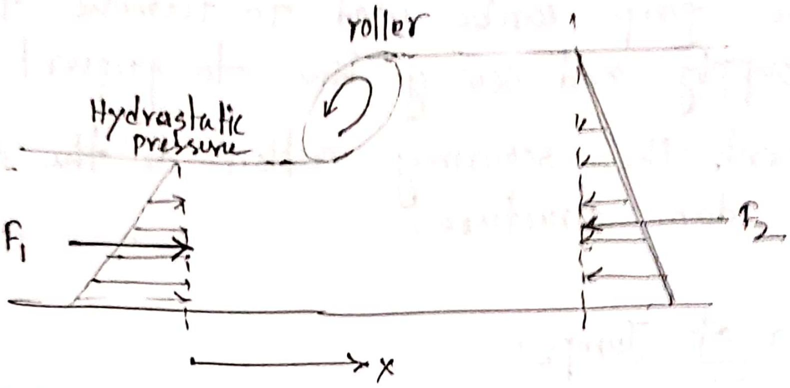
Classification of Jumps:

The most important factor that affects the hydraulic jump is the initial Froude number F1

$$F_1 = \frac{V_1}{\sqrt{gD}}$$ V_1 → longitudinal average velocity

Froude Number	Type of Jump
$F_r = 1$	Critical flow
$F_r = 1 \sim 1.7$	Undular jump
$F_r = 1.7 \sim 2.5$	Weak jump
$F_r = 2.5 \sim 4.5$	Oscilating Jump
$F_r = 4.5 \sim 9.0$	Steady jump
$F_r \geq 9.0$	Strong jump





Conservation of momentum

$$\sum F = \sum_{out} \text{Momentum} - \sum_{in} \text{Momentum}$$

$$\Rightarrow F_1 - F_2 = \rho Q (V_2 - V_1) \quad \text{———} \text{(I)}$$

If location (i) and (ii) are sufficiently away from the roller, we can assume the pressure distribution in hydrostatic

$$F_1 = P_c h_1 b = \left(\tfrac{1}{2}\rho g h_1\right) \cdot h_1 b = \tfrac{1}{2}\rho g h_1^2 b$$

$$F_2 = \tfrac{1}{2}\rho g h_2^2 b$$

$$\therefore \tfrac{1}{2}\cancel{\rho} g \cancel{b} (h_1^2 - h_2^2) = \cancel{\rho} V_1 h_1 \cancel{b} (V_2 - V_1)$$

$$\Rightarrow \tfrac{1}{2} g (h_1^2 - h_2^2) = V_1 h_1 (V_2 - V_1)$$

$$\Rightarrow \tfrac{1}{2} g (h_1 + h_2)(h_1 - h_2) = V_1 h_1 \left(\frac{V_1 h_1}{h_2} - V_1\right)$$

$$\Rightarrow \tfrac{1}{2} g (h_1 + h_2)\cancel{(h_1 - h_2)} = \frac{V_1^2 h_1}{h_2} \cancel{(h_1 - h_2)}$$

$$\Rightarrow \tfrac{1}{2}(h_1 + h_2) = \frac{V_1 h_1 V_2}{g h_2}$$

$$\Rightarrow (h_1 + h_2) = 2 F_{r_1}^2 \frac{h_1^2}{h_2}$$

$V_1 A_1 = V_2 A_2$

$V_1 h_1 = V_2 h_2$

$V_2 = \frac{V_1 h_1}{h_2}$

$F_{r_1}^2 = \frac{V_1^2}{g h_1}$

$$\Rightarrow \frac{h_2}{h_1^2}(h_1+h_2) = 2F_r^2$$

$$\Rightarrow \left(\frac{h_2}{h_1}\right)^2 + \left(\frac{h_2}{h_1}\right) - 2F_{r_1}^2 = 0 \rightarrow \text{[illegible]}\ ax^2+bx+c=0$$

$$F_{r_1}^2 = \frac{1}{2}\left[\left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1}\right]$$

$$\therefore \boxed{\frac{h_2}{h_1} = \frac{1}{2}\left(-1+\sqrt{1+8F_r^2}\right)}$$

Conservation of energy (to get head loss)

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} + h_L$$

$$\Rightarrow h_L = (h_1-h_2) + \frac{1}{2g}(v_1^2 - v_2^2)$$

$$= (h_1-h_2) + \frac{1}{2}F_r^2 h_1\left(1 - \frac{h_1^2}{h_2^2}\right)$$

$$\frac{h_L}{h_1} = \left(1 - \frac{h_2}{h_1}\right) + \frac{1}{2}F_r^2\left[1 - \left(\frac{h_1}{h_2}\right)^2\right]$$

$$\frac{1}{2g}(v_1^2 - v_2^2)$$

$$= \frac{1}{2g}\left(v_1^2 - \frac{v_1^2 h_1^2}{h_2^2}\right)$$

$$= \frac{v_1^2}{2g} \times \frac{h_1}{h_1}\left(1 - \frac{h_1^2}{h_2^2}\right)$$

$$= \frac{1}{2}F_r^2 h_1\left(1 - \frac{h_1^2}{h_2^2}\right)$$

Hence

$$\frac{h_L}{h_1} = (1-Y) + \frac{1}{4}(Y^2+Y)(1-Y^2)$$

$$= \frac{1}{4Y}(Y^3 - 3Y^2 + 3Y - 1)$$

$$= \frac{(Y-1)^3}{4Y}$$

$$= \frac{(h_2-h_1)^3}{4h_2h_1^2}$$

$$F_r^2 = \frac{1}{2}\left[\left(\frac{h_2}{h_1}\right)^2 + \frac{h_2}{h_1}\right]$$

$$Y = \frac{h_2}{h_1}$$

$$\therefore \frac{1}{2}F_r^2 = \frac{1}{4}\left[Y^2+Y\right]$$

$$\therefore \boxed{h_L = \frac{(h_2-h_1)^3}{4h_2h_1^2}}$$