Impact of Jets

1. Force exerted on a stationary plate
   a) Plate vertical
   b) " inclined
   c) " curved

2. force exerted on a moving plate
   a) Plate vertical
   b) " inclined
   c) " curved

Introduction:

Impact of jet = Change of momentum

$$= \frac{\Delta m V}{t}$$

$$= \dot{m} V$$

$$= \rho A V \cdot V \qquad [\because \dot{m} = \rho A V]$$

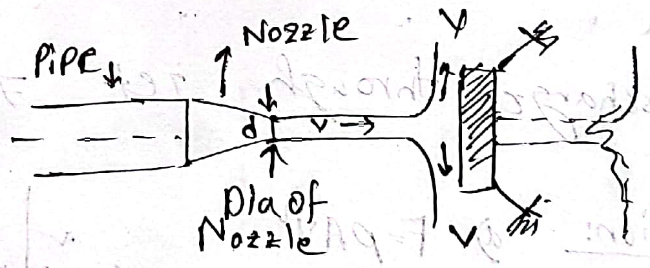
V — velocity at which strikes the jet

$$F = \rho A V^2$$

Unit: J/s or W

① Flat plates: Vertical

a) Stationary:



* Impact of jet $= \rho A V^2$
* Work done $= 0$

* Input power $= \frac{\text{kinetic energy of jet}}{1s}$

$$= \frac{1}{2} m v^2$$

$$= \frac{1}{2} (\rho A V) \times V^2$$

ii) moving:

* mass of water striking per sec $= m' = \rho A (V-u)$

* Force $= \rho A (V-u)^2$

* Work done by the jet on the plate, $W = F_x \times u$

ISO
t power. $\therefore W = \rho A (V-u)^2 u$

$IP = \frac{1}{2} \rho A V^3$
$OP = W$

V = of jet
u = of plate
V−u = when plate is moving away
V+u = plate moving towards

Problem 1 A jet of water of 50mm dia strikes a flat plate held normal to the direction of jet. Estimate the force exerted and work done by the jet if
a) The plate is stationary
b) " " " moving with a velocity 1 ms⁻¹ away from the jet along with the line of jet?
c) " " " " towards the " "
Discharge through jet 76 lps.

Solution: a) $F = \rho A V^2$
$= 1000 \times 1.96 \times 10^{-3} \times (38.07)^2$
$= 2934.47$ N

$Q = AV$
$\Rightarrow V = \frac{76 \times 10^{-3}}{1.96 \times 10^{-3}}$
$= 38.7$

$A = \frac{\pi}{4} (50 \times 10^{-3})^2$
$= 1.96 \times 10^{-3}$ m²

$\therefore u = 0$
$\therefore W = F \times u = 0$

$F_x = \rho A (V-u)^2$

$= 1000 \times 1.23 \times 10^{-3} (38.7-1)^2 = 2790 \text{ N}$

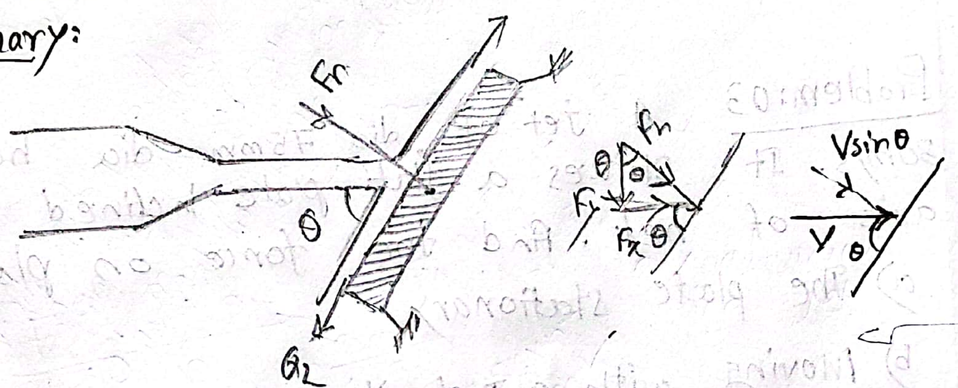
$W_x = F_x U = 2790 \times 1 = 2790 \text{ J/s} = 2790 \text{ W}$

$F_x = \rho A (V+u)^2 = 1000 \times 1.23 \times 10^{-3} (38.7+1)^2 = 3087.13 \text{ N}$

$W_x = F_x U = 3087.13 \times 1 = 3087.13 \text{ N/s} = 3087.13 \text{ W}$

Flat plates inclined:

i) Fixed, stationary:



* ~~$F_n = \rho A (V\sin\theta)$~~

* ~~$F_x = F$~~

* $F_n = (\rho A V)(V\sin\theta - 0)$ $\therefore$ $F_n = \rho A V^2 \sin\theta$

* $F_x = F_n \sin\theta =$

$\therefore$ $F_x = \rho A V^2 \sin^2\theta$

* $F_y = \rho A V^2 \sin\theta \cos\theta$

(b) Moving:

* $F_x = \rho A (V-u)^2 \sin\theta$

* $F_x = \rho A (V-u)^2 \sin^2\theta$

* $F_y = \rho A (V-u)^2 \sin\theta \cos\theta$

* $W = F_x \times u$

$\therefore W = \rho A (V-u)^2 \sin^2\theta \, u$ = output power = $F_x u$

Problem: 03 A jet of dia 75mm dia has a velocity of 30m/s. It strikes a flat plate inclined at 45° to the axis of jet. Find the force on plate when.

a) The plate stationary

b) Moving with a velocity 15m/s along and away from jet

c) Power and efficiency when moving.

Solution:

a) $\theta = 45°$

$A = 4.417 \times 10^{-3} \ m^2$

$V = 30 m/s$

$\therefore F_x = \rho A V^2 \sin^2\theta = 1987.65 \ N$

b) $F_x = \rho A (V-u)^2 \sin^2\theta = 497 \ N$ (away)

$F_x = \rho A (V+u)^2 \sin^2\theta = 2236 \ N$ (towards)

Power, or work done per sec = $W_x = \rho A (v-u)^2 \sin^2\theta \, u$

$\therefore$ Output power, $OP = W = F_x u$

$= 497 \times 15 = 7455.38 W$

Input ", IP = kinetic energy of jet per sec

$= \frac{1}{2} m v^2$

$= \frac{1}{2} \rho A v \cdot v^2$

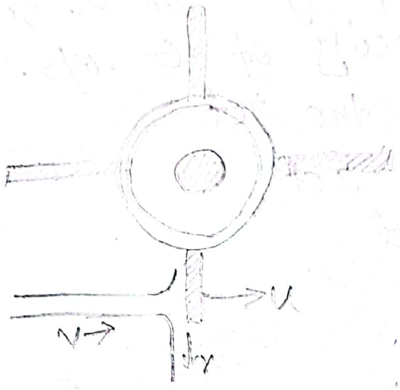
$= 59627.5 W$

$\therefore$ Efficiency $= \frac{~~59627.5~~ \; 7455.38}{59627.5} = 12.5\%$ Ans.

Problem: 4. A 75mm diameter jet ~~moving~~ having a velocity 12 m/s impinges a smooth flat plate, the normal of which is inclined 60° to the axis of jet. Find the ~~no~~ impact of jet at right angles to the plate when it's stationary

What will be the impact if the plate moves with a velocity of 6m/s in the direction of jet and away from it. What will be the force if the plate moves towards the plate.

Flat exerted by jet on series of flat vanes/plates:



Volume rate of water striking, $Q = AV$

mass " " " " $Q_m = \rho AV$ → because plates are stationary

V_{x_1} = initial velocity at point of impact $= (V-u)$

V_{x_2} = final velocity at exit $= 0$

$\therefore F = F_x = Q_m(-\Delta V_x) = \rho AV(V-u)$ $\quad \therefore$ $\boxed{F_x = \rho AV(V-u)}$

$\therefore W = F_x u = \rho AVu(V-u) = OP$

Energy provided by jet per sec $= \frac{1}{2} Q_m V^2$

$IP = \frac{1}{2}\rho AV^3$

$\therefore$ Efficiency $= \frac{OP}{IP} = \frac{\rho AV u(V-u)}{\frac{1}{2}\rho AV V^2}$

$\therefore \eta = \frac{2u(V-u)}{V^2}$

* A jet of water 50mm in diameter, moving with velocity of 15m/s, impringes on a series of vanes moving with a velocity of 6 m/s. Find
(a) force exerted by the jet
(b) Work done by the jet
(c) Efficiency of the jet

Solution:

(a) $A = \frac{\pi}{4} \times (50\times10^{-3})^2 = 1.73\times10^{-3}$

$V = 15\ m/s$

$u = 6$

$F_x = \rho A V(V-u) = 1.96\times10^{-3} \times 1000 \times 15 \times (15-6)$

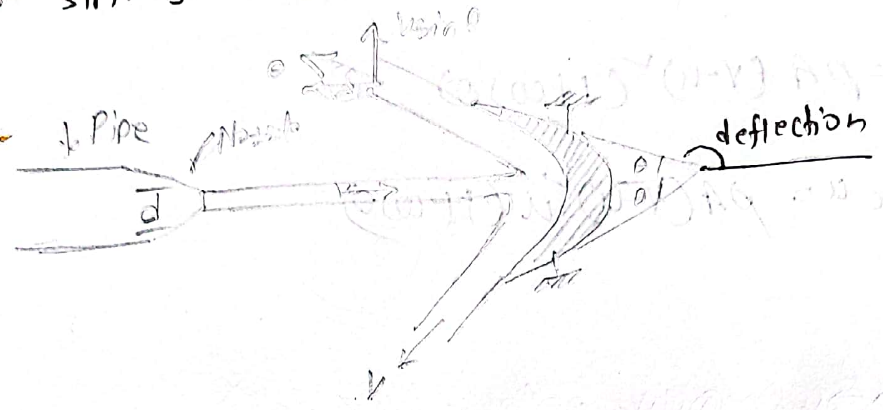
$= 265\ N$

(b) $W = F_x u = \rho A V(V-u) \times u = 265 \times 6 = 1590\ W$

(c) $\eta = \frac{2u(V-u)}{V^2} = \frac{2\times6(15-6)}{15^2} = 48\%$

Curved Vane: (stationary)

(a) Jet strikes at center:



$\therefore F_x = m\Delta V = m\,[V - (-V\cos\theta)]$

$\therefore F_x = m\,(V + V\cos\theta)$

$\boxed{\therefore F_x = \rho A V^2 (1 + \cos\theta)}$

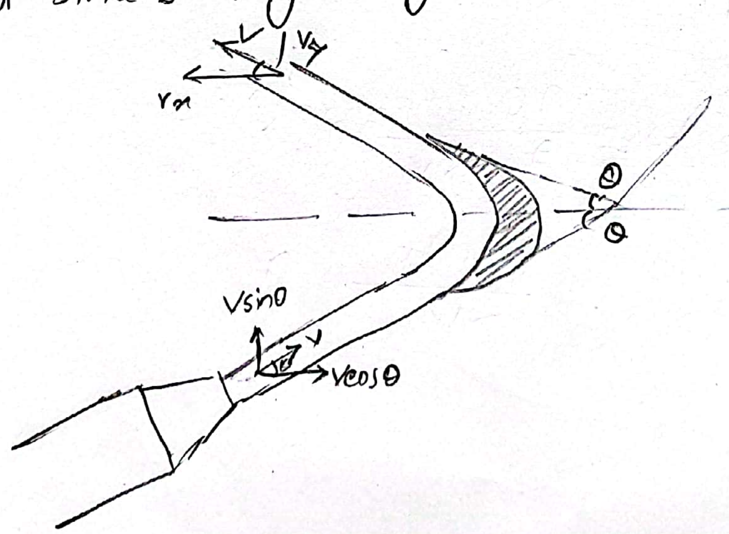
Angle of deflection = $180 - \theta$

$F_y = \rho A V\,[0 - V\sin\theta]$

$\boxed{F_y = -\rho A V^2 \sin\theta}$

$F_n = \sqrt{F_x^2 + F_y^2}$

(b) Jet strikes tangentially:



$F = \rho A V\,[V\cos\theta - (-V\cos\theta)]$

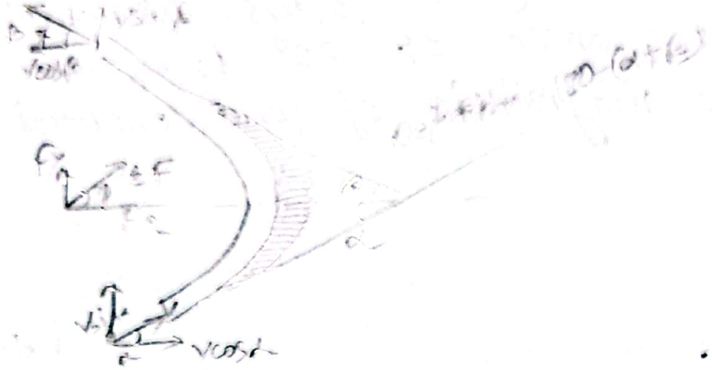
$\boxed{F_x = 2\rho A V^2 \cos\theta}$

deflection = $180 - 2\theta$

$F_y = 0$

Stationary Curved Vane: (unsymmetrical)

(i) Strikes at one end



$F_x = \rho A V [V\cos\alpha + V\cos\beta]$

$\therefore F_x = \rho A V^2 [\cos\alpha + \cos\beta]$

$F_y = \rho A V [V\sin\beta - V\sin\alpha]$

$\therefore F_y = \rho A V^2 [\sin\alpha - \sin\beta]$

Resultant force, $\Sigma F = \sqrt{F_x^2 + F_y^2}$

Angle of Resultant, $\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

Angle of deflection, $= 180 - (\alpha + \beta)$

Problem: A jet water strikes a stationary curved plate tangentially at one end at an angle 30°. The jet of 75mm diameter has a velocity of 30m/s. The jet leaves at the other end at angle of 20° to the horizontal. Determine the magnitude of force exerted along x and y directions.

Solution:

$\alpha = 30^\circ$
$\beta = 20^\circ$
$V = 30$
$A = \frac{\pi}{4} \times (75\times10^{-3})^2$
$= 4.417\times10^{-3}$

$F_x = \rho A V^2 (\cos\alpha + \cos\beta)$

$= 7178\ N$

$F_y = \rho A V^2 (\sin\alpha - \sin\beta)$

$= 628\ N$

Answer.

Problem: A jet of water of diameter 5cm moving with a velocity of 25 m/s impinge on a fixed curved plate tangentially at one end at an angle of 30° with the horizontal. Determine force of the jet on the plate in the horizontal and the vertical direction if the jet is deflected at an angle of 130°. Also find the direction of resultant force.

Solution: Here,

$A = \frac{\pi}{4} \times (5\times10^{-2})^2 = 1.76\times10^{-3}\ m^2$

$V = 25$

$\alpha = 30^\circ$

$\alpha + \beta = 180 - 130 = 50^\circ$

$\therefore \beta = 20^\circ$

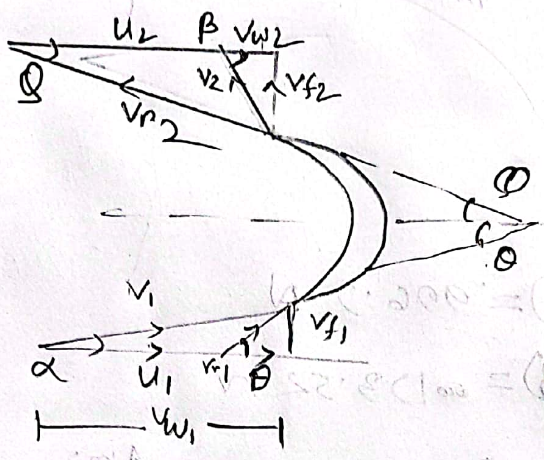
$F_x = \rho A V^2 (\cos\alpha + \cos\beta) = 996.7\ N$

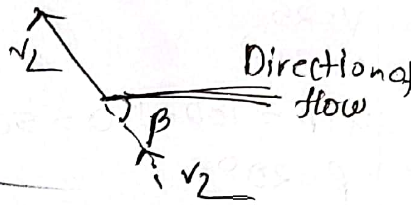
$F_y = \rho A V^2 (\sin\alpha - \sin\beta) = 173.52\ N$

$\therefore \phi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = -10^\circ$

Ans:

(ii) Unsymmetrical + Jet strikes at one of the tips:





Here, $V_{r_1} = V_{r_2}$

$$F_x = \rho A V_{r_1} \times [V_{w_1} \pm V_{w_2}]$$

$$W_x = F_x \times U$$

* W_x per unit weight $= \dfrac{F_x U}{(\rho A V_{r_1}) g}$ $\quad | \quad \rho A V_{r_1} = m$

$\rightarrow \dfrac{U(V_{w_1} \pm V_{w_2})}{g}$

* " " " $= \dfrac{F_x U}{(\rho A V_{r_1})}$

$\rightarrow U \times (V_{w_1} \pm V_{w_2})$

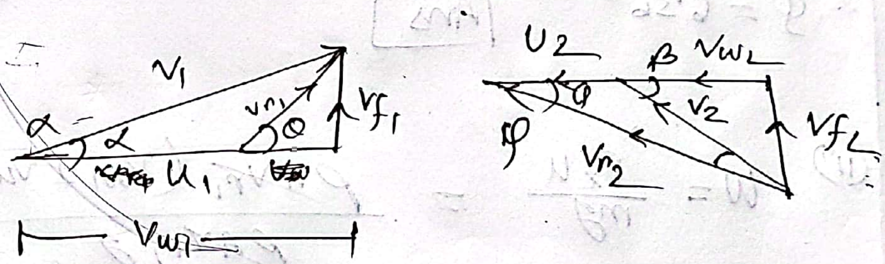
* $\eta = \dfrac{F_x U}{\frac{1}{2} \rho A V_1^3}$

#Problem 17.18 A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane of an outlet.

(i) Vane angles, so that the waters enters and leaves without shock

(ii) Work done per second per unit of water

Soln:

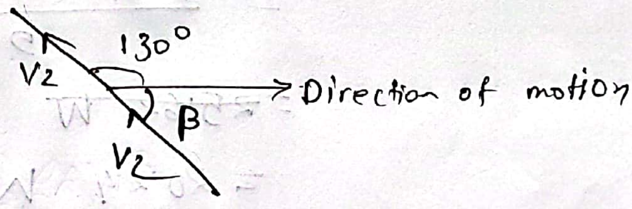


$U_1 = 10\ ms^{-1} = u_2$

$V_1 = 20$

$\alpha = 20°$

$\beta = 180 - 130° = 50°$



$\theta = \tan^{-1}\left(\frac{V_{f_1}}{V_{w_1} - u_1}\right)$

$= \tan^{-1}\left(\frac{20 \sin 20}{20 \cos 20 - 10}\right)$

$= 37.85°$

$\therefore V_{w_1} = V_1 \cos 20° = 18.79$

$V_{f_1} = V_1 \sin 20° = 6.84$

$V_{r_1} = \sqrt{V_{f_1}^2 + (V_{w_1} - u_1)^2} = 11.13$

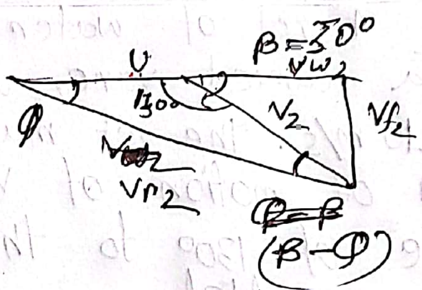
$u_1 = u_2 = u$

$V_{r_1} = V_{r_2}$

Now,

$V_{r_1} = V_{r_2} = 11.13$

Now, $u = 10$



sine rule,

$$\frac{V_{r_2}}{\sin(180-\beta)} = \frac{u}{\sin(\beta-\varphi)}$$

$\therefore \sin(\beta-\varphi) = 0.677$

$\Rightarrow$ ~~Sin~~ $\beta-\varphi =$ ~~42.68°~~ 43.44

$\therefore \varphi = 50 - 43.44 = 6.56°$

$\therefore \varphi = 6.56°$ **Ans**

$$\frac{V_2}{\sin 6.56} = \frac{V_{r2}}{\sin 130}$$

$\therefore V_2 =$ ~~3.884~~ 1.6

$V_{w_2} = V_2 \cos 50 = 2.49$

$V_{w_2} = V_{r_2} \cos 6.56 - u$
$= 1.226$

(iii) $$W = \frac{F \times u}{mg} = \frac{\rho A V_{r_1} (V_{w_1} \pm V_{w_2}) \times u}{\rho A V_{r_1} g}$$

$$= \frac{10 \times (18.79 + 1.226)}{9.8}$$

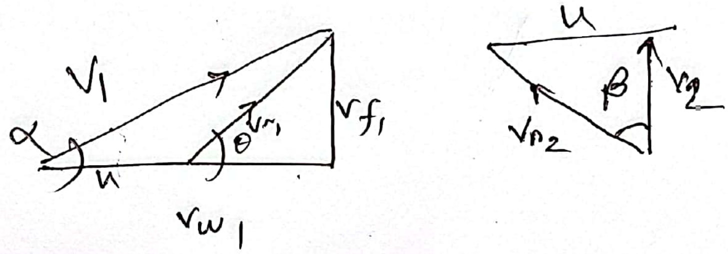
$=$ ~~25.62 W~~

$= 20.24$ W

Ans:

Problem 17:19 A jet of water having a velocity of 40m/s strikes a curved vane, which is moving with a velocity of 20m/s. The jet makes an angle of 30° with the direction of motion of vane at ~~out let~~ inlet and leaves at 90° to the direction of motion of vane at outlet. Draw the velocity triangles.

Ans:



$$\theta = \tan^{-1}\left(\frac{V_{f_1}}{V_{w_1}-u}\right)$$

$$= \tan^{-1}\left(\frac{40\sin 30^\circ}{40\cos 30^\circ - 20}\right) = 53.8^\circ$$

$$\therefore \theta = 53.8^\circ$$

$$V_{r_1} = \sqrt{V_{f_1}^2 + (V_{w_1}-u)^2} = 24.78$$

$$\therefore \beta = \cos^{-1}\left(\frac{u}{V_{r_2}}\right) = 36.18^\circ$$

Ans:

Problem 17.10

A jet of water of dia 25mm strikes a 20x20 cm^2 plate of uniform thickness with a velocity of 10 m/s at the center of the plate which is suspended vertically by a hinge on it's top horizontal edge. The weight of the plate 98.1N. The jet strikes normal to the plate (i) What force must be applied at the lower edge so that the plate is vertical?

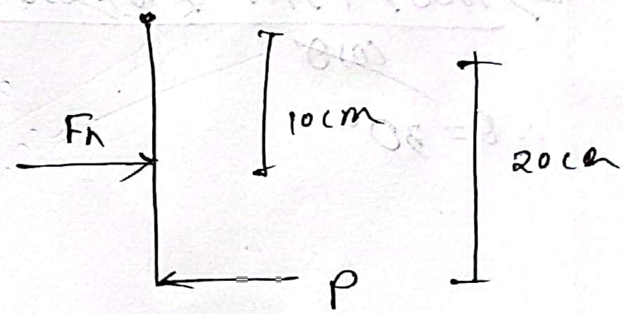
(ii) If the plate is to swing freely, what will be the inclination of the plate due to the force of water?

Solution:

$A = 4.9 \times 10^{-4}$

$V = 10$

$F_n = \rho A V^2$



① Taking moment at hinge,

$F_n \times 10 = P \times 20$

$\Rightarrow P = \frac{F_n}{2} = \frac{\rho A V^2}{2} = 24.5\,N$ □

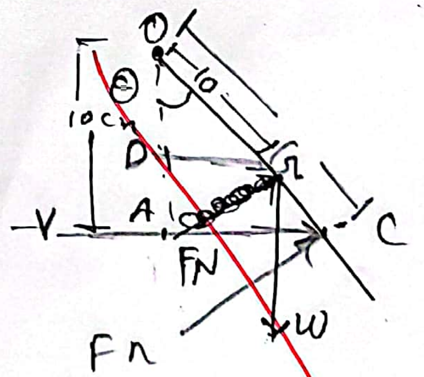
(ii) $OC = \frac{10}{\cos\theta}$

$DG = OG \cos\theta = 10 \sin\theta$ ← care

∴ Taking moment about hinge,

$F_N \times OC = W \times DG$

$\Rightarrow \frac{\rho A V^2 \times 10}{\cos\theta} = 98.1 \times 10 \sin\theta$

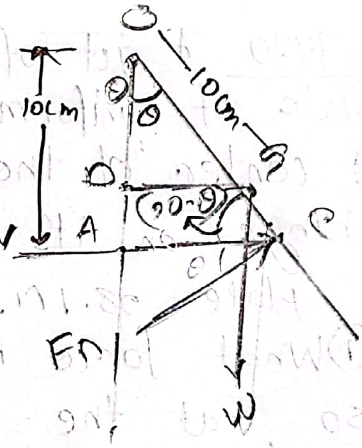


$\therefore OG = \frac{10}{\cos\theta}$

$DG = 10\sin\theta$

$F_N = \rho A V^2 \cos\theta$

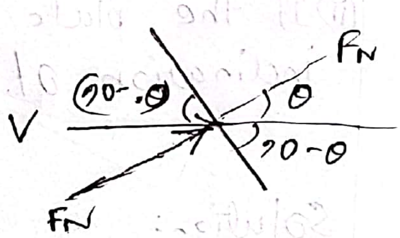
$= 1000 \times 4.7 \times 10^{-4} \times 10^2 \times \cos\theta$



Taking moment about Hinge O,

$F_N \times OC = W \times DG$

$\Rightarrow \frac{1000 \times 4.7 \times 10^{-4} \times 10^2 \times \cos\theta \times 10}{\cos\theta} = W \times 10\sin\theta$



$\therefore \theta = 30°$

Ans: