

Ch-02 Open-channel Flow and its classification

Open channel flow:

- is a type of liquid flow in a conduit that has a free surface, and the liquid is in atmospheric pressure and the flow happens due to gravity.

Pipe flow:

- is a type of flow with no atmospheric pressure but hydraulic pressure, has no free surface.

Difference between open channel & pipe flow:

	Open channel flow	Pipe flow
01.	Free surface	01. No free surface
02.	Atmospheric Pressure	02. No atmospheric pressure
03.	No hydraulic "	03. Has hydraulic "
04.	Flow condition complicated	04. Flow condition simple
05.	Reliable experiment data is hard to obtain	05. Easier to obtain
06.	Cross-section can be any shape and not fixed	06. Generally round fixed
07.	Maximum Velocity just below free surface	07. Max V at middle of flow
	Treatment of open channel flow is empirical	08. Analytical

$v = \text{velocity of flow}$

41 if $F < 1$, Sub critical

51

61

20, 17, 15, 13

* Why Open channel flow more complicated?

Ans: Flow conditions of open channel flow are complicated by the fact that depth of flow in and the position of free surface is to change with time and space.

- Also discharge, the slopes, free surface, depth of flow are interdependent

- Reliable experimental data is hard to obtain.
- Cross section varies
- Physical conditions change much more widely.
- Roughness is more wide ranged.

Therefore selection of the friction co-efficients is attended by greater uncertainty than pipes.

Classification of open channel flow:

A. Steady flow

1. Uniform
2. Varied
 - a. Gradually varied
 - b. Rapidly "

B. Unsteady flow

1. Unsteady uniform
2. Unsteady varied
 - a. Gradually varied unsteady
 - b. Rapidly " " "

CH-02 Open channels and properties

Types of Open channels:

A. According to origin

- (i) Natural
- (ii) Artificial

B. According to shape/Geometry

- (i) Prismatic — Unvarying cross-section, bed slope
- (ii) Non prismatic — Varying " , bed slope

Geometric elements of channel section:

* depth of flow, y — distance of the lowest point of a section to the free surface

* top width, T — Total width of section at free surface

* water area, A — Area of flow of section

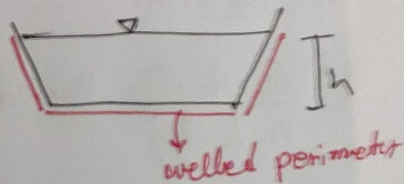
* Wetted perimeter, P —

* Hydraulic Radius, $R = \frac{A}{P}$

* Hydraulic Depth, $D = \frac{A}{T}$

* section factor, $Z = A\sqrt{D}$

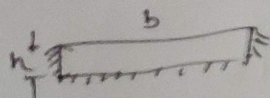
* h = Depth of flow



Wide channels:

when the width of the channel is very large compared to depth ($b \geq 10h$) the channel is called wide channel. Ex: Alluvial River

$$R = \frac{A}{P} = \frac{bh}{b+2h} \approx \frac{bh}{b} = h$$



Regimes of flow:

A combination of viscosity and gravity may cause 4 regimes in an open channel.

- 1. Subcritical laminar $\left\{ \begin{array}{l} F < 1, R < 200 \\ F > 1, R < 200 \end{array} \right.$
- 2. Supercritical " $\left\{ \begin{array}{l} F < 1, R < 200 \\ F > 1, R < 200 \end{array} \right.$
- 3. Subcritical turbulent $\left\{ \begin{array}{l} F < 1, R < 200 \\ F > 1, R < 200 \end{array} \right.$
- 4. Supercritical " $\left\{ \begin{array}{l} F < 1, R < 200 \\ F > 1, R < 200 \end{array} \right.$

(i) Steady flow: Flow characteristics does not change over time. $\frac{\partial v}{\partial t} = 0, \frac{\partial y}{\partial t} = 0, \frac{\partial Q}{\partial t} = 0$

(ii) Unsteady flow: $\frac{\partial v}{\partial t} \neq 0, \frac{\partial y}{\partial t} \neq 0, \frac{\partial Q}{\partial t} \neq 0$

(iii) Non uniform: Cross-section, depth, slope and velocity change over length of channel.

(iv) Uniform: " " " " remain constant

(v) Gradually Varied flow: " " " " changes gradually

(vi) Rapidly varied flow: " " " " rapidly.

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Effect of Gravity:

It's represented by the ratio of inertia forces to gravity forces. It's denoted as Froude Number,

$$F = \frac{V}{\sqrt{gL}}$$

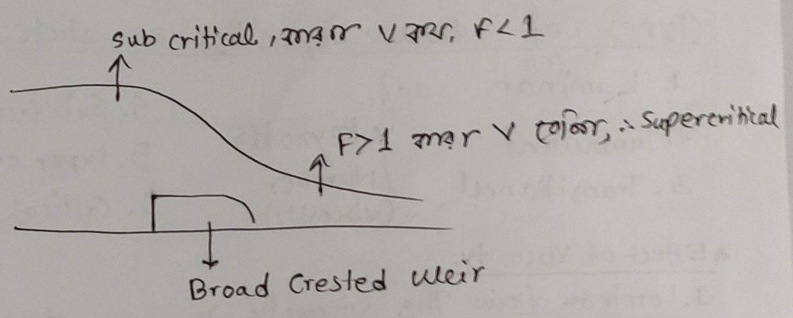
$$= \frac{V}{\sqrt{gD}}$$

; L = characteristic length = hydraulic flow depth, D
V = velocity of flow

4) if $F < 1$, Sub critical

5) $F > 1$, super "

6) $F = 1$, critical $\therefore V = \sqrt{gD}$ = critical velocity



* Celerity of Gravity Waves:

In the mechanics of water waves, \sqrt{gD} is denoted as ~~gavi~~ the celerity of gravity waves.

This occur in shallow water due to momentary change in local depth of water. Such change cause water to flow above or below average depth thus creating a wave that exerts a weight or gravity force.

Since the celerity is greater than subcritical flow thus ~~the~~ a gravity wave propagate in the upstream of subcritical flow

Time

- * Steady flow: Depth of flow doesn't change with Time
- * Unsteady flow: " " " Change " "

Space

- * Uniform Flow: Depth of flow same at every section of channel
- * Varied Flow: " " " not same " "

state of Flow 8

state of flow of open channel is governed by the effects of viscosity and gravity relative to the inertial force.

Types of flow depending on state:

- | | | | |
|-----------------|-------------------------------|-------------------|-----------------------------|
| 1. Laminar | } Reynolds Number (viscosity) | 4. Sub critical | } Froude's Number (Gravity) |
| 2. Turbulant | | 5. Super critical | |
| 3. Transitional | | 6. Critical | |

* Effect of Viscosity:

1. Laminar flow: The viscosity is so strong than inertial force is that the particles move in a line a path.
 $Re \leq 500-600$

2. Turbulent: Viscous forces are weak relative to inertial force, so particles move in irregular paths.
 $Re > 2000$

3. Transitional:

Between laminar & Turbulent state
 $600 < Re < 2000$

* The effect of viscosity relative to inertia can be represented by Reynolds number,

$$Re = \frac{VL}{\nu} = \frac{VR}{\nu}$$

$\nu = \text{velocity} = 10^{-6}$
 $L = \text{characteristic length} = \text{Hydraulic Radius} = R$
 $\nu = \text{kinematic viscosity ft}^2/\text{sec} = 10^{-6} \text{ m}^2/\text{s}$

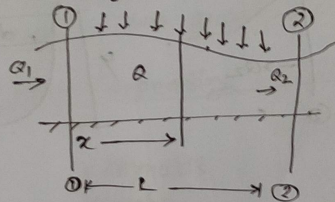
$\frac{\text{inertia force}}{\text{viscous force}}$

Continuity Equations:

(i) for steady flow:

In this case discharge is same for every section, $\therefore Q = A_1 V_1 = A_2 V_2 = \dots$

(ii) for steady spatially varied flow:



In spatially varied flow, discharge isn't same everywhere,

considering spatially varied flow with increasing discharge at section -1.

\therefore The rate of addition of discharge $\frac{dQ}{dx} = q$

\therefore The discharge at any section at a distance x from section -1,

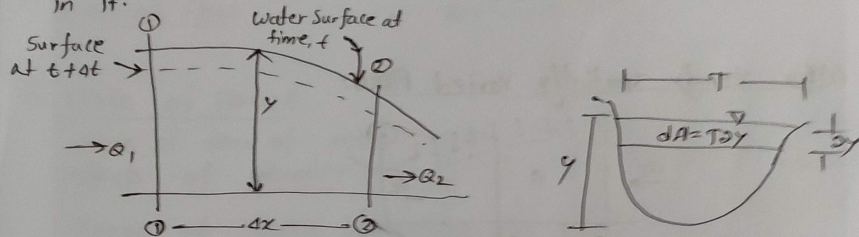
$$Q = Q_1 + \int_0^x q dx$$

$$\therefore Q = Q_1 + qx$$

$$\therefore Q_2 = Q_1 + qL$$

(iii) For Unsteady flow:

In a reach of a channel the continuity equation states the discharge going out of the boundary surface of the reach is equal to the depletion of the storage in it.



In figure $Q_2 > Q_1$. The excess volume in outflow surface in time Δt is made good by the depletion of storage. As a result the water surface will start falling.

If Δx is the distance between sec-1 & sec-2,

$$\therefore \text{The excess volume of outflow, } Q_2 - Q_1 = \frac{dQ}{dx} \Delta x$$

$$\text{" " " " " " in time } \Delta t, = \frac{dQ}{dx} \Delta x \cdot \Delta t \quad \text{--- (1)}$$

If T is top width of the channel at any depth y , then $\frac{dA}{dy} = T$

$$\text{Storage volume at for depth } y, = A \Delta x$$

Rate of decrease of storage at depth 'y',

$$= -Ax \frac{\partial A}{\partial y}$$

$$= -Ax \frac{\partial A}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$= -Ax \cdot T \cdot \frac{\partial y}{\partial t}$$

∴ Decrease in ~~time~~ ^{storage} ~~At~~ in time Δt , = $-Ax \cdot T \cdot \frac{\partial y}{\partial t} \cdot \Delta t$

①

By continuity, ① = ②

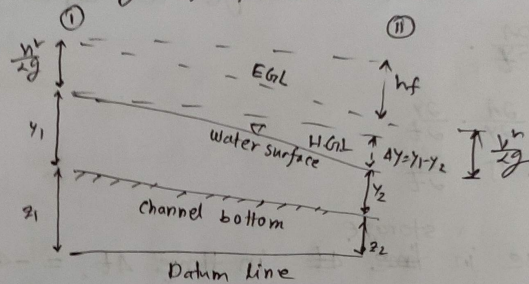
$$\frac{\partial Q}{\partial x} \cdot Ax \cdot \Delta t = -Ax \cdot T \cdot \frac{\partial y}{\partial t} \cdot \Delta t$$

$$\Rightarrow \frac{\partial Q}{\partial x} + T \frac{\partial y}{\partial t} = 0$$

Basic equation of continuity for unsteady incompressible flow. or

"The net change in discharge and the change in storage is zero"

④ Theoretical Discharge of open channel flow:



Total energy at section (I) $z_1 + y_1 + \frac{v_1^2}{2g} = E_1$

" " " " (II) $z_2 + y_2 + \frac{v_2^2}{2g} + h_f = E_2$

Now, $E_1 = E_2$

$\Rightarrow z_1 + y_1 + \frac{v_1^2}{2g} = z_2 + y_2 + \frac{v_2^2}{2g} + h_f$

$\Rightarrow z_1 + z_2 + y_1 - z_2 + \frac{v_1^2}{2g} = \frac{v_2^2}{2g} + h_f$

\therefore If the slope is small, $\therefore z_1 - z_2 = 0$ And $y_1 - y_2 = \Delta y$

$\Rightarrow \Delta y + \frac{v_1^2}{2g} = \frac{v_2^2}{2g} + h_f$

$\Rightarrow \frac{v_1^2}{2g} = \frac{v_2^2}{2g} + h_f - \Delta y$

$\therefore v_1 = \sqrt{v_2^2 + 2g(h_f - \Delta y)}$ ——— (I)

Again, from continuity equation, $v_1 A_1 = v_2 A_2$

$\therefore v_1 = \frac{v_2 A_2}{A_1}$ ——— (II)

\therefore (I) = (II)

$\frac{v_2 A_2}{A_1} = \sqrt{v_2^2 + 2g(h_f - \Delta y)}$

$\Rightarrow \frac{v_2^2 A_2^2}{A_1^2} = v_2^2 + 2g(h_f - \Delta y)$

$\Rightarrow v_2^2 \left(\frac{A_2^2}{A_1^2} - 1 \right) = 2g(h_f - \Delta y)$

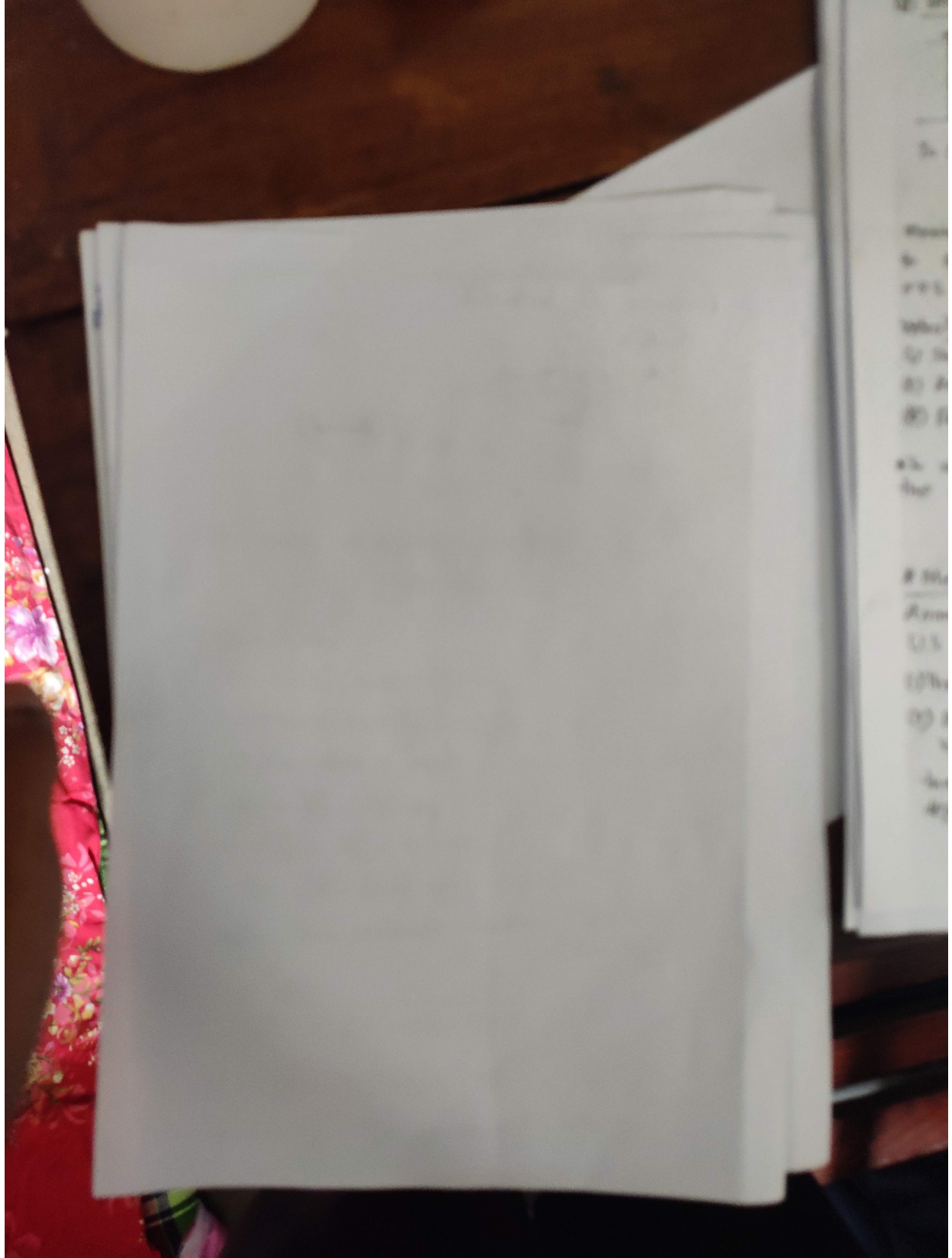
$\therefore v_2 = \sqrt{\frac{2g(h_f - \Delta y)}{1 - \frac{A_2^2}{A_1^2}}}$

∴ Discharge at section ②

G. L. W.

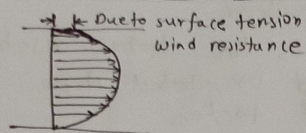
$$= \frac{4.2 \sqrt{0.9(9.8)}}{1.48}$$

(Answer)

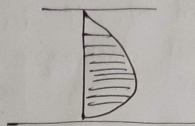


Ref: Vete chow

Velocity Distribution in a channel section:



In Open channel



In pipes

Maximum velocity in most channels usually appears to occur below the free surface at a distance 0.05 to 0.25 of the depth

Velocity Distribution depends on:

- (i) Shape of the section
- (ii) Roughness of the channel
- (iii) Presence of bends

* In shallow, ^(wide) broad, rapid stream, max velocity is at the free surface.

* Measurement of Velocity:

According to the stream-gaging procedure of the U.S Geological survey:

- (i) The channel cross-section is divided into vertical strips
- (ii) Mean velocities are determined by measuring velocity at 0.6 of the depth or by taking the average of velocities at 0.2 and 0.8 depth

* Velocity coef

Let, ΔA be
and w be

∴ The weight
the width

$$W = w \Delta A v$$

∴ The weight

$$E_k = \frac{1}{2} m v^2$$

$$= \frac{1}{2} w \Delta A v^2$$

$$\therefore E_k = \frac{1}{2} w \Delta A v^2$$

which is
and velocity

∴ The total

~~we know~~

∴ Correct

(iv) This velocity is then multiplied by the area of the strip and discharge is acquired.

(v) The sum of all discharge is the total discharge.

(vi) The total discharge divided by total Area is thus the mean velocity of section.

Velocity Distribution Co-efficients: (velocity Co-efficient)

* When the energy principle in computation the velocity head may be written as

$$\frac{\alpha V^2}{2g}$$

α = energy coefficient
= Coriolis coefficient

for straight prismatic channels $1.03 < \alpha < 1.36$

This value is higher for small channels lower for large channels

* Momentum Co-efficient:

momentum coefficient is β , = Boussinesq Co-eff

\therefore momentum of the fluid passing through a section per unit time

$$= \frac{\beta \rho Q V}{g}$$

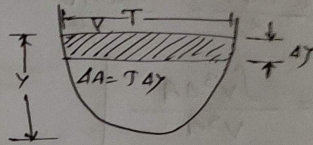
For straight prismatic channels $1.01 < \beta < 1.12$

* Mean Velocity:

$$U = \frac{Q}{A}$$

☐ Determination of velocity distribution co-effs:

* Velocity coefficient:



Let, ΔA be an elementary area of whole area A ,
and w the unit weight of water,

\therefore The weight of water passing through ΔA per unit time with a velocity v ,

$$W = w \Delta A v$$

\therefore the kinetic energy of water passing per unit time,

$$E_k = \frac{1}{2} m v^2$$

$$= \frac{1}{2} \frac{w \Delta A v}{g} \times v^2$$

$$\therefore E_k = \frac{w \Delta A v^3}{2g}$$

which is equivalent of the product of weight $w \Delta A v$
and velocity head $\frac{v^2}{2g}$

\therefore The total kinetic energy for the water Area

$$= \sum \frac{w \Delta A v^3}{2g} \quad \text{--- (i)}$$

$$= \frac{w A v^3}{2g}$$

~~we know~~ corrected velocity head = $\frac{2V^2}{2g}$

\therefore Corrected total kinetic energy = $\frac{2w A v^3}{2g}$ --- (ii)

equating (i) & (ii)

$$\frac{\sum w \Delta A v^3}{2g} = \frac{\alpha w A v^3}{2g}$$

$$\therefore \alpha = \frac{\int v^3 \Delta A}{v^3 A}$$

* Momentum Coefficient:

The momentum of water passing through ΔA per unit time,

$$= mv = \frac{w \Delta A v}{g} \times v = \frac{w \Delta A v^2}{g}$$

$$\therefore \text{Total momentum} = \frac{\sum w \Delta A v^2}{g} \quad \text{--- (i)}$$

Corrected momentum for the whole area,

$$= \frac{\beta w A v^2}{g} \quad \text{--- (ii)}$$

equating (i) & (ii)

$$\frac{\beta w A v^2}{g} = \frac{\sum w \Delta A v^2}{g}$$

$$\Rightarrow \beta = \frac{\int v^2 \Delta A}{v^2 A}$$

For Wide channel

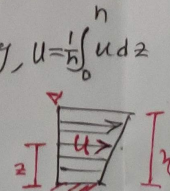
for unit width of channel

$$A = bh = h$$

\therefore Mean velocity, $u = \frac{1}{h} \int_0^h u dz$

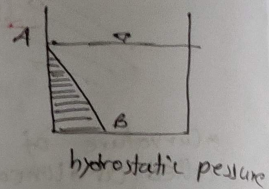
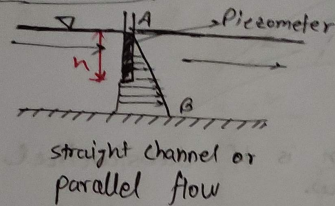
$$\alpha = \frac{\int_0^h u^3 dz}{u^3 h}$$

$$\beta = \frac{\int_0^h u^2 dz}{u^2 h}$$



Pressure Distribution in a channel Section:

(1) In straight channel:



The pressure of straight channel and static water container is same.

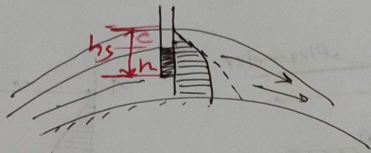
* Thus, the Hydrostatic Law of Pressure Distribution:

"The distribution of pressure over the cross section of the channel is the same as the distribution of hydrostatic pressure; that is the distribution is linear and can be represented by a line."

* It's only applicable when the flow doesn't have any acceleration. It's known as parallel flow. also stream lines don't have substantial curvature nor divergence.

* Uniform flow and gradually varied flow can also be regarded as parallel flow as the change in the depth of flow is so mild that the stream lines never cross each other and remain parallel; therefore no acceleration.

(i) Convex flow/curvilinear flow:



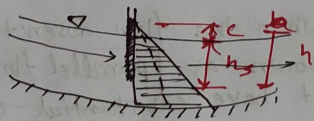
* Curvature of stream is if substantial then its called curvilinear flow.

* The effect of curvature produces centrifugal forces which deviates the pressure distribution from hydrostatic distribution

* In convex flow, centrifugal force acts upwards. Thus the pressure is lower than parallel flow.

$$h = h_s - c$$

(ii) Concave curvilinear flow:



* Centrifugal force downward

* Pressure is higher than parallel flow

Let, deviation from hydrostatic pressure, be c .

\therefore The true pressure/piezometric height

$$h = h_s + c$$

∴ The approximated centrifugal pressure

$$p = \frac{F}{A} = \frac{\frac{mvr}{n}}{A \cdot l} = \frac{mvr}{n} = \frac{mvr}{r} = \left(\frac{\omega d \cdot l}{g} \right) \frac{vr}{r}$$

∴ $p = \frac{\omega d \cdot l}{gr}$

Pressure head correction, $\frac{p}{w} = \frac{d \cdot l}{gr}$

$$\therefore C = \frac{d \cdot l}{gr}$$

(1) Given that

the approximated centrifugal pressure

$$P = \frac{F}{A} = \frac{m \cdot a}{A} = \frac{m \cdot \omega^2 \cdot r}{A}$$

$$= \frac{m \cdot \omega^2 \cdot r}{\pi \cdot r^2} = \frac{m \cdot \omega^2}{\pi \cdot r}$$

$$= \frac{m \cdot \omega^2}{\pi \cdot r} = \frac{m \cdot \omega^2}{\pi \cdot r}$$

Pressure head correction

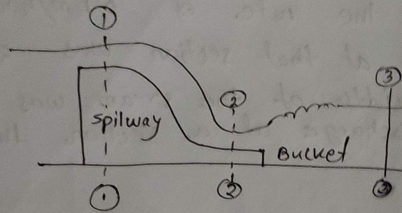
$$h_p = \frac{P}{\rho \cdot g} = \frac{m \cdot \omega^2}{\pi \cdot r \cdot \rho \cdot g}$$

E₂
ω

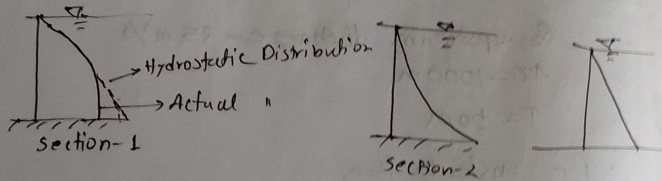
⇒ :
∴ x
∴ A
well
∴ D
R =

Problems

Pressure distribution diagram of different sections of a spillway flow.

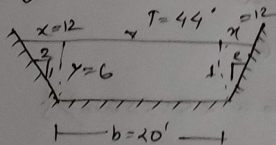


Answer:



Ex 2:1 Ven-Techow

compute the hydraulics, hydraulic depth, section factor?



$$\frac{1}{2} = \frac{y}{12} \therefore y = 6$$

$$\Rightarrow 2x + 20 = 44$$

$$\therefore x = 12$$

$$\therefore A = \frac{1}{2} \times 6 \times (20 + 44) = 172$$

$$\text{wetted perimeter, } P = 20 + 2x \sqrt{c^2 + 12^2} = 46.8$$

$$\therefore D = \frac{A}{P} = \frac{172}{46.8} = 4.36$$

$$R = \frac{A}{P} = \frac{172}{46.8} = 4.1$$

Ex: 1.6 Subramaniam

while measuring the discharge in a small stream it was found that the depth of flow increases at the rate of 0.1 m/hour . If the discharge at that section was $25 \text{ m}^3/\text{se}$ and the surface width of the stream was 20 m , estimate the discharge at a section 100 m upstream

Solution:

$$\frac{dy}{dt} = \frac{0.1}{60 \times 60}$$

$$Q_2 = \text{upstream discharge} = 25 \text{ m}^3/\text{s}$$

$$4x = 100 \text{ m}$$

$$T = 20 \text{ m}$$

\therefore for unsteady flow,

$$\frac{dQ}{dx} + T \frac{dy}{dt} = 0$$

$$\Rightarrow \frac{Q_2 - Q_1}{100} + 20 \times \frac{0.1}{60 \times 60} = 0$$

$$\Rightarrow Q_2 - Q_1 = -0.556$$

$$\therefore Q_1 = Q_2 + 0.556 = 25.556 \text{ m}^3/\text{s}$$

Ex. 11

[Faint handwritten text, likely bleed-through from the reverse side of the page]

$$2 \times 2 \quad \frac{1}{2} \left(41 - \frac{100}{h} \right) \times 2$$

$$\Rightarrow \frac{vD}{\nu} = 1 \Rightarrow \boxed{Fr = 1}$$

$$\Rightarrow \frac{vD}{\nu} = 1$$

$$\text{Again, } \frac{vD}{\nu} = 1 \Rightarrow \frac{v}{\nu} = \frac{1}{D}$$

velocity head hydraulic depth
 \rightarrow coefficient

Ex: 19 Abdul Atim (Niraa sir)

In a wide channel the velocity varies along a vertical as $u = 1 + \frac{3z}{h}$, where h is depth of flow and u is the mean velocity at a distance z from the channel bottom. Calculate. If $h = 5$

- (i) Compute mean velocity
- (ii) Discharge per unit width
- (iii) State of flow
- (iv) α, β

Solution:

(i) We know, mean velocity of wide channel,

$$u = \frac{1}{h} \int_0^h u dz = \frac{1}{h} \int_0^h \left(1 + \frac{3z}{h}\right) dz = \frac{1}{h} \left(h + \frac{3uL}{2h}\right) = 1 + \frac{3}{2} = 2.5 \text{ m/s}$$

(ii) Discharge per unit width, $q = Uh = 2.5 \times 5 = 12.50 \text{ m}^3/\text{s}$

(iii) State of flow:

$$Re = \frac{UR}{\nu} = \frac{2.5 \times 5}{10^{-6}} = 12.5 \times 10^6 > 2000$$

$$Fr = \frac{U^2}{gD} = \frac{2.5^2}{9.8 \times 5} = 0.36 < 1$$

\therefore state of flow subcritical turbulent

$$\begin{aligned} \text{(iv)} \alpha &= \frac{\int_0^h u^3 dz}{U^3 h} = \frac{\int_0^h \left(1 + \frac{3z}{h}\right)^3 dz}{U^3 h} = \frac{1}{U^3 h} \int_0^h \left(1 + \frac{3z}{h} + \frac{27z^2}{h^2} + \frac{27z^3}{h^3}\right) dz \\ &= \frac{1}{U^3 h} (h + 4.5h + 9h + 6.75h) \\ &= 1.36 \end{aligned}$$

$\beta =$

Problem:

Determine
wide open
shown in

Solution: fr

\therefore Mean vel

$$\alpha = \frac{\int_0^h u^3}{U^3 h}$$

$$= \frac{\int_0^h (4 + \dots)}{7^3 \times 2}$$

$$\beta = \frac{\int_0^h u^2 dz}{U^2 h} = \frac{\int_0^h (1 + \frac{3z}{h})^2 dz}{U^2 h} = 1.12$$

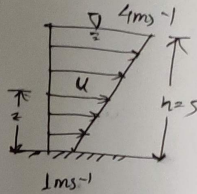
$$\therefore \text{by } u = (1 + \frac{3z}{h})$$

for depth of flow $h=0$, $u=1 \text{ m/s}$

" " " $h=$

for $z=0$, $u=1 \text{ m/s}$

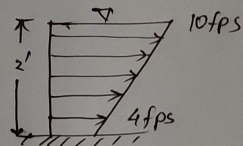
" $z=h$, $u=4 \text{ m/s}$



Answer:

Problem: Miraz Sir

Determine the velocity distribution co-efficients for a wide open channel flow with a velocity distribution shown in the figure below:



Solution: from figure, $u = 4 + \frac{6z}{h}$

$$\begin{aligned} \therefore \text{Mean velocity, } U &= \frac{1}{h} \int_0^h u dz = \frac{1}{h} \int_0^h (4 + \frac{6z}{h}) dz \\ &= \frac{1}{h} (4z + \frac{3z^2}{h}) \\ &= \frac{4h + 3h}{h} = 7 \text{ fps} \end{aligned}$$

$$\alpha = \frac{\int_0^h u^3 dz}{U^3 h}$$

$$= \frac{\int_0^h (4 + \frac{6z}{h})^3 dz}{7^3 \times h}$$

