

Hydraulic Jump

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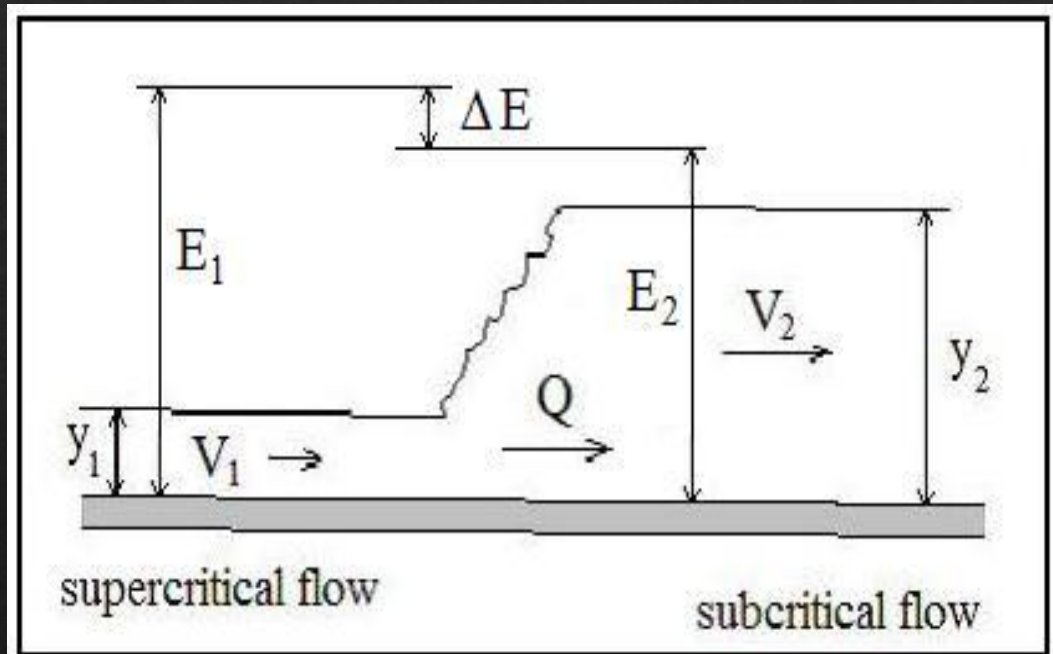
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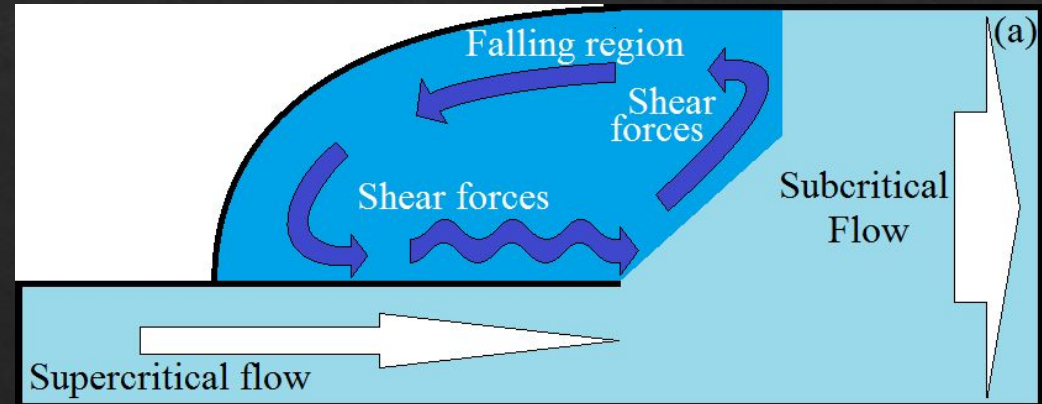
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Definition of Hydraulic Jump

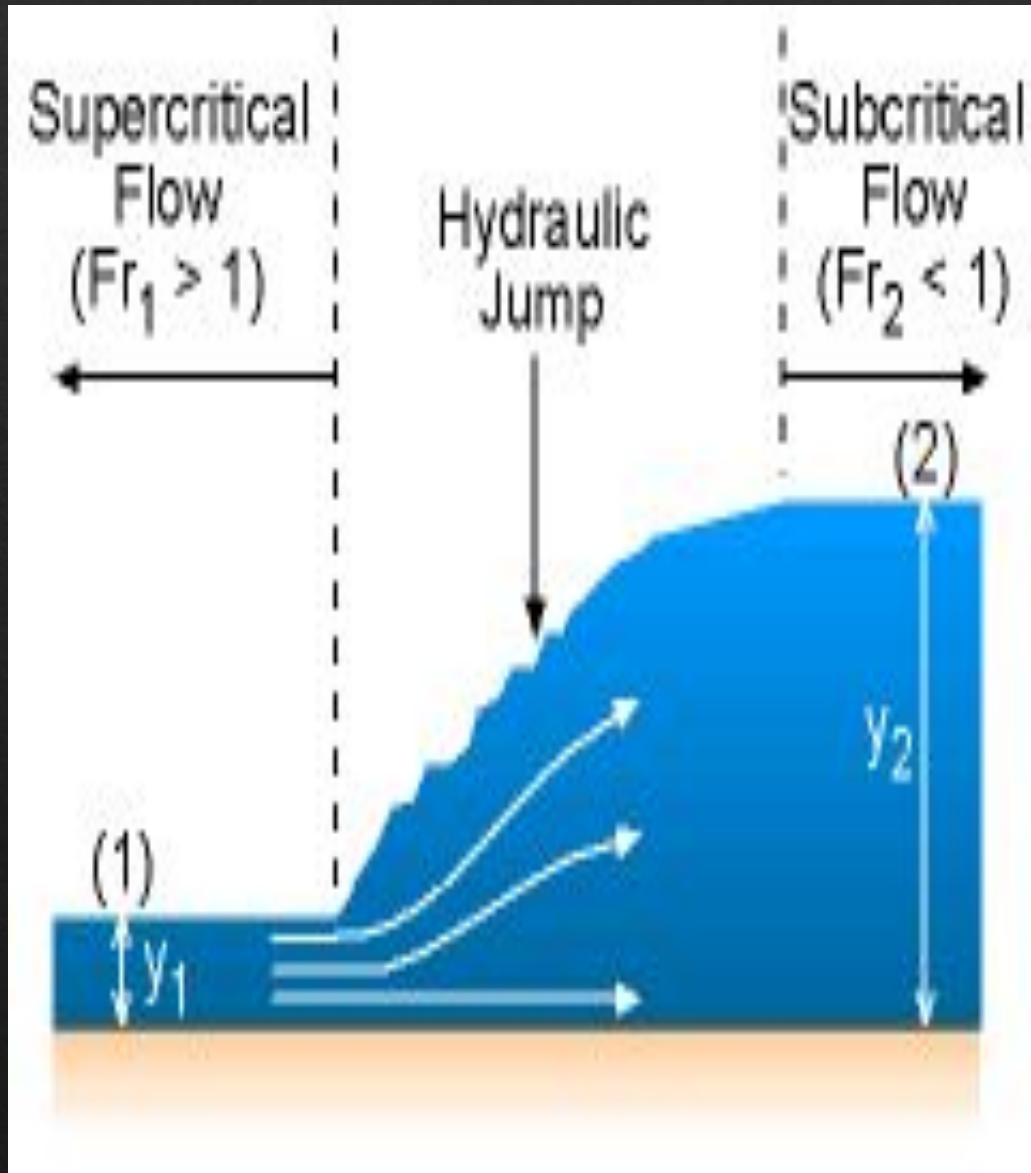
A **hydraulic jump** is a phenomenon in the science of **hydraulics** which is frequently observed in **open channel** flow such as **rivers and spillways**. When liquid at high velocity discharges into a zone of lower velocity, a rather abrupt rise occurs in the liquid surface.



Hydraulic Jump (with Heads and Head Loss)



Hydraulic Jump Cont...



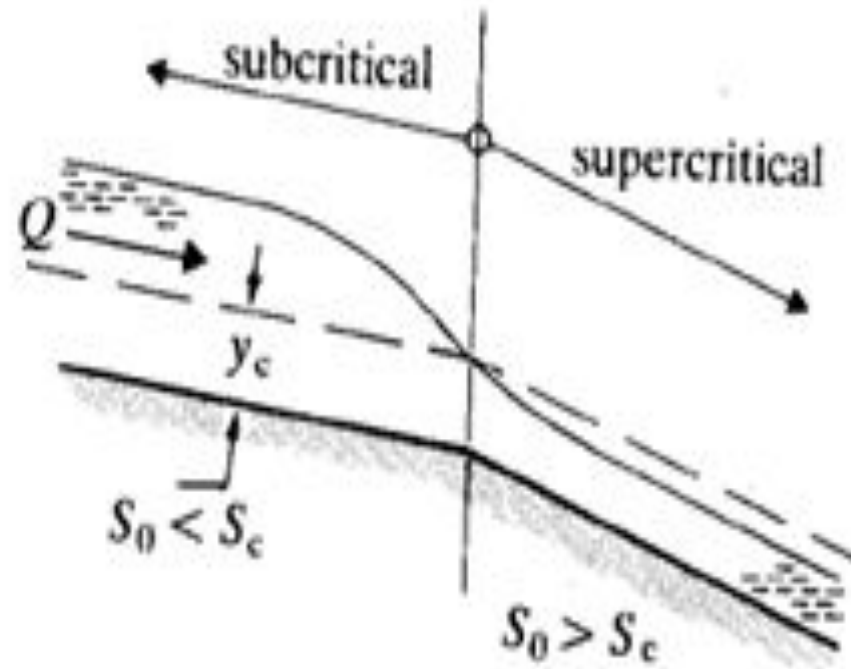
Hydraulic Jump Cont...



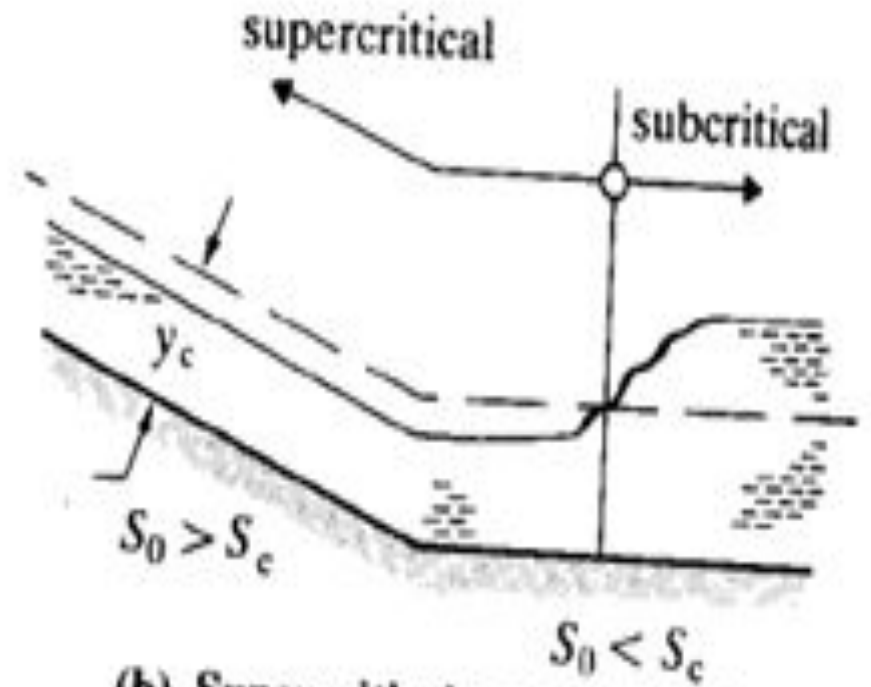
Hydraulic Jump Cont...



Hydraulic Drop and Jump



(a) Subcritical to supercritical



(b) Supercritical to subcritical

Figure 2. Hydraulic drop & hydraulic jump

Effect of Hydraulic Jump

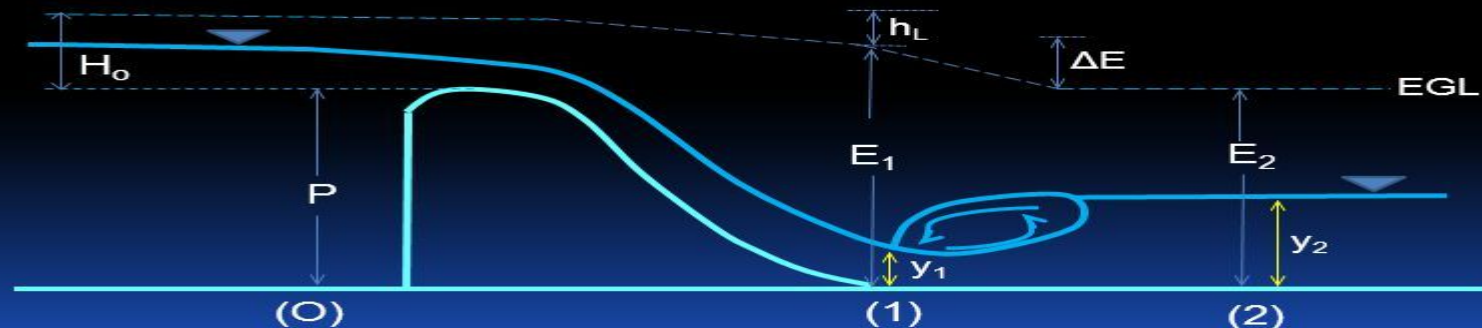
It is very common in the field of hydraulics to **use hydraulic jump**. It is used to perform different functions. Some of the effects of the hydraulic jump are as under:

- Actually the hydraulic jump usually acts as the **energy dissipater**. It distributes the surplus energy of water.

Energy Dissipation at the Toe of Overflow Spillway

➤ Excessive turbulent energy at the toe of an overflow spillway can be dissipated by a hydraulic jump, which is a phenomenon caused by the change in the stream regime from supercritical to subcritical with considerable energy dissipation.

➤ should be done to prevent scouring at the river bed.



Effect of Hydraulic Jump Cont..



Burrendong Dam spillway showing fully lined chute and full energy dissipator.

Effect of Hydraulic Jump Cont....

- Due to the hydraulic jump, many **noticeable disturbances** are created in the flowing water like eddies, reverse flow.
- Usually when the hydraulic jump takes place, the considerable amount of **air is trapped** in the water. That **air can be helpful** in **removing the wastes** in the streams that are causing pollution.
- Hydraulic jump also make the work of different hydraulic structures, effective like weirs, notches and flumes etc.

Applications of Hydraulic Jump:

01. Usually hydraulic jump **reverses the flow** of water. This phenomenon can be used to mix chemicals for **water purification**.



Weir in Riverfront Park, WA (left) and Hydraulic Jump in Coagulation Chamber (right)

Applications of Hydraulic Jump Cont..

02. Hydraulic jump usually maintains the **high water level** on the down stream side. This high water level can be used for **irrigation purposes**.

03. Hydraulic jump can be used to **remove the debris** from water supply and sewage lines to prevent the blocking.

04. It prevents the **scouring** action on the down stream side of the dam structure.

05. The reduction of uplift pressure under a structure by increasing weight on its apron.

Classification of Jumps

As mentioned earlier, the Froude number of supercritical flow influences the characteristics of the hydraulic jump. **Bradley and Peterka**, after extensive experimental investigations, have classified the hydraulic jump into **five categories** as shown in Figure 28.4. The hydraulic jump is the phenomenon that occurs where there is an abrupt transition from supercritical (inertia dominated) flow to sub critical (gravity dominated) flow. The most important factor that affects the hydraulic jump is the initial Froude number F_1 .

Classification of Jumps Cont..

$$F_1 = \frac{\bar{V}_1}{\sqrt{gD}}$$

in which \bar{V}_1 is the longitudinal average velocity at the initial section, g is the acceleration due to gravity and D is the hydraulic mean depth (ratio of area of flow at free surface width).

As mentioned above, it occurs in a straight prismatic horizontal channel of rectangular shape in which boundary friction is negligible (NHJ).

The hydraulic jump can be classified based on initial Froude number as Undular ($F_1 = 1 - 1.7$), weak ($F_1 = 1.7 - 2.5$), oscillating jet ($F_1 = 2.5 - 4.5$), steady ($F_1 = 4.5 - 9.0$) and strong ($F_1 > 9.0$).

Classification of Jumps Cont..

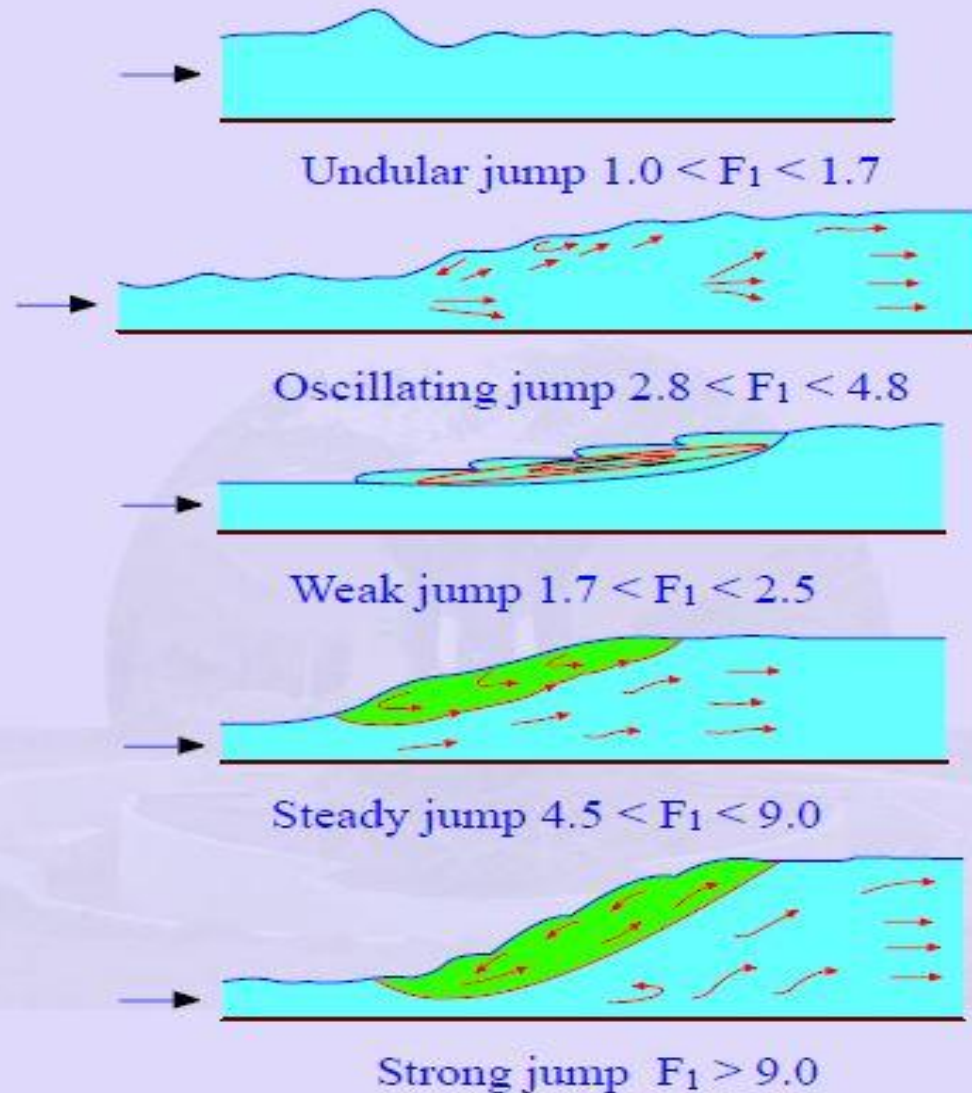


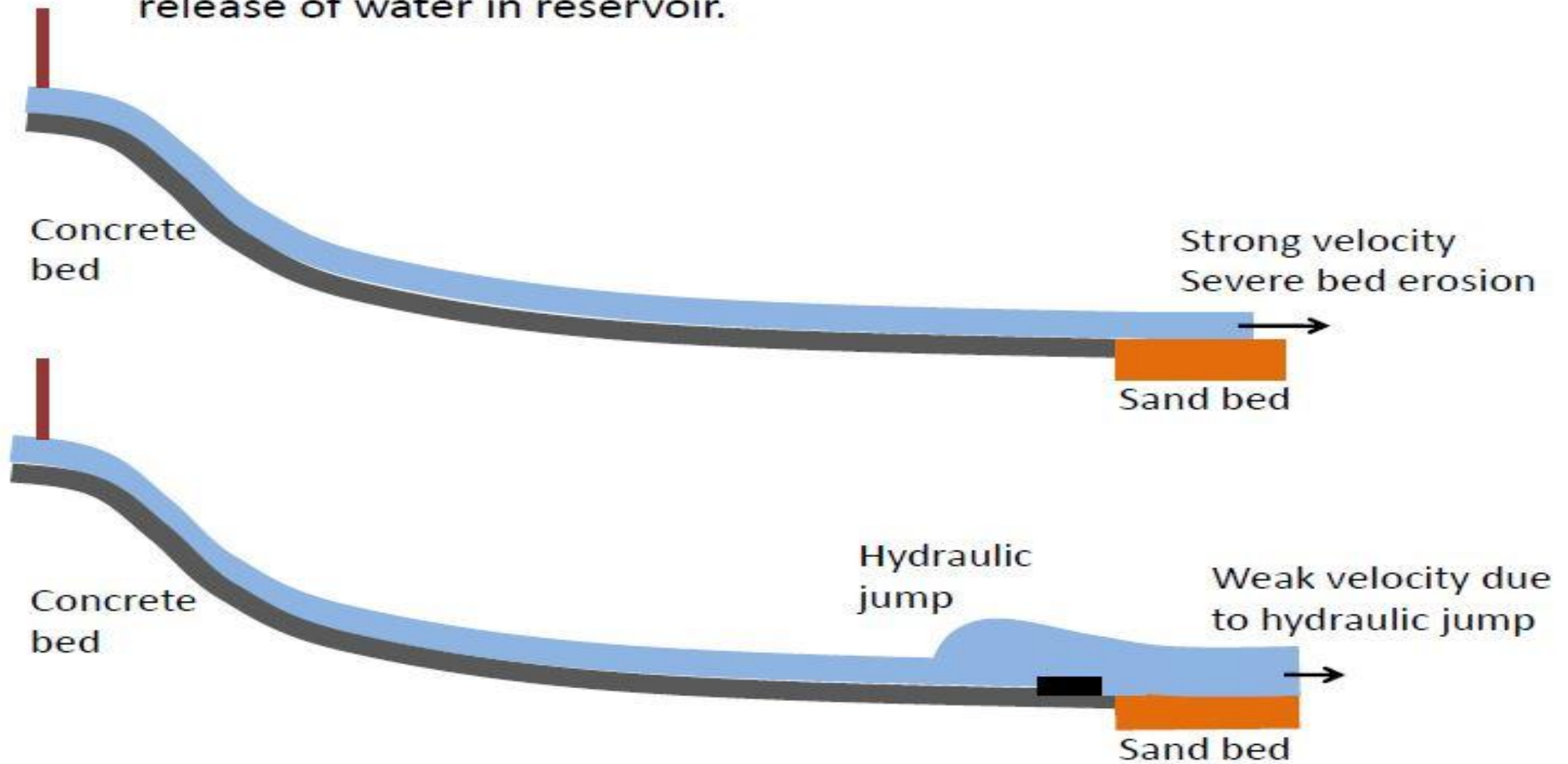
Fig. 28.4 - Classification of the Jump

Type of Jump	Froude Number	Remarks
Critical flow	$F_1 = 1$	Wavy surface , celerity $c = \sqrt{gy}$
Undular jump	$1 < F_1 < 1.7$	Undulations on the surface
Weak jump	$1.7 < F_1 < 2.5$	Small rollers, No baffles.
Oscillating Jump	$2.5 < F_1 < 4.5$	No periodicity. Rip rap may get damaged Canal drops, difficult to handle. Baffle blocks or appurtenances are of little value. Wave suppressors may be designed.
Steady jump	$4.5 < F_1 < 9.0$	Position, is sensitive to variation of Tail Water, Efficiency is 45 to 70 %.
Strong Jump	$F_1 > 9.0$	Efficiency is 85 %

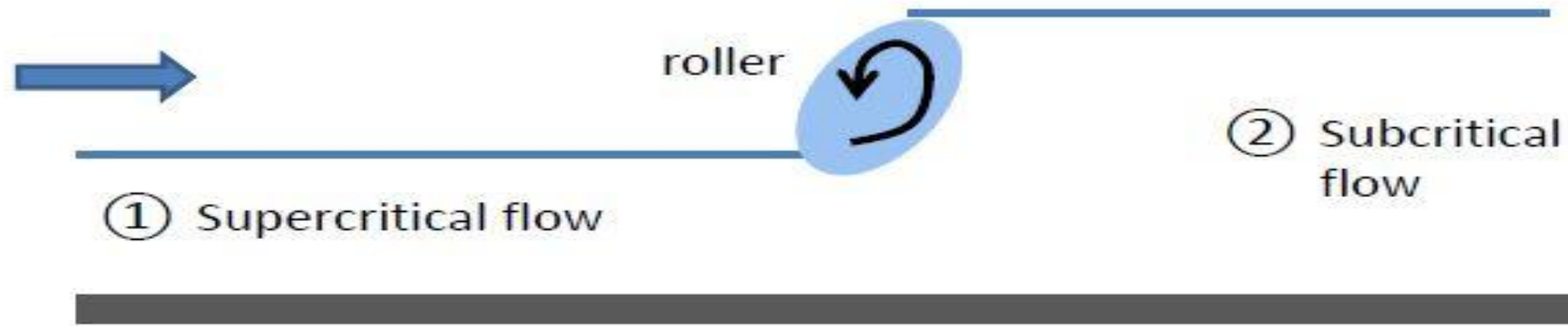
- Sub-critical flow also called Tranquil flow
- Super-critical flow also called Torrential flow
- Critical depth, Alternate depth.....

Hydraulic Jump

Hydraulic jump can be seen downstream of a spillway during the release of water in reservoir.



Basic Characteristics of Hydraulic Jump



Roller regime is similar to wave breaker— strong energy dissipation

The upstream Froude number:

$$Fr_1 = \frac{V_1}{\sqrt{gh_1}} > 1$$

For supercritical flow (low flow depth, high flow velocity)

The downstream Froude number:

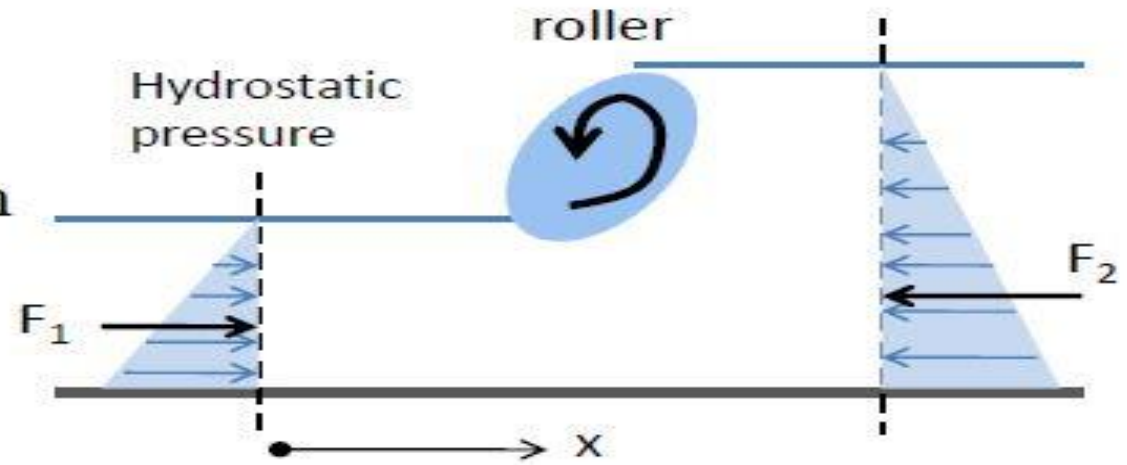
$$Fr_2 = \frac{V_2}{\sqrt{gh_2}} < 1$$

For subcritical flow (high flow depth, low flow velocity)

Conservation of momentum:

$$\sum_x F = \sum_{\text{out}} \text{Momentum} - \sum_{\text{in}} \text{Momentum}$$

$$\Rightarrow F_1 - F_2 = \rho Q (V_2 - V_1)$$



If locations (1) and (2) are sufficiently away from the roller, we can assume the pressure distribution is hydrostatic.

$$F_1 = P_c h_1 b = \left(\frac{1}{2} \rho g h_1 \right) \cdot h_1 b = \frac{1}{2} \rho g h_1^2 b$$

$$F_2 = \frac{1}{2} \rho g h_2^2 b$$

Recall: Force acting on a submerged surface is equal to pressure at centroid of the submerged area times area of submerged surface.

Momentum conservation $\Rightarrow \frac{1}{2} \rho g b (h_1^2 - h_2^2) = \rho V_1 h_1 b (V_2 - V_1)$

$$\Rightarrow \frac{1}{2} g (h_1^2 - h_2^2) = V_1 h_1 (V_2 - V_1)$$

Replace V_2 by continuity, i.e., $V_2 = V_1 h_1 / h_2$

$$\Rightarrow \frac{1}{2} (\cancel{h_1 - h_2})(h_1 + h_2) = \frac{V_1 h_1}{g} \frac{V_1}{h_2} (\cancel{h_1 - h_2})$$

$$\Rightarrow (h_1 + h_2) = 2 \text{Fr}_1^2 \frac{h_1^2}{h_2}$$

$$\Rightarrow \frac{h_2}{h_1^2} (h_1 + h_2) = 2 \text{Fr}_1^2$$

$$\Rightarrow \left(\frac{h_2}{h_1} \right)^2 + \left(\frac{h_2}{h_1} \right) - 2 \text{Fr}_1^2 = 0$$

$$\Rightarrow Y^2 + Y - 2 \text{Fr}_1^2 = 0$$

$$\therefore \frac{h_2}{h_1} = \frac{1}{2} \left(-1 + \sqrt{1 + 8 \text{Fr}_1^2} \right)$$

Recall: by definition:

$$\text{Fr}_1^2 = \frac{V_1^2}{g h_1}$$

set: $Y = \frac{h_2}{h_1}$

We can easily find solution of this 2nd order polynomial.
Discard negative root.

i.e., downstream flow depth of a hydraulic jump can be predicted by upstream flow information.

Conservation of energy (to get head loss): Note: $P_1 = P_2 = 0$

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_L \quad \Rightarrow \text{You can directly get } h_{L1} \text{ from measured upstream \& downstream flow depth, flow rate}$$

$$\Rightarrow h_L = (h_1 - h_2) + \underbrace{\frac{1}{2g}(V_1^2 - V_2^2)}$$

$$\frac{1}{2g} \left(V_1^2 - \frac{V_1^2 h_1^2}{h_2^2} \right) = \frac{V_1^2}{2g} \left(1 - \frac{h_1^2}{h_2^2} \right) = \frac{V_1^2}{2g} \frac{h_1}{h_1} \left(1 - \frac{h_1^2}{h_2^2} \right) = \frac{1}{2} Fr_1^2 h_1 \left(1 - \frac{h_1^2}{h_2^2} \right)$$

$$\Rightarrow h_L = (h_1 - h_2) + \frac{1}{2} Fr_1^2 h_1 \left(1 - \frac{h_1^2}{h_2^2} \right)$$

$$\Rightarrow \frac{h_L}{h_1} = \left(1 - \frac{h_2}{h_1} \right) + \frac{1}{2} Fr_1^2 \left[1 - \left(\frac{h_1}{h_2} \right)^2 \right]$$


Recall previously: $Fr_1^2 = \frac{1}{2} \left[\left(\frac{h_2}{h_1} \right)^2 + \left(\frac{h_2}{h_1} \right) \right]$ $Y = \frac{h_2}{h_1}$ $\therefore \frac{1}{2} Fr_1^2 = \frac{1}{4} (Y^2 + Y)$

Hence,
$$\frac{h_L}{h_1} = (1 - Y) + \frac{1}{4}(Y^2 + Y)\left(1 - \frac{1}{Y^2}\right)$$

$$= \frac{1}{4Y}(Y^3 - 3Y^2 + 3Y - 1)$$

$$= \frac{(Y - 1)^3}{4Y}$$

$$= \frac{(h_2 - h_1)^3}{4h_2h_1^2}$$


 Substitute h_2, h_1 back

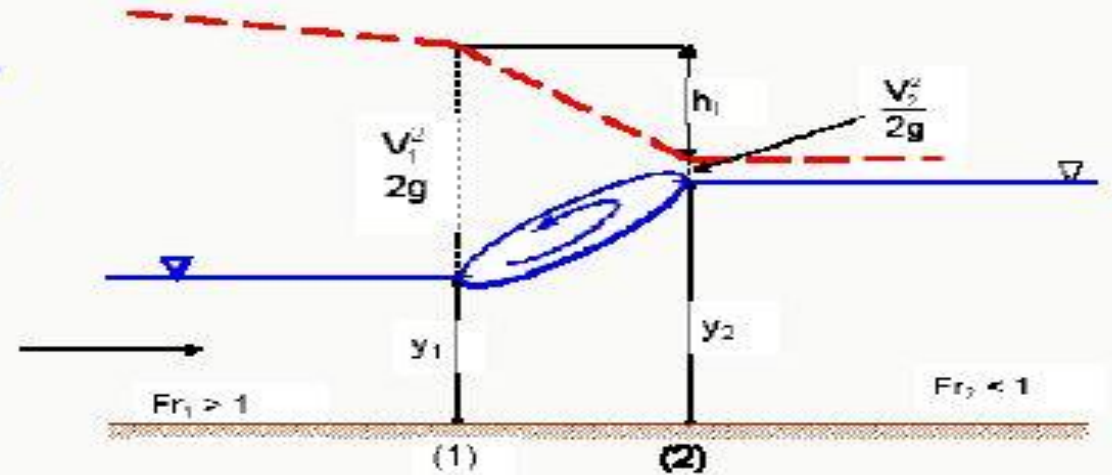
$$\therefore h_L = \frac{(h_2 - h_1)^3}{4h_2h_1}$$

$\Rightarrow h_L$ obtained theoretically can be compared with directly measured value h_{L1}

EXAMPLE 6 : HYDRAULIC JUMP

A rectangular horizontal channel 2m. wide, carries a flow of $4 \text{ m}^3/\text{s}$. The depth water on the downstream side of the hydraulic jump is 1m.

- What is the depth upstream?
- What is the loss of head?



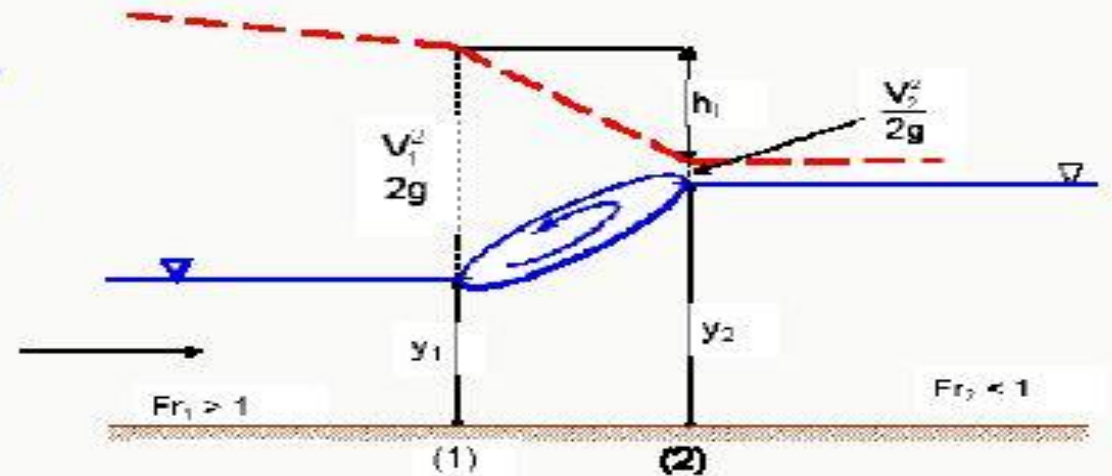
$$F_1 = F_2 \rightarrow \bar{y}_1 A_1 + \frac{Q^2}{gA_1} = \bar{y}_2 A_2 + \frac{Q^2}{gA_2}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

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$$F_1 = F_2 \rightarrow \bar{y}_1 A_1 + \frac{Q^2}{gA_1} = \bar{y}_2 A_2 + \frac{Q^2}{gA_2} \quad y_1 (2 y_1) + \frac{4^2}{g 2 y_1} = 1 * (2 * 1) + \frac{4^2}{g (2 * 1)}$$

$$2 y_1^2 + \frac{0.815}{y_1} = 2.815$$

$$y_1 = 0.311 \text{ m}$$

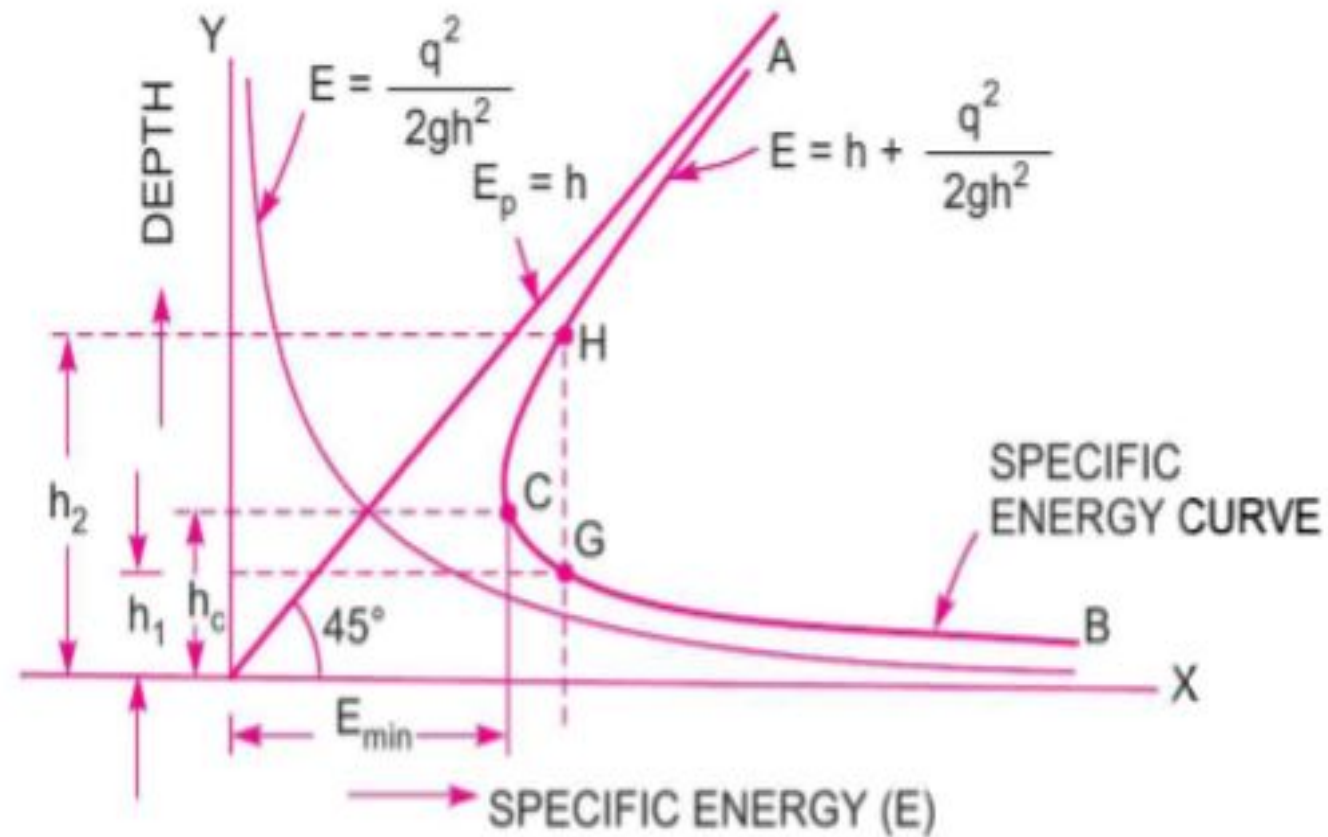
$$V_1 = 4 / (2 * 0.311) = 6.43 \text{ m/s}$$

$$V_2 = 4 / (2 * 1) = 2.0 \text{ m/s}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

$$h_{\text{loss}} = 2.42 - 1.20 = 1.22 \text{ m}$$

Water flows at a depth of 10 cm with a velocity of 6 m/s in a rectangular channel. Is the flow subcritical or supercritical? What is the alternate depth?



Water flows at a depth of 10 cm with a velocity of 6 m/s in a rectangular channel. Is the flow subcritical or supercritical? What is the alternate depth?

Solution:

Check Froude number

$$Fr = \frac{V}{\sqrt{gy}} = \frac{6 \text{ m/s}}{\sqrt{9.81 \text{ /s}^2 \cdot 0.1 \text{ m}}} = 6.06 > 1$$

so the flow is supercritical.

$$E = y + \frac{V^2}{2g} = 0.1 \text{ m} + \frac{(6 \text{ m/s})^2}{2 \cdot 9.81 \text{ m/s}^2} = 1.935 \text{ m}$$

Solving for the alternate depth for an $E = 1.935 \text{ m}$ yields $y_{\text{alt}} = 1.93 \text{ m}$

The spillway shown has a discharge of $1.2 \text{ m}^3/\text{s}$ per meter of width occurring over it. What depth y_2 will exist downstream of the hydraulic jump? Assume negligible energy loss over the spillway.

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Solution:

$$y_0 + \frac{q^2}{2gy_0^2} = y_1 + \frac{q^2}{2gy_1^2}$$

$$5 \text{ m} + \frac{(1.2 \text{ m}^3/\text{s}/\text{m})^2}{2 \cdot 9.81 \text{ m}/\text{s}^2 \cdot (5 \text{ m})^2} = y_1 + \frac{(1.2 \text{ m}^3/\text{s}/\text{m})^2}{2 \cdot 9.81 \text{ m}/\text{s}^2 \cdot y_1^2}$$

solving for y_1 we get $y_1 = 0.123 \text{ m}$.

$$\text{Fr}_1 = \frac{q}{\sqrt{gy_1^3}} = \frac{(1.2 \text{ m}^3/\text{s}/\text{m})^2}{\sqrt{9.81 \text{ m}/\text{s}^2 \cdot (0.123 \text{ m})^3}} = 8.88.$$

$$y_2 = \frac{y_1}{2} \left(\sqrt{1 + 8\text{Fr}_1^2} - 1 \right) = \frac{0.123 \text{ m}}{2} \left(\sqrt{1 + 8 \cdot 8.88^2} - 1 \right) = 1.48 \text{ m}.$$

Water is flowing as shown under the sluice gate in a horizontal rectangular channel that is 6 ft wide. The depths y_0 and y_1 are 65 ft and 1 ft respectively. What will be the horsepower lost in the hydraulic jump?

Problem: *The discharge of water through a rectangular channel of width 8 m, is $15 \text{ m}^3/\text{s}$ when depth of flow of water is 1.2 m. Calculate :*

- (i) Specific energy of the flowing water,*
- (ii) Critical depth and critical velocity,*
- (iii) Value of minimum specific energy.*

