CE 3121: ENGINEERING HYDRAULICS

Lecture: 4 hrs./week
Credit: 4.00
Prereq.: CE 2121

Miraz Ahamed
Lecturer
Department of Civil Engineering
Rajshahi University of Engineering & Technology (RUET).
Rajshahi-6204, Bangladesh.
Cell: +880-1771098551, +880-1521211236.
Email: miraz@ce.ruet.ac.bd
Website: https://www.ruet.ac.bd/miraz

Course Outline

Open channel flow and its classification, velocity and pressure distributions, energy equation, specific energy and transition problems, critical flow and control, principles of flow measurement and devices, concept of uniform flow, Chezy and Mannings equations, estimation of resistance coefficients and computation of uniform flow, momentum equation, hydraulic jump, stilling basin, dams and related structures. Theory and analysis of gradually varied flow, computation of flow profiles, design of channel. Impact of water jet, Principles of hydraulic machines: pumps.

Hydraulics

- The word "**hydraulics**" originates from the Greek word "*hydraulikos*" meaning *Study of pipe*.
- In applied engineering it is the study of water or other fluids at rest or in motion.
- Hydraulics is a very old science perhaps as old as human civilization.
- The main developments in hydraulics took place in last three hundred years when **Darcy, Bernoulli**, Bazin, Ganguillet, Kutter, **Manning, Chezy** and others gave their theories.

Complex pipe system

Water Transmission Systems

Dams, Spillways, Energy Dissipaters

Coastal Erosion

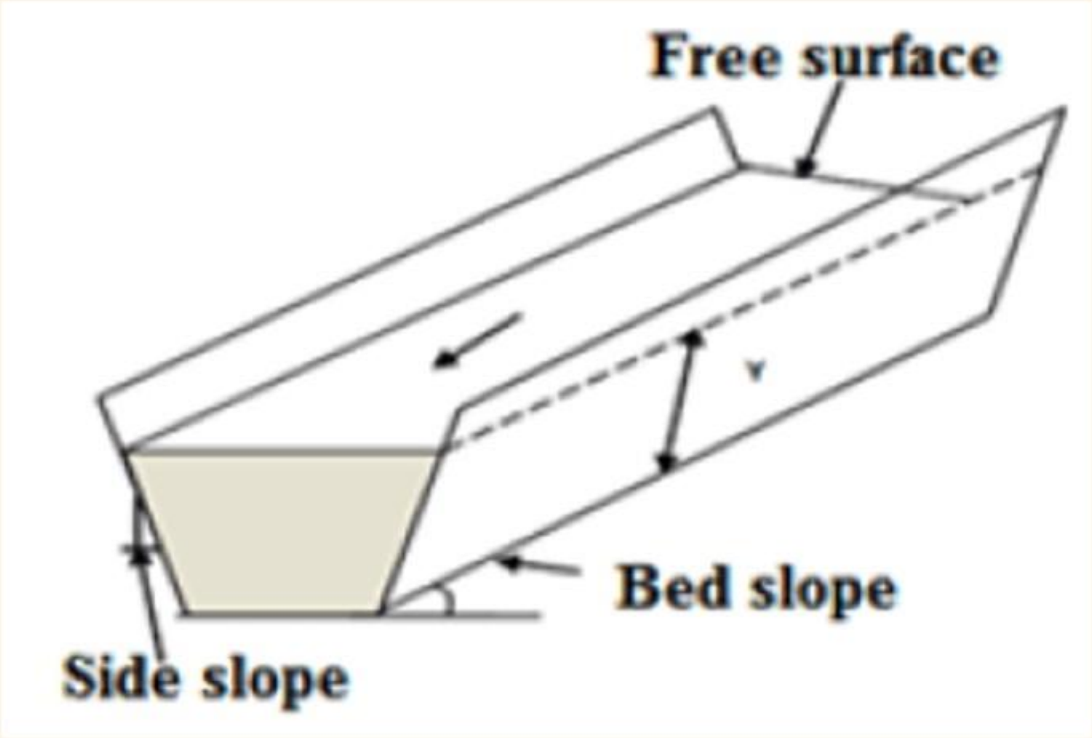
Rivers, estuaries, waterways,... etc.

Open Channel flow:

The flow of liquid through a channel with a free surface is defined as open channel flow. This free surface of the liquid is subjected to atmospheric pressure. The flow in an open channel takes place due to gravity that is achieved by providing a bed slope.

e.g. Rivers, natural and artificial canals, streams, channels etc. Partially filled pipes flow is also an example of open channel flow.



The flow of liquid through the open channel can be of several types like steady and unsteady flow, laminar or turbulent flow or uniform or non-uniform flow and finally sub-critical, critical and supercritical flow.

Classification of Open Channels:

❖ ***According to Origin:-***

Natural Channel & Artificial Channel

❑ A Channel which made by nature is call as natural channel.
Example – River

❑ Artificial open channels are the channels develops by men. They are usually designed with regular geometric shapes..
Example – Canal, Irrigation canals, laboratory flumes.

Classification of Open Channels:

❖ *According to Geometry:-*

Prismatic And Non-prismatic Channels:

- ❑ A channel in which the **cross sectional shape, size and the bottom slope** are constant is termed as prismatic channel.
- ❑ All natural channels generally have varying cross section and consequently are non-prismatic.
- ❑ Most of the man made channel are prismatic channels over long stretches. The rectangle, trapezoid, triangle and circle are commonly used shapes in manmade channels.

❖ *According to Boundary:-*

Rigid And Mobile Boundary Channels:

- Rigid channels are those in which the **boundary is not deformable**. The shape and roughness magnitudes are not functions of flow parameters. For example, lined canals and non erodible unlined canals.
- When the boundary of the channel is mobile and flow carries considerable amounts of sediment through suspension and is in contact with the bed. Such channels are classified as mobile channels.
- In the mobile channel, not only depth of flow but also bed width, longitudinal slope of channel may undergo changes with space and time depending on type of flow.
- The resistance to flow, quantity of sediment transported and channel geometry all depends on interaction of flow with channel boundaries.

CLASSIFICATION OF OPEN CHANNEL FLOW :

STEADY AND UNSTEADY FLOWS

- A *steady flow* occurs when the flow properties, such as the depth or discharge at a section do not change with time.
- If the depth or discharge changes with time, the flow is termed *unsteady*.
- Flood flows in rivers and rapidly varying surges in canals are some examples of unsteady flow.

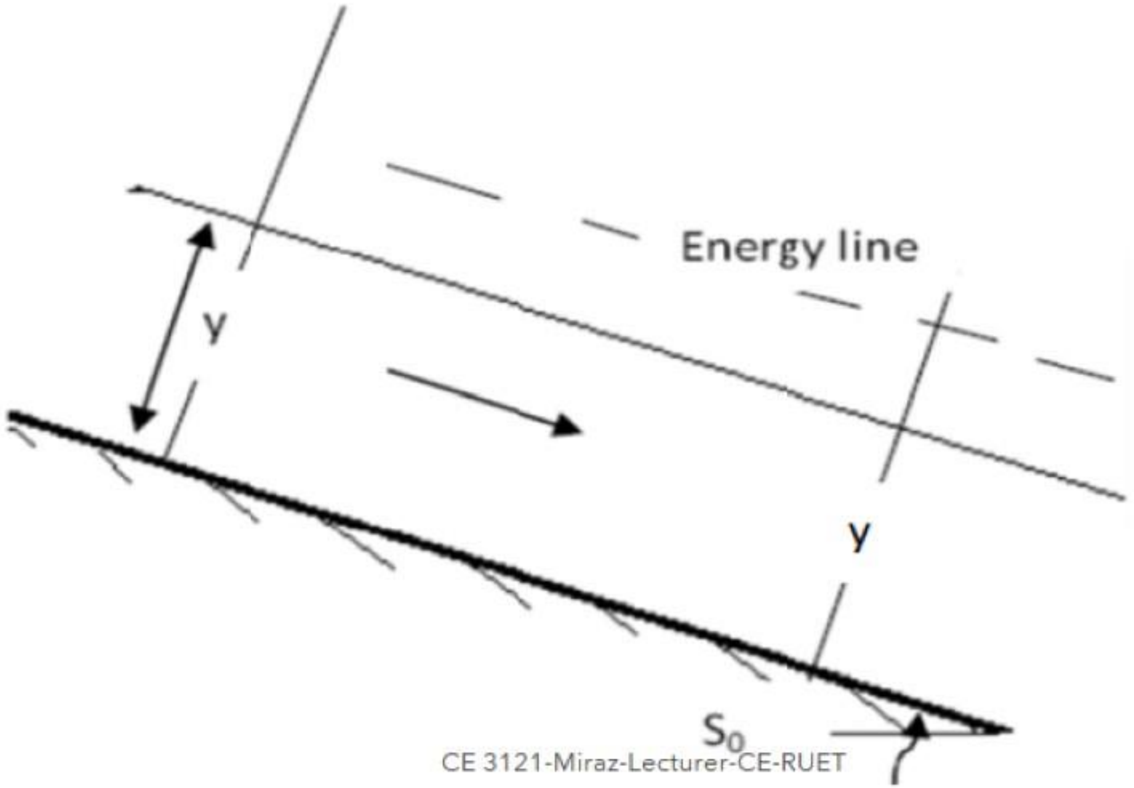
CLASSIFICATION OF OPEN CHANNEL FLOW :

UNIFORM AND NON-UNIFORM FLOWS

- If the flow properties, say the depth of flow, in an open channel remain constant along the length of the channel, the flow is said to be *uniform*.
- A flow in which the flow properties vary along the channel is termed as *non-uniform flow*.
- A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow.

CLASSIFICATION OF OPEN CHANNEL FLOW:

- In uniform flow, the gravity force on the flowing liquid balances the frictional force between the flowing fluid and inside surface of the channel, which is in contact with the fluid. In case of non-uniform flow, the friction and gravity force are not in balance.



CLASSIFICATION OF OPEN CHANNEL FLOW:

GRADUALLY VARIED AND RAPIDLY VARIED FLOW

- The non-uniform flow can be classified as gradually varied flow (GVF) and rapidly varied flow (RVF).

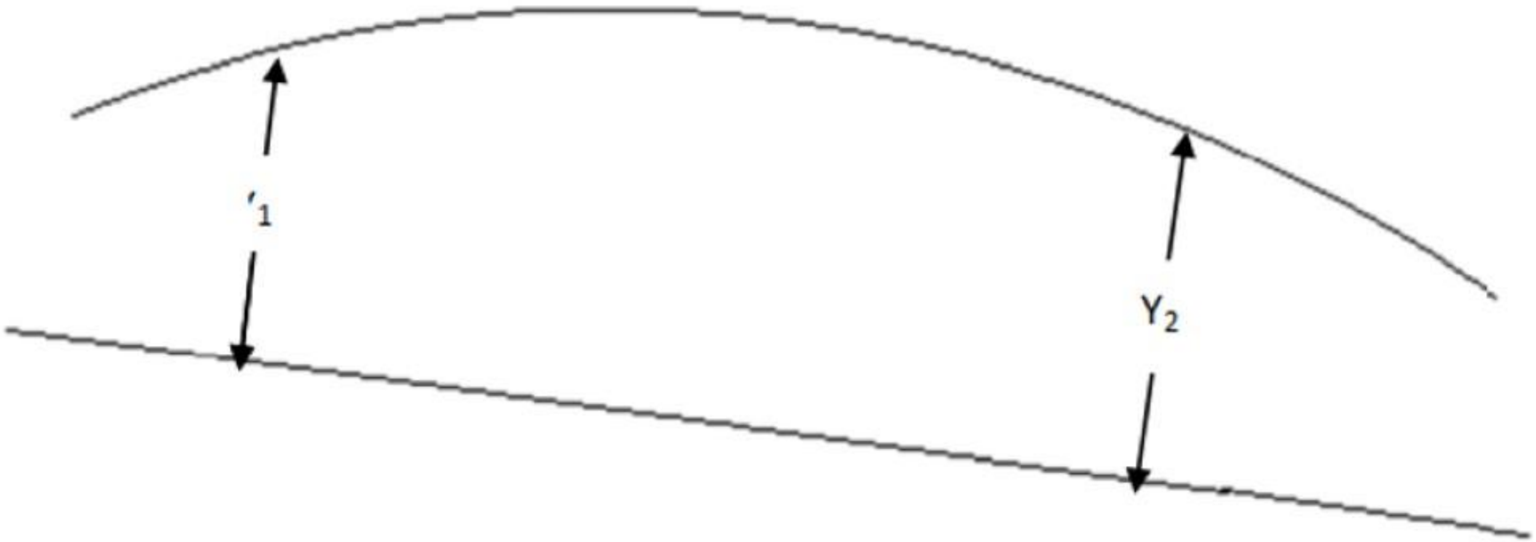
- Varied flow assumes that no flow is externally added to or taken out of channel system. The volume of water in a known time interval is conserved in the channel system

- If the change of depth in a varied flow is gradual so that the curvature of streamlines is not excessive, such a flow is said to be gradually *varied flow (GVF)*. Figure 1.3 shows water surface profile of a GVF; here y_1 and y_2 are the depth at section 1and 2 respectively

CLASSIFICATION OF OPEN CHANNEL FLOW:

- In GVF, the loss of energy is essentially due to boundary friction. Therefore, the distribution of pressure in the vertical direction may be taken as *hydrostatic*.

- If the curvature in a varied flow is large and the depth changes appreciably over short lengths, such a phenomenon is termed as *rapidly varied flow*.



CLASSIFICATION OF OPEN CHANNEL FLOW :

Spatially-varied flow (SVF): The discharge of a steady flow is non-uniform along a channel. This happens when water enters and/or leaves the channel along the course of flow. An example of flow entering a channel would be a road side gutter. An example of flow leaving a channel would be an irrigation channel.

State of Flow and Critical Depth In Open Channel

The state of open channel flow is mainly governed by the combined effect of viscous and gravity forces relative to the inertial forces. The state of flow is very important, as the flow behavior depends on it. In order to construct different structures in rivers and canals and to predict the river response, the state of flow must be known. Critical depth is very useful in determining the types of flow in practice.

State of Flow

Depending on the effect of viscosity relative to inertia, the flow may be **laminar, turbulent or transitional.** The effect of viscosity relative to the inertia is expressed by the Reynolds number, given by

Reynolds Number = Inertial Force / Viscous Force

$$Re = \frac{VR}{\nu}$$

Where,

V is the mean velocity of flow

R is the hydraulic radius (=A/P)

A is the wetted cross-sectional area

P is the wetted perimeter and υ is the kinematic viscosity of water.

Kinematic viscosity is a measure of a fluid's internal resistance to flow under gravitational forces. Kinematic viscosity varies with temperature. The values of kinematic viscosity of water at different temperatures are known values.

State of Flow

When, $Re < 500$ the flow is laminar

$500 \leq Re \leq 12{,}500$ the flow is transitional

$Re > 12{,}500$ the flow is turbulent.

Most open channel flows including those in rivers and canals are turbulent. The Reynolds number of most open channel flows is high, of the order of 10^6, indicating that the viscous forces are weak relative to the inertia forces and do not play a significant role in determining the flow behavior.

State of Flow

When the flow is dominated by the gravity, then the type of flow can be identified by a dimensionless number, known as **Froude Number**.

Given by

Froude Number= Inertial Force / Gravitational Forces

$$Fr = \frac{V}{\sqrt{gD}}$$

Where, V is the mean velocity of flow

D is the hydraulic depth (= A/T)

A is the cross-sectional area

T is the top width and g is the acceleration due to gravity

Depending on the effect of gravity relative to inertia, the flow may be subcritical, critical or supercritical

State of Flow

When, Fr<1	the flow is subcritical
Fr =1	the flow is critical
Fr>1	the flow is supercritical.

The flow in most rivers and canals is subcritical. Supercritical flow normally occurs downstream of a sluice gate and at the foot of drops and spillways. The Froude number of open channel flow varies over a wide range covering both subcritical and supercritical flows and the state or behavior of open channel flow is primarily governed by the gravity force relative to the inertia force. Therefore, the Froude number is the most important parameter to indicate the state or behavior of open channel flow.

Regime of Flow

Depending on the numerical values of Reynolds and Froude numbers, the following four states of flow are possible in an open channel:

i)	Subcritical laminar	$Fr<1$, $Re<500$
ii)	Supercritical laminar	$Fr>1$, $Re<500$
iii)	Subcritical turbulent	$Fr<1$, $Re>12{,}500$
iv)	Supercritical turbulent	$Fr>1$, $Re>12{,}500$

The first two states of flow, subcritical laminar and supercritical laminar, are not commonly encountered in applied open channel hydraulics. Since the flow is generally turbulent is open channel, the last two states of flow are encountered is engineering problems.

Critical Depth

Flow in an open channel is critical when the Froude number of the flow is equal to unity. Critical flow in a channel depends on the discharge and the geometry of channel section. For a rectangular section, the critical depth is given by

$$y_c = \sqrt[3]{\frac{Q^2}{gB^2}}$$

Where,

y_c is the critical depth,

Q is the discharge and B is the width of the channel.

When the depth is greater than the critical depth, the flow is subcritical. When the depth is less than the critical depth, the flow is supercritical.

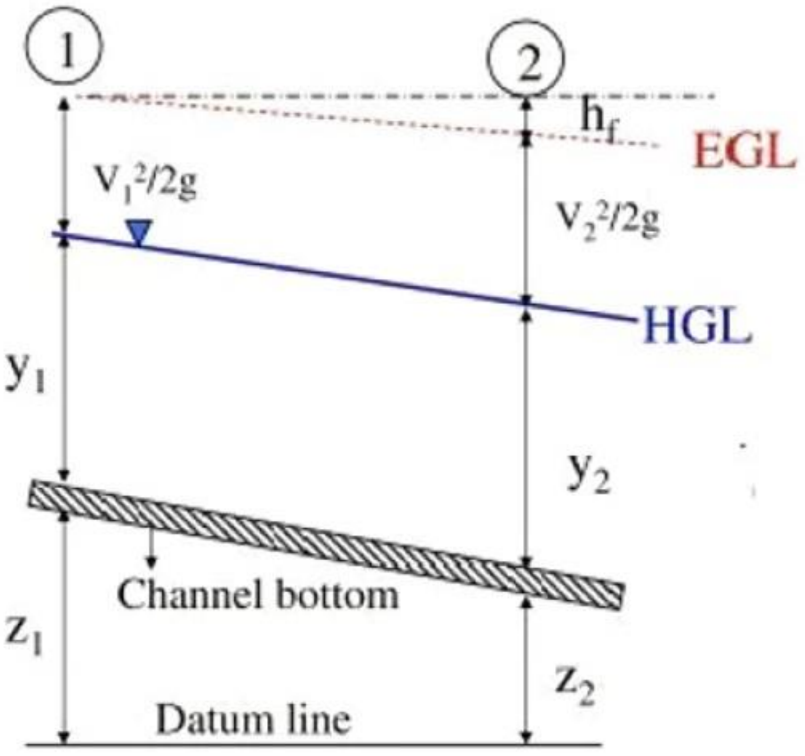
Difference between Open channel Flow & Pipe flow

Open Channel Flow	Pipe Flow
Open Channel Flow is a type of fluid flow in a conduit with a free surface open to the atmosphere.	The pipe flow is a type of flow within a closed conduit.
Open Channel Flow has a free surface	There is no Free surface in pipe flow
The pressure at the free surface remains constant	Pressure in the pipe is not constant
Flow Driven by Gravity	Flow Driven by Pressure
The maximum velocity occurs at a little distance below the water surface	The maximum velocity occurs at the center of the pipe.

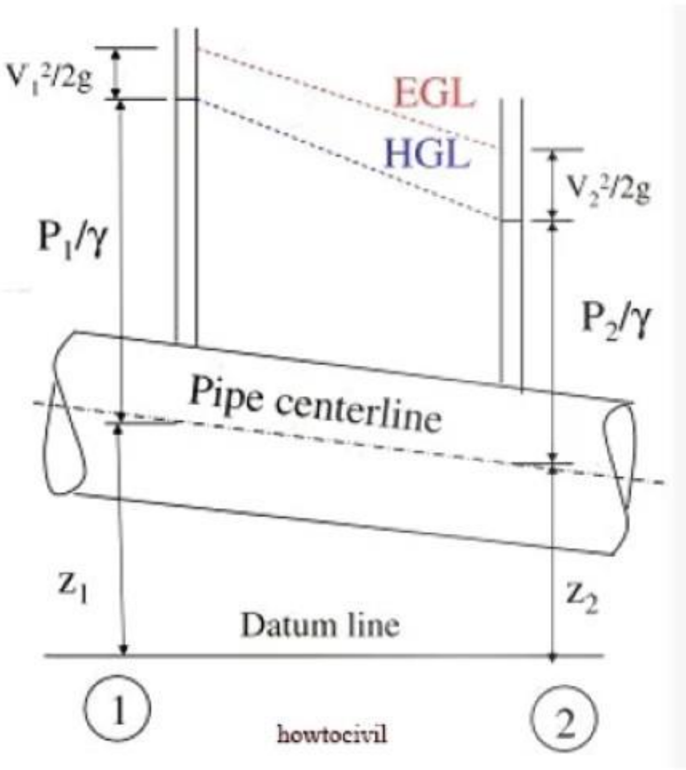
Difference between Open channel Flow & Pipe flow

Open Channel Flow	Pipe Flow
Surface roughness varies with depth of flow	Surface roughness varies with the type of pipe material
HGL(Hydraulic Gradient Line) coincides with the water surface line.	HGL(Hydraulic Gradient Line) do not coincide top surface of the water
The Cross-section of an open channel can be trapezoidal, triangular, rectangular, circular, etc.	The Cross-section of a pipe generally circular.
Piezometric Head = $z+y$ where y = depth of flow	Piezometric Head = $z+P/\gamma$, where p= pressure in the pipe.
Surface head negligible	Surface head dominant for small diameter

Difference between Open channel Flow & Pipe flow



Open channel Flow



Pipe flow

Why open channel flow is more complicated than pipe flow

The physical condition of open channels varies more widely than that of pipes. In pipe the cross section of flow is fixed. The cross section of a pipe is generally sound but in case of open channel may be of any shape from the circular to the irregular forms of natural streams.

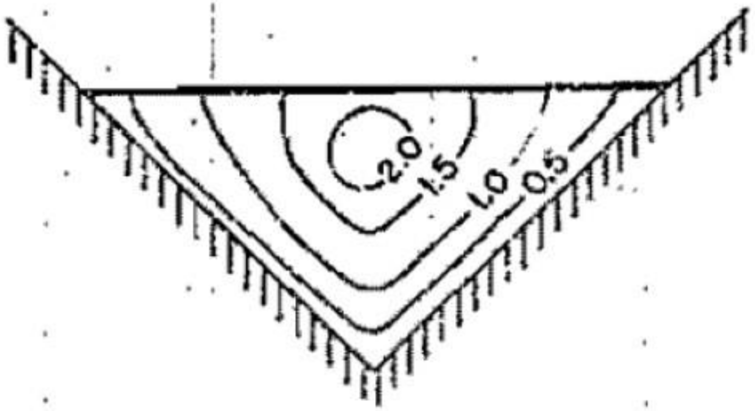
The interior surface in pipes ordinary ranges roughness for new and old corroded iron of steel pipes. In case of open channel, the surface varies from that of the polished metal used in testing flume to that of rough irregular river beds. The roughness in an open channel also vanes with the position of the free surface. Therefore, the section of friction co-efficient is attended by greater uncertainly for open channels than for pipes.

So open channel flow is more complicated than pipe flow.

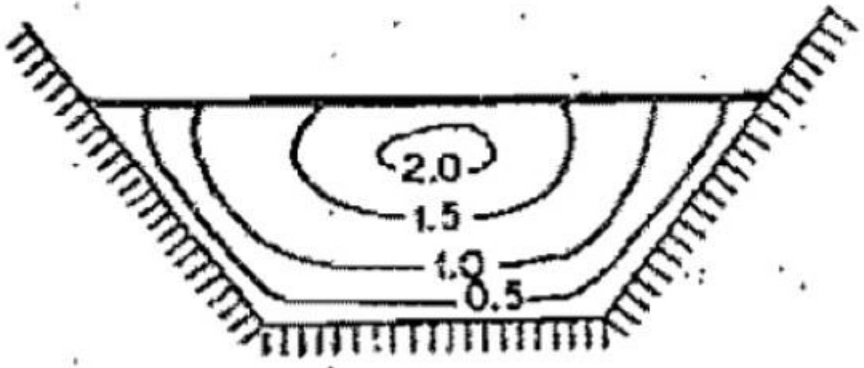
Velocity Distribution in a Channel Section:

- Owing to the presence of a free surface and to the friction along the channel wall, the velocities in a channel are not uniformly distributed in the channel section.
- The measured maximum velocity in ordinary channels usually appears to occur below the free su1face at a distance **of 0.5 to 0.25 of the depth;** the closer to the banks, the deeper is the maximum.
- The velocity distribution in a channel section depends also other factors, such as the unusual shape of the section, the roughness of the channel; and the presence of bends.

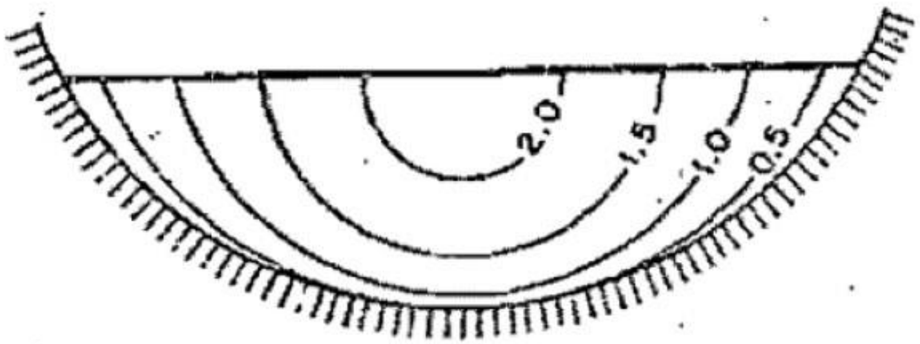
Velocity Distribution in a Channel Section:



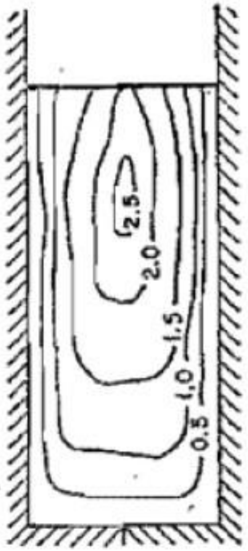
Triangular channel



Trapezoidal channel



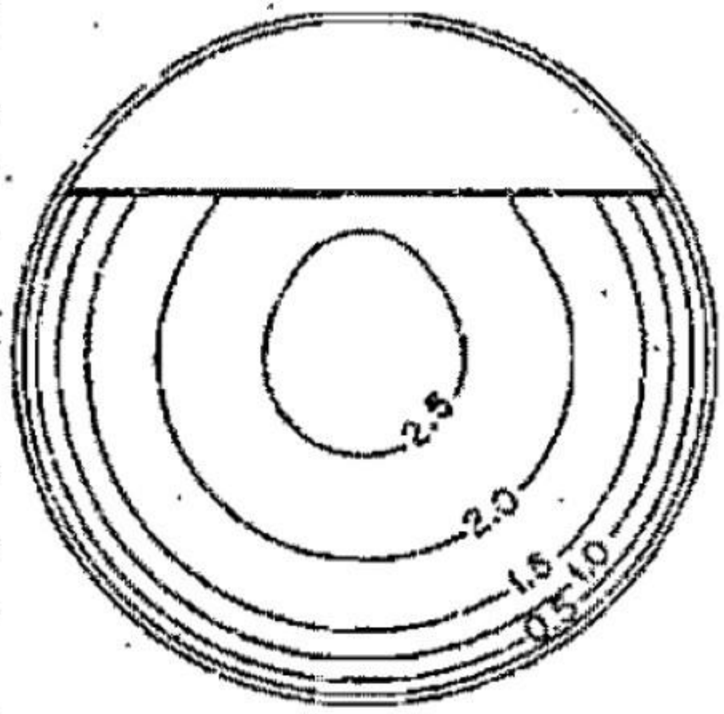
Shallow ditch



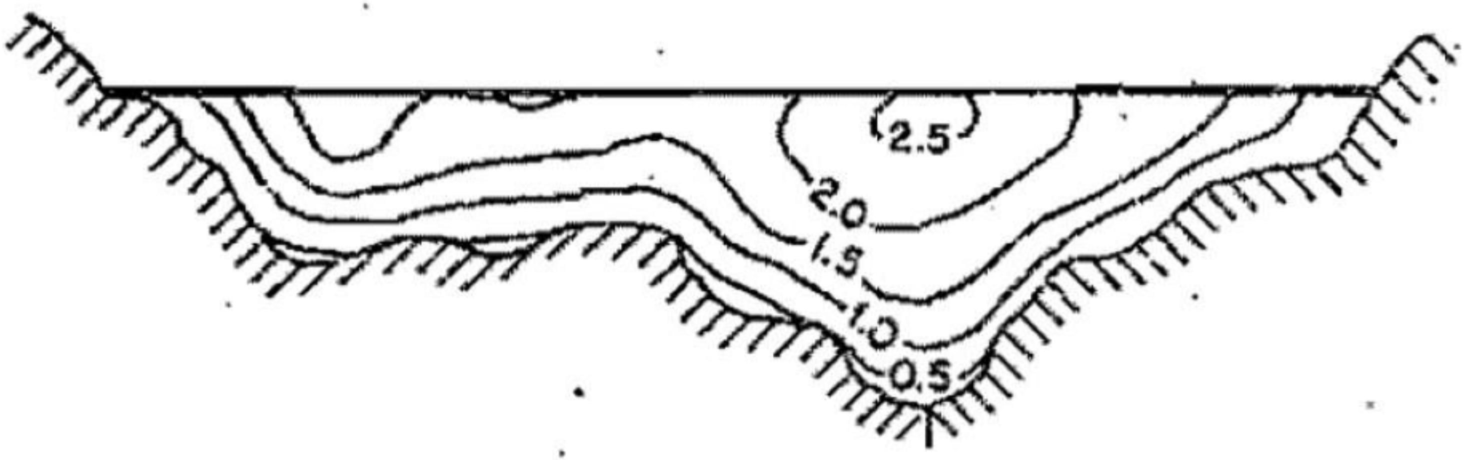
Narrow rectangular section

Typical curves of equal velocity in various channel sections.

Velocity Distribution in a Channel Section:



Pipe

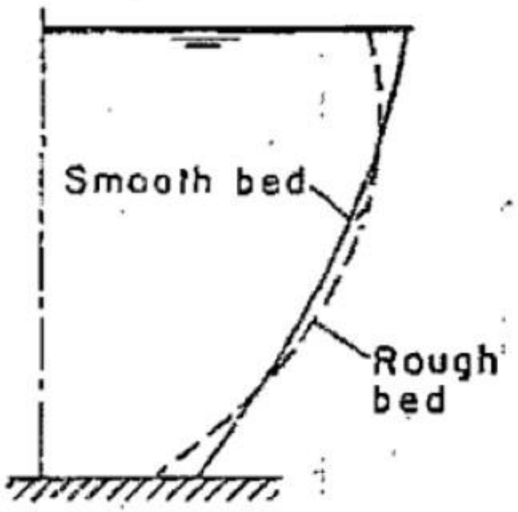


Natural irregular channel

Typical curves of equal velocity in various channel sections.

Velocity Distribution in a Channel Section:

- In a broad, rapid and shallow stream or in a very smooth channel, maximum velocity may often occur in the free surface.
- The roughness of the channel will cause the curvature of the vertical-velocity-distribution curve to increase (Fig).

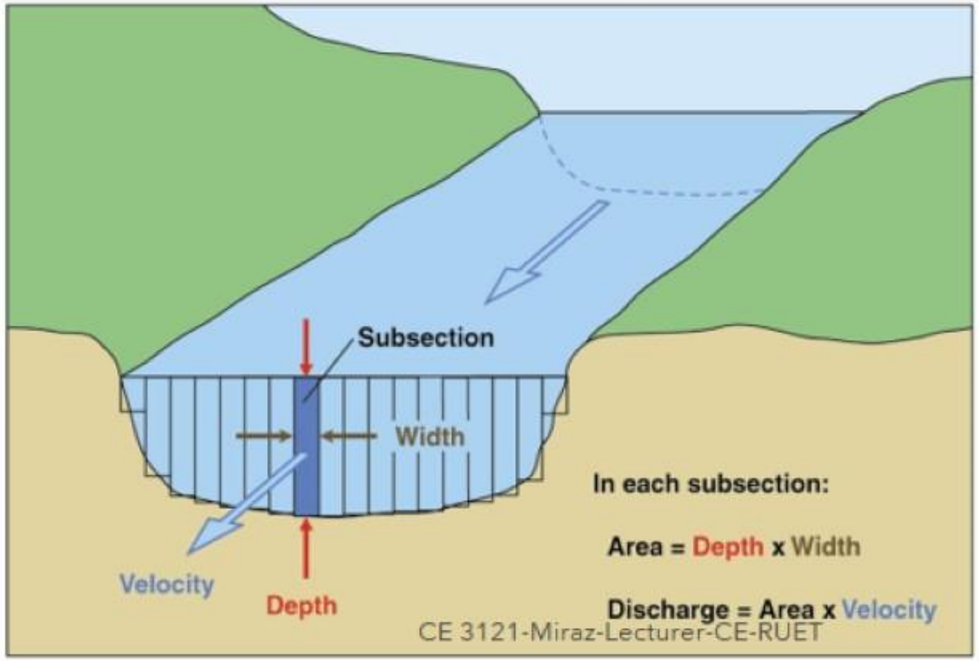


- On a bend the velocity increases greatly at the convex side, owing to the centrifugal action of the flow.
- Contrary to the usual belief, a surface wind has very little effect on velocity distribution.

Measurement of Velocity:

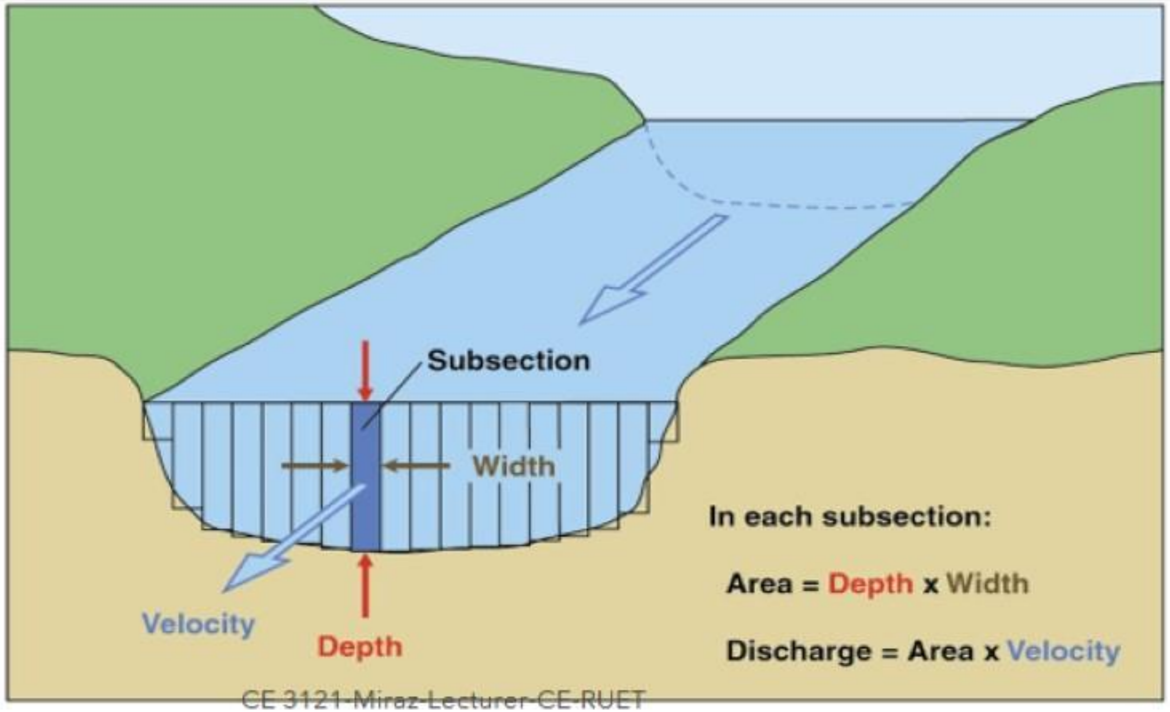
According to the stream-gaging procedure of the U.S. Geological Survey;

- The channel cross section is divided into vertical strips by a number of successive verticals.
- Mean velocities in verticals are determined by measuring the velocity at **0.6 of the depth** in each vertical, or, where more reliable results are required, by taking the average of the velocities at 0.2 and 0.8 of the depth.



Measurement of Velocity:

- The average of the mean velocities in any two adjacent verticals multiplied by the area between the verticals gives the discharge through this vertical strip of the cross-section.
- The sum of discharge through all strips is the total discharge.
- The **mean velocity** of the whole section is, therefore, equal to the total discharge divided by the whole area.



Velocity-Distribution Coefficients:

- As a result of non-uniform distribution of velocities over a channel section the velocity head of an open-channel flow is generally greater than the value computed according to the expression $\frac{V^2}{2g}$, where V is the mean velocity.
- When the energy principle is used in computation, the true velocity head may be expressed as $\frac{\alpha V^2}{2g}$, where α is known as the *energy coefficient* or *Coriolis coefficient*, in honor of G. Coriolis who first proposed it.
- Experimental data indicate that the value of **α varies from about 1.03 to 1.36** for fairly straight prismatic channels.
- The value is generally higher for small channels and lower for large streams of considerable depth.

Velocity-Distribution Coefficients:

- The non-uniform distribution of velocities also affects the computation of momentum in open-channel flow.

- From the principle of mechanics, the momentum of the fluid passing through a channel section per unit time is expressed by $\frac{\beta \omega Q V}{g}$, where β is known as ·the *momentum coefficient* or *Boussinesq coefficient*, after J. Boussinesq who first proposed it; ω is the unit weight of water; Q is the discharge; and V is the mean velocity.

- lt is generally found that the value of β for fairly straight prismatic channels varies approximately from **1.01 to 1.12**.

Determination of Velocity-distribution Coefficients:

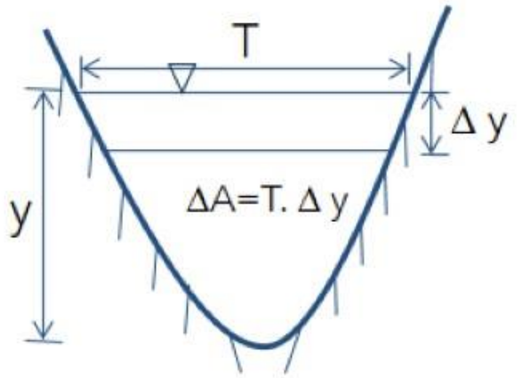
Let ΔA be an elementary area in the whole water area A, and ω the unit weight of water

Then, the weight of water passing ΔA per unit time with a velocity $v = \omega v \Delta A$.

The kinetic energy of water passing ΔA per unit time $= \dfrac{\omega v^3 \Delta A}{2g}$.

This is equivalent to the product of the weight $\omega v \Delta A$ and the velocity head $\dfrac{v^2}{2g}$.

The total kinetic energy for the whole water area $= \dfrac{\sum \omega v^3 \Delta A}{2g}$

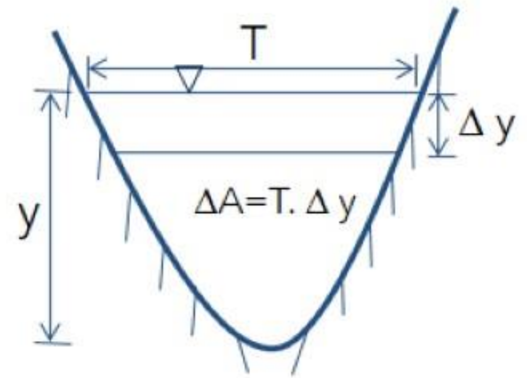


Determination of Velocity-distribution Coefficients:

Now, taking the whole area as A, the mean velocity as V and the corrected velocity head for the whole area as $\frac{\alpha V^2}{2g}$,

the total kinetic energy $= \frac{\alpha \omega V^3 A}{2g}$, Now, equating this quantity with $\frac{\sum \omega v^3 \Delta A}{2g}$ and reducing:

$$\alpha = \frac{\int v^3 \Delta A}{V^3 A} \approx \frac{\sum v^3 \Delta A}{V^3 A}$$



Determination of Velocity-distribution Coefficients:

The momentum of water passing ΔA per unit time is the product of the mass $\frac{\omega v \Delta A}{g}$ and the velocity $\mathbf{v} = \frac{\omega v^2 \Delta A}{g}$

The total momentum = $\frac{\sum \omega v^2 \Delta A}{g}$

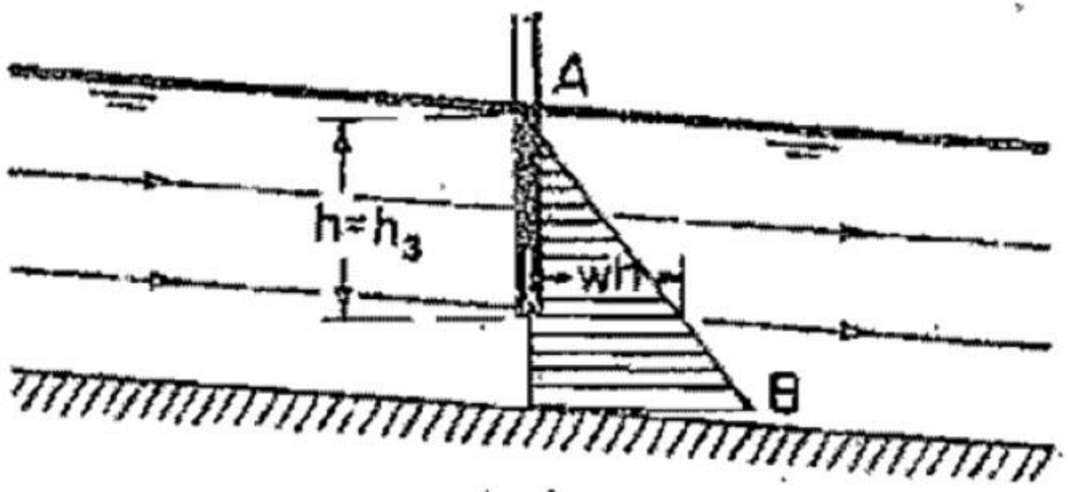
Equating this quantity with the corrected momentum for the whole area, or by $\frac{\beta \omega A V^2}{g}$, and reducing

$$\beta = \frac{\int v^2 dA}{V^2 A} \approx \frac{\sum v^2 \Delta A}{V^2 A}$$

Pressure Distribution in a Channel Section:

- The pressure at any point on the cross section of the flow in a channel of small slope can be measured by the height of the water column in a piezometer tube installed at the point.

- Ignoring minor disturbances due to turbulence, etc., it is apparent that this water column should rise from the point of measurement up to the hydraulic grade line or the water surface.

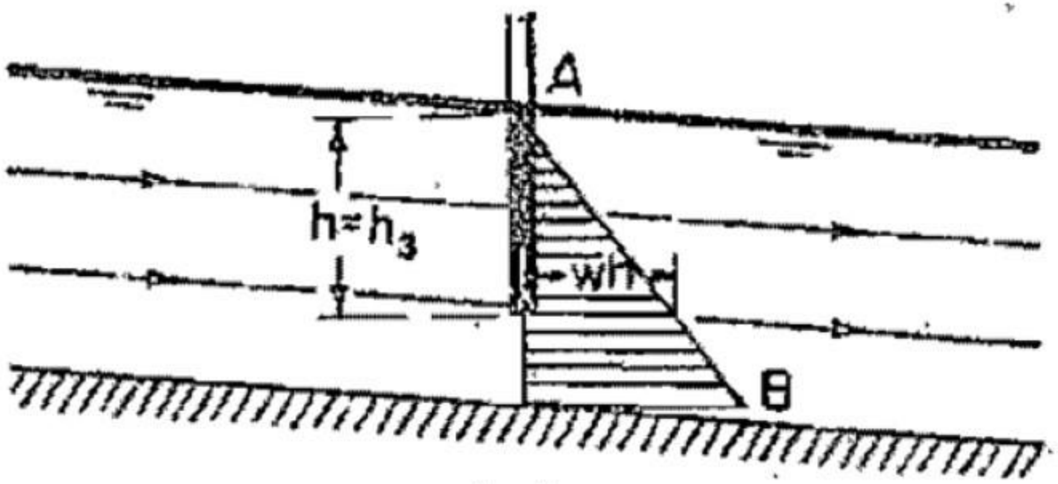


(a)

Pressure Distribution in a Channel Section:

- Therefore, the **pressure at any point** on the section is **directly proportional to the depth** of the point below the free surface and **equal to the hydrostatic pressure** corresponding to this depth.

- In other words, the distribution of pressure over the cross section of the channel is the same as the distribution of hydrostatic pressure; that is, the distribution is linear and can be represented by a straight line AB. This is known as the hydrostatic law of pressure distribution.



(a)

Pressure Distribution in a Channel Section:

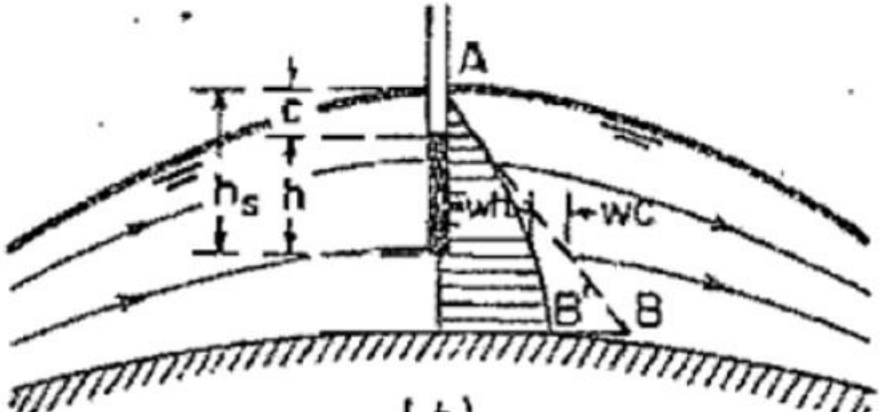
- Strictly speaking, the application of the hydrostatic law to the pressure distribution in the cross section of a flowing channel is valid only if the flow filaments have no acceleration components in the plane of cross section. This type of flow is theoretically known as **parallel flow**.
- In actual problems **uniform flow** is practically **parallel flow**.
- **Gradually varied flow** may also be regarded as **parallel flow**, since the change in depth of flow is so mild that the streamlines have **neither appreciable curvature nor divergence**; that is, the curvature and divergence are so small that the effect of the acceleration components in the cross-sectional plane is negligible.
- For practical purposes, therefore, the hydrostatic law of pressure distribution is applicable to gradually varied flow as well as to uniform flow.

Pressure Distribution in a Channel Section:

- If the curvature of streamlines is substantial, the flow is theoretically known as **curvilinear flow**.

- The effect of the curvature is to produce appreciable acceleration components or centrifugal forces normal to the direction of flow. Thus, the pressure distribution over the section deviates from the hydrostatic if curvilinear flow occurs in the vertical plane.

- Such curvilinear flow may be either convex or concave. In both cases the nonlinear pressure distribution is represented by ***AB′*** instead of straight distribution ***AB*** that would occur if the flow were parallel.



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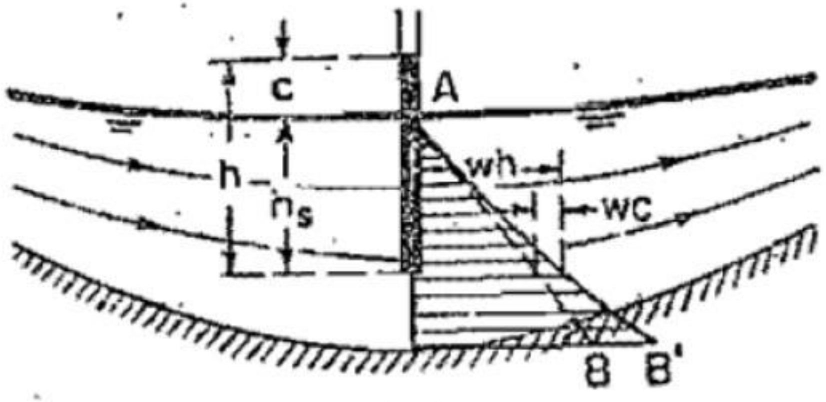
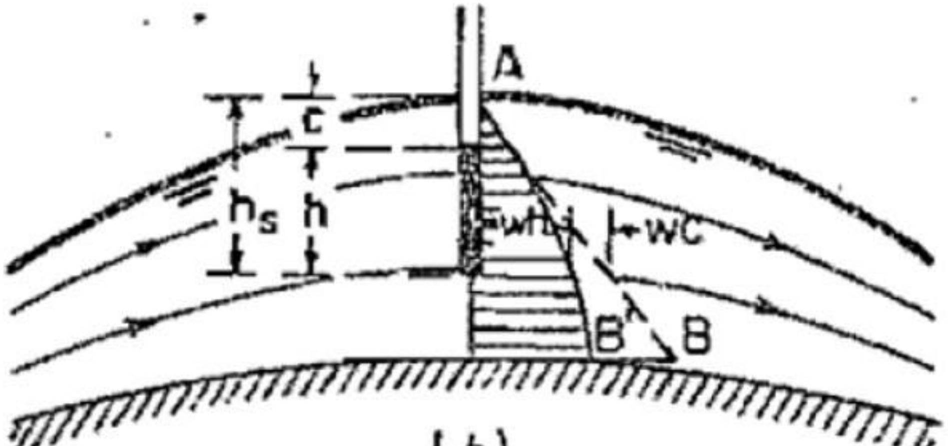


(c)

Pressure Distribution in a Channel Section:

- It is assumed that all streamlines are horizontal at the section under consideration.
- In concave flow the centrifugal forces are pointing downward to reinforcing the gravity action; so the resulting pressure is greater than the otherwise hydrostatic pressure of a parallel flow.
- In convex flow the centrifugal forces are acting upward against the gravity action; consequently, the resulting pressure is less than the otherwise hydrostatic pressure of a parallel flow.



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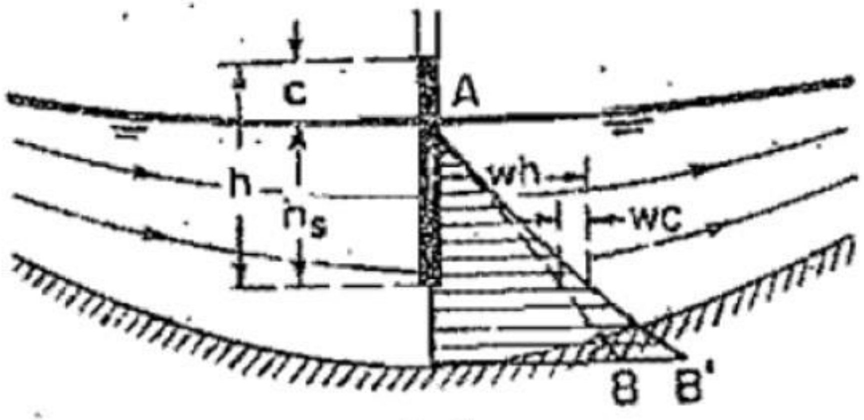


(c)

Pressure Distribution in a Channel Section:

Let the deviation from an otherwise hydrostatic pressure h, in a curvilinear flow be designated by c. Then the true pressure or the piezometric height

$h = h_s + c.$

If the channel has a curved longitudinal profile, the approximate centrifugal pressure may be computed, by Newton's law of acceleration, the product of the mass of water haying height ***d*** and cross section of **1 sq** ft that is, ***wd/g***, and the centrifugal acceleration ***v^2/r***; or

$$P = \frac{wdv^2}{gr}$$

Determination of Velocity-distribution Coefficients: Problem-01

The velocity distribution in a rectangular channel of width B and depth of flow y_0 was approximated as $v = k_1\sqrt{y}$ in which k_1 = a constant. Calculate the average velocity for the cross section and correction coefficients α and β.

Solution Area of cross section $A = By_0$

$$V = \frac{1}{A}\int_A v\,dA$$

Average velocity

$$V = \frac{1}{By_0}\int_0^{y_0} v(B\,dy)$$

$$= \frac{1}{y_0}\int_0^{y_0} k_1\sqrt{y}\,dy = \frac{2}{3}k_1\sqrt{y_0}$$

Determination of Velocity-distribution Coefficients: Problem-01

Kinetic energy correction factor

$$\alpha = \frac{\int v^3\,dA}{V^3 A}$$

$$\alpha = \frac{\int_0^{y_0} v^3 (B\,dy)}{V^3 B y_0} = \frac{\int_0^{y_0} k_1^3 y^{3/2} B\,dy}{\left(\frac{2}{3}k_1\sqrt{y_0}\right)^3 B y_0} = 1.35$$

Momentum correction factor

$$\beta = \frac{\int v^2 dA}{V^2 A}$$

$$\beta = \frac{\int_0^{y_0} v^2 (B\,dy)}{V^2 B y_0} = \frac{\int_0^{y_0} k_1^2 y B\,dy}{\left(\frac{2}{3}k_1\sqrt{y_0}\right)^2 B y_0} = 1.125$$

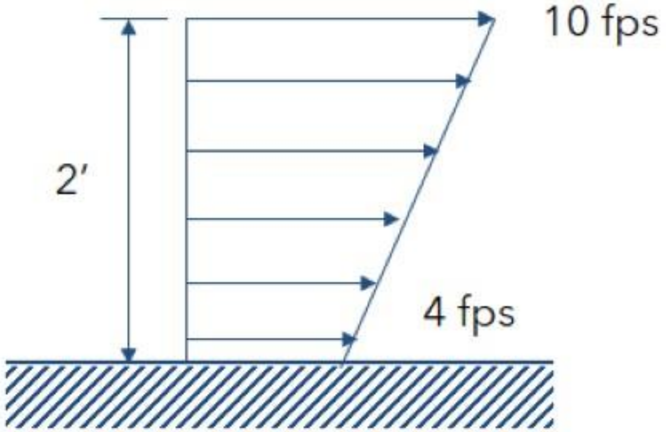
Determination of Velocity-distribution Coefficients: Assignment-01

The velocity distribution in a wide open channel may be approximated by the equation $v = 0.4 + \frac{0.6y}{h}$, where h = 1m. Find mean velocity *V*, velocity-distribution coefficients α & β.

Answer: $V = 0.7m/s$, α = 1.18 & β = 1.06.

Determination of Velocity-distribution Coefficients: Problem-02

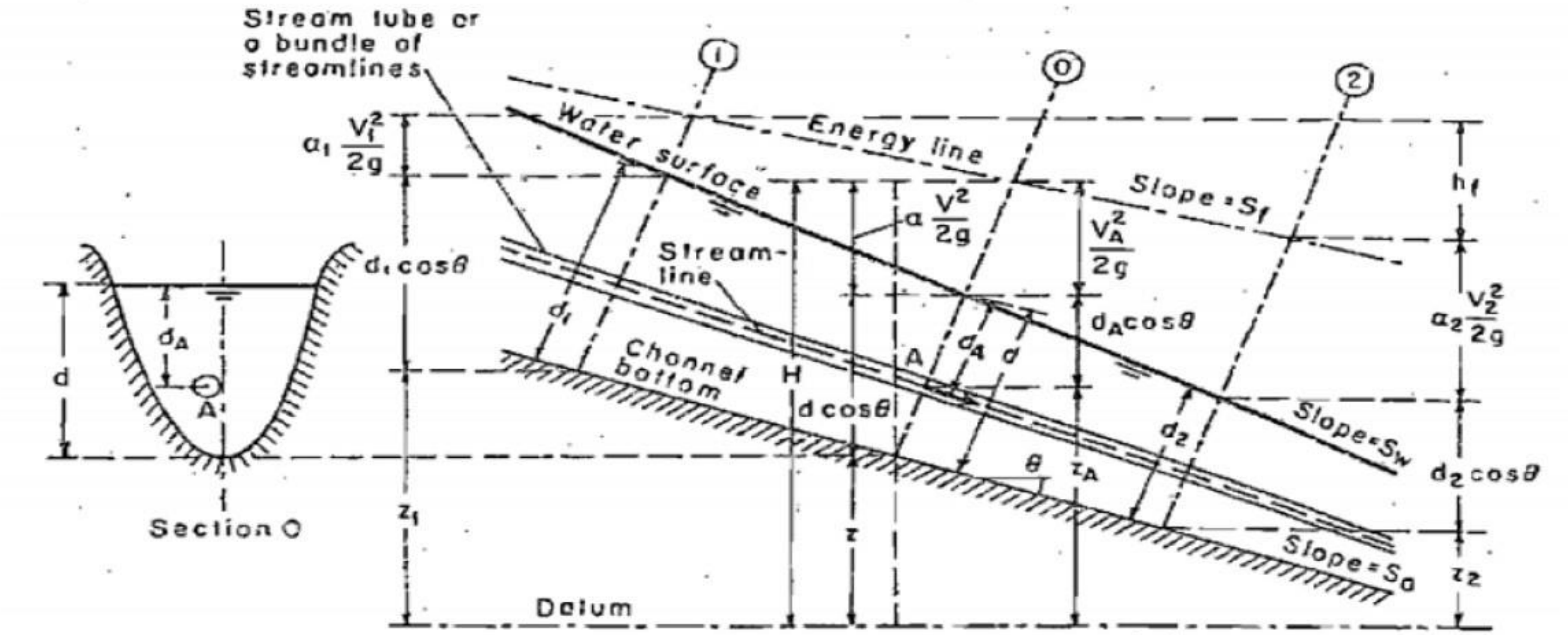
Determine the velocity distribution coefficients for a wide open channel flow with velocity distribution shown in the figure below.



Energy and Momentum Principles

Energy in Open Channel Flow:

It is known in elementary hydraulics that the total energy in foot-pounds per pound of water in any streamline, passing through a channel section may be expressed as the total head in feet of water, which is equal to the sum of the elevation above a datum, the pressure head, and the velocity head.

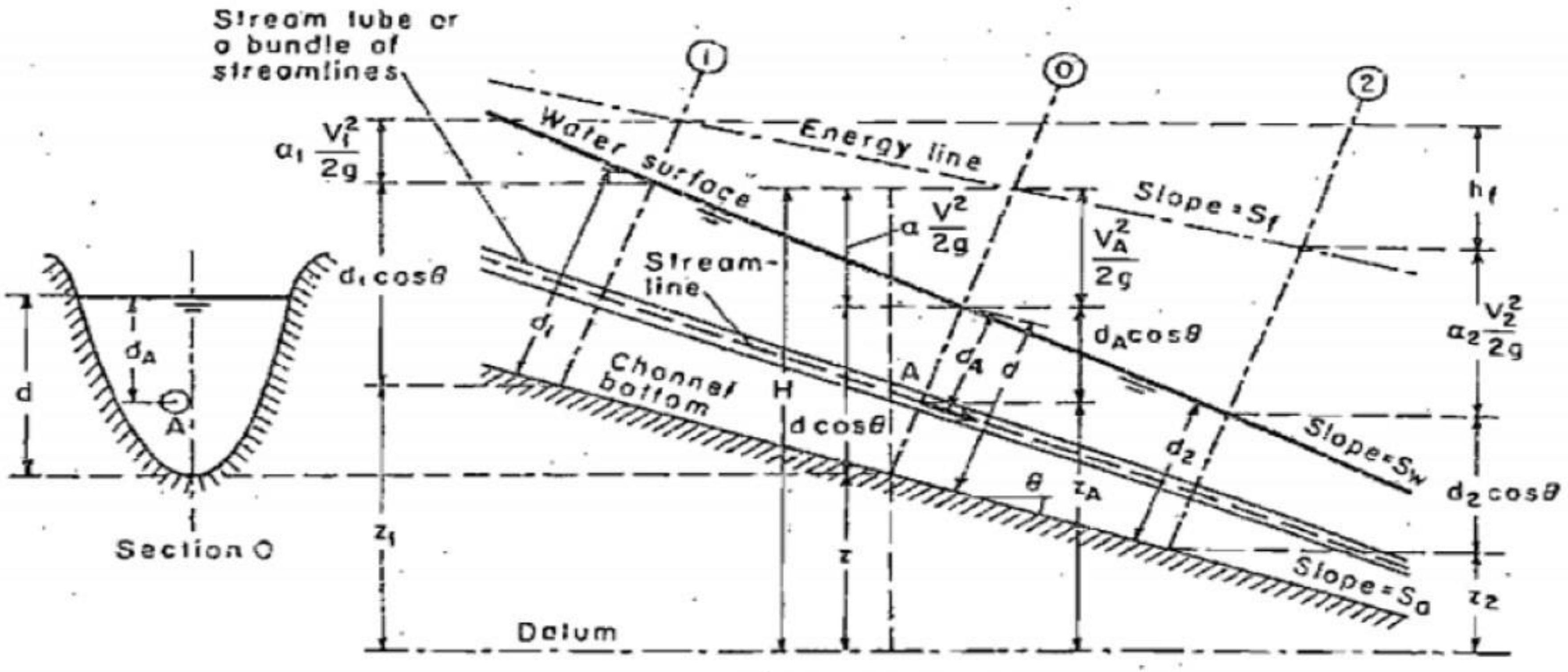


Energy and Momentum Principles

Energy in Open Channel Flow:

For example, with respect to the datum plane, the total head H at a section 0 containing point A on a streamline of flow in a channel, of large slope (Fig.) may be written

$$H = z_A + d_A \cos\theta + \frac{V_A^2}{2g}$$



Energy in Open Channel Flow:

- In general, every streamline passing through a channel section will have a different velocity head, owing to the nonuniform velocity distribution in actual flow.
- Only in an ideal parallel flow of uniform velocity distribution can the velocity head be truly identical for all, points on the cross section.
- In the case of gradually varied flow, however, it may be assumed, for practical purposes, that the velocity heads for all points on the channel section are equal, and the energy coefficient may be used to correct for the over-all effect of the nonuniform velocity distribution. Thus, the total energy at the channel section is,

$$H = z + d \cos \theta + \alpha \frac{V^2}{2g}$$

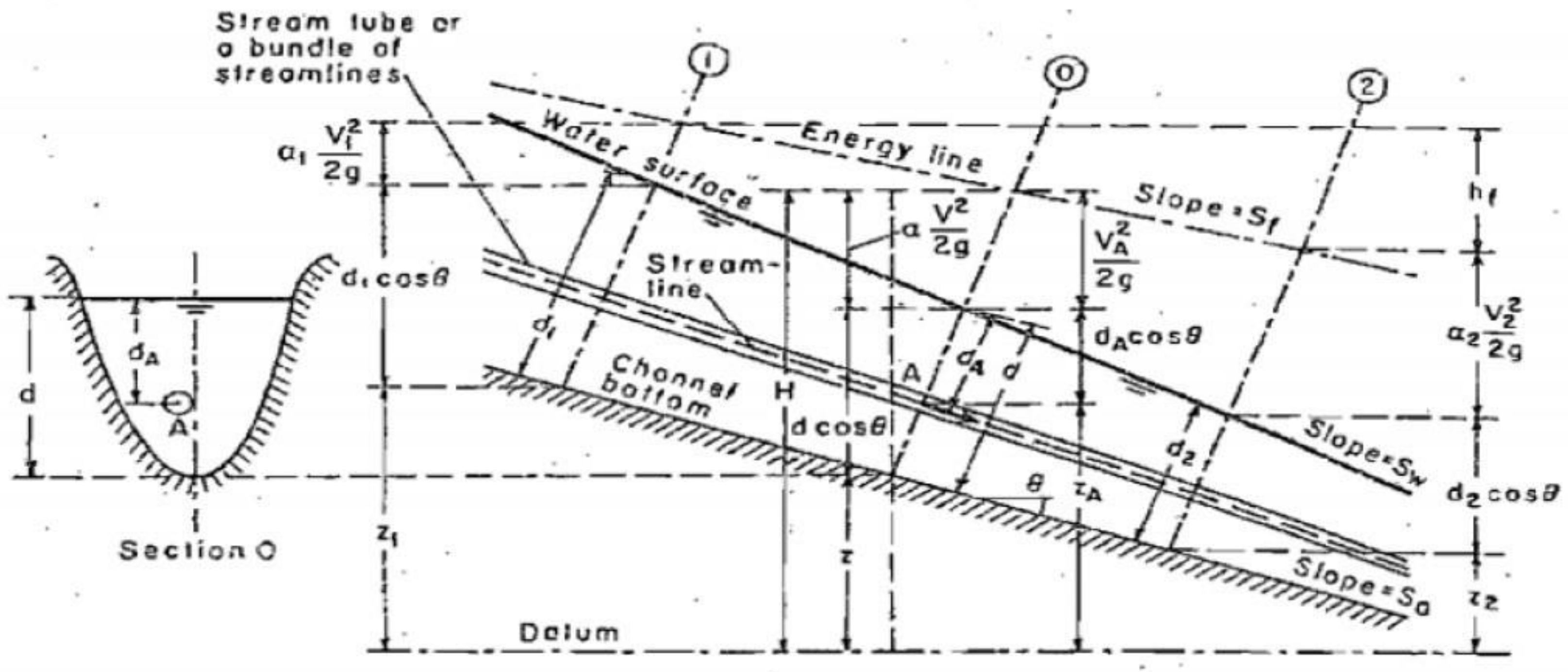
Energy in Open Channel Flow:

- For channels of small slope, $\theta \approx 0$. Thus, the total energy at the channel section is

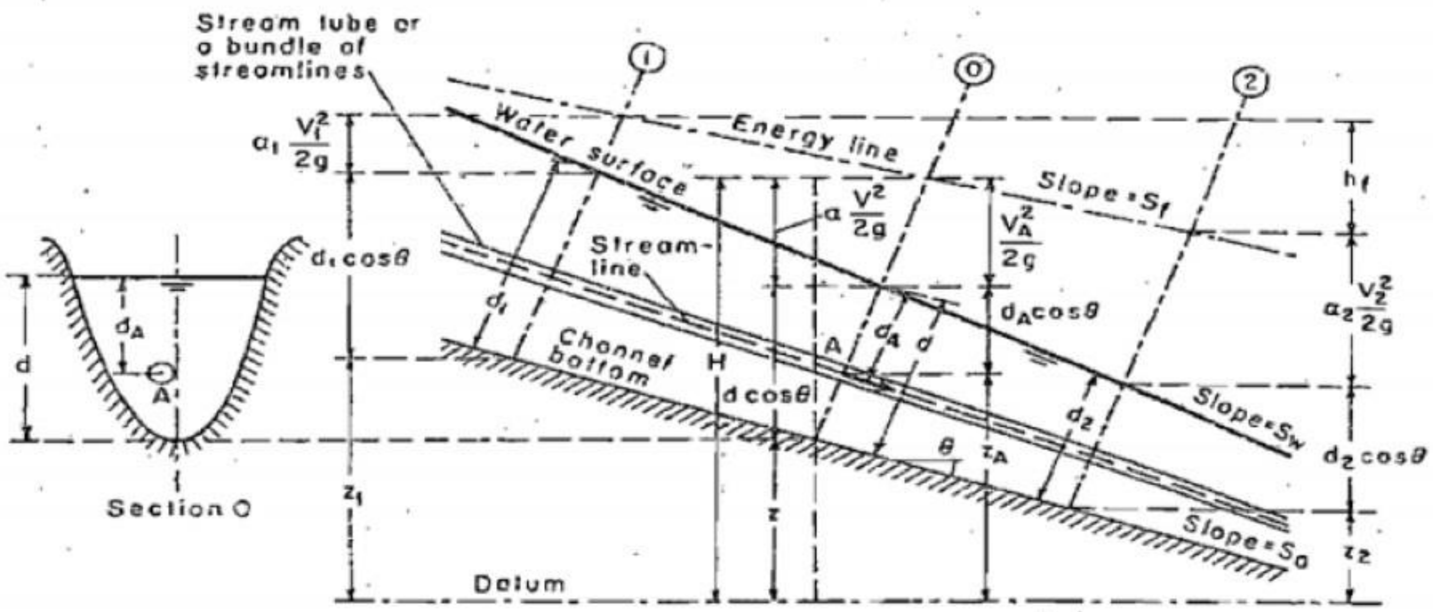
$$H = z + d + \alpha \frac{V^2}{2g}$$

Energy in Open Channel Flow:

- Consider uow a prismatic channel of large slope. The line representing the elevations of the total head of flow is the energy line.
- The slope of the line is known as the energy gradient, denoted by S_f. The slope of the water surface is denoted by S_w , and the slope of the channel bottom by $S_o = \sin\theta$. in uniform flow, $S_f = S_w = S_o = \sin\theta$



Energy in Open Channel Flow:



According to the principle of conservation of energy, the total energy head at the upstream section 1 should be equal to the total energy head at the downstream section 2 plus the loss of energy h_f between the two sections or

$$z_1 + d_1 \cos\theta + \alpha_1 \frac{V_1^2}{2g} = z_2 + d_2 \cos\theta + \alpha_2 \frac{V_2^2}{2g} + h_f$$

Energy in Open Channel Flow:

$$z_1 + d_1 \cos \theta + \alpha_1 \frac{V_1^2}{2g} = z_2 + d_2 \cos \theta + \alpha_2 \frac{V_2^2}{2g} + h_f$$

This equation applies to parallel or gradually varied flow. For a channel of small slope, it becomes

$$z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g} + h_f$$

Either of these two equations is known as the ***energy equation***. When,

$\alpha_1 = \alpha_2 = 1$ and $h_f = 0$,

$$z_1 + y_1 + \frac{V_1^2}{2g} = z_2 + y_2 + \frac{V_2^2}{2g} = \text{const}$$

This is the well-known ***Bernoulli energy equation***

Specific Energy:

Specific energy in a channel section is defined as the energy per pound of water at any section of a channel measured with respect to the channel bottom.

Thus, when, z = 0, the specific energy becomes

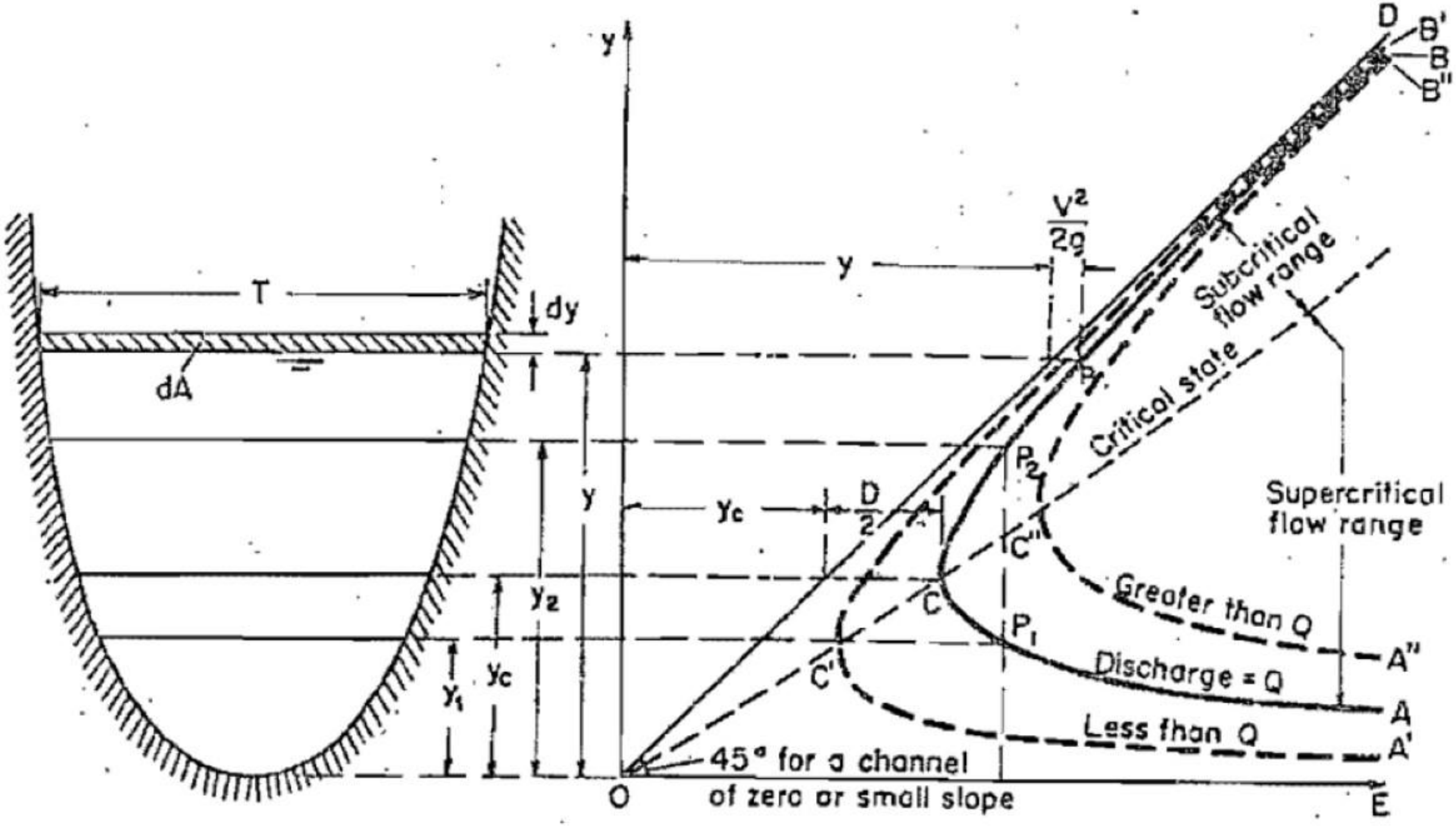
$$E = d \cos \theta + \alpha \frac{V^2}{2g}$$

or, for a channel of small slope and $\alpha = 1$,

$$E = y + \frac{V^2}{2g}$$

Which indicates that the specific energy is equal to the sum of the depth of water and the velocity head.

Specific Energy Curve:



Specific Force Curve:

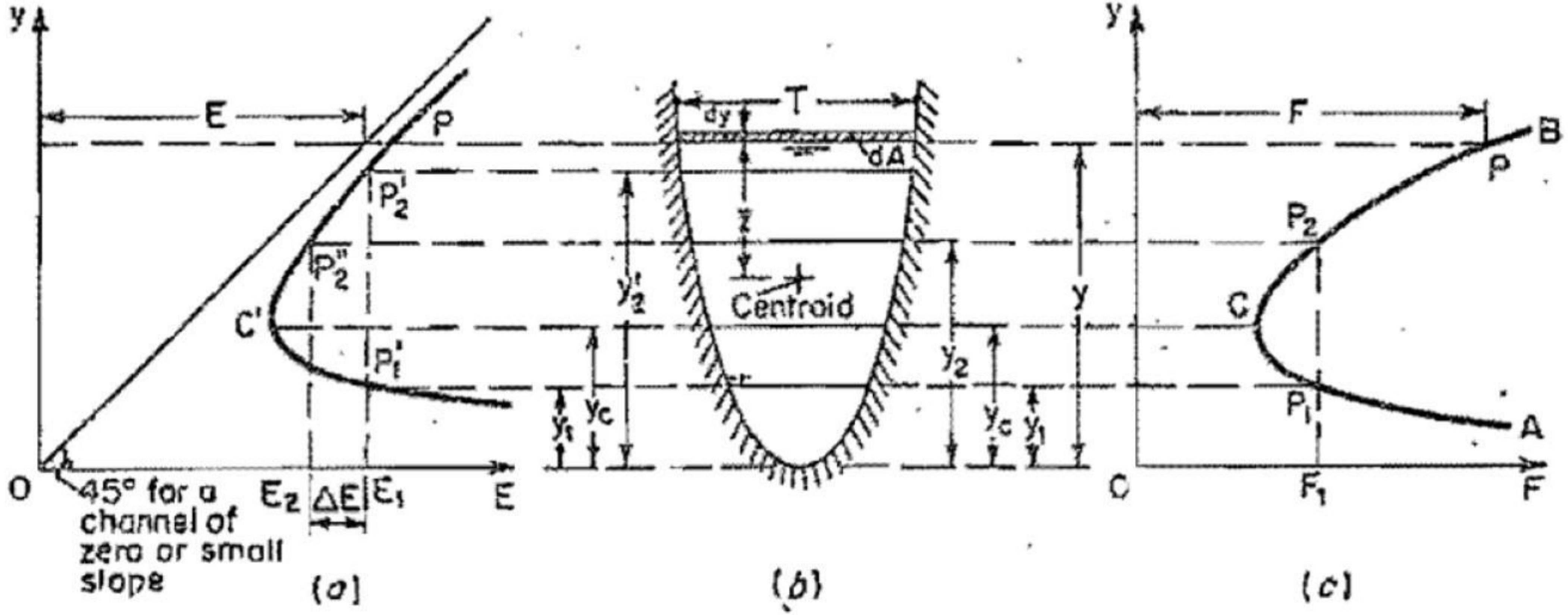


FIG. 3-9. Specific-force curve supplemented with specific-energy curve. (a) Specific-energy curve; (b) channel section; (c) specific-force curve.

Problem 3.3 (Chow):

Example 3-3. Derive a relationship between the initial depth and the sequent depth of a hydraulic jump on a horizontal floor in a rectangular channel.

Solution. The external forces of friction and the weight effect of water in the hydraulic jump on a horizontal floor are negligible, because the jump takes place in a relatively short distance and the slope angle of the horizontal floor is zero. The specific forces of sections 1 and 2 (Fig. 3-4), respectively, before and after the jump, can therefore be considered equal; that is,

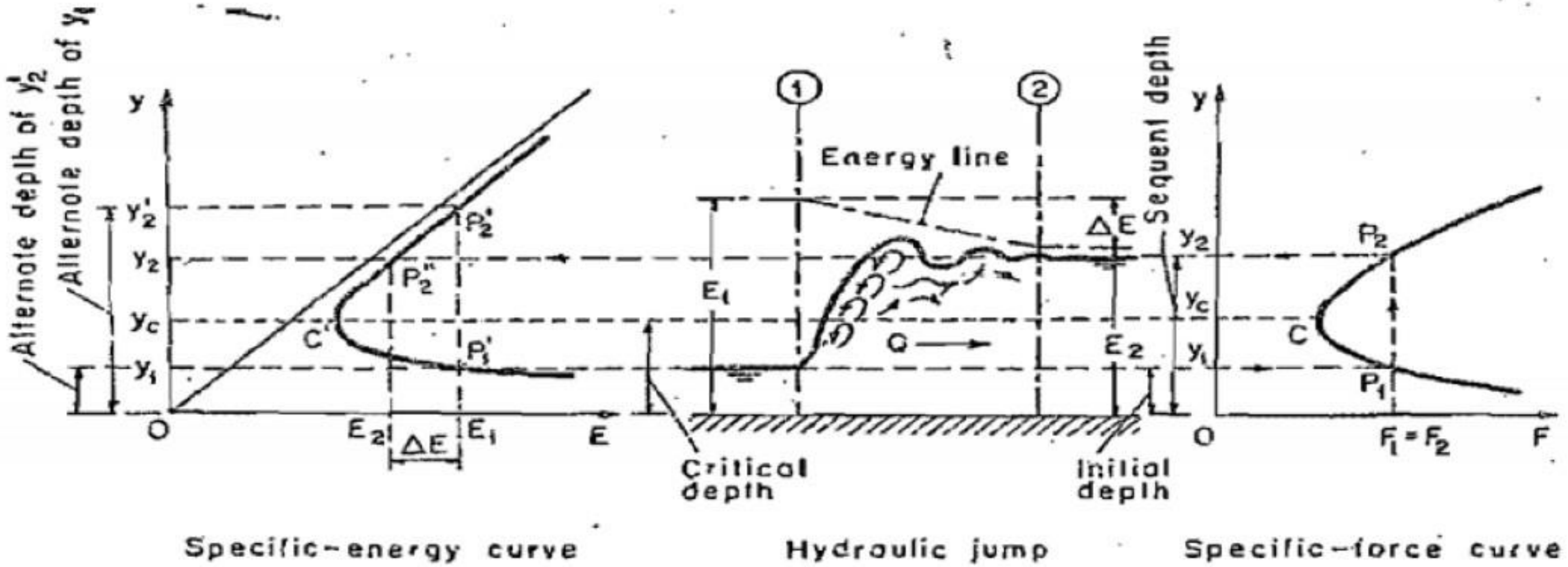


FIG. 3-4. Hydraulic jump interpreted by specific-energy and specific-force curves.

Problem 3.3 (Chow):

$$\frac{Q^2}{gA_1} + \bar{z}_1 A_1 = \frac{Q^2}{gA_2} + \bar{z}_2 A_2 \qquad (3\text{-}18)$$

For a rectangular channel of width b, $Q = V_1 A_1 = V_2 A_2$, $A_1 = by_1$, $A_2 = by_2$, $\bar{z}_1 = y_1/2$, and $\bar{z}_2 = y_2/2$. Substituting these relations and $\mathbf{F}_1 = V_1/\sqrt{gy_1}$ in the above equation and simplifying,

$$\left(\frac{y_2}{y_1}\right)^3 - (2\mathbf{F}_1^2 + 1)\left(\frac{y_2}{y_1}\right) + 2\mathbf{F}_1^2 = 0 \qquad (3\text{-}20)$$

Factoring,

$$\left[\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2\right]\left(\frac{y_2}{y_1} - 1\right) = 0$$

Then, let

$$\left(\frac{y_2}{y_1}\right)^2 + \frac{y_2}{y_1} - 2\mathbf{F}_1^2 = 0$$

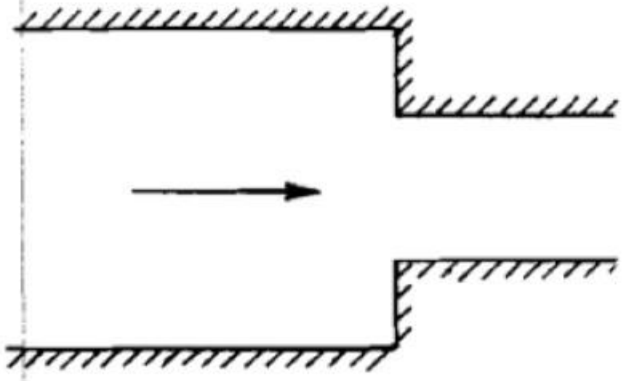
The solution of this quadratic equation is

$$\frac{y_2}{y_1} = \tfrac{1}{2}\left(\sqrt{1 + 8\mathbf{F}_1^2} - 1\right) \qquad (3\text{-}21)$$

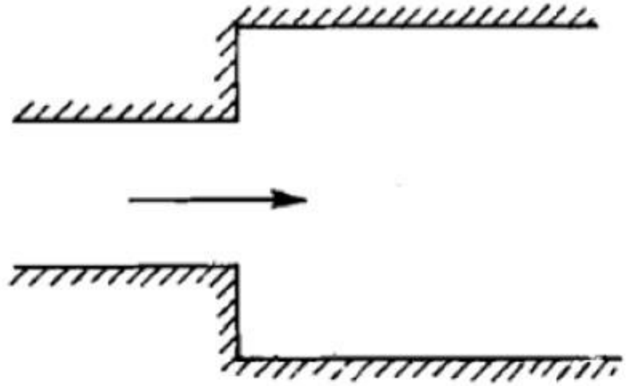
Channel transition

- **A transition** is that portion (with varying cross-section) of the channel which connects one prismatic channel to the other (which may or may not have the same cross sectional form or dimensions). The variation of the channel section may be caused either by reducing or increasing the width or by raising or lowering the bottom of the channel.
- Various channel transitions may be broadly **classified as sudden and gradual transitions.**
- **Sudden transitions** are those in which the **change of cross-sectional dimensions occur in a relatively short length.**
- On the other hand, in case of **gradual transitions** the **change of cross-sectional area takes place gradually in a relatively long length** of channel.

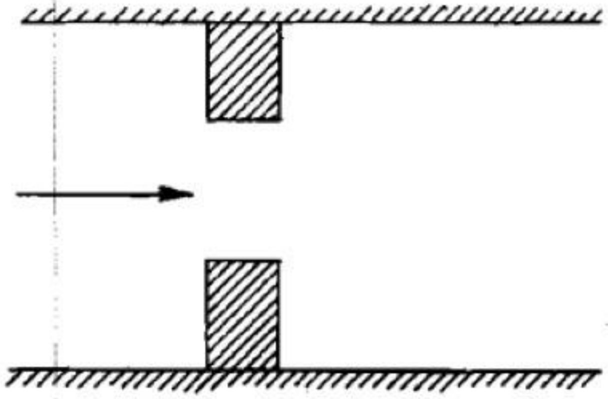
Channel transition



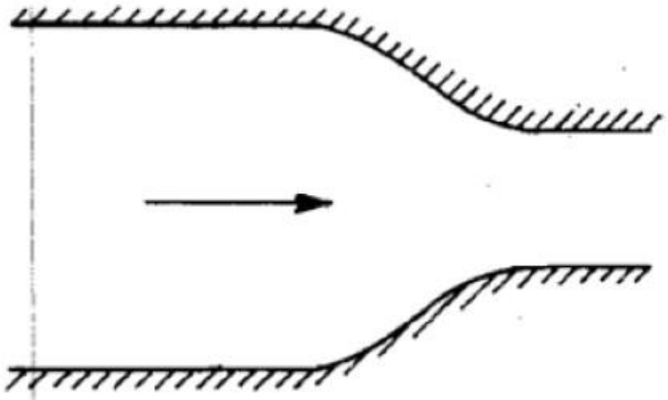
(i) SUDDEN CONTRACTION



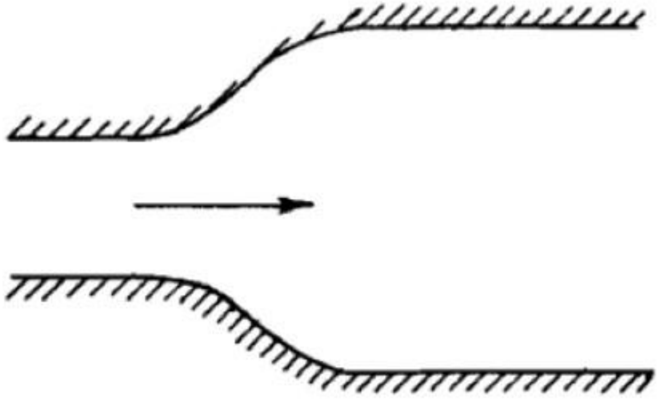
(ii) SUDDEN EXPANSION



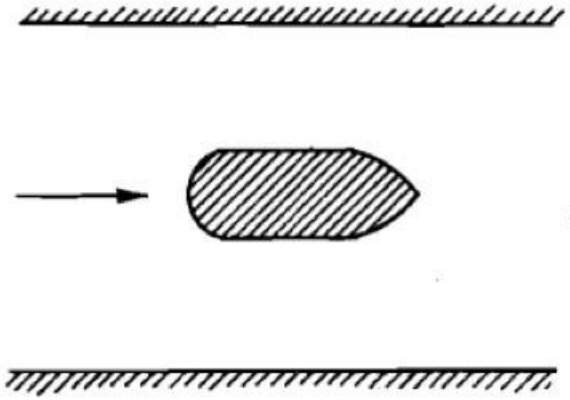
(v) CONSTRICTION



(iii) GRADUAL CONTRACTION



(iv) GRADUAL EXPANSION



(vi) BRIDGE PIER

Channel transition

Functions which channel transitions are made to serve are :

i) Metering of flow

ii) Dissipation of energy,

iii) Reduction or increase of velocities, and

iv) Change in channel section or alignment with a minimum of energy dissipation and least disturbance in the flow regime.

Some of the devices commonly used for measuring discharge, flowing in channels, are **venturi flume**, **standing wave flume** and **Parshall flume** which basically introduce a transition in the flow system.

Channel transition

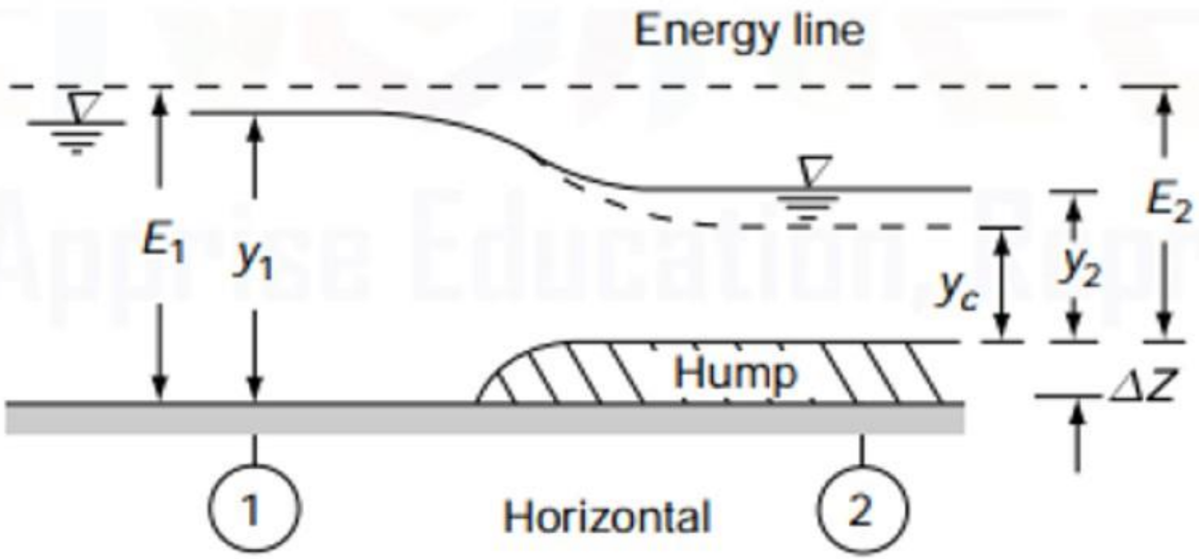
Channel with a Hump

(a) Subcritical Flow: Consider a horizontal, frictionless rectangular channel of width B carrying Q at a depth y1.

Let the flow be subcritical. At Section 2 a smooth hump of height ΔZ is built on the floor, since there are no energy losses between Sections 1 and 2, and construction of a hump causes the specific energy at Section 2 to decrease by ΔZ. Thus the specific energies at Sections 1 and 2 are given by

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

$$E_2 = E_1 - \Delta Z$$



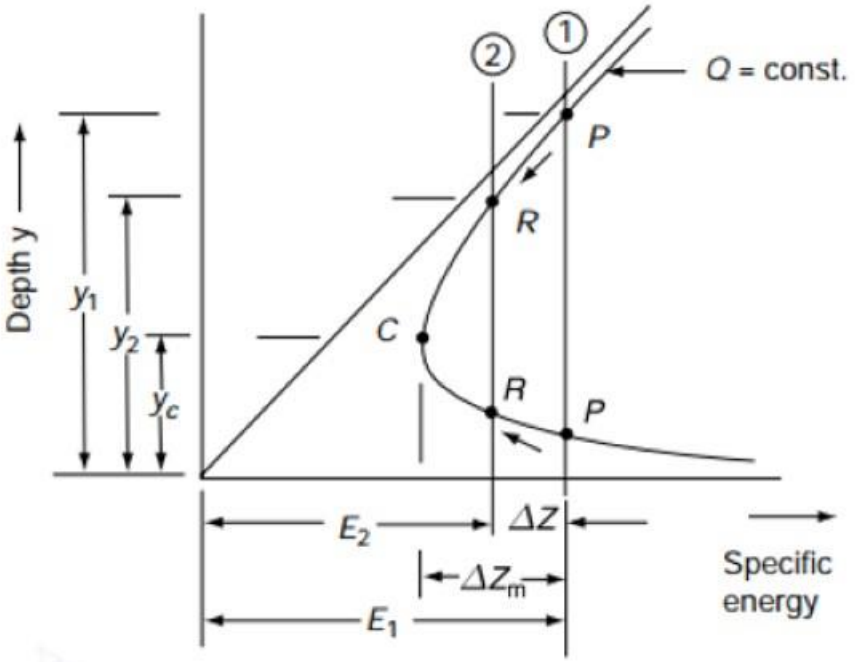
Channel transition

Channel with a Hump

(a) Subcritical Flow:

Since the flow is subcritical, the water surface will drop due to a decrease in the specific energy. In Fig., the water surface which was at P at Section 1 will come down to point R at Section 2. the depth y_2 will be given by

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2gB^2y_2^2}$$



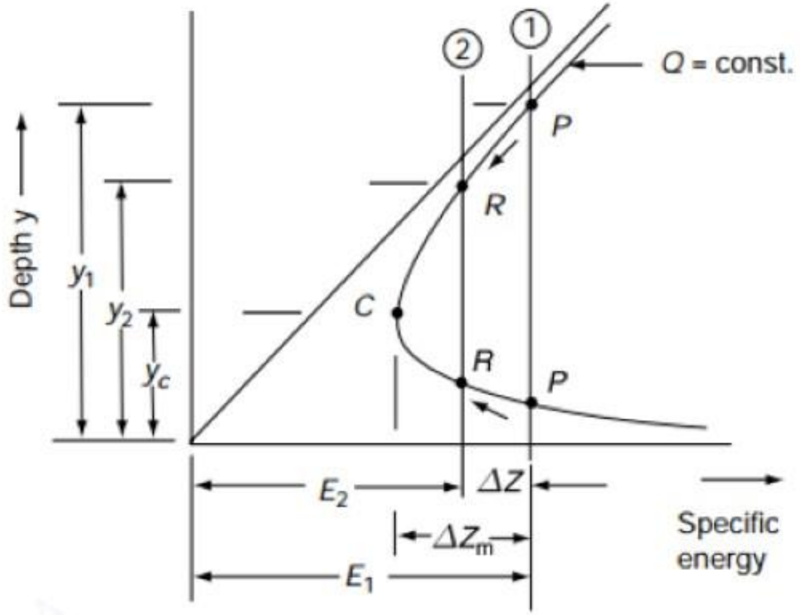
Channel transition

Channel with a Hump

(a) Subcritical Flow:

It is easy to see from Fig. that as the value of ΔZ is increased, the depth at Section 2, i.e. y_2, will decrease. The minimum depth is reached when the point R coincides with C, the critical depth point. At this point the hump height will be maximum, say = ΔZ m, $y_2 = y_c$ = critical depth and $E_2 = E_c$. Then condition at ΔZ m is given by the relation

$$E_1 - \Delta Z_m = E_2 = E_c = y_c + \frac{Q^2}{2g B^2 y_c^2}$$



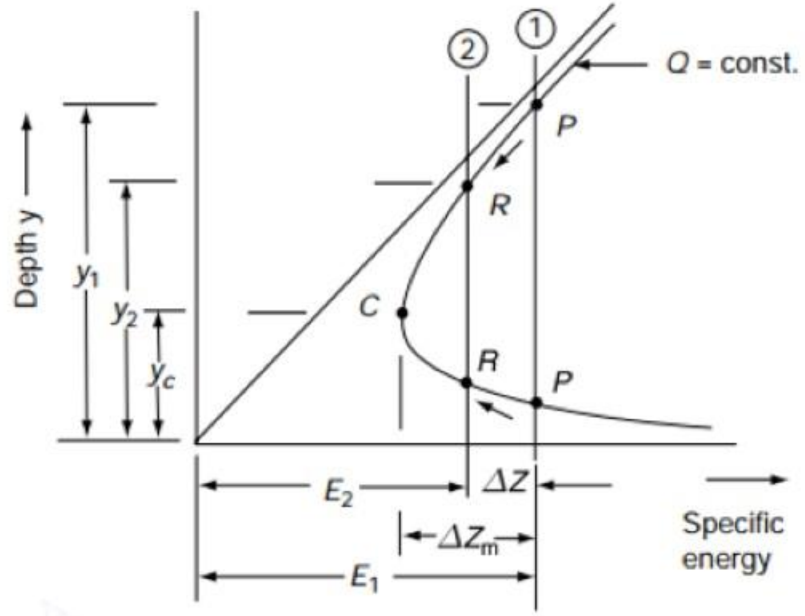
Channel transition

Channel with a Hump

(a) Subcritical Flow:

It is easy to see from Fig. that as the value of ΔZ is increased, the depth at Section 2, i.e. y_2, will decrease. The minimum depth is reached when the point R coincides with C, the critical depth point. At this point the hump height will be maximum, say = ΔZ_m, $y_2 = y_c$ = critical depth and $E_2 = E_c$. Then condition at ΔZ_m is given by the relation

$$E_1 - \Delta Z_m = E_2 = E_c = y_c + \frac{Q^2}{2gB^2 y_c^2}$$

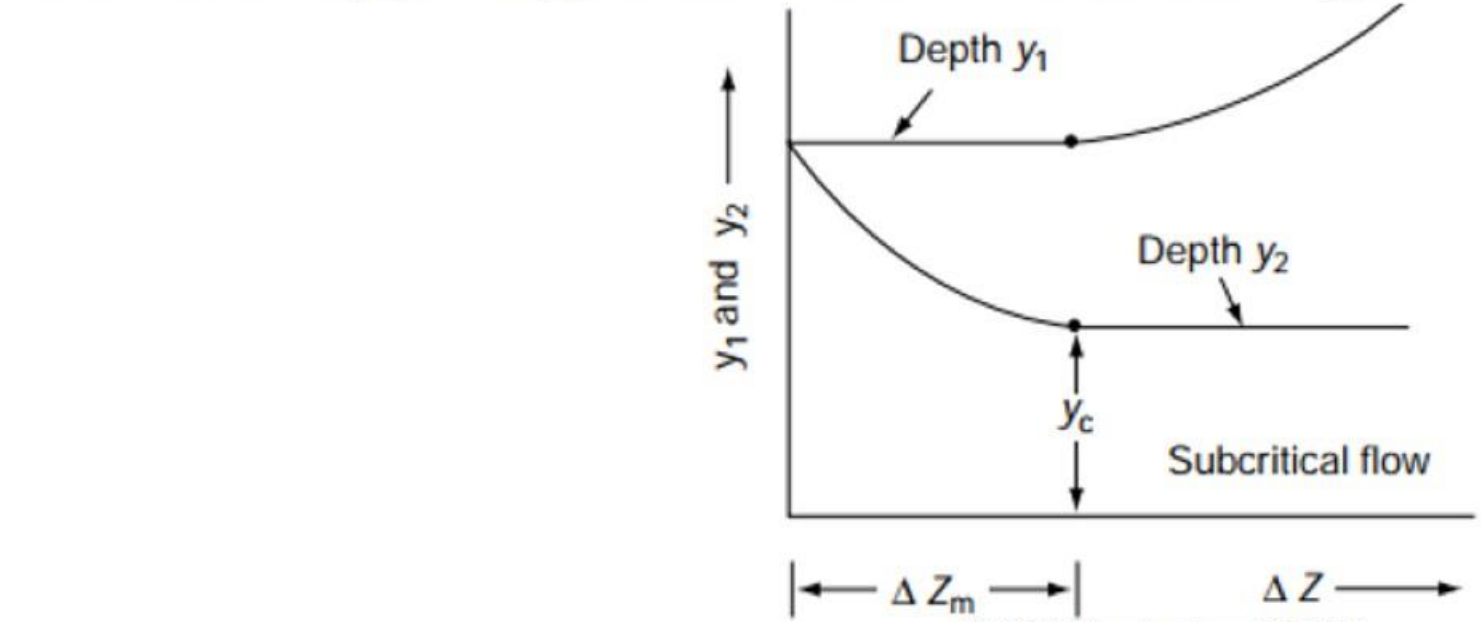


Channel transition

Channel with a Hump

(a) Subcritical Flow:

Recollecting the various sequences, when $0 < \Delta Z < \Delta Z_m$ the upstream water level remains stationary at y_1 while the depth of flow at Section 2 decreases with ΔZ reaching a minimum value of yc at $\Delta Z = \Delta Z_m$ .With further increase in the value of ΔZ, i.e. for $\Delta Z > \Delta Z_m$, y_1 will change to y'_1 while y_2 will continue to remain at y_c. The variation of y_1 and y_2 with ΔZ in the subcritical regime can be clearly noticed

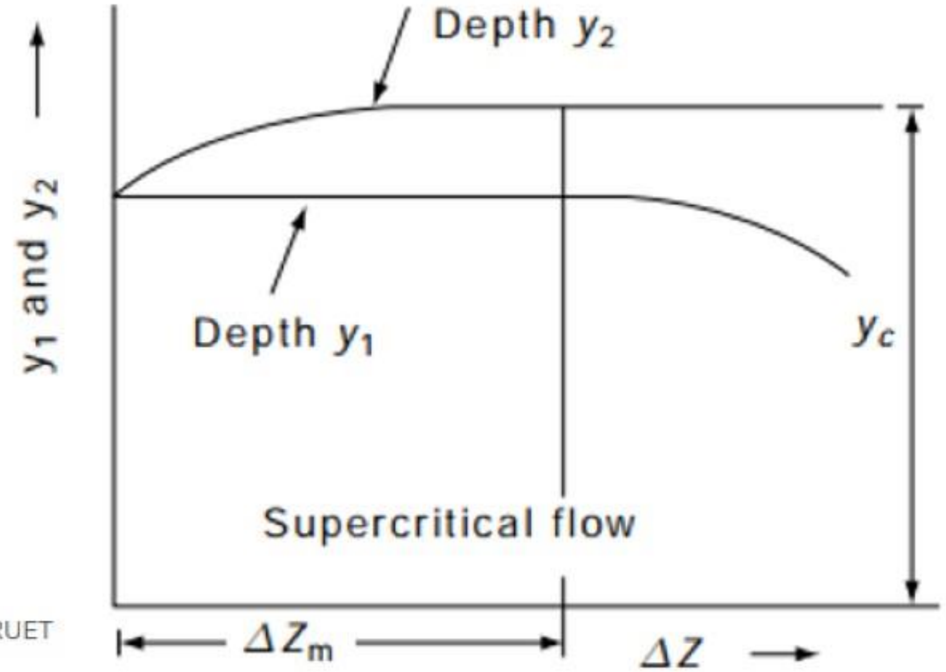


Channel transition

Channel with a Hump

(b) Supercritical Flow:

If y_1 is in the supercritical flow regime, Fig. shows that the depth of flow increases due to the reduction of specific energy. Up to the critical depth, y_2 increases to reach y_c at $\Delta Z = \Delta Z_m$. For $\Delta Z > \Delta Z_m$, the depth over the hump $y_2 = y_c$ will remain constant and the upstream depth y_1 will change. It will decrease to have a higher specific energy E′. The variation of the depths y_1 and y_2 with ΔZ in the supercritical flow is shown in Fig.

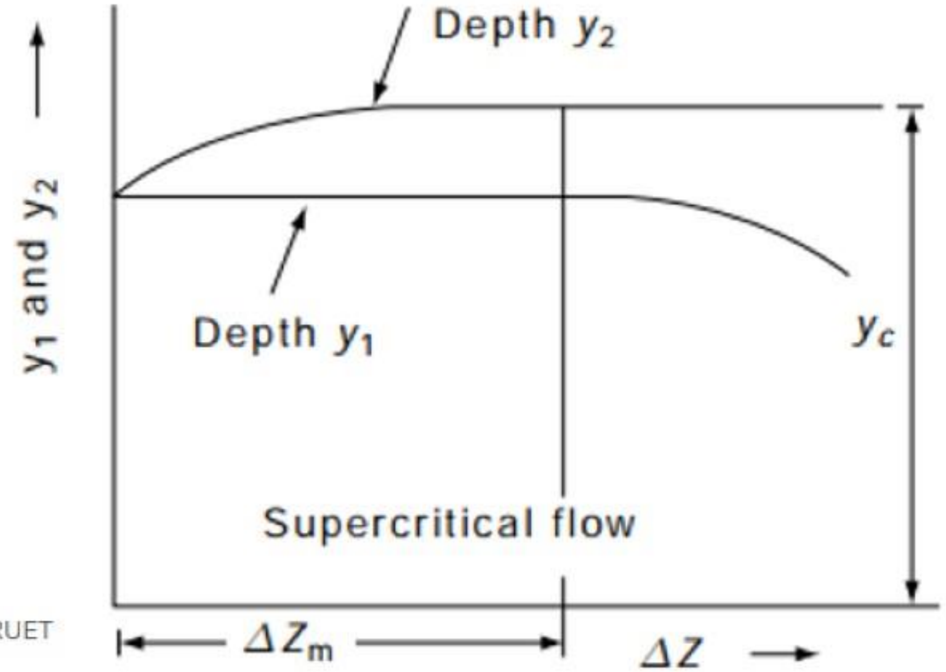


Channel transition

Channel with a Hump

(b) Supercritical Flow:

If y_1 is in the supercritical flow regime, Fig. shows that the depth of flow increases due to the reduction of specific energy. Up to the critical depth, y_2 increases to reach y_c at $\Delta Z = \Delta Z_m$. For $\Delta Z > \Delta Z_m$, the depth over the hump $y_2 = y_c$ will remain constant and the upstream depth y_1 will change. It will decrease to have a higher specific energy E′. The variation of the depths y_1 and y_2 with ΔZ in the supercritical flow is shown in Fig.



Channel transition

Example 2.10 *A rectangular channel has a width of 2.0 m and carriers a discharge of 4.80 m³/s with a depth of 1.60 m. at a certain section a small, smooth hump with a flat top and of height 0.10 m is proposed to be built. Calculate the likely change in the water surface. Neglect the energy loss.*

Solution Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively as in Fig. 2.9.

$$q = \frac{4.80}{2.0} = 2.40 \text{ m}^3\text{/s/m}$$

$$V_1 = \frac{2.40}{1.6} = 1.50 \text{ m/s}, \quad \frac{V_1^2}{2g} = 0.115 \text{ m}$$

$F_1 = V_1 / \sqrt{g y_1} = 0.379$, hence the upstream flow is subcritical and the hump will cause a drop in the watersurface elevation.

$$E_1 = 1.60 + 0.115 = 1.715 \text{ m}$$

Channel transition

$F_1 = V_1 / \sqrt{g y_1} = 0.379$, hence the upstream flow is subcritical and the hump will cause a drop in the watersurface elevation.

$$E_1 = 1.60 + 0.115 = 1.715 \text{ m}$$

At Section 2,

$$E_2 = E_1 - \Delta Z = 1.715 - 0.10 = 1.615 \text{ m}$$

$$y_c = \left(\frac{(2.4)^2}{9.81} \right)^{1/3} = 0.837 \text{ m}$$

$$E_c = 1.5\, y_c = 1.256 \text{ m}$$

Channel transition

The minimum specific energy at Section 2, E_{c2} is less then E_2, the available specific energy at that section. Hence $y_2 > y_c$ and the upstream depth y_1 will remain unchanged. The depth y_2 is calculated by solving the specific energy relation

$$y_2 + \frac{V_2^2}{2g} = E_2$$

i.e.

$$y_2 + \frac{(2.4)^2}{2 \times 9.81 \times y_2^2} = 1.615$$

Solving by trial-and-error, $y_2 = 1.481$ m.

Development of Uniform Flow and its Formula

Qualifications for Uniform Flow.

The uniform flow to be considered has the following main features:

- The depth, water area, velocity, and discharge at every section of the channel reach are constant; and
- The energy line, water surface, and channel bottom are all parallel; that is, their slopes are all equal, or $S_w = S_f = S_o = S$.

Development of Uniform Flow and its Formula

Establishment of Uniform Flow:

- When flow occurs in an open channel, **resistance is encountered** by the water as it flows downstream.
- This resistance is generally **counteracted** by the components of gravity forces acting on the body of the water in the direction of motion.
- A uniform flow will be developed if the **resistance is balanced by the gravity forces.**
- The **magnitude of the resistance**, when other physical factors of the channel are kept unchanged, **depends on the velocity of flow**.

Development of Uniform Flow and its Formula

Establishment of Uniform Flow:

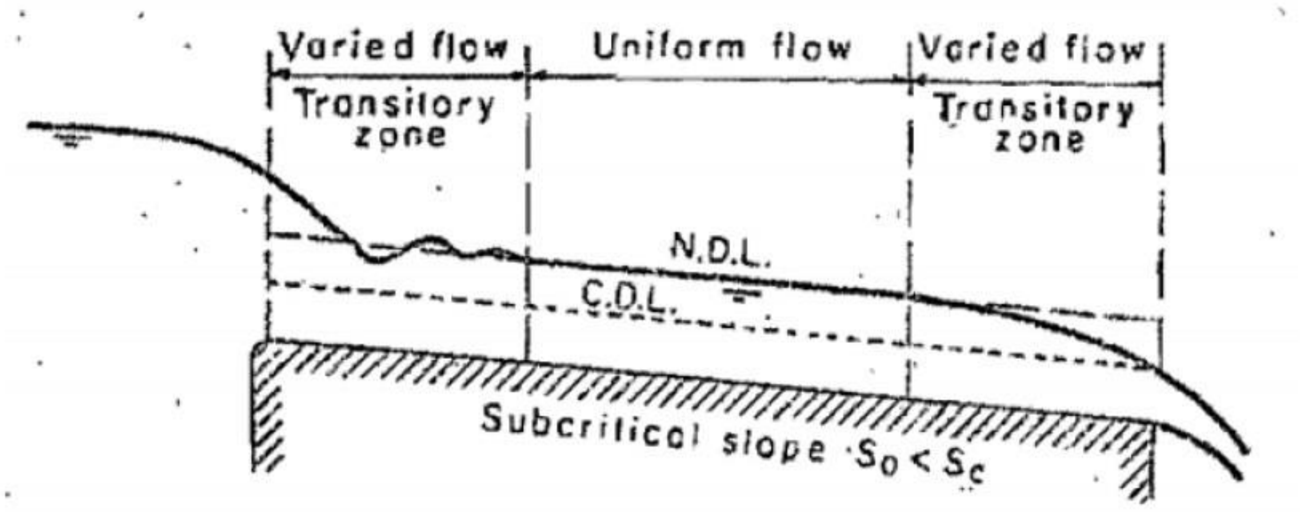
- If the water **enters the channel slowly**, the velocity and hence the resistance are, small, and the resistance is out balanced by the gravity forces, resulting-in an **accelerating flow in the upstream reach**.
- The velocity and the resistance will gradually increase until a balance between resistance and gravity forces are reached. At this moment and afterward the flow becomes uniform.
- The upstream reach, that is required for the establishment of uniform flow is known as the ***transitory zone***; In this zone the flow is accelerating and varied,
- If the channel is shorter than the transitory length required by the given conditions, uniform flow cannot be attained, Toward the downstream end of the channel the resistance may again be exceeded by gravity forces, and the flow may become varied again.

Development of Uniform Flow and its Formula

Establishment of Uniform Flow:

For purposes of explanation, a long channel is shown with three different slopes: subcritical, critical, and supercritical can be considered.

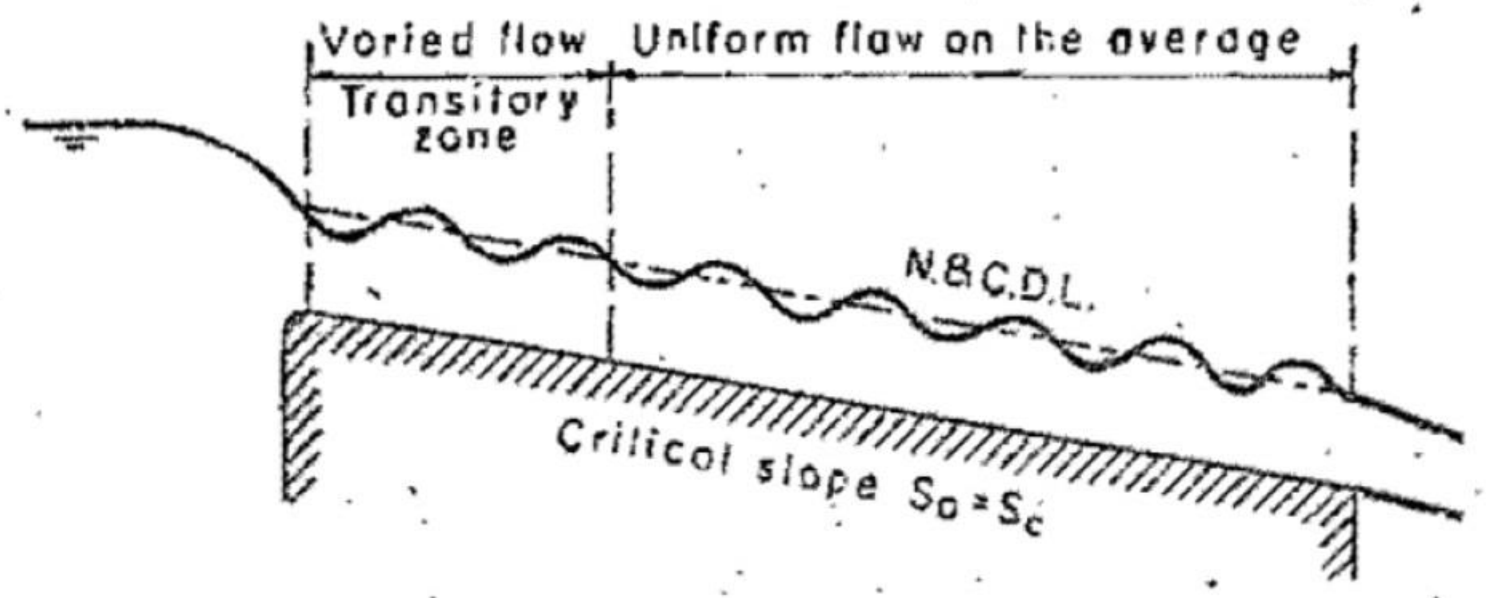
At the **subcritical slope** (in Fig.) the water surface in the transitory zone appears undulatory. The flow is uniform in the middle reach of the channel but varied at the two ends).



Development of Uniform Flow and its Formula

Establishment of Uniform Flow:

At the **critical slope** (in Fig.) the water surface of the critical flow is unstable. Possible undulations may occur in the middle reach, but on the average the depth is constant and the flow may be considered uniform.

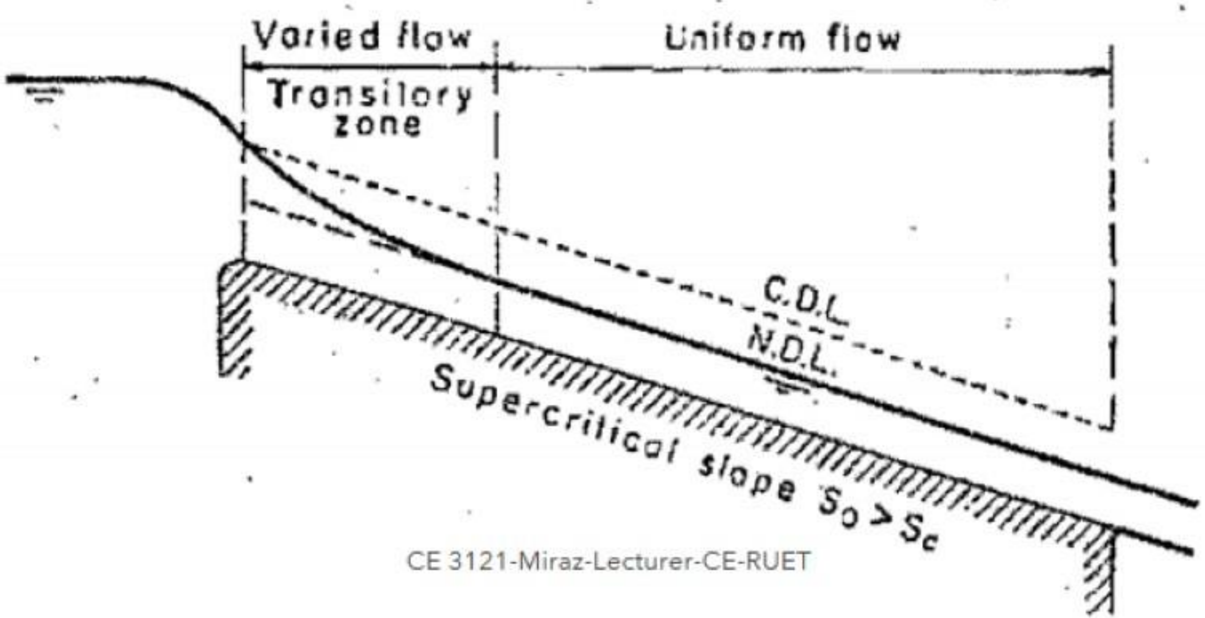


Development of Uniform Flow and its Formula

Establishment of Uniform Flow:

At the **supercritical slope** (bottom sketch in Fig. 5-1) the transitory water surface pasts from the subcritical stage to the supercritical stage through gradual hydraulic drop. Beyond the transitory zone the flow is approaching uniformity.

The depth of a uniform flow is called the **normal depth**. In all figures the long-dashed line represents the normal-depth liner abbreviated as N.D.L., and the short dashed or clotted line represents the critical-depth line. or C.D.L.



Development of Uniform Flow and its Formula

The Chezy Formula.

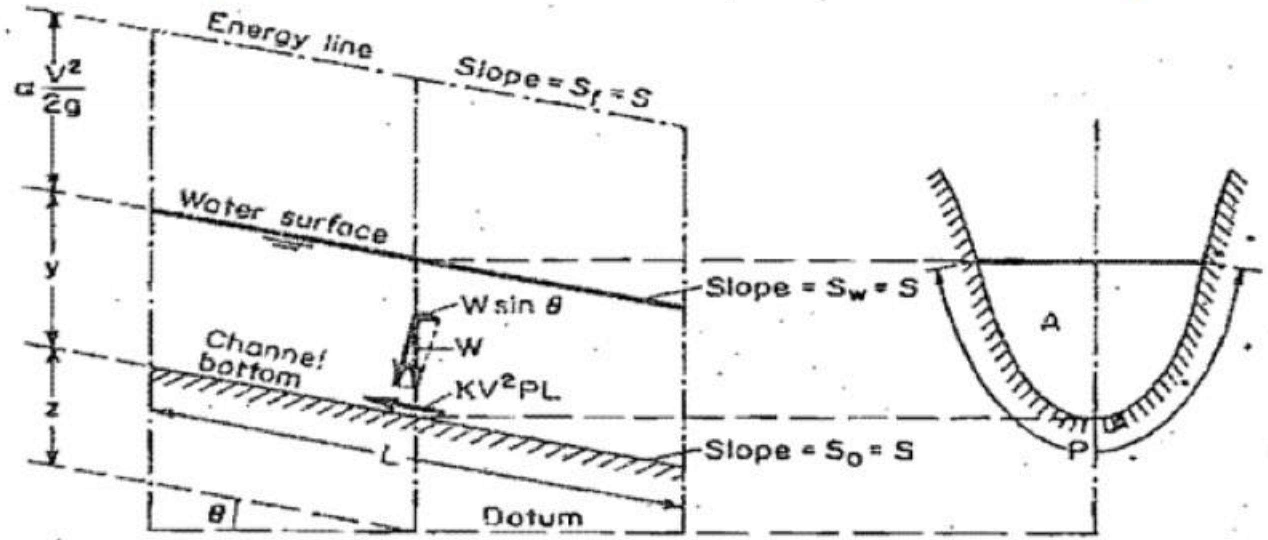
As early as 1769 the French engineer Antoine Chezy was developing probably the first uniform-flow formula, the famous Clezy formula which is usually expressed as follows:

$$V = C\sqrt{RS}$$

where *V* is the mean velocity in fps, *R* is the hydraulic radius in ft, *S* is the slope of the energy line, and *C* is a factor of flow resistance, called *Chezy's C*.

Development of Uniform Flow and its Formula

Derivation of The Chezy Formula for Uniform Flow in Open Channel



The Chezy formula can be derived **mathematically from two assumptions**.

The first assumption was made by Chezy.

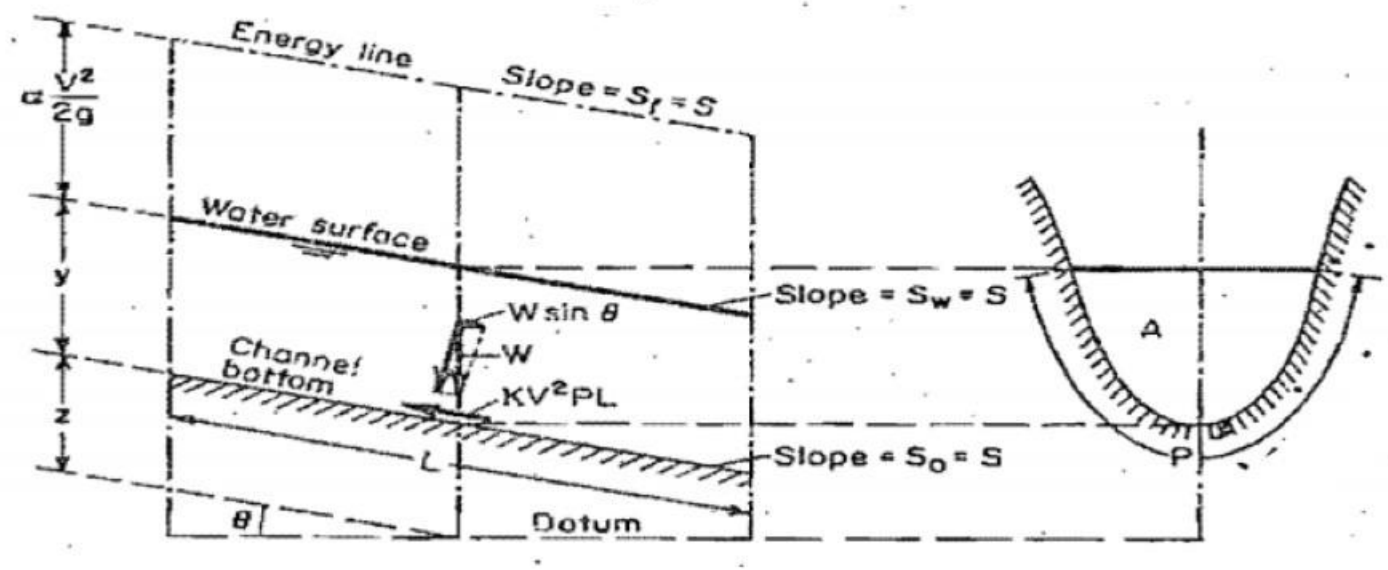
It states that the force resisting the flow per unit area of the stream bed is proportional to the square of the velocity;

That is, this resisting force is equal to ***KV^2***

Where, ***K*** is a constant of proportionality.

Development of Uniform Flow and its Formula

Derivation of The Chezy Formula for Uniform Flow in Open Channel



The surface of contact of the flow with the stream bed is equal to the product of the wetted perimeter and the length of the channel reach, or ***PL*** (Fig.) .

The total force resisting the flow is then equal to ***KV²PL***.

Development of Uniform Flow and its Formula

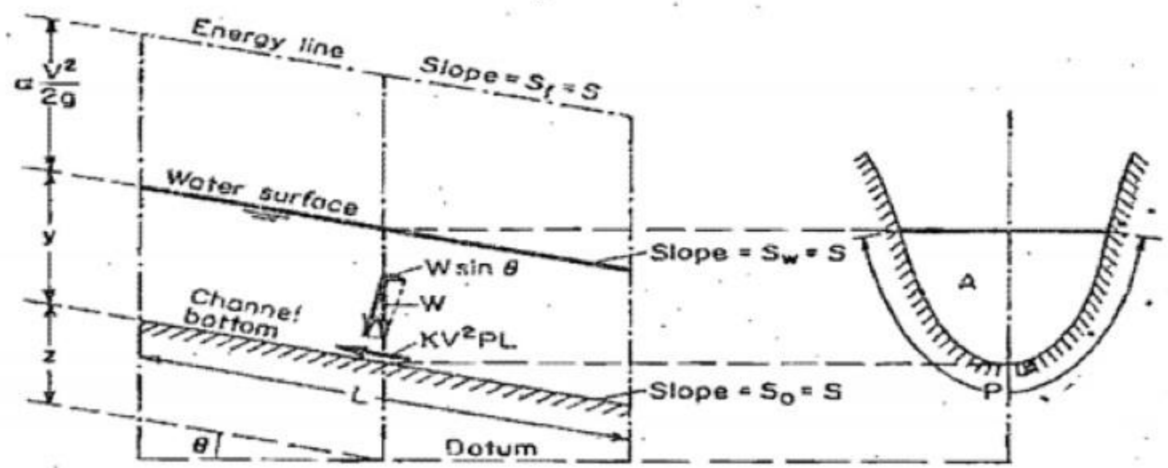
Derivation of The Chezy Formula for Uniform Flow in Open Channel

The second assumption is the basic principle of uniform flow, which is believed to have been claimed first by Brahms in 1754.

It states that, in uniform flow, the effective component of the gravity force causing the flow must be equal to the total force of resistance

Development of Uniform Flow and its Formula

Derivation of The Chezy Formula for Uniform Flow in Open Channel



The effective gravity-force component (Fig.) is parallel to the channel bottom and equal to $\omega AL\sin\theta = \omega ALS$,

where ω is the unit weight of water, **A** is the water area, θ is the slope angle, and ***S*** is the channel slope.

Thence, $\omega ALS = KV^2PL$, => $V^2 = \dfrac{\omega ALS}{KPL}$ => $V = \sqrt{\dfrac{\omega}{K} \times \dfrac{A}{P} \times S}$

Development of Uniform Flow and its Formula

Derivation of The Chezy Formula for Uniform Flow in Open Channel

Let $\frac{A}{P}$ = ***R*** and let $\sqrt{\frac{\omega}{K}}$ be replaced by a factor ***C***;

Then the previous equation is reduced to the Chezy formula,

$$\text{or } V = \sqrt{\frac{\omega}{K} \times \frac{A}{P} \times S} = C\sqrt{RS}$$

Many attempts have been made to determine the value of Chezy's C. Three important formulas developed for this purpose will be given in the next slide.

Development of Uniform Flow and its Formula

The Chezy Formula.

- ***Determination of Chezy's Resistance Factor.***

Three important formulas for the determination of *Chezy's C* are given as follows:

1. **Ganguillet -Kutter Formula**

$$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{S}\right)\frac{n}{\sqrt{R}}}$$

2. **Bazin Formula**

$$C = \frac{157.6}{1 + m/\sqrt{R}}$$

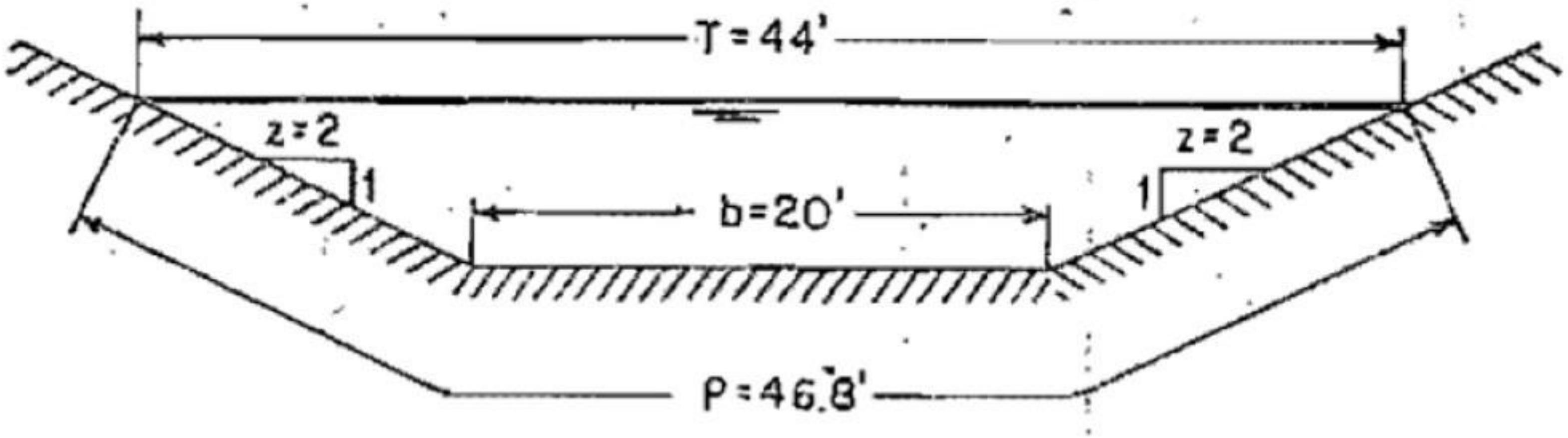
3. **Powell's Formula**

$$C = -42 \log\left(\frac{C}{4R} + \frac{\epsilon}{R}\right)$$

Development of Uniform Flow and its Formula

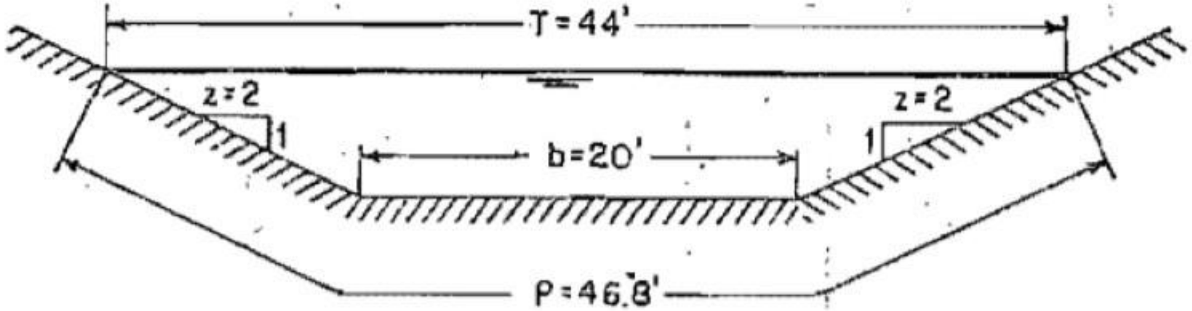
The Chezy Formula- Example 5.1:

Compute the velocity and discharge in the trapezoidal channel described in figure below, having a bottom width of 20 ft, side slopes 2:1, and a depth of water 6 ft. Given; Kutter's n = 0.015, and S = 0.005.



Development of Uniform Flow and its Formula

Example 5.1:



Solution. From Figure $A = 192.0\ \text{ft}^2$ and $R = 4.10$ ft. Using the G. K. formula, the value of Chézy's C is

$$C = \frac{41.65 + \dfrac{0.00281}{0.005} + \dfrac{1.811}{0.015}}{1 + \left(41.65 + \dfrac{0.00281}{0.005}\right)\dfrac{0.015}{\sqrt{4.10}}} = 124.2$$

Then, by the Chézy formula,

$$V = 124.2\sqrt{4.10 \times 0.005} = 17.8 \text{ fps}$$

Therefore,

$$Q = 192.0 \times 17.8 = 3{,}420 \text{ cfs}$$

Development of Uniform Flow and its Formula

The Manning Formula.

In 1889 the Irish engineer Robert Manning' presented a formula, which was later modified to its present well-known form

$$V = \frac{1.49}{n} \times R^{2/3} \times S^{1/2}$$

where V is the mean velocity in fps, R is the hydraulic radius in ft, S is the slope of energy line, and n is the coefficient of roughness, specifically known as ***Manning's n or Manning's roughness coefficient or rugosity coefficient.*** This formula was developed from seven different formulas, based on Bazin's experimental data, and further verified by 170 observations.

The Manning Formula can be written as-

$$V = \frac{1}{n} \times R^{2/3} \times S^{1/2}$$, In S.I units.

Development of Uniform Flow and its Formula

The Manning Formula.

Within the normal ranges of slope and hydraulic radius the values of Manning's n and Kutter's n are generally found to be numerically very, close. For practical purposes the two values may be considered identical when the slope is equal to or greater than 0.0001 and the hydraulic radius is between 1.0 and 30 ft.

Comparison of the Chezy and Manning formula

Comparing the Chezy formula with the Manning formula, it can be seen that

$$C = \frac{1.49}{n} \times R^{1/6} \text{ [In F.P.S Units]}$$

$$C = \frac{1}{n} \times R^{1/6} \text{ [In S.I Units]}$$

This equation provides an important relationship between Chezy's C and Manning's n.

Development of Uniform Flow and its Formula

Determination of Manning's Roughness Coefficient.

In applying the Manning formula or the G. K. formula., the greatest difficulty lies in the determination of the roughness coefficient ***n*,** for there is no exact method of selecting the ***n*** value.

At the present stage of knowledge, to select a value of ***n*** actually means to estimate the resistance to flow in a given channel, which is really a matter of intangibles.

To veteran engineers, this means the exercise of sound engineering judgment and experience; for beginners, it can be no more than a guess, and different individuals will obtain different results.

Development of Uniform Flow and its Formula

Determination of Manning's Roughness Coefficient.

In order to give guidance in the proper determination of the roughness coefficient, four general approaches will be discussed; namely,

(1) to understand the factors that affect the value of ***n*** and thus to acquire a basis knowledge of the problem and narrow the wide range of guesswork.

(2) to consult a table of typical ***n*** values for channels of various types.

(3) to examine and become acquainted with the appearance of some typical channels whose roughness coefficients are known, and

(4) to determine the value of ***n*** by an analytical procedure based on the theoretical velocity distribution in the channel cross section and on the data of either velocity or roughness measurement.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

It is not uncommon for engineers to think of a channel as having a single value of ***n*** for all occasions.

In reality, the value of n is highly variable and depends on a number of factors. In selecting a proper value of ***n*** for various design conditions, a basic knowledge of these factors should be found very useful.

The factors that exert the greatest influence upon the coefficient of roughness in both artificial and natural channels are therefore described below.

It should be noted that these factors are to a certain extent interdependent; hence discussion about one factor may be repeated in connection with another.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

A. Surface Roughness:

The surface roughness is represented by the size and shape of the grains of the material forming the wetted perimeter and producing a retarding effect on the flow.

Fine grains result in a relatively low value of n and coarse grains, in a high value of n.

In alluvial streams where the material is fine, the value of n is low and relatively unaffected by change in flow stage. When the material consists of gravels and boulders, the value of n is generally high.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

B. Vegetation.

Vegetation may be regarded as a kind of surface roughness, but it also markedly reduces the capacity of the channel and retards the flow. This effect depends mainly on height, density, distribution, and type of vegetation, and it is very important in designing small drainage channels.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

C. Channel Irregularity.

Channel irregularity comprises irregularities in wetted perimeter and variations in cross section, size, and shape along the channel length.

In natural channels, such irregularities are usually introduced by the presence of sand bars, sand waves, ridges and depressions, hales and humps on the channel bed. These irregularities definitely introduce, roughness in addition to that caused by surface roughness and other factors.

Generally speaking, a gradual and uniform change in cross section, size, and shape will not appreciably affect the value of n.

But abrupt changed or alternation of small and large sections necessitates the use of affect value of n.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

D. Channel Alignment.

Smooth curvature with large radius will give a relatively low value of n, whereas sharp curvature with severe meandering will increase n.

On the basis of flume tests, *Scobey* suggested that the value of n be increased 0.001 for each 20 degrees of curvature in 100 ft of channel.

Generally speaking, the increase of roughness in unlined channels carrying water at low velocities is negligible.

The meandering of natural streams, may increase the n value as high as 30 %.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

E. Silting and Scouring.

Generally speaking, silting may change a very irregular channel into a comparatively uniform one and decrease n, whereas scouring may do the reverse and increase n. However, the dominant, effect of silting will depend on the nature of the material deposited. Uneven deposits such as sand , bars, and sand waves are channel irregularities and will increase the roughness. The amount and uniformity of scouring will depend on the material forming the wetted perimeter.

F. Obstruction.

The presence of log jams, bridge piers, and the like tends to increase n. The amount of increase depends on the: nature of the obstructions, their size, shape, number, and distribution.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

G. Size and Shape of Channel.

There is no definite evidence about the size and shape of a channel a an important factor affecting the value of n. An increase in hydraulic radius may either increase, or decrease n, depending on the condition of the channel.

H. Stage and Discharge.

The n value in most streams decreases with increase in stage and in discharge. When the water is shallow, the irregularities of the channel bottom are exposed And their effects become pronounced. However, the n value duly be large at high stages if the banks are rough and grassy.

Development of Uniform Flow and its Formula

Factors Affecting Manning's Roughness Coefficient

I. Seasonal Changes.

Owing to the seasonal growth of aquatic plants, grass, weeds, willow, and trees in the channel or on the banks, the value of n may increase in the growing season and diminish in the dormant season. This seasonal change may cause changes in other factors.

J. Suspended Material and Bed Load.

The suspended material and the bed load, whether moving or not moving, would consume energy and cause head loss or increase the apparent channel roughness.

THANK YOU