

Soil Water

Soil water may be in the forms of free /gravitational water and held water

➤ The first type is free to move through the pore space of the soil mass under the influence of gravity.

➤ The second type is that which is held in the proximity of the surface of the soil grains by certain forces of attraction.

Gravitational water can be subdivided into

(a) Free water (bulk water) and (b) Capillary water.

(a) **Free water (bulk water).** It has the usual properties of liquid water. It moves at all times under the influence of gravity, or because of a difference in hydrostatic pressure head.

Free water may be further distinguished as

(i) Free surface water and (ii) Groundwater

Free surface water. Free surface water may be from precipitation, run-off, flood-water, melting snow, water from certain hydraulic operations. It is of interest when it comes into contact with a structure or when it influences the ground water in any manner. Rainfall and run-off are erosive agents which are capable of washing away soil and causing certain problems of strength and stability in the field of geotechnical engineering.

Ground water. Ground water is that water which fills up the voids in the soil up to the ground water table and translocates through them. It fills coherently and completely all voids. Ground water obeys the laws of hydraulics. The upper surface of the zone of full saturation of the soil, at which the ground water is subjected to atmospheric pressure, is called the 'Ground water table'.

(b) **Capillary water.** Water which is in a suspended condition, held by the forces of surface tension within the interstices and pores of capillary size in the soil, is called 'capillary water'.

Held Water is that water which is held in soil pores or void spaces because of certain forces of attraction. It can be further classified as-(a) **Structural water and** (b) **Absorbed water.**

Some-times, even ‘capillary water’ may be said to belong to this category of held water since the action of capillary forces will be required to come into play in this case.

(a) **Structural water.** Water that is chemically combined as a part of the crystal structure of the mineral of the soil grains is called ‘Structural water’. Under the loading encountered in geotechnical engineering, this water cannot be separated by any means. Even drying at $105^{\circ} - 110^{\circ}\text{C}$ does not affect it. Hence structural water is considered as part and parcel of the soil grains.

Adsorbed Water. This comprises, (i) **Hygroscopic moisture** & (ii) **Film-moisture**

(i) **Hygroscopic moisture.** Soils which appear quite dry contain, nevertheless, very *thin films of moisture* around the mineral grains, called ‘hygroscopic moisture’, which is also termed ‘*contact moisture*’ or ‘*surface bound moisture*’. This form of moisture is in a dense state, and surrounds the surfaces of the *individual soil grains as a very thin film.*

(ii) **Film moisture.** Film moisture forms *on the soil grains* because of the *condensation of aqueous vapour* ; this is attached to the surface of the soil particle as a film upon the layer of the hygroscopic moisture film. This film moisture is also held by molecular forces of high intensity but not as high as in the case of the hygroscopic moisture film.

Permeability

Lecture Outline:

1. Soil Permeability
2. Bernoulli's Equation
3. Darcy's Law
4. Hydraulic Conductivity
5. Permeability Test in the Field



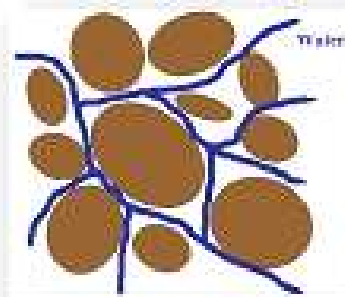
Cofferdam (U.S. Army Corps of Engineers 2004)

Textbook: Braja M. Das, "Principles of Geotechnical Engineering", 7th E. (Chapter 7).

Soil Permeability

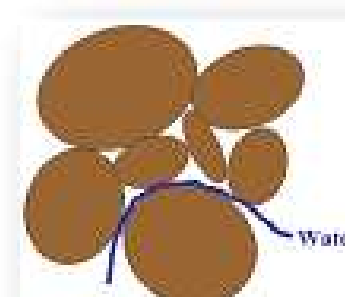
What is Permeability?

- Soils are assemblages of solid particles with interconnected voids where water can flow from a point of **high energy** to a point of **low energy**.
- **Permeability** is the measure of the soil's ability to permit water to flow through its pores or voids.
- It is one of the most important soil properties of interest to geotechnical engineers



Loose soil

Easy to flow - **High** permeability



Dense soil

Difficult to flow – **Low** permeability

Soil Permeability

Importance of permeability:

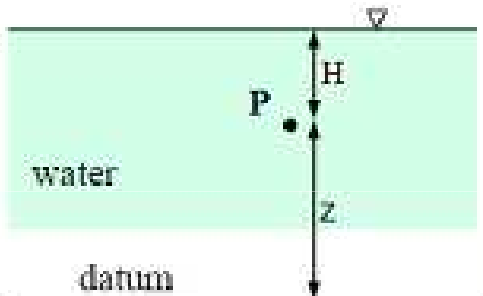
- Permeability influences the rate of settlement of a saturated soil under load.
- The design of earth dams is very much based upon the permeability of the soils used.
- The stability of slopes and retaining structures can be greatly affected by the permeability of the soils involved.
- Filters made of soils are designed based upon their permeability.

The study of permeability is important for:

- Estimating the quantity of underground seepage.
- Investigating problems involving pumping seepage of water for underground constructions.
- Analyzing the stability of earth dams and earth retaining walls subjected to seepage forces.

Bernoulli's Equation

- According to **Bernoulli's equation**, the total head at a point in water under motion can be expressed as the summation of the pressure, velocity, and elevation heads:



The diagram shows a cross-section of water above a datum. A point P is marked within the water. The vertical distance from the datum to point P is labeled 'z'. The vertical distance from point P to the free surface of the water is labeled 'H'. The total vertical distance from the datum to the free surface is labeled 'h'. The water is labeled 'water' and the datum is labeled 'datum'.

$$h = \frac{p}{\gamma_m} + \frac{v^2}{2g} + z$$

Pressure Head Velocity Head Elevation Head

h: total head (m)
 p: water pressure (Pa)
 v: velocity of water (m/s)
 z: elevation head (m)

- When water flows through soils, the seepage velocity is often very small. It is even smaller when squared, and the third component in Bernoulli's equation becomes negligible compared to the first two components. Therefore, the total head at any point can be adequately represented by :

$$h = \frac{p}{\gamma_m} + z$$

Bernoulli's Equation

- The heads of water at points A and B as the water flows from A to B are given as follows (with respect to a datum):

- Total head at A: $h_A = \frac{P_A}{\gamma_w} + z_A$

- Total head at B: $h_B = \frac{P_B}{\gamma_w} + z_B$

- The loss of head between A and B:

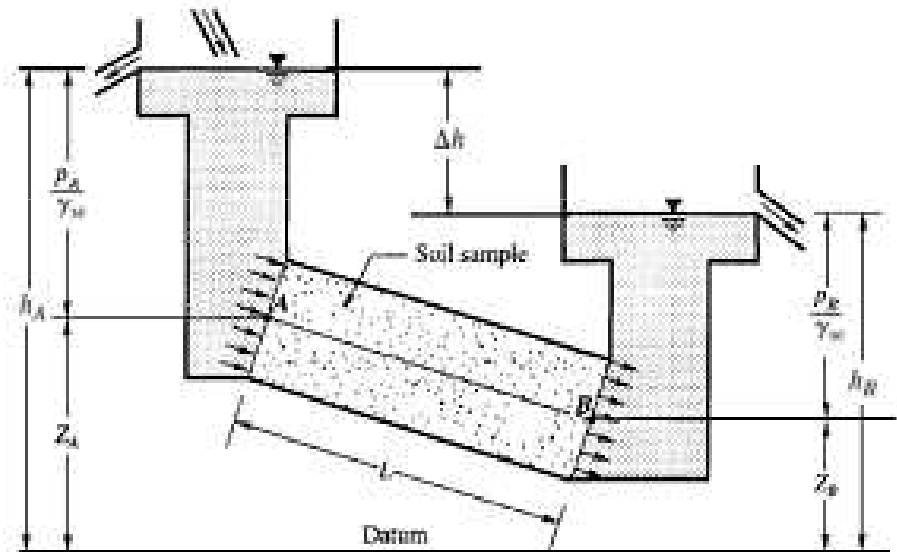
$$\Delta h = h_A - h_B = \left(\frac{P_A}{\gamma_w} + z_A \right) - \left(\frac{P_B}{\gamma_w} + z_B \right)$$

- The head loss may be expressed as:

$$i = \frac{\Delta h}{L}$$

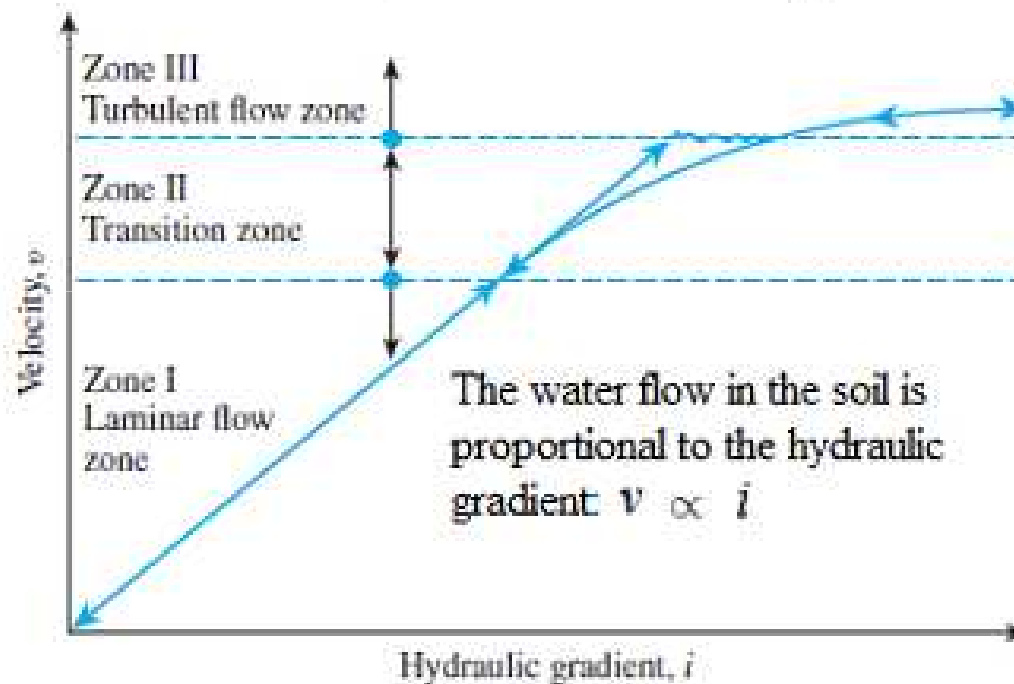
i : hydraulic gradient

L : distance between points A and B



Bernoulli's Equation

- the variation of the velocity (v) with the hydraulic gradient (i) may be divided into three main zones, as shown in the figure:



- In most soils, the flow of water through the void spaces can be considered laminar, thus: $v \propto i$

Darcy's Law

- Henri Darcy in 1856 derived an empirical formula for the behavior of flow through saturated soils. He found that the quantity of water (q) *per sec* flowing through a cross-sectional area (A) of soil under hydraulic gradient (i) can be expressed by the formula:

$$v = ki \quad \text{or} \quad q = \frac{Q}{t} = kiA$$

where,

v : discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil (cm/s).

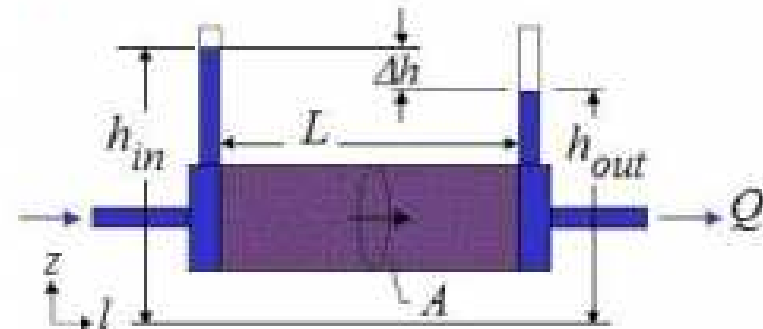
k : coefficient of permeability or hydraulic conductivity (cm/s).

q : flow rate (cm³/s).

Q : volume of collected water (cm³).

A : cross-sectional area (cm²).

i : hydraulic gradient.



Hydraulic conductivity is generally expressed in *cm/sec* or *m/sec* in SI units.

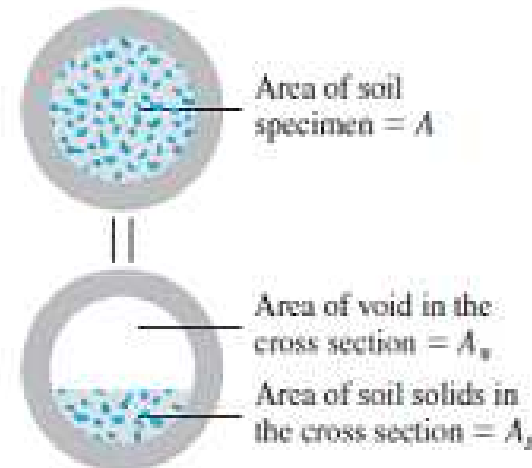
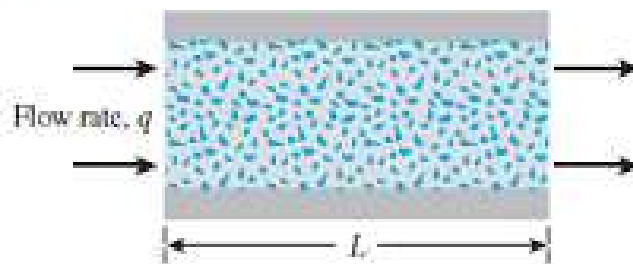
Darcy's Law

Assumption

Soil is Homogenous

Flow is Laminar

- Seepage velocity v_s : is the actual velocity of water through the void spaces.
- v_s is greater than v .



$$A = A_v + A_s$$

$$q = vA = v_s A_v$$

$$q = v(A_v + A_s) = A_v v_s$$

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)L}{A_v L} = \frac{v(V_v + V_s)}{V_v}$$

$$v_s = v \left[\frac{1 + \left(\frac{V_v}{V_s} \right)}{\frac{V_v}{V_s}} \right] = v \left(\frac{1 + e}{e} \right) = \frac{v}{n}$$

where,

V_v : volume of voids.

V_s : volume of solids.

e : void ratio.

n : porosity.

Hydraulic Conductivity

- The coefficient or permeability (k), also known as hydraulic conductivity, is a measure of soil permeability. It is generally expressed in cm/sec or m/sec in SI units.
- The hydraulic conductivity of soils depends on several factors:
 - Fluid viscosity
 - Pore size distribution
 - Grain size distribution
 - Void ratio
 - Degree of soil saturation
- k is determined in the lab using two methods:
 - **Constant-Head Test**
 - **Falling-Head Test**



Soil type	k (cm / sec)
Clean gravel	$10^0 - 10^2$
Coarse sand	$10^0 - 10^{-2}$
Fine sand	$10^{-2} - 10^{-3}$
Silty	$10^{-3} - 10^{-5}$
Clay	$< 10^{-5}$

Constant Head Test

- The constant head test is used primarily for **coarse-grained soils**.
- It is based on the assumption of laminar flow where k is independent of i (low values of i).
- This test applies a constant head of water to each end of a soil in a **“permeameter”** (ASTM D 2434).
- After a constant flow rate is established, water is collected in a graduated flask for a known duration.

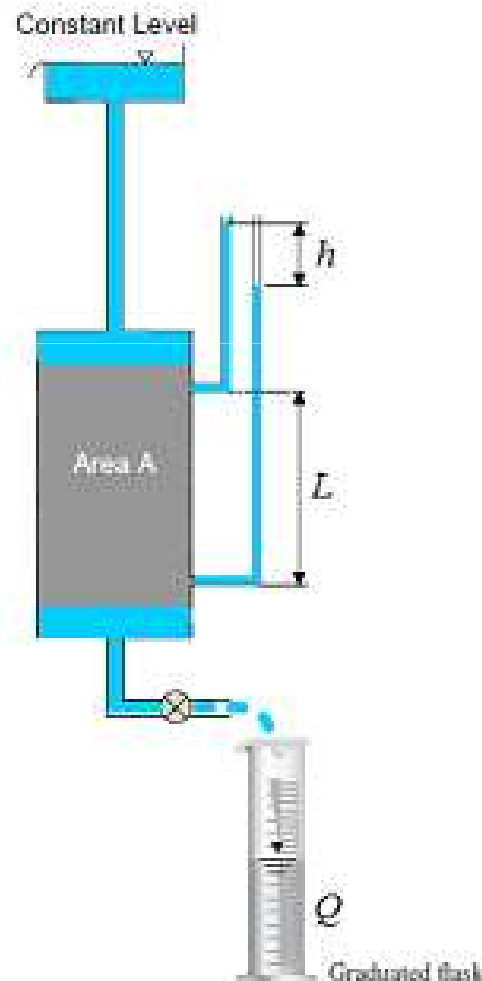


Permeameter cell



Constant Head Test

- The total volume of water collected may be expressed as:



$$Q = Avt$$

$$v = ki \quad \text{and} \quad i = \frac{h}{L}$$

$$Q = A \left(k \frac{h}{L} \right) t$$

therefore,

$$k = \frac{QL}{Aht} \quad [\text{m/s}]$$

Q : volume of water collected

A : area of cross section of the soil sample

t : duration of collection of water

Constant Head Test

- **Test procedure** (ASTM D 2434):
 1. Setup screens on the permeameter
 2. Measurements for permeameter, (D) , (L) , (H_1)
 3. Take 1000 g passing No.4 soil (M_1)
 4. Take a sample for M.C.
 5. Assemble the permeameter—***make sure seals are air-tight***
 6. Fill the mold in several layers and compact it as prescribed.
 7. Put top porous stone and measure (H_2)
 8. Weigh remainder of soil (M_2)
 9. Complete assembling the permeameter. (keep outlet valve closed)
 10. Connect Manometer tubes, but keep the valves closed.
 11. Apply vacuum to remove air for 15 minutes (through inlet tube at top)
 12. Run the Test (follow instructions in the lab manual)
 13. Take readings
 - Manometer heads (h_1) & (h_2)
 - Collect water at the outlet, Q (in ml) at time $t = 60$ sec.

Falling Head Test

- The falling head test is used for both coarse-grained soils as well as fine-grained soils.
- Same procedure in constant head test except:
 - Record initial head difference, h_1 at $t = 0$
 - Allow water to flow through the soil specimen
 - Record the final head difference, h_2 at time $t = t_2$
 - Collect water at the outlet, Q (in ml) at time $t \approx 60$ sec



Permeameter cell



Falling Head Test

- The rate of flow of the water through the specimen at any time t can be given by:

$$q = k \frac{h}{L} A = -a \frac{dh}{dt}$$

q : rate of flow

a : cross sectional area of standpipe

A : cross sectional area of the soil sample

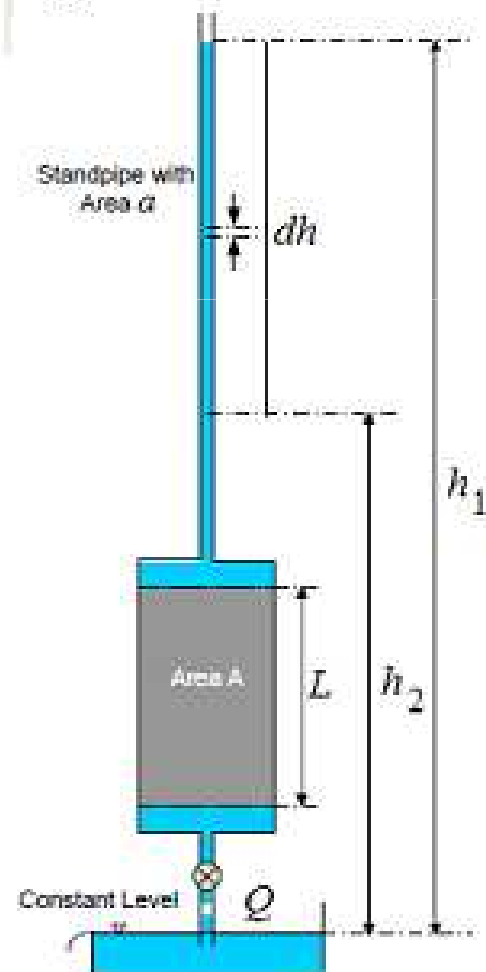
$$dt = \frac{aL}{Ak} \left(-\frac{dh}{h} \right)$$

$$\int dt = -\frac{aL}{Ak} \int \frac{dh}{h}$$

$$t = \frac{aL}{Ak} \ln \frac{h_1}{h_2}$$

$$k = \frac{aL}{At} \ln \frac{h_1}{h_2}$$

$$k = 2.33 \frac{aL}{At} \log \frac{h_1}{h_2}$$



h_1 : distance to bottom of the beaker before the test

h_2 : distance to bottom of the beaker after the test

Hydraulic Conductivity Relationships

- For fairly uniform sand (that is, sand with a small uniformity coefficient), **Hazen** (1930) proposed an empirical relationship for hydraulic conductivity in the form:

$$k \text{ (cm/s)} = cD_{10}^2$$

c : constant that varies from 1.0 to 1.5
 D_{10} : the effective size, in mm

- The **Kozeny-Carman** equation (Kozeny, 1927; Carman, 1938, 1956):

$$k = \frac{1}{C_s S_s^2 T^2} \frac{\gamma_w}{\eta} \frac{e^3}{1+2}$$

C_s : shape factor, which is a function of the shape of flow channels

S_s : specific surface area per unit volume of particles

T : tortuosity of flow channels

η : viscosity

e : void ratio

Hydraulic Conductivity Relationships

- Based on Kozeny-Carman equation, **Carrier** (2003) suggested the following equation:

$$k = 1.99 \times 10^{-4} \left[\frac{100\%}{\sum_i \frac{f_i}{D_{li}^{0.404} \times D_{si}^{0.595}}} \right]^2 \left(\frac{1}{SF} \right)^2 \frac{e^3}{1+e}$$

f_i : fraction of particles between a pair of two sieve sizes, li (larger) and si (smaller), in percent.
 SF : shape factor

- Chapuis** (2004) proposed the following empirical relationship for the hydraulic conductivity :

$$k \text{ (cm/s)} = 2.4622 \left[D_{10}^2 \frac{e^3}{1+e} \right]^{0.7625} \quad \text{where } D_{10} \text{ is in (mm)}$$

- Samarasinghe, Huang and Drnevich** (1982) suggested that the hydraulic conductivity of normally consolidated clays can be given by:

$$k = C \left(\frac{e^n}{1+e} \right) \quad \text{where } C \text{ and } n \text{ are constants to be determined experimentally}$$

typical values of coefficient of permeability for various soils

Material	Coefficient of permeability, mm/s
Coarse	$10 \text{ to } 10^3$
Fine gravel, coarse and medium sand	$10^{-2} \text{ to } 10$
Fine sand, clayey silt	$10^{-4} \text{ to } 10^{-4}$
Dense silt, clayey silt	$10^{-4} \text{ to } 10^{-4}$
Silty clay, clay	$10^{-5} \text{ to } 10^{-5}$

Validity of Darcy's Law

Darcy's law is true for laminar flow of water through the void spaces. A criterion for investigating the range can be furnished by Reynolds number. For flow through soils, Reynolds number can be given by the relation

$$R_n = \frac{vD\rho}{\mu}$$

Where,

v = discharge (superficial) velocity, cm/s

D = average diameter of the soil particle, cm

ρ = density of the fluid, g/cm³

μ = coefficient of viscosity, g/(cm · s)]

For laminar flow conditions in soils, experimental results show that

$$R_n = \frac{vD\rho}{\mu} \leq 1$$

Factor Affecting the Coefficient of Permeability

The coefficient of permeability depends on several factors, most of which are listed below:

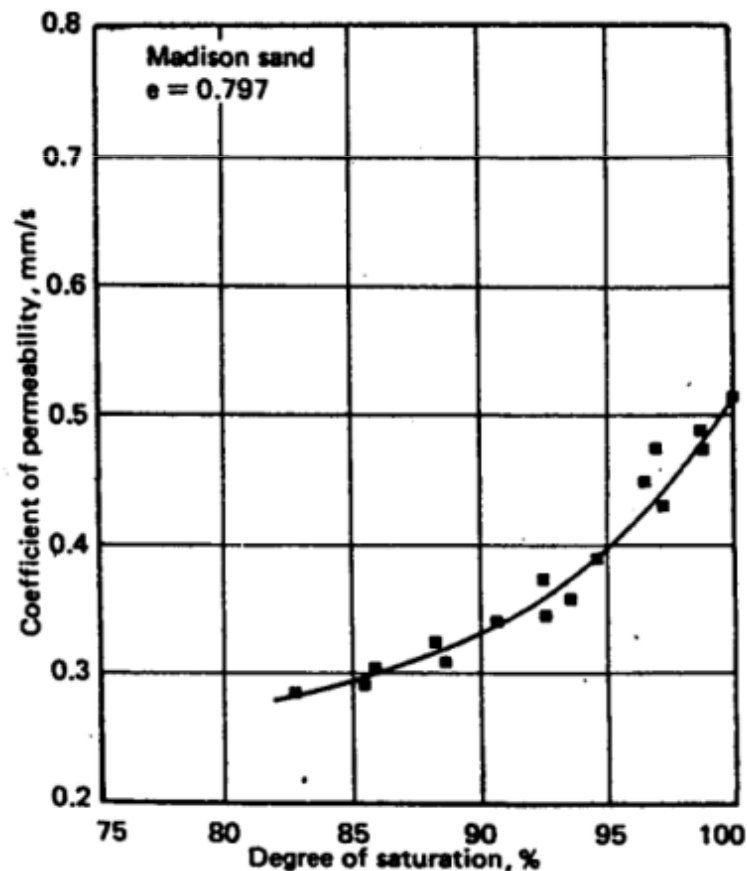
➤ Shape and size of the soil particles.

Void ratio : Permeability increases with increase of void ratio.

$$k \propto \frac{e^3}{1+e}$$
$$\text{Or } \frac{k_1}{k_2} = \frac{e_1^3/(1+e_1)}{e_2^3/(1+e_2)}$$

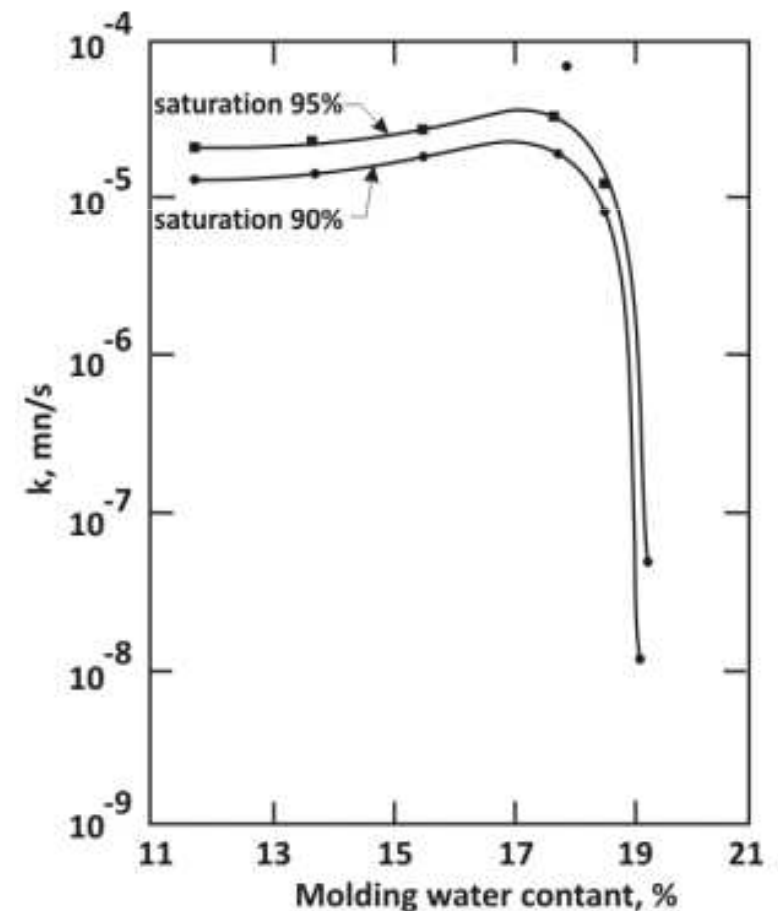
Degree of saturation : Permeability increases with increase of

degree of saturation. The variation of the value of k with degree of saturation for Madison sand is shown in Figure.



➤ **Composition of soil particles**: For sands and silts this is not important; however, for soils with clay minerals this is one of the most important factors. Permeability in this case depends on the *thickness of water held to the soil particles*, which is a function of the cation exchange capacity, valence of the cations, etc. Other factors remaining the same, the coefficient of permeability *decreases with increasing thickness of the diffuse double layer*.

➤ **Soil structure** : Fine-grained soils with a flocculated structure have a *higher coefficient of permeability* than those with a dispersed structure. This fact is demonstrated in Figure 2.4 for the case of a silty clay. The test specimens were prepared to a constant dry unit weight by kneading compaction. The molding moisture content was varied. Note that with the *increase of moisture content the soil becomes more and more dispersed*. With increasing degree of dispersion, the permeability decreases.



➤ Viscosity of permeant

➤ Density and concentration of permeant

The hydraulic conductivity of a soil is also related to the properties of the fluid flowing through it by the equation

$$k = \frac{\gamma_w}{\eta} \bar{K}$$

where γ_w = unit weight of water

η = viscosity of water

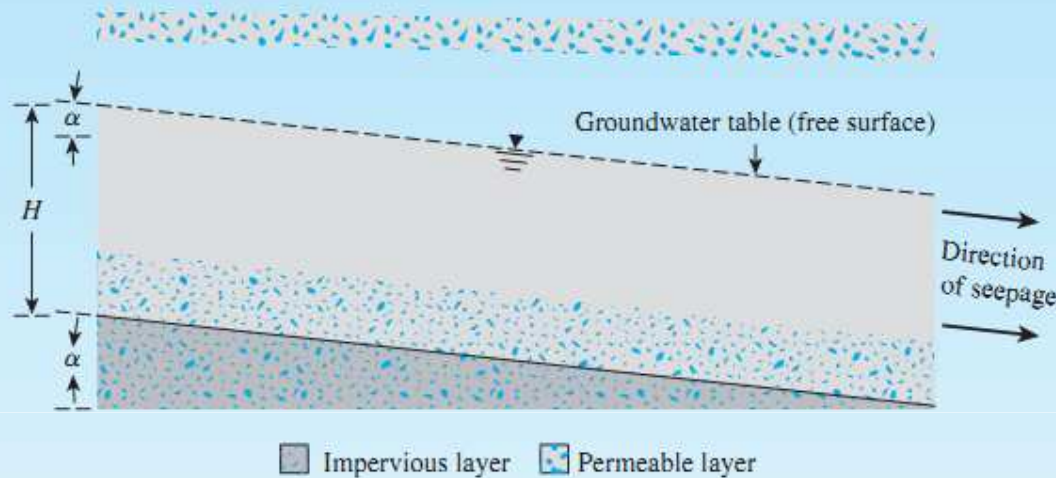
\bar{K} = absolute permeability

The absolute permeability \bar{K} is expressed in units of L^2 (that is, cm^2 , ft^2 , and so forth). Above equation showed that *hydraulic conductivity is a function of the unit weight and the viscosity of water*, which is in turn a function of the *temperature* at which the test is conducted. So, from above equ.,

$$k_{20^\circ C} = \left(\frac{\eta_{T^\circ C}}{\eta_{20^\circ C}} \right) k_{T^\circ C}$$

Example

A permeable soil layer is underlain by an impervious layer, as shown in Figure 7.9a. With $k = 5.3 \times 10^{-5}$ m/sec for the permeable layer, calculate the rate of seepage through it in $\text{m}^3/\text{hr}/\text{m}$ width if $H = 3$ m and $\alpha = 8^\circ$.



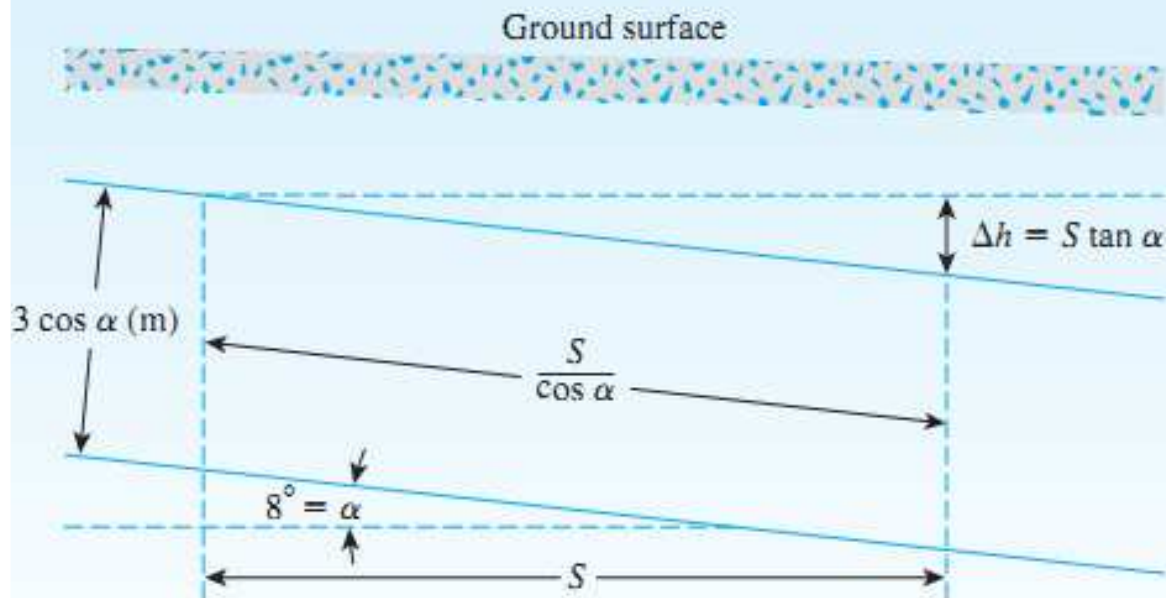
$$i = \frac{\text{head loss}}{\text{length}} = \frac{S \tan \alpha}{\left(\frac{S}{\cos \alpha}\right)} = \sin \alpha$$

$$q = kiA = (k)(\sin \alpha)(3 \cos \alpha) (1)$$

$$k = 5.3 \times 10^{-5} \text{ m/sec}$$

$$q = (5.3 \times 10^{-5})(\sin 8^\circ)(3 \cos 8^\circ) \times 3600$$

$$= 0.0789 \text{ m}^3/\text{hr}/\text{m}$$



m/hr

➤ **Permeability from consolidation test** The coefficient of permeability of clay soils is often determined by the consolidation test.

$$T_v = \frac{C_v t}{H^2}$$

Where

T_v = time factor

C_v = coefficient of consolidation

H = length of average drainage path

t = time

The coefficient of consolidation is

$$C_v = \frac{k}{\gamma_w m_v}$$

Where

γ_w = unit weight of water

m_v = volume coefficient of compressibility

$$\text{Also, } m_v = \frac{\Delta e}{\Delta \sigma (1+e)}$$

Where Δe = change of void ratio for incremental loading

$\Delta \sigma$ = incremental pressure applied

e = initial void ratio

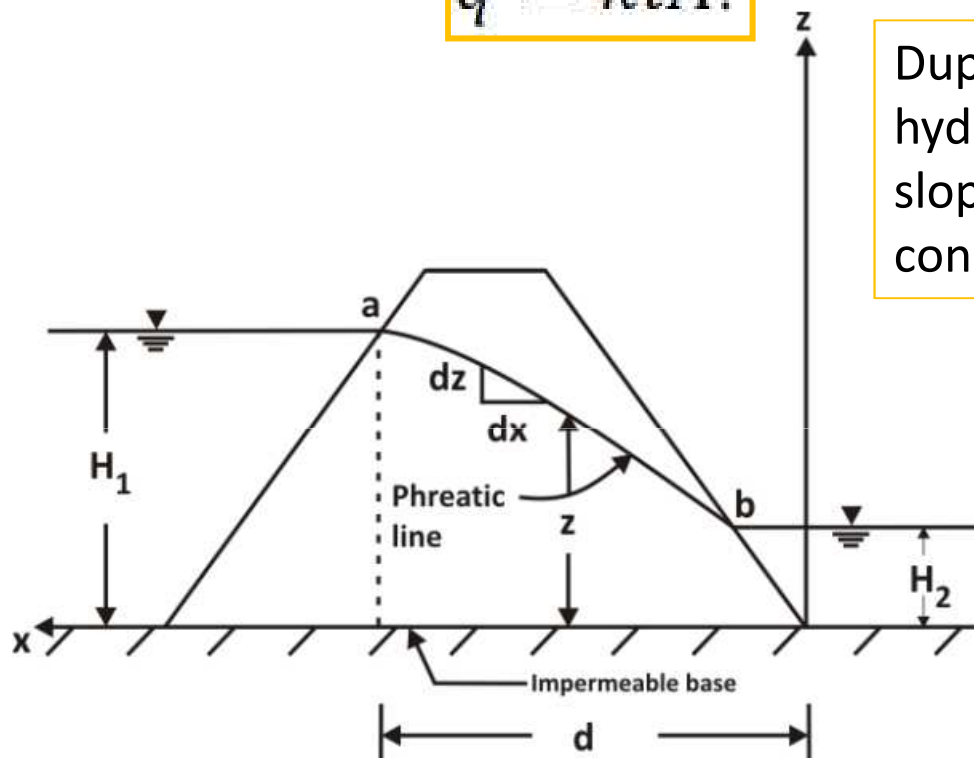
Combining these three equations, we have

$$k = \frac{T_v \gamma_w \Delta e H^2}{t \Delta \sigma (1+e)}$$

For 50% consolidation, $T_v = 0.198$; and the corresponding t_{50} can be estimated according to the procedure presented in section. 5.1.6.

$$\text{Hence, } k = \frac{0.198 \gamma_w \Delta e H^2}{t_{50} \Delta \sigma (1+e)}$$

Dupuit's solution : Figure 2.51 shows the section of an earth dam in which is the phreatic surface, i.e., the uppermost line of seepage. *The quantity of seepage* through a unit length at right angles to the cross section can be given the Darcy's law as $q = kiA$.



Dupuit (1863) assumed that the hydraulic gradient i is equal to the slope of the free surface and is constant with depth i.e., $i = dz/dx$. So,

$$q = k \frac{dz}{dx} [(z)(1)] = k \frac{dz}{dx} z$$

$$\int_0^d q dx = \int_{H_2}^{H_1} kz dz$$

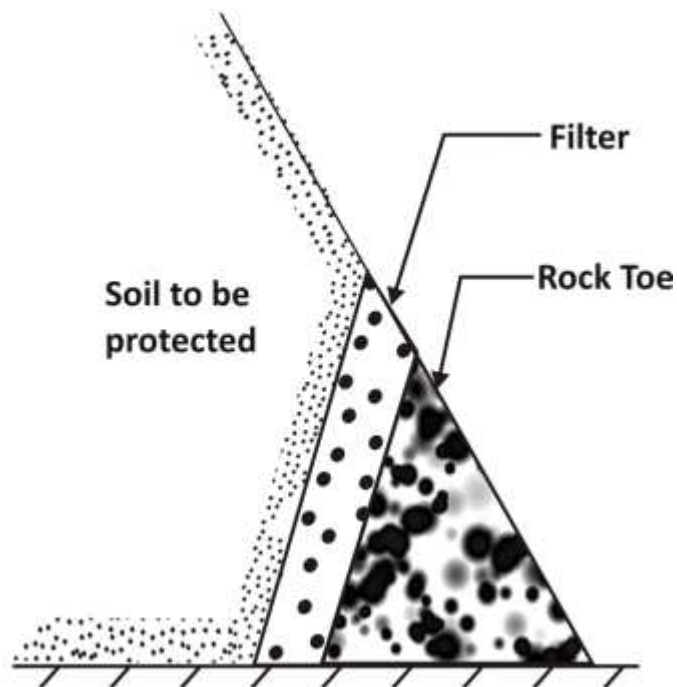
$$qd = \frac{k}{2} (H_1^2 - H_2^2)$$

$$\text{Or } q = \frac{k}{2d} (H_1^2 - H_2^2)$$

Equation represents a parabolic free surface. However, in the derivation of the equation, no attention has been paid to the entrance or exit conditions. Also note that if $H_2 = 0$ the phreatic line would intersect the impervious surface

Filter Design

When seepage water flows from a soil with relatively fine grains into a coarser material, there is a danger that the fine soil particles may wash away into the coarse material. Over a period of time, this process may clog the void spaces in the coarser material. Such a situation can be prevented by the use of a filter or protective filter between the two soils. For example, consider the earth dam section shown in Figure 2.66. If rock fills were only used at the toe of the dam, the seepage water would wash the fine soil grains into the toe and undermine the structure. Hence, for the safety of the structure, a filter should be placed between the fine soil and the rock toe (Figure 2.69). For the proper selection of the filter material, two conditions should be kept in mind:



1. The size of the voids in the filter material should be small enough to hold the larger particles of the protected material in place.
2. The filter material should have a high permeability to prevent building of large seepage forces and hydrostatic pressure in the filter.

Based on the experimental investigation of protective filters, Bertram (1940) provided the following criteria to satisfy the above condition:

$$\frac{D_{15(F)}}{D_{85(S)}} \leq 4 \text{ to } 5 \quad (\text{to satisfy condition 1}) \quad (2.200)$$

$$\frac{D_{15(F)}}{D_{15(S)}} \leq 4 \text{ to } 5 \quad (\text{to satisfy condition 2}) \quad (2.201)$$

Where

$D_{15(F)}$ = diameter through which 15% of filter material will pass

$D_{15(S)}$ = diameter through which 15% of soil to be protected will pass

$D_{85(S)}$ = diameter through which 85% of soil to be protected will pass

The proper use of equation (2.200) and (2.201) to determine the grain-size distribution of soils used as filters is shown in **Figure 2.70**. Consider the soil used for the construction of the earth dam shown in Figure 2.69. Let the grain-size distribution of this soil be given by curve *a* in Figure 2.70. We can

now determine $5D_{85(S)}$ and $5D_{15(S)}$ and plot them as shown in Figure 2.70. The acceptable grain-size distribution of the filter material will have to lie in the shaded zone.

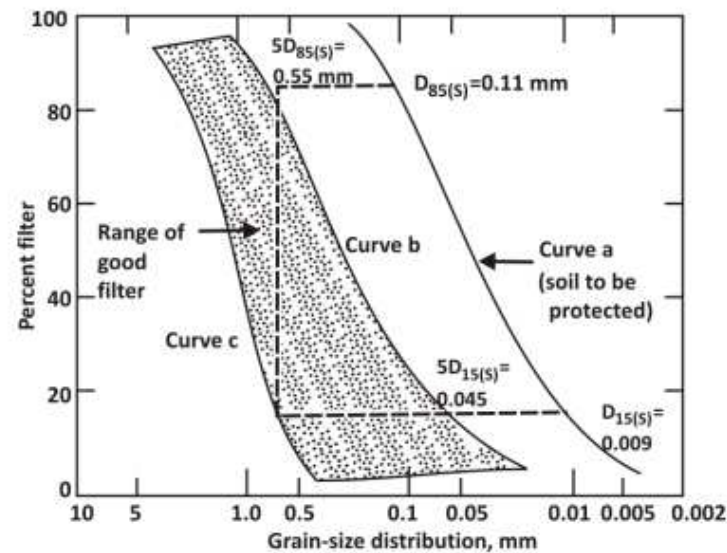


Figure 2.70 Determination of grain-size distribution of filter using eqs. (2.200) and (2.201)

The same principle can be adopted for determination of the size limits for the rock layer (Figure 2.69) to protect the filter material from being washed away.

The U. S. Navy (1971) requires the following conditions for the design of filters.

1. For avoiding the movement of the particles of the protected soil:

$$\frac{D_{15(F)}}{D_{85(S)}} < 5$$

$$\frac{D_{50(F)}}{D_{50(S)}} < 25$$

$$\frac{D_{15(F)}}{D_{15(S)}} < 20$$

If the uniformity coefficient C_u of the protected soil is less than 1.5, $D_{15(F)}/D_{85(S)}$ may be increased to 6. Also, if C_u of the protected soil is greater than 4, $D_{15(F)}/D_{15(S)}$ may be increased to 40.

2. For avoiding buildup of large seepage force in the filter,

$$\frac{D_{15(F)}}{D_{15(S)}} > 4$$

3. The filter material should not have grain sizes greater than 3 in (76.2 mm). (This is to avoid segregation of particles in the filter.)

4. To avoid internal movement of fines in the filter, it should have no more than 5% passing a No. 200 sieve.
5. When perforated pipes are used for collecting seepage water, filters are also used around the pipes to protect the fine-grained soil from being washed into the pipes. To avoid the movement of the filter material into the drain-pipe perforations, the following additional conditions should be met:

$$\frac{D_{15(F)}}{\text{slot width}} > 1.2 \text{ to } 1.4$$

$$\frac{D_{85(F)}}{\text{hole diameter}} > 1.0 \text{ to } 1.2$$

Thanikachalam and Sakthivadivel (1974) analyzed experimental results for filters reported by Karpoff (1955), U. S. Corps of Engineers (1953), Learhterwood and Peterson (1954), Dayaprakash and Gupta (1972), and Belyashevskii et al. (1972). Based on this analysis, they recommended that when the soil to be protected is of a granular nature, the stable filter design criteria may be given by the following equations:

$$\frac{D_{60(S)}}{D_{10(S)}} = 0.4 \frac{D_{10(F)}}{D_{10(S)}} - 2.0 \quad (2.202)$$

And

$$\frac{D_{60(F)}}{D_{10(F)}} = 0.941 \frac{D_{10(F)}}{D_{10(S)}} - 5.65 \quad (2.203)$$

Where $D_{60(S)}$ and $D_{10(S)}$ are, respectively, the diameters through which 60% and 10% of the soil to be protected in passing; and $D_{60(F)}$ and $D_{10(F)}$ are, respectively, the diameters through which 60% and 10% of the filter material is passing.

Cedergren (1960) constructed several flow nets, such as those shown in **Figure 2.71a, and b**, to study the condition of seepage into sloping filters placed at the downstream side of earth dams. Based on this work, he developed the chart given in **Figure 2.71c** which allows us to determine the minimum thickness of filter material, W , required on the downstream side of an earth dam. (Note that in **Figure 2.71**, k_F is the coefficient of permeability of the filter material, and k_S is the coefficient of permeability of the soil of the earth dam.)

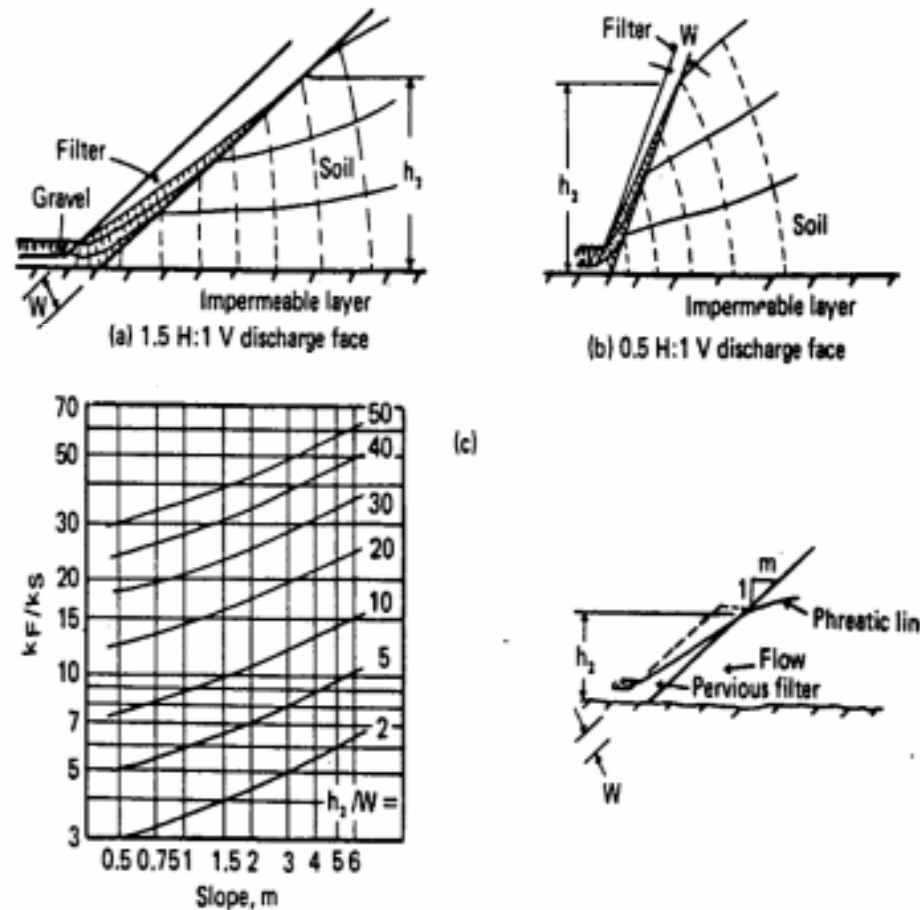


Figure 2.71 Thickness of filter material on the downstream side of an earth dam. [After H. R. Cedergren, Seepage Requirement of Filters and Pervious Bases, J. Soil Mech. Found. Div., ASCE, vol. 86, no. SM5 (part I), 1960

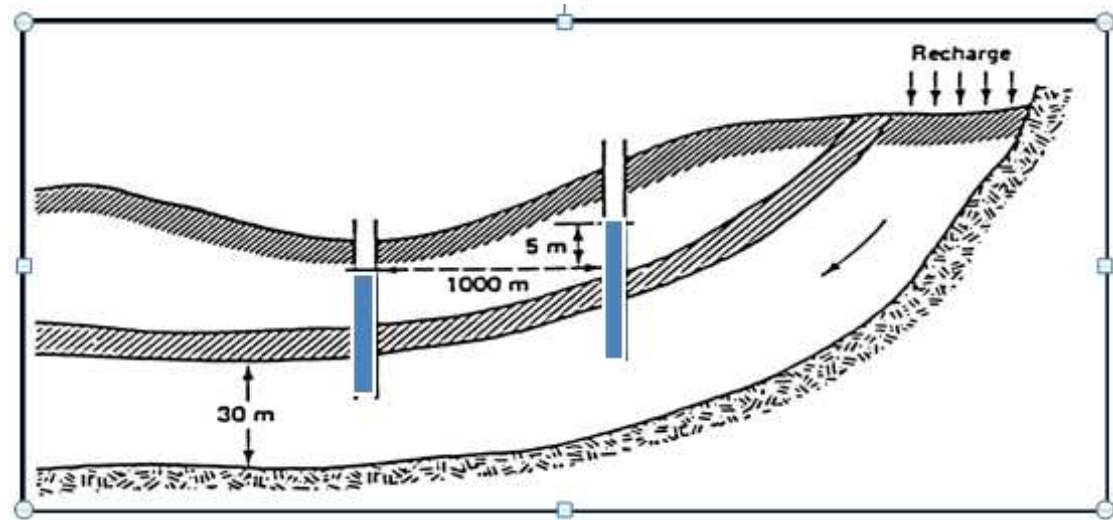
Example 1

$$Q = KA (dh/dL)$$

The hydraulic conductivity

K is a velocity, length / time

$$\text{and } n = \text{Vol}_{\text{voids}} / \text{Vol}_{\text{total}}$$



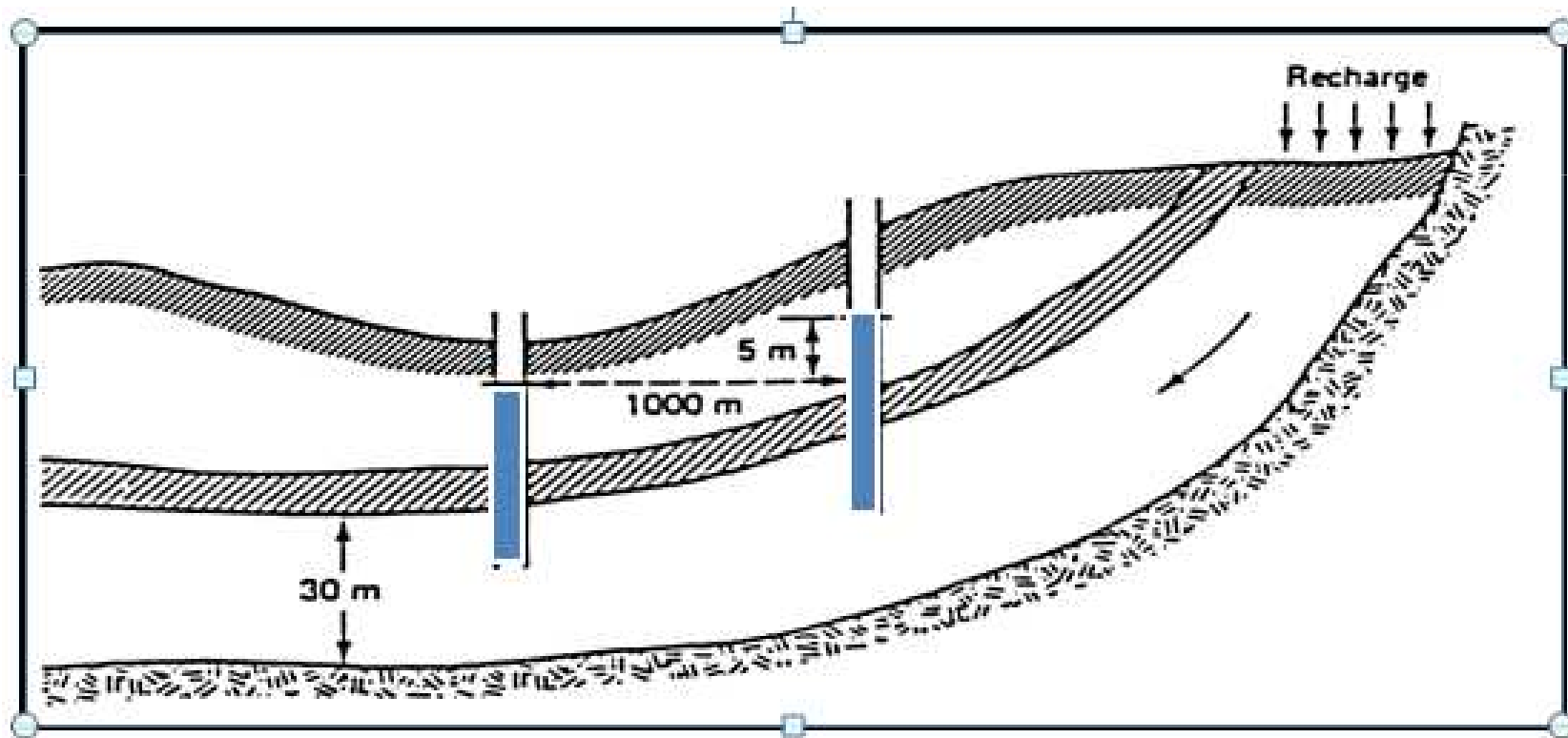
- A confined aquifer has a source of recharge.
- k for the aquifer is 50 m/day, and porosity n is 0.2.
- The piezometric head in two wells 1000 m apart is 55 m and 50 m respectively, from a common datum.
- The average thickness of the aquifer is 30 m, and the average width of the aquifer is 5 km = 5000m.

A **piezometer** is a small-diameter observation well used to measure the piezometric head of groundwater in aquifers.

Piezometric head is measured as a water surface elevation, expressed in units of length.

Compute:

- (a) the rate of flow through the aquifer
- (b) the average time of travel from the head of the aquifer to a point 4 km downstream



Solution

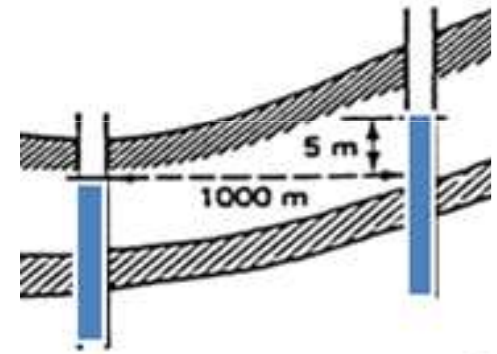
- Cross-Sectional area= $30(5000) = 1.5 \times 10^5 \text{ m}^2$
- Hydraulic gradient $dh/dL = (55-50)/1000 = 5 \times 10^{-3}$
- Find Rate of Flow for $k = 50 \text{ m/day}$

$$Q = (50 \text{ m/day}) (1.5 \times 10^5 \text{ m}^2) (5 \times 10^{-3})$$

$$Q = 37,500 \text{ m}^3/\text{day}$$

- Darcy Velocity: $V = Q/A$

- $= (37,500 \text{ m}^3/\text{day}) / (1.5 \times 10^5 \text{ m}^2) = \underline{0.25 \text{ m/day}}$

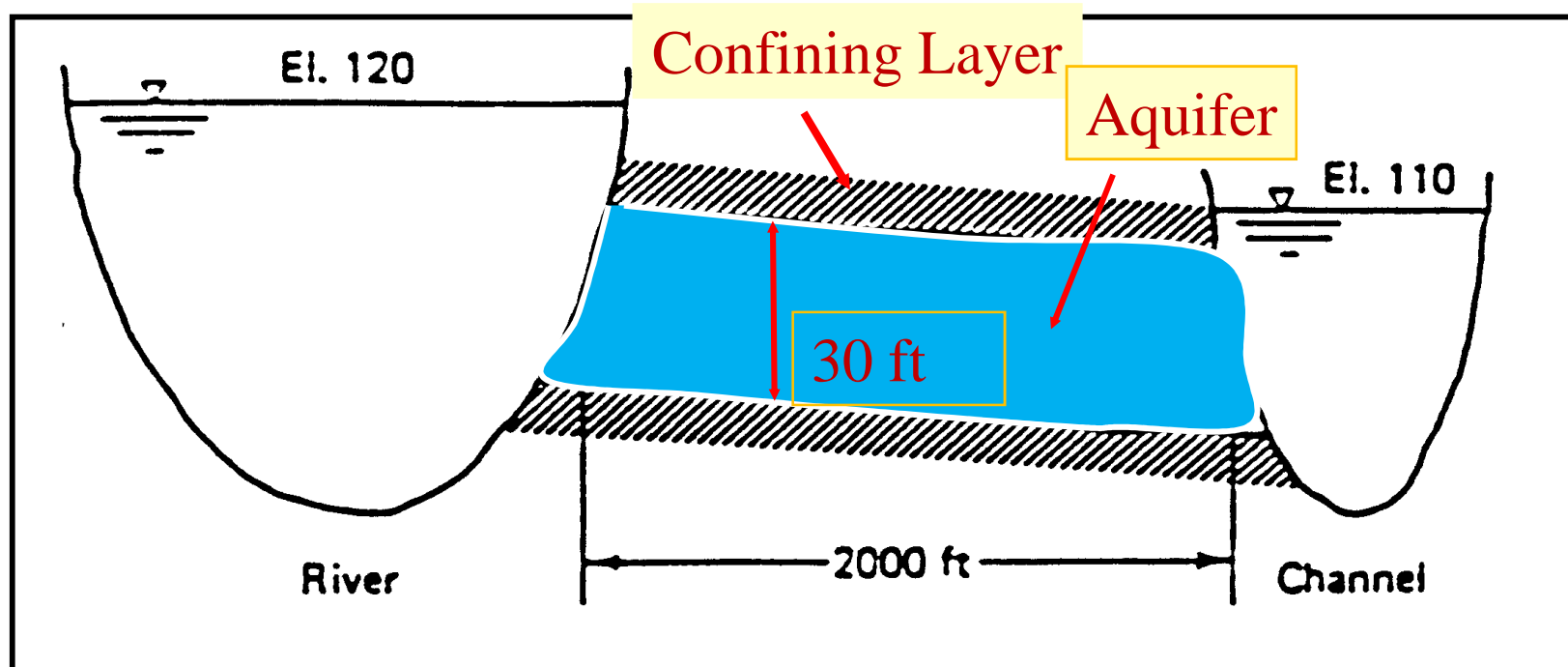


- Seepage Velocity:
 $V_s = V_D/n = (0.25) / (0.2) =$
1.25 m/day (about 4.1 ft/day)
- Time to travel 4 km downstream:
 $T = (4000\text{m}) / (1.25\text{m/day}) =$
3200 days or 8.77 years

Note: Groundwater moves very slowly

Example 2

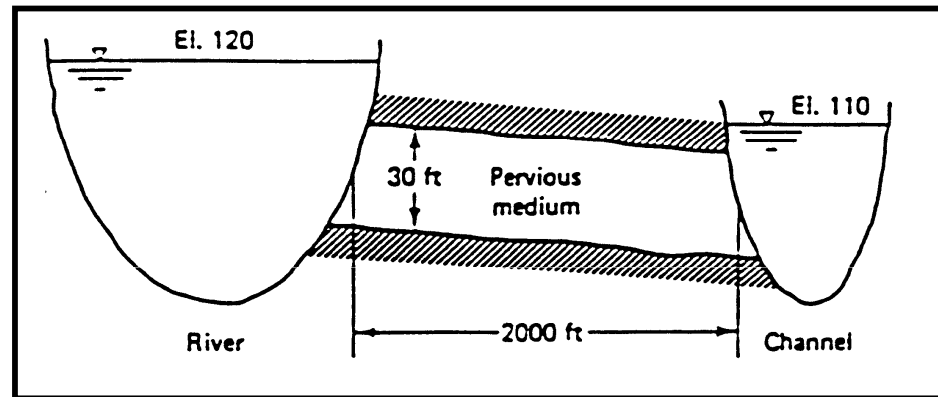
- A channel runs almost parallel to a river, and they are 2000 ft apart.
- The water level in the river is at an elevation of 120 ft . The channel is at an elevation of 110ft.
- A pervious formation averaging 30 ft thick and with hydraulic conductivity K of 0.25 ft/hr joins them.
- Determine the flow rate Q of seepage from the river to the channel.



Example 2: Confined Aquifer

- Consider 1-ft (i.e. unit) lengths of the river and small channel.

$$Q = KA [(h_1 - h_2) / L]$$



- Where:

$$A = (30 \times 1) = 30 \text{ ft}^2$$

$$K = (0.25 \text{ ft/hr}) (24 \text{ hr/day}) = 6 \text{ ft/day}$$

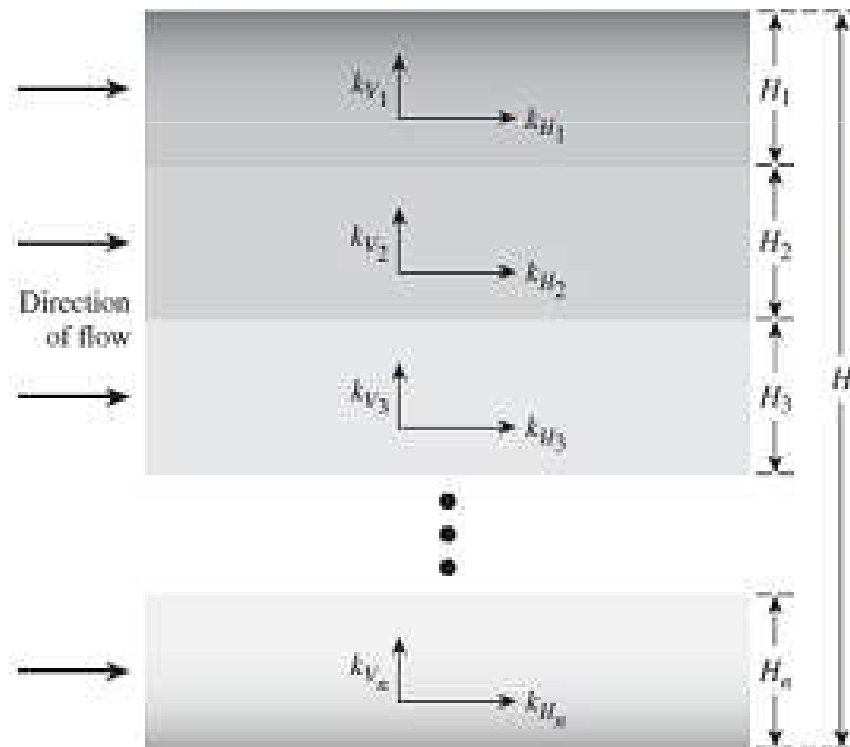
- Therefore,

$$Q = [6 \text{ ft/day} (30 \text{ ft}^2) (120 - 110 \text{ ft})] / 2000 \text{ ft}$$

$$Q = 0.9 \text{ ft}^3/\text{day} \text{ for each 1-foot length}$$

Equivalent Hydraulic Conductivity: Stratified Soil

- In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer, an equivalent hydraulic conductivity can be computed to simplify calculations:
- The equivalent hydraulic conductivity in the horizontal direction ($k_{H(eq)}$) is:

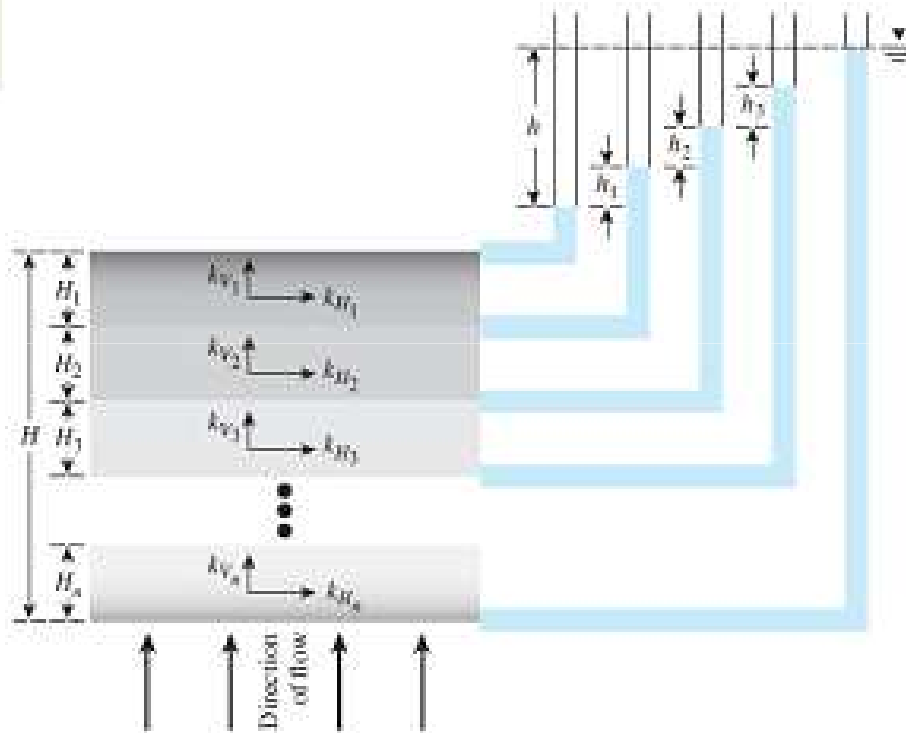


$$k_{H(eq)} = \frac{1}{H} (k_{H1} H_1 + k_{H2} H_2 + \dots + k_{Hn} H_n)$$

where $k_{H1}, k_{H2}, \dots, k_{Hn}$, are the hydraulic conductivities of the individual layers in the horizontal direction

Equivalent Hydraulic Conductivity: Stratified Soil

- The equivalent hydraulic conductivity in the vertical direction ($k_{V(eq)}$) is:

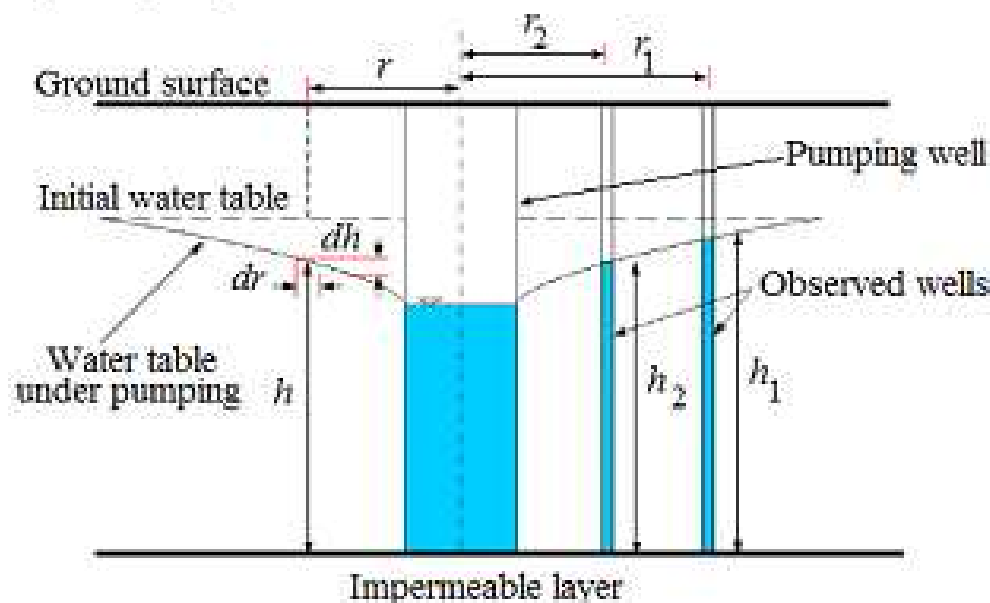


$$k_{V(eq)} = \frac{H}{\left(\frac{H_1}{k_{V1}}\right) + \left(\frac{H_2}{k_{V2}}\right) + \dots + \left(\frac{H_n}{k_{Vn}}\right)}$$

where $k_{V1}, k_{V2}, \dots, k_{Vn}$ are the hydraulic conductivities of the individual layers in the vertical direction.

Permeability Test in the Field: Pumping Well

- **Pumping test:** the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells.
- During the test, water is pumped out at a constant rate from a test well that has a perforated casing. Several observation wells at various radial distances are made around the test well.
- **Steady state:** the equilibrium state when the drawdown keeps no change at one particular location to the well, no further drawdown develops as pumping continues.



$$q = k \left(\frac{dh}{dr} \right) 2\pi r h$$

$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q} \right) \int_{h_2}^{h_1} h dh$$

$$k = \frac{2.303q \log \left(\frac{r_1}{r_2} \right)}{\pi (h_1^2 - h_2^2)}$$