

SEEPAGE of SOIL

Lecture Outline

1. Introduction
2. Laplace's Equation of Continuity
3. Flow Net
4. Seepage Calculation from Flownet



Failure of Teton Dam, June 5, 1976 (Teton River, USA)

Textbook: Braja M. Das, "Principles of Geotechnical Engineering", 7th E. (Chapter 8).

Introduction

Around 7:00 am on June 5, 1976 a leak about 30 m from the top of Teton dam was observed.

The Dam Broke at 11.59 AM

Post Failure Observation

- Seepage piping and internal erosion
- Seepage through rock openings
- Hydraulic fracture
- Differential settlement and cracking
- Settlement in bedrock

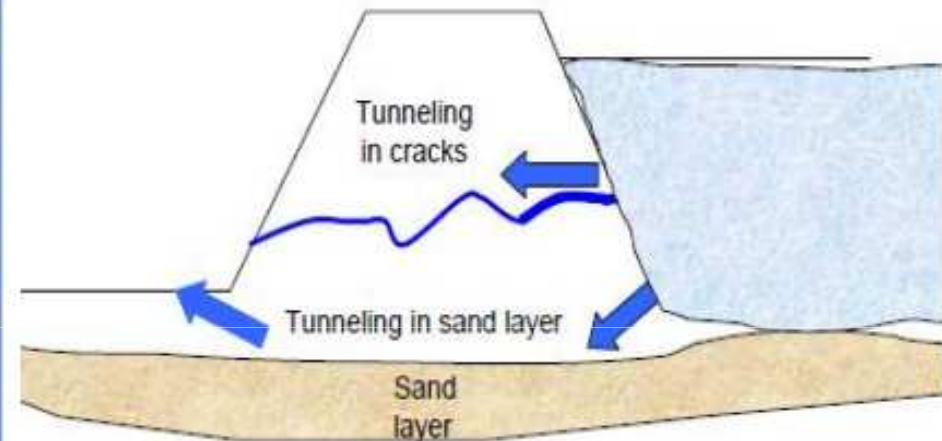


Introduction

Levee Stability: Seepage and Tunneling

Embankment : prevent the overflow

Seepage and tunneling have been the most common cause of levee failures in the system. Seepage occurs when the water seeps through the tiny soil pores and finds its way into some bigger cracks.

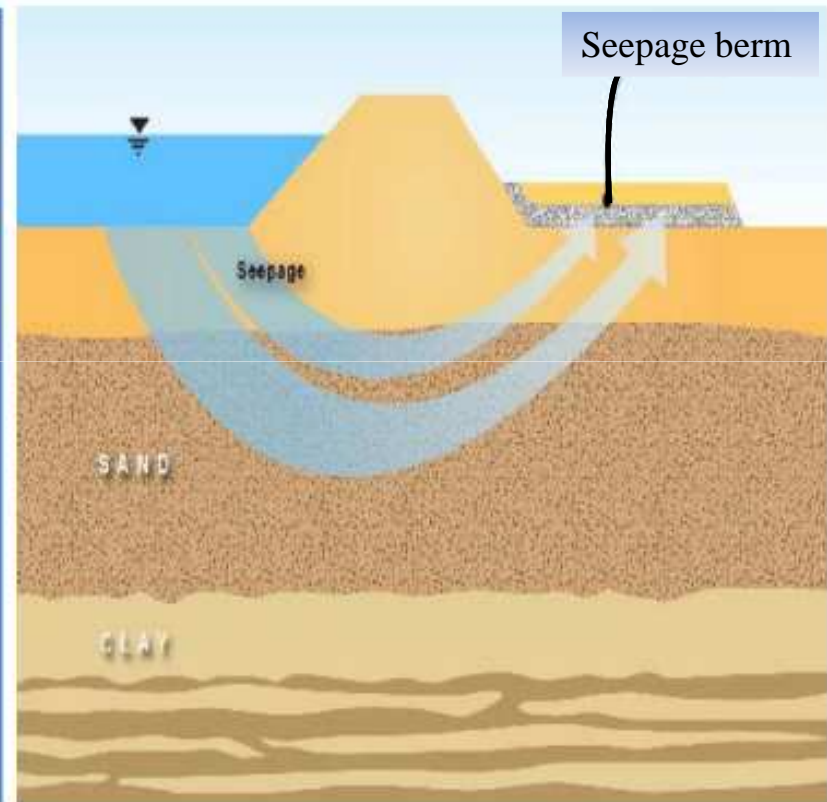


Once the water enters some of the larger cracks, the water meets less resistance and may start moving enough to entrain some of the surrounding soil particles, carrying them away and making the crack bigger. This starts a process of "tunneling" where a tiny crack becomes larger and larger as the water starts moving through it and carrying the surrounding soil particles away with it. Eventually the crack widens to the point where the water comes rushing through the levee and crumbles the entire structure.

Introduction

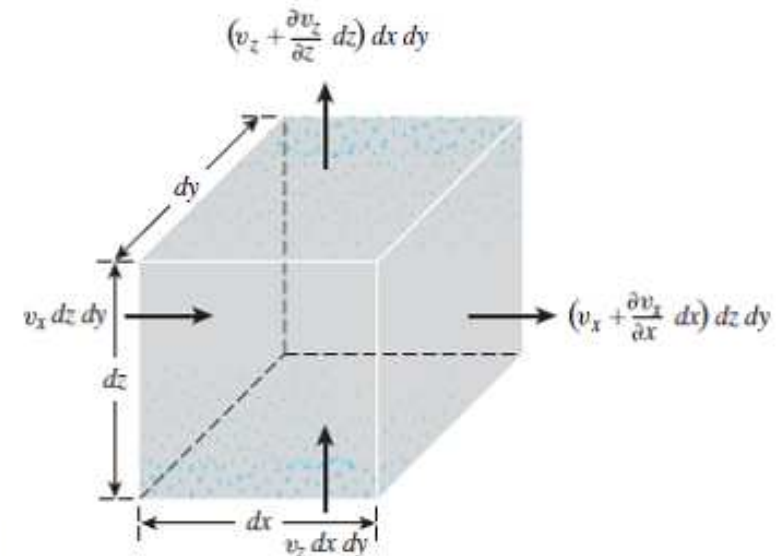
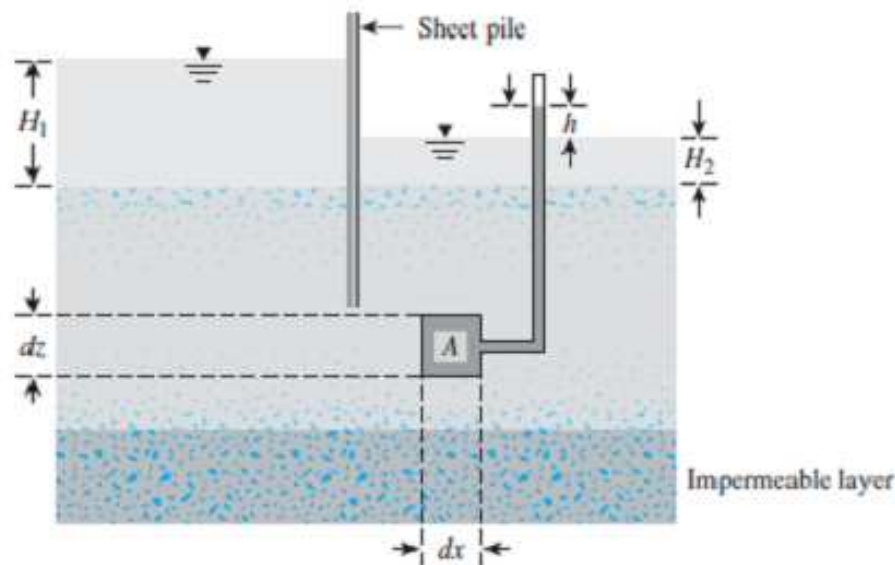
Levee Stability: Seepage and Tunneling

Another type of seepage and tunneling can occur underneath the foundation of the levee rather than through the levee. In some places, there may be a sand layer in the soils below the levee foundation. Because the sand lacks cohesion, water infiltration may cause tunneling in the sand layer. This can cause “boils” to come up on the land side of the levee and may eventually undermine the levee foundation causing failures.



Laplace's Equation of Continuity

- In reality, the flow of water through soil is not in one direction only, nor is it uniform over the entire area perpendicular to the flow. In such cases, the groundwater flow is generally calculated by the use of graphs referred to as flow nets. The concept of the flow net is based on Laplace's equation of continuity, which governs the steady flow condition for a given point in the soil mass.
- Laplace equation is the combination of the equation of continuity and Darcy's law.



Laplace's Equation of Continuity

Flow in: $v_x dx dz$, $v_y dx dz$, $v_z dx dy$

Flow out: $\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dy dz$, $\left(v_y + \frac{\partial v_y}{\partial y} dy\right) dx dz$, $\left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx dy$

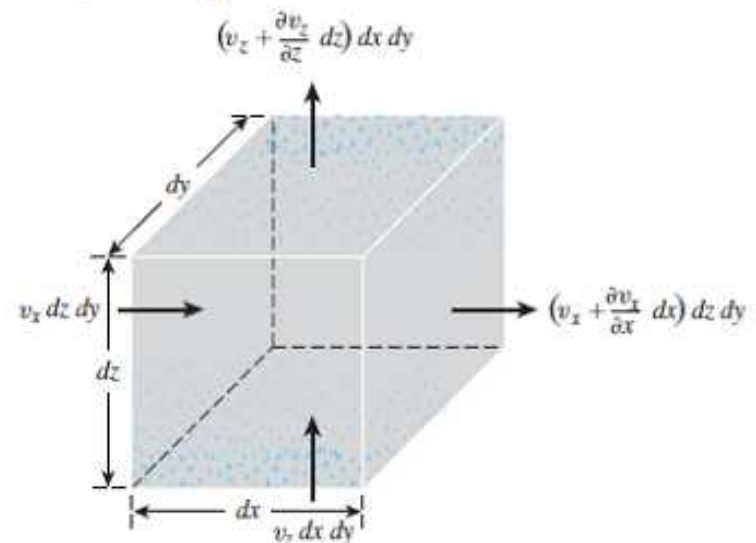
Flow in = Flow out

$$\left[\left(v_x + \frac{\partial v_x}{\partial x} dx\right) dy dz + \left(v_y + \frac{\partial v_y}{\partial y} dy\right) dx dz + \left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx dy \right] = v_x dy dz + v_y dx dz + v_z dx dy$$

We get the equation of continuity:

$$\frac{\partial v_x}{\partial x} dx dy dz + \frac{\partial v_y}{\partial y} dy dx dz + \frac{\partial v_z}{\partial z} dz dx dy = 0$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$



Laplace's Equation of Continuity

- From Darcy's law, we have:

$$v_x = k_x \frac{\partial h}{\partial x}, \quad v_y = k_y \frac{\partial h}{\partial y}, \quad v_z = k_z \frac{\partial h}{\partial z}$$

- Replace into the continuity relation we get:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

- If the soil is isotropic, we have:

$$k_x = k_y = k_z = k$$

- Then the preceding continuity equation simplifies to:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

(1)

← This is Laplace equation

Laplace's Equation of Continuity

Simple flow problems: 1D solution of Laplace equation

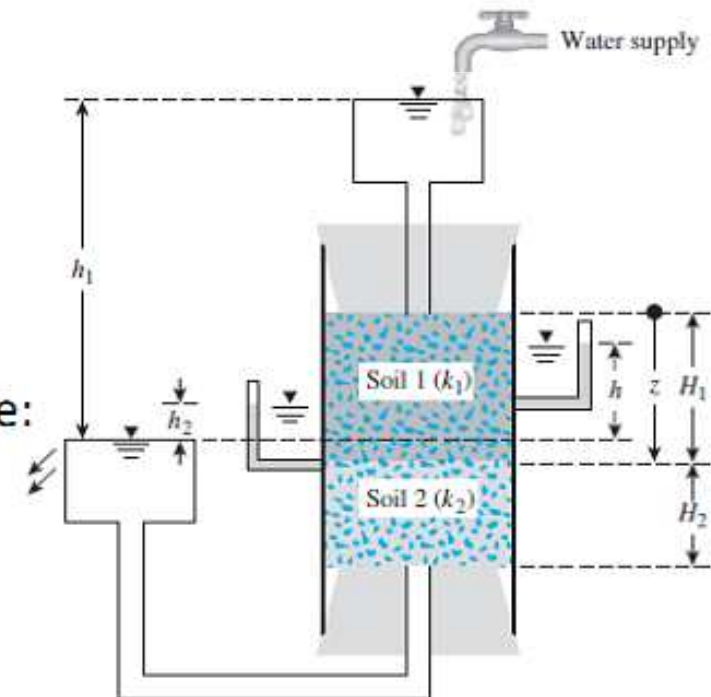
- We consider the case of only vertical flow as shown in the figure, in which a constant head is maintained across a two-layered soil for the flow of water.
- The head difference between the top of soil 1 and the bottom of soil 2 is h_1 .
- Because the flow is in only the z direction, the continuity equation is simplified to the form:

$$\frac{\partial^2 h}{\partial z^2} = 0 \quad (2)$$

- The solution of this equation is easy to get by having a integration of h with respect to z twice:

$$h = A_1 z + A_2 \quad (3)$$

where A_1 and A_2 are constants.



Laplace's Equation of Continuity

- The constants A_1 and A_2 can be determined by the boundary conditions.

For soil 1:

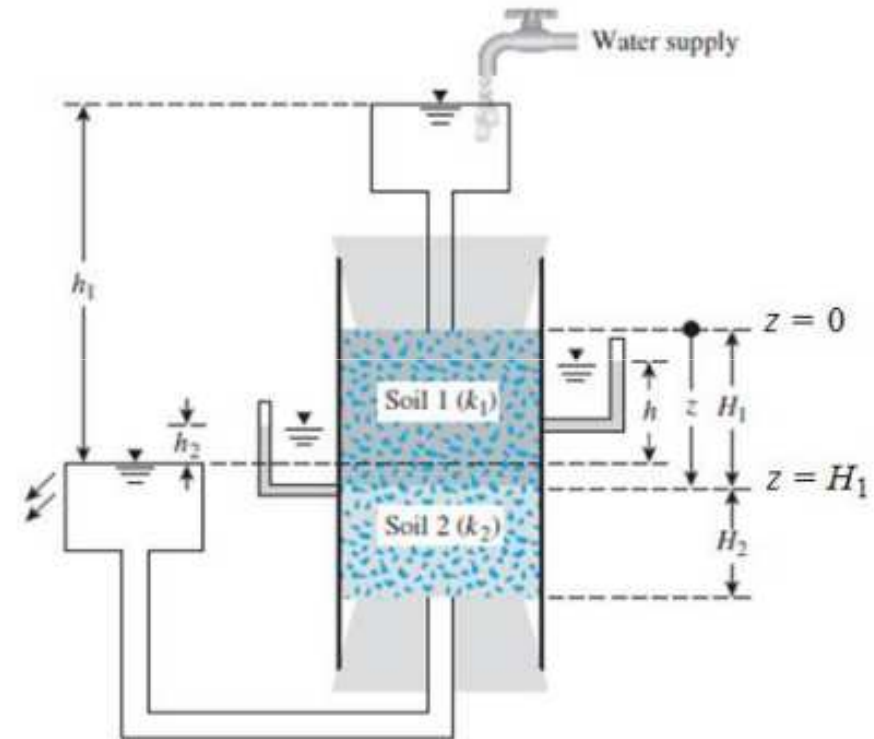
Condition 1: at $z = 0$, $h = h_1$

$$A_2 = h_1 \quad (4)$$

Condition 2: at $z = H_1$, $h = h_2$

$$h_2 = A_1 H_1 + h_1$$

$$A_1 = -\left(\frac{h_1 - h_2}{H_1}\right) \quad (5)$$



Combining Eqs. (3), (4), and (5), we obtain:

$$h = -\left(\frac{h_1 - h_2}{H_1}\right)z + h_1 \quad \text{for } 0 \leq z \leq H_1 \quad (6)$$

Laplace's Equation of Continuity

For soil 2:

Condition 1: at $z = H_1$, $h = h_2$

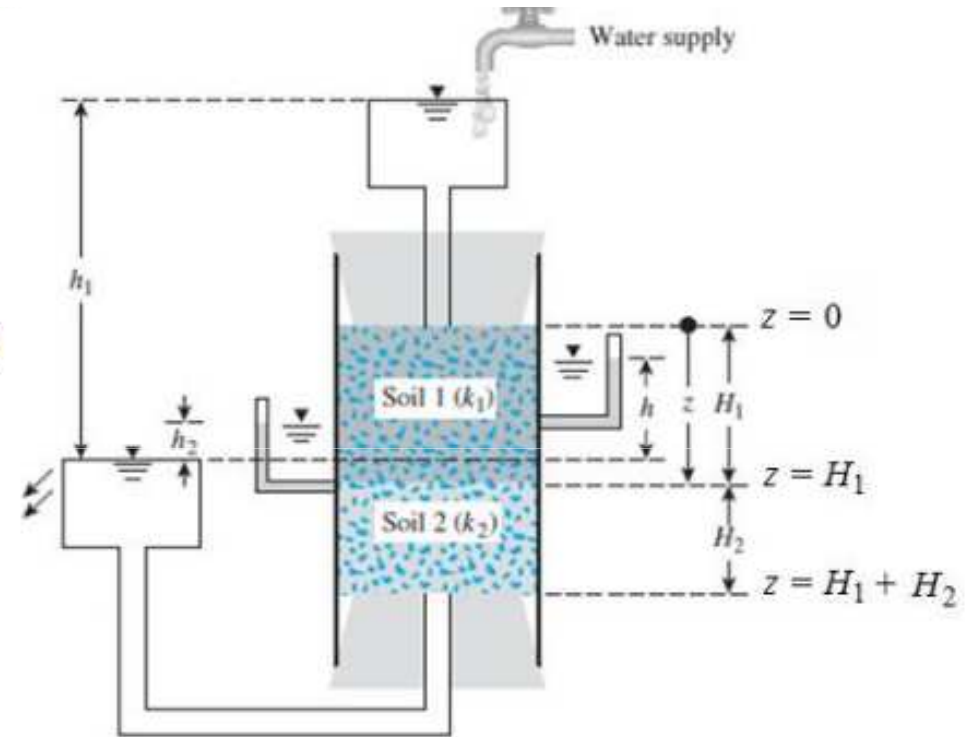
$$A_2 = h_2 - A_1 H_1 \quad (7)$$

Condition 2: at $z = H_1 + H_2$, $h = 0$

$$0 = A_1(H_1 + H_2) + (h_2 - A_1 H_1)$$

$$A_1 H_1 + A_1 H_2 + h_2 - A_1 H_1 = 0$$

or,
$$A_1 = -\frac{h_2}{H_2} \quad (8)$$



Combining Eqs. (3), (7), and (8), we obtain:

$$h = -\left(\frac{h_2}{H_2}\right)z + h_2\left(1 + \frac{H_1}{H_2}\right) \quad \text{for } H_1 \leq z \leq H_1 + H_2 \quad (9)$$

Laplace's Equation of Continuity

At any given time, flow through soil 1 equals flow through soil 2, so:

$$q_1 = q_2 = q \quad \text{then,} \quad q = k_1 \left(\frac{h_1 - h_2}{H_1} \right) A = k_2 \left(\frac{h_2 - 0}{H_2} \right) A$$

$$\text{or, } h_2 = \frac{h_1 k_1}{H_1 \left(\frac{k_1}{H_1} + \frac{k_2}{H_2} \right)} \quad (10)$$

where A : area of cross section of the soil
 k_1 : hydraulic conductivity of soil 1
 k_2 : hydraulic conductivity of soil 2

- Substituting Eq. (10) into Eq. (6), we obtain :

$$h = h_1 \left(1 - \frac{k_2 z}{k_1 H_2 + k_2 H_1} \right) \quad \text{for } 0 \leq z \leq H_1 \quad (11)$$

- Similarly, substituting Eq. (10) into Eq. (9), we get:

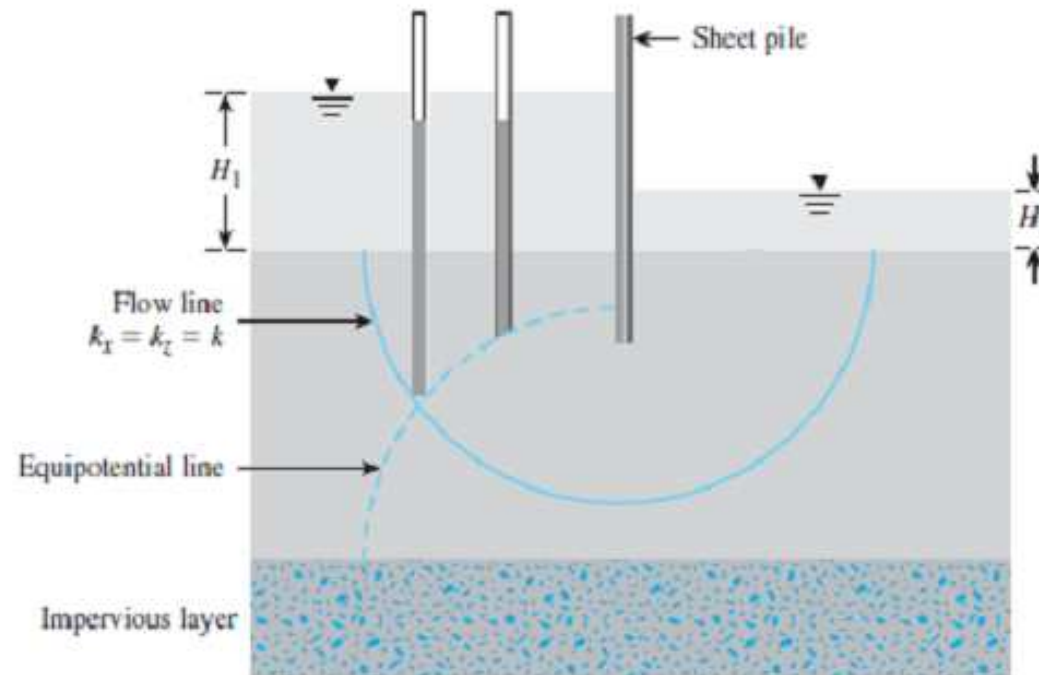
$$h = h_1 \left[\left(\frac{k_1}{k_1 H_2 + k_2 H_1} \right) (H_1 + H_2 - z) \right] \quad \text{for } H_1 \leq z \leq H_1 + H_2 \quad (12)$$

Flow Nets

In an isotropic medium The continuity equation represents two orthogonal families of curves:

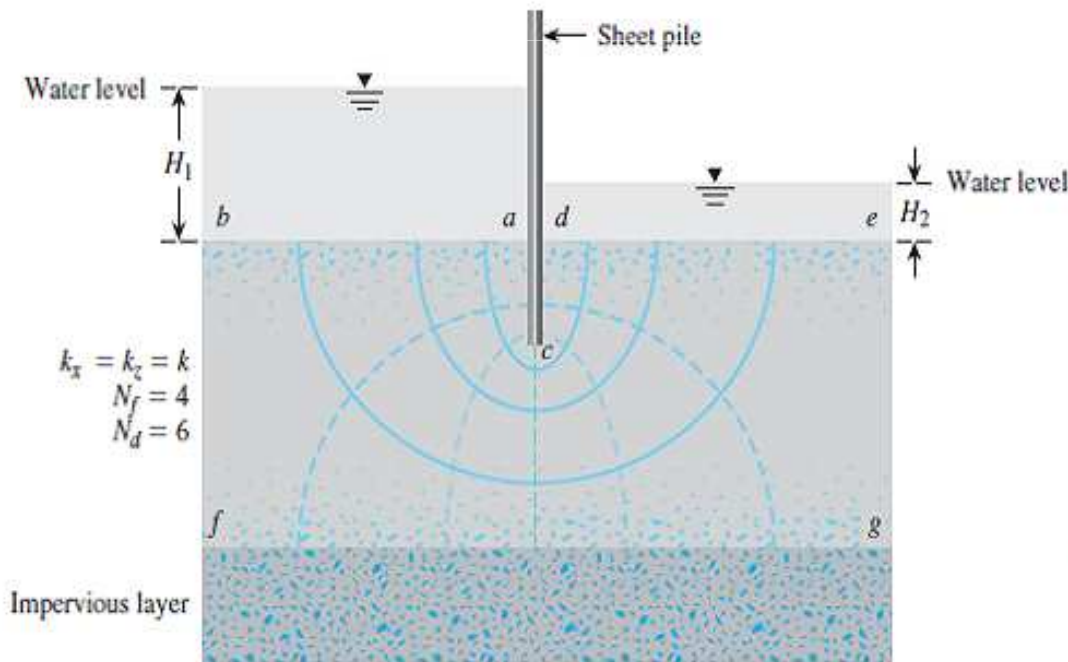
Flow lines: the line along which a water particle will travel from upstream to the downstream side in the permeable soil medium;

Equipotential lines: the line along which the potential (pressure) head at all points is equal.



Flow Nets

- 1. **Flow nets:** the combination of flow lines and equipotential lines.
- 2. To complete the graphic construction of a flow net, one must draw the flow and equipotential lines in such a way that:
 1. The equipotential lines intersect the flow lines at right angles.
 2. The flow elements formed are approximate squares.



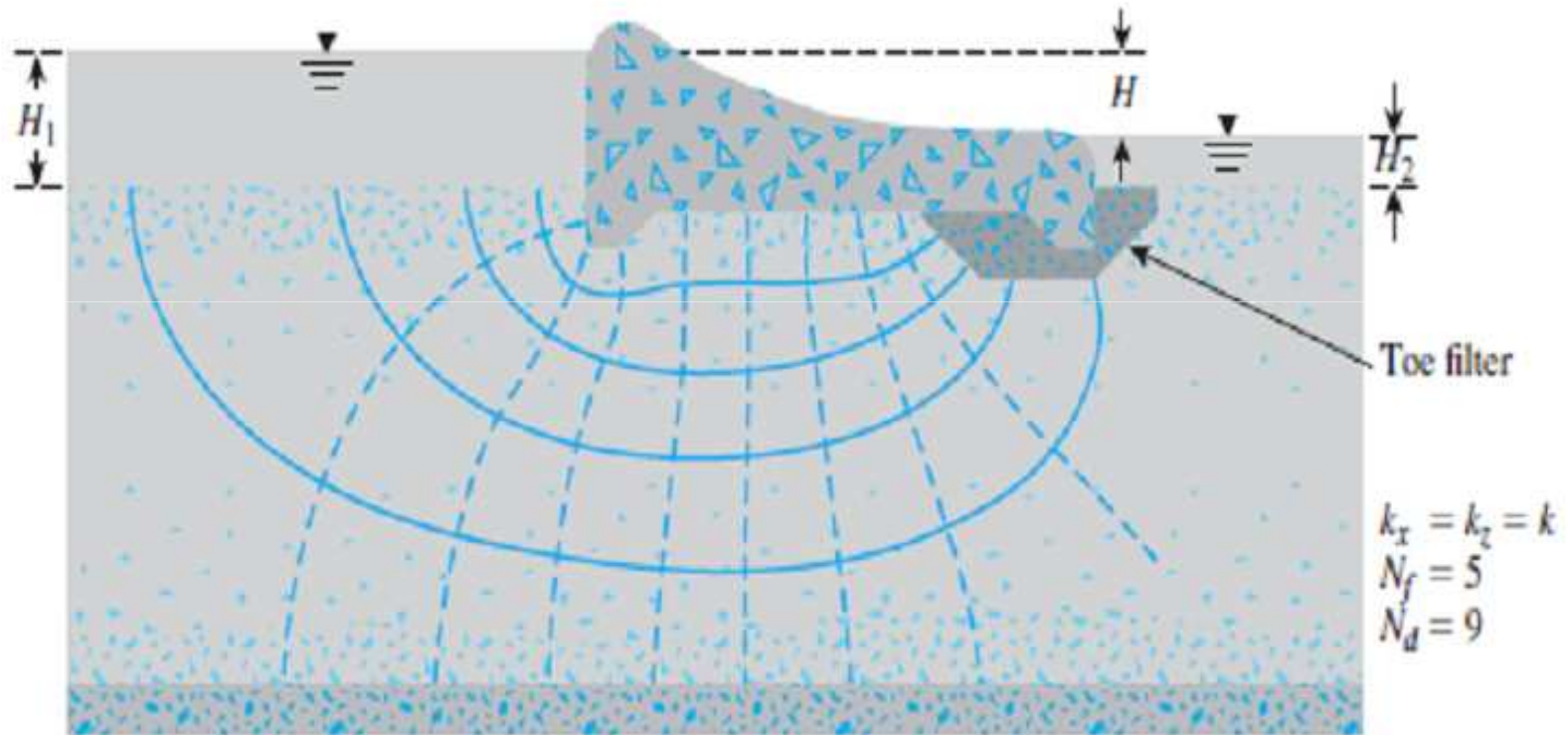
N_f : is the number of flow channels in the flow net.
 N_d : is the number of potential drops.

Flow Nets

- Drawing a flow net takes several trials. While constructing the flow net, keep the boundary conditions in mind. For the flow net shown in Figure above, the following four boundary conditions apply:
- **Condition 1:** The upstream and downstream surfaces of the permeable layer (lines *ab* and *de*) are equipotential lines.
 - **Condition 2:** Because *ab* and *de* are equipotential lines, all the flow lines intersect them at right angles.
 - **Condition 3:** The boundary of the impervious layer - that is, line *fg* - is a flow line, and so is the surface of the impervious sheet pile, line *acd*.
 - **Condition 4:** The equipotential lines intersect *acd* and *fg* at right angles.

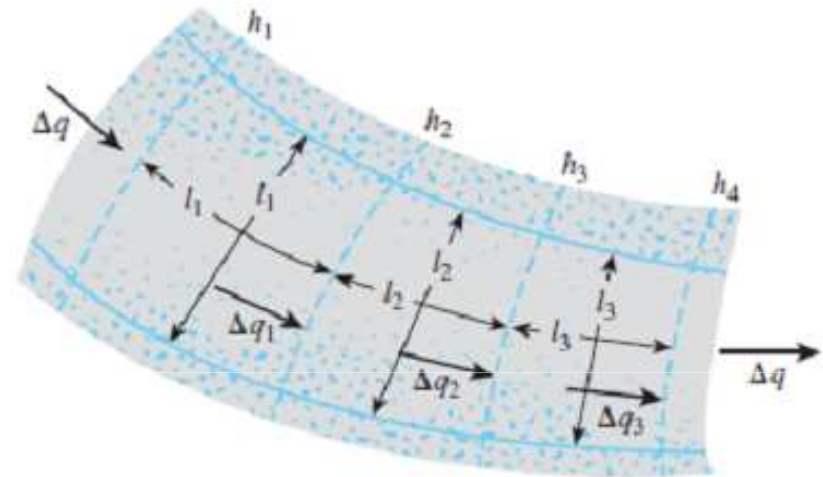
Flow Nets

Flow net under a dam with toe filter:



Seepage Calculation from Flow Nets

- In a flow net, the strip between any two adjacent flow lines is called a flow channel.
- The drop in the piezometric level between any two adjacent equipotential lines is the same and is called the potential drop.
- The flow rate in a element:



$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \dots = \Delta q$$

- From Darcy's law, the flow rate is equal to (kiA) . Thus,

$$\Delta q = k \left(\frac{h_1 - h_2}{l_1} \right) l_1 = k \left(\frac{h_2 - h_3}{l_2} \right) l_2 = k \left(\frac{h_3 - h_4}{l_3} \right) l_3 = \dots \quad (13)$$

Seepage Calculation from Flow Nets

Eq. (13) shows that if the flow elements are drawn as approximate squares, the drop in the piezometric level between any two adjacent equipotential lines is the same. This is called the potential drop. Thus,

$$h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \dots = \frac{H}{N_d} \longrightarrow \Delta q = k \frac{H}{N_d} \quad (14)$$

where H : head difference between the upstream and downstream sides.

N_d : number of potential drops.

- If the number of flow channels in a flow net is equal to N_f , the total rate of flow through all the channels per unit length can be given by:

$$q = k \frac{HN_f}{N_d} \quad (15)$$

Seepage Calculation from Flow Nets

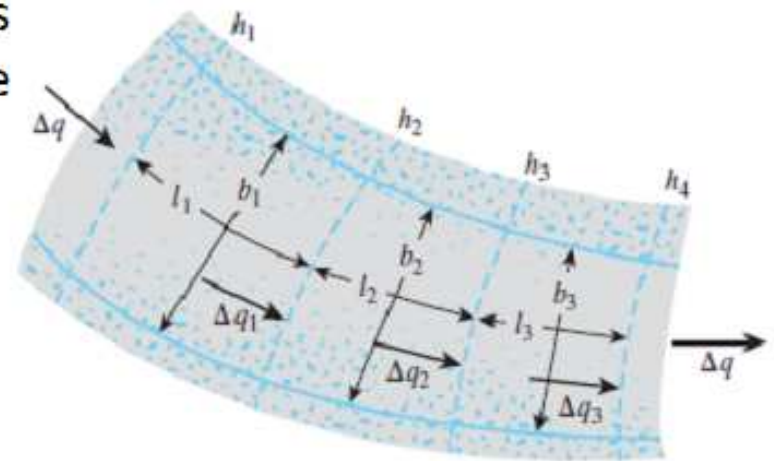
- Although drawing square elements for a flow net is convenient, it is not always necessary. Alternatively, one can draw a rectangular mesh for a flow channel, as shown in the figure, provided that the width-to-length ratios for all the rectangular elements in the flow net are the same.
- In this case, Eq. (13) for rate of flow through the channel can be modified to:

$$\Delta q = k \left(\frac{h_1 - h_2}{l_1} \right) b_1 = k \left(\frac{h_2 - h_3}{l_2} \right) b_2 = k \left(\frac{h_3 - h_4}{l_3} \right) b_3 = \dots \quad (16)$$

- If $b_1/l_1 = b_2/l_2 = b_3/l_3 = n$ (i.e., the elements are not square), Eqs. (14) and (15) can be modified to:

$$\Delta q = kH \left(\frac{n}{N_d} \right) \quad (17)$$

$$q = kH \left(\frac{N_f}{N_d} \right) n \quad (18)$$



Seepage Calculation from Flow Nets

- The figure below shows a flow net for seepage around a single row of sheet piles. Note that flow channels 1 and 2 have square elements. Hence, the rate of flow through these two channels can be obtained from Eq. (14):

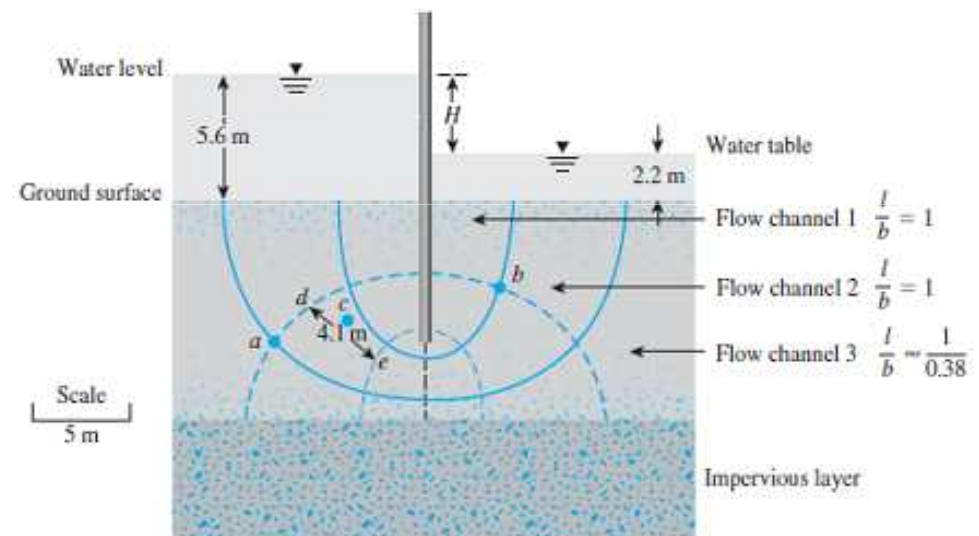
$$\Delta q_1 + \Delta q_2 = \frac{k}{N_d} H + \frac{k}{N_d} H = \frac{2kH}{N_d}$$

- However, flow channel 3 has rectangular elements. These elements have a width-to-length ratio of about 0.38; hence, from Eq. (17):

$$\Delta q_3 = \frac{k}{N_d} H(0.38)$$

- So, the total rate of seepage can be given as:

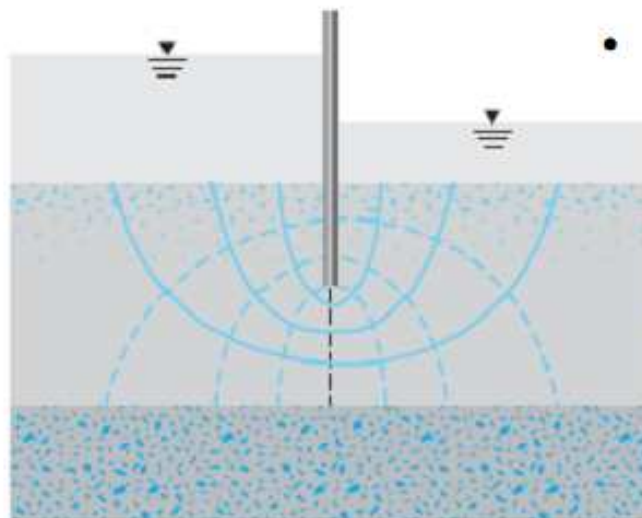
$$q = \Delta q_1 + \Delta q_2 + \Delta q_3 = 2.38 \frac{kH}{N_d}$$



Flow Net in Anisotropic Soil

To account for soil anisotropy with respect to hydraulic conductivity, we must modify the flow net construction. To construct the flow net, use the following procedure:

- Step 1:* Adopt a vertical scale (that is, z axis) for drawing the cross section.
- Step 2:* Adopt a horizontal scale (that is, x axis) such that horizontal scale = $\sqrt{k_z/k_x} \times$ vertical scale.
- Step 3:* With scales adopted as in Steps 1 and 2, plot the vertical section through the permeable layer parallel to the direction of flow.
- Step 4:* Draw the flow net for the permeable layer on the section obtained from Step 3, with flow lines intersecting equipotential lines at right angles and the elements as approximate squares.



- The rate of seepage per unit length can be calculated by modifying Eq. (15) to:

$$\frac{k_z}{k_x} = \frac{1}{6}$$

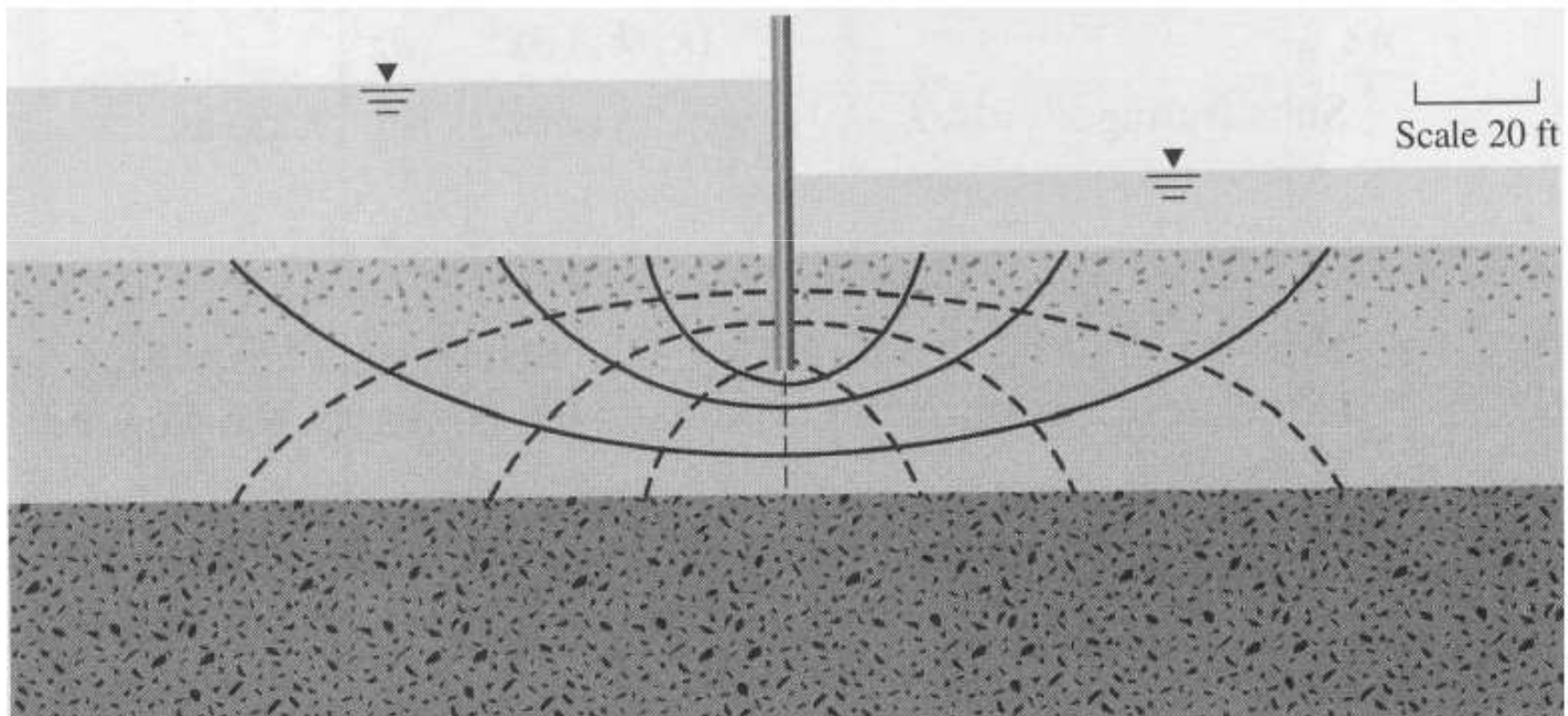
Vertical scale = 20 ft

Horizontal scale = $20(\sqrt{6}) = 49$ ft

$$q = \sqrt{k_x k_z} \frac{HN_f}{N_d} \quad (19)$$

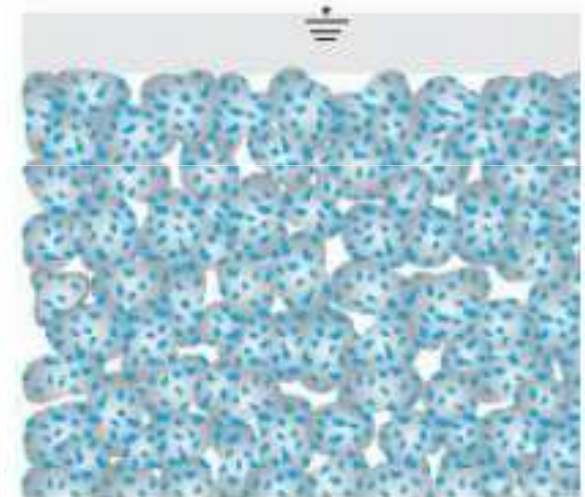
Flow Net in Anisotropic Soil

Flow element in anisotropic soil (in true section):



In-situ Stresses in Soil Mass

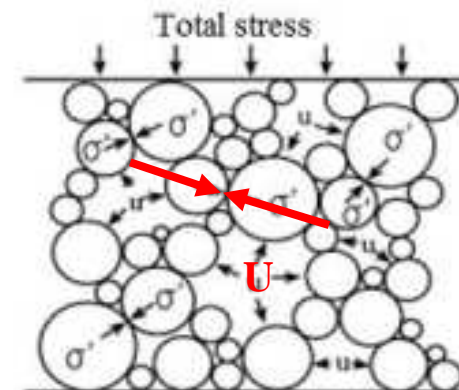
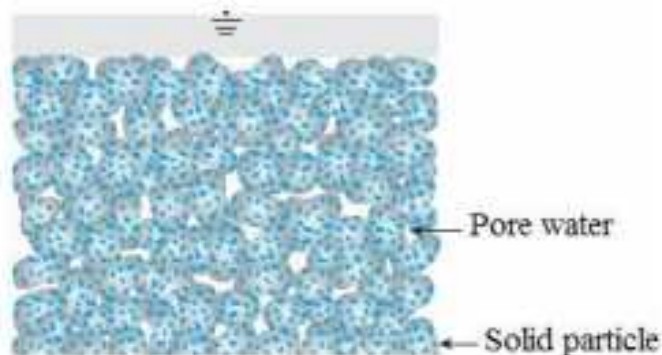
1. Effective Stress
2. Effective Stress and Pore Water Pressure
3. Stress in Saturated Soil without Seepage
4. Stress in Saturated Soil with Upward Seepage
5. Stress in Saturated Soil with Downward Seepage



Textbook: Braja M. Das, "Principles of Geotechnical Engineering", 7th E. (Chapter 9).

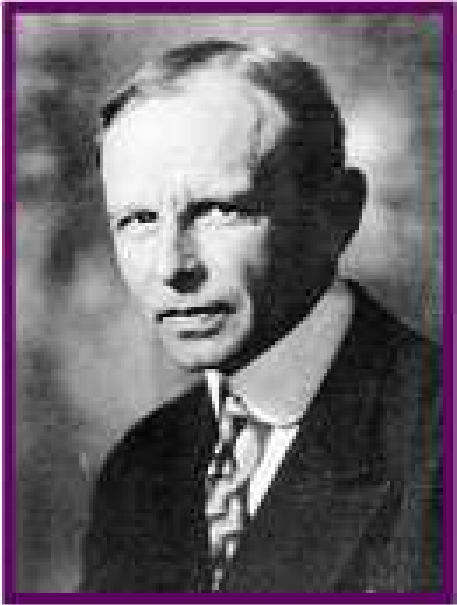
Effective Stress

- As in other materials, stresses may act in soils as a result of an external load and the volumetric weight of the material itself.
- The pressure transmitted through grain to grain at the contact points through a soil mass is termed as **intergranular or effective pressure (σ')**. It is known as effective pressure since this pressure is responsible for the decrease in the void ratio or increase in the frictional resistance of a soil mass.
- If the pores of a soil mass are filled with water and if a pressure induced into the pore water, tries to separate the grains, this pressure is termed as **pore water pressure (u) or neutral stress**. The effect of this pressure is to increase the volume or decrease the frictional resistance of the soil mass.



Effective stress

Father of Soil Mechanics

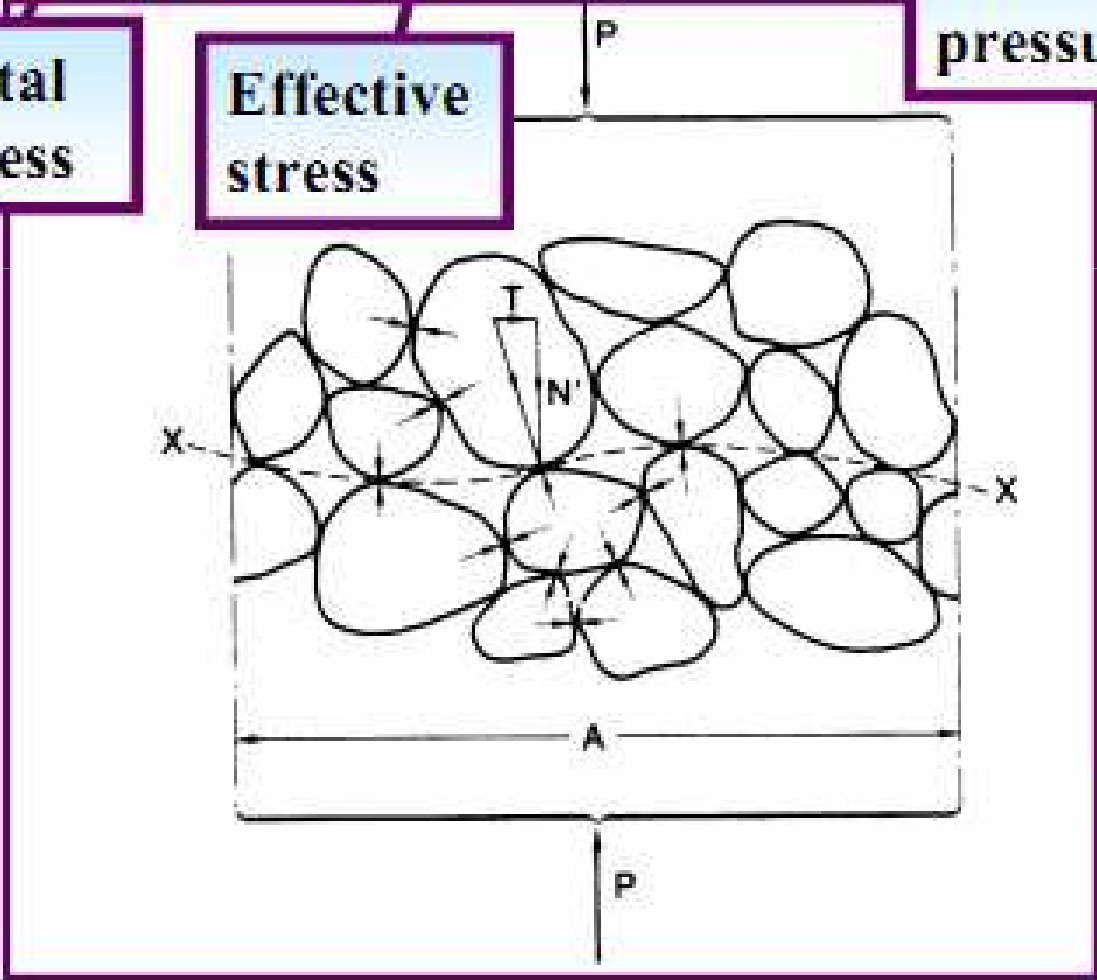


$$\sigma = \sigma' + u$$

Water pressure

Total stress

Effective stress



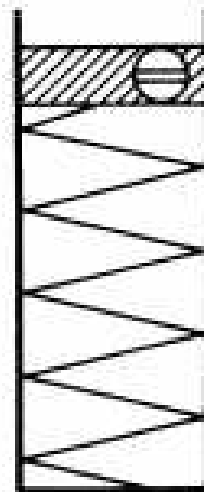
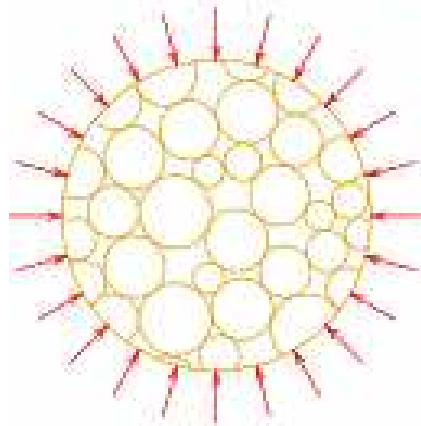
By Terzaghi (1923)

$$P = \sum N' + uA$$

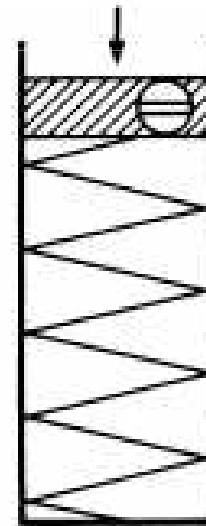
$$\frac{P}{A} = \frac{\sum N'}{A} + u$$

$$\sigma = \sigma' + u$$

Consolidation analogy: Transfer of stress to soil skeleton



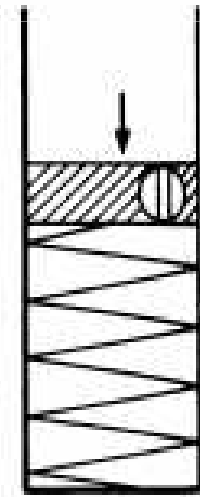
(a)



(b)



(c)



(d)

Effective Stress & Pore water Pressure

Case 1: Dry sand under load (Q)

The load applied at the surface of the soil is transferred to the soil grains in the mold through their points of contact.

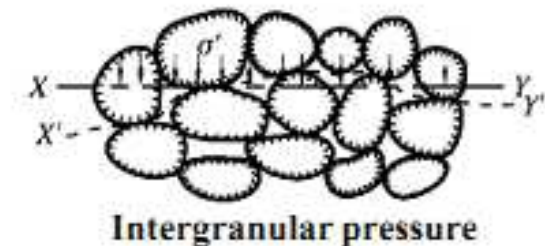
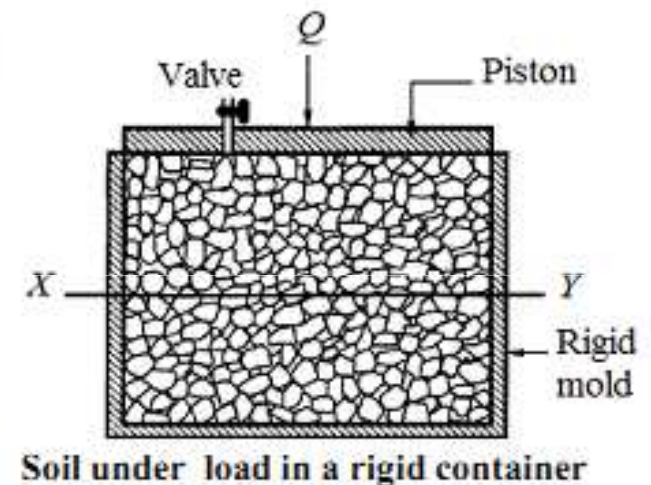
→ If the load is quite considerable, it would result in the compression of the soil mass in the mold.

→ If the sectional area of the cylinder is A , the average stress at any level $X-Y$ may be written as:

$$\sigma_a = \frac{Q}{A}$$

→ Since this stress is responsible for the deformation of the soil mass, it is termed the **intergranular** or **effective stress**. We may therefore write:

$$\sigma_a = \sigma'$$



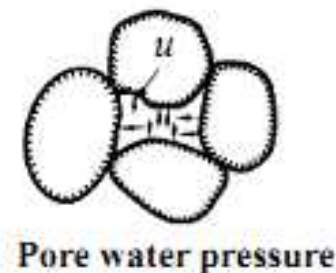
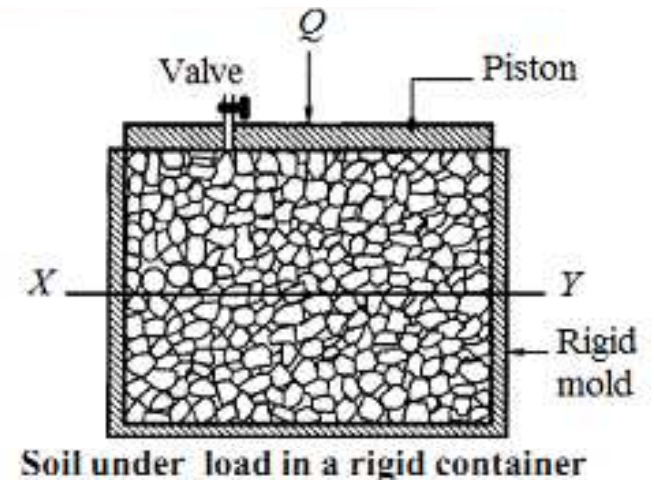
Effective Stress & Pore water Pressure

Case 2: Fully saturated sand under load (Q)

If the same load Q is placed on the piston, this load will not be transmitted to the soil grains as in the case 1.

- If we assume that water is incompressible, the external load will be transmitted to the water in the pores.
- The pressure that is developed in the water is called the pore water or neutral stress (u). This pore water pressure prevents the compression of the soil mass. The value of this pressure is:

$$u = \frac{Q}{A}$$



Effective Stress & Pore water Pressure

Case 2: Fully saturated sand under load (Q)

If the valve provided in the piston is opened, immediately there will be expulsion of water through the hole in the piston. The flow of water continues for some time and then stops.

- The expulsion of water from the pores decreases the pore water pressure and correspondingly increases the intergranular pressure. At any stage the total pressure Q/A is divided between water and the points of contact of grains. A new equation may therefore be written as:

$$\text{Total pressure } \sigma = \frac{Q}{A} = \text{Intergranular pressure} + \text{pore water pressure}$$

or

$$\sigma = \sigma' + u$$

- Final equilibrium will be reached when there is no expulsion of water. At this stage the pore water pressure $u = 0$. All the pressure will be carried by the soil grains. Therefore, we can write:

$$\sigma = \sigma'$$

Effective Stress & Pore water Pressure

In summary, the total stress is the sum of the effective stress and the pore pressure (neutral stress). The effective stress is carried by the soil skeleton. The pore pressure is carried by pore water.

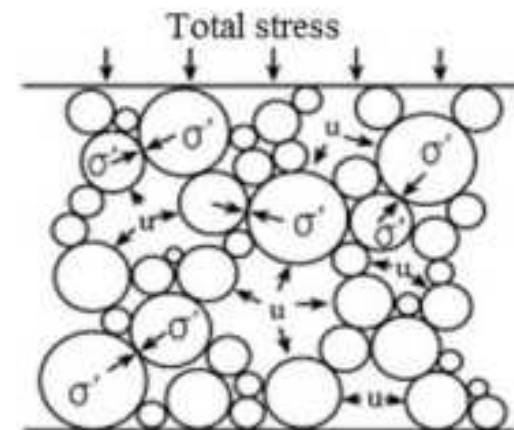
$$\sigma = \sigma' + u$$

or

$$\sigma' = \sigma - u$$

- The effective stress in a soil mass controls its volume change and strength. Increasing the effective stress (equivalent to reduce the pore pressure if the total stress is constant) induces soil to move into a denser state of packing.

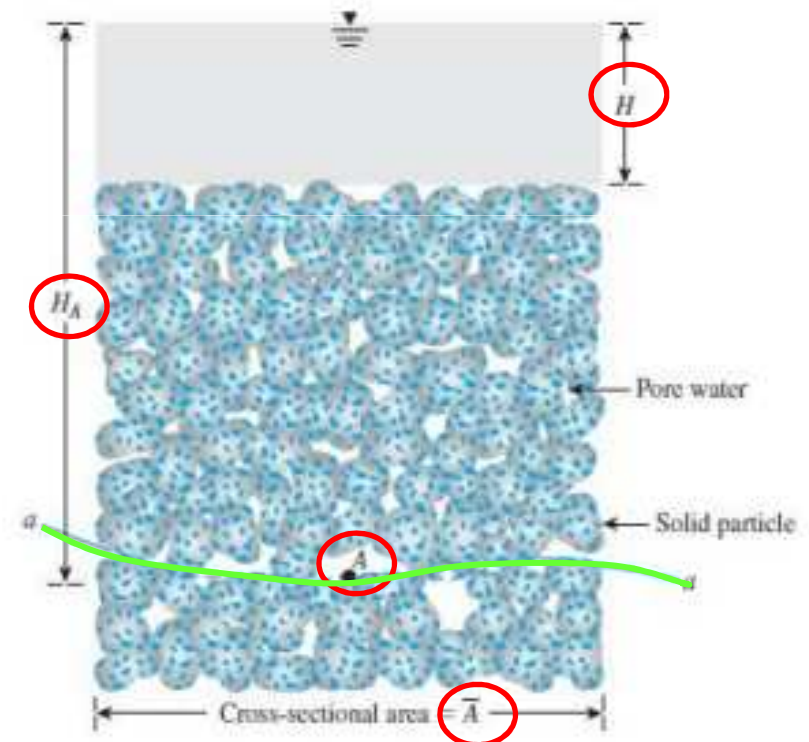
σ : Total stress;
 σ' : Effective stress;
 u : Pore water pressure.



Stresses in Saturated Soil without Seepage

The figure shows a column of saturated soil mass with no seepage of water in any direction. The total stress at the elevation of point **A** can be obtained from the saturated unit weight of the soil and the unit weight of water above it. Thus,

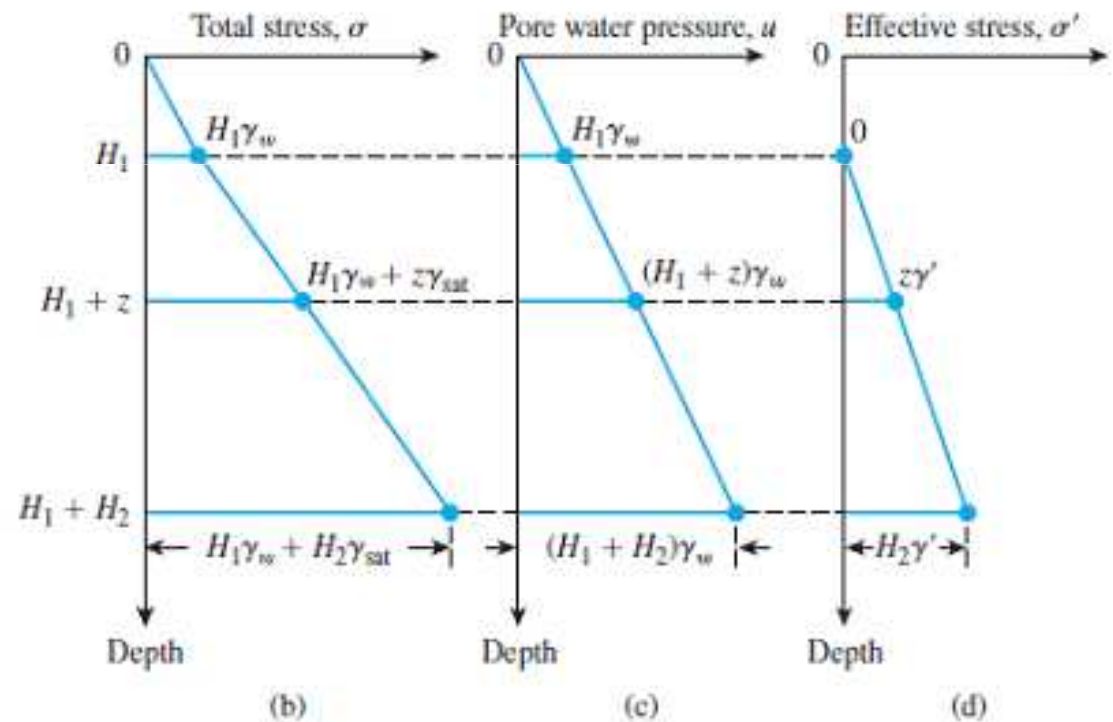
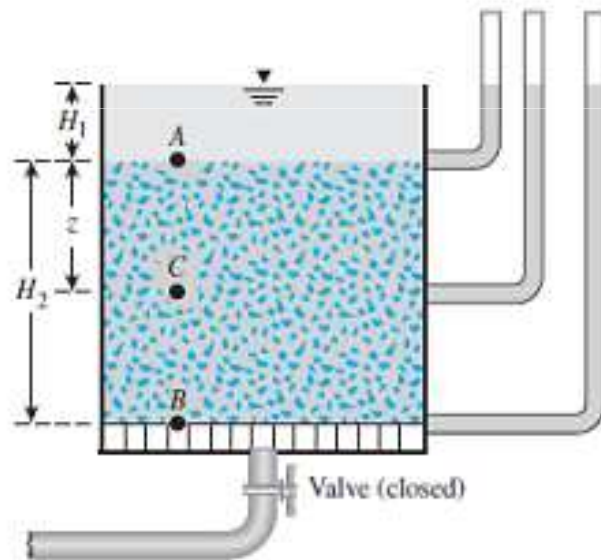
$$\begin{aligned}
 \sigma &= H\gamma_w + (H_A - H)\gamma_{sat} \\
 &= H\gamma_w + H_A\gamma_{sat} - H\gamma_{sat} \\
 &= H\gamma_w + H_A(\gamma_{sat} - \gamma_w) + H_A\gamma_w - H\gamma_{sat} \\
 &= H_A(\gamma_{sat} - \gamma_w) - H(\gamma_{sat} - \gamma_w) + H_A\gamma_w \\
 &= (H_A - H)(\gamma_{sat} - \gamma_w) + H_A\gamma_w \\
 &= (H_A - H)\gamma' + H_A\gamma_w \\
 &= \sigma' + u
 \end{aligned}$$



where $\gamma' = \gamma_{sat} - \gamma_w$ equals the submerged unit weight of soil.

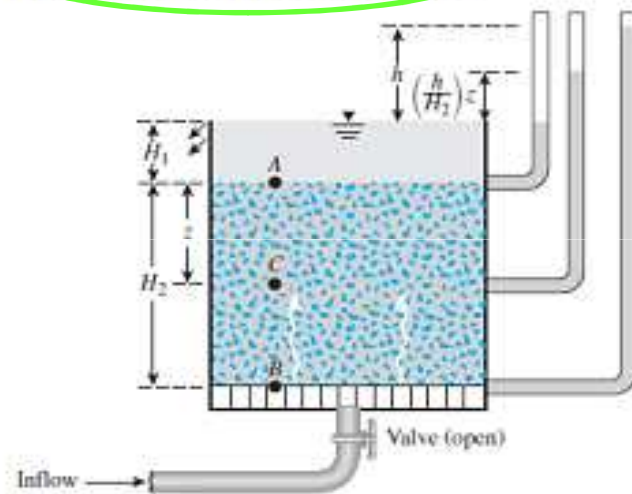
Stresses in Saturated Soil without Seepage

Plots of the variations of the total stress, pore water pressure, and effective stress, respectively, with depth for a submerged layer of soil placed in a tank with no seepage:



Stresses in Saturated Soil with Upward Seepage

If water is seeping, the effective stress at any point in a soil mass will differ from that in the static case. It will increase or decrease, depending on the direction of seepage.



At A,

- Total stress: $\sigma_A = H_1 \gamma_w$
- Pore water pressure: $u_A = H_1 \gamma_w$
- Effective stress: $\sigma'_A = \sigma_A - u_A = 0$

At B,

- Total stress: $\sigma_B = H_1 \gamma_w + H_2 \gamma_{sat}$
- Pore water pressure: $u_B = (H_1 + H_2 + h) \gamma_w$
- Effective stress: $\sigma'_B = \sigma_B - u_B$
 $= H_2 (\gamma_{sat} - \gamma_w) - h \gamma_w$
 $= H_2 \gamma' - h \gamma_w$

At C,

- Total stress: $\sigma_C = H_1 \gamma_w + z \gamma_{sat}$
- Pore water pressure: $u_C = \left(H_1 + z + \frac{h}{H_2} z \right) \gamma_w$
- Effective stress: $\sigma'_C = \sigma_C - u_C = z (\gamma_{sat} - \gamma_w) - \frac{h}{H_2} z \gamma_w = z \gamma' - \frac{h}{H_2} z \gamma_w$

Stresses in Saturated Soil with Upward Seepage

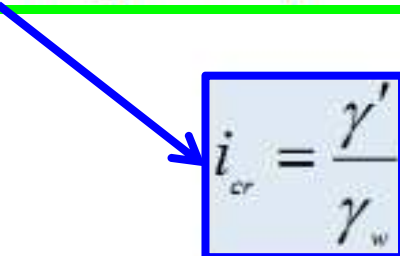
Note that (h/H_2) is the hydraulic gradient i caused by the flow, and therefore:

$$\sigma'_c = z\gamma' - iz\gamma_w$$

If the rate of seepage and thereby the hydraulic gradient gradually are increased, a limiting condition will be reached, at which point:

$$\sigma'_c = z\gamma' - i_{cr}z\gamma_w$$

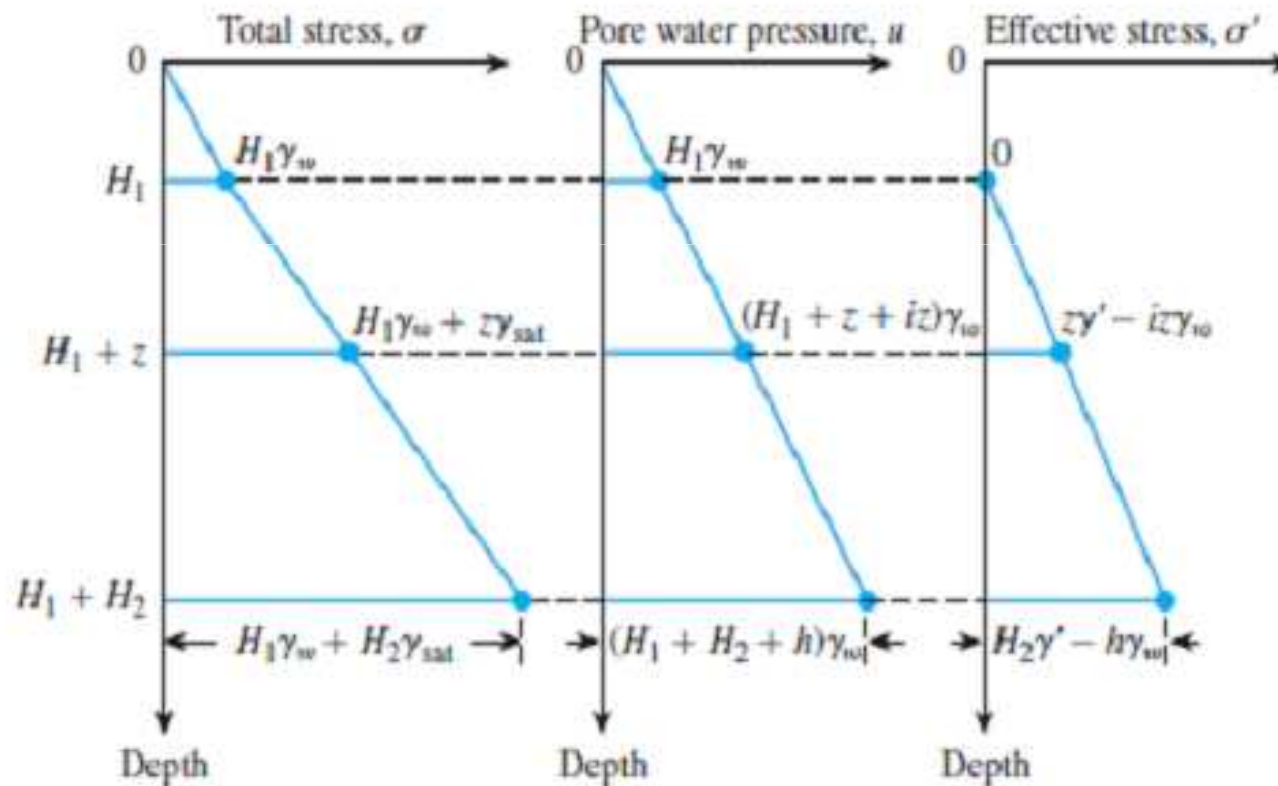
where i_{cr} : critical hydraulic gradient (for zero effective stress).


$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

- Under such a situation, soil stability is lost. This situation generally is referred to as boiling, or a quick condition.
- For most soils, the value of i_{cr} varies from 0.9 to 1.1, with an average of 1.

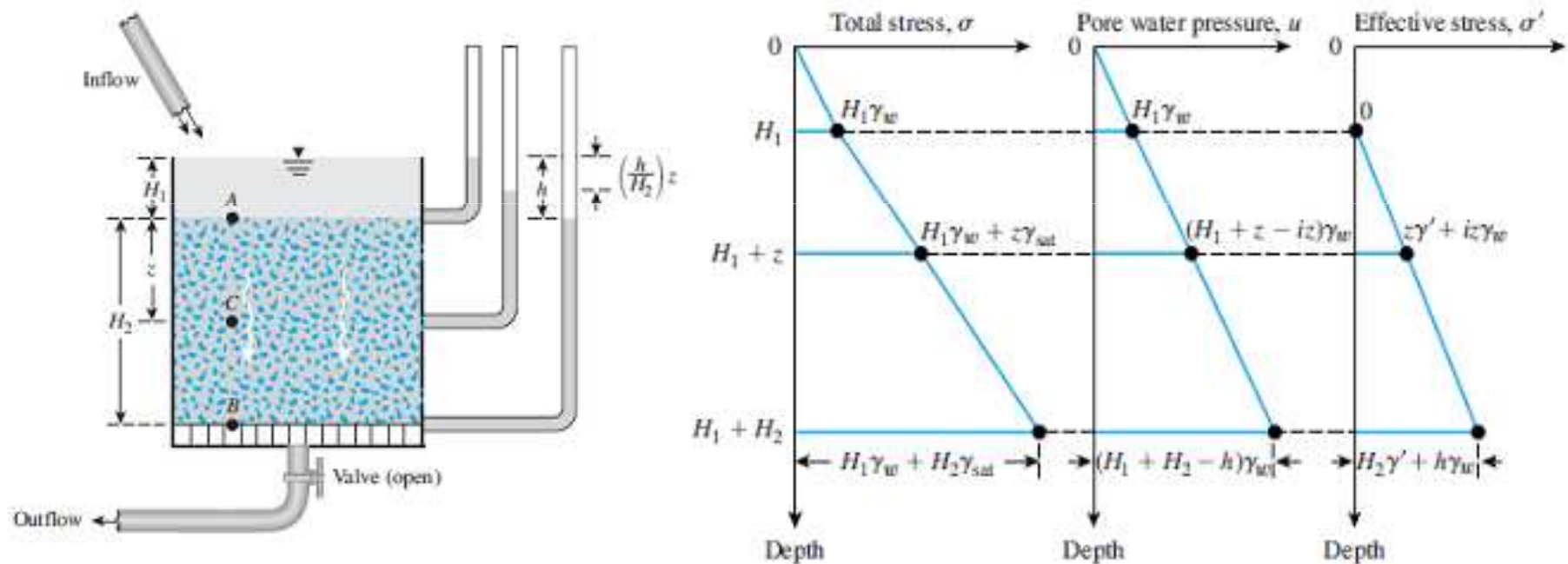
Stresses in Saturated Soil with Upward Seepage

Plots of the variations of the total stress, pore water pressure, and effective stress, respectively, with depth for a soil layer with upward seepage:



Stresses in Saturated Soil with Downward Seepage

variation of total stress, pore water pressure and effective stress with depth for a soil layer with downward seepage:

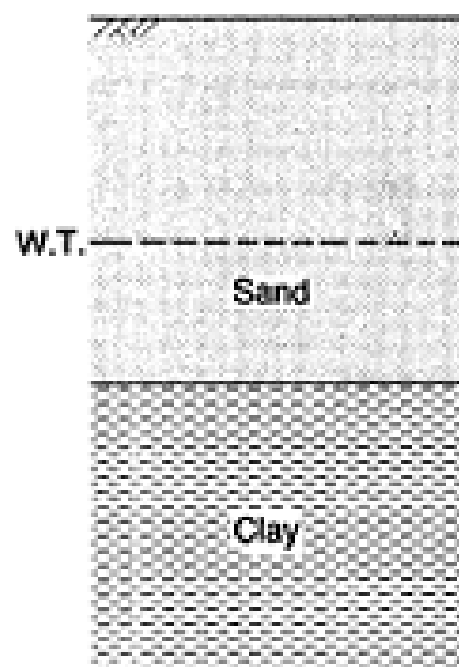


- The hydraulic gradient caused by the downward seepage equals $i = \frac{h}{H_2}$

Example

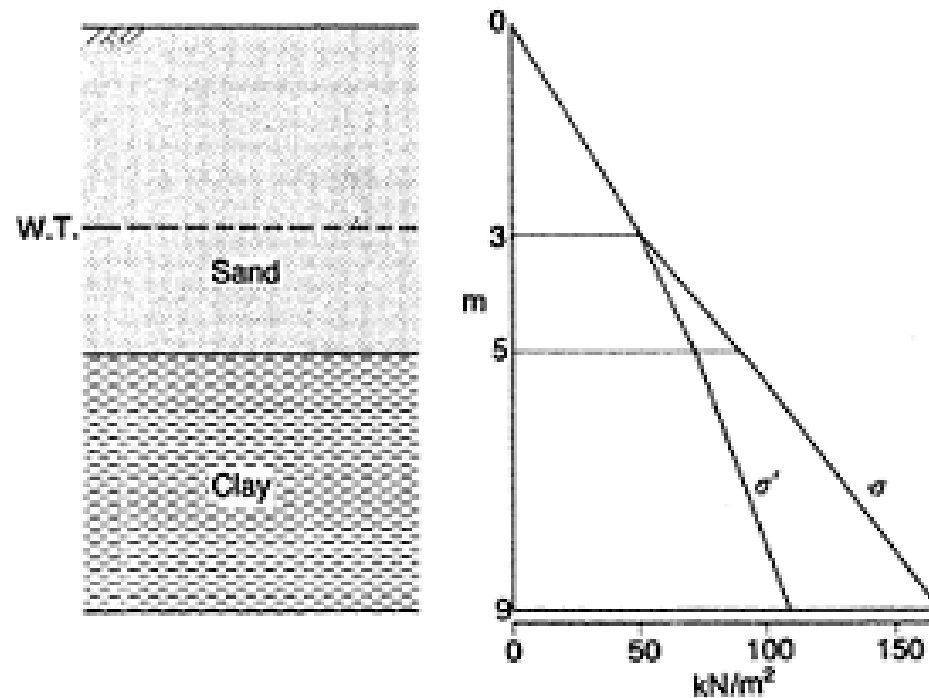
A layer of saturated clay 4m thick is overlain by sand 5m deep, the water table being 3m below the surface. The saturated unit weights of the clay and sand are 19 and 20kN/m³, respectively, above the water table the unit weight of the sand is 17kN/m³. Plot the values of total vertical stress and effective vertical stress against depth. If sand to a height of 1m above the water table is saturated with capillary water, how are the above stresses affected?

Solution:



The total vertical stress is the weight of all material (solids+water) per unit area over the depth in question. Pore water pressure is the hydrostatic pressure responding to the depth below the water table.

Depth (m)	σ_v (kN/m ²)	u (kN/m ²)	$\sigma'_v = \sigma_v - u$ (kN/m ²)
3 × 17	= 51.0	0	51.0
(3 × 17) + (2 × 20)	= 91.0	2 × 9.8 = 19.6	71.4
(3 × 17) + (2 × 20) + (4 × 19)	= 167.0	6 × 9.8 = 58.8	108.2



The **alternative** calculation of σ_v' at depths of 5 and 9m is as follows:

$$\text{Buoyant unit weight of sand} = 20 - 9.8 = 10.2 \text{ kN/m}^3$$

$$\text{Buoyant unit weight of clay} = 19 - 9.8 = 9.2 \text{ kN/m}^3$$

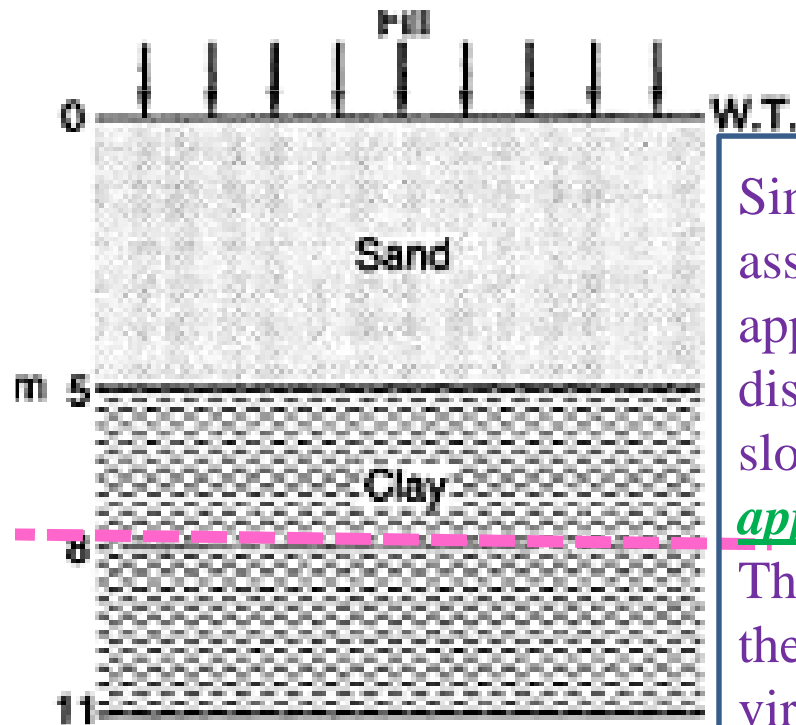
$$\text{At 5 m depth: } \sigma_v' = (3 \times 17) + (2 \times 10.2) = 71.4 \text{ kN/m}^2$$

$$\text{At 9 m depth: } \sigma_v' = (3 \times 17) + (2 \times 10.2) + (4 \times 9.2) = 108.2 \text{ kN/m}^2$$

The water table is the level at which pore water pressure is atmospheric (i.e. $u = 0$). Above the water table, water is held under negative pressure and, even if the soil is saturated above the water table, does not contribute to hydrostatic pressure below the water table. The only effect of the 1m capillary rise, therefore, is to increase the total unit weight of the sand between 2 and 3m depth from 17 to 20kN/m^3 , an increase of 3kN/m^3 . Both total and effective vertical stresses below 3m depth are therefore increased by the constant amount $3 \times 1 = 3.0\text{ kN/m}^2$, pore water pressures being unchanged.

Example

A 5m depth of sand overlies a 6m layer of clay, the water table being at the surface; the permeability of the clay is very low. The saturated unit weight of the sand is 19kN/m^3 and that of the clay is 20kN/m^3 . A 4m depth of fill material of unit weight 20kN/m^3 is placed on the surface over an extensive area. Determine the effective vertical stress at the centre of the clay layer (a) immediately after the fill has been placed, assuming this to take place rapidly and (b) many years after the fill has been placed.



Since the fill covers an extensive area it can be assumed that the condition of zero lateral strain applies. As the permeability of the clay is very low, dissipation of excess pore water pressure will be very slow immediately after the rapid placing the fill, no appreciable dissipation will have taken place.

Therefore, the effective vertical stress at the centre of the clay layer immediately after placing will be virtually unchanged from the original value.

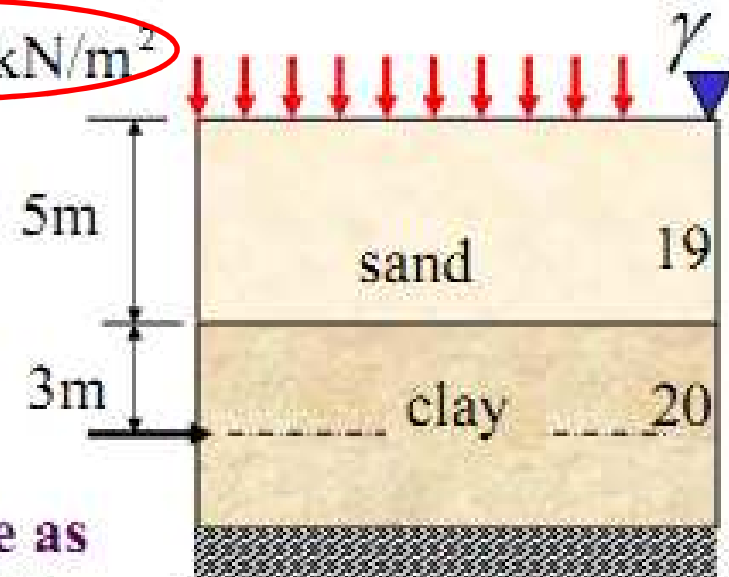
$$4 \times 20 = 80 \text{ kN/m}^2$$

Immediately after placing the fill

($t \rightarrow 0^+$) Approach 1

$$\sigma'_v = 5 \times 9.2 + 3 \times 10.2 = 76.6 \text{ kN/m}^2$$

Same as before



Approach 2

$$\sigma'_v = 5 \times 19 + 3 \times 20 + 80 - (8 \times 9.8 + 80) = 76.6 \text{ kN/m}^2$$

increase right the way

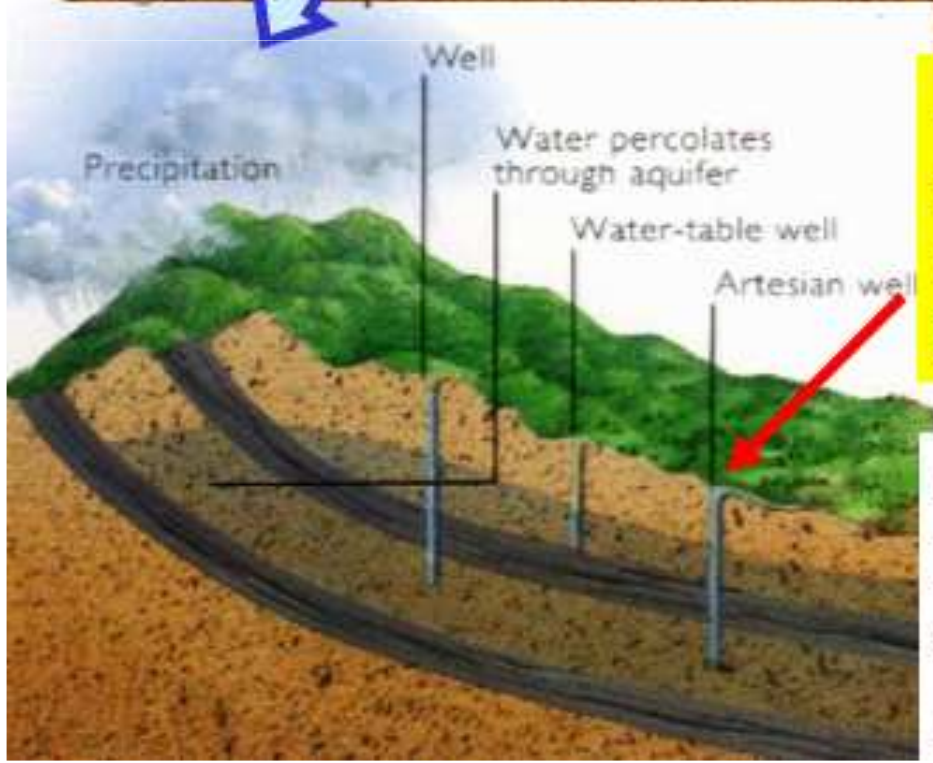
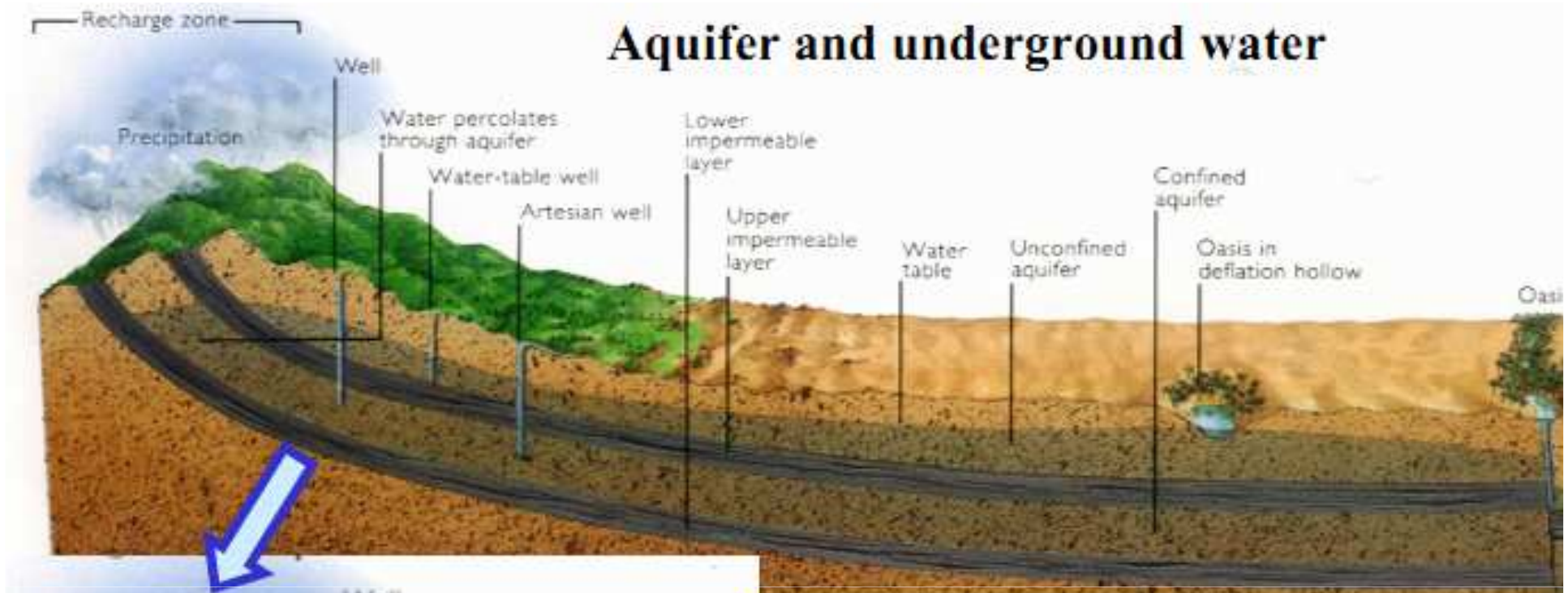
Pore pressure increment

Many years after placing the fill ($t \rightarrow \infty$)

$$\sigma'_v = 80 + 5 \times 9.2 + 3 \times 10.2 = 156.6 \text{ kN/m}^2$$

	u(kN/m ²)	σ' (kN/m ²)
$t \rightarrow 0^-$	78.4	76.6
$t \rightarrow 0^+$	158.4	76.6
$t \rightarrow \infty$	78.4	156.6

Aquifer and underground water

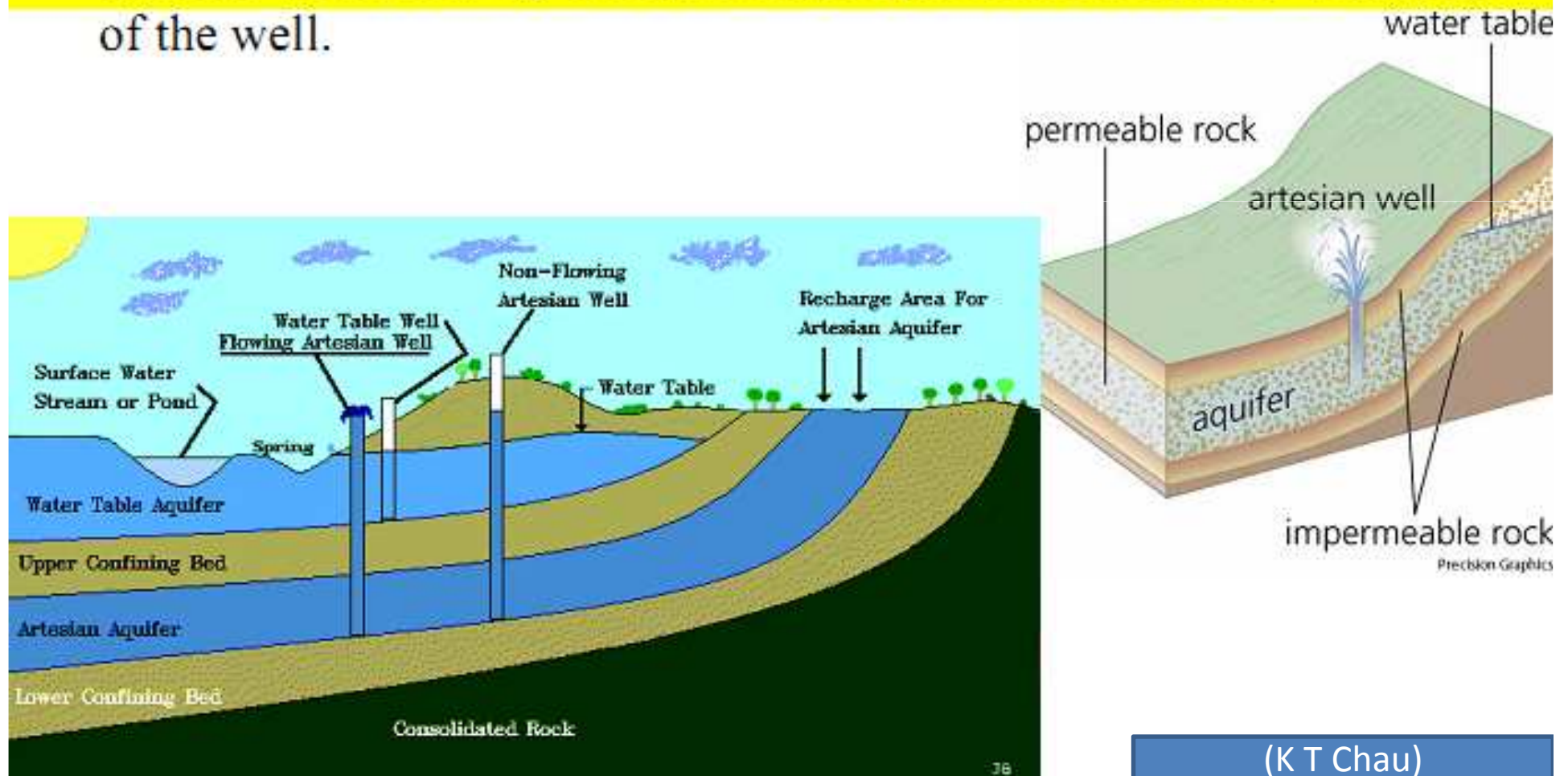


Artesian well= A well drilled through impermeable strata to reach water capable of rising to the surface by internal hydrostatic pressure.

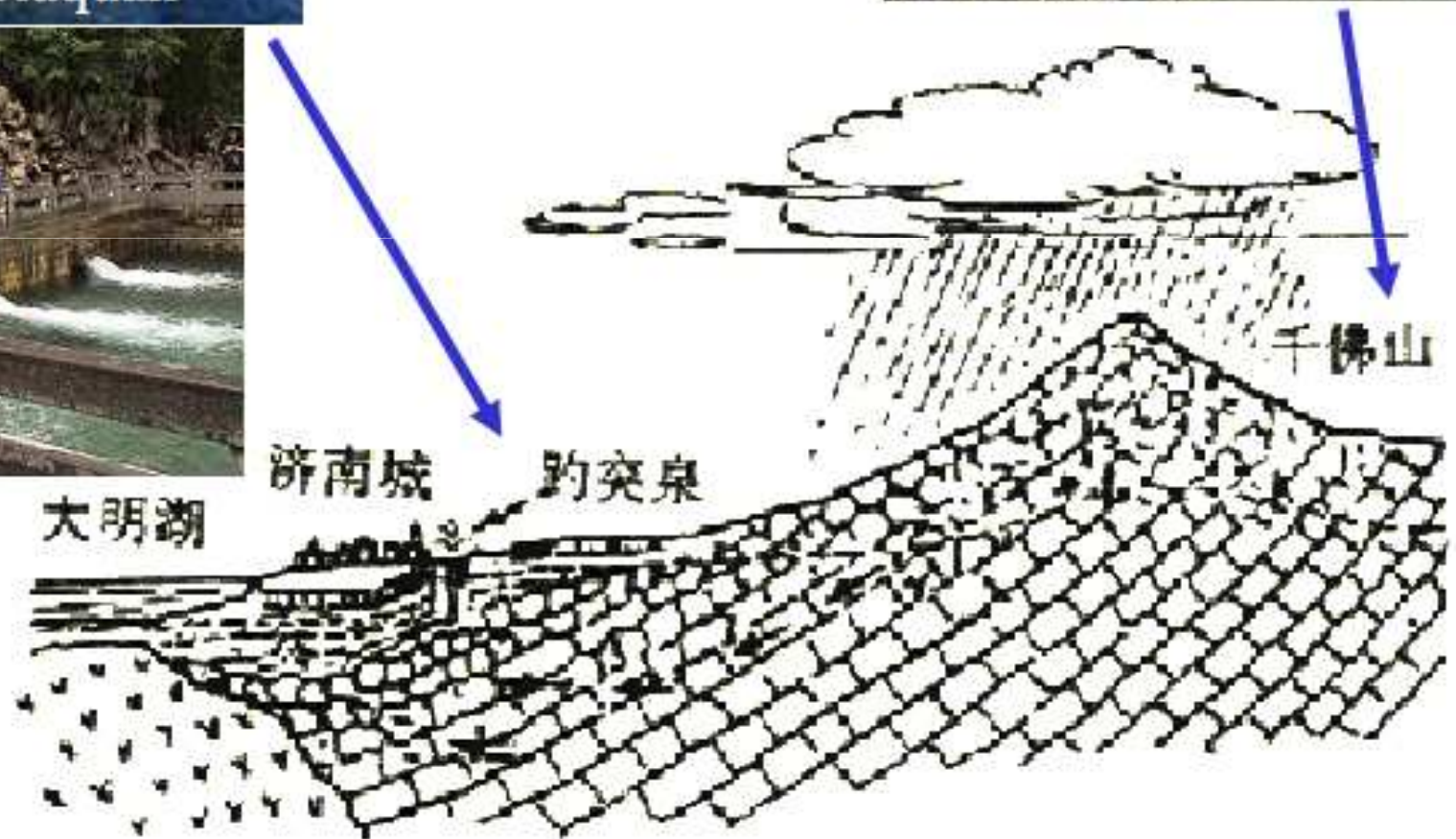
ETYMOLOGY:
French artésien, from Old French artésien, of Artois, from *Arteis Artois, France*

Artesian Aquifer

An aquifer that has pressure built up inside. This pressure is the result of the recharge area of the aquifer being at a higher level than the rest of the aquifer region. The force of gravity pulls the higher water down which creates extra pressure inside the aquifer. This is why artesian wells flow by themselves; the pressure forces the water out of the well.

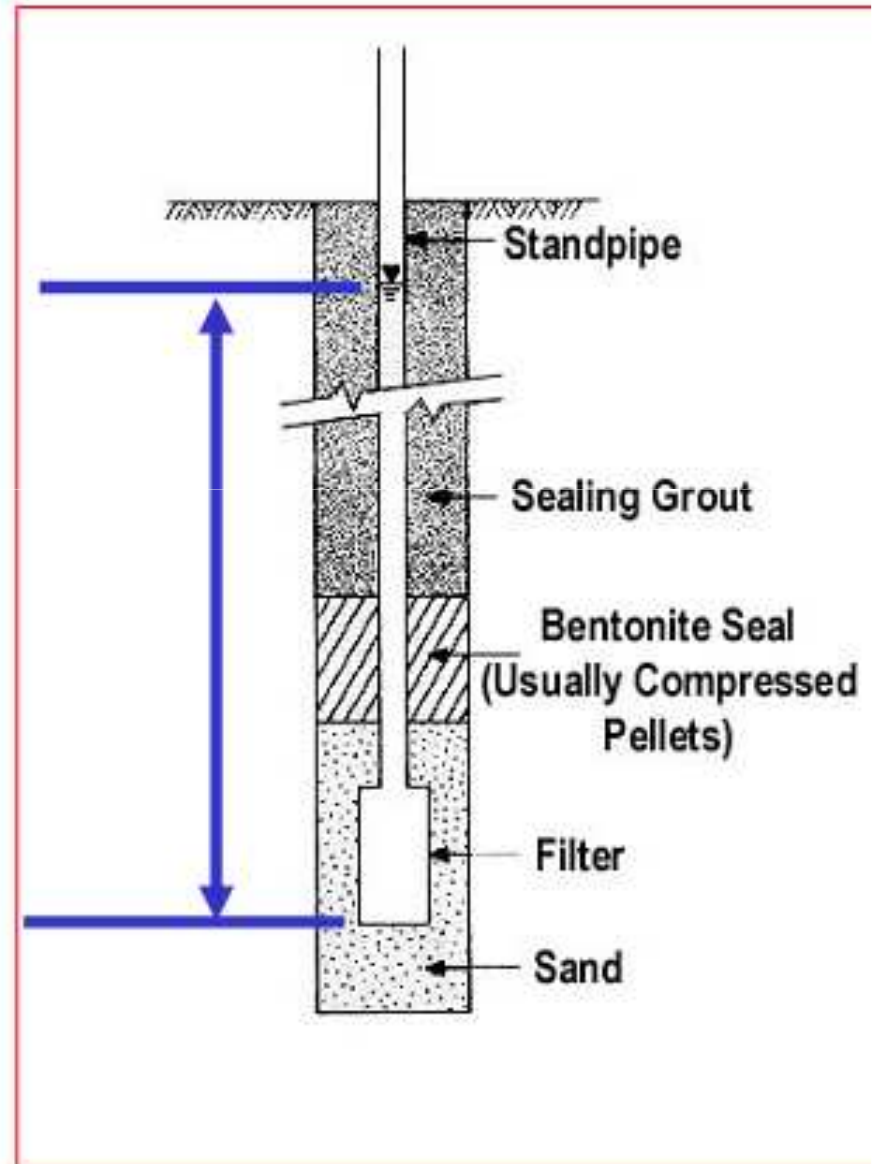
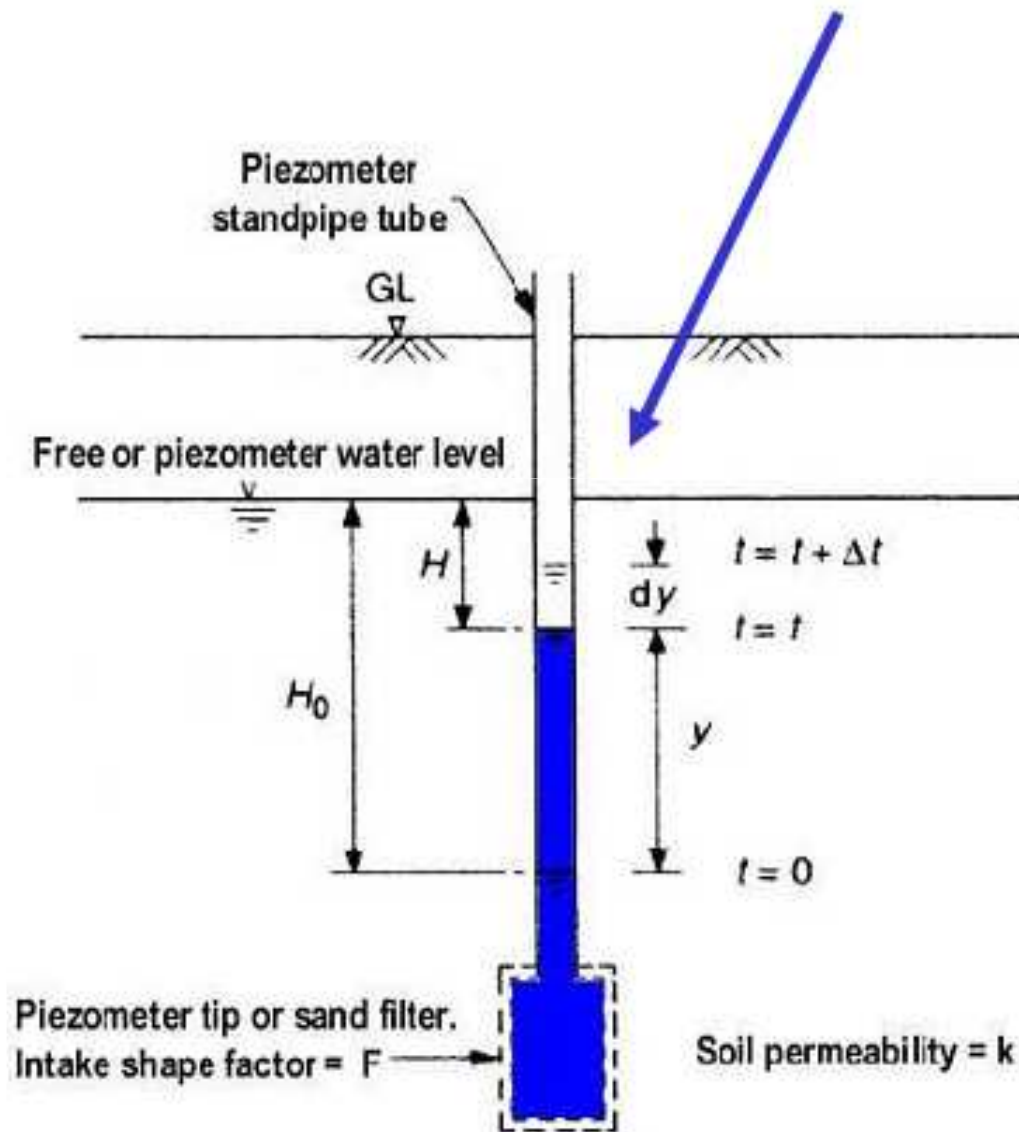


Example of Artesian Aquifer in China (JINAN)

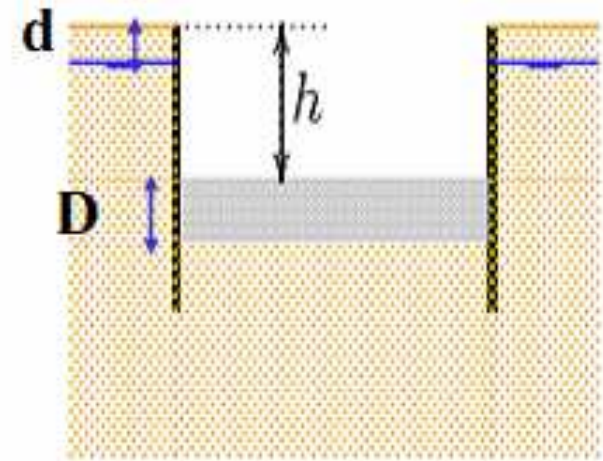
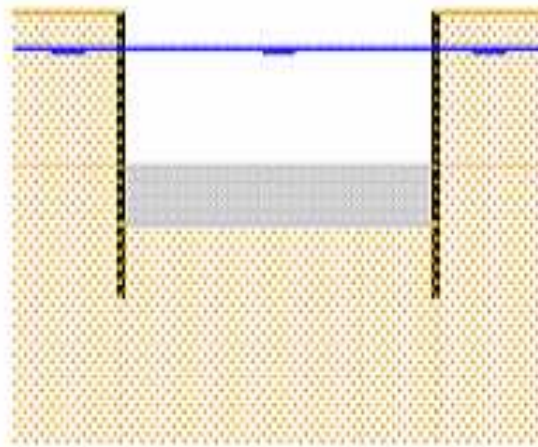
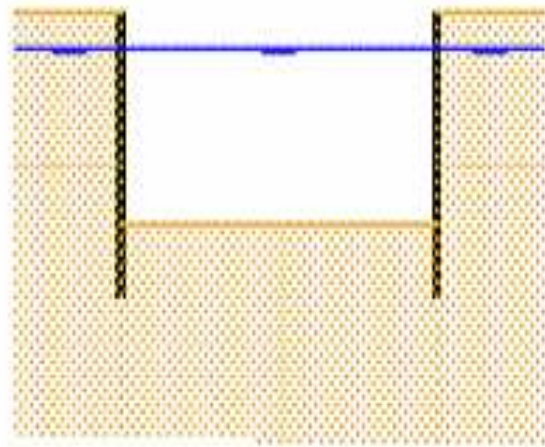


Piezometer, standpipe

Ground water table



Floatation effect of foundation in saturated soil



$$\sigma = \gamma_c D,$$

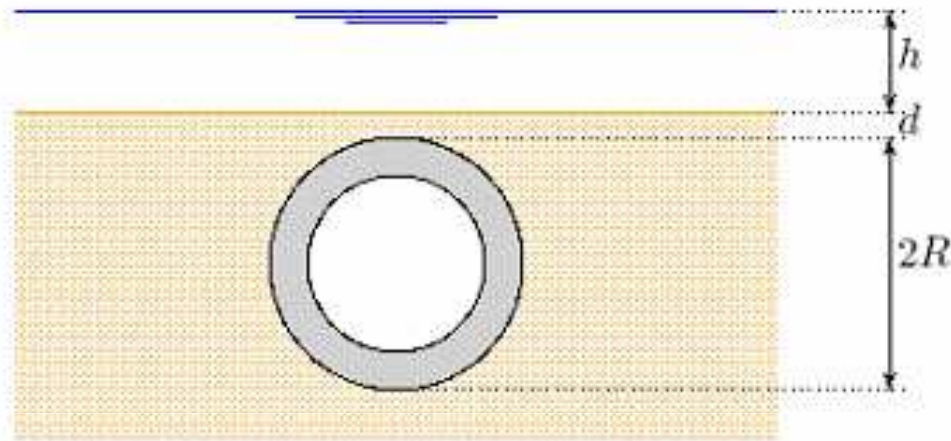
$$p = (h - d + D)\gamma_w,$$

Effective stress > 0

$$\sigma'_{zz} = \sigma_{zz} - p = \gamma_c D - \gamma_w(h - d + D) = (\gamma_c - \gamma_w)D - \gamma_w(h - d).$$

$$D > (h - d) \frac{\gamma_w}{\gamma_c - \gamma_w}.$$

Flotation of a Pipe



A pipe in the ground.

$$\gamma_p = G/\pi R^2.$$

$$F = \gamma_w \pi R^2,$$

G=weight of pipe/m

F=flotation force/m

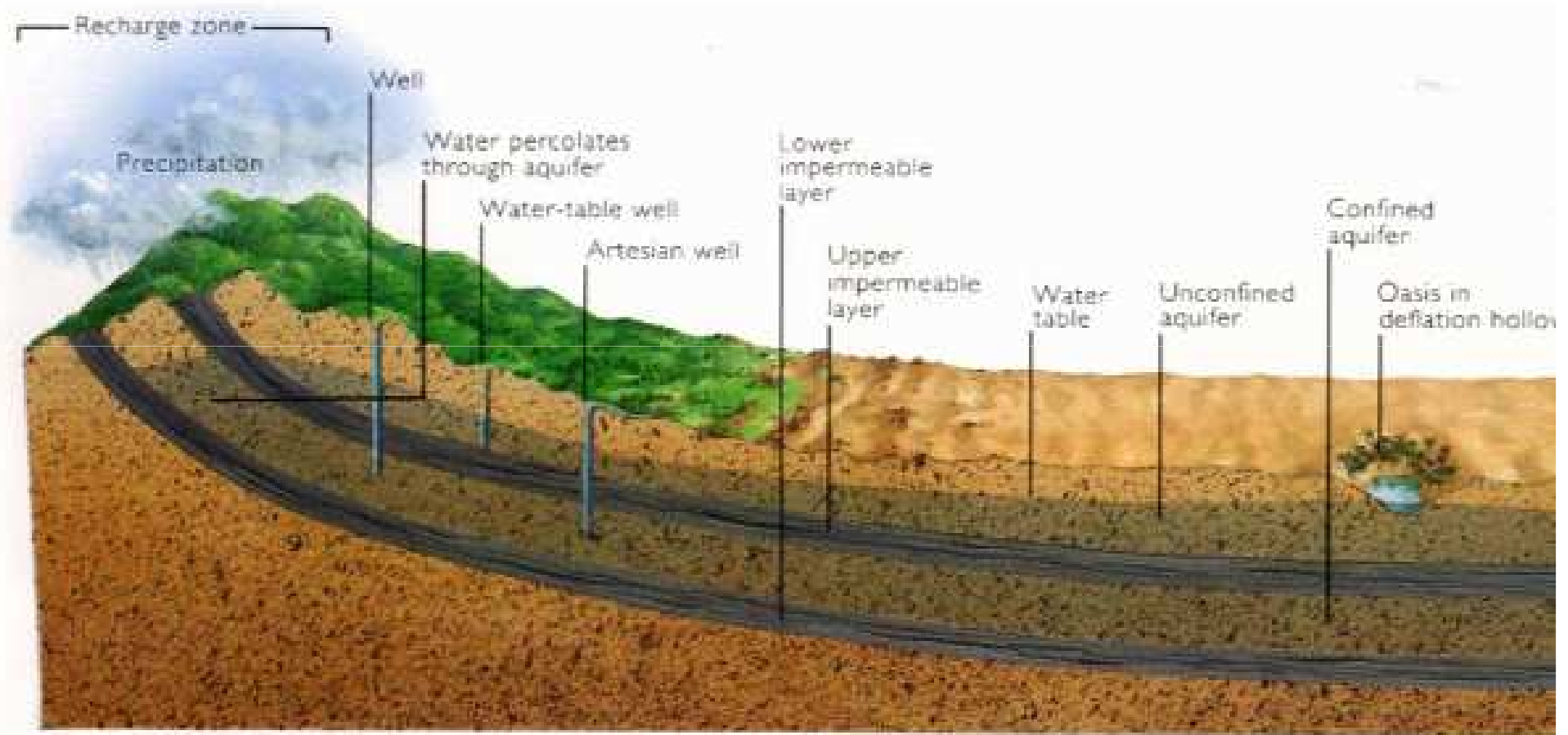
Stability criterion

$$\gamma_p > \gamma_w$$

FOS against floatation

$$F = \frac{\gamma_p}{\gamma_w}$$

Artesian water pressure in the ground



$$h = \zeta + \frac{u}{\gamma_w}$$

Effect of seepage

$$u = \gamma_w h - \gamma_w \zeta$$

$$\Rightarrow \frac{du}{dz} = \gamma_w \frac{dh}{dz} + \gamma_w$$

Hydrostatic case

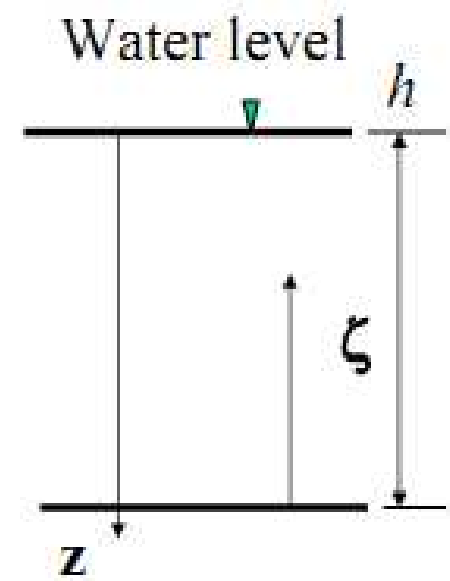
$$\sigma'_v = \gamma' z$$

$$\frac{d\sigma'_v}{dz} = \gamma'$$

$$\frac{du}{dz} = \gamma_w \pm i\gamma_w$$

+ve = upward flow

-ve = downward flow



ζ = elevation head

$$\sigma'_v = \sigma_v - u$$

$$\frac{d\sigma'_v}{dz} = -\frac{du}{dz}$$

Steady-state seepage case

$$\frac{d\sigma'_v}{dz} = \gamma' \pm i\gamma_w$$

$i\gamma_w$ = volumetric seepage force

+ve = downward flow
-ve = upward flow

Critical Hydraulic Gradient

$$\sigma'_v = z\gamma' - i_c z\gamma_w = 0$$

Zero effective stress

$$i_c = \frac{\gamma'}{\gamma_w}$$

Soil particles separate

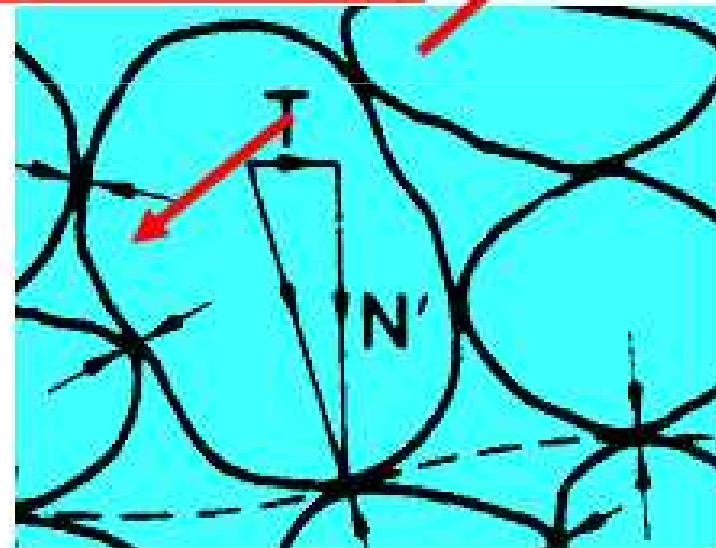
Quick condition

Boiling instability

Factor of safety against boiling instability

$$F_{exit} = \frac{i_c}{i_e}$$

Exit gradient in seepage



Example 2.3

$$q = 0.25 \text{ m}^3/\text{h per m}$$

$$N_d = 10$$

$$N_f = 6$$

$$k = q / \left(h \frac{N_f}{N_d} \right)$$
$$= \frac{0.25}{4.5 \times 6 / 10 \times 60^2} = 2.6 \times 10^{-5} \text{ m/s}$$

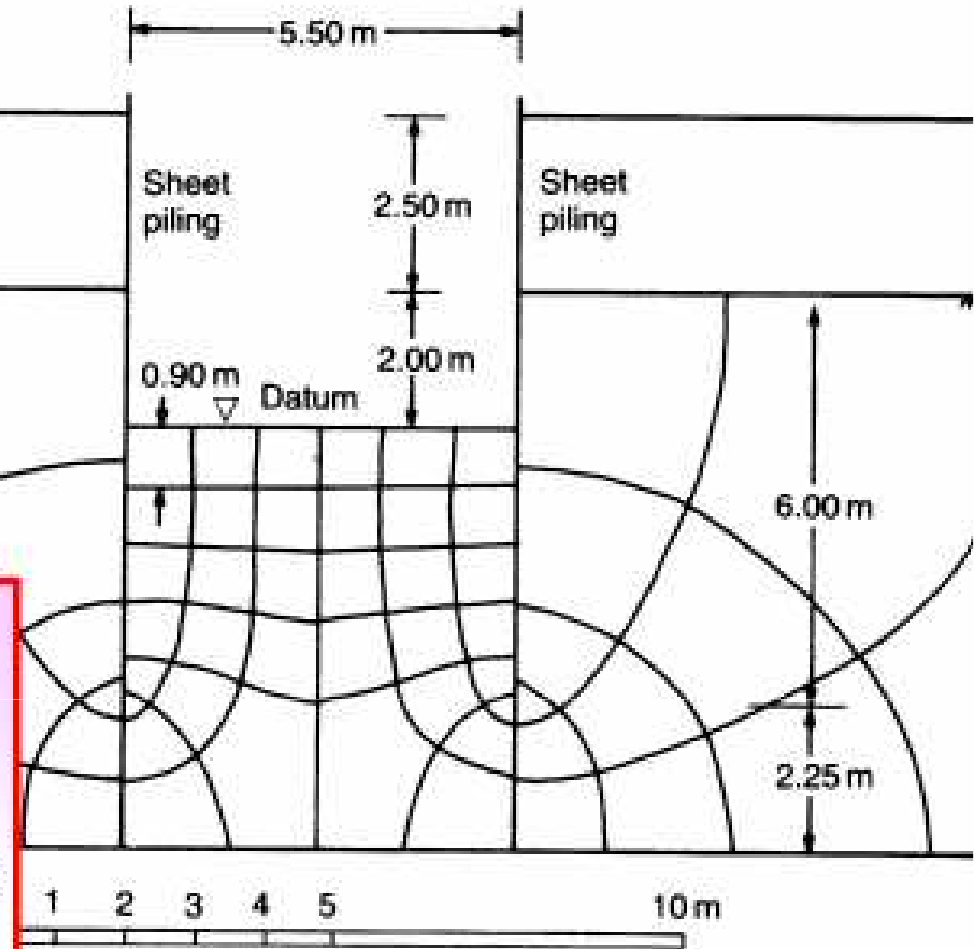


Fig. 2.10

EXIT GRADIENT

$$i_e = \frac{\Delta h}{\Delta s} = \frac{4.5}{10 \times 0.9} = 0.5$$

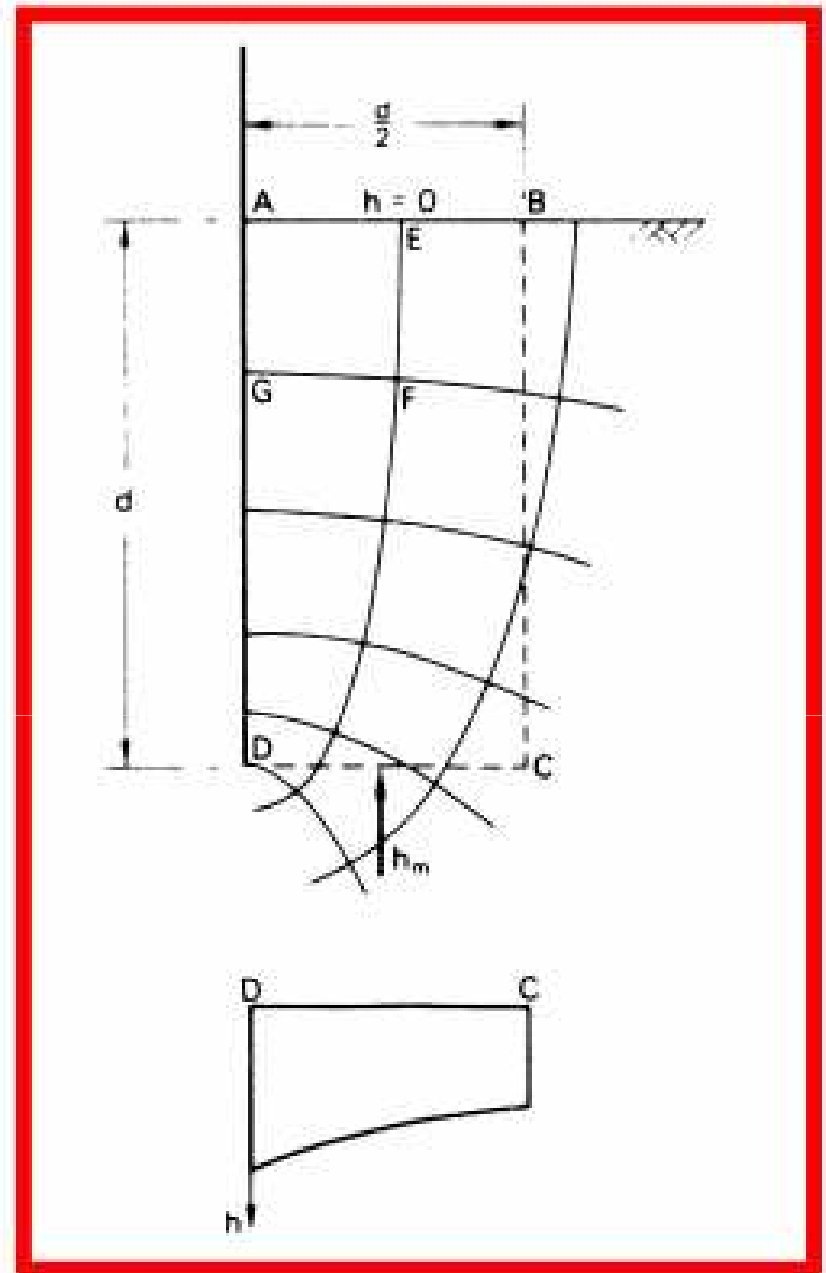
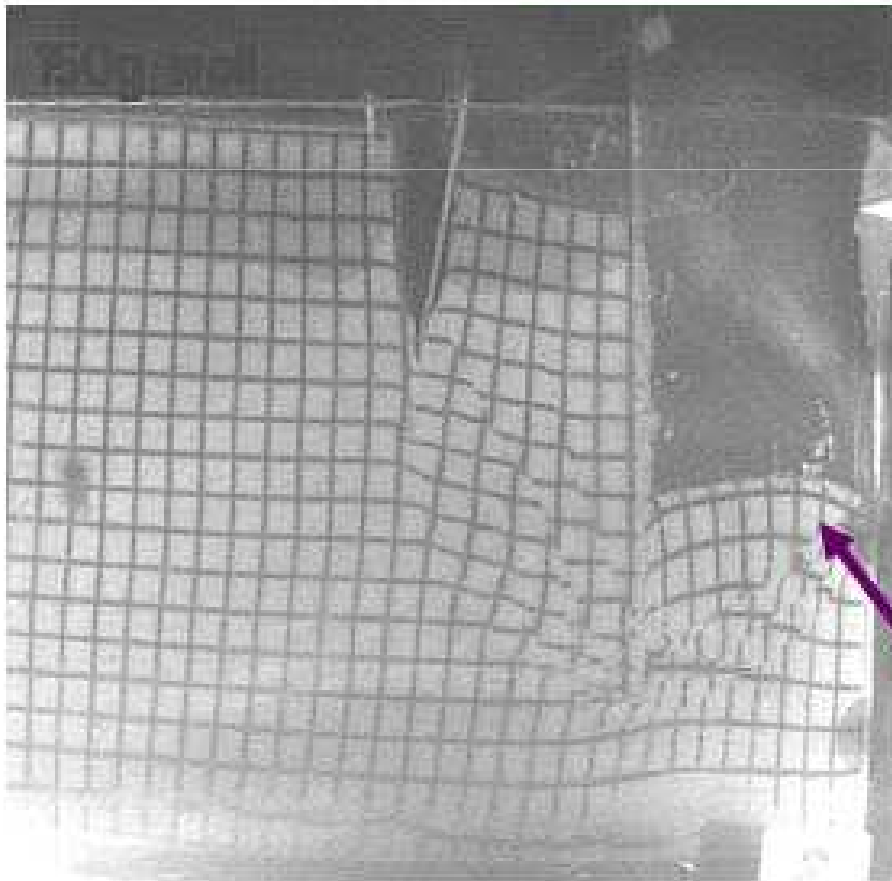
Factor of safety against boiling

$$F_{exit} = \frac{i_c}{i_e} \approx \frac{1}{0.5} = 2$$

Heave Instability

$$i_m = \frac{h_m}{d}$$

$$F_{heave} = \frac{i_c}{i_m}$$



Soil moves up (heave)

Summary: 2 types of instabilities

Factor of safety against boiling

$$i_c = \frac{\gamma'}{\gamma_w}$$

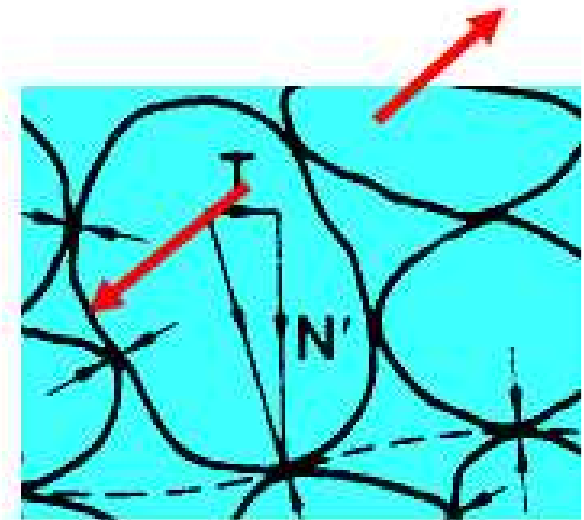
$$F_{exit} = \frac{\text{critical hydraulic gradient}}{\text{exit gradient}} = \frac{i_c}{i_e}$$

Factor of safety against heaving

$$F_{heave} = \frac{\text{critical hydraulic gradient}}{\text{hydraulic gradient for soil block}} = \frac{i_c}{i_m}$$

Summary:

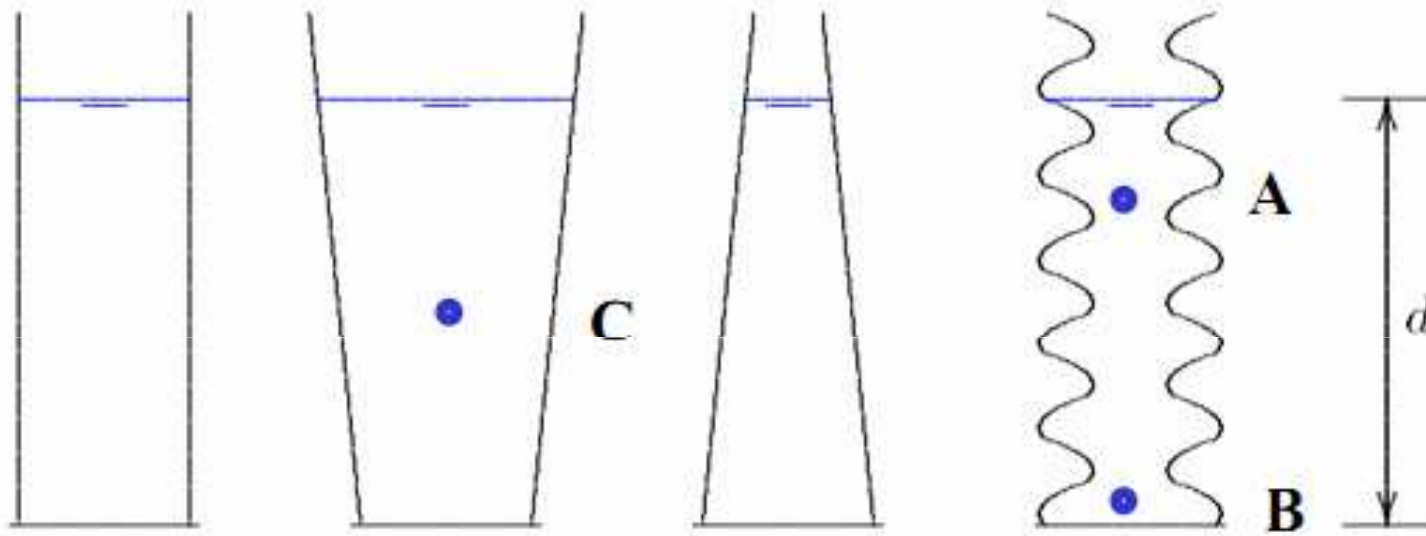
$$\text{critical hydraulic gradient} = i_c = \frac{\gamma'}{\gamma_w}$$



When hydraulic gradient equals the critical hydraulic gradient, effective stress will become zero. Thus, soil particles may separate!! The soil mass is no longer a solid, or it is called unstable or “instability”.

Total head within a continuous water mass is always constant

$$h_T^A = h_T^B = h_T^C$$



Hydrostatic water pressure depends upon depth only.

**Total head change \Rightarrow energy loss
(when water flow across soil)**