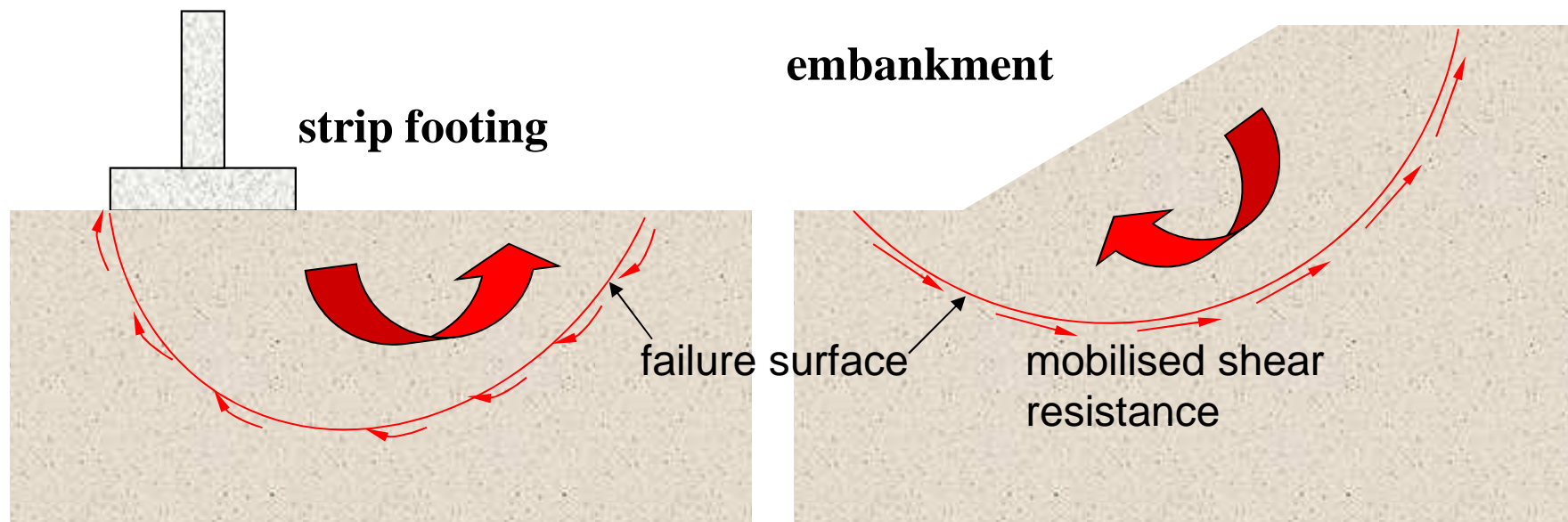


# **Shear Strength of Soils**

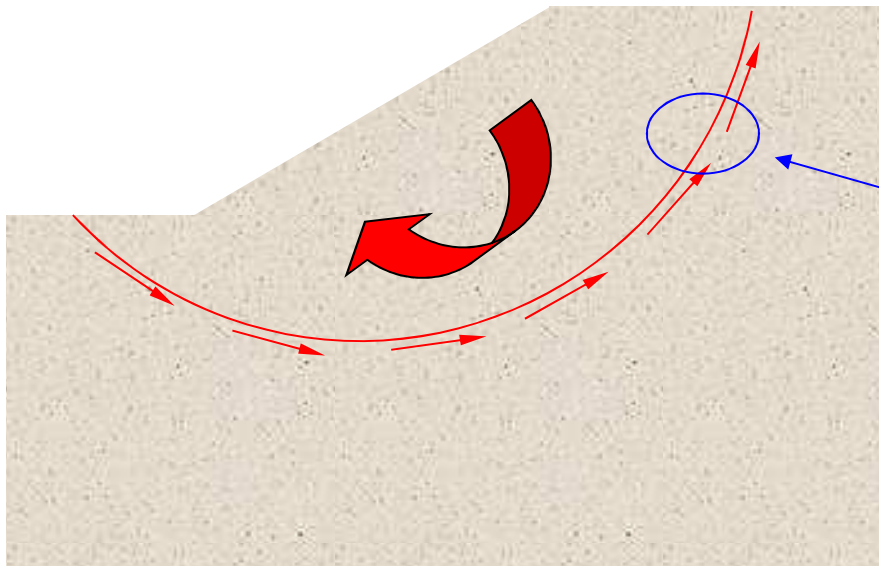
# Shear failure

Soils generally fail in shear



At failure, shear stress along the failure surface reaches the shear strength.

# Shear failure

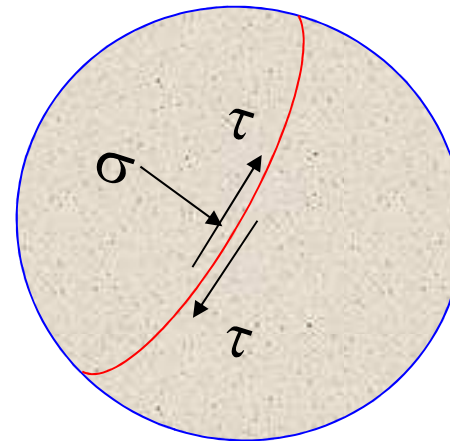
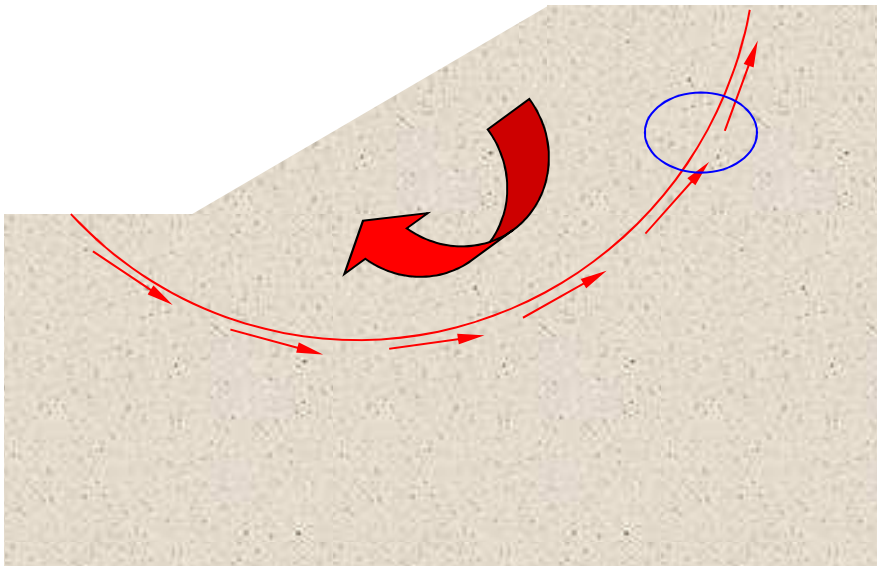


failure surface

The soil grains slide over each other along the failure surface.

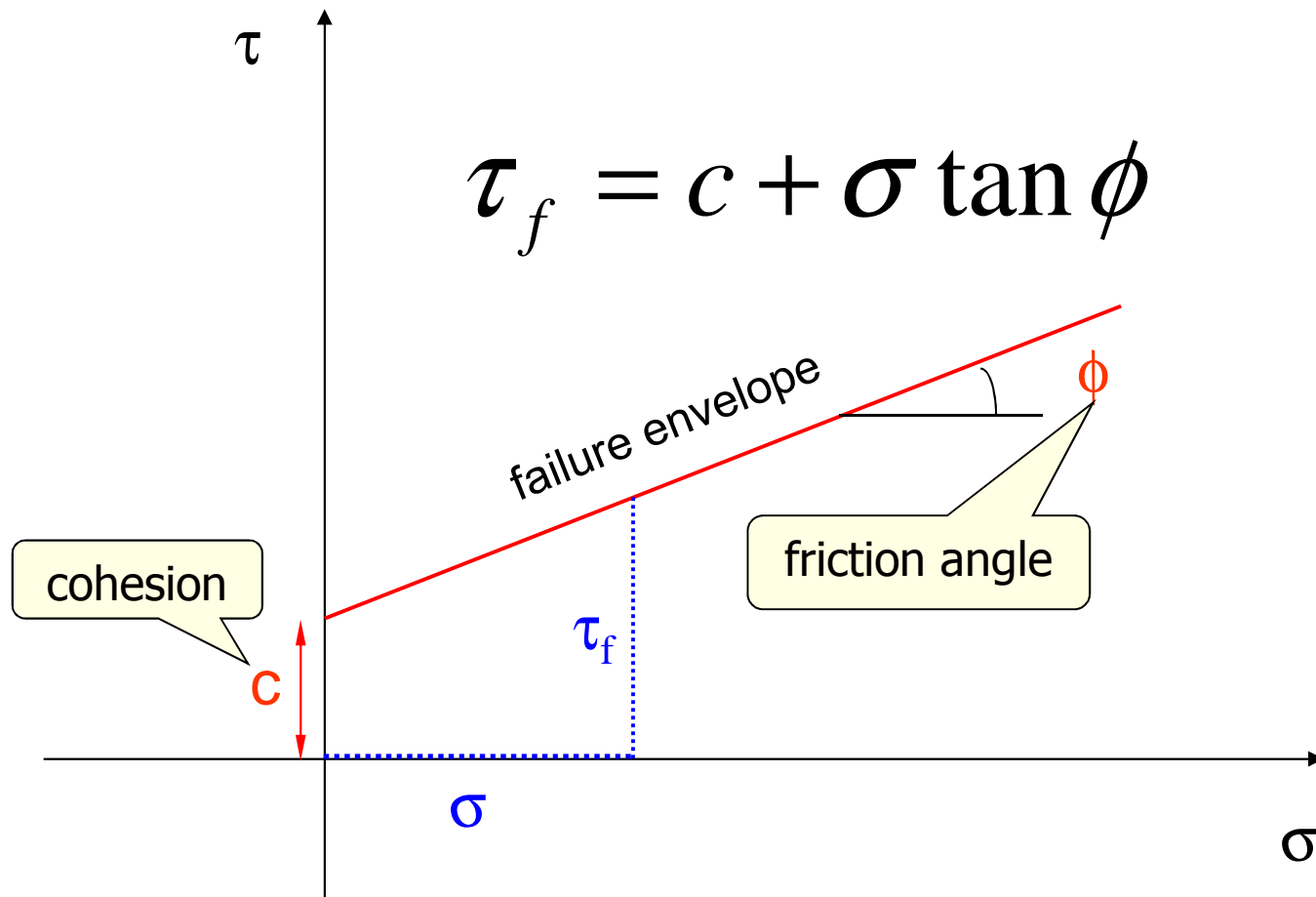
No crushing of individual grains.

# Shear failure



At failure, shear stress along the failure surface ( $\tau$ ) reaches the shear strength ( $\tau_f$ ).

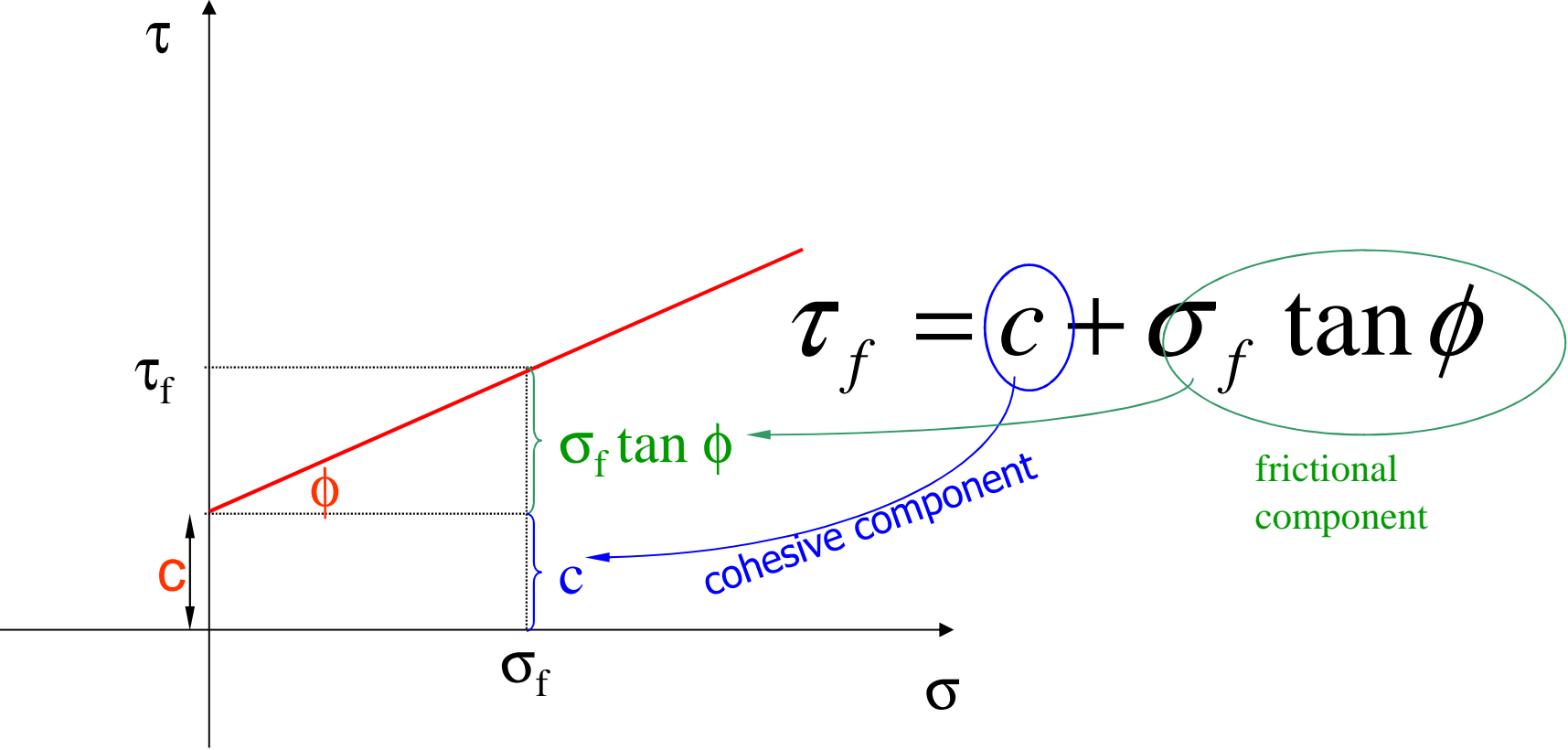
# Mohr-Coulomb Failure Criterion



$\tau_f$  is the maximum shear stress the soil can take without failure, under normal stress of  $\sigma$ .

# Mohr-Coulomb Failure Criterion

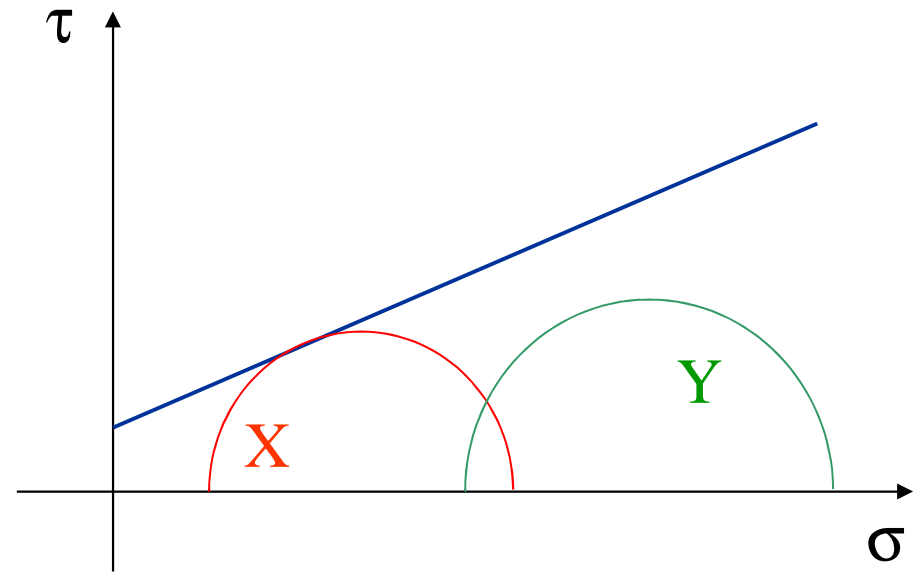
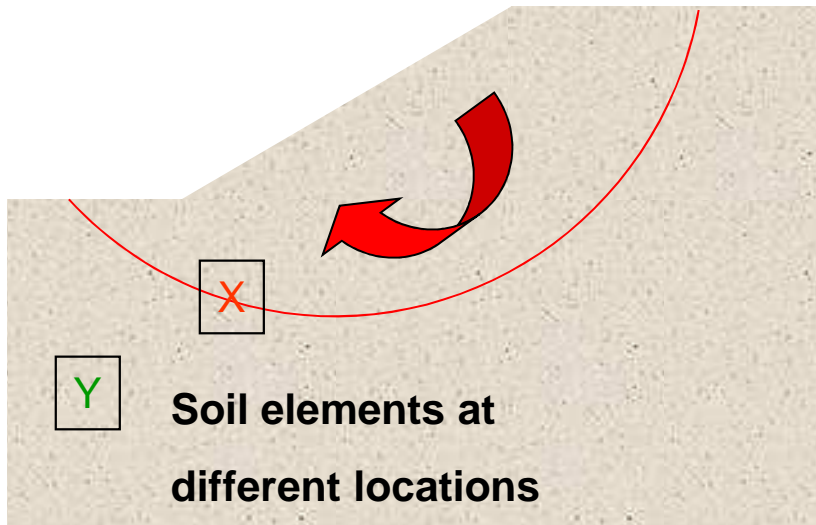
Shear strength consists of two components:  
**cohesive** and **frictional**.



➤ **c and  $\phi$  are measures of shear strength.**

➤ **Higher the values, higher the shear strength.**

# Mohr Circles & Failure Envelope

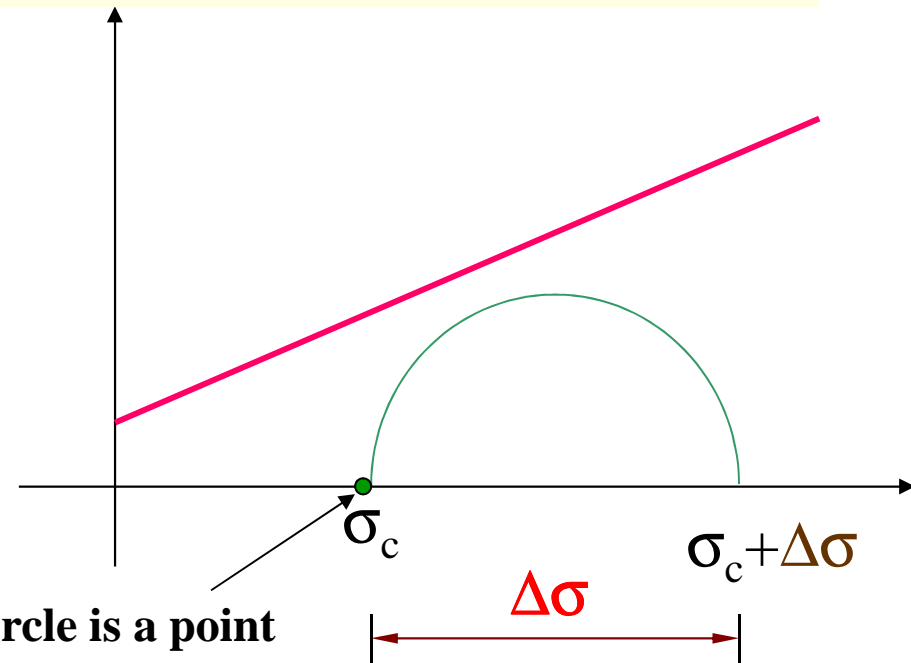
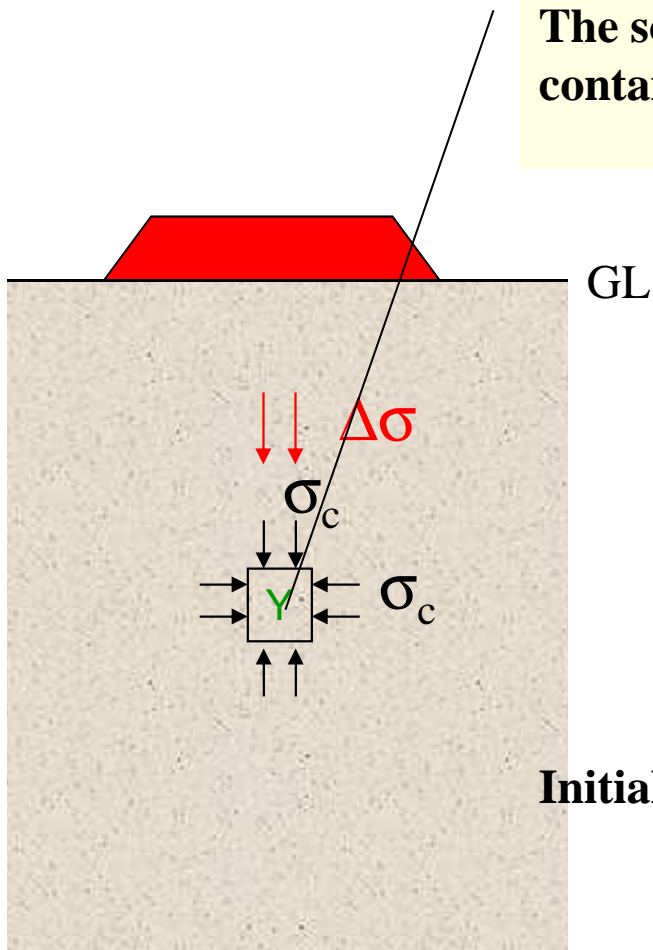


X  $\sim$  failure

Y  $\sim$  stable

# Mohr Circles & Failure Envelope

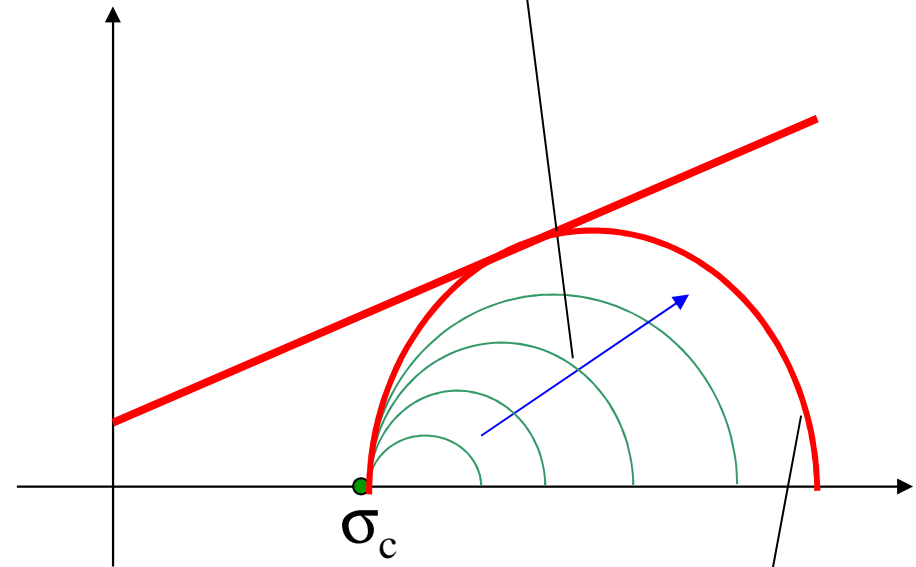
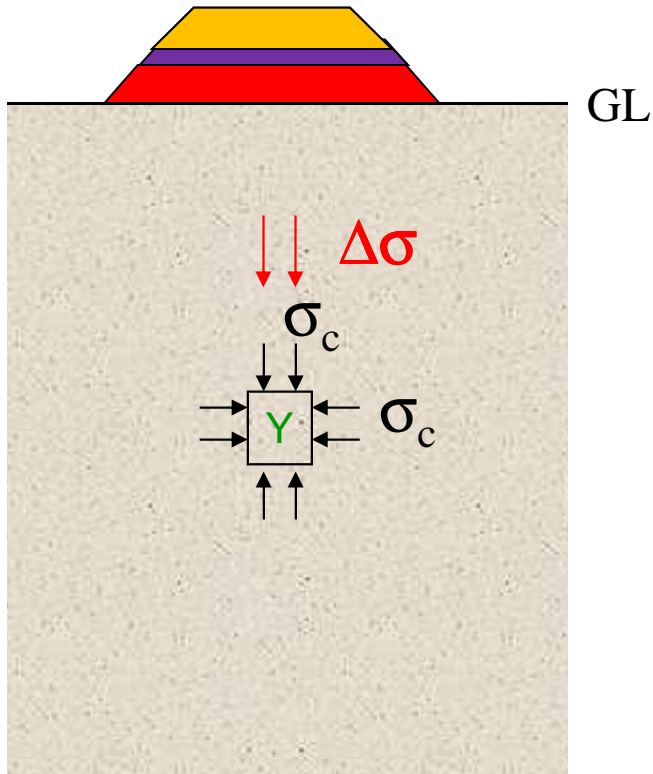
The soil element does not fail if the Mohr circle is contained within the envelope



Initially, Mohr circle is a point

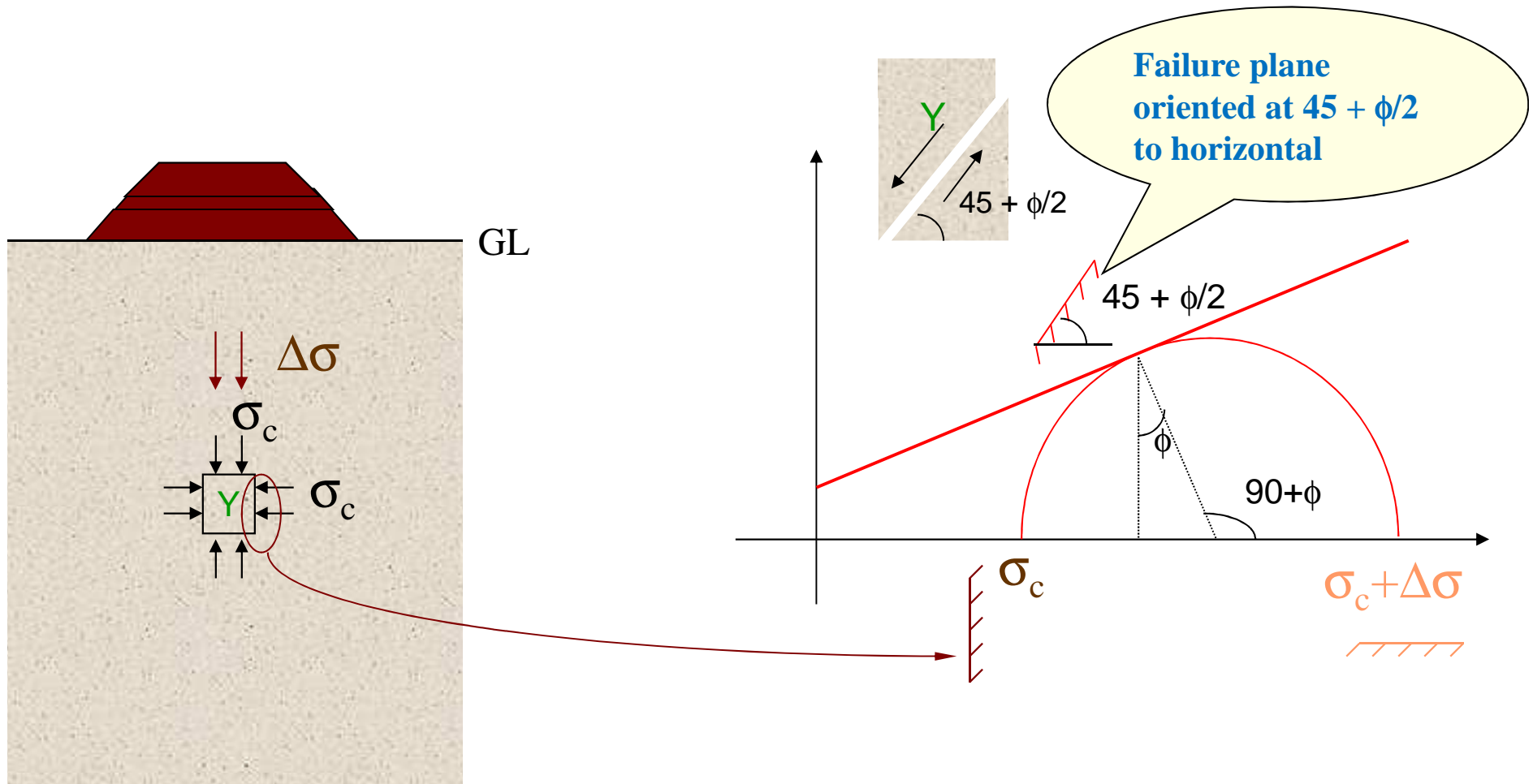
# Mohr Circles & Failure Envelope

As loading progresses, Mohr circle becomes larger...

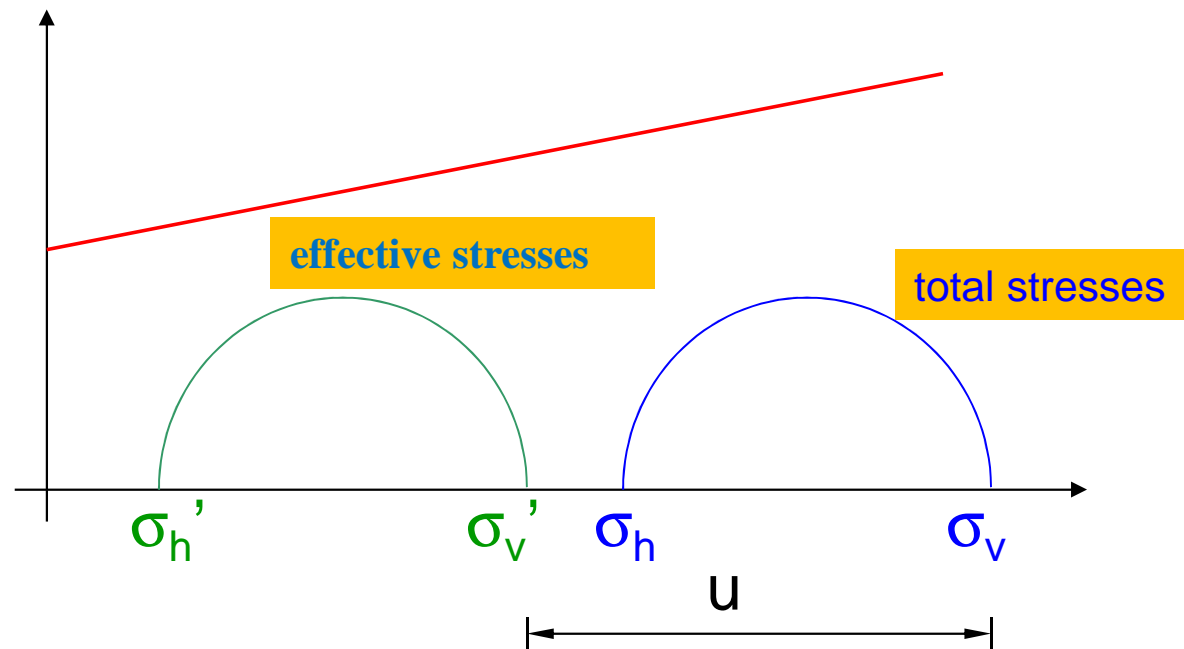
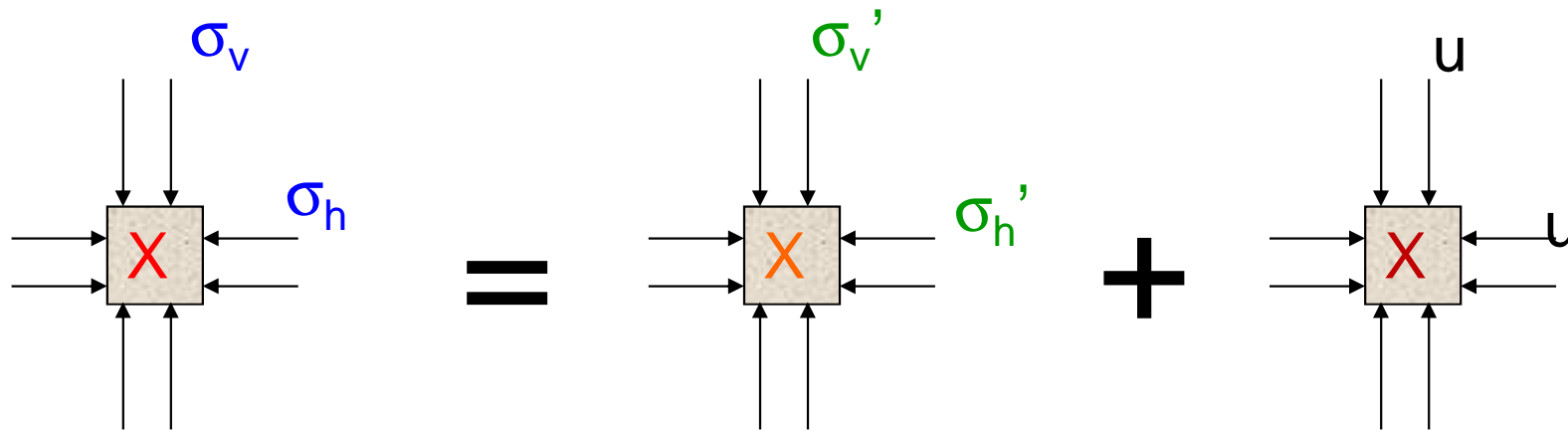


.. and finally failure occurs when Mohr circle touches the envelope

# Orientation of Failure Plane

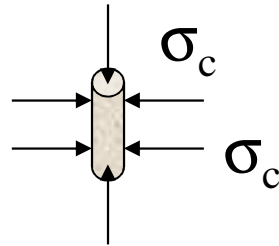
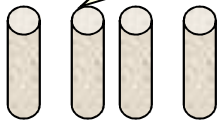


# Mohr circles in terms of $\sigma$ & $\sigma'$

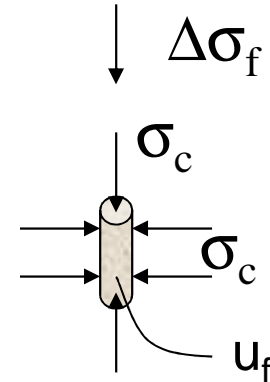


# Envelopes in terms of $\sigma$ & $\sigma'$

Identical specimens initially subjected to different isotropic stresses ( $\sigma_c$ ) and then loaded axially to failure



Initially...

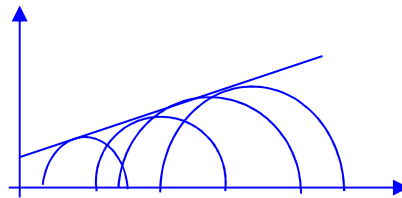


Failure

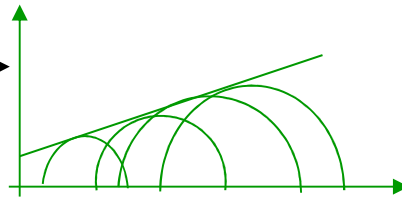
At failure,

$$\sigma_3 = \sigma_c; \quad \sigma_1 = \sigma_c + \Delta\sigma_f$$

$$\sigma_3' = \sigma_3 - u_f; \quad \sigma_1' = \sigma_1 - u_f$$



$c, \phi$   
in terms of  $\sigma$

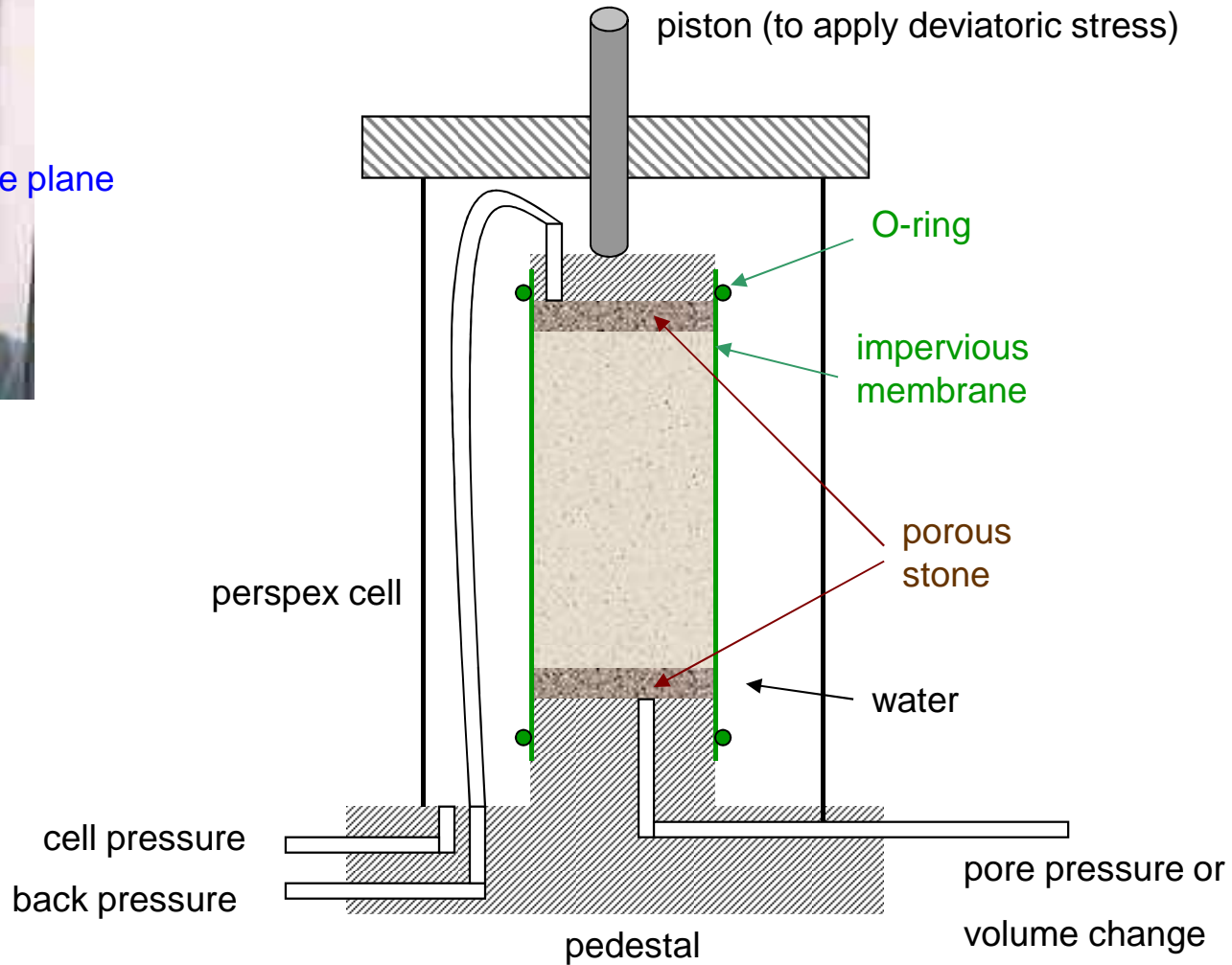


$c', \phi'$   
in terms of  $\sigma'$

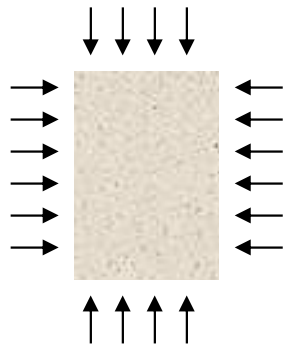
# Triaxial Test Apparatus



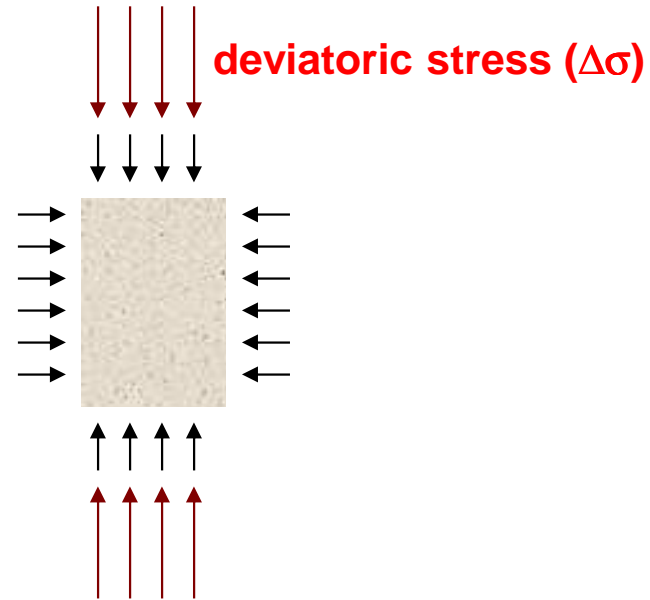
soil sample at failure



# Types of Triaxial Tests



Under all-around cell pressure  $\sigma_c$



Shearing (loading)

Is the drainage valve open?

yes

no

**C**onsolidated  
sample

**U**nconsolidated  
sample

Is the drainage valve open?

yes

no

**D**rained  
loading

**U**ndrained  
loading

# Types of Triaxial Tests

Depending on whether drainage is allowed or not during

❖ initial isotropic cell pressure application, and

❖ shearing,

there are three special types of triaxial tests that have practical significances. They are:

**Consolidated Drained (CD) test**

**Consolidated Undrained (CU) test**

**Unconsolidated Undrained (UU) test**

For unconsolidated  
undrained test, in  
terms of total  
stresses,  $\phi_u = 0$

Granular soils have  
no cohesion.  
 $c = 0$  &  $c' = 0$

For normally consolidated  
clays,  $c' = 0$  &  $c = 0$ .

# CD, CU and UU Triaxial Tests

## Consolidated Drained (CD) Test

- ❖ no excess pore pressure throughout the test
- ❖ very slow shearing to avoid build-up of pore pressure

Can be days!  
∴ not desirable

- ❖ **gives  $c'$  and  $\phi'$**

Use  $c'$  and  $\phi'$  for analysing fully drained situations (e.g., ***long term stability, very slow loading***)

# CD, CU and UU Triaxial Tests

## Consolidated Undrained (CU) Test

- ❖ pore pressure develops during shear

Measure →  $\sigma'$

- ❖ gives  $c'$  and  $\phi'$
- ❖ faster than CD ( $\therefore$  preferred way to find  $c'$  and  $\phi'$ )

# CD, CU and UU Triaxial Tests

## Unconsolidated Undrained (UU) Test

- ❖ pore pressure develops during shear

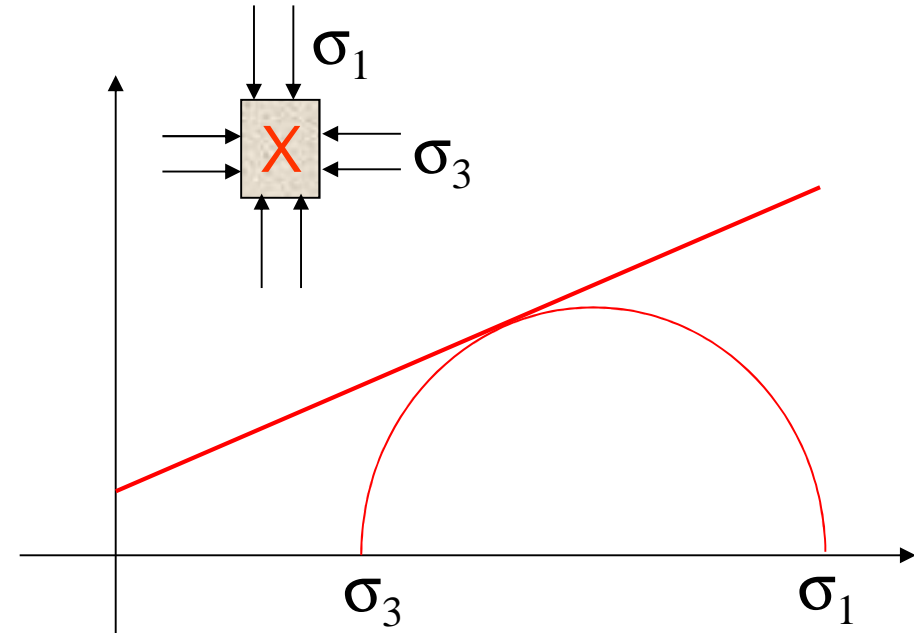
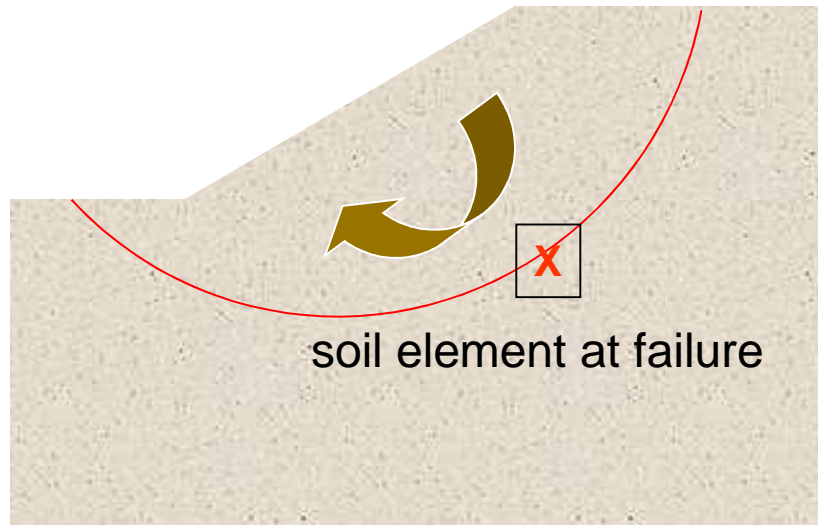
Not measured  
 $\therefore \sigma'$  unknown

= 0; i.e., failure envelope is horizontal

- ❖ analyse in terms of  $\sigma \rightarrow$  gives  $c_u$  and  $\phi_u$
- ❖ very quick test

Use  $c_u$  and  $\phi_u$  for analysing undrained situations (e.g., short term stability, quick loading)

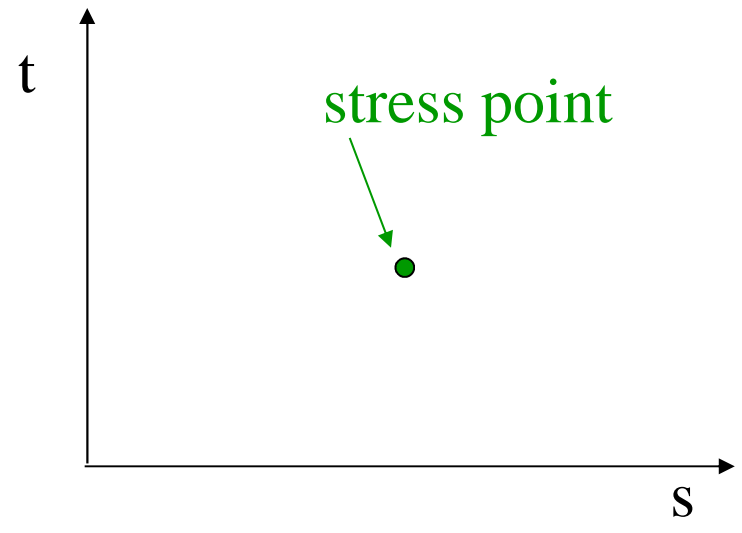
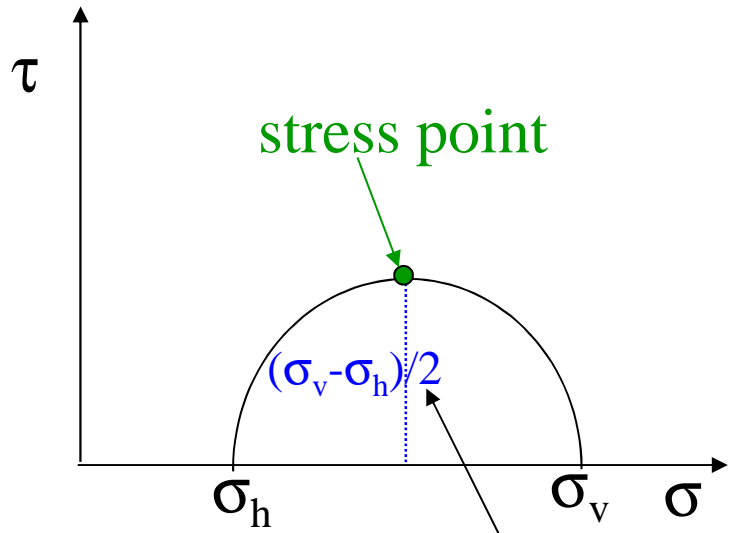
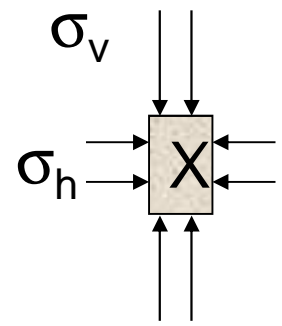
# $\sigma_1$ - $\sigma_3$ Relation at Failure



$$\sigma_1 = \sigma_3 \tan^2(45 + \phi / 2) + 2c \tan(45 + \phi / 2)$$

$$\sigma_3 = \sigma_1 \tan^2(45 - \phi / 2) - 2c \tan(45 - \phi / 2)$$

# Stress Point

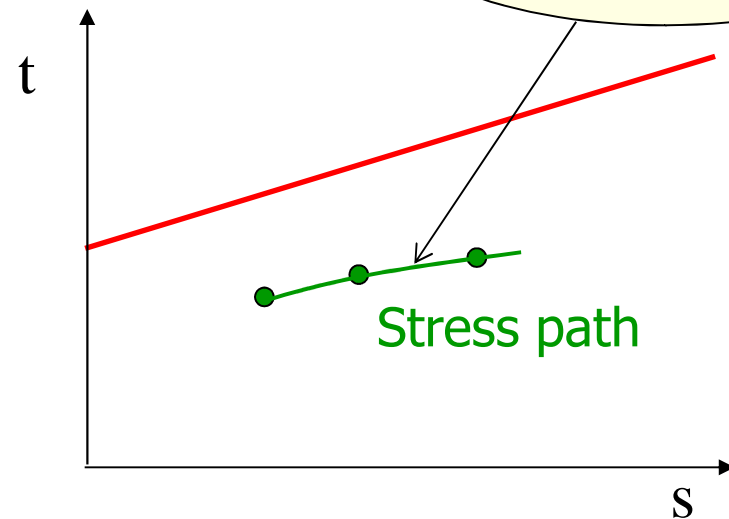
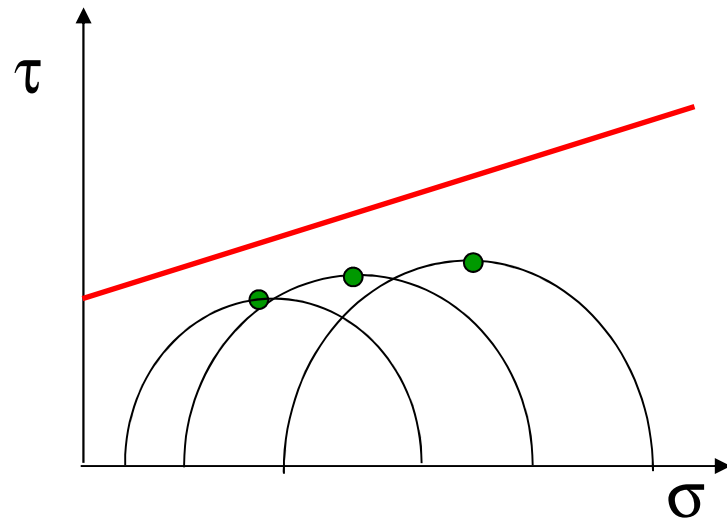


$$s = \frac{\sigma_v + \sigma_h}{2}$$

$$t = \frac{\sigma_v - \sigma_h}{2}$$

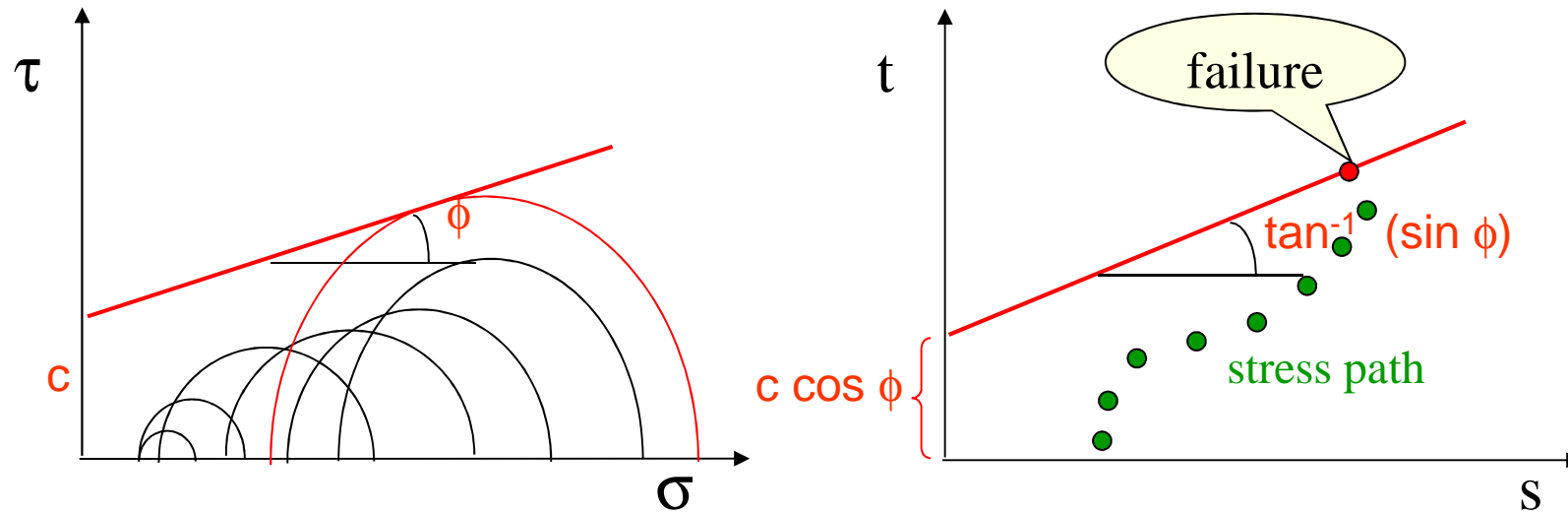
# Stress Path

During loading...



**Stress path is a convenient way to keep track of the progress in loading with respect to failure envelope.**

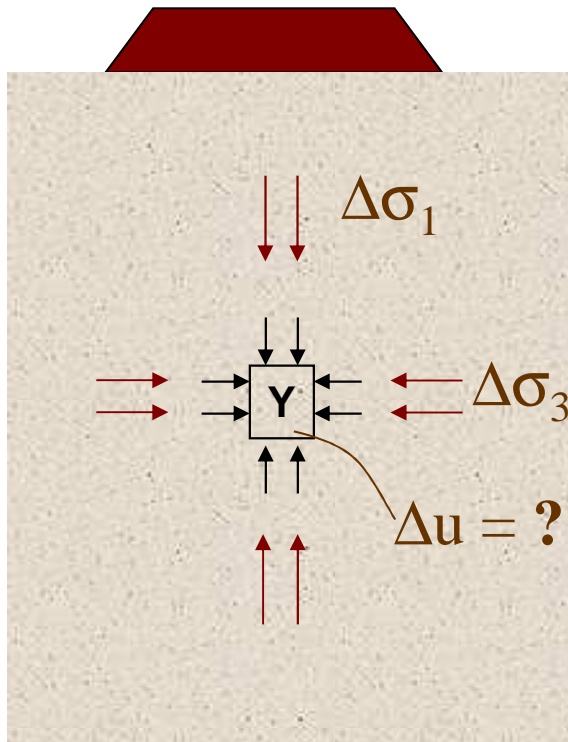
# Failure Envelopes



**During loading (shearing)....**

# Pore Pressure Parameters

A simple way to estimate the pore pressure change in **undrained loading**, in terms of total stress changes ~ after Skempton (1954)



$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

Skempton's pore pressure parameters **A** and **B**

# Pore Pressure Parameters

## B-parameter

$$B = f(\text{saturation,..})$$

For saturated soils,  $B \approx 1$ .

## A-parameter at failure ( $A_f$ )

$$A_f = f(\text{OCR})$$

For normally consolidated clays  $A_f \approx 1$ .

For heavily overconsolidated clays  $A_f$  is negative.

# Consolidated- drained test (CD Test)

Total,  $\sigma$

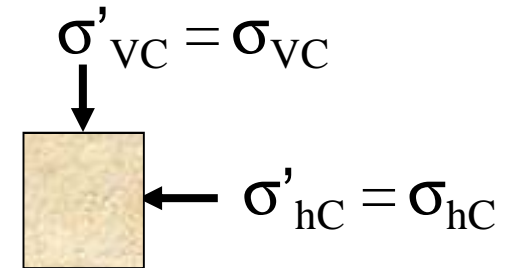
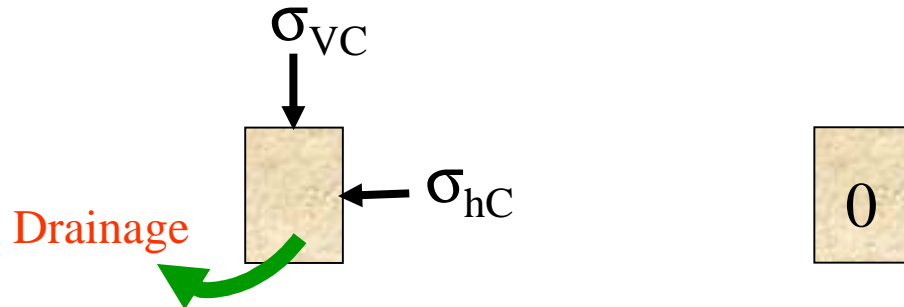
=

Neutral,  $u$

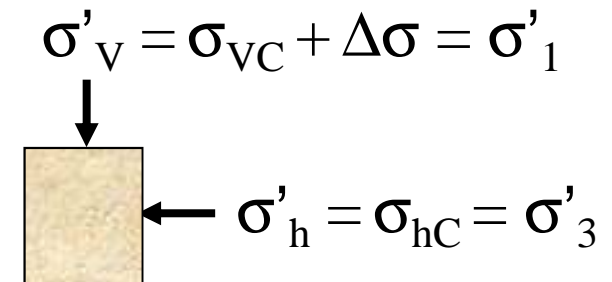
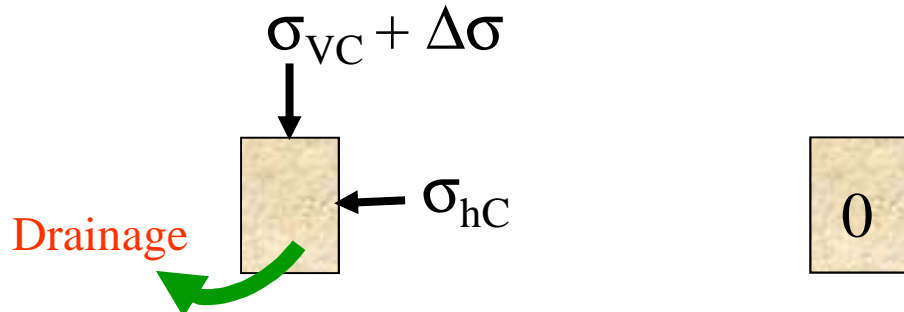
+

Effective,  $\sigma'$

Step 1: At the end of consolidation

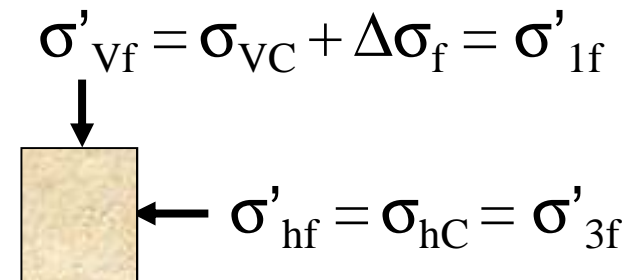
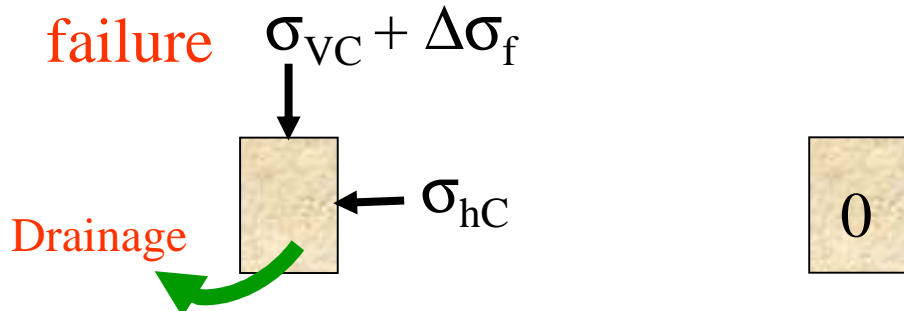


Step 2: During axial stress increase

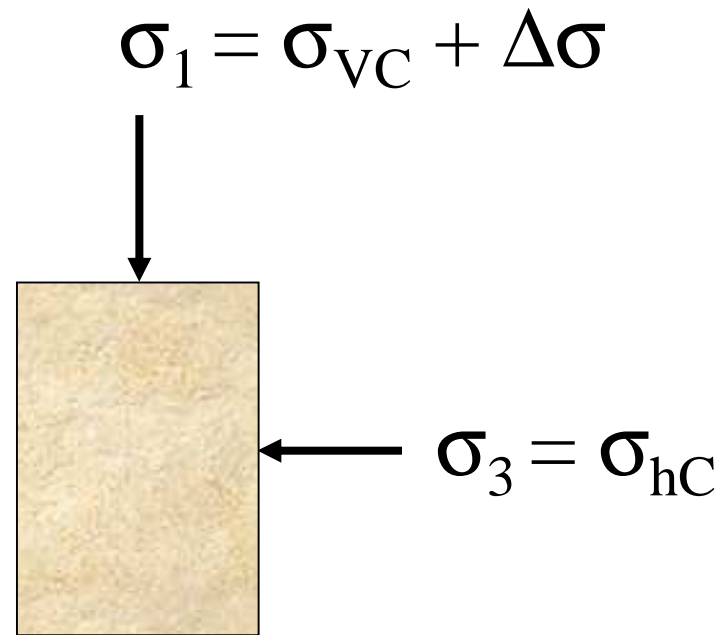


Step 3: At

failure

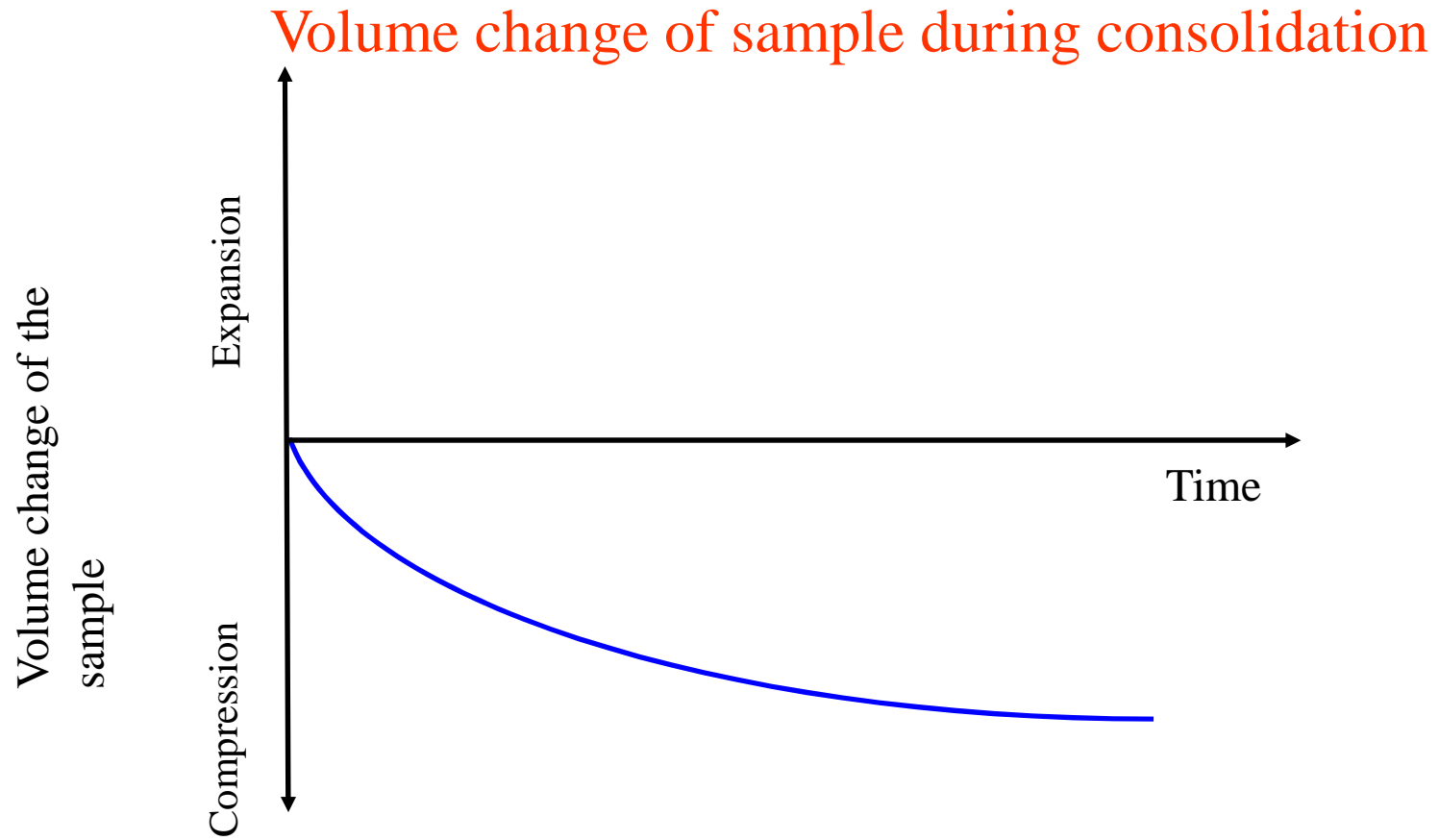


## Consolidated- drained test (CD Test)



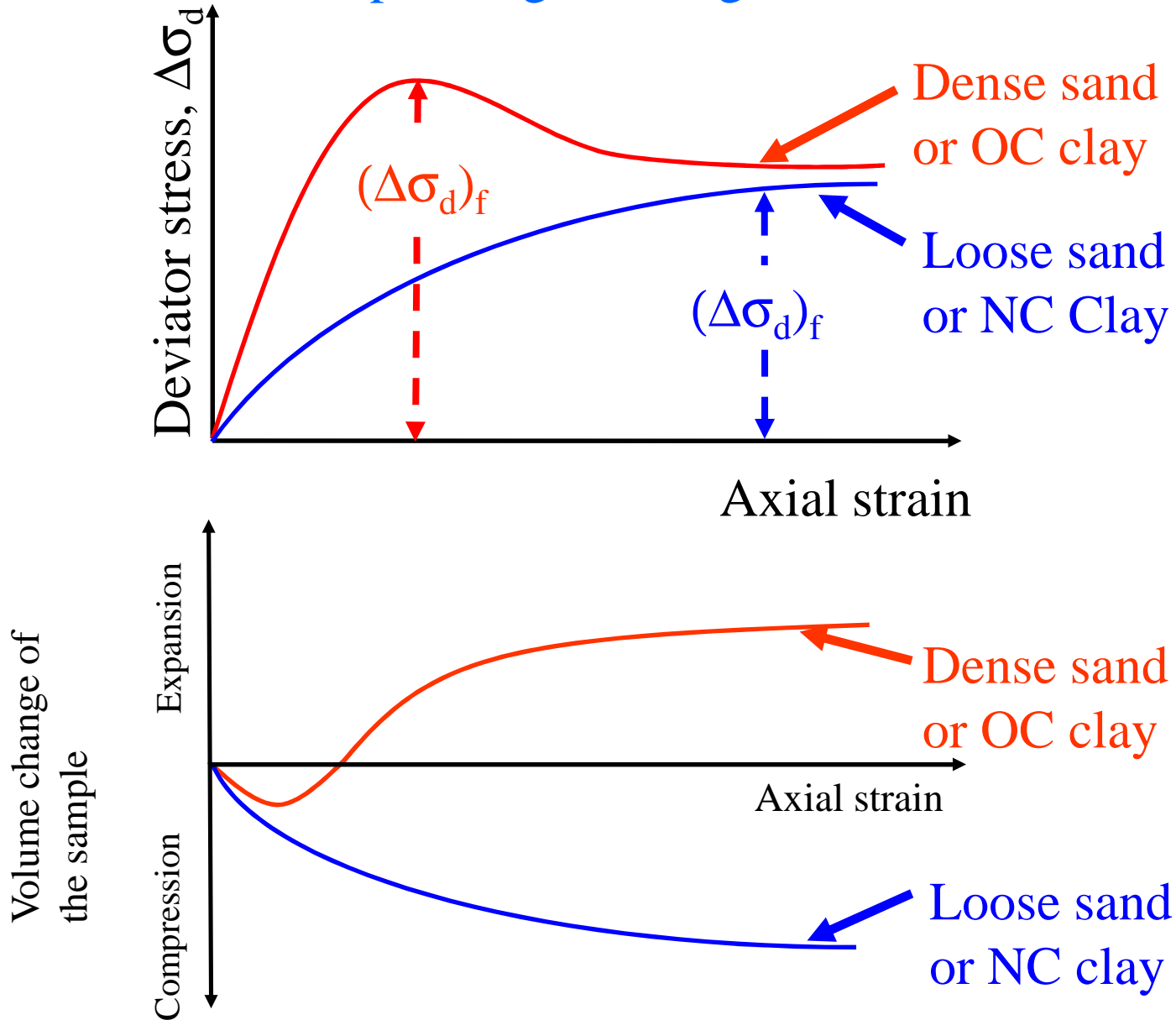
Deviator stress ( $q$  or  $\Delta\sigma_d$ ) =  $\sigma_1 - \sigma_3$

# Consolidated- drained test (CD Test)



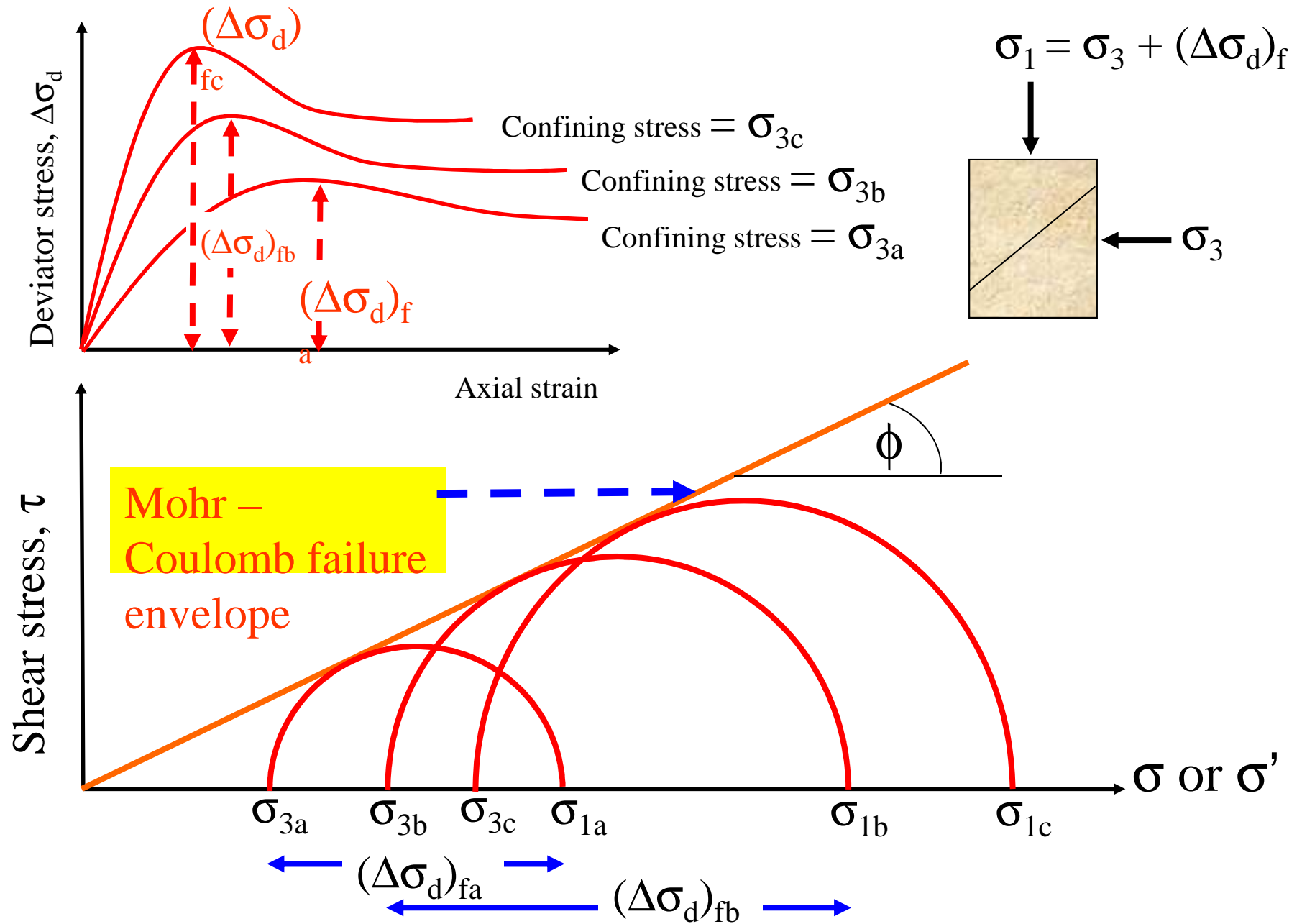
# Consolidated- drained test (CD Test)

## Stress-strain relationship during shearing



# CD tests

How to determine strength parameters  $c$  and  $\phi$



## CD tests

Strength parameters  $c$  and  $\phi$  obtained from CD tests

Since  $u = 0$  in CD tests,  $\sigma = \sigma'$

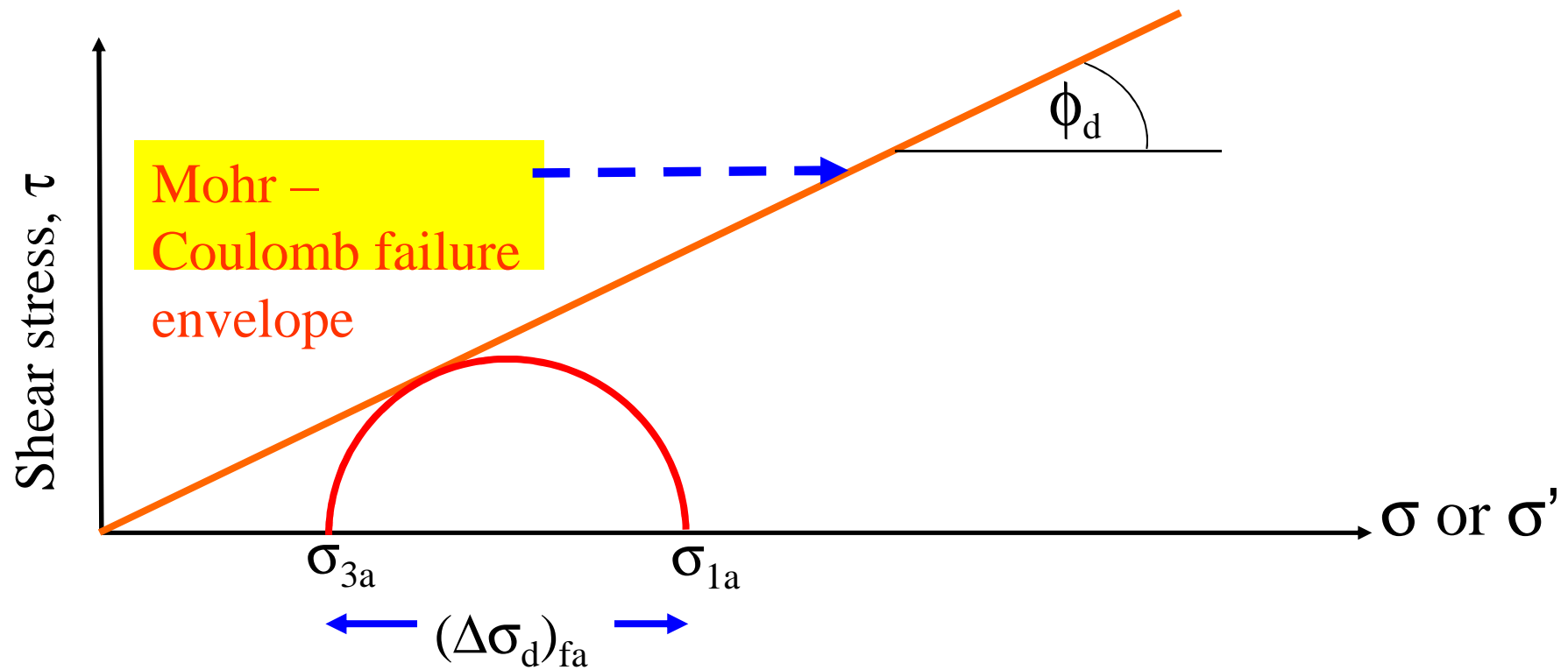
Therefore,  $c = c'$   
and  $\phi = \phi'$

$c_d$  and  $\phi_d$  are used to denote them

# CD tests

## Failure envelopes

For sand and NC Clay,  $c_d = 0$

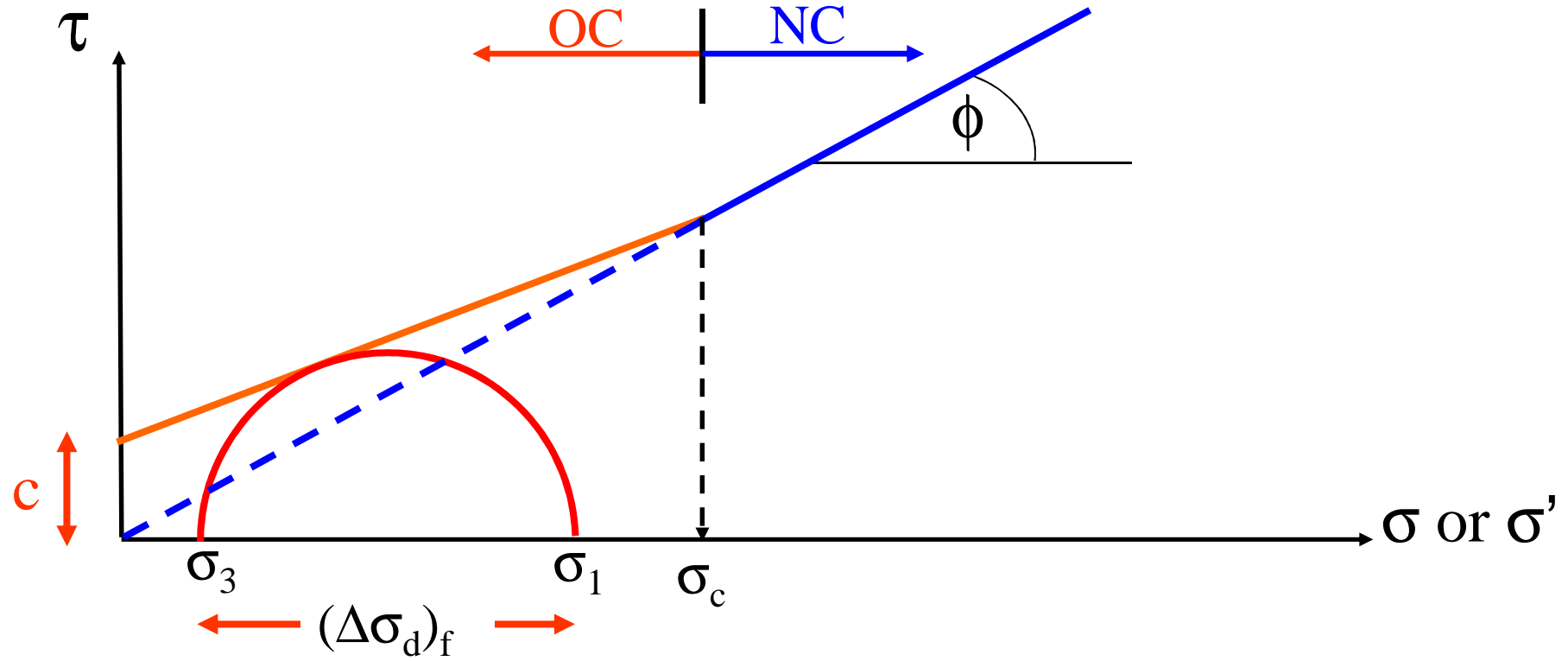


Therefore, one CD test would be sufficient to determine  $\phi_d$  of *sand or NC clay*

# CD tests

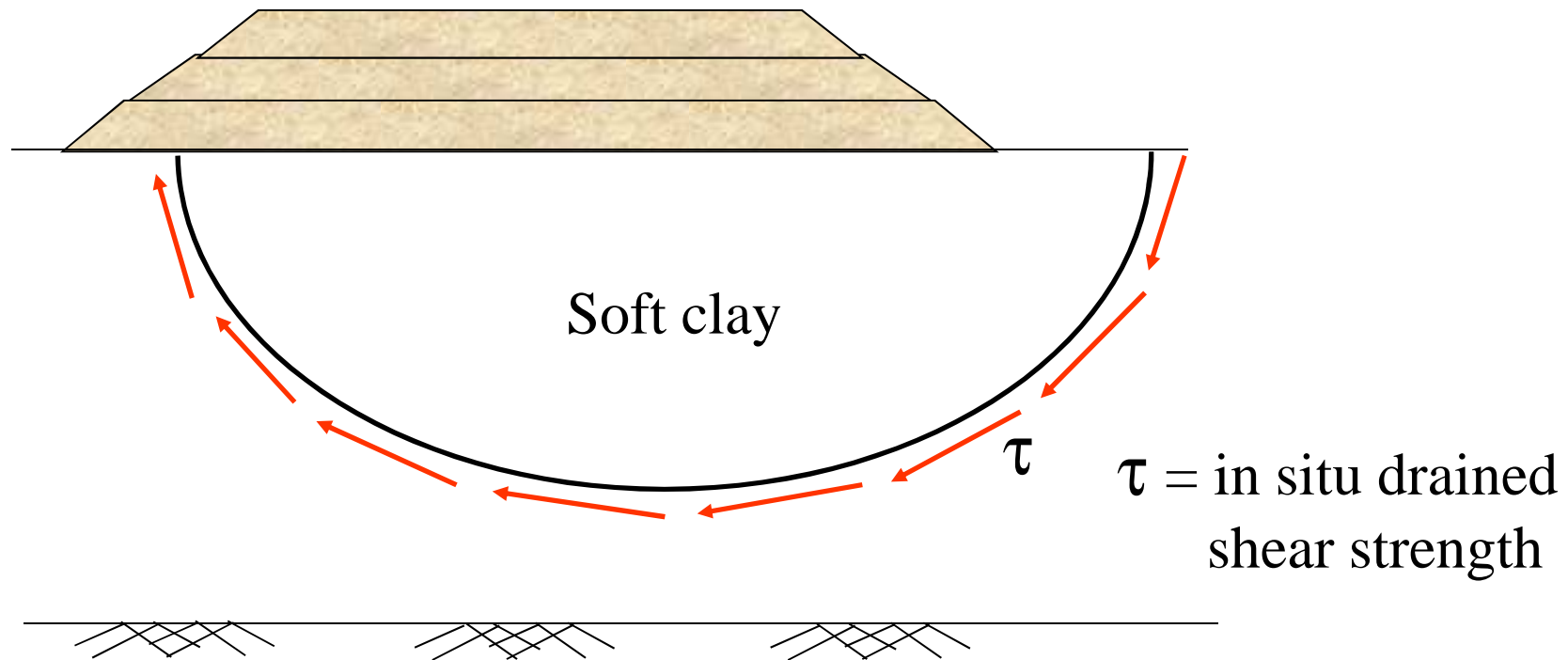
## Failure envelopes

For OC Clay,  $c_d \neq 0$



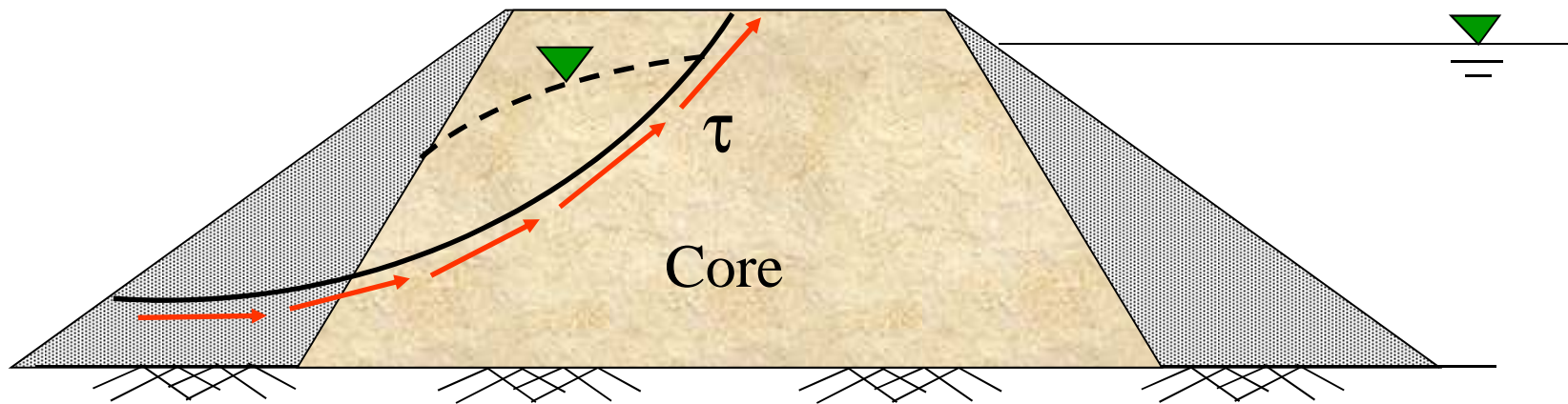
# Some practical applications of CD analysis for clays

1. Embankment constructed very slowly, in layers over a soft clay deposit



# Some practical applications of CD analysis for clays

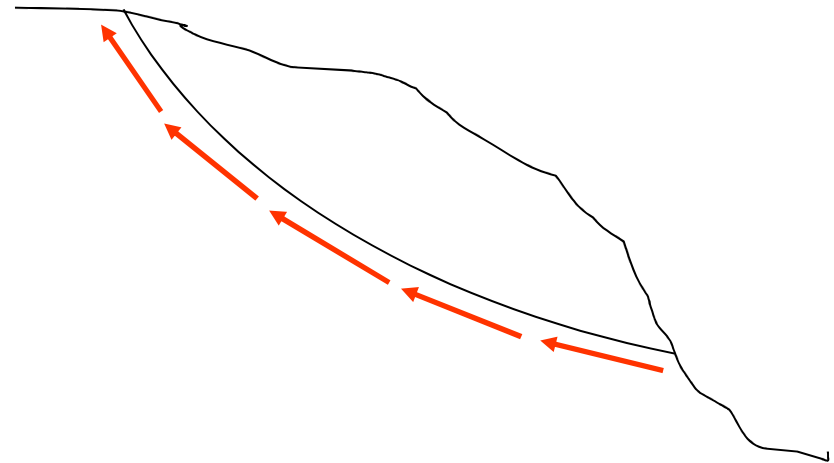
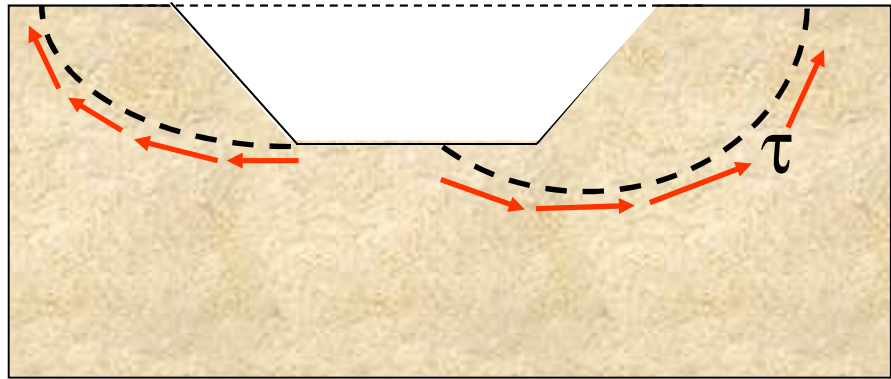
## 2. Earth dam with steady state seepage



$\tau$  = drained shear  
strength of clay core

# Some practical applications of CD analysis for clays

## 3. Excavation or natural slope in clay



$\tau$  = In situ drained shear strength

Note: CD test simulates the long term condition in the field. Thus,  $c_d$  and  $\phi_d$  should be used to evaluate the long term behavior of soils

# Consolidated- Undrained test (CU Test)

Total,  $\sigma$

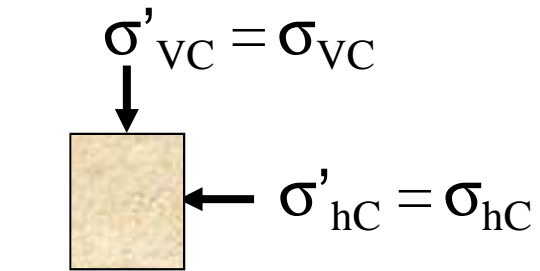
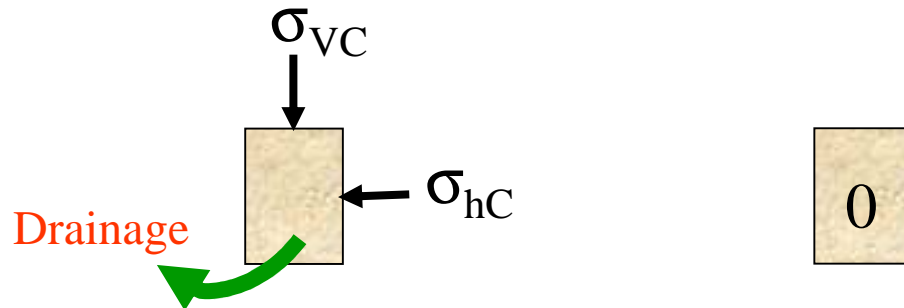
=

Neutral,  $u$

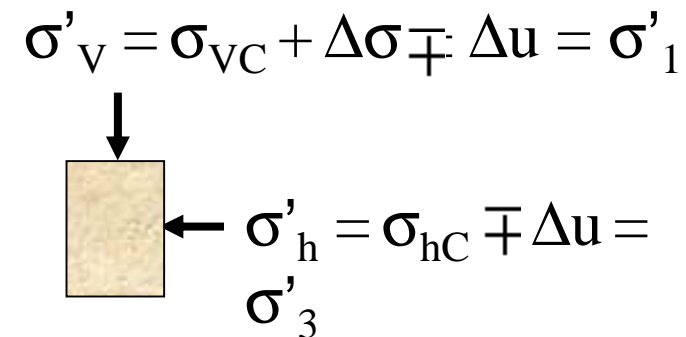
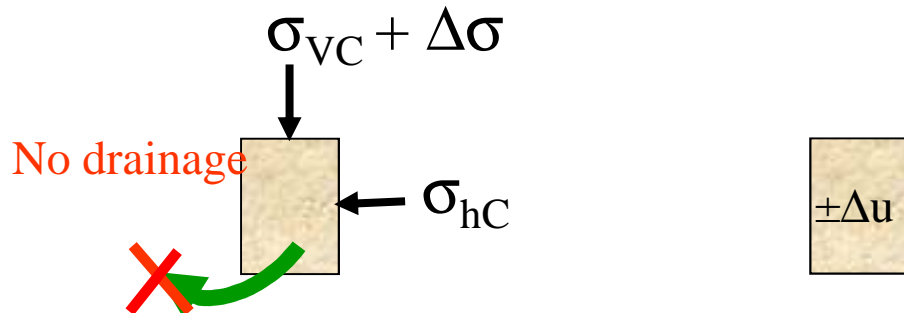
+

Effective,  $\sigma'$

Step 1: At the end of consolidation

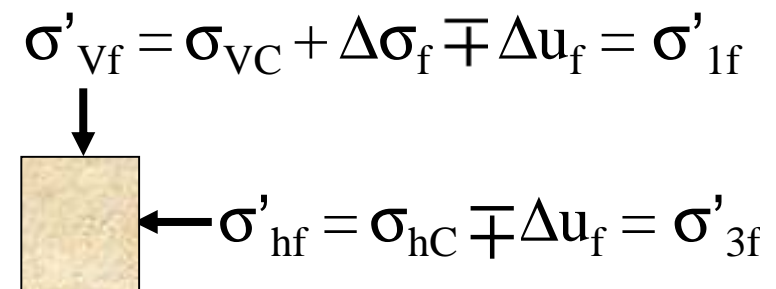
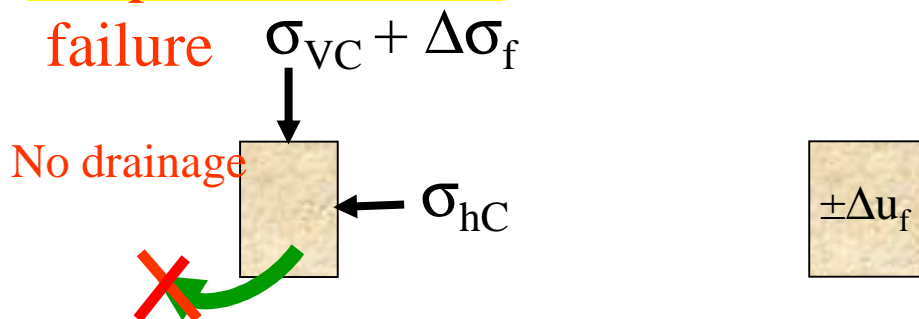


Step 2: During axial stress increase



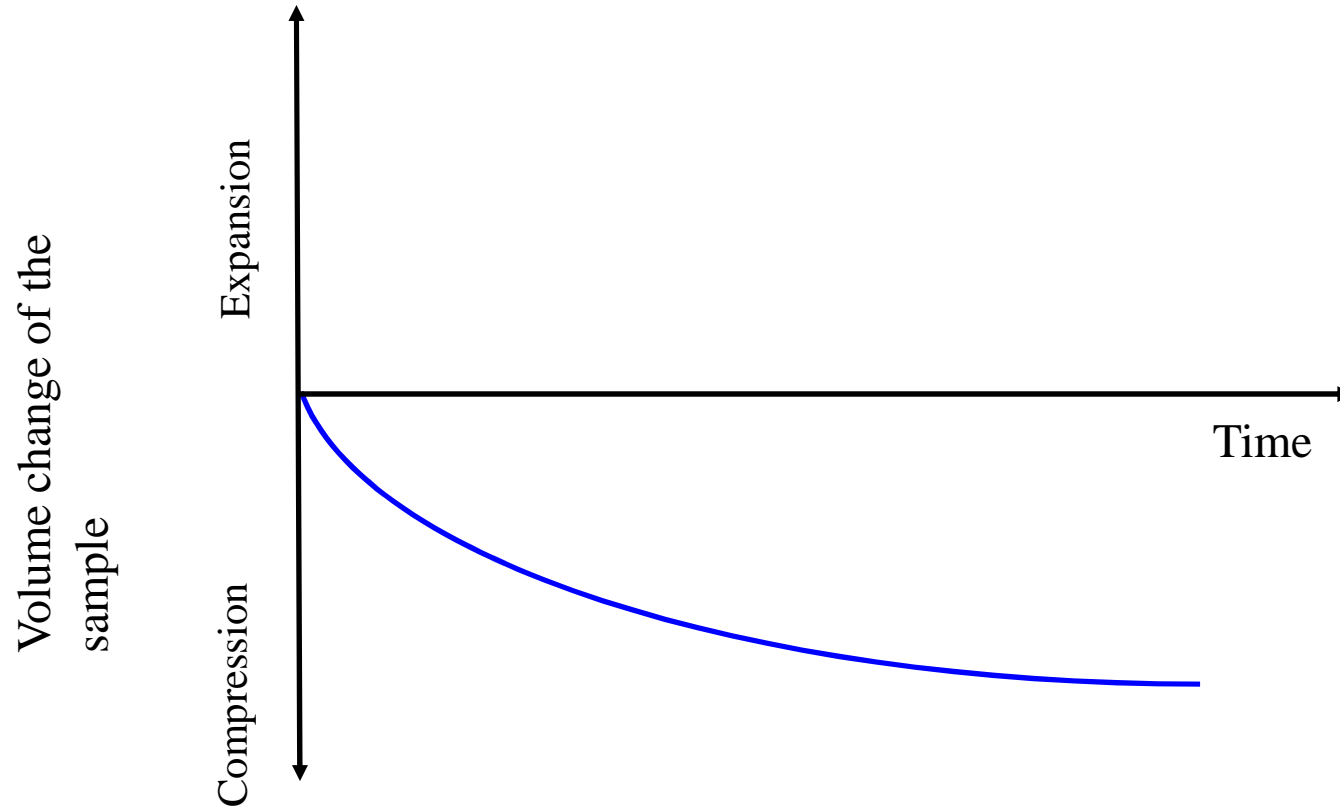
Step 3: At

failure



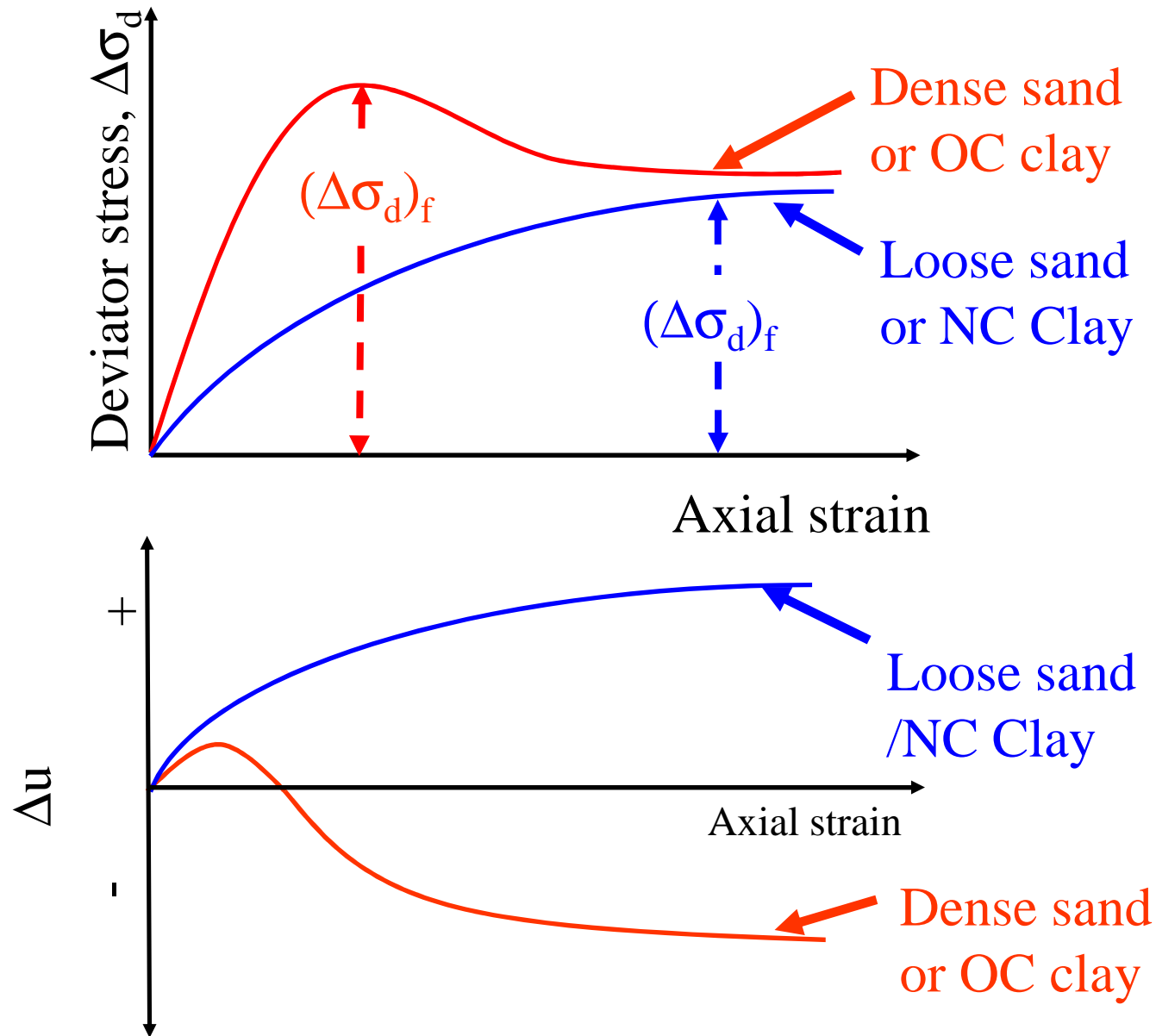
# Consolidated- Undrained test (CU Test)

## Volume change of sample during consolidation

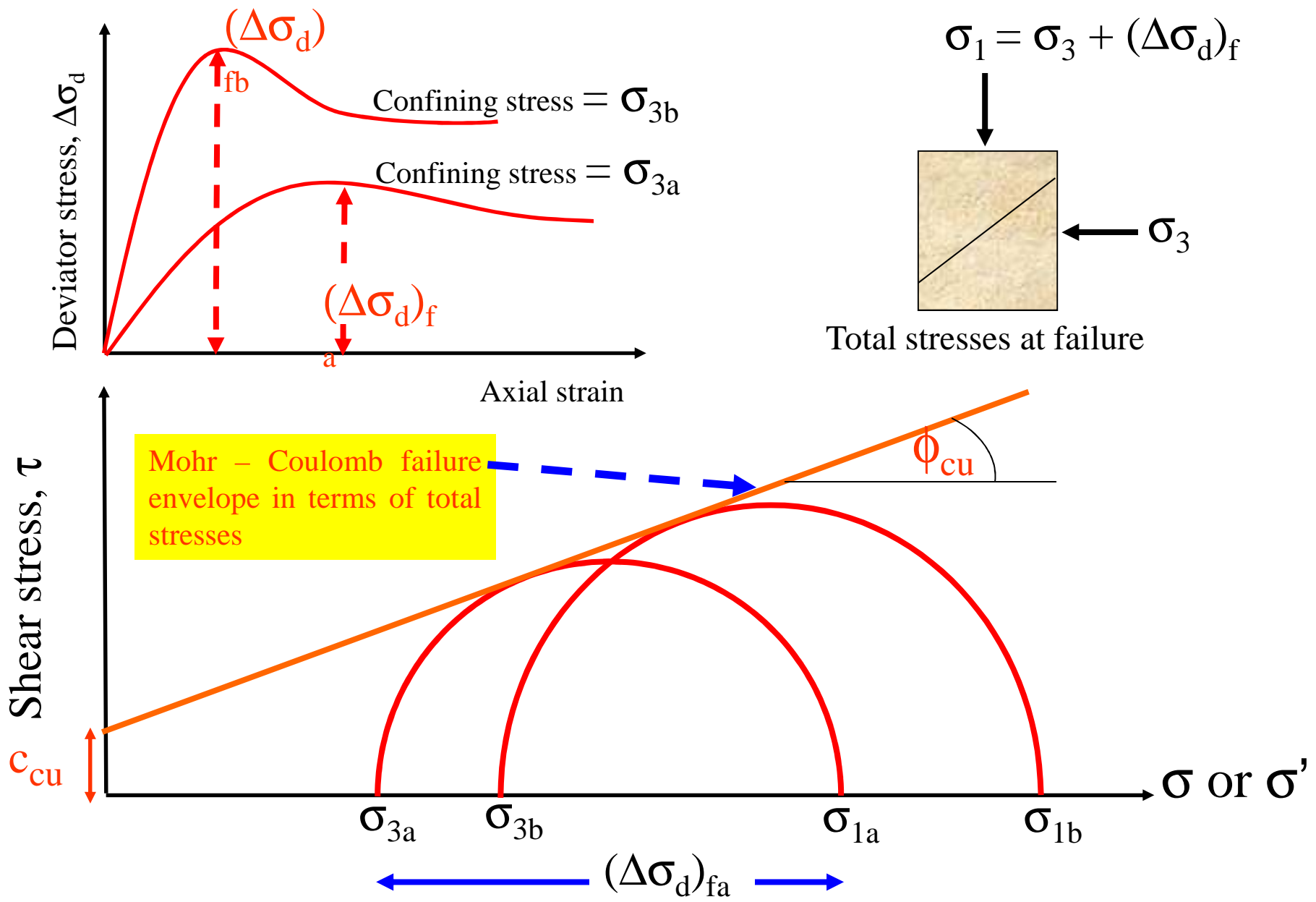


# Consolidated- Undrained test (CU Test)

## Stress-strain relationship during shearing



# CU tests      How to determine strength parameters $c$ and $\phi$





## CU tests

Strength parameters  $c$  and  $\phi$  obtained from CD tests

Shear strength parameters in terms of total stresses are  $c_{cu}$  and  $\phi_{cu}$

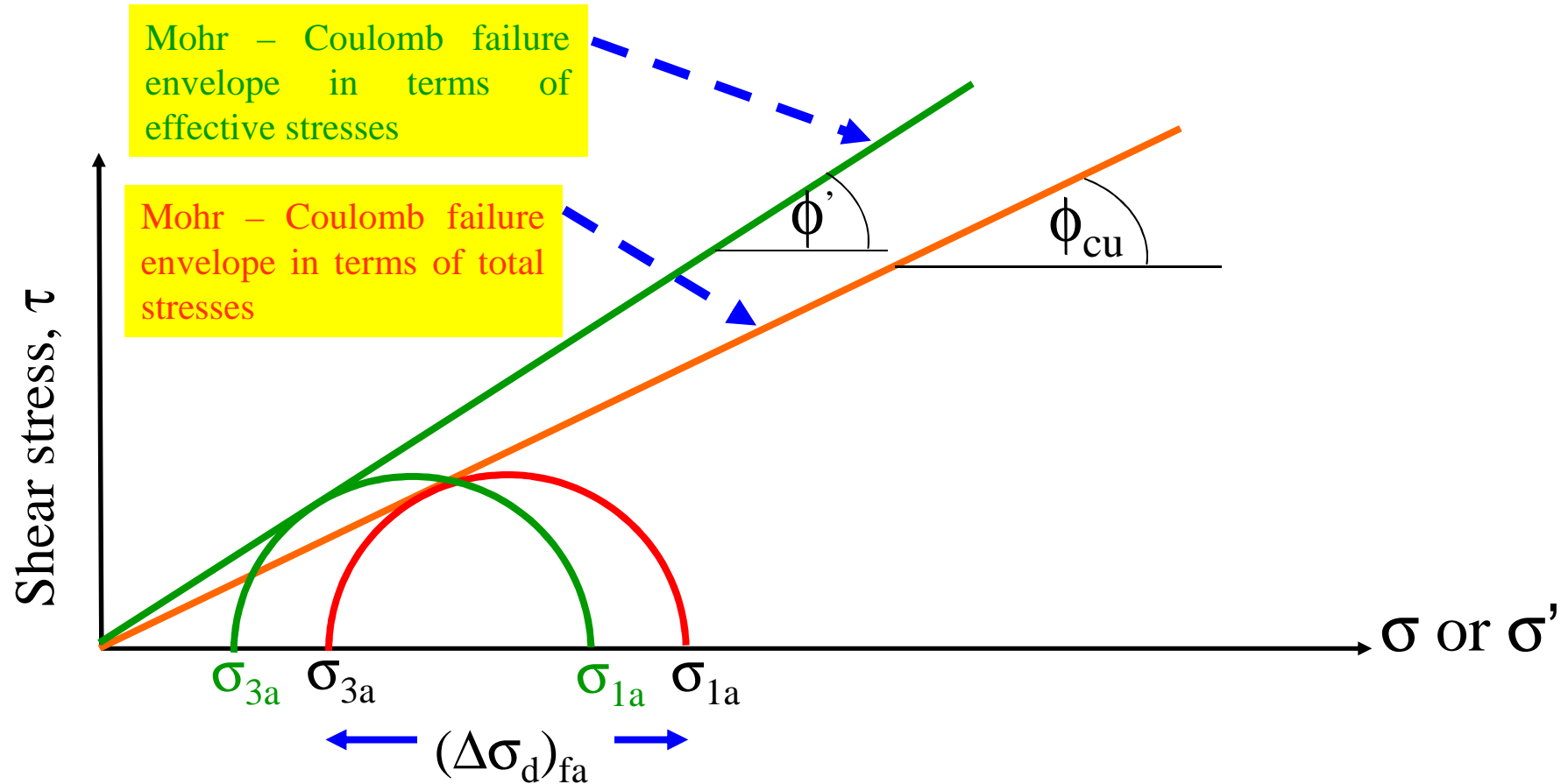
Shear strength parameters in terms of effective stresses are  $c'$  and  $\phi'$

$$c' = c_d \text{ and } \phi' = \phi_d$$

# CU tests

## Failure envelopes

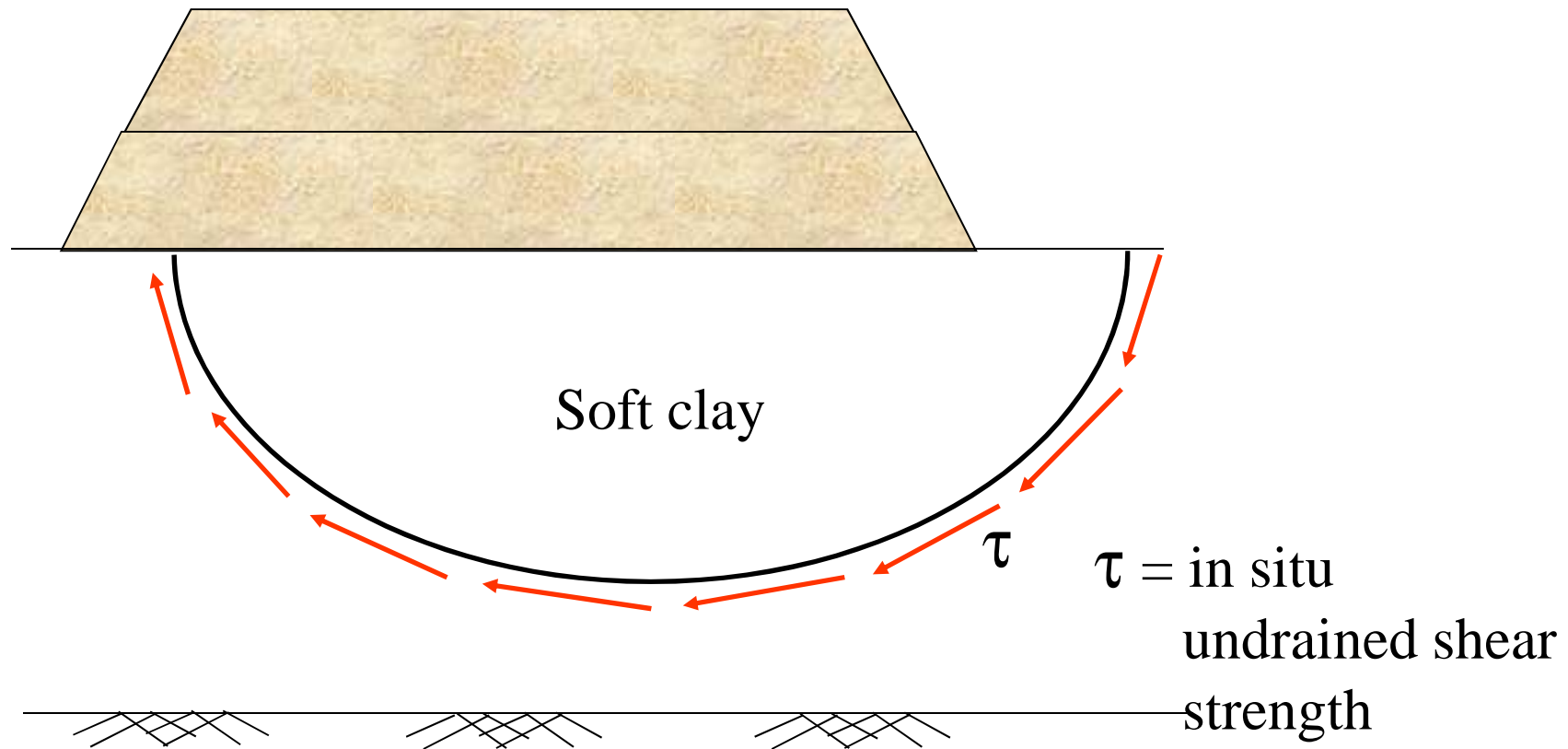
For sand and NC Clay,  $c_{cu}$  and  $c' = 0$



Therefore, one CU test would be sufficient to determine  $\phi_{cu}$  and  $\phi'$  ( $= \phi_d$ ) of sand or NC clay

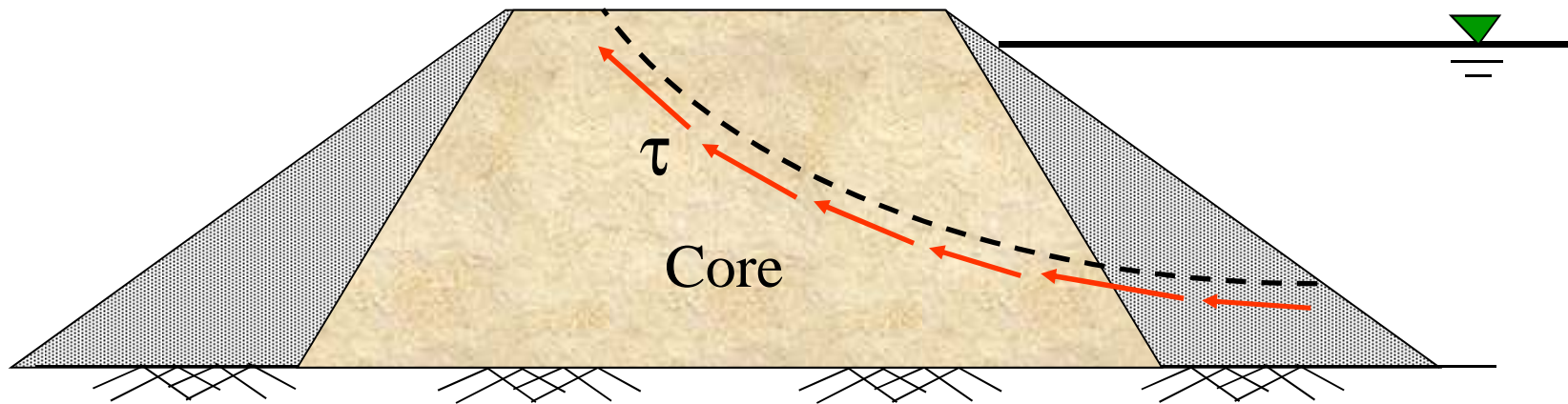
# Some practical applications of CU analysis for clays

## 1. Embankment constructed rapidly over a soft clay deposit



# Some practical applications of CU analysis for clays

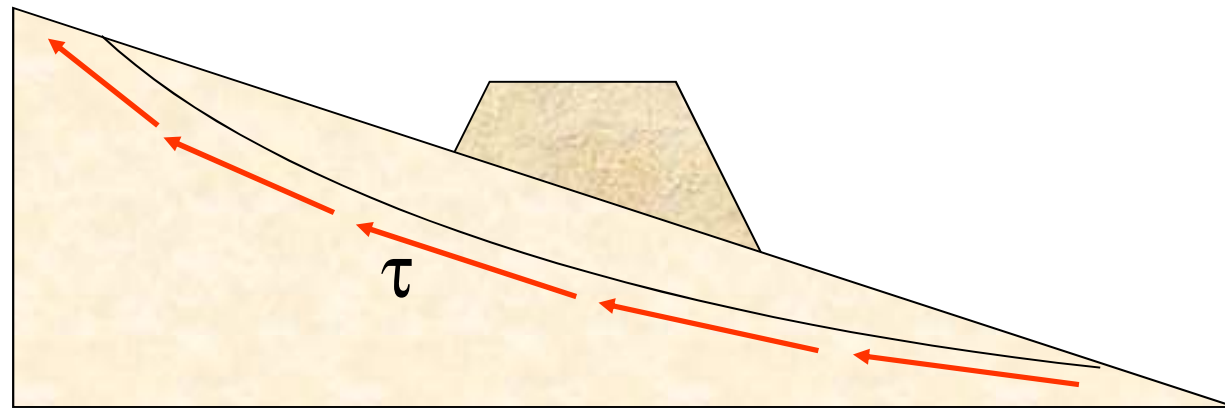
## 2. Rapid drawdown behind an earth dam



$\tau$  = Undrained shear strength of clay core

## Some practical applications of CU analysis for clays

### 3. Rapid construction of an embankment on a natural slope



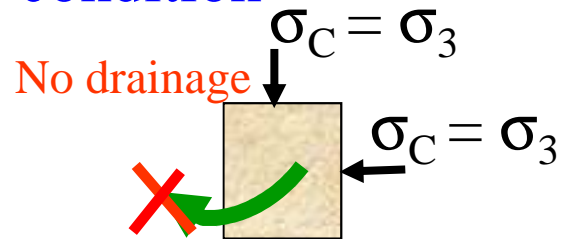
$\tau$  = In situ undrained shear strength

Note: Total stress parameters from CU test ( $c_{cu}$  and  $\phi_{cu}$ ) can be used for stability problems where, soil have become fully consolidated and are at equilibrium with the existing stress state; Then for some reason additional stresses are applied quickly with no drainage occurring

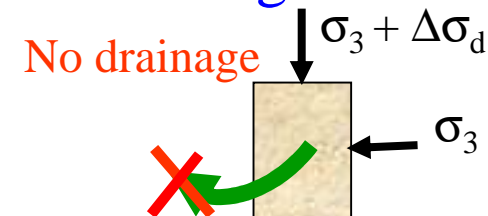
# Unconsolidated- Undrained test (UU Test)

## Data analysis

Initial specimen  
condition



Specimen  
condition during  
shearing



Initial volume of the sample =  $A_0 \times H_0$

Volume of the sample during shearing =  $A \times H$

Since the test is conducted under undrained condition,

$$A \times H = A_0 \times H_0$$

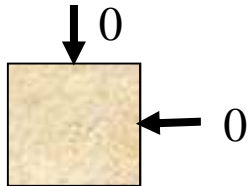
$$A \times (H_0 - \Delta H) = A_0 \times H_0$$

$$A \times (1 - \Delta H/H_0) = A_0$$

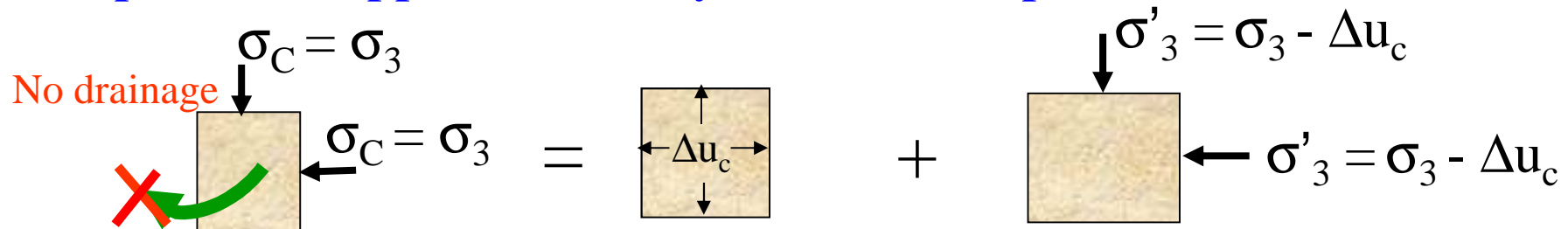
$$A = \frac{A_0}{1 - \varepsilon_z}$$

# Unconsolidated- Undrained test (UU Test)

Step 1: Immediately after sampling



Step 2: After application of hydrostatic cell pressure



$$\Delta u_c = B \Delta \sigma_3$$

Increase of pwp due to increase of cell pressure

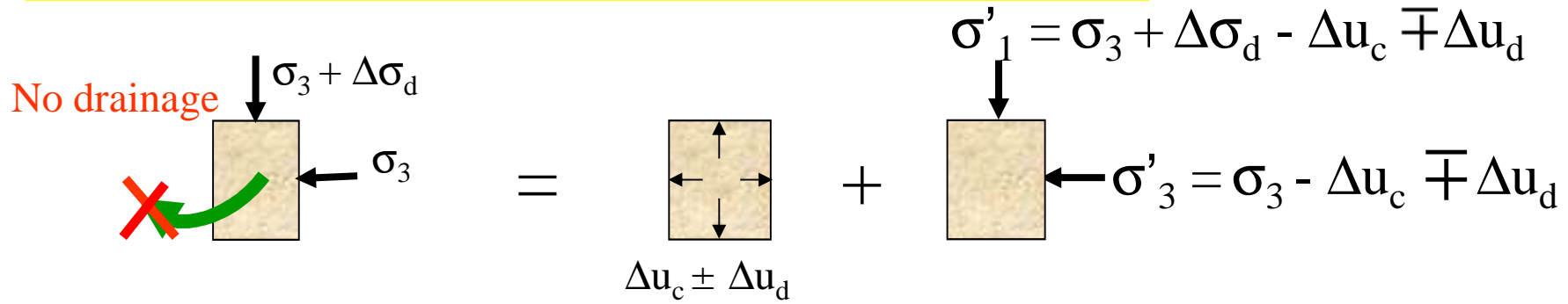
Skempton's pore water pressure parameter, B

Increase of cell pressure

Note: If soil is fully saturated, then  $B = 1$  (hence,  $\Delta u_c = \Delta \sigma_3$ )

# Unconsolidated- Undrained test (UU Test)

Step 3: During application of axial load



$$\Delta u_d = AB \Delta \sigma_d$$

Increase of pwp due to increase of deviator stress

Increase of deviator stress

Skempton's pore water pressure parameter, A

## Unconsolidated- Undrained test (UU Test)

Combining steps 2 and 3,

$$\Delta u_c = B \Delta \sigma_3$$

$$\Delta u_d = AB \Delta \sigma_d$$

Total pore water pressure increment at any stage,  $\Delta u$

$$\Delta u = \Delta u_c + \Delta u_d$$

$$\Delta u = B [\Delta \sigma_3 + A \Delta \sigma_d]$$

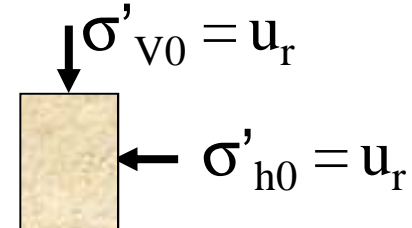
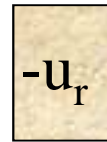
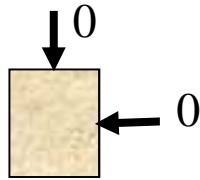
$$\Delta u = B [\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]$$

Skempton's pore  
water pressure  
equation

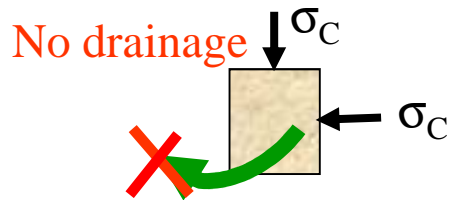
# Unconsolidated- Undrained test (UU Test)

$$\text{Total, } \sigma = \text{Neutral, } u + \text{Effective, } \sigma'$$

Step 1: Immediately after sampling

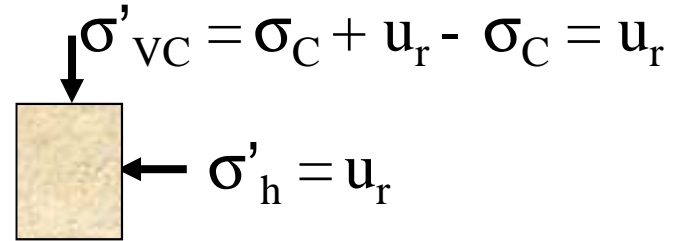
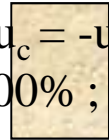


Step 2: After application of hydrostatic cell pressure

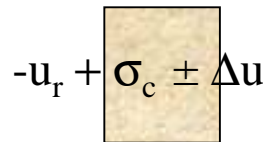
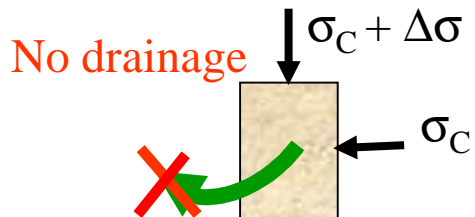


$$-u_r + \Delta u_c = -u_r + \sigma_c$$

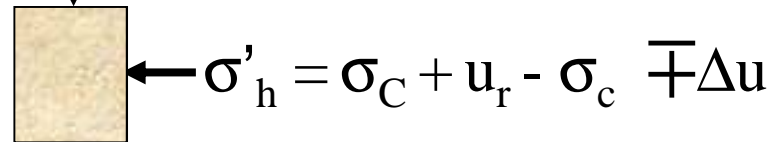
( $S_r = 100\%$ ;  $B = 1$ )



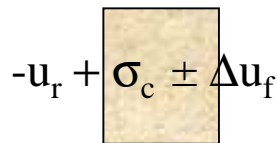
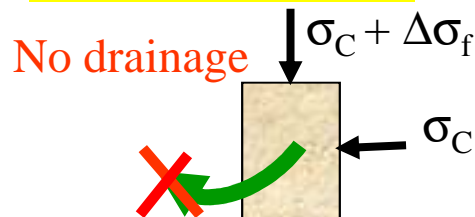
Step 3: During application of axial load



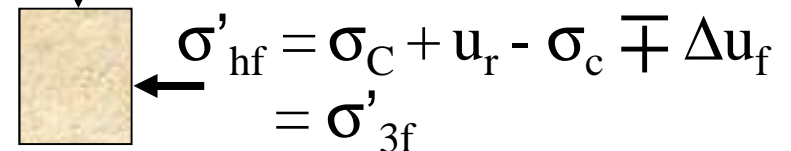
$$\sigma'_{v} = \sigma_c + \Delta\sigma + u_r - \sigma_c \mp \Delta u$$



Step 3: At failure



$$\sigma'_{vf} = \sigma_c + \Delta\sigma_f + u_r - \sigma_c \quad \Delta(\mp) = \sigma'_{1f}$$



# Unconsolidated- Undrained test (UU Test)

Total,

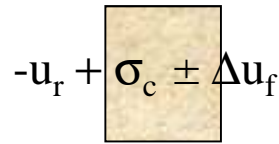
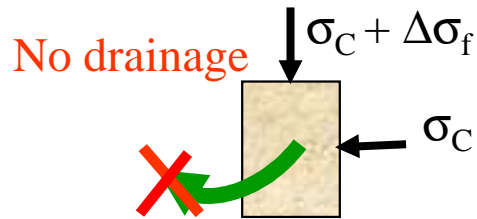
=

Neutral,  $u$

+

Effective,

Step 3: At failure

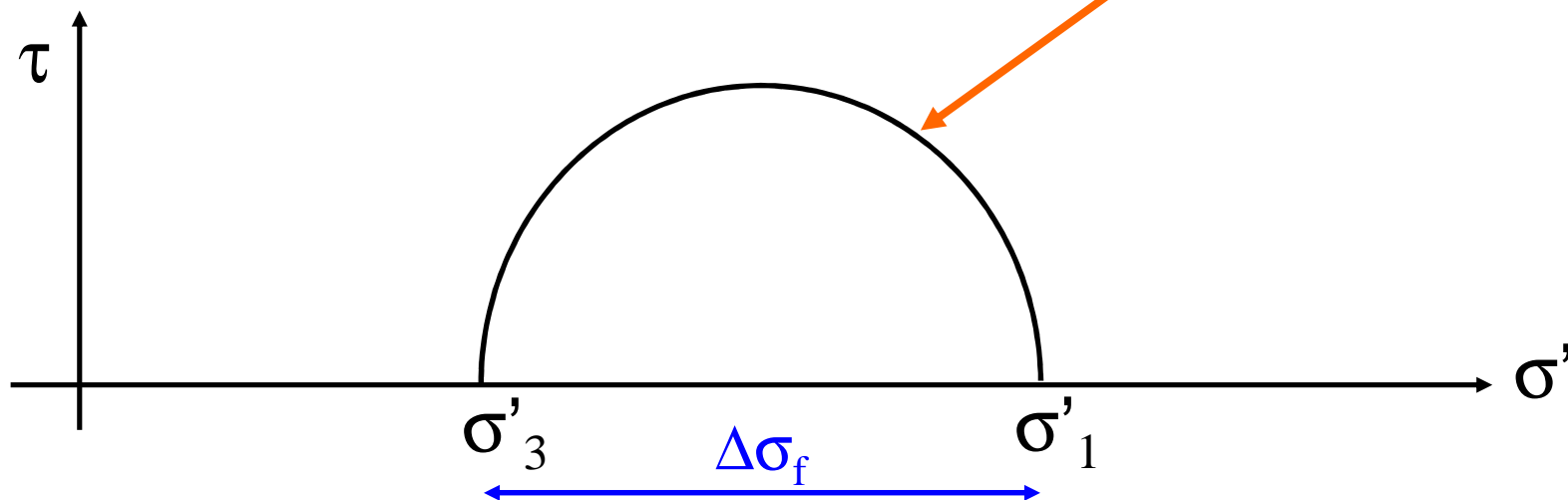


$$\sigma'_{vf} = \cancel{\sigma_c} + \Delta\sigma_f + u_r - \bar{\sigma}_c \quad \Delta u_f =$$

$$\sigma'_{lf} \quad \sigma'_{hf} = \cancel{\sigma_c} + u_r - \cancel{\sigma_c} \quad \Delta u_f = \sigma'_{3f}$$

Mohr circle in terms of effective stresses do not depend on the cell pressure.

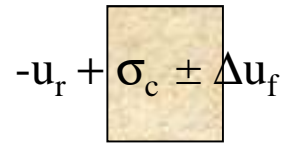
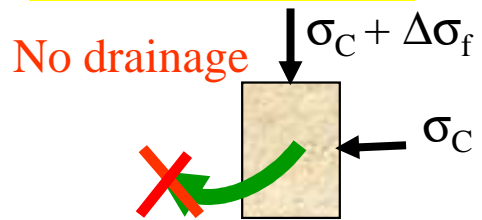
Therefore, we get only one Mohr circle in terms of effective stress for different cell pressures



# Unconsolidated- Undrained test (UU Test)

**Total,** = **Neutral, u** + **Effective,**

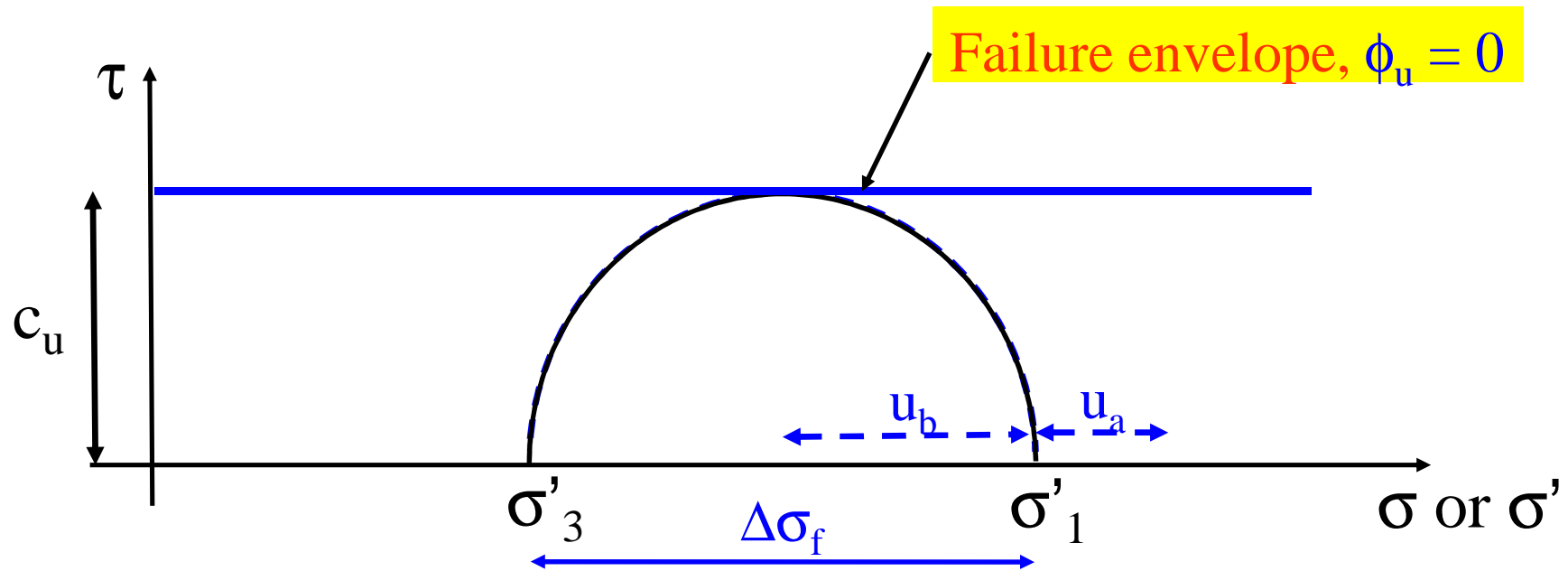
Step 3: At failure



$$\sigma'_{vf} = \cancel{\sigma_c} + \Delta\sigma_f + \cancel{u_r} - \cancel{\bar{u}_c} \quad \Delta u_f =$$

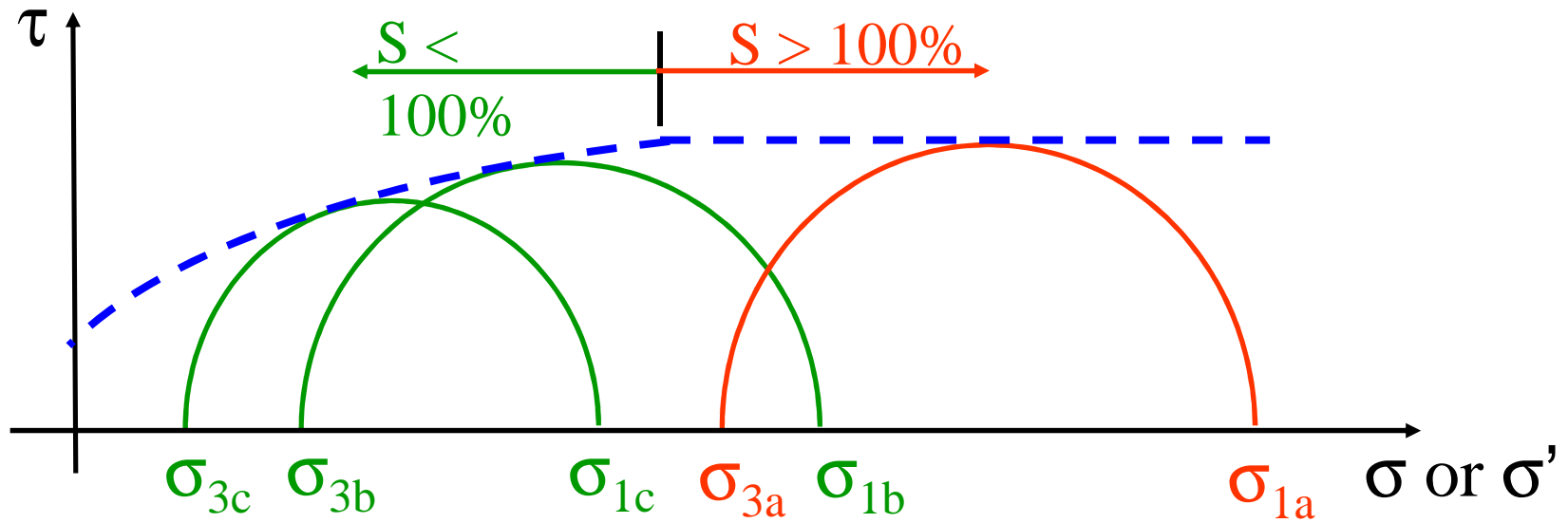
$$\sigma'_{lf} \quad \sigma'_{hf} = \cancel{\sigma_c} + \cancel{u_r} - \cancel{\sigma_c} \quad \Delta u_f = \sigma'_{3f}$$

Mohr circles in terms of total stresses



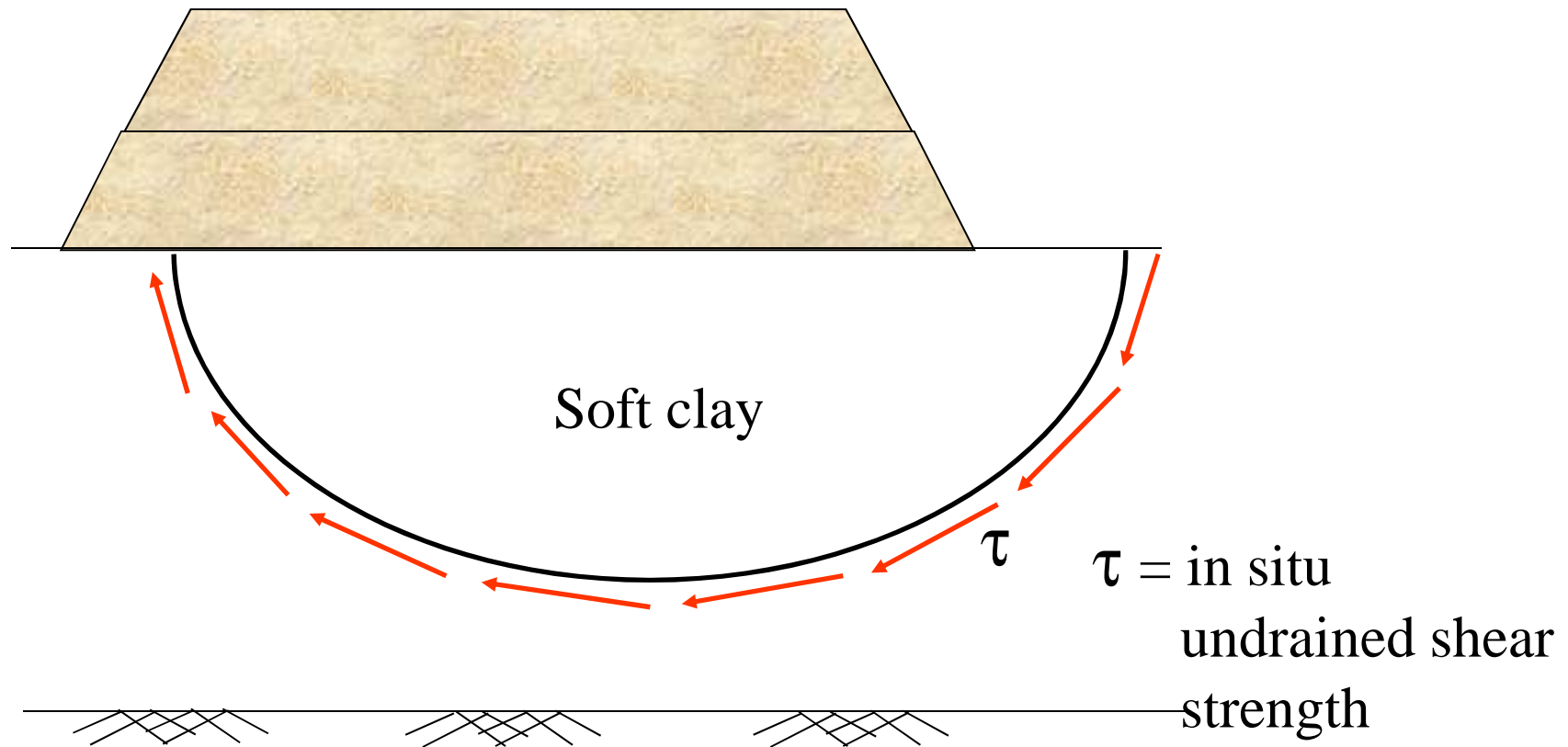
# Unconsolidated- Undrained test (UU Test)

Effect of degree of saturation on failure envelope



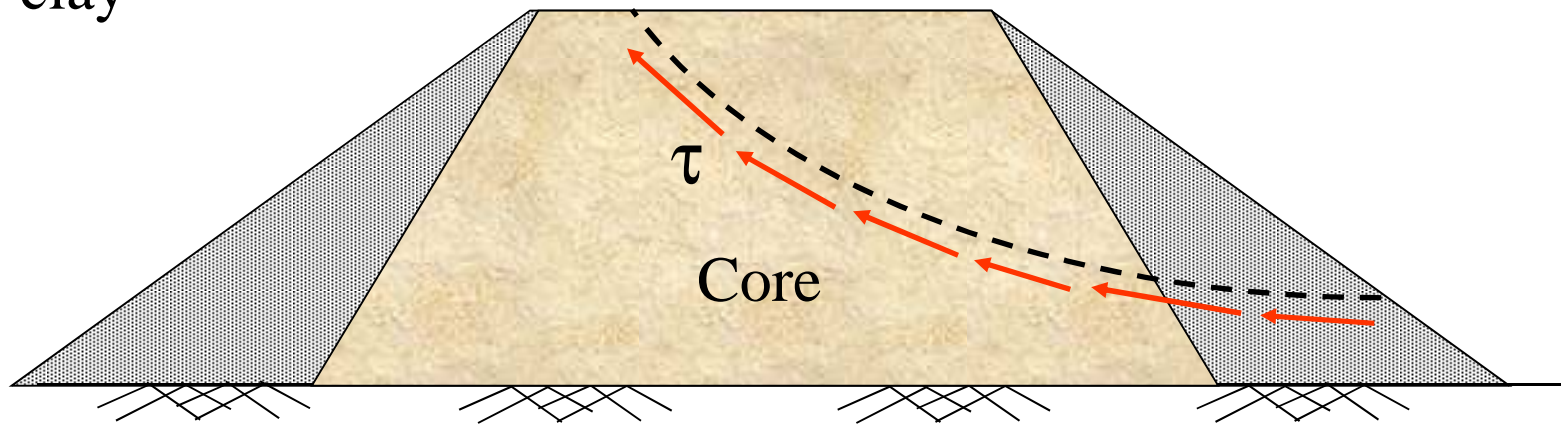
# Some practical applications of UU analysis for clays

1. Embankment constructed rapidly over a soft clay deposit



# Some practical applications of UU analysis for clays

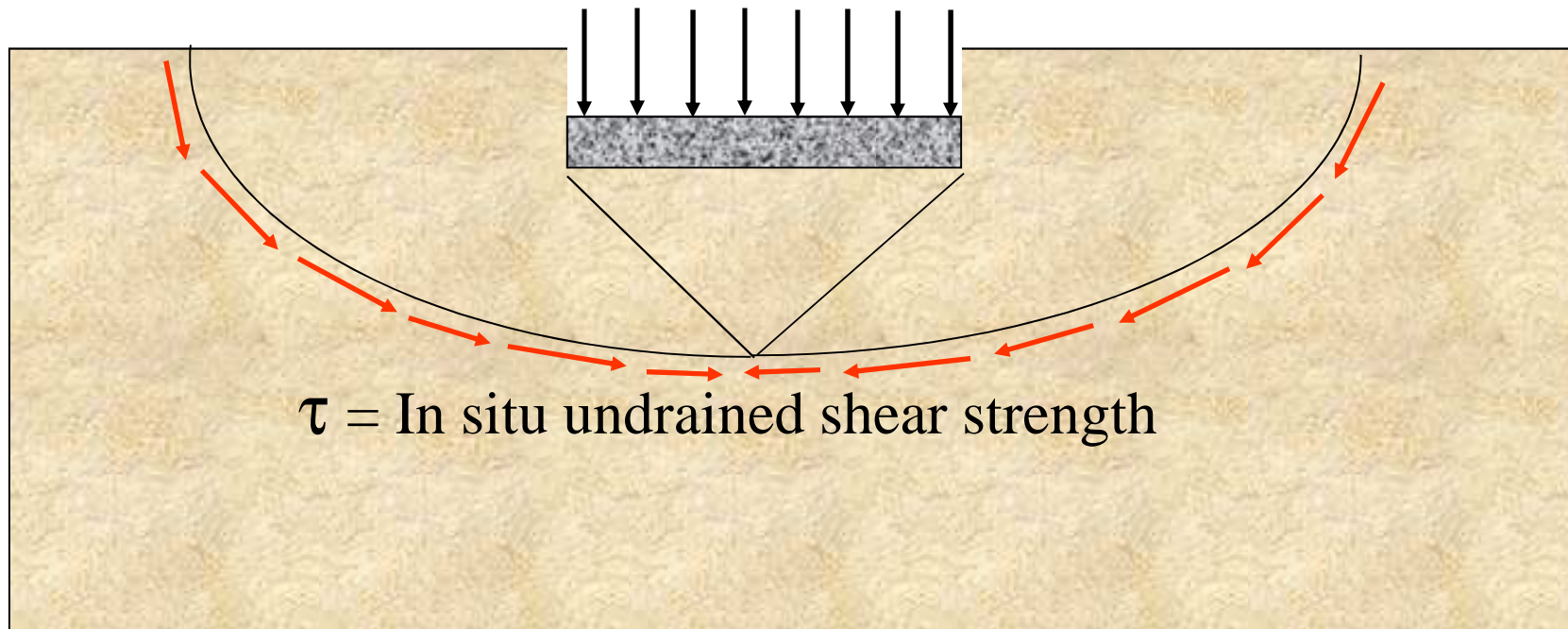
2. Large earth dam constructed rapidly with no change in water content of soft clay



$\tau$  = Undrained shear strength of clay core

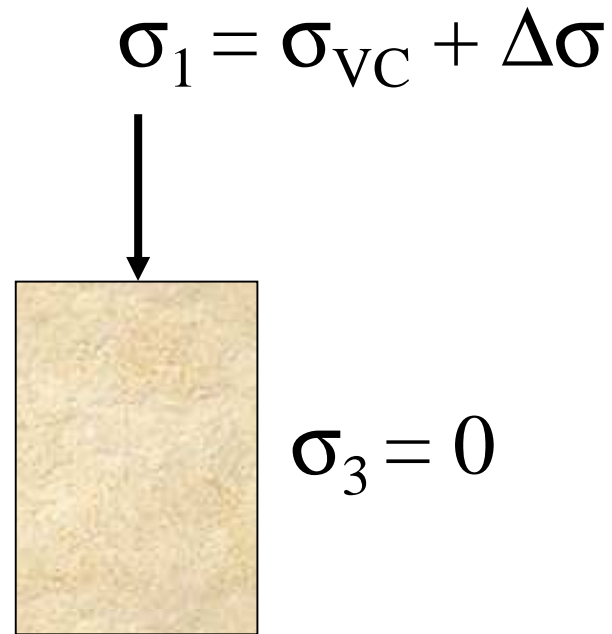
## Some practical applications of UU analysis for clays

### 3. Footing placed rapidly on clay deposit



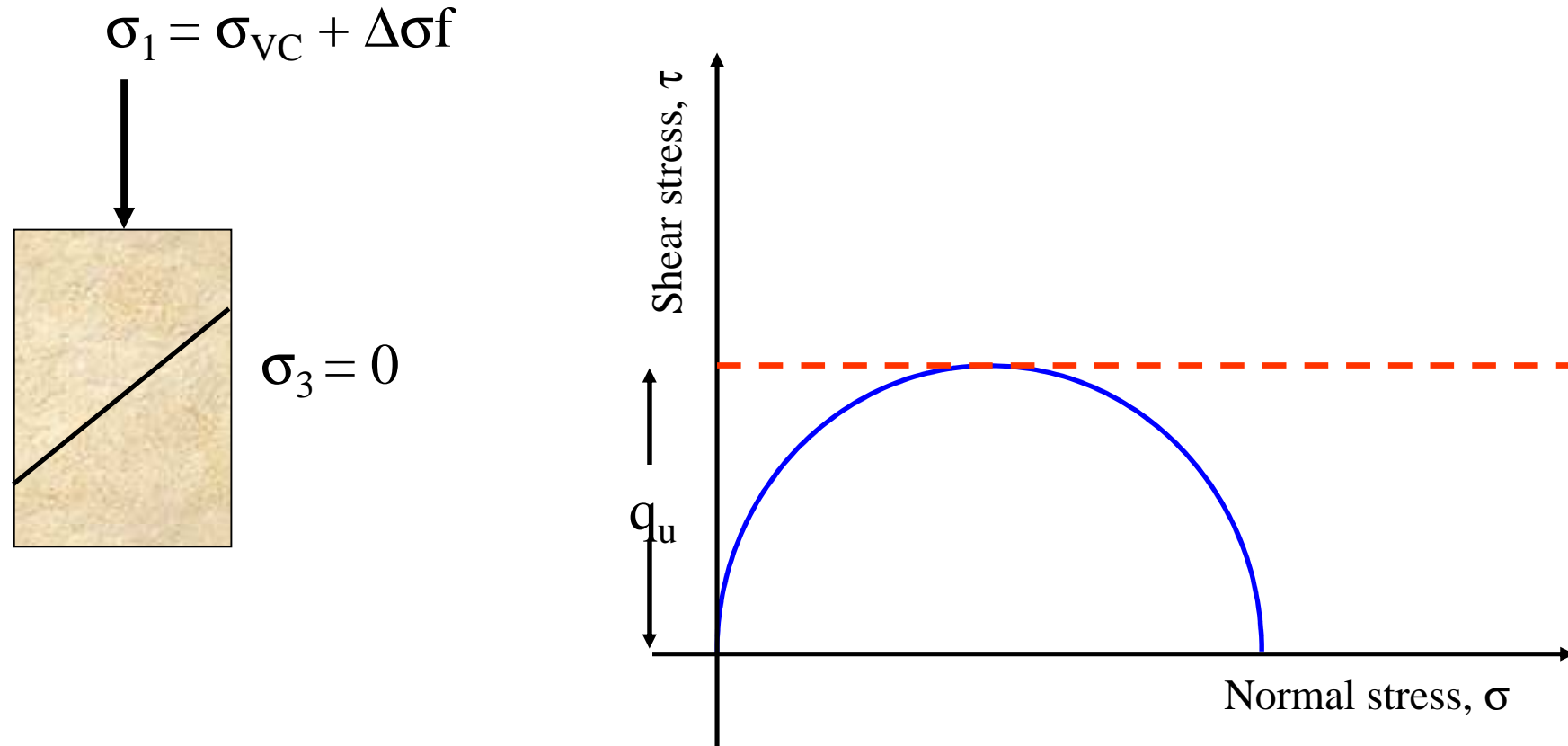
Note: UU test simulates the short term condition in the field. Thus,  $c_u$  can be used to analyze the short term behavior of soils

# Unconfined Compression Test (UC Test)



Confining pressure is zero in the UC test

# Unconfined Compression Test (UC Test)



$$\tau_f = \sigma_1/2 = q_u/2 = c_u$$

## Various correlations for shear strength

For NC clays, the undrained shear strength ( $c_u$ ) increases with the effective overburden pressure,  $\sigma'_0$

$$\frac{c_u}{\sigma'_0} = 0.11 + 0.0037(PI) \quad \text{Skempton (1957)}$$

Plasticity Index as a %

For OC clays, the following relationship is

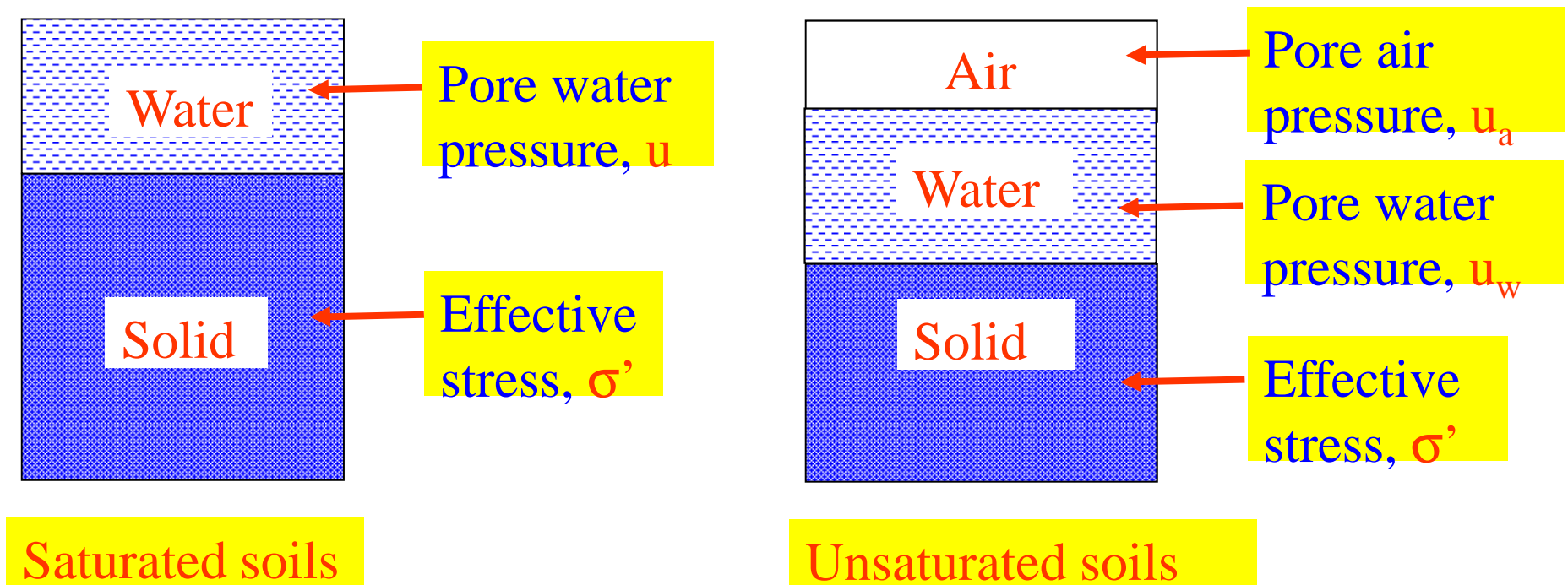
$$\left( \frac{c_u}{\sigma'_0} \right)_{\text{Overconsolidated}} / \left( \frac{c_u}{\sigma'_0} \right)_{\text{Normally Consolidated}} = (OCR)^{0.8} \quad \text{Ladd (1977)}$$

For NC clays, the effective friction angle ( $\phi'$ ) is related to  $PI$  as follows

$$\sin \phi' = 0.814 - 0.234 \log(IP) \quad \text{Kenny (1959)}$$

# Shear strength of partially saturated soils

In the previous sections, the shear strength of saturated soils is discussed. However, in most of the cases, it may encounter unsaturated soils



**Pore water pressure can be negative in unsaturated soils**

## Shear strength of partially saturated soils

Bishop (1959) proposed shear strength equation for unsaturated soils as follows

$$\tau_f = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)] \tan \phi'$$

Where,

$\sigma_n - u_a$  = Net normal stress

$u_a - u_w$  = Matric suction

$\chi$  = a parameter depending on the degree of saturation  
( $\chi = 1$  for fully saturated soils and 0 for dry soils)

Fredlund et al (1978) modified the above relationship as follows

$$\tau_f = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$$

Where,  $\tan \phi^b$  = Rate of increase of shear strength with matric suction

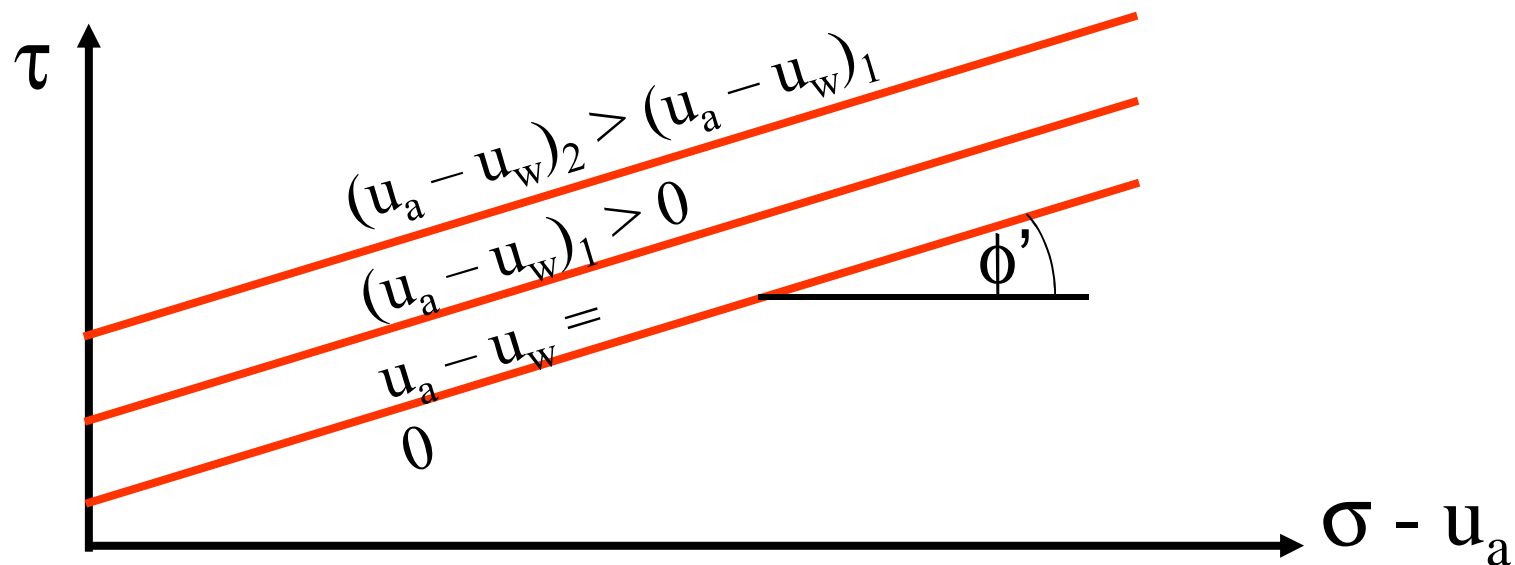
## Shear strength of partially saturated soils

$$\tau_f = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b$$

Same as saturated soils

Apparent cohesion due to matric suction

Therefore, strength of unsaturated soils is much higher than the strength of saturated soils due to matric suction



# Example 1

A drained triaxial compression test carried out on three specimens of the same soil yielded the following results:

Test No.	1	2	3
Cell pressure (kPa)	100	200	300
Deviator stress at failure (kPa)	210	438	644

Draw the shear strength envelop and determine the shear strength parameters,  $C'$  &  $\phi'$ , assuming that the pore water pressure remain constant during the axial loading stage.

## Example 2

Three consolidation undrained triaxial tests were carried out on 38mm diameter samples of the same clay. The applied axial force at failure of the samples were found to be as follows:-

<b>Test No.</b>	1	2	3
<b>Cell pressure (kN/m<sup>2</sup>)</b>	25	75	120
<b>Applied axial force at failure (kN)</b>	0.086	0.120	0.149

Determine the shear strength parameters of the clay in term of total stress.

## Example 3

The following results were obtained from undrained triaxial tests on specimens of a saturated normally consolidated clay.

<b>Test No.</b>	1	2	3
<b>Cell Pressure (kN/m<sup>2</sup>)</b>	100	200	300
<b>Ultimate Deviator Stress (kN/m<sup>2</sup>)</b>	137	210	283
<b>Ultimate Pore Pressure (kN/m<sup>2</sup>)</b>	28	86	147

Determine the shear strength parameters of the clay in term of total and effective stress.

## Example 4

The following results were obtained from undrained triaxial tests on specimens of an overconsolidated clay.

<b>Test No.</b>	1	2	3
<b>Cell Pressure (kN/m<sup>2</sup>)</b>	100	250	400
<b>Deviator Stress at failure (kN/m<sup>2</sup>)</b>	340	410	474
<b>Deviator Pore Pressure (kN/m<sup>2</sup>)</b>	-42	64	177

Determine the shear strength parameters of the clay in term of total and effective stress.

## Example 5

Referring to Example 2, if the shear strength parameters of the clay in term of effective stress were  $C' = 10 \text{ kN/m}^2$  and  $\phi' = 30^\circ$ , determine the pore water pressure in each sample at failure.

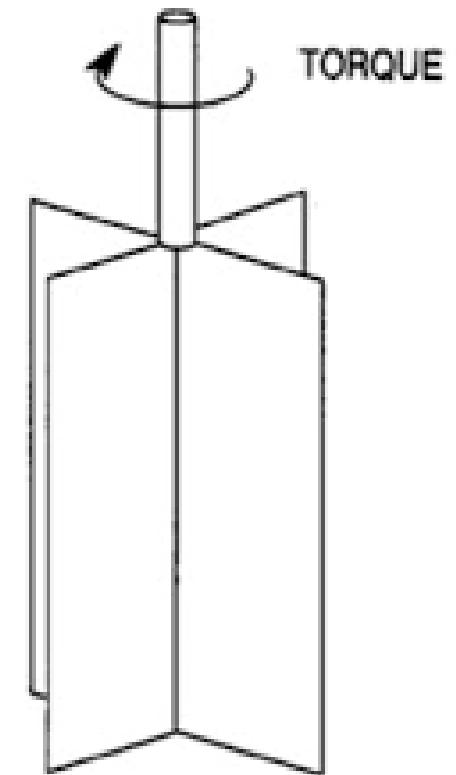
## Example 6

Consolidated undrained triaxial tested were carried out on 3 samples of the same clay soil and the following results were obtained at the point of failure:-

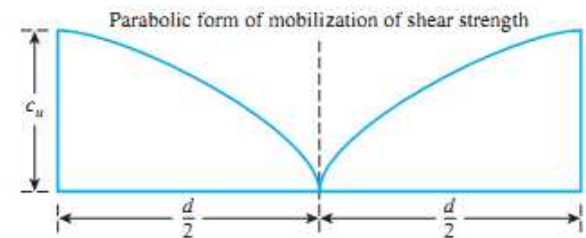
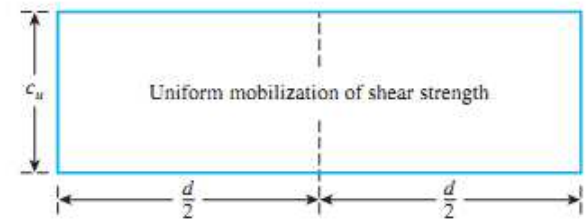
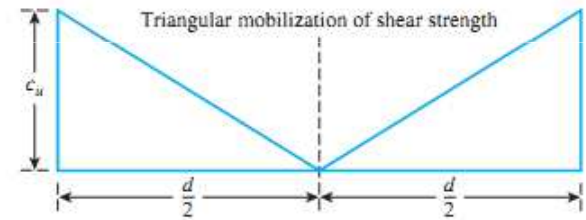
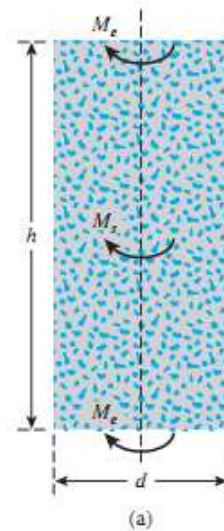
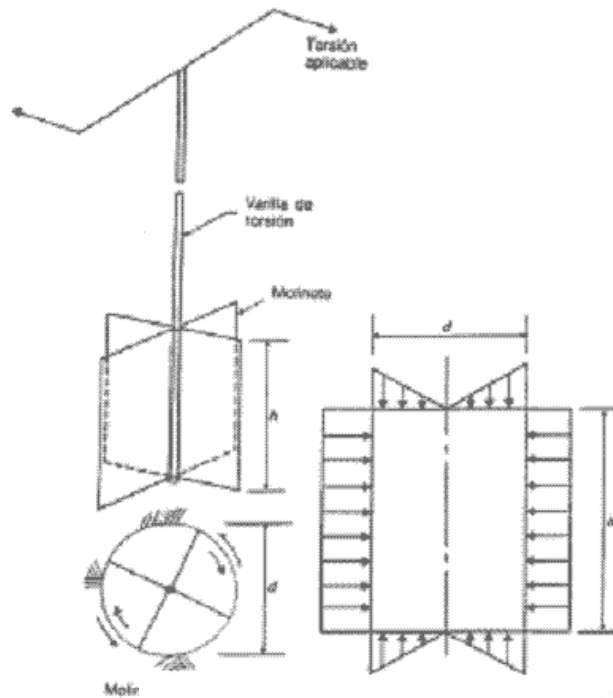
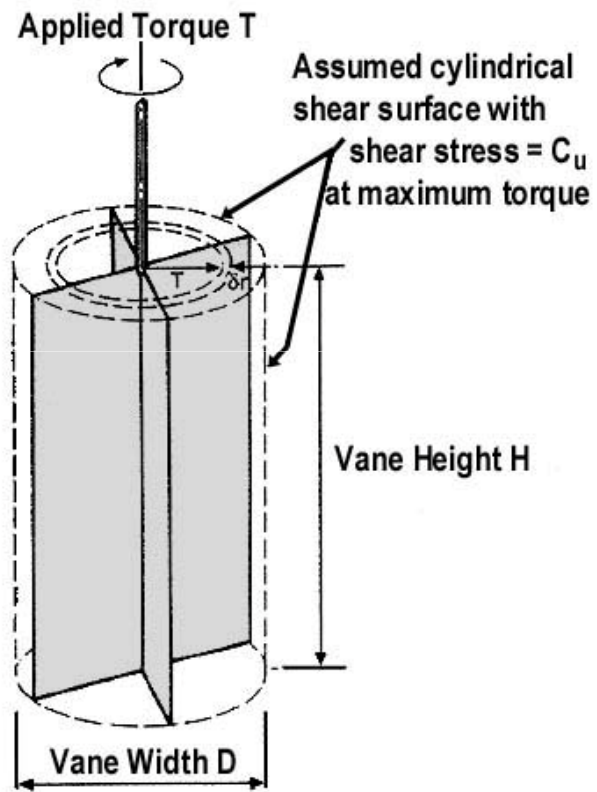
Sample No.	Cell Pressure (kN/m <sup>2</sup> )	Deviator Stress at failure (kN/m <sup>2</sup> )	Pore Water Pressure (kN/m <sup>2</sup> )	$C_u$ (kN/m <sup>2</sup> )	$\Phi_u$ (°)	$C'$ (kN/m <sup>2</sup> )	$\Phi'$ (°)
1	50	80.543	27.201	10	?	?	?
2	100	?	57.879				
3	?	158.514	?				

Determine the 6 unknown value (?) in the table by Calculation and Graphical method

# SHEAR-VANE TEST



**Vane shear apparatus**



(b)

If **T** is the maximum torque applied at the head of the torque rod to cause failure, it should be equal to the sum of the resisting moment of the shear force along the side surface of the soil cylinder (**M<sub>s</sub>**) and the resisting moment of the shear force at each end (**M<sub>e</sub>**)

$$T = M_s + \underbrace{M_e + M_e}_{\text{Two ends}}$$

$$M_s = \underbrace{(\pi dh)c_u}_{\text{Surface area}} \underbrace{(d/2)}_{\text{Moment arm}}$$

h = height of the shear vane

d = diameter of the shear vane

### For the calculation of *Me*

1. *Triangular*. Shear strength mobilization is  $c_u$  at the periphery of the soil cylinder and decreases linearly to zero at the center.
2. *Uniform*. Shear strength mobilization is constant (that is,  $c_u$ ) from the periphery to the center of the soil cylinder.
3. *Parabolic*. Shear strength mobilization is  $c_u$  at the periphery of the soil cylinder and decreases parabolically to zero at the center.

$$T = \pi c_u \left[ \frac{d^2 h}{2} + \beta \frac{d^3}{4} \right]$$



$$c_u = \frac{T}{\pi \left[ \frac{d^2 h}{2} + \beta \frac{d^3}{4} \right]}$$

where  $\beta = \frac{1}{2}$  for triangular mobilization of undrained shear strength

$\beta = \frac{2}{3}$  for uniform mobilization of undrained shear strength

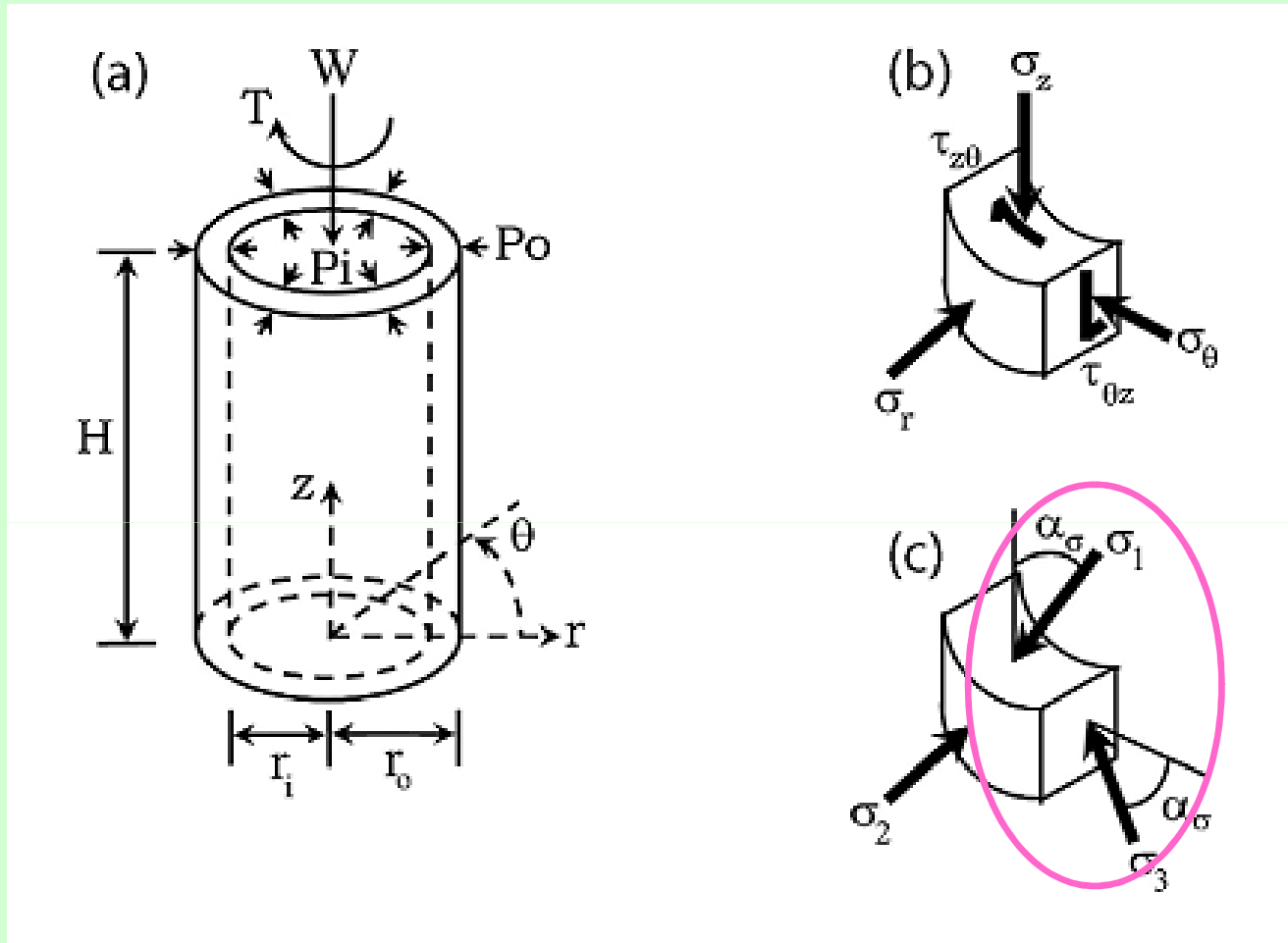
$\beta = \frac{3}{8}$  for parabolic mobilization of undrained shear strength

- ★ Suitable for determining the **in-situ** undrained shear strength of unfissured saturated clays and silts
- ★ The vane consists of **four rectangular blades** in a cruciform at the end of a steel rod
- ★ Shear strength is measure by pushing the vane into the soil and rotated by applying **a torque** at the surface end of the rod
- ★ The vane is first rotated at **6-12° per minute** to determine the **undisturbed** shear strength and then **the remoulded strength** is measured by rotating the vane rapidly

## **Example 1**

A shear vane used to test a soft clay had a diameter of 75mm and a length of 150mm. The average torques recorded after slow and then rapid rotations were 64 and 26 Nm respectively. Determine the undrained strength of the clay.

# Hollow cylinder test...



Moving Vehicles on road, Beds of flowing river,.....etc

# Truly Triaxial test, Plane strain test, simple shear test.....

