

GEOTECHNICAL ENGINEERING - ii



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A Handbook on

Geotechnical Engineering-ii

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SUBSURFACE EXPLORATION

BEARING CAPACITY OF SHALLOW
FOUNDATION

SETTLEMENT OF SHALLOW FOUNDATION

SLOPE STABILITY

LATERAL EARTH PRESSURE

STRESS DISTRIBUTION

PROBLEMS (SLOPE)

PROBLEMS(B.C.S.F)

SUBSOIL EXPLORATION

মো: রবিউল ইসলাম

Geotechnical Engineering - II

Md. AKHTER HOSSAIN (SIR).

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start.

শ্রী: রবিউল ইসলাম

রাজেশ্বরী অকোলেজ ও প্রযুক্তি বিশ্ববিদ্যালয়

পুরকোলেজ বিভাগ

রোল নং: ২০০২০

Geotechnical Engineering - II (MAH)

Subsurface Exploration

Q. What is subsoil/subsurface exploration? what are the purposes of subsoil/subsurface exploration?
2015, 14, 12, 11, 10, 09, 08, 07, 06.

Ans. Subsoil Exploration: The process of identifying the layers of deposits that underlie a proposed structure and their physical characteristics is generally referred to as subsurface exploration/subsoil exploration.

Purposes are given below:

- (I) To determining nature of soil at site and its stratification.
- (II) For obtaining disturbed and undisturbed soil samples for visual identification and appropriate laboratory tests.
↳ দুটি স্যাম্পল
- (III) To determining the depth and nature of bed rock if and when encountered.
- (IV) Selecting type and depth of foundation suitable for a given structure.
- (V) To evaluating load-bearing capacity of foundation.

- (VI) For estimating probable settlement of structure.
- (VII) To determining potential foundation problems (e.g. expansive soil, collapsible soil etc).
- (VIII) To determining location of water table.
- (IX) To observing drainage condition from an insitu site
- (X) For Predicting lateral earth pressure.

Q. what are the phases of planning for soil Exploration? ^{→ DRCC}

Answer: Planning for soil Exploration/subsurface Exploration:

(1) Compilation of existing information regarding the structure:

This phase includes information such as,

- (a) Type of structure to be constructed and its future use.
- (b) Requirement of existing local building codes.

(2) Collection of existing information for subsoil condition;

This type of information can be obtained from the following sources

- (a) Geological survey maps (GSM)
- (b) Country soil survey maps [prepared by the U.S. Department of Agriculture and the soil conservation service]
- (c) Soil Manuals [published by the state highway department]
- (d) Existing soil exploration reports prepared for construction of nearby structures.

(3) Reconnaissance of proposed construction site:

Engineers should always make a visual inspection of the site to obtain information about—

- (a) General topography of the site.
- (b) Soil stratification.
- (c) Type of vegetation at the site.
- (d) Ground water levels.
- (e) Types of construction nearby.

(4) Detailed site investigation: This phase consists of making several test borings at the site and collecting disturbed and undisturbed soil samples for various depths for visual observation and for laboratory test.

Q. How ^{2013, 2015} can you determine approximate minimum depth of Boring according to ASCE recommendation.

Answer: Engineers may use rules established by ASCE (1972).

ASCE Recommendation are given below:

- (a) Determine the net increase in effective stress, σ_v' under a foundation with depth as shown in figure (1).

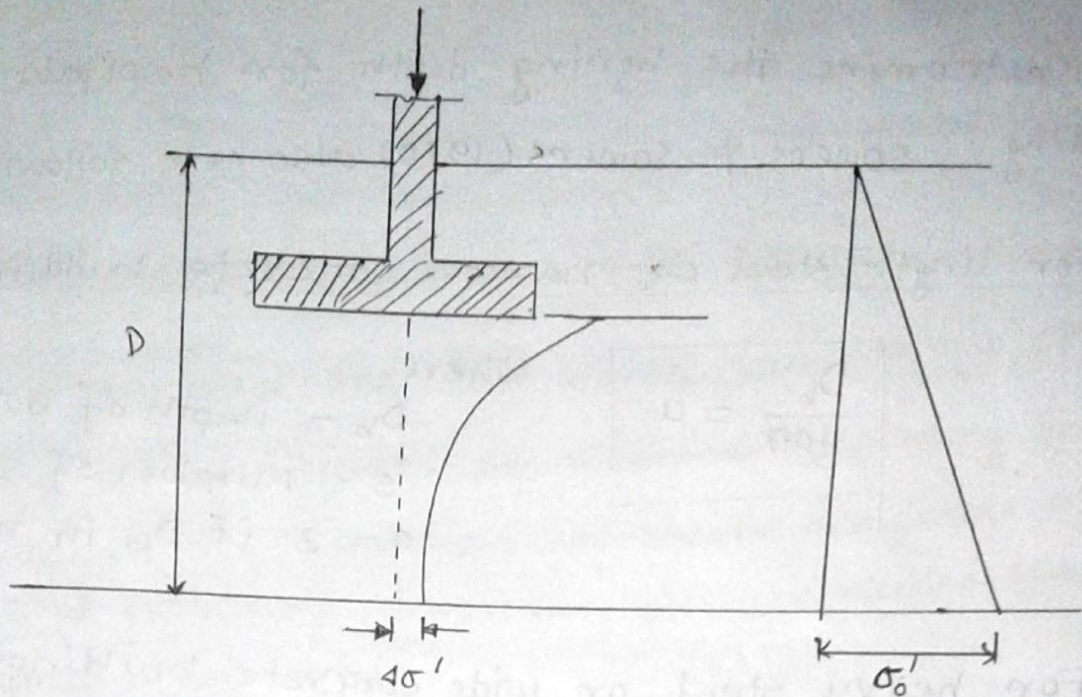


Fig. (1)

- (b) Estimate variation of vertical effective stress, σ'_0 with depth.
- (c) Determine the depth $D=D_1$, at which effective stress increase $4\sigma'$ is equal to $\frac{1}{10} q$ (q = estimated net stress on the foundation).
- (d) Determine the depth $D=D_2$ at which $\frac{4\sigma'}{\sigma'_0} = 0.05$.
- (e) Choose smaller of two depths D_1 and D_2 , just determined as approximate minimum depth of boring required, unless bedrock is encountered.

Boring depth According to Sowers and Sowers (1970).

⇒ If preceding rules are used, the depth of boring for a building with a width of 30m will be approximately as follows:-

Table: Approximate depths of borings for building with a width of 30m

No of stories	Boring depth (m)
1	3.5
2	6
3	10
4	16
5	24

⇒ To determine the boring depth for hospitals and office building, Sowers & Sowers (1970) also use following rules.

(i) For length steel or narrow concrete building:

$$\frac{D_b}{S^{0.7}} = a$$

Where,

D_b = Depth of boring
 S = Number of stories
 $a = 3$ if D_b in meters.

(ii) For heavy steel or wide concrete buildings:-

$$\frac{D_b}{S^{0.7}} = b$$

Where, D_b = Depth of boring
 S = Number of stories
 b → = 6, if D_b in meter
→ = 20, if D_b in feet.

When deep excavation are anticipated, depth of boring should be at least 1.5 times the depth of excavation.

Sometimes, subsoil conditions require that the foundation load be transmitted to the bed rock.

The minimum depth of core boring into the bed rock is about 3m.

If the bed rock is irregular or weathered, the core borings may have to be deeper.

Q. How can you determine spacing of Boring? Describe different Methods.

Answer: Determination of spacing of Boring:-

There are no hard-and fast rules for borehole spacing. The spacing can be increased or decreased depending on the subsoil condition. If various soil strata are more or less uniform and predictable, fewer boreholes are needed than in non-homogeneous soil strata.

Following Table gives some general guideline:-

Table → Approximate spacing of Boreholes.

Types of Project	Spacing (m)
Multistory Building	10-30
One-story industrial building	20-60
Highways	250-500
Residential subdivision	250-500
Dams and dikes	40-80

Methods of boring: Soil Borings can be made by several methods, such as,

- (I) Auger boring.
- (II) Wash boring.
- (III) Rotary drilling.
- (IV) Percussion drilling.

WARP

① Auger Boring :-

- (a) Auger boring is the simplest method of making exploratory boreholes. Two types of hand auger are present Post hole, helical.
- (b) Hand auger can not used for advancing holes to depth exceeding 3 to 5m. However, they can be used for soil exploration work on some highway and small structures.
- (c) Portable power-driven helical augers (30 to 75 mm dia) are available for making deeper boreholes. Soil samples obtained from such borings are highly disturbed.
- (d) In some non-cohesive soils or soil having low cohesion, the walls of boreholes will not stand unsupported. In such circumstances, a metal pipe is used as a casing to prevent the soil from caving in.

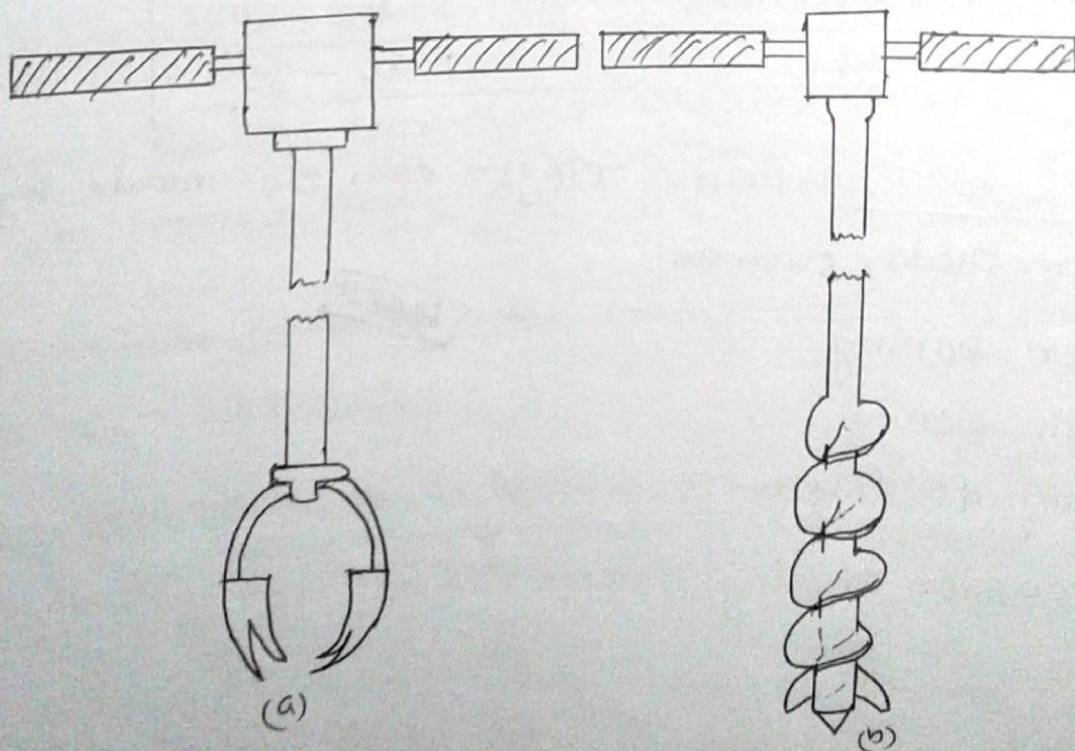


Fig: Hand tools (a) Post hole auger (b) helical auger.

(11) Wash Boring :- 2013

- (i) In this method a casing about 2 to 3 m long is driven into ground.
- (ii) Soil inside the casing is then removed by means of a chopping bit attached to a drilling rod.
- (iii) Water is forced through drilling rod and exits at a very high velocity through holes at bottom of chopping bit.
- (iv) The water and chopped soil particles rise in drill hole and overflow at the top of the casing through a T connection.
- (v) The wash water is collected in a container.
- (vi) The casing can be extended with additional pieces as the borehole progresses; however, that is not required if the borehole will stay open and not cave in.

* Wash borings are rarely used now in United States and other developed countries.

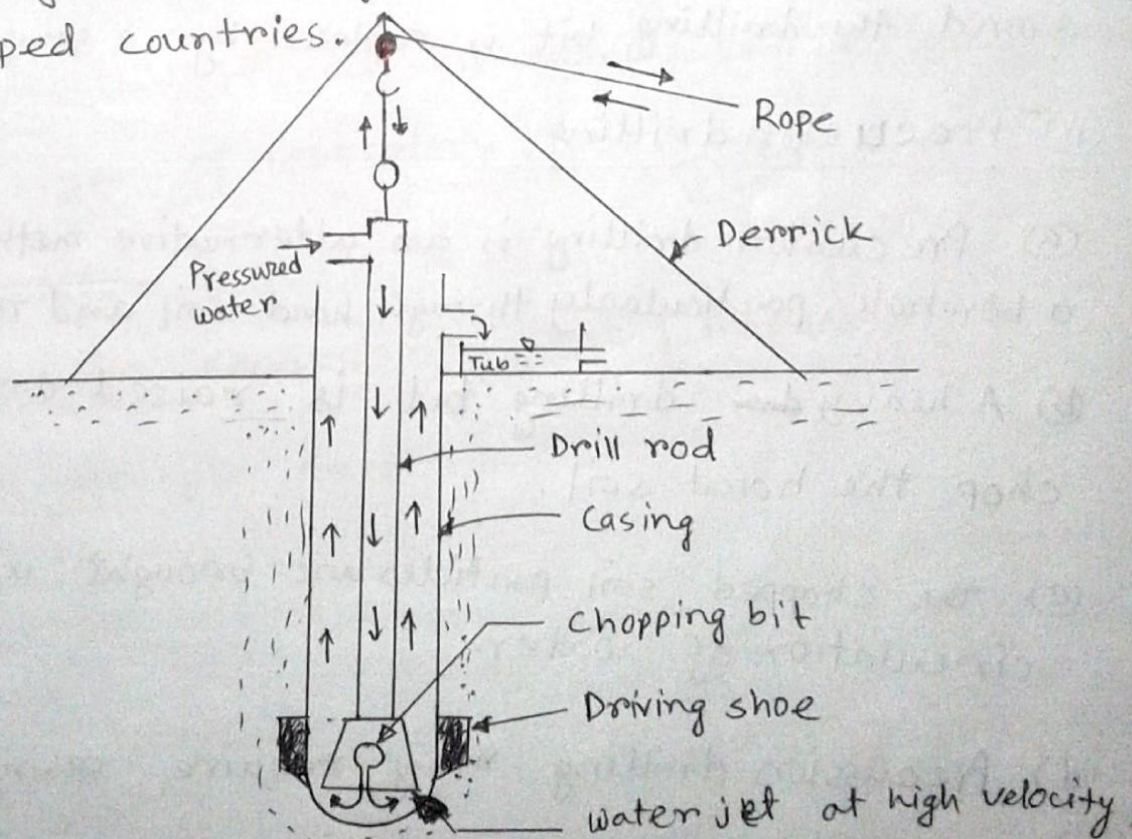


Fig. Wash boring.

(III) Rotary Drilling:-

(a) Rotary drilling is a procedure by which rapidly rotating drilling bits attached to bottom of drilling rods cut and grind the soil and advanced the borehole.

(b) Rotary drilling can be used in sand, clay and rocks.

(c) Water or drilling mud, is forced down ^{the} drilling rods to the bits, and the return flow forces the cuttings to the surface.

(d) Boreholes with diameters of 50-200mm can be made easily by this technique.

(e) The drilling mud is a slurry of water and bentonite.

(f) Generally, rotary drilling is used when soil that is encountered is likely to cave it.

(g) When soil samples are needed, the drilling rod is raised and the drilling bit is replaced by a sampler.

(IV) Precession drilling:-

(a) Precession drilling is an alternative method of advancing a borehole, particularly through hard soil and rock.

(b) A heavy ~~bit~~ drilling bit is raised and lowered to chop the hard soil.

(c) The chopped soil particles are brought up by the circulation of water.

(d) Precession drilling may require casing.

Q. what are the test that can be performed by disturbed and undisturbed soil sample.

Answer: Two types of soil samples can be obtained during subsurface exploration.

- (I) Disturbed soil.
- (II) Undisturbed soil.

Disturbed soil: Disturbed but representative soil samples can be generally used for the following types of laboratory test :-

- (a) Grain size analysis
- (b) Determination of Atterberg limits
- (c) Determination of organic content.
- (d) Specific gravity of soil solids.
- (e) Classification of soil

GACSO

Undisturbed soil: Undisturbed soil sample can be used for the following types of laboratory test.

- (a) Consolidation test.
- (b) Hydraulic conductivity test [Permeability test]
- (c) Shear strength test.

HSC

* Explain split-spoon sampling.

Answer: Split-spoon sampling :-

Split-spoon samplers can be used in the field to obtain soil samples that are generally disturbed but still representative.

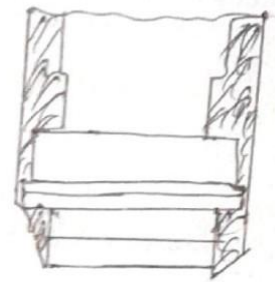
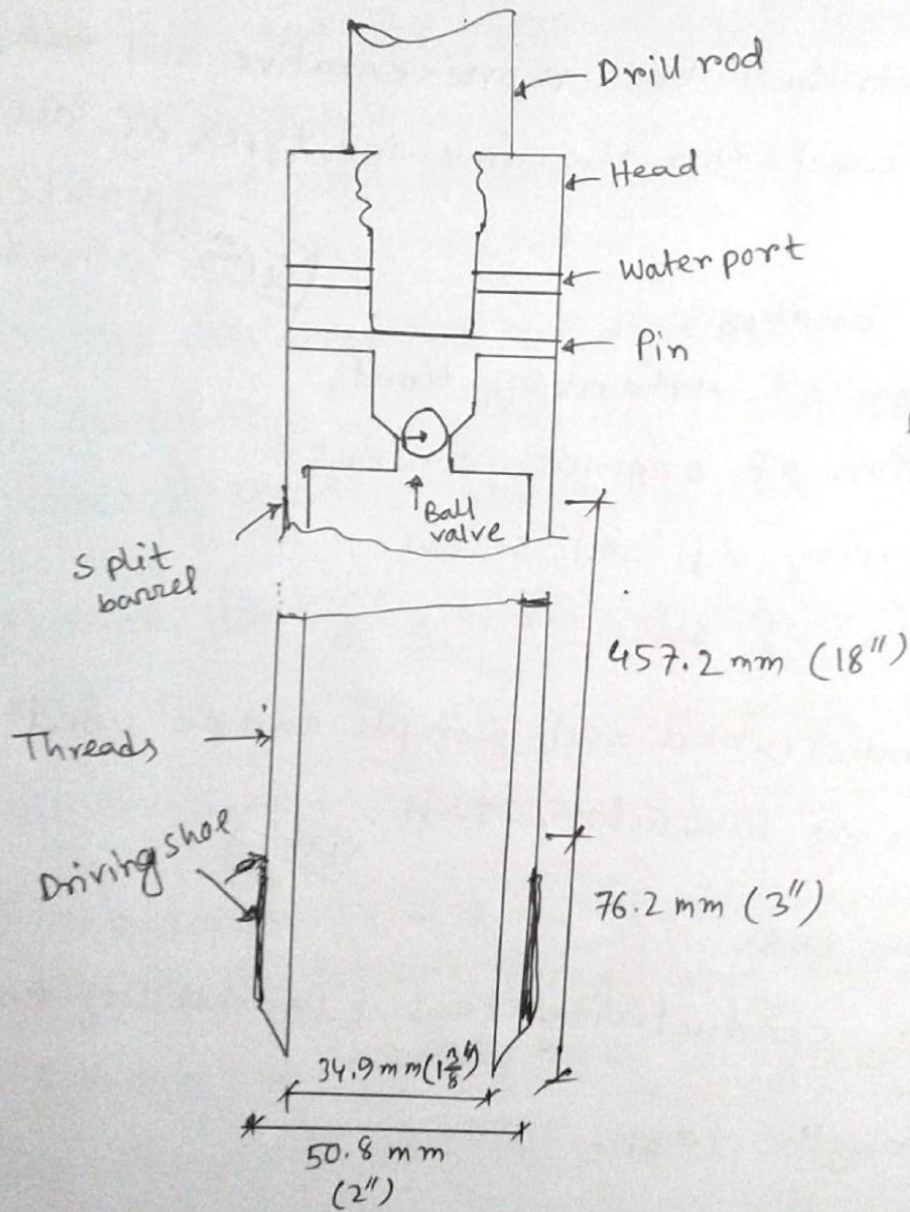


Fig (b) Spring core catcher.

Fig. (a) standard split-spoon sampler.

It consists of a tool-~~rod~~ steel driving shoe, a steel tube that is split longitudinally in half, and a coupling at the top. The coupling connects the sampler to the drill rod.

The standard split tube has an inside dia of 34.93mm and outside dia 50.8 mm; however samplers that have inside and outside dia up to 63.5 mm and 76.2mm, respectively, are also available.

Q. what do you mean by Standard Penetration Number or N value or SPT value/number?

Answer: Standard Penetration Number:-

When a borehole is extended to a predetermined depth, the drill tools are removed and sampler is lowered to the bottom of the borehole. The sampler is driven into soil by hammer blows to the top of the drill rod. The standard weight of hammer is 622.72 N (140 lb) and for each blow, the hammer drops a distance of 762 mm (30").

The number of blows required for a spoon penetration of three 152.4 mm intervals are recorded.

The number of blows required for the last two ^{6"} intervals (12") are added to give the standard penetration number, N, at that depth.

This number is generally referred to as 'N' value.

Q. what do you know by N_{60} ? explain.

Answer: N_{60} -Values: It is important to point out that several factors contribute to the variation of the standard penetration number 'N' at a given depth for similar soil profiles. Among these factors are.

- (a) SPT hammer efficiency.
- (b) Borehole diameter.
- (c) sampling method.
- (d) Rod length.

(Skempton; 1986; Seed, et al. 1985).

Two most common types of SPT hammers used in the field are safety hammer and donut hammer.

They are commonly dropped by a rope with two wraps around a pulley.

The SPT hammer energy efficiency can be expressed as,

$$E_r (\%) = \frac{\text{actual hammer energy to the sampler}}{\text{input energy}} \times 100$$

Theoretical input energy = Wh

where, W = weight of the hammer = 0.623 kN or 140 lb

h = height of drop = 0.76 m or 30 inch

$$\text{So, } Wh = 0.623 \times 0.76 = 0.474 \text{ kN-m (4200 in-lb)}$$

In the field, magnitude of E_r can vary from 30 to 90%. The standard practice now in the U.S is to express the N -value to an average energy ratio of 60% ($\approx N_{60}$)

Thus correcting for field procedures and on the basis of field observations, it appears reasonable to standardize in the field penetration number as a function of the input driving energy and its dissipation around the sampler into the surrounding soil, or.

$$N_{60} = \frac{N \eta_H \cdot \eta_B \cdot \eta_s \cdot \eta_R}{60}$$

where, N_{60} = standard penetration number, corrected for field conditions.

N = measured penetration number.

η_H = hammer efficiency (%)

η_B = correction for borehole diameter.

η_s = sampler correction.

η_R = correction for rod length.

Q. How you collect undisturbed soil from field?

Answer: Collection of Undisturbed sample from field:-

There are two ways to collection of undisturbed soil.

(I) Sampling by Thin wall Tube.

(II) Sampling by piston sampler.

Sampling by Thin wall Tube:

Thin wall Tube is used for obtaining fairly undisturbed soil samples. It is made of seamless steel (thin tubes) and commonly are referred to as Shelby tubes.

- (a) To collect samples at a given depth in a borehole, one first must remove drilling tools.
- (b) The sampler is attached to a drilling rod and lowered to the bottom of borehole.
- (c) After this, it is pushed hydraulically into the soil.
- (d) Then it is spun to shear off the base and is pulled out.
- (e) The sampler with soil inside is sealed and taken to laboratory for testing.
- (f) Most commonly used thin-wall tube samplers have outside diameter of 76.2mm (3 inch)

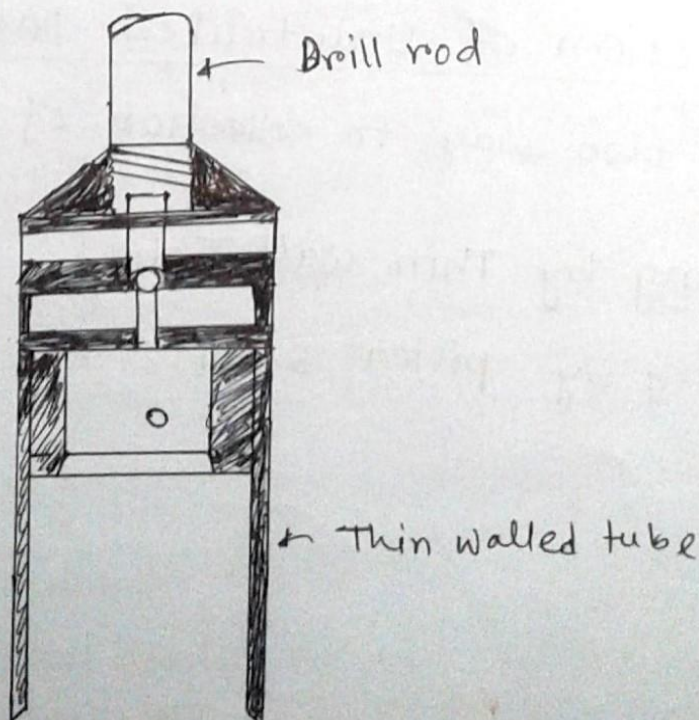


Fig. Thin-walled tube.

Sampling by Piston sampler :

Sampling by piston samplers is particularly useful when highly undisturbed samples are required.

- It consists of a thin-wall tube with a piston.
- Initially, the piston closes the end of the tube.
- The sampler is first lowered to bottom of the borehole.
- The thin wall tube is pushed into the soil hydraulically, past the piston.
- After this pressure is released through a hole in a piston rod.
- To a large extent, the presence of piston prevents distortion in the sample by not letting the soil squeeze into the sampling tube very fast and by not admitting excess soil.
- Consequently, samples obtained in this manner are less disturbed than those obtained by Shelby tubes.

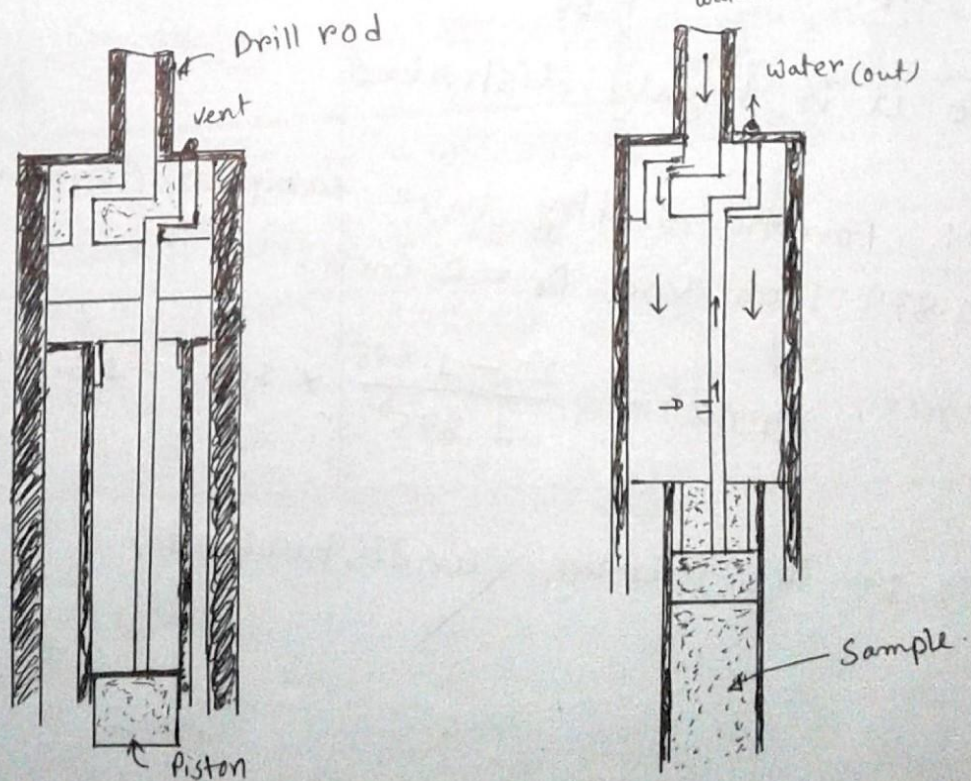


Fig. Piston sampler (a) sampler at the bottom of borehole
(b) Tube pushed into the soil hydraulically

Area ratio: The Degree of disturbance for a soil sample is collected by various methods can be expressed by a term called area ratio, which is given by,

$$A_R(\%) = \frac{D_o^2 - D_i^2}{D_o^2} \times 100$$

where, A_R = area ratio (ratio of disturbed area to total area of soil)

D_o = outside diameter of the sampling tube

D_i = inside diameter of the sampling tube.

A soil sample is generally can be considered to be undisturbed, when the area ratio is less than or equal to 10%.

Problem: For the standard split-spoon sampler, $D_i = 1.38$ inch and $D_o = 2$ inch. Here,

$$A_R(\%) = \frac{2^2 - 1.38^2}{2^2} \times 100 = 110\%$$

So it is highly disturbed.

Problem: For the Shelby tube sampler (2 inch diameter) $D_i = 1.875$ inch and $D_o = 2$ inch.

$$\text{Hence, } A_R(\%) = \frac{2^2 - 1.875^2}{2^2} \times 100 = 13.8\%$$

So, It is fairly undisturbed.

Correlation for Standard Penetration Test:

Cohesive Soil: The consistency of clay soils can be estimated from the standard penetration number N_{60} . In order to achieve that, Szechy and Vargi (1978) calculated the consistency Index (CI) as,

$$CI = \frac{LL - w}{LL - PL}$$

, where,

w = natural moisture content.

LL = Liquid limit.

PL = Plastic limit.

The approximate correlation between CI, N_{60} and the unconfined compression strength q_u is given in the following table.

Standard penetration number, N_{60}	Consistency	CI	Unconfined compression strength, q_u (KN/m^2) / (lb/ft^2)
< 2	Very soft	< 0.5	< 25 / 500
2-8	Soft to medium	0.5-0.75	25-80
8-15	Stiff	0.75-1.0	80-150
15-30	Very stiff	1.0-1.5	150-400
> 30	Hard	> 1.5	> 400

Hara, et al. (1971) also suggested the following correlation between the undrained shear strength of clay (c_u) and N_{60} .

$$\frac{c_u}{P_a} = 0.29 N_{60}^{0.72}$$

where, P_a = atmospheric pressure ($\approx 100 \text{ kN/m}^2 \approx 2000 \text{ lb/m}^2$)

The overconsolidation ratio, OCR of a natural clay deposit can also be correlated with the standard penetration number. On the basis of the regression analysis of 110 data points, Mayne and Kemper (1986) obtained the relationship.

$$\text{OCR} = 0.193 \left(\frac{N_{60}}{\sigma'_0} \right)^{0.689}$$

where, σ'_0 = effective vertical stress in MN/m^2 .

Correlation for Granular soil:

Granular soil: The standard penetration number N_{60} obtained from field needs to be corrected for following cases:-

- (I) Correction for overburden pressure.
- (II) Correction for submergence.

Correction for overburden pressure:-

In granular soils, the value of N is affected by the effective overburden pressure, σ_0' . For that reason, the value of N_{60} obtained from field exploration under different effective overburden pressure should be changed to correspond to a standard value of σ_0' .

That is,

$$(N_1)_{60} = C_N N_{60}$$

where, $(N_1)_{60}$ = value of N_{60} corrected to a standard value of σ_0' [100 kN/m² (2000 lb/ft²)]

C_N = correction factor.

N_{60} = value of N obtained from field exploration.

Several correlations has been developed over the years for the correction factor, C_N .

The Most commonly cited relationship are those of Liao and Whitman (1986) and Skempton (1986).

2013 Q. what are the various corrections needed in SPT value?
Liao and Whitman's relationship (1986): 2013

$$C_N = \left[\frac{1}{\frac{\sigma_o'}{P_a}} \right]^{0.5} \quad [\text{SI units}]$$

Where, σ_o' = effective over burden pressure in KN/m^2

P_a = atmospheric pressure (100 KN/m^2)

$$C_N = \sqrt{\frac{1}{\sigma_o'}} \quad [\text{English units}]$$

Skempton's relationship (1986): 2013

SI units:

$$C_N = \frac{2}{1 + \left(\frac{\sigma_o'}{P_a} \right)}$$

[For normally consolidated fine sand.]

$$C_N = \frac{3}{2 + \left(\frac{\sigma_o'}{P_a} \right)}$$

[For normally consolidated coarse sand]

$$C_N = \frac{1.7}{0.7 + \left(\frac{\sigma_o'}{P_a} \right)}$$

[For over consolidated sand]

Where, σ_o' = effective over burden pressure.

P_a = Atmospheric pressure (100 KN/m^2)

Skempton's relationship (1986): 2013

For English units:

$$C_N = \frac{2}{1 + \sigma_0'}$$

[For normally consolidated fine sand]

$$C_N = \frac{3}{2 + \sigma_0'}$$

[For normally consolidated coarse sand]

$$C_N = \frac{1.7}{0.7 + \sigma_0'}$$

[For over consolidated sand]

Peck et al relationship (1974): - 2013

$$C_N = 1 - 1.25 \log \left(\frac{\sigma_0'}{P_a} \right)$$

Bazaraa (1967): 2013

$$C_N = \frac{4}{1 + 4 \left(\frac{\sigma_0'}{P_a} \right)}$$

(for $\frac{\sigma_0'}{P_a} \leq 0.75$)

$$C_N = \frac{4}{3.25 + \left(\frac{\sigma_0'}{P_a} \right)}$$

[for $\frac{\sigma_0'}{P_a} > 0.75$]

Correction for submergence :-

In very fine, silty, saturated sand an apparent increase in resistance occurs.

Terzaghi and Peck have recommended use of an equivalent penetration resistance $(N_1)_{60}$ in place of actually observed value of N_{60} . When N_{60} is greater than 15.

$$(N_1)_{60} = 15 + \frac{1}{2} (N_{60} - 15)$$

N_{60} এর মান 15 এর কম হলে corrected $(N_1)_{60}$ এর মান বের করা
যাবে না। অবশিষ্ট মান 15.

Correlation between N_{60} and Relative Density of Granular soil:

⇒ Kulhawy and Mayne (1990) modified an empirical relationships for relative density that was given by Marcuson and Bieganski (1977). Which can be expressed as,

$$D_r (\%) = 12.2 + 0.75 \left[222 N_{60} + 2311 - 711 \cdot OCR - 779 \frac{\sigma'_0}{P_a} - 50 C_u \right]^{0.5}$$

Where, D_r = relative density.

σ'_0 = effective overburden pressure.

C_u = uniformity co-efficient of sand.

$OCR = \frac{\text{preconsolidation pressure, } \sigma'_e}{\text{effective overburden pressure, } \sigma'_0}$

P_a = atmospheric pressure.

⇒ Meyerhof (1957) developed a correlation between D_r and N_{60} as.

$$N_{60} = \left[17 + 24 \left(\frac{\sigma'_0}{P_a} \right) \right] D_r^2$$

$$\text{or, } D_r = \left\{ \frac{N_{60}}{\left[17 + 24 \left(\frac{\sigma'_0}{P_a} \right) \right]} \right\}^{0.5}$$

The above equation provides a reasonable estimate only for clean, medium fine sand.

⇒ Cubrinovski and Ishihara (1999) also proposed a correlation between N_{60} and the relative density of sand (D_r) that can be expressed as,

$$D_r (\%) = \left[\frac{N_{60} \left(0.23 + \frac{0.06}{D_{50}} \right)^{1.7}}{9} \left(\frac{1}{\frac{\sigma'_0}{P_a}} \right) \right]^{0.5} \times 100$$

Where, P_a = atmospheric pressure [$\approx 100 \text{ kN/m}^2$]

D_{50} = Diameter through which 50% soil will pass through (mm).

Table: Relation betⁿ the corrected $(N_1)_{60}$ values and the Relative Density in sands.

Standard Penetration number, $(N_1)_{60}$	Approximate relative density, $D_r(\%)$
0-5	0-5
5-10	5-30
10-30	30-60
30-50	60-95

⇒ Kulhawy and Mayne (1990) correlated the corrected standard penetration number and the relative density of sand in the form

$$D_r(\%) = \left[\frac{(N_1)_{60}}{C_p C_A C_{OCR}} \right]^{0.5} \times (100)$$

where,

C_p = Grain-size correlations factor = $60 + 25 \log D_{50}$

C_A = correlation factor for aging = $1.2 + 0.05 \log \left(\frac{t}{100} \right)$

C_{OCR} = correlation factor for overconsolidation = $OCR^{0.18}$

D_{50} = diameter through which 50% soil will pass through (mm)

t = age of soil since deposition (years)

OCR = overconsolidation ratio.

Correlation between Angle of Friction and Standard Penetration Number:-

The peak friction angle, ϕ' , of granular soil has also been correlated with N_{60} or $(N_1)_{60}$ by several investigators. Some of these correlations are as follows:-

1. Peck, Hanson and Thornburn (1974) give a correlation between N_{60} and ϕ' in a graphical form, which can be approximated as (Wolff, 1989).

$$\phi' \text{ (deg)} = 27.1 + 0.3 N_{60} - 0.00054 [N_{60}]^2$$

2. Schmertmann (1975) provided the correlation between N_{60} , σ'_0 and ϕ . Mathematically the correlation can be approximated as [Kulhawy and Mayne, 1990]

$$\phi' = \tan^{-1} \left[\frac{N_{60}}{12.2 + 20.3 \left(\frac{\sigma'_0}{P_a} \right)} \right]^{0.34}$$

Where, N_{60} = field standard penetration number.

σ'_0 = effective overburden pressure.

P_a = atmospheric pressure in the same units as σ'_0 .

ϕ' = soil friction angle.

3 Hatanaka and Uchida (1996) provided a simple correlation between ϕ' and $(N_1)_{60}$ that can be expressed as

$$\phi' = \sqrt{20(N_1)_{60}} + 20$$

The following qualifications should be noted when standard penetration resistance values are used in the preceding correlations to estimate soil parameters:-

- (I) The equations are approximate.
- (II) Because the soil is not homogeneous, the values of N_{60} obtained from a given bore-hole vary widely.
- (III) In soil deposits that contain large boulders and gravel, standard penetration numbers may be erratic and unreliable.

Correlation between Modulus of Elasticity and Standard Penetration Number :-

The modulus of elasticity of granular soils (E_s) is an important parameter in estimating the elastic settlement of foundations. A first order estimation for E_s was given by Kulhaway and Mayne (1990) as.

$$\frac{E_s}{P_a} = \alpha N_{60}$$

Where, P_a = atmospheric pressure (same unit as E_s)

$$\alpha = \begin{cases} 5 & \text{for sands with fines} \\ 10 & \text{for clean normally consolidated sand.} \\ 15 & \text{for clean overconsolidated sand.} \end{cases}$$

Sources of error in standard penetration Test :-

Although approximate, with correct interpretation standard penetration test provides a good evaluation of soil properties.

Primary sources of error in standard penetration tests are :-

- (I) Inadequate cleaning of bore-hole.
- (II) Careless measurement of blow count.
- (III) Eccentric hammer strikes and drill rod.
- (IV) Inadequate maintenance of water head in bore hole.

OTHER IN SITU TEST :-

* Describe vane shear Test (VST)?

Answer: Vane shear Test:-

The vane shear test (ASTM D-2573) may be used during the drilling operation to determine the in situ undrained shear strength (c_u) of clay soils - particularly soft clays. The vane shear apparatus consists of four blades on the end of a rod.

The height, H , of the vane is twice the diameter, D . The vane can be either rectangular or tapered.

The vanes of the apparatus are pushed into the soil at the bottom of a borehole without disturbing the soil appreciably.

Torque is applied at the top of the rod to rotate the vanes at a standard rate of $0.1^\circ/\text{sec}$. This rotation will induce failure in a soil of cylindrical shape

surrounding the vanes. The maximum torque, T , applied to cause failure is measured. Note that,

$$T = f(c_u, H, \text{ and } D) \text{ or } c_u = \frac{T}{k}$$

where, T is in Nm , c_u is kN/m^2 and,

k = a constant with a magnitude depending on the dimension and shape of the vane.

The constant, $k = \left(\frac{\pi}{106} \right) \left(\frac{DH}{2} \right) \left(1 + \frac{D}{3H} \right) \rightarrow \text{SI units.}$

where, D = diameter of vane in cm.

H = measured height of vane in cm.

In English Units;

$$k = \left(\frac{\pi}{1728} \right) \left(\frac{D^2 H}{2} \right) \left(1 + \frac{D}{3H} \right)$$

where, D = diameter of vane in inch

H = measured height of vane in inch.

Field vane shear tests are moderately rapid and economical and are used extensively in field soil-exploration programs. The test gives good results in soft and medium stiff clays and gives excellent results in determining the properties of sensitive clays.

Sources of significant error in the field vane shear test are poor calibration of torque measurement and damaged vanes. Other errors may be introduced if the rate of rotation of the vane is not properly controlled.

For actual design purposes, the undrained shear strength values obtained from field vane shear tests [$c_u(vst)$] are too high, and it is recommended that they be corrected according to the equation.

$$c_u(\text{corrected}) = \lambda c_u(vst)$$

where λ = correction factor.

Several correlations have been given previously for the correction factor λ . The most commonly used correlation for λ is that given by Bjerrum (1972), which can be expressed as.

$$\lambda = 1.7 - 0.54 \log [PI(\%)]$$

Morris and Williams (1994) provided the following correlations:-

$$\lambda = 1.18 e^{-0.08(PI)} + 0.57 \quad \rightarrow \quad \text{For } PI > 5$$

$$\lambda = 7.01 e^{-0.08(LL)} + 0.57 \quad \rightarrow \quad \text{Where } LL \text{ is in } \%$$

The field vane shear strength can be correlated with the preconsolidation pressure and the overconsolidation ratio of the clay. Using 343 data points,

Mayne and Mitchell (1988) derived the following empirical relationship for estimating the preconsolidation pressure of a natural clay deposits:-

$$\sigma_e' = 7.04 [c_u(\text{field})]^{0.83}$$

Here, σ_e' = preconsolidation pressure (KN/m²)

$c_u(\text{field})$ = field vane shear strength (KN/m²)

The overconsolidation ratio, OCR, also can be correlated to $c_u(\text{field})$ according to the equation

$$\text{OCR} = \beta \frac{c_u(\text{field})}{\sigma'_0}$$

where, σ'_0 = effective overburden pressure.

The magnitudes of β developed by various investigators are given below.

* Mayne and Mitchell (1988):

$$\beta = 22 [\text{PI}(\%)]^{-0.48}$$

* Hansbo (1957):

$$\beta = \frac{222}{\omega(\%)}$$

* Larsson (1980):

$$\beta = \frac{1}{0.08 + 0.0055(\text{PI})}$$

→ 2015, 14, 12, 11, 10, 09, 08

Question: what is N value? what is the significance of N-value in geotechnical Engineering?

Answer: N-value: In split spoon sampling, the sum of number of blows required for spoon penetration of the last two 6 inch intervals is referred to as standard penetration number at that depth. This number is generally referred to as 'N' value.

Significance of N-value:

(I) The consistency of clay soil can often be estimated from N-value.

N-value	consistency.
0-2	very soft.
2-5	soft -
5-10	Medium stiff.
10-20	stiff.
20-30	very stiff.
>30	Hard.

(II) The correlation between undrained shear strength of clay and N obtained from field, is given as,

$$C_u = k N_f.$$

where, $k = \text{constant} = 3.5 \text{ to } 6.5 \text{ kN/m}^2$

(III) The approximate relationship between corrected N and relative density of sand,

$$D_r(\%) = 11.7 + 0.76 (22.2 N_{60} + 1600 - 7.68 \sigma'_v - 50 C_u)^{0.5} \quad \text{--- (1)}$$

N_{cor}

$D_r(\%)$

0-5

0-5

5-10

5-30

10-30

30-60

30-50

60-95

(IV) The peak angle of friction of granular soil ϕ has correlation to the corrected N is given by Wolf, 1989,

$$\phi(\text{deg}) = 27.1 + 0.3 N_{cor} - 0.00054 N_{cor}^2$$

So, N is useful guideline in soil exploration and provide a good evaluation of soil properties.

(V) Net allowable bearing capacity can be calculated from N .

Sand, $B = \frac{\text{Depth}}{\text{width of the foundation}}$

(I) for $B \leq 4 \text{ ft}$, $q_{net} = \frac{N_{cor}}{4} \text{ k/ft}^2$

(II) for $B > 4 \text{ ft}$, $q_{net} = \frac{N_{cor}}{6} \left(\frac{B+1}{B}\right)^2 \text{ k/ft}^2$

Clay, $q_{net} = N \times 0.15 \text{ ton/ft}^2$

Q. when we use N_{corr} instead of N ? 2011

Answer: (I) N_{corr} will apply for sand and silty under submergence.

(II) N_{corr} will not apply for clay soil for having water.

(III) $(N_1)_{60} = N_{corr} = C_N \times N_{60}$

$C_N = \text{correction factor}$.

Q. write short notes on CPT (cone penetration test)

Answer: Cone penetration test:

This test is performed to

- (I) To Determine materials in a soil profile. and
- (II) Estimate their Engineering properties.

Procedure:

A 60° cone with a base area 10cm^2 is pushed into the ground at a steady rate of about 20 mm/sec. and resistance of penetration is measured as frictional and cone resistance and their ratio is the friction ratio.

$$Fr = \frac{\text{frictional resistance}}{\text{cone resistance}} = \frac{f_c}{q_c}$$

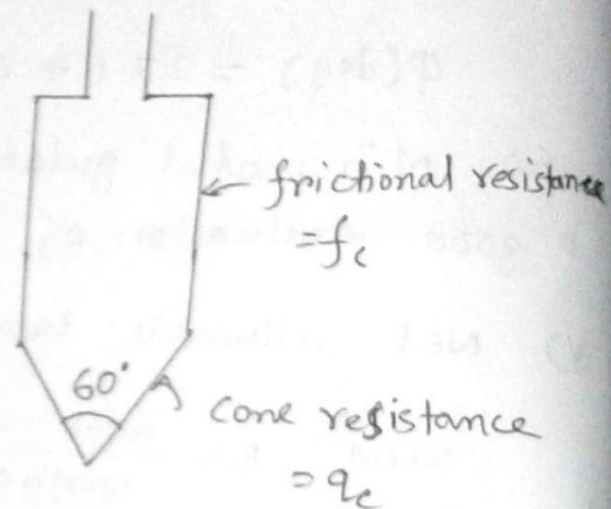


Fig. CPT apparatus.

By knowing c, ϕ , the bearing capacity of soil and D_r also determined by electrical and mechanical friction penetrometer. For there, it is also called Dutch cone penetration test.

Cone resistance: q_c To penetration developed by cone which is equal to vertical force applied to cone divided by its horizontally projected area.

Frictional resistance, f_c : Which is resistance measured by a sleeve located above cone with local soil surrounding it.

Frictional resistance is equal to vertical force applied to sleeve, divided by its surface area actually, sum of friction and adhesion.

* Write short note on PMT (Pressure Meter Test).

Pressure Meter Test: Strength and deformability of soil is determined by pressure meter test.

Procedure:

- (I) Measuring cell volume V_0 is measured.
- (II) The probe is inserted in the borehole.
- (III) Pressure is applied and volumetric expansion of cell is measured.
- (IV) Application of pressure is continued until the soil fails or pressure limit of device is reached.

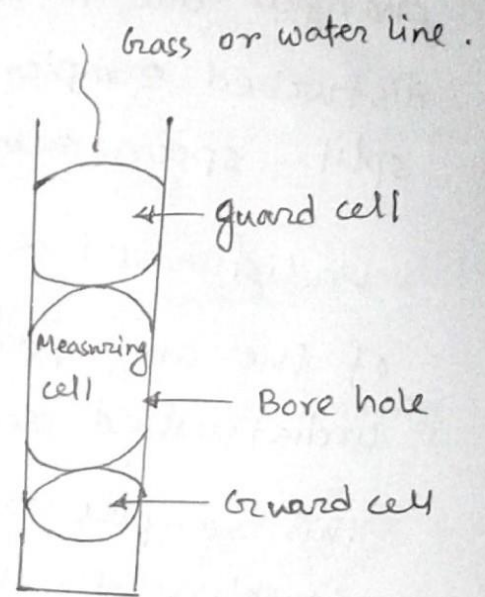


Fig. Pressure meter.

(V) Pressuremeter modulus, $E_p = 2(\mu + 1)$

$(V_0 + V_m) \times \frac{\Delta P}{\Delta V}$ is determined.

Here, $V_m = \frac{V_0 + V_f}{2}$, $\Delta P = P_f - P_0$

$\Delta V = V_f - V_0$, $\mu = \text{poisson's ratio.}$

2015, 2010, 2011
Q. What do you mean by disturbed and undisturbed sample.
How would you obtain undisturbed sample from field.

Answer: Disturbed sample: When there any parameter of soil such as specific gravity, grain size etc will be changed due to some reasons, then the sample is defined as disturbed sample. Disturbed samples are collected by the split spoon sampler.

Undisturbed sample: At in which sample, no change of the any parameter due to any reason is called undisturbed sample.

This samples are collected by piston sampling method, sampling by thin wall tube etc.

Collection of undisturbed sample:

Undisturbed samples are collected by forcing thin wall tube sampler into soil at the bottom of the bore hole. The penetration of the sampler into the soil should be continuous and rapid. The sampler should never be over driven so as to compress the sample.

Q. what is Boring log? 2013

Answer: Boring log: The detail informations are gathered from each bore hole is presented in a graphical form is called a Boring log.

where graphical representation of the boring hole is called the soil strata. It should have.

- (I) Drillers name
- (II) Date of Drilling.
- (III) Depth of GWT.
- (IV) Elevation of W.T.
- (V) Spoon penetration resistance.

Q. write down the subject matter of the soil exploration report. 2013

Answer: Soil Exploration Report: The followings are having seen in soil exploration report.

- (I) scope investigation.
- (II) Geologic conditions of the site.
- (III) Drainage facilities at the site.
- (IV) Details of boring.
- (V) Description of subsoil condition.
- (VI) Ground water table.
- (VII) Details of foundation recommendations and alternatives.
- (VIII) Any anticipated construction problems.
- (IX) Limitations of the investigation.
- (X) Site location map.
- (XI) Location of boring.
- (XII) Boring logs.
- (XIII) Laboratory test results.
- (XIV) Other special presentations if any.

SC 2274
S262 D4ABOL3

Q. what are the factors affecting N-value? why N-needed to be corrected? 2015

Answer: Factors affecting N-value: HET ITHE

- (I) Homogeneity of soil helps widely to get N.
- (II) Effective overburden pressure granular soil.
- (III) Type of soil - boulders, gravel, due to them, N will be unreliable.

Where (a) sandy soil resists the penetration and
(b) silty soil for it, the N will be varied for its physical significant.

That's why the correction of N value is needed.

ডাঃ ববির্চন ইসলাম
রাজশাহী প্রকৌশল ও প্রযুক্তি বিশ্ববিদ্যালয়
প্রকৌশল বিভাগ
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স্বাক্ষর

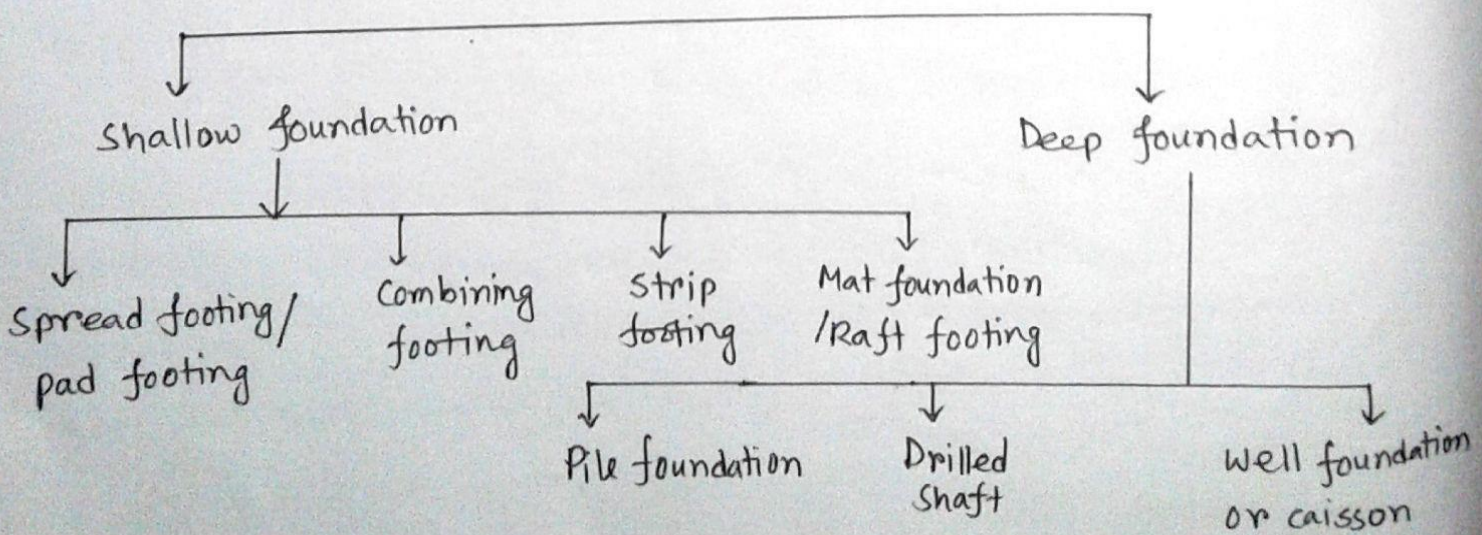
Bearing Capacity Of Shallow Foundation

Q. Define Foundation and Footing. Classify foundation.

Foundation: The lowest part of a structure which is transfer the load of the superstructure into the soil on which it is ~~rested~~ resting.

Footing: A footing is a foundation unit constructed in brick work masonry or concrete under base of a wall or a column for purpose of distributing load over a large area of soil on which it, resist on.

Types of Foundation:-



Spread footing: A spread footing is simply an enlargement of a load-bearing wall or column that makes it possible to spread the load of the structure over a larger area of the soil.

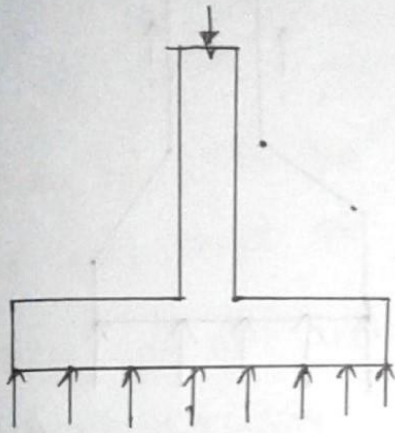


Fig. spread footing.

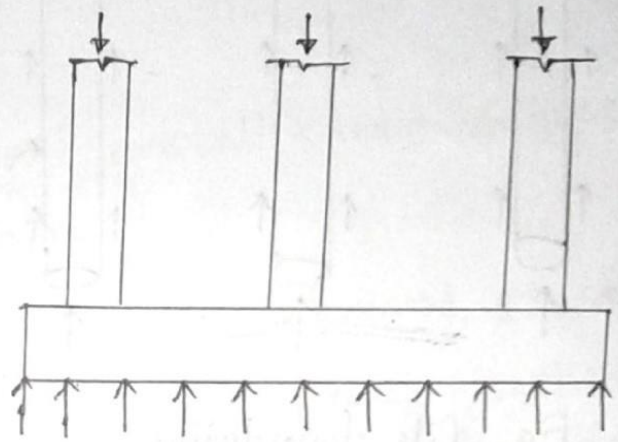


Fig. Mat foundation.

Mat foundation: In soil with low load bearing capacity, the size of the spread footings required is impracticably large. In that case, it is more economical to construct the entire structure over a concrete pad. This is called a mat foundation.

Pile foundation: Piles are structural members made of timber, concrete or steel that transmit the load of the superstructure to the lower layers of the soil.

According to how they transmit their load into the subsoil, piles can be divided into two categories:

- (i) Friction piles: In this case, the superstructure load is resisted by the shear stresses generated along the surface of the pile.
- (ii) End-Bearing piles: In this case, the load carried by the pile is transmitted at its tip to a firm stratum.

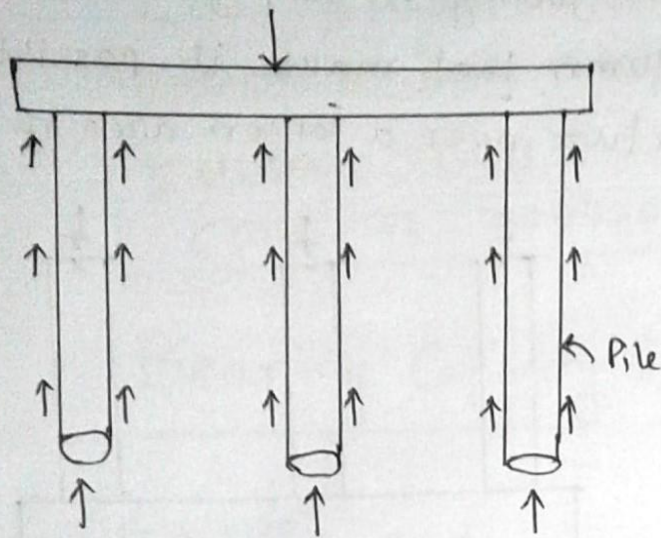


Fig. Pile foundation.

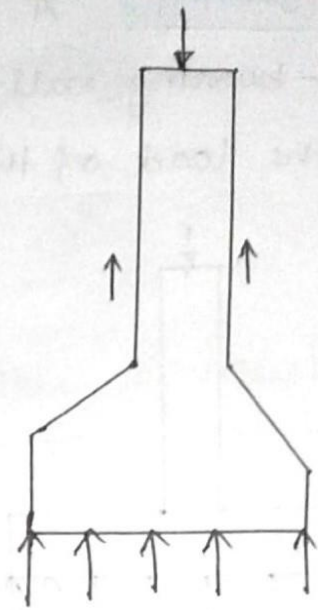


Fig. Drilled shaft foundation.

Drilled shaft foundation: In case of drilled shafts, a shaft is drilled into the subsoil and is then filled with concrete. A metal casing may be used while the shaft is being drilled. The casing may be left in place or may be withdrawn during the placing of concrete. Generally, the diameter of a drilled shaft is much larger than that of a pile. The distinction between piles and drilled shafts becomes hazy at an approximate diameter of 3 ft.

Well Foundation / Caisson: A large watertight retaining structure used to work on foundations of a bridge pier or for construction of a concrete dam. These are constructed such that water can be pumped out keeping the working environment dry and in which construction work may be carried out under water.

Q. what are the two main characteristics of shallow foundation to perform satisfactorily.

Answer; To perform satisfactorily, shallow foundation must have two main characteristics:—

- (I) They have to be safe against overall shear failure in the soil that support them.
- (II) They cannot undergo excessive displacement or settlement (The term excessive is relative, because degree of settlement allowed for a structure depends on several considerations).

Q. Define Ultimate Bearing capacity of soil. 2005

Answer; Ultimate Bearing capacity of soil: The maximum load per unit area of the foundation at which shear failure in soil occurs is called ultimate bearing capacity.

Shallow foundation: shallow foundations are foundations that have a depth of -embedment -to -width ratio of approximately less than four. ($\frac{B}{W} < 4$.)

Deep foundation: Deep foundations are foundations that have a depth of -embedment -to -width ratio of a foundation is greater than four ($\frac{B}{W} > 4$).

Q. Discuss with neat sketches the nature of bearing capacity failure of soil. 2015, 2014, 2013

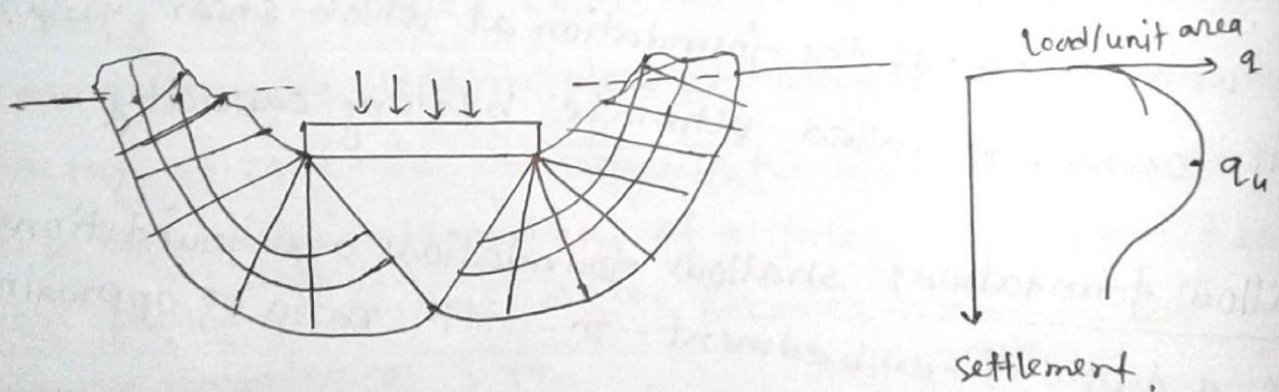
Answer: Modes of shear Failure of soil: Three modes are present.

- (I) General shear failure.
- (II) Local shear failure.
- (III) Punching shear failure.

GLP

General shear failure:

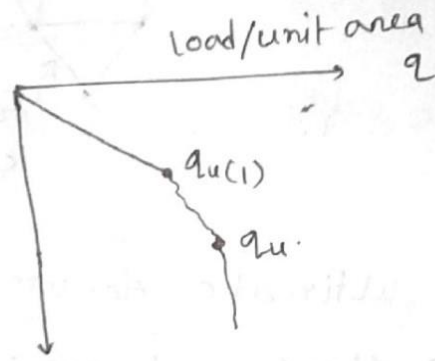
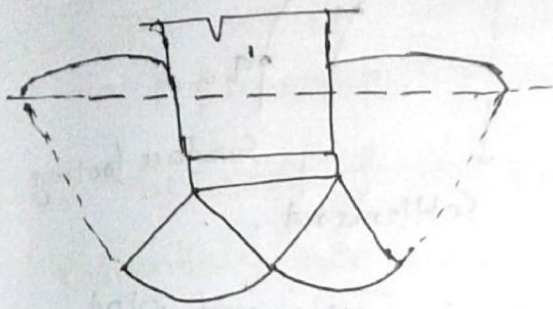
In this type of failure, Failure surface in soil will extend to the ground surface. It occurs in a dense ϕ sand or stiff cohesive soil.



If a load is gradually applied to the foundation, settlement will increase. At a certain point when load per unit area equals to q_u - a sudden failure in soil supporting foundation will take place. This load per unit area, is usually referred to as ultimate bearing capacity of the foundation. When such sudden failure in soil takes place, it is called general shear failure.

Local shear failure:-

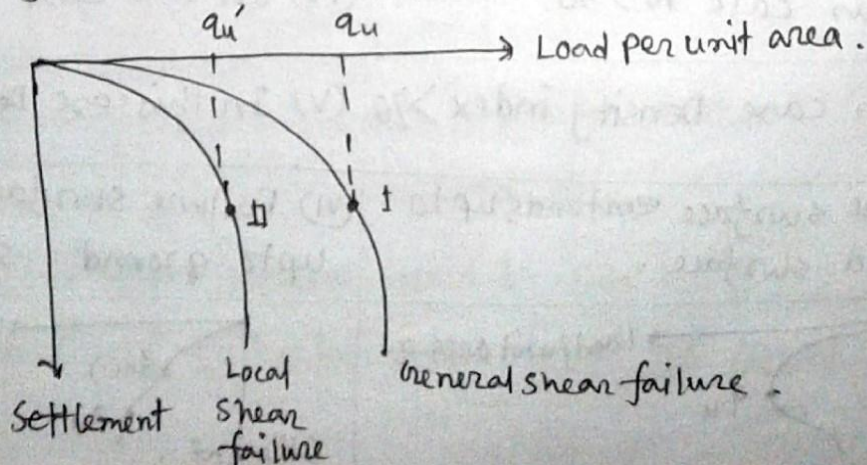
In this case, failure surface in soil will gradually extend outward from foundation and do not or little extend to ground surface. It occurs in sand in clayey soil of medium compaction.



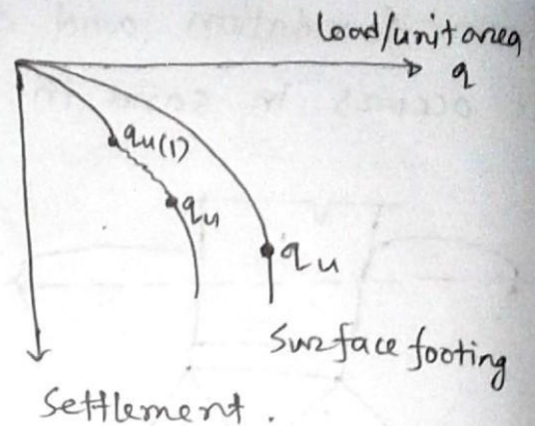
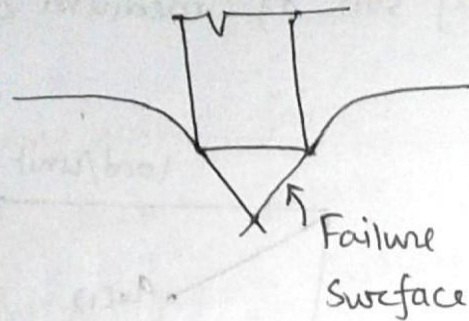
When load per unit area on foundation equals $q_{u(1)}$ movement of foundation will be accompanied by sudden jerks. A considerable movement of foundation is then required for failure surface in soil to extend to ground surface. Load per unit area at which this happens is ultimate bearing capacity q_u .

Beyond that point, an increase in load will be accompanied by a large increase in foundation settlement. Load per unit area of foundation $q_{u(1)}$ is referred to as first failure load.

A peak value of q is not realized in this type of failure.



Punching shear failure: In this case, failure surface in soil will not extend to ground surface. It occurs in a fairly loose soil.

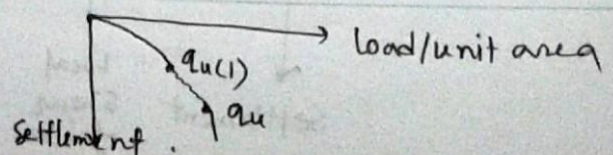
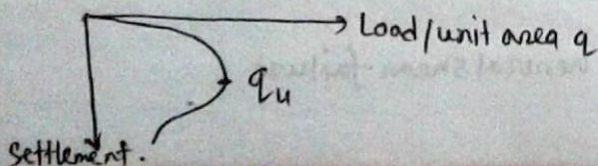


Beyond ultimate failure load q_u , load settlement plot will be steep and practically linear.

Q: Differentiate between general shear failure and local shear failure. 01, 02, 07, 08

Answer:

General shear failure	Local shear failure.
(I) It occurs in dense soil	(I) It occurs in fairly soft or loose soil.
(II) Downward movement is slight.	(II) Downward movement is much.
(III) In this case $\phi > 36^\circ$, ϕ = Angle of shearing resistance	(III) In this case $\phi < 28^\circ$
(IV) In this case $N > 30$.	(IV) In this case $N < 5$
(V) In this case Density index > 70	(V) In this case Density index < 2.0 <small>For</small>
(VI) Failure surface extends up to ground surface.	(VI) Failure surface does not extend up to ground surface.



Q. What are the assumptions made in the derivation of Terzaghi's bearing capacity theory?

Answer:

Assumption:

- (I) The soil is homogeneous and isotropic, and its shear strength is represented by Coulomb's equation.
- (II) The strip footing has a rough base, and the problem is essentially two dimensional.
- (III) The elastic zone has straight boundaries inclined at $\psi = \phi$ to the horizontal and the plastic zones fully develop.
- (IV) Passive pressure consists of three components which can be calculated separately.
- (V) Failure zones do not extend above the horizontal plane through the base of the footing.

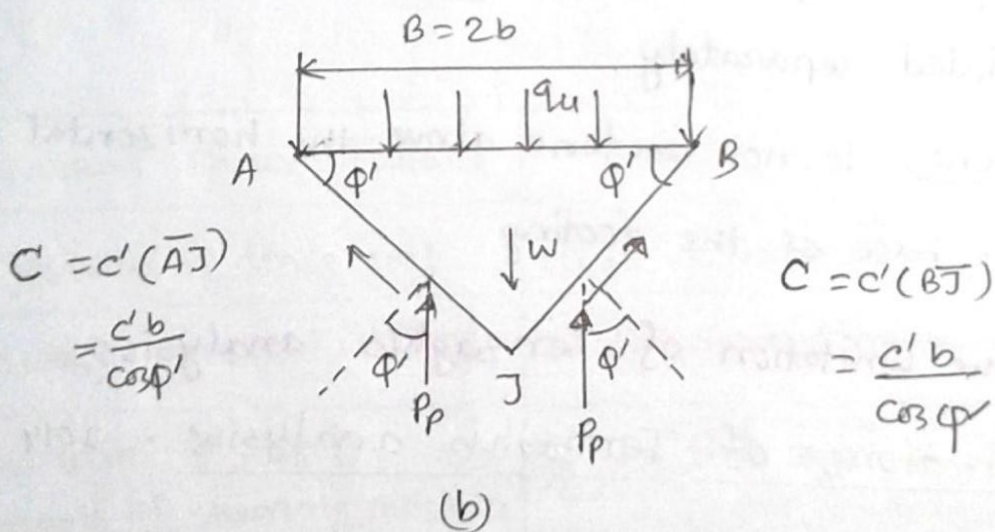
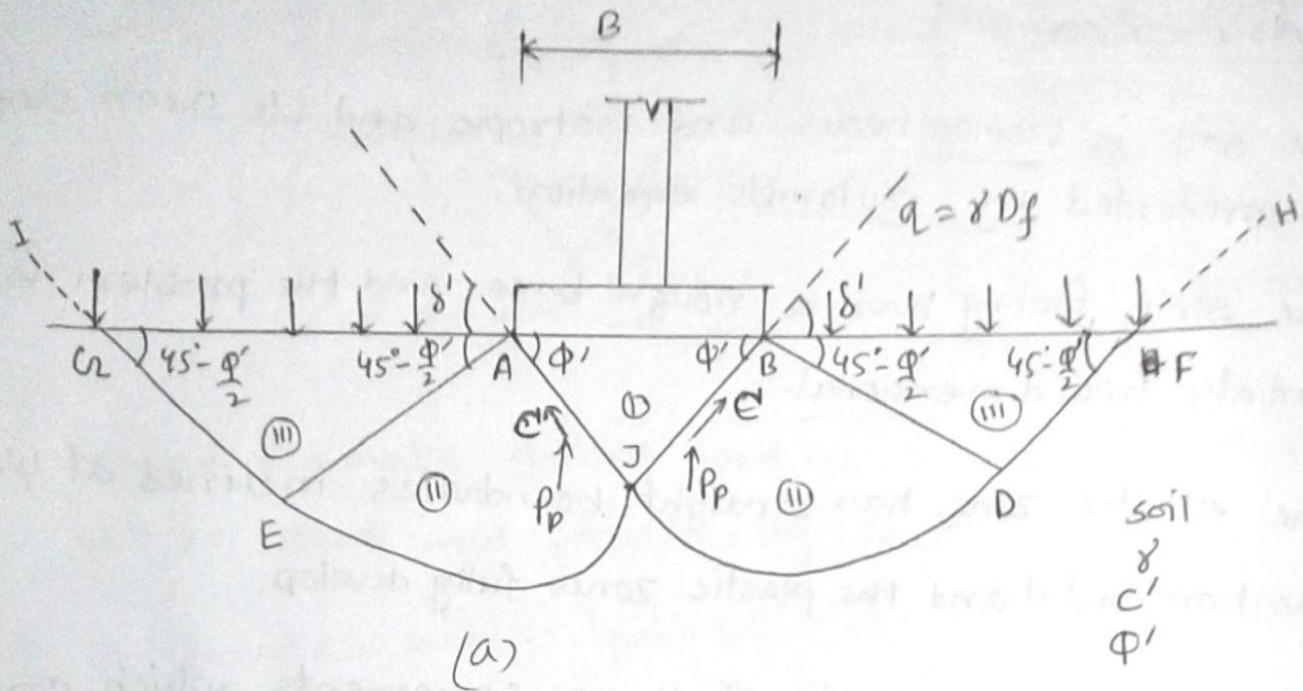
Q. What are the limitations of Terzaghi's analysis?

Answer: Limitations of Terzaghi's analysis: 2014

- (I) $D_f \leq B$.
- (II) No sliding between footing and soil.
- (III) Soil is a homogeneous semi-infinite mass.
- (IV) Failure plane angle is equal to ϕ' .
- (V) Not applicable for inclined load & rectangular foundation.
- (VI) No resistance of soil above the level of base foundation.

Q. Derive the Terzaghi's ultimate bearing capacity equation.

Answer:



Terzaghi suggested that for a continuous, or strip foundation failure surface in soil at ultimate load may be assumed to be similar to that shown in figure (a).

The effect of above the bottom of the foundation may assumed to be replaced by an equivalent surcharge, $q = \gamma D_f$.

The failure zone, under the foundation can be separated into three parts. :-

- (I) Triangular zone ABJ (immediately under foundation elastic zone)
- (II) Radial shear zones AJE and BJD with curves JE and JD being arcs of a logarithmic spiral.
- (III) Two triangular Rankine passive zones AEG and BDF.

Angles BAJ and ABJ are assumed to be equal to soil friction angle ϕ' .

With the replacement of soil above bottom of foundation by an equivalent surcharge q , shear resistance of soil along failure surface GF and FH was neglected.

The equation of arcs and of the logarithmic spirals JD and JE may be given as,

$$r = r_0 e^{\theta \tan \phi'}$$

If the load per unit area q_u is applied to the footing and general shear failure occurs, passive force P_p is acting on each of faces of soil wedge ABJ. P_p is inclined at an angle δ' to perpendicular drawn to wedge faces. In this case δ' should be equal to angle of friction of soil, ϕ' .

Now let us consider figure (b)

Considering unit length of the footing, we have for equilibrium.

$$(q_u)(2b)(1) = -W + 2C \sin \phi' + 2P_p$$

Where, $W =$ weight of soil wedge, $ABJ = \gamma b^2 \tan \phi'$

$C =$ cohesive force acting along each face $= \frac{c'b}{\cos \phi'}$

The passive pressure is sum of contribution of weight of soil γ , cohesion c' and surcharge q , and can be expressed as,

$$P_p = \frac{1}{2} \gamma (b \tan \phi')^2 K_p + c' (b \tan \phi') K_c + q (b \tan \phi') K_q$$

where, K_p , K_c , K_q are earth pressure co-efficient.

$$q_u = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma$$

where, $N_c = \tan \phi' (K_c + 1)$

$$N_q = K_q \tan \phi'$$

$$N_\gamma = \frac{1}{2} \tan \phi' (K_\gamma \tan \phi' - 1)$$

N_c , N_q and N_γ are respectively contributions of cohesion, surcharge and unit of soil to ultimate load bearing capacity.

If, $\gamma = 0$ (weightless soil), $c' = 0$ then,

$$q_u = q_q = q N_q.$$

By method of superimposition, when effects of unit weight of soil, cohesion and surcharge are considered we have

$$q_u = q_c + q_a + q_\gamma = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma$$

$$\therefore \boxed{q_u = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma}$$

This equation is referred to as Terzaghi's bearing capacity equation.

This equation is derived for general shear failure.

Value of q_u for different footing:

$$q_u = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma \rightarrow \text{strip footing}$$

$$q_u = 1.3 c' N_c + q N_q + 0.4 \gamma B N_\gamma \rightarrow \text{square footing}$$

$$q_u = 1.3 c' N_c + q N_q + 0.3 \gamma B N_\gamma \rightarrow \text{circular footing}$$

$$N_c = \cot \phi' \left[\frac{e^{2(3\pi/4 - \frac{\phi'}{2}) \tan \phi'}}{2 \cos^2(\frac{\pi}{4} + \frac{\phi'}{2})} - 1 \right] = \cot \phi' (N_q - 1)$$

$$N_q = \frac{e^{2(3\pi/4 - \frac{\phi'}{2}) \tan \phi'}}{2 \cos^2(45 + \frac{\phi'}{2})}$$

$$N_\gamma = \frac{1}{2} \left(\frac{k_p \gamma}{\cos^2 \phi'} - 1 \right) \tan \phi'$$

where, $k_p \gamma$ = passive pressure coefficient.

Q. How the ultimate bearing capacity in local shear is determined.
In case of local shear failure, we may assume that:

$$\bar{c}' = \frac{2}{3} c' \quad \text{and} \quad \tan \bar{\phi}' = \frac{2}{3} \tan \phi'$$

$$\therefore q_u' = \bar{c}' N_c' + q N_q' + \frac{1}{2} \gamma B N_\gamma' \rightarrow \text{strip footing}$$

$$q_u' = 1.3 \bar{c}' N_c' + q N_q' + 0.4 \gamma B N_\gamma' \rightarrow \text{square footing}$$

$$q_u' = 1.3 \bar{c}' N_c' + q N_q' + 0.3 \gamma B N_\gamma' \rightarrow \text{circular footing}$$

Substituting $\bar{\phi}' = \tan^{-1}(\frac{2}{3} \tan \phi')$ for ϕ'

General Bearing capacity equation of Meyerhof:

Meyerhof (1963) suggested following form of general bearing capacity equation,

$$q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

Where,

c' = cohesion,

q = effective stress at the level of the bottom of the foundation.

γ = unit weight of soil.

B = width of foundation (diameter of a circular foundation)

$F_{cs}, F_{qs}, F_{\gamma s}$ = shape factors,

$F_{cd}, F_{qd}, F_{\gamma d}$ = depth factors,

$F_{ci}, F_{qi}, F_{\gamma i}$ = load inclination factors,

N_c, N_q, N_γ = bearing capacity factors.

Bearing capacity factors:

$$N_q = \tan^2\left(45^\circ + \frac{\phi'}{2}\right) e^{\pi \tan \phi'} \rightarrow (\text{Reissner, 1924})$$

$$N_c = (N_q - 1) \cot \phi' \rightarrow (\text{Prandtl, 1921})$$

$$N_\gamma = 2(N_q + 1) \tan \phi' \rightarrow (\text{Vesic, 1973})$$

Shape factors:

$$F_{cs} = 1 + \left(\frac{B}{L}\right) \left(\frac{N_q}{N_c}\right)$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right) \tan \phi'$$

$$F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L}\right)$$

Depth factors:

For $\frac{D_f}{B} \leq 1$:

For, $\phi' = 0$;	For, $\phi' > 0$;
$F_{cd} = 1 + 0.4 \left(\frac{D_f}{L}\right)$	$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$
$F_{qd} = 1$	$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \left(\frac{D_f}{B}\right)$
$F_{\gamma d} = 1$	$F_{\gamma d} = 1$

For $\frac{D_f}{B} \geq 1$:

For $\phi' = 0$;	For $\phi' > 0$
$F_{cd} = 1 + 0.4 \underbrace{\tan^{-1} \left(\frac{D_f}{L}\right)}_{\text{radian } 4.229}$	$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$
$F_{qd} = 1$	$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \underbrace{\tan^{-1} \left(\frac{D_f}{B}\right)}_{\text{radian } 4.229}$
$F_{\gamma d} = 1$	$F_{\gamma d} = 1$

Inclination factor:

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)$$

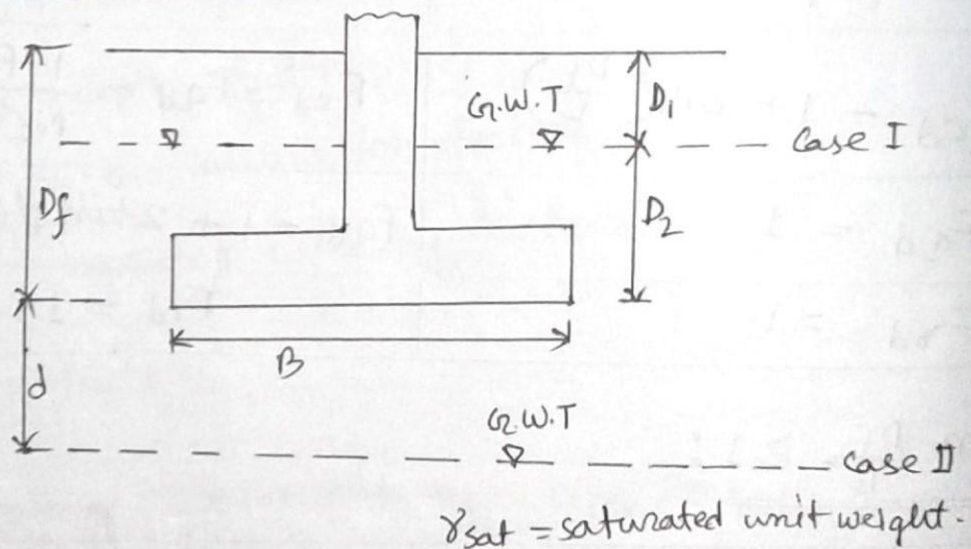
Meyerhof (1963);

Hanna and Meyerhof (1981)

β = inclination of the load on the foundation with respect to the vertical.

Question: What is the effect of the presence of water table at different stages of foundation? 2014, 2015, 2013,

Answer: Effect of water table at different stages of foundation:



In developing bearing capacity equations it is assumed that groundwater table is located at a depth much greater than width B of footing. However, if groundwater table is close to footing some changes are required in second and third terms of bearing capacity equations. Three different conditions can arise regarding location of groundwater table with respect to bottom of foundation.

Case I:

If the water table is located so that $0 \leq D_1 \leq D_f$. factor q in bearing capacity equation takes form,

$$q = \text{effective surcharge} = \gamma D_1 + D_2(\gamma_{\text{sat}} - \gamma_w)$$

where, $\gamma' = \gamma_{\text{sat}} - \gamma_w$ = effective unit weight of soil.

Also, the unit weight of soil γ , that appears in third term of bearing capacity equations should be replaced by γ' .

$$q_u = c' N_c + q N_q + \frac{1}{2} \gamma' B N_\gamma$$

Case II:

For a water table located so that $0 \leq d \leq B$, then.

$$q = \gamma D_f.$$

In this case, factor γ in last term of bearing capacity equation must be replaced by factor,

$$\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma')$$

$$q_u = c' N_c + \gamma D_f N_q + \frac{1}{2} \left\{ \gamma' + \frac{d}{B} (\gamma - \gamma') \right\} B N_\gamma$$

Case III:

When water table is located so that $d > B$, water will have no effect on ultimate bearing capacity.

दस्तावेज़: इन्फॉर्मेशन इंडिया
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पुस्तकालय विभाग
दस्तावेज़ नं: 260220.

Dr. R. K. Singh

Q. Write down the procedure to determine factor of safety (FS) in case of eccentrically loaded foundation.

Answer:

Calculating gross allowable load-bearing capacity of shallow foundations requires application of a factor of safety (FS) to gross ultimate bearing capacity, or,

$$q_{all} = \frac{q_u}{FS}$$

However, some practicing engineers prefer to use a factor of safety such that,

$$\text{Net stress increase on soil} = \frac{\text{net ultimate bearing capacity}}{FS}$$

Net ultimate bearing capacity is defined as ultimate pressure per unit area of foundation that can be supported by soil in excess of pressure caused by surrounding soil at foundation level.

If difference between unit weight of concrete used in foundation and unit weight of soil surrounding is assumed to be neglected,

then, $q_{net(u)} = q_u - q$, where,

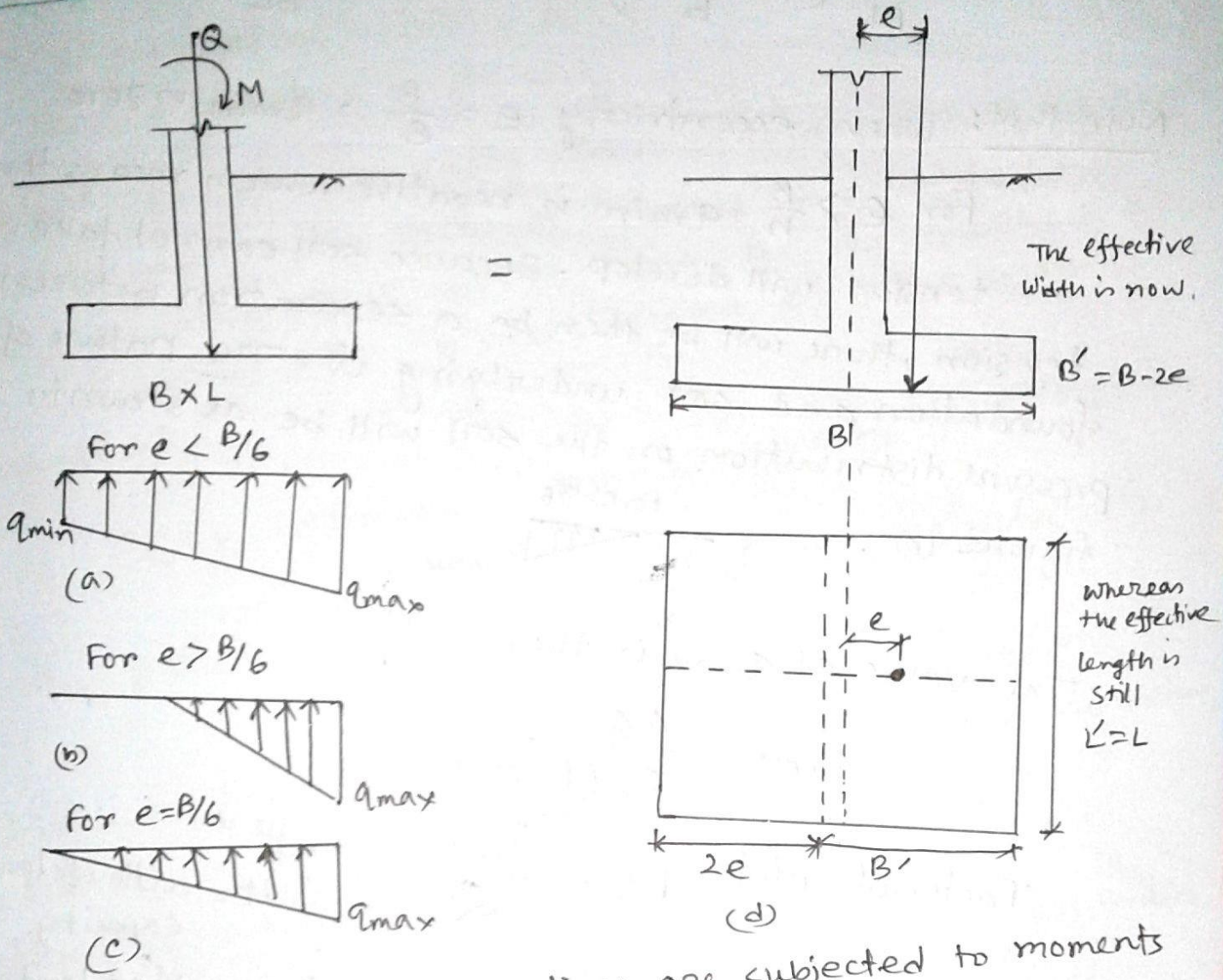
$q_{net(u)}$ = net ultimate bearing capacity,

$$q = \gamma D_f$$

So,
$$q_{all(net)} = \frac{q_u - q}{FS}$$

Factor of safety as defined should be at least 3 in all cases.

Eccentrically loaded Foundation:



In several instances, foundations are subjected to moments in addition to the vertical load as shown above picture/figure. In such cases distribution of pressure by the foundation upon soil is not uniform.

The nominal distribution of pressure is.

$$q_{max} = \frac{Q}{BL} + \frac{6M}{B^2L} \quad ; \quad q_{min} = \frac{Q}{BL} - \frac{6M}{B^2L}$$

where, Q = total vertical load.

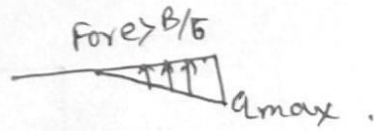
M = moment of the foundation

The distance, $e = \frac{M}{Q}$.

$$q_{\max} = \frac{Q}{BL} \left(1 + \frac{6e}{B}\right) ; q_{\min} = \frac{Q}{BL} \left(1 - \frac{6e}{B}\right)$$

Note that: when eccentricity $e = \frac{B}{6}$, q_{\min} is zero.

For $e > \frac{B}{6}$, q_{\min} is negative which means that tension will develop. Because soil cannot take any tension, there will be then be a separation between foundation and soil underlying it. The nature of the pressure distribution on the soil will be as shown in figure (b).



The value of q_{\max} is then,

$$q_{\max} = \frac{4Q}{3L(B-2e)}$$

Factor of safety, $FS = \frac{Q_{ult}}{Q}$

where,

Q_{ult} = ultimate load-carrying capacity.

Q = Applied load.

Ultimate Bearing capacity under Eccentric Loading:-

One-way Eccentricity:-

In 1953 Meyerhof proposed this theory:- (Effective area method)

The following is a step-by-step procedure for determining the ultimate load that the soil can support the factor of safety against bearing capacity failure.

Step-1: Determine the effective dimensions of the foundation:

$$B' = \text{effective width} = B - 2e$$

$$L' = \text{effective length} = L$$

Note that: If the eccentricity were in the direction of the length of the foundation, the value of L' would be equal to $L - 2e$, and the value of B' would equal B . The smaller of the two dimensions (i.e. B' and L') is the effective width of foundation.

Step-2: USE ultimate bearing capacity equation.

$$q_u' = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

To evaluate F_{cs} , F_{qs} and $F_{\gamma s}$ use relationships given earlier with effective length and effective width dimensions instead of L and B respectively.

To determine F_{cd} , F_{qd} , $F_{\gamma d}$ use relationships given earlier without replacing B with B' .

Step-3: calculate total ultimate load that the foundation can sustain is, $Q_{ult} = q_u' A' = q_u' B' L'$, where, $A' = \text{effective area}$.

Step-4: The factor of safety against bearing capacity failure is, $FS = \frac{Q_{ult}}{Q}$.

where, $Q = \text{total vertical load}$.

Plate-Load Test:

In some cases, conducting field-load tests to determine the soil-bearing capacity of foundations is desirable. The standard method for a field load test is given by ASTM under designation D-1194 (ASTM, 1997).

Circular steel bearing plates 162 to 760 mm (6 to 30 inch) in diameter and 305 mm x 305 mm (1 ft x 1 ft) square plates 162 to 760 mm are used for this type of test.

A diagram of the load test is shown in figure below.

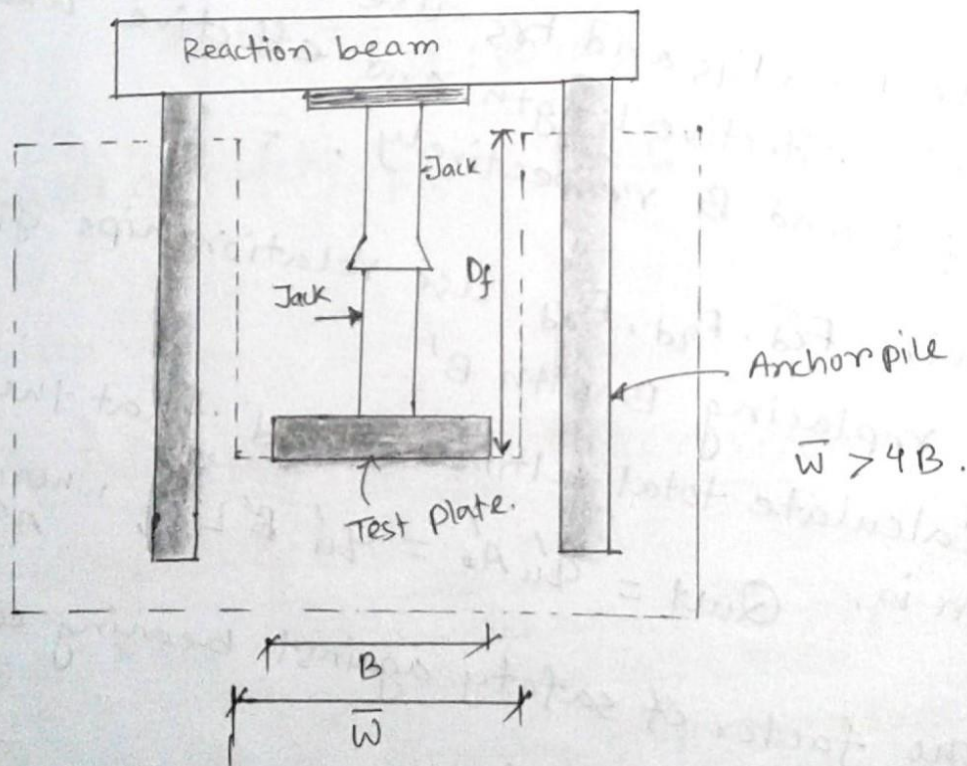
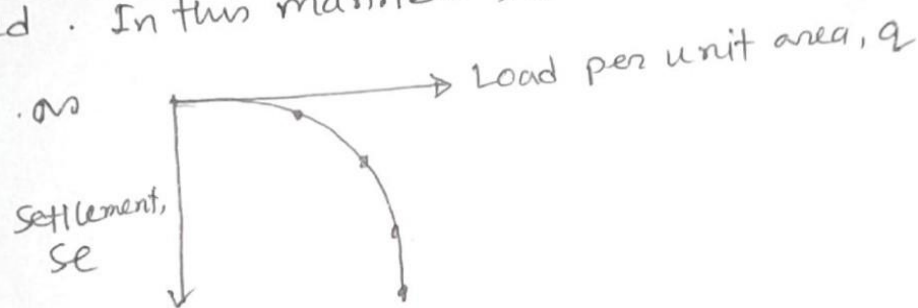


Fig. Diagram of plate load test.

To conduct the test, one must have a pit of depth D_f excavated. The width of the test pit should be at least four times the width of the bearing plate to be used for the test. The bearing plate is placed on the soil at the bottom of the pit, and an incremental load on the bearing plate is applied. After the application of an incremental load, enough time is allowed for settlement to occur. When the settlement of the bearing plate becomes negligible, another incremental load is applied. In this manner, a load-settlement plot can be obtained as



From the result of field load tests, the ultimate soil-bearing capacity of actual footings can be approximated as follows: -

For clays, $q_u(\text{footing}) = q_u(\text{plate})$

For sand soils, $q_u(\text{footing}) = q_u(\text{plate}) \frac{B(\text{footing})}{B(\text{plate})}$

For a given intensity of load q , the settlement of the actual footing also can be approximated from the following equations: -

In clay, $S_e(\text{footing}) = S_e(\text{plate}) \frac{B(\text{footing})}{B(\text{plate})}$

In sandy soil, $S_e(\text{footing}) = S_e(\text{plate}) \left[\frac{2B(\text{footing})}{B(\text{footing}) + B(\text{plate})} \right]$

Floating Foundation:

→ Where deep deposits of compressible & cohesive soil are present and piles are impractical, Building substructure is a combination mat and caisson to create a rigid box. Weight of earth displaced by foundation is equal to total weight of structure, thereby minimizing settlement from consolidation.

শ্রী: রবিউল ইসলাম
রাজশাহী সরকারি ও প্রযুক্তি বিশ্ববিদ্যালয়
পূর্বকোষাল বিভাগ
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Settlement of Shallow Foundation

* Define settlement and classify it?

Answer: Settlement: The deformation of soil particle or re-arrangement of soil particle due to application of load is called the settlement of soil. In another word, the total vertical deformation at surface resulting from external load & dewatering is known as settlement.

Classification: There are two types of settlement which are as given below: —

(I) Immediate or Elastic²⁰¹² settlement: Initial settlement of soil due to the application of load at immediately is called the immediate or elastic settlement, which is not time dependent.

(II) Consolidation settlement: The settlement is dependent on the time after the application of load is called the consolidation settlement. It is two kinds.

(a) Primary consolidation — due to dissipation of pore water pressure in case of clay soil.

(b) Secondary consolidation - It is also called creep. Long time rearrangement of soil particle is called the secondary consolidation settlement. The time is like 30-40 years.

Here, primary consolidation is more significant than secondary. Here also for organic clay, settlement is more significant than secondary on inorganic clay soil.

The total settlement, $S_T = S_e + S_s + S_c$.

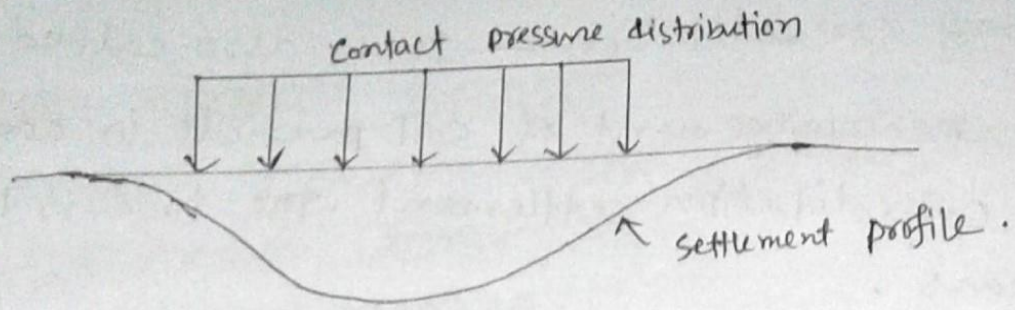
Purpose to study settlement:

- (I) Study settlement behavior.
- (II) Determine settlement value and time.
- (III) Study settlement influence to structure stability.

Describe about Immediate settlement?

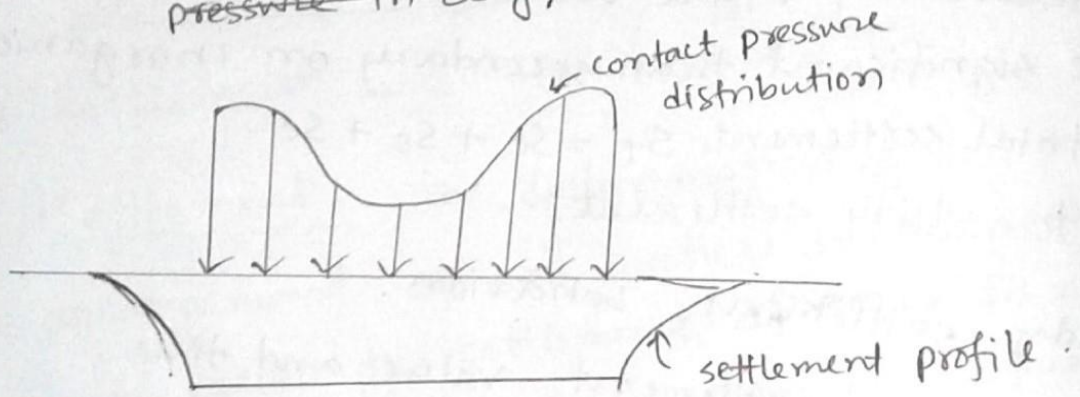
Answer: Immediate settlement: Immediate settlement defined as the settlement which occurred directly after application of a load, without a change in moisture content. and it is caused by soil elasticity behavior. The magnitude of the contact settlement will depend on the flexibility of the foundation and the type of material on which it is resting.

For clay, Immediate settlement generally very small comparing to consolidation settlement, therefore this immediate settlement mostly ignored.



(a) Flexible foundation.

~~● Elastic settlement profile and contact pressure in clay:-~~



(b) Rigid foundation.

Fig. Elastic settlement profile and contact pressure in clay.

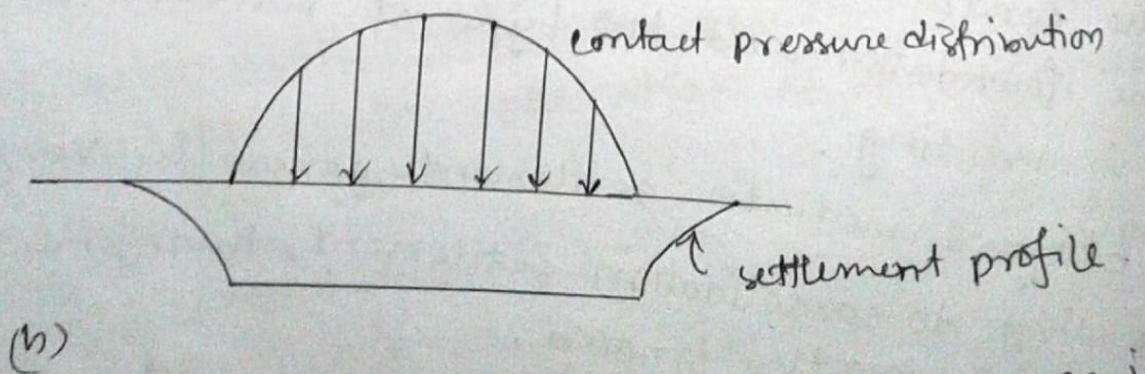
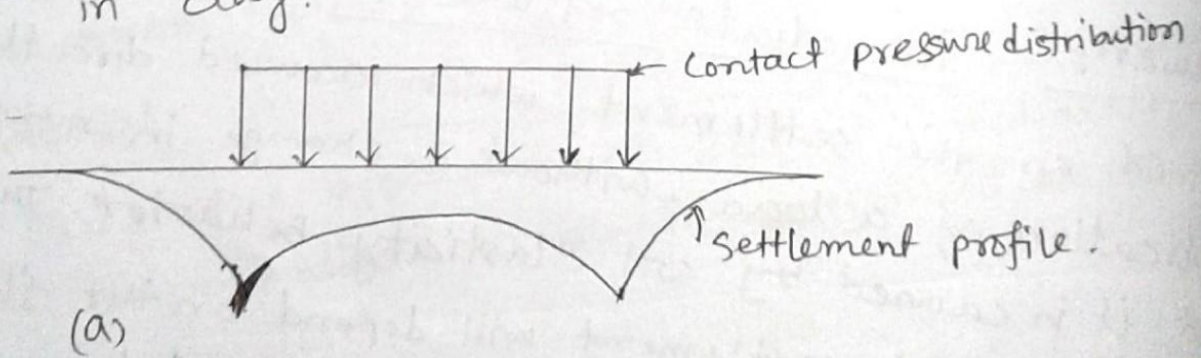


Fig. Elastic settlement profile and contact pressure in sand (a) Rigid foundation (b) Flexible foundation.

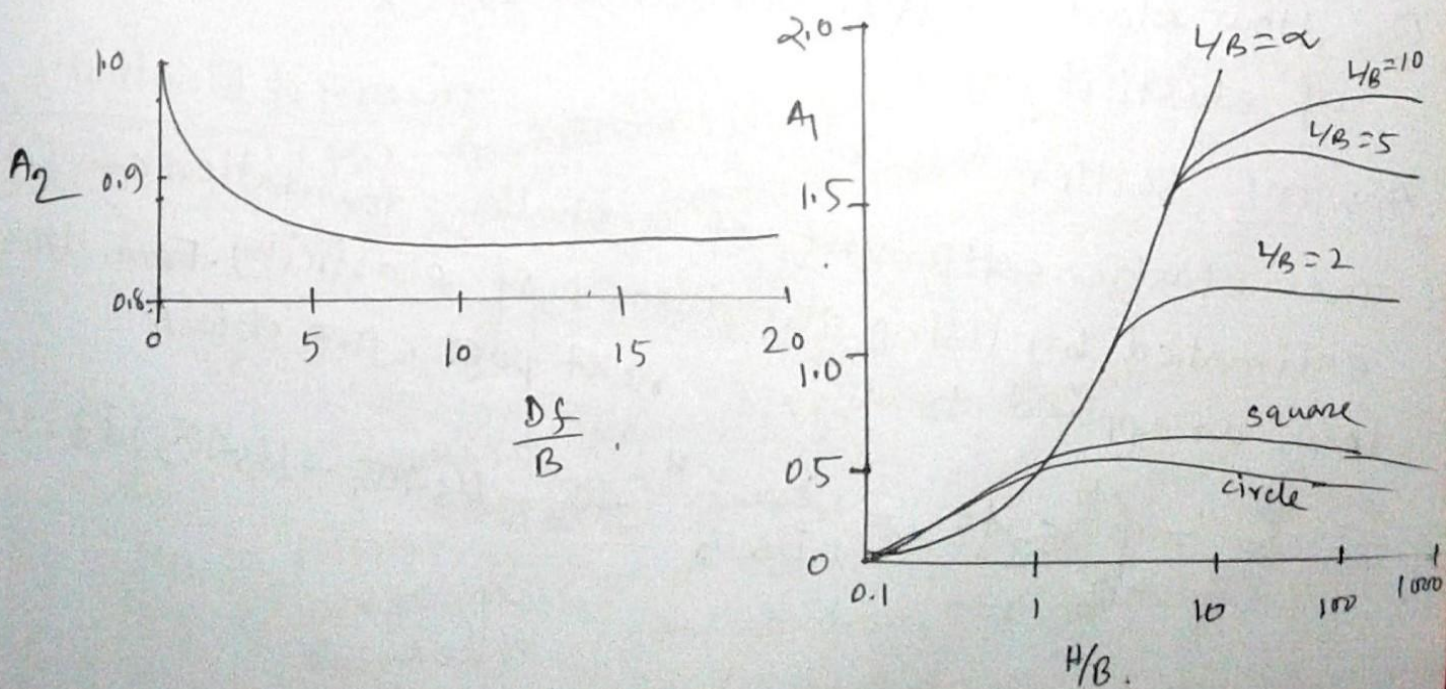
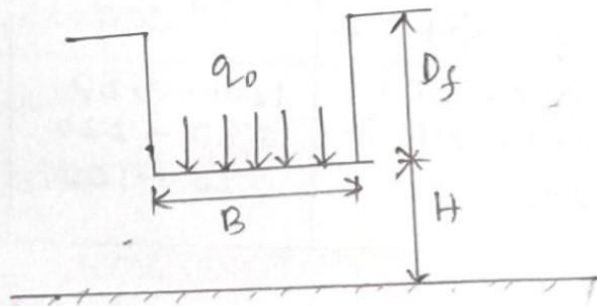
Describe Elastic settlement of foundation on saturated clay ($\mu_s = 0.5$).

Answer: Janbu et al (1956) proposed an equation for evaluating the average settlement of flexible foundations on saturated clay soils (Poisson's ratio, $\mu_s = 0.5$). For the notation

$$S_e = A_1 A_2 \frac{q_0 B}{E_s}$$

where, A_1 is a function of $\frac{H}{B}$ and $\frac{L}{B}$ and A_2 is a function of $\frac{D_f}{B}$.

Cristian and Carrier (1978) modified the values of A_1 and A_2 to some extent as presented figure below.



The modulus of elasticity (E_s) for clays can, in general be given as

$$E_s = \beta C_u$$

where, C_u = undrained shear strength.

The parameter β is primarily a function of the plasticity index and overconsolidation ratio. Following table provides a general range for β based on that proposed by Duncan and Buchignani (1976).

Table: Range of β for clay.

Plasticity index	β				
	OCR=1	OCR=2	OCR=3	OCR=4	OCR=5
<30	1500-600	1380-500	1200-580	950-380	730-300
30 to 50	600-300	550-270	580-220	380-180	300-150
>50	300-150	270-120	220-100	180-90	150-75

Q. How elastic settlement can be calculated by the theory of elasticity?

Answer: Settlement based on the Theory of Elasticity :-

The elastic settlement of a shallow foundation can be estimated by using the theory of elasticity. From Hook's law, as applied to figure next page, we obtain.

$$S_e = \int_0^H \frac{\sigma_z}{E_s} dz = \frac{1}{E_s} \int_0^H (\sigma_z - \mu_s \sigma_x - \mu_s \sigma_y) dz \quad \text{--- (1)}$$

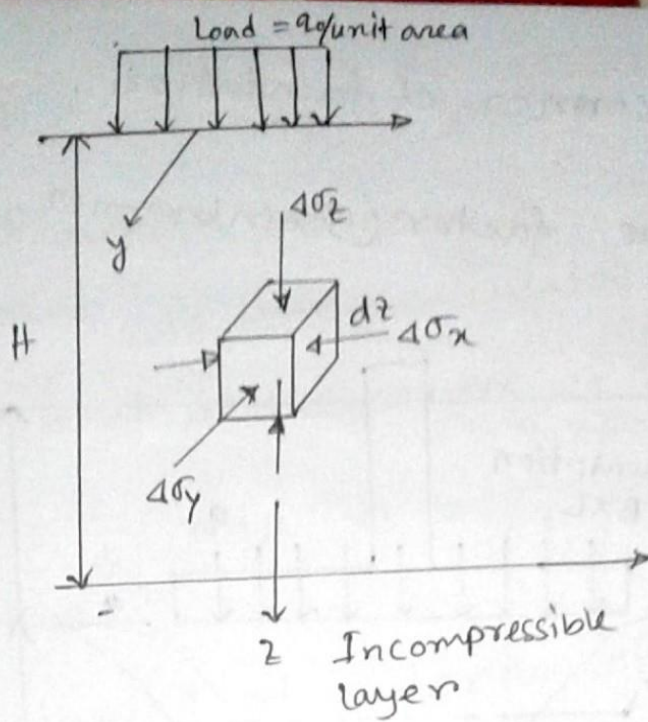


Fig. Elastic settlement of shallow foundation.

where, S_e = elastic settlement.

E_s = modulus of elasticity of soil.

H = thickness of the soil layer.

μ_s = Poisson's ratio of the soil.

$\Delta\sigma_x$, $\Delta\sigma_y$, $\Delta\sigma_z$ = stress increase due to the net applied foundation load in the x , y , z direction, respectively.

Theoretically, if the foundation is perfectly flexible (Bowels 1987), the settlement may be expressed as.

$$S_e = q_0 (\alpha \beta') \frac{1 - \mu_s^2}{E_s} I_s I_f$$

where, q_0 = net applied pressure on the foundation.

μ_s = Poisson's ratio of soil.

E_s = average modulus of elasticity of the soil under the foundation, measured from $z = 0$ to about $z = 5B$.

B' = $B/2$ for center of foundation.

$B' = B$ for corner of foundation.

$I_s =$ shape factor (Steinbrenner, 1934)

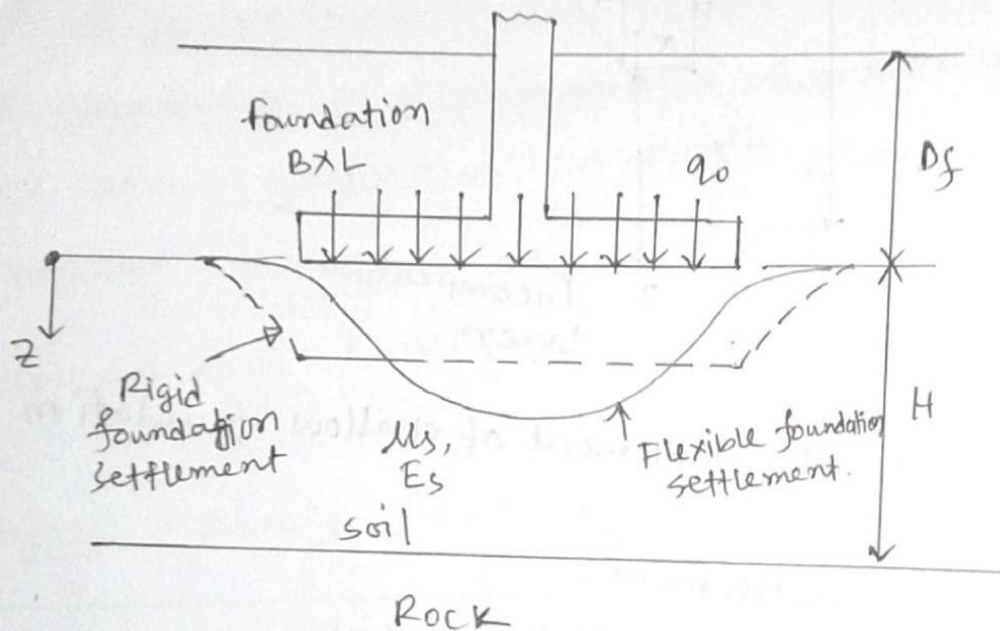


Fig. Elastic settlement of flexible and rigid foundations.

$$I_s = F_1 + \frac{1 - 2\mu_s}{1 - \mu_s} F_2$$

$$F_1 = \frac{1}{\pi} (A_0 + A_1)$$

$$F_2 = \frac{n'}{2\pi} \tan^{-1} A_2$$

$$A_0 = m' \ln \frac{(1 + \sqrt{m'^2 + 1}) \sqrt{m'^2 + n'^2}}{m' (1 + \sqrt{m'^2 + n'^2 + 1})}$$

$$A_1 = \ln \frac{(m' + \sqrt{m'^2 + 1}) \sqrt{1 + n'^2}}{m' + \sqrt{m'^2 + n'^2 + 1}}$$

$$A_2 = \frac{m'}{n' \sqrt{m'^2 + n'^2 + 1}}$$

I_f = depth factor (Fox, 1948) = $f\left(\frac{D_f}{B}, \mu_s \text{ and } \frac{L}{B}\right)$

α = a factor that depends on the location of the foundation where settlement is being calculated.

To calculate settlement at the centre of the foundation,

$$\alpha = 4, \quad m' = \frac{L}{B} \quad \text{and} \quad n' = \frac{H}{\left(\frac{B}{2}\right)}$$

To calculate settlement at a corner of the foundation,

$$\alpha = 1, \quad m' = \frac{L}{B} \quad \text{and} \quad n' = \frac{H}{B}$$

Variation of I_f with $\frac{D_f}{B}$, $\frac{B}{L}$, and μ_s is given below:

μ_s	D_f/B	B/L		
		0.2	0.5	1.0
0.3	0.2	0.95	0.93	0.90
	0.4	0.90	0.86	0.81
	0.6	0.85	0.80	0.74
	1.0	0.78	0.71	0.65
0.4	0.2	0.97	0.96	0.93
	0.4	0.93	0.89	0.85
	0.6	0.89	0.84	0.78
	1.0	0.82	0.75	0.69
0.6	0.2	0.99	0.98	0.96
	0.4	0.95	0.93	0.89
	0.6	0.92	0.87	0.82
	1.0	0.85	0.79	0.72

The elastic settlement of a rigid foundation can be estimated as,

$$S_e(\text{rigid}) = 0.93 S_e(\text{flexible, center})$$

Due to the non homogeneous nature of soil deposits, the magnitude of E_s may vary with depth. For that reason, Bowles (1987) recommended using a weighted average of E_s or,

$$E_s = \frac{\sum E_{s(i)} \Delta z}{\bar{z}}$$

where, $E_{s(i)}$ = soil modulus of elasticity within a depth Δz .
 $\bar{z} = H$ or $5B$, whichever is smaller.

Improved equation for Elastic settlement:

In 1999, Mayne and Poulos presented an improved formula for calculating the elastic settlement of foundations. The formula takes into account the rigidity of the foundation, the depth of embankment of the foundation, the increase in the modulus of elasticity of the soil with depth, and the location of rigid layers at a limited depth. To use Mayne and Poulos's equation, one needs to determine the equivalent diameter B_e of a rectangular foundation, or,

$$B_e = \sqrt{\frac{4BL}{\pi}}$$

where, B = width of foundation.
 L = length of foundation.

For circular foundations, $B_e = B$,
 where, $B =$ diameter of foundation.

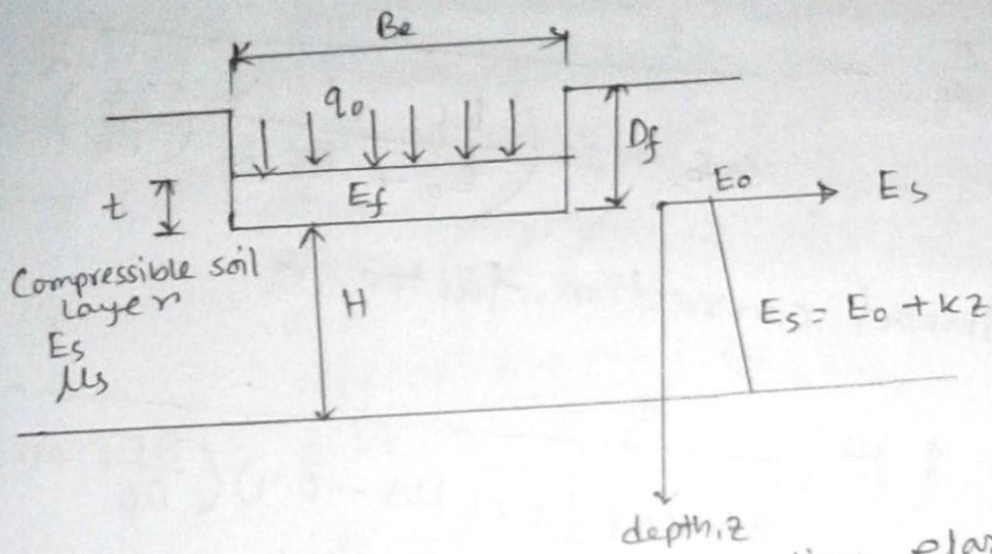


Fig. Improved equation for calculating elastic settlement general parameters.

The above figure shows a foundation with an equivalent diameter B_e located at a depth D_f below the ground surface. Let the thickness of the foundation be t and the modulus of elasticity of the foundation material be E_f . A rigid layer is located at a depth H below the bottom of the foundation, the modulus of elasticity of the compressible soil layer can be given as

$$E_s = E_0 + kz$$

With the preceding parameters defined, the elastic settlement below the center of the foundation is

$$S_e = \frac{q_0 B_e I_{az} I_F I_E}{E_0} (1 - \mu_s^2)$$

where, I_{az} = Influence factor for the variation of E_s with depth.
 $= f\left(\beta = \frac{E_0}{k B_e}, \frac{H}{B_e}\right)$

I_F = foundation rigidity correction factor.

I_E = foundation embedment correction factors.

The foundation rigidity correction factor can be expressed as,

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10 \left(\frac{E_f}{E_o + \frac{B e}{2} k} \right) \left(\frac{2t}{B e} \right)^3}$$

The embedment correction factor is,

$$I_E = 1 - \frac{1}{3.5 \exp(1.22 \mu_s - 0.4) \left(\frac{B e}{D_f} + 1.6 \right)}$$

⇒ Settlement of Foundation on sand Based on Standard penetration Resistance:—

Meyerhof's Method:

Meyerhof's (1956) proposed a correlation for the net bearing pressure for foundations with the standard penetration resistance, N_{60} . The net pressure has been defined as, $q_{net} = \bar{q} - \gamma D_f$.

where, \bar{q} = stress at the level of the foundation.

According to Meyerhof's theory, for 25mm (1 inch), of estimated maximum settlement,

$$q_{net} \text{ (KN/m}^2\text{)} = \frac{N_{60}}{0.08} \quad \text{[for } B \leq 1.22\text{m}]$$

$$\text{and, } q_{net} \text{ (KN/m}^2\text{)} = \frac{N_{60}}{0.125} \left(\frac{B + 0.3}{B} \right)^2 \quad \text{[for } B > 1.22\text{m}]$$

Since the time that Meyerhof proposed his original correlation researchers have observed that its results are rather conservative. Later Meyerhof (1965) suggested that the net allowable bearing pressure should be increased by about 50%. Bowles (1977) proposed that the modified form of the bearing pressure equations be expressed as,

$$q_{net} \text{ (kN/m}^2\text{)} = \frac{N_{60}}{0.05} F_d \left(\frac{S_e}{25} \right) \rightarrow \text{[for } B \leq 1.22\text{m]}$$

$$q_{net} \text{ (kN/m}^2\text{)} = \frac{N_{60}}{0.08} \left(\frac{B+0.3}{B} \right)^2 F_d \left(\frac{S_e}{25} \right) \rightarrow \text{[for } B > 1.22\text{m]}$$

where, F_d = depth factor = $1 + 0.33 \left(\frac{D_f}{B} \right)$

B = foundation width, in meters.

S_e = settlement, in mm.

Hence,

$$S_e \text{ (mm)} = \frac{1.25 q_{net} \text{ (kN/m}^2\text{)}}{N_{60} F_d} \quad \text{[for } B \leq 1.22\text{m]}$$

$$S_e \text{ (mm)} = \frac{2 q_{net} \text{ (kN/m}^2\text{)}}{N_{60} F_d} \left(\frac{B}{B+0.3} \right)^2 \quad \text{[for } B > 1.22\text{m]}$$

The N_{60} referred to in preceding equation is standard penetration resistance between bottom of foundation and $2B$ below bottom.

* Primary consolidation settlement Relationships:-

Consolidation settlement occurs over time in saturated clayey soils subjected to an increased load caused by construction of the foundation. On the basis of the one dimensional consolidation settlement equation, we write

$$S_c(P) = \int \epsilon_z dz \dots$$

where, $\epsilon_z = \text{vertical strain} = \frac{\Delta e}{1 + e_0}$

$\Delta e = \text{change of void ratio} = f(\sigma_0', \sigma_c' \text{ and } \Delta \sigma')$

So,
$$S_c(P) = \frac{C_c H_c}{1 + e_0} \log \frac{\sigma_0' + \Delta \sigma_{av}'}{\sigma_0'} \quad [\text{For normally consolidated clays}]$$

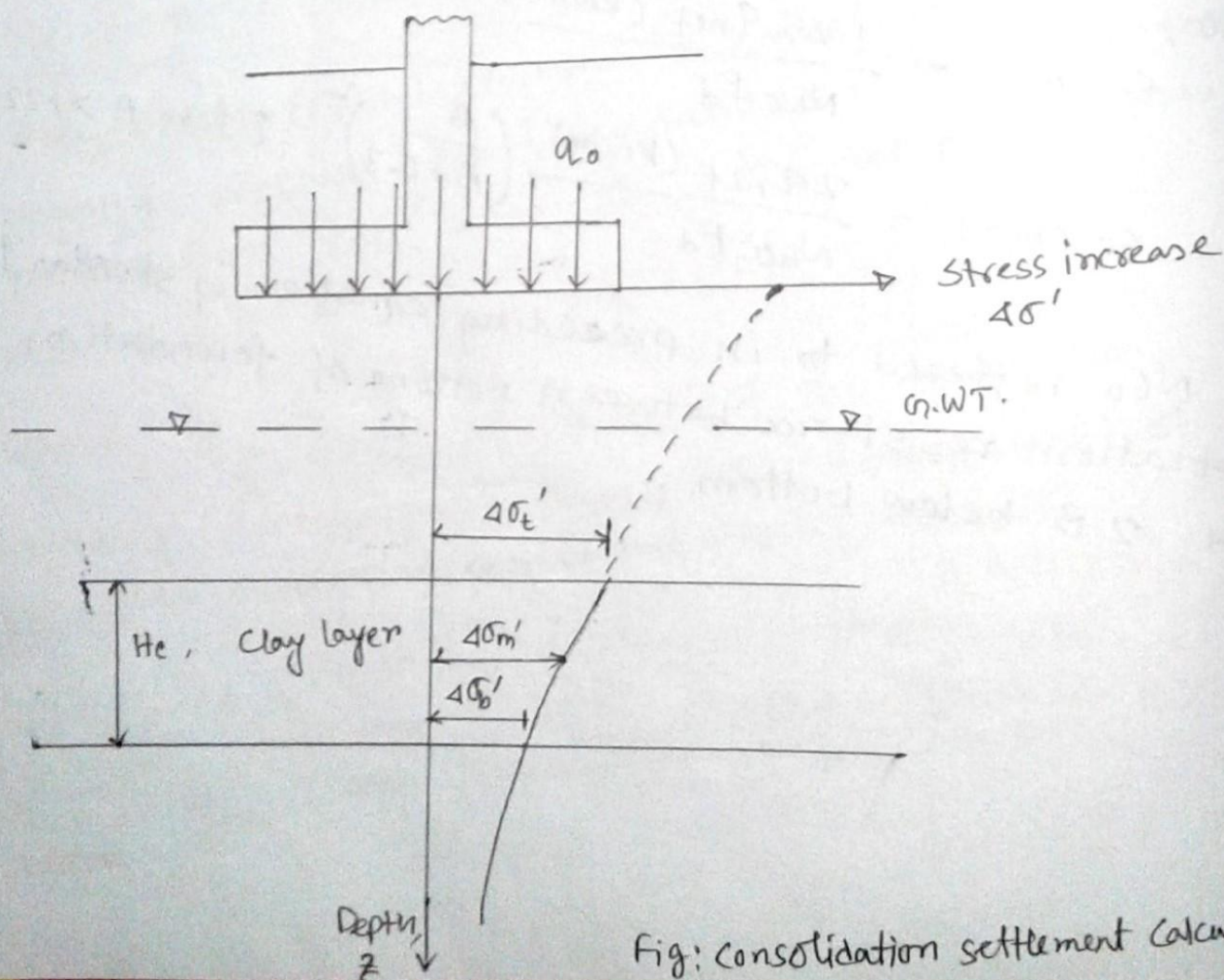


Fig: Consolidation settlement calculation

$$S_e(P) = \frac{C_s H_c}{1+e_0} \log \frac{\sigma_0' + 4\sigma_{av}'}{\sigma_0'} \quad \left[\text{for overconsolidated clays, with } \sigma_0' + 4\sigma_{av}' < \sigma_c' \right]$$

$$S_e(P) = \frac{C_s H_c}{1+e_0} \log \frac{\sigma_c'}{\sigma_0'} + \frac{C_c H_c}{1+e_0} \log \frac{\sigma_0' + 4\sigma_{av}'}{\sigma_c'} \quad \left[\text{for overconsolidated clays with } \sigma_0' < \sigma_c' < \sigma_0' + 4\sigma_{av}' \right]$$

where,

σ_0' = Average effective pressure on clay layer before the construction of the foundation.

$4\sigma_{av}'$ = Average increase in effective pressure on the clay layer caused by the construction of the foundation.

σ_c' = preconsolidation pressure.

e_0 = Initial void ratio of the clay layer.

C_c = compression index

C_s = swelling index.

H_c = Thickness of the clay layer.

Note that, the increase in effective pressure, $4\sigma'$, on the clay layer is not constant with depth. The magnitude of $4\sigma'$ will decrease with the increase in depth measured from the bottom of the foundation. However, the average increase in pressure may be approximated by.

$$4\sigma_{av}' = \frac{1}{6} (4\sigma_t' + 4\sigma_m' + 4\sigma_b')$$

where, $4\sigma_t'$, $4\sigma_m'$ and $4\sigma_b'$ are respectively the effective pressure increases at the top middle, and bottom of the clay layer that are caused by the construction of the foundation.

* Settlement due to secondary consolidation:

At the end of primary consolidation (i.e. After the complete dissipation of excess pore water pressure) some settlement is observed that is due to the plastic adjustment of soil ~~mat~~ fabrics. This stage of consolidation is called secondary consolidation. A plot of deformation against the logarithm of time during secondary consolidation is practically linear.

From the figure, the secondary compression index can be defined as,

$$C_{\alpha} = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log(t_2/t_1)}$$

where, C_{α} = secondary compression index
 Δe = change of void ratio.
 t_1, t_2 = time.

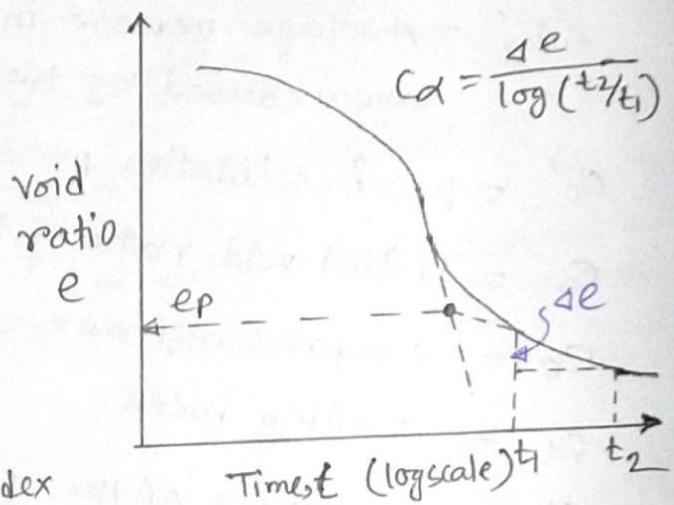


Fig. Variation of e with $\log t$.

The magnitude of the secondary consolidation can be calculated as,

$$S_c(s) = C_{\alpha}' H_c \log(t_2/t_1)$$

where, $C_{\alpha}' = \frac{C_{\alpha}}{1 + e_p}$

e_p = void ratio at the end of primary consolidation.
 H_c = Thickness of clay layer.

Mesri (1973) correlated C_{α} with the natural moisture content (w) of several soils from which it appears that,

$$C_{\alpha} = 0.0001 w$$

where, w = natural moisture content in percent.

For most overconsolidated soils, C_{α} varies betⁿ 0.0005 to 0.001

Mesri and Godlewski (1977) compiled the magnitude of $\frac{C_{\alpha}}{C_c}$ for a number of soils. Based on their compilation, it can be summarized that,

* For inorganic clays and silts: $\frac{C_{\alpha}}{C_c} \approx 0.04 \pm 0.01$

* For organic clays and silts: $\frac{C_{\alpha}}{C_c} = 0.05 \pm 0.01$

* For peats: $\frac{C_{\alpha}}{C_c} = 0.075 \pm 0.01$

Secondary consolidation settlement is more important in the case of all organic and highly compressible inorganic soils.

In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

Q. ^{2015, 2014} what is differential settlement? What are the causes of differential settlement? What are measuring to avoid it?

Answer: Differential settlement: Unequal setting of building foundation is called the differential settlement.

Causes: -

- (I) Ability of soil to expand, contract and get washed away by poor drainage and heavy down pour.
- (II) Change in water table.
- (III) Uneven drying of soil surface.

Measures: To avoid differential settlement following measure can be taken as follows:—

- (I) Proper drainage.
- (II) Soil contraction should be protected.
- (III) No uneven drying of soil surface.
- (IV) No change of water table immediately.
- (V) No difference between the size and shape of footing.

মো: রবিউল ইসলাম

রাজশাহী সরকারি ও প্রযুক্তি বিশ্ববিদ্যালয়

দূরকর্ষণ বিভাগ

রোল নং: ২৬০২২০

SLOPE STABILITY

Q: Define slope. classify it. what are the reasons of failure of it.

Answer: Slope: An exposed ground surface that stands at an angle with the horizontal is called an unrestrained slope.

Classification: It is two types in common.

- (a) Natural slope → In hilly areas it is obtained.
- (b) Artificial slope → Embankment slope.

Reasons for failure:

- (I) Action of gravitational and seepage force within soil.
- (II) Excavation or undercutting of its foot.
- (III) Gradual disintegration of structure of soil.

2013 Classification of the slope failure by Cruden and Varnes (1966):

(I) Fall: This is the detachment of soil and/or rock fragments that fall down a slope.

(II) Topple: This is a forward rotation of soil and/or rock mass about an axis below the center of gravity of mass being displaced.

(III) Slide: This is the downward movement of a soil mass occurring on a surface of rupture.

(W) Spread: This is a form of slide by translation. It occurs by "sudden movement of water bearing seams of sands or silts overlain by clays or loaded by fills."

(V) Flow: This is a downward movement of soil mass similar to a viscous fluid.

Q: Define slope stability analysis. what are the types of slope in practice distinguish between them.

Slope stability Analysis: The process of determining and comparing the shear stress developed along the most likely rupture surface with the shear strength of the soil is called the slope stability analysis.

Types of slope: Two types.

- (I) Finite slope.
- (II) Infinite slope.

Assumption:

- (I) Coloumb's equation of shear strength is used to determine c, ϕ .
- (II) Shear stress system is two dimensional.
- (III) Seepage condition and water level.
- (IV) Condition of plastic failure.

Differences: The differences between the infinite and finite slopes are given below:-

2012

Infinite slope	Finite slope
(I) If a slope represents the boundary surface of a semi-infinite soil mass and soil properties for all identical depth below the surface area are constant, it is called infinite slope.	(I) If the slope is of limited extent, it is called the finite slope.
(II) Slopes extending to infinity don't exist in nature.	(II) Inclined forces of dam, embankments and cuts are finite slope.

Q. Explain factor of safety of slope.

Answer: Factor of safety: The task of the engineer charged with analyzing slope stability is to determine the factor of safety. Generally factor of safety is defined as

$$F_s = \frac{\tau_f}{\tau_d} \quad \text{--- (I)}$$

where, F_s = Factor of safety with respect to strength.

τ_f = average shear strength of the soil.

τ_d = Average shear stress developed along the potential failure surface.

The shear strength of a soil consists of two components, cohesion and friction and may be written as,

$$\tau_f = c' + \sigma' \tan \phi' \quad \text{--- (II)}$$

where, c' = cohesion, ϕ' = angle of friction

σ' = normal stress on the potential failure surface.

In similar manner, we can write,

$$\tau_d = c'_d + \sigma'_d \tan \phi'_d \quad \text{--- (III)}$$

where, c'_d and ϕ'_d are, respectively the cohesion and the angle of friction that develop along the potential failure surface. Substituting the value from equation (II) & (III) in (I) we get,

$$F_s = \frac{c' + \sigma' \tan \phi'}{c'_d + \sigma'_d \tan \phi'_d} \quad \text{--- (IV)}$$

Factor of safety with respect to cohesion $F_{c'}$ and friction $F_{\phi'}$, they are defined as follows.

$$F_{c'} = \frac{c'}{c'_d} \quad \text{--- (V)} \quad \text{and} \quad F_{\phi'} = \frac{\tan \phi'}{\tan \phi'_d} \quad \text{--- (VI)}$$

When we compare eqn (IV) & (V) we can see that $F_{c'}$ becomes equal to $F_{\phi'}$. It gives the factor of safety with

respect to strength or if:

$$\frac{c'}{s_d} = \frac{\tan \phi'}{\tan \phi_d}$$

then we can write,

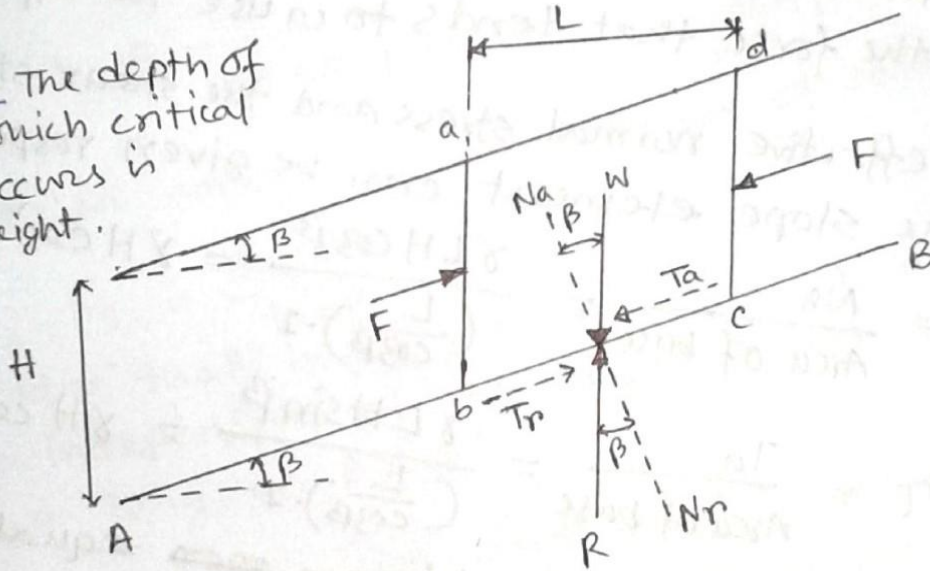
$$F_s = F_c' = F_{\phi}'$$

When F_s is equal to 1, the slope is in a state of impending failure. Generally a value of 1.5 for the factor of safety with respect to strength is acceptable for the design of a stable slope.

Q. Describe stability of infinite slopes.

Answer:

Critical height: The depth of plane along which critical equilibrium occurs is called critical height.



Considering infinite slope, the shear strength of the soil may be given by, $\tau_f = c' + \sigma' \tan \phi'$ — ①

Assuming that the pore water pressure is zero, we will evaluate the factor of safety against a possible slope failure along a plane AB located at a depth H below the ground surface. The slope failure can occur by the movement of soil above the plane AB from right to left.

Let us consider a slope element abcd that has a unit length perpendicular to the plane of the section. The forces F_1 that act on the faces ab and cd are equal and opposite and may be ignored. The weight of the soil element is,

$$W = (\text{volume of soil element}) \times (\text{unit of weight of soil})$$

$$\therefore W = \gamma L H$$

The weight W can be resolved into two components.

(i) The force perpendicular to the plane $AB = N_a = W \cos \beta$
 $\therefore N_a = \gamma L H \cos \beta$.

(ii) Force parallel to the plane $AB = T_a = W \sin \beta = \gamma L H \sin \beta$.
 this is the force that tends to cause the slip along the plane.

Thus, the effective normal stress and the shear stress at the base of the slope element can be given respectively as,

$$\sigma' = \frac{N_a}{\text{Area of base}} = \frac{\gamma L H \cos \beta}{\left(\frac{L}{\cos \beta}\right) \cdot 1} = \gamma H \cos^2 \beta \quad \text{--- (i)}$$

$$\text{and, } \tau = \frac{T_a}{\text{Area of base}} = \frac{\gamma L H \sin \beta}{\left(\frac{L}{\cos \beta}\right) \cdot 1} = \gamma H \cos \beta \sin \beta \quad \text{--- (ii)}$$

The reaction to the weight W is an ~~equal~~ equal and opposite force R . The normal and tangential components of R with respect to the plane AB are,

$$N_r = R \cos \beta = W \cos \beta$$

$$\text{and, } T_r = R \sin \beta = W \sin \beta$$

For the equilibrium, the resistive shear stress that develops at the base of the element is equal to τ_d

$$\tau_d = \frac{T_r}{\text{Area of base}} = \gamma H \sin \beta \cos \beta \quad \text{--- (iii)}$$

The resistive shear stress also may be written as,

$$\tau_d = c' + \sigma' \tan \phi_d' \quad \text{--- (iv)}$$

From (i) & (ii) we get,

$$\tau_d = c_d' + \gamma H \cos^2 \beta \tan \phi_d'$$

Thus, $\gamma H \sin \beta \cos \beta = c_d' + \gamma H \cos^2 \beta \tan \phi_d'$

or, $\frac{c_d'}{\gamma H} = \sin \beta \cos \beta - \cos^2 \beta \tan \phi_d'$

$$= \cos^2 \beta (\tan \beta - \tan \phi_d') \quad \text{--- (VI)}$$

The factor of safety with respect to strength has been defined as, $\tan \phi_d' = \frac{\tau_d}{F_s}$ and $c_d' = \frac{c'}{F_s}$

putting the above values in equation (VI) we get,

$$F_s = \frac{c'}{\gamma H \cos^2 \beta \tan \beta} + \frac{\tan \phi'}{\tan \beta} \quad \text{--- (VII)}$$

For granular soils, $c' = 0$, then factor of safety,

$$F_s = \frac{\tan \phi'}{\tan \beta}$$

This indicates that in an infinite slope in sand the value of F_s is independent of the height H and the slope is stable as long as $\beta < \phi'$.

If a soil possesses cohesion and friction, the depth of the plane along which critical equilibrium occurs may be determined by substituting $F_s = 1$ and $H = H_{cr}$, in eqn (VII).

$$\therefore H_{cr} = \frac{c'}{\gamma} \left[\frac{1}{\cos^2 \beta (\tan \beta - \tan \phi')} \right]$$

If there is steady state seepage through the soil and ground water table coincides with the ground surface, then factor of safety is given by,

$$F_s = \frac{c'}{\gamma_{sat} H \cos^2 \beta \tan \beta} + \frac{\gamma' \tan \phi'}{\gamma_{sat} \tan \beta}$$

where, γ_{sat} = saturated unit weight of soil

$\gamma' = \gamma_{sat} - \gamma_w$ = effective unit weight of soil.

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Geotechnical Engineering - II

LATERAL EARTH PRESSURE

Q Define Lateral Earth pressure. Write down the necessity of lateral earth pressure analysis.

Lateral Earth Pressure: "The word 'lateral' means sideway. The lateral earth pressure means pressure to the side or sideway pressure."

Necessity of lateral earth pressure analysis:

- (I) To design retaining wall. SAR
- (II) To design sheet pile
- (III) To design abutment,

Q: Discuss the factor on which lateral earth pressure depends.

Factors on which lateral earth pressure depends:-

- (I) Type and amount of movement of wall.
- (II) Shear strength parameters of a soil (c and ϕ)
- (III) Unit weight of soil.
- (IV) Drainage condition of the backfill.

Q. Classify lateral earth pressure and Define them. 2015
2009

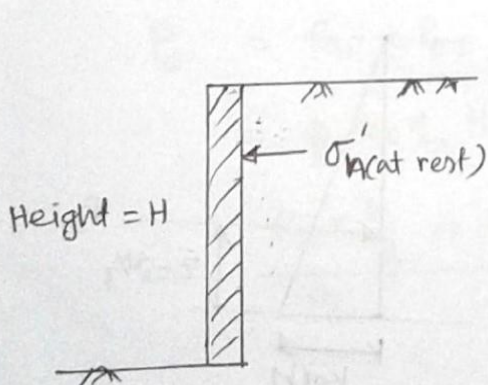
Answer: Types of Lateral earth pressure: Three types of lateral earth pressure are present.

- (I) Earth pressure at rest.
- (II) Active earth pressure.
- (III) Passive earth pressure.

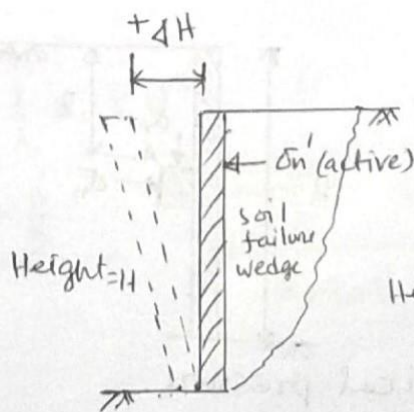
Earth pressure at rest: If the wall is restrain from moving, the lateral force/pressure at this condition is known as lateral earth pressure at rest.

Active earth pressure: If the wall moves away or tilt away from the soil retained, a triangular wedge behind the wall will fail. The lateral pressure at this condition is known as active earth pressure.

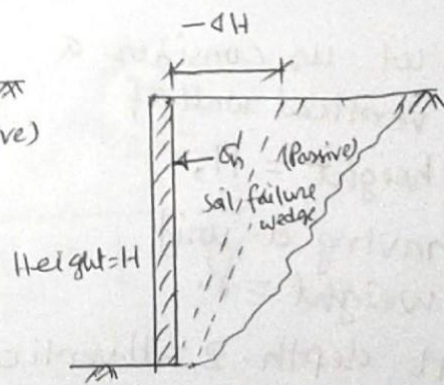
Passive earth pressure: If the wall moves towards the soil or the wall is pushed in the soil retain, a soil wedge will fail. The lateral pressure exerted on the wall at this stage of failure is referred to as passive earth pressure.



(a) Earth pressure at rest.



(b) Active earth pressure



(c) Passive earth pressure.

* Define Backfill, co-efficient of earth pressure.

Answer:-

Backfill: The material or soil retained or supported by the retaining wall is called backfill. It may have its top surface horizontal or inclined.

co-efficient of earth pressure: The ratio of horizontal stress to vertical stress is called the co-efficient of earth pressure. It is denoted by k .

$$k = \frac{\sigma_h}{\sigma_v}$$

At rest condition, $k_0 = \frac{\sigma_h}{\sigma_v}$

In Active condition, $k_a = \frac{1 - \sin \phi}{1 + \sin \phi}$

In passive condition, $k_p = \frac{1 + \sin \phi}{1 - \sin \phi}$.

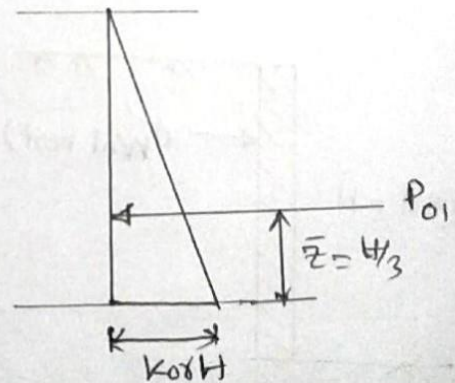
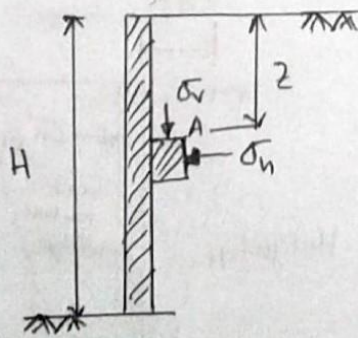
2008 Q. Derive an equation for determining the magnitude of earth pressure at rest condition.

(1) Without surcharge:

Let us consider a vertical wall of height = H , having a unit weight = γ .

At depth, $z = H$. vertical pressure,

$$\sigma_v = \gamma H.$$



Now, the co-efficient of earth pressure at rest,

$$K_0 = \frac{\sigma_h}{\sigma_v}$$

where, σ_h = lateral earth pressure.

$$\sigma_h = K_0 \sigma_v = K_0 \gamma H$$

$$P_0 = K_0 \gamma H$$

$$P_0 = \frac{1}{2} P_0 H = \frac{1}{2} K_0 \gamma H \cdot H$$

$$P_0 = \frac{1}{2} K_0 \gamma H^2$$

$$\bar{z} = \frac{H}{3}$$

(II) With surcharge:

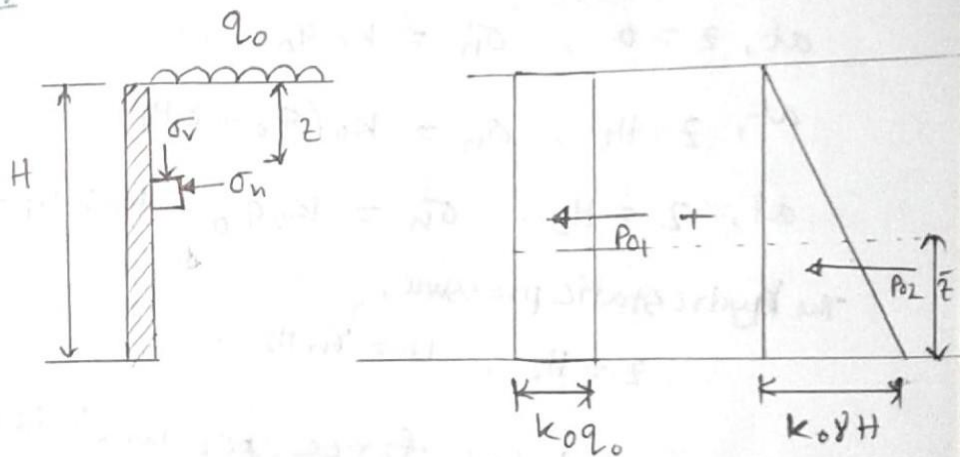
If a surcharge q_0 is applied on the ground surface the lateral earth pressure at rest at $z=H$ is given by,

$$\sigma_h = K_0 q_0 + K_0 \gamma H$$

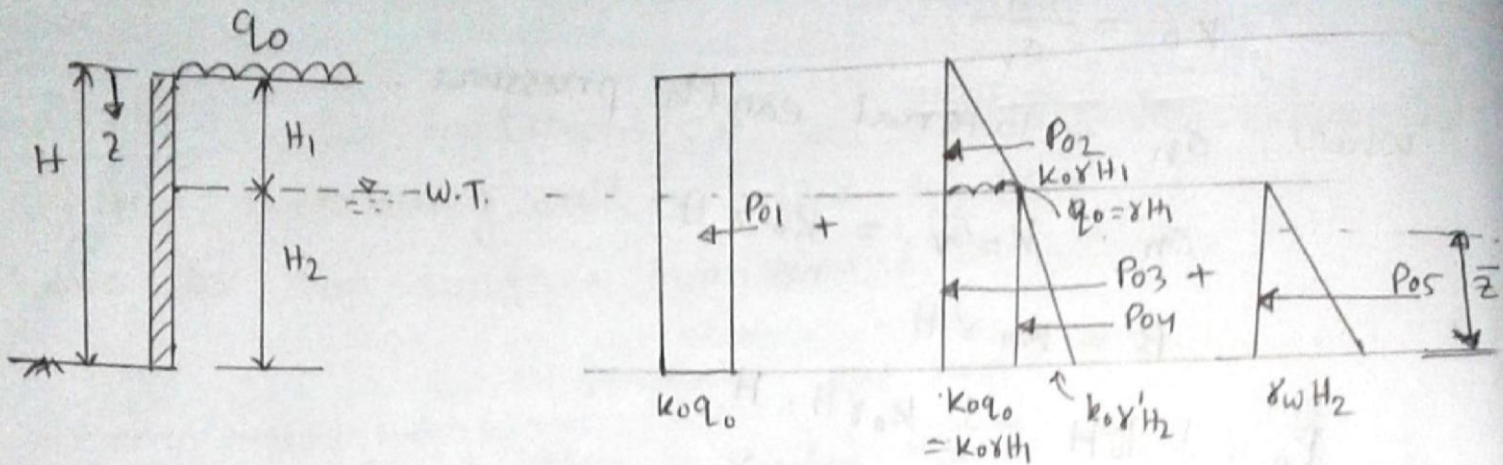
$$P_0 = P_{01} + P_{02}$$

$$= K_0 q_0 H + \frac{1}{2} K_0 \gamma H^2$$

$$\bar{z} = \frac{P_{01} \times \frac{H}{2} + P_{02} \times \frac{H}{3}}{P_0} = \frac{\frac{K_0 q_0 H}{2} + \frac{K_0 \gamma H^2}{3}}{K_0 q_0 H + \frac{1}{2} K_0 \gamma H^2}$$



(III) Lateral earth pressure at rest with water :-



If the water table is located at a depth $z < H$, then the above at-rest pressure diagram can be found. If the effective unit weight of soil below the water table equals γ' (i.e. $\gamma_{sat} - \gamma_w$), then,

$$\text{at, } z = 0, \quad \sigma_h = k_0 q_0$$

$$\text{at, } z = H_1, \quad \sigma_h = k_0 (q_0 + \gamma H_1)$$

$$\text{at, } z = H_2, \quad \sigma_h = k_0 q_0 + k_0 \gamma H_1 + k_0 \gamma' H_2$$

The hydrostatic pressure,

$$z = H_2, \quad u = \gamma_w H_2$$

\therefore The total force per unit length of the wall can be determined from the area of the pressure diagram.

$$\therefore P_{OT} = P_{01} + P_{02} + P_{03} + P_{04} + P_{05}$$

$$P_{OT} = k_0 q_0 H + \frac{1}{2} k_0 \gamma H_1^2 + k_0 \gamma H_1 H_2 + \frac{1}{2} k_0 \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

$$\bar{z} = \frac{\cancel{\frac{k_0 q_0 H^2}{2}} + \cancel{\frac{1}{2} k_0 \gamma H_1^2} + P_{01} \times \frac{H}{2} + P_{02} \left(H_2 + \frac{H_1}{3} \right) + P_{03} \times \frac{H_2}{2} + P_{04} \times \frac{H_2}{3} + P_{05} \times \frac{H_2}{3}}{P_{OT}}$$

Q How you obtain k_0 for different soil condition?

Answer: k_0 for different soil condition:-

(I) For normally consolidated coarse grained soil (Jaky, 1944) is,

$$k_0 = 1 - \sin \phi'$$

(II) For Normally consolidated Fine grained soil (MASSARCH, 1979) is,

$$k_0 = 0.44 + 0.42 \left[\frac{PI(\%)}{100} \right]$$

(III) For Normally consolidated clay (Brooker & Ireland 1965)

$$k_0 = 0.95 - \sin \phi'$$

(IV) For over consolidated clay.

$$k_{0(OC)} = k_{0(NC)} \sqrt{OCR}$$

Q. Write down the Assumptions of Rankine Active earth pressure & Derive, $P_a = K_a \gamma z$.

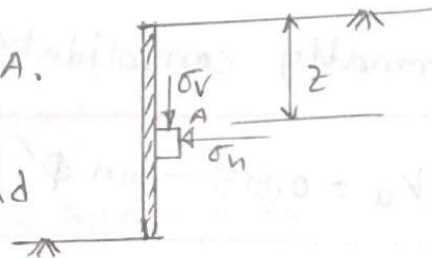
Answer: Assumptions: 2014, 2011

HS DC P SV PE

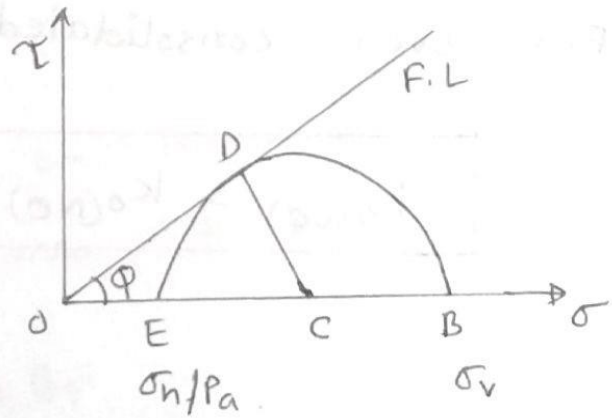
- (I) The soil mass is **homogeneous** and **semi-infinite**.
- (II) The soil mass is **dry** and **cohesionless**.
- (III) The **ground surface** is **plane**.
- (IV) The back of the retaining wall is **smooth** and **vertical**.
- (V) The soil element is in a state of **plastic equilibrium**.

$P_a = K_a \gamma z$:- cohesionless soil.

Let us consider a soil element A, in dry state and located at a depth of z below the ground surface as shown in figure.



If we plot τ vs σ graph then following points are obtained as shown in figure.



Point E represents the active earth pressure.

$$P_a = OE = OC - CE = OC - CD = OC - OC \sin \phi$$

$$\therefore P_a = OC(1 - \sin \phi) \quad \text{--- (I)}$$

Again, $\sigma_v = OB = OC + BC = OC + CD = OC + OC \sin \phi$

$$\therefore \sigma_v = OC(1 + \sin \phi) \quad \text{--- (II)}$$

(1) \div (II) we get,

$$\frac{P_a}{\sigma_v} = \frac{1 - \sin \phi}{1 + \sin \phi} = K_a$$

K_a = co-efficient of active earth pressure or Rankine's co-efficients of active earth pressure.

$$\therefore P_a = K_a \cdot \sigma_v$$

Since, $\sigma_v = \gamma z$.

$$\therefore \boxed{P_a = K_a \gamma z} \quad (\text{Derived})$$

\Rightarrow show that co-efficient of earth pressure at rest is larger than at active.

Answer: Let, $\phi = 30^\circ$,

We know, $K_0 = 1 - \sin \phi = 1 - \sin 30^\circ = 0.5$

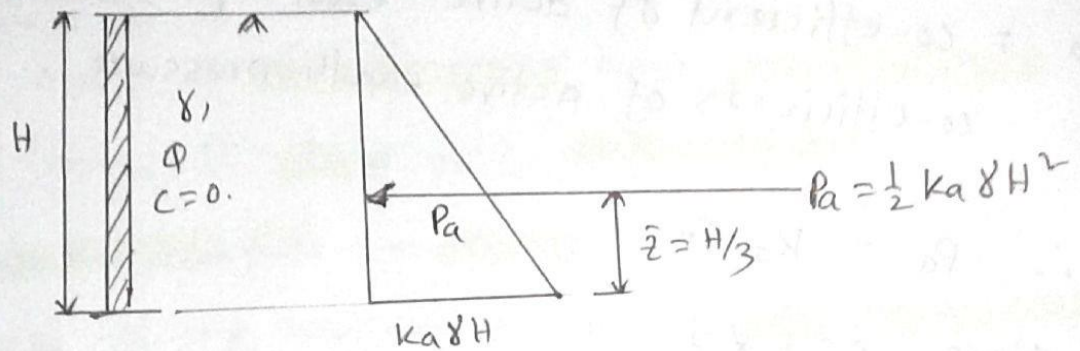
Again we know, $K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = 0.33$

Since, $K_0 > K_a$.

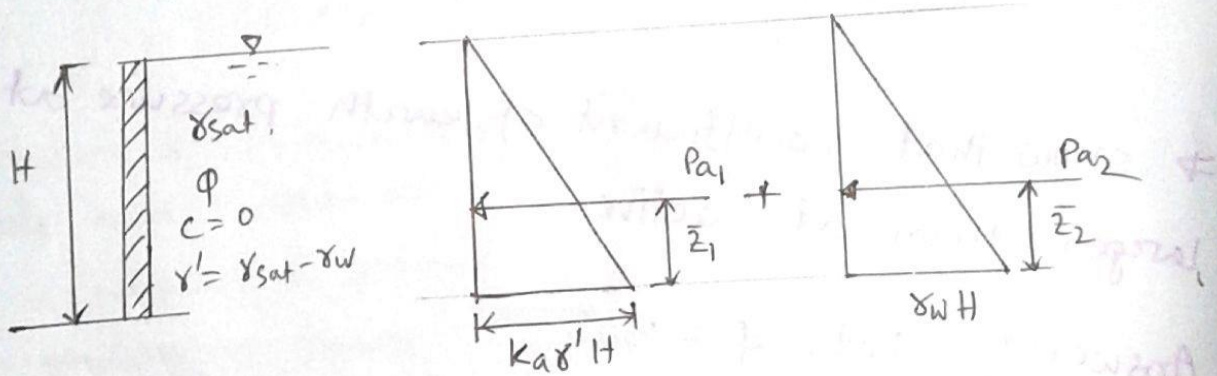
Hence, we can say that, co-efficient of earth pressure at rest is larger than at active.

⇒ Draw pressure distribution diagram for different cases.

Answers: Case I. Dry Backfill: -

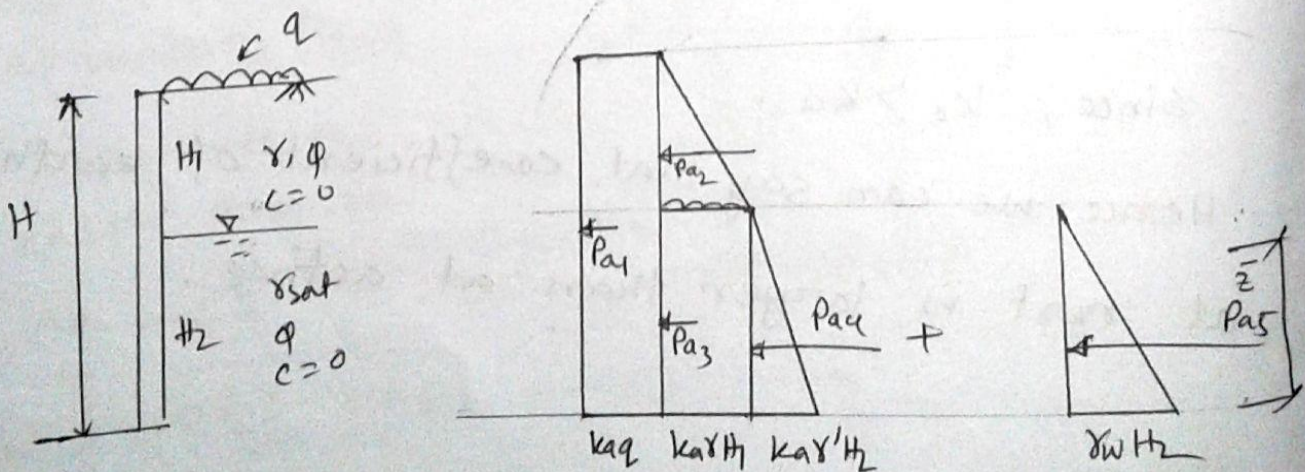


Case II: Submerged Backfill:



$$P_{Ta} = Pa_1 + Pa_2 = \frac{1}{2} Ka \gamma' H^2 + \frac{1}{2} \gamma_w H^2$$

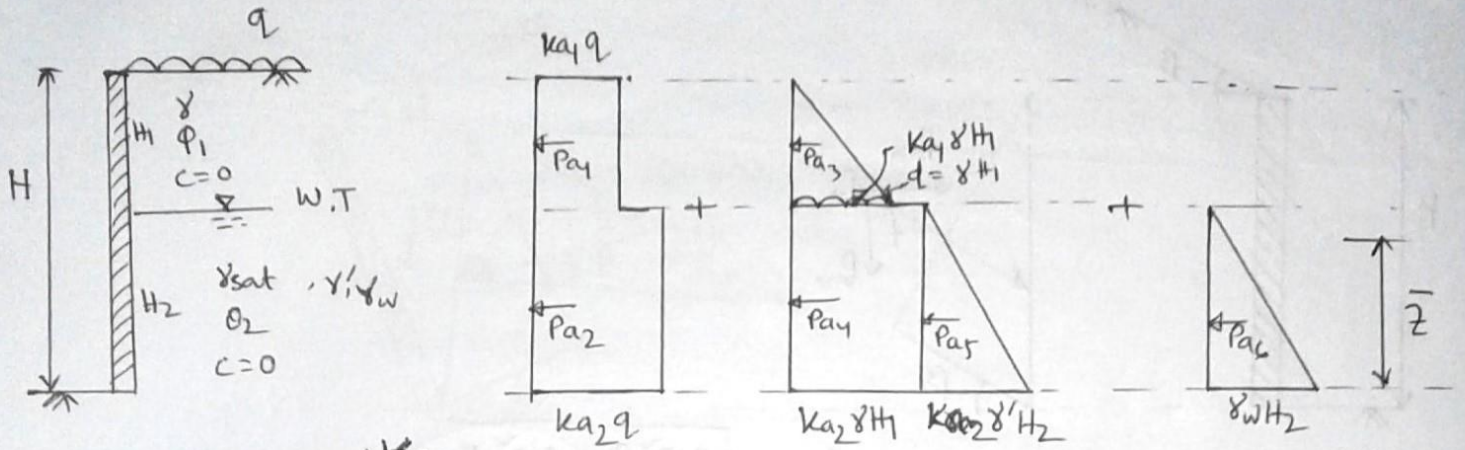
Case III: Submerged backfill with surcharge:



$$P_{Ta} = Pa_1 + Pa_2 + Pa_3 + Pa_4 + Pa_5$$

$$= KaqH + \frac{1}{2} Ka \gamma H_1^2 + Ka \gamma H_1 H_2 + \frac{1}{2} Ka \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

Case-IV: Partially submerged backfill with surcharge ($\phi_1 > \phi_2$):

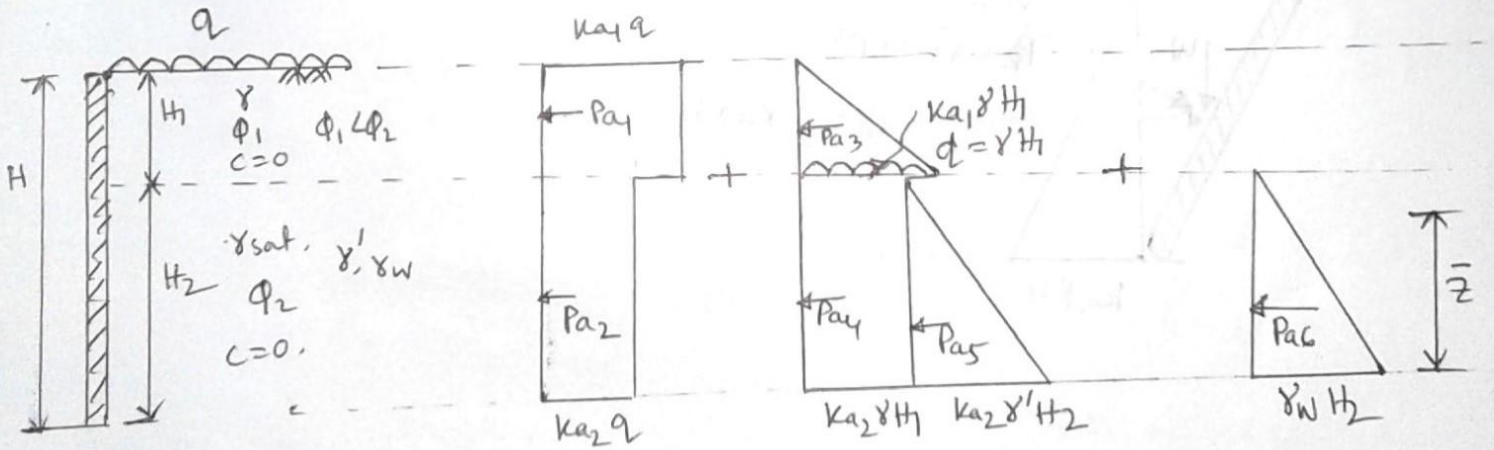


since $\phi_1 > \phi_2$, $ka_1 < ka_2$

$$P_{Ta} = P_{a1} + P_{a2} + P_{a3} + P_{a4} + P_{a5} + P_{a6}$$

$$= ka_1 q H_1 + ka_2 q H_2 + \frac{1}{2} ka_1 \gamma H_1^2 + ka_2 \gamma H_1 H_2 + \frac{1}{2} ka_2 \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

Case-V: Partially submerged backfill with surcharge ($\phi_1 < \phi_2$):

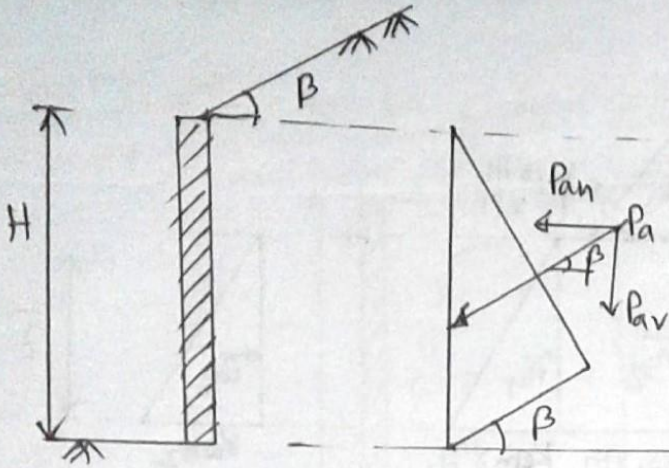


since $\phi_1 < \phi_2$, $ka_2 < ka_1$

$$P_{Ta} = P_{a1} + P_{a2} + P_{a3} + P_{a4} + P_{a5} + P_{a6}$$

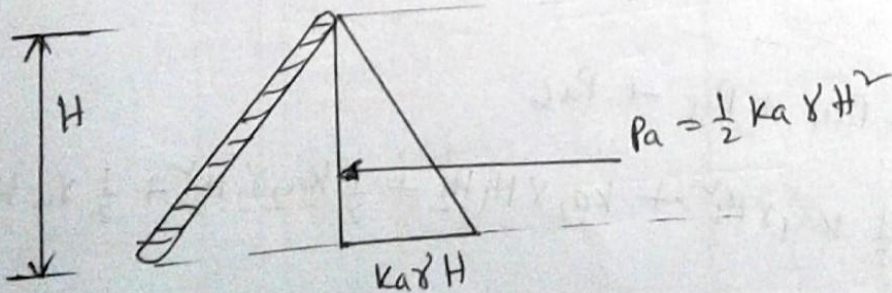
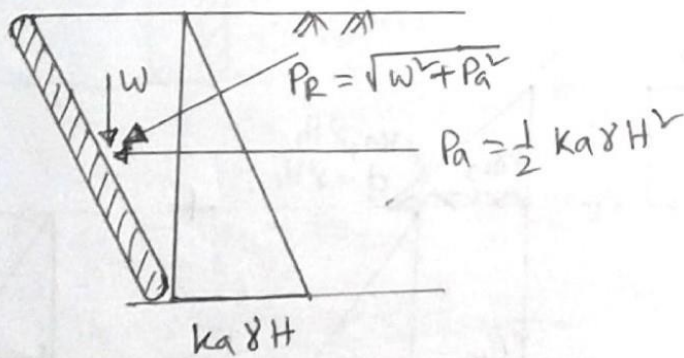
$$= ka_1 q + ka_2 q + \frac{1}{2} ka_1 \gamma H_1^2 + ka_2 \gamma H_1 H_2 + \frac{1}{2} ka_2 \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

Case VI: Backfill with slopping surface:

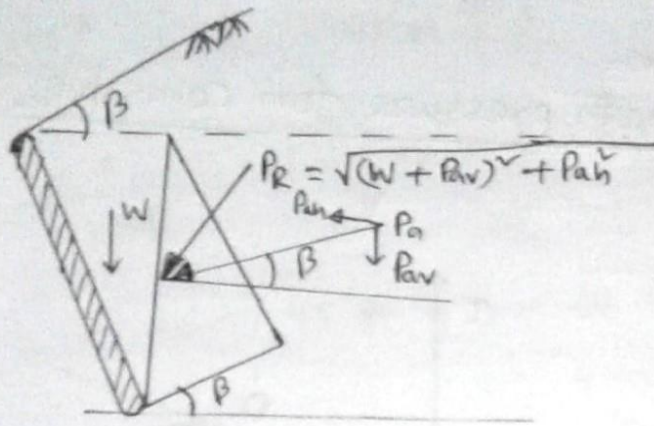


$$k_a = \cos \beta \cdot \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta + \cos^2 \phi}}$$

Case VII: Inclined back with surcharge:



Case VIII: Inclined back and surcharge with slopping surfaces



But, $\sigma_1 = \text{vertical effective overburden pressure} = \gamma z$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45^\circ - \frac{\phi}{2} \right)$$

$$\text{and, } \frac{\cos \phi}{1 + \sin \phi} = \tan \left(45^\circ - \frac{\phi}{2} \right) = \sqrt{K_a}$$

$$\therefore \sigma_3 = \sigma_1 \tan^2 \left(45^\circ - \frac{\phi}{2} \right) - 2c \tan \left(45^\circ - \frac{\phi}{2} \right)$$

$$P_a = \sigma_1 K_a - 2c \sqrt{K_a}$$

$$\therefore \boxed{P_a = K_a \gamma z - 2c \sqrt{K_a}} \quad \text{--- (1)}$$

This is the required equation.

We see that, $P_a = K_a \gamma z$ for cohesionless soil &
 $P_a = K_a \gamma z - 2c \sqrt{K_a}$ for cohesive soil.

From the above two equations we see that the force on the retaining wall is more of a cohesionless soil than cohesive soil.

So, we can use cohesive soil as a backfill material against cohesionless soil (sand).

$$\text{If, } z=0; P_a = -2c \sqrt{K_a} \quad [\text{from (1)}]$$

The negative sign indicates that the pressure is negative.

So, it tries to cause a pull on the wall.

If, $P_a = 0$;

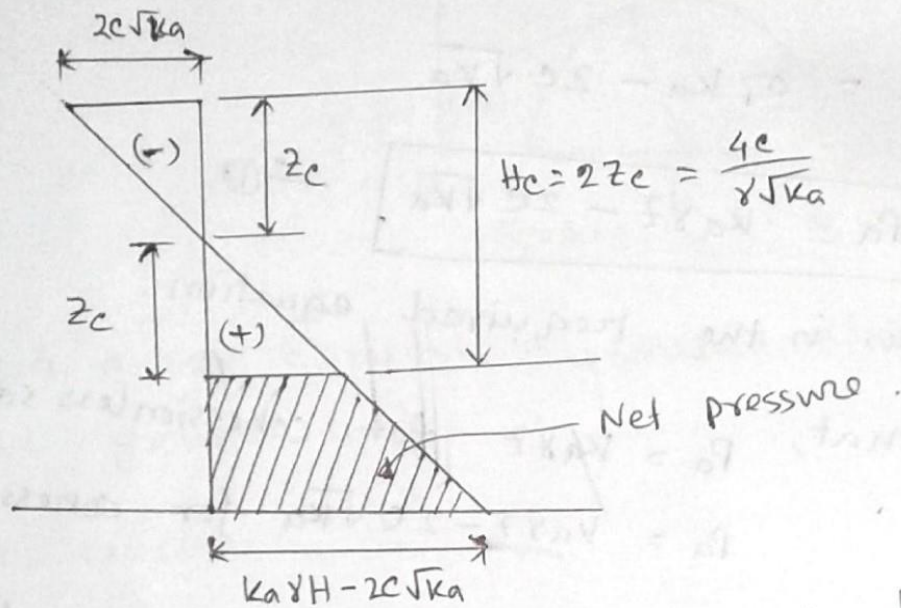
$$0 = K_a \gamma z - 2c \sqrt{K_a}$$

$$\therefore z = z_c = \frac{2c}{\gamma \sqrt{K_a}}$$

This $z = z_c$ is known as tension crack.

If $z = H$,

$$P_a = K_a \gamma H - 2c \sqrt{K_a}$$



H_c = critical height of unsupported vertical cut.

From this pressure diagram it reveals that a cohesive soil can stand for a height of H_c without any lateral support.

Q. Describe the effect of surcharge in Active earth pressure?

Answer: Effects of surcharge: We know that, 2014
earth pressure (with surcharge),

$$P_a = \gamma z K_a - 2c\sqrt{K_a} + qK_a \quad \text{--- (1)}$$

If $z=0$, then from (1) we get,

$$P_a = -2c\sqrt{K_a} + qK_a$$

This slope is stable as long as, $2c\sqrt{K_a} = qK_a$

$$\text{or, } q = \frac{2c\sqrt{K_a}}{K_a}$$

$$\therefore q = \frac{2c}{\sqrt{K_a}}$$

If $q > \frac{2c}{\sqrt{K_a}}$ this slope is unstable.

If $P_a = 0$, then from (1) we get,

$$0 = \gamma z K_a - 2c\sqrt{K_a} + qK_a$$

$$z = \frac{2c\sqrt{K_a} - qK_a}{\gamma K_a}$$

$$\therefore z = \frac{2c}{\gamma\sqrt{K_a}} - \frac{q}{\gamma}$$

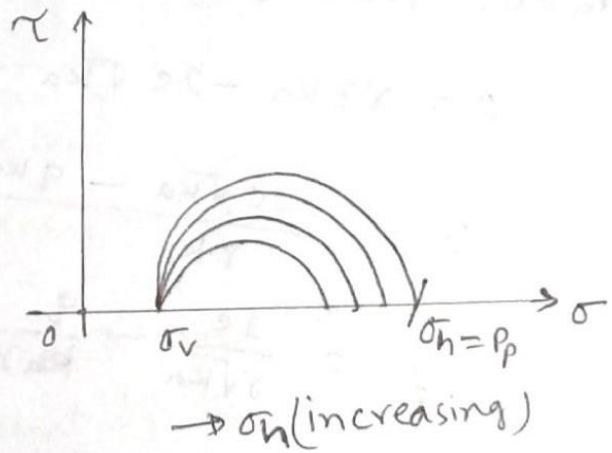
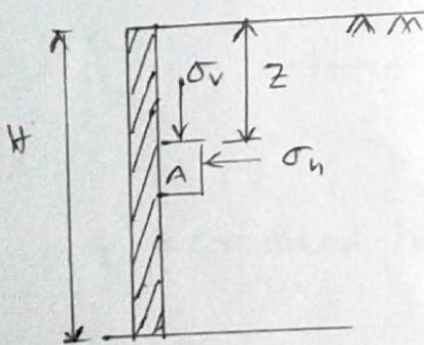
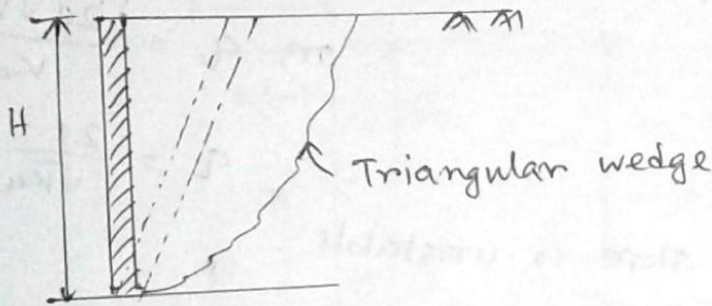
If $\frac{q}{\gamma} > \frac{2c}{\gamma\sqrt{K_a}}$, then there will be a negative pressure at the beginning.

If $\frac{q}{\gamma} < \frac{2c}{\gamma\sqrt{K_a}}$, then there will be a positive pressure at the beginning.

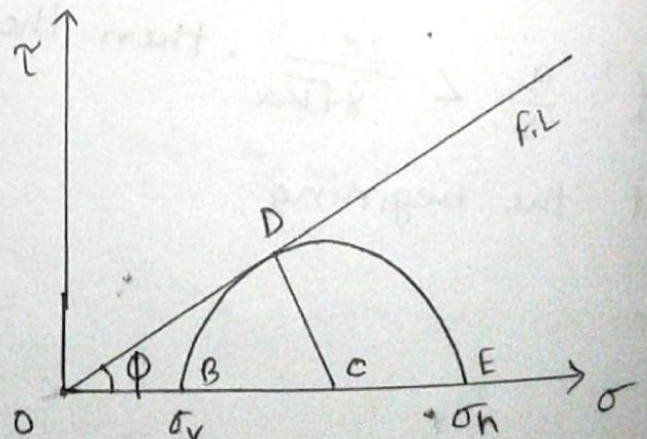
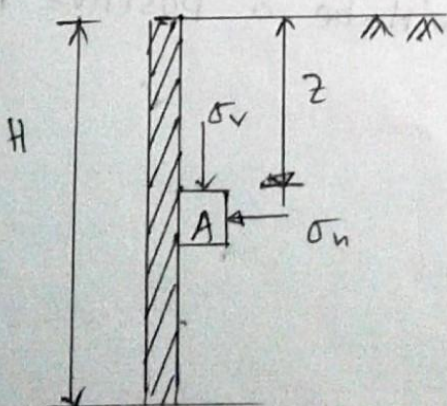
* Define passive earth pressure. Show that the co-efficient of passive earth pressure is more than active co-efficient of earth pressure.

Answer:

Passive earth pressure: If the wall moves towards the soil retain, a triangular wedge will fail. This pressure at this condition is known as passive earth pressure.



Derivation of co-efficient of passive earth pressure; -
(Cohesionless)



Let us consider a soil element 'A' located at a depth z below the ground surface.

From figure, point E represents passive earth pressure $= \sigma_h = P_p$

Now, $P_p = OE = OC + CE = OC + CD = OC + OC \sin \phi$

$$\therefore P_p = OC (1 + \sin \phi) \quad \text{--- (I)}$$

Again, $\sigma_v = OB = OC - BC = OC - OC \sin \phi$

$$\therefore \sigma_v = OC (1 - \sin \phi) \quad \text{--- (II)}$$

from (I) \div (II) we get,

$$\frac{P_p}{\sigma_v} = \frac{OC (1 + \sin \phi)}{OC (1 - \sin \phi)}$$

$$\therefore P_p = \sigma_v \frac{1 + \sin \phi}{1 - \sin \phi}$$

Again, $\sigma_v = \gamma z$.

$$\therefore P_p = \gamma z \frac{1 + \sin \phi}{1 - \sin \phi}$$

$$\therefore \boxed{P_p = \gamma z K_p} \quad \text{where, } \boxed{K_p = \frac{1 + \sin \phi}{1 - \sin \phi}}$$

= co-efficient of passive earth pressure.

Relation between k_a & k_p :

For $\phi = 30^\circ$, $k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1}{3}$

and, $k_p = \frac{1 + \sin \phi}{1 - \sin \phi} = 3$

$$\therefore \frac{k_p}{k_a} = 9, \quad \text{or } \boxed{k_p = 9 k_a}$$

So, we see that co-efficient of passive earth pressure is 9 times more than active co-efficient of earth pressure.

2009 CT R series

Passive earth pressure (Rankine's theory) cohesive backfill:

For the case of cohesive soil the principal stress relationship

Given below:-

From triangle FCD,

$$\sin \phi = \frac{CD}{FC} = \frac{CD}{FO+OC}$$

$$\therefore \sin \phi = \frac{CB}{FO+OC}$$

$$\text{Now, } OC = OB + CB$$

$$= \sigma_3 + \frac{\sigma_1 - \sigma_3}{2}$$

$$= \frac{\sigma_1 + \sigma_3}{2}$$

$$FO = c \cot \phi$$

$$\sin \phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{c \cot \phi + \frac{\sigma_1 + \sigma_3}{2}}$$

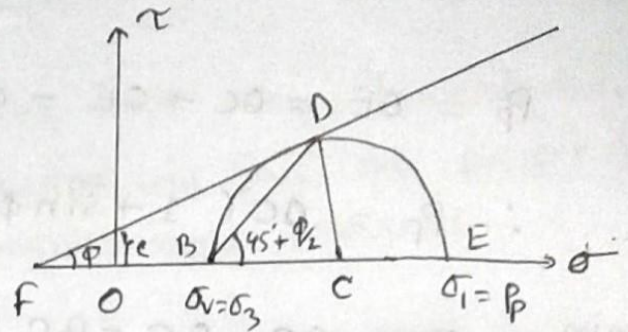
$$\sigma_1 = \sigma_3 \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2c \left(\frac{\cos \phi}{1 - \sin \phi} \right)$$

$$P_p = \sigma_v \tan^2 \left(45^\circ + \frac{\phi}{2} \right) + 2c \tan \left(45^\circ + \frac{\phi}{2} \right)$$

$$P_p = \gamma z K_p + 2c \sqrt{K_p}$$

$$\text{where, } K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = \tan^2 \left(45^\circ + \frac{\phi}{2} \right)$$

$$\sqrt{K_p} = \frac{\cos \phi}{1 - \sin \phi} = \tan \left(45^\circ + \frac{\phi}{2} \right)$$

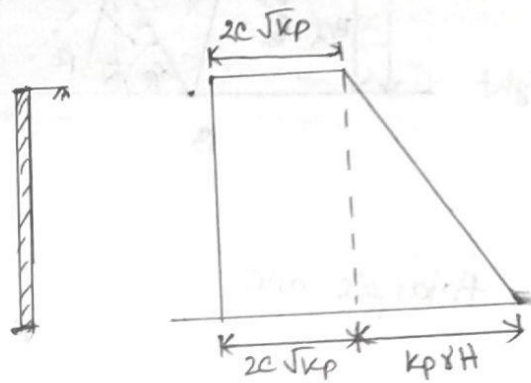


$$\text{At } z=0, P_p = 2c \tan(45^\circ + \frac{\phi}{2}) = 2c \sqrt{K_p}$$

$$\text{At } z=H, P_p = \gamma H K_p + 2c \sqrt{K_p}$$

$$\therefore \text{Total pressure, } P_p = \int_0^H (\gamma z K_p + 2c \sqrt{K_p}) dz$$

$$\therefore P_p = \frac{1}{2} K_p \gamma H^2 + 2c H \sqrt{K_p}$$



Variation of passive pressure cohesive backfill.

Question: Write down the Assumption of Coulomb's wedge theory. Derive an expression for active thrust against the retaining wall for a backfill using Coulomb's wedge theory. 2014, 2013, 2007

Answer: Assumption of Coulomb's wedge theory: Coulomb's (1776)

Proposed a theory. The following assumption were made :-

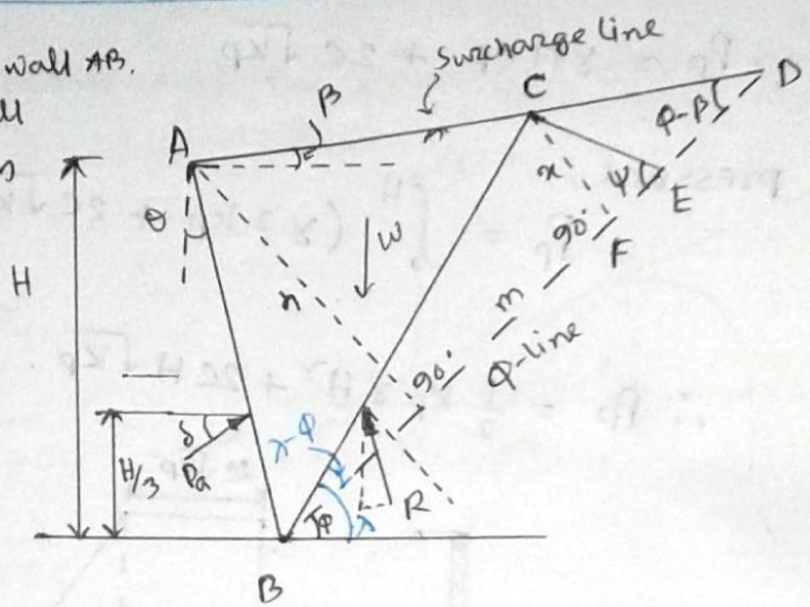
- (i) The backfill is dry and cohesionless, homogeneous, isotropic and plastic material.
- (ii) The slip surface is a plane surface passing through the heel of the wall.
- (iii) The wall surface is rough.
- (iv) The sliding wedge itself acts as a rigid body.

Coulomb's Active pressure in cohesionless soil:

Let us consider a retaining wall AB, inclined at angle θ . The wall supports the backfill which has an inclination angle β with the horizontal and, W = weight of soil.

R = Reaction as the inclined force on slip surface.
 P_a = Active pressure.

$\lambda - \phi$ = Angle between weight and reaction.



From geometry,

$\triangle BCE$ and force triangle are similar.

Hence,

$$\frac{P_a}{W} = \frac{CE}{BE}$$

$$\therefore P_a = W \times \frac{CE}{BE} \quad \text{--- (1)}$$

Now, from $\triangle CEF$,

$$\operatorname{cosec} \psi = \frac{CE}{CF}$$

$$CE = CF \operatorname{cosec} \psi$$

$$CE = x \operatorname{cosec} \psi = A_1 x \quad [\because A_1 = \operatorname{cosec} \psi, CF = x]$$

$$\therefore CE = A_1 x$$

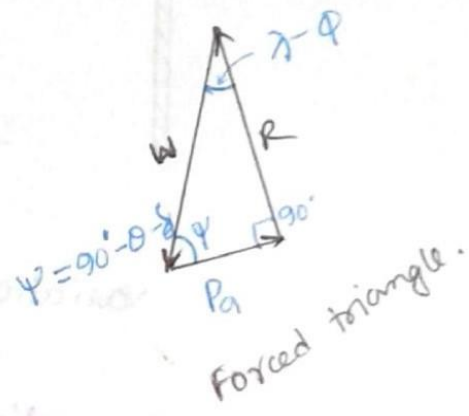
Again from $\triangle CDF$ & $\triangle CEF$

$$DF = x \cot(\phi - \beta) \text{ and } FE = x \cot \psi$$

$$BE = BD - DE = BD - (DF - FE)$$

$$= m - x [\cot(\phi - \beta) - \cot \psi]$$

$$\therefore BE = m - A_2 x \quad [\text{where, } BD = m, A_2 = \cot(\phi - \beta) - \cot \psi]$$



$$\text{Now, } W = 4ABC \times \gamma$$

$$= (4ABD - 4BCD) \gamma$$

$$= \left(\frac{1}{2} nm - \frac{1}{2} xm \right) \gamma$$

$$= \frac{1}{2} \gamma m (n - x)$$

Putting the values of CE, BE and W in equation (1), we get,

$$P_a = W \times \frac{CE}{BE} = \frac{1}{2} \gamma m (n - x) \frac{A_1 x}{m - A_2 x}$$

$$\therefore P_a = \frac{1}{2} \gamma m (n - x) \frac{A_1 x}{m - A_2 x} \quad \text{(According to theory)}$$

For, maxima,

$$\frac{dP_a}{dx} = 0.$$

$$\frac{d}{dx} \left\{ \frac{1}{2} \gamma m (n - x) \frac{A_1 x}{m - A_2 x} \right\} = 0$$

$$(n - 2x)(m - A_2 x) = -A_2 (nx - x^2)$$

$$\therefore mn - mx = x(m - A_2 x)$$

$$\Rightarrow \frac{mn}{2} - \frac{mx}{2} = \frac{x \cdot BE}{2}$$

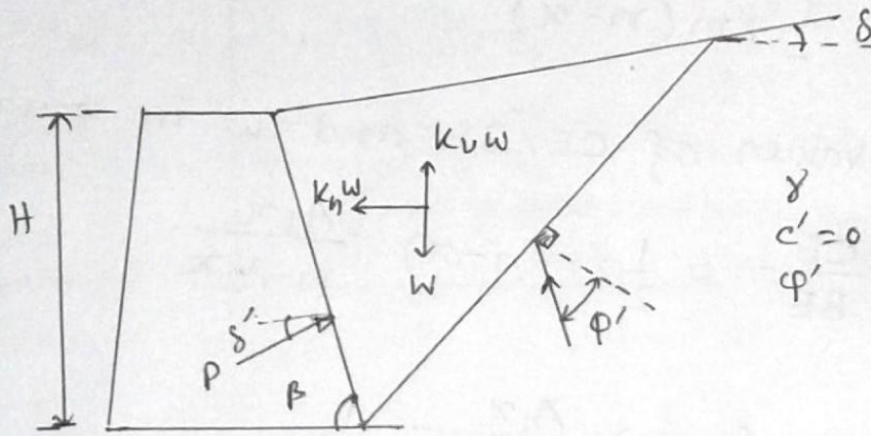
$$\Rightarrow 4ABD - 4BCD = 4BCE$$

$$\therefore 4ABC = 4BCE$$

The criteria for maximum active pressure is that the slip-plane is so chosen that $\triangle ABC$ and $\triangle BCE$ are equal in area.

Q. How earthquake can be incorporated with the Coulomb's active earth pressure? 2013, 2010, 09

Answer: Incorporation of earthquake with the Coulomb's active earth pressure :-



Considering the above figure, we get the active force per unit length of the wall due to the earthquake is given as follows,

$$P_{ae} = \frac{1}{2} K_{ae} \gamma H^2 (1 - k_v) \quad \text{--- (1)}$$

where, active pressure co-efficient due to earthquake,

$$K_{ae} = \frac{\sin^2(\phi' + \beta - \theta')}{\cos \theta' \sin^2 \beta \sin(\beta - \theta' - \delta') \left[1 + \sqrt{\frac{\sin(\phi' + \delta') \sin(\phi' - \theta' - \alpha)}{\sin(\beta - \delta' - \theta') \sin(\alpha + \beta)}} \right]^2}$$

$$\text{Here, } \theta' = \tan^{-1} \left(\frac{k_h}{1 - k_v} \right)$$

Now, $k_v = \frac{\text{vertical earthquake acceleration component}}{\text{acceleration due to gravity, } g}$.

$k_h = \frac{\text{horizontal earthquake acceleration component}}{\text{acceleration due to gravity, } g}$.

* Compare between Rankin's theory and Coulomb's wedge theory for earth pressure. 2014, 2012, 2007

Answer:

Rankin's theory	Coulomb's theory.
(I) Wall friction is neglected	(I) Wall friction is considered.
(II) The back of the wall is vertical and smooth.	(II) The wall surface is rough.
(III) The soil mass is semi-infinite	(III) The soil mass is isotropic.
(IV) The ground surface is plane.	(IV) The ground surface is irregular.
(V) Elementary soil mass is considered.	(V) A sliding wedge is considered.

2015 "The active earth pressure increases if a dry ~~and~~ soil becomes submerged" - Explain.

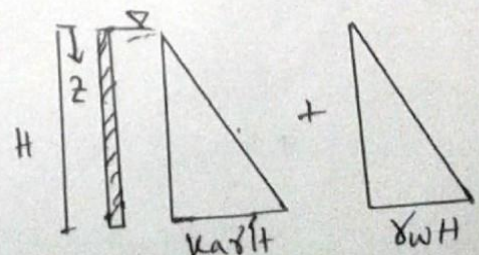
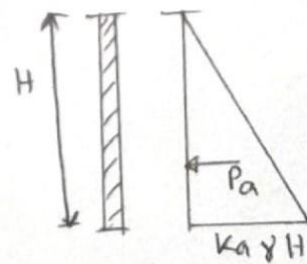
Answer:

For dry soil active pressure,

$$P_a = \frac{1}{2} K_a \gamma H \quad \text{--- (1)}$$

In submerged case, pressure made up two components.

- (1) Lateral pressure due to submerged weight γ' of the soil and.
- (2) Lateral pressure due to water w .



Thus any depth z below the surface,

$$P_a = K_a \gamma' z + \gamma_w z$$

The pressure at the base of the retaining wall is given by.

$$P_a = K_a \gamma' H + \gamma_w H \quad \text{---(II)}$$

from (I) & (II) we see that,

In case of submerged condition intensity of ~~active~~ active earth pressure is more than dry condition.

নাম: রাবিউল ইসলাম
রাজশাহী প্রকৌশল ও প্রযুক্তি বিশ্ববিদ্যালয়
পূর্বকৌশল বিভাগ

রোল নং: ১৬০১১০.

Rabiul → RUET

Stress Distribution

Q. Define Geostatic stress.

Answer: Geostatic stress: Geostatic stress can be defined as the stress on a soil element below the ground surface due to the self weight of the soil above the element.

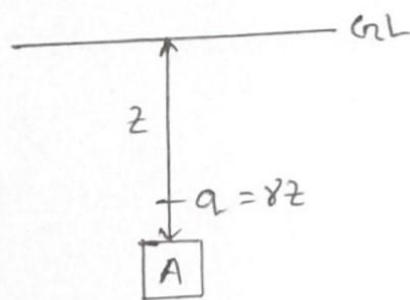


Fig. showing geostatic stress.

Subsoil / sub surface soil experiences two types of load:-

- (I) Geostatic stress (Self weight of soil)
- (II) Excess stress (From surcharge)

Q. How subsurface stress increase?

Answer: Subsurface stress increases as follows:-

- (I) Subsurface stresses in road pavements and airport runways are increased by wheel load on the surface.
- (II) Subsurface stresses is also increased due to surcharge from buildings.
- (III) Embankments and landfills cause to increase subsoil stress.

(iv) It is required to estimate the stress increase in the soil due to the applied loads on the surface.

Q. How you calculated subsurface stress increase?

Answer: Sub surface stress increase is calculated as follows:-

Sub surface stress increase -

- (i) Under point / concentrated loading.
- (ii) Under uniformly loaded circular area.
- (iii) Under line load.
- (iv) Under strip load.
- (v) Uniformly Loaded Rectangular area.

Q. Write down the assumption of the Boussinesq.

Answer: Subsurface stress increase under point load:-

Boussinesq proposed this method, where the following assumptions were made by him.

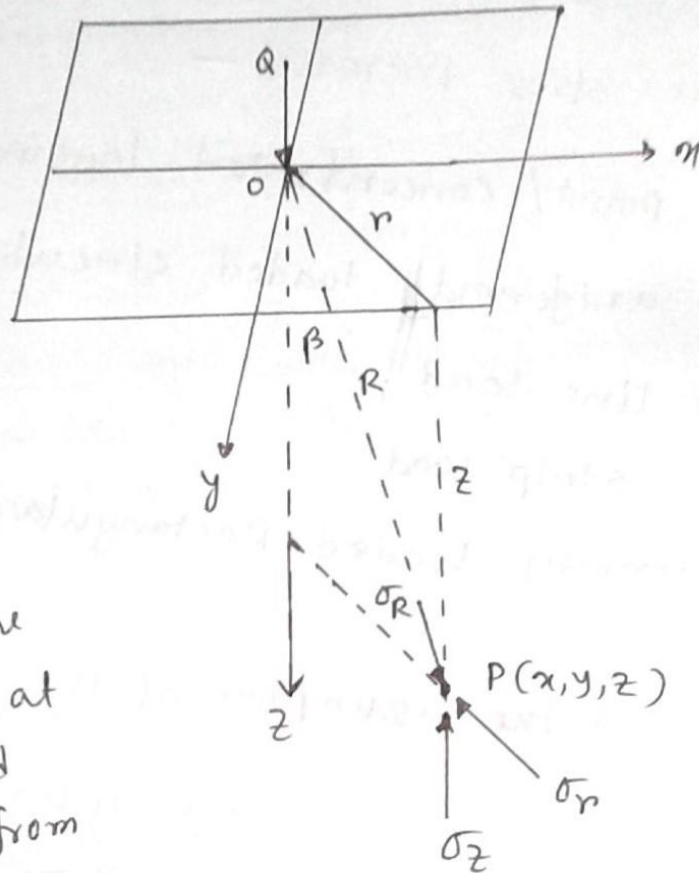
Boussinesq Assumptions:

- (i) The soil mass is an elastic media.
- (ii) The soil mass is homogeneous.
- (iii) The soil is isotropic.
- (iv) The soil mass is semi-infinite.

Q. Derive Boussinesq equation ?

Answer: Derivation of Boussinesq equation:-

Consider the figure, which showing the arrangement of soil and Q , concentrated load is applied on point O and its effect is shown in figure.



At the figure, due to the application of Q . load, at radial distance r and vertical distance z from above, at point P point. where. $P(x, y, z)$.

and, σ_z = vertical stress.

σ_R = polar radial stress

τ_{rz} = Tangential stress.

The polar radial stress is given by,

$$\sigma_R = \frac{3}{2} \frac{Q}{\pi} \frac{\cos \beta}{R^2} \quad \text{--- (1)}$$

Here,

$$R = \sqrt{r^2 + z^2}$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

$$\therefore [r^2 = x^2 + y^2]$$

$$\sin \beta = \frac{r}{R} \text{ and } \cos \beta = \frac{z}{R}$$

The vertical stress.

$$\sigma_z = \sigma_r \cos^3 \beta \quad \text{--- (1)}$$

$$= \frac{3}{2} \frac{Q}{\pi} \frac{\cos \beta}{R^2} \cos^3 \beta \quad [\text{from (1)}]$$

$$= \frac{3}{2} \frac{Q}{\pi} \frac{\cos^3 \beta}{R^2}$$

$$= \frac{3Q}{2\pi} * \frac{(z/R)^3}{R^2}$$

$$= \frac{3Q}{2\pi} * \frac{z^3}{R^5}$$

$$= \frac{3Q}{2\pi} * \frac{1}{z^2} \left[\frac{z^5}{(r^2+z^2)^{5/2}} \right] \quad [\because R = \sqrt{r^2+z^2}]$$

$$= \frac{3Q}{2\pi} * \frac{1}{z^2} \left[\frac{z^5}{z^5 \left\{ 1 + \left(\frac{r}{z} \right)^2 \right\}^{5/2}} \right]$$

$$= \frac{3Q}{2\pi} * \frac{1}{z^2} \left[\frac{1}{1 + \left(\frac{r}{z} \right)^2} \right]^{5/2}$$

$$\therefore \sigma_z = \frac{Q}{z^2} * k_B$$

$$\text{Where, } k_B = \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{r}{z} \right)^2} \right]^{5/2}$$

= Boussinesq's influence factor.

From the equation we can say that σ_z is not a function of E (Young's Modulus) and Poisson ratio (μ).

The tangential stress:

The tangential stress,

$$\tau_{rz} = \frac{1}{2} \sigma_r \sin 2\beta$$

$$= \frac{1}{2} \frac{3Q \cos \beta}{2\pi R^2} * z \sin \beta \cos \beta$$

$$= \frac{3Q}{2\pi} \frac{\cos^2 \beta \sin \beta}{R^2}$$

$$= \frac{3Q}{2\pi} * \frac{1}{R^2} * \left(\frac{z}{R}\right)^2 \left(\frac{r}{R}\right)$$

$$= \frac{3Q}{2\pi} * \frac{z^2 r}{R^5}$$

$$= \frac{3Q}{2\pi} * \frac{z^2 r}{(r^2 + z^2)^{5/2}} \quad [\because R = \sqrt{r^2 + z^2}]$$

$$= \frac{3Q}{2\pi} * \frac{1}{z^3} \frac{z^5 r}{z^3 \left\{ 1 + \left(\frac{r}{z}\right)^2 \right\}^{5/2}}$$

$$= \frac{3Qr}{2\pi z^3} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$$\therefore \tau_{rz} = \frac{Qr}{z^3} K_B$$

where, $K_B = \frac{3}{2\pi} * \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$

= Boussinesq influence factor.

From the above equation we can say that τ_{rz} does not depend on E and μ i.e. \uparrow Young Modulus & Poisson ratio.

$$\text{So, } \frac{\sigma_z}{\tau_{rz}} = \frac{Q}{z^2} \times \frac{z^3}{Qr} = \frac{z}{r}$$

$$\therefore \sigma_z = \left(\frac{z}{r}\right) \tau_{rz}$$

So, it shows that, σ_z is related with τ_{rz} by a non dimensional factor.

Vertical stress, σ_z directly below the point load (i.e. $r=0$;

$$\sigma_z = \frac{3Q}{2\pi} \times \frac{1}{z^2} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \left[\frac{1}{1+0} \right]^{5/2}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2}$$

$$\therefore \sigma_z = \frac{0.4775 Q}{z^2}$$

▣ Semi-infinite soil mass: If the soil mass is bounded by horizontal plane xy (ground surface) and z -axis is directed downward, it is called semi-infinite soil mass.

▣ Homogeneous soil mass: If all the elements in a soil mass are similar and it has identical properties at every points in it in ^{identical} direction, then it is called homogeneous soil mass.

▣ Isotropic soil mass: A soil mass can be defined as isotropic when this soil mass has identical elastic properties in all direction through any point of it.

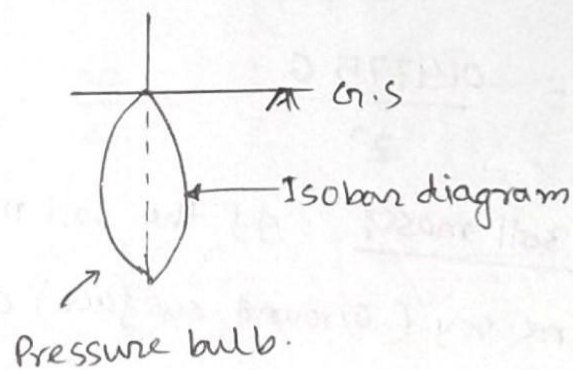
Q. What are the types of vertical pressure diagram.

Answer: Vertical pressure distribution Diagrams:-

- (I) Stress isobar or isobar diagram.
- (II) Vertical pressure distribution on a horizontal plane.
- (III) Vertical pressure distribution on a vertical plane.

Q. Define stress isobar. 2014

Answer: Stress isobar: An isobar is a curve or contour joining the points equal vertical stress below ground surface. Its shape is like an electric bulb. The zone bounded by an isobar is called a pressure bulb.



Q. What are the significances of pressure bulb? 2014

Answer: Significance of pressure bulb:

- (I) The vertical pressure at every point on the surface of pressure bulb is same.
- (II) The smaller the value of σ_z' higher the area of pressure bulb.

2015, 2014,

2. Prove that at a given depth when horizontal radial stress distance is equal to twice the depth the vertical pressure due to single concentrated load can be considered negligible.

Answer:

Let us determine the stresses at a depth $z = z$ unit.

Therefore,

$$\sigma_z = \frac{Q}{z^2} K_B$$

$$\therefore \sigma_z = \frac{Q}{z^2} K_B$$

$$\sigma_z = 0.25 Q K_B \quad \text{--- (1) where, } K_B = \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

r	r/z	K_B	σ_z	%
0	0	0.4775	0.1194 Q	100
0.5	0.25	0.4103	0.102575	86
1	0.5	0.2733	0.0683	57
1.5	0.75	0.1564	0.03911	32.7
2	1	0.0844	0.02110	17.67
2.5	1.25	0.0454	0.011356	9.5
3	1.5	0.025	0.006268	5.2
3.5	1.75	0.0143	0.003588	3
4	2	0.0085	0.002135	1.78

So, we can see that, when the horizontal distance = 2 × depth, the vertical pressure due to point load is negligible.

2015, 2013,

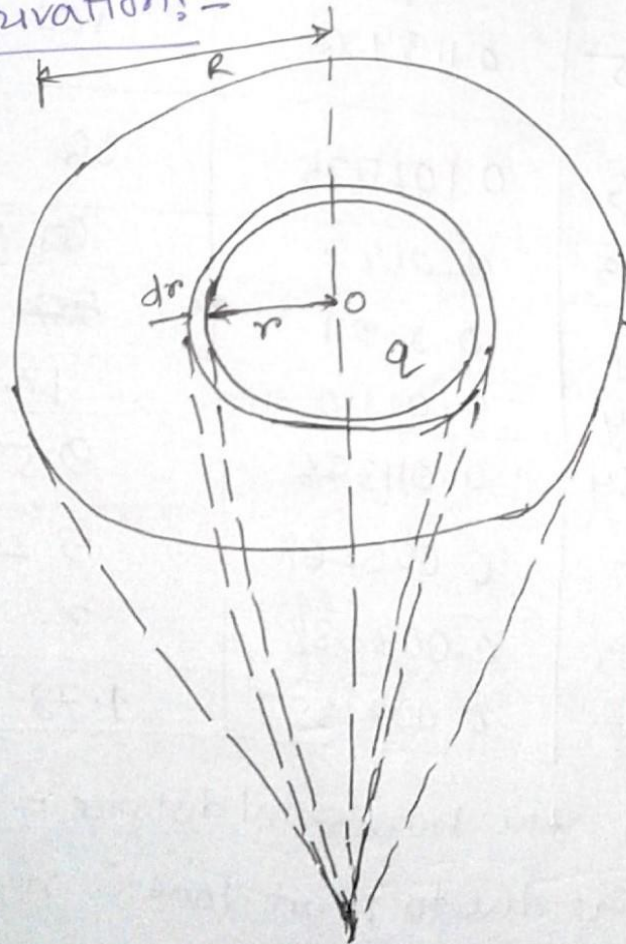
Q. what is an Influence diagram? what is its use in practice?

Answer: Influence diagram: An influence diagram is the vertical stress distribution diagram on a horizontal plane, at a given depth due to a unit concentrated load.

It is helpful to determine vertical stress at any point on that horizontal plane due to number of concentrated load.

Q. Derive expression for vertical stress due to uniformly loaded circular area.

Answer: Derivation: -



Consider radial loaded area,

q = intensity of load per unit area.

R = radius of loaded area

r = radius of elementary ring.

dr = width of elementary ring.

The load on elementary ring = $q + 2\pi r dr$

We know,

$$\sigma_z = \frac{3Q}{2\pi} \frac{1}{z^2} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

Following this equation,

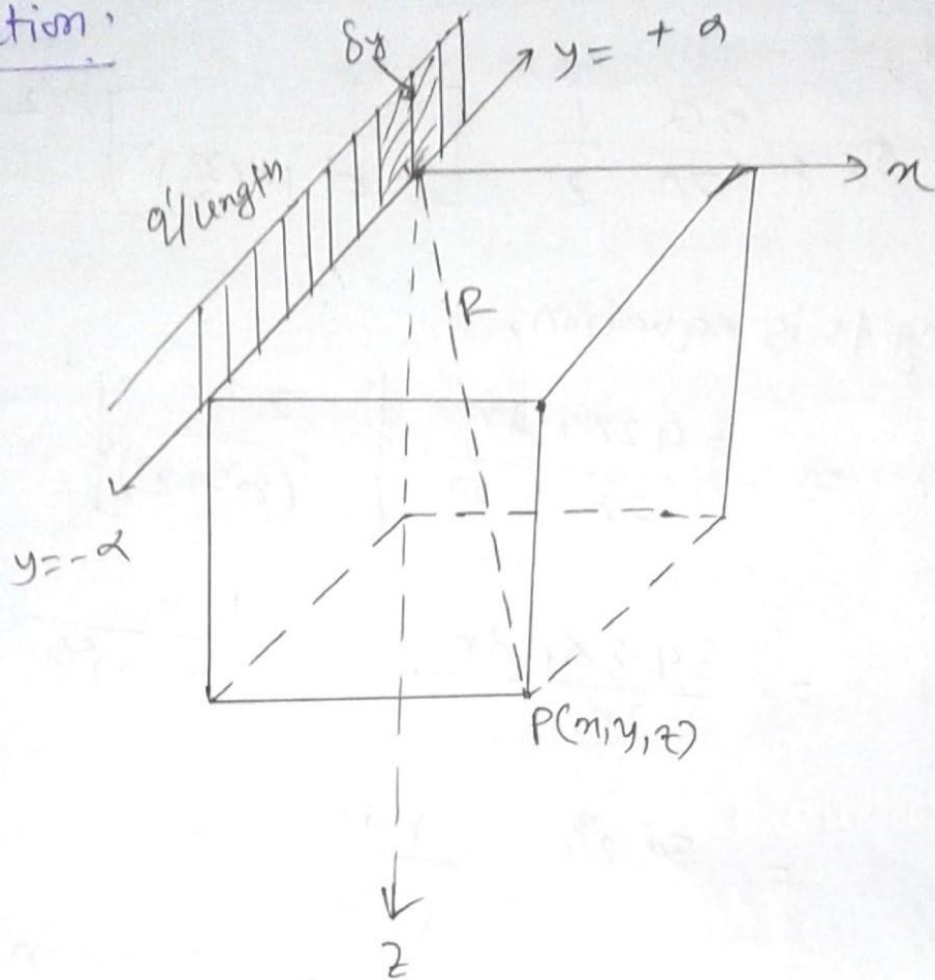
$$\begin{aligned} 4\sigma_z &= \frac{3Q 2\pi r dr}{2\pi z^2} \left[\frac{z^5}{(r^2 + z^2)^{5/2}} \right] \\ &= \frac{3Q 2\pi r dr}{2\pi} \cdot z^3 \cdot \frac{1}{(r^2 + z^2)^{5/2}} \\ &= 3Q z^3 \frac{r dr}{(r^2 + z^2)^{5/2}} \end{aligned}$$

Vertical load due to entire load is given by,

$$\begin{aligned} \sigma_z &= 3Q z^3 \int_0^R \frac{r dr}{(r^2 + z^2)^{5/2}} \quad \left| \begin{array}{l} \text{let,} \\ r^2 + z^2 = u \\ \therefore 2r dr = du \\ r dr = \frac{du}{2} \\ r=0, u=z^2 \\ r=R, u=z^2 + R^2 \end{array} \right. \\ &= 3Q z^3 \int_{z^2}^{z^2 + R^2} \frac{1}{u^{5/2}} \frac{du}{2} \\ &= \frac{3Q z^3}{2} \left(-\frac{2}{3} \right) \left[u^{-3/2} \right]_{z^2}^{z^2 + R^2} \\ &= -Q z^3 \left[\frac{1}{(z^2 + R^2)^{3/2}} - \frac{1}{z^3} \right] \\ &= Q z^3 \left[\frac{1}{z^3} - \frac{1}{z^3 \left\{ 1 + \left(\frac{R}{z}\right)^2 \right\}^{3/2}} \right] \\ &= Q \left[1 - \frac{1}{\left\{ 1 + \left(\frac{R}{z}\right)^2 \right\}^{3/2}} \right] \\ \sigma_z &= Q * k_c, \quad k_c = \left[1 - \frac{1}{\left\{ 1 + \left(\frac{R}{z}\right)^2 \right\}^{3/2}} \right] \\ \sigma_z &= Q k_c \quad \text{proved} \end{aligned}$$

Q. Derive expression for vertical stress due to line load.

Answer: Derivation.



Let,

$q' =$ intensity of load per unit length.

Let us considering the load acting on a small length δy

\therefore point load, $= q' \delta y$.

we know,

$$\sigma_z = \frac{3Q}{2\pi} \cdot \frac{1}{z^2} \left[\frac{1}{(r^2/z^2) + 1} \right]^{5/2}$$

Following this equation,

$$4\sigma_z = \frac{3q'\delta y}{2\pi} \cdot \frac{1}{z^2} \left[\frac{z^5}{(r^2 + z^2)^{5/2}} \right]$$

$$4\sigma_z = \frac{3q'\delta y}{2\pi} \frac{z^3}{(r^2 + z^2)^{5/2}}$$

Integrating we get,

$$\sigma_z = \frac{3a'z^3}{2\pi} \int_{-\alpha}^{\alpha} \frac{dy}{(r^2+z^2)^{5/2}}$$

$$= \frac{3a'z^3}{2\pi} * 2 \int_0^{\alpha} \frac{dy}{(r^2+y^2+z^2)^{5/2}} \quad [\because r^2 = r^2 + y^2]$$

$$= \frac{3a'z^3}{\pi} \int_0^{\alpha} \frac{dy}{(u^2+y^2)^{5/2}}$$

$$= \frac{3a'z^3}{\pi} \int_0^{\pi/2} \frac{u \sec^2 \theta d\theta}{(u^2 + u^2 \tan^2 \theta)^{5/2}}$$

$$= \frac{3a'z^3}{\pi} \int_0^{\pi/2} \frac{u \sec^2 \theta d\theta}{u^4 \sec^5 \theta}$$

$$= \frac{3a'z^3}{\pi u^4} \int_0^{\pi/2} \cos^3 \theta d\theta$$

$$= \frac{3a'z^3}{\pi u^4} \int_0^{\pi/2} \cos^2 \theta \cdot \cos \theta d\theta$$

$$= \frac{3a'z^3}{\pi u^4} \int_0^{\pi/2} (1 - \sin^2 \theta) \cos \theta d\theta$$

$$= \frac{3a'z^3}{\pi u^4} \int_0^1 (1 - t^2) dt$$

$$= \frac{3a'z^3}{\pi u^4} \left[t - \frac{t^3}{3} \right]_0^1$$

$$= \frac{3a'z^3}{\pi u^4} * \left(1 - \frac{1}{3} \right)$$

let,
 $r^2 + z^2 = u^2$

Again,

$$y = u \tan \theta$$

$$dy = u \sec^2 \theta d\theta$$

$$y = 0, \theta = 0$$

$$y = \alpha, \theta = \pi/2$$

Again let,

$$t = \sin \theta$$

$$dt = \cos \theta d\theta$$

$$\theta = 0, t = 0$$

$$\theta = \pi/2, t = 1$$

$$= \frac{2q'z^3}{\pi u^4} * \frac{2}{3}$$

$$\sigma_z = \frac{2q'z^3}{\pi (r^2+z^2)^2}$$

$$\sigma_z = \frac{2q'}{\pi z} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]$$

This is the required equation of vertical stress due to line load.

Q. 2015, 2014 Explain the applicability of Newmark's Influence chart?

Answer: USE of Newmark's Influence chart:

- (a) A plan of the loaded area is drawn on a tracing paper.
- (b) The scale is chosen such that unit length in Newmark's chart = depth (z) at point P below the surface.
- (c) The traced plan of the loaded area is placed over Newmark's chart such that point P coincides with the centre of the chart.
- (d) Count the no of small unit area in covered by the traced plan. Fractions of unit area should also be counted. Then, $\sigma_z = I \times n \times q$

Where,

$$n = n_1 + n_2$$

$n_1 =$ full area

$n_2 =$ full area by partial summation.

$I =$ influenced coefficient

$$I = 0.005$$

Q. What do you understand by contact pressure? what factors does it depend on?

Answer: Contact pressure: The upward pressure due to soil on the underside of the footing is termed contact pressure. So, far we assumed that the footing is flexible and contact pressure distribution is uniform.

However actual footings are not flexible as assumed.

Dependence of Contact pressure:

- (i) Elastic properties of footing material.
- (ii) Thickness of the footing.
- (iii) Relative rigidity K_r of footing soil system.

$$K_r = \frac{1}{6} \times \left(\frac{1 - \nu_s^2}{1 - \nu_f^2} \right) \left(\frac{E_f}{E_s} \right) \left(\frac{t}{b} \right)$$

$\nu_s, \nu_f =$ poisson's ratio for soil and footing materials.
 $E_s, E_f =$ Modulus of elasticity for soil and footing materials.
 $2b =$ width of footing.
 $t =$ thickness of footing.

If $k_r = 0$, it is a purely flexible footing.

If $k_r = \infty$, it is a perfectly rigid footing.

If k_r is $0 < k_r < \infty$ then it is not rigid nor flexible.

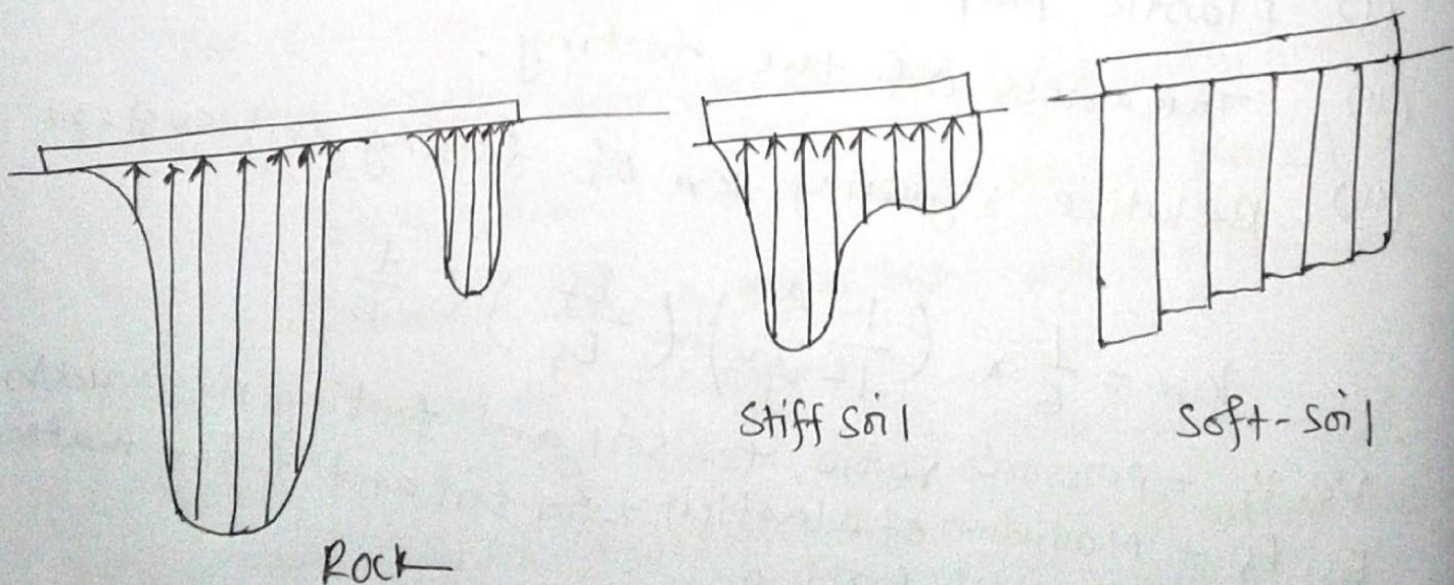
2015

Q. Draw and explain the contact pressure distribution below a mat foundation?

The contact pressure distribution for mat footing are quite different from those with spread footing.

Usually mat foundation have a much smaller thickness to width ratio and thus more flexible than spread footing. Therefore the assumption of rigidity is no longer rigid.

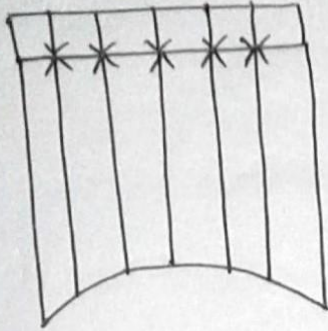
Also the assumptions of linear contact pressure distribution are erroneous.



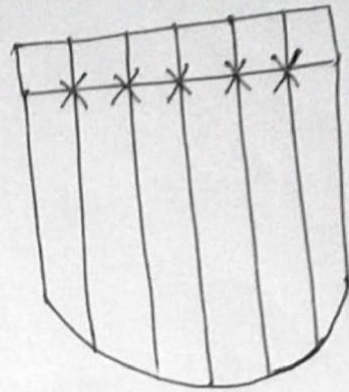
Draw contact pressure diagram for clay and sandy soil.

Answer:

Contact pressure on clay:

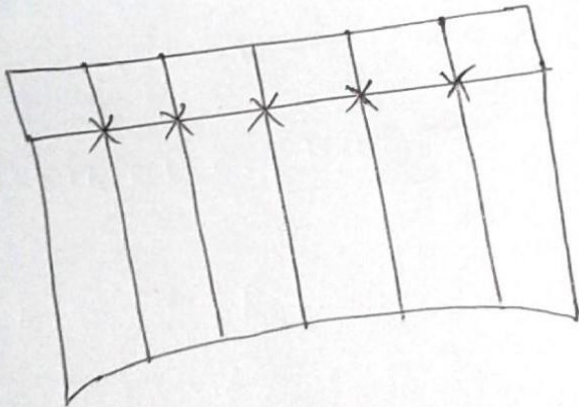


Rigid footing

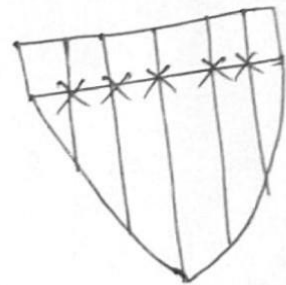


flexible footing.

Contact pressure on sand:



Flexible footing.



Rigid footing

শ্রী: রুবিউল ইসলাম

রাজশাহী প্রকৌশল ও প্রযুক্তি বিশ্ববিদ্যালয়

পুরাকৌশল বিভাগ

রোল নং: ২৬০২২০

Geotechnical Engineering - II

Slope Stability

Q. ²⁰¹³

Discuss the different modes of failure with sketches for the analysis of finite slopes with circular failure surface.

Answer: Modes of failure in finite slope:

There are two basic types of failure of finite slope.

(I) Slope failure (II) Base failure.

Slope failure: If the failure occurs along a surface of sliding that intersects the slope at or above its toe, the slide is known as slope failure.

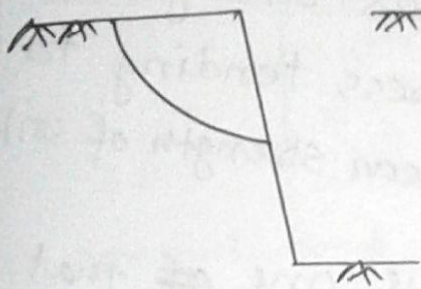
Slope failure is two types.

(a) Face failure and (b) Toe failure.

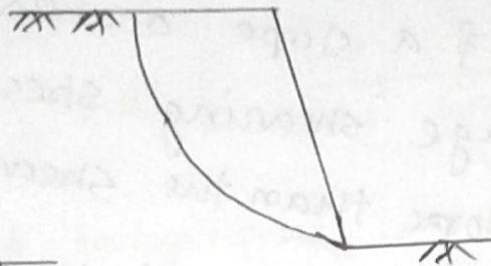
(a) Face Failure: If the failure surface passes above the toe, the slope failure is called the face failure.

(b) Toe failure: If the failure surface passes through the toe, the slope failure is called the toe failure.

(1) Base failure: If the failures occur along the surface of sliding that passes at some distance below the toe of slope, the slide is known as the base failure.



(a) Face failure



(b) Toe Failure



(c) Base failure.

Q: What are the methods of analysis of the finite slope?

Answer: Methods of analysis of finite slope are given below:-

- (I) Culman's method of planar failure surface.
- (II) Slip circle method.
- (III) The friction circle method.
- (IV) Bishop's method etc.

Q: Define critical slip circle/critical slip surface.

Answer: Critical slip circle/critical slip surface:-

The slip surface or circle have the least value of factor of the safety is called the critical slip circle or critical slip surface.

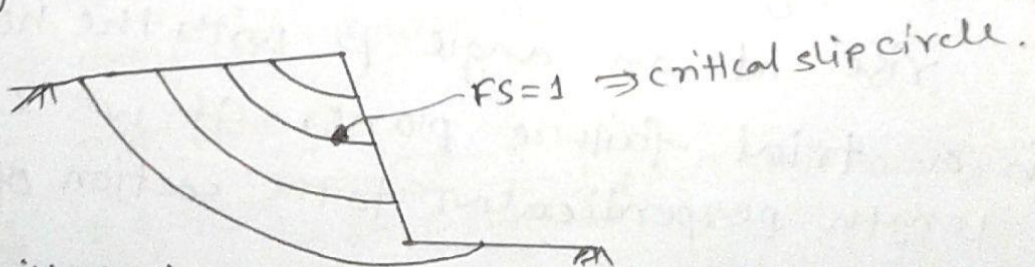


Fig. critical slip surface.

Q: Derive the equation for critical height of slope for finite slope with failure surface (Culmann's method)

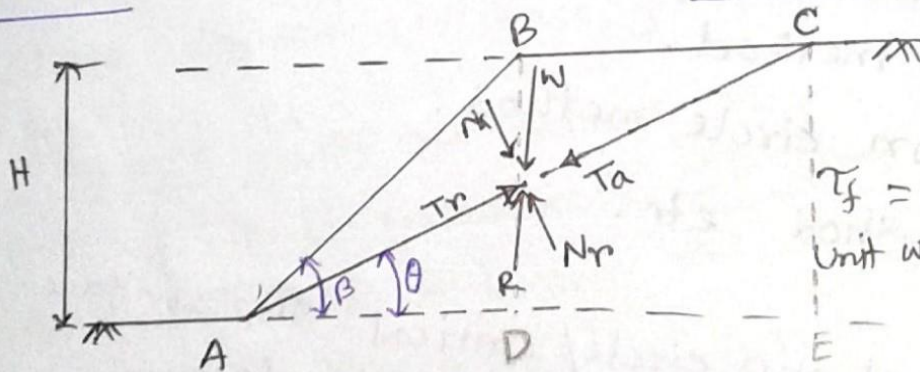
Answer:

Assumptions:

- (I) The failure of a slope occurs along a plane when the average shearing stress tending to cause slip is more than the shear strength of soil.
- (II) The most critical plane is the one that has a minimum ratio of the average shearing stress that tends to cause failure to the shear strength of soil.

Derivation:

$$\begin{aligned}
 H &= AC \sin \theta \Rightarrow AC = \frac{H}{\sin \theta} \\
 \frac{H}{AE} &= \tan \theta \Rightarrow AE = H \cot \theta \\
 \frac{H}{AD} &= \tan \beta \Rightarrow AD = H \cot \beta \\
 BC &= AE - AD \\
 &= H \cot \theta - H \cot \beta
 \end{aligned}$$



$$\begin{aligned}
 \tau_f &= c + \sigma' \tan \phi' \\
 \text{Unit weight of soil} &= \gamma
 \end{aligned}$$

Fig. Finite slope analysis - Culmann's method.

The above figure shows a slope of height H. The slope rise at an angle β with the horizontal, AC is a trial failure plane. If we consider a unit length perpendicular to the section of the slope,

We find the weight of the wedge ABC is,

$$W = \text{volume} \times \text{unit weight}$$

$$= \frac{1}{2} * (H) * (\overline{BC}) * 1 * \gamma$$

$$= \frac{1}{2} \gamma H (H \cot \theta - H \cot \beta) \quad [\because BC = H \cot \theta - H \cot \beta]$$

$$= \frac{1}{2} \gamma H^2 \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \quad \dots (I)$$

The normal and tangential components of W with respect to the plane AC are as follows: -

$$N_a = \text{Normal component} = W \cos \theta = \frac{1}{2} \gamma H^2 \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \cos \theta \quad \dots (II)$$

$$T_a = \text{tangential component} = W \sin \theta = \frac{1}{2} \gamma H^2 \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \sin \theta \quad \dots (III)$$

The average effective normal stress and the average shear stress on the plane AC are respectively,

$$\sigma' = \frac{N_a}{(\overline{AC}) \cdot 1} = \frac{N_a}{\frac{H}{\sin \theta}} = \frac{1}{2} \gamma H \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \cos \theta \cdot \sin \theta \quad \dots (IV)$$

$$\text{and, } \tau = \frac{T_a}{(\overline{AC}) \cdot 1} = \frac{T_a}{\frac{H}{\sin \theta}} = \frac{1}{2} \gamma H \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \sin^2 \theta \quad \dots (V)$$

The average resistive shearing stress developed along the plane AC also may be expressed as,

$$\tau_d = c_d' + \sigma' \tan \phi_d'$$

$$\tau_d = c_d' + \frac{1}{2} \gamma H \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \cos \theta \sin \theta \tan \phi_d' \quad \dots (VI)$$

From equation (V) & (VI) we get,

$$\frac{1}{2} \gamma H \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \sin^2 \theta = c_d' + \frac{1}{2} \gamma H \left[\frac{\sin(\beta - \theta)}{\sin \beta \cdot \sin \theta} \right] \cos \theta \sin \theta \tan \phi_d'$$

$$\text{or, } c_d' = \frac{1}{2} \gamma H \left[\frac{\sin(\beta - \theta) (\sin \theta - \cos \theta \tan \phi_d')}{\sin \beta} \right] \quad \text{--- (VI)}$$

For maxima, $\frac{\partial c_d'}{\partial \theta} = 0$.

$$\therefore \frac{\partial}{\partial \theta} [\sin(\beta - \theta) (\sin \theta - \cos \theta \tan \phi_d')] = 0$$

[since, γ , H and β are constant]

Solving above equation gives the critical value of θ , or

$$\theta_{cr} = \frac{\beta + \phi_d'}{2}$$

Substituting the value $\theta = \theta_{cr}$ in equation (VI) we get,

$$c_d' = \frac{1}{4} \gamma H \left[\frac{1 - \cos(\beta - \phi_d')}{\sin \beta \cos \phi_d'} \right] \quad \text{--- (VII)}$$

The above equation can be also written as,

$$\frac{c_d'}{\gamma H} = m = \frac{1 - \cos(\beta - \phi_d')}{4 \sin \beta \cos \phi_d'} \quad \text{, where, } m = \text{stability number}$$

The maximum height of the slope for which critical equilibrium occurs can be obtained by substituting $c_d' = c'$ and $\phi_d' = \phi'$ in equation (VII) we get,

$$H_{cr} = \frac{4c'}{\gamma} \left[\frac{\sin \beta \cos \phi'}{1 - \cos(\beta - \phi')} \right]$$

Q. what is stability number? what is its utility in the analysis of slope stability?

Answer: Stability number: The ratio of resistive cohesive force to force causing instability is called stability number. that is,

$$SN = \frac{cH}{F_c \gamma H^2}$$

, where, resistive cohesive force = cH
 force causing instability = $F_c \gamma H^2$
 $SN =$ Stability number.

$$SN = \frac{c}{F_c \gamma H}$$

$$SN = \frac{c}{\frac{Hc}{H} \gamma H}$$

$$[\because F_c = \frac{Hc}{H}]$$

$$\therefore SN = \frac{c}{\gamma Hc}$$

For non cohesive soil, $F = \frac{\tan \phi}{\tan i} \Rightarrow F_c = \frac{c}{c_m} = \frac{Hc}{H}$
 $\Rightarrow \frac{c}{Hc} = \frac{c_m}{H}$

$$\therefore SN = \frac{c}{\gamma Hc} = \frac{c_m}{\gamma H}$$

Significance:

- (1) From Taylor's stability number SN , slope angle i , and angle of internal friction ϕ can be determined.
- (2) For different slope angle depth factor D_f can be determined for different values of SN .

Modes of Failure of Finite Slope:

In general finite slope failure occurs two types.

(I) Slope failure $\left\{ \begin{array}{l} \rightarrow \text{Face failure} \\ \rightarrow \text{Toe failure} \end{array} \right.$

(II) Base failure.

Q. Describe the stability analysis procedure for

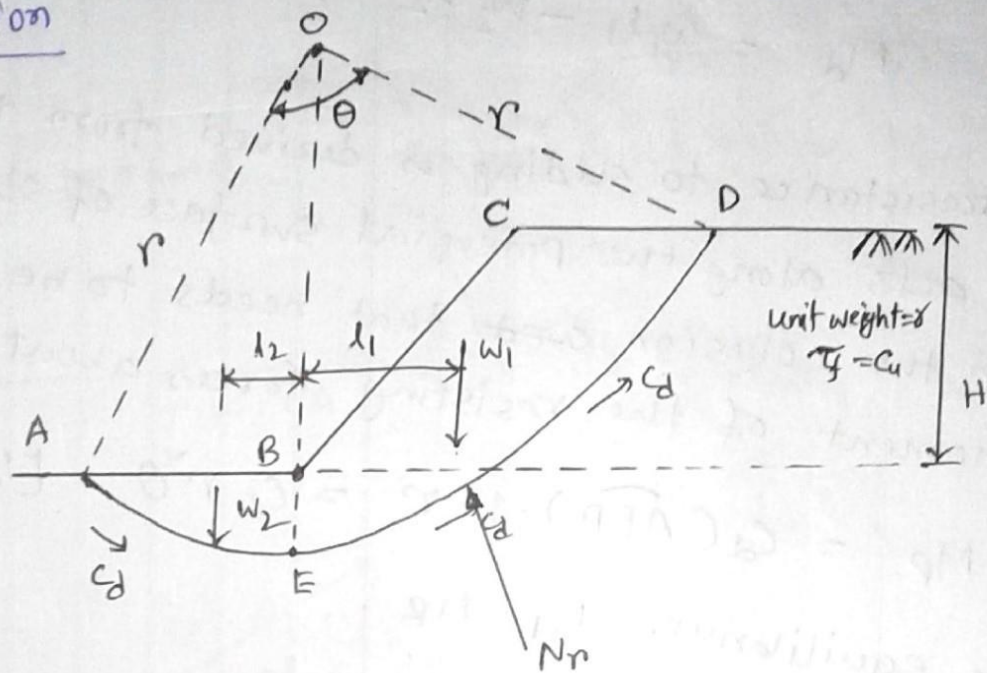
Answer: Various procedures of stability analysis may in generally divided into two major classes.

(I) Mass procedure: In this case, the mass of the soil above the surface of sliding is taken as a unit. This procedure is useful when the soil that forms the slope is assumed to be homogeneous, although this is not the case in most natural slopes.

(II) Method of slices: In this procedure, the soil above the surface of sliding is divided into a number of versatile parallel slices. The stability of each slice is calculated separately. This is a versatile technique in which the nonhomogeneity of the soils and pore water pressure can be taken into consideration. It also accounts for the variation of the normal stress along the potential failure surface.

Derive critical height by Mass procedure slopes in Homogeneous clay soil with $\phi = 0$ [Swedish slip circle method]

Answer: Derivation



The above figure shows a slope in a homogeneous soil.

The undrained shear strength of the soil is assumed to be constant with depth and may be given by $\tau_f = c_u$.

To perform the stability analysis we choose a trial potential curve of sliding AED , which is an arc of a circle that has a radius r and centre point is O .

Considering unit length perpendicular to the section.

The weight of soil above the curve AED is given as,

$$W = W_1 + W_2$$

where, $W_1 = (\text{area of } BCDEB) \gamma$

and, $W_2 = (\text{area of } ABEA) \gamma$

Failure of the slope may occur by sliding of the soil mass.

The moment of the driving force about O to cause slope instability is,

$$M_D = W_1 l_1 - W_2 l_2, \text{ where, } l_1, l_2 \text{ are moment arm.}$$

The resistance to sliding is derived from the cohesion that acts along the potential surface of sliding. If c_d is the cohesion and that needs to be developed, the moment of the resisting forces about O is,

$$M_R = c_d (\widehat{AED}) \cdot 1 \cdot r = c_d r^2 \theta, \quad [\because \widehat{AED} = r \theta]$$

For equilibrium, $M_D = M_R$.

$$\therefore c_d r^2 \theta = W_1 l_1 - W_2 l_2$$

$$\therefore c_d = \frac{W_1 l_1 - W_2 l_2}{r^2 \theta}$$

The factor of safety against sliding may now be found,

$$F_s = \frac{\tau_f}{c_d} = \frac{c_u}{c_d}$$

For the case of critical circle, the developed cohesion can be expressed by the relationship.

$$c_d = \gamma H m$$

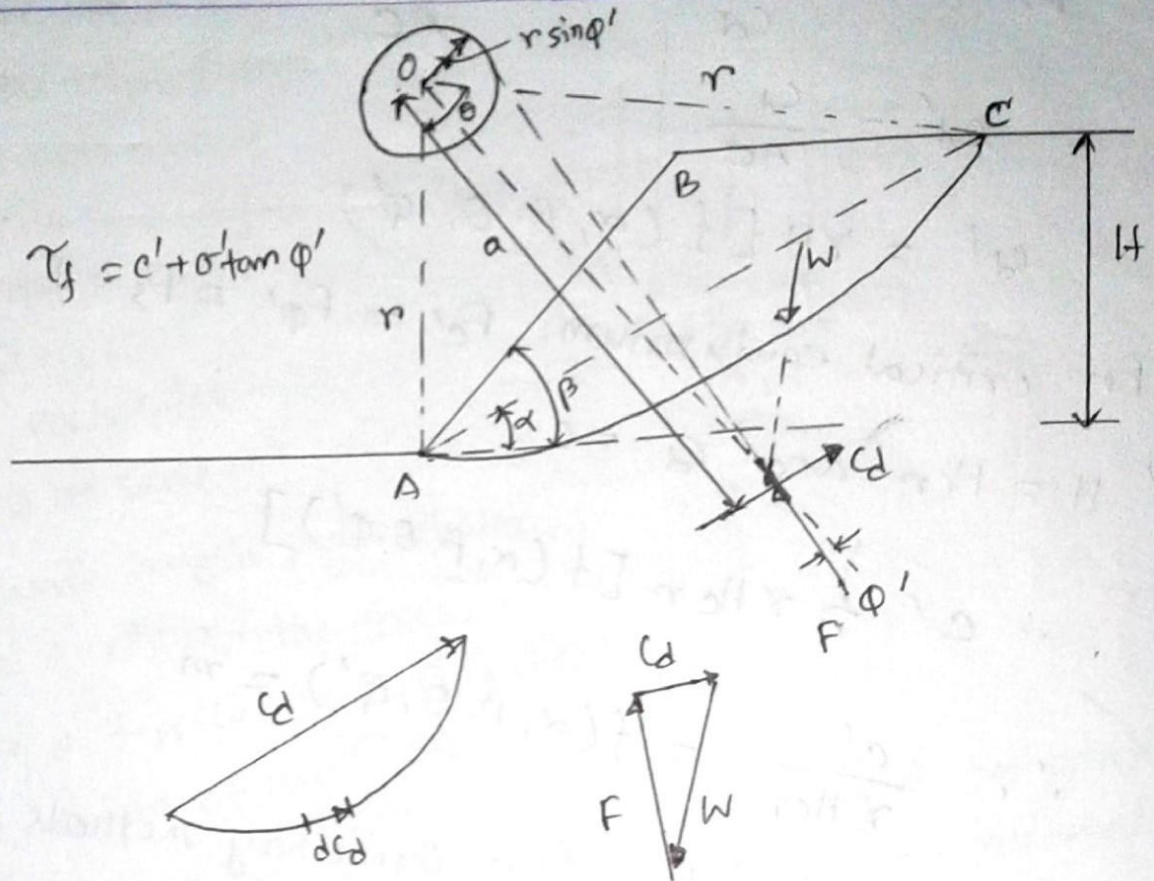
$$\text{or, } m = \frac{c_d}{\gamma H}$$

[where, m = stability number.]

The critical height of the slope can be evaluated by substituting $H = H_{cr}$ and $c_d = c_u$ into the preceding equation.

$$\text{Thus, } H_{cr} = \frac{c_u}{\gamma m}$$

Mass procedure - Slopes in Homogeneous $c'-\phi'$ Soil :-



A Slope in a homogeneous soil is shown above figure. The shear strength of the soil is given by,

$$\tau_f = c' + \sigma' \tan \phi'$$

Weight of soil wedge $ABC = W = (\text{Area of } ABC) \gamma$

For equilibrium the following other forces are acting on the wedge.

C_d — resultant of the cohesive force that is equal to the cohesion per unit arc developed times the length of the cord \overline{AC} . The magnitude of C_d is given by the following.

$$C_d = c' (\overline{AC})$$

C_d acts in a direction parallel to the cord (\overline{AC}) and at a distance a from the centre of the circle O such that,

$$C_d (a) = c' (\overline{AC}) r.$$

$$\therefore a = \frac{c'_d(\widehat{AC})r}{c_d} = \frac{\widehat{AC}}{AC} r$$

$$c'_d = \frac{c_d}{AC}$$

$$c'_d = \gamma H [f(\alpha, \beta, \theta, \phi')]$$

For critical equilibrium, $F_c' = F_\phi' = F_s = 1$.

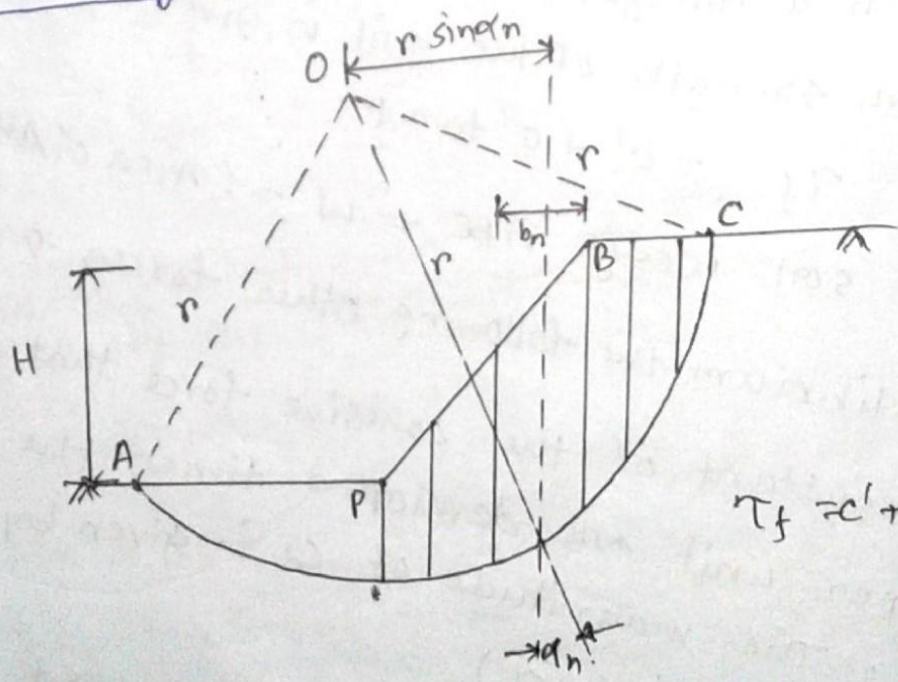
$$H = H_{cr} \text{ and } c'_d = c$$

$$\therefore c' = \gamma H_{cr} [f(\alpha, \beta, \theta, \phi')]$$

$$\therefore \frac{c'}{\gamma H_{cr}} = f(\alpha, \beta, \theta, \phi') = m$$

Derive equation of FS for ordinary methods of slices.

Answer: Ordinary Methods of slices:-

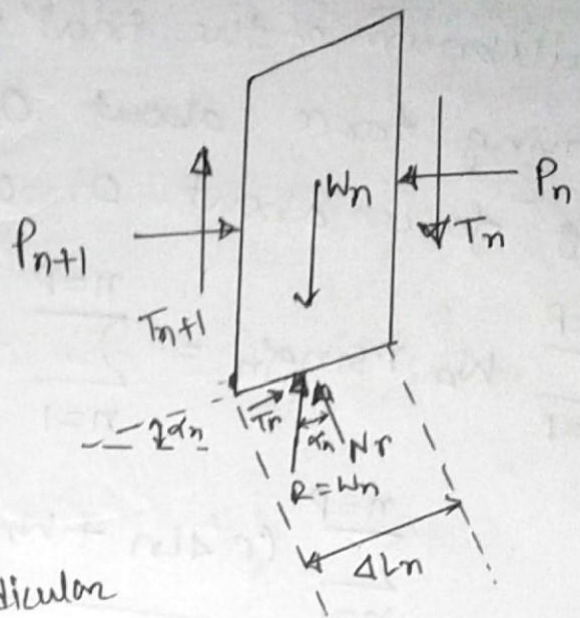


$$\tau_f = c' + \sigma' \tan \phi'$$

AC is an arc of a circle representing the trial failure surface. The soil above the trial failure surface is divided into several vertical slices.

The width of each slice is not need to be same.

Considering unit length perpendicular to the slope section, the forces that act on a typical slice (nth slice) W_n is the weight of the slice.



The forces N_r and T_r respectively normal and tangential components of the reaction R . P_n, P_{n+1} are the normal forces that act on the sides of the slice. T_n, T_{n+1} are the shearing forces that acts on the sides of slices.

The force, $P_n, P_{n+1}, T_n, T_{n+1}$ are difficult to determine.

For equilibrium consideration,

$$N_r = W_n \cos \alpha_n$$

The resisting shear force can be expressed as.

$$T_r = \tau_d (4L_n) = \frac{\tau_f (4L_n)}{F_s} = \frac{1}{F_s} [c' + \sigma' \tan \phi'] 4L_n$$

The normal stress, σ' , is equal to.

$$\sigma' = \frac{N_r}{4L_n} = \frac{W_n \cos \alpha_n}{4L_n}$$

For equilibrium of the trial wedge ABC, the moment of the driving force about O equals to the moment of resisting force about O, or,

$$\sum_{n=1}^{n=P} W_n r \sin \alpha_n = \sum_{n=1}^{n=P} \frac{1}{F_s} \left(c' + \frac{W_n \cos \alpha_n}{\Delta L_n} \tan \phi' \right) (\Delta L_n) r$$

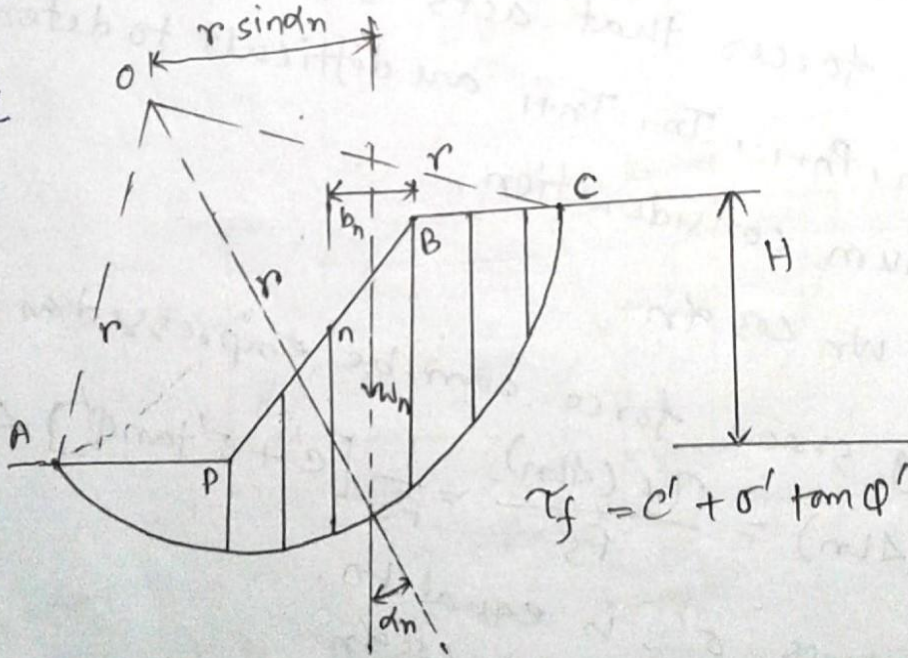
$$\therefore F_s = \frac{\sum_{n=1}^{n=P} (c' \Delta L_n + W_n \cos \alpha_n \tan \phi')}{\sum_{n=1}^{n=P} W_n \sin \alpha_n}$$

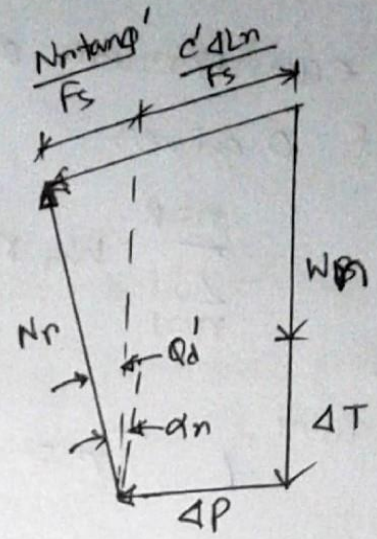
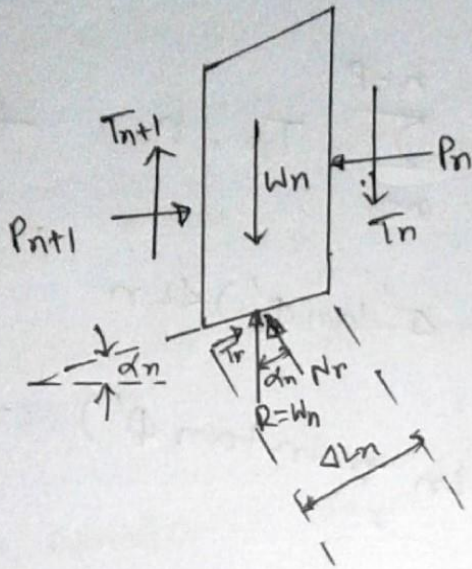
ΔL_n is approximately $= \frac{b_n}{\cos \alpha_n}$, $b_n =$ width of n th slice.

Q. 20/3

Derive equation Bishop's simplified method of slices.

Answer:





In 1995, Bishop proposed a more refined solution to the ordinary method of slices. In this method the effort of forces on the sides of the each slice are accounted for to some degree.

Now, $P_n - P_{n+1} = \Delta P$ and $T_n - T_{n+1} = \Delta T$ let;

Then we can write,

$$T_r = N_r \tan \phi'_d + c'_d \Delta L_n$$

$$\therefore T_r = N_r \left(\frac{\tan \phi'}{F_s} \right) + \frac{c' \Delta L_n}{F_s} \quad \text{--- (I)}$$

Summing the forces in vertical direction,

$$W_n + \Delta T = N_r \cos \alpha_n + \left[\frac{N_r \tan \phi'}{F_s} + \frac{c' \Delta L_n}{F_s} \right] \sin \alpha_n$$

$$\text{or, } N_r = \frac{W_n + \Delta T - \frac{c' \Delta L_n \sin \alpha_n}{F_s}}{\cos \alpha_n + \frac{\tan \phi' \sin \alpha_n}{F_s}} \quad \text{--- (II)}$$

For equilibrium of the wedge ABC, taking moment about O gives,

$$\sum_{n=1}^{n=P} W_n r \sin \alpha_n = \sum_{n=1}^{n=P} T_r \cdot r \quad \text{--- (ii)}$$

where,

$$T_r = \frac{1}{F_s} (c' + \sigma' \tan \phi') \Delta L_n$$

$$T_r = \frac{1}{F_s} (c' \Delta L_n + N_r \tan \phi') \quad \text{--- (iv)}$$

From (ii) & (iv) we get,

$$F_s = \frac{\sum_{n=1}^{n=P} (c' b_n + W_n \tan \phi' + \Delta T \tan \phi') \frac{1}{m_{\alpha}(n)}}{\sum_{n=1}^{n=P} W_n \sin \alpha_n}$$

where,

$$m_{\alpha}(n) = \cos \alpha_n + \frac{\tan \phi' \sin \alpha_n}{F_s}$$

For simplicity, $\Delta T = 0$.

$$\therefore F_s = \frac{\sum_{n=1}^{n=P} (c' b_n + W_n \tan \phi') \frac{1}{m_{\alpha}(n)}}{\sum_{n=1}^{n=P} W_n \sin \alpha_n}$$

Bishop's simplified method is probably the most widely used when incorporated into computer programs, it yields satisfactory results in most cases.

B. Distinguish between ϕ -circle method or Slice method:

Answer:

ϕ -circle method	Slice method.
(I) Entire circle is considered.	(I) Dividing the soil mass into number of slices.
(II) r, c, ϕ constant for entire soil mass.	(II) r, c, ϕ are different.
(III) $F_s = \frac{c}{c_m}$	(III) $F_s \neq \frac{c}{c_m}$

⇒ Write down the remedial measures against slope failure.

Answer: Remedial Measures:

- (I) Planting trees on slope surface.
- (II) Covering the surface area with polyethene.
- (III) Avoiding excavation or undercutting near the foot of slope.
- (IV) By proper settlement of soil mass.

প্রো: সুবিউন ইন্সটিটিউট
 রাজশাহী প্রকৌশল ও প্রযুক্তি বিশ্ববিদ্যালয়
 সুরকৌশল বিভাগ
 রোল নং: ২৩০২২০.

Bearing Capacity (Shallow foundation)

Problem solve

Important formula

For general shear failure:

Terzaghi's equation:-

$$q_u = c' N_c + q N_q + \frac{1}{2} \gamma B N_\gamma \rightarrow \text{strip footing.}$$

$$q_u = 1.3c' N_c + q N_q + 0.4 \gamma B N_\gamma \rightarrow \text{square footing}$$

$$q_u = 1.3c' N_c + q N_q + 0.3 \gamma B N_\gamma \rightarrow \text{circular footing.}$$

where,

$$N_c = \cot \phi' \left[\frac{e^{2(\frac{3\pi}{4} - \phi'/2) \tan \phi'}}{2 \cos^2(\frac{\pi}{4} + \frac{\phi'}{2})} - 1 \right] = \cot \phi' (N_q - 1)$$

$$N_q = \frac{e^{2(\frac{3\pi}{4} - \phi'/2) \tan \phi'}}{2 \cos^2(\frac{\pi}{4} + \frac{\phi'}{2})}$$

$$N_\gamma = \frac{1}{2} \left(\frac{K P \gamma}{\cos^2 \phi'} - 1 \right) \tan \phi'$$

where, $K P \gamma$ = passive pressure co-efficient.

For local shear failure:-

$$\bar{c}' = \frac{2}{3}c' \quad \text{and} \quad \tan \bar{\phi}' = \frac{2}{3} \tan \phi'$$

- * $q_u' = \bar{c}' N_c' + q N_q' + \frac{1}{2} \gamma B N_\gamma'$ → strip footing
- * $q_u' = 1.3 \bar{c}' N_c' + q N_q' + 0.4 \gamma B N_\gamma'$ → square footing
- * $q_u' = 1.3 \bar{c}' N_c' + q N_q' + 0.3 \gamma B N_\gamma'$ → circular footing.

where, $\bar{\phi}' = \tan^{-1} \left(\frac{2}{3} \tan \phi' \right)$ for ϕ'

যদি terzaghi-এর সূত্রের কথা যাচাই উল্লেখ নাহলে কোনাধিক
 তখনই আশ্রয় উসবের সূত্রগুলো ব্যবহার করতে পারব। R যদি
 দেওয়া না থাকে তখন নিজের সূত্রানুসারে করতে হবে।

For general shear failure, general bearing capacity of Meyerhof:

$$* q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

where,

$$* N_q = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) e^{\pi \tan \phi'}$$

$$* N_c = (N_q - 1) \cot \phi'$$

$$* N_\gamma = 2(N_q + 1) \tan \phi'$$

$$* F_{cs} = 1 + \left(\frac{B}{L} \right) \left(\frac{N_q}{N_c} \right)$$

$$* F_{qs} = 1 + \left(\frac{B}{L} \right) \tan \phi'$$

$$* F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L} \right)$$

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta}{90^\circ} \right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'} \right)$$

β = inclination of the load on the foundation with respect to the vertical.

Depth factors:

For $\frac{D_f}{B} \leq 1$:

For, $\phi' = 0$,

$$F_{cd} = 1 + 0.4 \left(\frac{D_f}{L} \right)$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

For $\frac{D_f}{B} \geq 1$:

$$F_{cd} = 1 + \underbrace{0.4 \tan^{-1} \left(\frac{D_f}{L} \right)}_{\text{radian}}$$

$$F_{qd} = 1$$

$$F_{\gamma d} = 1$$

For, $\phi' > 0$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B}$$

$$F_{\gamma d} = 1$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \tan^{-1} \left(\frac{D_f}{B} \right)$$

↑
radian

$$F_{\gamma d} = 1$$

For $0 \leq D_1 \leq D_f$

$$* q_u = \gamma D_1 + D_2 (\gamma_{sat} - \gamma_w)$$

γ is replaced by γ'

* For $0 < d < B$

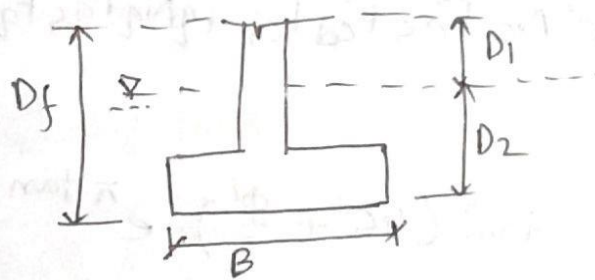
$$* q_u = \gamma d_f$$

$$\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma') \quad [\gamma \text{ is replaced by } \bar{\gamma}]$$

** Safe load/gross load, $Q_{all} = q_{all} \times \text{Area of footing.}$

$$** q_{all} = \frac{q_u}{F.S} \rightarrow \text{Factor of safety.}$$

$$* q_{net} = q_u - q$$



γ_{sat} = saturated unit weight

Problem-01: A square foundation $(2 \times 2) \text{ m}^2$ in plan. The soil supporting the foundation has a friction angle of $\phi' = 25^\circ$, $c = 20 \text{ kN/m}^2$. The unit weight of soil, γ is 16.5 kN/m^3 . Determine the allowable gross load on the foundation with a factor of safety (FS) of 3. Assume that the depth of the foundation (D_f) is 1.5 m and that general shear failure occurs in the soil.

Solution: We know that,

$$q_u = c' N_c F_{cd} F_{cs} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma d} F_{\gamma s} F_{\gamma i}$$

Since the load is vertical,

$$F_{ci} = F_{qi} = F_{\gamma i} = 1$$

$$N_q = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) e^{\pi \tan \phi'} \quad [\text{Given } \phi' = 25^\circ]$$

$$= 10.66$$

$$N_c = (N_q - 1) \cot \phi'$$

$$= 20.72$$

$$N_\gamma = 2(N_q + 1) \tan \phi'$$

$$= 10.87$$

$$F_{cs} = 1 + \left(\frac{B}{L} \right) \left(\frac{N_q}{N_c} \right) = 1 + \left(\frac{2}{2} \right) \left(\frac{10.66}{20.72} \right) = 1.514$$

$$F_{qs} = 1 + \left(\frac{B}{L} \right) \tan \phi' = 1 + \left(\frac{2}{2} \right) \tan 25^\circ = 1.47$$

$$F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L} \right) = 1 - 0.4 \left(\frac{2}{2} \right) = 0.6$$

$$q = \gamma D_f = 16.5 \times 1.5 = 24.75 \text{ kN/m}^2$$

Since, $\frac{D_f}{B} = 0.75 \leq 1,$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \left(\frac{D_f}{B}\right)$$

$$= 1 + 2 \tan 25^\circ (1 - \sin 25^\circ)^2 \times 0.75$$

$\therefore F_{qd} = 1.233$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'} = 1.233 - \frac{1 - 1.233}{20.72 \tan 25^\circ} = 1.257$$

$F_{yd} = 1.$

$$\therefore q_u = (20 \times 20.72 \times 1.257 \times 1.514 \times 1) + (24.75 \times 10.66 \times 1.47 \times 1.233 \times 1)$$

$$+ \left(\frac{1}{2} \times 16.5 \times 2 \times 10.87 \times 1 \times 0.6 \times 1\right)$$

$q_u = 1374.46 \text{ kN/m}^2.$

$\therefore q_{all} = \frac{q_u}{FS} = \frac{1374.46}{3} = 458.15 \text{ kN/m}^2$

Gross load $Q = q_{all} \times \text{Area of footing}$

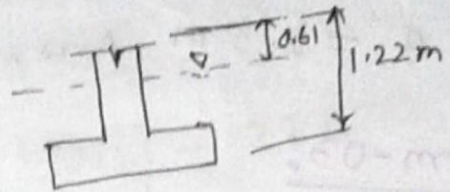
$= 458.15 \times 2 \times 2$

$= 1832.6 \text{ kN (Ans.)}$

Problem 02: A square foundation (BxB) has to be constructed as shown in figure. Assume that $\gamma = 16.5 \text{ kN/m}^3$, $\gamma_{\text{sat}} = 18.55 \text{ kN/m}^3$, $\phi' = 34^\circ$, $D_f = 1.22 \text{ m}$, $D_1 = 0.61 \text{ m}$. The gross allowable load, Q_{all} , with $FS = 3$, is 667.2 kN . Determine the size of the footing.

Soln

$$q_{\text{all}} = \frac{Q_{\text{all}}}{B^2} = \frac{667.2}{B^2} \text{ kN/m}^2 \quad \text{--- (1)}$$



$$q_{\text{all}} = \frac{q_u}{FS} = \frac{1}{3} \left(q N_q F_{qs} F_{qd} + \frac{1}{2} \gamma' B N_\gamma F_{\gamma s} F_{\gamma d} \right) \quad [c' = 0]$$

$$N_q = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) e^{\pi \tan \phi'} = 29.44 \quad [\text{Given, } \phi' = 34^\circ]$$

$$N_\gamma = 2(N_q + 1) \tan \phi' = 41.06$$

$$F_{qs} = (N_q + 1) \cot \phi' =$$

$$F_{qs} = 1 + \left(\frac{B}{L} \right) \tan \phi' = 1 + \tan 34 = 1.67 \quad \left[\frac{B}{L} = 1 \text{ m} \right]$$

$$F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L} \right) = 1 - 0.4 = 0.6$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B}$$

$$= 1 + \frac{1.05}{B}$$

$$F_{\gamma d} = 1$$

$$q = \gamma D_1 + D_2 (\gamma_{\text{sat}} - \gamma_w)$$

$$q = 0.61 \times 16.5 + 0.61 (18.55 - 9.81) = 15.4 \text{ kN/m}^2$$

$$\therefore q_{\text{all}} = \frac{1}{3} \left[(15.4 \times 29.44 \times 1.67) \left(1 + \frac{1.05}{B} \right) + \frac{1}{2} \times (18.55 - 9.81) \times B \times 41.06 \times 0.6 \right]$$

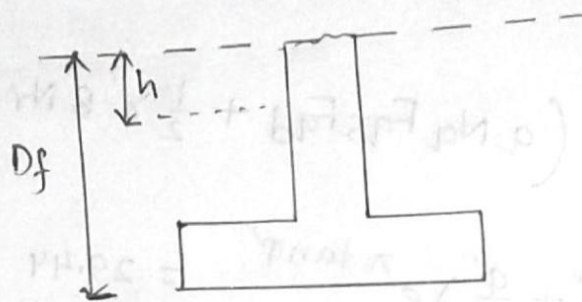
$$q_{\text{all}} = 252.38 + \frac{265}{B} + 35.89 B \quad \text{--- (2)}$$

From ① & ② equating,

$$\frac{667.2}{B^2} = 252.38 + \frac{265}{B} + 35.89 B$$

$$\therefore B = 1.3 \text{ m Am.}$$

Problem-03:



Given, $\gamma = 100 \text{ lb/ft}^3$, $\gamma_{\text{sat}} = 120 \text{ lb/ft}^3$, $c' = 0$, $\phi' = 30^\circ$, $D_f = 5 \text{ ft}$.

$Q_{\text{all}} = 40000 \text{ lb}$, $FS = 3$, Determine the size of footing

when $h = 0 \text{ ft}$, 2 ft , 5 ft .

Consider square footing of section $(B \times B)$.

Soln

$$q_{\text{all}} = \frac{Q_{\text{all}}}{B^2} = \frac{40000}{B^2} \text{ lb/ft}^2 \quad \text{--- ①}$$

$$q_{\text{all}} = \frac{q_u}{FS} = \frac{1}{3} \left[q N_q F_{qs} F_{qd} + \frac{1}{2} \gamma' B N_\gamma F_{\gamma s} F_{\gamma d} \right] \quad \text{--- ②}$$

$[c' = 0]$

$$N_q = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) e^{\pi \tan \phi'} = 18.40$$

$$N_\gamma = 2(N_q + 1) \tan \phi' = 22.4$$

$$F_{qs} = 1 + \frac{B}{L} \tan \phi' = 1 + \tan 30^\circ = 1.58 \quad \left[\frac{B}{L} = 1 \text{ ft} \right]$$

$$F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L} \right) = 1 - 0.4 = 0.6$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B} = 1 + \frac{1.44}{B} \quad \left[\frac{D_f}{B} \leq 1 \right]$$

$$F_{rd} = 1$$

when $h=0$,

$$Q = h\gamma + D_f \gamma'$$

$$Q = \gamma' D_f = (120 - 62.4) \times 5 = 288 \text{ lb/ft}^2$$

$$\therefore \frac{40000}{B^2} = \frac{1}{3} \left[288 \times 18.40 \times 1.58 \times \left(1 + \frac{1.44}{B}\right) + \frac{1}{2} (120 - 62.4) \times B \times 22.4 \times 0.6 \times 1 \right]$$

$$\Rightarrow \frac{40000 \times 3}{B^2} = \left\{ 8372.74 \left(1 + \frac{1.44}{B}\right) + 387.072 B \right\}$$

$$\therefore B = -2.07 \text{ (not ok)}$$

$$\therefore \frac{D_f}{B} > 1$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi') \tan^{-1} \left(\frac{D_f}{B} \right) \text{ radian}$$

$$= 1 + 0.289 \tan^{-1} \left(\frac{5}{B} \right)$$

$$F_{\gamma d} = 1$$

\therefore From (i) & (ii) we get,

$$\frac{40000}{B^2} = \frac{1}{3} \left[288 \times 18.40 \times 1.58 \left(1 + \frac{1.44}{B}\right) + \frac{1}{2} (120 - 62.4) \times B \times 22.4 \times 0.6 \times 1 \right]$$

$$\therefore \frac{40000 \times 3}{B^2} = 8372.736 \left(1 + 0.289 \tan^{-1} \left(\frac{5}{B} \right)\right) + 387.1 B$$

$$\therefore B = 3.25'$$

\therefore size of the footing = $(3.25' \times 3.25')$

For $h = 2$ ft,

$$q = h\gamma + (D_f - h)(\gamma_{sat} - \gamma_w)$$
$$= 2 \times 100 + 3(120 - 62.4)$$
$$= 372.8 \text{ lb/ft}^3$$

from ① & ⑩ we get,

$$\frac{40000}{B^2} = \frac{1}{3} \left[372.8 \times 18.40 \times 1.58 \times (1 + 0.289 \tan^{-1}(\frac{5}{B})) \right]$$
$$+ \frac{1}{2} (120 - 62.4) \times B \times 22.4 \times 0.6 \times 1$$

$$\therefore B = 2.80$$

Size of the footing, $(2.8' \times 2.8')$

For $h = 5$ ft,

$$q = \gamma D_f = 5 \times 100 = 500 \text{ lb/ft}^2$$

$$\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma') = \gamma' = 120 - 62.4 = 57.6 \text{ lb/ft}^3$$

\therefore from ① & ⑩ we get,

$$\frac{40000}{B^2} = \frac{1}{3} \left[500 \times 18.40 \times 1.58 (1 + 0.289 \tan^{-1}(\frac{5}{B})) + \frac{1}{2} \times 57.6 \right]$$
$$\times B \times 22.4 \times 0.6 \times 1$$

$$\therefore B = 2.44 \sim 2.5'$$

\therefore size of the footing $= (2.50' \times 2.50')$

One-way Eccentricity:

Step-1: Determination of Effective dimensions of foundations:-

$$B' = \text{effective width} = B - 2e$$

$$L' = \text{effective length} = B$$

যদি eccentricity length বরাবর হয় তবে $L' = L - 2e$ and $B' = B$
 B' এবং L' এর মধ্যে ছোটটো effective width হিসাবে গণ্য করা হবে।

Step-2: USE ultimate bearing capacity equation:-

$$q_u' = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

F_{cs} , F_{qs} , $F_{\gamma s}$ এর বরাবর অক্ষের সমীকরণে B' এর পরিবর্তে effective width B' and L এর পরিবর্তে effective length L' ব্যবহার করা হবে।

অন্যদিকে,
 F_{cd} , F_{qd} , $F_{\gamma d}$ এর বরাবর অক্ষের সমীকরণে relation হিসাবে
এর বরাবর হয় কোন পরিবর্তন (i.e. B & L ২ ব্যবহার) করা যাবে।

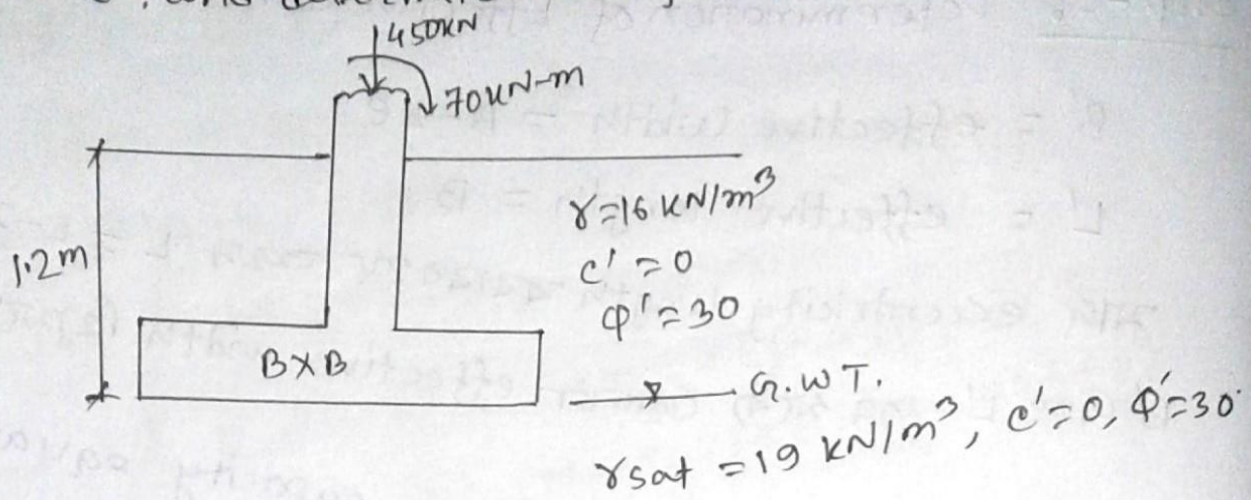
Step-3: Calculate total ultimate load foundation can sustain

$$Q_{ult} = q_u' A' = q_u' B' L', \text{ where, } A' = \text{effective area.}$$

Step-4: Determine factor of safety FS against bearing capacity failure.

$$FS = \frac{Q_{ult}}{Q}, \text{ where, } Q = \text{total vertical load}$$

Problem:- A square footing is shown in figure below.
 USE FS = 6, and determination of size of footing.



Solution:

Given,

$$Q = 450 \text{ kN}$$

$$M = 70 \text{ kN-m}$$

$$e = \frac{M}{Q} = \frac{70}{450} = 0.16$$

$$B' = \text{effective width} = B - 2e = B - 0.31$$

$$L' = \text{effective length} = L = B$$

Since, $c' = 0$.

$$q_u' = q_{Nq} F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma' B' N_{\gamma} F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

since, inclination ~~factor~~ $\beta = 0^\circ$,

$$\text{Hence, } F_{qi} = F_{\gamma i} = 1$$

$$N_q = \tan^2 \left(45 + \frac{\phi'}{2} \right) e^{\gamma \tan \phi'} = 18.40$$

$$N_{\gamma} = 2(N_q + 1) \tan \phi' = 22.4$$

$$F_{qs} = 1 + \frac{B'}{L'} \tan \phi' = 1 + \frac{(B - 0.31)}{B} \times 0.58$$

$$F_{\gamma s} = 1 - 0.4 \frac{B'}{L'} = 1 - 0.4 * \frac{(B - 0.31)}{B}$$

for $\frac{D_f}{B} \leq 1$,

$$F_{qd} = 1 + \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B}$$

$$F_{qd} = 1 + \frac{0.17}{B}$$

$$F_{\gamma d} = 1$$

$$q = \gamma h = 16 \times 1.2 = 19.2 \text{ kN/m}^2$$

$$\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma') = \gamma' = 19 - 9.81 = 9.19 \text{ kN/m}^3$$

$$q_u' = \left[19.2 \times 18.4 \left\{ 1 + \left(\frac{B-0.31}{B} \right) 0.58 \right\} \times 1 + \left\{ \frac{1}{2} \times 9.19 \times (B-0.31) \right\} \times 22.4 \times \left(1 - 0.4 \times \left(\frac{B-0.31}{B} \right) \right) \times 1 \times 1 \right]$$

$$q_u' = 353.28 + 204.91 \left(\frac{B-0.31}{B} \right) + \left\{ 102.928 - 41.17 \left(\frac{B-0.31}{B} \right) \right\} (B-0.31)$$

Now, $FS = \frac{Q_{ult}}{Q} \Rightarrow Q_{ult} = Q \times FS = 450 \times 6 = 2700$.

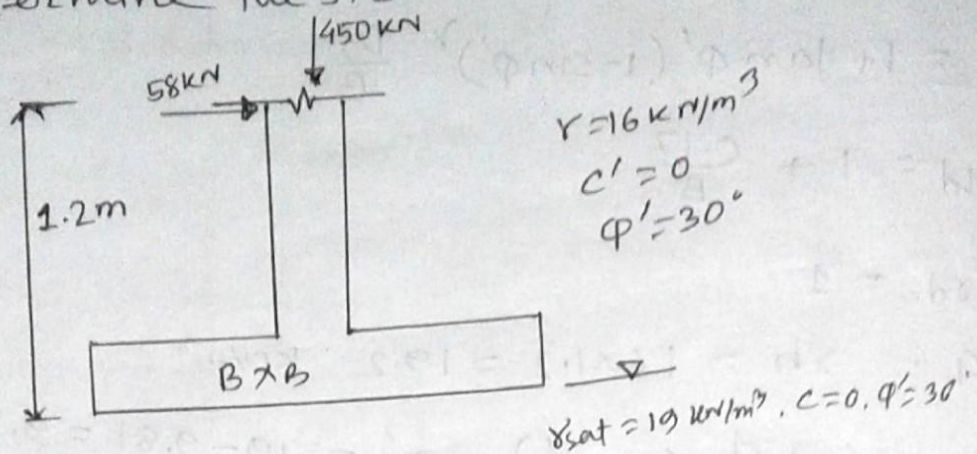
$$\therefore Q_{ult} = q_u' A' = q_u' B' L'$$

$$\Rightarrow 2700 = \left[353.28 + 204.91 \left(\frac{B-0.31}{B} \right) + \left\{ 102.928 - 41.17 \left(\frac{B-0.31}{B} \right) \right\} (B-0.31) \right] (B-0.31) B$$

$$\therefore B = 2.33 \text{ m} \quad \frac{D_f}{B} = \frac{1.2}{2.33} = 0.52 \leq 1 \text{ (OK)}$$

$$\therefore \text{size of the footing} = (2.33 \text{ m} \times 2.33 \text{ m}) \quad \underline{\text{Ans}}$$

Problem: A square footing is shown in figure below. USE FS=6. Determine the size of the footing.



Soln Since, $c' = 0$.

$$q_u' = q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma' B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

Given,

$$Q = 450 \text{ kN}$$

$$P_H = 58 \text{ kN}$$

$$M = P_H \times D_f = 58 \times 1.2 = 69.6 \text{ kN-m}$$

$$e = \frac{M}{Q} = \frac{69.6}{450} = 0.15 \text{ m}$$

$$b' = \text{effective width} = B - 2e = B - 2 \times 0.15 = B - 0.3$$

$$L' = \text{length} = L = B$$

Inclination angle $\beta = 0^\circ$.

Hence, $F_{qi} = F_{ri} = 1$

$$N_q = \tan^2 \left(45 + \frac{\phi'}{2} \right) e^{\pi \tan \phi'} = 18.40$$

$$N_\gamma = 2(N_q + 1) \tan \phi' = 22.4$$

$$F_{qs} = 1 + \frac{b'}{L'} \tan \phi' = 1 + \left(\frac{B - 0.3}{B} \right) \times 0.58$$

$$F_{\gamma s} = 1 - 0.4 \frac{b'}{L'} = 1 - 0.4 \left(\frac{B - 0.3}{B} \right)$$

For $\frac{D_f}{B} \leq 1$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \frac{D_f}{B}$$

$$= 1 + \frac{0.17}{B}$$

$$F_{\gamma d} = 1$$

$$q = \gamma h = 16 \times 1.2 = 19.2 \text{ kN/m}^2$$

$$\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma') = \gamma' = \gamma_{sat} - \gamma_w = 19 - 9.81 = 9.19 \text{ kN/m}^3$$

$$q_u' = [19.2 * 18.40 * \left\{ 1 + \left(\frac{B-0.3}{B} \right) * 0.58 \right\} * \left(1 + \frac{0.17}{B} \right) * 1]$$

$$+ [19.2 * 9.19 * (B-0.3) * 22.4 * \left\{ 1 - 0.4 \left(\frac{B-0.3}{B} \right) \right\} * 1 * 1]$$

$$q_u' = [353.28 \left\{ 1 + \left(\frac{B-0.3}{B} \right) * 0.58 \right\} * \left(1 + \frac{0.17}{B} \right)]$$

$$+ [3952.44 (B-0.3) \left\{ 1 - 0.4 \left(\frac{B-0.3}{B} \right) \right\}]$$

$$q_u' = [353.28 \left(\frac{B + 0.58B - 0.174}{B} \right) \left(\frac{B + 0.17}{B} \right)]$$

$$+ [3952.44 (B-0.3) \left(\frac{B - 0.4B + 0.12}{B} \right)]$$

Now, $FS = \frac{Q_{ult}}{Q} \Rightarrow Q_{ult} = FS \times Q = 6 \times 450 = 2700$

Again, $Q_{ult} = q_u' A' = q_u' B' L'$

$$\Rightarrow 2700 = \left[\left\{ 353.28 \left(\frac{B + 0.58B - 0.174}{B} \right) * \left(\frac{B + 0.17}{B} \right) \right\} \right. \\ \left. + \left\{ 3952.44 (B-0.3) \left(\frac{B - 0.4B + 0.12}{B} \right) \right\} \right] * (B-0.3)B$$

$$\Rightarrow 2700 = \left[\frac{353.28 [B^2 + 0.17B + 0.58B^2 + 0.0986B - 0.02958]}{B^2} \right. \\ \left. + \frac{3952.44 (B^2 - 0.4B^2 + 0.12B - 0.3B + 0.12B - 0.036)}{B} \right] * (B^2 - 0.3B)$$

$$\Rightarrow 2700 = \left[\frac{558.18B^{\checkmark} + 94.89B - 10.45}{B^{\checkmark}} + \frac{2371.46B^{\checkmark} - 237.14B - 142.29}{B} \right] (B^{\checkmark} - 0.3B)$$

$$\therefore 2700 B^{\checkmark} = [558.18B^{\checkmark} + 94.89B - 10.45 + 2371.46B^3 - 237.14B^{\checkmark} - 142.29B] (B^{\checkmark} - 0.3B)$$

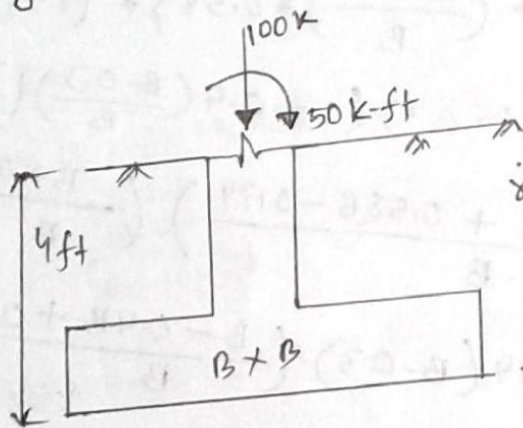
$$\therefore 2371.46B^3 - 2378.96B^{\checkmark} - 47.4B - 10.45 = 0$$

$$B = 1.12 \text{ m} \quad \text{Assm}$$

$$B = 1.02 \text{ m}$$

2015 (6c)
Problem:

A square footing is shown in figure below. Use an factor of safety for the soil $FOS = 3$ and determine the size of the footing.



$$\gamma = 100 \text{ lb/ft}^3, \quad \phi = 30^\circ, \quad c = 0$$

$$\gamma_{sat} = 120 \text{ lb/ft}^3, \quad \phi = 30^\circ, \quad c = 0$$

Soln

Given, $M = 50 \text{ k-ft}$

$Q = 100 \text{ k}$

$$\therefore e = \frac{M}{Q} = \frac{50}{100} = 0.5 \text{ ft}$$

$$B' = \text{effective width} = B - 2e = B - 2 \times 0.5 = B - 1$$

$$L' = \text{effective length} = L = B$$

Since $e = 0$.

$$q_u' = q N_q f_{qd} f_{qs} f_{qi} + \frac{1}{2} \gamma' B' N_\gamma f_{\gamma s} f_{\gamma d} f_{\gamma i}$$

Since, inclination, $\beta = 0'$,

$$f_{qi} = f_{\gamma i} = 1.$$

Now, $N_q = \tan^2(45^\circ + \frac{\phi}{2}) e^{\gamma \tan \phi} = 18.40$

$$N_\gamma = 2(N_q + 1) \tan \phi' = 22.4$$

$$f_{qs} = 1 + \frac{B'}{L'} \tan \phi = 1 + \left(\frac{B-1}{B}\right) \times 0.58$$

$$= \frac{B + 0.58B - 0.58}{B}$$

$$= \frac{1.58B - 0.58}{B}$$

$$f_{\gamma s} = 1 - 0.4 \frac{B'}{L'} = 1 - 0.4 \times \frac{B-1}{B}$$

$$= \frac{B - 0.4B + 0.4}{B} = \frac{0.6B + 0.4}{B}$$

For, $\frac{D_f}{B} \leq 1$.

$$f_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 * \frac{D_f}{B}$$

$$= 1 + 2 \tan 30^\circ (1 - \sin 30^\circ)^2 * \frac{4}{B}$$

$$= 1 + \frac{1.15}{B} = \frac{B + 1.15}{B}$$

$$f_{\gamma d} = 1.$$

$$q = \gamma h = 100 * 4 = 400 \text{ lb/ft}^2$$

$$\bar{\gamma} = \gamma' + \frac{d}{B} (\gamma - \gamma') = \gamma' = \gamma_{\text{sat}} - \gamma_w = 120 - 62.4 = 57.6 \text{ lb/ft}^3$$

Now,

$$FS = \frac{Q_{ult}}{Q} \Rightarrow Q_{ult} = FS \times Q = 3 \times 100 \times 10^3 = 3 \times 10^5 \text{ lb.}$$

$$\therefore Q_{ult} = q_u' A' = q_u' b' L' = q_u' \cdot (B - 1) B$$

$$300000 = q_u' (B^2 - B)$$

$$300000 = \left[400 \times 1840 \times \left(\frac{B+0.58}{B} \right) \left(\frac{1.58B - 0.58}{B} \right) \cdot 1 \right] + \left[\frac{1}{2} \times 57.6 \times (B-1) \times 22.4 \times \left(\frac{0.6B+0.4}{B} \right) \times 1 \times 1 \right] (B^2 - B)$$

$$\Rightarrow 300000 = \left[7360 \left(\frac{1.58B^2 + 0.58B + 0.92B - 0.34}{B^2} \right) + 645.12 \left(\frac{0.6B^2 + 0.4B - 0.6B - 0.4}{B} \right) \right] (B^2 - B)$$

$$\Rightarrow 300000 = \left[\frac{11628.8B^2 + 2502.4B - 2502.4}{B^2} + \frac{387.072B^2 - 192.024B - 258.048}{B} \right] (B^2 - B)$$

$$\Rightarrow 300000 B^2 = (11628.8B^2 + 2502.4B - 2502.4 + 387.072B^2 - 192.024B - 258.048) (B^2 - B)$$

$$\Rightarrow 300000 B^2 = (387.072B^3 + 11436.76B^2 - 2244.35B - 2502.4) (B^2 - B)$$

$$\therefore B = 8.34$$

After 5.5 m $\sigma'_0 = 5.5 \times 18 + (\text{at } 5.5) \times 9.69 =$
at 5.5 depth.

Depth	σ'_0 (kN/m ²)	C_N	N_{60}	$(N_1)_{60}$	ϕ'
1.5	27	1.57	5	7.87 ~ 8	29.45°
3.0	54	1.30	7	9.1 ~ 9	29.76°
4.5	81	1.105	9	9.945 ~ 10	30.045°
6.0	103.845	0.98	8	7.85 ~ 8	29.45°
7.5	118.38	0.92	13	11.9 ~ 12	30.62°
9.0	132.915	0.86	12	10.30 ~ 10	30.045°
10.5	147.45	0.81	14	11.31 ~ 11	30.33°
$n=7$					$\Sigma \phi' = 209.7$

Average value of $\phi = \frac{\Sigma \phi'}{n} = \frac{209.7}{7} = 29.96^\circ$ Ans.

2014 Solve & Find corrected N value by Liao and whitman's relationship.

Soln Given, $\gamma_d = 18 \text{ kN/m}^3$, $\gamma_{sat} = 19.5 \text{ kN/m}^3$, $\gamma' = \gamma_{sat} - \gamma_w = 9.69 \text{ kN/m}^3$

Liao and whitman's, $\sigma'_0 = \gamma' h$ $P_a = 100 \text{ kN/m}^2$

Depth	σ'_0 (kN/m ²)	$C_N = \left(\frac{1}{\frac{\sigma'_0}{P_a}} \right)^{0.5}$	N_{60}	$(N_1)_{60}$	ϕ'
1.5	27	1.92	5	9.62 ~ 10	30.046°
3.0	54	1.36	7	9.52 ~ 10	30.046°
4.5	81	1.11	9	9.99 ~ 10	30.046°
6	103.845	0.98	8	7.84 ~ 8	29.45°
7.5	118.38	0.92	13	11.96 ~ 12	30.62°
9.0	132.915	0.87	12	10.44 ~ 10	30.046°
10.5	147.45	0.83	14	11.62 ~ 12	30.62°
$n=7$					$\Sigma \phi' = 210.87^\circ$

\therefore Average $\phi = \frac{\Sigma \phi'}{n} = \frac{210.87^\circ}{7} = 30.12^\circ$ Ans

2013 5(c) The N_{60} value at a depth of 4.5 m for a sandy soil is 10. Determine the soil friction angle ϕ' .

Soln

Peck, Hanson and Thornburn :

$$\phi' = 27.1 + 0.3 N_{60} - 0.00054 (N_{60})^2 \quad \left| \quad N_{60} = 10 \right.$$

$$= 30.046^\circ \quad \text{Ans}$$

Schmertman :

$$\phi' = \tan^{-1} \left[\frac{N_{60}}{12.2 + 20.3 \left(\frac{\sigma'_0}{P_a} \right)} \right]^{0.34}$$

$$\phi' = \tan^{-1} \left[\frac{10}{12.2 + 20.3 \left(\frac{81}{100} \right)} \right]^{0.34}$$

$$= 34.96^\circ \quad \text{Ans}$$

$\gamma = 18 \text{ kN/m}^3 \text{ (wt)}$
 $\sigma'_0 = \gamma h$
 $= 18 \times 4.5$
 $= 81 \text{ kN/m}^2$
 $P_a = 100 \text{ kN/m}^2$
 $N_{60} = 10$

Hatanaka and Uchida

$$\phi' = \sqrt{20 (N_1)_{60} + 20}$$

$$= \sqrt{20 \times 10 + 20}$$

$$= 34.14^\circ \quad \text{Ans}$$

$$(N_1)_{60} = 10$$

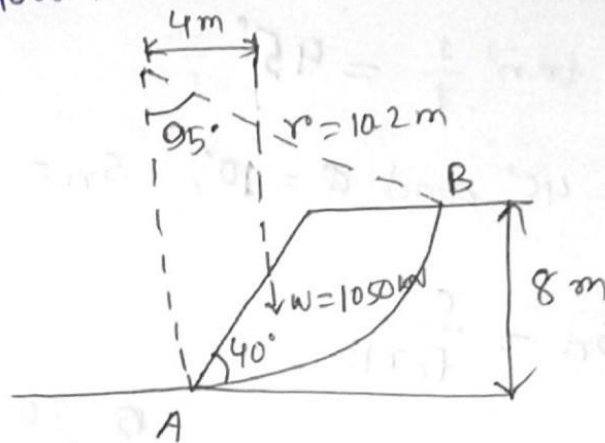
Stability of Slopes

2015 7(c) A 40° slope is excavated to a depth of 8 m in a deep layer of saturated clay ($c = 70 \text{ kN/m}^2$, $\phi = 0$, $\gamma = 19 \text{ kN/m}^3$) Determine the factor of safety for the trial failure surface as shown in figure below:

Soln

Length of arc AB,

$$\begin{aligned}\hat{L} &= \frac{2\pi r \theta}{360^\circ} \\ &= \frac{2\pi \times 10.2 \times 95^\circ}{360^\circ} \\ &= 16.91 \text{ m}\end{aligned}$$



$$\text{Disturbing moment} = M_D = W \bar{x} = 1050 \times 4 = 4200 \text{ kN-m}$$

$$\begin{aligned}\text{Resisting moment} &= M_R = c \hat{L} \cdot R_{\text{radius}} = 70 \times 16.91 \times 10.2 \\ &= 12073.74 \text{ kN-m.}\end{aligned}$$

$$\therefore \text{Factor of safety } FS = \frac{M_R}{M_D} = \frac{12073.74}{4200} = 2.86 \text{ Ans}$$

$$FS = \frac{M_R}{M_D} = \frac{12073.74}{4200}$$

2073 A 6 m deep cut is to be made in cohesive soil with a slope of 1:1. The soil has $c_u = 30 \text{ kN/m}^2$, $\phi_u = 10^\circ$ and $\gamma = 18 \text{ kN/m}^3$. Find the factor of safety with respect to cohesion. What will be the critical height of the slope in this soil?

Soln $i = \tan^{-1} \frac{1}{1} = 45^\circ$,

For, $i = 45^\circ$ and $\phi = 10^\circ$, $S_n = 0.108$.

But, $S_n = \frac{c}{F_c \gamma H}$

$$\therefore F_c = \frac{c}{S_n \gamma H} = \frac{30}{0.108 \times 18 \times 6} = 2.57$$

The critical height is given by,

$$F_c = \frac{H_c}{H}$$

$$\therefore H_c = F_c \times H = 2.57 \times 6 = 15.43 \text{ m}$$

Ans

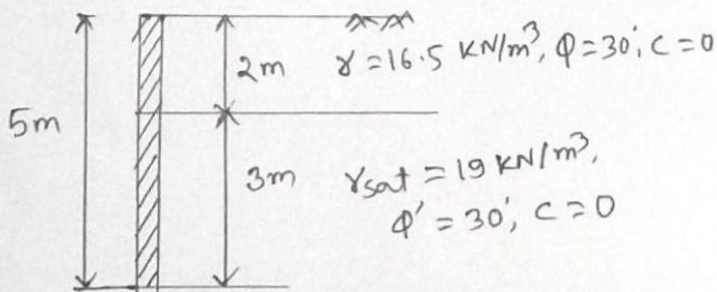
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 রাজশাহী প্রকৌশল ও প্রযুক্তি বিশ্ববিদ্যালয়
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Geotechnical Engineering - II

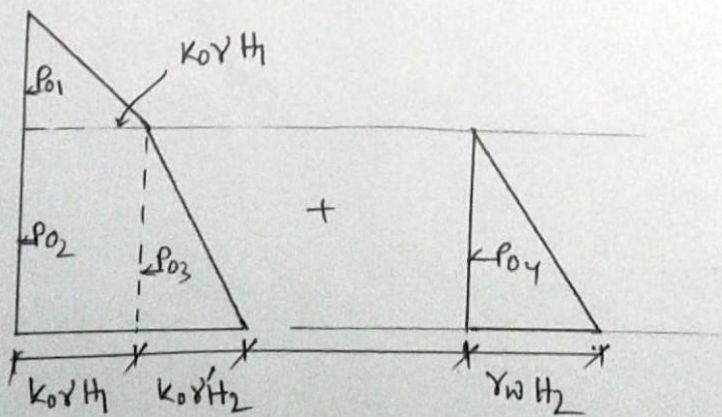
Mathematical problem solve.

Lateral Earth Pressure

Problem 1 For the retaining wall as shown in the figure. Determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force.



Solution: Pressure diagram.



Here,

$$k_0 = 1 - \sin \phi' = 1 - \sin 30^\circ = 0.50$$

$$k_0 \gamma H_1 = 0.50 \times 16.5 \times 2 = 16.5 \text{ kN/m}^2$$

$$k_0 \gamma' H_2 = 0.50 (19 - 9.81) \times 3 = 13.785 \text{ kN/m}^2$$

$$\gamma_w H_2 = 9.81 \times 3 = 29.43 \text{ kN/m}^2$$

$$P_{0T} = P_{01} + P_{02} + P_{03} + P_{04}$$

$$\begin{aligned} &= \left(\frac{1}{2} \times 16.5 \times 2\right) + (16.5 \times 3) + \left(\frac{1}{2} \times 3 \times 13.785\right) + \left(\frac{1}{2} \times 3 \times 29.43\right) \\ &= 16.5 + 49.5 + 20.6775 + 44.145 \\ &= 130.8225 \text{ kN/m}^2 \end{aligned}$$

Location of resultant force from below surface of the wall.

$$\bar{z} = \frac{1}{P_{0T}} \left[P_{01} \times \left(H_2 + \frac{H_1}{3}\right) + \left(P_{02} \times \frac{H_2}{2}\right) + \left(P_{03} \times \frac{H_2}{3}\right) + \left(P_{04} \times \frac{H_2}{3}\right) \right]$$

$$= \frac{1}{130.8225} \left[16.5 \times \left(3 + \frac{2}{3}\right) + \left(49.5 \times \frac{3}{2}\right) + \left(20.6775 \times \frac{3}{3}\right) + \left(44.145 \times \frac{3}{3}\right) \right]$$

$$= \frac{199.5725}{130.8225}$$

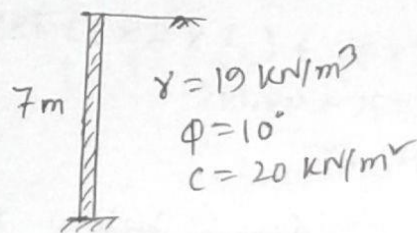
$$= 1.53 \text{ m from below } \uparrow \text{ surface of the wall.}$$

\therefore From the ground surface force will be act at,

$$\bar{z} = 5 - 1.53 = 3.47 \text{ m below from Top surface.}$$

Problem 01: A retaining wall as shown in the figure supports a clay soil. Determine,

- (I) The depth of tension crack.
- (II) The depth of unsupported vertical cut.
- (III) Lateral earth force before the tension crack.
- (IV) Lateral earth force after the tension crack.



Solution: we know that for clay.

$$P_a = K_a \gamma z - 2c \sqrt{K_a}$$

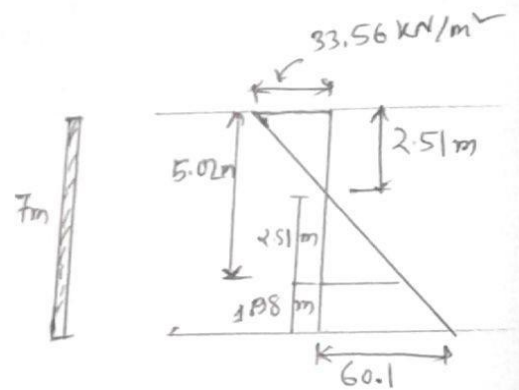
$$\therefore P_a = 0.704 \times 19 z - 2 \times 20 \times \sqrt{0.704}$$

$$= 13.38 z - 33.56$$

At, $z=0$, $P_a = -33.56 \text{ kN/m}^2$

At, $P_a = 0$, $0 = 13.38 z - 33.56$
 $z = 2.51 \text{ m}$

Here,
 $K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$
 $\therefore K_a = 0.704$



\therefore (I) The depth of tension crack, $z = 2.51 \text{ m}$

(II) The depth of unsupported vertical cut = $2z = 2 \times 2.51$
 $= 5.02 \text{ m}$

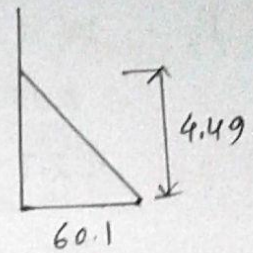
When, $z = 7 \text{ m}$, $P_a = (13.38 \times 7) - 33.56 = 60.1 \text{ kN/m}^2$

(III) Lateral earth force before tension crack = $\frac{1}{2} \times 60.1 \times (1.98 + 2.51) - \frac{1}{2} \times 33.56 \times 2.51$
 $= 92.81 \text{ kN}$

(iv) Lateral force after tension crack,

$$= \frac{1}{2} \times 60.1 \times 4.49$$

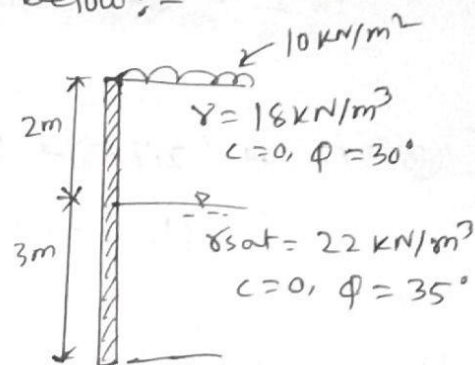
$$= 134.92 \text{ kN (Ans.)}$$



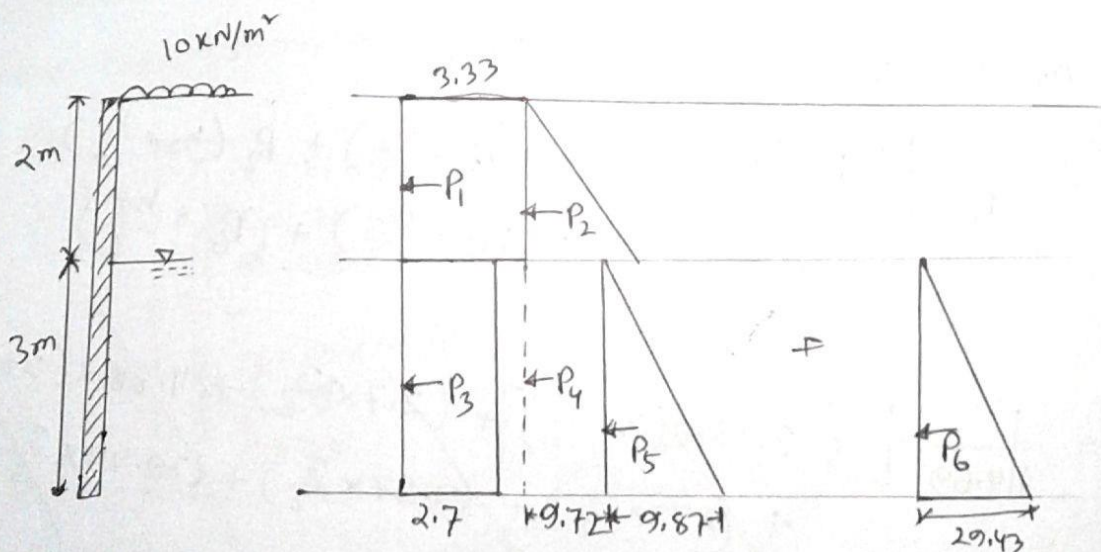
Pressure diagram after tension crack.

Problem 02: A retaining wall support a sandy soil, the properties of which are shown in the figure below:-

Calculate the resultant active earth force and its location from the top of the retaining wall.



Soln



For upper layer,

$$P_1 = k_a q = \frac{1}{3} \times 10 = 3.33 \text{ kN/m}^2$$

$$P_2 = k_a \gamma h_1 = \frac{1}{3} \times 18 \times 2 = 11.88 \text{ kN/m}^2$$

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\therefore k_{a1} = \frac{1}{3}$$

$$\therefore k_{a2} = 0.27$$

For lower layer,

$$P_3 = K a_2 q$$

$$= 0.27 \times 10$$

$$= 2.7 \text{ KN/m}^2$$

→ surcharge due to upper layer

$$q = \gamma h_1$$

$$P_4 = K a_2 q$$

$$= K a_2 \gamma h_1$$

$$= 0.27 \times 18 \times 2$$

$$= 9.72 \text{ KN/m}^2$$

$$P_5 = K a_2 \gamma' h_2 = 0.27 \times (22 - 9.81) \times 3$$

$$P_5 = 9.87$$

Due to water,

$$P_6 = \gamma_w h_2 = 9.81 \times 3 = 29.43 \text{ KN/m}^2$$

$$P_{0T} = \left[(3.3 \times 2) + (2.7 \times 3) + (9.72 \times 3) + \left(\frac{1}{2} \times 9.87 \times 3\right) + \left(\frac{1}{2} \times 11.88 \times 2\right) + \left(\frac{1}{2} \times 29.43 \times 3\right) \right]$$

$$= 114.69 \text{ KN/m}^2$$

Location of the force from base,

$$\bar{z} = \frac{1}{P_{0T}} \left[\left\{ P_1 \times \left(h_2 + \frac{h_1}{2} \right) \right\} + \left(P_2 \times \frac{h_2}{2} \right) + P_2 \left(h_2 + \frac{h_1}{3} \right) + \left(P_4 \times \frac{h_2}{2} \right) + \left(P_5 \times \frac{h_2}{3} \right) + \left(P_6 \times \frac{h_2}{3} \right) \right]$$

$$= \frac{1}{114.69} \left[\left\{ 3.33 \times 2 \left(3 + \frac{2}{2} \right) \right\} + \left(2.7 \times 3 \times \frac{3}{2} \right) + \left\{ 11.88 \times \left(3 + \frac{2}{3} \right) \right\} + \left(9.72 \times \frac{3}{2} \right) + \left(9.87 \times \frac{3}{2} \right) + \left(29.43 \times \frac{3}{3} \right) \right]$$

$$= \frac{1}{114.69} \times \left[(13.32) + 4.05 + 43.56 + 14.73 + 14.805 + 29.43 \right]$$

$$= 1.045 \text{ m}$$

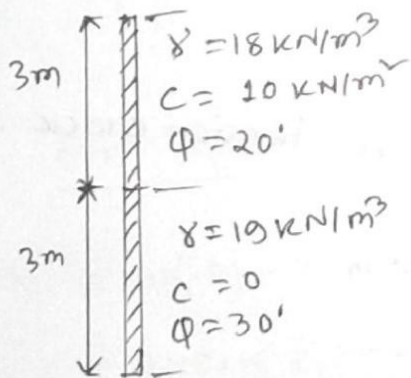
$$= \frac{1}{114.69} \left[\left\{ 3.33 \times 2 \left(3 + \frac{2}{2} \right) \right\} + \left(2.7 \times 3 \times \frac{3}{2} \right) + \left\{ \frac{1}{2} \times 11.88 \times 2 \left(3 + \frac{2}{3} \right) \right\} \right. \\ \left. + \left(2.72 \times 3 \times \frac{3}{2} \right) + \left(\frac{1}{2} \times 9.87 \times 3 \times \frac{3}{3} \right) + \left(\frac{1}{2} \times 29.43 \times 3 \times \frac{3}{3} \right) \right]$$

$$= \frac{1}{114.69} [26.64 + 12.15 + 43.56 + 43.74 + 14.805 + 44.145]$$

$$= 1.61 \text{ m. from base.}$$

\therefore Location of the resultant surface force from top surface
 $= 5 - 1.61 = 3.39 \text{ m}$ Ans.

Problem-03: A retaining wall supports a multilayer of soil.
 (I) Draw the earth pressure envelop before and after formation of crack.
 (II) Calculate the active thrust on the retaining wall and its position after the formation of crack.



Solution:

For layer 1

$$P_a = k_{a1} \gamma_1 z - 2c \sqrt{k_{a1}}$$

$$P_a = 0.49 \times 18 \times z - 2 \times 10 \times \sqrt{0.49}$$

$$P_a = 8.82z - 14 \quad \text{--- (1)}$$

$$k_{a1} = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 20^\circ}{1 + \sin 20^\circ}$$

$$\therefore k_{a1} = 0.49$$

When, $P_a = 0, z = 1.59 \text{ m}$

$z = 0, P_a = -14 \text{ kN/m}^2$

$z = 3, P_a = 12.46 \text{ kN/m}^2$

For layer 2

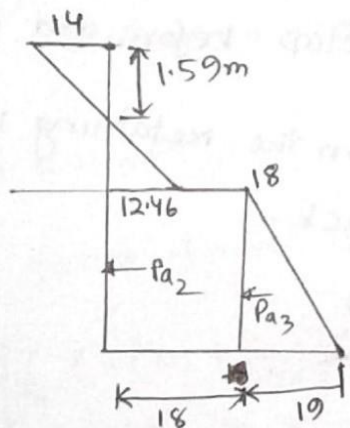
Swrchange due to upper layer soil,

$q = \gamma_1 H_1 = 18 \times 3 = 54 \text{ kN/m}^2$

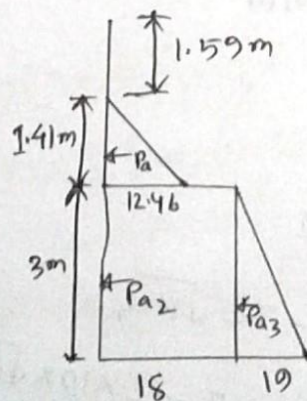
$k_{a2} = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ} = \frac{1}{3}$

$P_{a2} = k_{a2} q = \frac{1}{3} \times 54 = 18 \text{ kN/m}^2$

$P_{a3} = k_{a2} \gamma_2 H_2 = \frac{1}{3} \times 19 \times 3 = 19 \text{ kN/m}^2$



pressure envelop before tension crack .



pressure envelop after tension crack.

Active thrust after formation of tension crack.

$$P_{0T} = \left(\frac{1}{2} \times 12.46 \times 1.41\right) + (18 \times 3) + \left(\frac{1}{2} \times 19 \times 3\right)$$

$$= 91.2843 \text{ kN/m}^2$$

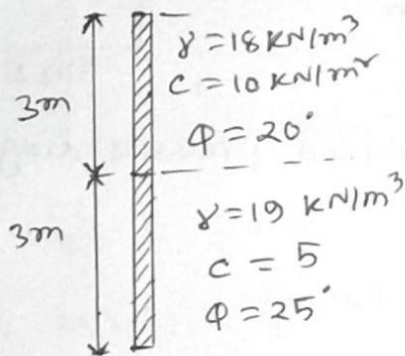
Location of the resultant force from below ^{base} surface.

$$\bar{z} = \frac{1}{P_{0T}} \left[\left\{ \frac{1}{2} \times 12.46 \times 1.41 \times \left(3 + \frac{1.41}{3}\right) \right\} + \left(18 \times 3 \times \frac{3}{2}\right) + \left(\frac{1}{2} \times 19 \times 3 \times \frac{3}{3}\right) \right]$$

$$= 1.53 \text{ m}$$

Location from top surface = $6 - 1.53 = 4.47 \text{ m}$ Ans. \leftarrow

Problem: 04: Determine the total active pressure after the develop of tension crack. Also draw the pressure diagram.



Solution: For layer 1:

$$P_a = k_a \gamma z_1 - 2c\sqrt{k_a}$$

$$= 0.49 \times 18 z_1 - 2 \times 10 \times \sqrt{0.49}$$

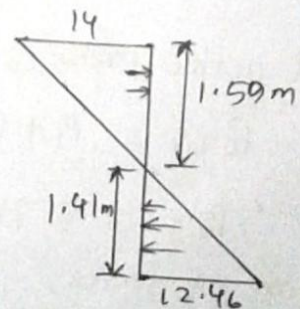
$$= 8.82 z_1 - 14$$

When, $P_a = 0$, $z = 1.59 \text{ m}$

$z = 0$, $P_a = -14 \text{ kN/m}^2$

$z = 3$, $P_a = 12.46 \text{ kN/m}^2$

$$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.49$$



For bottom layer:

For bottom layer, the weight of top soil will act as surcharge at the end of upper layer.

$$\therefore q = \gamma_1 H_1 = 18 \times 3 = 54 \text{ kN/m}^2$$

At any depth z_2 below the end of upper layer.

$$P_{a2} = \gamma'_2 z_2 k_{a2} - 2c_2 \sqrt{k_{a2}} + q k_{a2}$$

$$= 19 z_2 \times \frac{1}{3} - 2 \times 5 \times \sqrt{\frac{1}{3}} + (54 \times \frac{1}{3})$$

$$= 6.33 z_2 + 12.23$$

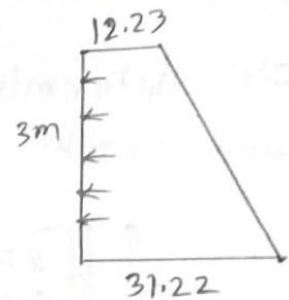
$$k_{a2} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$= \frac{1}{3}$$

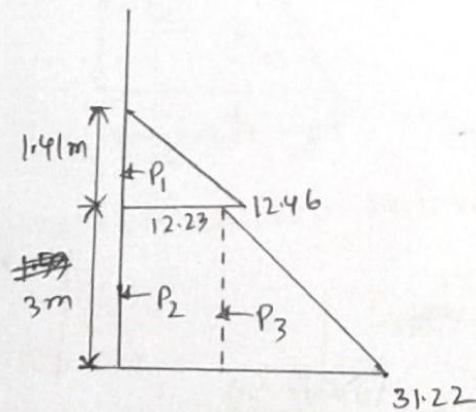
$$c_2 = c = 5 \text{ kN/m}^2$$

Now, $z_2 = 0$, $P_{a2} = 12.23 \text{ kN/m}^2$

$z_2 = 3 \text{ m}$, $P_{a2} = 31.22 \text{ kN/m}^2$



After tension crack develop, the combined pressure diagram is given by,



Total active pressure,

$$P_{oT} = P_1 + P_2 + P_3 = \left(\frac{1}{2} \times 12.46 \times 1.41 \right) + (12.23 \times 3) + \left(\frac{1}{2} \times 31.22 \times 3 \right)$$

$$\therefore P_{oT} = 8.7843 + 36.69 + 46.83 = 92.3043 \text{ kN/m}^2$$

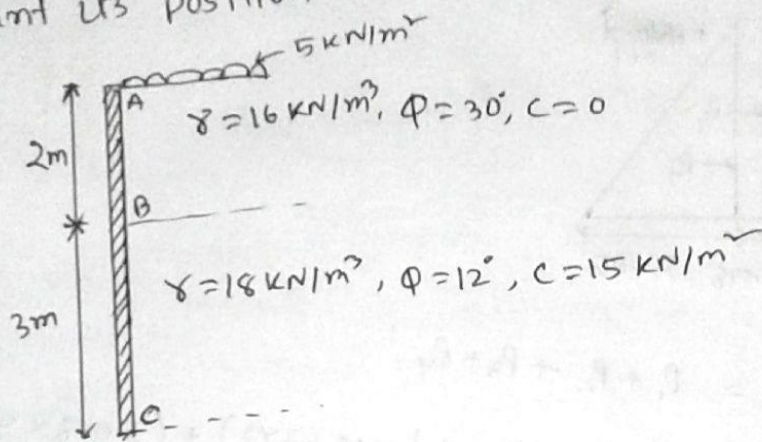
Location of force,

$$\bar{z} = \frac{1}{P_{oT}} \left[P_1 \times \left(3 + \frac{1.41}{3} \right) + \left(P_2 \times \frac{3}{2} \right) + \left(P_3 \times \frac{3}{3} \right) \right]$$

$$= \frac{1}{92.3043} \left[8.784 \times \left(3 + \frac{1.41}{3} \right) + \left(36.69 \times \frac{3}{2} \right) + \left(46.83 \times \frac{3}{3} \right) \right] = 1.43 \text{ m from base}$$

\therefore Location of force from top surface = $6 - 1.43 = 4.57 \text{ m}$ Ans

2015 3(c) A retaining wall with stratified backfill and surcharge is shown in figure below. Draw the earth pressure diagram detailing the values at the critical points. Also estimate the resultant thrust on the wall and its position.



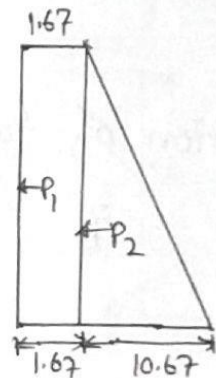
Solution: $K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$, $\therefore K_{a1} = \frac{1}{3}$, $K_{a2} = 0.66$

For top soil:

$$P_a = K_{a1} q + K_{a1} \gamma_1 z_1$$

$$P_1 = K_{a1} q = \frac{1}{3} \times 5 = 1.67 \text{ kN/m}^2$$

$$P_2 = K_{a1} \gamma_1 z_1 = \frac{1}{3} \times 16 \times 2 = 10.67 \text{ kN/m}^2$$



For bottom layer:

The weight of top layer is surcharge for bottom layer.

$$q_2 = (q + \gamma_1 H_1) = 5 + (16 \times 2) = 37 \text{ kN/m}^2$$

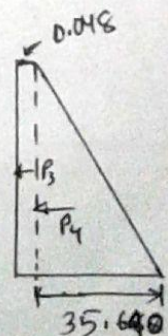
$$\text{Now, } P_a = K_{a2} q_2 + K_{a2} \gamma_2 z_2 - 2c \sqrt{K_{a2}}$$

$$= (0.66 \times 37) + (0.66 \times 18 \times z_2) - (2 \times 15 \times \sqrt{0.66})$$

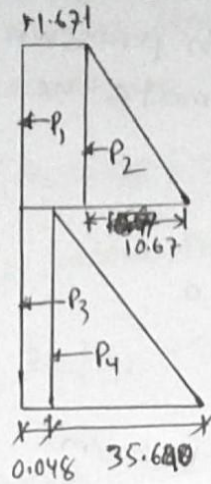
$$= 11.88 z_2 + 0.048$$

$$\text{For, } z_2 = 0 \text{ m, } P_a = 0.048 \text{ kN/m}^2$$

$$\text{For, } z_2 = 3 \text{ m, } P_a = 35.688 \text{ kN/m}^2$$



The composite pressure diagram is given below.



$$\begin{aligned} \text{Total force, } P_{OT} &= P_1 + P_2 + P_3 + P_4 \\ &= (1.67 \times 2) + \left(\frac{1}{2} \times 10.67 \times 2\right) + (0.048 \times 3) + \left(\frac{1}{2} \times 35.640 \times 3\right) \\ &= 67.686 \text{ kN/m} \end{aligned}$$

Location of force,

$$\bar{x} = \frac{1}{P_{OT}} \left[\left\{ 1.67 \times 2 \times \left(3 + \frac{2}{2}\right) \right\} + \left\{ \frac{1}{2} \times 10.67 \times 2 \times \left(3 + \frac{2}{3}\right) \right\} + \left(0.048 \times 3 \times \frac{3}{2}\right) + \left(\frac{1}{2} \times 35.640 \times 3 \times \frac{3}{3}\right) \right]$$

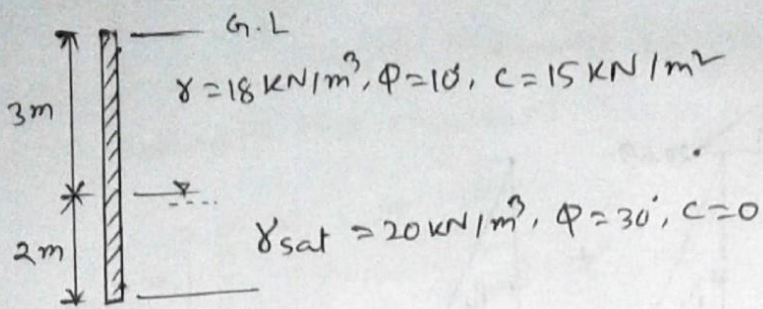
$$= \frac{1}{67.686} [6.68 + 39.12 + 0.216 + 53.532]$$

$$= 1.47 \text{ m from base.}$$

\therefore Location of the resultant thrust from top surface

$$= 5 - 1.47 = 3.53 \text{ m Ans.}$$

2015 4(c) calculate the total active earth thrust and its location from ground level following the figure below.



Solution: $K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$, $K_{a1} = 0.704$, $K_{a2} = \frac{1}{3}$

For top soil:

$$P_a = K_{a1} \gamma_1 z_1 - 2c\sqrt{K_{a1}}$$

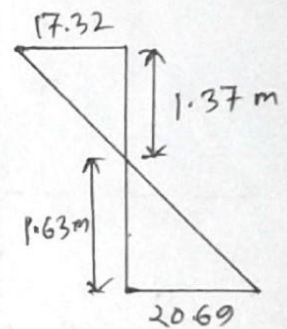
$$= 0.704 \times 18 \times z_1 - 2 \times 15 \times \sqrt{\frac{1}{3}}$$

$$= 12.67 z_1 - 17.32$$

When, $z_1 = 0$, $P_a = -17.32 \text{ kN/m}^2$

$P_a = 0$, $z_1 = 1.37 \text{ m}$

$z_1 = 3 \text{ m}$, $P_a = 20.69 \text{ kN/m}^2$



For bottom layer:

The weight of top layer soil is surcharge for bottom layer,

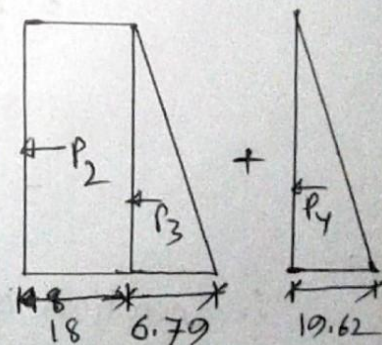
$$q = \gamma_1 H_1 = 18 \times 3 = 54 \text{ kN/m}^2$$

$$P_2 = K_{a2} q = \frac{1}{3} \times 54 = 18 \text{ kN/m}^2$$

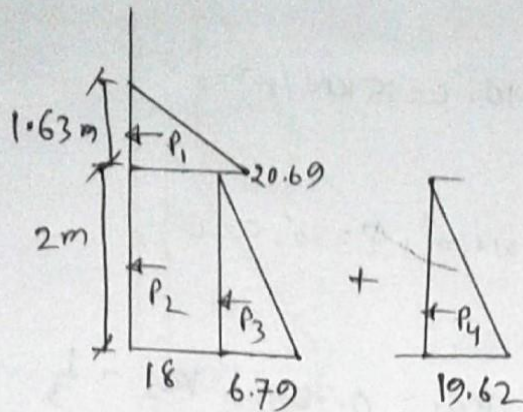
$$P_3 = K_{a2} \gamma'_2 H_2 = \frac{1}{3} \times (20 - 9.81) \times 2$$

$$= 6.79 \text{ kN/m}^2$$

$$P_4 = \gamma_w H_2 = 9.81 \times 3 = 19.62 \text{ kN/m}^2$$



The combined active pressure diagram is shown in figure below:-



Since the tension crack develop, active earth pressure from $Z_1 = 0$ to 1.37m will be zero.

$$\therefore P_{OT} = P_1 + P_2 + P_3 + P_4$$

$$= \left(\frac{1}{2} \times 20.69 \times 1.63\right) + (18 \times 2) + \left(\frac{1}{2} \times 6.79 \times 2\right) + \left(\frac{1}{2} \times 19.62 \times 2\right)$$

$$= 16.86 + 36 + 6.79 + 19.62$$

$$= 79.27 \text{ kN/m}^2$$

Location of force,

$$\bar{z} = \frac{1}{P_{OT}} \left[P_1 \times \left(2 + \frac{1.63}{3}\right) + \left\{ P_2 \times \frac{2}{2} + P_3 \times \frac{2}{3} + P_4 \times \frac{2}{3} \right\} \right]$$

$$= \frac{1}{79.27} \left[\left\{ 16.86 \times \left(2 + \frac{1.63}{3}\right) \right\} + \left(36 \times \frac{2}{2} + 6.79 \times \frac{2}{3} + 19.62 \times \frac{2}{3} \right) \right]$$

$$= 1.22 \text{ m from base,}$$

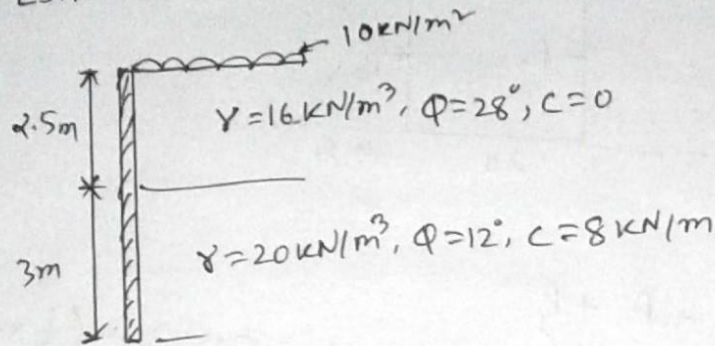
\therefore Location of force from top surface.

$$\bar{z} = 5 - 1.22 = 3.78 \text{ m} \quad \underline{\underline{\text{Ans}}}$$

2014 4(c)

A retaining wall with stratified backfill and a surcharge load is shown in figure below:-

- (I) Draw the earth pressure diagram detailing the contact point.
- (II) Estimate the resultant thrust and its location.



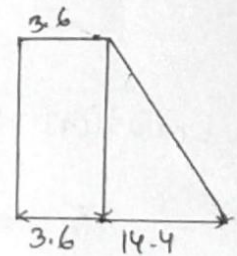
Solution: $K_a = \frac{1 - \sin \phi}{1 + \sin \phi}$, $K_{a1} = 0.36$, $K_{a2} = 0.66$

For top layer:

$$P_a = K_{a1} q + K_{a1} \gamma_1 z_1$$

$$P_1 = K_{a1} q = 0.36 \times 10 = 3.6 \text{ kN/m}^2$$

$$P_2 = K_{a1} \gamma_1 z_1 = 0.36 \times 16 \times 2.5 = 14.4 \text{ kN/m}^2$$



For bottom layer:

The weight of top layer soil in surcharge for bottom layer.

$$q_2 = (q + \gamma_1 H) = (10 + 16 \times 2.5) = 50 \text{ kN/m}^2$$

Now, $P_a = K_{a2} q_2 + K_{a2} \gamma_2 H_2 - 2c \sqrt{K_{a2}}$

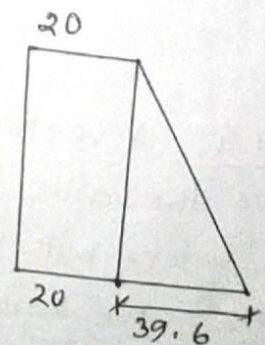
$$= (0.66 \times 50) + (0.66 \times 20 \times 3) - 2 \times 8 \times \sqrt{0.66}$$

$$= 50.6 \text{ kN/m}^2$$

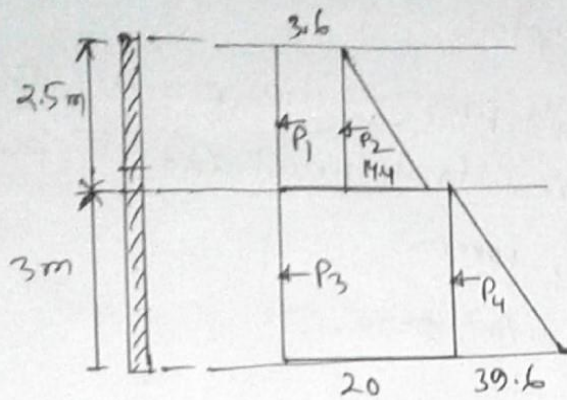
$$P_a = 20 + 13.2 z_2$$

For, $z_2 = 0$, $P_a = 20 \text{ kN/m}^2$

For, $z_2 = 3\text{m}$, $P_a = 50.6 \text{ kN/m}^2$



The combined active earth pressure ^{diagram} is given below:-



$$\begin{aligned}
 P_{OT} &= P_1 + P_2 + P_3 + P_4 \\
 &= (3.6 \times 2.5) + \left(\frac{1}{2} \times 14.4 \times 2.5\right) + (20 \times 3) + \left(\frac{1}{2} \times 39.6 \times 3\right) \\
 &\rightarrow 9 + 18 + 60 + 59.4 \\
 &= 146.4 \text{ kN/m}^2
 \end{aligned}$$

Location of resultant force,

$$\begin{aligned}
 \bar{x} &= \frac{1}{P_{OT}} \left[P_1 \times \left(3 + \frac{2.5}{2}\right) + P_2 \times \left(3 + \frac{2.5}{3}\right) + \left(P_3 \times \frac{3}{2}\right) + \left(P_4 \times \frac{3}{3}\right) \right] \\
 &= \frac{1}{146.4} \left[9 \times \left(3 + \frac{2.5}{2}\right) + 18 \times \left(3 + \frac{2.5}{3}\right) + \left(60 \times \frac{3}{2}\right) + \left(59.4 \times \frac{3}{3}\right) \right] \\
 &= 1.75 \text{ m from base surface.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Location of resultant force from top surface,} \\
 &= 5.5 - 1.75 = 3.75 \text{ m } \underline{\text{Ans.}}
 \end{aligned}$$

CT-12 series

Problem 4: A vertical cut of given height has $c = 10 \text{ kN/m}^2$ and $\phi = 15^\circ$. Calculate the value of surcharge for which the vertical cut that was stable before will be unstable?

Solution: We know at stable stage surcharge q is given by,

$$q = \frac{2c}{\sqrt{K_a}} = \frac{2 \times 10}{\sqrt{0.59}}, \quad \left| \begin{array}{l} \text{Here, } c = 10 \text{ kN/m}^2 \\ K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.59 \end{array} \right.$$

$$q = 26.06 \text{ kN/m}^2 \underline{\text{Ans.}}$$

Problem 2013, 1(c) A vertical excavation was made in a clay deposit having unit weight of 21 kN/m^3 . It caved-in after the digging reached 4m depth. Assuming $\phi = 0^\circ$, calculate the magnitude of cohesion.

Solution: Given, $\phi = 0^\circ$
 $\therefore K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = 1$

We know,
 $Z_0 = \frac{2c}{\gamma \sqrt{K_a}}$
 $Z_0 = \frac{2c}{21 \sqrt{1}} = 9.5 \times 10^{-2} c$

\therefore safe depth of cut off $= 2Z_0 = 1.9 \times 10^{-1} c$

$\therefore 1.9 \times 10^{-1} c = 4$

$\therefore c = 21 \text{ kN/m}^2$ Ans.

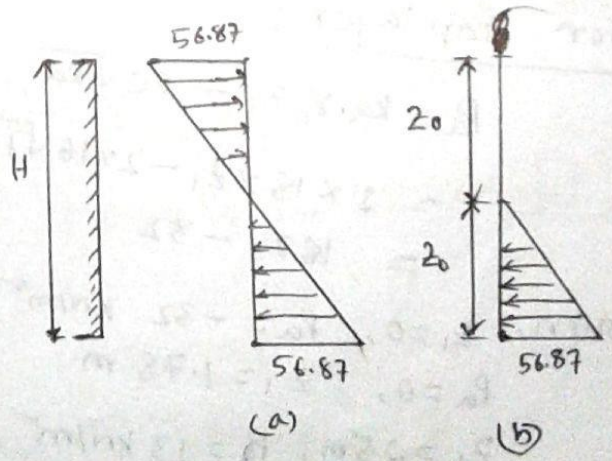
2011 2(c) An unsupported excavation is to be made in a clay layer. If $\gamma_1 = 19 \text{ kN/m}^3$, $c = 35 \text{ kN/m}^2$ and $\phi = 12^\circ$.

- (i) Calculate the depth of tension crack.
- (ii) Calculate the maximum possible unsupported depth.
- (iii) Draw the pressure diagram before the occurrence of crack.
- (iv) Draw the pressure diagram after the formation of crack.

Solution: $K_a = \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1 - \sin 12^\circ}{1 + \sin 12^\circ} = 0.66$

$P_a = \gamma z K_a - 2c \sqrt{K_a}$
 $= 19z \times 0.66 - 2 \times 35 \sqrt{0.66}$
 $= 12.54z - 56.87$ — (1)

At top, $z = 0$
 $P_a = -56.87 \text{ kN/m}^2$



P_a is zero at $z = z_0 = \frac{56.87}{12.54} = 4.53 \text{ m}$

Hence the depth of tension crack = 4.53 m .

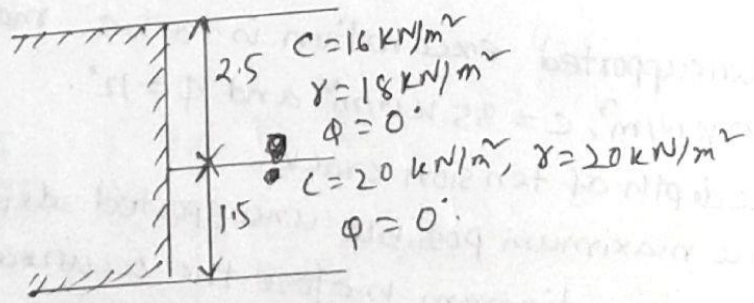
Maximum possible unsupported depth = $2z_0 = 2 \times 4.53 = 9.06 \text{ m}$

Pressure at bottom .

$P_a = 12.54 \times 9.06 - 56.87 = 56.87 \text{ kN/m}^2$

The active pressure diagram before the occurrence of crack is shown in figure (a) and after the formation of tension crack , the pressure diagram is shown in figure (b) .

Problem 2011 7 (b) For an earth retaining structure shown in figure below - determine the total active earth pressure on the wall. Also draw the earth pressure distribution diagram before and after the formation of tension crack .



Solution:

$k_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 1$

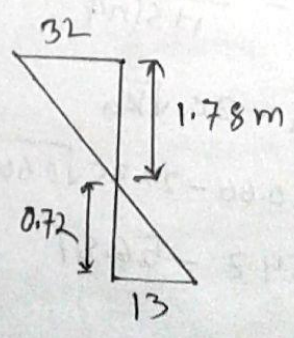
For top layer:

$$P_a = k_a \cdot \gamma_1 \cdot z_1 - 2c \sqrt{k_a}$$

$$= 1 \times 18 \times z_1 - 2 \times 16 \sqrt{1}$$

$$= 18z_1 - 32$$

when, $z_1 = 0, P_a = -32 \text{ kN/m}^2$
 $P_a = 0, z_1 = 1.78 \text{ m}$
 $z_1 = 2.5 \text{ m}, P_a = 13 \text{ kN/m}^2$



for bottom layer:

The weight of top layer soil is surcharge for bottom layer.

$$q = \gamma_1 H_1 = 18 \times 2.5 = 45 \text{ kN/m}^2$$

$$P_2 = K_a q = 1 \times 45 = 45 \text{ kN/m}^2$$

$$P_3 = K_a \gamma_2' H_2 = 1 \times (?)$$

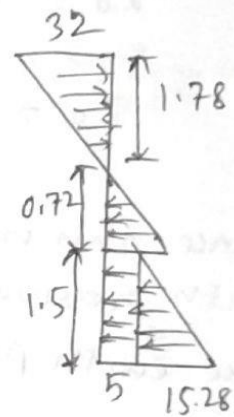
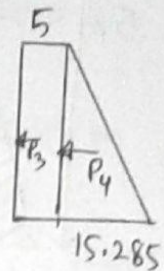
$$P_a = K_a q + K_a \gamma_2' z_2 - 2c \sqrt{K_a}$$

$$= 1 \times 45 + 1 \times (20 - 9.81) z_2 - 2 \times 20 \sqrt{1}$$

$$= 10.19 z_2 + 45 - 40$$

for, $z_2 = 0$, $P_a = 5 \text{ kN/m}^2$

$z_2 = 1.5$, $P_a = 20.285 \text{ kN/m}^2$



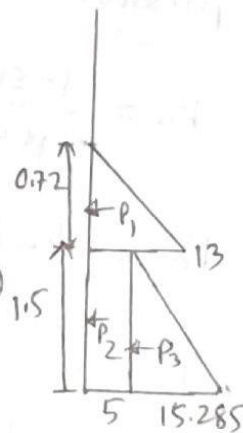
The combined pressure diagram given below:-
since the tension crack developed, the pressure at the depth 0 to 1.78 m will be zero.

$$P_{OT} = P_1 + P_2 + P_3$$

$$= \left(\frac{1}{2} \times 13 \times 0.72 \right) + (5 \times 1.5) + \left(\frac{1}{2} \times 15.285 \times 1.5 \right)$$

$$= 4.68 + 7.5 + 11.46$$

$$= 23.64 \text{ kN/m}^2$$



(after formation crack)

Location,

$$\bar{z} = \frac{1}{P_{OT}} \left[P_1 \times \left(1.5 + \frac{0.72}{3} \right) + \left(P_2 \times \frac{1.5}{2} \right) + \left(P_3 \times \frac{1.5}{3} \right) \right]$$

$$= \frac{1}{23.64} \left[4.68 \times \left(1.5 + \frac{0.72}{3} \right) + \left(7.5 \times \frac{1.5}{2} \right) + \left(11.46 \times \frac{1.5}{3} \right) \right]$$

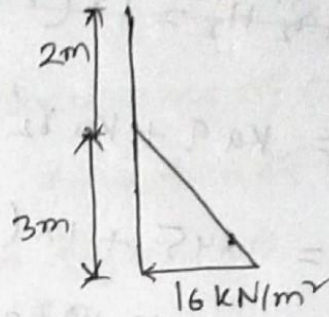
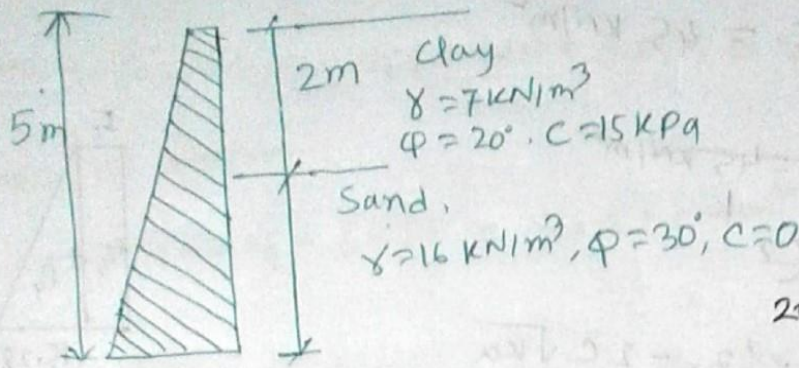
$$= 0.82 \text{ m from base.}$$

$$\therefore \text{From top surface} = 2.5 + 1.5 - 0.82 = 3.18 \text{ m}$$

B.C Punmia

20.28

Plot the distribution of active earth pressure after tensile crack occur.



Soln

For clay,

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.49,$$

$$z_0 = \frac{2c}{\gamma \sqrt{K_a}} = \frac{2 \times 15}{7 \times \sqrt{0.49}} = 2.52 \text{ m}$$

Since this is more than height ($H_1 = 2 \text{ m}$) of clay, there will be no earth pressure due to clay.

The earth pressure will be entirely due to sand.

For sand, $K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = \frac{1}{3}, c = 0.$

$$\therefore P_a = K_a \gamma_2 H_2 = \frac{1}{3} \times 16 \times 3 = 16 \text{ kN/m}^2$$

Ans

20.34 Determine the stresses at the top and bottom of a vertical cut, 4.5m deep in soil with $\phi' = 16^\circ, c' = 19.1 \text{ kN/m}^2$ and $\gamma = 18.5 \text{ kN/m}^3$. what could be the depth of the potential crack? what is the maximum depth of excavation that can be left unsupported.

Answer: For a $c-\phi$ soil, the stress at any depth z is given by

$$P_a = \gamma z K_a - 2c' \sqrt{K_a}$$

$$K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = 0.5679$$

$$P_a = 18.5 z (0.5679) - 2 \times 19.1 \sqrt{0.5679}$$

$$P_a = 10.505 z - 28.787$$

At top, $z=0$, $P_a = -28.787 \text{ kN/m}^2$ (i.e. tension)

At bottom, $z=4.5 \text{ m}$, $P_a = 18.489 \text{ kN/m}^2$

Depth of potential crack is at z_0 where $P_a = 0$.

$$\therefore z_0 = \frac{2c'}{\gamma \sqrt{K_a}} = \frac{2 \times 19.1}{18.5 \times \sqrt{0.5979}} = 2.74 \text{ m}$$

\therefore Max. depth of unsupported excavation, $= 2z_0 = 5.48 \text{ m}$ Ans.

20.36 A vertical bank was formed. $\dots \phi = 15^\circ$, $\gamma = 1800 \text{ kg/m}^3$
when the depth of excavation reached 5.5 m the bank failed. What was the cohesion of clay.

Soln We know that,

$$z_0 = \frac{2c}{\gamma \sqrt{K_a}}$$

$$= \frac{2c}{1800 \sqrt{0.589}}$$
$$= 1.448 \times 10^{-3} c$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.589$$

\therefore Safe depth of cutoff $= 2z_0 = 2.896 \times 10^{-3} c$

$$\therefore 2.896 \times 10^{-3} c = 5.5$$

$$\therefore c = 1899 \text{ kg/m}^2 \text{ Ans.}$$

20.46 Compute P_p at depth 8 m where $c=0$, $\phi=30^\circ$, $\gamma_{\text{sat}} = 21 \text{ kN/m}^3$.

Soln Given, $\gamma_{\text{sat}} = 21 \text{ kN/m}^3$, $\phi = 30^\circ$,

$$K_p = \frac{1 + \sin \phi}{1 - \sin \phi} = 3$$

$$P_p = K_p \gamma' H + \gamma_w H = 3 * (21 - 9.81) * 8 + (9.81 * 8)$$
$$P_p = 347.04 \text{ kN/m}^2 \text{ Ans.}$$

2017 A clay having $\gamma = 22 \text{ kN/m}^3$, It caved in after the digging reached 4m depth. Assume $\phi = 0$. calculate $c = ?$

Answers:

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 1$$

$$z_0 = \frac{2c}{\gamma \sqrt{K_a}} = \frac{2c}{22 \times \sqrt{1}} = \frac{c}{11}$$

\therefore Maximum unsupported depth, $= 2z_0 = 2 \times \frac{c}{11} = 4$ (given)

$$\therefore c = 4 \times \frac{11}{2} = 22 \text{ kN/m}^2 \text{ Ans.}$$

2014 3(c)

Given,

$$G_s = 2.66$$

$$e = 0.6$$

$$\phi = 32^\circ$$

$$K_a = \frac{1 - \sin \phi}{1 + \sin \phi} = 0.307$$

$$\gamma = \frac{G_s \cdot \gamma_w}{1 + e} = \frac{2.66 \times 9.81}{1 + 0.6} = 16.31 \text{ kN/m}^3$$

$$\gamma_{\text{sat}} = \frac{(G_s + e) \gamma_w}{1 + e} = \frac{(2.66 + 0.6) \cdot 9.81}{1 + 0.6} = 19.99 \sim 20 \text{ kN/m}^3$$

(i) Total vertical stress, $\sigma_v = \gamma h_1 + \gamma' h_2$ (effective)

$$= (16.31 \times 1.5) + (20 - 9.81) \times 2$$

Total vertical stress.

$$\sigma_v = \gamma h_1 + \gamma_{\text{sat}} h_2 = (16.31 \times 1.5) + (20) \times 2 = 64.465 \text{ kN/m}^2$$

(ii) ~~Total horizontal stress,~~

Pore pressure $= \gamma_w h_2 = 9.81 \times 2 = 19.62 \text{ kN/m}^2$

(iii) $\sigma_H = K_a \gamma h_1 + K_a \gamma' h_2 + \gamma_w h_2$

$$= (0.307 \times 16.31 \times 1.5) + 0.307 \times (20 - 9.81) \times 2 + 9.81 \times 2$$

$$= 7.51 + 6.26 + 19.62 = 33.39 \text{ kN/m}^2$$

STRESS DISTRIBUTION

Md. Rabiul Islam RUET CE'130110

Q: Prove that the maximum vertical stress on a vertical line at a constant radial distance r from the axis of a vertical load is induced at the point of intersection of the vertical line with a radial line at $\beta = 39.15^\circ$ from the point of application of concentrated load. What will be the value of shear stress at the point? Hence, or otherwise, find the maximum vertical stress on a line situated at $r = 2\text{m}$, from the axis of a concentrated load of value 20 kN .

Solution: we know that,

$$\sigma_z = \frac{3Q}{2\pi} \left[\frac{z^3}{(r^2+z^2)^{5/2}} \right] \quad \text{--- (1)}$$

$$= \frac{3Q}{2\pi} z^r \left[\frac{1}{1+(r/z)^2} \right]^{5/2} \quad \text{--- (2)}$$

For the maximum value of σ_z (where r is constant),

$$\frac{d\sigma_z}{dz} = 0$$

$$\Rightarrow \frac{d}{dz} \left(\frac{3Q}{2\pi} \left[\frac{z^3}{(r^2+z^2)^{5/2}} \right] \right) = 0$$

$$\therefore \frac{3Q}{2\pi} \frac{(r^2+z^2)^{5/2} \cdot 3z^2 - z^3 \cdot \frac{5}{2} (z^2+r^2)^{3/2} \cdot 2z}{(r^2+z^2)^5} = 0$$

$$\Rightarrow 3z^2(r^2+z^2)^{5/2} - 5z^4(r^2+z^2)^{3/2} = 0$$

$$\therefore \Rightarrow 3(r^2+z^2) - 5z^2 = 0 \quad \left[\because \text{divided by } (r^2+z^2)^{3/2} \text{ both side} \right]$$

$$\Rightarrow 3r^2 + 3z^2 - 5z^2 = 0$$

$$\therefore \frac{r^2}{z^2} = \frac{2}{3}$$

$$\Rightarrow \frac{r}{z} = 0.817$$

$$\Rightarrow \tan \beta = 0.817$$

$$\therefore \beta = 39^\circ 15'$$

From equation (1) we put, $z = \frac{r}{0.817}$ we get,

$$(\sigma_z)_{\max} = \frac{3Q}{2\pi} \frac{1}{\left(\frac{r}{0.817}\right)^2} \left[\frac{1}{1 + (0.817)^2} \right]^{5/2}$$

$$(\sigma_z)_{\max} = 0.0888 \frac{Q}{r^2} \quad \left| \begin{array}{l} Q = 20 \text{ kN} \\ r = 2 \text{ m} \end{array} \right.$$

$$= 0.0888 \times \frac{20}{2^2}$$

$$= 0.444 \text{ kN/m}^2$$

Again,

$$\tau_{rz} = \frac{3Q}{2\pi} \frac{r z^2}{(r^2 + z^2)^{5/2}} = \frac{3Q}{2\pi} \frac{r}{z^3} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$$\tau_{rz} = (\sigma_z)_{\max} \frac{r}{z}$$

$$= 0.444 \times 0.817$$

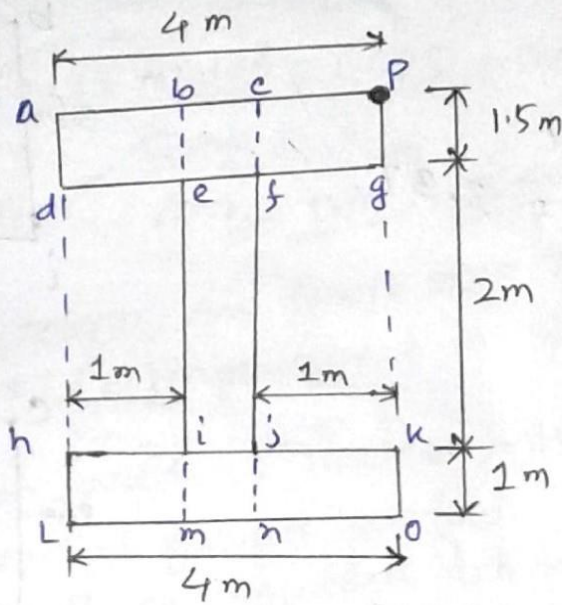
$$= 0.362 \text{ kN/m}^2 \quad \underline{\text{Ans}}$$

2015 1(c) A uniformly loaded area is shown in figure. It carries a uniform load of 60 kN/m^2 at ground surface. Find the vertical pressure at point P below 3 m depth using Newmark solution.

Solution:

given,
 $z = 3 \text{ m}$

$q = 60 \text{ kN/m}^2$



$$\text{Area} = alop - ahkp + bpik - cjkp + adgp - begp + cftg$$

Now

we know,

$$KN = \frac{1}{2\pi} \left[\frac{mn}{\sqrt{f}} \left(\frac{f+1}{f+m^2n^2} + \sin^{-1} \left(\frac{mn}{f+m^2n^2} \right) \right) \right]$$

$$m = \frac{L}{z}, \quad n = \frac{B}{z}, \quad f = m^2 + n^2 + 1$$

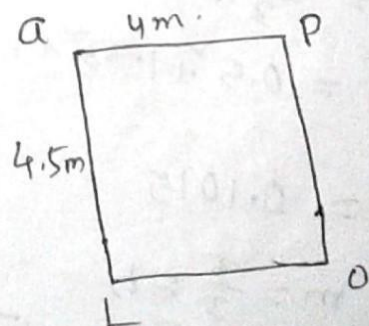
Now,
(1)

$$m = \frac{L}{z} = \frac{4}{3} = 1.33$$

$$n = \frac{4.5}{3} = 1.5$$

$$f = 5.018$$

$$KN_1 = 0.1303$$



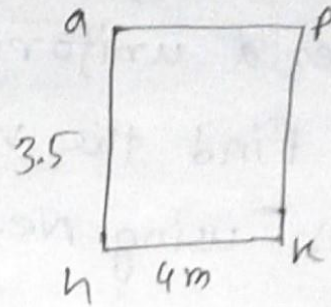
(i)

$$m = \frac{4}{3} = 1.33$$

$$n = \frac{3.5}{3} = 1.17$$

$$f = 1.33^2 + 1.17^2 + 1 = 4.14$$

$$KN_2 = 0.133$$



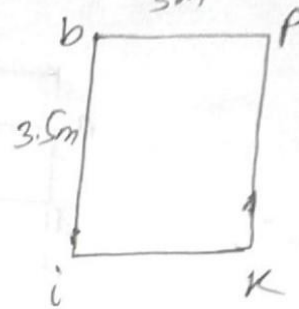
(ii)

$$m = \frac{3}{3} = 1$$

$$n = \frac{3.5}{3} = 1.17$$

$$f = 1^2 + 1.17^2 + 1 = 3.37$$

$$KN_3 = 0.133$$



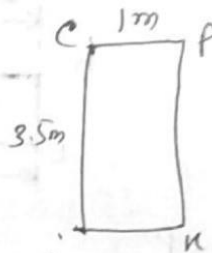
(iii)

$$m = \frac{1}{3} = 0.33$$

$$n = \frac{3.5}{3} = 1.17$$

$$f = 0.33^2 + 1.17^2 + 1 = 2.48$$

$$KN_4 = 0.075$$



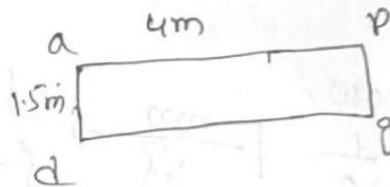
(iv)

$$m = \frac{4}{3} = 1.33$$

$$n = \frac{1.5}{3} = 0.5$$

$$f = 0.5^2 + 1.33^2 + 1 = 3.02$$

$$KN_5 = 0.1015$$



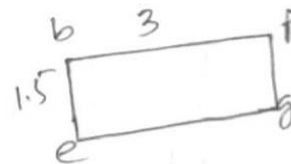
(v)

$$m = \frac{3}{3} = 1$$

$$n = \frac{1.5}{3} = 0.5$$

$$f = 0.5^2 + 1^2 + 1 = 2.25$$

$$KN_6 = 0.101$$



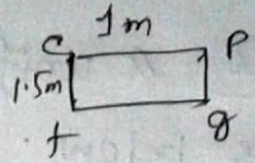
(VII)

$$m = \frac{1}{3} = 0.33$$

$$n = \frac{1.5}{3} = 0.5$$

$$f = 0.33^2 + 0.5^2 + 1 = 1.36$$

$$KN_7 = 0.057$$



Now,

$$\sigma_z = q (KN_1 - KN_2 + KN_3 - KN_4 + KN_5 - KN_6 + KN_7)$$

$$= 60 \times (0.1303 - 0.133 + 0.133 - 0.075 + 0.1015 - 0.101 + 0.057)$$

$$= 6.768 \text{ KN/m}^2 \text{ Ans. Done}$$

उत्तर सही है, उत्तर, Ans मिलान करे 25 अंक का
 ब्रह्मणेन सत्यं वाचं। Feelings sweet in big problem.

2015 2(c) A circular area is loaded with a uniform load intensity of 80 KN/m^2 at ground surface. Calculate the vertical pressure at a point P so situated on the vertical line through the centre of the loaded area that the area subtends an angle 90° at P. Use Boussinesq Analysis.

Solution:

$$\text{Here, } z = R, \quad q = 80 \text{ KN/m}^2.$$

For vertical pressure we know,

$$\sigma_z = q \left[1 - \left\{ \frac{1}{1 + (R/z)^2} \right\}^{3/2} \right]$$

$$= 80 \left[1 - \left\{ \frac{1}{1 + (1)^2} \right\}^{3/2} \right]$$

$$= 51.72 \text{ KN/m}^2 \text{ Ans}$$



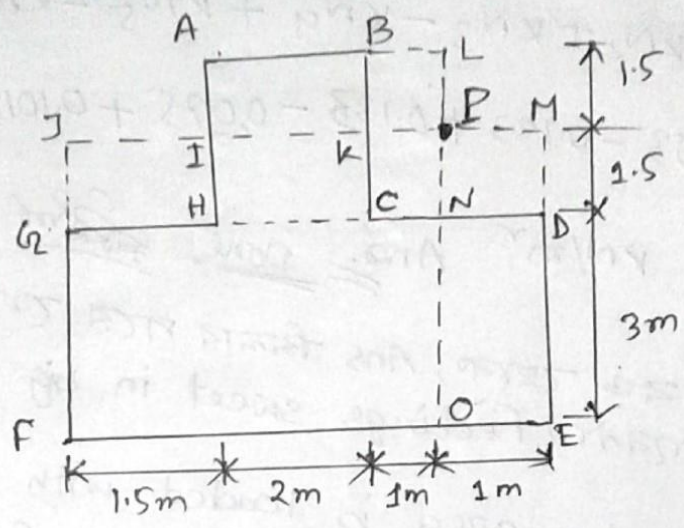
Punmia - 324 page - 2 No problem $80 \text{ AD } 200 \times 200 \text{ } 100 \text{ cm}^2$

$$\sigma_z = 100 * \left[1 - \left\{ \frac{1}{1 + (1)^2} \right\}^{3/2} \right] = 64.6 \text{ KN/m}^2 \text{ Ans}$$

2014 1(c) A uniformly loaded area is shown in figure below. It carries a uniform load of 80 kN/m^2 at ground surface. Find the vertical pressure at point P below 5m of ground surface using Newmark solution.

Solution:

Given,
 $z = 5 \text{ m}$
 $q = 80 \text{ kN/m}^2$



$$\text{Area} = \text{JPOF} - \text{JPNC} + \text{POEM} - \text{PNMD} + \text{IPNH} - \text{KPNC} + \text{ALPI} - \text{BKPL}$$

We know that,

$$m = \frac{L}{z}, \quad n = \frac{B}{z}, \quad f = m^2 + n^2 + 1$$

$$KN = \frac{1}{2\pi} \left[\frac{mn}{\sqrt{f}} * \frac{f+1}{f+m^2n^2} + \sin^{-1} \left(\frac{mn}{f+m^2n^2} \right) \right] \quad \left[\text{radian} \frac{2\pi}{360} \right]$$

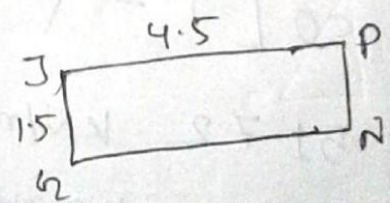
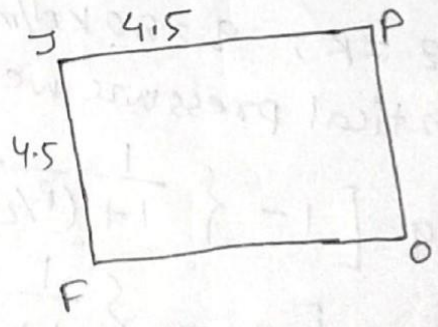
Now,

(i) $m = \frac{4.5}{5} = 0.9$
 $n = \frac{4.5}{5} = 0.9$
 $f = 0.9^2 + 0.9^2 + 1$
 $f = 2.62$

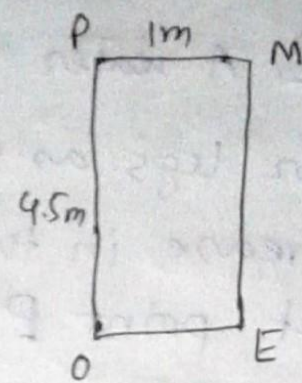
$KN_1 = 0.128$

(ii) $m = \frac{4.5}{5} = 0.9, \quad n = \frac{1.5}{5} = 0.3$
 $f = 0.9^2 + 0.3^2 + 1 = 1.9$

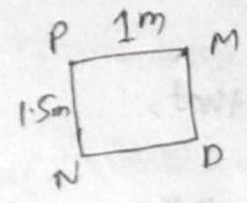
$KN_2 = 0.0677$



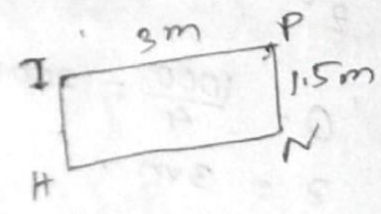
(iii) $m = \frac{1}{5} = 0.2$
 $n = \frac{4.5}{5} = 0.9$
 $f = 4.85$
 $KN_3 = 0.047$
 ~~$KN_3 = 0.078$~~



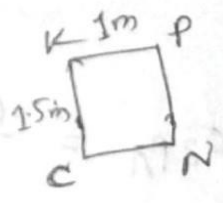
(iv) $m = \frac{1}{5} = 0.2$
 $n = \frac{1.5}{5} = 0.3$
 $f = 1.13$



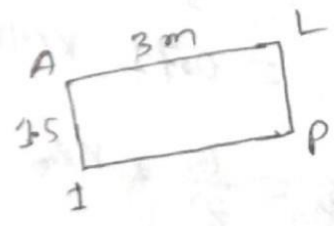
$KN_4 = 0.025$



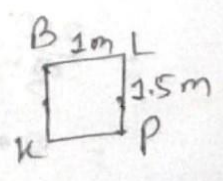
(v) $m = \frac{3}{5} = 0.6$
 $n = \frac{1.5}{5} = 0.3$
 $f = 1.45$
 $KN_5 = 0.0587$



(vi) $m = \frac{1}{5} = 0.2, n = \frac{1.5}{5} = 0.3$
 $f = 1.13$
 $KN_6 = 0.025$



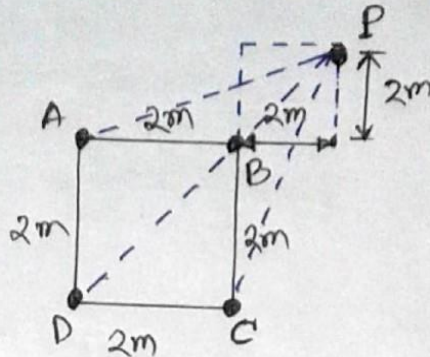
(vii) $m = \frac{3}{5} = 0.6, n = \frac{1.5}{5} = 0.3$
 $f = 1.45$
 $KN_7 = 0.0587$



(viii) $m = \frac{1}{5} = 0.2, n = \frac{1.5}{5} = 0.3$
 $f = 1.13$
 $KN_8 = 0.025$

$\therefore \sigma_2 = q (KN_1 - KN_2 + KN_3 - KN_4 + KN_5 - KN_6 + KN_7 - KN_8)$
 $= 80 (0.128 - 0.0677 + 0.047 - 0.025 + 0.0587 - 0.025 + 0.0587 - 0.025)$
 $= 11.976 \text{ kN/m}^2$ Ans

2014 2(c) A water tank of weight 1000 kN is supported by four legs as shown in figure below. Determine the increase in the vertical stress in the soil below 3m at point P.



Solution

We know that,

$$\sigma_z = \frac{Q}{z^2} * K_B$$

Where, $Q = \frac{1000}{4} = 250 \text{ kN}$.

$z = 3 \text{ m}$.

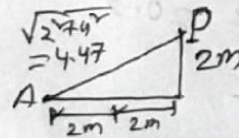
and, $K_B = \frac{3}{2\pi} \left[\frac{1}{1 + (r/z)^2} \right]^{5/2}$

Now,

$$\begin{aligned} \text{(i)} \quad \sigma_{zPA} &= \frac{Q}{z^2} * K_{BA} \\ &= \frac{250}{3^2} * 0.0257 \\ &= 0.71 \text{ kN/m}^2 \end{aligned}$$

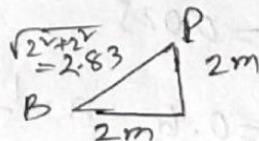
$$\begin{aligned} \text{(ii)} \quad \sigma_{zPB} &= \frac{Q}{z^2} * K_{BB} \\ &= \frac{250}{3^2} * 0.097 \\ &= 2.70 \text{ kN/m}^2 \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad \sigma_{zPC} &= \frac{Q}{z^2} * K_{BC} \\ &= \frac{250}{3^2} * 0.0257 \\ &= 0.71 \text{ kN/m}^2 \end{aligned}$$



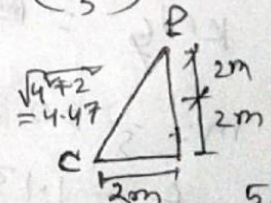
$r = 4.47 \text{ m}$

$$\begin{aligned} K_{BA} &= \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{4.47}{3}\right)^2} \right]^{5/2} \\ &= 0.0257 \end{aligned}$$



$r = 2.83$

$$\begin{aligned} K_{BB} &= \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{2.83}{3}\right)^2} \right]^{5/2} \\ &= 0.097 \end{aligned}$$

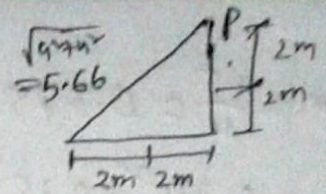


$r = 4.47$

$$\begin{aligned} K_{BC} &= \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{4.47}{3}\right)^2} \right]^{5/2} \\ K_{BC} &= 0.0257 \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad \sigma_{zPD} &= \frac{Q}{z^2} * K_{BD} \\ &= \frac{250}{3^2} * 0.0108 \\ &= 0.298 \text{ kN/m}^2 \end{aligned}$$

$$\begin{aligned} \therefore \sigma_{zPT} &= 0.71 + 2.70 + 0.71 + 0.298 \\ &= 4.42 \text{ kN/m}^2 \quad \underline{\underline{\text{Ans}}} \end{aligned}$$



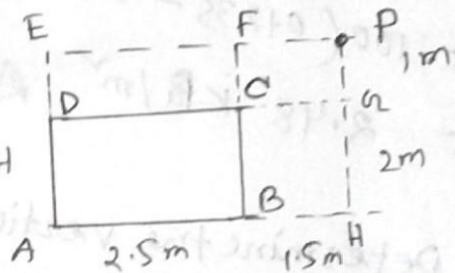
$$r = 5.66$$

$$K_{BD} = \frac{3}{2r} \left[\frac{1}{1 + \left(\frac{5.66}{3}\right)^2} \right]^{5/2}$$

2013 2(c) A rectangular loaded area (shaded) is shown in figure below, which carries a load of 100 kPa. Determine the vertical stress at point 'P' at a depth of 4m.

Soln: $z = 4\text{m}$,
 $q = 100 \text{ kPa}$.

$$\text{Area} = AEPH - DEPG_2 - BFP_2 + CFP_2$$



We know that,

$$m = \frac{L}{z}, \quad n = \frac{B}{z}$$

$$f = m^2 + n^2 + 1$$

$$K_{BD} = \frac{1}{2r} \left[\frac{mn}{\sqrt{f}} * \frac{f+1}{f+m^2n^2} + \sin^{-1} \left(\frac{mn}{f+m^2n^2} \right) \right]$$

NOW

$$\text{(i)} \quad m = \frac{4}{4} = 1, \quad n = \frac{3}{4} = 0.75$$

$$f = m^2 + n^2 + 1 = 2.5625$$

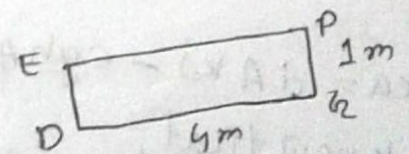
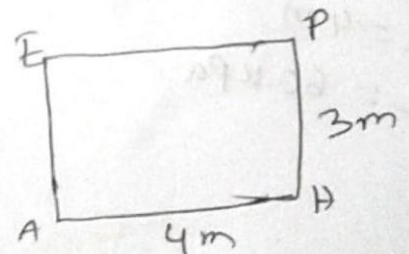
$$K_{N1} = 0.1235$$

(ii)

$$m = \frac{4}{4} = 1\text{m}, \quad n = \frac{1}{4} = 0.25\text{m}$$

$$f = 1^2 + 0.25^2 + 1 = 2.0625$$

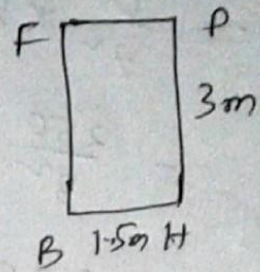
$$K_{N2} = 0.05869$$



$$(iii) \quad m = \frac{1.5}{4} = 0.375, \quad n = \frac{3}{4} = 0.75$$

$$f = 0.375^2 + 0.75^2 + 1 = 1.703$$

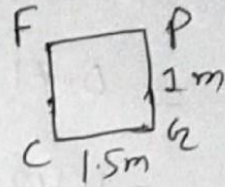
$$KN_3 = 0.077$$



$$(iv) \quad m = \frac{1.5}{4} = 0.375, \quad n = \frac{1}{4} = 0.25$$

$$f = 0.375^2 + 0.25^2 + 1 = 1.203$$

$$KN_4 = 0.037$$



Vertical stress at point P.

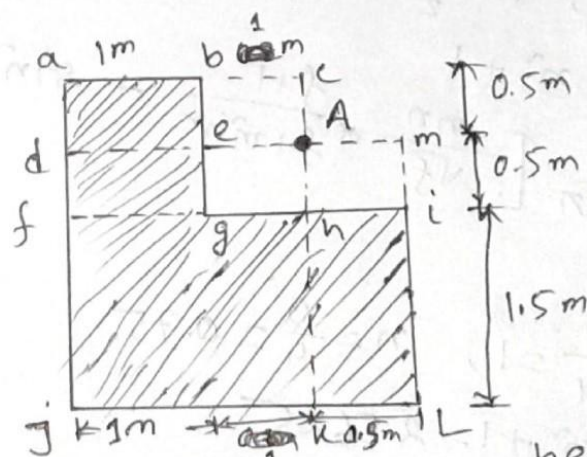
$$\begin{aligned} \sigma_{zP} &= q (KN_1 - KN_2 - KN_3 + KN_4) \\ &= 100 (0.1235 - 0.05869 - 0.077 + 0.037) \\ &= 2.48 \text{ kPa/m}^2 \end{aligned}$$

2013 5(c) Determine the vertical stress at point A below 4m of the following figure. The foundation given in the figure below carries a uniform load of 60 kPa.

Soln:

$$z = 4m$$

$$q = 60 \text{ kPa}$$



$$\text{Area} = dAki - eghA + AKLm - Ahim + Adab - beAc$$

We know that,

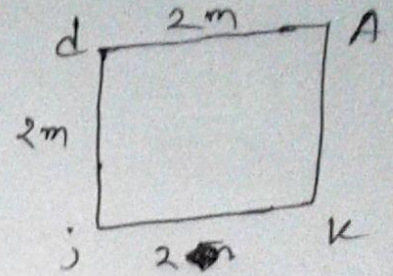
$$m = \frac{L}{z}, \quad n = \frac{B}{z}, \quad f = m^2 + n^2 + 1$$

$$KN = \frac{1}{2\pi} \left[\frac{mn}{\sqrt{f}} + \frac{f+1}{f+m^2n^2} + \sin^{-1} \left(\frac{mn}{f+m^2n^2} \right) \right]$$

$$(i) \quad m = \frac{2}{4} = 0.5, \quad n = \frac{2}{4} = 0.5$$

$$f = 1.5$$

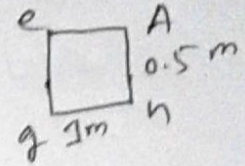
$$KN_1 = 0.077$$



$$(ii) \quad m = \frac{1}{4} = 0.25, \quad n = \frac{0.5}{4} = 0.125$$

$$f = 1.078$$

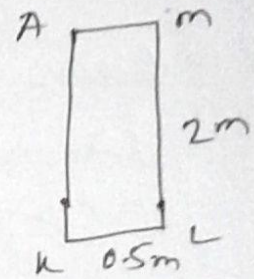
$$KN_2 = 0.0138$$



$$(iii) \quad m = \frac{0.5}{4} = 0.125, \quad n = \frac{2}{4} = 0.5$$

$$f = 1.2656$$

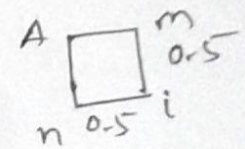
$$KN_3 = 0.0236$$



$$(iv) \quad m = \frac{0.5}{4} = 0.125, \quad n = \frac{0.5}{4} = 0.125$$

$$f = 1.03125$$

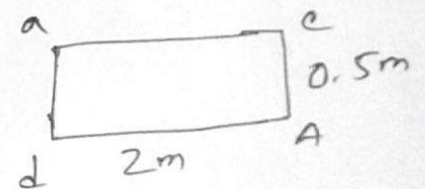
$$KN_4 = 0.007$$



$$(v) \quad m = \frac{2}{4} = 0.5, \quad n = \frac{0.5}{4} = 0.125$$

$$f = 1.2656$$

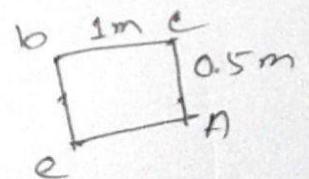
$$KN_5 = 0.0236$$



$$(vi) \quad m = \frac{1}{4} = 0.25, \quad n = \frac{0.5}{4} = 0.125$$

$$f = 1.078$$

$$KN_6 = 0.0138$$



$$\begin{aligned} \therefore \sigma_z &= q (KN_1 - KN_2 + KN_3 - KN_4 + KN_5 - KN_6) \\ &= 60 * (0.077 - 0.0138 + 0.0236 - 0.007 + 0.0236 - 0.0138) \\ &= 5.38 \text{ kPa/m}^2 \end{aligned}$$

Ans