Structural Analysis & Design - II

Written By :

Ahmed Hossain.
Civil '09
ahmed_coda@yahoo.com

A Hand-note On

STRUCTURAL ANALYSIS & DESIGN - II

CE 313

Written By :

Ahmed Hossain.
Civil '09
ahmed_coda@yahoo.com

PDF By :

Md. Oli ur rahman
Civil'11.
oli110064@gmail.com

Md. imran hossain
Civil'11.
Imranhossain65@ymail.com

Mominul islam
Civil'11.
sejance@gmail.com

Topics

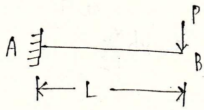
Deflection – 1
Deflection – 2
Portal frame
Two hinged arch
Truss

RON 090001

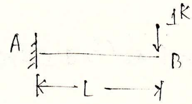
DEFLECTION

AHMED HOSSh 090001.

☐ Find deflection at point B. EI constant.



Solve: Unit load diagram:



portion : AB

origin : B

limit : 0 to L

M : $-px$

m : $-1.x$

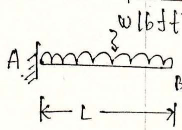
$$EI\Delta_B = \int_0^L (-px).(-x)\,dx$$

$$= \int_0^L px^2\,dx$$

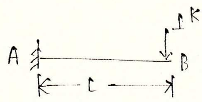
$$= p\left[\frac{x^3}{3}\right]_0^L = \frac{pL^3}{3}$$

$$\Delta_B = \frac{pL^3}{3EI}\ (\downarrow).$$

☐ Find deflection at point B. EI constant



Solve: Unit load diagram:



portion: AB

origin: B

Limit: 0 to L

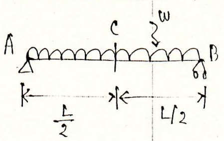
M : $-w.x.\frac{x}{2}$

m : $-1.x$.

$$EI\Delta_B = \int_0^L \frac{wx^2}{2}.x\,dx = \int_0^L \frac{wx^3}{2}dx = \frac{w}{2}\left[\frac{x^4}{4}\right]_0^L$$

$$\Delta_B = \frac{wL^4}{8EI}.$$

□ <u>Find deflection at point C. EI constant.</u>



$\boxed{\text{Solve:}}$ <u>Unit load diagram:</u>

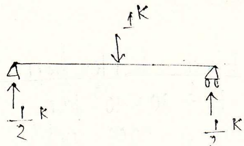


Fig: 2

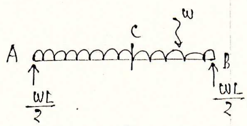


Fig: 1

portion :	AC	BC.
origin :	A	B
Limit :	0 to $\frac{L}{2}$	0 to $\frac{L}{2}$
M :	$\frac{wL}{2}.x - wx.\frac{x}{2}$	$\frac{wLx}{2} - \frac{wx^2}{2}$
m :	$\frac{1}{2}.x$	$\frac{1}{2}x$

$$EI\Delta_c = 2\int_0^{\frac{L}{2}}\left[\frac{wLx}{2}-\frac{wx^2}{2}\right]\cdot\frac{x}{2}\cdot dx$$

$$= \frac{1}{2}\cdot\int_0^{L/2}\left(wLx^2 - wx^3\right)dx$$

$$= \frac{1}{2}\cdot\left[\frac{wLx^3}{3}-\frac{wx^4}{4}\right]_0^{\frac{L}{2}}$$

$$= \frac{1}{2}\left[\frac{wL}{3}\cdot\frac{L^3}{8}-\frac{w}{4}\cdot\frac{L^4}{16}\right]$$

$$= \frac{1}{2}\cdot\left[\frac{wL^4}{24}-\frac{wL^4}{64}\right]$$

$$= \frac{wL^4}{2}\left[\frac{1}{24}-\frac{1}{64}\right]$$

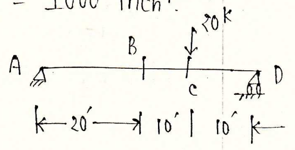
$$= \frac{5}{384}\cdot wL^4.$$

$$\Delta_c = \frac{5}{384}\,\frac{wL^4}{EI}.$$

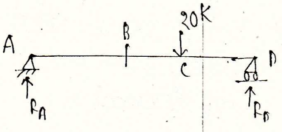
□ <u>Find deflection at point B.</u>

$E = 30\times10^3$ ksi.

$I = 1000$ inch^4.



Solve:



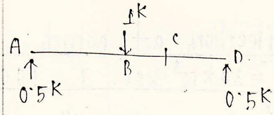
$\Sigma M_D = 0.$

$\Rightarrow\ R_A\times40 - 20\times10 = 0.$

$\therefore\ R_A = 5^k$

$R_D = 15^k.$

Unit load diagram:



portion:	AB	BC	DC
origin:	A	A	D
Limit:	0 to 20	20 to 30	0 to 10
M:	$5x$	$5x$	$15x$
m:	$0.5x$	$0.5x-1(x-20)$	$0.5x$

$$EI\Delta_B = \int_0^{20} 2.5x^2\,dx + \int_{20}^{30} 5x\,(0.5x - x + 20)\,dx + \int_0^{10} 7.5x^2\,dx$$

$$= \frac{2.5}{3} \times 20^3 + \int_{20}^{30} 5x(-0.5x+20) + 7.5\left[\frac{x^3}{3}\right]_0^{10}$$

$$= 6666.67 + \int_{20}^{30} (-2.5x^2 + 100x)\,dx + 2500$$

$$= 9166.67 + \left[-2.5\frac{x^3}{3} + 100\frac{x^2}{2}\right]_{20}^{30}$$

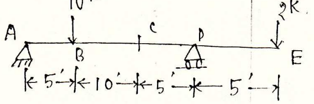
$$= 9166.67 + [-22500 + 45000 - (-6666.67 + 20000)]$$

$$= 18333.34.$$

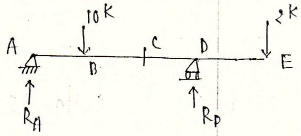
$$\Delta_B = \frac{18333.34}{30 \times 10^3 \times 1000} \times 1728$$

$$\Delta_B = 1.056 \text{ inch.}$$

□ Find deflection at point C and E of the follow structure. $E = 30 \times 10^3$ ksi, $I = 1000$ inch^4



Solve:



$\Sigma M_D = 0.$

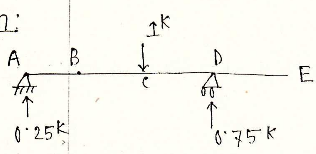
$\Rightarrow R_A \times 20 - 10 \times 15 + 2 \times 5 = 0.$

$\therefore R_A = 7^K$

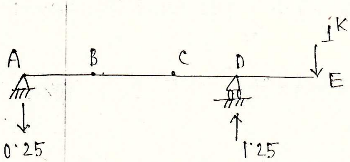
$R_D = 5^K$

Unit load diagram:

For Δ_C:



For Δ_E:



portion	AB	BC	CD	DE
origin	A	A	E	E
Limit	0-5	5-15	5-10	0-5
M	$7x$	$7x-10(x-5)$	$-2x+5(x-5)$	$-2x$
m_c	$0.25x$	$0.25x$	$0.75(x-5)$	0
m_E	$-0.25x$	$-0.25x$	$-x+1.25(x-5)$	$-x$

$$EI\Delta_C = \int_0^5 1.75x^2 + \int_5^{15} (7x - 10x + 50) \times 0.25x + \int_5^{10} (-2x + 5x - 25) \times (0.75x - 3.75)$$

$$= 1.75\frac{5^3}{3} + \int_5^{15} (-3x + 50) \times 0.25x + \int_5^{10} (3x - 25) \times (0.75x - 3.75)$$

$$= 72.92 + \int_5^{15} [-0.75x^2 + 12.5x] + \int_5^{10} (2.25x^2 - 11.25x - 18.75x + 93.75)$$

$$= 72.92 + \left[-0.75\frac{x^3}{3} + 12.5\frac{x^2}{2}\right]_5^{15} + \left[2.25\frac{x^3}{3} - 30\frac{x^2}{2} + 93.75x\right]_5^{10}$$

$$= 72.92 + 437.5 + 0.$$

$$= 510.42.$$

$$\Delta_C = \frac{510.42}{30 \times 10^3 \times 1000} \times 1728 = 0.0294 \text{ inch (Downward)}$$

$$EI\Delta_E = \int_0^5 -1.75x^2 + \int_5^{15} 0.75x^2 - 12.5x + \int_5^{10} (3x - 25) \times (-x + 1.25x - 6.25)$$

$$+ \int_0^5 2x^2$$

$$= -72.92 - 437.5 + \int_5^{10} (3x - 25)(0.25x - 6.25) + 2\frac{5^3}{3}.$$

$$= -510.42 + \int_5^{10} (0.75x^2 - 18.75x - 6.25x + 156.25) + 83.33.$$

$$= -427.09 + \left[0.75\frac{x^3}{3} - 25\frac{x^2}{2} + 156.25x\right]_5^{10}$$

$$= -427.09 + 62.5.$$

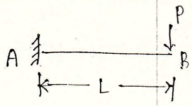
$$= -364.59.$$

$$\Delta_E = -\frac{364.59}{30 \times 10^3 \times 1000} \times 1728 = 0.021 \text{ inch (Downward)}.$$

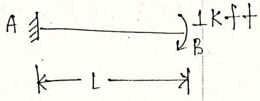
PART-B

Formula: $EI\theta = \int Mm\,dx$

□ Find slope θ_B for the for the following beam



Solve



portion: AB.

origin : B.

Limit: 0 to L.

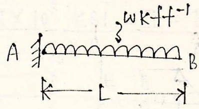
M : $-px$.

m : -1.

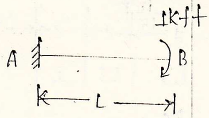
$$EI\theta_B = \int_0^L px\,dx = \int_0^L \frac{px^2}{2} = \frac{PL^2}{2}$$

$$\theta_B = \frac{PL^2}{2EI}$$

□ Find slope θ_B for the following beam.



Solve:



portion: AB

origin: B

Limit: 0 to L

M : $-\frac{wx^2}{2}$

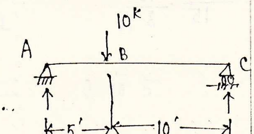
m : -1

$$EI\theta_B = \int_0^L \frac{wx^2}{2}\,dx = \frac{w}{2}\left[\frac{x^3}{3}\right]_0^L = \frac{wL^3}{6}$$

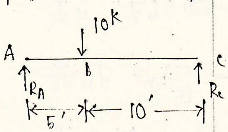
$$\theta_B = \frac{wL^3}{6EI}$$

□ Find slope θ_A, θ_C for the following beam.

$E = 30\times10^3$ ksi, $I = 1000$ inch⁴



Solve:

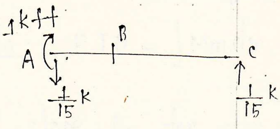


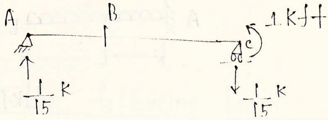
$\Sigma M_C = 0$.

$\Rightarrow R_A \times 15 - 10\times10 = 0$.

$\therefore R_A = 6.67^k$ $\quad R_C = 3.33^k$

Applying unit rotation: at A and C





portion:	AB	BC
origin:	A	C
Limit:	0 to 5	0 to 10
M :	$6.67x$	$3.33x$
m_{θ_A} :	$1-\frac{1}{15}x$	$\frac{1}{15}x$
m_{θ_B} :	$\frac{1}{15}x$	$1-\frac{1}{15}x$.

$$EI\theta_A = \int_0^5 (6.67x - 0.44x^2)dx + \int_0^{10} \frac{3.33}{15}x^2\,dx$$

$$= \left[6.67\frac{x^2}{2} - 0.44\frac{x^3}{3}\right]_0^5 + \frac{3.33}{15}\cdot\frac{10^3}{3}$$

$$= 65.042 + 74.$$

$$\theta_A = \frac{139.042}{30\times10^3\times1000}\times 144 = 0.00067$$

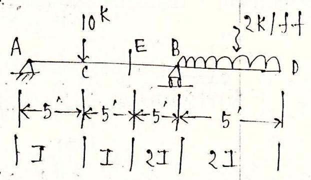
$$EI\theta_C = \int_0^5 \frac{6.67}{15}\cdot x^2 dx + \int_0^{10} 3.33x - \frac{3.33}{15}x^2\,dx$$

$$= \frac{6.67}{15}\cdot\frac{5^3}{3} + \left[3.33\frac{x^2}{2} - \frac{3.33}{15}\cdot\frac{x^3}{3}\right]_0^{10}$$

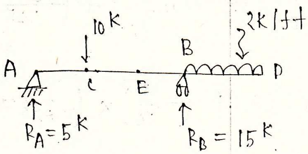
$$= 18.53 + 166.5 - 74.$$

$$\theta_C = \frac{111.03}{30\times10^3\times1000}\times 144 = 0.00053.$$

Find θ_B and θ_D for the following beam.
$E = 30 \times 10^3$ ksi, $I = 1000$ inch^4



$\boxed{\text{Solve:}}$



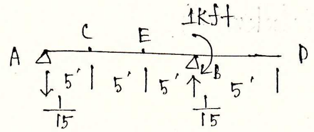
$\sum M_B = 0$

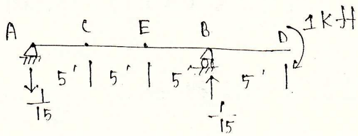
$\Rightarrow R_A \times 15 - 10 \times 10 + 2\frac{5^2}{2} = 0$

$R_A = 5^k$

$R_B = 15^k$

Applying unit rotation at B and D.





Portion:	AC	CE	BE	BD
origin:	A	A	A	D
Limit:	0 to 5	5 to 10	10 to 15	0 to 5
M :	$5x$	$5x-10(x-5)$	$5x-10(x-5)$	$-2\frac{x^2}{2}$
m_{θ_B} :	$-\frac{x}{15}$	$-\frac{x}{15}$	$-\frac{x}{15}$	0
m_{θ_D} :	$-\frac{x}{15}$	$-\frac{x}{15}$	$-\frac{x}{15}$	-1
I :	I	I	2I	2I

$$E\,\theta_B = \int_0^5 -\frac{x^2}{3I}\,dx + \int_5^{10} \frac{(5x-10x+50)}{I}\left(-\frac{x}{15}\right) + \int_{10}^{15}\left(\frac{-5x+50}{2I}\right)\cdot\left(-\frac{x}{15}\right.$$

$$= -\frac{1}{3I}\left[\frac{x^3}{3}\right]_0^5 + \left(-\frac{1}{15I}\right)\int_5^{10}\left(-5x^2+50x\right)dx + \left(-\frac{1}{30I}\right)\int_{10}^{15}\left(-5x^2+50x\right)dx$$

$$= -\frac{1}{3I}\cdot\frac{5^3}{3} + \left(-\frac{1}{15I}\right)\cdot\left[-5\frac{x^3}{3}+50\frac{x^2}{2}\right]_5^{10} + \left(-\frac{1}{30I}\right)\left[-5\frac{x^3}{3}+50\frac{x^2}{2}\right]_{10}^{15}$$

$$EI\theta_B = -13.89 + \left(-\frac{1}{15}\right)\times 416.67 + \left(-\frac{1}{30}\right)\times\left(-833.33\right)$$

$$\Rightarrow EI\theta_B = -13.89 - 27.78 + 27.78.$$

$$\Rightarrow \theta_B = -\frac{13.89}{30\times10^3\times1000}\times 144.$$

$$\therefore \theta_B = 6.67\times10^{-5}$$ radian (anticlockwise).

$$EI\theta_D = -13.89 + \frac{1}{2}\int_0^5 x^2\,dx$$

$$= -13.89 + \frac{1}{2}\left[\frac{x^3}{3}\right]_0^5$$

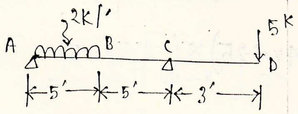
$$EI\theta_D = -13.89 + \frac{5^3}{6}.$$

$$= 6.94.$$

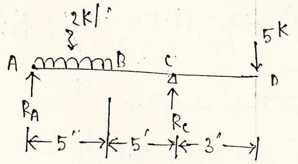
$$\theta_D = \frac{6.94}{30\times10^3\times1000}\times 144$$

$$= 3.33\times10^{-5}$$ radian (clockwise).

Find θ_{CD}, Δ_B, Δ_D for the following beam.
$E = 30 \times 10^3$ ksi, $I = 1000$ inch^4



Solve:



$\sum M_C = 0$

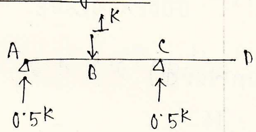
$\Rightarrow R_A \times 10 - 10 \times (5 + \frac{5}{2}) + 3 \times 5 = 0$

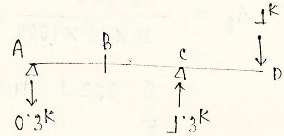
$R_A = 6^k$

$R_C = 9^k$

Unit load diagram:

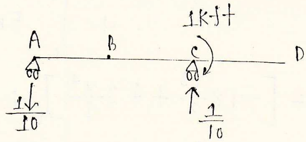
For Δ_B





For Δ_D

Applying unit rotation at C:



portion :	AB	BC	CD
origin :	A	A	D
Limit :	0-5	5-10	0 to 3
M :	$6x - 2\cdot\frac{x^2}{2}$	$6x - 10\left(x - \frac{5}{2}\right)$	$-5x$
m_{Δ_B} :	$0.5x$	$0.5x - 1(x-5)$	0
m_{Δ_D} :	$-0.3x$	$-0.3x$	$-x$
$m_{\theta_{CD}}$:	$-\frac{x}{10}$	$-\frac{x}{10}$	0

$$EI\Delta_B = \int_0^5 (3x^2 - 0.5x^3) + \int_5^{10} (6x - 10x + 25) \times (0.5x - x + 5)$$

$$= \left[3\frac{x^3}{3} - 0.5\frac{x^4}{4}\right]_0^5 + \int_5^{10} (-4x + 25) \times (-0.5x + 5)$$

$$= 125 - 78.125 + \int_5^{10} (2x^2 - 20x - 12.5x + 125)\,dx$$

$$= 46.875 + \left[\frac{2x^3}{3} - 32.5\frac{x^2}{2} + 125x\right]_5^{10}$$

$$= 46.875 + (-10.42)$$

$$= 36.455$$

$$\Delta_B = \frac{36.455}{30 \times 10^3 \times 1000} \times 1728$$

$$= 0.0021 \text{ inch (Downward)}$$

$$EI\Delta_D = \int_0^5 (-1.8x^2 + 0.3x^3)\,dx + \int_5^{10} (-1.8x^2 + 3x^2 - 7.5x)\,dx$$

$$+ \int_0^3 5x^2\,dx$$

$$= \left[-1.8\frac{x^3}{3} + 0.3\frac{x^4}{4}\right]_0^5 + \left[1.2\frac{x^3}{3} - 7.5\frac{x^2}{2}\right]_5^{10} + 5\left[\frac{x^3}{3}\right]_0^3$$

$$= -28.125 + 68.75 + 45$$

$$\Delta_D = \frac{85.625}{30 \times 10^3 \times 1000} \times 1728$$

$$= 0.0049 \text{ inch (Downward)}$$

$$EI\theta_{CD} = \int_0^5 (6x - x^2)\left(\frac{-x}{10}\right) dx + \int_5^{10} (6x - 10x - 1.25) \times \left(-\frac{x}{10}\right).$$

$$= -\frac{1}{10}\int_0^5 \left[6x^2 - x^3\right] dx - \frac{1}{10}\cdot\int_5^{10}\left(-4x^2 + 25x\right)$$

$$= -\frac{1}{10}\cdot\left[\frac{6x^3}{3} - \frac{x^4}{4}\right]_0^5 - \frac{1}{10}\cdot\left[-4\frac{x^3}{3} + 25\frac{x^2}{2}\right]_5^{10}$$

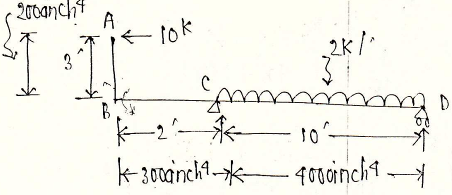
$$= -9.375 + 22.92.$$

$$= 13.545.$$

$$\theta_{CD} = \frac{13.545}{30\times 10^3 \times 1000} \times 144.$$

$$= 6.5 \times 10^{-5} \text{ radian (clockwise)}.$$

□ Find deflection at point A and rotation at point B for the following beam.



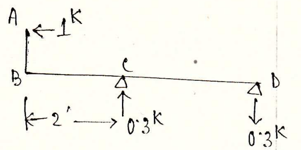
Solve. $\Sigma M_D = 0.$

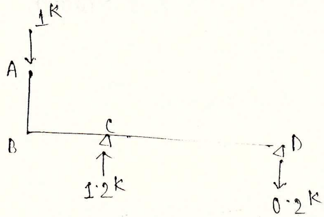
$\Rightarrow R_C \times 10 - 20 \times 5 - 10 \times 3 = 0.$

$\therefore R_C = 13^k$

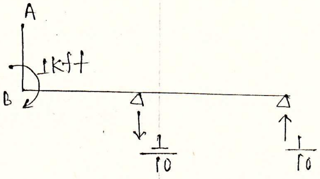
$R_D = 7^k$

unit load diagram:





Applying unit rotation:



portion	AB	BC	CD
origin	A	B	D
Limit	0-3	0-2	0-10
M	$-10x$	-30	$7x-2\cdot\frac{x^2}{2}$
$m_{\Delta_{AH}}$	$-x$	-3	$-\frac{3}{10}x$
$m_{\Delta_{AV}}$	0	$-1.x$	$-0.2x$
m_{θ_B}	0	1	$\frac{1}{10}x$
I	2I	3I	4I (I=1000)

$$E\Delta_{AH} = \int_0^3 \frac{10x^2}{2I}dx + \int_0^2 \frac{90}{3I}\cdot dx + \int_0^{10} \frac{-2.1x^2+0.3x^3}{4I}\cdot dx$$

$$\Rightarrow EI\Delta_{AH} = \frac{10}{2}\cdot\frac{3^3}{3} + \frac{1}{3}\cdot 90\times 2 + \frac{1}{4}\left[0.3\frac{x^4}{4} - 2.1\frac{x^3}{3}\right]_0^{10}$$

$$\Rightarrow EI\Delta_{AH} = 45 + 60 + 12.5$$

$$\Delta_{AH} = \frac{117.5}{EI} \text{ ——— (i)}$$

$$E\Delta_{AV} = \int_0^2 \frac{30x}{3I}dx + \int_0^{10} \frac{-1.4x^2+0.2x^3}{4I}dx$$

$$\Rightarrow EI\Delta_{AV} = \frac{30}{3}\cdot\frac{2^2}{2} + \frac{1}{4}\cdot\left[0.2\frac{x^4}{4} - 1.4\frac{x^3}{3}\right]_0^{10}$$

$$\Rightarrow \Delta_{AV} = \frac{53.33}{EI} \text{ ——— (ii)}$$

Deflection at $A = \sqrt{\Delta_{AH}^2 + \Delta_{AV}^2}$

$$= \sqrt{\left(\frac{117.5}{EI}\right)^2 + \left(\frac{53.33}{EI}\right)^2}$$

$$= \frac{1}{EI}\sqrt{117.5^2 + 53.33^2}$$

$$= \frac{129.036}{30\times10^3\times1000}\times1728.$$

$$= 0.0074 \text{ inch.}$$

$$E\theta_B = \int_0^2 -\frac{30}{3I}\,dx + \int_0^{10} (7x - x^2)\times\frac{x}{10}\times\frac{1}{4I}\,dx.$$

$$\Rightarrow EI\theta_B = \left[-10x\right]_0^2 + \frac{1}{40}\int_0^{10}\left[7x^2 - x^3\right]$$

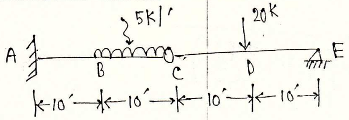
$$\Rightarrow EI\theta_B = -20 + \frac{1}{40}\left[\frac{7x^3}{3} - \frac{x^4}{4}\right]_0^{10}$$

$$\Rightarrow EI\theta_B = -20 - 4.17.$$

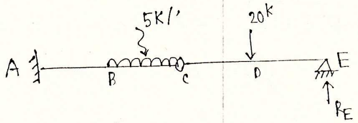
$$\theta_B = \frac{-24.17}{30\times10^3\times1000}\times144.$$

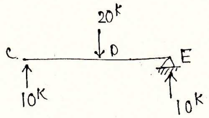
$$= 1.17\times10^{-4} \text{ radian (anticlockwise).}$$

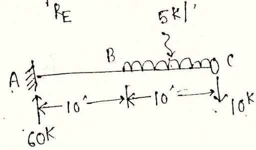
□ Find θ_E and θ_{CD} for the following beam



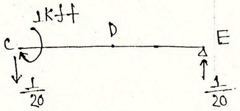
Ans

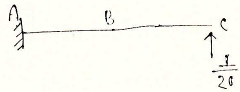




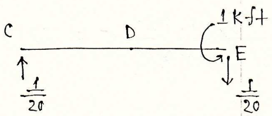


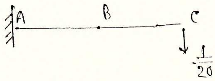
Applying unit rotation at C:





Applying unit rotation at E:





Portion	AB	BC	CD	DE
Origin	C	C	C	E
Limit	10-20	0-10	0-10	0-10
M	$-10x-50(x-5)$	$-10x-5.\frac{x^2}{2}$	$10x$	$10x$
$m_{\theta_{CD}}$	$\frac{1}{20}x$	$\frac{1}{20}x$	$-\frac{1}{20}x+1$	$\frac{1}{20}x$
m_{θ_E}	$-\frac{1}{20}x$	$-\frac{1}{20}x$	$\frac{1}{20}x$	$-\frac{1}{20}x+1$

$$EI\theta_{CD} = \int_{10}^{20}(-10x-50x+250)\times\frac{1}{20}x\,dx + \int_{0}^{10}(-10x-2.5x^2)\times\frac{1}{20}x\,dx$$

$$+\int_0^{10} 10x\times\left(1-\frac{x}{20}\right)dx + \int_0^{10} 10x\cdot\frac{x}{20}\,dx$$

$$= \frac{1}{20}\int_{10}^{20}(-60x^2+250x)\,dx + \frac{1}{20}\int_0^{10}(-10x^2-2.5x^3)\,dx + 10\int_0^{10}\left(x-\frac{x^2}{20}\right)dx$$

$$+\frac{1}{2}\left[\frac{x^3}{3}\right]_0^{10}$$

$$= \frac{1}{20}\left[-60.\frac{x^3}{3}+250.\frac{x^2}{2}\right]_{10}^{20} + \frac{1}{20}\left[-10.\frac{x^3}{3}-2.5.\frac{x^4}{4}\right]_0^{10}$$

$$+10\left[\frac{x^2}{2}-\frac{x^3}{60}\right]_0^{10} + \frac{1}{6}\times10^3$$

$$= -5125 - 479.17 + 333.33 + 166.67$$

$$= -5104.17$$

$$\theta_{CD} = -\frac{5104.17}{30\times10^3\times1000}\times144 = 0.0245 \text{ radian (anticlockwise)}$$

$$EI\theta_E = \int_{10}^{20} (-10x - 50x + 250)\left(-\frac{1}{20}x\right)dx + \int_0^{10} (10x + 2.5x^2) \times \frac{1}{20}x \, dx$$

$$+ \int_0^{10} \frac{x^2}{2} dx + \int_0^{10} \left(1 - \frac{x}{20}\right).10x \, dx$$

$$= -\frac{1}{20}\int_{10}^{20} (-60x^2 + 250x)\, dx + \frac{1}{20}\int_0^{10} (10x^2 + 2.5x^3)\, dx + \frac{1}{2}\left[\frac{x^3}{3}\right]_0^{10}$$

$$+ 10 \int_0^{10} \left(x - \frac{x^2}{20}\right) dx$$

$$= -\frac{1}{20}.\left[-60.\frac{x^3}{3} + 250.\frac{x^2}{2}\right]_{10}^{20} + \frac{1}{20}.\left[10.\frac{x^3}{3} + 2.5\frac{x^4}{4}\right]_0^{10}$$

$$+ 166.67 + 10\left[\frac{x^2}{2} - \frac{x^3}{60}\right]_0^{10}$$

$$= 5125 + 479.167 + 166.67 + 333.33$$

$$= 6104.17$$

$$\theta_E = \frac{6104.17}{30 \times 10^3 \times 1000} \times 144$$

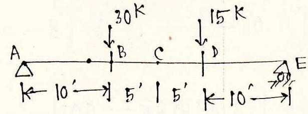
$$= 0.0293 \text{ radian}$$

AHMED HOSSAIN
090001
25 th Feb. 2013.

AHMED HOSSAIN
090001.

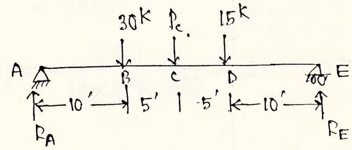
PART-2.

□ Find deflection at c and rotation at A for the following structure. $E = 30\times10^3$ ksi. $I = 1000$ inch^4



Apply castigliano's theorem.

Ans: Applying load point c.



$\Sigma M_E = 0.$

$\Rightarrow R_A \times 30 - 30\times20 - P\times15 - 15\times10 = 0.$

$R_A = 25 + 0.5 P_c.$

$\Sigma F_y = 0.$

$\Rightarrow 25 + 0.5P_c + R_E = P_c + 45.$

$R_E = P_c + 45 - 0.5P_c - 25.$

$R_E = 0.5P_c + 20.$

Portion	AB	BC	ED	DC
origin	A	A	E	E
Limit	0–10	10–15	0–10	10–15
M	$R_A x$	$R_A x - 30(x-10)$	$R_E x$	$R_E x - 15(x-10)$
$\frac{dM}{dP_c}$	$0.5x$	$0.5x$	$0.5x$	$0.5x.$

Where, $R_A = 25 + 0.5 P_c.$

$R_E = 20 + 0.5 P_c.$

$$EI\Delta_c = \int_0^{10} (25+0.5P_c)\times 0.5x^2\,dx + \int_{10}^{15} \{(25+0.5P_c)x - (30x-300)\} \times 0.5x$$

$$+\int_0^{10} (20+0.5P_c)\times 0.5x^2 + \int_{10}^{15} \{(20+0.5P_c)x - 15(x-10)\} \times 0.5x$$

Applying the condition that $P_c = 0.$

$$EI\Delta_c = 12.5\left[\frac{x^3}{3}\right]_0^{10} + \int_{10}^{15} (25x - 30x + 300)\times 0.5x + 10\left[\frac{x^3}{3}\right]_0^{10}$$

$$+\int_{10}^{15} (20x - 15x + 150)\times 0.5x$$

$$= 4166.67 + \left[12.5\frac{x^3}{3} - 15.\frac{x^3}{3} + 150.\frac{x^2}{2}\right]_{10}^{15} + 3333.33$$

$$+\left[12.5\frac{x^3}{3} + 75\frac{x^2}{2}\right]_{10}^{15}$$

$= 4166.67 + 7395.83 + 3333.33 + 6666.67$

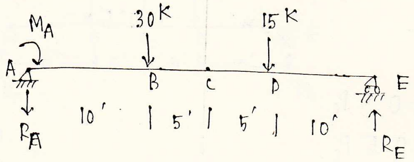
$= \cancel{17604.16}\ 21562.5$

$$\Delta_c = \frac{\cancel{17604.16}\ 21562.5}{30\times10^3 \times 1000} \times 12^3$$

$\Delta_c = 1.24''$ [Downward]

<u>Rotation at point A:</u>

Applying moment at point A:



$\Sigma M_E = 0.$

$\Rightarrow -R_A \times 30 + M_A - 30\times 20 - 15\times 10 = 0$

$$R_A = -25 + \frac{M_A}{30}$$

$\Sigma F_y = 0.$

$$\Rightarrow 45 - 25 + \frac{M_A}{30} = R_E$$

$$R_E = 20 + \frac{M_A}{30}$$

Portion	AB	BD	DE
origin	A	A	E
Limit	0–10	10–20	0–10
M.	$M_A - \left(\frac{M_A}{30} - 25\right)x$	$M_A - \left(\frac{M_A}{30} - 25\right)x - 30\times(x-10)$	$\left(\frac{M_A}{30} + 20\right)x$
$\frac{dM}{dM_A}$	$1 - \frac{x}{30}$	$1 - \frac{x}{30}$	$\frac{x}{30}$

$$EI\theta_A = \int_0^{10} \left\{M_A - \left(\frac{M_A}{30} - 25\right)x\right\} \times \left(1 - \frac{x}{30}\right) dx + \int_{10}^{20} \left\{M_A - \left(\frac{M_A}{30} - 25\right)x - 30(x-10)\right\} \times \left(1 - \frac{x}{10}\right) dx$$

$$+ \int_0^{10} \left(\frac{M_A}{30} + 20\right)x \times \frac{x}{30}$$

Putting, $M_A = 0$.

$$EI\theta_A = \int_0^{10} 25x\left(1 - \frac{x}{30}\right) dx + \int_{10}^{20} (25x - 30x + 300) \times \left(1 - \frac{x}{1010}\right) dx + \int_0^{10} 0.67\,x^2 dx$$

$$= \int_0^{10} (25x - 0.83x^2)\,dx + \int_{10}^{20} (-5x + 300)\left(1 - \frac{x}{30}\right) dx + .67\left[\frac{x^3}{3}\right]_0^{10}$$

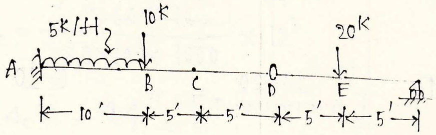
$$= \left[25\frac{x^2}{2} - 0.83\frac{x^3}{3}\right]_0^{10} + \int_{10}^{20}\left(300 - \cancel{30}x - 5x + \frac{5x^2}{30}\right) dx + 223.33$$

$$= 973.33 + \left[300x - \cancel{15}\frac{x^2}{2} + \frac{5x^3}{30\,3}\right]_{10}^{20} + 223.33.$$

$$= 973.33 + \overset{1138.89}{\cancel{1083.33}} + 223.33 = 113.33.$$

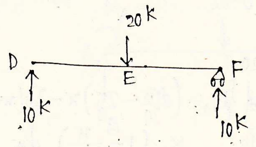
$$\theta_A = \frac{113.33}{30\times10^3\times1000} \times 12^2 = 0.011 \text{ Radian [Clockwise]}$$

☐ Find deflection at point C and rotation at point D for the following structure.
$E = 30 \times 10^3$ ksi. $I = 1000$ inch^4

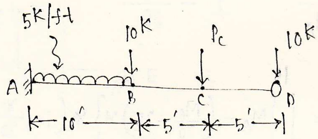


Use castigliano's method.

Ans Separating DF and AD part and applying load P_c at point C.



(A)



(B)

Portion	DC	CB	BA
origin	D	D	D
Limit	0-5	5-10	10-20
M	$-10x$	$-10x - P_c(x-5)$	$-10x - P_c(x-5)$ $-10(x-10) - 5\cdot\frac{(x-10)^2}{2}$
$\frac{dM}{dP}$	0	$-x+5$	$-x+5$

$$EI\Delta_c = \int_5^{10} \{-10x - P_c(x-5)\} \times (5-x)\,dx + \int_{10}^{20} \{-10x - P_c(x-5) - 10(x-10) - 5\frac{(x-10)^2}{2}\} \times (5-x)\,dx$$

Applying the condition that $P_c = 0$.

$$EI\Delta_c = \int_5^{10} (-10x) \times (5-x)\,dx + \int_{10}^{20} \{-10x - 10(x-10) - 2.5(x-10)^2\}(5-x)$$

$$= \int_5^{10} (-10x \times 5 + 10x^2)\,dx + \int_{10}^{20} \{-10x - 10x + 100 - 2.5(x^2 - 20x + 100)\}(5-x)\,dx$$

$$= \left[10\frac{x^3}{3} - 50\frac{x^2}{2}\right]_5^{10} + \int_{10}^{20} (-20x + 100 - 2.5x^2 + 50x - 250) \times (5-x)\,dx$$

$$= 1041.67 + \int_{10}^{20} (30x - 2.5x^2 - 150)(5-x)\,dx$$

$$= 1041.67 + \int_{10}^{20} (150x - 12.5x^2 - 750 - 30x^2 + 2.5x^3 + 150x)\,dx$$

$$= 1041.67 + \int_{10}^{20} (300x - 42.5x^2 + 2.5x^3 - 750)\,dx$$

$$= 1041.67 + \left[300\frac{x^2}{2} - 42.5\frac{x^3}{3} + 2.5\frac{x^4}{4} - 750.x\right]_{10}^{20}$$

$$= 1041.67 + 32083.33$$

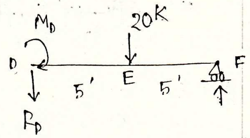
$$= 33125.00$$

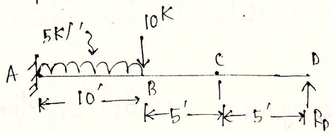
$$\Delta_c = \frac{33125.00}{30 \times 10^3 \times 1000} \times 12^3$$

$$= \underline{1.91} \text{ inch (Downward)}.$$

Rotation at point D:

Applying moment M_D at point D:





$\Sigma M_F = 0.$

$\Rightarrow -R_D \times 10 + M_D - 20 \times 5 = 0.$

$R_D = \dfrac{M_D - 100}{10}$

$R_D = \dfrac{M_D}{10} - 10.$

$\Sigma F_y = 0$

$\Rightarrow \dfrac{M_D}{10} - 10 + 20 = R_F.$

$R_F = \dfrac{M_D}{10} + 10$

portion	DB	BA	FE	ED
origin	D	D	F	F
Limit	0–10	10–20	0–5	5–10
M.	$\left(\frac{M_D}{10}-10\right)x$	$\left(\frac{M_D}{10}-10\right)x - 10(x-10) - 5\frac{(x-10)^2}{2}$	$R_F \times x$	$R_F x - 20(x-5)$
$\frac{dM}{dM_D}$	$\frac{x}{10}$	$\frac{x}{10}$	$\frac{x}{10}$	$\frac{x}{10}$

$$EI\theta_D = \int_0^{10}\left(\frac{M_D}{10}-10\right)\frac{x^2}{10}\,dx + \int_{10}^{20}\left\{\left(\frac{M_D}{10}-10\right)x - 10(x-10) - \frac{5}{2}(x-10)^2\right\}\times\frac{x}{10}$$

$$+\int_0^5\left(\frac{M_D}{10}+10\right)\frac{x^2}{10}\,dx + \int_5^{10}\left\{\left(\frac{M_D}{10}+10\right)x - 20(x-5)\right\}\times\frac{x}{10}\,dx$$

putting $M_D = 0$.

$$EI\theta_D = \int_0^{10} -x^2\,dx + \int_{10}^{20} \left\{-10x - 10x + 100 - 2.5\left(x^2 - 20x + 100\right)\right\} \times \frac{x}{10}$$

$$+ \int_0^5 x^2\,dx + \int_5^{10} \left(10x - 20x + 100\right) \times \frac{x}{10}\,dx$$

$$= -\frac{10^3}{3} + \int_{10}^{20} \left(-20x + 100 - 2.5x^2 + 50x - 250\right) \times \frac{x}{10}\,dx$$

$$+ \frac{5^3}{3} + \int_5^{10} \left(-10x + 100\right) \frac{x}{10}\,dx$$

$$= -333.33 + 41.67 + \int_{10}^{20} \left(3x^2 - 0.25x^3 - 15x\right) dx + \int_5^{10} \left(-x^2 + 10x\right) dx$$

$$= -291.67 + \int_{10}^{20} \left[3 \cdot \frac{x^3}{3} - 0.25\frac{x^4}{4} - 15\frac{x^2}{2}\right] dx + \left[10 \cdot \frac{x^2}{2} - \frac{x^3}{3}\right]_5^{10}$$

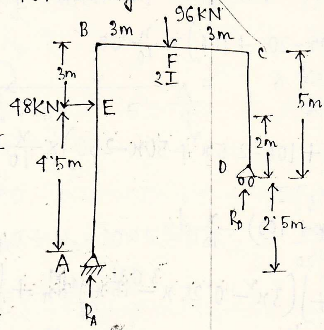
$$= -291.67 - 4625 + 83.33.$$

$$= -4833.34.$$

$$\theta_D = -\frac{4833.34}{30 \times 10^3 \times 1000} \times 12^2 = -0.023 \text{ Radian}.$$

$\boxed{\text{Ans}}$ $\theta_D = 0.023$ radian (Anticlockwise)

□ Using unit load method, find $\theta_A, \theta_B, \theta_C, \theta_D$ for the following structure: $E = 200\times10^6\ KNm^{-2}$, $I = 160\times10^{-6}\ m^4$

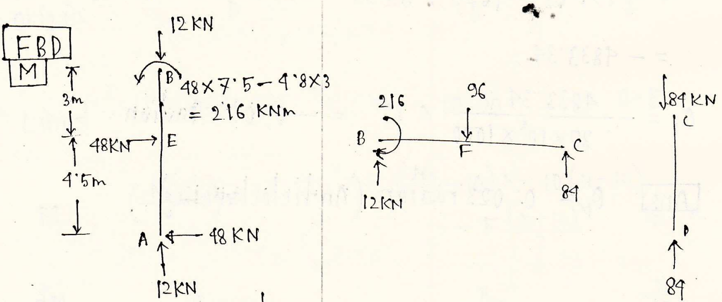


$\Sigma M_A = 0.$

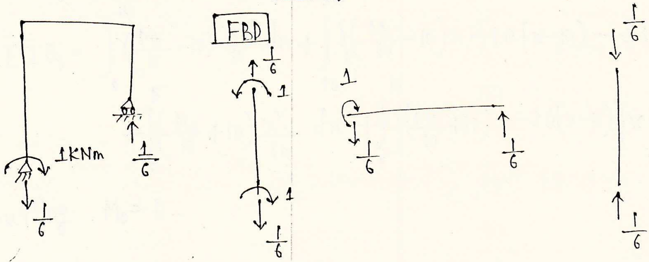
$\Rightarrow -R_D\times 6 + 96\times3 + 48\times4.5 = 0.$

$R_D = 84\ KN$

$R_A = 12\ KN.$

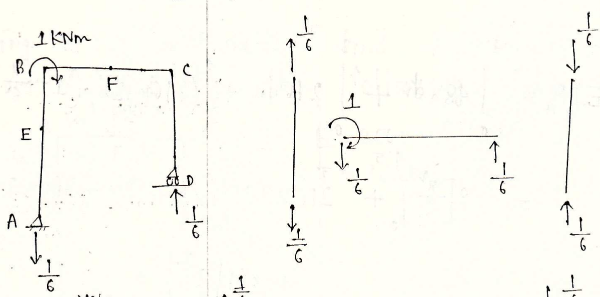


Applying unit ~~load~~ moment: $\boxed{M_{\theta_A}}$

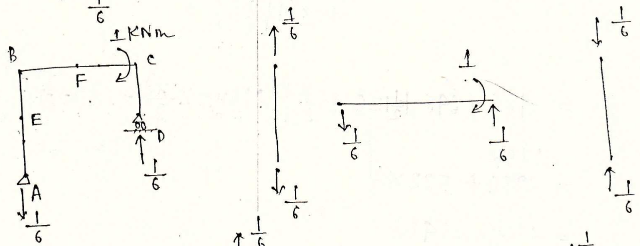


FBD

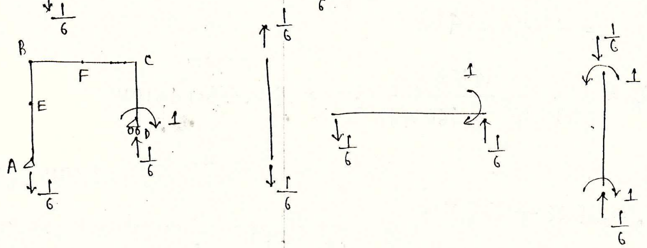
m_{θ_B}



m_{θ_C}



m_{θ_D}



Portion	AE	BE	BF	CF	CD
Origin	A	B	B	C	C
Limit	0-4.5	0-3	0-3	0-3	0-5
M	$48x$	216	$216+12x$	$84x$	0
m_{θ_A}	1	1	$1-\frac{1}{6}x$	$\frac{1}{6}x$	0
m_{θ_B}	0	0	$1-\frac{x}{6}$	$\frac{x}{6}$	0
m_{θ_C}	0	0	$-\frac{x}{6}$	$\frac{x}{6}-1$	0
m_{θ_D}	0	0	$-\frac{x}{6}$	$\frac{x}{6}-1$	1
I	I	I	2I	2I	I

$$EI\theta_A = \int_0^{4.5} 48x\,dx + \int_0^{3} 216\,dx + \frac{1}{2}\int_0^{3}(216+12x)\left(1-\frac{x}{6}\right)dx + \frac{1}{2}\int_0^{3} 84x\cdot\frac{x}{6}\,dx$$

$$= 48\left[\frac{x^2}{2}\right]_0^{4.5} + 216\times 3 + \int_0^3 (216+12x-36x-2x^2)\,dx$$

$$+\frac{1}{2}\times 14\left[\frac{x^3}{3}\right]_0^3$$

$$= 486 + 648 + \frac{1}{2}126 + \frac{1}{2}\left[216x - 2\cdot\frac{x^3}{3} - 24\cdot\frac{x^2}{2}\right]_0^3$$

$$= \overset{1197}{\cancel{1260}} + 522\times\frac{1}{2}$$

$$\cancel{= 1782} = 1458$$

$$\theta_A = \frac{1458}{200\times 10^6 \times 160\times 10^{-6}} = \overset{0.045}{\cancel{0.056}}\ \text{radian}$$

$$EI\theta_B = \frac{1}{2}\times 522 + 126\times\frac{1}{2}$$

$$\theta_B = \frac{324}{200\times 160} = \overset{0.0101}{\cancel{0.0203}}\ \text{radian.}$$

$$EI\theta_C = \frac{1}{2}\int_0^3 (216+12x)\times\left(-\frac{x}{6}\right)dx + \frac{1}{2}\int_0^3 84x\times\left(\frac{x}{6}-1\right)$$

$$= \frac{-1}{2\times 6}\int_0^3 (216x+12x^2)\,dx + \frac{84}{2}\int_0^3\left(\frac{x^2}{6}-x\right)dx$$

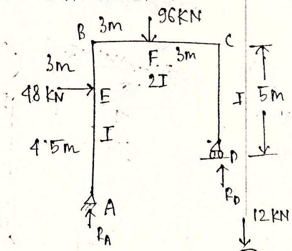
$$= -\frac{1}{12}\times\left[216\cdot\frac{x^2}{2}+12\cdot\frac{x^3}{3}\right]_0^3 + 42\cdot\left[\frac{x^3}{18}-\frac{x^2}{2}\right]_0^3.$$

$$= -90 + (-126) = 216.$$

$$\theta_C = \frac{216}{200\times 160} = 6.75\times 10^{-3}\ \text{radian.}$$

$$\theta_D = 6.75\times 10^{-3}\ \text{radian.}$$

□ Applying unit load method, find the value of deflection at, A, B, C, D for the following frame.

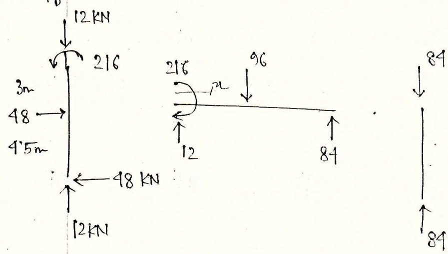


$E = 200 \times 10^6 \ KN m^{-2}$

$I = 160 \times 10^{-6} \ m^4$

Solve: $R_A = 12$ KN

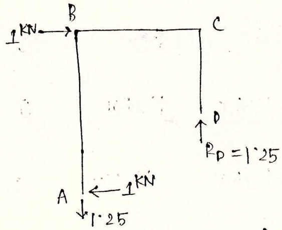
$R_D = 84$ KN

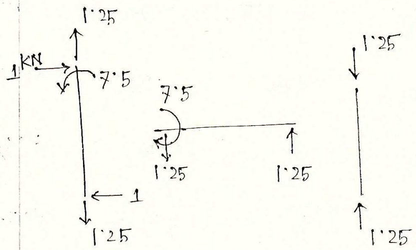


FBD

Applying unit load at:

at B

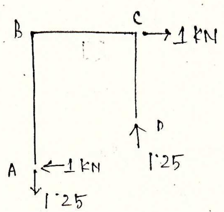


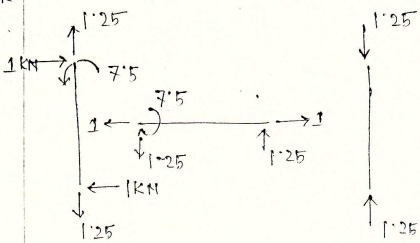


$\Sigma M_A = 0$; $1 \times 7·5 - R_D \times 6 = 0$. $R_D = 1·25^k$

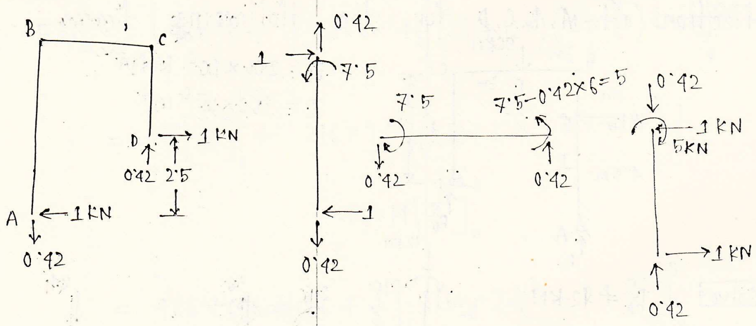
$R_A = 1·25^k$

at C





$\boxed{\text{at D}}$ $\Sigma M_A = 0;$ $1 \times 2.5 - P_D \times 6 = 0;$ $P_D = 0.42$ kN



portion	AE	BE	BF	CF	CD
origin	A	B	B	C	C
Limit	0-4.5	0-3	0-3	0-3	0-5
M	$48x$	216	$216+12x$	$84x$	0
m_{4B}	x	$7.5-x$	$7.5-1.25x$	$1.25x$	0
m_{4C}	x	$7.5-x$	$7.5-1.25x$	$1.25x$	0
m_{4D}	x	$7.5-x$	$7.5-0.42x$	$0.42x+5$	$x-5$.
I	I	I	2I	2I	I

$$EI\Delta_B = \int_0^{4.5} 48x^2\,dx + \int_0^{3} 216\times(7.5-x)\cdot dx + \frac{1}{2}\int_0^{3}(216+12x)(7.5-1.25x)$$

$$+\frac{1}{2}\int_0^{3} 84x\times 1.25x\,dx$$

$$= 48\left[\frac{x^3}{3}\right]_0^{4.5} + 216\left[7.5x-\frac{x^2}{2}\right]_0^{3} + \frac{1}{2}\int_0^{3}(1620-270x+90x-15x^2)\,dx$$

$$+\frac{1}{2}\,105\left[\frac{x^3}{3}\right]_0^{3}$$

$$= 1458 + 3888 + 472.5 + \frac{1}{2}\left[1620x - 180\frac{x^2}{2} - 15\frac{x^3}{3}\right]_0^{3}$$

$$= 5818.5 + 3915\times\frac{1}{2}$$

$$\Delta_B = \frac{\cancel{9733.5}\ 7776}{200\times 160} = \cancel{0.304}\ 0.243\ \text{radian}$$

$$\Delta_C = 0.243\ \text{radian}$$

$$EI\Delta_D = 1458 + 3888 + \frac{1}{2}\int_0^{3}(216+12x)(7.5-0.42x)\,dx + \frac{1}{2}\int_0^{3} 84x\times(0.42x+5)\,dx$$

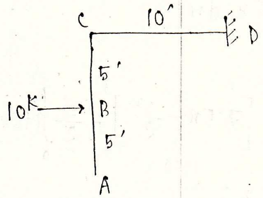
$$= 5346 + \frac{1}{2}\int_0^{3}(1620 - 90.72x + 90x - 5.04x^2)\,dx + \frac{84}{2}\left[0.42\frac{x^3}{3} + 5.\frac{x^2}{2}\right]_0^{3}$$

$$= 5346 + \frac{1}{2}\left[1620x - 0.72\frac{x^2}{2} - 5.04\frac{x^3}{3}\right]_0^{3} + 1103.76$$

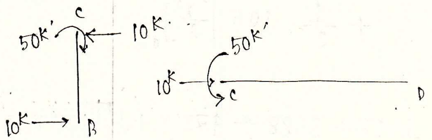
$$= 6449.76 + 4811.4\times\frac{1}{2}$$

$$\Delta_D = \frac{\cancel{11261.16}\ 8855.46}{200\times 160} = \cancel{0.352}\ 0.276\ \text{radian}$$

□ Find the rotation and deflection at point A for the following beam. Apply unit load method.
$E = 30 \times 10^3$ ksi, $I = 1000$ inch^4



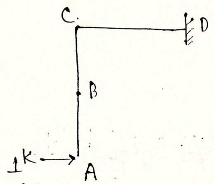
$\boxed{\text{Solve:}}$ $\boxed{\text{FBD}}$

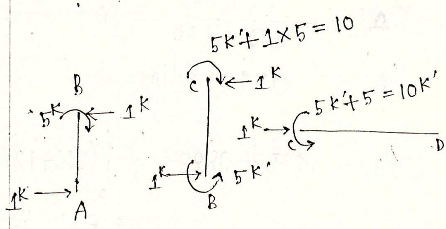


Applying unit load: $\boxed{\text{at A}}$

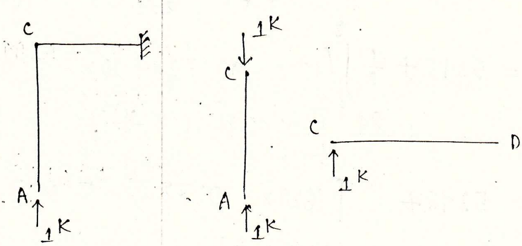
$\boxed{\text{Horizontal:}}$

$\boxed{\text{FBD}}$

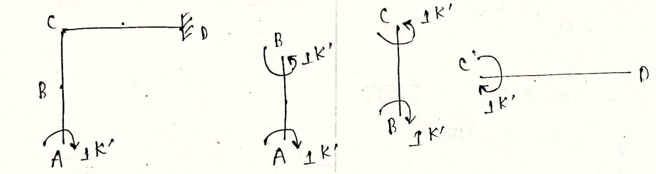




$\boxed{\text{Vertical}}$



Applying unit rotation at A.



Portion:	AB	BC	CD
origin:	A	B	C
Limit:	0–5	0–5	0–10
M	0	$-10x$	-50
$m_{\Delta AH}$	$-x$	$-x-5$	-10
$m_{\Delta AV}$	0	0	x
$m_{\theta A}$	1	1	1.

Deflection at A. $= \sqrt{(\Delta_A)_H^2 + (\Delta_A)_V^2}$ ——— (1).

$$EI\Delta_{AH} = \int_0^5 (10x^2 + 50x)\,dx + \int_0^{10} 500\,dx$$

$$= \left[10 \cdot \frac{x^3}{3} + 50 \cdot \frac{x^2}{2}\right]_0^5 + \left[500x\right]_0^{10}$$

$$= 1041.67 + 5000.$$

$$= 6041.67.$$ $\Delta_{AH} = 0.348$ inch.

$$EI\Delta_{AV} = \int_0^{10} -50x\,dx$$

$$= -50.\left[\frac{x^2}{2}\right]_0^{10}$$

$$= -50 \times 50 = -2500.$$

$$\Delta_{AH} = 0.144 \text{ inch}.$$

$$\Delta_A = \cancel{\frac{1}{EI}\sqrt{6041.67^2 + 2500^2}} = \sqrt{0.348^2 + 0.144^2}$$

$$\cancel{= 0.0002 \text{ inch}} = 0.3766 \text{ inch}.$$

$$EI\theta_A = \int_0^5 -10x\,dx + \int_0^{10} -50\,dx$$

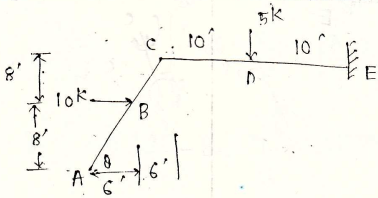
$$= -10.\left[\frac{x^2}{2}\right]_0^5 - 50.\left[x\right]_0^{10}$$

$$= -120 - 500.$$

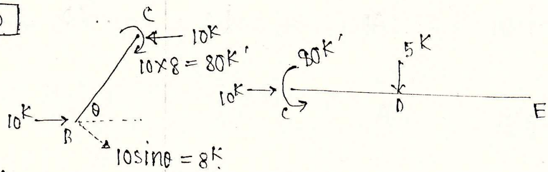
$$= -620$$

$$\theta_A = -\frac{620}{30\times10^3\times1000} \times 12^2 = 0.0029 \text{ radian (anticlockwise)}$$

□ Find the rotation and deflection at A:
Apply unit load method. $E = 30\times10^3$ ksi, $I = 1000$ inch^4



Solve: FBD



$\theta = \tan^{-1}\left(\frac{8}{6}\right)$

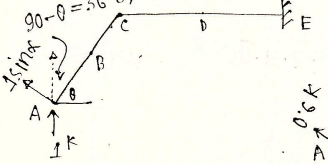
$= 53.13^\circ$

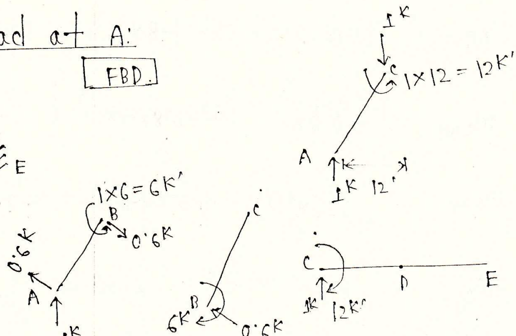
Applying unit load at A:

Vertical:

FBD

$90-\theta = 36.87^\circ = \alpha$

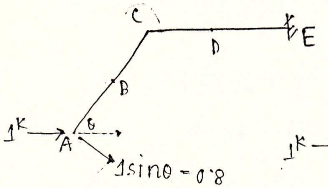


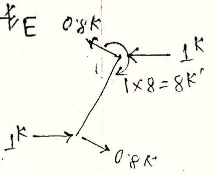


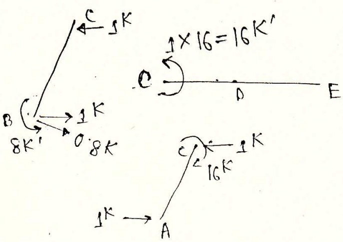
$1\sin\alpha = 0.6$

Horizontal

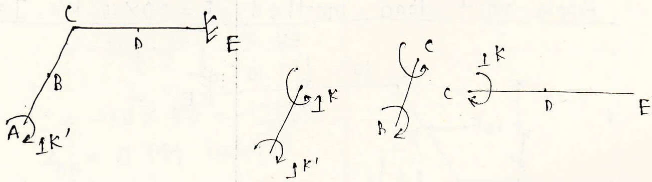
FBD







Applying unit rotation:



portion	AB	BC	CD	DE
origin	A	B	C	C
Limit	0–10	0–10	0–10	10–20
M	0	$-8x$	-80	$-80-5(x-10)$
$m_{\Delta AH}$	$-0.8x$	$-0.8x-8$	-16	-16
$m_{\Delta AV}$	$0.6x$	$0.6x+6$	$x+12$	$x+12$
m_{θ_A}	1	1	1	1

$$EI\Delta_{AH} = \int_0^{10} (-8x)\times(-0.8x-8)\,dx + \int_0^{10} 1280\,dx + \int_{10}^{20}(-80-5x+50)\times(-16)\,dx$$

$$= -8\left[-0.8\frac{x^3}{3} - 8\frac{x^2}{2}\right]_0^{10} + 1280\times 10 + (-16)\times\left[-5\frac{x^2}{2}-30x\right]_{10}^{20}$$

$$= (-8)\times(-666.67) + 12800 + (-16)\times(-1050)$$

$$= 5333.36 + 12800 + 16800.$$

$$\Delta_{AH} = \frac{34933.36}{30\times 10^3\times 1000}\times 12^3 = 2.01 \text{ inch}.$$

$$EI\Delta_{AV} = \int_0^{10}(-8x)\times(0.6x+6)\,dx + \int_0^{10}(-80)\times(x+12)\,dx + \int_{10}^{20}(-80-5x+50)\times(x+12)\,dx.$$

$$= -8\times\left[0.6\frac{x^3}{3} + 6\frac{x^2}{2}\right]_0^{10} - 80\times\left[12x+\frac{x^2}{2}\right]_0^{10} + \int_{10}^{20}(-5x-30)\times(x+12)\,dx$$

$$= -4000 - 13600 + \int_{10}^{20}(-5x^2-30x-60x-360)\,dx$$

$$= -17600 + \left[-5\frac{x^3}{3} - 90.\frac{x^2}{2} - 360x\right]_{10}^{20}$$

$$= -17600 + (-28766.67)$$

$$= 46366.67.$$

$$\Delta_{AH} = 2.67 \text{ inch}.$$

$$EI\theta_A = \int_0^{10} -8x\,dx + \int_0^{10} -80\,dx + \int_{10}^{20} (-80-5x+50)\,dx$$

$$= -8\left[\frac{x^2}{2}\right]_0^{10} + \left[-80.x\right]_0^{10} + \int_{10}^{20} (-5x-30)\,dx$$

$$= -400 - 80\times 10 + \left[-5.\frac{x^2}{2} - 30x\right]_{10}^{20}$$

$$= -1200 - 1050$$

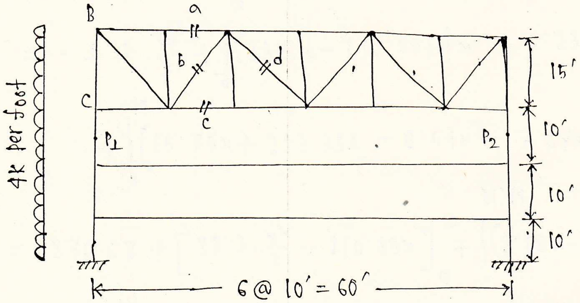
$$= 2250.$$

$$\theta_A = \frac{2250}{30\times 10^3 \times 1000} \times 12^2 = 0{\cdot}0108 \text{ radian.}$$

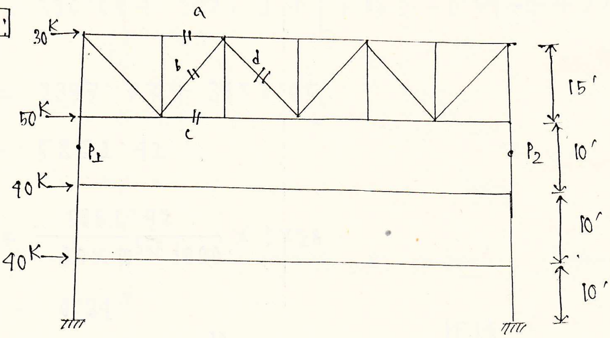
Portal Frame

AHMED HOSSAIN
090001.

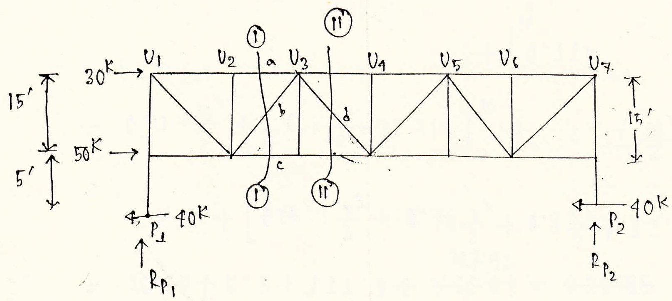
☐ Determine stress in member a, b, c and d. Draw SFD & BMD for left column and girder.



Solve:



considering free body above P_1P_2,



$\Sigma M_{P_1} = 0.$

$\Rightarrow 50 \times 5 + 30 \times 20 - R_{P_2} \times 60 = 0.$

$R_{P_2} = 14.17^K$

$R_{P_1} = 14.17^K$

considering ~~left~~ right portion of section ①-①.

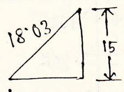
$\Sigma M_{U_3} = 0.$

$\Rightarrow C \times 15 + 40 \times 20 - 14.17 \times 40 = 0.$

$C = -15.55^K = 15.55^K\ (C).$

$\Sigma F_y = 0.$

$\Rightarrow R_{P_2} - b \times \frac{15}{18.03} = 0.$



$\therefore b = \frac{14.17 \times 18.03}{15} = 17.03^K\ (T).$

$\Sigma F_x = 0.$

$\Rightarrow a + c + b_x + 40 = 0.$

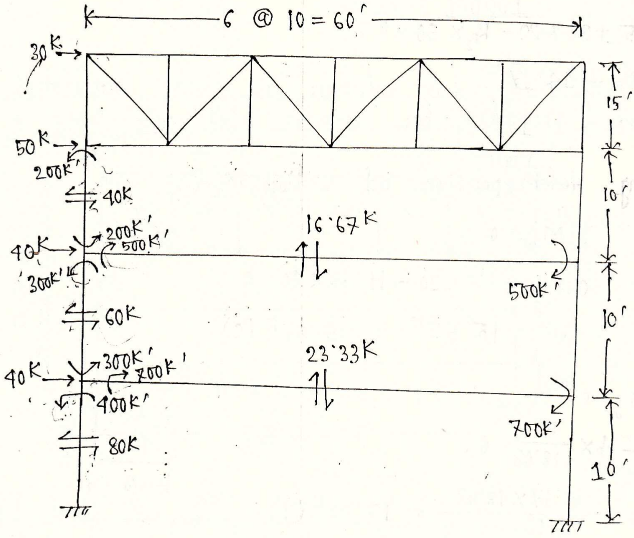
$\Rightarrow a - 15.55 + b \times \frac{10}{18.03} + 40 = 0.$

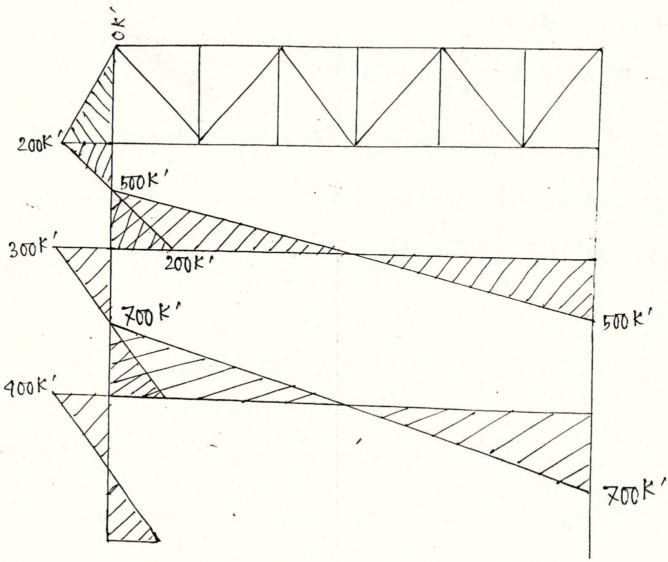
$\therefore a = -33.9^K = 33.9^K\ (C)$

considering right of section (II)-(II).

$\Sigma F_y = 0, \quad 14.17 = d \times \frac{15}{18.03} \quad \Rightarrow d = 17.03^K.$

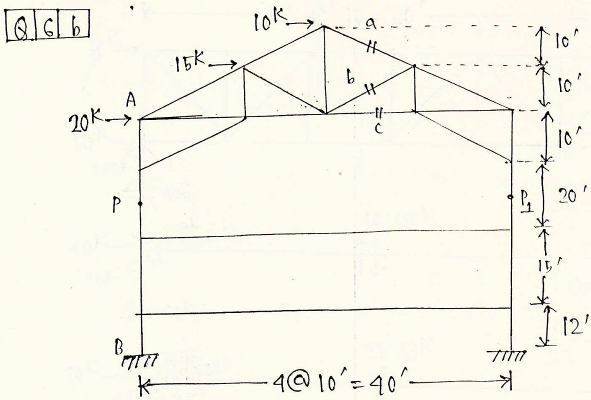
Bending moment by portal method:



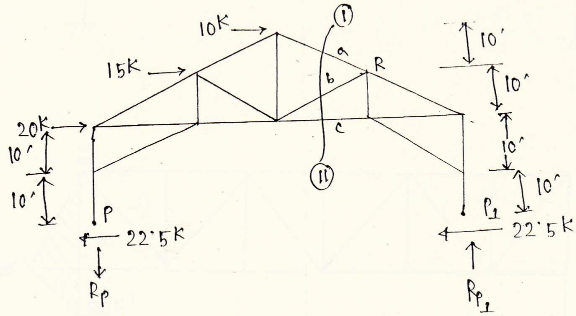


$\boxed{2010.}$

$\boxed{Q.6.b}$



$\boxed{\text{Solution:}}$ considering uper portion of point PP_1.



$\Sigma M_P = 0$.

$\Rightarrow 20 \times 20 + 15 \times 30 + 10 \times 40 = R_{P_1} \times 40$.

$\therefore R_{P_1} = 31.25^K (\uparrow)$

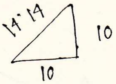
$R_P = 31.25^K (\downarrow)$

considering Right portion of section (I)–(II).

$\Sigma M_R = 0$.

$\Rightarrow C \times 10 + 22.5 \times 30 = 31.25 \times 10$.

$\therefore C = -36.25^K = 36.25^K (C)$



$\sum F_x = 0.$

$\Rightarrow c + 22.5 + b_x + a_x = 0.$

$\Rightarrow -36.25 + 22.5 + b \times \frac{10}{14.14} + a \times \frac{10}{14.14} = 0$

$\therefore 0.71a + 0.71b = 13.75$ ——— (I)

$\sum F_y = 0.$

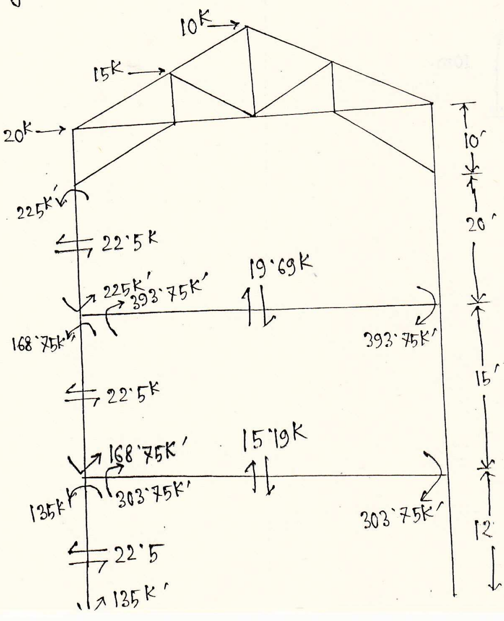
$\Rightarrow 31.25 + a_y - b_y = 0.$

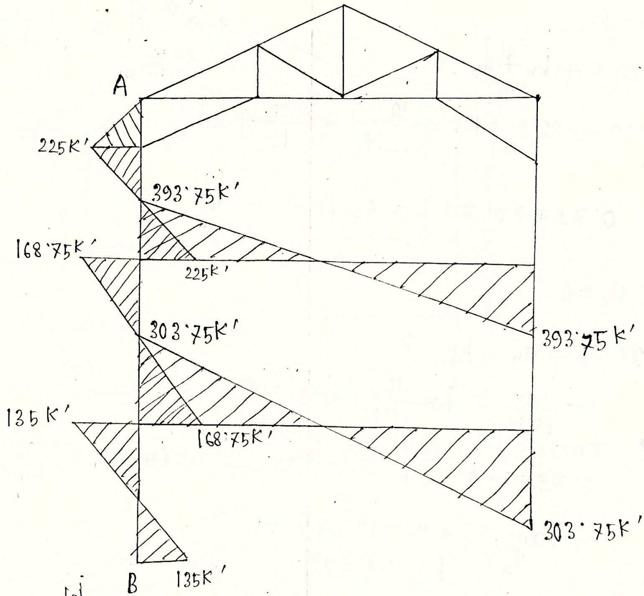
$\Rightarrow a \times \frac{10}{14.14} - b \times \frac{10}{14.14} = -31.25.$

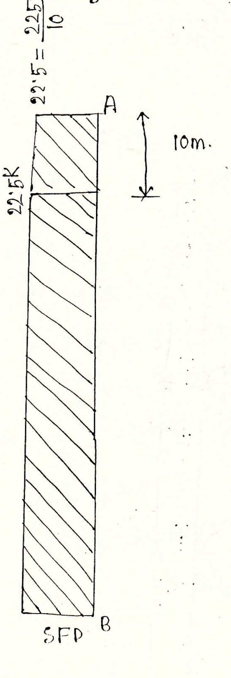
$0.71a - 0.71b = -31.25$ ——— (II).

solving, $a = -13.32^K$
$b = 31.69^K.$

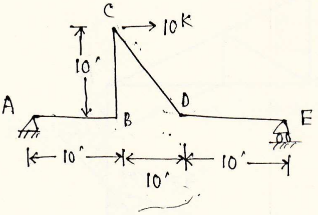
Bending moment by portal method:

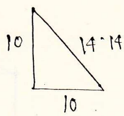



A
225K'
393'75K'
168'75K'
225K'
393'75K'
303'75K'
135K'
168'75K'
303'75K'
B
135K'


22'5 = 225/10
A
10m.
22'5K
B
SFD

Class test-2



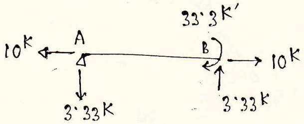


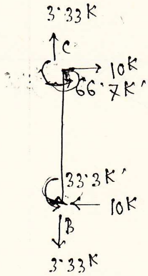
$\Sigma M_A = 0.$

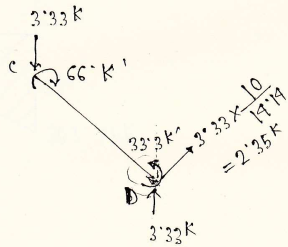
$\Rightarrow R_A \times 30 + 10 \times 10 = 0.$

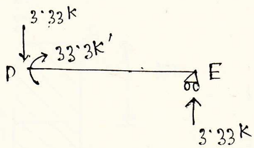
$R_A = -3.33^K (\downarrow)$. $\quad R_E = 3.33^K (\uparrow)$

FBD.



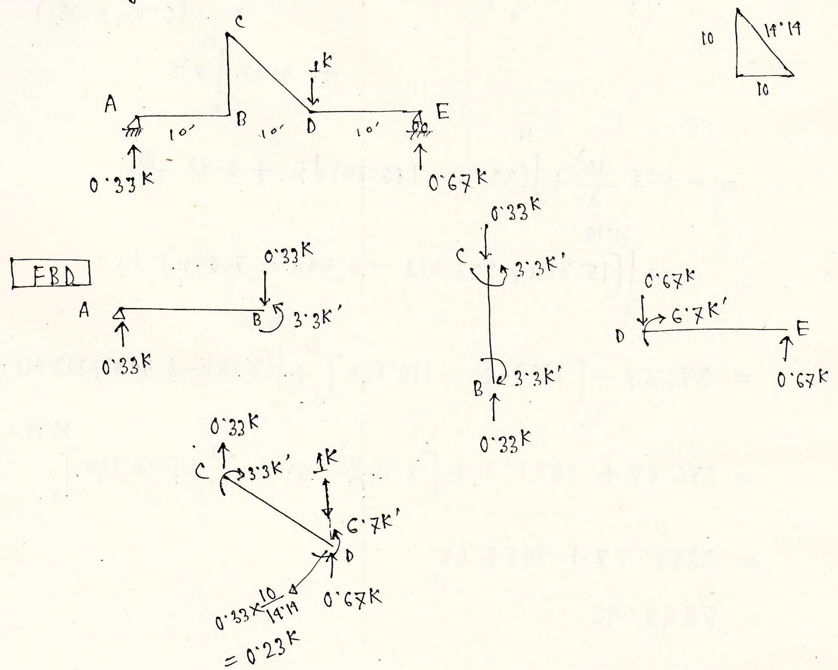




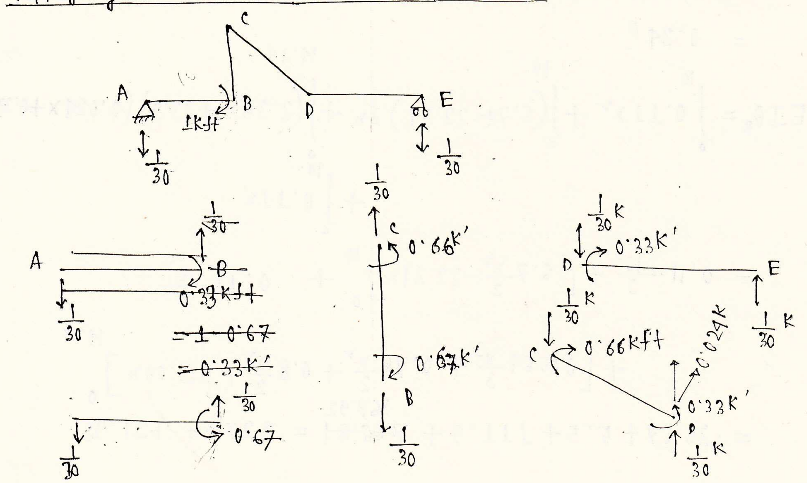


portion	AB	BC	CD	DE
origin	A	B	D	E
Limit	0-10	0-10	0-14.14	0-10
M	$-3.33x$	$10x-33.3$	$2.35x+33.3$	$3.33x$
m_{4D}	$0.33x$	3.33	$6.7-0.23x$	$0.67x$
m_{0D}	$-\frac{1}{30}x$	0.67	$0.024x+0.33$	$\frac{1}{30}x$

Applying unit load vertically at point D.



Applying unit rotation at point B.



$$EI\Delta_D = -\int_0^{10} 1.1x^2 + \int_0^{10} (10x - 33.3)\times 3.33 + \int_0^{14.14} (2.35x + 33.3)\times(6.7 - 0.23x) + 2.23\int_0^{10} x^2$$

$$= -1.1\,\frac{10^3}{3} + \int_0^{10} (33.3x - 110.89)\,dx + 2.23\cdot\frac{10^3}{3}$$

$$+\int_0^{14.14} (15.75x + 223.311 - 0.54x^2 - 7.67x)\,dx$$

$$= 376.67 + \left[33.3\,\frac{x^2}{2} - 110.89x\right]_0^{10} + \int_0^{14.14} (8.08x - 0.54x^2 + 223.311)\,dx$$

$$= 376.67 + 2021.1 + \left[8.08\,\frac{x^2}{2} - 0.54\,\frac{x^3}{3} + 223.11x\right]_0^{14.14}$$

$$= 2397.77 + 3453.65.$$

$$= 5851.42.$$

$$\Delta_D = \frac{5851.42}{30\times 10^3 \times 1000}\times 1728.$$

$$= 0.34''$$

$$EI\theta_B = \int_0^{10} 0.11x^2 + \int_0^{10} (6.7x - 22.31)\,dx + \int_0^{14.14} (2.35x + 33.3)(0.024x + 0.33) + \int_0^{10} 0.11x$$

$$= 0.11\cdot\frac{10^3}{3} + \left[6.7\,\frac{x^2}{2} - 22.31x\right]_0^{10} + 0.11\,\frac{10^2}{2}$$

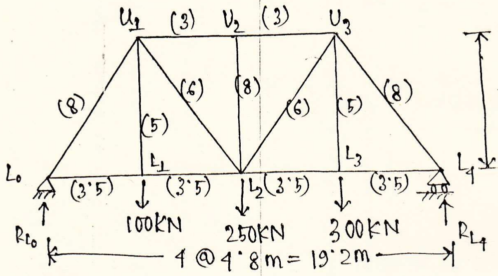
$$+\left[0.0564\,\frac{x^3}{3} + 0.78\,\frac{x^2}{2} + 0.8\,\frac{x^2}{2} + 11.09x\right]_0^{14.14}$$

$$= 36.67 + 5.5 + 111.9 + \overset{367.92}{\cancel{766.81}} = \cancel{920.88}\ 521.98.$$

Truss

- Calculate Horizontal and vertical deflection at L_3.

$E = 200 \times 10^{+6}\ kNm^{-2}$, $A = (N \times 10^{-3})\ m^2$.



Solve:

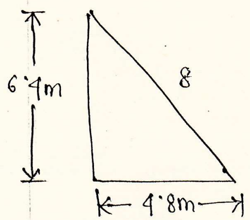
$$\Delta = \frac{\Sigma SUL}{AE}.$$

$\Sigma M_{L4} = 0.$

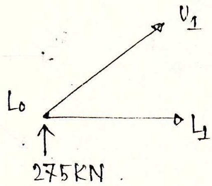
$\Rightarrow R_{L_0} \times 19.2 - 100 \times 14.4 - 250 \times 9.6 - 300 \times 4.8 = 0.$

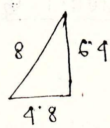
$\therefore R_{L_0} = 275\ KN.$

$R_{L_4} = 375\ KN.$



Joint L_0





$\Sigma F_y = 0.$

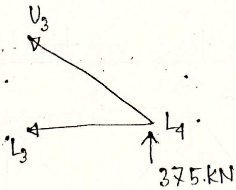
$\Rightarrow 275 + L_0U_1 \times \frac{6.4}{8} = 0.$

$L_0U_1 = -343.75$ KN (c).

$\Sigma F_x = 0, \quad L_0U_1 \times \frac{4.8}{8} + L_0L_1 = 0.$

$L_0L_1 = 206.25$ KN (T).

Joint L_4.



$\Sigma F_y = 0.$

$\Rightarrow 375 + U_3L_4 \times \frac{6.4}{8} = 0.$

$\therefore U_3L_4 = -468.75$ KN (c).

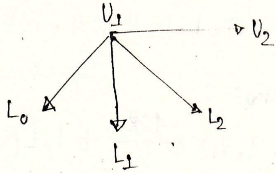
$\Sigma F_x = 0.$

$\Rightarrow \quad U_3L_4 \times \frac{4.8}{8} + L_3L_4 = 0.$

$L_3L_4 = 281.25$ KN (T).

Joint U_1.

$\Sigma F_y = 0$



$\Rightarrow (-343.75) \times \frac{6.4}{8} + 100 + U_1L_2 \times \frac{6.4}{8} = 0.$

$\therefore U_1L_2 = 218.75$ KN (T).

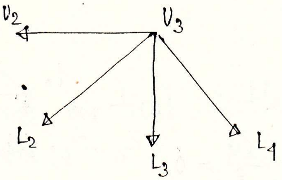
$U_1L_1 = 100$ KN.

$\Sigma F_x = 0.$

$\Rightarrow U_1U_2 + U_1L_2 \times \frac{4.8}{8} - U_1L_0 \times \frac{4.8}{8} = 0.$

$\therefore \quad U_1U_2 = -337.5$ KN (c).

Joint U_3



$\Sigma F_y = 0.$

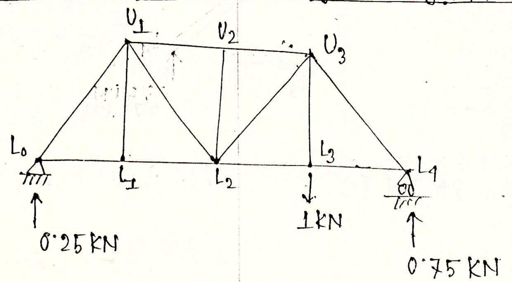
$\Rightarrow U_3L_2 \times \frac{6.4}{8} + 300 + U_3L_4 \times \frac{6.4}{8} = 0.$

$U_3L_3 = 300\,KN.$

$U_3L_4 = -468.75.$

$U_3L_2 = +93.75\,KN\ (T).$

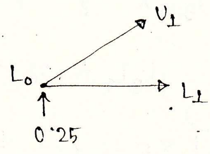
Applying unit load vertically at joint L_3:



Joint. L_0. $\Sigma F_y = 0$.

$\Rightarrow 0.25 + L_0U_1 \times \frac{6.4}{8} = 0.$

$\therefore L_0U_1 = -0.3125\,KN\ (C)$

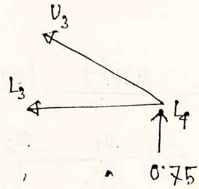


$\Sigma F_x = 0.$

$\Rightarrow L_0U_1 \times \frac{4.8}{8} + L_0L_1 = 0.$

$L_0L_1 = +0.19\,KN\ (T)$

$\boxed{\text{Joint } L_4}$ $\Sigma F_y = 0.$



$$\Rightarrow 0.75 + U_3L_4 \times \frac{6.4}{8} = 0.$$

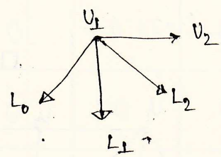
$$U_3L_4 = -0.94 \text{ KN (c)}.$$

$$\Sigma F_x = 0.$$

$$\Rightarrow U_3L_4 \times \frac{4.8}{8} + L_3L_4 = 0.$$

$$\therefore L_3L_4 = 0.56 \text{ KN (T)}.$$

$\boxed{\text{Joint } U_1}$



$$\Sigma F_y = 0.$$

$$\Rightarrow L_0U_1 \times \frac{6.4}{8} + U_1L_2 \times \frac{6.4}{8} = 0$$

$$U_1L_2 = -L_0U_1 = 0.3125 \text{ KN (T)}$$

$$L_0U_1 = -0.3125 \text{ KN (c)}.$$

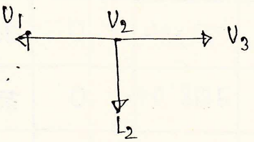
$$U_1L_1 = 0.$$

$$\Sigma F_x = 0.$$

$$\Rightarrow U_1U_2 + U_1L_2 \times \frac{4.8}{8} - U_1L_0 \times \frac{4.8}{8} = 0.$$

$$\Rightarrow U_1U_2 = \frac{4.8}{8}(-0.3125 - 0.3125) = -0.38 \text{ KN (c)}.$$

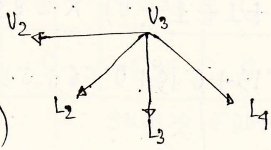
$\boxed{\text{Joint } U_2}$



$$U_1U_2 = U_2U_3 = -0.38 \text{ KN(c)}.$$

$$U_2L_2 = 0.$$

$\boxed{\text{joint } U_3}$ $\Sigma F_y = 0$



$$1 + U_3L_4 \times \frac{6.4}{8} + U_3L_2 \times \frac{6.4}{8} = 0$$

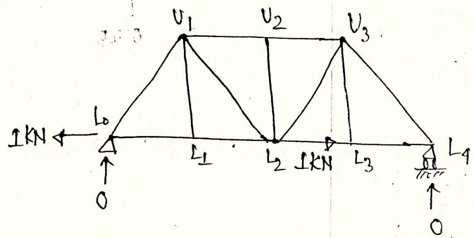
$$\Rightarrow U_3L_2 = \frac{8}{6.4} \times \left(-1 + 0.94 \times \frac{6.4}{8}\right)$$

$$\therefore U_3L_2 = -0.31 \text{ KN (c)}$$

$$U_3L_3 = 1 \text{ KN}$$

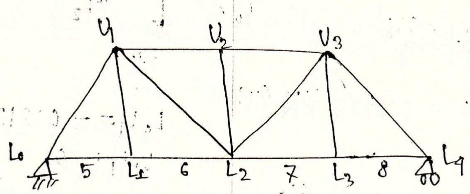
$$U_3L_4 = -0.94 \text{ KN}.$$

Applying unit load horizontally at joint L_3.



So, $L_0L_1 = L_1L_2 = L_2L_3 = 1$ KN.

Given,
$\alpha = 11.7 \times 10^{-6}$.



If temperature drops 50°C at lower chord only,

$$(\Delta L)_5 = (\Delta L)_6 = (\Delta L)_7 = (\Delta L)_8 = \alpha L \Delta t.$$

$$= 11.7 \times 10^{-6} \times 4800 \times (-50)$$

$$= -2.81 \text{ mm}.$$

$$\Delta H_{L_3} = (1+1+1+0) \times 2.81 = 8.43 \text{ mm}$$

$$\Delta V_{L_3} = (0.19+0.19+0.56+0.56) \times 2.81 = 4.22 \text{ mm}.$$

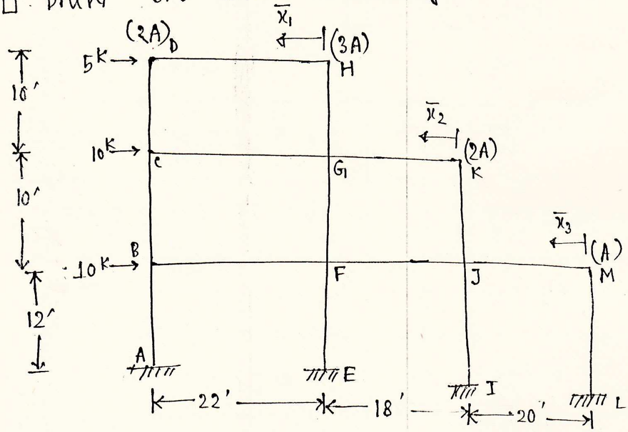
$E = 200 \times 10^{+6}\ kNm^{-2}$

Member	Area $10^{-3}(m^2)$	Length (m)	Stress (S) (KN)	$U_{H_{L_3}}$	$U_{V_{L_3}}$	$\frac{SU_H L}{AE}$	$\frac{SU_V L}{AE}$
L_0L_1	3.5	4.8	+206.25	1	+0.19	0.0014	0.00027
L_1L_2	3.5	4.8	+206.25	1	+0.19	0.0014	0.00027
L_2L_3	3.5	4.8	+281.25	1	+0.56	0.0019	0.00108
L_3L_4	3.5	4.8	+281.25	0	+0.56	0	0.00108
U_1L_1	5	6.4	+100	0	0	0	0
U_2L_2	8	6.4	0	0	0	0	0
U_3L_3	5	6.4	+300	0	+1	0	0.00192
U_1U_2	3	4.8	-337.5	0	-0.38	0	0.001
U_2U_3	3	4.8	-337.5	0	-0.38	0	0.001
L_0U_1	8	8	-343.75	0	-.3125	0	0.00054
U_1L_2	6	8	+218.75	0	+0.3125	0	0.00046
L_2U_3	6	8	+93.75	0	-0.31	0	-0.00019
U_3L_4	8	8	-468.75	0	-0.94	0	0.0022
					sum ⇒	0.0047	0.009

Cantilever method

Ahmed Hossain
090001.

□ Draw SFD and BMD diagram.



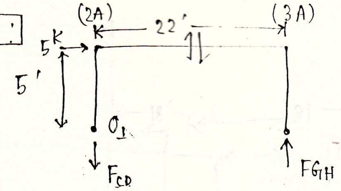
Solution: Location of centroid:

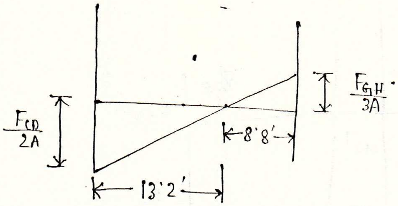
$$\bar{x}_1 = \frac{2A \times 22}{2A + 3A} = 8.8'$$

$$\bar{x}_2 = \frac{2A \times 40 + 3A \times 18}{2A + 3A + 2A} = 19.14'$$

$$\bar{x}_3 = \frac{2A \times 20 + 3A \times 38 + 2A \times 60}{2A + 3A + 2A + A} = 34.25'$$

1st step:





From similar triangle,

$$\frac{\frac{F_{CD}}{2A}}{13.2} = \frac{\frac{F_{GH}}{3A}}{8.8}$$

$$\Rightarrow \frac{F_{CD}}{2 \times 13.2} = \frac{F_{GH}}{3 \times 8.8}$$

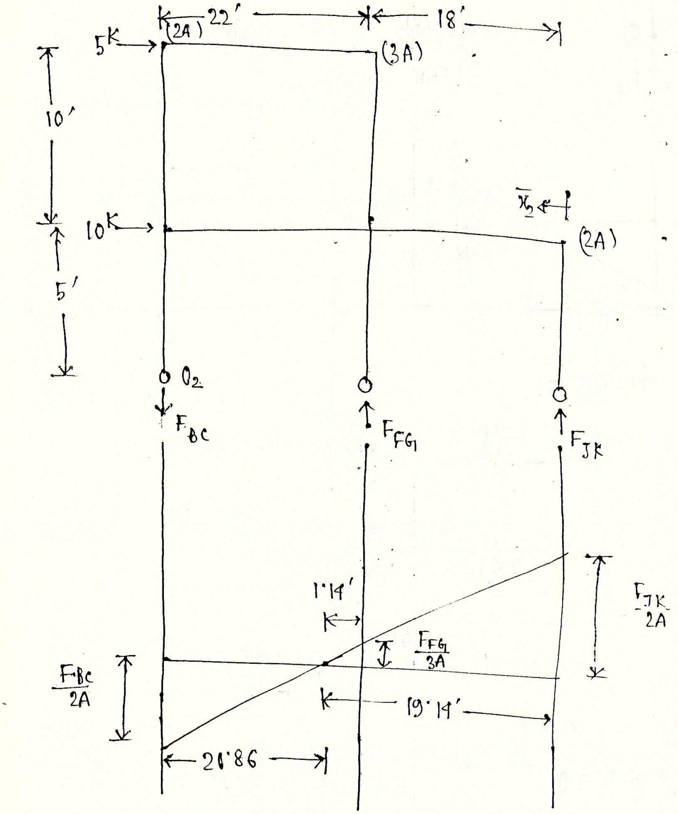
$$\therefore F_{CD} = F_{GH}.$$

$$\Sigma M_{O_1} = 0.$$

$$\Rightarrow F_{GH} \times 22 - 5 \times 5 = 0.$$

$$\therefore F_{GH} = 1.14^{k} = F_{CD}.$$

$\boxed{\text{2nd step:}}$ $\bar{x}_2 = 19{\cdot}14'$



$$\frac{F_{BC}}{2 \times 20{\cdot}86} = \frac{F_{FG}}{3 \times 1{\cdot}14} = \frac{F_{JK}}{2 \times 19{\cdot}14}$$

$F_{FG} = 0{\cdot}082\, F_{BC}$ $\qquad$ $F_{JK} = 0{\cdot}918\, F_{BC}$

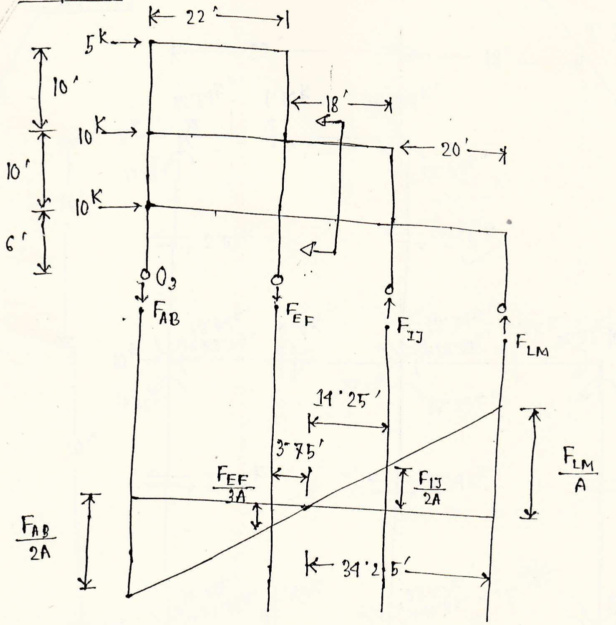
$\Sigma M_{O_2} = 0,$ $\qquad F_{FG} \times 22 + F_{JK} \times 40 = 10 \times 5 + 5 \times 15$

$F_{BC} = 3{\cdot}24^K$

$F_{FG} = 0{\cdot}27^K$

$F_{JK} = 2{\cdot}97^K$

Step-3. $\bar{x}_3 = 34.25'$



$$\frac{F_{AB}}{2 \times 25.75} = \frac{F_{EF}}{3 \times 3.75} = \frac{F_{IJ}}{14.25 \times 2} = \frac{F_{LM}}{1 \times 34.25}$$

$F_{EF} = 0.218\,F_{AB}$.

$F_{ij} = 0.55\,F_{AB}$.

$F_{LM} = 0.665 F_{AB}$.

$\Sigma M_{O_3} = 0$.

$\Rightarrow 10 \times 6 + 10 \times 16 + 5 \times 26 + F_{EF} \times 22 = F_{IJ} \times 40 + F_{LM} \times 60$.

$\cancel{F_{AB} = 6.16^K}$. $F_{AB} = 6.129^K$.

$F_{EF} = 1.34^K$

$F_{IJ} = 3.37^K$

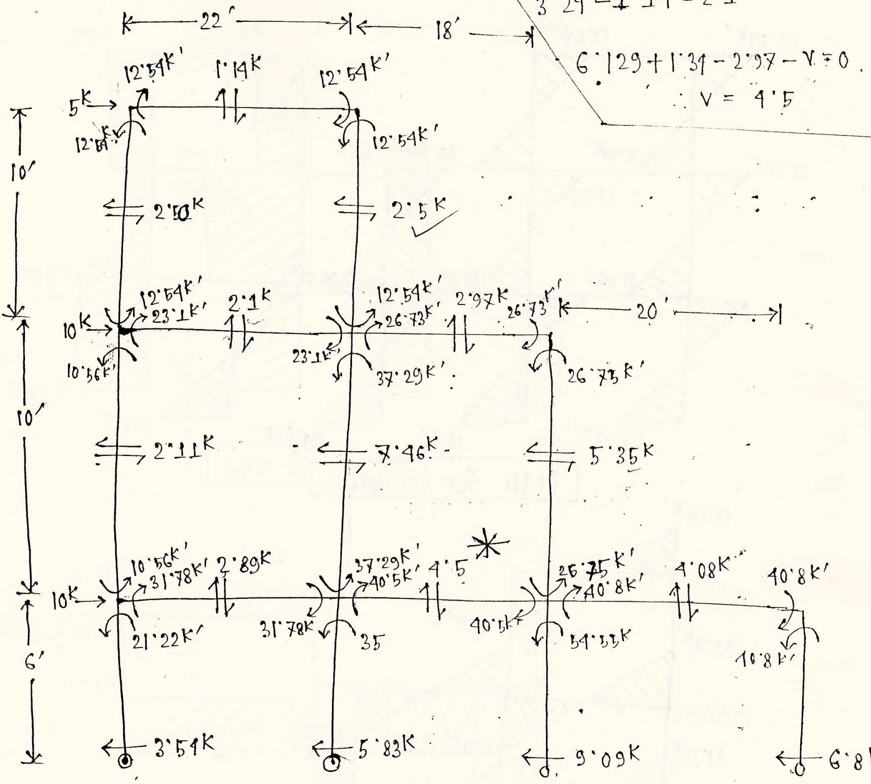
$F_{LM} = 4.08^K$

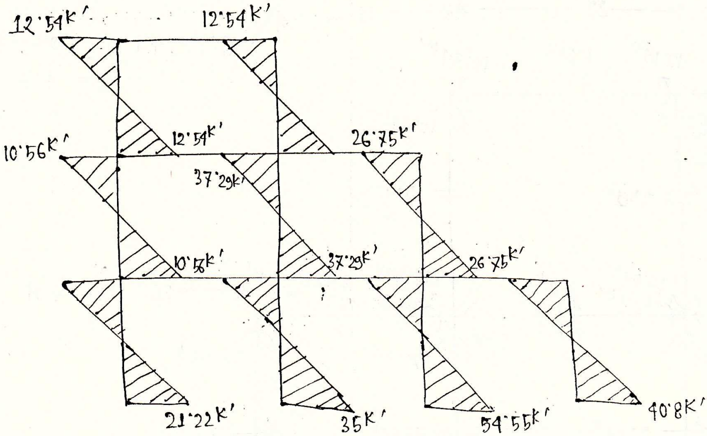
Step-4

$3.21 - 1.14 = 2.1^{K}$

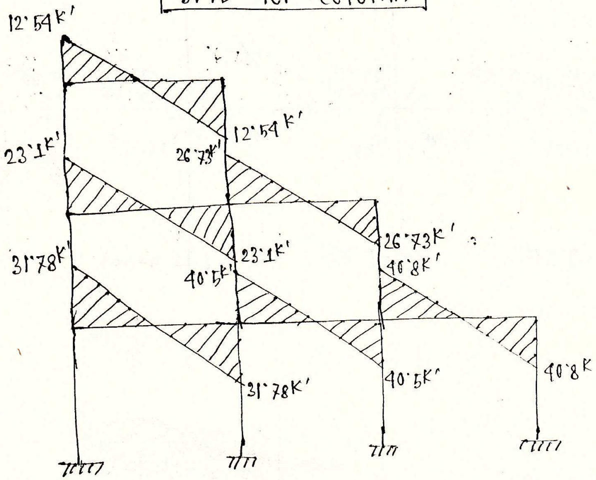
$6.129 + 1.31 - 2.97 - V = 0$

$\therefore V = 4.5$

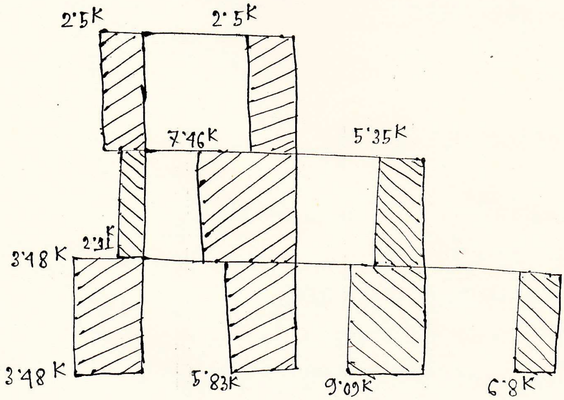




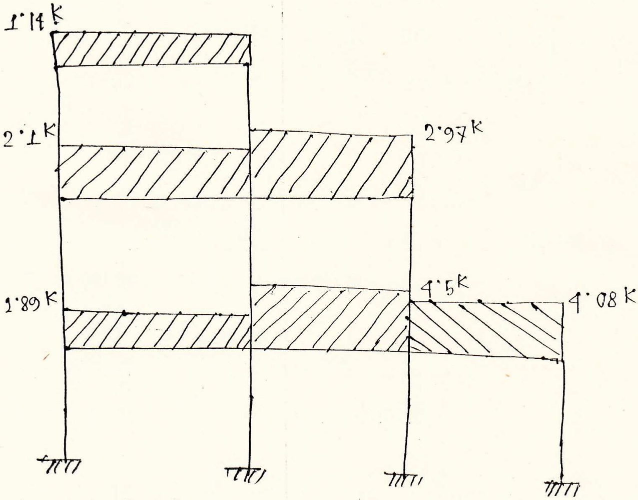
BMD for column



BMD for Girder



SFD for column



SFD for Girder

Design Wind pressure & Earthquake load.

Ahmed Hossain
090001

1. Design wind pressure, $P_z = C_G C_p q_z$.

   C_G = Gust co-efficient ⇒ G_z, G_h or G.
   C_p = pressure co-efficient.
   q_z = sustained wind pressure.

2. Sustained wind pressure, $q_z = C_c C_I C_z . V_b^2$.

   C_c = Velocity to pressure convertion co-efficient.
   C_I = structure importance co-efficient
   C_z = combined height and exposure co-efficient.
   V_b = Basic wind speed (kmh^{-1}).
   q_z = wind pressure (kNm^{-2}).

3. <u>Given Table:</u>

z (metre)	C_z	G_h.
0-4·5	0·801	1·321
6	0·866	1·294
9	0·972	1·258
12	1·055	1·233
15	1·125	1·215

Earthquake load

1. $$F_x = \frac{(V - F_t) \times W_x h_x}{\sum_{i=1}^{n} W_i h_i}$$

2. $$V = \frac{ZICW}{R}$$

Z = seismic zone co-efficient.
I = structure importance co-efficient.
W = seismic dead load.
R = Response modification co-efficient.
C = Numerical co-efficient.

3. $$C = \frac{1.25S}{T^{2/3}}$$

S = site co-efficient for soil characteristics.

4. $$T = C_t (h_n)^{3/4}$$

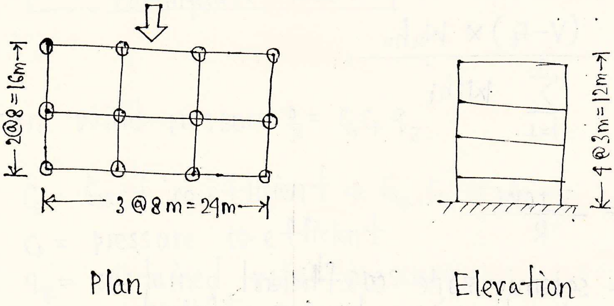
C_t = structure type co-efficient.
h_n = total elevation.
T = period of vibration.

5. Condition:
   (i) When $T > 0.7$ second, $F_t = 0.07TV \leq 0.25V$.
   (ii) When $T \leq 0.7$ second, $F_t = 0$

6. F_t will act at the top point of elevation.

2010

Q. 1.



Given,

seismic zone co-efficient, $Z = 0.075$

structure importance co-efficient, $I = 1$.

Response modification co-efficient, $R = 8$.

structure type co-efficient, $C_t = 0.049$.

site co-efficient for soil characteristics, $S = 1.50$

Seismic dead load, $W = 1500$ kN / floor.

velocity to pressure conversion co-efficient, $C_c = 47 \times 10^{-6}$

Basic wind speed, $V_b = 165$ kmh^{-1}.

pressure co-efficient, $C_p = 1.8$.

Z (meters)	C_z	G_h
0 - 4.5	0.801	1.321
6	0.866	1.294
9	0.972	1.258
12	1.055	1.233

Design wind pressure: $P_z = C_G C_p \times q_z$ ——— (I)

$q_z = C_c C_I C_z \cdot V_b^2$.

$\Rightarrow q_z = 47 \times 10^{-6} \times 1 \times 165^2 \times C_z$.

$\therefore q_z = 1.279 C_z$ ——— (II)

$q_1 = 1.279 \times 0.801 = 1.02$

$q_2 = 1.279 \times 0.866 = 1.11$

$q_3 = 1.279 \times 0.972 = 1.24$

$q_4 = 1.279 \times 1.055 = 1.35$.

From (I) we get, $P_z = C_G C_p q_z$.

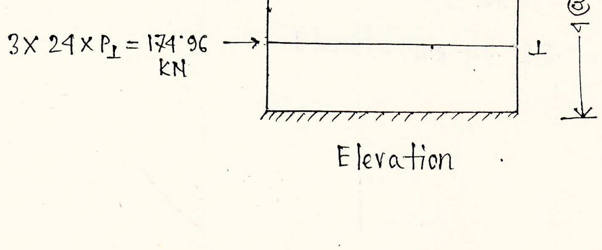
$\Rightarrow P_z = 1.8 \times C_G \times q_z$.

$P_1 = 1.8 \times 1.321 \times 1.02 = 2.43 \text{ KNm}^{-2}$

$P_2 = 1.8 \times 1.294 \times 1.11 = 2.59 \text{ KNm}^{-2}$

$P_3 = 1.8 \times 1.258 \times 1.24 = 2.81 \text{ KNm}^{-2}$

$P_4 = 1.8 \times 1.233 \times 1.35 = 3 \text{ KNm}^{-2}$.



Elevation.

Earthquake load:

$$F_x = \frac{(V - F_t) \times W_x h_x}{\sum_{i=1}^{n} W_i h_i} \quad \text{——— (I)}$$

$$V = \frac{ZICW}{R}$$

$$= \frac{0.075 \times 1 \times C}{8} \times 1500 \times 4.$$

$$V = 56.25C. \quad \text{——— (II)}$$

$$C = \frac{1.25S}{T^{2/3}} \quad \text{——— (III)}.$$

$$T = C_t \times (h_n)^{\frac{3}{4}} = 0.049 \times (12)^{\frac{3}{4}} = 0.31s.$$

$$\therefore C = \frac{1.25 \times 1.50}{(0.31)^{2/3}} = 4.042.$$

$$V = 56.25 \times 4.042 = 227.37 \text{ KN}.$$

since, $T < 0.7$ second, $F_t = 0$.

From (I), $$F_x = \frac{V \times W_x h_x}{\sum_{i=1}^{n} W_i h_i}.$$

$$\Rightarrow F_1 = \frac{227.37 \times 1500 \times 3}{1500 \times (3+6+9+12)}$$

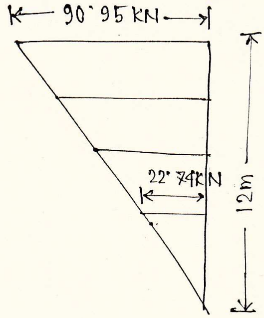
$$\Rightarrow F_1 = \frac{227.37 \times 3}{30}.$$

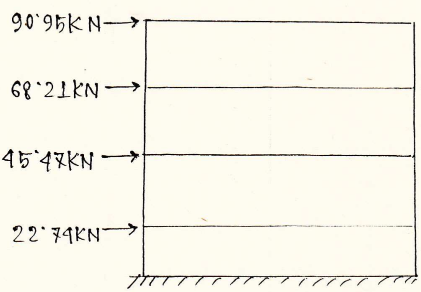
$$\therefore F_1 = 22.74 \text{ KN}.$$

$$F_2 = \frac{227.37 \times 6}{30} = 45.47 \text{ KN}.$$

$$F_3 = \frac{227.37 \times 9}{30} = 68.21 \text{ KN}$$

$$F_4 = \frac{227.37 \times 12}{30} = 90.95 \text{ KN}.$$





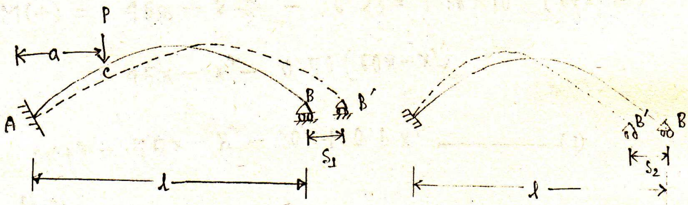
Elevation.

Two Hinged Arch

Ahmed Hossain
090001

□ Prove that $H = \dfrac{\int_0^l M_s y\,dx}{\int_0^l y^2 dx}$ where symbols indicate their usual meaning.

Ans: Let us consider the following figure:



Let M_s be the bending moment at any section of the determinate arch. Applying unit load at roller end,

Horizontal movement, $S_1 = \int_0^l \dfrac{M_s y\,ds}{EI}$.

Horizontal movement, $S_2 = \int_0^l \dfrac{y^2 ds}{EI}$.

If H is the horizontal thrust at hinge, the deflection of roller end due to force H is $H \int_0^l \dfrac{y^2 ds}{EI}$.

since supports do not yield,

$$H \int_0^l \frac{y^2 ds}{EI} = \int_0^l \frac{M_s y\,ds}{EI}.$$

$$\Rightarrow \quad H = \frac{\int_0^l \frac{M_s y\,ds}{EI}}{\int_0^l \frac{y^2 ds}{EI}} \qquad (1)$$

putting, $I = I_c \sec\theta$. [I_c = moment of inertia at crown]

and; $\cos\theta = \frac{dx}{ds}$.

$$\Rightarrow ds = \sec\theta . dx .$$

putting these values in (1),

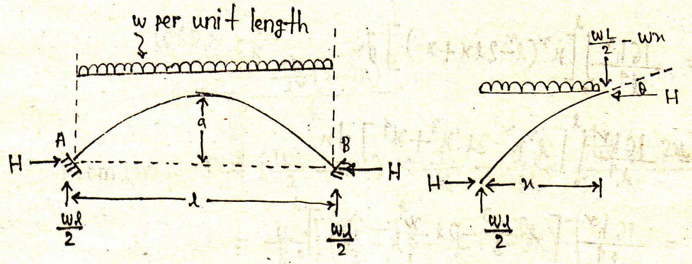
$$H = \frac{\int_0^l \frac{M_s y \sec\theta . dx}{E I_c \sec\theta}}{\int_0^l \frac{y^2 \sec\theta . dx}{E I_c \sec\theta}}$$

$$\therefore H = \frac{\int_0^l M_s y dx}{\int_0^l y^2 dx} .$$

□ For a parabolic arch loaded with uniformly distributed load, prove that –

1. Bending moment at any section is zero.
2. Shear force at any section is zero
3. Normal thrust at any section, $N = \dfrac{w\left[\frac{2h}{l^2}(l-2x)^2 + \frac{l^2}{8h}\right]}{\sqrt{\frac{10h^2}{l^4}(l-2x)^2 + 1}}$

$\boxed{\text{Ans:}}$ Let us consider the following figure:



Bending moment at any section, $M_x = M_s - H \times y$ ——— (i)

$$H = \frac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx} \quad \text{——— (ii)}$$

$$\int_0^l M_s y\,dx = \int_0^l \left(\frac{wl}{2}x - \frac{wx^2}{2}\right) y\,dx.$$

$$= \frac{w}{2}\int_0^l (lx - x^2)\,\frac{4hx(l-x)}{l^2}\,dx \quad \left[y = \frac{4hx(l-x)}{l^2}\right]$$

$$= \frac{w}{2} \times \frac{4h}{l^2}\int_0^l (lx - x^2)^2\,dx$$

$$= \frac{2wh}{l^2}\int_0^l (l^2x^2 - 2.lx.x^2 + x^4)\,dx$$

$$= \frac{2wh}{l^2}\left[l^2\frac{x^3}{3} - 2l\frac{x^4}{4} + \frac{x^5}{5}\right]_0^l$$

$$= \frac{2wh}{l^2}\left[\frac{l^5}{3} - \frac{l^5}{2} + \frac{l^5}{5}\right]$$

$$= \frac{2wh}{l^2} \times \frac{l^5}{30}.$$

$$= \frac{whl^3}{15}.$$

$$\int_0^l y^2\,dx = \int_0^l \left[\frac{4hx(l-x)}{l^2}\right]^2 dx$$

$$= \frac{16h^2}{l^4}\int_0^l \left[x^2(l^2 - 2lx + x^2)\right] dx$$

$$= \frac{16h^2}{l^4}\int_0^l \left[l^2x^2 - 2l.x^3 + x^4\right] dx$$

$$= \frac{16h^2}{l^4}\left[l^2\frac{x^3}{3} - 2l\frac{x^4}{4} + \frac{x^5}{5}\right]_0^l$$

$$= \frac{16h^2}{l^4}\left[\frac{l^5}{3} - \frac{l^5}{2} + \frac{l^5}{5}\right] = \frac{16h^2}{l^4} \times \frac{l^5}{30} = \frac{8}{15}h^2 l.$$

From (II) we get, $H = \dfrac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx}$

$$\Rightarrow H = \frac{\frac{whl^3}{15}}{\frac{8h^2l}{15}}$$

$$\Rightarrow H = \frac{whl^3}{15} \times \frac{15}{8h^2l} = \frac{wl^2}{8h}$$

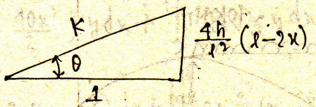
From (I) we get, $M_x = M_s - H \times y$.

$$= \frac{wl}{2}.x - \frac{wx^2}{2} - \frac{wl^2}{8h} \times \frac{4hx(l-x)}{l^2}$$

$$= \frac{wlx}{2} - \frac{wx^2}{2} - \frac{wx}{2}(l-x)$$

$$= 0.$$

Shear force at any section, $F = V\cos\theta - H\sin\theta$ ——— (IV)



$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left\{\frac{4hx}{l^2}(l-x)\right\} = \frac{4h}{l^2}(l-2x)$$

$$K = \sqrt{\frac{16h^2}{l^4}(l-2x)^2 + 1}$$

$$\cos\theta = \frac{1}{K}.$$

$$\sin\theta = \frac{\frac{4h}{l^2}(l-2x)}{K}$$

From (IV), $F = \left(\frac{wl}{2} - wx\right) \times \frac{1}{K} - \frac{wl^2}{8h} \cdot \frac{\frac{4h}{l^2}(l-2x)}{K}$

$$= \frac{w}{K}\left[\left(\frac{l}{2} - x\right) - \frac{1}{2}(l-2x)\right]$$

$$= 0.$$

Normal thrust at any section.

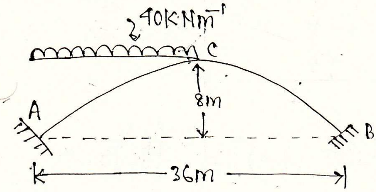
$$N = V\sin\theta + H\cos\theta$$

$$= \left(\frac{wl}{2} - wx\right) \times \frac{\frac{4h}{l^2}(l-2x)}{K} + \frac{wl^2}{8h} \times \frac{1}{K}$$

$$= \frac{w}{K}\left[\frac{2h}{l^2}\cdot(l-2x)^2 + \frac{l^2}{8h}\right]$$

$$= \frac{w\left[\frac{2h}{l^2}(l-2x)^2 + \frac{l^2}{8h}\right]}{\sqrt{\frac{16h^2}{l^4}(l-2x)^2+1}}$$

□ A uniformly distributed load of $40\,KNm^{-1}$ covers left hand half of the span of a parabolic arch, span 36m and central rise 8m. Determine the position and magnitude of maximum bending moment. Also find shear force and normal thrust at that section.



$\Sigma M_B = 0$.

$$R_A \times 36 - (40 \times 18) \times \left(18 + \frac{18}{2}\right) = 0.$$

$$R_A = 540\ KN.$$

$$R_B = 180\ KN.$$

$$l = 36m.$$
$$h = 8m.$$

$$M_{AC} = 540x - 40\frac{x^2}{2}.$$

$$M_{BC} = 180x.$$

Horizontal thrust, $H = \dfrac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx}$ ——— (1).

$$\int_0^{36} y^2\,dx = \int_0^{36} \left[\frac{4hx}{l^2}(l-x)\right]^2 dx$$

$$= \int_0^{36} \left\{\frac{4\times 8\times x}{36^2}(36-x)\right\}^2 dx$$

$$= \frac{4}{6561}\int_0^{36} (36x-x^2)^2\,dx$$

$$= \frac{4}{6561}\int_0^{36} \left[36^2x^2 - 2\times 36x\times x^2 + x^4\right] dx$$

$$= \frac{4}{6561}\cdot\left[36^2\frac{x^3}{3} - 72\frac{x^4}{4} + \frac{x^5}{5}\right]_0^{36}$$

$$= 1228.8.$$

$$\int_0^{36} M_s y\,dx = \int_0^{18}\left(540x - \frac{40x^2}{2}\right) y\,dx + \int_0^{18} 180x\,y\,dx$$

$$= \int_0^{18}(540x-20x^2)\frac{2}{81}(36x-x^2)\,dx + \int_0^{18} 180x\frac{2}{81}(36x-x^2)\,dx$$

$$= \frac{2}{81}\left[\int_0^{18}(540x-20x^2).(36x-x^2)\,dx + \int_0^{18} 180(36x^2-x^3)\,dx\right]$$

Here,

$$y = \frac{4hx}{l^2}(l-x) = \frac{4\times 8x(36-x)}{36^2} = \frac{2}{81}(36x-x^2)$$

$$= \frac{2}{81}\left[\int_0^{18}(19440.x^2 - 540x^3 - 720x^3 + 20x^4)\,dx + 180.\left[36\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{18}\right]$$

$$= \frac{2}{81}\left\{\left[19440\frac{x^3}{3} - 540.\frac{x^4}{4} - 720.\frac{x^4}{4} + 20\frac{x^5}{5}\right]_0^{18} + 7873200\right\}$$

$$= \frac{2}{81}\cdot(12282192 + 7873200)$$

$$= 497664.$$

$$H = \frac{497.664}{1228.8} = 405 \text{ KN}.$$

Maximum +ve bending moment occurs in AC portion.

$$M(+) = M_s - H \times y$$

$$\Rightarrow M(+) = 540x - \frac{40x^2}{2} - 405 \times \frac{2x}{81}(36-x).$$

$$= 540x - 20x^2 - 10(36x - x^2)$$

$$= 540x - 20x^2 - 360x + 10x^2 \quad \text{———— (I)}$$

condition

$$\frac{dM}{dx} = 0.$$

$$\Rightarrow 540 - 40x - 360 + 20x = 0.$$

$$\Rightarrow -20x + 180 = 0.$$

$$\therefore x = 9m.$$

$$M_{max}(+) = 810 \text{ KNm}.$$

Maximum negative bending moment occurs in BC portion.

$$M(-) = M_s - H \times y.$$

$$= 180x - 405.y.$$

$$= 180x - 405.\frac{2}{81}(36x - x^2)$$

$$= 180x - 360x + 10x^2. \quad \text{———— (II)}$$

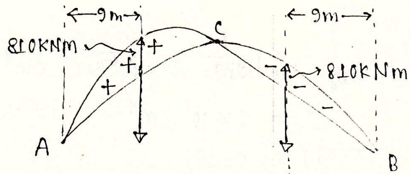
condition

$$\frac{dM}{dx} = 0.$$

$$\Rightarrow 180 - 360 + 20x = 0.$$

$$\therefore x = 9m.$$

$M_{max}(-) = -810.\ \text{KNm}.$



Bending moment diagram.

Shear force, $S.F = -H\sin\theta + V\cos\theta$ ——— (I)

Normal thrust, $N.T = H\cos\theta + V\sin\theta$ ——— (II)

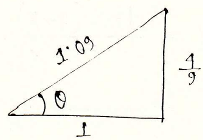
Given, $y = \frac{4hx}{l^2}(l-x)$.

$= \frac{4\times 8x}{36^2}(l-x)$. [$h = 8$ m, $l = 36$m) .

$= \frac{2}{81}(36x - x^2)$.

$\tan\theta = \frac{dy}{dx} = \frac{2}{81}\cdot(36-2x)$

* At, $\boxed{x = 9,}$ $\tan\theta = \frac{4}{9}$.



$\sin\theta = \frac{\frac{4}{9}}{1.09} = 0.41$

$\cos\theta = \frac{1}{1.09} = 0.92$.

$H = 405$ KN.

From left.
$S.F = -405\times 0.41 + (540 - 40\times 9)\times 0.92 = 0$.
$N.T = 405\times 0.92 + (540 - 40\times 9)\times 0.41 = 446.4$ KN.

From right
$SF = -405\times 0.41 + 180\times 0.92 = 0$.
$N.T = 405\times 0.92 + 180\times 0.41 = 446.4$ KN.

$\boxed{\text{Exmple} - 11.5.}$

□ Given data,

$l = 60\,m.$

$h = 10\,m.$

Moment of inertia at any section $= 6 \times 10^6 \sec\theta\ cm^4.$

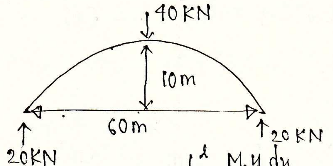
$I_c = 6 \times 10^6\ cm^4 = 6 \times 10^6 \times 100^{-4}\ m^4$

point load $= 40$ KN.

$E = 10\ KN mm^{-2} = 10 \times 10^{+6}\ KNm^{-2}$

$\boxed{\text{condition}}$

Bending moment at quarter point = BM at crown —— (I)



Horizontal thrust. $H = \dfrac{\int_0^l \dfrac{M_s y\, dx}{EI_c} - 4}{\int_0^l \dfrac{y^2 dx}{EI_c}}$ —— (II)

$$y = \frac{4hx(l-x)}{l^2} = \frac{4 \times 10 \times x \times (60-x)}{60^2}$$

$$= \frac{1}{90}(60-x) \times x$$

$$\int_0^l \frac{y^2 dx}{EI_c} = \frac{1}{EI_c} 2 \times \int_0^{30} \left[\frac{x}{90}(60-x)\right]^2 dx$$

$$= \frac{1}{EI_c} 2 \times \frac{1}{90^2} \int_0^{30} x^2 [60^2 - 2 \cdot \times 60 \times x + x^2]$$

$$= \frac{1}{EI_c} \cdot \frac{1 \times 2}{90^2} \int_0^{30} (60^2 x^2 - 120 \cdot x^3 + x^4)$$

$$= \frac{1}{EI_c} \cdot \frac{1 \times 2}{90^2} \left[60^2 \frac{x^3}{3} - 120 \frac{x^4}{4} + \frac{x^5}{5}\right]_0^{30}$$

$$\int_0^l \frac{y^2 dx}{EI_c} = \frac{1}{EI_c} \times 1600 \times 2 = \frac{3200}{EI_c}$$

$$\therefore \int_0^l \frac{M_s y\, dx}{EI_c} = \frac{1}{EI_c} \times 2 \int_0^{30} 20x \left[\frac{x(60-x)}{90}\right] dx \quad [M_s = 20x]$$

$$= \frac{1}{EI_c} \cdot \frac{20}{90} \times 2 \int_0^{30} x^2 (60-x)\, dx$$

$$= \frac{4}{9} \times \frac{1}{EI_c} \int_0^{30} (60x^2 - x^3)\, dx$$

$$= \frac{4}{9} \times \frac{1}{EI_c} \left[60 \cdot \frac{x^3}{3} - \frac{x^4}{4}\right]_0^{30}$$

$$= \frac{1}{EI_c} \cdot \frac{4}{9} \times 337500.$$

$$= \frac{150000}{EI_c}.$$

From (II) $\Rightarrow$ $$H = \frac{\frac{150000}{EI_c} - 4}{\frac{3200}{EI_c}} \quad \text{———— (III)}.$$

BM at centre of span $= 20 \times 30 - H \times 10.$

$$M(+) = 600 - 10H \text{ ———— (IV)}$$

BM at quarter span $= 20 \times 15 - H \times y.$

$$\left[y = \frac{4hx(l-x)}{l^2} = \frac{4 \times 10 \times 15 \times (60-15)}{60^2} = 7 \cdot 5 \right]$$

$$M(-) = 300 - 7 \cdot 5H \text{ ———— (V)}.$$

Equating (IV) & (V) $\quad 600 - 10H = (300 - 7 \cdot 5H) \times (-1)$

$$\Rightarrow 600 - 10H = -300 + 7 \cdot 5H.$$

$$H = 51 \cdot 43 \text{ KN}.$$

From (III) we get,

$$\frac{\frac{150000}{E.I_c} - \Delta}{\frac{3200}{EI_c}} = 51.43$$

$$\Rightarrow \frac{150000}{EI_c} - \Delta = \frac{164576}{EI_c}$$

$$\Rightarrow 150000 - EI_c\Delta = 164576$$

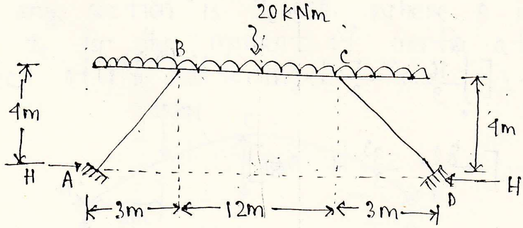
$$\Rightarrow \Delta = \frac{150000 - 164573}{EI_c}$$

$$\Rightarrow \Delta = \frac{150000 - 164573}{6\times10^6 \times 100^{-4} \times 10\times10^6}$$

$$\Rightarrow \Delta = -0.024\ m$$

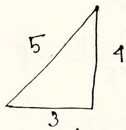
$$\Delta = -2.42\ cm\ (Ans)$$

□ Draw Bending moment diagram for the following arch.



Solve: For AB and CD portion, $\frac{y}{x} = \frac{4}{3}$.

$y = \frac{4}{3}x$.



so, $\frac{ds}{dx} = \frac{5}{3}$.

$$H = \frac{\int_0^l \frac{M_s y\,ds}{EI}}{\int_0^l \frac{y^2 ds}{EI}} \quad \text{---(1)}$$

$\Sigma M_D = 0$.

$\Rightarrow R_A \times 18 - 18 \times 20 \times \frac{18}{2} = 0$.

$\therefore R_A = 180 KN$.

$R_D = 180 KN$.

$$\int_0^l \frac{M_s y\,ds}{EI} = \frac{2\times}{EI}\int_0^3 \left(180x - 20 \cdot \frac{x^2}{2}\right) \times \left(\frac{4x}{3}\right) \times \left(\frac{5}{3}\,dx\right)$$

$$+ \frac{2\times}{EI}\int_3^9 \left(180x - 10x^2\right) \times (4)(dx) \quad [y=4,\ ds = dx]$$

$$= \frac{2}{EI}\left[\frac{20}{9}\int_0^3 (180x^2 - 10x^3)\,dx + 4\int_3^9 \left[180 \cdot \frac{x^2}{2} - 10 \cdot \frac{x^3}{3}\right] dx\right]$$

$$= \frac{2}{EI} \cdot \left\{\frac{20}{9} \cdot \left[180 \cdot \frac{x^3}{3} - 10 \cdot \frac{x^4}{4}\right]_0^3 + 4 \times 4140\right\}$$

$$= \frac{2}{EI} \times 19710$$

$$= \frac{39420}{EI}$$

$$\int_0^l \frac{y^2 ds}{EI} = \frac{1}{EI} \cdot \left[2\times \int_0^3 \left(\frac{4x}{3}\right)^2 \frac{5}{3} dx + 2\times \int_3^9 4^2 dx. \right]$$

$$= \frac{2}{EI} \cdot \left[\cdot \int_0^3 \frac{16}{9} \cdot x^2 \cdot \frac{5}{3} dx . + \quad 16(9-3) \cdot \right]$$

$$= \frac{2}{EI} \cdot \left[\cdot \frac{80}{27} \cdot \frac{3^3}{3} + 96 \right]$$

$$= \frac{245.33}{EI}.$$

$$H = \frac{39420}{245.33} = 160.68 \text{ kN}.$$

Bending moment diagram:

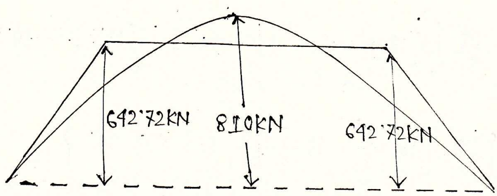
At point B = 160.68 × 4 = 642.72 kNm

At point C = 160.68 × 4 = 642.72 kNm

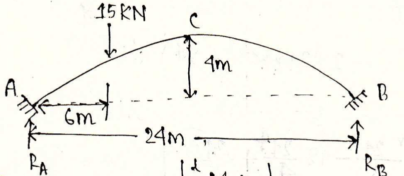
At crown, $M = 180x - 20\frac{x^2}{2}$.

$= 180 \times 9 - 10 \times 9^2$

$= 810$ kN



□ Find the horizontal thrust for the two hinged parabolic arc shown in figure. The moment of inertia at any section is $I_c \sec\theta$. where θ is the slope and I_c is the moment of inertia at crown. Neglect effect of rib shortening.



Solution:

$$H = \frac{\int_0^l M_s y \, dx}{\int_0^l y^2 dx} \quad \text{———} (1)$$

where, $y = \frac{4hx(l-x)}{l^2}$.

$$= \frac{4 \times 4 \times x \times (24-x)}{24^2} \quad [l = 24, h = 4]$$

$$y = \frac{1}{36}(24x - x^2).$$

$\Sigma M_B = 0.$

$\Rightarrow R_A \times 24 - 15 \times 18 = 0.$

$\therefore R_A = 11.25 KN$

$R_B = 3.75 KN$

$$\int_0^l M_s y \, dx = \int_0^6 (11.25x) y \, dx + \int_0^{18} (3.75x) y \, dx.$$

$$= \int_0^6 11.25 . x \times \frac{(24x - x^2)}{36} dx + \int_0^{18} 3.75 x \frac{(24x - x^2)}{36} . dx.$$

$$= \frac{5}{16} . \int_0^6 (24x^2 - x^3) \, dx + \frac{5}{48} . \int_0^{18} (24x^2 - x^3) \, dx$$

$$= \frac{5}{16} . \left[24 \frac{x^3}{3} - \frac{x^4}{4}\right]_0^6 + \frac{5}{48} . \left[24 . \frac{x^3}{3} - \frac{x^4}{4}\right]_0^{18}$$

$$= \frac{5}{16} \times 1404 + \frac{5}{48} \times 20412$$

$$= 2565$$

$$\int_0^l y^2 dx = \int_0^{24} \left\{ \frac{1}{36} (24x - x^2) \right\}^2 dx.$$

$$= \frac{1}{36^2} \cdot \int_0^{24} (24^2 x^2 - 2 \times 24 x \times x^2 + x^4)\, dx$$

$$= \frac{1}{36^2} \cdot \left[24^2 \frac{24^3}{3} - 48 \cdot \frac{24^4}{4} + \frac{24^5}{5} \right]$$

$$= 204.8.$$

$$H = \frac{2565}{204.8} = 12.52 \text{ KN}.$$

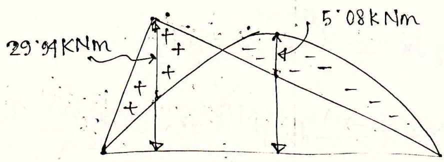
Bending moment diagram:

Rise of arch at 6m from right. $(x = 6m)$

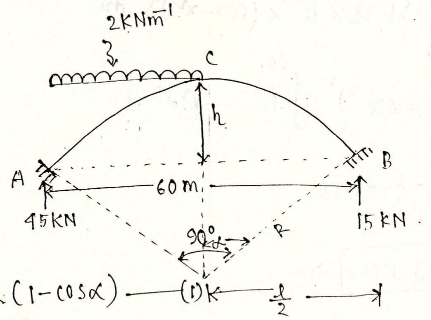
$$y = \frac{4hx(l-x)}{l^2} = \frac{4 \times 4 \times 6 \times (24-6)}{24^2} = 3m.$$

BM under the load, $= 11.25 \times 6 - 12.52 \times 3. = 29.94$ KNm

BM at crown $= 3.75 \times 12 - 12.52 \times 4 = -5.08$ KNm.



« Segmental Arch »



$h = R(1-\cos\alpha)$ ——— (i)

$\alpha = \frac{90^\circ}{2} = 45^\circ$

Radius, $R = \frac{\frac{l}{2}}{\sin\alpha} = 42.43\,m$

$h = 12.43\,m$

$\Sigma M_B = 0$, $\quad R_A \times 60 - 2\times 30 \times \left(30 + \frac{30}{2}\right) = 0$

$\therefore R_A = 45\,kN$

$$H = \frac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx} \quad \text{——— (1)}$$

$$y = \frac{4hx(l-x)}{l^2} = \frac{4\times 12.43x(60-x)}{60^2} = 1.38\times 10^{-2}(60x-x^2)$$

$$\int_0^l M_s y\,dx = \int_0^{30}\left(45x - 2\cdot\frac{x^2}{2}\right)\cdot 1.38\times 10^{-2}(60x-x^2)\,dx + \int_0^{30} 15x \times 1.38\times 10^{-2}(60x-x^2)\,dx$$

$$= 1.38\times 10^{-2}\left[\int_0^{30}(45x-x^2)(60x-x^2)\,dx + 15\int_0^{30}(60x-x^2)\cdot x\,dx\right]$$

$$= 1.38\times 10^{-2}\left(7897500 + \overset{5062500}{\cancel{7965000}}\right)$$

$$= 178848.$$

$$\int_0^l y^2\,dx = \int_0^{60} \left\{1.38\times10^{-2}\times(60x-x^2)\right\}^2 dx$$

$$= (1.38\times10^{-2})^2 \int_0^{60} (60x-x^2)^2\,dx$$

$$= 4936.2048.$$

$$\boxed{H = 36.23\ \text{KN.}}\ *$$

$$M(+) = 45x - 2\frac{x^2}{2} - 36.23\times1.38\times10^{-2}(60x-x^2).$$

$$= 45x - x^2 - 0.50(60x-x^2)$$

$$M(+) = 45x - x^2 - 30x + 0.5x^2 \text{ ——— (I)}$$

$$\frac{dM}{dx} = 0$$

$$\Rightarrow 45 - 2x - 30 + x = 0$$

$$\Rightarrow -x + 15 = 0.$$

$$\therefore x = 15'$$

$$M(+) = 112.5\ \text{kNm}.$$

$$M(-) = 15x - 36.25\times1.38\times10^{-2}(60x-x^2)$$

$$= 15x - 0.50(60x-x^2)$$

$$= 15x - 30x + 0.5x^2 \text{ ——— (II)}$$

$$\frac{dM}{dx} = 0 \Rightarrow 15 - 30 + x = 0 \quad \therefore x = 15'$$

$$M(-) = -112.5\ \text{kNm}.$$