Structural Analysis & Design - II

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A Hand-note On

STRUCTURAL ANALYSIS & DESIGN - II

CE 313

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Topics

Deflection
Approximate method
Portal frame
Cntilever method
Castiglianos method
Moment distribution method
Vitrual work method
Two hinged arch
Truss
3D

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Deflection

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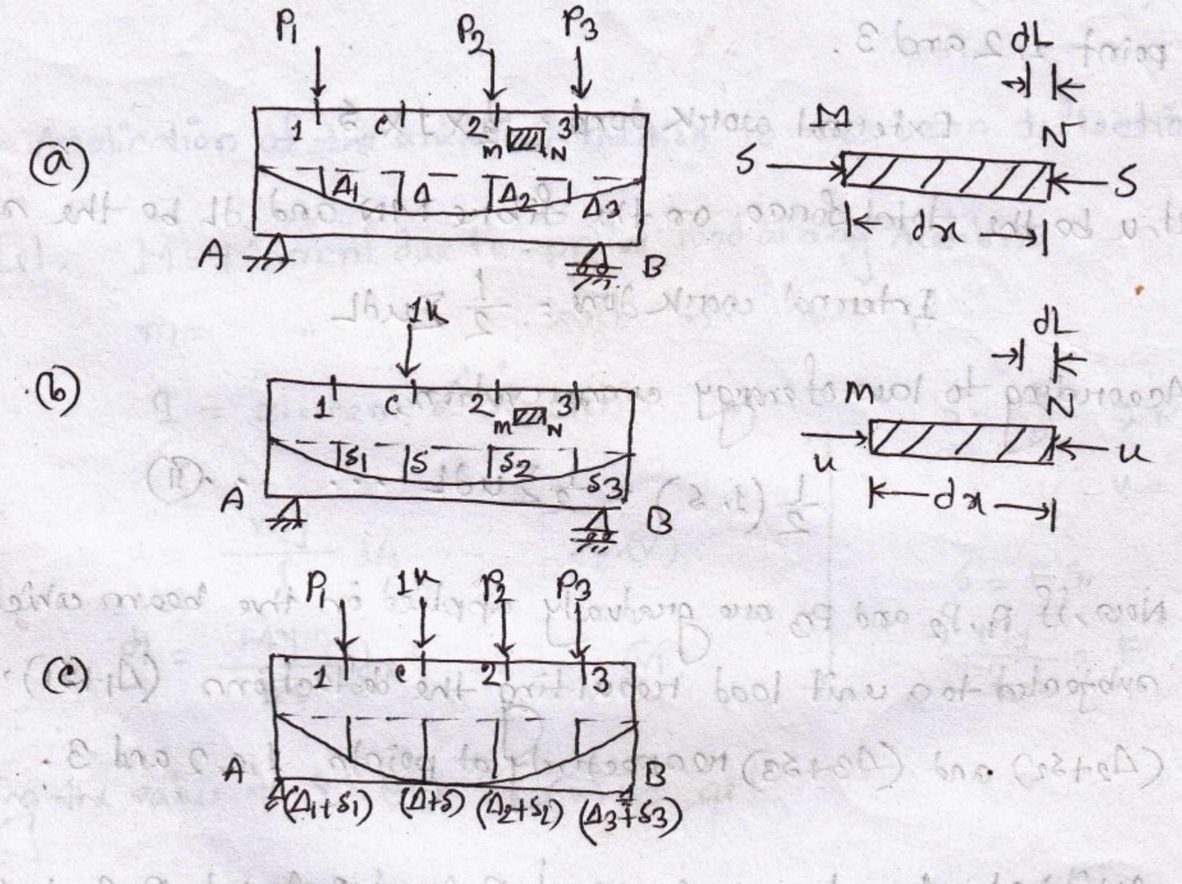
05

Q-1 : Derive the basic formula of unit load method.



Let us consider a simply supported beam AB supported to applied loads P_1, P_2, P_3 at points 1, 2 and 3 respectively. Let, $\Delta_1, \Delta_2, \Delta_3$ are the deflections at point 1, 2 and 3. We have to find out the deflection at point C.

If P_1, P_2 and P_3 are gradually applied,

$$\text{External work done} = \frac{1}{2} P_1\Delta_1 + \frac{1}{2} P_2\Delta_2 + \frac{1}{2} P_3\Delta_3$$

If S be the total force on any fibre MN of length dx and area dA, dL be the shortening,

$$\text{Internal work done} = \frac{1}{2} \sum S\,dL \quad \cdots\cdots (I)$$

According to law of energy conservation,

External work done = Internal work done

$$\Rightarrow \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 = \frac{1}{2}\sum S\,dL \quad \cdots \cdots (i)$$

Let, a unit load is applied first at point C and $\delta_1, \delta_2, \delta_3$ are the deflection at point 1, 2 and 3.

$$\text{External work done} = \frac{1}{2} \times 1 \times \delta$$

Let, u be the total force on the fibre MN and dL be the shortening,

$$\text{Internal work done} = \frac{1}{2}\sum u\,dL$$

According to law of energy conservation,

$$\frac{1}{2}(1.\delta) = \frac{1}{2}\sum u\,dL \quad \cdots \cdots (ii)$$

Now, if P_1, P_2 and P_3 are gradually applied on the beam which is already subjected to a unit load resulting the deflections $(\Delta_1+\delta_1)$, $(\Delta+\delta)$, $(\Delta_2+\delta_2)$ and $(\Delta_3+\delta_3)$ respectively at points 1, C, 2 and 3.

$$\text{Additional external work done} = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + 1.\Delta$$

$$\text{Additional internal work done} = \frac{1}{2}\sum S\,dL + \sum u\,dL$$

$$\text{Total external work done} = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + 1.\Delta + \frac{1}{2}(1.\delta)$$

$$\text{Total internal work done} = \frac{1}{2}\sum S\,dL + \sum u\,dL + \frac{1}{2}\sum u\,dL$$

According to law of energy conservation,

$$\frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + 1.\Delta + \frac{1}{2}(1.\delta) = \frac{1}{2}\sum S\,dL + \sum u\,dL + \frac{1}{2}\sum u\,dL \quad \cdots \cdots (iii)$$

Now, (iii) − [(i) + (ii)]

$$1.\Delta = \sum u\,dL$$

$$\therefore \Delta = \sum u\,dL \quad \cdots \cdots \text{(iv)}$$

This is the basic formula of unit load method,

(Derived).

Q-2: Application of the unit load method to the beam deflection.

Let, M = Moment due to applied load at any section

m = " " " unit " " "

I = Moment of inertia

$s = \frac{P}{A} \Rightarrow P = sA$

$\therefore u = \frac{my}{I} dA$

$s = E\delta$

$\Rightarrow \frac{My}{I} = E.\frac{dL}{dx}$

Now,

$$u = \frac{my}{I} dA \quad \cdots \cdots \text{(v)}$$

$$dL = \frac{My}{EI} dx \quad \cdots \cdots \text{(vi)}$$

Putting the value of (v), (vi) in (iv) we get,

$$1.\Delta = \sum \left(\frac{my}{I} dA\right)\left(\frac{My}{EI} dx\right)$$

$$= \int_0^L \int_0^A \frac{Mmy^2}{EI^2} dA\,dx$$

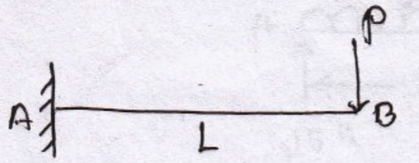
$$= \int_0^L \frac{Mm}{EI^2} dx \int_0^A y^2 dA$$

$$= \int_0^L \frac{Mm}{EI^2} dx . I$$

$$\therefore \Delta = \int_0^L \frac{Mm}{EI} dx .$$

Q-1. Find $\Delta_B = ?$

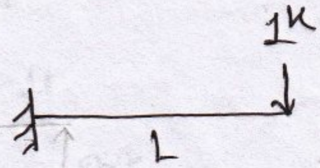
EI constant.



Portion : AB

Limit : 0-L

Origin : B



$M : -Px$

$m : -x$

$$EI\,\Delta_B = \int_0^L (-Px)(-x)\,dx$$

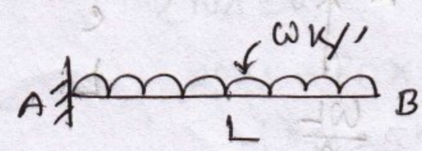
$$= P \cdot \left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{PL^3}{3} \; (\downarrow)$$

$$\therefore \Delta_B = \frac{PL^3}{3EI} \quad \text{(Ans)}.$$

Q-2. Find $\Delta_B = ?$

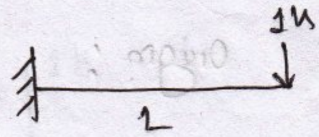
EI constant



Portion : AB

Origin : B

Limit : 0-L



$M : -\frac{\omega x^2}{2}$

$m : -x$

$$EI\,\Delta_B = \int_0^L \left(-\frac{\omega x^2}{2}\right)(-x)\,dx$$

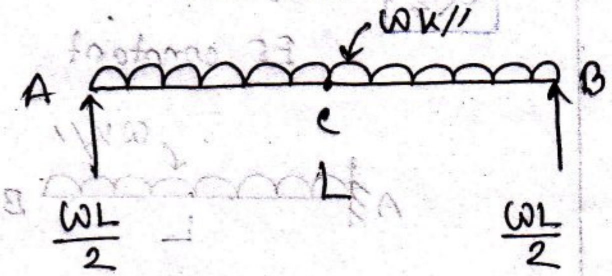
$$= \int_0^L \frac{\omega x^3}{2}\,dx$$

$$= \left[\frac{\omega x^4}{8}\right]_0^L$$

$$= \frac{\omega L^4}{8} \; (\downarrow)$$

(Ans).

Q-3: Find $\Delta c = ?$ EI constant



Portion: AC | BC

Origin: A | B

Limit: $0 - \frac{L}{2}$ | $0 - \frac{L}{2}$

M : $\frac{\omega L}{2} \cdot x - \frac{\omega x^2}{2}$ | $\frac{\omega L}{2} x - \frac{\omega x^2}{2}$

m : $\frac{x}{2}$ | $\frac{x}{2}$

$$EI \, \Delta c = 2\int_0^{\frac{L}{2}} \left(\frac{\omega L}{2} x - \frac{\omega x^2}{2}\right)\left(\frac{x}{2}\right) dx$$

$$= 2\int_0^{\frac{L}{2}} \left(\frac{\omega L x^2}{4} - \frac{\omega x^3}{4}\right) dx$$

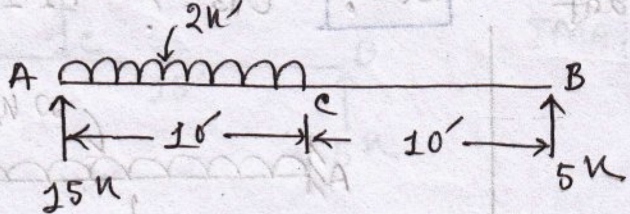
$$2 \cdot \left[\frac{\omega L x^3}{12} - \frac{\omega x^4}{16}\right]_0^{L/2}$$

$$= 2\left(\frac{\omega L \cdot L^3}{96} - \frac{\omega L^4}{256}\right)$$

$$= \frac{5\,\omega L^4}{384} \ (\downarrow)$$

(Ans).

Q-4: $E = 30000$ ksi, $I = 1000$ in^4, $\Delta_C = ?$

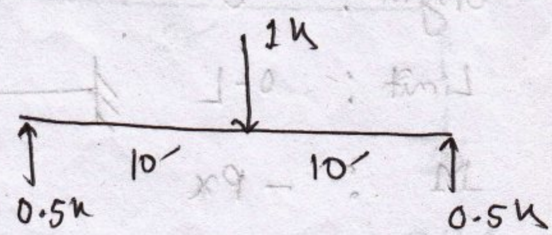


$\Sigma M_A = 0,$

$20 \times 5 = R_B \times 20$

$\therefore R_B = 5k$

Portion :	AC	BC
Origin :	A	B
Limit :	0-10	0-10
M :	$15x - \frac{2x^2}{2}$	$5x$
m :	$\frac{x}{2}$	$\frac{x}{2}$



$$\Delta_C EI = \int_0^{10} (15x - x^2)\left(\frac{x}{2}\right) dx + \int_0^{10} 5x \cdot \frac{x}{2}\, dx$$

$$= \int_0^{10} \left(\frac{15x^2}{2} - \frac{x^3}{2}\right) dx + \int_0^{10} \frac{5x^2}{2}\, dx$$

$$= \left[\frac{15x^3}{6} - \frac{x^4}{8}\right]_0^{10} + \left[\frac{5x^3}{6}\right]_0^{10}$$

$$= \frac{15 \times 1000}{6} - \frac{(10)^4}{8} + \frac{5 \times 1000}{6}$$

$$= 2500 - 1250 + 833.33$$

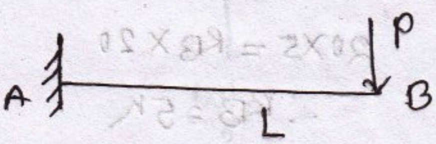
$$= 2083.33$$

$$\Rightarrow \Delta_C = \frac{2083.33 \times 1728}{30000 \times 1000} = 0.12 \ (\downarrow)$$

(Ans).

Rotation

Q-5: $\theta_B = ?$ $EI =$ constant



Portion: AB

Origin: B

Limit: 0-L

M : $-Px$

m : -1

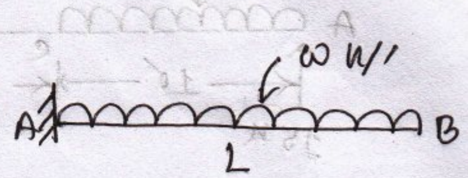
$$EI\,\theta_B = \int_0^L (-Px)(-1)\,dx$$

$$= \left[\frac{Px^2}{2}\right]_0^L$$

$$= \frac{PL^2}{2}$$ (↓) clockwise

(Ans)

Q-6: $\theta_B = ?$ $EI =$ constant



Portion: AB

Origin: B

Limit: 0-L

M : $-\frac{\omega x^2}{2}$

m : -1

$$EI\,\theta_B = \int_0^L \left(-\frac{\omega x^2}{2}\right)(-1)\,dx$$

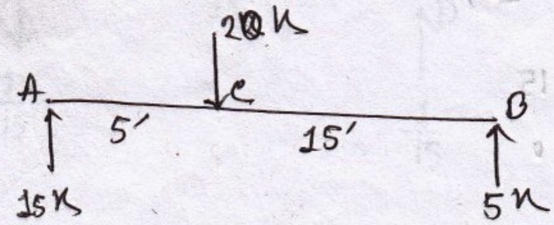
$$= \int_0^L \frac{\omega x^2}{2}\,dx$$

$$\left[\frac{\omega x^3}{6}\right]_0^L$$

$$= \frac{\omega L^3}{6}$$ (clockwise)

(Ans)

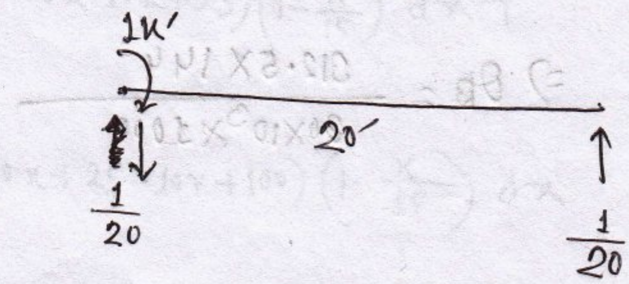
$\theta_A, \theta_B = ?$ $E = 30 \times 10^3$ ksi $I = 1000$ in^4

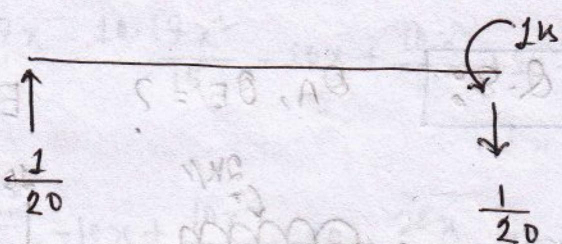


$\Sigma M_A = 0$, $20 \times 5 = R_B \times 20$

$\therefore R_B = 5k$

Portion	:	AC	BC
Origin	:	A	B
Limit	:	0-5	0-15
M	:	$15x$	$5x$
$m\theta_A$	:	$1-\frac{x}{20}$	$\frac{x}{20}$
$m\theta_B$	:	$\frac{x}{20}$	$1-\frac{x}{20}$





$$EI\theta_A = \int_0^5 15x\left(1-\frac{x}{20}\right)dx + \int_0^{15} 5x \cdot \frac{x}{20}\,dx$$

$$= \left[\frac{15x^2}{2} - \frac{15x^3}{60}\right]_0^5 + \left[\frac{5x^3}{60}\right]_0^{15}$$

$$= 187.5 - 31.25 + 281.25$$

$$= 437.5$$

$$\Rightarrow \theta_A = \frac{437.5 \times 144}{30 \times 10^3 \times 1000} = 0.002 \text{ rad (clockwise)}$$

$$EI\theta_B = \int_0^5 15x \cdot \frac{x}{20}\,dx + \int_0^{15} 5x\left(1-\frac{x}{20}\right)dx$$

$$= \left[\frac{15x^3}{60}\right]_0^5 + \left[\frac{5x^2}{2} - \frac{5x^3}{60}\right]_0^{15}$$

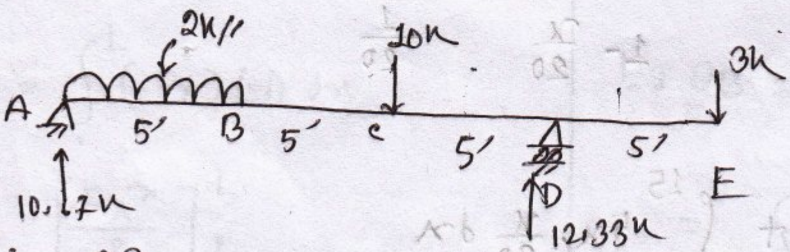
$$= 31.25 + 562.5 - 281.25$$

$$= 312.5$$

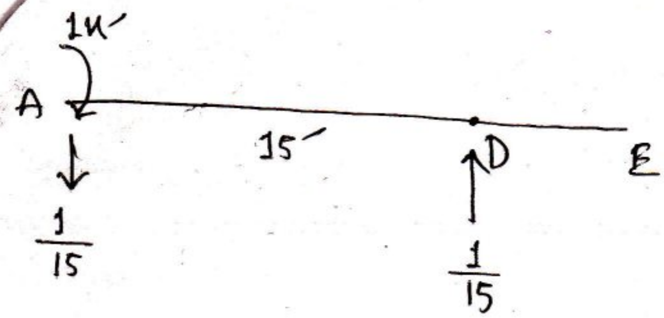
$$\Rightarrow \theta_B = \frac{312.5 \times 144}{30\times10^3 \times 1000} = 0.0015 \text{ rad (counter-clockwise)}$$

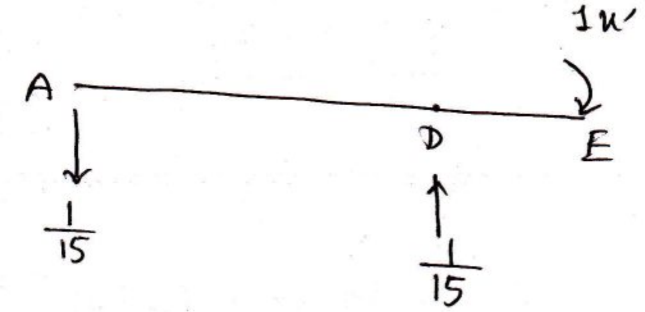
(Ans).

Q-8: $\theta_A, \theta_E = ?$ $E = 30\times10^3$ ksi $I = 1000$ in^4



Portion:	AB	BC	CD	DE
Origin:	A	A	A	E
Limit:	0-5	5-10	10-15	0-5
M:	$10.67x - \frac{2x^2}{2}$	$10.67x - 10(x-2.5)$	$10.67x - 10(x-2.5) - 10(x-10)$	$-3x$
m_{θ_A}:	$1-\frac{x}{15}$	$1-\frac{x}{15}$	$1-\frac{x}{15}$	0
m_{θ_E}:	$-\frac{x}{15}$	$-\frac{x}{15}$	$-\frac{x}{15}$	-1





$$EI\theta_A = \int_0^5 (10.67x - x^2)\left(1-\frac{x}{15}\right)dx + \int_5^{10}(10.67x - 10x + 25)\left(1-\frac{x}{15}\right)dx +$$

$$\int_{10}^{15}(10.67x - 10x + 25 - 10x + 100)\left(1-\frac{x}{15}\right)dx$$

$$= \int_0^5\left(10.67x - \frac{10.67x^2}{15} - x^2 + \frac{x^3}{15}\right)dx + \int_5^{10}\left(10.67x - \frac{10.67x^2}{15} - 10x + \frac{10x^2}{15} + 25 - \frac{25x}{15}\right)dx$$

$$+ \int_{10}^{15}\left(10.67x - \frac{10.67x^2}{15} - 10x + \frac{10x^2}{15} + 25 - \frac{25x}{15} - 10x + \frac{10x^2}{15} + 100 - \frac{100x}{15}\right)dx$$

$$= \left[\frac{10.67x^2}{2} - \frac{10.67x^3}{45} - \frac{x^3}{3} + \frac{x^4}{60}\right]_0^5 + \left[\frac{10.67x^2}{2} - \frac{10.67x^3}{45} - \frac{10x^2}{2} + \frac{10x^3}{45} + 25x - \frac{25x^2}{30}\right]_5^{10}$$

$$+ \left[\frac{10.67x^2}{2} - \frac{10.67x^3}{45} - \frac{10x^2}{2} + \frac{10x^3}{45} + 25x - \frac{25x^2}{30} - \frac{10x^2}{2}\right]_{10}^{15}$$

$$= \cancel{133.375} - \cancel{29.64} - 41.67 + 10.42 + \cancel{533.5} - \cancel{237.11} - \cancel{500} + \cancel{222.22} + \cancel{250} - \cancel{83.33}$$
$$- \cancel{133.375} + \cancel{29.64} + 125 - 27.78 - 125 + 20.83 + 1200.38 - 800.25 - 1125$$
$$+ 750 + 375 - 187.5 - 1125 - \cancel{533.5} + \cancel{237.11} + \cancel{500} - \cancel{222.22} - \cancel{250} + \cancel{83.33}$$
$$- 500$$

$$= -1450.57 \qquad \therefore \theta_A = -0.007 \text{ radian}$$

06

CE-313

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Elementary Structural Analysis by Approximate method

Q. Write down the assumptions for vertical load analysis. (Portal method)

Assumptions for vertical load analysis:

i) The axial force in the girder is assumed to be zero.

ii) There is a hinge point at a distance 0.1L from both the left and right support.

Q. Write down the assumptions for lateral load analysis (Portal method)

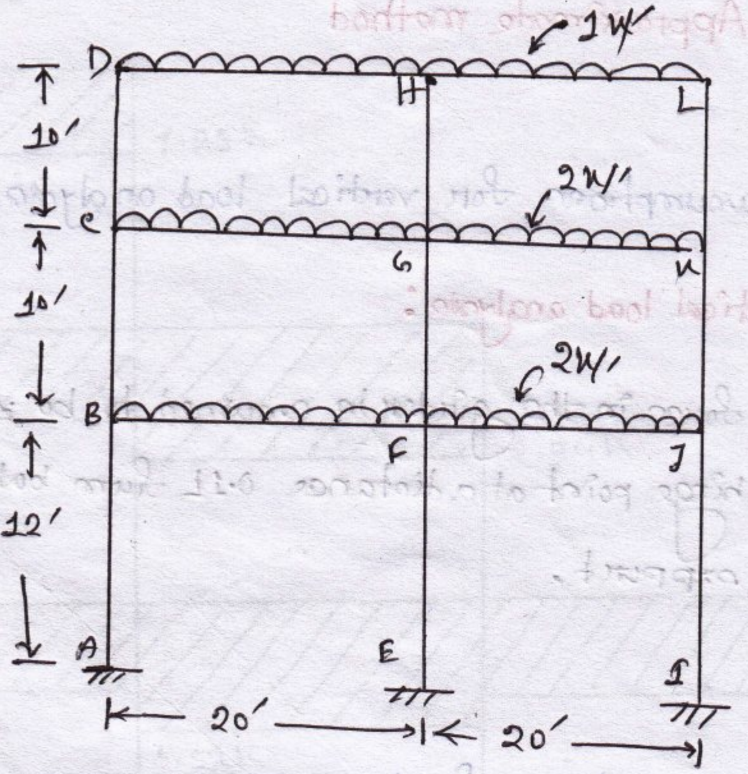
Assumption for lateral load analysis:

i) The total horizontal shear in all columns of a given story is equal and opposite to the sum of all horizontal loads acting above the story.

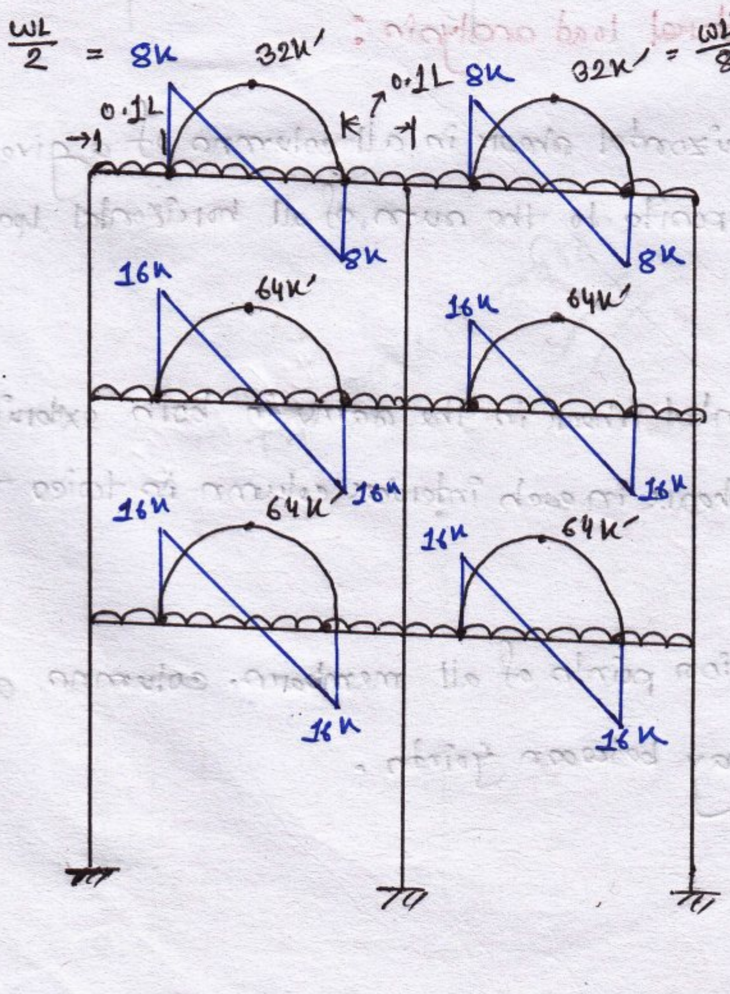
ii) The horizontal shear is the same in both exterior columns, the horizontal shear in each interior column is twice that in a exterior column.

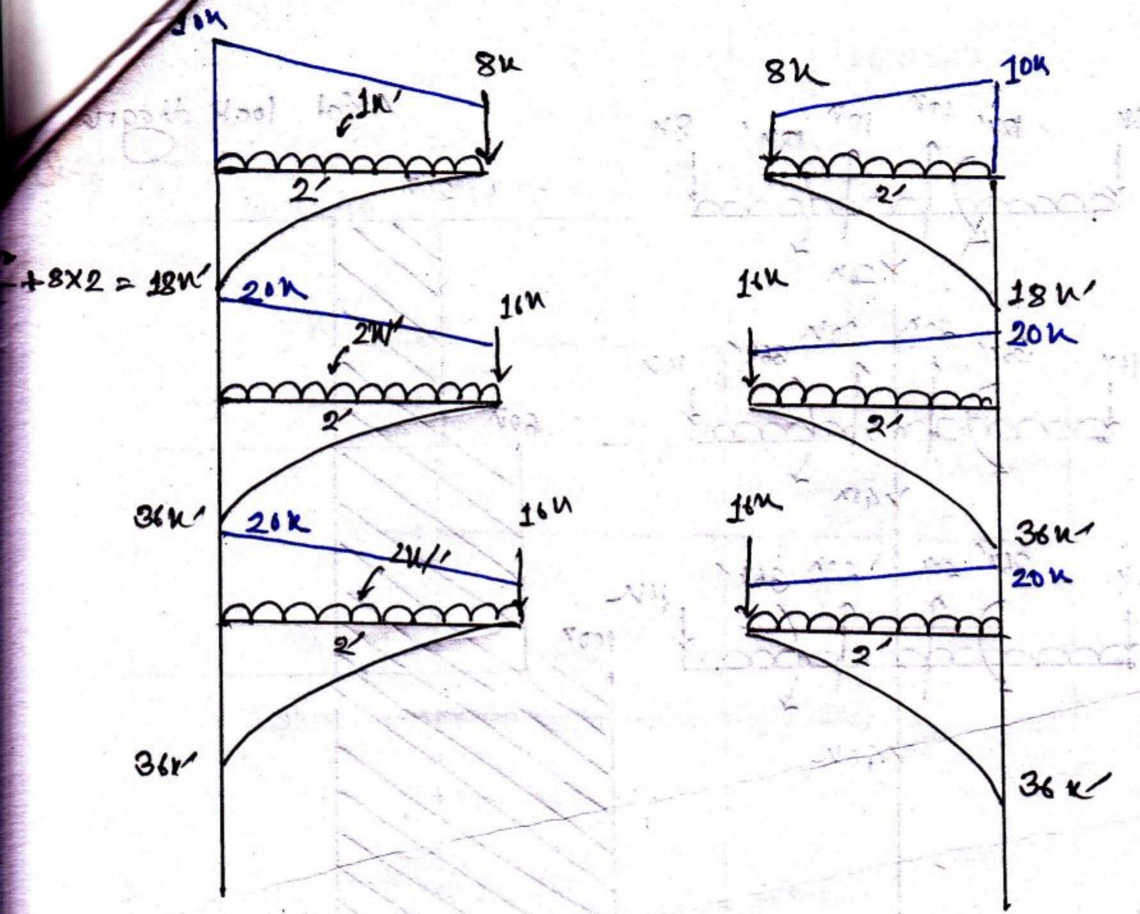
iii) The inflection points of all members, columns and girders are located midway between joints.

Q-1: Draw SFD & BMD for girder & column for the following.

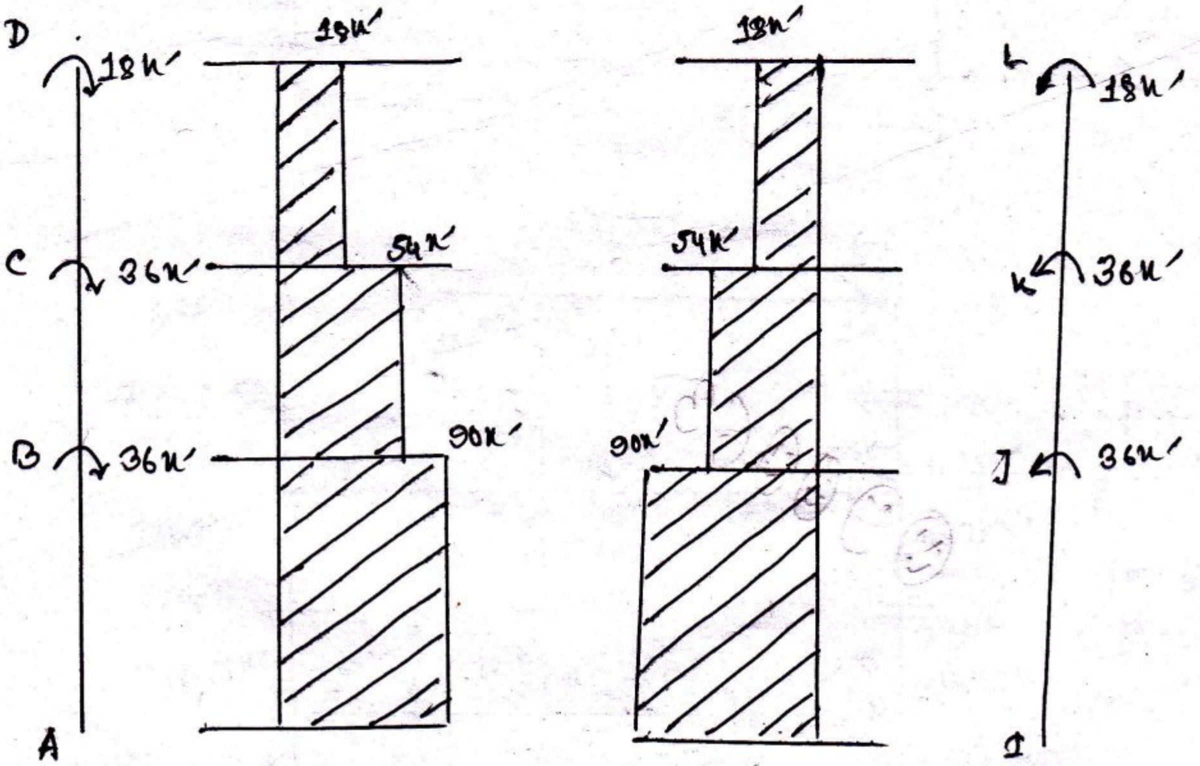


Solution:

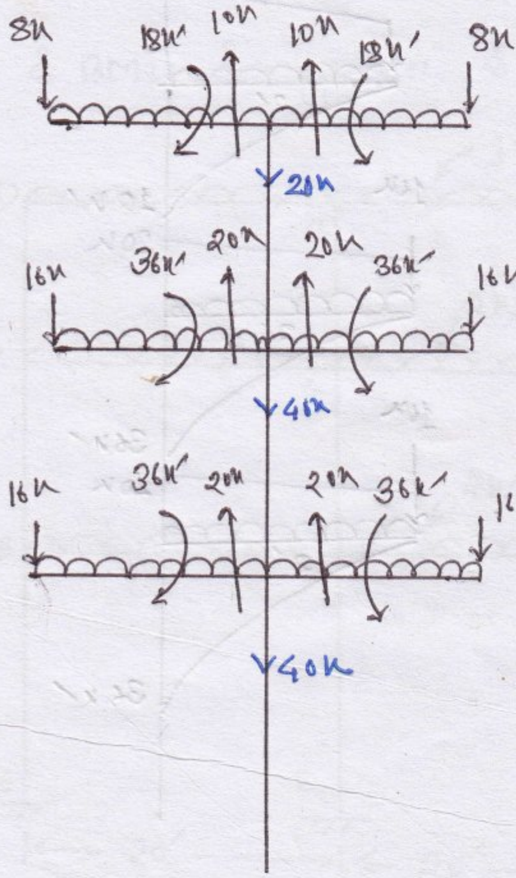




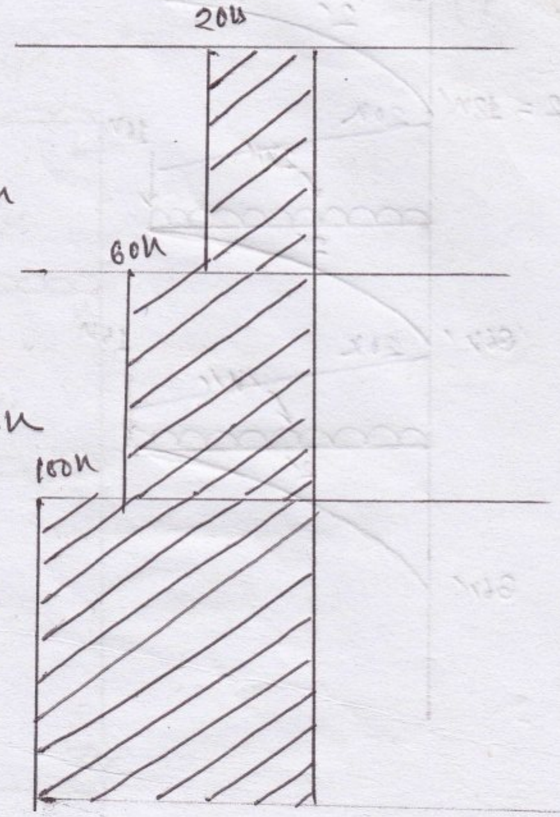
10k
8k
1k'
2'
+8x2 = 18k'
20k
16k
2k'
2'
36k'
20k
16k
2k'
2'
36k'
8k
10k
2'
16k
18k'
20k
2'
16k
36k'
20k
2'
36k'



D
18k'
18k'
C
36k'
54k'
B
36k'
90k'
A
18k'
54k'
90k'
L
18k'
K
36k'
J
36k'
I

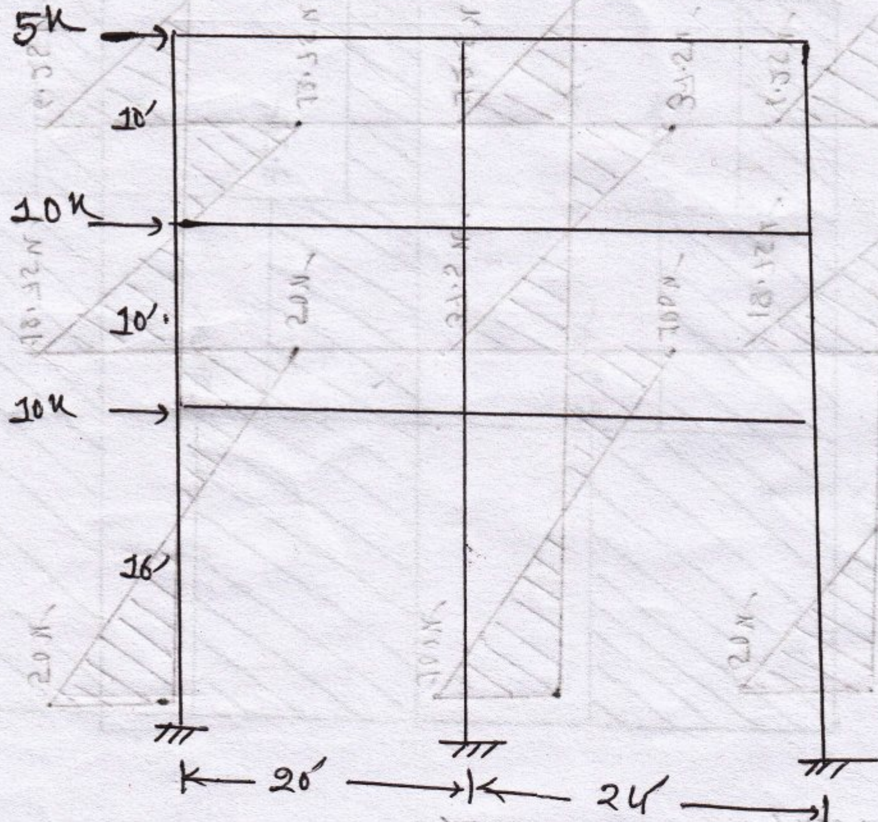


Axial load diagram

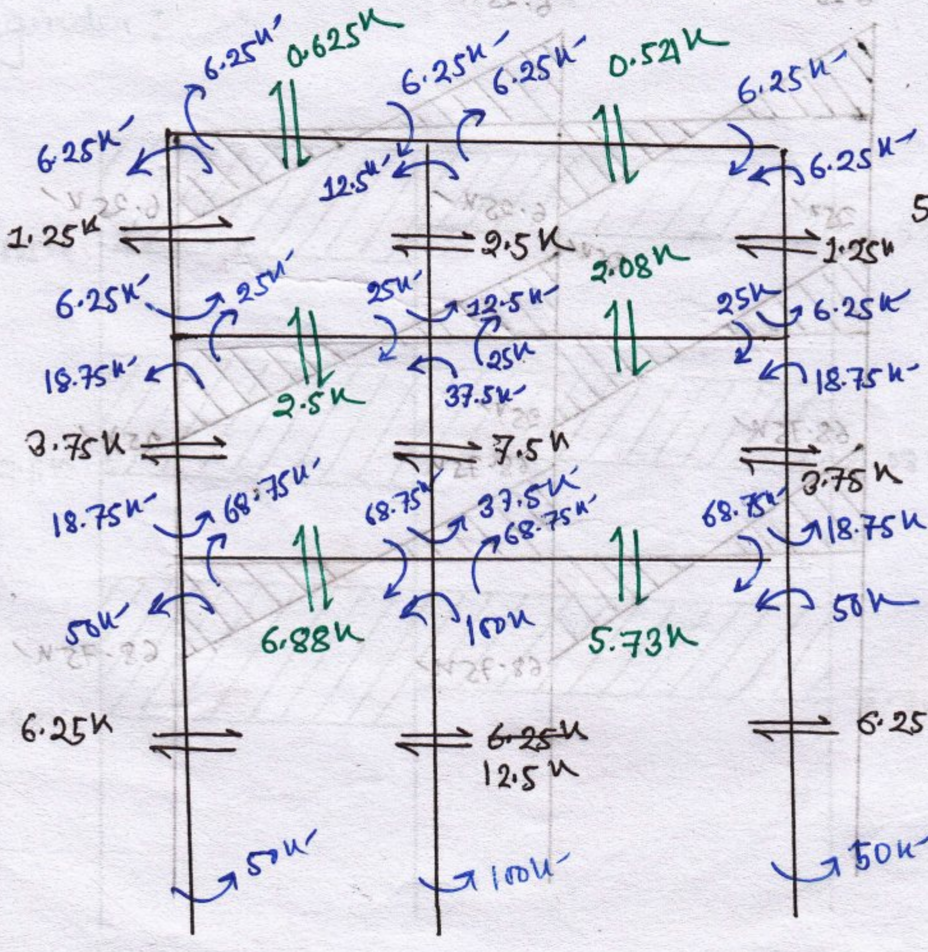


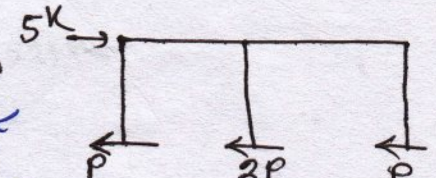
Portal Method

Q-2: Draw SFD and BMD



Solution:

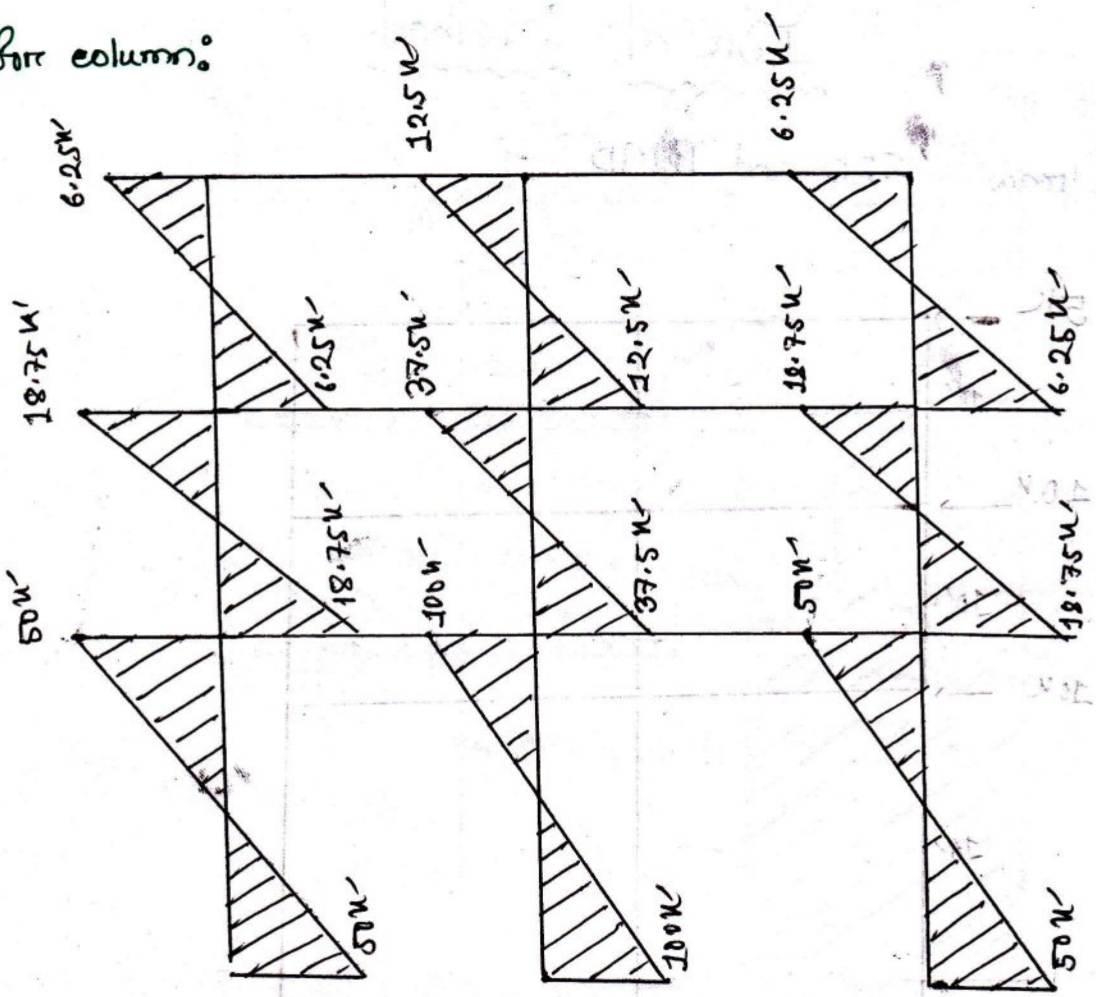




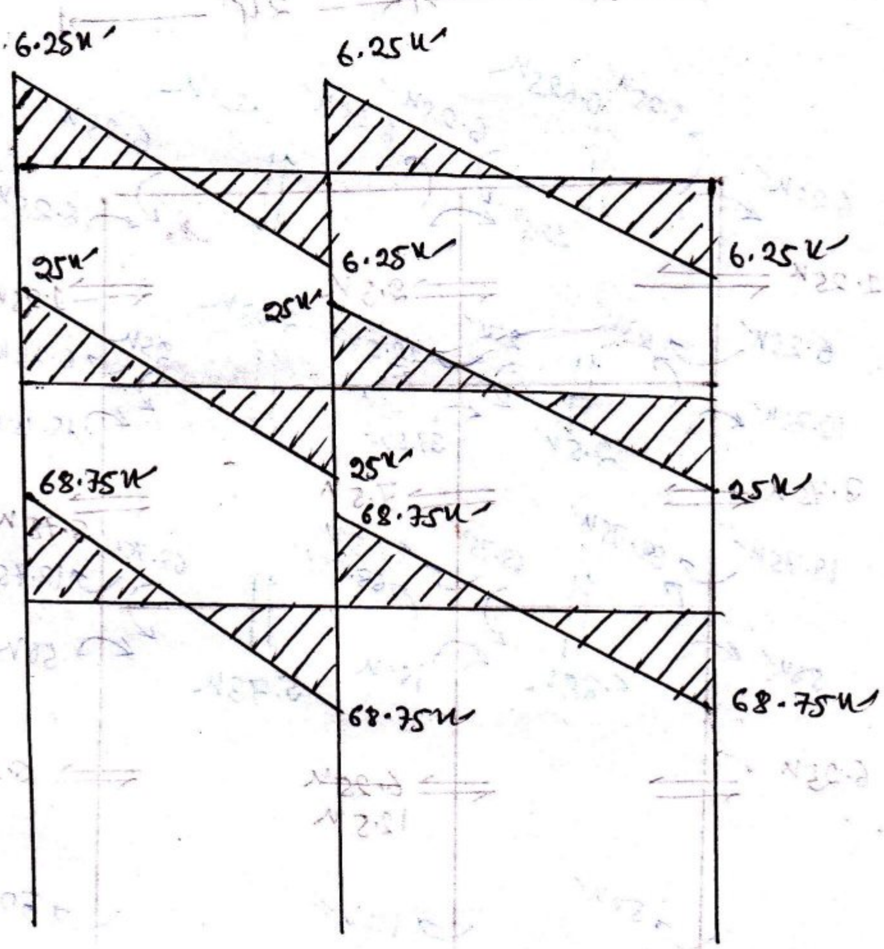
$4P = 5$

$\therefore P = 1.25$

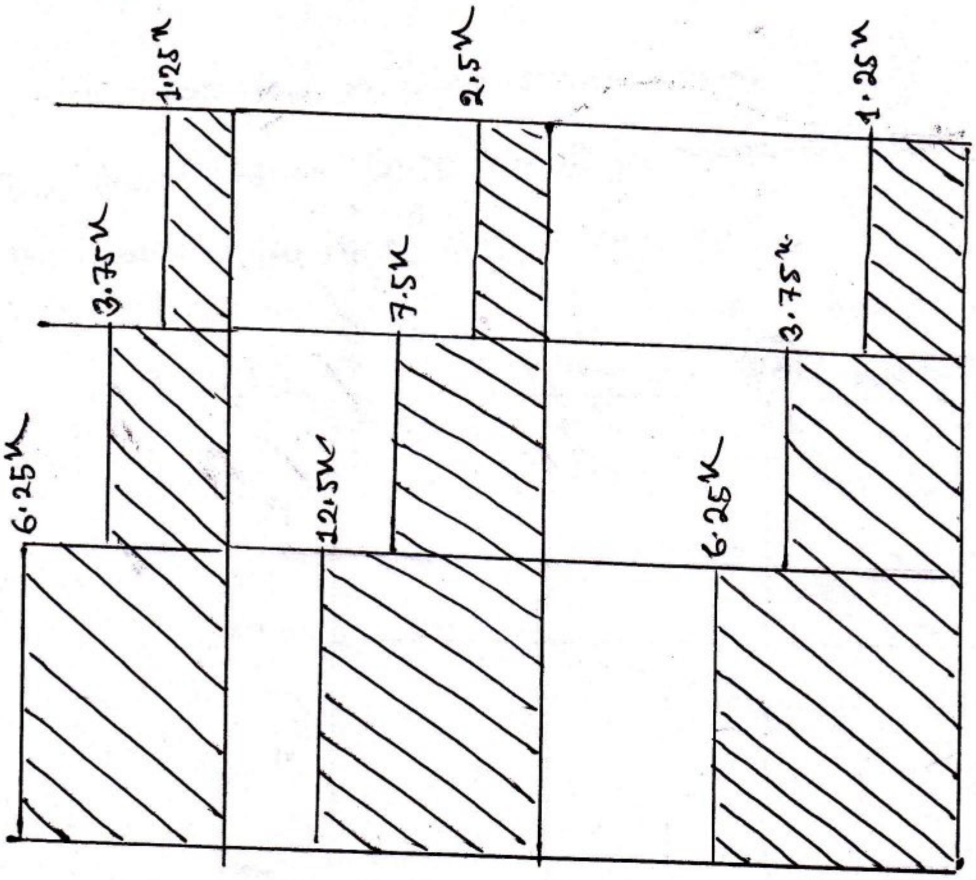
BMD for column:



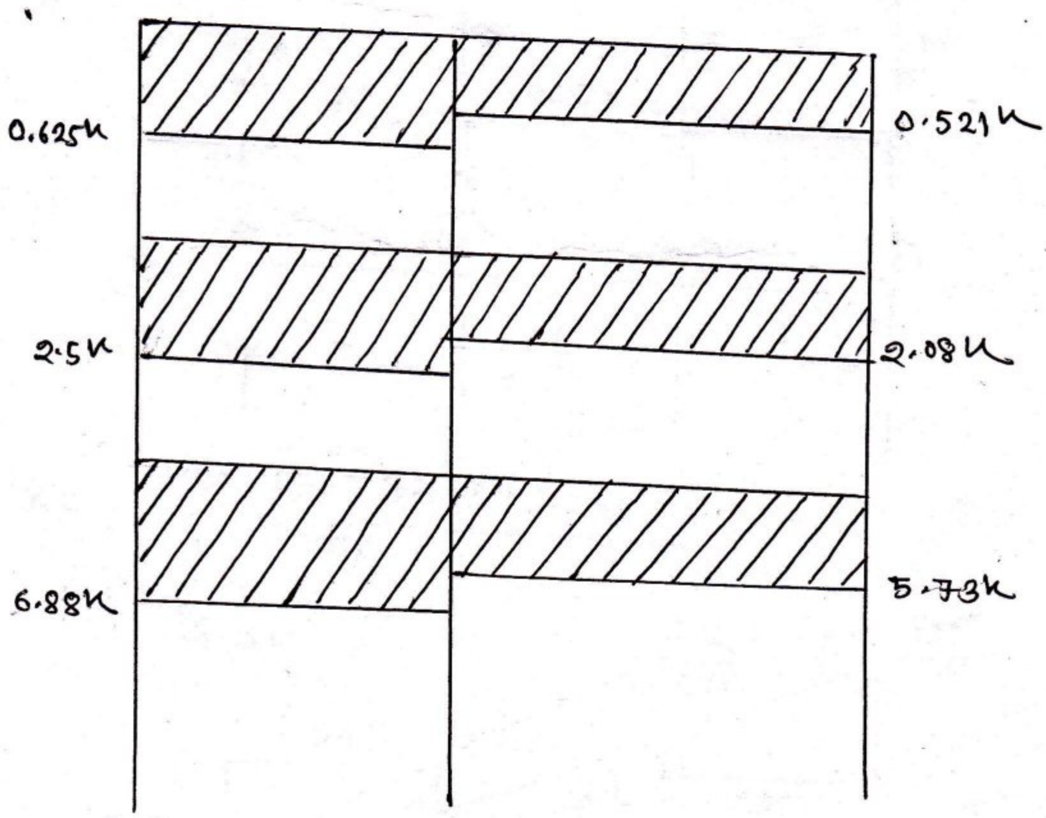
BMD for Girder:



SFD for column:

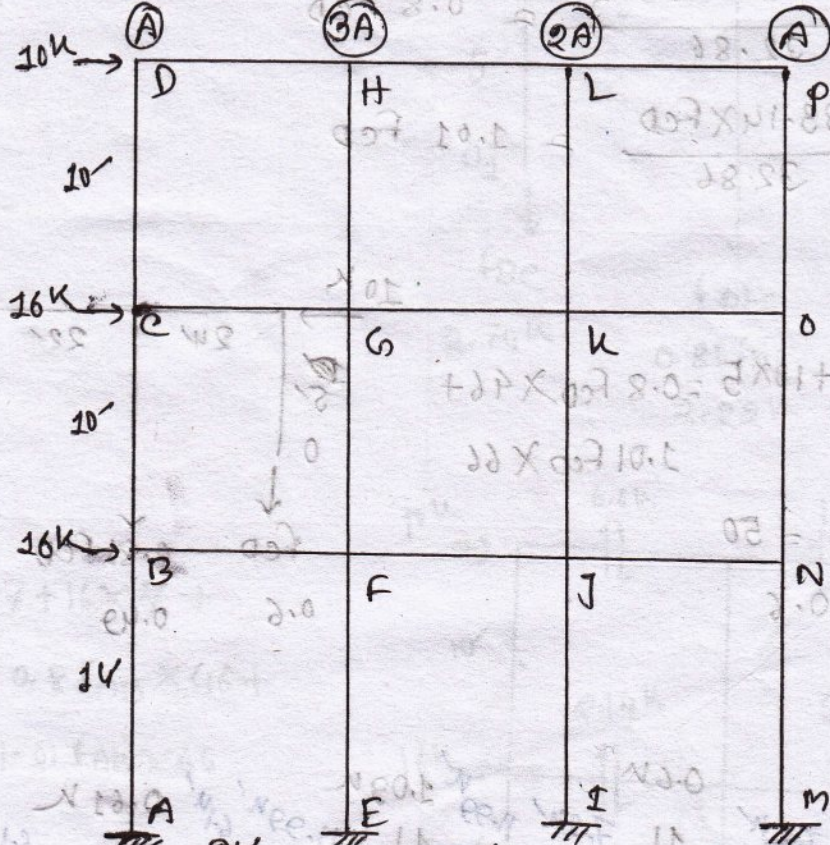


SFD for girder:

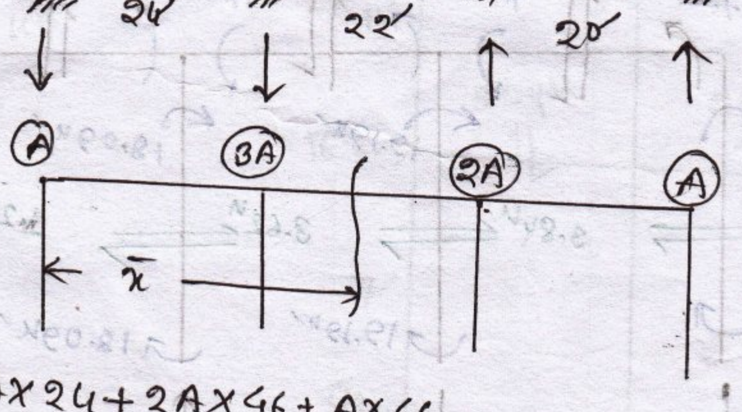


Cantilever Method

Q. Analyze the building frame shown in fig below by cantilever method. Also draw SFD and BMD for all the members. Cross-section areas of column are shown at the top of each column.

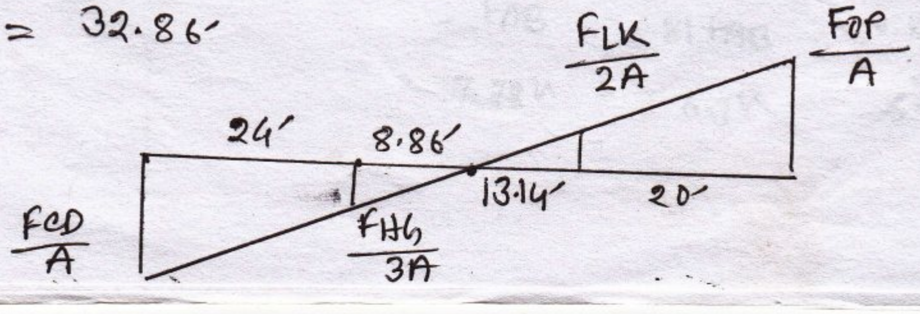


Solution:



$$\bar{x} = \frac{3A \times 24 + 2A \times 46 + A \times 66}{7A}$$

$$= 32.86'$$



$$\frac{\frac{F_{CD}}{A}}{32.86} = \frac{\frac{F_{HG}}{3A}}{8.86} = \frac{\frac{F_{LK}}{2A}}{13.14'} = \frac{\frac{F_{OP}}{A}}{33.14'}$$

$$F_{CD} = \frac{32.86 \times F_{HG}}{3 \times 8.86} = 1.24\ F_{HG} \quad \therefore F_{HG} = 0.81\ F_{CD}$$

$$F_{LK} = \frac{13.14 \times F_{CD} \times 2}{32.86} = 0.8\ F_{CD}$$

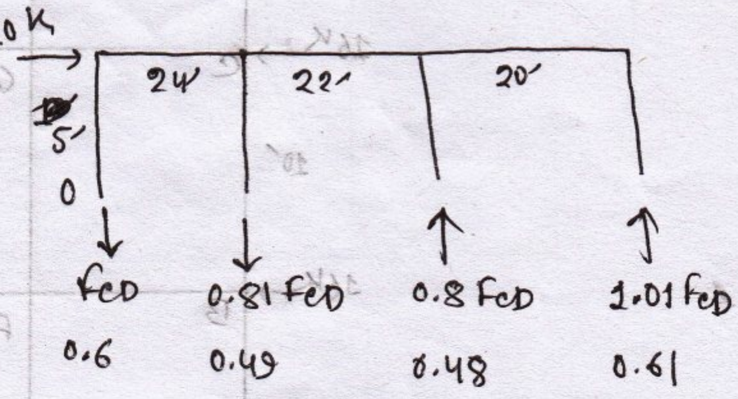
$$F_{OP} = \frac{33.14 \times F_{CD}}{32.86} = 1.01\ F_{CD}$$

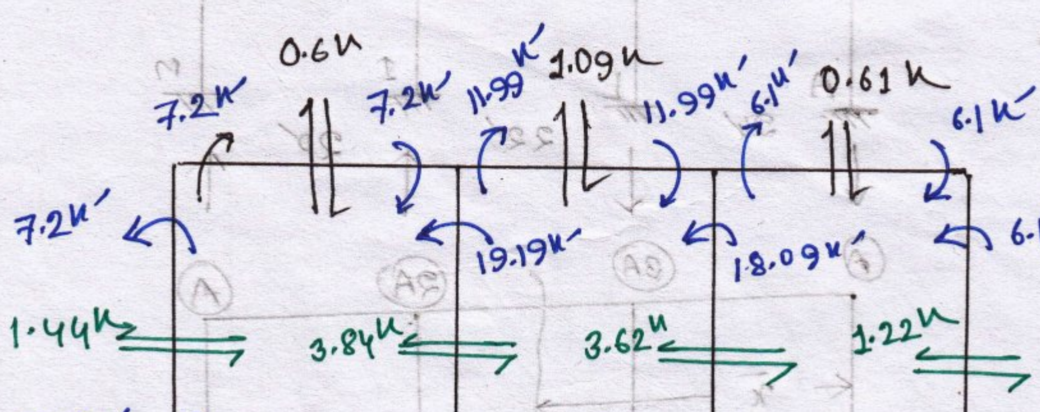
$\Sigma M_O = 0,$

$0.81 F_{CD} \times 24 + 10 \times 5 = 0.8 F_{CD} \times 46 + 1.01 F_{CD} \times 66$

$\Rightarrow 84.02 F_{CD} = 50$

$\therefore F_{CD} = 0.6$



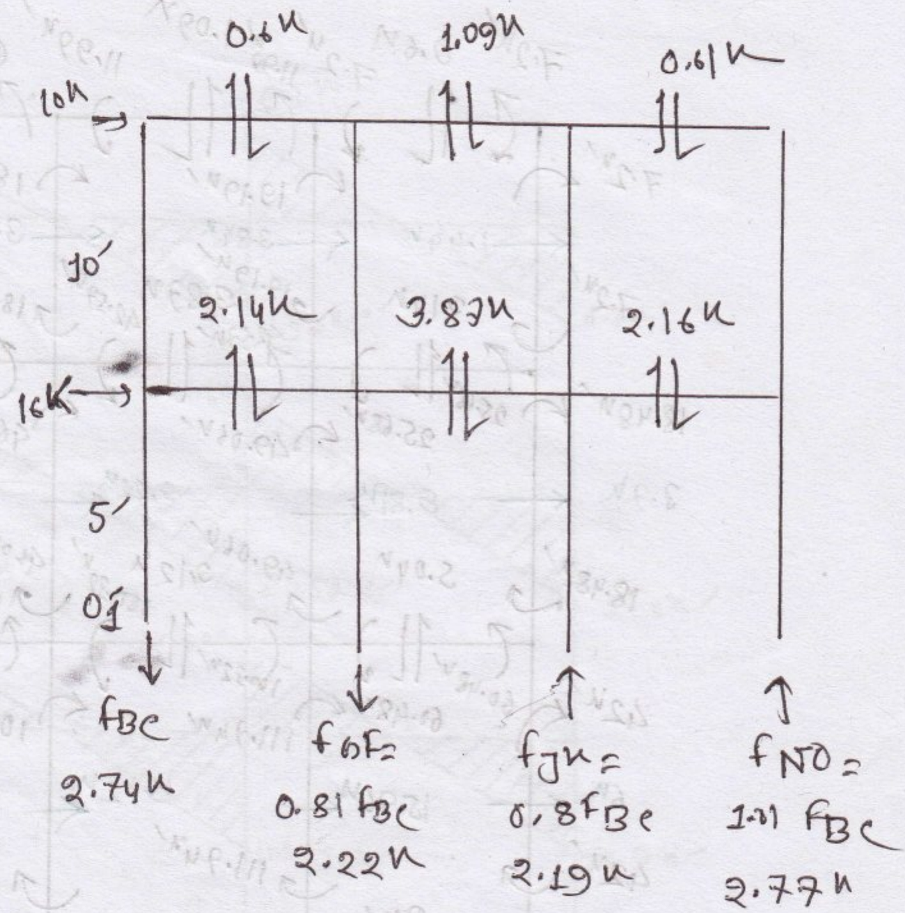


$\Sigma M_{O_1} = 0,$

$10 \times 15 + 16 \times 5 + 0.81 F_{BC} \times 24 =$

$0.8 F_{BC} \times 46 + 1.01 F_{BC} \times 66$

$\therefore F_{BC} = 2.74^k$



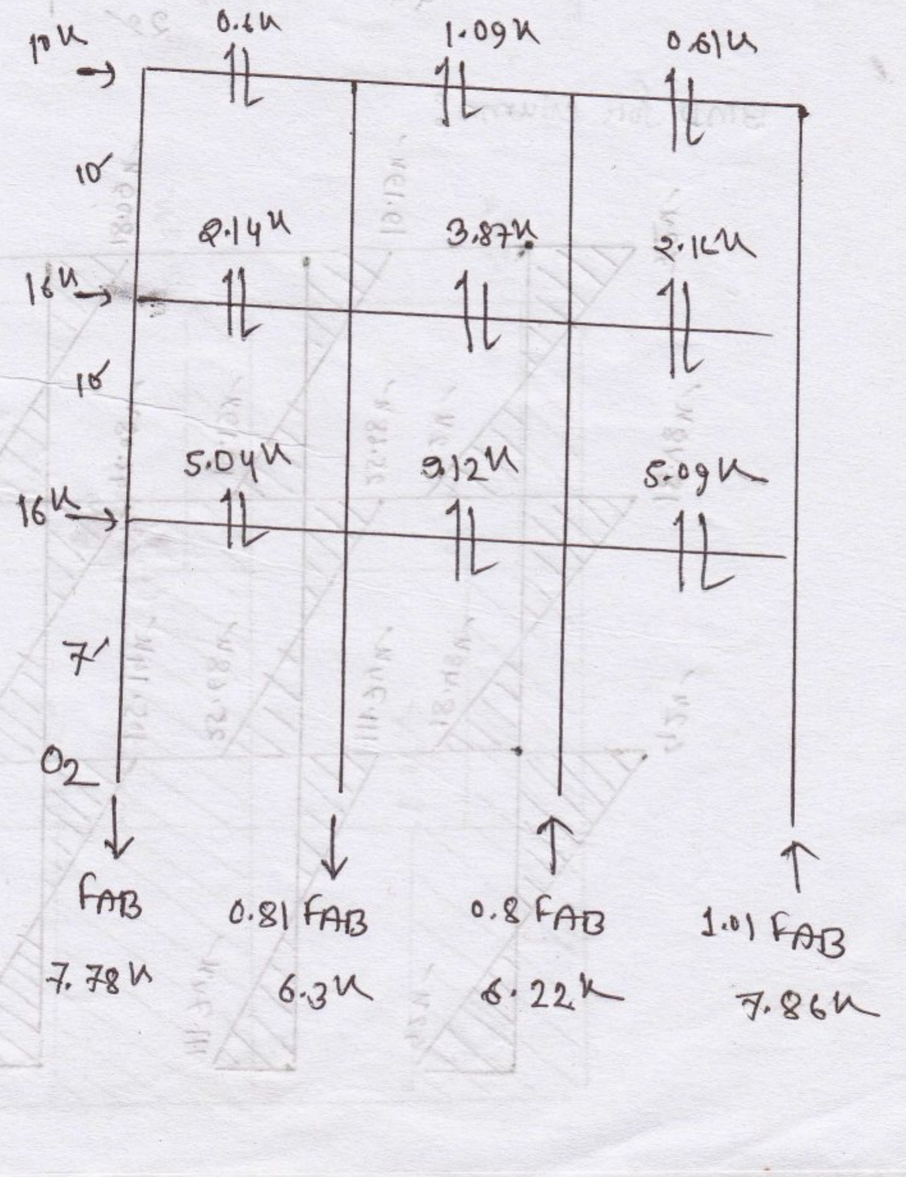
$\Sigma M_{O_2} = 0,$

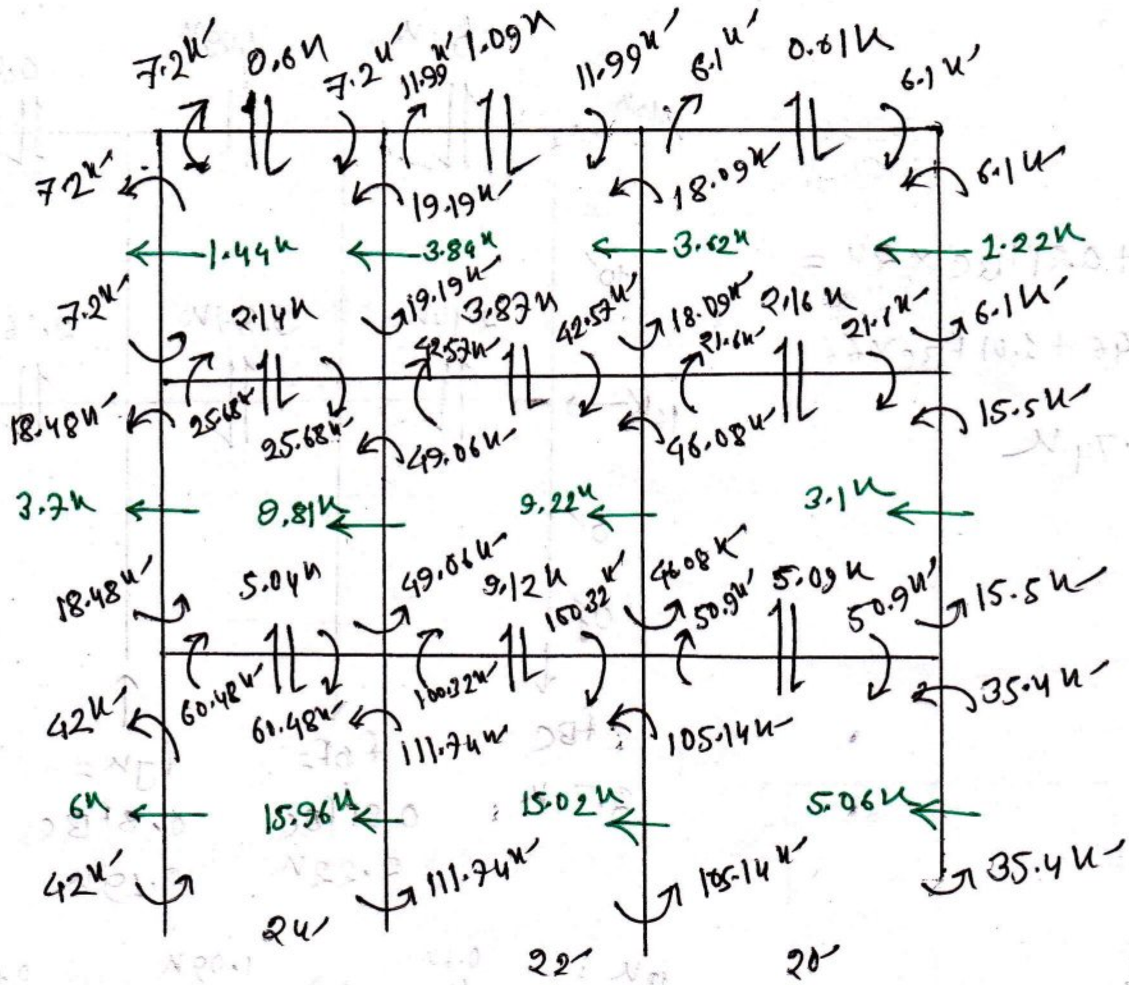
$10 \times 27 + 16 \times 17 + 16 \times 7 +$

$0.81 F_{AB} \times 24 = 0.8 F_{AB} \times 46 +$

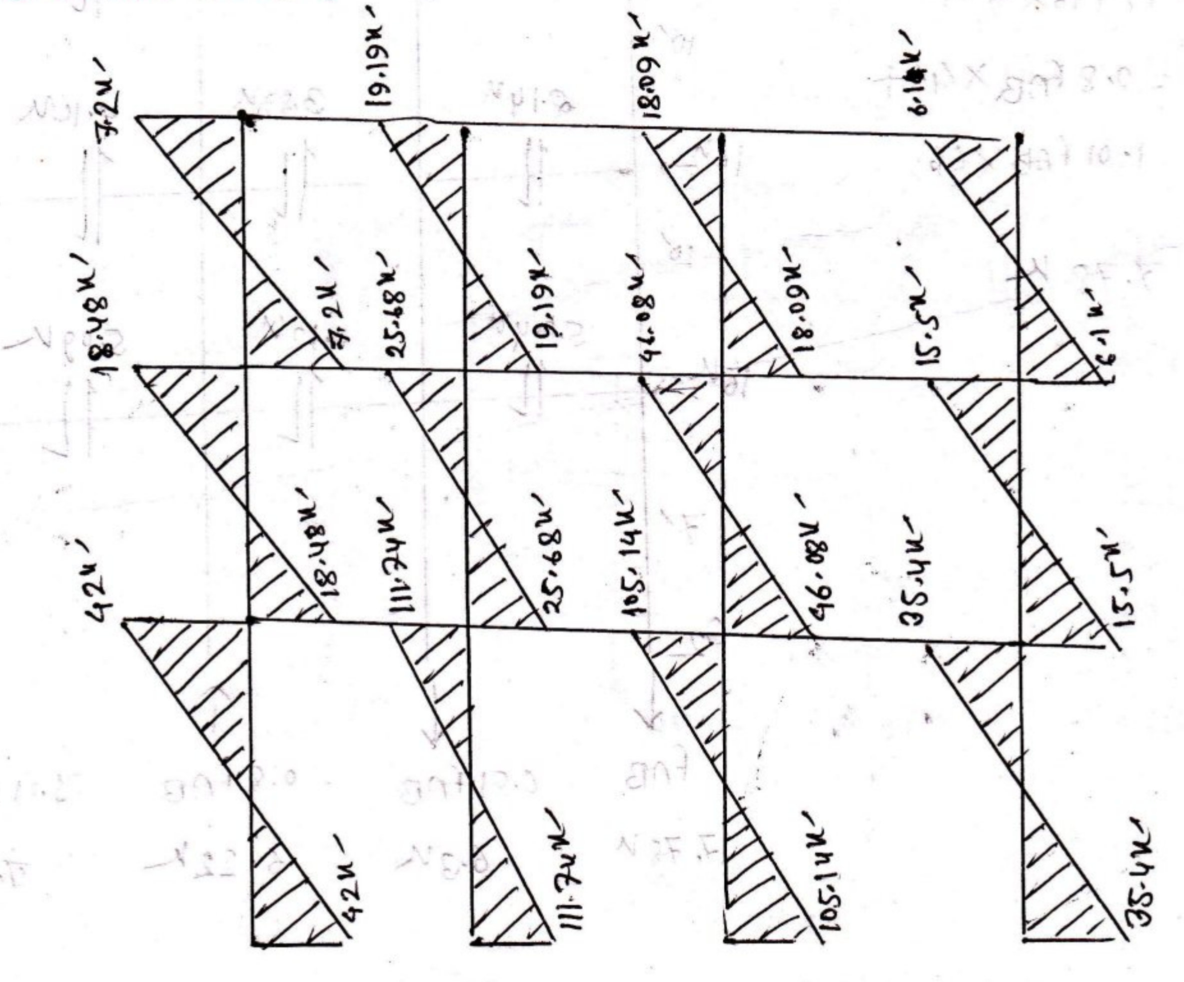
$1.01 F_{AB} \times 66$

$\therefore F_{AB} = 7.78^k$

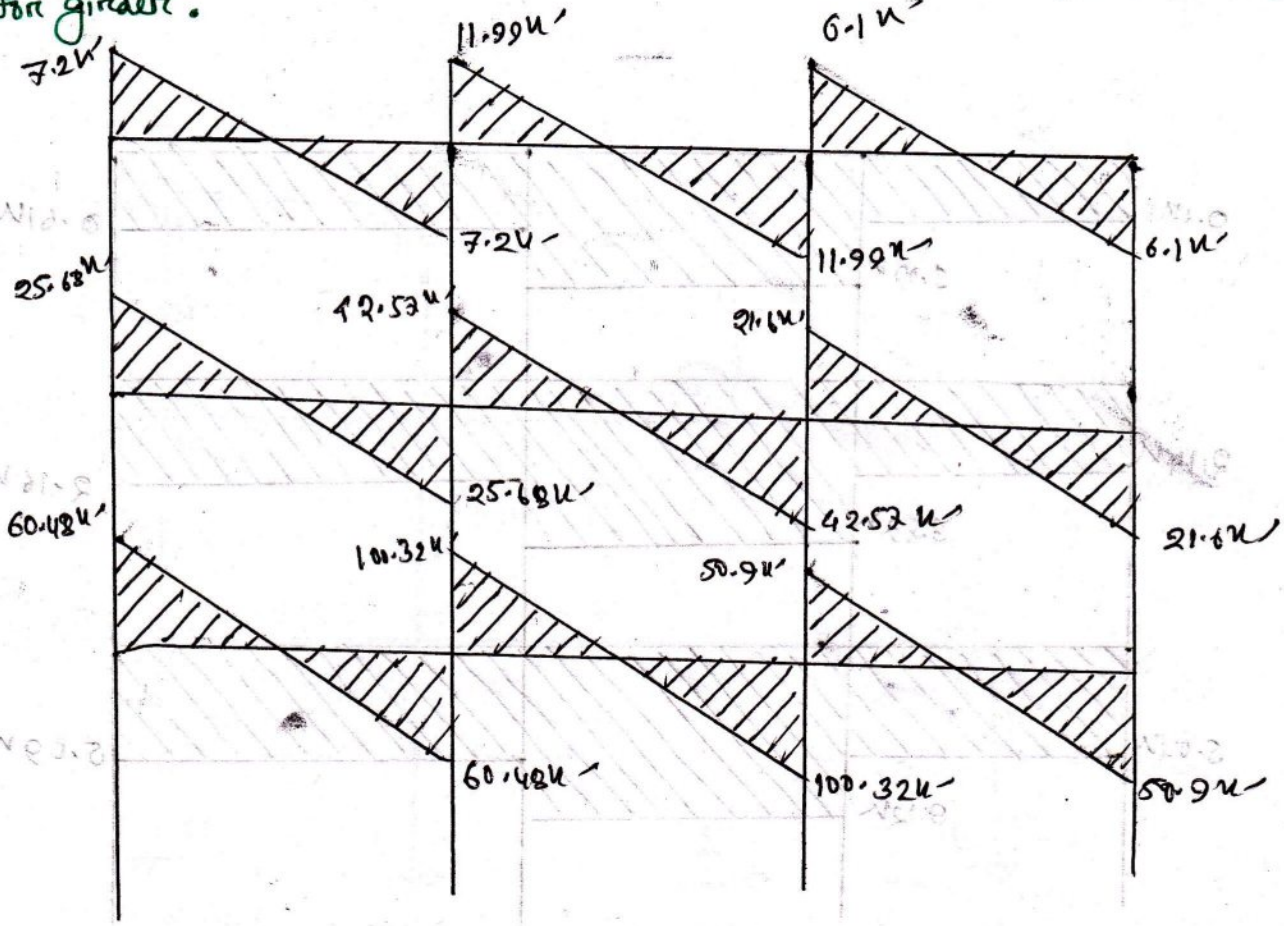




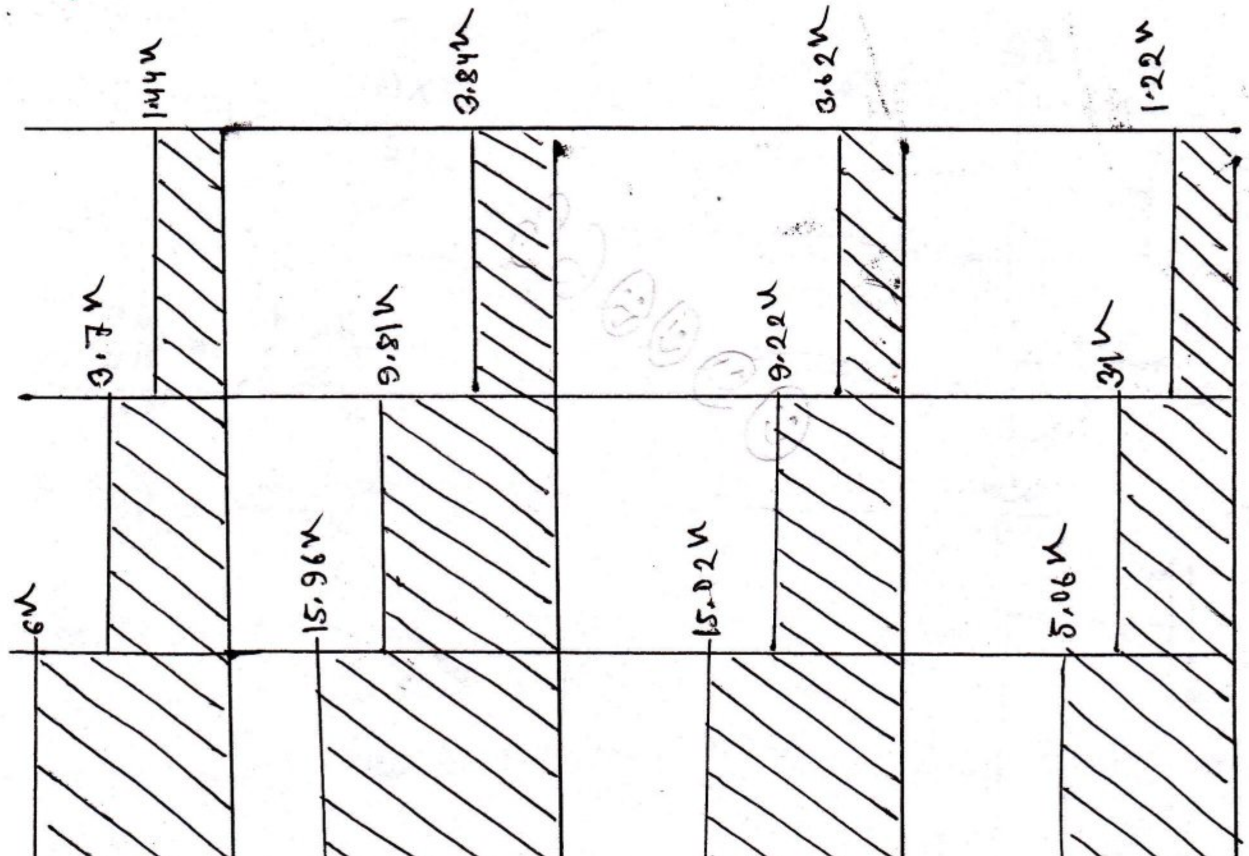
BMD for column:



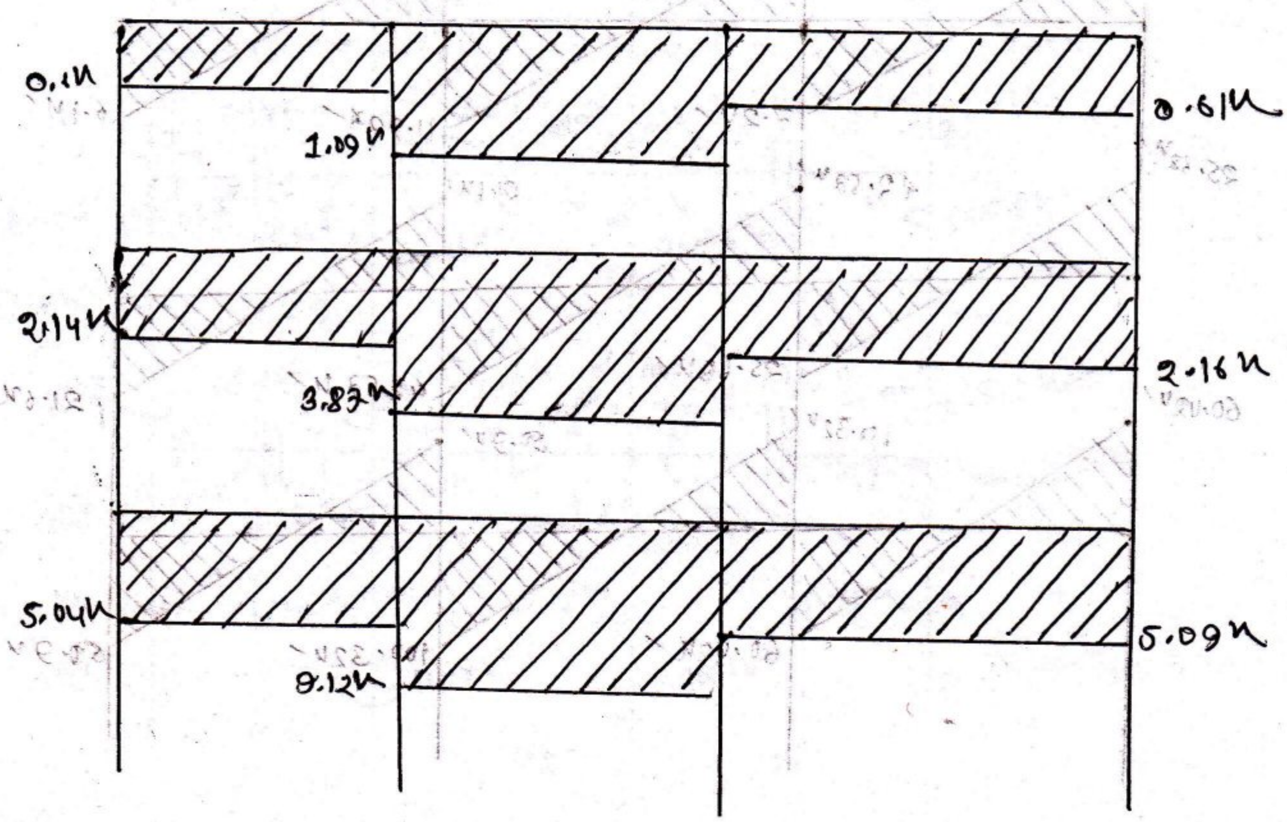
BMD for girder:



SFD for column:

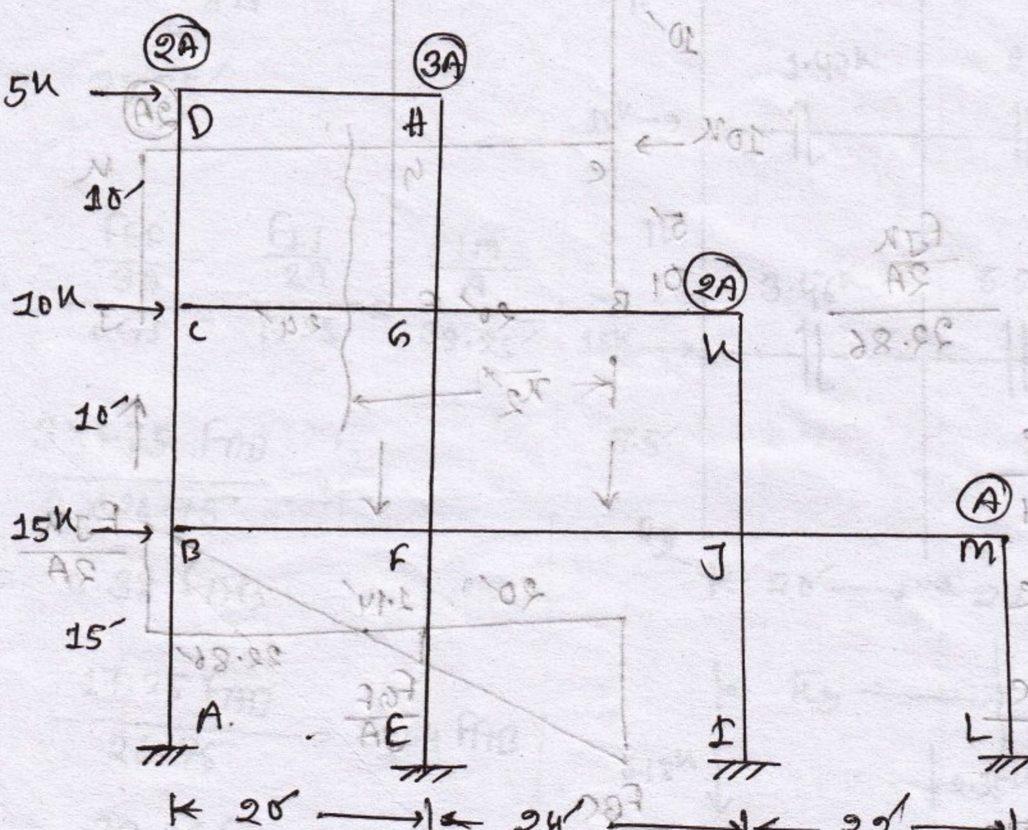


SFD for girder:



$\boxed{08}$

Q. 3



Solution:

$$\bar{x}_1 = \frac{3A \times 20}{5A} = 12'$$

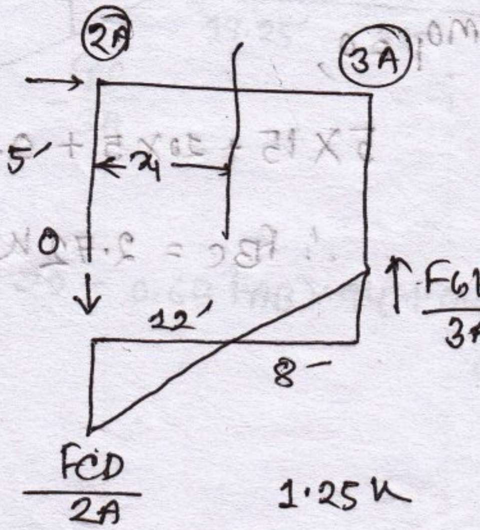
$$\frac{\frac{F_{CD}}{2A}}{12} = \frac{\frac{F_{GH}}{3A}}{8}$$

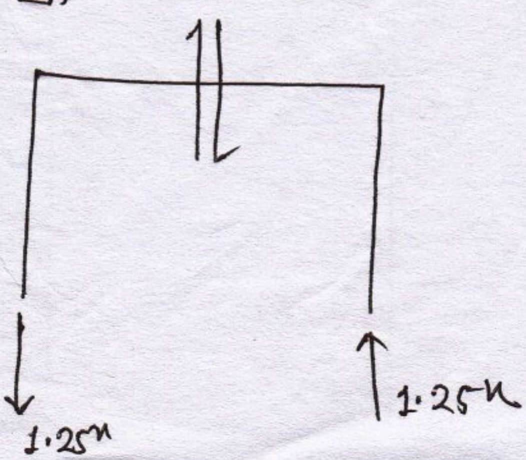
$$\Rightarrow F_{GH} = \frac{24 F_{CD}}{24} = F_{CD}$$

$\Sigma M_O = 0,$

$$F_{CD} \times 20 = 5 \times 5$$

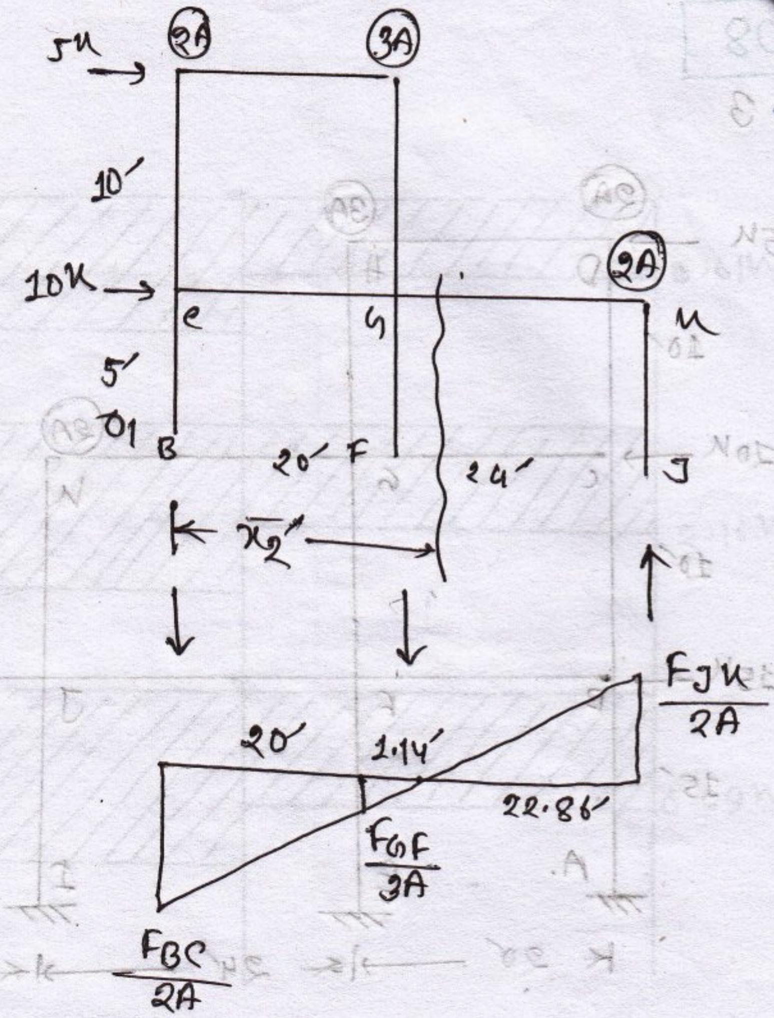
$$\therefore F_{CD} = 1.25^k$$





$$\bar{x}_2 = \frac{3A \times 20 + 2A \times 44}{7A}$$

$$= 21.14'$$



$$\frac{\frac{F_{BC}}{2A}}{21.14} = \frac{\frac{F_{GF}}{3A}}{1.14} = \frac{\frac{F_{JK}}{2A}}{22.86}$$

$$F_{GF} = \frac{3 \times 1.14\, F_{BC}}{2 \times 21.14}$$

$$= 0.08\, F_{BC}$$

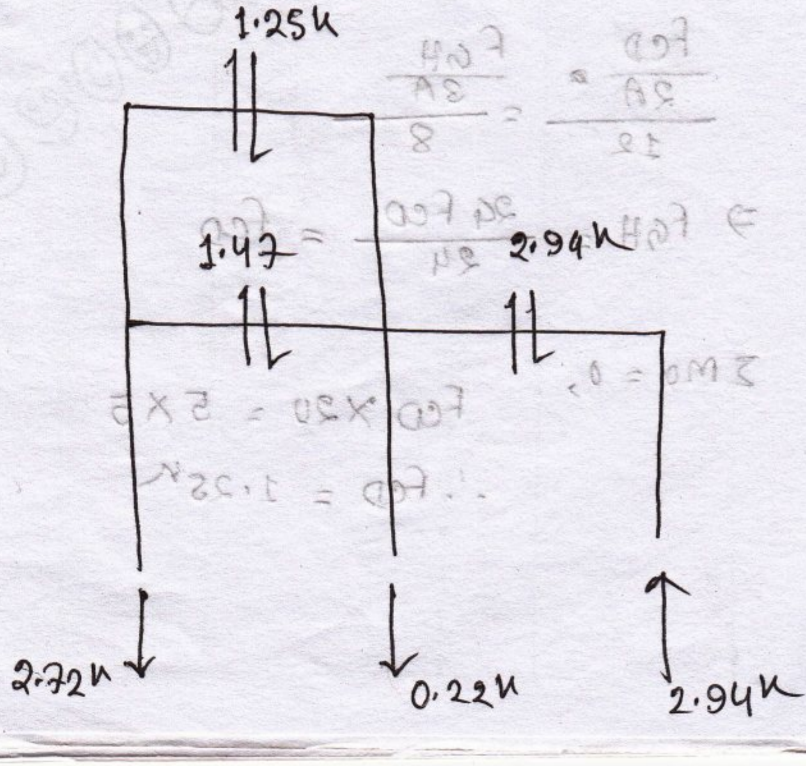
$$F_{JK} = \frac{22.86\, F_{BC}}{21.14}$$

$$= 1.08\, F_{BC}$$

$\Sigma M_{O_1} = 0,$

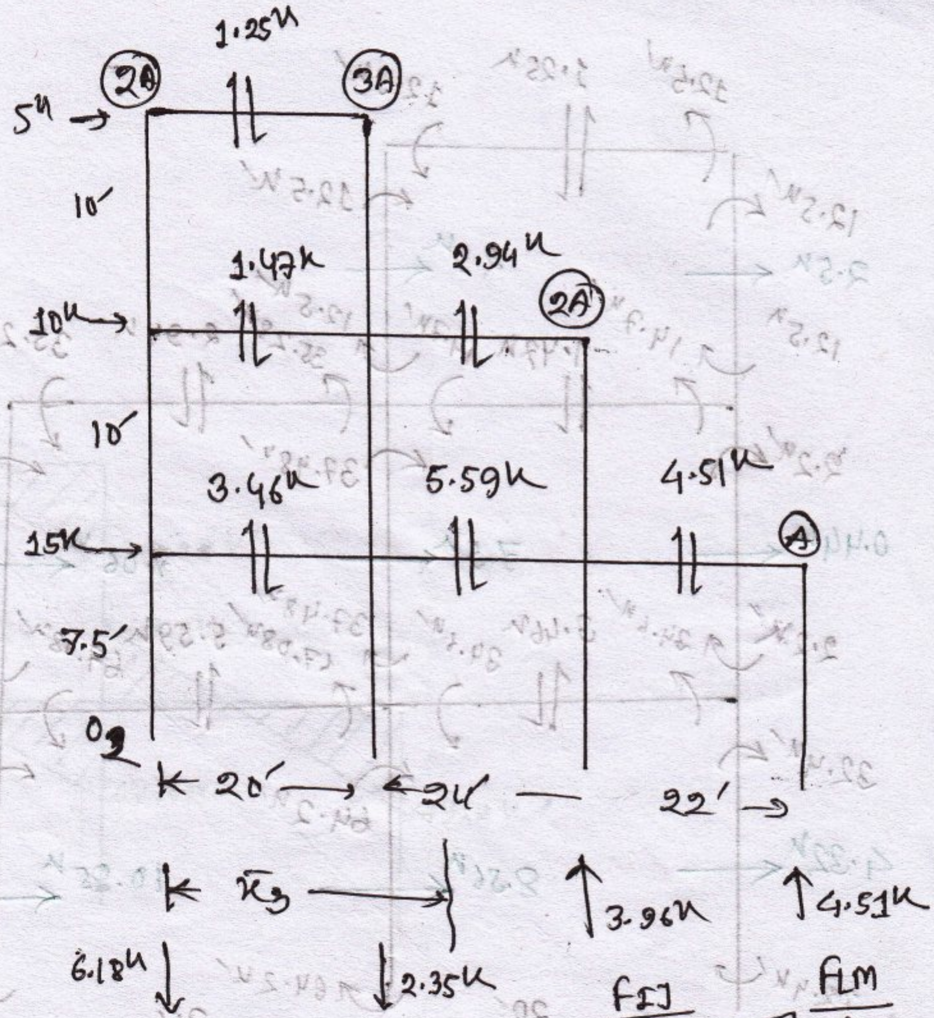
$$5 \times 15 + 10 \times 5 + 0.08\, F_{BC} \times 20 = 1.08\, F_{BC} \times 44$$

$$\therefore F_{BC} = 2.72\text{k}$$



$$\bar{x}_3 = \frac{3A\times20+2A\times44+A\times66}{8A}$$

$$= 26.75'$$



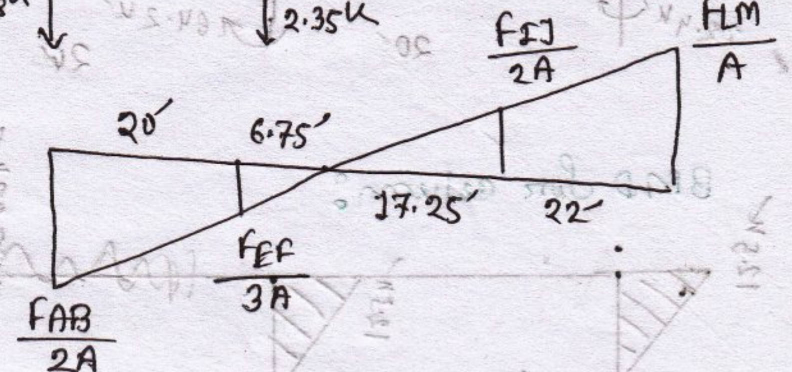
$$\frac{F_{AB}/2A}{26.75} = \frac{F_{EF}/3A}{6.75} = \frac{F_{IJ}/2A}{17.25} = \frac{F_{LM}/A}{39.25'}$$

$$F_{EF} = \frac{3\times6.75\,F_{AB}}{2\times26.75}$$

$$= 0.38\,F_{AB}$$

$$F_{IJ} = \frac{17.25\,F_{AB}}{26.75} = 0.64\,F_{AB}$$

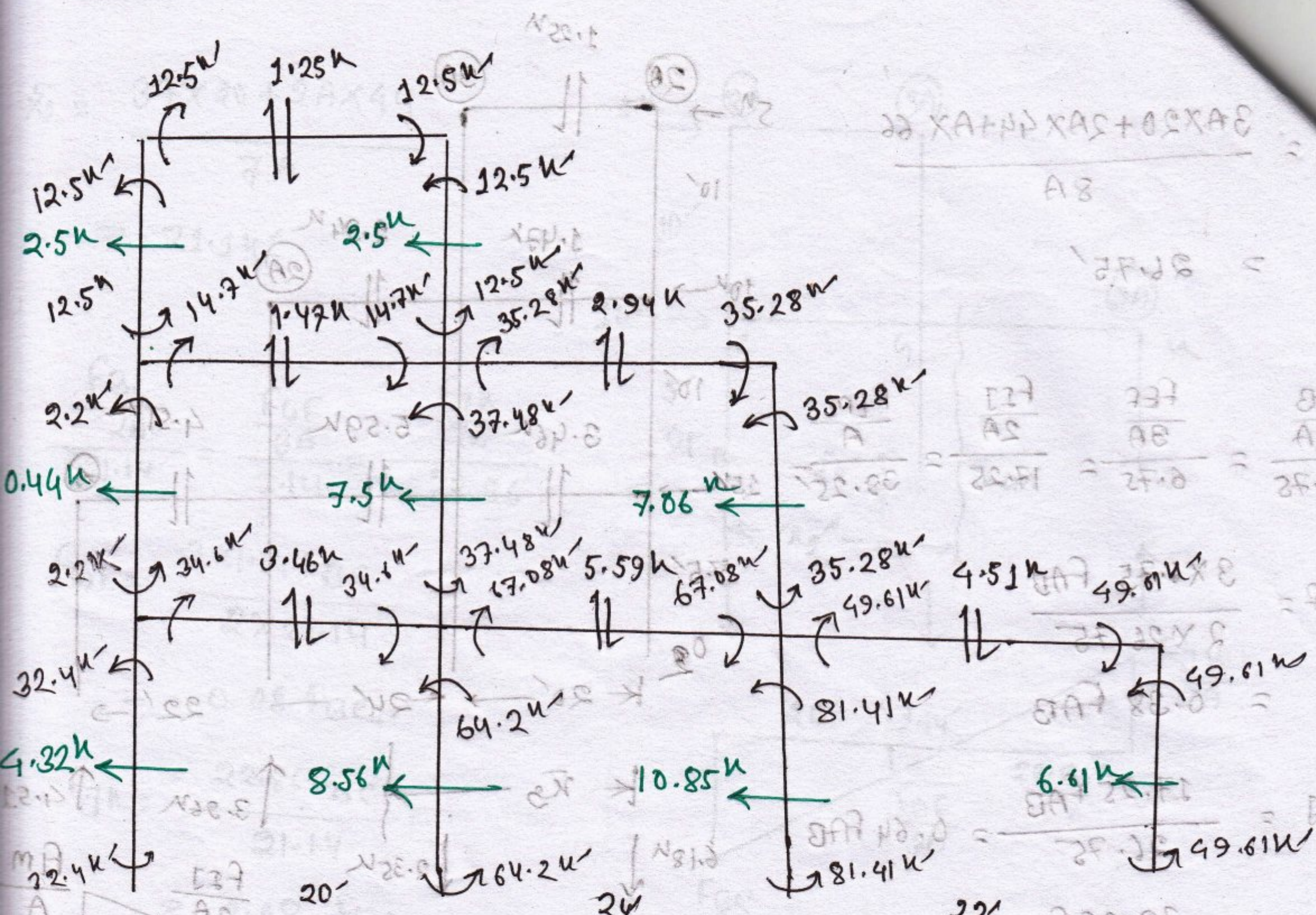
$$F_{LM} = \frac{39.25\,F_{AB}}{2\times26.75} = 0.73\,F_{AB}$$



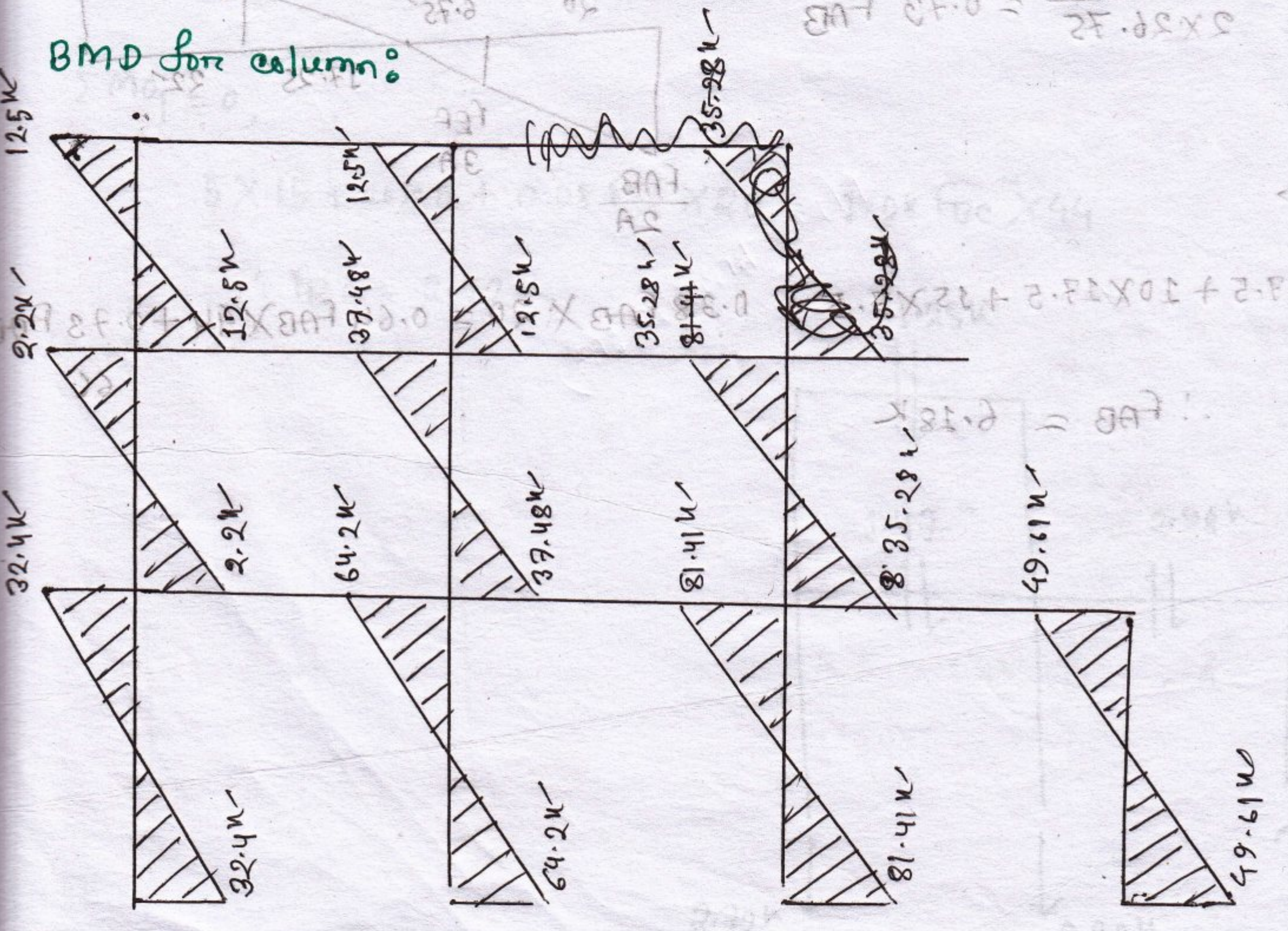
$\Sigma M_{O_2} = 0$,

$$5\times27.5+10\times17.5+15\times7.5+0.38\,F_{AB}\times20 = 0.64\,F_{AB}\times44+0.73\,F_{AB}\times66$$

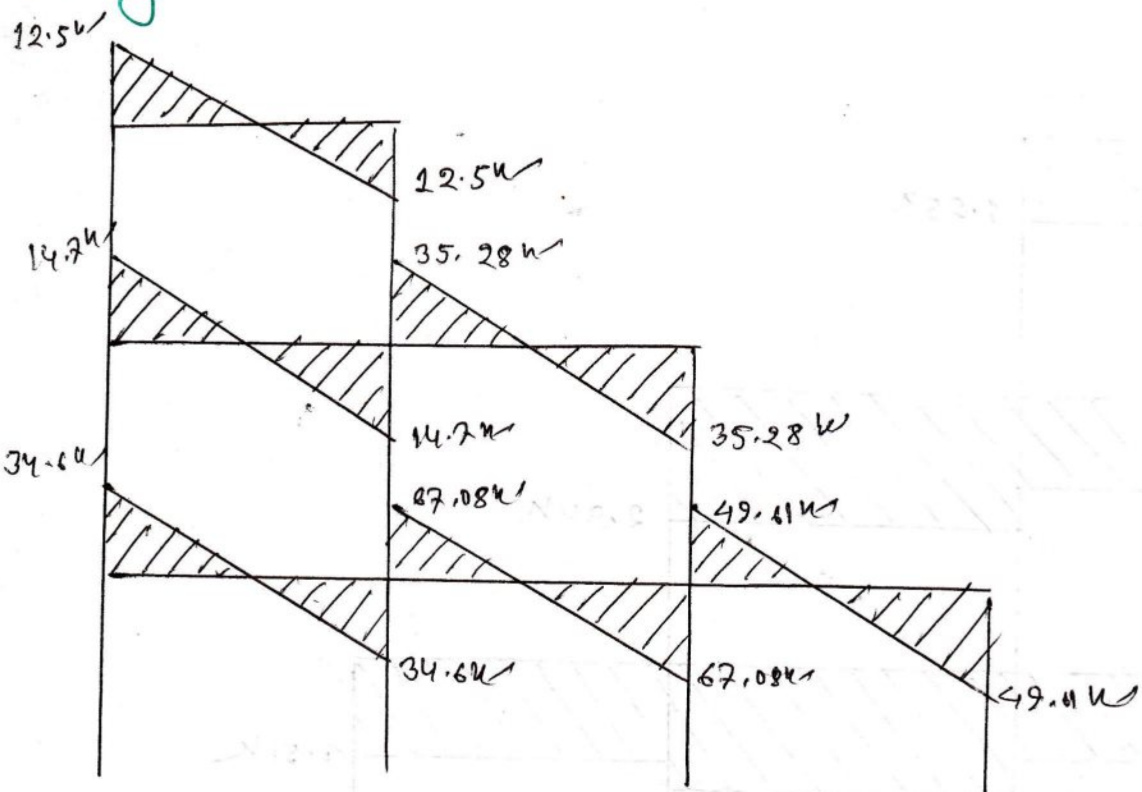
$$\therefore F_{AB} = 6.18^k$$



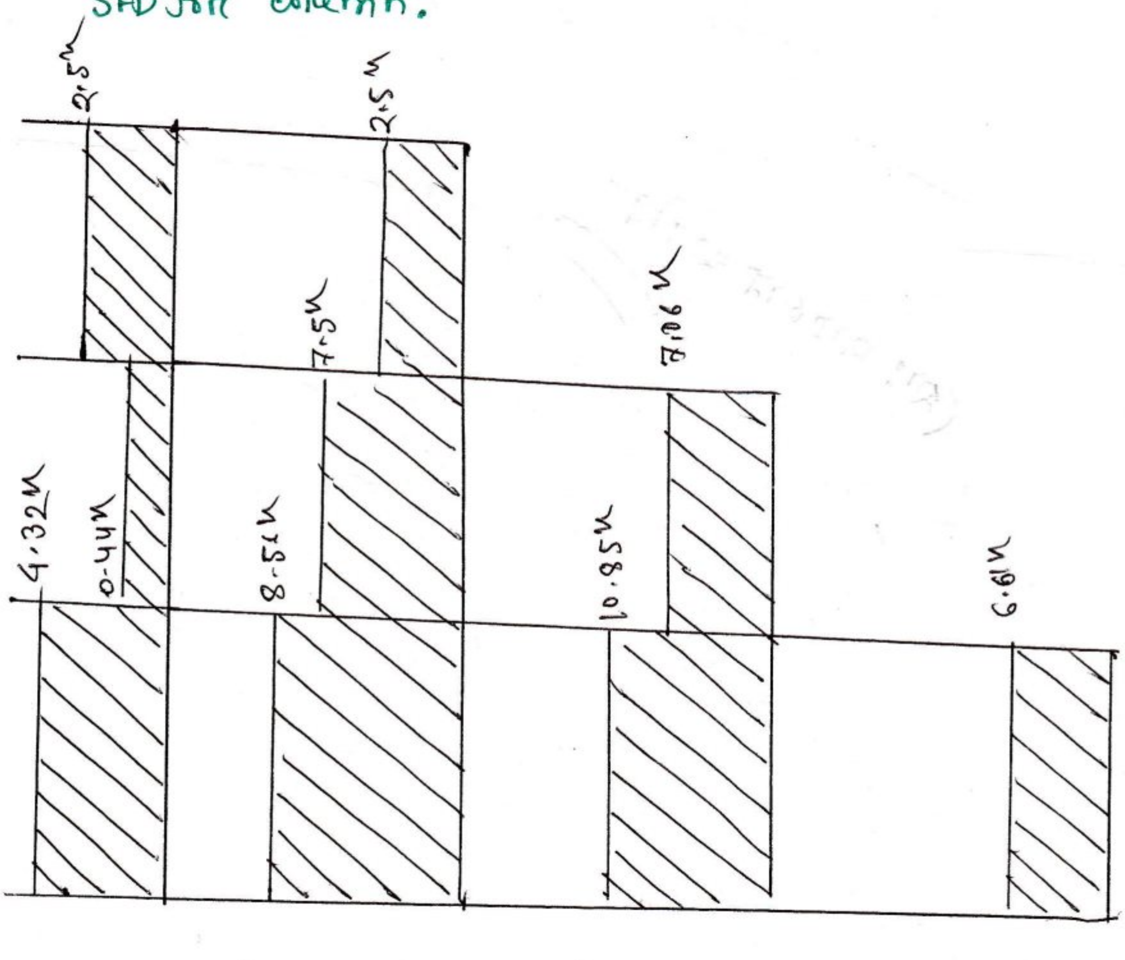
BMD for column:



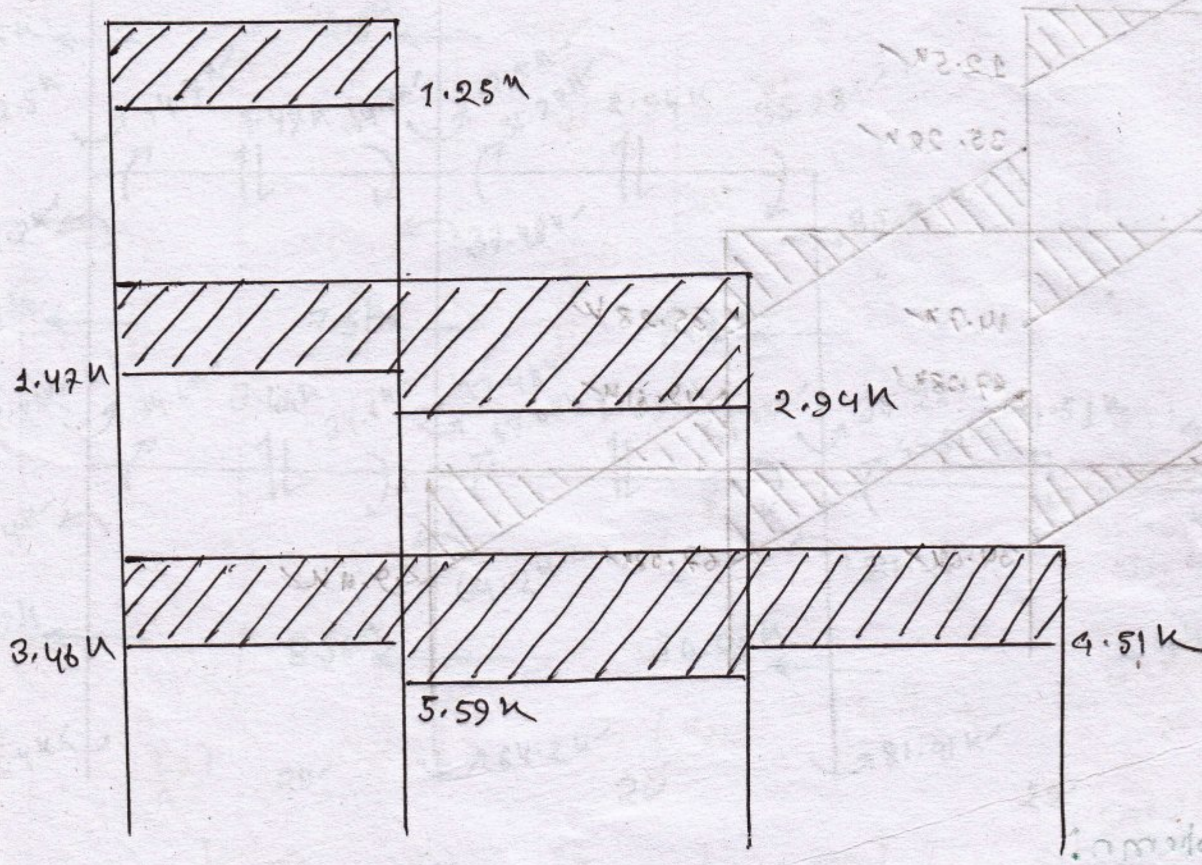
SFD for girder:



SFD for column:



SFD for girder:

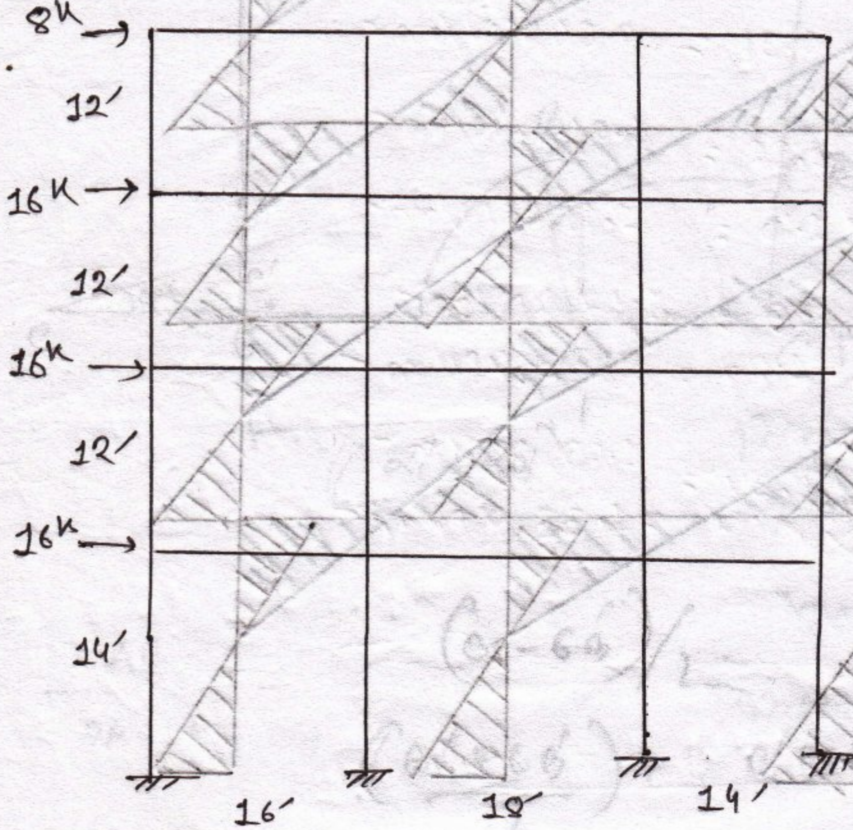


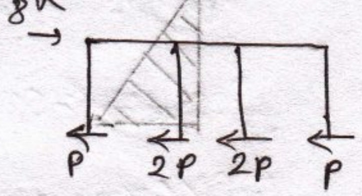
মো: আসাদুজ্জামান কবীর

Portal Method

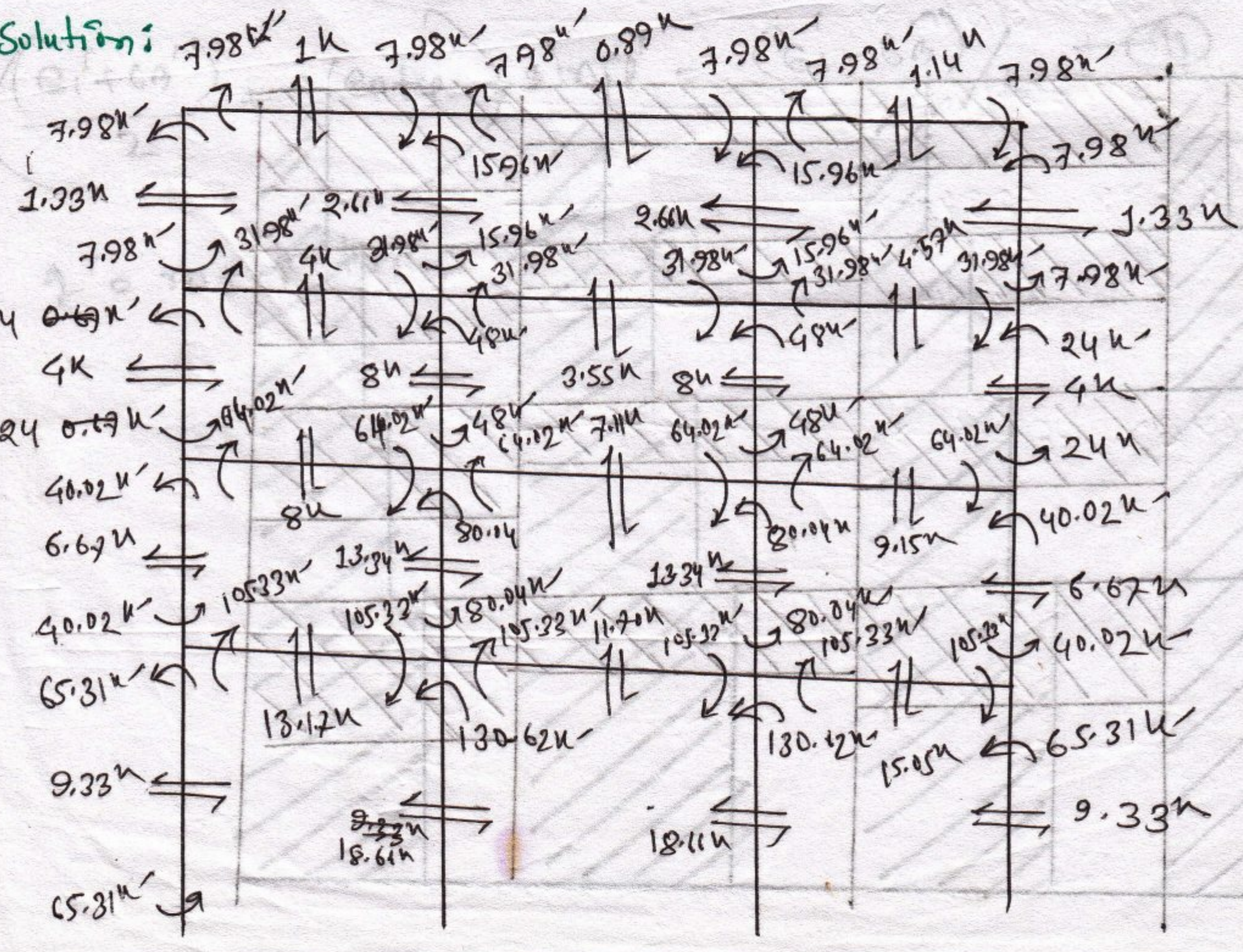
090063
AHSAN

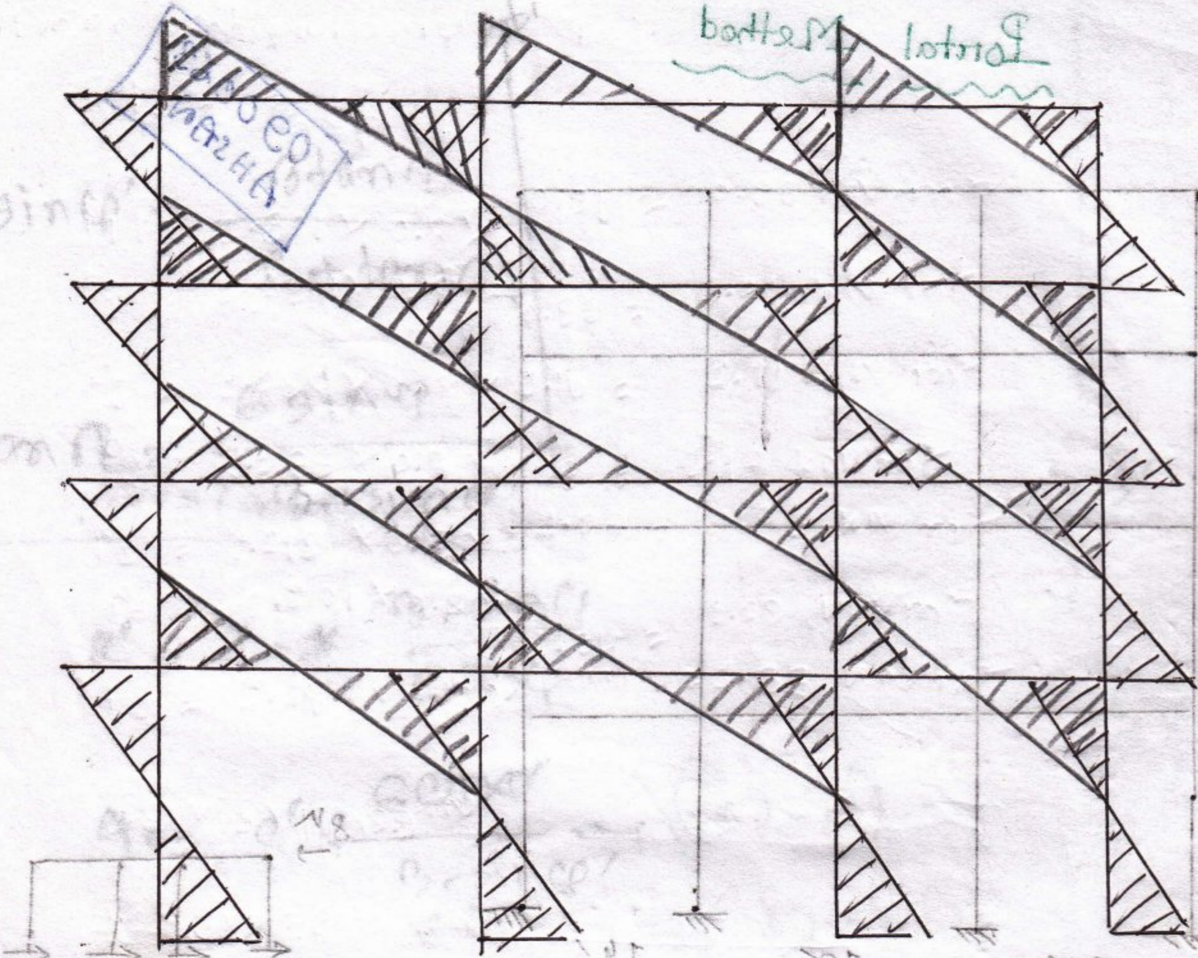
1.

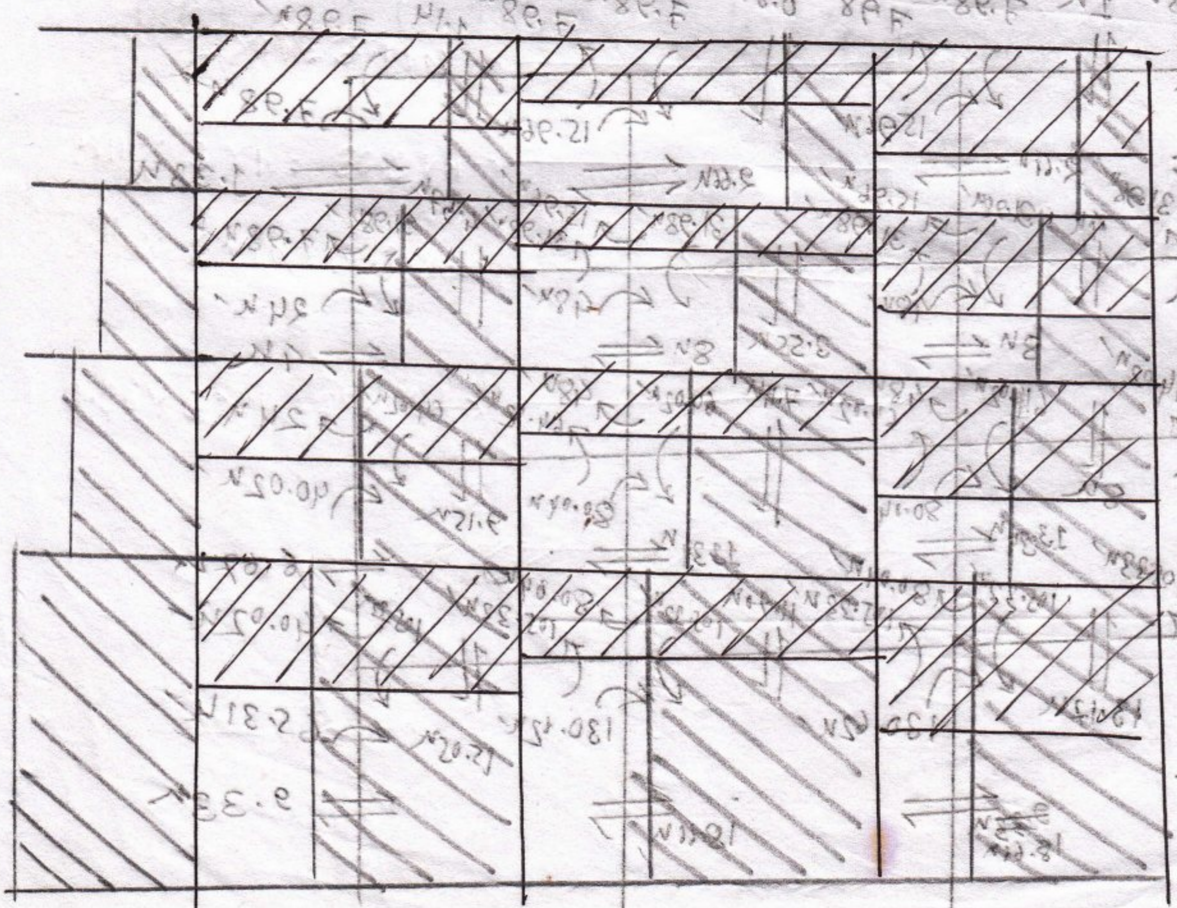




Solution:



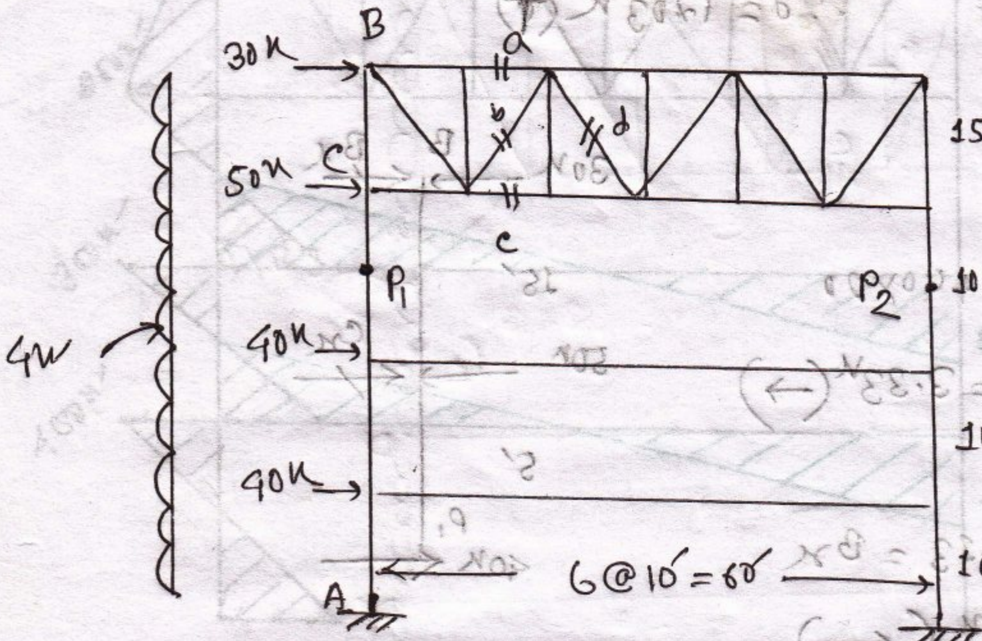




03 / 1/ 02

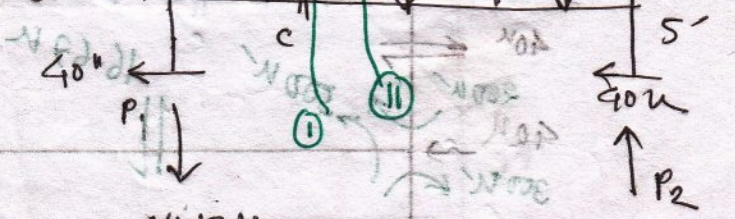
Portal Frame

1. Determine stress in members a, b, c and d. also draw SFD & BMD for left column and girder.



Solution:

Considering free body above $P_1 P_2$,



$\sum M_{P_1} = 0,$

$30 \times 20 + 50 \times 5 = R_{P_2} \times 60$

$\therefore R_{P_2} = 14.17^k$

Considering right part of sec ①①

$\sum M_{U_3} = 0,$

$C \times 15 + 40 \times 20 = 14.17 \times 40$

$\therefore C = -15.55^k = 15.55^k (C)$

$\sum f_y = 0, \quad b_V = 14.17 \Rightarrow b \times \frac{15}{\sqrt{15^2 + 10^2}} = 14.17 \quad \therefore b = 17.03^k (T)$

$\sum f_x = 0, \quad a + b_x + c + 40 = 0$

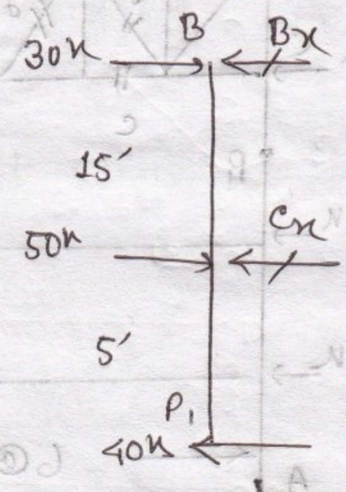
$\Rightarrow a + 17.03 \times \frac{10}{18.03} - 15.55 + 40 = 0 \quad \therefore a = -33.9^k$

Considering right of section (11) (11) $\Rightarrow$

$$\Sigma F_y = 0, \quad dv = 14.17 \Rightarrow d \times \frac{15}{\sqrt{15^2+10^2}} = 14.17$$

$$\therefore d = 1703\,k \;(T)$$

Freebody of BP_1,



$\Sigma M_B = 0$,

$$50 \times 15 = C_x \times 15 + 40 \times 20$$

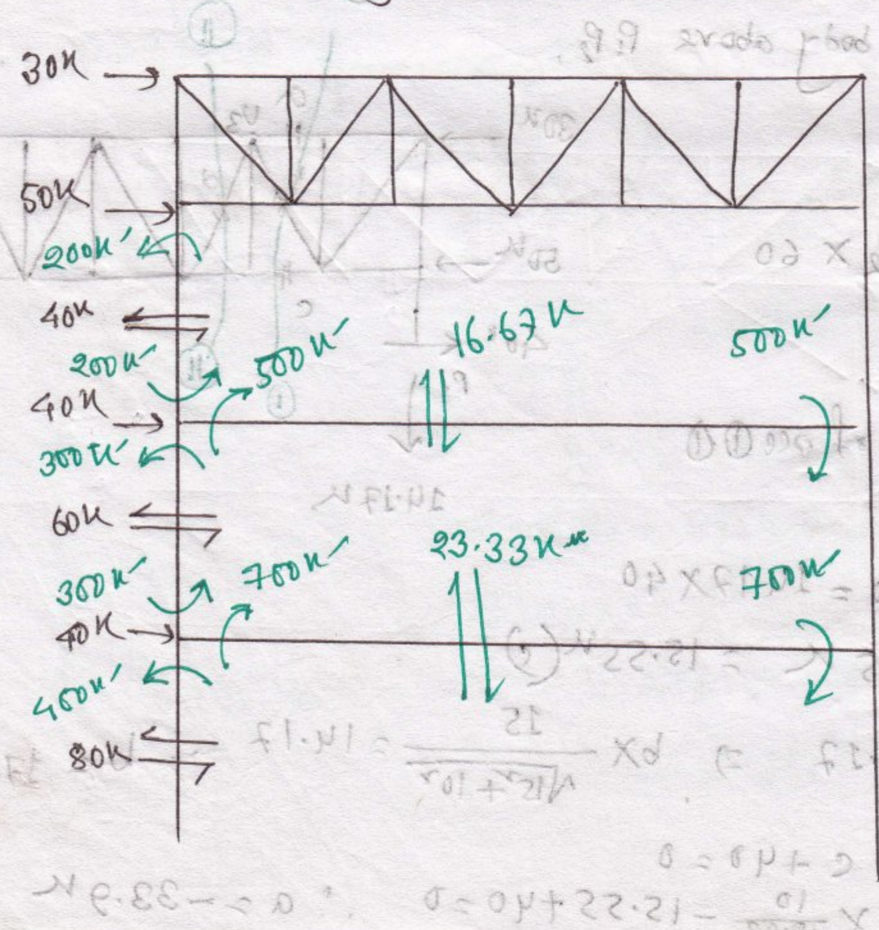
$$\therefore C_x = -3.33\,k = 3.33\,k\,(\rightarrow)$$

$\Sigma F_x = 0$,

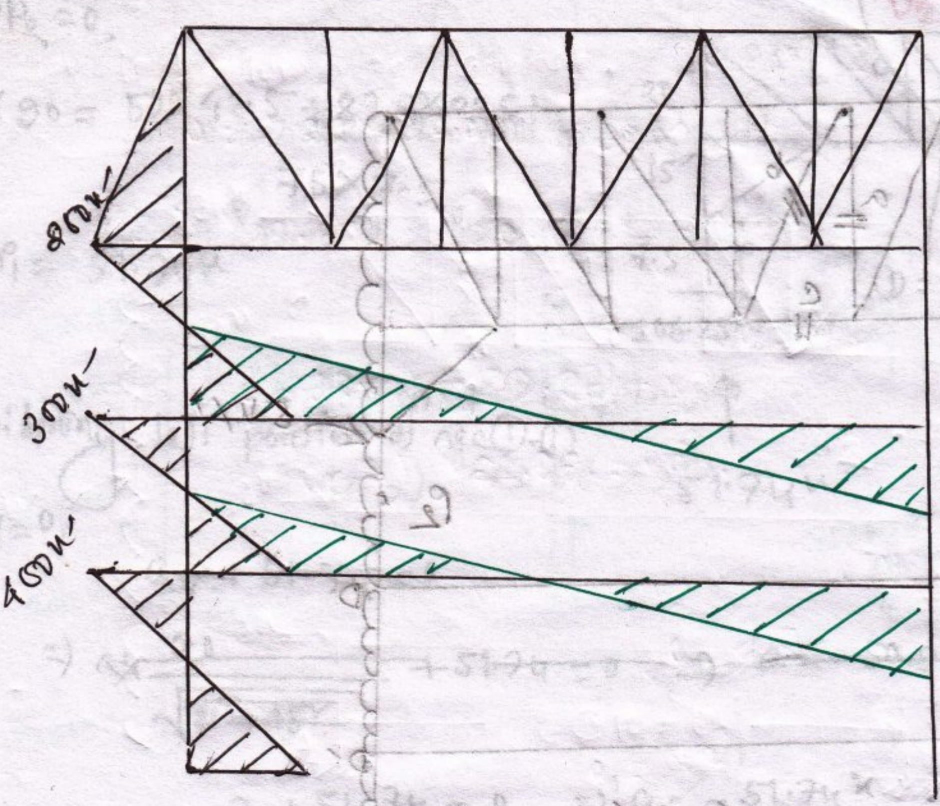
$$30 + 50 - 40 + 3.33 = B_x$$

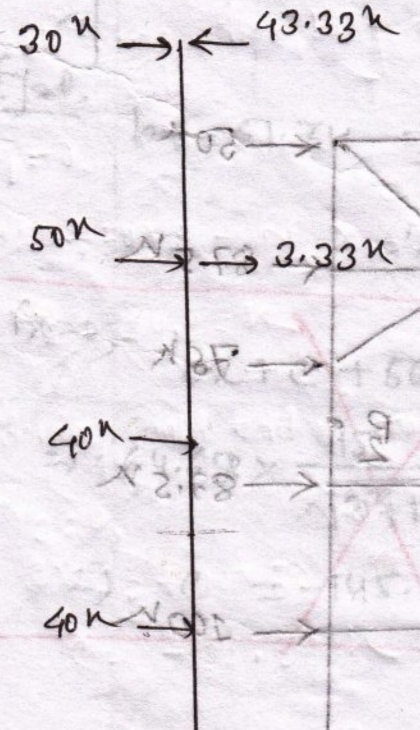
$$\therefore B_x = 43.33\,k\,(\leftarrow)$$

Now, By Portal method we get,

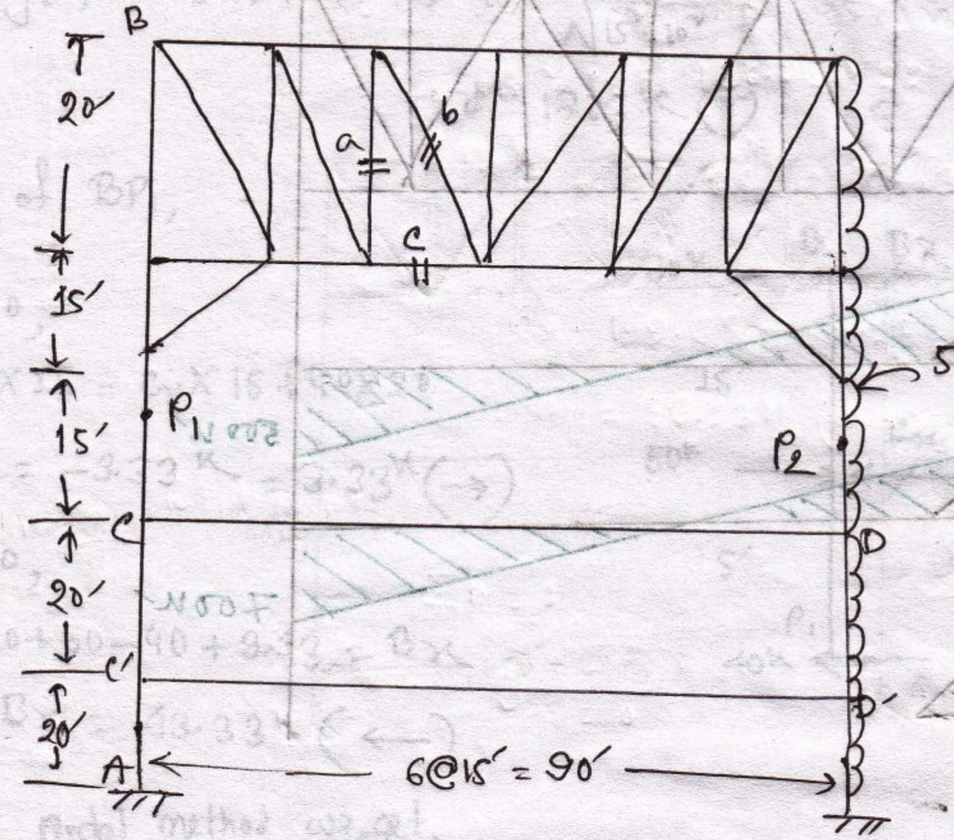


for column & girder:

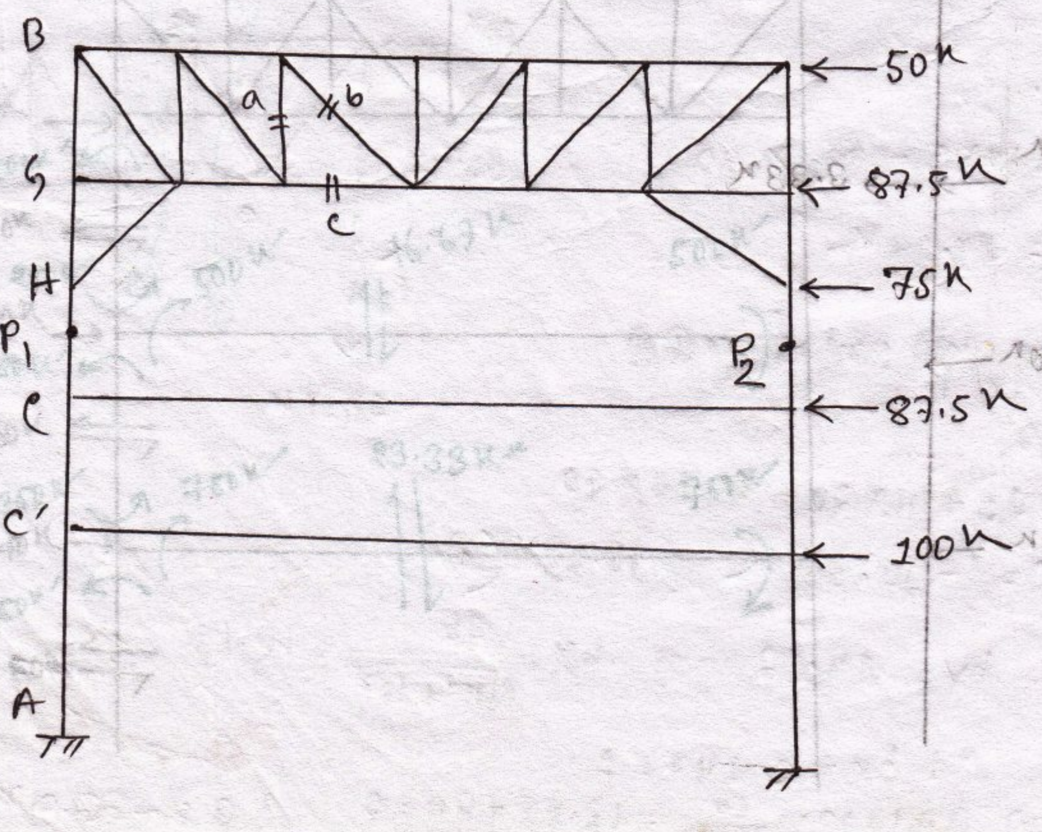




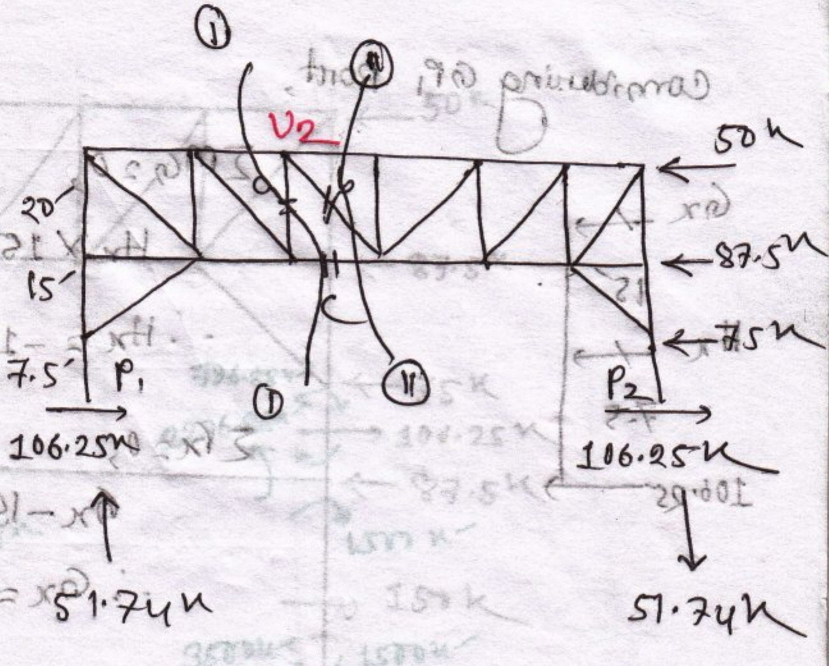
02. Find stresses in a, b, c and Draw SFD and BMD of AB & CD



Solution:



Considering upper part of P_1P_2.



$\Sigma M_{P_2} = 0,$

$R_{P_1} \times 90 = 50 \times 42.5 + 87.5 \times 22.5 + 75 \times 7.5$

$\therefore R_{P_1} = 51.74k$

Considering left portion of sec I-I

$\Sigma F_y = 0,$

$\cancel{a_v + 51.74 = 0}$

$\cancel{\Rightarrow a \times \frac{20}{\sqrt{20^2+15^2}} + 51.74 = 0 \Rightarrow a = -50.94^k = 50.94^k (C)}$

$a + 51.74 = 0 \quad \Rightarrow a = -51.74^k = 51.74^k (C)$

Considering left portion of sec II-II

$\Sigma F_y = 0,$

$b_v = 51.74 \Rightarrow b \times \frac{20}{\sqrt{20^2+15^2}} = 51.74$

$\therefore b = 64.68^k (T)$

$\Sigma F_x = 0,$

$\cancel{b_x + c + 50 + 87.5 + 75 - 106.25 = 0}$

$\cancel{\Rightarrow 64.68 \times \frac{15}{\sqrt{20^2+15^2}} + c + 106.25 = 0}$

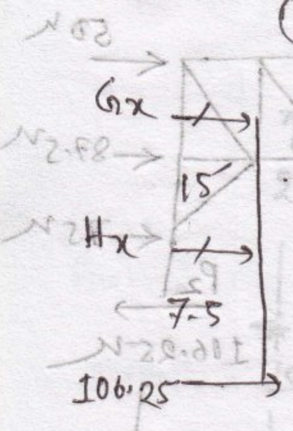
$\cancel{\Rightarrow c = -145.06k = 145.06^k (C)}$

$\Sigma M_{U_2} = 0,$

$C \times 20 + 106.25 \times 42.5 = 51.74 \times 30$

$C = 148.17k$

Considering GP_1 part,



$\Sigma M_G = 0,$

$H_x \times 15 + 106.25 \times 22.5 = 0$

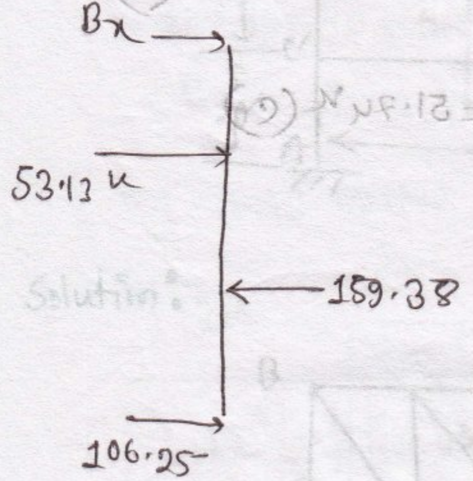
$\therefore H_x = -159.38^k = 159.38^k (\leftarrow)$

$\Sigma F_x = 0,$

$G_x - 159.38 + 106.25 = 0$

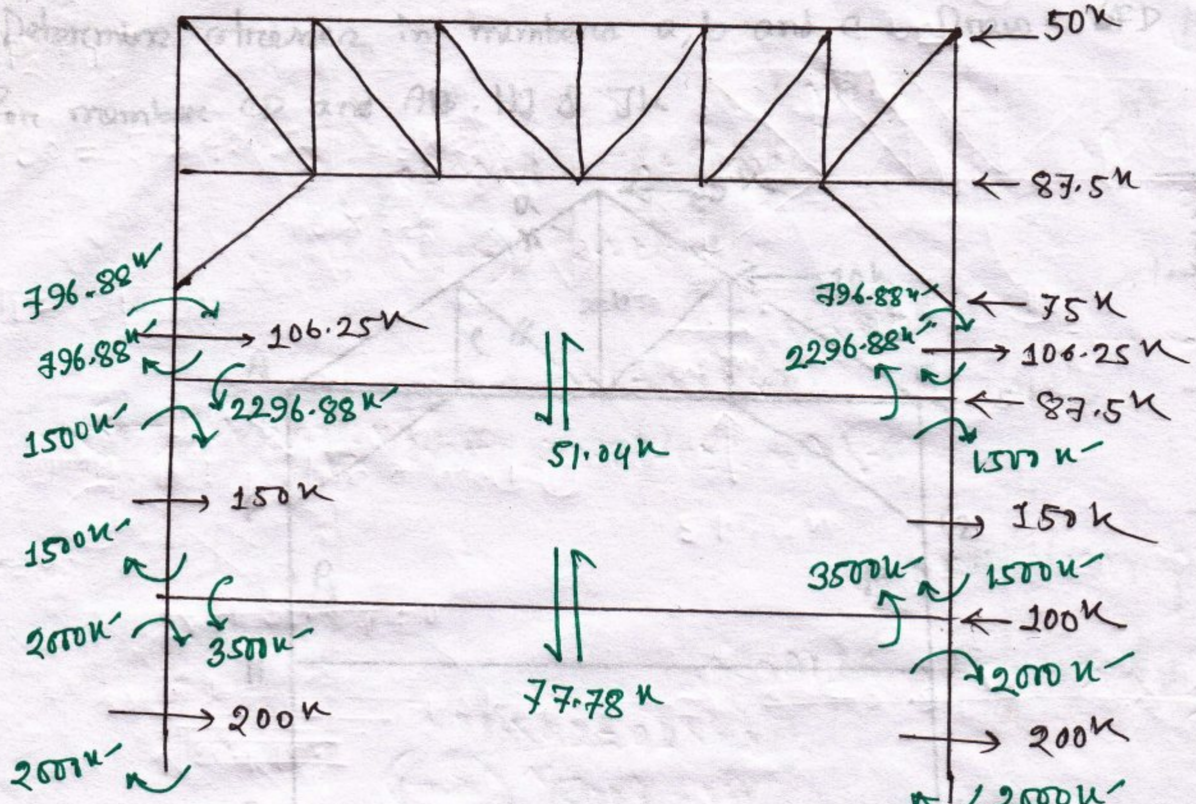
$\therefore G_x = 53.13^k (\rightarrow)$

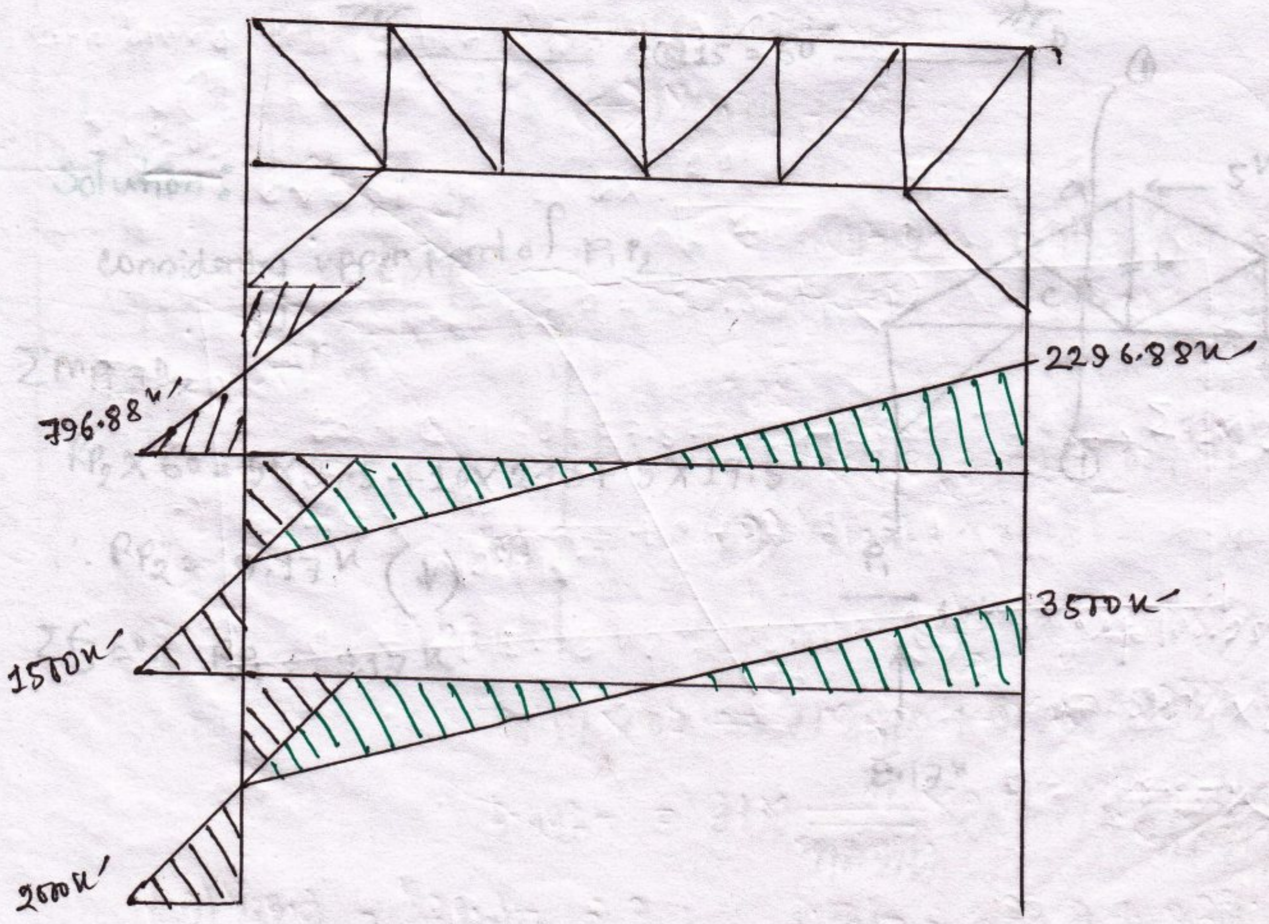
Considering BP_1 part,



$\Sigma F_x = 0,$

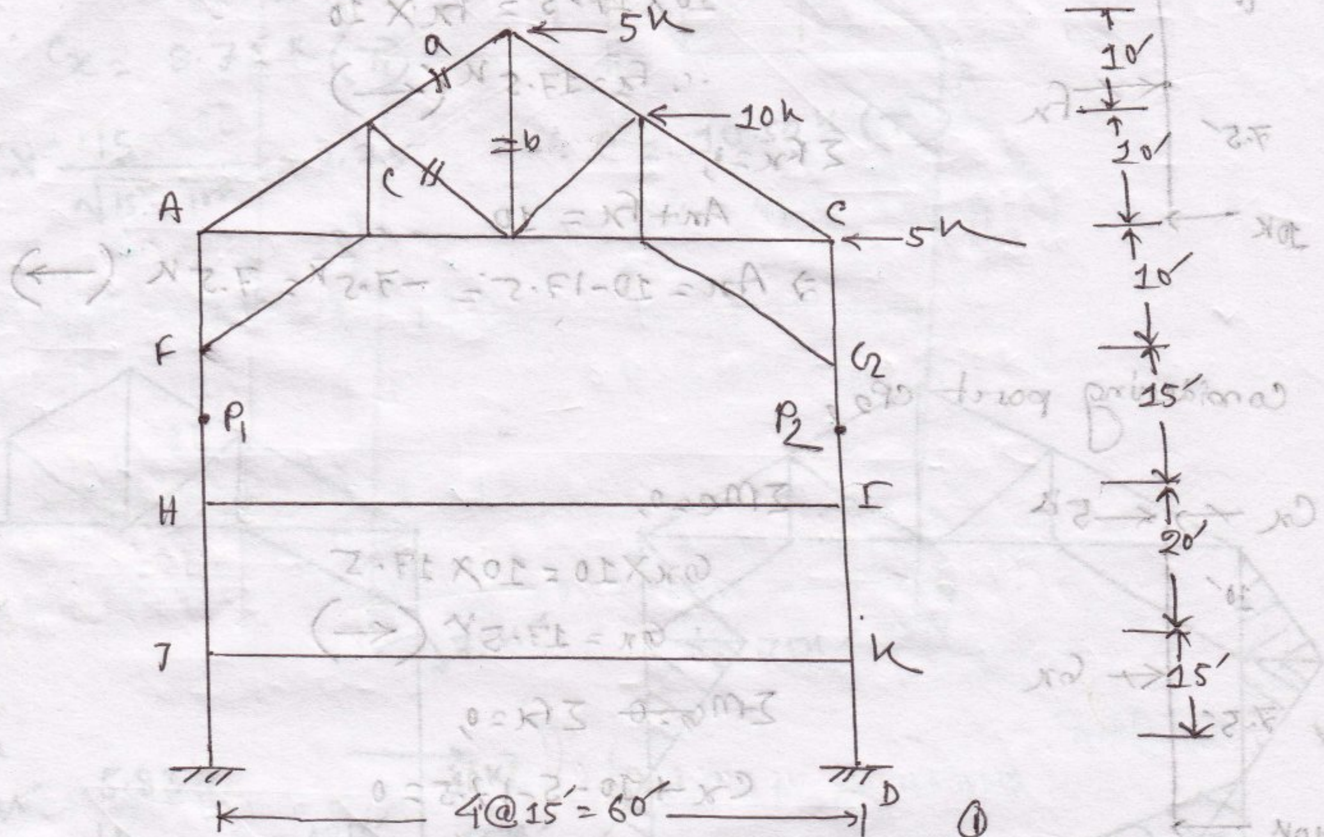
$B_x = 0$


50k
87.5k
796.88k'
796.88k'
106.25k
75k
2296.88k'
106.25k
2296.88k'
87.5k
1500k'
51.04k
1500k'
150k
150k
1500k'
3500k'
1500k'
100k
2000k'
3500k'
77.78k
2000k'
200k
200k
2000k'
2000k'


2296.88k'
796.88k'
3500k'
1500k'
2000k'

07

3. Determine stresses in members a, b and c. Draw SFD & BMD for member CD and AB. HI & JK



Solution:

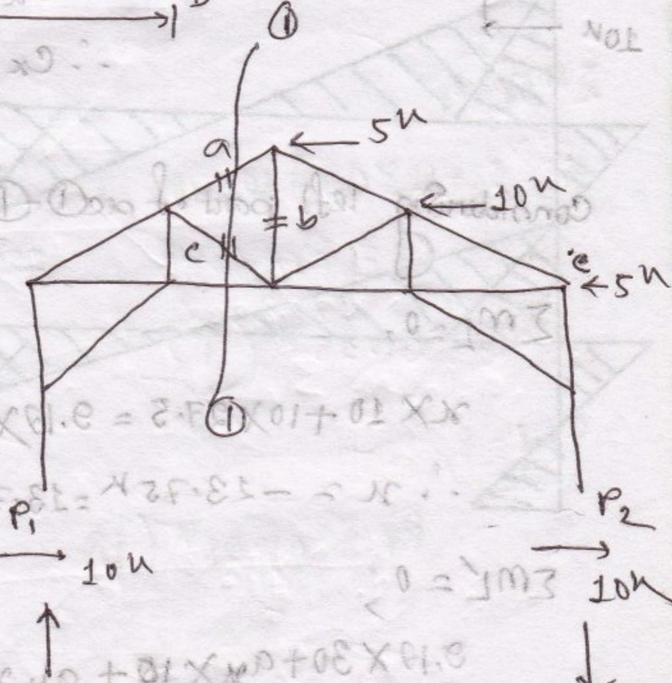
considering upper part of $P_1 P_2$

$\Sigma M_{P_1} = 0,$

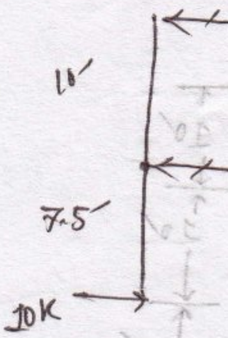
$R_{P_2} \times 60 = 5 \times 37.5 + 10 \times 27.5 + 5 \times 17.5$

$\therefore R_{P_2} = 9.17^k \; (\downarrow)$

$\Sigma f_y = 0, \quad R_{P_1} = 9.17^k$



Considering part AP_1,



$\Sigma M_A = 0$,

$10 \times 17.5 = F_x \times 10$

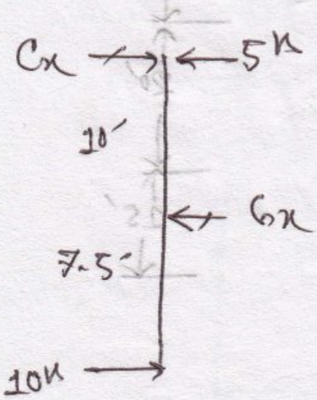
$\therefore F_x = 17.5^k (\leftarrow)$

$\Sigma F_x = 0$,

$A_x + F_x = 10$

$\Rightarrow A_x = 10 - 17.5 = -7.5^k = 7.5^k (\rightarrow)$

Considering part CP_2,



$\Sigma M_C = 0$,

$G_x \times 10 = 10 \times 17.5$

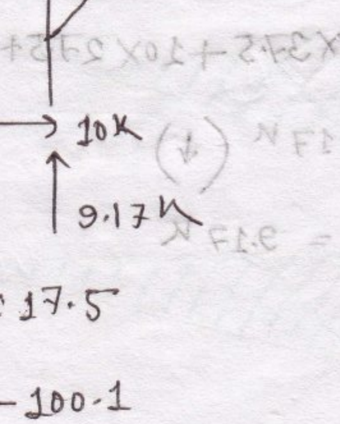
$\therefore G_x = 17.5^k (\leftarrow)$

$\cancel{\Sigma M_G = 0}$ $\Sigma F_x = 0$,

$C_x + 10 - 5 - 17.5 = 0$

$\therefore C_x = 12.5^k (\rightarrow)$

Considering left part of sec ①-①,



$\Sigma M_L = 0$,

$x \times 10 + 10 \times 27.5 = 9.17 \times 15$

$\therefore x = -13.75^k = 13.75^k (C)$

$\Sigma M_{L'} = 0$,

$9.17 \times 30 + a_x \times 10 + a_y \times 15 = 10 \times 17.5$

$\Rightarrow a \times \frac{15}{\sqrt{15^2+10^2}} \times 10 + a \times \frac{10}{\sqrt{15^2+10^2}} \times 15 = -100.1$

$\Rightarrow 8.32a + 8.32a = -100.1$ $\therefore a = -6.02^k = 6.02^k (C)$

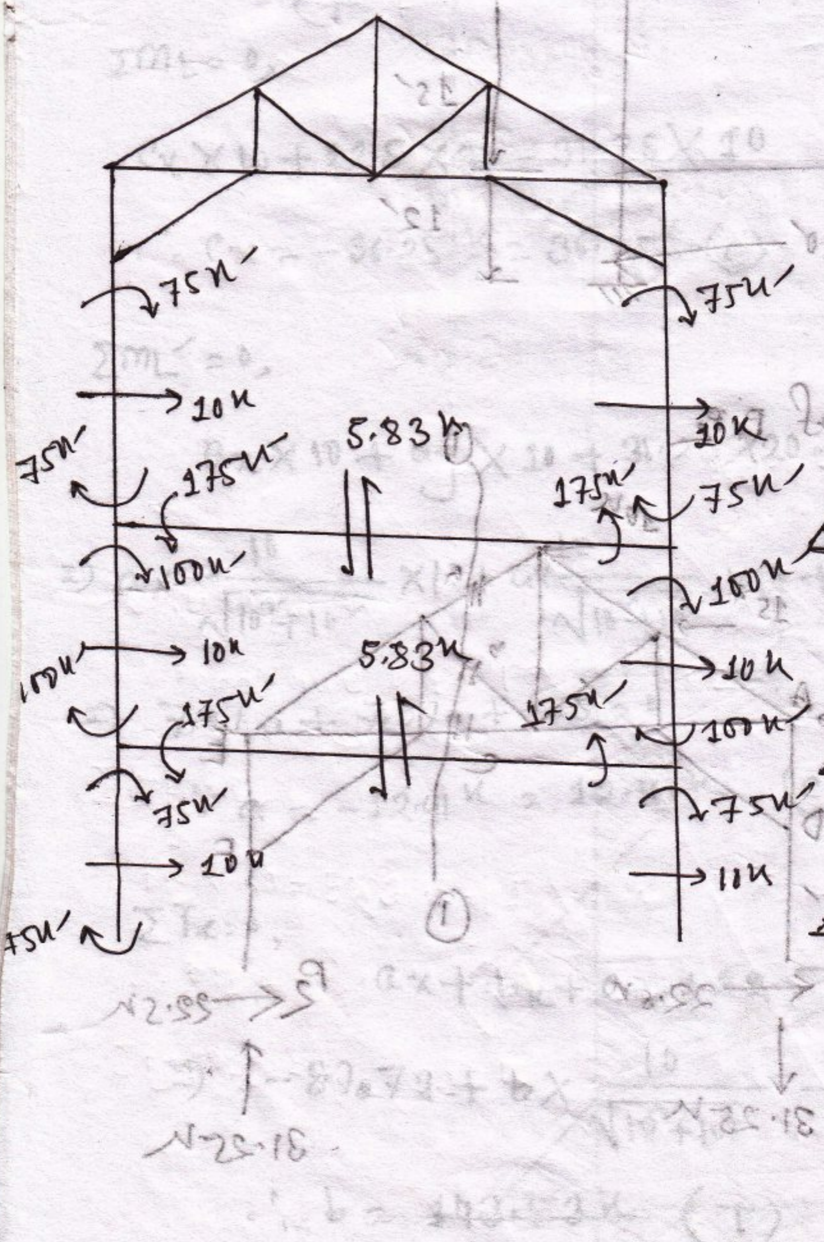
$\Sigma F_x = 0,$

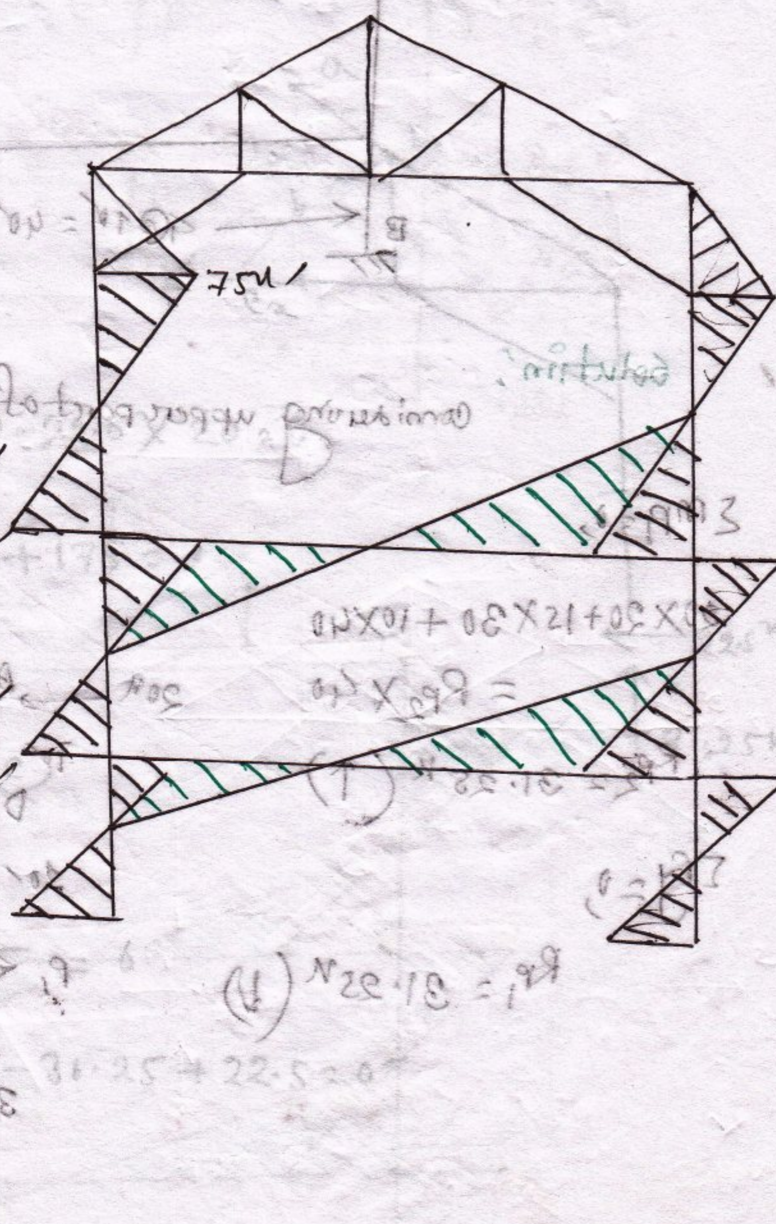
$x + ax + cx + 10 = 0$

$\Rightarrow -13.75 - 5 + c_x + 10 = 0$

$\therefore c_x = 8.75^k\,(T)$

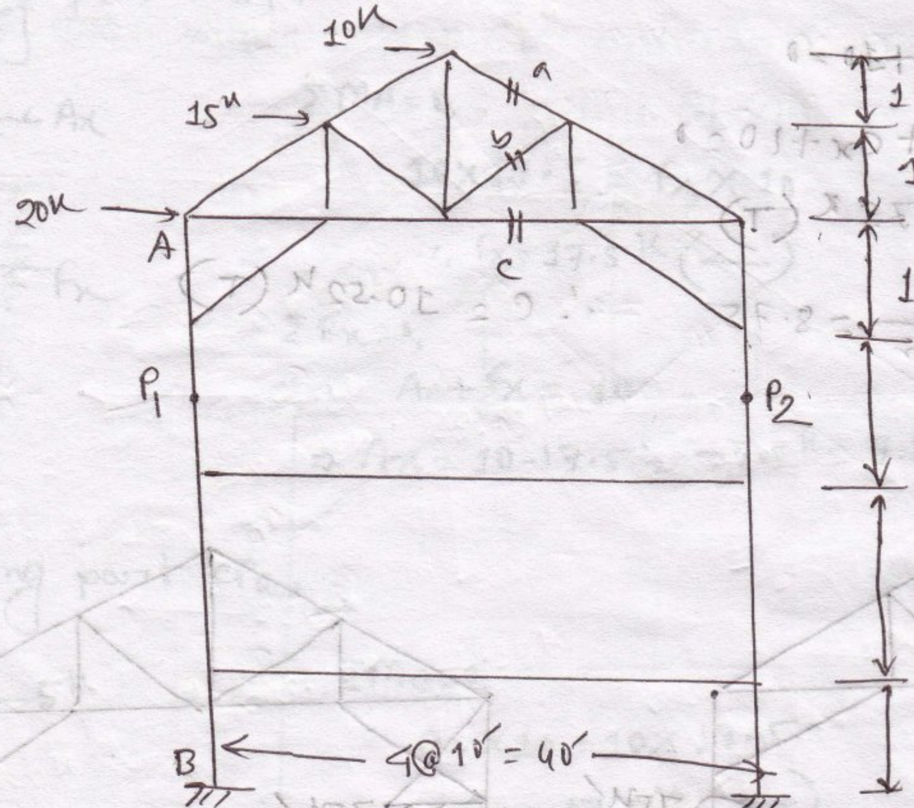
$C \times \dfrac{15}{\sqrt{15^2+10^2}} = 8.75 \qquad \therefore C = 10.52^k\,(T)$





10

6.(b)



Solution:

Considering upper part of P_1P_2

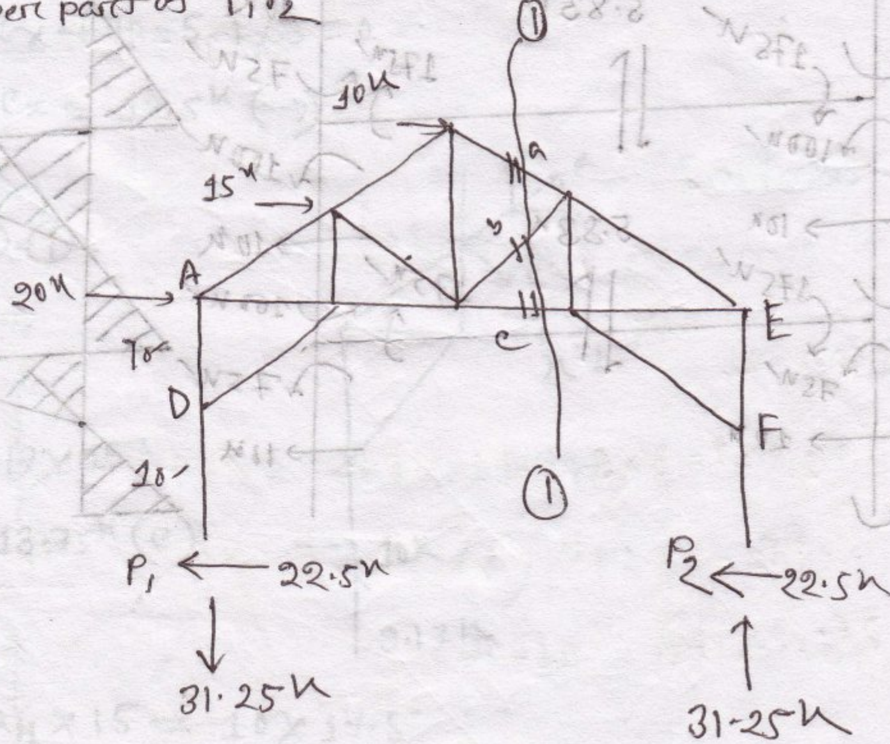
$\sum M_{P_1} = 0,$

$20 \times 20 + 15 \times 30 + 10 \times 40 = R_{P_2} \times 40$

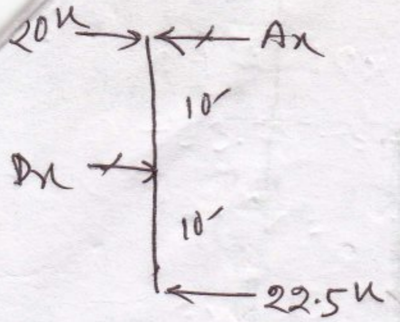
$\therefore R_{P_2} = 31.25^k \ (\uparrow)$

$\sum F_y = 0,$

$R_{P_1} = 31.25^k \ (\downarrow)$



onsider part : AP1,



$\sum M_A = 0,$

$22.5 \times 20 = D_x \times 10$

$\therefore D_x = 45^k \; (\rightarrow)$

$\sum F_x = 0,$

$D_x + 20 - A_x - 22.5 = 0$

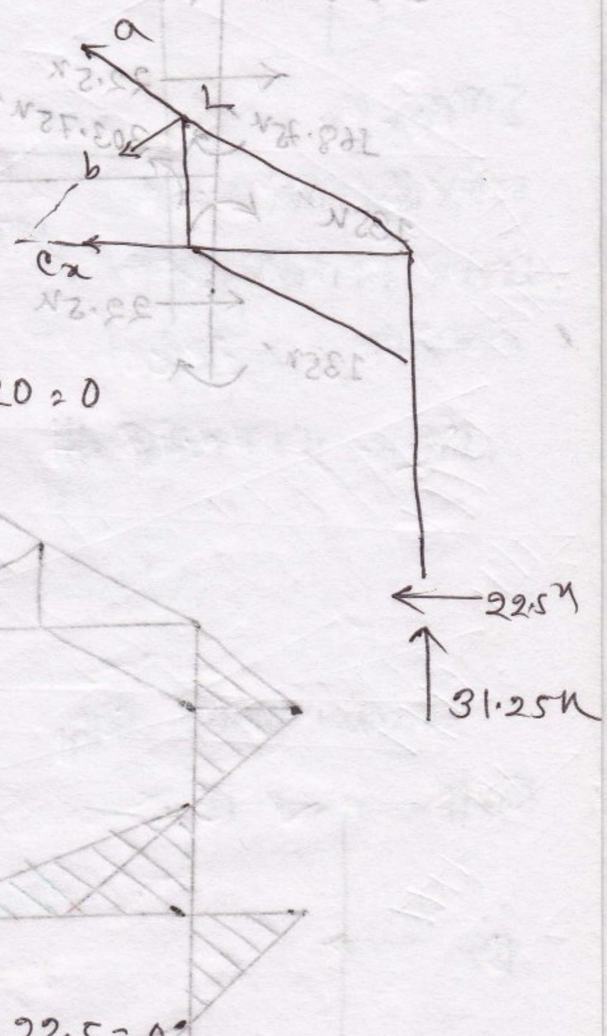
$\therefore \cancel{D_x} \; \therefore A_x = 42.5^k \; (\leftarrow)$

consider right portion of sec 1-1

$\sum M_L = 0,$

$C_x \times 10 + 22.5 \times 30 = 31.25 \times 10$

$\therefore C_x = -36.25^k = 36.25^k$ (C)



$\sum M_{L'} = 0,$

$a_x \times 10 + a_y \times 10 + 31.25 \times 20 - 22.5 \times 20 = 0$

$\Rightarrow a \times \frac{10}{\sqrt{10^2+10^2}} \times 10 + a \times \frac{10}{\sqrt{10^2+10^2}} \times 10 + 175 = 0$

$\Rightarrow 7.07a + 7.07a + 175 = 0$

$\therefore a = -12.41^k = 12.41^k$ (C)

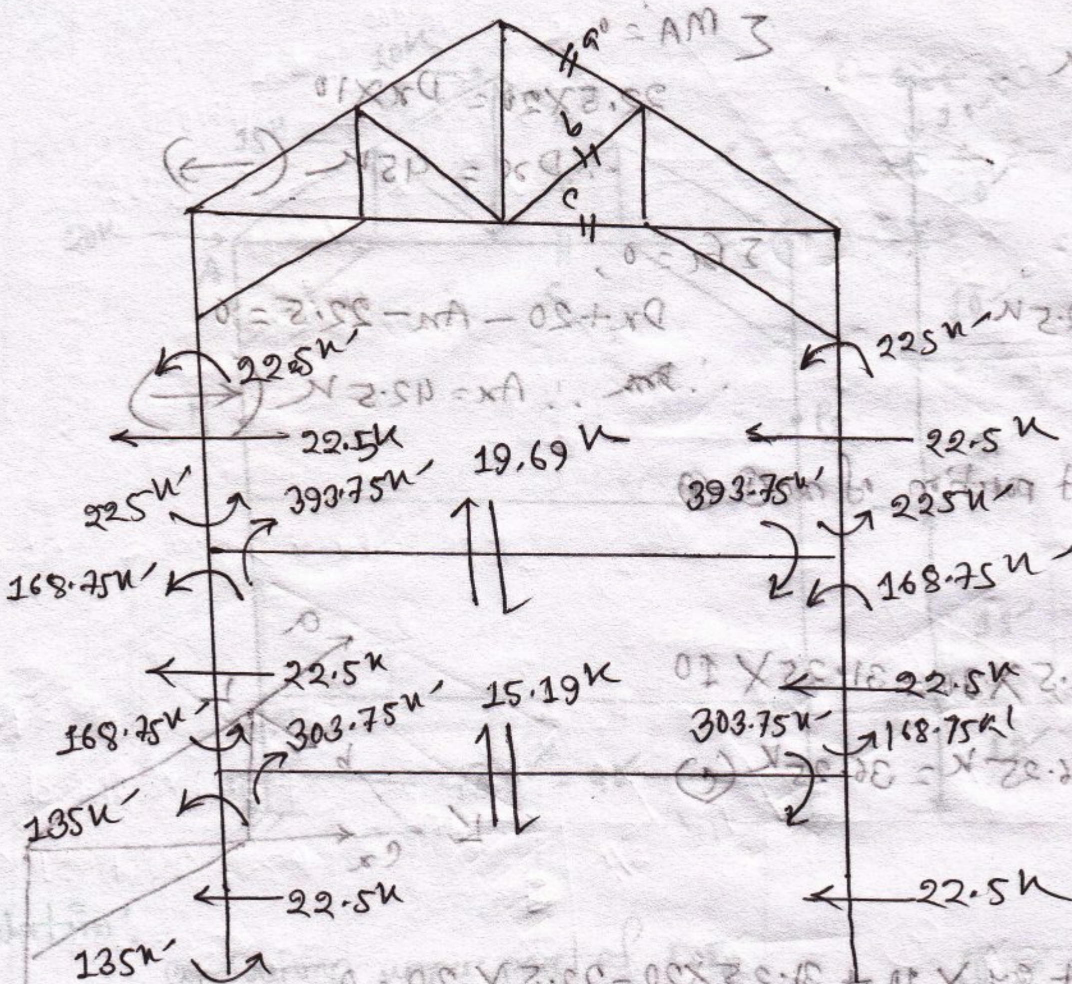
$\sum F_x = 0,$

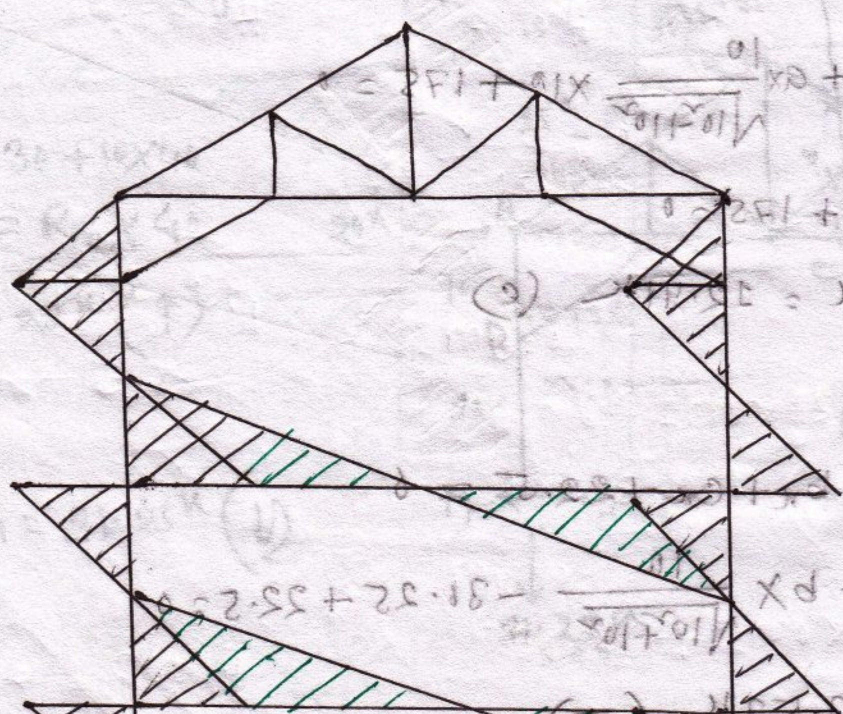
$a_x + b_x + C_x + 22.5 = 0$

$\Rightarrow -8.774 + b \times \frac{10}{\sqrt{10^2+10^2}} - 36.25 + 22.5 = 0$

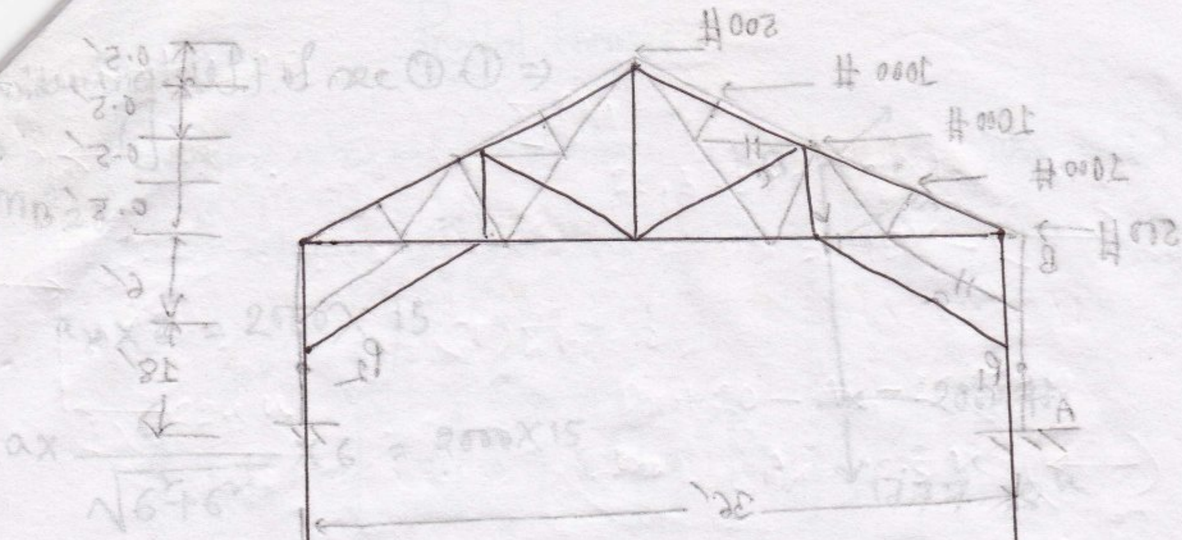
$\therefore b = \cancel{143.53^k}$ (T)

$= 31.78^k$

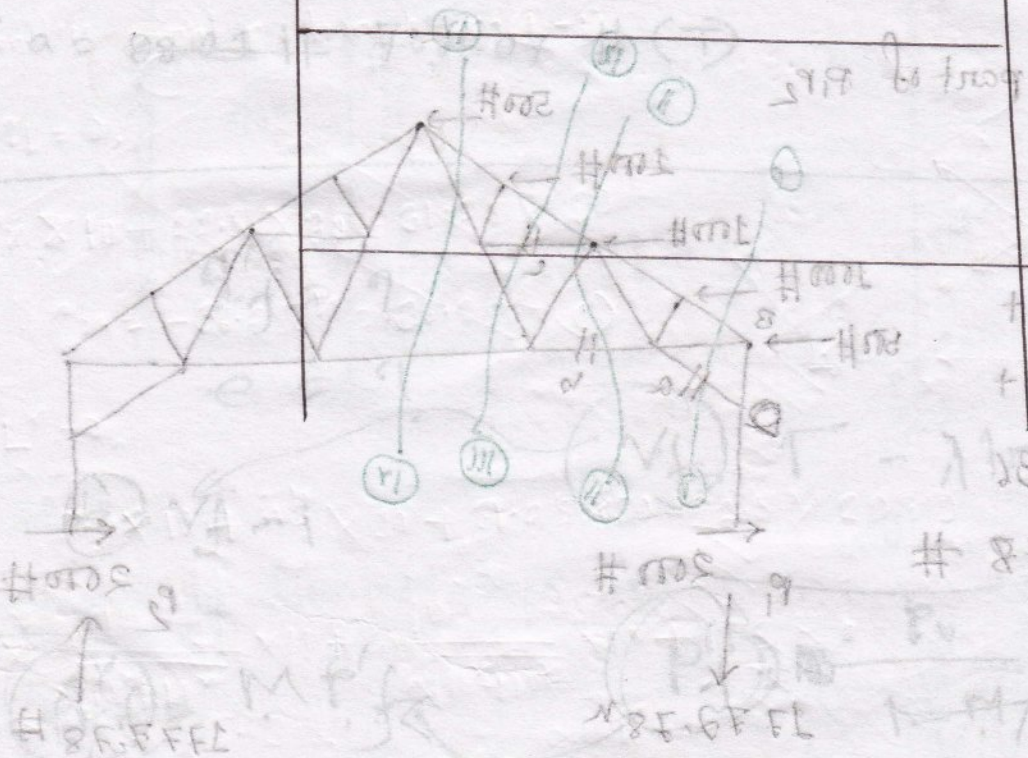

a
b
c
225k'
22.5k
22.5k
19.69k
225k'
393.75k'
393.75k'
225k'
168.75k'
168.75k'
22.5k
22.5k
15.19k
168.75k'
303.75k'
303.75k'
168.75k'
135k'
22.5k
22.5k
135k'



1.



Considering upper part of RP_2

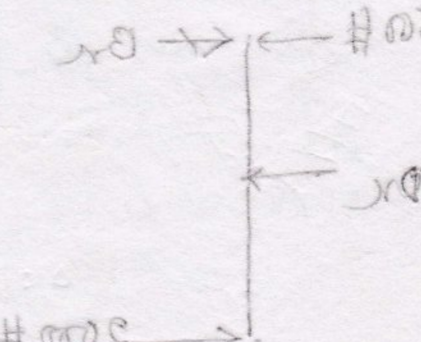


$\Sigma M_{P_1} = 0,$

$500\times15 + 1000\times15.5 + 1000\times16 + 1000\times16.5 + 500\times17 = R_{P_2}\times36$

$\therefore R_{P_2} = 1777.78\#$

Consider part DP_1



$\Sigma M_B = 0,$

$D_x\times6 = 2000\times15$

$\therefore D_x = 5000\#\ (\leftarrow)$

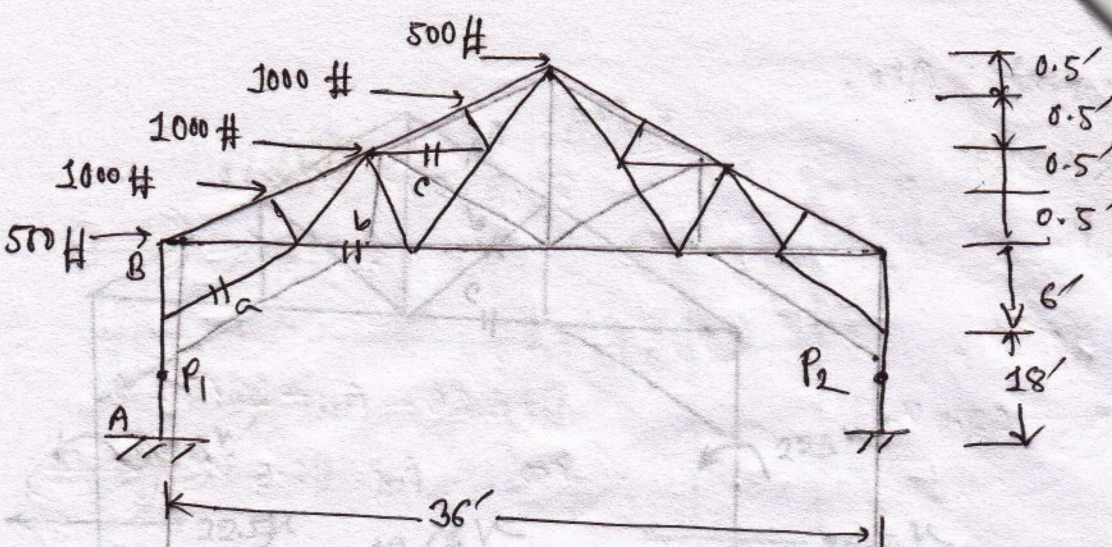
$\Sigma M_D = 0,$

$500\times6 + 2000\times9 = B_x\times6$

$\therefore B_x = 3500\#\ (\rightarrow)$

04

1.

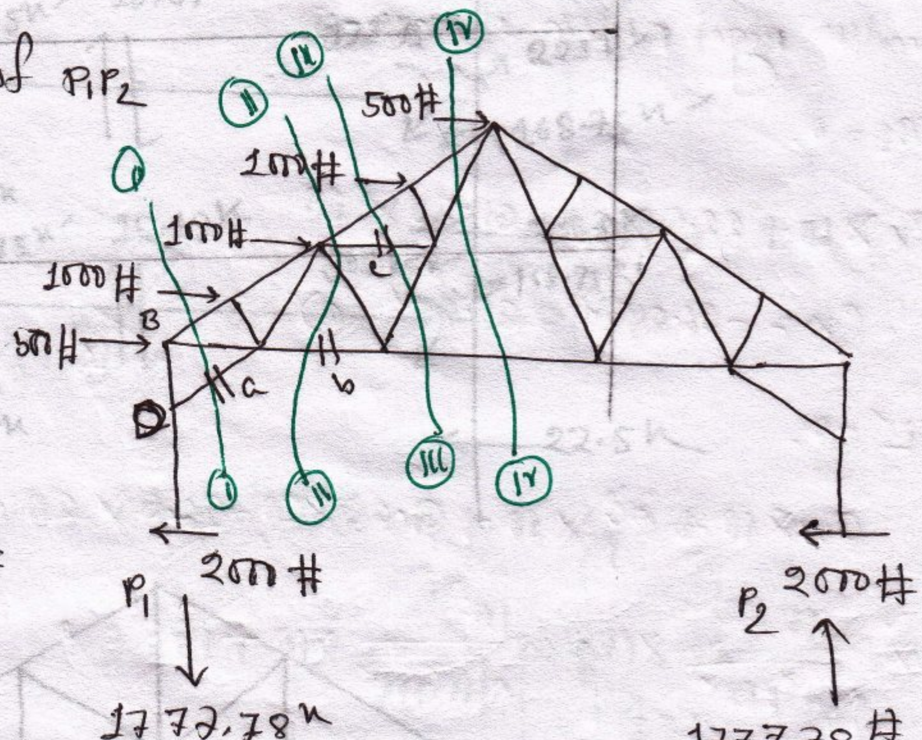


Considering upper part of P_1P_2

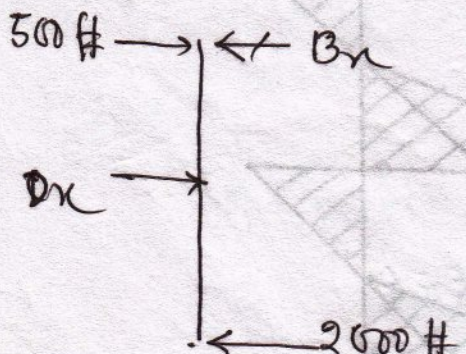
$\Sigma M_{P_1} = 0,$

$500 \times 15 + 1000 \times 15.5 + 1000 \times 16 + 1000 \times 16.5 + 500 \times 17 = R_{P_2} \times 36$

$\therefore R_{P_2} = 1777.78$ #



Consider part BP_1



$\Sigma M_B = 0,$

$D_x \times 6 = 2000 \times 15$

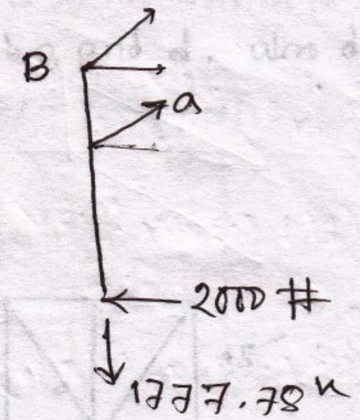
$\therefore D_x = 5000$ # $(\rightarrow)$

$\Sigma M_D = 0,$

$500 \times 6 + 2000 \times 9 = B_x \times 6$

$\therefore B_x = 3500$ # $(\leftarrow)$

sidering left of sec ①-① ⇒



$\Sigma M_B = 0$,

$a_x \times 6 = 2000 \times 15$

$\Rightarrow a \times \frac{6}{\sqrt{6^2+6^2}} \times 6 = 2000 \times 15$

$\therefore a = \cancel{98.21 \#}$ $7071.07 \# \ (T)$

$q'_f = ?$

$e = ?$

$v_f = T - \lambda \ln p'_f$

$e = v_f - 1$

$q'_f = M p'_f$

$p'_f = \frac{p_0}{1 - M/B}$

Cantilever Method

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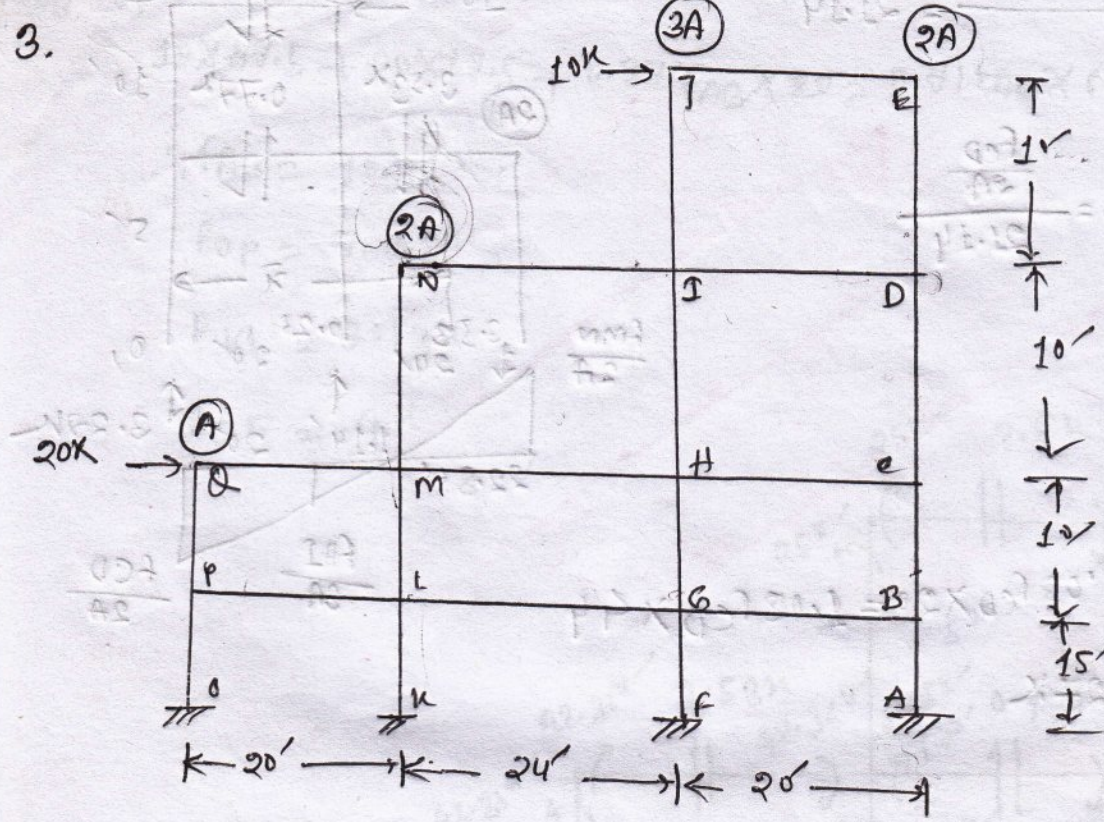
07

3.



Solution:

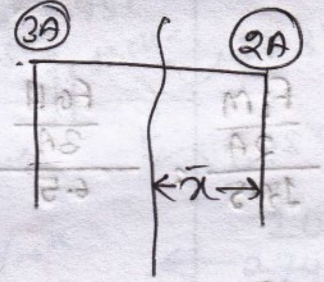
$$\frac{F_{DE}/2A}{12} = \frac{F_{IJ}/3A}{8}$$

$\Rightarrow F_{IJ} = F_{DE}$

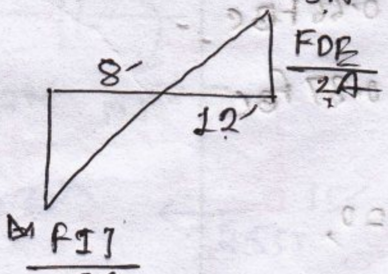
$\Sigma M_O = 0$

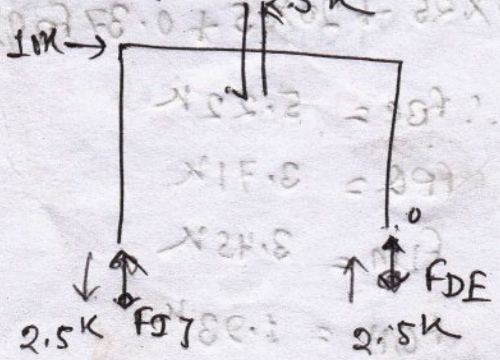
$5 \times 10 \times 10 = F_{IJ} \times 20$

$\therefore F_{IJ} = \cancel{5^k}\ 2.5^k$

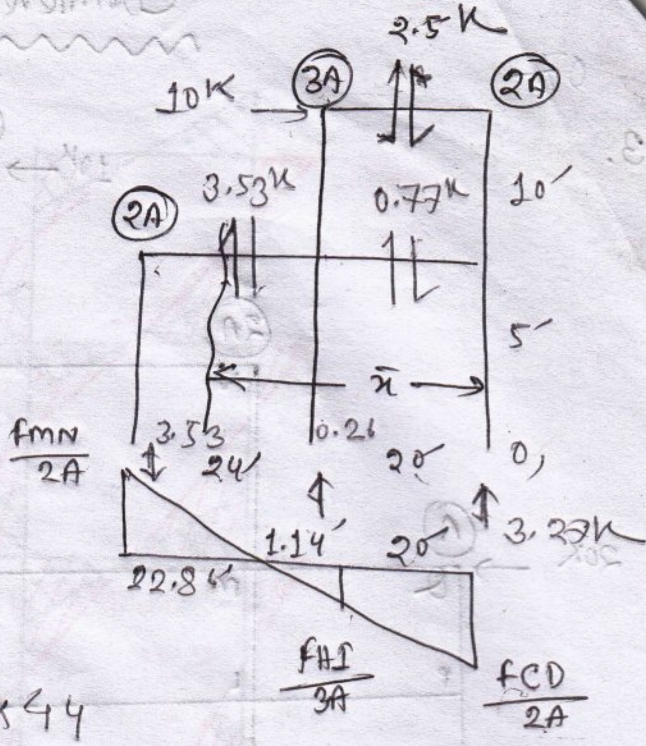


$$\bar{x} = \frac{3A \times 20}{5A} = 12$$





$$\bar{x} = \frac{3A\times 20 + 2A\times 44}{7A} = 21.14'$$



$$\frac{\frac{f_{MN}}{2A}}{22.86} = \frac{\frac{f_{HI}}{3A}}{1.14} = \frac{\frac{f_{CD}}{2A}}{21.14}$$

$\therefore f_{MN} = 1.08\, f_{CD}$

$f_{HI} = 0.08\, f_{CD}$

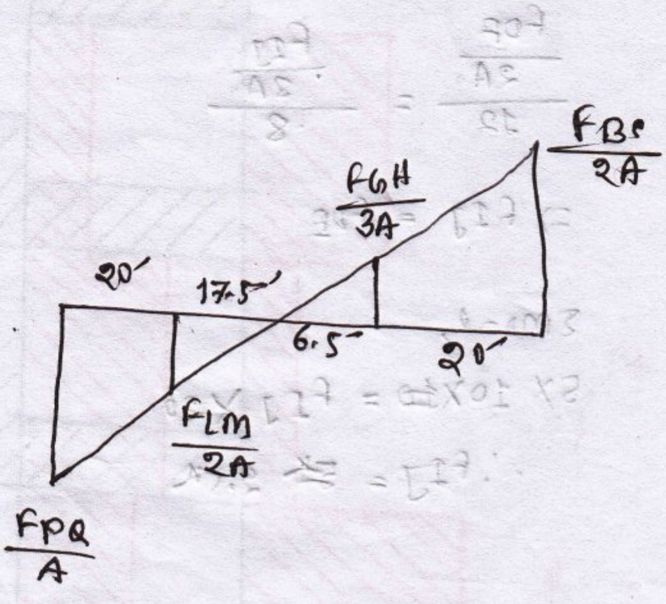
$\Sigma M_{O_1} = 0,$

$$10\times 15 + 0.08 f_{CD}\times 20 = 1.08 f_{CD}\times 44$$

$\therefore f_{CD} = 3.27^K$

$$\bar{x} = \frac{3A\times 20 + 2A\times 44 + A\times 64}{8A} = 26.5'$$

$$\frac{\frac{f_{PQ}}{A}}{37.5} = \frac{\frac{f_{LM}}{2A}}{17.5} = \frac{\frac{f_{GH}}{3A}}{6.5} = \frac{\frac{f_{BC}}{2A}}{26.5}$$



$f_{PQ} = 0.71\, f_{BC}$

$f_{LM} = 0.66\, f_{BC}$

$f_{GH} = 0.37\, f_{BC}$

$\Sigma M_{O_2} = 0,$

$$10\times 25 + 20\times 5 + 0.37 f_{BC}\times 20 = 0.71 f_{BC}\times 64 + 0.66 f_{BC}\times 44$$

$\therefore f_{BC} = 5.22^K$

$f_{PQ} = 3.71^K$

$f_{LM} = 3.45^K$

$f_{GH} = 1.93^K$

Mo3 20,

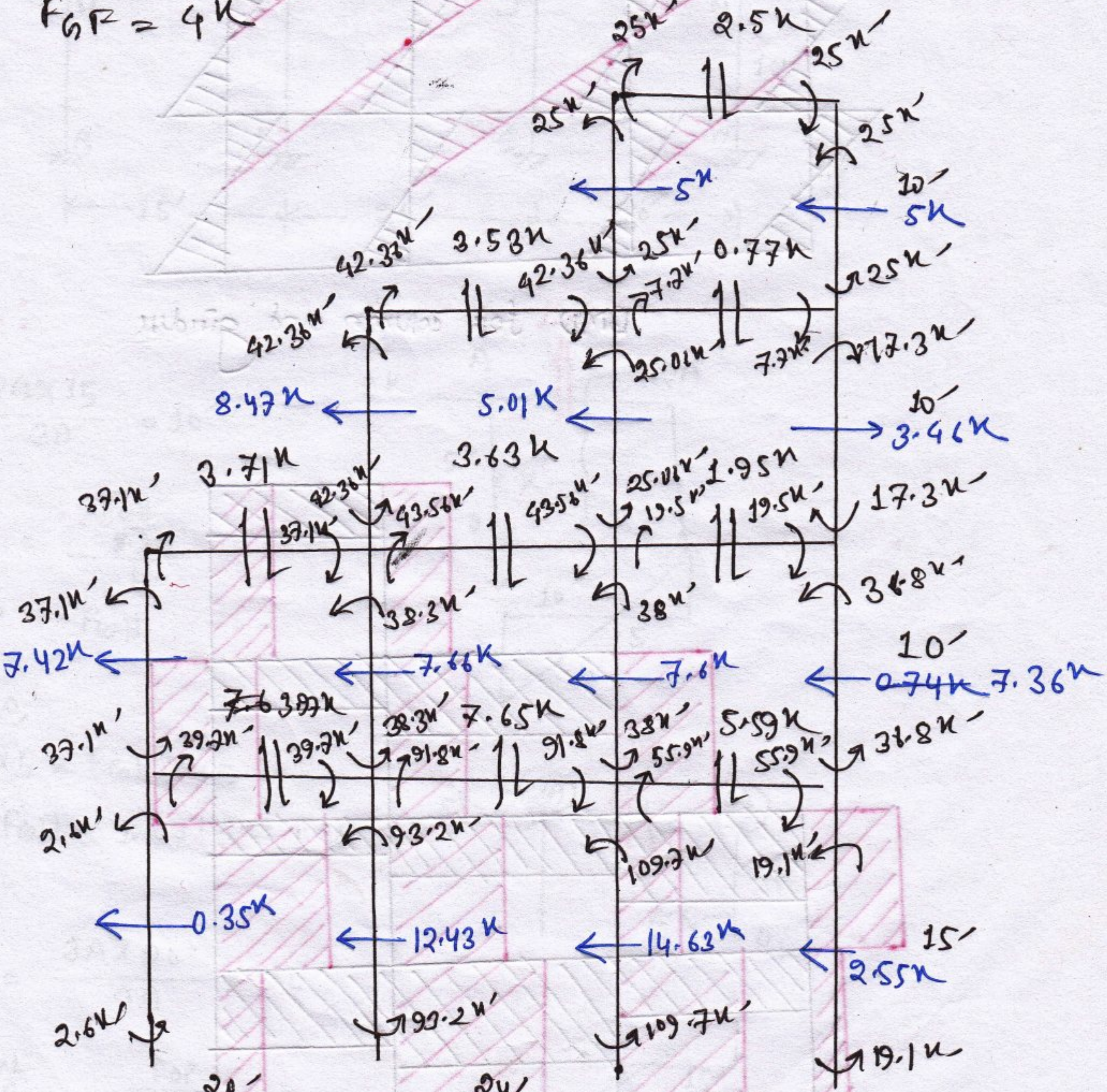
$$10 \times 37.5 + 20 \times 17.5 + 0.37\, F_{AB} \times 20 = 0.71 F_{AB} \times 64 + 0.66\, F_{AB} \times 44$$

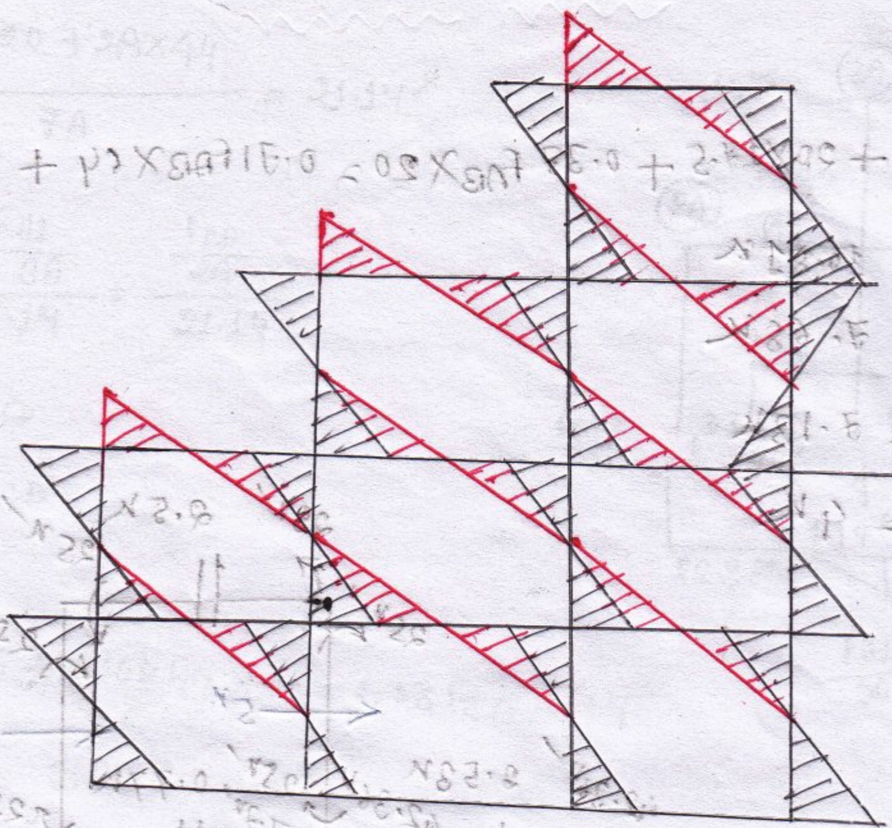
$\therefore F_{AB} = 10.81^k$

$F_{OP} = 7.68^k$

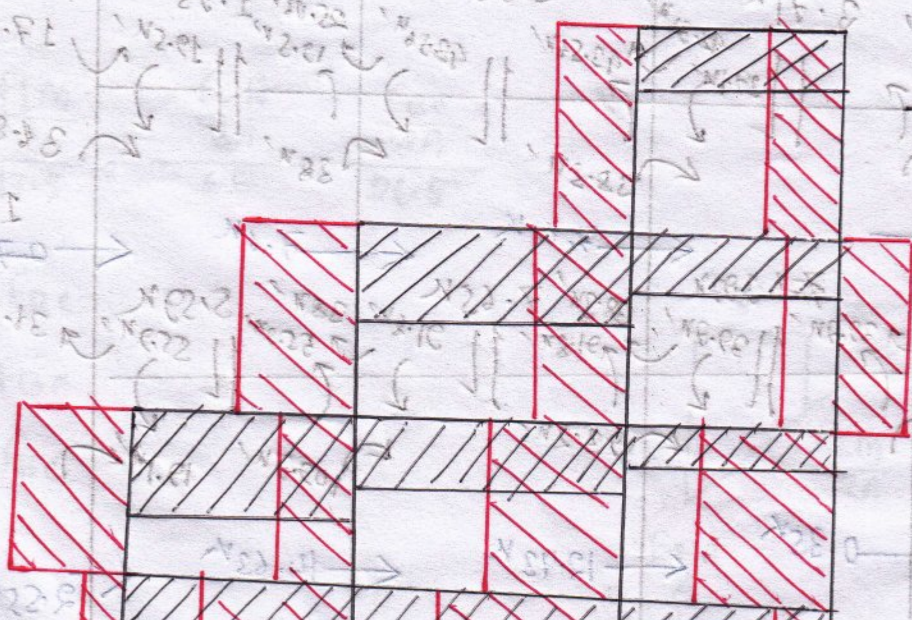
$F_{KL} = 7.13^k$

$F_{GF} = 4^k$





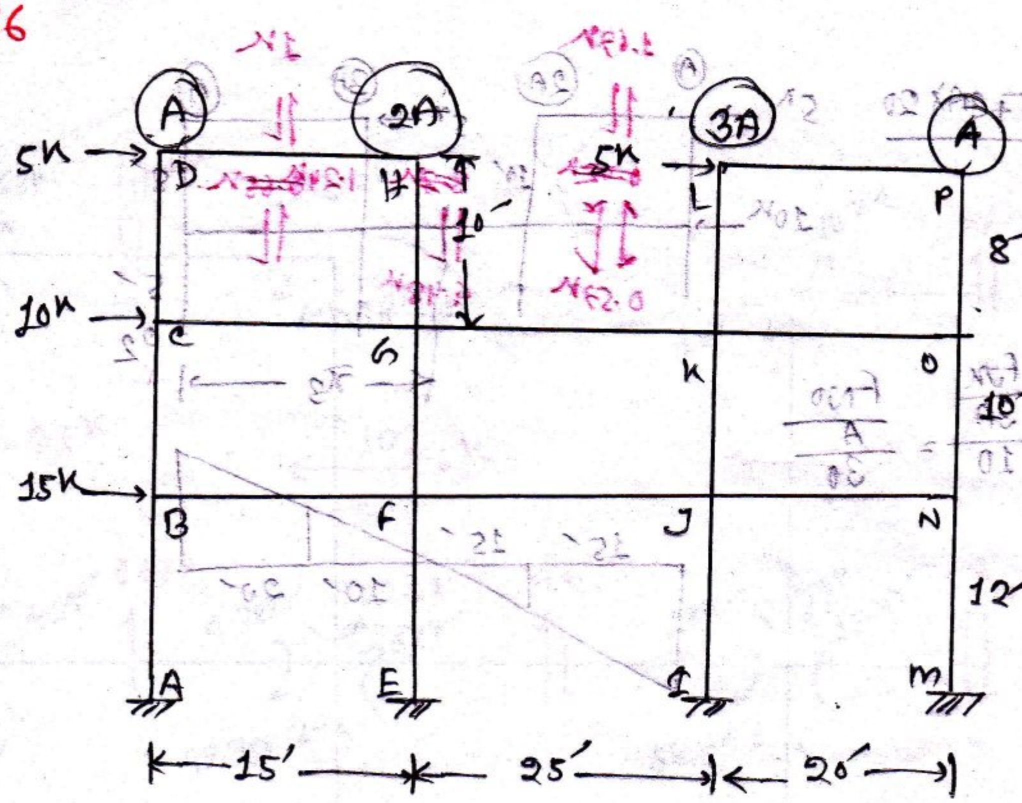
BMD for column and girder



SFD for column and girder

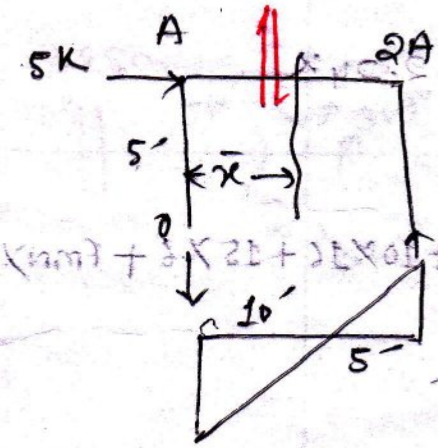
06

5.



Solution:

$$\bar{x} = \frac{2A \times 15}{3A} = 10'$$



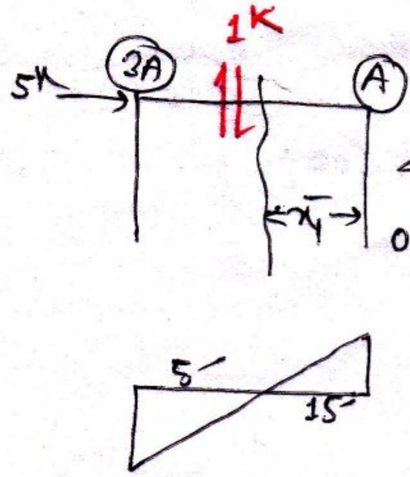
$$\frac{\frac{F_{CD}}{A}}{10} = \frac{\frac{F_{GH}}{2A}}{5}$$

$\therefore F_{CD} = F_{GH}$

$\Sigma M_o = 0,$

$5 \times 5 = F_{GH} \times 15$

$\therefore F_{GH} = 1.67^K = F_{CD}$



$$\bar{x}_1 = \frac{3A \times 20}{4A} = 15'$$

$$\frac{\frac{F_{KL}}{3A}}{5} = \frac{\frac{F_{OP}}{A}}{15}$$

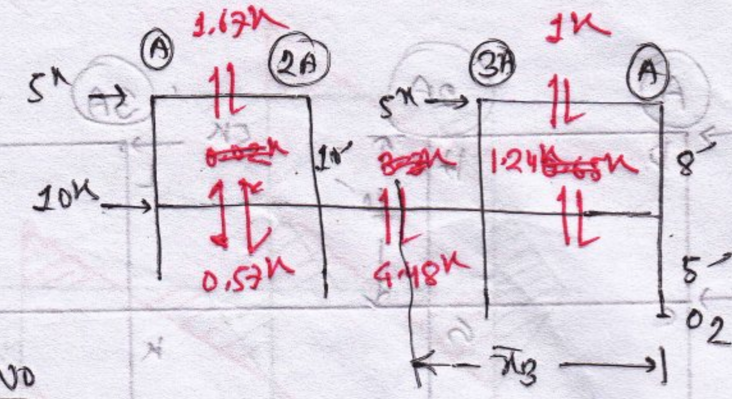
$\therefore F_{KL} = F_{OP}$

$\Sigma M_{O_1} = 0, \quad 5 \times 4 = F_{KL} \times 20$

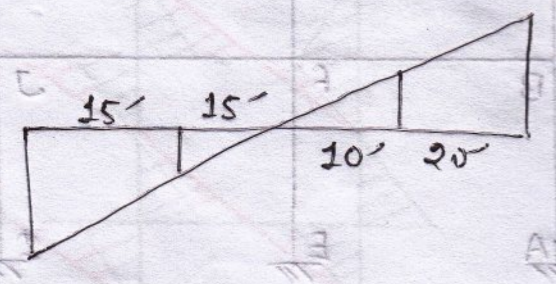
$\therefore F_{KL} = 1^K = F_{OP}$

$$\bar{x}_3 = \frac{A\times 60 + 2A\times 45 + 3A\times 20}{7A}$$

$$= 30K$$



$$\frac{\frac{F_{BC}}{A}}{30} = \frac{\frac{F_{GF}}{2A}}{15} = \frac{\frac{F_{JK}}{3A}}{10} = \frac{\frac{F_{NO}}{A}}{30}$$



$F_{BC} = F_{NO}$

$F_{GF} = F_{NO}$

$F_{JK} \approx F_{NO}$

$\Sigma M_{O_2} = 0,$

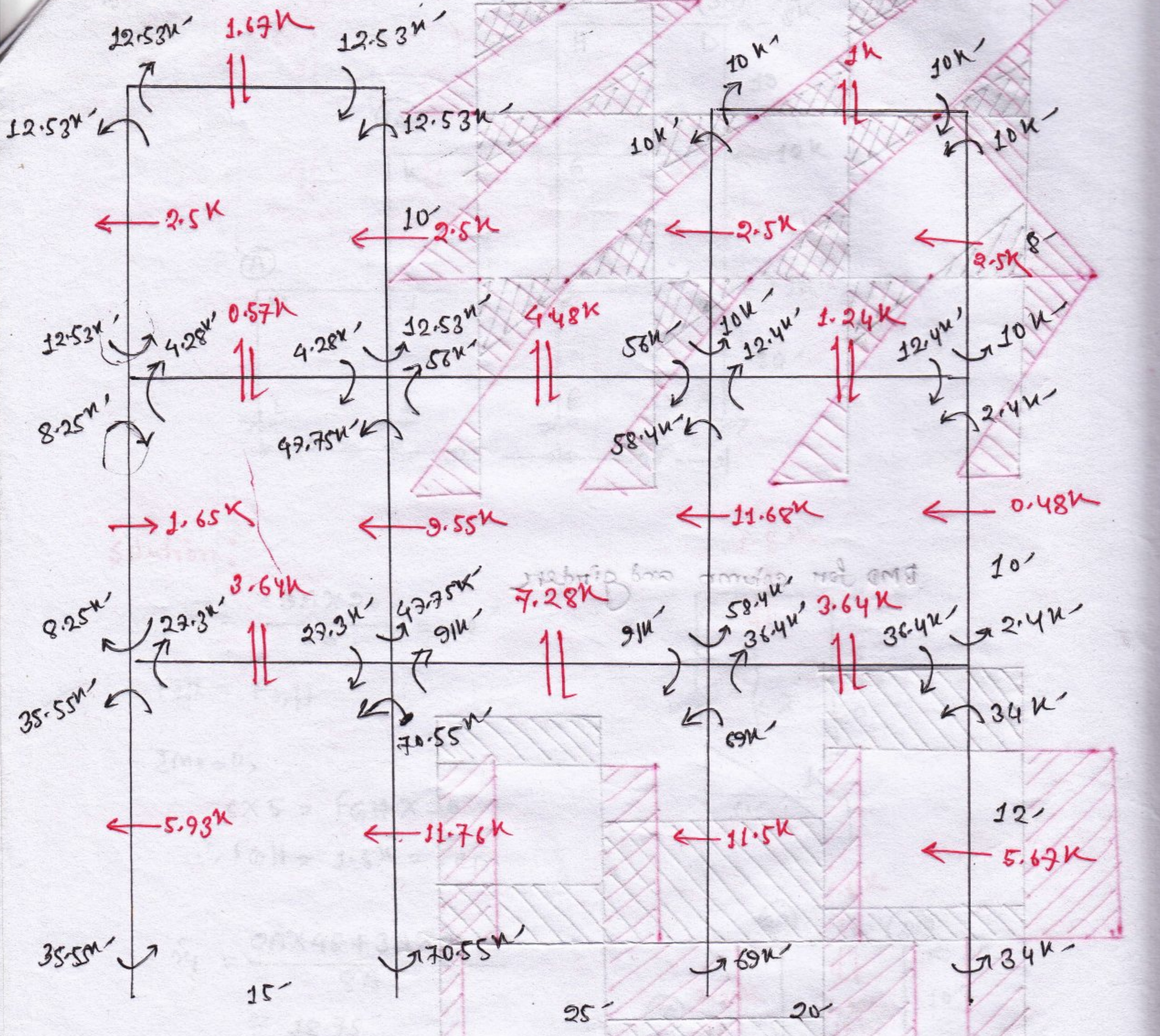
$$10\times5 + 5\times15 + 5\times13 + F_{NO}\times20 = F_{NO}\times60 + F_{NO}\times45$$

$\therefore F_{NO} = \cancel{1.65K}\ 2.24K$

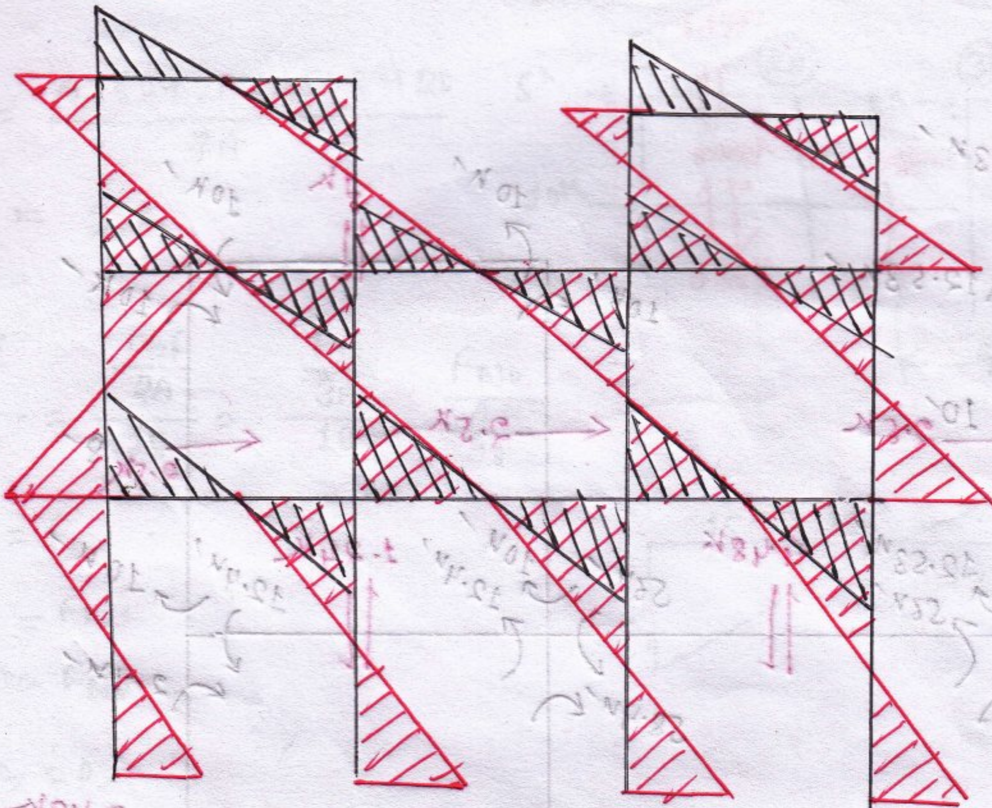
$\Sigma M_{O_3} = 0,$

$$5\times26 + 5\times24 + 10\times16 + 15\times6 + F_{MN}\times20 = F_{MN}\times60 + F_{MN}\times45$$

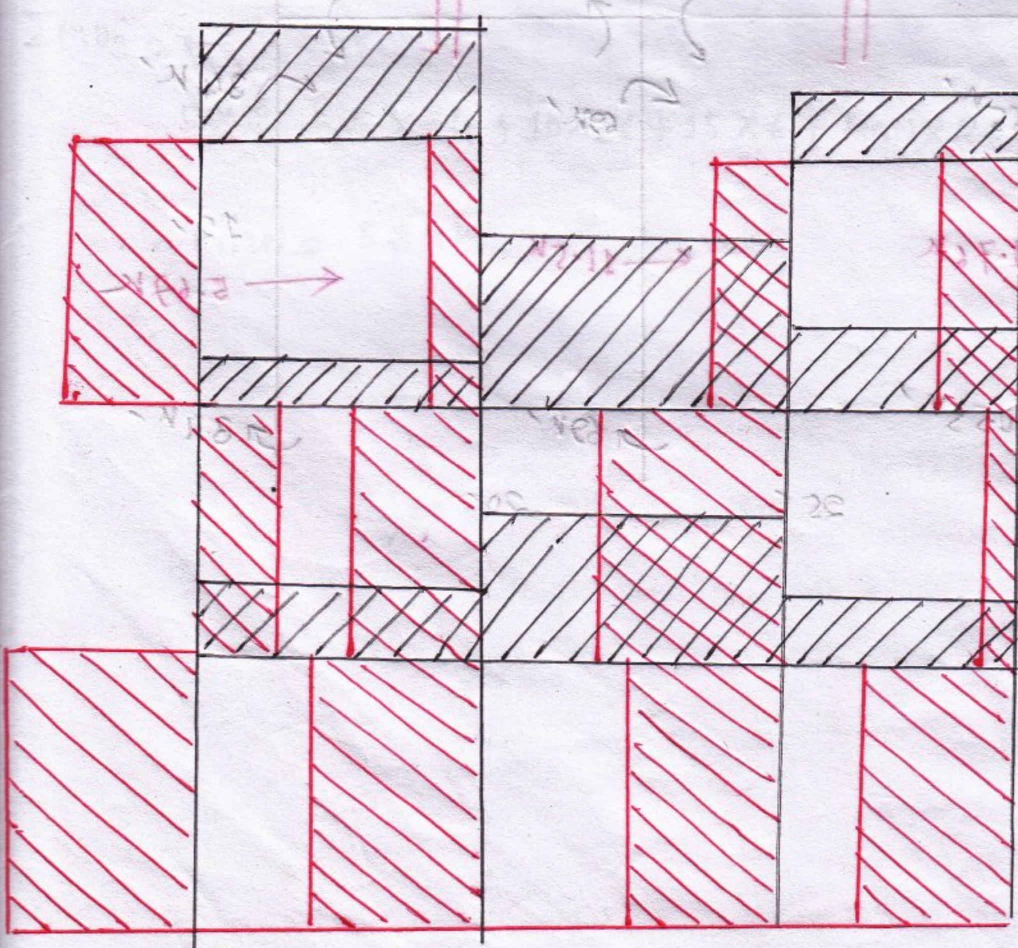
$\therefore F_{MN} = 5.88K$



12.53k'
1.67k
12.53k'
12.53k'
12.53k'
2.5k
10'
2.5k
10k'
1k
10k'
10k'
10k'
2.5k
2.5k
8'
12.53k'
4.28k'
0.57k
4.28k'
12.53k'
4.48k
58k'
58k'
10k'
12.4k'
1.24k
12.4k'
10k'
8.25k'
47.75k'
58.4k'
2.4k'
1.65k
9.55k
11.68k
0.48k
10'
8.25k'
27.3k'
3.64k
27.3k'
47.75k'
91k'
7.28k
91k'
58.4k'
36.4k'
3.64k
36.4k'
2.4k'
35.55k'
70.55k'
69k'
34k'
5.93k
11.76k
11.5k
12'
5.67k
35.55k'
70.55k'
69k'
34k'
15'
25'
20'



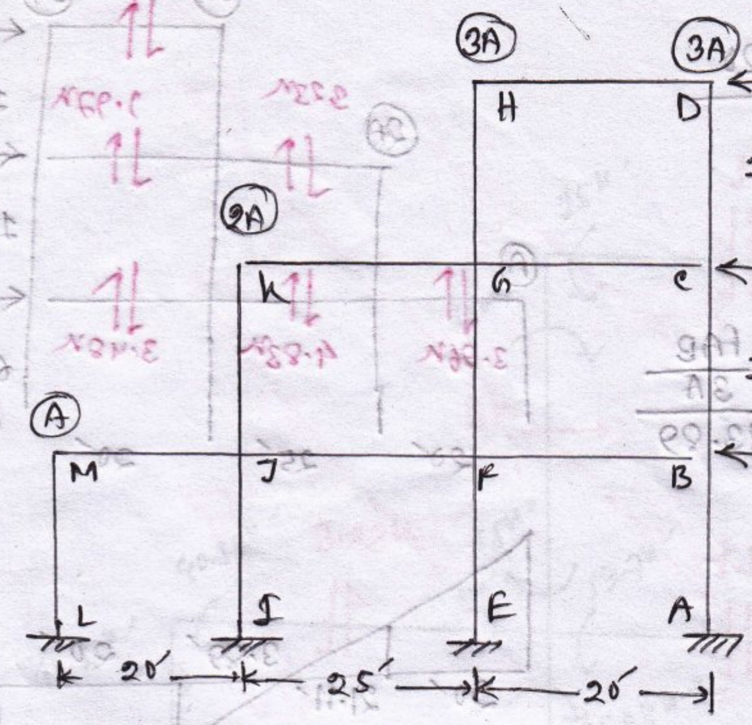
BMD for column and girder



SFD for column and girder

04

5.



Solution:

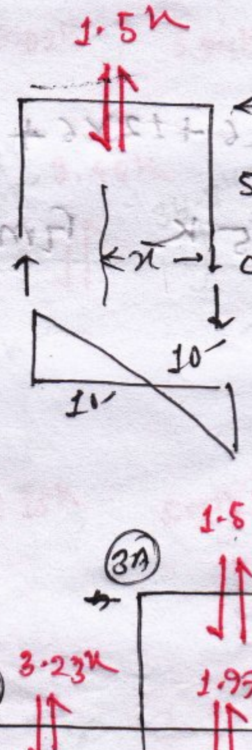
$$\bar{x} = \frac{3A \times 20}{6} = 10'$$

$F_{CD} = F_{GH}$

$\Sigma M_O = 0,$

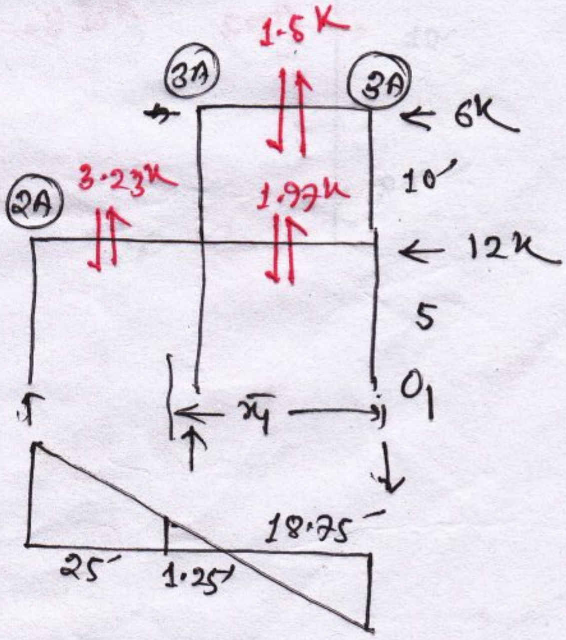
$6 \times 5 = F_{GH} \times 20$

$\therefore F_{GH} = 1.5^k = F_{CD}$



$$\bar{x}_1 = \frac{2A \times 45 + 3A \times 20}{8A}$$

$$= 18.75'$$



$$\frac{\frac{F_{BC}}{3A}}{18.75} = \frac{\frac{F_{GF}}{3A}}{1.25} = \frac{\frac{F_{JK}}{2A}}{26.25}$$

$\therefore F_{GF} = 0.07\, F_{BC}$

$F_{JK} = 0.93\, F_{BC}$

$\Sigma M_{O_1} = 0,$

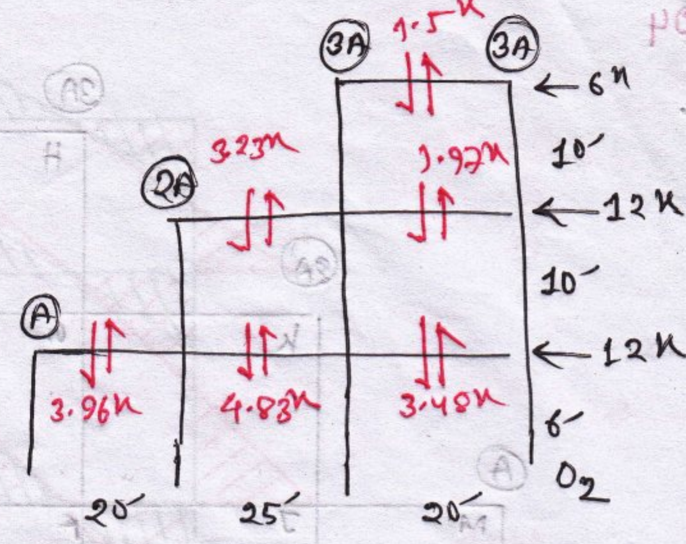
$6 \times 15 + 12 \times 5 = 0.07\, F_{BC} \times 20 + 0.93 \times 45\, F_{BC}$

$\therefore F_{BC} = 3.47^k$ $\quad F_{GF} = 0.24^k$ $\quad F_{JK} = 3.23^k$

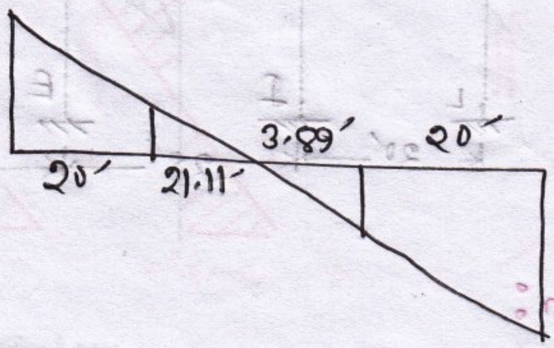
ΣM

$$\bar{x}_2 = \frac{A \times 65 + 2A \times 45 + 3A \times 20}{9A}$$

$$= 23.89'$$



$$\frac{\frac{F_{LM}}{A}}{41.11} = \frac{\frac{F_{IJ}}{2A}}{21.11} = \frac{\frac{F_{EF}}{3A}}{3.89} = \frac{\frac{F_{AB}}{3A}}{23.89}$$



$F_{LM} = 0.57\ F_{AB}$

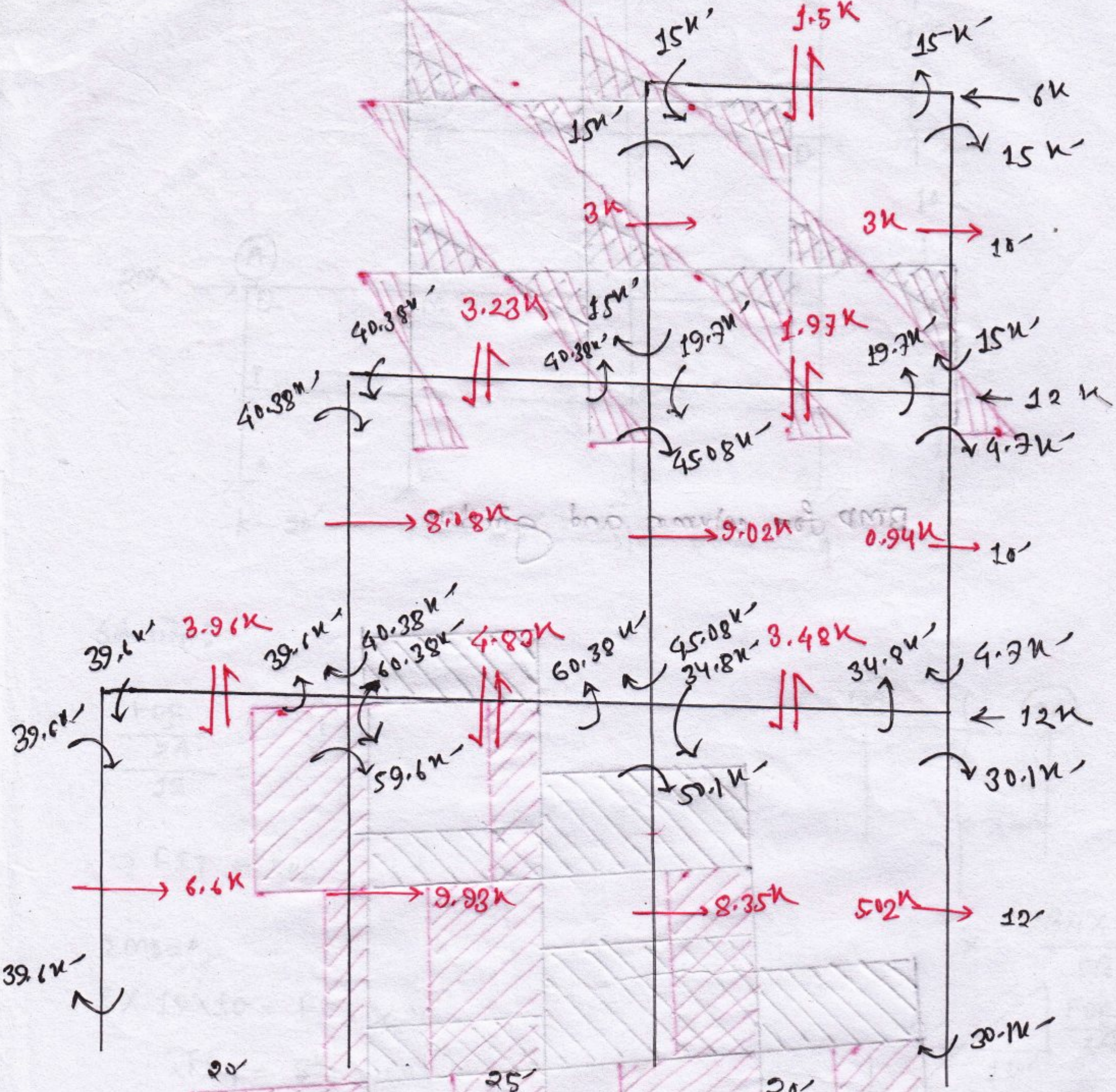
$F_{IJ} = 0.59\ F_{AB}$

$F_{EF} = 0.16\ F_{AB}$

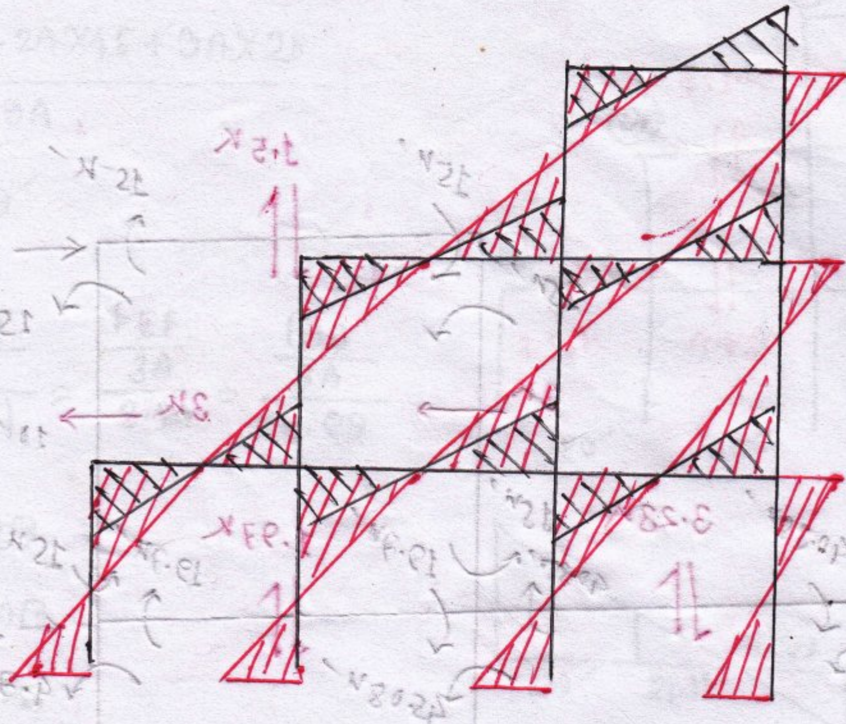
$\Sigma M_{O_2} = 0,$

$$6 \times 26 + 12 \times 16 + 12 \times 6 + 0.16 F_{AB} \times 20 = 0.57 F_{AB} \times 65 + 0.59 F_{AB} \times 45$$

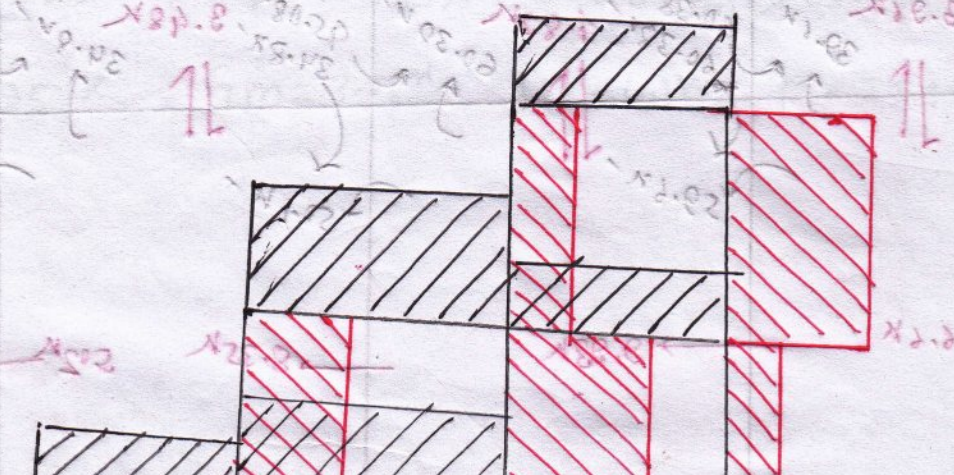
$\therefore F_{AB} = 6.95\ K \quad F_{LM} = 3.96^K \quad F_{IJ} = 4.1^K \quad F_{EF} = 1.11^K$



15k'
1.5k
15k'
6k
15k'
15k'
3k
3k
10'
40.38k'
3.23k
15k'
40.38k'
19.7k'
1.97k
19.7k'
15k'
40.38k'
12k
45.08k'
4.7k'
8.18k
9.02k
0.94k
10'
39.6k'
3.96k
39.6k'
40.38k'
60.38k'
4.82k
60.38k'
45.08k'
34.8k'
3.48k
34.8k'
4.7k'
39.6k'
12k
59.6k'
50.1k'
30.1k'
6.6k
9.93k
8.35k
5.02k
12'
39.6k'
30.1k'
20'
25'
20'



BMD for column and girder



SFD for column and girders

(02)

Castigliano's Theorem / Partial Derivative Method

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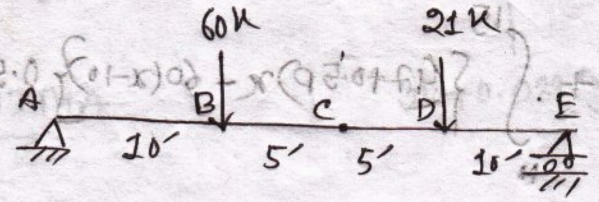
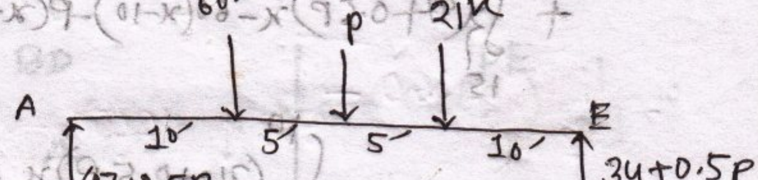
Q-1:



$E = 30\times10^3$ ksi

$I = 1000$ in^4

$\Delta_C, \theta_A = ?$



$\Sigma M_A = 0,\ 60\times10 + P\times15 + 21\times20 = R_E\times30$

$\therefore R_E = 34 + 0.5P$

Solution:

Portion:	AB	BC	CD	DE
Origin:	A	A	A	E
Limit:	0–10	10–15	15–20	0–10
M:	$(47+0.5P)x$	$(47+0.5P)x - 60(x-10)$	$(47+0.5P)x - 60(x-10) - P(x-15)$	$(34+0.5P)x$
$\frac{\partial M}{\partial P}$:	$0.5x$	$0.5x$	$0.5x-(x-15)$ $= 15-0.5x$	$0.5x$

$$\Delta_C = \int_0^L \frac{M}{EI} \cdot \frac{\partial m}{\partial P} \cdot dx$$

$$= \frac{1}{EI}\left[\int_0^{10} (47 + 0.5P)x(0.5x)\,dx + \int_{10}^{15} \{(47+0.5P)x - 60(x-10)\}\, 0.5x\,dx \right.$$

$$+ \int_{15}^{20} \{(47+0.5P)x - 60(x-10) - P(x-15)\}(15-0.5x)\,dx +$$

$$\left. \int_0^{10} (34+0.5P)x(0.5x)\,dx \right] \quad (P=0)$$

$$= \frac{1}{EI}\left[\int_0^{10} 23.5x^2\,dx + \int_{10}^{15} (600-13x)\,0.5x\,dx + \int_{15}^{20} (600-13x)(15-0.5x)\,dx \right.$$

$$\left. + \int_0^{10} 17x^2\,dx \right]$$

$$= \frac{1}{EI}\left\{ \left[\frac{23.5x^3}{3}\right]_0^{10} + \left[\frac{300x^2}{2} - \frac{6.5x^3}{3}\right]_{10}^{15} + \left[9000x - \frac{300x^2}{2} - \frac{195x^2}{2} + 6.5\frac{x^3}{3}\right]_{15}^{20} \right.$$

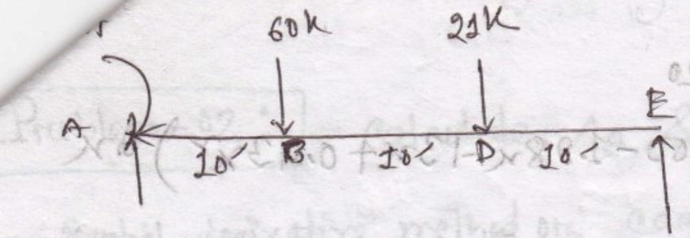
$$\left. + \left[\frac{17x^3}{3}\right]_0^{10} \right\}$$

$$= \frac{1}{EI}\big(7833.33 + 33750 - 7312.5 - 15000 + 2166.67 + 180000 -$$

$$60000 - 39000 + 17333.33 - 135000 + 33750 + 21937.5 -$$

$$7312.5 + 5666.67\big)$$

$$= \frac{1}{30 \times 10^3 \times 1000} \times 38812.5 \times 1728 = 2.23 \text{ in } (\downarrow)$$



$\Sigma M_A = 0,$

$M_A + 60 \times 10 + 21 \times 20 = R_E \times 30$

$\therefore R_E = 34 + 0.033 M_A$

$\underline{47 - 0.033 M_A}$ $\quad$ $34 + 0.033 M_A$

Portion:	AB	BD	DE
Origin:	A	A	E
Limit:	0-10	10-20	0-10
M:	$(47-0.033M_A)x + M_A$	$(47-0.033M_A)x + M_A - 60(x-10)$	$(34+0.033M_A)x$
$\frac{\partial m}{\partial M_A}$:	$1 - 0.033x$	$1-0.033x$	$0.033x$

$$\theta_A = \frac{1}{EI}\int_0^L M \cdot \frac{\partial m}{\partial M_A} \cdot dx$$

$$= \frac{1}{EI}\left[\int_0^{10}\{(47-0.033M_A)x + M_A\}(1-0.033x)\,dx + \int_{10}^{20}\{(47-0.033M_A)x + M_A - 60(x-10)\}(1-0.033x)\,dx + \int_0^{10}(34+0.033M_A)x\cdot(0.033x)\,dx\right]$$

$$= \frac{1}{EI}\left[\int_0^{10}(47x - 1.551x^2)\,dx + \int_{10}^{20}(47x - 60x + 600)(1-0.033x)\,dx + \int_0^{10} 34x\ 1.122x^2\,dx\right]$$

$$= \frac{1}{EI}\left\{\left[\frac{47x^2}{2} - \frac{1.551x^3}{3}\right]_0^{10} + \int_{10}^{20}(600 - 1.98x - 13x + 0.93x^2)\,dx + \left[\frac{1.22x^3}{3}\right]_0^{10}\right\}$$

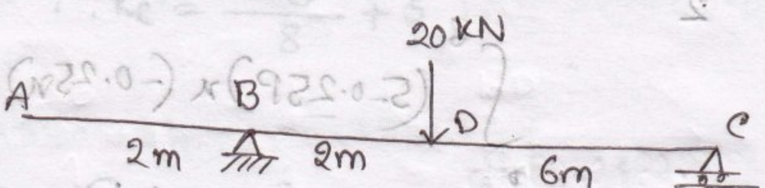
$$= \frac{1}{EI}\left(2350 - 517 + 12000 - 2996 + 34.67 - 6000 + 749 - 4.33 + 406.67\right)$$

$$= \frac{1}{30\times10^3\times 1000} \times 6023.01 \times 144$$

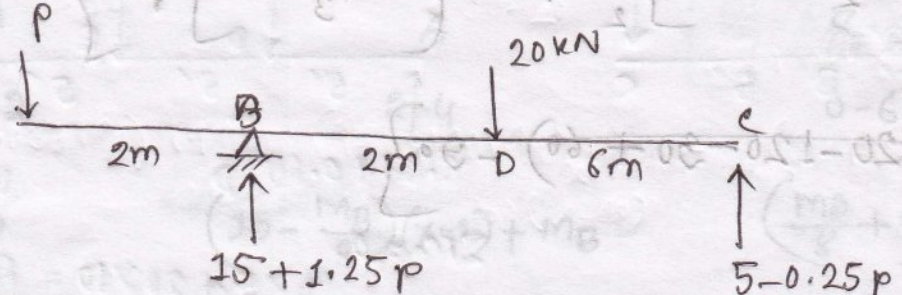
$$= 0.03_{rad} (\circlearrowright)$$

(Am

Problem-02: Calculate Δ_A & θ_B of the following structure by Partial derivative method or Castigliano's method? $E = 200 \times 10^6\ kN/m^2$, $I = 160 \times 10^{-6}\ m^4$.



Solution:



$\sum M_B = 0,$

$$P \times 2 + R_c \times 8 = 20 \times 2$$

$$\therefore R_c = 5 - 0.25\ P$$

Portion:	AB	BD	DC
Origin:	A	A	C
Limit:	0–2	2–4	0–6
M:	$-Px$	$-Px + (15 + 1.25P)(x-2)$	$(5 - 0.25P)x$
$\frac{dM}{dP}$:	$-x$	$-x + 1.25(x-2)$	$-0.25x$

$$\Delta_A = \frac{1}{EI}\int_0^L m \cdot \frac{\partial m}{\partial P}\, dx$$

$$= \frac{1}{EI}\left\{\int_0^2 Px^2\, dx + \int_2^4 (-Px + 15x - 30 - 1.25Px + 2.5P)(0.25x - 2.5)\, dx + \int_0^6 (5 - 0.25P)x\,(-0.25x)\, dx\right\}$$

$$= \frac{1}{EI}\left\{\left[\frac{15x^2}{2} - 30x\right]_2^4 - \left[\frac{1.25x^3}{3}\right]_0^6\right\}$$

$$= \frac{1}{EI}\left\{(120 - 120 - 30 + 60) - 90\right\}$$

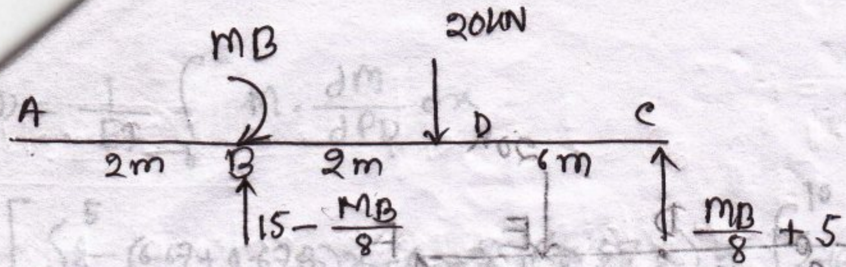
$$= \frac{1}{EI}\left\{\int_2^4 (3.75x^2 - 37.5x - 7.5x + 75)\, dx - \left[\frac{1.25x^3}{3}\right]_0^6\right\}$$

$$= \frac{1}{EI}\left\{\left[\frac{3.75x^3}{3} - \frac{45x^2}{2} + 75x\right]_2^4 - 90\right\}$$

$$= \frac{1}{EI}(80 - 360 + 300 - 10 + \cancel{90} - 150 - \cancel{90})$$

$$= \frac{1}{200\times10^6 \times 160\times10^{-6}} \times (-140)$$

$= -0.00437$ m $= 0.00437$ m (↑)



$\Sigma M_B = 0, \quad M_B + 20 \times 2 = R_C \times 8$

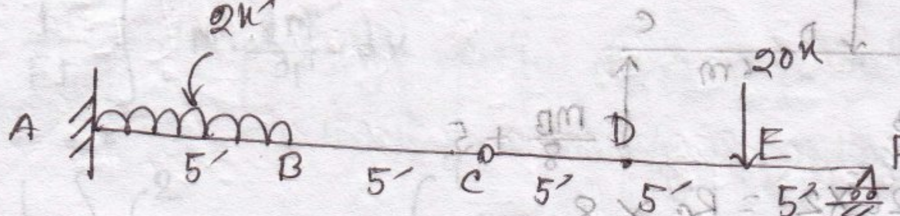
$\therefore R_C = \frac{M_B}{8} + 5$

Portion :	AB	BD	DC
Origin :	A	A	C
Limit :	0 - 2	2 - 4	0 - 6
M :	0	$\left(15-\frac{M_B}{8}\right)(x-2)+M_B$	$\left(\frac{M_B}{8}+5\right)x$
$\frac{dm}{dm_B}$:		$1-\frac{x-2}{8}$	$\frac{x}{8}$

$$\theta_B = \frac{1}{EI}\int_0^2 M . \frac{dm}{dm_B}\, dx$$

=

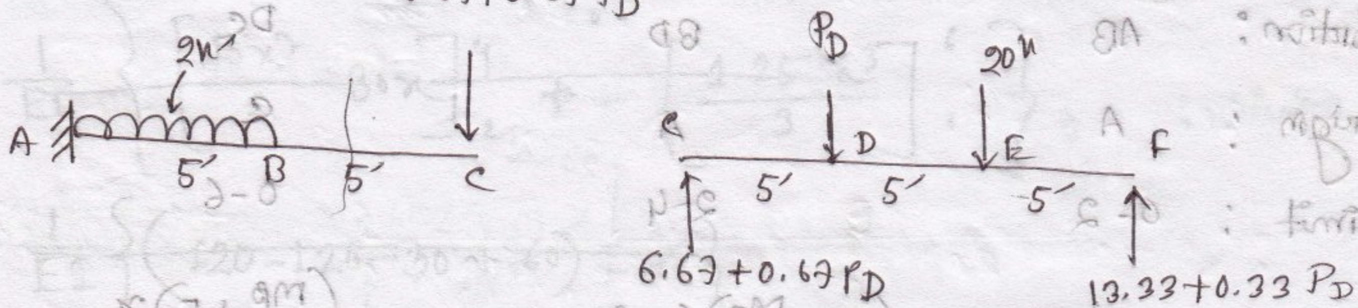
Problem-03:



$\Delta_D = ?$

$\theta_{CD} = ?$

Solution:



$\Sigma M_C = 0;$

$P_D \times 5 + 20 \times 10 = R_F \times 15$

$R_F = 0.33 P_D + 13.33$

Portion:	AB	BC	CD	DE	EF
Origin:	C	C	C	F	F
Limit:	0-5	5-10	0-5	5-10	0-5
M:	$-(6.67+0.67P_D)x - (x-5)^2$	$-(6.67+0.67P_D)x$	$(6.67+0.67P_D)x$	$\cancel{(6.67+0.67P_D)x}$ $\cancel{-P_D(x-5)}$ $(13.33+0.33P_D)x - 20(x-5)$	$(13.33+0.33P_D)x$
$\frac{\partial M}{\partial P_D}$:	$-0.67x$	$-0.67x$	$0.67x$	$0.33x$	$0.33x$

$$\Delta_D = \frac{1}{EI}\int_0^L M \cdot \frac{\partial M}{\partial P_D}\, dx$$

$$= \frac{1}{EI}\Big[\int_0^5 \{-(6.67+0.67P_D)x - (x-5)^2\}(-0.67x)\,dx + \int_5^{10}\{-(6.67+0.67P_D)x\}(-0.67x)\,dx$$

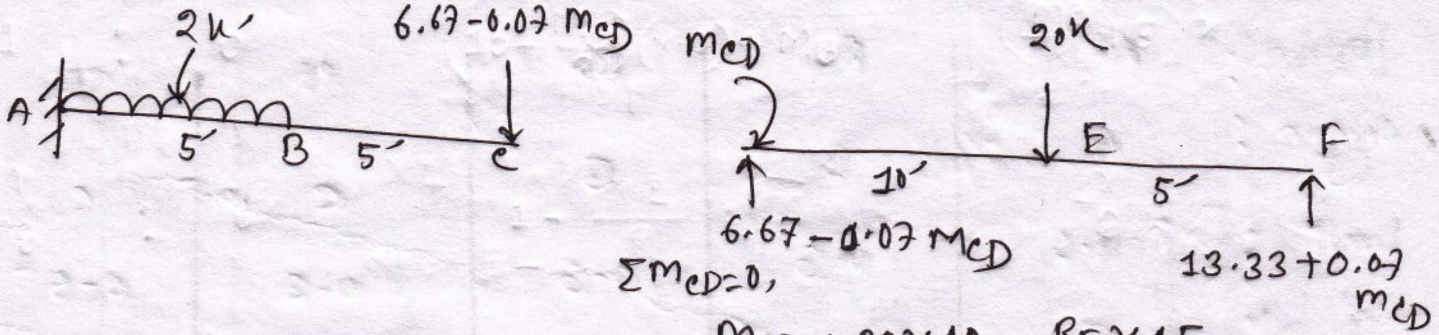
$$+ \int_0^5 (6.67+0.67P_D)x \times 0.67x\,dx + \int_5^{10}\{(13.33+0.33P_D)x - 20(x-5)\}\,0.33x\,dx$$

$$+ \int_0^5 (13.33+0.33P_D)x \times 0.33x\,dx \Big\}$$

= --- — ---

--- ---

[$P_D = 0$ বসালে হবে]



$\Sigma M_{CD} = 0,$

$M_{CD} + 20\times 10 = R_F \times 15$

$\therefore R_F = 0.07\, M_{CD} + 13.33$

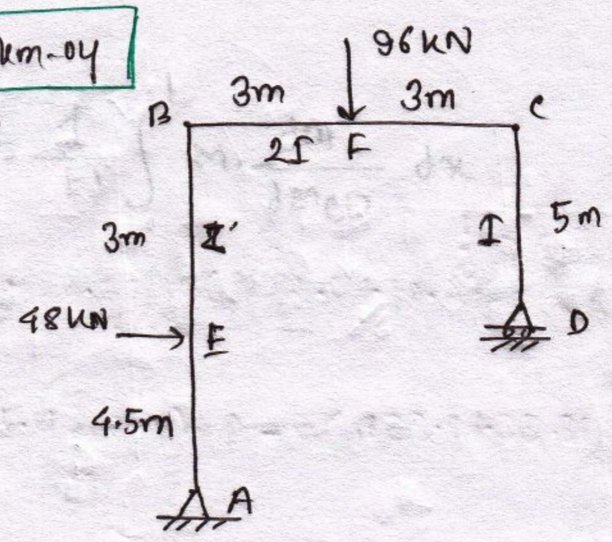
Portion:	AB	BC	CE	EF
Origin:	C	C	F	F
Limit:	0-5	5-10	5-15	0-5
M:	$-(6.67-0.07M_{CD})x - (x-5)^2$	$-(6.67-0.07M_{CD})x$	$(13.33+0.07M_{CD})x - 20(x-5)$	$(13.33+0.07M_{CD})x$
$\frac{\partial M}{\partial M_{CD}}$:	$0.07x$	$0.07x$	$0.07x$	$0.07x$

$$\theta_{CD} = \frac{1}{EI}\int_0^L M \cdot \frac{\partial m}{\partial M_{CD}}\, dx$$

$(M_{CD} = 0)$

Problem-04

Q.



$E = 200 \times 10^6$ kN/m $\quad I = 160 \times 10^{-6}$ m^4

$\theta_A, \theta_B, \theta_C, \theta_D = ?$

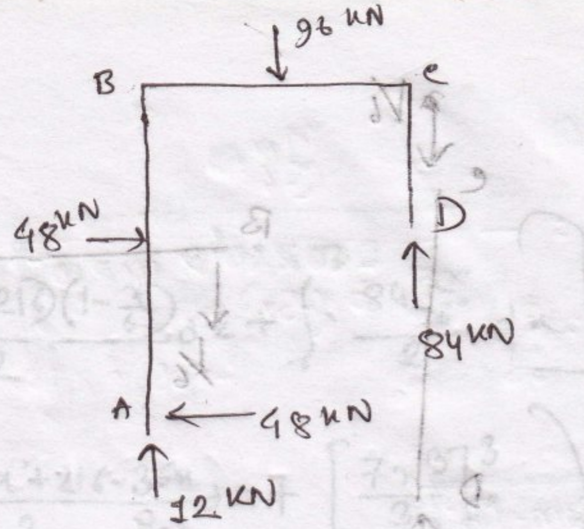
Solution:

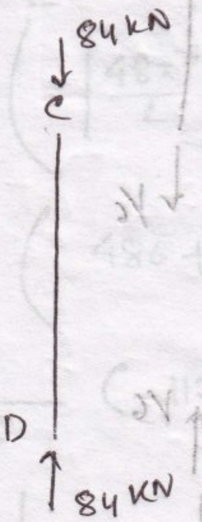
Portion :	AE	BE	BF	CF	CD
Origin :	A	B	B	C	C
Limit :	0-4.5	0-3	0-3	0-3	0-5
I :	I	I	2I	2I	I
M :	$48x$	216	$12x+216$	$84x$	0
$m\theta_A$:	1	1	$1-\frac{x}{6}$	$\frac{x}{6}$	0
$m\theta_B$:	0	0	$1-\frac{x}{6}$	$\frac{x}{6}$	0
$m\theta_C$:	0	0	$-\frac{x}{6}$	$\frac{x}{6}-1$	0
$m\theta_D$:	0	0	$-\frac{x}{6}$	$\frac{x}{6}-1$	-1

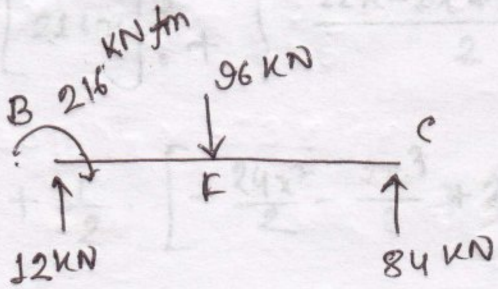
$\Sigma M = 0,\quad 48 \times 4.5 + 96 \times 3 = R_D \times 6$

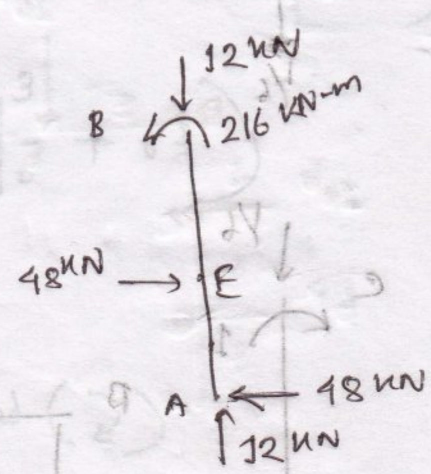
$\therefore R_D = 84\ kN$

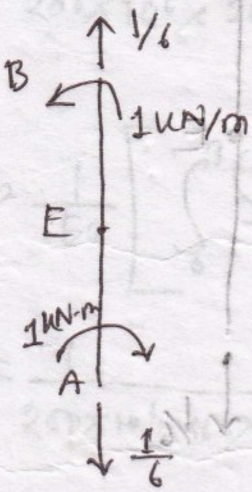
$\Sigma F_y = 0,\quad R_D = 96 - 84 = 12\ kN$

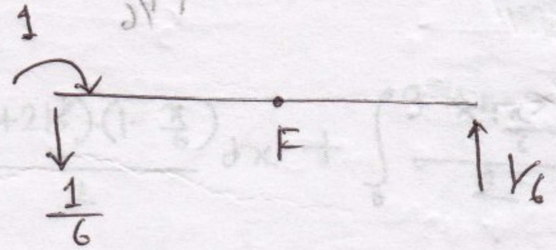


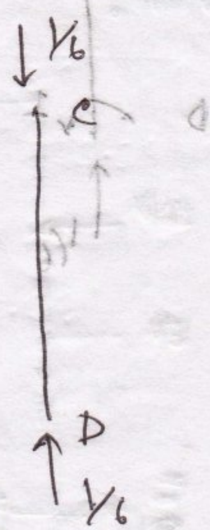


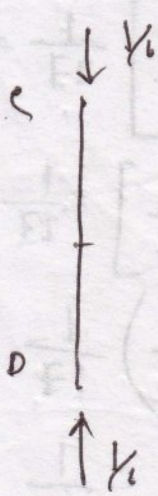


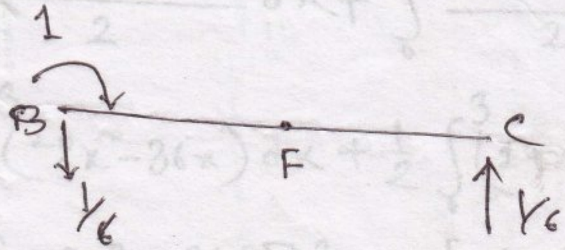


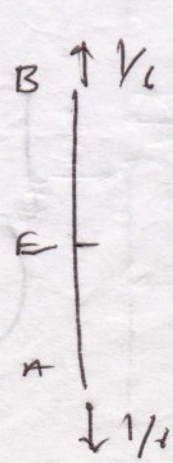


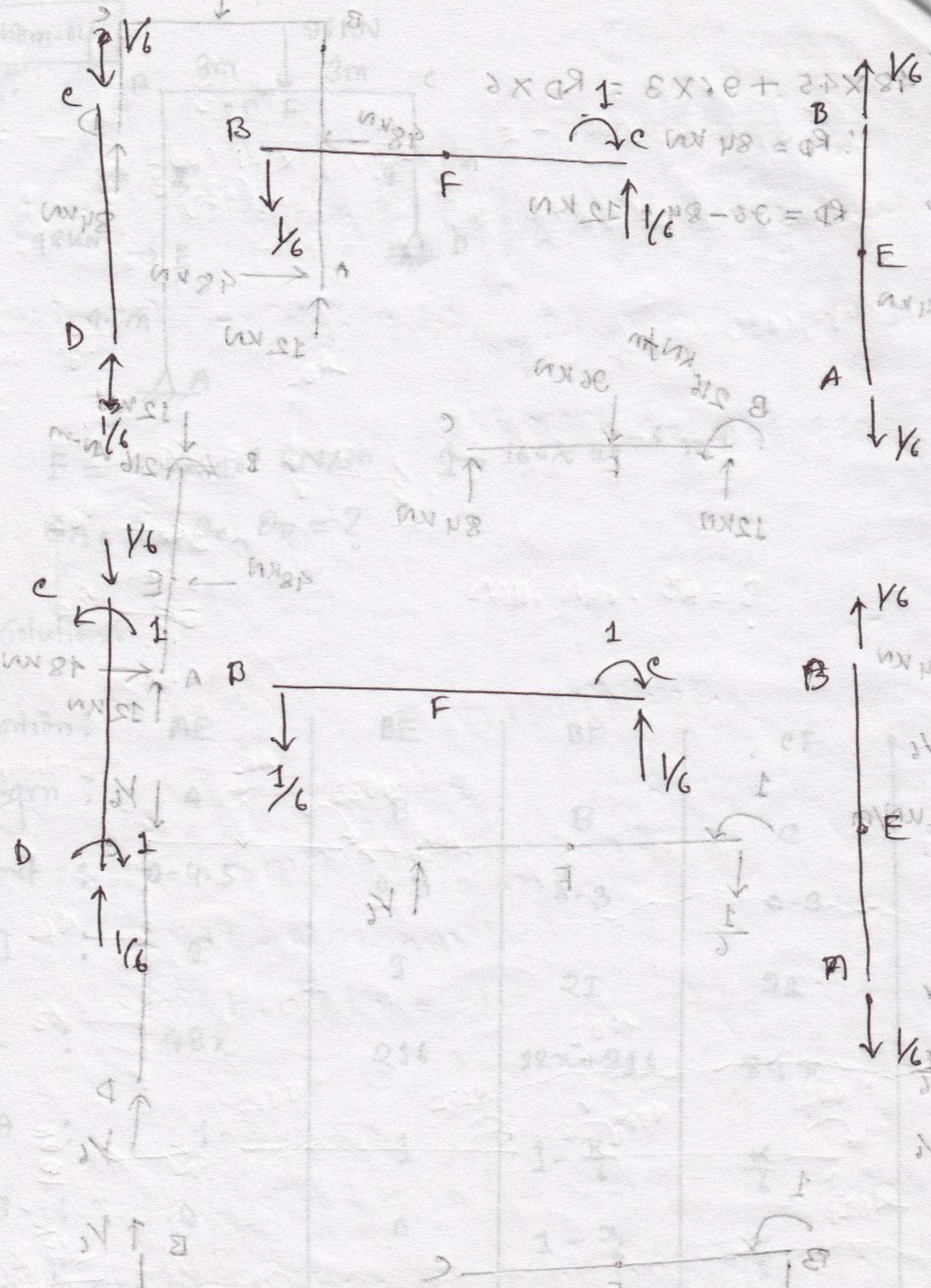












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1/6
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$$\frac{1}{EI}\int_0^L Mm\,dx$$

$$= \frac{1}{EI}\left[\int_0^{4.5} 48x\,dx + \int_0^3 216\,dx + \int_0^3 \frac{(12x+216)(1-\frac{x}{6})}{2}\,dx + \int \frac{84\frac{x^2}{6}}{2}\,dx\right]$$

$$= \frac{1}{EI}\left(\left[\frac{48x^2}{2}\right]_0^{4.5} + \left[216x\right]_0^3 + \int_0^3 \frac{12x-2x^2+216-36x}{2}\,dx + \left[\frac{7x^3}{3}\right]_0^3\right)$$

$$= \frac{1}{EI}\left(486+648+\frac{1}{2}\cdot\left[-\frac{24x^2}{2}-\frac{2x^3}{3}+216x\right]_0^3 + 63\right)$$

$$= \frac{1}{EI}\left(1134+261+63\right)$$

$$= \frac{1}{200\times10^6\times160\times10^{-6}}\times1458 = 0.046 \text{ rad } (\circlearrowright)$$

$$\theta_B = \frac{1}{EI}\left[\int_0^3 \frac{(12x+216)(1-\frac{x}{6})}{2}\,dx + \int_0^3 \frac{84\frac{x^2}{6}}{2}\,dx\right]$$

$$= \frac{1}{200\times10^6\times160\times10^{-6}}\times(261+63) = 0.010 \text{ rad } (\circlearrowright)$$

$$\theta_C = \frac{1}{EI}\left[\int_0^3 \frac{(12x+216)(-\frac{x}{6})}{2}\,dx + \int_0^3 \frac{84x(\frac{x}{6}-1)}{2}\,dx\right]$$

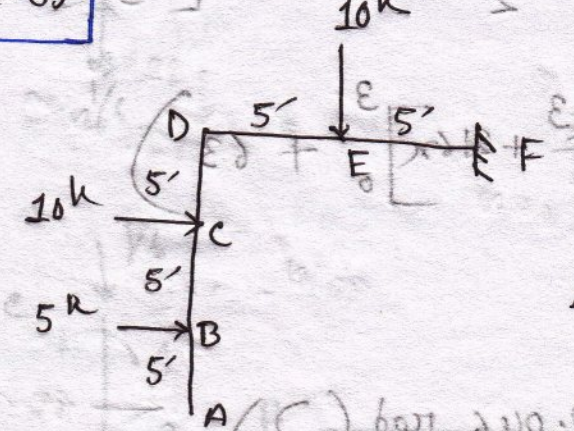
$$= \frac{1}{EI}\left[\frac{1}{2}\int_0^3(-2x^2-36x)\,dx + \frac{1}{2}\int_0^3(14x^2-84x)\,dx\right]$$

$$= \frac{1}{EI}\left(\frac{1}{2}\left[-\frac{2x^3}{3}-\frac{36x^2}{2}\right]_0^3 + \frac{1}{2}\left[\frac{14x^3}{3}-\frac{84x^2}{2}\right]_0^3\right)$$

$$= \frac{1}{EI}\left(\cancel{360}\ 90 + -126\right)$$

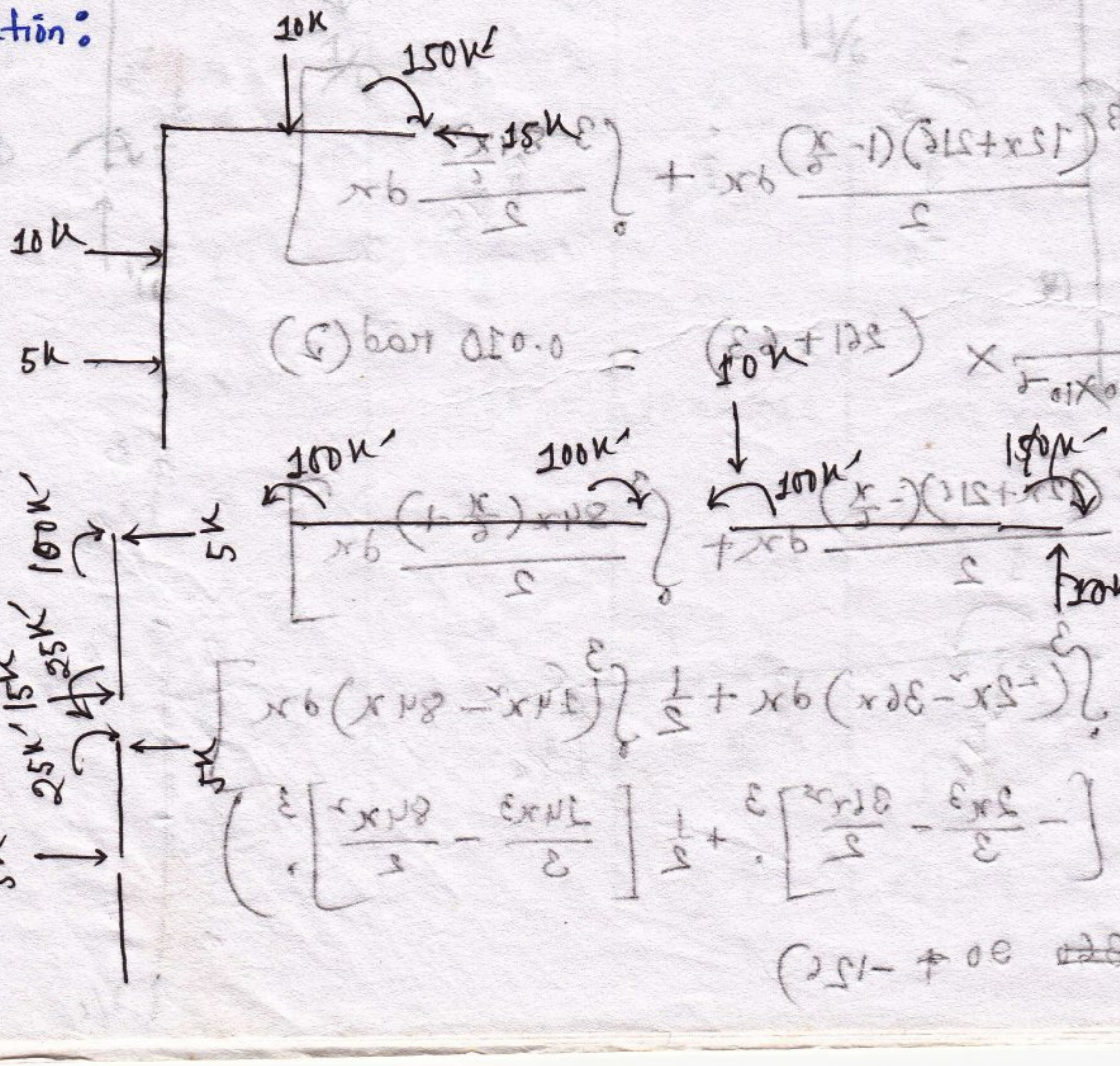
$$= -\frac{36}{200\times10^{6}\times160\times10^{-6}} = -0.0011 \text{ rad}$$

$\boxed{\text{Problem-05}}$

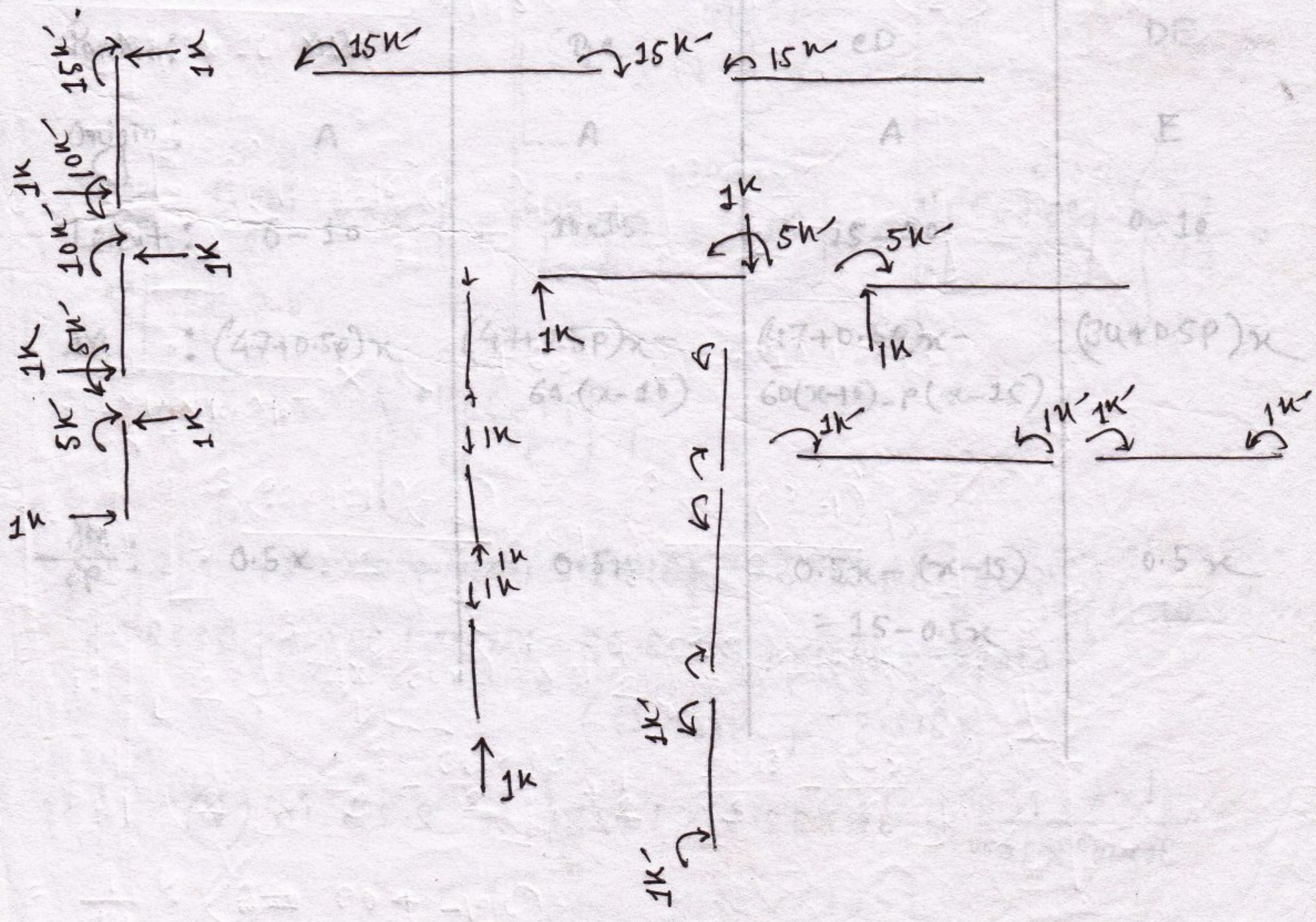


$\Delta_{AH},\ \Delta_{AV},\ \theta_A = ?$

Solution:



Portion:	AB	BC	CD	DE	EF
Origin:	A	B	C	D	E
Limit:	0-5	0-5	0-5	0-5	0-5
M:	~~-65x~~ 0	~~-15x-25~~ -5x	-15x-25	-100	-10x-100
m_{AH}:	-x	-x-5	-x-10	-15	-15
m_{AV}:	0	0	0	-x	x-5
θ_A:	1	1	1	1	1



$$\Delta_{AH} = \int_0^L M m_{AH} \, dx$$

$$= \frac{1}{EI}\left[\int_0^5 (-5x)(-x-5)\,dx + \int_0^5 (-15x-25)(-x-10)\,dx + \int_0^5 (-100)(-15)\,dx + \int_0^5 (-10x-100)(-15)\,dx\right]$$

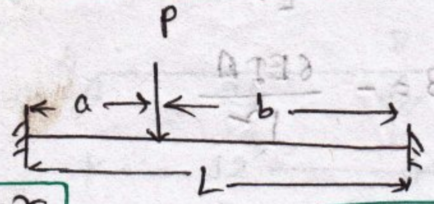
$=$

Moment-distrabution Method

AHSAN
090063

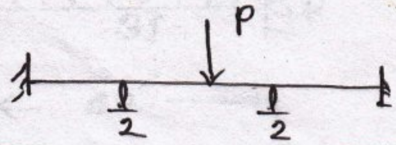
□ Fixed-end moment:

1.



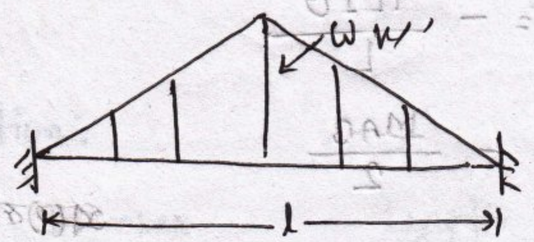
$\boxed{\dfrac{Pab^2}{L^2}}$ $\boxed{\dfrac{Pa^2b}{L^2}}$

2.



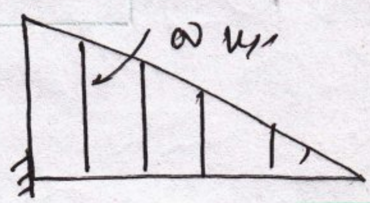
$\boxed{\dfrac{PL}{8}}$ $\boxed{\dfrac{PL}{8}}$

3.



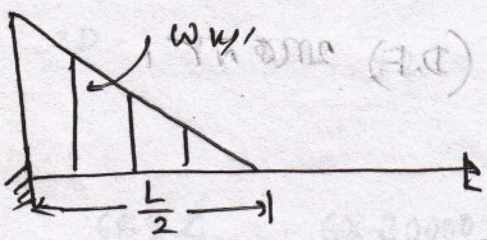
$\boxed{\dfrac{5\omega L^2}{96}}$ $\boxed{\dfrac{5\omega L^2}{96}}$

4.



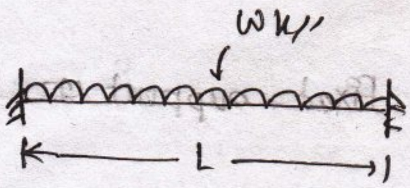
$\boxed{\dfrac{\omega L^2}{20}}$ $\boxed{\dfrac{\omega L^2}{30}}$

5.



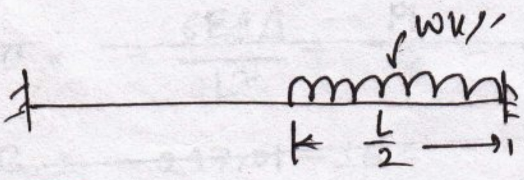
$\boxed{\dfrac{23\omega L^2}{960}}$ $\boxed{\dfrac{7\omega L^2}{960}}$

6.



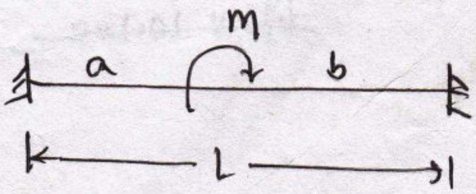
$\boxed{\dfrac{\omega L^2}{12}}$ $\boxed{\dfrac{\omega L^2}{12}}$

7.

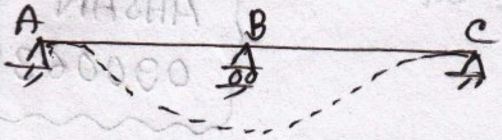


$\boxed{\dfrac{5\omega L^2}{192}}$ $\boxed{\dfrac{11\omega L^2}{192}}$

8.



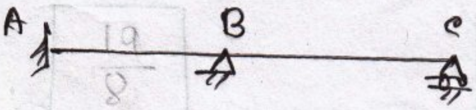
$\boxed{\dfrac{Mb}{L^2}(3a-L)}$ $\boxed{\dfrac{Ma}{L^2}(3b-L)}$



যদি B point এ settlement হয়

$$M_{AB} = M_{BA} = \frac{6EI\Delta}{L^2}$$

$$M_{BC} = M_{CB} = -\frac{6EI\Delta}{L^2}$$



$M_{AB} =$

যদি A point এ rotation হয়,

$$M_{AB} = -\frac{4EI\theta}{L}$$

$$M_{BA} = -\frac{M_{AB}}{2}$$

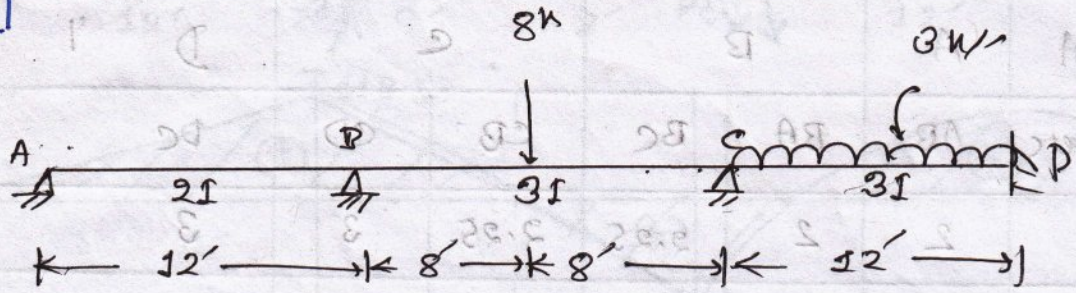
Relative stiffness (K) বের করার সময় I neglect করা হয় এবং প্রত্যেকটি $\frac{1}{L}$ কে এমন একটি একই সংখ্যা দ্বারা গুণ করা হয়।

Fixed support এ Distribution factor (D.F) থাকে না।

090063, 3H

01

Problem: Draw SFD & BMD



$E = 30000\ ksi \qquad I = 300\ in^4$

consider B point is settle $= \frac{1}{2}''$

Solution:

Member	Length	R.S
AB	12	$\frac{2I}{12} \times 12 = 2$
BC	16	$\frac{3I}{16} \times 12 = 2.25$
CD	12	$\frac{3I}{12} \times 12 = 3$

$$M_{AB} = \frac{6EI\Delta}{L^2} = \frac{6 \times 30000 \times 2 \times 300 \times 0.5}{(12 \times 12)^2 \times 12} = 217.01\ k\text{-}ft$$

$M_{BA} = 217.01\ k\text{-}ft$

$$\cancel{M_{BC} = -\frac{6EI\Delta}{L^2} + \frac{PL}{8} = -217.01 + 16 = -201.01\ k\text{-}ft}$$

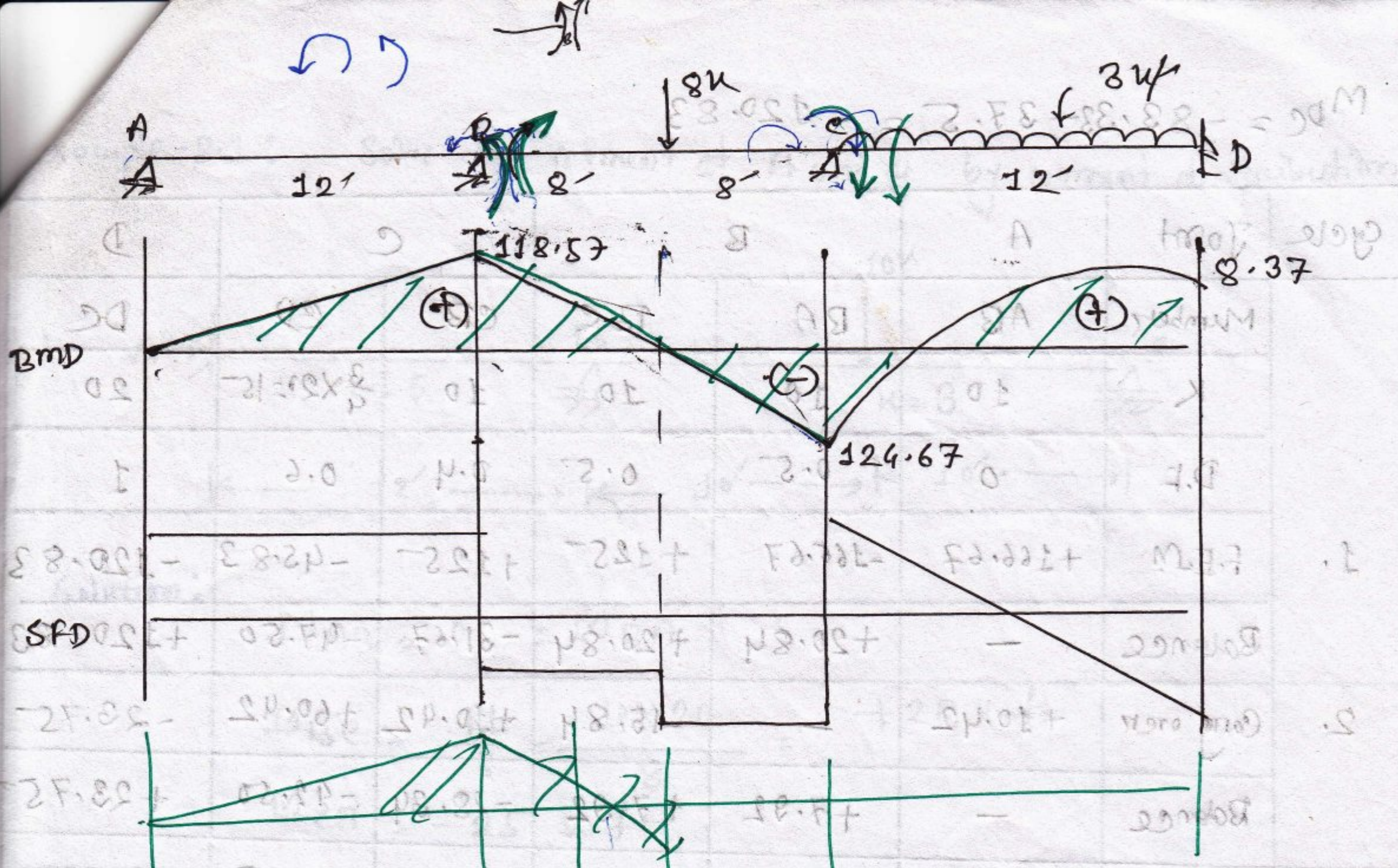
$$\cancel{M_{CB} = -217.01 - 16 = -233.01\ k\text{-}ft}$$

$$M_{BC} = -\frac{6EI\Delta}{L^2} + \frac{PL}{8} = -\frac{6 \times 30000 \times 3 \times 300 \times 0.5}{(16 \times 12)^2 \times 12} + \frac{8 \times 16}{8} = -167.11$$

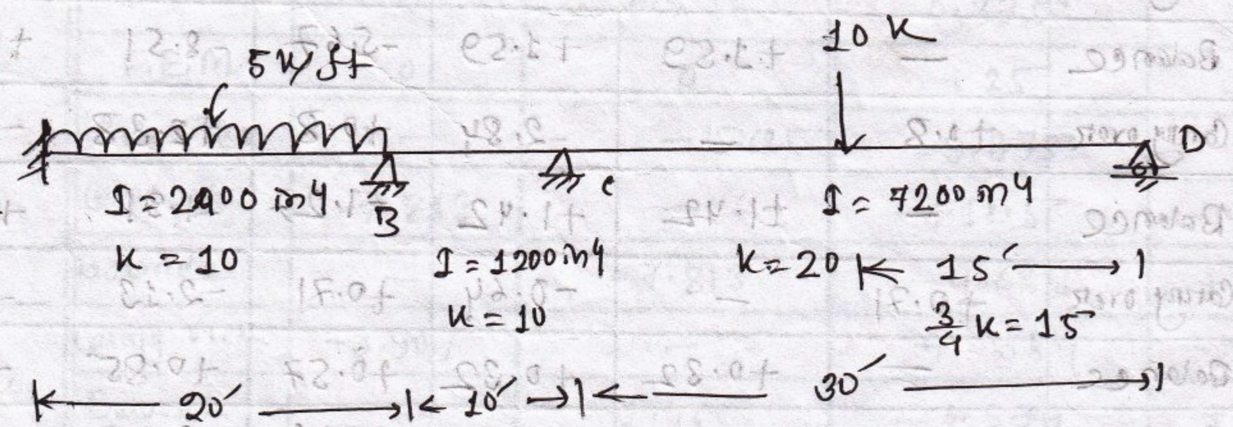
$M_{CB} = -183.11 - 16 = -199.11\ k\text{-}ft$

$$M_{CD} = \frac{wL^2}{12} = \frac{3 \times (12)^2}{12} = 36\ k\text{-}ft \qquad M_{DC} = -36\ k\text{-}ft$$

Cycle	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	2	2	2.25	2.25	3	3
	D.F	1	0.47	0.53	0.43	0.57	0
1.	F.E.M	+217.01	+217.01	-167.11	-199.11	+36	-36
	Balance	-217.01	-23.45	-26.95	+70.14	+92.97	—
2.	Carry over	-11.73	-108.51	+35.07	-13.23	—	+46.49
	Balance	+11.73	+34.52	+38.92	+5.69	+7.54	—
3.	Carry over	+17.26	+5.87	+2.85	+19.46	—	+3.77
	Balance	-17.26	-4.1	-4.62	-8.37	-11.09	—
4.	Carry over	-2.05	-8.63	-4.19	-2.31	—	-5.55
	Balance	+2.05	+6.03	+6.79	+0.99	+1.32	—
5.	Carry over	+3.02	+1.03	+0.5	+3.4	—	+0.66
	Balance	-3.02	-0.72	-0.81	-1.46	-1.94	—
6.	Carry over	-0.36	-1.51	-0.73	-0.41	—	-0.97
	Balance	+0.36	+1.05	+1.19	+0.18	+0.23	—
7	Carry over	+0.53	+0.18	+0.09	+0.6	—	+0.12
	Balance	-0.53	-0.13	-0.14	-0.26	-0.34	—
8	Carry over	-0.07	-0.27	-0.13	-0.07	—	-0.17
	Balance	+0.07	+0.19	+0.21	+0.03	+0.04	—
9	Carry over	+0.1	+0.04	+0.02	+0.11	—	+0.02
	Balance	-0.1	-0.03	-0.03	-0.05	-0.06	—
	Σ	0	+118.57	-118.57	-124.67	+124.67	+8.37



Example - 8.3 : From the beam of fig find the moments at A, B, and C by moment distribution. The support at C settles by 0.1 in. use $E = 30000\ k/in^2$



Solution :

$$M_{AB} = \frac{wL^2}{12} = \frac{5 \times (20)^2}{12} = +166.67\ k\text{-}ft \qquad M_{BA} = -166.67\ k\text{-}ft$$

$$M_{BC} = \frac{6EI\Delta}{L^2} = \frac{6 \times 30000 \times 2400 \times 0.1}{(10 \times 12)^2 \times 12} = \cancel{+250}\ k\text{-}ft \quad 125\ k\text{-}ft$$

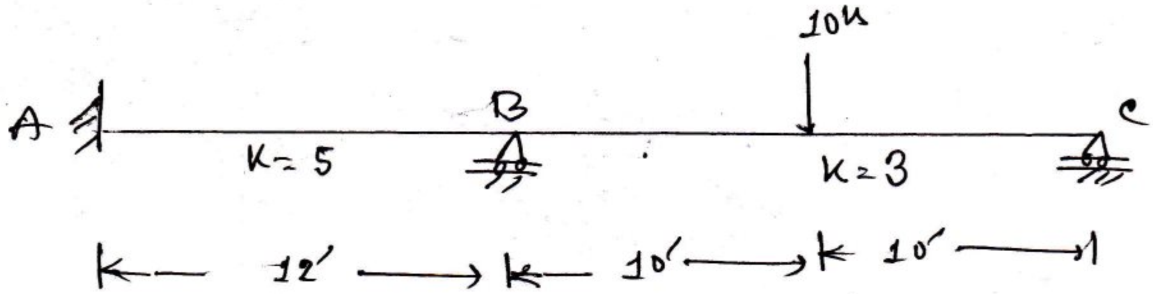
$$M_{CB} = +125\ k\text{-}ft$$

$$M_{CD} = \cancel{-125} + \frac{-6 \times 30000 \times 7200 \times 0.1}{(30 \times 12)^2 \times 12} + \frac{10 \times 30}{8} = -45.83\ k\text{-}ft$$

$M_{DC} = -83.33 - 37.5 = -120.83$

Cycle	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	10	10	10	10	$\frac{3}{4}\times 20 = 15$	20
	D.F	0	0.5	0.5	0.4	0.6	1
1.	F.E.M	+166.67	-166.67	+125	+125	-45.83	-120.83
	Balance	—	+20.84	+20.84	-31.67	-47.50	+120.83
2.	Carry over	+10.42	—	-15.84	+10.42	+60.42	-23.75
	Balance	—	+7.92	+7.92	-28.34	-42.50	+23.75
3.	Carry over	+3.96	—	-14.17	+3.96	+11.88	-21.25
	Balance	—	+7.09	+7.09	-6.34	-9.5	+21.25
4.	Carry over	+3.55	—	-3.17	+3.55	+10.63	-4.75
	Balance	—	+1.59	+1.59	-5.67	-8.51	+4.75
5.	Carry over	+0.8	—	-2.84	+0.8	+2.38	-4.26
	Balance	—	+1.42	+1.42	-1.27	-1.91	+4.26
6.	Carry over	+0.71	—	-0.64	+0.71	-2.13	-0.96
	Balance	—	+0.32	+0.32	+0.57	+0.85	+0.96
7.	Carry over	+0.16	—	+0.29	+0.16	+0.48	+0.43
	Balance	—	-0.15	-0.15	-0.26	-0.38	-0.43
	Σ	+186.27	-127.64	+127.66	71.62	-71.62	0

Example-8.1 : Solve for moment at A and B by moment distribution.



Solution:

$M_{AB} = 0 = M_{BA}$

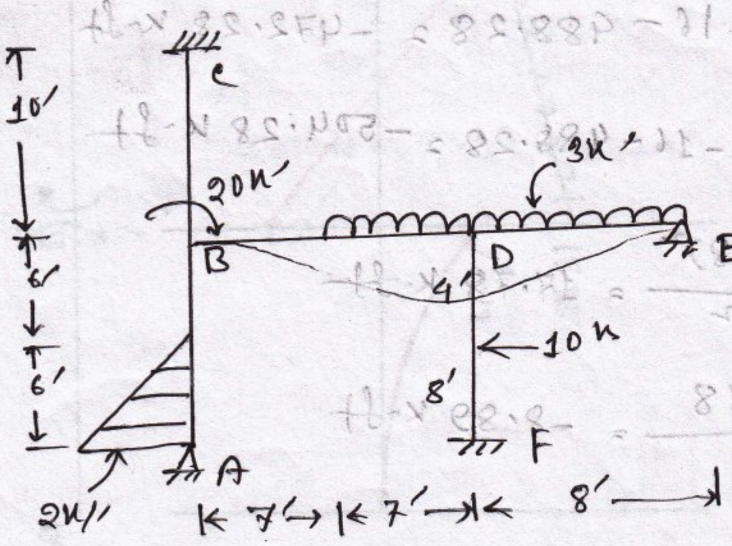
$M_{BC} = \frac{PL}{8} = \frac{10 \times 20}{8} = +25$ k-ft

$M_{CB} = -25$ k-ft

Cycle	Joint	A	B		C
	Member	AB	BA	BC	CB
	K	5	5	3	3
	D.F	0	0.625	0.375	1
1.	F.E.M	0	0	+25	−25
	Balance	—	−15.625	−9.375	+25
2.	Carry over	−7.813	—	+12.5	−4.688
	Balance	—	−7.813	−4.688	+4.688
3.	Carry over	−3.907	—	+2.344	−2.344
	Balance	—	−1.465	−0.879	+2.344
4.	Carry over	−0.733	—	+1.172	−0.44
	Balance	—	−0.733	−0.44	+0.44
	Σ	−12.45	−25.64	+25.64	0

Settlement

Problem: Solve for moment and draw SFD, BMD.



$E = 30000$ ksi

$I = 300 in^4$

$\Delta F = 1''$

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Solution:

Member	Length	R. S
AB	12'	$\frac{I}{12} \times 14 = 1.17$
BC	10'	$\frac{I}{10} \times 14 = 1.4$
BD	14'	$\frac{I}{14} \times 14 = 1$
DE	8'	$\frac{I}{8} \times 14 = 1.75$
DF	12'	$\frac{I}{12} \times 14 = 1.17$

$$M_{AB} = \frac{23 wL^2}{960} = \frac{23 \times 2 \times (12)^2}{960} = 6.9 \text{ k-ft}$$

$$M_{BA} = \frac{7 wL^2}{960} = \frac{7 \times 2 \times (12)^2}{960} = -2.1 \text{ k-ft}$$

$$M_{BC} = M_{CB} = 0$$

$$M_{BD} = \frac{5wL^2}{192} + \frac{6EI\Delta}{L^2} = \frac{5 \times 3 \times (14)^2}{192} + \frac{6 \times 30000 \times 300 \times 1}{(14)^2 \times (12)^2 \times 12}$$

$$= 15.31 + 159.44 = +174.75 \text{ k-ft}$$

$$M_{DB} = -\frac{11 wL^2}{192} + \frac{6EI\Delta}{L^2} = -\frac{11 \times 3 \times (14)^2}{192} + 159.44 = 125.75 \text{ k-ft}$$

$$M_{DE} = \frac{wL^2}{12} - \frac{6EI\Delta}{L^2} = \frac{3\times(8)^2}{12} - \frac{6\times 30000\times 300\times 1}{(8\times12)^2\times 12}$$

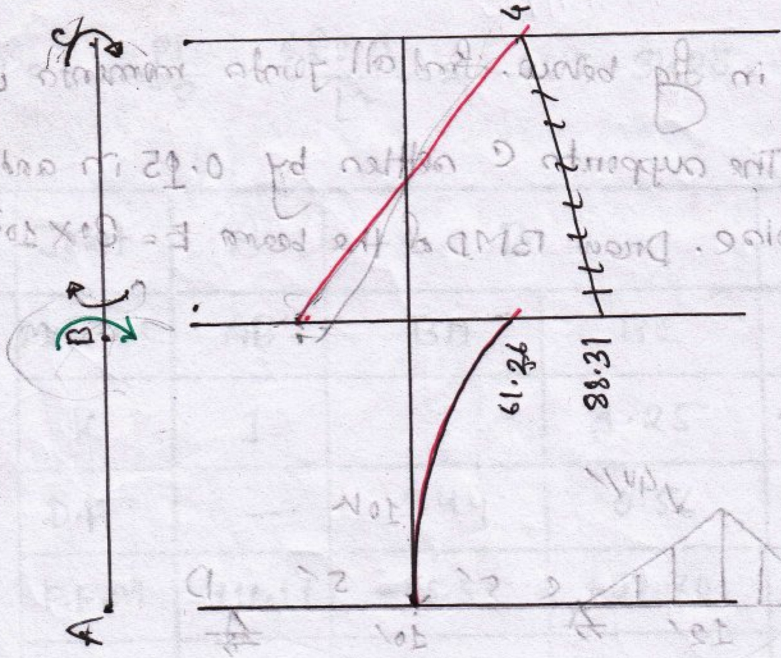
$$= 16 - 488.28 = -472.28 \text{ k-ft}$$

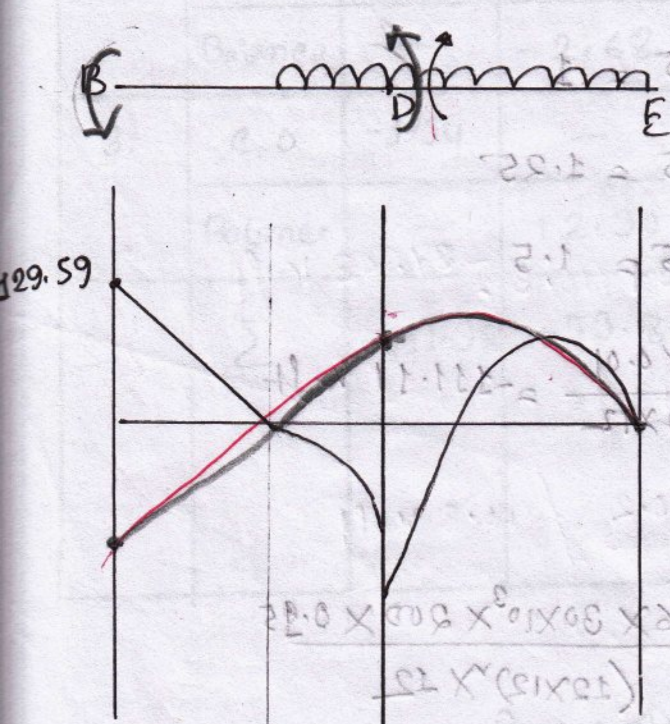
$$M_{ED} = -\frac{wL^2}{12} - \frac{6EI\Delta'}{L^2} = -16 - 488.28 = -504.28 \text{ k-ft}$$

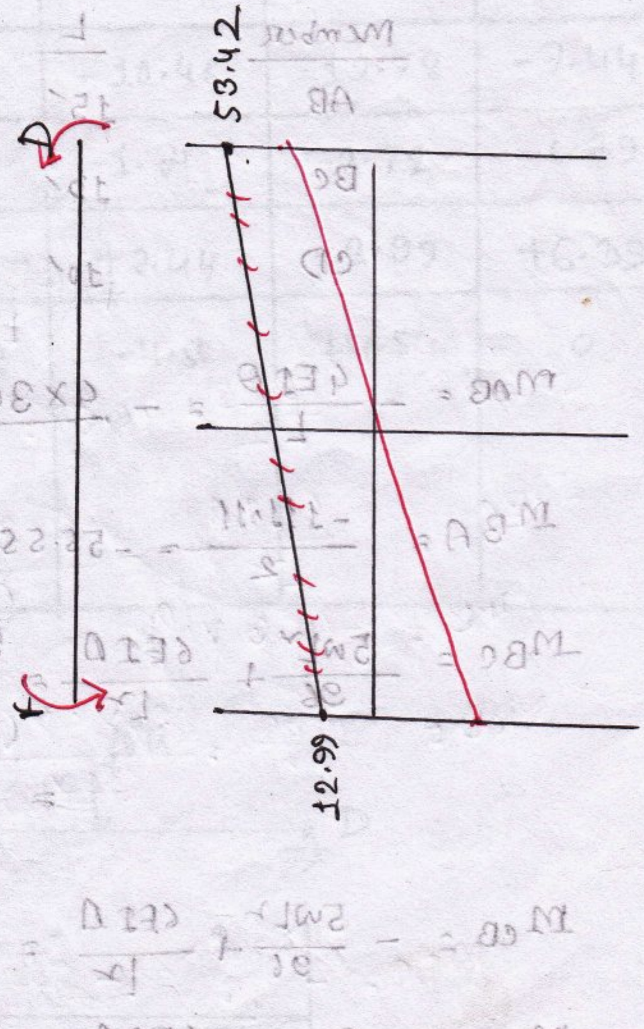
$$M_{DF} = \frac{Pab^2}{L^2} = \frac{10\times4\times(8)^2}{(12)^2} = 17.78 \text{ k-ft}$$

$$M_{FD} = \frac{Pa^2b}{L^2} = \frac{10\times4^2\times8}{(12)^2} = -8.89 \text{ k-ft}$$

Cycle	Joint	A	B				C	D			E	F
	Member	AB	BA	BC	BD		CB	DB	DE	DF	ED	FD
	K	1.17	1.17	1.4	1		1.4	1	1.75	1.17	1.75	1.17
	D.F	1	0.33	0.39	0.28		—	0.25	0.45	0.3	1	—
1.	F.E.M	+6.9	-2.1	0	+174.75	+20	0	+125.75	-472.28	+17.78	-504.28	-8.89
	Balance	-6.9	-63.57	-75.13	-53.94		—	+82.19	+147.94	+98.63	+504.28	—
2.	C.O	-31.79	-3.45	—	+41.10		-37.57	-26.97	+252.14	—	+73.97	+49.32
	Balance	+31.79	-12.42	-14.68	-10.54		—	-56.29	-101.33	-67.55	-73.97	—
3.	C.O	-6.21	+15.9	—	-28.15		-7.34	-5.27	-36.99	—	-50.67	-33.78
	Balance	+6.21	+4.04	+4.78	+3.43		—	+10.57	+19.02	+12.68	+50.67	—
4.	C.O	+2.02	+3.11	—	+5.29		+2.39	+1.72	+25.34	—	+9.51	+6.34
	Balance	-2.02	-2.77	-3.28	-2.35		—	-6.77	-12.18	-8.12	-9.51	—
Cycle আরো continue করতে হবে →	Σ	0	-61.26	-88.31	+129.59		-42.52	+124.93	-178.34	+53.42	0	+12.99
							—					—


42.52
61.76
88.31
B
A

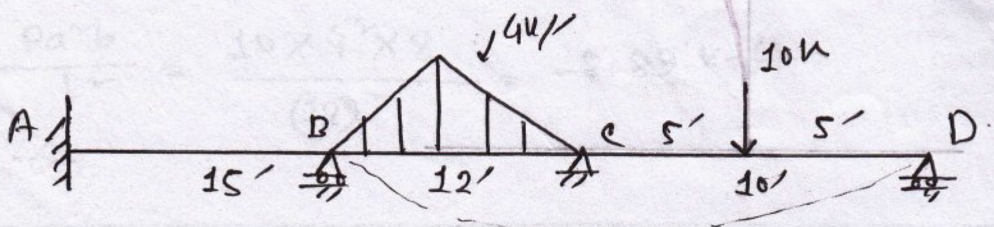

B
D
E
129.59


53.42
D
F
12.99

Settlement + Rotation

Problem:

For the beam shown in fig below. Find all joints moments using moment distribution method. The supports C settles by 0.15 in and support A rotates by 0.01 rad clockwise. Draw BMD of the beam $E = 30\times10^3$ ksi and $I = 200$ in^4.



Solution:

Member	L	K
AB	15'	$\frac{I}{15}\times15 = 1$
BC	12'	$\frac{I}{12}\times15 = 1.25$
CD	10'	$\frac{I}{10}\times15 = 1.5$

$$M_{AB} = -\frac{4EI\theta}{L} = -\frac{4\times30\times10^3\times200\times0.01}{(15\times12)\times12} = -111.11 \text{ k-ft}$$

$$M_{BA} = \frac{-111.11}{2} = -55.55\text{k}$$

$$M_{BC} = \frac{5wL^2}{96}+\frac{6EI\Delta}{L^2} = \frac{5\times4\times12^2}{96}+\frac{6\times30\times10^3\times200\times0.15}{(12\times12)^2\times12}$$

$$= 30+\cancel{7.23}\ 21.70 = 50.70 \text{ k-ft}$$

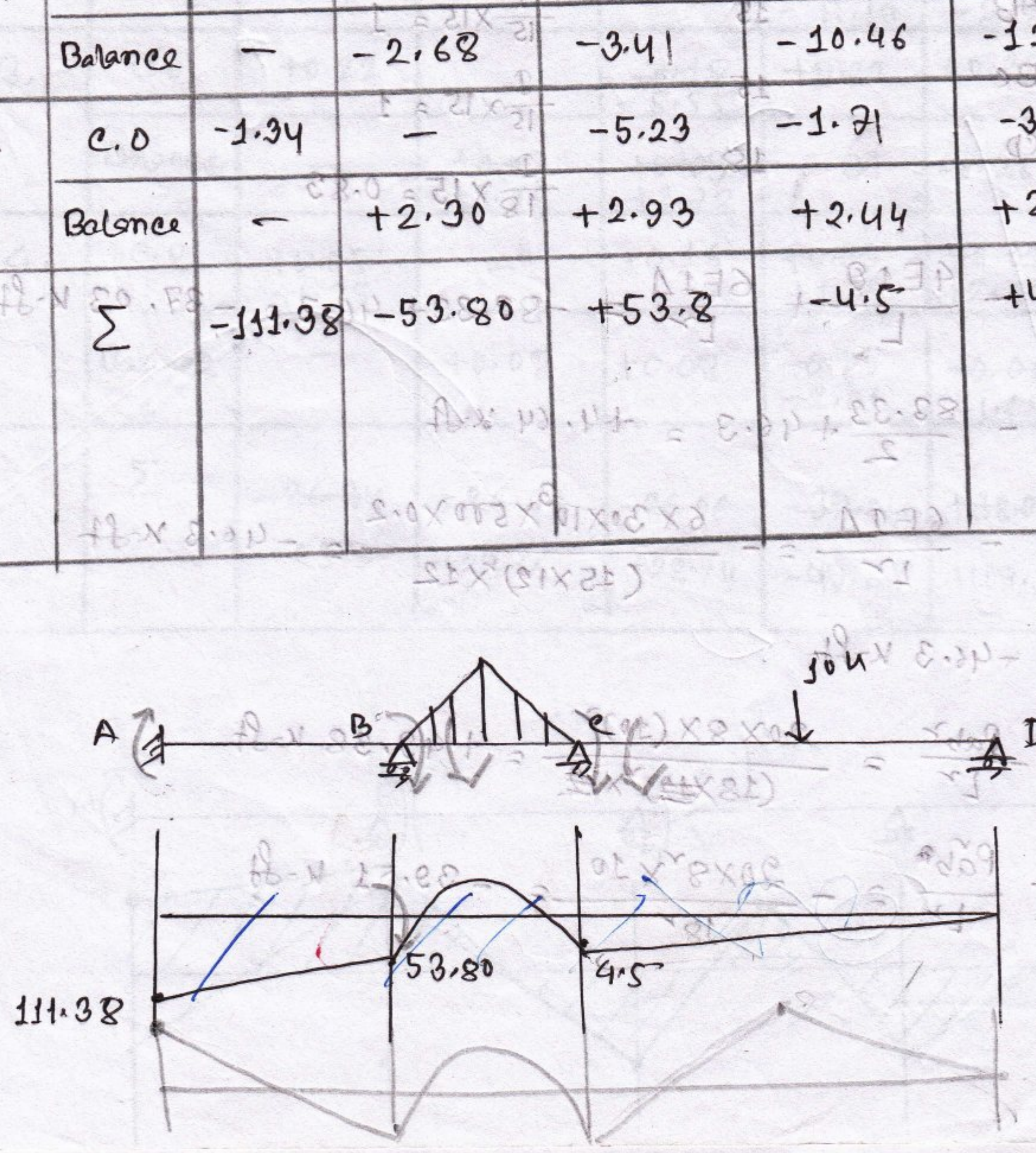
$$M_{CB} = -\frac{5wL^2}{96}+\frac{6EI\Delta}{L^2} = -30+21.70 = -8.3\text{K}$$

$$M_{CD} = \frac{PL}{8}-\frac{6EI\Delta}{L^2} = \frac{10\times10}{8}-\frac{6\times30\times10^3\times200\times0.15}{(10\times12)^2\times12}$$

$$= 12.5-31.25 = -18.75 \text{ k-ft}$$

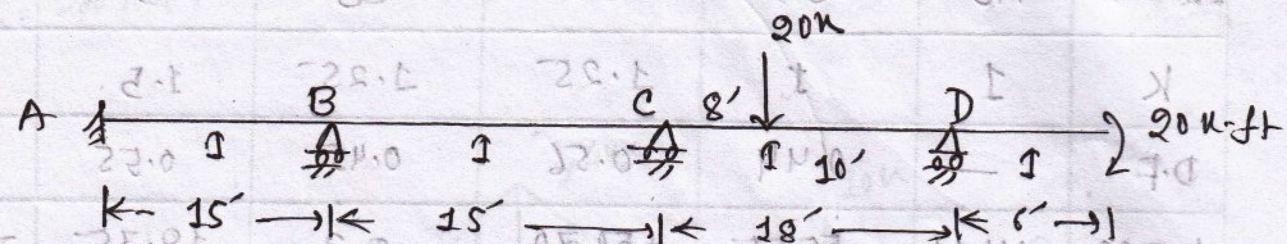
$$M_{DC} = -\frac{PL}{8} - \frac{6EI\Delta}{L^2} = -12.5 - 31.25 = -43.75 \text{ k-ft}$$

Cycle	Joint	A	B		C		D
	Member	AB	BA	BC	CB	CD	DC
	K	1	1	1.25	1.25	1.5	1.5
	D.F	—	0.44	0.56	0.45	0.55	1
1.	F.E.M	-111.11	-55.55	+50.70	-8.3	-18.75	-43.75
	Balance	—	+2.134	+2.716	+12.17	+14.88	+43.75
2.	C.O	+1.07	—	+6.09	+1.36	+21.88	+7.44
	Balance	—	-2.68	-3.41	-10.46	-12.78	-7.44
3.	C.O	-1.34	—	-5.23	-1.71	-3.72	-6.39
	Balance	—	+2.30	+2.93	+2.44	+2.99	+6.39
	Σ	-111.38	-53.80	+53.8	-4.5	+4.5	0



07

Q-2: For the beam shown in fig below. find the moments at A, B, C a[illegible] by moment distribution method. The support B settle by 0.2 inch and supp[illegible] A rotates by 0.003 radian clockwise. Draw BM diagram of the beam. Use $E = 30\times10^6$ psi $I = 500$ in^4.



Solution:

Member	L	K
AB	15	$\frac{I}{15}\times15 = 1$
BC	15	$\frac{I}{15}\times15 = 1$
CD	18	$\frac{I}{18}\times15 = 0.83$

$$M_{AB} = -\frac{4EI\theta}{L} + \frac{6EI\Delta}{L^2} = -83.33 + 46.3 = -37.03 \text{ k-ft}$$

$$M_{BA} = -\frac{83.33}{2} + 46.3 = +4.64 \text{ k-ft}$$

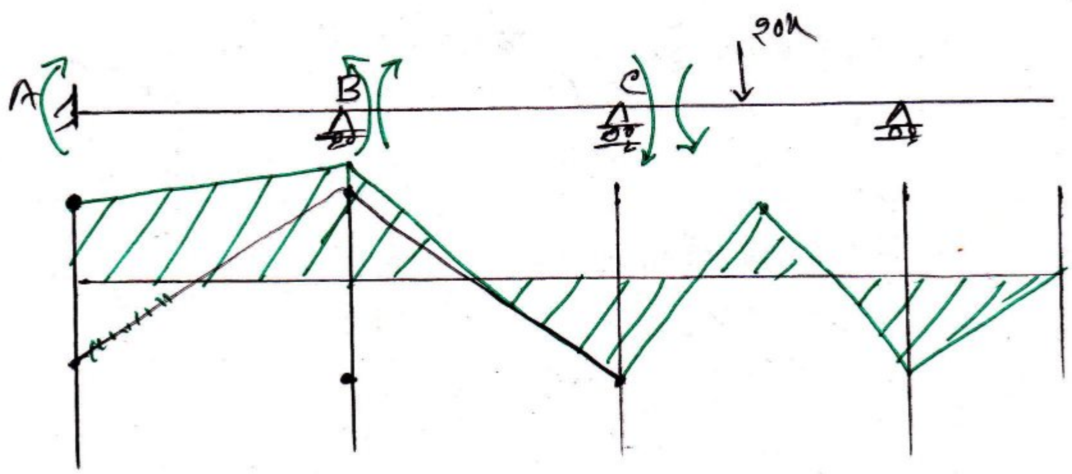
$$M_{BC} = -\frac{6EI\Delta}{L^2} = -\frac{6\times30\times10^3\times500\times0.2}{(15\times12)^2\times12} = -46.3 \text{ k-ft}$$

$$M_{CB} = -46.3 \text{ k-ft}$$

$$M_{CD} = \frac{Pab^2}{L^2} = \frac{20\times8\times(10)^2}{(18\cancel{\times12})^2\times\cancel{12}} = +49.38 \text{ k-ft}$$

$$M_{DC} = -\frac{Pa^2b}{L^2} = -\frac{20\times8^2\times10}{18^2} = -39.51 \text{ k-ft}$$

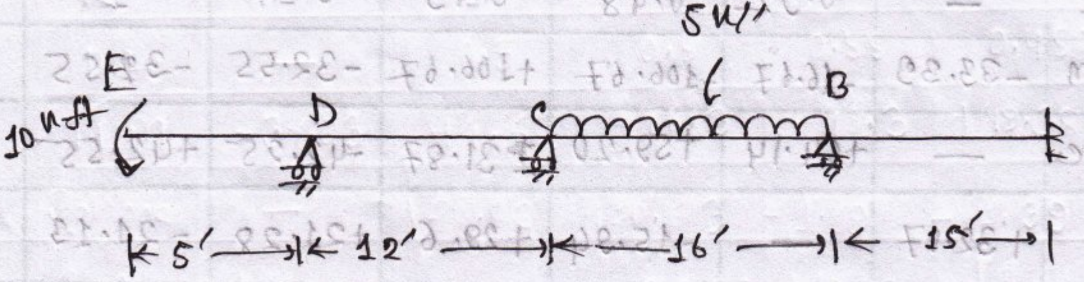
Cycle	Joint	A	B		C		D	
	Member	AB	BA	BC	CB	CD	DC	
	K	1	1	1	1	0.83	0.83	
	D.F	—	0.5	0.5	0.55	0.45	1	
1.	F.E.M	-37.03	+4.64	-46.3	-46.3	+49.38	-39.51	+20
	Balance	—	+20.83	+20.83	-1.69	-1.39	+19.51	
2.	C.O	+10.42	—	-0.85	+10.42	+9.76	-0.70	
	Balance	—	+0.43	+0.43	~~+0.36~~ -11.10	~~-0.30~~ -9.08	+0.70	
3.	C.O	+0.22	—	~~-0.18~~ -5.55	+0.22	+0.35	~~-0.15~~ -4.54	
	Balance	—	~~+0.09~~ +2.78	~~+0.09~~ +2.78	-0.31	-0.26	~~+0.15~~ 4.54	
4.	C.O	~~+0.05~~ +1.39	—	-0.16	~~+0.05~~ +1.39	~~+0.08~~ +2.27	-0.13	
	Balance	—	+0.08	+0.08	~~-0.07~~ -2.01	~~-0.06~~ -1.65	+0.13	
	Σ	~~-26.34~~ -25	~~+26.07~~ +28.76	~~-26.06~~ -28.74	~~-38.04~~ -49.38	~~+38.04~~ +49.38	-20	



05

Q-3. For the beam shown in fig below, find all joints moments using moment distribution method. The support D settles by 0.15 inch and support A rotates by 0.002 radian clockwise. Draw BMD of the beam. $E = 30\times10^3$ ksi

$I = 300$ in^4.



Solution:

আয়মান জাবীর

Member	L	k
AB	15'	$\frac{I}{15}\times16 = 1.07$
BC	16'	$\frac{I}{16}\times16 = 1$
CD	12'	$\frac{I}{12}\times16 = 1.33$
DE	5	$\frac{I}{5}\times16 = 3.2$

$$M_{AB} = -\frac{4EI\theta}{L} = -\frac{4\times30\times10^3\times300\times0.002}{15\times12\times12} = -33.33 \text{ k-ft}$$

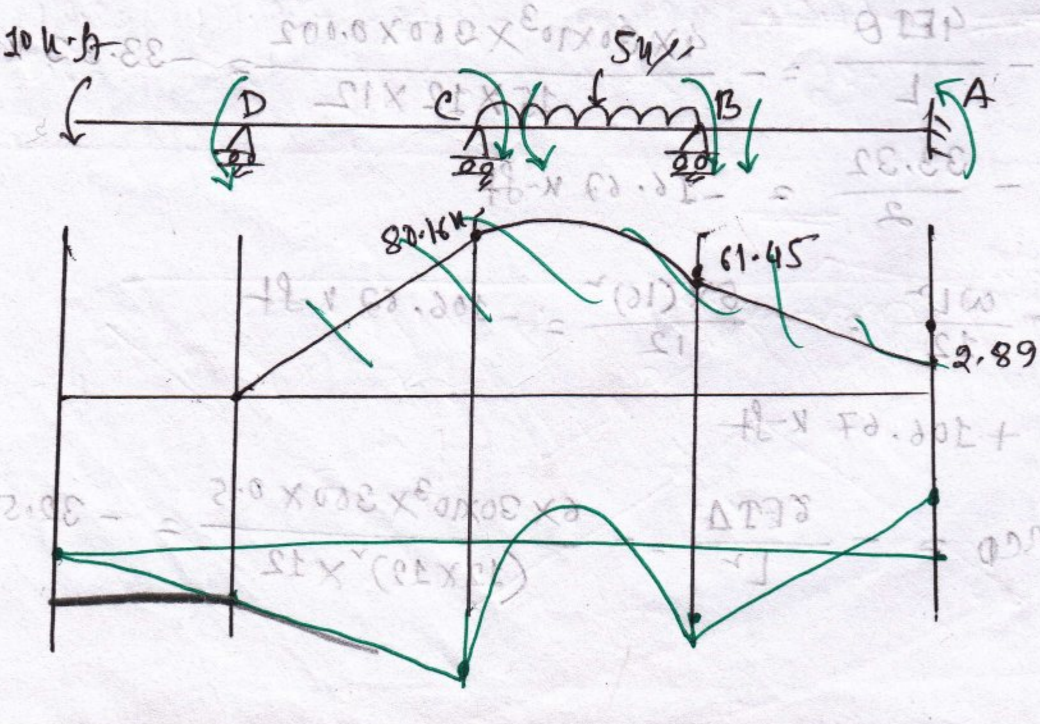
$$M_{BA} = -\frac{33.32}{2} = -16.67 \text{ k-ft}$$

$$M_{BC} = -\frac{wL^2}{12} = -\frac{5\times(16)^2}{12} = -106.67 \text{ k-ft}$$

$$M_{CB} = +106.67 \text{ k-ft}$$

$$M_{DC} = M_{CD} = -\frac{6EI\Delta}{L^2} = -\frac{6\times30\times10^3\times300\times0.15}{(12\times12)^2\times12} = -32.55 \text{ k-ft}$$

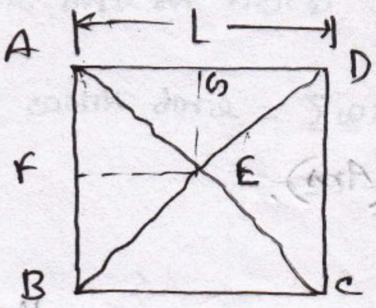
Cycle	Joint	A	B		C		D	
	Member	AB	BA	BC	CB	CD	DC	DE
	K	1.07	1.07	1	1	1.33	1.33	—
	D.F	—	0.52	0.48	0.43	0.57	1	-
1.	F.E.M	−33.39	−16.67	−106.67	+106.67	−32.55	−32.55	—10
	Balance	—	+64.14	+59.20	±31.87	−42.25	+42.55	
2.	C.O	+32.07	—	−15.94	+29.6	+21.28	−21.13	
	Balance	—	+8.29	+7.65	−21.88	−29	+21.13	
3.	C.O	+4.15	—	−10.94	+3.83	+10.57	−14.5	
	Balance	—	+5.69	+5.25	−6.19	−8.21	+14.5	
	Σ	+2.89	+61.45	−61.45	+80.16	−80.16	0 +10	

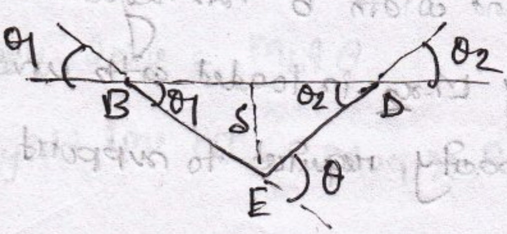


(10) # Yield Line (Virtual work method)

AHSAN
090063

Q. A simply supported square slab has moment capacity $M_x = M_y = M$. Compute M, support uniform load ω per unit area.





External work:

Consider plate ABE

Total load on ABE, $W = \frac{1}{2} \times L \times \frac{L}{2} \times \omega = \frac{\omega L^2}{4}$

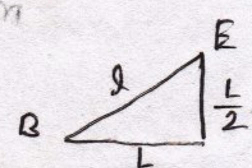
Deflection at the centroid of plane AB, $\Delta_e = \frac{\delta}{3}$

$\therefore W.\Delta e = \frac{\omega L^2}{4} . \frac{\delta}{3} = \frac{\omega L^2 \delta}{12}$

$\therefore$ Total external work done $= \Sigma W \Delta c = 4 . \frac{\omega L^2 \delta}{12} = \frac{\omega L^2 \delta}{3}$

Internal work;

length of line BE, $l = \frac{L}{\sqrt{2}}$



$l^2 = \frac{L^2}{4} + \frac{L^2}{4}$

$= \frac{L^2}{2}$

$l = \frac{L}{\sqrt{2}}$

$\theta_1 = \frac{\delta}{\frac{L}{\sqrt{2}}} = \frac{\sqrt{2}\delta}{L} = \theta_2$

$\therefore \theta = \frac{2\sqrt{2}\delta}{L}$

Internal work done on line AE, $= M b l \theta$

$= M . \frac{L}{\sqrt{2}} . \frac{2\sqrt{2}\delta}{L} = 2M\delta$

Total internal work done $= \sum M_b L\theta$

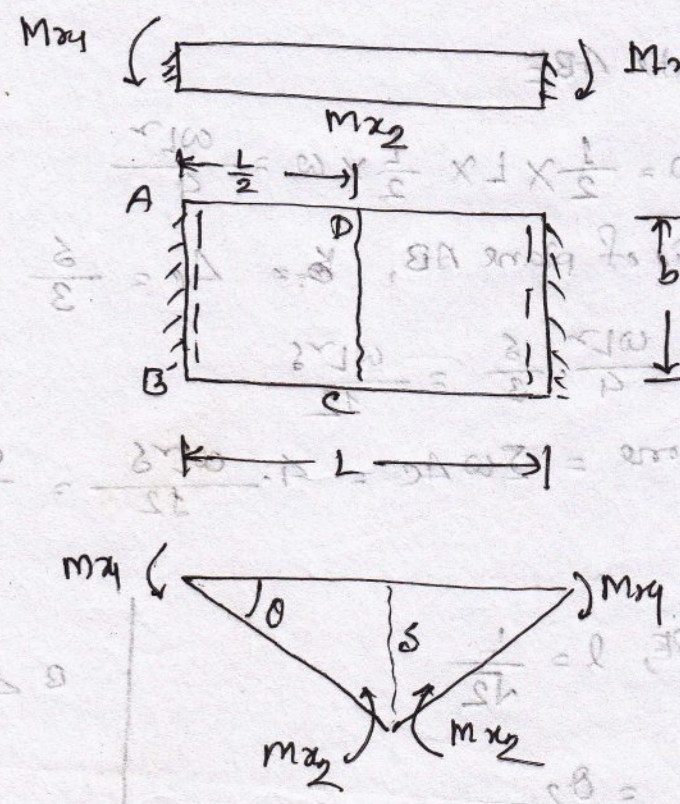
$= 4 \times 2M\delta = 8M\delta$

Now, Total external work done = Total internal work done

$$\frac{\omega L^2 \delta}{3} = 8M\delta$$

$$\therefore M = \frac{\omega L^2}{24} \text{ (Ans).}$$

Q. One way slab of length l and width b has negative moment capacity M_{x_1} and positive moment capacity M_{x_2} is loaded with uniform load ω per unit area. Compute the moment capacity required to support ω.



External work:

Total load on segment ABCD, $W = \frac{L}{2} \times B \times \omega$

Deflection at the centroid of ABCD, $\Delta_c = \frac{\delta}{2}$

$\therefore$ External work done on ABCD, $= W\Delta_c = \frac{L}{2} \times B \times \omega \times \frac{\delta}{2}$

$\therefore$ Total external work done $= \Sigma W \Delta_c$

$$= 2 \cdot \frac{L}{2} \times B \times \omega \times \frac{\delta}{2} = \frac{LB\omega\delta}{2}$$

Internal work:

Internal work done $= M_b l \theta$

[Inte]rnal work done by yield line AB $= Mx_1 \times B \times \frac{\delta}{\frac{L}{2}}$

$\therefore$ Total internal work done $= 2\left(Mx_1 \times B \times \frac{2\delta}{L}\right) + 2\left(Mx_2 \times B \times \frac{2\delta}{L}\right)$

$$= \frac{4B\delta}{L}(Mx_1 + Mx_2)$$

External work done = Internal work done

$$\frac{LB\omega\delta}{2} = \frac{4B\delta}{L}(Mx_1 + Mx_2)$$

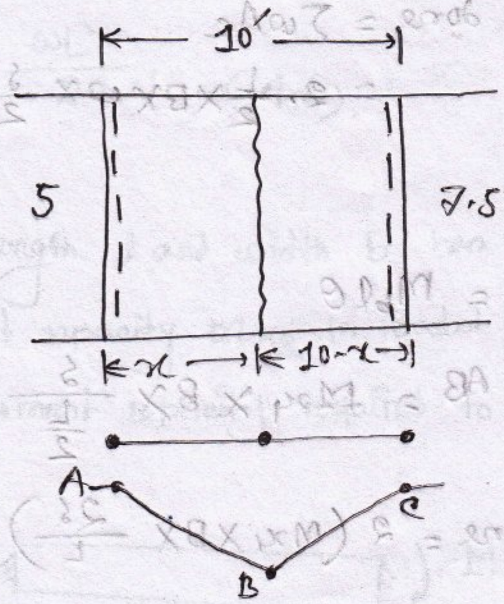
$\therefore Mx_1 + Mx_2 = \frac{\omega L^2}{8}$, (Ans).

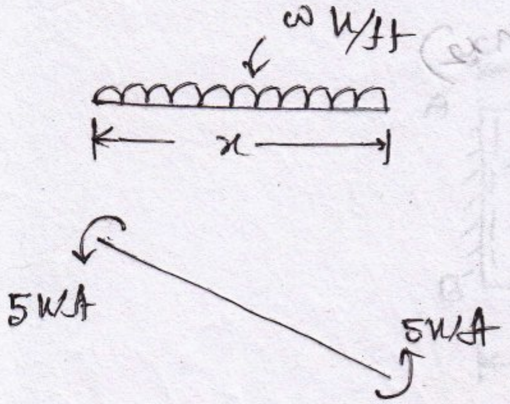
Equilibrium method

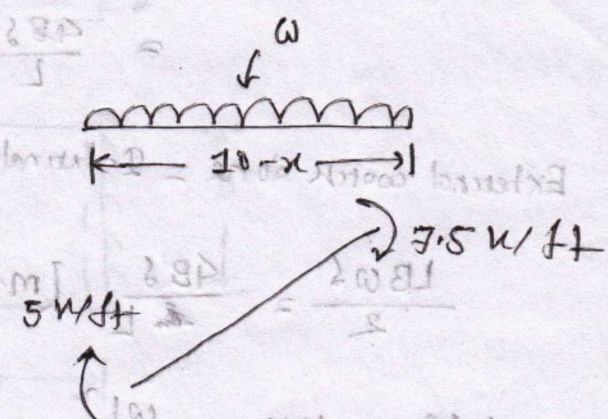
Problem:

A slab has a 10 ft span and is reinforced to provide a resistance to positive bending of 5 k/ft through the span. In addition the slab has negative moment capacities of 5 k/ft and 7.5 k/ft respectively over left and right supports. Determine the ultimate load capacity of the slab. (By equilibrium method)

Solution:







$$\frac{wx^2}{2} - 10 = 0$$

$$\Rightarrow wx^2 = 20$$

$$\Rightarrow w = \frac{20}{x^2} \quad -(1)$$

$$\frac{w(10-x)^2}{2} - 12.5 = 0$$

$$\Rightarrow \frac{20(10-x)^2}{2x^2} - 12.5 = 0$$

$$\Rightarrow 20(100-20x+x^2) - 25x^2 = 0$$

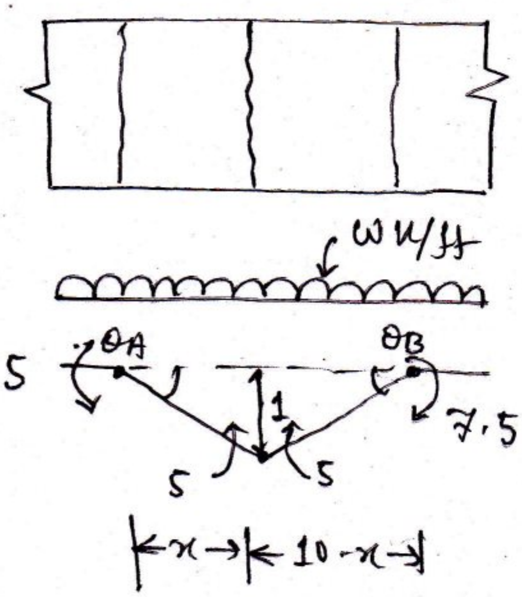
$$\Rightarrow 2000 - 400x + 20x^2 - 25x^2 = 0$$

$$\Rightarrow 5x^2 + 400x - 2000 = 0$$

$$\therefore x = 4.72'$$

$$\omega = \frac{20}{(4.72)^2} = 0.89 \text{ K/ft}^2 \text{ (Ans)}.$$

Problem; Above problem by Virtual work method.



External work done $= \frac{1}{2} \times w \times x + \frac{1}{2} \times w \times (10-x)$

$\theta A = \frac{1}{x}$ $\qquad \theta B = \frac{1}{10-x}$

Internal work done =

$$5 \times \frac{1}{x} \times 2 + 5 \times \frac{1}{10-x} \times 1 + 7.5 \times \frac{1}{10-x}$$

Now,

External work done = Internal work done

$$\frac{wx}{2} + 5w - \frac{wx}{2} = \frac{10}{x} + \frac{5}{10-x} + \frac{7.5}{10-x}$$

$$\Rightarrow 5w = \frac{10}{x} + \frac{12.5}{10-x}$$

$$\Rightarrow w = \frac{2}{x} + \frac{2.5}{10-x}$$

For minimum value of w, $\frac{dw}{dx} = 0$

$$\frac{dw}{dx} = -\frac{2}{x^2} + \frac{2.5}{(10-x)^2} = 0$$

$$\Rightarrow \frac{2}{x^2} = \frac{2.5}{100 - 20x + x^2}$$

$\therefore x = 4.72$ ft

$w = \cancel{0.08}$ 0.89 k/ft^2 (Ans).

Two Hinge Arch

{ AHSAN
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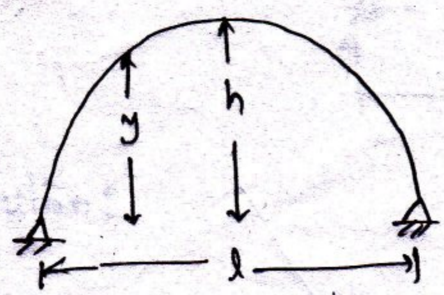
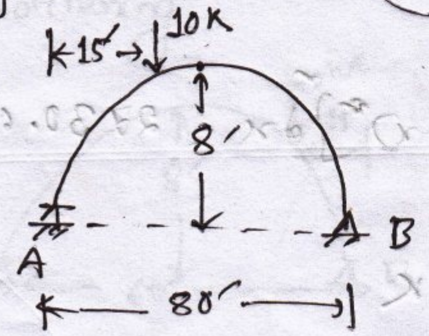


* Equation of parabolic arch, $y = \dfrac{4hx(l-x)}{l^2}$

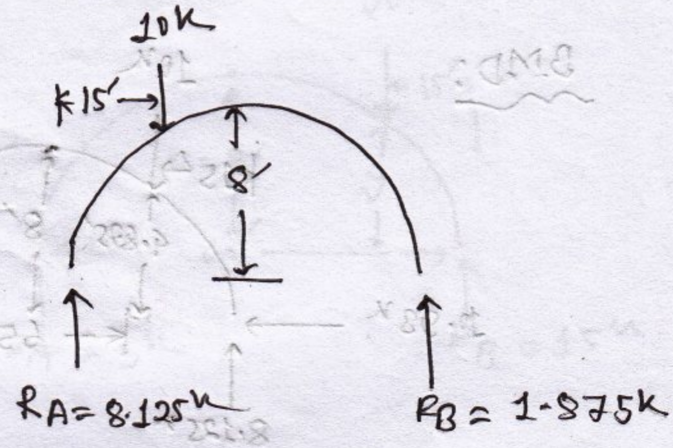
* Horizontal thrust, $H = \dfrac{\int_0^l M_s y \, dx}{\int_0^l y^2 \, dx}$

[Bayezid vai এর নোট থেকে]

Problem: Find the horizontal thrust for the two hinged parabolic arch shown in fig and draw BMD. Neglect the effect of rib shortening.



Solution:



$\Sigma M_A = 0,$

$10 \times 15 = R_B \times 80$

$\therefore R_B = 1.875^K$

$\Sigma F_y = 0, \quad R_A = 8.125^K$

We know, equation of parabolic arch,

$$y = \frac{4hx(l-x)}{l^2}$$

$$= \frac{4 \times 8 \times x(80-x)}{(80)^2} = 5 \times 10^{-3} x(80-x)$$

$$H = \frac{\int_0^l M_s \cdot y \, dx}{\int_0^l y^2 \, dx} \quad \cdots (1)$$

Now,

$$\int_0^l M_s y \, dx = \int_0^{15} 8.125x \times 5\times10^{-3} x(80-x)\,dx + \int_0^{65} 1.875x \times 5\times10^{-3} x(80-x)\,dx$$

$$= \int_0^{15}(3.25x^2 - 0.04x^3)\,dx + \int_0^{65}(0.75x^2 - 0.0094x^3)\,dx$$

$$= \left[\frac{3.25x^3}{3} - \frac{0.04x^4}{4}\right]_0^{15} + \left[\frac{0.75x^3}{3} - \frac{0.0094x^4}{4}\right]_0^{65}$$

3142.09

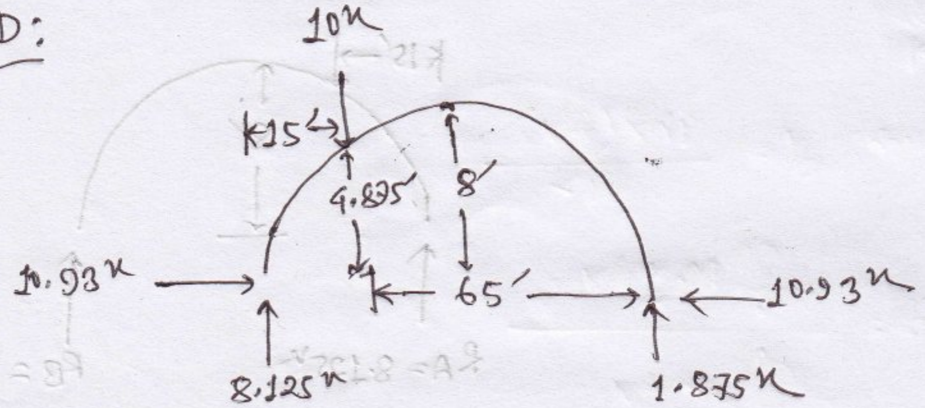
$$= 3651.25 - 506.25 + 68656.25 - 41948.97$$

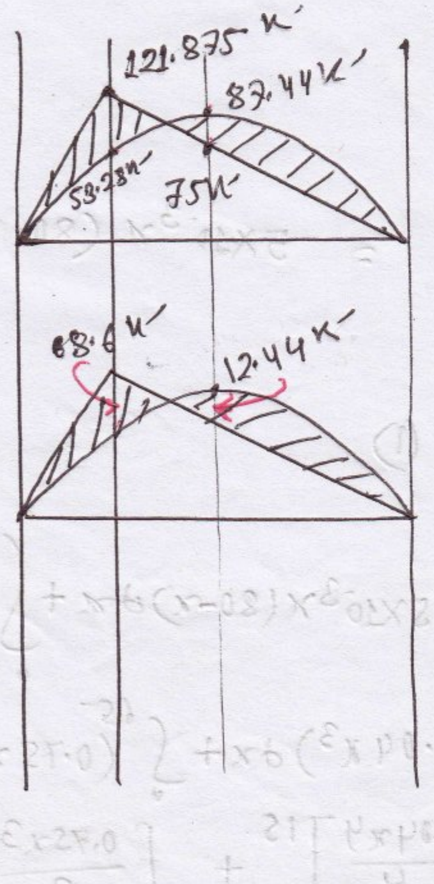
$$= 29857.28$$

$$\int_0^l y^2 dx = \int_0^{80} \left\{5\times10^{-3} x(80-x)\right\}^2 dx = 2730.67$$

$$H = \frac{29857.28}{2730.67} = 10.93^k$$

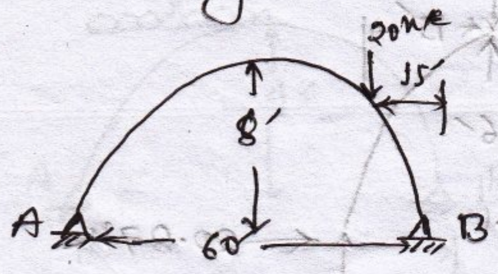
B.M.D:



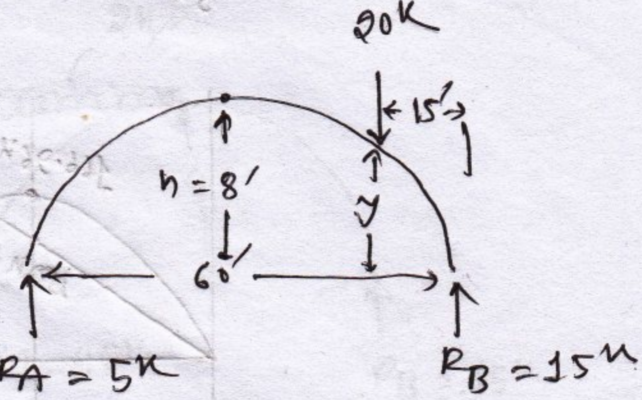


04

Problem : Find the horizontal thrust for the two hinged parabolic arch shown in fig below due to a 20K concentrated load and Draw B.M.D. Neglect the effect of rib shortening.



Solution :



$\Sigma M_A = 0,$

$20 \times 45 = R_B \times 60$

$\therefore R_B = 15^K$

$\Sigma F_y = 0, \quad R_A = 5^K$

$$y = \frac{4hx(l-x)}{l^2}$$

$$= \frac{4 \times 8 \times x(60-x)}{(60)^2} = 8.89 \times 10^{-3}\, x(60-x)$$

$$H = \frac{\int_0^l M_s y \, dx}{\int_0^l y^2 \, dx}$$

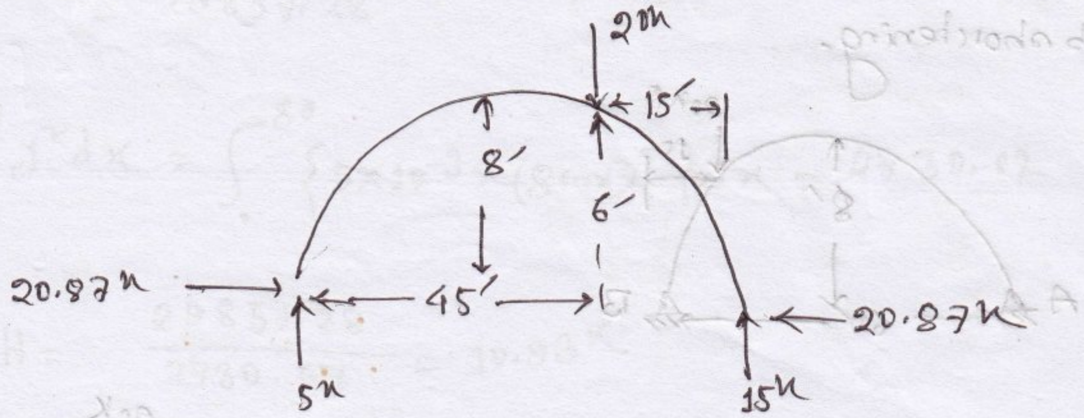
Now,

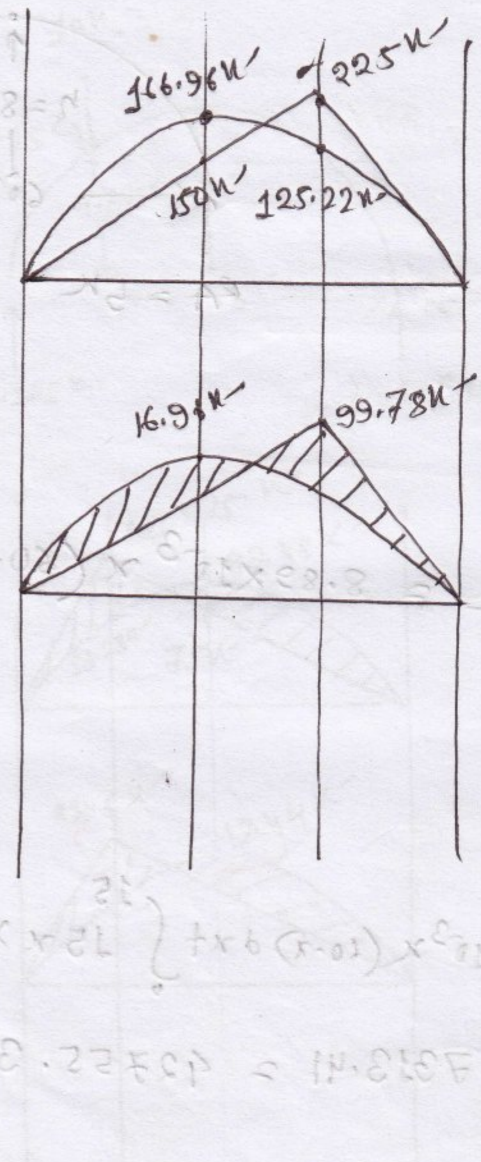
$$\int_0^l M_s y \, dx = \int_0^{45} 5x \times 8.89 \times 10^{-3} x(60-x)\, dx + \int_0^{15} 15x \times 8.89 \times 10^{-3} x(60-x)\, dx$$

$$= 35441.93 + 7313.41 = 42755.34$$

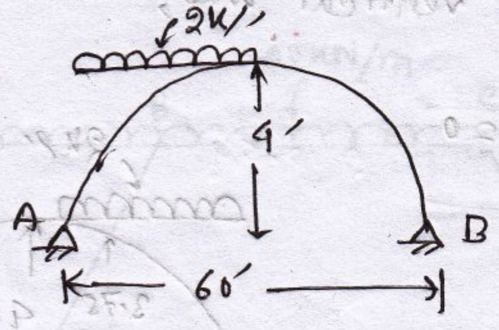
$$\int_0^l y^2 \, dx = \int_0^{60} \{8.89 \times 10^{-3} x(60-x)\}^2 dx = 2048.51$$

$$\therefore H = \frac{42755.34}{2048.51} = 20.87^{k}$$

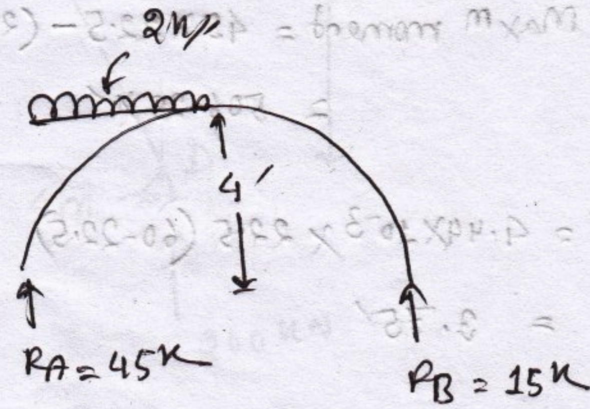




Problem : Find the horizontal thrust for the two hinged arch shown in fig below and also draw BMD. Neglect the effect of rib shortening.



Solution :



$\Sigma M_A = 0,$

$2 \times 30 \times 15 = R_B \times 60$

$\therefore R_B = 15K$

$\Sigma f_y = 0, \quad R_A = 45^K$

$$y = \frac{4hx(l-x)}{l^2} = \frac{4 \times 4 \times x(60-x)}{(60)^2} = 4.44 \times 10^{-3} x(60-x)$$

$$H = \frac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx}$$

Now,
$$\int_0^l M_s y\,dx = \int_0^{30} \left(45x - \frac{2x^2}{2}\right) \times 4.44 \times 10^{-3} x(60-x)\,dx + \int_0^{30} 15x \times 4.44 \times 10^{-3} x(60-x)\,dx$$

$$= 35065 + 22477.5 = 57542.5$$

$$\int_0^l y^2\,dx = \int_0^{60} \{4.43 \times 10^{-3} x(60-x)\}^2\,dx$$

$$= 508.68$$

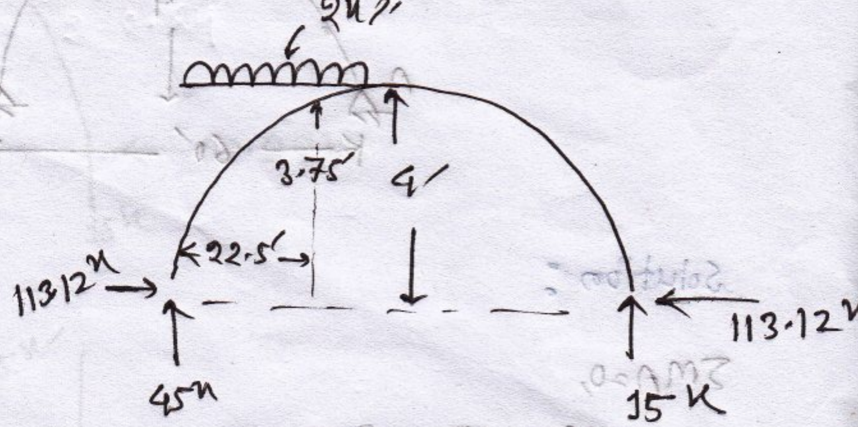
$$H = \frac{57542.5}{508.68} = 113.12^{K}$$

Point of max^m moment due to vertical load,

$$\frac{d}{dx}\left(45x - 2\cdot\frac{x^2}{2}\right) = 0$$

$$\Rightarrow 45 - 2x = 0$$

$$\therefore x = 22.5'$$

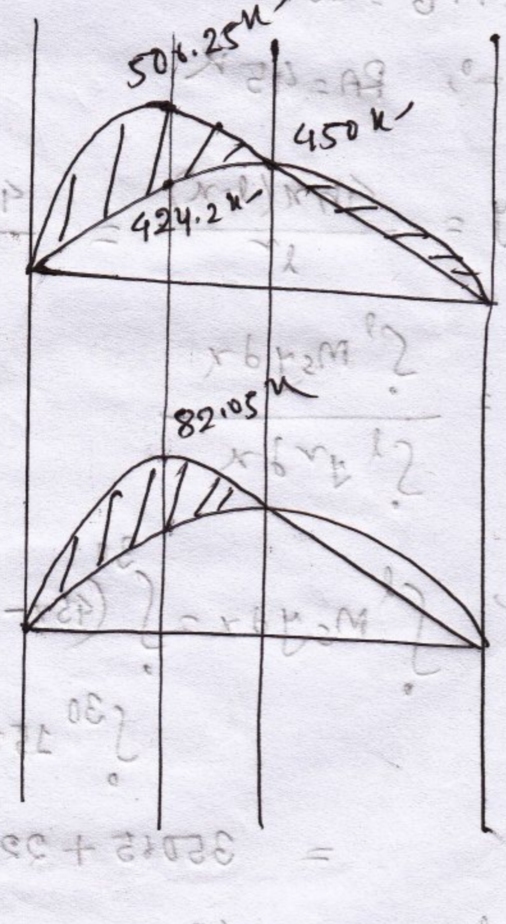


$$\therefore Max^m \text{ moment} = 45 \times 22.5 - (22.5)^2$$

$$= 506.25\,K'$$

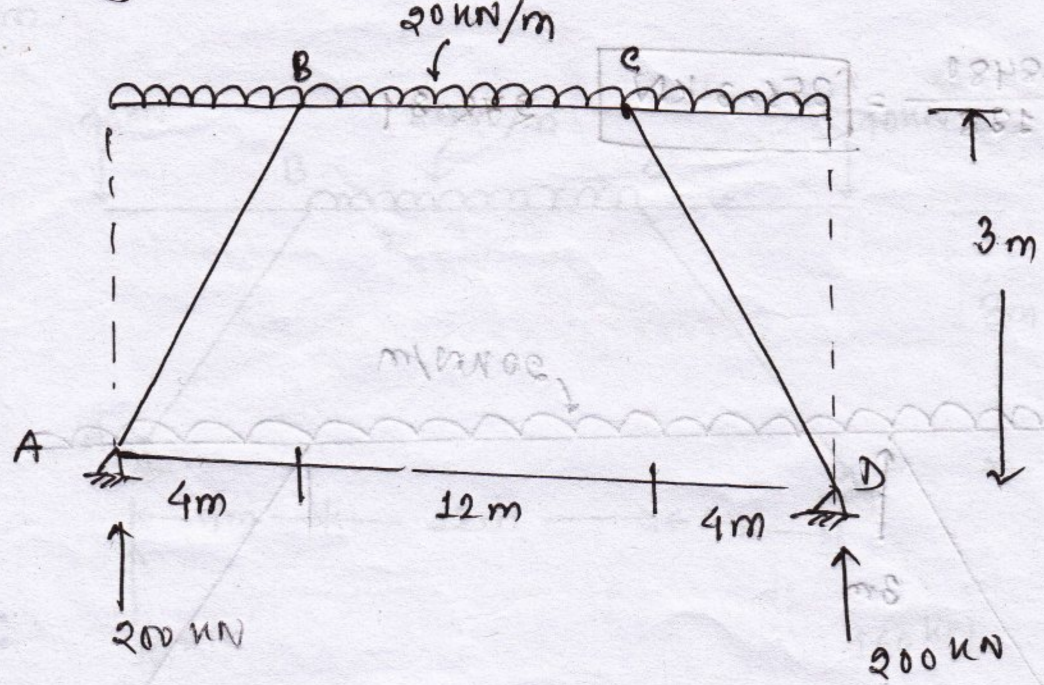
$$y = 4.44 \times 10^{-3} \times 22.5\,(60 - 22.5)$$

$$= 3.75'$$



05, 08

Problem : Two hinged arch of the form shown in fig below has constant section throughout. Determine the horizontal thrust. Draw BM diagram.



Solution :

$\Sigma M_A = 0,\quad 20 \times 20 \times 10 = R_D \times 20$

$\therefore R_D = 200$ KN

$\Sigma f_y = 0,\quad R_A = 200$ KN

$ds = \frac{5}{4} dx$

$= 1.25 dx$

$200 \times 4 - 20 \times \cancel{10 \times 5}\ 4 \times 2$

For AB and DC, $y = \frac{3}{4} x$ For BC, $y = 3m$

Now,

$$\int_0^l M_s y\, dx = \int_0^4 \left(200x - 20 \cdot \frac{x^2}{2}\right) \frac{3}{4} x\, dx \times 1.25 + \int_0^{12} \left(120x - 20 \cdot \frac{x^2}{2} + 640\right) \times 3\, dx$$

$$+ \int_0^4 \left(200x - 20 \cdot \frac{x^2}{2}\right) \frac{3}{4} x\, dx \times 1.25$$

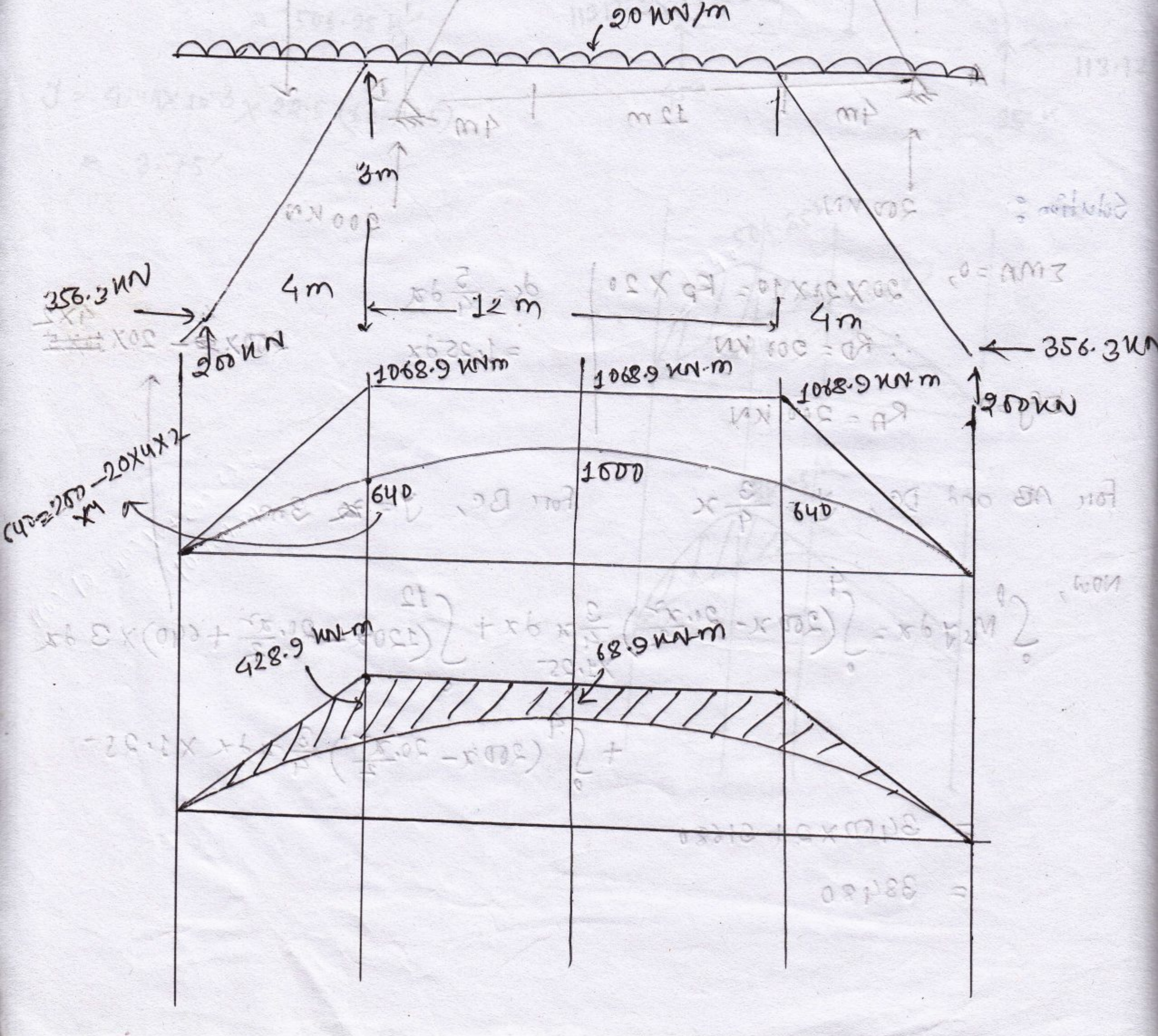
$= 3400 \times 2 + 31680$

$= 38480$

$$\int_0^l y^2\,dx = 2\int_0^4 \left(\frac{3}{4}x\right)^2 \times 1.25\,dx + \int_0^{12} 3^2\,dx$$

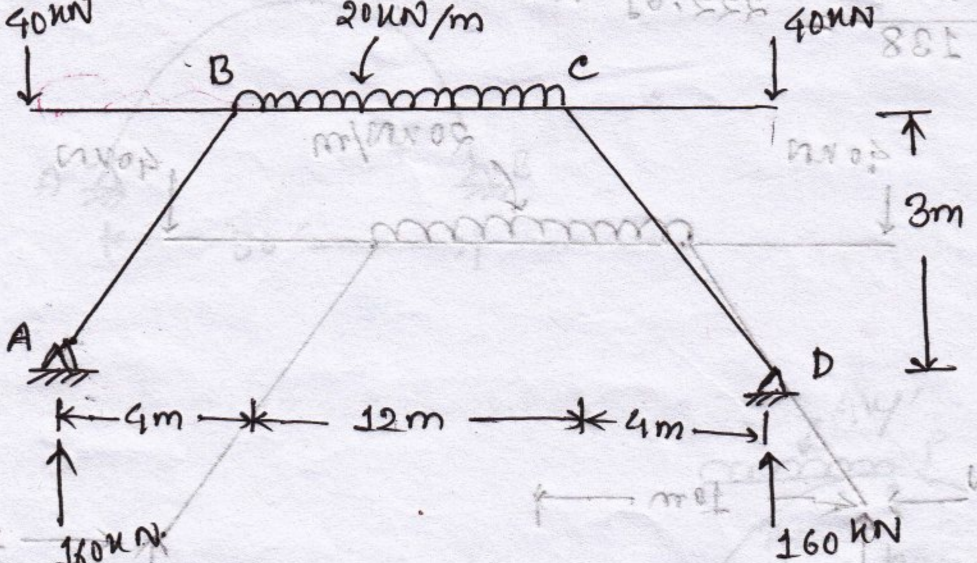
$$= 30 + 108 = 138$$

$$H = \frac{38480}{138} = \boxed{356.3\ \text{KN}} \qquad 278.84$$



05

Problem : Two hinge arch of the form shown in figure below has constant section throughout. Determine the horizontal thrust. Draw BM diagram.



Solution :

$\Sigma M_A = 0$,

$40 \times 20 + 20 \times 12 \times (4+6) = R_D \times 20$

$\therefore R_D = 160\,KN$

$\Sigma F_y = 0, \quad R_A = 160\,KN$

For AB and CD, $y = \frac{3}{4}x$ and BC $y = 3\,m$

For AB and CD, $ds = \frac{\sqrt{4^2+3^2}}{4}dx = \frac{5}{4}dx = 1.25\,dx$

$y = \cancel{\frac{4h x(l-x)}{l^2}} \quad \cancel{4x}$

$\int_0^l M_1 y\,dx = 2\int_0^4 (160x - 40x) \times \frac{3}{4}x \times 1.25\,dx + \int_0^{12} \left(120x - 20 \cdot \frac{x^2}{2} + 480\right) \times 3\,dx$

$= 2 \times 2400 + 25920$

$= 30720$

$2\int_4^{10} \left(160x - 20\frac{(x-4)^2}{2}\right) \times 3\,dx$

$160 \times 4 - 40 \times 4$

$160 \times 10 - 40 \times 10 - 20 \times 6 \times 3$

$= 920\,kN\text{-}m$

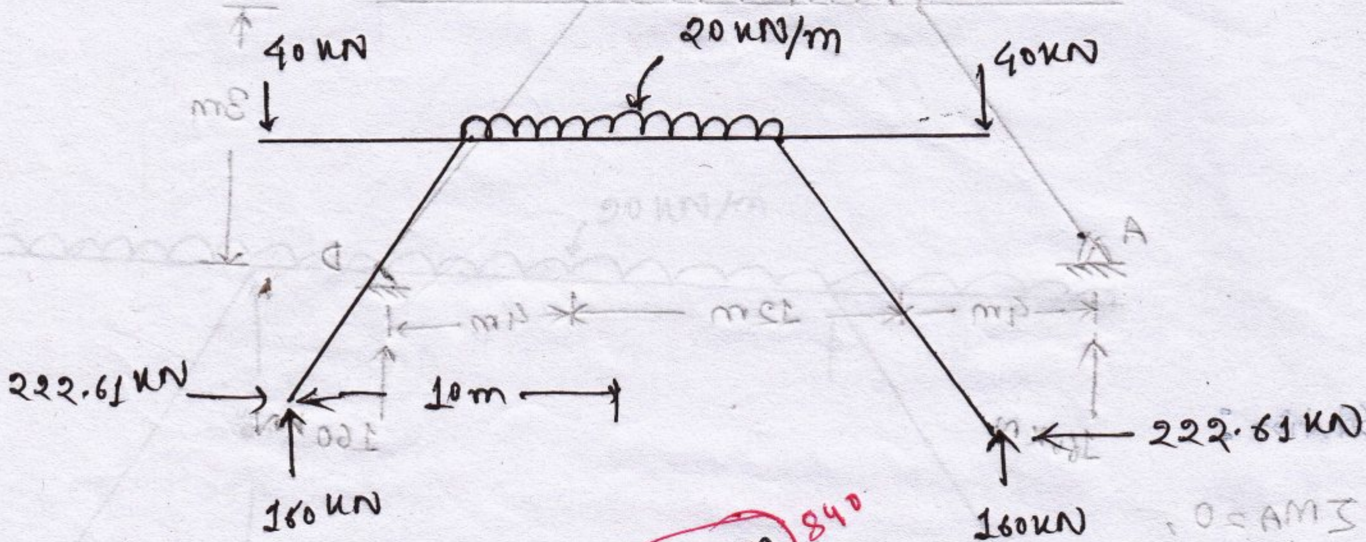
$160 \times 10 - 40 \times 10 + 3 \times 6 \times 3$

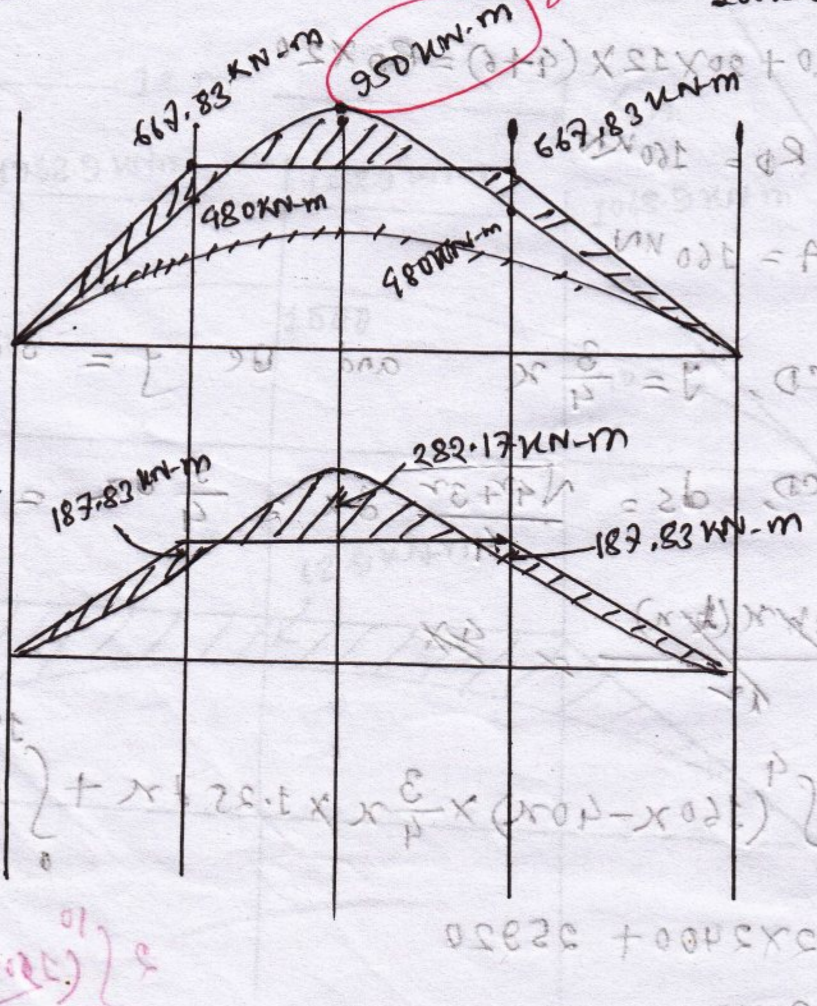
$= 840\,kN\text{-}m$

$$\int_0^l y^2 dx = 2\int_0^4 \left(\frac{3}{4}x\right)^2 \times 1.25\, dx + \int_0^{12} (3)^2 dx$$

$$= 2\times 15 + 108 = 138$$

$$H = \frac{30720\cancel{0}}{138} = 222.61 \text{ kN}$$





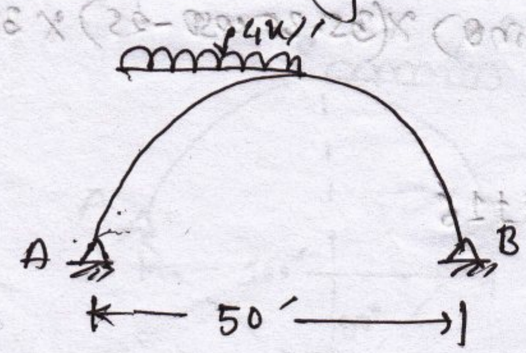
$160\times10 - 40\times10 - 20\times5\times2.5$

$= 950$ kN-m

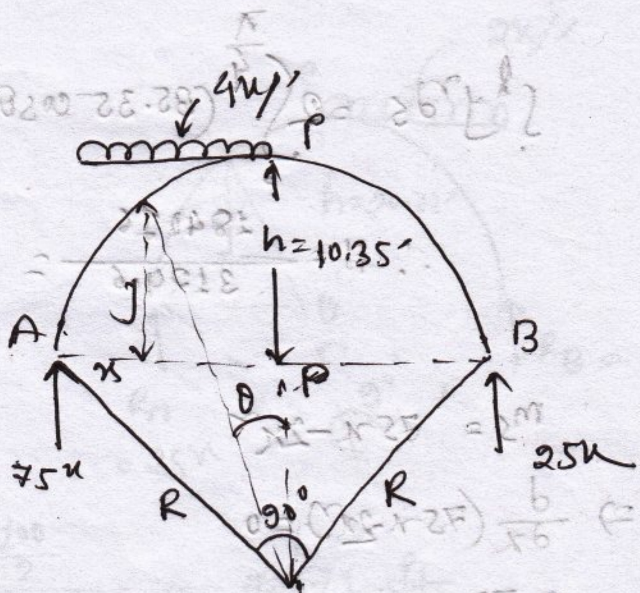
$160\times10 - 40\times10 + 20\times6\times3$

$= 840$ kN-m

Problem: The segmental arch shown in the fig subtends an angle of 90° at the centre, and supports a uniformly distributed load of 4k/ft over half of the span. Draw BM diagram.



Solution:



$\Sigma M_A = 0,$

$4 \times 25 \times 12.5 = R_B \times 50$

$\therefore R_B = 25k$

$\Sigma F_y = 0, \; R_A = 75k$

Radius of circle, $R = \dfrac{l/2}{\sin\alpha}$

$= \dfrac{50/2}{\sin 45°} = 35.35'$

$h = R - R\cos\alpha = 35.35 - 35.35\cos 45° = 10.35'$

$y = R(\cos\theta - \cos\alpha) = 35.35(\cos\theta - \cos 45°) = 35.35\cos\theta - 25$

$x = R(\sin\alpha - \sin\theta) = 35.35(\sin 45° - \sin\theta) = 25 - 35.35\sin\theta$

M_S for section AP $= 75x - 4 \cdot \dfrac{x^2}{2} = 75x - 2x^2$

M_S " " BP $= 25x$

$$\int_0^l M_s y\, ds = \int_0^{\frac{\pi}{4}} \left\{75(25-35.35\sin\theta) - 2(25-35.35\sin\theta)^2\right\} \times (35.35\cos\theta - 25) \times 35.35\, d\theta$$

$$+ \int_0^{\frac{\pi}{4}} 25(25-35.35\sin\theta) \times (35.35\cos\theta - 25) \times 35.35\, d\theta$$

$$= 112900 + 71116$$

$$= 184016.$$

$$\int_0^l y^2 ds = 2\int_0^{\frac{\pi}{4}} (35.35\cos\theta - 25)^2 \times 35.35\, d\theta = 2 \times 1562.3$$

$$= 3124.6$$

$$\therefore H = \frac{184016}{3124.6} = 58.89\,k$$

$$M_s = 75x - 2x^2$$

$$\Rightarrow \frac{d}{dx}(75x - 2x^2) = 0$$

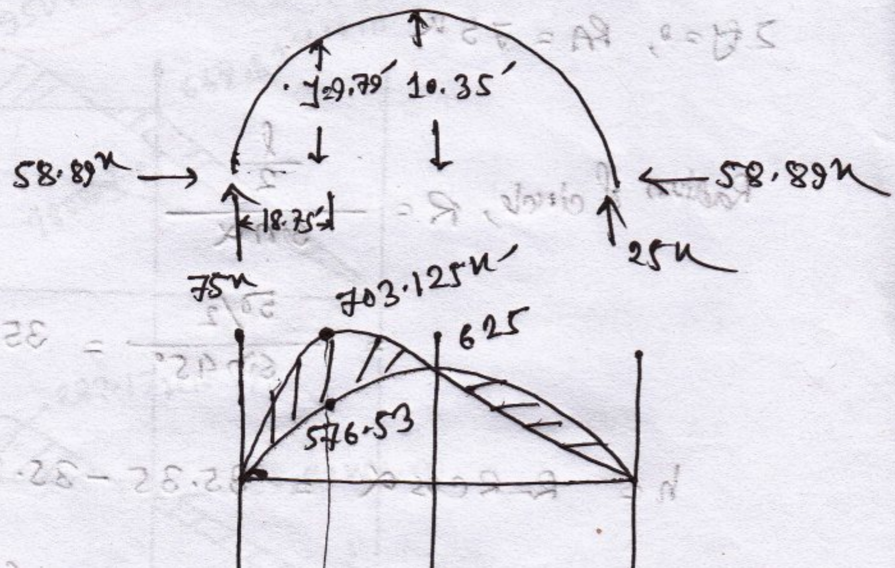
$$\Rightarrow 75 - 4x = 0$$

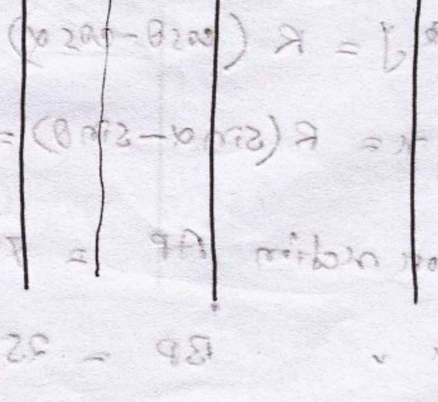
$$\therefore x = 18.75$$

$$18.75 = 25 - 35.35\sin\theta$$

$$\therefore \theta = 10.18^\circ$$

$$y =$$



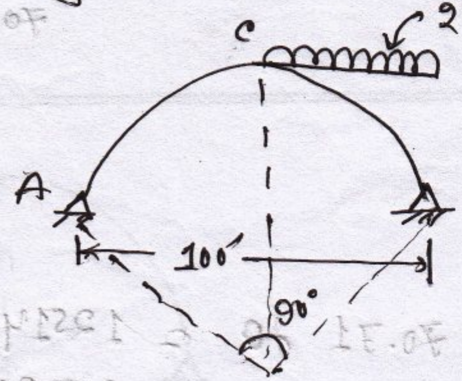


$$M_{max} = 75 \times 18.75 - 2(18.75)^2$$

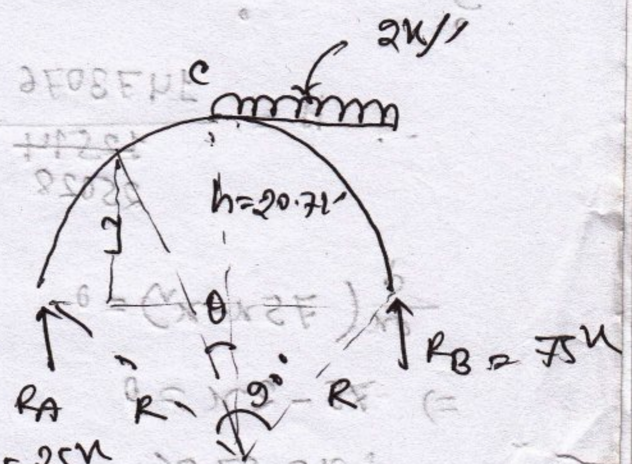
$$= 703.125\,k$$

07

Problem: A two hinged segmental arch of span 100ft subtends an angle of 90 degree at the centre as shown in fig below. Draw BM and normal thrust diagram for the arch.



Solution:



$\Sigma M_A = 0,$

$2 \times 50 \times 75 = R_B \times 100$

$\therefore R_B = 75^k$

$\Sigma f_y = 0, \quad R_A = 25^k$

Radius of circle $R = \dfrac{\frac{l}{2}}{\sin\alpha} = \dfrac{\frac{100}{2}}{\sin 45} = 70.71$ ft

$h = R - R\cos\alpha = 70.71 - 70.71\cos 45 = 20.71'$

$y = R(\cos\theta - \cos\alpha) = 70.71(\cos\theta - \cos 45) = 70.71\cos\theta - 50$

$x = R(\sin\alpha - \sin\theta) = 70.71(\sin 45 - \sin\theta) = 50 - 70.71\sin\theta$

M_s for segment AC $= 25x$

M_s for segment BC $= 75x - 2 \cdot \dfrac{x^2}{2} = 75x - x^2$

$$\int_0^l M_s y\,ds = \int_0^{\pi/4} 25(50-70.71\sin\theta)(70.71\cos\theta-50)\times 70.71\,d\theta +$$

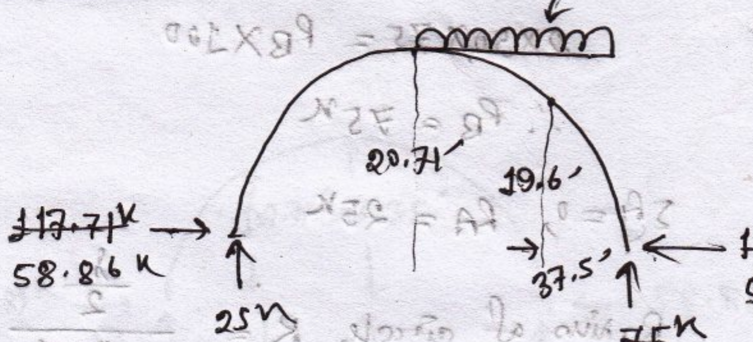
$$\int_0^{\pi/4}\left\{75(50-70.71\sin\theta)-(50-70.71\sin\theta)^2\right\}\times(70.71\cos\theta-50)\times 70.71\,d\theta$$

$= 569266 + 903810$

$= 1473076$

$$\int_0^l y^2\,ds = 2\int_0^{\pi/4}(70.71\cos\theta-50)^2\times 70.71\,d\theta = 12514\times 2 = 25028$$

$$\therefore H = \frac{1473076}{\cancel{12514}\ 25028} = \cancel{117.71}^{k}\ 58.86$$



$\frac{d}{dx}(75x-x^2)=0$

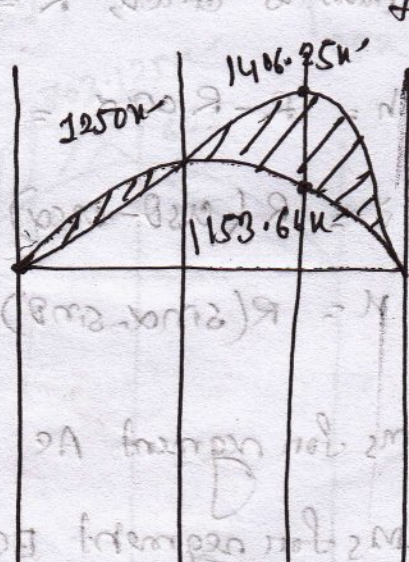
$\Rightarrow 75-2x=0$

$\therefore x = 37.5'$

$37.5 = 50 - 70.71\sin\theta$

$\therefore \theta = 10.18°$

$y = 70.71\cos 10.18 - 50 = 19.6'$



$M_{max} = 75\times 37.5 - (37.5)^2$

$= 1406.25\ k'$

03

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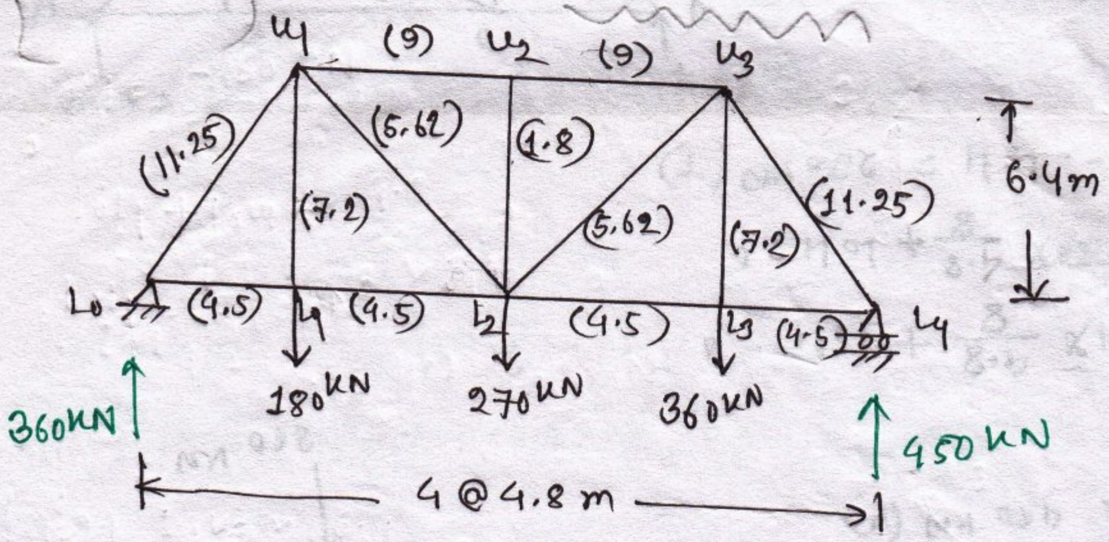


$E = 200 \times 10^6 \text{ KN/m}^2$

Area = ( ) $\times 10^{-3} m^2$

$\Delta H_{L3} = ?$, $\Delta V_{L3} = ?$

Solution:

$\Sigma M_{L0} = 0,$

$180 \times 4.8 + 270 \times 2 \times 4.8 + 360 \times 3 \times 4.8 = R_{L4} \times 4 \times 4.8$

$\therefore R_{L4} = 450 \text{ KN}$

$R_{L0} = 360 \text{ KN}$

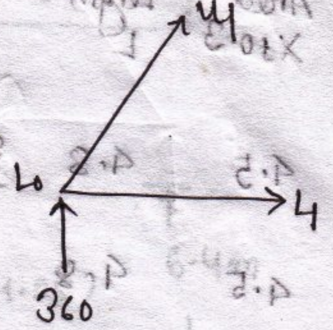
Member	Area $\times 10^{-3}$	Length L	S	u_{HL_3}	u_{VL_3}	$\frac{S u_{HL_3} L}{AE}$	$\frac{S u_{VL_3} L}{AE}$
L_0L_1	4.5	4.8	270	1	0.19	0.0014 0.014	0.0027 0.00027
L_1L_2	4.5	4.8	270	1	0.19	0.014	0.0027
L_2L_3	4.5	4.8	~~180~~ 337.5	1	~~0.75~~ 0.56	0.018	0.01
L_3L_4	4.5	4.8	337.5	1	~~0.75~~ 0.56	0.018	0.01
L_0u_1	11.25	8	-450	0	-0.31	0	0.005
L_1u_1	7.2	6.4	180	0	0	0	0
L_2u_1	5.62	8	225	0	0.31	0	0.005
u_1u_2	9	4.8	-405	0	-0.37	0	0.004
u_2u_3	9	4.8	-405	0	-0.37	0	0.004
L_2u_2	1.8	6.4	0	0	0	0	0
L_2u_3	5.62	8	112.5	0	-0.31	0	-0.002
L_3u_3	7.2	6.4	360	0	1	0	0.002
L_4u_3	11.25	8	-562.5	0	-0.99	0	0.002
						0.069	0.0454

0.0064

$\Sigma F_y = 0,$

$$360 + L_0U_1 \times \frac{6.4}{8} = 0$$

$$\therefore L_0U_1 = -450 \text{ kN}$$



$\Sigma F_x = 0,$

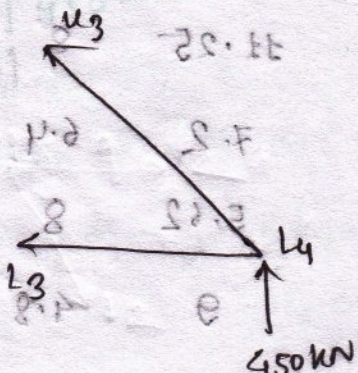
$$L_0L_1 + L_0U_1 \times \frac{4.8}{8} = 0$$

$$\therefore L_0L_1 = 270 \text{ kN}$$

$\Sigma F_y = 0,$

$$450 + L_4U_3 \times \frac{6.4}{8} = 0$$

$$\therefore L_4U_3 = -562.5 \text{ kN}$$



$\Sigma F_x = 0,$

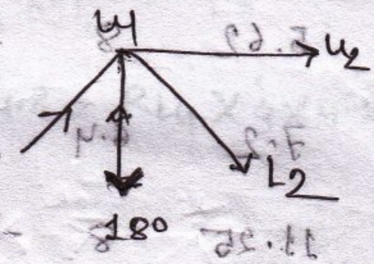
$$L_3L_4 + L_4U_3 \times \frac{4.8}{8} = 0$$

$$\therefore L_3L_4 = 337.5 \text{ kN}$$

$\Sigma F_y = 0,$

$$180 + L_2U_1 \times \frac{6.4}{8} - L_0U_1 \times \frac{6.4}{8} = 0$$

$$\therefore L_2U_1 = 225 \text{ kN}$$



$\Sigma F_x = 0,$

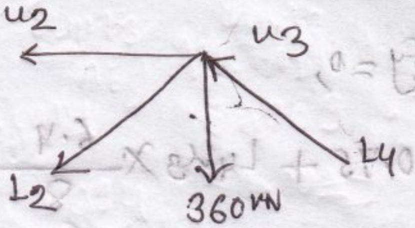
$$U_1U_2 + L_2U_1 \times \frac{4.8}{8} + L_0U_1 \times \frac{4.8}{8} = 0$$

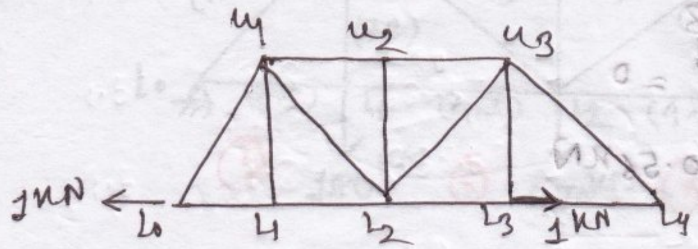
$$\therefore U_1U_2 = -135 - 270 = -405 \text{ kN}$$

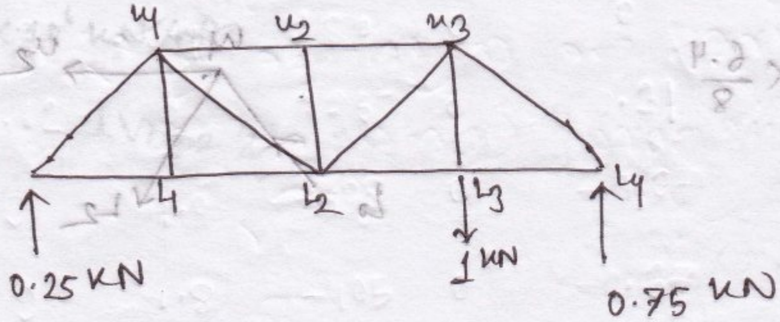
$\Sigma f_y = 0,$

$$360 + L_2u_3 \times \frac{6.4}{8} = L_4u_3 \times \frac{6.4}{8}$$

$\therefore L_2u_3 = 112.5\ kN$



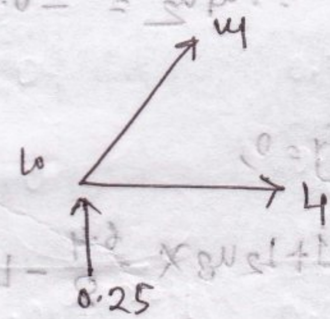




$\Sigma M_{L0} = 0,$

$$1 \times 3 \times 4.8 = R_{L4} \times 4 \times 4.8$$

$\therefore R_{L4} = 0.75\ kN$



$\Sigma f_y = 0,$

$$0.25 + L_0u_1 \times \frac{6.4}{8} = 0$$

$\therefore L_0u_1 = -0.31\ kN$

$\Sigma f_x = 0,$

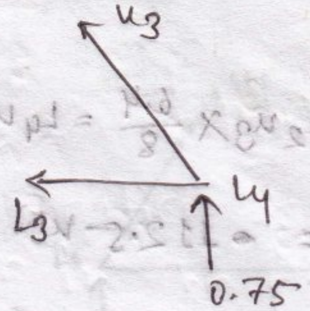
$$L_0L_1 + L_0u_1 \times \frac{4.8}{8} = 0$$

$\therefore L_0L_1 = 0.19\ kN$

$\Sigma F_y = 0,$

$$0.75 + L_4U_3 \times \frac{6.4}{8} = 0$$

$$\therefore L_4U_3 = -0.94 \text{ kN}$$



$\Sigma F_x = 0,$

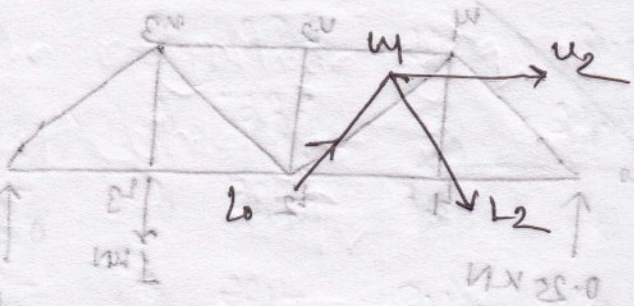
$$L_3L_4 + L_4U_3 \times \frac{4.8}{8} = 0$$

$$\therefore L_3L_4 = \cancel{0.75 \text{ kN}} \;\; 0.56 \text{ kN}$$

$\Sigma F_y = 0,$

$$L_0U_4 \times \frac{6.4}{8} = L_2U_4 \times \frac{6.4}{8}$$

$$\therefore L_2U_4 = 0.31 \text{ kN}$$



$\Sigma F_x = 0,$

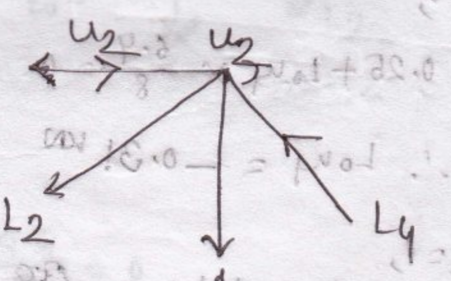
$$U_4U_2 + L_2U_4 \times \frac{4.8}{8} + L_0U_4 \times \frac{4.8}{8} = 0$$

$$\therefore U_4U_2 = -0.186 - 0.186 = -0.37$$

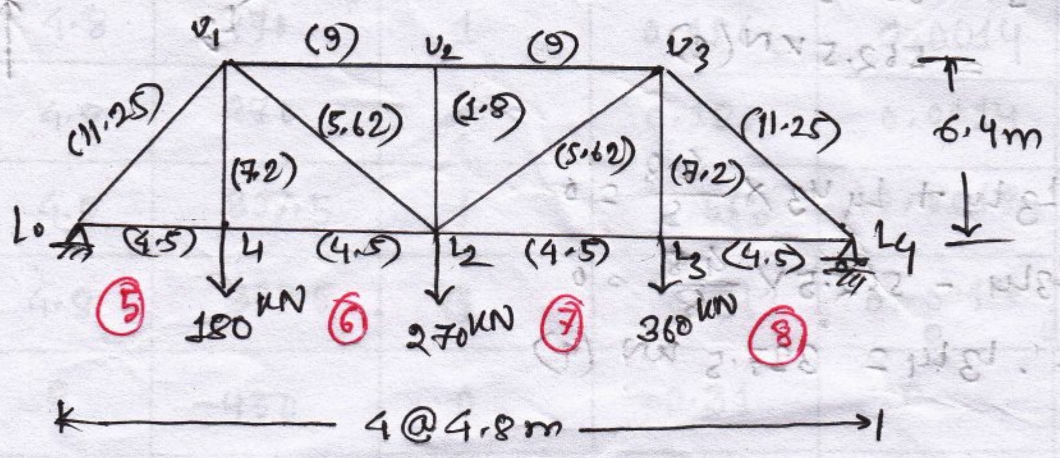
$\Sigma F_y = 0,$

$$1 + L_2U_3 \times \frac{6.4}{8} - L_4U_3 \times \frac{6.4}{8} = 0$$

$$\therefore L_2U_3 = -0.31 \text{ kN}$$



Problem-01:



$E = 200\times10^6$ kN/m²

$\Delta H_{L_3} = ?$ $\quad \Delta V_{L_3} = ?$

Solution:

$\Sigma M_{L_0} = 0,$

$180\times4.8 + 270\times(2\times4.8) + 360\times(3\times4.8) = R_{L_4}\times4\times4.8$

$\therefore R_{L_4} = 450$ kN

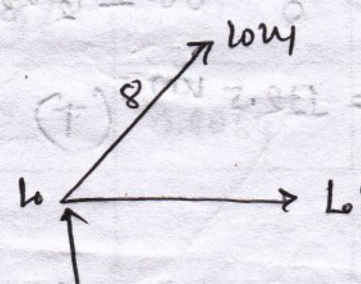
$\Sigma f_y = 0, \quad R_{L_0} = 810-450 = 360$ kN.

$\Sigma f_y = 0,$

$L_0U_1 \times \frac{6.4}{8} + 360 = 0$

$\Rightarrow L_0U_1 = -450$ kN

$= 450$ kN (C)



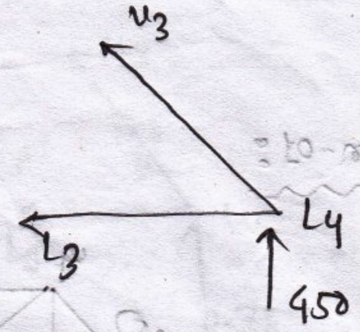
$\Sigma f_x = 0,$

$L_0U_1 \times \frac{4.8}{8} + L_0L_1 = 0$

$\Rightarrow -450\times\frac{4.8}{8} + L_0L_1 = 0$

$\therefore L_0L_1 = 270$ kN (T)

$\Sigma F_y = 0,$



$$450 + L_4U_3 \times \frac{6.4}{8} = 0$$

$$\therefore L_4U_3 = -562.5 \text{ KN}$$

$$= 562.5 \text{ KN (C)}$$

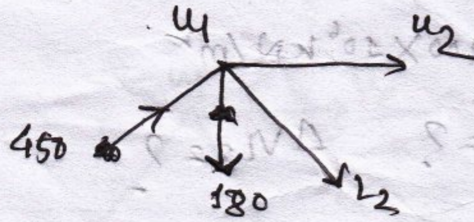
$\Sigma F_x = 0,$

$$L_3L_4 + L_4U_3 \times \frac{4.8}{8} = 0$$

$$\Rightarrow L_3L_4 - 562.5 \times \frac{4.8}{8} = 0$$

$$\therefore L_3L_4 = 337.5 \text{ KN (T)}$$

$\Sigma F_y = 0,$



$$450 \times \frac{6.4}{8} - 180 - L_2U_1 \times \frac{6.4}{8} = 0$$

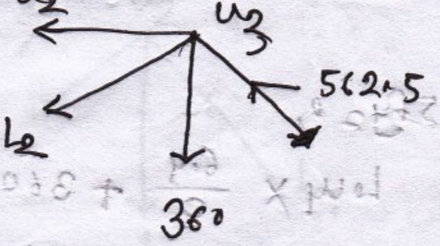
$$\therefore L_2U_1 = 225 \text{ KN (T)}$$

$\Sigma F_x = 0,$

$$450 \times \frac{4.8}{8} + L_2U_1 \times \frac{4.8}{8} + U_1U_2 = 0$$

$$\therefore U_1U_2 = -405 \text{ KN} = 405 \text{ KN (C)}$$

$\Sigma F_y = 0,$



$$562.5 \times \frac{6.4}{8} - 360 - L_2U_3 \times \frac{6.4}{8} = 0$$

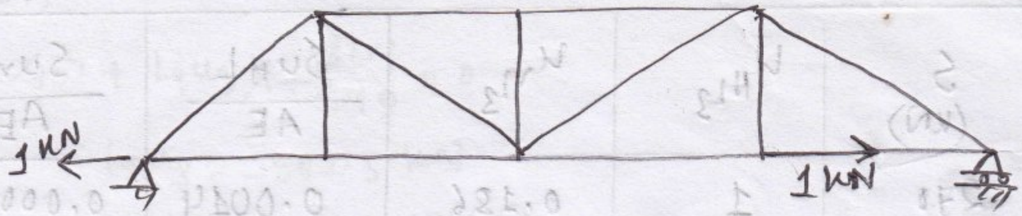
$$\therefore L_2U_3 = 112.5 \text{ KN (T)}$$

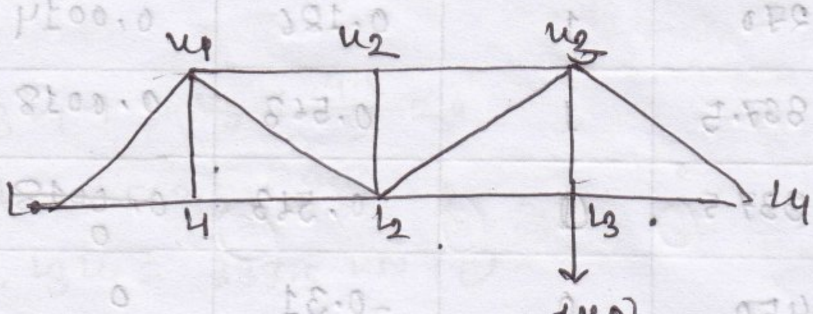
$\Sigma F_x = 0,$

$$U_2U_3 + L_2U_3 \times \frac{4.8}{8} + 562.5 \times \frac{4.8}{8} = 0$$

$$\therefore U_2U_3 = -405 \text{ KN} = 405 \text{ KN (C)}$$

M	A $\times 10^{-3}$	L (m)	S (kN)	u_{HL_3}	u_{VL_3}	$\frac{Su_HL}{AE}$	$\frac{Su_VL}{AE}$
L_0L_1	4.5	4.8	270	1	0.186	0.0014	0.00027
L_1L_2	4.5	4.8	270	1	0.186	0.0014	0.00027
L_2L_3	4.5	4.8	337.5	1	0.563	0.0018	0.0010
L_3L_4	4.5	4.8	337.5	0	0.563	~~0.0018~~ 0	0.0010
L_0U_1	11.25	8	−450	0	−0.31	0	0.0005
L_1U_1	7.2	6.4	180	0	0	0	0
L_2U_1	5.62	8	225	0	0.31	0	0.0005
L_2U_2	1.8	6.4	0	0	0	0	0
L_2U_3	5.62	8	112.5	0	−0.312	0	−0.00025
L_3U_3	7.2	6.4	360	0	1	0	0.0016
L_4U_3	11.25	8	−562.5	0	−0.938	0	0.0019
U_1U_2	9	4.8	−405	0	−0.372	0	0.0004
U_2U_3	9	4.8	−405	0	−0.376	0	0.0004
					Σ	0.0064	0.015



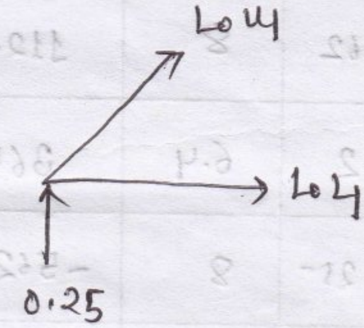


$\Sigma M_{L_0} = 0,$

$1 \times 3 \times 4.8 = R_{L_4} \times 4 \times 4.8$

$\therefore R_{L_4} = 0.75 \text{ kN}$

$\Sigma F_y = 0, \quad R_{L_0} = 0.25 \text{ kN}$



$\Sigma F_y = 0,$

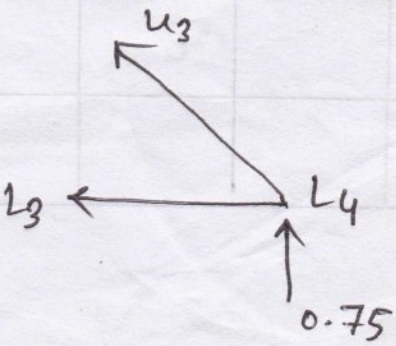
$0.25 + L_0U_1 \times \frac{6.4}{8} = 0$

$\therefore L_0U_1 = -0.31 \text{ kN} = 0.31 \text{ kN (C)}$

$\Sigma F_x = 0,$

$L_0L_1 + L_0U_1 \times \frac{4.8}{8} = 0$

$\therefore L_0L_1 = \cancel{0.186 \text{ kN}}\ 0.186 \text{ kN (T)}$



$\Sigma F_y = 0,$

$0.75 + L_4U_3 \times \frac{6.4}{8} = 0$

$\therefore L_4U_3 = -0.938 \text{ kN} ($

$= 0.938 \text{ kN (C)}$

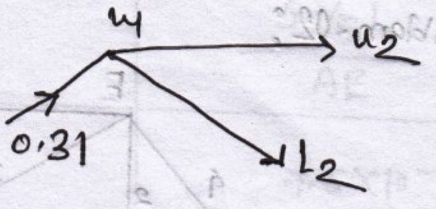
$\Sigma F_x = 0, \quad L_3L_4 + L_4U_3 \times \frac{4.8}{8} = 0$

$\therefore L_3L_4 = 0.563 \text{ kN (T)}$

$\Sigma F_y = 0,$

$$0.31 \times \frac{6.4}{8} = L_2u_4 \times \frac{6.4}{8}$$

$\therefore L_2u_4 = 0.31$ KN (T)



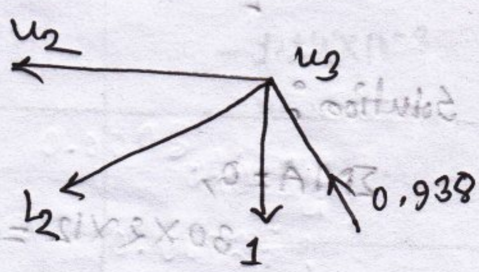
$\Sigma F_x = 0,$

$$u_4u_2 + 0.31 \times \frac{4.8}{8} + L_2u_4 \times \frac{4.8}{8} = 0$$

$\therefore u_4u_2 = -0.372$ KN (T)

$= 0.372$ KN (C)

$\Sigma F_y = 0,$



$$0.938 \times \frac{6.4}{8} = 1 + L_2u_3 \times \frac{6.4}{8}$$

$\therefore L_2u_3 = -0.312$

$\Sigma F_x = 0,$

$$u_2u_3 + L_2u_3 \times \frac{4.8}{8} + 0.938 \times \frac{4.8}{8} = 0$$

$\therefore u_2u_3 = -0.376$

2. Find the horizontal and vertical deflection of joint L_3 due to temperature drop of 50°c in the lower chord only. Coefficient of thermal expansion or contraction = 11.7×10^{-6} per degree centigrade.

$$(\Delta L)_5 = (\Delta L)_6 = (\Delta L)_7 = (\Delta L)_8 = \alpha L t$$

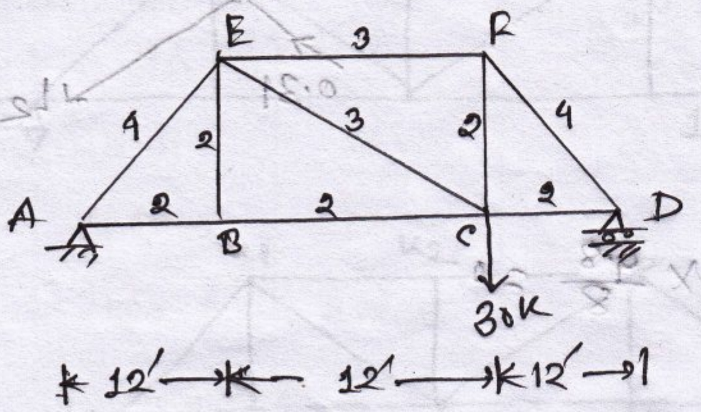
$$= 11.7 \times 10^{-6} \times 4.8 \times 10^{3} \times (-50) = -2.81 \text{ mm}$$

$$\Delta HL_3 = \Sigma u \Delta L = (1+1+1+0) \times (-2.81) = -8.43 \text{ mm}$$

$$\Delta VL_3 = \Sigma v \Delta L = (0.186 + 0.186 + 0.563 + 0.563) \times (-2.81) = 4.21 \text{ mm}$$

(Ans).

Problem-02:



$E = 30 \times 10^3$ ksi

$\Delta_{VB} = ?$

Solution:

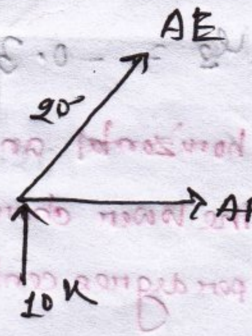
$\Sigma M_A = 0,$

$30 \times 2 \times 12 = R_D \times 3 \times 12$

$\therefore R_D = 20K$

$\Sigma F_y = 0, \quad R_A = 10K$

$\Sigma F_y = 0,$



$10 + AE \times \frac{16}{20} = 0$

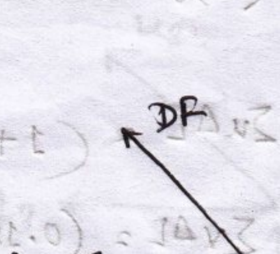
$\therefore AE = -12.5^K = 12.5^K (C)$

$\Sigma F_x = 0,$

$AB + AE \times \frac{12}{20} = 0$

$\therefore AB = 7.5^K (T)$

$\Sigma F_y = 0,$



$20 + DF \times \frac{16}{20} = 0$

$\therefore DF = -25^K = 25^K (C)$

$\Sigma F_x = 0,$

$CD + DF \times \frac{12}{20} = 0$

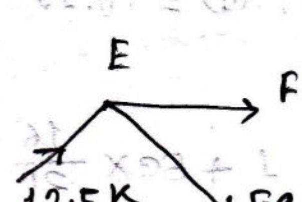
$\therefore CD = 15^K (T)$

M	A (in²)	L (ft)	S (K)	V_E	$\frac{S V_E L}{AE}$
AB	2	12	7.5	0.50	7.5×10^{-4}
AE	4	20	-12.5	-0.84	1.75×10^{-3}
BE	2	16	0	0	0
BC	2	12	7.5	0.50	7.5×10^{-4}
CE	3	20	12.5	-0.41	-1.14×10^{-3}
CF	2	16	30	0	0
CD	2	12	15	0.25	7.5×10^{-4}
DF	4	20	-25	-0.41	1.71×10^{-3}
EF	3	12	15	-0.26	-5.2×10^{-4}
					0.0041

$\Sigma F_y = 0,$

$$12.5\times\frac{16}{20} = EC\times\frac{16}{20}$$

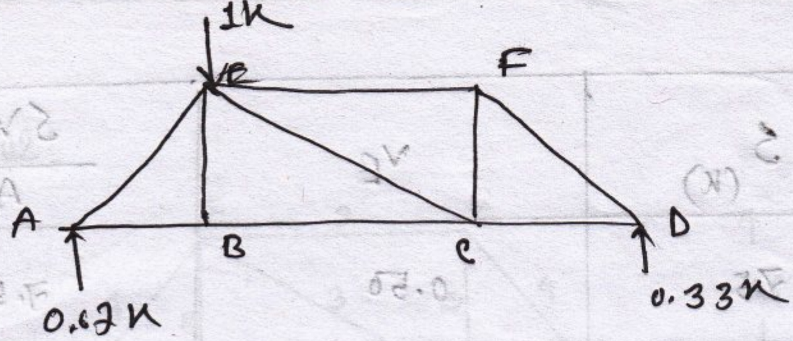
$\therefore EC = 12.5^K\ (T)$



$\Sigma F_x = 0,$

$$EF + EC\times\frac{12}{20} + 12.5\times\frac{12}{20} = 0$$

$\therefore EF = -15^K = 15^K\ (C)$



$\Sigma M_A = 0,$

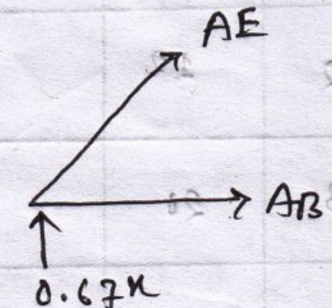
$1 \times 12 = R_D \times 36 \quad \therefore R_D = 0.33k$

$\Sigma F_y = 0, \quad R_A = 0.67k$

$\Sigma F_y = 0,$

$0.67 + AE \times \frac{16}{20} = 0$

$\therefore AE = -0.84 = 0.84^k\,(C)$



$\Sigma F_x = 0,$

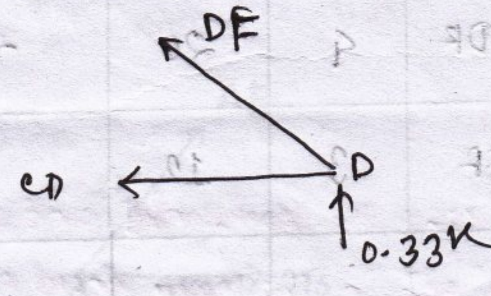
$AB + AE \times \frac{12}{20} = 0$

$\therefore AB = 0.50^k\,(T)$

$\Sigma F_y = 0,$

$DF \times \frac{16}{20} + 0.33 = 0$

$\therefore DF = -0.41 = 0.41^k\,(C)$



$\Sigma F_x = 0,$

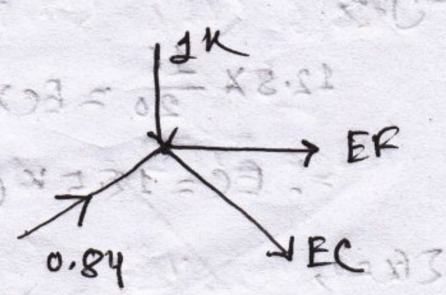
$CD + DF \times \frac{12}{20} = 0$

$\therefore CD = 0.25^k\,(T)$

$\Sigma F_y = 0,$

$1 + EC \times \frac{16}{20} = 0.84 \times \frac{16}{20}$

$\therefore EC = -0.41 = 0.41^k\,(C)$



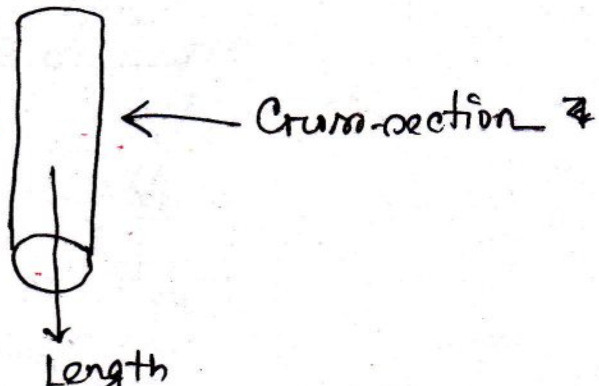
$\Sigma F_x = 0, \quad EF + 0.84 \times \frac{12}{20} - \cancel{0.4} \; 0.41 \times \frac{12}{20} = 0$

$\therefore EF = -0.21 = 0.26^k\,(C)$

3D

AHSAN
090063

* Just Mind it carefully :

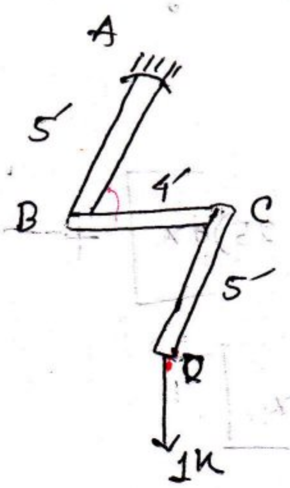


1. Cross-section বরাবর যে গুলো ফোর্স করে তাদের কারণে Torque ঘটে
2. Length " " " " " " Moment "
3. যেটাকে origin ধরবো সব সময় সেই দিকে তাকাবো M, T লেখার সময়।
4. একটা Member থেকে অন্য member এ যাওয়ার সময় সব কিছুকেই নিয়ে যেতে হবে, কেউ যেন বাদ না পড়ে।
5. Torque এর ক্ষেত্রে anti-clockwise কে আমি positive ধরছি।

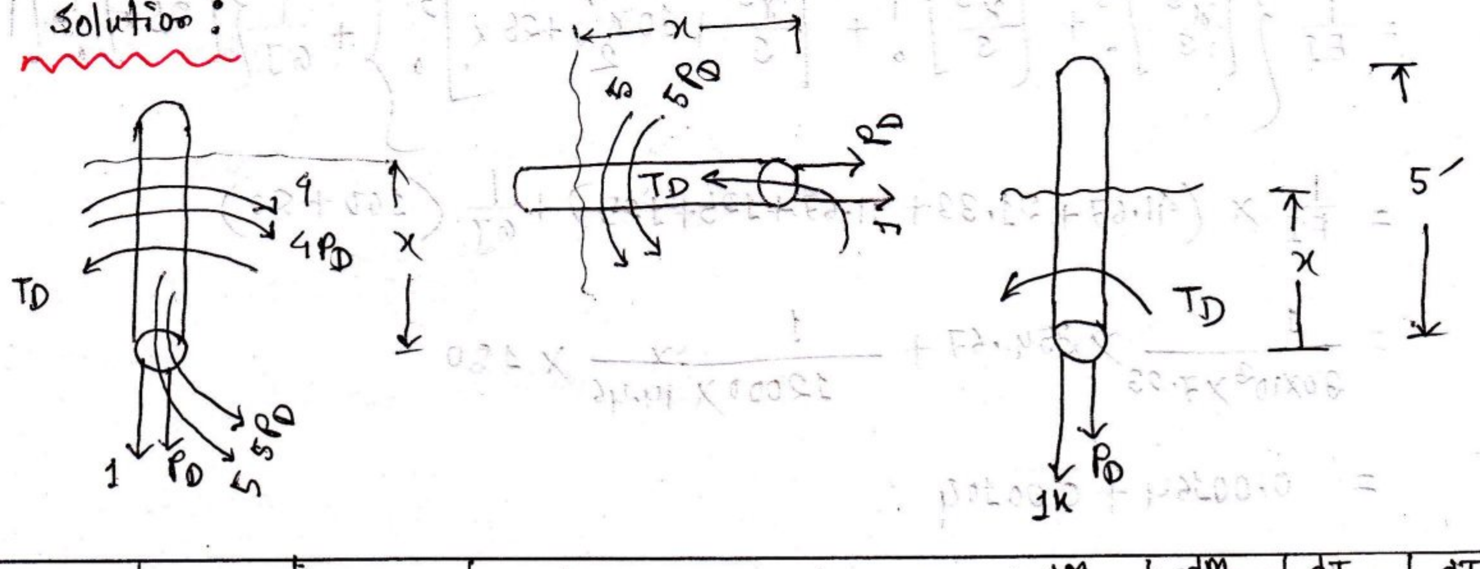
3D

AHSAN
090063

01. 4 in standard pipe bracket have 90° angle at B and C and located at a horizontal plane. Find out a) vertical deflection at D. b) rotational deflection at D in the plane normal to the axis of CD. Plane moment of inertia = 7.23 in⁴, $G = 12000$ ksi, $E = 30\times10^3$ ksi, $J = 14.46$ in⁴. Use Castigliano's theorem.



Solution :



Portion	Origin	Limit	M	T	$\frac{\partial M}{\partial P}$	$\frac{\partial M}{\partial T}$	$\frac{\partial T}{\partial P}$	$\frac{\partial T}{\partial T}$
DC	D	0–5	$x + P_D x$	T_D	x	0	0	1
CB	C	0–4	$x + P_D x - T_D$	$5 + 5P_D$	x	−1	5	0
BA	B	0–5	$x + P_D x + \cancel{}\, 5 + 5P_D$	$T_D - 4 - 4P_D$	$x+5$	0	−4	1

$$\Delta v_D = \frac{\int_0^l M \cdot \frac{dM}{dP}}{EI} dx + \frac{\int_0^l T \cdot \frac{dT}{dP}}{GJ} dx$$

$$= \frac{1}{EI}\left[\int_0^5 (x + P_D x)\, x \, dx + \int_0^4 (x + P_D x - T_D)\, x \, dx + \int_0^5 (x + P_D x + 5 + 5P_D)(x+5)\, dx\right]$$

$$+ \frac{1}{GJ}\left[\int_0^5 T_D \cdot 0 \, dx + \int_0^4 (5 + 5P_D)(5)\, dx + \int_0^5 (T_D - 4 - 4P_D)(-4)\, dx\right]$$

Now, $P_D = 0$, $T_D = 0$

$$= \frac{1}{EI}\left[\int_0^5 x^2 dx + \int_0^4 x^2 dx + \int_0^5 (x^2 + 10x + 25)\, dx\right] +$$

$$\frac{1}{GJ}\left[0 + \int_0^4 25\, dx + \int_0^5 16\, dx\right]$$

$$= \frac{1}{EI}\left\{\left[\frac{x^3}{3}\right]_0^5 + \left[\frac{x^3}{3}\right]_0^4 + \left[\frac{x^3}{3} + 10\frac{x^2}{2} + 25x\right]_0^5\right\} + \frac{1}{GJ}\left\{\left[25x\right]_0^4 + \left[16x\right]_0^5\right\}$$

$$= \frac{1}{EI} \times (41.67 + 21.33 + 41.67 + 125 + 125) + \frac{1}{GJ}(100 + 80)$$

$$= \frac{1}{30 \times 10^3 \times 7.23} \times 354.67 + \frac{1}{12000 \times 14.46} \times 180$$

$$= 0.00164 + 0.00104$$

$$= 0.00268 \times 1728$$

$$= 4.63 \text{ in } (\downarrow)$$

(Ans).

$$\theta_D = \frac{\int_0^l M \cdot \frac{dm}{dT}}{EI} + \frac{\int_0^l T \cdot \frac{dT}{dT}}{GJ}$$

$$= \frac{1}{EI}\left[\int_0^5 (x+P_D x)\cdot 0 + \int_0^4 (x+P_D x - T_D)(-1)\,dx + \int_0^5 (x+P_D x+5+5P_D)\cdot 0\right] +$$

$$\frac{1}{GJ}\left[\int_0^5 T_D \cdot 1\,dx + \int_0^4 (5+5P_D)\cdot 0\,dx + \int_0^5 (T_D-4-4P_D)\,1\,dx\right]$$

Now, $P_D=0$, $T_D=0$,

$$= \frac{1}{EI}\left[0+\int_0^4 -x\,dx+0\right] + \frac{1}{GJ}\left[\int_0^5 0\,dx + 0 + \int_0^5 -4\,dx\right]$$

$$= \frac{1}{EI}\left[-\frac{x^2}{2}\right]_0^4 + \frac{1}{GJ}\left[-4x\right]_0^5$$

$$= \frac{1}{30\times10^3 \times 7.23}\left[-\frac{4^2}{2}\right] + \frac{1}{12000 \times 14.46}(-20)$$

$$= -0.000037 - 0.000115$$

$$= -0.000152 \times (12)^2$$

$$= \cancel{-0.0018\text{ rad}}\ -0.0219\text{ rad}$$

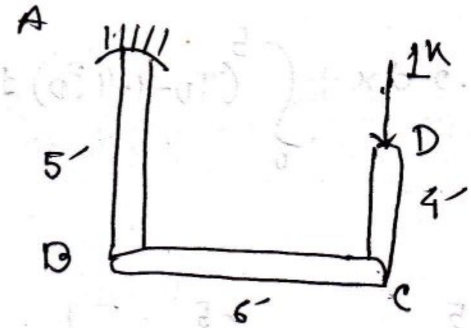
$$= \cancel{0.0018\text{ rad}}\ (\curvearrowright)$$

$$= 0.0219\text{ rad}$$

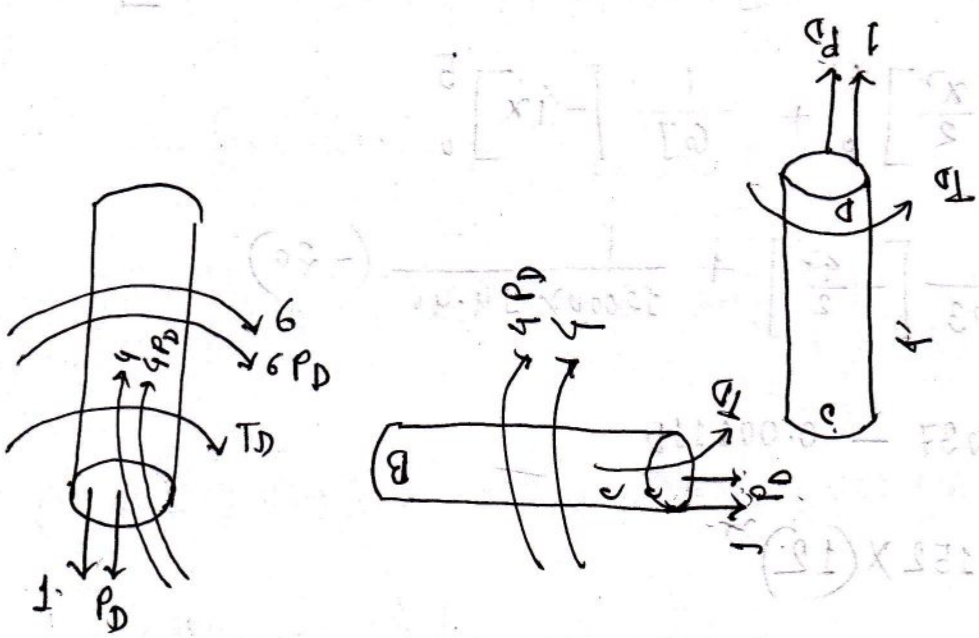
(Ans).

2011

2. The bracket have 90° angle at B and C and located on horizontal pla[ne]
Find a) vertical deflection of D, b) rotational deflection of D in the pla[ne]
normal to the axis of CD, $I = 10\ in^4$ $G = 12000\ ksi$, $E = 30000\ ksi$, $J = 14.46\ in^4$



Solution:



Portion	Origin	Limit	M	T	$\frac{\partial M}{\partial P}$	$\frac{\partial M}{\partial T}$	$\frac{\partial T}{\partial P}$	$\frac{\partial T}{\partial T}$
DC	D	0-4	$x+P_D x$	T_D	x	0	0	1
CB	C	0-6	$x+P_D x+T_D$	$-4-4P_D$	x	1	-4	0
BA	B	0-5	$x+P_D x-4-4P_D$	$-T_D-6-6P_D$	$x-4$	0	-6	-1

$$\Delta V_D = \frac{\int M \frac{\partial M}{\partial P}}{EI} + \frac{\int T \cdot \frac{\partial T}{\partial P}}{GJ}$$

$$= \frac{1}{EI}\left[\int_0^4 (x+P_D x)\, x\, dx + \int_0^6 (x+P_D x+T_D)\, x\, dx + \int_0^5 (x+P_D x-4-4P_D)(x-4)\, dx\right]$$

$$+ \frac{1}{GJ}\left[\int_0^4 T_D \cdot 0\, dx + \int_0^6 (-4-4P_D)(-4)\, dx + \int_0^5 (-T_D-6-6P_D)(-6)\, dx\right]$$

Now, $P_D = 0$, $T_D = 0$

$= - - - - - -$

$= - - - - -$

$$\Delta V_D = 0.00197'' \times 1728 = 3.40'' \ (\downarrow)$$

$$\theta_{VD} = \frac{\int M \cdot \frac{\partial M}{\partial T}}{EI} + \frac{\int T \cdot \frac{\partial T}{\partial T}}{GJ}$$

$$= \frac{1}{EI}\left[\int_0^4 (x+P_D x) \cdot 0\, dx + \int_0^6 (x+P_D x+T_D) \cdot 1\, dx + \int_0^5 (x+P_D x-4-4P_D) \cdot 0\, dx\right]$$

$$+ \frac{1}{GJ}\left[\int_0^4 T_D \cdot 1\, dx + \int_0^6 (-4-4P_D) \cdot 0\, dx + \int_0^5 (-T_D-6-6P_D) \cdot (-1)\, dx\right]$$

Now, $P_D = 0$, $T_D = 0$

$= - \ \_\_\_\_$

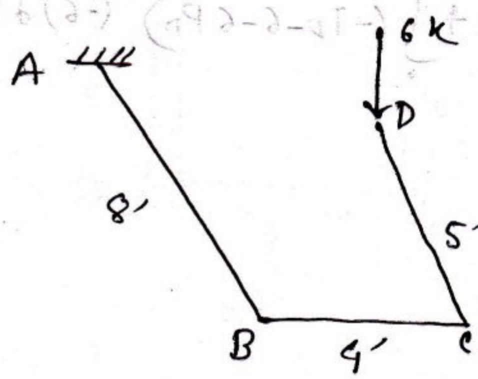
$= - \ \_\_\_\_$

$= 0.0023 \times (12)^2$ (↻)

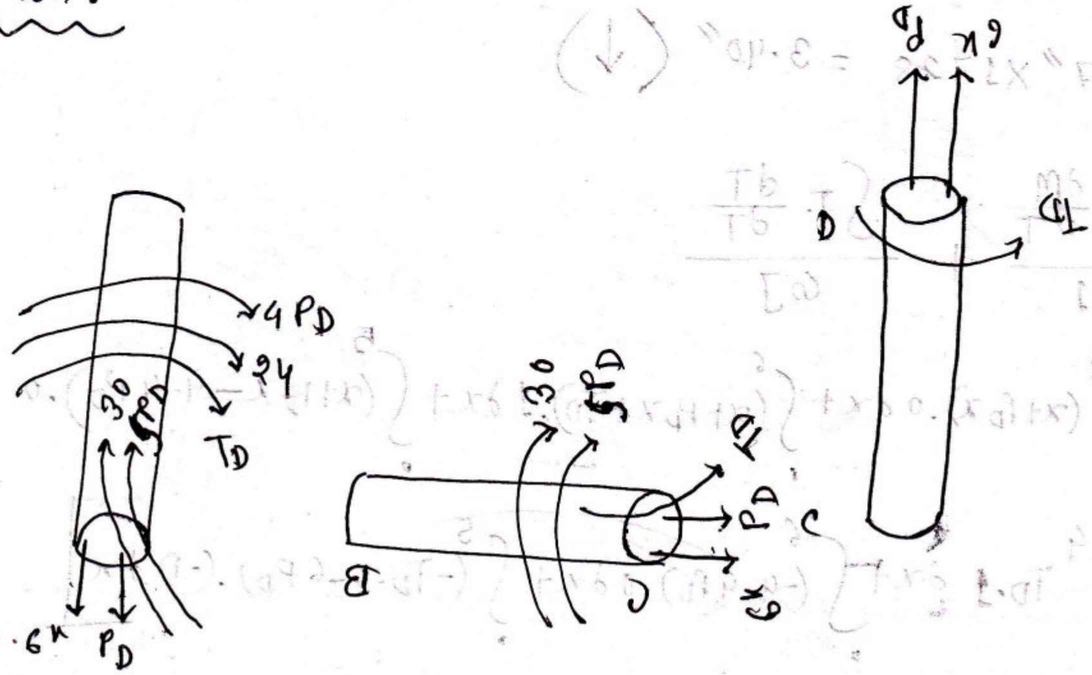
$= 0.033$ rad. (Ans).

2010

03. Given a standard pipe bracket hanging at 90° angle at B and C located in a horizontal plane. Find the vertical deflection component and rotational deflection component of D in the axis of CD. $I = 40\ in^4$, $G = 12\times10^6$ psi and $E = 30\times10^6$ psi.



Solution:



Portion	Origin	Limit	M	T	$\frac{\partial M}{\partial P}$	$\frac{\partial M}{\partial T}$	$\frac{\partial T}{\partial P}$	$\frac{\partial T}{\partial T}$
DC	D	0-5	$6x + P_D x$	T_D	x	0	0	1
CB	C	0-4	$6x + P_D x + T_D$	$-30 - 5P_D$	x	1	-5	0
BA	B	0-8	$6x + P_D x - 30 - 5P_D$	$-T_D - 24 - 4P_D$	$x-5$	0	-4	-1

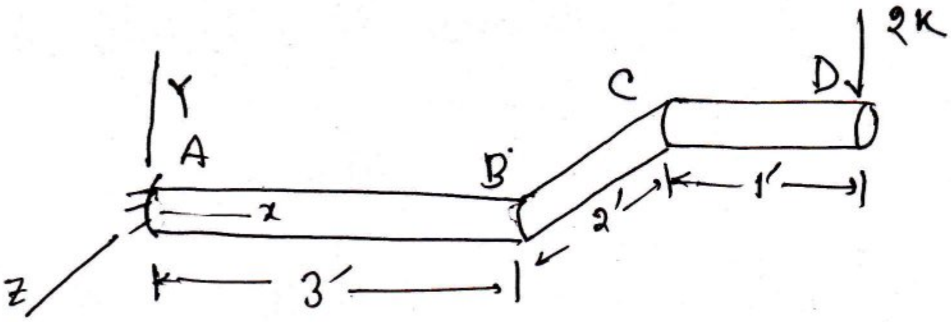
$\Delta v_D =$

$\theta_D =$ Integration ⟶ এগুলো নিজে করি

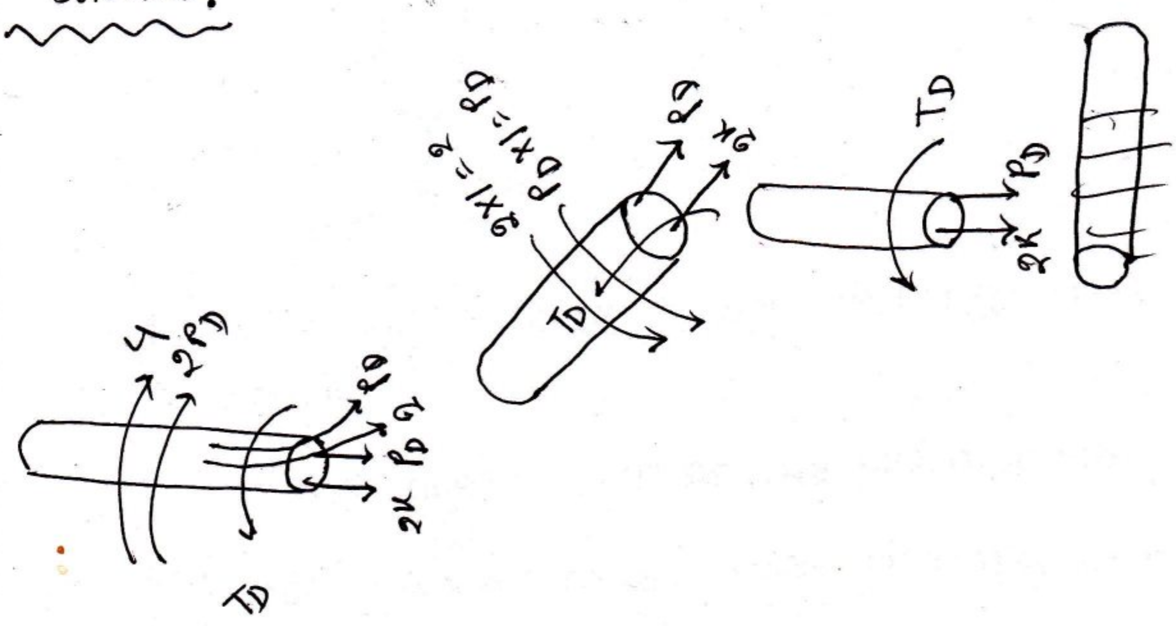
মোঃ আহসান হাবিব

2008

04.



Solution:



Portion	origin	Limit	M	T	$\frac{\partial M}{\partial P}$	$\frac{\partial M}{\partial T}$	$\frac{\partial T}{\partial P}$	$\frac{\partial T}{\partial T}$
DC	D	0-1	$2x+P_D x$	T_D	x	0	0	1
CB	C	0-2	$2x+P_D x-T_D$	$2+P_D$	x	-1	1	0
BA	B	0-3	$2x+P_D x+2+P_D$	T_D-2P_D-4	$x+1$	0	-2	1

$\Delta v_D = 0.55''$ (↓) $\theta_D = 0.011$ rad (↻) (Ans).