STRUCTURAL ANALYSIS & DESIGN

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CE 15, 1500045

"Are those who have knowledge and those who have no knowledge alike? Only the men of understanding are mindful. " (Quran, 39:9)

Special THANKS to-

My friend

SAYEM AHAMEED

CE 15, 1500119

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Tohur Sir

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Approximate Analysis of Statically Indeterminate Structure

Introduction: From a broad view point, the analysis of every structure is approximate, for it is necessary to make certain assumptions in order to carry out the analysis. For example, in computing the stresses in a pin-connected truss, it is assumed that the pins are frictionless, so that the truss members carry axial force only. It is, of cource, impossible to build a pin connection that is frictionless and as a result the stress analysis of a pin-connected truss is approximate. It may therefore be said that there is no such thing as an "exact" analysis.

Importance of Approximate methods in Analysing statically Indeterminate structures: The analysis of statically determinate structure does not depend on the elastic properties of its member.

In case of analysis of statically indeterminate structure stress analysis depends on the elastic properties of members. These elastic properties include modulus of elasticity, cross-sectional area, cross sectional moment of inertia and length of the member.

Approximate analysis of statically indeterminate structures are therefore important in preliminary design stages.

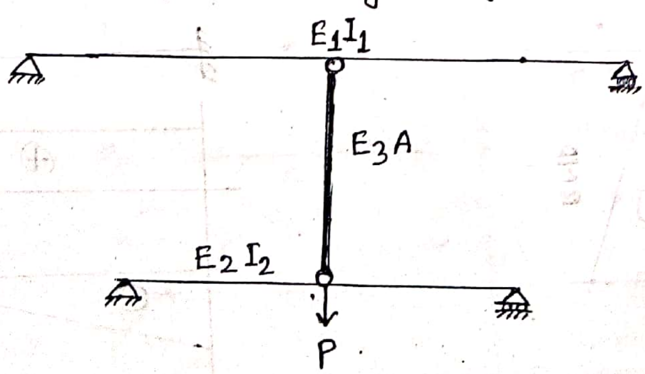

Fig. Effect of elastic properties on stress analysis.

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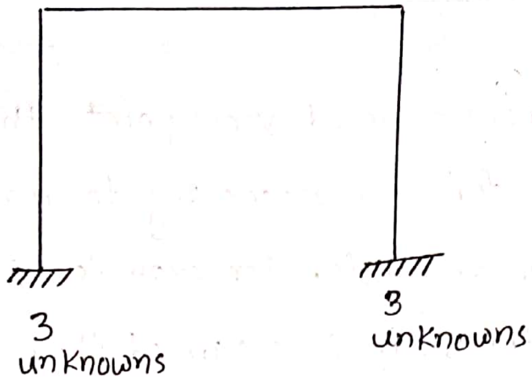
Number of Assumption required:

Total no. of unknown

= (3+3) = 6

Equation of statics = 3

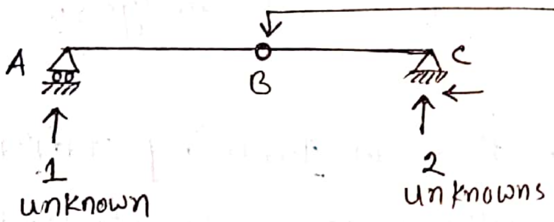


Hence, statically indeterminate structure to the (6−3) = 3rd degree. It will then be necessary to make 3 independent assumptions, each of which supplies an independent equation.

Unstable structure: when Equation of statics > Total no. of unknown

Example:

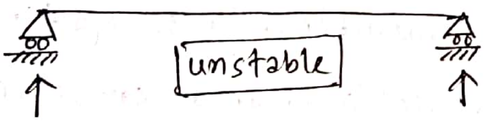
internal link means



* Moment at that point is zero
information

No. of equation = 3 + (1) = 4

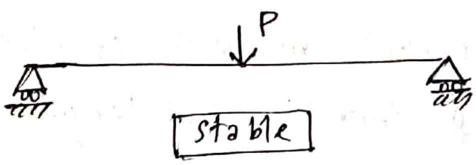
unknown = (2+1) = 3



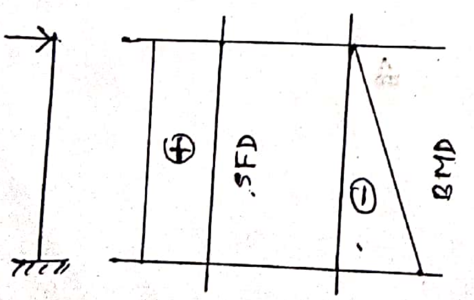
unknown = 2

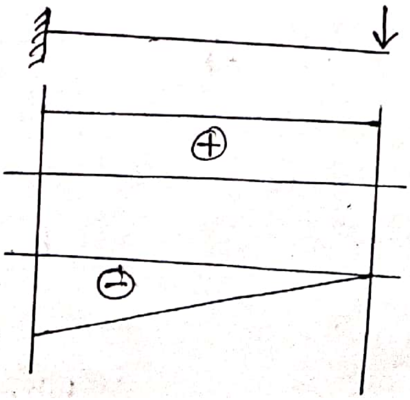
Equation = 3

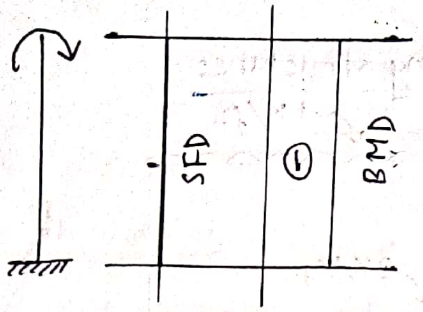
But, for a particular load, This structure is stable

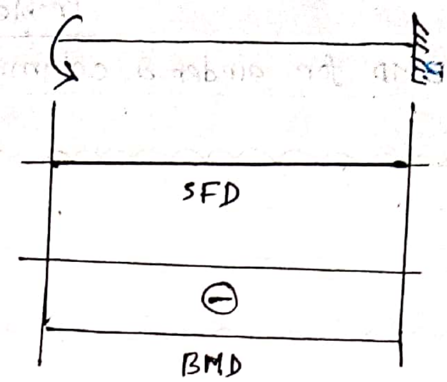


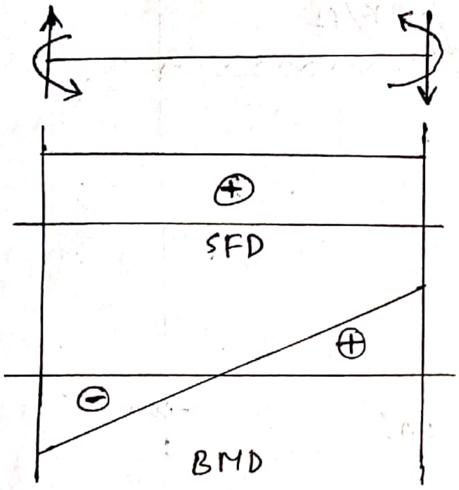
SFD & BMD:

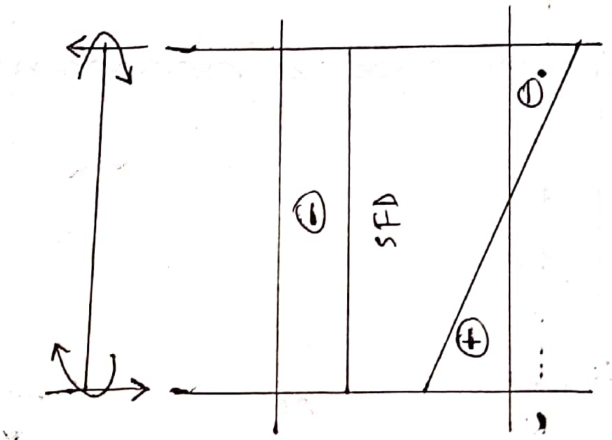








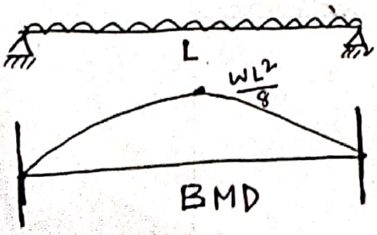


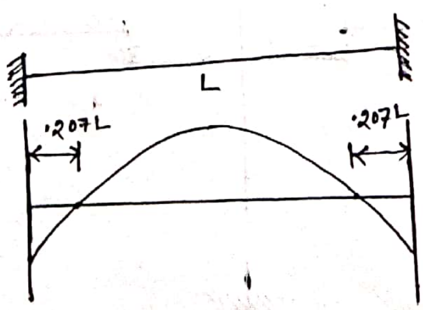


Vertical Load Analysis: (Assumptions) 15, 13, 11

The following three assumptions will be made for each girder, in analysing a building bent acted upon by vertical loads:

1. The axial force in the girder is zero.
2. A point of inflection occurs at the one-tenth point measured along the span from left support.
3. A point of inflection occurs at the one-tenth point measured along the span from right support.





* Actually civil engineer can not provide perfectly hinged and perfectly fixed support.

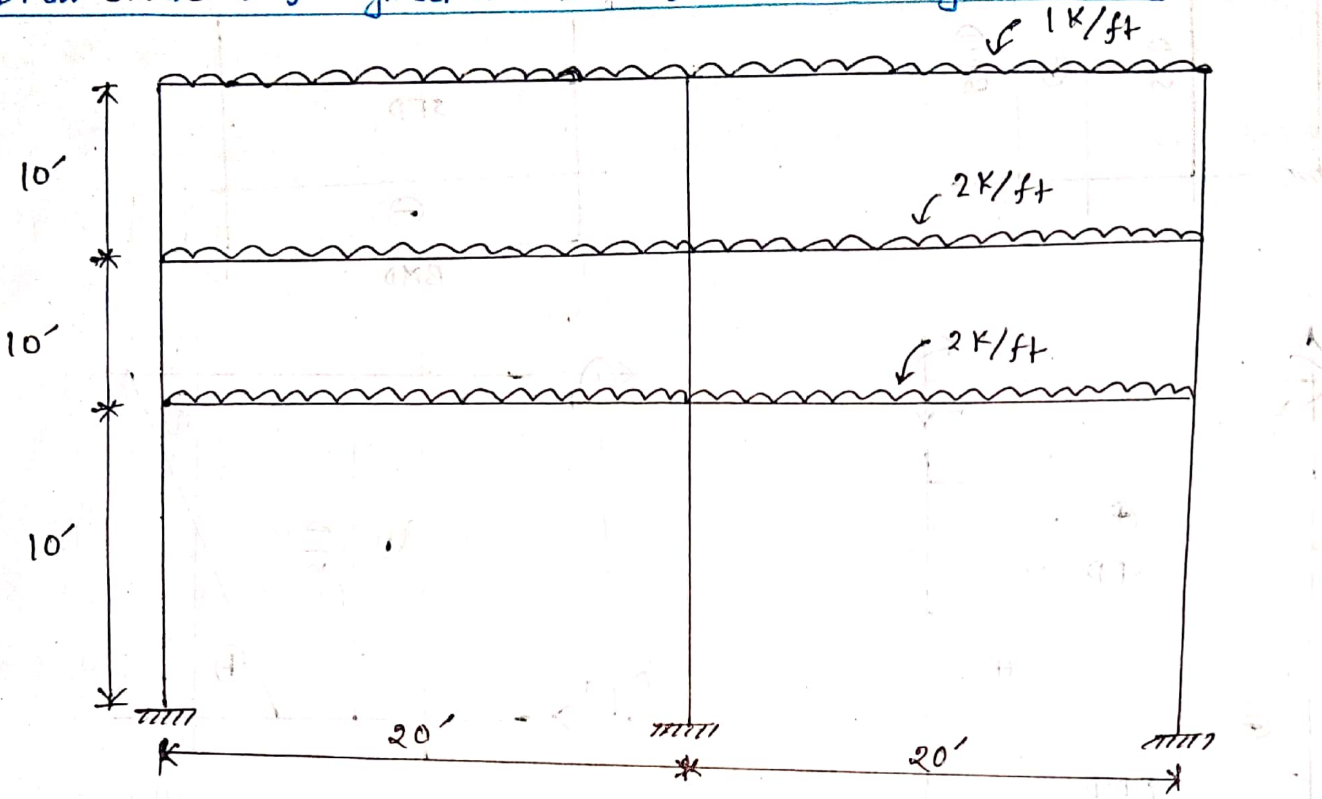
Hence,

$$\text{point of inflection} = \frac{(0 + 0.207L)}{2}$$

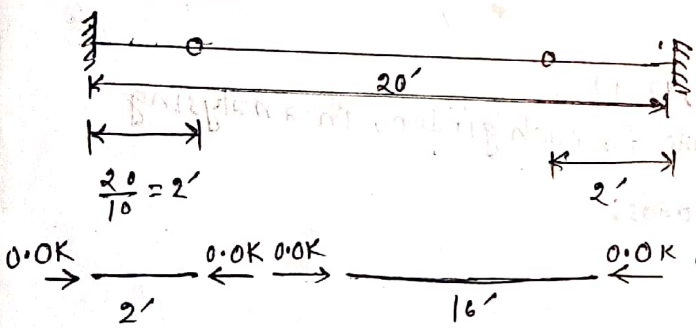
$$= 0.1035 \simeq \frac{1}{10}$$

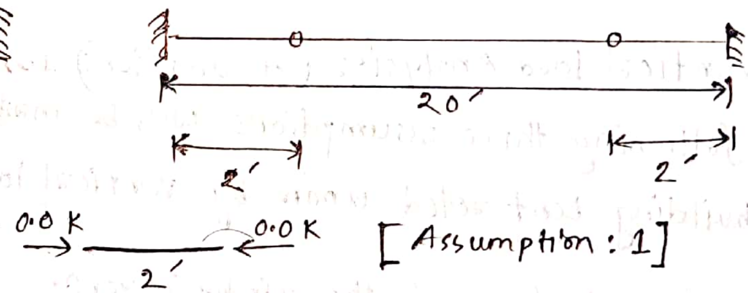
Problem: 01

Draw SFD&BMD for girder & column for the following structure:

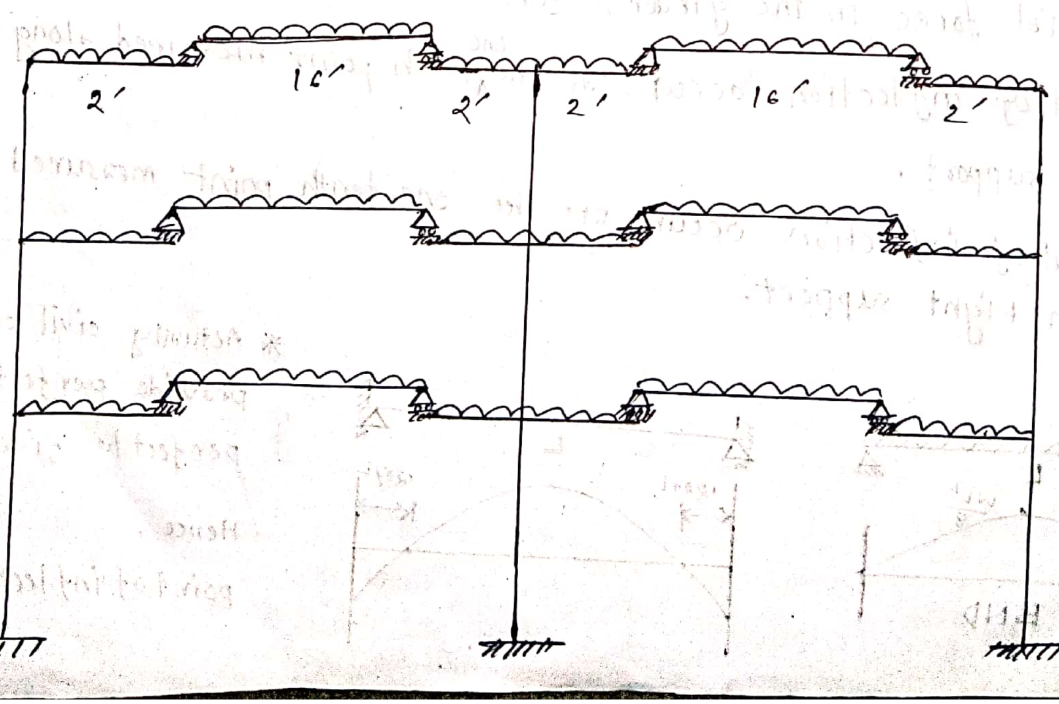


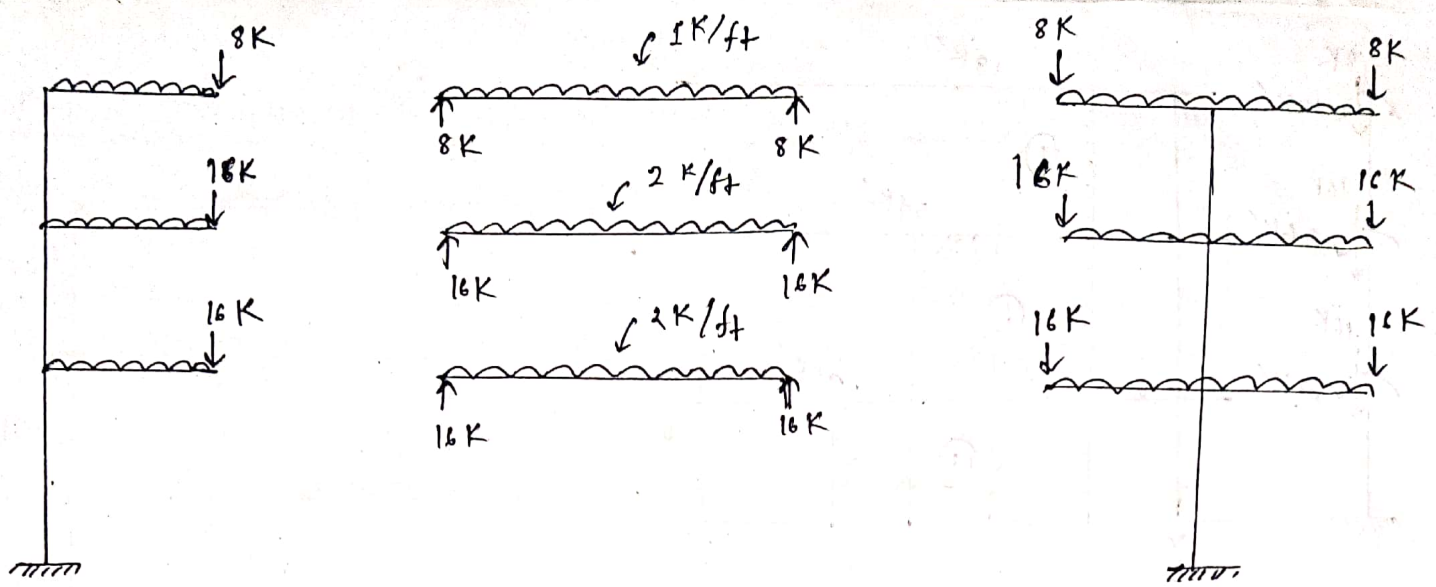
Left girders: **Right girders:**





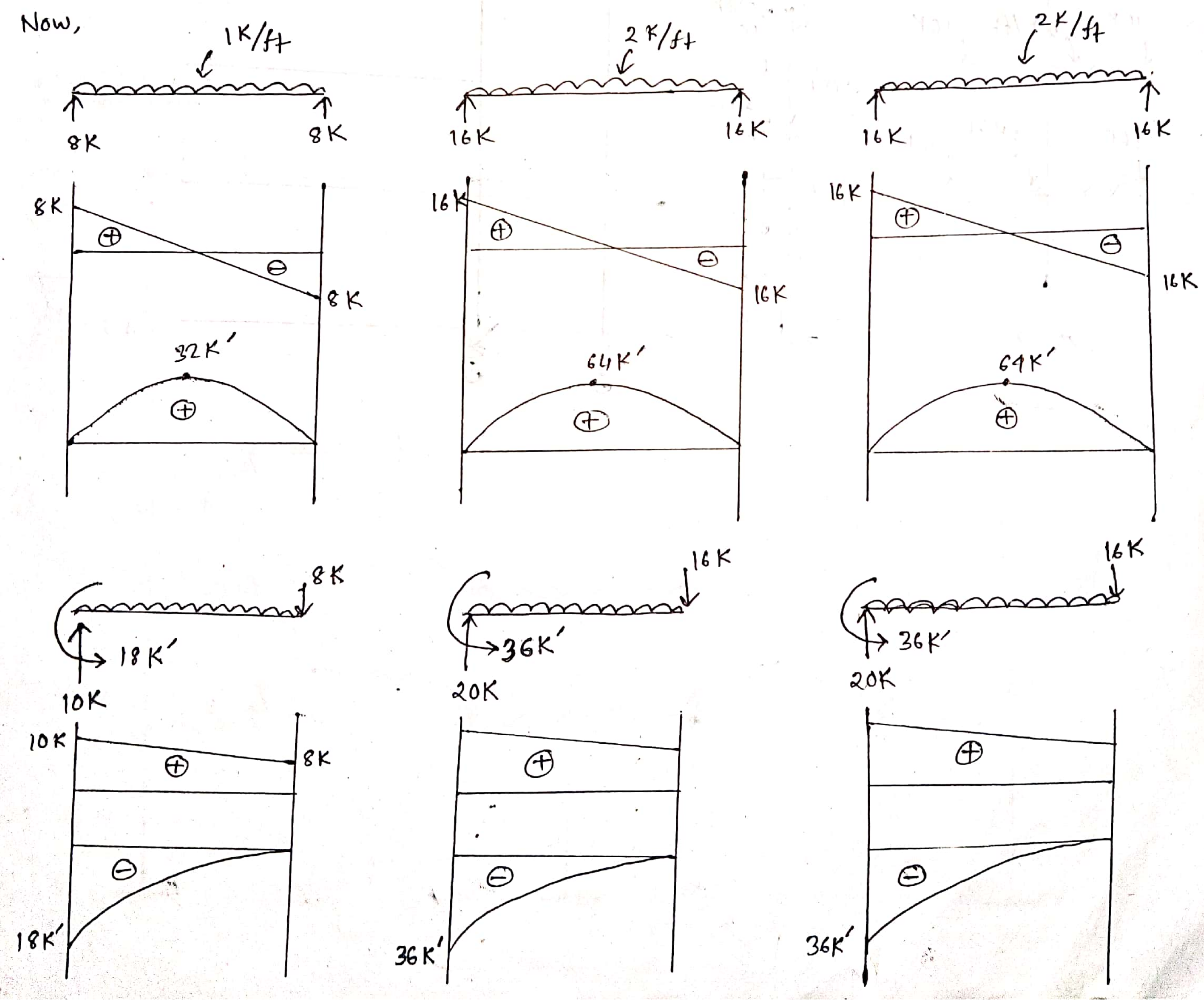
[Assumption : 1]

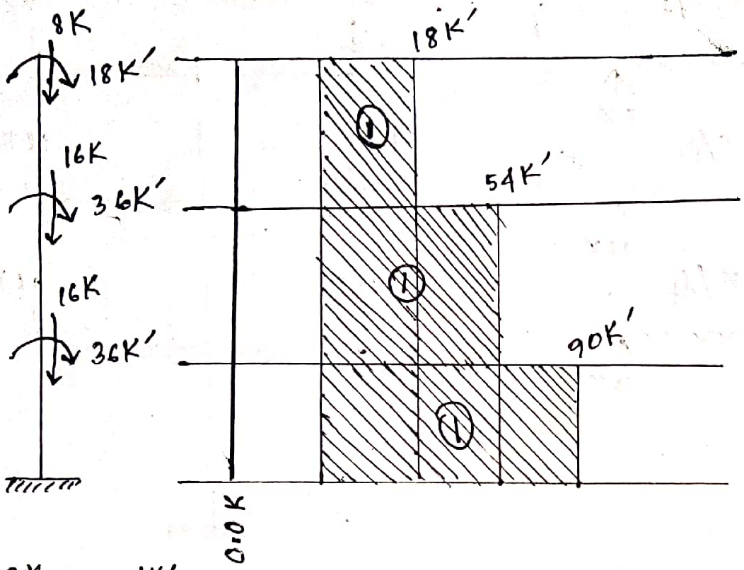


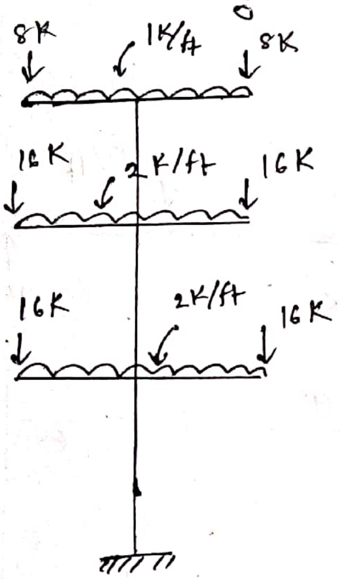


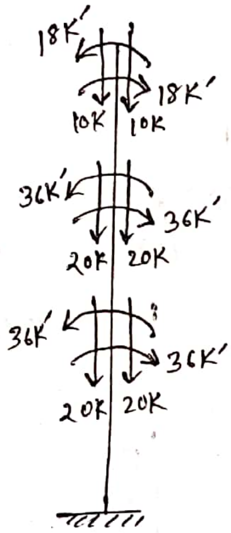
[Right portion is same as Left Portion]

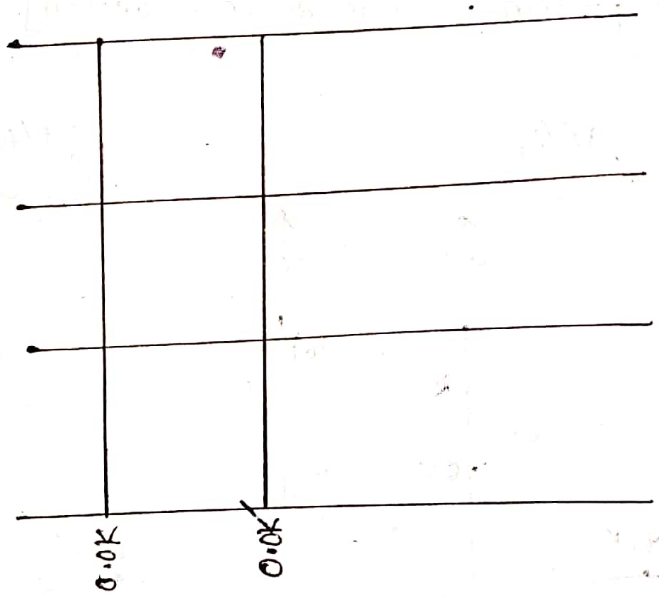
Now,




8K
18K'
16K
36K'
16K
36K'
18K'
54K'
90K'
0.0K

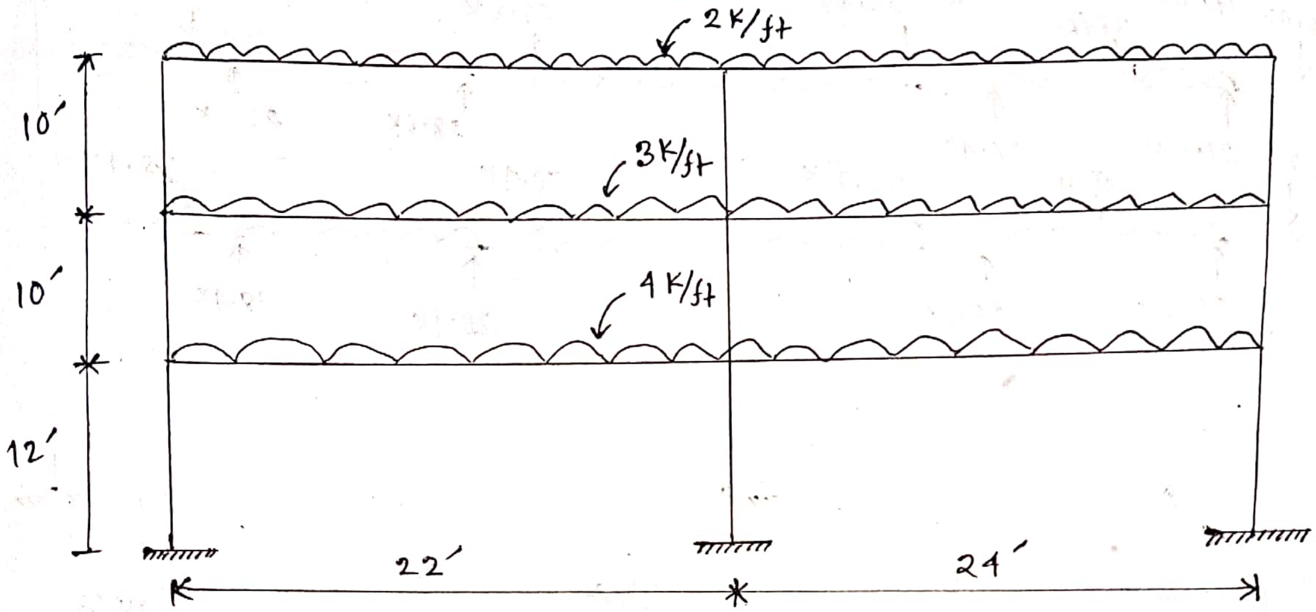

8K
1K/ft
8K
16K
2K/ft
16K
16K
2K/ft
16K


18K'
18K'
10K
10K
36K'
36K'
20K
20K
36K'
36K'
20K
20K

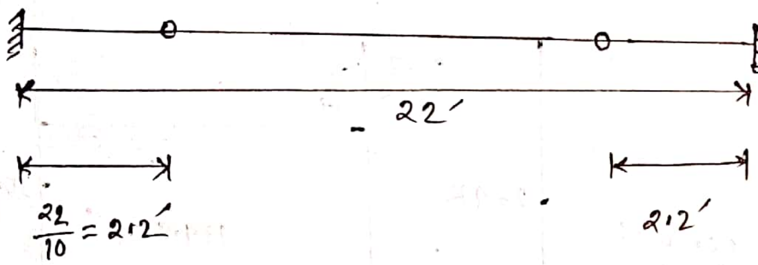

0.0K
0.0K

Assignment : 01

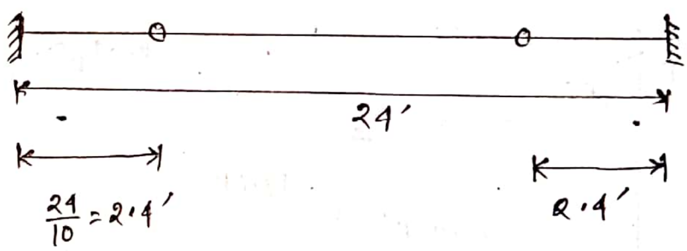
Draw SFD & BMD for girder & column for the following structure:

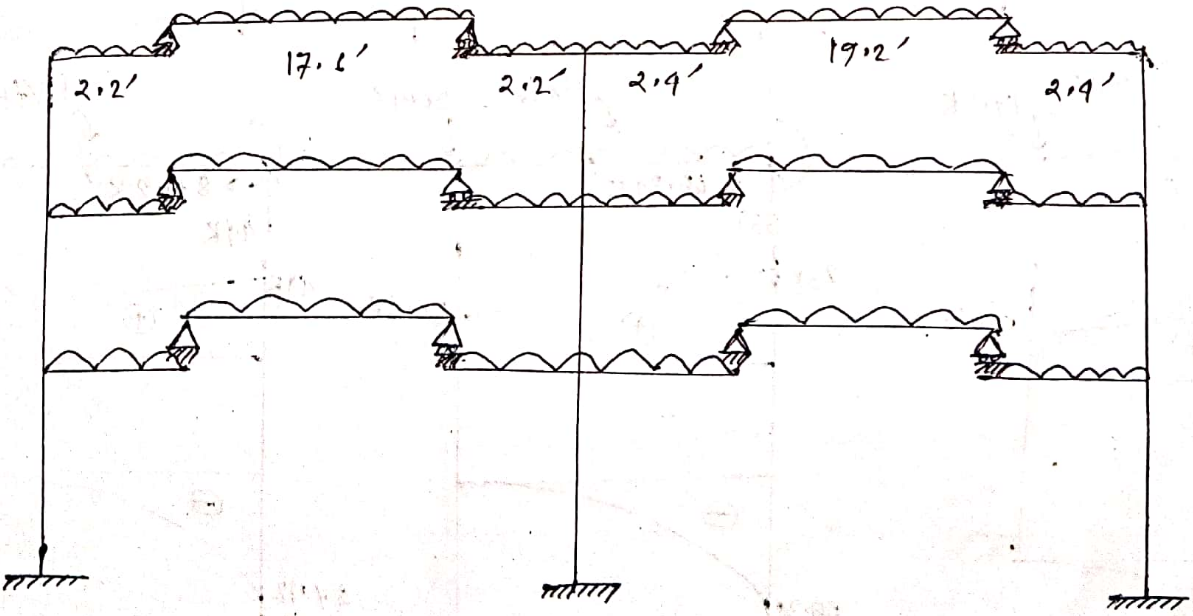


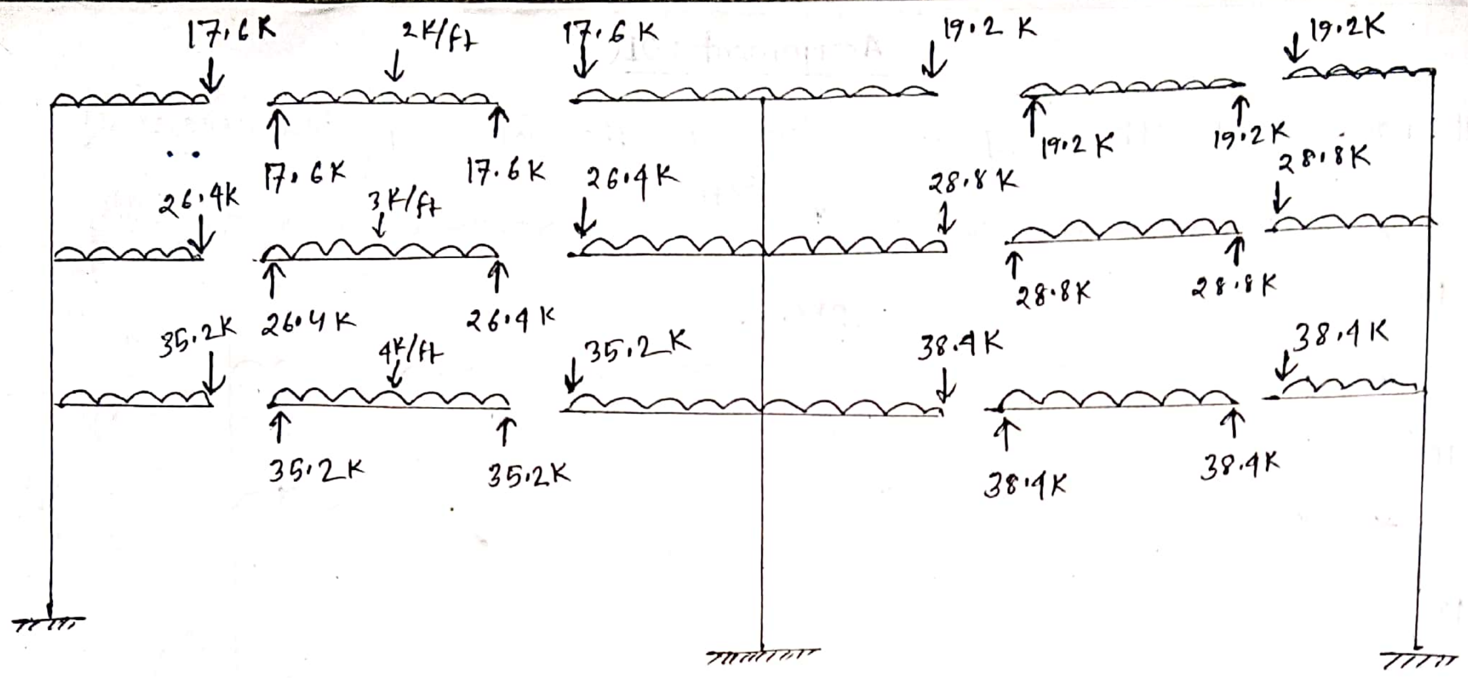
Left girders:



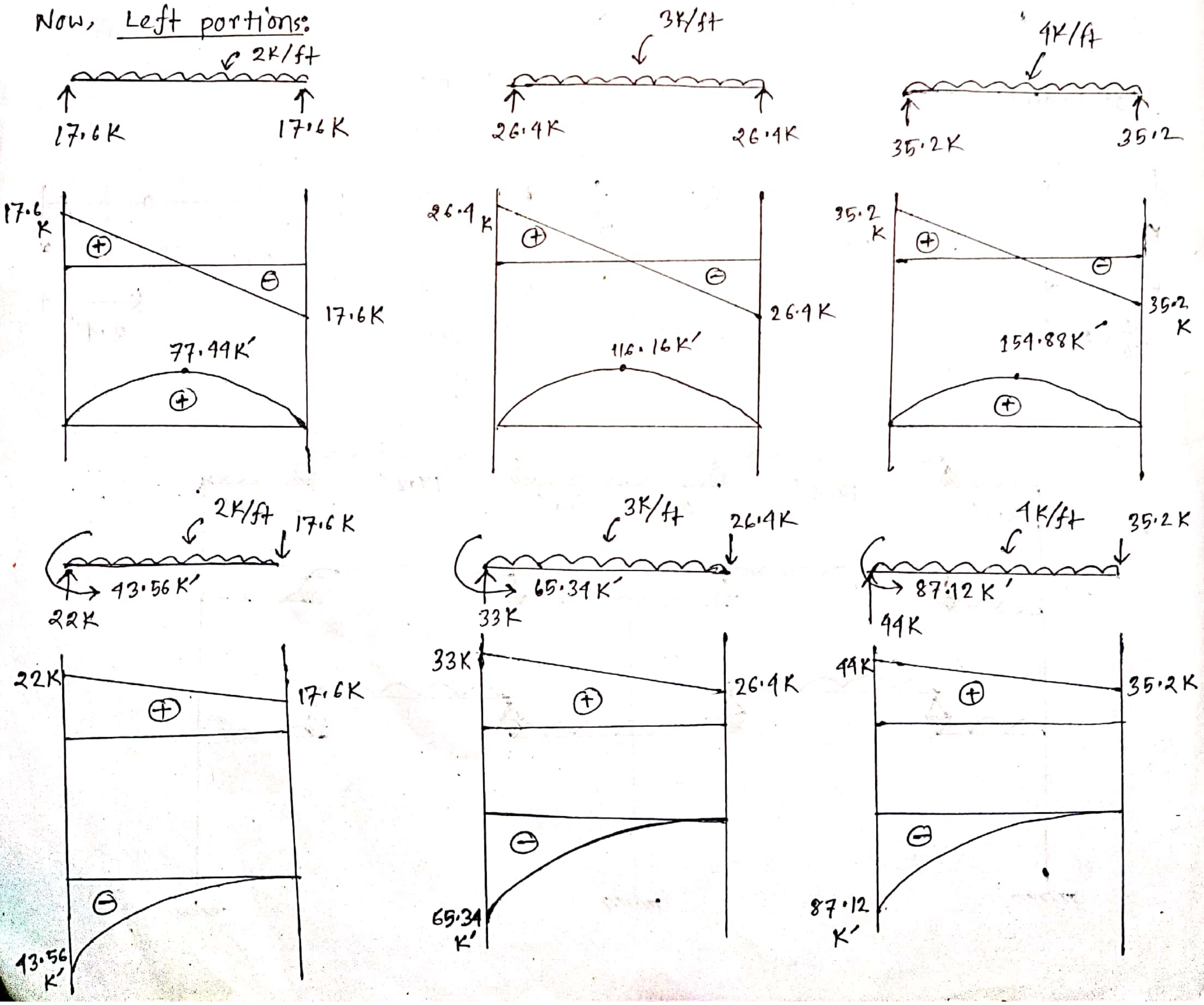
Right girders:



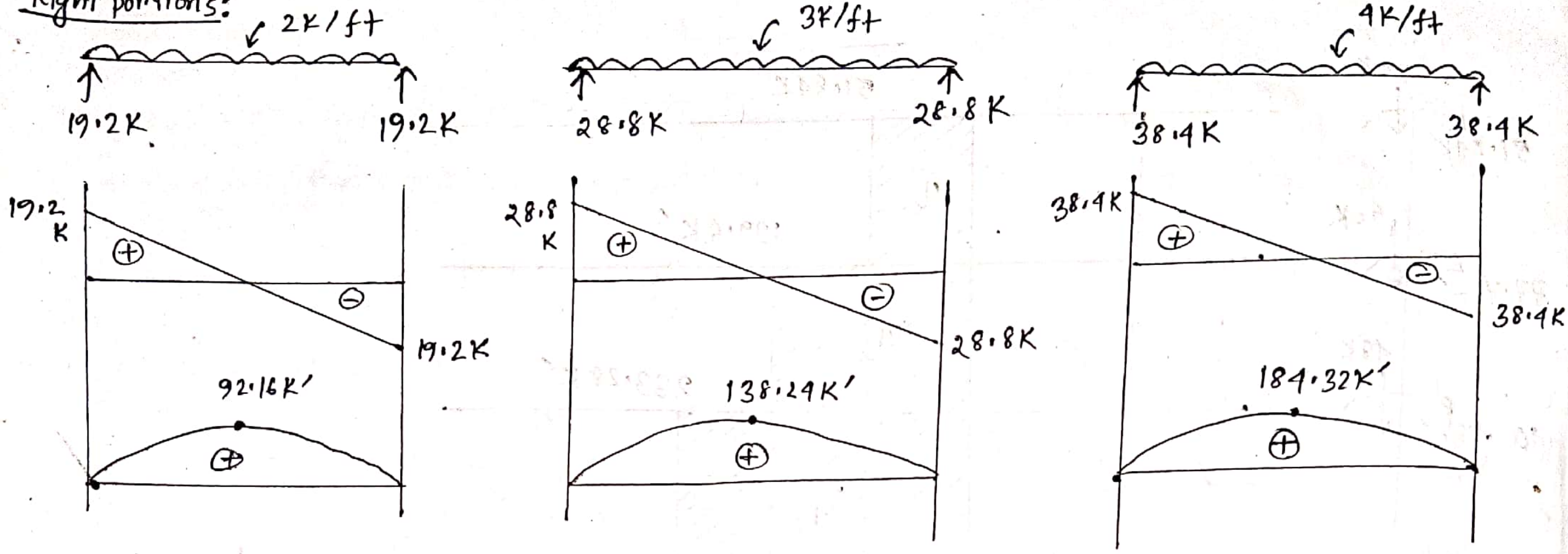


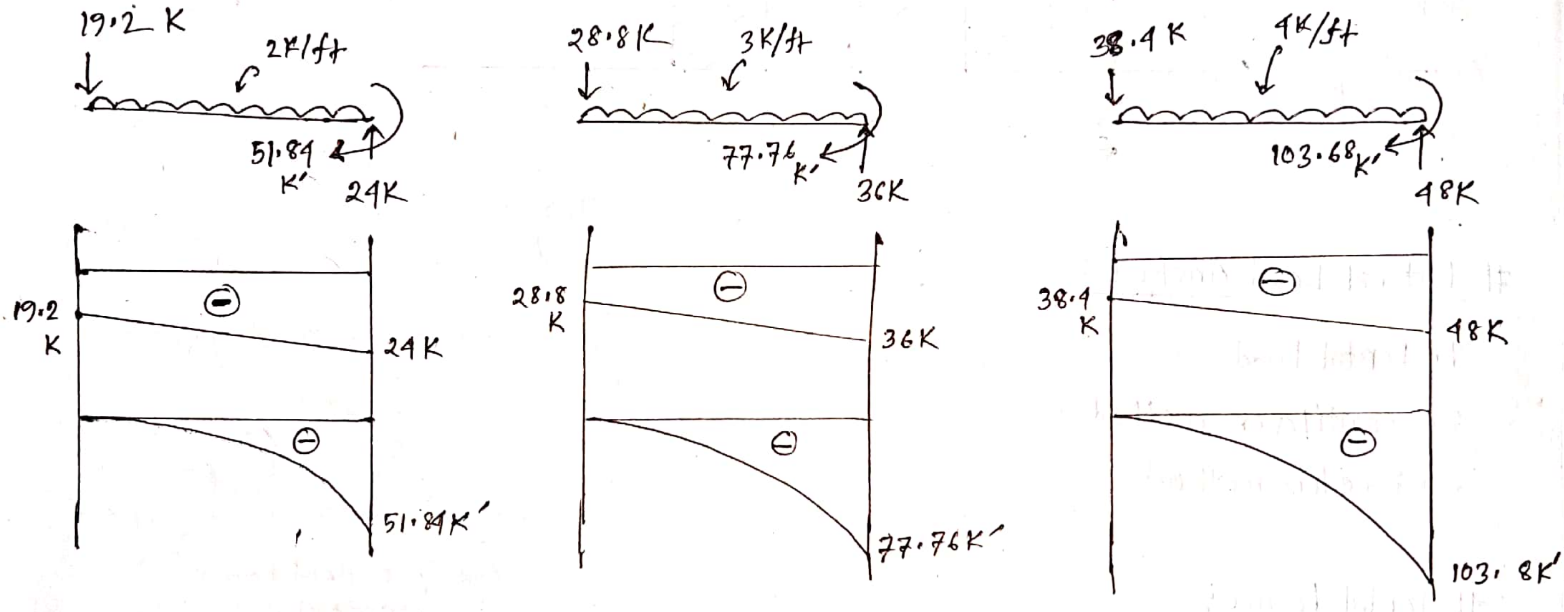


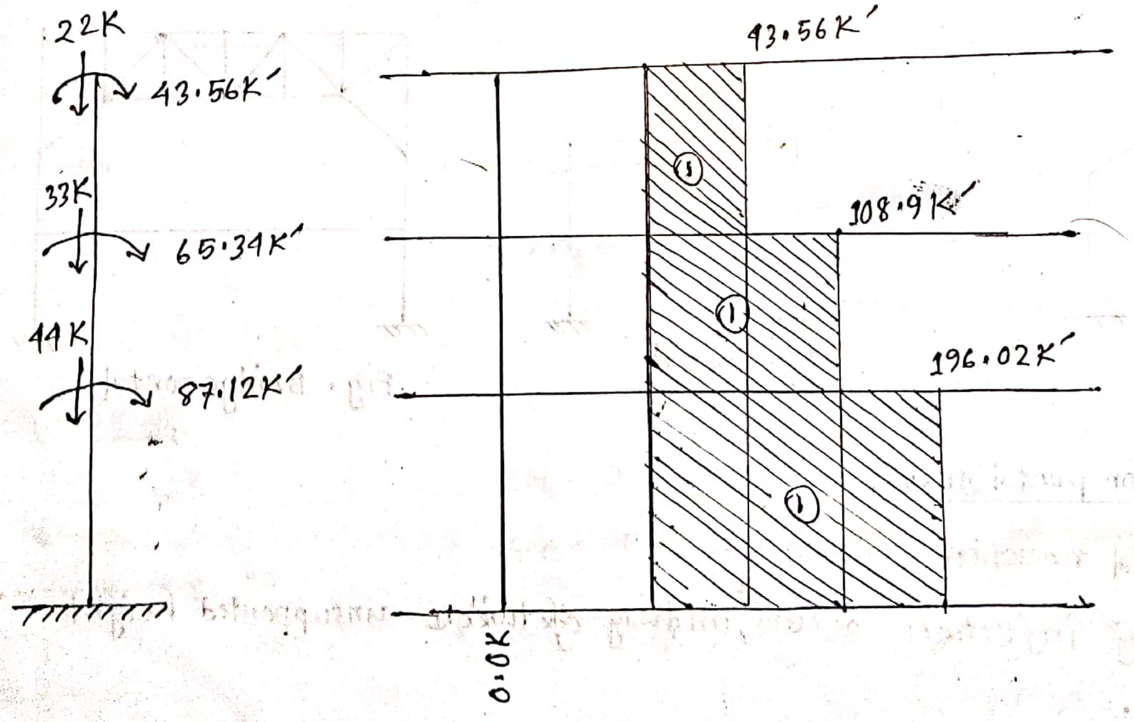
Now, Left portions:

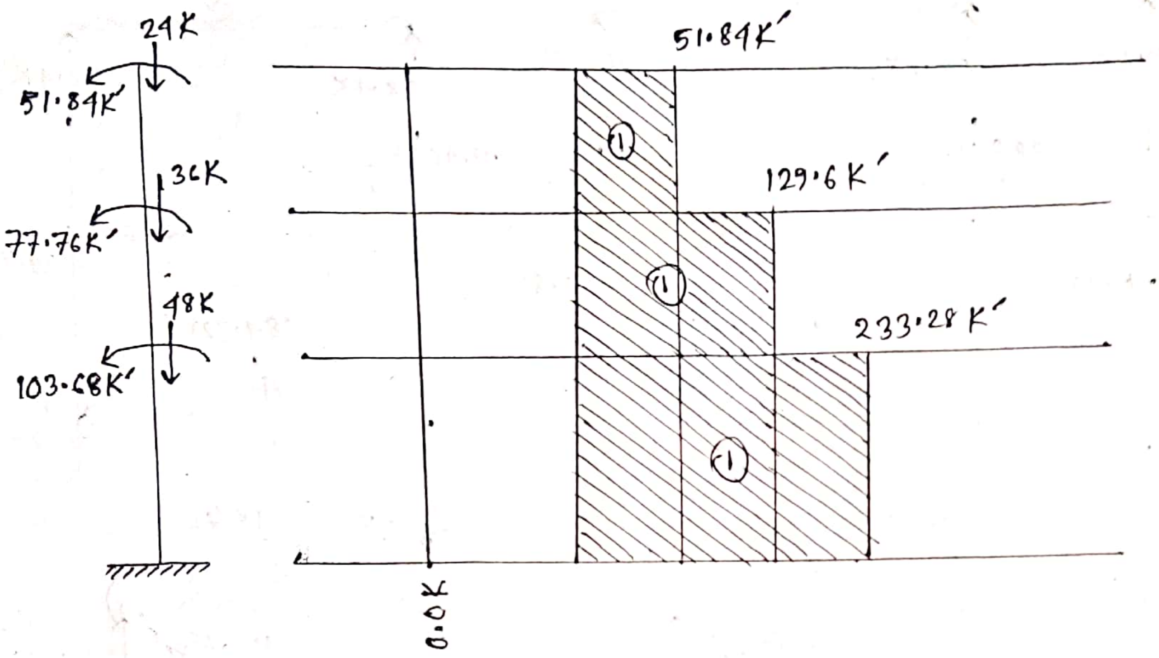


Right portions:









Lateral Load Analysis:

1. Portal Load
2. cantilever method
3. Factor method

Portal Frame:

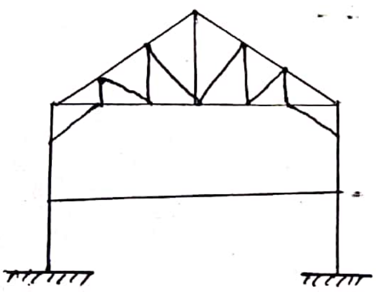


Fig. Mill Bents

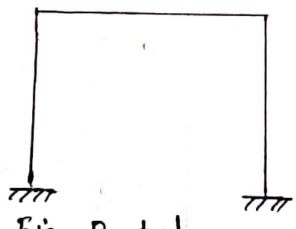


Fig. Portal

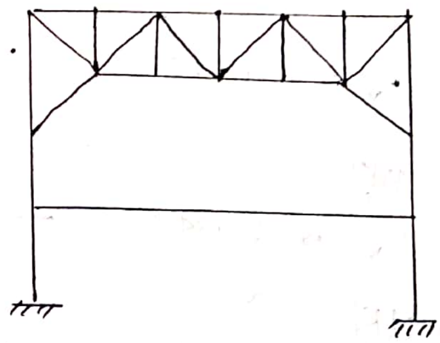


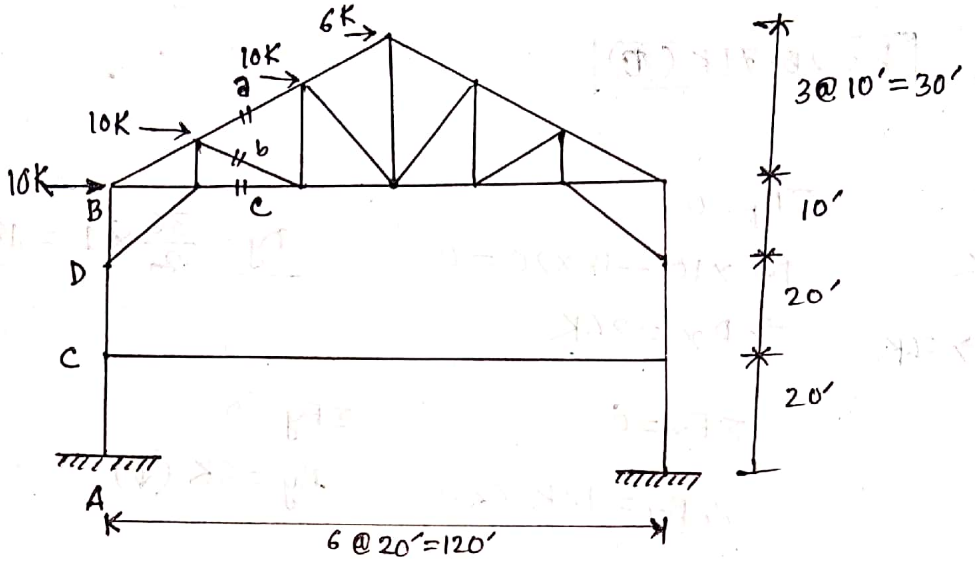
Fig. Bridge portal

Assumptions for portal frame:

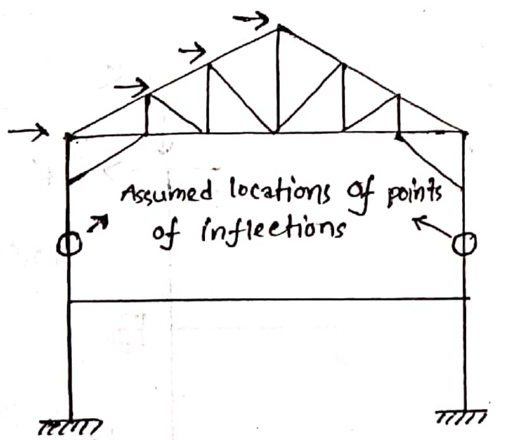
1. The horizontal reactions are equal.
2. A point of inflection occurs at midway of the unsupported height each column.

Problem: 02

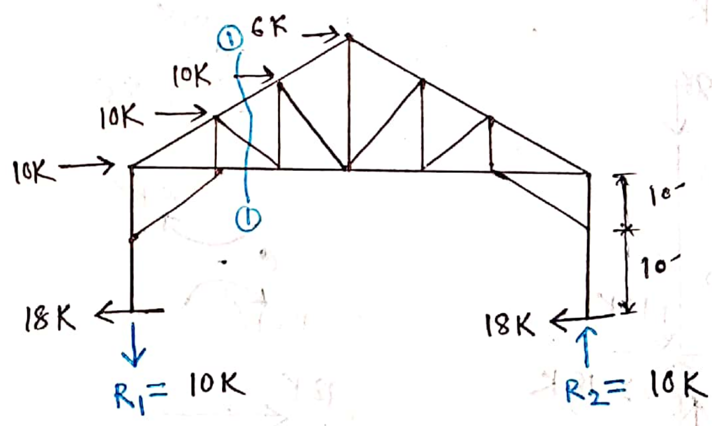
Draw SFD and BMD of girder and column for the following structure. also determine the stress in member a, b and c.



According to second assumption,



According to first assumption,

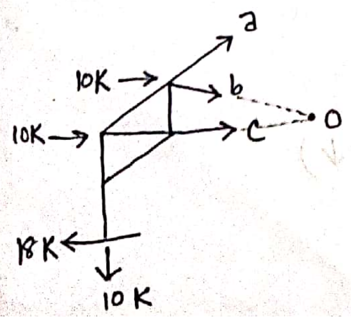


$\Sigma M_1 = 0$

$10\times20 + 10\times30 + 10\times40 + 6\times50 - 120\times R_2 = 0$

$\therefore R_2 = \frac{1200}{120} = 10K (\uparrow) \quad \therefore R_1 = 10K (\downarrow)$

Section ①-①:



$\Sigma M_o = 0$

$a_x \times 10 + a_y \times 20 + 10\times10 + 18\times20 - 10\times40 = 0$

$\Rightarrow \frac{2a}{\sqrt{5}}\times10 + \frac{a}{\sqrt{5}}\times20 = -60 \quad \Rightarrow a = -60\times\frac{\sqrt{5}}{20}\times\frac{1}{2} = -3.354K$

$\therefore a = 3.354 K (c)$

$\Sigma F_y = 0$

$a_y - b_y - 10 = 0$

$\Rightarrow -\frac{3.354}{\sqrt{5}} \times 1 - b \times \frac{1}{\sqrt{5}} - 10 = 0$

$\Rightarrow b = -11.5 \times \sqrt{5} = -25.71\ K$

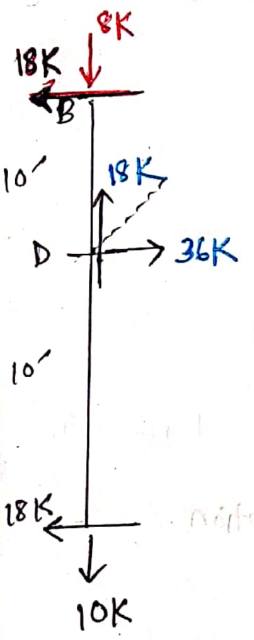
$\boxed{b = 25.71 K\ (C)}$

$\Sigma F_x = 0$

$10 + 10 + c + a_x + b_x - 18 = 0$

$\Rightarrow c = -2 - 3.354 \times \frac{2}{\sqrt{5}} + 25.71 \times \frac{2}{\sqrt{5}}$

$\Rightarrow \boxed{c = 24\ K\ \ (T)}$



$\Sigma M_B = 0$

$D_x \times 10 - 18 \times 20 = 0$

$\Rightarrow D_x = 36K$

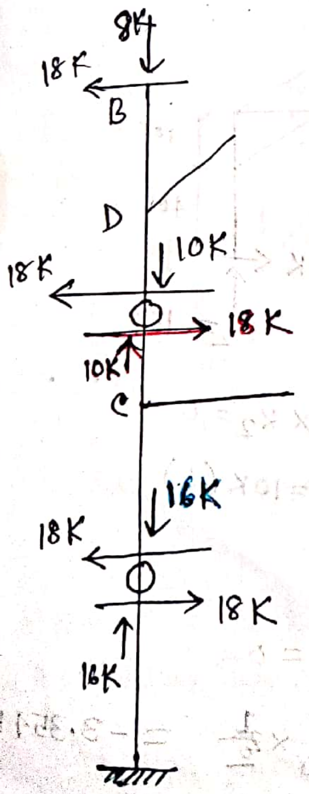
$D_y = \frac{36}{2} \times 1 = 18\ K$

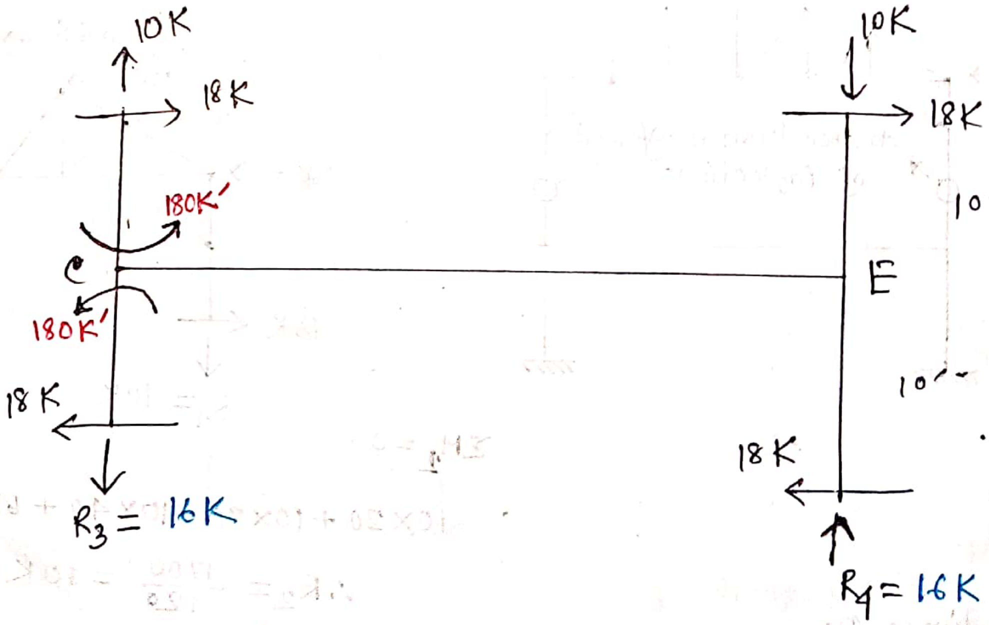
$\Sigma F_x = 0$

$\therefore B_x = 18K\ (\leftarrow)$

$\Sigma F_y = 0$

$B_y = 8K\ (\downarrow)$

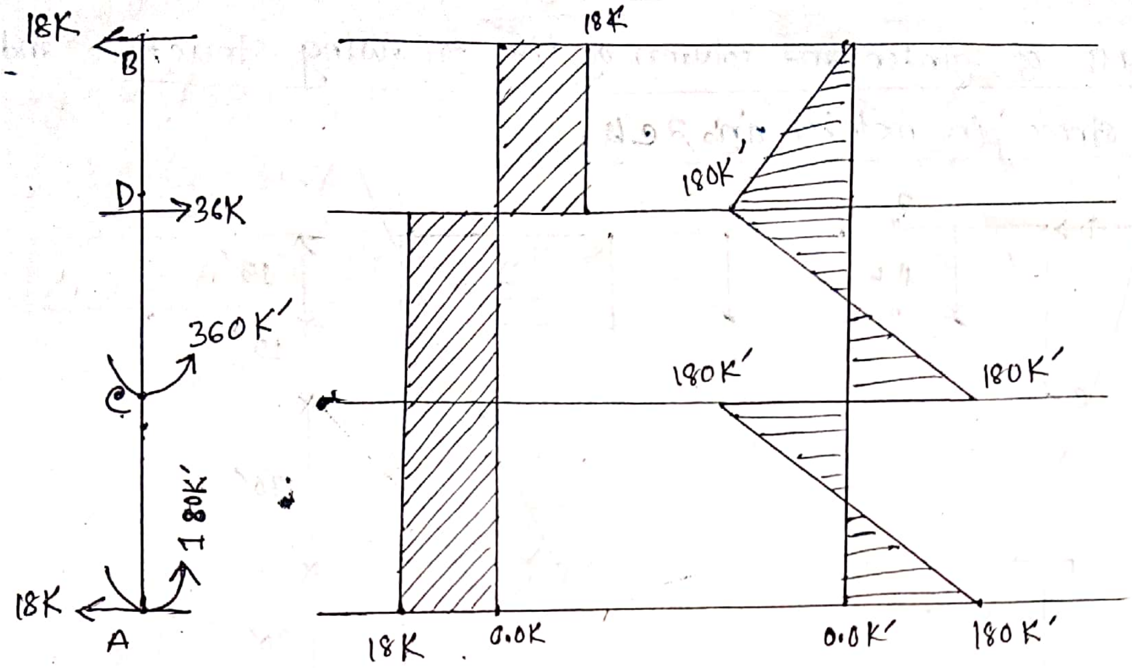




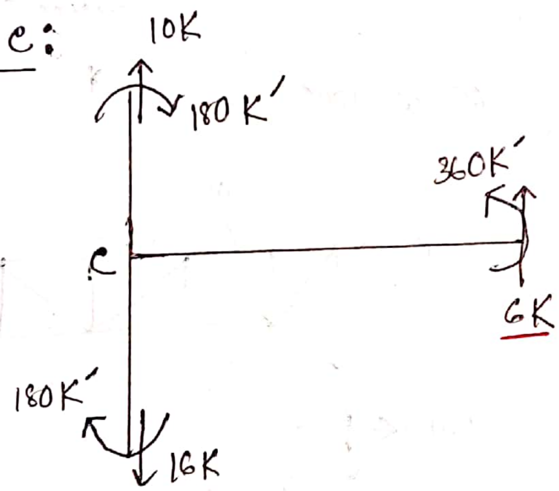
$\Sigma M_3 = 0$

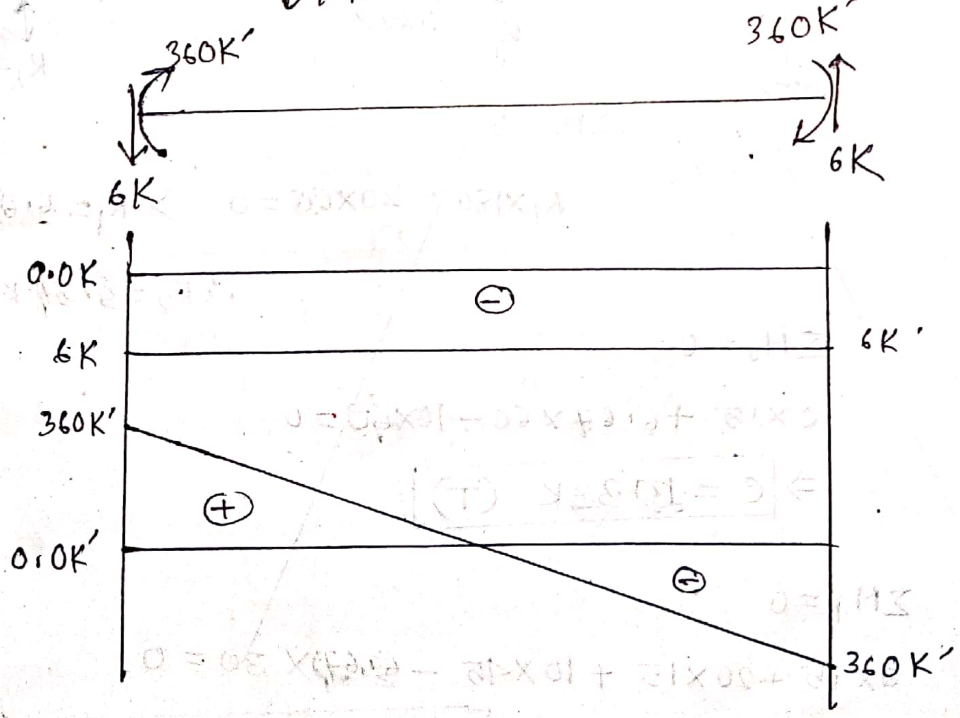
$2 \times 18 \times 20 + 10 \times 120 - R_4 \times 120 = 0$

$\therefore R_4 = 16\ K\ (\uparrow)$ $\quad \therefore R_3 = 16K\ (\downarrow)$



considering point C:





Portal Method:

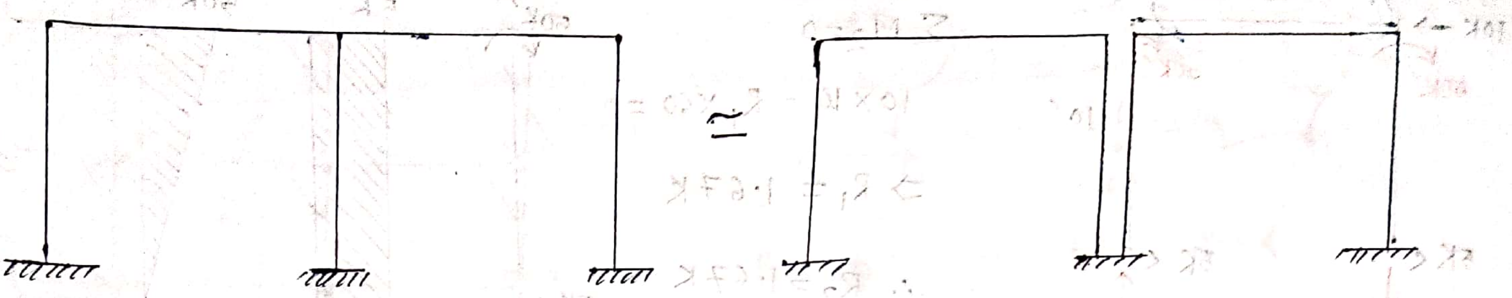
Assumption for portal method: 10

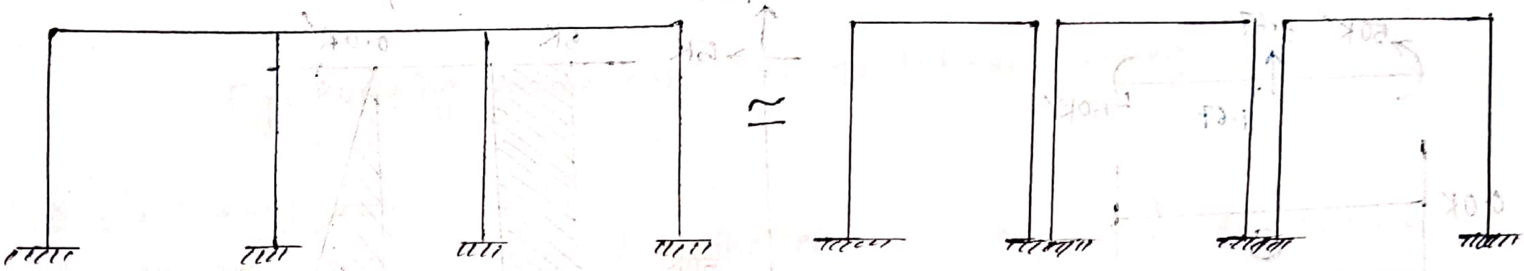
1. There is a point of inflection at centre of each girder.

2. There is a point of inflection at centre of each column.

3. The total horizontal shear on each story is divided between the columns of that story so that each interior column carries twice as much shear as each exterior column.

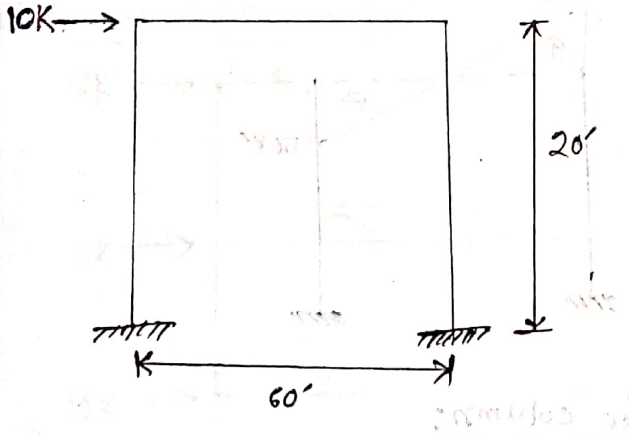
(Assumption : 3)





Problem:03

Analyze the rigid frame shown in figure below using portal method. Draw SFD and BMD for all member :



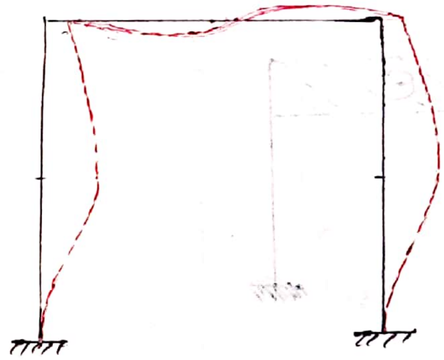
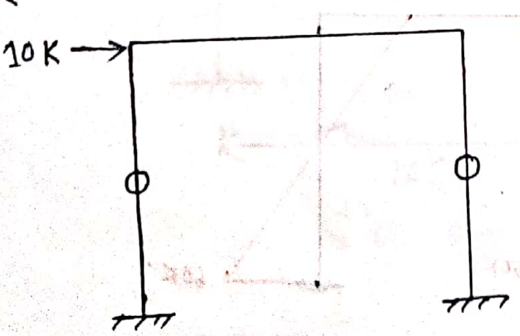
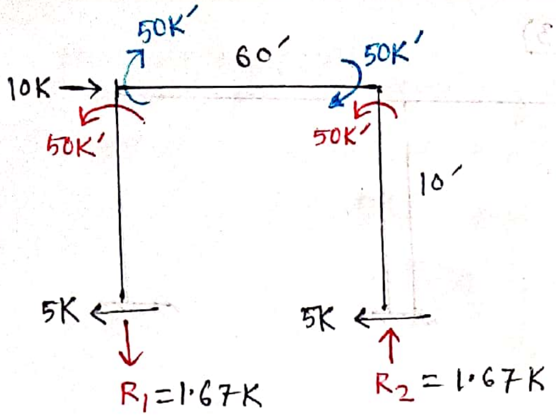


Fig. deformed shape of portal frame under the action of the applied load.

(Second assumption)



Scanned by CamScanner

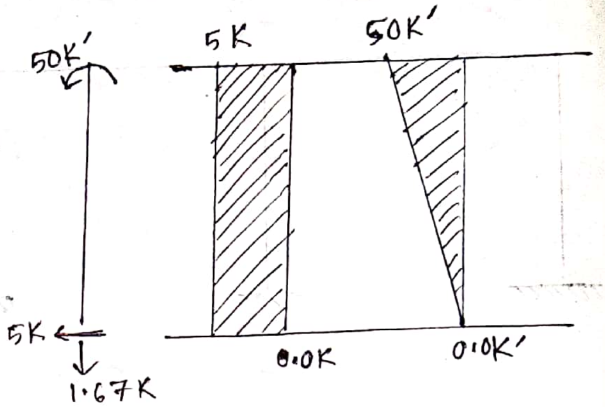


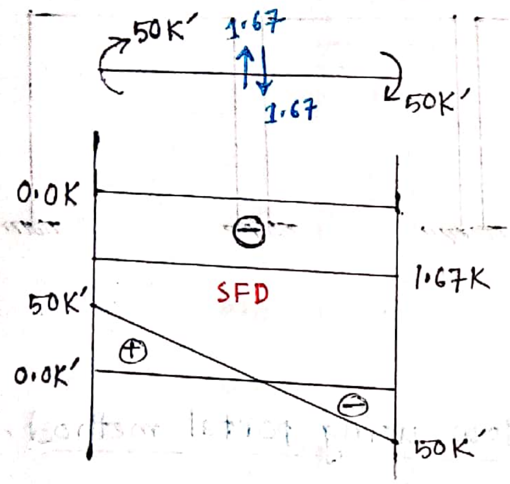
$\Sigma M_2 = 0$

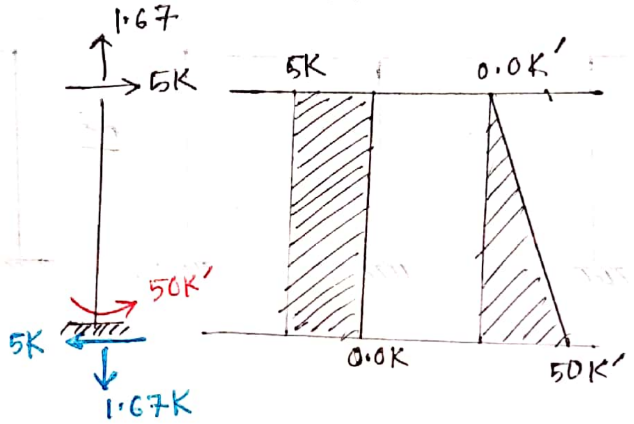
$10 \times 10 - R_1 \times 60 = 0$

$\Rightarrow R_1 = 1.67K$

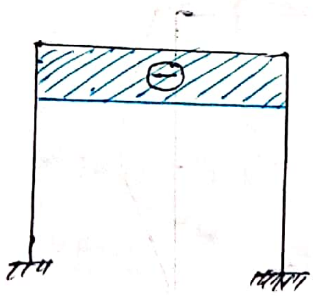
$\therefore R_2 = 1.67K$



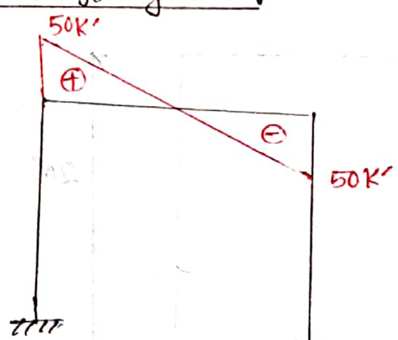




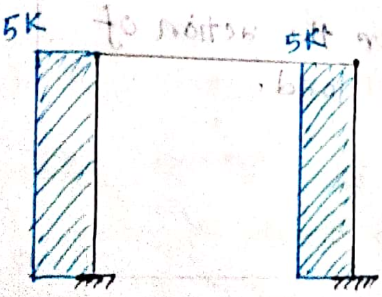
SFD for girder:



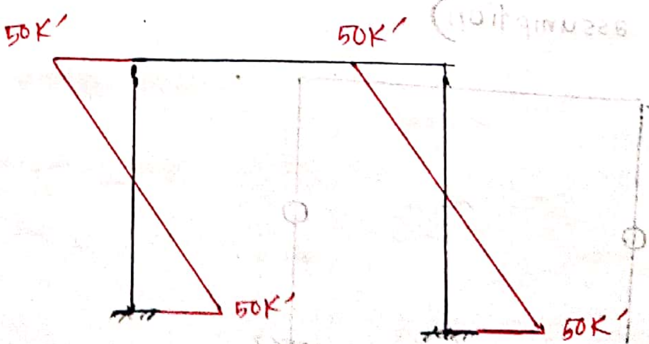
BMD for girder:



SFD for column:



BMD for column:



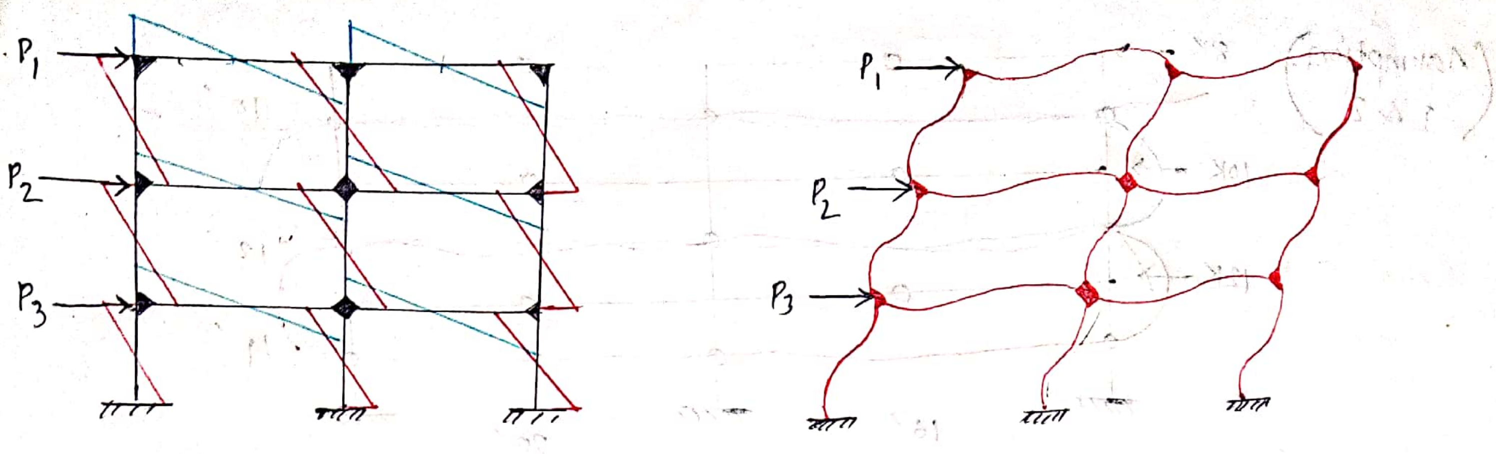


Fig. Building frame subjected to Lateral loading.

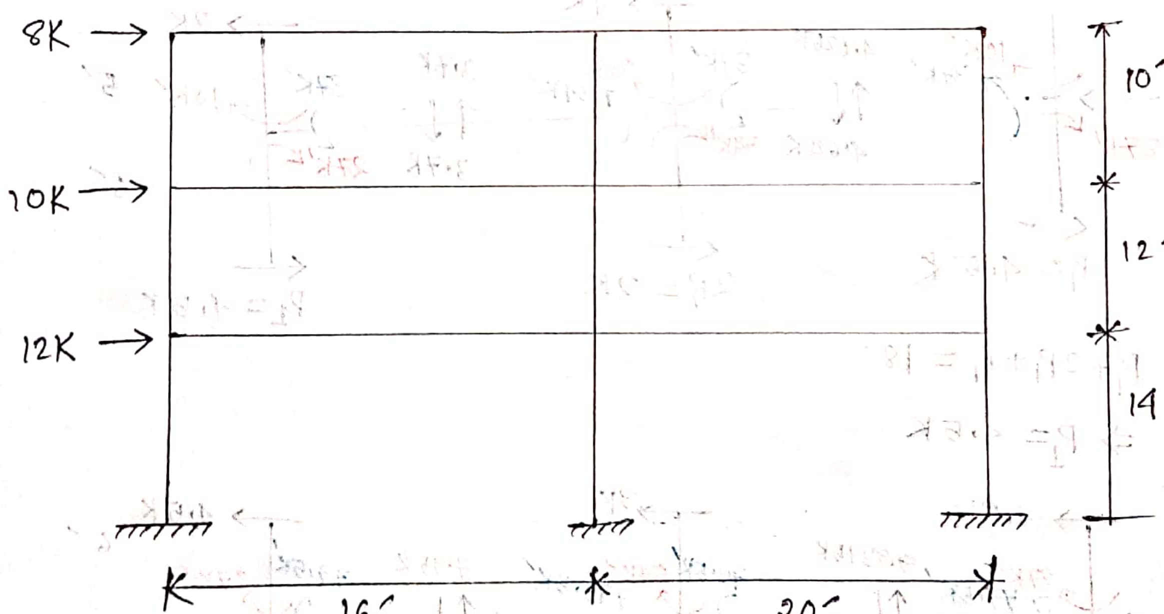
Rigid Frame:

Girders are rigidly connected to the columns so that all the members can carry bending moment, shear and axial force. Such a frame can carry bending moment, shear and axial force is called a rigid frame. It is also referred to as 'Building Bent'.

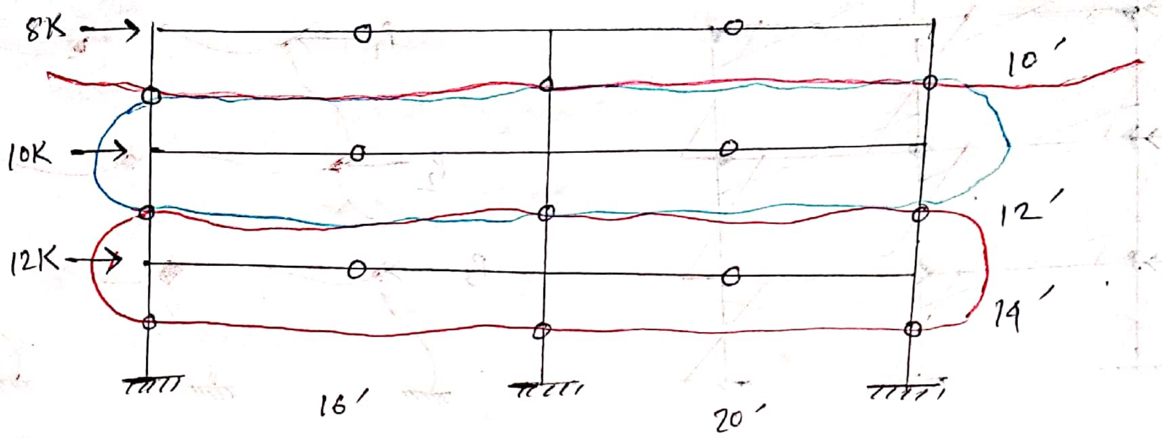
Truss is not rigid frame. Because truss can carry only axial force.

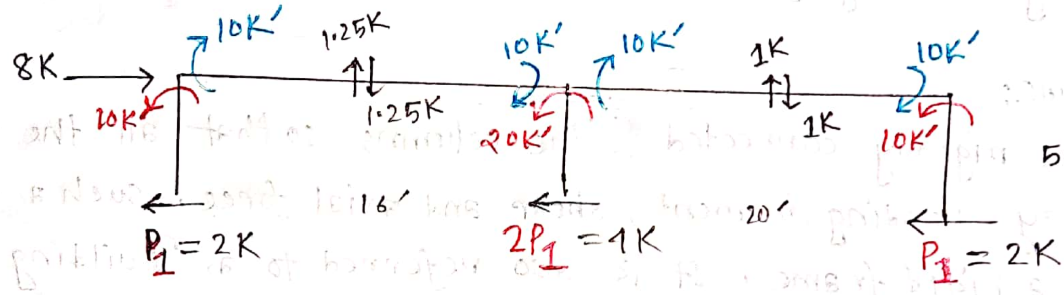
Problem: 04

Draw SFD and BFD for all members:



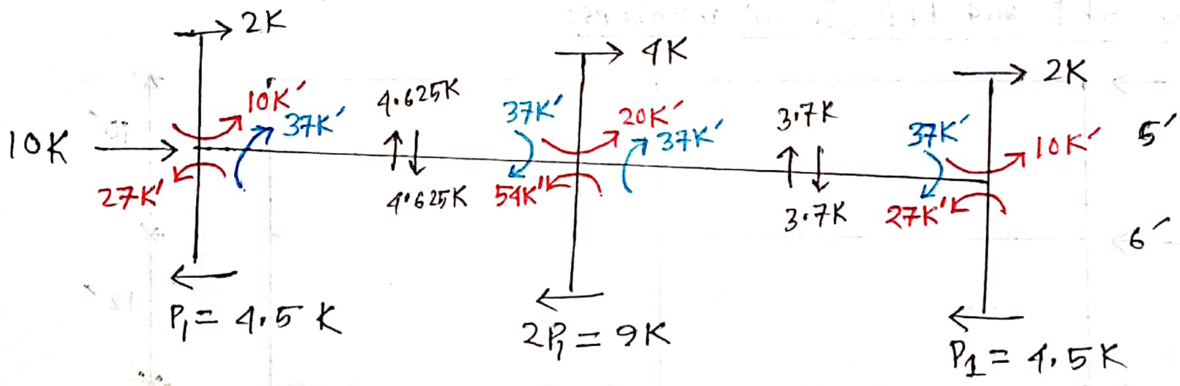
(Assumption 1 & 2)





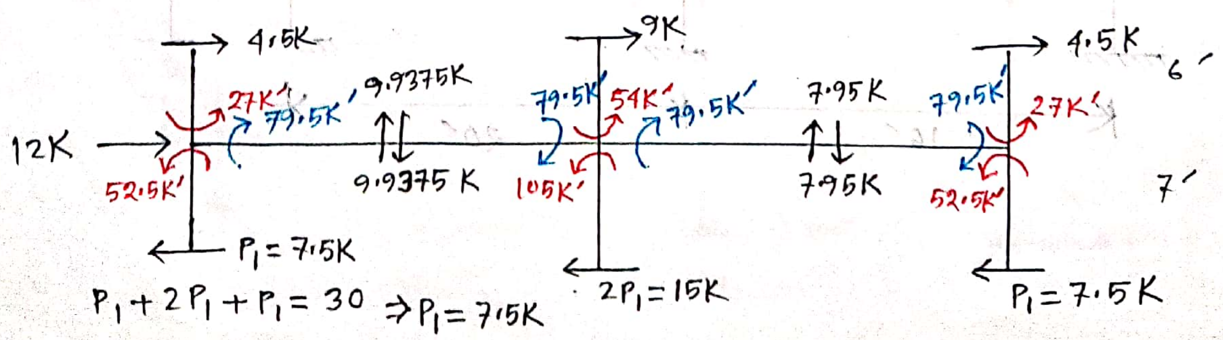
$$P_1 + 2P_1 + P_1 = 8$$

$$\Rightarrow P_1 = 2K$$

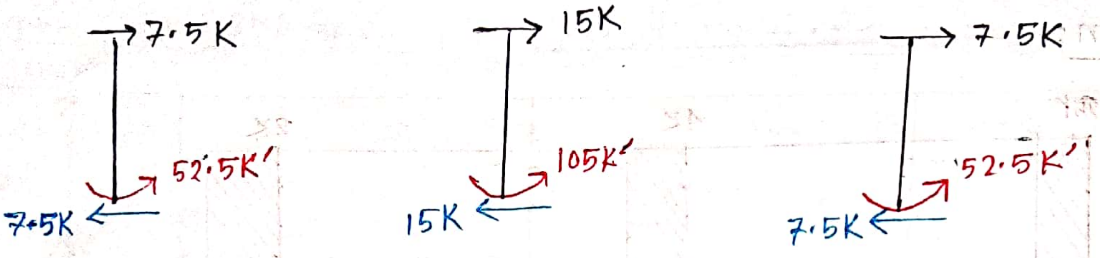


$$P_1 + 2P_1 + P_1 = 18$$

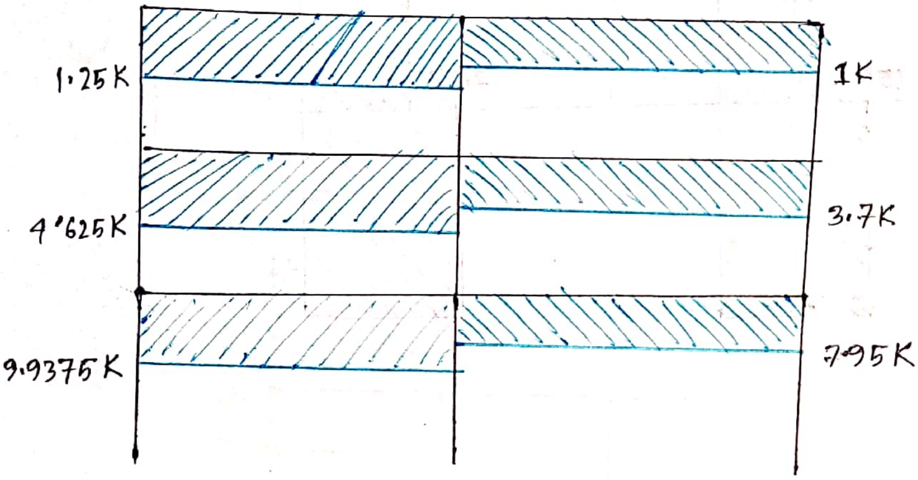
$$\Rightarrow P_1 = 4.5K$$



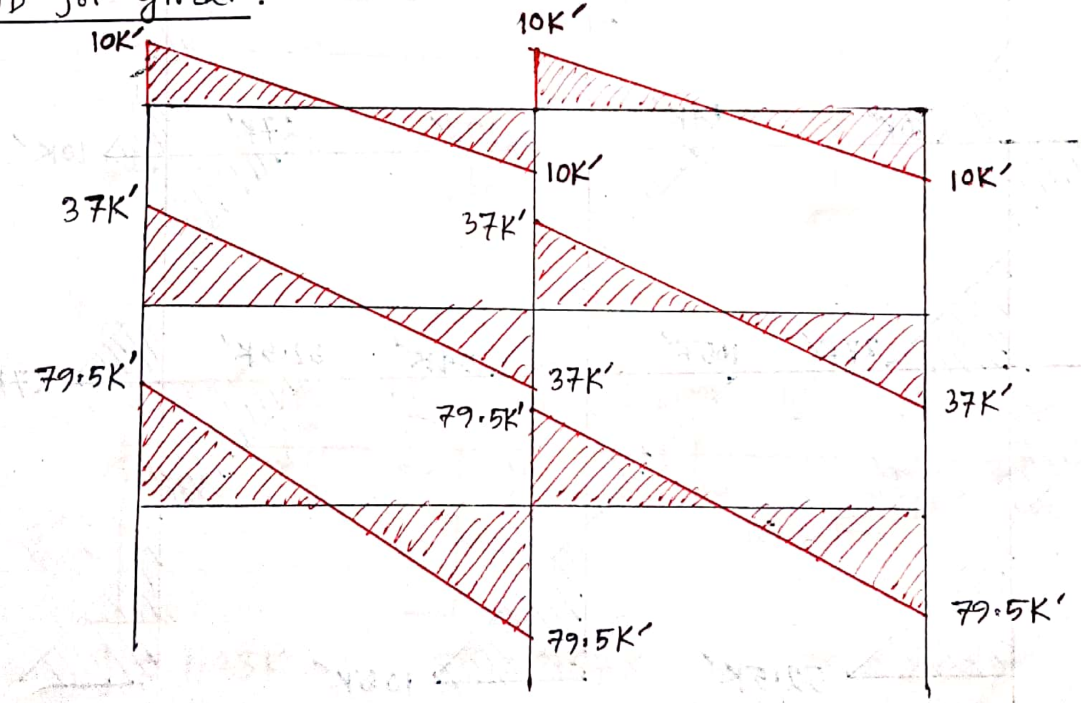
$$P_1 + 2P_1 + P_1 = 30 \Rightarrow P_1 = 7.5K$$



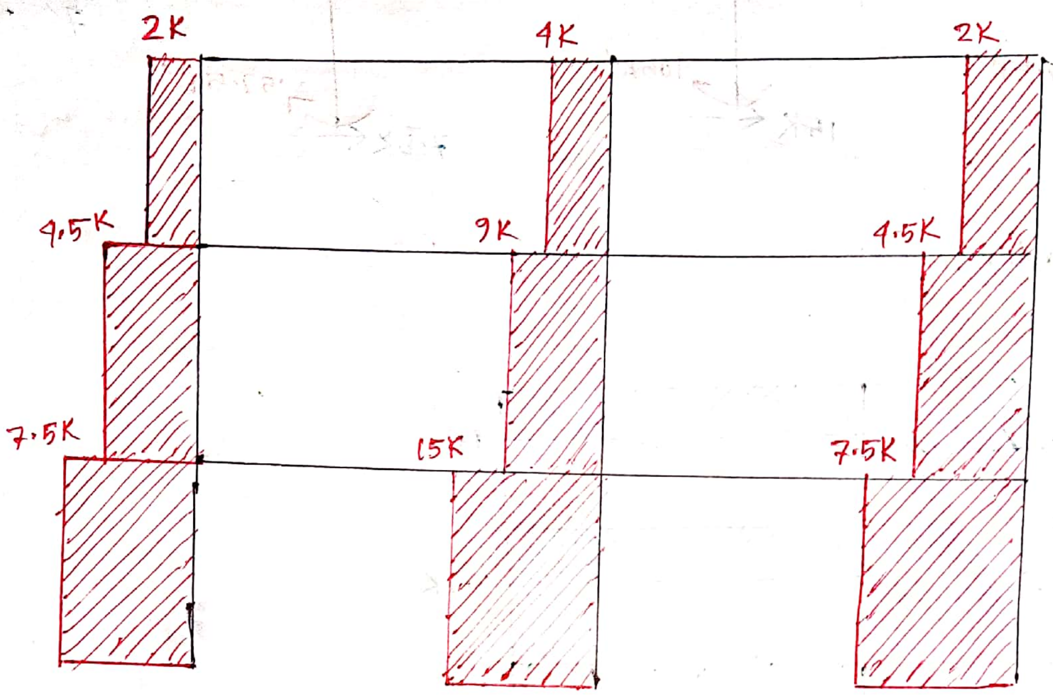
SFD for girder:



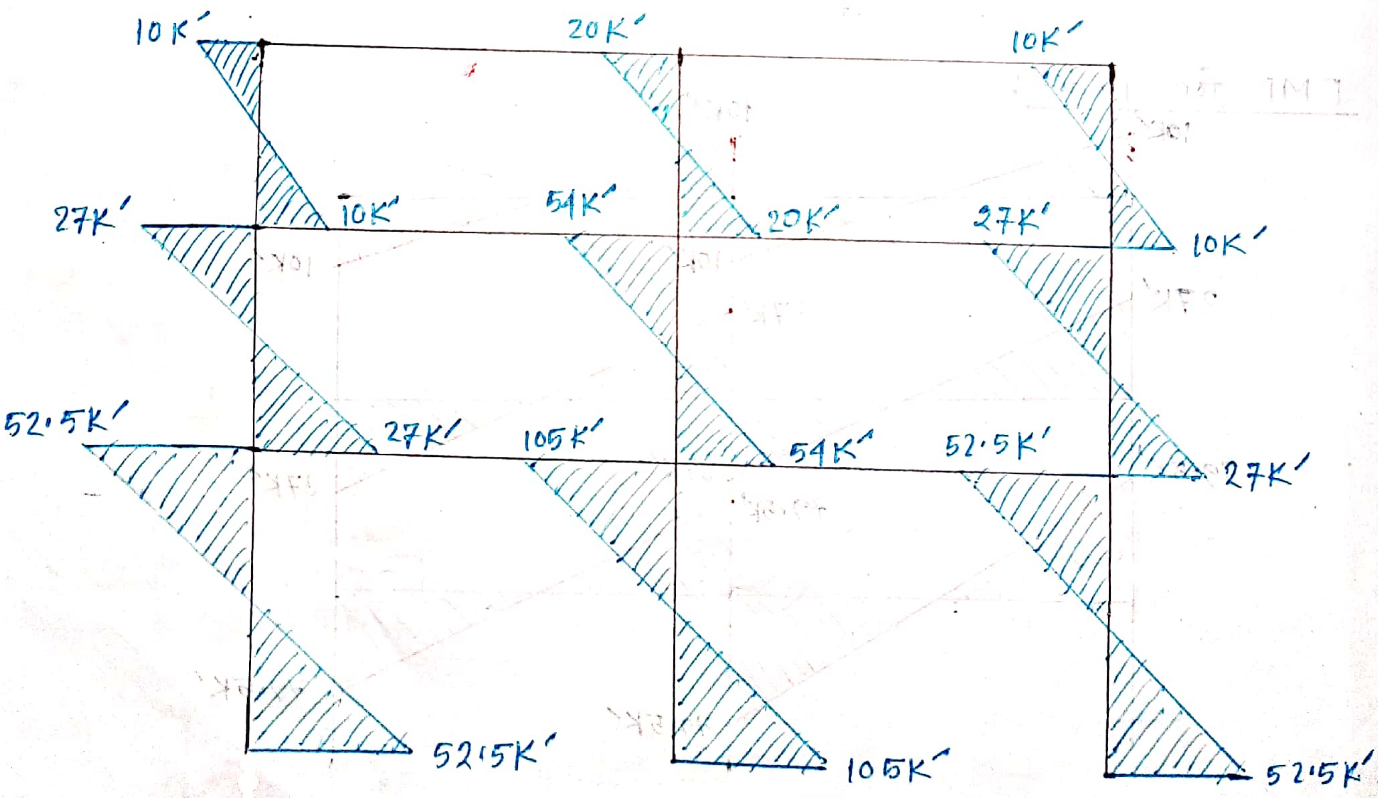
BMD for girder:



SFD for column:



BMD for column:



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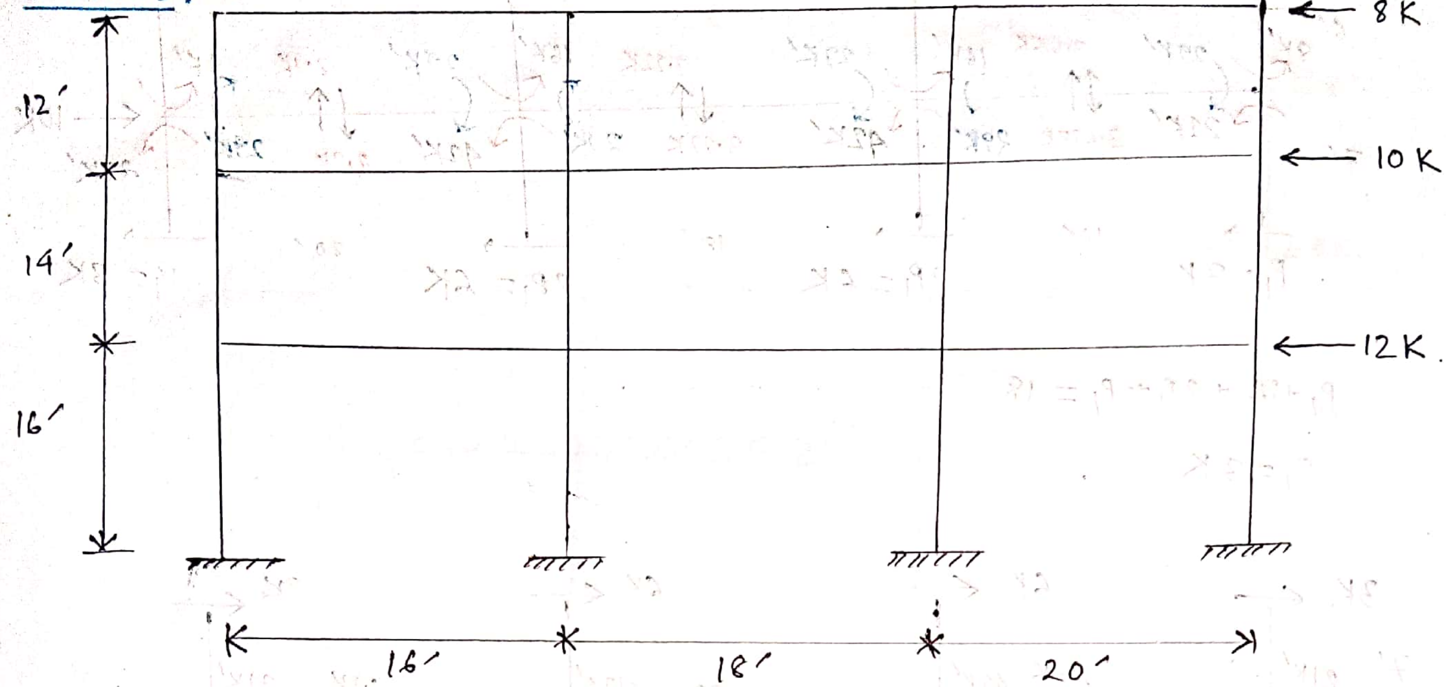
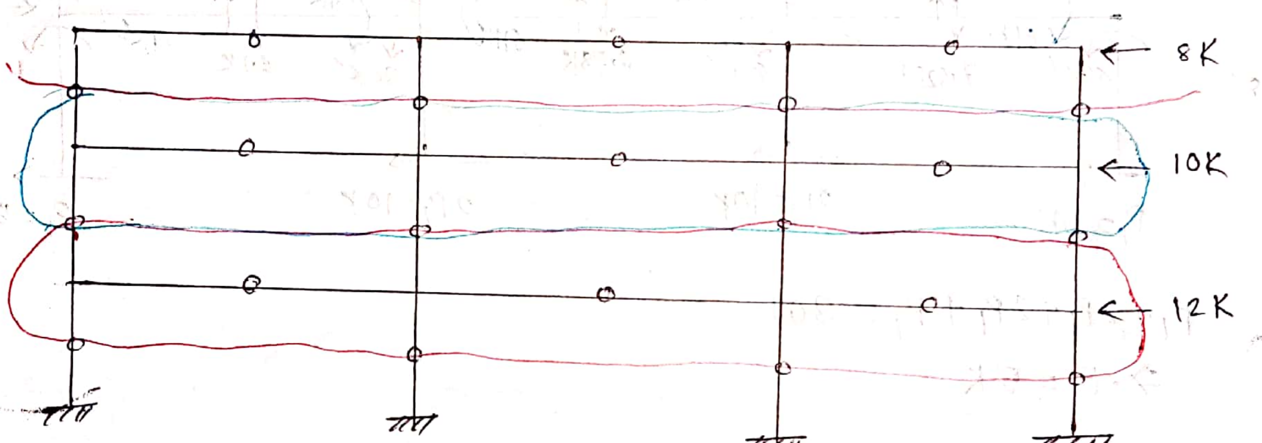
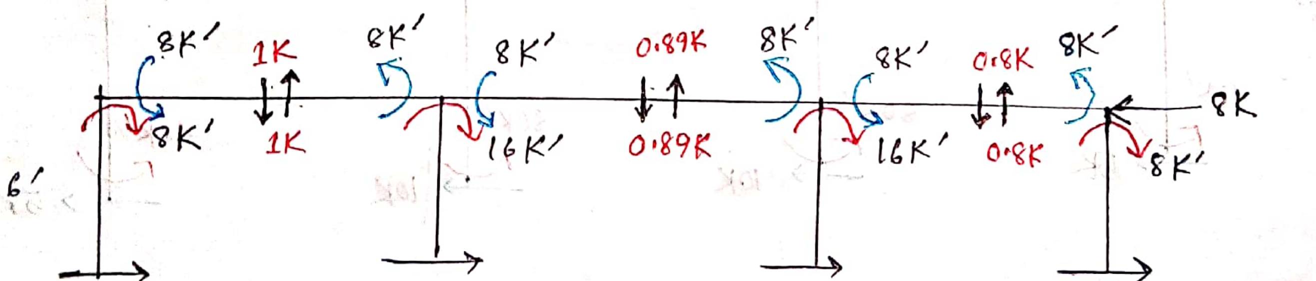
Draw SFD & BMD for all members:

Assignment: 03



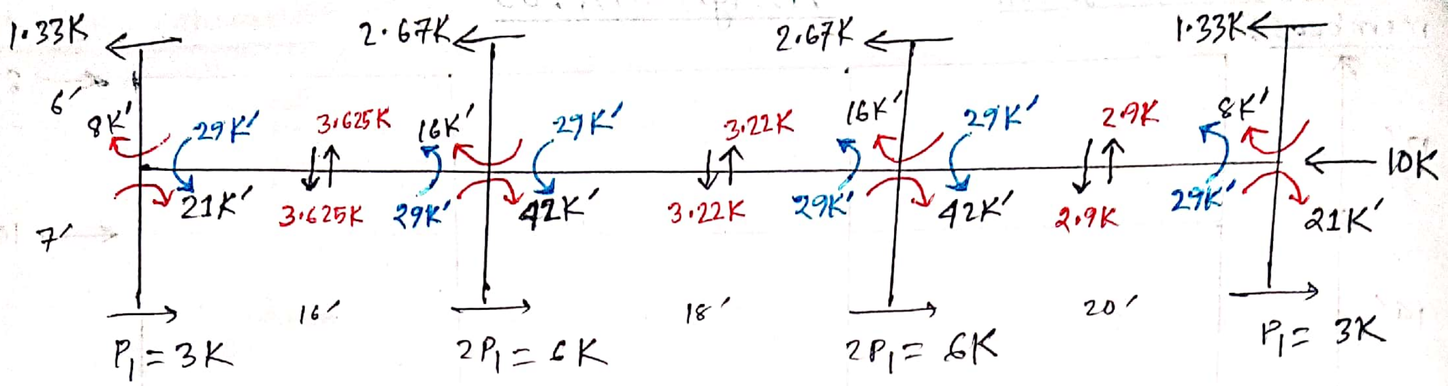




$P_1 = 1.33K$ $2P_1 = 2.67K$ $2P_1 = 2.67K$ $P_1 = 1.33K$

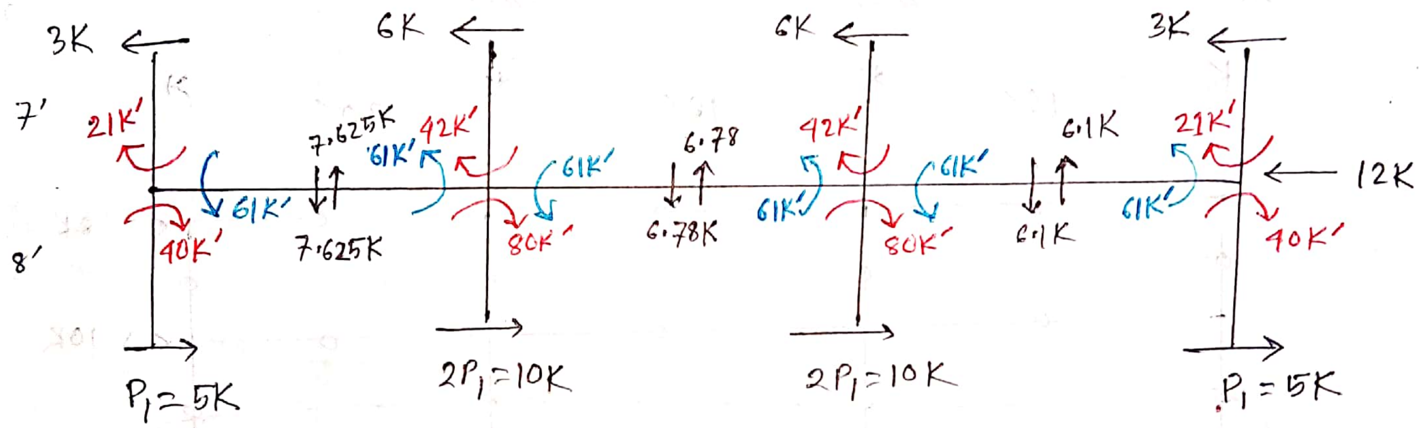
$$P_1 + 2P_1 + 2P_1 + P_1 = 8$$

$$\Rightarrow P_1 = 1.33K$$



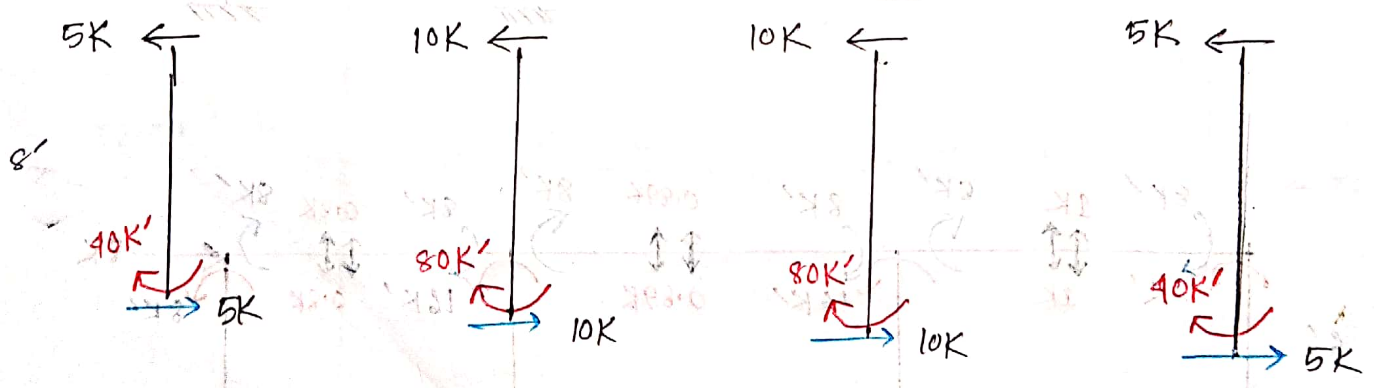
$P_1 + 2P_1 + 2P_1 + P_1 = 18$

$P_1 = 3K$



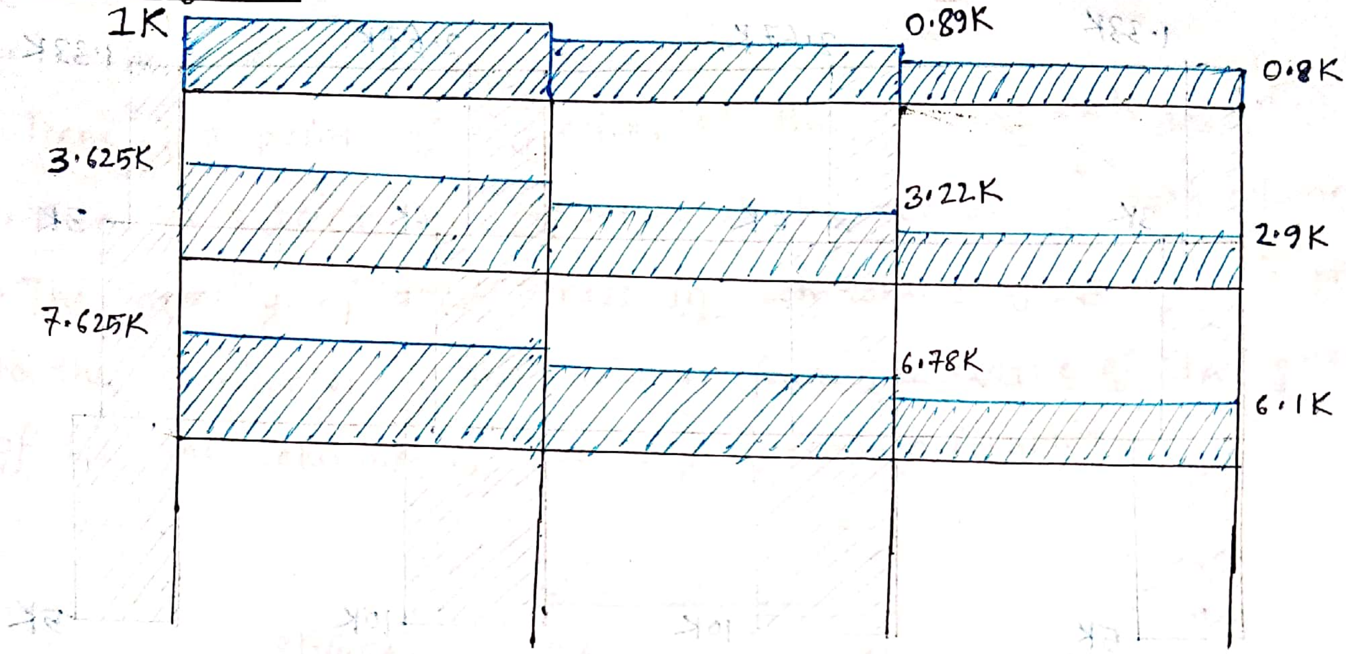
$P_1 + 2P_1 + 2P_1 + P_1 = 30$

$\Rightarrow P_1 = 5K$

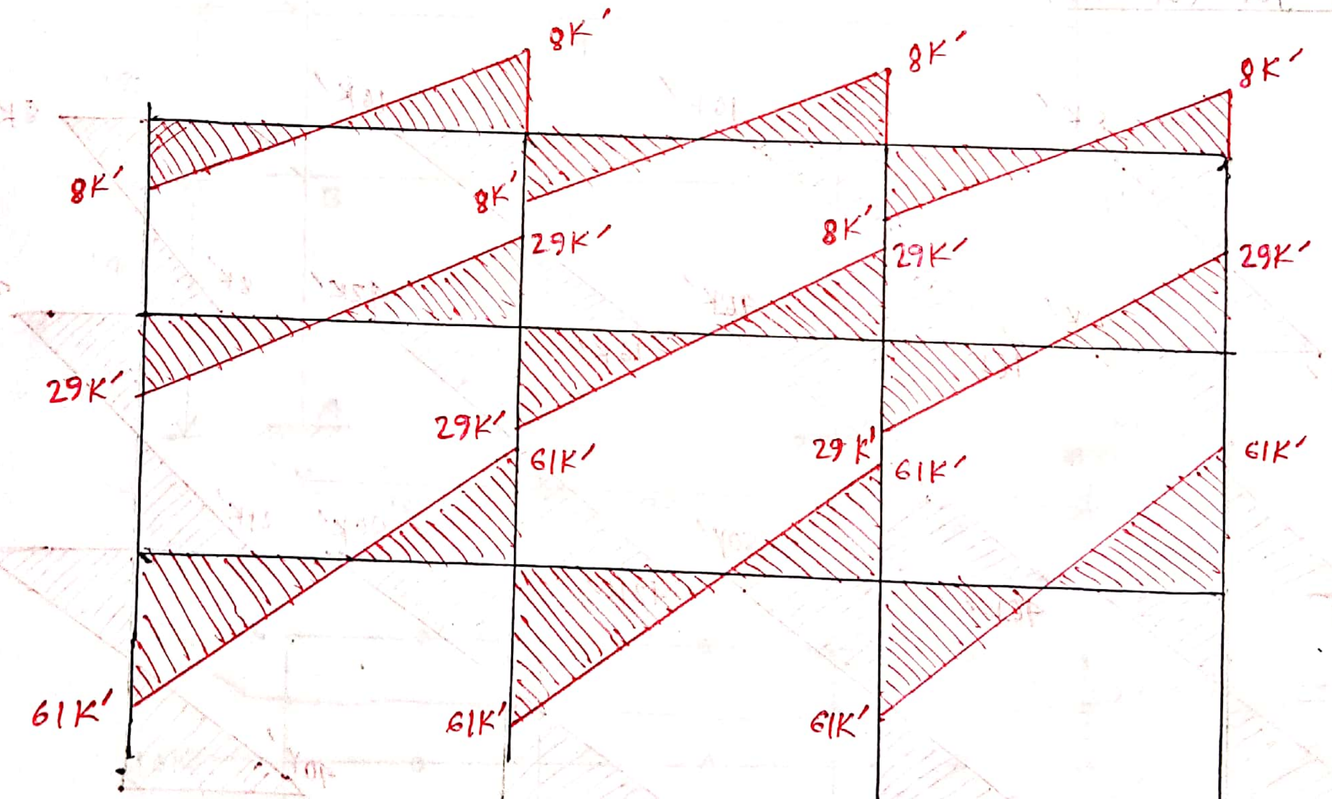


Scanned by CamScanner

SFD for girder:

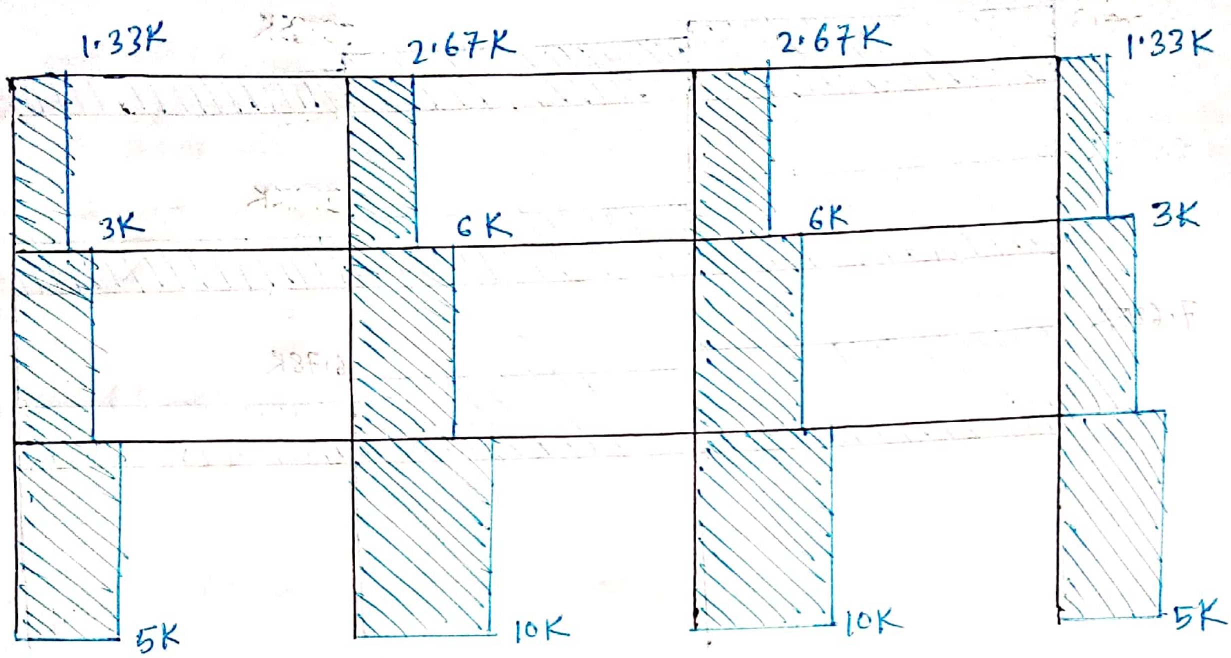


BMD for girder:

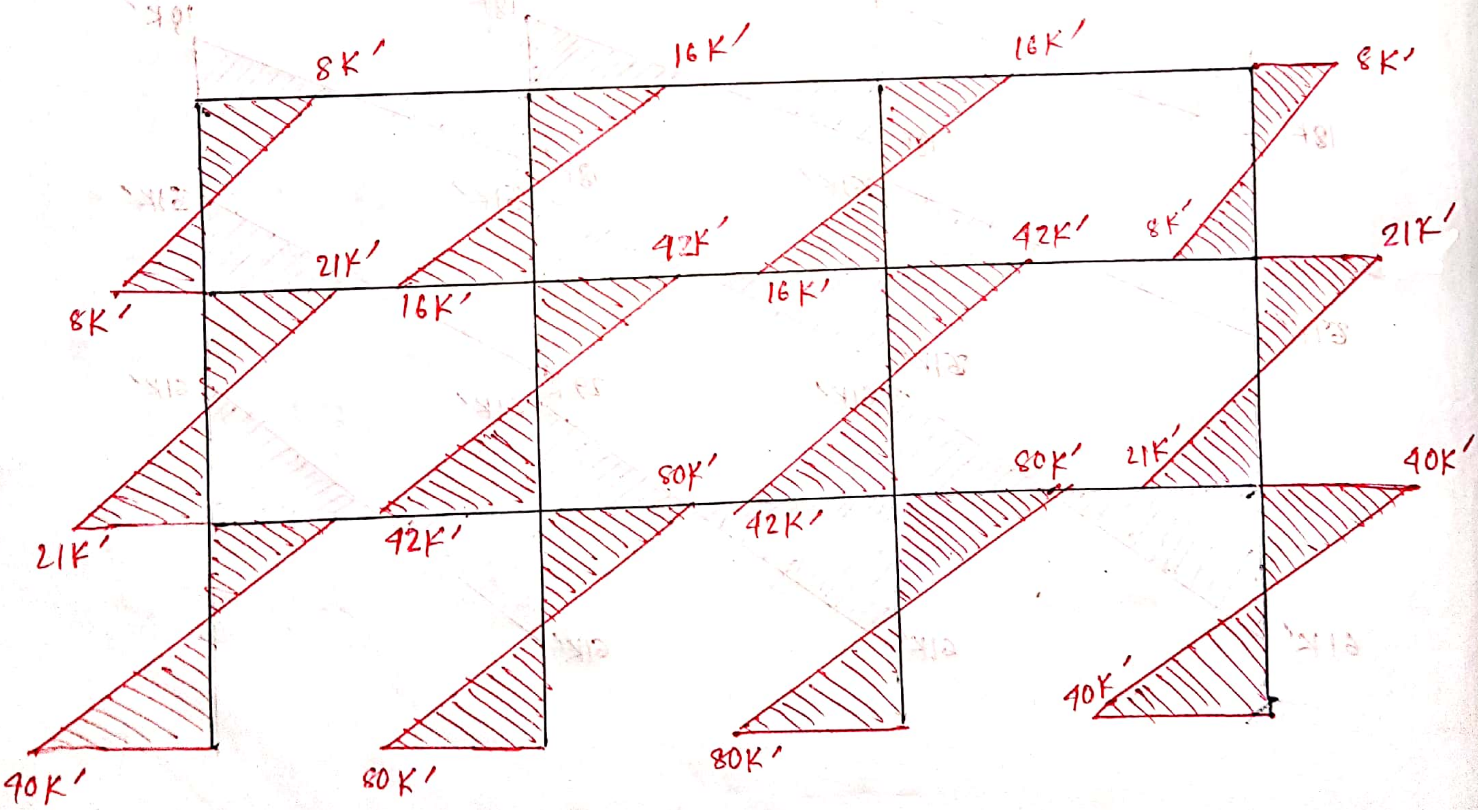


Scanned by CamScanner

SFD for Column:



BMD for column:

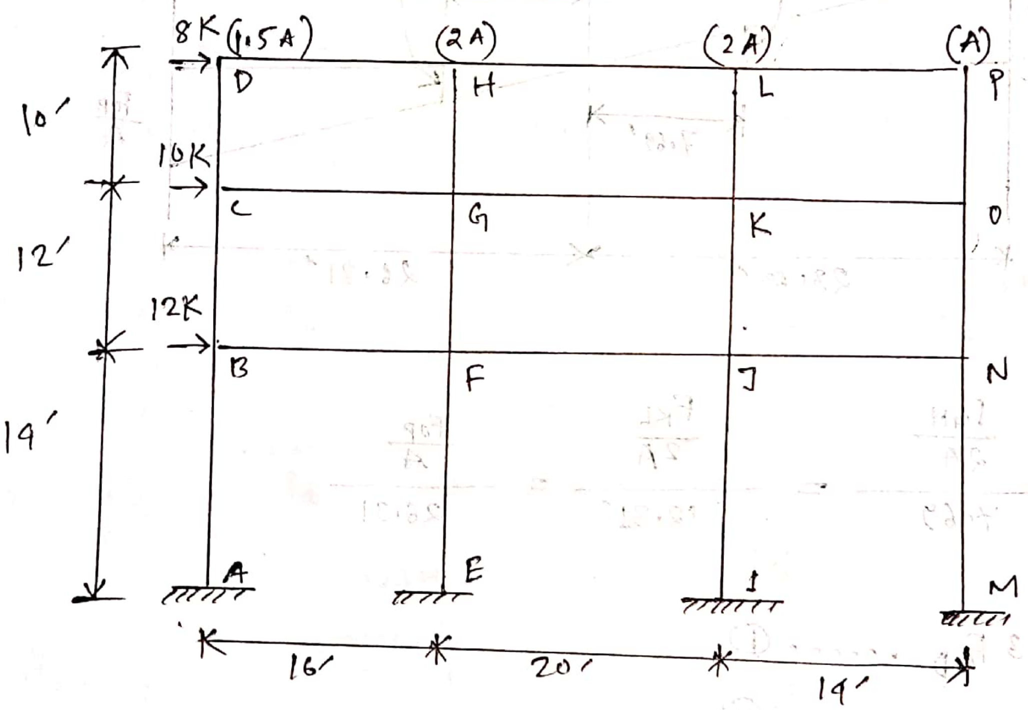


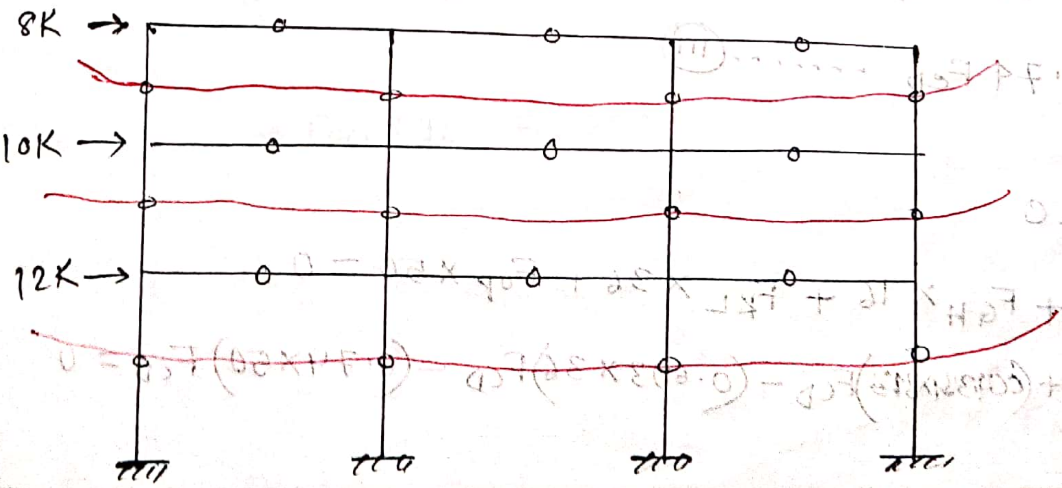
Cantilever Method

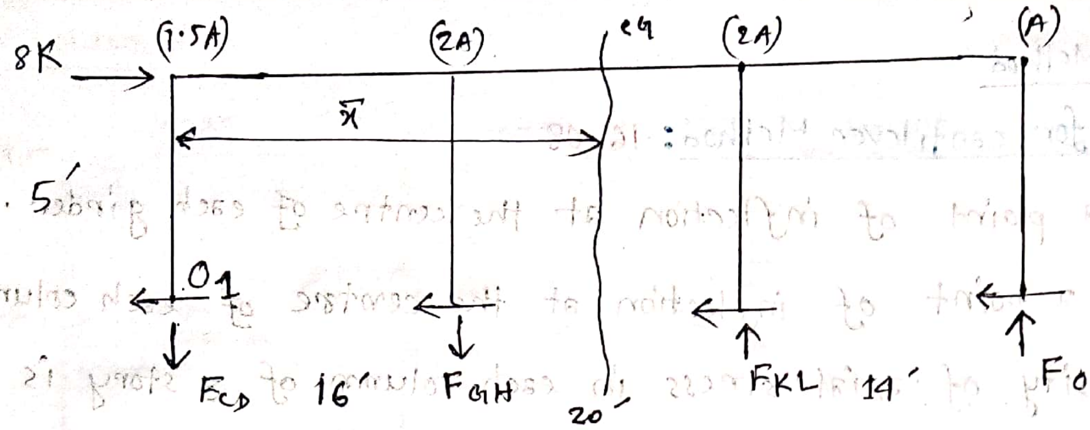
Assumptions for cantilever Method: 16/08

1. There is a point of inflection at the centre of each girder.
2. There is a point of inflection at the centre of each column.
3. The intensity of axial stress in each column of a story is proportional to the horizontal of that column from the centre of the gravity of all the columns of the story under consideration.

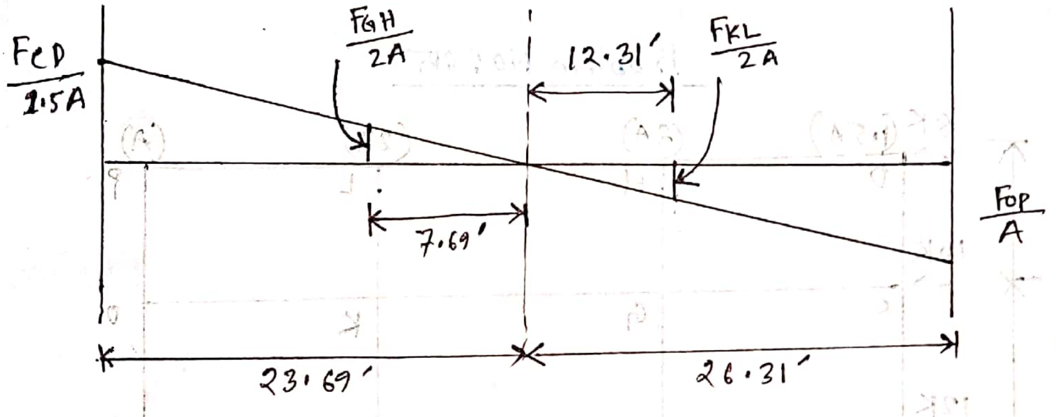
Problem No: 05







$$\bar{x} = \frac{2A \times 16 + 2A \times 36 + A \times 50}{(1.5 + 2 + 2)A} = \frac{154A}{6.5A} = 23.69'$$



Now,

$$\frac{\frac{F_{CD}}{1.5A}}{23.69} = \frac{\frac{F_{GH}}{2A}}{7.69} = \frac{\frac{F_{KL}}{2A}}{12.31} = \frac{\frac{F_{OP}}{A}}{26.31}$$

$F_{GH} = 0.433 F_{CD}$ ...... (i)

$F_{KL} = 0.693 F_{CD}$ ...... (ii)

$F_{OP} = 0.74 F_{CD}$ ...... (iii)

$\Sigma M_{O_1} = 0$

$8 \times 5 + F_{GH} \times 16 - F_{KL} \times 36 - F_{OP} \times 50 = 0$

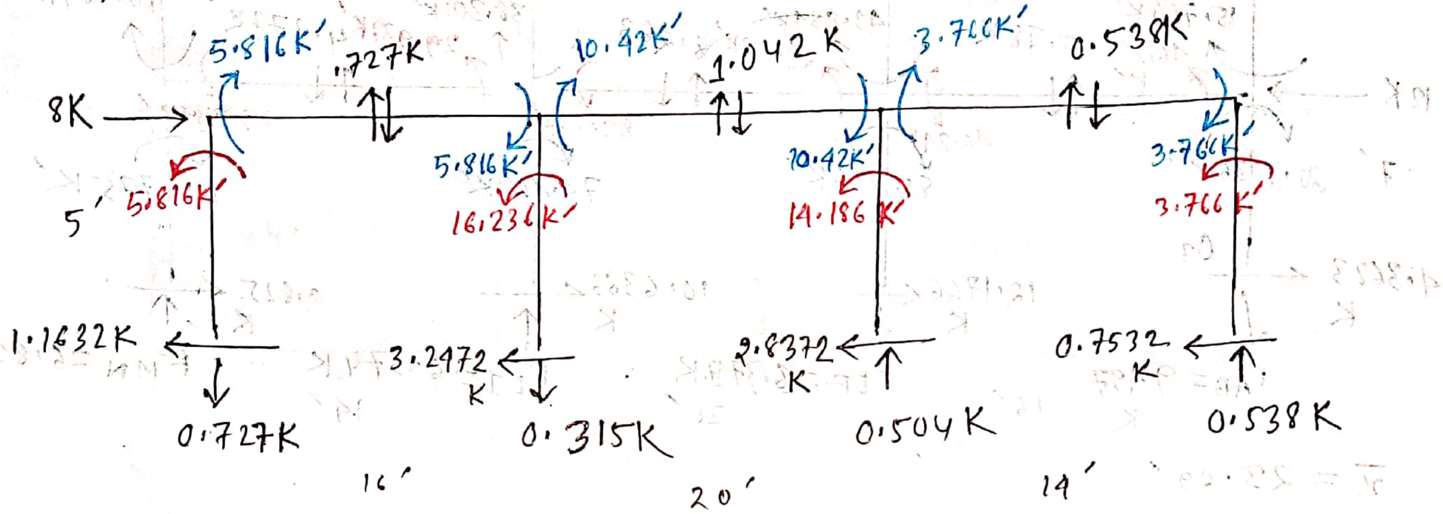
$\Rightarrow 40 + (0.433 \times 16) F_{CD} - (0.693 \times 36) F_{CD} - (0.74 \times 50) F_{CD} = 0$

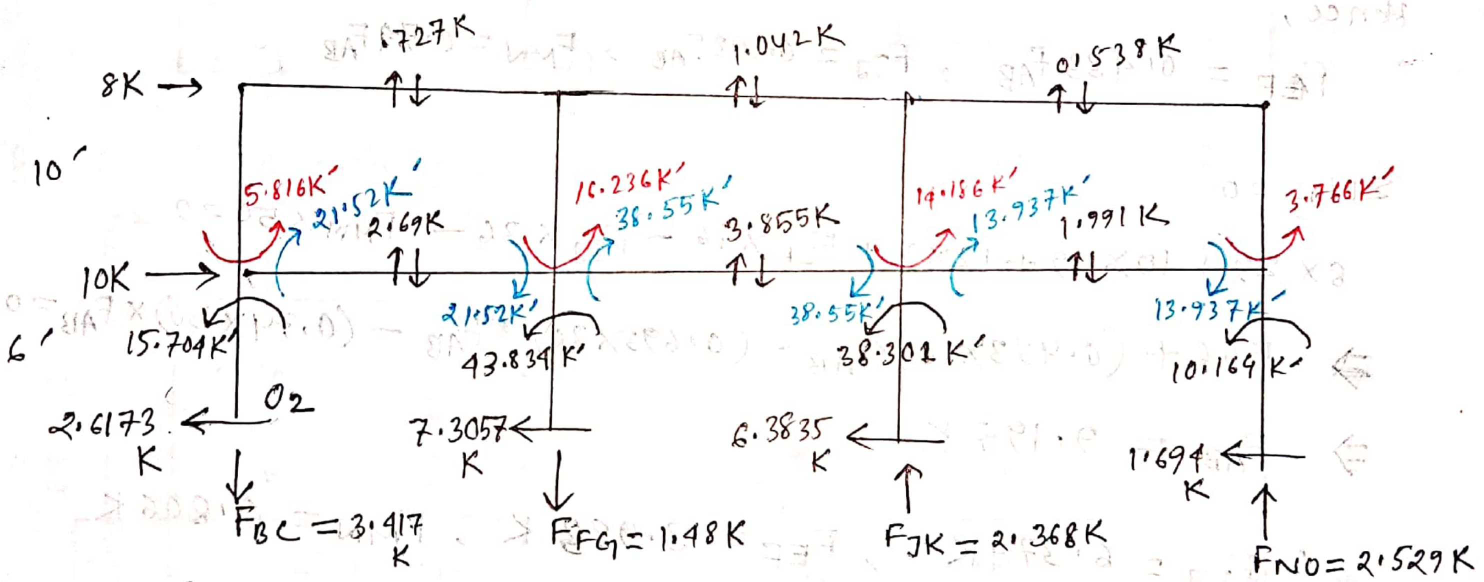
$\Rightarrow F_{CD} = 0.727\ K$

$\therefore F_{GH} = (0.433 \times 0.727) = 0.315\ K$

$F_{KL} = (0.693 \times 0.727) = 0.504\ K$

$F_{OP} = (0.74 \times 0.727) = 0.538\ K$





$\Sigma M_{O_2} = 0$

$8 \times 16 + 10 \times 6 + F_{FG} \times 16 - F_{JK} \times 36 - F_{NO} \times 50 = 0$

$\Rightarrow 188 + (.433 \times 16) F_{BC} - (.693 \times 36) F_{BC} - (.74 \times 50) \times F_{BC} = 0$

$\Rightarrow F_{BC} = 3.417\ K$

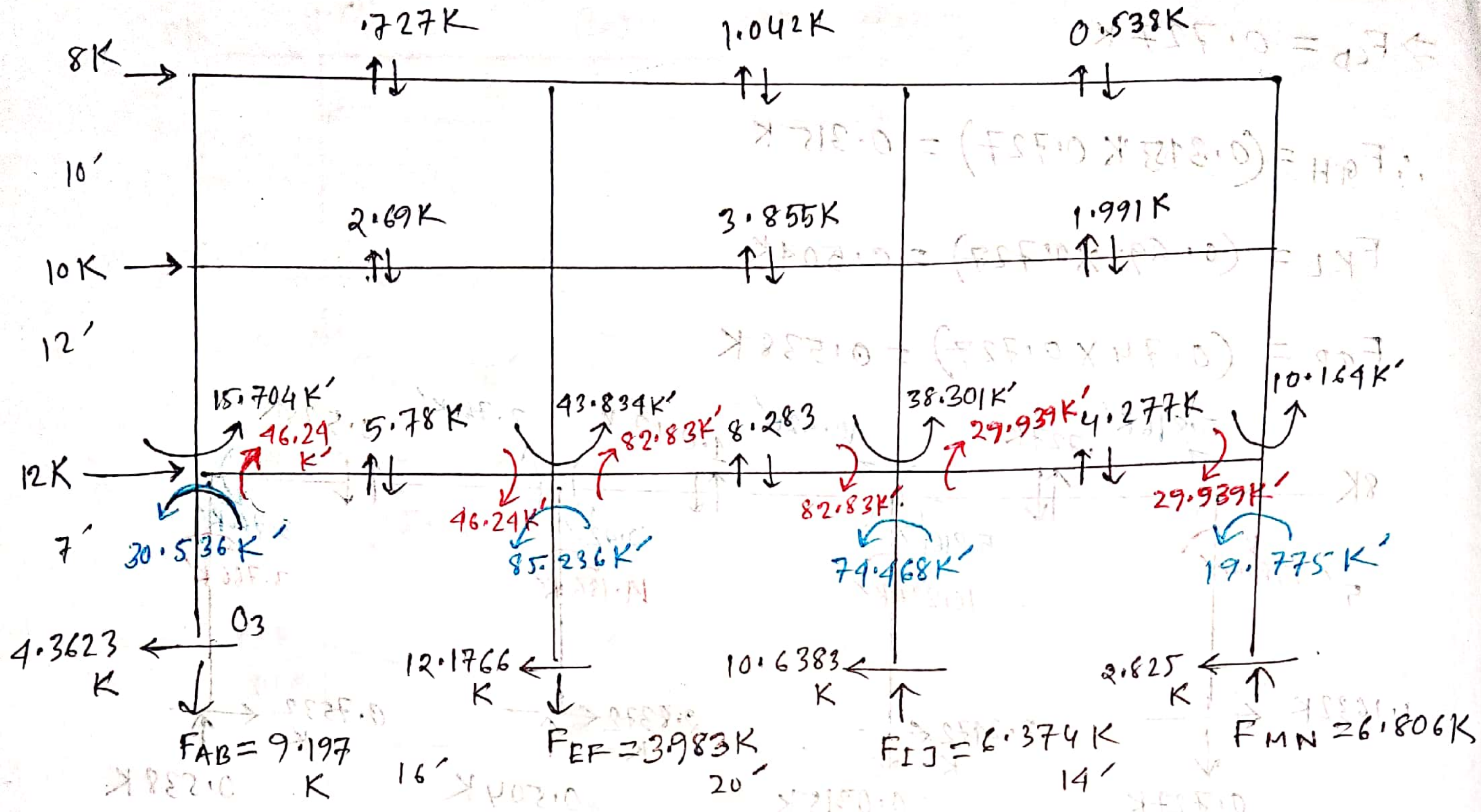
Here, $\bar{x} = 23.69'$, Hence

$F_{FG} = 0.433 F_{BC}$

$F_{JK} = 0.693 F_{BC}$

$F_{NO} = 0.74 F_{BC}$

$\therefore F_{FG} = 1.48\ K$, $F_{JK} = 2.368\ K$, $F_{NO} = 2.529\ K$



$\bar{x} = 23.69'$

Hence,

$F_{EF} = 0.433\,F_{AB}$, $F_{IJ} = 0.693\,F_{AB}$, $F_{MN} = 0.74\,F_{AB}$

$\Sigma M_{O_3} = 0$

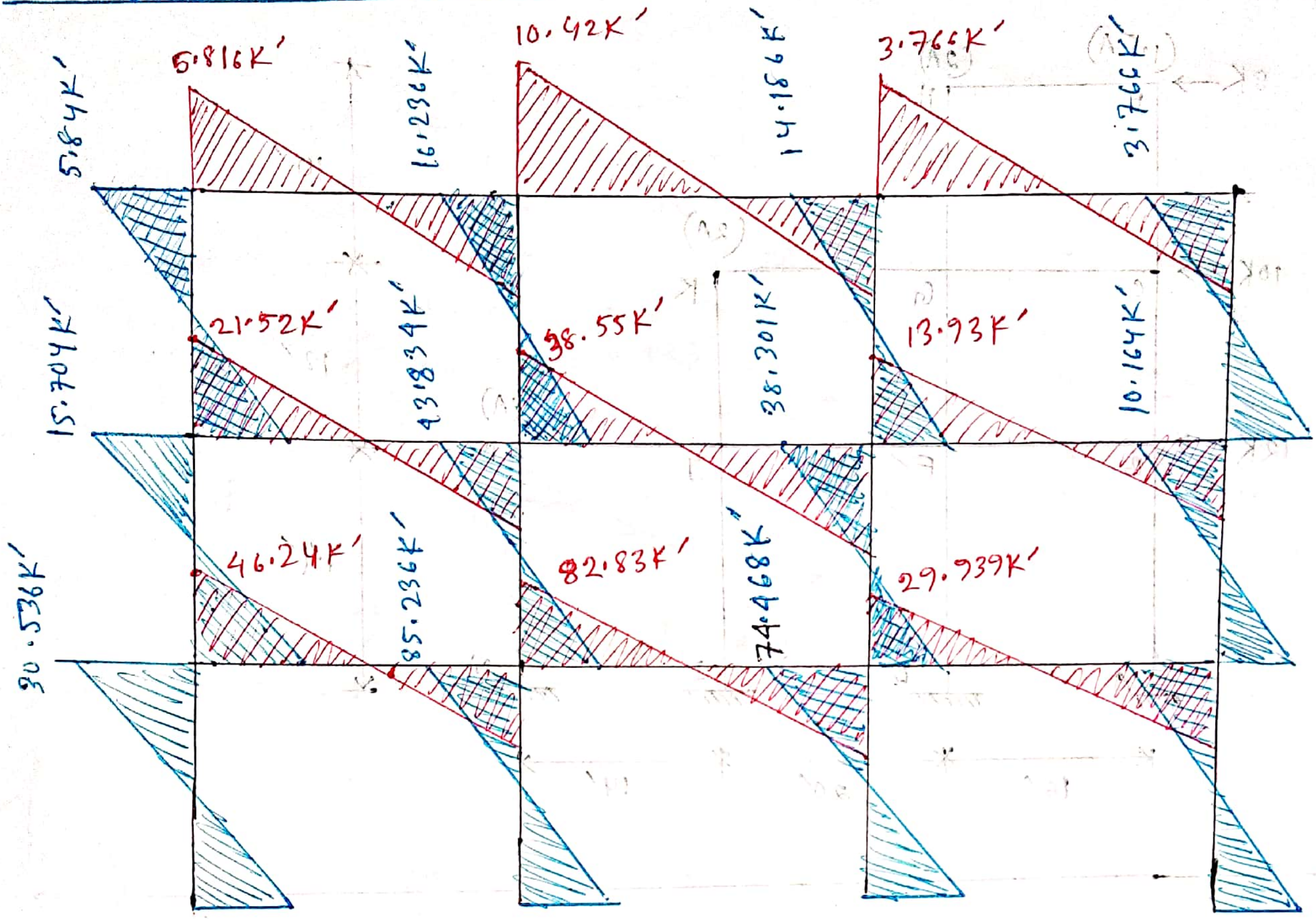
$8 \times 29 + 10 \times 19 + 12 \times 7 + F_{EF} \times 16 - F_{IJ} \times 36 - F_{MN} \times 50 = 0$

$\Rightarrow 506 + (0.433 \times 16) \times F_{AB} - (0.693 \times 36) \times F_{AB} - (0.74 \times 50) \times F_{AB} = 0$

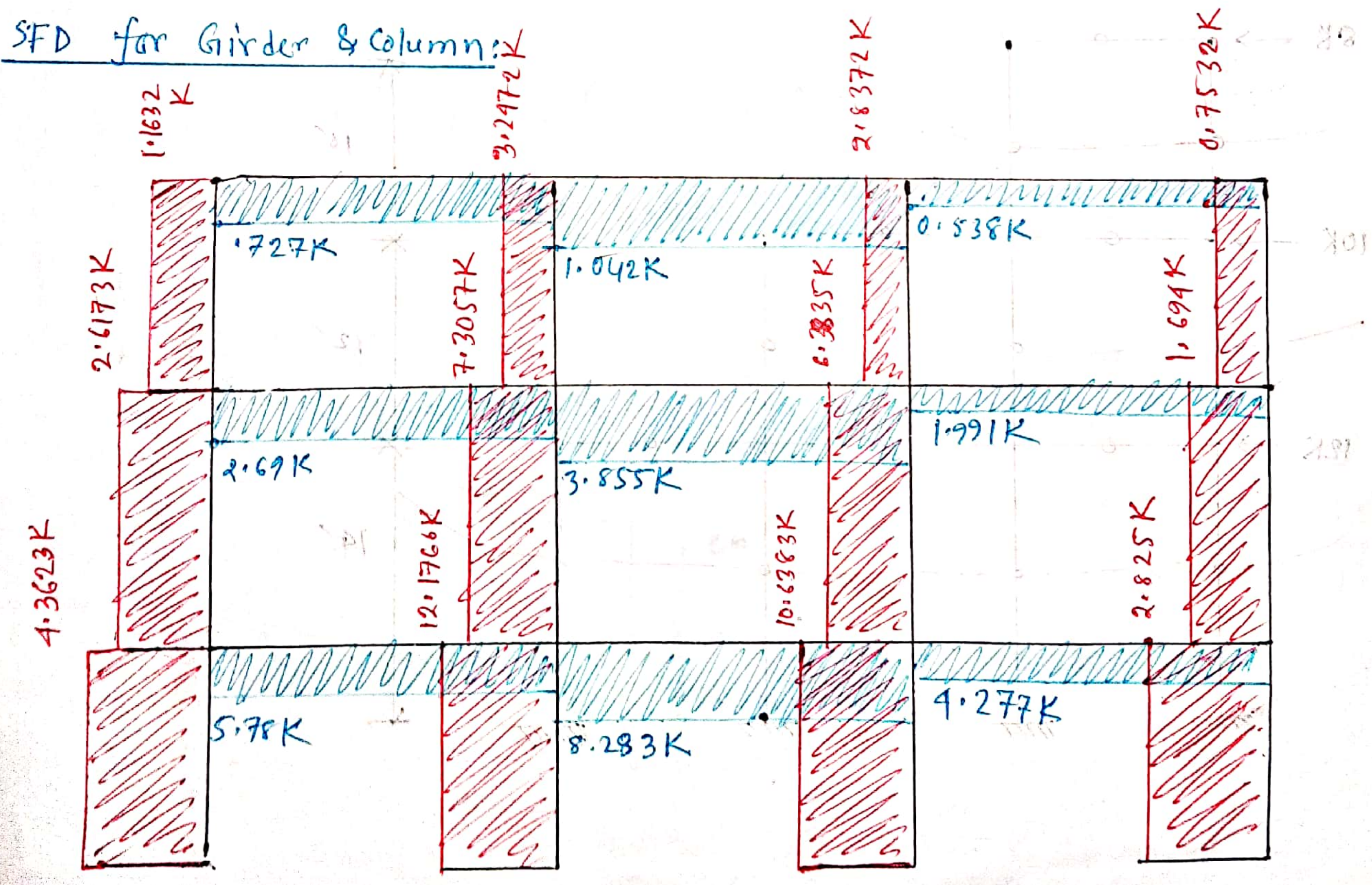
$\Rightarrow F_{AB} = 9.197\text{ K}$

$\therefore F_{IJ} = 6.374\text{K}$, $F_{EF} = 3.983\text{ K}$, $F_{MN} = 6.806\text{ K}$

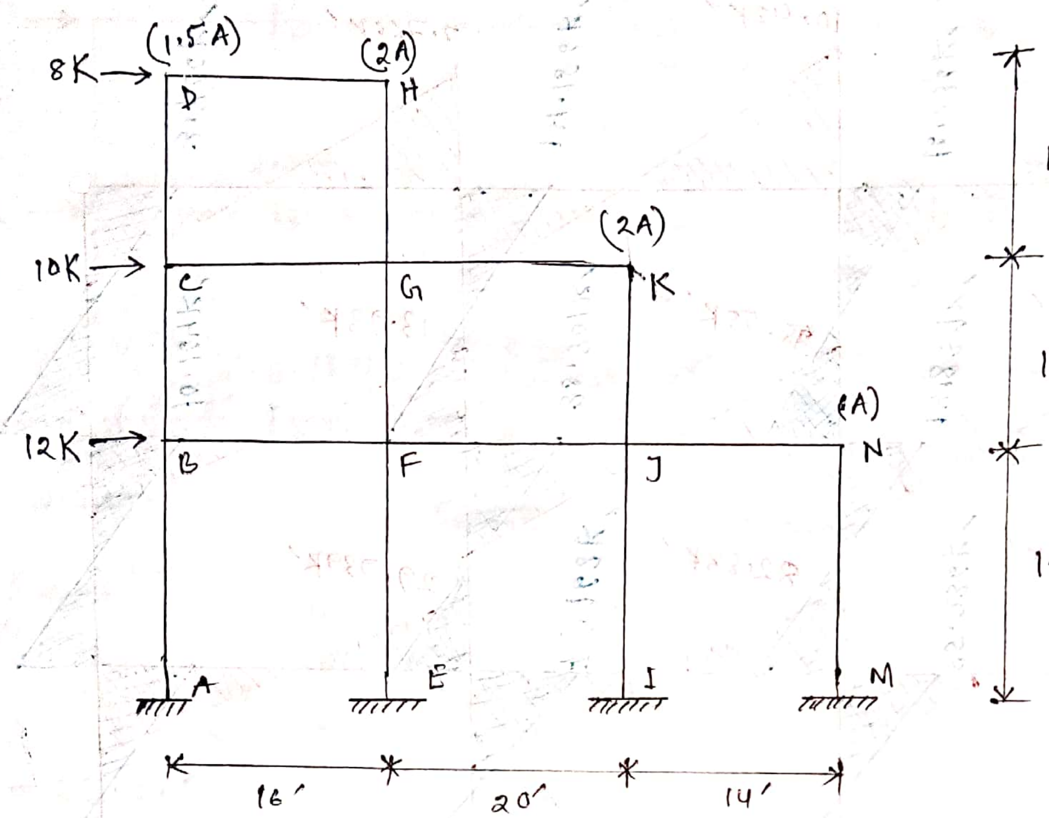
BMD for Girder and Column:

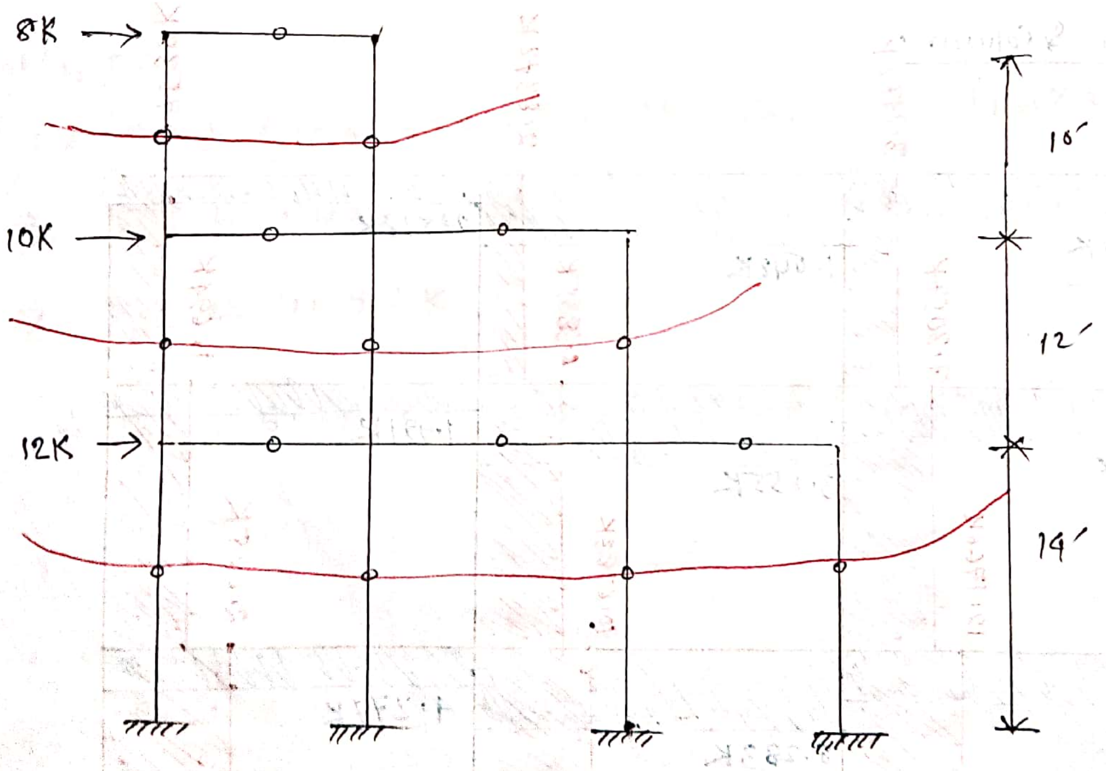


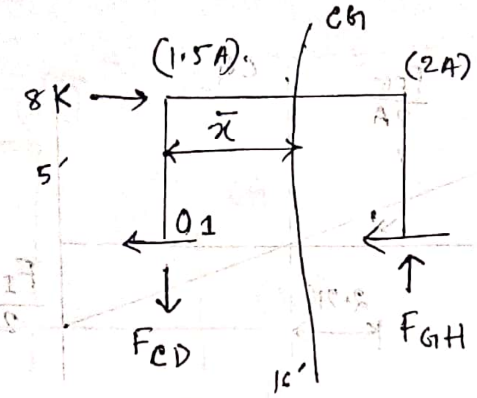
SFD for Girder & Column:



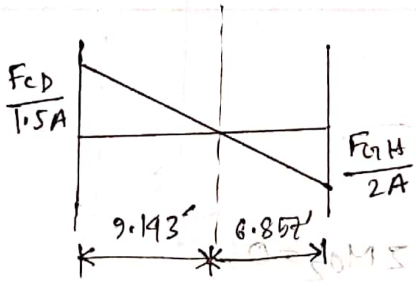
Assignment No: 04







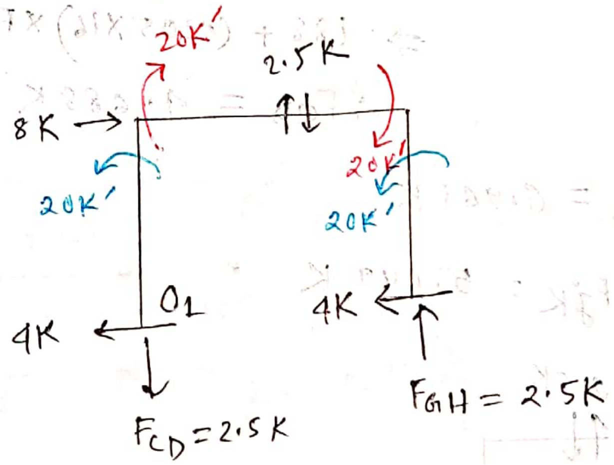
$$\bar{x} = \frac{2A \times 16}{(1.5+2)A} = 9.143'$$



Now,

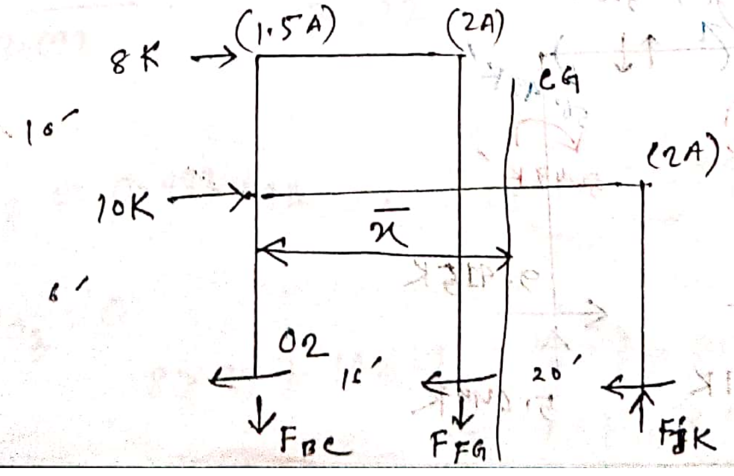
$$\frac{\frac{F_{CD}}{1.5A}}{9.143'} = \frac{\frac{F_{GH}}{2A}}{6.857}$$

$\therefore F_{GH} = F_{CD}$



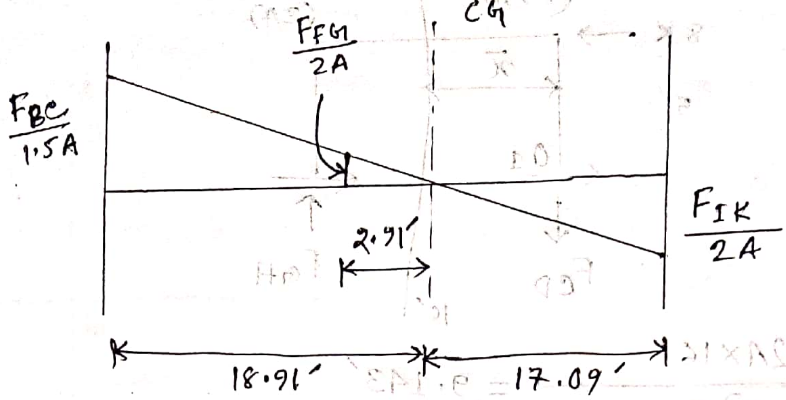
$\Sigma M_{O_1} = 0$

$8 \times 5 - F_{GH} \times 16 = 0 \Rightarrow 40 - 16 F_{CD} = 0 \quad \therefore F_{CD} = 2.5\ K$



$$\bar{x} = \frac{2A \times 16 + 2A \times 36}{(1.5+2+2)A}$$

$\therefore \bar{x} = 18.91'$



Now,

$$\frac{\frac{F_{BC}}{1.5A}}{18.91} = \frac{\frac{F_{FG}}{2A}}{2.91} = \frac{\frac{F_{JK}}{2A}}{17.09}$$

$\therefore F_{FG} = 0.205 F_{BC}$

& $F_{JK} = 1.205 F_{BC}$

$\Sigma M_{O_2} = 0$

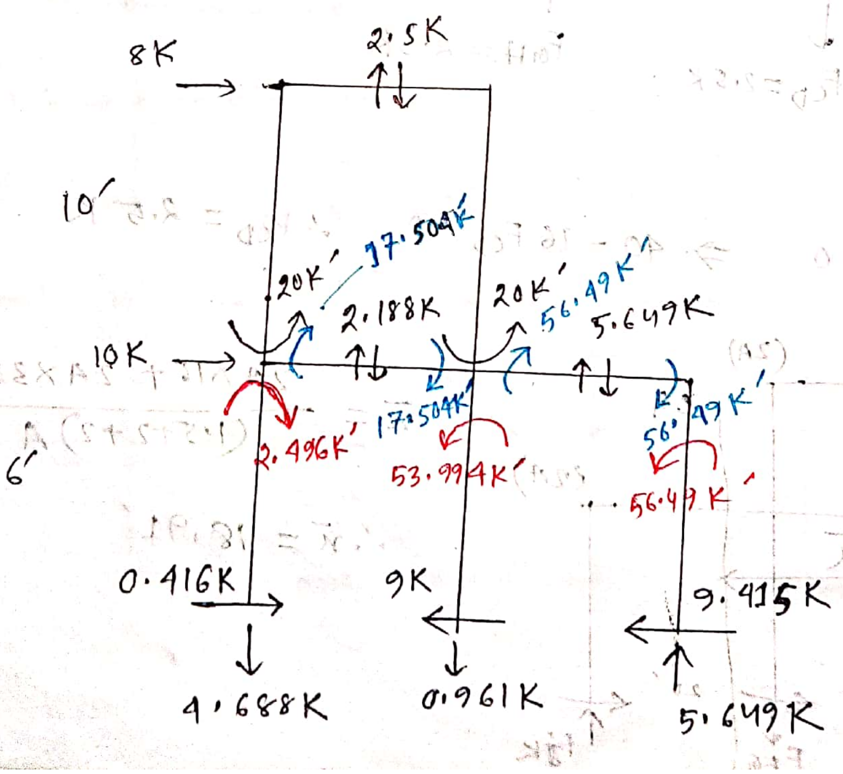
$8 \times 16 + 10 \times 6 + F_{FG} \times 16 - F_{JK} \times 36 = 0$

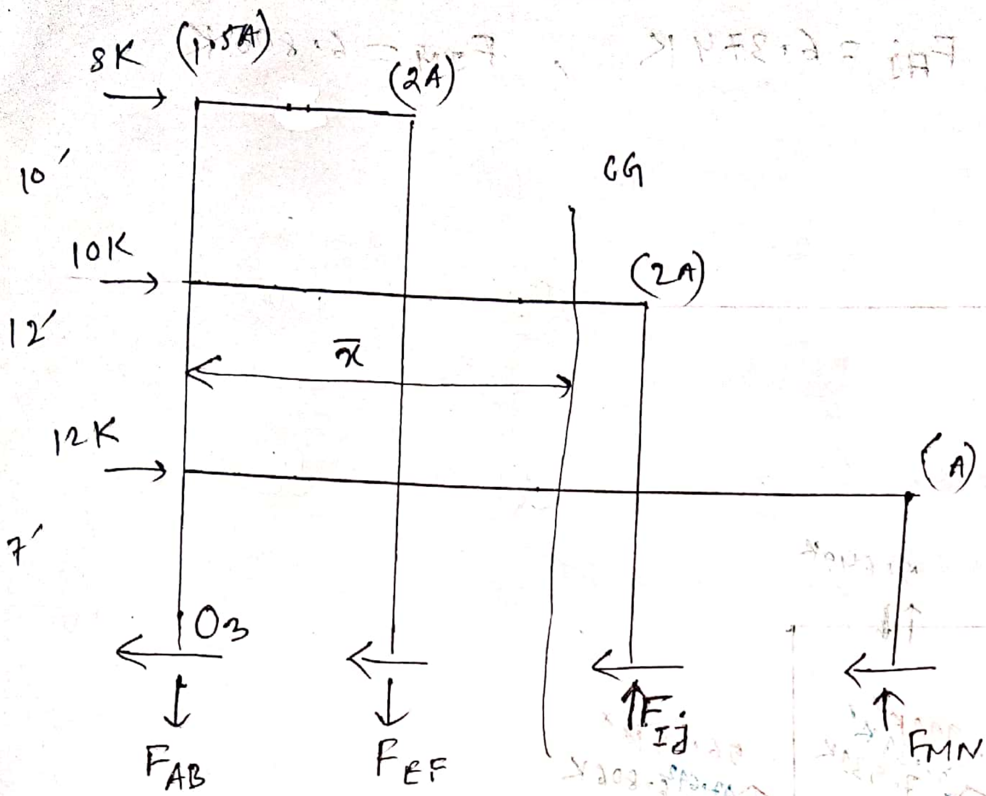
$\Rightarrow 188 + (.205 \times 16) \times F_{BC} - (1.205 \times 36) \times F_{BC} = 0$

$\therefore F_{BC} = 4.688\text{ K}$

Hence, $F_{FG} = 0.961\text{ K}$

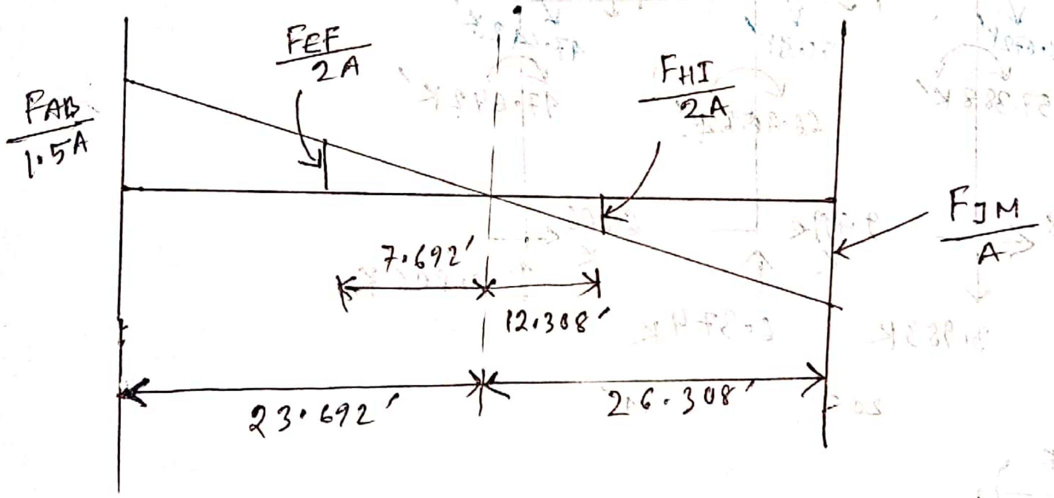
and, $F_{JK} = 5.649\text{ K}$





$$\bar{x} = \frac{2A \times 16 + 2A \times 36 + 1A \times 50}{(1.5+2+2+1)A}$$

$$\therefore \bar{x} = 23.692 \text{ ft}$$



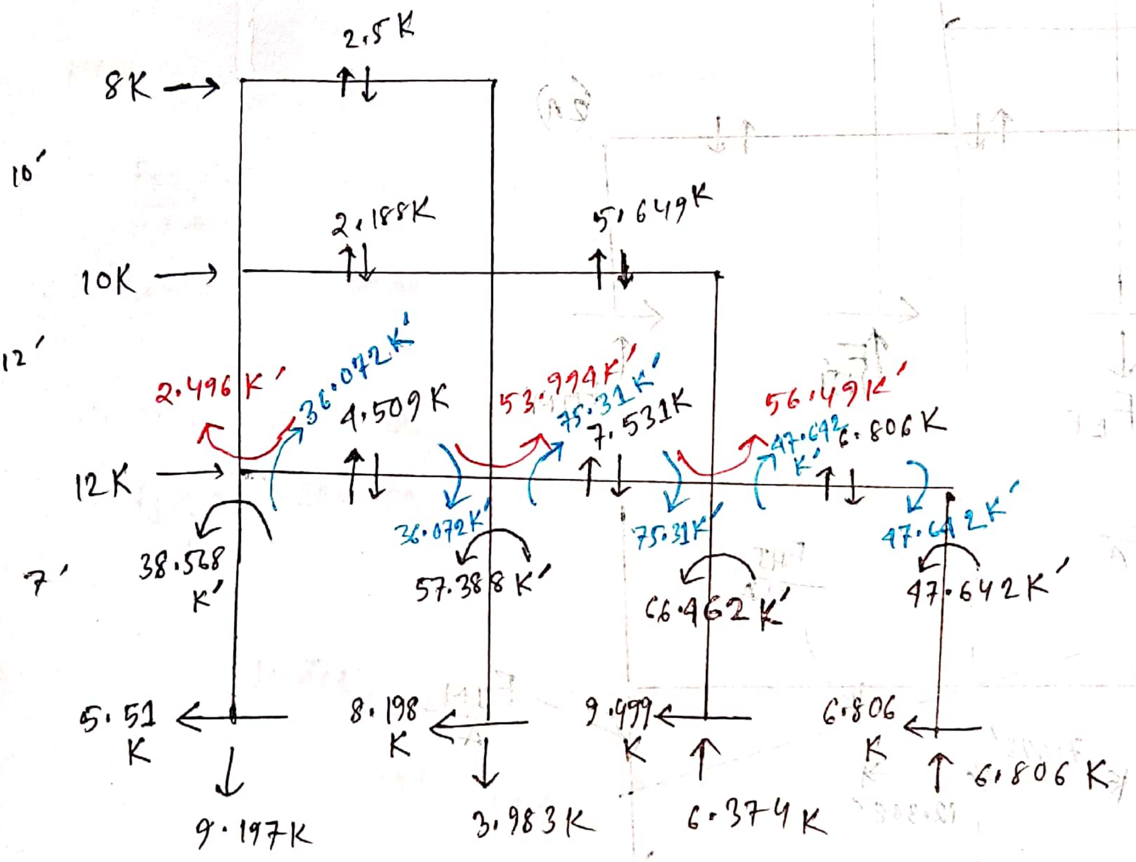
$$\frac{\frac{F_{AB}}{1.5A}}{23.692} = \frac{\frac{F_{EF}}{2A}}{7.692} = \frac{\frac{F_{IJ}}{2A}}{12.308} = \frac{\frac{F_{MN}}{A}}{26.308}$$

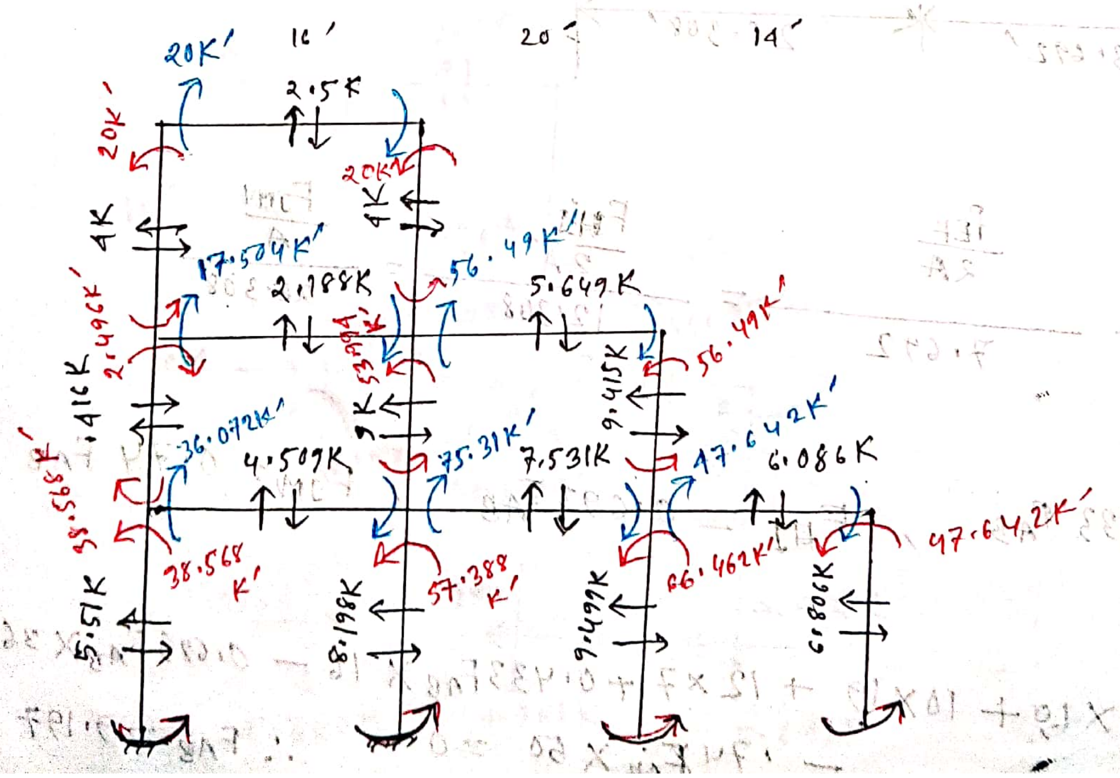
$F_{EF} = 0.433 F_{AB}$, $F_{IJ} = 0.693 F_{AB}$, $F_{MN} = 0.74 F_{AB}$

$\Sigma M_{O_3} = 0$

$$8 \times 29 + 10 \times 19 + 12 \times 7 + 0.433 F_{AB} \times 16 - 0.693 F_{AB} \times 36 - .74 F_{AB} \times 50 = 0 \quad \therefore F_{AB} = 9.197 \text{ K}$$

$\therefore F_{EF} = 3.983\ K$, $F_{IJ} = 6.374\ K$, $F_{MN} = 6.806\ K$





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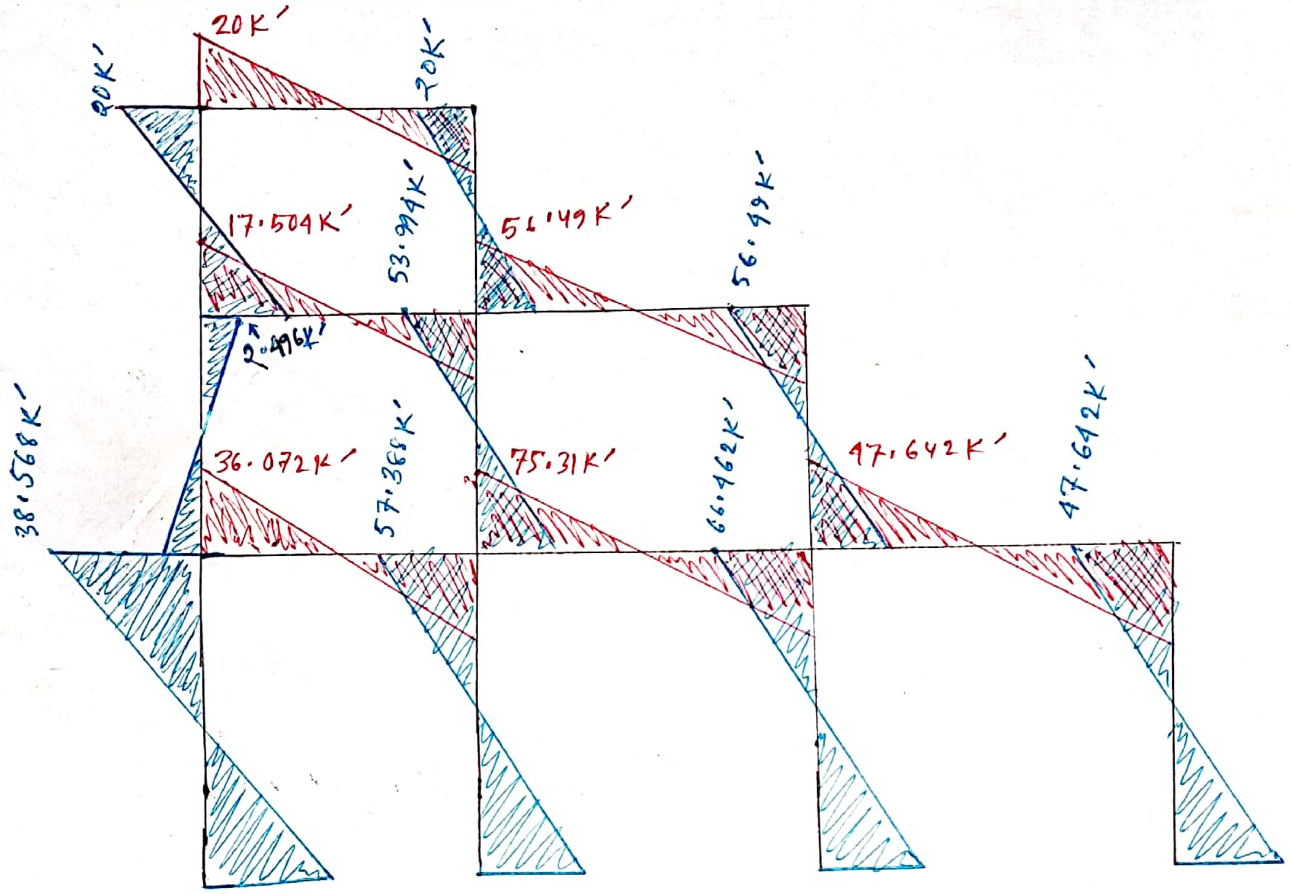
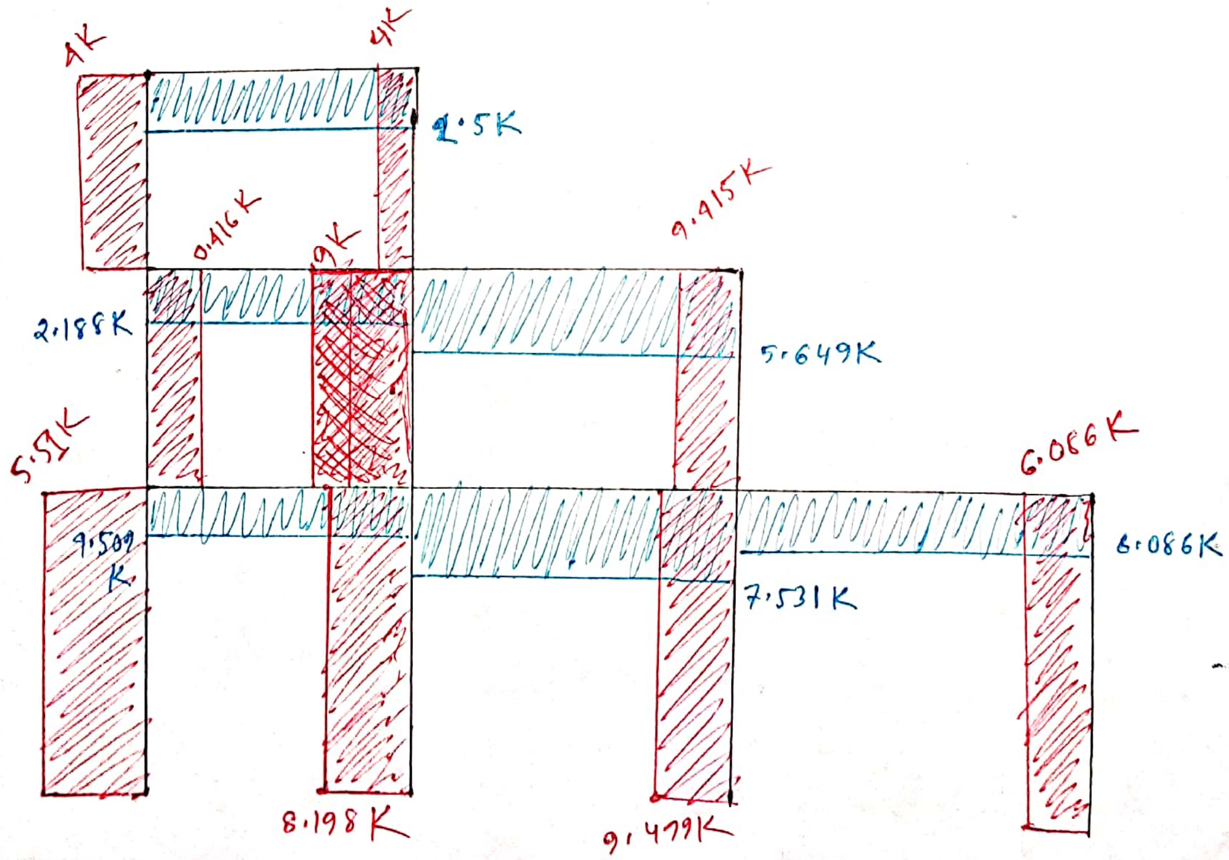
BMD for Girder & Column:



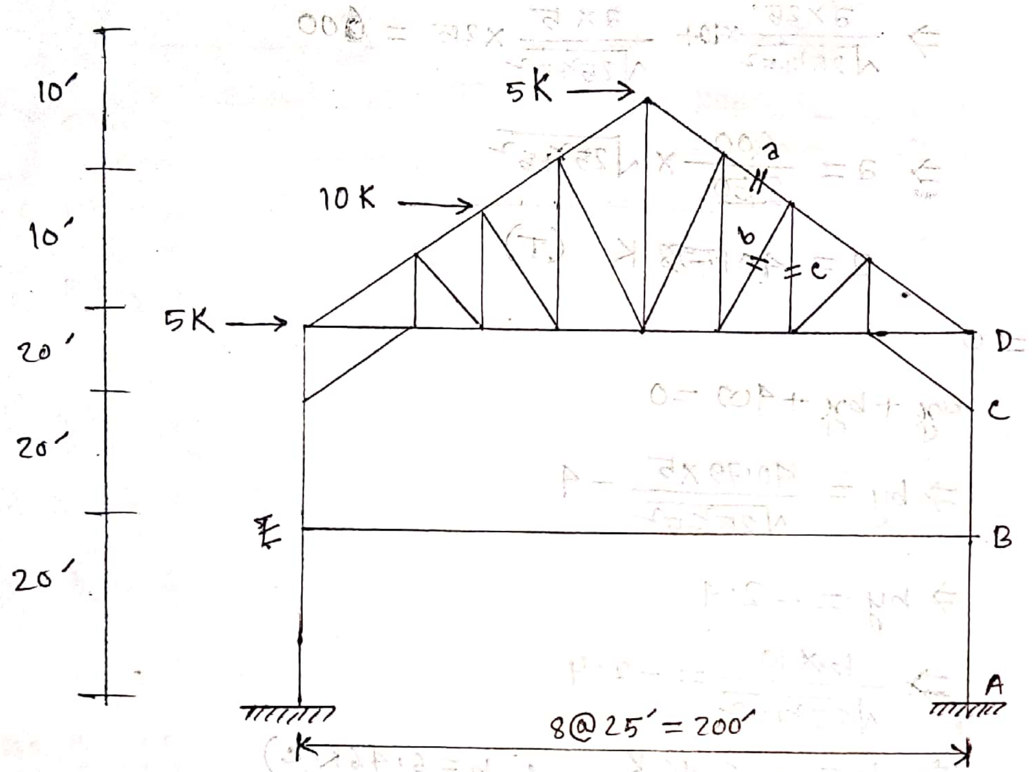
SMD for Girder & Column:

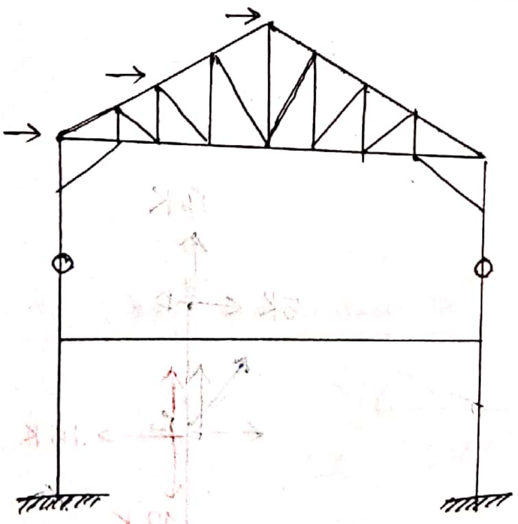


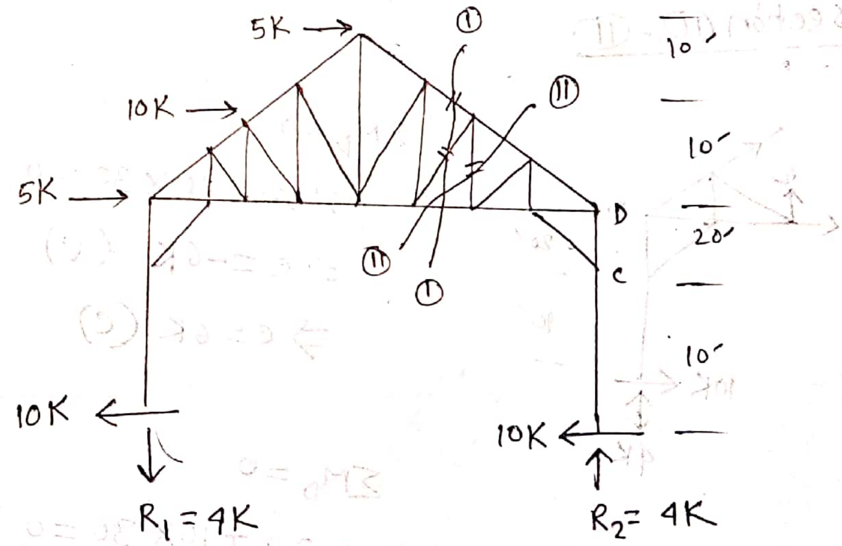
2016

Portal Frame

Determine the stresses in the member a, b and c of the mill bents as shown in figure below. Draw SFD and BMD for the members AD and BE





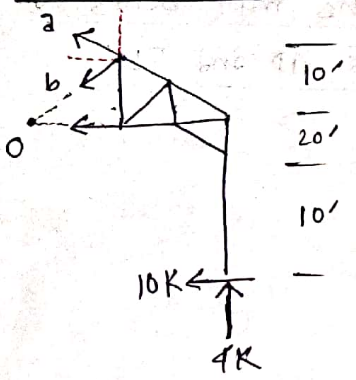


$\Sigma M_1 = 0$

$5 \times 30 + 10 \times 40 + 5 \times 50 - R_2 \times 200 = 0 \Rightarrow R_2 = 4K\ (\uparrow)$

$\therefore R_1 = 4K\ (\downarrow)$

section ① - ①



$\Sigma M_o = 0$

$a_x \times 10 + a_y \times 25 - 10 \times 30 + 4 \times 75 = 0$

$\Rightarrow \frac{a \times 25}{\sqrt{25^2+5^2}} \times 10 + \frac{a \times 5}{\sqrt{25^2+5^2}} \times 25 = 0$

$\Rightarrow a = 0K$

$\Sigma F_y = 0$

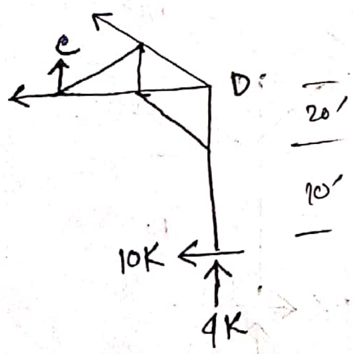
$a_y - b_y + 4 = 0$

$\Rightarrow b_y = 4K$

$\Rightarrow \frac{b \times 10}{\sqrt{25^2+10^2}} = 4$

$\therefore b = 10.77K$ (T)

section (II) - (II)

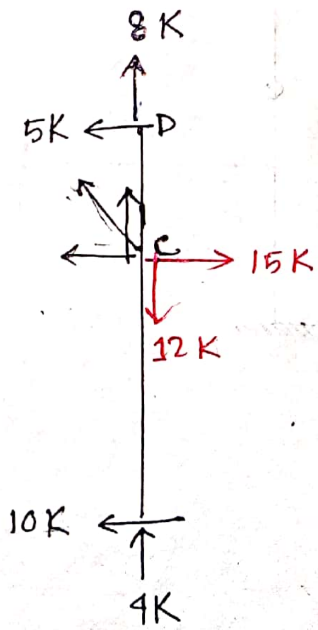


$\Sigma M_D = 0$

$c \times 50 + 10 \times 30 = 0$

$\Rightarrow c = -6K$

$\Rightarrow c = 6K$ (C)



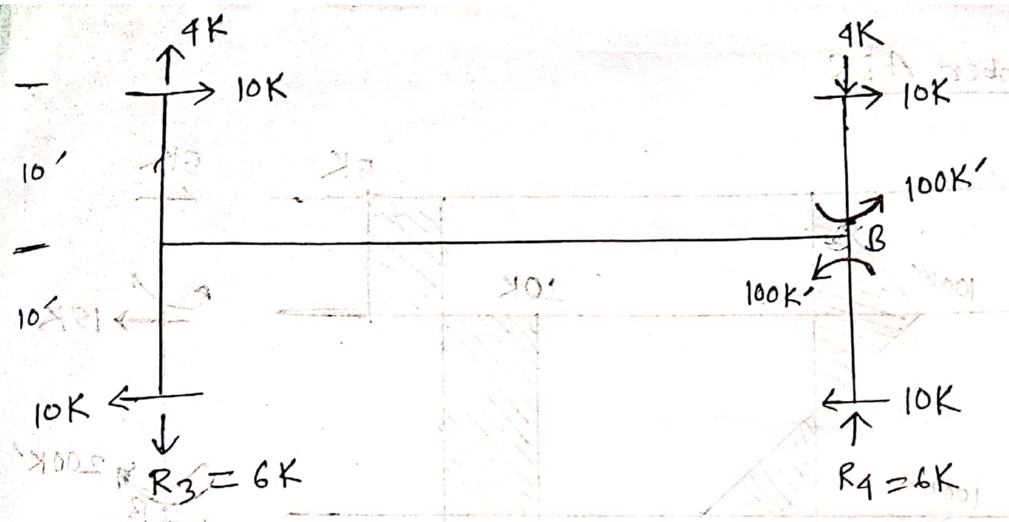
$\Sigma M_D = 0$

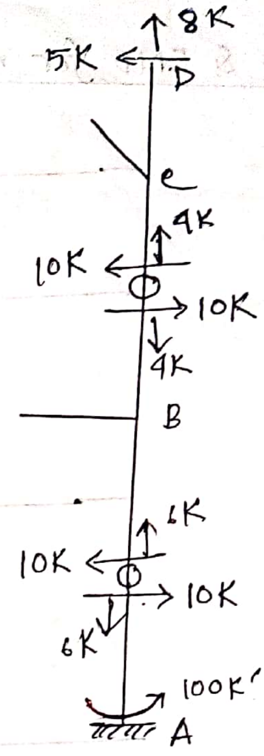
$c_x \times 20 + 10 \times 30 = 0$

$\Rightarrow c_x = -15K$ $\therefore c_x = 15K$ (→)

$c_y = \frac{-15}{25} \times 20 = -12K$

$\therefore c_y = 12K$ (↓)



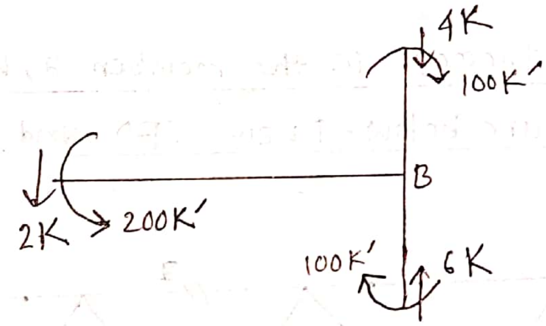


$\Sigma M_3 = 0$

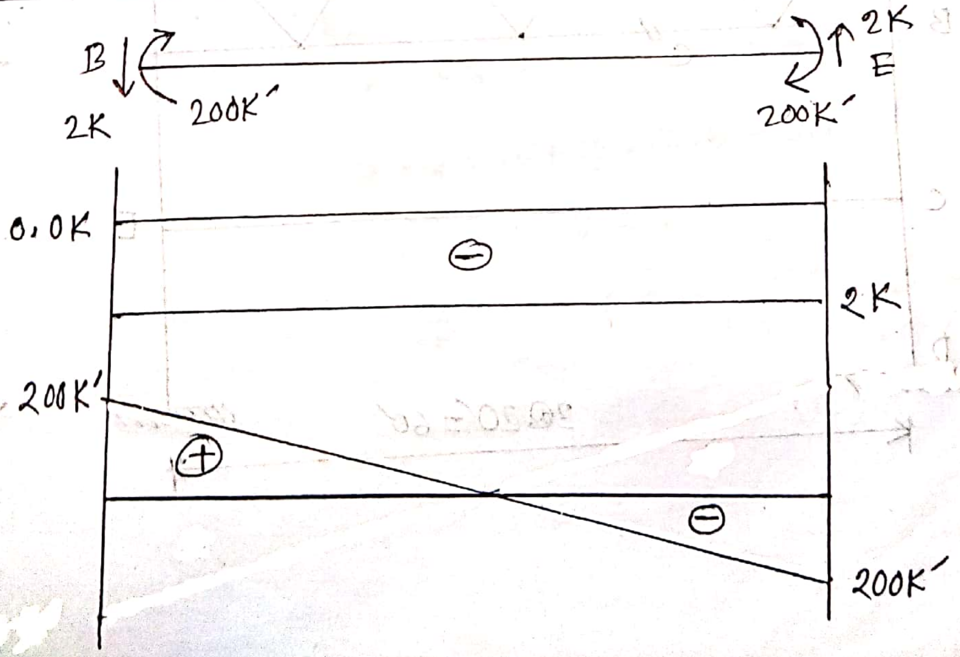
$2 \times 10 \times 20 + 4 \times 200 - R_4 \times 200 = 0$

$\Rightarrow R_4 = 6K$ (↑) $\therefore R_3 = 6K$ (↓)

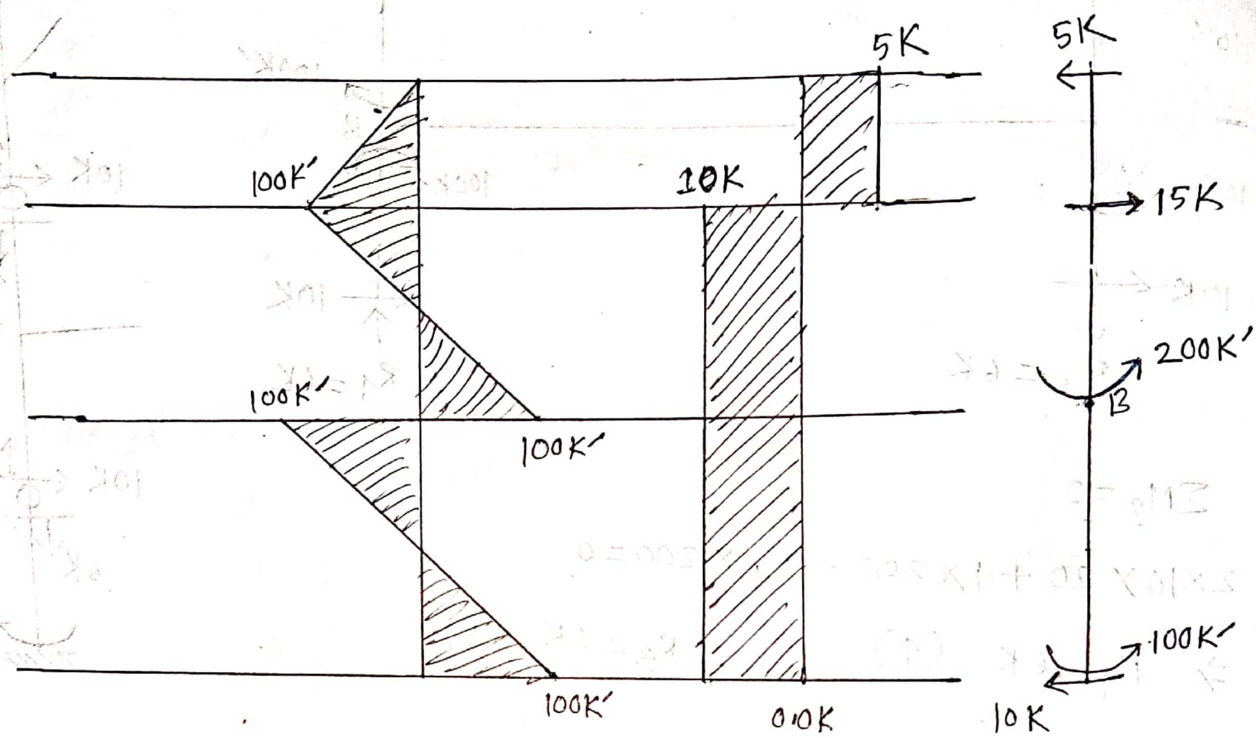
considering point B:



SFD & BMD for member BE:

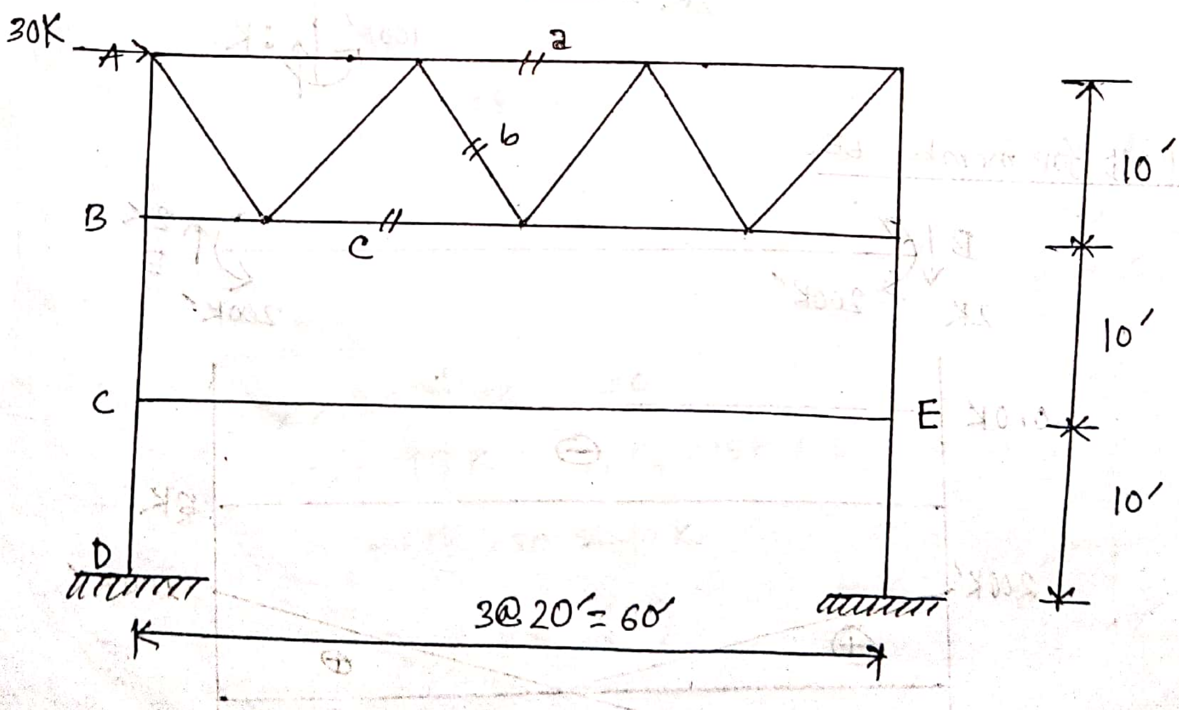


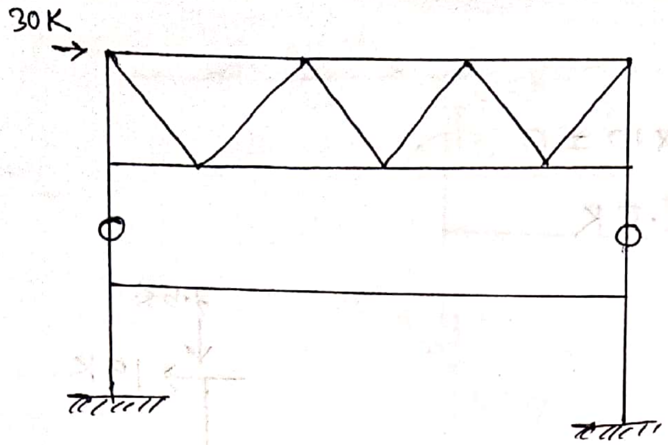
SFD & BMD for member AD:

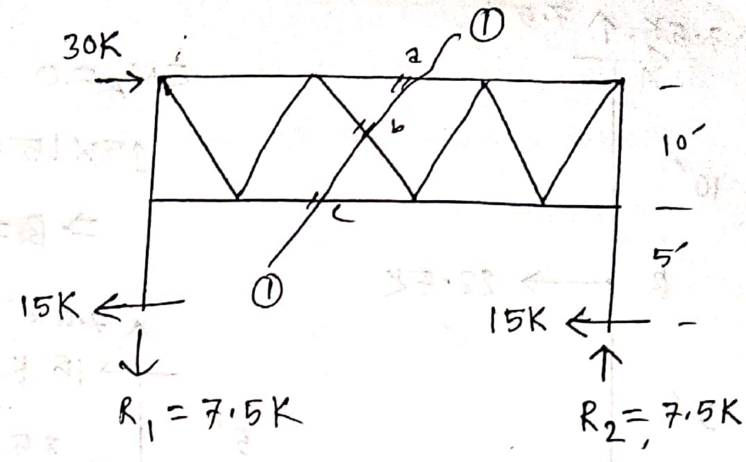


2015

Determine the stresses in the member a, b & c of the portal frame as shown in figure below. Draw SFD and BMD for the members AD and CE





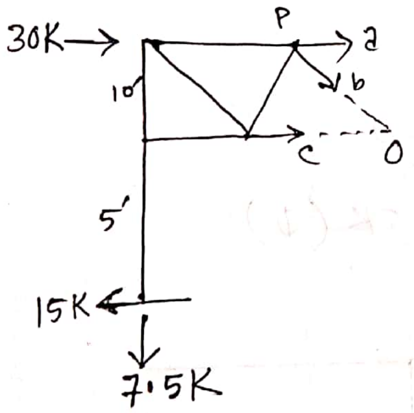


$\Sigma M_1 = 0$

$30 \times 15 - R_2 \times 60 = 0$

$\Rightarrow R_2 = 7.5K \ (\uparrow)$ $\quad \therefore R_1 = 7.5K \ (\downarrow)$

<u>Section ①-①</u>



$\Sigma M_O = 0$

$a \times 10 + 15 \times 5 - 7.5 \times 30 + 30 \times 10 = 0$

$\Rightarrow a = -15\ K$

$\therefore a = 15K\ (C)$

$\Sigma M_P = 0$

$c \times 10 \quad 15 \times 15 \quad 7.5 \times 20 = 0$

$\Rightarrow c = 7.5\ K\ (T)$

$\Sigma F_x = 0$

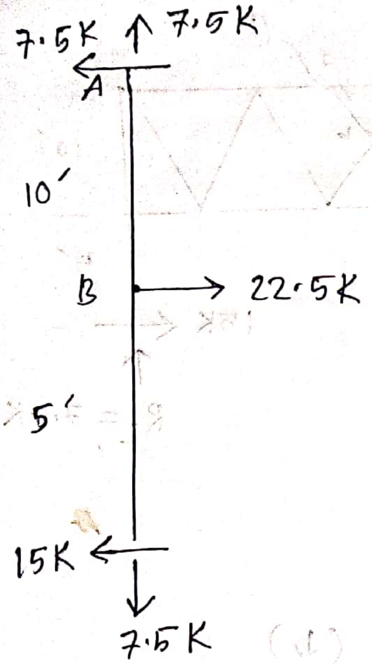
$b_x + a + c + 30 - 15 = 0$

$\Rightarrow b_x = 15 + 15 - 7.5 - 30$

$\Rightarrow b_x = -7.5\ K$

$\Rightarrow b = -7.5 \times \dfrac{\sqrt{10^2 + 10^2}}{10} = -10.6066\ K$

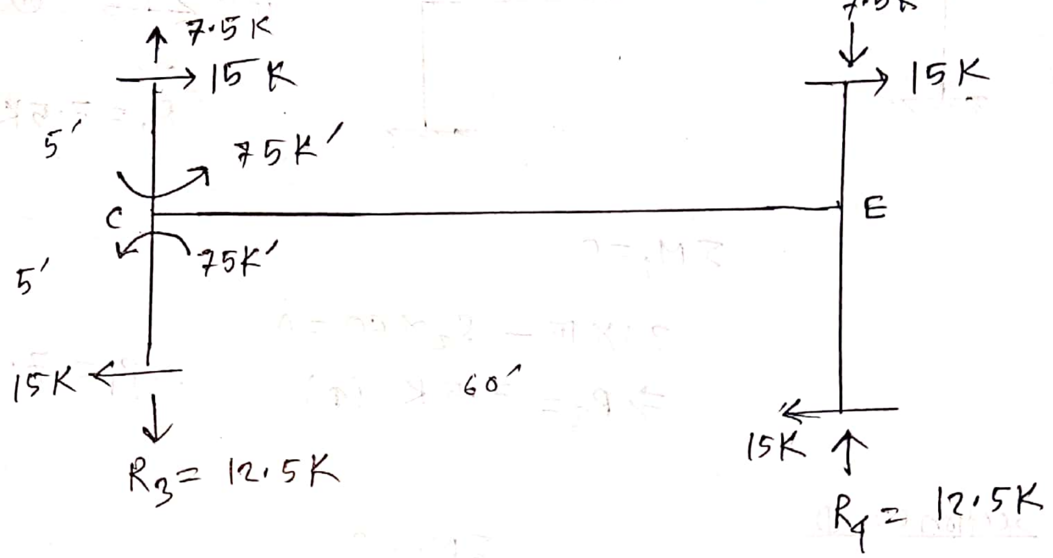
$\therefore b = 10.6066K\ (C)$

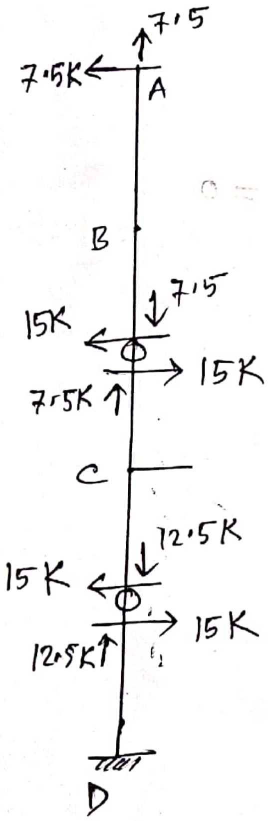


$\Sigma M_A = 0$

$15 \times 15 - B \times 10 = 0$

$\Rightarrow B = 22.5\,K$



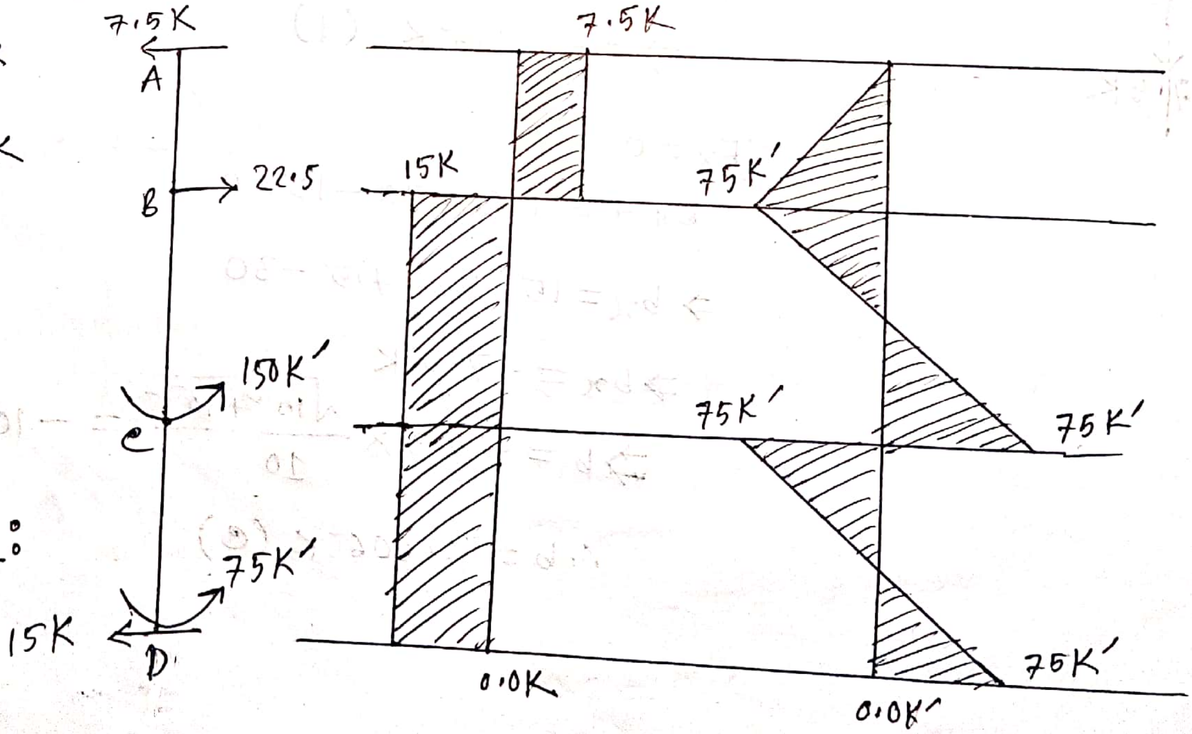


$\Sigma M_3 = 0$

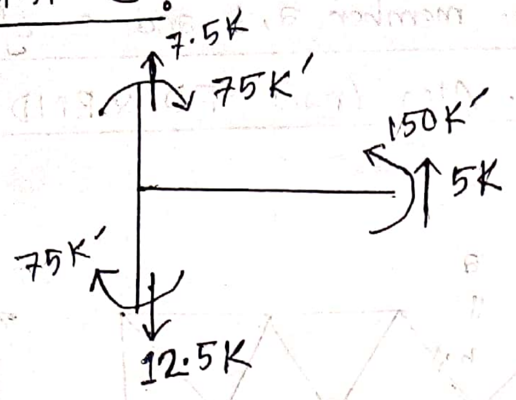
$2 \times 15 \times 10 + 7.5 \times 60 - R_4 \times 60 = 0$

$\Rightarrow R_4 = 12.5K\,(\uparrow) \quad \therefore R_3 = 12.5K\,(\downarrow)$

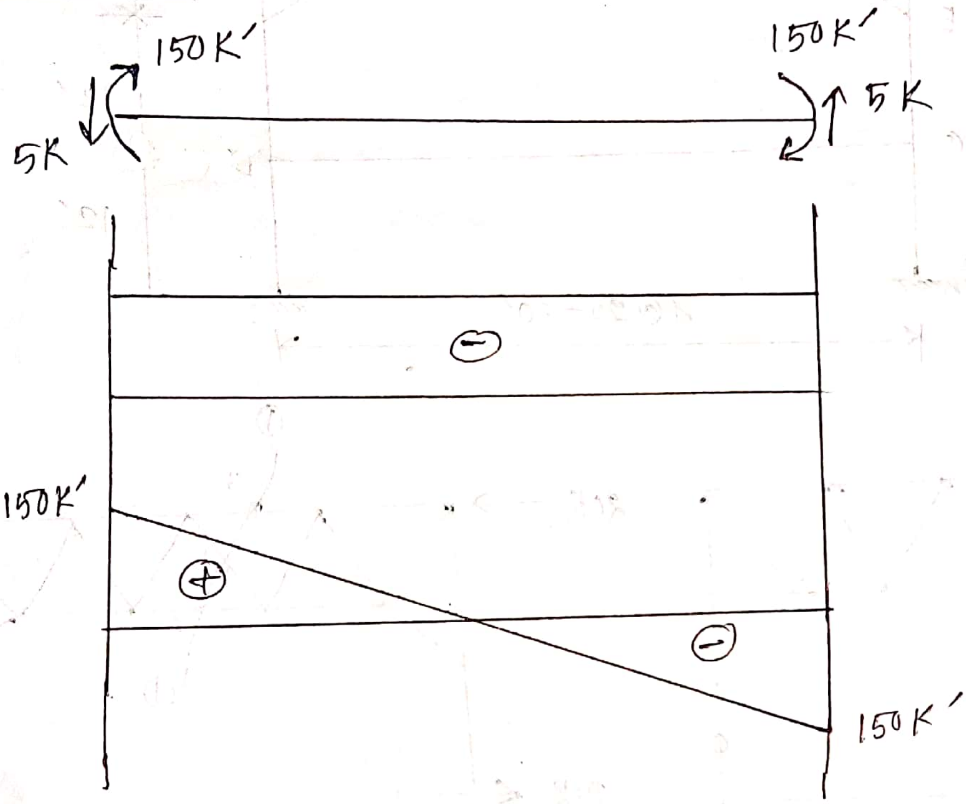
<u>SFD & BMD</u>
<u>for</u>
<u>Member AD:</u>



considering point c:

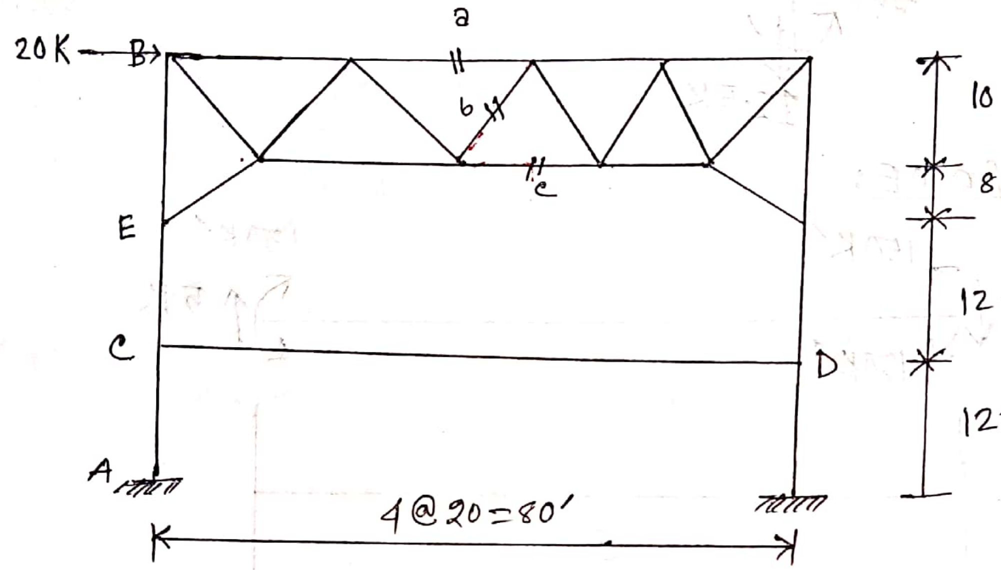


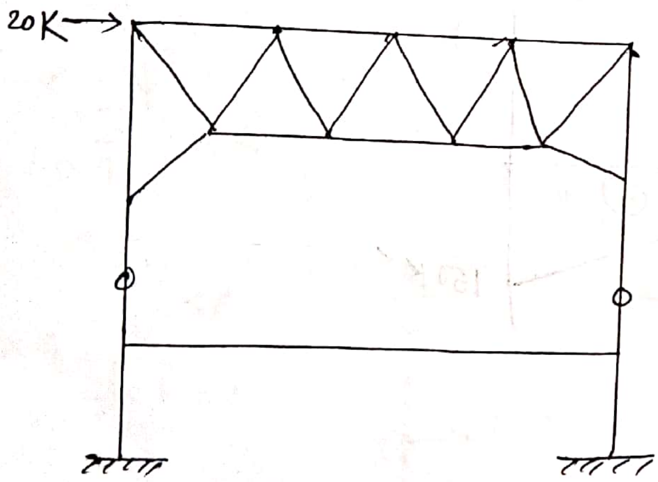
SFD & BMD for CE:

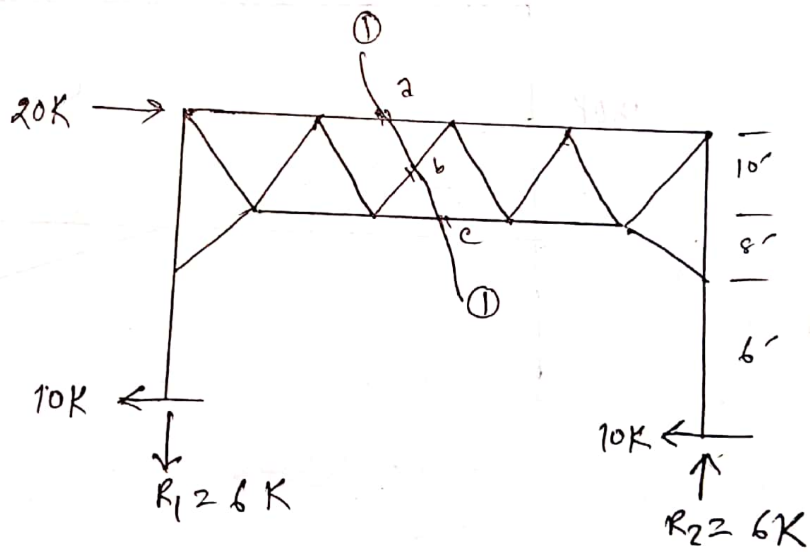


2011

Determine the stresses in the member a, b and c of the portal frame shown in figure below. Also draw SFD & BMD for the vertical column AB.



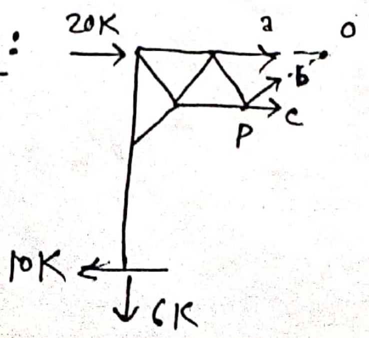




$\Sigma M_1 = 0$

$20 \times 24 - R_2 \times 80 = 0 \Rightarrow R_2 = 6K\ (\uparrow)\ \therefore R_1 = 6K\ (\downarrow)$

Section ①-①:



$\Sigma M_o = 0$

$c \times 10 - 10 \times 24 + 6 \times 40 = 0$

$\Rightarrow c = 0\ K$

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$\Sigma M_P = 0$

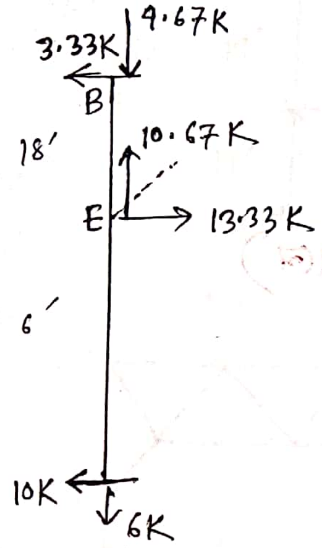
$a \times 10 + 20 \times 10 + 10 \times 14 - 6 \times 30 = 0$

$\Rightarrow a = -16K \qquad \therefore a = 16K \; (C)$

$\Sigma F_x = 0$

$b_x + c + a + 20 - 10 = 0$

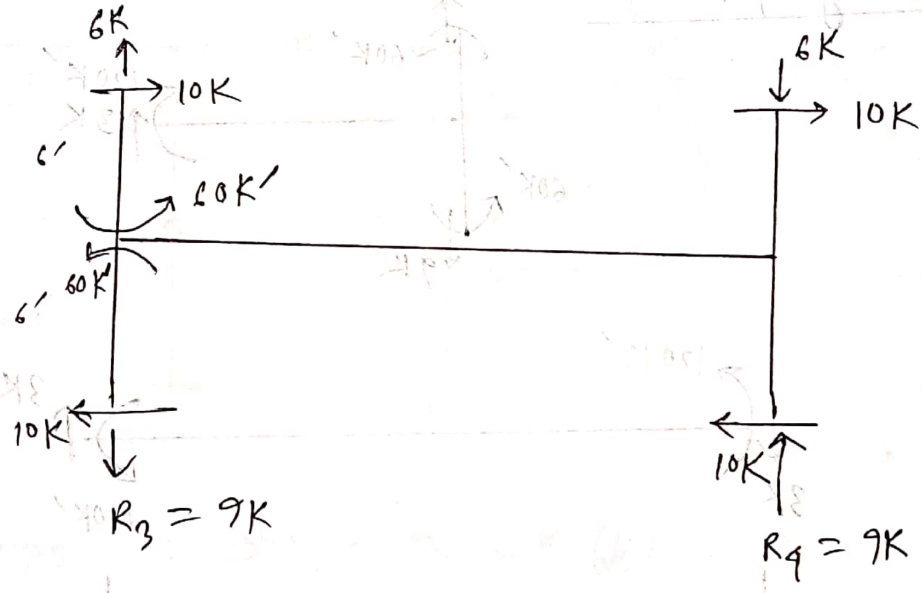
$\Rightarrow b_x = 6K \qquad \therefore b = 6 \times \frac{\sqrt{10^2 + 10^2}}{10} = 8.485 \, K \; (T)$

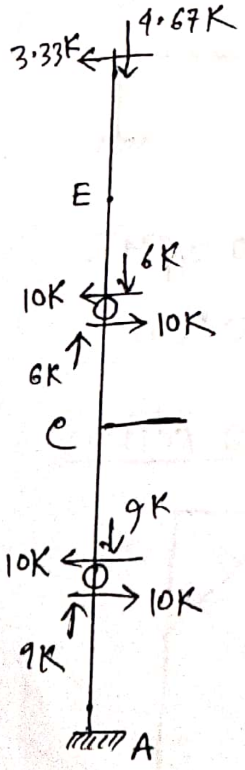


$\Sigma M_B = 0$

$E_x \times 18 - 10 \times 24 = 0 \quad \Rightarrow E_x = 13.33K$

$\therefore E_y = \frac{13.33}{10} \times 8 = 10.67K$



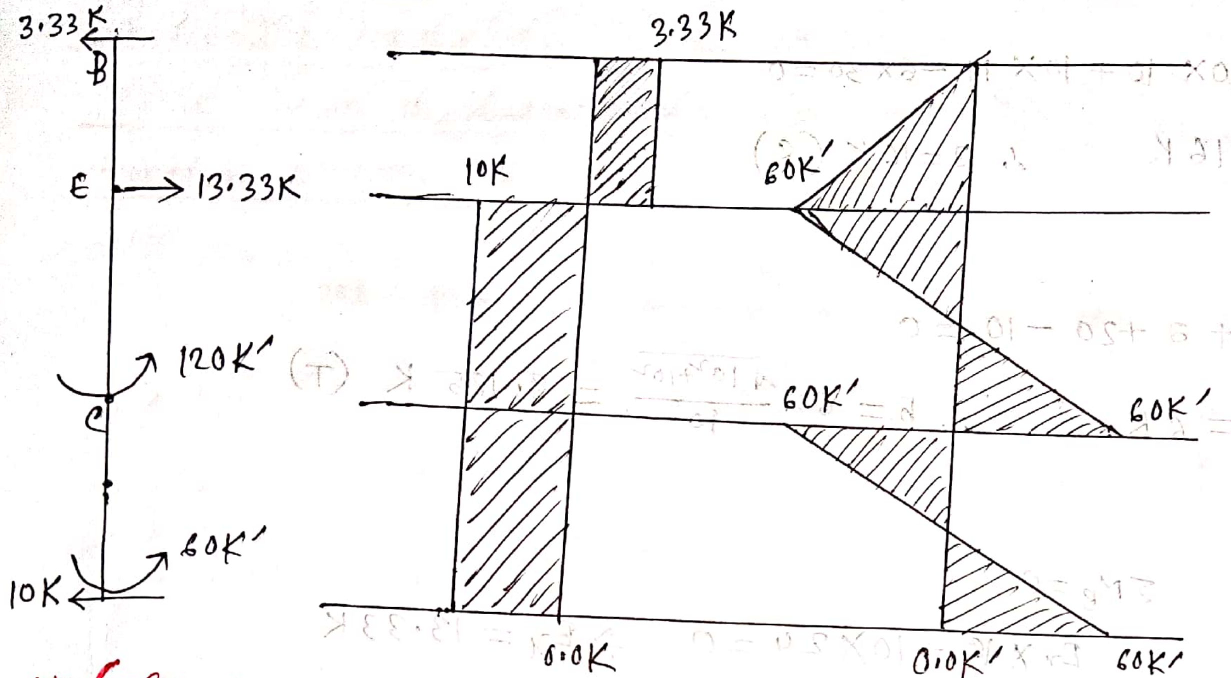


$\Sigma M_3 = 0$

$2 \times 10 \times 12 + 6 \times 80 - R_4 \times 80 = 0 \qquad \therefore R_4 = 9K \; (\uparrow)$

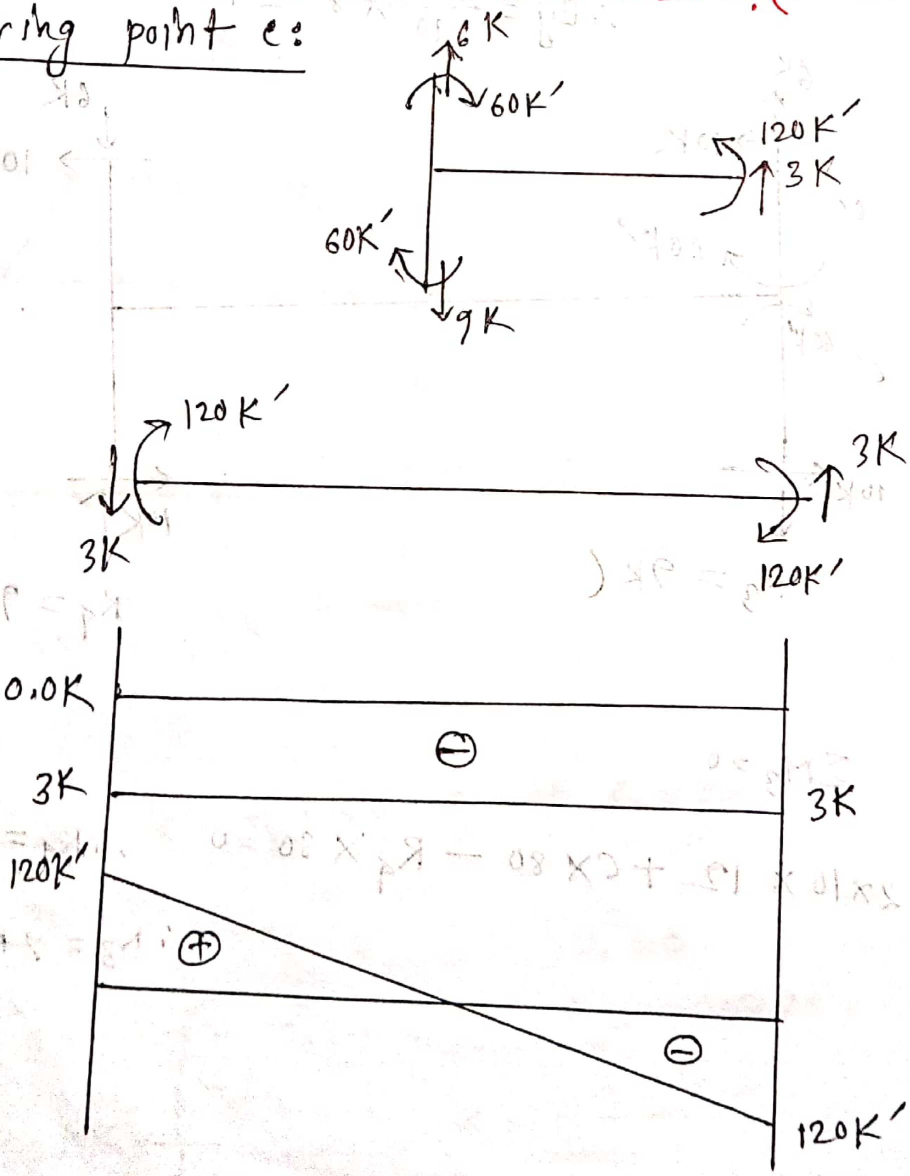
$\therefore R_3 = 9K \; (\downarrow)$

SFD & BMD for AB:



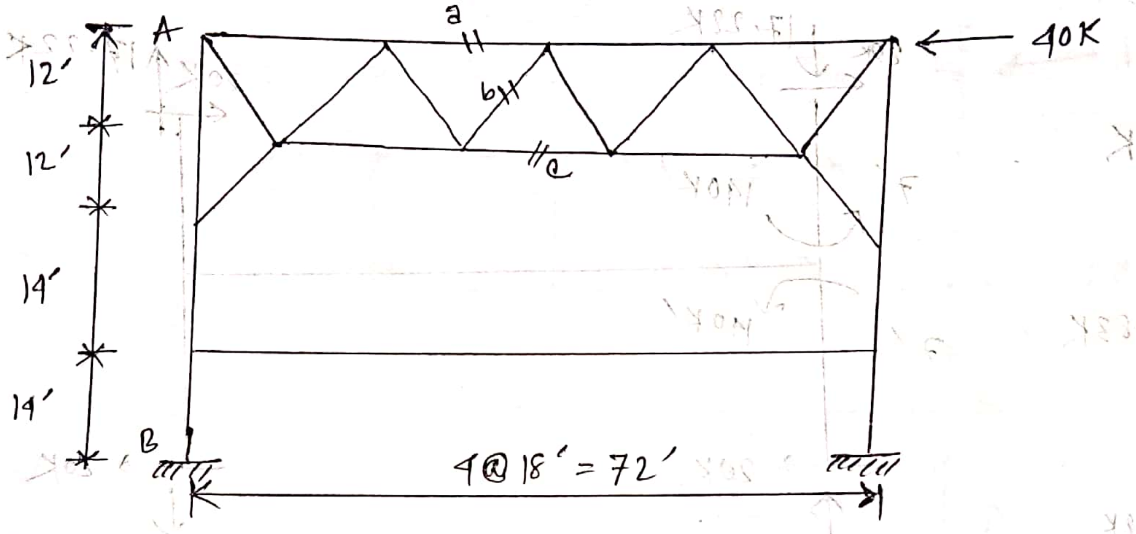
(যদি CD member এর SFD & BMD draw করতে বলে তাহলে)

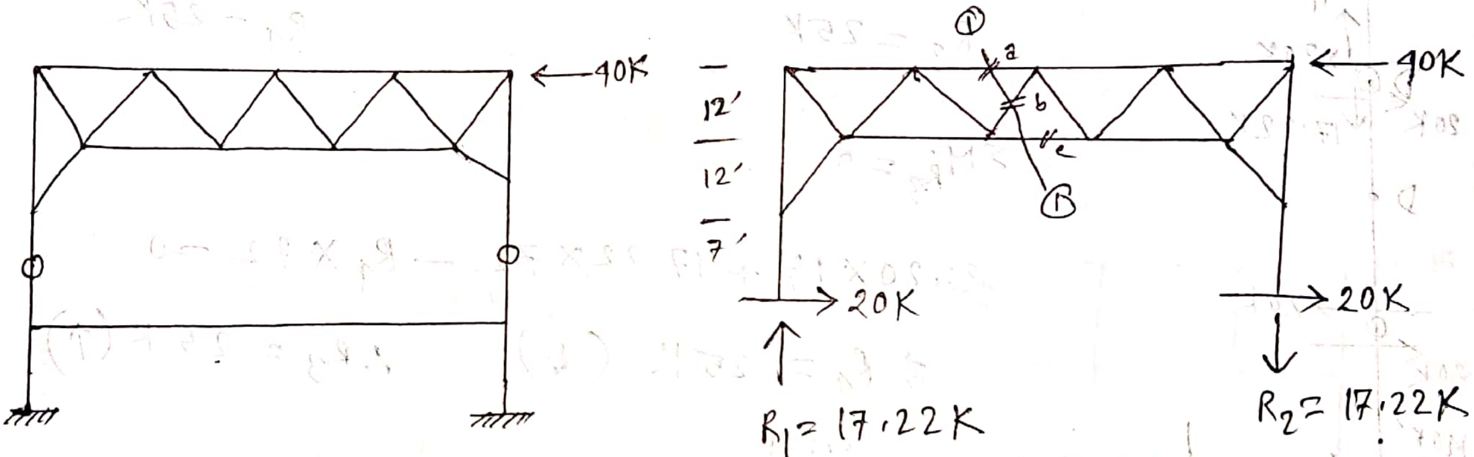
considering point C:



2008

Determine the stresses in the members a, b and c of the portal frame shown in figure below. Also draw the SFD and BFD for vertical column AB

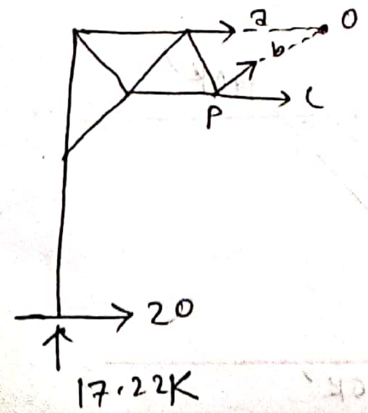




$\Sigma M_1 = 0$

$40 \times 31 - R_2 \times 72 = 0 \Rightarrow R_2 = 17.22 K (\downarrow)$ $\therefore R_1 = 17.22 K (\uparrow)$

section ①-①



$\Sigma M_0 = 0$

$c \times 12 + 20 \times 31 - 17.22 \times 36 = 0$

$\Rightarrow c = 0 K$

$\Sigma M_P = 0$

$a \times 12 - 20 \times 21 + 17.22 \times 27 = 0$

$\Rightarrow a = -3.75 K$ $\therefore a = 3.75 K (C)$

$\Sigma F_x = 0$

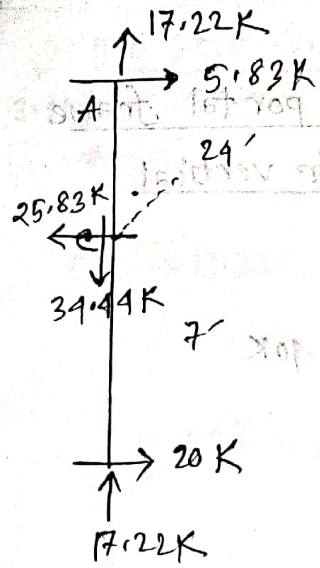
$a + b_x + c + 20 = 0$

$\Rightarrow b_x = -16.25$

$\Rightarrow b = -16.25 \times \frac{\sqrt{12^2+9^2}}{9}$

$\Rightarrow b = -27.083 K$

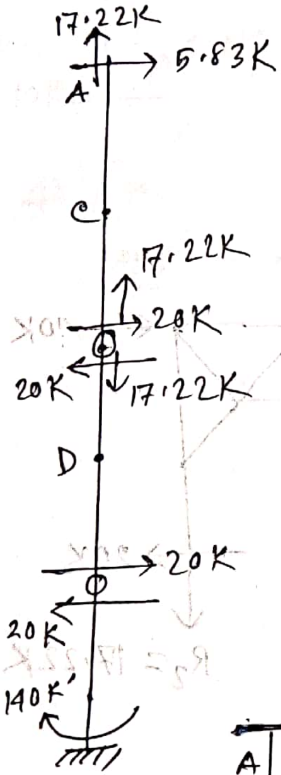
$\therefore b = 27.083 K (C)$

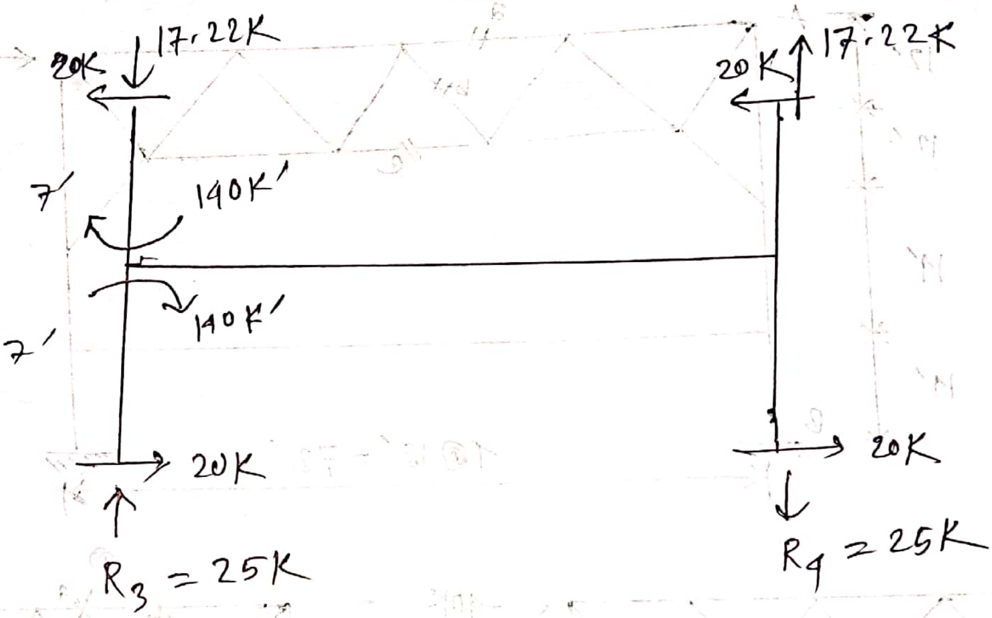


$\Sigma M_A = 0$

$C_x \times 24 - 20 \times 31 = 0 \Rightarrow C_x = 25.83\,K$

$\therefore C_y = \frac{25.83}{9} \times 12 = 34.44\,K$



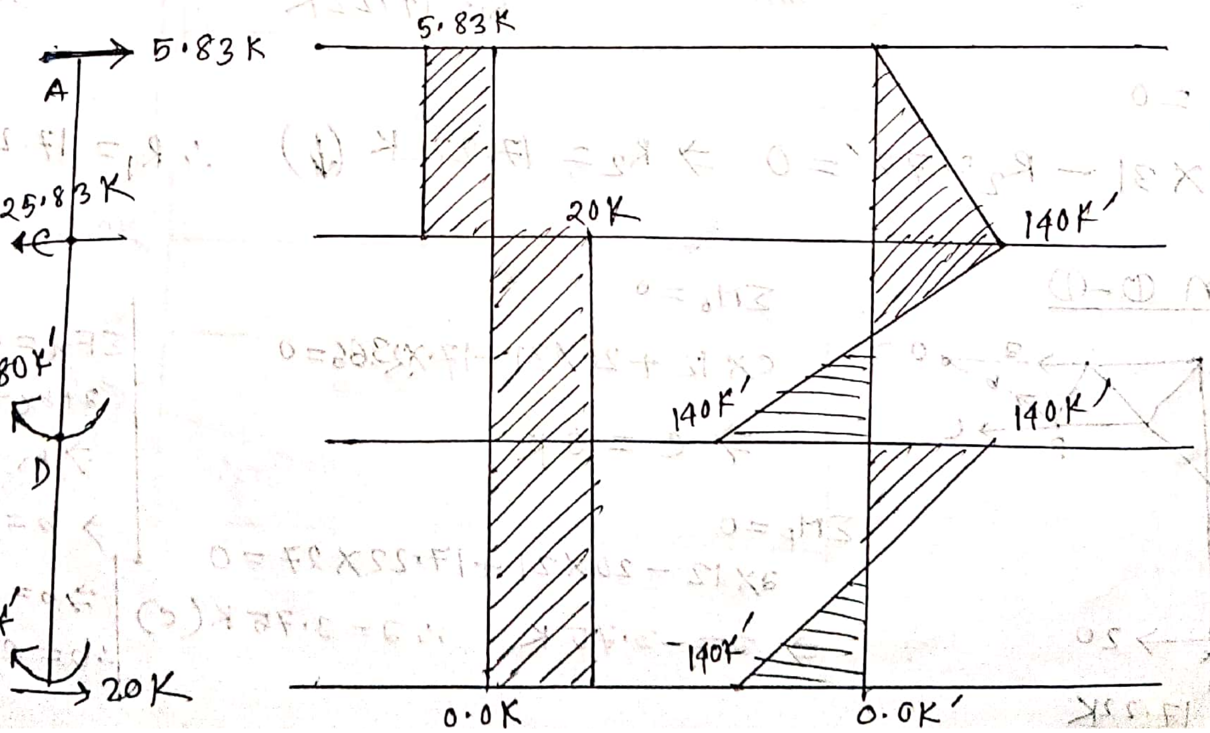


$\Sigma M_3 = 0$

$2 \times 20 \times 14 + 17.22 \times 72 - R_4 \times 72 = 0$

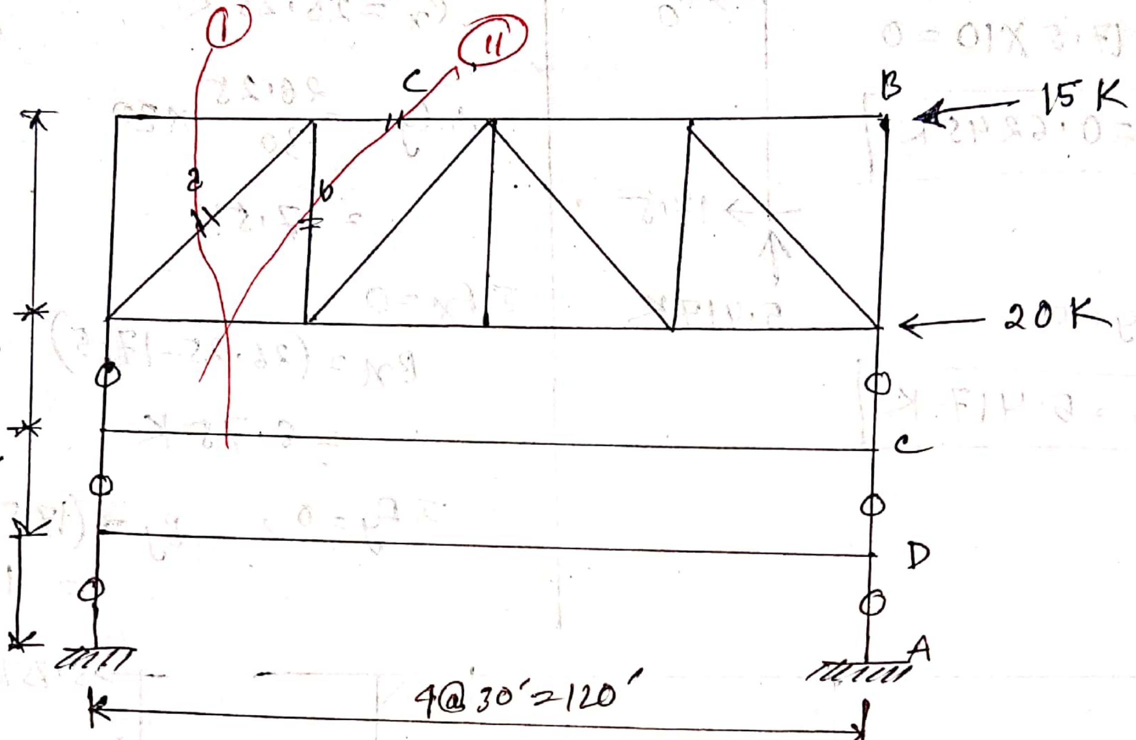
$\Rightarrow R_4 = 25\,K\ (\downarrow) \qquad \therefore R_3 = 25\,K\ (\uparrow)$

SFD & BMD for AB



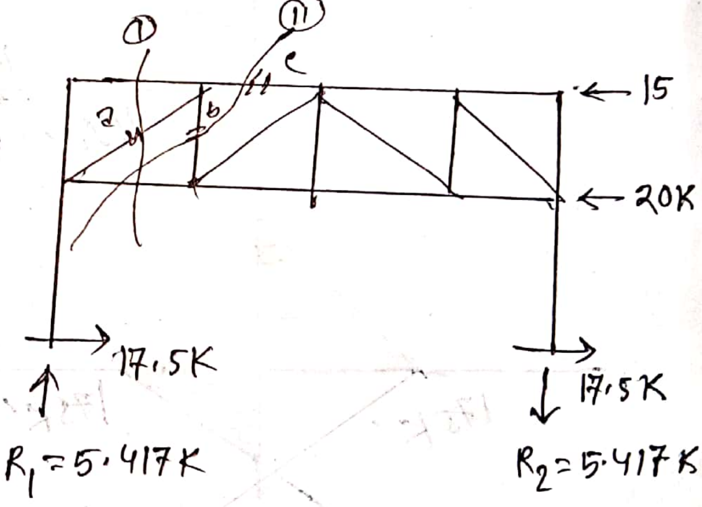
2017

Determine the stresses in the member a, b and c of the portal frame shown in figure below. Also draw SFD and BMD for the column AB.



Solution:

considering section,



$\Sigma F_y = 0$

$a_y + 5.417 = 0$

$\Rightarrow a_y = -5.417$

$\Rightarrow a = -5.417 \times \frac{\sqrt{20^2+30^2}}{20}$

$\therefore a = -9.766K$

$\therefore a = 9.766K(C)$

$\Sigma M_1 = 0$

$15 \times 30 + 20 \times 10 = R_2 \times 120$

$\therefore R_2 = 5.417K$

$\therefore R_1 = 5.417K$

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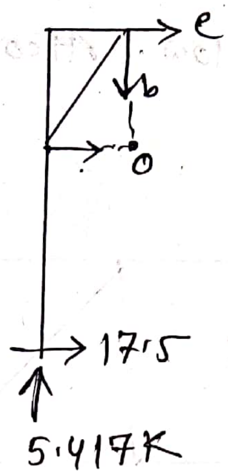
considering section (11) - (11)

$\Sigma M_o = 0$

$e \times 20 + 5.417 \times 30 - 17.5 \times 10 = 0$

$\Rightarrow e = 0.6245 K$



$\Sigma F_y = 0$

$\therefore b = 5.417 K$

$\Sigma M_B = 0$

$C_x \times 20 = 17.5 \times 30$

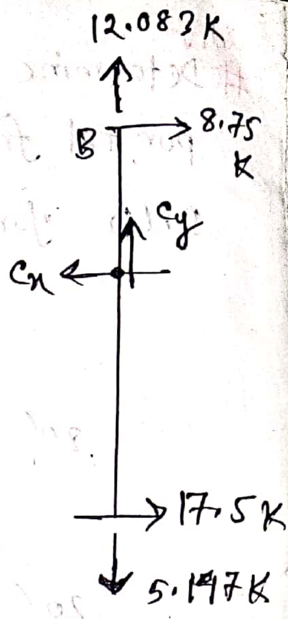
$C_x = 26.25 K$

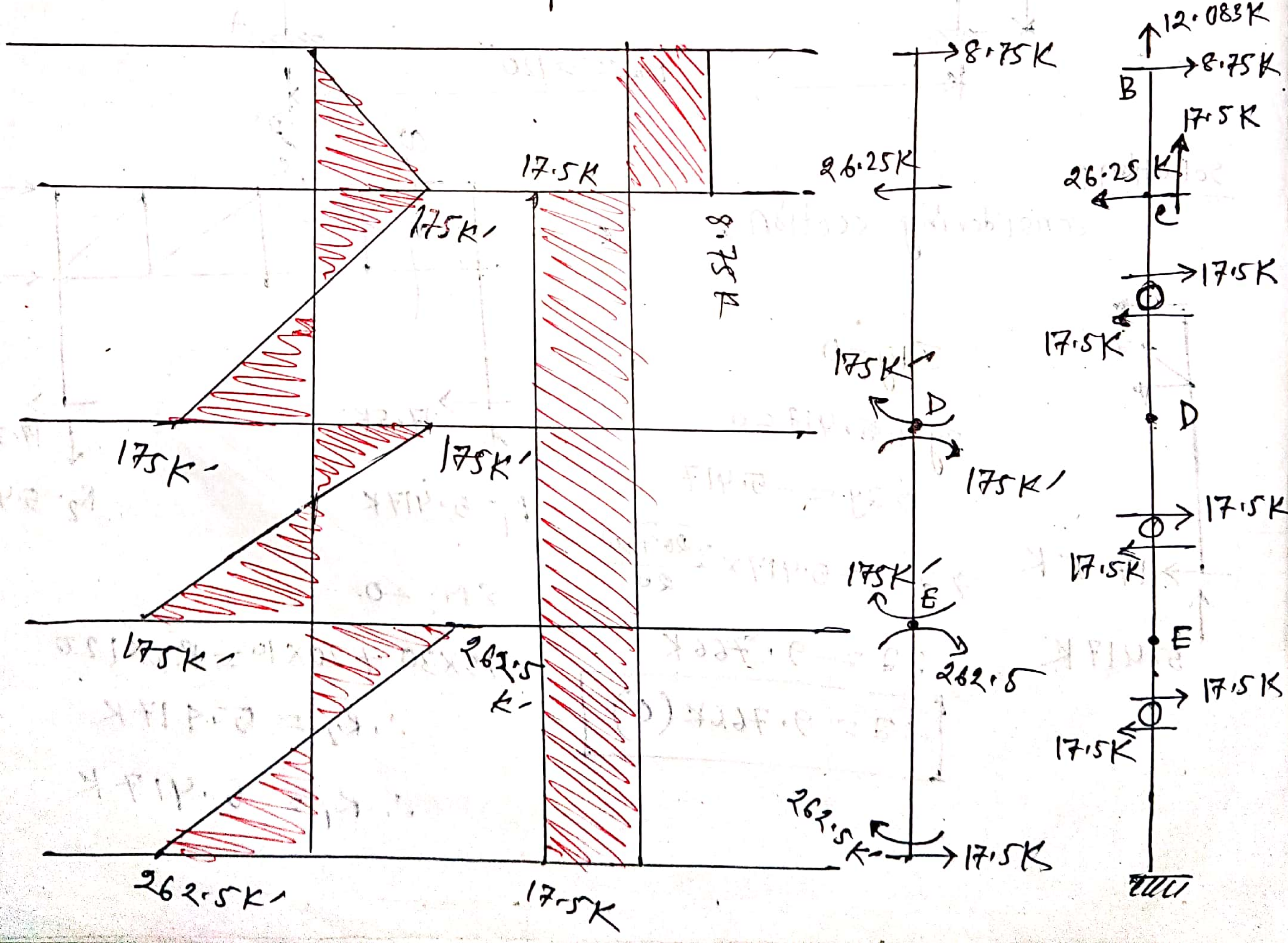
$\therefore C_y = \frac{26.25}{30} \times 20 = 17.5 K$

$\Sigma F_x = 0$

$B_x = (26.25 - 17.5) = 8.75 K$

$\Sigma F_y = 0, \quad B_y = (17.5 - 5.417) = 12.083 K$





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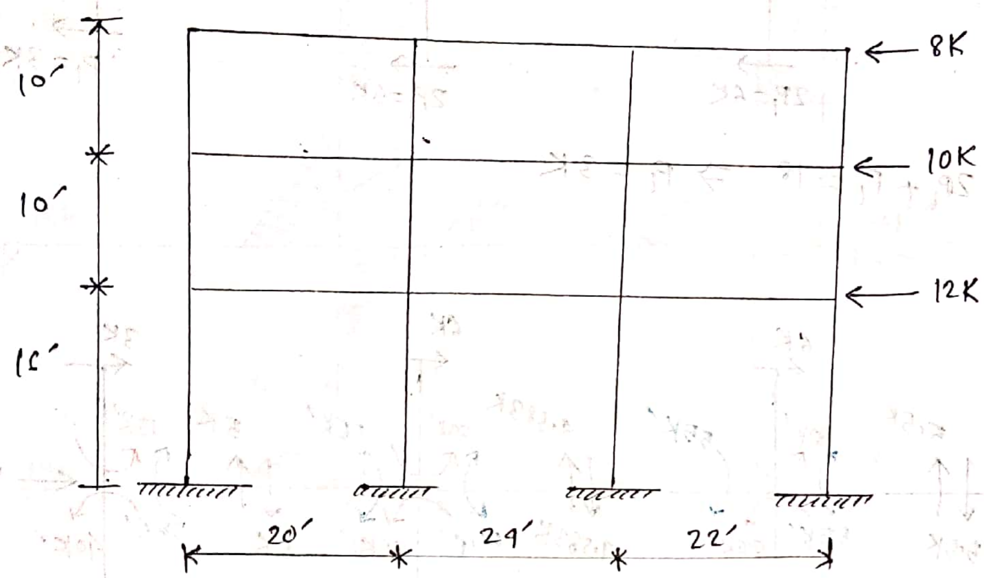
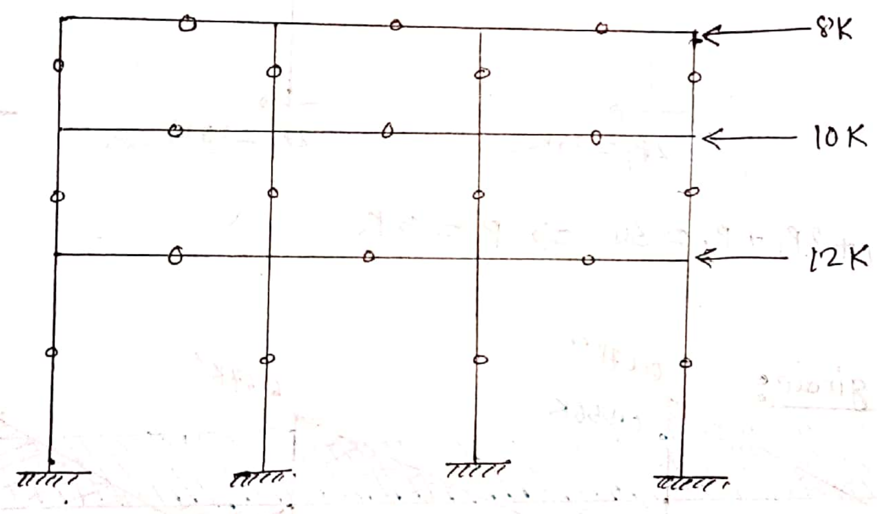
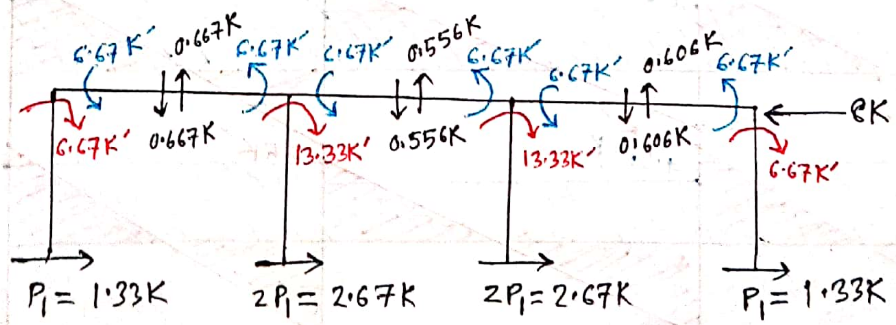
Portal Method

2019

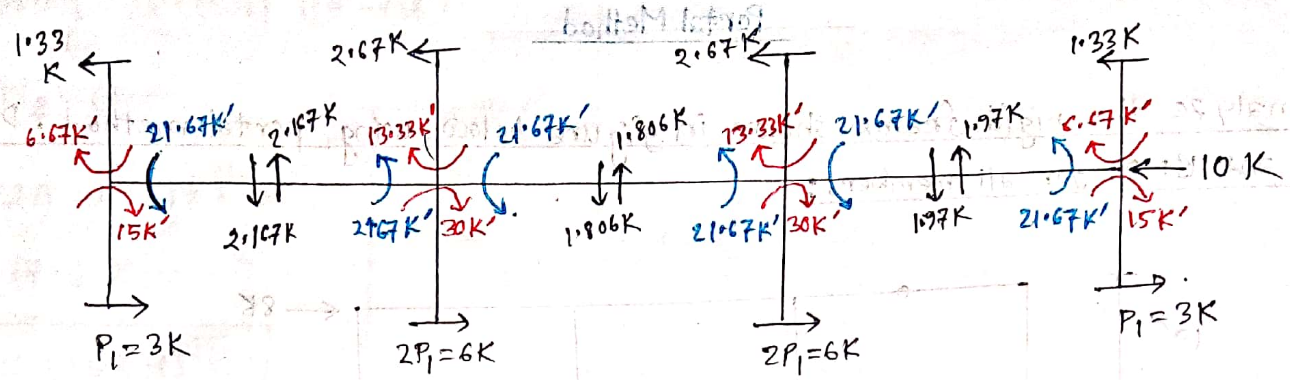
Analyze the rigid frame shown in figure below using portal method. Draw SFD and BMD for all members:



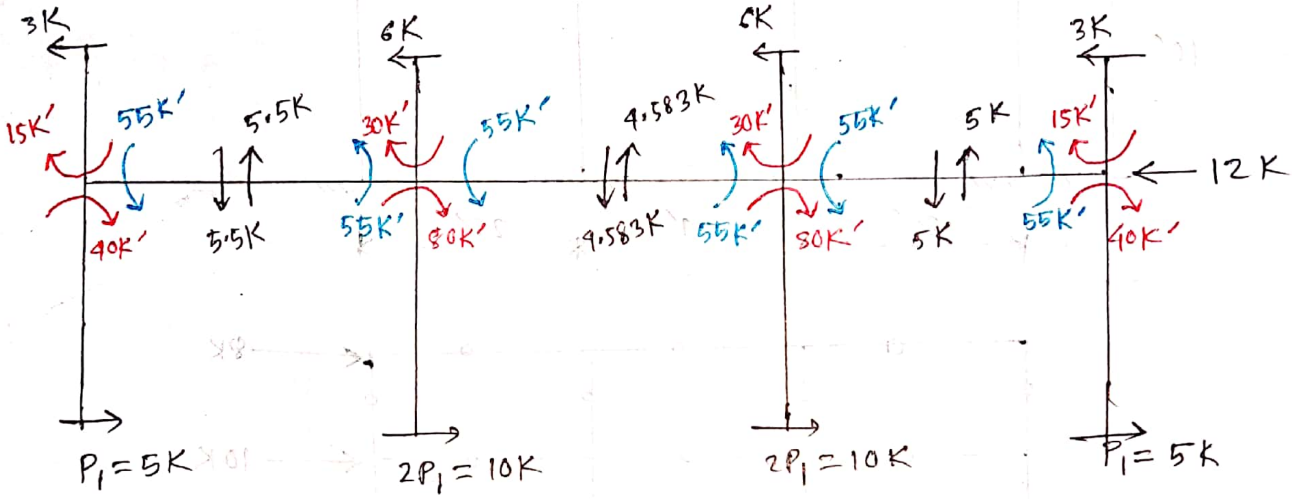




$P_1 + 2P_1 + 2P_1 + P_1 = 8 \Rightarrow P_1 = 1.33K$

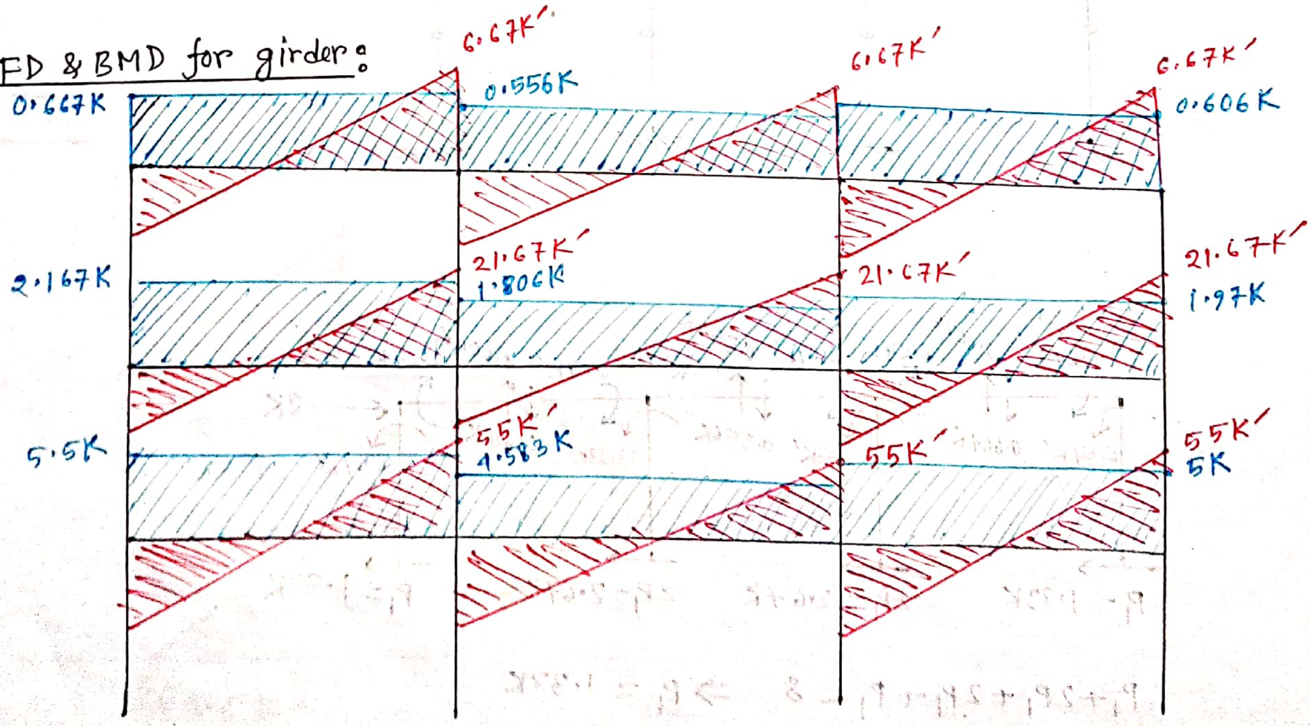


$P_1 + 2P_1 + 2P_1 + P_1 = 18 \Rightarrow P_1 = 3K$

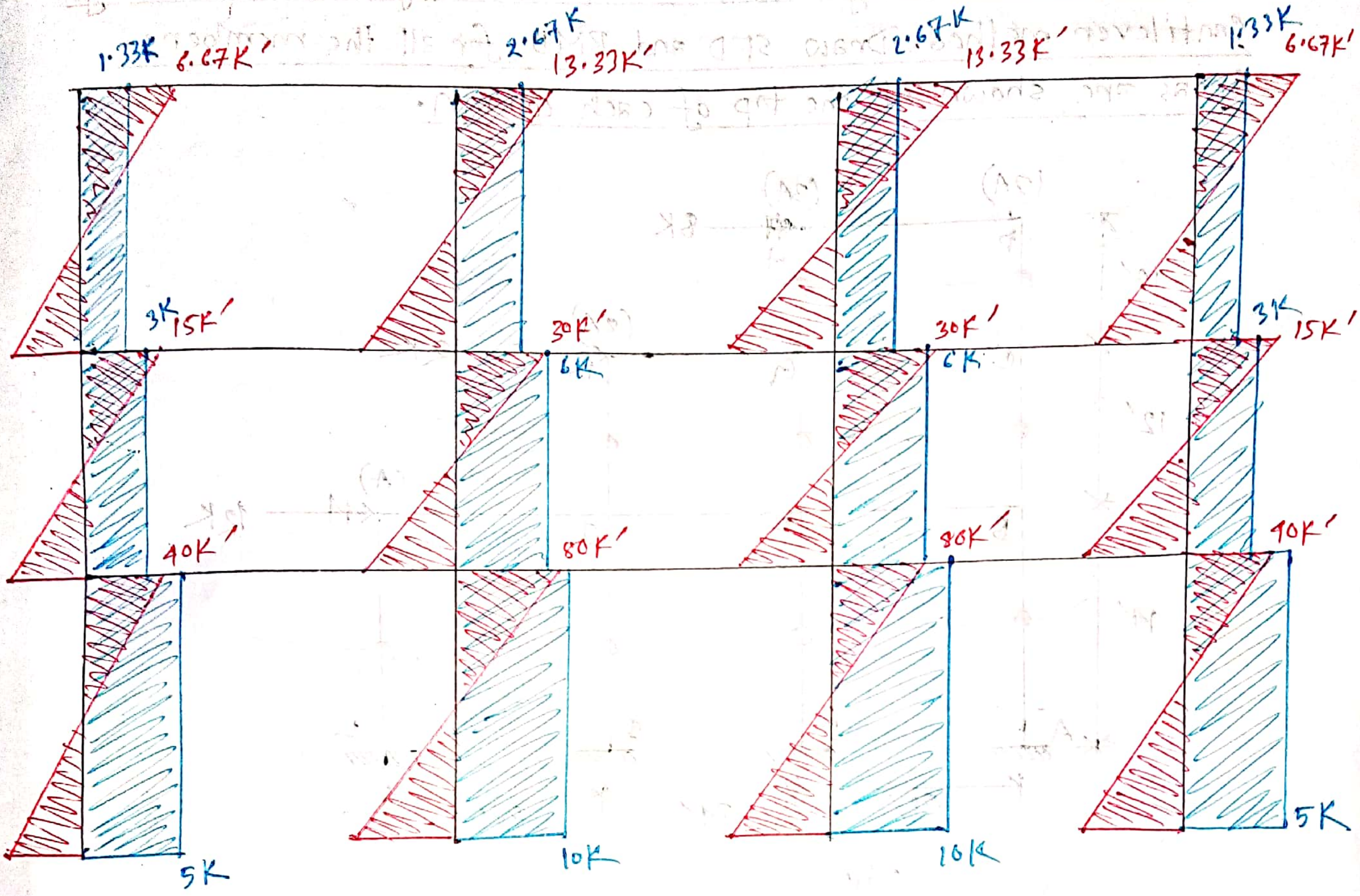


$P_1 + 2P_1 + 2P_1 + P_1 = 30 \Rightarrow P_1 = 5K$

SFD & BMD for girder:



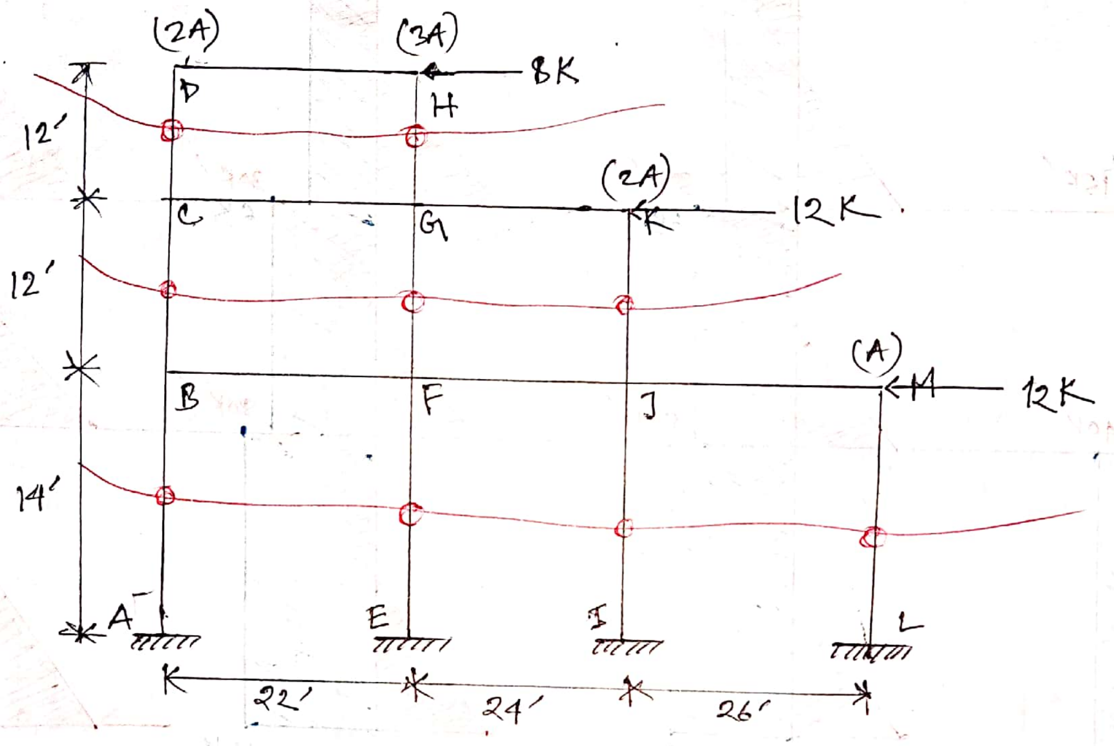
SFD & BMD for column:

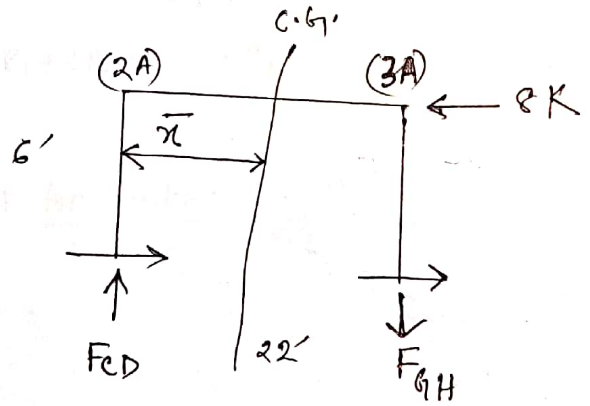


Cantilever Method

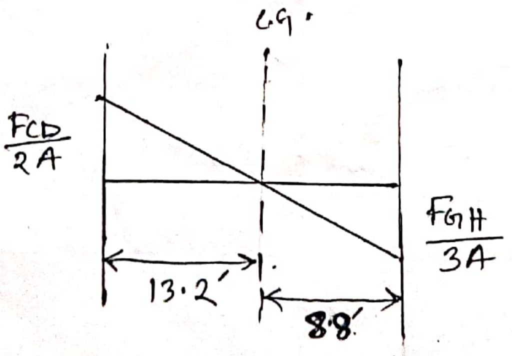
2016

Analyze the building frame shown in Figure below using Cantilever method. Draw SFD and BMD for all the members. Areas are shown at the top of each column.





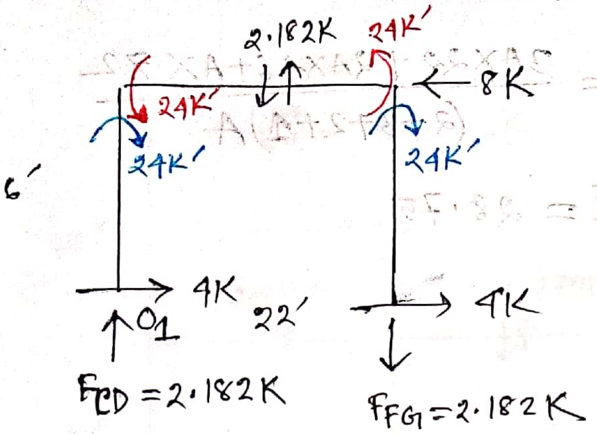
$$\bar{x} = \frac{3A \times 22}{(2+3)A} = 13.2$$



Now,

$$\frac{\frac{F_{CD}}{2A}}{13.2} = \frac{\frac{F_{GH}}{3A}}{8.8}$$

$$\Rightarrow F_{GH} = F_{CD}$$

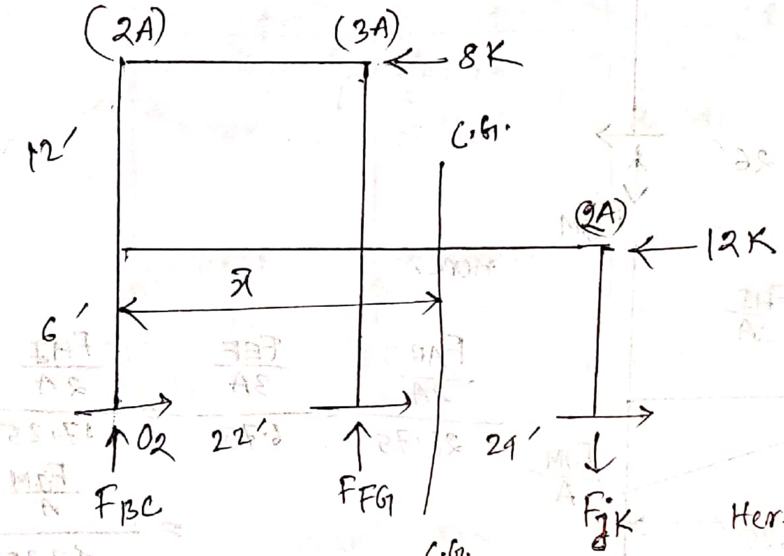


$\Sigma M_{O_1} = 0$

$8 \times 6 - F_{GH} \times 22 = 0$

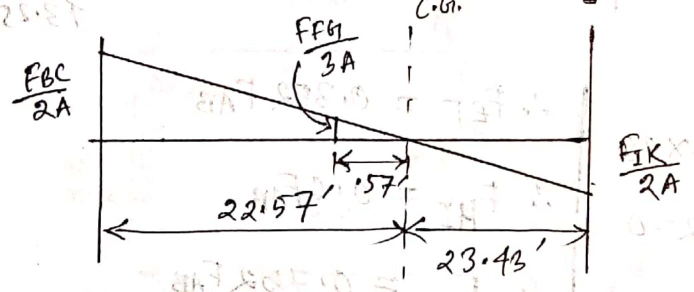
$\therefore F_{GH} = 2.182K$

$\therefore F_{CD} = 2.182K$



$$\bar{x} = \frac{3A \times 22 + 2A \times (22+24)}{(2+3+2)A}$$

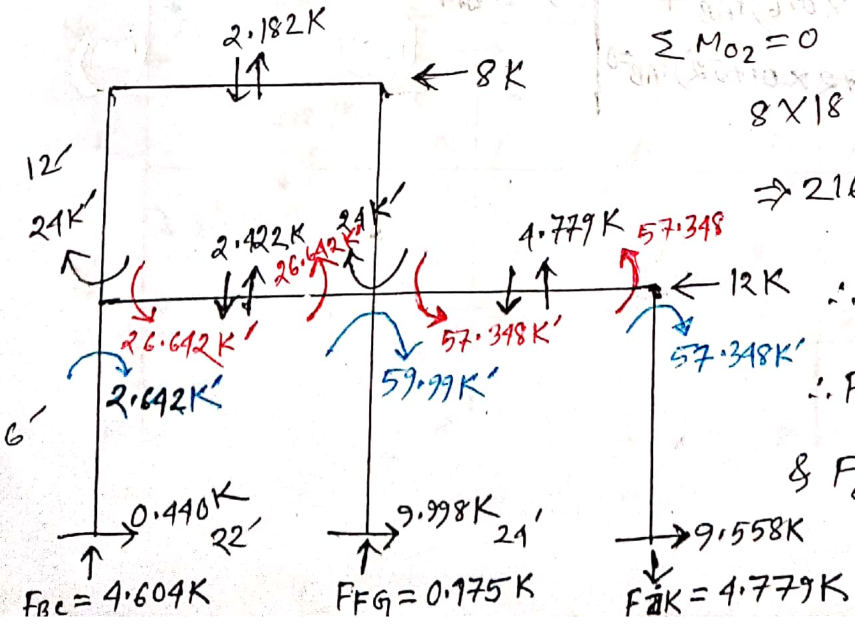
$\therefore \bar{x} = 22.57'$



Here,

$$\frac{\frac{F_{BC}}{2A}}{22.57} = \frac{\frac{F_{FG}}{3A}}{0.57} = \frac{\frac{F_{JK}}{2A}}{23.43}$$

$\therefore F_{FG} = 0.038 F_{BC}$ & $F_{JK} = 1.038 F_{BC}$



$\Sigma M_{O_2} = 0$

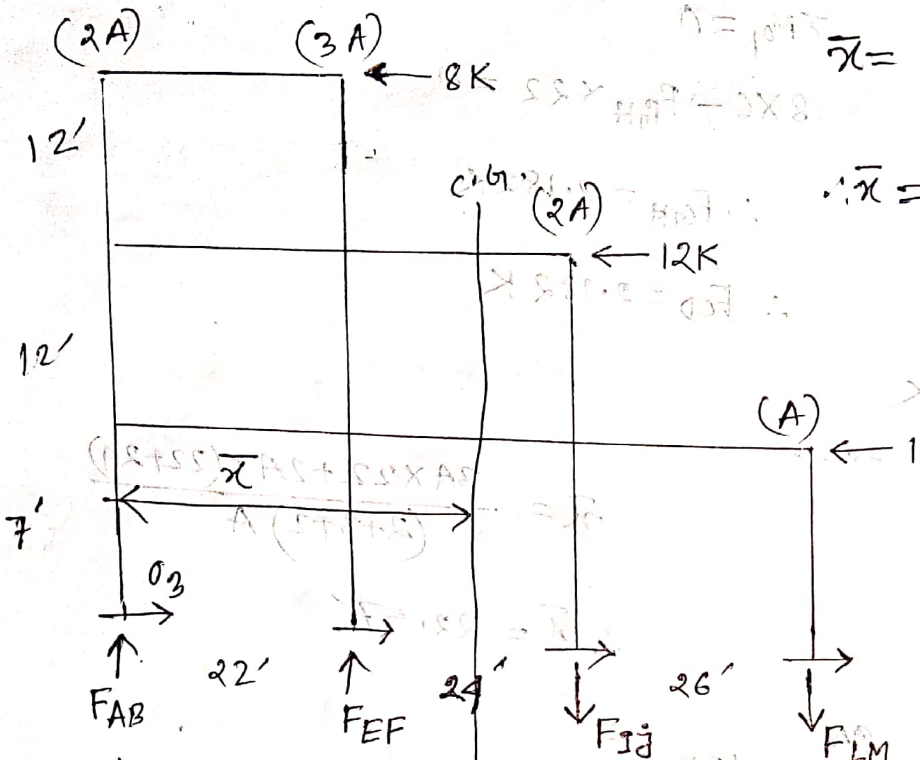
$8 \times 18 + 12 \times 6 + F_{FG} \times 22 - F_{JK} \times 46 = 0$

$\Rightarrow 216 + (0.038 \times 22) \times F_{BC} - (46 \times 1.038) F_{BC} = 0$

$\therefore F_{BC} = 4.604K$

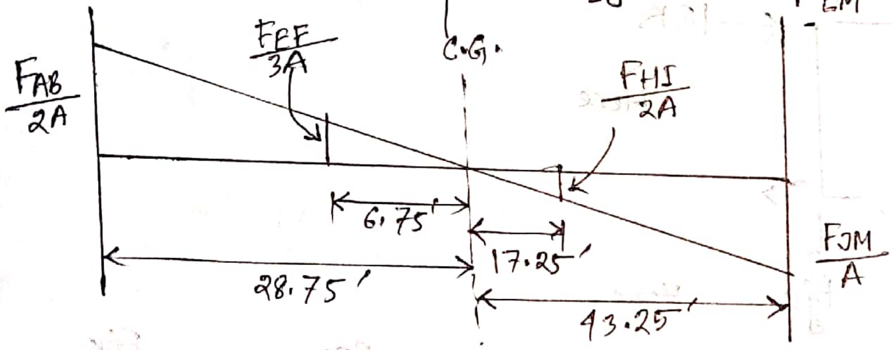
$\therefore F_{FG} = 0.175K$

& $F_{JK} = 4.779K$



$$\bar{x} = \frac{3A \times 22 + 2A \times 46 + A \times 72}{(2+3+2+1)A}$$

$$\therefore \bar{x} = 28.75$$



Here,

$$\frac{\frac{F_{AB}}{2A}}{28.75} = \frac{\frac{F_{EF}}{3A}}{6.75} = \frac{\frac{F_{IJ}}{2A}}{17.25} = \frac{\frac{F_{LM}}{A}}{43.25}$$

$\Sigma M_{O_3} = 0$

$8 \times 31 + 12 \times 19 + 12 \times 7 - F_{EF} \times 22 + F_{IJ} \times 46 + F_{LM} \times 72 = 0$

$\Rightarrow 560 - (0.352 \times 22) \times F_{AB} + (46 \times 0.6) F_{AB} + (72 \times 0.752) F_{AB} = 0$

$\therefore F_{EF} = 0.352 F_{AB}$

$\therefore F_{IJ} = 0.6 F_{AB}$

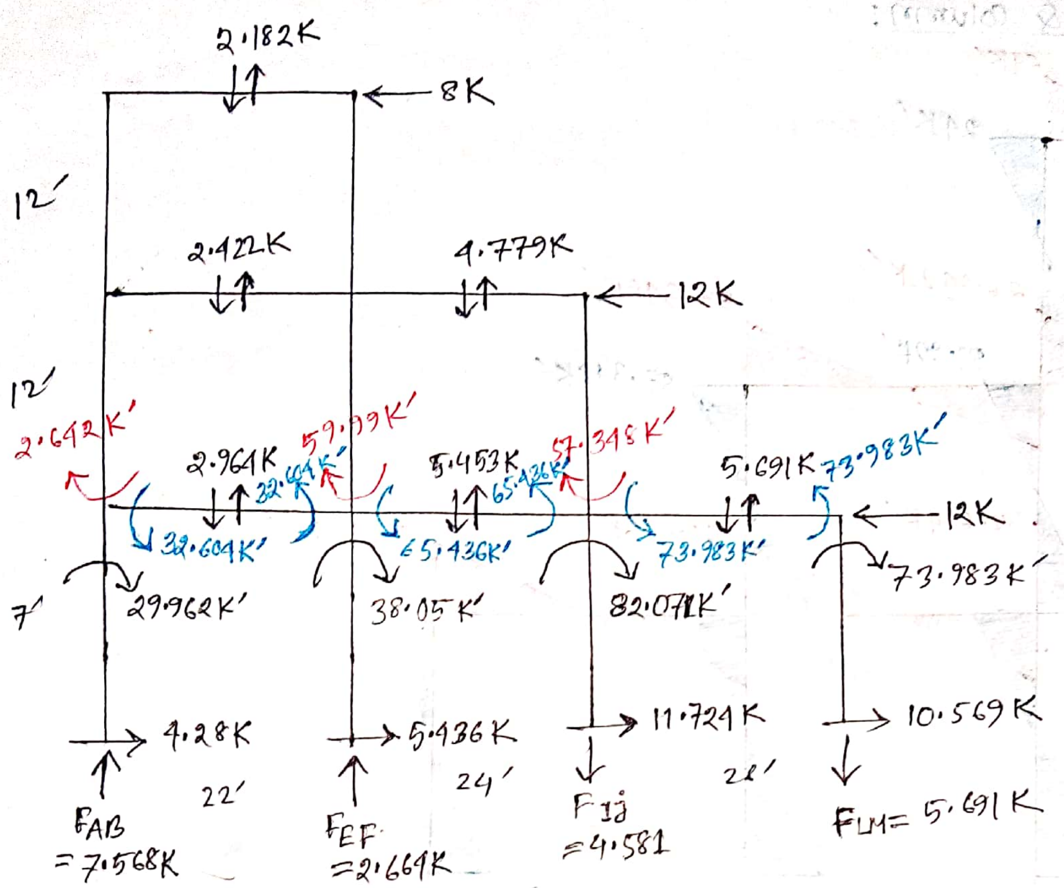
& $F_{LM} = 0.752 F_{AB}$

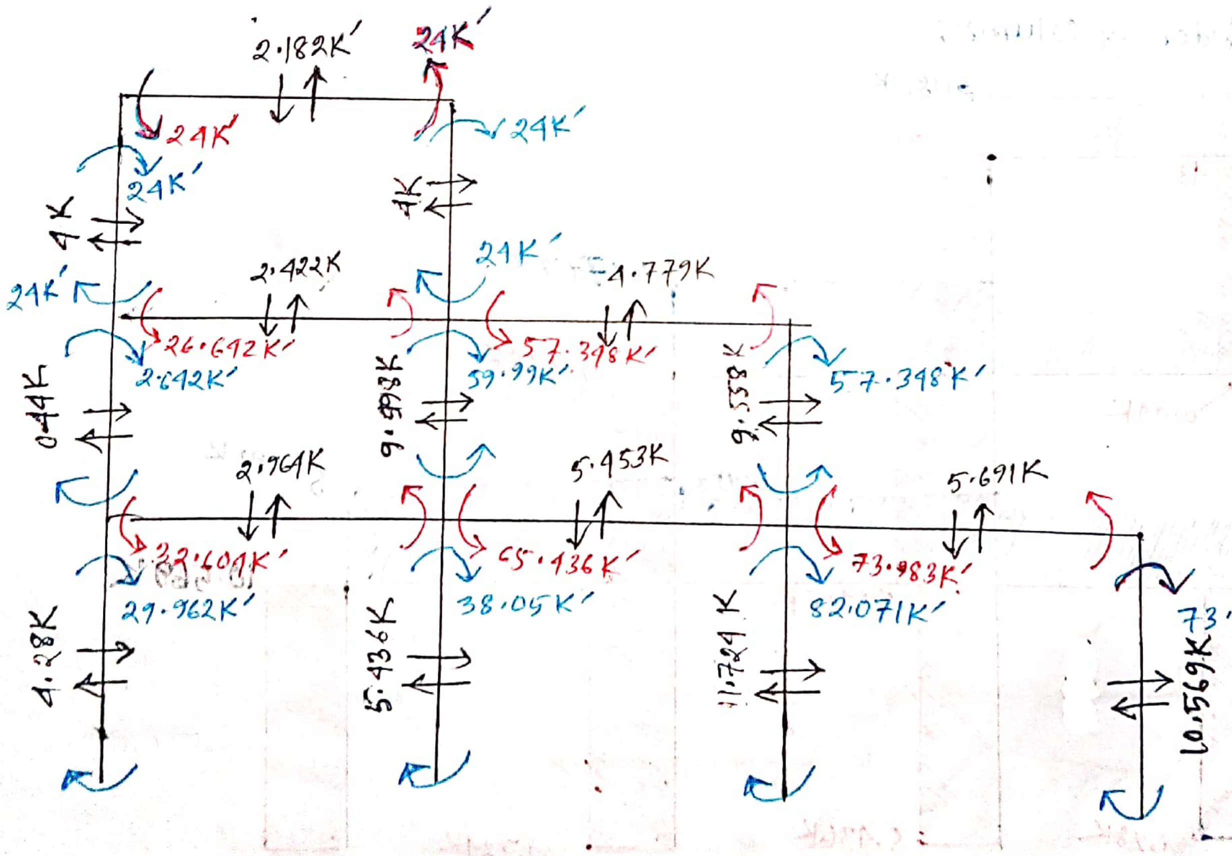
$\therefore F_{AB} = 7.568$ K

$F_{EF} = 2.664$ K

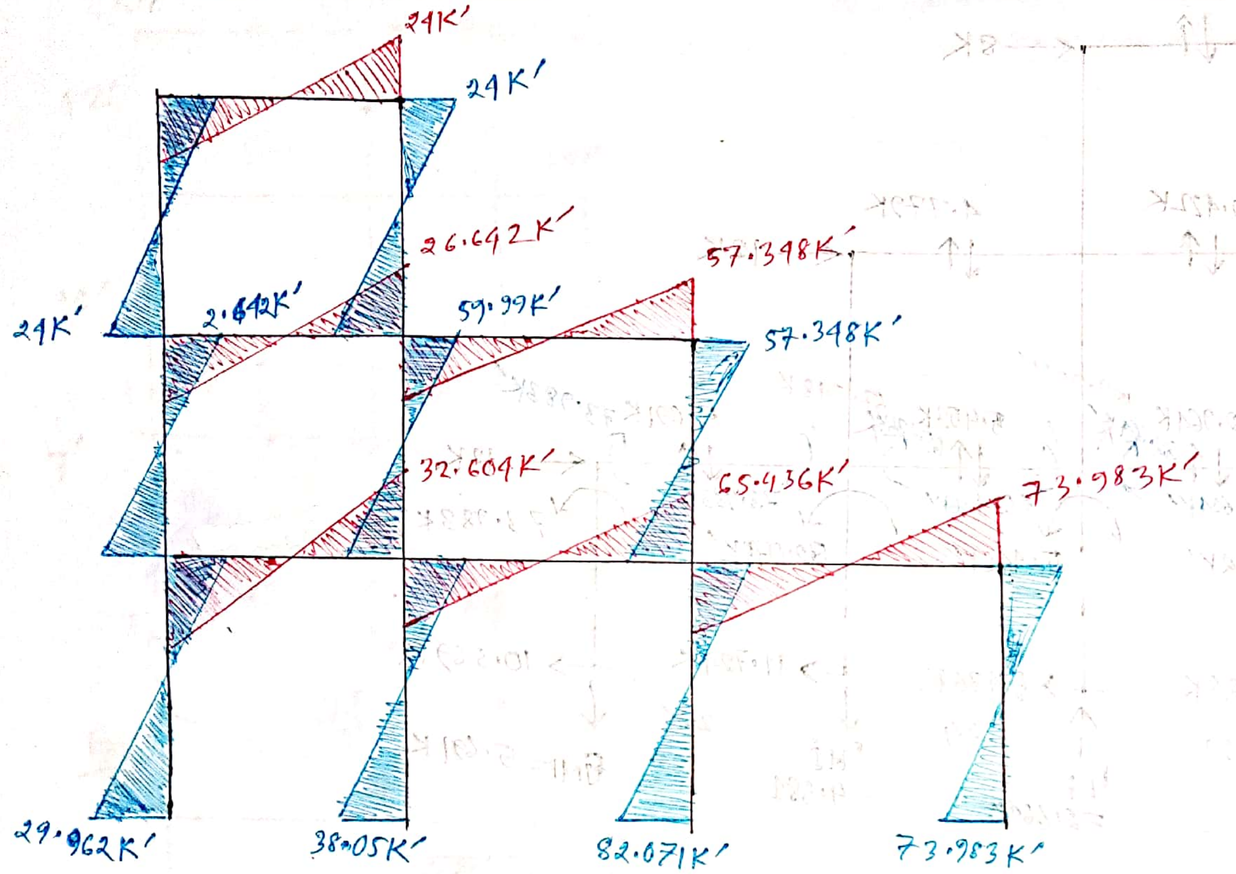
$F_{IJ} = 4.581$ K

$F_{LM} = 5.691$ K

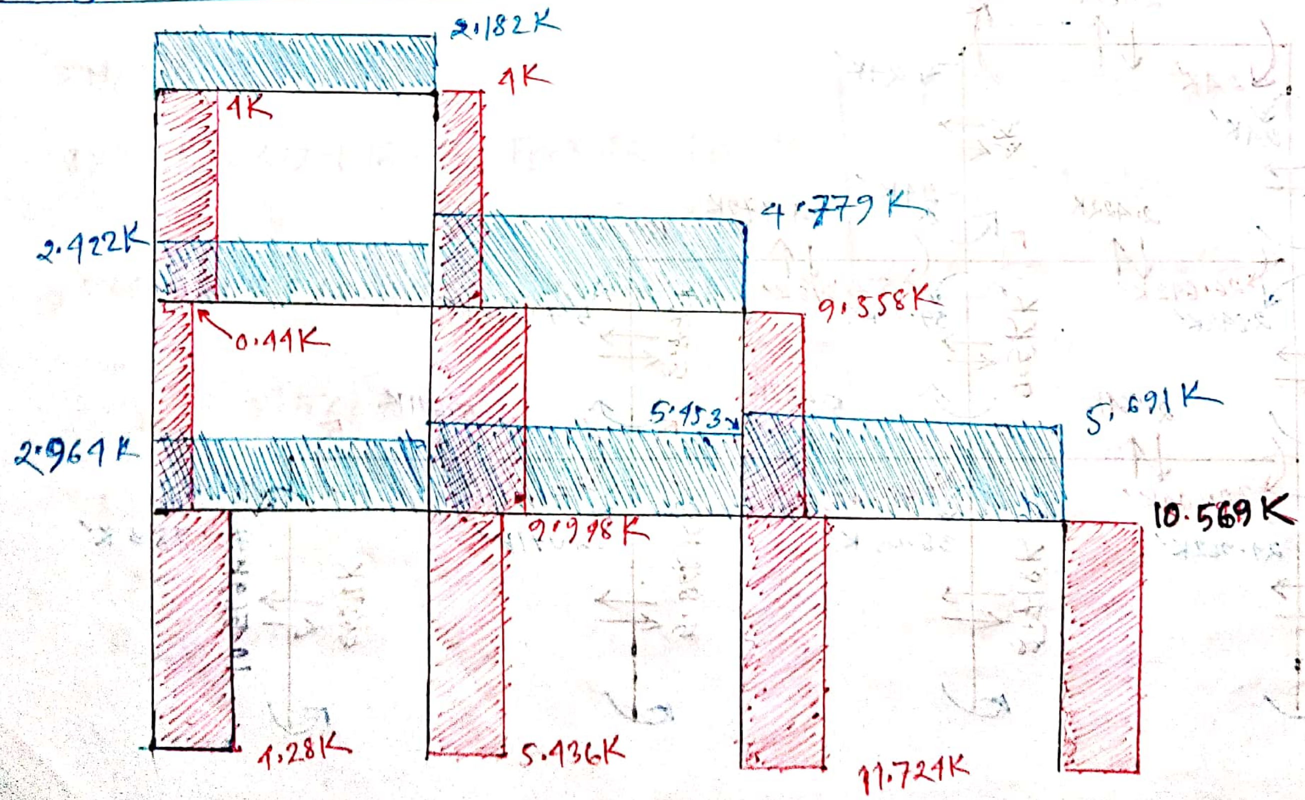

2.182K
8K
12'
2.422K
4.779K
12K
12'
2.642K'
2.964K
59.99K'
5.453K
57.348K'
5.691K
73.983K'
12K
32.604K'
65.436K'
73.983K'
73.983K'
7'
29.962K'
38.05K'
82.071K'
4.28K
5.436K
11.724K
10.569K
22'
24'
22'
F_{AB} = 7.568K
F_{EF} = 2.664K
F_{IJ} = 4.581
F_{LM} = 5.691K


2.182K'
24K'
24K'
24K'
24K'
4K
4K
24K'
2.422K
24K'
4.779K
26.642K'
2.642K'
57.348K'
59.99K'
57.348K'
0.44K
9.558K
9.558K
2.964K
5.453K
5.691K
32.604K'
65.436K'
73.983K'
29.962K'
38.05K'
82.071K'
73.983K'
4.28K
5.436K
11.724K
10.569K

BMD for Girder & Column:



SMD for Girder & Column:



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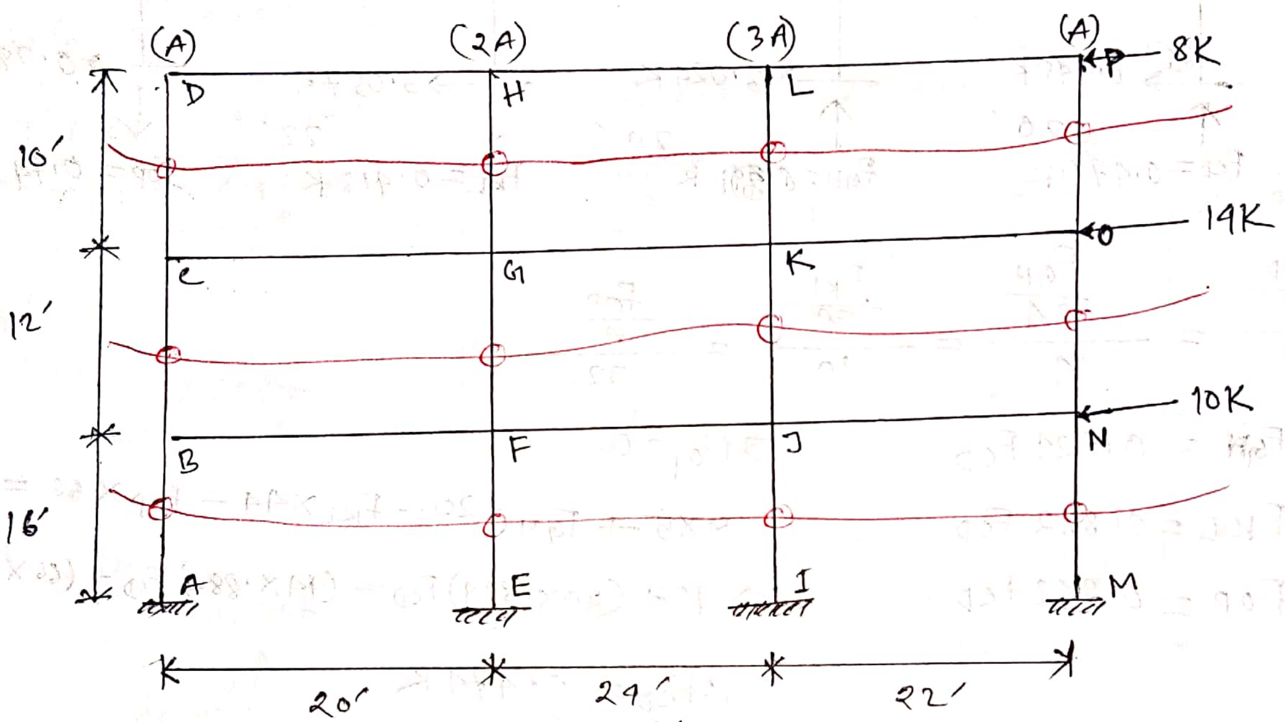
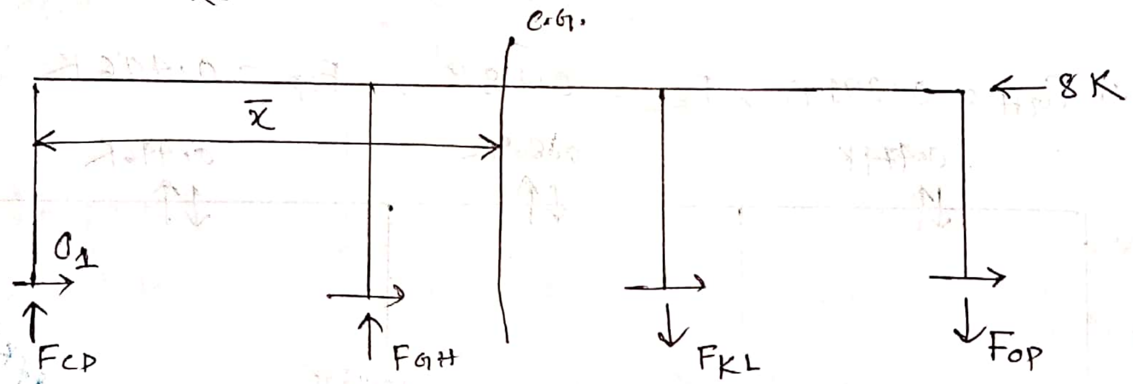
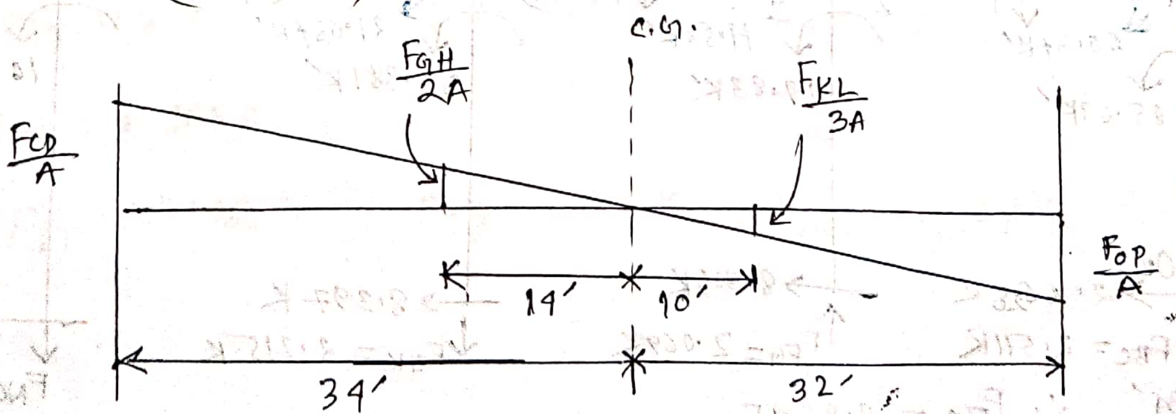
2013

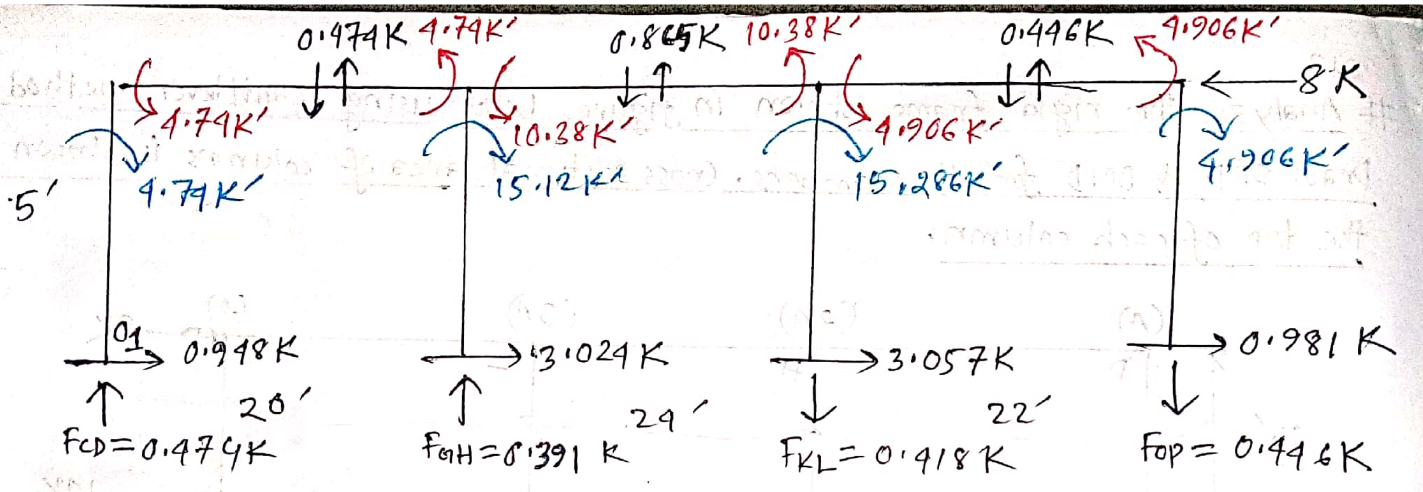
Analyze the rigid frame shown in figure below using cantilever method. Draw SFD & BMD for all members. Cross sectional area of columns is shown at the top of each column.





$$\bar{x} = \frac{2A \times 20 + 3A \times 44 + A \times 66}{(1+2+3+1)A} = 34'$$





$$\frac{\frac{F_{CD}}{A}}{34} = \frac{\frac{F_{GH}}{2A}}{19} = \frac{\frac{F_{KL}}{3A}}{10} = \frac{\frac{F_{OP}}{A}}{32}$$

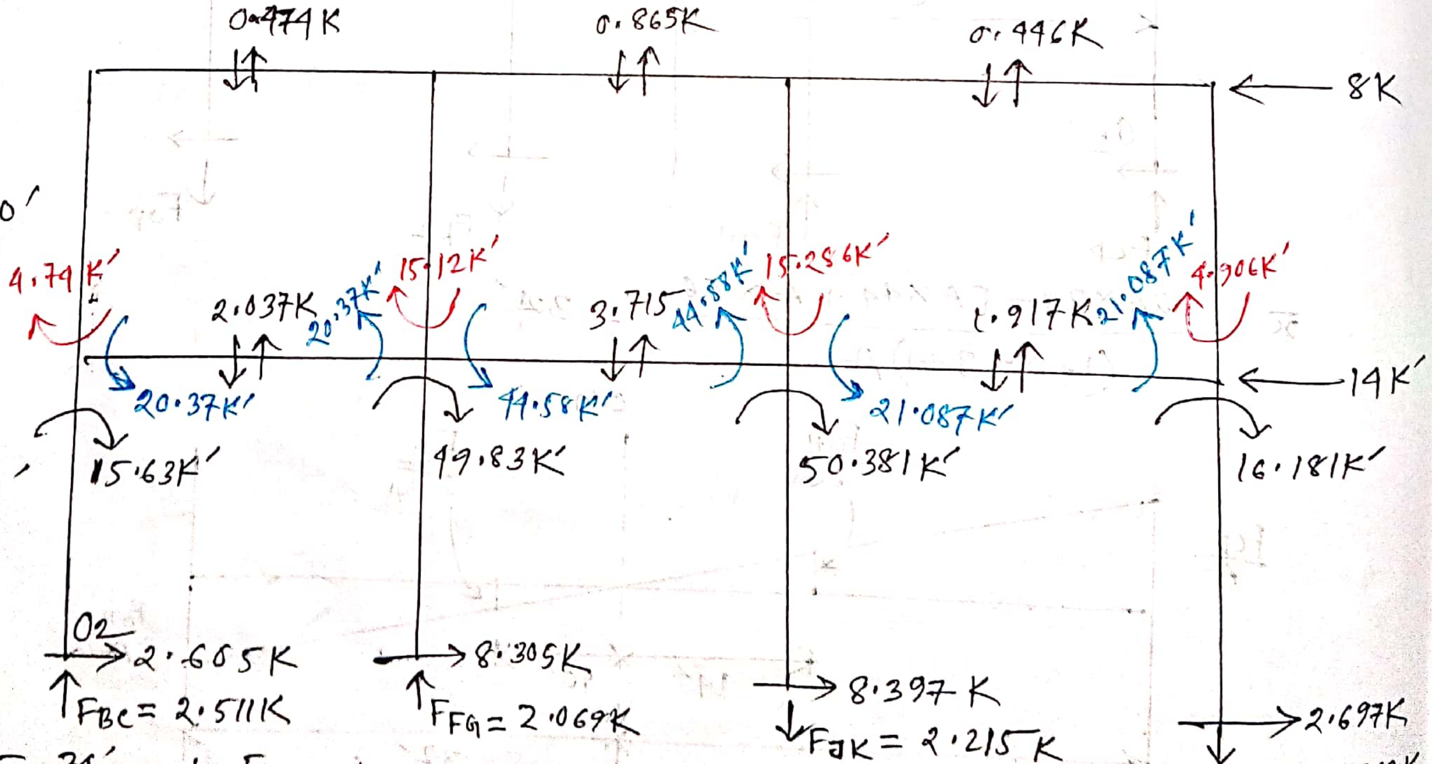
$\therefore F_{GH} = 0.824\, F_{CD}$ $\qquad \Sigma M_{O_1} = 0$

$F_{KL} = 0.882\, F_{CD}$ $\qquad 8 \times 5 + F_{GH} \times 20 - F_{KL} \times 44 - F_{OP} \times 66 = 0$

$\& \; F_{OP} = 0.941\, F_{CD}$ $\qquad \Rightarrow 40 + (20 \times .824) F_{CD} - (44 \times .882) F_{CD} - (66 \times .941) F_{CD} = 0$

$\therefore F_{CD} = 0.474\,K$

$\therefore F_{GH} = 0.391\,K, \quad F_{KL} = 0.418\,K, \quad F_{OP} = 0.446\,K$



Here, $\bar{x} = 34'$ $\quad \therefore F_{FG} = 0.824 F_{BC}, \quad F_{JK} = 0.882 F_{BC} \quad \& \; F_{NO} = 0.941 F_{BC}$

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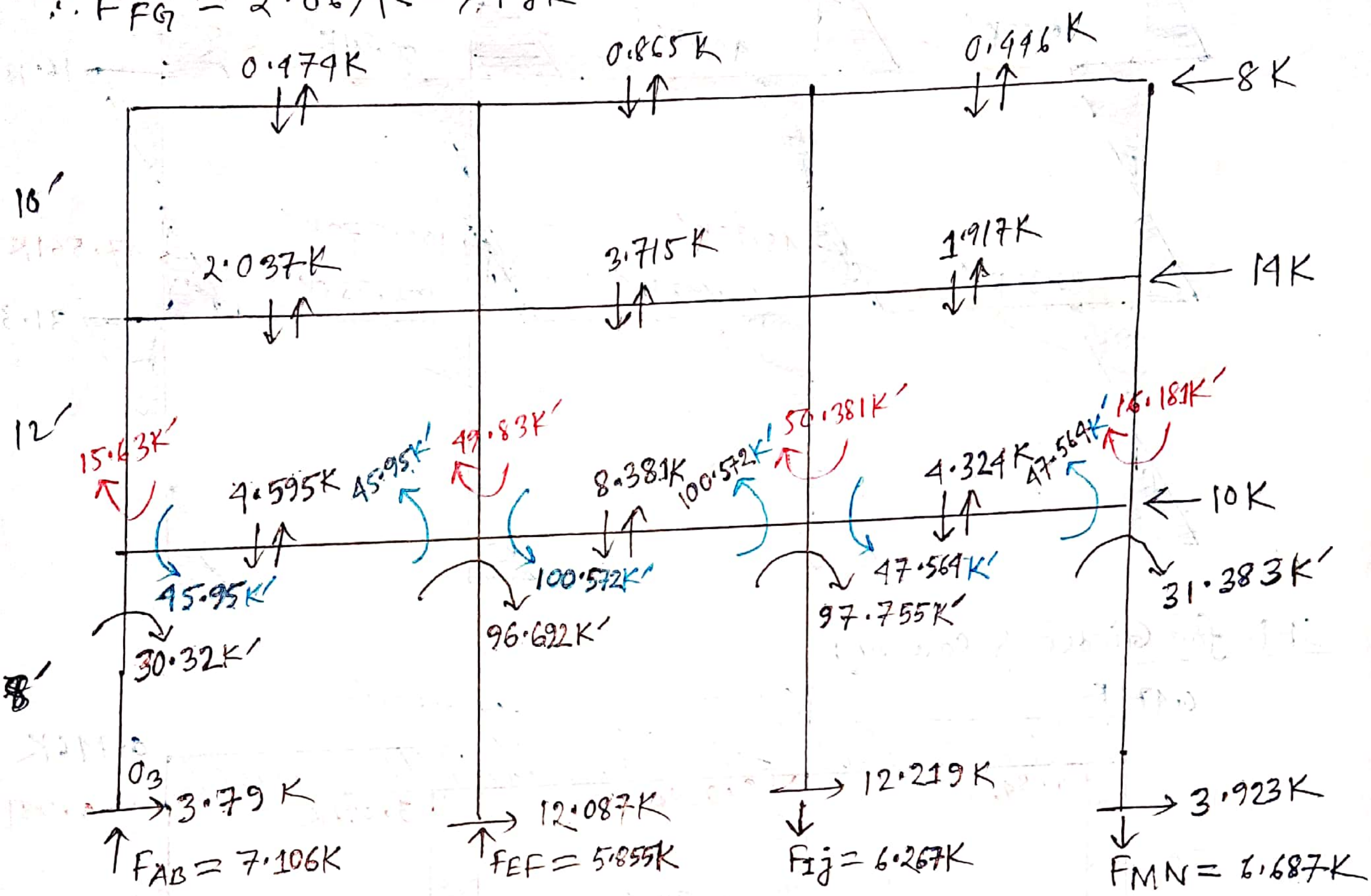
$\Sigma M_{O_2} = 0$

$8 \times 16 + 14 \times 6 + F_{FG} \times 20 - F_{JK} \times 44 - F_{NO} \times 66 = 0$

$\Rightarrow 212 + (20 \times 0.824) F_{BC} - (44 \times 0.882) F_{BC} - (66 \times 0.941) F_{BC} = 0$

$\therefore F_{BC} = 2.511\,K$

$\therefore F_{FG} = 2.069\,K$, $F_{JK} = 2.215\,K$, $F_{NO} = 2.363\,K$



$\bar{x} = 34'$

$\therefore F_{EF} = 0.824 F_{AB}$, $F_{IJ} = 0.882 F_{AB}$ & , $F_{MN} = 0.941 F_{AB}$

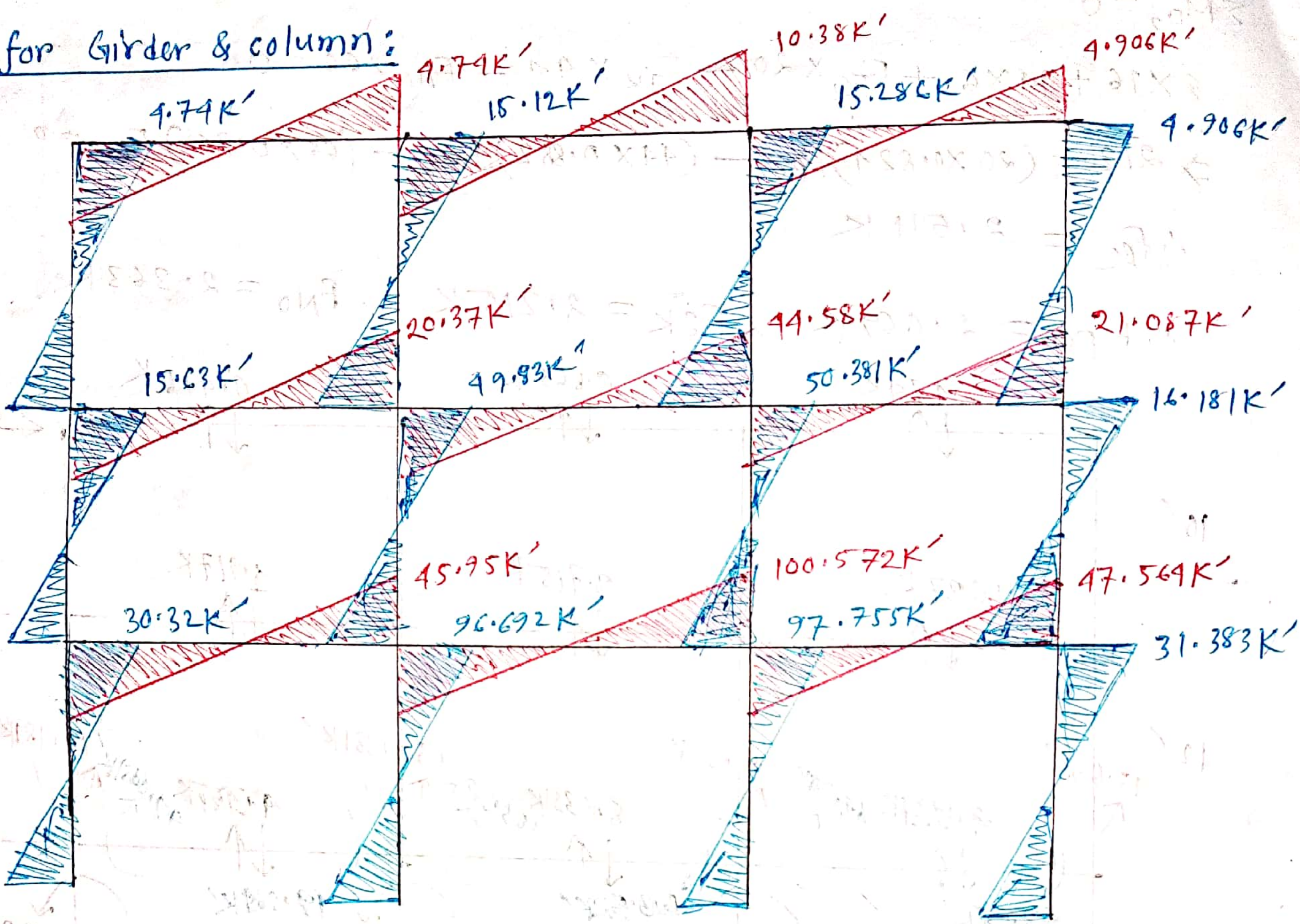
$\Sigma M_{O_3} = 0$

$8 \times 30 + 14 \times 20 + 10 \times 8 + (20 \times .824) F_{AB} - (44 \times 0.882) F_{AB} - (66 \times 0.941) F_{AB} = 0$

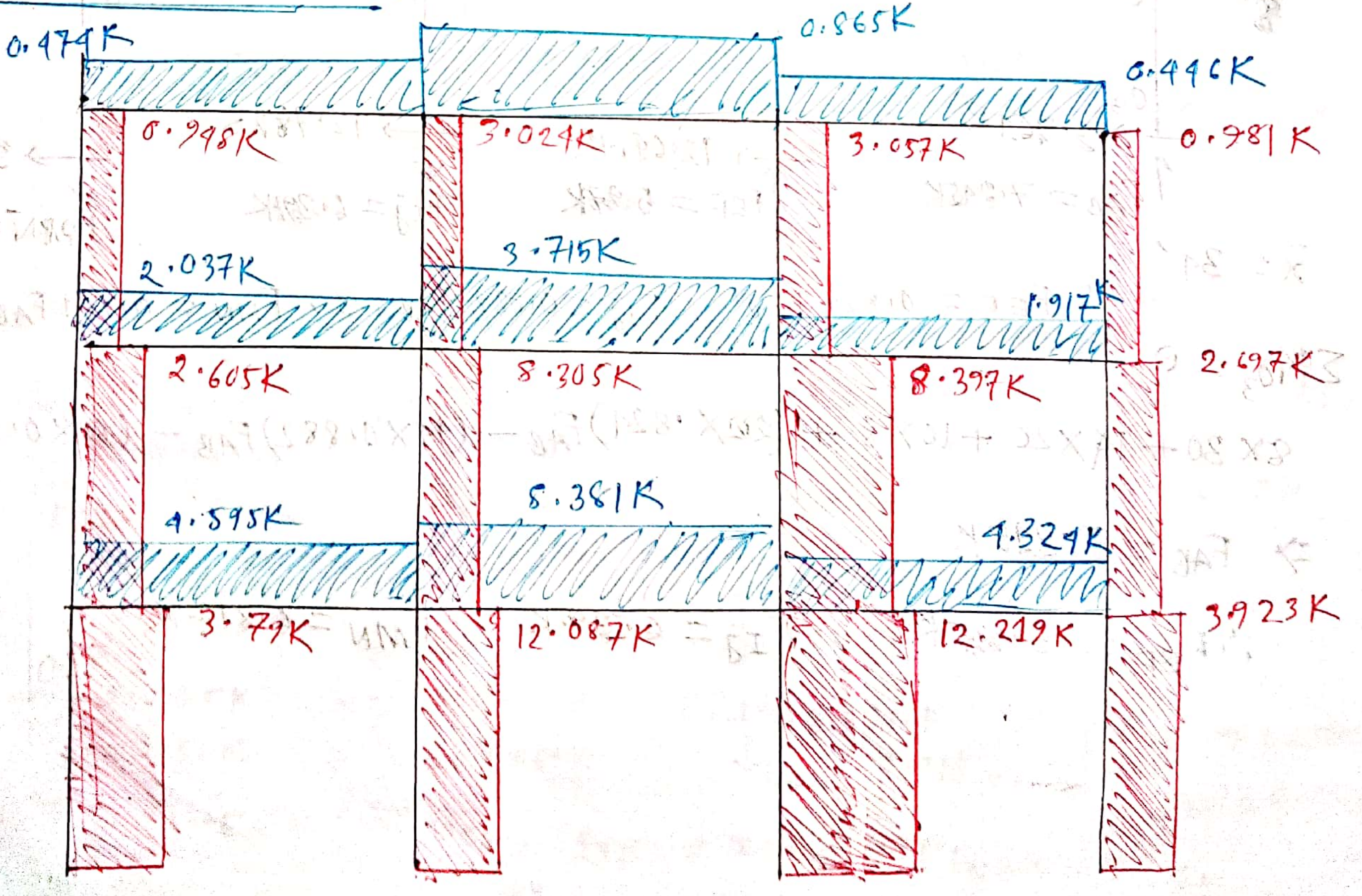
$\Rightarrow F_{AB} = 7.106\,K$

$\therefore F_{EF} = 5.855\,K$, $F_{IJ} = 6.267\,K$, $F_{MN} = 6.687\,K$

BMD for Girder & column:



SFD for Girder & Column:



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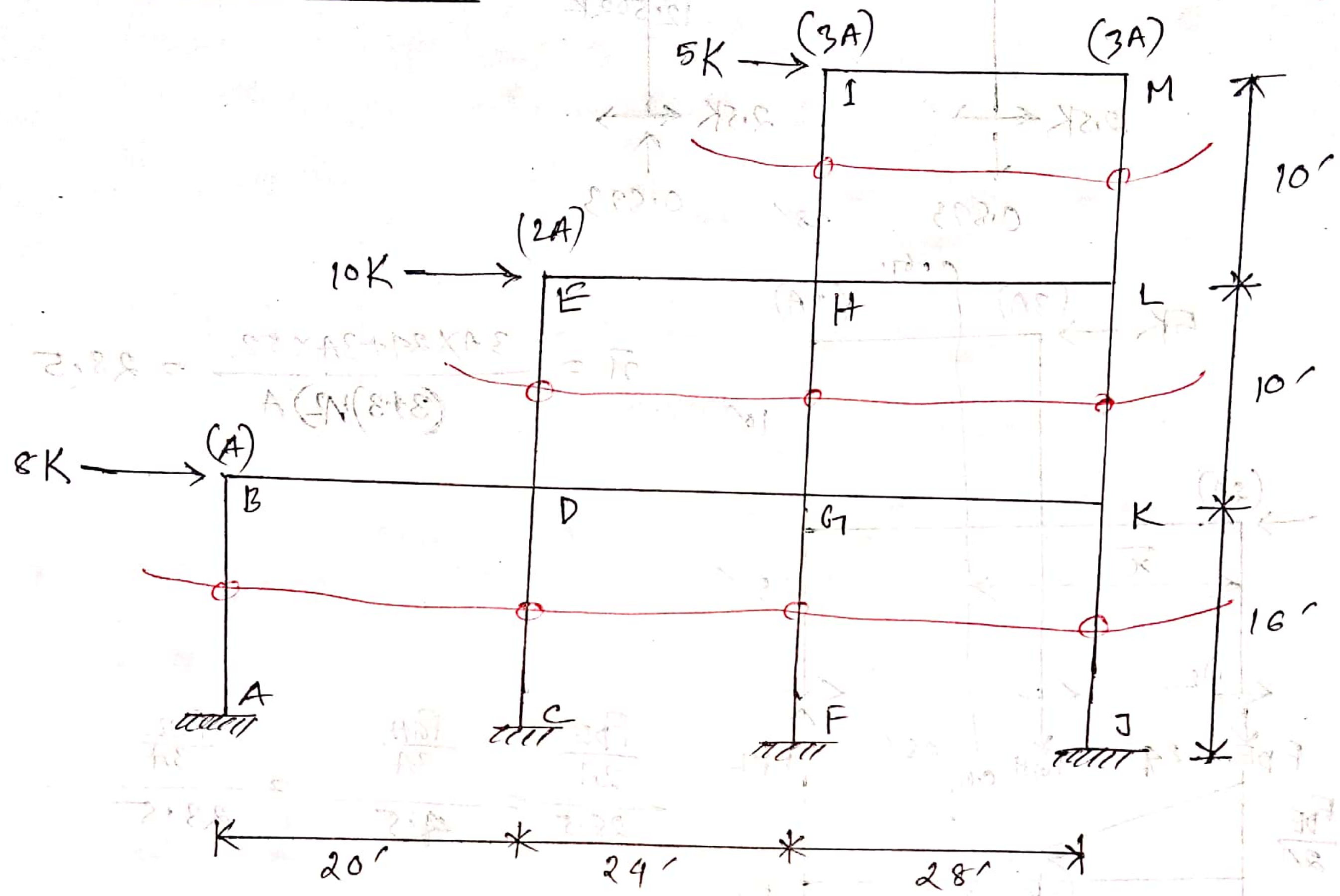
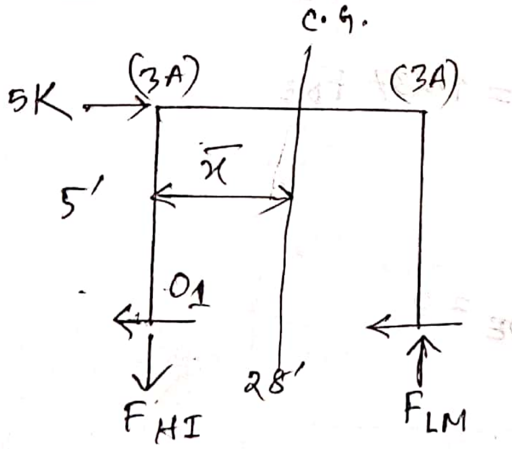
2011

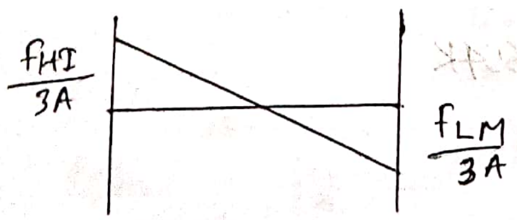
Analyze the frame shown in figure below using cantilever method. Draw SFD and BMD for all members. Areas of columns are shown at the top of each column.





$$\bar{x} = \frac{3A \times 28}{(3+3)A} = 14'$$

$$\frac{\frac{F_{HI}}{3A}}{14} = \frac{\frac{F_{LM}}{3A}}{14}$$

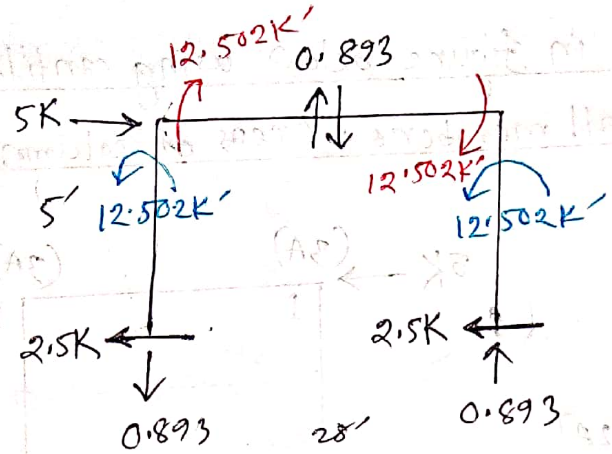


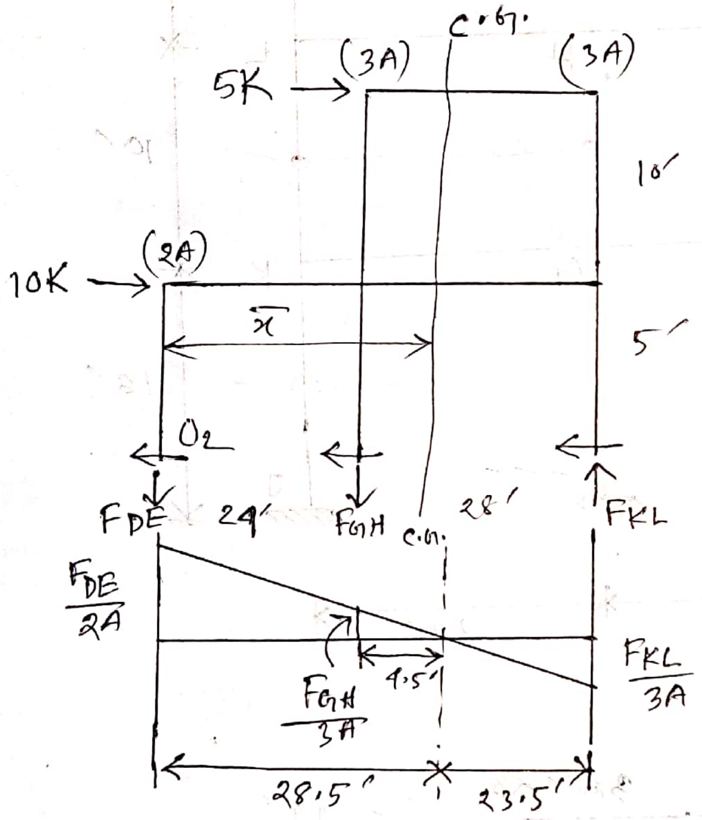
$\therefore F_{HI} = F_{LM}$

$\Sigma M_{O_1} = 0$

$5 \times 5 - F_{LM} \times 28 = 0 \quad \therefore F_{LM} = 0.893\,K$

$\therefore F_{HI} = 0.893\,K$





$$\bar{x} = \frac{3A \times 24 + 3A \times 52}{(3+3+2)A} = 28.5$$

$$\frac{\frac{F_{DE}}{2A}}{28.5} = \frac{\frac{F_{GH}}{3A}}{4.5} = \frac{\frac{F_{KL}}{3A}}{23.5}$$

$\therefore F_{GH} = 0.237\, F_{DE}$

$\&\ F_{KL} = 1.237\, F_{DE}$

$\Sigma M_{O_2} = 0$

$5 \times 15 + 10 \times 5 + F_{GH} \times 24 - F_{KL} \times 52 = 0$

$\Rightarrow 125 + (24 \times 0.237)\, F_{DE} - (52 \times 1.237)\, F_{DE} = 0$

$\therefore F_{DE} = 2.132K$

$\therefore F_{GH} = 0.505\,K\ \&\ F_{KL} = 2.637K$

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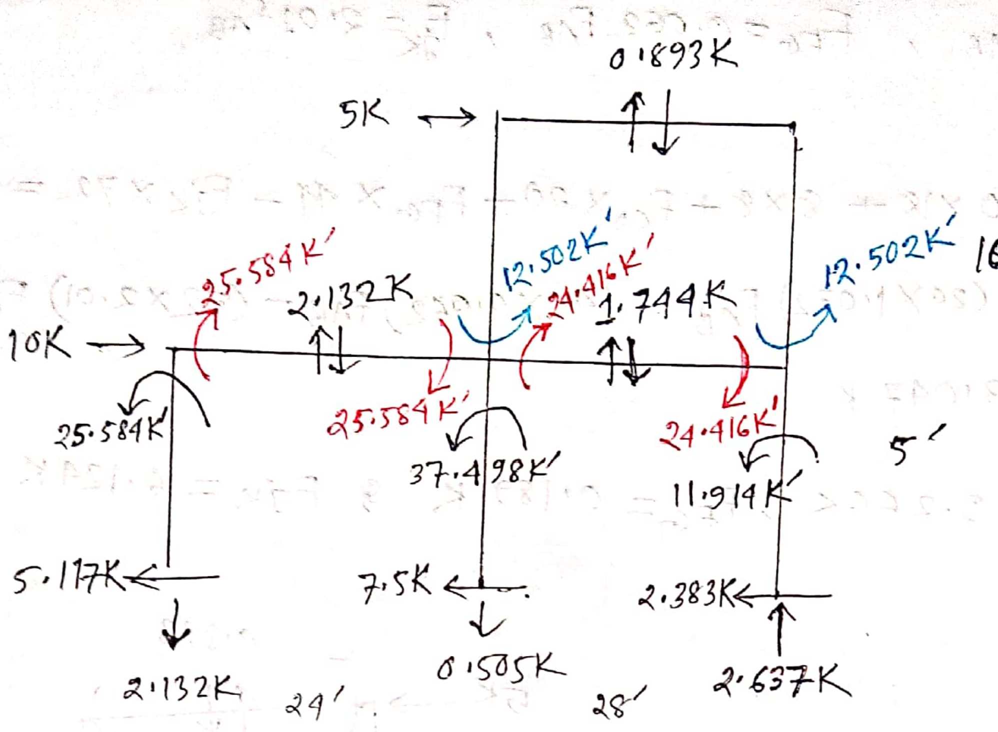
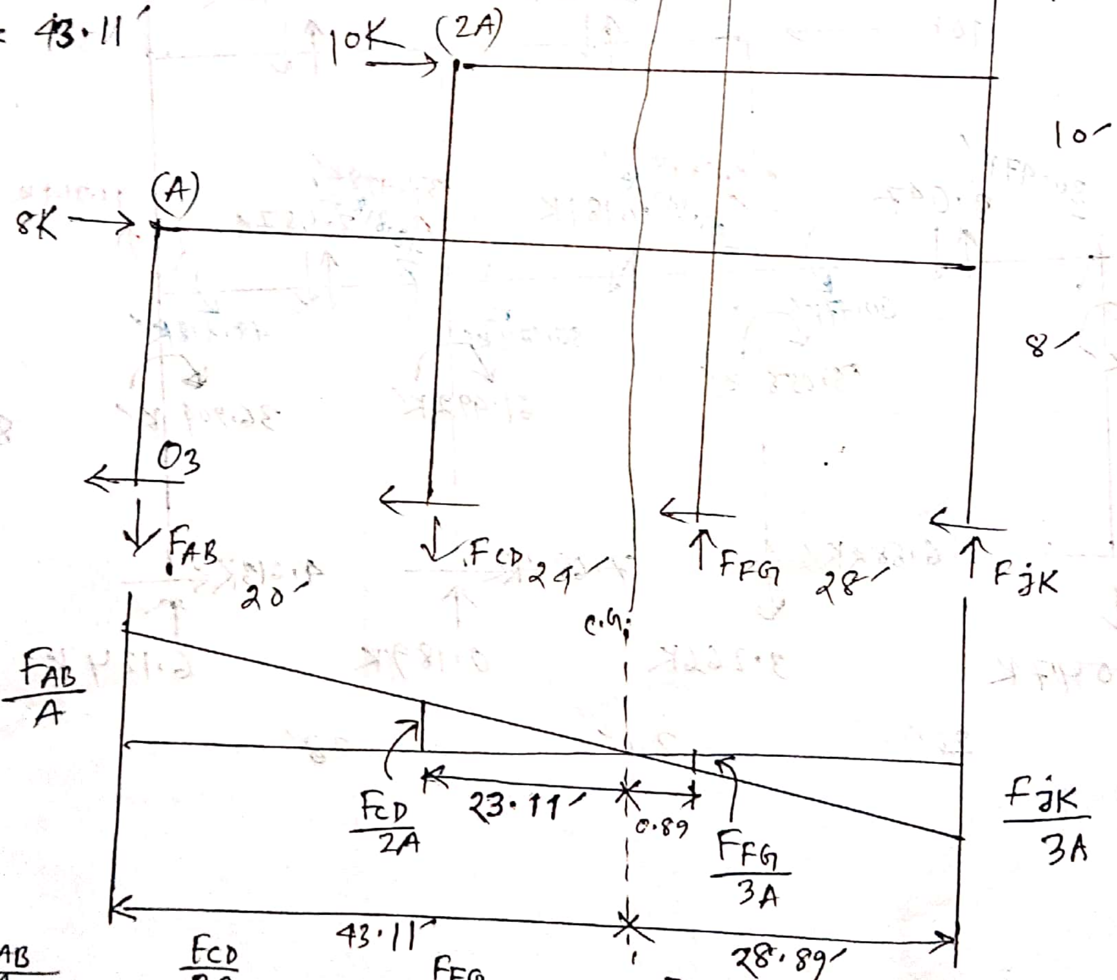


$$\bar{x} = \frac{2A \times 20 + 3A \times 44 + 3A \times 72}{(1+2+3+3)A}$$

$$= 43.11'$$



$$\frac{\frac{F_{AB}}{A}}{43.11} = \frac{\frac{F_{CD}}{2A}}{23.11} = \frac{\frac{F_{FG}}{3A}}{0.89} = \frac{\frac{F_{JK}}{3A}}{28.89}$$

$F_{CD} = 1.072 F_{AB}$, $F_{FG} = 0.062 F_{AB}$, $F_{JK} = 2.01 F_{AB}$

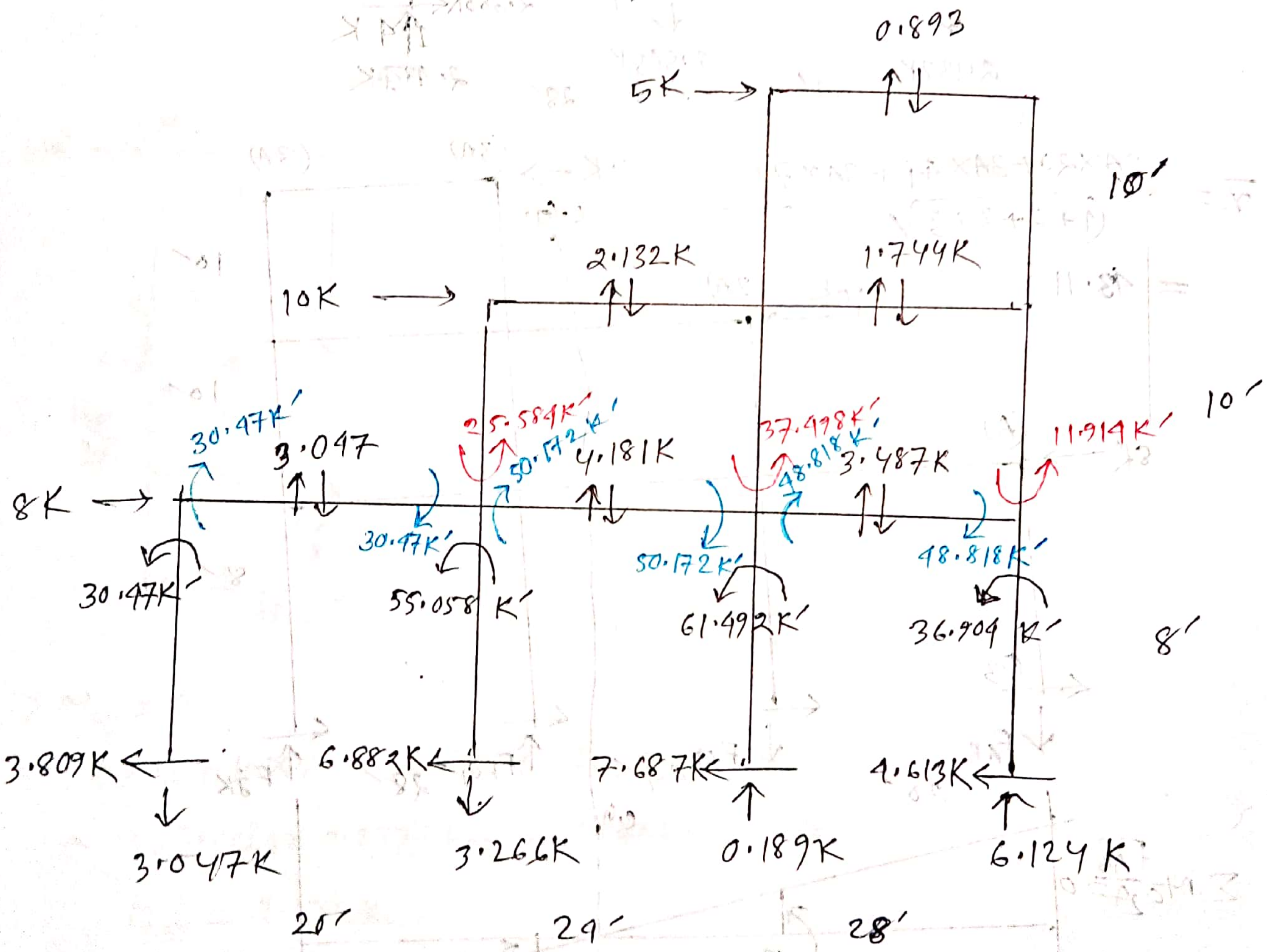
$\Sigma M_{O_3} = 0$

$5 \times 28 + 10 \times 18 + 8 \times 8 + F_{CD} \times 20 - F_{FG} \times 44 - F_{JK} \times 72 = 0$

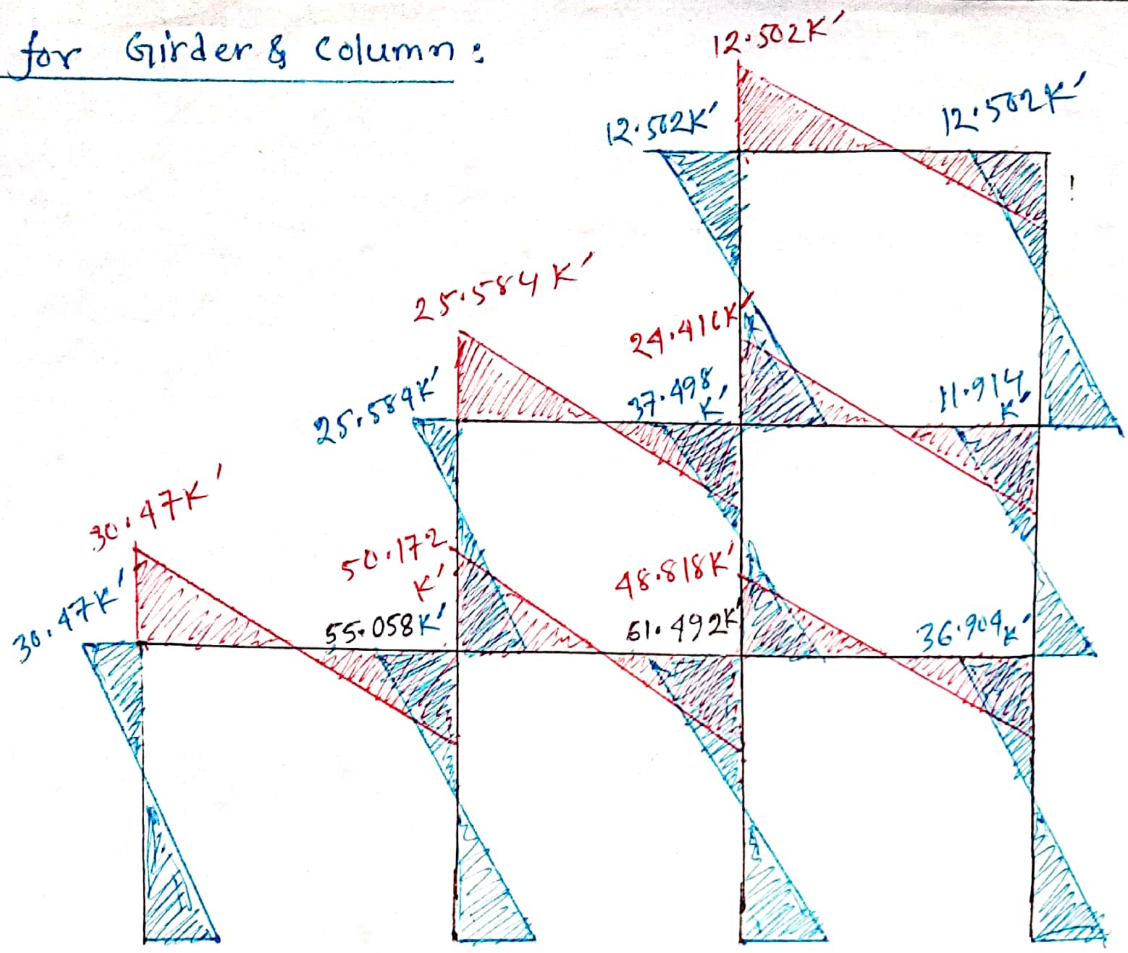
$\Rightarrow 384 + (20 \times 1.072) F_{AB} - (44 \times 0.062) F_{AB} - (72 \times 2.01) F_{AB} = 0$

$\therefore F_{AB} = 3.047$ K

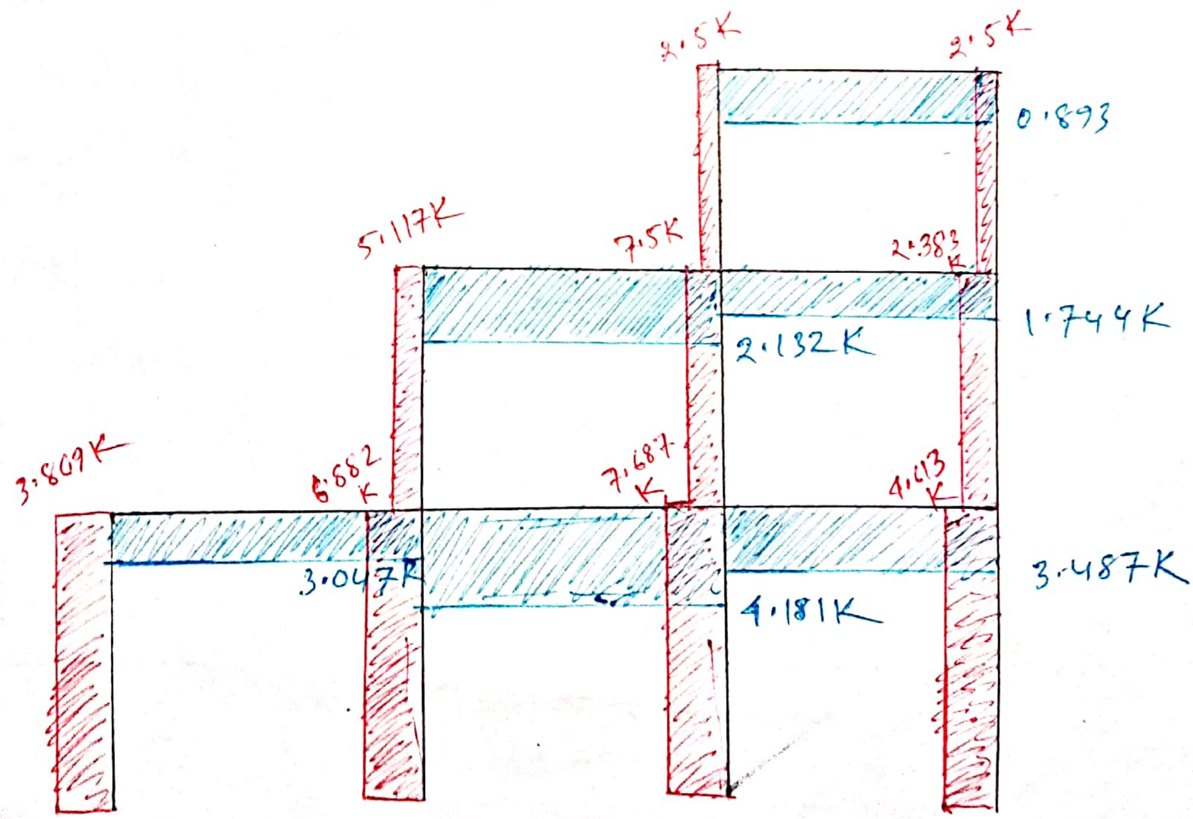
$\therefore F_{CD} = 3.266$K , $F_{FG} = 0.189$ K & $F_{JK} = 6.124$ K



BMD for Girder & Column:



SFD for Girder & Column:

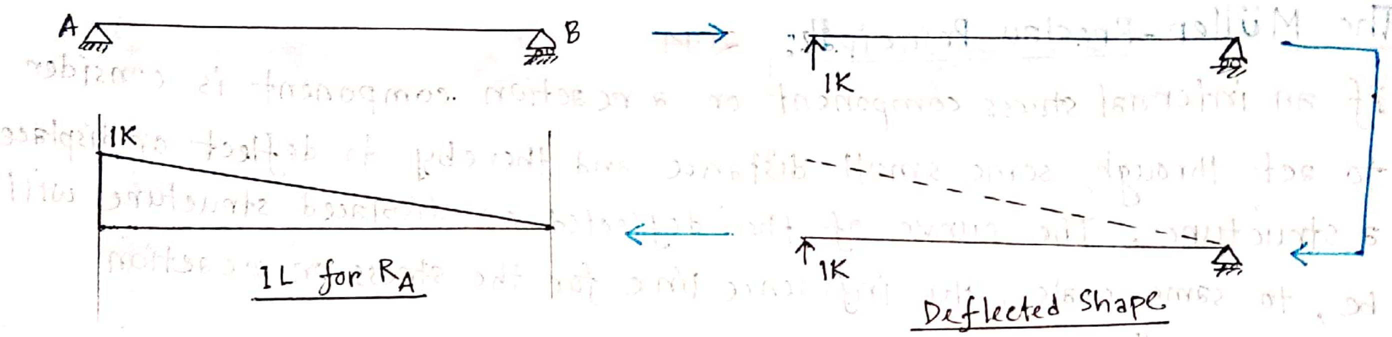


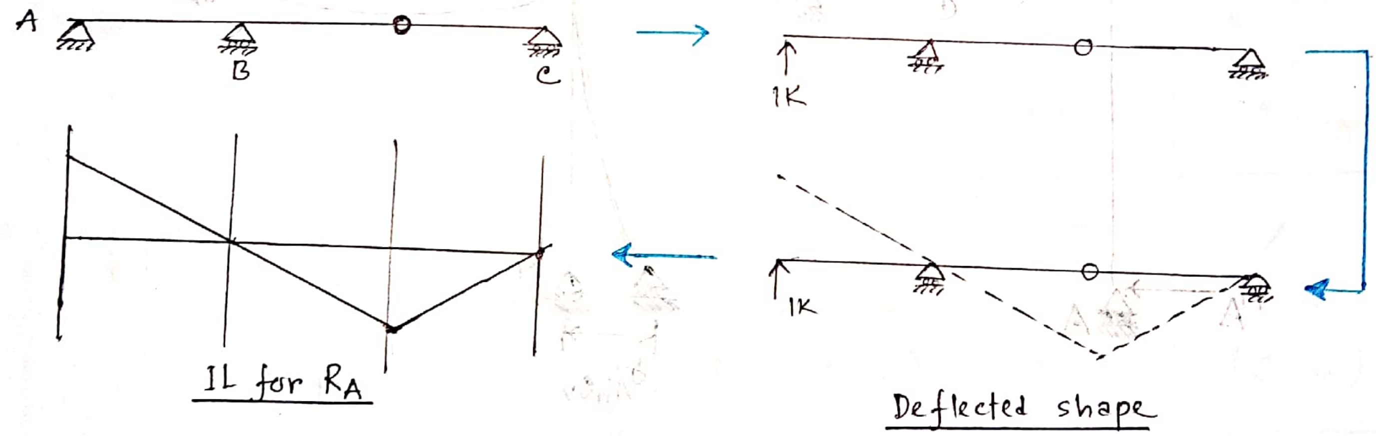
Influence Line for Statically Indeterminate Structure

IL for Reaction

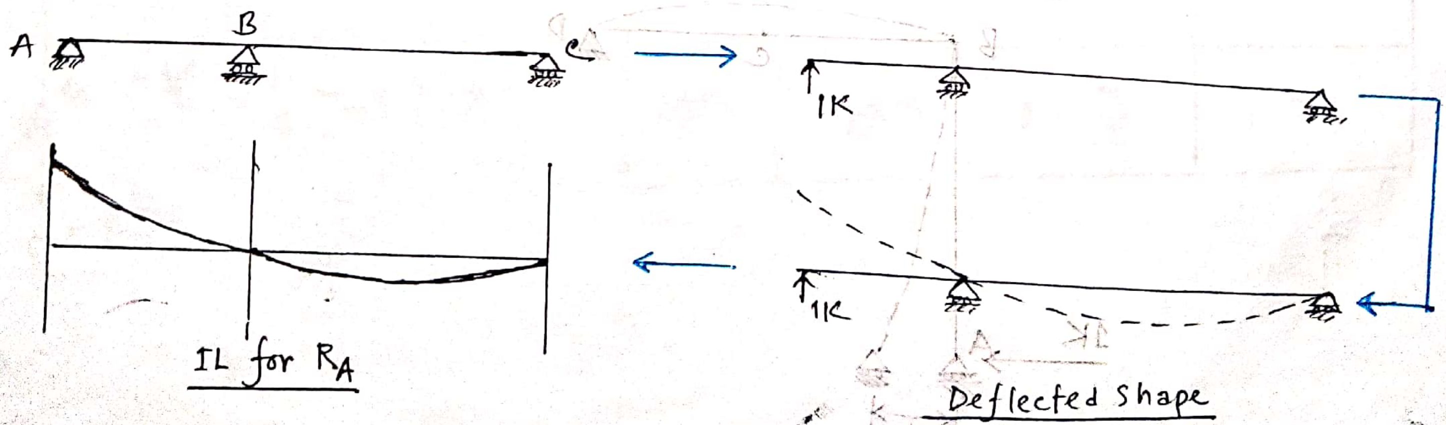
Draw IL for Reaction at 'A':

Determinate Structures:





Indeterminate Structures:



Note that,

* Deflected Shape = Influence Line.

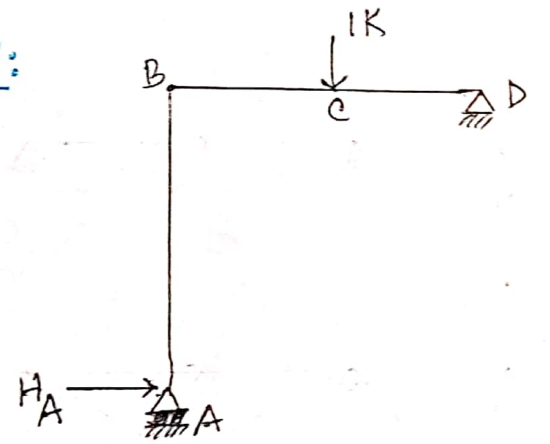
* For determinate structure, Influence Line is straight Line.

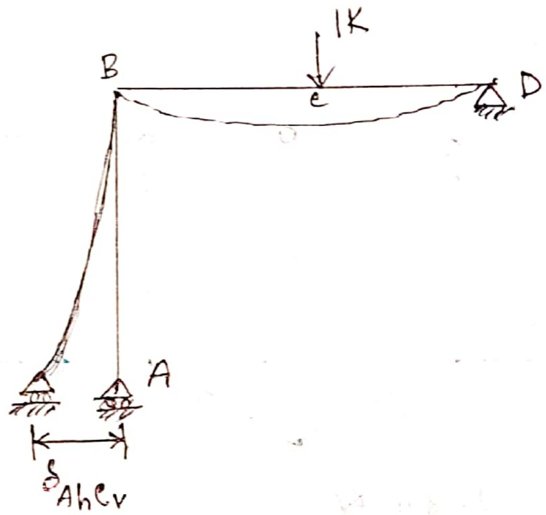
* For Indeterminate structure, Influence Line is curve Line.

The Müller-Breslau Principle: 2014

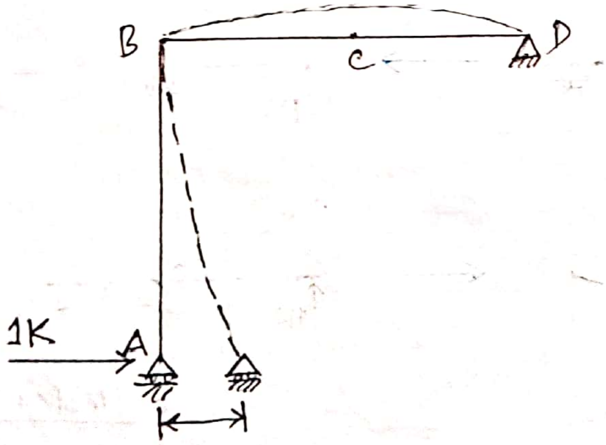
"If an internal stress component or a reaction component is consider to act through some small distance and thereby to deflect or displace a structure, The curve of the deflected or displaced structure will be, to same scale, the influence line for the stress or reaction component."

Proof:





Here, $\delta_{A_hC_v}$ = Horizontal deflection at point A due to Vertical load at C.

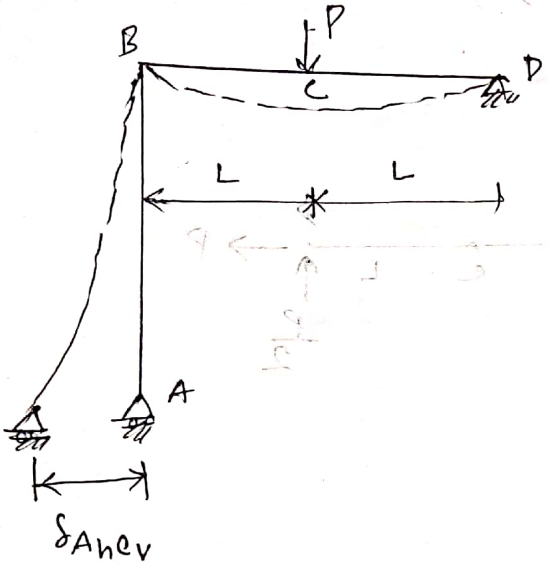


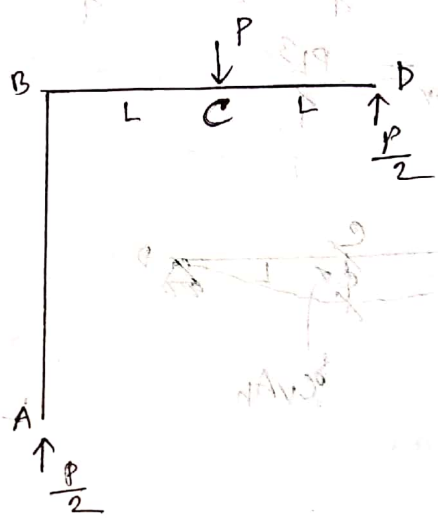
Similarly,

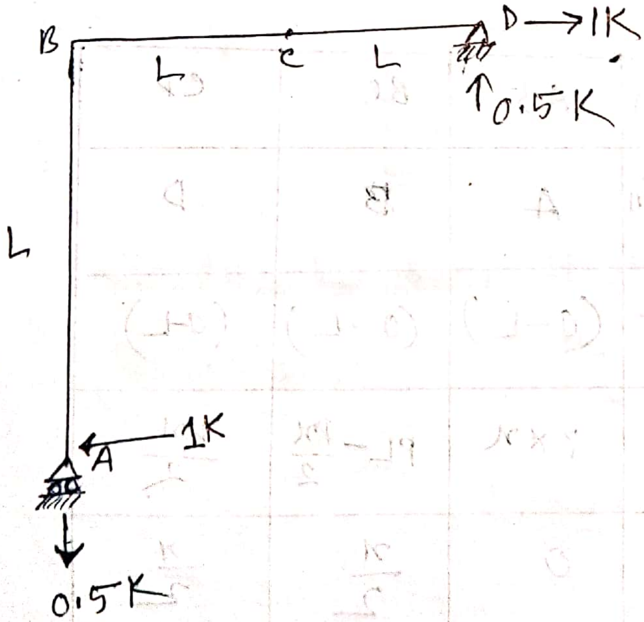
$\delta_{A_hA_h}$ = Horizontal deflection at point A due to Horizontal load at A.

For H_A, $\delta_{A_h C_v} = H_A \times \delta_{A_h A_h}$

$$\therefore H_A = \frac{\delta_{A_h C_v}}{\delta_{A_h A_h}}$$ (General Method)







Portion	CD	BC	AB
Origin	D	B	A
Limit	(0–L)	(0–L)	(0–L)
M	$\frac{P}{2} \times x$	$\frac{P}{2} \times x$	0
m	$\frac{1}{2} \times x$	$L - \frac{x}{2}$	x

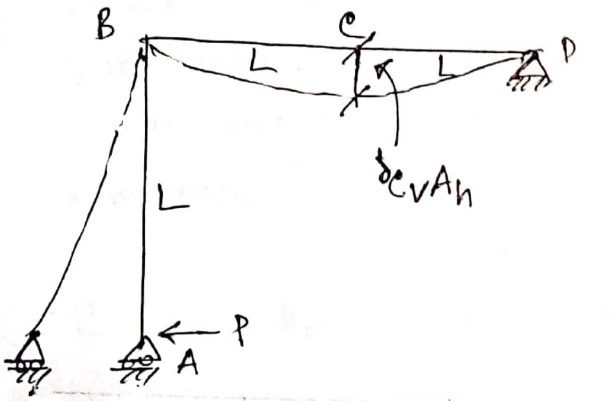
$$EI \cdot \Delta_{AhCv} = \int_0^L \left(\frac{Px}{2} \times \frac{x}{2}\right) dx + \int_0^L \left[\frac{Px}{2} \times \left(L - \frac{x}{2}\right)\right] dx$$

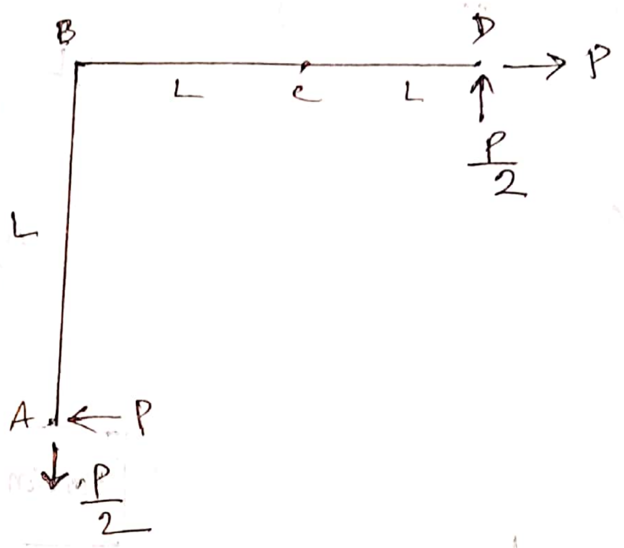
$$= \frac{P}{4}\left[\frac{x^3}{3}\right]_0^L + \frac{PL}{2}\left[\frac{x^2}{2}\right]_0^L - \frac{P}{4}\left[\frac{x^3}{3}\right]_0^L$$

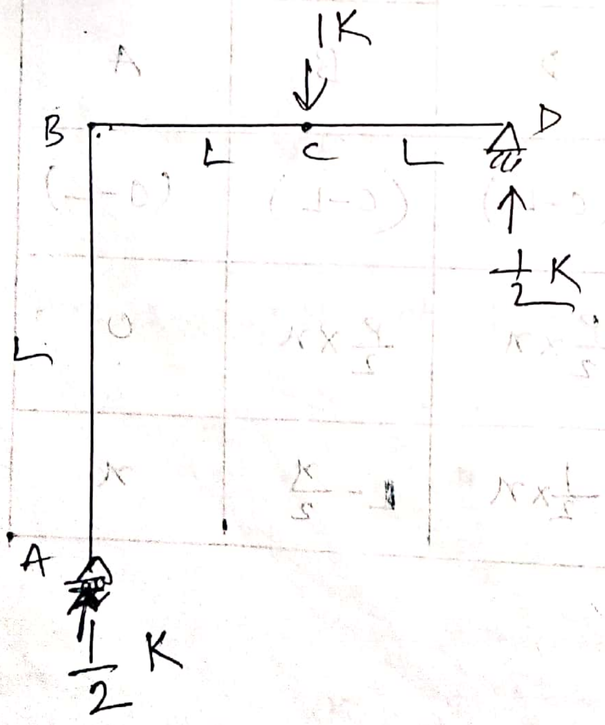
$$= \frac{P}{4} \times \frac{L^3}{3} + \frac{PL^3}{4} - \frac{P}{4} \times \frac{L^3}{3}$$

$$EI \cdot \Delta_{AhCv} = \frac{PL^3}{4}$$

Now,







Portion	AB	BC	CD
Origin	A	B	D
Limit	(0 − L)	(0 − L)	(0 − L)
M	$P \times x$	$PL - \frac{Px}{2}$	$\frac{Px}{2}$
m	0	$\frac{x}{2}$	$\frac{x}{2}$

$$EI \cdot \Delta_{C_V A_h} = \int_0^L \left[\frac{x}{2} \times \left(PL - \frac{Px}{2}\right)\right] dx + \int_0^L \left(\frac{Px}{2} \times \frac{x}{2}\right) dx$$

$$= \frac{PL}{2}\left[\frac{x^2}{2}\right]_0^L - \frac{P}{4}\left[\frac{x^3}{3}\right]_0^L + \frac{P}{4} \times \left[\frac{x^3}{3}\right]_0^L$$

$$= \frac{PL}{2} \times \frac{L^2}{2} - \frac{P}{4} \times \frac{L^3}{3} + \frac{P}{4} \times \frac{L^3}{3}$$

$$\therefore EI \cdot \Delta_{C_V A_h} = \frac{PL^3}{4}$$

Hence, $EI \cdot \Delta_{A_h C_V} = EI \cdot \Delta_{C_V A_h}$

$$\therefore \Delta_{A_h C_V} = \Delta_{C_V A_h}$$

(Maxwell's reciprocal deflection theorem)

But, we know,

$$H_A = \frac{\delta_{A_h C_V}}{\delta_{A_h A_h}}$$

$$\therefore H_A = \frac{\delta_{C_V A_h}}{\delta_{A_h A_h}}$$

From this equation, It is apparent that the deflection curve of the member BD is the influence line for H_A for a vertical load on BD.

(Proved)

Conjugate Beam

1. The Length of a conjugate beam = The length of the real beam

2. The load of the conjugate beam = $\frac{M}{EI}$ diagram of the real beam

3. The shear at any section of the conjugate beam = The slope of the corresponding section of the real beam.

4. The moment at any section of the conjugate beam = The deflection of the corresponding section of the real beam.

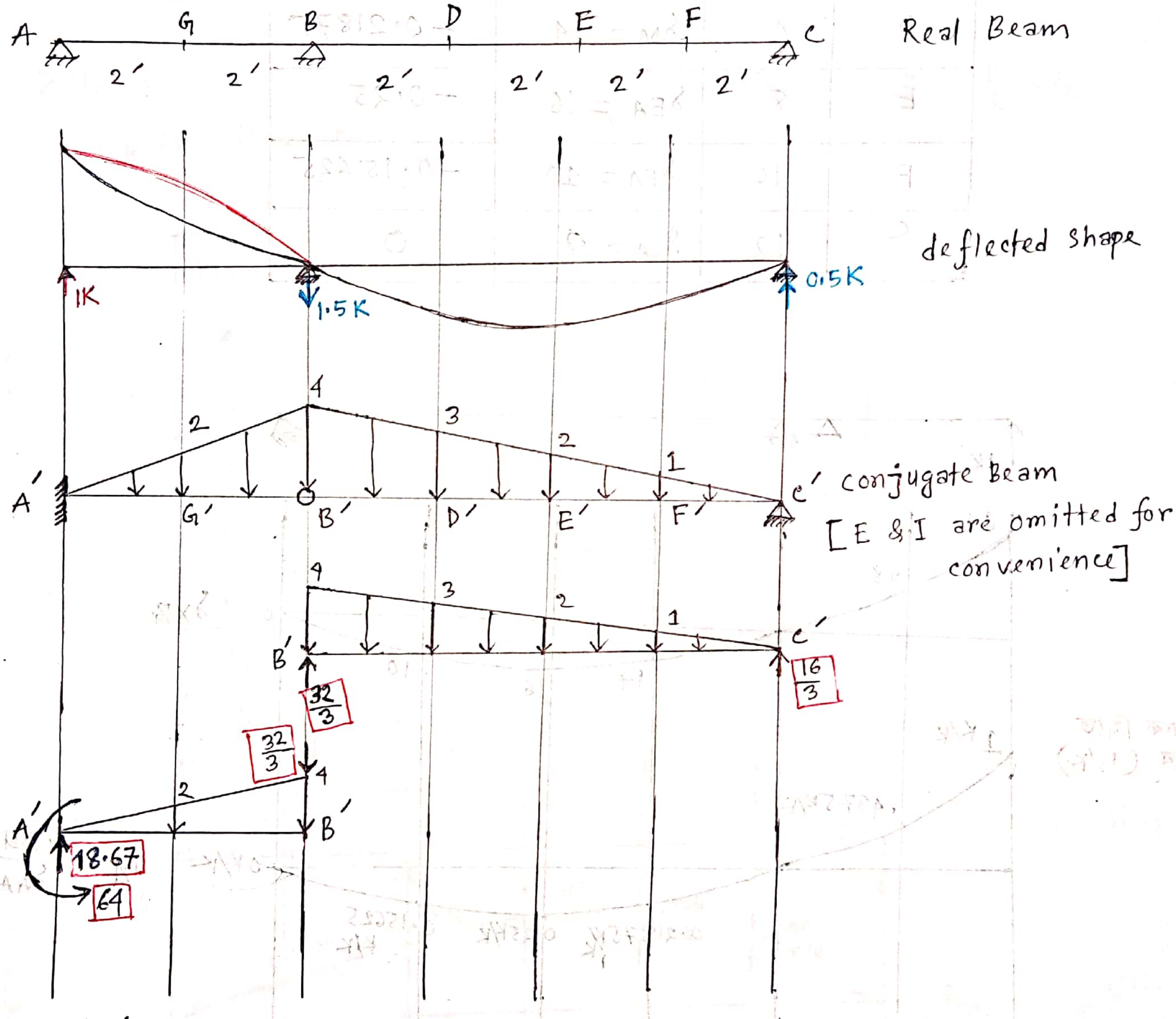
Support Change :

Real	Conjugate	Why provide
Fixed	Free	Real beam-এ fixed support থাকলে, deflection & slope = 0। এজন্য এমন একটি support দিতে হবে যেখানে Moment & Shear = 0। যেহেতু এমন কোনো support নেই যেখানে Moment & shear উভয়ই zero হয়, তাই Free end.
Free	Fixed	Real beam-এ free end থাকলে, যেখানে deflection & slope উভয়ই থাকবে। এজন্য Fixed support দিতে হবে। কারণ Fixed support-এ Moment & Shear উভয় থাকে।
External Hinge	External Hinge	Real beam এ external hinge থাকলে, slope থাকবে যেখানে, তাই এমন support দিতে হবে যেখানে Shear থাকে। এজন্য External hinge-ই দেওয়া হয় কারণ Shear থাকে। Another reason : (deflection zero)
Internal	Link	Real beam-এ internal hinge থাকলে, deflection = 0 & slope থাকবে এজন্য এমন support দিতে হবে যেখানে Moment = 0 & Shear থাকে। এমন support হলো Link support.
Link	Internal hinge	Real beam-এ Link থাকলে, যেখানে Slope থাকবে & deflection থাকবে এজন্য এমন support দিতে হবে যেখানে Moment & Shear থাকে। এমন support হলো Internal hinge

Problem: 01

Draw IL for R_A and R_B:

IL for R_A:



$M_A' = -64$

$M_G' = -64 + 18.67 \times 2 - .5 \times 2 \times 2 \times (\frac{1}{3} \times 2) = -28$

$M_B' = 0$

$M_D' = \frac{16}{3} \times 6 - .5 \times 6 \times 3 \times (\frac{1}{3} \times 6) = 14$

$M_E' = \frac{16}{3} \times 4 - .5 \times 4 \times 2 \times (\frac{1}{3} \times 4)$

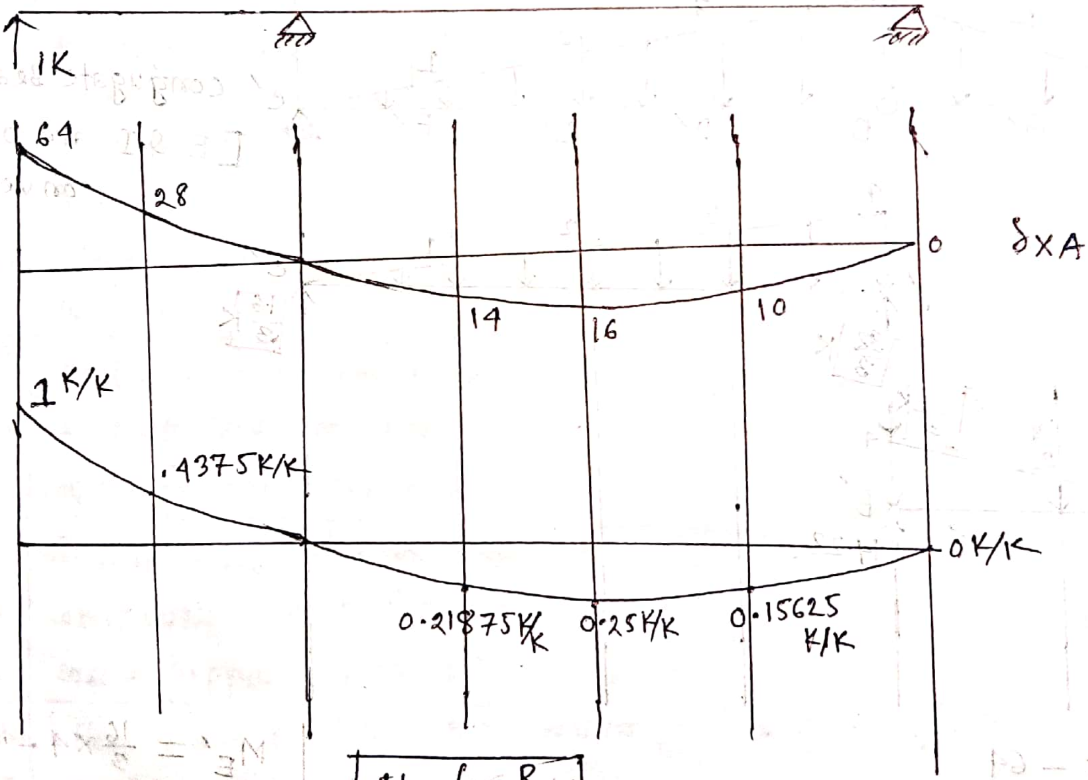
$= 16$

$M_F' = \frac{16}{3} \times 2 - .5 \times 2 \times 1 \times (\frac{1}{3} \times 2)$

$= 10$

$M_C' = 0$

point	x (ft)	δ_{XA}	δ_{XA}/δ_{AA}
A	0	$\delta_{AA} = -64$	1
G	2	$\delta_{GA} = -28$	0.4375
B	4	$\delta_{BA} = 0$	0
D	6	$\delta_{DA} = 14$	−0.21875
E	8	$\delta_{EA} = 16$	−0.25
F	10	$\delta_{FA} = 10$	−0.15625
C	12	$\delta_{CA} = 0$	0



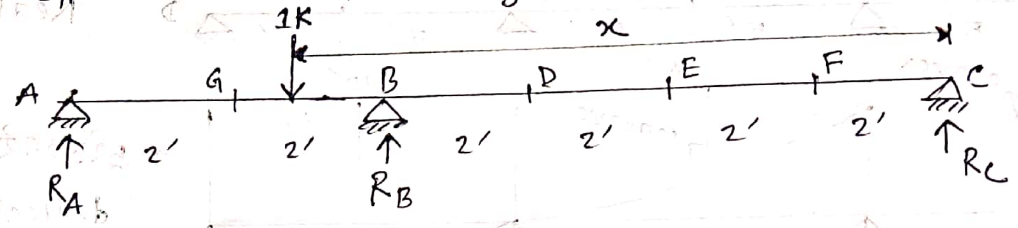
* একক দিতে হবে (K/K)

$$R_A = \frac{\delta_{XA}}{\delta_{AA}}$$

IL for R_A

IL for R_B:

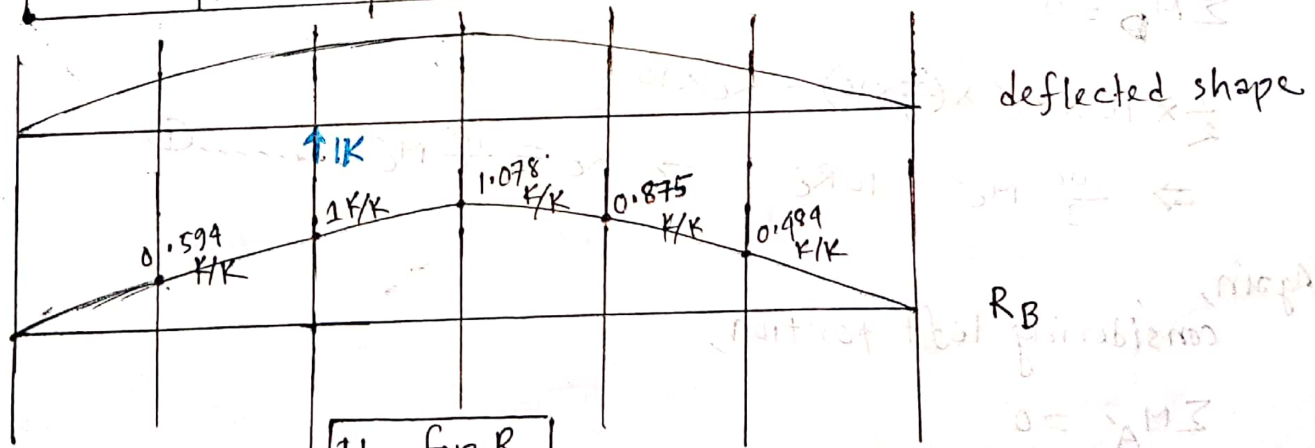
using equilibrium condition for Influence line of R_B.



$\Sigma M_C = 0$

$R_B \times 8 - 1 \times x + R_A \times 12 = 0 \Rightarrow R_B = \frac{x - 12R_A}{8} = \frac{x}{8} - \frac{12}{8} \times R_A$

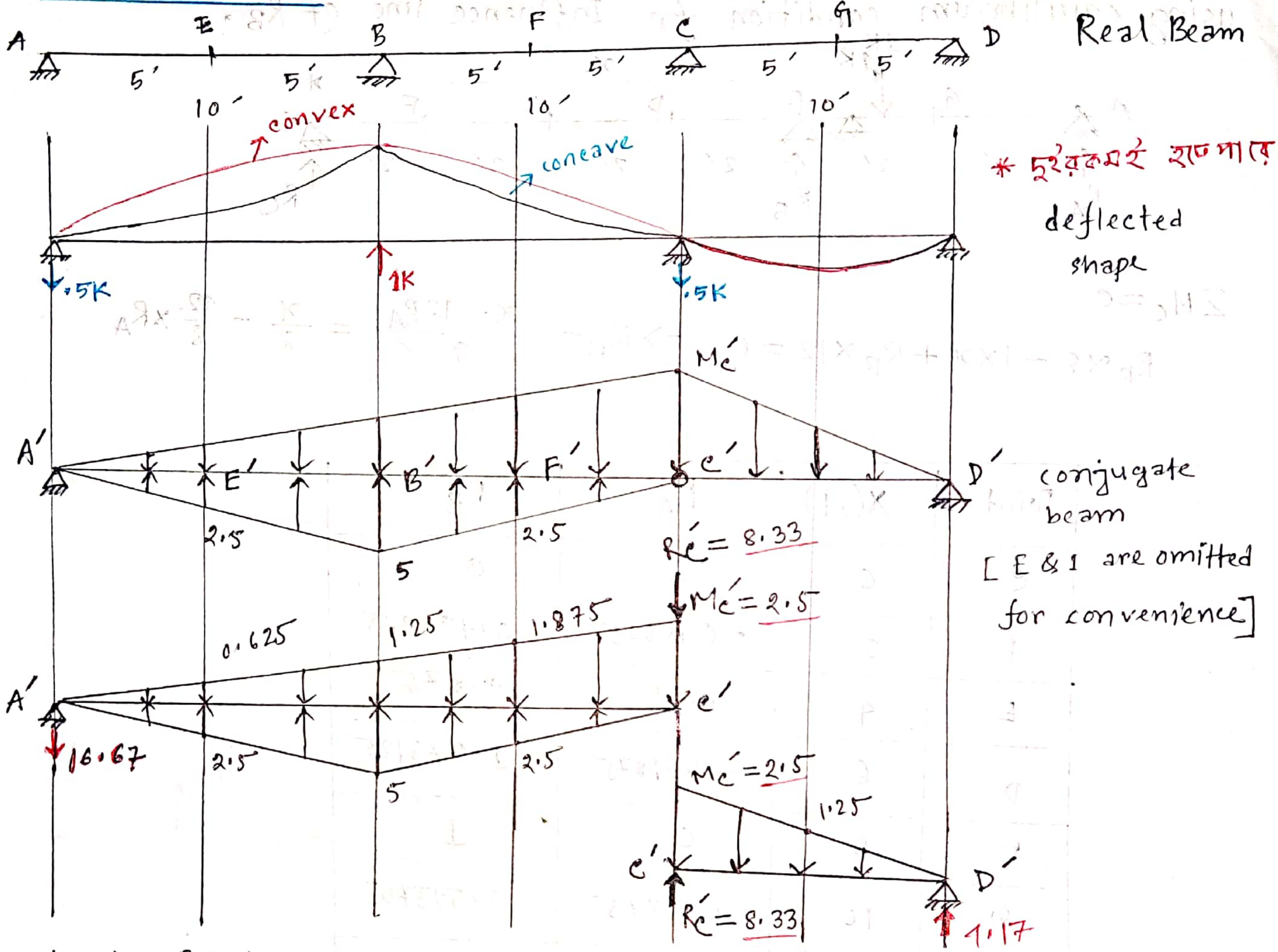
Point	X(ft)	R_A	R_B
c	0	0	0
F	2	-0.15625	0.484375
E	4	-0.25	0.875
D	6	-0.21875	1.078125
B	8	0	1
G	10	0.4375	0.59375
A	12	1	0



IL for R_B

Assignment : 01

Draw IL for R_B:



considering Right portion,

$\Sigma M_{D'} = 0$

$\frac{1}{2} \times 10 \times M_c' \times (\frac{2}{3} \times 10) - R_c' \times 10 = 0$

$\Rightarrow \frac{100}{3} M_c' = 10 R_c' \quad \Rightarrow R_c' = \frac{10}{3} M_c' \quad \text{.......(i)}$

Again,

considering left portion,

$\Sigma M_{A'} = 0$

$\frac{1}{2} \times 20 \times M_c' \times (\frac{2}{3} \times 20) + R_c' \times 20 - \frac{1}{2} \times 20 \times 5 \times 10 = 0$

$\Rightarrow 20 R_c' + \frac{400}{3} M_c' = 500 \quad \text{......(ii)}$

From equation ① and ⑪ we obtain,

$$M_C' = 2.5$$

$$R_C' = 8.33$$

Now, $M_A' = 0$

$M_E' = -16.67 \times 5 - .5 \times .625 \times 5 \times (\frac{1}{3} \times 5) + .5 \times 2.5 \times 5 \times (\frac{1}{3} \times 5) = -75.5375$

$M_B' = -16.67 \times 10 - .5 \times 1.25 \times 10 \times (\frac{1}{3} \times 10) + .5 \times 5 \times 10 \times (\frac{1}{3} \times 10) = -104.2$

$M_F' = -16.67 \times 15 - .5 \times 1.875 \times 15 \times (\frac{1}{3} \times 15) + .5 \times 10 \times 5 \times (5 + \frac{1}{3} \times 10)$

$+ 2.5 \times 5 \times \frac{5}{2} + \frac{1}{2} \times 5 \times (5 - 2.5) \times (\frac{2}{3} \times 5) = -59.946$

$M_C' = \boxed{-16.67 \times 20 - .5 \times 2.5 \times 20 \times (\frac{1}{3} \times 20) + .5 \times 20 \times 5 \times 10} = 0$

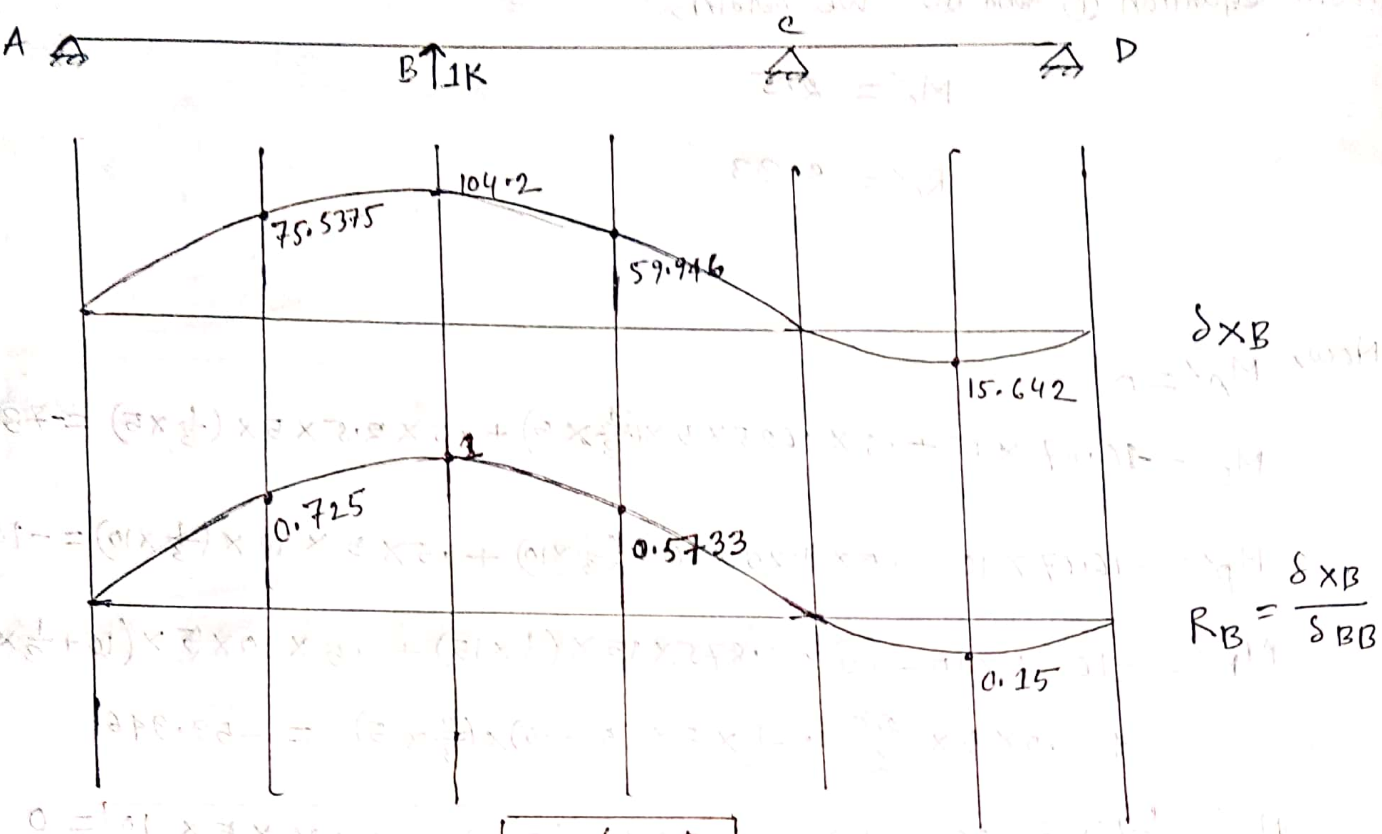
↳ সমাধান নেই

* [Link-এ moment = 0]

$M_G' = 4.17 \times 5 - .5 \times 1.25 \times 5 \times (\frac{1}{3} \times 5) = 15.642$

$M_D' = 0$

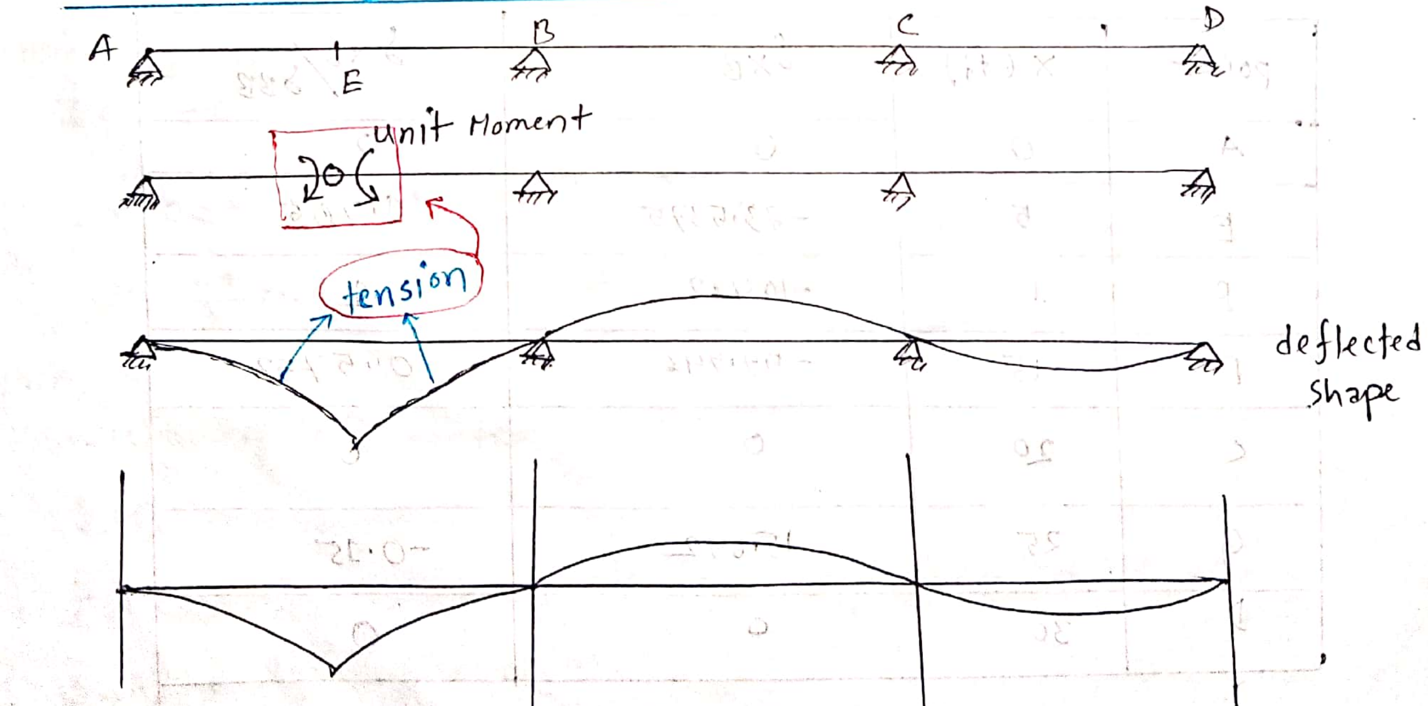
point	x (ft)	δ_{XB}	δ_{XB}/δ_{BB}
A	0	0	0
E	5	-75.5375	0.725
B	10	-104.2	1
F	15	-59.946	0.5753
C	20	0	0
G	25	15.642	-0.15
D	30	0	0



IL for R_B

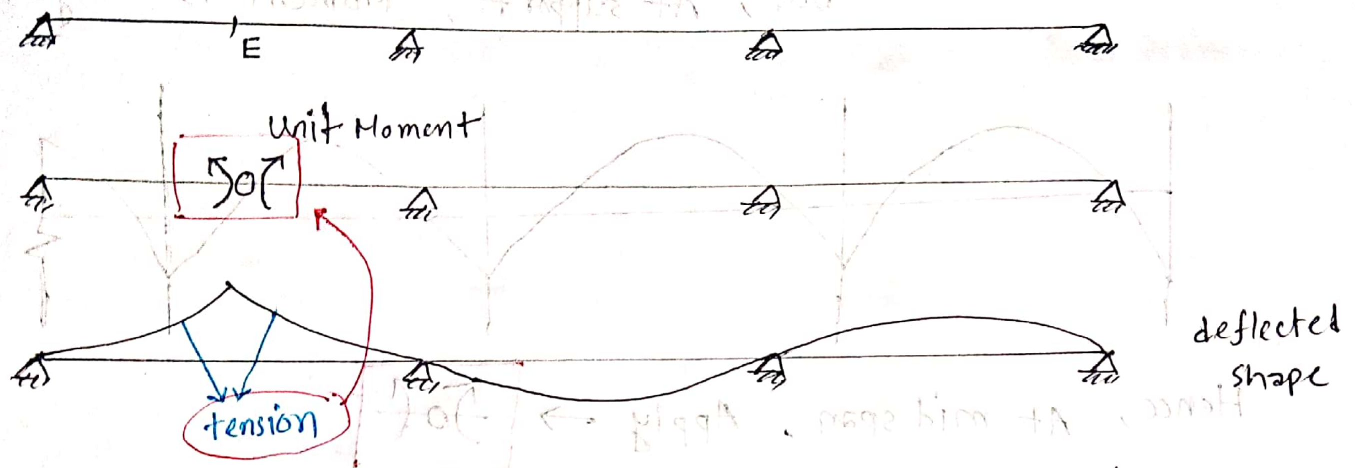
IL for Moment

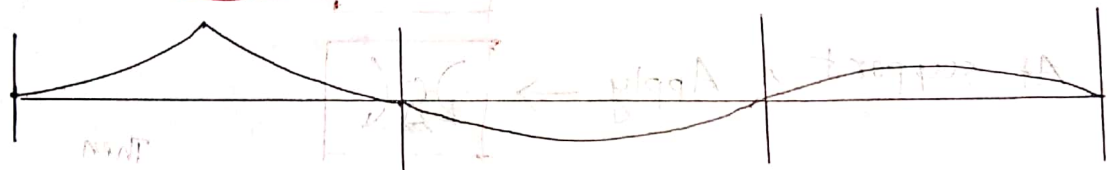
Draw IL for M_E for tension on upper* fibre:



IL for M_E

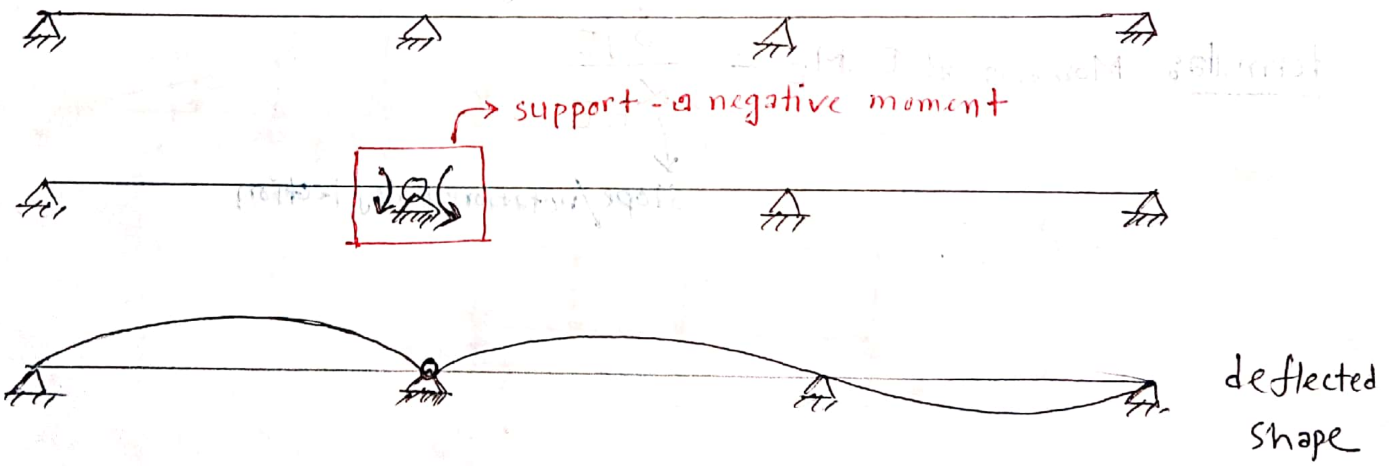
Draw IL for M_E for tension on *lower fibre:

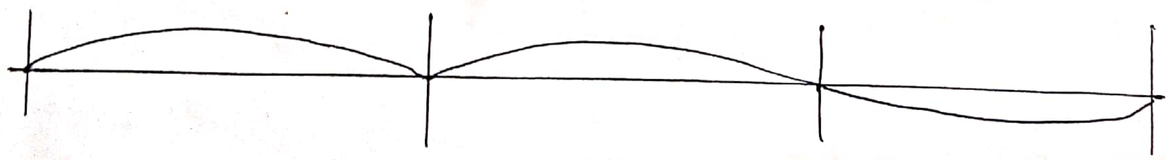




IL for M_E

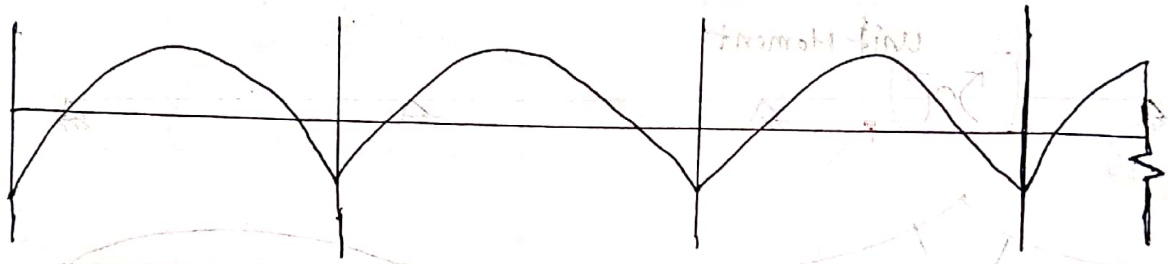
Draw IL for M_B:

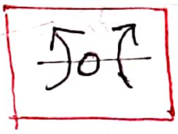


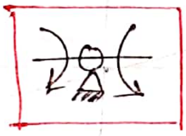


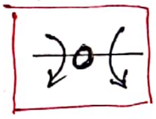
IL for M_B

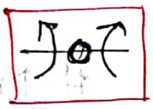
* **Note:** For Continuous Structure, At mid span, Moment is positive but, At support, Moment is Negative.



Hence, At mid span, Apply → 

At support, Apply → 

But, if specificaly asked for tension on upper fibre, Then Apply 

and, for tension on lower fibre, Apply 

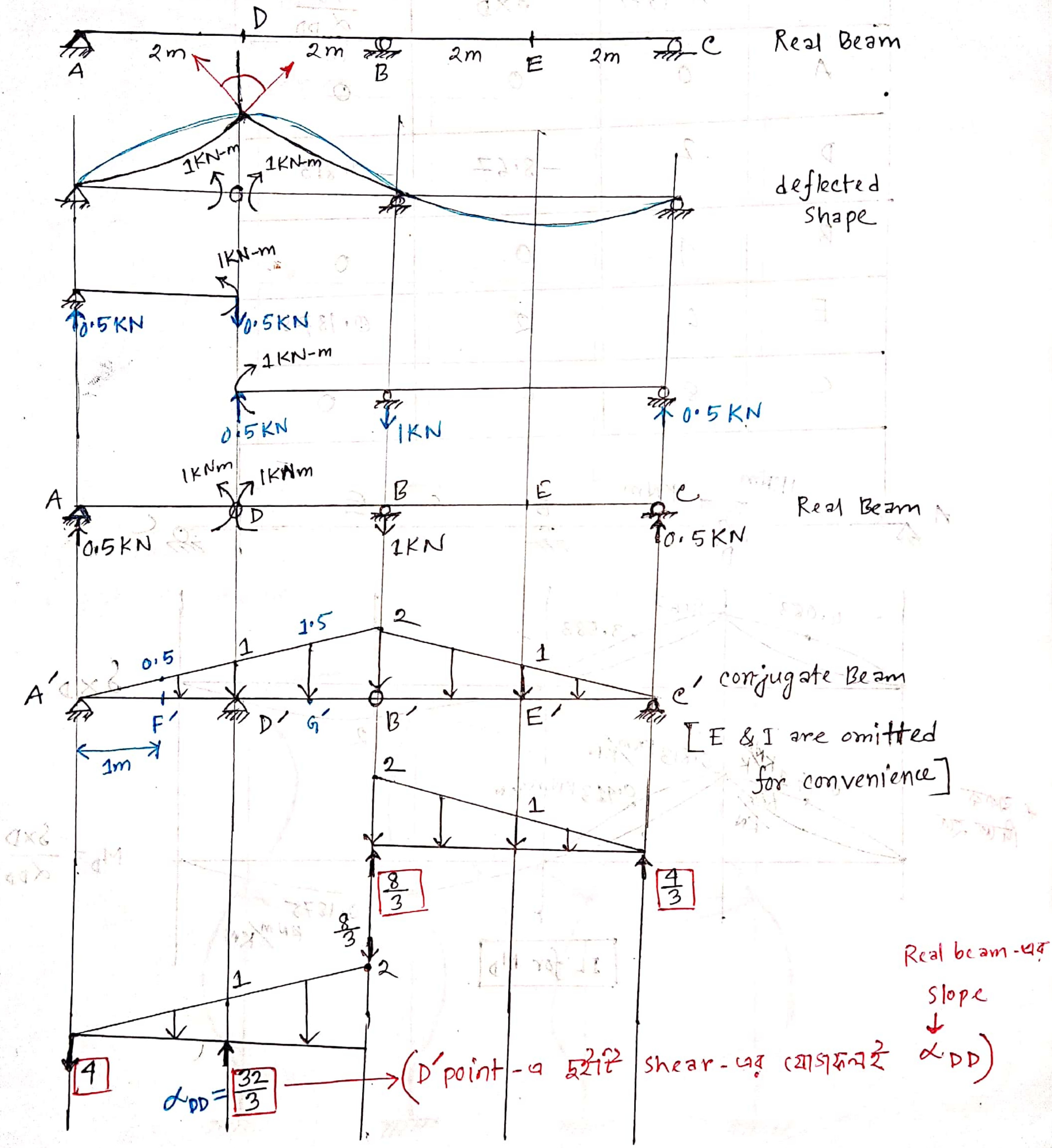
Formula: Moment at E, $M_E = \dfrac{\delta_{JE}}{\alpha_{EE}}$

δ_{JE} → deflection

α_{EE} → slope/rotational deflection

Problem: 02

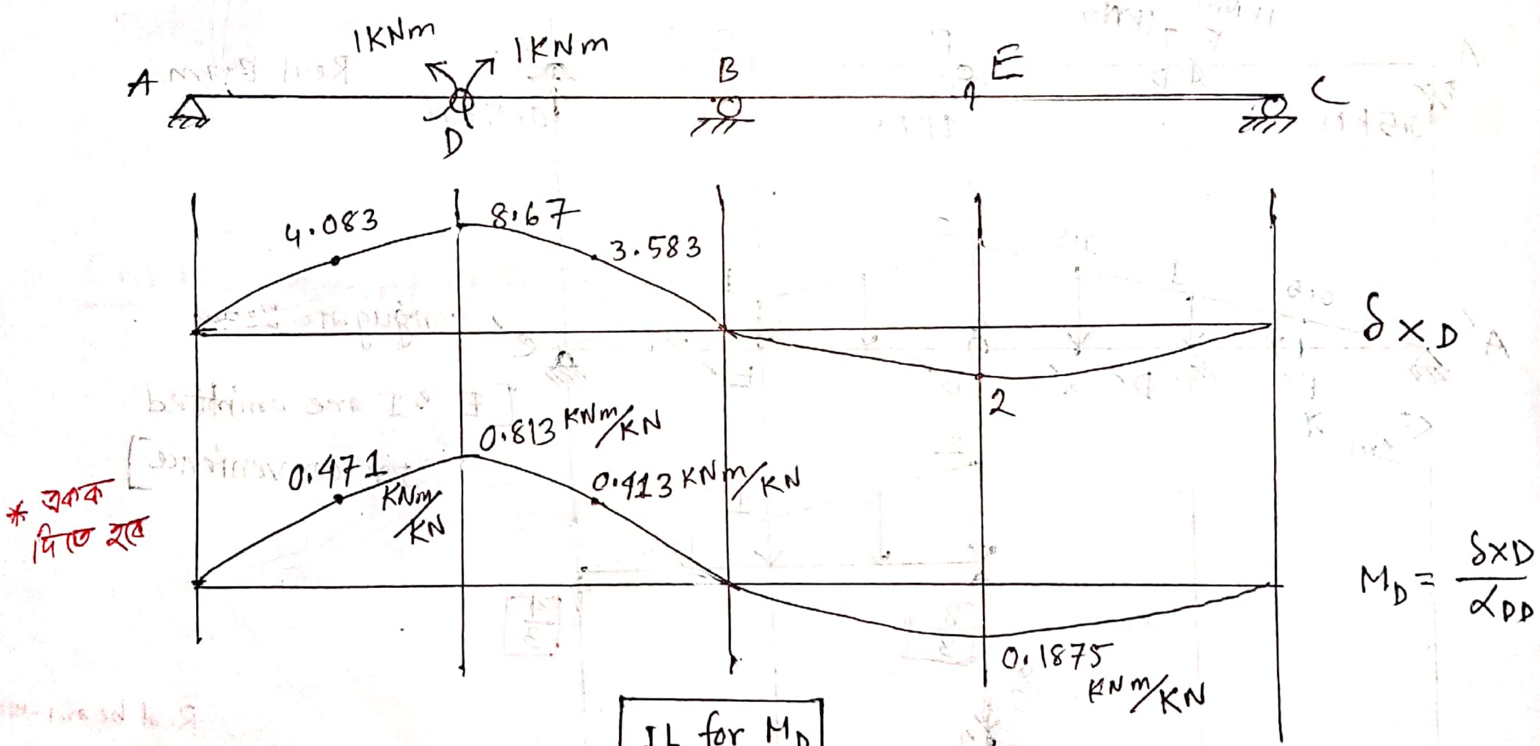
Draw IL for M_D: 2016



$M_{A'} = 0$ $\quad$ $M_{B'} = 0$ $\quad$ $M_{C'} = 0$

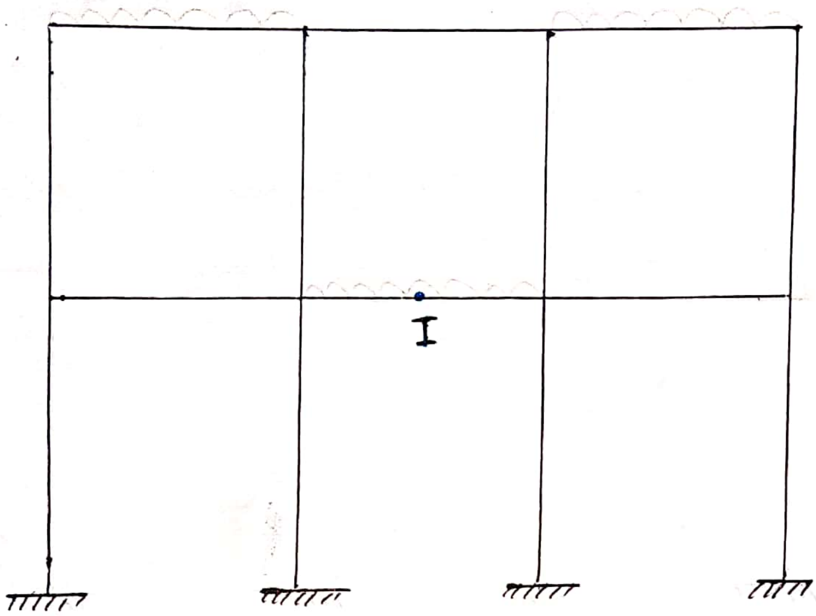
$M_{D'} = -8.67$ $\quad$ $M_{E'} = 2$ $\quad$ * $M_{F'} = -4.083$ $\quad$ * $M_{G'} = -3.583$

point	X (ft)	δ_{XD}	$\frac{\delta_{XD}}{\alpha_{DD}}$
A	0	0	0
D	2	−8.67	−0.813
B	4	0	0
E	6	2	0.1875
C	8	0	0

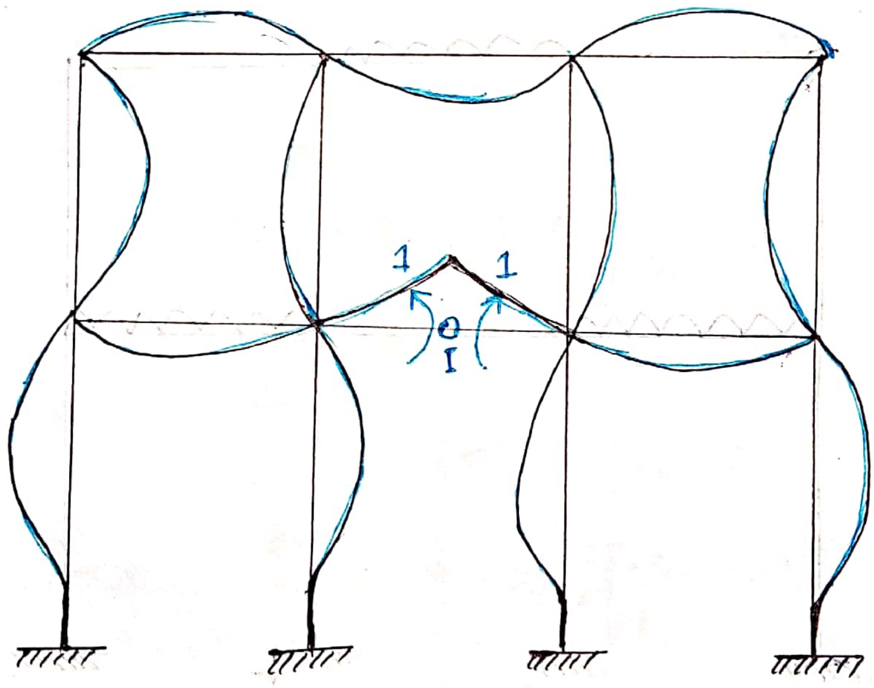


IL for M_D

Draw qualatative diagram of Influence Line for M_I of the following frame:

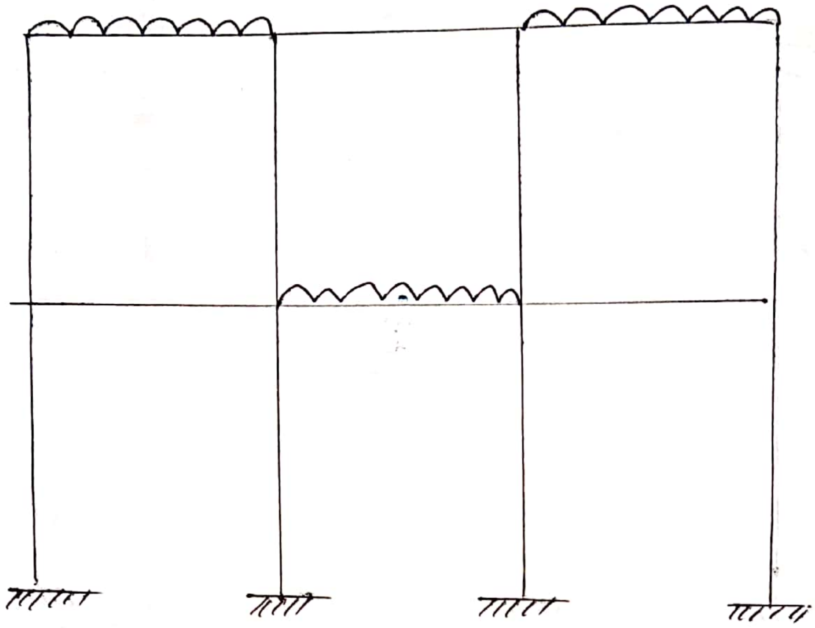


* Also draw loading for Maximum positive Moment and Maximum negative moment.

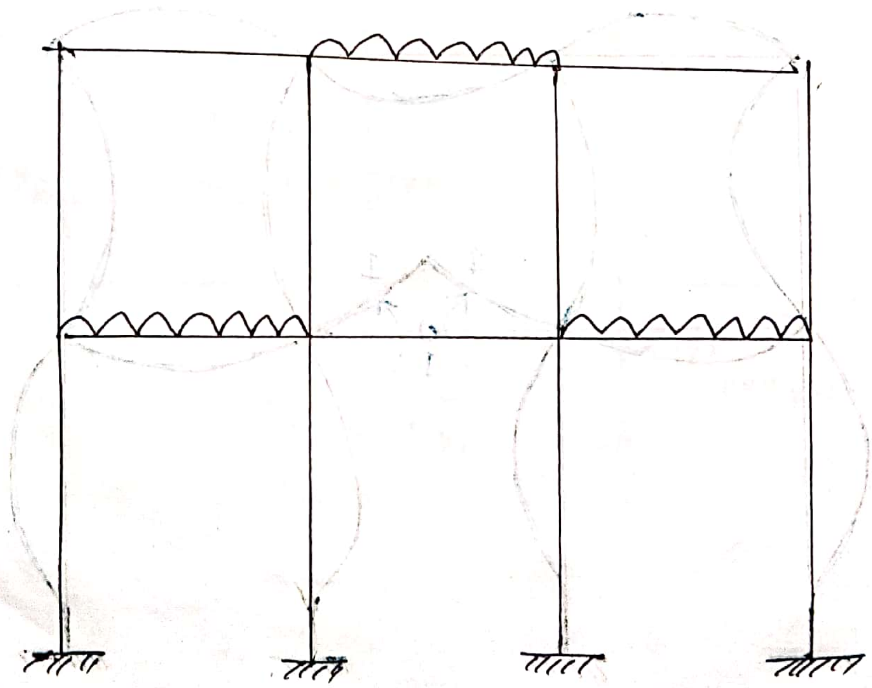


IL for M_I

Loading → For Maximum +ve Moment

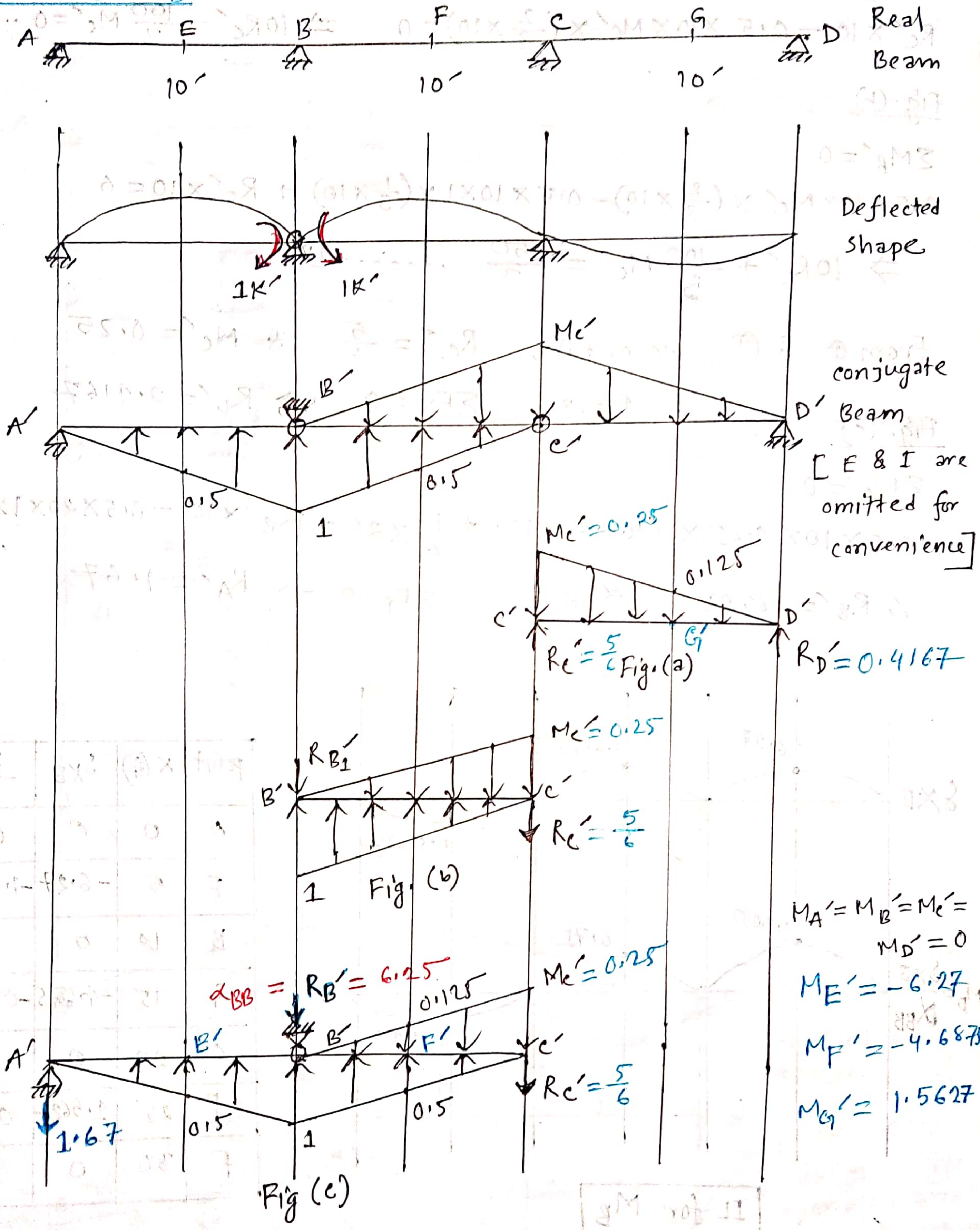


Loading → For Maximum -ve Moment



Assignment : 02

Draw IL for M_B : 2008



$M_A' = M_B' = M_C' = M_D' = 0$

$M_E' = -6.27$

$M_F' = -4.6875$

$M_G' = 1.5627$

Fig. (a)

$\Sigma M_D' = 0$

$R_C' \times 10 - 0.5 \times 10 \times M_C' \times (\frac{2}{3} \times 10) = 0 \Rightarrow 10R_C' - \frac{100}{3} M_C' = 0$ ....(I)

Fig. (b)

$\Sigma M_B' = 0$

$0.5 \times 10 \times M_C' \times (\frac{2}{3} \times 10) - 0.5 \times 10 \times 1 \times (\frac{1}{3} \times 10) + R_C' \times 10 = 0$

$\Rightarrow 10R_C' + \frac{100}{3} M_C' = \frac{50}{3}$ ....(II)

From (I) & (II), we obtain, $R_C' = \frac{5}{6}$ & $M_C' = 0.25$

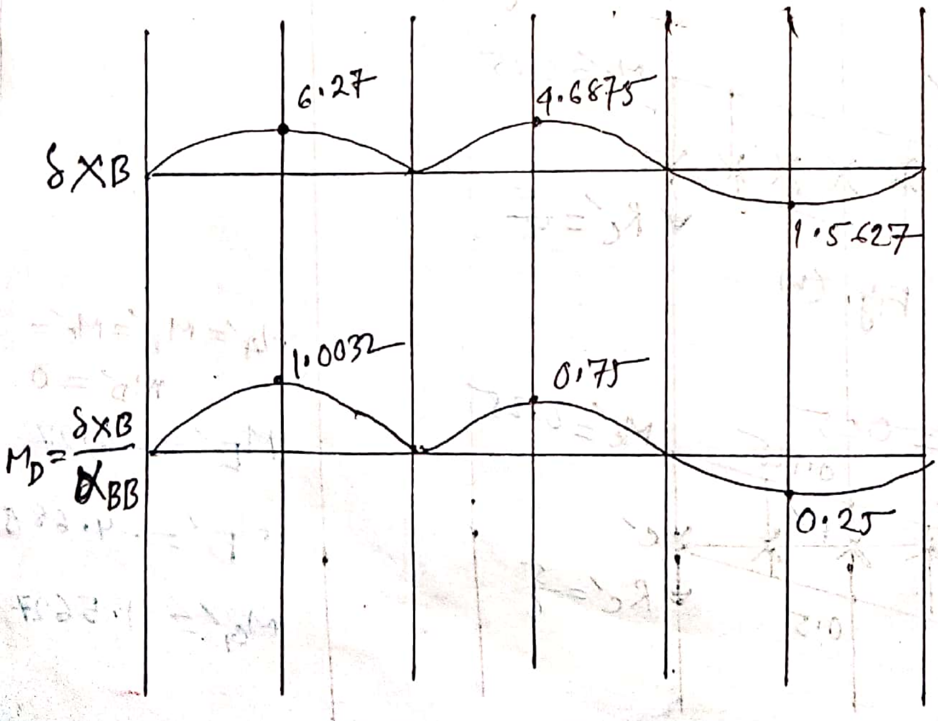
Fig (a) $\Sigma F_Y = 0 \rightarrow R_D' = 0.4167$

Fig. (c)

$\Sigma M_A' = 0$

$0.5 \times 10 \times 0.25 \times (10 + \frac{2}{3} \times 10) + \frac{5}{6} \times 20 + R_B' \times 10 - 0.5 \times 20 \times 1 \times 10 = 0$

$\therefore R_B' = 6.25 = \alpha_{BB}$ $\Sigma F_y = 0 \rightarrow R_A' = 1.67$



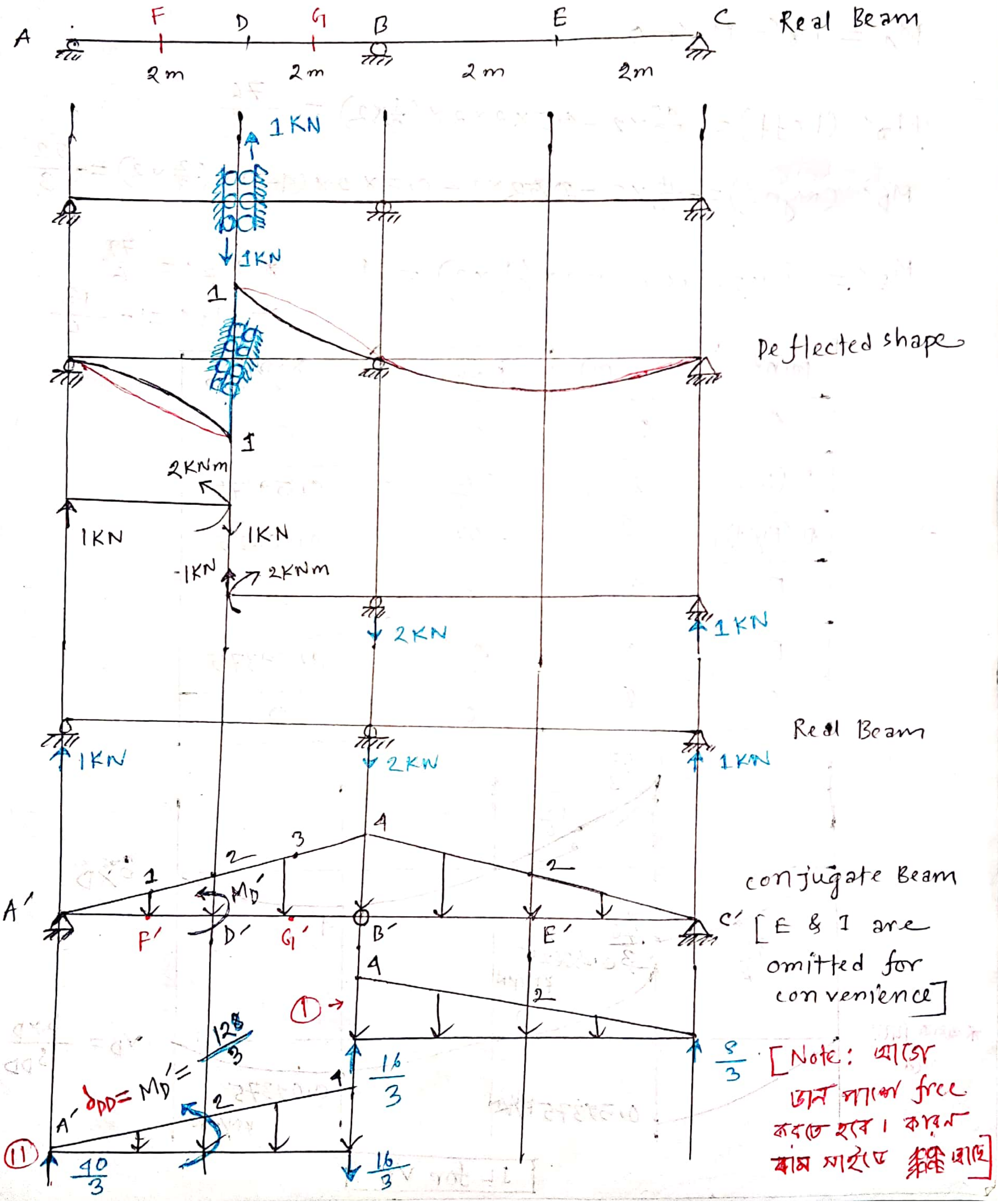
IL for M_B

point	x (ft)	δ_{XB}	$\frac{\delta_{XB}}{\delta_{BB}}$
A	0	0	0
F	5	-6.27	-1.0032
B	10	0	0
G	15	-4.6875	-0.75
C	20	0	0
F	25	1.5627	0.25
F	30	0	0

Influence Line for Shear

Problem : 03

Draw IL for V_D : 2014, 2010



$\Sigma M_{A}' = 0$

$-M_{D}' + .5 \times 4 \times 4 \times (\frac{2}{3} \times 4) + \frac{16}{3} \times 4 = 0$

$\Rightarrow M_{D}' = \frac{128}{3} = \delta_{DD}$

$M_{A}' = M_{B}' = M_{C}' = 0$

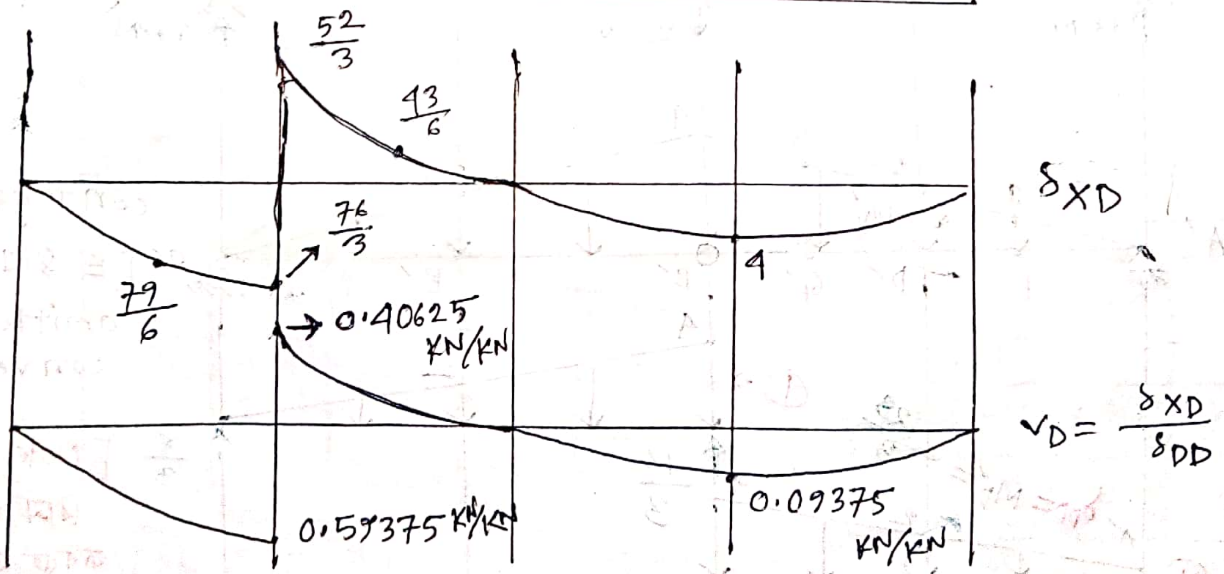
$M_{D}' \text{ (Left)} = \frac{40}{3} \times 2 - 0.5 \times 2 \times 2 \times (\frac{1}{3} \times 2) = \frac{76}{3}$

$M_{D}' \text{ (Right)} = -\frac{16}{3} \times 2 - 2 \times 2 \times 1 - 0.5 \times 2 \times (4-2) \times (\frac{2}{3} \times 2) = -\frac{52}{3}$

$M_{E}' = \frac{8}{3} \times 2 - .5 \times 2 \times 2 \times (\frac{1}{3} \times 2) = 4$

* $M_{F}' = \frac{79}{6}$
* $M_{G}' = -\frac{43}{6}$

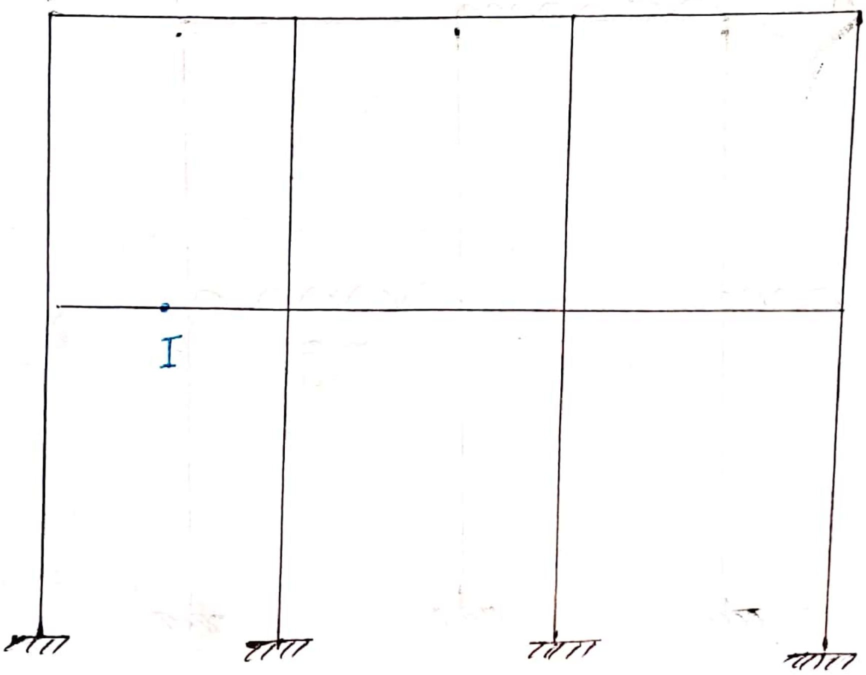
point	x (m)	δ_{XD}	δ_{XD}/δ_{DD}
A	0	0	0
D (Left)	2	$\frac{76}{3}$	0.59375
D (Right)	2	$-\frac{52}{3}$	-0.40625
B	4	0	0
E	6	4	0.09375
C	8	0	0



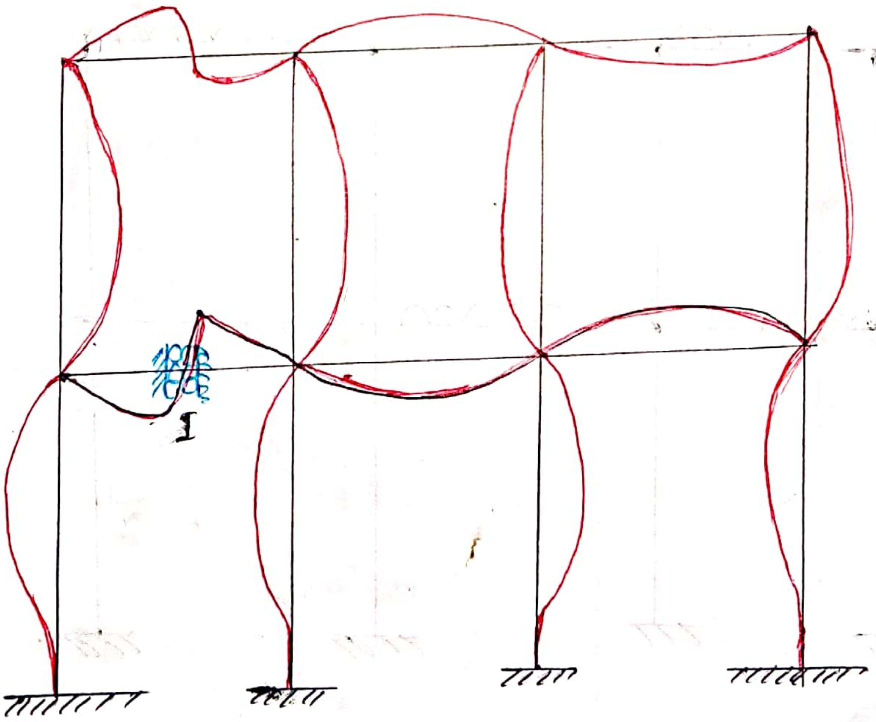
*একক দিতে হবে

IL for V_D

Draw Qualatative Influence Line for V_I for the following frame:

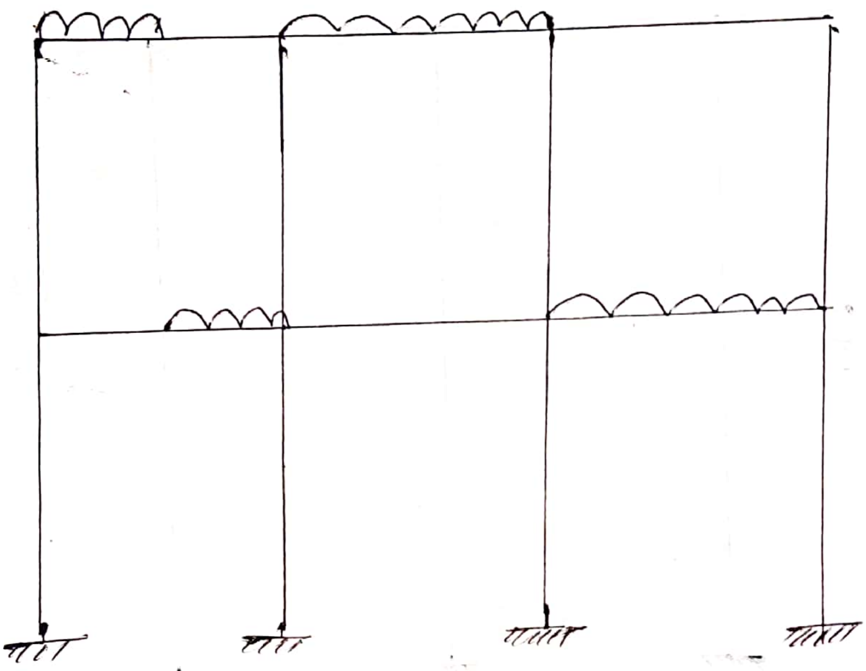


Also draw loading for Maximum positive shear and Maximum negative shear.

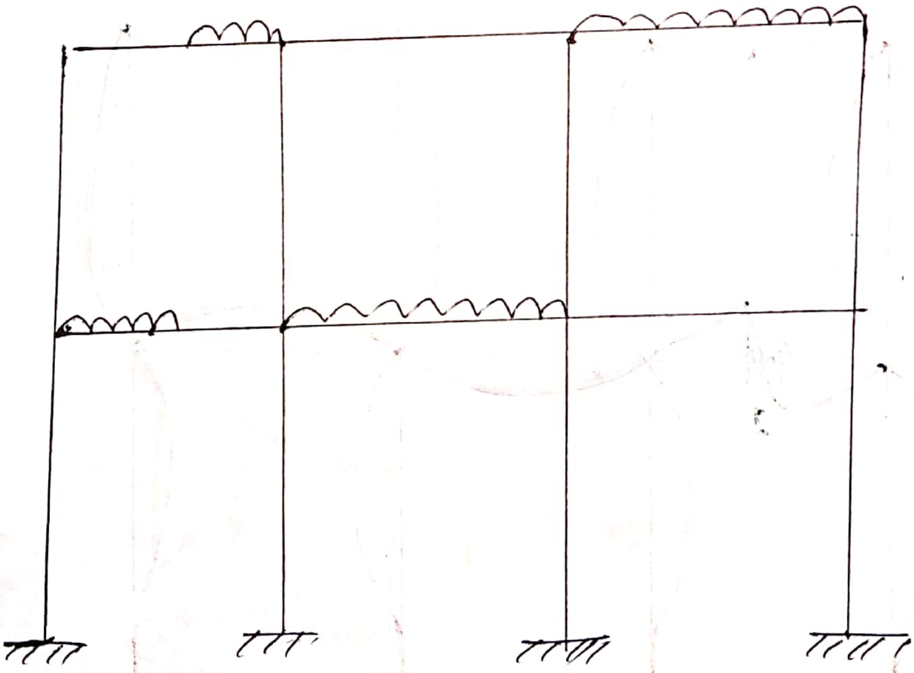


IL for V_I

Loading for Maximum +ve Shear



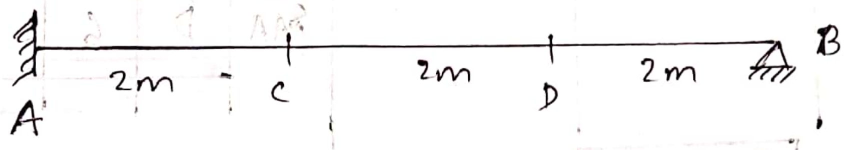
Loading for Maximum -ve shear



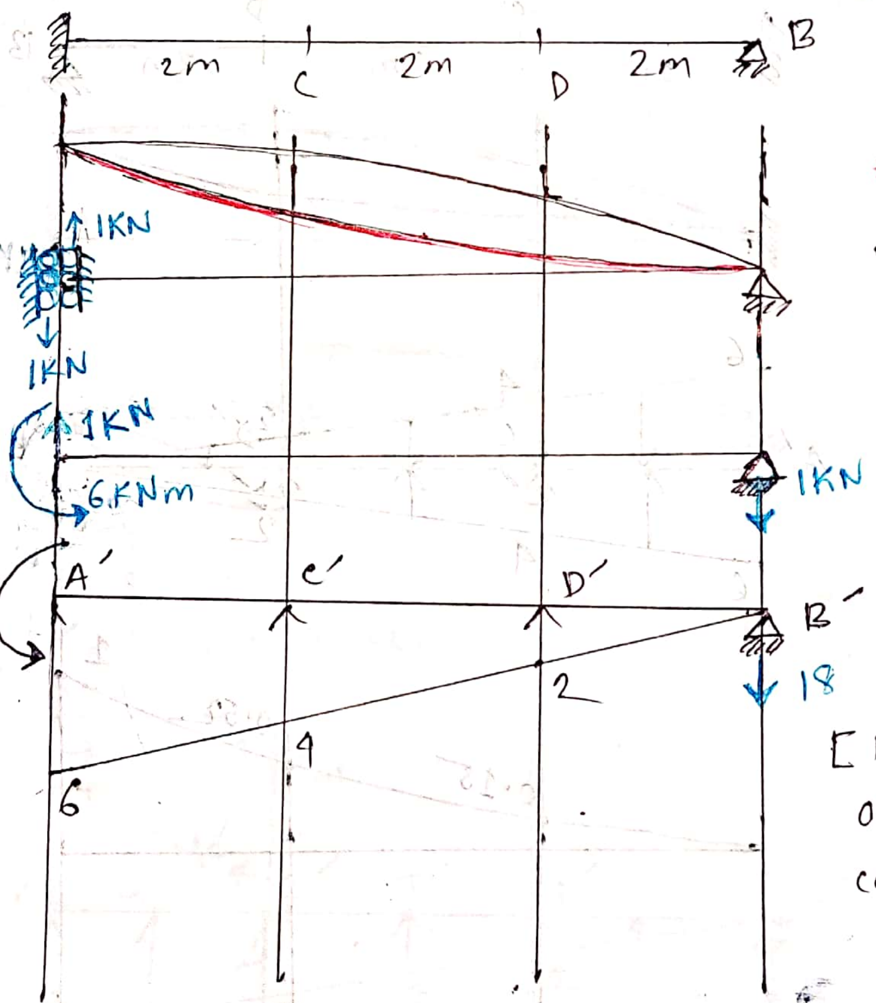
Assignment: 03

Draw Influence Line for — the vertical reaction at A and B
— Shear at C
— Bending moment at A and C

EI is constant. Plot Numerical values for every 2m.



(i) IL for R_A:



convex/concave
* দুই ধরনের হতে পারে।

Deflected shape

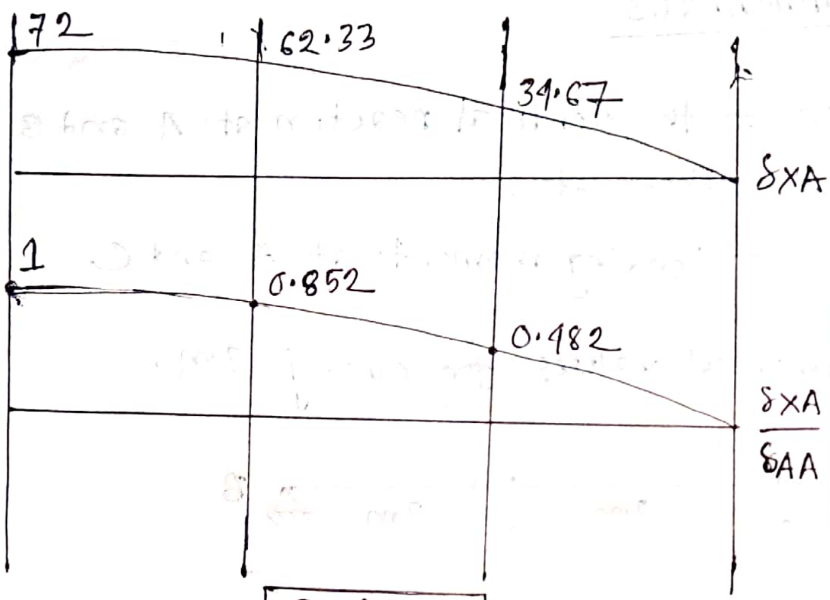
Real Beam

conjugate Beam

[E & I are omitted for convenience]

$$M_D' = -18\times 2 + .5\times 2\times 2\times\left(\tfrac{1}{3}\times 2\right) = -34.67$$

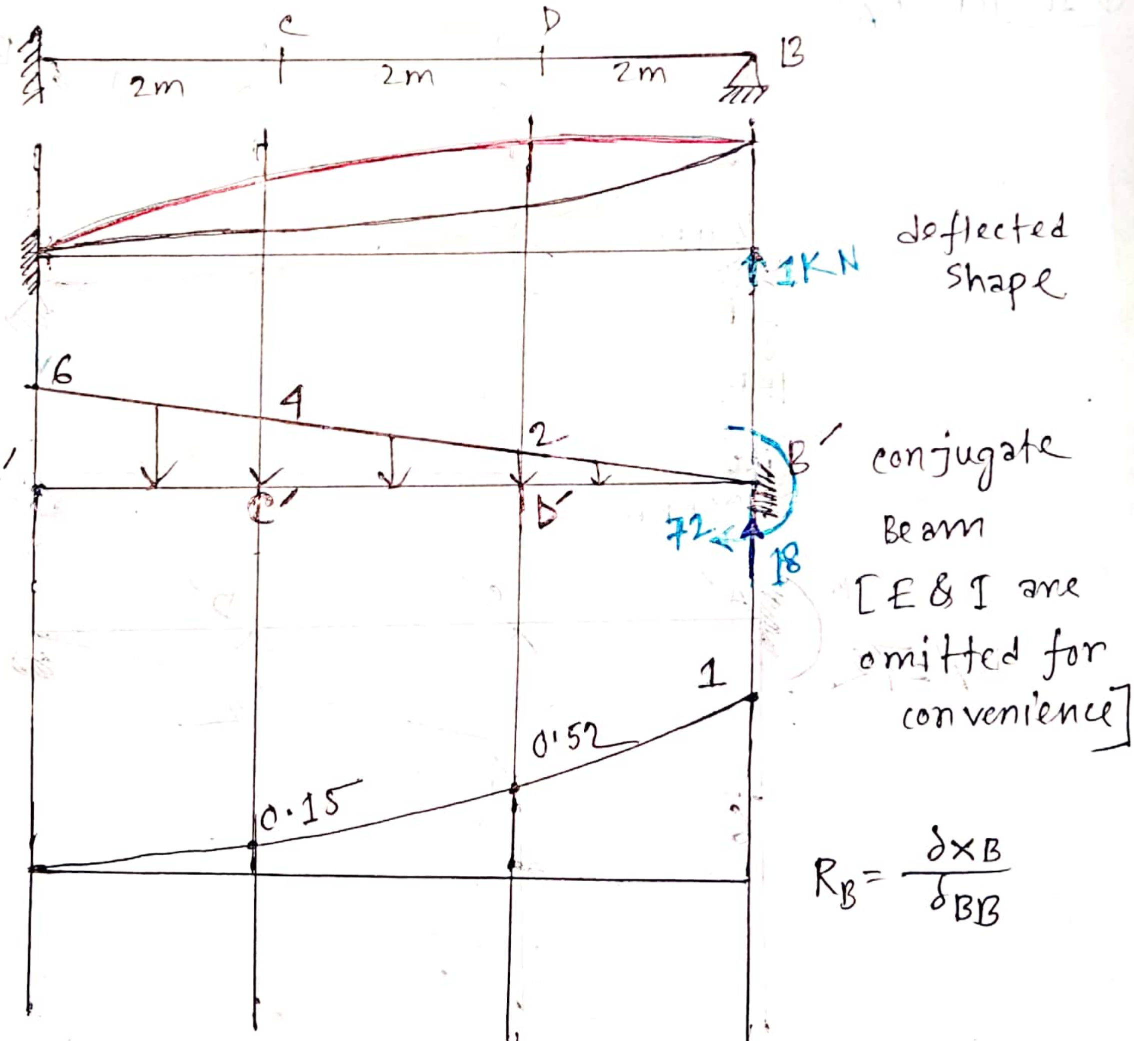
$$M_C' = -18\times 4 + .5\times 4\times 4\times\left(\tfrac{1}{3}\times 4\right) = -61.33$$



Point	X(m)	δ_{XA}	$\frac{\delta_{XA}}{\delta_{AA}}$
A	0	-72	1
C	2	-61·33	0·852
D	4	-34·67	0·482
D	6	0	0

IL for R_A

(ii) IL for R_B: 2013



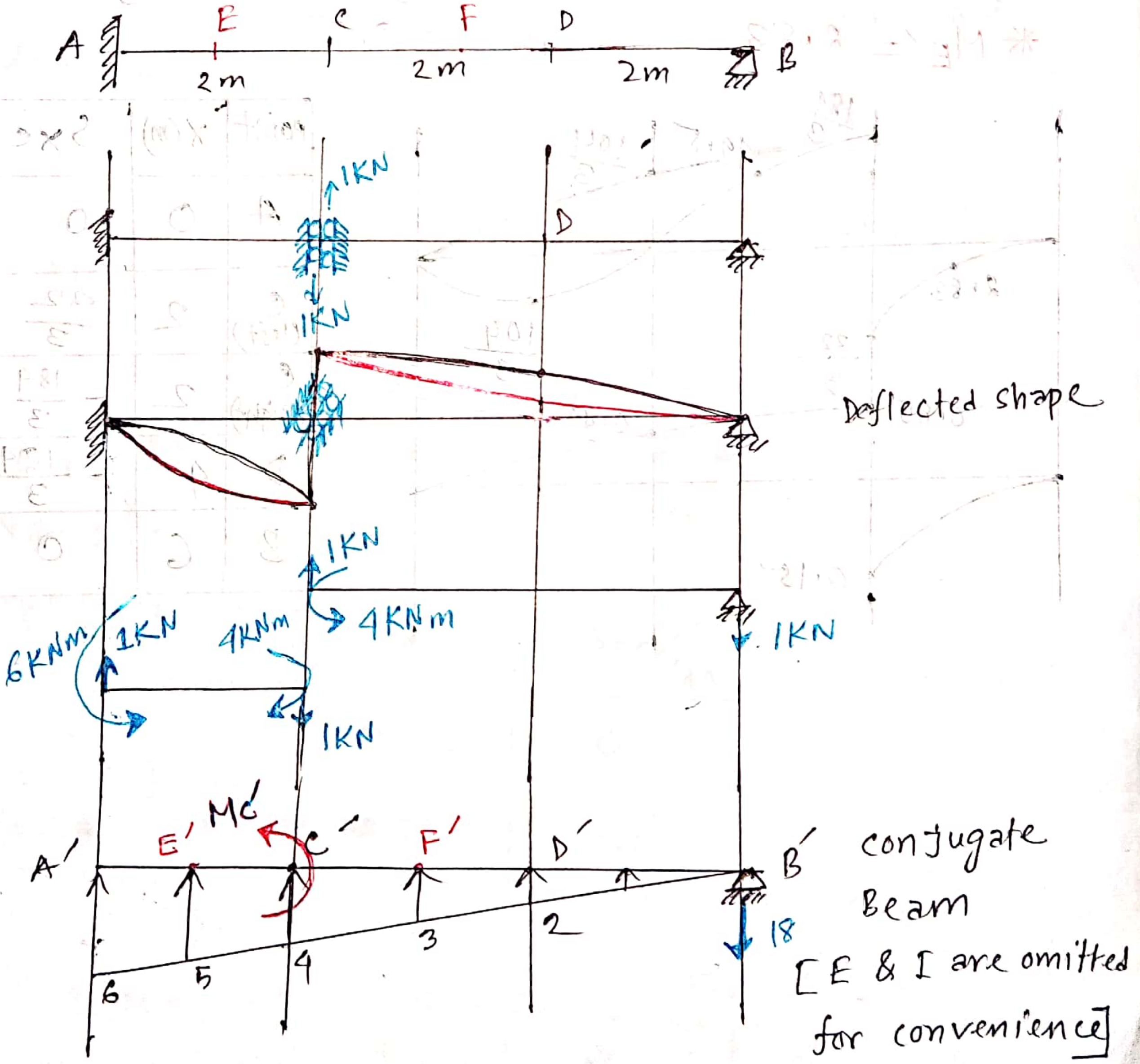
$$R_B = \frac{\delta_{XB}}{\delta_{BB}}$$

$$M_D' = -72 - \cdot 5 \times 2 \times 2 \times \left(\tfrac{1}{3} \times 2\right) + 18 \times 2 = -37 \cdot 33$$

$$M_C' = -72 - \cdot 5 \times 4 \times 4 \times \left(\tfrac{1}{3} \times 4\right) + 18 \times 4 = -10 \cdot 67$$

Point	x (m)	δ_{xB}	δ_{xB}/δ_{BB}
B	0	-72	1
D	2	-37.33	0.52
C	4	-10.67	0.15
A	6	0	0

(iii) IL for V_C :



$\Sigma M_A' = 0$

$.5 \times 6 \times 6 \times (\frac{1}{3} \times 6) + M_c' - 18 \times 6 = 0$

$\therefore M_c' = 72 = \delta_{cc}$

Now,

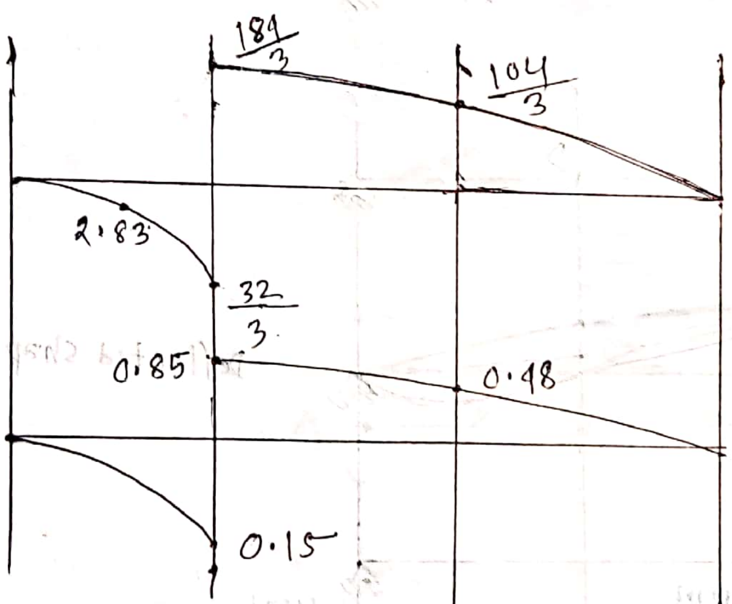
$M_A' = M_B' = 0$

$M_c'(\text{left}) = 4 \times 2 \times 1 + 0.5 \times 2 \times (6-4) \times (\frac{2}{3} \times 2) = \frac{32}{3}$

$M_c'(\text{Right}) = -18 \times 4 + .5 \times 4 \times 4 \times (\frac{1}{3} \times 4) = -\frac{184}{3}$

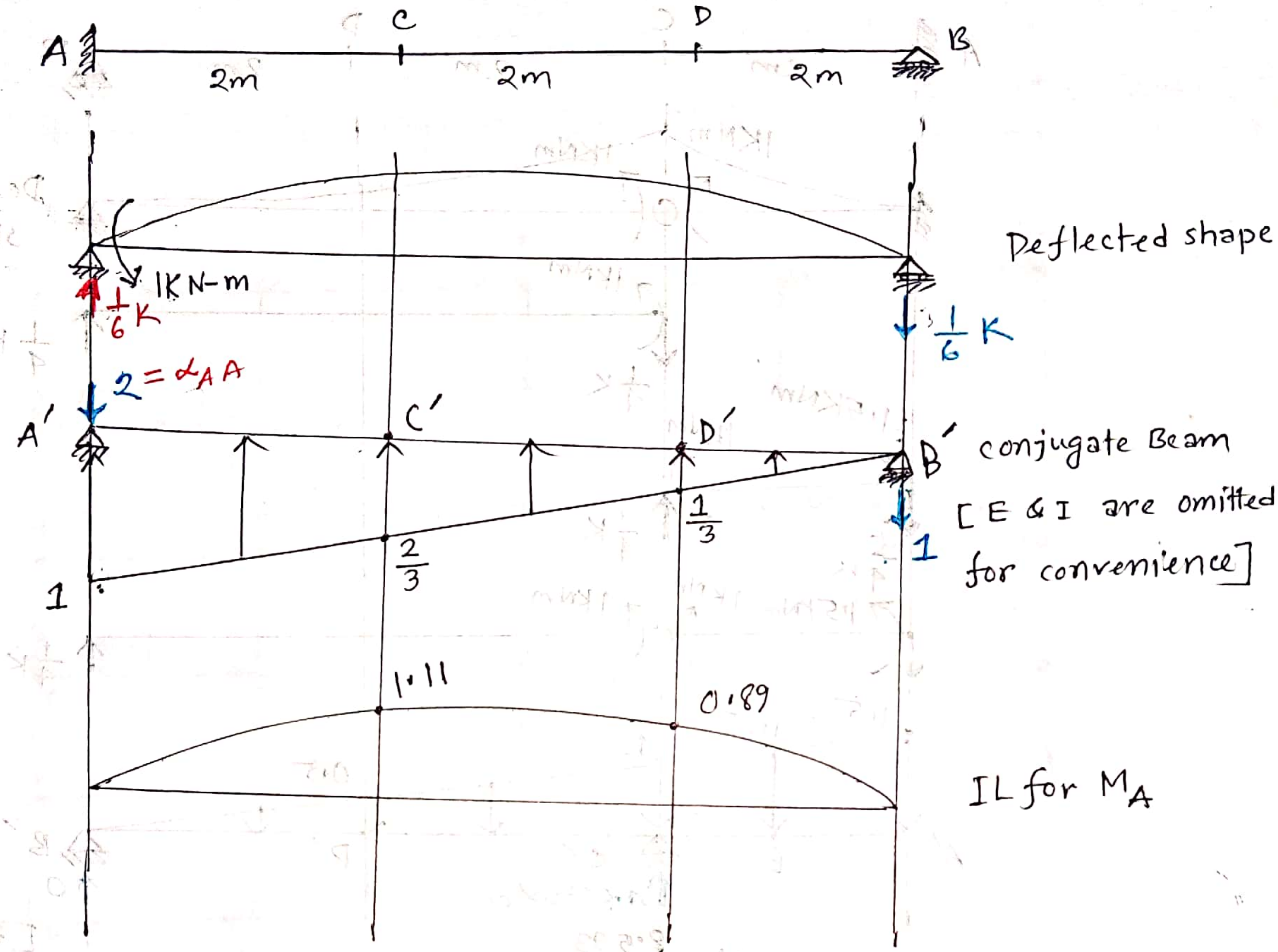
$M_D' = -18 \times 2 + .5 \times 2 \times 2 \times (\frac{1}{3} \times 2) = -\frac{104}{3}$

$* M_E' = 2.83$



point	x (m)	δ_{xc}	δ_{xc}/δ_{cc}
A	0	0	0
C (left)	2	$\frac{32}{3}$	0.15
C (Right)	2	$-\frac{184}{3}$	-0.85
D	4	$-\frac{104}{3}$	-0.48
B	6	0	0

(iv) IL for Moment M_A:

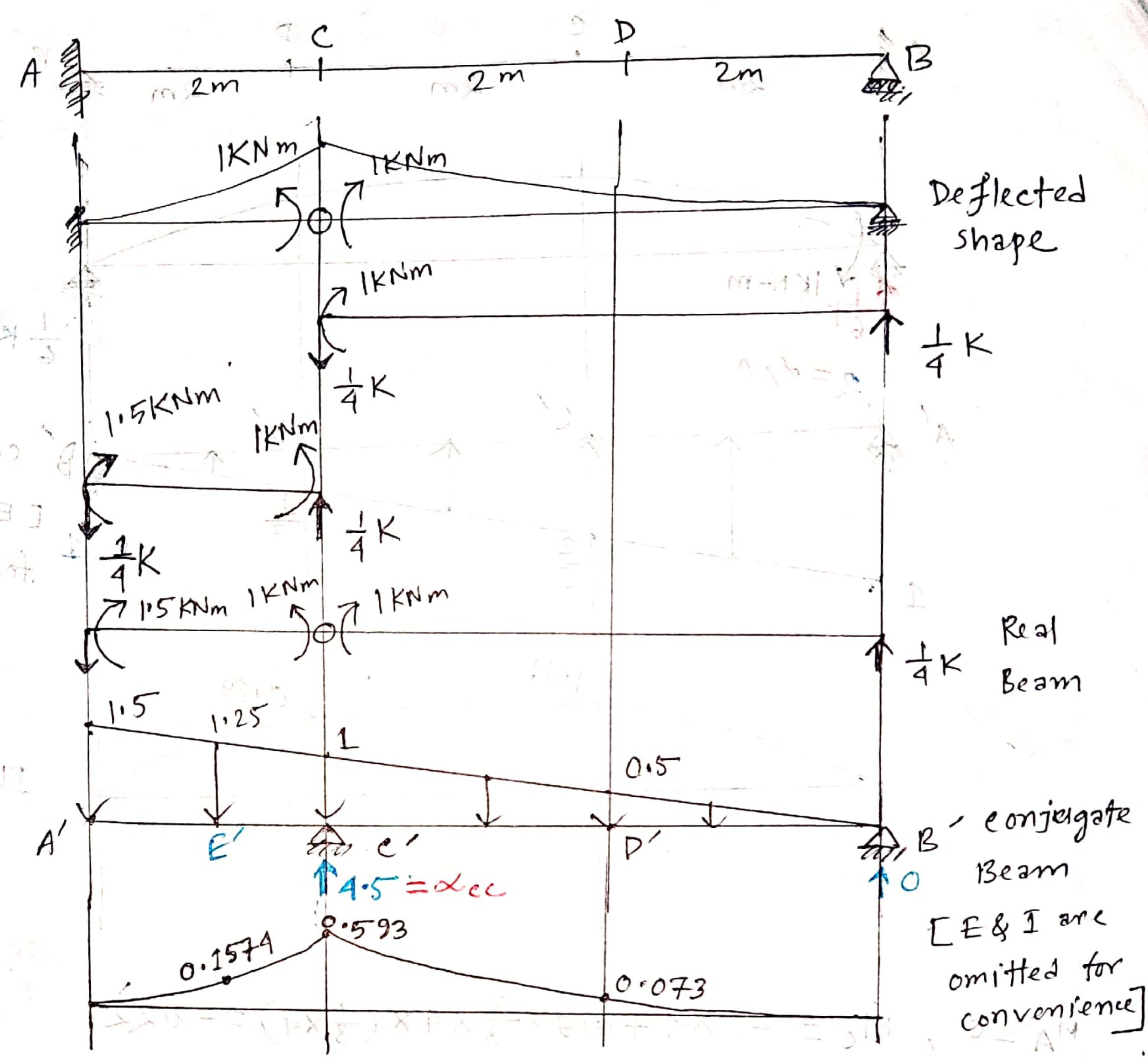


$M_{A}' = 0$, $M_{C}' = -1\times4 + 0.5\times\frac{2}{3}\times4\times(\frac{1}{3}\times4) = -2.22$

$M_{D}' = -1\times2 + 0.5\times\frac{1}{3}\times2\times(\frac{1}{3}\times2) = -1.78$, $M_{B}' = 0$

point	x (m)	δ_{xA}	δ_{xA}/α_{AA}
A	0	0	0
C	2	-2.22	-1.11
D	4	-1.78	-0.89
B	6	0	0

(v) IL for Moment M_C:



$M_{A'} = 0$, $M_{B'} = 0$

$M_{C'} = -.5 \times 4 \times 1 \times (\frac{1}{3} \times 4) = -2.67$

$M_{D'} = -.5 \times 2 \times 0.5 \times (\frac{1}{3} \times 2) = -0.33$

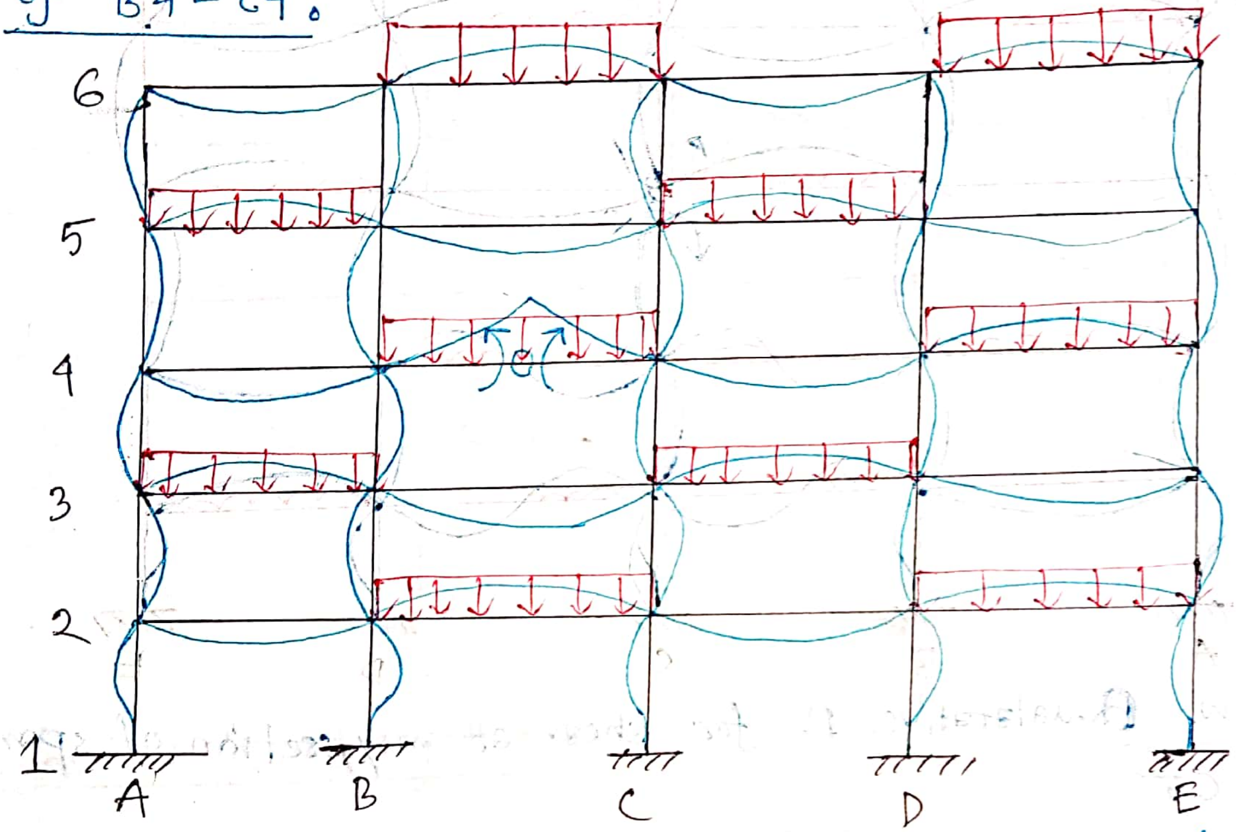
$M_{E'} = -0.5 \times 5 \times 1.25 \times (\frac{1}{3} \times 5) + (1.5 \times 1) = -0.7083$

point	x (m)	δxc	δxc/δcc
A	0	0	0
C	2	-2.67	-0.593
D	4	-0.33	-0.073
B	6	0	0

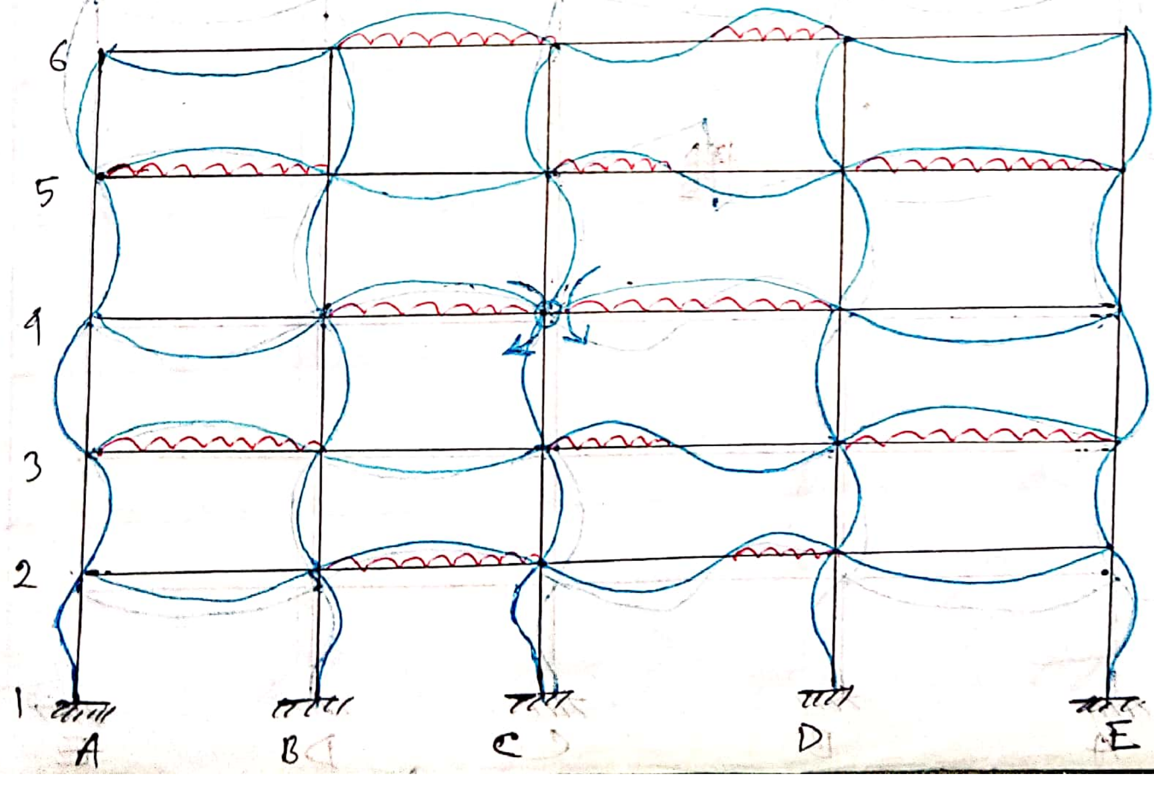
Qualatative Influence Line Diagram

IL Diagram for Frame

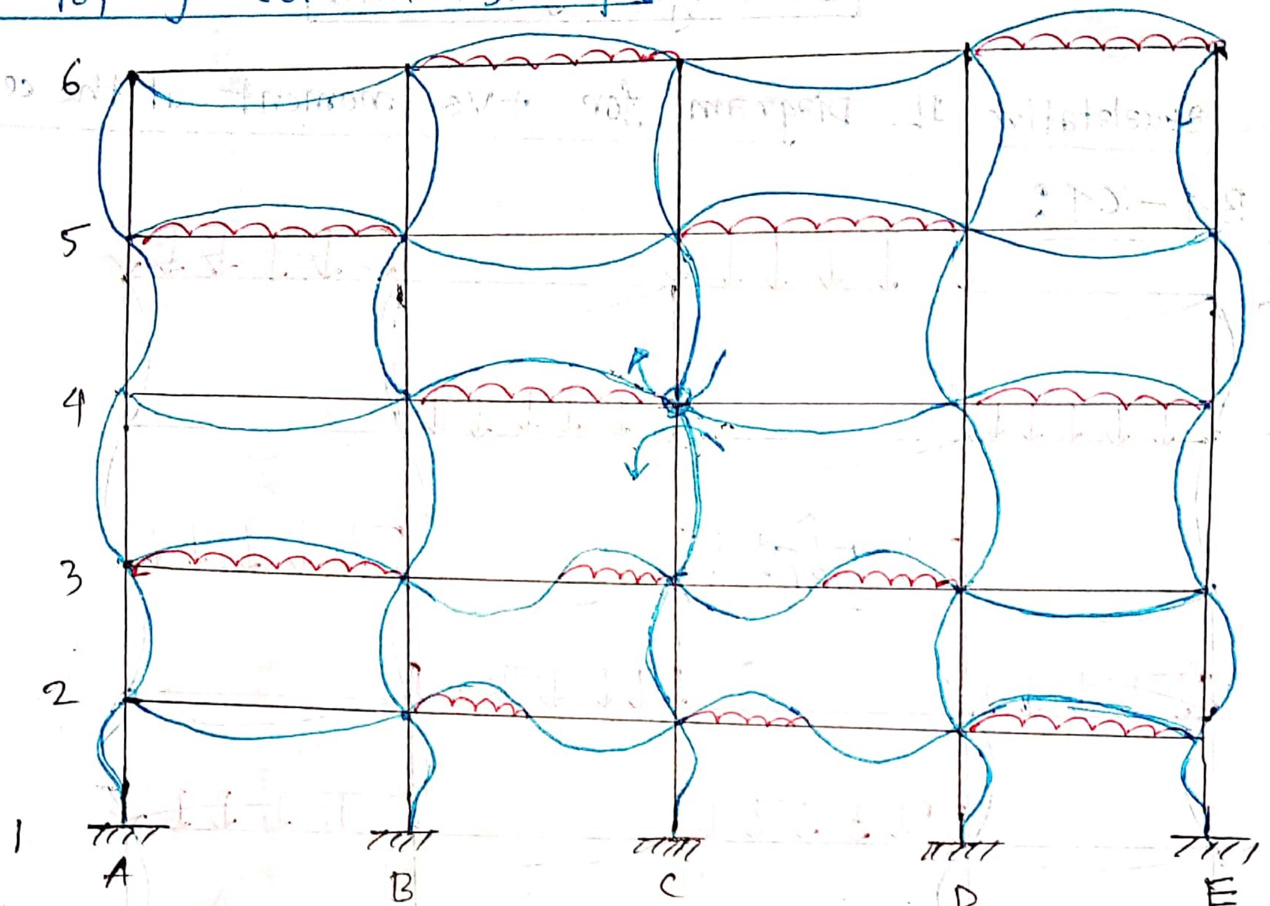
Draw Qualatative IL Diagram for +ve moment at the centre of B4 – C4:



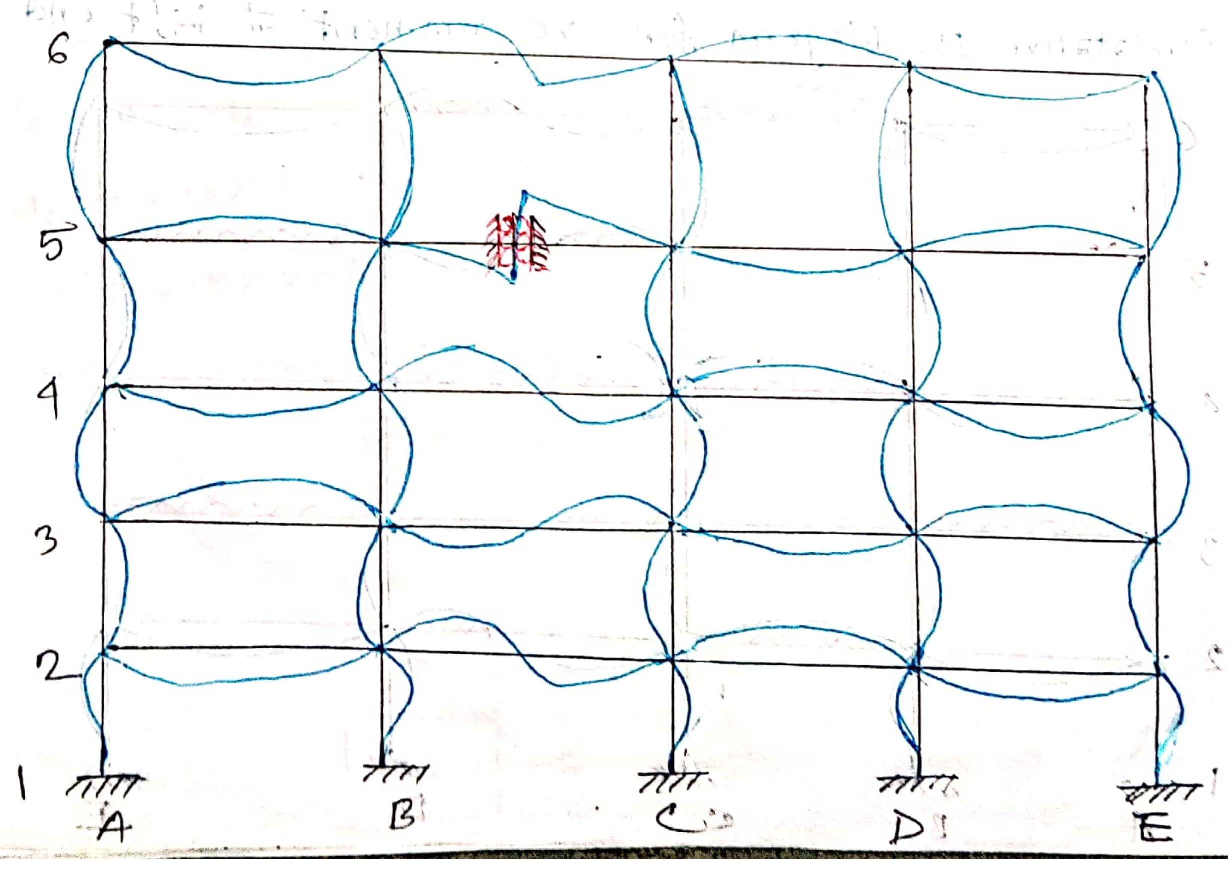
Draw Qualatative IL Diagram for –ve moment at left end of C^4D^4:



Draw Qualatative IL for causing tension on right side at top of column C3 – C4:

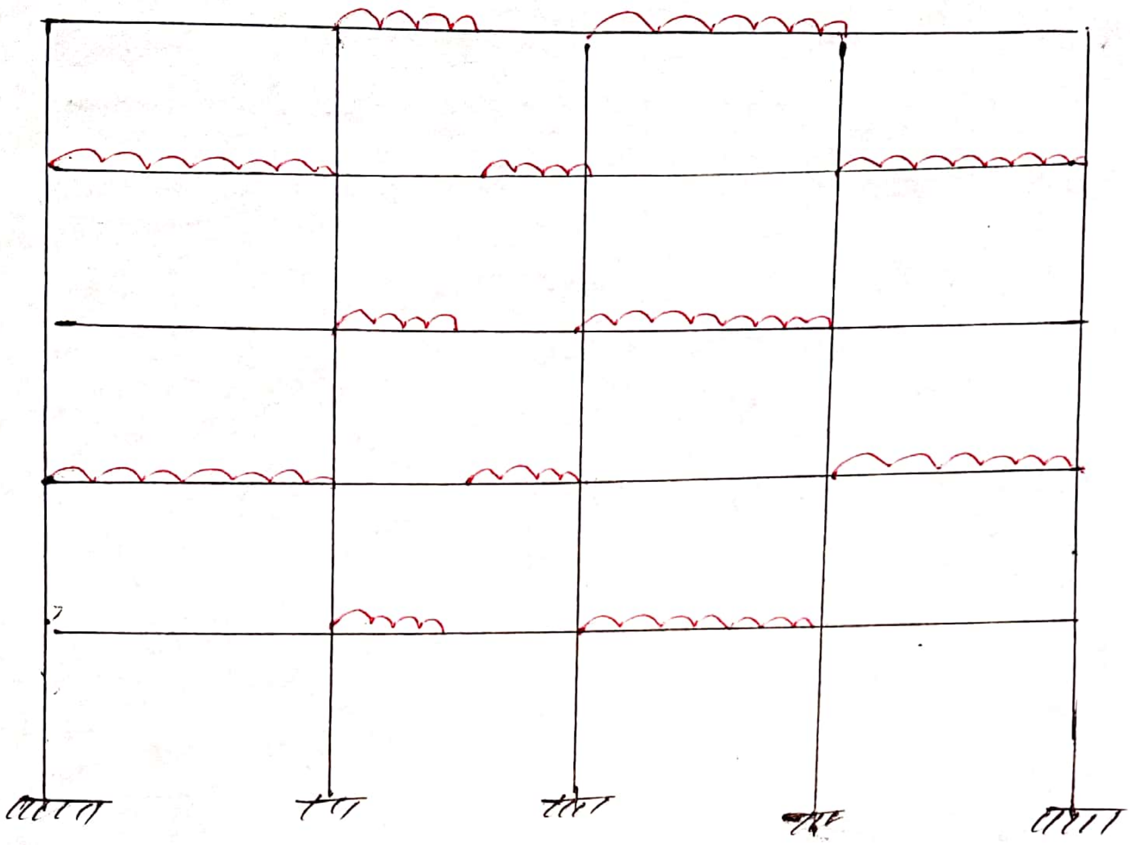


Draw Qualatative IL for shear at any section of span B5 – C5

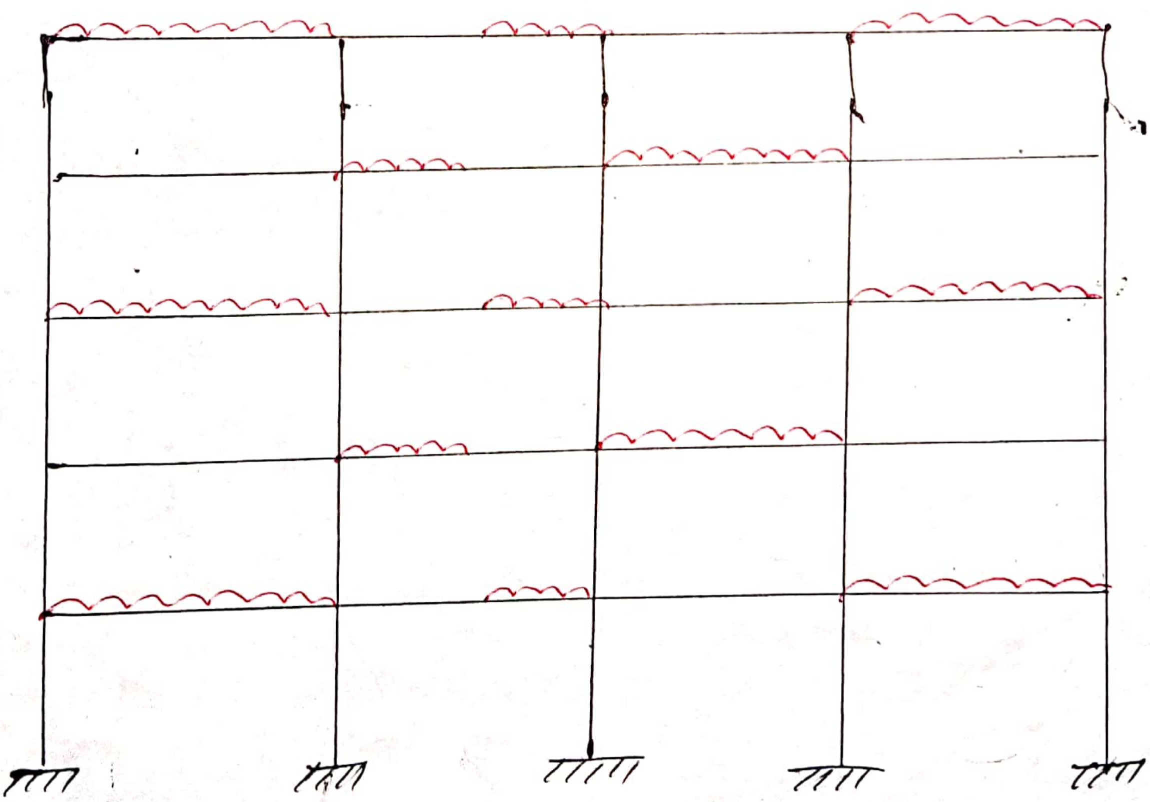


Scanned by CamScanner

for maximum positive shear:



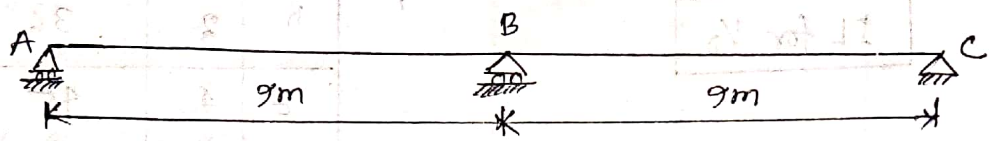
for Maximum negative shear:



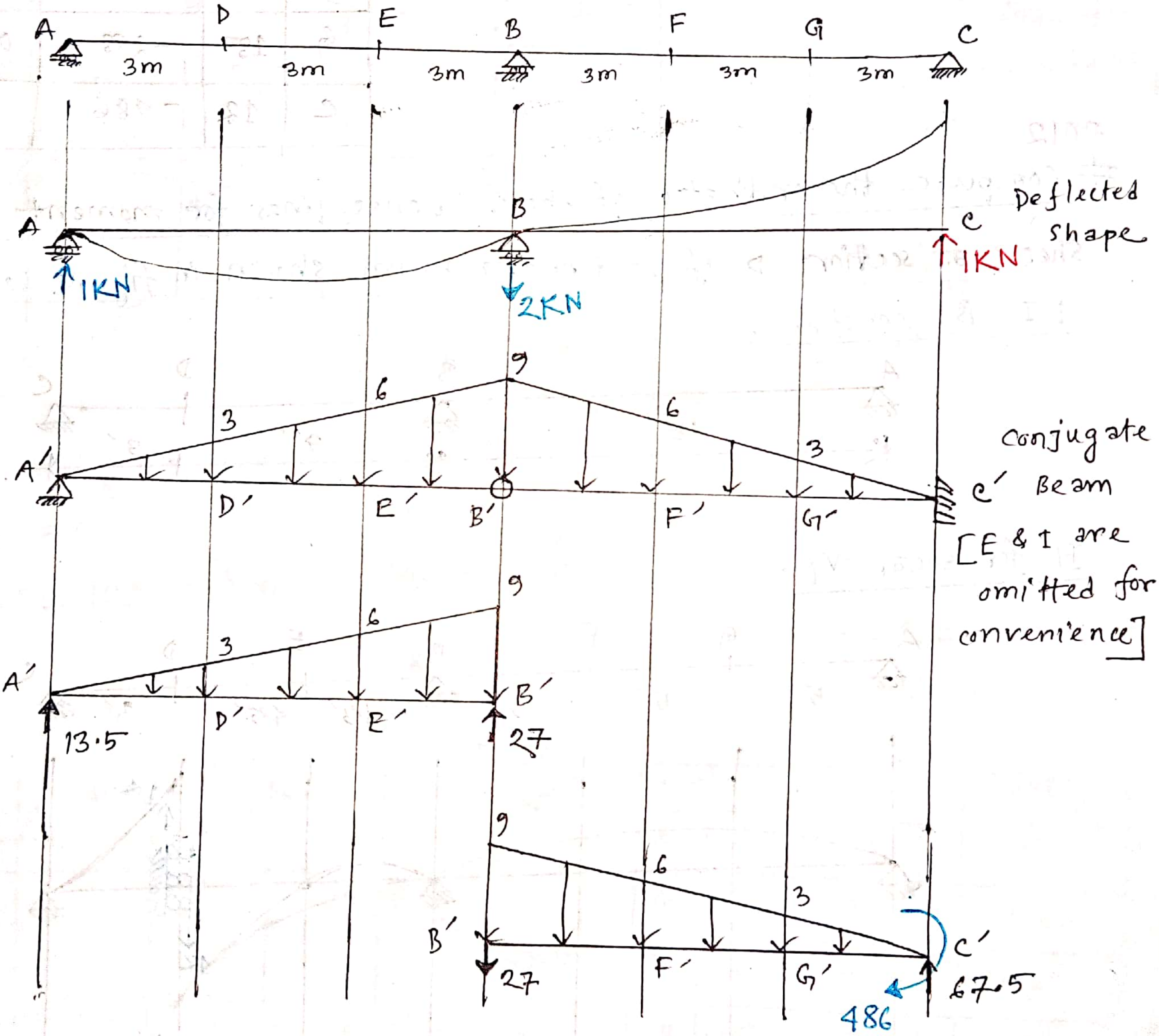
Influence Line.

2015

\# Compute the ordinates of the influence line for reaction at C of the continuous beam shown in fig. below. Use intervals of 3 m. EI is constant



IL for R_c:



$M_{A'} = 0$, $M_{D'} = 36$, $M_{E'} = 45$

$M_{B'} = 0$, $M_{F'} = -117$, $M_{G'} = -288$, $M_{C'} = -486 = \delta_{cc}$

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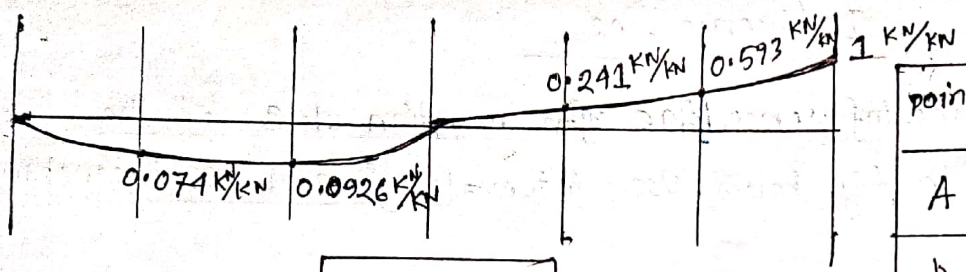
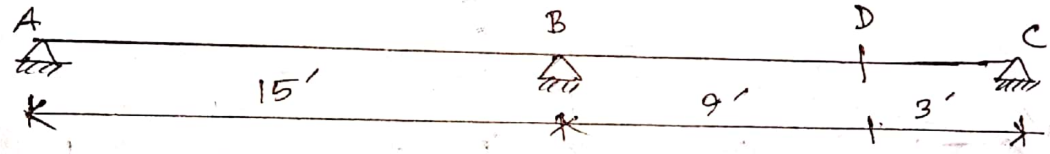


IL for V_D

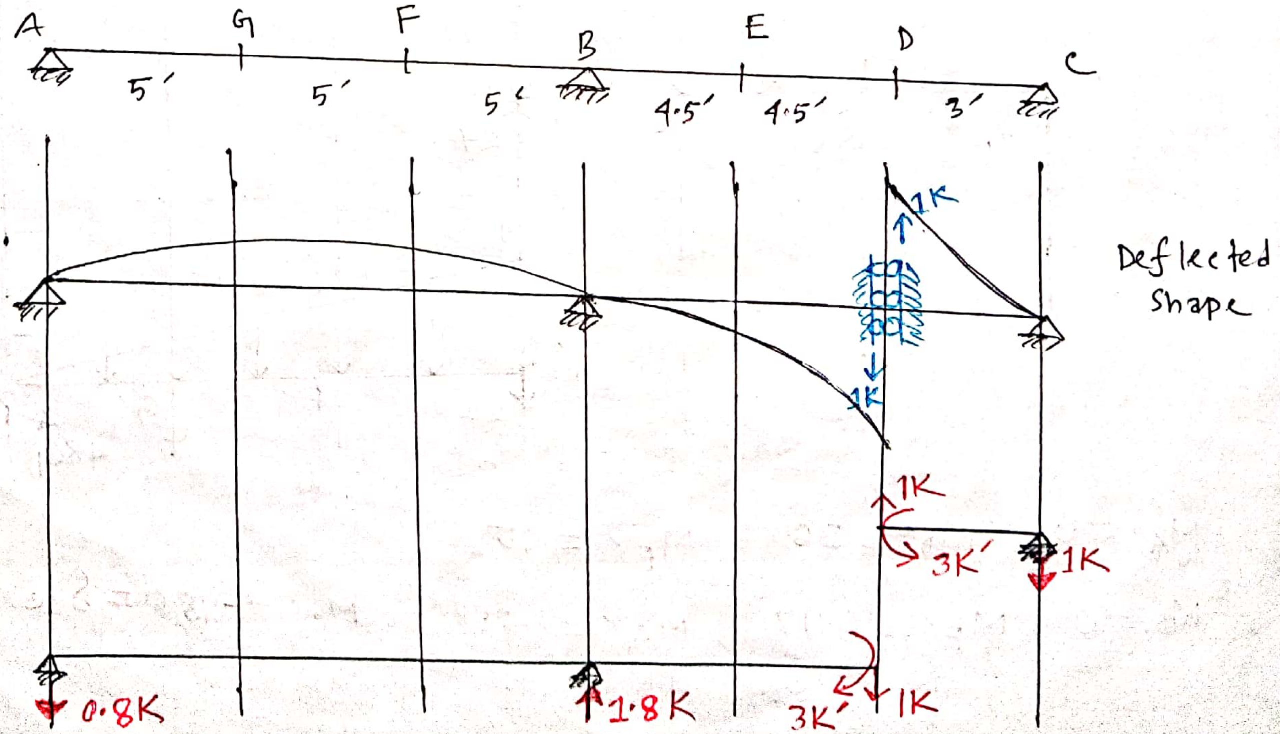
point	x (m)	δxc	$\delta xc/\delta cc$
A	0	0	0
D	3	36	-0.074
E	6	45	-0.0926
B	9	0	0
F	12	-117	0.241
G	15	-288	0.593
C	18	-486	1

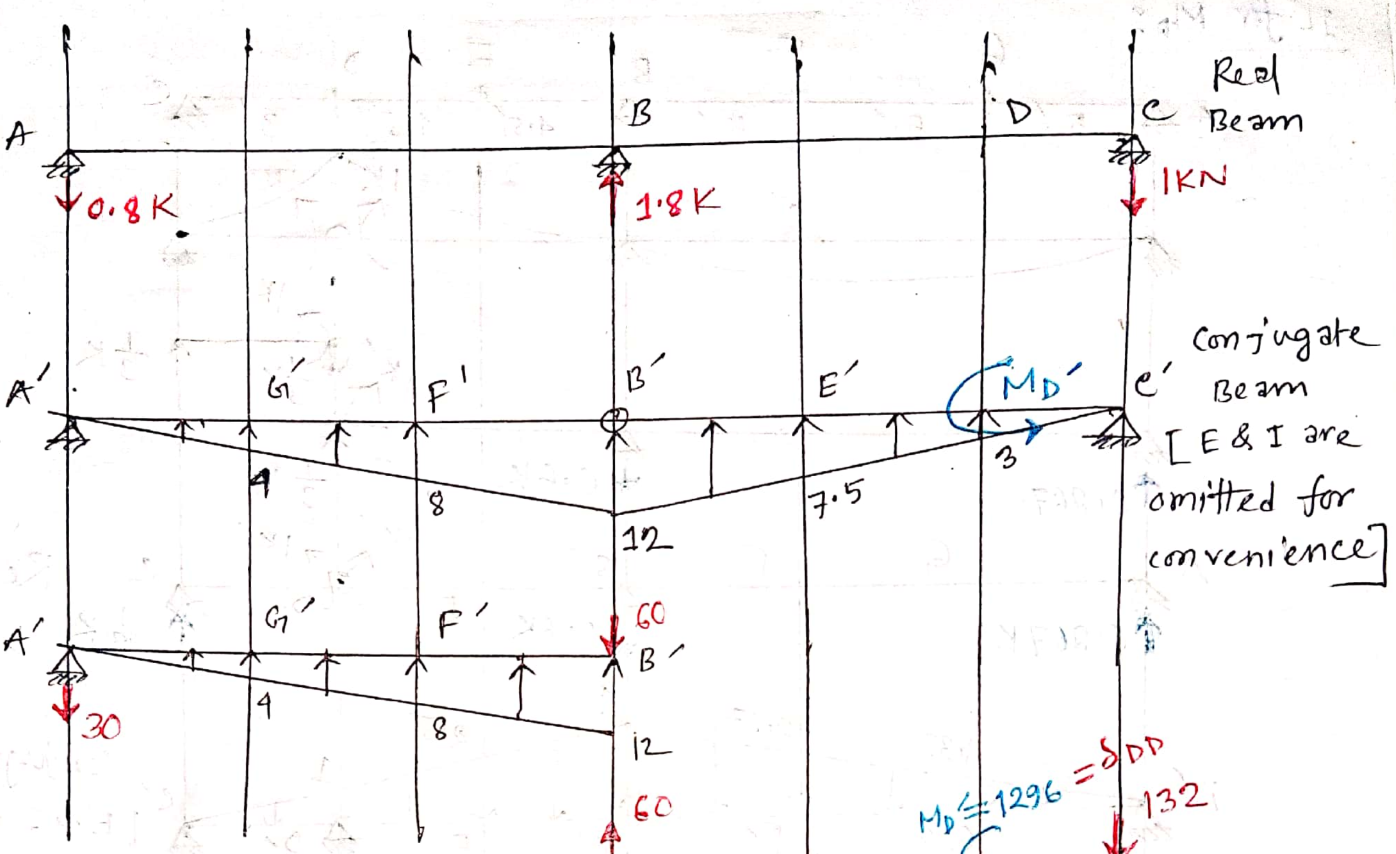
2012

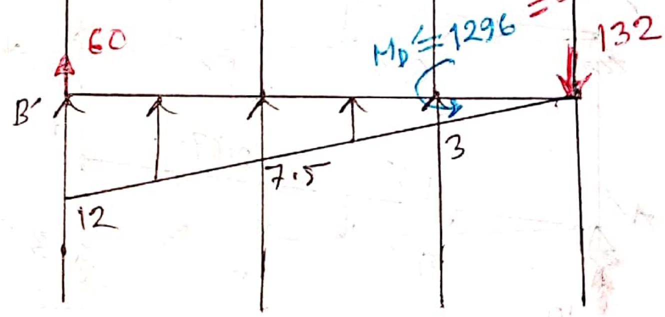
Compute the ordinates of the influence lines for moment and Shear at section D of continuous beam shown in figure below. EI is constant.



IL for shear V_D:







$\Sigma M_C' = 0$

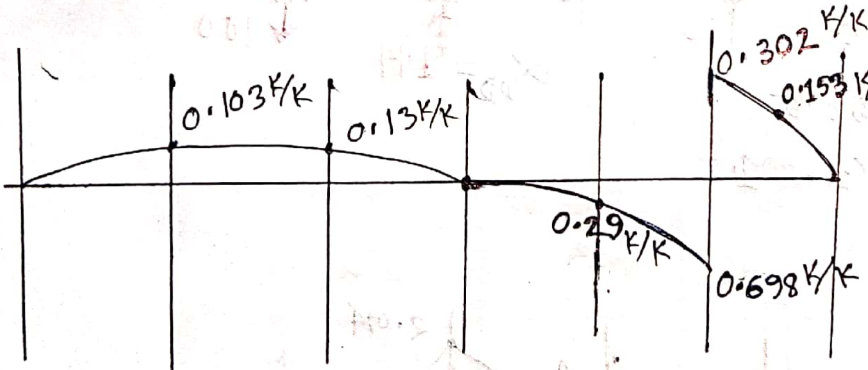
$60 \times 12 + .5 \times 12 \times 12 \times (\frac{2}{3} \times 12) - M_D' = 0$

$\therefore M_D' = 1296 = \delta_{DD}$

$M_A' = 0$, $M_G' = -133.33$, $M_F' = -166.67$

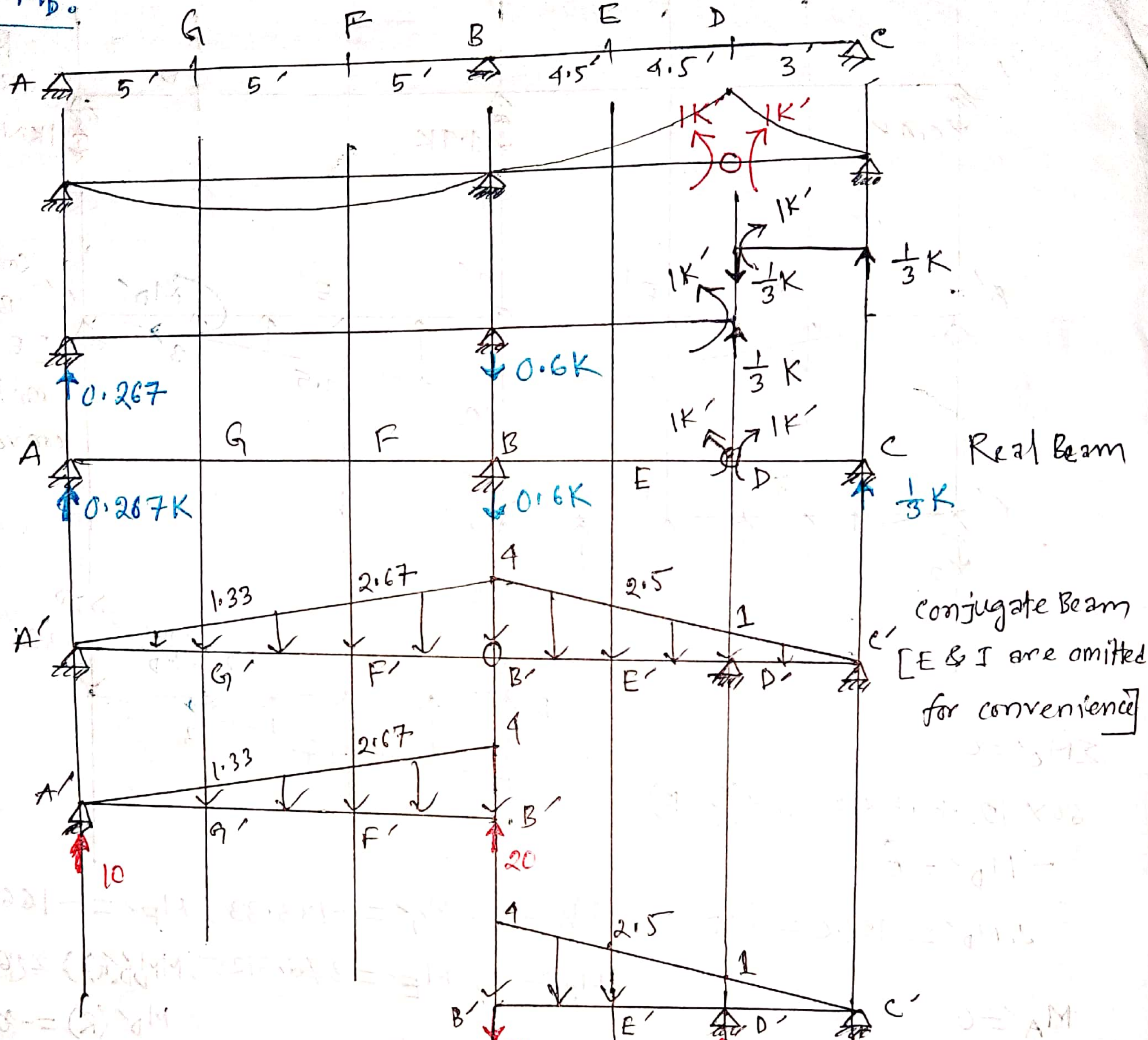
$M_B' = 0$, $M_E' = 376.3125$, $M_D'(L) = 904.5$, $M_C' = 0$

$M_D'(R) = -391.5$



point	X (ft)	δ_{XD}	δ_{XD}/δ
A	0	0	0
G	5	−133.33	−0.103
F	10	−166.67	−0.13
B	15	0	0
E	19.5	376.3125	0.29
D(L)	24	904.5	0.698
D(R)	24	−391.5	−0.302
C	27	0	0

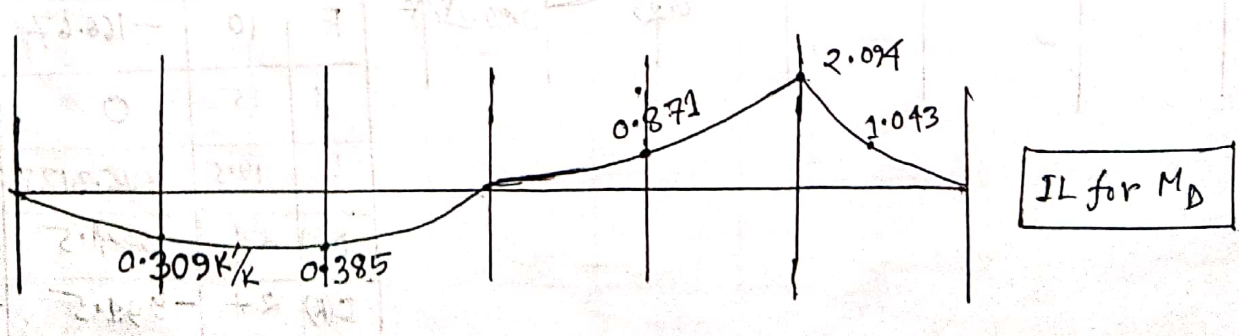
IL for M_D:



$M_A' = 0$, $M_B' = 0$, $M_C' = 0$

$M_G' = 44.46$, $M_F' = 55.5$

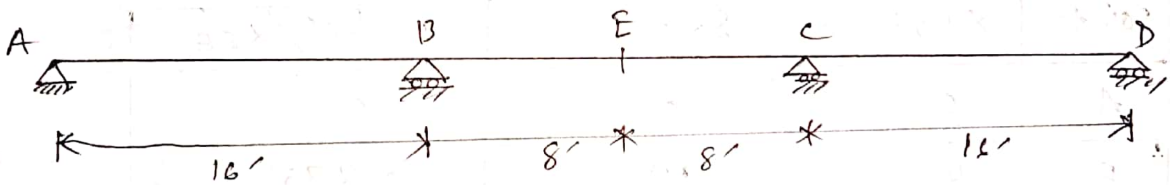
$M_E' = -125.4375$, $M_D' = -301.5$



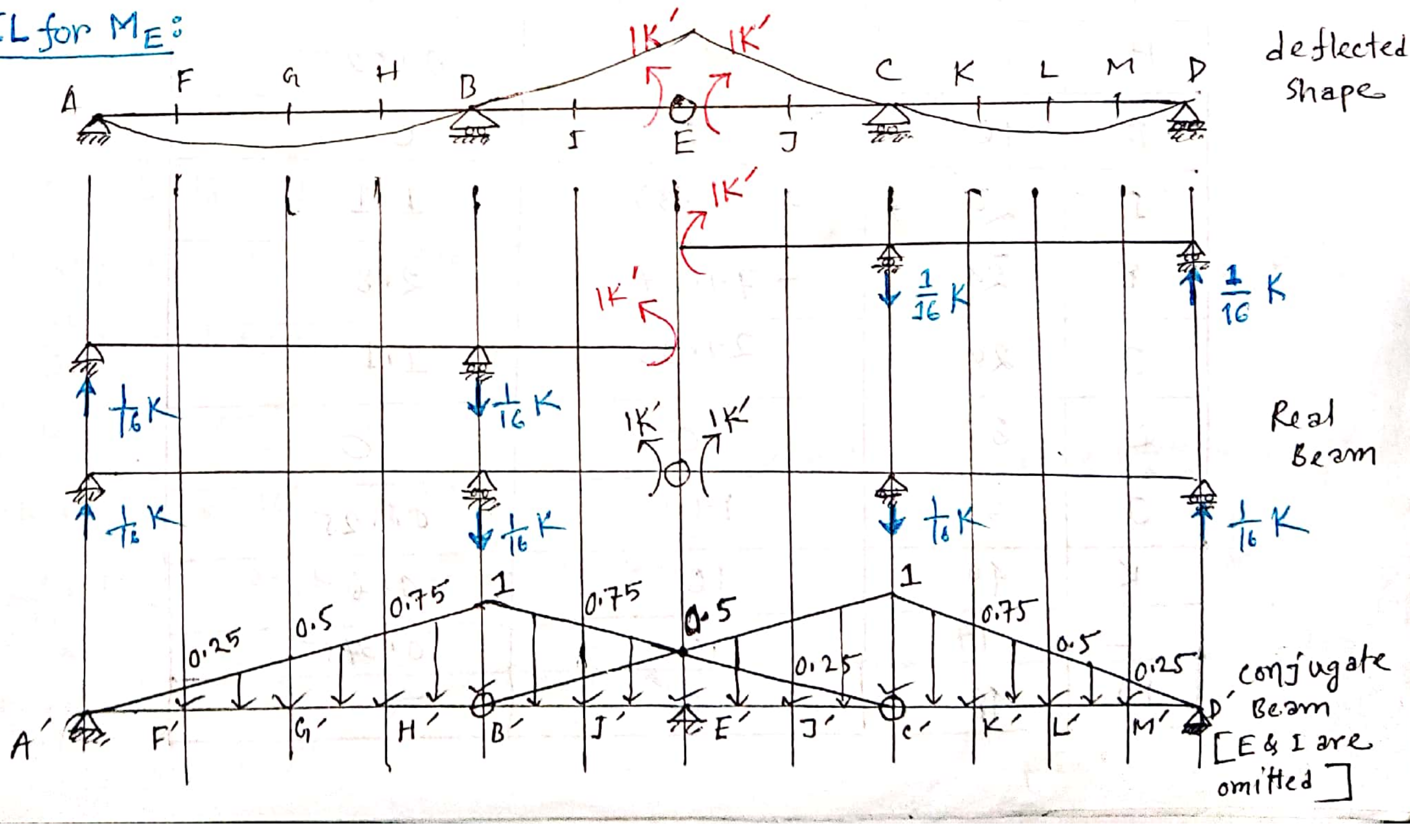
point	X (ft)	δXD	δXD/αDD
A	0	0	0
G	5	44.46	0.309
F	10	55.5	0.385
B	15	0	0
E	19.5	−125.4375	−0.871
D	24	−301.5	−2.094
C	27	0	0

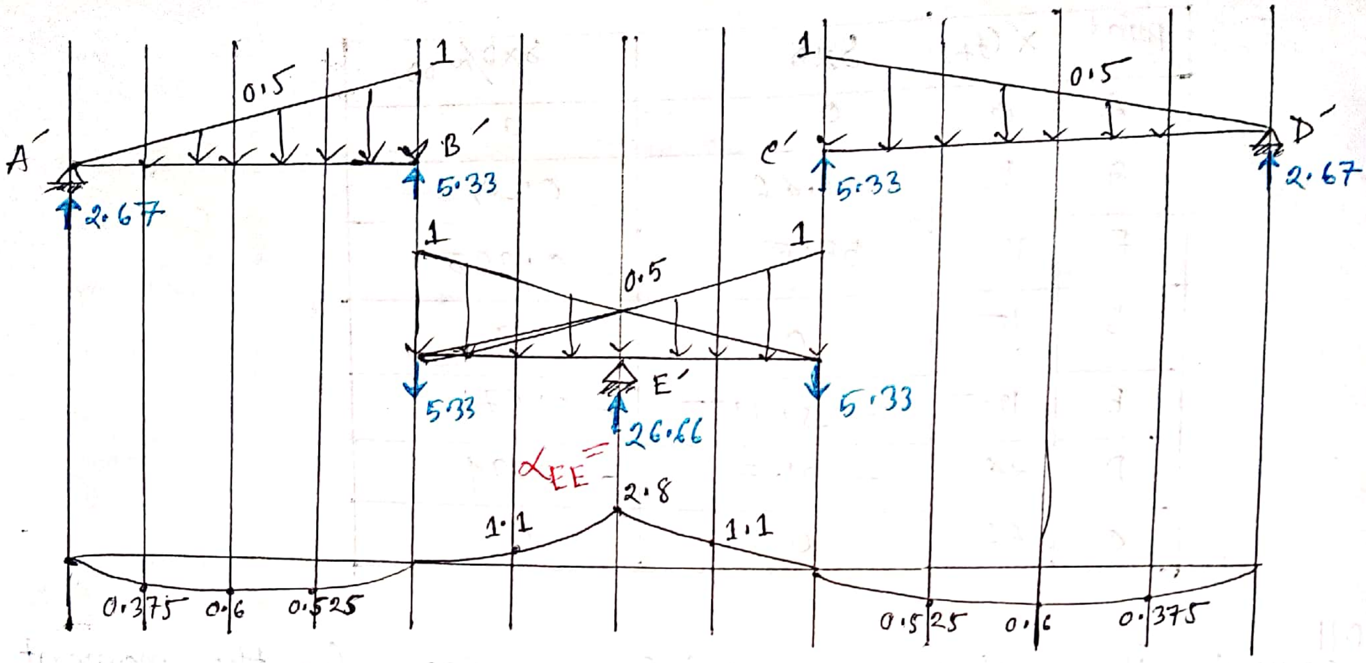
2011

Compute the ordinates of the influence line for the moment at E and the reaction at D of the continuous beam shown in figure below. Use intervals of 4 ft for moment and 8 ft for reaction. EI is constant.



IL for M_E:





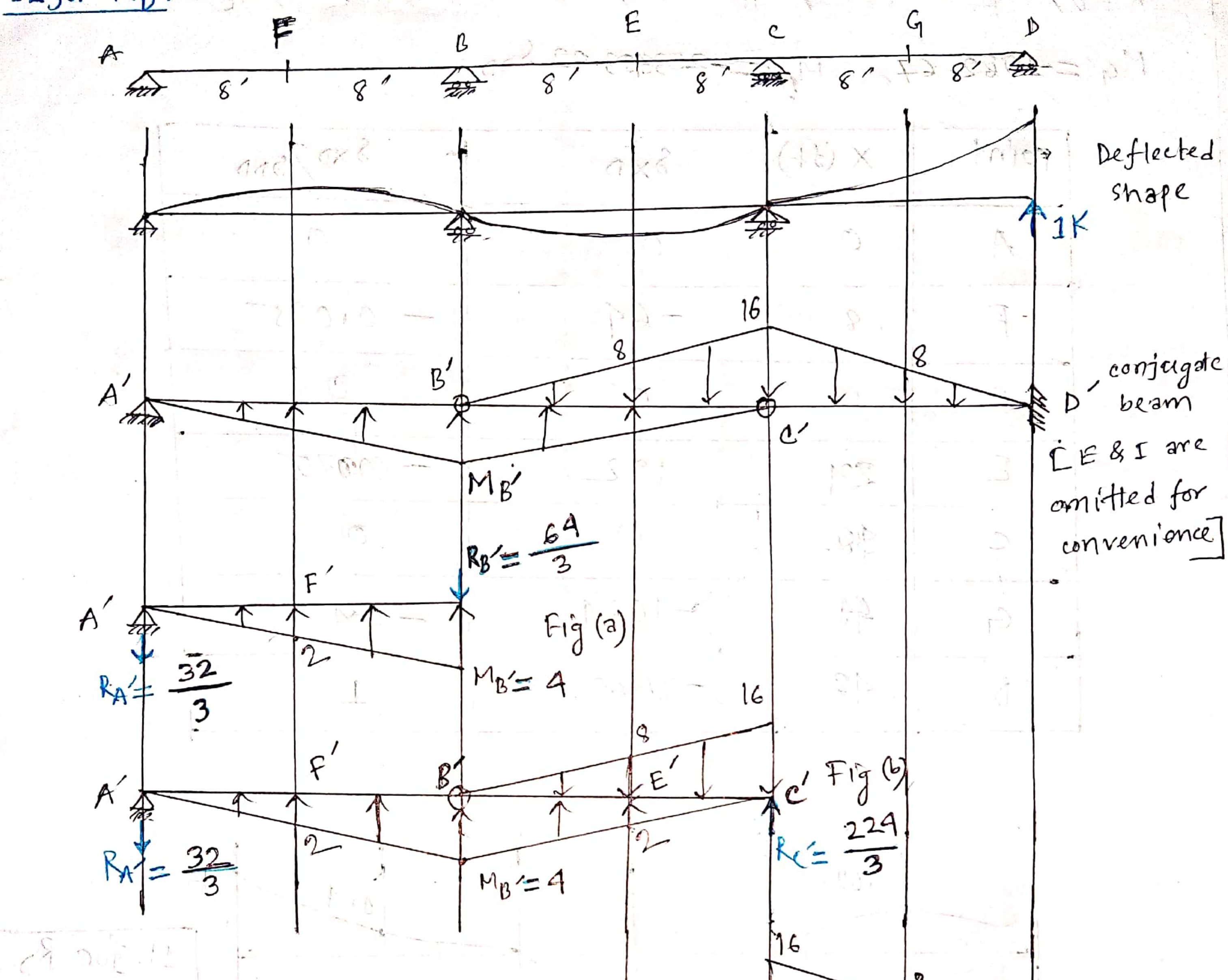
$M_{A'} = 0$, $M_{F'} = 10$, $M_{G'} = 16$, $M_{H'} = 14$, $M_{B'} = 0$, $M_{I'} = -29.33$

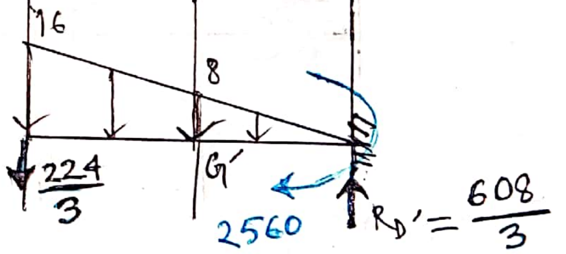
$M_{E'} = -74.67$, $M_{J'} = -29.33$, $M_{K'} = 14$, $M_{L'} = 16$, $M_{M'} = 10$, $M_{D'} = 0$

$M_{C'} = 0$

Point	x (ft)	δ_{xE}	δ_{xE}/α_{EE}
A	0	0	0
F	4	10	0.375
G	8	16	0.6
H	12	14	0.525
B	16	0	0
I	20	-29.33	1.1
E	24	-74.67	2.8
J	28	-29.33	1.1
C	32	0	0
J	36	14	0.525
K	40	16	0.6
L	44	10	0.375
M	48	0	0

IL for R_D:





$\Sigma M_{B'} = 0$ [Fig. (a)]

$\frac{1}{2} \times 16 \times M_B' \times (\frac{1}{3} \times 16) - R_A' \times 16 = 0$

$\Rightarrow \frac{128}{3} M_B' - 16 R_A' = 0$ ......(I)

$\Sigma M_{C'} = 0$ [Fig. (b)]

$\frac{1}{2} \times 16 \times M_B'(16 + \frac{1}{3} \times 16) - R_A' \times 32 + \frac{1}{2} \times 16 \times M_B' \times (\frac{2}{3} \times 16) - \frac{1}{2} \times 16 \times 16 \times (\frac{1}{3} \times 16) = 0$

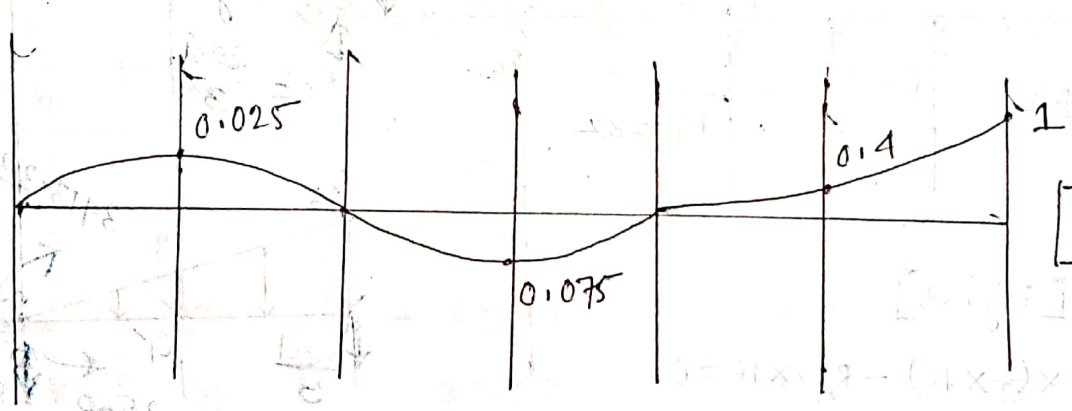
$\Rightarrow 256 M_B' - 32 R_A' = \frac{2048}{3}$ ......(II)

From eq^n (I) & (II) $\Rightarrow M_B' = 4$ and $R_A' = \frac{32}{3}$

$M_{A'} = 0$, $M_{F'} = -64$, $M_{B'} = 0$, $M_{E'} = 192$, $M_{C'} = 0$

$M_{G'} = -1024$, $M_{D'} = -2560 = \delta_{DD}$

Point	X (ft)	δ_{XD}	δ_{XD}/δ_{DD}
A	0	0	0
F	8	-64	0.025
B	16	0	0
E	24	192	-0.075
C	32	0	0
G	40	-1024	0.4
D	48	-2560	1



IL for R_D

2009

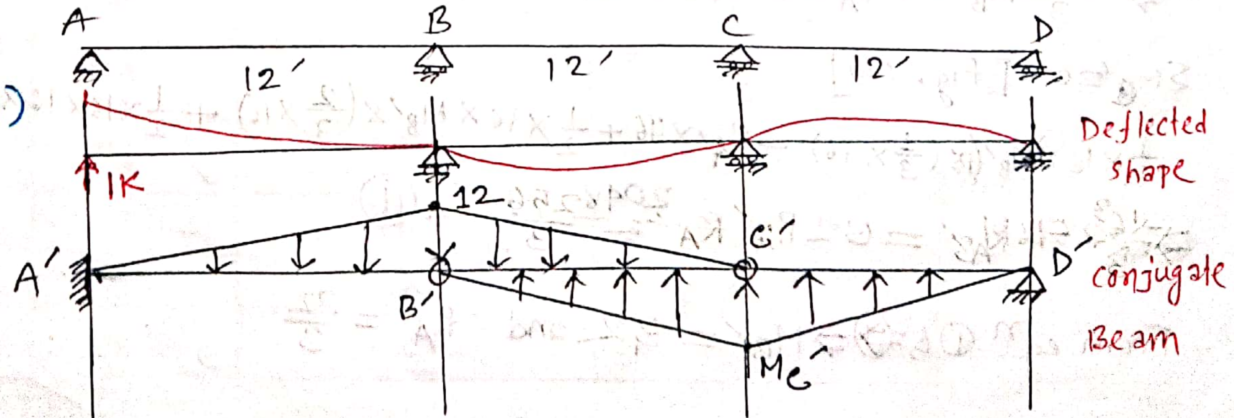
IL for R_A:

(4 ft interval)

Solve

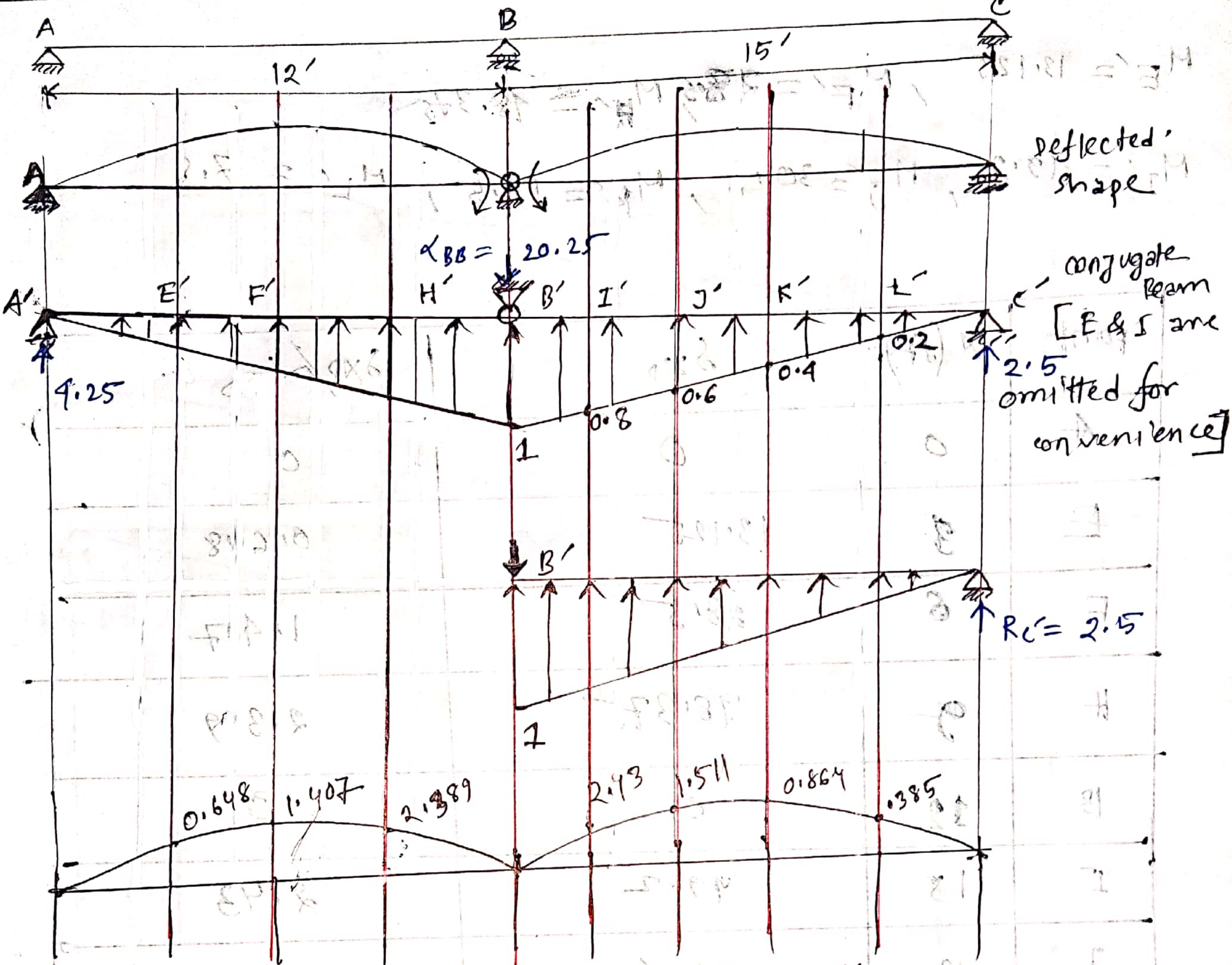
↓

same procedure



2010

compute the influence line ordinates for moment at B of the beam ABC shown in the figure at an interval of 3 ft.



$\Sigma M_{B'} = 0$

$\frac{1}{2} \times 15 \times 1 \times (\frac{1}{3} \times 15) = R_C' \times 15 \quad \therefore R_C' = 2.5$ K

$\Sigma M_{A'} = 0$

$\frac{1}{2} \times 12 \times 1 \times (\frac{2}{3} \times 12) + \frac{1}{2} \times 15 \times 1 \times (12 + \frac{1}{3} \times 15) + 2.5 \times 27 = R_B' \times 12$ (R_C')

$\therefore R_B' = 20.25$ K

$\Sigma F_y = 0$

$M_A' = 0$

$M_B' = 0$

$M_C' = 0$

$M_E' = 13.125$, $M_F' = 28.5$, $M_H' = 48.375$

$M_I' = 49.2$, $M_J' = 30.6$, $M_K' = 17.5$, $M_L' = 7.8$

point	x (ft)	δ_{XD}	δ_{XD}/α_{DD}
A	0	0	0
E	3	13.125	0.648
F	6	28.5	1.407
H	9	48.375	2.389
B	12	0	0
I	15	49.2	2.43
J	18	30.6	1.511
K	21	17.5	0.864
L	24	7.8	0.385
C	27	0	0

2009

compute the ordinates of the influence line for the moment at E of the continuous beam shown in figure below. Use intervals of 4 ft. EI is constant.

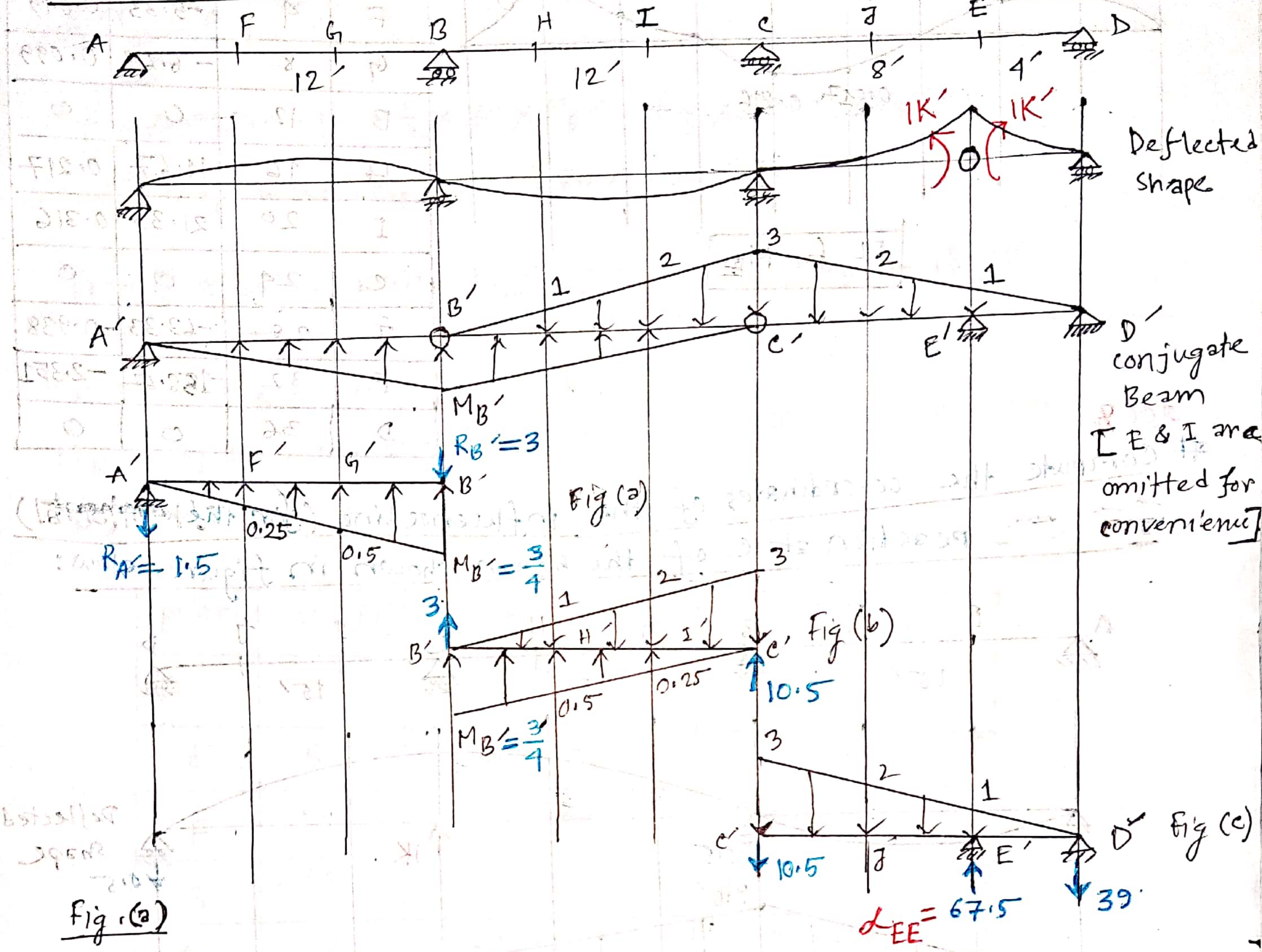


Fig.(a)

$\Sigma M_{A'} = 0$

$-\frac{1}{2} \times 12 \times M_{B'} \times \left(\frac{2}{3} \times 12\right) + 12 R_{B'} = 0 \Rightarrow 12R_{B'} - 48M_{B'} = 0$ ......(I)

$\Sigma M_{C'} = 0$ [Fig.(b)]

$\frac{1}{2} \times 12 \times M_{B'} \times \left(\frac{2}{3} \times 12\right) + 12R_{B'} - \frac{1}{2} \times 12 \times 3 \times \left(\frac{1}{3} \times 12\right) = 0 \Rightarrow 12R_{B'} + 48M_{B'} = 72$ ......(II)

from (I) & (II) we obtain, $R_{B'} = 3$ & $M_{B'} = \frac{3}{4}$

$M_{A'} = 0$, $M_{F'} = -5.33$, $M_{G'} = -6.67$, $M_{B'} = 0$, $M_{H'} = 14.67$,

$M_{I'} = 21.33$, $M_{C'} = 0$, $M_{J'} = -63.33$, $M_{E'} = -158.67$, $M_{D'} = 0$

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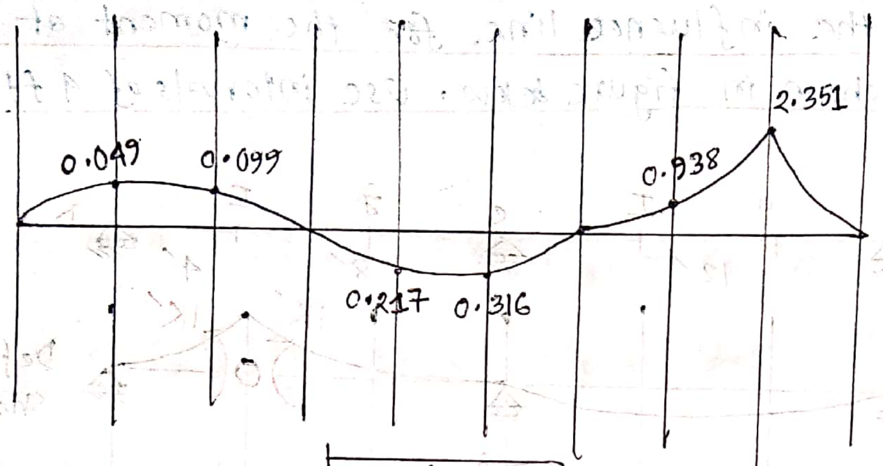
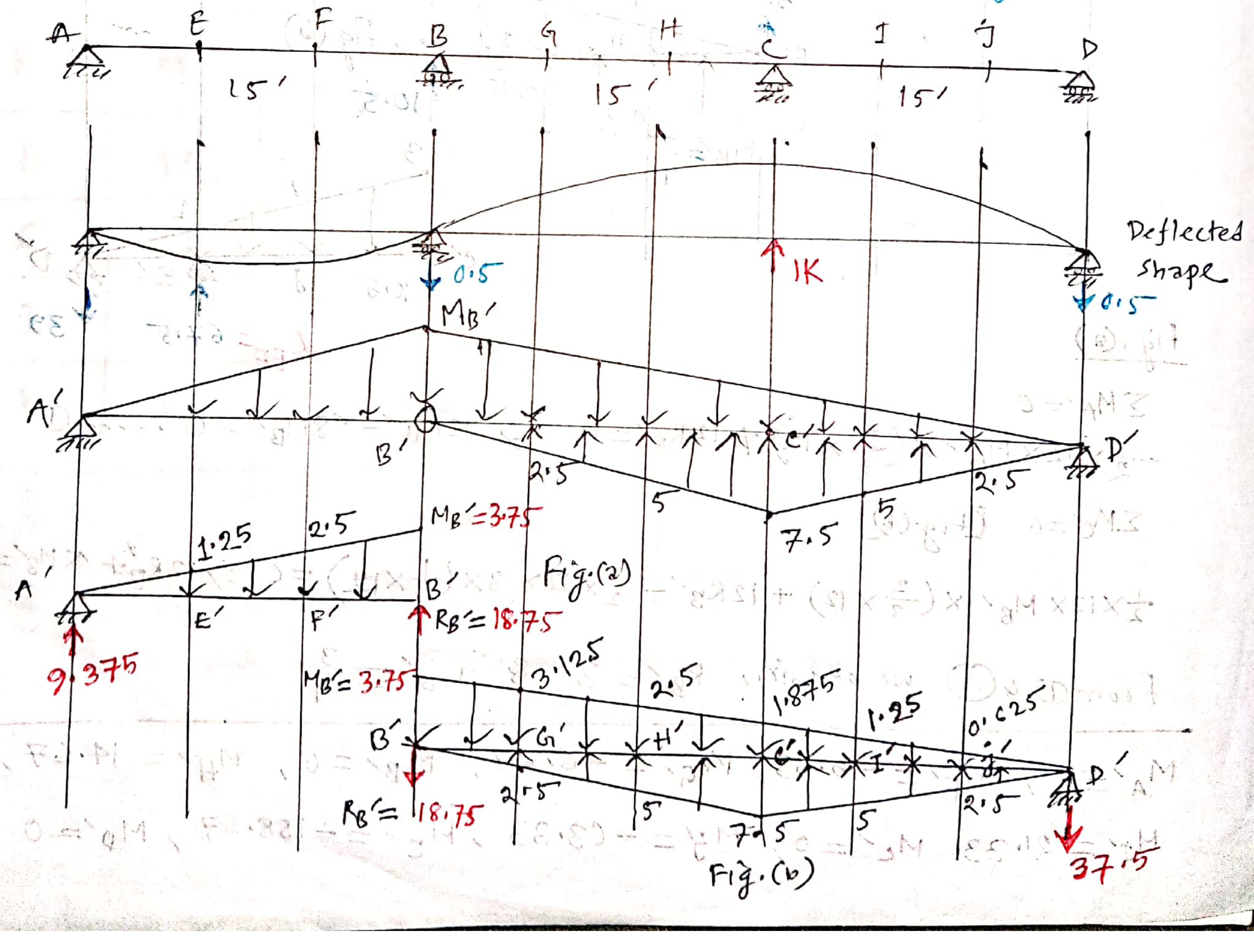


IL for M_E

point	X (ft)	δ_{XE}	δ_{XE}/α_{EE}
A	0	0	0
F	4	−5.33	0.049
G	8	− 6.67	0.099
B	12	0	0
H	16	14.67	0.217
I	20	21.33	0.316
C	24	0	0
J	28	−63.33	−0.938
E	32	−158.67	−2.351
D	36	0	0

2008

Compute the co-ordinates of the influence line (Use 5ft interval) for the reaction at C of the beam shown in figure below:



$\Sigma M_A' = 0$ [Fig. (a)]

$\frac{1}{2} \times 15 \times M_B' \times (\frac{2}{3} \times 15) - 15 R_B' = 0 \Rightarrow 75 M_B' - 15 R_B' = 0 \quad \ldots \text{(i)}$

$\Sigma M_D' = 0$ [Fig. (b)]

$\frac{1}{2} \times 7.5 \times 30 \times 15 - \frac{1}{2} \times 30 \times M_B' \times (\frac{2}{3} \times 30) - 30 R_B' = 0$

$\Rightarrow 30 R_B' + 300 M_B' = 1687.5 \quad \ldots \text{(ii)}$

From (i) & (ii) we obtain, $M_B' = 3.75$, $R_B' = 18.75$

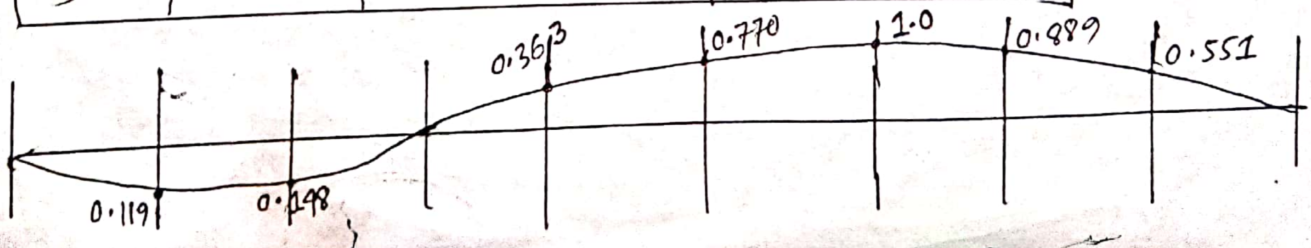
$M_A' = 0$, $M_B' = 0$, $M_D' = 0$

$M_E' = 41.67$, $M_F' = 52.083$, $M_G' = -127.604$, $M_H' = -270.83$

$M_C' = -351.5625$, $M_I' = -312.5$, $M_J' = -179.6875$

point	x (ft)	δ_{xc}	δ_{xc}/δ_{cc}
A	0	0	0
E	5	41.67	0.119
F	10	52.083	0.148
B	15	0	0
G	20	−127.604	0.363
H	25	−270.83	0.770
C	30	−351.5625	1
I	35	−312.5	0.889
J	40	−179.6875	0.511
D	45	0	0

IL for R_C



Lateral Load Analysis

BNBC (1993)

Live Load:

Live load is the load superimposed by the use of the buiding not including the environmental loads such as Wind load, Earthquake load.

Wind Load:

Wind velocity varies at various distances from the ground. Wind speed increases with structural height.

Wind Velocity is most unpredictable closer to the ground, because it is affected by interacting with things on the ground.

This unpredicability makes it difficult to make accurate wind calculations.

High winds can be very destructive. The speed of the wind or wind velocity acts as pressure when it meets with a structure. The intensity of that pressure is the wind load. Calculating wind load is necessary for the design and construction of safer, more wind resistant buildings. There are many factors that need to be considered when calculating wind load.

Wind Load Analysis:

1. Sustained wind pressure (q_z):

$$q_z = C_c C_I C_z V_b^2 \quad \ldots\ldots (1)$$

Where, q_z = sustained wind pressure at height z, (KN/m²)

C_I = Structural importance co-efficient as given in Table 6.2.9

C_c = velocity to pressure conversion co-efficient = 47.2×10^{-6}

C_z = combined height and exposure co-efficient as given in Table 6.2.10

V_b = Basic wind speed in km/h obtained from Table 6.2.8

- **Exposure category:**

(a) Exposure A: Urban and Sub-urban areas, industrial areas, wooded areas, hilly or other terrain covering at least 20 percent of the area with obstructions of 6 meters or more in height and extending from the site at least 500 meters or 10 times the height of the structure, which ever is greater.

(b) Exposure B: Open terrain with scattered obstructions having height generally less than 10 m extending 800 m or more from the site in any full quardant. This category includes air fields, open park lands, sparsely built up outskirts of towns, flat open country and glass lands.

(c) Exposure C: Flat and unobstructed open terrain, coastal areas and river-sides facing large bodies of water, over 1.5 Km or more in width. Exposure C extends island from the shoreline 400 m or 10 times the height of the structure, whichever is greater.

2. Design Wind Pressure, (P_z):

$$P_z = C_G C_p q_z$$

where, P_z = Design wind pressure at height z, KN/m^2

C_G = Gust co-efficient which shall be G_z, G_h or ... as set forth in section 2.4.6.6 (Table - 6.2.11)

C_p = pressure co-efficient for structures or components as set forth sec. 2.4.6.7 (Table - 6.2.15)

q_z = sunstained wind pressure obtained from equation - ①

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Table 6.2.8
Basic Wind Speeds for Selected Locations in Bangladesh

Location	Basic Wind Speed (km/h)	Location	Basic Wind Speed (km/h)
Angarpota	150	Lalmonirhat	204
Bagerhat	252	Madaripur	220
Bandarban	200	Magura	208
Barguna	260	Manikganj	185
Barisal	256	Meherpur	185
Bhola	225	Moheshkhali	260
Bogra	198	Moulvibazar	168
Brahmanbaria	180	Munshiganj	184
Chandpur	160	Mymensingh	217
Chapai Nawabganj	130	Naogaon	175
Chittagong	260	Narail	222
Chuadanga	198	Narayanganj	195
Comilla	196	Narsinghdi	190
Cox's Bazar	260	Natore	198
Dahagram	150	Netrokona	210
Dhaka	210	Nilphamari	140
Dinajpur	130	Noakhali	184
Faridpur	202	Pabna	202
Feni	205	Panchagarh	130
Gaibandha	210	Patuakhali	260
Gazipur	215	Pirojpur	260
Gopalganj	242	Rajbari	188
Habiganj	172	Rajshahi	155
Hatiya	260	Rangamati	180
Ishurdi	225	Rangpur	209
Joypurhat	180	Satkhira	183
Jamalpur	180	Shariatpur	198
Jessore	205	Sherpur	200
Jhalakati	260	Sirajganj	160
Jhenaidah	208	Srimangal	160
Khagrachhari	180	St. Martin's Island	260
Khulna	238	Sunamganj	195
Kutubdia	260	Sylhet	195
Kishoreganj	207	Sandwip	260
Kurigram	210	Tangail	160
Kushtia	215	Teknaf	260
Lakshmipur	162	Thakurgaon	130

d) configuration and dynamic response characteristics of the building or structure,

e) occupancy importance of the building,

f) magnification of the mean wind pressure due to the effect of the fluctuating component of wind speed, i.e. gusts, and

g) additional load amplification resulting from the dynamic wind-structure interaction effects due to gusts on slender buildings and structures.

2.4.6.2 Sustained Wind Pressure : The sustained wind pressure, q_z on a building surface at any height z above ground shall be calculated from the following relation :

$$q_z = C_c C_I C_z V_b^2 \qquad (2.4.1)$$

where, q_z = sustained wind pressure at height z, kN/m^2
C_I = structure importance coefficient as given in Table 6.2.9

C_c = velocity-to-pressure conversion coefficient = 47.2×10^{-6}
C_z = combined height and exposure coefficient as given in Table 6.2.10
V_b = basic wind speed in km/h obtained from Sec 2.4.5

If a structure is located within a local topographic zone, q_z shall be modified in accordance with Sec 2.4.6.8.

Table 6.2.9
Structure Importance Coefficients, C_I for Wind Loads

Structure Importance Category (see Table 6.1.1 for Occupancy)	Structure Importance Coefficient, C_I
I Essential facilities	1.25
II Hazardous facilities	1.25
III Special occupancy structures	1.00
IV Standard occupancy structures	1.00
V Low-risk structures	0.80

Table 6.2.10
Combined Height and Exposure Coefficient, C_z

Height above ground level, z (metres)	Coefficient, C_z (1)		
	Exposure A	Exposure B	Exposure C
0-4.5	0.368	0.801	1.196
6.0	0.415	0.866	1.263
9.0	0.497	0.972	1.370
12.0	0.565	1.055	1.451
15.0	0.624	1.125	1.517
18.0	0.677	1.185	1.573
21.0	0.725	1.238	1.623
24.0	0.769	1.286	1.667
27.0	0.810	1.330	1.706
30.0	0.849	1.371	1.743
35.0	0.909	1.433	1.797
40.0	0.965	1.488	1.846
45.0	1.017	1.539	1.890
50.0	1.065	1.586	1.930
60.0	1.155	1.671	2.002
70.0	1.237	1.746	2.065
80.0	1.313	1.814	2.120
90.0	1.383	1.876	2.171
100.0	1.450	1.934	2.217
110.0	1.513	1.987	2.260
120.0	1.572	2.037	2.299
130.0	1.629	2.084	2.337
140.0	1.684	2.129	2.371
150.0	1.736	2.171	2.404
160.0	1.787	2.212	2.436
170.0	1.835	2.250	2.465
180.0	1.883	2.287	2.494
190.0	1.928	2.323	2.521
200.0	1.973	2.357	2.547
220.0	2.058	2.422	2.596
240.0	2.139	2.483	2.641
260.0	2.217	2.541	2.684
280.0	2.910	2.595	2.724
300.0	2.362	2.647	2.762

Note : (1) Linear interpolation is acceptable for intermediate values of z.

Table 6.2.11
Gust Response Factors, G_h and G_z (1)

Height above ground level (metres)	G_h (2) and G_z		
	Exposure A	Exposure B	Exposure C
0-4.5	1.654	1.321	1.154
6.0	1.592	1.294	1.140
9.0	1.511	1.258	1.121
12.0	1.457	1.233	1.107
15.0	1.418	1.215	1.097
18.0	1.388	1.201	1.089
21.0	1.363	1.189	1.082
24.0	1.342	1.178	1.077
27.0	1.324	1.170	1.072
30.0	1.309	1.162	1.067
35.0	1.287	1.151	1.061
40.0	1.268	1.141	1.055
45.0	1.252	1.133	1.051
50.0	1.238	1.126	1.046
60.0	1.215	1.114	1.039
70.0	1.196	1.103	1.033
80.0	1.180	1.095	1.028
90.0	1.166	1.087	1.024
100.0	1.154	1.081	1.020
110.0	1.114	1.075	1.016
120.0	1.134	1.070	1.013
130.0	1.126	1.065	1.010
140.0	1.118	1.061	1.008
150.0	1.111	1.057	1.005
160.0	1.104	1.053	1.003
170.0	1.098	1.049	1.001
180.0	1.092	1.046	1.000
190.0	1.087	1.043	1.000
200.0	1.082	1.040	1.000
220.0	1.073	1.035	1.000
240.0	1.065	1.030	1.000
260.0	1.058	1.026	1.000
280.0	1.051	1.022	1.000
300.0	1.045	1.018	1.000

Note : (1) For main wind-force resisting systems, use building or structure height *h* for z.
(2) Linear interpolation is acceptable for intermediate values of z.

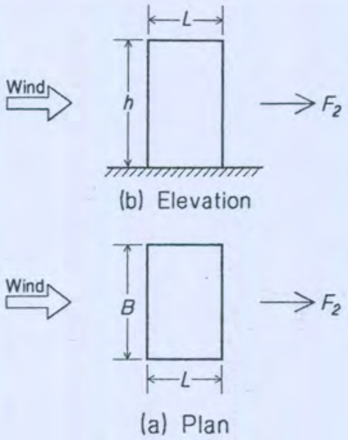
c) Gust Response Factor, $\overline{G}$ for Slender Buildings and Structures : Gust response factor, $\overline{G}$ for the primary framing systems of slender buildings and structures shall be calculated by a rational analysis incorporating the dynamic properties of the primary framing system as given by the following relations.

$$\overline{G} = 0.65+\sqrt{\left(\frac{P}{\beta}+\frac{11.0T_I^2S}{1+kc}\right)} \qquad (2.4.8)$$

where,

$$P = \overline{f}J\,Y \qquad (2.4.9)$$

$$\overline{f} = \frac{55.44fh}{sV_b} \qquad (2.4.10)$$



Rectangular Building

Table 6.2.15 (1)

Overall Pressure Coefficients, $\bar{C}_p^{(2)}$ for Rectangular Buildings with Flat Roofs

h/B	*L/B*					
	0.1	0.5	0.65	1.0	2.0	≥ 3.0
≤0.5	1.40	1.45	1.55	1.40	1.15	1.10
10.0	1.55	1.85	2.00	1.70	1.30	1.15
20.0	1.80	2.25	2.55	2.00	1.40	1.20
≥40.0	1.95	2.50	2.80	2.20	1.60	1.25

Note:(1) These coefficients are to be used with Method-2 given in Sec 2.4.6.6a(ii). Use $\bar{C}_p = \pm 0.7$ for roof in all cases.
(2) Linear interpolation may be made for intermediate values of *h/B and L/B.*

Table 6.2.16

Overall Pressure Coefficient, $\bar{C}_p$ for Buildings and Structures such as Chimneys, Tanks, etc.

Shape	Type of surface	$\bar{C}_p$ for h/D values of		
		1	7	25
Square (wind normal to a face)	All	1.3	1.4	2.0
Square (wind along diagonal)	All	1.0	1.1	1.5
Hexagonal or octagonal: $(D\sqrt{q_z} > 0.167)$	All	1.0	1.2	1.4
Round $(D\sqrt{q_z} > 0.167)$:	Moderately smooth	0.5	0.6	0.7
	Rough ($D'/D \approx 0.02$)	0.7	0.8	0.9
	Very rough($D'/D \approx 0.08$)	0.8	1.0	1.2
Round $(D\sqrt{q_z} \leq 0.167)$:	All	0.7	0.8	1.2

Notes: 1) The design wind force shall be calculated based on the area of the structure projected on a plane normal to the wind direction. The force shall be assumed to act parallel to the wind direction.
2) Linear interpolation may be used for *h/D* values other than those shown.
3) Notation :
D: diameter or least horizontal dimension, metres.
D': depth of protruding elements such as ribs and spoilers, metres.
h: height of structure, metres.

Table 6.2.17

Overall Pressure Coefficients $\bar{C}_p$ for Monoslope Roofs Over Unenclosed Buildings and Structures

θ (degrees)	*L/B*						
	5	3	2	1	1/2	1/3	1/5
10	0.2	0.25	0.3	0.45	0.55	0.7	0.75
15	0.35	0.45	0.5	0.7	0.85	0.9	0.85
20	0.5	0.6	0.75	0.9	1.0	0.95	0.9
25	0.7	0.8	0.95	1.15	1.1	1.05	0.95
30	0.9	1.0	1.2	1.3	1.2	1.1	1.0

	Location of centre of pressure, X/L, for L/B values of :		
	2 to 5	1	1/5 to 1/2
10 to 20	0.35	0.3	0.3
25	0.35	0.35	0.4
30	0.35	0.4	0.45

Note: 1) Wind forces act normal to the surface and shall be directed inward or outward.
2) Wind shall be assumed to deviate by ± 10 degrees from horizontal.
3) Notation :
B: dimension of roof measured normal to wind direction, metres
L: dimension of roof measured parallel to wind direction, metres
X: distance to centre of pressure from windward edge of roof, metres
Q: angle of plane of roof from horizontal, degrees.

2.5.6 Equivalent Static Force Method

This method may be used for calculation of seismic lateral forces for all structures specified in Sec 2.5.5.1(a)

2.5.6.1 Design Base Shear : The total design base shear in a given direction shall be determined from the following relation :

$$V = \frac{ZIC}{R} W \qquad (2.5.1)$$

where, Z = Seismic zone coefficient given in Table 6.2.22
I = Structure importance coefficient given in Table 6.2.23
R = Response modification coefficient for structural systems given in Table 6.2.24
W = The total seismic dead load defined in Sec 2.5.5.2
C = Numerical coefficient given by the relation :

$$C = \frac{1.25S}{T^{2/3}} \qquad (2.5.2)$$

S = Site coefficient for soil characteristics as provided in Table 6.2.25
T = Fundamental period of vibration in seconds, of the structure for the direction under consideration as determined by the provisions of Sec 2.5.6.2.

The value of C need not exceed 2.75 and this value may be used for any structure without regard to soil type or structure period. Except for those requirements where Code prescribed forces are scaled up by $0.375R$, the minimum value of the ratio C/R shall be 0.075.

Table 6.2.22
Seismic Zone Coefficients, Z

Seismic Zone (see Fig 6.2.10)	Zone Coefficient
1	0.075
2	0.15
3	0.25

Table 6.2.23
Structure Importance Coefficients *I*, *I'*

Structure Importance Category (see Table 6.1.1 for occupancy)	Structure Importance Coefficient	
	I	I'
I Essential facilities	1.25	1.50
II Hazardous facilities	1.25	1.50
III Special occupancy structures	1.00	1.00
IV Standard occupancy structures	1.00	1.00
V Low-risk Structures	1.00	1.00

2.5.6.2 Structure Period : The value of the fundamental period, T of the structure shall be determined from one of the following methods :

a) **Method A** : For all buildings the value of T may be approximated by the following formula :

$$T = C_t (h_n)^{3/4} \qquad (2.5.3)$$

where, C_t = 0.083 for steel moment resisting frames
= 0.073 for reinforced concrete moment resisting frames, and eccentric braced steel frames
= 0.049 for all other structural systems
h_n = Height in metres above the base to level n.

Alternatively, the value of C_t for buildings with concrete or masonry shear walls may be taken as $0.031/\sqrt{A_c}$. The value of A_c shall be obtained from the relation :

$$A_c = \sum A_e \left[0.2 + (D_e/h_n)^2\right] \qquad (2.5.4)$$

where, A_c = The combined effective area, in square metres, of the shear walls in the first storey of the structure.
A_e = The effective horizontal cross-sectional area, in square metres of a shear wall in the first storey of the structure.
D_e = The length, in metre of a shear wall element in the first storey in the direction parallel to the applied forces.

The value of D_e/h_n for use in Eq (2.5.4) shall not exceed 0.9.

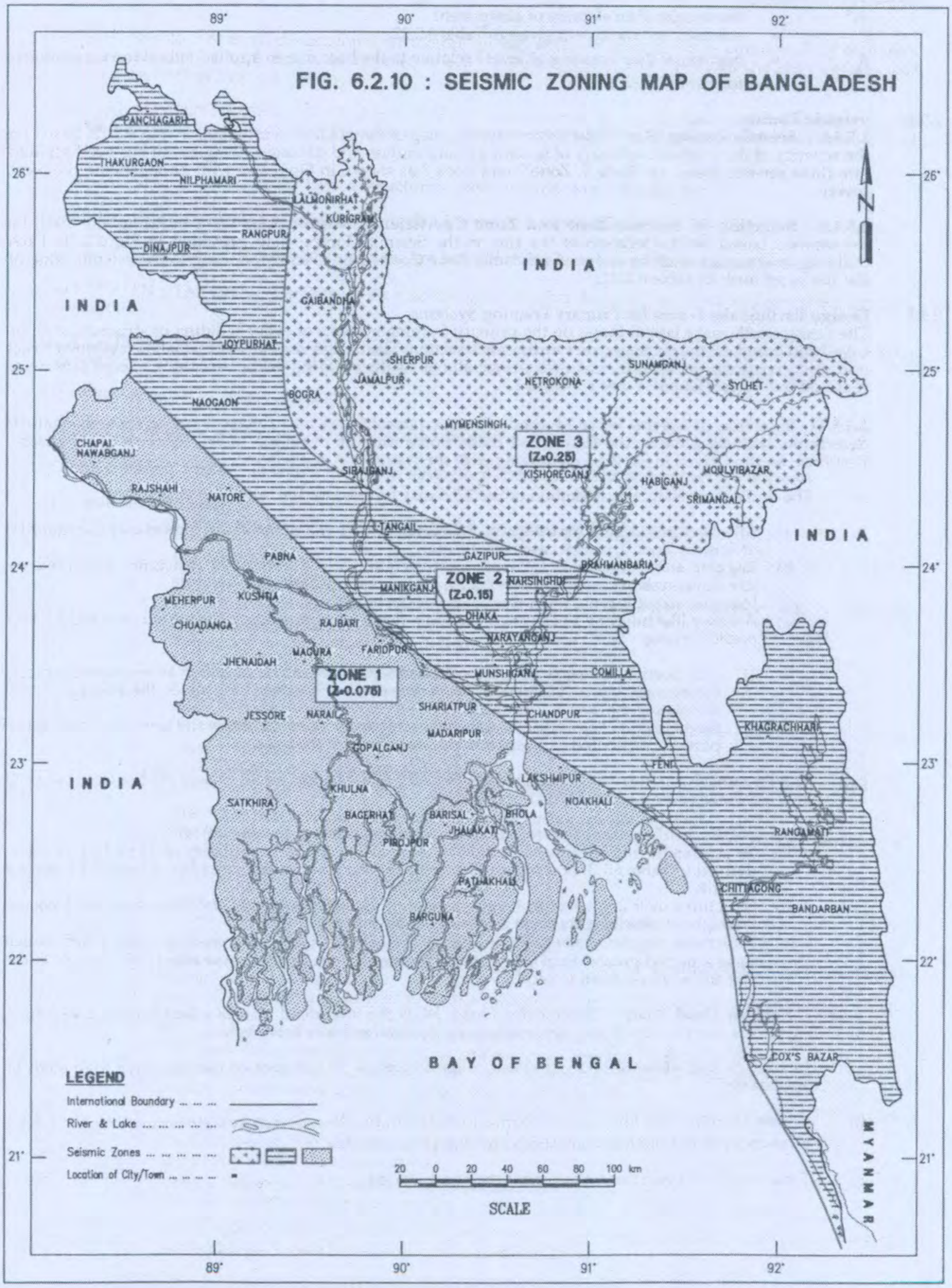


FIG. 6.2.10 : SEISMIC ZONING MAP OF BANGLADESH

Table 6.2.24
Response Modification Coefficient for Structural Systems, *R*

Basic Structural System[1]	Description of Lateral Force Resisting System	R [2]
a. Bearing Wall System	1. Light framed walls with shear panels	
	i) Plywood walls for structures, 3 storeys or less	8
	ii) All other light framed walls	6
	2. Shear walls	
	i) Concrete	6
	ii) Masonry	6
	3. Light steel framed bearing walls with tension only bracing	4
	4. Braced frames where bracing carries gravity loads	
	i) Steel	6
	ii) Concrete [3]	4
	iii) Heavy timber	4
b. Building Frame System	1. Steel eccentric braced frame (EBF)	10
	2. Light framed walls with shear panels	
	i) Plywood walls for structures 3-storeys or less	9
	ii) All other light framed walls	7
	3. Shear walls	
	i) Concrete	8
	ii) Masonry	8
	4. Concentric braced frames (CBF)	
	i) Steel	8
	ii) Concrete [3]	8
	iii) Heavy timber	8
c. Moment Resisting Frame System	1. Special moment resisting frames (SMRF)	
	i) Steel	12
	ii) Concrete	12
	2. Intermediate moment resisting frames (IMRF), concrete[4]	8
	3. Ordinary moment resisting frames (OMRF)	
	i) Steel	6
	ii) Concrete [5]	5
d. Dual System	1. Shear walls	
	i) Concrete with steel or concrete SMRF	12
	ii) Concrete with steel OMRF	6
	iii) Concrete with concrete IMRF [4]	9
	iv) Masonry with steel or concrete SMRF	8
	v) Masonry with steel OMRF	6
	vi) Masonry with concrete IMRF [3]	7
	2. Steel EBF	
	i) With steel SMRF	12
	ii) With steel OMRF	6
	3. Concentric braced frame (CBF)	
	i) Steel with steel SMRF	10
	ii) Steel with steel OMRF	6
	iii) Concrete with concrete SMRF [3]	9
	iv) Concrete with concrete IMRF [3]	6
e. Special Structural Systems	See Sec 1.3.2, 1.3.3, 1.3.5	

Notes : (1) Basic Structural Systems are defined in Sec 1.3.2, Chapter 1.
(2) See Sec 2.5.6.6 for combination of structural systems, and Sec 1.3.5 for system limitations.
(3) Prohibited in Seismic Zone 3.
(4) Prohibited in Seismic Zone 3 except as permitted in Sec 2.5.9.3.
(5) Prohibited in Seismic Zones 2 and 3. Sec 1.7.2.6.

Table 6.2.25
Site Coefficient, S for Seismic Lateral Forces (1)

Site Soil Characteristics		Coefficient, S
Type	**Description**	
S_1	A soil profile with either : a) A rock-like material characterized by a shear-wave velocity greater than 762 m/s or by other suitable means of classification, or b) Stiff or dense soil condition where the soil depth is less than 61 metres	1.0
S_2	A soil profile with dense or stiff soil conditions, where the soil depth exceeds 61 metres	1.2
S_3	A soil profile 21 metres or more in depth and containing more than 6 metres of soft to medium stiff clay but not more than 12 metres of soft clay	1.5
S_4	A soil profile containing more than 12 metres of soft clay characterized by a shear wave velocity less than 152 m/s	2.0

Note : (1) The site coefficient shall be established from properly substantiated geotechnical data. In locations where the soil properties are not known in sufficient detail to determine the soil profile type, soil profile S_3 shall be used. Soil profile S_4 need not be assumed unless the building official determines that soil profile S_4 may be present at the site, or in the event that soil profile S_4 is established by geotechnical data.

b) **Method B** : The fundamental period T may be calculated using the structural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. This requirement may be satisfied by using the following formula :

$$T = 2\pi \sqrt{\sum_{i=1}^{n} w_i \delta_i^2 \bigg/ g \sum_{i=1}^{n} f_i \delta_i} \qquad (2.5.5)$$

The values of f_i represent any lateral force distributed approximately in accordance with the principles of Eq (2.5.6), (2.5.7) and (2.5.8) or any other rational distribution. The elastic deflections, δ_i shall be calculated using the applied lateral forces, f_i. The value of T determined from Eq (2.5.5) shall not exceed that calculated using Eq (2.5.3) by more than 40%.

2.5.6.3 Vertical Distribution of Lateral Forces : In the absence of a more rigorous procedure, the total lateral force, which is the base shear V, shall be distributed along the height.of the structure in accordance with Eq (2.5.6), (2.5.7) and (2.5.8):

$$V = F_t + \sum_{i=1}^{n} F_i \qquad (2.5.6)$$

where, F_i = Lateral force applied at storey level -i and
F_t = Concentrated lateral force considered at the top of the building in addition to the force F_n.

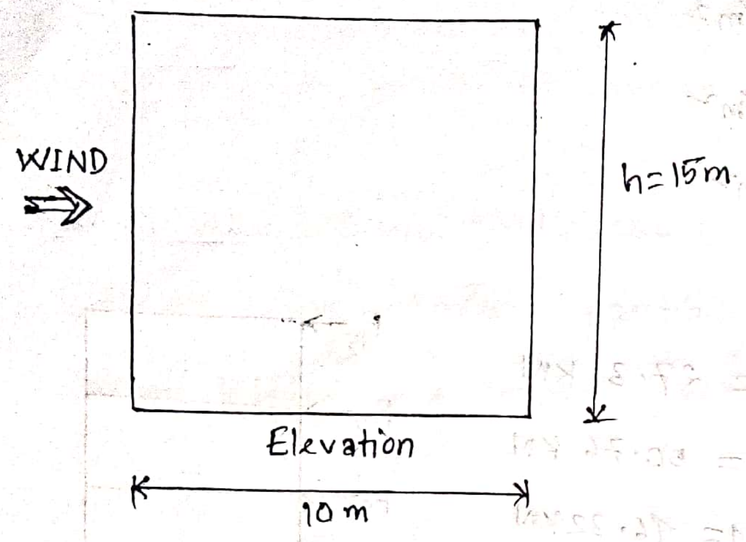
The concentrated force, F_t acting at the top of the building shall be determined as follows:

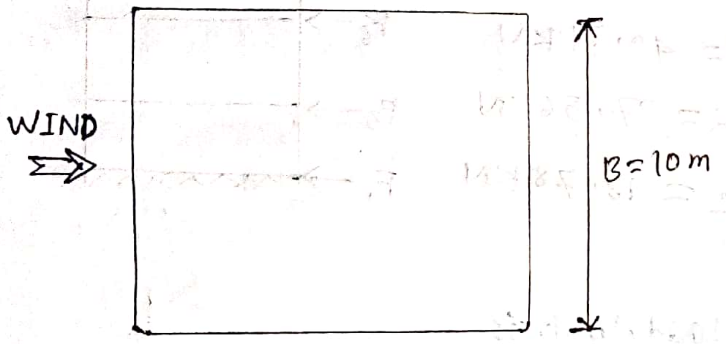
$$F_t = 0.07\,TV \leq 0.25\,V \qquad \text{when } T > 0.7 \text{ second} \qquad (2.5.7a)$$
$$F_t = 0.0 \qquad \text{when } T \leq 0.7 \text{ second} \qquad (2.5.7b)$$

The remaining portion of the base shear $(V\text{-}F_t)$, shall be distributed over the height of the building, including level-n, according to the relation :

$$F_x = \frac{(V - F_t) w_x h_x}{\sum_{i=1}^{n} w_i h_i} \qquad (2.5.8)$$

Problem: 01





solution:

(1) sustained wind pressure:

$q_z = C_c C_I C_z V_b^2$

Let, A hospital building will to be built in Rajshahi.

Hence, $V_b = 155$ Km/h

$C_I = 1.25$

$C_c = 47.2 \times 10^{-6}$

$\therefore q_z = 47.2 \times 10^{-6} \times 1.25 \times C_z \times 155^2$

$\Rightarrow q_z = 1.4175 \times C_z$

Now, From table - 6.2.10

$q_0 = 1.4175 \times 0.368 = 0.522$ KN/m²

$q_3 = 1.4175 \times 0.368 = 0.522$ KN/m²

$q_6 = 1.4175 \times 0.415 = 0.588$ KN/m²

$q_9 = 1.4175 \times 0.497 = 0.7045$ KN/m²

$q_{12} = 1.4175 \times 0.565 = 0.801$ KN/m²

$q_{15} = 1.4175 \times 0.624 = 0.885$ KN/m²

(2) Design wind pressure:

$P_z = C_G C_p q_z$

Here, h=15, B=10, L=10

$h/B = 1.5$ & $L/B = 1$

Hence, From Table - 6.2.15

$C_p = 1.45$ (Let)

Now, From Table - 6.2.11

$P_0 = 1.654 \times 1.45 \times 0.522 = 1.252$ KN/m²

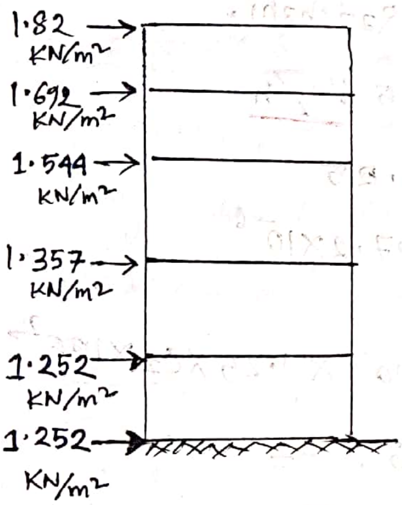
$P_3 = 1.654 \times 1.45 \times 0.522 = 1.252$ KN/m²

$P_6 = 1.592 \times 1.45 \times 0.588 = 1.357\ KN/m^2$

$P_9 = 1.511 \times 1.45 \times 0.7045 = 1.544\ KN/m^2$

$P_{12} = 1.457 \times 1.45 \times 0.801 = 1.692\ KN/m^2$

$P_{15} = 1.418 \times 1.45 \times 0.885 = 1.82\ KN/m^2$



Now,

$F_{15} = \left(\frac{3\times10}{2}\right) \times 1.82 = 27.3\ KN$

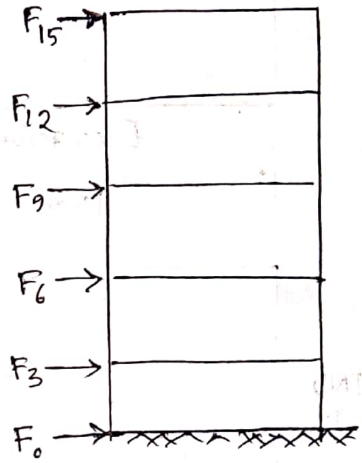
$F_{12} = (3\times10) \times 1.692 = 50.76\ KN$

$F_9 = (3\times10) \times 1.544 = 46.32\ KN$

$F_6 = (3\times10) \times 1.357 = 40.71\ KN$

$F_3 = (3\times10) \times 1.252 = 37.56\ KN$

$F_0 = \left(\frac{3\times10}{2}\right) \times 1.252 = 18.78\ KN$



Earth Quake Load Analysis

Seismic loading: Seismic loading is one of the basic concepts of earthquake engineering which means applications of an earthquake generated agitation to a structure.

It happens at contact surface of a structure either with the ground or with adjacent structures, or with gravity waves from tsunami.

Seismic loading depends, primarily on:

* Anticipated earthquake's parameter at the site - Known as seismic hazard.

* Geotechnical parameters of the site.

* Structure's parameter
* characterestics of the anticipated gravity waves from tsunami (if applicable)

Sometimes, seismic load exceeds ability of a structure to resist it without being broken, partially or completely. Due to their interaction seismic loading and seismic performance of a structure are intimately related.

▶ Earthquake Load Analysis: (Equivalent Static Force Method)

1. Design Base shear:

$$V = \frac{ZIC}{R} W$$

Where,

Z = Seismic zone co-efficient given in Table 6.2.22

I = Structure importance co-efficient given in Table 6.2.23

R = Response modification co-efficient for structural systems given in Table 6.2.24

W = The total seismic dead load defined in Section 2.5.5.2

C = Numerical co-efficient given by the relation:

$$C = \frac{1.25S}{T^{2/3}}$$

Here, S = site co-efficient for soil characterestics as provided in Table 6.2.25

T = Fundamental period of vibration in seconds, of the structure for the direction under consideration as determined by the following:

For all buildings,

$$T = C_t (h_H)^{3/4}$$

where, $C_t = 0.083$ for steel moment resisting frame.

$= 0.073$ for reinforced concrete moment resisting frames and Eccentric braced steel frame

$= 0.049$ for all other structural systems.

h_H = Height in meter above the base to level n

* Note: The value of C need not exceed 2.75 and this value may be used for any structure with regard to soil type or structure period.

2. <u>Vertical Load distribution</u>:

$$V = F_t + \sum_{i=1}^{n} F_i$$

where, F_i = Lateral force applied at storey level-i and

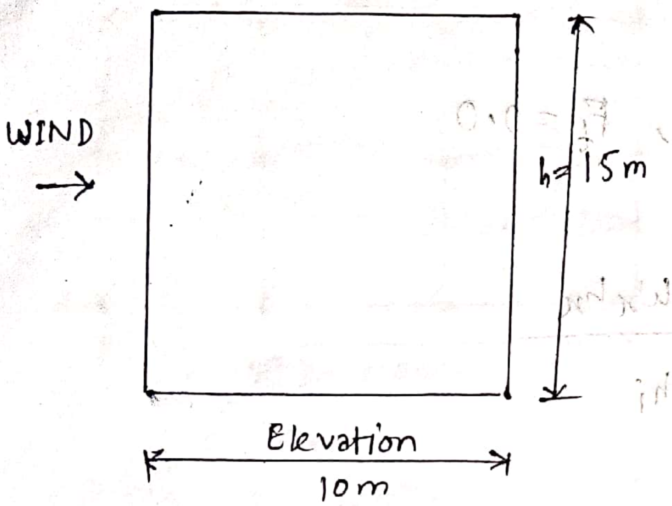
F_t = Concentrated lateral force considered at the top of the building in addition to the force F_n.

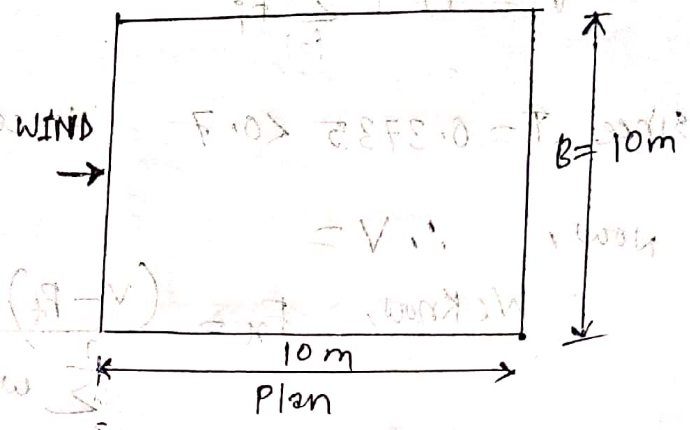
$F_t = 0.07\,TV \le 0.25V$ when $T > 0.7$ second.

$F_t = 0.0$ when $T \le 0.7$ second.

Then, $$F_x = \frac{(V - F_t)\, w_x h_x}{\sum_{i=1}^{n} w_i h_i}$$

Problem: 02 A hospital building in Rajshahi.





Solution:

We know,

Design base shear,

$$V = \frac{ZIC}{R} W$$

Here,

$Z = 0.075$ (Rajshahi - Zone 1)

$I = 1.25$

$R = 8$

Assume, siesmic dead load,

, $W = 3000$ KN/ floor

Now, $C = \dfrac{1.25 \times S}{T^{\frac{2}{3}}} = \dfrac{1.25 \times 2}{T^{\frac{2}{3}}} = \dfrac{2.5}{T^{\frac{2}{3}}}$

Again,

$$T = C_t (h_n)^{3/4}$$

$= 0.049 \times (15)^{\frac{3}{4}}$

$= 0.3735$

$\therefore C = \dfrac{2.5}{(.3735)^{\frac{2}{3}}} = 4.82 > 2.75$

Hence, $C = 2.75$

Now, $V = \dfrac{ZICxW}{R} = \dfrac{0.075 \times 1.25 \times 2.75}{8} \times (3000 \times 6)$ (5 stored + Ground floor)

$\therefore V = 580$ KN

Vertical distribution of Lateral force:

We know,

$$V = F_t + \sum_{i=1}^{n} F_i$$

since $T = 0.3735 < 0.7$ Hence, $F_t = 0.0$

Now,

We Know, $$F_x = \frac{(V - F_t)\, w_x h_x}{\sum_{i=1}^{n} w_i h_i}$$

$$\therefore F_0 = \frac{(580-0) \times 3000 \times 0}{3000 \times (0+3+6+9+12+15)} = 0$$

(45)

$$F_3 = \frac{580 \times 3000 \times 3}{3000 \times 45} = 38.67 \text{ KN}$$

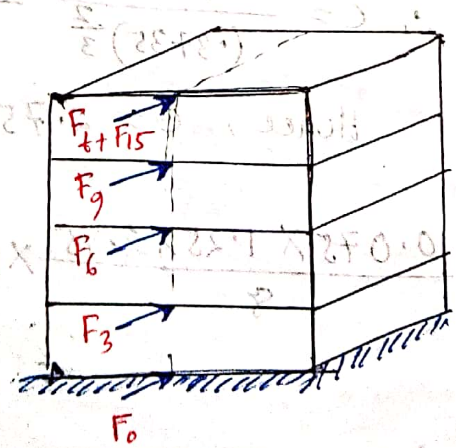
$$F_6 = \frac{580 \times 3000 \times 6}{3000 \times 45} = 77.33 \text{ KN}$$

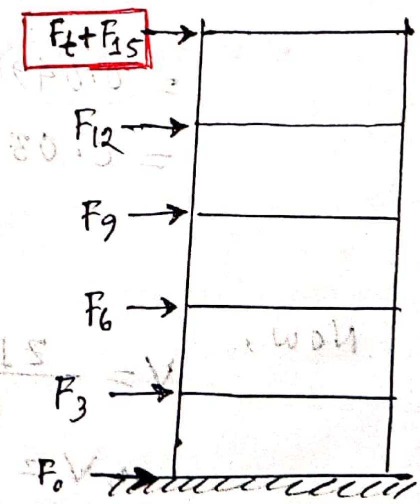
$$F_9 = \frac{580 \times 3000 \times 9}{3000 \times 45} = 116 \text{ KN}$$

$$F_{12} = \frac{580 \times 3000 \times 12}{3000 \times 45} = 154.66 \text{ KN}$$

$$F_{15} = \frac{580 \times 3000 \times 15}{3000 \times 45} = 193.33 \text{ KN}$$

এই Fig.
[Exam-এ দেওয়ার প্রয়োজন নেই]





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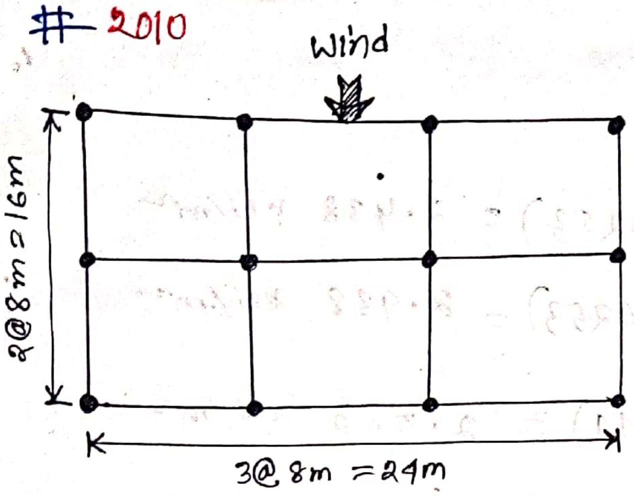

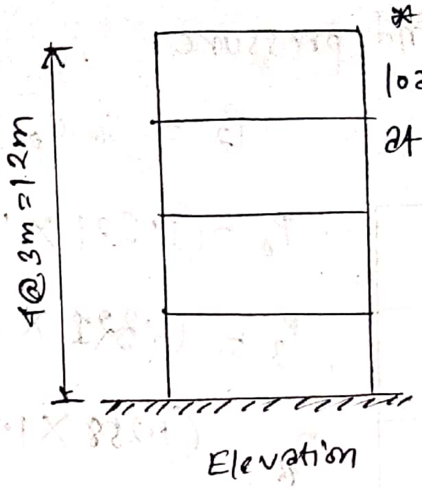

* Calculate earthquake load and wind force at each floor level.

Given, $Z = 0.075$ $W = 1500$ KN/floor

$I = C_I = 1.00$ $C_c = 47 \times 10^{-6}$

$C_t = 0.049$ $V_b = 165$ Km/h

$R = 8$, $S = 1.50$ $C_p = 1.80$

Z(m)	C_z	C_G
0–4.5	0.801	1.321
6.0	0.866	1.294
9.0	0.972	1.258
12.0	1.055	1.233

Solution:

Wind Load Analysis

(I) sustained Wind pressure, $q_z = C_c C_I C_z V_b^2$

$= 47 \times 10^{-6} \times 1 \times C_z \times (165)^2$

$= 1.28\, C_z$

$\therefore q_0 = (1.28 \times 0.801) = 1.0253$ KN/m²

$q_3 = (1.28 \times 0.801) = 1.0253$ KN/m²

$q_6 = (1.28 \times 0.866) = 1.11$ KN/m²

$q_9 = (1.28 \times 0.972) = 1.244$ KN/m²

$q_{12} = (1.28 \times 1.055) = 1.35$ KN/m²

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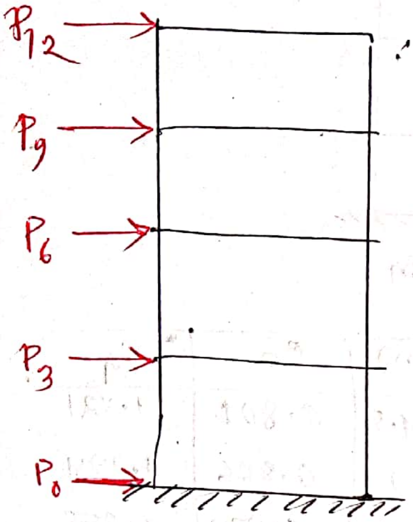
(ii) design wind pressure,

$$P_z = G_G\, C_p\, q_z$$



$\therefore P_0 = (1.321 \times 1.8 \times 1.0253) = 2.438$ KN/m²

$P_3 = (1.321 \times 1.8 \times 1.0253) = 2.438$ KN/m²

$P_6 = (1.294 \times 1.8 \times 1.11) = 2.585$ KN/m²

$P_9 = (1.258 \times 1.8 \times 1.244) = 2.817$ KN/m²

$P_{12} = (1.233 \times 1.8 \times 1.35) = 3.0$ KN/m²

Now,

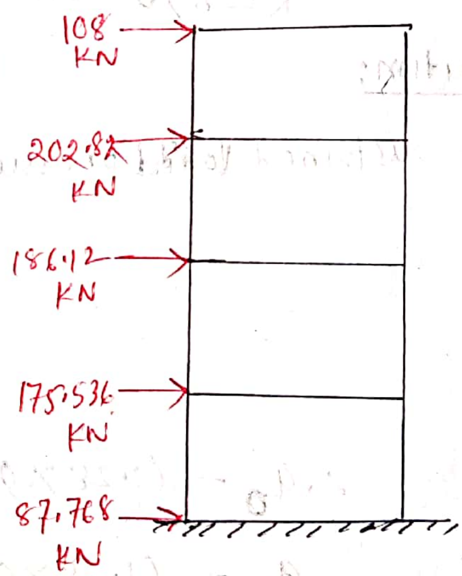
$F_0 = \left(\frac{3\times24}{2}\right)\times 2.438 = 87.768$ KN

$F_3 = (3\times24)\times 2.438 = 175.536$ KN

$F_6 = (3\times24)\times 2.585 = 186.12$ KN

$F_9 = (3\times24)\times 2.817 = 202.824$ KN

$F_{12} = \left(\frac{3\times24}{2}\right)\times 3 = 108$ KN



Earthquake Load Analysis

We know,

Design base shear, $V = \frac{ZIC}{R} W$

Here

$C = \frac{1.25S}{T^{\frac{2}{3}}}$; $T = C_t (h_n)^{\frac{3}{4}} = 0.049 \times (12)^{\frac{3}{4}} = 0.316$

$\therefore C = \frac{1.25\times1.5}{(.316)^{2/3}} = 4.04 > 2.75$ $\therefore C = 2.75$

Scanned by CamScanner

∴ Design base shear, $V = \frac{0.075 \times 1 \times 2.75}{8} \times (1500 \times 5)$

$\therefore V = 193.36\ KN$

Now,

$T = 0.316 < 0.7 \quad \therefore F_t = 0$

we know,

$$F_n = \frac{(V - F_t)\, W_n h_n}{\sum_{n=1}^{n} W_i h_i}$$

$$\text{Here, } \sum_{i=1}^{n} W_i h_i = 1500 \times (0 + 3 + 6 + 9 + 12) = 45000\ KN\text{-}m$$

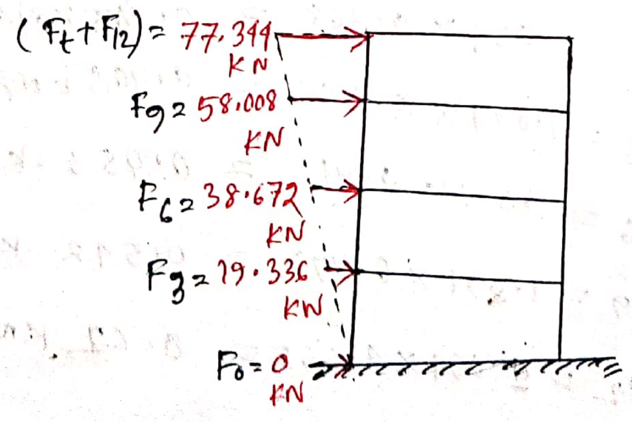
$\therefore F_0 = 0$

$$F_3 = \frac{(193.36 - 0) \times 1500 \times 3}{45000} = 19.336\ KN$$

$$F_6 = \frac{193.36 \times 1500 \times 6}{45000} = 38.672\ KN$$

$$F_9 = \frac{193.36 \times 1500 \times 9}{45000} = 58.008\ KN$$

$$F_{12} = \frac{193.36 \times 1500 \times 12}{45000} = 77.344\ KN$$



2013

calculate the earth quake load and design wind force at each floor level.

Given, $Z = 0.075$

$I = C_I = 1.00$

$R = 8.00$

$C_t = 0.049$

$C_p = 1.40$

$S = 1.50$

$W = 1400$ KN/floor

$C_c = 47.2 \times 10^{-6}$

$V_b = 152$ Km/h

Z(m)	C_z	C_G
(0-4.5)	0.368	1.654
6	0.415	1.592
9	0.497	1.511
12	0.565	1.457

Solution:

Wind Load Analysis

(1) sustained wind pressure, $q_z = C_z C_I C_c V_b^2$

$= 1 \times 47.2 \times 10^{-6} \times (152)^2 \times C_z$

$= 1.091 C_z$

$\therefore q_0 = 1.091 \times 0.368 = 0.4015$ KN/m²

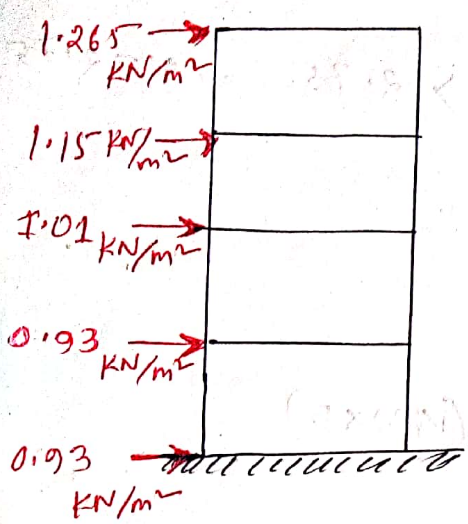
$q_3 = 1.091 \times 0.368 = 0.4015$ KN/m²

$q_6 = 1.091 \times 0.415 = 0.453$ KN/m²

$q_9 = 1.091 \times 0.497 = 0.542$ KN/m²

$q_{12} = 1.091 \times 0.565 = 0.62$ KN/m²

(iii) Design wind pressure, $P_z = C_G C_p q_z$



$\therefore P_0 = 1.40 \times 1.654 \times 0.4015 = 0.93 \ KN/m^2$

$P_3 = 1.40 \times 1.654 \times 0.4015 = 0.93 \ KN/m^2$

$P_6 = 1.40 \times 1.592 \times 0.453 = 1.01 \ KN/m^2$

$P_9 = 1.40 \times 1.511 \times 0.542 = 1.15 \ KN/m^2$

$P_{12} = 1.40 \times 1.457 \times 0.62 = 1.265 \ KN/m^2$

For both side portion,

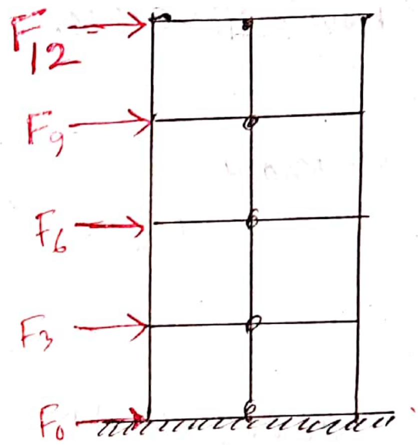
$\therefore F_0 = \left(\frac{8 \times 3}{2}\right) \times 0.93 = 11.16 \ KN$

$F_3 = (8 \times 3) \times 0.93 = 22.32 \ KN$

$F_6 = (8 \times 3) \times 1.01 = 24.24 \ KN$

$F_9 = (8 \times 3) \times 1.15 = 27.6 \ KN$

$F_{12} = \left(\frac{8 \times 3}{2}\right) \times 1.265 = 15.18 \ KN$



For middle portion,

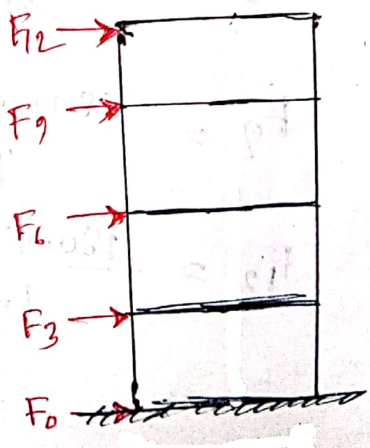
$F_0 = \left(\frac{6 \times 3}{2}\right) \times 0.93 = 8.37 \ KN$

$F_3 = (6 \times 3) \times 0.93 = 16.74 \ KN$

$F_6 = (6 \times 3) \times 1.01 = 18.18 \ KN$

$F_9 = (6 \times 3) \times 1.15 = 20.7 \ KN$

$F_{12} = \left(\frac{6 \times 3}{2}\right) \times 1.265 = 11.385 \ KN$



Earthquake load Analysis

we know,

$$T = C_t (h_n)^{\frac{3}{4}} = 0.049 \times (12)^{\frac{3}{4}} = 0.316$$

$$\therefore C = \frac{1.25\,S}{T^{\frac{2}{3}}} = \frac{1.25 \times 1.5}{(.316)^{\frac{2}{3}}} = 4.04 > 2.75 \quad \therefore C = 2.75$$

$\therefore$ Design base shear, $V = \frac{ZI\,C \times W}{R}$

$$= \frac{0.075 \times 1 \times 2.75}{8} \times (1400 \times 5)$$

$$\therefore V = 180.47 \text{ KN}$$

Now, $T = 0.316 < 0.7 \quad \therefore F_t = 0$

We know,

$$F_x = \frac{(V - F_t)\, W_x h_n}{\sum_{i=1}^{n} w_i h_i}$$

Here, $\sum_{i=1}^{n} w_i h_i = 1400(0+3+6+9+12) = 42000$ KN-m

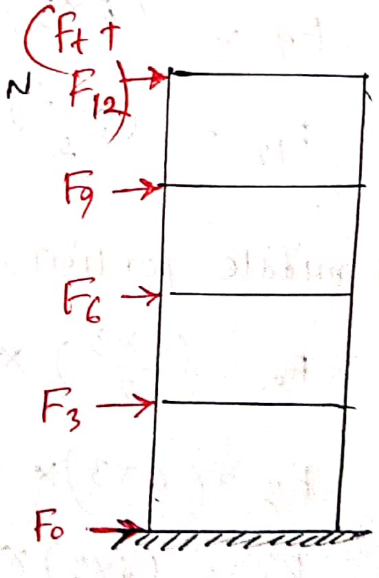
$\therefore F_0 = 0$ KN

$$F_3 = \frac{(180.47 - 0) \times 1400 \times 3}{42000} = 18.047 \text{ KN}$$

$$F_6 = \frac{180.47 \times 1400 \times 6}{42000} = 36.094 \text{ KN}$$

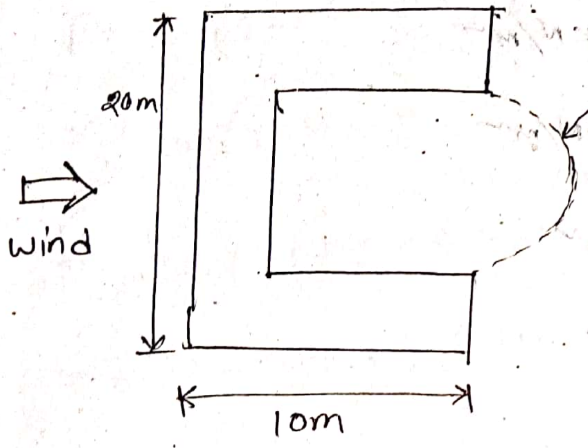
$$F_9 = \frac{180.47 \times 1400 \times 9}{42000} = 54.141 \text{ KN}$$

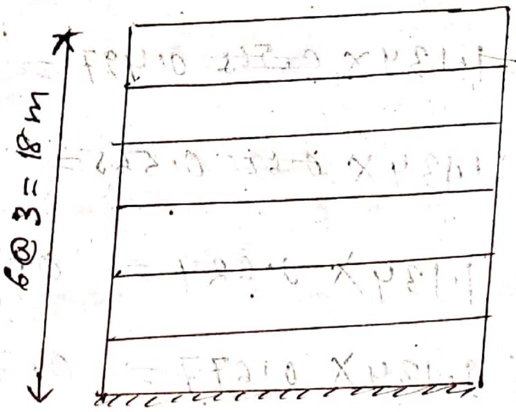
$$F_{12} = \frac{180.47 \times 1400 \times 12}{12000} = 72.188 \text{ KN}$$



2015

Calculate the design wind forces and the earth quake loads at each floor level for following six storied building constructed with in Rajshahi City Corporation.





Elevation

Structure Importance co-efficient = 1.00, Responce modification co-efficient = 9.00, structure type co-efficient = 0.049, site co-efficient for soil character = 2.00, siesmic dead load = 1400 KN/floor and pressure co-efficient = 1.50

Z (meter)	0-4.5	6.0	9.0	12.0	15.0	18.0
co-efficient, C_z	0.368	0.415	0.497	0.565	0.624	0.677
co-efficient, G_h	1.654	1.594	1.511	1.457	1.418	1.388

Solution: sustained wind pressure,

$$q_z = C_c C_I C_z V_b^2$$

Rajshahi ↑

$$= 47.2 \times 10^{-6} \times 1.00 \times (155)^2 \times$$

$$= 1.134\, C_z$$

Assume, $C_I = 1.00$ for standard occupancy structure

$q_0 = 1.134 \times 0.368 = 0.42$ KN/m²

$q_3 = 1.134 \times$ ~~0.415~~ $0.368 = 0.42$ KN/m²

$q_6 = 1.134 \times$ ~~0.497~~ $0.415 = 0.471$ KN/m²

$q_9 = 1.134 \times$ ~~0.565~~ $0.497 = 0.564$ KN/m²

$q_{12} = 1.134 \times$ ~~0.62~~ $0.565 = 0.641$ KN/m²

$q_{15} = 1.134 \times 0.624 = 0.71$ KN/m²

$q_{18} = 1.134 \times 0.677 = 0.77$ KN/m²

Design wind pressure, $P_z = C_G C_p q_z$

$= 1.50 \times C_G \times q_z$

$\therefore P_0 = 1.5 \times 0.42 \times 1.654 = 1.04$ KN/m²

$P_3 = 1.5 \times 0.42 \times 1.654 =$ ~~1.125~~ 1.04 KN/m²

$P_6 = 1.5 \times 0.471 \times 1.592 = 1.125$ KN/m²

$P_9 = 1.5 \times 0.564 \times 1.511 = 1.28$ KN/m²

$P_{12} = 1.5 \times 0.641 \times 1.457 = 1.4$ KN/m²

$P_{15} = 1.5 \times 0.71 \times 1.418 = 1.51$ KN/m²

$P_{18} = 1.5 \times 0.77 \times 1.388 = 1.60$ KN/m²

$F_0 = \left(20 \times \frac{3}{2}\right) \times 1.04 = 31.2$ KN

$F_3 = (20 \times 3) \times 1.04 = 62.4$ KN

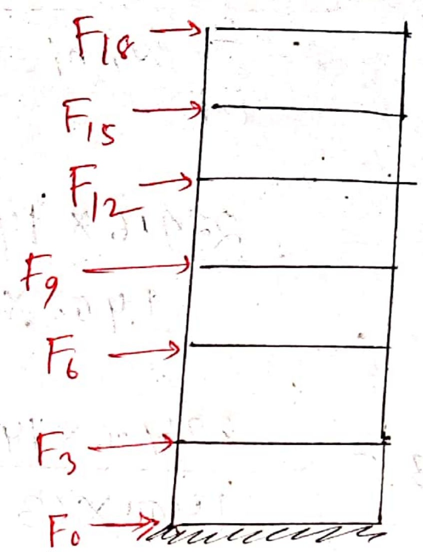
$F_6 = (20 \times 3) \times 1.125 = 67.5$ KN

$F_9 = (20 \times 3) \times 1.28 = 76.8$ KN

$F_{12} = (20 \times 3) \times 1.4 = 84$ KN

$F_{15} = (20 \times 3) \times 1.51 = 90.6$ KN

$F_{18} = \left(20 \times \frac{3}{2}\right) \times 1.c = 30$ KN



Earth Quake load Analysis

We Know,

design base shear, $V = \frac{ZIC}{R} W$

Here, $C = \frac{1.25\,S}{T^{\frac{2}{3}}}$ and, $T = C_t (h_n)^{3/4} = (0.049) \times (18)^{\frac{3}{4}}$

$\therefore T = 0.428$

$\therefore C = \frac{1.25 \times 2.00}{(0.428)^{\frac{2}{3}}} = 4.4 > 2.75$

$\therefore C = 2.75$

$V = \frac{0.075 \times 1.00 \times 2.75}{9.0} \times (1400 \times 7) = 224.6$ KN

$T = 0.428 < 0.7 \quad \therefore F_t = 0$

we know,

$$\therefore F_x = \frac{(V - F_t)\, W_x h_x}{\sum_{i=1}^{n} W_i h_i}$$

$$\therefore F_0 = \frac{224.6 \times 1400 \times 0}{1400 \times (0+3+6+9+12+15+18)} = 0$$

$$F_3 = \frac{224.6 \times 1400 \times 3}{1400 \times 63} = 10.7 \text{ KN}$$

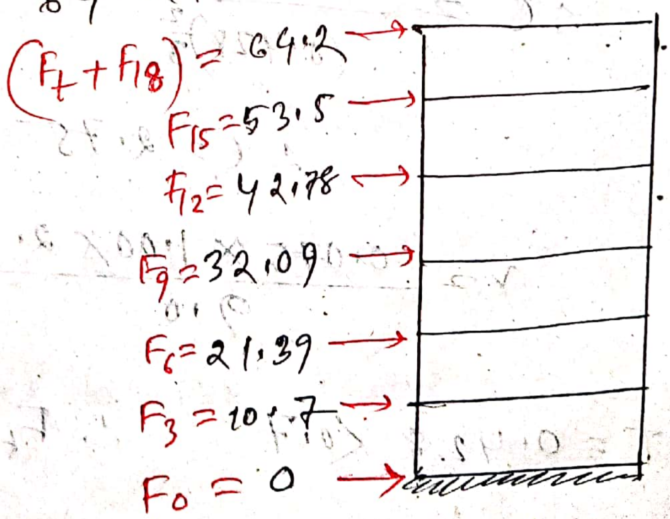
$$F_6 = \frac{224.6 \times 1400 \times 6}{1400 \times 63} = 21.39 \text{ KN}$$

$$F_9 = \frac{224.6 \times 1400 \times 9}{1400 \times 63} = 32.09 \text{ KN}$$

$$F_{12} = \frac{224.6 \times 1400 \times 12}{1400 \times 63} = 42.78 \text{ KN}$$

$$F_{15} = \frac{224.6 \times 1400 \times 15}{1400 \times 63} = 53.5 \text{ KN}$$

$$F_{18} = \frac{224.6 \times 1400 \times 18}{1400 \times 63} = 64.2 \text{ KN}$$



Deflection – (BEAM)

Unit Load Method

Derive the basic formula of unit load method.

Fig. (a)

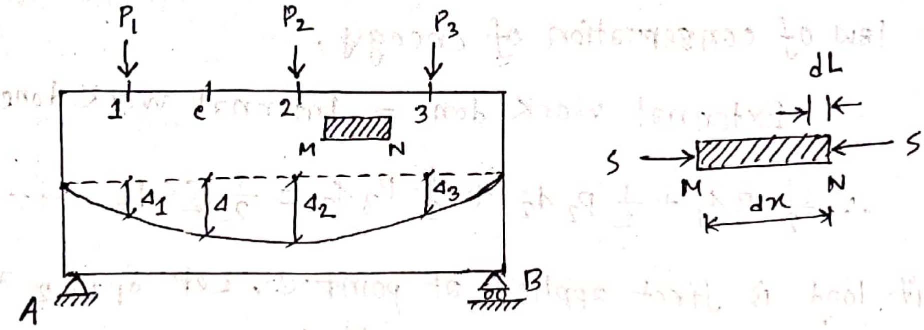


Fig. (b)

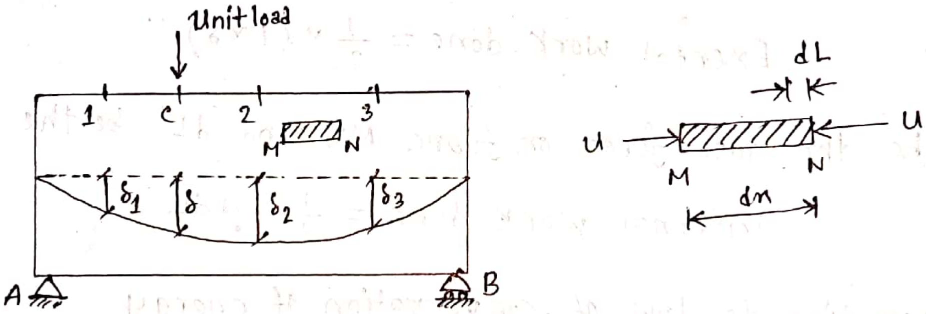
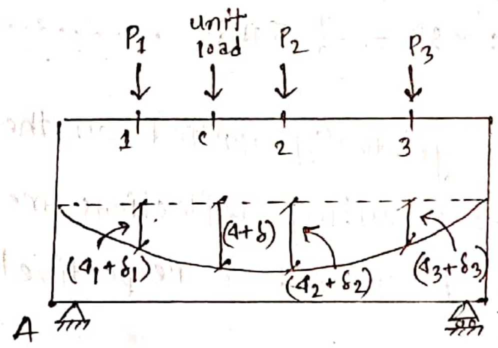


Fig. (c)



Let us consider a simply supported beam AB subjected to the applied loads P_1, P_2 and P_3 at the points 1, 2 and 3 respectively. Let Δ_1, Δ_2 and Δ_3 are the deflections at points 1, 2 and 3 respectively. We have to find out the deflection at c. [Fig. (a)]

If P_1, P_2 and P_3 are gradually applied,

$$\text{External Work done} = \frac{1}{2} \times P_1\Delta_1 + \frac{1}{2} \times P_2\Delta_2 + \frac{1}{2} \times P_3\Delta_3$$

if 'S' be the total force on any fibre MN, of the length 'dx' and area 'dA' and 'dL' be the shortening.

$$\text{Internal Work done} = \frac{1}{2}\Sigma S\,dL$$

According to law of conservation of energy,

External Work done = Internal Work done

$$\therefore \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 = \frac{1}{2}\Sigma S\,dL \quad \text{.........(1)}$$

Now, a unit load is first applied at point C. Let δ_1, δ_2 and δ_3 are the deflection at points 1, 2 and 3 respectively.

$$\text{Exertal work done} = \frac{1}{2}\times(1\times\delta)$$

Let 'u' be the total force on fibre MN and 'dL' be the shortening.

$$\text{Internal work done} = \frac{1}{2}\Sigma u\,dL$$

Again, According to law of conservation of energy,

$$\frac{1}{2}\times(1\times\delta) = \frac{1}{2}\Sigma u\,dL \quad \text{..........(2)}$$

Now, if the load P_1, P_2 and P_3 are gradually applied on the beam which is already subjected to unit load, the resulting deflections are $(\Delta_1+\delta_1)$, $(\Delta+\delta)$, $(\Delta_2+\delta_2)$ and $(\Delta_3+\delta_3)$ at points 1, 2 and 3 respectively.

$$\text{Additional External work done} = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + (1\cdot\Delta)$$

$$\text{Additional internal work done} = \frac{1}{2}\Sigma S\,dL + \Sigma u\,dL$$

$$\therefore \text{Total external work done} = \frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + (1\cdot\Delta) + \frac{1}{2}\times(1\times\delta)$$

$$\text{and Total internal work done} = \frac{1}{2}\Sigma S\,dL + \Sigma u\,dL + \frac{1}{2}\Sigma u\,dL$$

According to law of conservation of energy,

$$\frac{1}{2}P_1\Delta_1 + \frac{1}{2}P_2\Delta_2 + \frac{1}{2}P_3\Delta_3 + (1\cdot\Delta) + \frac{1}{2}\times(1\times\delta) = \frac{1}{2}\Sigma S\,dL + \Sigma u\,dL + \frac{1}{2}\Sigma u\,dL \quad \text{..........(3)}$$

Now, From ③ − [② + ①] we obtain,

$$(1 \cdot \Delta) = \Sigma u \, dL$$

This is the basic formula of unit load method.

Application to the beam deflection of unit load method:

Let, M = moment due to applied load at any section

m = moment due to unit load at any section

I = Moment of Inertia of the section

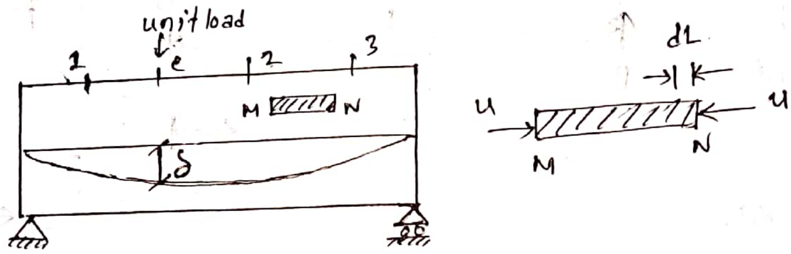


Fig. (a)

From figure (a),

$$u = \frac{my}{I} dA \quad \dots (i) \quad \left[\because \sigma = \frac{P}{A} \Rightarrow P = \sigma A \therefore u = \frac{my}{I} dA \right]$$

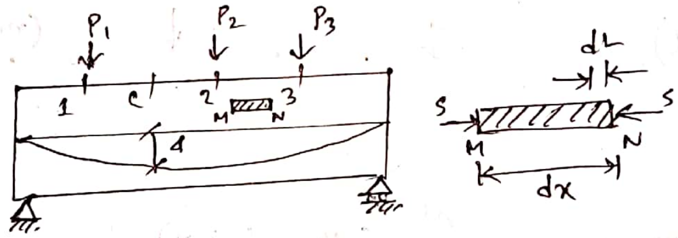


Fig (b)

From figure (b),

$$dL = \frac{My}{EI} dx \quad \dots (ii) \quad \left[\because \sigma = E\delta \Rightarrow \frac{My}{I} = E \cdot \frac{dL}{dx} \therefore dL = \frac{My}{EI} dx \right]$$

Now, we know,

$$1 \cdot \Delta = \Sigma u \cdot dL$$

$$\Rightarrow 1 \cdot \Delta = \Sigma \left(\frac{my}{I} \cdot dA \right) \times \left(\frac{My}{EI} dx \right)$$

$$\Rightarrow 1 \cdot \Delta = \int_0^L \int_0^A \frac{Mmy^2}{EI^2} dA \cdot dx$$

$$\Rightarrow 1\cdot\Delta = \int_0^L \frac{Mm}{EI^2}\,dx \times \int_0^A y^2\,dA$$

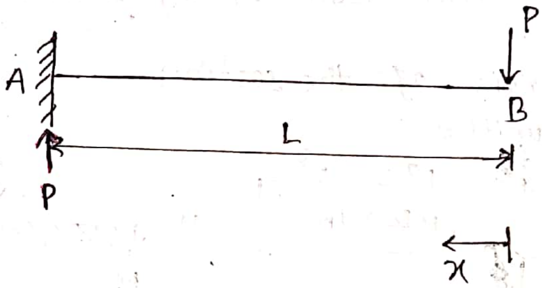
$$\Rightarrow 1\cdot\Delta = \int_0^L \frac{Mm}{EI^2}\,dx \times I$$

$$\therefore 1\cdot\Delta = \int_0^L \frac{Mm}{EI}\,dx$$

Problem: 01

Find $\Delta_B = ?$

$EI = \text{constant}$



$\vdash\!\rightarrow x$	clock wise (+) Anti clock wise (−)
$x \leftarrow\!\dashv$	clockwise (−) Anticlockwise (+)

Portion — AB

Origin — B

Limit — (0 to L)

M — $(-P\cdot x)$

m — $(-1\cdot x)$

(must আঁকতে হবে)

Now,

$$\Delta = \int_0^L \frac{Mm}{EI}\,dx = \int_0^L \frac{(-Px)\cdot(-1\cdot x)}{EI}\,dx = \int_0^L \frac{Px^2}{EI}\,dx$$

$$\Rightarrow \Delta = \frac{P}{EI}\int_0^L x^2\,dx$$

$$\Rightarrow \Delta = \frac{P}{EI}\left[\frac{x^3}{3}\right]_0^L$$

$$\therefore \Delta = \frac{PL^3}{3EI} \ (\downarrow)$$

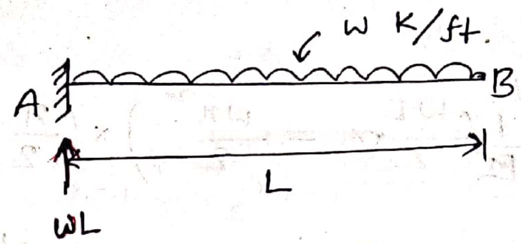
(Ans.)

[deflection (+ve), Hence যেদিকে unit load apply করা হয়েছে deflection সেই দিকে হবে]

Problem : 02

Find $\Delta_B = ?$

$EI = \text{constant}$



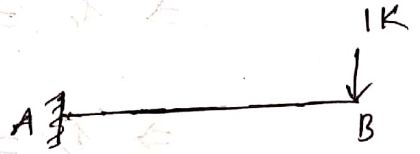
Portion — AB

origin — B

Limit — (0 to L)

M — $\left(-\frac{wx^2}{2}\right)$

m — $(-1 \cdot x)$



Now,

$$\Delta_B = \int_0^L \frac{Mm}{EI}\,dx = \frac{1}{EI}\int_0^L \left(\frac{-wx^2}{2}\right) \times (-1 \times x) \cdot dx$$

$$\Rightarrow \Delta_B = \frac{w}{2EI}\int_0^L x^3\,dx = \frac{w}{2EI}\left[\frac{x^4}{4}\right]_0^L = \frac{w}{2EI} \times \frac{L^4}{4}$$

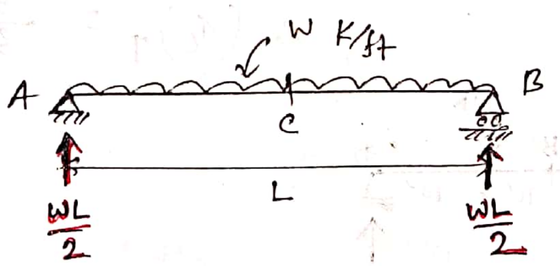
$$\therefore \Delta_B = \frac{wL^4}{8EI} \ (\downarrow)$$

(Ans.)

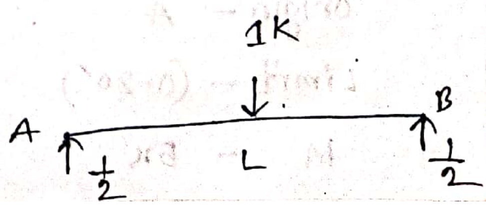
Problem : 03

Find $\Delta_C = ?$

$EI = \text{constant}$



portion	AC	BC
Origin	A	B
Limit	$\left(0 \text{ to } \frac{L}{2}\right)$	$\left(0 \text{ to } \frac{L}{2}\right)$
M	$\left(\frac{wL}{2} \cdot x - \frac{wx^2}{2}\right)$	$\left(\frac{wL}{2} \cdot x - \frac{wx^2}{2}\right)$
m	$\frac{1}{2}x$	$\frac{1}{2}x$



AC Portion:

$$\Delta_c = \int_0^L \frac{Mm}{EI}\,dx = \int_0^{L} \frac{1}{EI}\left(\frac{WL}{2}\times x - \frac{Wx^2}{2}\right)\times\left(\frac{x}{2}\right)\cdot dx$$

$$\Rightarrow \Delta_c = \frac{1}{EI}\int_0^{\frac{L}{2}}\left(\frac{WLx^2}{4} - \frac{Wx^3}{4}\right)dx = \frac{W}{4EI}\int_0^{\frac{L}{2}}(Lx^2 - x^3)\,dx$$

$$\Rightarrow \Delta_c = \frac{W}{4EI}\left[\frac{L\times x^3}{3} - \frac{x^4}{4}\right]_0^{\frac{L}{2}}$$

$$\Rightarrow \Delta_c = \frac{W}{4EI}\left[\frac{L^4}{24} - \frac{L^4}{64}\right]$$

$$\Rightarrow \Delta_c = \frac{W}{4EI}\times\frac{5L^4}{192}$$

$$\therefore \Delta_c = \frac{5WL^4}{768}\ (\downarrow)$$

Similarly,

for BC portion,

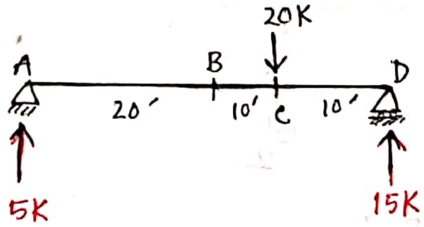
$$\Delta_c = \frac{5WL^4}{768}\ (\downarrow)$$

Hence, Total deflection at C, $(\Delta_c)_T = 2\times\frac{5WL^4}{768}\ (\downarrow)$

$$\therefore (\Delta_c)_T = \frac{5}{384}WL^4\ (\downarrow)$$

(Ans.)

Problem: 04



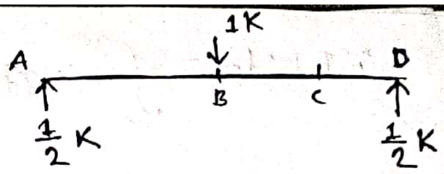
$E = 30000$ ksi

$I = 1000\ in^4$

$\Delta_B = ?$

portion	AB	BC	CD
origin	A	A	D
Limit	(0-20')	(20'-30')	(0-10')
M	5x	5x	15x.

portion —	AB	BC	CD
m —	$\frac{x}{2}$	$\left[\frac{x}{2} - 1(x-20)\right]$	$\frac{x}{2}$



$$EI \cdot \Delta_B = \int_0^{20} (5x)\cdot\left(\frac{x}{2}\right) dx + \int_{20}^{30} (5x)\cdot\left[\frac{x}{2} - (x-20)\right]\cdot dx + \int_0^{10} (15x)\cdot\left(\frac{x}{2}\right) dx$$

$$\Rightarrow EI\Delta_B = \frac{5}{2}\times\left[\frac{x^3}{3}\right]_0^{20} + \frac{5}{2}\times\left[\frac{x^3}{3}\right]_{20}^{30} - 5\times\left[\frac{x^3}{3}\right]_{20}^{30} + 100\times\left[\frac{x^2}{2}\right]_{20}^{30} + \frac{15}{2}\times\left[\frac{x^3}{3}\right]_0^{10}$$

$$\Rightarrow EI\Delta_B = 6666.67 + 15833.33 - 31666.67 + 25000 + 2500$$

$$\Rightarrow EI\cdot\Delta_B = 18333.33$$

$$\Rightarrow \Delta_B = \frac{18333.33 \times 1728}{30000 \times 1000 \times 1} = 1.056 \text{ in.}$$

$$\therefore \Delta_B = 1.056 \text{ in } (\downarrow)$$

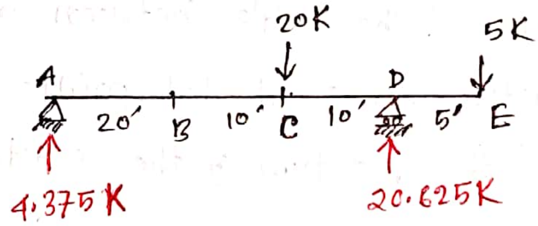
(Ans.)

Problem : 05

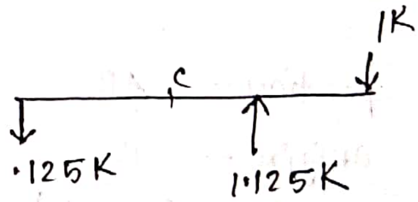
$E = 30000$ ksi

$I = 1000$ in^4

$\Delta_E = ?$



portion —	AC	CD	DE
origin —	A	A	E
Limit —	(0–30')	(30'–40')	(0–5')
M —	$4.375x$	$[4.375x - 20(x-30)]$	$-5x$
m —	$(-0.125x)$	$(-0.125x)$	$(-x)$



$$\therefore EI \cdot \Delta_E = \int_0^{30} (4.375x)(-0.125x)\,dx + \int_{30}^{40} (4.375x - 20x + 600)x \cdot (-0.125x)\,dx + \int_0^5 (-5x)\cdot(-x)\,dx$$

$$= -0.546875 \times \left[\frac{x^3}{3}\right]_0^{30} - 0.546875 \times \left[\frac{x^3}{3}\right]_{30}^{40} + 2.5 \times \left[\frac{x^3}{3}\right]_{30}^{40} - 75 \times \left[\frac{x^2}{2}\right]_{30}^{40} + 5 \times \left[\frac{x^3}{3}\right]_0^5$$

$$= -4921.875 - 6744.79 + 30833.33 - 26250 + 208.33$$

$$\therefore EI \cdot \Delta_E = -6875.005$$

$$\Rightarrow \Delta_E = \frac{-6875.005 \times 1728}{30000 \times 1000 \times 1} = -0.396 \text{ in}$$

$$\therefore \Delta_E = 0.396 \text{ in } (\uparrow)$$

(Ans.)

Application of the unit load to the beam slope:

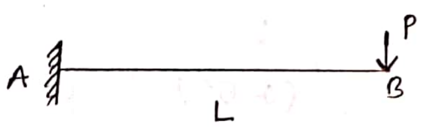
Tangent or the median measure of the angle between original beam axis and tangent to the elastic curve at that point. This is the angle which is considered as the rotation of the elastic curve.

$$1 \cdot \theta = \int_0^L \frac{Mm}{EI}\,dx$$

problem: 06

Find $\theta_B = ?$

EI constant.



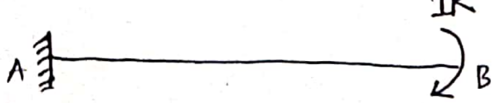
portion - AB

origin - B

Limit - (0 to L)

M — $(-P \times x)$

m — (-1)



$$\therefore \theta_B = \int_0^L \frac{M m}{EI} dx = \frac{1}{EI}\int_0^L (-P.x).(-1).dx$$

$$\Rightarrow \theta_B = \frac{P}{EI}\int x\,dx = \frac{P}{EI} \times \left[\frac{x^2}{2}\right]_0^L = \frac{P}{EI} \times \frac{L^2}{2}$$

$$\therefore \theta_B = \frac{PL^2}{2EI} \quad (2)$$

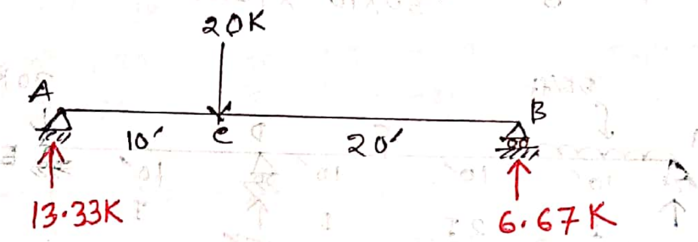
(Ans.)

Problem: 07

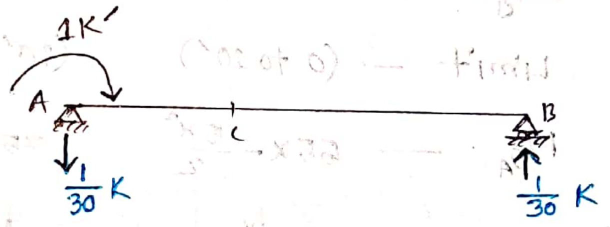
$\theta_A, \theta_B = ?$

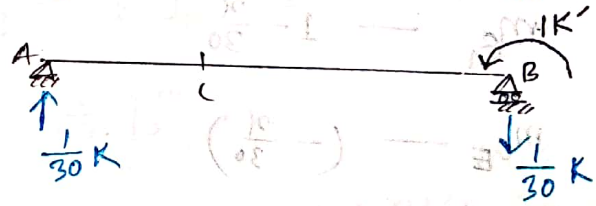
$E = 30000$ ksi

$I = 1000$ in^4



	AC	BC
portion	AC	BC
origin	A	B
Limit	(0 to 10')	(0 to 20')
M	$13.33x$	$6.67x$
$m_{\theta A}$	$\left(1 - \frac{x}{30}\right)$	$\frac{x}{30}$
$m_{\theta B}$	$\frac{x}{30}$	$\left(1 - \frac{x}{30}\right)$





Now,

$$EI.\theta_A = \int_0^{10} (13.33x).\left(1-\frac{x}{30}\right).dx + \int_0^{20} (6.67x).\frac{x}{30}.dx$$

$$= 13.33 \times \left[\frac{x^2}{2}\right]_0^{10} - \frac{13.33}{30} \times \left[\frac{x^3}{3}\right]_0^{10} + \frac{6.67}{30} \times \left[\frac{x^3}{3}\right]_0^{20}$$

$$= 13.33 \times \frac{10^2}{2} - \frac{13.33}{30} \times \frac{10^3}{3} + \frac{6.67}{30} \times \frac{20^3}{3}$$

$$= 666.5 - 148.11 + 592.89$$

$$EI\theta_B = 1111.28 \Rightarrow \theta_B = \frac{1111.28 \times 144}{30 \times 10^3 \times 1000} = 5.33 \times 10^{-3} \text{ rad.} \quad (2)$$

(Ans.)

And,

$$EI\cdot\theta_B = \int_0^{10}(13.33x)\cdot\left(\frac{x}{30}\right)dx + \int_0^{20}(6.67x)\cdot\left(1-\frac{x}{30}\right)\cdot dx$$

$$= \frac{13.33}{30}\times\left[\frac{x^3}{3}\right]_0^{10} + 6.67\times\left[\frac{x^2}{2}\right]_0^{20} - \frac{6.67}{30}\times\left[\frac{x^3}{3}\right]_0^{20}$$

$$= 148.11 + 1334 - 592.89$$

$$\Rightarrow EI\cdot\theta_B = 889.22 \quad \Rightarrow \theta_B = \frac{889.22\times144}{30\times10^3\times1000} = 4.27\times10^{-3} \text{ rad. } (\circlearrowleft)$$

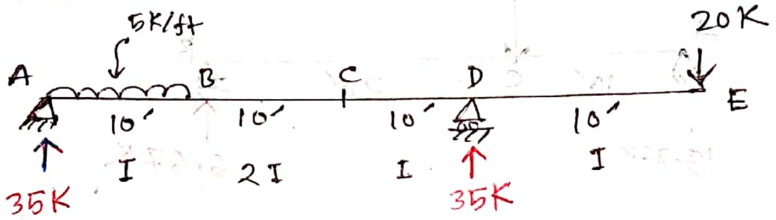
(Ans.)

Problem: 08

$\theta_A, \theta_E = ?$

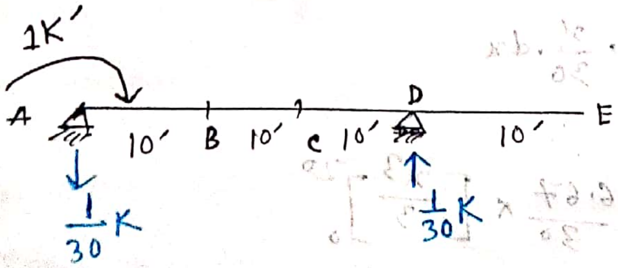
$E = 30\times10^3$ ksi

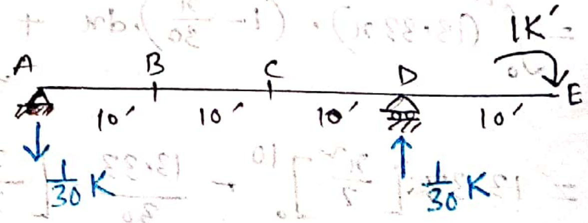
$I = 1000$ in^4



$\Sigma M_A = 0 \Rightarrow 5\times10\times5 + 20\times40 = R_D\times30 \Rightarrow R_D = 35K$

portion	AB	BC	CD	DE
origin	A	A	A	E
Limit	(0 to 10')	(10' to 20')	(20' to 30')	(0 to 10')
M	$55x-\frac{5x^2}{2}$	$35x-50(x-5)$	$35x-50(x-5)$	$-20x$
m_{θ_A}	$1-\frac{x}{30}$	$1-\frac{x}{30}$	$1-\frac{x}{30}$	0
m_{θ_E}	$\left(-\frac{x}{30}\right)$	$-\frac{x}{30}$	$-\frac{x}{30}$	-1





$$EI\cdot\theta_A = \int_0^{10}\left(55x-\frac{5x^2}{2}\right)\cdot\left(1-\frac{x}{30}\right)dx + \frac{1}{2}\int_{10}^{20}(35x-50x+250)\cdot\left(1-\frac{x}{30}\right)\cdot dx$$

$$+\int_{20}^{30}(35x-50x+250)\cdot\left(1-\frac{x}{30}\right)dx + \int_0^{10}(-20x)\cdot 0\cdot dx$$

$$\Rightarrow EI\cdot\theta_A = \int_0^{10}\left(55x - \frac{5x^2}{2} - \frac{55}{30}x^2 + \frac{1}{12}\times x^3\right)dx + \frac{1}{2}\int_{10}^{20}\left(250 - 15x - \frac{25}{3}x + \frac{x^2}{2}\right)dx$$

$$+ \int_{20}^{30}\left(250 - 15x - \frac{25}{3}x + \frac{x^2}{2}\right)dx$$

$$\Rightarrow EI\cdot\theta_A = 55\times\left[\frac{x^2}{2}\right]_0^{10} - \frac{5}{2}\times\left[\frac{x^3}{3}\right]_0^{10} - \frac{55}{30}\times\left[\frac{x^3}{3}\right]_0^{10} + \frac{1}{12}\times\left[\frac{x^4}{4}\right]_0^{10} + \frac{1}{2}\times 250\times[x]_{10}^{20}$$

$$- \frac{1}{2}\times 15\times\left[\frac{x^2}{2}\right]_{10}^{20} - \frac{1}{2}\times\frac{25}{3}\times\left[\frac{x^2}{2}\right]_{10}^{20} + \frac{1}{2}\times\frac{1}{2}\times\left[\frac{x^3}{3}\right]_{10}^{20} + 250\times[x]_{20}^{30}$$

$$- 15\times\left[\frac{x^2}{2}\right]_{20}^{30} - \frac{25}{3}\times\left[\frac{x^2}{2}\right]_{20}^{30} + \frac{1}{2}\times\left[\frac{x^3}{3}\right]_{20}^{30}$$

$$\Rightarrow EI\cdot\theta_A = 2750 - 833.33 - 611.11 + 208.33 + 1250 - 1125 - 625 + 583.33$$

$$+2500 - 3750 - 2083.33 + 3166.67$$

$$\Rightarrow EI\cdot\theta_A = 1430.56 \qquad \Rightarrow \theta_A = \frac{1430.56\times 144}{30\times10^3\times1000} = 6.87\times10^{-3}\ \text{rad.}$$

$$\therefore \theta_A = 6.87\times10^{-3}\ \text{rad.}\ (\curvearrowright) \quad \text{(Ans).}$$

Now,

$$EI\cdot\theta_E = \int_0^{10}\left(55x - \frac{5x^2}{2}\right)\cdot\left(-\frac{x}{30}\right)\cdot dx + \int_{10}^{20}(250 - 15x)\left(-\frac{x}{30}\right)\cdot dx + \int_{20}^{30}(250-15x)\cdot\left(-\frac{x}{30}\right)dx$$

$$+ \int_0^{10}(-20x)\cdot(-1)\,dx$$

$$= -\frac{55}{30}\times\left[\frac{x^3}{3}\right]_0^{10} + \frac{1}{12}\times\left[\frac{x^4}{4}\right]_0^{10} - \frac{25}{3}\times\left[\frac{x^2}{2}\right]_{10}^{20} + \frac{1}{2}\times\left[\frac{x^3}{3}\right]_{10}^{20}$$

$$- \frac{25}{3}\times\left[\frac{x^2}{2}\right]_{20}^{30} + \frac{1}{2}\times\left[\frac{x^3}{3}\right]_{20}^{30} + 20\times\left[\frac{x^2}{2}\right]_0^{10}$$

$$= -611.11 + 208.33 - 1250 + 1166.67 - 2083.33 + 3166.67$$

$$+ 1000$$

$$\Rightarrow EI\cdot\theta_E = 1597.23 \qquad \Rightarrow \theta_E = \frac{1597.23\times144}{30\times10^3\times1000} = 7.67\times10^{-3}\ \text{rad.}\ (\curvearrowright)$$

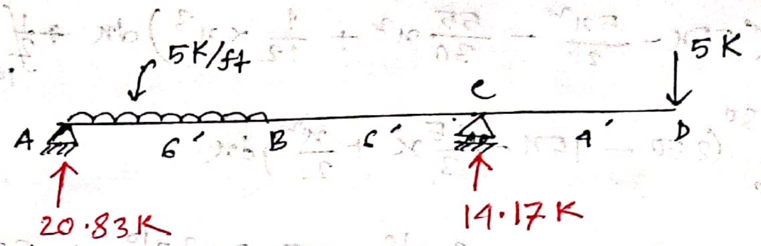
(Ans.)

Problem: 09

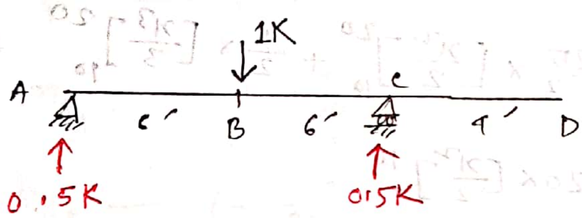
$\Delta_B, \Delta_D, \theta_A = ?$

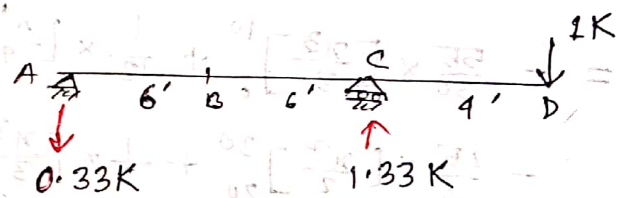
$E = 30 \times 10^3$ ksi

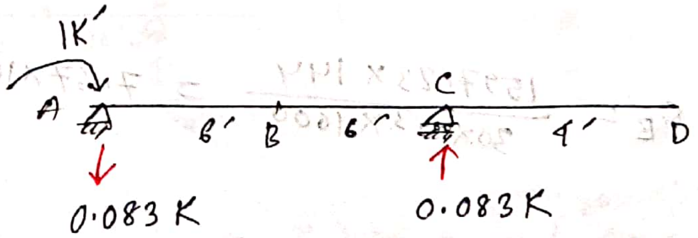
$I = 1000$ in^4



portion	AB	BC	CD
Origin	A	D	D
Limit	(0–6')	(4–10')	(0–4')
M	$\left(20.83x - \frac{5x^2}{2}\right)$	$[-5x + 14.17(x-4)]$	$(-5x)$
$m_{\Delta B}$	$(0.5x)$	$[0.5(x-4)]$	0
$m_{\Delta D}$	$(-0.33x)$	$[-x + 1.33(x-4)]$	$(-x)$
$m_{\theta A}$	$(1 - 0.083x)$	$[.083(x-4)]$	0







$$EI\Delta_B = \int_0^6 (20.83x - 2.5x^2)\cdot(0.5x)\cdot dx + \int_4^{10} [-5x + 14.17(x-4)] \times [0.5(x-4)]\cdot dx$$

$$+ \int_0^4 (-5x)\cdot 0 \cdot dx$$

$$= \int_0^6 (10.415x^2 - 1.25x^3)\,dx + \int_4^{10} (9.17x - 56.68)\cdot(.5x - 2)\,dx + 0$$

$$= 10.415 \times \left[\frac{x^3}{3}\right]_0^6 - 1.25 \times \left[\frac{x^4}{4}\right]_0^6 + \int_4^{10} (4.585x^2 - 46.68x + 113.36)\,dx$$

$$= 749.88 - 405 + 4.585 \times \left[\frac{x^3}{3}\right]_4^{10} - 46.68 \times \left[\frac{x^2}{2}\right]_4^{10} + 113.36 \times [x]_4^{10}$$

$$= 344.88 + 1430.52 - 1960.56 + 680.16$$

$$\Rightarrow EI\Delta_B = 495 \Rightarrow \Delta_B = \frac{495 \times 1728}{30 \times 10^3 \times 1000 \times 1} = 0.0285'' \;(\downarrow)$$

Now,

$$EI\Delta_D = \int_0^6 (20.83x - 2.5x^2)\cdot(-0.33x)\cdot dx + \int_4^{10} (9.17x - 56.68)\cdot(0.33x - 5.32)\,dx$$

$$+ \int_0^4 (-5x)\cdot(-x)\cdot dx$$

$$= \int_0^6 (-6.8739x^2 + 0.825x^3)\,dx + \int_4^{10} (3.0261x^2 - 67.49x + 301.54)\,dx$$

$$+ \int_0^4 5x^2\,dx$$

$$= -6.8739 \times \left[\frac{x^3}{3}\right]_0^6 + 0.825 \times \left[\frac{x^4}{4}\right]_0^6 + 3.0261 \times \left[\frac{x^3}{3}\right]_4^{10}$$

$$- 67.49 \times \left[\frac{x^2}{2}\right]_4^{10} + 301.54 \times [x]_4^{10} + 5 \times \left[\frac{x^3}{3}\right]_0^4$$

$$= -494.921 + 267.3 + 944.1432 - 2834.58 + 1809.24 + 106.67$$

$$\Rightarrow EI\Delta_D = -202.1478 \quad \Rightarrow \Delta_D = -\frac{202.1478 \times 1728}{30 \times 10^3 \times 1000 \times 1} = -0.011644''$$

$$\therefore \Delta_D = 0.012'' \;(\uparrow)$$

$$EI\theta_A = \int_0^6 (20.83x - 2.5x^2)\cdot(1-0.083x)\cdot dx + \int_4^{10} (9.17x - 56.68)\cdot(0.083x - .332)\cdot dx$$

$$+ \int_0^4 (-5x)\cdot 0 \; dx$$

$$= \int_0^6 (0.2075x^3 - 4.23x^2 + 20.83x)\,dx + \int_4^{10} (0.76111x^2 - 7.75x + 18.82)\,dx + 0$$

$$= 0.2075\times\left[\frac{x^4}{4}\right]_0^6 - 4.23\times\left[\frac{x^3}{3}\right]_0^6 + 20.83\times\left[\frac{x^2}{2}\right]_0^6 + 0.76111\times\left[\frac{x^3}{3}\right]_4^{10}$$

$$- 7.75\times\left[\frac{x^2}{2}\right]_4^{10} + 18.82\times[x]_4^{10}$$

$$= 67.23 - 304.56 + 374.94 + 237.47 - 325.5 + 112.92$$

$$\Rightarrow EI\theta_B = 162.50 \Rightarrow \theta_B = \frac{162.50\times 144}{30\times10^3\times 1000} = 7.80\times10^{-4}\ \text{rad}$$

$$\therefore \theta_B = 7.80\times10^{-4}\ \text{rad}\ (\curvearrowright)$$

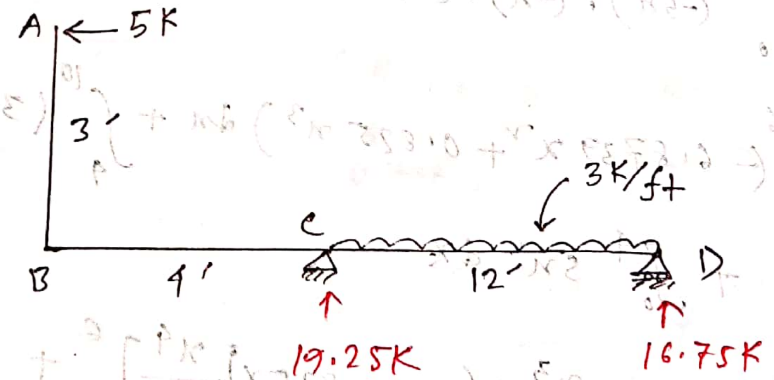
(Ans.)

Problem: 10

$\theta_B,\ \Delta_{AH},\ \Delta_{AV} = ?$

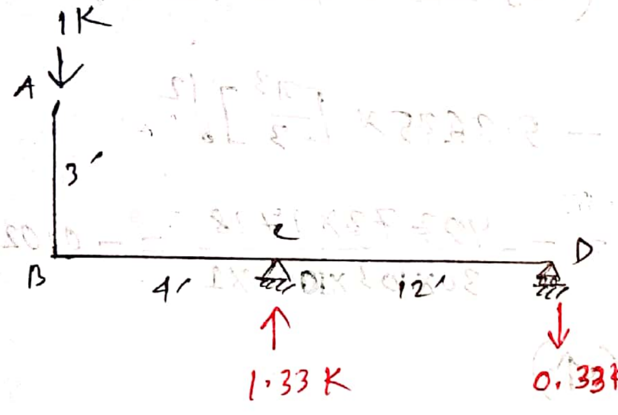
$E = 30\times10^3$ ksi

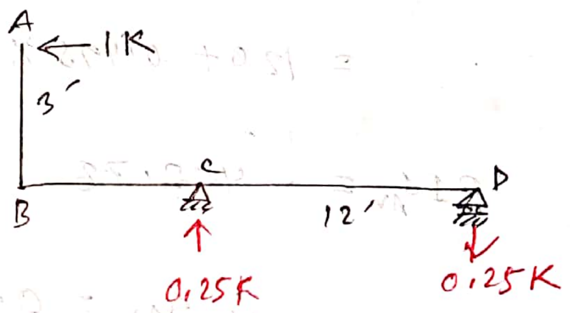
$I = 1000\ in^4$

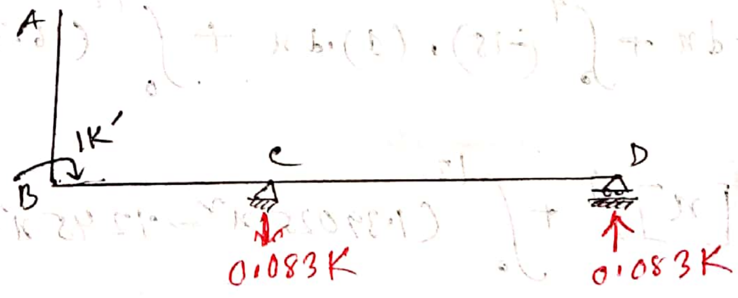


portion —	AB	BC	CD
origin —	A	B	D
Limit —	(0-3')	(0-4')	(0-12')

portion	AB	BC	CD
M	$(-5x)$	(-15)	$[16.75x - \frac{3}{2}x^2]$
$m_{\Delta AH}$	$(-1x)$	(-3)	$(-.25x)$
$m_{\Delta AV}$	0	$(-x)$	$(-0.33x)$
$m_{\theta B}$	0	1	$(0.083x)$







Now,

$$EI\Delta_{AH} = \int_0^3 (-5x)\cdot(-x)\cdot dx + \int_0^4 (-15)\cdot(-3)\cdot dx + \int_0^{12} (16.75x - 1.5x^2)\cdot(-.25x)\cdot dx$$

$$= 5\times\left[\frac{x^3}{3}\right]_0^3 + 45\times[x]_0^4 + \int_0^{12}(0.375x^3 - 4.1875x^2)\,dx$$

$$= 45 + 180 + 0.375\times\left[\frac{x^4}{4}\right]_0^{12} - 4.1875\times\left[\frac{x^3}{3}\right]_0^{12}$$

$$\Rightarrow EI\Delta_{AH} = 225.375 + 1944 - 2412 = -242.625$$

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$$\therefore \Delta_{AH} = \frac{-242.625 \times 1728}{30\times10^3 \times 1000 \times 1} = -0.014''$$

$$\therefore \Delta_{AH} = 0.014'' \ (\rightarrow)$$

Again,

$$EI\Delta_{AV} = \int_0^3 (-5x)\cdot 0\cdot dx + \int_0^4 (-15)(-x)\,dx + \int_0^{12} (16.75x - 1.5x^2)\cdot(-0.33x)\,dx$$

$$= 0 + 15\left[\frac{x^2}{2}\right]_0^4 + \int_0^{12} (-5.5275x^2 + 0.495x^3)\,dx$$

$$= 120 + 0.495\times\left[\frac{x^4}{4}\right]_0^{12} - 5.5275\times\left[\frac{x^3}{3}\right]_0^{12}$$

$$EI\Delta_{AV} = -497.76 \Rightarrow \Delta_{AV} = -\frac{497.76\times1728}{30\times10^3\times1000\times1} = -0.0287''$$

$$\therefore \Delta_{AV} = 0.0287'' \ (\uparrow)$$

And,

$$EI\theta_B = \int_0^3 (-5x)\cdot 0\cdot dx + \int_0^4 (-15)\cdot(1)\cdot dx + \int_0^{12} (16.75x - 1.5x^2)\cdot(0.083x)\cdot dx$$

$$= 0 - 15\times[x]_0^4 + \int_0^{12} (1.39025x^2 - .1245x^3)\,dx$$

$$= -60 + 1.39025\times\left[\frac{x^3}{3}\right]_0^{12} - 0.1245\times\left[\frac{x^4}{4}\right]_0^{12}$$

$$= -60 + 800.784 - 645.408$$

$$\Rightarrow EI\theta_B = 95.376 \Rightarrow \theta_B = \frac{95.376\times144}{30\times10^3\times1000} = 4.58\times10^{-4}\ \text{rad.}$$

$$\therefore \theta_B = 4.58\times10^{-4}\ \text{rad}\ (\curvearrowright)$$

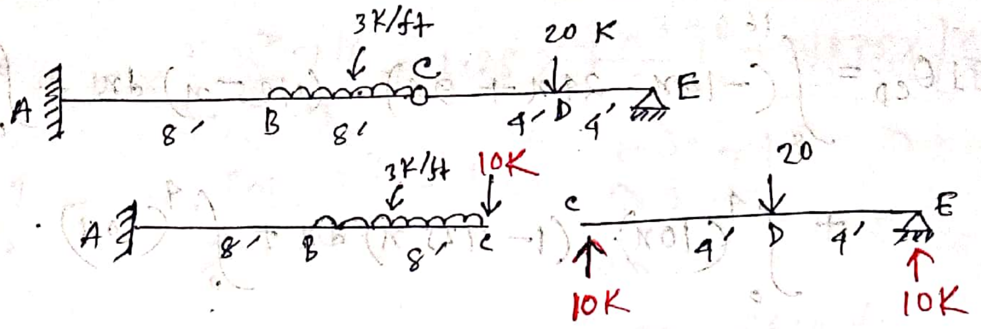
(Ans.)

Problem: 11

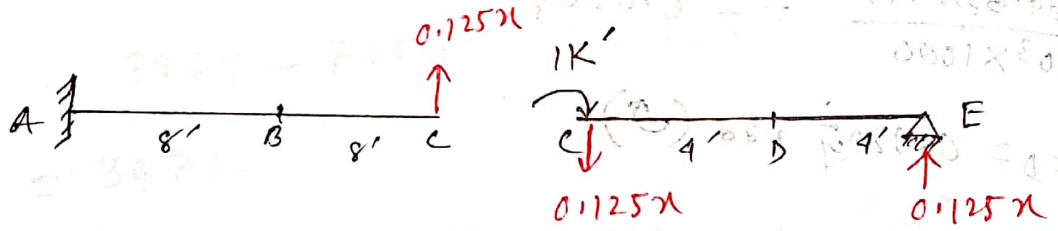
$\theta_{CD}, \theta_E, \theta_{CB} = ?$

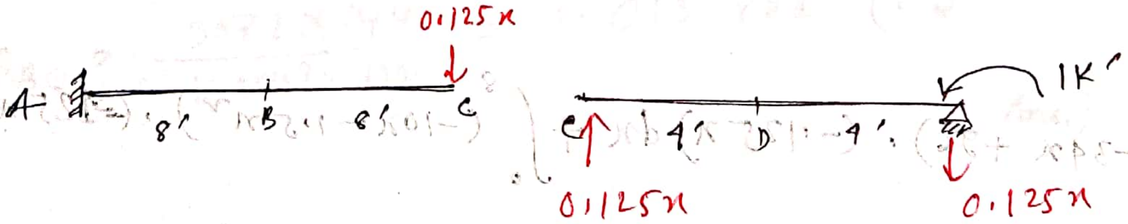
$E = 30 \times 10^3$ ksi

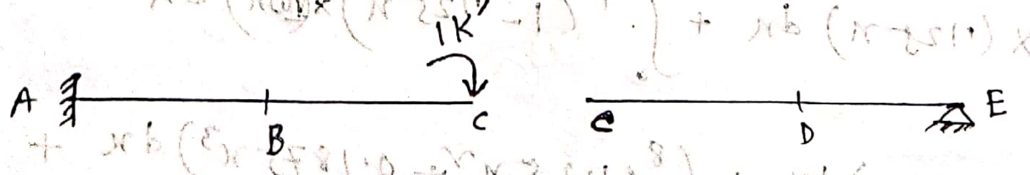
$I = 1000$ in^4



portion	AB	BC	CD	DE
origin	C	C	C	E
Limit	(8-16')	(0-8')	(0-4')	(0-4')
M	$[-10x - 24(x-4)]$	$\left(-10x - \frac{3x^2}{2}\right)$	$(10x)$	$(10x)$
$m_{\theta CD}$	$(0.125x)$	$(0.125x)$	$(1 - .125x)$	$(0.125x)$
$m_{\theta E}$	$(-0.125x)$	$(-0.125x)$	$(0.125x)$	$(1 - 0.125x)$
$m_{\theta CB}$	(-1)	(-1)	0	0







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$$EI\theta_{CD} = \int_8^{16} (-10x - 24x + 96)\cdot(.125x)\,dx + \int_0^8 \left(-10x - \frac{3x^2}{2}\right)\cdot(.125x)\,dx$$

$$+ \int_0^4 (10x)\cdot(1 - .125x)\,dx + \int_0^4 (10x)\cdot(.125x)\cdot dx$$

$$= \int_8^{16} (-4.25x^2 + 12x)\,dx + \int_0^8 (-1.25x^2 - 0.1875x^3)\,dx$$

$$+ \int_0^4 (10x - 1.25x^2)\,dx + \int_0^4 (1.25x^2)\,dx$$

$$= -4.25\times\left[\frac{x^3}{3}\right]_8^{16} + 12\times\left[\frac{x^2}{2}\right]_8^{16} - 1.25\times\left[\frac{x^3}{3}\right]_0^8 - 0.1875\times\left[\frac{x^4}{4}\right]_0^8$$

$$+ 10\times\left[\frac{x^2}{2}\right]_0^4 - 1.25\times\left[\frac{x^3}{3}\right]_0^4 + 1.25\times\left[\frac{x^3}{3}\right]_0^4$$

$$= -5077.33 + 1152 - 213.33 - 192 + 80 - 26.67 + 26.67$$

$$EI\theta_{CD} = -4250.67$$

$$\Rightarrow \theta_{CD} = \frac{-4250.67\times 144}{30\times 10^3\times 1000} = -0.0204 \text{ rad}$$

$$\therefore \theta_{CD} = 0.0204 \text{ rad } (\circlearrowleft)$$

Now,

$$EI\theta_E = \int_8^{16} (-34x + 96)\cdot(-.125x)\,dx + \int_0^8 (-10x - 1.5x^2)\cdot(-.125x)\,dx$$

$$+ \int_0^4 (10x)\times(.125x)\,dx + \int_0^4 (1 - .125x)\times(10x)\,dx$$

$$= \int_8^{16} (4.25x^2 - 12x)\,dx + \int_0^8 (1.25x^2 + 0.1875x^3)\,dx +$$

$$\int_0^4 1.25x^2\,dx + \int_0^4 (10x - 1.25x^2)\,dx$$

$$= 4.25 \times \left[\frac{x^3}{3}\right]_8^{16} - 12 \times \left[\frac{x^2}{2}\right]_8^{16} + 1.25 \times \left[\frac{x^3}{3}\right]_0^8 + 0.1875 \times \left[\frac{x^4}{4}\right]_0^8$$

$$+ 1.25 \times \left[\frac{x^3}{3}\right]_0^4 + 10 \times \left[\frac{x^2}{2}\right]_0^4 - 1.25 \times \left[\frac{x^3}{3}\right]_0^4$$

$$= 5077.33 - 1152 + 213.33 + 192 + 26.67 + 80 - 26.67$$

$$= 4410.66$$

$$\therefore \theta_E = \frac{4410.66 \times 144}{30 \times 10^3 \times 1000} = 0.0212 \text{ rad} \quad (\circlearrowleft)$$

And,

$$EI \cdot \theta_{BC} = \int_8^{16} (-34x + 96) \cdot (-1)\, dx + \int_0^8 (-10x - 1.5x^2)\, (-1)\, dx +$$

$$+ \int_0^4 (10x) \cdot (0) \cdot dx + \int_0^4 (10x) \cdot (0) \cdot dx$$

$$= 34 \times \left[\frac{x^2}{2}\right]_8^{16} - 96 \times [x]_8^{16} + 10 \times \left[\frac{x^2}{2}\right]_0^8 + 1.5 \times \left[\frac{x^3}{3}\right]_0^8$$

$$= 3264 - 768 + 320 + 256$$

$$= 3072$$

$$\therefore \theta_{BC} = \frac{3072 \times 144}{30 \times 10^3 \times 1000} = 0.015 \text{ rad} \quad (\circlearrowright)$$

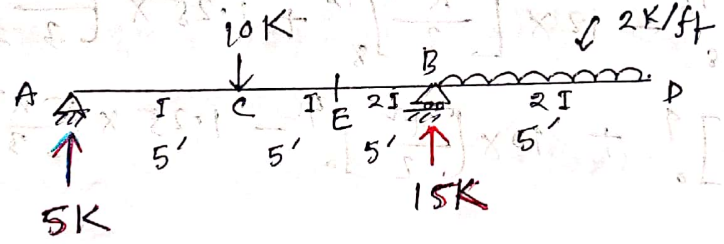
(Ans.)

Problem:12

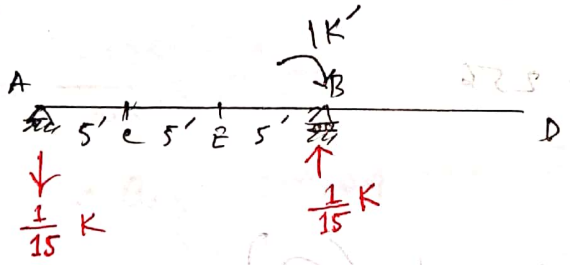
θ_B & θ_D = ?

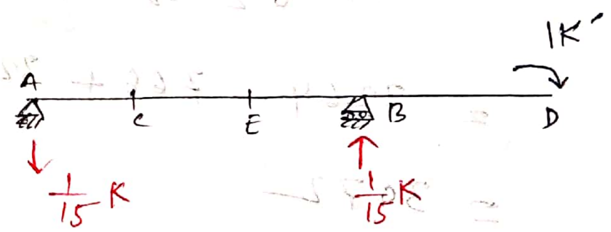
$E = 30\times10^3$ ksi

$I = 1000$ in^4



portion –	AC	CE	EB	BD
Origin –	A	A	A	D
Limit –	(0–5')	(5'–10')	(10'–15')	(0–5')
M –	(5x)	[5x − 10(x−5)]	[5x − 10(x−5)]	$\left(-2\cdot x\cdot\frac{x}{2}\right)$
$m_{\theta B}$ –	$\left(-\frac{x}{15}\right)$	$\left(-\frac{x}{15}\right)$	$\left(-\frac{x}{15}\right)$	0
$m_{\theta E}$ –	$\left(-\frac{x}{15}\right)$	$\left(-\frac{x}{15}\right)$	$\left(-\frac{x}{15}\right)$	(−1)





$$EI\cdot\theta_B = \int_0^5 (5x)\cdot\left(-\frac{x}{15}\right)\cdot dx + \int_5^{10} (5x-10x+50)\cdot\left(-\frac{x}{15}\right)\cdot dx + \frac{1}{2}\times\int_{10}^{15}(-5x+50)\cdot\left(-\frac{x}{15}\right)\cdot dx$$

$$+\frac{1}{2}\times\int_0^5 (-x^2)\cdot 0\, dx$$

$$= -\frac{1}{3}\times\left[\frac{x^3}{3}\right]_0^5 + \frac{1}{3}\left[\frac{x^3}{3}\right]_5^{10} - \frac{50}{15}\times\left[\frac{x^2}{2}\right]_5^{10} + \frac{1}{2}\times\left\{\frac{1}{3}\times\left[\frac{x^3}{3}\right]_{10}^{15}\right.$$

$$\left. - \frac{50}{15}\times\left[\frac{x^2}{2}\right]_{10}^{15}\right\} + 0$$

$= -13.89 + 97.22 - 125 + \frac{1}{2} \times (263.89 - 208.33)$

$= -13.89$

$\therefore \theta_B = \frac{-13.89 \times 144}{30 \times 10^3 \times 1000} = -6.67 \times 10^{-5}$ rad

$\therefore \theta_B = 6.67 \times 10^{-5}$ rad (↶)

Now,

$$EI \cdot \theta_D = \int_0^5 (5x) \cdot \left(-\frac{x}{15}\right) dx + \int_5^{10} (-5x+50) \cdot \left(\frac{-x}{15}\right) \cdot dx + \frac{1}{2}\int_{10}^{15} (-5x+50) \cdot \left(-\frac{x}{15}\right) \cdot dx$$

$$+ \frac{1}{2}\int_0^5 (-x^2) \times (-1) \cdot dx$$

$= -13.89 + \frac{1}{2} \times \left[\frac{x^3}{3}\right]_0^5$

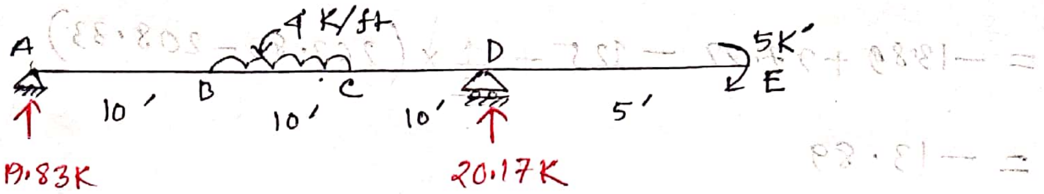
$= -13.89 + 20.833$

$= 6.94$

$\therefore \theta_D = \frac{6.94 \times 144}{30 \times 10^3 \times 1000} = 3.3312 \times 10^{-5}$ rad. (↷)

(Ans.)

Problem: 13

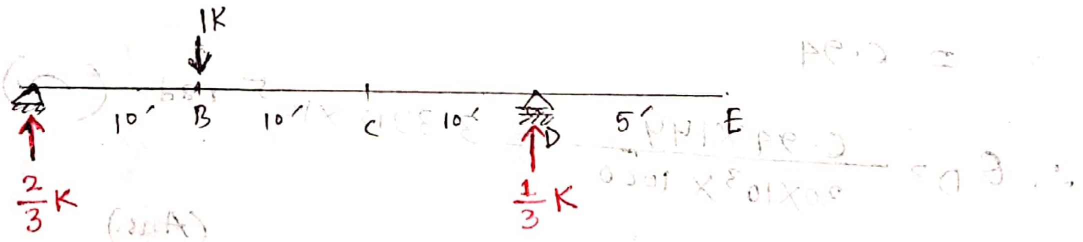


$\Delta_B = ?$

$E = 30 \times 10^3$ Ksi

$I = 1000$ in^4

portion	—	AB	BC	CD	DE
origin	—	A	A	A	E
Limit	—	(0–10')	(10'–20')	(20'–30')	(0–5')
M	—	$(19.83x)$	$\left[19.83x - 4\cdot\frac{(x-10)^2}{2}\right]$	$[19.83x - 40(x-15)]$	(-5)
$m_{\Delta B}$	—	$\left(\frac{2}{3}x\right)$	$\left\{\frac{2}{3}x - 1\times(x-10)\right\}$	$\left\{\frac{2}{3}x - (x-10)\right\}$	0



$$EI.\Delta_B = \int_0^{10} (19.83x)\cdot\left(\frac{2}{3}x\right)\cdot dx + \int_{10}^{20} \left\{19.83x - 2(x-10)^2\right\}\left\{\frac{2}{3}x - (x-10)\right\}\cdot dx$$

$$+ \int_{20}^{30} \left\{19.83x - 40(x-15)\right\}\cdot\left\{\frac{2}{3}x - (x-10)\right\}\cdot dx + \int_0^5 (-5)\cdot 0\cdot dx$$

$$= 19.83x\cdot\frac{2}{3}\times\left[\frac{x^3}{3}\right]_0^{10} + \int_{10}^{20} (-2x^2 + 59.83x - 200)\cdot(-0.33x+10)\cdot dx + \int_{20}^{30} (600 - 20.17x)\cdot(-0.33x+10)\cdot dx$$

$= 4406.67 + 11543.89 + 2156.11$

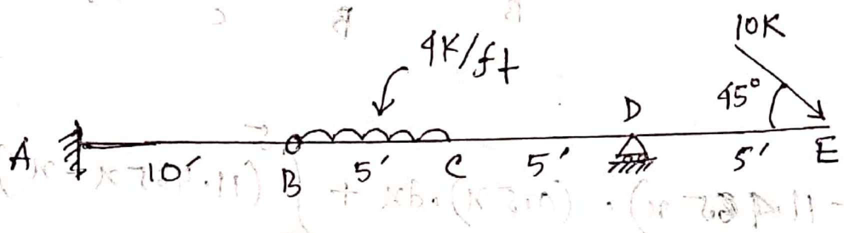
$= 18106.67$

$\therefore \Delta_B = \dfrac{18106.67 \times 1728}{30 \times 10^3 \times 1000} = 1.043$ in $(\downarrow)$

(Ans)

Problem: 14

$\Delta_E = ?$

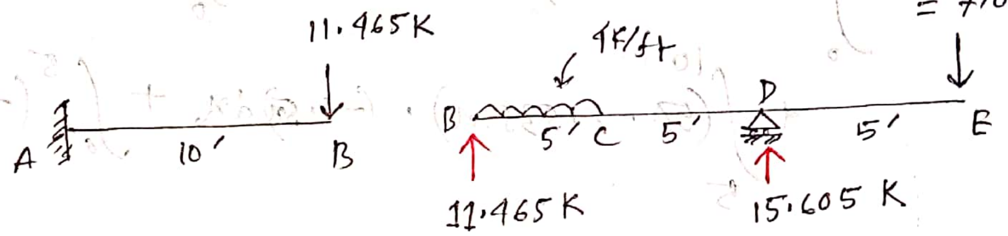


$10 \sin 45° = 7.07$ K

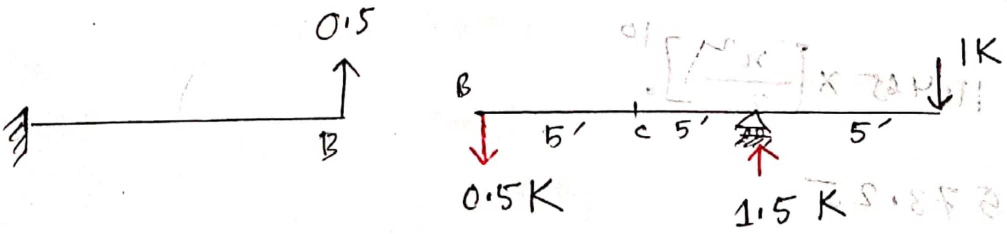
$\theta_{BA} = ?$

$E = 30 \times 10^3$ Ksi

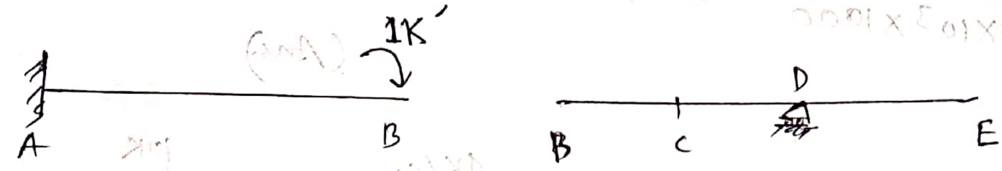
$I = 1000$ in^4



portion —	AB	BC	CD	DE
origin —	B	B	B	E
Limit —	(0–10')	(0–5')	(5'–10')	(0–5')
M —	$(-11.465x)$	$(11.465x - 4 \cdot x \cdot \frac{x}{2})$	$\{11.465x - 20(x-2.5)\}$	$(-7.07x)$
$m_{\Delta E}$ —	$0.5x$	$(-0.5x)$	$(-0.5x)$	$(-x)$



portion	AB	BC	CD	DE
$m_{\theta BA}$	(−1)	0	0	0



$$EI \cdot \Delta_E = \int_0^{10} (-11.465x) \cdot (0.5x) \cdot dx + \int_0^{5} (11.465x - x^2) \cdot (-.5x) dx$$

$$+ \int_5^{10} (50 - 8.535x) \cdot (-0.5x) dx + \int_0^{5} (-7.07x) \cdot (-x) \cdot dx$$

$$= -1910.83 - 160.73 + 307.1875 + 294.583$$

$$= -1469.79$$

$$\therefore \Delta_E = \frac{-1469.79 \times 1728}{30 \times 10^3 \times 1000} = -0.08466 \text{ in.}$$

$$\therefore \Delta_E = 0.08466 \text{ in } (\uparrow)$$

$$EI \cdot \theta_{BA} = \int_0^{10} (-11.465x) \cdot (-1) \cdot dx + 0 + 0 + 0$$

$$= 11.465 \times \left[\frac{x^2}{2}\right]_0^{10}$$

$$= 573.25$$

$$\therefore \theta_{BA} = \frac{573.25 \times 144}{30 \times 10^3 \times 1000} = 2.7516 \times 10^{-3} \text{ rad. } (\curvearrowright)$$

(Ans.)

Castigliano's Theorem

First Theorem: The partial derivative of the total internal energy in a structure with respect to the deflection at any point is equal to the load applied at that point.

Proof:

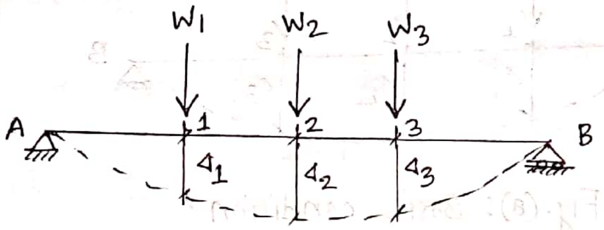


Fig.(a): Basic Condition.

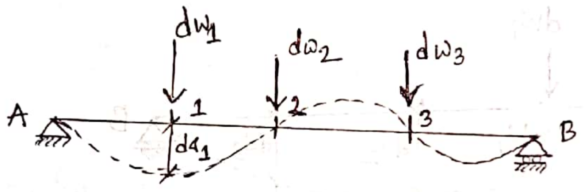


Fig.(b): Incremental Condition.

W_1, W_2, W_3 cause deflection $\Delta_1, \Delta_2, \Delta_3$ at point 1, 2, 3 respectively.

Incremental loads dw_1, dw_2, and dw_3 are in such proportions that they will cause additional deflection $d\Delta_1$ at point 1 and $d\Delta_2 = 0$, $d\Delta_3 = 0$ at point 2 and 3 respectively.

Incremental external work done or, internal energy,

$$du = W_1 d\Delta_1 + \frac{1}{2} dw_1 d\Delta_1$$

neglecting $\frac{1}{2} dw_1 d\Delta_1$ we obtain,

$$du = W_1 d\Delta_1$$

$$\therefore \boxed{\frac{du}{d\Delta_1} = W_1}$$

(Proved)

Second Theorem: The partial derivative of the total internal energy in a structure with respect to the load at any point is equal to deflection at that point.

Proof:

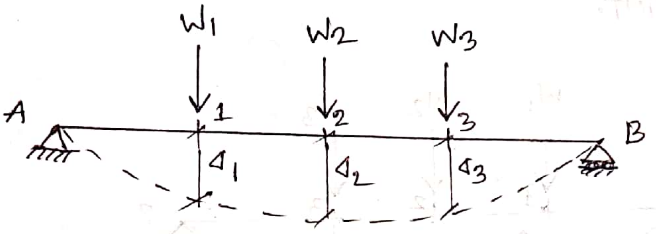


Fig. (a): Basic condition.

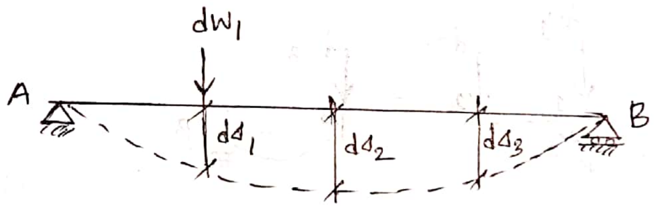


Fig.(b): Incremental condition.

A single incremental load dW_1 is added at point 1 causes deflections $d\Delta_1$, $d\Delta_2$, $d\Delta_3$ at point 1, 2 and 3 respectively.

Incremental external work done or internal energy,

$$du = W_1 d\Delta_1 + W_2 d\Delta_2 + W_3 d\Delta_3 + \frac{1}{2} dW_1 d\Delta_1$$

neglecting $\frac{1}{2} dW_1 d\Delta_1$ we obtain,

$$du = W_1 d\Delta_1 + W_2 d\Delta_2 + W_3 d\Delta_3 \quad \text{......(I)}$$

Now,

Total internal work done,

$$(U + du) = \frac{1}{2}(W + dW_1)(\Delta_1 + d\Delta_1) + \frac{1}{2} W_2 (\Delta_1 + d\Delta_2) +$$

$$\frac{1}{2} W_3 (\Delta_3 + d\Delta_3) \quad \text{.....(II)}$$

Basic condition is,

$$U = \frac{1}{2}W_1\Delta_1 + \frac{1}{2}W_2\Delta_2 + \frac{1}{2}W_3\Delta_3 \quad \cdots\cdots (3)$$

From (2) − (3) we obtain,

$$dU = \frac{1}{2}\Delta_1 dW_1 + \frac{1}{2}W_1 d\Delta_1 + \frac{1}{2}dW_1 d\Delta_1 + \frac{1}{2}W_2 d\Delta_2 + \frac{1}{2}W_3 d\Delta_3$$

neglecting $\frac{1}{2}dW_1 d\Delta_1$,

$$dU = \frac{1}{2}\Delta_1 dW_1 + \frac{1}{2}W_1 d\Delta_1 + \frac{1}{2}W_2 d\Delta_2 + \frac{1}{2}W_3 d\Delta_3$$

$$\Rightarrow dU = \frac{1}{2}\Delta_1 dW_1 + \frac{1}{2}dU \quad [\text{from eq}^n \text{ (1)}, \; dU = W_1 d\Delta_1 + W_2 d\Delta_2 + W_3 d\Delta_3]$$

$$\Rightarrow dU = \Delta_1 dW_1$$

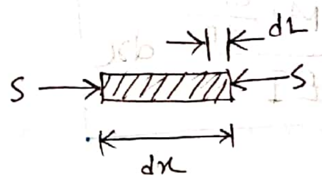
$$\therefore \boxed{\frac{dU}{dW_1} = \Delta_1}$$

(Proved)

Application of Castigliano's Theorem to the beam deflection:

$$\Delta = \frac{dW}{dP}$$ where, dW = work done, dP = Applied load

Internal work done, $W = \frac{1}{2}\sum S\,dL$



Now,

$$\frac{\frac{S}{dA}}{\frac{dL}{dx}} = E$$

$$\Rightarrow \frac{S}{E}\cdot\frac{1}{dA} = \frac{dL}{dx}$$

$$\Rightarrow \frac{dL}{dx} = \frac{1}{E}\cdot\frac{My}{I}\cdot dA\cdot\frac{1}{dA} \quad [\because S = \frac{My}{I}dA]$$

$$\Rightarrow dL = \frac{My}{EI}dx$$

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$$W = \frac{1}{2} \sum S \, dL$$

$$\Rightarrow W = \frac{1}{2} \sum \left(\frac{My}{I} dA \right) \cdot \left(\frac{My}{EI} dx \right)$$

$$\therefore W = \frac{1}{2} \sum \frac{M^2 y^2}{EI^2} dA \cdot dx$$

Now, $\Delta = \dfrac{dW}{dP}$

$$= \frac{d \left\{ \frac{1}{2} \sum \frac{M^2 y^2}{EI^2} dA \cdot dx \right\}}{dM} \cdot \frac{dM}{dP}$$

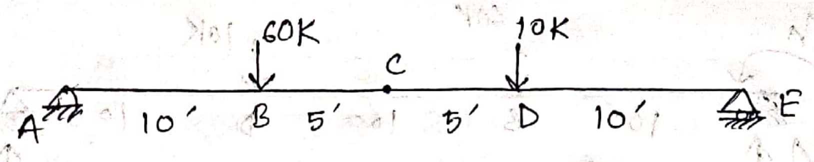
$$= \sum \frac{My^2}{EI^2} dA \cdot dx \cdot \frac{dM}{dP}$$

$$= \int_0^L \int_0^A \frac{M}{EI^2} \times y^2 dA \times dx \cdot \frac{dM}{dP}$$

$$= \int_0^L \frac{M}{EI^2} \times I \times dx \cdot \frac{dM}{dP} \qquad \left(\because \int_0^A y^2 dA = I \right)$$

$$\therefore \boxed{\Delta = \int_0^L \frac{M \cdot \frac{dM}{dP}}{EI} dx}$$

Problem: 01

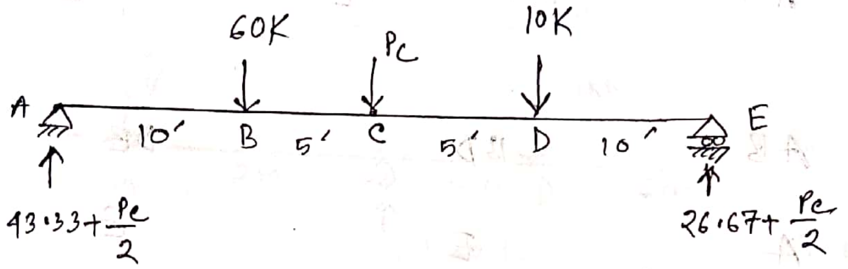


$E = 30 \times 10^3$ ksi

$I = 1000\ in^4$

$\Delta_c,\ \theta_A = ?$

Solution:



$\Sigma M_A = 0,\quad 60 \times 10 + P_c \times 15 + 10 \times 20 = R_E \times 30 \quad \therefore R_E = 26.67 + \frac{P_c}{2}$

Part —	AB	BC	CD	DE
origin —	A	A	E	E
Limit —	(0 – 10)	(10 – 15)	(10 – 15)	(0 – 10)
M —	$(43.33 + \frac{P_c}{2})x$	$(43.33 + \frac{P_c}{2})x - 60(x-10)$	$(26.67 + \frac{P_c}{2})x - 10(x-10)$	$(26.67 + \frac{P_c}{2})x$
$\frac{dM}{P_c}$ —	$\frac{x}{2}$	$\frac{x}{2}$	$\frac{x}{2}$	$\frac{x}{2}$

Now,

$$EI \cdot \Delta_c = \int_0^{10} (43.33 + \tfrac{P_c}{2})x \times (\tfrac{x}{2})\,dx + \int_{10}^{15} \left[(43.33 + \tfrac{P_c}{2})x - 60(x-10)\right] \times (\tfrac{x}{2})\,dx$$

$$+ \int_{10}^{15} \left[(26.67 + \tfrac{P_c}{2})x - 10(x-10)\right] \times (\tfrac{x}{2})\,dx + \int_0^{10} (26.67 + \tfrac{P_c}{2})x \times (\tfrac{x}{2})\,dx$$

(Now, $P_c = 0$)

$$\therefore EI \cdot \Delta_c = \int_0^{10} \frac{43.33x^2}{2}\,dx + \int_{10}^{15} (600 - 16.67x) \times \frac{x}{2}\,dx + \int_{10}^{15} (16.67x + 100)\frac{x}{2}\,dx$$

$$+ \int_0^{10} \frac{26.67}{2}x^2\,dx.$$

$$= 7221.67 + 12151.46 + 9723.54 + 4445$$

$$EI \cdot \Delta_c = 33541.67 \qquad \therefore \Delta_c = \frac{33541.67 \times 1728}{30 \times 10^3 \times 1000 \times 1} = 1.932\ in.\ (\downarrow)$$

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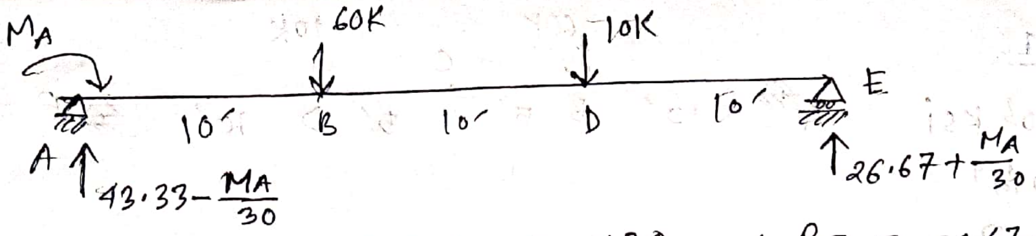
Again,



$\Sigma M_A = 0$, $M_A + 60 \times 10 + 10 \times 10 = R_E \times 30$ $\therefore R_E = 26.67 + \frac{M_A}{30}$

Portion	AB	BD	DE
origin	A	E	E
L	(0-10)	(10-20)	(0-10)
M	$\left(43.33 - \frac{M_A}{30}\right)x + M_A$	$\left(26.67 + \frac{M_A}{30}\right)x - 10(x-10)$	$\left(26.67 + \frac{M_A}{30}\right)x$
$\frac{dM}{M_A}$	$\left(1 - \frac{x}{30}\right)$	$\left(\frac{x}{30}\right)$	$\left(\frac{x}{30}\right)$

$$EI \cdot \theta_C = \int_0^{10} \left[\left(43.33 - \frac{M_A}{30}\right)x + M_A\right] \times \left(1 - \frac{x}{30}\right) dx + \int_{10}^{20} \left[\left(26.67 + \frac{M_A}{30}\right)x - 10(x-10)\right] \times \frac{x}{30}\, dx + \int_0^{10} \left[\left(26.67 + \frac{M_A}{30}\right)x\right] \times \frac{x}{30}\, dx$$

Now $[M_A = 0]$

$$EI \cdot \theta_C = \int_0^{10} 43.33x \times \left(1 - \frac{x}{30}\right) dx + \int_{10}^{20} \left[26.67x - 10(x-10)\right] \frac{x}{30}\, dx + \int_0^{10} 26.67x \times \frac{x}{30}\, dx$$

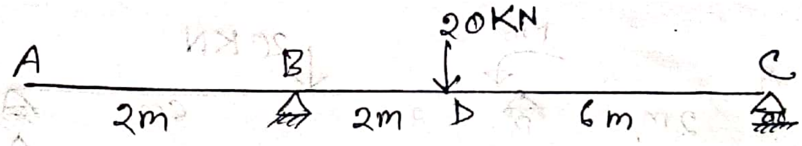
$= 1685.056 + 1796.56 + 296.33$

$EI \cdot \theta_C = 3777.946$

$$\therefore \theta_C = \frac{3777.946 \times 144}{30 \times 10^3 \times 1000} = 0.018 \text{ rad.}$$ (2)

(Ans.)

Problem: 02:

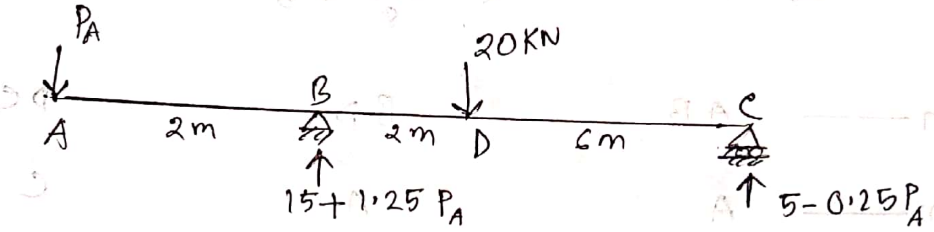


$E = 200 \times 10^6 \, KN/m^2$

$I = 160 \times 10^{-6} \, m^4$

$\Delta_A, \theta_B = ?$

Solution:



$\Sigma M_B = 0$

$-P_A \times 2 + 20 \times 2 = R_C \times 8 \quad \therefore R_C = 5 - 0.25P$

portion –	AB	BD	DC
origin –	A	A	C
Limit –	(0–2)	(2–4)	(0–6)
M	$-P_A x$	$-P_A x + (15 + 1.25P_A) \times (x-2)$	$(5 - 0.25P_A) x$
$\frac{dM}{dP_A}$	$-x$	$-x + 1.25(x-2)$	$-0.25x$

$$EI\,\Delta_A = \int_0^2 (-P_A x)(-x)\,dx + \int_2^4 [-P_A x + (15 + 1.25P_A)] \times (x-2) \times [.25x - 2.5]\,dx + \int_0^6 (5 - 0.25P_A) \times (-0.25x)$$

Now, $(P_A = 0)$

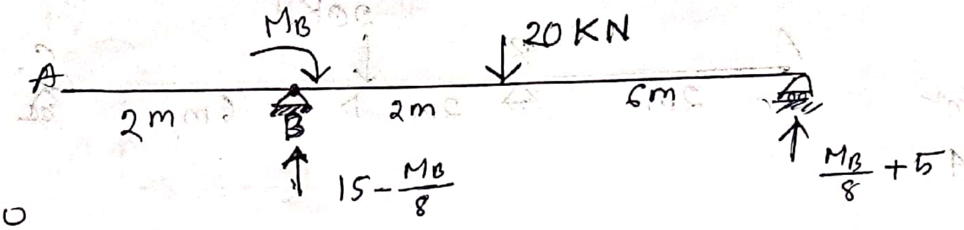
$$EI \cdot \Delta_A = \int_2^4 15 \times (x-2) \times (0.25x - 2.5)\,dx + \int_0^6 5 \times (-0.25x)\,dx$$

$$= 3.75\left[\frac{x^3}{3}\right]_2^4 - 7.5\left[\frac{x^2}{2}\right]_2^4 - 37.5\left[\frac{x^2}{2}\right]_2^4 + 75\left[x\right]_2^4 - 1.25\left[\frac{x^3}{3}\right]_0^6$$

$$= 70 - 45 - 225 + 150 - 90$$

$$EI\,\Delta_A = -140$$

$$\therefore \Delta_A = \frac{-140}{200 \times 10^6 \times 100 \times 10^{-6}} = -7 \times 10^{-3} \, m \quad \therefore \Delta_A = 7 \times 10^{-3} \, m \; (\uparrow)$$



$\Sigma M_B = 0$

$M_B + 20\times 2 = R_c \times 8 \qquad \therefore R_c = \frac{M_B}{8} + 5$

portion	AB	BD	DC
Origin	A	A	C
Limit	(0−2)	(2−4)	(0−6)
M	0	$(15-\frac{M_B}{8})(x-2)+M_B$	$(\frac{M_B}{8}+5)x$
$\frac{dM}{dM_B}$	0	$-\frac{(x-2)}{8}+1$	$\frac{1}{8}\times x$

$$EI\cdot\theta_B = \int_0^2 0\times 0\,dx + \int_2^4 \left[(15-\frac{M_B}{8})\times(x-2)+M_B\right]\times\left[1-\frac{x-2}{8}\right]dx + \int_0^6 \left[(\frac{M_B}{8}+5)x\right]\times\frac{x}{8}\,dx$$

Now, $(M_B = 0)$

$$\therefore EI\cdot\theta_B = \int_2^4 15\,(x-2)\times(1-\frac{x-2}{8})\,dx + \int_0^6 5x\times\frac{x}{8}\,dx$$

$$= \int_2^4 \left\{15x - 30 - \frac{15}{8}\times(x^2-2x) + \frac{30}{8}(x-2)\right\}dx + \int_0^6 \frac{5}{8}x^2\,dx$$

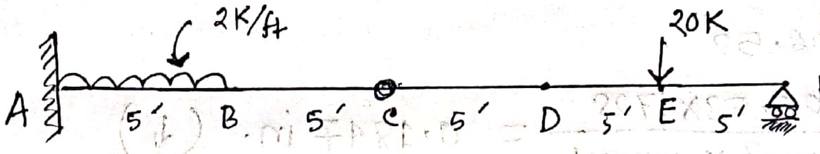
$= 25 + 45$

$\Rightarrow EI\cdot\theta_B = 70$

$$\therefore \theta_B = \frac{70}{200\times10^6\times100\times10^{-6}} = 3.5\times10^{-3}\,\text{rad}\,(\circlearrowright)$$

(Ans.)

Problem: 03

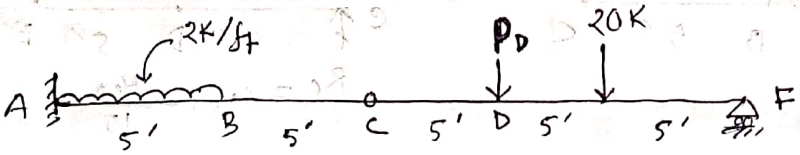


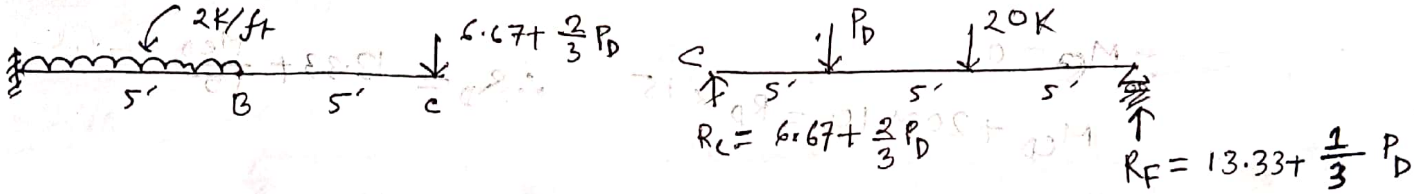
$E = 30\times10^3$ Ksi

$I = 1000\ in^4$

$\Delta_D = ?\quad \theta_{CD} = ?$

Solution:





Portion	AB	BC	CD	DE	EF
Origin	C	C	C	F	F
Limit	(5–10)	(0–5)	(0–5)	(5–10)	(0–5)
M	$-(6.67+\frac{2}{3}P_D)x-\frac{2(x-5)^2}{2}$	$-(6.67+\frac{2}{3}P_D)x$	$(6.67+\frac{2}{3}P_D)x$	$(13.33+\frac{P_D}{3})x-20(x-5)$	$(13.33+\frac{P_D}{3})x$
$\frac{dM}{dP_D}$	$-\frac{2}{3}x$	$-\frac{2}{3}x$	$\frac{2}{3}x$	$\frac{1}{3}x$	$\frac{1}{3}x$

$$EI.\Delta_D = \int_5^{10}\left\{-(6.67+\tfrac{2}{3}P_D)x-(x-5)^2\right\}\times\left(-\tfrac{2}{3}x\right)dx + \int_0^5\left\{-(6.67+\tfrac{2}{3}P_D)x\right\}\times\left(-\tfrac{2}{3}x\right)dx$$

$$+\int_0^5\left\{(6.67+\tfrac{2}{3}P_D)x\right\}\times\left(\tfrac{2}{3}x\right)dx + \int_5^{10}\left\{(13.33+\tfrac{P_D}{3})x\right\}\times\left(\tfrac{1}{3}x\right)dx + \int_0^5\left\{(13.33+\tfrac{P_D}{3})x\times\left(\tfrac{1}{3}x\right)dx\right.$$

Now, $(P_D = 0)$

$$\therefore EI.\Delta_D = \int_5^{10}\left\{-6.67x-(x-5)^2\right\}\times\left(-\tfrac{2}{3}x\right)dx + \int_0^5(-6.67x)\times\left(-\tfrac{2}{3}x\right)dx \;+$$

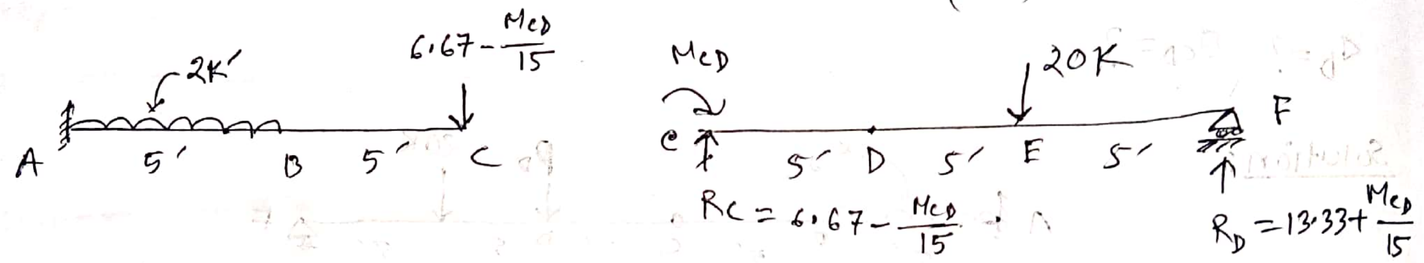
$$\int_0^5(6.67x)\times\left(\tfrac{2}{3}x\right)dx + \int_5^{10}(13.33x)\times\left(\tfrac{1}{3}x\right)dx$$

$$= +1540 + 185.28 + 185.28 + 1295.97$$

$\Rightarrow EI \cdot \Delta_D = 3206.53$

$\therefore \Delta_D = \dfrac{3206.53 \times 1728}{30 \times 10^3 \times 1000 \times 1} = 0.1847 \text{ in.} \ (\downarrow)$

(Ans.)



$\Sigma M_C = 0$

$M_{CD} + 20 \times 10 = R_D \times 15 \qquad \therefore R_D = 13.33 + \dfrac{M_{CD}}{15}$

portion	AB	BC	CE	EF
origin	C	C	C	F
Limit	(5−10)	(0−5)	(0−10)	(0−5)
M	$-\left(6.67 - \frac{M_{CD}}{15}\right)x - (x-5)^2$	$-\left(6.67 - \frac{M_{CD}}{15}\right)x$	$M_{CD} + \left(6.67 - \frac{M_{CD}}{15}\right)x$	$\left(13.33 + \frac{M_{CD}}{15}\right)x$
$\frac{dM}{M_{CD}}$	$+\frac{x}{15}$	$+\frac{x}{15}$	$1 - \frac{x}{15}$	$\frac{x}{15}$

$$EI\theta_{CD} = \int_5^{10} \left\{-\left(6.67 - \frac{M_{CD}}{15}\right)x - (x-5)^2\right\} \times \left(\frac{x}{15}\right) dx + \int_0^5 \left\{-\left(6.67 - \frac{M_{CD}}{15}\right)x\right\} \times \left(\frac{x}{15}\right) dx$$

$$+ \int_0^{10} \left\{M_{CD} + \left(6.67 - \frac{M_{CD}}{15}\right)x\right\} \times \left(1 - \frac{x}{15}\right) dx + \int_0^5 \left\{\left(13.33 + \frac{M_{CD}}{15}\right)x\right\} \left(\frac{x}{15}\right) dx$$

Now, ($M_{CD} = 0$)

$$\therefore EI \cdot \theta_{CD} = \int_5^{10} \left\{-6.67x - (x-5)^2\right\} \times \left(\frac{x}{15}\right) dx + \int_0^5 (-6.67x)\left(\frac{x}{15}\right) dx +$$

$$\int_0^{10} (6.67x) \times \left(1 - \frac{x}{15}\right) dx + \int_0^5 (13.33x) \times \left(\frac{x}{15}\right) dx$$

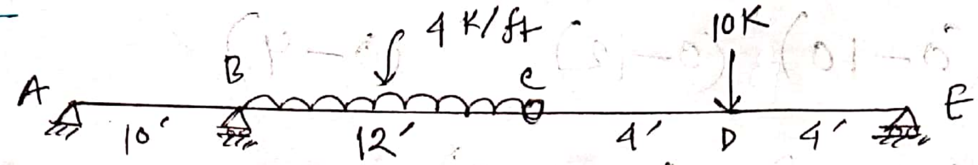
$= -154 - 18.53 + 185.28 + 37.03$

$\Rightarrow EI\,\theta_{CD} = 49.78$

$\Rightarrow \theta_{CD} = \dfrac{49.78 \times 144}{30\times10^3 \times 1000} = 2.39\times10^{-4}\ \text{rad.}\ (\circlearrowright)$ (Ans.)

Class Test

#

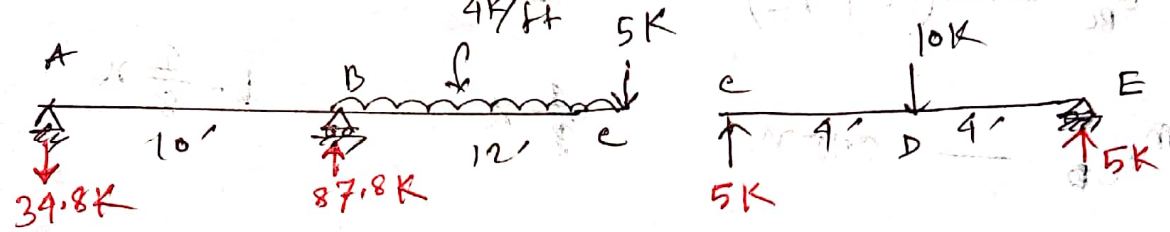


$E = 30\times10^3$ ksi

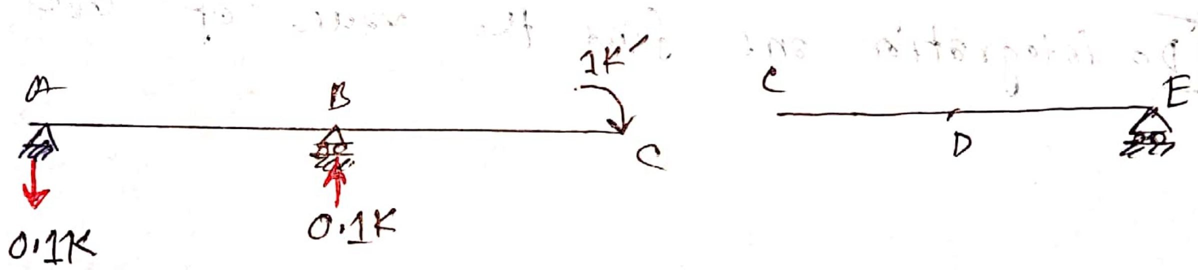
$I = 1000$ in^4

Find rotation θ_{CB} and θ_{CD} by unit load Method.

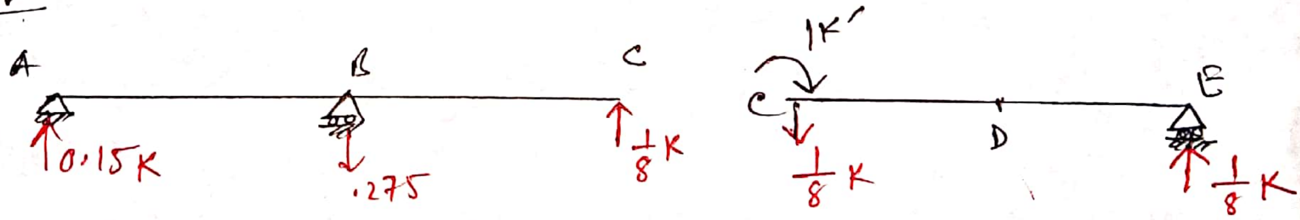
Solution:



For θ_{CB}:



For θ_{CD}:



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portion —	AB	BC	CD	DE
origin —	A	C	C	E
Limit —	(0–10)	(0–12)	(0–4)	(0–4)
M —	$(-34.8x)$	$\left(-5x - 4 \cdot x \cdot \frac{x}{2}\right)$	$5x$	$5x$
$m_{\theta_{BC}}$ —	$(-0.1x)$	-1	0	0
$m_{\theta_{CD}}$ —	$0.15x$	$\frac{1}{8}x$	$1 - \frac{1}{8}x$	$\frac{1}{8}x$

[Do integration and find the value of θ_{BC} & θ_{CD}.]

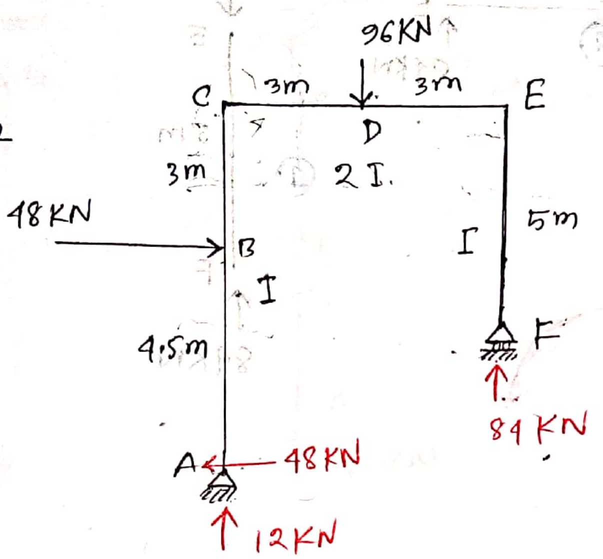
FRAME

Problem: 01

$E = 200 \times 10^6 \ KNm^{-2}$

$I = 160 \times 10^{-6} m^4$

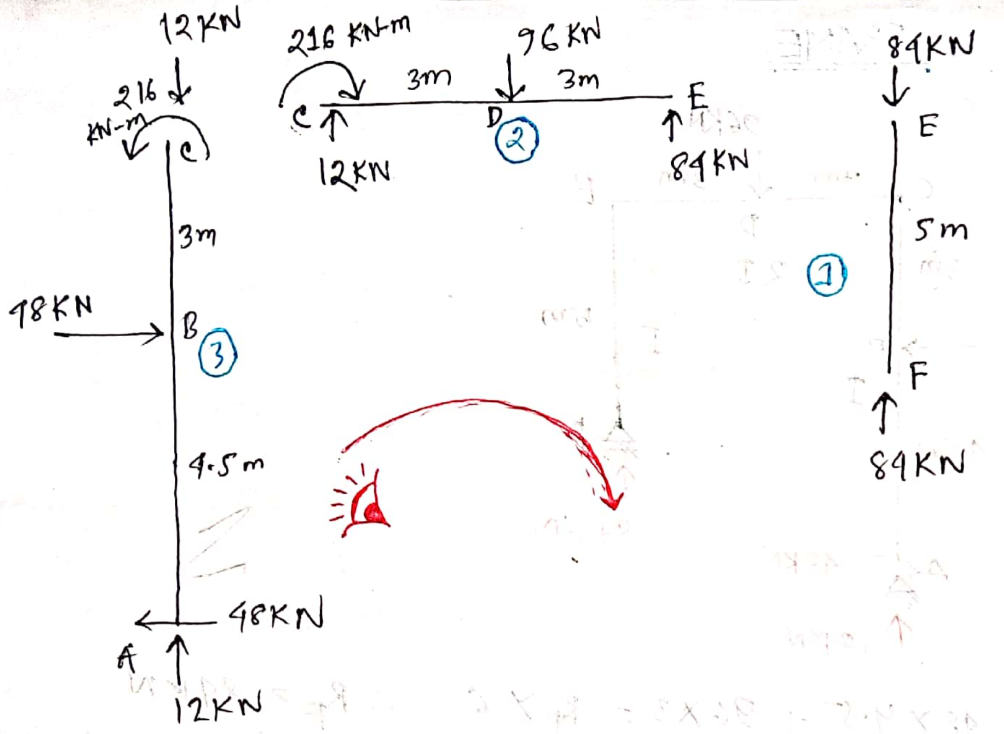
$\theta_A, \theta_C, \theta_E, \theta_F = ?$



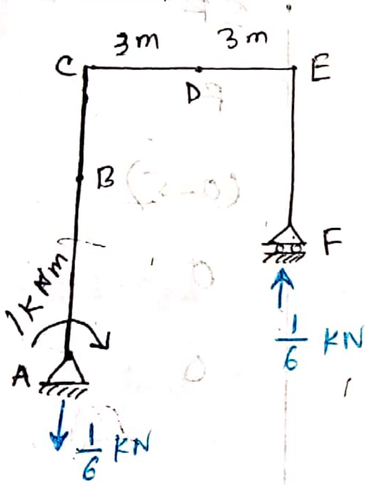
Solution: $\Sigma M_A = 0$, $48 \times 4.5 + 96 \times 3 = R_F \times 6$ $\therefore R_F = 84 KN$

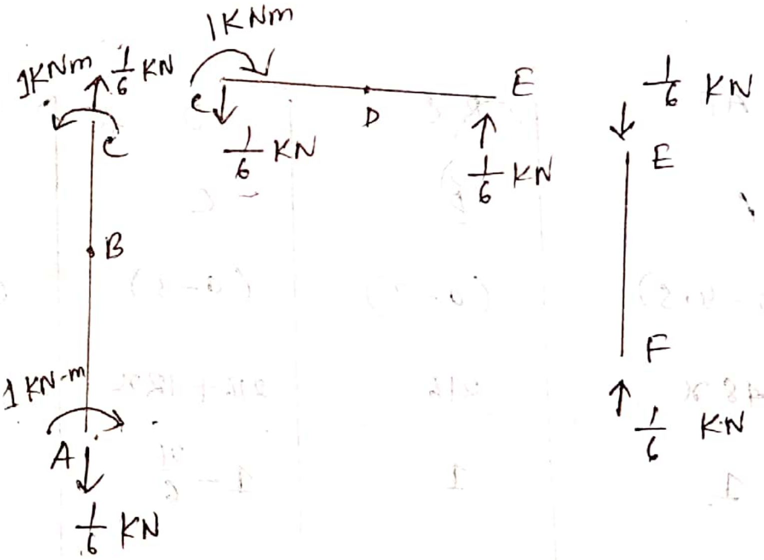
$\therefore R_A = 12 KN$

Portion —	AB	BC	CD	DE	EF
Origin —	A	B	C	E	F
Limit —	(0–4.5)	(0–3)	(0–3)	(0–3)	(0–5)
M —	$48x$	216	$216 + 12x$	$84x$	0
m_{θ_A} —	1	1	$1 - \frac{x}{6}$	$\frac{x}{6}$	0
m_{θ_C} —	0	0	$1 - \frac{x}{6}$	$\frac{x}{6}$	0
m_{θ_E} —	0	0	$-\frac{x}{6}$	$\frac{x}{6} - 1$	0
m_{θ_F} —	0	0	$-\frac{x}{6}$	$\frac{x}{6} - 1$	-1
I —	I	I	2I	2I	I

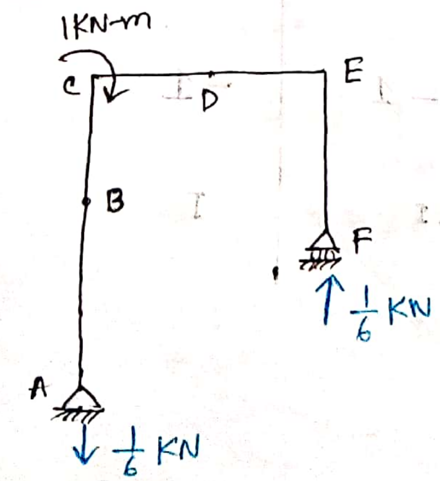


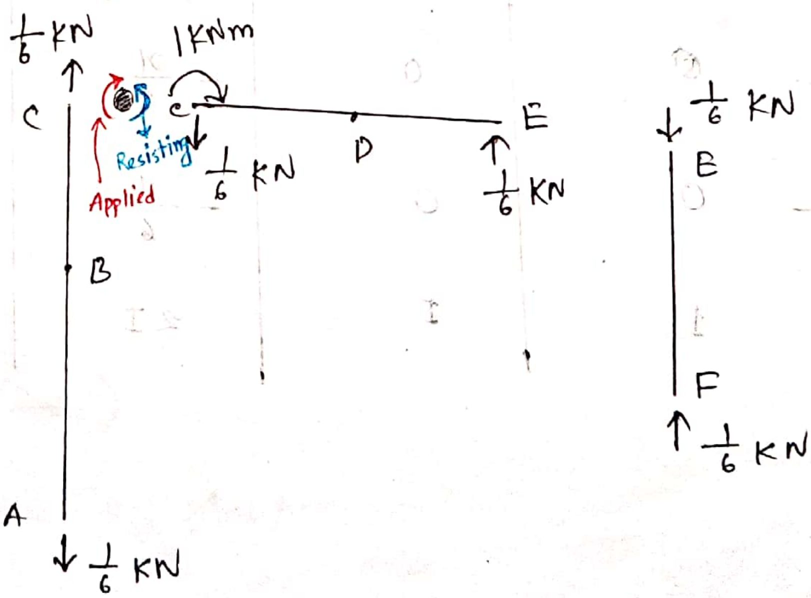
For θ_A:



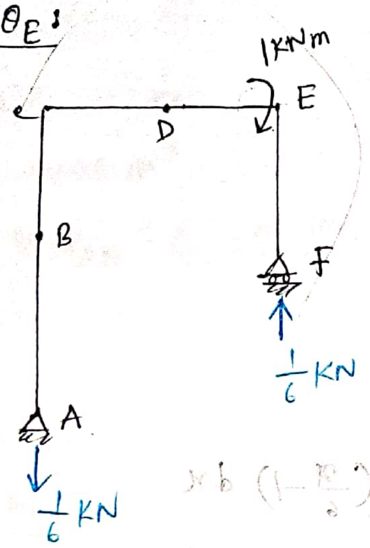


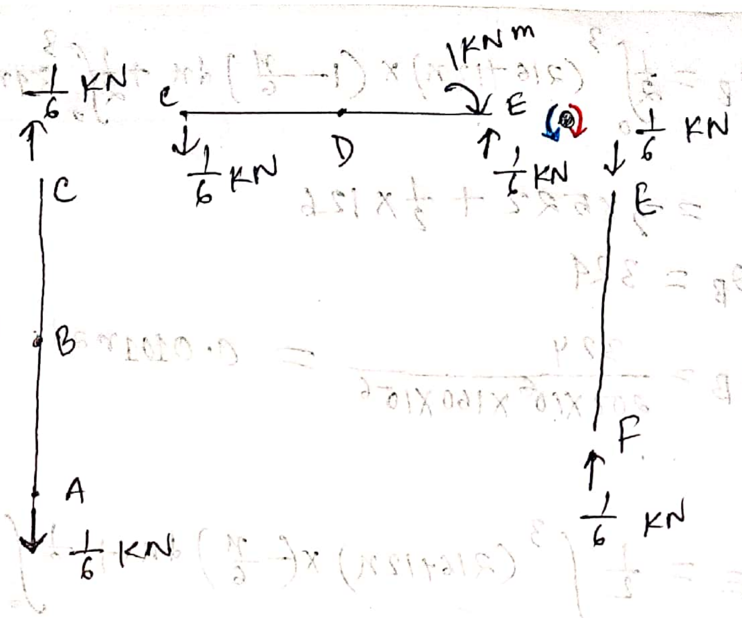
For θ_C:



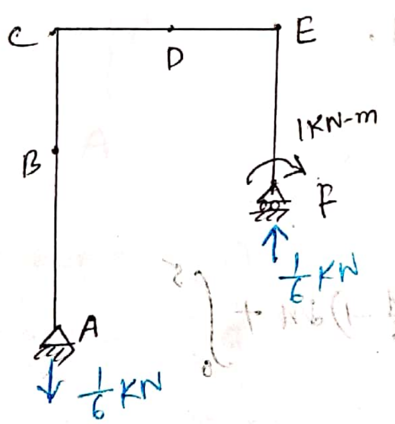


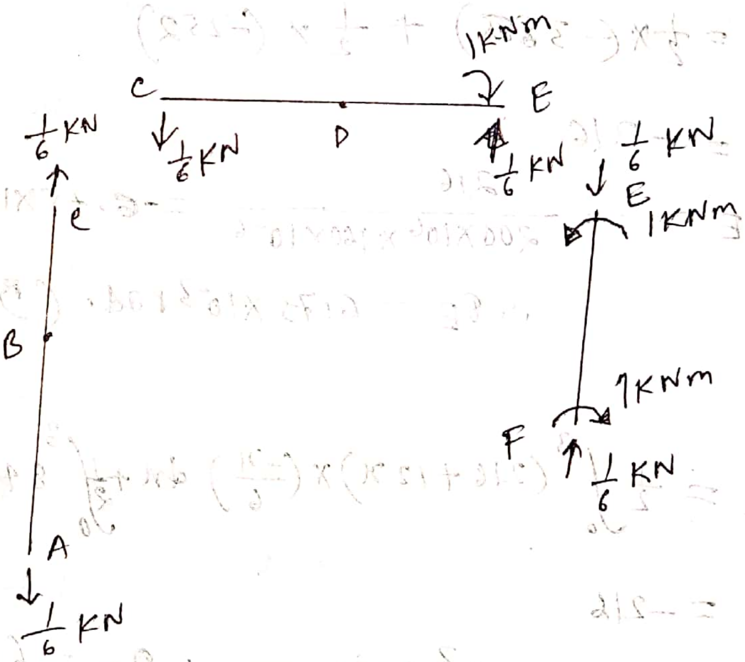
For θ_E :





For θ_F :





$$EI\cdot\theta_A = \int_0^{4.5} 48x\,dx + \int_0^3 (216\times 1)\,dx + \frac{1}{3}\int_0^3 (216+12x)\times\left(1-\frac{x}{6}\right)dx + \frac{1}{3}\int_0^3 84x\times\frac{x}{6}\,dx$$

$$= 486 + 648 + \frac{1}{2}\times 522 + \frac{1}{2}\times 126$$

$$EI\cdot\theta_A = 1458$$

$$\theta_A = \frac{1458}{200\times 10^6\times 160\times 10^{-6}} = 0.0456 \text{ rad.} \quad (\circlearrowright)$$

$$EI \cdot \theta_B = \frac{1}{2}\int_0^3 (216+12x) \times \left(1-\frac{x}{6}\right) dx + \frac{1}{2}\int_0^3 84x \times \frac{x}{6}\, dx$$

$$= \frac{1}{2} \times 522 + \frac{1}{2} \times 126$$

$$\Rightarrow EI \cdot \theta_B = 324$$

$$\therefore \theta_B = \frac{324}{200 \times 10^6 \times 160 \times 10^{-6}} = 0.0101 \text{ rad. } (\curvearrowleft)$$

Now,

$$EI \cdot \theta_E = \frac{1}{2}\int_0^3 (216+12x) \times \left(-\frac{x}{6}\right) dx + \frac{1}{2}\int_0^3 84x \times \left(\frac{x}{6}-1\right) dx$$

$$= \frac{1}{2} \times (-180) + \frac{1}{2} \times (-252)$$

$$EI \cdot \theta_E = -216$$

$$\therefore \theta_E = -\frac{216}{200 \times 10^6 \times 160 \times 10^{-6}} = -6.75 \times 10^{-3} \text{ rad.}$$

$$\therefore \theta_E = 6.75 \times 10^{-3} \text{ rad. } (\circlearrowright)$$

And,

$$EI \cdot \theta_F = \frac{1}{2}\int_0^3 (216+12x) \times \left(\frac{-x}{6}\right) dx + \frac{1}{2}\int_0^3 84x \times \left(\frac{x}{6}-1\right) dx$$

$$EI \cdot \theta_F = -216$$

$$\therefore \theta_F = -6.75 \times 10^{-3} \text{ rad.} \qquad \therefore \theta_F = 6.75 \times 10^{-3} \text{ rad. } (\circlearrowright)$$

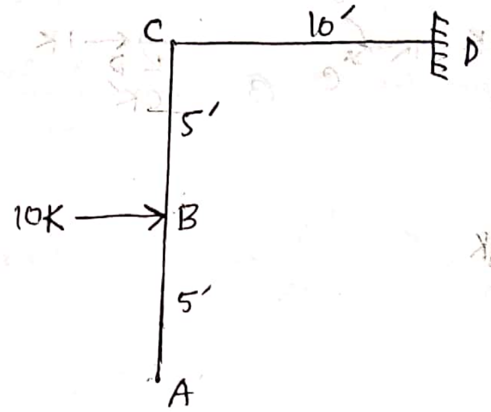
(Ans.)

Problem: 02

$E = 30\times10^3$ ksi

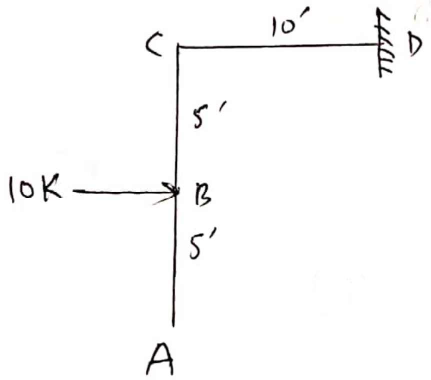
$I = 1000$ in⁴

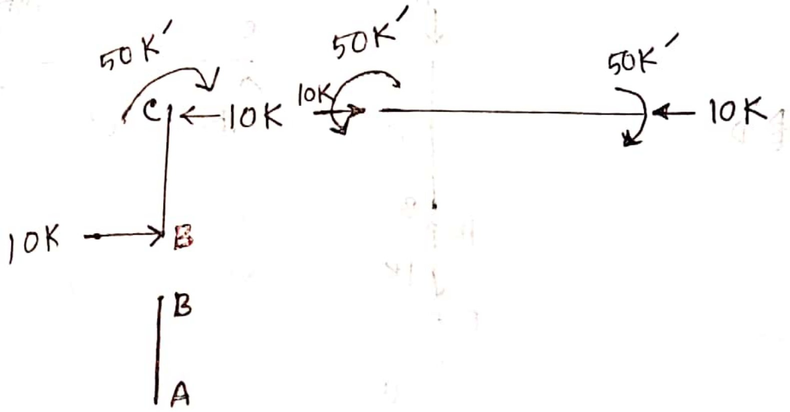
θ_A & Δ_{AH}, Δ_{AV} = ?



Solution:

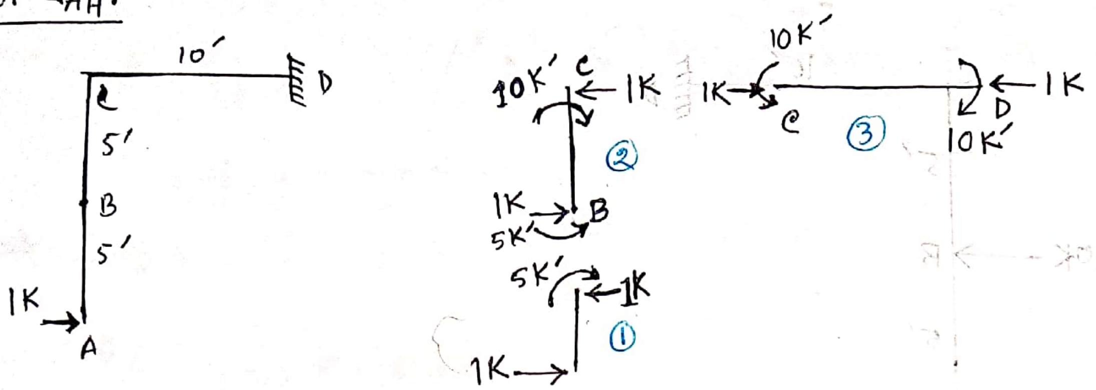
For M:



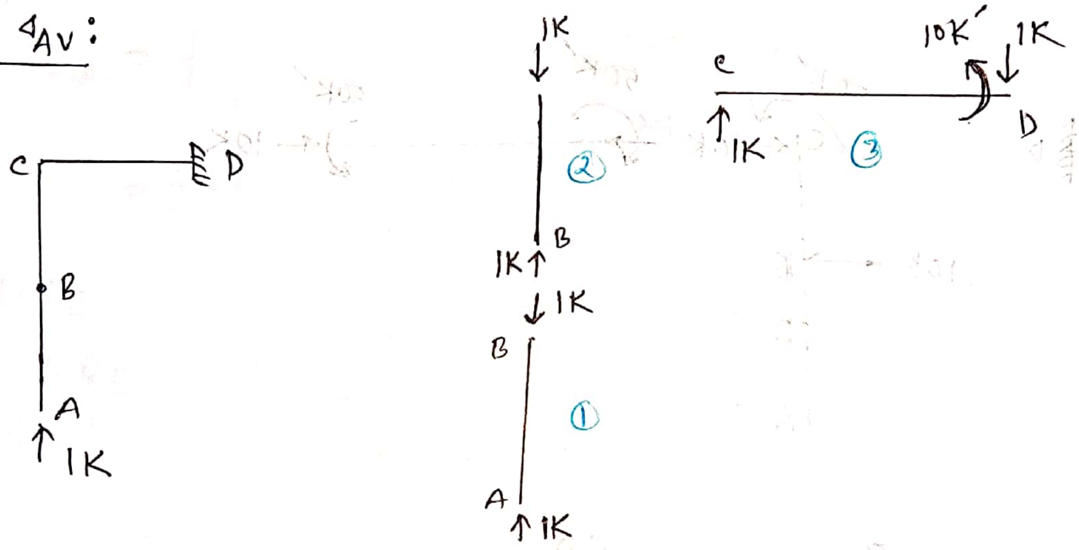


Portion	AB	BC	CD
origin	A	B	C
Limit	(0-5)	(0-5)	(0-10)
M	0	$-10x$	-50
$m_{\Delta AH}$	$-x$	$-5-x$	-10
$m_{\Delta AV}$	0	0	x
$m_{\theta A}$	1	1	1

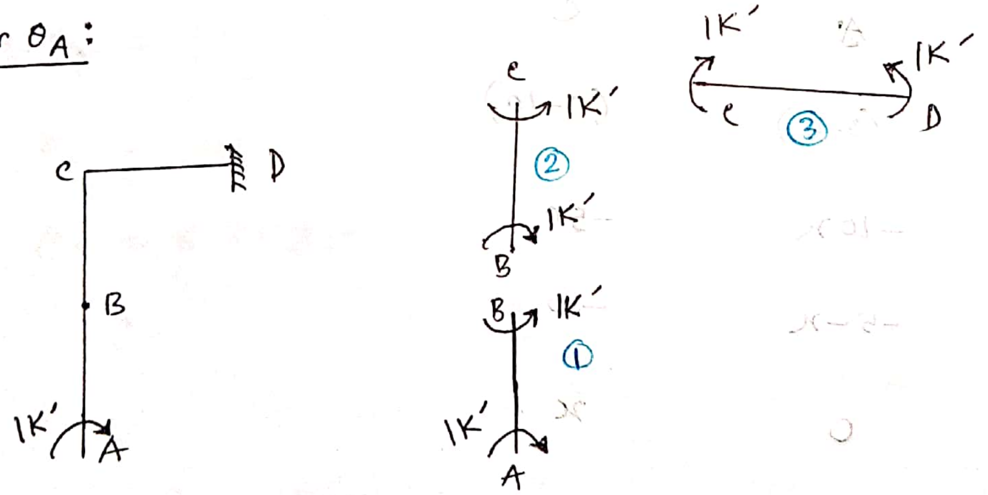
For Δ_{AH}:



For Δ_{AV}:



For θ_A:



Now,

$$EI \cdot \Delta_{AV} = \int_0^5 (-10x) \times (-x-5)\,dx + \int_0^{10} (-50) \times (-10)\,dx$$

$$= 1041.67 + 5000$$

$$\Delta_{AV} = \frac{6041.67 \times 1728}{30 \times 10^3 \times 1000 \times 1} = 0.35 \text{ in } (\rightarrow)$$

$$EI \cdot \Delta_{AV} = \int_0^{10} (-50) \times (x)\, dx = -2500$$

$$\therefore \Delta_{AV} = \frac{2500 \times 1728}{30 \times 10^3 \times 1000} = 0.144 \text{ inch } (\downarrow)$$

And,

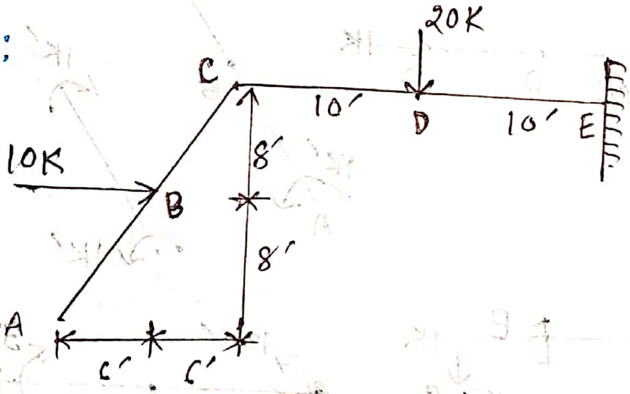
$$EI \cdot \theta_A = \int_0^{5} (-10x) \cdot 1 \cdot dx + \int_0^{10} (-50) \times 1 \cdot dx$$

$$= -125 - 500$$

$$\therefore \theta_A = \frac{-625 \times 144}{30 \times 10^3 \times 1000} = -3 \times 10^{-3} \text{ rad.}$$

$$\therefore \theta_A = 3 \times 10^{-3} \text{ rad } (\circlearrowright)$$

Problem : 03 :



$E = 30 \times 10^3$ ksi

$I = 1000$ in^4

$\Delta_{AH} = ?$

$\Delta_{AV} = ?$

$\theta_A = ?$

Portion	AB	BC	CD	DE
origin	A	B	C	D
Limit	(0-10)	(0-10)	(0-10)	(0-10)
M	0	$-8x$	-80	$-80-20x$
Δ_{AH}	$-0.8x$	$-8-0.8x$	-16	-16
Δ_{AV}	$0.6x$	$6+0.6x$	$12+x$	$22+x$
θ_A	1	1	1	1

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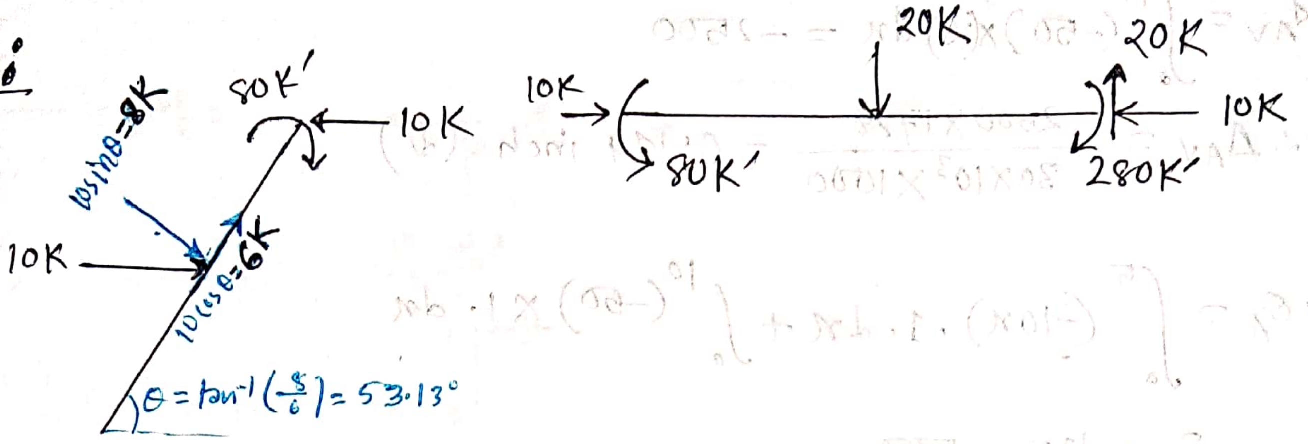
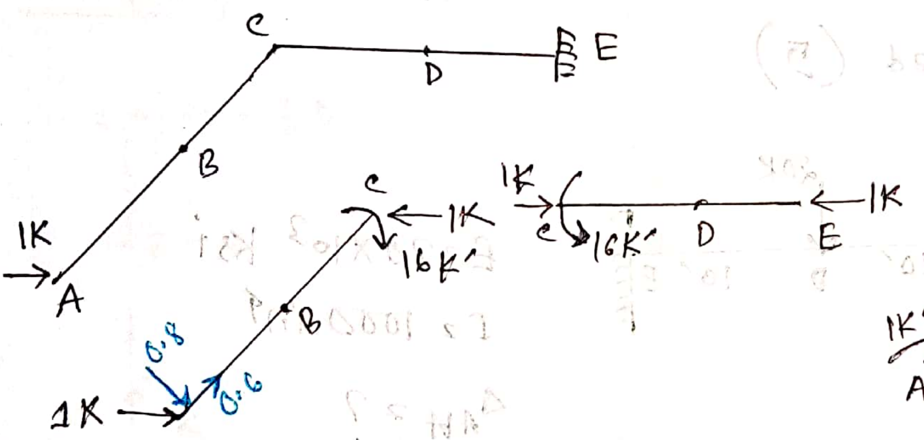
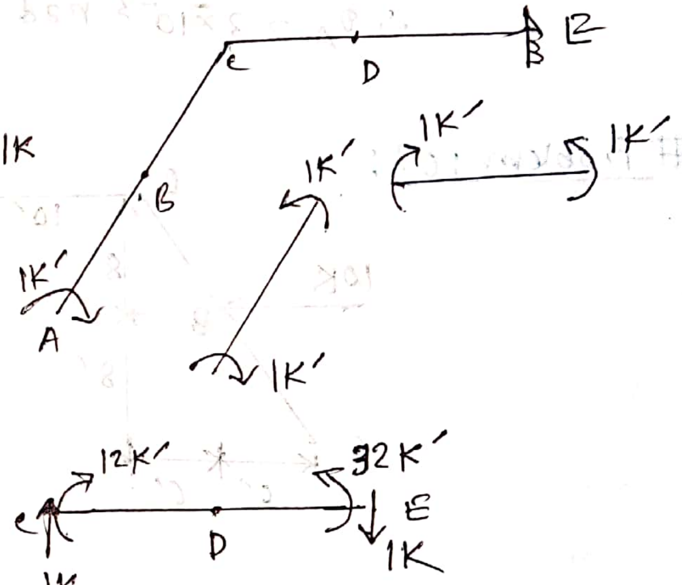
For M_0:



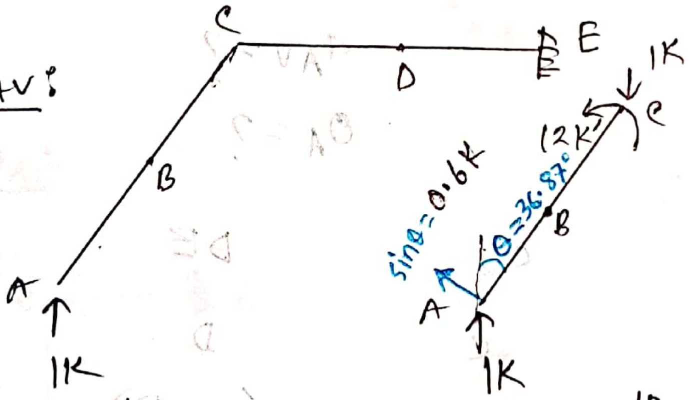
For Δ_{AH}:



For θ_A:



For Δ_{AV}:



$$EI \cdot \Delta_{AH} = \int_0^{10} (-8x) \times (-8 - 0.8x)\, dx + \int_0^{10} (-80) \times (-16)\, dx + \int_0^{10} (-80 - 20x) \times (-16)\, dx$$

$$= 5333.33 + 12800 + 28800$$

$$EI \cdot \Delta_{AH} = 46933.33 \qquad \therefore \Delta_{AH} = \frac{46933.33 \times 1728}{30 \times 10^3 \times 1000} = 2.7 \text{ inch.} (\rightarrow)$$

$$EI \cdot \Delta_{AV} = \int_0^{10} (-8x) \times (6+0.6x)\,dx + \int_0^{10} (-80) \times (12+x)\,dx + \int_0^{10} (-80-20x) \times (22+x)\,dx$$

$$\Rightarrow EI \cdot \Delta_{AV} = -4000 + (-13600) + (-50266.67)$$

$$\Rightarrow EI \cdot \Delta_{AV} = -67866.67$$

$$\therefore \Delta_{AV} = \frac{67866.67 \times 1728}{30 \times 10^3 \times 1000} = 3.91 \text{ inch } (\downarrow)$$

$$EI \cdot \theta_A = \int_0^{10} (-8x) \times (1)\,dx + \int_0^{10} (-80) \times (1)\,dx + \int_0^{10} (-80-20x) \times (1)\,dx$$

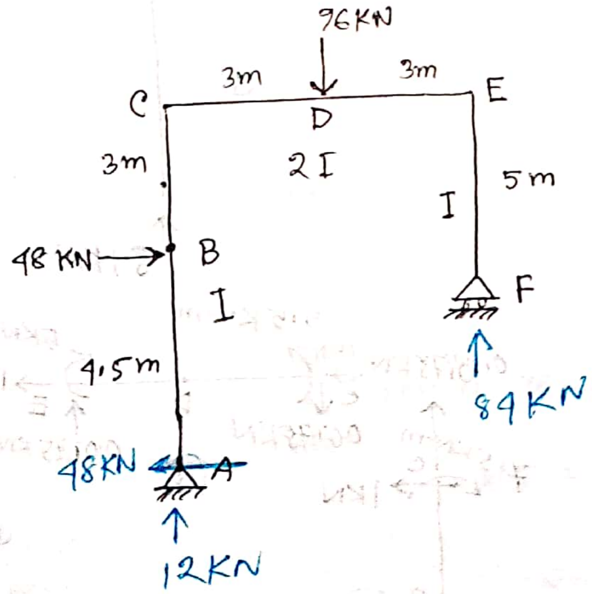
$$= -400 - 800 - 1800$$

$$EI \cdot \theta_A = -3000$$

$$\therefore \theta_A = \frac{3000 \times 144}{30 \times 10^3 \times 1000} = 0.0144 \; (\circlearrowright)$$

(Ans.)

Problem: 04:



$E = 200 \times 10^6 \; KN/m^2$

$I = 160 \times 10^{-6} \; m^4$

$\Delta_{HF}, \Delta_{HD}, \Delta_{HC}, \Delta_{HA} = ?$

Solution:

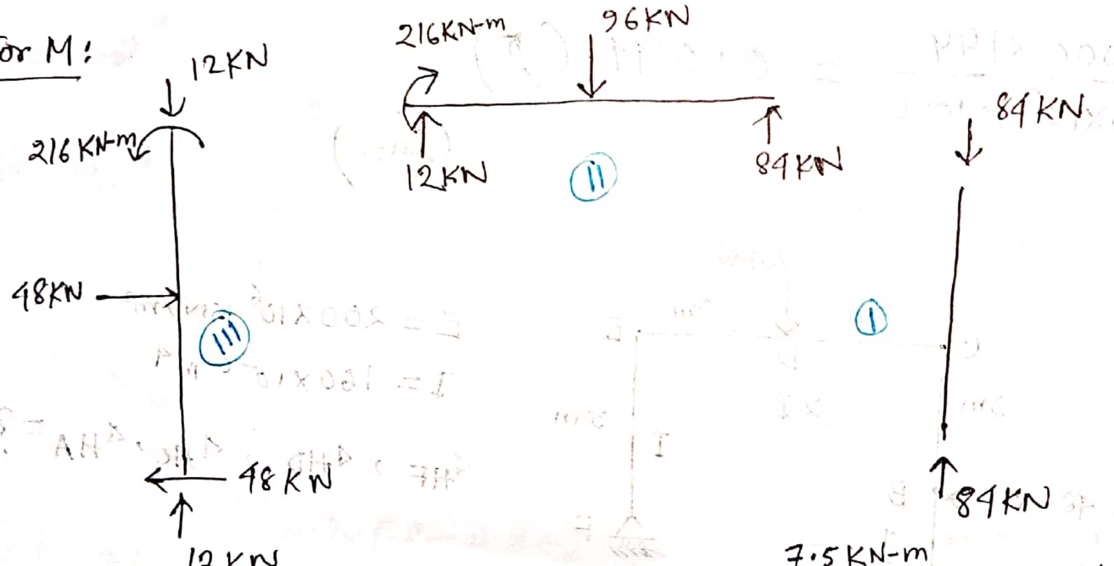
$\Sigma M_A = 0$

$48 \times 4.5 + 96 \times 3 = R_F \times 6 \quad \therefore R_F = 84 KN$

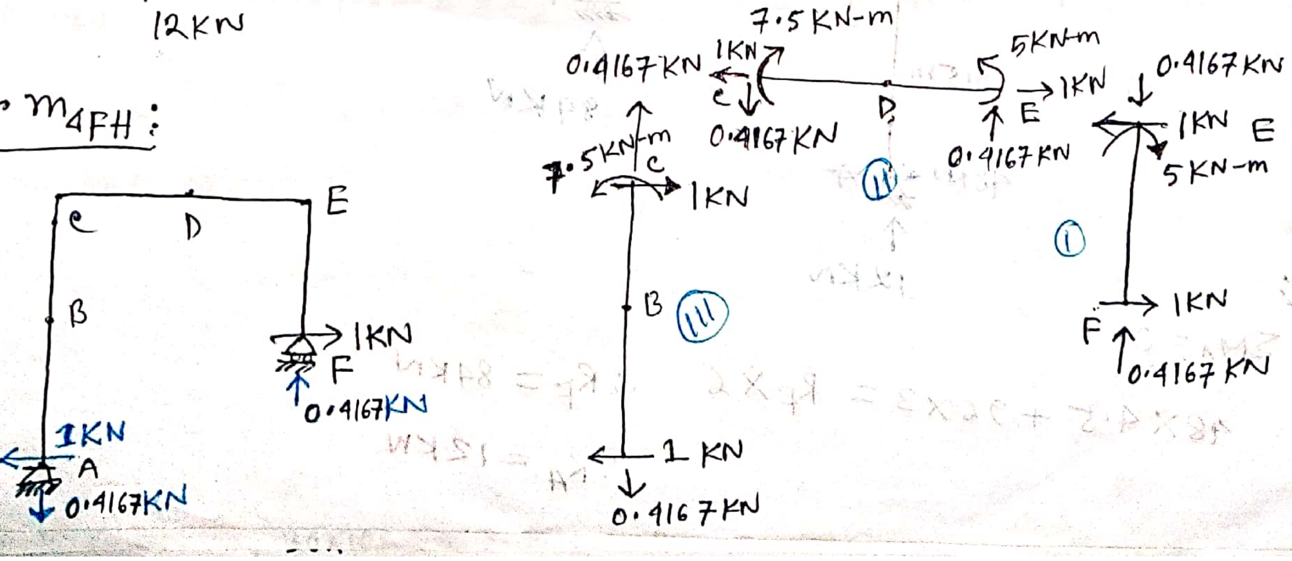
$\therefore R_A = 12 KN$

portion	AB	BC	CD	DE	EF
origin	A	B	C	E	F
Limit	0–4.5	0–3	0–3	0–3	0–5
M	$48x$	$(48\times4.5)=216$	$216+12x$	$84x$	0
$m_{\Delta FH}$	x	$4.5+x$	$7.5-0.4167x$	$5+0.4167x$	x
$m_{\Delta EH}$	x	$4.5+x$	$7.5-1.25x$	$1.25x$	0
$m_{\Delta CH}$	x	$4.5+x$	$7.5-1.25x$	$1.25x$	0
$m_{\Delta AH}$	–	–	–	–	–

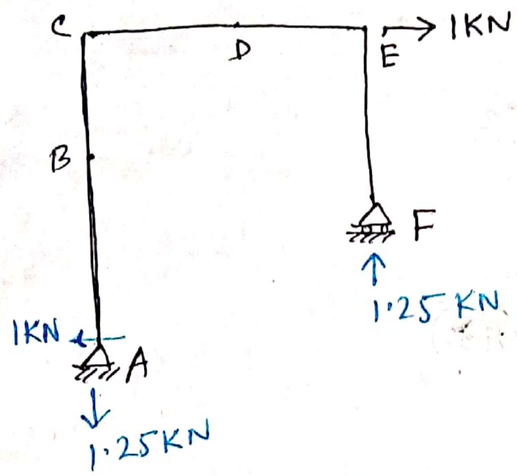
For M:

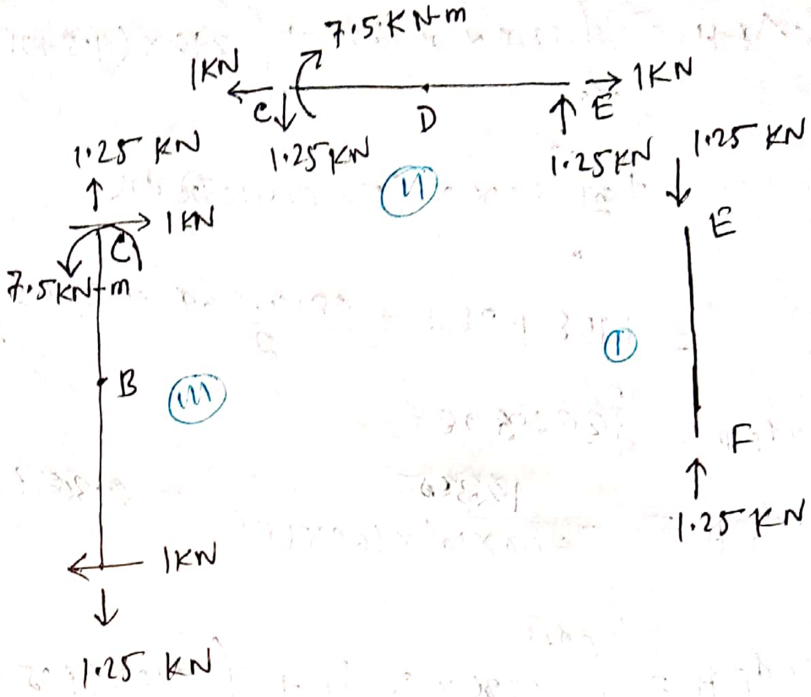


For $m_{\Delta FH}$:

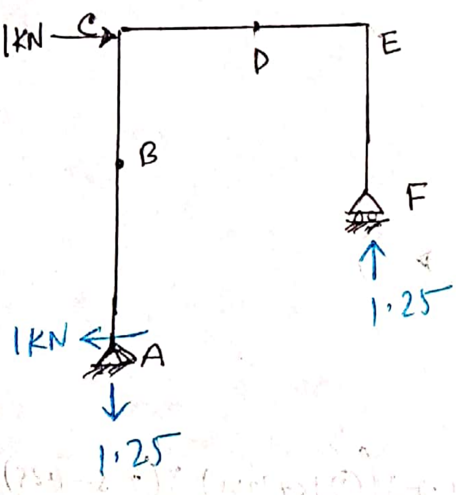


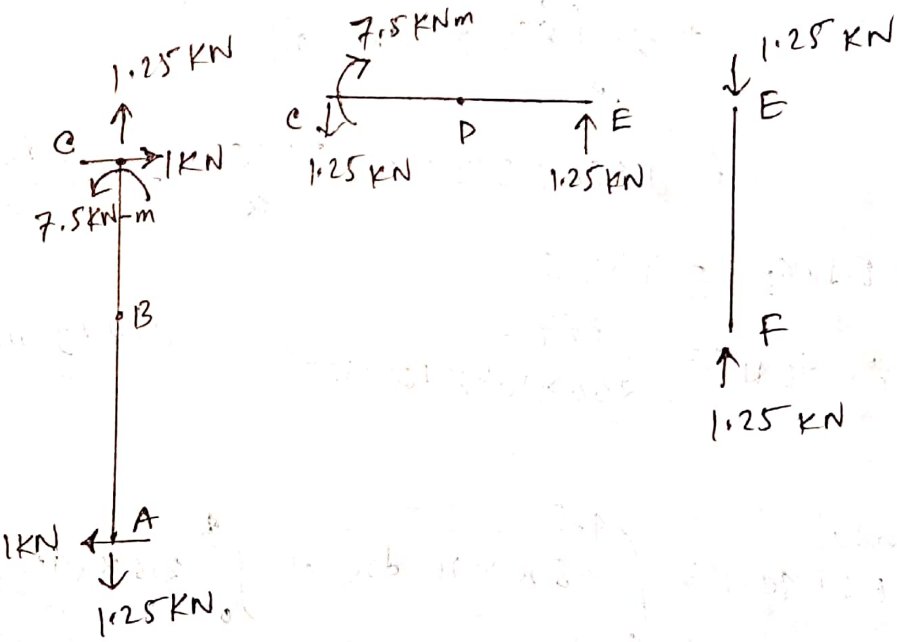
For $m_{\Delta EH}$:



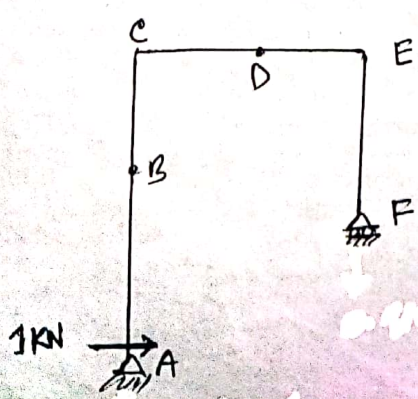


For $m_{\Delta CH}$:





For $m_{\Delta AH}$:



For hinge,

$\Delta_{AH} = 0$

$$EI \cdot \Delta_{FH} = \int_0^{4.5} 48x \times x \, dx + \int_0^3 216 \times (4.5 + x)\, dx + \frac{1}{2}\int_0^3 (216 + 12x) \times (7.5 - 0.4167x)\, dx$$

$$+ \frac{1}{2}\int_0^3 84x \times (5 + 0.4167x)\, dx.$$

$$= 1458 + 3888 + \frac{4814.964}{2} + \frac{2205.0252}{2}$$

$$\Rightarrow EI \cdot \Delta_{FH} = 8856$$

$$\therefore \Delta_{FH} = \frac{8856}{200 \times 10^6 \times 160 \times 10^{-6}} = 0.277 \text{ m } (\rightarrow)$$

Now,

$$EI \cdot \Delta_{EH} = \int_0^{4.5} 48x \times x \, dx + \int_0^3 216 \times (4.5 + x)\, dx + \frac{1}{2}\int_0^3 (216 + 12x)(7.5 - 1.25x)\, dx$$

$$+ \frac{1}{2}\int_0^3 84x \times (1.25x)$$

$$= 1458 + 3888 + \frac{3915}{2} + \frac{945}{2}$$

$$\Rightarrow EI \cdot \Delta_{EH} = 7776$$

$$\therefore \Delta_{EH} = \frac{7776}{200 \times 10^6 \times 160 \times 10^{-6}} = 0.243 \text{ m } (\rightarrow)$$

And,

$$EI \cdot \Delta_{CH} = \int_0^{4.5} 48x \times x \, dx + \int_0^3 216 \times (4.5 + x)\, dx + \frac{1}{2}\int_0^3 (216 + 12x) \times (7.5 - 1.25x)\, dx$$

$$+ \frac{1}{2}\int_0^3 (84x) \times (1.25x)\, dx$$

$$= 7776$$

$$\therefore \Delta_{CH} = 0.243 \text{ m } (\rightarrow)$$

(Ans.)

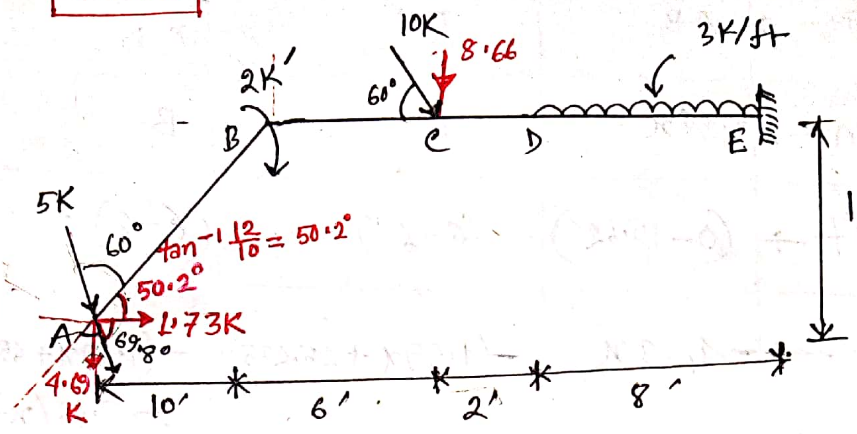
Frame

2017 #

$E = 30 \times 10^3$ ksi

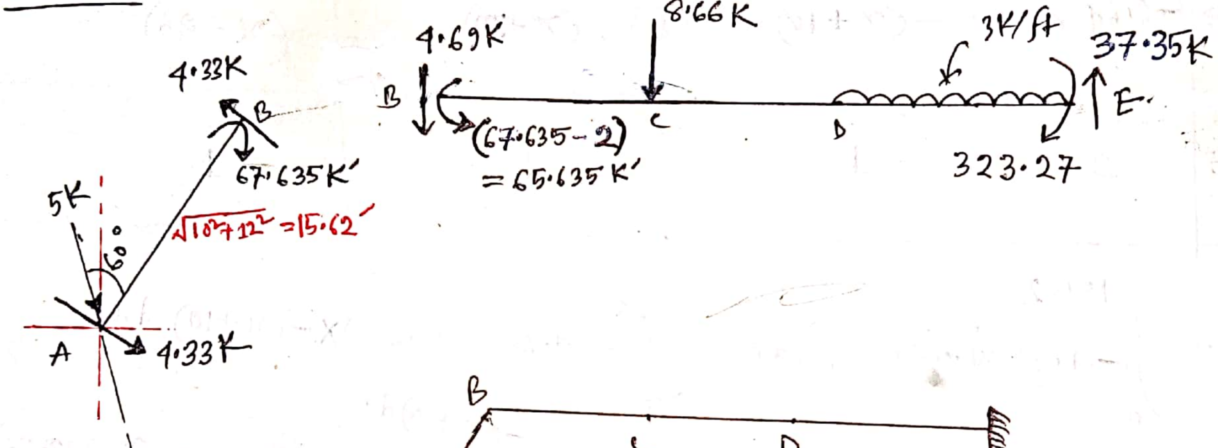
$I = 200$ in^4

Δ_{AV}, θ_B = ?

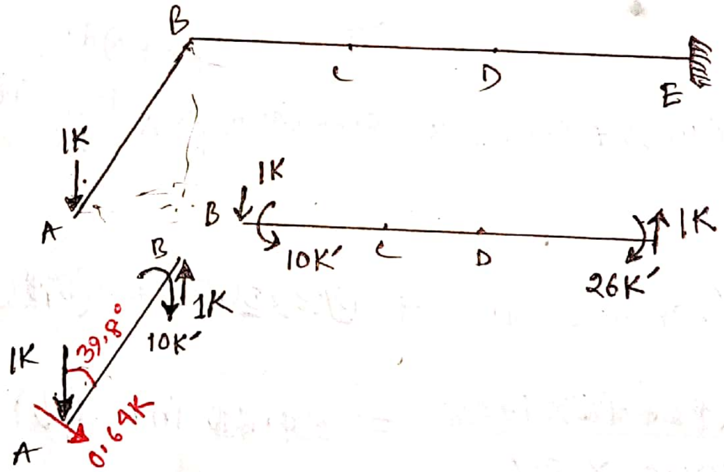


Solution:

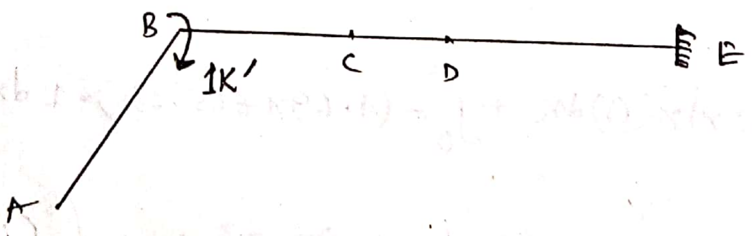
For M:



For Δ_{AV}:



For θ_B:



Portion →	AB	BC	CD	DE
Origin →	A	B	B	E
Limit →	(0 – 15.62)	(0 – 6)	(6 – 8)	(0 – 8)
M →	$(-4.33x)$	$-(4.69x+65.635)$	$-(4.69x+65.635) - 8.66(x-6)$	$37.35x - 3\cdot x\cdot\frac{x}{2} - 323.27$
$m_{\Delta AV}$ →	$(-0.64x)$	$-(x+10)$	$-(x+10)$	$(x-26)$
$m_{\theta B}$ →	0	1	1	−1

$$EI\cdot\Delta_{AV} = \int_0^{15.62}(-4.33x)\times(-0.64x)\,dx + \int_0^6 -(4.69x+65.635)\cdot -(x+10)\,dx$$
$$+ \int_6^8 \{-(4.69x+65.635) - 8.66(x-6)\}\times -(x+10)\,dx + \int_0^8 (37.35x - 1.5x^2 - 323.27)\times(x-26)\,dx$$

$$EI\cdot\Delta_{AV} = 3520.38 + 6301.41 + 3651.15 + 37314.72$$

$$\therefore \Delta_{AV} = \frac{50787.66\times 1728}{30\times10^3\times 200} = 14.63 \text{ in.} \ (\downarrow)$$

$$EI\cdot\theta_B = \int_0^{15.62}(-4.33x)\times(0)\,dx + \int_0^6 -(4.69x+65.635)\cdot 1\,dx +$$
$$\int_6^8 \{-(4.69x+65.635) - 8.66(x-6)\}\cdot 1\,dx + \int_0^8 (37.35x - 1.5x^2 - 323.27)\times 1\,dx$$

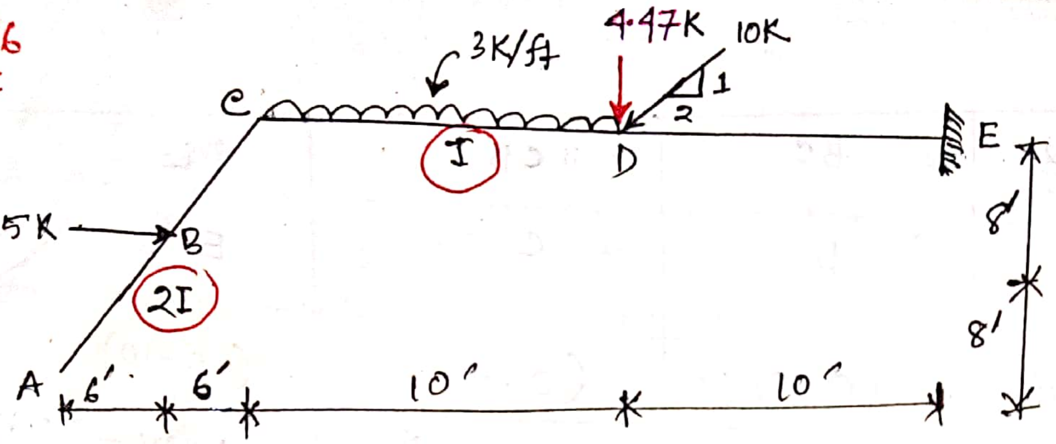
$$= (-478.23 - 213.68 - 1646.96) = -2338.87$$

$$\theta_B = \frac{-2338.87\times 144}{30\times10^3\times 200} = -0.056 \text{ rad} \qquad \therefore \theta_B = 0.056 \text{ rad } (\circlearrowright)$$

(Ans.)

2016

#



$E = 30 \times 10^3$ ksi

$I = 200$ in^4

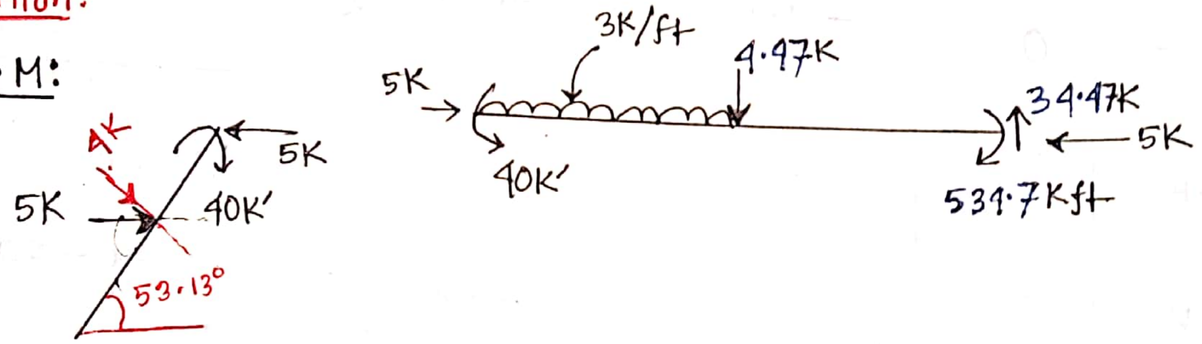
$\Delta_{CH} = ?$

$\Delta_{CV} = ?$

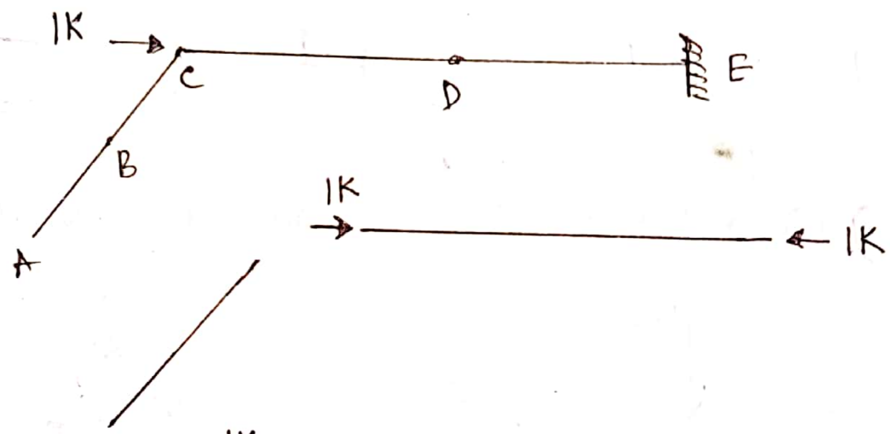
$\theta_A = ?$

Solution:

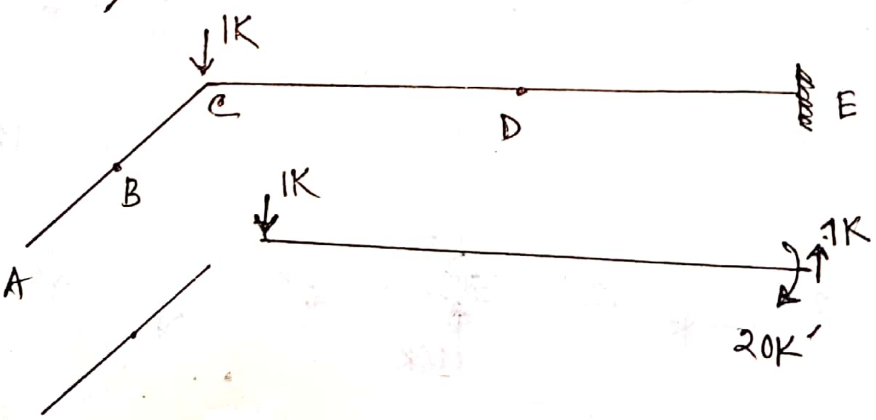
For M:



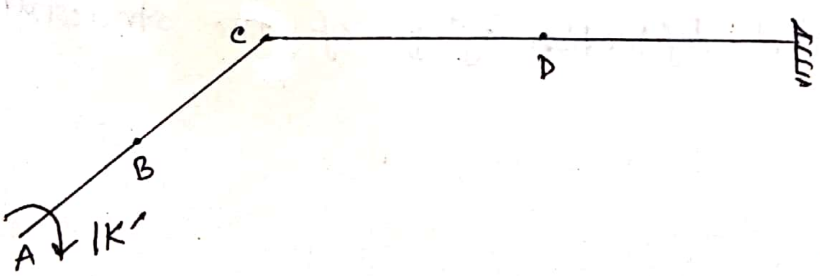
For Δ_{CH}:



For Δ_{CV}:



For θ_A:



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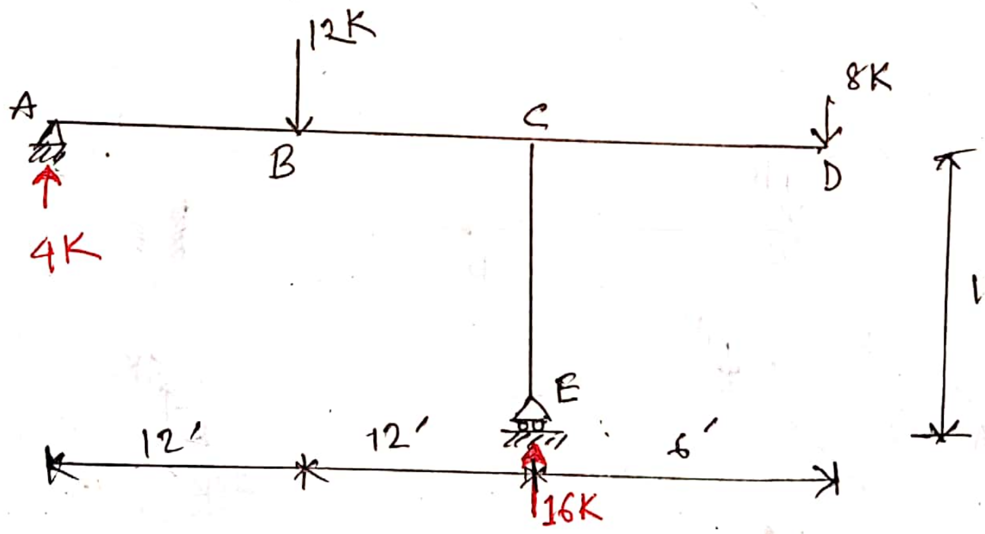
Portion	AB	BC	CD	DE
origin	A	B	C	E
Limit	(0–10)	(0–10)	(0–10)	(0–10)
M	0	$-4x$	$-40-3\cdot x\cdot\frac{x}{2}$	$34.47x-534.7$
m_{dcH}	0	0	0	0
m_{dcV}	0	0	$-x$	$x-20$
$m_{\theta A}$	+1	+1	+1	−1

(Do integration) ☺ * Notice: Moment of Inertia is different for each member.

2015

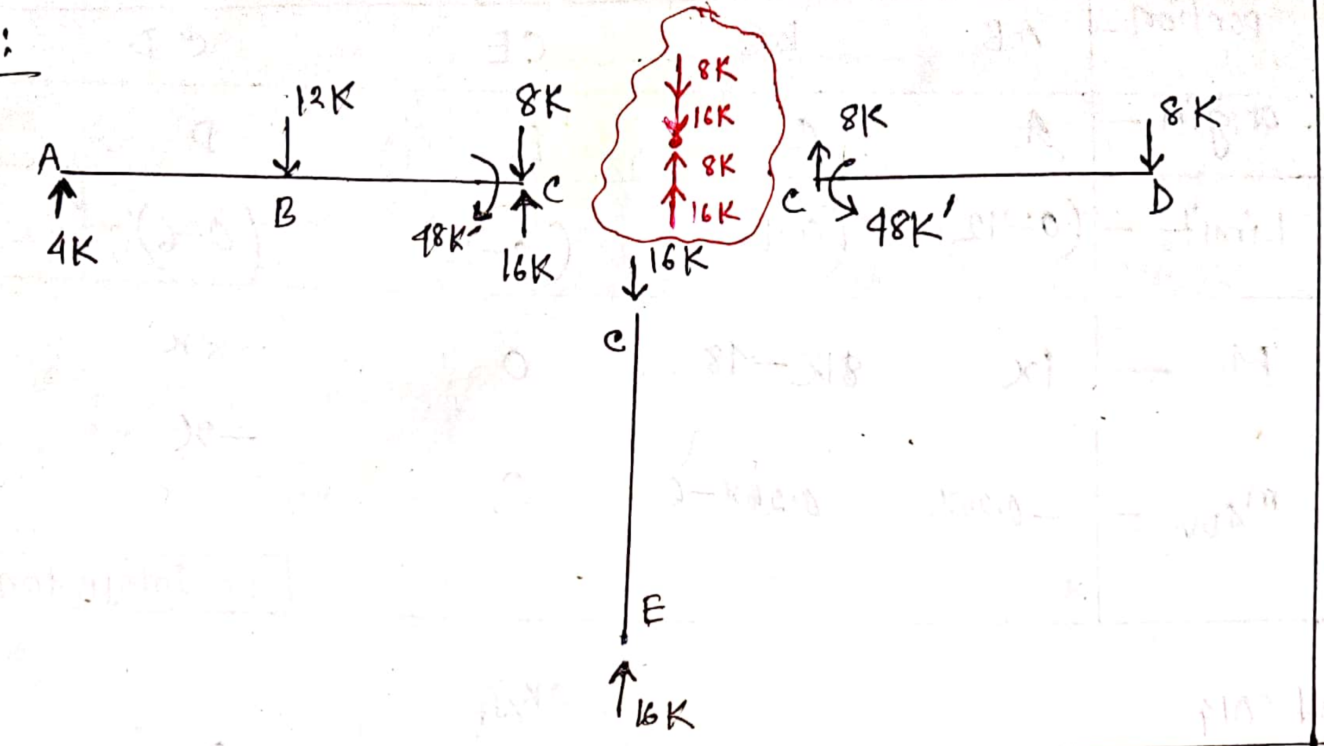


Calculate the vertical deflection of D of the structure.

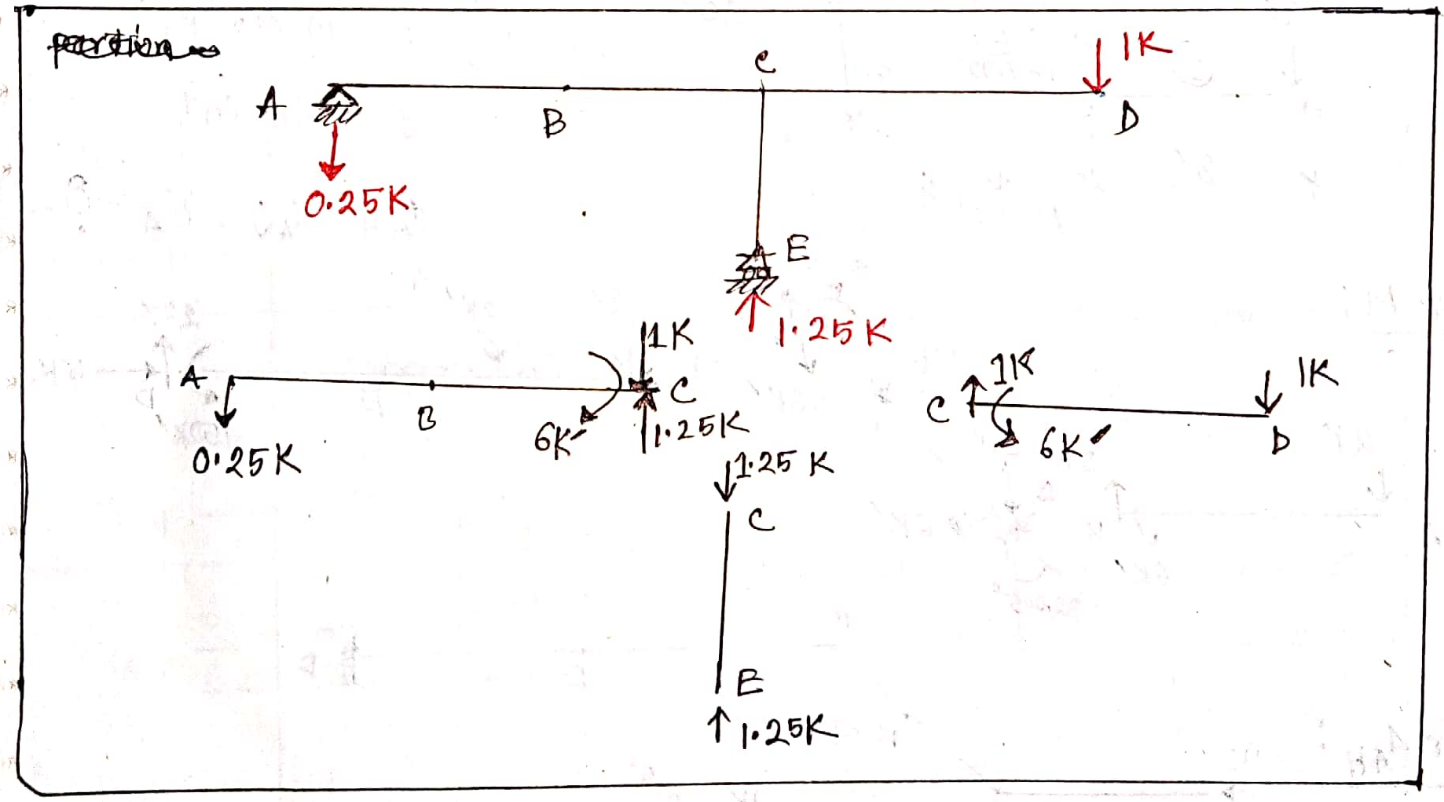
$E = 30\times10^6$ psi

$I = 500\ in^4$

For M:



For $m_{\Delta DV}$:

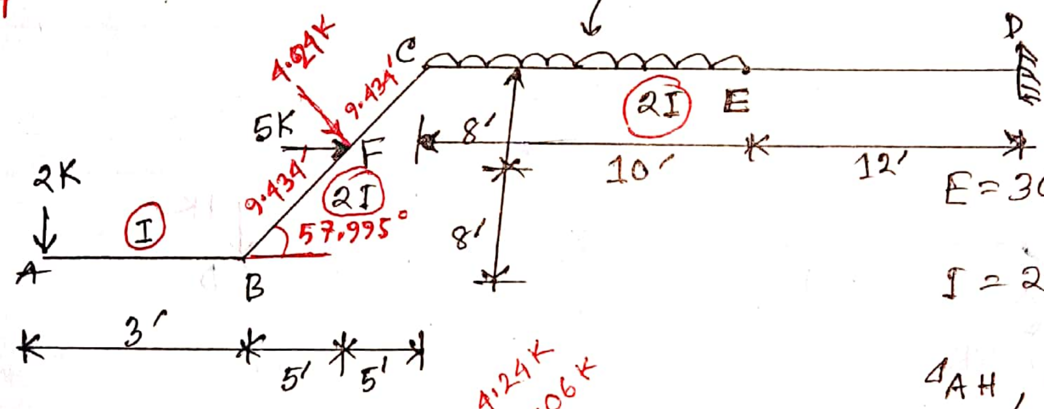


Scanned by CamScanner

portion	AB	BC	CE	CD
origin	A	C	E	D
Limit	(0-12)	(0-12)	(0-12)	(0-6)
M	$4x$	$8x-48$	0	$-8x$
				$-x$
$m_{\Delta DV}$	$-0.25x$	$0.25x-6$	0	

Do Integration

2014

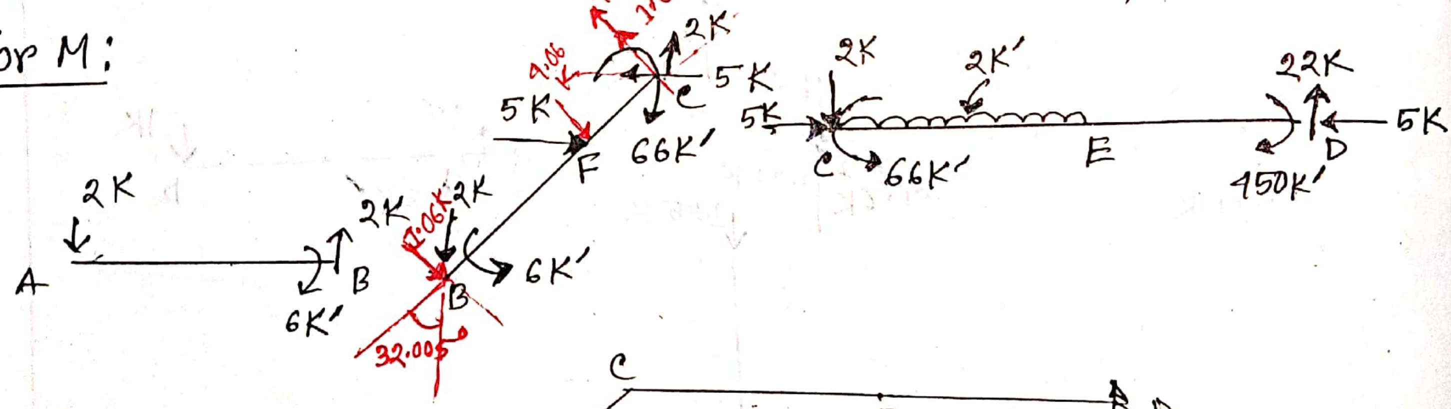


$E = 30 \times 10^3$ ksi

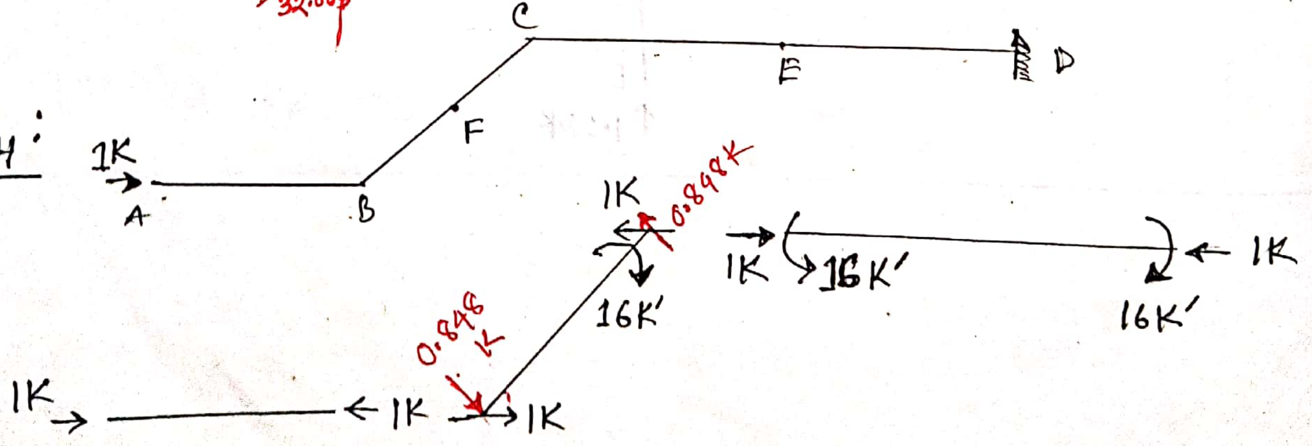
$I = 200$ in^4

$\Delta_{AH}, \Delta_{AV}, \theta_A = ?$

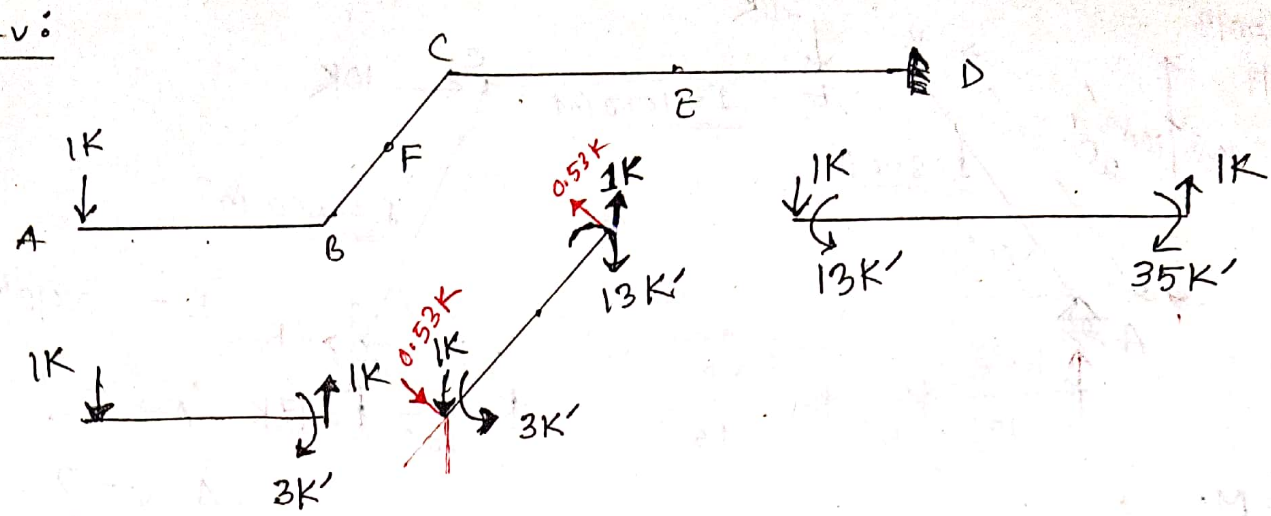
For M:



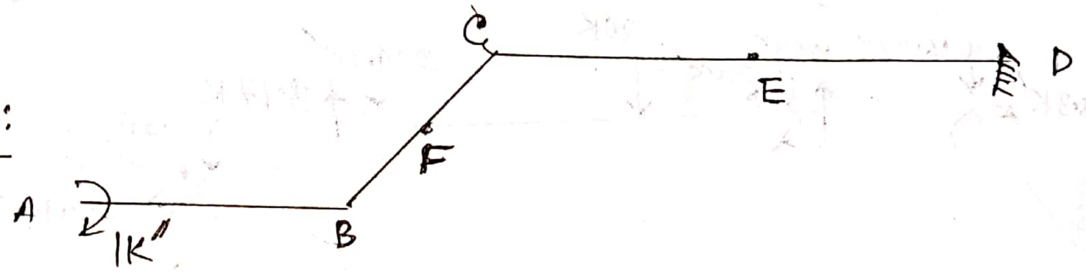
For Δ_{AH}:



For Δ_{AV}:



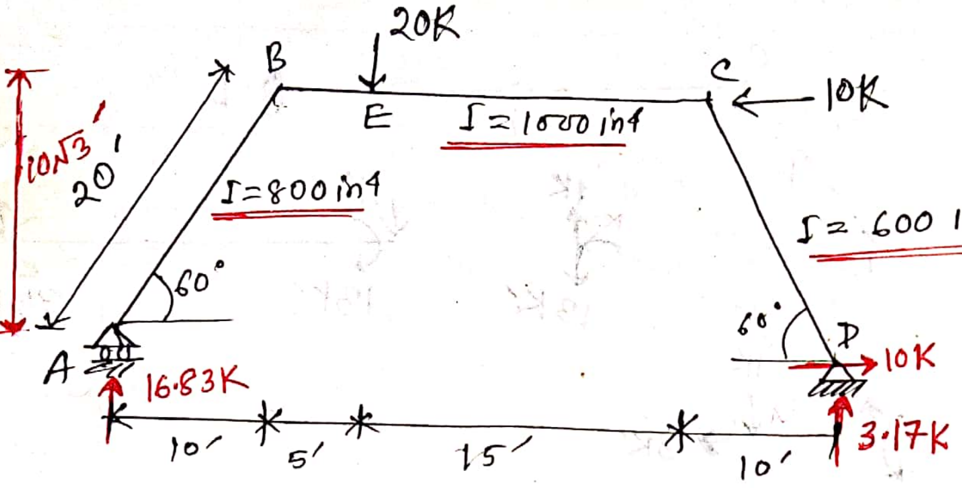
For θ_A:



Portion —	AB	BF	FC	CE	ED
origin —	A	B	C	C	D
Limit —	(0–3)	(0–9.43)	(0–9.43)	(0–10)	(0–12)
M —	(−2x)	−1.06x − 6	5.3x − 66	$\left(-2x - 2\cdot x\cdot\frac{x}{2} - 66\right)$	22x − 450
$m_{\Delta AH}$ —	0	−0.848x	0.848x − 16	−16	−16
$m_{\Delta AV}$ —	−x	−0.53x − 3	0.53x − 13	−x − 13	x − 35
$m_{\theta A}$ —	1	1	−1	1	−1

(Do Integration)

2013 #

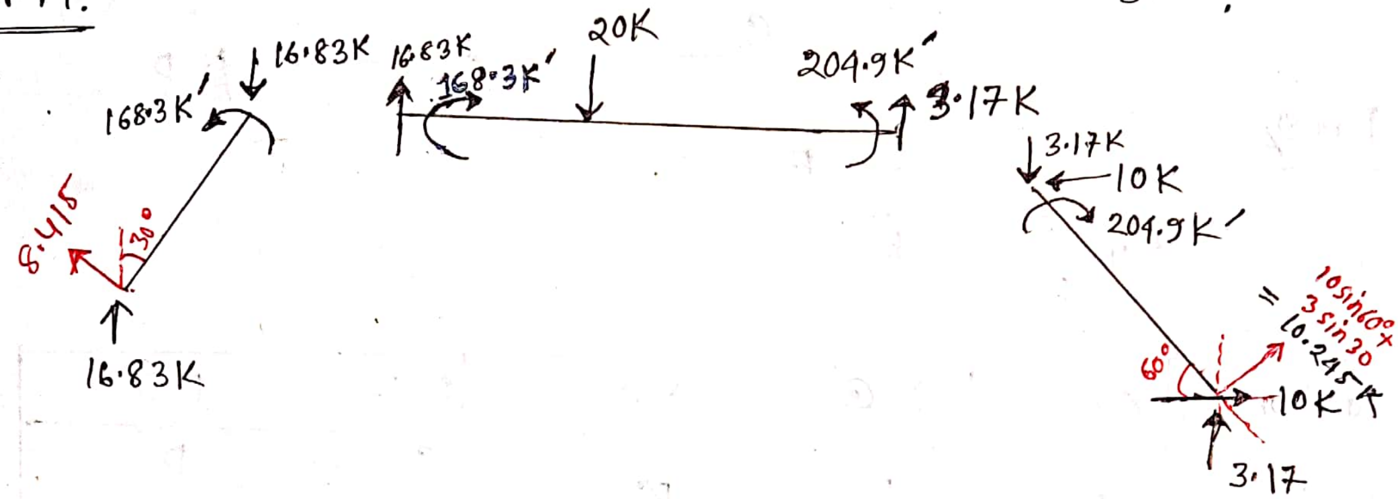


$E = 30 \times 10^6$ psi

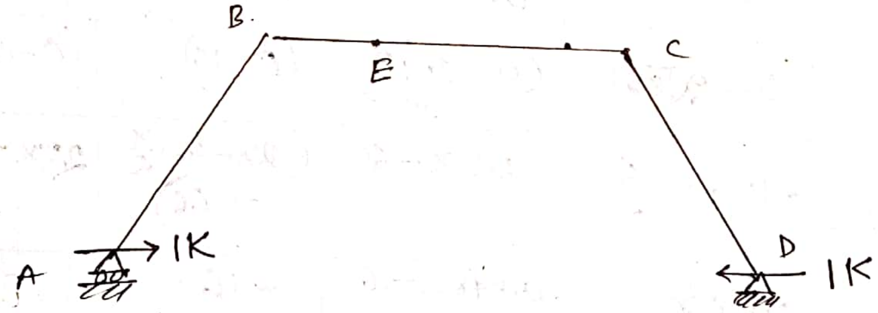
$\Delta_{AH} = ?$

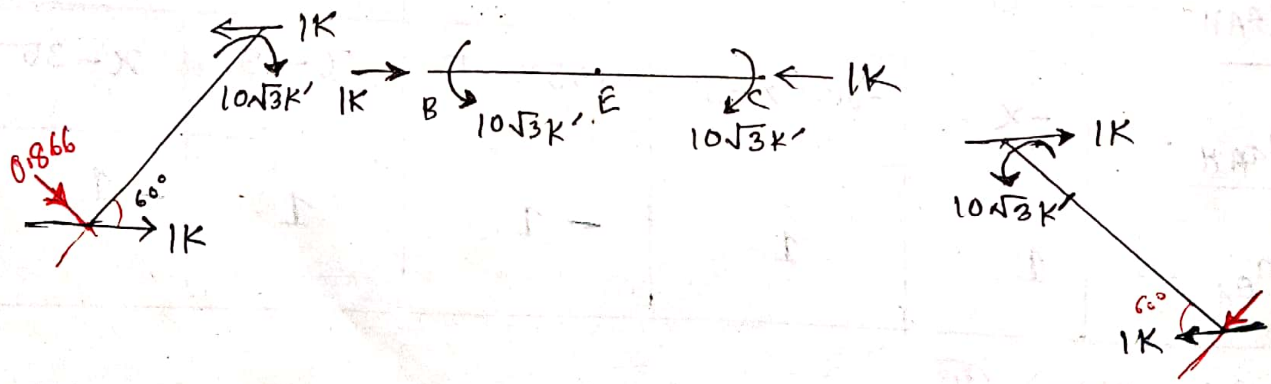
$\Delta_{BV} = ?$

For M:

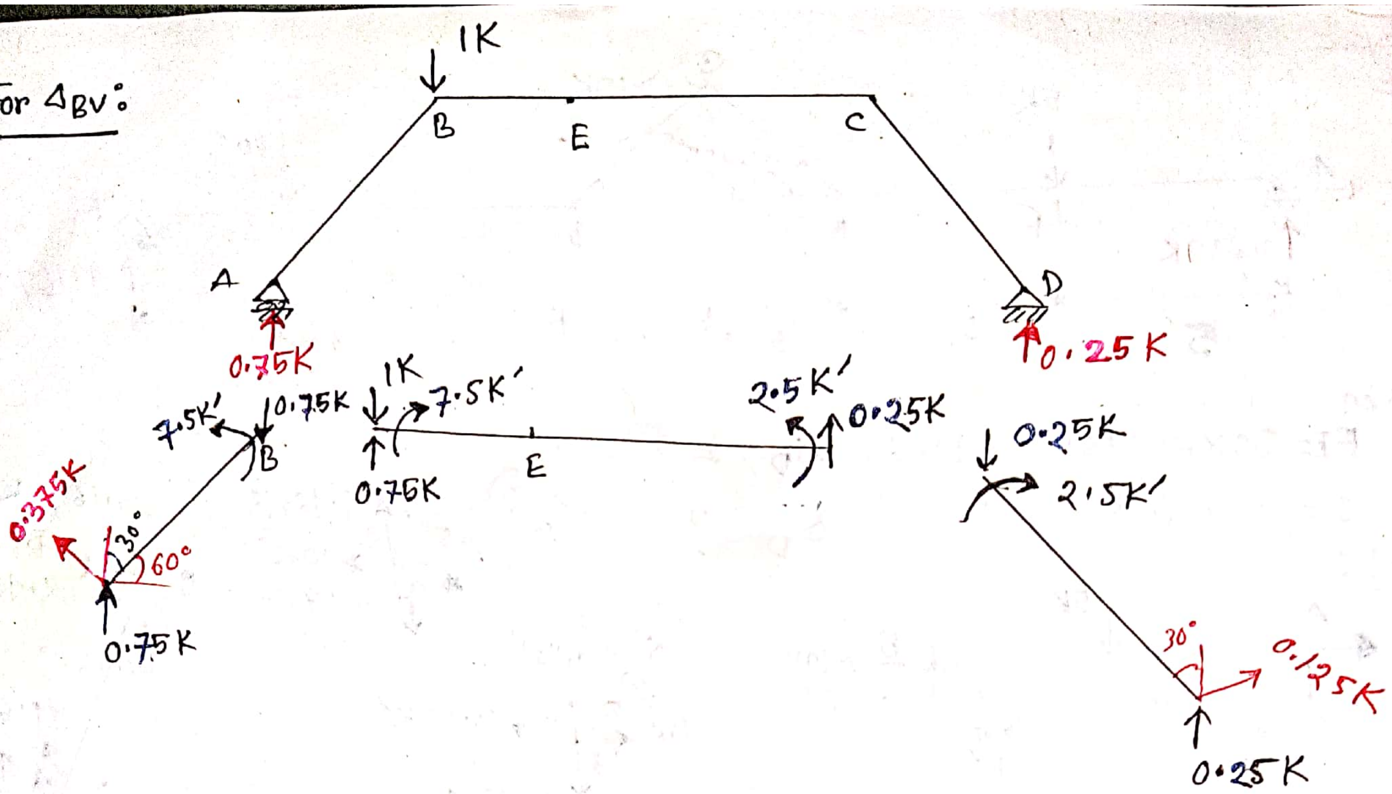


For Δ_{AH}:





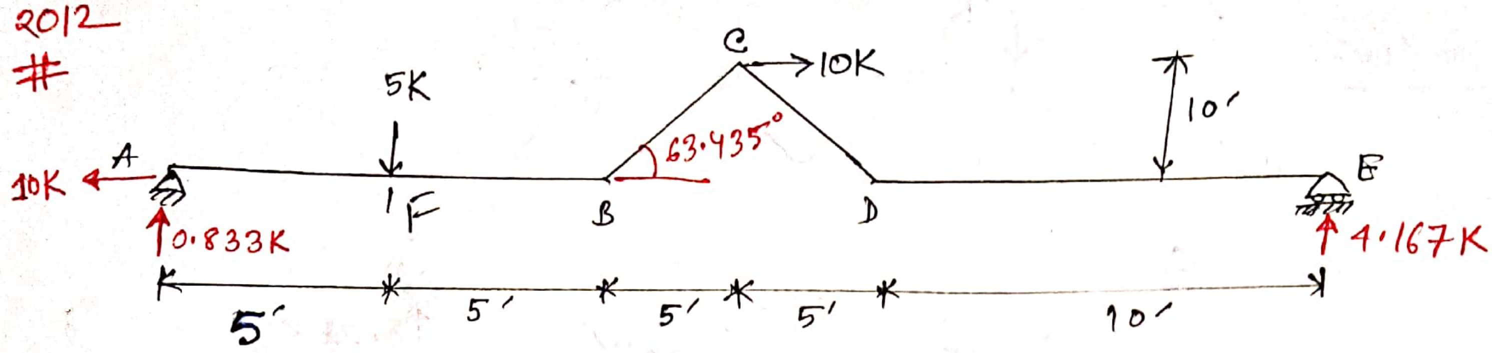
For Δ_{BV}:



Portion	AB	BE	EC	CD
origin	A	B	C	D
Limit	0 – 20	0 – 5	0 – 15	0 – 10
M	8.415 x	16.83x + 168.3	3.17x + 204.9	10.245x
$m_{\Delta AHA}$	−0.866 x	$10\sqrt{3}$	$10\sqrt{3}$	−0.866 x
$m_{\Delta BV}$	0.375x	−.25 x + 7.5	.25 x + 2.5	0.125 x

2012

#

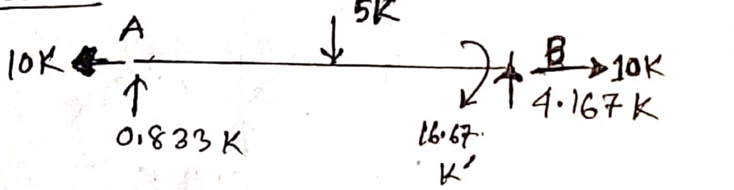


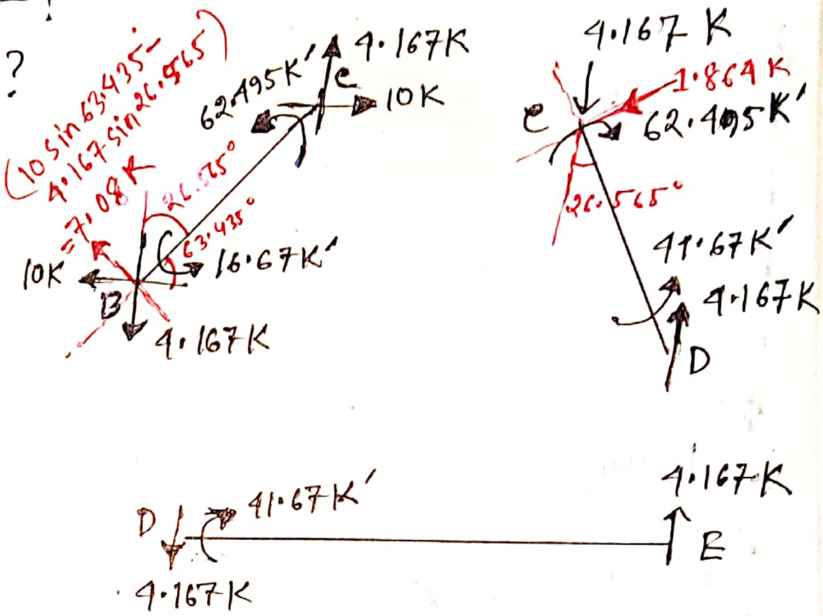
Given

$E = 30 \times 10^6$ psi Find, $\Delta_{DV} = ?$

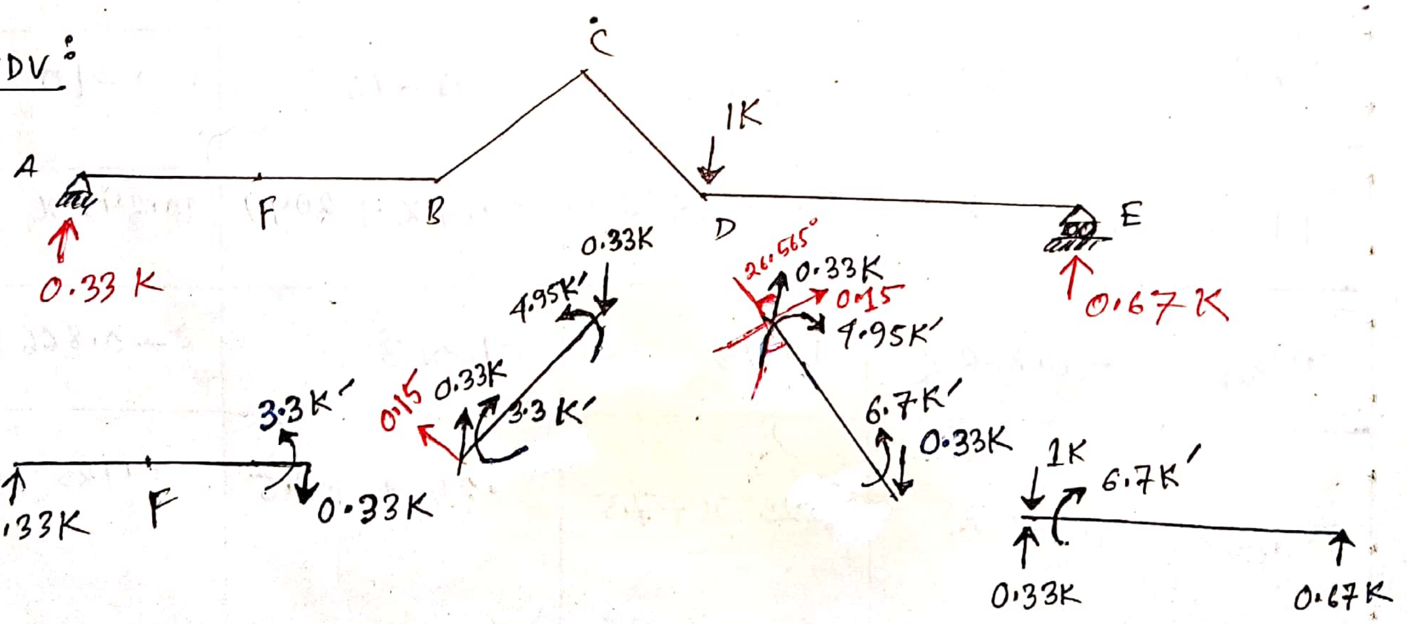
$I = 200$ in^4 $\theta_D = ?$

For M :

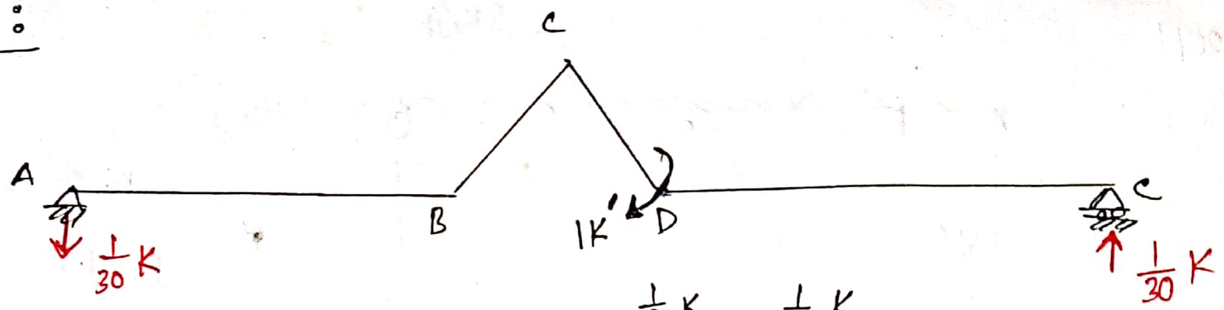


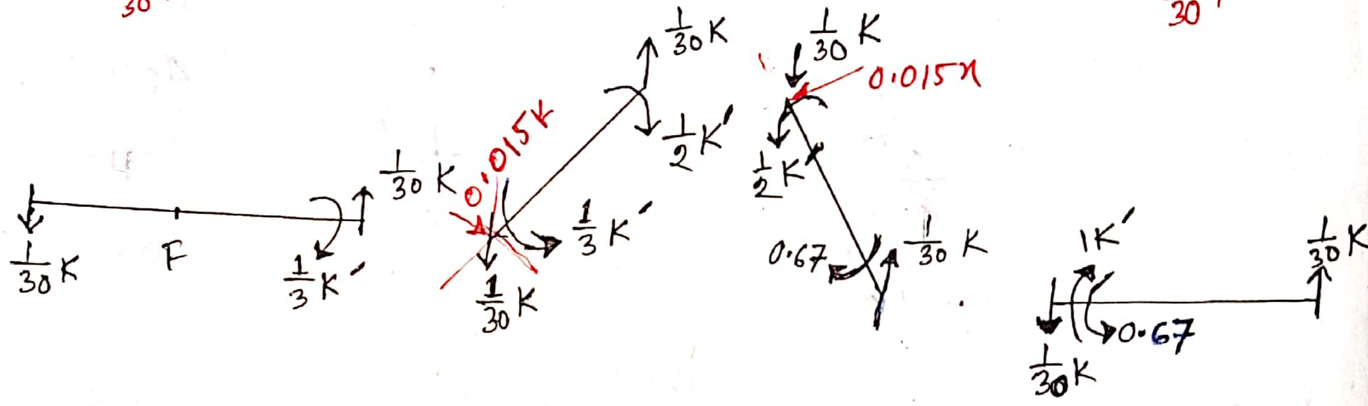


For Δ_{DV} :



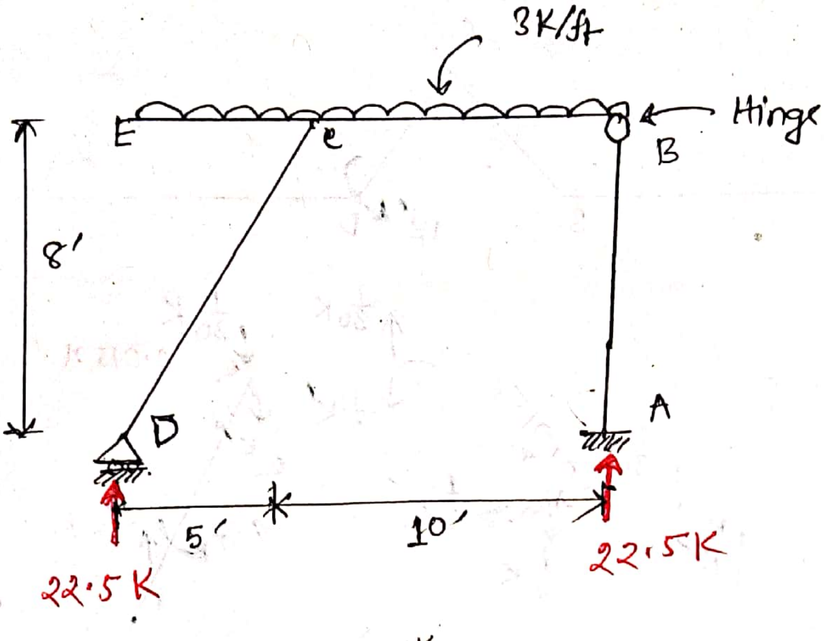
For θ_D :





portion	AF	FB	BC	CD	DE
origin	A	B	B	C	E
Limit	0 − 5	0 − 5	$0 - 5\sqrt{5}$	$0 - 5\sqrt{5}$	0 − 10
M	$0.833x$	$4.167x - 16.67$	$7.08x - 16.67$	$1.864x - 62.495$	$4.167x$
$m_{\Delta DV}$	$0.33x$	$3.3 - .33x$	$0.15x + 3.3$	$0.15x + 4.95$	$0.67x$
$m_{\theta D}$	$\left(-\frac{1}{30}x\right)$	$\frac{1}{30} \times x - \frac{1}{3}$	$-0.015x - \frac{1}{3}$	$-0.015x - \frac{1}{2}$	$\frac{1}{30}x$

2011



EI is constant,

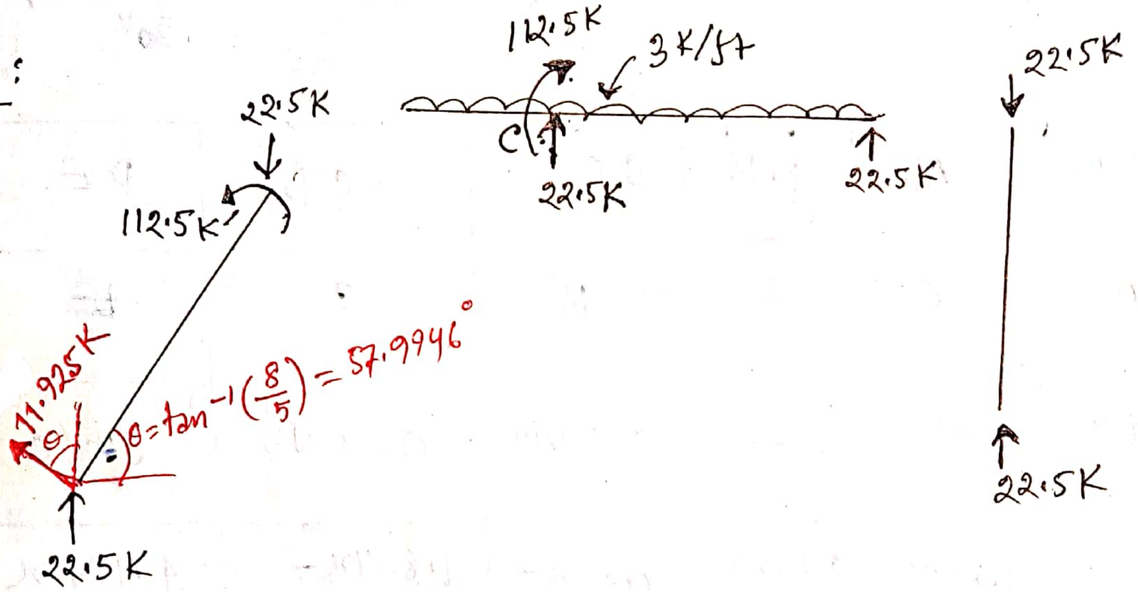
Find,

$\Delta_{EV} = ?$

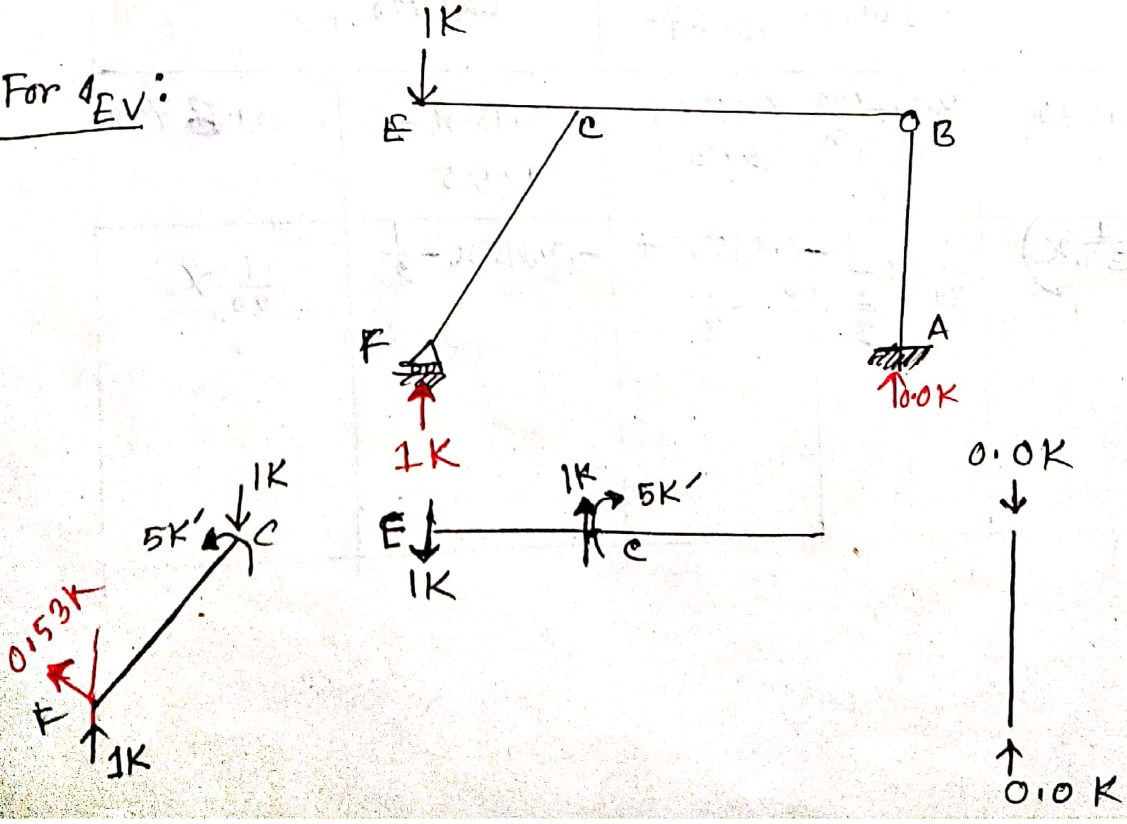
$\Delta_{BH} = ?$

$\Delta_{DH} = ?$

For M:



For Δ_{EV}:



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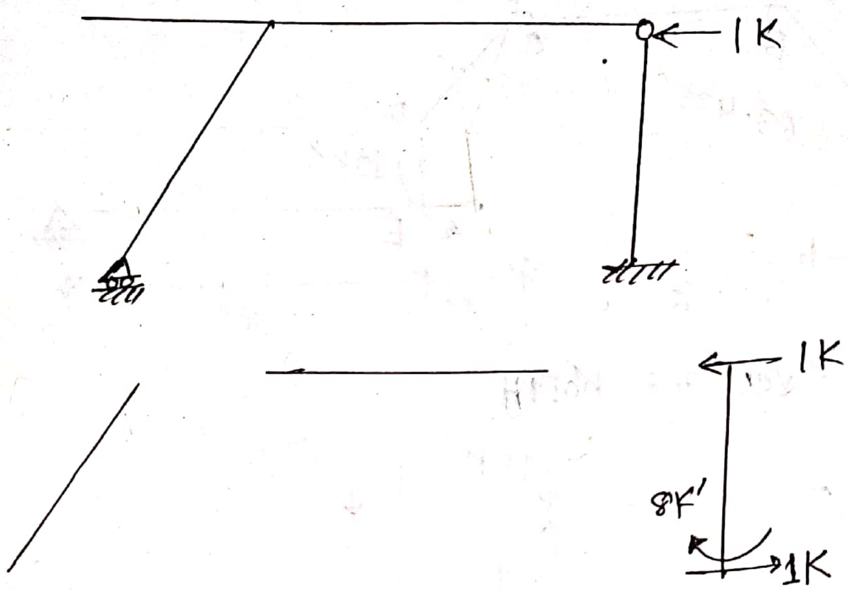
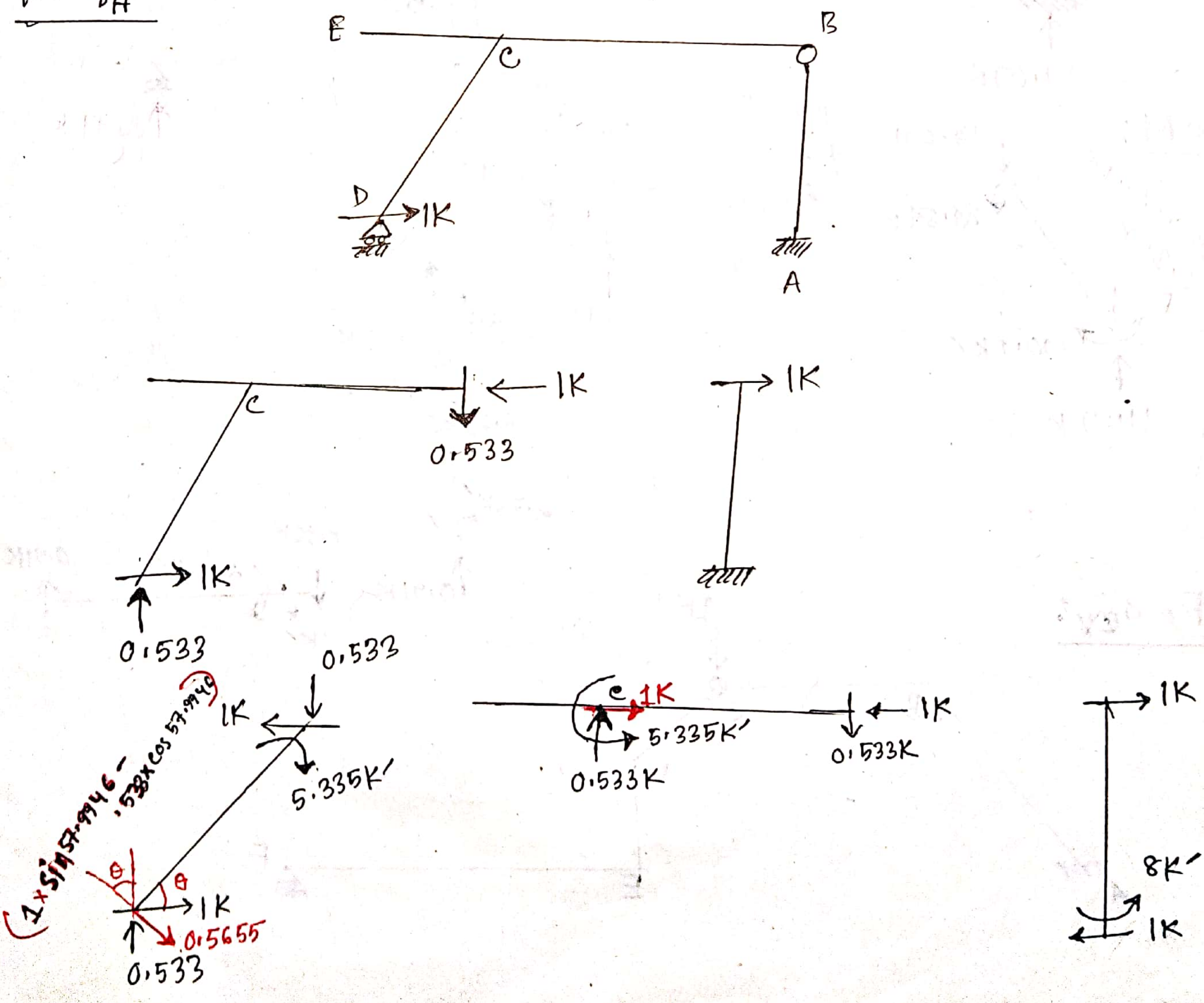
For Δ_{BH}:



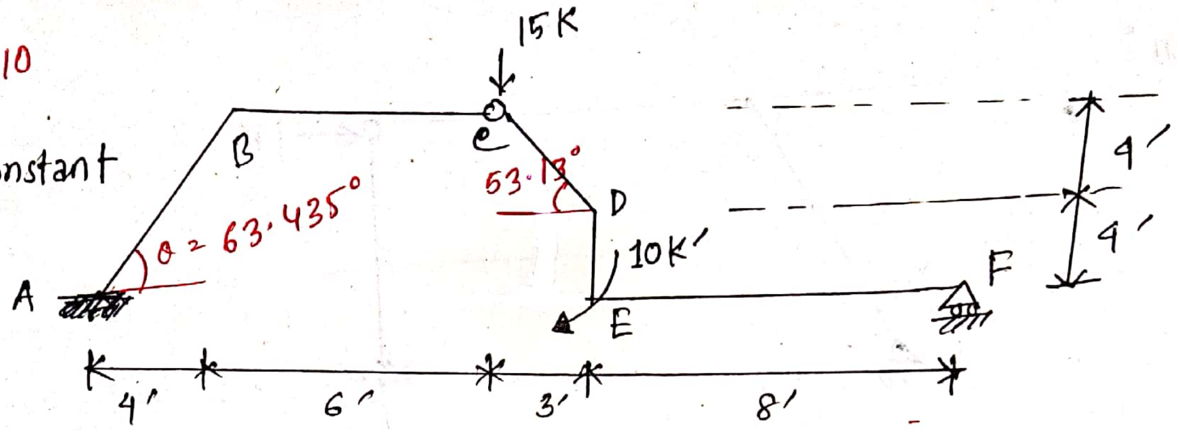
For Δ_{DH}:



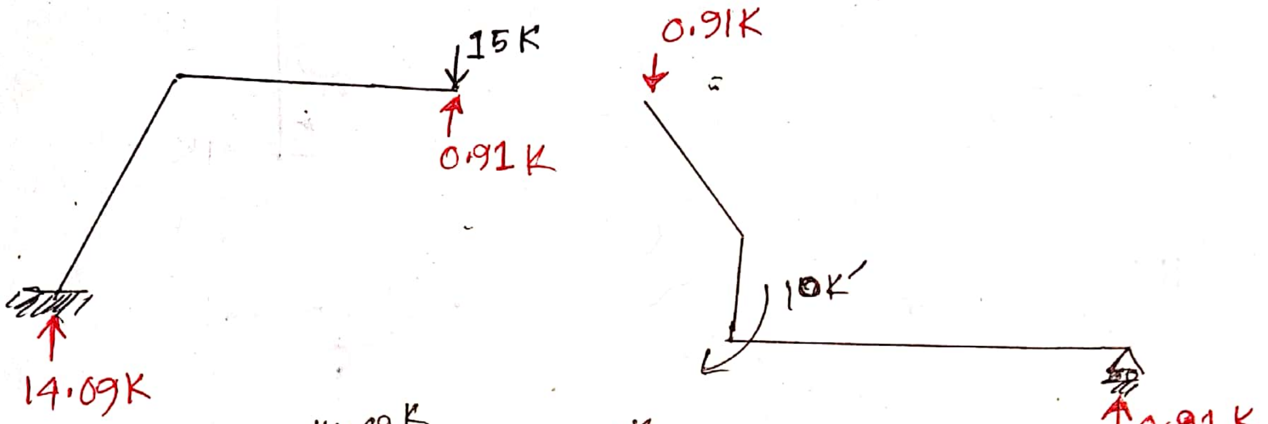
Portion	DC	EC	CB	BA
origin	D	E	B	A
Limit	0 – 9.434	0 – 5	0 – 10	0 – 8
M	$11.92x$	$-\frac{3x^2}{2}$	$22.5x - \frac{3x^2}{2}$	0
$m_{\Delta EV}$	$0.53x$	$-x$	0	0
$m_{\Delta BH}$	0	0	0	$-8+x$
$m_{\Delta DH}$	$-.5655x$	0	$-0.533x$	$8-x$

2010

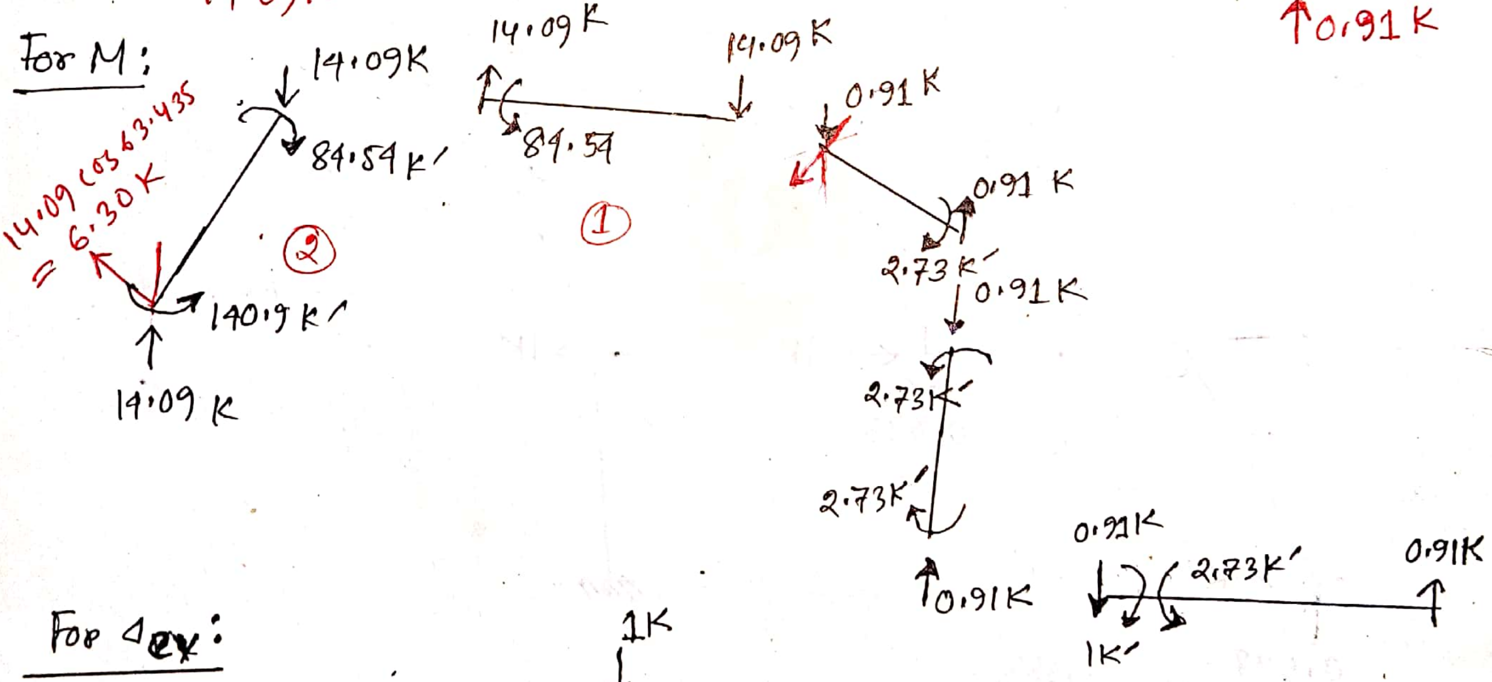
EI is constant



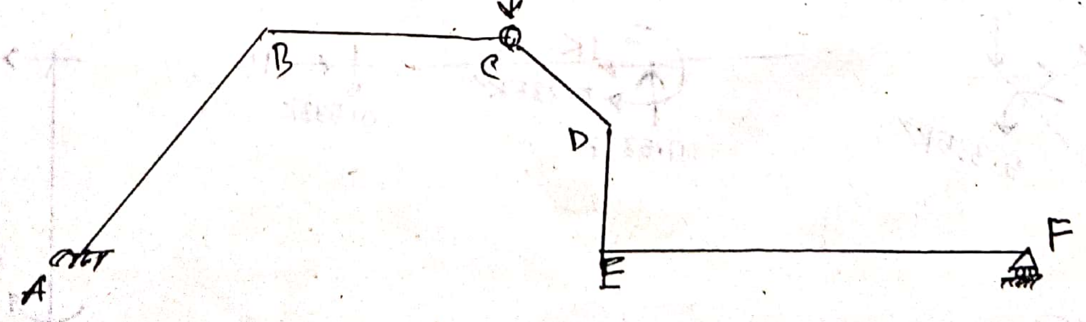
* Calculate the Δ_{cv} and Δ_{FH}

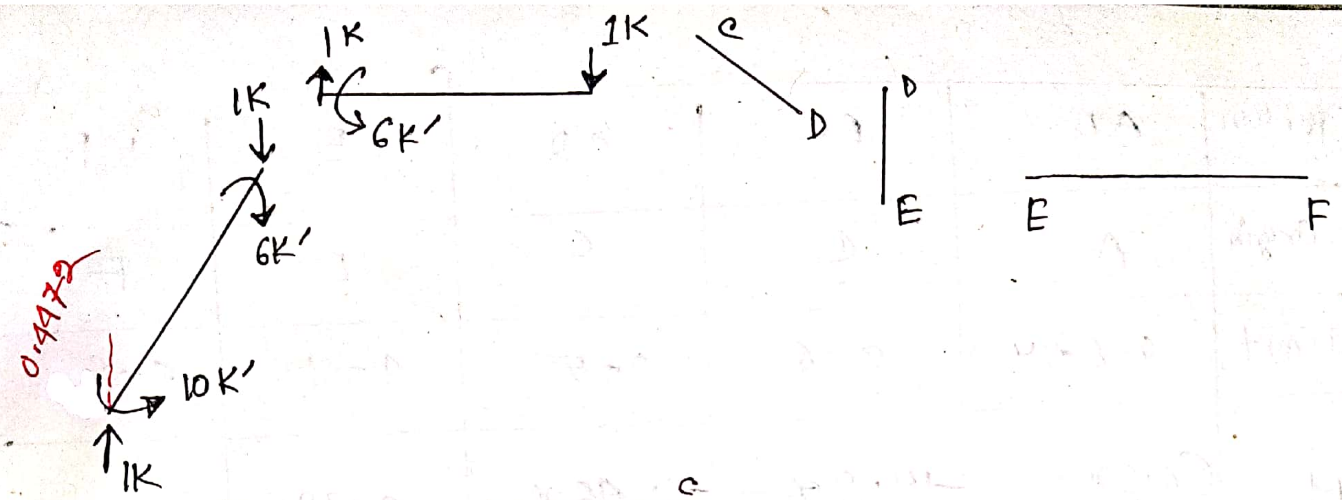


For M:

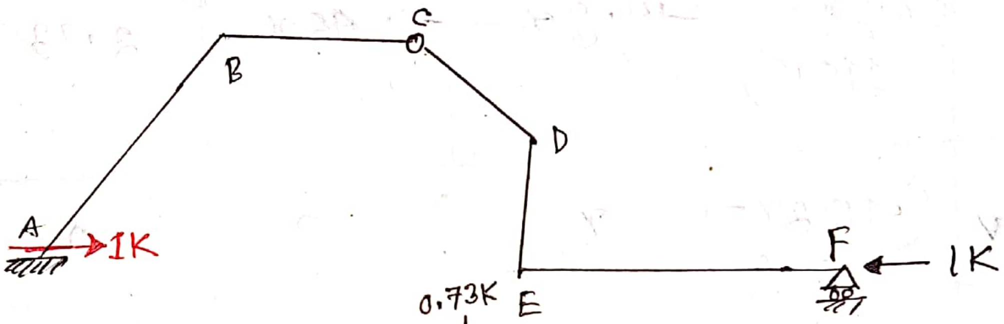


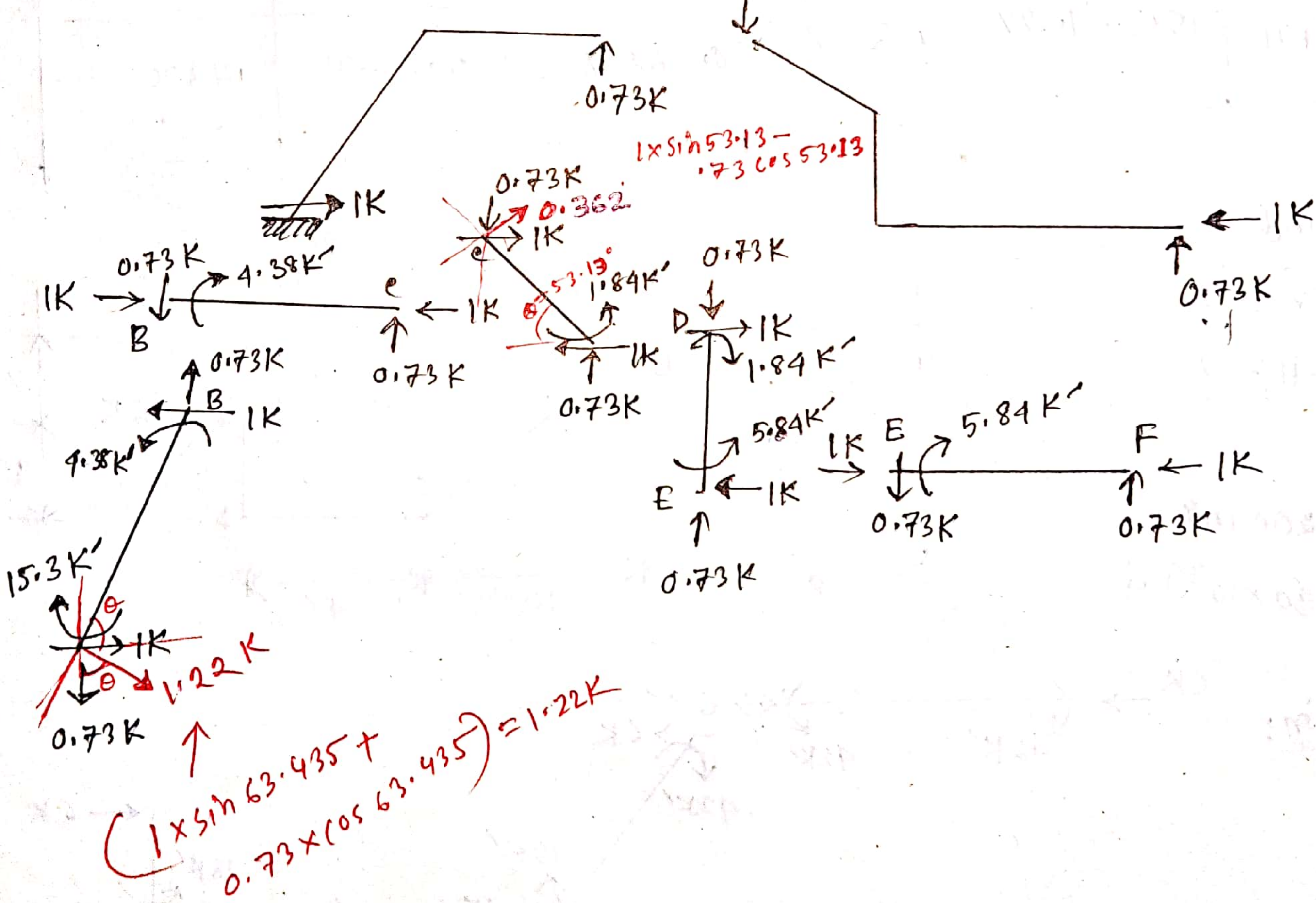
For Δ_{cv}:




1K
1K
C
D
D
E
E
F
1K
6K'
6K'
0.4472
10K'
1K

For Δ_{FH}:


C
B
D
A
1K
F
1K
0.73K
E


0.73K
1K
1x sin53.13 −
.73 cos 53.13
0.73K
0.362
1K
0.73K
4.38K'
C
1K
53.13°
1.84K'
0.73K
1K
1K
B
0.73K
1K
D
1K
1.84K'
0.73K
0.73K
B
1K
0.73K
0.73K
4.38K'
5.84K'
1K
E
5.84K'
F
1K
E
1K
0.73K
0.73K
0.73K
15.3K'
θ
1K
θ
1.22K
0.73K
(1 x sin 63.435 +
0.73 x cos 63.435) = 1.22K

portion	AB	BC	CD	DE	EF
Origin	A	C	C	E	F
Limit	0-8.94	0-6	0-5	0-4	0-8
M	$(6.3x - 140.9)$	$-14.09x$	$-0.546x$	2.73	$0.91x$
m_{4EV}	$(0.4472x - 10)$	x	0	0	0
m_{4EH}	$15.3 - 1.22x$	$.73x$	$0.362x$	$5.84 - x$	$.73x$

2008

Find

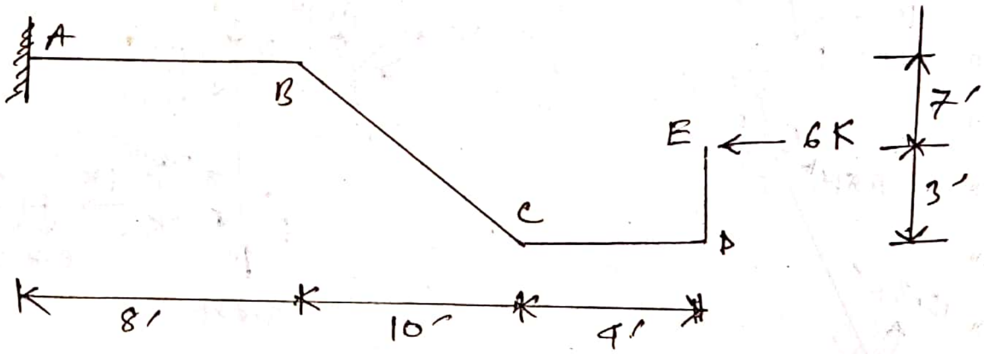
$\Delta_{EV} = ?$

$\Delta_{EH} = ?$

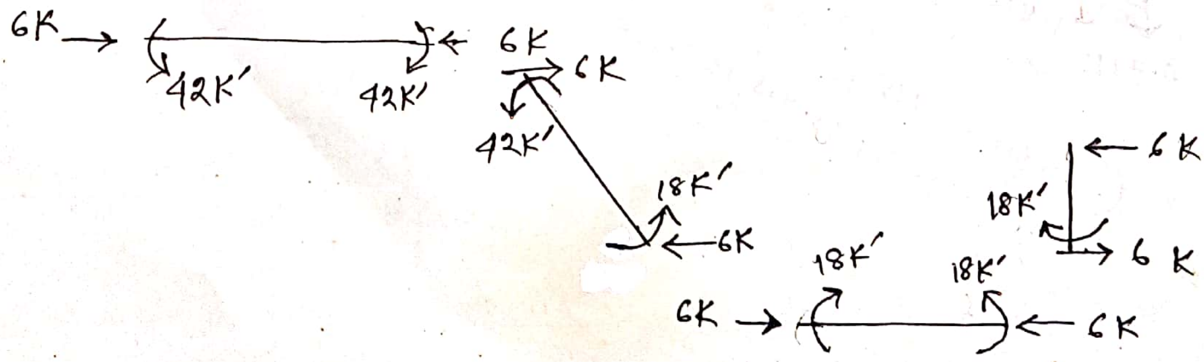
$\theta_E = ?$

$I = 200 \text{ in}^4$

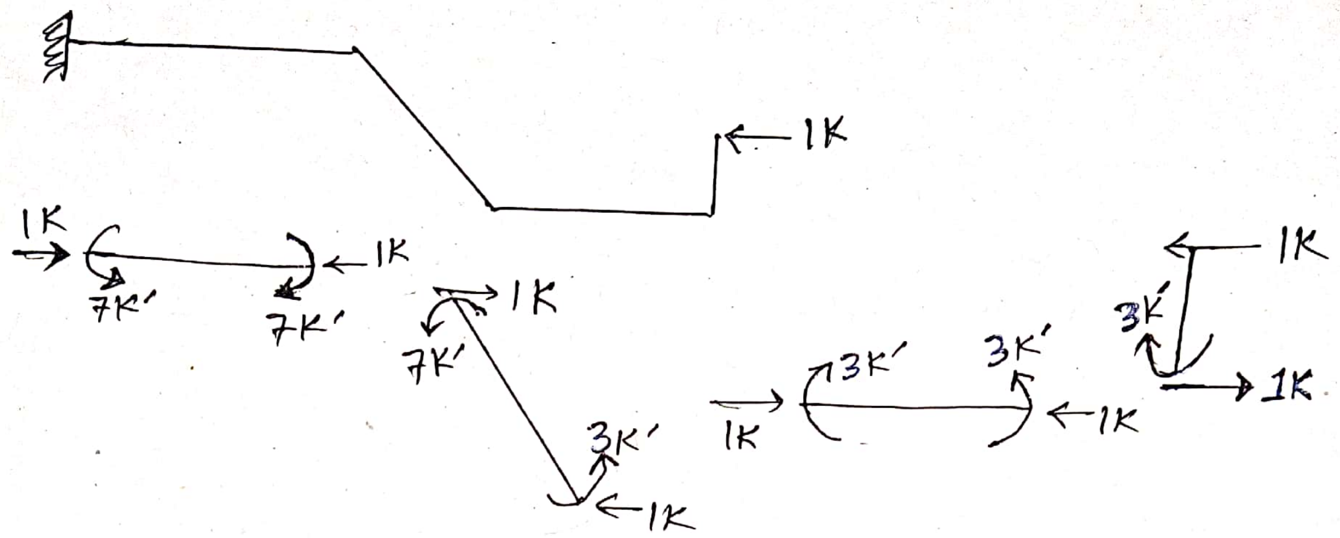
$E = 30 \times 10^3$ ksi



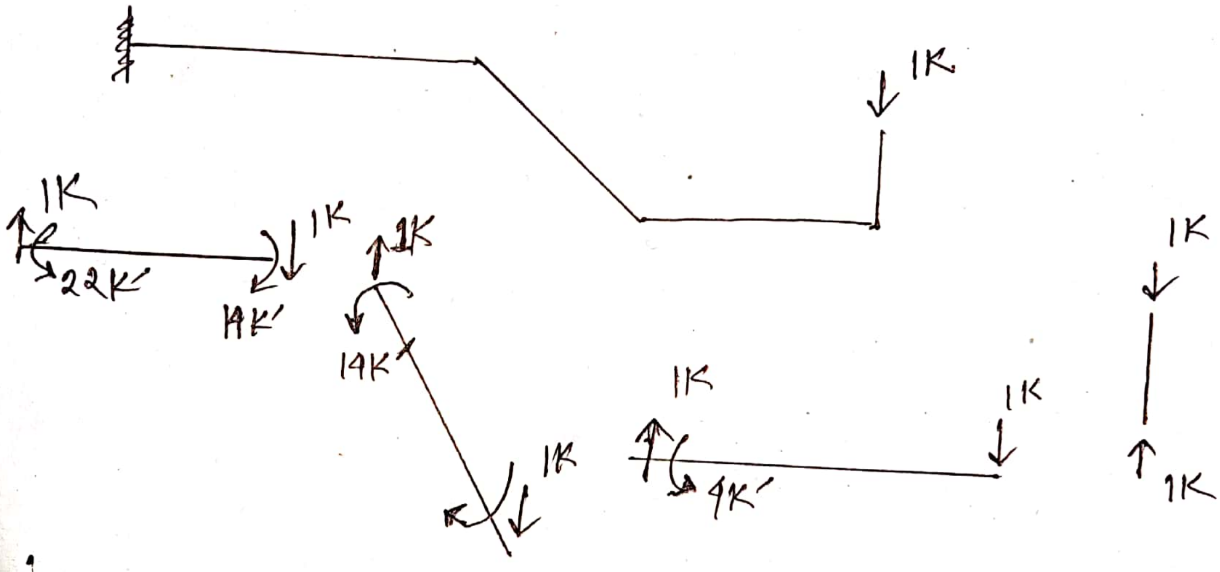
Solution:



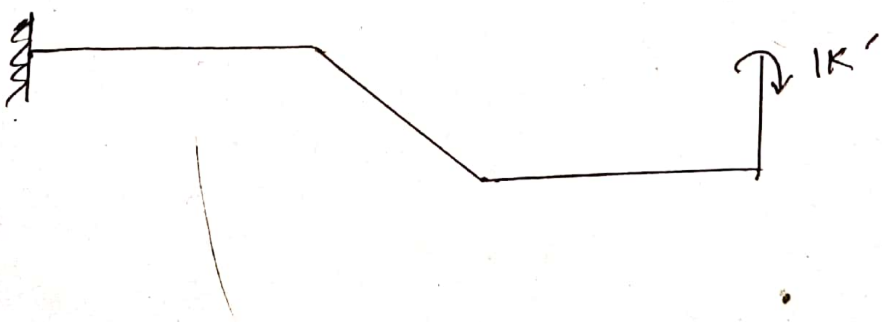
δ_{EV}:



Δ_{EH}:



θ_B:



TRUSS

We know, $\Delta = \sum u dL = \sum u \cdot \frac{SL}{AE} = \sum \frac{SuL}{AE}$

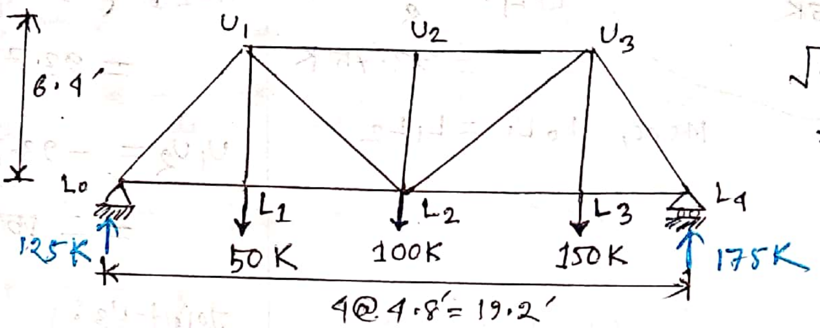
Problem: 01

$E = 30 \times 10^3$ ksi

$A = 2 in^2$

$\Delta H_{L_3} = ?$

$\Delta V_{L_3} = ?$



$\sqrt{6.4^2 + 4.8^2} = 8$

Solution:

$\sum M_{L_0} = 0,$

$50 \times 4.8 + 100 \times 4.8 \times 2 + 150 \times 4.8 \times 3 = R_{L_4} \times 19.2$ $\therefore R_{L_4} = 175$ K

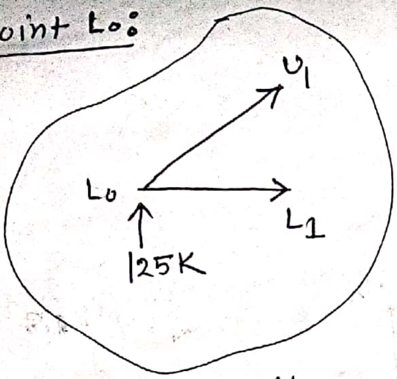
$\therefore R_{L_0} = 125$ K

Member	Area	Length	S (K)	$u_{H_{L_3}}$	$u_{V_{L_3}}$	$\frac{Su_{H_{L_3}} L}{AE}$	$\frac{Su_{V_{L_3}} L}{AE}$
$L_0 L_1$	2	4.8	93.75	1	0.1875	7.5×10^{-3}	1.41×10^{-3}
$L_0 U_1$	2	8	-156.25	0	-0.3175	0	6.61×10^{-3}
$L_1 L_2$	2	4.8	93.75	1	0.1875	7.5×10^{-3}	1.41×10^{-3}
$L_1 U_1$	2	6.4	50	0	0	0	0
$L_2 L_3$	2	4.8	131.25	1	0.5625	0.0105	5.91×10^{-3}
$L_2 U_1$	2	8	93.75	0	0.3125	0	3.91×10^{-3}
$L_2 U_2$	2	6.4	0	0	0	0	0
$L_2 U_3$	2	8	31.25	0	-0.3125	0	-1.31×10^{-3}
$L_3 L_4$	2	4.8	131.25	0	0.5625	0	5.91×10^{-3}
$L_3 U_3$	2	6.4	150	0	1	0	0.016
$L_4 U_3$	2	8	-218.75	0	-0.9375	0	0.027
$U_1 U_2$	2	4.8	-150	0	-0.375	0	4.5×10^{-3}
$U_2 U_3$	2	4.8	-150	0	-0.375	0	4.5×10^{-3}
						Σ = 0.0255	Σ = 0.07585

Scanned by CamScanner

For S:

joint L_0:



$L_0U_1 = \frac{-125}{6.4} \times 8$

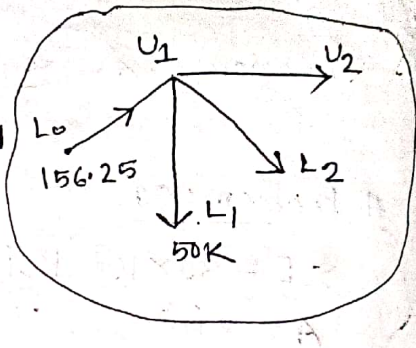
$= -156.25\ K$

$L_0L_1 = \frac{156.25}{8} \times 4.8$

$= 93.75\ K$

Here, $L_0L_1 = L_1L_2$

joint U_1



$(L_2U_1)_V = \left(\frac{156.25}{8} \times 6.4 - 50\right)$

$\therefore L_2U_1 = \left(75 \times \frac{8}{6.4}\right)$

$= 93.75\ K$

$U_1U_2 = -93.75 \times \frac{4.8}{8} - 156.25 \times \frac{4.8}{8}$

$= -150\ K$

joint L_4:

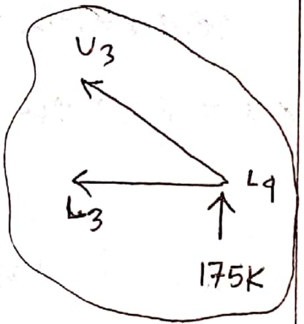
$L_4U_3 = -\frac{175}{6.4} \times 8$

$= -218.75\ K$

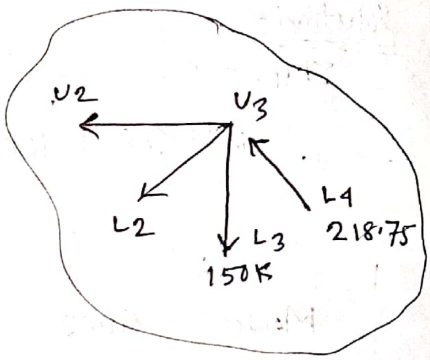
$L_3L_4 = \frac{218.75}{8} \times 4.8$

$= 131.25\ K$

Here, $L_2L_3 = L_3L_4$



joint U_3:



$(L_2U_3)_V = \left(\frac{218.75}{8} \times 6.4 - 150\right)$

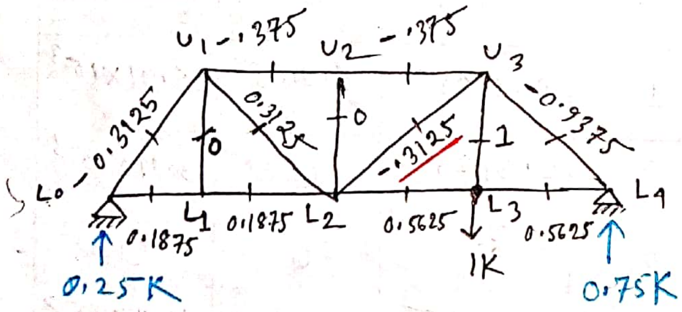
$\therefore L_2U_3 = \left(25 \times \frac{8}{6.4}\right)$

$= 31.25\ K$

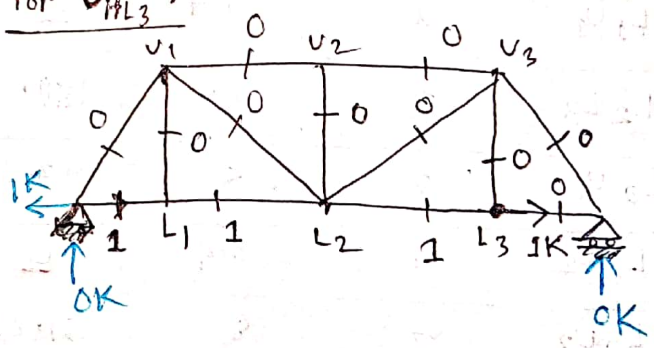
$U_2U_3 = -31.25 \times \frac{4.8}{8} - 218.75 \times \frac{4.8}{8}$

$= -150\ K$

For U_{VL_3}:



For U_{HL_3}:



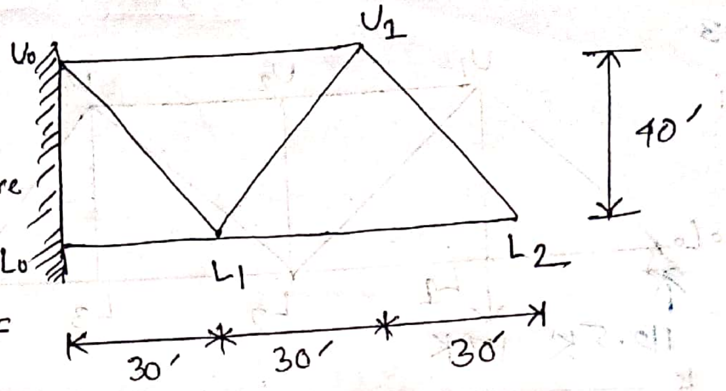
$\therefore \Delta_{HL_3} = 0.0255\ ft\ (\rightarrow)\ \&\ \Delta_{VL_3} = 0.07585\ ft\ (\downarrow)$

(Ans)

Scanned by CamScanner

Problem: 02

Find deflection of L_2 due to rise in temparature $50°F$ in the Lower chord. co-efficient, $\alpha = 6.5 \times 10^{-6} /°F$

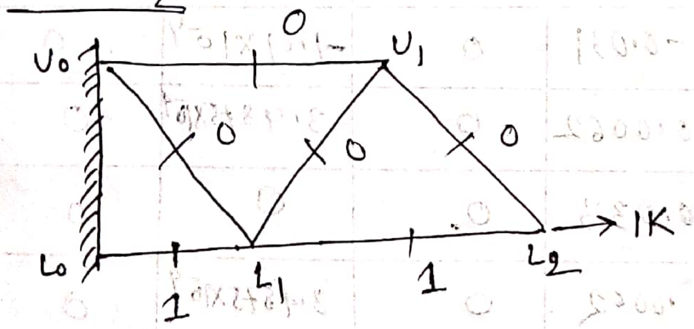


$\sqrt{40^2+30^2} = 50$

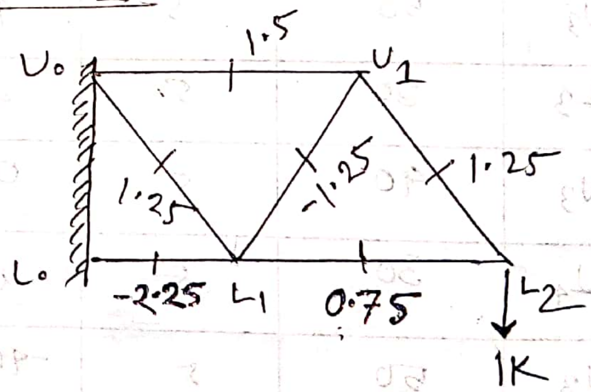
Solution: We know, $\Delta = \Sigma U dL = \Sigma U \cdot \alpha LT = \alpha \Sigma ULT$

Member	Length	T (°F)	U_{HL_2}	U_{VL_2}	$U_{HL_2}LT$	$U_{VL_2}LT$
L_0L_1	30	50	1	-2.25	1500	-3375
L_1U_0	50	0	0	1.25	0	0
L_1U_1	50	0	0	-1.25	0	0
L_1L_2	60	50	1	-0.75	3000	-2250
L_2U_1	50	0	0	1.25	0	0
U_0U_1	60	0	0	1.5	0	0
					$\Sigma = 4500$	$\Sigma = -5625$

For U_{HL_2}:



For U_{VL_2}:



$\Delta_{HL_2} = \alpha \Sigma U_{HL_2} LT$

$= (6.5 \times 10^{-6} \times 4500) = +0.029\ ft\ (\rightarrow)$

$\Delta_{VL_2} = \alpha \Sigma U_{VL_2} LT$

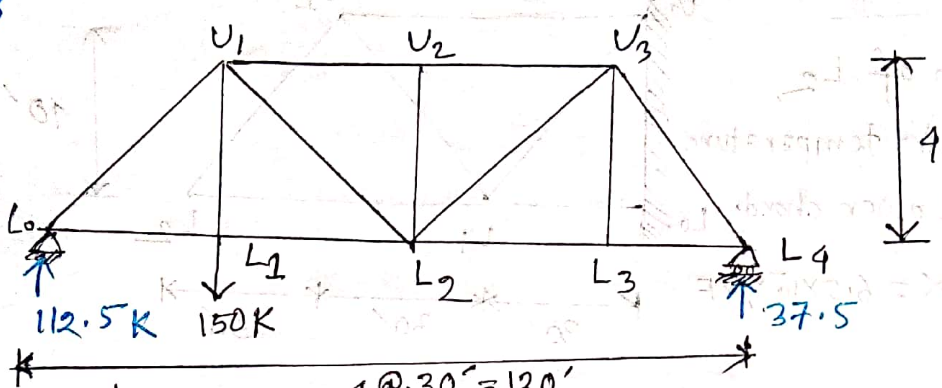
$= (6.5 \times 10^{-6} \times -5625) = -0.0366\ ft$ $\quad \therefore \Delta_{VL_2} = 0.0585\ ft\ (\uparrow)$

Problem:03

$E = 30 \times 10^3$ ksi

$A = 5$ in²

Find Rotational deflection L_2L_3 and, Relative displacement L_1U_2



$\sqrt{40^2+30^2} = 50$

Solution: $\Sigma M_{L_0} = 0$

$50 \times 30 = R_{L_4} \times 120$ $\therefore R_{L_4} = 37.5$ K

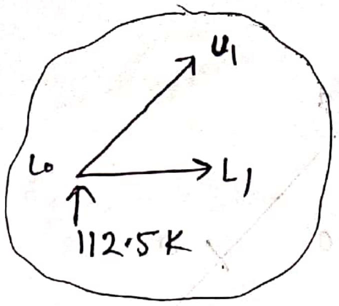
$\therefore R_{L_0} = 112.5$ K

Member	length	A	S	$U_{L_2L_3}$	$U_{L_1U_2}$	$\frac{SU_{L_2L_3}L}{AE}$	$\frac{SU_{L_1U_2}L}{AE}$
L_0L_1	30	5	+84.375	−0.0062	0	-1.05×10^{-4}	0
L_0U_1	50	5	−140.625	.0103	0	-4.83×10^{-4}	0
L_1U_1	40	5	150	0	−0.8	0	−0.032
L_1L_2	30	5	84.375	−0.0062	−0.6	-1.05×10^{-4}	−0.01013
L_2U_1	50	5	−46.875	−0.0103	1	1.61×10^{-4}	−0.0156
L_2U_2	40	5	0	0	−0.8	0	0
L_2U_3	50	5	46.875	−0.031	0	-1.61×10^{-4}	0
L_2L_3	30	5	28.125	0.0062	0	3.4875×10^{-4}	0
L_3U_3	40	5	0	0.033	0	0	0
L_3L_4	30	5	28.125	.0062	0	3.4875×10^{-4}	0
L_4U_3	50	5	−46.875	−0.0103	0	1.61×10^{-4}	0
U_1U_2	30	5	−56.25	.01236	0	-1.3905×10^{-4}	0
U_2U_3	30	5	−28.125	.01236	−0.6	-6.9525×10^{-4}	$+3.375\times10^{-3}$

$\Sigma = -6.688\times10^{-4}$ ft $\Sigma = 0.061$ ft

For S:

joint L_0:



$L_0U_1 = \frac{-112.5}{40} \times 50$

$= -140.625K$

$L_0L_1 = \frac{140.625}{50} \times 30$

$\therefore L_0L_1 = 84.375 = L_1L_2$

joint L_4:

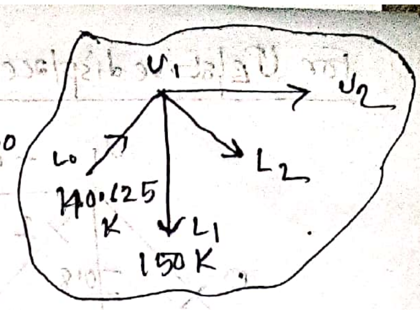
$L_4U_3 = \frac{-37.5}{40} \times 50$

$= -46.875K$

$L_3L_4 = \frac{46.875}{50} \times 30$

$\therefore L_3L_4 = 28.125K = L_2L_3$

joint U_1:



$(L_2U_1)_v = \left(\frac{140.625}{50} \times 40 - 150\right)$

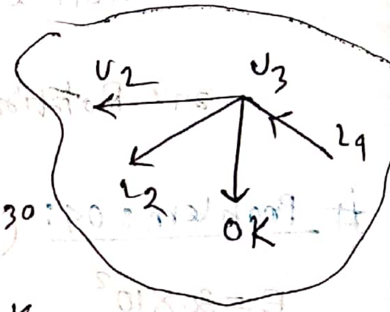
$= -37.5K$

$\therefore L_2U_1 = \left(\frac{-37.5}{40} \times 50\right) = -46.875K$

$U_1U_2 = \left(-\frac{140.625}{50} \times 30 + \frac{46.875}{50} \times 30\right)$

$= -56.25K$

Joint U_3:

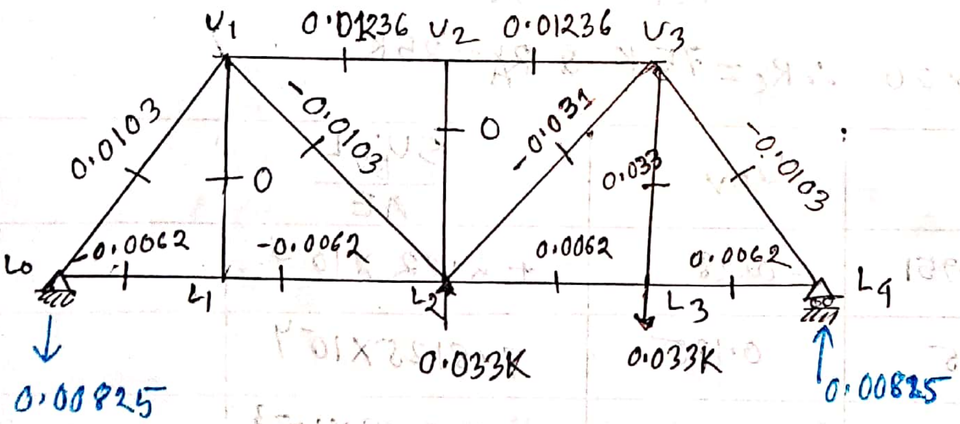


$L_2U_3 = 46.875$

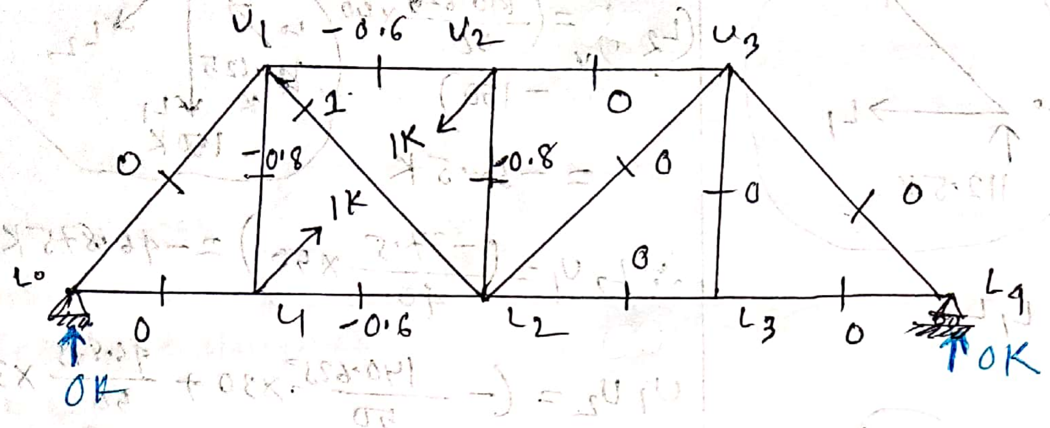
$U_2U_3 = -\frac{46.875}{50} \times 30$

$= -28.125K$

For Rotational deflection L_2L_3:



For Relative displacement $U_{L_1L_2}$:



∴ Relative displacement $L_1L_2 = 0.061$ ft (↗)

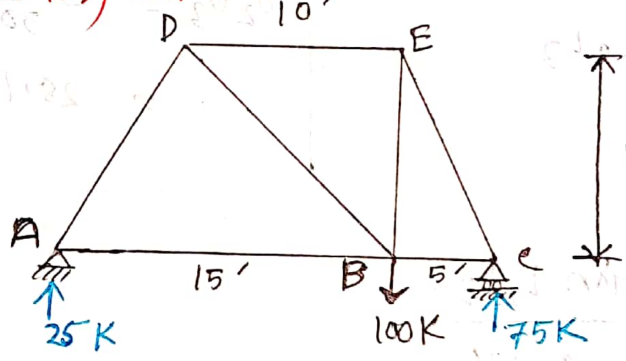
and Rotational deflection, $L_2L_3 = 6.688\times10^{-4}$ rad. (↻)

Problem: 04 (14 series) CT

$E = 30\times10^3$

$A = 1\ in^2$

Find $\Delta_{BV} = ?$



$\sqrt{10^2+5^2} = 5\sqrt{5}$

$\sqrt{10^2+10^2} = 10\sqrt{2}$

Solution: $\Sigma M_A = 0$

$100\times15 = R_C\times20$ ∴ $R_C = 75K$ & $R_A = 25K$

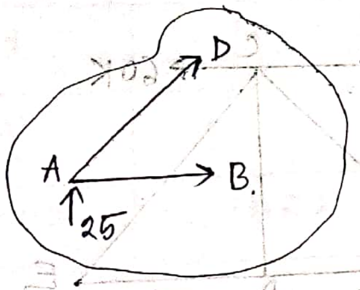
Member	Length	A	S	U_{BV}	$\frac{SU_{BV}L}{AE}$	
AD	$5\sqrt{5}$	1	-27.951	-0.28	$+2.92\times10^{-3}$	
AB	15	1	12.5	0.125	7.8125×10^{-4}	
BD	$10\sqrt{2}$	1	35.355	0.354	5.89×10^{-3}	
BE	10	1	75	0.75	0.01875	
BC	5	1	37.5	0.375	2.344×10^{-3}	
CE	$5\sqrt{5}$	1	-83.85	-0.84	0.026	
DE	10	1	37.5	0.375	4.6875×10^{-3}	$\Sigma = 0.0614$

For S:

Joint A:

$$AD = \frac{-25}{10} \times 5\sqrt{5}$$

$$= -27.951\ K$$



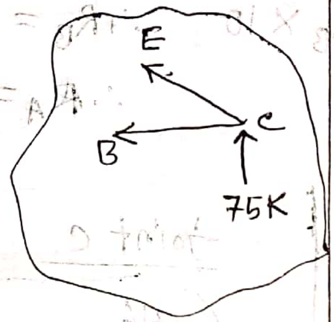
$$AB = \frac{27.951}{5\sqrt{5}} \times 5$$

$$= 12.5\ K$$

Joint C:

$$CE = \frac{-75}{10} \times 5\sqrt{5}$$

$$= -83.85\ K$$



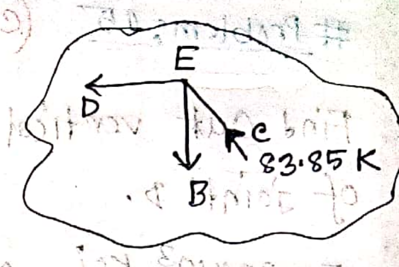
$$CB = \frac{83.85}{37.5} \times 5$$

$$= 37.5\ K$$

Joint E:

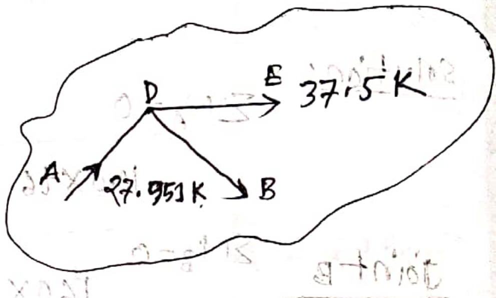
$$BE = \frac{83.85}{5\sqrt{5}} \times 10$$

$$= 75\ K$$



$$DE = \frac{83.85}{5\sqrt{5}} \times 5 = 37.5$$

Joint D:

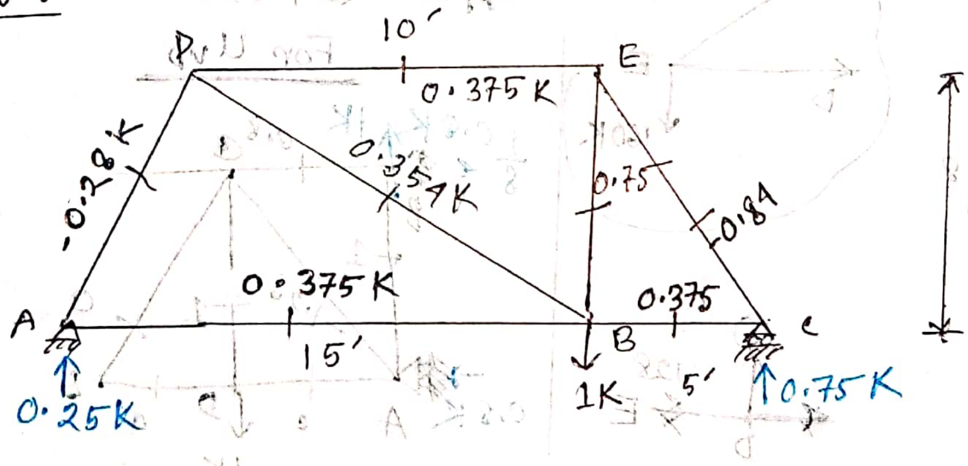


$$(DB)_v = \frac{27.951}{5\sqrt{5}} \times 10$$

$$= 25\ K$$

$$DB = \frac{25}{10} \times 10\sqrt{2} = 35.355\ K$$

For U_{BV}:

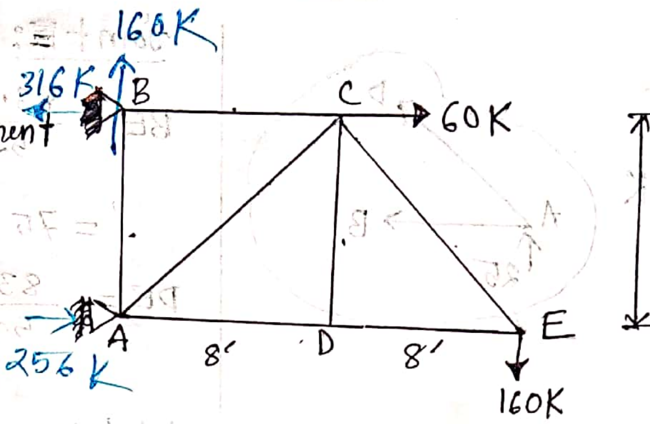


∴ vertical deflection at B, $\Delta_{BV} = 0.0614$ ft (↓)

Problem: 05 (CT-2012 Series)

Find out vertical displacement of joint D.

$E = 30 \times 10^3$ ksi, $A = 5\ in^2$



$\sqrt{10^2+8^2}$
$= 12.806'$

Solution:

$\sum M_A = 0$

$160 \times 16 + 60 \times 10 = R_B \times 10 \quad \therefore R_B = 316K$

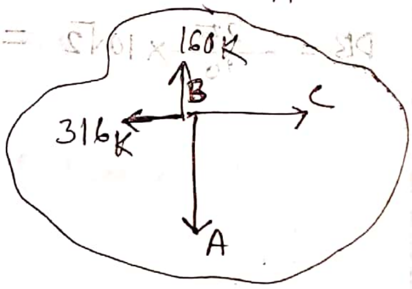
$\sum M_B = 0$

$160 \times 16 = R_A \times 10 \quad \therefore R_A = 256K$

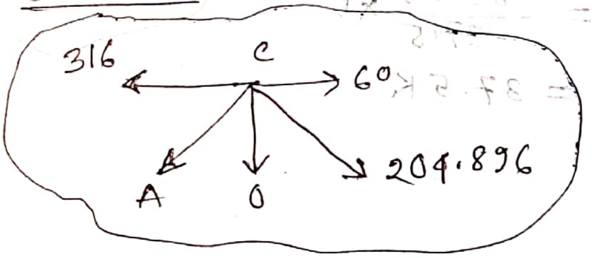
Joint B

BC = 316K

AB = 160K



Joint C



$CA = -204.896$

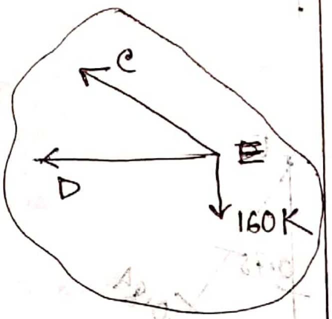
Joint E

$CE = \frac{160}{10} \times 12.806$

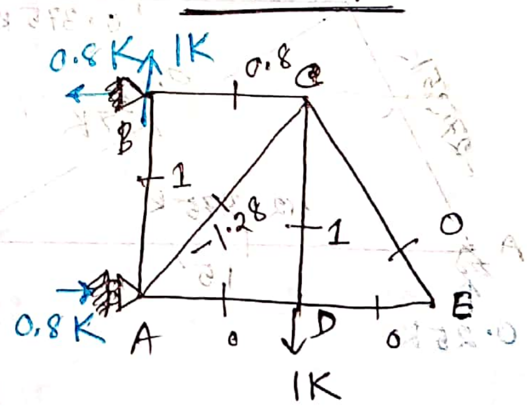
$= 204.896\ K$

$DE = \frac{-204.896}{12.806} \times 8$

$= -128\ K$



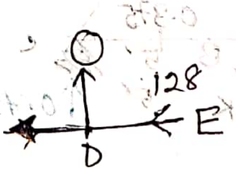
For U_{VD}



Joint D

DC = 0

AD = -128



Member	S	U_{Dv}	length	Area	SU_DL	
AB	160	1	10	5	1600	
AD	-128	0	8	5	0	
AC	-204.896	-1.28	12.806	5	3358.59	
BC	316	0.8	8	5	2528	
CD	0	1	10	5	0	
DE	-128	0	8	5	0	
CE	204.896	0	12.806	5	0	$\Sigma = 7486.59$

∴ Vertical displacement of joint D, $\Delta_{Dv} = \frac{7486.59}{30 \times 10^3 \times 5} = 0.05$ ft

(Ans.)

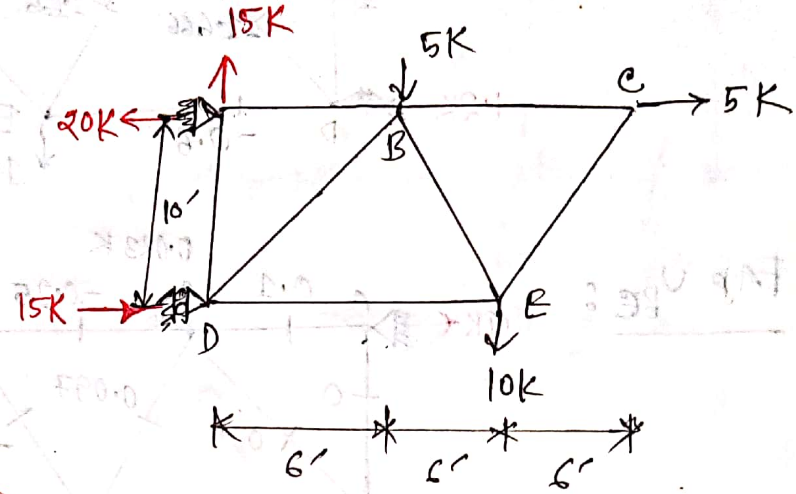
Truss

2017

\# Find vertical deflection of E and rotational deflection of BC member.

Given $A = 2\ in^2$ (all members)

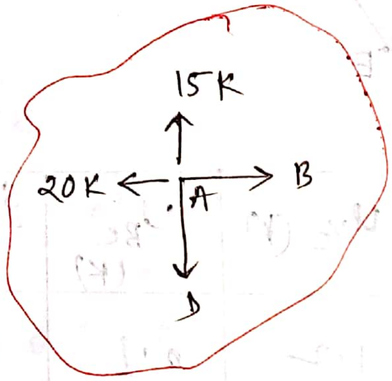
$E = 30 \times 10^3\ ksi$



Solution:

For S:

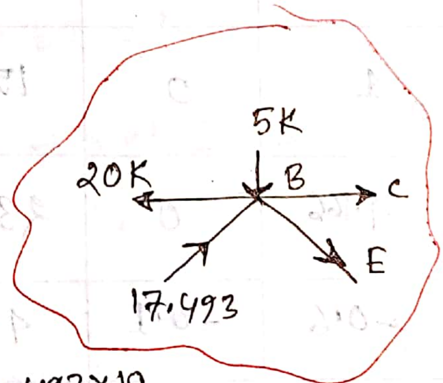
① joint A:



$AB = 20K$

$AD = 15K$

③ joint B:



$(BE)_v + 5 = \frac{17.493 \times 10}{\sqrt{10^2+6^2}}$

$\Rightarrow (BE)_v = (15-5) = 10K$

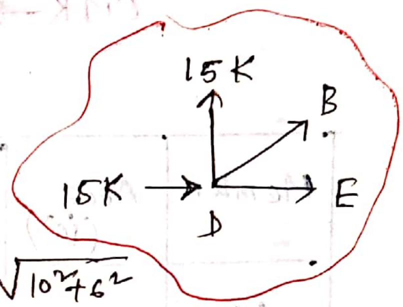
$\therefore BE = \frac{10}{10} \times \sqrt{10^2+6^2} = 11.66\ K$

Again

$BC = 20 - \frac{17.493}{\sqrt{10^2+6^2}} \times 6 - \frac{11.66 \times 6}{\sqrt{10^2+6^2}}$

$\therefore BC = (20+9-6) = 5K$

② joint D:



$(DB)_y + 15 = 0$

$(DB) = -\frac{15}{10} \times \sqrt{10^2+6^2}$

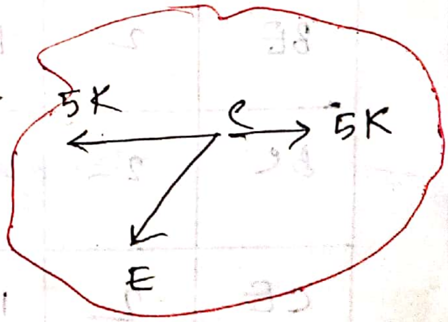
$= -17.493\ K$

$\therefore DE + 15 = (DB)_x$

$\Rightarrow DE = -15 + \frac{17.493 \times 6}{\sqrt{10^2+6^2}}$

$\therefore DE = -6\ K$

④ joint C

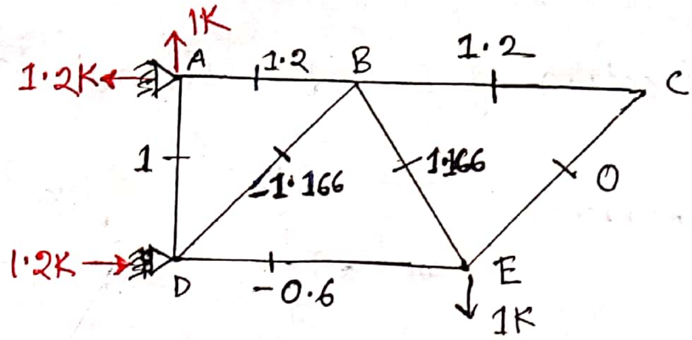


$\Sigma F_y = 0$

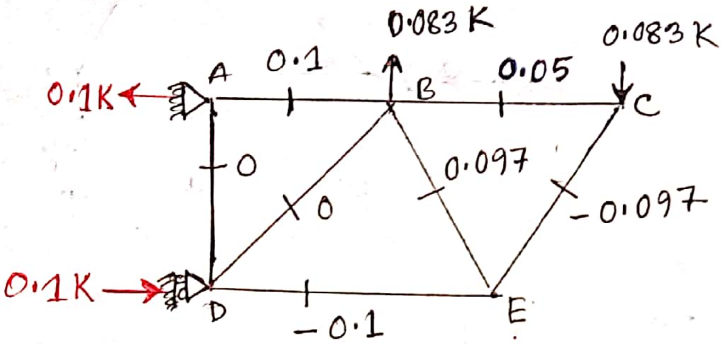
$(CE)_y = 0$

$\therefore CE = 0$

For U_{VE}:



For U_{BC}:



Member	Area (in^2)	Length (ft)	S (K)	U_{VE} (K)	U_{BC} (K)	$SU_{VE}L$	$SU_{BC}L$
AB	2	6	20	1.2	0.1	144	12
AD	2	10	15	1	0	150	0
BD	2	$\sqrt{10^2+6^2}$ =11.66	-17.496	-1.166	0	237.87	0
DE	2	12	-6	-0.6	-0.1	43.2	7.2
BE	2	11.66	11.66	1.166	0.097	158.52	13.19
BC	2	12	5	1.2	0.05	72	3
CE	2	11.66	0	0	-0.097	0	0
						Σ= 805.59	Σ= 35.41

$$\therefore \Delta_{EV} = \frac{\Sigma S U_{VE} L}{AE} = \frac{805.59}{2\times30\times10^3} = 0.0134 \text{ ft } (\downarrow)$$

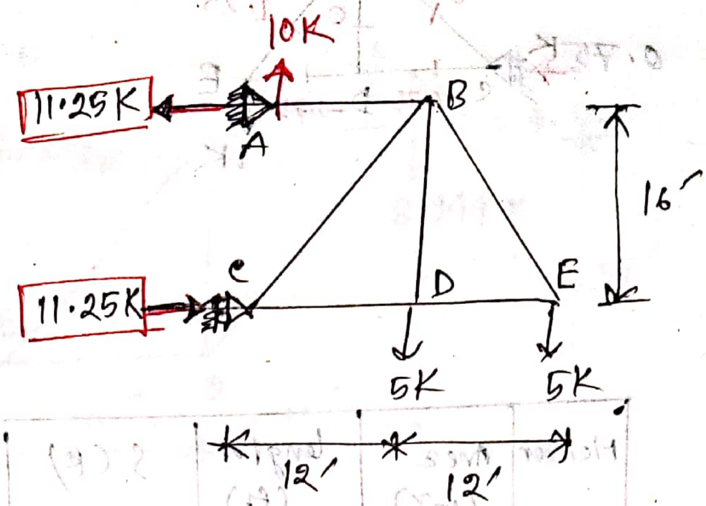
$$\theta_{BC} = \frac{\Sigma S U_{BC} L}{AE} = \frac{35.41}{2\times30\times10^3} = 0.0006 \text{ (rad.) } (\curvearrowright)$$

2015

Find vertical deflection of E and rotational deflection of member DE.

A = 3 in² (in all member)

E = 30×10⁶ psi

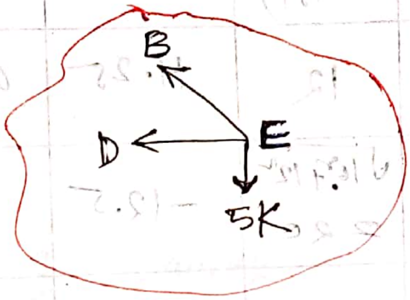


Solution:

For S:

(i) Joint A: $AB = 11.25K$

(ii) Joint E:



$(BE)_y = 5K$

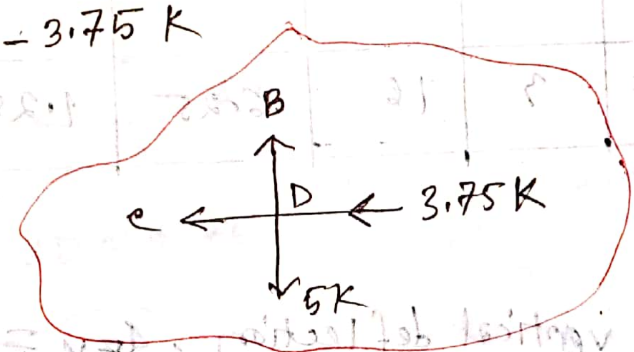
$\Rightarrow BE = \frac{5 \times \sqrt{16^2+12^2}}{16}$

$\therefore BE = 6.25\ K$

$(BE)_x + DE = 0$

$\Rightarrow DE = -\frac{6.25 \times 12}{\sqrt{16^2+12^2}} = -3.75\ K$

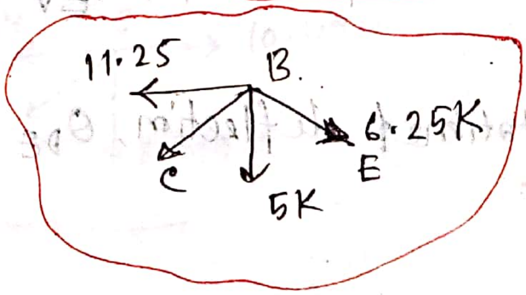
(iii) Joint D:



$DB = 5K$

$DC = -3.75K$

(iv) Joint B:

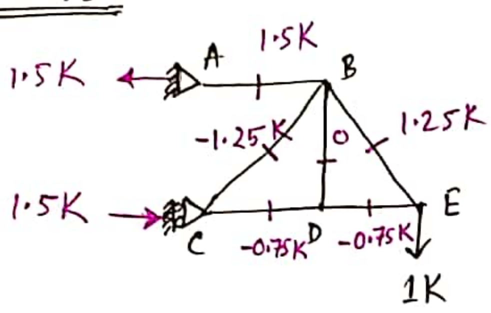


$(BC)_x + 11.25 = (BE)_x$

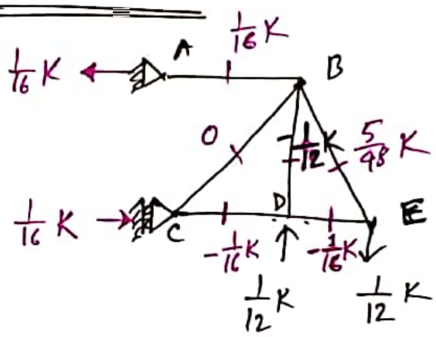
$\Rightarrow (BC)_x = \frac{6.25 \times 12}{\sqrt{16^2+12^2}} - 11.25$

$\therefore (BC)_x = -7.5$ $\quad \therefore BC = -\frac{7.5}{12} \times \sqrt{16^2+12^2} = -12.5$

For U_{VE} :



For U_{DE} :



Member	Area	Length (ft)	S (K)	U_{VE} (K)	U_{DE} (K)	$SU_{VE}L$	$SU_{DE}L$
AB	3	12	11.25	1.5	$\frac{1}{16}$	202.5	8.4375
BC	3	$\sqrt{16^2+12^2} = 20$	-12.5	-1.25	0	312.5	0
CD	3	12	-3.75	-0.75	$-\frac{1}{16}$	33.75	+2.8125
BD	3	16	5	0	$-\frac{1}{12}$	0	-6.6667
DE	3	12	-3.75	-0.75	$-\frac{1}{16}$	33.75	+2.8125
BE	3	16	6.25	1.25	$\frac{5}{48}$	125	10.4167

$\Sigma = 707.5$ $\quad \Sigma = 17.8125$

$\therefore$ vertical deflection, $\Delta_{VE} = \frac{\Sigma S U_{VE} L}{AE} = \frac{707.5}{3 \times 30 \times 10^3} = 0.0078\,6ft$ $(\downarrow)$

Rotational deflection, $\theta_{DE} = \frac{\Sigma S U_{DE} L}{AE} = \frac{17.8125}{3 \times 30 \times 10^3} = 0.00198$ rad $(\curvearrowright)$

(Ans.)

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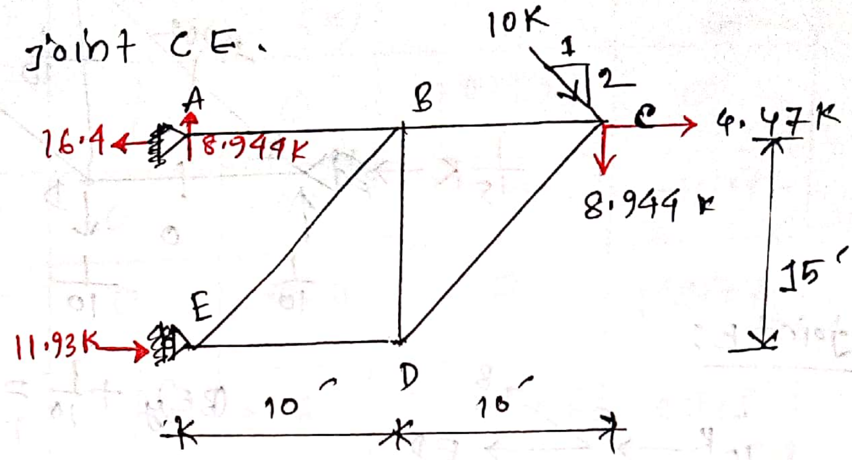
2014

Determine the rotational deflection of member DE and relative displacement between joint C E.

Given, $A = 2\ in^2$

$E = 30 \times 10^3\ ksi$

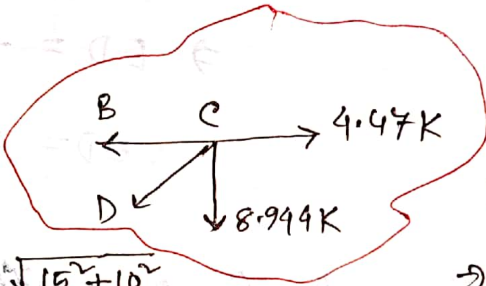


Solution:

For S:

(i) joint A: $AB = 16.4K$

(ii) joint C:



$(CD)_y + 8.944 = 0$

$\Rightarrow CD = -8.944 \times \frac{\sqrt{15^2+10^2}}{15}$

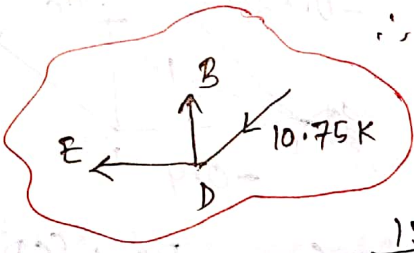
$\therefore CD = -10.75K$

$BC = (CD)_x + 4.47$

$\Rightarrow BC = 10.75 \times \frac{10}{\sqrt{15^2+10^2}} + 4.47$

$\therefore BC = 10.43\ K$

(iii) joint D:

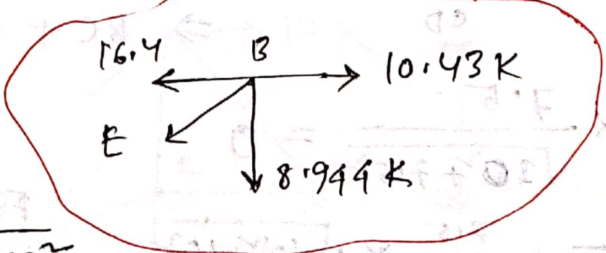


$DE = -10.75 \times \frac{10}{\sqrt{10^2+15^2}}$

$\therefore DE = -5.963\ K.$

$BD = 10.75 \times \frac{15}{\sqrt{15^2+10^2}} = 8.944\ K$

(iv) joint B:

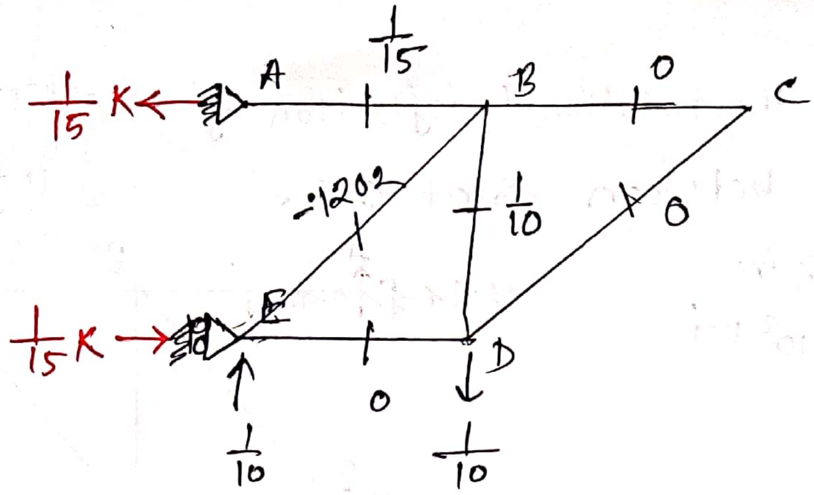


$(BE)_y - BD = 0$

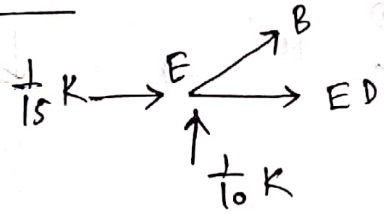
$\Rightarrow BE = -8.944 \times \frac{\sqrt{15^2+10^2}}{15}$

$\therefore BE = -10.75\ K$

For U_{DE} :



joint E:



$$(BE)_y + \frac{1}{10} = 0$$

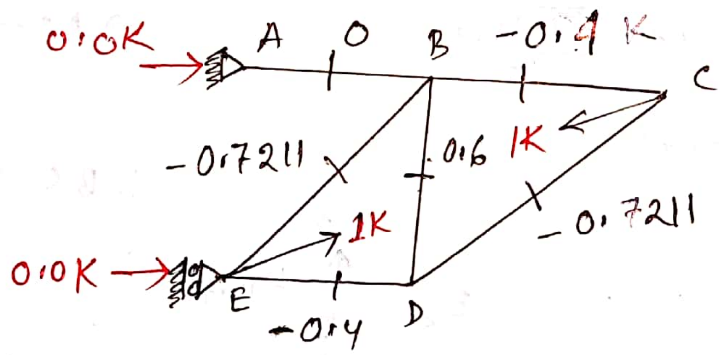
$$\Rightarrow BE = -\frac{1}{10} \times \frac{\sqrt{15^2+10^2}}{15} = -0.1202$$

$$(BE)_x + ED + \frac{1}{15} = 0$$

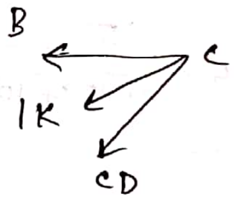
$$\Rightarrow ED = -\frac{1}{15} + 0.1202 \times \frac{10}{\sqrt{10^2+15^2}}$$

$$\therefore ED = 0 \text{ K}$$

For U_{CE} :



Joint C



$$BC + (CD)_x + 1 \times \frac{10}{\sqrt{7.5^2+10^2}} = 0$$

$$\Rightarrow BC + CD \times \frac{10}{\sqrt{15^2+10^2}} = -\frac{10}{12.5} \quad \text{......(1)}$$

$$(CD)_y + 1 \times \frac{7.5}{\sqrt{10^2+7.5^2}} = 0$$

$$\therefore CD = -\frac{7.5}{12.5} \times \frac{\sqrt{10^2+15^2}}{15}$$

$$\therefore CD = -0.7211$$

From eqn (1),

$$BC = -\frac{10}{12.5} + 0.7211 \times \frac{10}{18.03}$$

$$= -0.4$$

Member	Area (in²)	Length (ft)	S (K)	u_{DE} (K)	u_{CE} (K)	$S u_{DE} L$	$S u_{CE} L$
AB	2	10	16.4	$\frac{1}{15}$	0	10.933	0
BC	2	10	10.43	0	-0.4	0	-41.72
CD	2	$\sqrt{10^2+15^2}$ = 18.03	-10.75	0	-.7211	0	139.77
DB	2	15	8.944	$\frac{1}{10}$	0.6	13.416	80.5
DE	2	10	-5.963	0	-0.4	0	23.85
BE	2	18.03	-10.75	-.1202	-.7211	23.3	139.77

$\Sigma = 47.65$ $\Sigma = 342.17$

∴ Rotational deflection, $\theta_{DE} = \frac{\Sigma S u_{DE} L}{AE} = \frac{47.65}{30\times10^3 \times 2} = 7.94\times10^{-4}$ rad (↻)

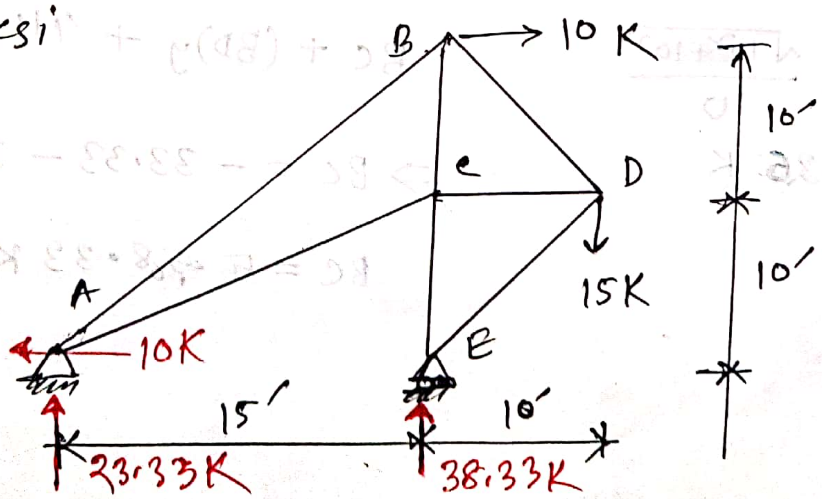
Relative displacement $= \frac{\Sigma S u_{CE} L}{AE} = \frac{139.77}{30\times10^3 \times 2} = 2.33\times10^{-3}$ ft (↖↘) (Ans.)

2012, 2006

Find horizontal deflection of point B, vertical deflection of joint D and rotational deflection of member CD.

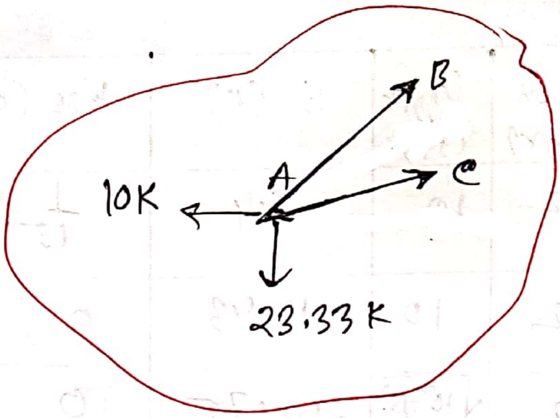
$A = 3\,in^2$

$E = 30\times10^3$ Ksi



Solution: For S:

① Joint A:



~~$(AB)_y = 23.33$~~

~~$\Rightarrow AB = 23.33 \times \frac{\sqrt{20^2+15^2}}{20}$~~

~~$\therefore AB =$~~

$(AB)_y + (AC)_y = 23.33$

$\Rightarrow AB \times \frac{20}{\sqrt{20^2+15^2}} + AC \times \frac{10}{\sqrt{10^2+15^2}} = 23.33 \quad \cdots\cdots (i)$

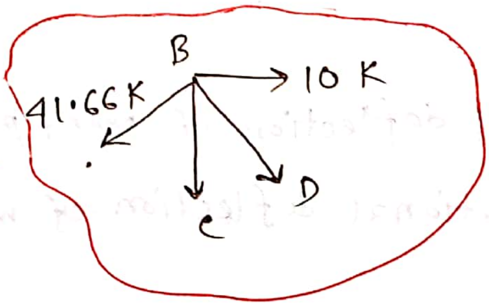
$(AB)_x + (AC)_x = 10$

$\Rightarrow AB \times \frac{15}{\sqrt{20^2+15^2}} + AC \times \frac{15}{\sqrt{10^2+15^2}} = 10 \quad \cdots\cdots (ii)$

Solving (i) & (ii), $AB = 41.66\ K$

$AC = -18.024\ K$

(ii) Joint B:



$(BD)_x + 10 = 41.66 \times \frac{15}{\sqrt{15^2+20^2}}$

$\Rightarrow (BD)_x = 25 - 10$

$\Rightarrow BD = 15 \times \frac{\sqrt{10^2+10^2}}{10}$

$\therefore BD = 21.21\ K$

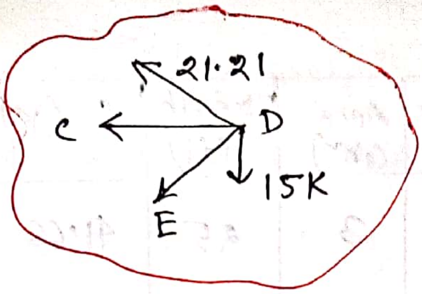
$BC + (BD)_y + 41.66 \times \frac{20}{\sqrt{15^2+20^2}} = 0$

$\Rightarrow BC = -33.33 - 21.21 \times \frac{10}{\sqrt{10^2+10^2}}$

$BC = -48.33\ K$

(iii) joint D:

$CD + (DE)_x + 21.21 \times \frac{10}{\sqrt{10^2+10^2}} = 0$



$\Rightarrow CD + DE \times \frac{10}{\sqrt{10^2+10^2}} = -15$ ..... (1)

$(DE)_y + 15 = 21.21 \times \frac{10}{\sqrt{10^2+10^2}}$

$\Rightarrow (DE)_y = 0$

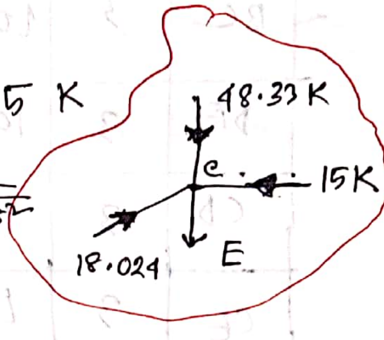
$\therefore DE = 0$

Now, From eqn (1),

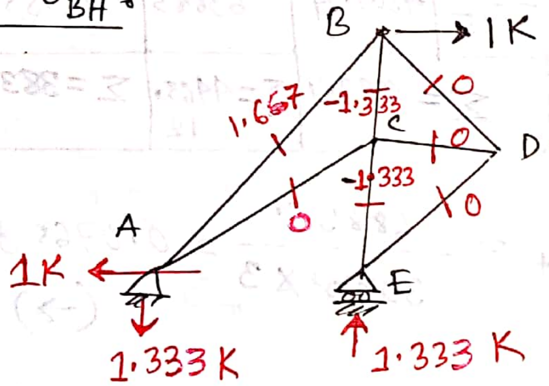
$CD = -15$ K

(iv) joint e: $CE + 48.33 = 18.024 \times \frac{10}{\sqrt{10^2+15^2}}$

$\therefore CE = -38.33$ K



For U_{BH}:



$\sqrt{20^2+15^2} = 25$

$\sqrt{10^2+15^2} = 18.03$

* For solving quickly follow joint free of main Truss:

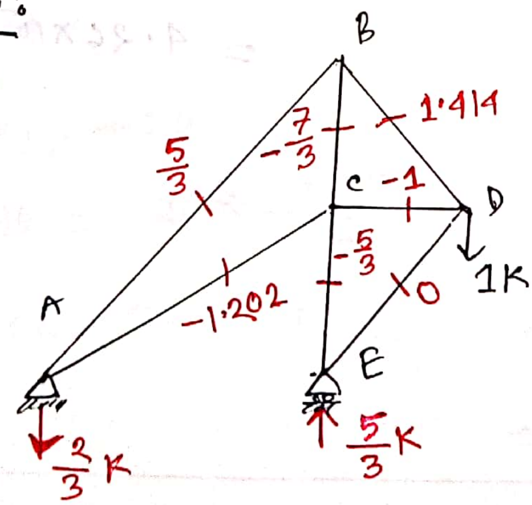
[Like:

(1) joint A: same

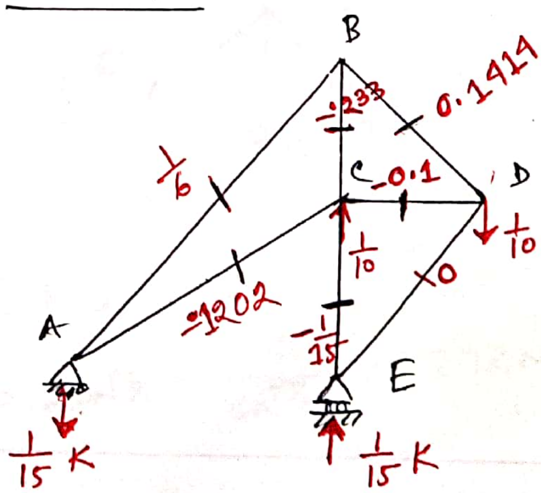
$AB \times \frac{20}{\sqrt{20^2+15^2}} + AC \times \frac{10}{\sqrt{10^2+15^2}} = 1.33$

উত্তর আগের 23.33 এর জায়গায়

For U_{DV}:



For U_{CD}:



Member	Area (in²)	Length (ft)	S (k)	U_{BH}	U_{DV}	U_{CD}	$SU_{BH}L$	$SU_{DV}L$	$SU_{BH}L$
AB	3	25	41.66	1.667	5/3	$\frac{1}{6}$	1736.2	1735.83	148.8
AC	3	18.03	-18.024	0	-1.202	-0.1202	0	390.62	39.062
BD	3	$10\sqrt{2}$	21.11	0	1.414	0.1414	0	422.14	42.214
BC	3	10	-48.33	-1.333	$-\frac{7}{3}$	-0.233	644.24	1127.7	112.61
DE	3	$10\sqrt{2}$	0	0	0	0	0	0	0
CD	3	10	-15	0	-1	-0.1	0	150	15
CE	3	10	-38.33	-1.333	$-\frac{5}{3}$	$-\frac{1}{15}$	4509.94	63883	25.553
							Σ = 6883.4	Σ = 4465.12	Σ = 383.24

Horizontal deflection at B, $\Delta_{BH} = \frac{6883.4}{30\times10^3\times3} = 0.0765$ ft (→)

Vertical deflection at C, $\Delta_{CV} = \frac{4465.12}{30\times10^3\times3} = 0.05$ ft (↓)

Rotational deflection at CD member, $\theta_{CD} = \frac{383.24}{30\times10^3\times3}$

$= 4.26\times10^{-3}$ rad. (↻)

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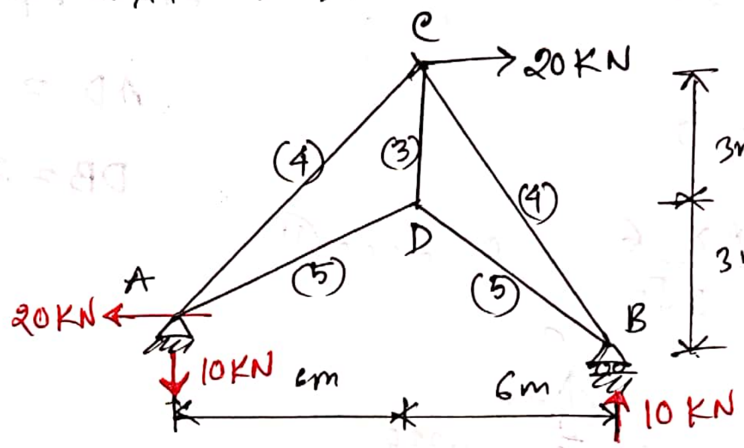
2011

By unit load method, determine the horizontal deflection at support B and vertical deflection at D due to the applied load shown in figure; Number in parentheses are areas in $\times 10^{-4}$ m^2 and $E = 20 \times 10^6$ KN/m^2



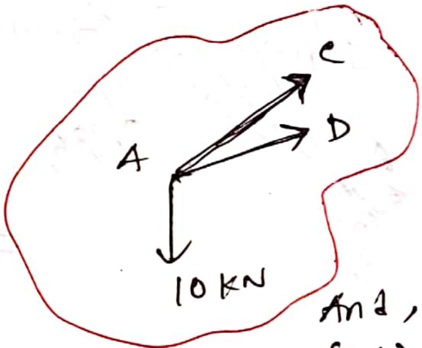
$\sqrt{6^2+6^2} = 6\sqrt{2}$

$\sqrt{6^2+3^2} = 3\sqrt{5}$

Solution:

For s :

① Joint A:



$(AD)_x + (AC)_x = 20$

$\Rightarrow AD \times \frac{6}{3\sqrt{5}} + AC \times \frac{6}{6\sqrt{2}} = 20 \ldots\ldots ①$

And,

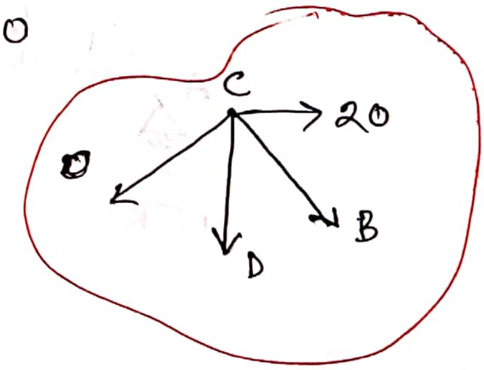
$(AD)_y + (AC)_y = 10$

$\Rightarrow AD \times \frac{3}{3\sqrt{5}} + AC \times \frac{6}{6\sqrt{2}} = 10$ ......(II)

From eqn ① & (II) we obtain, $AD = 22.36$ KN

$AC = 0$

(II) Joint C:



$(CB)_x + 20 = 0$

$\Rightarrow CB = -20 \times \frac{6\sqrt{2}}{6}$

$\therefore CB = -20\sqrt{2}$ KN

$CD = (CB)_y$

$\Rightarrow CD = 20\sqrt{2} \times \frac{6}{6\sqrt{2}} = 20$ KN

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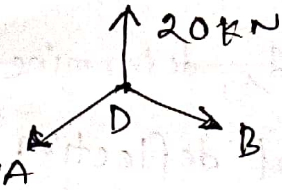
(iv) joint D:



$(AD)_y + (DB)_y = 20$

$\Rightarrow AD \times \frac{3}{3\sqrt{5}} + DB \times \frac{3}{3\sqrt{5}} = 20 \quad \cdots\cdots (I)$

And,

$(AD)_x - (DB)_x = 0$

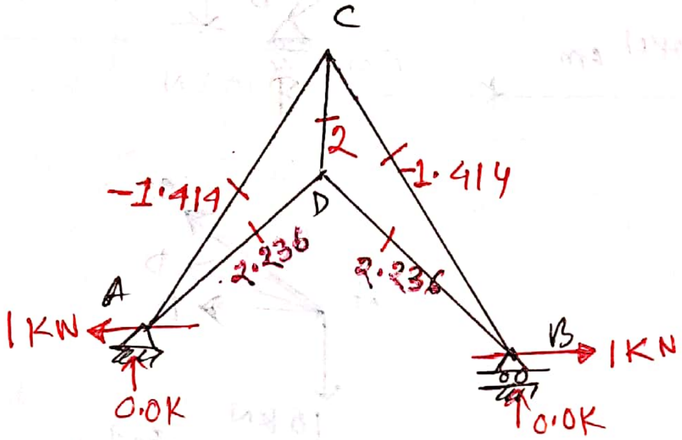
$\Rightarrow AD \times \frac{6}{3\sqrt{5}} - \frac{DB \times 6}{3\sqrt{5}} = 0 \quad \cdots\cdots (II)$

From eqn (I) & (II)

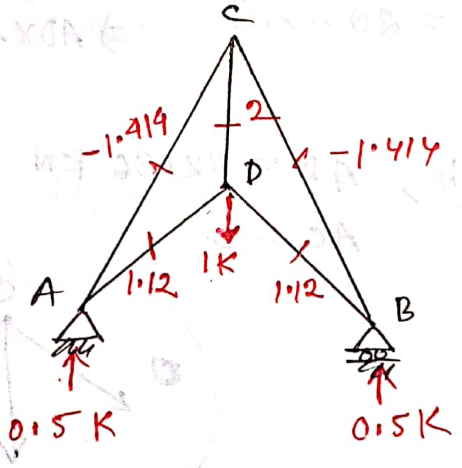
$AD = 22.36$ KN

$DB = 22.36$ KN

For U_{BH}:



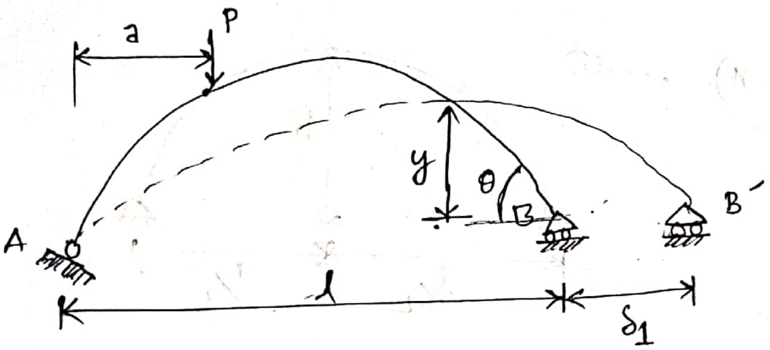
For U_{DV}:

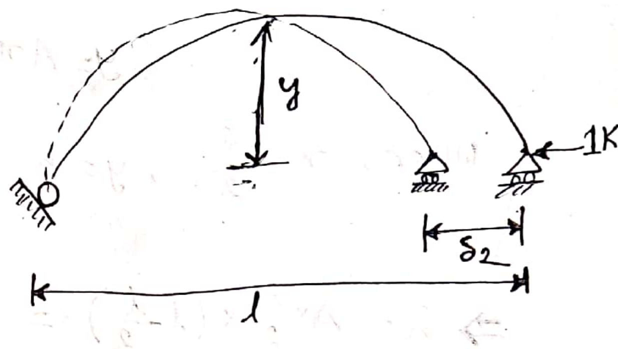


DO Table & calculate deflections.

Two-Hinged Arch

Find horizontal thrust for the parabolic two hinged Arch.





We know,

$$\delta = \int_0^l \frac{Mm}{EI}\,dx$$

$$\delta_1 = \int_0^l \frac{M_s \cdot y \cdot 1}{EI}\,ds$$

$$\delta_2 = \int_0^l \frac{y^2}{EI}\,ds \quad (\text{for } 1K)$$

$\therefore$ for H, $\quad \delta_2 = H\int_0^l \frac{y^2}{EI}\,ds$

For equilibrium, $\quad \delta_1 = \delta_2$

$$\int_0^l \frac{M_s\,y}{EI}\,ds = H\int_0^l \frac{y^2}{EI}\,dy$$

$$\Rightarrow H = \frac{\int_0^l \frac{M_s y}{EI}\,ds}{\int_0^l \frac{y^2}{EI}\,ds}$$

$$\Rightarrow H = \frac{\int_0^l \frac{M_s y}{EI_c \sec\theta} \times dx\sec\theta}{\int_0^l \frac{y^2}{EI_c\sec\theta} \times dx\sec\theta}$$

$\cos\theta = \frac{dx}{ds}$

$\therefore ds = dx\sec\theta$

$I = I_c \sec\theta$

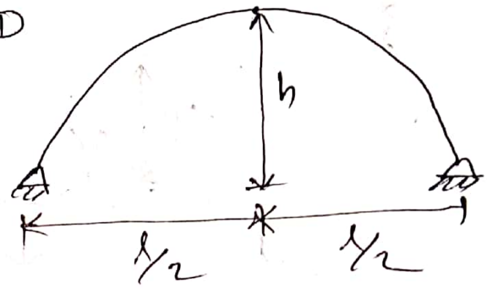
I_c = moment of inertia at crown

$$\therefore H = \frac{\int_0^l M_s\, y\, dx}{\int_0^l y^2\, dx}$$

Find out the height at any point of the two hinged Arch:

We know, parabolic equation,

$$y = Ax(l-x) \quad \ldots (1)$$



when, $x = \frac{l}{2}$, $y = h$

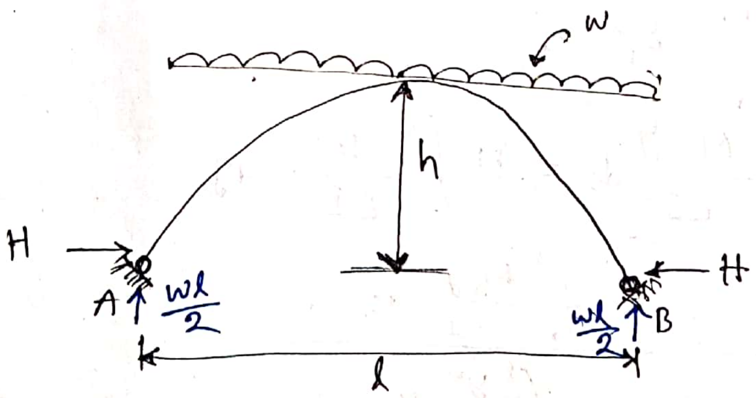
$$\Rightarrow h = A \times \frac{l}{2} \times \left(l - \frac{l}{2}\right) = \frac{Al^2}{4}$$

$$\therefore A = \frac{4h}{l^2}$$

The value of A is replaced in equation (1)

$$y = \frac{4hx}{l^2}(l-x)$$

Find out the horizontal thrust of Parabolic two hinge arch which subjected to uniform distributed load.



We know,

$$H = \frac{\int_0^l M_s\, y\, dx}{\int_0^l y^2\, dx}$$

$$\int_0^l M_s\, y\, dx = \int_0^l \left(\frac{wl}{2}x - \frac{wx^2}{2}\right) y\, dx$$

$$= \int_0^l \left(\frac{wl}{2}x - \frac{wx^2}{2}\right) \times \frac{4hx(l-x)}{l^2}\, dx$$

$$= \frac{2hw}{l^2} \times \int_0^l (lx - x^2)\,(lx - x^2) \cdot dx$$

$$= \frac{2hw}{l^2} \times \int_0^l (l^2x^2 - 2lx^3 + x^4)\, dx$$

$$= \frac{2hw}{l^2} \times \left[l^2 \times \frac{x^3}{3} - 2l \times \frac{x^4}{4} + \frac{x^5}{5} \right]_0^l$$

$$= \frac{2hw}{l^2} \times \left[l^2 \times \frac{l^3}{3} - 2l \times \frac{l^4}{4} + \frac{l^5}{5} \right]_0^l$$

$$= \frac{2hw}{l^2} \times \left[\frac{l^5}{3} - \frac{l^5}{2} + \frac{l^5}{5} \right]$$

$$= \frac{2hw}{l^2} \times \left[\frac{10l^5 - 15l^5 + 6l^5}{30} \right]$$

$$= \frac{2hw}{l^2} \times \frac{l^5}{30}$$

$$\therefore \int_0^l M_s\, y\, dx = \frac{hwl}{15}$$

$$\int_0^l y^2\,dx = \int_0^l \left[\frac{4hx(l-x)}{l^2}\right]^2 dx$$

$$= \frac{16h^2}{l^4}\int_0^l \left[x^2(l-x)^2\right] dx$$

$$= \frac{16h^2}{l^4}\times\int_0^l \left[x^2l^2 - 2lx^3 + x^4\right]$$

$$= \frac{16h^2}{l^4}\times\left[l^2\times\frac{x^3}{3} - 2l\times\frac{x^4}{4} + \frac{x^5}{5}\right]$$

$$= \frac{16h^2}{l^4}\times\left[\frac{l^5}{3} - \frac{l^5}{2} + \frac{l^5}{5}\right]$$

$$= \frac{16h^2}{l^4}\times\frac{l^5}{30}$$

$$\int_0^l y^2\,dx = \frac{8h^2l}{15}$$

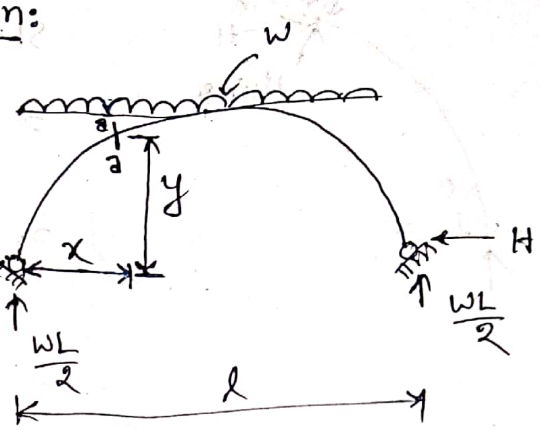
$$\therefore H = \frac{\int_0^l M_s \cdot y\,dx}{\int_0^l y^2\,dx}$$

$$H = \frac{hwl^3/15}{8h^2l/15} = \frac{wl^2}{8h}$$

$$\boxed{\therefore H = \frac{wl^2}{8h}}$$

Find out Bending moment at any section:

$$M_{a-a} = \frac{wl}{2} \times x - \frac{wl^2}{8h} \times y - \frac{wx^2}{2}$$



$$= \frac{wl}{2} \times x - \frac{wl^2}{2 \cdot 8h} \times \frac{4hx(l-x)}{l^2} - \frac{wx^2}{2}$$

$$= \frac{wlx - wx(l-x) - wx^2}{2}$$

$$= \frac{wlx - wlx + wx^2 - wx^2}{2}$$

$\therefore M_{a-a} = 0$

Hence, Bending moment at any section due to uniformly distributed o is zero.

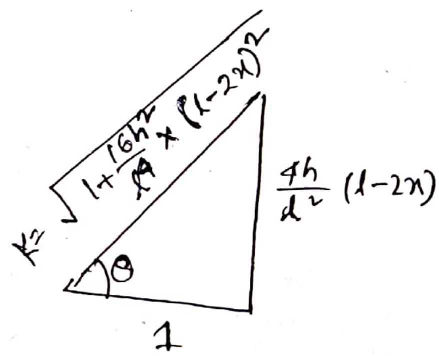
Find out shear force at any section:

We know,

$$y = \frac{4hx}{l^2}(l-x)$$

$$\frac{dy}{dx} = \frac{4h}{l^2}\int_0^l (lx - x^2)$$

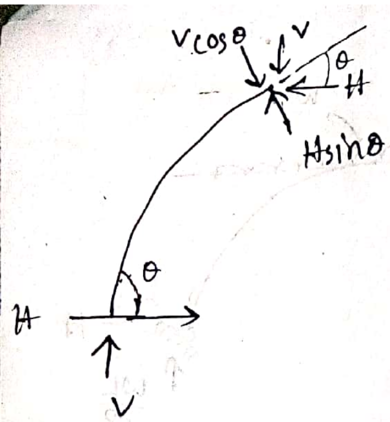
$$\Rightarrow \tan\theta = \frac{4h}{l^2} \times (l-2x) = \frac{\text{লম্ব}}{\text{ভূমি}}$$



$$\therefore \text{অতিভুজ} = \sqrt{1 + \left[\frac{4h}{l^2} \times (l-2x)\right]^2}$$

$$= \sqrt{1 + \frac{16h^2}{l^4} \times (l-2x)^2}$$

$$\therefore \sin\theta = \frac{\frac{4h}{l^2} \times (l-2x)}{K} \quad \text{and} \quad \cos\theta = \frac{1}{K}$$



Shear force at that section

$$= V\cos\theta - H\sin\theta$$

$$= \left(\frac{wl}{2} - wx\right) \times \frac{1}{K} - \frac{wl^2}{8h} \times \frac{4h \times (l-2x)}{l^2 K}$$

$$= \frac{wl - 2wx - wl + 2wx}{2K}$$

$$= 0$$

Hence, shear force at any section due to uniformly distributed load is zero.

Normal thrust at any section:

Normal thrust at that section $= V\sin\theta + H\cos\theta$

$$= \left(\frac{wl}{2} - wx\right) \times \frac{4h(l-2x)}{K} + \frac{wl^2}{8h} \times \frac{1}{K}$$

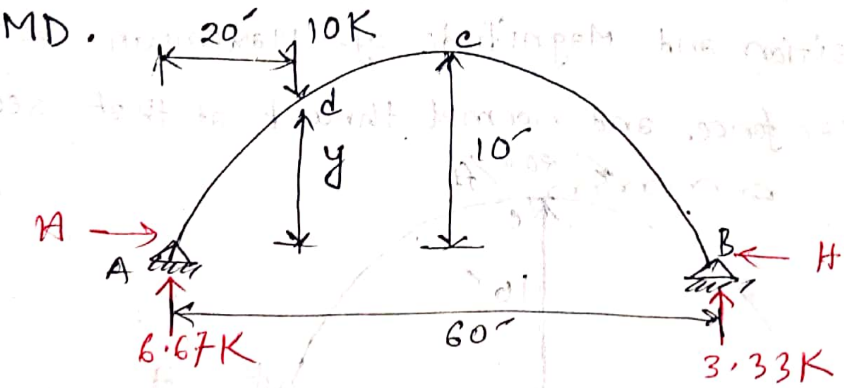
$$= \frac{(wl - 2wx) \times 4h(l-2x)}{2K} + \frac{wl^2}{8hK}$$

$$= \frac{16h^2(wl^2 - 2wx)(l-2x) + wl^2}{8hK}$$

$$= \frac{16h^2(wl^3 - 2wlx - 2wl^2x + 4wx^2) + wl^2}{8hK}$$

$$= \frac{16wl^3h^2 - 32wlh^2x - 32wl^2hx + 64wh^2x^2 + wl^2}{K}$$

Draw BMD.



Solution:

Here, $y = \frac{4hx}{l^2}(l-x) = \frac{4 \times 10 \times x}{60^2} \times (60-x)$

$\therefore y = \frac{x(60-x)}{90}$

$$\int_0^l M_s y\,dx = \int_0^{20} 6.67x \times \frac{x(60-x)}{90}\,dx + \int_0^{40} 3.33x \times \frac{x(60-x)}{90}\,dx$$

$= 8893.33 + 23680$

$= 32573.33$

$$\int_0^l y^2\,dx = \int_0^{60} \left[\frac{x(60-x)}{90}\right]^2 dx$$

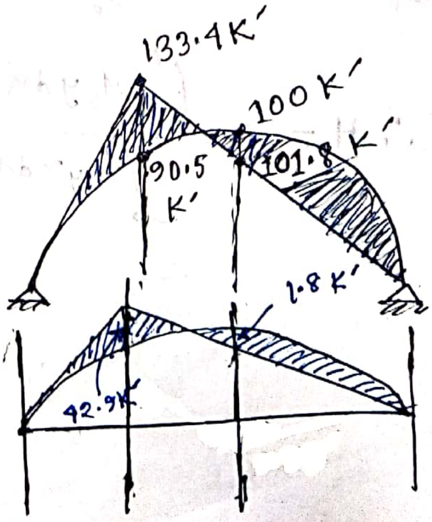
$= 3200$

$$\therefore H = \frac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx} = \frac{32573.33}{3200} = 10.18\text{ K}$$

Now, when $x = 20'$, $y = \frac{20 \times (60-20)}{90} = 8.89'$

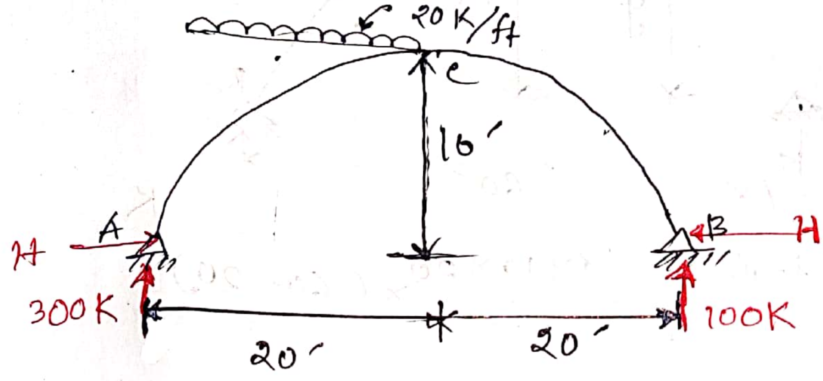
$\therefore M_d = (6.67 \times 20 - 10.18 \times 8.89) = 42.9\text{ K}'$

$\therefore M_c = (3.33 \times 30 - 10.18 \times 10) = -1.9\text{ K}'$



BMD

Find the position and Magnitude of Maximum bending Moment. and Find shear force, and normal thrust at that section



<u>Solution:</u> Here, $y = \frac{4hx}{l^2}(l-x) = \frac{4\times10\times x}{40^2}\times(40-x)$

$$\therefore y = \frac{x(40-x)}{40}$$

$$\int_0^l M_s\, y\, dx = \int_0^{20}\left(300x - 20\times\frac{x^2}{2}\right)\times\frac{x(40-x)}{40}\,dx + \int_0^{20} 100x \times \frac{x(40-x)}{40}\,dx$$

$$= 260000 + 166666.6667$$

$$= 426666.6667$$

$$\int_0^l y^2\, dx = \int_0^{40}\left[\frac{x(40-x)}{40}\right]^2 dx = 2133.33$$

$$\therefore H = \frac{\int_0^l M_s\, y\, dx}{\int_0^l y^2\, dx} = \frac{426666.6667}{2133.33} = 200\ K$$

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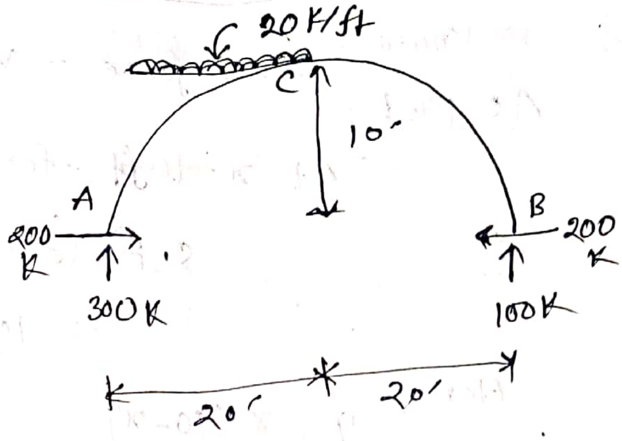
$$M_{AC} = 300x - 20\frac{x^2}{2} - 200y$$



$$\therefore M_{AC} = 300x - 10x^2 - 200 \times \frac{x(40-x)}{40}$$

$$\Rightarrow \frac{dM_{AC}}{dx} = 300 - 20x - 5(40-2x)$$

For maximum moment, $\frac{dM_{AC}}{dx} = 0$

$$300 - 20x - 5(40-2x) = 0$$

$$\Rightarrow 300 - 20x - 200 + 10x = 0$$

$$\Rightarrow 10x = 100$$

$$\therefore x = 10'$$

$$\therefore M_{AC} = 300 \times 10 - 10 \times 10^2 - 200 \times \frac{10 \times (40-10)}{40}$$

$$= 500 \text{ K-ft} \quad (+\text{ve maximum})$$

Again,

$$M_{BC} = 100x - 200y = 100x - \frac{200 \times x(40-x)}{40}$$

$$\frac{dM_{BC}}{dx} = 100 - 5(40-2x)$$

For Maximum moment, $\frac{dM_{BC}}{dx} = 100 - 5(40-2x) = 0$

$$\Rightarrow 100 - 200 + 10x = 0$$

$$\therefore x = 10'$$

$$\therefore M_{BC} = 100 \times 10 - 200 \times \frac{10 \times (40-10)}{40} = -500 \text{ K-ft} \quad (-\text{ve maximum})$$

We know, Shear force $= V\cos\theta - H\sin\theta$ and, Normal thrust $= V\sin\theta + H\cos\theta$

AC part:

$\therefore$ At $x = 10$ ft from left

$$S.F = (300 - 20\times10)\times\cos\theta - 200\times\sin\theta$$

$$= 100\cos\theta - 200\sin\theta$$

and, Normal thrust $= 100\sin\theta + 200\cos\theta$

Here, $y = \frac{x(40-x)}{40}$

$$\frac{dy}{dx} = \tan\theta = \frac{40-2x}{40}$$

$$\therefore \theta = \tan^{-1}\frac{40-2\times10}{40} = 26.565^\circ$$

$\therefore \sin\theta = 0.447$ and $\cos\theta = 0.894$

$$\therefore S.F = (100\times.894 - 200\times0.447) = 0\text{ K}$$

Now, Normal thrust $= (100\times0.447 + 200\times0.894) = 223.5$ K [(300 − 20×10) ↑ 100]

Again,

BC Part: At $x = 10$ ft from right

$$S.F = V\cos\theta - H\sin\theta$$

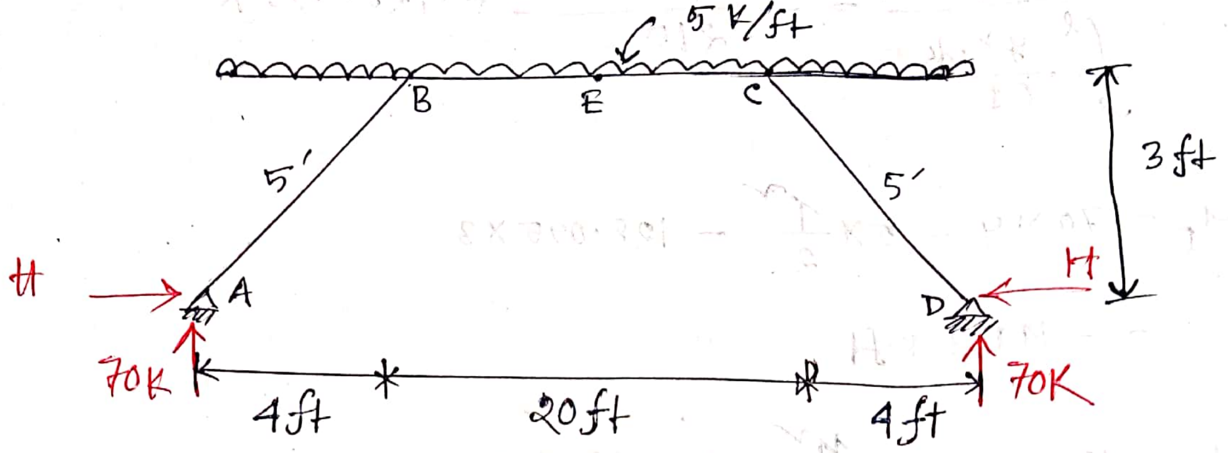
$$S.F = (100\times0.894 - 200\times0.447) = 0\text{ K}$$

Now, Normal thrust $= V\sin\theta + H\cos\theta = (100\times0.447 + 200\times.894)$

$$N.T = 223.5\text{ K}$$

(Ans.)

Find the Horizontal thrust and Draw BMD



Solution:

AB, CD: $\frac{y}{x} = \frac{3}{4}$

$\Rightarrow y = \frac{3x}{4}$

BE, CE: $y = 3$

and, $ds = dx$

again,

$\frac{ds}{dx} = \frac{5}{4} \Rightarrow ds = \frac{5}{4} dx$

$$\int_0^l \frac{M_s y}{EI} ds = \frac{1}{EI} \times 2 \int_0^4 \left(70x - \frac{5x^2}{2}\right) \times \left(\frac{3x}{4}\right) \times \left(\frac{5}{4} dx\right) +$$

$$\frac{1}{EI} \times 2 \int_4^{14} \left(70x - \frac{5x^2}{2}\right) \times 3 \times dx$$

$$= \frac{2}{EI} \times \left[1250 + 12200\right]$$

$$= \frac{2 \times 13450}{EI} = \frac{26900}{EI}$$

$$\int_0^l \frac{y^2 ds}{EI} = \frac{1}{EI} \times 2 \int_0 \left(\frac{3}{4}x\right)^2 \times \left(\frac{5}{4} dx\right) + \frac{1}{EI} \times 2 \int_4^{14} (3)^2 \times dx$$

$$= \frac{2}{EI} \times (15 + 90) = \frac{210}{EI}$$

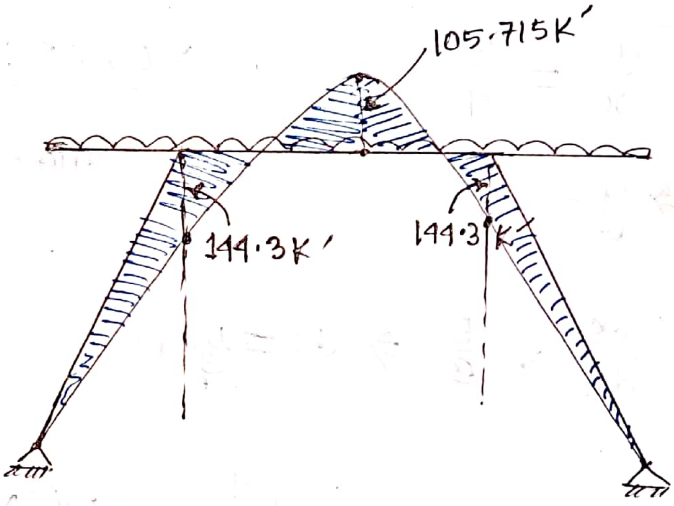
$$H = \frac{\int_0^l \frac{M_s y \, ds}{EI}}{\int_0^l \frac{y^2 \cdot ds}{EI}} = \frac{26900}{210} = 128.095 \text{ K}$$

$$M_B = 70 \times 4 - 5 \times \frac{4^2}{2} - 128.095 \times 3$$

$$= -144.3 \text{ K-ft}$$

$$M_E = 70 \times 14 - 5 \times \frac{14^2}{2} - 128.095 \times 3$$

$$= 105.715 \text{ K-ft}$$



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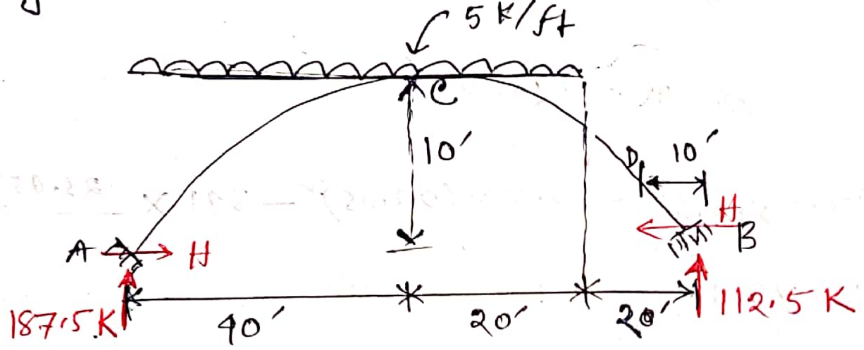
2011

Determine the position and magnitude of maximum bending for the parabolic arch shown below. Also find out the shear force and Normal thrust of section 10 ft from right support. Neglect the rib shortening.



Solution:

$$y = \frac{4hx(l-x)}{l^2} = \frac{4\times 10\times x(80-x)}{80^2} = \frac{x(80-x)}{160}$$

$$\int_0^l M_s y\,dx = \int_0^{60}(187.5x - 5\frac{x^2}{2})\times(\frac{80x-x^2}{160})\,dx + \int_0^{20} 112.5x \times (\frac{80x-x^2}{160})\,dx$$

$$= 1333125 + 121875$$

$$= 1455000$$

$$\int_0^l y^2\,dx = \int_0^{80}(\frac{80-x^2}{160})^2\,dx = 4266.67$$

$$\therefore H = \frac{\int_0^l M_s y\,dx}{\int_0^l y^2\,dx} = \frac{1455000}{4266.67} = 341\text{ K}$$

For AC Part: $M_{AC} = 187.5x - \frac{5x^2}{2} - 341\times y$

$$= 187.5x - 2.5x^2 - 341\times\frac{x(80-x)}{160}$$

For maximum moment, $\frac{dM_{AC}}{dx} = 187.5 - 5x - 341 \times \frac{80-2x}{160} = 0$

$\Rightarrow 5x = 187.5 - 2.13125(80-2x)$

$\Rightarrow 5x - 4.2625x = 17$

$\Rightarrow x = 23.05'$

$\therefore M_{AC} = 187.5 \times 23.05 - 2.5 \times (23.05)^2 - 341 \times \frac{23.05 \times (80-23.05)}{160}$

$= 195.93\ K'$ (+ve maximum)

For BC part:

$M_{BC} = 112.5x - \frac{5(x-20)^2}{2} - 341 \times \frac{80x - x^2}{160}$

For maximum moment, $\frac{dM_{BC}}{dx} = 0$

$\therefore 112.5 - 2.5(2x-40) - 341 \times \frac{80-2x}{160} = 0$

$\Rightarrow x = 56.95'$

$\therefore M_{BC} = 112.5 \times 56.95 - \frac{5(56.95-20)^2}{2} - 341 \times \frac{80 \times 56.95 - 56.95^2}{160}$

$= 195.93\ K'$

Now,

$y = \frac{80x - x^2}{160}$

$\Rightarrow \frac{dy}{dx} = \tan\theta = \frac{80-2x}{160}$

At 10ft from Right. $\therefore \tan\theta = \frac{80-2\times10}{160} = 0.375$

$\therefore \theta = 20.556$ $\therefore \sin\theta = 0.35$

and, $\cos\theta = 0.94$

∴ Shear force at that section,

$S.F = V\cos\theta - H\sin\theta$

$= 112.5 \times 0.94 - 341 \times 0.35$

$= -13.6$ K

and

Normal thrust, $N.T = V\sin\theta + H\cos\theta$

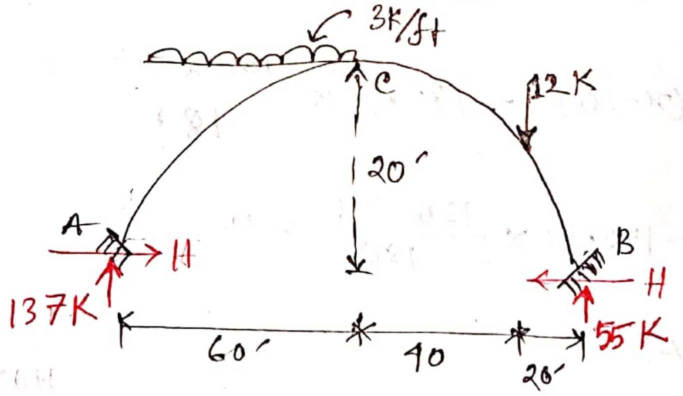
$= 112.5 \times 0.35 + 341 \times 0.94$

$= 359.915$ K

(Ans.)

2012

Determine position and magnitude of maximum bending moment. Find shear force and normal thrust at the position of applied concentrated load. Neglect the rib shortening.



Solution:

$$y = \frac{4hx(l-x)}{l^2} = \frac{4 \times 20 \times x(120-x)}{120^2} = \frac{120x - x^2}{180}$$

$$\int_0^l M_s\, y\, dx = \int_0^{60}\left(137x - \frac{3x^2}{2}\right) \times \left(\frac{120x-x^2}{180}\right)dx + \int_0^{20} 55x \times \left(\frac{120x-x^2}{180}\right)dx$$

$$+ \int_{20}^{60}\left[55x - 12(x-20)\right] \times \left(\frac{120x-x^2}{180}\right)dx$$

$$= (2166000 + 85555.56 + 1386666.67) = 3638222.226$$

$$\int_0^l y^2 dx = \int_0^{120} \left(\frac{120x - x^2}{180}\right)^2 dx = 25600$$

$$\therefore H = \frac{\int_0^l M_s y \, dx}{\int_0^l y^2 dx} = \frac{3638222.226}{25600} = 142.12 \text{ K}$$

<u>For AC part :</u>

$$M_{AC} = 137x - \frac{3}{2}x^2 - 142.12 \times \frac{120x - x^2}{180}$$

For maximum,

$$\frac{dM_{AC}}{dx} = 137 - 1.5 \times 2x - 142.12 \times \frac{120 - 2x}{180} = 0$$

$$\Rightarrow x = 29.74'$$

$$\therefore M_{AC} = 137 \times 29.74 - 1.5 \times (29.74)^2 - 142.12 \times \frac{120 \times 29.74 - (29.74)^2}{180}$$

$$= 628.25 \text{ K-ft}$$ (+ve maximum)

<u>For BC Part:</u>

$$M_{BC} = 55x - 12(x - 20) - 142.12 \times \frac{120x - x^2}{180}$$

For maximum,

$$\frac{dM_{BC}}{dx} = 55 - 12 - 142.12 \times \frac{120 - 2x}{180} = 0$$

$$\Rightarrow x = 32.77'$$

$$\therefore M_{BC} = 55 \times 32.77 - 12 \times (32.77 - 20) - 142.12 \times \frac{120 \times 32.77 - 32.77^2}{180}$$

$$= -607.86 \text{ K ft}$$ (−ve maximum)

Now, $y = \frac{120x - x^2}{180}$; At 20 ft from Right

$$\tan\theta = \frac{dy}{dx} = \frac{120 - 2x}{180} \Rightarrow \theta = \tan^{-1}\frac{120 - 2 \times 20}{180} = 23.75°$$

$$\therefore \sin\theta = 0.406$$

and, $\cos\theta = 0.914$

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∴ Shear force at that section,

$S.F = V\cos\theta - H\sin\theta$

$= (55 \times 0.914 - 142.12 \times 0.406)$

$= -7.43$ K

Normal thrust at that section,

$N.T = V\sin\theta + H\cos\theta$

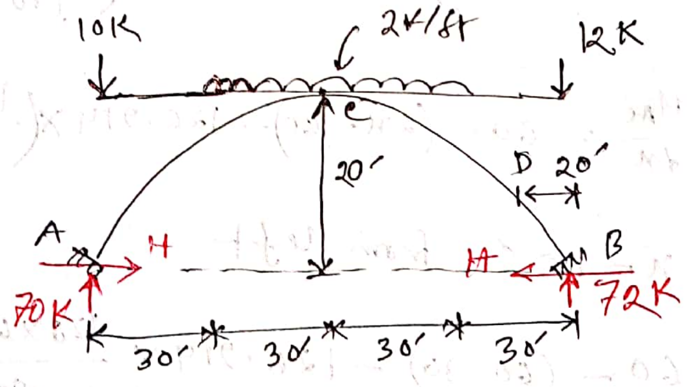
$= (55 \times 0.406 + 142.12 \times 0.914)$

$= 152.23$ K

(Ans.)

2013

Determine position and magnitude of maximum bending moment. Find shear force and normal thrust at position 20' distance from right support. Neglect the rib shortening.



Solution:

$$y = \frac{4hx}{l^2}(l-x) = \frac{4 \times 20 \times x}{(120)^2} \times (120-x) = \frac{120x - x^2}{180}$$

$$\tan\theta = \frac{dy}{dx} = \frac{120-2x}{180}$$

At 20' from Right, $\theta = \tan^{-1}\frac{120-2\times 20}{180}$

$\therefore \theta = 23.96°$

$\therefore \sin\theta = 0.406$ and $\cos\theta = 0.914$

$$\int_0^l M_s\, y\, dx = \int_0^{30} (70x - 10x) \times \left(\frac{120x - x^2}{180}\right) dx + \int_{30}^{90} \left[70x - 10x - \frac{2\times(x-30)^2}{2}\right] \times \left(\frac{120x - x^2}{180}\right) dx + \int_0^{30} (72x - 12x) \times \left(\frac{120x - x^2}{180}\right) dx$$

$= 292500 + 2664000 + 292500$

$= 3249000$

$$\int_0^l y^2\, dx = \int_0^{120} \left(\frac{120x - x^2}{180}\right)^2 dx = 25600$$

$$\therefore H = \frac{\int_0^l M_s\, y\, dx}{\int_0^l y^2\, dx} = \frac{3249000}{25600} = 126.914\ K$$

For AC part:

$$M_{AC} = 70x - 10x - \frac{2(x-30)^2}{2} - 126.914 \times \left(\frac{120x - x^2}{180}\right)$$

For maximum, $\frac{dM_{AC}}{dx} = 60 - (2x - 60) - 126.914 \times \left(\frac{120 - 2x}{180}\right)$

$\Rightarrow x = 60'$ from left

$$\therefore M_{AC} = 60\times 60 - (60-30)^2 - 126.914 \times \frac{120\times 60 - 60^2}{180}$$

$= 161.72$ K-ft

For BC part:

$$M_{BC} = 72x - 12x - 2x\frac{(x-30)^2}{2} - 126.914 \times \frac{120x - x^2}{180}$$

For maxmum, $\frac{dM_{BC}}{dx} = 60 - (2x-60) - 126.914 \times \frac{120-2x}{180}$

$\Rightarrow x = 60$ ft from Right

$$\therefore M_{BC} = 60 \times 60 - (60-30)^2 - 126.914 \times \frac{120 \times 60 - 60^2}{180}$$

$$= 161.72 \text{ K-ft}$$

Shear force at 20' distance from right

$$S.F = V\cos\theta - H\sin\theta = (72-12) \times 0.914 - 126.914 \times 0.406$$

$$\therefore S.F = 3.313 \text{ K}$$

and

Normal thrust at that section,

$$N.T = V\sin\theta + H\cos\theta$$

$$= (72-12) \times 0.406 + 126.914 \times 0.914$$

$$= 140.36 \text{ K}$$

(Ans.)

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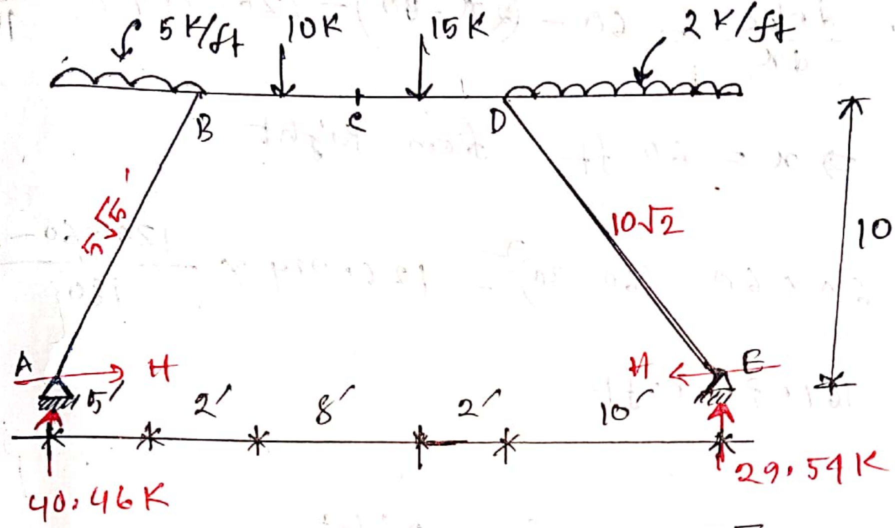
2017

~~Find vertical deflection of B and rotational deflection of B~~

Find Horizontal thrust and draw BMD for two hinged arch as shown below. $E = 30\times10^6$, $I = 300\ in^4$



Solution:

For AB: $\frac{y}{x} = \frac{10}{5} \Rightarrow y = 2x$ and $\frac{ds}{dx} = \frac{5\sqrt{5}}{5} = \sqrt{5} \Rightarrow ds = \sqrt{5}\,dx$

For BC, CD: $y = 10'$, $ds = dx$

For DE: $\frac{y}{x} = \frac{10}{10} \Rightarrow y = x$ and, $\frac{ds}{dx} = \frac{10\sqrt{2}}{10} = \sqrt{2} \Rightarrow ds = \sqrt{2}\,dx$

Now, We Know,

$$H = \frac{\int_0^l M_s\, y\, ds/EI}{\int_0^l \frac{y^2}{EI}\, ds}$$

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$$\frac{1}{EI}\int_0^l M_s y\,ds = \int_0^5 \left(40.46x - 5\cdot x\cdot\frac{x}{2}\right)(2x)\cdot(\sqrt{5}\,dx) + \int_5^7 \left[40.46x - 5\times5\times(x-2.5)\right]\times10\times dx + \int_7^{15}\left[40.46x - 5\times5\times(x-2.5) - 10(x-7)\right]\times10\times dx + \int_0^{10}\left(29.54x - \frac{2\cdot x^2}{2}\right)\times(x)\times(\sqrt{2}\,dx) + \int_{10}^{12}\left[29.54x - 2\times10\times(x-5)\right]\times10\times dx$$

$$= 5792.35 + 3105.2 + 15404.8 + 10389.76 + 4098.8$$

$$= 38790.91$$

$$\frac{1}{EI}\int_0^l y^2\,ds = \int_0^5 (2x)^2\cdot(\sqrt{5})\,dx + \int_5^{17}(10)^2\cdot dx + \int_0^{10}(x)^2\cdot\sqrt{2}\,dx$$

$$= 2044.08$$

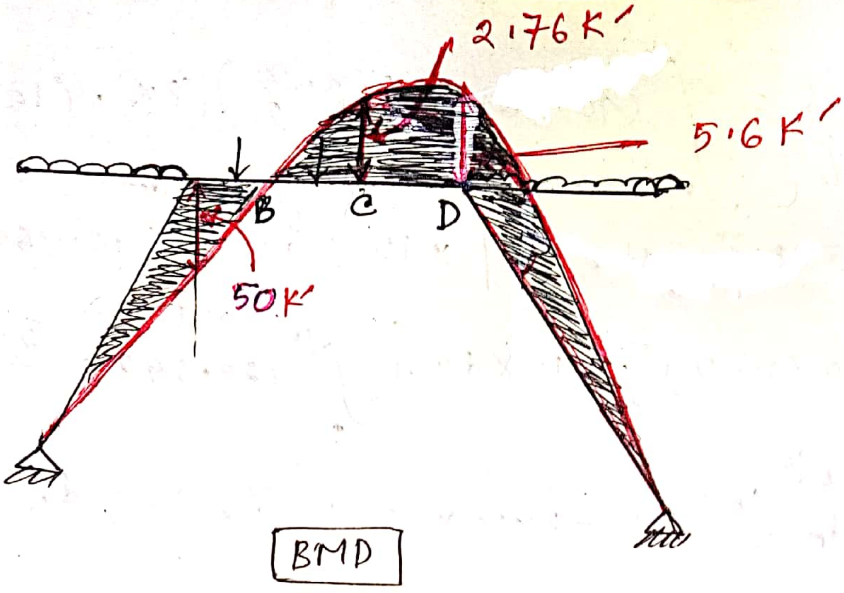
$$\therefore H = \frac{\frac{1}{EI}\int_0^l M_s y\,ds}{\frac{1}{EI}\int_0^l y^2\,ds} = \frac{10389.76}{2044.08} = 18.98\,K$$

Now,

$$M_B = 40.46\times5 - 5\times5\times2.5 - 18.98\times10 = -50.0\ \text{k-ft}$$

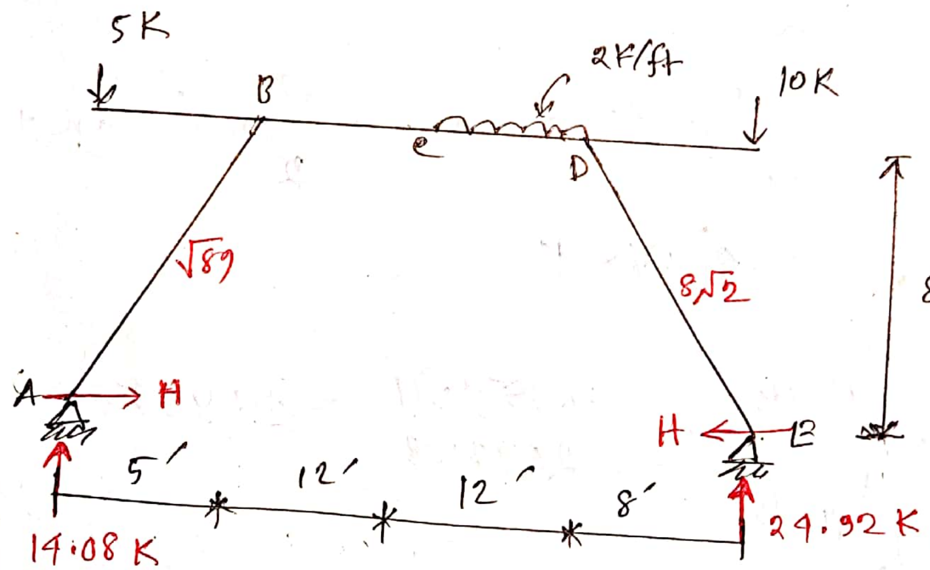
and, $M_E = 40.46\times11 - 5\times5\times8.5 - 18.98\times10 - 10\times4 = 2.76\ \text{K-ft}$

$$M_D = 29.54\times10 - 2\times10\times5 - 18.98\times10 = 5.6\ \text{K-ft}$$



2016

Determine horizontal thrust and draw BMD for the two hinged arch below given; $E = 30 \times 10^3$ ksi, $I = 400$ in^4



Solution:

For, AB: $\frac{y}{x} = \frac{8}{5} \Rightarrow y = \frac{8}{5}x$ and, $\frac{ds}{dx} = \frac{\sqrt{89}}{5}$

$\therefore ds = \frac{\sqrt{89}}{5} dx$

For BD: $y = 8'$; $ds = dx$

For DE: $\frac{y}{x} = \frac{8}{8} = 1 \quad \therefore y = x$ and $ds = \frac{8\sqrt{2}}{8} dx = \sqrt{2} dx$

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Now,

$$\frac{1}{EI}\int_0^l M_s\, y\, ds = \int_0^5 (14.08x - 5x)\times\left(\frac{8}{5}x\right)\times\frac{\sqrt{89}}{5}\,dx + \int_0^8 (24.92x - 10x)\cdot x(x)\times(\sqrt{2})\,dx + \int_8^{20}\left(24.92x - 10x - 2\times\frac{(x-8)^2}{2}\right)\times 8\times dx + \int_5^{17}(14.08x - 5x)\times 8\times dx$$

$$= 1142.14 + 3601.08 + 15444.48 + 9588.48$$

$$= 29776.18$$

$$\frac{1}{EI}\int_0^l y^2\, ds = \int_0^5 \left(\frac{8}{5}x\right)^2\times\frac{\sqrt{89}}{5}\,dx + \int_5^{29}(8)^2\times dx + \int_0^8 x^2\times\sqrt{2}\,dx$$

$$= 1978.62$$

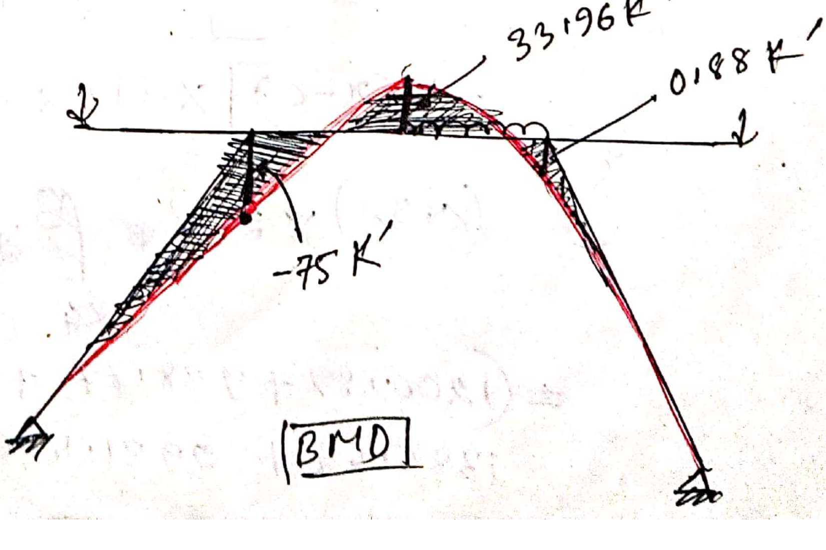
$$\therefore H = \frac{29776.18}{1978.62} = 15.05\ K$$

Now,

$M_B = 14.08\times 5 - 5\times 5 - 15.05\times 8 = -75\ K'$

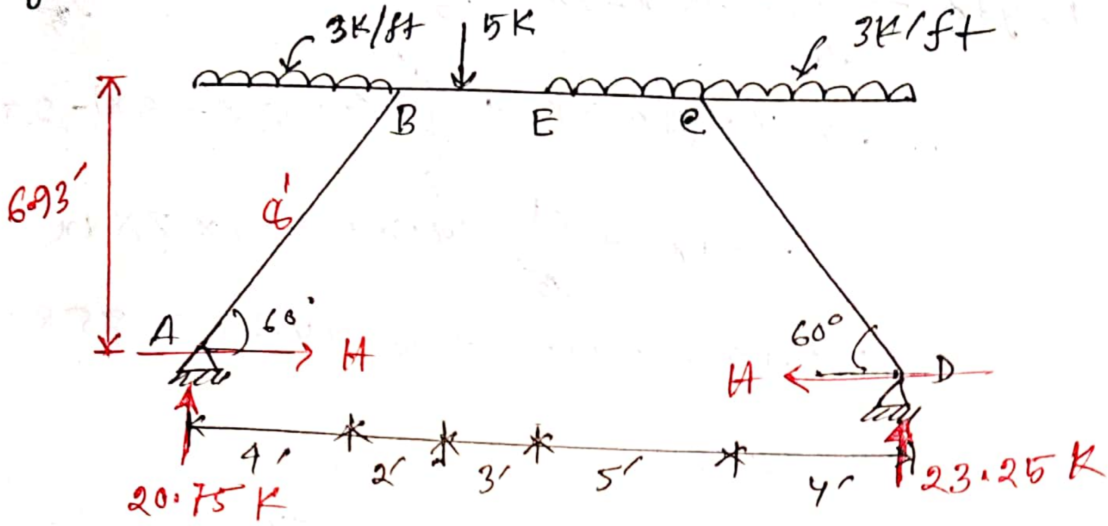
$M_C = 14.08\times 17 - 5\times 17 - 15.05\times 8 = 33.96\ K'$

$M_D = 24.94\times 8 - 10\times 8 - 15.05\times 8 = -0.88\ K'$



2015

Determine horizontal thrust developed and draw BMD of two hinge arch shown below:



Solution:

For AB & CD: $\frac{y}{x} = \frac{6.93}{4} = \sqrt{3} \Rightarrow y = \sqrt{3}x$

$\frac{ds}{dx} = \frac{8}{4} = 2 \Rightarrow ds = 2dx$

For BE, CE: $ds = dx$, $y = 4\sqrt{3} = 6.93'$

Now,

$$\frac{1}{EI}\int_0^l M_s y\, ds = \int_0^4 \left(20.75x - 3\cdot x\cdot\frac{x}{2}\right)\times(\sqrt{3}x)\times 2\,dx + \int_4^6 \left[20.75x - 3\times 4\times(x-2)\right]\times 4\sqrt{3}\times dx + \int_6^9 \left[20.75x - 3\times4\times(x-2) - 5(x-6)\right]\times 4\sqrt{3}\times dx + \int_0^4 \left(23.25x - 3\cdot x\cdot\frac{x}{2}\right)\times(\sqrt{3}x)\times 2\,dx + \int_4^9 \left(23.25x - 3\times4\times(x-2) - 3\frac{(x-4)^2}{2}\right)\times 4\sqrt{3}\times dx$$

$$= (1200.89 + 938.77 + 1706.936 + 1385.64 + 2931.496) = 8163.732\ K'$$

$$\frac{1}{EI}\int_0^l y^2\,ds = 2\times\int_0^4 (\sqrt{3}x)^2 \times 2\,dx + \int_4^{14} (6.93)^2\,dx$$

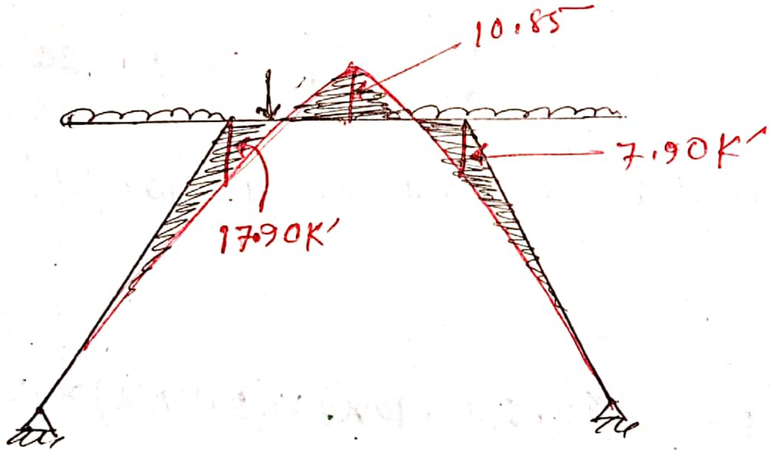
$$\simeq 736.$$

$$\therefore H = \frac{8163.732}{736} = 11.1\text{ K}$$

$M_B = 20.75\times4 - 3\times4\times2 - 11.1\times4\sqrt{3} = -17.90$ K'

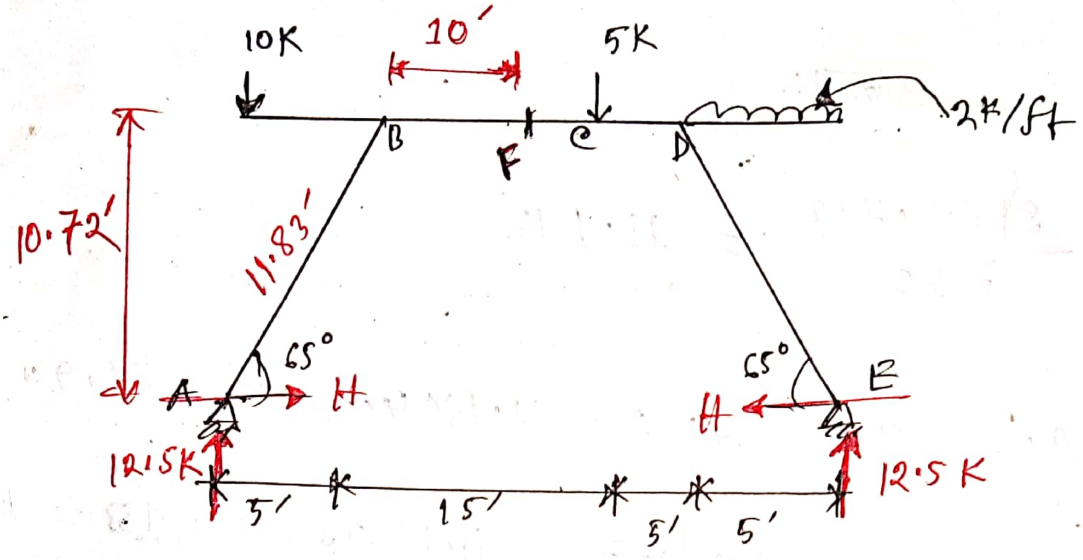
$M_E = 20.75\times9 - 3\times4\times7 - 11.1\times4\sqrt{3} - 5\times3 = 10.85$ K'

$M_C = 23.25\times4 - 3\times4\times2 - 11.1\times4\sqrt{3} = -7.90$ K'



2014

Determine the thrust and draw BMD of the two hinged arch shown below:



Solution:

For AB and DE: $\frac{y}{x} = \frac{10.72}{5} \Rightarrow y = 2.14x$

$\frac{ds}{dx} = \frac{11.83}{5} = 2.366 \Rightarrow ds = 2.366\,dx$

For BC & CD: $ds = dx$, $y = 10.72'$

$$\frac{1}{EI}\int_0^l M_s\, y\, ds = \int_0^5 (12.5x - 10x)\times(2.14x)\times(2.366)dx + \int_5 (12.5x - 10x)\times 10.72 \times dx + \int_0^5 \left(12.5x - 2\cdot x\cdot \frac{x}{2}\right)\times(2.14x)\times(2.366)dx + \int_5^{10}\left[12.5x - 2\times5\times(x-2.5)\right]\times 10.72\,dx$$

$= 527.42 + 5025 + 1845.97 + 2345$

$= 9743.4$

$$\frac{1}{EI}\int_0^l y^2 ds = 2\times\int_0^5 (2.14x)^2 \times 2.366 dx + \int_5^{25} (10.72)^2 \times dx$$

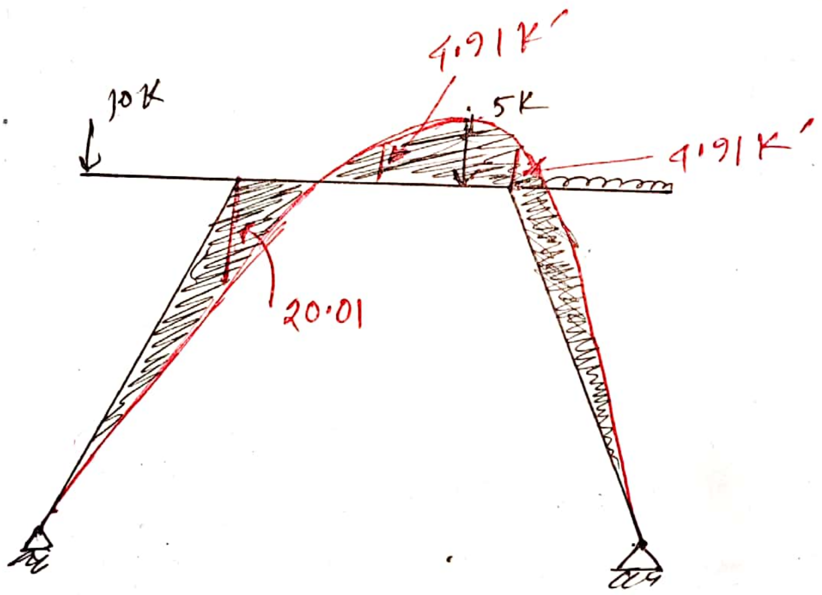
$$= 3201.31$$

$$\therefore H = \frac{9743.4}{3201.31} = 3.04 K$$

$$M_B = 12.5\times 5 - 10\times 5 - 3.04\times 10.72 = -20.01 K'$$

$$M_F = 12.5\times 15 - 10\times 15 - 3.04\times 10.72 = 4.91 K'$$

$$M_D = 12.5\times 5 - 2\times 5\times 2.5 - 3.04\times 10.72 = 4.91 K$$

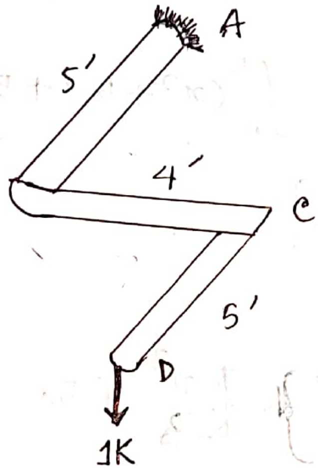


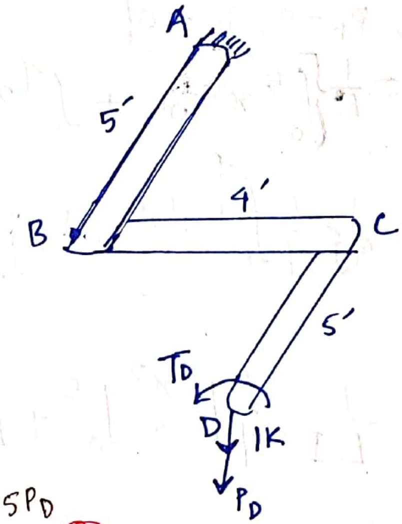
3D

\# A 4 inch Ø standard pipe bracket having an angle 90° at B & C And is located in horizontal plane. Find (i) vertical deflection ~~in horizontal plane~~ component of D (ii) Rotational deflection component of B in the plane normal to the axis of CD.

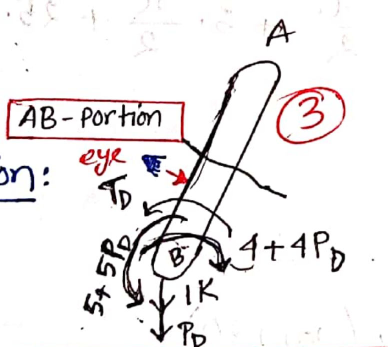
Plane moment of inertia = 10 in⁴, $G = 12000$ ksi, $E = 30 \times 10^3$ ksi

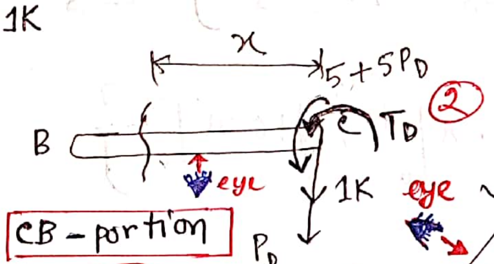
$J = 2I$ in⁴

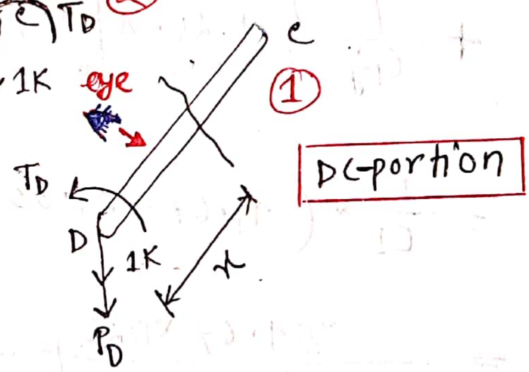




Solution:







* considering Anti-clockwise (+ve) → T

* right section, clockwise (+ve) → M

Portion	Origin	Limit	Moment, M	T, (Torque)	$\frac{dM}{dP_D}$	$\frac{dM}{dT_D}$	$\frac{dT}{dP_D}$	$\frac{dT}{dT_D}$
DC	D	0–5	$1 \cdot x + P_D \cdot x$	T_D	x	0	0	1
CB	C	0–4	$1 \cdot x + P_D \cdot x - T_D$	$5+5P_D$	x	−1	5	0
BA	B	0–5	$1 \cdot x + P_D \cdot x + 5 + 5P_D$	$T_D - (4+4P_D)$	$x+5$	0	−4	1

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$$\therefore \Delta_{V_D} = \int_0^l \frac{M \cdot \frac{dM}{dP_D}}{EI} dx + \int_0^l \frac{T \cdot \frac{dT}{dP_D}}{GJ} dx$$

GJ → Polar moment; G → Modulus of rigidity

$$= \frac{1}{EI}\left[\int_0^5 (x+P_D x)\cdot x\,dx + \int_0^4 (x+P_D x - T_D)\cdot x\,dx + \int_0^5 (x+P_D x+5+5P_D)\cdot(x+5)\,dx\right]$$

$$+ \frac{1}{GJ}\left[\int_0^5 T_D \cdot 0\,dx + \int_0^4 (5+5P_D)\cdot 5\,dx + \int_0^5 (T_D - 4 - 4P_D)\cdot(-4)\,dx\right]$$

[Now, $P_D = 0$, $T_D = 0$]

$$\therefore \Delta_{V_D} = \frac{1}{EI}\left\{\int_0^5 x^2\,dx + \int_0^4 x^2\,dx + \int_0^5 (x^2+5x+5x+25)\,dx\right\} +$$

$$\frac{1}{GJ}\left\{\int_0^4 25\,dx + \int_0^5 16\,dx\right\}$$

$$= \frac{1}{EI}\left\{\left[\frac{x^3}{3}\right]_0^5 + \left[\frac{x^3}{3}\right]_0^4 + \left[\frac{x^3}{3} + 5\cdot\frac{x^2}{2} + 5\cdot\frac{x^2}{2} + 25\cdot x\right]_0^5\right\}$$

$$+ \frac{1}{GJ}\left\{\left[25\cdot x\right]_0^4 + \left[16x\right]_0^5\right\}$$

$$= \frac{1}{EI}\times(41.67 + 21.33 + 291.67) + \frac{1}{GJ}\times(100+80)$$

$$= \frac{354.67\times 1728}{30\times 10^3\times 10} + \frac{180\times 1728}{12000\times(2\times 10)}$$

$$= 3.34 \text{ in. } (\downarrow)$$

$$\theta_D = \int_0^l \frac{M \cdot \frac{dM}{dT_D}\, dx}{EI} + \frac{\int_0^l T \cdot \frac{dT}{dT_D}\, dx}{GJ}$$

$$= \frac{1}{EI}\left[\int_0^5 (x + P_D \cdot x) \cdot 0 \cdot dx + \int_0^4 (x + P_D \cdot x - T_D) \cdot (-1)\, dx + \int_0^5 (x + P_D x + 5 + 5P_D) \cdot 0\right]$$

$$+ \frac{1}{GJ}\left[\int_0^5 T_D \cdot 1\, dx + \int_0^4 (5 + 5P_D) \cdot 0 \cdot dx + \int_0^5 (T_D - 4 - 4P_D) \cdot 1\, dx\right]$$

Now, $P_D = 0$, $T_D = 0$

$$= \frac{1}{EI} \times \left[0 + \int_0^4 -x\, dx + 0\right] + \frac{1}{GJ} \times \left[\int_0^5 0 \cdot dx + 0 + \int_0^5 -4\, dx\right]$$

$$= \frac{1}{EI} \times \left[-\frac{x^2}{2}\right]_0^4 + \frac{1}{GJ}\left[-4x\right]_0^5$$

$$= \frac{-8 \times 144}{30 \times 10^3 \times 10} + \frac{-20 \times 144}{12000 \times (2 \times 10)}$$

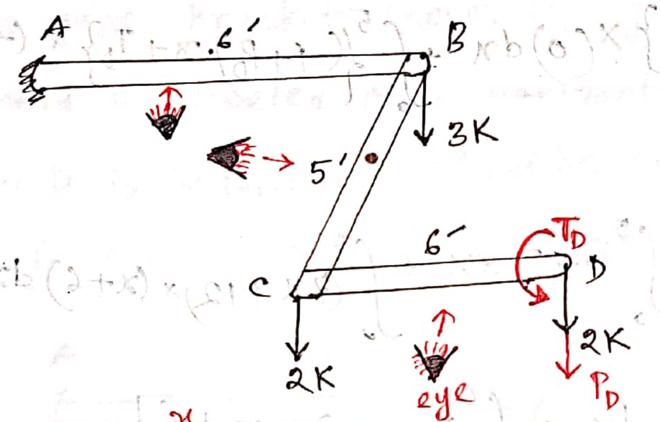
$$= -0.016 \text{ rad.}$$

$\theta_D = 0.016$ rad (↻) (Ans.)

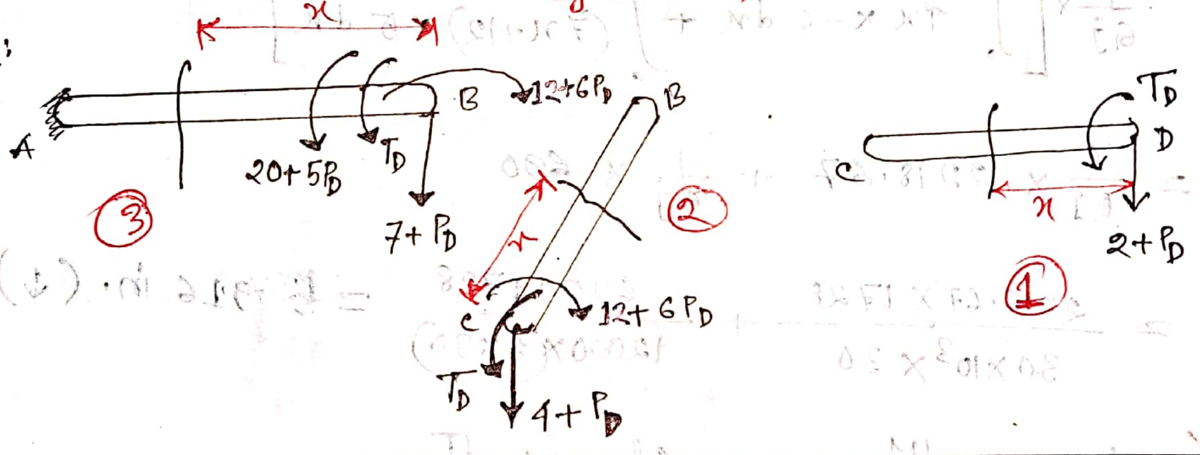
3D

2017

The standard pipe bracket shown in figure below having 90° angle at B and C located in a horizontal plane. Find vertical deflection of D, rotational deflection of D about the axis CD, $I = 30 \text{ in}^4$, $G = 12000$ ksi, $E = 30 \times 10^6$ psi.



Solution:



Portion	Origin	Limit	Moment, (M)	Torque (T)	$\frac{dM}{dP_D}$	$\frac{dM}{dT_D}$	$\frac{dT}{dP_D}$	$\frac{dT}{dT_D}$
CD	D	0–6	$(2+P_D)x$	T_D	x	0	0	1
BC	C	0–5	$(4+P_D)\cdot x + T_D$	$-(12+6P_D)$	x	1	−6	0
AB	B	0–6	$(7+P_D)\cdot x + (12+6P_D)$	$T_D + 20 + 5P_D$	$x+6$	0	5	1

$$\Delta_{VD} = \int_0^l \frac{M \cdot \frac{dM}{dP_D}}{EI} dx + \int_0^{l} \frac{T \cdot \frac{dT}{dP_D}}{GJ}$$

$$= \frac{1}{EI}\left[\int_0^6 \{(2+P_D)x\} \times x \, dx + \int_0^5 \{(4+P_D)\cdot x + T_D\} \times x \, dx + \int_0^6 \{(7+P_D)\cdot x + (12+6P_D)\} \times (x+6) dx\right]$$

$$+ \frac{1}{GJ}\left[\int_0^6 \{(2+P_D)x\} \times (0) \, dx + \int_0^5 \{(4+P_D)\cdot x + T_D\} \times (-6) dx + \int_0^6 \{(7+P_D)x + (12+6P_D)\} \times 5 \, dx\right]$$

[Now $P_D = 0, T_D = 0$]

$$= \frac{1}{EI}\left[\int_0^6 2x^2 dx + \int_0^5 4x^2 dx + \int_0^6 (7x+12) \times (x+6) \, dx\right] +$$

$$\frac{1}{GJ} \times \left[\int_0^5 4x \times -6 \, dx + \int_0^6 (7x+12) \times 5 \, dx\right]$$

$$\Delta_{VD} = \frac{1}{EI} \times 2218.67 + \frac{1}{GJ} \times 690$$

$$\therefore \Delta_{VD} = \frac{2218.67 \times 1728}{30 \times 10^3 \times 30} + \frac{690 \times 1728}{12000 \times (2 \times 30)} = 5.916 \text{ in. } (\downarrow)$$

Now,

$$\theta_D = \int_0^l \frac{M \cdot \frac{dM}{dT_D}}{EI} dx + \int_0^l \frac{M \cdot \frac{dT}{dT_D}}{GJ} dx$$

$$= \frac{1}{EI} \times \left[\int_0^6 \{(2+P_D)x\} \times (0) dx + \int_0^5 \{(4+P_D) \times x + T_D\} \times (1) dx + \int_0^6 \{(7+P_D)\cdot x + (12+6P_D)\} \times (0) \, dx\right] + \frac{1}{GJ}\left[\int_0^6 \{(2+P_D)\cdot x\} \times (1) dx\right.$$

$$\left. + \int_0^5 \{(4+P_D)\cdot x + T_D\} \times (0) \, dx + \int_0^6 \{(7+P_D)x + (12+6P_D)\} \times (1) \, dx\right]$$

(Now $P_D = 0$, $T_D = 0$)

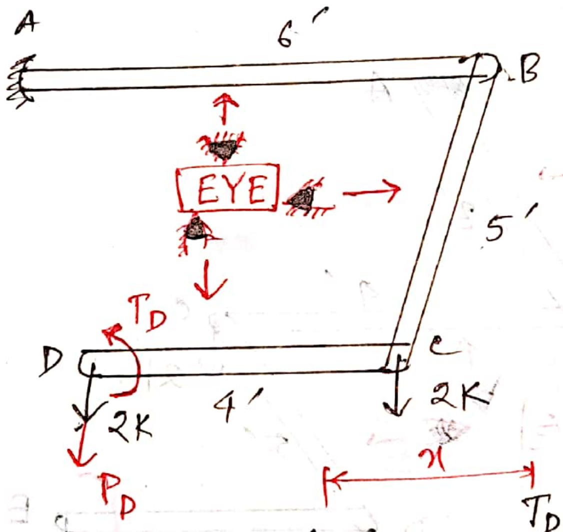
$$= \frac{1}{EI} \times \left[\int_0^5 4x \, dx \right] + \frac{1}{GJ} \left[\int_0^6 2x \, dx + \int_0^6 (7x+12) \, dx \right]$$

$$\therefore \theta_D = \frac{50 \times 144}{30 \times 10^3 \times 30} + \frac{234 \times 144}{12000 \times 2 \times 30} = 0.0548 \text{ rad } (\curvearrowleft)$$

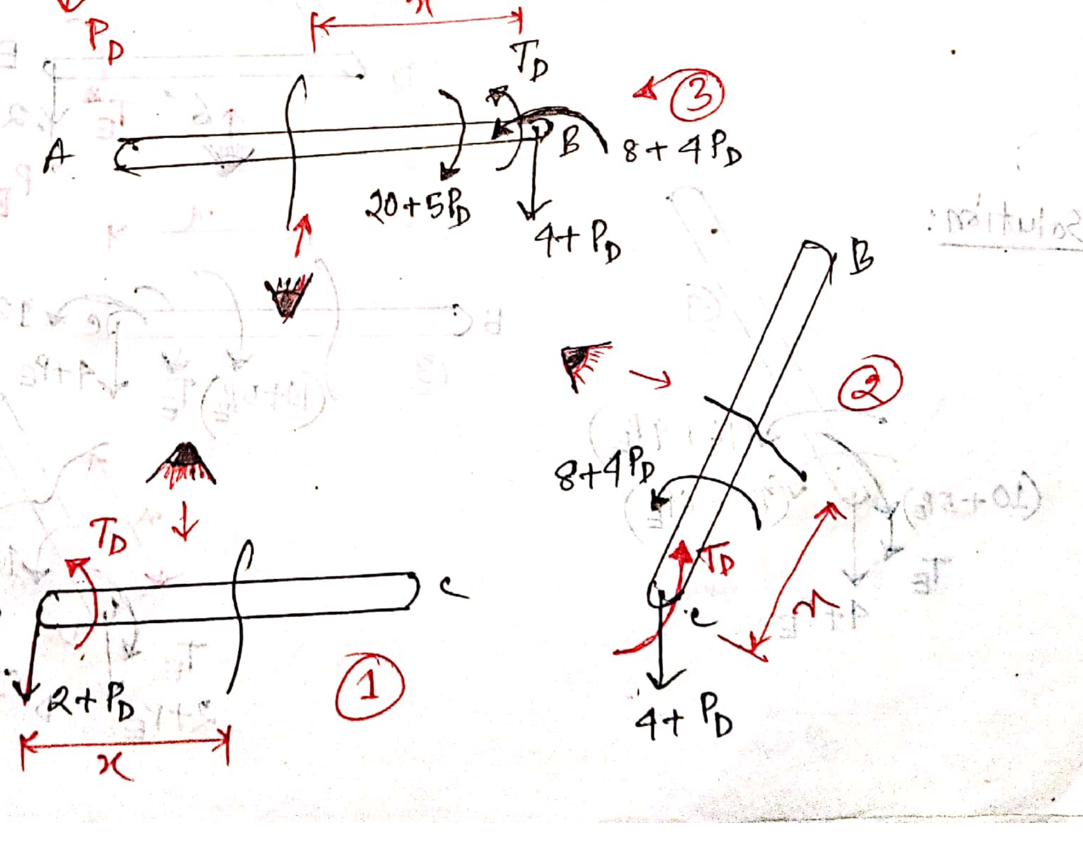
(Ans.)

2016

The standard pipe Bracket shown in figure below having 90° angle at B and C leveled in a horizontal plane. Find (i) vertical deflection of D (ii) Rotational deflection of D in plane normal to the axis of CD. [$I = 20 \text{ in}^4$, $G = 12000$ ksi, $E = 30 \times 10^3$ ksi]



Solution:



Portion	Origin	Limit	M	T	$\frac{dM}{dP_D}$	$\frac{dM}{dT_D}$	$\frac{dT}{dP_D}$	$\frac{dT}{dT_D}$
CD	D	0-4	$(2+P_D)x$	T_D	x	0	0	1
BC	C	0-5	$(4+P_D)x - T_D$	$8+4P_D$	x	-1	4	0
AB	B	0-6	$(4+P_D)x - (8+4P_D)$	$-T_D + (20+5P_D)$	$x-4$	0	5	-1

Do integration yourself and find Δ_{VD} & θ_D ☺

2015 #

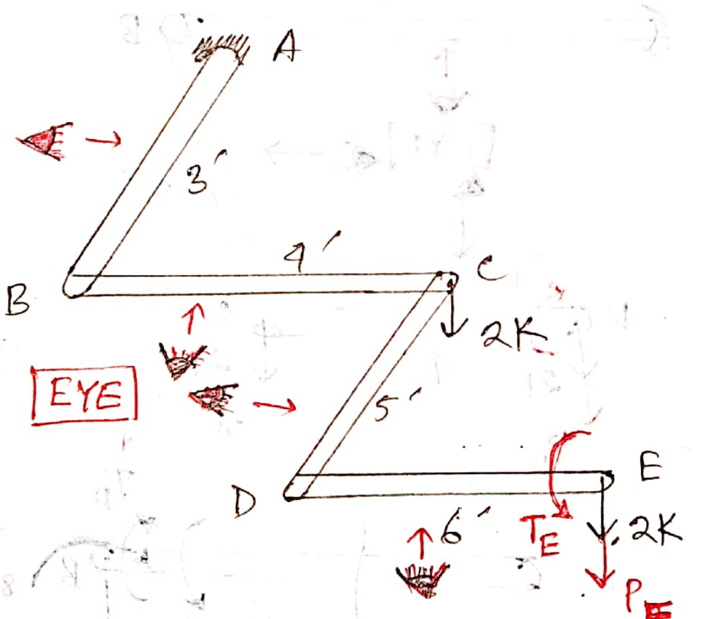
$E = 30\times10^3$ ksi

$I = 10.6\ in^4$

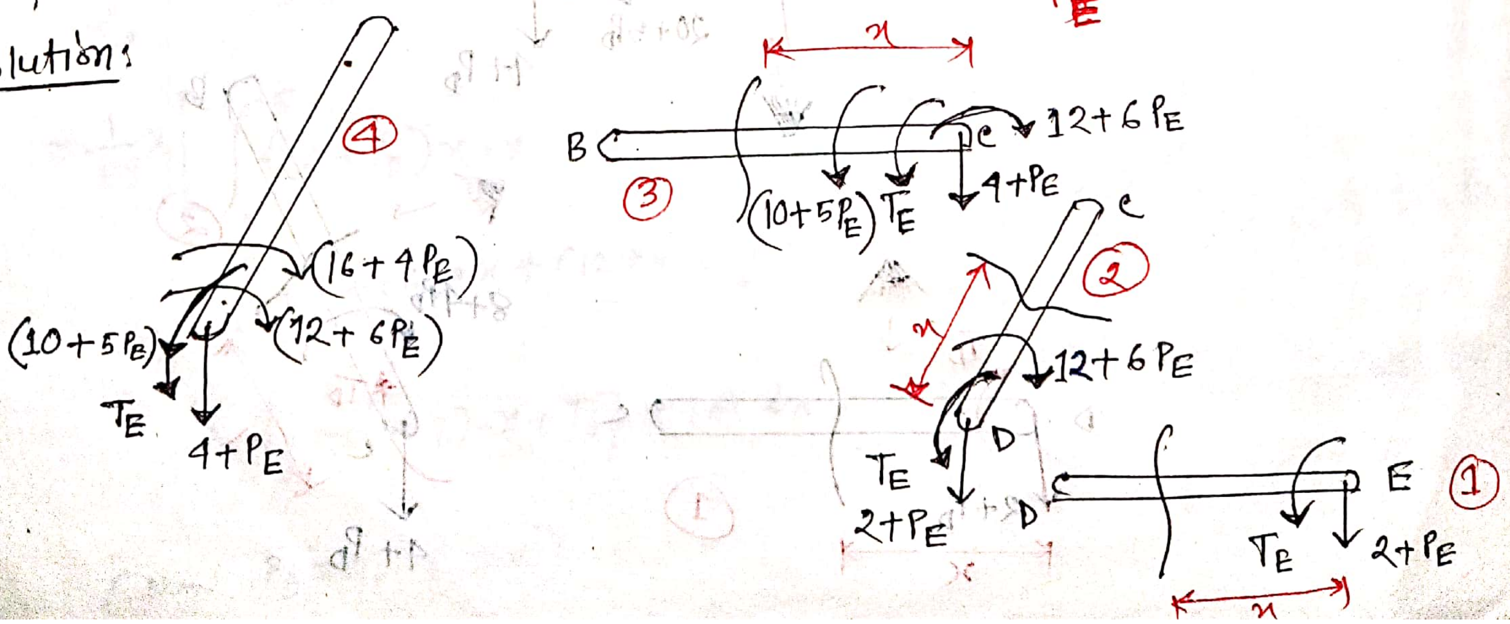
$G = 12000$ ksi

Find $\Delta_{VE} = ?$

$\theta_E = ?$



Solution:



Portion	Origin	Limit	M	T	$\frac{dM}{dP_E}$	$\frac{dT}{dP_E}$	$\frac{dM}{dT_E}$	$\frac{dT}{dT_E}$
DE	E	0-6	$(2+P_E)\cdot x$	T_E	x	0	0	1
CD	D	0-5	$(2+P_E)\cdot x + T_E$	$-(12+6P_E)$	x	-6	1	0
BC	C	0-4	$(4+P_E)\cdot x + (12+6P_E)$	$T_E + (10+5P_E)$	$x+6$	5	0	1
AB	B	0-3	$(4+P_E)\cdot x + T_E + (10+5P_E)$	$-(12+6P_E) - (16+4P_E)$	$x+5$	-6-4	1	0

(Do Integration)

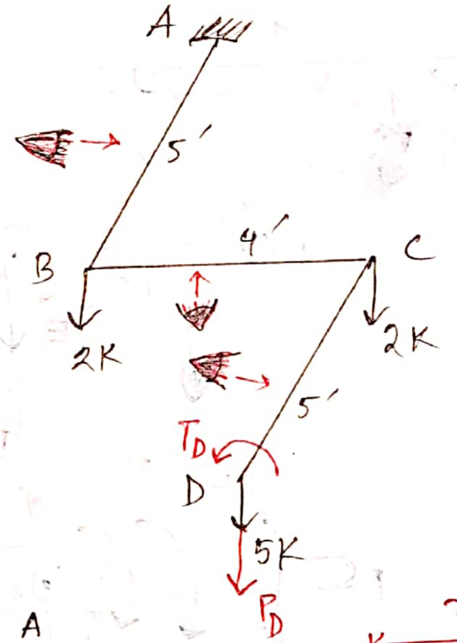
2014 #

$E = 30000$ ksi

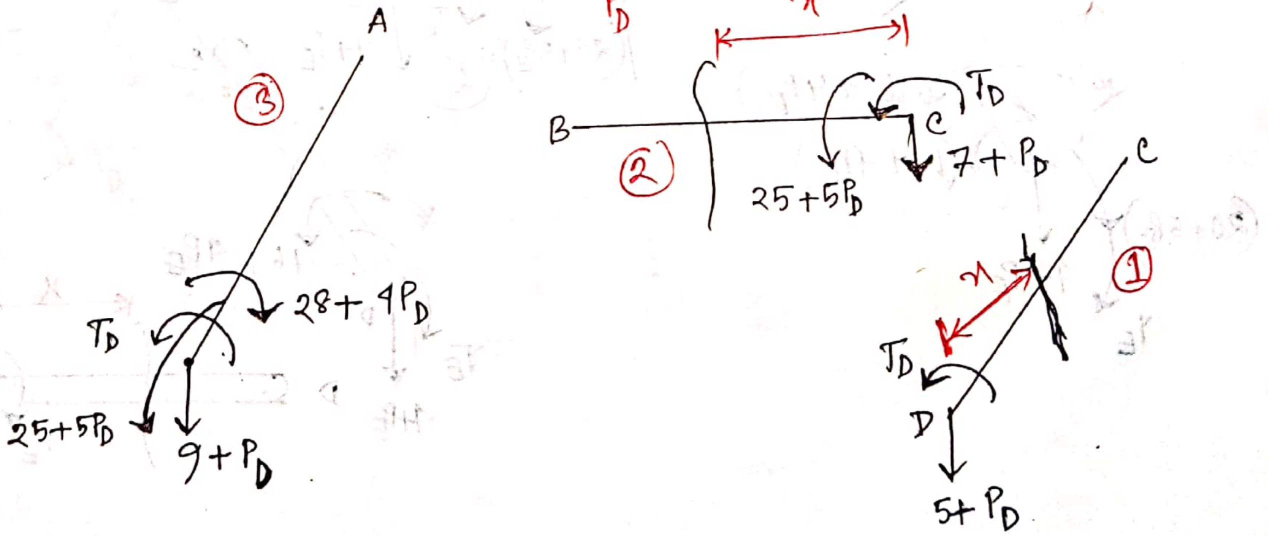
$G = 12000$ ksi

$I = 10$ in^4

Find Δv_D & θ_D.



Solution:



Portion	Origin	Limit	M	T	$\frac{dM}{dP_D}$	$\frac{dT}{dP_D}$	$\frac{dM}{dT_D}$	$\frac{dT}{dT_D}$
CD	D	0–5	$(5+P_D)\cdot x$	T_D	x	0	0	1
BC	C	0–4	$(7+P_D)\cdot x - T_D$	$25+5P_D$	x	5	−1	0
AB	B	0–5	$(9+P_D)\cdot x + 25+5P_D$	$T_D-(28+4P_D)$	$x+5$	−4	0	1

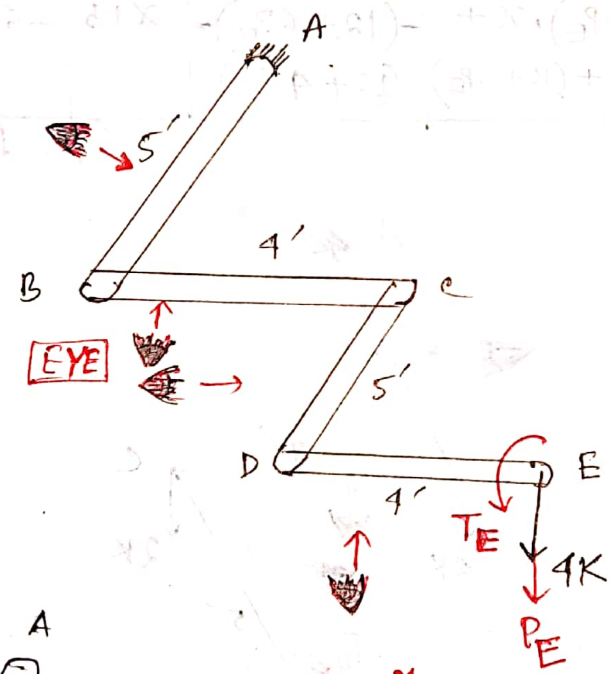
2012
#

$I = 10.25\ in^4$

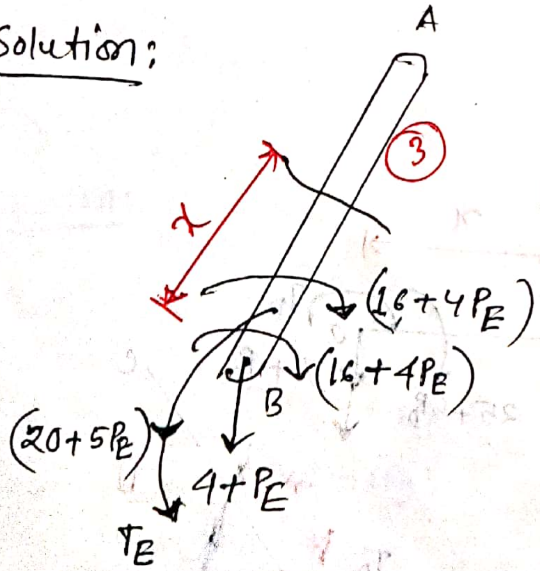
$G = 12000\ ksi$

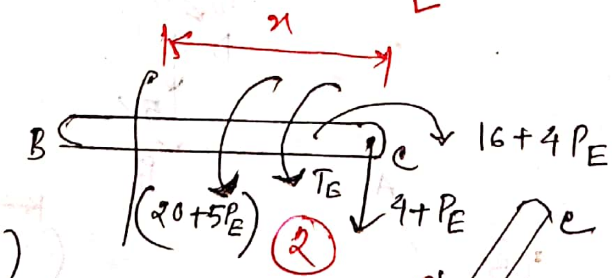
$E = 30\times10^3\ ksi$

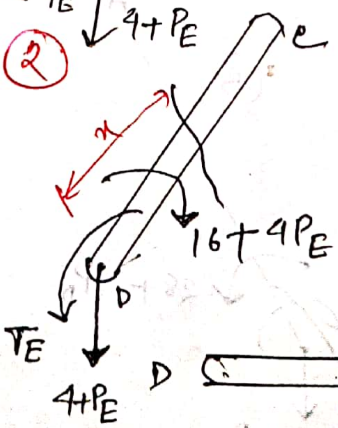
Find, Δ_{VE}, $\theta_E = ?$

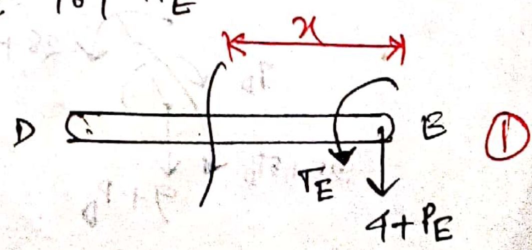


Solution:









Portion	Origin	Limit	M	T	$\frac{dM}{dP_E}$	$\frac{dT}{dP_E}$	$\frac{dM}{dT_E}$	$\frac{dT}{dT_E}$
DE	E	0-4	$(4+P_E)\cdot x$	T_E	x	0	0	1
DC	D	0-5	$(4+P_E)\cdot x + T_E$	$-(16+4P_E)$	x	-4	1	0
CB	C	0-4	$(4+P_E)\cdot x + (16+4P_E)$	$T_E + (20+5P_E)$	$x+4$	5	0	1
AB	B	0-5	$(4+P_E)\cdot x + T_E + (20+5P_E)$	$-2(16+4P_E)$	$x+5$	8	1	0

2011, 2010 (same)

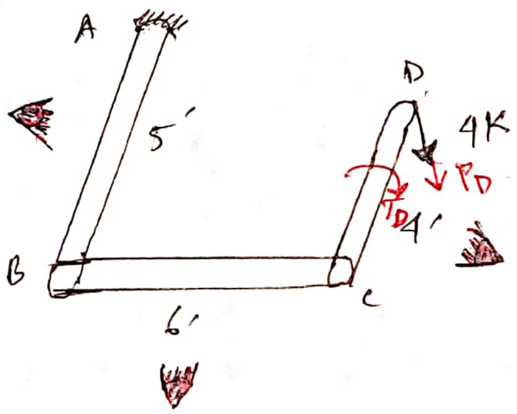
#

$I = 10\ in^4$

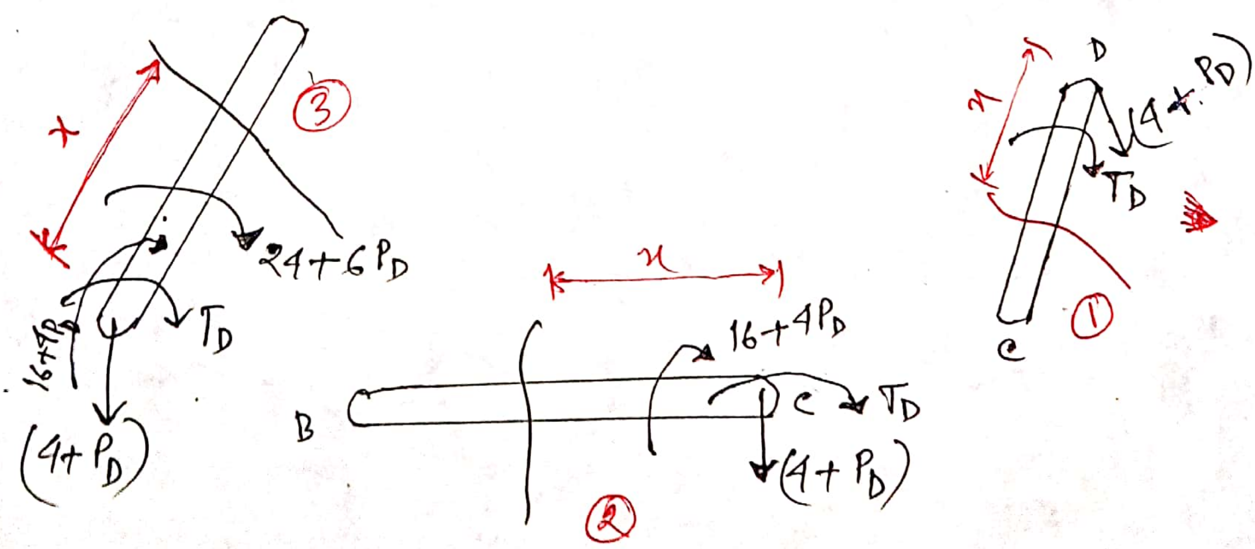
$G = 12000\ ksi$

$E = 30000\ ksi$

Find $\Delta v_D, \theta_B = ?$



solution:



portion	origin	Limit	M	T	$\frac{dM}{dP_D}$	$\frac{dT}{dP_D}$	$\frac{dM}{dT_D}$	$\frac{dT}{dT_D}$
CD	D	0–4	$(4+P_D)\cdot x$	T_D	x	0	0	1
BC	C	0–6	$(4+P_D)\cdot x+T_D$	$-(16+4P_D)$	x	-4	1	0
AB	B	0–5	$(4+P_D)\cdot x-(16+4P_D)$	$-T_D-(24+6P_D)$	$x-4$	-6	0	-1

(Do integration)