CONJUGATE BEAM METHOD

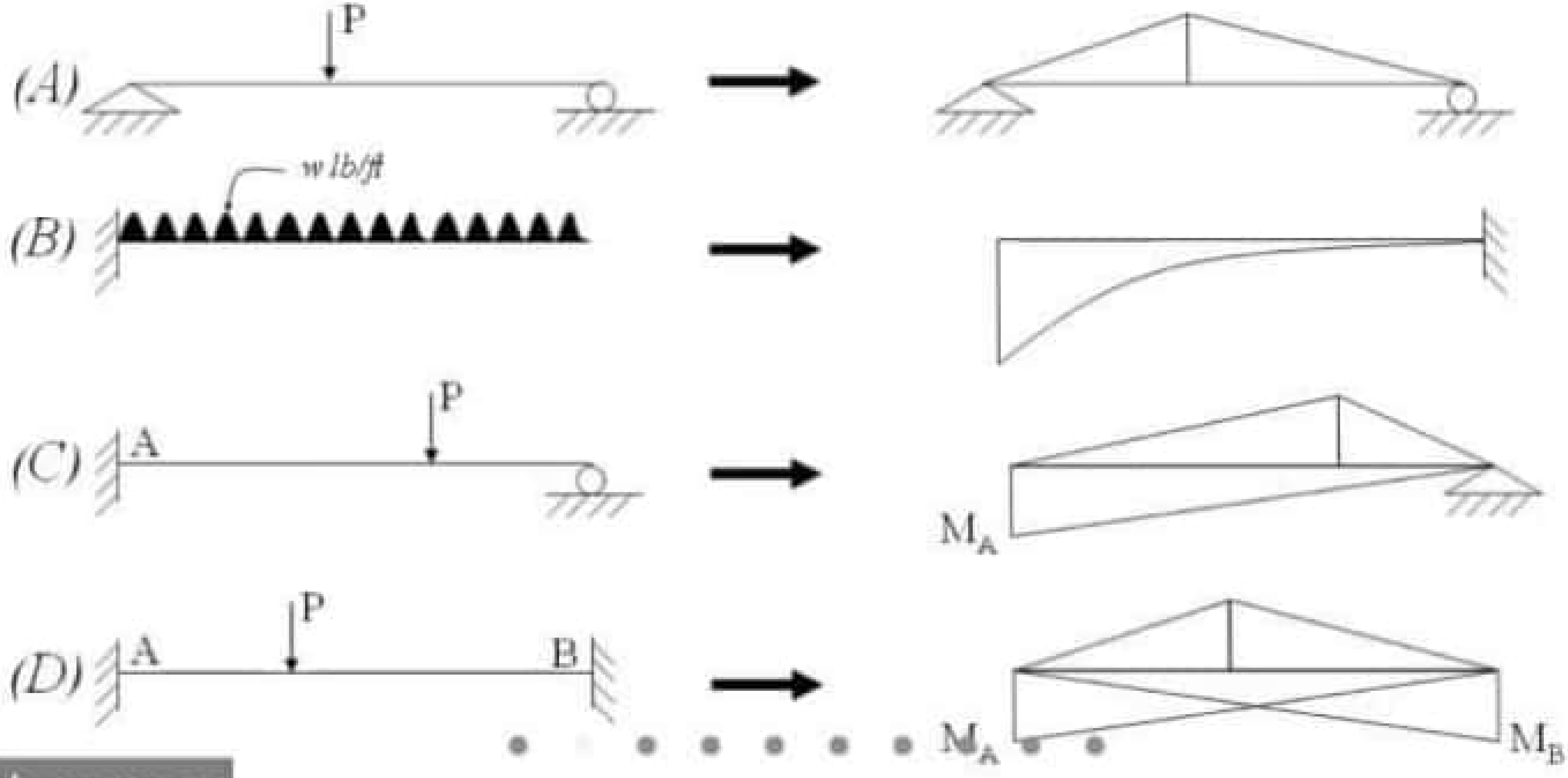
PROPOSITIONS OF CONJUGATE BEAM:

- Span of the conjugate beam is equal to the span of the real beam.
- The load of the conjugate beam is the M/EI diagram of the real beam.
- The shear at any section of the conjugate beam is equal to the slope of the corresponding section of the real beam.
- The moment at any section of the conjugate beam is equal to the deflection of the corresponding section of the real beam.

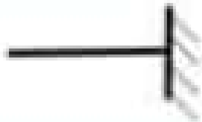
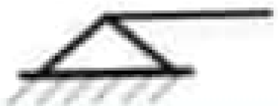
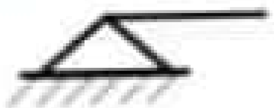
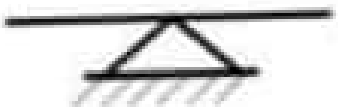
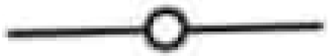
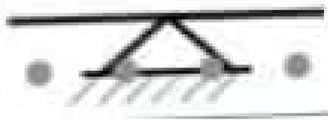
n's screen

RELATION BETWEEN CONJUGATE BEAM AND REAL BEAM

REAL BEAM	CONJUGATE BEAM



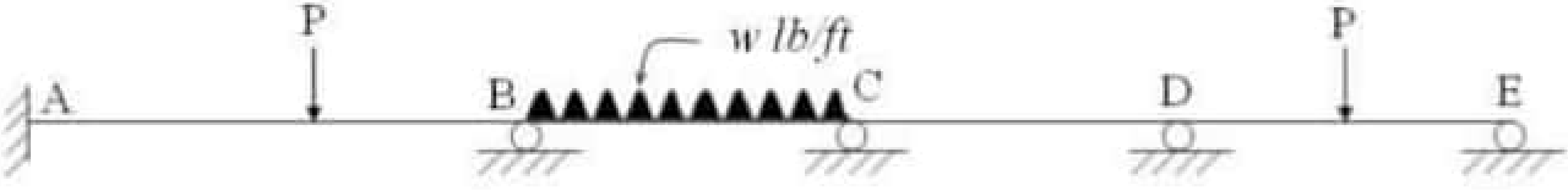
CONVERSION OF SUPPORTS

No.	REAL BEAM		CONJUGATE BEAM	
(1)		Fixed		Free
(2)		Free		Fixed
(3)		Ex. Roller		Ex. Roller
(4)		Ex. Hinge		Ex. Hinge
(5)		Int. Hinge		Int. Link
(6)		Int. Link		Int. Hinge

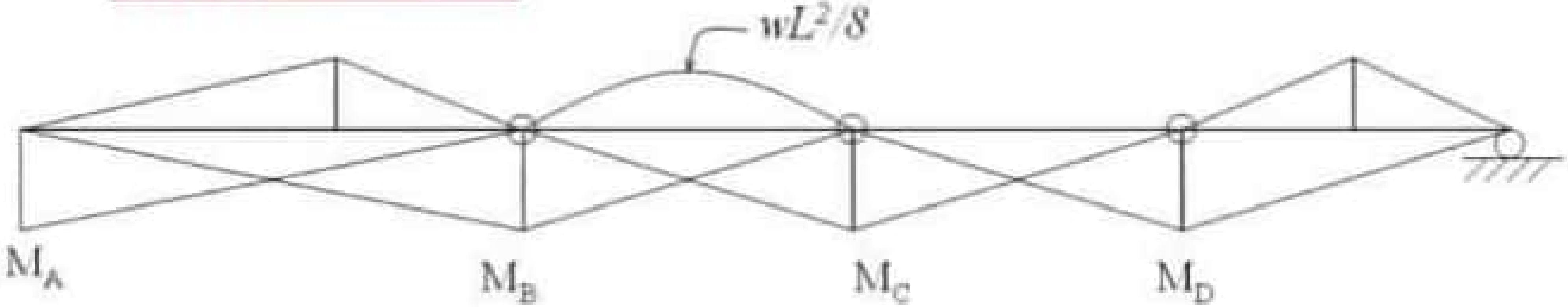
17.8KB/

CONVERSION OF REAL BEAM TO CONJUGATE BEAM

REAL BEAM

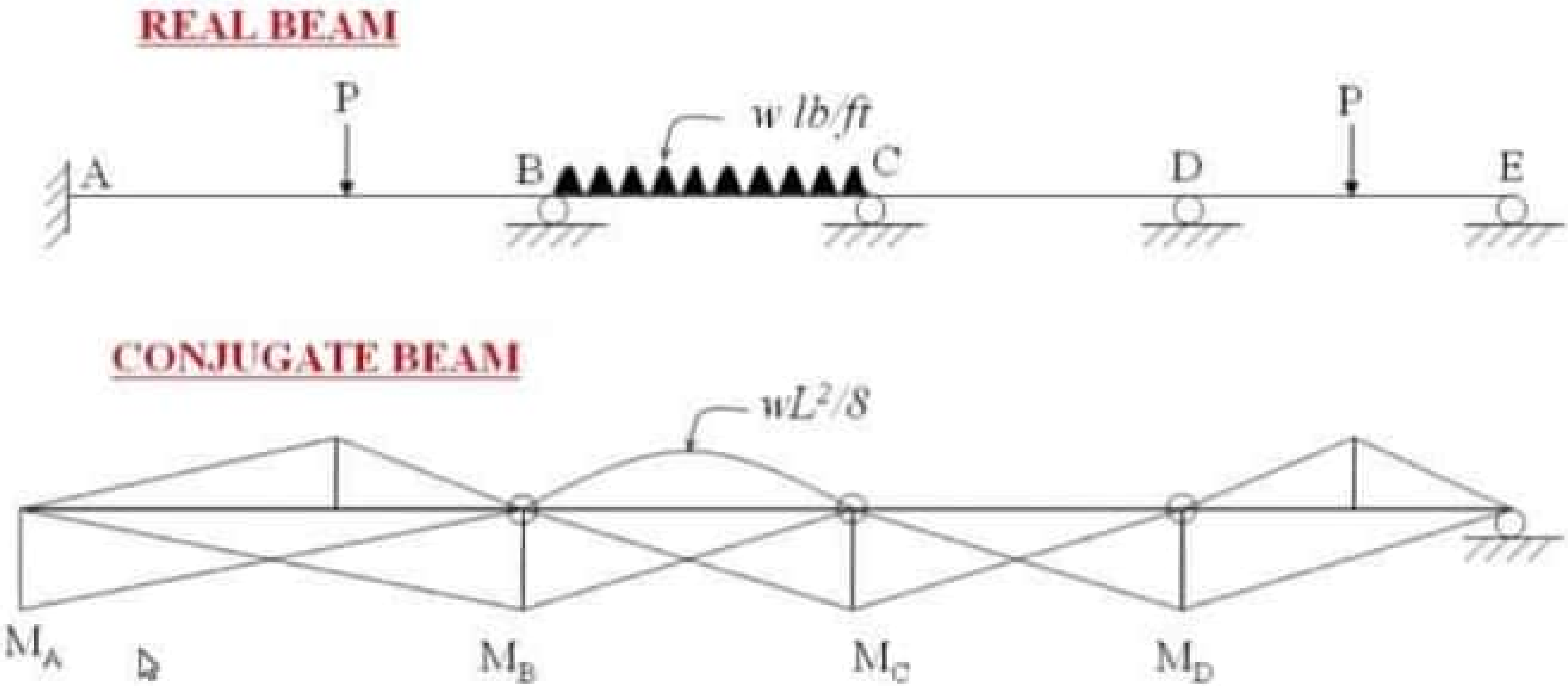


CONJUGATE BEAM



n's screen

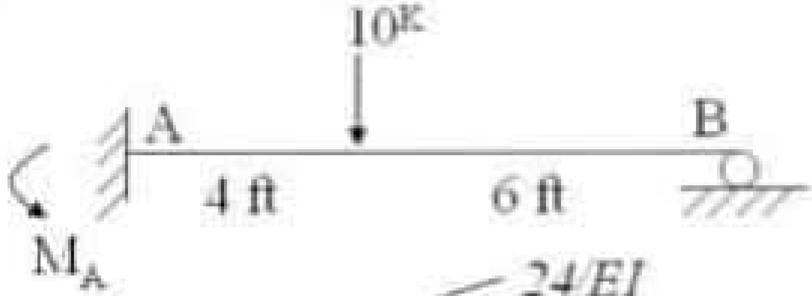
CONVERSION OF REAL BEAM TO CONJUGATE BEAM



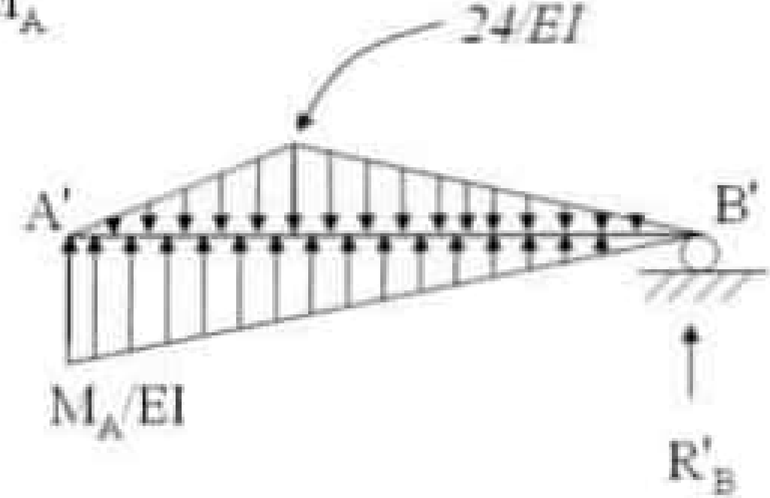
n's screen

SOLUTION – 1

REAL BEAM



CONJUGATE BEAM



$$\sum M'_B = 0$$

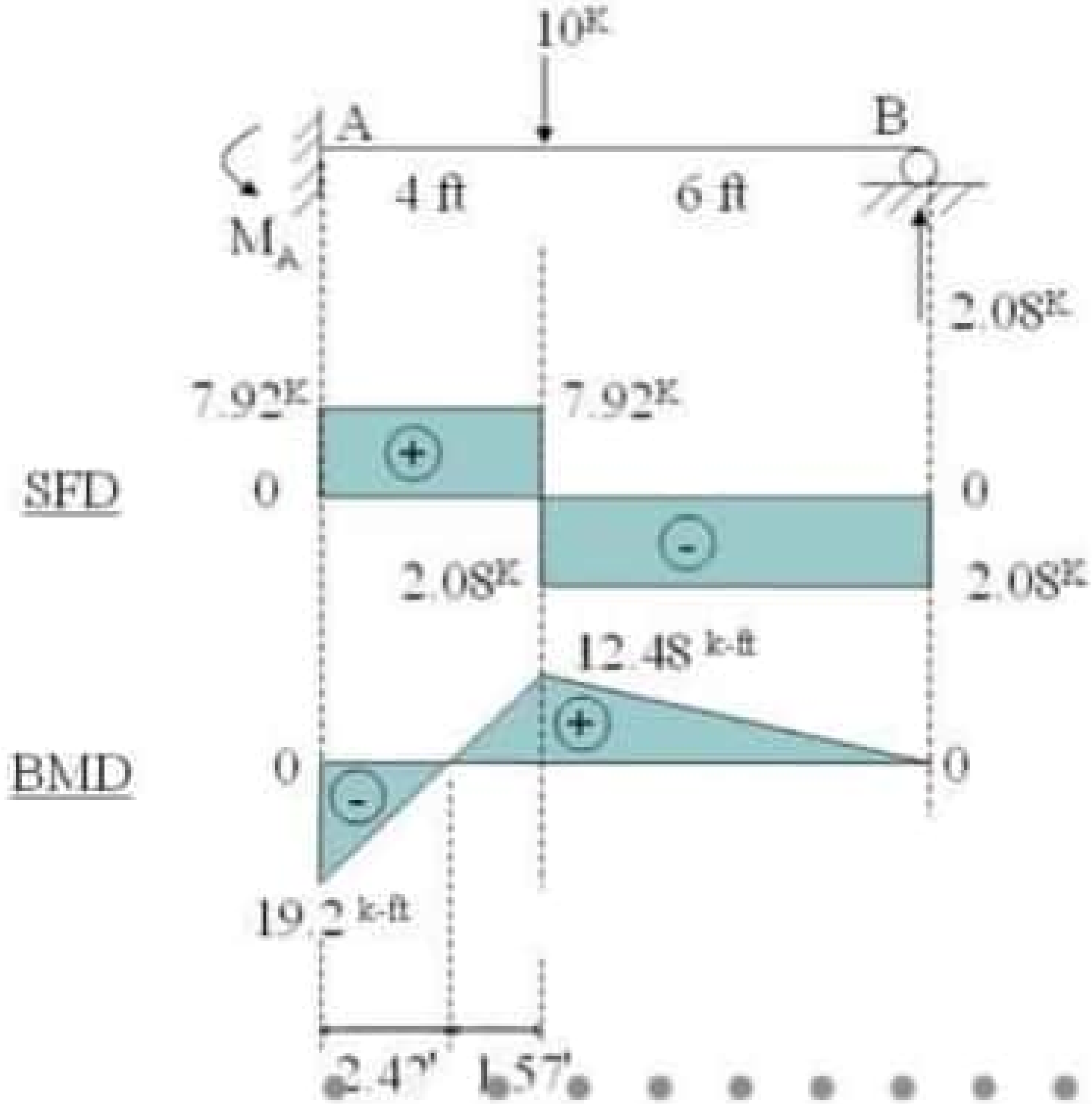
$$\Rightarrow \frac{1}{2} \times M_A \times \frac{2}{3} \times 10 = (\frac{1}{2} \times 6 \times 24 \times 4) + \{\frac{1}{2} \times 4 \times 24 \times (6 + \frac{4}{3})\}$$

$$\Rightarrow M_A = 19.2\ k - ft$$

From the real beam; $\sum M_A = 19.2$

or; $19.2 = 40 - R_B \times 10$ *or,* $R_B = 2.08\ k$

SOLUTION – 1

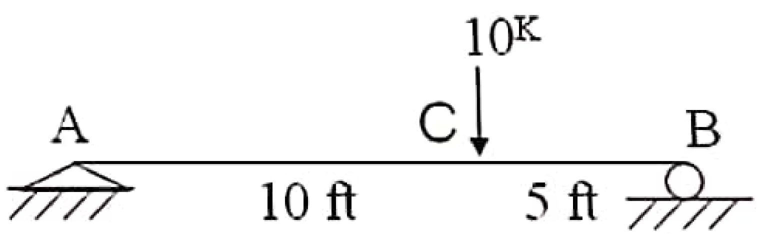


h's screen

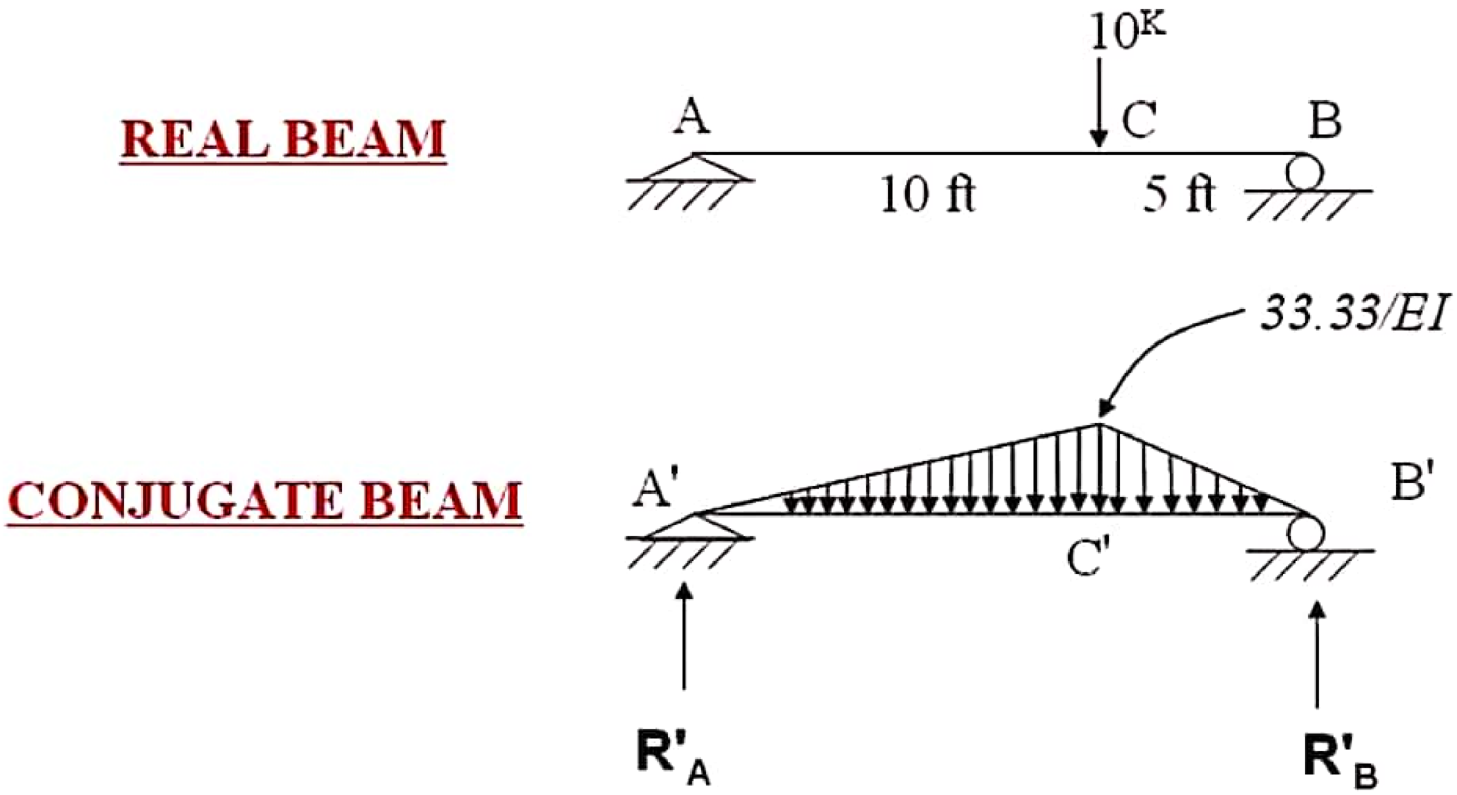
PROBLEM – 2

Problem: Using the conjugate beam method; find –

1. Slope at A.
2. Deflection at point C.
3. The section of maximum deflection.
4. The value of maximum deflection.
   (E = 30,000 k/in^2 & I = 200 in^4)



SOLUTION – 2



SOLUTION – 2

(1) From the Conjugate Beam; $\sum M'_B = 0$

$$or;\ \frac{1}{2} \times \frac{33.33}{EI} \times 10 \times \left(5 + \frac{10}{3} \right) + \frac{1}{2} \times 5 \times \frac{33.33}{EI} \times \frac{2}{3} \times 5 = R'_A \times 15$$

$$R'_A = \frac{111.11}{EI}$$

So; Slope at A; $\theta_A = \frac{111.11 \times 144}{30000 \times 200} = 0.00266\ Rad.$

SOLUTION – 2

(2) *Deflection at 'C' –*

$$\Rightarrow \Delta_C = \frac{111.11}{EI} \times 10 - \frac{1}{2} \times 10 \times \frac{33.33}{EI} \times \frac{10}{3}$$

$$\Rightarrow \Delta_C = \frac{555.6}{EI} \quad \Rightarrow \quad \Delta_C = \frac{555.6 \times 1728}{30000 \times 200} \quad \Rightarrow \quad \Delta_C = 0.16 \text{ inch.}$$

(3) *The Section of Maximum Deflection –*

$$\text{Point of Zero Shear} \Rightarrow \frac{111.11}{EI} - \frac{1}{2} \times x \times \frac{33.33}{10EI} \times x = 0$$

$$\Rightarrow x = 8.16 \text{ ft}$$

SOLUTION – 2

(4) *Maximum Deflection,* Δ_{Max}

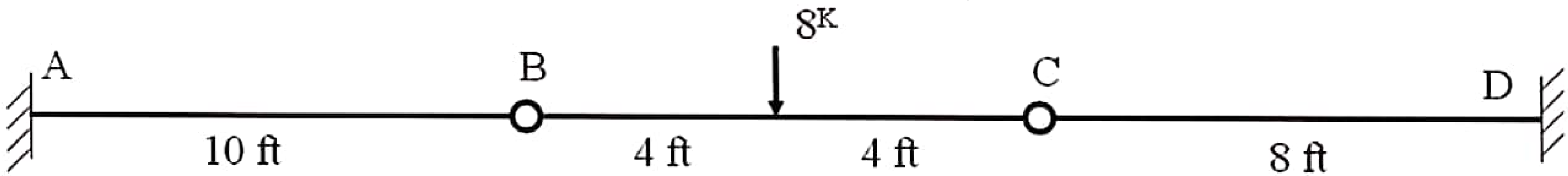
$$\Rightarrow \Delta_{Max} = \frac{111.11}{EI} \times x - \frac{1}{2} \times x \times \frac{33.33}{EI} \times x \times \frac{x}{3}$$

$$= \frac{111.11}{EI} \times 8.16 - \frac{1}{2} \times \frac{33.33}{10EI} \times \frac{8.16^3}{3} = \frac{605}{EI} = 0.17 \text{ inch. (Ans.)}$$

PROBLEM – 3

- Problem: Draw the conjugate beam and then calculate deflection at B & C.

(E= 30000 k/in^2; I = 500 in^4)

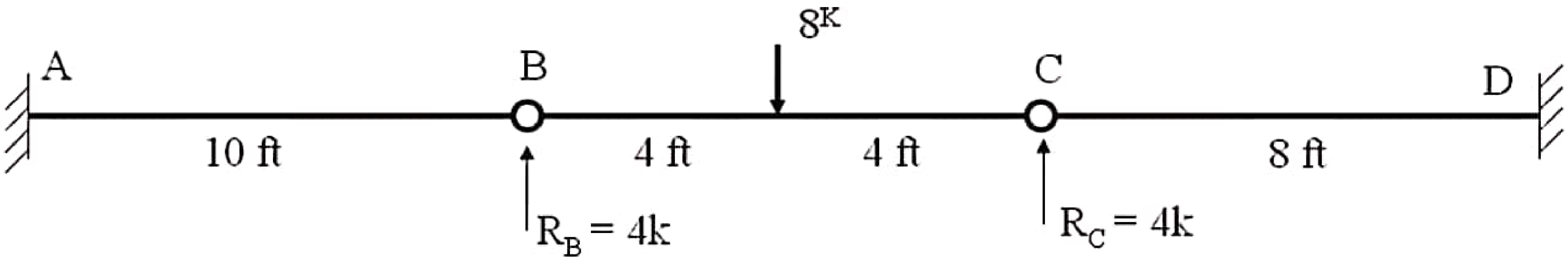


Note:

1. *Internal Link will not carry any moment and structure will be discontinuous.*

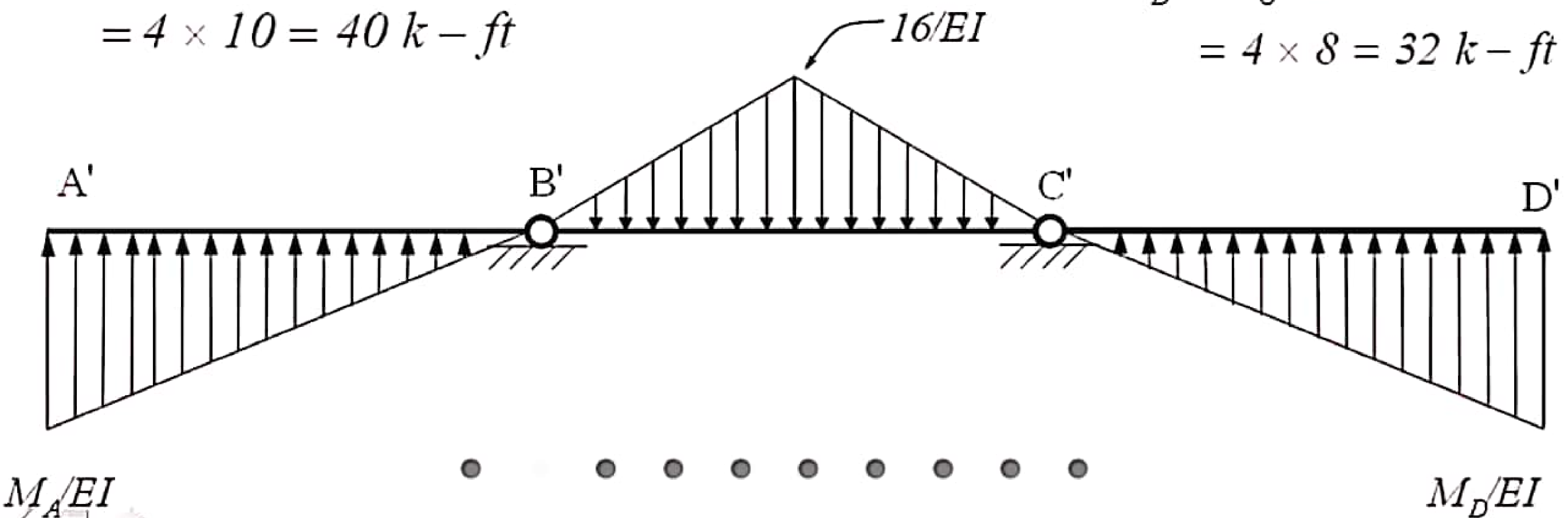
2. *Internal Hinge will carry moment and structure will be continuous.*

SOLUTION – 3



$$M_A = R_B \times 10$$
$$= 4 \times 10 = 40\ k-ft$$

$$M_D = R_C \times 8$$
$$= 4 \times 8 = 32\ k-ft$$



SOLUTION – 3

From the Real Beam:

$$M_A = R_B \times 10 = 4 \times 10 = 40\ k-ft\ \&$$

$$M_D = R_C \times 8 = 4 \times 8 = 32\ k-ft$$

Deflection at 'C' =

$$\frac{1}{2} \times \frac{32}{EI} \times 8 \times \frac{2}{3} \times 8 = \frac{682.267 \times 1728}{30000 \times 500} = 0.078\ in.$$

Deflection at 'B' =

$$\frac{1}{2} \times \frac{40}{EI} \times 10 \times \frac{2}{3} \times 10 = \frac{1333.33 \times 1728}{30000 \times 500} = 0.15\ in.\ (Ans.)$$