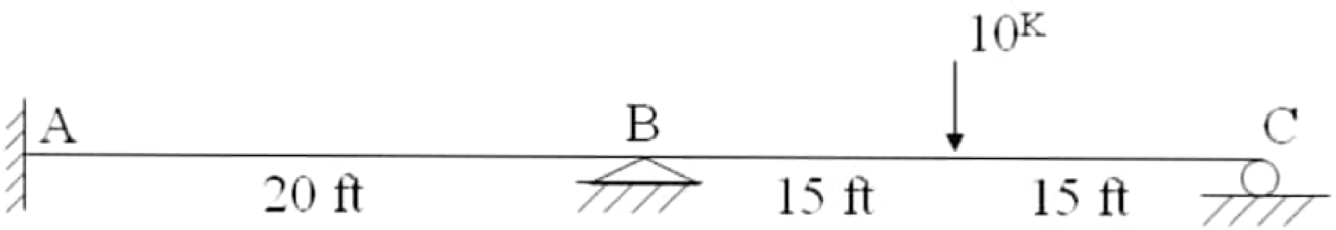
# PROBLEM – 4

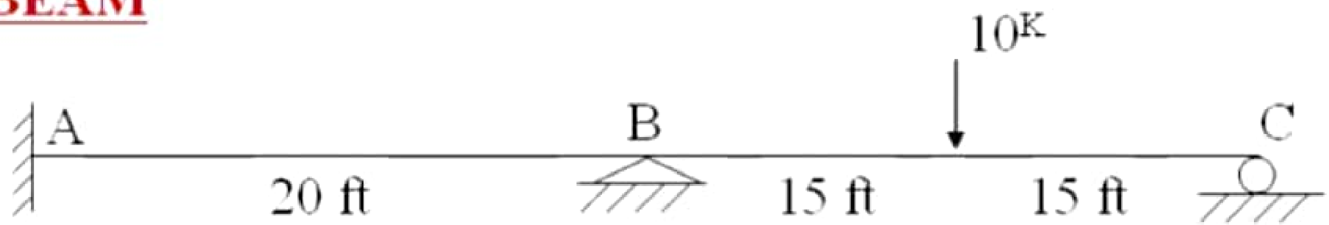
Problem: Using the conjugate beam method; calculate –

1. $M_A$ and $M_B$.
2. Distance from C to the point of maximum deflection in BC.
3. Magnitude of Maximum deflection in span BC.
4. The slope at B. ($E = 30{,}000\ k/in^2$ & $I = 500\ in^4$)

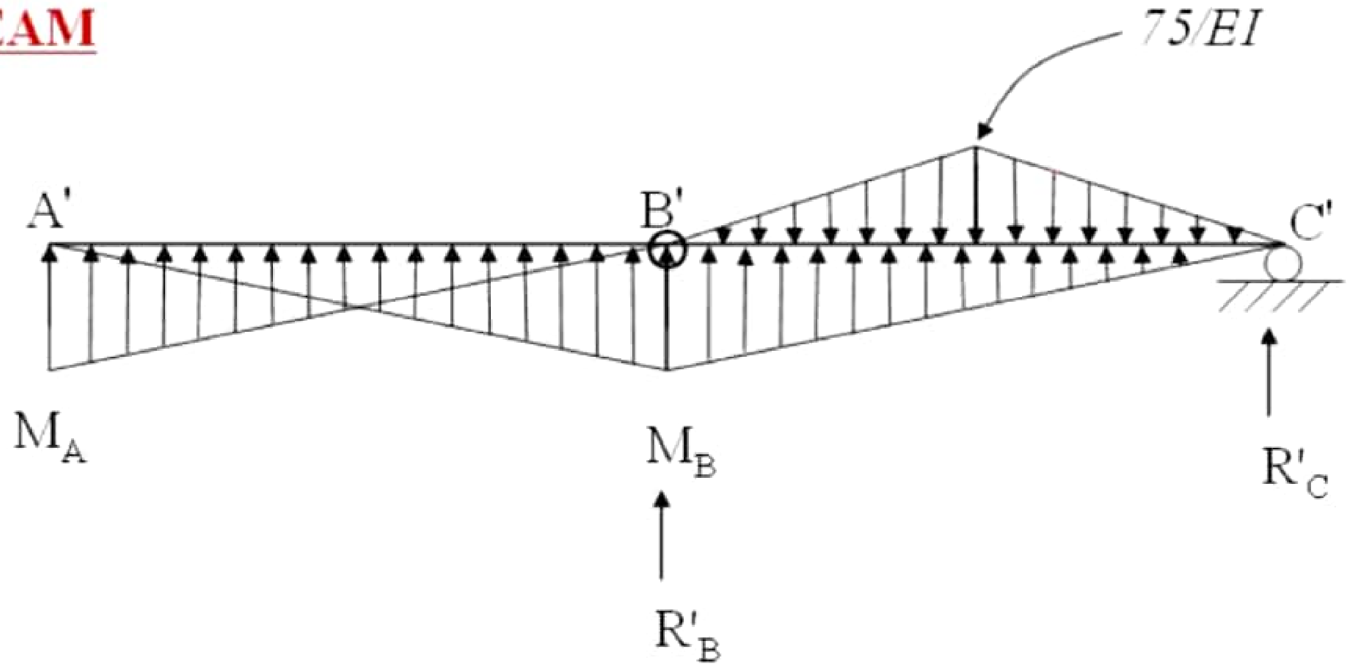


# SOLUTION – 4

**REAL BEAM**




**CONJUGATE BEAM**



# SOLUTION – 4

$$(1)\ \sum M_B = 0\ (Considering\ Left\ Portion\ of\ A'B')$$

$$\Rightarrow \frac{1}{2} \times 20 \times M_A \times \frac{2}{3} \times 20 + \frac{1}{2} \times 20 \times M_B \times \frac{20}{3} = 0$$

$$\Rightarrow \frac{400}{3} M_A + \frac{400}{6} M_B = 0 \quad \Rightarrow M_A = -0.5\, M_B \ldots\ldots (1)$$

# SOLUTION – 4

$$\sum M_C = 0 \ (Considering\ Left\ Portion\ of\ C'A')$$

$$\Rightarrow \frac{1}{2} \times 20 \times M_A \times (30 + \frac{2}{3} \times 20) + \frac{1}{2} \times 20 \times M_B \times (30 + \frac{20}{3})$$

$$+ \frac{1}{2} \times 30 \times M_B \times \frac{2}{3} \times 30 = \frac{1}{2} \times 15 \times 75 \times \frac{2}{3} \times 15 + \frac{1}{2} \times 15 \times 75 \times (15 + \frac{15}{3})$$

$$\Rightarrow 433.33\ M_A + 666.67\ M_B = 16875 \ldots\ldots\ldots(2)$$

$$From\ (1)\ \&\ (2) \Rightarrow M_B = 37.5\ k - ft$$

$$\&\ M_A = -18.75\ k - ft$$

# SOLUTION – 4

$$(2) \quad \sum M_B = 0 \ (Considering\ Right\ Portion\ of\ B'C')$$

$$\Rightarrow \quad R'_C \times 30 + \frac{1}{2} \times 37.5 \times 30 \times \frac{30}{3} =$$

$$\frac{1}{2} \times 15 \times 75 \times 10 + \frac{1}{2} \times 15 \times 75 \times 20$$

$$\Rightarrow \quad R'_C = 375^K$$

*Point of Zero Shear in Section B'C' –*

$$375 + \frac{1}{2} \times x \times \frac{37.5x}{30} - \frac{1}{2} \times x \times \frac{75x}{15} = 0$$

$$\Rightarrow \quad x = 14.14\ ft$$

# SOLUTION – 4

(3) *Maximum Deflection in Span BC –*

$$\Rightarrow \quad EI\Delta_{Max} = 375x + \frac{1}{2} \times x \times \frac{37.5}{30} \times x \times \frac{x}{3} - \frac{1}{2} \times x \times \frac{75}{15} \times x \times \frac{x}{3}$$

$$\Rightarrow \quad EI\Delta_{Max} = 375 \times 14.14 + \frac{1}{2} \times \frac{37.5}{30} \times \frac{14.14^3}{3} - \frac{1}{2} \times 5 \times \frac{14.14^3}{15}$$

$$\Rightarrow \quad \Delta_{Max} = \frac{3535.53 \times 1728}{30000 \times 500} \qquad \Rightarrow \quad \Delta_{Max} = 0.41 \ in$$

en

# SOLUTION – 4

$$(4)\ \sum V = 0 \quad (Considering\ Right\ Portion\ of\ B'C')$$

$$\Rightarrow \quad \frac{1}{2} \times 75 \times 30 - \frac{1}{2} \times 37.5 \times 30 - 375 = R'_B$$

$$\Rightarrow \quad R'_B = 187.5^{K}$$

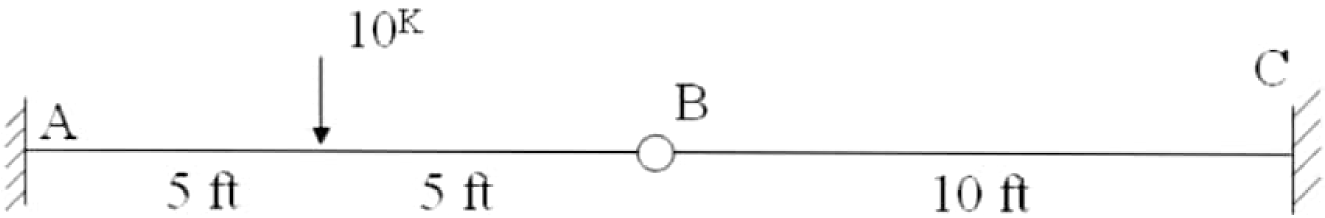
$$\Rightarrow \quad \theta_A = \frac{187.5 \times 144}{30000 \times 500} = 0.0018\ Rad \ldots\ldots\ldots\ldots(Ans.)$$

# PROBLEM – 5

Problem:

Using the conjugate beam method; calculate –

1. $M_A$ & $M_C$ and deflection at 'B'.
   ($E = 30{,}000\ k/in^2$ & $I = 200\ in^4$)



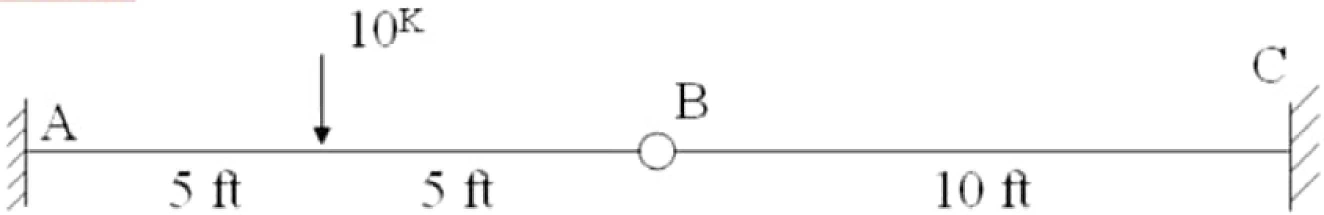

***Note:***

***•Internal Link will not carry any moment and structure will be discontinuous.***

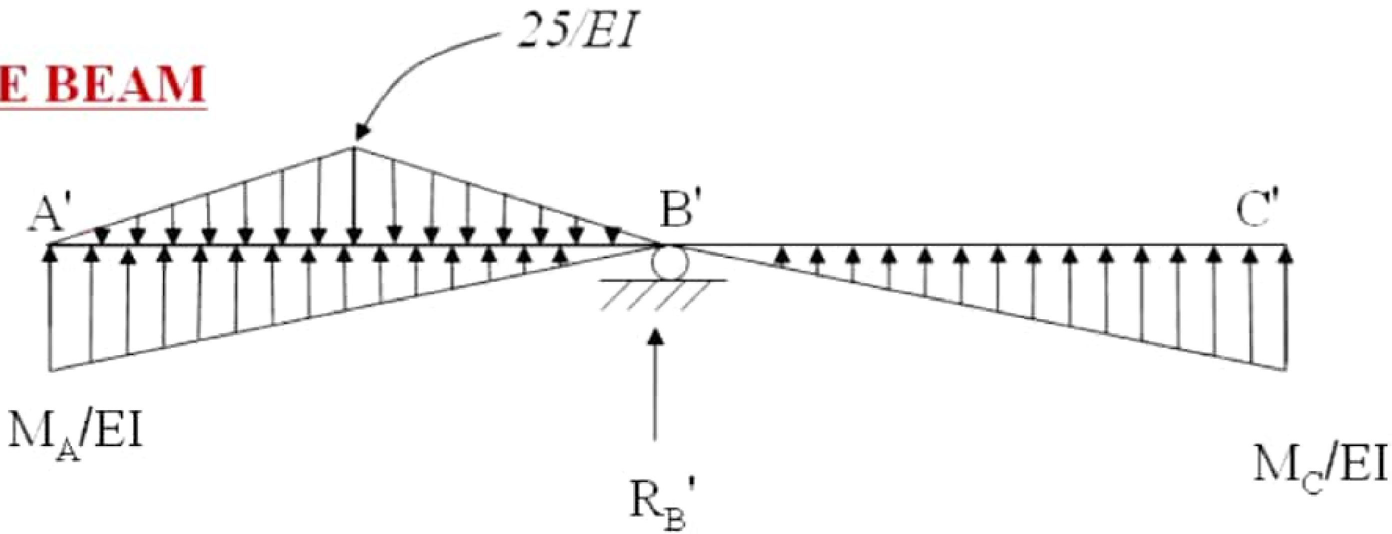
***•Internal Hinge will carry moment and structure will be continuous.***

# SOLUTION – 5

**REAL BEAM**




**CONJUGATE BEAM**



# SOLUTION – 5

$$(1)\ \sum B' = 0\ (\text{Considering Left Portion of } A'B')$$

$$\Rightarrow\ \frac{1}{2} \times 10 \times \frac{25}{EI} \times 5 = \frac{1}{2} \times 10 \times \frac{M_A}{EI} \times \frac{2}{3} \times 10$$

$$\Rightarrow\ M_A = 18.75\ k-ft$$

$$\text{Again; } \sum M_A = 18.75 = 50 - 10R_B \ \Rightarrow\ R_B = 3.125^K$$

$$\therefore\ M_C = 3.125 \times 10 = 31.25\ k-ft$$

# SOLUTION – 5

$(2)\ Deflection\ at\ 'B' -$

$$= \frac{1}{2} \times 10 \times \frac{31.25}{EI} \times \frac{2}{3} \times 10$$

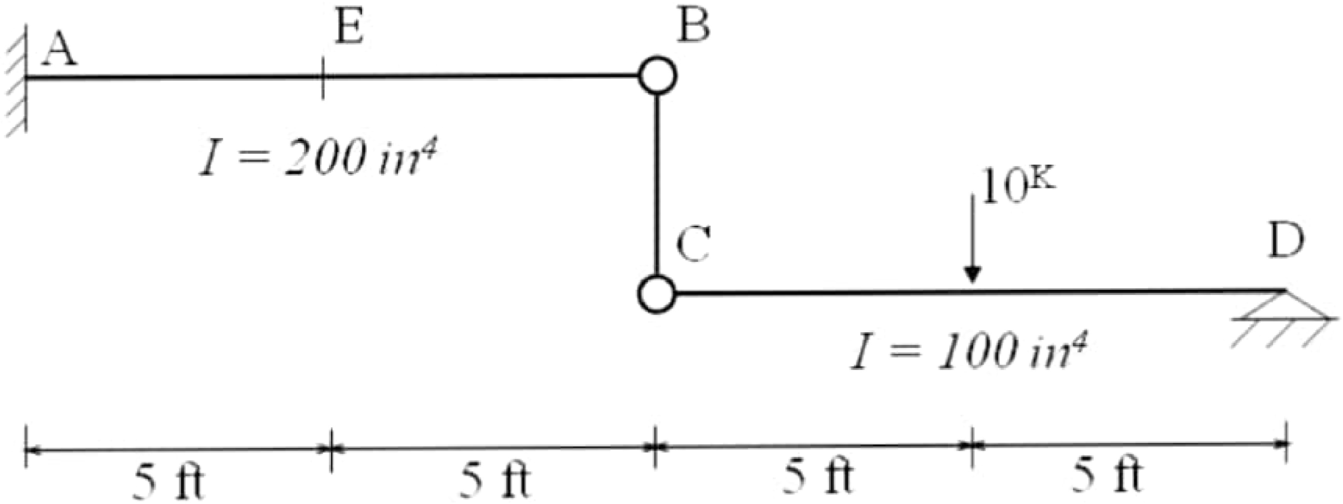
$$= \frac{1041.67}{EI}$$

$$= \frac{1041.67 \times 1728}{30000 \times 200} = 0.3\ in. \qquad (Ans)$$

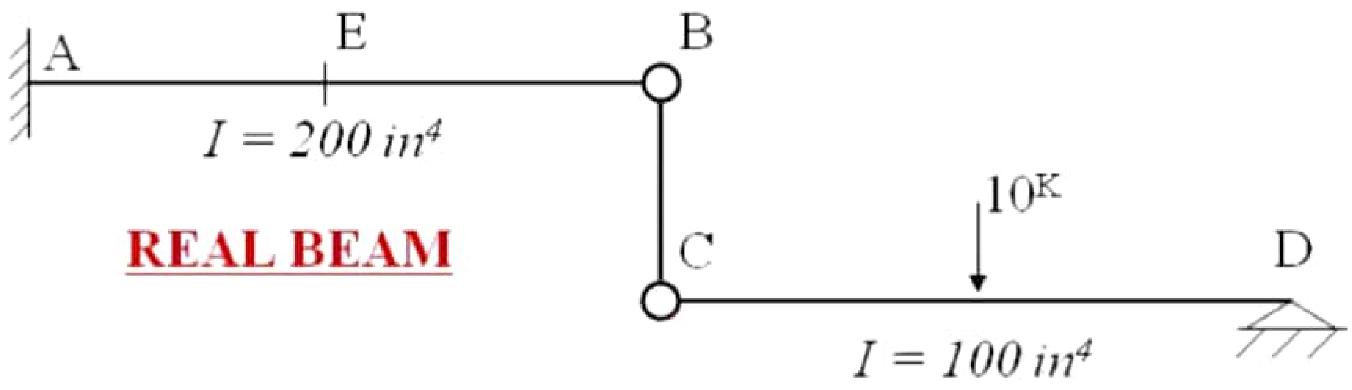
# PROBLEM – 6

Problem: Using the conjugate beam method; calculate –

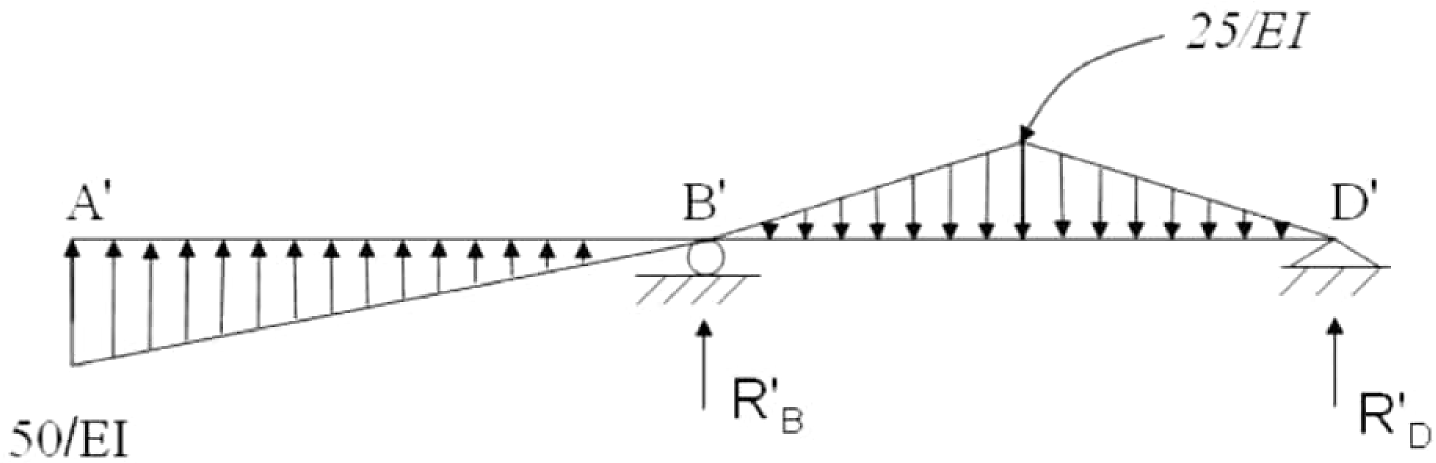
1. Deflection at B and C.
2. Deflection at E.
3. Slope at B.
4. Slope at D. (Given - E = 30,000 k/in$^2$)



# SOLUTION – 6




**REAL BEAM**




**CONJUGATE BEAM**

# SOLUTION – 6

(1) Deflection at B and C –

$$= \frac{1}{2} \times \frac{50}{EI} \times 10 \times \frac{2}{3} \times 10 = \frac{1666.67}{EI}$$

$$= \frac{1666.67 \times 1728}{30000 \times 200}$$

$$= 0.48 \text{ inch.}$$

# SOLUTION – 6

*(2) Deflection at E –*

$$= \frac{25}{EI} \times 5 \times 2.5 + \frac{1}{2} \times 5 \times \frac{25}{EI} \times \frac{2}{3} \times 5 = \frac{520.83}{EI}$$

$$= \frac{520.83 \times 1728}{30000 \times 200}$$

$$= 0.15 \text{ inch.}$$

# SOLUTION – 6

$(3)$ *Slope at B* –

$$= \frac{1}{2} \times \frac{50}{EI} \times 10 = \frac{250}{EI}$$

$$\theta_B = \frac{250 \times 144}{30000 \times 200}$$

$$\theta_B = 0.006 \ Rad.$$

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# SOLUTION – 6

$(4)$ *Slope at D;* $\sum M'_B = 0$ *(Right Side)*

$$\Rightarrow R'_D \times 10 = \frac{1}{2} \times 10 \times \frac{50}{E \times 100} \times 5 + \frac{1}{2} \times 10 \times \frac{50}{E \times 200} \times \frac{2}{3} \times 10$$

$$\Rightarrow \quad R'_D = \frac{1.458}{E}$$

$$\theta_D = \frac{1.458 \times 144}{30000} = 0.007\ Rad. \qquad (Ans.)$$