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outline

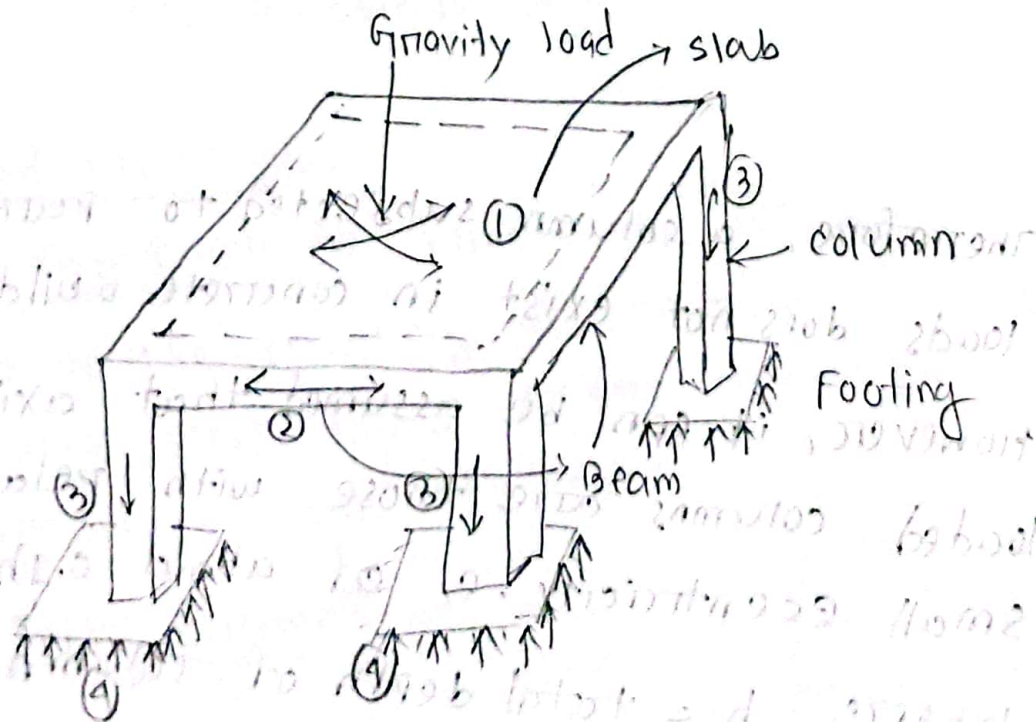
08-10-2022

- Two way slabs
- columns
- Retaining walls

column:

column → Axial load supporting members.

- Have a ratio of height to the least lateral dimension of 3 or greater.



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In reinforced concrete buildings concrete beams, floors and columns are cast monolithically, causing some moments in the columns due to end restraint.

Moreover, perfect vertical alignment of columns

therefore, a column subjected to pure axial loads does not exist in concrete buildings.

however, it can be assumed that axially loaded columns are those with relatively small eccentricity,  $e$ , of about  $0.1h$  or less.

where,  $h$  = total depth of column

$e$  = eccentric distance from the center ~~to~~ of the column.

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Because concrete has a high compressive strength and is an expensive material.

Types of columns:

Based on loading, columns may be classified as,

i) Axially loaded columns

ii) Eccentrically "

iii) Bi-axially "

Based on length,

i) Short column: where the column's failure is due to the crushing of concrete or the yielding of the steel bars under the full load capacity of the column.

ii) Long column: where

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Based on the shape of the cross-section:

→ Square

→ Rectangular

→ Round

→ L shaped

→ Octagonal etc

→ any desired shape with an adequate side width etc dimensions. (i)

Based on column ties

i) Tied columns: containing steel ties to confine the main longitudinal bars in the columns. Ties are normally spaced uniformly along the height of the column.

ii) Spiral columns: containing spirals (spring type reinforcement) to hold the main longitudinal reinforcement and to help the column ductility before failure.

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Based on frame bracing:

columns may be part of a frame that is braced against sideways etc & unbraced against sideways. Bracing may be achieved by using shear walls etc bracing in the building frame.

In braced frames, columns resist mainly gravity loads, ~~and~~ shear well as axial force and bending moment in column. Also braced frame resist lateral displacement, seismic forces and wind, much more than non-braced building.

There are different kinds of bracing:

- I) Single diagonal
- II) Cross bracing
- III) K-bracing
- IV) V-bracing
- V) Eccentric bracing.

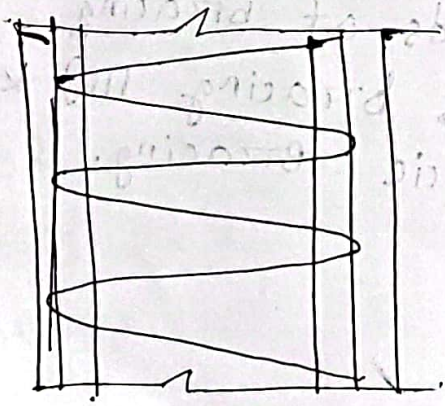
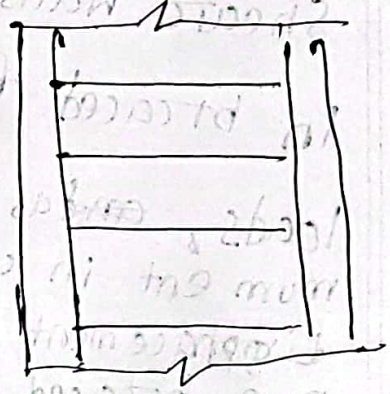
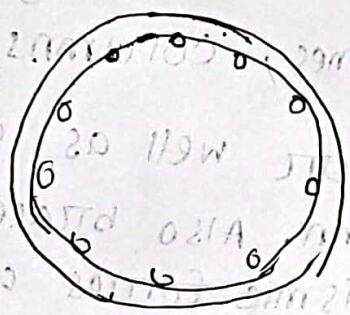
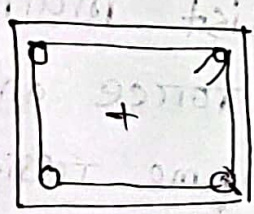
Based on materials

- Reinforced
- Prestressed
- Composite
- A combination of rolled steel sections and reinforcing bars.

concrete columns reinforced with longitudinal reinforcing bars are the most common type used in concrete building.

10-10-2022

Types of columns:



- reinforced
- precast
- composite
- A combination of precast and cast-in-place concrete

$$\text{MPa} \rightarrow \text{PSI}$$
$$\text{MPa} \times 145 = \text{PSI}$$

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## Why Reinforcement?

In members that sustain chiefly or exclusively axial compression loads, such as building columns. It is economical to make the concrete carry most of the load. Still, some reinforcement is always provided for various reasons. For,

1. Very few members are truly axially loaded; steel is essential for resisting any bending that may exist.
2. If part of the total load is carried by steel with its much greater strength, the cross sectional dimensions of the members can be reduced.

$$P = \frac{F}{A} \Rightarrow A = \frac{F}{P}$$

Let,

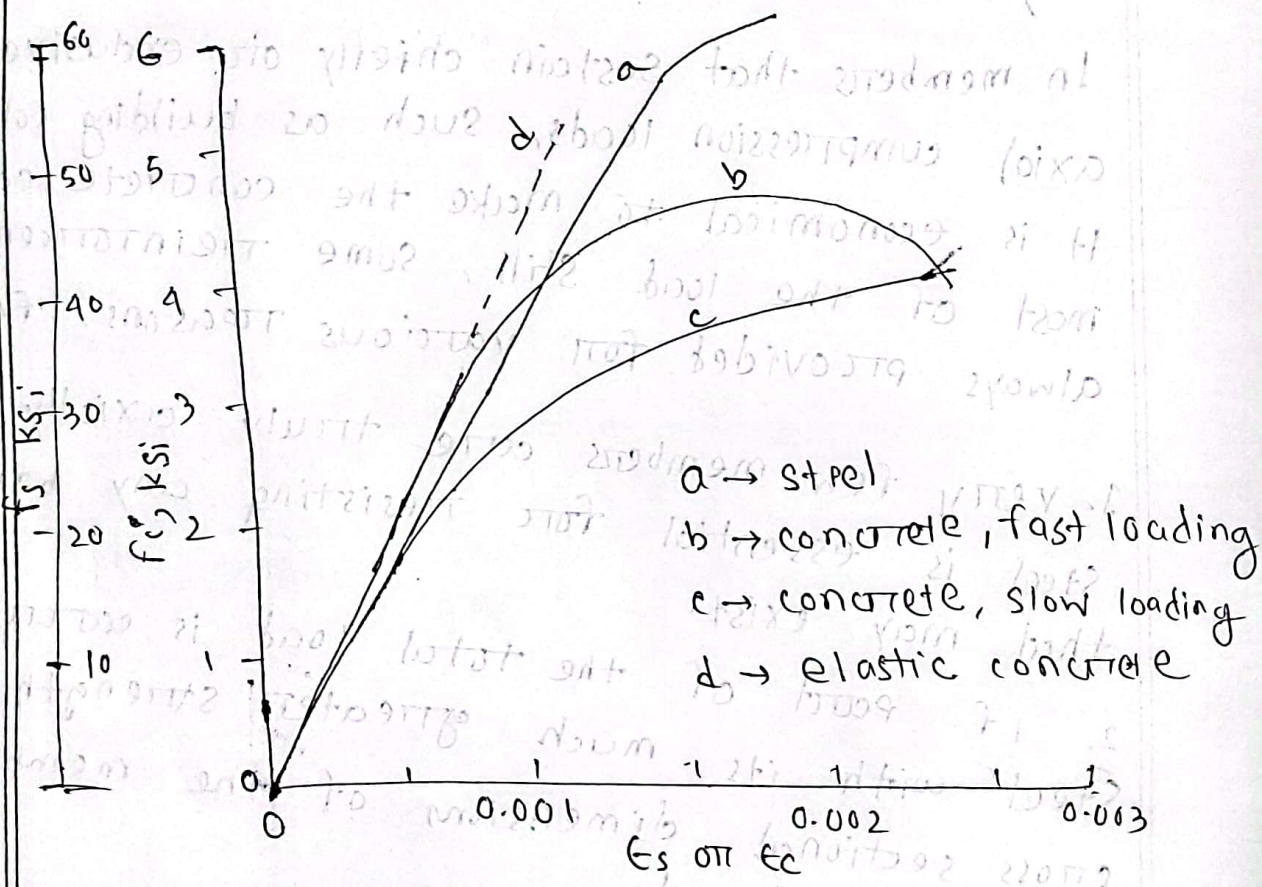
$$\text{Load} = 10 \times 10^8 \text{ N}$$

$$\text{Strength, } \sigma = 36 \times 10^6 \text{ N/mm}^2$$

129 ← 09M  
 129 = 09M

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Stress-strain curve of concrete



b. cylinder test

c. Real structure

Max reliable compressive strength of concrete would be  $0.85 f_c'$

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Elastic behaviour:

→ At low stresses, up to about  $f_c/2$  or  $(\frac{f_c'}{3})$ , the concrete is seen to behave nearly elastically, i.e. stresses and strains are quite closely proportional, the straight line  $d$  represents this range of behaviour with little error for both rates of loading.

→ The compression strain in the concrete, at any given load, is equal to the compression strain in the steel.

→ ~~The compression strain in the concrete, at~~

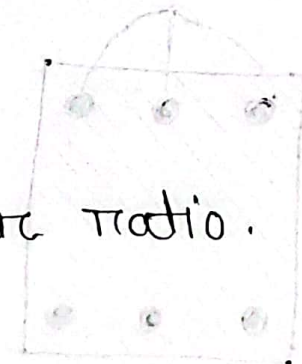
$$\epsilon_c = \frac{f_c}{E_c} = \epsilon_s = \frac{f_s}{E_s}$$

$$\Rightarrow \frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$\Rightarrow f_s = \frac{E_s}{E_c} f_c$$

$$\Rightarrow f_s = n f_c$$

where  $n = \text{modular ratio.}$



Let,

$A_g$  = gross area

$A_c$  = net area of concrete, i.e., gross area minus area occupied by reinforcing bars

$A_{st}$  = total area of reinforcing bars

$P$  = Axial load.

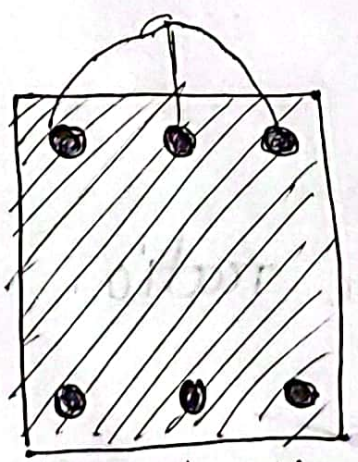
Then, 
$$P = f_c A_c + f_s A_{st}$$

$$= f_c A_c + n f_c A_{st}$$

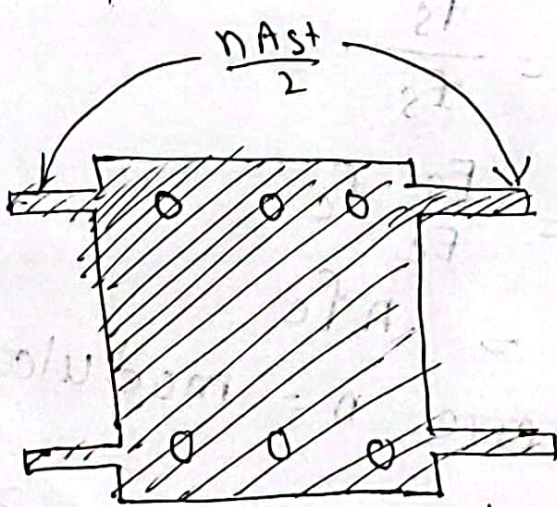
or, 
$$P = f_c (A_c + n A_{st})$$

$$\Rightarrow P = f_c \{ (A_g - A_{st}) + n A_{st} \}$$

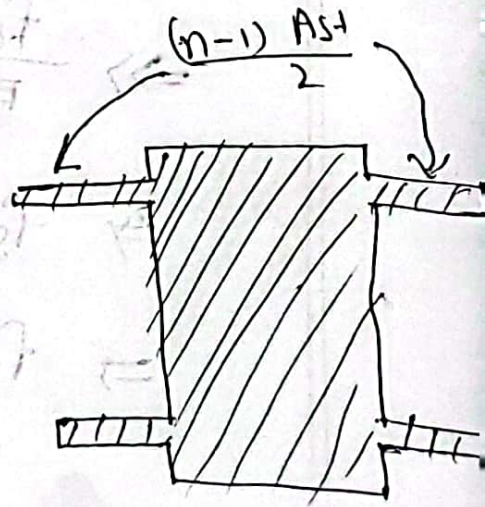
$$\Rightarrow P = f_c \{ A_g + (n-1) A_{st} \}$$



Actual section (a)



Transformed section  
 $A_t = A_c + n A_{st}$



Transformed section  
 $A_t = A_g + (n-1) A_{st}$

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Normal strength of the axially loaded concrete column can be found from,

$$P_n = 0.85f_c' A_c + f_y A_{st}$$

$$P_n = 0.85f_c' (A_g - A_{st}) + f_y A_{st}$$

The design strength of an axially loaded column is to be found based on eqn  $(P_n = 0.85f_c' (A_g - A_{st}) + f_y A_{st})$  with the introduction of certain strength reduction factor.

Why strength reduction factor of column is smaller?

→ column has a greater importance in a structure.

→ A beam failure would normally affect only a local region, whereas a column failure could result in the collapse of the entire section.

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Strength reduction factor of column:

Spirally reinforced column,  $\phi = 0.75$

Tied column,  $\phi = 0.65$

Beam  $\phi = 0.9$

A further limitation on column strength is imposed by ACI code to allow for accidental eccentricities of loading not considered in the analysis.

This is done by imposing an upper limit on the axial load that is less than the calculated design strength.

The upper limit for spiral column is 0.85 times the design strength.

The upper limit for tied column is 0.80 times the design strength.

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columns

Strength reduction  
factor,  $\phi$

Upper limit of  
axial load,  $k$

spiral

$\phi = 0.75$

0.85

columns

0.80

tied columns

ultimate strength of column:-

for spiral column,  $\phi = 0.75$

$$\phi P_n(\max) = \frac{0.85 \phi}{k} [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$$

for tied column,  $\phi = 0.65$

$$\phi P_n(\max) = \frac{0.80 \phi}{k} [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$$

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Problem-01

15-10-2022

Determine the allowable design axial load on a 12-in. square, short tied column reinforced with four #9 bars. ties are #3 spaced at 12 in. use  $f_c' = 4 \text{ ksi}$  and  $f_y = 60 \text{ ksi}$ .

Solution:

1. Eqn.  $P_u = \phi P_n$   
 $= \phi [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$

Here,  $A_g = 12 \times 12 = 144 \text{ in}^2$

$A_{st} = 4 \times 1 = 4 \text{ in}^2$

for tied column,  $\phi = 0.65$  and  $k = 0.80$

$\therefore P_u = 0.80 \times 0.65 [0.85 \times 4000 (144 - 4) + 60,000 \times 4]$

$= 372320 \text{ lb}$

$= 372.32 \text{ kip}$

(Ans)

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Problem-02:

Determine the allowable design axial load on a 16-in dia, short spiral column reinforced with ~~four~~ <sup>six</sup> #8 bars. Spirals are #3 spaced at 12 in. Use  $f'_c = 4 \text{ ksi}$  and  $f_y = 60 \text{ ksi}$

Solution:

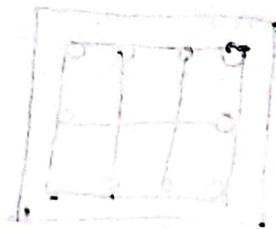
Here,  $A_g = \frac{\pi}{4} (16)^2 = 201.06 \text{ in}^2$

$A_{st} = 6 \times (0.79) = 4.74 \text{ in}^2$

Now,  $P_u = \phi P_n$   
 $= \phi [0.85 f'_c (A_g - A_{st}) + f_y A_{st}]$

For ~~short~~ spiral column,  
 $\phi = 0.75$  and  $k = 0.85$

$P_u = 0.85 \times 0.75 [0.85 \times 4 \times (201.06 - 4.74) + 60 \times 4.74]$   
 $= 606.83 \text{ k (ANS)}$



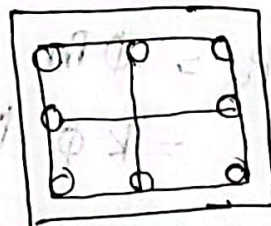
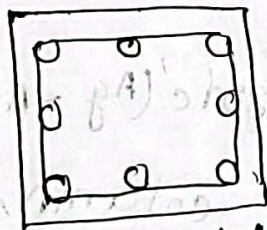
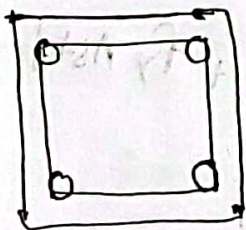
### Lateral Ties:

Bending moments are large  $\rightarrow$  "much of the longitudinal steel at the faces of highest compression or tension.

Heavily loaded columns  $\rightarrow$  large steel percentages

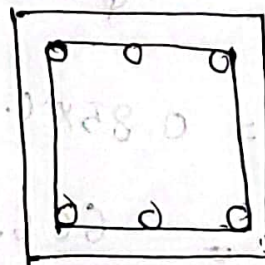
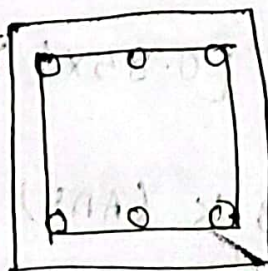
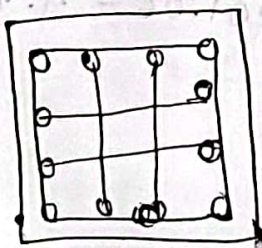
$\rightarrow$  large number of bars

$\rightarrow$  each of them positioned and held individually by ties.



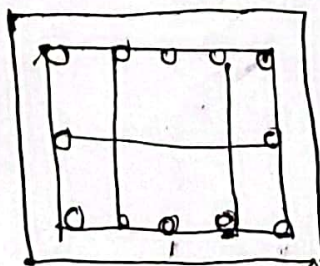
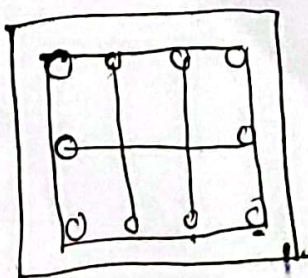
spacing  $< 6"$

spacing  $> 6"$



spacing  $< 6"$

spacing  $> 6"$



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→ Large steel percentages and consequent ties may cause steel congestion which leads to difficulties in placing the concrete.

→ In such cases, bundled bars are frequently employed.

\* Function of Lateral ties and spirals: Why ties are provided in RC column?

→ To hold the longitudinal bars in position in the forms while the concrete is being placed.

→ For this purpose, longitudinal and transverse steel is welded together to form cages, which are then moved into the forms and properly positioned before placing the concrete.

→ To prevent the highly stressed, slender longitudinal bars from buckling outward by bursting the thin concrete cover.

→ The spacing must be sufficiently small to prevent buckling between ties and that, in any tie plane.

→ A sufficient number of ties must be provided to position and hold all bars.

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## Bar Arrangement Rule (Aci code 7.10.5)

→ All bars of tied column shall be enclosed by lateral ties, at least No. 3 (10mm) for longitudinal bars up to No. 10 (32 mm)

→ At least No. 4 (13mm) in size for Nos. 11, 14 and 18 (36, 43 and 57 mm) and bundled longitudinal bars.

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## Steel Requirement :

→ the minimum longitudinal steel percentage is 1% and

→ the maximum percentage is 8% of the gross area of the section.

$$\text{e.g. } \boxed{1\% \text{ of } A_g < P < 8\% A_g}$$

→ Minimum reinforcement is necessary to

provide resistance to

- bending which may exist and

- to reduce the effects of creep and shrinkage of the concrete under sustained compressive stresses.

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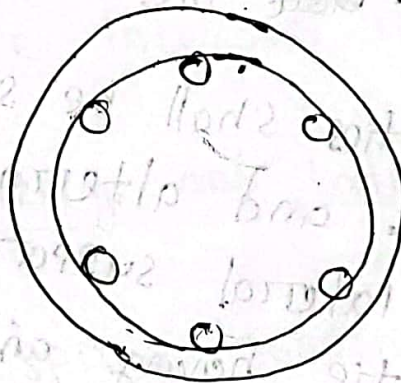
→ practically, it is very difficult to fit more than 8% of steel reinforcement into a column and maintain sufficient space for concrete to flow between bars.

→ At least four bars are required for tied circular and rectangular members and

→ six bars are needed for circular members enclosed by spirals.



4 bars



6 bars

→ For other shapes,

- One bar should be provided at each corner
- Proper lateral reinforcement must be provided.

→ For tied triangular columns, at least three bars are required.

→ Bars shall not be located at a distance greater than 6in, clear on either side from

a laterally supported bar :-  
 → The minimum concrete cover in columns is 1.5 inch.

Bar arrangement Rule:-

→ the spacing of the ties shall not exceed

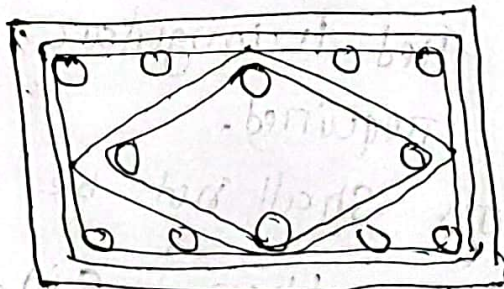
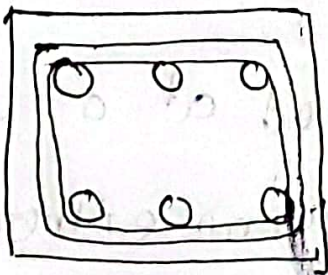
$s = 16 \times \text{diameter of longitudinal bars}$

$s = 48 \times \text{diameter of tie bars}$

$s = \text{the least dimension of the column.}$

→ The ties shall be so arranged that every corner and alternate longitudinal bars shall have lateral support provided by the corner of a tie having an included angle of not more than  $135^\circ$ .

→ No bar shall be further than 16 in clear on either side from such a laterally supported bar.



→ spiral shall consist of a continuous bar or wire not less than  $\frac{3}{8}$  inch diameter.

→ The clear spacing between turns of the spiral must not exceed 3 inch.

→ Not be less than 1 inch.

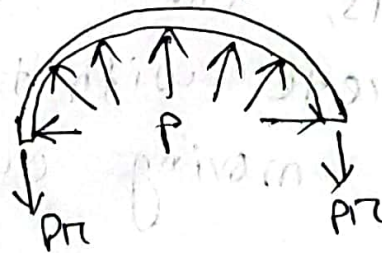
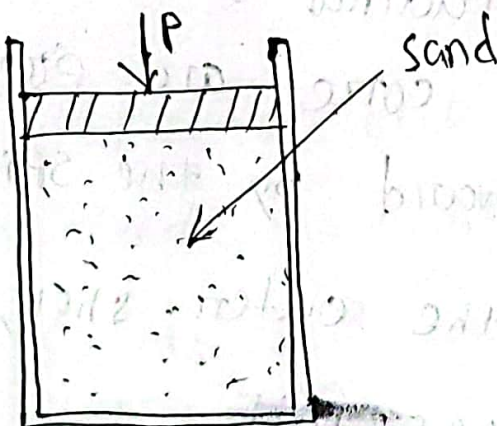
Structural effect of a spiral:-

→ Lateral pressure is exerted by the sand causes hoop tension in the wall.

→ The load can be increased until the drum bust.

→ longitudinal shortening and laterally expansion occurs.   
soil drum

→ A closely spaced spiral confining the column counteracts the expansion, as did the steel drum in the model.



→ This causes hoop tension in the spiral, while the carrying of the confined concrete in the core is greatly increased.

→ Failure occurs only when the spiral steel yields, which greatly reduces its confining effect, or when it fractures.

~~→~~

Behaviour of tied vs spiral column:-

→ At  $P_n$  load, the concrete in tied column fails by crushing and shearing outward along inclined planes.

→ And the longitudinal steel by buckling outward between ties.

→ In a spirally reinforced column at same load is, the longitudinal steel and the concrete with the core are prevented from moving outward by the spiral.

→ The concrete in the outer shell, not being so confined does fail.

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→ i.e. the outer shell spalls off when the load  $P_n$  is reached.

→ In contrast to the practice, any excess capacity beyond the spalling load of the shell is wasted the member. although not actually failed, would no longer be considered serviceable.

→ For this reason, the ACI code provides a minimum spiral reinforcement of such an amount that ~~its~~ its contribution to the carrying capacity is just slightly larger than that of the concrete in the shell.

→ The concrete in the outer shell, not being so confined does fail.

→ i.e. the outer shell

→ Spalling load of a spiral column equal to the ultimate load of the tied column.

→ The failure of the tied column is abrupt and complete.

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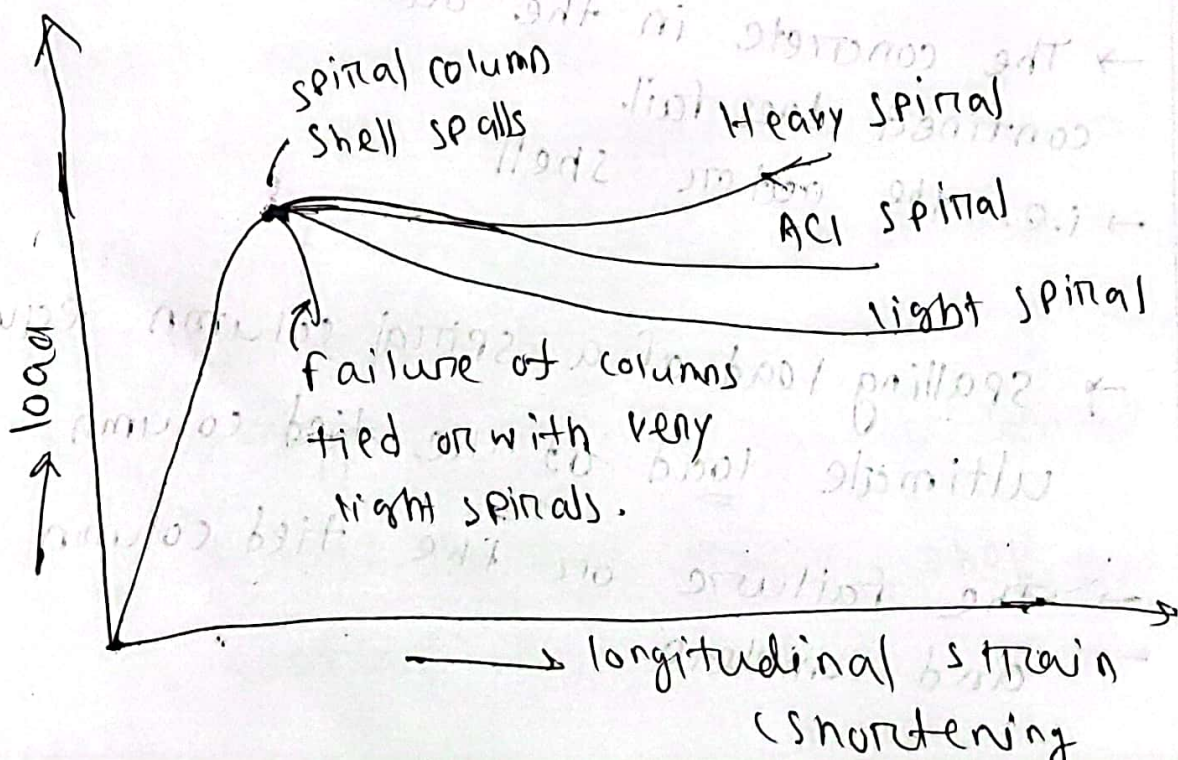
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→ Light spiral column has strength contribution is considerably less than the strength lost in the spalled shell.

→ With a heavy spiral the reverse is true.

→ The "ACI spiral", its strength contribution axial compensating for that lost in the spalled shell, hardly increases the ultimate load.

→ However, by preventing instantaneous crushing of concrete and buckling of steel it produces a more gradual and ductile failure. a tougher column



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## ~~Spiral Reinforcement Ratio~~

Strength contribution of the shell =  $0.85 f_c' (A_g - A_{ch})$ ,

where,  $A_g$  = gross area of concrete

$A_{ch}$  = core area

• In spirally reinforced column, spiral steel is at least twice as effective longitudinal bars;

therefore, strength provided by spiral =  $2 \rho_s f_y A_{ch}$

where,  $\rho_s$  = the ratio of volume of spiral reinforcement to total volume of core.

• The basis for the design of the spiral is that the strength gain provided by the spiral should be at least equal to that lost when the shell spalls

$$0.85 f_c' (A_g - A_{ch}) = 2 \rho_s f_y A_{ch}$$

$$\Rightarrow \rho_s = 0.425 \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f_c'}{f_y}$$

According to ACI code, this result is rounded upward slightly, and ACI code states that the ratio of spiral reinforcement shall not be less than  $\rho_s \text{ min.}$

$$\rho_s = 0.45 \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f_c'}{f_y}$$

→ The design relationship of spittals may be obtained as follows,

$$P_s = A_{dc} A_{sp} \times \frac{4}{A_{dc} S}$$

$$A_{sp} = \frac{P_s d_e S}{4}$$

$A_{sp}$  = cross-sectional area of spittal wire.

$d_e$  = outside diameter of spittal

$S$  = spacing of pitch of spittal wire

$P_s > P_{s \text{ min}}$

$$\frac{1}{G} \left( 1 - \frac{A}{A_{dc}} \right) = P_s$$

$$\frac{1}{G} \left( 1 - \frac{A}{A_{dc}} \right) = P_s$$

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Problem:

Design a tied column to support an axial load of 400 kip and live load 232 kip.  $f_c' = 5000$  psi,  $f_y = 60$  ksi = 60,000 psi. steel ratio is of 5% about 5% design necessary ~~the~~ ties.

Solution:-

Given, DL = 400 k  
LL = 232 k

$$\text{Factored design load} = 1.2 \text{ DL} + 1.6 \text{ LL} \\ = (1.2 \times 400) + (1.6 \times 232) \\ = 851.2 \text{ k}$$

$$P_u = \phi K [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$$

$$\rho_s = \frac{A_{st}}{A_g}$$

According to given condition,

$$5\% \rho_s = \frac{A_{st}}{A_g}$$

$$\Rightarrow A_{st} = \frac{5}{100} A_g = 0.05 A_g$$

$$\text{Now, } 851.2 = 0.65 \times 0.80 [0.85 \times 5 (A_g - 0.05 A_g) + 60 \times 0.05 A_g]$$

$$\Rightarrow A_g = 232.6 \text{ in}^2$$

$$\therefore A_{st} = 0.05 \times 232.6 = 11.63 \text{ in}^2$$

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Round up the value

$$b = \sqrt{232.6} = 15.25 \text{ inch}$$

Assume, ~~15.25~~ square column of (15.25 x 15.25) in

$$\phi \text{ So, } A_g = 15.25 \times 15.25 = 232.56 \text{ in}^2$$

Now,

$$A_{st} = 0.05 \times 232.56 = 11.63 \text{ in}^2$$

$$A_{st} = 0.05 \times 232.56 = 11.63 \text{ in}^2$$

Now,

$$851.2 = 0.65 \times 0.80 \left( 0.85 \times 5 \left( 232.56 - 11.63 \right) + 60 \times 11.63 \right)$$

$$\Rightarrow A_{st} = 11.04 \text{ in}^2$$

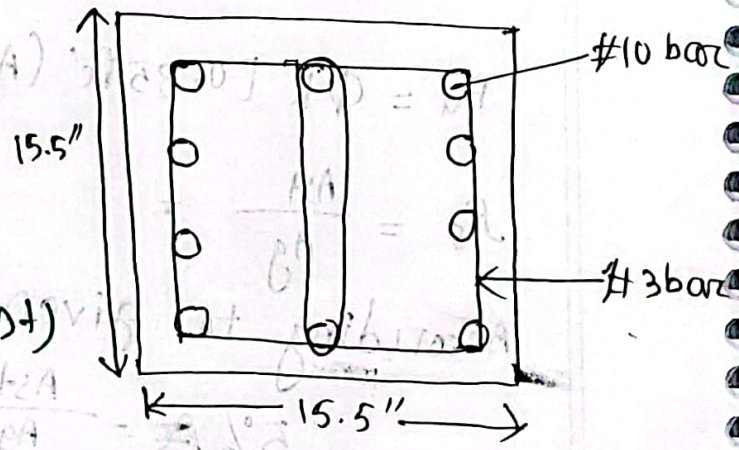
(eg #10 bars)

#3 bar (tie)

$$S = 1.48 \times \frac{3}{8} = 18''$$

$$S = 2.16 \times \frac{30}{8} = 20''$$

$$S = 3 \times 15.5 = 15.5'' \text{ (least)}$$



यदि 6" पर जेम्स (वर्क) र्थे जे 2.16 बाबत पकटा हे

(पिले शक)

\* Same design 14" width पर करे करे शक (H.W)

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Prb-02

Design a circular spiral column to support axial dead load of 475 kip and live load of 250 kip. using  $f_c' = 4 \text{ ksi}$  and  $f_y = 60 \text{ ksi}$ . Steel ratio is about 3%. Also design necessary ~~ties~~ spirals.

Solution:

$$\begin{aligned} \text{Factored design load} &= 1.2 \text{ D.L.} + 1.6 \text{ L.L.} \\ &= (1.2 \times 475) + (1.6 \times 250) \\ &= 970 \text{ kip} \end{aligned}$$

Here, 3% =  $\frac{A_{st}}{A_g}$   
 $\Rightarrow A_{st} = 0.03 A_g$

$$P_u = \phi K [0.85 f_c' (A_g - A_{st}) + f_y A_{st}]$$

$$970 = 0.75 \times 0.85 [0.85 \times 4 (A_g - 0.03 A_g) + 60 \times 0.03 A_g]$$

$$\Rightarrow A_g = 298.46 \text{ in}^2$$

$$A_{st} = 298.46 \times \frac{3}{100} = 8.95 \text{ in}^2$$

~~$b = \sqrt{298.46}$~~  = NOW,  $\pi \times \frac{d^2}{4} = 298.46$

$$\Rightarrow d = 19.49 \approx 19.5 \text{ inch}$$

$$\text{So, } A_g = \pi \times \frac{19.5^2}{4} = 298.64 \text{ in}^2$$

$$\text{NOW, } 970 = 0.75 \times 0.85 [0.85 \times 4 (298.64 - A_{st}) + 60 \times A_{st}]$$

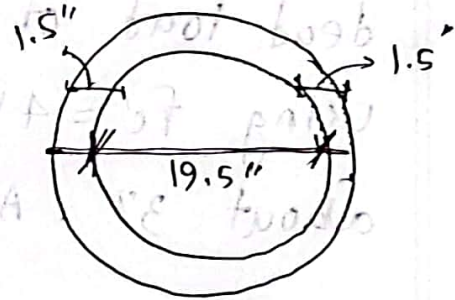
$$\Rightarrow A_{st} = 8.94 \text{ in}^2 \text{ (9 \#9 bars)}$$

for spiral column,

$$\text{Steel ratio } P_{sp} = \frac{4 A_{sp}}{D_e S}$$

$$P_{s(\min)} = 0.45 \left( \frac{A_g}{A_{ch}} - 1 \right) \frac{f_c'}{f_y}$$

$$= 0.45 \times \left( \frac{298.64}{213.83} - 1 \right) \times \frac{4}{60}$$



$$D_e = 19.5 - (1.5 \times 2)$$

$$= 16.5 \text{ inch}$$

$$A_{ch} = \frac{\pi}{4} (16.5)^2$$

$$= 213.83 \text{ in}^2$$

Now,  $P_{sp} = \frac{4 A_{sp}}{D_e S}$

Area of #3 bar

$$\Rightarrow 0.0119 = \frac{4 \times (0.11)}{16.5 \times S}$$

$$\Rightarrow S = 2.24 \text{ inch}$$

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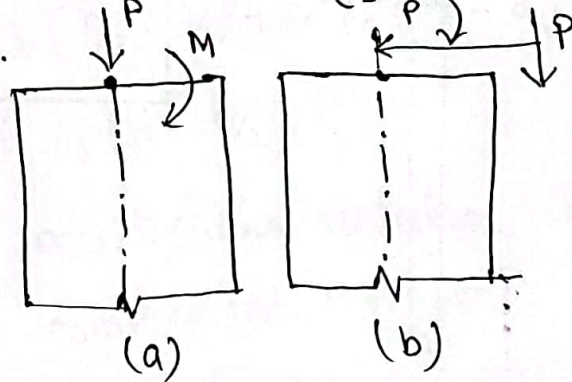
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Reinforced concrete column (compression + Bending):-

Compression plus Bending of rectangular columns:-

When a member is subjected to combined axial compression  $P$  and moment  $M$ , it is usually convenient to replace the axial load and moment with an equal load  $P$  applied at eccentricity  $e = M/P$ .

The two loadings are statically equivalent.



Strain compatibility and Analysis and interaction Diagram:-

compressive force,  $P_0$

Eccentricity,  $e$  (measured from the centerline).

The ultimate concrete strain is  $\epsilon_u$ ,

The strain in the bars nearest the load is,  $\epsilon'_s$ .

While that in the tension bars at the far side is  $\epsilon_s$ .

compression steel with  $a_{sc}$ ,  $a'_s$

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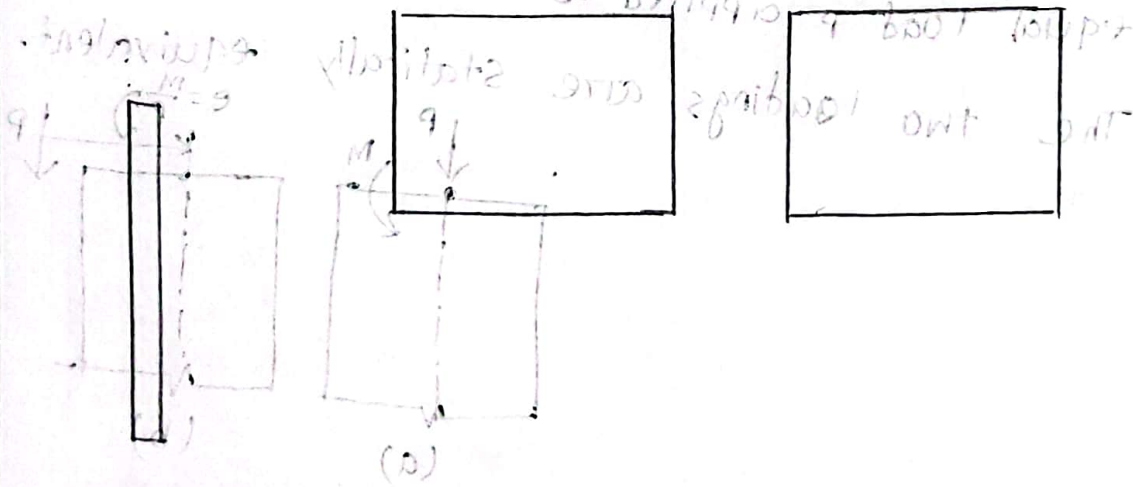
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Tension steel with area,  $a_s$ .

Compression steel location from compression face,  $d'$

Tension steel location from compression face,  $d$



just as for simple bending, the actual concrete compressive stress distribution is replaced by an equivalent rectangular distribution having depth

$$a = \beta_1 c$$

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Equilibrium between external and internal axial forces in fig. c.

$$P_n = 0.85 f_c' ab + f_c' A_c' - f_s A_s$$

Also, the moment about the centerline of the section of the internal stresses and forces must be equal and opposite to the moment of the external force  $P_n$ ,

So that,

$$M_n = P_n e = 0.85 f_c' ab \left( h/2 - a/2 \right) + f_c' A_c' \left( h/2 - d' \right) + f_s A_s \left( d - h/2 \right)$$

These are the two basic equilibrium relations for rectangular eccentrically compressed members.

The fact that the presence of the compression reinforcement  $A_s$  has displaced a corresponding amount of concrete of area  $A_s'$  is neglected in writing these equations.

If necessary, particularly for large reinforcement ratios, this excess force can be removed in both equations by multiplying  $A_s'$  by  $(f_s' - 0.85 f_c')$  rather than by  $f_s'$ .

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For large eccentricities, failure is initiated by yielding of the tension steel  $A_s$ .

Hence for this case,  $f_s = f_y$ .

When the concrete reaches its ultimate strain  $\epsilon_u$ , the compression steel may or may not have yielded; this must be determined based on compatibility of strains.

For a given eccentricity determined from frame analysis (that is,  $e = \frac{M_u}{P_u}$ )

In both cases, equations,  $f_s'$ ,  $f_s$ , and  $a$  can be expressed in terms of a single unknown  $c$ , the distance to the neutral axis,

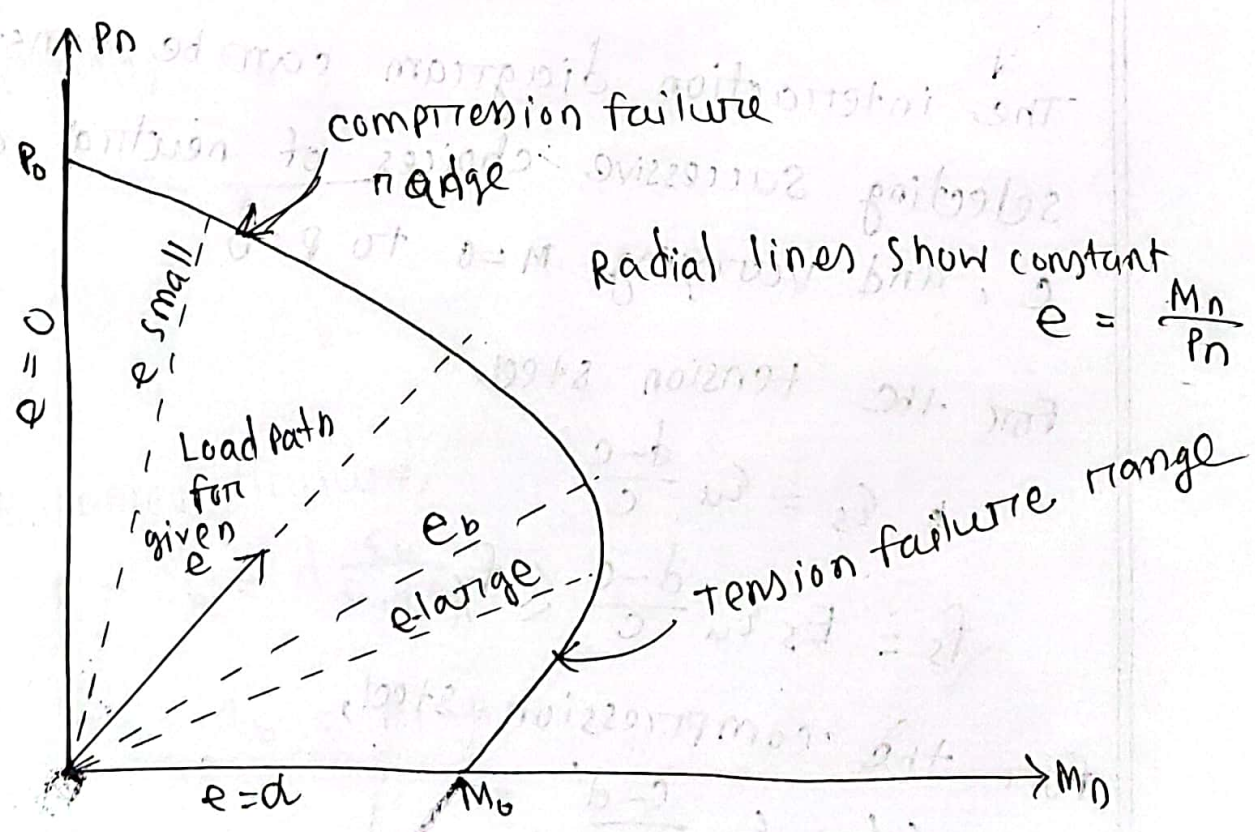
from geometry of the strain diagram, with  $\epsilon_u$  taken equal to 0.003 it can be done.

The two equations will contain only two unknowns,  $P_n$  and  $c$ , and can be solved.

However, to do so in practice would be complicated algebraically.

A better approach, providing the basis for practical design, is to construct a strength interaction diagram defining the failure load and failure moment for a given column for the full range of eccentricities from zero to infinity.

For any eccentricity there is a unique pair of values of  $P_n$  and  $M_n$  that will produce the state of incipient failure. That pair of values can be plotted as a point of graph  $P_n$  and  $M_n$



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Radial line represents a particular eccentricity  $e = \frac{M}{P}$ .  
When gradually increasing the load will reaches the limit curve, failure will result.  
The vertical axis corresponds to  $e=0$  and  $P_0$  is the capacity of the column if concentrically loaded.

The horizontal axis corresponds to an  $e$  of infinite value of  $e$ , that is, pure bending.

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The interaction diagram can be constructed by selecting successive choices of neutral axis distance  $c$ , and varying  $M=0$  to  $P=0$ .

For the tension steel,

$$\epsilon_s = \epsilon_u \frac{d-c}{c}$$

$$f_s = E_s \epsilon_u \frac{d-c}{c} \leq f_y$$

For the compression steel,

$$\epsilon_s' = \epsilon_u \frac{c-d'}{c}$$

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$$f_s' = E_s \epsilon_u \frac{c-d'}{c} \leq f_y$$

The concrete stress block has depth

$$a = \beta_1 c \leq h$$

The concrete compressive resultant is

$$C = 0.85 f_c' a b$$

Balanced Failure:

The interaction curve is divided into a compression failure and a tension failure range.

When the load  $P_b$  and Moment  $M_b$  and corresponding eccentricity  $e_b$

for balanced failure,

$$c = c_b = d \frac{\epsilon_u}{\epsilon_u + \epsilon_y}$$

$$a = a_b = \beta_1 c_b$$

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### Problem

A  $12 \times 20$  in column is reinforced with four NO. 9 <sup>in mm</sup> (NO. 29) bars of area  $10 \text{ in}^2$  each, one in each corner as shown in fig. The concrete cylinder strength is  $f_c' = 4000 \text{ Psi}$  and steel yield strength is  $60 \text{ ksi}$ . Determine -

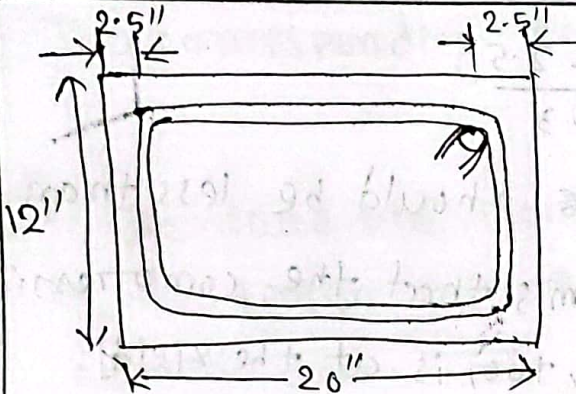
- (a) the load  $P_b$ , moment  $M_b$ , and corresponding eccentricity  $e_b$  for balanced failure.
- (b) The load and moment for a representative point in the tension failure region of the interaction curve;
- (c) the load and moment for a representative point in the compression failure region, and
- (d) the axial load strength for zero eccentricity
- (e) sketch the strength interaction diagram for this column, finally
- (f) design the transverse reinforcement, (based on ACI code provisions)

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Solution:

(a) the load  $P_b$ , moment  $M_b$  and corresponding eccentricity  $e_b$  for balanced failure:

For the balanced failure condition,

$$\epsilon_u = 0.003 \text{ and}$$

$$\text{effective depth} = 20 - 2.5 \\ d = 17.5$$

$$\epsilon_y = \frac{60,000}{29,000,000} = 0.0021$$

$$c_b = d \frac{\epsilon_u}{\epsilon_u + \epsilon_y} = 17.5 \times \frac{0.003}{0.003 + 0.0021} = 10.3 \text{ inch}$$

Stress block depth,  $a = \beta_1 c_b$

$$= 0.85 \times 10.3 = 8.76 \text{ in.}$$

For the balanced failure condition, by definition,

$$f_s = f_y$$

The compressive steel stress is:

$$f_s' = E_s \epsilon_u \frac{c - d'}{c} \leq f_y$$

$$\therefore f_s' = 0.003 \times 29,000 \times \frac{10.3 - 2.5}{10.3}$$

= 65.9 ksi but ~~it~~ should be less than 60 ksi

[confirms that the compression steel, too, is at the yield]

The compressive resultant is,  $c = 0.85 f_c' a_b$

$$= 0.85 \times 4 \times 876 \times 12$$

$$= 357 \text{ kips.}$$

The balanced load,  $P_b$  is,

$$P_b = 0.85 f_c' a_b + f_s' A_s' - f_s A_s$$

$$\therefore P_b = 357 + 60 \times 2 \times 1 - 60 \times 2 \times 1$$

compression side of steel

Tension side of steel

$$= 357 \text{ kips}$$

The balanced moment is,

$$M_n = P_n e = 0.85 f_c' a_b (h/2 - a/2) + f_s' A_s' (h/2 - d')$$

$$+ f_s A_s (d - h/2)$$

$$= 0.85 \times 357 (10 - 4.38) + 60 \times 2 \times 1 (10 - 2.5) +$$

$$60 \times 2 \times 1 (17.5 - 10)$$

$$= 3806 \text{ in-kips} = 317 \text{ ft-kips}$$

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The corresponding eccentricity of load is  $= \frac{3806}{352} = 10.66$  in

(b) The load and moment for a representative point in the tension failure region of the interaction curve:-

$c$  smaller than  $c_b$  will give a point in the tension failure region of the interaction curve, with eccentricity larger than  $e_b$ .

Let's choose a  $c$  smaller than  $c_b = 10.3$  in.

Choose  $c = 5.0$  in.

By definition,  $f_s = f_y$ .

The compressive steel stress is found to be

$$f_s' = E_s \epsilon_u \frac{c-d'}{c} \leq f_y$$

$$\Rightarrow f_s' = 0.003 \times 29000 \times \frac{5.0-2.5}{5.0} = 43.5 \text{ ksi}$$

with the stress-block depth,  $a = 0.85 \times 5.0 = 4.25$ .

the compressive resultant is,  $C = 0.85 \times 4 \times 4.25 \times 12 = 173$  kips.

$$P_n = 173 + 43.5 \times 2.0 - 60 \times 2.0 = 140 \text{ kips.}$$

The moment capacity is,

$$M_n = 173(10 - 2.12) + 43.5 \times 2.0(10 - 2.5) + 60 \times 2.0(17.5 - 10) = 2916 \text{ in-kips} = 243 \text{ ft-kips.}$$

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$$e \text{ eccentricity, } e = \frac{2916}{140} = 20.83 \text{ in.}$$

(c) the load and moment for a representative point in the compression failure region :-

$$\text{Let choose, } c = 18.0 \text{ in}$$

$$a = 0.85 \times 18.0 = 15.3 \text{ inch}$$

The compressive concrete resultant is  $C = 0.85 \times 4 \times 15.3 \times 12 = 624 \text{ kips}$

The stress in the steel at the left side of the column is,

$$f_s = E_s \epsilon_u \frac{d-e}{c} \leq f_y$$

$$f_s = 0.003 \times 29,000 \times \frac{17.5 - 18.0}{18.0} = -2 \text{ ksi [As is in}$$

compression if  $e$  is greater than  $d$ ]

The compressive steel stress is,

$$f_s' = E_s \epsilon_u \frac{c-d'}{c} \leq f_y$$

$$f_s' = 0.003 \times 29,000 \times \frac{18.0 - 2.5}{18.0} = 75 \text{ ksi but } \leq 60 \text{ ksi}$$

(c) The load and moment for a representative point in the compression failure region:-

Then the column capacity is,

$$P_n = 624 + 60 \times 2.0 + 2 \times 2.0 = 748 \text{ kips}$$

$$M_n = 624(10 - 7.65) + 60 \times 2.0(10 - 2.5) - 2 \times 2.0(17.5 - 10) \\ = 2336 \text{ in-kips} = 195 \text{ ft-kips}$$

$$\text{Eccentricity, } e = \frac{2336}{748} = 3.12 \text{ in}$$

(d) The axial load and moment strength for zero eccentricity:-

The axial strength of the column if concentrically loaded corresponds to,

$$e = d \text{ and } e = 0$$

For this case,

$$P_n = 0.85 f_c' A_g + f_y A_{st}$$

$$= 0.85 \times 4 \times 12 \times 20 + 60 \times 4.0 = 1056 \text{ kips}$$

considering the concrete replaced by the steel,

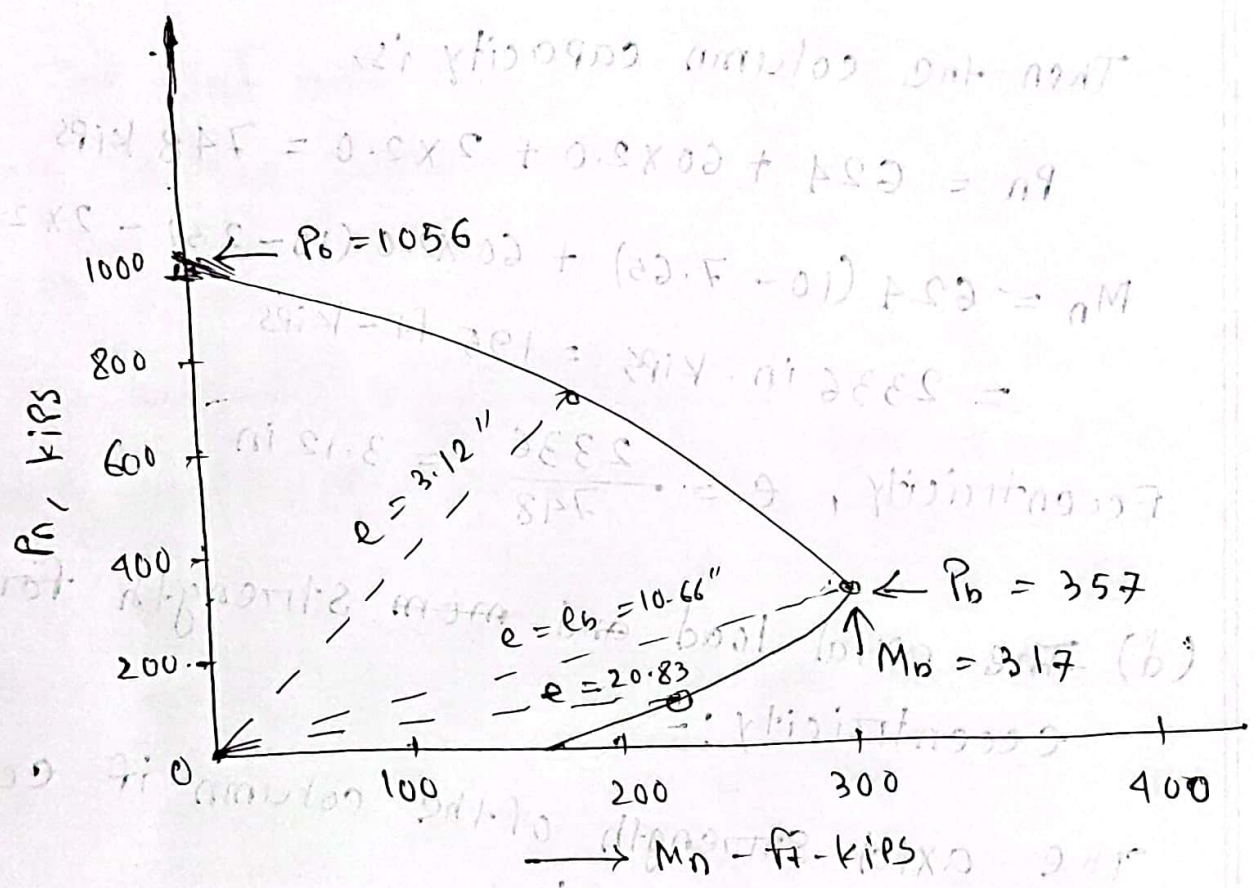
$$P_n = 0.85 f_c' (A_g - A_{st}) + f_y A_{st}$$

$$= 0.85 \times 4 (12 \times 20 - 4) + (60 \times 4.0)$$

$$= 1042 \text{ kips}$$

The error in neglecting this deduction is only 1% in this case.

(e) sketch the strength interaction diagram for this column:-



(f) design the transverse reinforcement:-

As No. 9 (No. 29) longitudinal bars is used so

# 3 bars can be used as tie.

The tie spacing is not to exceed.

$$48 \times \frac{3}{8} = 18 \text{ in}$$

$$16 \times 1.128 = 18.05 \text{ inches}$$

$$b = 12 \text{ inch}$$

# 3 bars should be provided @ 12" c/c.