REINFORCED CONCRETE

MD FARHAD HOSSAIN

CE 15, 1500045

"Whoever travels a path in search of knowledge, Allah makes easy for him a path to Paradise" (Sahih Muslim)

Special THANKS to-

My friend

SAYEM AHAMEED

CE 15, 1500119

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ZIA SIR

SARKAR SIR

Footings and Foundation

Define 11,12,14,15,08

Foundation:

The foundation or substructure is the part of a structure that is usually placed below the surface of the ground and that transmit the load to the underlying soil or rock.

State the general requirments for foundation design. 16,15,09,12

The two essential requirements in the design of foundations are:

1. The total settlement of the structure should be limited to a tolerably small amount.
2. Differential settlement of the various parts of the structure be eliminated as nearly as possible.

What preventing measures to be taken to limit settlement? 14

To limit settlement, it is necessary:

(1) to transmit the load of the structure to a soil stratum of sufficient strength

(2) to spread the load over sufficiently large area of that stratum to minimize bearing pressure.

(3) to apply proper compaction to the substructure before construction.

Or, Describe the functions of foundation. 15

Write down the objectives of footing: 2001

Every structures are provided with foundation at the base to fulfill the following objectives:

1. To distribute the load of the structure over large bearing area.
2. To prevent unequal settlement.
3. To prevent the lateral movement of the supporting material.
4. To increase the stability of the structure as a whole.

Explain-
Different types of spread footing with neat sketch: 12, 07

If satisfactory soil directly under lies the structure, it is merely necessary to spread the load, by footings or other means, such substructures are known as spread foundations.

Spread footings can be classified as:
(i) wall footings and (ii) column footings.

Wall footings: A wall footing is a continuous strip of reinforced concrete, wider than the wall that distributes its pressure.

Single-column footings: They are usually square, sometimes rectangular and represent the simplest and most economical type. When footing is provided to support an individual column, it is called isolated or single column footing.

Combined footings: The use of single column footing under exterior columns meets with difficulties if property rights prevent the use of footings projecting beyond the exterior walls. In this case combined footing or strap footings are used. Combined footings are also used when two column are so close to each other that their individual footings would overlap.

Strap footing: It consists of two single-column footing connected with a structural strap or a lever. The strap connects the footing such that they behave as one unit. A strap footing is more economical than a combined footing when the allowable soil pressure is relatively high and distance between the columns is large.

Mat or Raft footing: It is a large solid reinforced concrete slab that extends under the entire building and consequently, distributes the load of the structure over the maximum avaiable area. Such foundation minimizes differential settlement. A mat is required when the allowable soil pressure is low or where the columns and walls are so close that individual footings overlap or nearly touch each other.

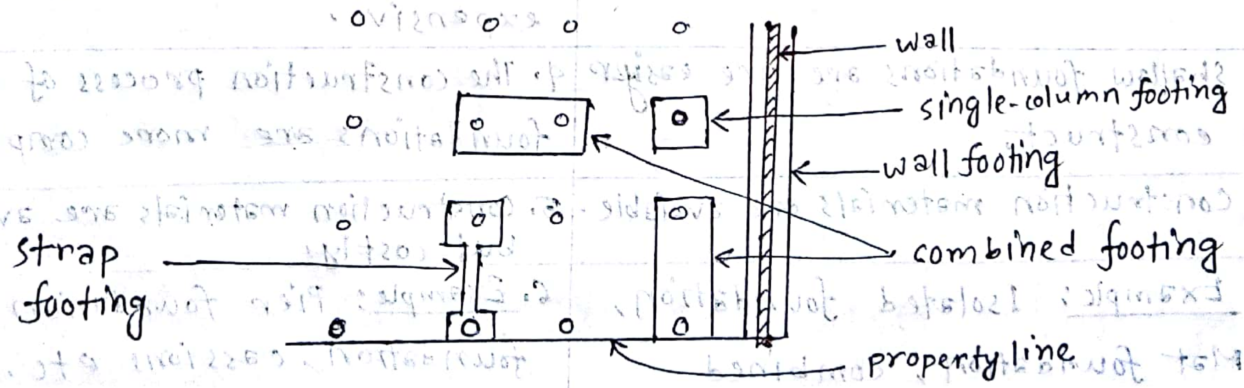


Fig. Types of spread footing.

Describe

Types of foundation:

Foundations may be two types:

1. Shallow foundation.
2. Deep foundation.

Shallow foundation: A foundation is shallow if its depth is equal or less than its width.

types of shallow foundation: single-column footing, combined footing, Mat or Raft footing, strap footing.

Deep foundation: Deep foundation are those in which the depth of foundation is very large in comparision to its width.

Types of deep foundation: pile foundation, pier foundation, cassions, Shaft foundation etc.

Distinguish between shallow foundation and deep foundation. 2016

Shallow foundation	Deep foundation
1. It transfers the loads at a shallow depth.	1. It transfers the loads to deep strata.
2. The depth of shallow foundation is generally about 3 m or less than the footing width.	2. The depth of the deep foundation is greater than shallow foundation.
3. Shallow foundation is cheaper.	3. Deep foundations are more expensive.
4. shallow foundations are are easier to construct.	4. The construction process of deep foundations are more complex.
5. Construction materials are avaiable.	5. Construction materials are avaiable but costly.
6. Example: Isolated foundation, Mat foundation, combined foundation, strap foundation.	6. Example: Pier foundation, Pile foundation, cassions etc.

Explain the pressure distribution under different types of soil.

In ordinary construction, the load on a wall or column is transmitted vertically to the footing, which in turn is supported by the upward pressure of the soil on which it rests.

If the load is symmetrical with respect to the bearing area, the bearing pressure is assumed to be uniformly distributed. Fig. (a)

Under footings resting on coarse grained soils, the pressure is larger at the center of the footing and decreases toward the perimeter. Fig. (b)

In constrast, in clay soil pressures are higher near the edge than at the center of footing, since in such soils the load produces a shear resistance around the perimeter that odds to the upward pressure. Fig. (c)

On compressible soils, footings should be loaded concentrically to avoid tilting, which will result if bearing pressures are significantly larger under one side of the footing than under the opposite side. Fig. (d)

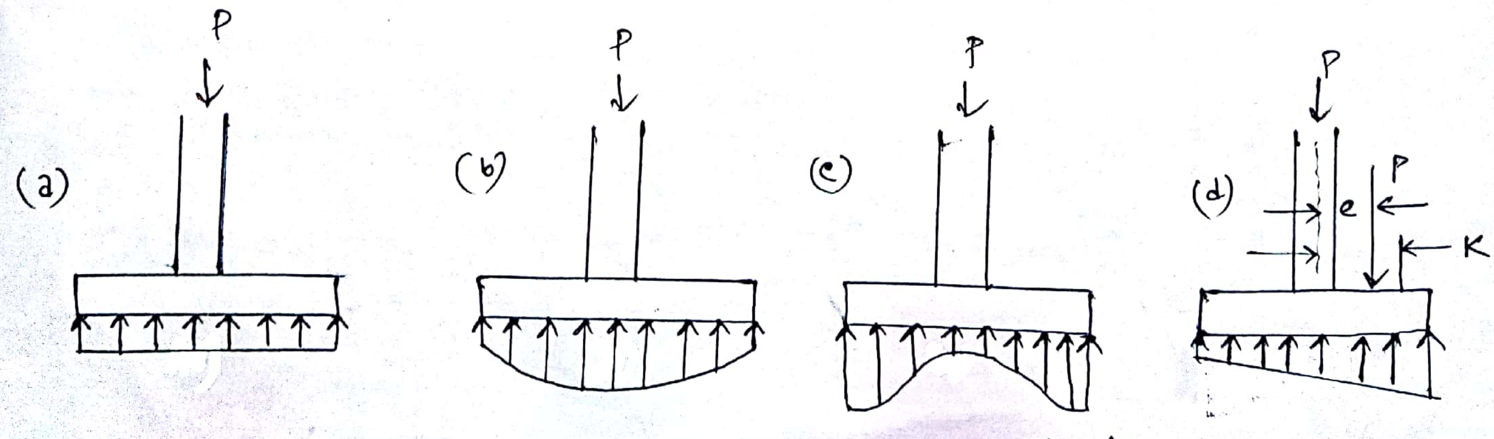


Fig. Bearing pressure distribution

USD

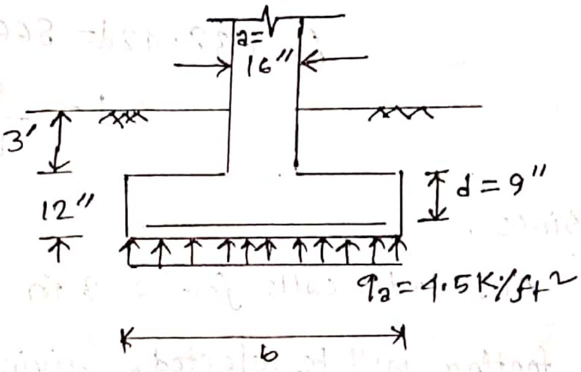
Wall Footing

Example: 16.1 2012

A 16 in. concrete wall supports a dead load $D = 14$ Kips/ft and a live load $L = 10$ Kips/ft. The allowable bearing pressure is $q_a = 4.5$ Kips/ft^2 at the level of the bottom of the footing, which is 4 ft below grade. Design a footing for this wall using 4000 psi concrete and grade 60 steel.

Solution: Let, thickness of footing = 12″

∴ Fill on top of footing = 3′



∴ weight of soil on top of the footing = (3 × 100) = 300 psf

and weight of footing = 150 psf

∴ Net pressure on footing, $q_e = 4.5 \times 1000 - (300 + 150)$

$\therefore q_e = 4050$ psf

∴ The required width of footing, $b = \dfrac{\text{Total Load}}{q_e} = \dfrac{(14000 + 10000)}{4050}$ ft

$\therefore b = 5.93$ ft $\simeq 6$ ft

∴ Ultimate bearing pressure of footing, $q_u = \dfrac{(1.2 \times 14000 + 1.6 \times 10000)}{6}$

$\therefore q_u = 5466.67$ psf

Moment calculation:

∴ ultimate moment, $M_u = \frac{1}{8} \times q_u (b-a)^2 = \frac{1}{8} \times 5466.67 \times (6 - \frac{16}{12})^2$

$\therefore M_u = 14881.5$ lb-ft/ft

Depth Check:

Assuming $d = 9$ in,

The shear, $V_u = q_u \left(\frac{b-a}{2} - d\right) = 5466.67 \times \left[\frac{1}{2}(6-1.33) - \frac{9}{12}\right]$

$\therefore V_u = 8664.67$ lb/ft

The design shear, $\phi V_c = 2\phi\sqrt{f_c'}\, bd$

$\Rightarrow \phi V_c = 2 \times .75 \times \sqrt{4000} \times 12 \times d = 1138.42d$ lb/ft

From which,

$1138.42d = 8669.95$

$\Rightarrow d = \frac{8664.67}{1138.42} = 7.61$ in. $< 9''$

$\therefore$ design is OK.

Since,

ACI code calls for a 3 in clear cover on bars; a 12 in thick footing will be selected, giving $d = 8.5''$. $\therefore t = (8.5+3) = 11.5''$

This is sufficiently close to the assumed values and calculations need not revised.

Reinforcement calculation:

The required steel area, $A_s = \frac{M_u}{\phi f_y (d - \frac{a}{2})}$

$\Rightarrow A_s = \frac{14881.5 \times 12}{0.9 \times 60000 \times (8.5 - \frac{1}{2} \times 1.47 A_s)}$

$\Rightarrow A_s = 0.40$ in^2/ft

Here,

$a = \frac{A_s f_y}{.85 f_c' b}$

$= \frac{A_s \times 60000}{.85 \times 4000 \times 12}$

$= 1.47 A_s$.

provide #5 bar @ $\frac{0.31}{0.40} \times 12 = 9.3''$ c/c

if $f_y < 60$ ksi

$A_{st(min)} = 0.0020bt$

Distribution reinforcement, $A_{st(min)} = 0.0018\, bt$ $(f_y = 60$ ksi$)$ $= 0.0018 \times 12 \times 11.5$

$= 0.25$ in^2/ft

provide #3 bar @ $\frac{0.11}{0.25} \times 12 = 5.28''$ c/c

Development Length: As No. 5 bar is used, $d_b = \frac{5}{8}$ in.

The required development length, $l_d = \frac{\alpha \beta \lambda f_y}{25\sqrt{f_c'}} \times d_b$ [using, $\alpha = 1.3$, $\beta = 1.5$, $\lambda = 1.0$]

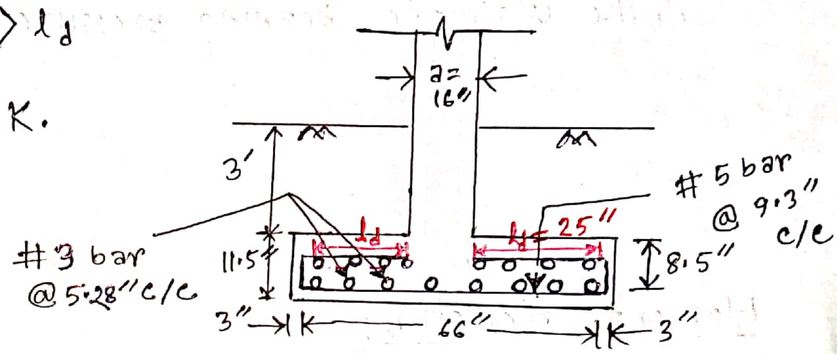
$$= \frac{1.3 \times 1.5 \times 1 \times 60000}{25\sqrt{4000}} \times \frac{5}{8}$$

$$\therefore l_d = 46.25 \text{ in.}$$

The actual development length

$= (\underset{b}{6 \times 12} - \underset{c.c}{2 \times 3} - \underset{a}{16}) = 50 \text{ in.} > l_d$

$\therefore$ design is ok.

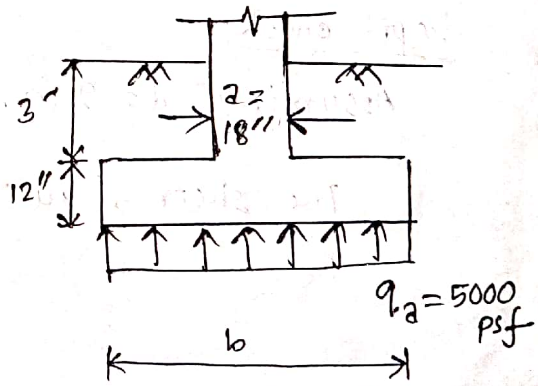


2011 2017 2015 USD

\# Design a wall footing for an 18 in. brick wall carrying a service load of 14 K/ft dead load and 16 K/ft live load using $f_c' = 3000$ psi, $f_y = 60000$ psi and allowable pressure, $q_a = 5000$ psf.

Solution: Let, thickness of the footing = 12"

and, Fill on top of footing = 3'



$\therefore$ weight of soil on top of the footing,

= (3×100) psf = 300 psf

and weight of footing = 150 psf

$\therefore$ Net pressure on footing = 5000 − (300+150) psf

= 4550 psf

$\therefore$ The required width of the footing $= \frac{\text{Total load}}{q_e}$

$$= \frac{(14000 + 16000)}{4550} \text{ ft}$$

$$= 6.6 \text{ ft} \simeq 7 \text{ ft}$$

$\therefore$ The ultimate bearing pressure of footing, $q_u = \frac{1.2 D.L + 1.6 L.L}{b}$

$$= \frac{1.2 \times 14000 + 1.6 \times 16000}{7}$$

$$= 6057.143 \text{ psf}$$

Moment calculation:

Ultimate moment, $M_u = \frac{1}{8} q_u (b-a)^2$

$$= \frac{1}{8} \times 6057.143 \times \left(7 - \frac{18}{12}\right)^2$$

$$= 22903.6 \text{ lb-ft/ft}$$

Depth check:

Assuming $d = 9$ in.

The shear, $V_u = q_u \left(\frac{b-a}{2} - d\right) = 6057.143 \times \left(\frac{7-1.5}{2} - \frac{9}{12}\right)$

$\therefore V_u = 12114.3$ lb/ft

The design shear, $\phi V_c = 2\phi\sqrt{f_c'}\, bd$.

$\Rightarrow \phi V_c = 2 \times .75 \times \sqrt{3000} \times 12 \times d = 985.9\, d$

From which,

$985.9\, d = 12114.3 \quad \Rightarrow d = 12.29 > 9''$

design is not OK.

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Assuming $d = 12''$

The shear, $V_u = 6057.143 \times \left(\frac{7-1.5}{2} - \frac{12}{12}\right) = 10600$ lb/ft

$\therefore 985.90\,d = 10600 \Rightarrow d = 10.75$ in $< 12''$

$\therefore t = (12+3) = 15''$ design is ok.

Reinforcement calculation:

The required steel area, $A_s = \dfrac{M_u}{\phi f_y (d - \frac{a}{2})}$

$\Rightarrow A_s = \dfrac{22903.6 \times 12}{.9 \times 60000 \times (12 - \frac{1.96 A_s}{2})}$

$\Rightarrow A_s = 0.44$ in^2/ft

Here $a = \dfrac{A_s f_y}{.85 f_c' b}$

$\Rightarrow a = \dfrac{A_s \times 60}{.85 \times 3 \times 12}$

$\therefore a = 1.96 A_s$

provide #5 @ $\frac{.31}{.44} \times 12 = 8.45''$ c/c

Distribution reinforcement, $A_{s+(min)} = 0.0018\, bt$

$= 0.0018 \times 12 \times 15$

$= 0.324$ in^2/ft

provide # 3 bar @ $\frac{0.11}{0.324} \times 12 = 4.07'' \simeq 4''$ c/c

* for 6 no. bar or smaller bar, $l_d = \dfrac{\alpha \beta \lambda f_y}{25\sqrt{f_c'}} d_b$

Development length: As No. 5 bar is used,

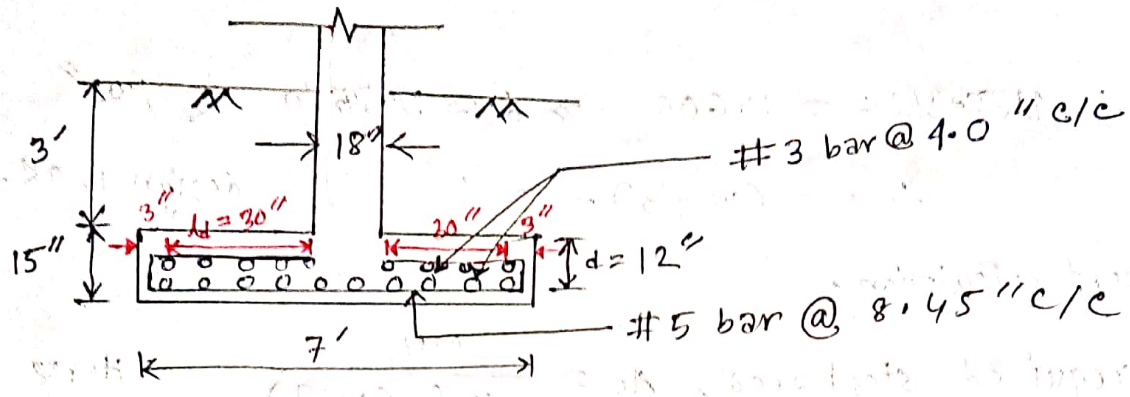
The required development length, $l_d = \dfrac{\alpha \beta \lambda f_y}{25\sqrt{f_c'}} \times d_b$

using $\alpha = 1.3$, $\beta = 1.5$, $\lambda = 1.0$; $l_d = \dfrac{1.3 \times 1.5 \times 1 \times 60000}{25 \times \sqrt{3000}} \times \frac{5}{8}$

$= 53.40$ in.

The actual development, = $(7\times12-2\times3-18) = 60$ in $> l_d$

$\therefore$ design is OK

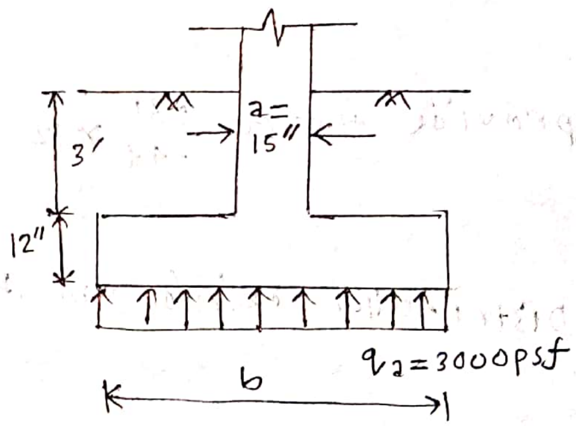


2013
2015 WSD

Design a reinforced concrete footing for a masonry wall 15 inch thick carrying a total load of 20 K/ft. The safe bearing capacity of soil is 3000 psf. Assume $f_c' = 1000$ psi and $f_s = 29000$ psi.

Solution: Let, thickness of footing = 12"

and, Fill on top of footing = 3'



$\therefore$ weight of soil on top of footing.

$= (3\times100)$ psf

$= 300$ psf

and, weight of footing = 150 psf

$\therefore$ Net pressure on footing $= 3000-(300+150)$ psf

$= 2550$ psf

$\therefore$ The required width of the footing, $b = \frac{\text{Total load}}{q_e}$

$= \frac{(20\times1000)}{2550}$

$= 7.84$ ft $\simeq 8$ ft

∴ The allowable bearing pressure of footing, $P = \frac{20000}{8}$ psf

$= 2500$ psf

Moment Calculation:

Maximum bending moment, $M = \frac{1}{8} p (b-a)^2$

$= \frac{1}{8} \times 2500 \times \left(8 - \frac{15}{12}\right)^2$

$= 14238.28$ lb-ft/ft

Depth Check: (for moment)

Assuming clear cover $= 3''$ ∴ $d_{eff} = (12-3) = 9''$

Now,

$r = \frac{f_s}{f_c} = \frac{24000}{.45 \times 4000} = 13.33$

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$

∴ $K = \frac{n}{n+r} = \frac{8}{8+13.33} = 0.375$

$j = 1 - \frac{K}{3} = \left(1 - \frac{.375}{3}\right) = 0.875$

$R = \frac{1}{2} f_c j K = \frac{1}{2} \times (.45 \times 4000) \times .875 \times .375 = 295.3125$

$d_{req} = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{14238.28 \times 12}{295.3125 \times 12}} = 6.944'' < d_{eff}$

Hence, design is ok.

(for shear).

The shear, $V_s = p\left(\frac{b-a}{2} - d\right) = 2500 \times \left(\frac{8 - \frac{15}{12}}{2} - \frac{9}{12}\right)$

∴ $V_s = 6562.5$ lb/ft

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The nominal shear, $V_c = 1.1\sqrt{f_c'}\,bd$

$\therefore V_c = 1.1\sqrt{4000} \times 12 \times d = 834.84\,d$

From which,

the effective depth required for shear, $d_{req} = \dfrac{6562.5}{834.84}$ in.

$\therefore d_{req.} = 7.86$ in $< d_{eff}$

Hence, design is OK.

<u>Reinforcement calculation:</u>

The required steel area, $A_s = \dfrac{M}{f_s jd} = \dfrac{14238.28 \times 12}{24000 \times .875 \times 9}$ in^2

$\therefore A_s = 0.904\ in^2/ft$

provide # 6 bar @ $\dfrac{.44}{0.904} \times 12 = 5.84''$ c/c

Distribution reinforcement, $A_{st(min)} = 0.0018\,bt$

$= (0.0018 \times 12 \times 12)\ in^2$

$\therefore A_{st(min)} = 0.26\ in^2/ft$

provide # 3 bars @ $\dfrac{.11}{0.26} \times 12 = 5''$ c/c

<u>Development length:</u> As No. 6 bar is used, $d_b = \frac{6}{8}''$

The required development length, $l_d = \dfrac{f_s d_b}{4u}$ Here, $u = \dfrac{4.8\sqrt{f_c'}}{d_b}$

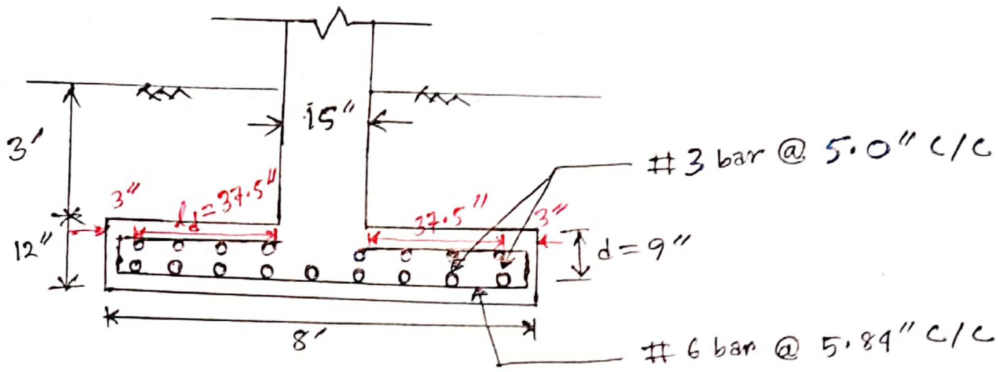
$\Rightarrow l_d = \dfrac{24000 \times \frac{6}{8}}{4 \times 404.77}$ in $= \dfrac{4.8\sqrt{4000}}{6/8}$

$\therefore l_d = 11.12$ in $= 404.77$ psi

The actual development length for one side of the section,

$$\frac{1}{2} \times (8 \times 12 - 2 \times 3 - 15) = 37.5 \text{ in} > l_d$$

Hence, design is OK.



USD

Square Footing

Example : 16·2

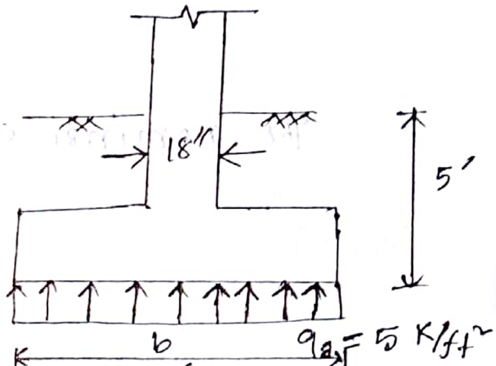
2015, 2014

A column 18 in. square, with $f_c' = 4$ Ksi, reinforced with eight No. 8 bars of $f_y = 60$ Ksi, support a dead load of 225 Kips. and a live load of 175 Kips. The allowable soil pressure q_a is 5 Kips/ft². Design a square footing with base 5 ft below grade, using $f_c' = 4$ Ksi and $f_y = 60$ Ksi

Solution:

Since The space between the bottom of the footing and the surface will be occupied partly by concrete and partly by soil,

Assume, an average unit weight of concrete and soil = 125 pcf



The pressure of this materials at 5ft depth,

$$= (5 \times 125) \text{ psf}$$

$$= 625 \text{ psf}$$

∴ Net pressure on footing, $q_e = (5 \times 1000 - 625)$

$$= 4375 \text{ psf} = 4.375 \text{ Ksf}$$

The required area of the footing, $A_{req} = \dfrac{\text{Total Load}}{q_e}$

$$\Rightarrow b^2 = \frac{225 + 175}{4.375} \text{ ft}^2$$

$$\Rightarrow b^2 = 91.43 \text{ ft}^2$$

$$\therefore b = 9.56 \text{ ft} \simeq 9.5 \text{ ft}$$

∴ ultimate bearing pressure,

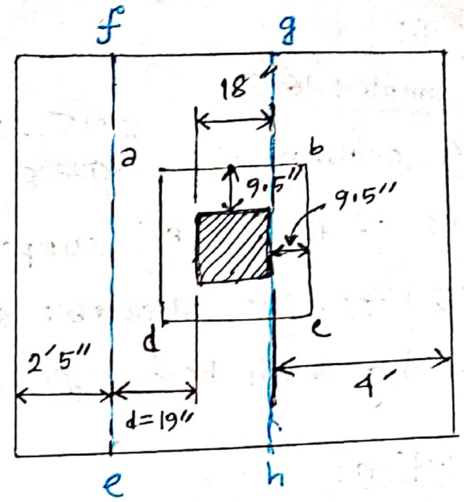
$$q_u = \frac{(1.2 \times 225 + 1.6 \times 175)}{9.5 \times 9.5} = 6.1 \text{ K/ft}^2$$

punching shear check:

Assume, $d = 19''$. Hence the length of the critical perimeter is

$b_o = 4\times(18+d) = 4\times(18+19)$

$\therefore b_o = 148$ in.



The shear acting on this perimeter,

$$V_{u_1} = q_u\left[b^2 - \left(\frac{18+d}{12}\right)^2\right]$$

$$= 6.1\times\left[9.5^2 - \left(\frac{18+19}{12}\right)^2\right]$$

$$= 492.5 \text{ Kips.}$$

The nominal shear, $V_c = 4\sqrt{f_c'}\times b_o d$

$$= 4\times\sqrt{4000}\times 148\times 19$$

$$= 711386 \text{ lb}$$

$$= 711.4 \text{ Kips}$$

design shear, $\phi V_c = (0.75\times 711.4) = 533.55 \text{ Kips} > V_{u_1}$

$\therefore$ design is OK.

Beam shear check: (beam shear on section ef)

The factored shear force, $V_{u_2} = q_u b d_1$ $\longrightarrow (d_1 = 2'5'')$

$$= 6.1\times 9.5\times\frac{29}{12}$$

$$= 140 \text{ Kips.}$$

The nominal shear strength, $V_c = 2\sqrt{f_c'}\, bd$

$$= 2\sqrt{4000}\times(9.5\times 12)\times 19$$

$$= 273979.7 \text{ lb} = 274 \text{ Kips.}$$

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∴ design shear, $\phi V_c = (0.75 \times 274) = 205.5$ kips $> V_{u2}$

∴ design is OK

Moment calculation: (bending moment on section gh)

The ultimate moment, $M_u = \frac{1}{2} b q_u L^2$ $\rightarrow (w = b q_u)$

$= \frac{1}{2} \times 9.5 \times 6.1 \times 4^2$ $\rightarrow (d + d_1)$

$= 463.3$ K-ft

Reinforcement calculation: The required steel area,

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

Here,

$$a = \frac{A_s f_y}{.85 f_c' b}$$

$$\Rightarrow a = \frac{A_s \times 60000}{.85 \times 4000 \times (9.5 \times 12)}$$

$$\therefore a = 0.155 A_s$$

$$\Rightarrow A_s = \frac{463.3 \times 12000}{0.9 \times 60000 \times \left(19 - \frac{1}{2} \times .155 A_s\right)}$$

$$\therefore A_s = 5.54 \text{ in}^2$$

The minimum reinforcement ratio,

$$A_{s(min)} = \frac{3\sqrt{f_c'}\, bd}{f_y} = \frac{3\sqrt{4000} \times (9.5 \times 12) \times 19}{60000} = 6.84 \text{ in}^2$$

and, not less than,

$$A_{s(min)} = \frac{200 bd}{f_y} = \frac{200 \times (9.5 \times 12) \times 19}{60000} = 7.22 \text{ in}^2$$

Hence, the required steel area $= 7.22$ in^2

provide 12 # 7 bars @ $\frac{.60}{7.22} \times (9.5 \times 12) = 9.47''$ c/c

Development Length: As No. 7 bar is used, $d_b = \frac{7}{8}$ in.

The required development length, $l_d = \frac{\alpha \beta \lambda f_y}{20\sqrt{f_c'}} \times d_b$

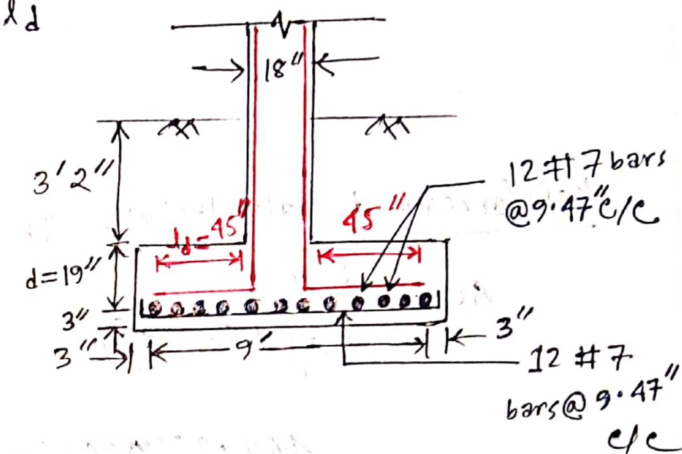
$$\Rightarrow l_d = \frac{1.3 \times 1.5 \times 1 \times 60000}{20\sqrt{4000}} \times \frac{7}{8} \quad \left[\text{Using, } \alpha = 1.3,\ \beta = 1.5,\ \lambda = 1.0\right]$$

$\therefore l_d = 80.93$ in.

The actual development length,

$= (9.5 \times 12 - 2 \times 3 - 18) = 90$ in $> l_d$

$\therefore$ design is OK.



USD

2016

An interior column for a tall concrete structure carries total service loads D=500 Kips and L=514 Kips. The column is 22×22 in. in cross section and is reinforced with twelve No. 11 bars centred 3 in. from the column faces (equal number of bars in each face). The column will be supported on a square footing with the bottom of the footing 6 ft below grade. Design the footing considering the allowable soil bearing pressure is 8000 psf and materials strengths for the footing are $f_c' = 3000$ psi and $f_y = 60000$ psi.

Solution: Assume, an average unit weight of concrete and soil = 125 pcf

The pressure of the material at 6 ft depth = (6×125) psf

= 750 psf

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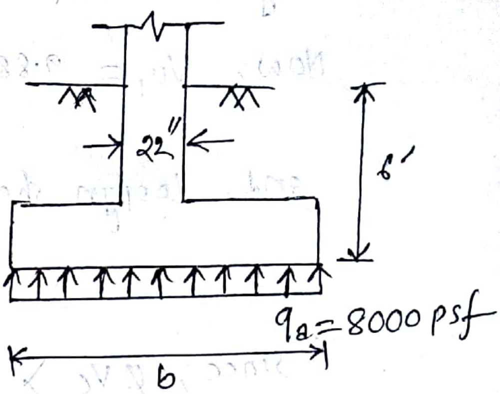
$\therefore$ Net pressure on footing, $q_e = (8000-750)$ psf

$= 7250$ psf

$\therefore q_e = 7.25$ Ksf



The required area of the footing,

$$A_{req.} = \frac{\text{Total Load}}{q_e} = \frac{500+514}{7.25} \, ft^2$$

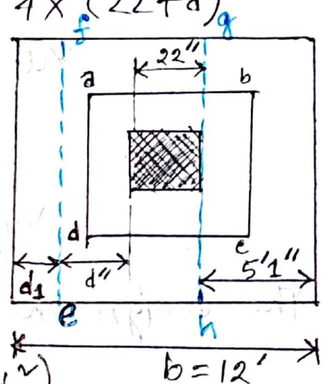
$$\Rightarrow b^2 = 139.862 \, ft^2 \quad \therefore b = 11.83 \, ft \simeq 12 \, ft$$

$\therefore$ Ultimate bearing pressure, $q_u = \dfrac{(1.2 \times 500)+(1.6 \times 514)}{12 \times 12} = 9.88 \, K/ft^2$

Punching Shear Check:

The length of the critical perimeter, $b_o = 4 \times (22+d)$

The shear acting on this perimeter,



$$V_{u_1} = q_u \left[b^2 - \left(\frac{22+d}{12} \right)^2 \right]$$

$$= 9.88 \times \left[12^2 - \left(\frac{22+d}{12} \right)^2 \right]$$

$$= 1422.72 - \frac{9.88}{12^2} \times (22^2 + 44d + d^2)$$

$$= 1422.72 - 33.21 - 3.02d - 0.0686d^2$$

$$\therefore V_{u_1} = 1389.51 - 3.02d - 0.0686d^2$$

The design shear, $\phi V_c = 4\phi \sqrt{f_c'} \, b_o d = \dfrac{4 \times .75 \times \sqrt{3000}}{1000} \times 4.(22+d) \times d$

$$\therefore \phi V_c = 14.46d + 0.6573d^2$$

From which,

$$1389.51 - 3.02d - 0.0686d^2 = 14.46d + 0.6573d^2$$

$$\Rightarrow 0.7259d^2 + 17.48d - 1389.51 = 0$$

$$\Rightarrow d = 33.34'' \simeq 34 \text{ inch.}$$

$\therefore$ Taking $d = 34$ in.

Now, $V_{u_1} = 9.88 \times \left[12^2 - \left(\frac{22+34}{12}\right)^2\right] = 1207.56$ Kips.

and, design shear, $\phi V_c = \frac{4 \times .75 \times \sqrt{3000} \times 4 \times (22+34) \times 34}{1000}$ Kips.

$= 1251.44$ Kips.

Since, $\phi V_c > V_{u_1}$. The depth $d = 34$ in. is adequate for punching shear or two way shear.

$\therefore$ Design is OK.

Beam shear check: (beam shear on section ef)

The factored shear force acting on that section,

$V_{u_2} = q_u b d_1 \longrightarrow \left(d_1 = (144 - 61 - 22 - 34) = 27''\right)$

$= 9.88 \times 12 \times \frac{27}{12}$

$V_{u_2} = 266.76$ Kips.

The design shear, $\phi V_c = 2\phi\sqrt{f_c'}\,bd = \frac{2 \times .75 \times \sqrt{3000}}{1000} \times (12 \times 12) \times 34$

$\therefore \phi V_c = 402.25$ Kips

Since, $\phi V_c > V_{u_2}$. The depth $d = 34$ in. is also adequate for one way shear or beam shear.

$\therefore$ Design is OK.

Moment Calculation: (bending moment on gh section)

The ultimate moment, $M_u = \frac{1}{2} b q_u L^2 = \frac{1}{2} \times 12 \times 9.88 \times \underbrace{(5.083)}_{(d+d_1)}^2$

$\therefore M_u = 1531.61$ K-ft

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Reinforcement calculation:

The required steel area,

$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

Here,

$$a = \frac{A_s f_y}{.85 f_c' b}$$

$$= \frac{A_s \times 60000}{.85 \times 3000 \times (12 \times 12)}$$

$$\therefore a = 0.1634\, A_s$$

$$\Rightarrow A_s = \frac{1531.61 \times 12000}{0.9 \times 60000 \times \left(34 - \frac{0.1634}{2} A_s\right)}$$

$$\Rightarrow A_s = 10.264\ in^2$$

The minimum reinforcement ratio,

$$A_{s\,(min)} = \frac{3 \times \sqrt{f_c'}\ bd}{f_y} = \frac{3 \times \sqrt{3000} \times (12 \times 12) \times 34}{60000}$$

$$\therefore A_{s\,(min)} = 13.41\ in^2$$

but not less than,

$$A_{s\,(min)} = \frac{200\, bd}{f_y} = \frac{200 \times (12 \times 12) \times 34}{60000} = 16.32\ in^2$$

Hence, the required steel area is $= 16.32\ in^2$

provide 28 #7 bars @ $\frac{0.60}{16.32} \times (12 \times 12) = 5.29''$ c/c Or, $\frac{12}{28} \times 12 = 5.14''$ c/c

b

Development length: As No. 7 bar is used, $d_b = \frac{7}{8}$ in.

$\therefore$ The required development length, $l_d = \frac{\alpha \beta \lambda f_y}{20\sqrt{f_c'}} \times d_b$

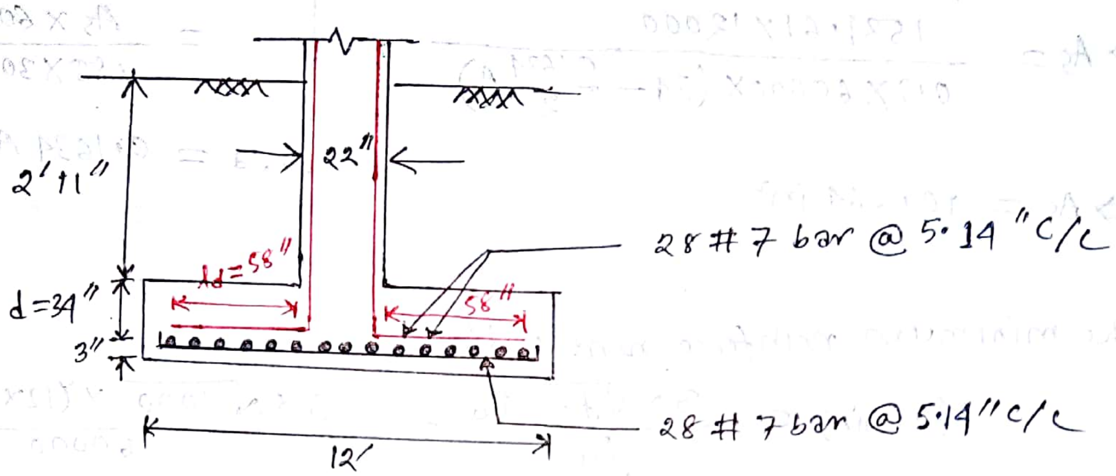
using, $\alpha = 1.3$, $\beta = 1.5$ and, $\lambda = 1.0$,

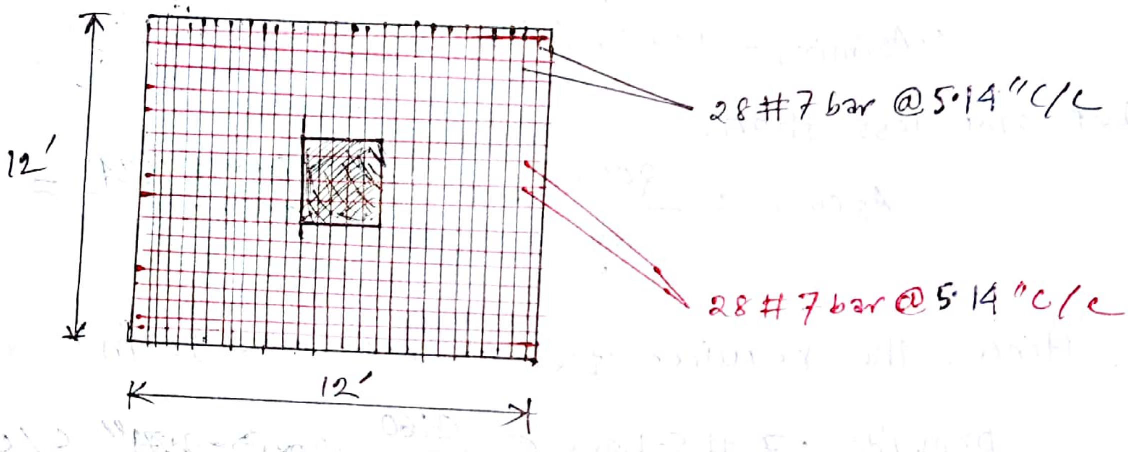
$$l_d = \frac{1.3 \times 1.5 \times 1.0 \times 60000}{20 \times \sqrt{3000}} \times \frac{7}{8}$$

$$\therefore l_d = 93.46\ in.$$

The actual development length = $(12\times12 - 2\times3 - 22) = 116$ in $> l_d$

$\therefore$ design is OK.





Combined Footing

Example: 16.3

An exterior 24 × 18 in. column with D = 170 Kips, L = 130 Kips and an interior 24 × 24 in. column with D = 250 Kips, L = 200 Kips are to be supported on a combined rectangular footing whose outer end cannot protrude beyond the outer face of the exterior column. The distance center to center of column is 18 ft and the allowable bearing pressure of the soil is 6000 psf. The bottom of the footing is 6 ft below grade and a surcharge of 100 psf is specified on the surface. Design the footing for $f_c' = 3000$ psi, $f_y = 60000$ psi.

Solution: Assume, an average unit weight of concrete and soil = 125 psf

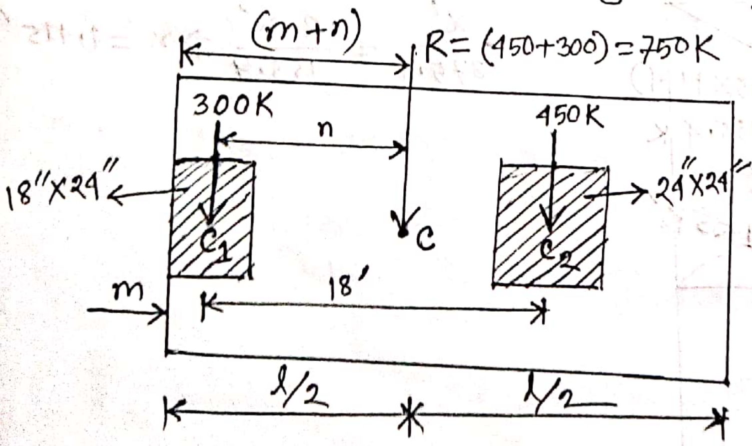
The pressure of this materials at the 6 ft depth $= (125 \times 6)$ psf
$= 750$ psf

Given, a surcharge = 100 psf

$\therefore$ Net pressure on footing, $q_e = q_a - (750 + 100)$ psf
$= (6000 - 850)$ psf

$\therefore q_e = 5150$ psf $= 5.15$ ksf

The required area of the footing, $A_{req} = \dfrac{\text{sum of column loads}}{q_e}$



$= \dfrac{170 + 130 + 250 + 200}{5.15}$

$= \dfrac{750}{5.15} = 145.63$ ft^2

Σ Moment of Resultant loads = Σ Moment of Individual loads

$750 \times n = 0 \times 300 + 18 \times 450$

$\therefore n = \dfrac{18 \times 450}{750} = 10.8$ ft

∴ The resultant loads, R is located from the center of the exterior column = 10.8 ft.

Hence, The length of of the footing, $l = 2(m+n)$

$$= 2\times\left(\frac{1}{2}\times\frac{18}{12} + 10.8\right) \text{ ft}$$

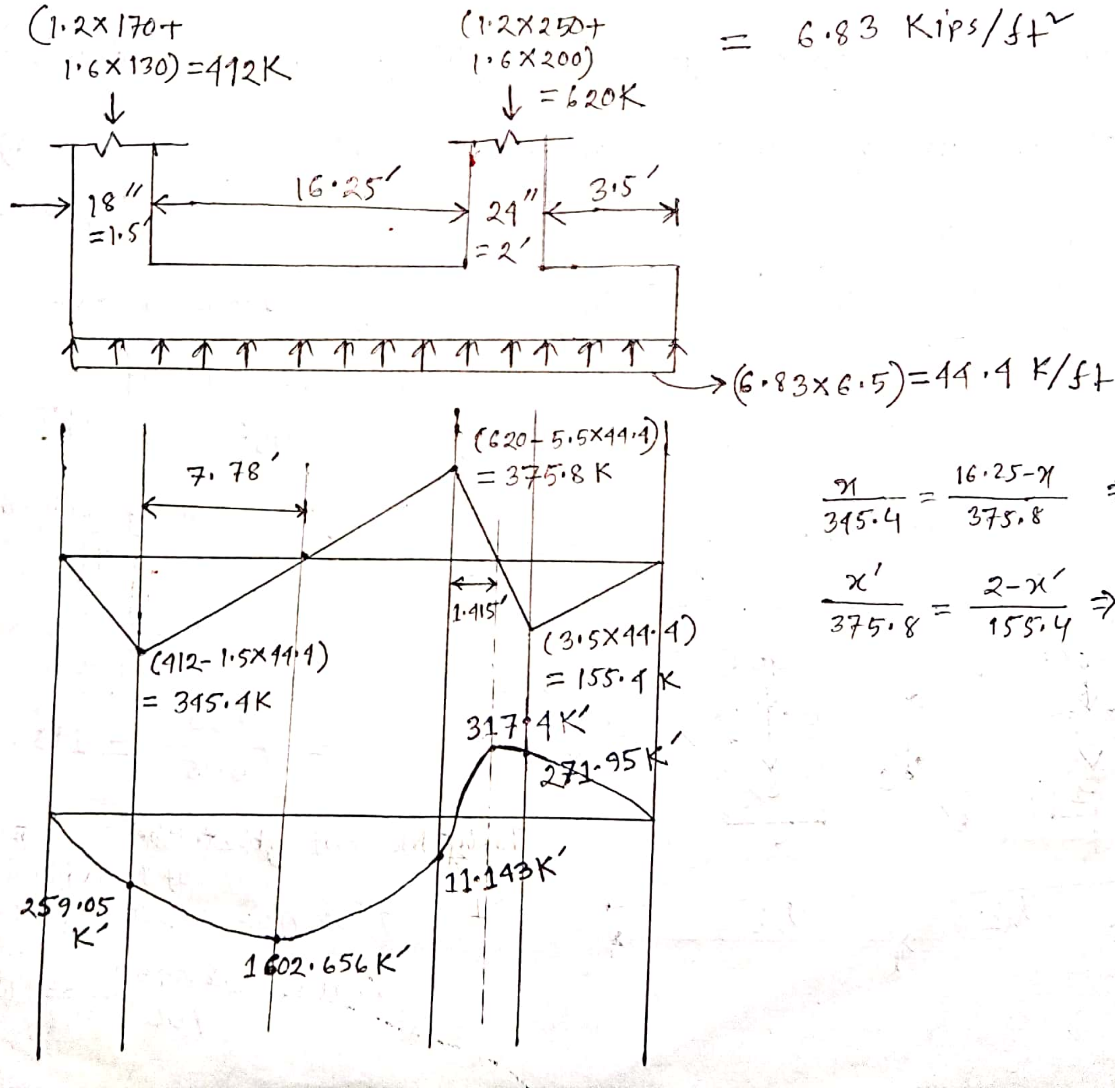
$$\therefore l = 23.1 \text{ ft} \simeq 23.25 \text{ ft}$$

Then,

The required width, $b = \dfrac{145.63}{23.25} = 6.26 \text{ ft} \simeq 6.5 \text{ ft}$

The ultimate bearing pressure, $q_u = \dfrac{1.2\times(170+250) + 1.6\times(130+200)}{23.25\times6.5}$

$$= 6.83 \text{ Kips/ft}^2$$



$$\frac{x}{345.4} = \frac{16.25-x}{375.8} \Rightarrow x = 7.78'$$

$$\frac{x'}{375.8} = \frac{2-x'}{155.4} \Rightarrow x' = 1.415'$$

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From diagram, zero shear is at a distance $= (1.5 + 7.78)$ ft

$= 9.28$ ft

$\therefore$ The maximum negative moment between the column occurs at 9.28 ft. The moment at this section,

$$M_u = -1602.656 \, K'$$

And,

The moment at the right edge of the interior column,

$$M_u = 271.95 \, K'$$

Beam Shear Check:

The Critical section for flexural shear occurs at a distance 'd' to the left of the left face of the interior column.

At that point, The factored shear, $V_u = \left(375.8 - \frac{d}{12} \times 44.4\right) \times 1000$

and, the design shear, $\phi V_c = 2\phi \sqrt{f_c'} \, b \, d$

$$= 2 \times 0.75 \times \sqrt{3000} \times (6.5 \times 12) \times d$$

$$\therefore \phi V_c = 6408.354 \, d$$

From which,

$$1000 \times \left(375.8 - \frac{d}{12} \times 44.4\right) = 6408.354 d$$

$$\Rightarrow 10108.354 \, d = 375800$$

$$\therefore d = 37.18 \text{ in} \simeq 37.5 \text{ in.}$$

Taking $d = 37.5$ in, we obtain,

$$V_u = \left(375.8 - \frac{37.5}{12} \times 44.4\right) \times 1000 = 237050 \text{ lb}$$

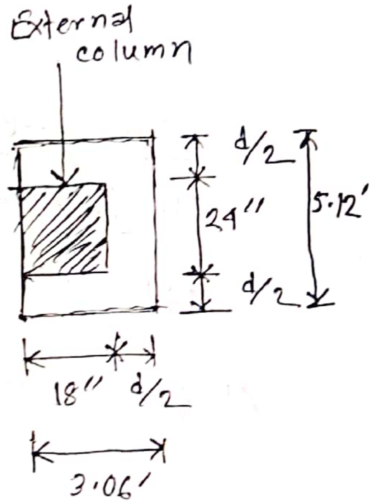
and, design shear, $\phi V_c = 2 \times .75 \times \sqrt{3000} \times (6.5 \times 12) \times 37.5$

$$\therefore \phi V_c = 240313.27 \text{ lb}$$

Since, $\phi V_c > V_u$. The depth, $d = 37.5$ in. is adequate for beam shear.

$\therefore$ Design is OK.

<u>Punching shear check:</u>



The length of the critical perimeter,

$$b_o = 2 \times \left(\overbrace{18 + \frac{37.5}{2}}^{3.06'}\right) + \left(\overbrace{24 + 2 \times \frac{37.5}{2}}^{5.12'}\right)$$

$$\therefore b_o = 135 \text{ in} = 11.25 \text{ ft}$$

$\therefore$ The shear acting on this perimeter,

$$V_u = 412 - (3.06 \times 5.12) \times 6.83$$

$$= 305 \text{ K}$$

and, design shear, $\phi V_c = 4\phi \sqrt{f_c'}\, b_o d = 4 \times 0.75 \times \sqrt{3000} \times (11.25 \times 12) \times 37.5$

$$\phi V_c = 831853.6 \text{ lb} = 831.85 \text{ K}$$

Since $\phi V_c > V_u$. Depth, $d = 37.5$ in. is adequate for punching shear.

$\therefore$ Design is OK

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Reinforcement Calculation:

longitudinal reinforcement:

The required steel area, $A_s = \dfrac{M_u}{\phi f_y j d}$

$$= \frac{1602.656 \times 12000}{0.9 \times 60000 \times 0.95 \times 37.5}$$

$\therefore A_s = 10 \text{ in}^2$

$\therefore$ providing 10 #9 bars

For cantilever portion,

$$A_s(\min) = \frac{3\sqrt{f_c'}}{f_y} \times bd = \frac{3 \times \sqrt{3000}}{60000} \times (6.5 \times 12) \times 37.5$$

$\therefore A_s(\min) = 8.01 \text{ in}^2$

but not less than,

$$A_s(\min) = \frac{200}{f_y} \times bd = \frac{200}{60000} \times (6.5 \times 12) \times 37.5$$

$\therefore A_s(\min) = 9.75 \text{ in}^2$

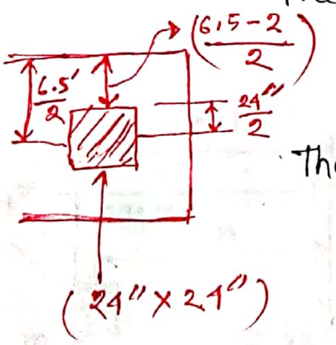
$\therefore$ provide 17 #7 bars

Transverse reinforcement:

The width of the transverse beam $= 24 + 2 \times \frac{d}{2}$

$= (24 + 37.5)$

$= 61.5 \text{ in}$



The moment at the edge of the column, 24''

$$M_u = W\frac{L^2}{2} = \frac{620}{6.5} \times \frac{1}{2} \times \left(\frac{6.5-2}{2}\right)^2 = 241.43 \text{ K-ft}$$

W L

$$\therefore A_s = \frac{M_u}{\phi f_y j d} = \frac{241.43 \times 12000}{0.9 \times 60000 \times .95 \times 37.5} = 1.51 \text{ in}^2$$

but not less than,

(i) under interior column, $A_s(min) = \frac{200}{f_y} bd$

$= \frac{200}{60000} \times (24 + 37.5) \times 36.5$

$\simeq (37.5 - \phi)$

since transverse bars are placed on top of longitudinal bars.

$= 7.48 \ in^2$

∴ provide 13 # 7 bars.

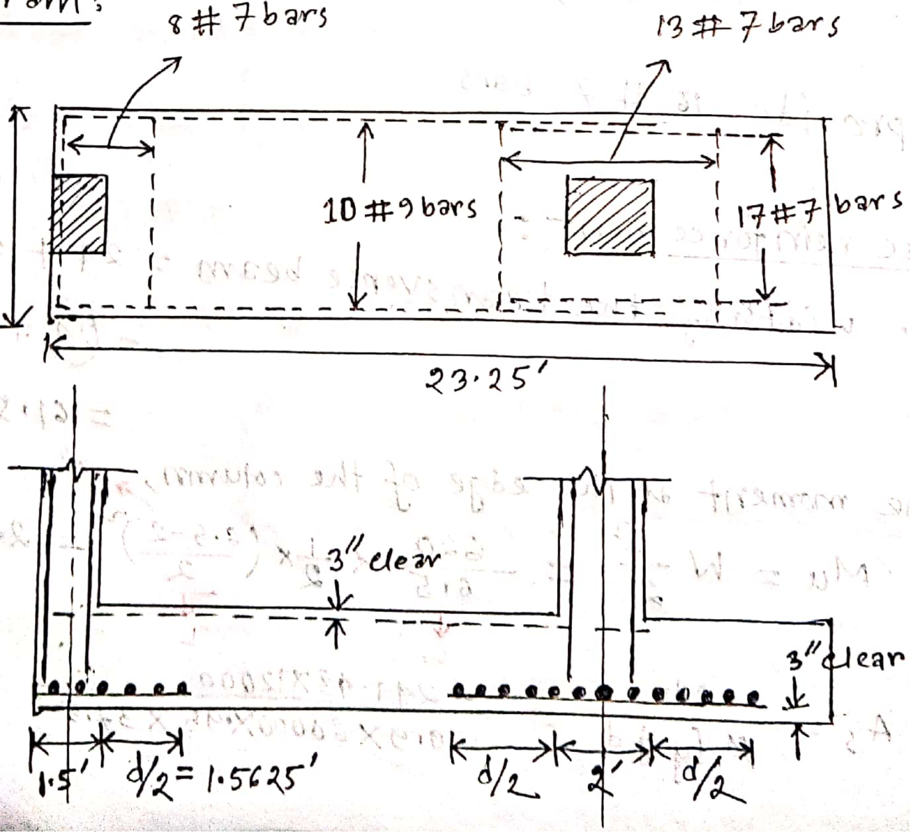
(ii) under exterior column, $A_s(min) = \frac{200}{f_y} bd$

$= \frac{200}{60000} \times \left(18 + \frac{37.5}{2}\right) \times 37.5$

$= 4.6 \ in^2$

∴ provide 8 # 7 bars.

working diagram:



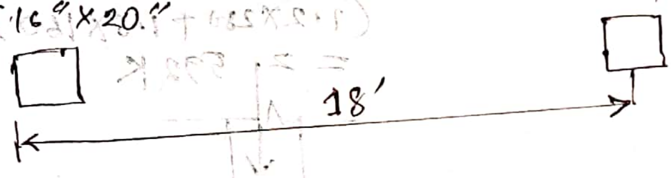
2017

Problem: A combined rectangular footing supports two column loadings as shown in figure.

The loadings are.

column	Dead load (Kips)	Live load (Kips)
A	180	110
B	280	160

Assume depth of footing 5 ft below grade, $q_a = 4500$ psf, $f_c' = 3000$ psi and $f_y = 60000$ psi. Find reinforcement in longitudinal direction.



solution: Assume, the average unit weight of soil and concrete = 125 pcf

$\therefore$ the pressure of this material on footing $= (5 \times 125)$ psf

$= 625$ psf

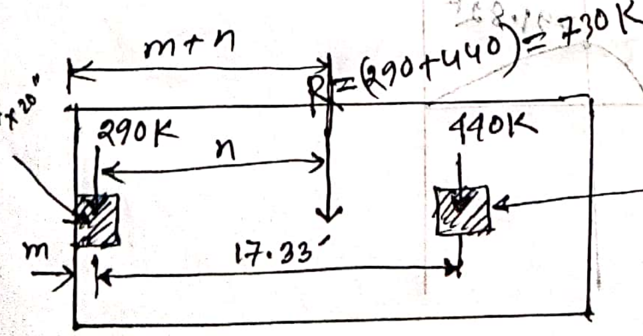
$\therefore$ The net pressure on footing, $q_e = (q_a - 625)$ psf

$= (4500 - 625)$ psf

$= 3875$ psf

$\therefore$ The required area of the footing $= \dfrac{(180+110+280+160)\times 1000}{3875}$

$= 188.4\ ft^2$



Σ Moment of individual loads = Moment of Resultant load

$\Rightarrow 0 \times 290 + 440 \times 17.33 = 730 \times n$

$\Rightarrow n = 10.45$ ft

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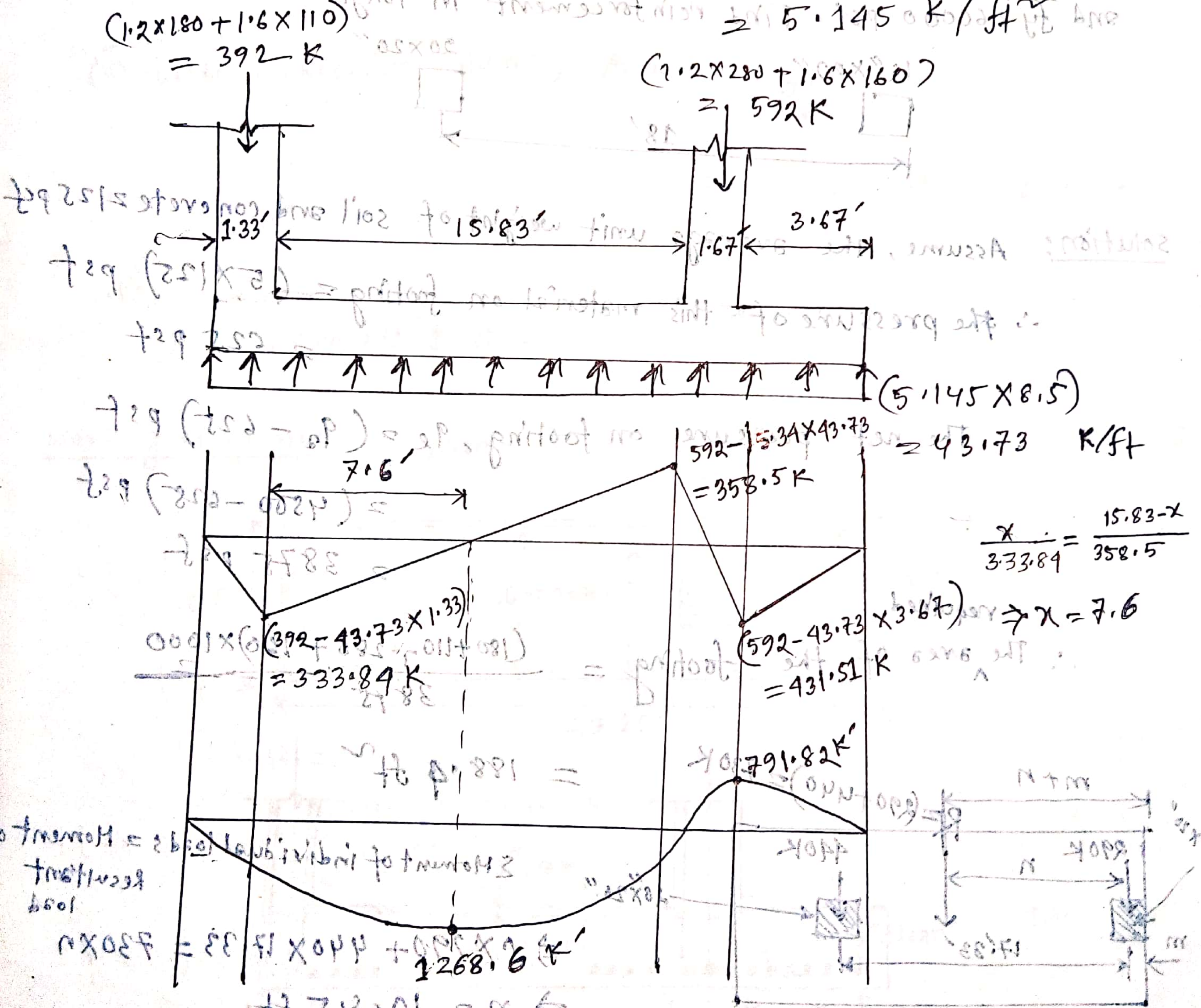
∴ length of the footing, $l = 2(m+n) = 2\left(\frac{1}{2}\times\frac{16}{12} + 10.45\right)$

$= 22.23 \simeq 22.5$ ft

∴ Required width of the footing, $b = \frac{188.4}{22.5} = 8.37 \simeq 8.5$ ft

∴ The ultimate bearing pressure, $q_u = \frac{1.2\times(180+280)+1.6\times(110+160)}{22.5\times 8.5}$

$= 5.145$ K/ft²



The zero shear is at a distance $(1.33+7.8) = 9.13$ ft from the left face of the exterior column.

$\therefore$ The maximum moment at this section $= -1268.6$ K'

Beam shear check:

The critical section for flexural shear occurs at a 'd' distance to the left of left face of the interior column.

$\therefore$ The factored shear, $V_u = \left[358.5 - \left(\frac{d}{12} \times 43.73\right)\right] \times 1000$

and, design shear $\phi V_c = 2\phi\sqrt{f_c'}\, bd$

$= 2 \times 0.75 \times \sqrt{3000} \times (8.5 \times 12) \times d$

From which.

$\left[358.5 - \left(\frac{d}{12} \times 43.73\right)\right] \times 1000 = 2 \times 0.75 \times \sqrt{3000} \times (8.5 \times 12) \times d$

$\Rightarrow 12024.32\, d = 358500$

$\therefore d = 29.8'' \simeq 30$ in.

Now,

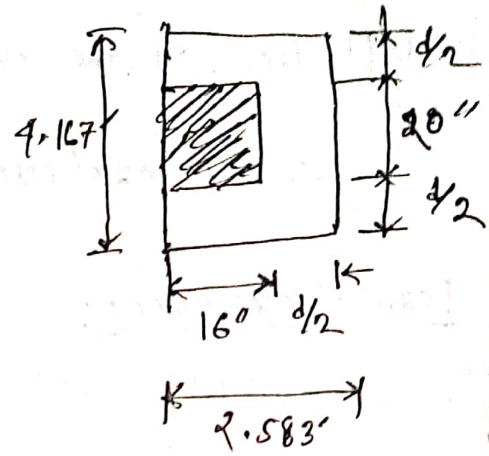
$V_u = \left[358.5 - \left(\frac{30}{12} \times 43.73\right)\right] \times 1000 = 249175$ lb

$\phi V_c = 2 \times 0.75 \times \sqrt{3000} \times 8.5 \times 12 \times 30 = 251404.6 > V_u$

(OK)

punching shear check:

The perimeter of the critical section,



$$b_o = 2 \times \left(16 + \frac{30}{2}\right) + (20 + 30)$$

$= 112$ inch.

$$V_u = 392 - (4.167 \times 2.583) \times 5.145$$

$= 336.62$ K

$$\phi V_c = 4\phi\sqrt{f_c'}\, b_o d = 4 \times .75 \times \sqrt{3000} \times 112 \times 30$$

$= 552104.34$

$= 552.104$ K $> V_u$

(OK)

Reinforcement Calculation:

In longitudinal direction,

$$A_s = \frac{M_u}{\phi f_y \, jd}$$

~~Here, a = $\frac{A_s f_y}{0.85 f_c' b}$ = $\frac{60 A_s}{.85 \times 3 \times (8.5 \times 12)}$ = 0.23 A_s~~

$$A_s = \frac{1268.6 \times 12000}{0.9 \times 60000 \times .95 \times 30}$$

$= 9.89$ in^2

provide 10 #9 bar

For cantilever portion,

$$A_{s(min)} = \frac{3\sqrt{f_c'}}{f_y} bd$$

$$= \frac{3\sqrt{3000}}{60000} \times (8.5 \times 12) \times 30$$

$$= 8.38 \text{ in}^2$$

but not less than,

$$A_{s(min)} = \frac{200\, bd}{f_y} = \frac{200 \times (8.5 \times 12) \times 30}{60000}$$

$$= 10.2 \text{ in}^2$$

∴ provide 17 # 7 bar

<u>Tranverse reinforcement:</u>

width of the transverse beam $= (20 + d)$

$= (20 + 30) = 50$ in.

The moment at the edge of the interior column,

$$M_u = \frac{1}{2} \times \frac{592}{8.5} \times \left(\frac{8.5 - 1.67}{2}\right)^2 = 406.12 \text{ k-ft}$$

$$\therefore A_s = \frac{406.12 \times 12000}{.9 \times 60000 \times .95 \times 30} = 3.17 \text{ in}^2$$

but not less than,

(i) under interior column: $A_{(min)} = \frac{200}{60000} \times \underbrace{(20+30)}_{b} \times \underbrace{29}_{d=(30-1)}$

$$= 4.83 \text{ in}^2$$

provide 9 # 7 bar.

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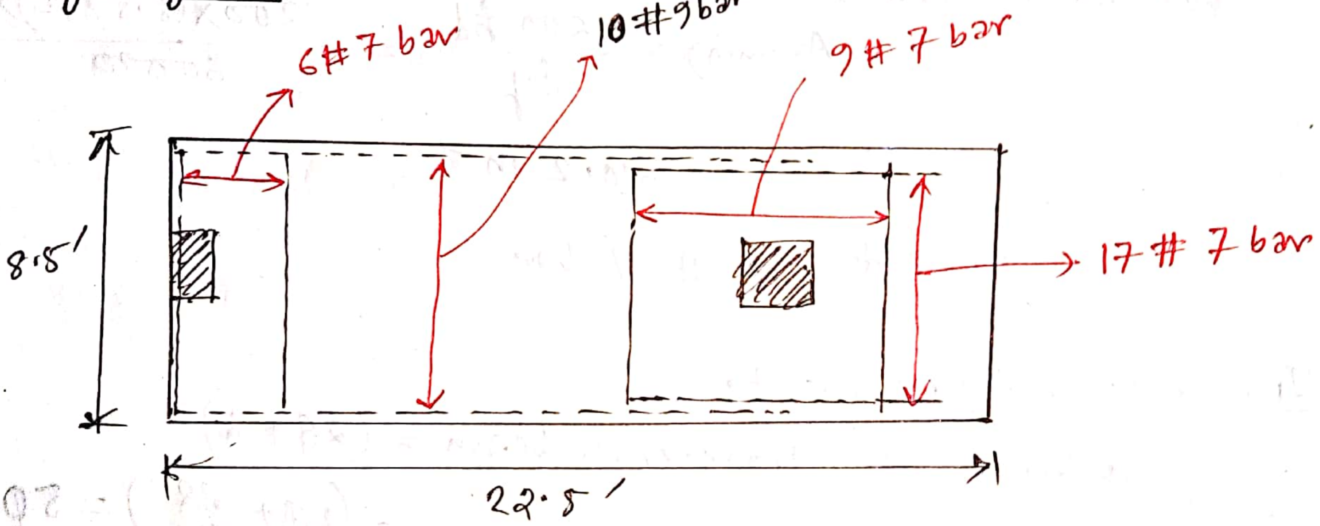
(ii) under exterior column: $A_s(min) = \frac{200}{60000} \times (\underset{b}{16+15}) \times \underset{d}{30}$

$= 3.1 in^2$

provide 6 # 7 bar.

working diagram:



Flat Slab

Define Flat Slab: 14

A flat slab is one consisting of a reinforced concrete floor slab built monolithically with the columns and supported directly by the columns without the aid of beams and girders.

Define Drop Panel: 17, 13, 11

The slab has uniform thickness throughout the entire area, or a part of it, symmetrical about the column, is made somewhat thicker than the rest of the slab, the thicked portion of slab thus formed constituting what is known as Drop Panel.

Write the functions of Drop panel: 17, 16, 15, 13, 11

(i) Drop panels are used to reduce the shearing stress in the slab.

(ii) To increase the effective slab thickness.

(iii) To decrease the compression stresses in the concrete.

(iv) To reduce the amount of steel required over the column heads.

(v) To reduce deflection by stiffening the flat slab.

Define Column Capital: 15, 13, 11

The columns in practically all cases flare out toward the top, forming ~~a~~ a shape somewhat similar to an inverted truncated cone, is known as column capital.

#Write the function of column capitals: 13, 11

1. Column capital gives wider support for the floor slab.
2. It helps to decrease the bending moment.
3. It decreases the shearing stress around the perimeter of the column.
4. It reduces the effective span.

Types of Flat slab:

The column tends to punch through the slab in Flat slabs, which can be treated by Three methods:

1. Using a drop panel and a column capital in flat slab.
2. Using a drop panel without a column capital in flat flab.
3. Using a column capital without drop panel in flat slab.

These are three types of Flat slab.

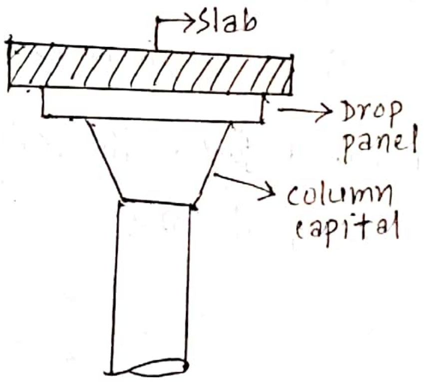


Fig. Flat slab with column capital & Drop panel

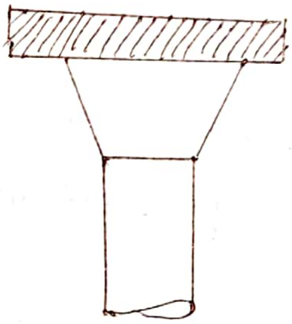


Fig. Flat slab with column capital

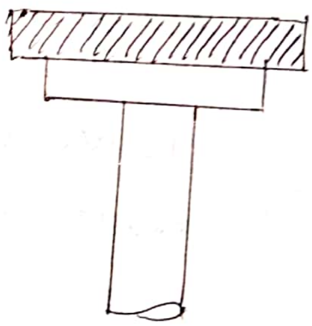


Fig. Flat slab with drop panel

Advantages of Flat Slab:

1. For ordinary spans with heavy loads, under average condition, the flat slab is more economical than beam and girder floor.

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2. As No beam is used, floor height can be reduced and consequently building height will be reduced.

3. The flat slab form work is much simpler.

4. The flat slab, owing to the lack of many sharp corners, is better able to resist continued exposure to fire than other slabs.

5. Automatic sprinkler protection may be made more complete under a flat slab.

6. More light may be admitted into the building if desired.

7. Reinforcement placement is easier.

Flat plate Slab:

Flat plate slab is a two-way reinforced concrete framing system utilizing a slab of uniform thickness, the simplest of structural shapes.

Flat Slab

A parking garage, to be designed as a flat slab structure, is to carry a working live load of 200 psf. Drop panels 8 ft square will be used and each column capital will consist of a 90° truncated cone with 4 ft diameter at the intersection of the capital with the bottom of the drop panel. Columns are spaced 22 ft on centers in each direction. Design the slab reinforcement, and determine concrete dimensions for a typical interior panel. Given, $f_c' = 3000$ psi and $f_s = 20000$ psi.

Solution:

(1) Thickness Calculation: The minimum thickness of the slab is,

$$t_2 = \frac{L}{40} = \frac{22\times12}{40} = 6.6 \text{ inch} \simeq 8 \text{ in.}$$

or, $t_2 \geq 4$ in.

$$\text{or, } t_2 = 0.024L\left(1 - \frac{2c}{3L}\right)\sqrt{\frac{w'}{f_c'/2000}} + 1$$

(0.15L to 0.25L) ⇒ range

Here, $c = 4$ ft , $L = 22$ ft

$$D.L = \frac{t}{12} \times 150 = \frac{8}{12} \times 150 = 100 \text{ psf}$$

$$L.L = 200 \text{ psf}$$

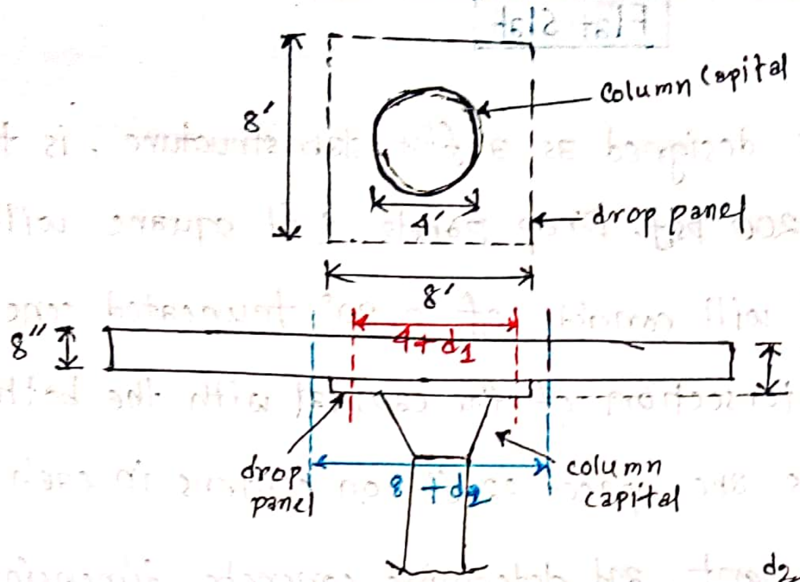
$$\therefore w' = D.L + L.L = (100+200) = 300 \text{ psf}$$

$$\therefore t_2 = 0.024 \times 22 \times \left(1 - \frac{2\times4}{3\times22}\right)\sqrt{\frac{300}{3000/2000}} + 1 = 7.56 \text{ in}$$

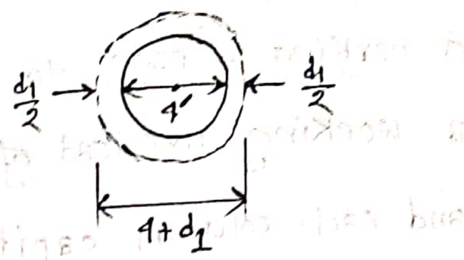
Hence, Slab thickness, $t_2 = 8$ inch.

Now, $t_1 \leq 1.5t_2$ $\therefore t_1 = (1.5\times8) = 12'' \simeq 11$ in.

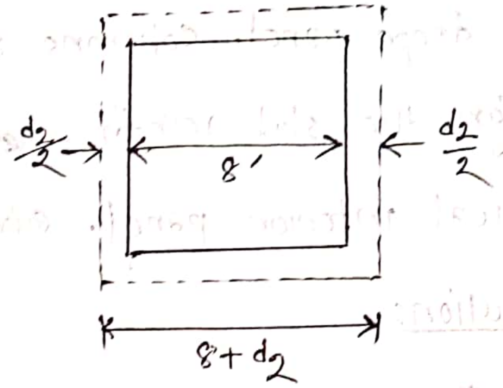
and, Thickness of Drop panel $= \frac{t_2}{3} = \frac{8}{3} = 2.67 \simeq 3$ in



Critical section



(i) within drop panel



(ii) outside of drop panel

(ii)
Punching shear check:

Relative to punching shear,

(a) The first critical section in the drop panel is at a distance $\frac{d_1}{2}$ from the edge of the column capital.

$d_1 = t_1 -$ Clear cover $= (11 - 1.5) = 9.5''$

$\therefore$ The diameter of the crical section $= (4 + d_1) = \left(4 + \frac{9.5}{12}\right) = 4.79'$

The floor area included within critical section $= \frac{\pi}{4} \times (4.79)^2$

$= 18\ ft^2$

The developed shear on the critical section, $V_{dev} = (\underbrace{22 \times 22}_{L \times L} - 18) \times \underbrace{300}_{w'}$

$= 139800\ lb$

$\therefore$ The nominal shear stress, $v = \frac{V}{b_o d} = \frac{139800}{3.1416 \times (4.79 \times 12) \times 9.5}$

$b_o \to \boxed{\pi(4+d_1)}$ (perimeter)

$= 81.5$ psi

But, The allowable shear stress on that section, $V_{all} = 2\sqrt{f_c'} = 2\sqrt{3000}$

$= 109.5\ psi > v$

(OK)

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(b) The second critical section is at a distance $\frac{d_2}{2}$ from the edge of the drop panel.

$d_2 = t_2 - \text{clear cover} = (8 - 1.5) = 6.5''$

∴ The side dimension of the critical section $= (8 + d_2) = \left(8 + \frac{6.5}{12}\right)$

$= 8.54'$

∴ The included area of the critical section $= (8.54 \times 8.54)\ ft^2$

$= 72.93\ ft^2$

$\simeq 73\ ft^2$

∴ The developed shear on the critical section, $V_{dev} = (22 \times 22 - 73) \times 300$

$= 123300\ lb$

∴ The nominal shear stress, $v = \frac{V}{b_o d}$ $\quad b_o \rightarrow \boxed{4(8+d_2)}$ (perimeter)

$\therefore v = \frac{123300}{4 \times (8.54 \times 12) \times 6.5} = 46.3\ psi$

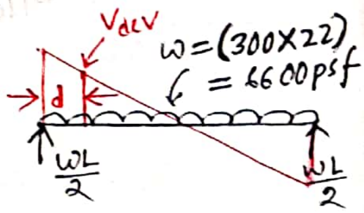
But, the allowable shear stress, $v_{all} = 109.5\ psi > v$

(OK)

(iii) Beam Shear Check: Beam shear is checked at a distance 'd' past the face of the support.

$d = \sqrt{\frac{\pi}{4} C^2}/2 + d_2 = \left(\sqrt{\frac{\pi}{4} \times (4)^2}/2 + \frac{6.5}{12}\right) = 2.31\ ft$

C: dia of capital



At 'd' distance, the developed shear, $V_{dev} = \frac{WL}{2} - d \times W$

$= 6600 \times \frac{22}{2} - 2.31 \times 6600$

$= 57,354\ lb$

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$\therefore$ The nominal shear stress, $v = \frac{V}{bd} = \frac{57354}{(22\times12)\times6.5} = 33.4$ psi

But, The allowable shear stress, $V_{all} = 1.1\sqrt{f_c'} = 1.1\times\sqrt{3000}$

$= 60.25$ psi $> v$

(OK)

(iv) <u>Moment Calculation:</u>

Column strip	Middle strip	Column strip
$-0.5M_0$ drop panel	$-0.15\times M_0$	$-0.5M_0$ drop panel
$0.2M_0$	$+0.15\times M_0$	$0.2M_0$
$-0.5M_0$ drop panel	$-0.15\times M_0$	$-0.5M_0$ drop panel

The design moment,

$M_0 = 0.09WLF\left(1-\frac{2c}{3L}\right)^2$

Here, $W = (300\times22\times22) = 145200$

and, $F = 1.15 - \frac{c}{L} \geq 1$

$\therefore F = \left(1.15 - \frac{4}{22}\right) = 0.97 \simeq 1$

$\therefore M_0 = 0.09\times145200\times22\times1\times\left(1-\frac{2\times4}{3\times22}\right)^2 = 222024$ lb-ft

Now, Column strip $-$ve Moment $= 0.50M_0$

$= (0.50\times222024)$ lb-ft

$= 111012$ lb-ft

Column strip +ve Moment $= +0.2M_0$

$= (0.2\times222024)$ lb-ft

$= 44404.8$ lb-ft

Middle strip +ve and $-$ve Moment $= 0.15M_0$

$= (0.15\times222024)$ lb-ft

$= 33303.6$ lb-ft

v) Depth Check:

(a) In the portion of slab with drop panel:

$$d_1 = \sqrt{\frac{M}{Rb_1}}$$

$$= \sqrt{\frac{111012 \times 12}{223 \times 72}}$$

$\therefore d_1 = 9.11$ in. $< d_{eff} = 9.5$ in (OK)

Here, $n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$

$r = \frac{f_s}{f_c} = \frac{20000}{3000 \times 0.45} = 14.8$

$K = \frac{n}{n+r} = \frac{9}{9+14.8} = 0.378$

$j = 1 - \frac{K}{3} = 0.874$

$R = \frac{1}{2} f_c j K = \frac{1}{2} \times .45 \times 3000 \times 0.874 \times 0.378$

$\therefore R = 223$

$\boxed{b_1 = 0.75 \times \text{drop panel width}}$

$\therefore b_1 = (0.75 \times 8 \times 12) = 72$ in.

and $b_2 = (0.75 \times 11 \times 12) = 99$ in

$\frac{L}{2}$

(b) In the portion of slab without drop panel:

$$d_2 = \sqrt{\frac{M}{Rb_2}}$$

$$= \sqrt{\frac{44404.8 \times 12}{223 \times 99}}$$

$\therefore d_2 = 4.91$ in. $< d_{eff} = 6.5''$ (OK)

vi) Reinforcement Calculation:

At column strip: (for −ve Moment) $A_s = \frac{M}{f_s j d} = \frac{111012 \times 12}{20000 \times 0.874 \times 9.5}$

$= 8.02$ in²

providing 19 # 6 bars.

(for +ve Moment) $A_s = \frac{M}{f_s j d} = \frac{44404.8 \times 12}{20000 \times 0.874 \times 6.5}$

$= 4.69$ in²

providing 11 # 6 bars.

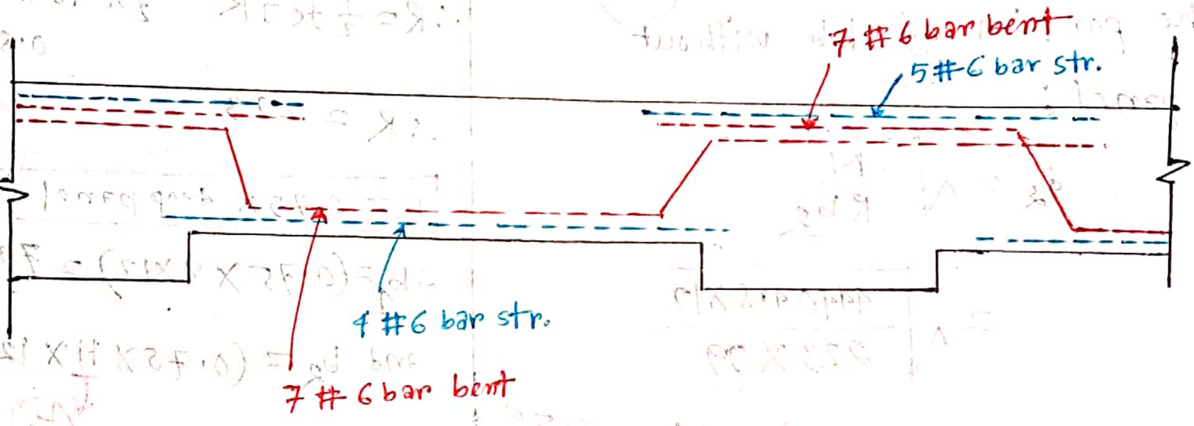
At Middle strip, $A_s = \frac{M}{f_s j d} = \frac{33303.6 \times 12}{20000 \times 0.879 \times 6.5}$

$= 3.52 \text{ in}^2$

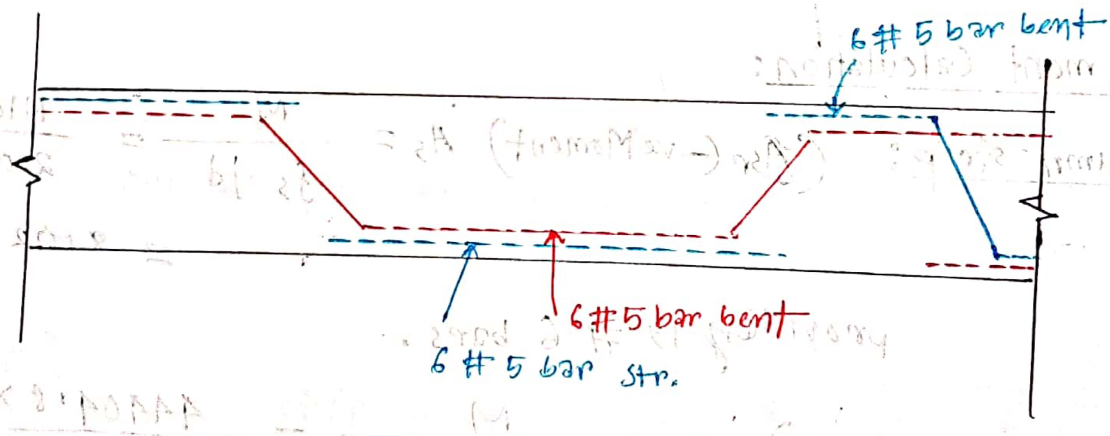
providing 12 # 5 bars.

(vii) Working Diagram:

(a) column strip steel: 33% bars will be straight $= (0.33 \times 11) = 3.63 \simeq 4$ bars



(b) Middle strip steel: 50% bars will be straight & 50% bent



2017

Design an interior panel of flat slab with drop panels having column spacing of 24 ft on centers in both direction. The live load on the flat slab is 130 psf. Use $f_c' = 3$ ksi and $f_y = 60$ ksi.

Solution:

(i) Thickness calculation: The minimum thickness of the slab is,

$$t_2 = \frac{L}{40} = \frac{24\times12}{40} = 7.2 \text{ in.} \simeq 9 \text{ in}$$

or, $t_2 \geq 4$ in.

$$\text{or, } t_2 = 0.024L\left(1 - \frac{2c}{3L}\right)\sqrt{\frac{w'}{f_c'/2000}} + 1$$

Here, $c = 4'$, $L = 24'$, $D.L. = \frac{9}{12}\times150 = 112.5$ psf

$L.L = 130$ psf $\therefore w' = (112.5 + 130) = 242.5$ psf

$$\therefore t_2 = 0.024\times24\times\left(1 - \frac{2\times4}{3\times24}\right)\sqrt{\frac{242.5}{3000/2000}} + 1 = 7.51 \text{ in}$$

Hence, slab thickness, $t_2 = 9$ in.

Now, $t_1 \leq 1.5t_2$ $\therefore t_1 = (1.5\times9) = 13.5\text{ in} \simeq 12$ in.

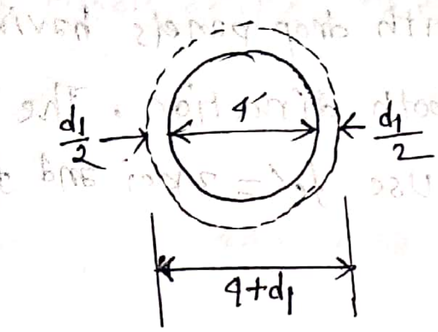
and, thickness of the drop panel $= \frac{t_2}{3} = \frac{9}{3} = 3$ in.

(ii) puching shear check:

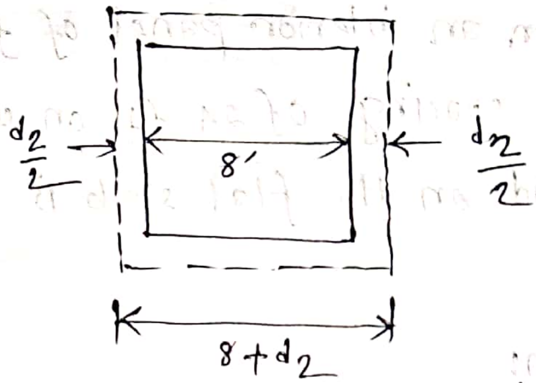
Assume, the diameter of the column capital $= 4$ ft.

and the length of the drop panel $= \frac{L}{3} = \frac{24}{3} = 8$ ft

if $\boxed{L > 20 \text{ ft} \text{ or, } LL > 130\text{psf}}$ drop panel have to be used.



(a) with in Drop panel

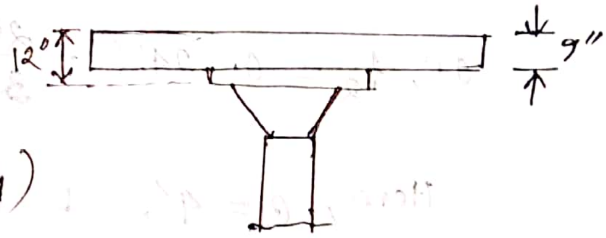


(b) outside of droppanel.

Relative to punching shear,

(a) The first critical section in the drop panel is at a distance $\frac{d_1}{2}$ from the edge of the column capital.

$\therefore d_1 = (12-1.5) = 10.5''$



$\therefore$ diameter of critical section $= (4+d_1)$

$= \left(4+\frac{10.5}{12}\right) = 4.875'$

$\therefore V_{dev} = \left\{24\times 24 - \frac{\pi}{4}(4.875)^2\right\}\times 242.5 = 135153.61$ lb

$\therefore v = \frac{V}{b_o d} = \frac{135153.61}{3.1416\times(4.875\times 12)\times 10.5} = 70$ psi

But, $v_{all} = 2\sqrt{f_c'} = 2\times\sqrt{3000} = 109.5$ psi $> v$

(OK)

(b) The second critical section is at a distance $\frac{d_2}{2}$ from the edge of the drop panel.

$\therefore d_2 = (9-1.5) = 7.5$ in.

$\therefore$ The side dimension of the critical section $= (8+d_2)$

$= \left(8+\frac{7.5}{12}\right)$ ft

$= 8.625'$

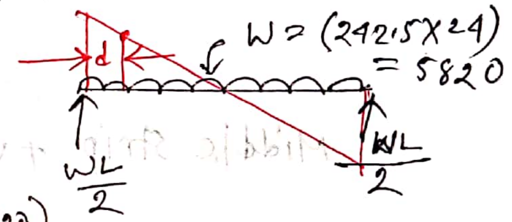
$\therefore V_{dev} = (24 \times 24 - 8.625 \times 8.625) \times 242.5 = 121640.3\ lb$

$\therefore v = \dfrac{V}{b_o d} = \dfrac{121640.3}{4 \times (8.625 \times 12) \times 7.5} = 39.18\ psi$

But, $v_{all} = 109.5\ psi > v$ (OK)

(iii) Beam shear check: Beam shear is checked at a distance 'd' past the face of the support.

$\therefore d = \sqrt{\frac{\pi}{4} c^2} / 2 + d_2 = \sqrt{\frac{\pi}{4} \times (4)^2} / 2 + \frac{7.5}{12} = \left(\frac{3.54}{2} + \frac{7.5}{12}\right) = 2.395'$



$\therefore$ At 'd' distance, $V_{dev} = \frac{WL}{2} - d\,W$

$= \left(5820 \times \frac{24}{2}\right) - (2.395 \times 5820)$

$= 55{,}901\ lb$

$\therefore$ The nominal shear stress, $v = \dfrac{V}{bd} = \dfrac{55901}{(24 \times 12) \times 7.5}$

$= 25.88\ psi$

But, $v_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{3000} = 60.25\ psi > v$ (OK)

(iv) Moment Calculation:

The design moment, $M_o = 0.09\,WLF\left(1 - \frac{2c}{3L}\right)^2$

Here, $W = (242.5 \times 24 \times 24) = 139680\ lb$

and, $F = 1.15 - \frac{c}{L} = \left(1.15 - \frac{4}{24}\right) = 0.983 \simeq 1$

$\therefore M_o = 0.09 \times 139680 \times 24 \times 1 \times \left(1 - \frac{2 \times 4}{3 \times 24}\right)^2 = 238387.2$ lb-ft

Now, column strip -ve Moment $= 0.5\ M_o$

$= (0.5 \times 238387.2)$ lb-ft

$= 119193.6$ lb-ft

column strip +ve Moment $= 0.2\ M_o$

$= (0.2 \times 238387.2)$ lb-ft

$= 47677.44$ lb-ft

Middle strip +ve and -ve Moment $= 0.15\ M_o$

$= (0.15 \times 238387.2)$ lb-ft

$= 35758$ lb-ft

(v) Depth check:

$n = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$

$r = \frac{.4 \times 60000}{.45 \times 3000} = 17.78$

$\therefore K = \frac{n}{n+r} = \frac{9}{9 + 17.78} = 0.336$

$\therefore j = 1 - \frac{0.336}{3} = 0.888$

$R = \frac{1}{2} f_c j K = \frac{1}{2} \times .45 \times 3000 \times 0.888 \times 0.336 = 201.4$

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(a) in the portion of slab with drop panel:

$$d_1 = \sqrt{\frac{M}{Rb_1}} = \sqrt{\frac{119193.6 \times 12}{201.4 \times 0.75 \times (8 \times 12)}} = 9.93 \text{ in.}$$

$d_{eff} = 10.5$ in $>$ 9.93 in.

(OK)

(b) in the portion of slab with out drop panel:

$$d_2 = \sqrt{\frac{M}{Rb_2}} = \sqrt{\frac{47677.44 \times 12}{201.4 \times 0.75 \times (12 \times 12)}} = 5.12 \text{ in}$$

$d_{eff} = 7.5$ in $>$ 5.12 in.

(OK)

(vi) <u>Rein force ment Calculation:</u>

<u>At column strip:</u> for (-ve) moment,

$$A_s = \frac{M}{f_s j d} = \frac{119193.6 \times 12}{24000 \times 0.888 \times 10.5}$$

$$= 6.39 \text{ in}^2$$

providing, 15 # 6 bars

for (+ve) moment,

$$A_s = \frac{M}{f_s j' d} = \frac{47677.44 \times 12}{24000 \times .888 \times 7.5}$$

$$= 3.58 \text{ in}^2$$

providing 9 # 6 bars.

At Middle strip, $A_s = \dfrac{M}{f_s j d} = \dfrac{38758 \times 12}{24000 \times 0.888 \times 7.5}$

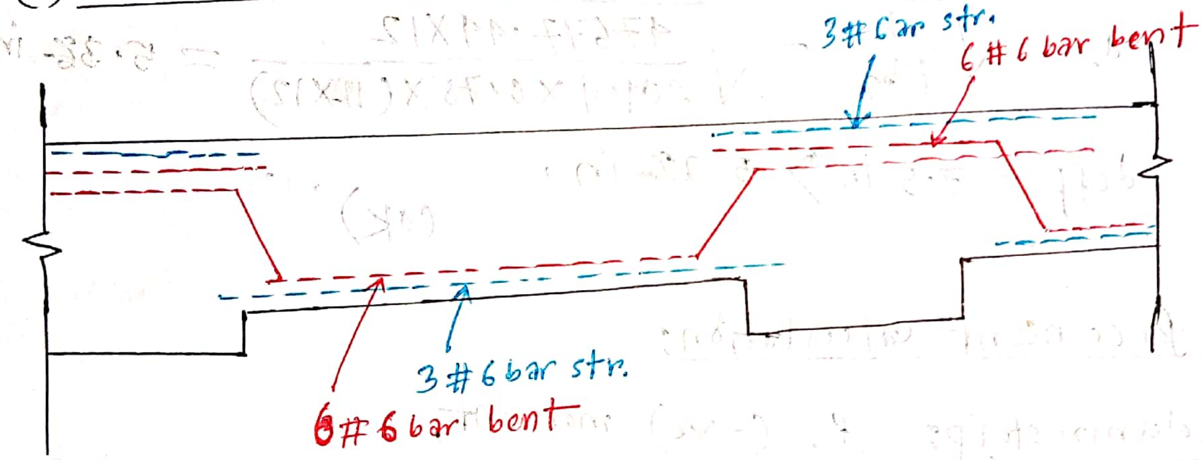
$= 2.68 \text{ in}^2$

providing 9 # 5 bars.

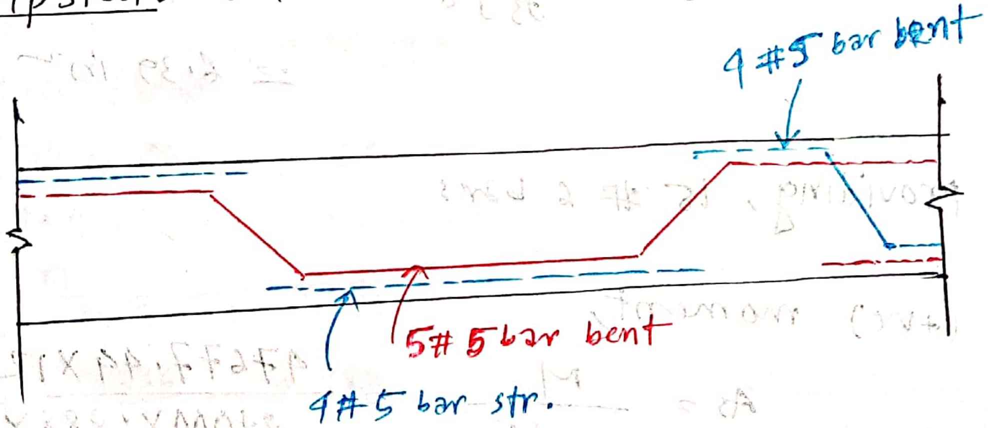
(vii) Working Diagram:

(a) Column strip steel: 33% bars will be straight $= (0.33 \times 9)$

$= 2.97 \simeq 3$ bars



(b) Middle strip steel: 50% bars will be straight & 50% bent.



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2015

A flat slab floor system is to support a dead load of 125 psf including its self weight and a service live load of 225 psf having column spacing of 23 ft on centers in both directions. Using 8-ft drop panel, design the slab. Assume $f_c' = 3000$ psi, $f_y = 50000$ psi

Solution:

(i) Thickness calculation:

$$t_2 = \frac{L}{40} = \frac{23 \times 12}{40} = 6.9 \simeq 8 \text{ in.}$$

or, $t_2 \geq 4$ in.

$$t_2 = 0.024 L \left(1 - \frac{2c}{3L}\right) \sqrt{\frac{w'}{f_c'/2000}} + 1$$

Here, D.L = 125 psf (in addition to its own weight থাকলে $\frac{t}{12} \times 150$ যোগ করব)

L.L = 225 psf

$\therefore w' = (125 + 225) = 350$ psf ; Let $c = 4$ ft

$$\therefore t_2 = 0.024 \times 23 \times \left(1 - \frac{2 \times 4}{3 \times 23}\right) \times \sqrt{\frac{350}{3000/2000}} + 1 = 8.45 \text{ in.} \simeq 9 \text{ in.}$$

Hence, The thickness of the slab, $t_2 = 9$ in

$t_1 = 1.5\, t_2 = (1.5 \times 9) = 13.5 \simeq 13$ in

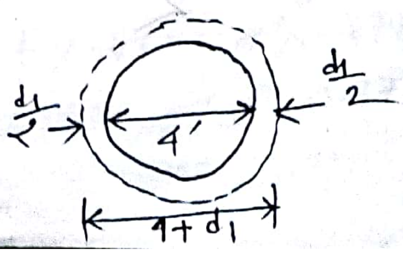
and the thickness of the drop panel $= \frac{t_2}{3} = \frac{9}{3} = 3$ in.

But, $(t_1 - t_2) = (13 - 9) = 4$ in (selected)

(ii) punching shear check: Relative to punching shear,

(a) The first critical section is at $\frac{d_1}{2}$ distance from the edge of column capital in the drop panel.

$\therefore d_1 = (13 - 1.5) = 11.5$ in



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∴ The dia meter of the critical section $= \left(4 + \frac{11.5}{12}\right) = 4.96'$

$$\therefore V_{dev} = \left\{23 \times 23 - \frac{\pi}{4}(4.96)^2\right\} \times 350 = 178387 \text{ lb}$$

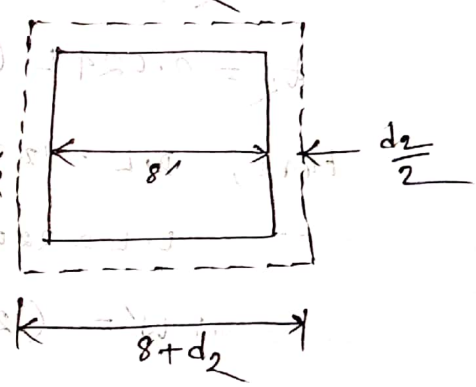
$$\therefore v = \frac{V}{b_o d} = \frac{178387}{3.1416 \times (4.96 \times 12) \times 11.5} = 82.96 \text{ psi}$$

But, $v_{all} = 2\sqrt{f_c'} = 2 \times \sqrt{3000} = 109.5 \text{ psi} > v$

(OK)

(b) The second critical section is at a distance $\frac{d_2}{2}$ from the edge of the drop panel.

$\therefore d_2 = (9 - 1.5) = 7.5$ in.



∴ The side dimension of the critical section $= \left(8 + \frac{7.5}{12}\right) = 8.625'$

$$\therefore V_{dev} = (23 \times 23 - 8.625 \times 8.625) \times 350 = 159113.3 \text{ lb}$$

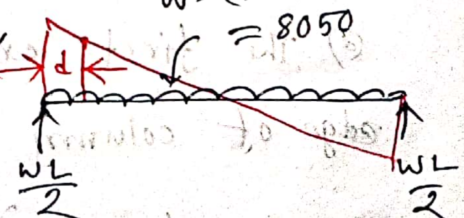
$$\therefore v = \frac{V}{b_o d} = \frac{159113.3}{4 \times (8.625 \times 12) \times 7.5} = 51.24 \text{ psi}$$

But, $v_{all} = 109.5 \text{ psi} > v$

(OK)

(iii) <u>Beam shear check:</u> Beam shear is checked at a distance d from the face of the support

$$d = \frac{\sqrt{\frac{\pi}{4} D^2}}{2} + d_2 = \frac{\sqrt{\frac{\pi}{4} \times (4)^2}}{2} + \frac{7.5}{12} = 2.395'$$



At 'd' distance, $V_{dev} = \frac{WL}{2} - dW = \left(\frac{8050 \times 23}{2} - 2.395 \times 8050\right)$

$= 73295.25$ lb

$\therefore v = \frac{V}{bd} = \frac{73295.25}{(23 \times 12) \times 7.5} = 35.40$ psi

But, $v_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{3000} = 60.25$ psi $> v$

(OK)

(iv) Moment Calculation:

The design moment, $M_o = 0.09\, WLF\left(1 - \frac{2c}{3L}\right)^2$

Here, $W = (350 \times 23 \times 23) = 185150$ lb

$F = 1.15 - \frac{c}{L} = \left(1.15 - \frac{4}{23}\right) = 0.976 \simeq 1$

$\therefore M_o = 0.09 \times 185150 \times 23 \times 1 \times \left(1 - \frac{2 \times 4}{3 \times 23}\right)^2 = 299540.5$ lb-ft

$\therefore$ Column strip $-$ve Moment $= 0.5 \times M_o = (0.5 \times 299540.5)$

$= 149770.25$ lb ft

Column strip $+$ve Moment $= 0.2 \times M_o = (0.2 \times 299540.5)$

$= 59908$ lb ft

Middle strip $-$ve & $+$ve Moment $= 0.15\, M_o = (0.15 \times 299540.5)$

$= 44931$ lb ft

(v) Depth check: $n = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$

$r = \frac{0.4 \times 50}{0.45 \times 3} = 14.8$; $\therefore K = \frac{n}{n+r} = \frac{9}{9+14.8} = 0.378$

$\therefore j = \left(1 - \frac{0.378}{3}\right) = 0.874$; $\therefore R = \frac{1}{2} f_c jK = \frac{1}{2} \times .45 \times 3000 \times 0.874 \times 0.378$

$\therefore R = 223$

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(a) with drop panel: $d_1 > \sqrt{\frac{M}{Rb_1}} = \sqrt{\frac{149770.25 \times 12}{223 \times 0.75 \times 8 \times 12}}$

$\therefore d_1 = 10.58$ in

But, $d_{eff} = 11.5 > 10.58$ in. (OK)

(b) without drop panel: $d_2 = \sqrt{\frac{M}{Rb_2}} = \sqrt{\frac{59908 \times 12}{223 \times 0.75 \times \frac{23}{2} \times 12}}$

$\therefore d_2 = 5.58$ in.

$d_{eff} = 7.5$ in $> d_2$ (OK)

(vi) Reinforcement calculation:

At column strip, for −ve moment $A_s = \frac{M}{f_s j d} = \frac{149770.25 \times 12}{20000 \times .874 \times 11.5}$

$= 8.94$ in^2

providing 21 #6 bars.

for +ve moment $A_s = \frac{59908 \times 12}{20000 \times 0.874 \times 7.5} = 5.48$ in^2

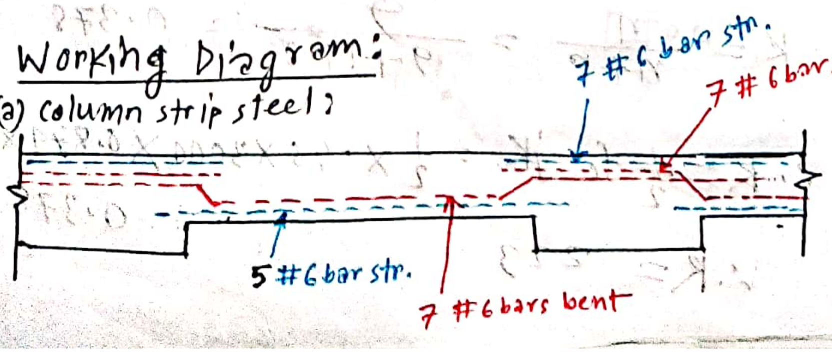
providing 13 #6 bars.

At middle strip, $A_s = \frac{44931 \times 12}{20000 \times 0.874 \times 7.5} = 4.11$ in^2

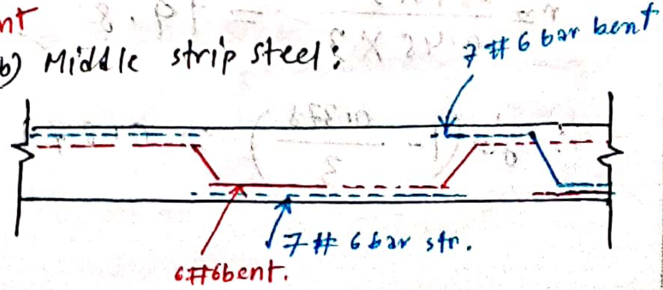
providing 13 #5 bars.

Working Diagram:

(a) column strip steel:



(b) Middle strip steel:



2012

\# A flat slab floor system with panels 20ft by 22 ft is supported on columns in both directions. Drop panel is to be provided. Design a typical interior panel for a live load of 180 psf. Assume, $f_c' = 4000$ psi and $f_s = 20000$ psi.

<u>Solution:</u>

(i) <u>Thickness calculation:</u>

$$t_2 = \frac{L}{40} = \frac{22 \times 12}{40} = 6.6 \simeq 8 \text{ in.}$$

or, $t_2 \geq 4$ in.

$$t_2 = 0.024\, L \left(1 - \frac{2c}{3L}\right)\sqrt{\frac{w'}{f_c'/2000}} + 1, \text{ Assume } c = 4'$$

Here, $w' = D.L + L.L = \left(\frac{8}{12} \times 150 + 180\right) = 280$ psf

$$\therefore t_2 = 0.024 \times 22 \times \left(1 - \frac{2 \times 4}{3 \times 22}\right) \times \sqrt{\frac{280}{4000/2000}} + 1 = 6.49$$

Hence, thickness of the slab = 8 in.

$t_1 = 1.5\, t_2 = (1.5 \times 8) = 12$ in $\simeq 11$ in.

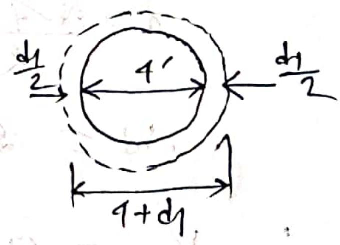
∴ the thickness of the drop panel = 3 in.

(ii) <u>Punching shear check:</u>

Relative to punching shear,

(a) The first critical section is at a distance '$\frac{d_1}{2}$' from the edge of the column capital in the drop panel.

$$\therefore d_1 = (11 - 1.5) = 9.5 \text{ in.}$$



∴ The dia. of critical section = $(4 + d_1)$

$$= \left(4 + \frac{9.5}{12}\right)$$

$$= 4.79'$$

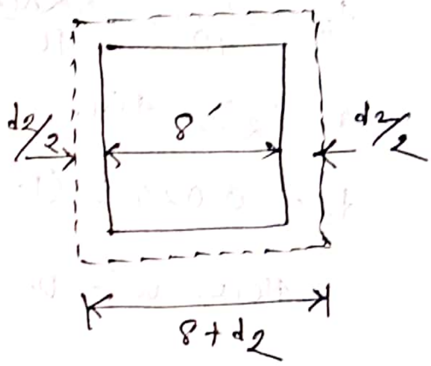
$\therefore V_{dev} = \{20 \times 22 - \frac{\pi}{4} \times (4.79)^2\} \times 280 = 118159.3171$ lb

$\therefore v = \frac{V_{dev}}{b_o d} = \frac{118159.3171}{\pi \times (4.79 \times 12) \times 9.5} = 68.87$ psi

But, $v_{all} = 2\sqrt{f_c'} = 2\sqrt{4000} = 126.5$ psi $> v$ (ok)

(ii) The second critical section is at a distance $\frac{d_2}{2}$ from the edge of the drop panel.

$d_2 = (8 - 1.5) = 6.5$ in



$\therefore$ the side dimension of the critical section

$= (8 + d_2)$

$= (8 + \frac{6.5}{12})$ in

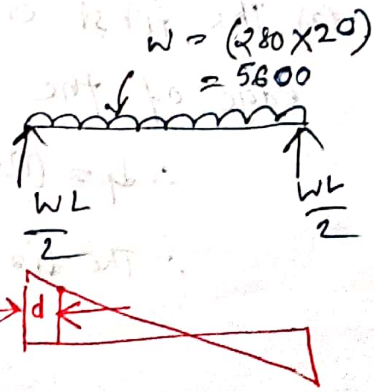
$= 8.542$ in.

$\therefore V_{dev} = (20 \times 22 - 8.542 \times 8.542) \times 280 = 102769.5861$ lb

$\therefore v = \frac{V_{dev}}{b_o d} = \frac{102769.5861}{4 \times (8.542 \times 12) \times 6.5} = 38.56$ psi

But, $V_{all} = 2\sqrt{f_c'} = 126.5$ psi $> v$ (ok)

(iii) Beam shear check; Beam shear checked at a distance 'd' from face of the support.



$d = \frac{\sqrt{\frac{\pi}{4} c^2}}{2} + d_2 = \frac{\sqrt{\frac{\pi}{4} \times (4)^2}}{2} + \frac{6.5}{12}$

$\therefore d = 2.31'$

$\therefore V_{dev} = 5600 \times \frac{22}{2} - 2.31 \times 5600$

$= 48664.0$ lb

$$v = \frac{V_{dev}}{bd} = \frac{48664}{(20\times12)\times6.5} = 31.2 \text{ psi}$$

$$V_{all} = 1.1\sqrt{f_c'} = 1.1\times\sqrt{4000} = 69.57 \text{ psi} > v$$

(OK)

(iv) Moment calculation: The design moment, $M_o = 0.09\, WLF\left(1-\frac{2c}{3L}\right)^2$

Here, $W = 280\times22\times20$; $L = 22'$; $F = 1.15 - \frac{c}{L} = 1.15 - \frac{4}{22}$

$= 123200$ $\therefore F = 0.96 \simeq 1$

$$M_o = 0.09\times123200\times22\times1\times\left(1-\frac{2\times4}{3\times22}\right)^2 = 188384 \text{ lb-ft}$$

Now, column strip +ve Moment $= 0.5M_o = (0.5\times188384)$ lb-ft

$= 94192$ lb-ft

column strip −ve Moment $= 0.2M_o = (0.2\times188384)$ lb-ft

$= 37676.8$ lb-ft

strip −ve &+ve Moment $= (0.15\times188384)$ lbft

$= 28257.6$ lb-ft

(v) depth Check:

(a) with drop panel: $d_1 = \sqrt{\frac{M}{Rb_1}}$

Here, $n = \frac{29\times10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$; $r = \frac{20000}{.45\times4000} = 11.11$

$K = \frac{8}{8+11.11} = 0.42$; $j = 1-\frac{0.42}{3} = 0.86$

$\therefore R = \frac{1}{2}f_c jK = \frac{1}{2}\times0.45\times4000\times.86\times.42 = 325.08$

$$d_1 = \sqrt{\frac{94192\times12}{325.08\times8\times12\times0.75}} = 6.24 < d_{eff} = 9.5 \text{ in.}$$

(OK)

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(b) With out drop panel: $d_2 = \sqrt{\dfrac{M}{R\,b_2}} = \sqrt{\dfrac{37676.8 \times 12}{385.08 \times .75 \times \frac{22}{2} \times 12}}$

$\therefore d_2 = 3.44 < d_{eff} = 6.5$ in. (OK)

(vi) Reinforcement Calculation:

At column strip: for −ve moment,

$$A_s = \frac{M}{f_s\, j\, d} = \frac{94192 \times 12}{20000 \times 0.86 \times 9.5} = 6.92\ in^2$$

providing 16 #6 bars.

for +ve moment,

$$A_s = \frac{37676.8 \times 12}{20000 \times 0.86 \times 6.5} = 4.04\ in^2$$

providing 10 # 6 bars.

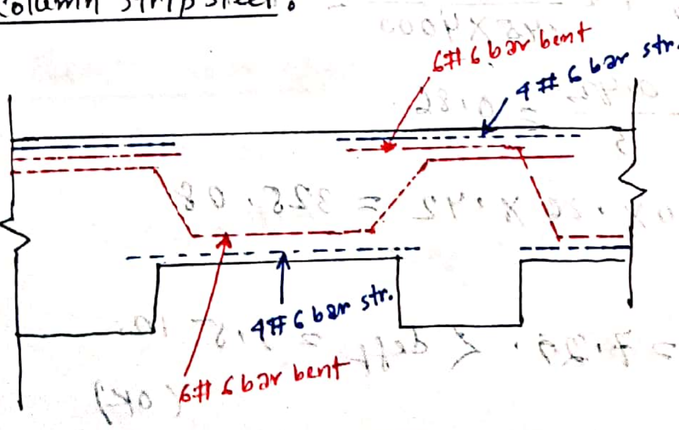
At middle strip.

$$A_s = \frac{28257.6 \times 12}{20000 \times 0.86 \times 6.5} = 3.03\ in^2$$

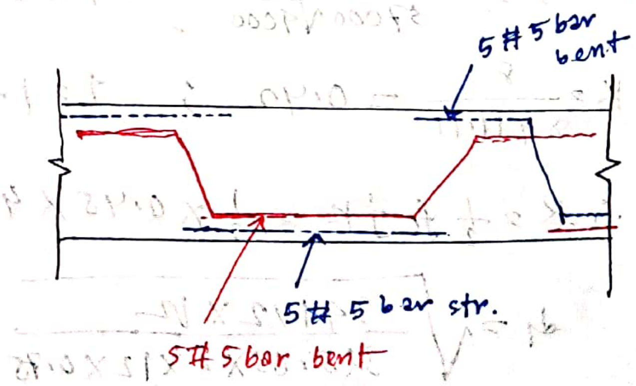
providing 10 # 5 bar

(vii) working diagram.

(a) Column strip Steel:



(b) Middle strip steel:



Class Test

Problem: Design a flat slab of columns are spaced 18 ft center in each direction and column capital 3.5 ft diameter, carries a live load 120 psf and dead load 100 psf. Use $f_c' = 3000$ psi and $f_y = 60000$ psi.

Solution:

(i) Thickness calculation:

$$t = \frac{L}{40} = \frac{18 \times 12}{40} = 5.4 \text{ in} \simeq 7 \text{ in.}$$

or, $t \geq 4$ in.

$$t = 0.028\, L \left(1 - \frac{2c}{3L}\right) \sqrt{\frac{w'}{f_c'/2000}} + 1.5$$ (without droppanel)

Here, $w' = (D.L + L.L) = (100 + 120) = 220$ psf

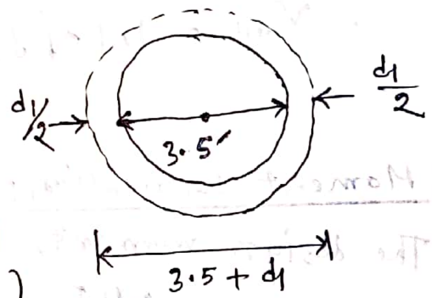
$c = 4'$ (Assume)

$$\therefore t = 0.028 \times 18 \times \left(1 - \frac{2 \times 4}{3 \times 18}\right) \sqrt{\frac{220}{3000/2000}} + 1.5 = 6.7''$$

$\therefore$ Thickness of the slab, $t_2 = 7$ in.

(ii) Punching shear check:

The critical section is at a distance $\frac{d_1}{2}$ from the edge of the column capital.



$d_1 = (t - 1.5) = (7 - 1.5) = 5.5''$

$\therefore$ diameter of the critical section $= (3.5 + d_1)$

$= \left(3.5 + \frac{5.5}{12}\right)$

~~$\therefore$ The floor area~~

$= 3.96'$

$\therefore$ The floor area of the critical section $= \frac{\pi}{4}(3.96)^2 = 12.31 \text{ ft}^2$

$\therefore$ developed shear on the critical section,

$V_u = (18 \times 18 - 12.31) \times 220 = 68572.7$ lb

$\therefore$ Nominal shear stress, $v = \frac{V}{b_o d} = \frac{68572.7}{\pi \times (3.96 \times 12) \times 5.5}$

$= 83.51$ psi

Allowable shear stress, $V_{all} = 2\sqrt{f_c'} = 2\sqrt{3000} = 109.5 \text{ psi} > v$ (ok)

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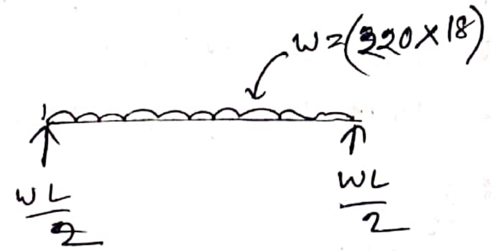
(iii) Beam shear check: It is checked at a distance 'd' from the face of the support.



$$d = \frac{\sqrt{\frac{\pi}{4}c^2}}{2} + d_1$$

$$= \frac{\sqrt{\frac{\pi}{4} \times 4^2}}{2} + \frac{5.5}{12}$$

$$= 2.23 \text{ ft}$$

At a distance 'd', the developed shear, $V_d = \frac{WL}{2} - \cancel{dW}$

$$= \frac{(220 \times 18)}{2} \times 18 - 2.23 \times (220 \times 18)$$

$$= 26809.2$$

$\therefore$ nominal shear stress,

$$v = \frac{V_d}{bd} = \frac{26809.2}{(18 \times 12) \times 5.5}$$

$$= 22.57 \text{ psi}$$

Allowable shear stress,

$$v_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{3000} = 60.25 \text{ psi} > v$$

(iv) Moment calculation:

The design moment,

$$M_o = 0.09\, WLF \left(1 - \frac{2c}{3L}\right)^2$$

Here, $W = (220 \times 18 \times 18) = 71280$ lb

$$F = 1.15 - \frac{c}{L} = 1.15 - \frac{4}{18} = 0.92 \simeq 1$$

$$\therefore M_o = 0.09 \times 71280 \times 18 \times 1 \times \left(1 - \frac{2 \times 4}{3 \times 18}\right)^2 = 83793.6$$

Now, column strip $-$ve Moment $= 0.46\,M_o$

$= (0.46 \times 83793.6)$

$= 38545.056$ lb-ft

column strip $+$ve Moment $= 0.2\,M_o$

$= (0.2 \times 83793.6)$

$= 16758.72$ lb-ft

Middle strip $+$ve and $-$ve Moment $= 0.16\,M_o$

$= (0.16 \times 83793.6)$

$= 13406.976$ lb-ft

(v) depth check: $\boxed{j = 0.888 \;\&\; R = 201.4}$

$$d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{38545.056 \times 12}{0.75 \times 201.4 \times (\frac{18}{2} \times 12)}} = 5.32'' < d_{eff} = 5.5''$$

(OK)

(vi) Reinforcement Calculation: (Column Strip):-

(for $-$ve Moment) $A_s = \dfrac{M}{f_s j d} = \dfrac{38545.056 \times 12}{24000 \times .888 \times 5.5} = 3.946 \text{ in}^2$

providing, 13 #5 bars.

(for $+$ve Moment) $A_s = \dfrac{M}{f_s j d} = \dfrac{16758.72 \times 12}{24000 \times .888 \times 5.5} = 1.72 \text{ in}^2$

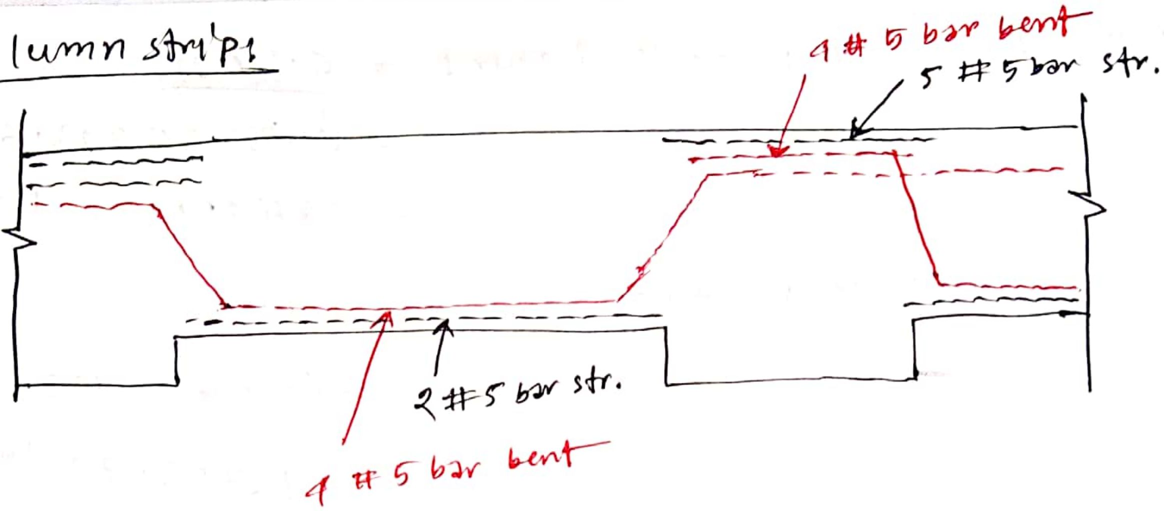
providing, 6 #5 bars.

(At middle strip):- $A_s = \dfrac{M}{f_s j d} = \dfrac{13406.976 \times 12}{24000 \times 0.888 \times 5.5} = 1.4 \text{ in}^2$

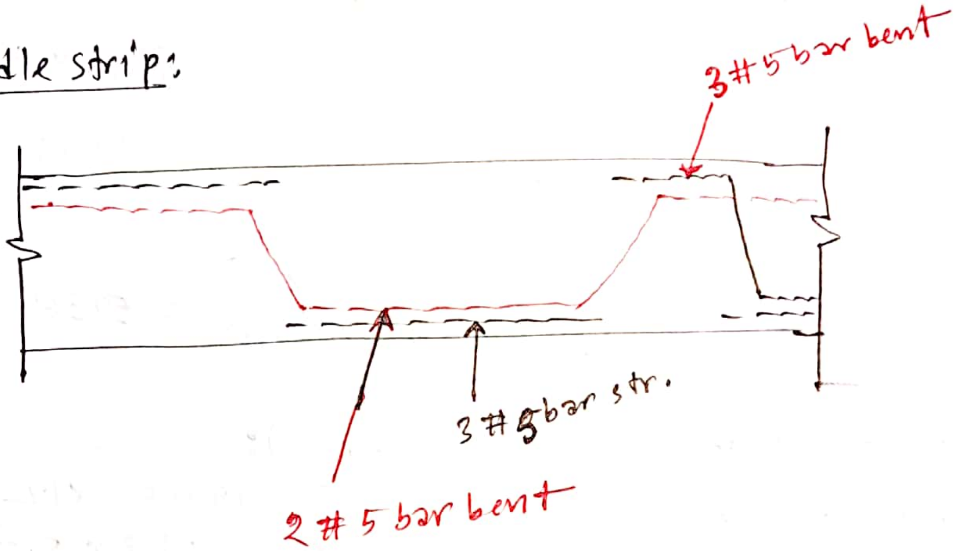
providing 5 #5 bars.

(vii) Reinforcement working diagram:

Column strip:



Middle strip:



(14'x16') FP & 15" sq. column

Flat Plate Slab

$\boxed{2016, 2010}$ → (17'x20') FP & 18" sq. column

\# A 18'x20' interior panel of flat plate floor system carries 60 psf live load and 100 psf dead load including its self weight. Design the panel assuming column size = 15"x15". Given $f_c' = 4$ ksi and $f_y = 60000$ psi.

Solution:

(i) Thickness of slab:

(i) $t = 5$ in.

(ii) $t = \dfrac{l_n \times (800 + 0.005 f_y)}{36000}$ Here, $l_n = 20' - \left(\frac{15}{12}\right)' = 18.75' = 225''$

$= \dfrac{225 \times (800 + 0.005 \times 60000)}{36000}$

$= 6.875$ in.

∴ Assume, the thickness of the slab = 7 in.

(ii) Load calculation:

Given, D.L = 100 psf

L.L = 60 psf

∴ $W_u = (1.2 \times 100 + 1.6 \times 60) = 216$ psf

(iii) Punching shear check: The critical section is at a distance $d/2$ from the edge of column.

$\therefore d = t - c.c - \phi/2 = 7 - 0.75 - \frac{4}{2 \times 8} = 6''$

∴ The perimeter of the critical section, $b_o = 4(15 + d) = 4 \times (15+6) = 84''$

The area of the crical section $= \left(\frac{15+d}{12}\right)^2 = \left(\frac{21}{12}\right)^2 = 3.0625$ ft²

The shear on this critical section, $V_u = [(18 \times 20) - 3.0625] \times 216$

$= 77098.5$ lb

and, $V_{c4} = 4\phi\sqrt{f_c'}\ b_o d$

$= 4 \times 0.75 \times \sqrt{4000} \times 84 \times 6$

$= 95627.3$ lb $> V_u$

(OK)

(iv) Moment calculation:

$d_{long} = 7 - 0.75 - \frac{4}{2\times8} = 6''$

$d_{short} = 6 - \frac{4}{8} = 5.5''$

$d_{long} - \phi$

$M_{o\,(long)} = 0.125\ W_u\, l_s\, l_l^2 = 0.125 \times 216 \times 18 \times (20 - \frac{15}{12})^2$

$= 170859.375$ lb-ft

$M_{o\,(short)} = 0.125\ W_u\, l_l\, l_s^2 = 0.125 \times 216 \times 20 \times (18 - \frac{15}{12})^2$

$= 151503.75$ lb-ft

(v) Depth check: $d = \sqrt{\frac{M}{\phi R b}}$

Here $R = \rho f_y (1 - \frac{\beta}{\alpha} \times \frac{\rho f_y}{f_c'})$

But, $\rho_b = 0.85\ \beta_1 \times \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = 0.85 \times 0.85 \times \frac{4}{60} \times \frac{0.003}{0.003+0.005}$

$\therefore \rho_b = 0.018$ $\quad\therefore \rho = 0.75\rho_b = 0.75 \times 0.018 = 0.0135$

$\therefore R = 0.0135 \times 60000 \times (1 - 0.59 \times \frac{0.0135 \times 60}{4}) = 713.23$

$\therefore d = \sqrt{\frac{170859.375 \times 12}{0.9 \times 713.23 \times (\frac{18}{2} \times 12)}} = 5.44'' < d_{eff} = 6''$

$b = \frac{\text{short length}}{2}$

(OK)

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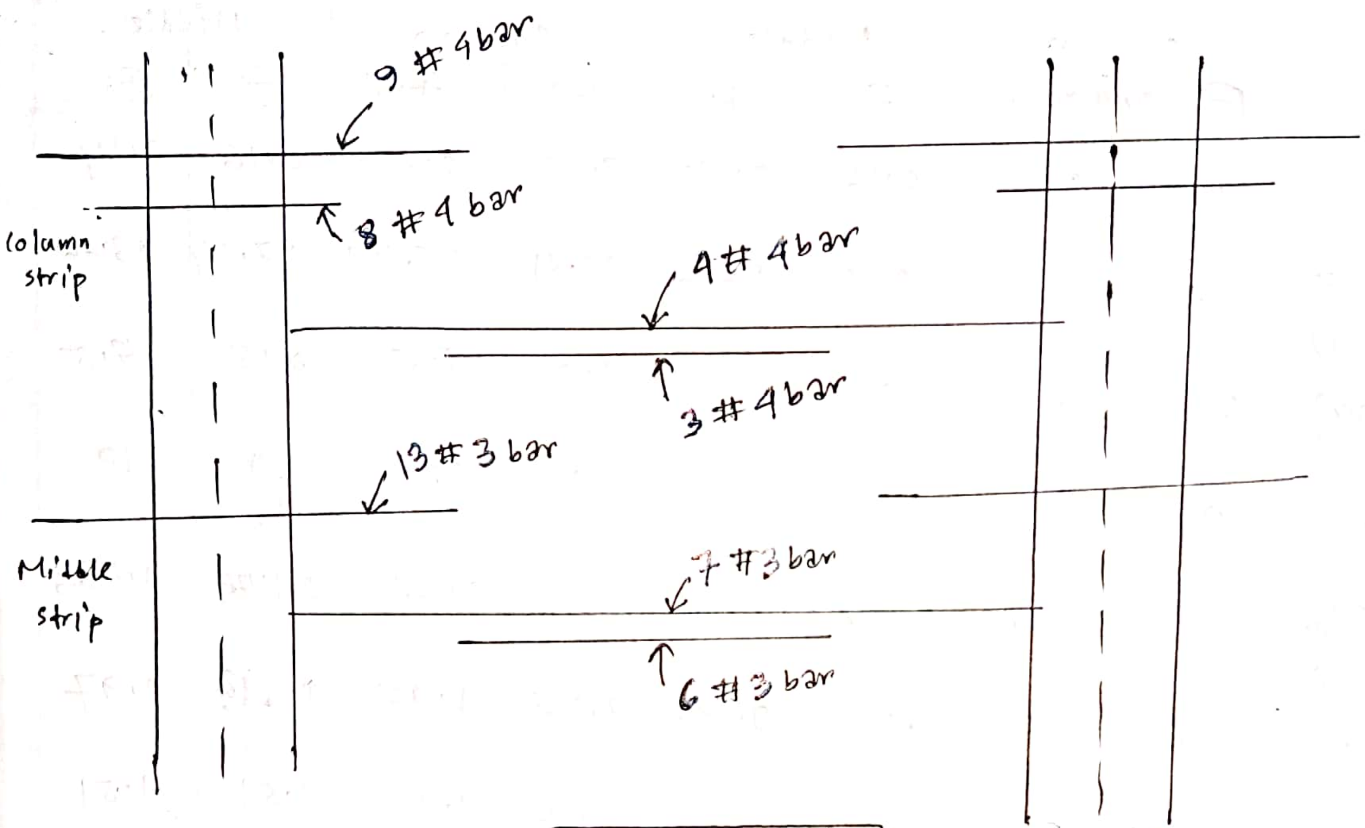
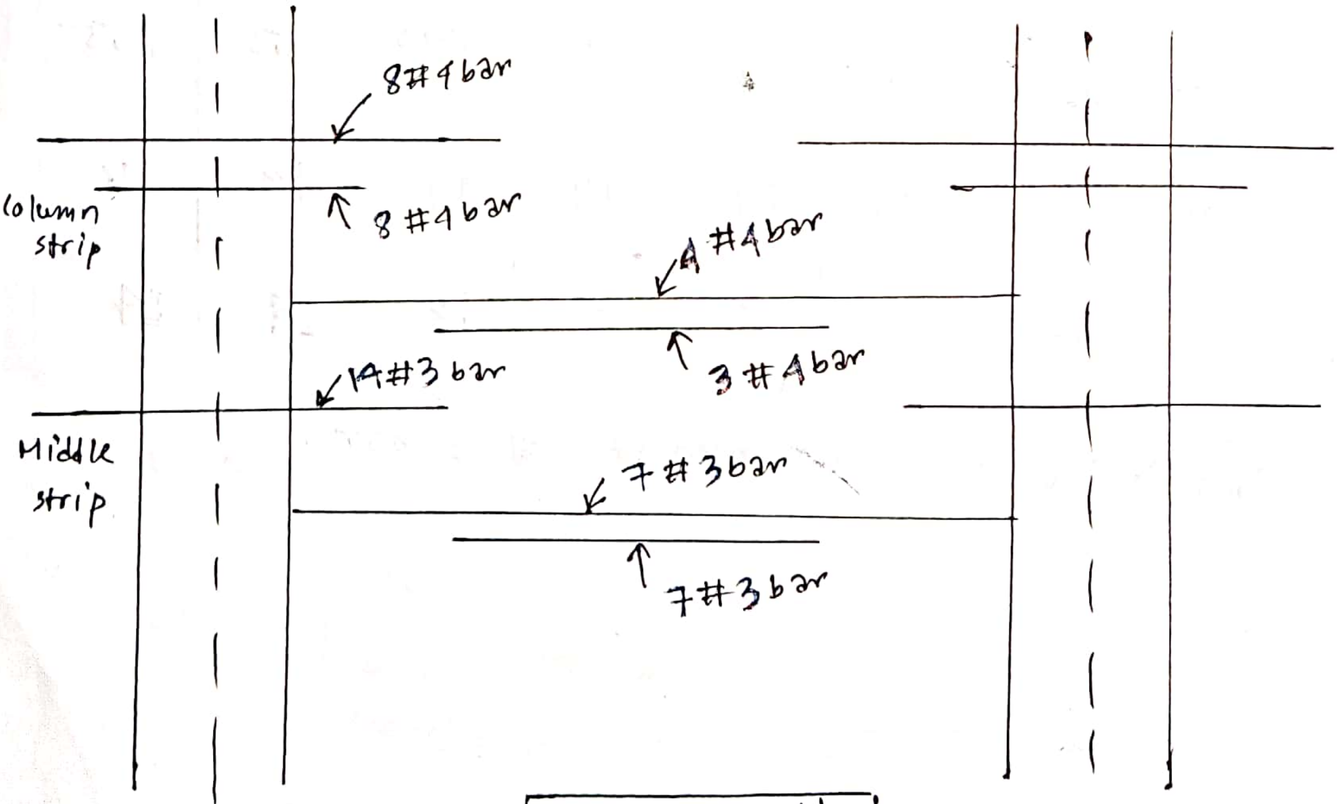
Direction	longer, $M_n = \frac{171}{0.9} = 190$ K-ft				shorter, $M_n = \frac{152}{0.9} = 169$ K-ft			
Strip	column		Middle		Column		Middle	
sign	⊖	⊕	⊖	⊕	⊖	⊕	⊖	⊕
K	0.49	0.21	0.16	0.14	0.49	0.21	0.16	0.14
$M_p = M_n \times K$ (K ft)	93.1	39.9	30.4	26.6	82.81	35.49	27.04	23.66
d (inch)	6	6	6	6	5.5	5.5	5.5	5.5
b (ft)	9 ($l_s/2$)	9	9	9	9	9	10 ($l_1/2$)	10
$a = \frac{f_y}{0.85 f_c'}$ $a = 17.65 \times \frac{A_s}{b}$ (in.)	0.16 As	0.16 As	0.16 As	0.16 As	0.16 As	0.16 As	0.147As	0.147As
$A_s = \frac{M_r}{f_y (d - a/2)}$ in²	3.24	1.35	1.02	0.89	3.15	1.32	0.997	0.87
$A_{s(min)} = 0.0018bt$ (in²)	1.36	1.36	1.36	1.36	1.36	1.36	1.57	1.57
using # 4 bar (number)	17	7	7	7	16	7	8	8
$S_{av} = \frac{b}{n}$ (in)	6.35	15.43	15.43	15.43	6.75	15.43	15	15
$S_{max} = 2t$ (in)	14	14	14	14	14	14	14	14
Using # 3 bar (No.)	—	—	13	13	—	—	14	14

* if $S_{max} > S_{av}$ then no need of # 3 bar.

Reinforcement Details:



Longer Direction



Shorter Direction

2008, 2006

Problem: A 19′ X 22′ interior panel of a flat plate floor is to carry Live load of 130 psf. Design the panel assuming column diameter of 20 inch. Given $f_c' = 3$ ksi and $f_y = 60$ ksi

Solution: $a = \sqrt{\frac{\pi}{4}(20)^2} = 17.72'' = 1.48'$

(i) Thickness of slab: (i) $t = 5$ in.

(ii) $t = \dfrac{l_n\,(800 + 0.005 f_y)}{36000}$ here $l_n = 22 - a$
$= (22 - 1.48)$
$= 20.52'$

$= \dfrac{20.52 \times 12 \times (800 + 0.005 \times 60000)}{36000}$

$= 7.524$ in.

∴ Assume, Thickness of the slab $= 12$ in

(ii) load calculation: self weight of slab $= \frac{t}{12} \times 150 = (\frac{12}{12} \times 150) = 150$ psf

Given, Live load $= 130$ psf

∴ $W_u = (1.2 \times 150 + 1.6 \times 130) = 388$ psf

(iii) punching shear check: The critical section is at a $d/2$ distance from the edge of the column,

∴ $d = t - c.c - \frac{\phi}{2} = 12 - 0.75 - \frac{4}{2 \times 8} = 11$ in.

∴ $b_o = \pi(D + d) = \pi(20 + 11) = 97.4$ in.

$V_u = \left[19 \times 22 - \frac{\pi}{4}\left(\frac{20+11}{12}\right)^2\right] \times 388 = 160150.3$ lb

and,

$V_{all} = 4\phi\sqrt{f_c'}\,b_o d = 4 \times 0.75 \times \sqrt{3000} \times 97.4 \times 11$

∴ $V_{all} = 176048.98$ lb $> V_u$

(OK)

(iv) Moment Calculation:

$$d_{long} = (12-1) = 11''$$

$$d_{long} = (11 - \tfrac{4}{8}) = 10.5''$$

$$M_o\,(long) = 0.125 \times 388 \times 19 \times \left(22 - \frac{1.48}{2}\right)^2$$

$$= 388016.4 \text{ lb-ft}$$

$$M_o\,(short) = 0.125 \times 388 \times 22 \times \left(19 - \frac{1.48}{2}\right)^2$$

$$= 327516.08 \text{ lb-ft}$$

(v) Depth check:

$$d = \sqrt{\frac{M}{\phi R b}}$$

$$\rho_b = 0.85\,\beta_1 \frac{f_c'}{f_y} \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005}$$

$$\therefore \rho_b = 0.014 \qquad \therefore \rho = 0.75\rho_b = (0.75 \times 0.014) = 0.0105$$

$$R = \rho f_y \left(1 - \frac{\beta}{\alpha} \frac{\rho f_y}{f_c'}\right) = 0.0105 \times 60000 \times \left(1 - 0.59 \times \frac{0.0105 \times 60}{3}\right)$$

$$\therefore R = 551.94$$

$$\therefore d = \sqrt{\frac{388016.4 \times 12}{0.9 \times 551.94 \times \left(\frac{19}{2} \times 12\right)}} = 9.07 < d_{eff} = 11''$$

(OK)

* $a = \dfrac{A_s f_y}{.85 f_c' b} = \dfrac{60000\,A_s}{.85 \times 3000 \times b} = 23.53 \dfrac{A_s}{b}$

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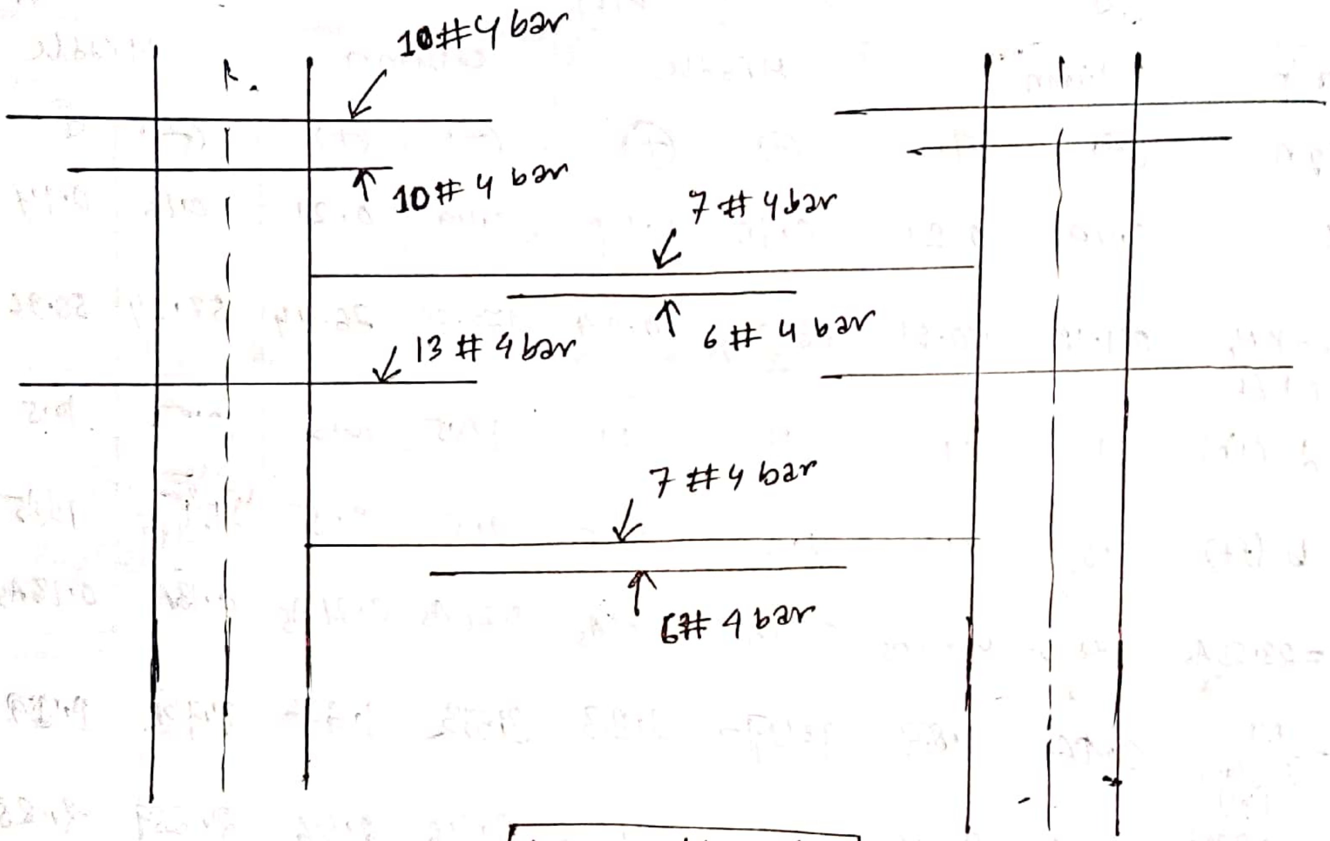
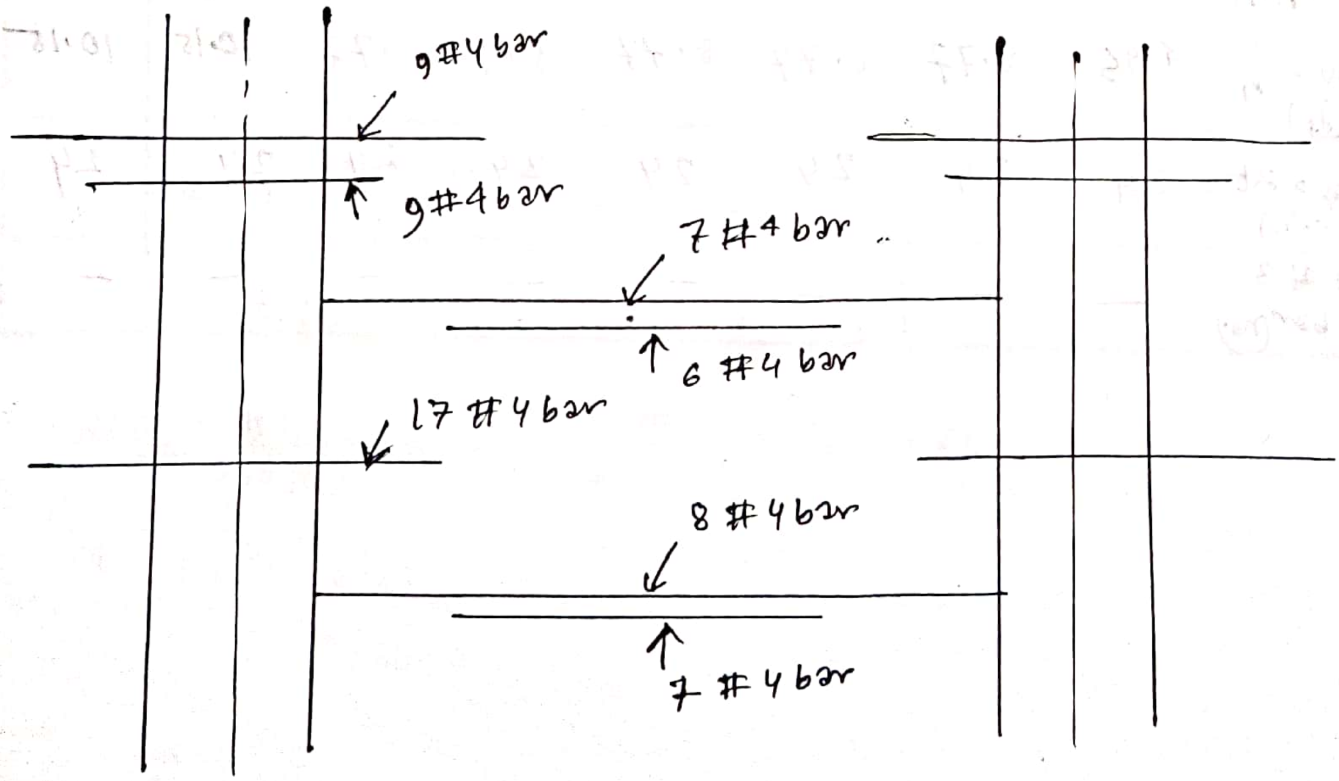
Direction	long direction ($M_n = \frac{M_o}{\phi} = \frac{388}{0.9} = 431$ K ft)				short direction ($M_n = \frac{328}{0.9} = 364$ K-ft)			
Strip	Column		Middle		Column		Middle	
sign	⊖	⊕	⊖	⊕	⊖	⊕	⊖	⊕
K	0.49	0.21	0.16	0.14	0.49	0.21	0.16	0.14
$M_r = K M_n$ (K ft)	211.19	90.51	68.96	60.34	178.36	76.44	58.24	50.96
d (in)	11	11	11	11	10.5	10.5	10.5	10.5
b (ft)	9.5 $\frac{A_s}{2}$	9.5	9.5	9.5	9.5	9.5	$A_s - \frac{A_s}{2}$ 12.5	12.5
* $a = 23.53 \frac{A_s}{b}$	$0.21 A_s$	$0.21 A_s$	$0.21 A_s$	$0.21 A_s$	$0.21 A_s$	$0.21 A_s$	$0.16 A_s$	$0.16 A_s$
$A_s = \frac{M_r}{f_y(d - \frac{a}{2})}$ (in²)	3.99	1.67	1.27	1.10	3.52	1.47	1.12	0.98
$A_{s(min)} = 0.0018\, bt$ (in²)	2.46	2.46	2.46	2.46	2.46	2.46	3.24	3.24
#4 bar (no.)	20	13	13	13	18	13	17	17
$S_{av} = \frac{b}{n}$ (in)	4.95	8.77	8.77	8.77	5.7	8.77	10.15	10.15
$S_{max} = 2t$ (in)	24	24	24	24	24	24	24	24
using #3 bar (no.)	—	—	—	—	—	—	—	—

Reinforcement details:



longer direction



Yield Line Analysis for Slab

Upper Bound Theorem: 17

If, for a small increment of displacement, The internal work done by slab, assuming that the moment at every plastic hinge is equal to the yield moment and that boundary conditions are satisfied, is equal to the external work done by the given load for that same small increment of displacement, then that load is an upper bound of the true carrying capacity.

Lower bound theorem:

If for a given external load, it is possible to find a distribution of moment that satisfies equilibrium requirements, with the moment that not exceeding the yield moment at any location, and the boundary conditions are satisfied, then the given load is a lower bound of the true bearing capacity.

Yield Line Theory:

The method of analysing the reinforced concrete slab which utilize the plastic deformation is called the yield line theory.

Yield Line:

Yield lines are typical crack pattern generated when ultimate moment is reached.

Characterestics of Yield Line:

(i) Yield lines are straight

(ii) Yield lines end at supporting edges of slab.

(iii) Yield lines passes through intersection of axis of rotation of adjacent slab element.

(iv) Axis of rotation lies along lines of supports and passes over column.

Assumptions of Yield Line theory: 2015,12,10

1. At the collapse stages, the steel reinforcement will be fully yielded along the yield lines.

2. During the collapse, the slab is deformed plastically and they gets separated into segments. Elastic behaviour is followed by the individual segments.

3. In yield lines theory, only plastic deformations are taken into consideration. The elastic deformations are neglected.

4. Along the yield lines, there is uniform distribution of bending and twisting moments.

5. The yield lines are lines of intersection of two planes, hence they will remain straight.

Guidelines for establishing axes of rotation and yield lines: 2016

1. Yield lines are straight lines because they represent the intersection of two planes.

2. Yield lines represent axes of rotation.

3. The supported edges of the slab will also establish axes of rotation.
If the edge is fixed, a negative yield line may form providing constant resistance to rotation.

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If the edge is simply supported, the axis of rotation provides zero restraint

4. An axis of rotation will pass over any column support. Its orientation depends on other consideration.

5. Yield lines form under concentrated loads, radiating outward from the point of application.

6. A yield line between two slab segment must pass through the point of intersection of the axes of rotation of the adjacent slab segments,

Limitations of Yield Line Theory: 2017, 15, 12, 10

1. Yield lines theory do not account elastic deformations.

2. Yield line theory provides a method for determining the capacity of designs. rather than for determining the amount and spacing of reinforcement.

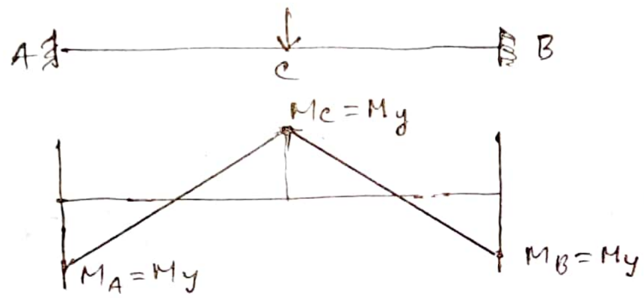
3. In applying yield line analysis to slabs, the analysis is predicted upon avaiable rotation capacity at the yield lines.

4. The yield line analysis focuses entirely on the flexural capacity of the slab. It is presumed that earlier failure will not occur due to shear or torsion.

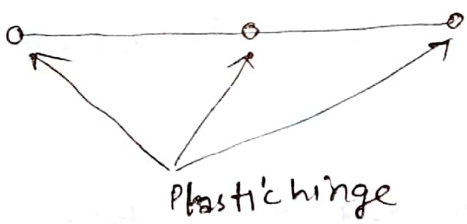
Collapse Mechanism: 2011, 2010

If a structure is subjected to progressive loading, moment at some section increases as if magnitude of load increases. The section with maximum moment is frusted and plastic hinge is formed at that section. Plastic hinge is also formed at another section, depending on the boundary conditions and loading pattern.

For a certain number of plastic hinge at the structure and it reaches a mechanism for which it collapses. This mechanism is known as collapse mechanism.

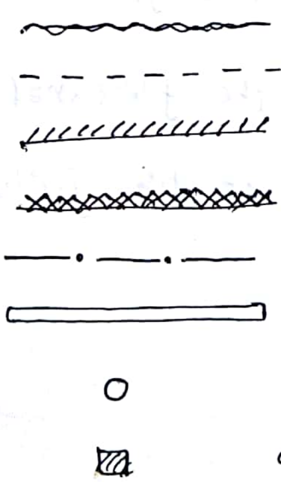


P >>> higher



collapse will occur

Notations for Yield line and Supports:



positive yield line

Negative yield line

simply supported edge

continuous or fixed edge

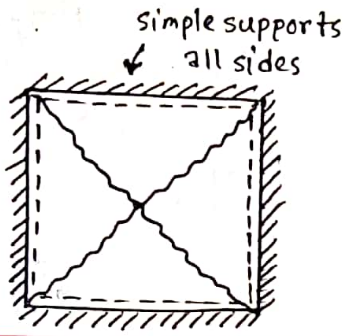
axis of rotation

Beam support

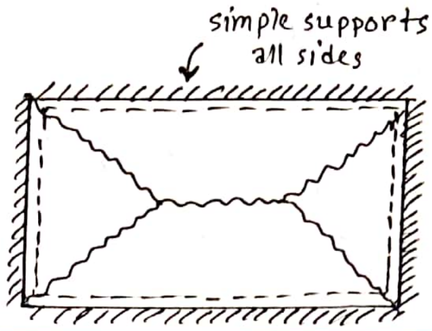
point load

column support

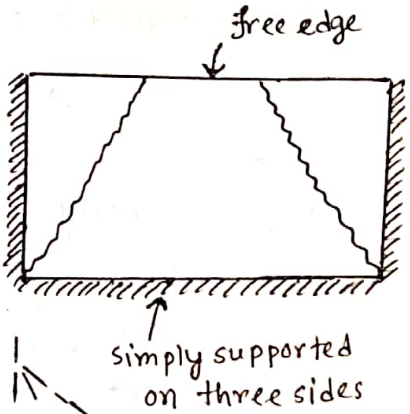
Typical Yield Line Pattern:



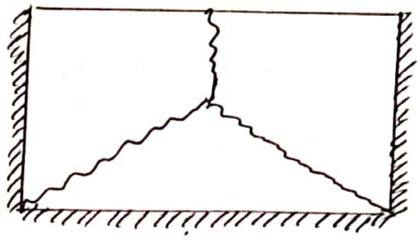
Simply supported Square Slab

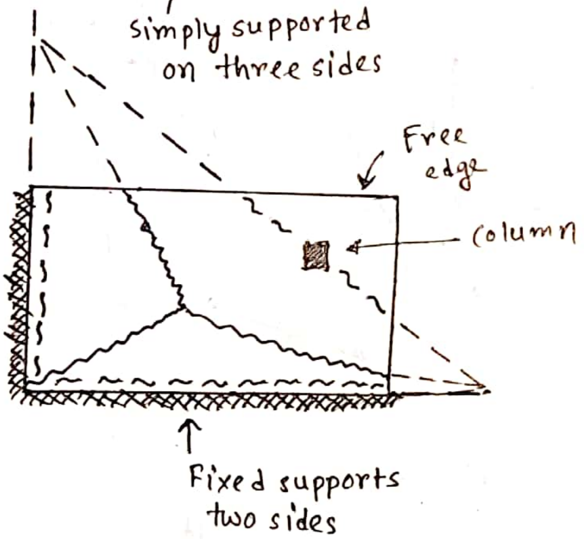


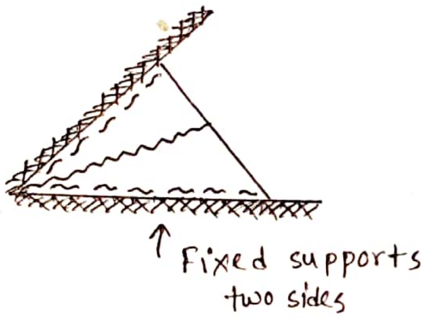
Simply Supported Rectangular Slab

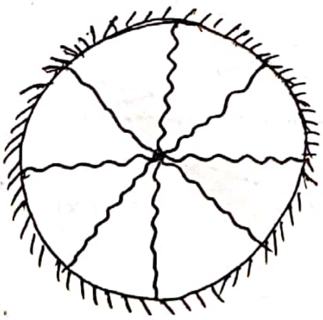


or,

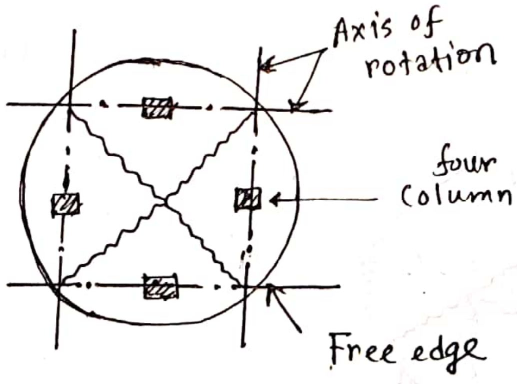








circular Slab



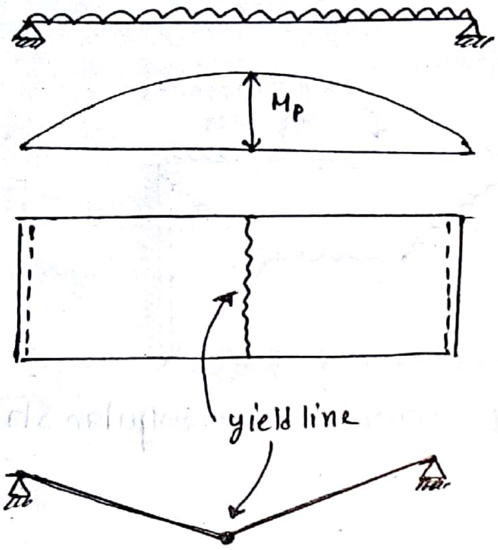


Fig. Simply Supported, uniformly loaded one-way slab

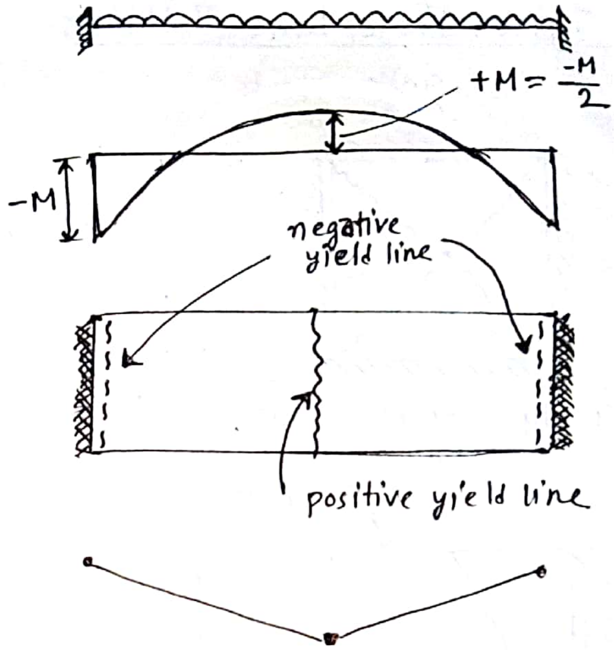
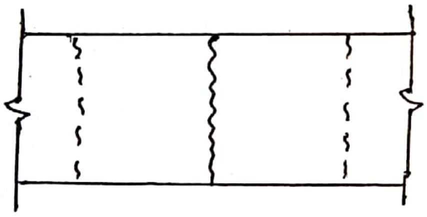
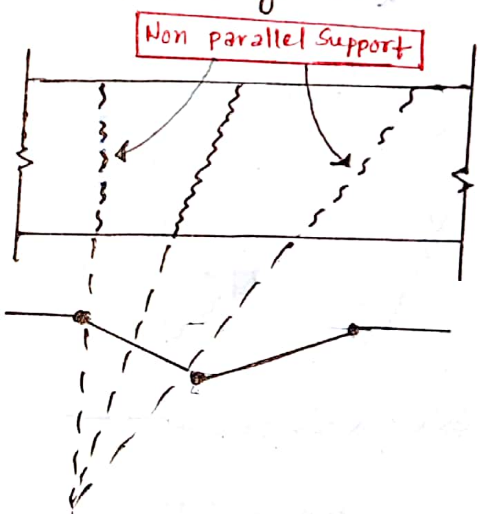
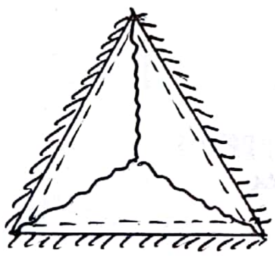


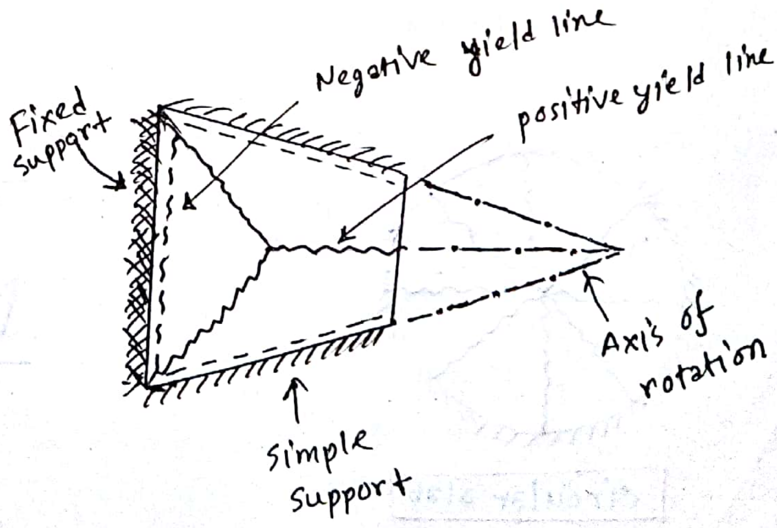
Fig. Fixed end, uniformly loaded One-way slab.

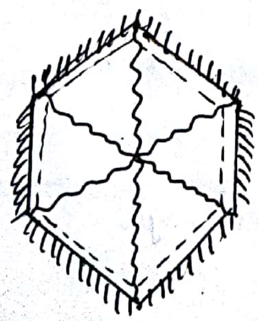




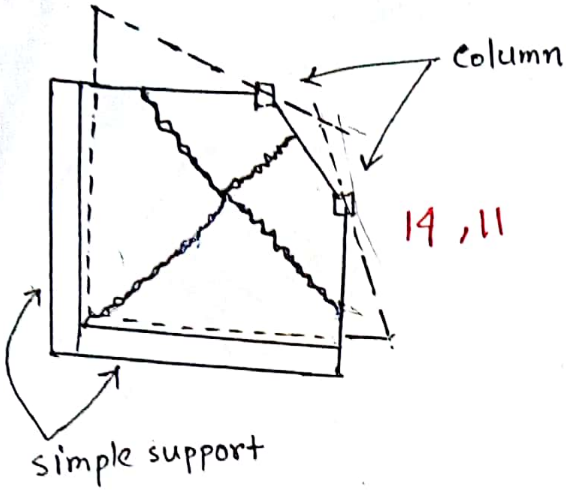


Triangular Slab

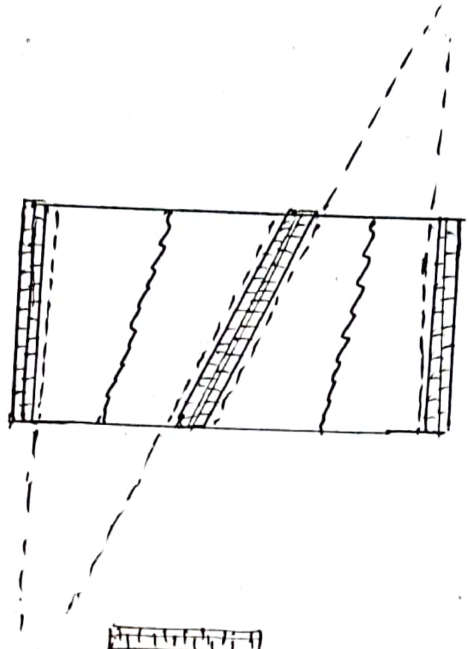




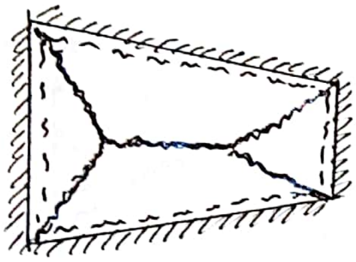
Hexagonal Slab



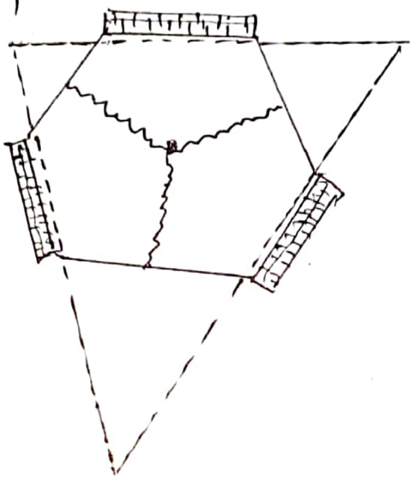
14, 11



14



14, 11



14, 11

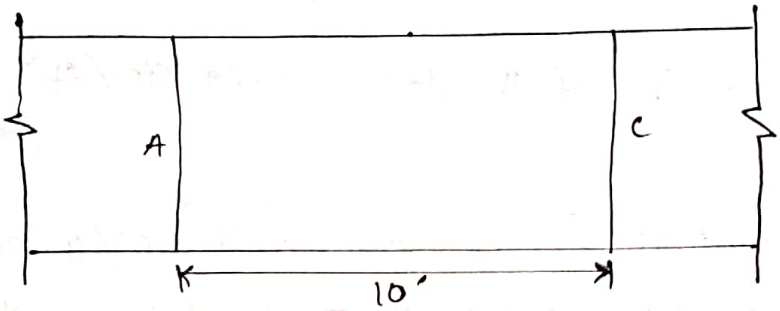
Isotropically Reinforced:
If a slab is reinforced in orthogonal directions so that the resisting moment is same in these two directions, the moment capacity of slab will be same same along any other line, regardless of direction. Such a slab is said to be isotropically reinforced.

Orthogonally anistropic: If however, the strengths are different in two perpendicular directions, the slab is called orthogonally ~~reinforced~~ anistropic or simply orthogonic.

Yield Line Analysis

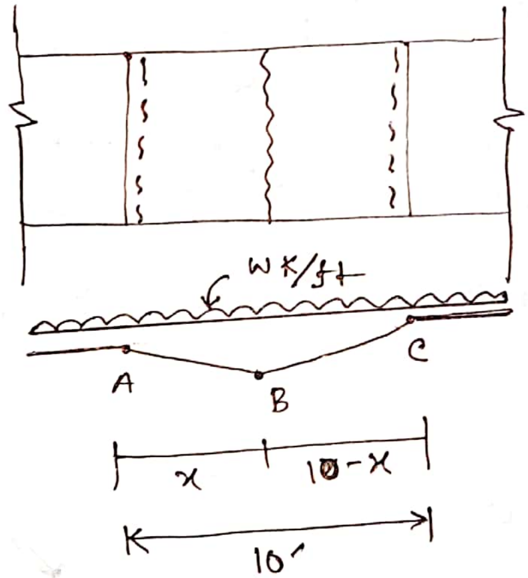
Example: 14.1

A slab is one-way, uniformly loaded & continuous as shown in figure:

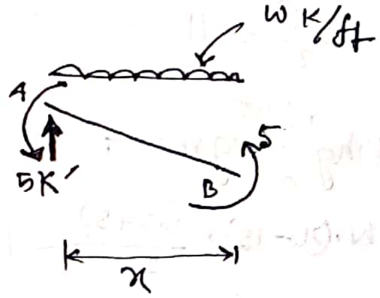


The slab has a 10 ft span and is reinforced to provide a resistance to positive bending $\phi m_n = 5\ K'/ft$ through the span. In addition, negative steel over the supports provides moment capacity of $5\ K'/ft$ at A and $7\ K'/ft$ at C. Determine the load capacity of the slab.

Solution:



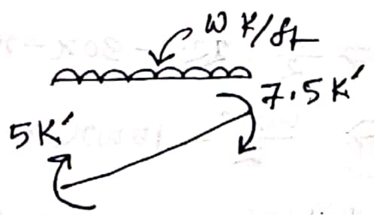
Taking the left segment of the slab,



$$wx \cdot \frac{x}{2} - 10 = 0$$

$$\Rightarrow \frac{w}{2}x^2 - 10 = 0 \quad \cdots\cdots (I)$$

Similarly, for right segment of the slab,



$$w \times (10-x) \times \frac{(10-x)}{2} - 12 = 0$$

$$\Rightarrow \frac{w}{2}(100 - 20x + x^2) - 12.5 = 0$$

$$\Rightarrow \frac{w}{2}x^2 - 10wx + 50w - 12.5 = 0 \quad \cdots\cdots (II)$$

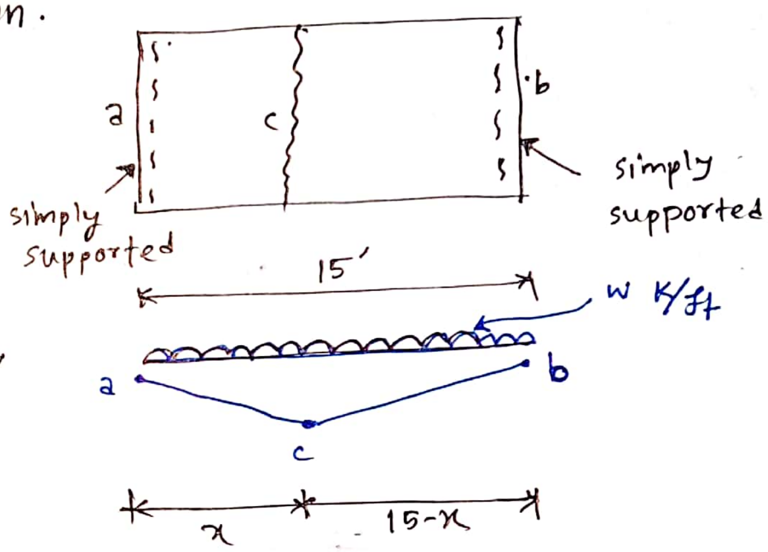
Solving Eqn ① & ⑪ we obtain, $w = 0.897\ K/ft^2$

$x = 4.7\ ft$

∴ The load capacity of the slab $= 0.89\ Kips/ft^2$ (Ans.)

2016

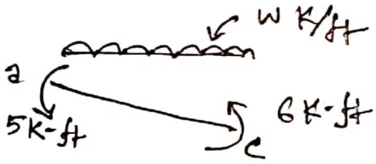
The yield moment at 'a' is $m_a = -5\ ft\text{-}Kips/ft$ at 'b' is $m_b = -4\ ft\text{-}Kip/ft$ and for positive moment at 'c' is $m_c = 6\ ft\text{-}Kip/ft$ as shown is figure: By yield line Method, calculate the ultimate uniform load that the slab will support on a 15 ft span.



Solution:

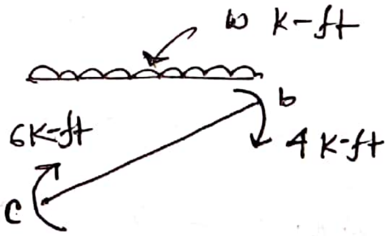
Let, the slab will support a uniform load w - K/ft^2

Taking left segment of the slab,



$$w \cdot x \cdot \frac{x}{2} - 11 = 0$$

$$\Rightarrow \frac{wx^2}{2} = 11 \quad \text{------} ①$$



Taking right segment of the slab,

$$w \cdot (15-x) \cdot \frac{(15-x)}{2} - 10 = 0$$

$$\Rightarrow \frac{w}{2}(225 - 30x + x^2) = 10$$

$$\Rightarrow \frac{wx^2}{2} - 15wx + 112.5w = 10 \quad \text{-----} ⑪$$

Solving Eqn ① & ⑪ we obtain, $w = 0.373\ Kips/ft^2$

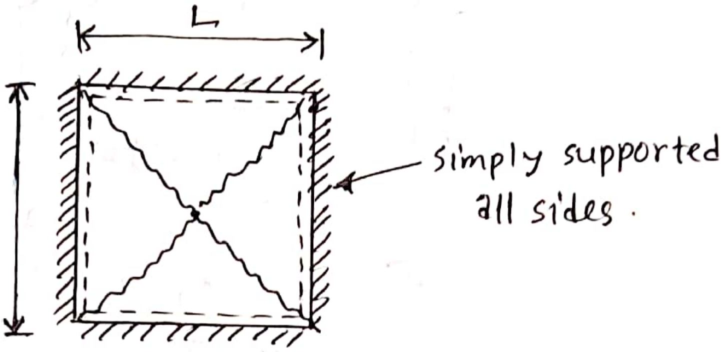
$x = 7.68\ ft$

(Ans.)

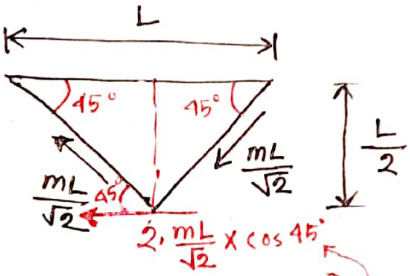
Example : 14.2 2014 , 2010

A square slab is simply supported along all sides and is to be isotropically reinforced. Determine the resisting moment $\boxed{m = \phi m_n}$ (or, $M_x = M_y$) per linear foot required just to sustain a uniformly distributed factored load of w psf.

Solution: (1) Segment equilibrium analysis:



Considering moment equilibrium of any one of identical slab segments about its support,

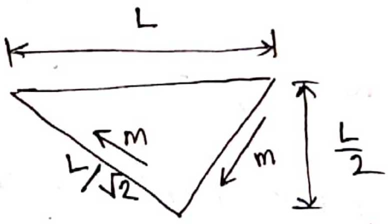


$$\left[\left(\frac{1}{2} \times L \times \frac{L}{2}\right) \times w\right] \times \left(\frac{1}{3} \times \frac{L}{2}\right) - 2 \times \frac{mL}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{wL^3}{24} = mL$$

$$\boxed{\therefore m = \frac{wL^2}{24}}$$

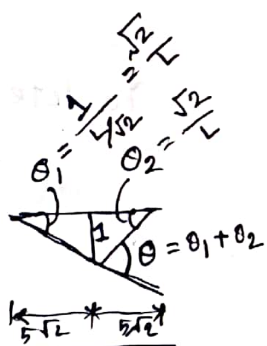
Solution: (2) Virtual work method:



considering one segment,

$$W_{ex.} = \left[\left(\frac{1}{2} \times L \times \frac{L}{2}\right) \times w\right] \times \frac{1}{3} \times 4$$ → for equal four segment

$$= \frac{wL^2}{3}$$



$$W_{int.} = \left(m \times \frac{L}{\sqrt{2}} \times \theta\right) \times 4 \quad \text{Here, } \theta = \frac{2\sqrt{2}}{L}$$

$$= m \times \frac{L}{\sqrt{2}} \times \frac{2\sqrt{2}}{L} \times 4 = 8m$$

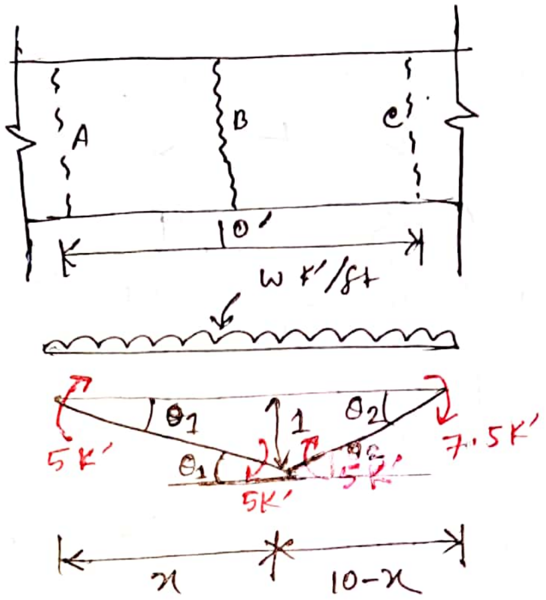
Now, $W_{ex.} = W_{int}$

$$\Rightarrow \frac{wL^2}{3} = 8m \Rightarrow \boxed{m = \frac{wL^2}{24}}$$

Example : 14.3

Determine the load capacity of the one way ~~sto~~ uniformly distributed continuous slab as shown in below: using method of virtual work. The resisting moments of the slab are 5.0, 5.0 and 7.5 K'/ft at A, B and C.

Solution:



$$\theta_1 = \frac{1}{x}$$

$$\theta_2 = \frac{1}{10-x}$$

$$W_{int} = \left(5 \times \frac{1}{x}\right) \times 2 + 5 \times \frac{1}{10-x} + 7.5 \times \frac{1}{10-x} \quad \cdots\cdots (i)$$

$$W_{ex.} = w \times x \times \frac{1}{2} + w \times (10-x) \times \frac{1}{2} \quad \cdots\cdots (ii)$$

Equating (i) & (ii)

$$\frac{wx}{2} + 5w - \frac{wx}{2} = \frac{10}{x} + \frac{5}{10-x} + \frac{7.5}{10-x}$$

$$\Rightarrow 5w = \frac{10}{x} + \frac{12.5}{10-x}$$

$$\Rightarrow w = \frac{2}{x} + \frac{2.5}{10-x} \quad \cdots\cdots (iii)$$

To determine minimum value of w,

$$\frac{dw}{dx} = -\frac{2}{x^2} + \frac{2.5}{(10-x)^2} = 0$$

$$\Rightarrow x = 4.75 \text{ ft}$$

From equation (iii), we obtain, $w = \frac{2}{4.75} + \frac{2.5}{10-4.75} = 0.89 \text{ K}/\text{ft}^2$

(Ans)

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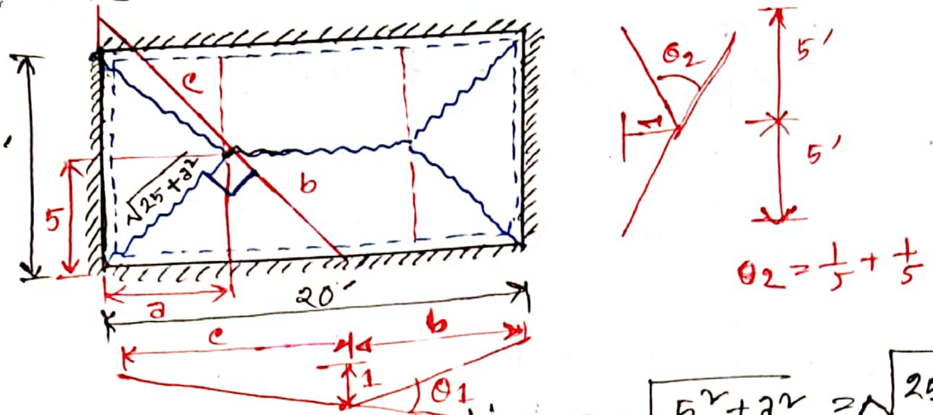
Example: 14.4

The two way slab shown in figure is simply supported on all four ~~edges~~ sides and carries a uniformly distributed load of w psf. Determine the required moment resistance for the slab, which is to be isotropically reinforced.



Solution: The length of the diagonal yield line $= \sqrt{5^2+a^2} = \sqrt{25+a^2}$

From simlar triangle,

$$\frac{\sqrt{25+a^2}}{a} = \frac{b}{5} \qquad \frac{c}{a} = \frac{\sqrt{25+a^2}}{5}$$

$$\Rightarrow b = 5 \times \frac{\sqrt{25+a^2}}{a} \qquad \Rightarrow c = a \times \frac{\sqrt{25+a^2}}{5}$$

The rotation of the plastic hinge at diagonal yield line corresponding to a unit deflection at the center of the slab,

$$\theta_1 = \frac{1}{b} + \frac{1}{c} = \frac{a}{5\sqrt{25+a^2}} + \frac{5}{a\sqrt{25+a^2}}$$

$$\theta_1 = \frac{1}{\sqrt{25+a^2}}\left(\frac{a}{5} + \frac{5}{a}\right) = \frac{(a^2+25)}{5a\sqrt{25+a^2}} = \frac{\sqrt{25+a^2}}{5a}$$

and, The rotation of the yield line parallel to the long edges of the slab,

$$\theta_2 = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$$

$$W_{int} = \left[m \underbrace{\left(\frac{\sqrt{25+a^2}}{5a}\right)}_{\theta} \times \underbrace{\sqrt{25+a^2}}_{l}\right] \times \underset{\substack{\downarrow \\ \text{four yield line}}}{4} + m \times \underbrace{\frac{2}{5}}_{\theta} \times \underbrace{(20-2a)}_{l}$$

and,

$$W_{ex} = \left[\left(\cancel{\tfrac{1}{2}} \times \overset{5}{\cancel{10}} \times a\right) \times w \times \frac{1}{3}\right] \times 2 + \left[\{(20-2a) \times 5\} \times w \times \cancel{\tfrac{1}{2}}\right] \times \cancel{2}$$

$$+ \left[\left(\cancel{\tfrac{1}{2}} \times \cancel{2}a \times 5\right) \times w \times \frac{1}{3}\right] \times 2$$

$$= \frac{10}{3} aw + (20-2a) \times 5w + \frac{10}{3} aw$$

$$= \frac{20}{3} aw + 100w - 10aw$$

$$W_{ex} = 100w - \frac{10}{3} aw$$

Successive trials for different values of a result in following data.

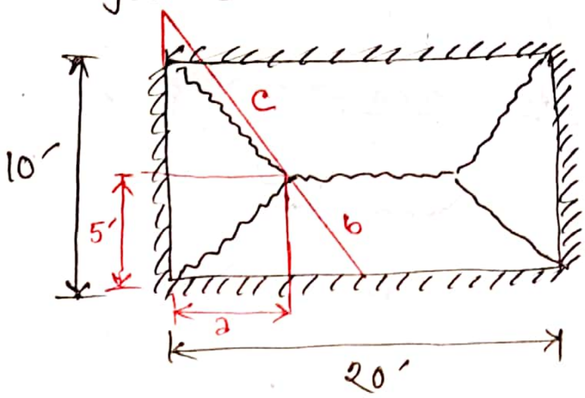
a	W_{int}	W_{ex}	m
6.0	11.36 m	80.0 w	7.05 w
6.5	11.08 m	78.4 w	7.08 w
7.0	10.87 m	76.6 w	7.04 w
7.5	10.69 m	75.0 w	7.02 w

It is evident that the yield line pattern defined by a = 6.5 ft is critical.

Hence, the required resisting moment for the given slab is 7.08 w.

2017

The two way slab shown in figure below is simply supported on all four sides and carries a uniformly load of 80 psf. Determine the required moment resistance for the slab, which is to be isotropically reinforced.



Solution:

same procedure like Example 14.4,

we obtain, $W_{int} = \left[m \times \left(\frac{\sqrt{25+a^2}}{5a} \right) \times \sqrt{25+a^2} \right] \times 4 + m \times \frac{2}{5} \times (20-2a)$

and,

$W_{ex} = 100w - \frac{10}{3} aw$

Given, $w = 80 psf$

$\therefore W_{ex} = 8000 - \frac{800}{3} a$

successive trials:

a	W_{int}	W_{ex}	m
6.0	11.36m	6400	563.4
6.5	11.08m	6266.67	565.6
7.0	10.87m	6133.33	564.2
7.5	10.69m	6000	561.3

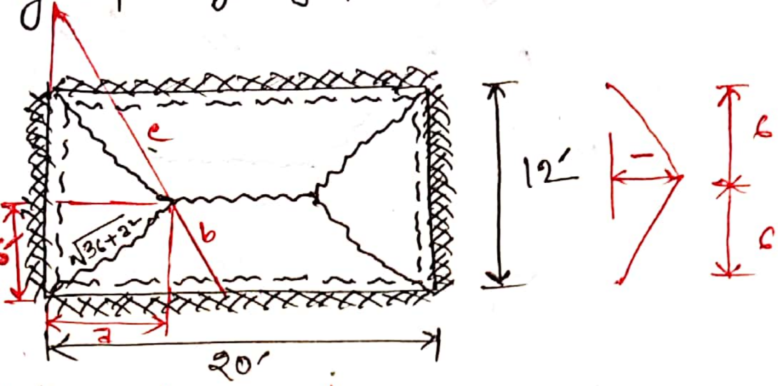
$\therefore$ a = 6.5 is critical. The required resisting moment

m = 565.6 K' (Ans)

2013 Same type – 2015 (14′×21′), 2012 (20′×25′)

A rectangular concrete slab of 12 ft × 20 ft is continuous on all four sides has a yield moment capacity 4000 lb-ft for positive moment and 5000 lb-ft for negative moment. Compute ultimate load carrying capacity of the slab.

Solution:



Given,

+ve moment = 4000 lb-ft
= 4 K-ft

−ve moment = 5000 lb-ft
= 5 K-ft

$$W_{ex} = \left[\left(\tfrac{1}{2}\times 12 \times a\right)\times w \times \tfrac{1}{3}\right]\times 2 + \left[\{(20-2a)\times 6\}\times w \times \tfrac{1}{2}\right]\times 2$$

$$+\left[\left(\tfrac{1}{2}\times 2a \times 6\right)\times w \times \tfrac{1}{3}\right]\times 2$$

$$= 4aW + 120W - 12aW + 4aW$$

$$= 120W - 4aW$$

$$\theta_1 = \frac{1}{b}+\frac{1}{c}$$

$$b = \frac{\sqrt{36+a^2}}{a}\times 6$$

$$c = \frac{\sqrt{36+a^2}}{c}\times a$$

$$\therefore \theta_1 = \frac{\sqrt{36+a^2}}{6a}$$

$$\theta_2 = \frac{1}{6}+\frac{1}{6} = \frac{1}{3}$$

$$W_{int} = \left(4\times\frac{\sqrt{36+a^2}}{6a}\times\sqrt{36+a^2}\right)\times 4 + 4\times\tfrac{1}{3}\times(20-2a)$$
$$+\left(5\times 12\times\tfrac{1}{6}\right)\times 2 + \left(5\times 20\times\tfrac{1}{6}\right)\times 2$$

successive trials for different values:

a	Wint	Wex	W
3.0	88	108 W	0.815
3.5	85.24	106 W	0.804
4.0	83.33	104 W	0.801
4.5	82	102 W	0.804
5	81.066	100 W	0.810

. a=4.0 is critical. The ultimate load carrying capacity of the slab, $w = 0.801$ K/ft

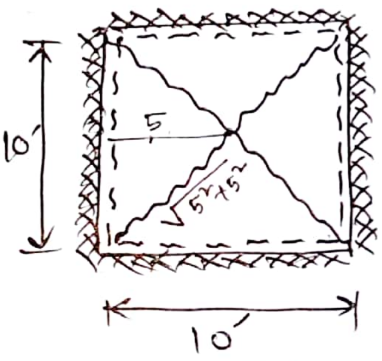
(Ans.)

2011

A 10 ft x 10 ft slab is fixed on four sides and has -ve moment capacity of 3 K-ft and +ve moment capacity of 2 K-ft. Compute the load 'w' corresponding to failure of the slab.

<u>Solution:</u>

four -ve yield line



(+ ve) M = 2K'
(- ve) M = 3K'

$$W_{ex} = \left[\left(\underbrace{\tfrac{1}{2} \times 10 \times 5}_{w}\right) \times w \times \underbrace{\tfrac{1}{3}}_{\Delta}\right] \times 4$$

$$= \frac{100w}{3}$$

$$W_{int} = \left(\underset{m}{3} \times \underset{\lambda}{\overset{2}{\cancel{10}}} \times \underset{\theta}{\frac{1}{\cancel{5}}}\right) \times 4 + \left(\underset{m}{2} \times \underset{\lambda}{\sqrt{5^2+5^2}} \times \underset{\theta}{\frac{\sqrt{2}}{5}}\right) \times 4 \leftarrow \text{four +ve yield line.}$$

$$\left(\theta = \frac{1}{\sqrt{5^2+5^2}} + \frac{1}{\sqrt{5^2+5^2}}\right)$$

$$= 24 + 16$$

$$= 40$$

$\therefore$ $W_{ex} = W_{int}$

$$\Rightarrow \frac{100w}{3} = 40$$

$$\therefore w = \frac{120}{100} = 1.2 \text{ k/ft}$$

(Ans.)

Column

Define column. 16,15,12

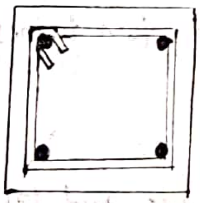
Answer: Column is a vertical structural member supporting axial compressive loads with or without moment.

* The cross sectional dimensions of a column are generally considerable less than its height.
* columns support vertical loads from the floors and roof, and transmit these loads to the foundation.
* Many columns carry tension and their axial forces are often accompanied by bending moment.
* The main reinforcements in Reinforced concrete Columns are longitudinal, parallel to the direction of the load and consists of bars arranged in a square, rectangular or circular pattern.

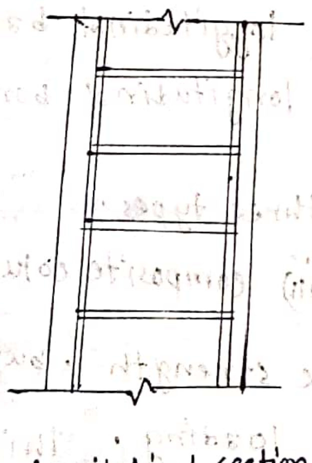
Draw longitudinal and cross-section of column.

Answer:

Tied Column

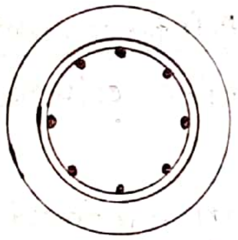


cross-section

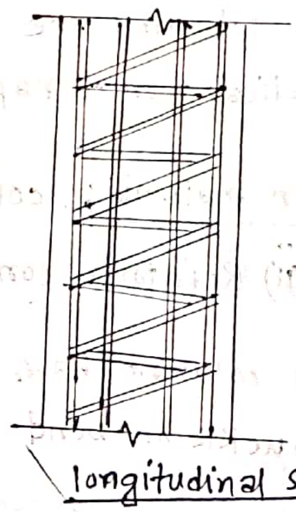


longitudinal section

Spiral Column



cross-section



longitudinal section

Describe different types of column.(with diagram) - 16, 12, 08

Answer: * Based on slenderness ratio, column may be two types:

(i) Short Column. (ii) Slender column.

Short column: A column is considered to be short column, when the ratio of its effective length to its least lateral dimension does not exceed 12.

The strength of the short columns is controlled by the strength of the material and the geometry of the section.

Slender column: If the ratio of the effective length to its least lateral dimension exceeds 12, the column is considered to be slender column. slender columns are limited by their geometry and ill buckle before the concrete or steel reinforcement yields. The strength of the slender column may be significantly reduced by lateral deflections.

* Based on the types of reinforcement, column may be two types:

(i) Tied column (ii) spiral column.

Tied column: A column reinforced with longitudinal bars and lateral ties is called Tied column.

Spiral column: Spiral columns are cylindrical with longitudinal bars and a continuous helical bar wrapped around the longitudinal bars.

* Based on construction material, column may be three types:

(i) steel column (ii) Reinforce concrete column (iii) composite column

Steel column: steel columns have good compressive strength, but have a tendency to buckle or bend under extreme loading. This is due to : length, cross sectional area, method of fixing, shape of the section.

Reinforced concrete column: Reinforced concrete columns have an embedded steel mesh to provide reinforcement. The design of reinforcement can be either spiral or tied.

composite column: composite columns are constructed using various combinations of structural steel and concrete in an attempt to utilize the benificial properties of each material.

* Based on shape, columns may be following types:
(i) Rectangular column (ii) square column (iii) circular column
(iv) L-shape column (v) T-shape column

* Based on types of Loading, columns may be:
(i) Axially loaded column (ii) column with uniaxial eccentric loading
(iii) columns with Biaxial eccentric loading.

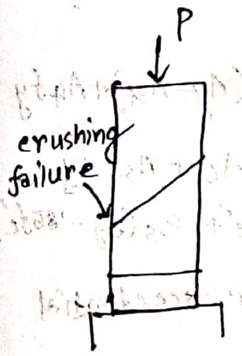


Fig. short column

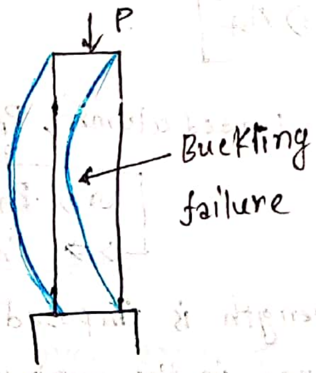


Fig. slender column

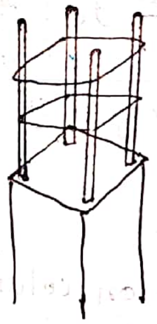


Fig. Tied column

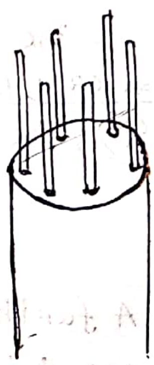


Fig. Spiral column

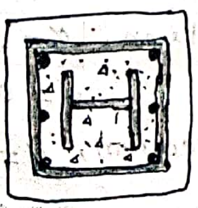


Fig. composite column.

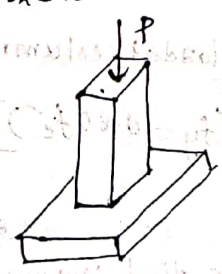


Fig. axially loaded

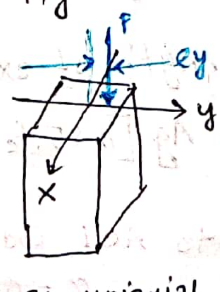


Fig. uniaxial eccentric loading

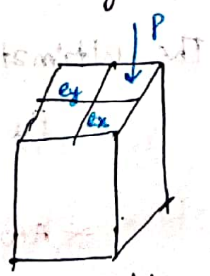


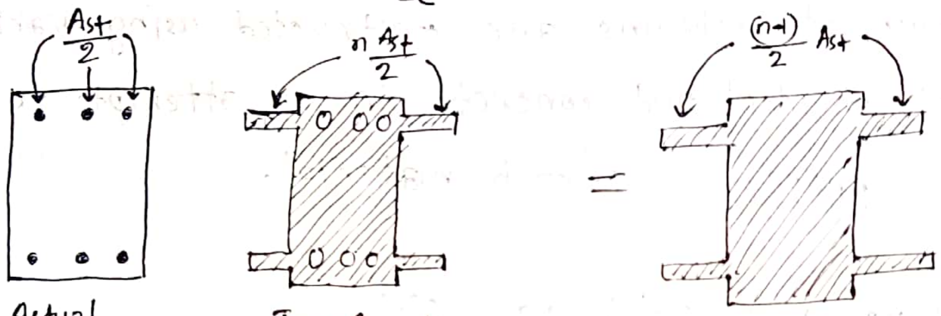
Fig. biaxial eccentric loading.

Elastic Behavior of column subject to axial loads:

At low stresses, up to about $\frac{f_c'}{2}$, the concrete is seen to behave nearly elastically i.e., stress $\propto$ strain.

$$\varepsilon_c = \frac{f_c}{E_c} = \epsilon_s = \frac{f_s}{E_s}$$

$$\Rightarrow f_s = \frac{E_s}{E_c} f_c \quad \therefore f_s = n f_c \quad \text{where, } n = \frac{E_s}{E_c} = \text{modular ratio.}$$



Actual section

Transformed section
$A_t = A_c + n A_{st}$

Transformed section
$A_t = A_g + (n-1) A_{st}$

$$P = P_c + P_s = f_c A_c + f_s A_{st}$$

$$= f_c A_c + n f_c A_{st}$$

$$= f_c (A_c + n A_{st})$$

The axial load, $P = f_c \left[A_g + (n-1) A_{st} \right]$

Load carrying capacity of column:

In (USD), The nominal strength of axially loaded column; $P_n = 0.85 f_c' (A_g - A_{st}) + A_{st} f_y$

or, $P_n = 0.85 f_c' A_c + A_{st} f_y$

$\Rightarrow P_n = A_g [.85 f_c' + e_s (f_y - .85 f_c')]$

A further limitation on column strength is imposed to allow for accedential eccentricities of loading not considered in the analysis.

The ultimate strength of axially loaded column,

$$P_u = \alpha \phi A_g [0.85 f_c' + e_s (f_y - 0.85 f_c')]$$

For tied column, $\boxed{\phi = 0.70}$
$\alpha = 0.80$

and for spiral column, $\boxed{\phi = 0.75}$
$\alpha = 0.85$

* According to ACI Code,
$\phi = 0.65$ for tied column
$\phi = 0.70$ for spiral column

in (WSD)

The allowable load, $P_{all} = \phi'(0.25 f_c' A_g + f_{s(all)} A_s)$

$\Rightarrow P_{all} = \phi' A_g [0.25 f_c' + \rho_s f_{s(all)}]$

For tied column, $\phi' = 0.85$

For spiral column, $\phi' = 1$

Explain the behavior of the reinforced concrete tied and spiral column with load-deformation curves. 15, 10, 16, 12, 08

Answer:

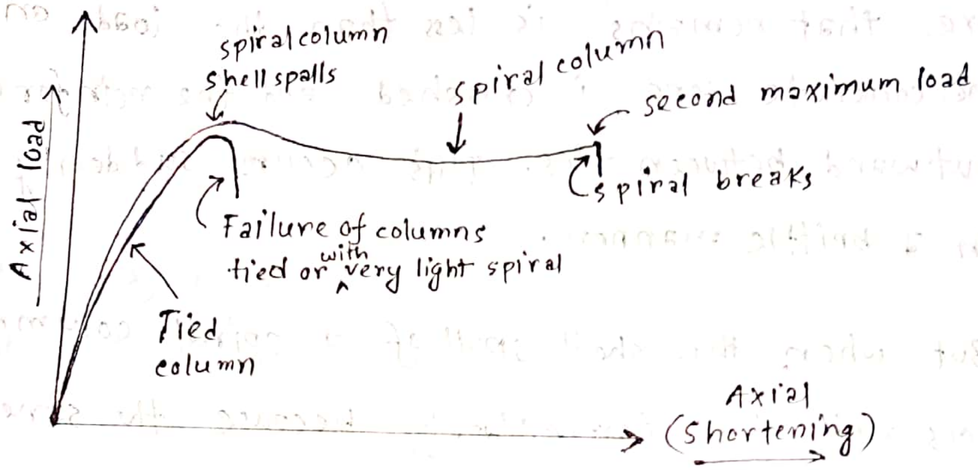


Fig. Behavior of spirally reinforced and tied columns.

Under a compressive load, A spiral column concrete shortens longitudinally and expands laterally, depending on Poisson's ratio. The lateral expansion of the concrete inside the spiral (referred to as core) is restrained by the spiral. This causes hoop tension in the spiral, while the carrying capacity of the confined concrete in the core is greatly increased. Failure occurs only when the spiral steel yields which greatly reduces its confining effect.

In a tied column, the ties provide relatively little lateral restraint to the core. Outward pressure on the sides of the ties due to lateral expansion of the core merely bends them outward, developing an insignificant hoop stress effect.

As, the maximum load is reached, vertical cracks and crushing develop in the concrete shell outside ties or spiral and this concrete spalls off.

When this occurs in a tied column, the capacity of the core that remains is less than the load on the column. The concrete core is crushed and the reinforcement buckles outward between ties. This occurs suddenly, without warning, in a brittle manner.

But when the shell spall of a spiral column, the column does not fail immediately because the strength of the core is enhanced by the triaxial stresses resulting from the effect of the spiral reinforcement. As a result, the column can undergo large deformations, eventually reaching a second maximum load, when the spirals yield and the column finally collapses. Such a failure is much more ductile than that of a tied column and gives warning of the impending failure

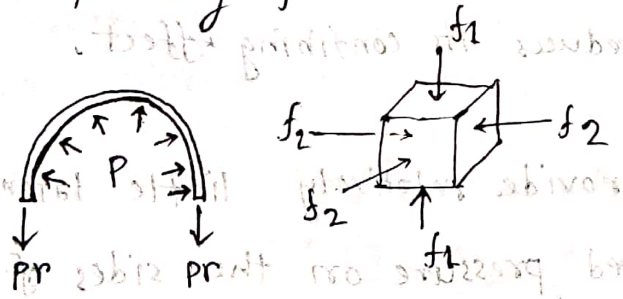


Fig. Traxial stress in core of spiral columns

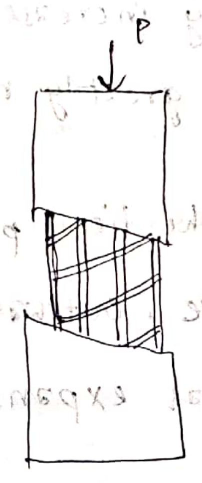


Fig. Failure of spiral column

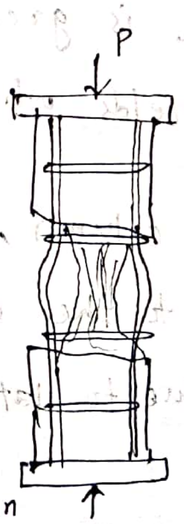


Fig. failure of a tied column

Explain why strength reduction factor for tied column is less than spiral column.

Answer: Spiral column gives warning of impending failure and such a failure is more ductile than tied column. But In a tied column, failure occurs suddenly without warning, in a brittle manner. Due to this, spiral column carry little more load than the tied column.

Because of greater ductility of spiral columns, compression-controlled failures of spiral column are assigned a strength reduction factor, ϕ of 0.70, rather than the value of $\phi = 0.65$ used for tied column.

Describe the functions of lateral reinforcement used in RC column.

Answer: (Or, why ties are provided in RC column) 04, 05, 07, 11, 12, 13, 16

Functions:

1. Hold the longitudinal reinforcement in position.
2. Resist shear force.
3. Increase strength and ductility by providing confinement.
4. prevent outward bulking of longitudinal reinforcement.
5. Maintain proper distance between bars.

What are design considerations for column?

Answer: or, (ACI recommendation for column design):

1. Ratio of A_s and A_g to be in the range of (1–8) %
2. Ratios of (2–4) % are more commonly used.

3. The lower limit is necessary to ensure minimum resistance to bending and reduce effects to creep and shrinkage.

4. Ratios higher than 0.08 are uneconomical and would cause conges of reinforcement.

5. ACI further specifies that a minimum of four longitudinal bars (size at least # 5) should be used for rectangular column and six longitudinal bars for spiral column.

6. Every column should be designed for a minimum eccentricity
For tied column, $e_{min} = 0.10t$ where, t = largest dimension of column.
For spiral column, $e_{min} = 0.05t$ where, t = spiral diameter.

Write specifications for ties and spiral in column:

Answer:

Specifications for Ties: 14, 12, 11, 09, 05

Arrangment: 1. The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle $\leq 135°$.

2. No bar shall be further than 6 in. clear on either side from such a laterally supported bar.

spacing: 1. $s = 16 d_b$ (d_b = dia. of longitudinal bar)

2. $s = 48 d_t$ (d_t = dia. of tie bar)

3. s = least dimension of column

Minimum spacing should be used.

Tie bar size: At least No. 3 ties for longitudinal bar No. 10 or less,
No. 4 ties for No. 11 bar or larger and bundled bars.

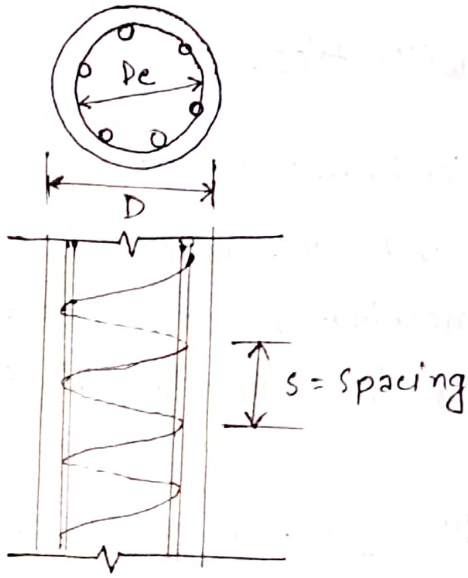
Specifications for spiral:

1. Spirals may not have diameter less than $\frac{3}{8}$ in. (#3 bar)
2. Spacing : (i) $s_{max} = 3$ in.

   (ii) $s_{max} = \frac{1}{6} \times D_c$ (D_c = dia. of core column)

   (iii) $s_{min} = 1$ in.

[Core : core is that part of the section located with in the outer circumference of the spiral]



Spiral Reinforce ratio:

$$\rho_{sp} = \frac{\text{Volume of spiral}}{\text{Volume of core}} = \frac{A_{sp} \times l_{sp}}{\frac{\pi}{4} D_c^2 \times s}$$

$$= \frac{A_{sp} \times \pi D_c}{\frac{\pi}{4} D_c^2 \times s}$$

$$\therefore \rho_{sp} = \frac{4 A_{sp}}{s D_c}$$

According to ACI code,

$$\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_c'}{f_y}$$

Why reinforcement is required for column?

Reasons for providing reinforcement in column :

(i) Minimum reinforcement should be provided in a column to carry the moment, which is not accounted in design or analysis during the construction.

(ii) To reduce the diameter, cross section of column the steel should be provided due to fault of construction.

(iii) To resist creep and shrinkage of concrete in the construction.

Why spiral column can support more loads than tied column?

concrete column reinforced with spiral reinforcement can withstand more loads than tied column. This phenomenon happens because;

when load eccentricities are small, spirally reinforced column shows greater toughness, greater ductility than columns reinforced with ties.

Slenderness ratio:

Slenderness ratio is a geometrical property of a compression member which is related to the ratio of its effective length to its least lateral dimension.

Distinguish between short column and long column:

Short Column	Long Column
1. Length / least dimension < 12	1. length / least dimension > 12
2. Fails by crushing.	2. Fails by buckling
3. Slenderness ratio < 45	3. Slenderness ratio > 45
4. Subjected to compressive stress.	4. Subjected to bending stress.
5. Radius of gyration is more.	5. Radius of gyration is less.
6. More load capacity.	6. Less load capacity.

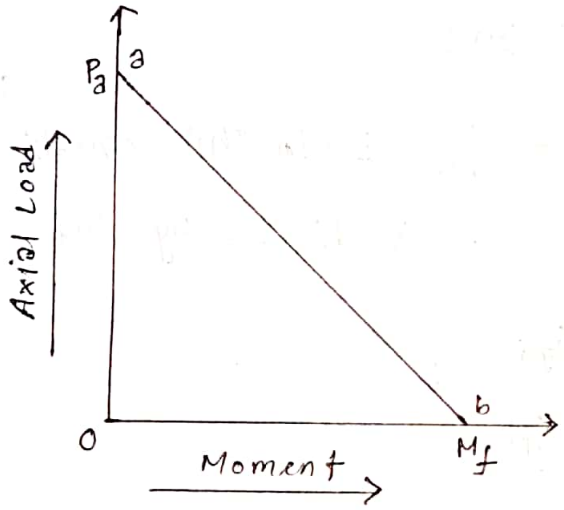
What is Interaction diagram: 14, 15

An interaction diagram is a diagram which is used for design of column. Every point on this diagram represents a combination of design loads and moments. Any point beyond this line represent the combination of load and moment that the column is not capable to carry.

(Interaction Diagram for compression plus bending)

Describe different points of interaction diagram with equation 14

A plot of simple interaction equation is shown in figure:



In an elastic member under combression and bending, but not not subject to tension cracking, the maximum fibre stress is found from

$$\frac{P}{A} + \frac{Mc}{I} = f_{max}$$

$$\Rightarrow \frac{P}{A} + \frac{M}{z} = f_{max}$$

$$\Rightarrow \frac{P}{A f_{max}} + \frac{M}{z f_{max}} = 1$$

$$\Rightarrow \frac{P}{P_a} + \frac{M}{M_f} = 1 \qquad \text{where, } P_a = A f_{max}$$
$$M_f = z f_{max}$$

The axial load P_a acts alone without simultaneous bending and the flexural moment M_f acts alone without simultaneous compression.

When Moment $M = 0$ then, $P = P_a$ at point a, and

when Axial load $P = 0$ then, $M = M_f$ at point b.

Describe different points of interaction diagram with equation for WSD design method: 15

The working stress design for members in compression with bending is described below:

We know, $\frac{P}{P_a} + \frac{M}{M_f} = 1$. In this equation, the first term is divided by A_g and the second term is divided by S_{ut}

$$\Rightarrow \frac{P/A_g}{P_a/A_g} + \frac{M/S_{ut}}{M_f/S_{ut}} = 1$$

$$\Rightarrow \frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

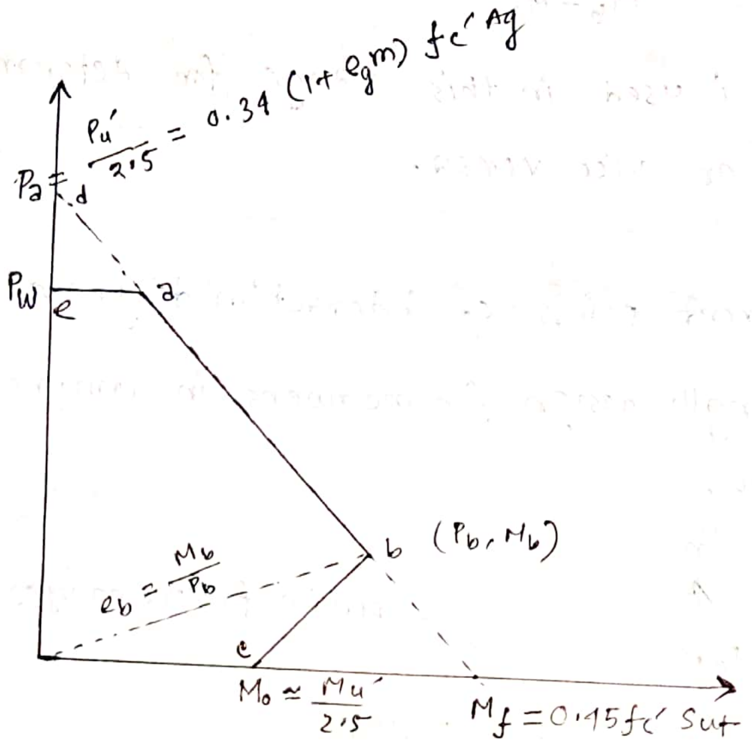
where,

$$F_a = 0.34\,(1 + \rho_g m) f_c' \quad \text{and} \quad m = \frac{f_y}{0.85 f_c'}$$

$$F_b = 0.45 f_c'$$

$$S_{ut} = \frac{I_{ut}}{c} = \frac{1}{c} \times \left[\frac{bt^3}{12} + 2(2n-1) A_s \left(\frac{t}{2} - d'\right) \right]$$

For zero or small eccentricities (interval ea), the allowable load is, $P = \phi' A_g (0.25 f_c' + f_s \rho_g)$



For moderate eccentricities, when compression governs in the interval ab, the following equation applies:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

Point b determines the boundary between members governed by compression and those governed by tension. It is found by calculating e_b from equation,

$$e_b = (0.67 \rho_g m + 0.17) d \quad \text{for tied columns}$$

$$= (0.43 \rho_g m D_c + 0.14 t) \quad \text{for spiral columns.}$$

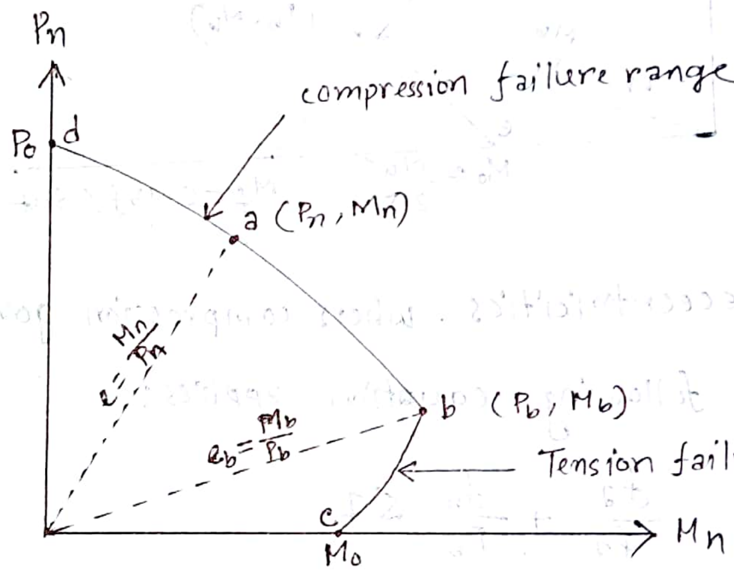
For large eccentricities, when tension governs, a linear variation is assumed between the moment M_b at the balance point b and M_o for simple flexure. (shown by straight line bc)

$$\frac{P}{P_b} = \frac{M - M_o}{M_b - M_o} \qquad \text{where } M_o = 0.4\, A_s f_y (d - d')$$

This equation is used in this range for determining P if M is given or vice versa.

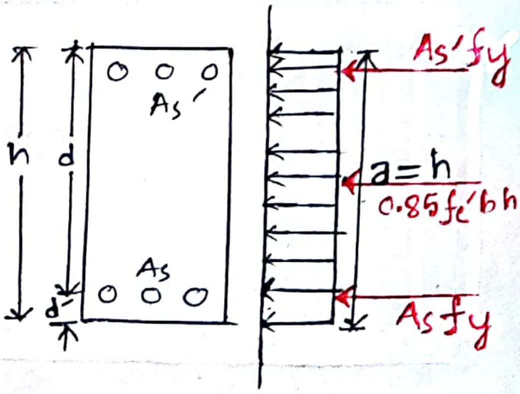
Describe different points of interaction diagram for USD method

The ultimate strength design for members in compression with bending is described below:



For concentric compression ($M_n = 0$) the curve starts at points 'd' with strength P_o of a concentrically loaded member. It is computed as follows:

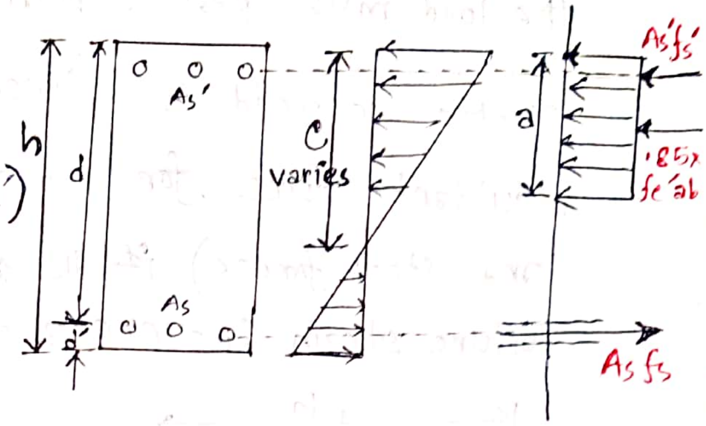
$$P_o = 0.85 f_c' bh + A_s' f_y + A_s f_y$$



The portion 'db' pertains to that range of small eccentricities in which failure is initiated by crushing of the concrete. The corresponding computations are:

$$P_n = 0.85 f_c' ab + A_s' f_s' - A_s f_s$$

and, $$M_n = 0.85 f_c' ab\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) - A_s f_s \left(d - \frac{h}{2}\right)$$



where, $f_s' = E_s \varepsilon_s' = E_s \varepsilon_u \times \frac{c-d'}{c}$

and $f_s = E_s \varepsilon_s = E_s \varepsilon_u \times \frac{d-c}{c}$

Point 'b' represents represents the balanced condition i.e under the simultaneous action of the load P_b and the corresponding moment M_b, the concrete will reach its limiting strain (0.003); simultaneously with the tension steel reaching its yield stress f_y.

For balanced condition, $c_b = \frac{\varepsilon_u}{\varepsilon_y + \varepsilon_u}$ where, $\varepsilon_u = 0.003$

and $\varepsilon_y = \frac{f_y}{E_s}$

and, $f_s = f_y$

The portion 'be' represents that range of large eccentricities in which failure is initiated by yielding of tension steel.

Finally, the end point 'e' refers to that moment capacity M_o in simple bending i.e., when $P_n = 0$.

Define Plastic Centroid:

For an unsymmetrical Reinforced column to be loaded concentrically, the load must pass a point known as plastic centrod. Plastic centroid is defined as the point of application of the resultant force for the column cross section (including concrete and steel forces) if the column is compressed uniformly to the failure strain $\varepsilon_u = 0.003$ over its entire cross section.

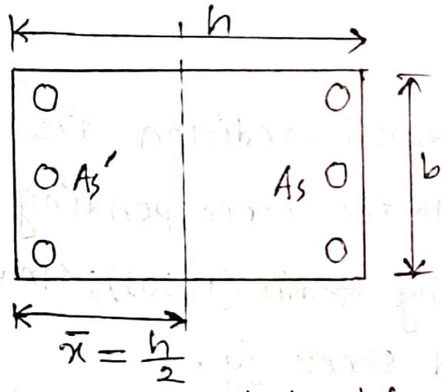


Fig. symmetrical reinforced cross section.

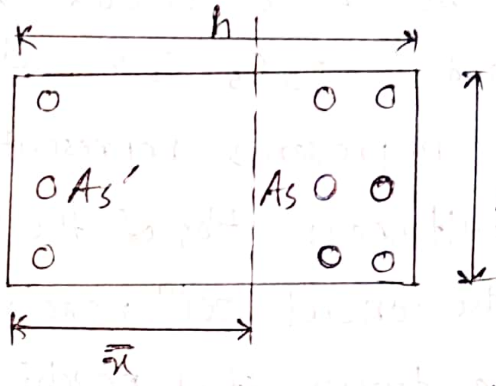


Fig. unsymmetrically reinforced cross section.

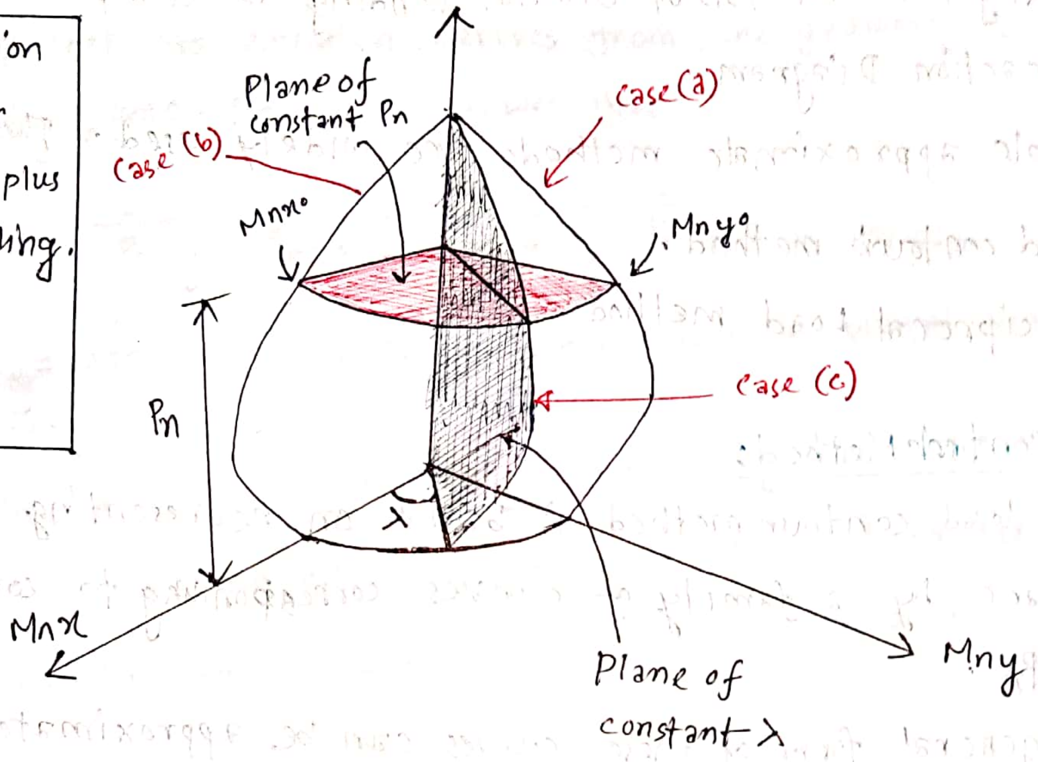
From left face.

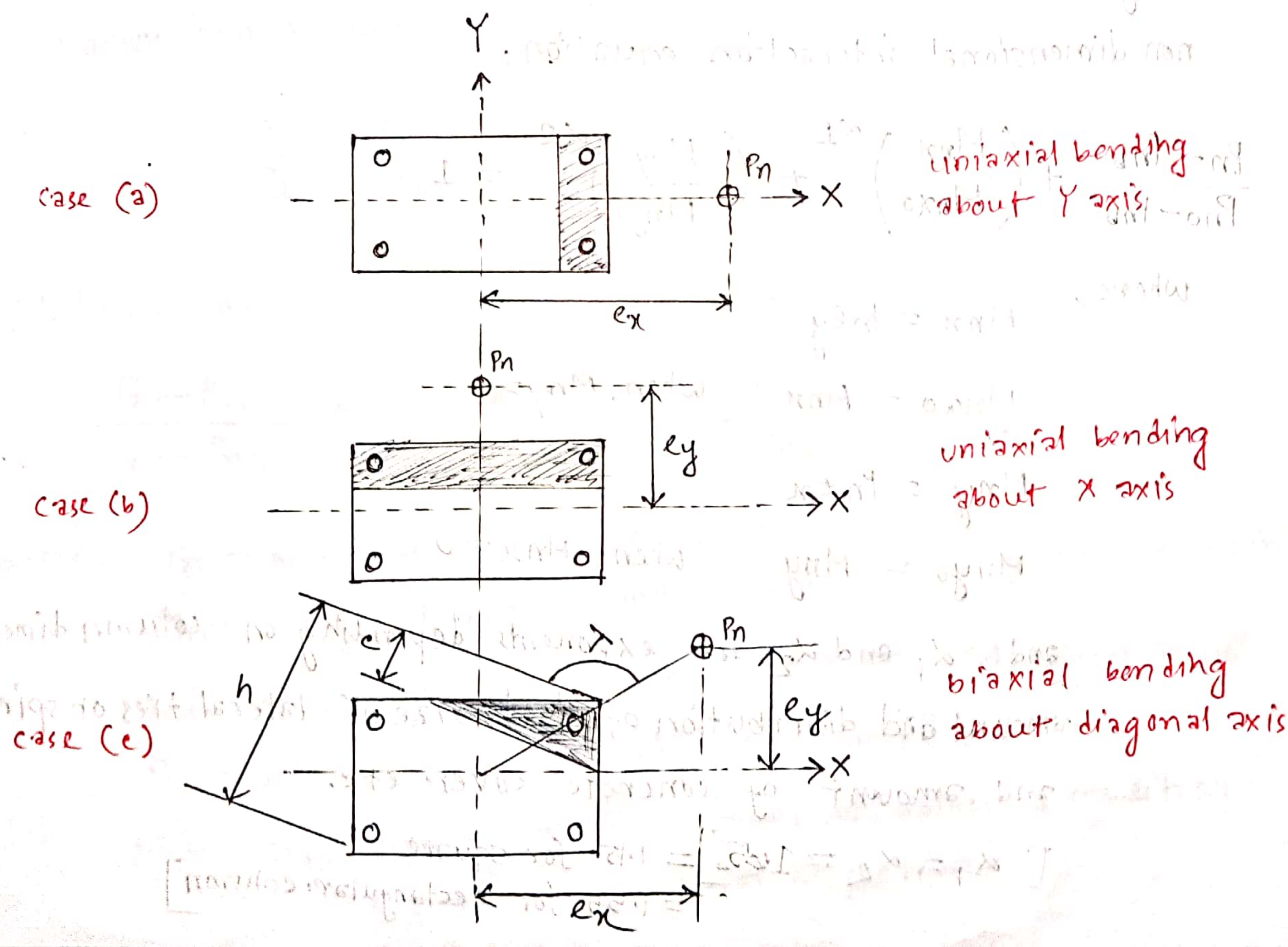
$$\bar{x} = \frac{0.85 f_c' bh \times \frac{h}{2} + A_s f_s (h-d') + A_s' f_s' d'}{0.85 f_c' bh + A_s f_s + A_s' f_s'}$$

if, $A_s = A_s'$ then, $\bar{x} = \frac{h}{2}$.

Draw Interaction diagram for compression plus biaxial bending:

Fig. Interaction surface for compression plus biaxial bending.





Method of Analysis of Biaxial Bending:

Manually, The analysis of Biaxial Bending is complicated with Interaction Diagram.

Simple approximate methods are widely used. These are:

1. Load contour method.
2. Reciprocal Load method.

Load Contour Method:

The load contour method is based on representing the failure surface by a family of curves corresponding to constant values of P_n.

The general form of these curves can be approximated by a non dimensional interaction equation:

$$\left(\frac{M_{nx}}{M_{nxo}}\right)^{\alpha_1} + \left(\frac{M_{ny}}{M_{nyo}}\right)^{\alpha_2} = 1$$

where,

$M_{nx} = P_n e_y$

$M_{nxo} = M_{nx}$ when, $M_{ny} = 0$

$M_{ny} = P_n e_x$

$M_{nyo} = M_{ny}$ when, $M_{nx} = 0$

and, α_1 and α_2 are exponents depending on column dimensions, amount and distribution of steel, size of lateral ties or spiral and amount of concrete cover etc.

[$\alpha_1 = \alpha_2 = \alpha = 1.15$ for square
$= 1.55$ for rectangular column]

Reciprocal Load Method:

A simple, approximate design method was developed by Bresler. Bresler's reciprocal load equation derives from the geometry of the approximating plane. It can be shown that

$$\frac{1}{P_{nxy}} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_o}$$

where, P_{nxy} = approximate value of nominal load in biaxial bending with eccentricities

P_{nyo} = nominal load when only eccentricity e_x is present ($e_y = 0$)

P_{nxo} = nominal load when only eccentricity e_y is present ($e_x = 0$)

P_o = nominal load for concentrically loaded column.

Another form of Reciprocal Load Method:

$$\frac{P}{P_a} + \frac{M_x}{M_{fx}} + \frac{M_y}{M_{fy}} \leq 1$$

Modified Load contour Method: The equation is as follows:

$$\frac{P_n - P_{nb}}{P_{no} - P_{nb}} + \left(\frac{M_{nx}}{M_{nbx}}\right)^{1.5} + \left(\frac{M_{ny}}{M_{nby}}\right)^{1.5} \leq 1$$

where, P_n = Nominal axial compression

M_{nx}, M_{ny} = Nominal bending about x and y axis respectively

P_{no} = Maximum axial compression = $0.85 f_c' (A_g - A_{st}) + A_{st} f_y$

P_{nb} = Nominal axial compression at balanced condition

M_{nbx}, M_{nby} = Nominal bending moment at balanced condition about x and y axis respectively.

Bar Splicing in Columns (13th edition - Nilson - Page 281)

The main vertical reinforcement in columns is usually spliced above each floor. This permits the column steel area to be reduced progressively at the higher levels in a building, where loads are smaller.

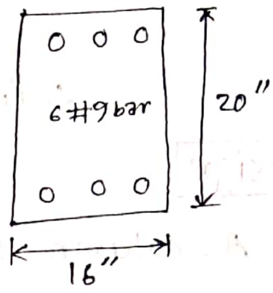
column steel may be spliced by lapping, by butt welding, by mechanical connections or by directly end bearing.

The most common method of splicing column steel is the simple lapped bar splice.

ANALYSIS | Axially loaded Column

A column has a cross section 16×20 in. and is reinforced by six No. 9 bars. Determine the axial load that will stress the concrete to 1200 psi.

<u>Solution:</u> $n = \frac{E_s}{E_c} = \frac{29\times10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$



$A_g = (16 \times 20)\ in^2$

$\Rightarrow A_g = 320\ in^2$

$A_{st} = (6\times1) = 6\ in^2$

The total load on the column, $P = f_c\ [A_g + (n-1)A_{st}]$

$= 1200 \times [320 + (8-1)\times 6]$

$= 434400\ lb$

of this total load,

The concrete carry, $P_c = f_c A_c = f_c\ (A_g - A_{st})$

$= 1200 \times (320-6)$

$= 376800\ lb$

and, The steel carry, $P_s = f_s A_s$

$= n f_c A_s = (8\times1200\times6) = 57600\ lb$

(Ans.)

A column section 10"×10" is reinforced with four No. 5 bars. calculate: (i) working load and (ii) ultimate load to be carried by the column. $f_c' = 3000$ psi and $f_y = 60000$ psi.

<u>Solution:</u> (i) working load to be carried by column,

column section 10"×10", Hence it is tied column ($\phi' = 0.85$)

$P_{all} = \phi' A_g\ (0.25 f_c' + \rho_s f_{s(all)})$

$= 0.85 \times 10\times10 \times [.25\times3000 + \frac{4\times.31}{10\times10}\times0.4\times60000]$

$\therefore P_{all} = 89046\ lb$

ultimate load to be carried by column,

$$P_u = \alpha \phi A_g [.85 f_c' + \rho_s (f_y - 0.85 f_c')]$$

$$= 0.80 \times 0.70 \times (10 \times 10) \times [.85 \times 3000 + \frac{4 \times .31}{10 \times 10} \times (60000 - .85 \times 3000)]$$

$$\therefore P_u = 182693.28 \text{ lb}$$

(Ans.)

Design

A column section 20" × 30" is reinforced with $\rho_s = 1\%$. Design the tie.

<u>Solution:</u> $\rho_s = \frac{A_s}{A_g} \Rightarrow \frac{1}{100} = \frac{A_s}{20 \times 30} \Rightarrow A_s = 6 \text{ in}^2$

provide 6 #9 bars as main reinforcement.

And, using #3 bar as tie.

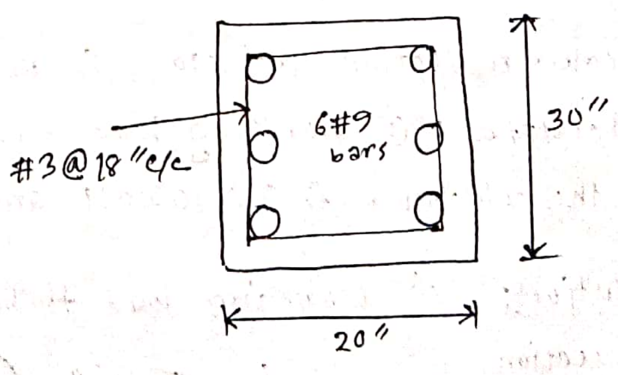
Now, <u>spacing of tie bar:</u>

$S_{max} = 16 d_b = (16 \times \frac{9}{8}) = 18''$ → #10 bar dia.

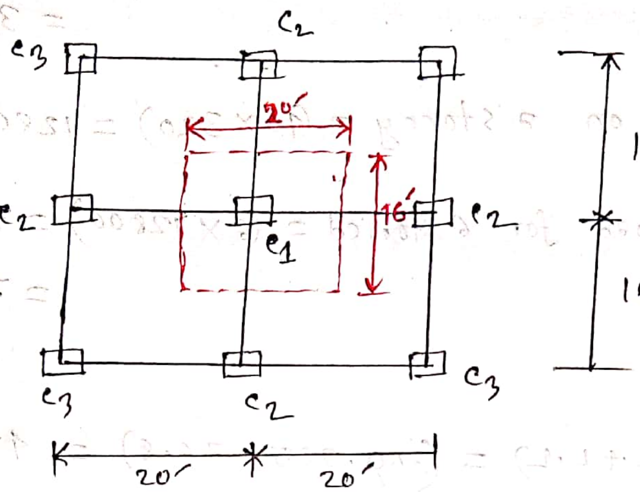
$S_{max} = 48 d_t = (48 \times \frac{3}{8}) = 18''$ → tie dia.

$S_{max} = \underline{20''}$ (least dimension of column)

$\therefore$ using #3 bar @ 18" c/c



Plan of six storied building is shown in figure below:



Design column 1. Assume thickness of slab = 6", beam size = 12" × 18", partition wall = 5", floor finish = 30 psf, Live load = 40 psf, $f_c' = 3$ ksi, $f_y = 60$ ksi.

Loaded area $= (20' \times 16') = 320$ sq. ft

Solution: Dead load calculation: weight of slab $= \frac{t}{12} \times 150 = \left(\frac{6}{12} \times 150\right)$ psf

$= 75$ psf

$\therefore$ Load on column from slab $= (75 \times 320) = 24000$ lb.

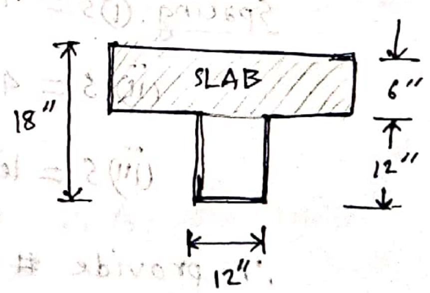
weight of floor finish $= (30 \times 320) = 9600$ lb.

Height of one storey

weight of partition wall $= \left\{\left(\frac{5}{12} \times 120\right) \times \left[10 \times (20+16)\right]\right\} = 18000$ lb

weight of beam $= \left(\frac{12}{12} \times \frac{(18-6)}{12} \times 150\right) \times (20+16) = 5400$ lb

Assume; column section = 18" × 18"



$\therefore$ weight of column $= \frac{18}{12} \times \frac{18}{12} \times (10-1) \times 150$

$= 3037.5$ lb

Total dead load for one storey building $= (24000 + 9600 + 18000 + 5400 + 3037.5)$ lb

$= 60037.5$ lb

$\therefore$ Total Dead load for six storied building $= (6 \times 60037.5) = 360225$ lb

$= 360.225$ K

Live load calculation:

Live load on a storey $= (40 \times 320) = 12800$ lb

$\therefore$ Total live load for 6 storied $= (6 \times 12800) = 76800$ lb

$= 76.8$ K

WSD Method:

Total Load $= (D.L + L.L) = (360.225 + 76.8) = 437.025$ K

1. consider tied column, $\phi' = 0.85$

$$P_{all} = \phi' A_g [0.25 f_c' + \rho_s f_s]$$

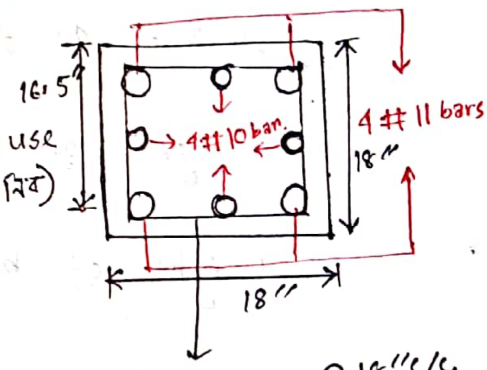
$$\Rightarrow 437.025 = 0.85 \times (18 \times 18) \times [0.25 \times 3 + \rho_s \times (0.4 \times 60)]$$

$$\Rightarrow 437.025 = 275.4 (.75 + 24 \rho_s)$$

$$\Rightarrow \rho_s = 0.035$$

$\rho_s = \frac{A_s}{A_g} \Rightarrow A_s = \rho_s \times A_g = 0.035 \times (18 \times 18) = 11.34$ in^2

Provide 4 # 11 bars in corner and 4 # 10 bars in middle.



#10 bar (দুই ধরনের bar use করলে ছোটটা দিব)

Tie design:

using # 4 bar

Spacing: (i) $s = 16 d_b = (16 \times \frac{10}{8}) = 20''$

(ii) $s = 48 d_t = (48 \times \frac{4}{8}) = 24''$

(iii) s = least dimension of column $= 18''$

$\therefore$ provide # 4 ties @ 18'' c/c.

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2. consider spiral column, $\phi' = 1$

Let, $\rho_s = 0.02 \rightarrow$ [ACI recommendation: $\rho_s = (0.01 \sim 0.08)$]

$$P_{all} = \phi' A_g [0.25 f_c' + \rho_s f_s]$$

$$\Rightarrow 437.025 = 1 \times A_g \times [0.25 \times 3 + 0.02 \times 24]$$

$$\Rightarrow A_g = 355.3 \text{ in}^2$$

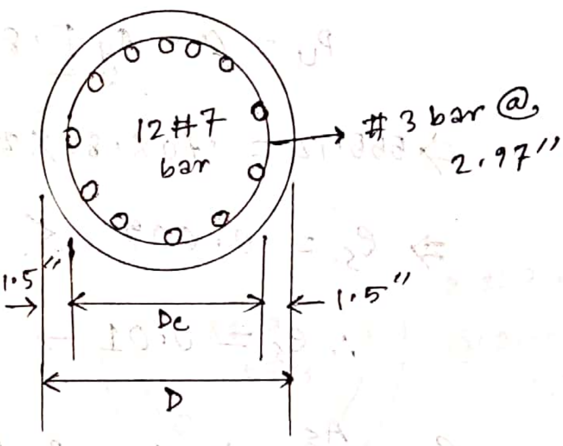
$$\Rightarrow \frac{\pi}{4} D^2 = 355.3 \text{ in}^2$$

$$\therefore D = 21.27 \text{ in} \simeq 21.5 \text{ in.}$$

Hence, $A_g = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (21.5)^2 = 363 \text{ in}^2$

$\rho_s = \frac{A_s}{A_g} \Rightarrow A_s = \rho_s A_g = (0.02 \times 363) = 7.26 \text{ in}^2$

$\therefore$ Provide 12 #7 bars



Spiral Design:

$D_c = D - (2 \times \underbrace{1.5}_{\text{clear cover}}) = (21.5 - 3) = 18.5 \text{ in.}$

Minimum spiral ratio,

$$\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_c'}{f_y}$$

$$\Rightarrow \rho_{sp} = 0.45 \times \left(\frac{D^2}{D_c^2} - 1\right) \times \frac{f_c'}{f_y} = 0.45 \times \left(\frac{21.5^2}{18.5^2} - 1\right) \times \frac{3}{60} = 0.008$$

$$\therefore \rho_{sp} = 0.008$$

Now, $\rho_{sp} = \frac{4 A_{sp}}{S \times D_c}$, Let provide #3 bar $\therefore A_{sp} = 0.11 \text{ in}^2$

$$\therefore \text{Spacing, } S = \frac{4 A_{sp}}{\rho_{sp} \times D_c} = \frac{4 \times .11}{0.008 \times 18.5} = 2.97 \text{ in.}$$

Again, spacing:

(i) $S_{max} = 3\ in.$

(ii) $S_{max} = \frac{1}{6} \times D_c = \frac{1}{6} \times 18.5 = 3.08\ in$

(iii) $S_{min} = 1\ in.$

$\therefore$ provide #3 bar @ 2.97" c/c

USD Method:

Ultimate load, $P_u = 1.2\ D.L + 1.6 \times L.L$

$$= (1.2 \times 360.2) + (1.6 \times 76.8)$$

$$= 555.12\ K$$

1. Consider tied column,

$$\phi = 0.70 \text{ and } \alpha = 0.80$$

$$P_u = \phi \alpha A_g [.85 f_c' + \rho_s (f_y - .85 f_c')]$$

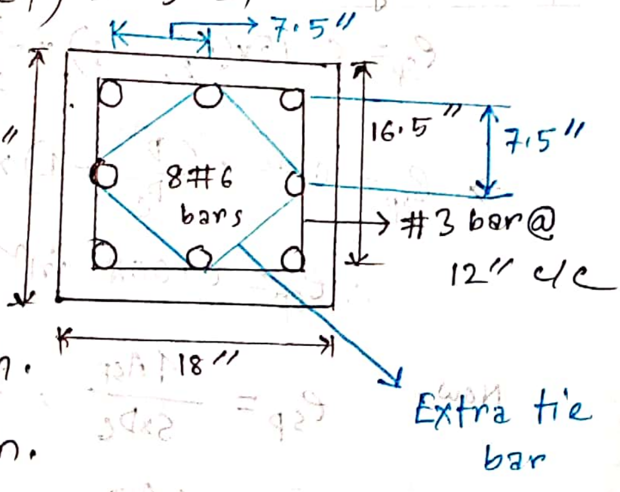
$$\Rightarrow 555.12 = .70 \times .80 \times 324 \times [.85 \times 3 + \rho_s (60 - .85 \times 3)]$$

$$\Rightarrow \rho_s = 0.009 < 0.01$$

$\therefore \rho_s = 0.01 \rightarrow$ [ACI recommendation, $\rho_s = (0.01 - 0.08)$]

$\rho_s = \frac{A_s}{A_g} \Rightarrow A_s = \rho_s A_g = (0.01 \times 324) = 3.24\ in^2$

provide 8 #6 bars.



Design of Tie: using #3 bar

Spacing: (i) $s = 16 d_b = (16 \times \frac{6}{8}) = 12\ in.$

(ii) $s = 48 d_t = (48 \times \frac{3}{8}) = 18\ in.$

(iii) s = least dimension of column = 18 in.

$\therefore$ provide #3 ties @ 12" c/c

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spacing between two main bar $= 7.5'' > 6''$

Hence, Extra tie bar should be provided.

2. consider spiral columns,

$\phi = .75$ and $\alpha = 0.85$

Let, $\rho_s = 0.02$

$$P_u = \phi \alpha A_g [.85 f_c' + \rho_s (f_y - .85 f_c')]$$

$$\Rightarrow 555.12 = .75 \times .85 \times A_g \times [.85 \times 3 + .02 \times (60 - .85 \times 3)]$$

$$\Rightarrow A_g = 235.41 \text{ in}^2$$

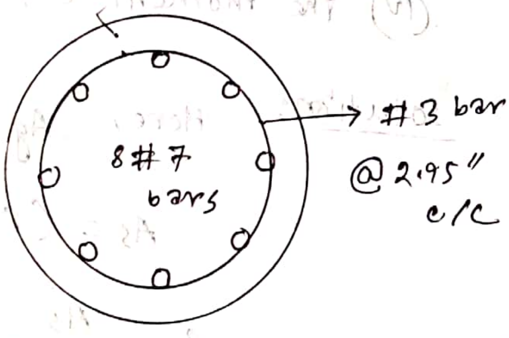
$$\Rightarrow \frac{\pi}{4} D^2 = 235.41 \text{ in}^2$$

$$\therefore D = 17.31 \text{ in} \simeq 17.5 \text{ in.}$$

$$\therefore A_g = \frac{\pi}{4} \times (17.5)^2 = 240.53 \text{ in}^2$$

$$\therefore A_s = A_g \times \rho_s = (.02 \times 240.53) = 4.81 \text{ in}^2$$

$\therefore$ provide 8 # 7 bars



Design of spiral:

$$D_c = D - (2 \times 1.5) = (17.5 - 3) = 14.5 \text{ in.}$$

Minimum spiral ratio,

$$\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_c'}{f_y}$$

$$\Rightarrow \rho_{sp} = 0.45 \times \left(\frac{D^2}{D_c^2} - 1\right) \times \frac{f_c'}{f_y} = 0.45 \times \left(\frac{17.5^2}{14.5^2} - 1\right) \times \frac{3}{60}$$

$$\therefore \rho_{sp} = 0.0103$$

Now, $\rho_{sp} = \frac{4 A_{sp}}{s \times D_c}$, Let provide # 3 bar, $\therefore A_{sp} = .11 \text{ in}^2$

$$\therefore \text{spacing, } s = \frac{4 \times .11}{.0103 \times 14.5} = 2.95''$$

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Again,

Spacing: (i) $S_{max} = 3$ in.

(ii) $S_{max} = \frac{1}{6} \times D_c = (\frac{1}{6} \times 14.5) = 2.42$ in.

(iii) $S_{min} = 1$ in.

$\therefore$ provide #3 bar @ 2.95" c/c

(Analysis)

WSD Method

$\boxed{\text{Axially loaded column + bending in one axis}}$

A column section 16 in. x 25 in. is reinforced with 8 #11 bars. Calculate,

$f_c' = 4000$ psi

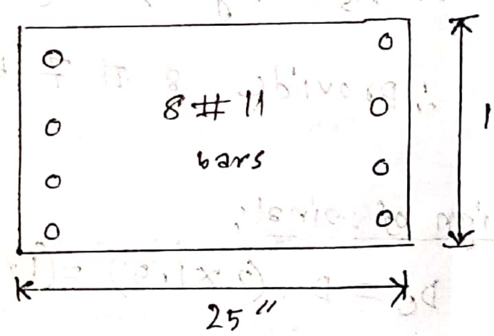
$f_y = 50000$ psi

(i) the allowable load.

(ii) the moment capacity when $P = 360$ kips.

(iii) the balanced load and balanced moment.

(iv) the moment capacity when $p = 100$ kips.



Solution: Here, $A_g = (16 \times 25) = 400$ in²

$A_s = (8 \times 1.56) = 12.48$ in²

$$\rho_g = \frac{A_s}{A_g} = \frac{12.48}{400} = 0.0312$$

$$m = \frac{f_y}{.85 f_c'} = \frac{50000}{.85 \times 4000} = 14.7$$

$$F_a = 0.34(1 + \rho_g m) f_c' = 0.34 \times (1 + 0.0312 \times 14.7) \times 4 = 1.98 \text{ ksi}$$

$$F_b = 0.45 f_c' = (0.45 \times 4) = 1.8 \text{ ksi}$$

$$I_{ut} = \frac{16 \times 25^3}{12} + 2 \times (2 \times 8 - 1) \times \underset{4 \times 1.56}{6.24} \times (12.5 - \underset{d'}{2.5})^2 = 39553.33 \text{ in}^4$$

$$\hookrightarrow \frac{bt^3}{12} + 2(2n-1) \times A_s \times (\frac{t}{2} - d')^2$$

$$S_{ut} = \frac{I_{ut}}{c} = \frac{39553.33}{12.5} = 3164.27 \text{ in}^3$$

$\downarrow \frac{t}{2}$

(i) <u>Allowable load</u>: $P_{all} = 0.85\, A_g\, (.25 f_c' + \rho_g f_s)$

$$= 0.85 \times 400 \times (.25 \times 4 + .0312 \times .4 \times 50)$$

$$\therefore P_{all} = 552.16 \text{ K}$$

(ii) <u>When, P=360K, M=?</u>

P=360K, Assume compression governs.

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P}{A_g} = \frac{360}{400} = 0.9 \text{ Ksi}$

$f_b = \frac{M}{S_{ut}} = \frac{M}{3164.27}$

$F_a = 1.98$ Ksi

$F_b = 1.8$ Ksi

$$\Rightarrow \frac{0.9}{1.98} + \frac{M/3164.27}{1.8} = 1$$

$$\Rightarrow M = 3106.74 \text{ K-in.}$$

$$\therefore e = \frac{M}{P} = \frac{3106.74}{360} = 8.63 \text{ in.}$$

and, $e_b = (.67\, \rho_g m + 0.17)\, d = (0.67 \times .0312 \times 14.7 + 0.17) \times 22.5$

$$\therefore e_b = 10.74 \text{ in.} > e$$

Hence, compression will govern.

(iii) <u>Balance load and Balanced Moment:</u>

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P_b}{A_g} = \frac{P_b}{400}$

$f_b = \frac{M_b}{S_{ut}} = \frac{P_b e_b}{3164.27}$

$e_b = 10.74$

$F_a = 1.98$ Ksi

$F_b = 1.8$ Ksi

$$\Rightarrow \frac{P_b/400}{1.98} + \frac{\frac{P_b \times 10.74}{3164.27}}{1.8} = 1$$

$$\Rightarrow P_b = 317.64 \text{ Kips.}$$

$\therefore$ balanced load $= 317.64\,K$

and balanced moment, $M_b = P_b e_b$

$$= (317.64 \times 10.74)$$

$$\therefore M_b = 3411.45 \text{ K-in}$$

(IV) When $P = 100\,K$, $M = ?$

$P = 100\,K$, Assume compression governs.

Now,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P}{A_g} = \frac{100}{400} = 0.25$

$f_b = \frac{M}{S_{ut}} = \frac{M}{3164.27}$

$F_a = 1.98$ Ksi

$F_b = 1.8$ Ksi

$$\Rightarrow \frac{0.25}{1.98} + \frac{M/3164.27}{1.8} = 1$$

$$\Rightarrow M = 4976.53 \text{ K-in}$$

$$\therefore e = \frac{M}{P} = \frac{4976.53}{100} = 49.77 \text{ in.} > e_b$$

Hence, tension governs.

For tension region,

$$\frac{P}{P_b} = \frac{M - M_o}{M_b - M_o}$$

Here, $M_o = 0.4\, A_s f_y\, (d - d')$

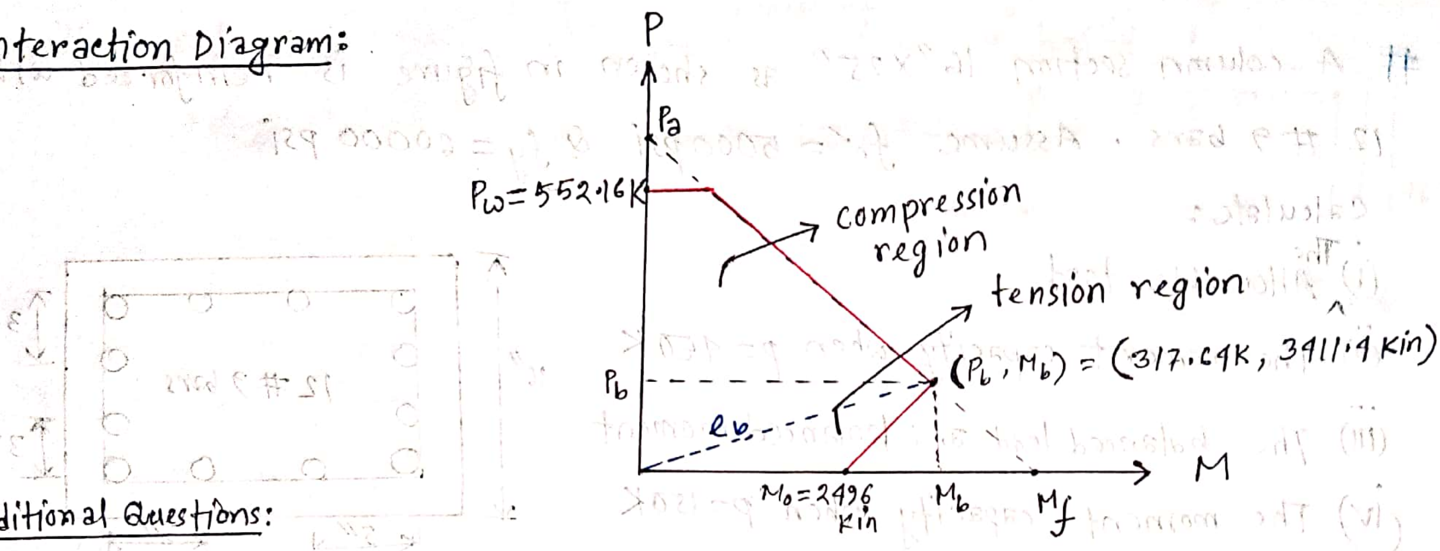
$\Rightarrow M_o = 0.4 \times 6.24 \times 50 \times (22.5 - 2.5)$

$\therefore M_o = 2496$ K-in.

$$\Rightarrow \frac{100}{317.64} = \frac{M - 2496}{3411.45 - 2496}$$

$$\Rightarrow M = 2784.2 \text{ K-in.}$$

Interaction Diagram:



Additional Questions:

(V) check whether the section can carry $P = 100K$ & $M = 4000$ K-in.

Solution: The combination of 100 K load and 4000 K-in moment is (100K, 4000 K-in) point which is outside of interaction line.

Hence, The section can not carry 100 K load with 4000 K-in moment.

(vi) check whether the section can carry $P = 100K$ & $M = 2000$ K-in

solution: The combination of 100 K load and 2000 K-in moment is (100K, 2000 K-in.) point which is inside of interaction line.

Hence, The section can carry 100K load with 2000 K-in moment.

A column section 16"X25" as shown in figure is reinforced with 12 #9 bars. Assume $f_c' = 5000$ psi & $f_y = 60000$ psi

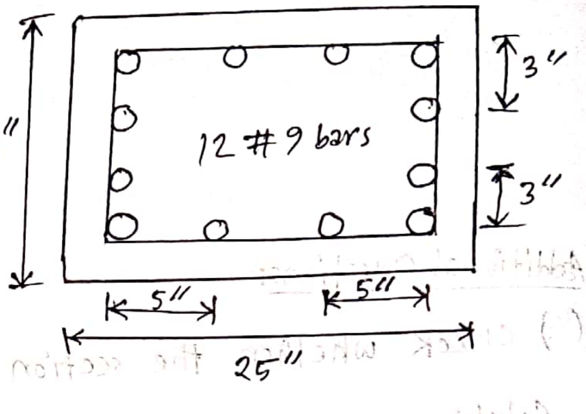
Calculate:

(i) Allowable load

(ii) The moment capacity when $P = 450$ K

(iii) The balanced load and balanced moment

(iv) The moment capacity when $P = 150$ K



Solution:

(i) Allowable load:

$$A_g = (16 \times 25)\ in^2 = 400\ in^2$$

$$A_s = (12 \times 1)\ in^2 = 12\ in^2$$

$$\therefore \rho_g = \frac{A_s}{A_g} = \frac{12}{400} = 0.03$$

Now,

$$P_{all} = 0.85\, A_g\, (.25 f_c' + \rho_g f_s)$$

$$\therefore P_{all} = 0.85 \times 400 \times (.25 \times 5 + 0.03 \times 4 \times 60) = 669.8\ K$$

(iii) Balanced load and Balance Moment:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{5000}} = 7.19 \simeq 7$$

$$m = \frac{f_y}{0.85 f_c'} = \frac{60000}{.85 \times 5000} = 14.12$$

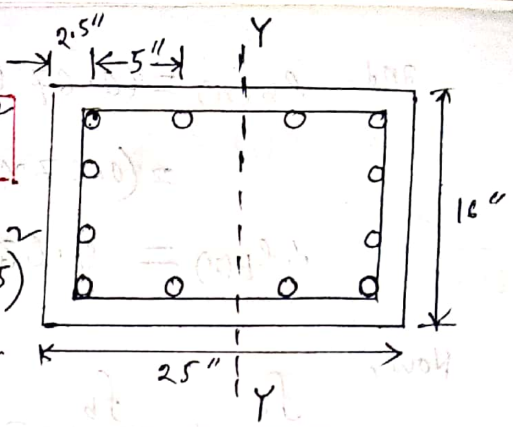
$$F_a = 0.34\,(1 + \rho_g m)\, f_c' = 0.34 \times (1 + 0.03 \times 14.12) \times 5$$

$$\therefore F_a = 2.42\ \text{Ksi}$$

and, $F_b = 0.45 f_c' = (0.45 \times 5) = 2.25$ Ksi with respect to y axis:

for every column of steel

$$I_{ut(y)} = \frac{bt^3}{12} + \boxed{2\cdot(2n-1)\cdot A_s\cdot\left(\frac{t}{2}-d'\right)^2}$$



$$\Rightarrow I_{ut(y)} = \frac{16\times25^3}{12} + 2\times(2\times7-1)\times4\times(12.5-2.5)^2$$
$$+ 2\times(2\times7-1)\times2\times(12.5-7.5)^2$$

$\therefore I_{ut(y)} = 32533.33 \text{ in}^4$

$$\therefore S_{ut(y)} = \frac{I_{ut(y)}}{c} = \frac{32533.33}{12.5} = 2602.67 \text{ in}^3$$

and, $e_{b(y)} = (0.67\,\rho_g m + 0.17)d = (0.67\times0.03\times14.12+0.17)\times22.5 = 10.21 \text{ in}$

We know,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P_b}{A_g} = \frac{P_b}{400}$

$f_b = \frac{M_b}{S_{ut(y)}} = \frac{P_b e_b}{2602.27}$

$$\Rightarrow \frac{P_b/400}{2.42} + \frac{\frac{P_b\times10.21}{2602.27}}{2.25} = 1$$

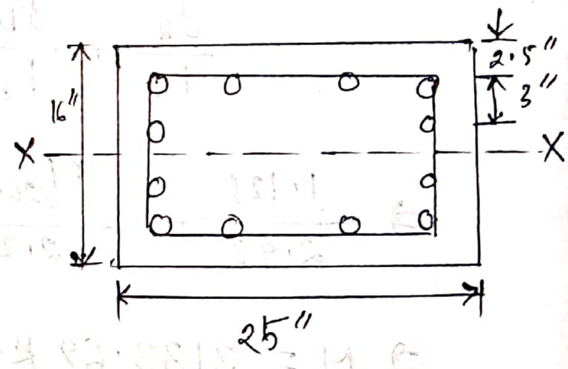
$\therefore P_{b(y)} = 360.12 \text{ K}$

$\therefore M_{b(y)} = P_b e_b = (360.12\times10.21) = 3676.85 \text{ K in.}$

with respect to X axis:

for every row of steel

$$I_{ut(x)} = \frac{tb^3}{12} + \boxed{2\,(2n-1)\cdot A_s\cdot\left(\frac{b}{2}-d'\right)^2}$$



$$= \frac{25\times16^3}{12} + 2\times(2\times7-1)\times4\times(8-2.5)^2 +$$
$$2\times(2\times7-1)\times2\times(8-5.5)^2$$

$\therefore I_{ut(x)} = 12004.33 \text{ in}^4$

$$\therefore S_{ut(x)} = \frac{I_{ut(x)}}{c} = \frac{12004.33}{8} = 1500.54 \text{ in}^3$$

and, $e_{b(x)} = (0.67\ e_g m + 0.17)\, d$

$= (0.67 \times 0.03 \times 14.12 + 0.17) \times 13.5$

$\therefore e_{b(x)} = 6.13$

Now,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P_b}{400}$

$f_b = \frac{M_b}{S_{ut}(x)} = \frac{P_b e_b}{1500.54}$

$$\Rightarrow \frac{P_b/400}{2.42} + \frac{\frac{P_b \times 6.13}{1500.54}}{2.25} = 1$$

$\Rightarrow P_{b(x)} = 351.04$ K

$\therefore M_{b(x)} = P_b e_b = (351.04 \times 6.13) = 2151.88$ K-in.

(ii) When $P = 450$ K, $M = ?$:

Assume, compression governs,

with respect to Y axis:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P}{A_g} = \frac{450}{400} = 1.125$ Ksi

$f_b = \frac{M}{S_{ut}(y)} = \frac{M}{2602.67}$

$$\Rightarrow \frac{1.125}{2.42} + \frac{M/2602.67}{2.25} = 1$$

$\Rightarrow M = 3133.69$ K-in.

$\therefore e = \frac{M}{P} = \frac{3133.69}{450} = 6.96$ in $< e_{b(y)} (= 10.21)$

Hence, compression will govern.

with respect to x-axis:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = 1.125$ ksi

$f_b = \frac{M}{S_{ut(x)}} = \frac{M}{1500.54}$

$$\Rightarrow \frac{1.125}{2.42} + \frac{M/1500.54}{2.25} = 1$$

$$\Rightarrow M = 1806.69 \text{ k-in.}$$

$$\therefore e = \frac{M}{P} = \frac{1806.69}{450} = 4.015 \text{ in.} < e_{b(x)} (= 6.13)$$

Hence, compression will govern.

(iv) When $P = 150$ K, $M = ?$

Assume, compression governs.

with respect to Y axis:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = \frac{P}{A_g} = \frac{150}{400} = 0.375$ ksi

$f_b = \frac{M}{S_{ut(y)}} = \frac{M}{2602.67}$

$$\Rightarrow \frac{0.375}{2.42} + \frac{M/2602.67}{2.25} = 1$$

$$\Rightarrow M = 4948.57 \text{ k-in.}$$

$$\therefore e = \frac{M}{P} = \frac{4948.57}{150} = 33 \text{ in} > e_{b(y)} (= 10.21)$$

Hence, tension will govern.

Now, For tension region,

$$\frac{P}{P_b} = \frac{M - M_o}{M_b - M_o}$$

Here, $M_o = 0.4 A_s f_y (d - d')$

$= 0.4 \times 6 \times 60 \times (22.5 - 2.5)$

$= 2880$ k-in.

$$\Rightarrow \frac{150}{360.12} = \frac{M-2880}{3676.85-2880}$$

$$\Rightarrow M = 3211.91 \text{ K-in.}$$

with respect to x-axis:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

Here, $f_a = 0.375$ Ksi

$$f_b = \frac{M}{S_{uf(x)}} = \frac{M}{1500.54}$$

$$\Rightarrow \frac{0.375}{2.42} + \frac{M/1500.54}{2.25} = 1$$

$$\Rightarrow M = 2853.04 \text{ K-in.}$$

$$\therefore e = \frac{M}{P} = \frac{2853.04}{150} = 19.02 \text{ in.} > e_{b(x)} (= 6.13)$$

Hence, tension will govern.

Now, For tension region,

$$\frac{P}{P_b} = \frac{M-M_o}{M_b-M_o}$$

Here, $M_o = 0.4 A_s f_y (d-d')$

$= 0.4 \times 6 \times 60 \times (13.5-2.5)$

$= 1584$ K-in

$$\Rightarrow \frac{150}{351.04} = \frac{M-1584}{2151.88-1584}$$

$$\Rightarrow M = 1826.66 \text{ K-in.}$$

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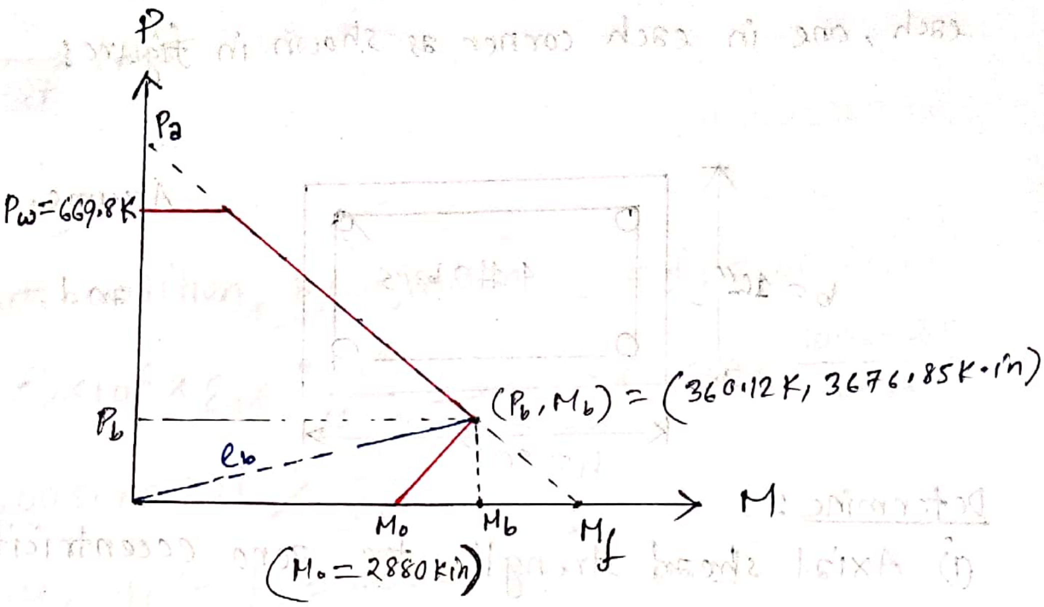
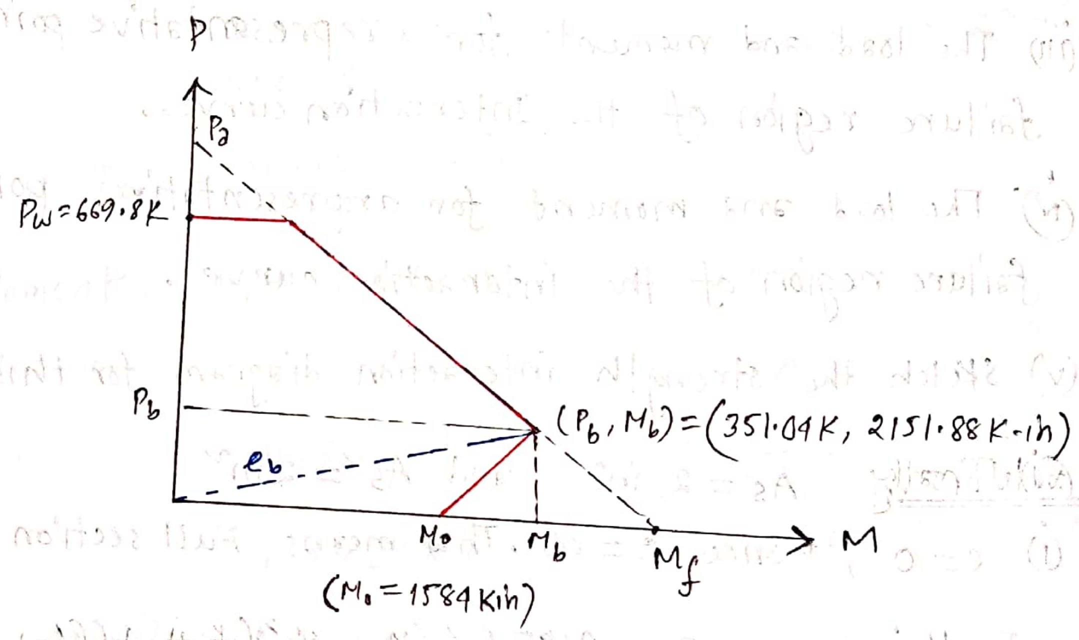
Interaction Diagram:

with respect to Y-axis:



with respect to X-axis:

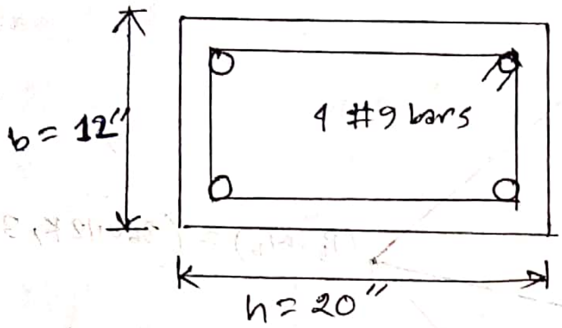


(Analysis)

USD Method:

Example: 8.1

A 12 × 20 in. column is reinforced with four No. 9 bars of area 1.0 in² each, one in each corner as shown in figure:



Assume, $f_c' = 4000$ psi

and, $f_y = 60000$ psi

Determine:

(i) Axial load strength for zero eccentricity

(ii) The balance load P_b, Moment M_b and corresponding eccentricity e_b for balanced failure.

(iii) The load and moment for a representative point in the tension failure region of the Interaction curve.

(iv) The load and moment for a representative point in the compression failure region of the interaction curve.

(v) Sketch the strength interaction diagram for this column.

Solution: $A_s = 2$ in² and $A_s' = 2$ in²

(i) $e = 0$, Hence $c = \infty$. That means Full section is in compression.

For this case, $P_n = 0.85 f_c' \underbrace{a}_{a=h} b + A_s' f_y + A_s f_y$

$= 0.85 \times 4 \times 20 \times 12 + 2 \times 60 + 2 \times 60$

$= 1056$ Kips.

(ii) we know,

$$c_b = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \times d \qquad \text{Here, } \varepsilon_y = \frac{f_y}{E_s} = \frac{60000}{29\times10^6} = 0.00207$$

$$= \frac{0.003}{0.003 + 0.00207} \times 17.5$$

$$\therefore c_b = 10.36 \text{ in}$$

For the balance failure condition, by definition $f_s = f_y = 60000$ psi

$$f_s' = E_s \varepsilon_s' = 29\times10^6 \times \varepsilon_u \times \frac{c-d'}{c} = 29\times10^6 \times 0.003 \times \frac{10.36-2.5}{10.36}$$

$$\therefore f_s' = 66005.79 \text{ psi} > f_y$$

Hence, $f_s' = f_y = 60000$ psi

The balance load, $P_b = 0.85 f_c' ab + A_s' f_s' - A_s f_s$

$$= 0.85 \times 4 \times (.85 \times 10.36) \times 12 + 2\times60 - 2\times60$$

$$\therefore P_b = 359.285 \text{ Kips.}$$

The balanced Moment, $M_b = 0.85 f_c' ab\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s'\left(\frac{h}{2} - d'\right) + A_s f_s\left(d - \frac{h}{2}\right)$

$$= 0.85 \times 4 \times (\underbrace{.85 \times 10.36}_{a}) \times 12 \times \left(\frac{20}{2} - \frac{.85\times10.36}{2}\right) + 2\times60\times\left(\frac{20}{2} - 2.5\right) + 2\times60\times(17.5-10)$$

$$\therefore M_b = 3810.92 \text{ K-in}$$

The corresponding eccentricity of load, $e_b = \frac{M_b}{P_b} = \frac{3810.92}{359.285}$

$$\therefore e_b = 10.607 \text{ in.}$$

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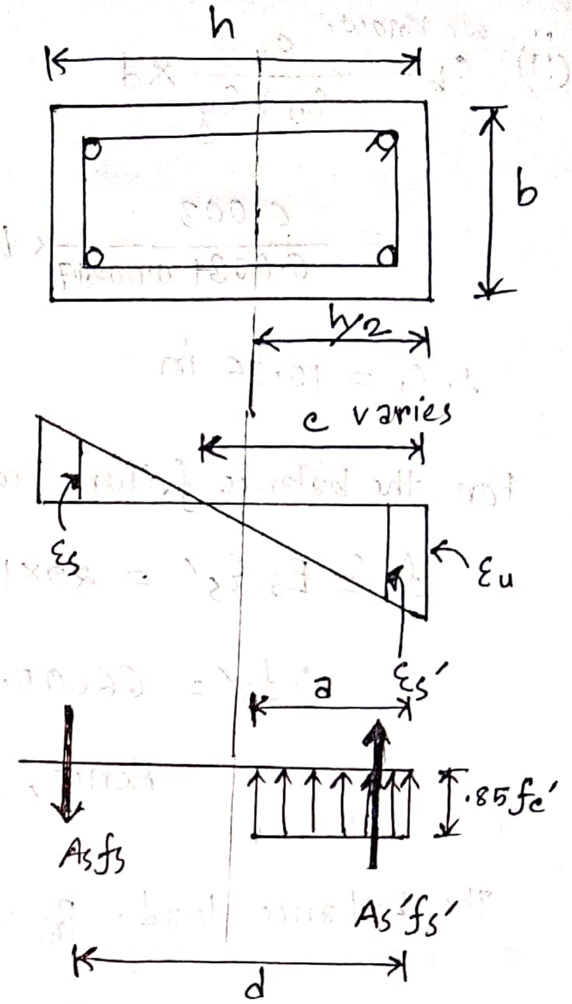
(iii) if $c < c_b$, it will give a point in the tension failure region of the interaction curve, with $e > e_b$



Let, $c = 5$ in.

$$f_s' = E_s \varepsilon_s' = E_s \times \varepsilon_u \times \frac{c-d'}{c}$$

$$= 29000 \times .003 \times \frac{5-2.5}{5}$$

$$\therefore f_s' = 43.5 \text{ ksi} < f_y$$

$$\therefore a = \beta_1 c = (5 \times .85) = 4.25$$

Now,

$$P_n = 0.85 f_c' a b + A_s' f_s' - A_s f_s$$

$$= .85 \times 4 \times 4.25 \times 12 + 2 \times 43.5 - 2 \times 60$$

$$\therefore P_n = 140.4 \text{ kips.}$$

and,

$$M_n = .85 f_c' a b \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + A_s f_s \left(d - \frac{h}{2}\right)$$

$$= .85 \times 4 \times 4.25 \times 12 \times \left(10 - \frac{4.25}{2}\right) + 2 \times 43.5 \times (10 - 2.5) + 2 \times 60 \times (17.5 - 10)$$

$$= 2922.36 \text{ k-in.}$$

$$\therefore e = \frac{M_n}{P_n} = \frac{2922.36}{140.4} = 20.8 \text{ in.} > e_b$$

(iv) if $c > c_b$. It will give a point in the compression failure region of the interaction diagram.

Let, $c = 15$ in. $\therefore a = (\beta_1 \times c) = (.85 \times 15) = 12.75$

$$f_s = E_s \times \varepsilon_u \times \underbrace{\frac{d-c}{c}}_{\varepsilon_s} = 29\times10^3 \times \left(\frac{17.5-15}{15}\right) \times 0.003 = 14.5 \text{ ksi} < f_y$$

$\therefore f_s = 14.5$ ksi

$$f_s' = E_s \varepsilon_s' = E_s \times \varepsilon_u \times \frac{c-d'}{c} = 29\times10^3 \times .003 \times \left(\frac{15-2.5}{15}\right) = 72.5 \text{ ksi} > f_y$$

$\therefore f_s' = f_y = 60$ ksi

Now,

$$P_n = .85 f_c' ab + A_s' f_s' - A_s f_s$$
$$= 0.85 \times 4 \times 12.75 \times 12 + 2\times60 - 2\times14.5$$

$\therefore P_n = 611.2$ kips

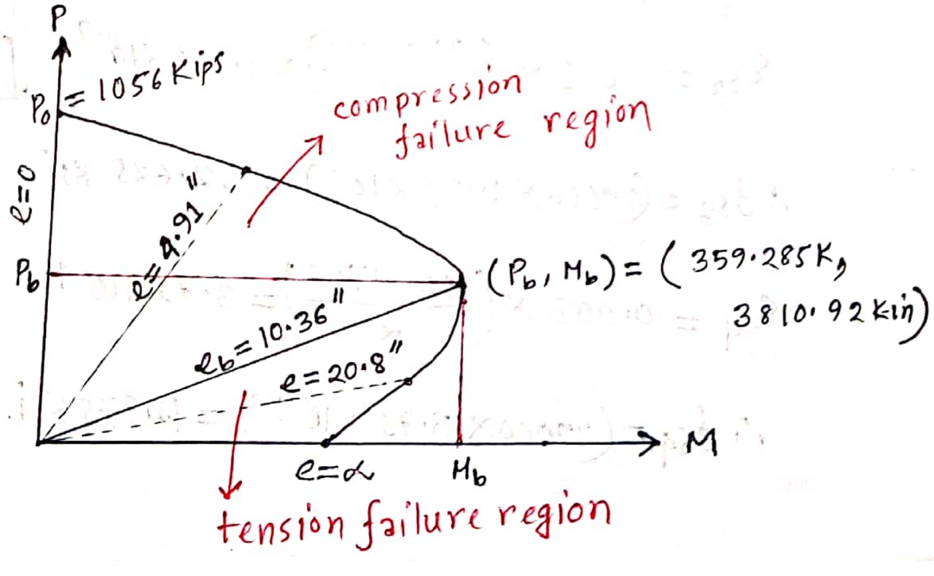
and,

$$M_n = .85 f_c' ab \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + A_s f_s \left(d - \frac{h}{2}\right)$$
$$= .85 \times 4 \times 12.75 \times 12 \times \left(10 - \frac{12.75}{2}\right) + 2\times60\times(10-2.5) + 2\times14.5 \times (17.5-10)$$

$\therefore M_n = 3003.225$ k-in.

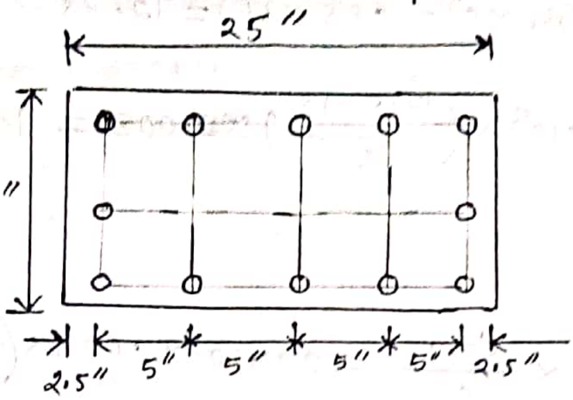
$$\therefore e = \frac{M_n}{P_n} = \frac{3003.225}{611.2} = 4.91 \text{ in.} < e_b$$

(v) Interaction Diagram:



Example : 8.2

The column as shown in figure below is reinforced with 12 No. 9 bars distributed around the perimeter as shown.

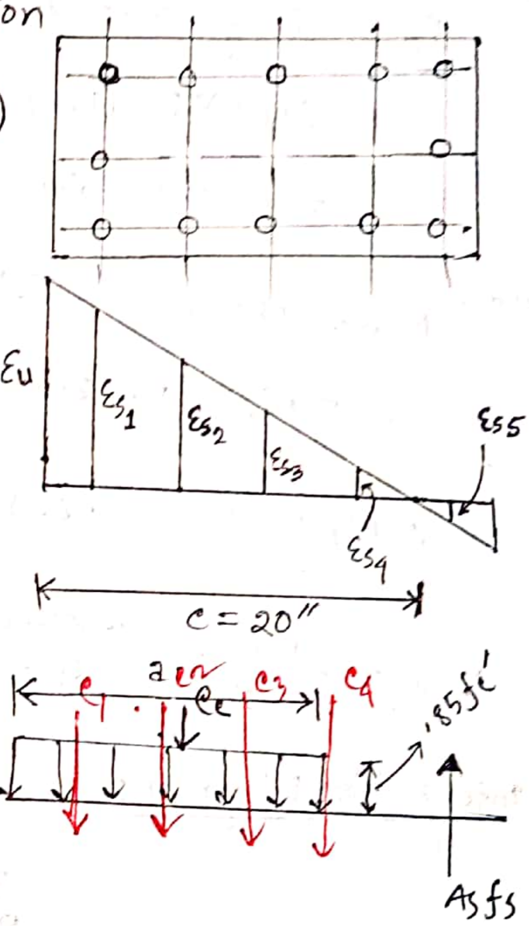


Assume,

$f_c' = 5000$ psi

$f_y = 60000$ psi

Find the load and moment corresponding to a failure point with neutral axis $c = 20$ in. from the Left face



Solution: $\varepsilon_1, \varepsilon_2, \varepsilon_3$ & ε_4 are in compression

We Know, $\varepsilon_s' = \varepsilon_u \times \dfrac{c-d'}{c}$ (for compression)

$\varepsilon_{s_1} = .003 \times \left(\dfrac{20-2.5}{20}\right) = 2.625 \times 10^{-3}$

$\therefore f_{s_1} = E_s \varepsilon_s = (29000 \times 2.625 \times 10^{-3})$

$= 76.125$ Ksi $> f_y$

$\therefore f_{s_1} = 60$ Ksi

$\varepsilon_{s_2} = 0.003 \times \left(\dfrac{20-7.5}{20}\right) = 1.875 \times 10^{-3}$

$\therefore f_{s_2} = (29000 \times 1.875 \times 10^{3})$ Ksi

$= 54.375$ Ksi

$\varepsilon_{s_3} = .003 \times \left(\dfrac{20-12.5}{20}\right) = 1.125 \times 10^{-3}$

$\therefore f_{s_3} = (29000 \times 1.125 \times 10^{-3}) = 32.625$ Ksi

$\varepsilon_{s_4} = 0.003 \times \left(\dfrac{20-17.5}{20}\right) = 3.75 \times 10^{-4}$

$\therefore f_{s_4} = (29000 \times 3.75 \times 10^{-4}) = 10.875$ Ksi

ε_{s5} is in tension.

We Know, $\varepsilon_s = \varepsilon_u \times \frac{d-c}{c}$

$\therefore \varepsilon_{s5} = 0.003 \times \frac{22.5-20}{20} = 3.75 \times 10^{-4}$

$\therefore f_{s5} = (29000 \times 3.75 \times 10^{-4}) = 10.875$ ksi

For $f_c' = 5000$ psi $\beta_1 = 0.80$

$\therefore a = \beta_1 c = (.80 \times 20) = 16$ in.

Now,

$P_n = 0.85 f_c' ab + \Sigma A_s' f_s' - \Sigma A_s f_s$

$= (0.85 \times 5 \times 16 \times 12) + (3 \times 60) + (2 \times 54.375) + (2 \times 32.625) + (2 \times 10.875) - (3 \times 10.875)$

$= 1159.125$ Kips.

and,

$M_n = 0.85 f_c' ab (\frac{h}{2} - \frac{a}{2}) + \Sigma A_s' f_s' (\frac{h}{2} - d') + \Sigma A_s f_s (d - \frac{h}{2})$

$= .85 \times 5 \times 16 \times 12 \times (12.5 - 8) + 3 \times 60 \times (12.5 - 2.5) + 2 \times 54.375 \times (12.5 - 7.5) + 2 \times 32.625 \times (12.5 - 12.5) + 2 \times 10.875 \times (12.5 - 17.5) + 3 \times 10.875 \times (22.5 - 12.5)$

$= 6233.25$ K-in.

The corresponding eccentricity, $e = \frac{6233.25}{1159.125} = 5.38$ in.

WSD Method (Design)

\# Design a column to carry an axial load of 300 Kip and simultaneous moment of 200 Kip-ft. Assume $f_c' = 4000$ psi and $f_y = 50000$ psi

Solution: Assume, the column section $= b \times h$

where, $h = 1.5b$

$$\therefore A_g = (b \times 1.5b) = 1.5b^2$$

Let, $\rho_s = 0.02$ $\quad \therefore A_s = \rho_s A_g = (0.02 \times 1.5b^2) = 0.03b^2$

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$$

$$I_{ut} = \frac{bh^3}{12} + 2(2n-1)A_s\left(\frac{h}{2} - d'\right)^2$$

$$= \frac{b \times (1.5b)^3}{12} + 2 \times (2 \times 8 - 1) \times \frac{0.03b^2}{2} \times \left(\frac{1.5b}{2} - 2.5\right)^2$$

$$= \frac{9b^4}{32} + 0.45b^2(0.75b - 2.5)^2$$

Now,

$$S_{ut} = \frac{I_{ut}}{c} = \frac{1}{1.5b/2} \times \left[\frac{9b^4}{32} + 0.45b^2(0.75b - 2.5)^2\right]$$

$$\therefore S_{ut} = \frac{3b^3}{8} + \frac{3}{5}b(0.75b - 2.5)^2$$

$$F_a = 0.34 f_c'(1 + \rho_g m) = 0.34 \times 4 \times \left(1 + 0.02 \times \frac{50}{0.85 \times 4}\right) = 1.76$$

$$F_b = 0.45 f_c' = (0.45 \times 4) = 1.8$$

$$f_a = \frac{P}{A_g} = \frac{300}{1.5b^2} \quad \text{and,} \quad f_b = \frac{M}{S_{ut}} = \frac{200 \times 12}{S_{ut}} = \frac{2400}{S_{ut}}$$

Now, $$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

$$\Rightarrow \frac{300/1.5b^2}{1.76} + \frac{2400/S_{ut}}{1.8} = 1$$

$$\Rightarrow \frac{300}{1.76 \times 1.5 b^2} + \frac{2400}{1.80 \times \left[\frac{3b^3}{8} + \frac{3}{5} b (0.75b - 2.5)^2\right]} = 1$$

$\Rightarrow b = 16.00''$

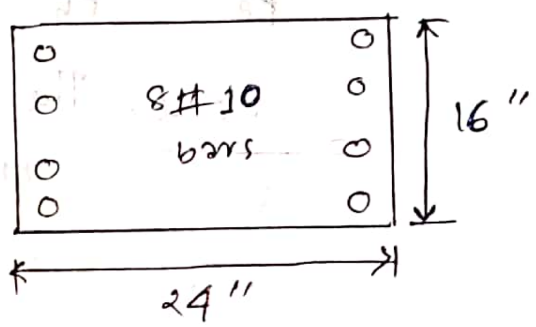
and, $h = 1.5 \times b = 1.5 \times 16.00 = 24.00''$

Hence, Taking the column section, $b = 16''$ and $h = 24$ in

$\therefore A_g = (16 \times 24) = 384.0 \ in^2$

Now, $A_s = \rho_s A_g = (0.02 \times 384.0) \ in^2 = 7.68 \ in^2$

$\therefore$ provide 8 # 10 bars.



same type: (Alterate Solution)

Design a column section $P = 500$ kip and $M = 300$ k-ft. $f_c' = 4$ ksi and $f_y = 60$ ksi. Follow WSD method.

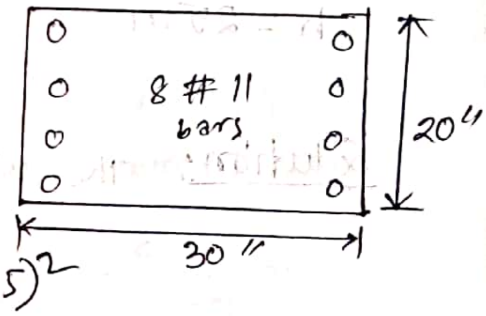
Solution: Assume, the cross section of column = 20 in × 30 in.

and reinforced with 8 # 11 bars.

$A_g = (20 \times 30) = 600 \ in^2$ and $A_s = (8 \times 1.56) = 12.48 \ in^2$

$\therefore \rho_g = \frac{A_s}{A_g} = \frac{12.48}{600} = 0.0208$

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$



$$I_{ut} = \frac{20 \times 30^3}{12} + 2 \times (2 \times 8 - 1) \times (4 \times 1.56) \times (15 - 2.5)^2$$

$= 74250 \ in^4$

$$S_{ut} = \frac{I_{ut}}{c} = \frac{74250}{15} = 4950\ in^3$$

$$F_a = 0.34 f_c' (1 + \rho_g m) = 0.34 \times 4 \times \left(1 + 0.0208 \times \frac{60}{.85 \times 4}\right) = 1.86$$

$$F_b = 0.45 f_c' = (0.45 \times 4) = 1.8$$

$$f_a = \frac{P}{A_g} = \frac{500}{600} = 0.83$$

$$f_b = \frac{M}{S_{ut}} = \frac{300 \times 12}{4950} = 0.73$$

Now,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{0.83}{1.86} + \frac{0.73}{1.8} = 0.85 < 1$$

(Design is OK)

Hence, The assumed section is safe for $P = 500\ K$ and $M = 300\ K\text{-}ft$.

Design

USD Method:

Example: 8.3

Design a column using the following data:

D.L = 222 Kips	D.L moment = 162 K-ft
L.L = 333 Kips	L.L moment = 166 K-ft
b = 20 in	$f_c' = 4000$ psi
h = 25 in	$f_y = 60000$ psi

Solution: The ultimate load, $P_u = 1.2\,D.L + 1.6\,L.L$

$$= (1.2 \times 222) + (1.6 \times 333)$$

$$= 799.2\ K \simeq 800\ Kips$$

The ultimate moment, $M_u = (1.2 \times 162 + 1.6 \times 166) = 460$ K-ft

$A_g = (20 \times 25)\ in^2 = 500$

$$\gamma = \frac{h - 2d'}{h} = \frac{25-5}{25} = 0.8$$

Given, $f_c' = 4$ Ksi

$f_y = 60$ Ksi

$$K_n = \frac{P_u}{\phi f_c' A_g} = \frac{800}{0.65 \times 4 \times (20 \times 25)} = 0.615$$

0.65 (tied)　0.70 (spiral)

$$R_n = \frac{M_u}{\phi f_c' A_g h} = \frac{460 \times 12}{0.65 \times 4 \times (20 \times 25) \times 25} = 0.17$$

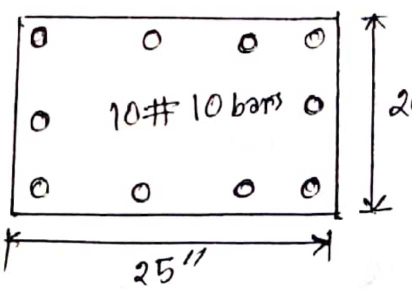
From Graph A.7 (Next Page)

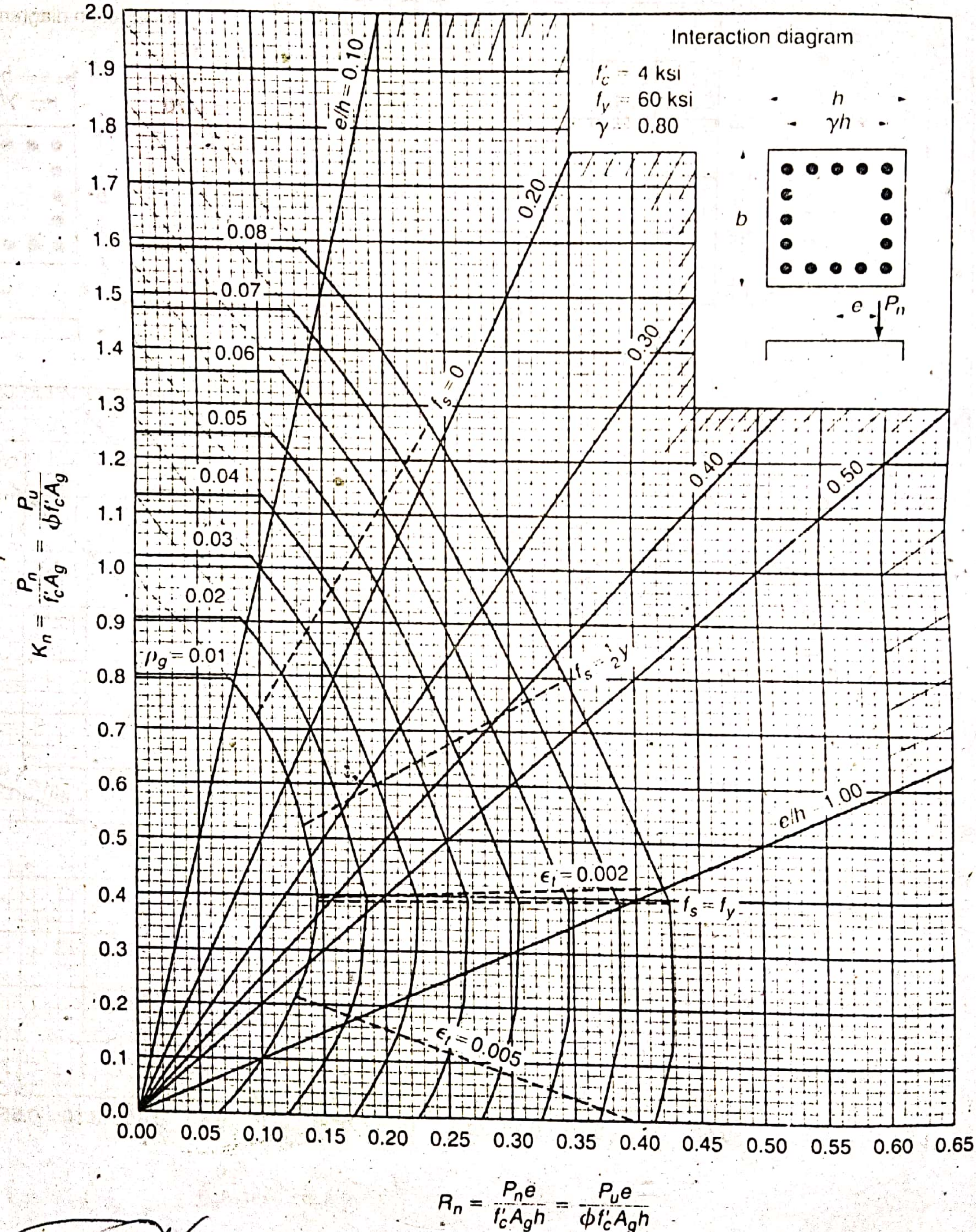
we obtain, The required reinforcement ratio, $\rho_g = (0.02 - 0.03)$

Hence, $\rho_g = 0.025$

The steel area, $A_s = \rho_g \times A_g = 0.025 \times (20 \times 25) = 12.5\ in^2$

Provide 10 # 10 bars.





GRAPH A.7
Column strength interaction diagram for rectangular section with bars on four faces and γ = 0.80 (for instructional use only).

~~Compression Biaxial Bending~~

Example: 8.4

Design a column using following data:

$P_u = 481$ Kips $\quad f_c' = 4000 psi$

$M_u = 492$ Kip-ft $\quad f_y = 60000$ psi

$e_g = 0.03$

$\gamma = 0.80$

Solution: Assume the height of the column, $h = 25$ in.

Now, $e = \dfrac{M_u}{P_u} = \dfrac{492 \times 12}{481} = 12.27$ in

$\dfrac{e}{h} = \dfrac{12.27}{25} = 0.49$

From graph A.11 with $\frac{e}{h} = 0.49$ and $e_g = 0.03$

we obtain, $K_n = \dfrac{P_u}{\phi f_c' A_g} = 0.51$

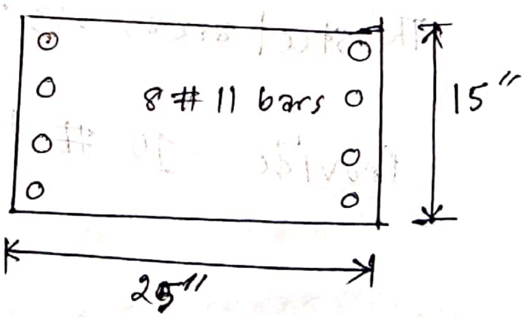
$\Rightarrow \dfrac{P_u}{\phi f_c' bh} = 0.51$

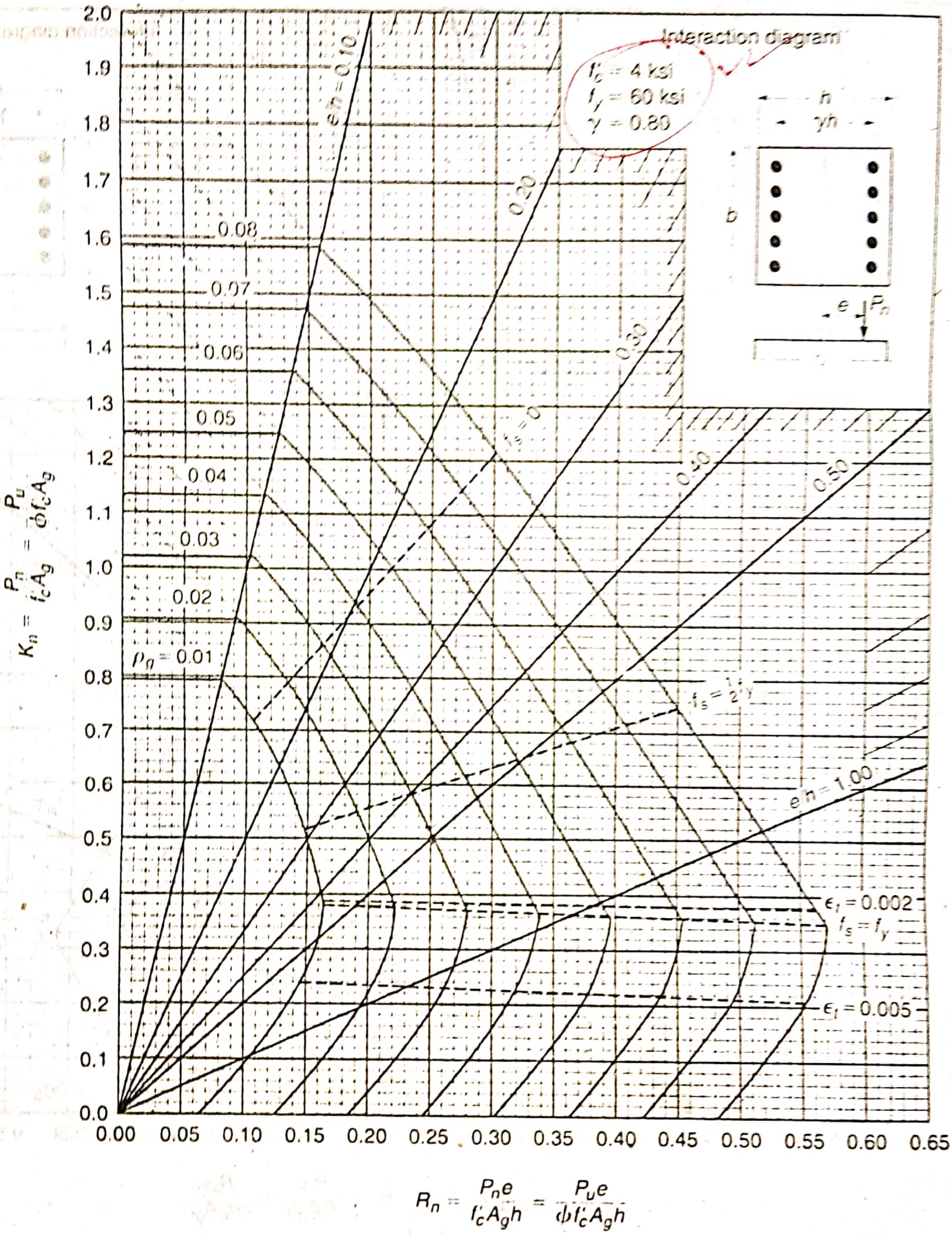
$\Rightarrow b = \dfrac{481}{.65 \times 4 \times 0.51 \times 25} = 14.5$ in.

Hence, A column section 15 in × 25 in will be used.

$A_s = e_g \times A_g = 0.03 \times (15 \times 25) = 11.25$

Provide 8 # 11 bars.





GRAPH A.11

Column strength interaction diagram for rectangular section with bars on end faces and γ = 0.80 (for instructional use only).

(WSD) $\boxed{\text{Design}}$ compression + bending in one axis

Design a spiral reinforced, circular column for P = 300 Kips, with an eccentricity e = 6 in. using concrete with $f_c' = 3$ Ksi and steel with $f_y = 40$ Ksi.

Solution: We know, $P_{all} = \emptyset' A_g(0.25 f_c' + \rho_g f_s)$

$$\Rightarrow 300 = 1 \times A_g \times (0.25 \times 3 + 0.02 \times 0.4 \times 40) \quad (\text{Assume, } \rho_g = 0.02)$$

$$\therefore A_g = 280.37 \text{ in}^2$$

$$\Rightarrow \frac{\pi}{4} D^2 = 280.37 \quad \therefore D = 18.89 \text{ in} \simeq 20 \text{ in}$$

$$A_s = \rho_g A_g = (0.02 \times 280.37) = 5.60 \text{ in}^2$$

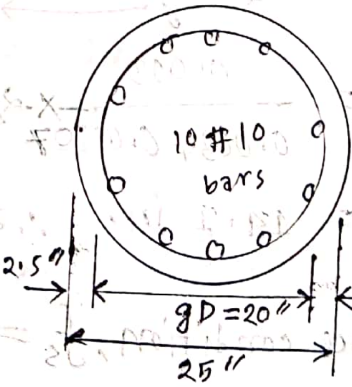
This column section will be adequate for axial load alone.

Hence, with a eccentricity, a larger section will be required to accomodate the additional moment, Pe.

Assume The required diameter of the column for additional moment, D = 25 in. and reinforced with 10 No. 10 bars. $\therefore A_s = 10 \times 1.27 = 12.7$ in^2

$$A_g = \frac{\pi}{4} \times (25)^2 = 490.875 \text{ in}^2$$

$$\therefore \rho_g = \frac{12.7}{490.875} = 0.026$$



$$* \; I_{ut} = \frac{\pi D^4}{64} + (2n-1) A_s \frac{(gD)^2}{8}$$

$$= \frac{3.1416 \times 25^4}{64} + (2 \times 9 - 1) \times 12.7 \times \frac{20^2}{8}$$

$$\therefore I_{ut} = 29969.8 \text{ in}^4 \qquad \therefore S_{ut} = \frac{I_{ut}}{c} = \frac{29969.8}{12.5} = 2397.6 \text{ in}^3$$

$$F_a = 0.34\, f_c' (\rho_g m + 1) = 0.34 \times 3 \times \left(0.026 \times \frac{40}{.85 \times 3} + 1\right) = 1.436 \text{ Ksi}$$

$$F_b = .45 \times 3 = 1.34 \text{ Ksi}$$

$$f_a = \frac{P}{A_g} = \frac{300}{490.875} = 0.61$$

$$f_b = \frac{M}{S_{ut}} = \frac{300 \times 6}{2397.6} = 0.75$$

Hence,

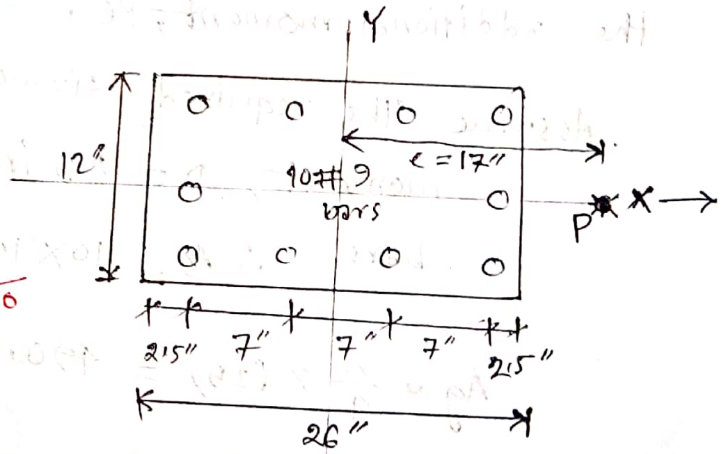
$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = \frac{0.61}{1.436} + \frac{0.75}{1.34} = 0.98 < 1$$

(Design is OK)

So, The assumed section is adequate for applied load and moment.

(USD) $\boxed{\text{Analysis}}$ compression + bending in one axis

\# The column as shown in figure, is loaded eccentrically with $e = 17$ in. $f_c' = 3.0$ Ksi and $f_y = 60$ Ksi. Determine the ultimate load and corresponding moment.



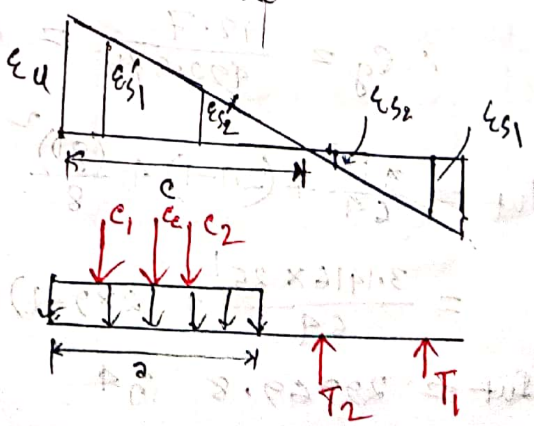
Solution:

we know,

$$c_b = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \times d \qquad \frac{f_y}{E_s} = \frac{60}{29000}$$

$$= \frac{0.003}{0.003 + 0.00207} \times 23.5$$

$\therefore c_b = 13.9$ in. $\quad \therefore a = (.85 \times 13.9)$ in. $= 11.82$ in.



For balance condition, $f_s = f_y$

$$\therefore f_{s_1} = 60 \text{ Ksi}$$

$f_{s2} = E_s \varepsilon_{s2} = 29000 \times 0.003 \times \frac{16.5-13.9}{13.9} = 16.27$ ksi

$f_{s1}' = E_s \varepsilon_{s1}' = 29000 \times 0.003 \times \frac{13.9-2.5}{13.9} = 71.35$ ksi > 60 ksi

$\therefore f_{s1}' = 60$ ksi

$f_{s2}' = E_s \varepsilon_2' = 29000 \times \frac{13.9-9.5}{13.9} \times 0.003 = 27.54$ ksi

Now,
$P_b = C_c + C_1 + C_2 - T_2 - T_1$

$= 0.85 f_c' ab + A_{s1}' f_{s1}' + A_{s2}' f_{s2}' - A_{s2} f_{s2} - A_{s1} f_{s1}$

$= 0.85 \times 3 \times 11.82 \times 12 + 3 \times 60 + 2 \times 27.54 - 2 \times 16.27 - 3 \times 60$

$= 384.232$ Kips.

$M_b = C_c \left(\frac{h}{2} - \frac{a}{2}\right) + C_1 \left(\frac{h}{2} - d_1'\right) + C_2 \left(\frac{h}{2} - d_2'\right) + T_1 \left(d_1 - \frac{h}{2}\right) + T_2 \left(d_2 - \frac{h}{2}\right)$

$= 361.692 \times \left(13 - \frac{11.82}{2}\right) + 180 \times (13-2.5) + 55.08 \times (13-9.5) + 180 \times (23.5-13) + 32.54 \times (16.5-13)$

$= 6651.07$ kip-in.

$e_b = \frac{M_b}{P_b} = \frac{6651.07}{384.232} = 17.3$ in $> e = 17$ in.

Hence compression region.

c will be greater than c_b. Assume $c = 14$ in.

$\therefore a = (\beta_1 \times c) = (0.85 \times 14) = 11.9$ in.

$f_{s1}' = 29000 \times 0.003 \times \left(\frac{14-2.5}{14}\right) = 71.46$ ksi > 60 ksi

$\therefore f_{s1}' = 60$ ksi

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$f_{s_2}' = 29000 \times 0.003 \times \left(\frac{14-9.5}{14}\right) = 27.96$ ksi

$f_{s_1} = E_s \varepsilon_{s_1} = 29000 \times 0.003 \times \left(\frac{23.5-14}{14}\right) = 59.04$ ksi

$f_{s_2} = 29000 \times 0.003 \times \left(\frac{16.5-14}{14}\right) = 15.536$ ksi

$C_c = .85 f_c' ab = 0.85 \times 3 \times 11.9 \times 12 = 364.14$ K

$C_1 = A_{s_1}' f_{s_1}' = (3 \times 60) = 180$ K

$C_2 = A_{s_2}' f_{s_2}' = (2 \times 27.96) = 55.92$ K

$T_1 = A_{s_1} f_{s_1} = (3 \times 59.04) = 177.12$ K

$T_2 = A_{s_2} f_{s_2} = (2 \times 15.536) = 31.072$ K

$\therefore P_n = C_c + C_1 + C_2 - T_2 - T_1 = (364.14 + 180 + 55.92 - 177.12 - 31.072)$

$\therefore P_n = 391.87$

$M_n = C_c\left(\frac{h}{2} - \frac{a}{2}\right) + C_1\left(\frac{h}{2} - d_1'\right) + C_2 \times \left(\frac{h}{2} - d_2'\right) + T_1\left(d_1 - \frac{h}{2}\right) + T_2\left(d_2 - \frac{h}{2}\right)$

$= 364.14 \times \left(13 - \frac{12.9}{2}\right) + 180 \times (13 - 2.5) + 55.92 \times (13 - 9.5) + 177.12 \times (23.5 - 13) + 28.18 \times (16.5 - 13)$

$= 6621.42$ Kip-in.

$\therefore e = \frac{M_n}{P_n} = \frac{6621.42}{391.87} = 16.9$ in. which is very close to the given eccentricity $e = 17$.

$\therefore$ The ultimate load, $P_u = (0.65 \times 391.87) = 254.72$ Kips.

and ultimate moment, $M_u = (0.65 \times 6621.42) = 4303$ Kip-in.

(Ans)

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Design

Compression + Biaxial Bending

Design a column using following data: (Modified Load Contour Method)

$P_n = 340$ Kips, $M_{nx} = 185$ K-ft, $M_{ny} = 120$ K-ft

$f_c' = 4000$ psi and $f_y = 60000$ psi

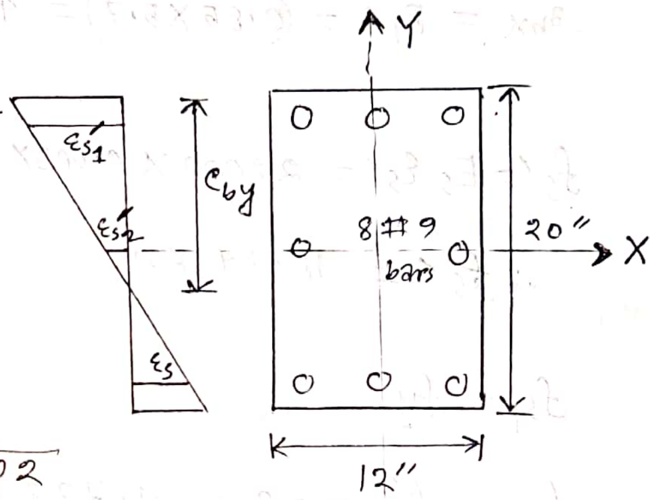
Solution: Assume, the column section = 12 in × 20 in and reinforced with 8 #9 bars

Here, $A_g = (12 \times 20) = 240\ in^2$, $A_s = 9\ in^2$

$P_{no} = 0.85 f_c' (A_g - A_{st}) + A_{st} f_y$

$= 0.85 \times 4 \times (240 - 8) + 8 \times 60$

$\therefore P_{no} = 1268.8$ Kips.

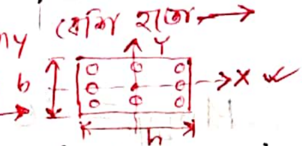


$$c_{by} = d \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} = 17.5 \times \frac{0.003}{0.003 + 0.002}$$

$\therefore c_{by} = 10.5$ in.

Hence, $a_{by} = \beta_1 c_b = (0.85 \times 10.5) = 8.925$ in.

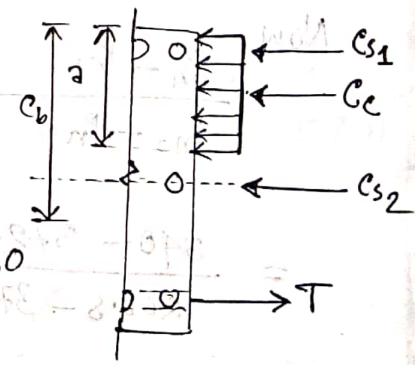
* (যদি Mny বেশি হতো → Arragement



$$f_{s1}' = E_s \varepsilon_{s1}' = 29000 \times 0.003 \times \frac{10.5 - 2.5}{10.5} = 66.3\ ksi > 60\ ksi$$

$\therefore f_{s1}' = 60$ ksi

$$f_{s2}' = E_s \varepsilon_{s2}' = 29000 \times 0.003 \times \frac{10.5 - 10}{10.5} = 4.143\ ksi$$



Now, $P_{nbx} = C_c + C_{s1} + C_{s2} - T$

$\Rightarrow P_{nbx} = 0.85 f_c' ab + A_{s1}' f_{s1}' + A_{s2}' f_{s2}' - A_s f_y$

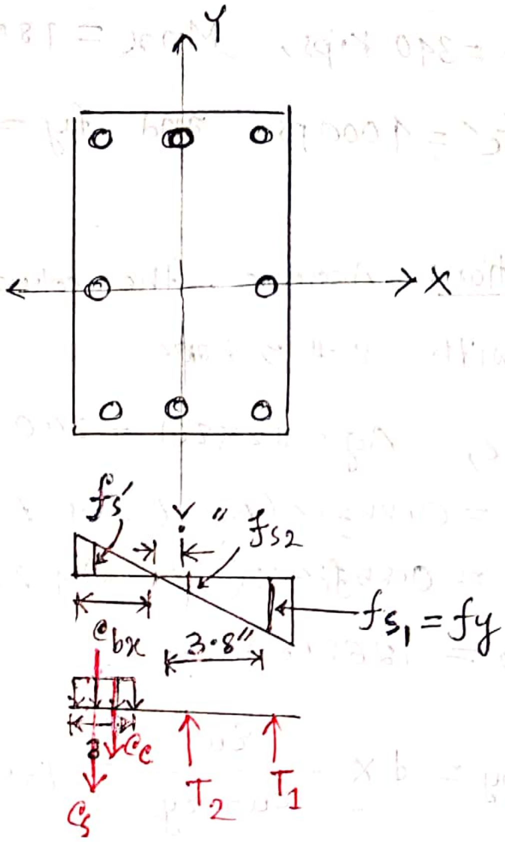
$\Rightarrow P_{nbx} = 0.85 \times 4 \times 8.925 \times 12 + 3 \times 60 + 2 \times 4.143 - 3 \times 60$

$\therefore P_{nbx} = 372.426$ K $= P_{nb}$

$$M_{nbx} = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{s_1} \times \left(\frac{h}{2} - d'\right) + T\left(d - \frac{h}{2}\right)$$

$$= 0.85 \times 4 \times 8.925 \times 12 \times \left(10 - \frac{8.925}{2}\right) + 3 \times 60 \times (10 - 2.5) + 3 \times 60 \times (17.5 - 10)$$

$\therefore M_{nbx} = 4716.425$ Kip-in



Now,

$$c_{bx} = 9.5 \times \frac{0.003}{0.003 + 0.002} = 5.7 \text{ in.}$$

$$a_{bx} = \beta_1 c = (0.85 \times 5.7) = 4.845 \text{ in.}$$

$$f_s' = E_s \varepsilon_s' = 29000 \times 0.003 \times \frac{5.7 - 2.5}{5.7}$$

$\therefore f_s' = 48.84$ Ksi

$f_{s_1} = f_y$

$$f_{s_2} = \frac{60}{3.8} \times 0.3 = 4.737 \text{ Ksi}$$

$$M_{nby} = C_c\left(\frac{b}{2} - \frac{a}{2}\right) + C_s\left(\frac{b}{2} - d'\right) + T_1\left(d - \frac{b}{2}\right)$$

$$= 0.85 \times 4 \times 4.845 \times 20 \times \left(6 - \frac{4.845}{2}\right) + 3 \times 48.84 \times (6 - 2.5) + 3 \times 60 \times (9.5 - 6)$$

$= 2321.46$ K-in.

Now,

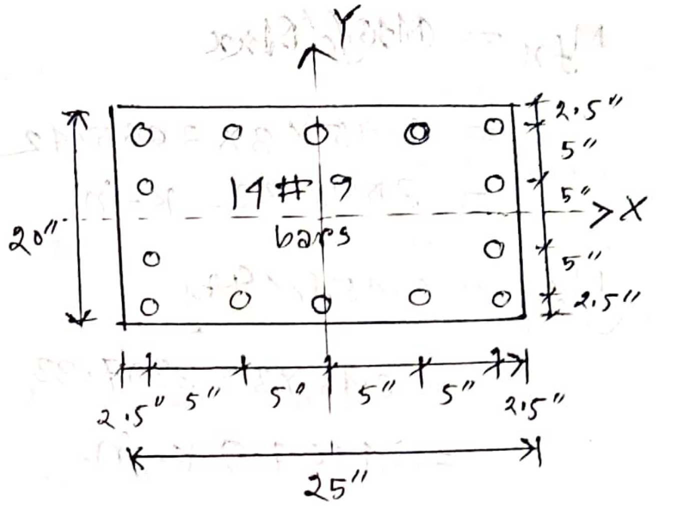
$$\frac{P_n - P_{nb}}{P_{no} - P_{nb}} + \left(\frac{M_{nx}}{M_{nbx}}\right)^{1.5} + \left(\frac{M_{ny}}{M_{nby}}\right)^{1.5}$$

$$= \frac{340 - 372.424}{1268.8 - 372.424} + \left(\frac{185 \times 12}{4716.425}\right)^{1.5} + \left(\frac{120 \times 12}{3294.54}\right)^{1.5}$$

$= 0.78 < 1$

Hence, The assumed section will be safe.

A column section is shown in figure: verify the capacity of the section if $P = 250\,K$, $M_x = 1000$ K-in., $M_y = 1500$ K-in. Given, $f_c' = 3$ Ksi and, $f_y = 60$ Ksi



(a) using Reciprocal load Method.

(b) using Bresler equation.

Solution:

(a) Reciprocal Load Method:

Here, $P = 250\,K$, $A_g = (25 \times 20) = 500$ in², $A_s = 14$ in²

$\therefore \rho_g = \frac{14}{500} = .028$

$$P_a = 0.34 f_c' A_g (\rho_g m + 1) = 0.34 \times 3 \times 500 \times \left(0.028 \times \frac{60}{.85 \times 3} + 1\right)$$

$\therefore P_a = 846\,K$

$$n = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$$

Now, $$I_{utx} = \frac{25 \times 20^3}{12} + 2 \times (2 \times 9 - 1) \times 5 \times (10 - 2.5)^2 + 2 \times (2 \times 9 - 1) \times 2 \times (10 - 7.5)^2$$

$\therefore I_{utx} = 26654.17$ in⁴

$$\therefore S_{utx} = \frac{I_{utx}}{c} = \frac{26654.17}{10} = 2665.42 \text{ in}^3$$

$$I_{uty} = \frac{20 \times 25^3}{12} + 2 \times (2 \times 9 - 1) \times 4 \times (12.5 - 2.5)^2 + 2 \times (2 \times 9 - 1) \times 2 \times (12.5 - 7.5)^2$$

$\therefore I_{uty} = 41341.67$ in⁴

$$\therefore S_{uty} = \frac{I_{uty}}{c} = \frac{41341.67}{12.5} = 3307.33 \text{ in}^3$$

$$M_{fx} = 0.45 f_c' S_{utx}$$

$$= 0.45 \times 3 \times 2665.42$$

$$= 3598.32 \text{ K-in.}$$

$$M_{fy} = 0.45 f_c' S_{uty}$$

$$= 0.45 \times 3 \times 3307.33$$

$$= 4464.9 \text{ K-in.}$$

Now, $\frac{P}{P_a} + \frac{M_x}{M_{fx}} + \frac{M_y}{M_{fy}} = \frac{250}{846} + \frac{1000}{3598.32} + \frac{1500}{4464.9}$

$= 0.91 < 1$ (Design is OK)

Hence, according to reciprocal load method, The column section will be safe.

(b) <u>Bresler Equation</u>: The equation is, $\frac{1}{P_{xy}} = \frac{1}{P_x} + \frac{1}{P_y} - \frac{1}{P_o}$

Here, $P_o = P_a = 846$ Kip

$$P_x = \left(1 - \frac{M_x}{M_{fx}}\right) \times P_a = \left(1 - \frac{1000}{3598.32}\right) \times 846 = 610.89 \text{ K}$$

$$P_y = \left(1 - \frac{M_y}{M_{fx}}\right) \times P_a = \left(1 - \frac{1500}{4464.9}\right) \times 846 = 561.78 \text{ K}$$

$$\therefore \frac{1}{P_{xy}} = \frac{1}{610.89} + \frac{1}{561.78} - \frac{1}{846}$$

$$\therefore P_{xy} = 447.43 \text{K} > P = 250 \text{Kip}$$

(Design is OK)

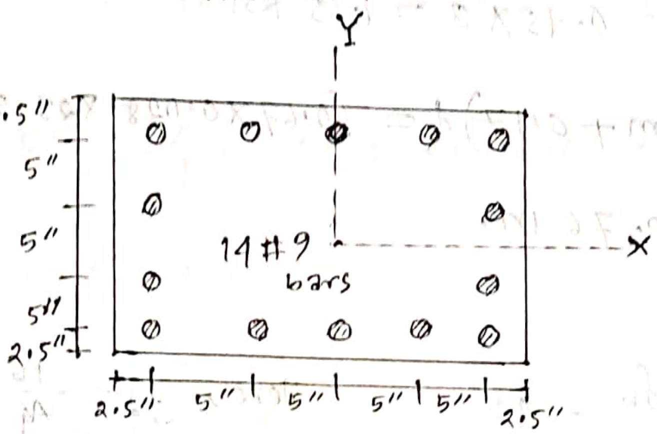
Hence, according to bresler equation, The column section will be safe.

[Column]

[WSD]

2017

For the column section shown in figure below. Draw the interaction diagram about x axis. Follow WSD method. Use $f_c' = 3$ ksi and $f_y = 60$ ksi



Solution:

$A_g = (20 \times 25) = 500$ in²

$A_s = (14 \times 1) = 14$ in²

$\rho_g = \frac{A_s}{A_g} = \frac{14}{500} = 0.028$

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.29 \simeq 9$

$$I_{ut}(x) = \frac{25 \times 20^3}{12} + 2 \times (2 \times 9 - 1) \times \overset{(5\times1)}{5} \times (10 - 2.5)^2 + 2 \times (2 \times 9 - 1) \times \overset{(2\times1)}{2} \times (10 - 5 - 2.5)^2$$

$= 26654.167$ in⁴

$\therefore S_{ut}(x) = \frac{I_{ut}}{c} = \frac{26654.167}{10} = 2665.42$ in

$P_{all} = 0.85\, A_g (.25 f_c' + \rho_g f_s) = 0.85 \times 500 \times (.25 \times 3 + 0.028 \times .4 \times 60)$

$\therefore P_{all} = 604.35$ K

$$m = \frac{f_y}{.85 f_c'} = \frac{60}{.85 \times 3} = 23.53$$

$$F_a = 0.34(1 + \rho_g m) f_c' = 0.34 \times (1 + 0.028 \times 23.53) \times 3 = 1.692 \text{ Ksi}$$

$$F_b = 0.45 f_c' = 0.45 \times 3 = 1.35 \text{ Ksi}$$

$$e_{bx} = (0.67 \rho_g m + 0.17) d = (0.67 \times 0.028 \times 23.53 + 0.17) \times 17.5$$

$$\therefore e_{bx} = 10.7 \text{ in.}$$

Now, $\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$ Here, $f_a = \frac{P_b}{A_g} = \frac{P_b}{500}$

$$f_b = \frac{M}{S_{ut}} = \frac{P_b e_b}{2665.42}$$

$$\frac{P_b/500}{1.692} + \frac{\frac{10.7 \times P_b}{2665.42}}{1.35} = 1$$

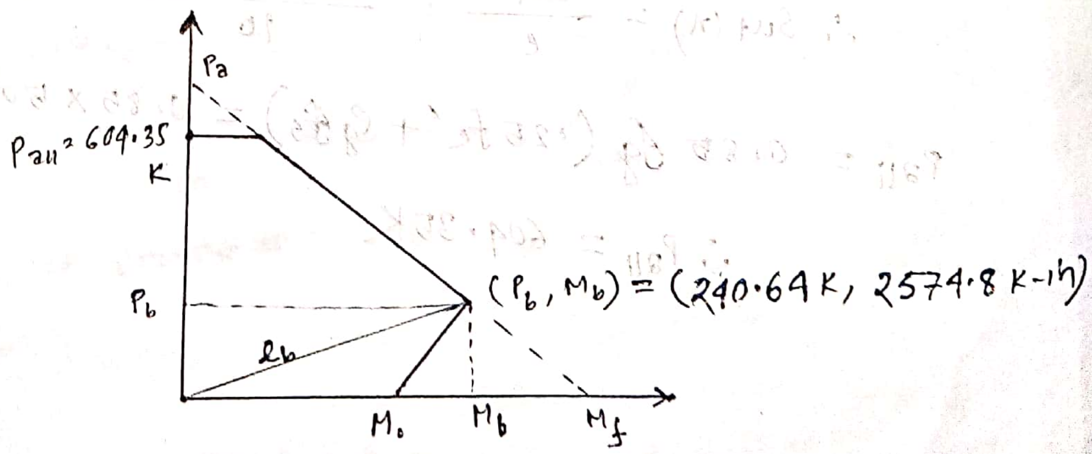
$$\Rightarrow P_b = 240.64 \text{ K}$$

$$\therefore M_b = P_b e_b = (240.64 \times 10.7) = 2574.8 \text{ K-in.}$$

$$M_o = 0.4 A_s f_y (d - d') = 0.4 \times (7 \times 1) \times 60 \times (17.5 - 2.5)$$

$$\therefore M_o = 2520 \text{ K-in}$$

Interaction Diagram:



2016 , 2015

A column section 16 in. diameter is reinforced with 8 # 10 bars. Determine the allowable ultimate load using $f_c' = 4$ ksi and $f_y = 60$ ksi. Also design the lateral reinforcemeement and draw the section.

Solution:

$$A_g = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (16)^2 = 201.0624 \text{ in}^2$$

$$A_{st} = (8 \times 1.27) = 10.16 \text{ in}^2$$

$$\rho_g = \frac{A_s}{A_g} = \frac{10.16}{201.0629} = 0.051$$

For spiral column, $\alpha = 0.85$ and $\phi = 0.75$

$\therefore$ The ultimate load, $P_u = \alpha \phi A_g \times [.85 f_c' + \rho_g (f_y - .85 f_c')]$

$$= .85 \times .75 \times 201.0624 \times [.85 \times 4 + 0.051 \times (60 - .85 \times 4)]$$

$$= 805.8 \text{ K}$$

Design of spiral:

$$D_c = (D - 2 \times c.c) = (16 - 2 \times 1.5) = 13 \text{ in.}$$

Minimum spiral ratio,

$$\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_c'}{f_y}$$

$$= 0.45 \times \left(\frac{\frac{\pi}{4} \times 16^2}{\frac{\pi}{4} \times 13^2} - 1\right) \times \frac{4}{60}$$

$$\rho_{sp} = 0.015444$$

Now,

$$\rho_{sp} = \frac{4 A_{sp}}{S D_c} \Rightarrow S = \frac{4 A_{sp}}{\rho_{sp} D_c}$$

provide # 3 bar as spiral , $\therefore$ Spacing, $S = \frac{4 \times .11}{.015444 \times 13} = 2.19''$

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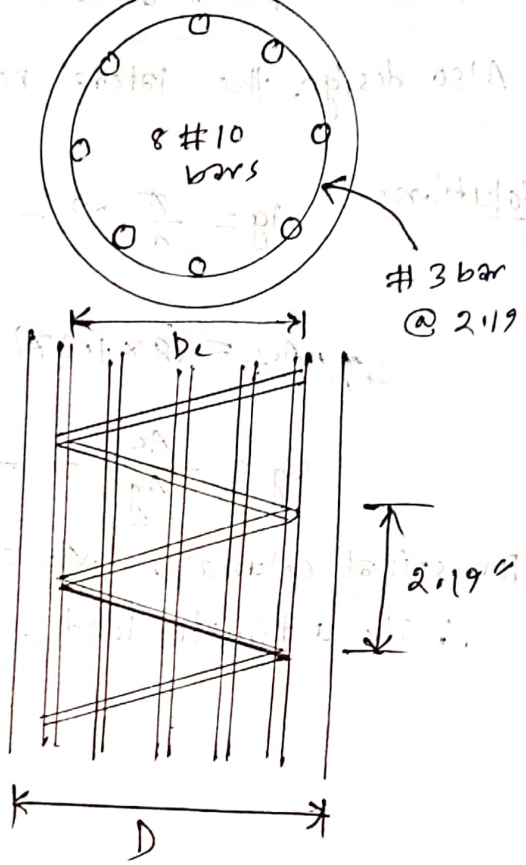
Again, spacing, (i) $s_{max} = 3''$

(ii) $s_{max} = \frac{1}{6} \times D_c = \frac{1}{6} \times 13 = 2.17''$

(iii) $s_{min} = 1''$.

$\therefore$ provide #3 bar @ $2.19''$.



2014, 2011

Design completely by WSD method a circular column to carry an axial load of 75 kips. Assume $f_c' = 4000$ psi and $f_s = 24000$ psi

Solution:

Let, $\rho_s = 0.02$ $\therefore$ for circular column, $\phi' = 1$

We know, $P_{all} = \phi' A_g (.25 f_c' + \rho_s f_s)$

$\Rightarrow 75 = 1 \times A_g (.25 \times 4 + 0.02 \times 24)$

$\therefore A_g = 50.68 \text{ in}^2$

$\Rightarrow \frac{\pi}{4} D^2 = 50.68$ $\therefore D = 8.03'' < 10''$

According to ACI, Diameter of circular column should not less than $10''$

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Hence, $D = 10''$ $\therefore A_g = \frac{\pi}{4} \times 10^2 = 78.54 \text{ in}^2$

Now, $P_{all} = \phi' A_g (.25 f_c' + \rho_s f_s)$

$\Rightarrow 75 = 1 \times 78.54 \, (.25 \times 4 + \rho_s \times 24)$

$\Rightarrow \rho_s = -0.002$

But, According to ACI code, $\rho_s = (0.01 - 0.08)$

Hence, $\rho_s = 0.01$

Now, $A_s = \rho_s A_g = (0.01 \times 78.54) = 0.7854 \text{ in}^2$

provide # 5 bar. $\therefore$ No of bar $= \frac{.7854}{.31} = 2.53 \simeq 3$

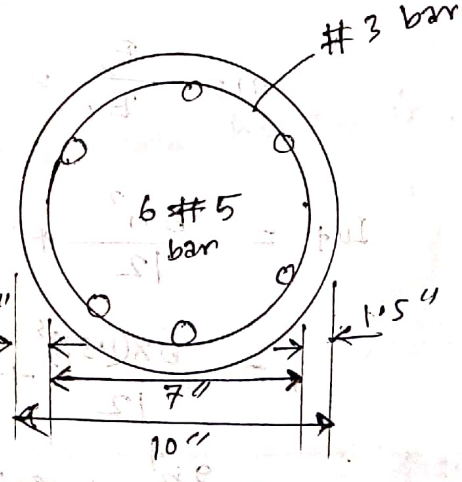
But, According to ACI recommendation,

A minimum of 6 longitudinal bars should be used in spiral column.

Hence provide 6 # 5 bar.

Design of spiral:

$D_c = (10 - 2 \times 1.5) = 7.0$ in



Minimum spiral ratio,

$\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_c'}{f_y}$

$= 0.45 \times \left(\frac{10^2}{7^2} - 1\right) \times \frac{4}{60}$

$= 0.03$ Let, provide # 3 bar.

Now,

spacing, $s = \frac{4 A_{sp}}{\rho_{sp} D_c}$

$\therefore s = \frac{4 \times .11}{.03 \times 7} = 2.095'' \simeq 2''$

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Hence, $D = 10''$ $\therefore A_g = \frac{\pi}{4} \times 10^2 = 78.54 \ in^2$

Now, $P_{all} = \phi' A_g (.25 f_c' + \rho_s f_s)$

$\Rightarrow 75 = 1 \times 78.54 (.25 \times 4 + \rho_s \times 24)$

$\Rightarrow \rho_s = -0.002$

But, According to ACI code, $\rho_s = (0.01 - 0.08)$

Hence, $\rho_s = 0.01$

Now, $A_s = \rho_s A_g = (0.01 \times 78.54) \simeq 0.7854 \ in^2$

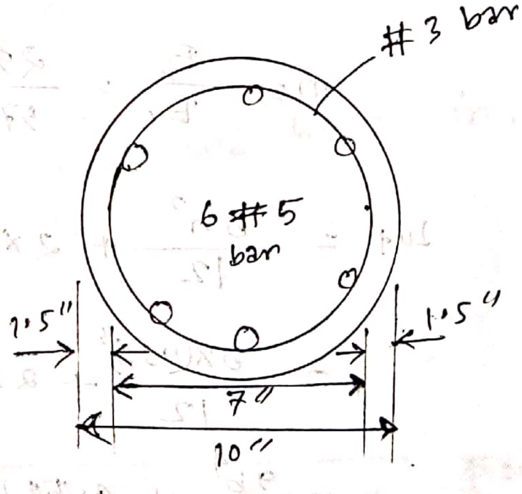
provide # 5 bar. $\therefore$ No of bar $= \frac{.7854}{.31} = 2.53 \simeq 3$

But, According to ACI recommendation, A minimum of 6 longitudinal bars should be used in spiral column.

Hence provide 6 # 5 bar.

<u>Design of spiral:</u>

$D_c = (10 - 2 \times 1.5) = 7.0$ in



Minimum spiral ratio,

$\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \times \frac{f_c'}{f_y}$

$= 0.45 \times \left(\frac{10^2}{7^2} - 1\right) \times \frac{4}{60}$

$= 0.03$ Let, provide # 3 bar.

Now,

spacing, $s = \frac{4 A_{sp}}{\rho_{sp} D_c}$

$\therefore s = \frac{4 \times .11}{.03 \times 7} = 2.095'' \simeq 2''$

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$F_a = 0.34 f_c' (1 + \rho_g m) = 0.34 \times 3 \times \left(1 + .02 \times \frac{40}{.85 \times 3}\right) = 1.34 \text{ ksi}$

$F_b = 0.45 f_c' = (0.45 \times 3) = 1.35$

$f_a = \frac{P}{A_g} = \frac{250}{1.5b^2}$

$f_b = \frac{M}{S_{ut}} = \frac{Pe}{S_{ut}} = \frac{250 \times 8}{S_{ut}} = \frac{2000}{S_{ut}}$

Now,

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} = 1$$

$$\Rightarrow \frac{250}{1.5b^2 \times 1.34} + \frac{2000}{1.35 \times \left[\frac{3b^3}{8} + 0.68\,b(.75b - 2.5)^2\right]} = 1$$

$\Rightarrow b = 16.44''$

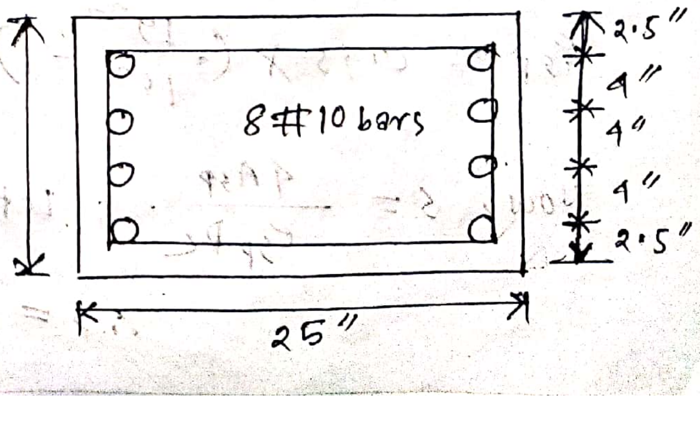
and, $h = 1.5b = (1.5 \times 16.44) = 24.66''$

Hence, Taking the column section, $b = 17$ in and $h = 25$ in.

$\therefore A_g = (17 \times 25) = 425 \text{ in}^2$

Now, $A_s = \rho_s A_g = (0.02 \times 425) \text{ in}^2 = 8.5 \text{ in}^2$

$\therefore$ provide 8 #10 bars.



USD

2012, 2010

Design completely a spirally reinforced circular column to support an axial dead load of 450 Kips and a live load of 200 Kips using $f_c' = 4000$ psi, $f_y = 60000$ psi and steel ratio of about 2.78%. Apply USD method.

Solution: The ultimate load, $P_u = 1.2 D.L + 1.6 L.L$

$$= (1.2 \times 450 + 1.6 \times 200)$$

$$\therefore P_u = 860 \text{ Kips.}$$

We know,

$$P_u = \alpha \phi A_g [.85 f_c' + \rho_s (f_y - .85 f_c')]$$

$$\Rightarrow 860 = .85 \times .75 \times [.85 \times 4 + .0278 \times (60 - 0.85 \times 4)] \times A_g$$

$$\Rightarrow A_g = 271.243$$

$$\Rightarrow \frac{\pi}{4} D^2 = 271.243 \quad \therefore D = 18.584'' \simeq 19''$$

$$\therefore A_g = \frac{\pi}{4}(19)^2 = 283.53 \text{ in}^2$$

$$\rho_s = \frac{A_s}{A_g} \Rightarrow A_s = \rho_s A_g = (0.0278 \times 283.53) = 7.88 \text{ in}^2$$

$\therefore$ Provide 8 # 9 bar

Design of spiral:

$$D_c = D - 2 \times c.c = 19 - (2 \times 1.5) = 16''$$

$$\rho_{sp} = 0.45 \times \left(\frac{19^2}{16^2} - 1\right) \times \frac{4}{60} = 0.0123$$

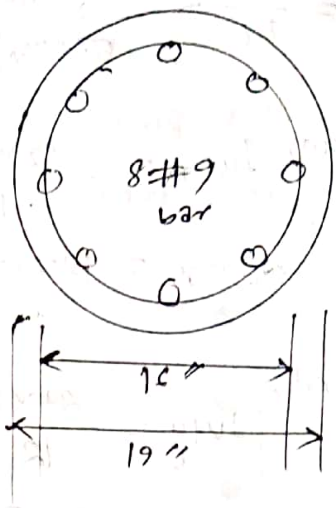
Now, $s = \dfrac{4 A_{sp}}{\rho_{sp} D_c}$ Let, provide # 3 bar as spiral

$$\therefore s = \frac{4 \times .11}{.0123 \times 16} = 2.236 \text{ in.}$$

Again, spacing, $S_{max} = 3''$

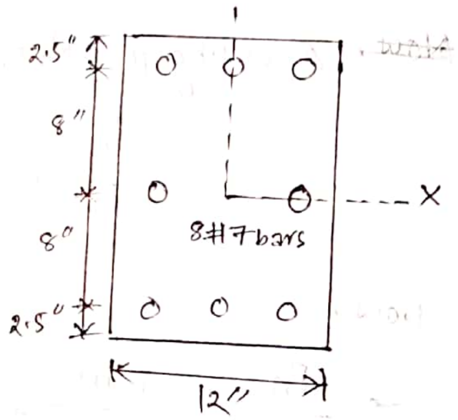
$S_{min} = 1''$

$\therefore$ provide #3 bar @ 2.236'' in.



2017

A corner column section is shown in figure below. The column is designed to carry a factored compressive load, $P_u = 200K$, a factored bending moment ~~about~~ $M_{ux} = 100$ K-ft about the x axis and a factored bending moment $M_{uy} = 75$ K-ft about y axis. Check whether the section is adequate if $f_c' = 4$ Ksi and $f_y = 60$ Ksi.



Solution:

Here,

$P_u = 200K = \phi P_n$

$\therefore P_n = \frac{200}{0.65} = 307.7$ K

$M_{ux} = \phi M_{nx} = 100$ K-ft

$\therefore M_{nx} = \frac{100}{0.65} = 153.85$ K-ft

and, $M_{uy} = \phi M_{ny} = 75$ K-ft

$\therefore M_{ny} = \frac{75}{0.65} = 115.4$ K-ft

$A_g = (21 \times 12) = 252$ in^2

$A_s = (8 \times 0.6) = 4.8$ in^2 $\qquad \therefore \rho_g = \frac{A_s}{A_g} = 0.019$

$P_a = 0.34 f_c' A_g (\rho_g m + 1) = 0.34 \times 4 \times 252 \times (0.019 \times \frac{60}{.85 \times 4} + 1)$

$\therefore P_a = 457.632$ K

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$$

Now, $$I_{utx} = \frac{12 \times 21^3}{12} + 2 \times (2\times 8 - 1) \times (3 \times 0.6) \times (10.5 - 2.5)^2$$

$\therefore I_{utx} = 12717 \text{ in}^4$ $\quad\therefore S_{utx} = \frac{12717}{10.5} = 1211.143 \text{ in}^3$

and, $$I_{uty} = \frac{21 \times 12^3}{12} + 2 \times (2 \times 8 - 1) \times (3 \times 0.6) \times (6 - 2.5)^2$$

$\therefore I_{uty} = 3685.5 \text{ in}^4$ $\quad\therefore S_{uty} = \frac{3685.5}{6} = 614.25 \text{ in}^3$

$$M_{fx} = 0.45 f_c' S_{utx} = (0.45 \times 4 \times 1211.143) = 2180.06 \text{ K-in.}$$

$$M_{fy} = 0.45 f_c' S_{uty} = (0.45 \times 4 \times 614.25) = 1105.65 \text{ K-in.}$$

According to Reciprocal load method, we know that,

$$\frac{P_n}{P_a} + \frac{M_{nx}}{M_{fx}} + \frac{M_{ny}}{M_{fy}} \leq 1$$

Now,

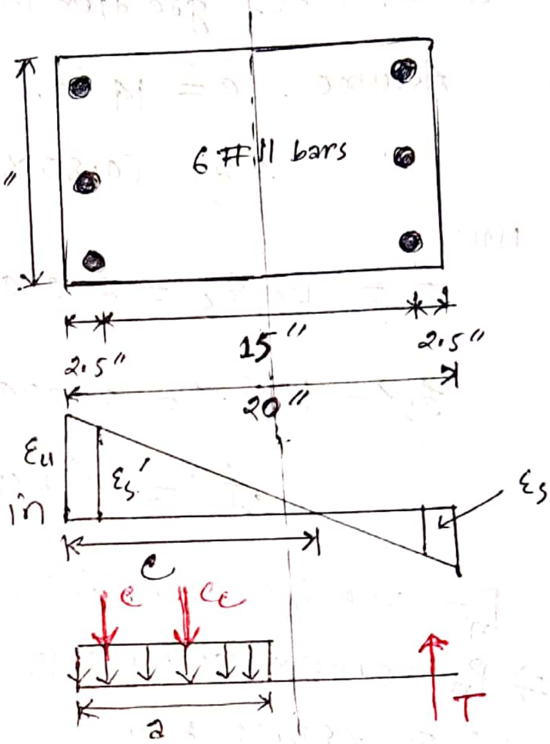
$$\frac{P_n}{P_a} + \frac{M_{nx}}{M_{fx}} + \frac{M_{ny}}{M_{fy}} = \frac{307.7}{457.632} + \frac{(153.85 \times 12)}{2180.06} + \frac{(115.4 \times 12)}{1105.65}$$

$$= 2.77 > 1$$

Hence, The column section is not adequate.

2015, 2013

A 15″×20″ column is reinforced with 6 #11 bars as shown in figure below. Find the ultimate load and corresponding moment for an eccentricity of e=8 inch. Assume $f_c' = 4$ ksi and $f_y = 60$ ksi



Solution:

We know,

$$c_b = \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \times d$$

$$\Rightarrow c_b = \frac{0.003}{0.003 + 0.002} \times 17.5$$

$$\therefore c_b = 10.5 \text{ in.}$$

$f_c' = 4$ ksi

$$a = \beta_1 c = (0.85 \times 10.5) = 8.925 \text{ in}$$

For balanced condition, $f_s = f_y$

$$\therefore f_s = 60 \text{ ksi}$$

$$f_s' = E_s \varepsilon_s' = 29000 \times \varepsilon_u \times \frac{c - d'}{c} = 29000 \times 0.003 \times \frac{10.5 - 2.5}{10.5}$$

$$\therefore f_s' = 66.29 \text{ ksi} > 60 \text{ ksi}$$

Hence, $f_s' = 60$ ksi

Now, $P_b = (C_c + C - T) = 0.85 f_c' a b + A_s' f_s' - A_s f_s$

$$\Rightarrow P_b = 0.85 \times 4 \times 8.925 \times 15 + (3 \times 1.56) \times 60 - (3 \times 1.56) \times 60$$

$$\therefore P_b = 455.175 \text{ K}$$

$$M_b = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C \times \left(\frac{h}{2} - d'\right) + T\left(d - \frac{h}{2}\right)$$

$$= 455.175 \times \left(10 - \frac{8.925}{2}\right) + 3 \times 1.56 \times 60 \times (10 - 2.5) + 3 \times 1.56 \times 60 \times (17.5 - 10)$$

$$\therefore M_b = 6732.53 \text{ K-in.}$$

$e_b = \frac{M_b}{P_b} = \frac{6732.53}{455.175} = 14.79$ in. $> e = 8$ in

Hence, compression region.

c will be greater than c_b.

Assume, $c = 14$ in.

$\therefore a = (0.85 \times 14) = 11.9$ in

Now,
$f_s = E_s \varepsilon_s = 29000 \times \frac{17.5-14}{14} \times 0.003 = 21.75$ ksi

$f_s' = E_s \varepsilon_s' = 29000 \times \frac{14-2.5}{14} \times 0.003 = 71.46$ ksi > 60 ksi

$\therefore f_s' = 60$ ksi

$P_n = 0.85 f_c' ab + A_s' f_s' - A_s f_s$

$\Rightarrow P_n = 0.85 \times 4 \times 11.9 \times 15 + 3 \times 1.56 \times 60 - 3 \times 1.56 \times 21.75$

$\therefore P_n = 785.91$ Kips.

and, $M_n = 0.85 f_c' ab \times (\frac{h}{2} - \frac{a}{2}) + A_s' f_s' (\frac{h}{2} - d') + A_s f_s (d - \frac{h}{2})$

$\Rightarrow M_n = 0.85 \times 4 \times 11.9 \times 15 \times (10 - \frac{11.9}{2}) + 3 \times 1.56 \times 60 \times (10-2.5) + 3 \times 1.56 \times 21.75 \times (17.5-10)$

$\therefore M_n = 5327.37$ K-in.

$\therefore e = \frac{M_n}{P_n} = \frac{5327.37}{785.91} = 6.78$ in $< e = 8$ in.

Hence, c will be less than 14.

Again, Assume, $c = 13$ in.

$\therefore a = (.85 \times 13) = 11.05$ in

Now, $f_s = E_s \varepsilon_s = 29000 \times 0.003 \times \frac{17.5-13}{13} = 30.115$ ksi

$f_s' = 60$ ksi

$P_n = 0.85 \times 4 \times 11.05 \times 15 + 3 \times 1.56 \times 60 - 3 \times 1.56 \times 30.11$

$= 703.412$ Kips.

and $M_n = 0.85 \times 4 \times 11.05 \times 15 \times (10 - 0.5 \times 11.05) + 3 \times 1.56 \times 60 \times 7.5$
$+ 3 \times 1.56 \times 30.11 \times 7.5$

$= 5684.92$ kin

$\therefore e = \dfrac{M_n}{P_n} = \dfrac{5684.92}{703.412} = 8.08$ which is very close to the given eccentricity $e = 8$ in.

Hence,

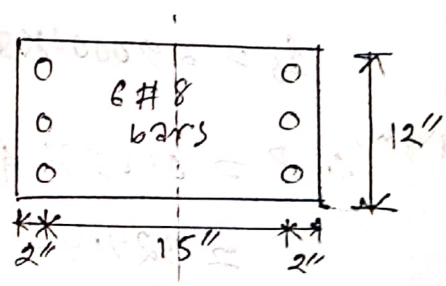
ultimate load, $P_u = (0.65 \times 703.412) = 457.22$ Kips.

and ultimate moment, $M_u = (0.65 \times 5684.92) = 3695.2$ K-in.

2014

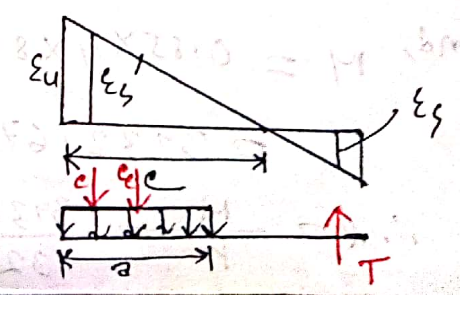
The column section shown in figure is reinforced with six No. 8 bars. Estimate by USD method, the ultimate load of the column for an eccentricity of 12 inch. Data given: $f_c' = 4$ Ksi and $f_y = 60$ Ksi

Solution: We know,



$C_b = \dfrac{\varepsilon_u}{\varepsilon_u + \varepsilon_y} \times d$

$\therefore C_b = \dfrac{0.003}{0.003 + 0.002} \times 17 = 10.2$ in.



$\therefore a = (0.85 \times 10.2) = 8.67$ in.

For balanced condition,

$f_s = f_y \quad \therefore f_s = 60$ Ksi

$f_s' = E_s \varepsilon_s' = 29000 \times 0.003 \times \frac{10.2-2}{10.2} = 69.94$ Ksi > 60 Ksi

$\therefore f_s' = 60$ Ksi

Now, $P_b = \underbrace{0.85 \times 4 \times 8.67 \times 12}_{C_c} + \underbrace{3 \times 0.79 \times 60}_{C} - \underbrace{3 \times 0.79 \times 60}_{T}$

$= 353.736$ K

$M_b = 0.85 \times 4 \times 8.67 \times 12 \times (9.5 - 0.5 \times 8.67) + 3 \times 0.79 \times 60 \times (9.5-2)$
$+ 3 \times 0.79 \times 60 \times (17 - 9.5)$

$= 3960.05$ Kin

$\therefore e_b = \frac{M_b}{P_b} = \frac{3960.05}{353.736} = 11.19$ in. $< e = 12''$

Hence, tension region.

c will be less than $c_b = 10.2$ in.

Assume, $c = 9.5$ in

$\therefore a = (0.85 \times 9.5) = 8.075$ in.

$f_s' = 29000 \times .003 \times \frac{9.5-2}{9.5} = 68.68$ Ksi > 60 Ksi $\therefore f_s' = 60$ Ksi

$f_s = 29000 \times 0.003 \times \frac{17-9.5}{9.5} = 68.68$ Ksi > 60 Ksi, $\therefore f_s = 60$ Ksi

Now, $\therefore P = 0.85 \times 4 \times 8.075 \times 12 + 3 \times 0.79 \times 60 - 3 \times 0.79 \times 60$

$= 329.46$ Kips.

and, $M = 0.85 \times 4 \times 8.075 \times 12 \times (9.5 - 0.5 \times 8.075) + 3 \times .79 \times 60 \times (9.5-2) +$
$3 \times .79 \times 60 \times (17 - 9.5)$

$= 3932.675$ K-in.

$\therefore e = \frac{M}{P} = \frac{3932.675}{329.46} = 11.94$ in $\simeq e = 12$in $\quad \therefore P_u = (0.65 \times 329.46)$ Kips
$= 214.15$ Kips.

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Class Test

Problem: In a six storied building frame columns are spaced at 15 ft on centers. Design an interior spiral column considering the following data and assumptions (i) slab thickness = 6 in. (ii) LL on slab = 100 psf (iii) columns are located at the intersections of beam of size 12 in x 24 in. (iv) The column can be designed as an axially loaded column (v) $f_c' = 5000$ psi and $f_y = 60000$ psi.
Design the column using USD and WSD methods and compare your results.

Solution :

load calculation : loaded area $= (15' \times 15') = 225 \text{ ft}^2$

(i) self weight of slab $= (\frac{t}{12} \times 150) = (\frac{6}{12} \times 150) = 75$ psf

$\therefore$ load from slab on column $= (75 \times 225) = 16875$ lb

(ii) load from beam on column $= (\frac{12}{12} \times \frac{24-6}{12} \times 150) \times (15+15)$ lb

$= 6750$ lb

Assume,

beam size (12"x24")

Diameter of spiral column = 24 in

$\therefore$ self weight of the column $= \frac{\pi}{4} \times (\frac{24}{12})^2 \times (10 - \frac{24}{12}) \times 150$

$= 3769.92$ lb

$\therefore$ Total dead load for one storey building $= (16875 + 6750 + 3769.92)$

$= 27394.92$ lb

$\simeq 27395$ lb

for six storey building, Total D.L $= (27395 \times 6) = 164370$ lb

$= 164.37$ K

Live load on a storey = (100 × 225) = 22500 lb

∴ Total live load for six storey building = (6 × 22500) lb
= 135000 lb
= 135 K

USD Method:

Ultimate load, $W_u = 1.2 \times D.L + 1.6 \times L.L$
$= (1.2 \times 164.37 + 1.6 \times 135) = 413.244$ K

We know,

$$P_u = \alpha \phi A_g [0.85 f_c' + \rho_s (f_y - 0.85 f_c')]$$

$$\Rightarrow 413.244 = 0.85 \times 0.75 \times [0.85 \times 5 + \rho_s (60 - 0.85 \times 5)] \times \underbrace{\frac{\pi}{4} \times (24)^2}_{A_g\ (in^2)}$$

$$\Rightarrow \rho_s = -0.05$$

But, ACI recommendation $\rho_s = (0.01 - 0.08)$

Hence, $\rho_s = 0.01$

$\therefore \frac{A_s}{A_g} = 0.01 \Rightarrow A_s = 0.01 \times \frac{\pi}{4} \times 24^2 = 4.52\ in^2$

Provide 8 #6 bars.

Spiral design:

$D_c = D - 2 \times c.c = (24 - 2 \times 1.5) = 21$

Minimum spiral ratio, $\rho_{sp} = 0.45 \times \left(\frac{A_g}{A_c} - 1\right) \frac{f_c'}{f_y}$

$= 0.45 \times \left(\frac{24^2}{21^2} - 1\right) \times \frac{5}{60}$

$= 0.0115$

Let, provide #3 bar,

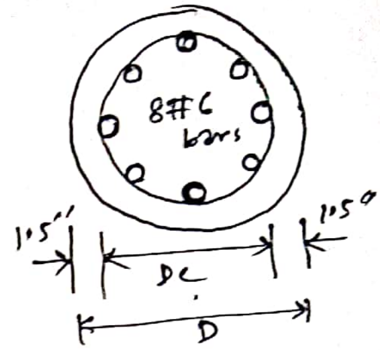
Now, $\rho_{sp} = \frac{4 A_{sp}}{s \times D_c} \Rightarrow$ spacing, $s = \frac{4 \times 0.11}{0.0115 \times 21} = 1.82'' \simeq 1.75''$

But, (i) $S_{max} = 3 in.$

(ii) $S_{max} = \frac{1}{6} \times D_c = \frac{1}{6} \times 21 = 3.5''$

(iii) $S_{min} = 1 in.$

$\therefore$ provide #3 bar @ $1.75''$ c/c



WSD Method:

working load, $W = (D.L + L.L) = (164.37 + 135)$

$= 299.37 \simeq 300$ lb

we know,

$$P_{au} = \phi' A_g [.25 f_c' + \rho_s f_s]$$

$$\Rightarrow 300 = 1 \times \frac{\pi}{4} \times 24^2 \times [.25 \times 5 + \rho_s \times 0.4 \times 60]$$

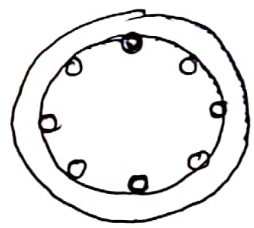
$$\Rightarrow \rho_s = -0.02$$

But ACI code recommend, $\rho_s = (0.01 - 0.08)$

$\therefore \rho_s = 0.01$

$\therefore \frac{A_s}{A_g} = 0.01$ $\therefore A_s = 0.01 \times \frac{\pi}{4} \times 24^2 = 4.52 in^2$

$\therefore$ provide 8 #6 bars.



Spiral Design: (Same)

provide #3 bar @ $1.75''$ c/c

Retaining Wall

Retaining wall: Retaining walls are structures used to hold back masses of earth or other loose materials where conditions make it impossible to let those masses assume their natural slopes.

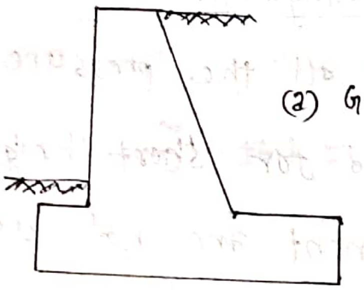
Types of Retaining wall:

There are three types of retaining walls:

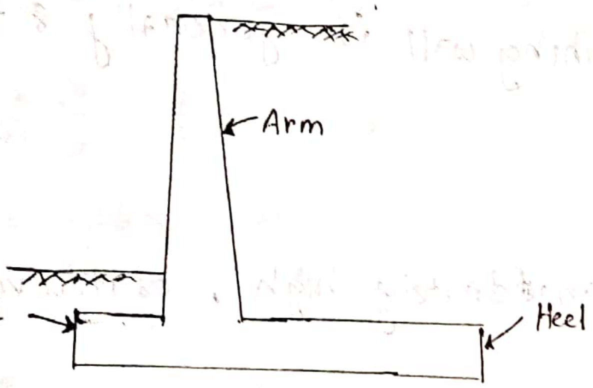
(i) Gravity retaining wall

(ii) cantilever retaining wall

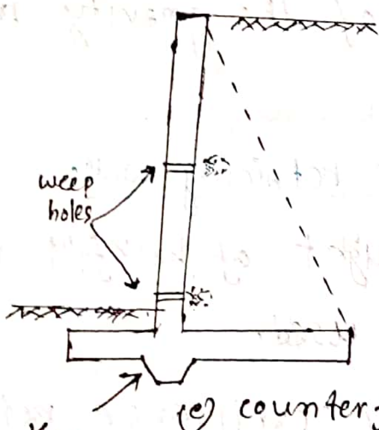
(iii) counter fort wall



(a) Gravity retaining wall



(b) cantilever retaining wall



(c) counter fort wall

Fuction of Retaining wall:

1. To Provide functional support for keeping soil in place
2. To prevent sink holes and eliminate the eye sore of dirt piles and hills
3. To prevent flooding.
4. To reduce maintenance and to prevent erosion
5. To prevent damage to property or sorrounding structures.

Advantages / functions of Key:

(i) It helps to provide reinforcement

(ii) Friction occurs between soil layers which increases sliding resistance.

(iii) It increases passive pressure.

Gravity retaining wall:

(i) It resist all the pressure with its own weight.

(ii) It is used for low height of backfill.

(iii) Reinforcement are not used in gravity retaining wall.

(iv) Height of the gravity retaining wall is generally 8 to 10 ft.

Cantilever retaining wall:

(i) If the height of backfill is moderately high, cantilever retaining wall is used.

(ii) Height of cantilever retaining wall is generally 12 ft to 20 ft.

(iii) Reinforcement are used in cantilever retaining wall.

Gravity Retaining wall

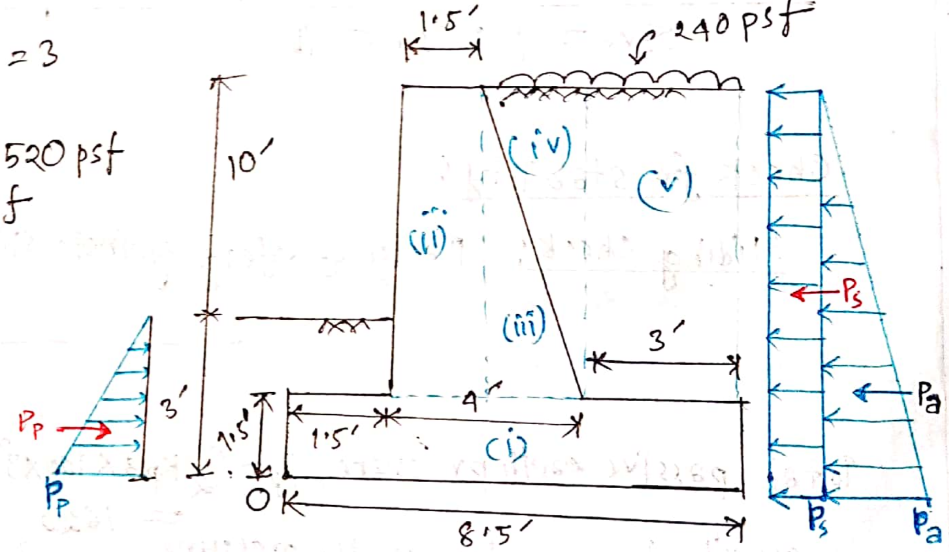
\# Design a gravity retaining wall to retain a 10 ft high backfill. Assume (i) Unit weight of soil 120 lb/ft^3 (ii) Angle of internal friction, $\phi = 30°$ (iii) co-efficient of friction at base between concrete and soil, $f = 0.5$ (iv) Bearing capacity of soil = 3000 psf (v) surcharge = 240 psf.

Solution: Considering overall height, $H = (10+3) = 13$ ft

$\therefore$ width of the base, $B = \frac{2}{3} \times H = \left(\frac{2}{3} \times 13\right) = 8.67$ ft $\simeq 8.5$ ft

$\left(\text{or}, \frac{1}{8} H\right) \leftarrow$ Depth of the base $= \frac{1}{10} H = \frac{1}{10} \times 13 = 1.3$ ft $\simeq 1.5$ ft

$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}$ $\therefore K_p = 3$



$P_a = K_a \gamma H = \left(\frac{1}{3} \times 120 \times 13\right) = 520$ psf

$P_s = q K_a = \left(\frac{1}{3} \times 240\right) = 80$ psf

$\therefore$ Active pressure,

$P_a = \left(\frac{1}{2} \times 520 \times 13\right)$ lb

$= 3380$ lb

and, $P_s = (80 \times 13)$ lb

$= 1040$ lb

$\therefore$ Total lateral earth pressure $= P_a + P_s = (3380 + 1040) = 4420$ lb

Over turning moment, $M_o = P_a \times \frac{H}{3} + P_s \times \frac{H}{2} = \left(3380 \times \frac{13}{3} + 1040 \times \frac{13}{2}\right)$

$= 21406.67$ lb-ft

Section	W (lb)	Moment Arm (ft)	Resisting moment (lb-ft)
(i)	$(8.5 \times 1.5 \times 150) = 1912.5$ plain concrete	4.25	8128.125
(ii)	$(1.5 \times 11.5 \times 120) = 2070$ Masonry	$\left(1.5 + \frac{1.5}{2}\right) = 2.25$	4657.5
(iii)	$\left(\frac{1}{2} \times 2.5 \times 11.5 \times 120\right) = 1725$ Masonry	$\left(3 + \frac{2.5}{3}\right) = 3.833$	6612.5
(iv)	$\left(\frac{1}{2} \times 2.5 \times 11.5 \times 120\right) = 1725$ soil	$\left(3 + \frac{2.5 \times 2}{3}\right) = 4.667$	8050
(v)	$(3 \times 11.5 \times 120) = 4140$ soil	$\left(5.5 + \frac{3}{2}\right) = 7$	28980

$\Sigma W = 11572.5$ lb $\qquad \Sigma M_R = 56428.125$ lb-ft

Check for stability:

Sliding check: Factor of safety against sliding, $F.S = \frac{\text{Resisting force}}{\text{Sliding force}} = \frac{W \times f}{(P_a + P_s)}$

$= \frac{11572.5 \times 0.5}{4420}$

$= 1.309 < 1.5$ (not OK)

Total passive earth pressure, $P_p = \left(\frac{1}{2} \times 3 \times 120 \times 3^2\right) = 1620$

considering passive earth pressure,

$$F.S = \frac{W \times f + P_p}{P_a + P_s} = \frac{11572.5 \times 0.5 + 1620}{4420}$$

$\therefore F.S. = 1.68 > 1.50$

(OK)

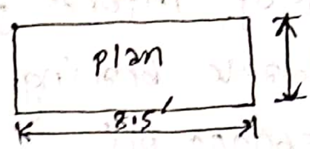
Over turning check:

Factor of safety against overturning, $f.s. = \dfrac{\text{resisting moment}}{\text{over turning moment}} = \dfrac{M_R}{M_o}$

$$= \frac{56428.125}{21406.67}$$

$$= 2.636 > 1.5 \text{ (OK)}$$

Check for soil pressure:



$A = (B \times 1) = 8.5\ ft^2$

$I = \dfrac{1 \times B^3}{12} = \dfrac{8.5^3}{12} = 51.177\ ft^4$

$a = \dfrac{M_R - M_o}{W} = \dfrac{56428.125 - 21406.67}{11572.5} = 3.026\ ft$

$e = \left|\dfrac{B}{2} - a\right| = \left|\dfrac{8.5}{2} - 3.026\right| = 1.224\ ft$

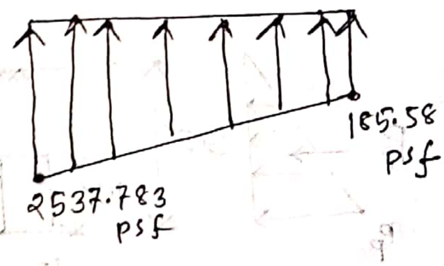
$c = \dfrac{B}{2} = \dfrac{8.5}{2} = 4.25$; $M = W \times e = (11572.5 \times 1.224)$ lb-ft

$\therefore M = 14164.74$ lb-ft

Now, $\sigma_1 = \dfrac{W}{A} + \dfrac{Mc}{I}$

$$= \frac{11572.5}{8.5} + \frac{14164.74 \times 4.25}{51.177}$$

$$= 2537.783 \text{ psf} < 3000 \text{ psf (OK)}$$



$\sigma_2 = \dfrac{W}{A} - \dfrac{Mc}{I}$

$$= \frac{11572.5}{8.5} - \frac{14164.74 \times 4.25}{51.177}$$

$$= 185.158 \text{ psf} < 3000 \text{ psf (OK)}$$

Cantilever Retaining Wall

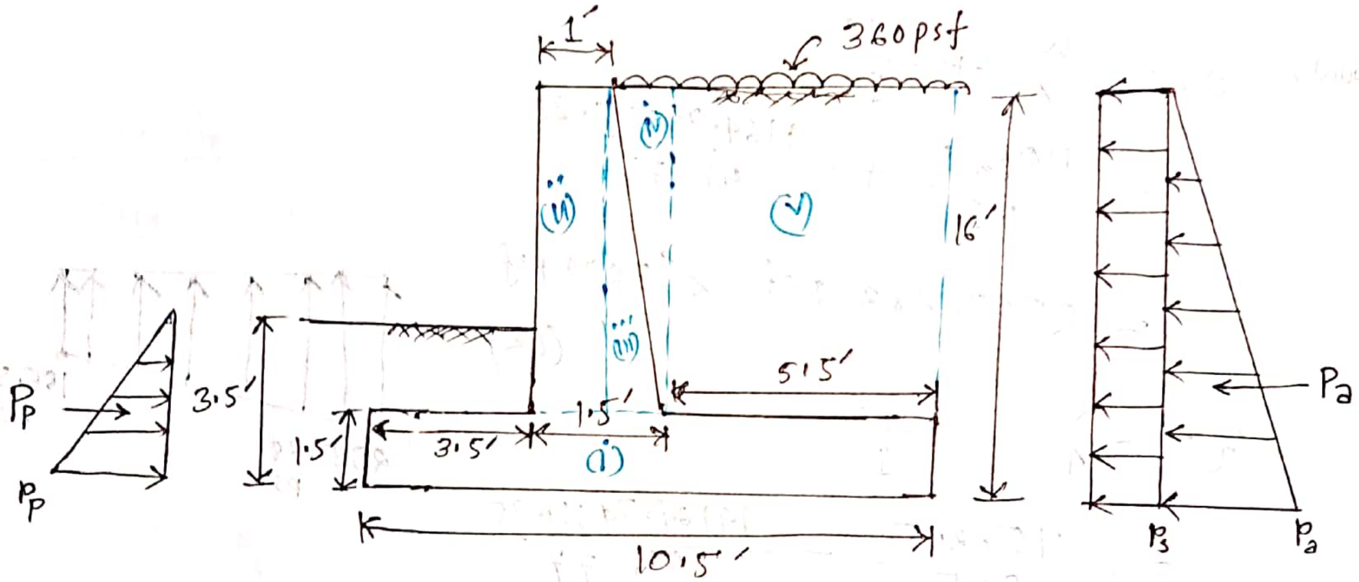
A cantilever retaining wall is to be designed for an overall height of 16'. Design the wall using the following data:

(i) Live load surcharge = 360 psf (ii) unit weight of soil = 120 pcf
(iii) Angle of interal friction, $\phi = 30°$ (iv) co-efficient of friction = 0.5
(v) Allowable bearing capacity = 8000 psf. (vi) $f_c' = 4000$ psi
(vii) $f_y = 60000$ psi

Solutions: Given, overall height, $H = 16$ ft

Width of the base, $B = \frac{2}{3} \times H = \frac{2}{3} \times 16 = 10.67 \simeq 10.5$ ft

Depth of the base, $B = \frac{1}{10} H = \left(\frac{1}{10} \times 16\right) = 1.6 \simeq 1.5$ ft



$K_a = \frac{1 - \sin 30}{1 + \sin 30} = \frac{1}{3}$ ∴ $K_p = 3$

$P_s = q K_a = (360 \times \frac{1}{3}) = 120$ psf

$P_a = K_a \gamma H = (\frac{1}{3} \times 120 \times 16) = 640$ psf

$P_p = K_p \gamma H_1 = (3 \times 120 \times 3.5) = 1260$ psf

Total active pressure $= (P_a + P_s) = (\frac{1}{2} \times 640 \times 16) + (120 \times 16)$

$= (5120 + 1920) = 7040$ lb

Total passive pressure $= (\frac{1}{2} \times 1260 \times 3.5) = 2205$ lb

Total overturning moment, $M_o = P_a \times \frac{H}{3} + P_s \times \frac{H}{2}$

$= 5120 \times \frac{16}{3} + 1920 \times \frac{16}{2}$

$= 42666.67$ lb-ft

Section	W (lb)	Moment Arm, $\bar{x}$ (ft)	Resisting Moment, M_R (lb-ft)
(i)	$10.5 \times 1.5 \times 150 = 2362.5$	$\frac{10.5}{2}$	12403.125
(ii)	$2 \times 14.5 \times 150 = 2175$	$(3.5 + 0.5)$	8700
(iii)	$\frac{1}{2} \times 0.5 \times 14.5 \times 150 = 543.75$	$(4.5 + \frac{0.5}{3})$	2537.5
(iv)	$\frac{1}{2} \times 0.5 \times 14.5 \times 120 = 435$	$(4.5 + \frac{2}{3} \times 0.5)$	2102.5
(v)	$(5.5 \times 14.5 \times 120) = 9570$	$(5 + \frac{5.5}{2})$	74167.5
	$\Sigma W = 15086.25$		$\Sigma = 99910.625$

Check for stability:

overturning check:

Factor of safety against overturning, $F.S = \frac{M_R}{M_o}$

$= \frac{99910.625}{42666.67}$

$= 2.34 > 1.5$

(OK)

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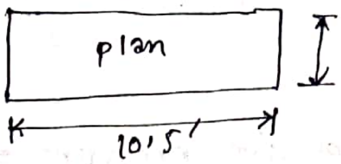
Check for soil pressure:

$A = (1 \times B) = (1 \times 10.5) = 10.5\, ft^2$



$I = \frac{1 \times B^3}{12} = \frac{1 \times (10.5)^3}{12} = 96.47\, ft^4$

$a = \frac{M_R - M_o}{W} = \frac{99910.625 - 42666.67}{15086.25} = 3.794\ ft$

$e = \left(\frac{B}{2} - a\right) = \left(\frac{10.5}{2} - 3.794\right) = 1.456\ ft$

$M = (W \times e) = (15086.25 \times 1.456) = 21956.58\ lb\text{-}ft$

$\therefore \sigma_1 = \frac{W}{A} + \frac{MC}{I} = \frac{15086.25}{10.5} + \frac{21965.58 \times \frac{10.5}{2}}{96.47} = 2632.2\ psf < 8000\ psf$ (OK)

and, $\sigma_2 = \frac{W}{A} - \frac{MC}{I} = \frac{15086.25}{10.5} - \frac{21965.58 \times \frac{10.5}{2}}{96.47} = 241.4\ psf < 8000\ psf$ (OK)

Sliding check:

Factor of safety against sliding, $F.S = \frac{W \times f}{P_a + P_s} = \frac{15086.25 \times 0.5}{7040}$

$= 1.07 < 1.5$ (Not OK)

Now, considering passive earth pressure,

$F.S = \frac{15086.25 \times 0.5 + 2205}{7040} = 1.385 < 1.5$ (Not OK)

Hence, Key should be provided to increase sliding resistance.

After providing Key, friction at base occurs in two ways. Along abe friction occurs between soil layers. Then $f = \tan\phi$

Along cdef, friction occurs between concrete and soil. Then $f = 0.5$

$PQ = \frac{(2632.2 - 241.4)}{10.5} \times (10.5 - 3.75) = 1536.94\ psf$

$\therefore OQ = (1536.94 + 241.4) = 1778.34\ psf$

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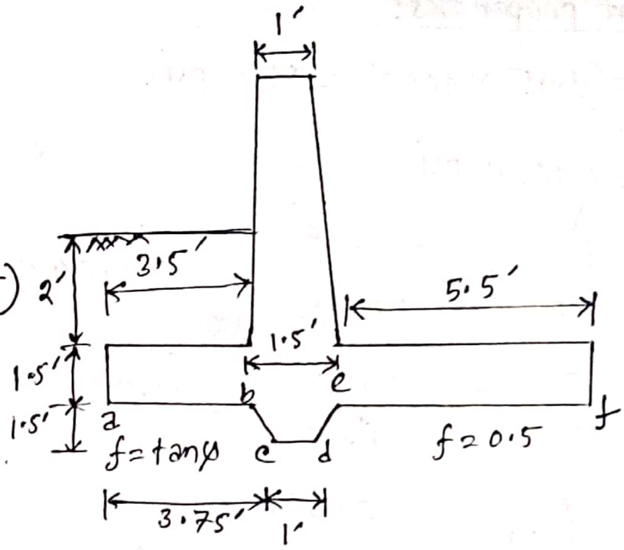
$$W_1 = \frac{(2632.2 + 1778.34)}{2} \times 3.75$$

$$= 8269.7625 \text{ psf}$$

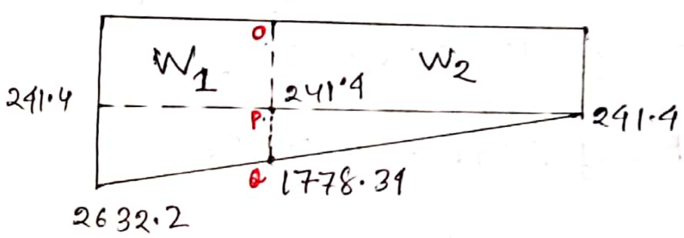


$$W_2 = \frac{(1778.34 + 241.4)}{2} \times (10.5 - 3.75)$$

$$= 6816.63 \text{ psf}$$

Now, Factor of safety against

Sliding, $F.S. = \dfrac{W_1 \tan\phi + W_2 f}{P_a + P_s}$



$$= \frac{6816.63 \times .5 + 8269.7625 \times \tan 30^\circ}{7040}$$

$$= 1.16 < 1.5$$

(Not ok)

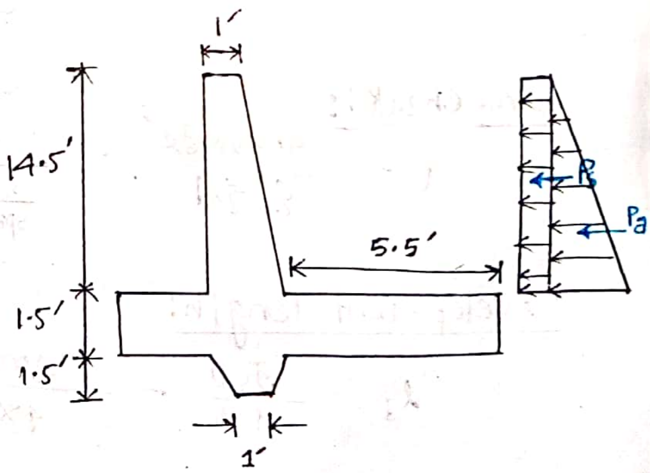
considering passive earth pressure,

$$P_p = \frac{1}{2} \times 3 \times 120 \times 5^2 = 4500 \text{ psf}$$

$$F.S = \frac{8264.7625 \times \tan 30^\circ + 6816.63 \times 0.5 + 4500}{7040} = 1.80 > 1.5$$

(ok)

Design of stem:



$$V_{max} = \left[\underbrace{\frac{1}{3} \times 360 \times 14.5}_{P_s} + \frac{1}{2} \times \underbrace{\left(\frac{1}{3} \times 120 \times 14.5\right) \times 14.5}_{P_a}\right]$$

$$\therefore V_{max} = 5945 \text{ lb}$$

$$\text{Max. Moment} = \left(\frac{1}{3} \times 360 \times 14.5 \times \frac{14.5}{2}\right) + \left[\frac{1}{2} \times \left[\frac{1}{3} \times 120 \times 14.5\right] \times 14.5 \times \frac{14.5}{3}\right]$$

$$= 32939.167 \text{ lb-ft}$$

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Relevant properties:

$f_c = (0.45 \times 4000) = 1800$ psi

$f_s = 24000$ psi

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$

$r = \frac{f_s}{f_c} = \frac{24000}{1800} = 13.33$

$K = \frac{n}{n+r} = 0.375$

$j = 1 - \frac{K}{3} = 0.875$

$R = \frac{1}{2} f_c' j K = 295.3125$

depth check:

$d = \sqrt{\frac{M}{Rb}}$

$= \sqrt{\frac{32939.167 \times 12}{295.3125 \times 12}}$

$= 10.56$ in.

$d_{eff} = t - c.c. - \frac{\phi}{2}$

$= 18 - 3 - \frac{7}{2 \times 8}$

$= 14.56$ in > 10.56 in.

(OK)

Reinforcement Calculation:

$A_s = \frac{M}{f_s j d} = \frac{32939.167 \times 12}{24000 \times 0.875 \times 14.56} = 1.3$ in²/ft

provide # 7 bar @ $\frac{0.6 \times 12}{1.3} = 5.54'' \simeq 5.5''$ c/c

A_s (min) $= 0.0018\,bt = 0.0018 \times 12 \times 18 = 0.39$ in²/ft

provide # 4 bar @ $\frac{0.2 \times 12}{0.39} = 6.46'' \simeq 6''$ c/c

Bond stress:

$U = \frac{V_{max}}{\Sigma_o\, jd} = \frac{V_{max}}{\frac{b}{spacing} \times \pi \times \phi \times jd} = \frac{5945}{\frac{12}{5.5} \times \pi \times \frac{7}{8} \times 0.875 \times 14.56}$

$= 77.80$ psi

Development length:

$l_d = \frac{f_s D}{4U} = \frac{24000 \times 7/8}{4 \times 77.80} = 67.5$ in.

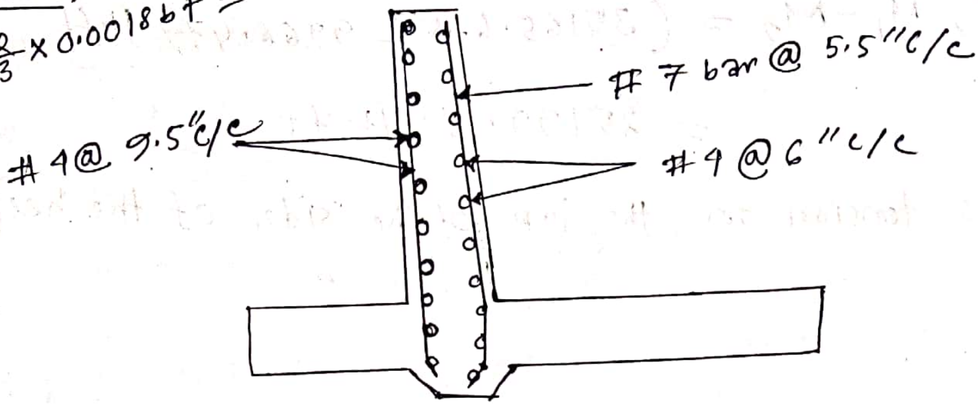
Shear check: $V_{dev} = \frac{V_{max}}{bd} = \frac{5945}{12 \times 14.56} = 34$ psi

$V_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{4000} = 69.57$ psi $> V_{dev}$

(OK)

Working diagram:

$\frac{2}{3} \times 0.0018bt = 0.26 \text{ in}^2$



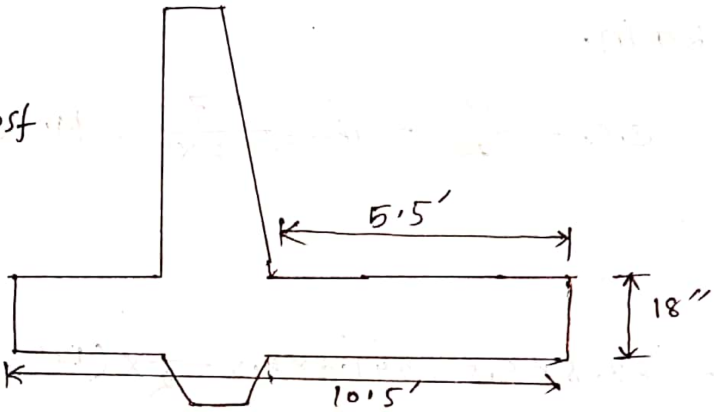
Design of Base:

Heel Slab design:

(i) self weight $= \left(\frac{t}{12} \times 150\right)$

$= \left(\frac{18}{12} \times 150\right)$ psf

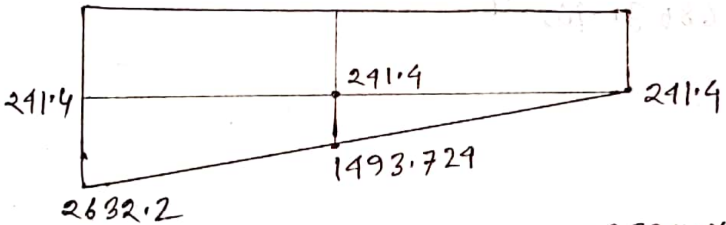
$= 225$ psf



(ii) weight of soil

$= (14.5 \times 120) = 1740$ psf

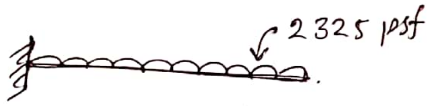
(iii) surcharge $= 360$ psf



Total downward force

$= (225 + 1740 + 360)$ psf

$= 2325$ psf.



Moment, $M_1 = \frac{2325 \times (5.5)^2}{2} = 35165.625$ lb-ft

$M_2 = 241.5 \times 5.5 \times \frac{5.5}{2} + \frac{1}{2} \times (1493.729 - 241.4) \times 5.5 \times \frac{5.5}{3}$

$= 9966.47$ lb-ft

Net Moment, $M_1 - M_2 = (35165.625 - 9966.47)$ lb-ft

$= 25199.89$ lb-ft

(creats tension on the top of the side of the heel)

depth check:

$$d = \sqrt{\frac{M}{Rb}}$$

$$= \sqrt{\frac{25199.89 \times 12}{295.3125 \times 12}}$$

$= 9.24$ in.

$d_{eff} = t - c.c. - \frac{\phi}{2} = 18 - 3 - \frac{7}{2 \times 8} = 14.56$ in > 9.24 in.

(OK)

$\rightarrow$

* $V_{max} = 2325 \times 5.5 - 241.4 \times 5.5 - \frac{1}{2} \times (1493.724 - 241.4) \times 5.5$

$= 6853.409$ lb

Reinforcement calculation:

$$A_s = \frac{25199.89 \times 12}{24000 \times 0.875 \times 14.5} = 0.993 \text{ in}^2$$

provide # 7 bar @ $\frac{0.6 \times 12}{0.993} = 7.25'' \ c/c$

$A_{s(min)} = 0.0018bt = 0.0018 \times 12 \times 18 = 0.39 \text{ in}^2$

provide # 4 bar @ $\frac{.21 \times 12}{.39} = 6.46'' \simeq 6.0'' \ c/c$

Bond stress: $U = \frac{V_{max}}{\Sigma_o jd} = \frac{*V_{max}}{\frac{b}{spacing} \times \pi \times \phi \times jd} = \frac{6853.409}{\frac{12}{7.25} \times \pi \times \frac{7}{8} \times 0.875 \times 14.56}$

$\therefore U = 118.23$ psi

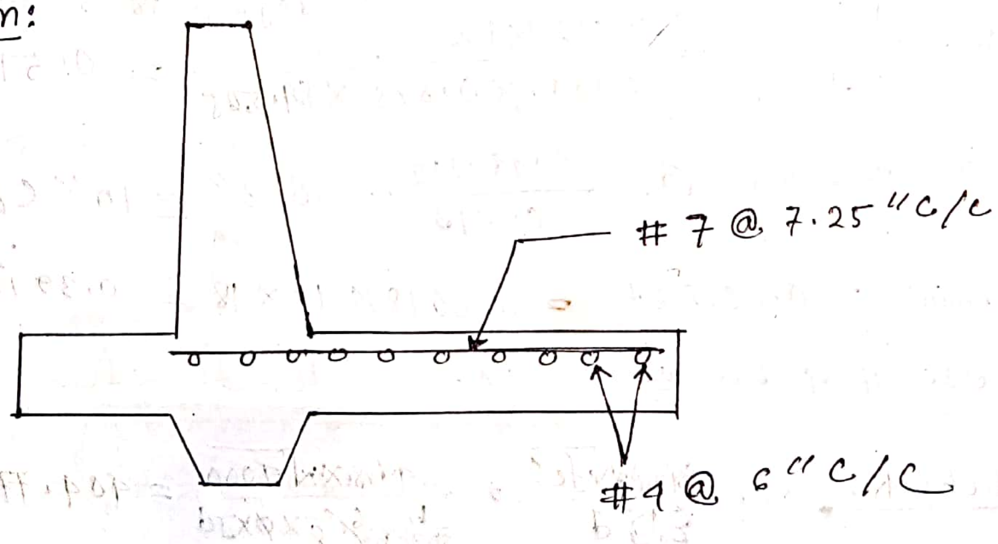
Development length: $l_d = \frac{f_s D}{4U} = \frac{24000 \times 7/8}{4 \times 118.23} = 44.4$ in.

Shear check: $v_{dev} = \frac{V_{max}}{bd} = \frac{V_{max}}{12 \times 14.56} = \frac{6853.409}{12 \times 14.56} = 39.2$ psi

$v_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{4000} = 69.57 \text{ psi} > v_{dev}$

(OK)

working diagram:



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Design of toe slab:

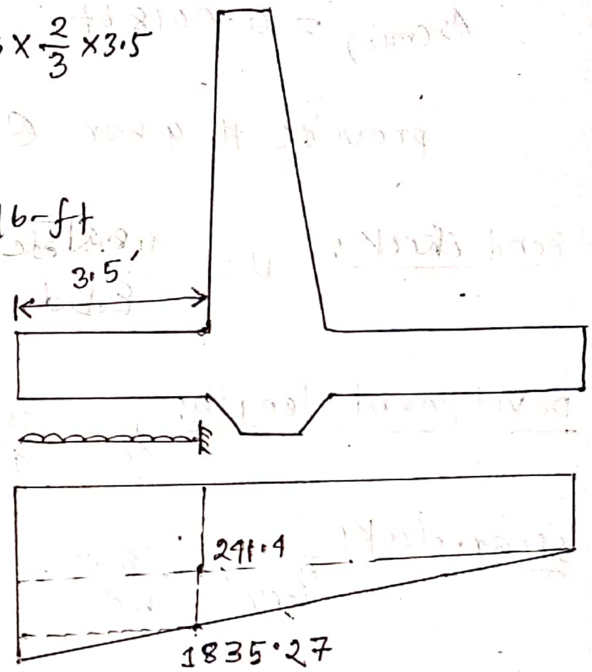
load calculation: (i) self weight of toe $= \left(\frac{t}{12} \times 150\right) = \left(\frac{8}{12} \times 150\right) = 225$ psf.

(ii) weight of soil above toe may be neglected.

$$\therefore M_1 = 1835.27 \times 3.5 \times \frac{3.5}{2} + \frac{1}{2} \times (2632.2 - 1835.27) \times 3.5 \times \frac{2}{3} \times 3.5$$

$$= 14495.1596 \text{ lb-ft}$$

$$\therefore M_2 = \frac{wL^2}{2} = \frac{225 \times 3.5^2}{2} = 1378.125 \text{ lb-ft}$$



Net moment,

$$\therefore M_1 - M_2 = (14495.1596 - 1378.125) \text{ lb-ft}$$

$$= 13117 \text{ lb-ft}$$

creats tension on the bottom of toe.

depth check: $d = \sqrt{\frac{13117 \times 12}{295.3125 \times 12}} = 6.66$ in.

Reinforcement calculation:

$d_{eff} = 18 - 3 - \frac{6}{2 \times 8} = 14.625'' > d$ (OK)

$$A_s = \frac{M}{f_s j d} = \frac{13117 \times 12}{24000 \times 0.875 \times 14.625} = 0.513 \text{ in}^2$$

use # 6 bar @ $\frac{0.44 \times 12}{0.513} = 10.3'' \simeq 10''$ C/C

$A_{s(min)} = 0.0018\, bt = 0.0018 \times 12 \times 18 = 0.39$ in²

use # 4 bar @ 6'' C/C

Bond stress: $U = \frac{V_{max}}{\Sigma_o j d} = \frac{V_{max}}{\frac{b}{spacing} \times \pi \times \phi \times jd}$

Here,

$V_{max} = 1835.27 \times 3.5 + \frac{1}{2} \times (2632.2 - 1835.27) \times 3.5 - 225 \times 3.5 = 7030.6$ lb

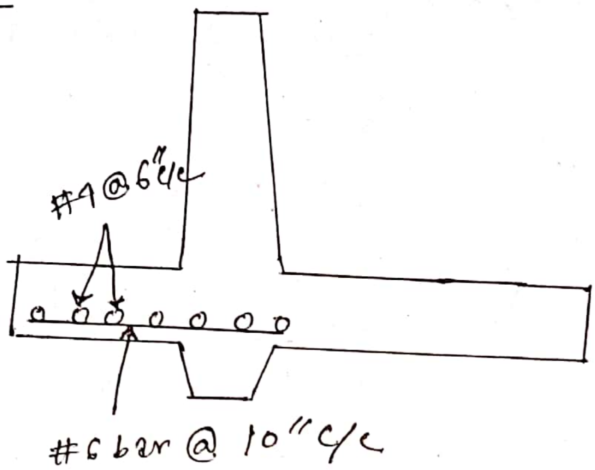
$$\therefore U = \frac{7030.6}{\frac{12}{10} \times \pi \times \frac{6}{8} \times 0.875 \times 14.625} = 194.3 \text{ psi}$$

development length: $l_d = \frac{f_s D}{4U} = \frac{24000 \times 6/8}{4 \times 194.3} = 23.16$ in.

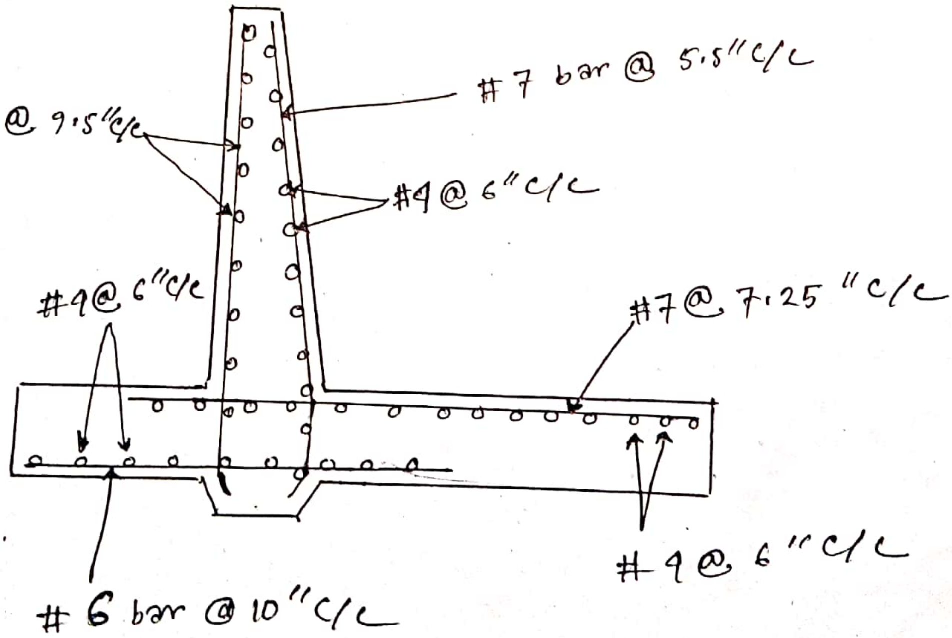
shear check: $V_{dev} = \frac{V_{max}}{bd} = \frac{7030.6}{12 \times 14.625} = 40.06$ psi

$v_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{4000} = 69.57$ psi > V_{dev} (OK)

working diagram:



Reinforcement Details:



Retaining Wall

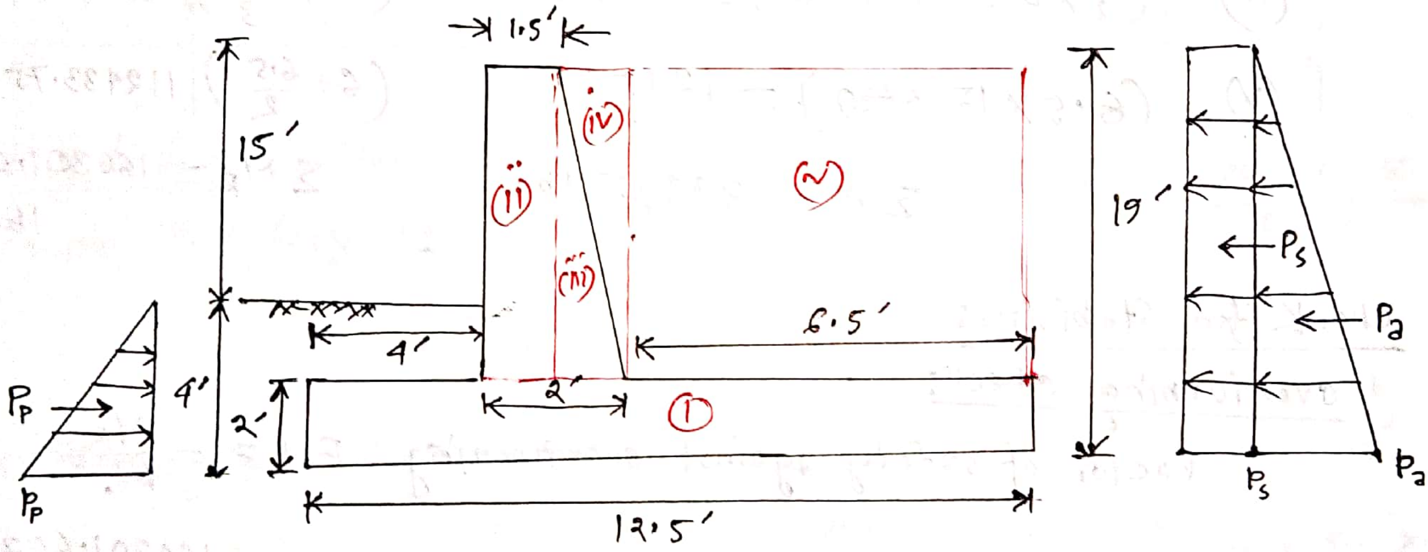
2017

Problem: Make a preliminary design of a cantilever retaining wall to retain a backfill 15 ft high. Also design the stem. Assume unit weight of backfill = 110 pcf, $\varphi = 30°$, allowable soil bearing capacity = 4 ksf, f = 0.5, surcharge = 275 psf, $f_c' = 3$ ksi and $f_s = 24$ ksi

Solution: considering the overall height, $H = (15+4) = 19$ ft

width of the base, $B = \frac{2}{3} \times H = \frac{2}{3} \times 19 = 12.7$ ft $\simeq 12.5$ ft

Depth of the base, $B = \frac{1}{10} \times H = \frac{1}{10} \times 19 = 1.9$ ft $\simeq 2$ ft



$$K_a = \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$$

$P_s = q K_a = (275 \times \frac{1}{3}) = 91.67$ psf

$P_a = K_a \gamma H = (\frac{1}{3} \times 110 \times 19) = 696.67$ psf

$\therefore$ Total active pressure $= (P_a + P_s) = (\frac{1}{2} \times 696.67 \times 19) + (91.67 \times 19)$
$= (6618.37 + 1741.73) = 8360$ lb

$P_p = K_p \gamma H = (3 \times 110 \times 4) = 1320$ psf

$\therefore$ Total passive pressure $= (\frac{1}{2} \times 1320 \times 4) = 2640$ lb

$\therefore$ Total overturning moment, $M_o = \dfrac{6618.37 \times 19}{3} + 1741.73 \times \dfrac{19}{2}$

$= 58462.78$ lb-ft

section	W (lb)	Moment Arm $\bar{x}$ (ft)	Resisting Moment, M_R (lb-ft)
(i)	$(12.5 \times 2 \times 150) = 3750$	$\frac{12.5}{2}$	23437.5
(ii)	$(1.5 \times 17 \times 150) = 3825$	$(4 + \frac{1.5}{2})$	18168.75
(iii)	$(\frac{1}{2} \times 0.5 \times 17 \times 150) = 637.5$	$(5.5 + \frac{0.5}{3})$	3612.5
(iv)	$(\frac{1}{2} \times 0.5 \times 17 \times 110) = 467.5$	$(5.5 + \frac{2 \times 0.5}{3})$	2649.167
(v)	$(6.5 \times 17 \times 110) = 12155$	$(6 + \frac{6.5}{2})$	112433.75

$\Sigma W = 20835$ lb $\qquad \Sigma M_R = 160301.667$ lb-ft

Check for Stability:

overturning check:

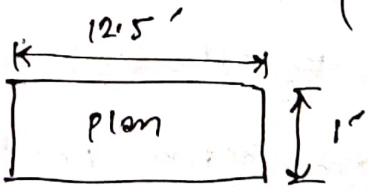
Factor of safety against overturning, $F.S = \dfrac{M_R}{M_o}$

$= \dfrac{160301.667}{58462.78}$

$= 2.742 > 1.5$

(OK)

Check for soil pressure:



$A = 1 \times B = 1 \times 12.5 = 12.5\ ft^2$

$I = \dfrac{1 \times B^3}{12} = \dfrac{1 \times 12.5^3}{12} = 162.76\ ft^4$

$a = \dfrac{M_R - M_o}{W} = \dfrac{160301.667 - 58462.78}{20835} = 4.888$

$e = \left|\frac{B}{2} - a\right| = \left|\frac{12.5}{2} - 4.888\right| = 1.362$ ft

$\therefore M = W \times e = (20835 \times 1.362) = 28377.27$ lb-ft

$$\therefore \sigma_1 = \frac{W}{A} + \frac{Mc}{I} = \frac{20835}{12.5} + \frac{28377.27 \times \frac{12.5}{2}}{162.76}$$

$\therefore \sigma_1 = 2786.5$ psf < 4000 psf

(OK)

and,

$$\sigma_2 = \frac{W}{A} - \frac{Mc}{I} = \frac{20835}{12.5} - \frac{28377.27 \times \frac{12.5}{2}}{162.76}$$

$\therefore \sigma_2 = 577$ psf < 4000 psf

(OK)

Sliding check: without considering passive pressure,

Factor of safety against sliding, $F.S = \frac{W \times f}{P_a + P_s} = \frac{20835 \times 0.5}{8360}$

$= 1.246 < 1.5$

(not ok)

considering passive pressure,

$$F.S = \frac{W \times f + P_p}{P_a + P_s} = \frac{20835 \times .5 + 2640}{8360} = 1.56 > 1.5$$

(OK)

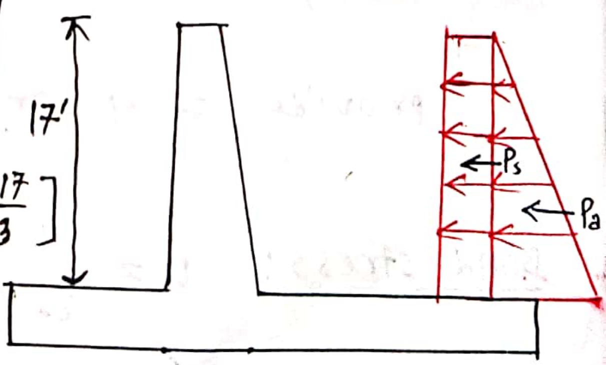
Design of Stem:

$V_{max} = \left[\frac{1}{3} \times 275 \times 17 + \frac{1}{2} \times \left(\frac{1}{3} \times 110 \times 17\right) \times 17\right]$

$= 6856.67$ lb

$M_{max} = \left[\frac{1}{3} \times 275 \times 17 \times \frac{17}{2} + \frac{1}{2} \times \left(\frac{1}{3} \times 110 \times 17\right) \times 17 \times \frac{17}{3}\right]$

$= 43269.72$ lb-ft



depth check:

$$n = \frac{29\times10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$$

$$r = \frac{24000}{.45\times3000} = 17.78$$

$$\therefore K = \frac{n}{n+r} = \frac{9}{9+17.78} = 0.336$$

$$j = 1 - \frac{0.336}{3} = 0.888$$

$$\therefore R = \frac{1}{2} f_c\, j K = \frac{1}{2}\times(.45\times3000)\times.888\times.336 = 201.4 \text{ psi}$$

$$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{43269.72\times12}{201.4\times12}} = 14.66 \text{ in}$$

$$d_{eff} = (2\times12) - 3 - \frac{7}{8} = 20.125 \text{ in.}$$

$$t \quad - c/c \quad - \phi$$

Reinforcement calculation:

$$A_s = \frac{M}{f_s j d} = \frac{43269.72\times12}{24000\times.888\times20.125} = 1.21 \text{ in}^2/\text{ft}$$

provide #7 bar @ $\frac{0.6\times12}{1.21} = 5.95'' \simeq 5.75''$ c/c

$$A_{s\,(min)} = 0.0018\, bt = 0.0018\times12\times24 = 0.52 \text{ in}^2/\text{ft}$$

provide #4 bar @ $\frac{0.20\times12}{0.52} = 4.61'' \simeq 4.5''$ c/c

Bond stress: $$u = \frac{V_{max}}{\Sigma_o\, jd} = \frac{6856.67}{\frac{12}{5.75}\times\pi\times\frac{7}{8}\times0.888\times20.125} = 66.88 \text{ psi}$$

Development length:

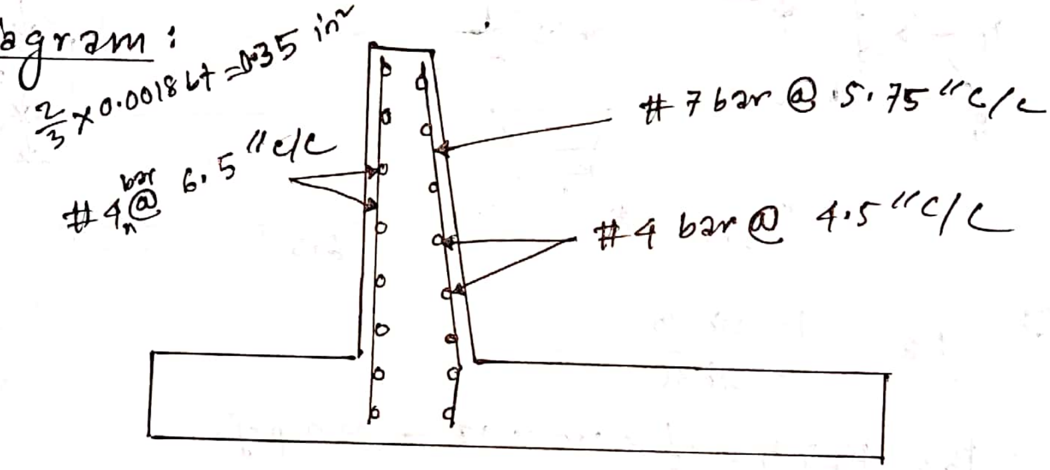
$$l_d = \frac{f_y D}{4U} = \frac{24000 \times \frac{7}{8}}{4 \times 66.88} = 78.5 \text{ in.}$$

Shear check: $v_{dev} = \frac{V_{max}}{bd} = \frac{6856.67}{12 \times 20.125} = 28.4$ psi

$$v_{all} = 1.1\sqrt{f_c'} = 1.1 \times \sqrt{3000} = 60.25 \text{ psi} > v_{dev}.$$

(ok)

Working diagram:



2016, 2012

* Check the $\boxed{\text{adequacy}}$ of the retaining wall with regard to — overturning, sliding and allowable soil pressure.

2015, 2014, 2011

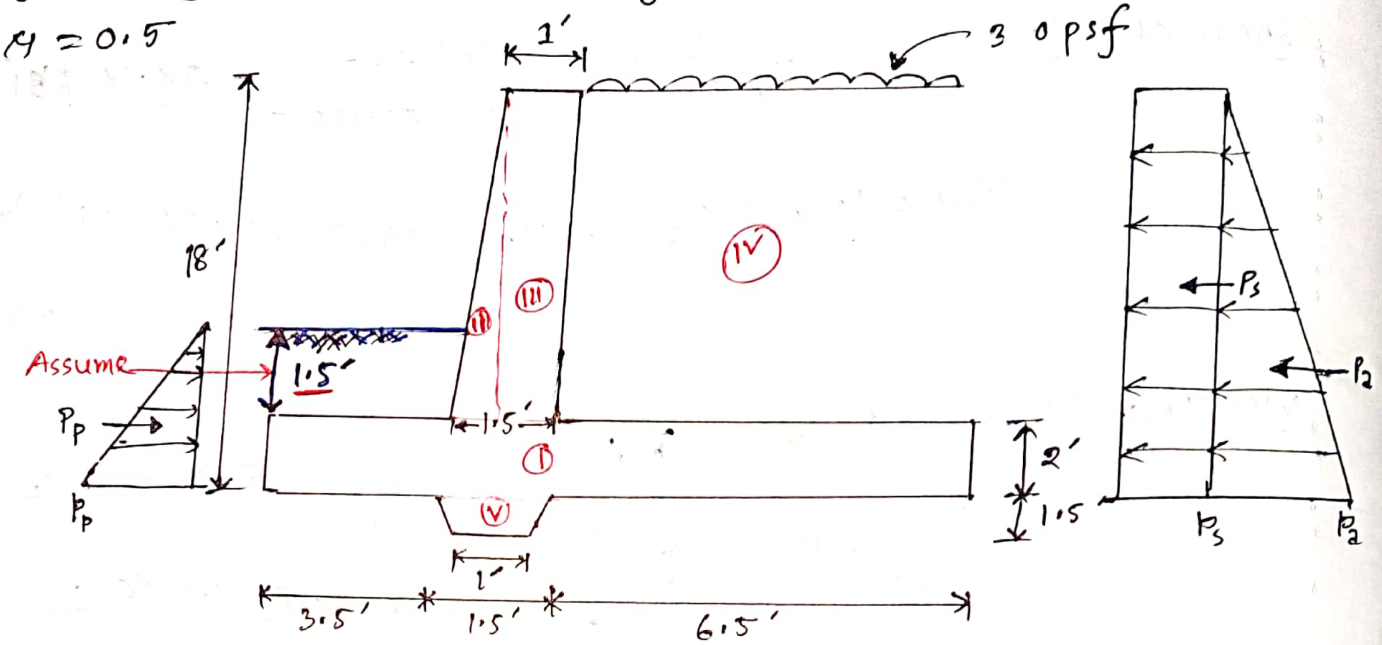
sliding & Overturning & soil pressure check

* Check $\boxed{\text{external stability}}$ and design the stem of the retainingwall

Scanned by CamScanner

2016

Problem: check the adequacy of the retaining wall shown in figure below with regard to overturning, sliding and allowable soil pressure. Assume, weight of backfill = 110 pcf, Angle of friction, $\phi = 30°$. Allowable bearing capacity 3 ksf, coefficient of friction between concrete and soil $\mu = 0.5$



Solution:

$$K_a = \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$$

$$\therefore P_a = K_a \gamma H = \frac{1}{3} \times 110 \times 18 = 660 \text{ psf}$$

$$P_s = q K_a = 330 \times \frac{1}{3} = 110.00 \text{ psf}$$

$\therefore$ Total active pressure $= P_a + P_s$

$$= \left(\frac{1}{2} \times 660 \times 18\right) + (110.00 \times 18)$$

$$= (5940 + 1980) = 7920 \text{ lb}$$

$$P_p = K_p \gamma H = (3 \times 110 \times 3.5) = 1155.0 \text{ psf}$$

$\therefore$ Total passive pressure, $P_p = \left(\frac{1}{2} \times 1155.0 \times 3.5\right) = 2021.25 \text{ lb}$

Over turning moment, $M_o = \times\frac{H}{3} + P_s \times \frac{H}{2}$

$$= \left(5940 \times \frac{18}{3} + 1920 \times \frac{18}{2}\right)$$

$$= 52920 \text{ lb-ft}$$

section	W (lb)	$\bar{x}$ (ft)	M_R (lb-ft)
(i)	$(2 \times 11.5 \times 150) = 3450$	$\frac{11.5}{2}$	19837.5
(ii)	$(\frac{1}{2} \times 0.5 \times 16) \times 150 = 600$	$(3.5 + \frac{2}{3} \times 0.5)$	2300
(iii)	$(1 \times 16 \times 150) = 2400$	$(4 + \frac{1}{2})$	10800
(iv)	$(6.5 \times 16 \times 110) = 11440$	$5 + \frac{6.5}{2}$	94380
(v)	$0.5 \times (1.5+2) \times 1.5 \times 150 = 281.25$	$(3.5 + \frac{1.5}{2})$	1195.3125
	$\Sigma W = 18171.25$		$\Sigma M_R = 128512.8125$

<u>check for stability:</u>

<u>Overturning check:</u>

F.S agaist overturing $= \frac{M_R}{M_o} = \frac{128512.8125}{52920} = 2.43 > 1.5$ (ok)

<u>check for sliding:</u>

without considering passive earth pressure, $F.S = \frac{W \times f}{P_a + P_s}$

$= \frac{18171.25 \times 0.5}{7860}$

$= 1.16 < 1.5$ (not ok)

considering passive earth pressure,

$F.S = \frac{W \times f + P_p}{P_a + P_s}$

$= \frac{18171.25 \times 0.5 + 2021.25}{7920}$

$= 1.40 < 1.5$ (not ok)

~~Hence Key should be provided.~~

Check for soil pressure:

$A = (1 \times B) = 1 \times 11.5 = 11.5 \ ft^2$

$I = \frac{1 \times B^3}{12} = \frac{1 \times 11.5^3}{12} = 126.74 \ ft^4$

$a = \frac{M_R - M_o}{W} = \frac{128572.8125 - 52920}{18171.25} = 4.16$

$e = \left|\frac{B}{2} - a\right| = \left|\frac{11.5}{2} - 4.16\right| = 1.59$

$\therefore M = W \times e = (18171.25 \times 1.59) = 28892.3 \ lb\text{-}ft$

$c = \frac{11.5}{2} = 5.75$

$\therefore \sigma_1 = \frac{W}{A} + \frac{Mc}{I} = \frac{18171.25}{11.5} + \frac{28892.3 \times 5.75}{126.74}$

$\therefore \sigma_1 = 2890.91 \ psf < 3000 \ psf$

(OK)

$\sigma_2 = \frac{W}{A} - \frac{Mc}{I} = \frac{18171.25}{11.5} - \frac{28892.3 \times 5.75}{126.74}$

$\therefore \sigma_2 = 269.31 \ psf < 3000 \ psf.$

(OK)

Hence, the cross section of the given retaining wall is adequate for overturning and soil pressure but not for sliding.

Two Way Slab

2015, 2013

\# Prove that a slab can be designed as a one way slab if the long to short span ratio of the slab is larger than 2.

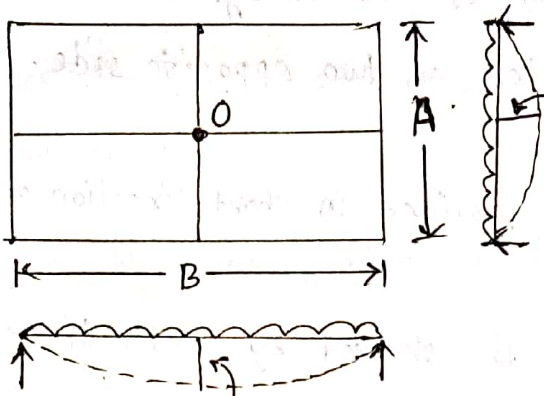
consider an isolated slab supported on brick walls on all four sides at shown in figure:

Assume,

Total uniformly distributed load on slab = W

W_A = load acting along direction A

W_B = load acting along direction B



$$\Delta_{A(max)} = \frac{5}{384} \times \frac{W_A A^4}{EI}$$

$$\Delta_{B(max)} = \frac{5}{384} \times \frac{W_B B^4}{EI}$$

$$\therefore W = W_A + W_B$$

Equating deflection at point O, $\Delta_{A(max)} = \Delta_{B(max)}$

$$\Rightarrow \frac{5}{384} \times \frac{W_A A^4}{EI} = \frac{5}{384} \times \frac{W_B B^4}{EI}$$

$$\Rightarrow \frac{W_A}{W_B} = \left(\frac{B}{A}\right)^4$$

If $B > A$ then, $W_A > W_B$. That is Maximum load is carried by shorter direction. (2012)

Now, Assume $\frac{B}{A} = 2$ Then, $\frac{W_A}{W_B} = 2^4 = 16$

$$\therefore W_A = 16\,W_B$$

That is about $\left(\frac{16}{17} \times 100\right) = 94.12\%$ load is carried by shorter direction, (Almost Total Load)

As we know, if load distribution is shared by one direction only it is one way slab.

Hence, if $\frac{B}{A} > 2$, then slab can be designed as one way slab.

One way slab:

* Load distribution is shared by one direction
* slab is supported on two opposite side.
* $\frac{B}{A} > 2$
* Reinforcement are provided in short direction only.

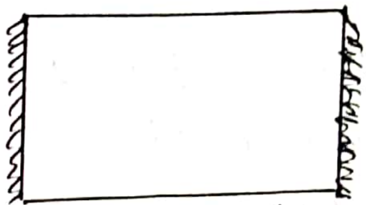


Fig. One way slab

Two way Slab:

* load distribution is shared by both direction
* Slab is supported on two or more adjacent side.
* $\frac{B}{A} < 2$
* Reinforcement are provided in both direction.

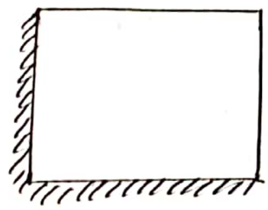


Fig. Two way slab

2014

Distinguish Between one way slab and Two way slab.

(Above four points)

corner reinforcement:

corner reinforcement are also called as torsional reinforcement. Torsional reinforcement shall be provided at corner of two way slab. The torsional moment are high near the corner therefore torsional reinforcement is essential to prevent corner slab from lifting and prevent cracks.

* Twisting moments at exterior corners of a two-way slab

system tend to crack the slab at the bottom along the panel diagonal and at the top perpendicular to the panel diagonal.

* Special reinforcement should be provided at exterior corners in both the bottom and top of the slab, for a distance in each direction from the corner equal to one fifth the longer span of the corner panel.

* The reinforcement at the bottom of the slab should be perpendicular to the diagonal from the corner.

* These reinforcement can also be placed in two bands parallel to the side of the slab.

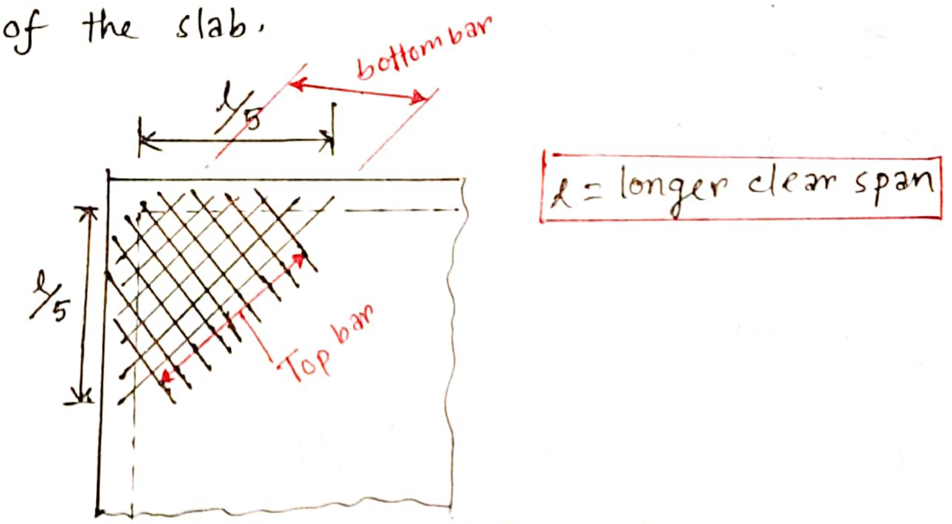


Fig. corner reinforcement

Problem: consider $A = 20'$, $B = 25'$, Total load $w = 200$ psf. calculate w_A, w_B and M_A, M_B. Also calculate reinforcement if $f_c' = 3000$ psi, $f_y = 60000$ psi.

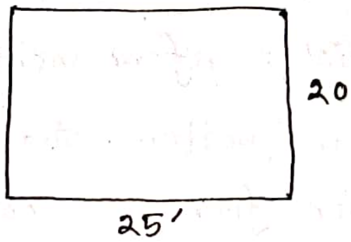


Fig. Isolated slab supported on brick walls on all four side

Solution: For this slab,

$$\frac{w_A}{w_B} = \left(\frac{B}{A}\right)^4$$

$$= \left(\frac{25}{20}\right)^4 = 2.44$$

$$\therefore w_A = 2.44\, w_B$$

Given, $w = 200$

$$\Rightarrow w_A + w_B = 200$$

$$\Rightarrow 2.44 w_B + w_B = 200$$

$$\Rightarrow w_B = 58.14 \text{ psf} \quad \therefore w_A = 141.88 \text{ psf}$$

$$M_A = \frac{w_A A^2}{8} = \frac{141.88 \times 20^2}{8} = 7094 \text{ lb-ft/ft}$$

and, $$M_B = \frac{w_B B^2}{8} = 4542.2 \text{ lb-ft/ft}$$

Thickness calculation:

Minimum thickness (i) $t_{min} = \dfrac{\text{perimeter}}{180} = \dfrac{2 \times (20+25) \times 12}{180}$

$$\therefore t_{min} = 6''$$

again (ii) $t_{min} = 3.5''$

$\therefore$ thickness of the slab, $t = 7$ inch. (Assume)

$$\therefore d_{eff} = (7-1) = 6 \text{ in.}$$

$t - (c/c + \frac{\phi}{2})$

depth check:

We know, $d = \sqrt{\dfrac{M}{Rb}}$

Here, $n = \dfrac{E_s}{E_c} = \dfrac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$

$R = \frac{1}{2} f_c j K$

$= \frac{1}{2} \times 0.45 \times 3000 \times 0.888 \times 0.336$

$= 201.4$

$r = \dfrac{f_s}{f_c} = \dfrac{.4 \times 60000}{.45 \times 3000} = 17.77$

$K = \dfrac{n}{n+r} = \dfrac{9}{9+17.77} = 0.336$

$j = 1 - \dfrac{K}{3} = 1 - \dfrac{0.336}{3} = 0.888$

$\therefore d = \sqrt{\dfrac{7094 \times 12}{201.4 \times 12}}$

$= 5.935$ inch $< d_{eff} = 6$ in. (OK)

Reinforcement calculation:

In short direction, $A_s = \dfrac{M_A}{f_s j d} = \dfrac{7094 \times 12}{.4 \times 60000 \times 0.888 \times 6}$

$= 0.67$ in^2 $> A_{s\,min} = 0.0018\,bt = 0.13$ in^2

$\therefore$ use # 4 bar @ $\dfrac{0.20 \times 12}{0.67} = 3.58''$ c/c $\simeq 3.5''$ c/c

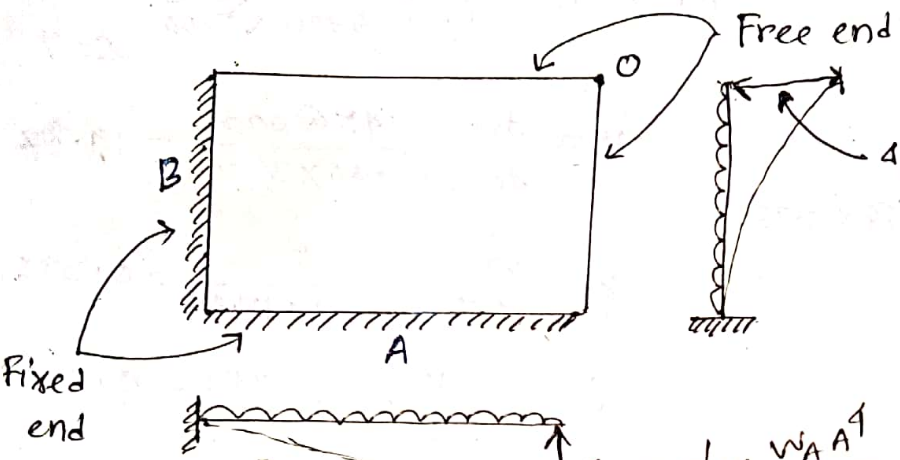
In long direction, $A_s = \dfrac{M_B}{f_s j d} = \dfrac{4542.2 \times 12}{.4 \times 60000 \times 0.888 \times (6 - 4/8)}$

$d_{long} = d_{short} - \phi$

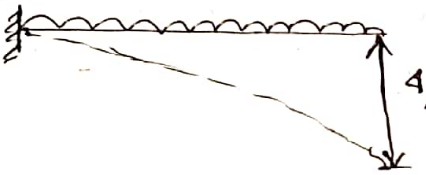
$= 0.465$ in^2

use # 4 bar @ $\dfrac{0.20 \times 12}{0.465} = 5.16'' \simeq 5''$ c/c

Design of Balcony Slab:



$$\Delta_B = \frac{1}{8}\frac{W_B B^4}{EI}$$



$$\Delta_A = \frac{1}{8} \times \frac{W_A A^4}{EI}$$

Equating the deflection at point O,

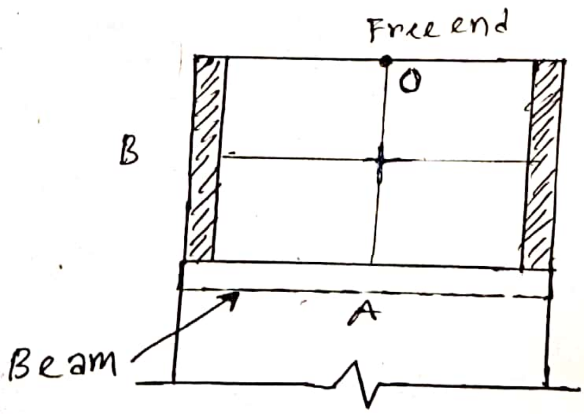
$$\Delta_A = \Delta_B$$

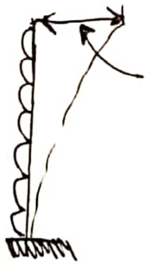
$$\Rightarrow \frac{1}{8}\frac{W_A A^4}{EI} = \frac{1}{8} \times \frac{W_B B^4}{EI}$$

$$\Rightarrow \frac{W_A}{W_B} = \left(\frac{B}{A}\right)^4$$

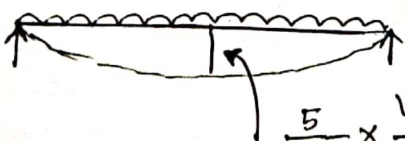
$$M_A = \frac{W_A A^2}{2} \quad \text{and} \quad M_B = \frac{W_B B^2}{2}$$

Design of veranda slab:





$$\frac{1}{8} \times \frac{W_B B^4}{EI}$$



$$\frac{5}{384} \times \frac{W_A A^4}{EI}$$

Equating the deflection at point O,

$$\frac{1}{8}\times\frac{W_B B^4}{EI}=\frac{5}{384}\times\frac{W_A A^4}{EI}$$

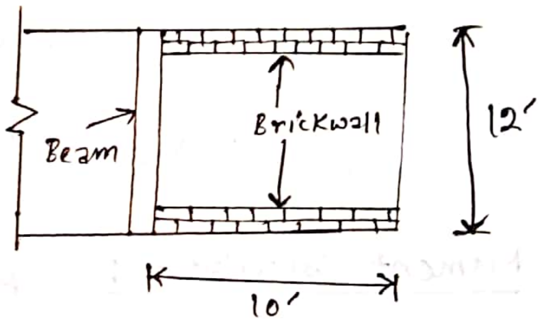
$$\Rightarrow \frac{W_A}{W_B}=\frac{384}{8\times5}\times\left(\frac{B}{A}\right)^4$$

$$\therefore \frac{W_A}{W_B}=9.6\times\left(\frac{B}{A}\right)^4$$

(10' by 15')

(2014), 2015, 2011 → (10' by 10') 2016 → (8' by 15')

Problem: A 10 ft by 12 ft slab is supported on brick walls on two opposite sides and on a beam on the third side as shown in figure below. The slab is to be designed for a live load of 100 psf. Assume $f_c' = 3000$ psi and $f_s = 24000$ psi.



Solution:

Here, $\frac{B}{A}=\frac{12}{10}=1.2<2$

∴ The slab is to be designed as two-way slab.

Thickness of Slab;

(i) $t_{min}=\frac{\text{perimeter}}{180}=\frac{2\times(10+12)\times12}{180}=2.93$ inch

(ii) $t_{min}=3.5''$

Assume, the thickness of the slab = 4.5 inch

Load calculation: Dead load $=\frac{t}{12}\times150=\left(\frac{4.5}{12}\times150\right)=56.25$ psf

Given, Live load = 100 psf

∴ Total load, $W_T=(56.25+100)=156.25$ psf.

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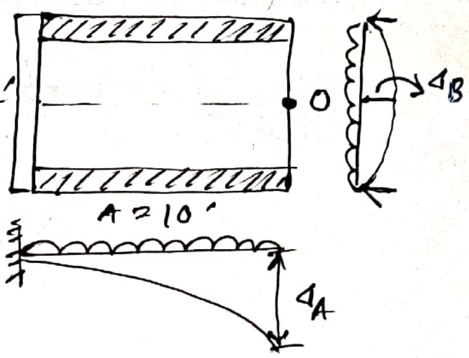
In direction A, $\Delta_A = \frac{1}{8}\frac{W_A A^4}{EI}$

In direction B, $\Delta_B = \frac{5}{384}\frac{W_B B^4}{EI}$



Equating deflection at point O,

$$\frac{W_A}{W_B} = \frac{8 \times 5}{384} \times \left(\frac{B}{A}\right)^4$$

$$\Rightarrow W_A = \frac{40}{384} \times (1.2)^4 \times W_B$$

$$\therefore W_A = 0.216\, W_B$$

Now, $W_T = W_A + W_B = 156.25$

$$\Rightarrow 0.216\, W_B + W_B = 156.25$$

$$\therefore W_B = 128.5 \text{ psf}$$

$$\therefore W_A = (156.25 - 128.5) = 27.75 \text{ psf}$$

Moment calculation: $M_A = \frac{1}{2} \times W_A \times A^2 = \frac{1}{2} \times 27.75 \times 10^2 = 1387.5$ lb-ft/ft

and, $M_B = \frac{1}{8} \times W_B \times B^2 = \frac{1}{8} \times 128.5 \times 12^2 = 2313$ lb-ft/ft

Depth check:

$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$$

$$r = \frac{f_s}{f_c} = \frac{24000}{0.45 \times 3000} = 17.78$$

$$K = \frac{n}{n+r} = \frac{9}{9+17.78} = 0.336$$

$$j = 1 - \frac{K}{3} = 1 - \frac{0.336}{3} = 0.888$$

$\therefore R = \frac{1}{2} f_c j K = \frac{1}{2} \times (0.45 \times 3000) \times 0.888 \times 0.336 = 201.4$

$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{2313 \times 12}{201.4 \times 12}} = 3.39$ in.

$d_{eff} = (t-1) = (4.5-1) = 3.5$ in $> d$ (OK)

Reinforcement Calculation:

In direction A, $A_s = \frac{M_A}{f_s j d} = \frac{1387.5 \times 12}{24000 \times 0.888 \times 3.5}$

$\therefore A_s = 0.223$ in^2

$A_{s(min)} = 0.0018\, bt = (0.0018 \times 12 \times 4.5) = 0.0972$ in^2

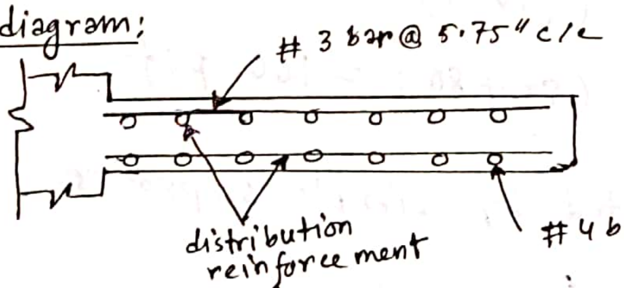
$\therefore$ Use #3 bar @ $\frac{0.11 \times 12}{0.223} = 5.9'' \simeq 5.75''$ c/c

In direction B, $A_s = \frac{M_B}{f_s j d} = \frac{2313 \times 12}{24000 \times 0.888 \times (3.5 - \frac{3}{8})}$

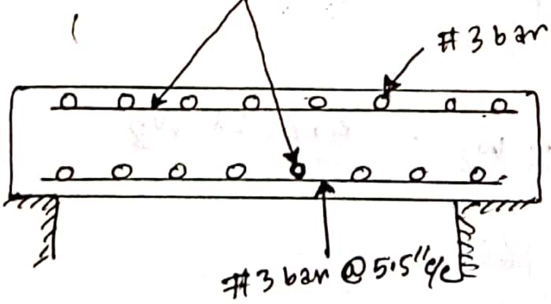
$= 0.42$ in^2

$\therefore$ Use #4 bar @ $\frac{0.20 \times 12}{0.42} = 5.7'' \simeq 5.5''$ c/c.

working diagram:



(in A direction)



(in B direction)

2010

Problem: Design a two-way isolated slab panel of size 12 ft by 18 ft. Assume (i) All the sides are supported on simple supports (ii) Dead load = 30 psf (iii) Live load = 80 psf (iv) $f_c' = 4$ ksi (v) $f_y = 60$ ksi

Solution: Here, $\frac{B}{A} = \frac{18}{12} = 1.5 < 2$

∴ The slab is to be designed as two-way slab.

Thickness of slab: (i) $t_{min} = \frac{\text{perimeter}}{180} = \frac{2 \times (12+18) \times 12}{180}$

$= 4$ inch

(ii) $t_{min} = 3.5''$

∴ Thickness of the slab = 4 inch.

Load calculation:

∴ self weight of slab $= \frac{t}{12} \times 150 = \left(\frac{4}{12} \times 150\right) = 50$ psf

∴ Total dead load $= (50 + 30) = 80$ psf

Live load = 80 psf (given)

∴ Total Load, $W_T = (80+80) = 160$ psf

All sides are supported by simple supports.

$$\therefore \frac{W_A}{W_B} = \left(\frac{B}{A}\right)^4$$

$$W_A = \left(\frac{18}{12}\right)^4 \times W_B = 5.0625\, W_B$$

$$W_T = W_A + W_B = 160$$

$\Rightarrow 5.0625\, W_B + W_B = 160$ ∴ $W_B = 26.39$ psf

Scanned by CamScanner

$\therefore W_A = (160 - 26.39) = 133.61$ psf

Moment calculation:

$M_A = \frac{1}{8} \times W_A \times A^2 = \frac{1}{8} \times 133.61 \times 12^2 = 2404.98$ lb-ft/ft

$M_B = \frac{1}{8} \times W_B \times B^2 = \frac{1}{8} \times 26.39 \times 18^2 = 1068.795$ lb-ft/ft

Depth check:

$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{57000\sqrt{4000}} = 8.04 \simeq 8$

$r = \frac{f_s}{f_c} = \frac{0.4 \times 60}{.45 \times 4} = 13.33$

$k = \frac{n}{n+r} = \frac{8}{8+13.33} = 0.375$

$j = 1 - \frac{k}{3} = 1 - \frac{0.375}{3} = 0.875$

$\therefore R = \frac{1}{2} f_c j k = \frac{1}{2} \times 0.45 \times 4000 \times 0.875 \times 0.375 = 295.3125$

$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{2404.98 \times 12}{295.3125 \times 12}} = 2.854''$

$d_{eff} = (t-1) = (4-1) = 3$ in $> d$ (OK)

Reinforcement calculation:

In short direction (A), $A_s = \frac{M_A}{f_s j d} = \frac{2404.98 \times 12}{24000 \times .875 \times 3}$

$\therefore A_s = 0.46$ in^2

$A_{s(min)} = 0.0018\,bt = (0.0018 \times 12 \times 4) = 0.0864$ in^2

$\therefore$ use #4 bar @ $\frac{0.20 \times 12}{0.46} = 5.2'' \simeq 5''$ c/c

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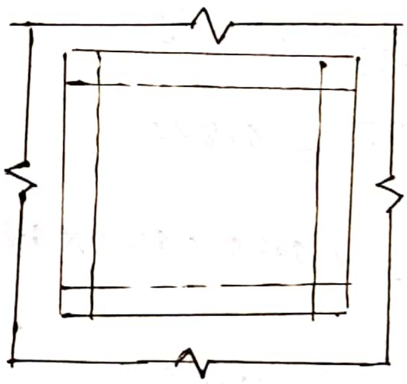
In long direction (B), $A_s = \frac{M_B}{f_s j d} = \frac{1068.795 \times 12}{24000 \times 0.875 \times (3 - \frac{4}{8})}$

$\therefore A_s = 0.25 \text{ in}^2$

$\therefore$ use # 3 bar @ $\frac{.11 \times 12}{0.25} = 5.28'' \simeq 5.25''$ C/C

2012

Problem: A 8′ × 10′ slab panel shown in figure is to be designed to support its own weight only. Design the slab and draw reinforcement details. Assume, $f_c' = 3$ ksi, $f_s = 20$ ksi, beam width = 12″



Solution: clear span, $B = (10 - \frac{12}{12}) = 9'$

$A = (8 - \frac{12}{12}) = 7'$

$\therefore \frac{B}{A} = \frac{9}{7} = 1.286 < 2$; Hence the slab is to be designed as two way slab.

Thickness of slab: (i) $t_{min} = \frac{2 \times (9+7) \times 12}{180} = 2.13''$

(ii) $t_{min} = 3.5''$

$\therefore$ Thickness of the slab $= 3.5''$

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Load calculation:

self weight of the slab $= \left(\frac{3.5}{12} \times 150\right) = 43.75$ psf

Assume, weight of floor furnish and plaster = 30 psf

$\therefore$ Total dead load, ~~W_T~~ $= (43.75 + 30) = 73.75$ psf

Moment calculation:

$$m = \frac{A}{B} = \frac{7}{9} = 0.78$$

Case-2:

Tabulated Moment Co-efficient: (Nilson 7th edition - Page - 200, 201, 202 & 203)

m	$-C_A$	$-C_B$	C_{ADL}	C_{BDL}	—
0.75	0.069	0.022	0.028	0.009	—
0.80	0.065	0.027	0.026	0.011	—

For $m = 0.78$,

$$-C_A = 0.069 - \frac{0.03}{0.05} \times (0.069 - 0.065) = 0.0666$$

$$-C_B = 0.022 + \frac{0.03}{0.05} \times (0.027 - 0.022) = 0.025$$

$$C_{ADL} = 0.028 - \frac{0.03}{0.05} \times (0.028 - 0.026) = 0.0268$$

$$C_{BDL} = 0.009 + \frac{0.03}{0.05} \times (0.011 - 0.009) = 0.0102$$

Now, In short direction,

$$(-) M_A = C_A \times W_T \times A^2 = 0.0666 \times 73.75 \times 7^2 = 240.675 \text{ lb-ft/ft}$$

In long direction,

$$(-) M_B = C_B \times W_T \times B^2 = 0.025 \times 73.75 \times 9^2 = 149.34375 \text{ lb-ft/ft}$$

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In short direction, $(+) M_A = C_{A DL} \times W_{DL} \times A^2 + C_{A LL} \times W_{LL} \times A^2$

$= 0.0268 \times 73.75 \times 7^2 + 0$

$= 96.848$ lb-ft/ft

In long direction, $(+) M_B = C_{B DL} \times W_{DL} \times B^2 + C_{B LL} \times W_{LL} \times B^2$

$= 0.0102 \times 73.75 \times 9^2 + 0$

$= 60.93$ lb-ft/ft

__Depth Check:__

$$n = \frac{29 \times 10^6}{57000\sqrt{3000}} = 9.28 \simeq 9$$

$$r = \frac{20000}{.45 \times 3000} = 14.81$$

$$\therefore K = \frac{n}{n+r} = \frac{9}{9+14.81} = 0.378$$

$$j = 1 - \frac{K}{3} = 1 - \frac{0.378}{3} = 0.874$$

$$\therefore R = \frac{1}{2} f_c jK = \frac{1}{2} \times (0.45 \times 3000) \times .874 \times 0.378 = 223$$

$$\therefore d = \sqrt{\frac{M}{Rb}} = \sqrt{\frac{240.675 \times 12}{223 \times 12}} = 1.04 \text{ inch}$$

$d_{eff} = (3.5 - 1) = 2.5'' > d$

(ok)

__Reinforcement Calculation:__

In short direction, $(+) A_s = \frac{(+) M_A}{f_s j d} = \frac{96.848 \times 12}{20000 \times 0.874 \times 2.5}$

$\therefore (+) A_s = 0.0271 \text{ in}^2/\text{ft}$

But, $A_{s(min)} = \boxed{0.0020\, bt} = (0.0020 \times 12 \times 3.5) = 0.084 \text{ in}^2/\text{ft}$

$f_y < 60000$

Hence, $(+) A_s = 0.084 \text{ in}^2$. $S_{max} = 3t = (3 \times 3.5) = 10.5''$

$\therefore$ use # 3 bar @ $\dfrac{0.11 \times 12}{0.084} = 15.7'' \simeq 15.5''$ c/c $> S_{max}$

Hence, use # 3 bar @ $10.5''$ c/c

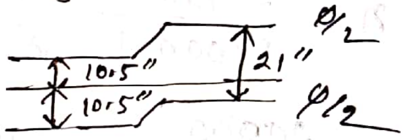
$$(-) A_s = \frac{(-) M_A}{f_s j d} = \frac{240.675 \times 12}{20000 \times .874 \times 2.5} = 0.066 \text{ in}^2 < A_{s(min)}$$

$\therefore (-) A_s = 0.084 \text{ in}^2/\text{ft}$

Assume 50% positive reinforcement will be crank up.

Therefore, negative reinforcement available from crank bar:

$$\frac{0.11 \times 12}{2 \times 10.5} = 0.063 \text{ in}^2/\text{ft}$$



$\therefore$ Additional reinforcement required $= (0.084 - 0.063) \text{ in}^2/\text{ft}$
$= 0.021 \text{ in}^2/\text{ft}$

$\therefore$ use # 3 bar @ $\dfrac{0.11 \times 12}{0.021} = 60'' > S_{max}$

Hence, # 3 bar @ $10.5''$ c/c

In long direction,

$$(+) A_s = \frac{(+) M_B}{f_s J d} = \frac{60.93 \times 12}{20000 \times 0.875 \times (2.5 - \frac{3}{8})} = 0.02 \text{ in}^2/\text{ft} < A_{s(min)}$$

$\therefore (+) A_s = 0.084 \text{ in}^2$

use # 3 bar @ $10.5''$ C/C

$$(-) A_s = \frac{(-) M_B}{f_s j d} = \frac{149.34375 \times 12}{20000 \times 0.875 \times (2.5 - \frac{3}{8})} = 0.05 \text{ in}^2/\text{ft} < A_{s(min)}$$

$\therefore (-) A_s = 0.084 \text{ in}^2/\text{ft}$

Assume 50% positive reinforcement will be crank up.

Therefore, negative reinforcement available from crank bar:

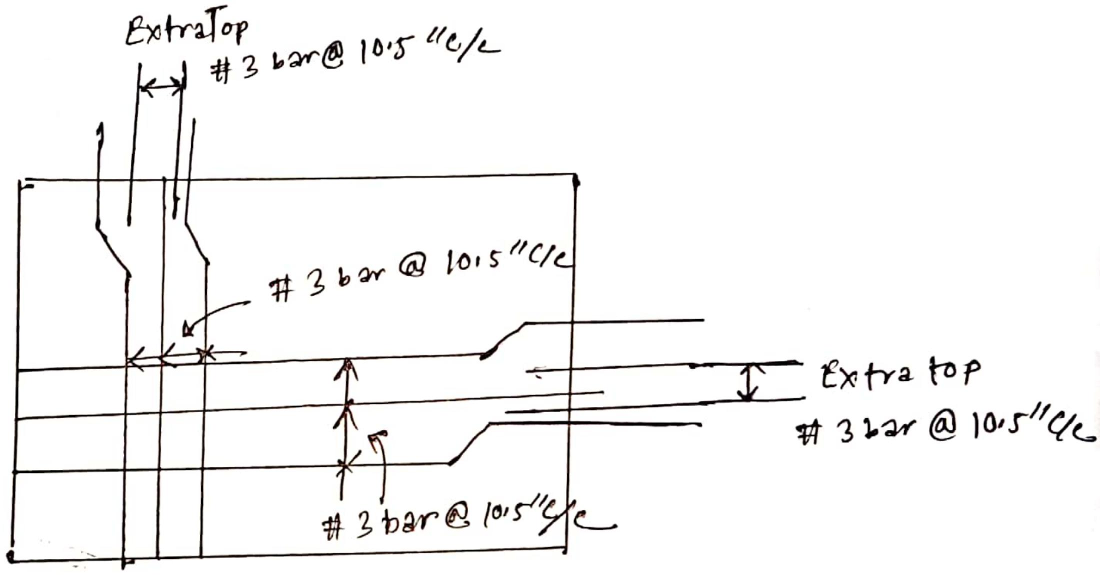
$$\frac{0.11 \times 12}{2 \times 10.5} = 0.063 \ in^2/ft$$

$\therefore$ Addition reinforcement required $= (0.084 - 0.063)\ in^2/ft$

$= 0.021\ in^2/ft$

Use #3 bar @ 10.5" c/c

Working Diagram:



Bond check: $$V_d = \frac{V_{max}}{\Sigma_o\, j\, d} = \frac{0.5 \times 73.75 \times 9}{\frac{12}{10.5} \times \underset{j}{0.875} \times \underset{d}{(2.5 - 3/8)} \times \pi \times \underset{\phi}{3/8}} = 132.57 \text{ psi}$$

$$V_{all} = \frac{3.4\sqrt{f_c'}}{D} = \frac{3.4 \times \sqrt{3000}}{3/8} = 496.6 \text{ psi} > V_d$$

(ok)

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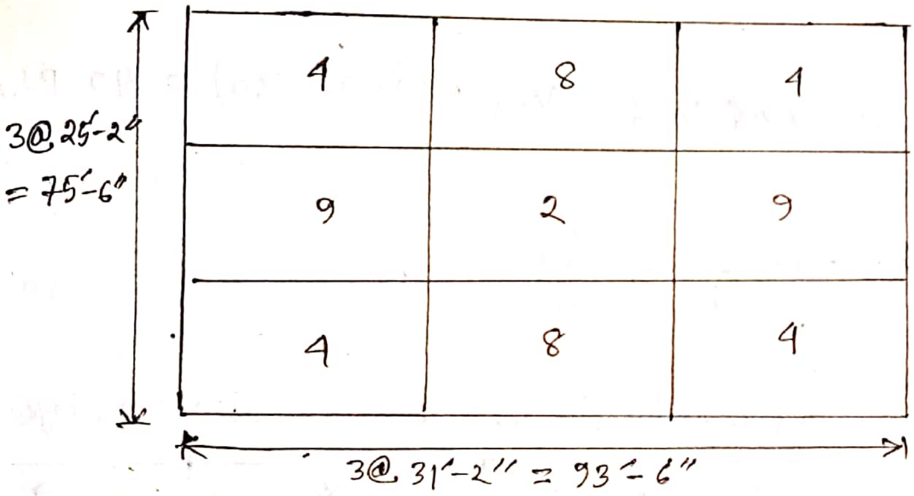
Problem:

Design a beam-column supported floor slab (93'-6" × 75'-6") is to carry service live load of 100 psf in addition to its own weight, $\frac{1}{2}$" thick plaster and $\frac{3}{2}$" thick floor furnish. Supporting columns of 14" square are spaced orthogonally at an interval at 31'-2" and 25'-2" on centers along longitudinal and transverse direction respectively. width of the beam is 14 in. Using BNBC/ ACI code of Moment co-efficient design the slab by USD method if $f_c' = 3000$ psi and $f_y = 60000$ psi.



Solution: Clear Span, B = 31'-2" – (1'-2") = 30'-0"

beam width

A = 25'-2" – 1'-2" = 24'-0"

Thickness of Slab:

(i) $t_{min} = \dfrac{\text{perimeter}}{180}$

$$= \frac{2(A+B)}{180} = \frac{2(24+30)\times 12}{180} = 7.2'' \simeq 8''$$

again,

(ii) $t_{min} = 3.5''$

Hence, Thickness of the slab = 8 inch.

Load calculation:

self weight of slab $= \left(\frac{t}{12} \times 150\right) = \left(\frac{8}{12} \times 150\right) = 100$ psf

weight of Floor furnish $= \frac{3/2}{12} \times 150 = 18.75$ psf

weight of Plaster $= \frac{1/2}{12} \times 150 = 6.25$ psf

$\therefore$ Total Dead load, $W_{DL} = (100 + 6.25 + 18.75) = 125$ psf

$\therefore W_{UDL} = (1.2 \times 125) = 150$ psf

Given, $W_{LL} = 100$ psf

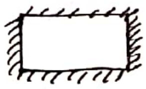
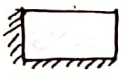
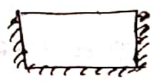
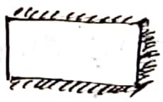
$\therefore W_{ULL} = (1.6 \times 100) = 160$ psf

$\therefore$ Total ultimate ~~dead~~ load, $W_u = (150 + 160) = 310$ psf

Moment Calculation:

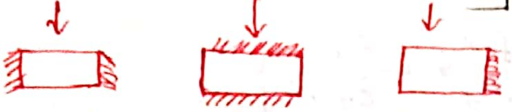
$m = \frac{A}{B} = \frac{24}{30} = 0.8$

* Tabulated Moment co-efficient: (From Book - 7th edition Nilson, Page-200, 201, 202, 203)

Case	2	4	8	9
for m=0.8				
$-C_A$	0.065	0.071	0.055	0.075
$-C_B$	0.027	0.029	0.041	0.017
$C_{A\,DL}$	0.026	0.039	0.032	0.029
$C_{B\,DL}$	0.011	0.016	0.015	0.010
$C_{A\,LL}$	0.041	0.048	0.044	0.042
$C_{B\,LL}$	0.017	0.020	0.019	0.017

Note: Negative moment at discontinuous ends/edges not given in table. [Table -9.2, page-200, Book - Nilson (7th)]

It shall be assumed equal to $\frac{1}{3} \times$ (+ve) Moment for same direction. [case-3, case-5, case-6]



Design of Panel-2: (case-2)

In short direction, (−ve) $M_A = C_A \cdot W_u A^2$ at continuous edges

$= (0.065 \times 310 \times 24^2)$ lb ft/ft

$= 11606.4$ lb-ft/ft

(+ve) $M_A = C_{A\,DL} W_{uDL} A^2 + C_{A\,LL} W_{uLL} A^2$

$= (0.026 \times 150 \times 24^2 + 0.041 \times 160 \times 24^2)$

$= 6024.96$ lb-ft/ft

In long direction, (−ve) $M_B = C_B W_u B^2$ at continuous edges.

$= (0.027 \times 310 \times 30^2)$

$= 7533$ lb-ft/ft

(+ve) $M_B = C_{B\,DL} W_{uDL} B^2 + C_{B\,LL} W_{uLL} B^2$

$= 0.011 \times 150 \times 30^2 + 0.017 \times 160 \times 30^2$

$= 3933$ lb-ft/ft

Depth check: We know, $d = \sqrt{\frac{M}{\phi R b}}$

Here, $R = \rho \cdot f_y \left(1 - \frac{\beta}{\alpha} \times \frac{\rho f_y}{f_c'}\right)$

But, $\rho = 0.85 \beta_1 \frac{f_c'}{f_y} \times \frac{\varepsilon_u}{\varepsilon_u + \varepsilon_y}$

*(if $f_c' \leq 4000$ psi then, $\beta_1 = 0.85$)

$f_c' = 3000$ psi

$= 0.85 \times 0.85 \times \frac{3}{60} \times \frac{0.003}{0.003 + 0.005}$

$= 0.013$

$\therefore R = 0.013 \times 60000 \times \left(1 - 0.59 \times \frac{0.013 \times 60}{3}\right)$

$= 660.35$

$\therefore d = \sqrt{\frac{11606.4 \times 12}{0.9 \times 660.35 \times 12}} = 4.42''$

$d_{eff} = t - c/c - \phi/2 = (8 - 1) = 7$ inch $> (d = 4.42'')$

(OK)

Reinforcement calculation:

In short direction,

for (+ve) Moment, (+) $A_s = \frac{M_u}{\phi f_y (d - a/2)}$

Here,

$a = \frac{A_s f_y}{0.85 f_c' b}$

$= \frac{A_s \times 60}{0.85 \times 3 \times 12}$

$\therefore a = 1.96 A_s$.

$\Rightarrow A_s = \frac{6024.96 \times 12}{0.9 \times 60000 \times (7 - \frac{1.96 A_s}{2})}$

$\Rightarrow A_s = 0.197$ in^2/ft

But, $A_{s(min)} = 0.0018\, bt = (0.0018 \times 12 \times 8)$ in^2/ft

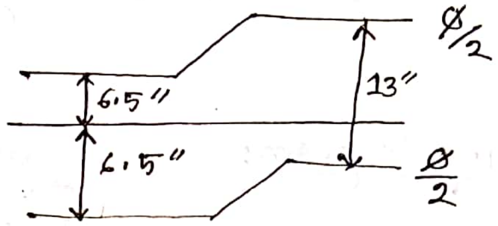
$= 0.174$ in^2/ft

$\therefore$ use # 3 bar @ $\frac{0.11 \times 12}{0.197} = 6.7'' \simeq 6.5''$ c/c for (-ve) moment,

$$(-)A_s = \frac{11606.4 \times 12}{0.9 \times 60000 \times (7 - \frac{0.386}{2})} = 0.379 \text{ in}^2/\text{ft}$$

Assume 50% (+ve) Reinforcement will be crank up. Therefore negative reinforcement avaiable from crank bar:

$$\frac{0.11 \times 12}{2 \times 6.5} = 0.102 \text{ in}^2/\text{ft}$$



∴ Additional reinforcement required $= (0.379 - 0.102) \text{ in}^2/\text{ft}$

$= 0.277 \text{ in}^2/\text{ft}$

use # 4 bar @ $\frac{.20 \times 12}{0.277} = 8.66" \simeq 8.5"$ c/c

In long direction,

$$d_{eff} = d_{short} - \phi = (7 - \frac{4}{8}) = 6.5"$$

for (+ve) moment, $(+)A_s = \frac{3933 \times 12}{0.9 \times 60000 \times (6.5 - \frac{0.386}{2})} = 0.14 \text{ in}^2/\text{ft}$

∴ $(+)A_s = 0.14 \text{ in}^2/\text{ft} < A_{s\,min} = 0.174 \text{ in}^2/\text{ft}$

Hence, $(+)A_s = 0.174 \text{ in}^2/\text{ft}$

∴ use # 3 bar @ $\frac{0.11 \times 12}{.174} = 7.58" \simeq 7.5"$ c/c

for (-ve) moment, $(-)A_s = \frac{7533 \times 12}{.9 \times 60000 \times (6.5 - \frac{0.386}{2})} = 0.27 \text{ in}^2/\text{ft}$

Assume 50% positive reinforcement will be crank up.

Therefore, negative reinforcement avaiable from crank bar:

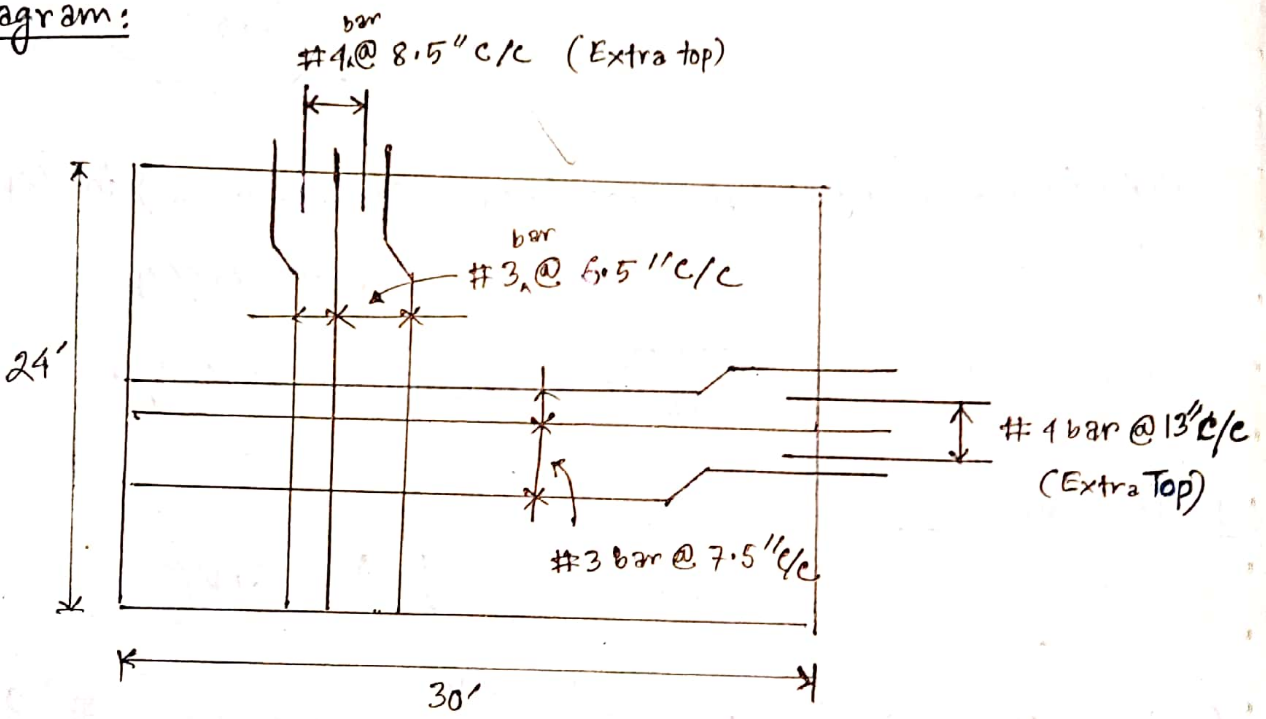
$$\frac{0.11 \times 12}{2 \times 7.5} = 0.088 \text{ in}^2$$

$\therefore$ Additional reinforcement required $= (0.27 - 0.088)$

$= 0.182 \text{ in}^2/\text{ft}$

$\therefore$ use # 4 bar @ $\frac{0.20 \times 12}{0.182} = 13.2'' \simeq 13''$ c/c

Working Diagram:



Bond Check:

$$V_d = \frac{V_{max}}{\Sigma_0 \left(d - \frac{a}{2}\right)} = \frac{0.5 \times 310 \times 30}{\frac{12}{7.5} \times \pi \times \frac{3}{8} \times \left(6.5 - \frac{0.386}{2}\right)} = 391.14 \text{ lb}$$

$$V_{all} = \frac{6.7 \times \sqrt{f_c'}}{D} = \frac{6.7 \times \sqrt{3000}}{3/8} = 978.6 \text{ lb} > V_d$$

(OK)

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