Yield Lines

Yield Line: For slabs, a location along a slab, upon overloading, there would be large enelastic rotation at at essentially a constant resisting moment. This mechanism is the yield line.

~~Lower~~ Upper Bound Theorem: If, for a small increament of displacement the internal work done by the slab, (assuming that the moment at every plastic hinge is equal to the yield moment and the boundary conditions are satisfied) is equal to the external work done for a given load for that same displacement, then that load is known as the upper bound of the true carrying capacity.

Lower Bound Theorem: If for a given external load

- it is possible to find a distribution of moments that satisfies boundary conditions, equilibrium requirements.
- With the moment not exceeding the yield moment at any location,
- and if the boundary conditions are satisfied

Then the given load is the lower bound of the true capacity.

Assumptions of Yield line theory:

(i) At the collapse stage, the steel reinforcement will be fully yielded along yield lines.

(ii) At the collapse stage, the slab is deformed plastically & and the get seperated into segments. Elastic behaviour is followed by each segment.

(iii) Only plastic deformations are taken into consideration. The elastic deformations are neglected.

(iv) Along the yield line, there's uniform distribution of bending & twisting moment.

(v) The yield lines are intersection of two planes, hence they'll remain straight.

Guidelines for establishing axes of Rotation and yield lines:

(i) A yield line is a straight line because they represent the intersection of two planes.

(ii) Yield lines represents axis of rotation

(iii) The support edges will cause an axis of rotation. If the edge is fixed, then a negative yield line will occur, providing moment resistance to ration. If the edge is simply supported then the axis of ratation will provide zero renistance.

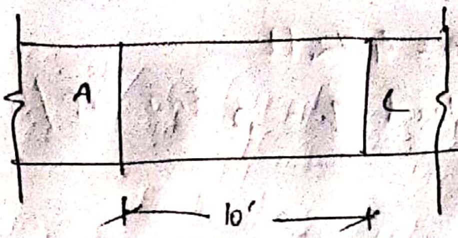
(iv) Axis of rotation will pass over any column sup

(v) Yield lines under concentrated loads will radiate outward from the point of application.

(vi) An yield line between two slab segments should through the intersection of the axes of rotation of the adjacant two slab segment.

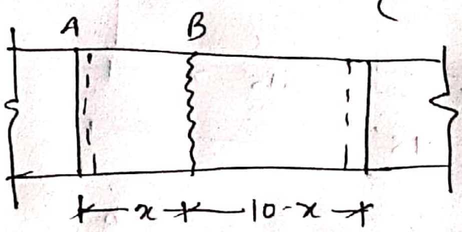
Equilibrium Method:

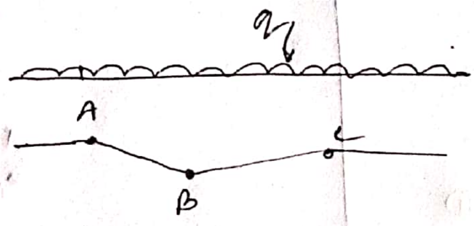
Example: 14.1 A slab is one-way, uniformly loaded & continuous as shown in figure:

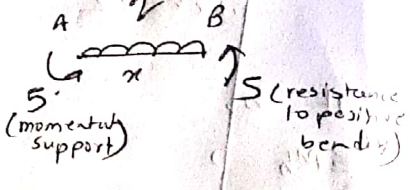


The slab is 10' in span and is reinforced to provide a resistance to positive bending $\phi m_n = 5$ kip ft/ft through the span. In addition, negative steel over the supports provides moment capacities of 5 kip ft/ft at A and 7.5 ft.kip/ft at C. Determine the load capacity of slab and location of yield line.

Solⁿ:

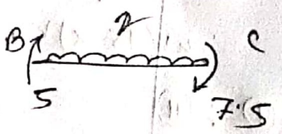






Taking moment about A, $M_A = 0$

$$\Rightarrow \frac{q x^2}{2} - (5+5) = 0$$



Taking moment about C, $M_C = 0$

$$\Rightarrow \frac{q(10-x)^2}{2} - (5+7.5) = 0$$

$\therefore$ Solving, $q = 0.82$ kip/ft^2

location of yield line $x = 4.75$ ft

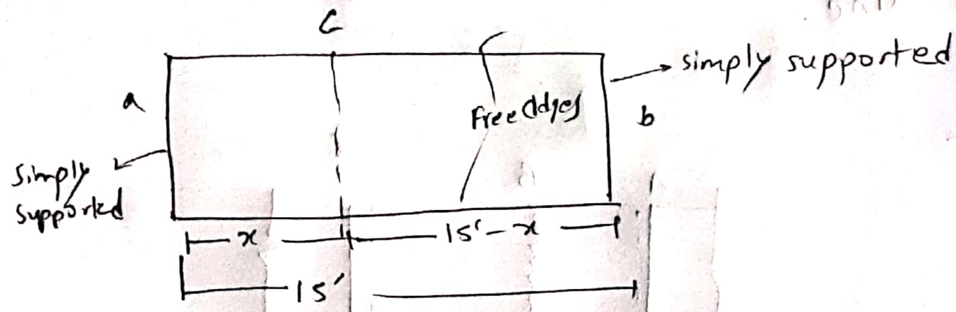
$$\frac{\frac{x^2}{2}}{\frac{(10-x)^2}{2}} = \frac{10}{12.5}$$

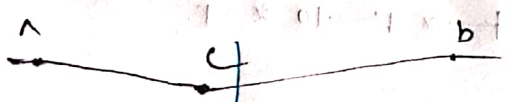
$$\Rightarrow \frac{x^2}{(10-x)^2} = \frac{10}{12.5} \qquad \therefore x = 4.72 \text{ ft}$$

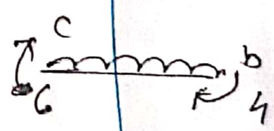
$\therefore$ load $q = 0.897$ kip/ft²

Ans:

2016 The yield moment at a is $m_a = -5$ ft-kips/ft, at b is $m_b = -4$ ft-kips/ft and for positive moment is $m_c = 6$ kips-ft/ft shown in figure below. By the yield line method, calculate the ultimate uniform load the slab will support on a 15' span.







At a,

$\Sigma m_a = 0$

$$\Rightarrow \frac{w x^2}{2} - 11 = 0$$

$$\therefore w x^2 = 22 \quad \text{—— (i)}$$

At b,

$\Sigma m_b = 0$

$$\Rightarrow \frac{w(15-x)^2}{2} - 10 = 0$$

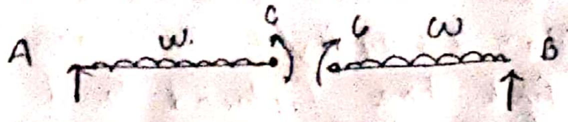
$$\therefore w(15-x)^2 = 20 \quad \text{—— (ii)}$$

(i) ÷ (ii)

$$\frac{\cancel{w} x^2}{\cancel{w}(15-x)^2} = \frac{22}{20}$$

$\therefore$ location of yield = 7.68 ft

Uniform load capacity $w = 0.373$ kip/ft²



$\Sigma M_A = 0$

$\Rightarrow \frac{w x^2}{2} - 6 = 0$

$\therefore w x^2 = 12$

$\Sigma M_B = 0$

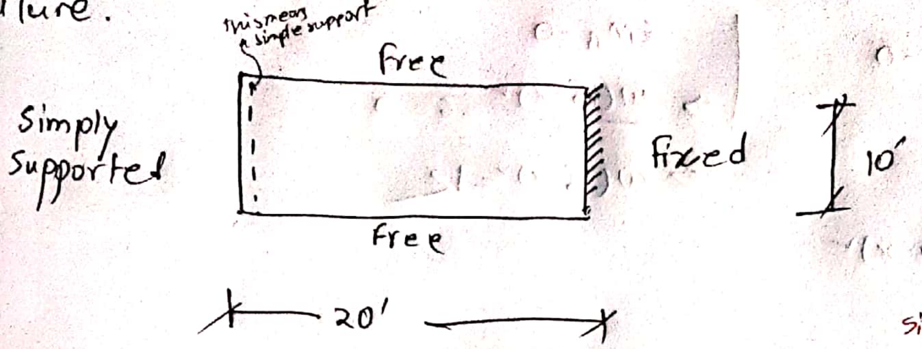
$\Rightarrow \frac{w(15-x)^2}{2} - 6 = 0$

$\Rightarrow w(15-x)^2 = 12$

$\therefore \cancel{w} x^2 = \cancel{w}(10 \cdot x)^2$

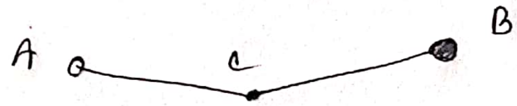
$\Rightarrow$

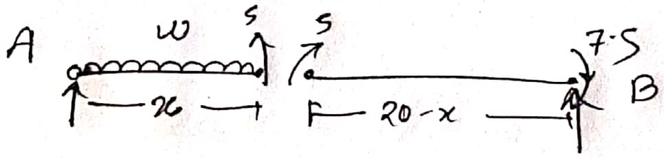
* **Problem: 14.3** The one-way reinforced concrete slab spans 20'. It's simply supported at it's left edge, fully fixed at it's right edge, and free of support along the two long sides. Reinforcement provides design strength $\phi m_n = 5$ ft-kips/ft in positive bending and $\phi m_n = 7.5$ kip-fts/ft in negative bending at the right edge. Find the factored load q_u uniformly distributed over the surface that would cause flexure failure.



simply support এ নেই resisting moment তাই yield line ও নেই

Soln:





$\Sigma M_A = 0;$ $\quad \dfrac{w x^2}{2} = 5$

$\Rightarrow w x^2 = 10$ —— (i)

$\Sigma M_B = 0,$

$\dfrac{w(20-x)^2}{2} - 5 - 7.5 = 0$

$w(20-x)^2 = 25$ —— (ii)

(i) ÷ (ii)

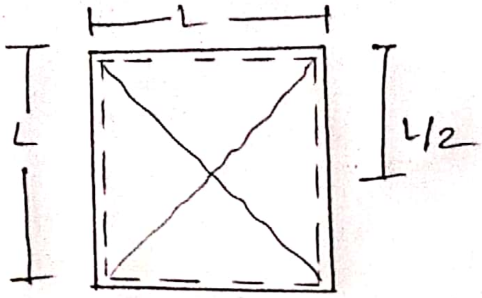
$\dfrac{x^2}{(20-x)^2} = \dfrac{10}{25}$ $\quad \therefore x = 7.748'$

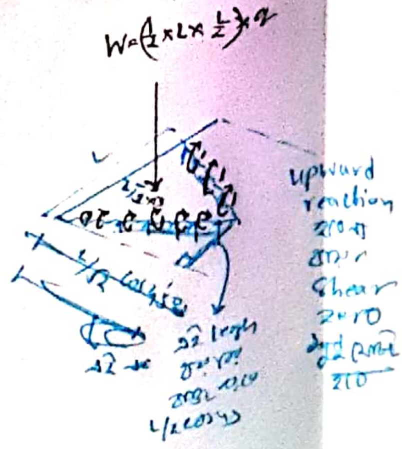
$\therefore w = 0.167 \text{ k/ft}^2$

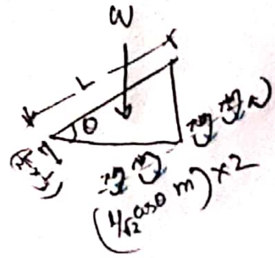
Virtual Work Analysis:

Ex 14.2: A square slab is simply supported along all sides and is to be isotropically reinforced. Determine the resisting moment $m = \varphi m_n$ per linear foot required just to sustain a uniformly distributed factored load of q psf.

soln: we know for a simply supported square slab







Total load of each isometric section, $w = \left(\frac{1}{2} \times L \times L/2\right) \times q$

∴ moment about support of length L,

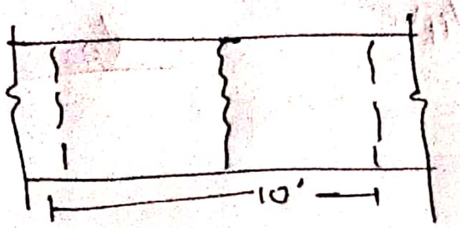
$\Sigma M_L = 0$

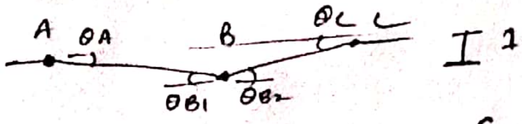
$\Rightarrow w \times L/3 \times 2 - \left(L/\sqrt{2} \cos 45° \times m\right) \times 2 = 0$

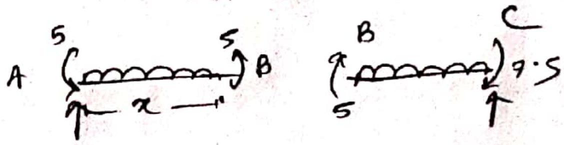
$\Rightarrow \frac{q L^2}{4} \times L/3 \times 2 - \frac{L}{\sqrt{2}} \frac{1}{\sqrt{2}} m \times 2 = 0$

$\Rightarrow m = \frac{qL^2}{24}$

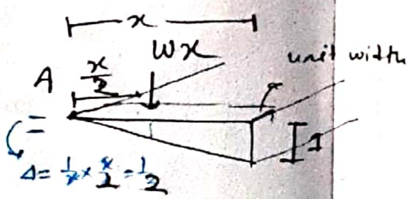
Ex. 14.3 Determine the load capacity of the one-way uniformly loaded continuous slab, using method of virtual work. The resisting moment of the slab are 5, 5, 7.5 kipft/ft at A, B, C.







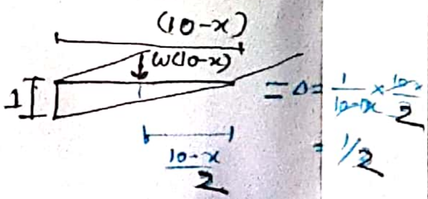
External work:



W_e = Total load on section x deflection at point of resultant load

$$= (x \times 1) w \times \frac{1}{2} + ((10-x) \times 1 \times w) \times \frac{1}{2}$$

$$= \frac{wx}{2} + \frac{(10-x)w}{2} \quad \text{—— (I)}$$

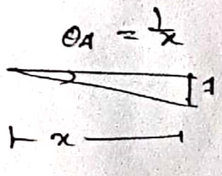


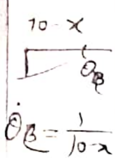
Internal work:

$$W_i = m_{AB} \times \theta_A + m_{BC}\,\theta_C$$

$$= (5+5) \times \frac{1}{x} + (5+7.5)\frac{1}{10-x}$$

$$= \frac{10}{x} + \frac{12.5}{10-x} \quad \text{—— (II)}$$





∴ External work = internal work

$$\frac{wx}{2} + \frac{(10-x)w}{2} = \frac{10}{x} + \frac{12.5}{10-x}$$

$$\Rightarrow w\,\frac{(x+10-x)}{2} = \frac{10}{x} + \frac{12.5}{10-x}$$

$$\Rightarrow w = \frac{2}{x} + \frac{2.5}{10-x}$$

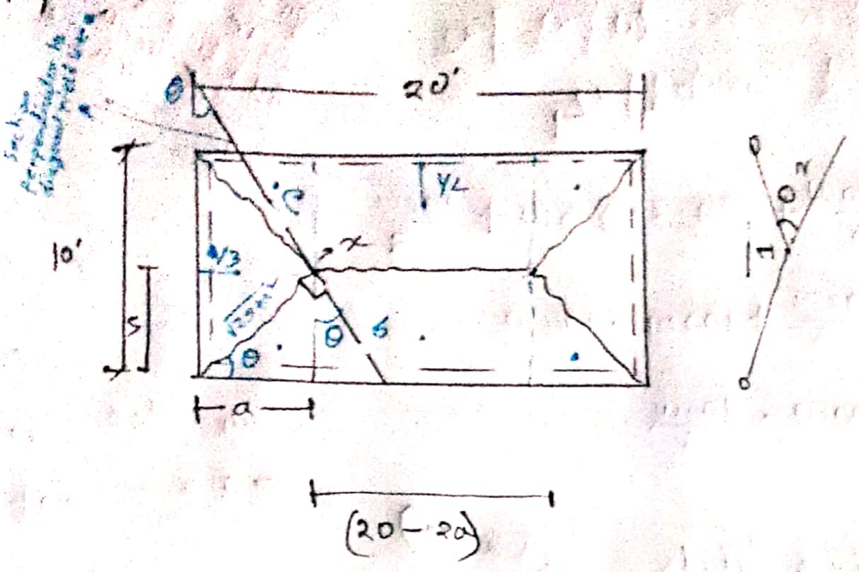
Now, $\frac{dw}{dx} = -\frac{2}{x^2} + \frac{2.5}{(10-x)^2} = 0$

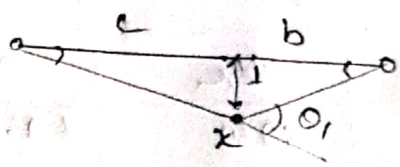
$\therefore x = 4.72\ ft$

$\therefore w = 0.827\ kip/ft^2$

Ans:

Example 14.4: The two way slab below is simply supported on all four sides and carries a distributed load of Z psf. Determine required moment resistance for the slab.





$; \; b = 5 \times \frac{\sqrt{25+a^2}}{a}$

$; \; c = a \times \frac{\sqrt{25+a^2}}{5}$

Rotation of the plastic hinge (x) at the diagonal yield line, $\theta_1 = \frac{1}{c} + \frac{1}{b}$

$$= \frac{5}{a\sqrt{25+a^2}} + \frac{a}{5\sqrt{25+a^2}} = \frac{\sqrt{25+a^2}}{5a}$$

And rotation at the hinge of yield line parallel to the length

$$\theta_2 = \frac{1}{5} + \frac{1}{5} = \frac{2}{5} = 0.40$$

$\therefore W_i$ = Work by diagonal + Work by parallel yield line

4ta Diagonal YL or total length

$$= (4ml) \times \theta_1 + (ml)\theta_2$$

$$= 4 \times m \times \sqrt{25+a^2} \times \frac{\sqrt{25+a^2}}{5a} + m \times (20-2a) \times 0.40$$

$$W_i = \frac{4m(25+a^2)}{5a} + 0.40\, m(20-2a) = \frac{20m + 8am}{3}$$

২টা Triangle

$$W_{ex} = \left\{\left(\tfrac{1}{2} \times 10 \times a\right) w \times \tfrac{1}{3}\right\} \times 2 + \cancel{\tfrac{1}{2} \times 5 \times (20 + 20 - 2a) \times w} + \left\{(20-2a)\, w \times 5 \times \tfrac{1}{2}\right\} \times 2$$

$$+ \left[\left\{(20-2a) \times 5 \times w\right\} \times \tfrac{1}{2}\right] \times 2$$

$$+ \left[\left(\tfrac{1}{2} \times a \times 5 \times w\right) \times \tfrac{1}{3}\right] \times 4$$

[illegible] area Yield line [illegible] parallel line [illegible] Area [illegible]

$$W_e = \frac{10aw}{3} + (20-2a)\,5w + \frac{10aw}{3}$$

$$= \frac{20aw}{3} + 100w - 10aw$$

$$\therefore W_{ex} = 100w - \frac{10aw}{3}$$

$\therefore W_i = W_{ex}$

$$\Rightarrow m = \frac{\dfrac{300w - 10aw}{3}}{\dfrac{20 + 8a}{a}}$$

Double Calculate

$\therefore$ Successive trials (use function)

a	W_i	W_{ex}	m
6	11.33 m	80 w	7.069 w
6.5	11.0767 m	78.33 w	7.072 w
7	10.857 m	76.67 w	7.061 w
7.5	10.67 m	75 w	7.027 w

$\therefore$ Yield line denoted by 6.5 is critical for it'll have the most moment.