$$Q = \text{Area} \times \text{Velocity} = (b \times 1) \times K\left(-\frac{dh}{dl}\right)$$

$$Q = Kb\left(-\frac{dh}{dl}\right) = T\left(-\frac{dh}{dl}\right) \qquad \text{... (9.12)}$$

where $T = Kb$ is the transmissivity. When the hydraulic gradient equals unity, the discharge is equal to T itself. In other words, the transmissivity of an aquifer may be defined as the discharge through the aquifer of one metre width under a unit hydraulic gradient. The value of T ranges from about 12 m^2/day to about 1200 m^2/day depending on the type and thickness of the aquifer. It is usually evaluated from the pumping tests.

Example 9.1. *An undisturbed rock sample has an oven dry weight of 0.655 kg. After saturation with kerosene its weight is 0.732 kg. It is then immersed in kerosene and found to displace 0.301 kg. What is the porosity of the sample ?*

Solution. Oven-dry weight $w_1 = 0.655$ kg

Weight of saturated sample $w_2 = 0.732$ kg

Weight of kerosene required to saturate the sample

$$= (w_2 - w_1)$$
$$= 0.732 - 0.655 = 0.077 \text{ kg}$$

Weight of kerosene displaced by the saturated sample

$$w_3 = 0.301 \text{ kg}$$

Porosity of the sample n

$$n = \frac{(w_2 - w_1)}{w_3} = \frac{0.077}{0.301} = 0.2558 = 25.58\%$$

Example 9.2. *When 3.68 million m^3 of water was pumped out from an unconfined aquifer of 6.2 km^2 areal extent, the water table was observed to go down by 2.6 m. What is the specific yield of the aquifer ? During a monsoon season if the water table of the same aquifer goes up by 10.8 m what is the volume of recharge ?*

Solution.

Water released from the aquifer $= (6.2 \times 10^6) \times 2.6 \times s_y$

Water pumped out $= 3.68 \times 10^6$ m^3

Equating these two quantities $s_y = \dfrac{3.68}{6.2 \times 2.6} = 0.2283$

$\therefore$ Specific yield of the aquifer $= 0.2283$ or 22.83%

Volume of recharge $= 6.2 \times 10^6 \times 10.8 \times s_y$

$= 6.2 \times 10^6 \times 10.8 \times 0.2283$

$= 15.287$ million m^3.

Example 9.3. *The water table levels in two observation wells 350 m apart are + 210.5 and + 206.25 m respectively. If the hydraulic conductivity and porosity of the aquifer are 12.5 m/day and 15 per cent, what is the actual velocity of flow in the aquifer ?*

Solution. $v = K\left(-\frac{dh}{dl}\right)$

$$v = 12.5 \times \left[-\frac{(206.25 - 210.5)}{350}\right]$$

$$= 0.1518 \text{ m/day}$$

$$v_a = \frac{v}{n} = \frac{0.1518}{0.15} = 1.012 \text{ m/day}$$

∴ The actual velocity in the aquifer = 1.012 m/day.

Example 9.4. *A sample has a hydraulic conductivity of 10 m/day. What would be its intrinsic permeability ? What is its hydraulic conductivity in cm/s ? What would be its hydraulic conductivity at 30°C ?*

Solution. $k = \frac{K.\mu}{\rho g}$

$K = 10$ m/day $= 0.011574$ cm/s

$\mu = 0.01$ gm-cm/s

$\rho = 1$ gm/c.c.

$g = 981$ cm/s^2

$$\therefore \quad k = \frac{0.011574 \times 0.01}{981} = 1.17982 \times 10^{-7} \text{ cm}^2$$

But one darcy $= 0.987 \times 10^{-12}$ m^2 $= 0.987 \times 10^{-8}$ cm^2

$$\therefore \quad k = \frac{1.17982 \times 10^{-7}}{0.987 \times 10^{-8}} = 11.954 \text{ darcys}$$

Intrinsic permeability of the sample = 11.954 darcys

Hydraulic conductivity in cm/s = 0.011574.

From Eq. (9.10), $K = \frac{cd^2 \rho g}{\mu}$

$$= \frac{cd^2 g}{(\mu/\rho)} = \frac{cd^2 g}{\nu}$$

where ν is the kinematic viscosity. Thus for a given pours medium the hydraulic conductivity is inversely proportional to the kinematic viscosity of the fluid. Therefore, we have

$$\frac{K}{K_t} = \frac{\nu_t}{\nu} \quad \text{or} \quad K_t = K \cdot \frac{\nu}{\nu_t}$$

where K_t and ν_t are the hydraulic conductivity and kinematic viscosity at some temperature t and K and ν are the hydraulic conductivity and kinematic viscosity at standard temperature of 20°C.

$$K_{30} = K \cdot \frac{\nu}{\nu_{30}} = 10 \times \frac{0.01}{0.008} = 12.5 \text{ m/day}$$

(Since kinematic viscosity for water at 20°C is 0.01 cm^2/s and at 30°C, it is 0.008 cm^2/s)

∴ Hydraulic conductivity of the sample at 30°C = 12.5 m/day.

9.4 GROUNDWATER MOVEMENT

Groundwater moves very slowly in the aquifers under differential head. A groundwater basin in filled and the excess water is discharged by several ways until a quasi-equilibrium is reached. All the perennial streams are generally effluent through at least a portion of their length gathering contributions from groundwater. When the water table is close to the surface, groundwater may be discharged by direct evaporation and transpiration. The groundwater movement in aquifers is governed by the established hydraulic principles. The theory of groundwater movement is based on Darcy's law. The study of groundwater movement helps in predicting the yields from a well and the effects of future pumping, evaluating aquifer parameters, defining the aquifer boundaries, analysing the salt water intrusion problems etc. The theory of groundwater movement around wells is commonly termed the *well hydraulics*. There are two cases to be considered ; *steady state* and *unsteady state* flows. The steady state case rarely occurs in practice but the conditions after a prolonged pumping from a well may be approximated as steady state. The unsteady state case, or the *transient flow* case is of more practical significance. The following sections deal with steady and unsteady radial groundwater flow towards wells.

9.5 STEADY RADIAL FLOW TO WELLS

The analysis of groundwater movement will be greatly simplified if the vertical flow components are neglected and the groundwater is assumed to move primarily in a lateral direction. Accordingly the following assumption are made in the theory of groundwater movement.

1. The flow is horizontal and uniformly distributed in a vertical section.
2. The velocity of flow is proportional to the tangent of the hydraulic gradient i...

$$Q = \frac{2\pi T s_w}{\ln\left(\frac{R}{r_w}\right)} \quad ...(9.15)$$

The following empirical equation may sometimes be useful to predict the radius of influence R.

$$R = 3000\, s_w \sqrt{K} \quad ...(9.16)$$

where s_w is in m and K is in m/s.

Equation (9.13) or (9.14) can be used (for a given drawdown) to estimate the yield from a well when the hydraulic conductivity is known, or to estimate the hydraulic conductivity of the aquifer if the discharge from the well is known.

This equation was derived by Dupuit in 1863 and modified by Thiem in 1906. It fails to describe accurately the drawdown curve near the well because the streamlines there are markedly curved, with large vertical flow components contradicting the assumption. Another serious drawback of this equation is that the true steady state equilibrium condition required for its applicability occurs only after a very long pumping period because of the low velocities of groundwater flow.

Example 9.5. *An unconfined aquifer has a thickness of 30 m. A fully penetrating 20 cm diameter well in this aquifer is pumped at a rate of 35 lit/s. The drawdown measured in two observation wells located at distances of 10 m and 100 m from the well are 7.5 m and 0.5 m respectively. Determine the average hydraulic conductivity of the aquifer. At what distance from the well the drawdown is insignificant ?*

Solution. $$Q = \frac{\pi K(h_1^2 - h_2^2)}{\ln\left(\frac{r_1}{r_2}\right)}$$

or $$K = \frac{Q \cdot \ln\left(\frac{r_1}{r_2}\right)}{\pi(h_1^2 - h_2^2)}$$

Here $Q = 35$ lit/s $= 0.035$ m^3/s

$r_1 = 100$ m ; $r_2 = 10$ m ; $\ln\left(\frac{r_1}{r_2}\right) = 2.302585$

$H = 30$ m ; $s_1 = 0.5$ m and $s_2 = 7.5$ m

$h_1 = H - s_1 = 30 - 0.5 = 29.5$ m

CamScanner

$$h_2 = H - s_2 = 30 - 7.5 = 22.5 \text{ m}$$

$$\therefore \quad K = \frac{0.035 \times 2.302585}{\pi(29.5 + 22.5)(29.5 - 22.5)}$$

$$= 7.04745 \times 10^{-5} \text{ m/s}$$

$$= 6.089 \text{ m/day}$$

Let R be the radius of influence where the drawdown is zero.

$$\text{Then} \quad Q = \frac{\pi K(H^2 - h_1^2)}{\ln\left(\frac{R}{r_1}\right)}$$

$$\ln\left(\frac{R}{r_1}\right) = \frac{\pi K(H^2 - h_1^2)}{Q}$$

$$= \frac{\pi \times 7.04745 \times 10^{-5}(30^2 - 29.5^2)}{0.035}$$

$$= 0.1882$$

$$\frac{R}{r_1} = 1.207$$

$$\therefore \quad R = 1.207 \times 100 = 120.7 \text{ m.}$$

Example 9.6. *A well with a radius of 0.5 m, completely penetrates an unconfined aquifer of thickness 50 m and K = 30 m/day. The well is pumped so that the water level in the well remains at 40 m above the bottom. Assuming that pumping has essentially no effect on water table at r = 500 m, what is the steady-state discharge.*

Solution. $Q = \dfrac{\pi K(H^2 - h_w^2)}{\ln\left(\frac{R}{r_w}\right)}$

Here $H = 50$ m ; $h_w = 40$ m ; $R = 500$ m ; $r_w = 0.50$ m ; $K = 30$ m/day.

$$\therefore \quad Q = \frac{\pi \times 30 \times (50^2 - 40^2)}{\ln\left(\frac{500}{0.5}\right)} = 12279.387 \text{ m}^3\text{/day}$$

$$= 0.1421225 \text{ m}^3\text{/s}$$

$$= 142.12 \text{ lit/s.}$$

Steady Radial Flow to a Well in a Confined Aquifer. The flow around a well penetrating fully into a confined aquifer of thickness '*b*' under steady-state conditions is shown in Fig. 9.8. In this case the depth of flow

Example 9.7. *In an artesian aquifer of 8 m thick, a 10 cm diameter well is pumped at a constant rate of 100 lit/minute. The steady state drawdown observed in two wells located at 10 m and 50 m distances from the centre of the well are 3 m and 0.05 m respectively, compute the transmissivity and the hydraulic conductivity of the aquifer.*

Solution. $$Q = \frac{2\pi T (s_2 - s_1)}{\ln\left(\frac{r_1}{r_2}\right)}$$

$$Q = 100 \text{ lit/min} = 0.00167 \text{ m}^3\text{/s}$$

$$r_2 = 10 \text{ m} \; ; \text{ and } r_1 = 50 \text{ m}$$

$$\therefore \quad \ln\left(\frac{r_1}{r_2}\right) = 1.60944$$

$$(s_2 - s_1) = 3 - 0.05 = 2.95 \text{ m}$$

$$\therefore \quad T = \frac{0.00167 \times 1.60944}{2\pi \times 2.95} = 1.4472 \times 10^{-4} \text{ m}^2\text{/s}$$

$$= 12.5 \text{ m}^2\text{/day}$$

The permeability of the aquifer

$$K = \frac{T}{b} = \frac{1.4472 \times 10^{-4}}{8}$$

$$= 1.809 \times 10^{-5} \text{ m/s}$$

$$= 1.563 \text{ m/day.}$$

Example 9.8. *A well of 0.5 m diameter penetrates fully into a confined aquifer of thickness 20 m and hydraulic conductivity 8.2×10^{-4} m/s. What is the maximum yield expected from this well if the drawdown in the well is not to exceed 3 m. The radius of influence may be taken as 260 m.*

Solution. $$Q = \frac{2\pi T s_w}{\ln\left(\frac{R}{r_w}\right)}$$

$$T = K \cdot b$$

$$= 8.2 \times 10^{-4} \times 20$$

$$= 0.0164 \text{ m}^2/\text{s}$$

$$s_w = 3 \text{ m} ;$$

$$\frac{R}{r_w} = \frac{260}{0.25}$$

$$= 520$$

$$\ln\left(\frac{R}{r_w}\right) = 6.25383$$

$$\therefore \quad Q = \frac{2\pi \times 0.0164 \times 3}{6.25383}$$

$$= 0.04943 \text{ m}^3/\text{s}$$

$$= 49.43 \text{ lit/s.}$$

9.13 Describe the Recovery test method to determine the transmissivity of the aquifer.

9.14 How do you compare the results of aquifer parameters obtained from Theis, Cooper and Jacob, and Chow's methods ?

9.15 What do you understand by *safe yield* of a groundwater basin ? What are the factors influencing the safe yield ? What is meant by *overdraft* and mining ?

9.16 Explain how the yield of an open well can be determined using recuperation test.

9.17 Define the terms critical depression head, working head, and specific capacity in the case of an open well.

9.18 What are the advantages of groundwater compared to surface water.

Analytical

9.19 An undisturbed rock sample has an oven-dry weight of 1305 gm. When it is completely saturated with kerosene it weighed 1463 gm.The saturated sample, when immersed in kerosene, displaced 605 gm of kerosene. What is the porosity of the sample.

9.20 An unconfined aquifer has an areal extent of 15 km^2. When 9.5 million m^3 of water was pumped out, the water table was observed to go down by 2.4 m. What is the specific yield of the aquifer ? If the water table of the same aquifer rises by 12.5 m during a monsoon season, what is the volume of recharge ?

9.21 The elevation of water table in an unconfined aquifer at two locations separated by a distance of 100 m is 1026.2 m and 1025.0 m respectively. If the permeability of the aquifer is 12 m/day and porosity is 15 per cent, what is the actual velocity of flow in the aquifer ?

9.22 A soil sample has a hydraulic conductivity of 15 m/day. What would be its intrinsic permeability ? What would be its hydraulic conductivity at 30°C ?

9.23 In order to determine the permeability of an aquifer, a tracer is introduced in an observation well and it is traced in another downstream well 78 m away from the first after 46.5 h. If the elevations of water levels in the two wells differ by 2.9 m and the porosity of the aquifer is 18%, calculate the coefficient of permeability of the aquifer.

9.24 An unconfined aquifer has an area of 325 km^2, thickness of 24.5 m and a porosity of 30%. What is its specific retention if it can yield 1890 million m^3 of free-draining water ?

9.25 What is the hydraulic conductivity of a porus medium with fluid density 1 gm/c.c. fluid viscosity 1 centipoise and intrinsic permeability 7.5 darcys.

9.26 In a water table aquifer of 50 m thickness, a 20 cm diameter well is pumped at a uniform rate of 0.05 m^3/s. If the steady state drawdown measured in the observation wells located at 10 m and 100 m distances from the well are 6.5 m and 0.25 m respectively, determine the hydraulic conductivity of the aquifer.

9.27 Determine the radius of influence and the drawdown in the well of Exercise 9.26.

9.28 Determine the yield from a 30 cm diameter well under a drawdown of 10 m in the well, if the radius of influence and hydraulic conductivity are 150 m and 5.0 m/day respectively. The aquifer is unconfined with a thickness of 60 m.

9.29 In Exercise 9.28, if all other conditions remain unchanged, what is the percentage change in the yield under the following cases

(*a*) The diameter of the well is increased to 45 cm

(*b*) The drawdown is limited to 6 m

(*c*) The permeability of the aquifer becomes 12.5 m/day.

9.30 A 20 cm diameter well penetrates fully a confined aquifer of thickness 25 m. When the well is pumped at a rate of 200 litres/minute the steady state drawdown in the two observations wells located at 10 m and 100 m distance from the pumping well are found to be 3.5 m and 0.05 m respectively. Calculate the permeability and the transmissivity of the aquifer.

9.31 Determine the radius of influence and the drawdown in the well for the well of Exercise 9.30.

9.32 A well with a radius of 0.5 m penetrates completely a confined aquifer of thickness 40 m and permeability 30 m/day. The well is pumped so that the water level in the well remains at 7.5 m below the original piezometric surface. Assuming that the radius of influence is 500 m, compute the steady state discharge from the well.

9.33 For the well in Exercise 9.32, what is the percentage decrease in drawdown for the same discharge if the well diameter is doubled ? What is the percentage increase in discharge for the same drawdown if the well diameter is doubled ? Assume the radius of influence to be same in all the cases.

9.34 A well is pumped at the constant rate of 0.004 m^3/s in a confined aquifer of transmissivity T = 0.004 m^2/s and storativity S = 0.0005. Calculate the drawdown 24 h after the start of pumping in an observation well located at a distance of 250 m from the pumped well.

9.35 A well is pumped at the constant rate of 0.008 m^3/s. A match of the well function [u versus $W(u)$] with drawdown versus time data from an observation well located at a distance of 430 m from the pumped well has produced the following matching values : u = 1, $W(u)$= 1, s = 0.21 m and r^2/t = 2055 m^2/minute. Calculate the transmissivity and storativity of the aquifer.

Chapter Problems:

9.19/

Dry weight, $w_1 = 1305$ gm
Saturated " , $w_2 = 1963$ gm
displaced kerosine, $w_3 = 605$ gm
∴ porosity, $n = \frac{w_2 - w_1}{w_3} = 26.11\%$.

Ans:

9.20/

Water drained out $= 7.5 \times 10^6 \, m^3$
Volume of available water, $= 15 \times 10^6 \times 2.4 \times S_y$
∴ available water = water drained out
⇒ $15 \times 10^6 \times 2.4 \times S_y = 7.5 \times 10^6$
∴ $S_y = 26.38\%$.

Answer:

Volume of available water in monsoon $= 15 \times 10^6 \times 12.5 \times S_y$
Water recharged $= V$
∴ $V = 15 \times 10^6 \times 12.5 \times 0.2638 = 49.5 \times 10^6 \, m^3$

Answer

9.21 Here, from Darcy's law,

Discharge velocity, $V = ki$
$= 12 \times \frac{1026.2 - 1025}{100} = 0.144 \, m/day$

∴ we know, Actual velocity, $V_a = \frac{V}{n} = \frac{0.144}{0.15}$
$= 0.96 \, m/day$

Ans.

9.22| Here, $K = 15\ m/day = \frac{15 \times 100}{86400} = 0.01736\ cm/s$

viscosity, $\mu = 0.01\ gm\cdot cm/s$
$\rho = 1\ g/cc$
$g = 981\ cm/s^2$

we know, $k = \frac{K\mu}{\rho g}$

$= \frac{0.01736 \times 0.01}{1 \times 981}$

$= 1.7696 \times 10^{-7}\ cm/s$

$= \frac{1.7696 \times 10^{-7}}{0.987 \times 10^{-8}}$ Darcy 1 Darcy $= 0.987 \times 10^{-8}\ cm^2$

$= 17.93$ darcys Ans:

we know,
$K_{20^\circ} = 0.01736$

$\nu_{20^\circ} = \frac{\mu}{\rho} = \frac{0.01}{1} = 0.01\ cm^2/s$

$\nu_{30^\circ} = 0.008\ cm^2/s$

$K_{30^\circ} = ?$

Direct 15m/dy x factor ← easy

$\therefore \frac{K_{20}}{K_{30}} = \frac{\nu_{30}}{\nu_{20}} \Rightarrow K_{30} = \frac{\nu_{20} \times K_{20}}{\nu_{30}} = \frac{0.01 \times 0.01736}{0.008}$

$= 2.17 \times 10^{-2}\ cm/s$

$= 18.75\ m/day$

Ans.

9.23| $V_a = 78/46.5 = 1.677\ m/hr$
$n = 0.18$

$\therefore V_a = \frac{V}{n}$

$\Rightarrow V = n V_a = 0.15 \times 1.677 = ~~0.2516~~\ 0.30186\ m/hr$

Now,
$V = ki$

$\Rightarrow k = \frac{V}{i} = \frac{~~0.2516~~\ 0.30186}{2.0/78} = 8.118\ m/hr$ Ans

9.24 Here, specific yield, $S_y = \dfrac{\text{Water drained}}{\text{water available}}$

$$= \frac{1870\times10^6}{325\times10^6\times24.5} = 0.23736$$

∴ We know,

$$S_y + S_r = n$$

$$\Rightarrow S_r = n - S_y = 0.30 - 0.23736 = 0.0626$$

Ans.

9.25 We know,

$$K = \frac{k\rho g}{\mu}$$

$$\Rightarrow k = \frac{K\mu}{\rho g} = \frac{7.5\times10^{-8}\times 0.987\times10^{-8}\times1\times981}{0.01}$$

$$= 6.27 \text{ m/day}$$

Answer.

Well এর Math: Example 4টির [illegible]

9.26

$H = 50$ m, $r = 0.1$ m, $Q = 0.05$ m³/s

$r_1 = 10$ m, $r_2 = 100$ m

$s_1 = 6.5$, $s_2 = 0.25$

$h_1 = H - s_1 = 43.5$ m

$h_2 = H - s_2 = 49.75$ m

$$\therefore Q = \frac{\pi K (h_2^2 - h_1^2)}{2.303 \log(r_2/r_1)}$$

$$\Rightarrow 0.05 = \frac{\pi K (49.75^2 - 43.5^2)}{2.303 \log\left(\frac{100}{10}\right)}$$

$\therefore K = 5.437$ m/day

Ans.

Let, draw down of well = h, Radius of influence = R

$$86400\times0.05 = \frac{\pi K (H^2 - h_1^2)}{2.303 \log \frac{R}{r_1}}$$

$$= \frac{\pi\times5.4377\,(50^2 - 43.5^2)}{2.303 \log \frac{R}{10}}$$

$\therefore R = 110.547$ m

$R = 110.547$

$$\therefore Q = \frac{\pi k (H-h)}{2.303 \log\left(\frac{R}{r}\right)}$$

$$\Rightarrow 86400 \times 0.05 = \frac{\pi \times 5.4377 \times (50-h)}{2.303 \log\left(\frac{100}{0.1}\right)} \quad 110.547$$

$\therefore h = 26.94 \, m$

$s = 50 - 26.94 = 23.05$ Ans.

9.28)

$H = 60 m$

$r = 0.15 m$

$S = 10 m$

$R = 150$

$k = 5 m/day$

$\therefore h = H - S = 60 - 10 = 50 m$

$$\therefore Q = \frac{\pi k (H^2 - h^2)}{2.303 \log\left(\frac{R}{r}\right)}$$

$$= \frac{\pi \times 5 (60^2 - 50^2)}{2.303 \log\left(\frac{150}{0.15}\right)} = 2500.9 \, m^3/day$$

Ans.

9.30)

$r = 0.1 m$

$b = 25 m$

$Q = 200 \times 10^{-3} \times 60 \times 24 = 288 \, m^3/day$

$r_1 = 10 m \quad s_1 = 3.5 m$

$r_2 = 100 m \quad s_2 = 0.05 m$

$$\therefore Q = \frac{2\pi k b (s_1 - s_2)}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi k \times 25 (3.5 - 0.05)}{\ln\left(\frac{100}{10}\right)}$$

$\therefore K = 1.2238 \, m/day$

$\therefore T = kb = 1.2238 \times 25 = 30.59 \, m^2/day$

$$Q = \frac{2\pi kb(S_1 - 0)}{\ln\left(\frac{R}{r_1}\right)}$$

$$\Rightarrow 288 = \frac{2\pi \times 1.2238 \times 3.5}{\ln\left(\frac{R}{10}\right)} \quad \therefore R = 103.41 m$$

Again,

$$Q = \frac{2\pi kb \times S}{\ln\left(\frac{R}{r}\right)}$$

$$\Rightarrow 288 = \frac{2\pi \times 1.2238 \times 25 \times S}{\ln\left(\frac{103.41}{0.1m}\right)} \quad \therefore S = 10.4m$$

Ans:

9.32)

$r = 0.5m$

$b = 40m$

$k = 30m/day$

$S = 7.5m$

$R = 500$

$Q = ?$

$$Q = \frac{2\pi kb\, S}{\ln\left(\frac{R}{r}\right)}$$

$$= \frac{2\pi \times 30 \times 40 \times 7.5}{\ln\left(\frac{500}{0.5}\right)} = 0.0947 \, m^3/s$$

9.34)