Heaven's Light is Our Guide
Department of Civil Engineering
Rajshahi University of Engineering & Technology

CE 3233
Geotechnical Engineering II

Credit: 3 ***Contact Hours : 3 hours/week***

Lecture by-
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Prof. Dept. of CE, RUET

Course Content

Indicative Syllabus (my part)

- Lateral Earth Pressure
- Stress Distribution

Text Books

Reference Books:

i. Principles of Foundation Engineering - by Braja M. Das

ii. Principles of Geotechnical Engineering - by Braja M. Das, Khaled Sobhan

iii. Foundation Analysis and Design – by Joseph E. Bowles

iv. Foundation design and construction –by M.J Tomlinson

v. Soil Mechanics and Foundations - by B. C. Punmia, Ashok Kumar Jain

vi. Soil Mechanics And Foundation Engineering – by K. R. Arora

Learning Outcome

Intended Learning Outcome

Upon completion of the chapter, students will be able to:

- Know the lateral earth pressure and its type.
- Draw lateral earth pressure distribution diagrams according to Rankine and Coulomb theories.
- Calculate the lateral earth pressure exerted on a retaining structure.
- Analyze and design retaining structures

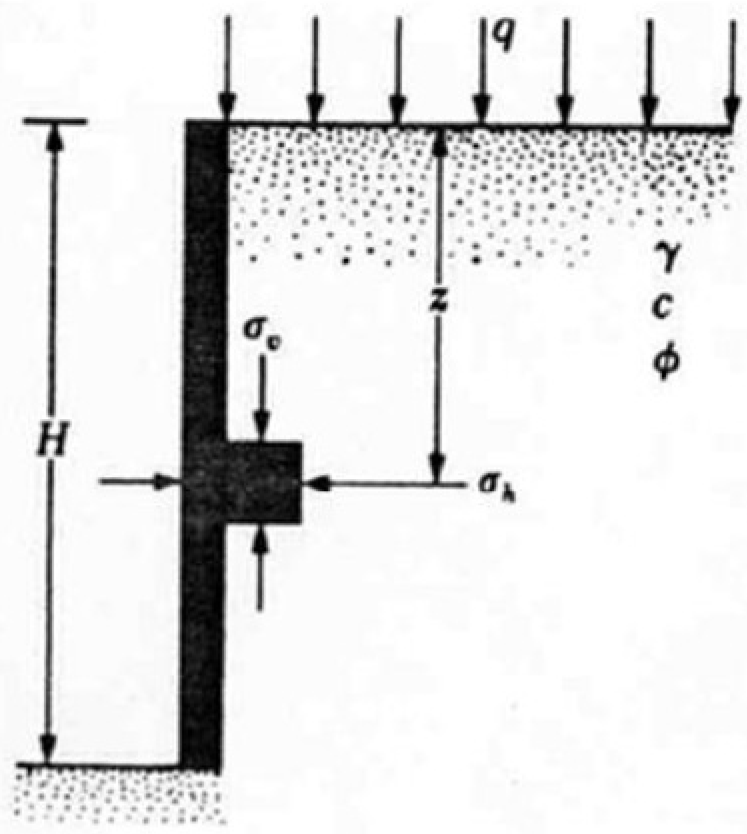
Lateral Earth Pressure

Introduction:

- Different structures such as retaining wall often subjected to lateral pressures.

Types:

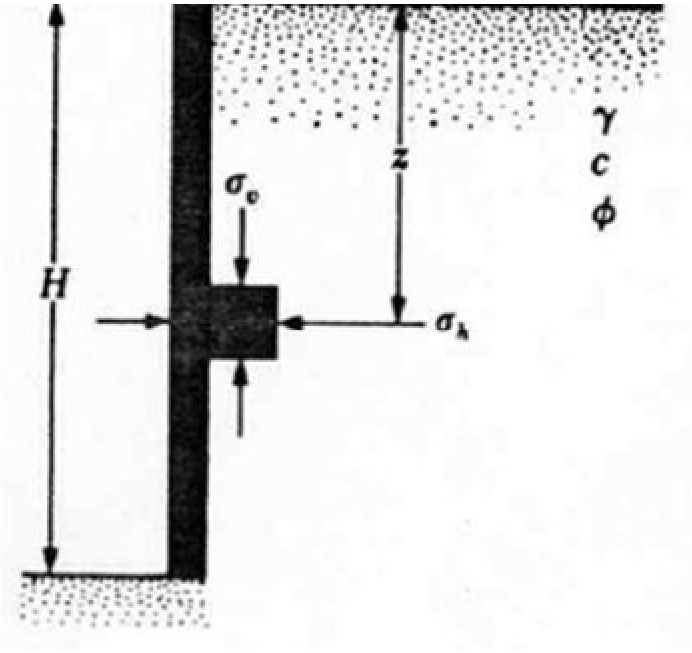
- Earth Pressure at rest
- Active earth pressure
- Passive earth pressure



Lateral Earth Pressure

Lateral Earth Pressure at Rest:

- ❑ Wall is rigid
- ❑ Wall is at rest
- ❑ Does not move with lateral pressure
- ❑ Zero horizontal strain



Coefficient of at–rest earth pressure, K_o

$$K_o = \sigma_h / \sigma_v$$

$$\Rightarrow \quad \sigma_h = K_o \sigma_v$$

$$\Rightarrow \quad \sigma_h = K_o \gamma H$$

Lateral Earth Pressure

Coefficient of Earth Pressure at Rest:

- For normally consolidated coarse-grained soils (Jaky 1944)

$$K_o \approx 1 - \sin\phi'$$

- For normally consolidated fine-grained soils (Massarch 1979)

$$K_o = 0.44 + 0.42\left[\frac{PI(\%)}{100}\right]$$

- For normally consolidated clays, (Brooker and Ireland, 1965)

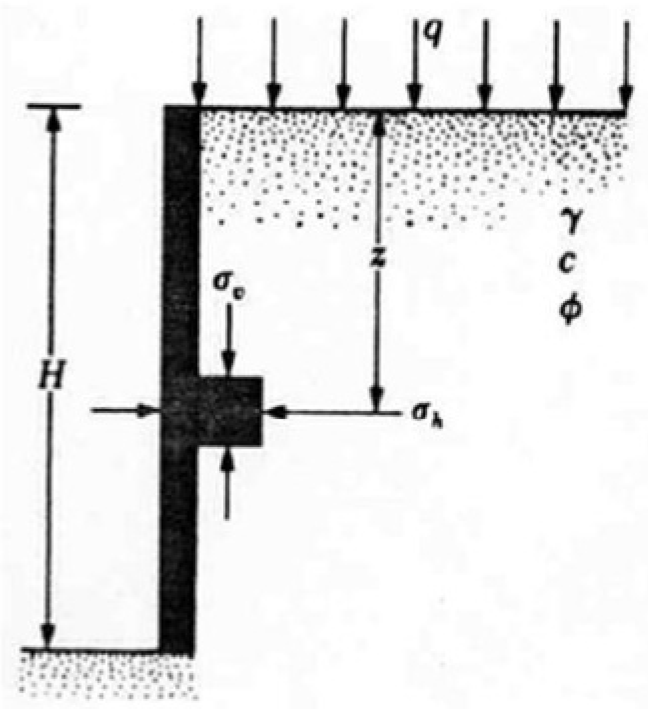
$$K_o = 0.95 - \sin\phi'$$

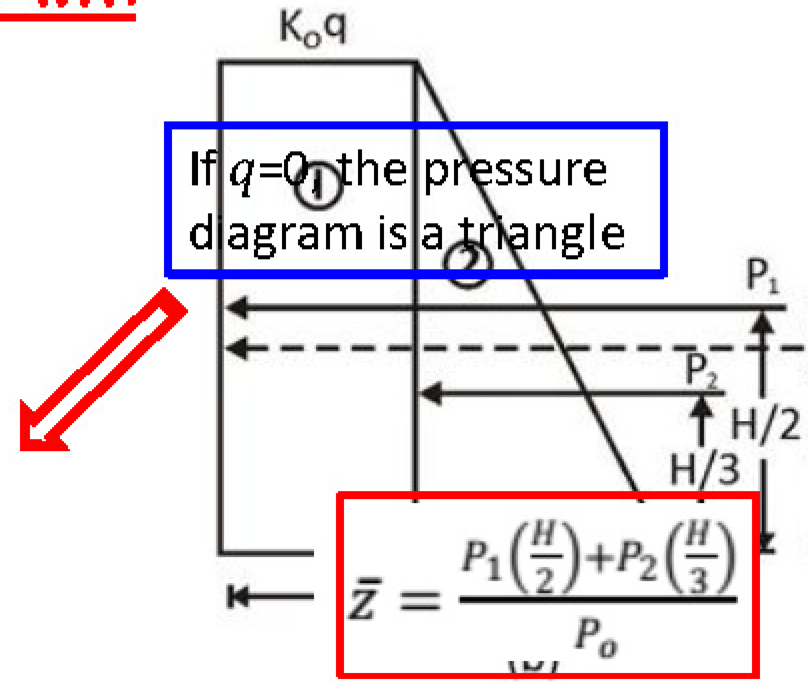
- For over consolidated clays

$$K_{o(OC)} = K_{o(NC)}\sqrt{OCR}$$

Lateral Earth Pressure

Lateral Earth Pressure at Rest witl





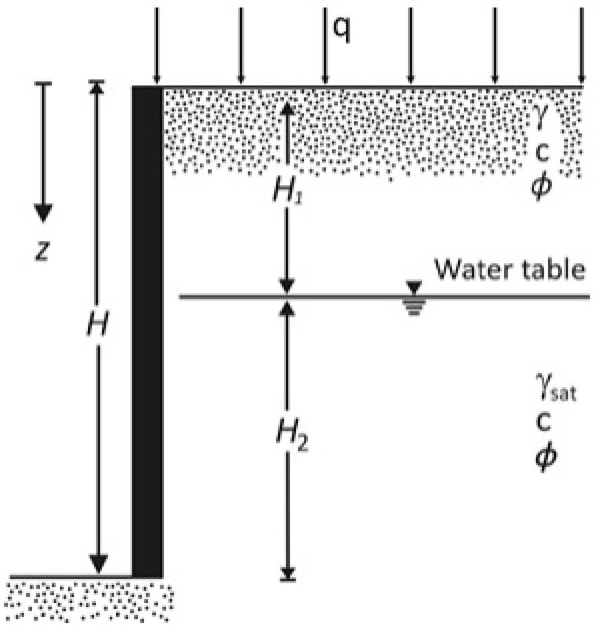
$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2 K_o$$

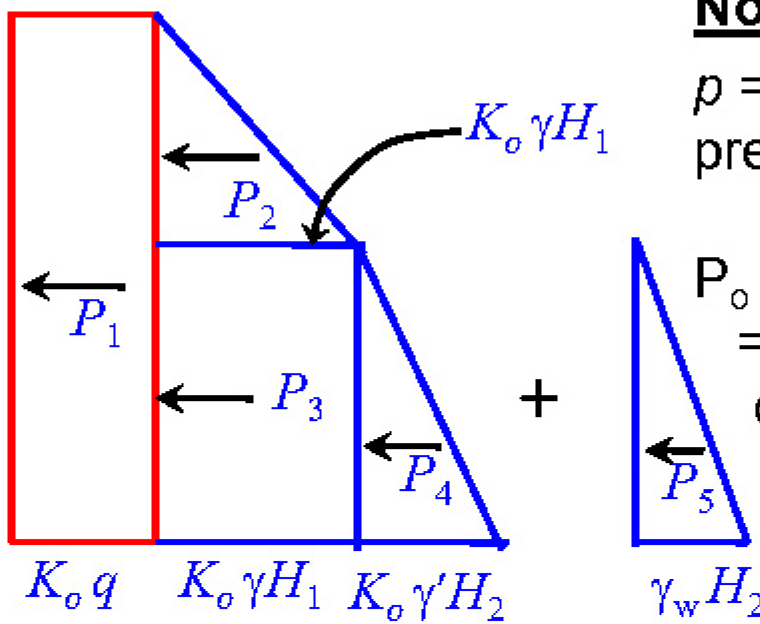
Pressure distribution diagram

Lateral Earth Pressure

Lateral Earth Pressure at Rest with Partially Submerged Backfill and Surcharge:



(a)



Note:

p = intensity of pressure

P_o = Force/thrust =Area of diagram

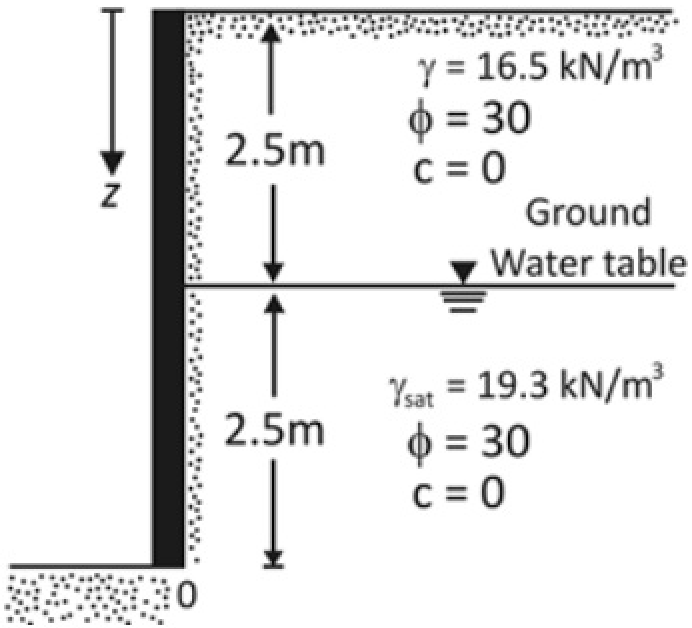
$$P_o = P_1 + P_2 + P_3 + P_4 + P_5$$

$$P_o = K_o q H_1 + \tfrac{1}{2} K_o \gamma H_1^2 + K_o (q + \gamma H_1) H_2 + \tfrac{1}{2} K_o \gamma' H_2^2 + \tfrac{1}{2} \gamma_w H_2^2$$

Lateral Earth Pressure

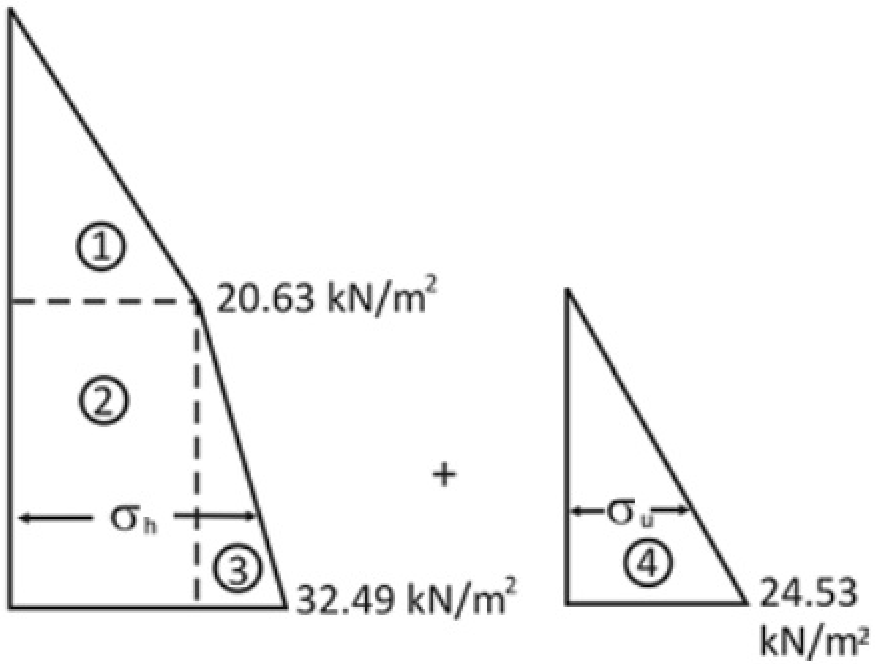
Example-01:

- For the retaining wall shown in figure below, determine the lateral earth force at rest per unit length of the wall. Also determine the location of the resultant force.



Lateral Earth Pressure

Solution-01:



$K_o = 1 - \sin\phi = 1 - \sin 30° = 0.5$

$At\ z = 2.5,\ p_1 = K_o \gamma H_1 = 20.63\ kN/m^2$

$At\ z = 5,\quad p_2 = K_o \gamma H_1 + K_o \gamma' H_2$
$= 32.49\ kN/m^2$

$At\ z = 5,\quad p_3 = \gamma_W H_2 = 24.53\ kN/m^2$

$P_o = P_1 + P_2 + P_3$

$= \frac{1}{2}(2.5)(20.63) + (2.5)(20.63) + \frac{1}{2}(2.5)(32.49 - 20.63) + \frac{1}{2}(2.5)(24.53) =$
$122.85\ \text{kN/m}$

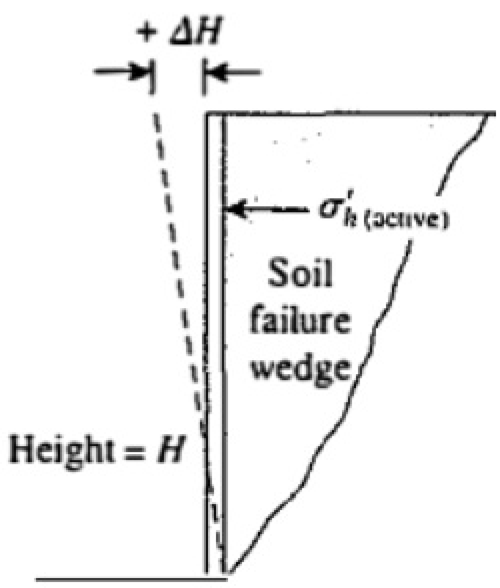
$$\bar{z} = \frac{(25.788)(3.33)+(51.575)(1.25)+(14.825+30.663)(0.833)}{122.85} = 1.53\ \text{m}$$

Lateral Earth Pressure

Active Earth Pressure:

Definition:

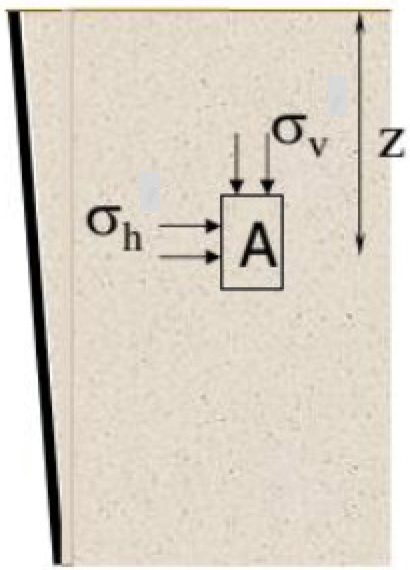
If the wall moves away or tilt away from the soil retained, a triangular soil wedge behind the wall is fail. The lateral pressure at this condition is known as active earth pressure.



Lateral Earth Pressure

Active Earth Pressure:

Granular Soil:



$$\sigma_v = \gamma z$$

Initially, there is no lateral movement

$$\sigma_h = K_0\,\sigma_v = K_0 \gamma z$$

As the wall moves away from the soil

σ_v remains the same; but

σ_h decreases till failure occur

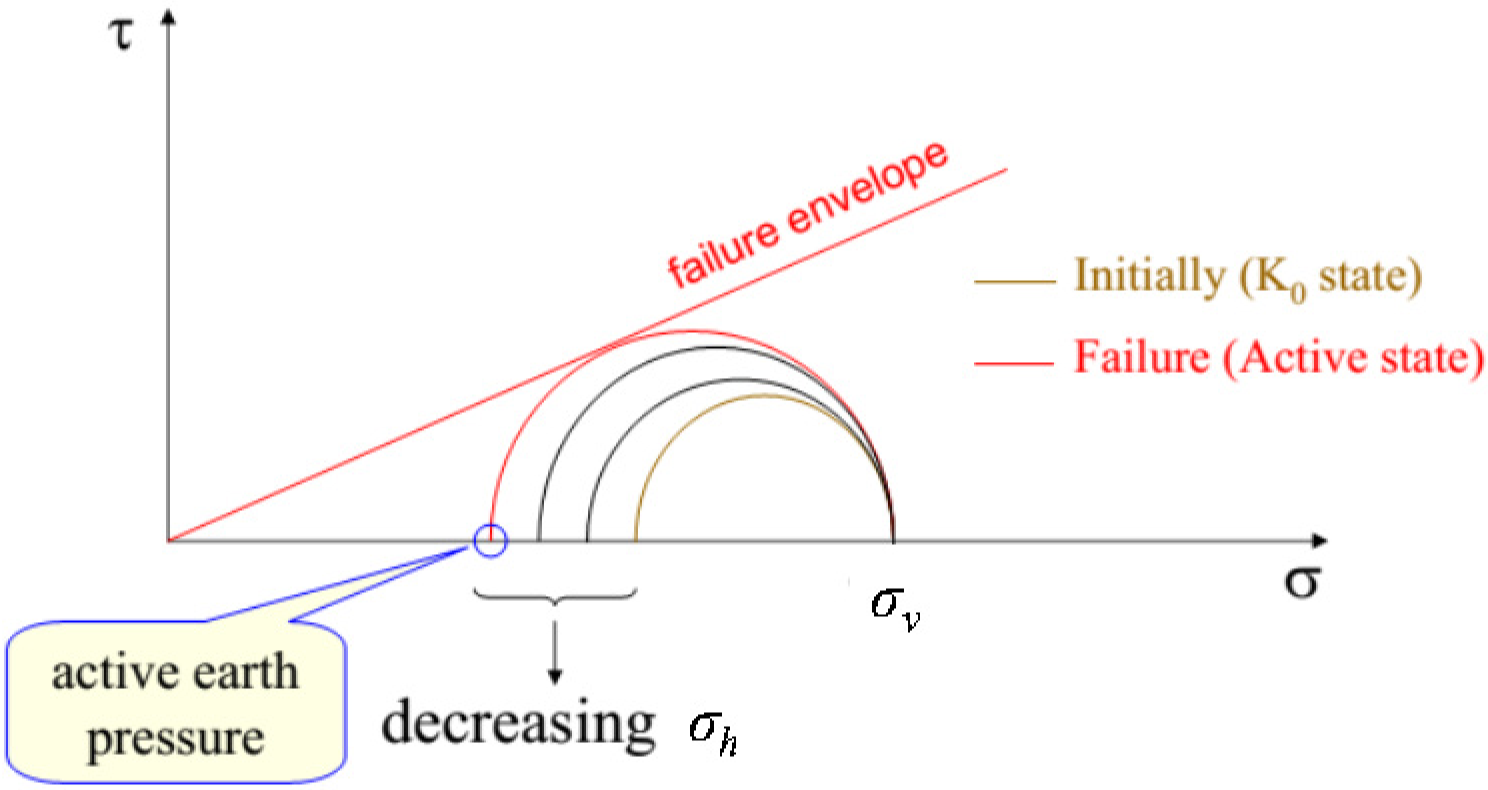
Active state

Lateral Earth Pressure

Active Earth Pressure:

Granular Soil:

As the wall moves away from the soil,



Lateral Earth Pressure

Rankine (1857) Active Earth Pressure:

Assumptions:

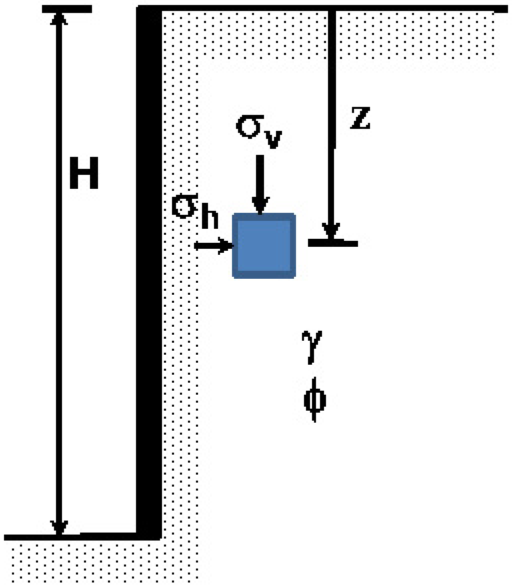
Rankine (1857) made the following assumptions:

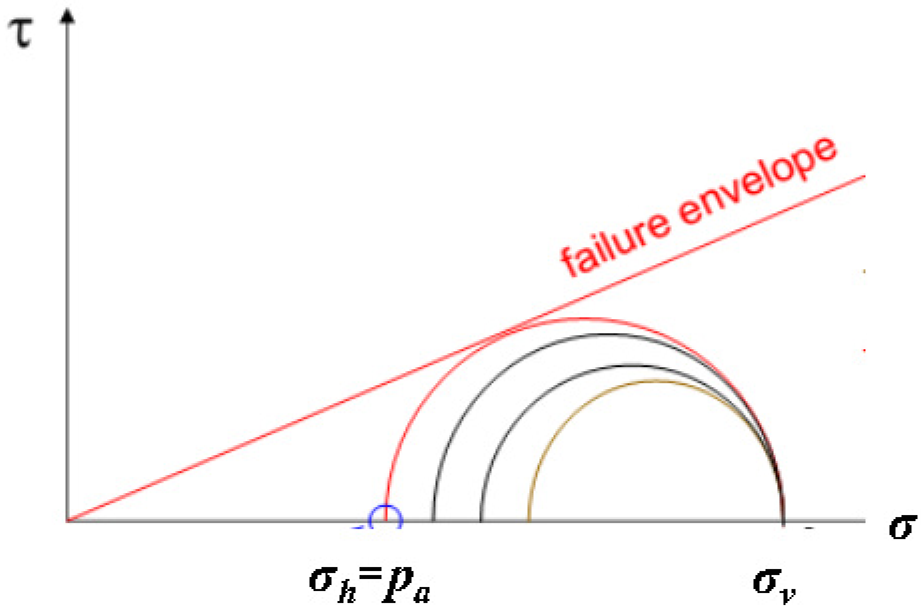
1) The soil mass is homogeneous and semi-infinite
2) The soil mass is dry and cohesionless
3) The ground surface is plane
4) The back of the retaining wall is smooth and vertical
5) The soil element is in a state of plastic equilibrium

Lateral Earth Pressure

Rankine Active Earth Pressure:

Let us consider an element of dry soil located at a depth z as shown in Figure below.





Lateral Earth Pressure

Rankine Active Earth Pressure:

Point E represents the active condition

p_a = OE=OC-CE

=OC-CD

=OC-OC $\sin\phi$

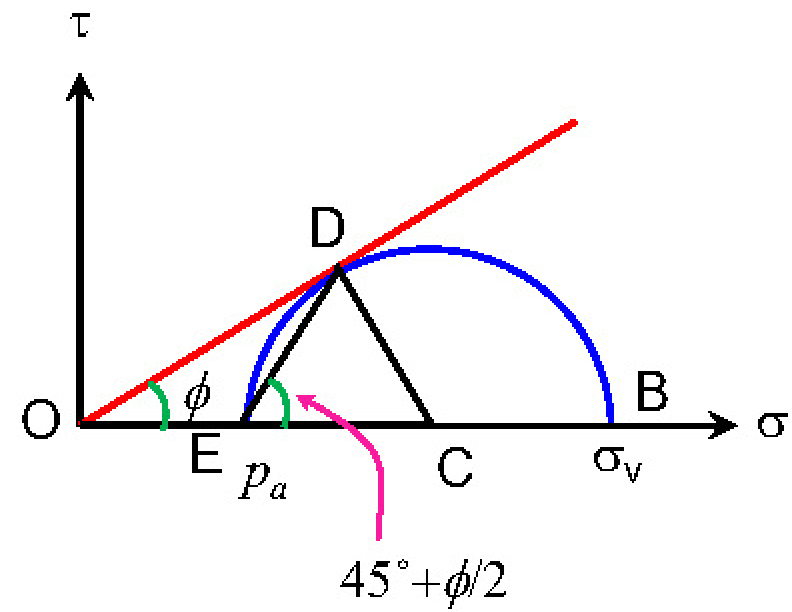
= OC(1-$\sin\phi$)

σ_v=OB=OC+CB

=OC+CD

=OC+OC $\sin\phi$

= OC(1+$\sin\phi$)



$$\frac{p_a}{\sigma_v} = \frac{1-\sin\phi}{1+\sin\phi} \Rightarrow p_a = \left[\frac{1-\sin\phi}{1+\sin\phi}\right]\sigma_v \Rightarrow p_a = K_a\sigma_v \Rightarrow p_a = K_a\gamma Z$$

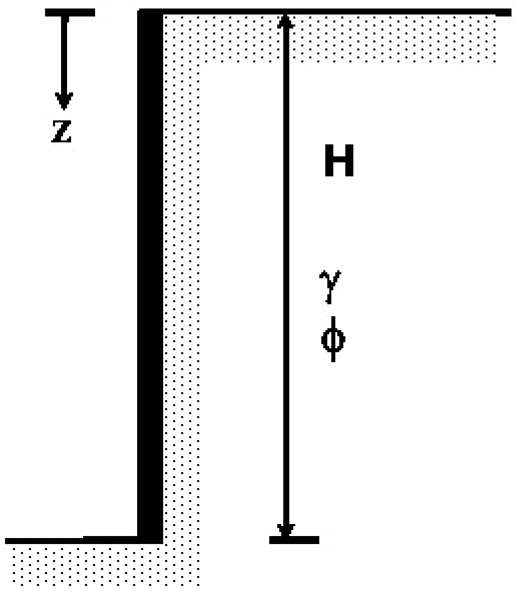
K_a = coefficient of active earth pressure= $\left[\dfrac{1-\sin\phi}{1+\sin\phi}\right]$

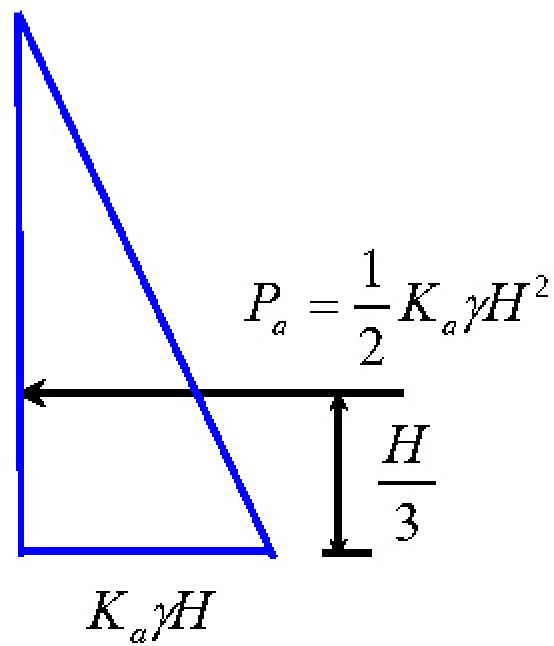
Lateral Earth Pressure

Rankine Active Earth Pressure:

Several Cases:

❑ **Case-01:** Dry Backfill

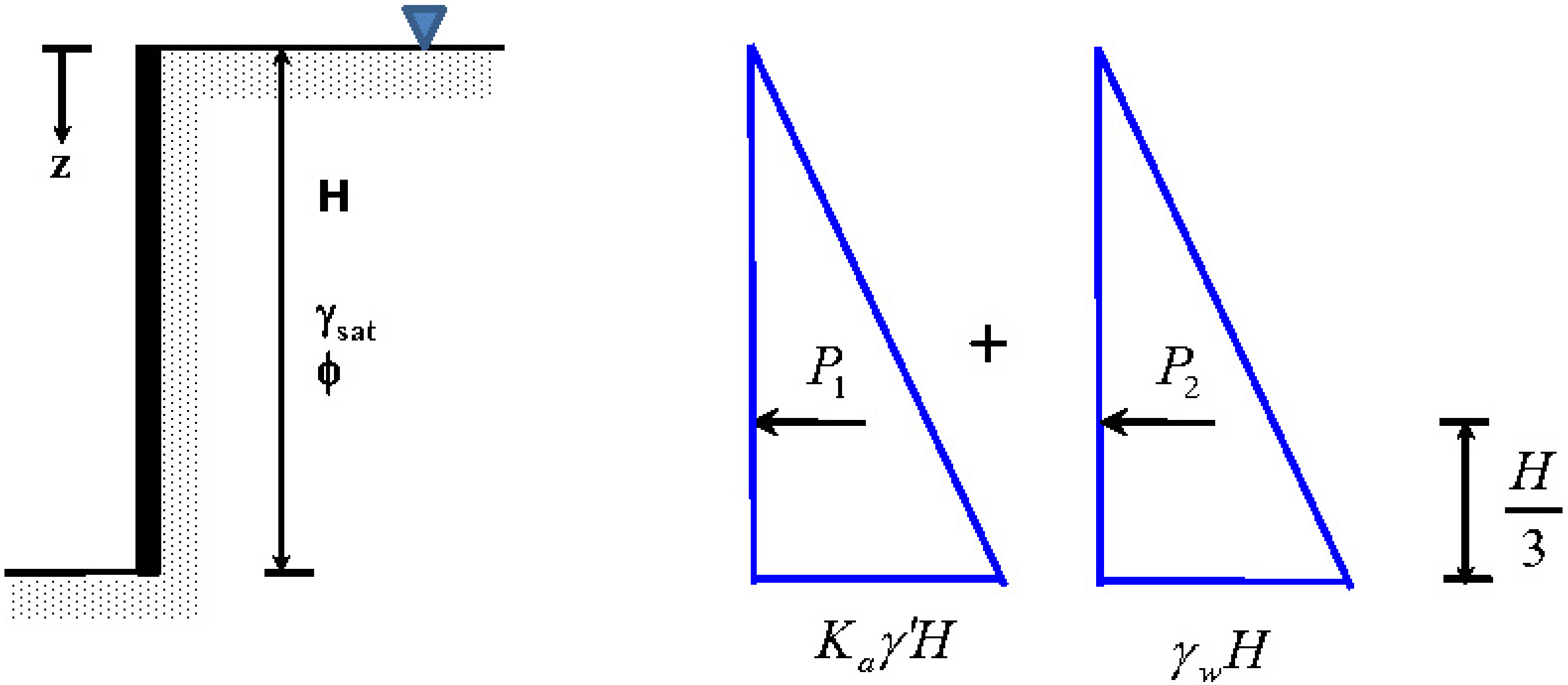




Lateral Earth Pressure

Rankine Active Earth Pressure:

❑ **Case-02:** Submerged Backfill

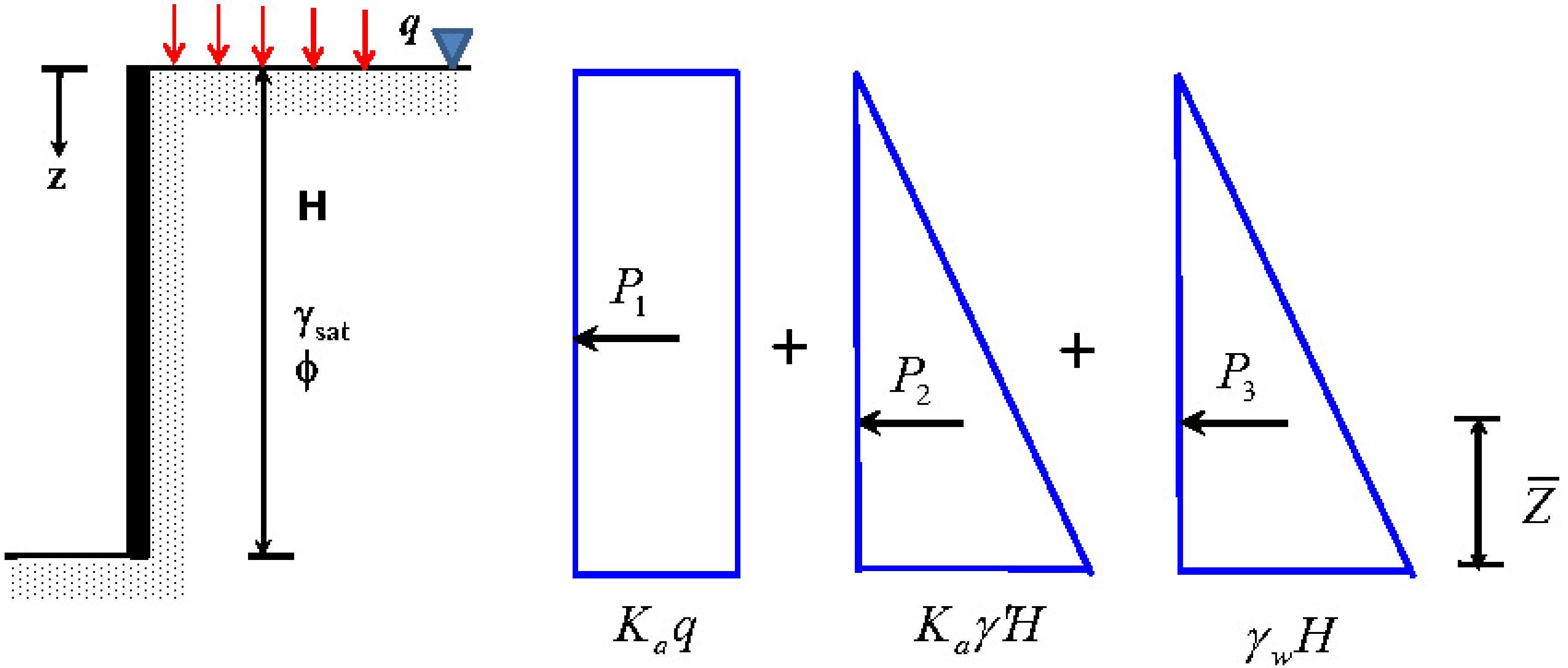


$$P_a = P_1 + P_2 = \frac{1}{2}K_a\gamma' H^2 + \frac{1}{2}\gamma_w H^2$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

- **Case-03:** Submerged Backfill with Surcharge



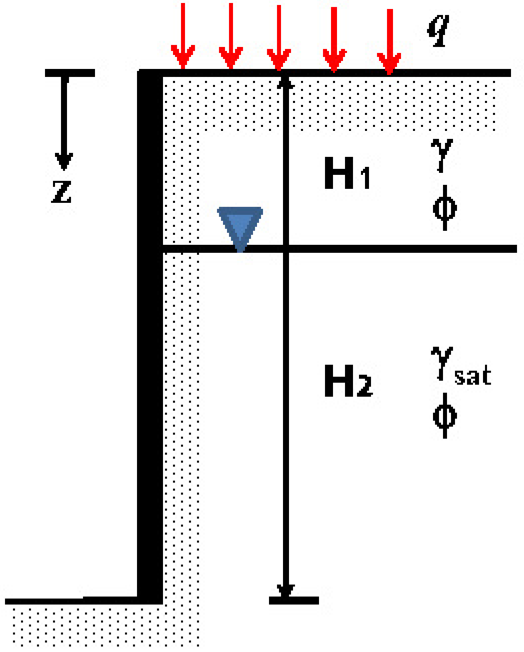
$$P_a = P_1 + P_2 + P_3 = K_a qH + \frac{1}{2} K_a \gamma' H^2 + \frac{1}{2} \gamma_w H^2$$

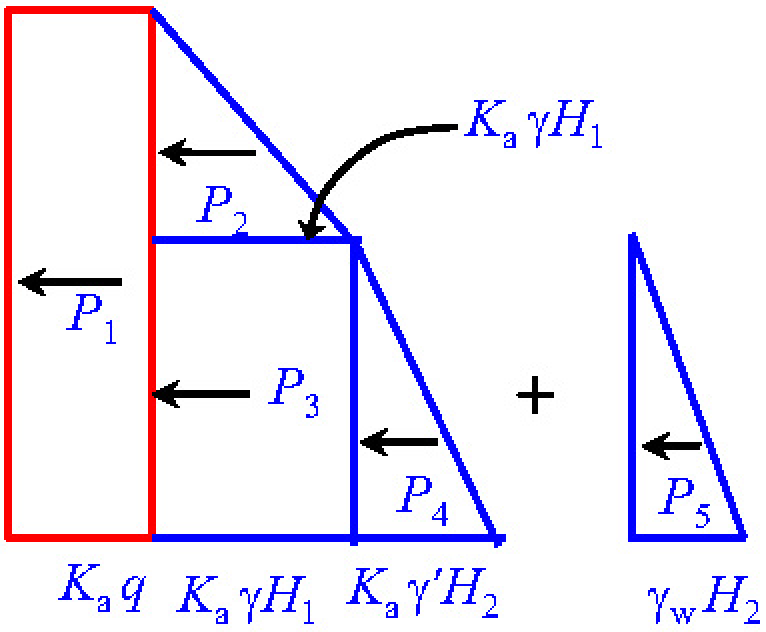
$$\bar{Z} = \frac{P_1 \times (H/2) + P_2 \times (H/3) + P_3 \times (H/3)}{P_a}$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

❑ **Case-04:** Partially Submerged Backfill with Surcharge





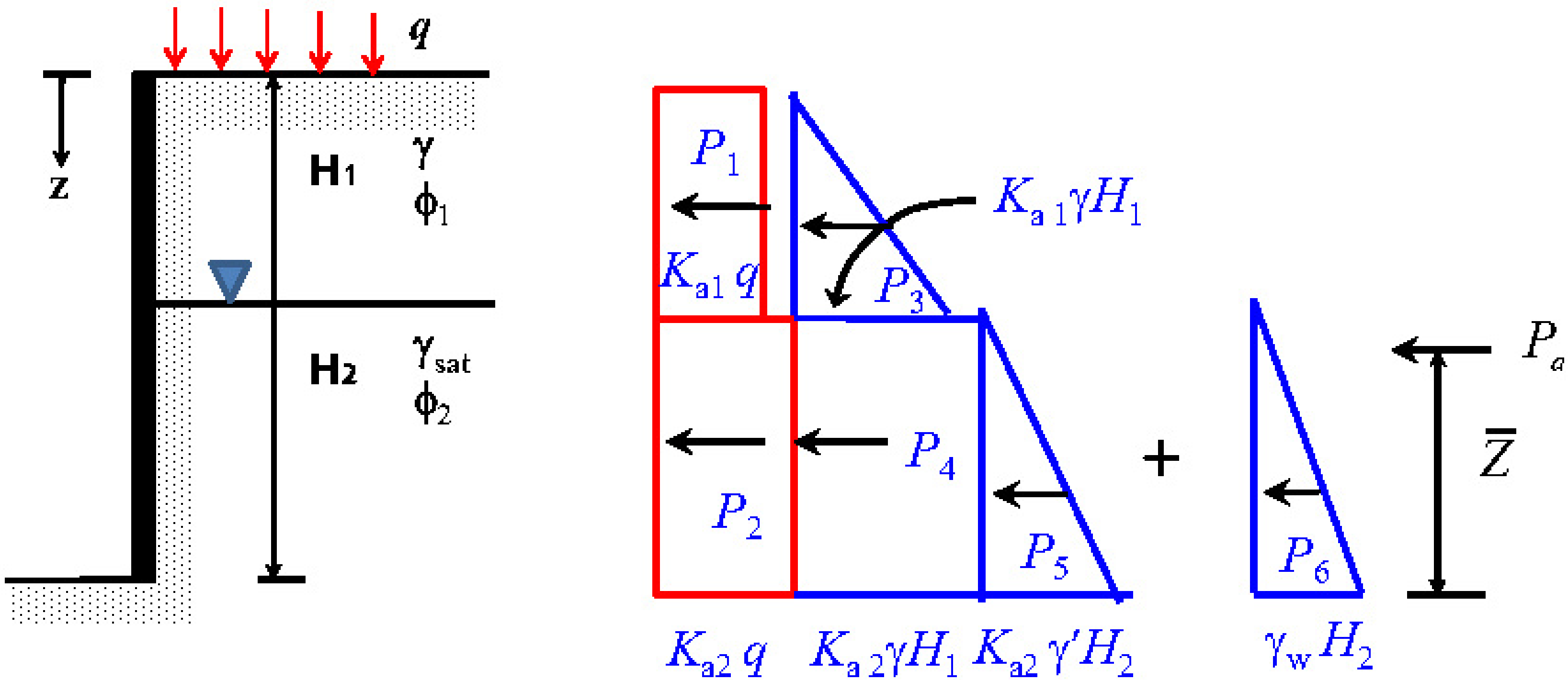
$$P_a = P_1 + P_2 + P_3 + P_4 + P_5$$

$$= K_a q (H_1 + H_2) + \frac{1}{2} K_a \gamma H_1^2 + K_a \gamma H_1 H_2 + \frac{1}{2} K_a \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

❑ **Case-05:** Partially Submerged Backfill with Surcharge ($\phi_1 > \phi_2$)



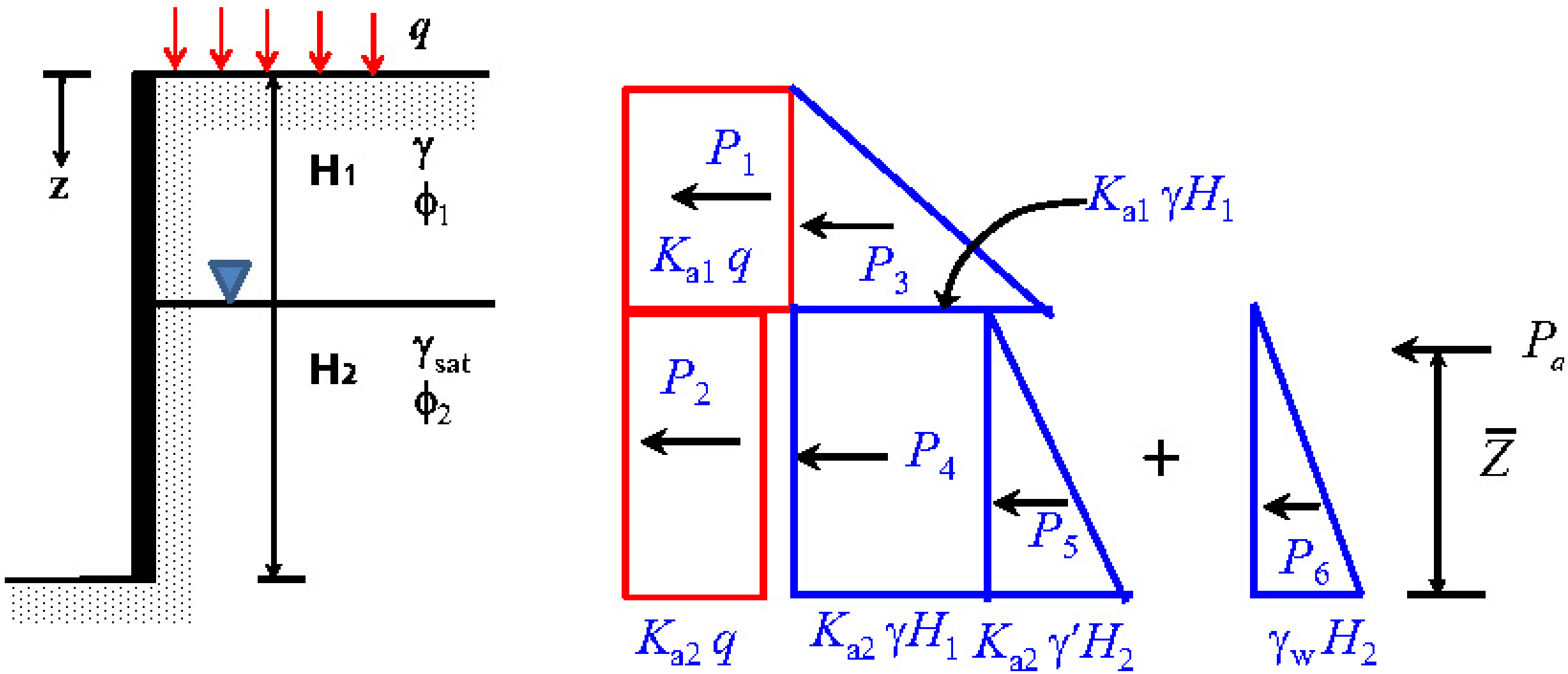
$$P_a = P_1 + P_2 + P_3 + P_4 + P_5 + P_6$$

$$= K_{a1} q H_1 + K_{a2} q H_2 + \frac{1}{2} K_{a1} \gamma H_1^2 + K_{a2} \gamma H_1 H_2 + \frac{1}{2} K_{a2} \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

❑ **Case-06:** Partially Submerged Backfill with Surcharge ($\phi_1 < \phi_2$)



$$P_a = P_1 + P_2 + P_3 + P_4 + P_5 + P_6$$

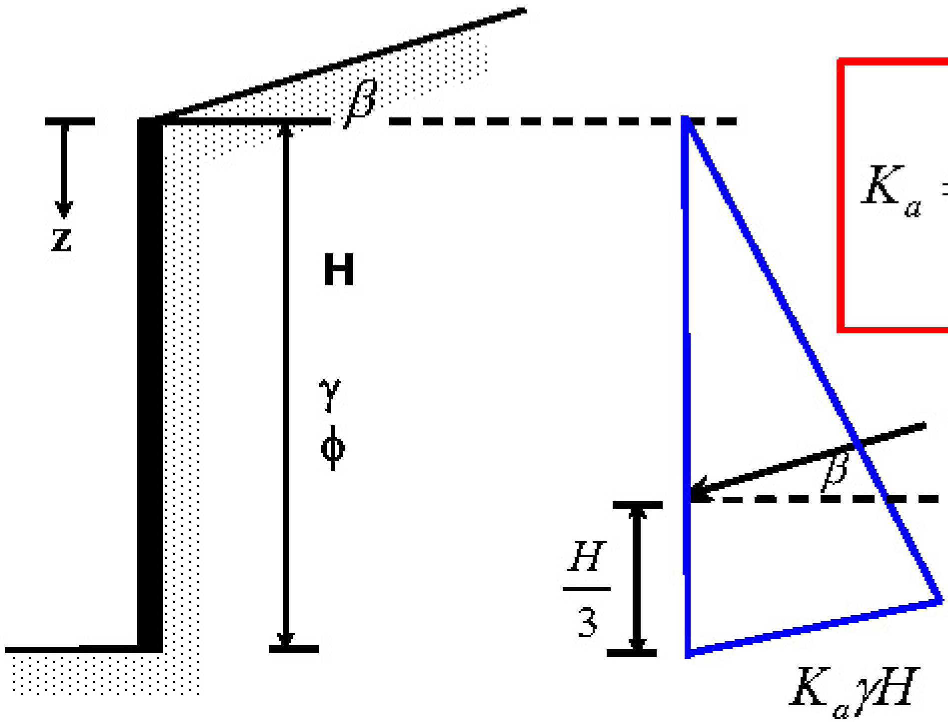
$$= K_{a1} q H_1 + K_{a2} q H_2 + \frac{1}{2} K_{a1} \gamma H_1^2 + K_{a2} \gamma H_1 H_2 + \frac{1}{2} K_{a2} \gamma' H_2^2 + \frac{1}{2} \gamma_w H_2^2$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

Several Cases:

❑ **Case-07:** Backfill with sloping surface



$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

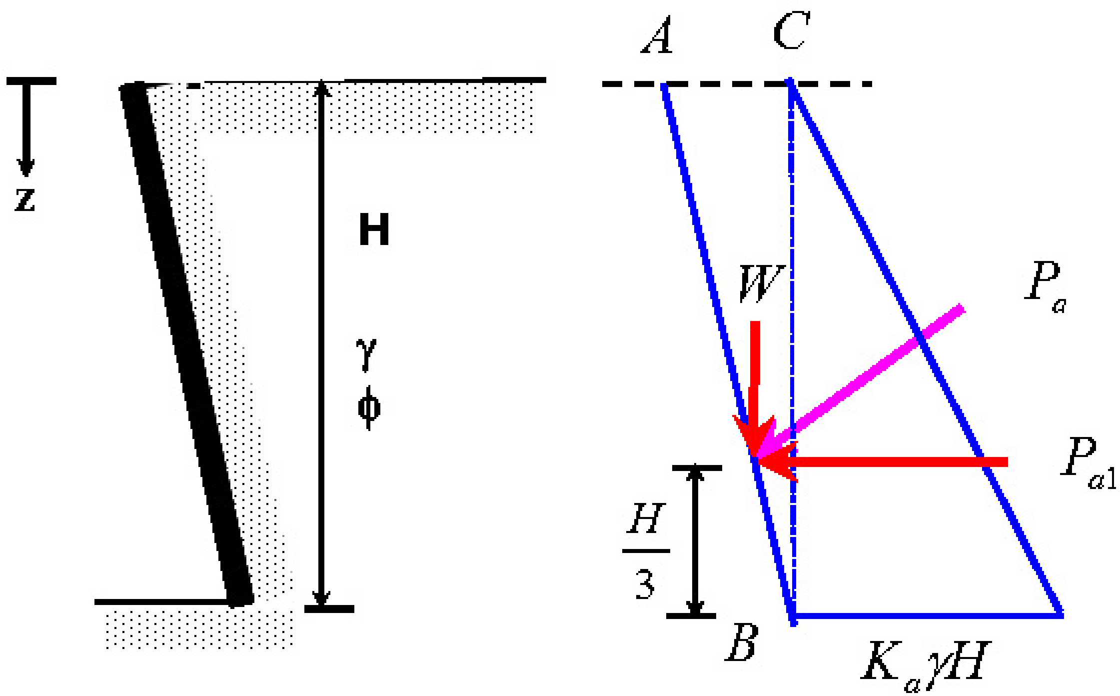
$$P_a = \frac{1}{2} K_a \gamma H^2$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

Several Cases:

❑ **Case-08:** Inclined back and surcharge



$$P_{a1} = \frac{1}{2} K_a \gamma H^2$$

$$W = weight\ of\ ABC$$

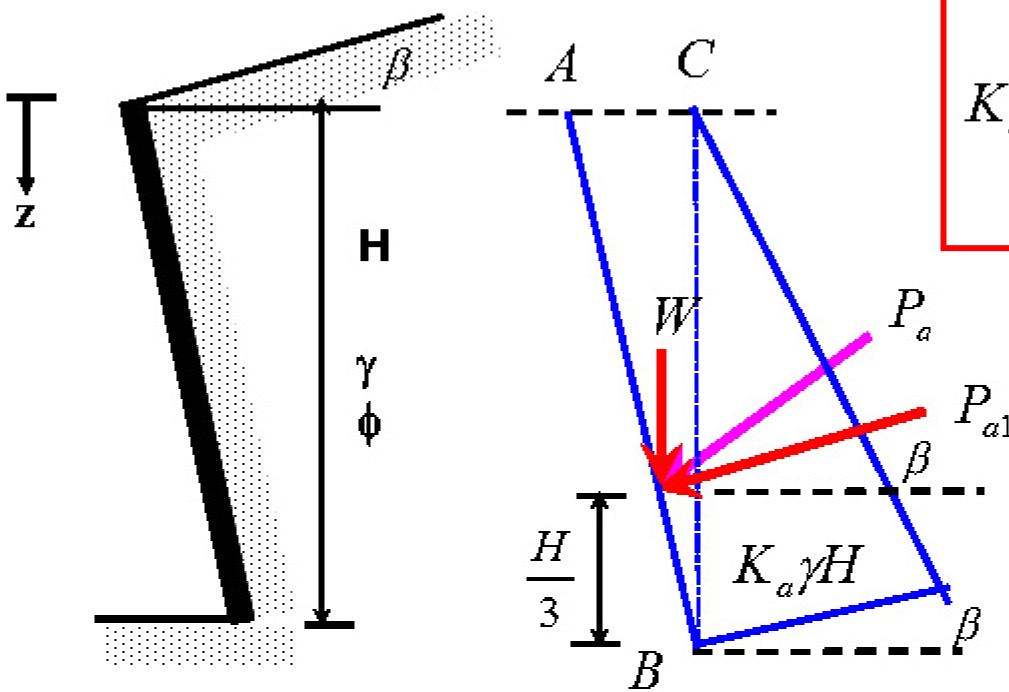
$$P_a = \sqrt{P_{a1}^2 + W^2}$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

Several Cases:

❑ **Case-09:** Inclined back and surcharge with sloping surface



$$K_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$P_{a1} = \frac{1}{2} K_a \gamma H^2$$

$$W = weight\ of\ ABC$$

$$P_a = \sqrt{P_{a1}^2 + W^2}$$

Lateral Earth Pressure

Rankine Active Earth Pressure:

Cohesive Soil:

Rankine's original theory was for cohesionless soils. It was extended by Resel (1910) and Bell (1915) for cohesive soil.

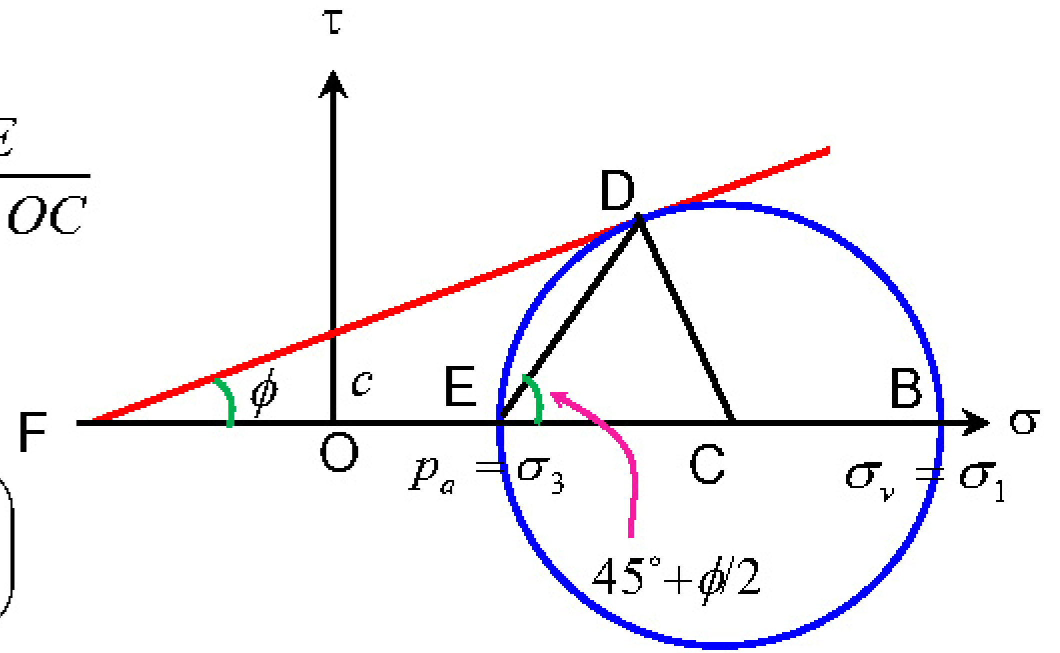
From triangle FCD

$$\sin\phi = \frac{CD}{FC} = \frac{CD}{FO+OC} = \frac{CE}{FO+OC}$$

$$\sin\phi = \frac{(\sigma_1 - \sigma_3)/2}{c\cot\phi + (\sigma_1 + \sigma_3)/2}$$

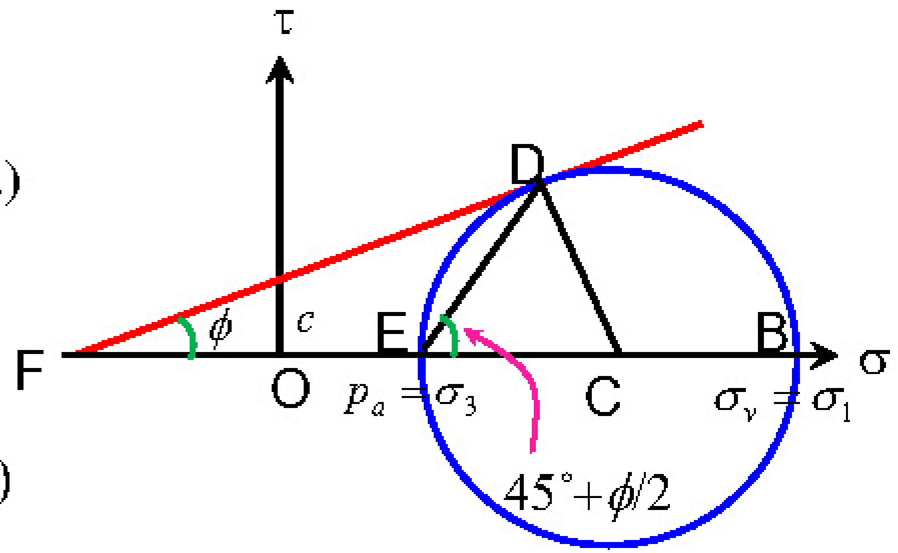
$$\sigma_3 = \sigma_1\left(\frac{1-\sin\phi}{1+\sin\phi}\right) - 2c\left(\frac{\cos\phi}{1+\sin\phi}\right)$$

$$\sigma_3 = \sigma_1 \tan^2(45^\circ - \phi/2) - 2c\tan(45^\circ - \phi/2)$$



Lateral Earth Pressure

As $\sigma_h=\sigma_3=p_a$ and $\sigma_1=\gamma Z$ (see Fig.)



$$p_a=\gamma Z\tan^2(45^\circ-\phi/2)-2c\tan(45^\circ-\phi/2)$$

$$p_a=K_a\gamma Z-2c\sqrt{K_a}$$

where $K_a=\dfrac{1-\sin\phi}{1+\sin\phi}=\tan^2(45^\circ-\phi/2)$

$$\sqrt{K_a}=\frac{\cos\phi}{1+\sin\phi}=\tan(45^\circ-\phi/2)$$

If Z=0; $p_a=-2c\tan(45^\circ-\phi/2)=-2c\sqrt{K_a}$

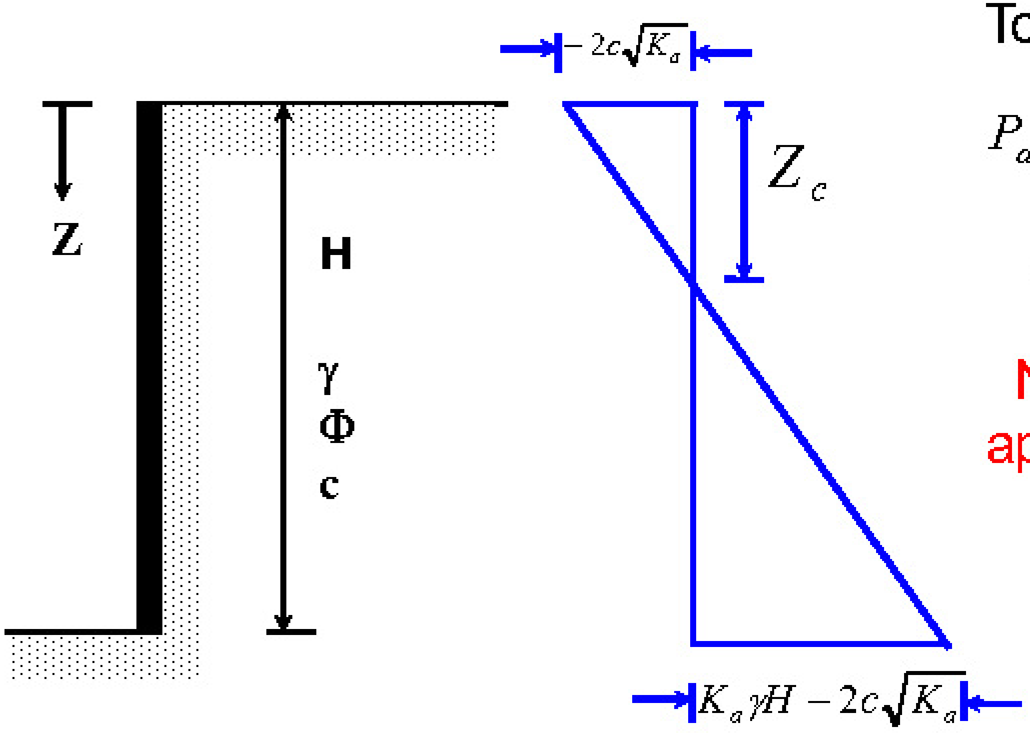
The negative sign indicates that the pressure is negative. So, it tries to cause a pull on the wall.

Lateral Earth Pressure

If p_a=0; $0 = K_a \gamma Z - 2c\sqrt{K_a} \Rightarrow Z = Z_c = \dfrac{2c}{\gamma\sqrt{K_a}}$

This depth $Z=Z_c$ is known as the depth of tension crack.

If Z=H; $p_a = K_a \gamma H - 2c\sqrt{K_a}$



Total pressure is given by

$$P_a = \int_0^H (K_a \gamma H - 2c\sqrt{K_a})\, dZ$$

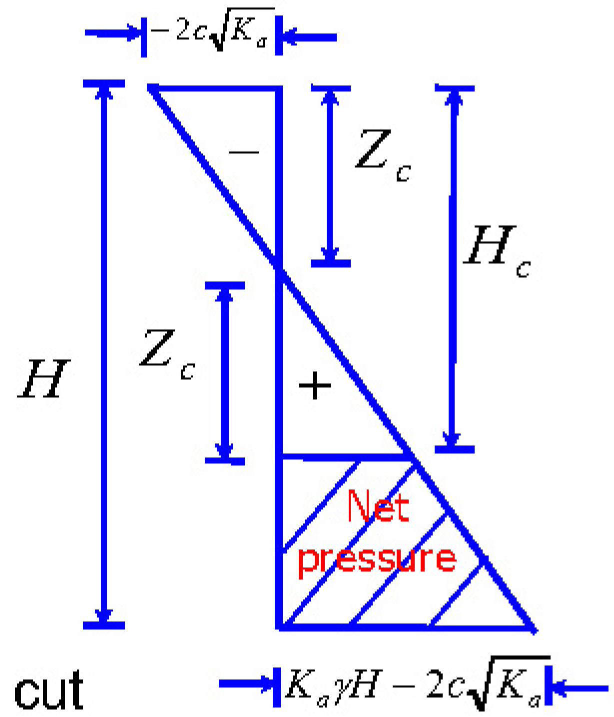
$$= K_a \frac{\gamma H^2}{2} - 2c\sqrt{K_a}\, H$$

Note that this pressure is applicable before formation of crack.

Lateral Earth Pressure

After the formation of tension crack, the force on the wall is caused only by the pressure from $Z=Z_c$ to $Z=H$.

Thus, $$P_a = \frac{1}{2}(H - Z_C)(K_a \gamma H - 2c\sqrt{K_a})$$



Critical height of unsupported vertical cut:

From Fig. it is clear that the total net pressure upon a depth of $2Z_c$ is zero.

It reveals that a cohesive soil should be able to stand with a vertical face upto a depth of $2Z_c$ ***without any lateral support.***

The critical height H_c of the unsupported vertical cut in cohesive soil can be given by-

$$H_c = 2Z_c = 2 \times \frac{2c}{\gamma\sqrt{K_a}} = \frac{4c}{\gamma\sqrt{K_a}}$$

Lateral Earth Pressure

Backfill with Surcharge:

If the backfill carries a surcharge *q*,

$$p_a = K_a \gamma Z - 2c\sqrt{K_a} + qK_a$$

At Z=0,

$$p_a = -2c\sqrt{K_a} + qK_a$$

When p_a=0,

$$0 = K_a \gamma Z - 2c\sqrt{K_a} + qK_a$$

$$\Rightarrow \quad 0 = \sqrt{K_a}\gamma Z - 2c + q\sqrt{K_a}$$

$$\Rightarrow \quad \sqrt{K_a}\gamma Z = 2c - q\sqrt{K_a}$$

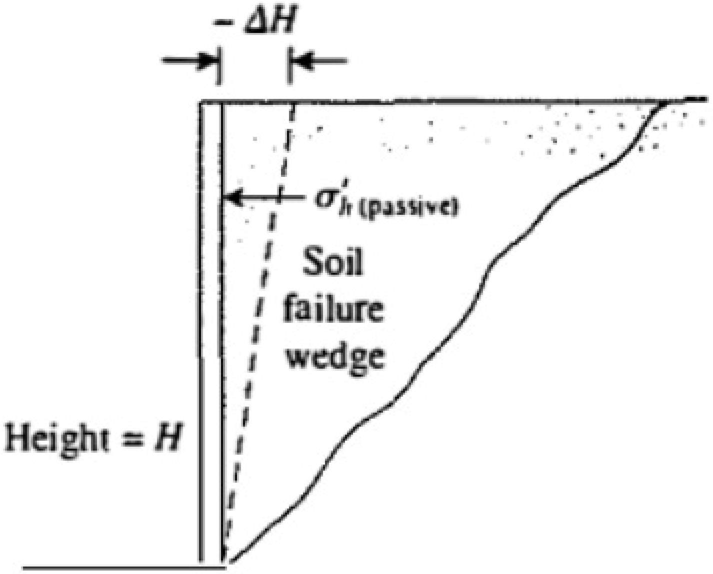
$$Z = Z_c = \frac{2c}{\gamma\sqrt{K_a}} - \frac{q}{\gamma}$$

Lateral Earth Pressure

Passive Earth Pressure:

Definition:

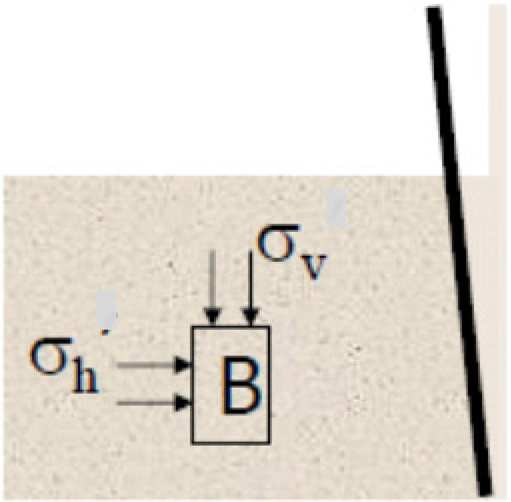
If the wall moves towards or pushed into the soil retained, a triangular soil wedge behind the wall is fail. The lateral pressure at this condition is known as passive earth pressure.



Lateral Earth Pressure

Passive Earth Pressure:

Granular Soil:



$$\sigma_v = \gamma Z$$

Initially, there is no lateral movement

$$\sigma_h = K_0\,\sigma_v = K_0 \gamma Z$$

As the wall moves towards the soil

σ_v remains the same; and

σ_h increases till failure occur

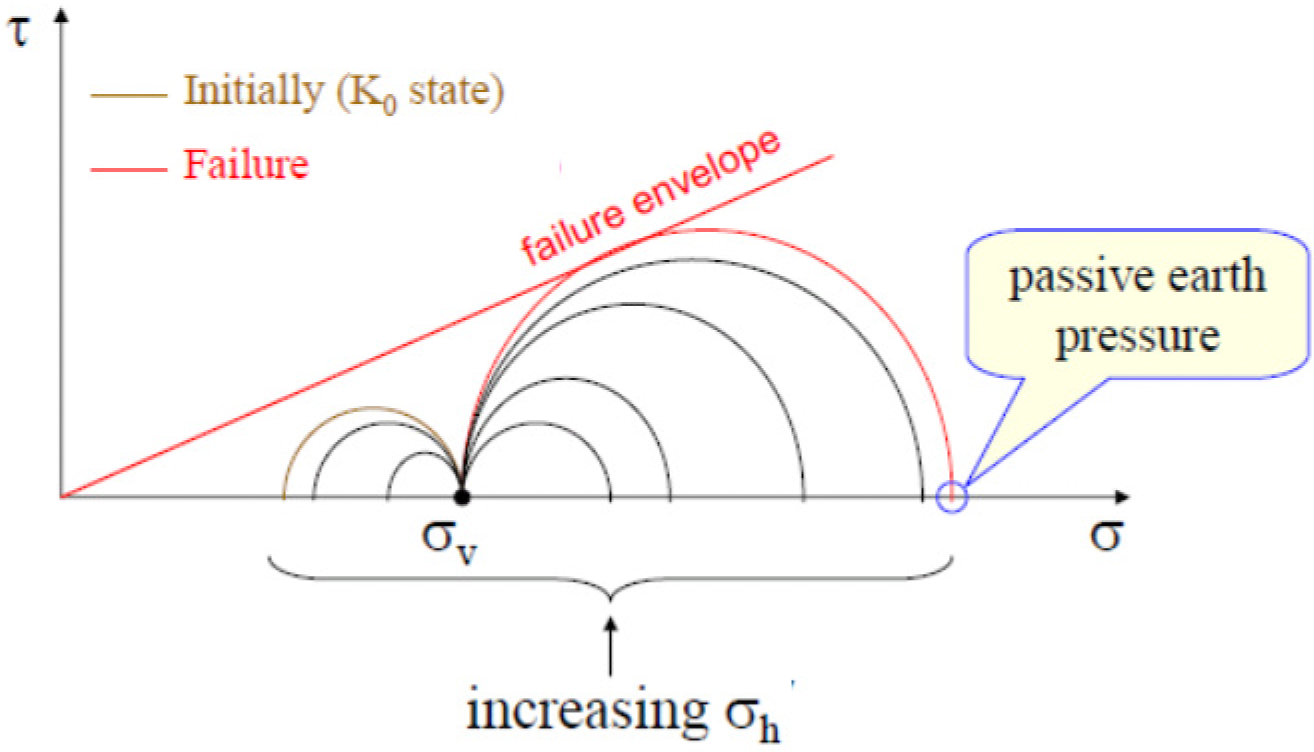
Passive state

Lateral Earth Pressure

Passive Earth Pressure:

Granular Soil:

As the wall moves towards the soil,



Lateral Earth Pressure

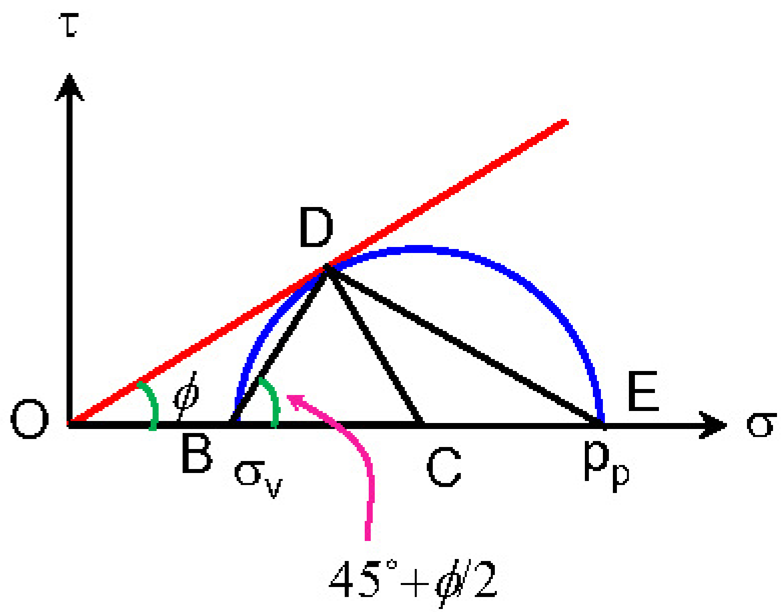
Rankine Passive Earth Pressure:

Granular Soil:

Point E represents the passive condition

$$p_p = OE = OC + CE$$
$$= OC + CD$$
$$= OC + OC \sin\phi$$
$$= OC(1 + \sin\phi)$$

$$\sigma_v = OB = OC - CB$$
$$= OC - CD$$
$$= OC - OC \sin\phi$$
$$= OC(1 - \sin\phi)$$



$$\frac{p_p}{\sigma_v} = \frac{1+\sin\phi}{1-\sin\phi} \Rightarrow p_p = \left[\frac{1+\sin\phi}{1-\sin\phi}\right]\sigma_v \Rightarrow p_p = K_p \sigma_v \Rightarrow p_p = K_p \gamma Z$$

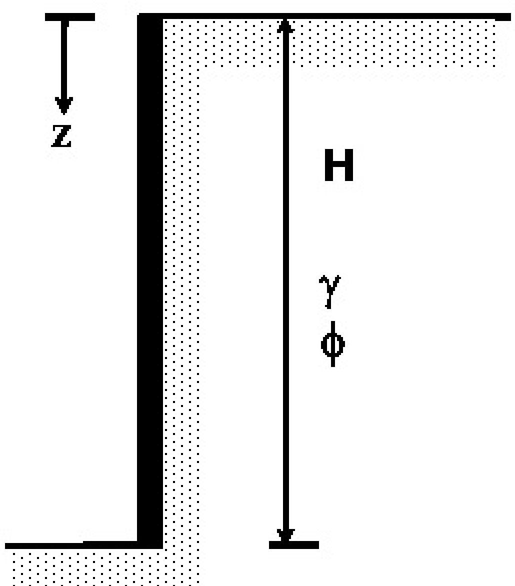
K_p = coefficient of passive earth pressure= $\left[\frac{1+\sin\phi}{1-\sin\phi}\right]$

Lateral Earth Pressure

Rankine Passive Earth Pressure:

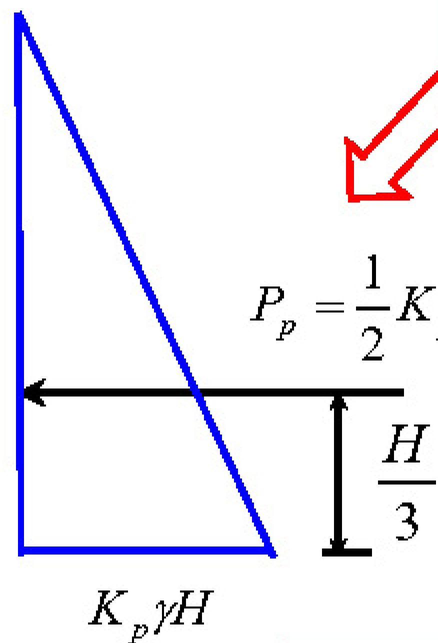
Several Cases:

❑ **Case-01:** Dry Backfill



Similar to active pressure.

$$P_p = \frac{1}{2} K_p \gamma H^2$$



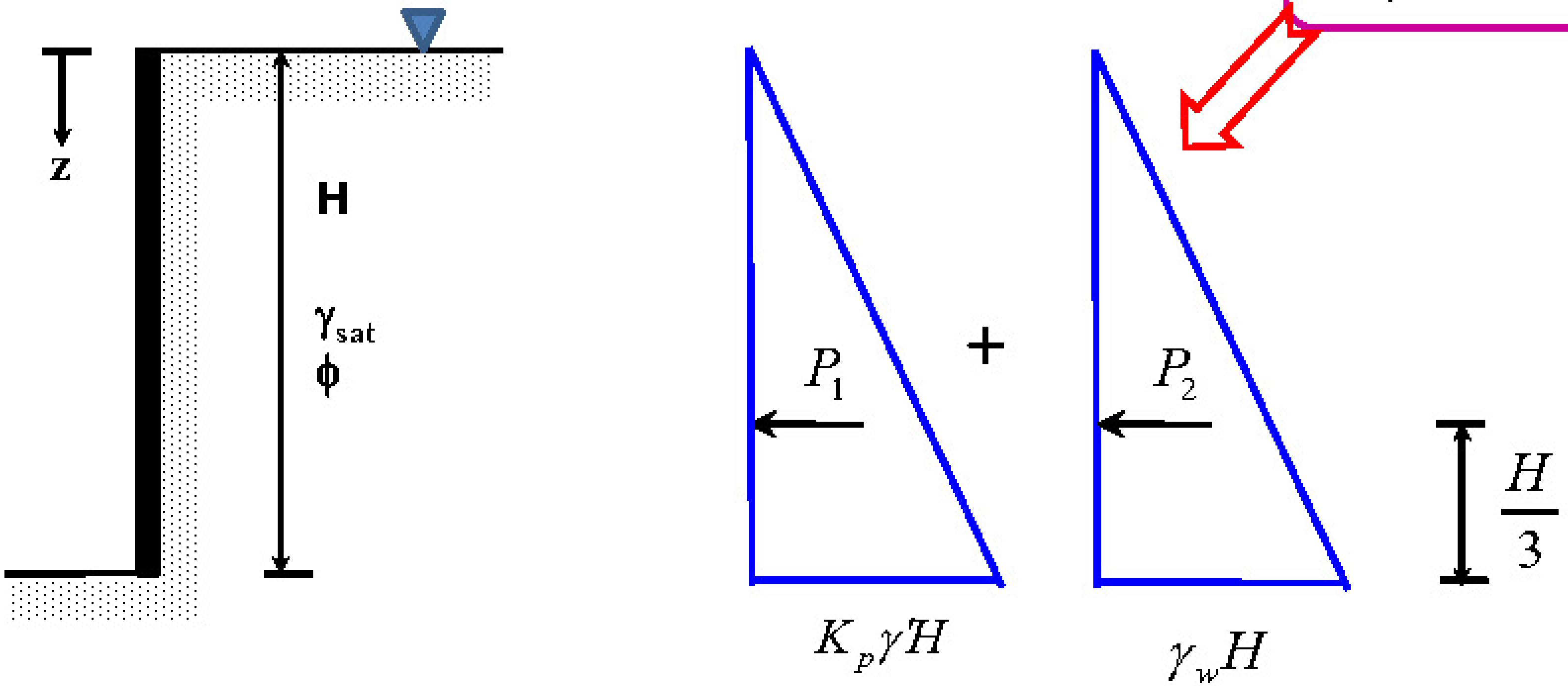
Difference: K_a is replaced by k_p

Lateral Earth Pressure

Rankine Passive Earth Pressure:

❑ Case-02: Submerged Backfill

Similar to active pressure.

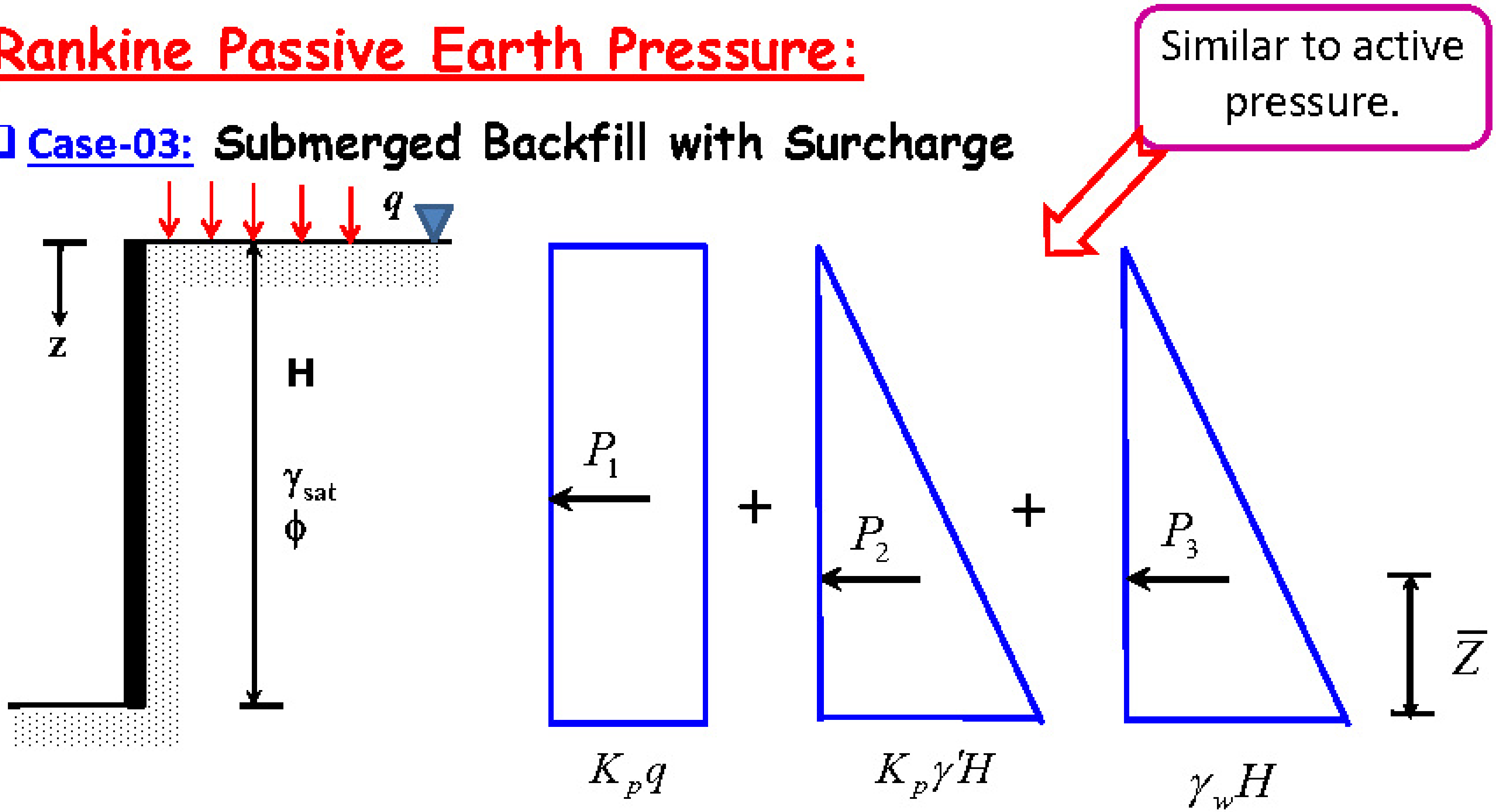


$$P_p = P_1 + P_2 = \frac{1}{2} K_p \gamma H^2 + \frac{1}{2} \gamma_w H^2$$

Lateral Earth Pressure

Rankine Passive Earth Pressure:

❑ **Case-03:** Submerged Backfill with Surcharge



$$P_p = P_1 + P_2 + P_3 = K_p q H + \frac{1}{2} K_p \gamma' H^2 + \frac{1}{2} \gamma_w H^2$$

$$\bar{Z} = \frac{P_1 \times (H/2) + P_2 \times (H/3) + P_3 \times (H/3)}{P_p}$$

Lateral Earth Pressure

Rankine Passive Earth Pressure:

Cohesive Soil:

From triangle FCD

$$\sin\phi = \frac{CD}{FC} = \frac{CD}{FO+OC} = \frac{CB}{FO+OC}$$

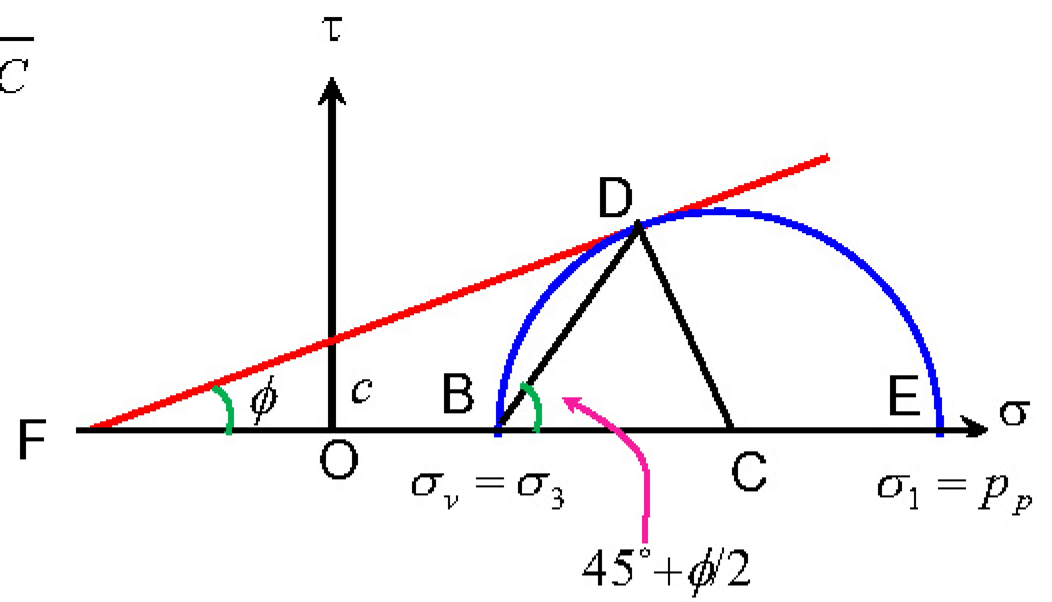
Now, $OC = OB + CB$

$$= \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2}$$

$$\sin\phi = \frac{(\sigma_1 - \sigma_3)/2}{c\cot\phi + (\sigma_1 + \sigma_3)/2}$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\left(\frac{\cos\phi}{1-\sin\phi}\right)$$

$$p_p = \sigma_v \tan^2(45^\circ + \phi/2) + 2c\tan(45^\circ + \phi/2)$$



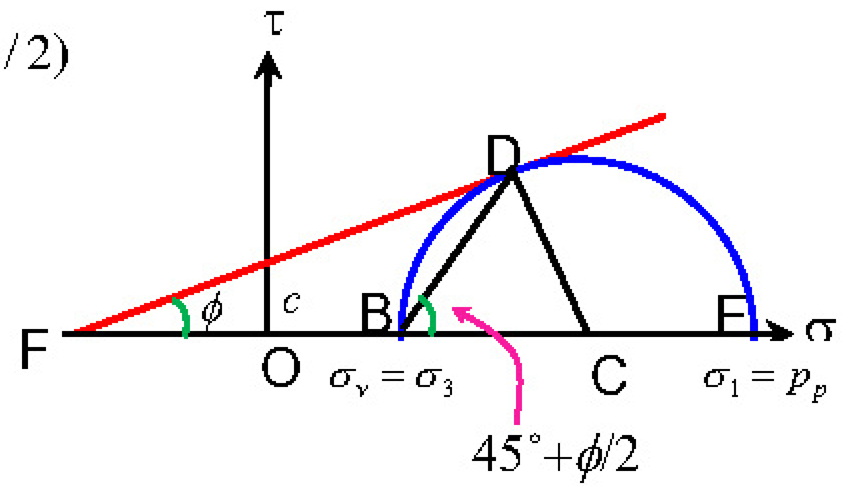
Lateral Earth Pressure

$$p_p = \sigma_v \tan^2(45^\circ + \phi/2) + 2c\tan(45^\circ + \phi/2)$$

$$p_p = \gamma Z K_p + 2c\sqrt{K_p}$$

where $K_p = \dfrac{1+\sin\phi}{1-\sin\phi} = \tan^2(45^\circ + \phi/2)$

$$\sqrt{K_p} = \frac{\cos\phi}{1-\sin\phi} = \tan(45^\circ + \phi/2)$$

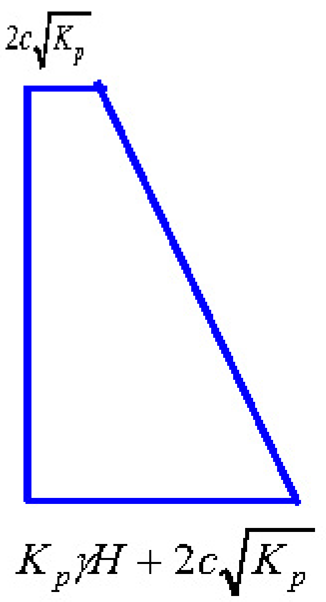


If Z=0; $p_p = 2c\tan(45^\circ + \phi/2) = 2c\sqrt{K_p}$

If Z=H; $p_p = \gamma H K_p + 2c\sqrt{K_p}$

Total pressure $P_p = \int_0^H (\gamma Z K_p + 2c\sqrt{K_p})dZ$

$$\Rightarrow \quad P_p = \frac{1}{2}K_p \gamma H^2 + 2cH\sqrt{K_p}$$



Lateral Earth Pressure

Coulomb's Wedge Theory:

Assumptions:

- Coulomb (1776) proposed a theory. The following assumptions were made:

1) The backfill is dry and cohesionless, homogeneous, isotropic and plastic material.
2) The slip surface is a plane surface passing through the heel of the wall.
3) The wall surface is rough.
4) The sliding wedge itself acts as a rigid body.

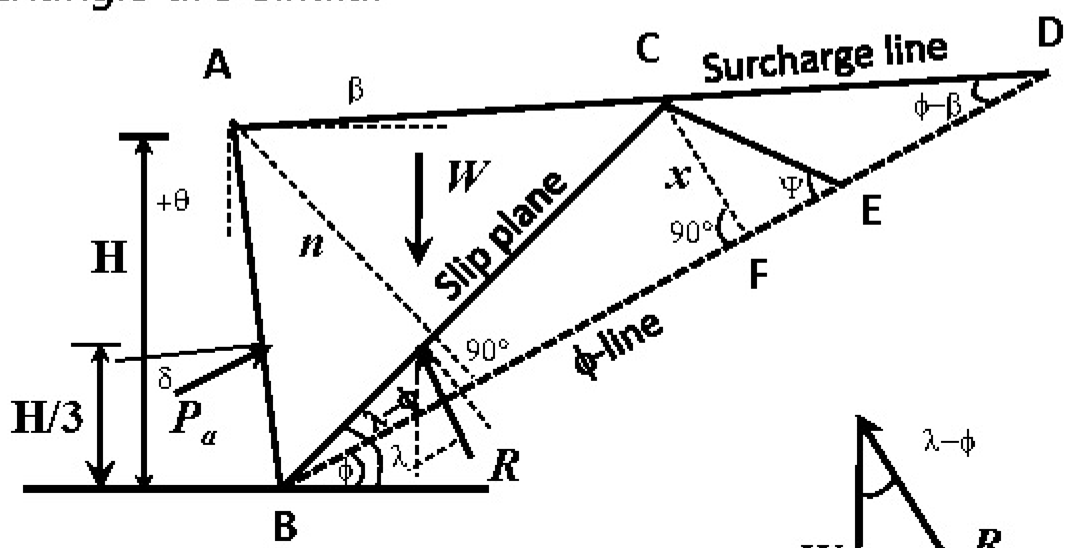
Lateral Earth Pressure

Coulomb's Active Pressure in Cohesionless Soil:

From geometry, Δ BCE and force triangle are similar

Hence, $\dfrac{P_a}{W} = \dfrac{CE}{BE}$

$$\Rightarrow P_a = W \times \frac{CE}{BE} \qquad (i)$$

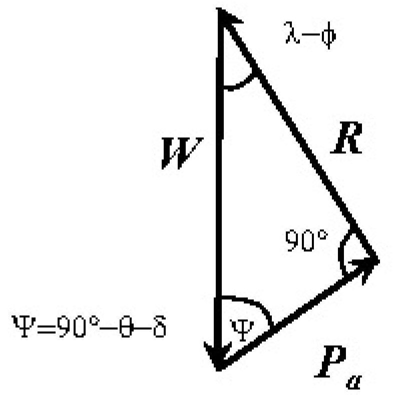


Now, from Δ CEF

$$\text{cosec}\,\psi = \frac{CE}{CF}$$

$$\Rightarrow CE = CF\,\text{cosec}\,\psi$$

$$\Rightarrow CE = x\,\text{cosec}\,\psi = A_1 x \;[\because A_1 = \text{cosec}\,\psi]$$



Force Triangle

Again, from Δ CDF & Δ CEF

$$DF = x\cot(\phi - \beta) \;\; and \;\; FE = x\cot\psi$$

Lateral Earth Pressure

Coulomb's Active Pressure in Cohesionless Soil:

$$BE = BD - DE = BD - (DF - FE)$$

$$= m - x[\cot(\phi - \beta) - \cot\psi]$$

$$= m - A_2 x$$

where, $BD = m;$

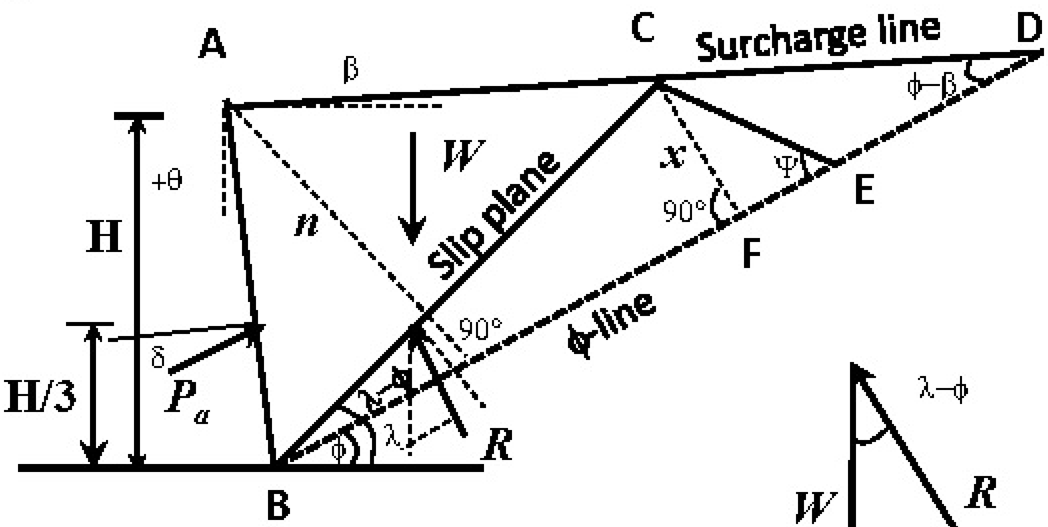
$$A_2 = \cot(\phi - \beta) - \cot\psi$$

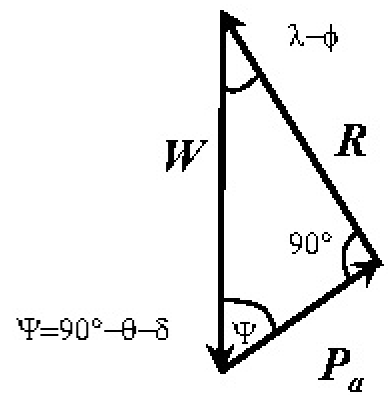
Now, $W = \Delta ABC \times \gamma$

$$= (\Delta ABD - \Delta BCD)\gamma$$

$$= \left(\frac{1}{2}nm - \frac{1}{2}xm\right)\gamma$$

$$= \frac{1}{2}\gamma m(n - x)$$





Force Triangle

Putting the values of CE, BE and W in equation (i), we get

$$P_a = W \times \frac{CE}{BE} = \frac{1}{2}\gamma m(n - x)\frac{A_1 x}{m - A_2 x}$$

Lateral Earth Pressure

Coulomb's Active Pressure in Cohesionless Soil:

For maxima,

$$\frac{dP_a}{dx} = 0$$

$$\frac{d}{dx}\left(\frac{1}{2}\gamma m(n-x)\frac{A_1 x}{m - A_2 x}\right) = 0$$

$$\frac{d}{dx}\left(\frac{1}{2}\gamma m A_1 \frac{nx - x^2}{m - A_2 x}\right) = 0$$

$$(n-2x)(m - A_2 x) = -A_2(nx - x^2)$$

$$mn - mx = x(m - A_2 x)$$

$$\frac{mn}{2} - \frac{mx}{2} = \frac{x.BE}{2}$$

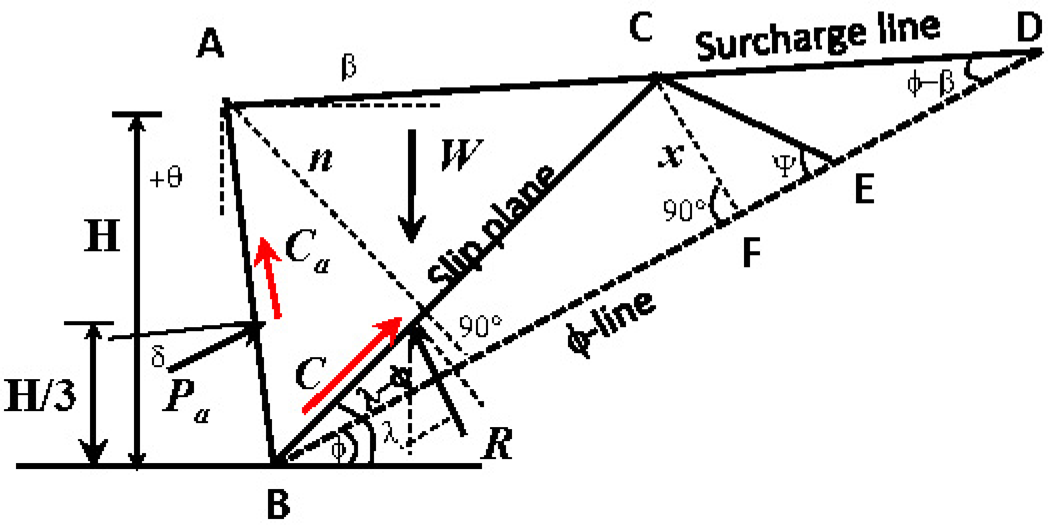
$$\Delta ABD - \Delta BCD = \Delta BCE$$

$$\boxed{\Delta ABC = \Delta BCE}$$

The criteria for maximum active pressure is that the slip-plane is so chosen that ΔABC and ΔBCE are equal in area.

Lateral Earth Pressure

Coulomb's Active Pressure in Cohesive Soil:

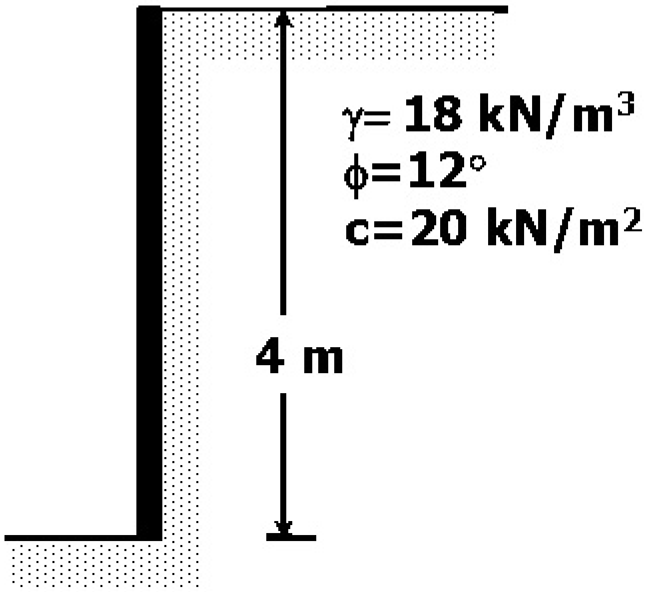


C_a=Adhesive force

C=Cohesive force

Lateral Earth Pressure

Example-02:



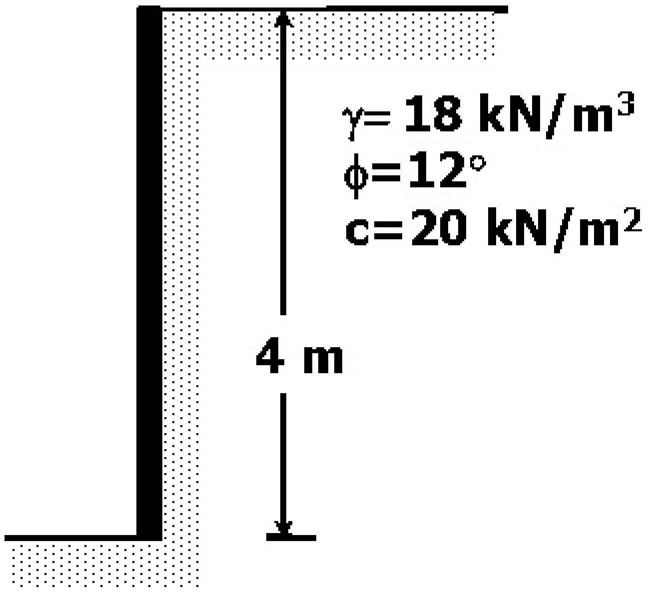
(i) Draw the earth pressure diagram.

(ii) Maximum depth of potential crack

(iii) Maximum depth of unsupported excavation

Lateral Earth Pressure

Solution-02:



$$K_a = \frac{1-\sin 12^\circ}{1+\sin 12^\circ} = 0.656$$

$$p_a = K_a \gamma Z - 2c\sqrt{K_a}$$

$$= 0.656 \times 18Z - 2\times 20 \times \sqrt{0.656}$$

$$= 11.81Z - 32.4$$

$$At\ Z = 0,\ p_a = -32.40\ \ kN/m^2$$

$$At\ Z = 4,\ p_a = 14.80\ \ kN/m^2$$

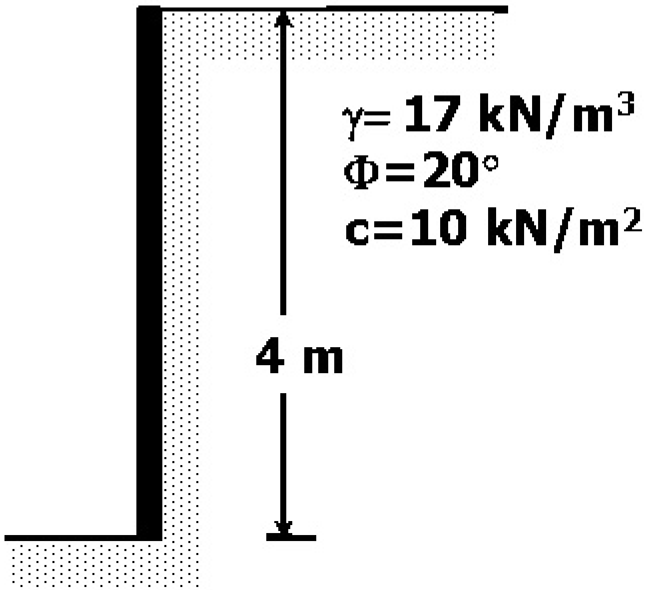
Depth of tension crack

$$Z_c = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2\times 20}{18\times\sqrt{0.656}} = 2.745$$

$$H_c = 2Z_c = 2\times 2.745 = 5.49\,m$$

Lateral Earth Pressure

Example-03:



(i) Draw the earth pressure diagram before tension crack.

(ii) Active earth pressure (force) on the wall before tension crack

(iii) Location of active pressure before tension crack

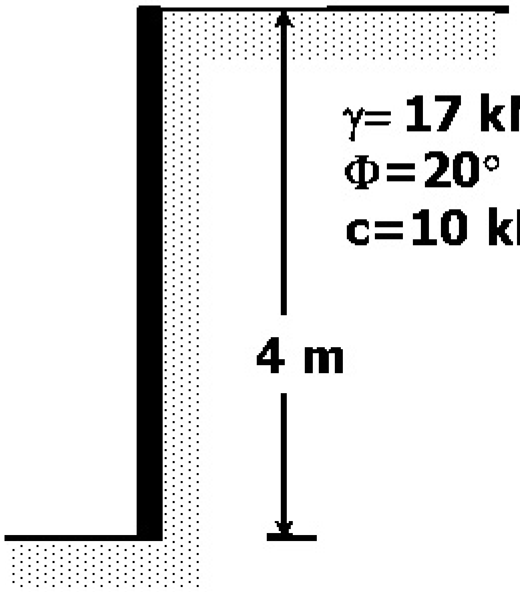
(iv) Draw the earth pressure diagram after tension crack.

(v) Active earth pressure (force) on the wall after tension crack

(vi) Location of active pressure after tension crack

Lateral Earth Pressure

Solution-03:



$$K_a = \frac{1-\sin 20^\circ}{1+\sin 20^\circ} = 0.49$$

$$p_a = K_a \gamma Z - 2c\sqrt{K_a}$$

$$= 0.49 \times 17Z - 2 \times 10 \times \sqrt{0.49}$$

$$= 8.33Z - 14$$

$$At\ Z = 0,\ \ p_a = -14\ \ kN/m^2$$

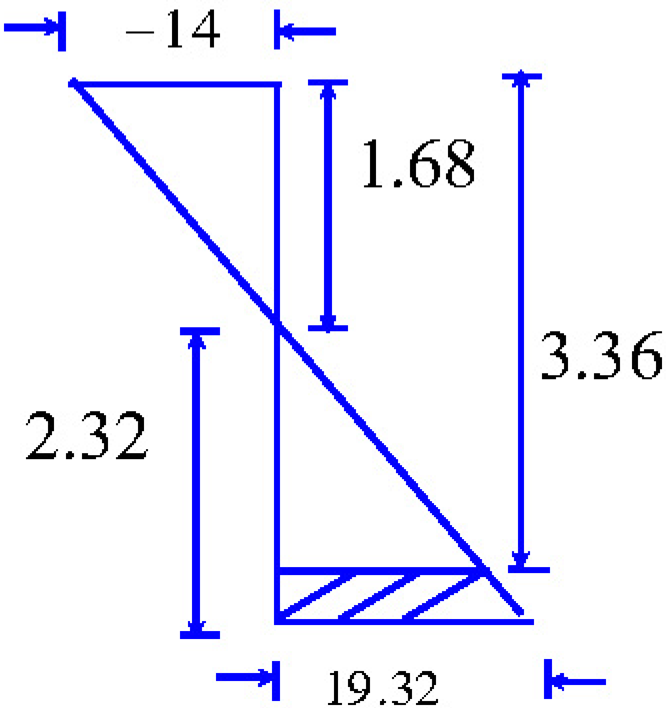
$$At\ Z = 4,\ \ p_a = 19.32\ \ kN/m^2$$

Depth of tension crack

$$Z_c = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2 \times 10}{17 \times \sqrt{0.49}} = 1.68$$

$$H_c = 2Z_c = 2 \times 1.68 = 3.36m$$

Lateral Earth Pressure



Before Tension Crack

Before Tension Crack

$$P_a = \frac{1}{2}\times 17\times 4^2\times 0.49 - 2\times 10\times 4\times\sqrt{0.49}$$

$$= 66.64 - 56 = 10.64\ kN/m$$

Alternatively

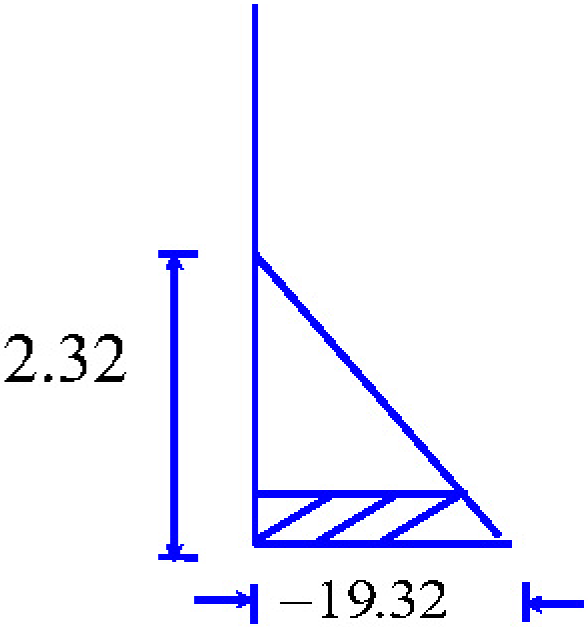
$$P_a = \frac{1}{2}\times 19.32\times 2.32 - \frac{1}{2}\times 14\times 1.68$$

$$= 22.41 - 11.76 = 10.65\ kN/m$$

$$\bar{Z} = \frac{1}{2}\times 19.32\times 2.32\times\frac{2.32}{3} - \frac{1}{2}\times 14\times 1.68\times\left(2.32 + \frac{2\times 1.68}{3}\right)\Big/(22.41 - 11.76)$$

$$\bar{Z} = (17.33 - 40.45)/10.65 = 2.17\,m$$

Lateral Earth Pressure



After Tension Crack

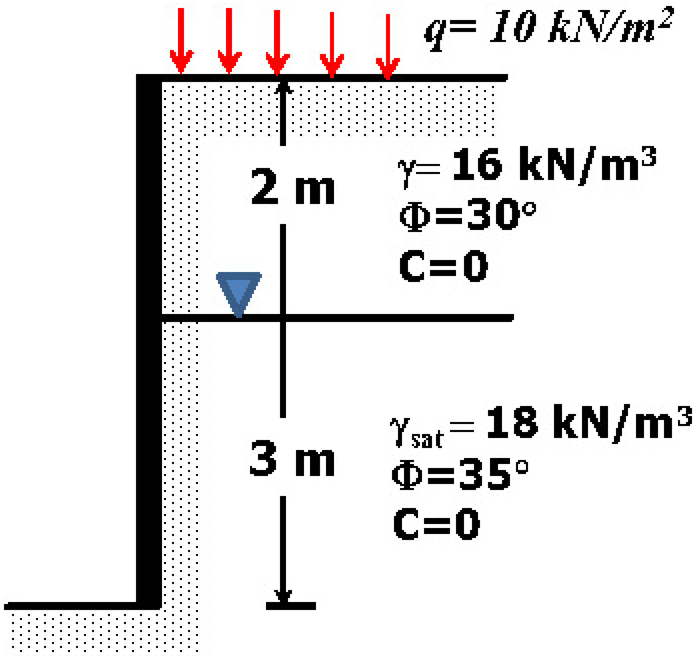
After Tension Crack

$$P_a = \frac{1}{2} \times 19.32 \times 2.32$$
$$= 22.41\ kN/m$$

$$\bar{z} = \frac{2.32}{3} = 0.77$$

Lateral Earth Pressure

Example-04:



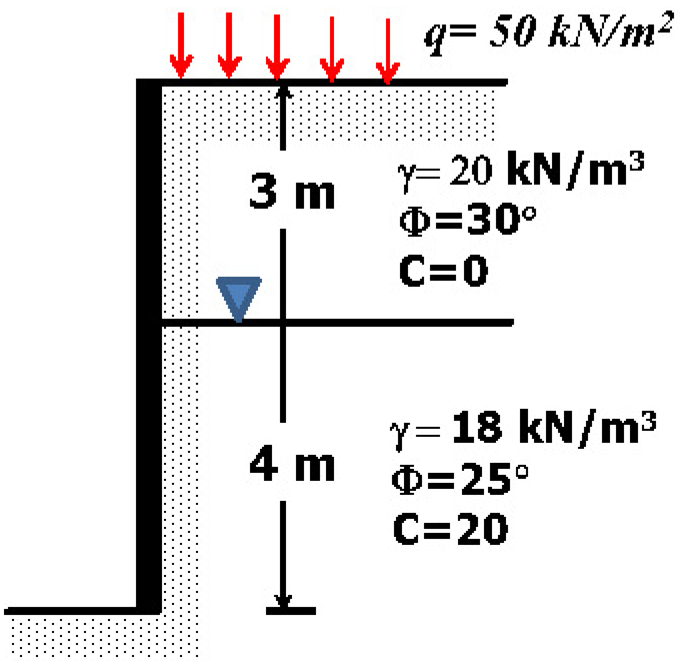
(i) Draw the earth pressure distribution diagram

(ii) Active earth pressure

(iii) Location of active pressure

Lateral Earth Pressure

Example-05:



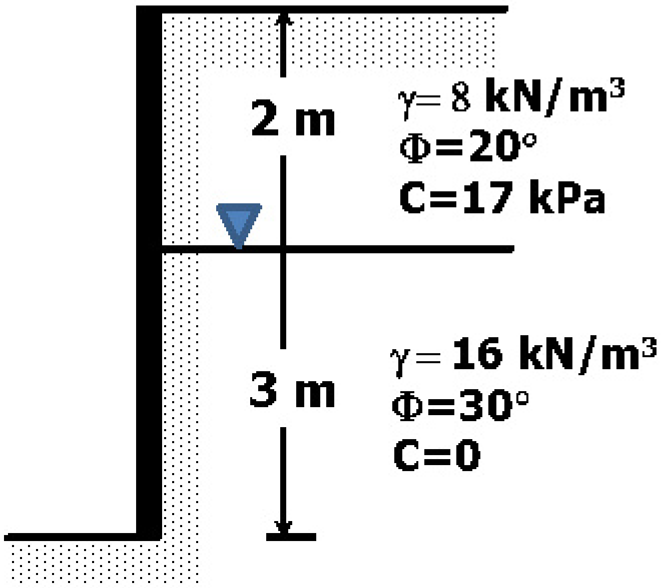
(i) Draw the earth pressure distribution diagram

(ii) Active earth pressure

(iii) Location of active pressure

Lateral Earth Pressure

Example-06:



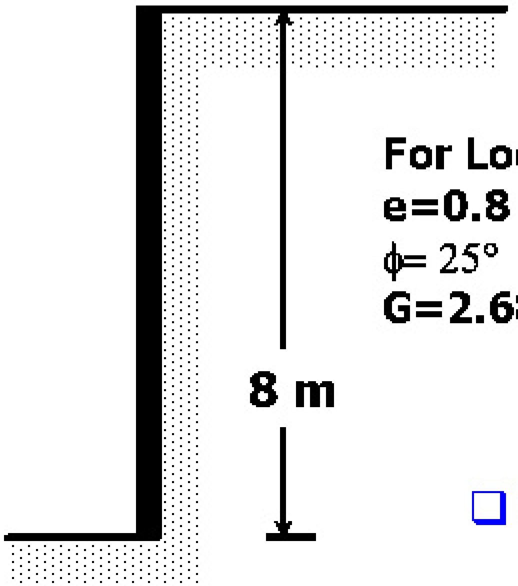
(i) Draw the earth pressure distribution diagram after occurrence of tensile crack

(ii) Determine active earth pressure

(iii) Find out the location of active pressure

Lateral Earth Pressure

Example-07:



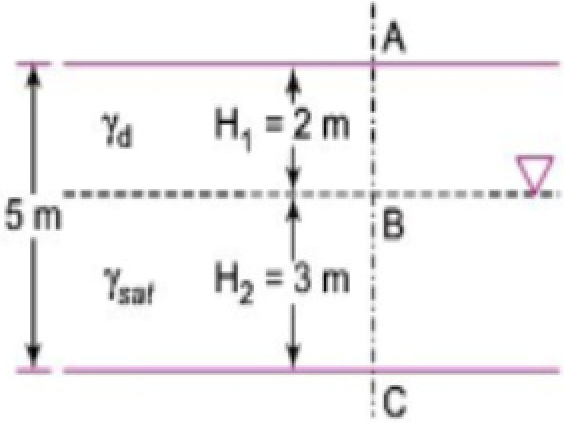
For Loose state
e=0.8
$\phi = 25°$
G=2.68 kPa

For Dense state
e=0.45
$\phi = 35°$
G=2.68 kPa

- Compute and compare active and passive earth pressure in both cases. Also give your comments on the results you obtained from the comparison.

Lateral Earth Pressure

Example-08:



- Compute: (i) Effective vertical stress at C
  (ii) Total vertical stress stress at C
  (iii) Pore pressure at C
  (iv) Active earth pressure at C
  (v) Active earth force on the wall

if void ratio=0.50, G=2.65 and ϕ=30 deg.

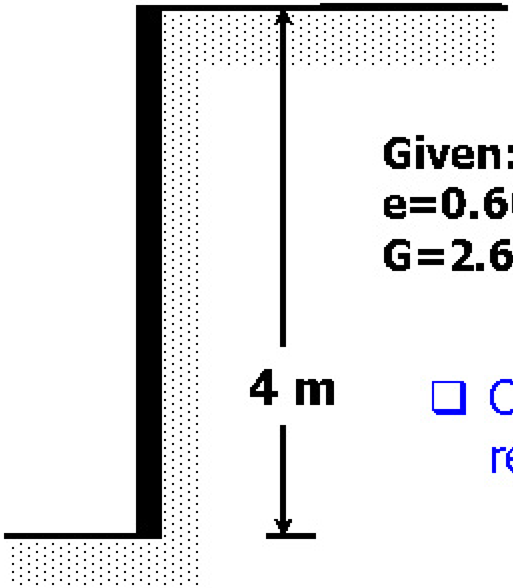
$$\gamma_d = \frac{G\gamma_w}{1+e} \qquad \gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

Lateral Earth Pressure

Example-09:

The observation data of triaxial tests on a silty sand at peak state are as follow:

$\sigma_3\ (kN/m^2)$	4.5	9	13.8
$\sigma_1\ (kN/m^2)$	39.5	53	66.8



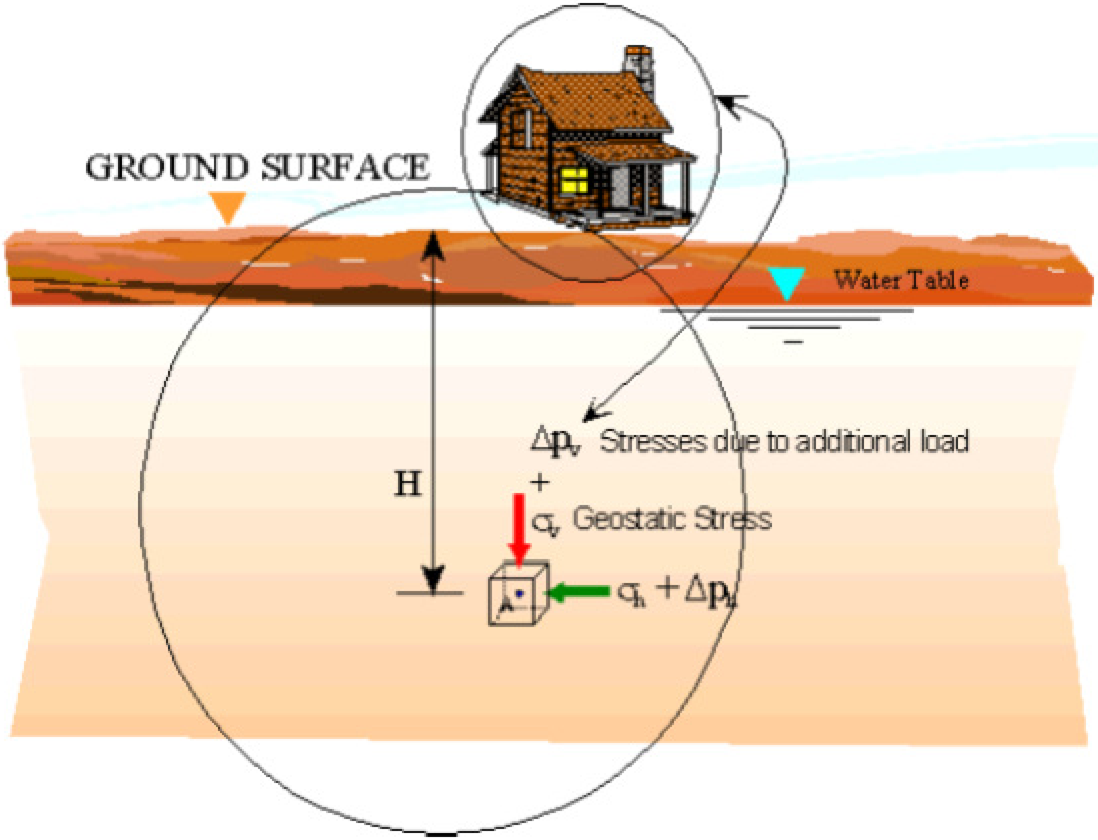
Given:
e=0.60
G=2.66 kPa

- Compute the lateral earth force and its location on the retaining wall.

Stress Distribution

Introduction:

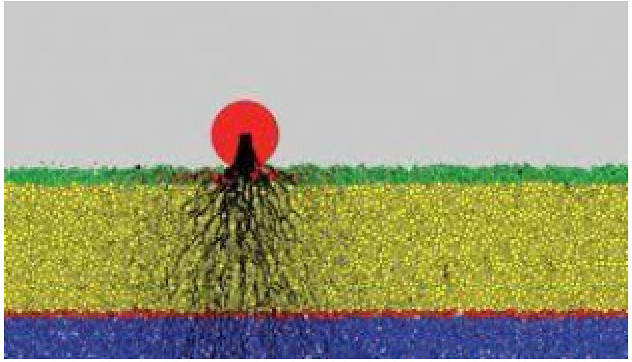
- Sub-surface soil experiences two types of load: (i) Geostatic stress (self weight of soil) (ii) Excess stress (from surcharge)

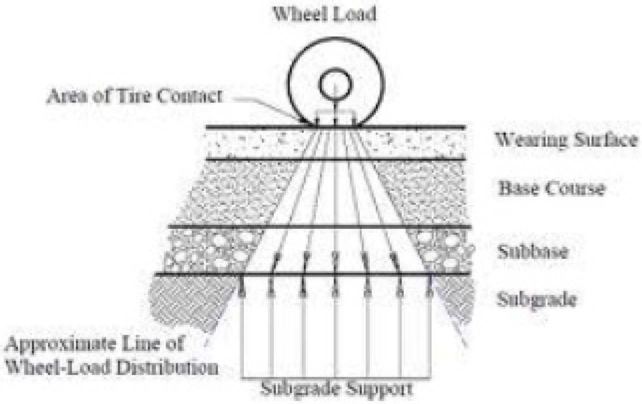


Stress Distribution

Introduction:

- Subsurface stresses in road pavements and airport runways are increased by wheel load on the surface.





Stress Distribution

Introduction:

- Subsurface stresses is also increased due to surcharge from buildings.

Stress Distribution

Introduction:

❖ Embankments and landfills cause to increase subsoil stresses.

Stress Distribution

Importance:

- It is required to estimate the stress increase in the soil due to the applied loads on the surface.

Stress Distribution

Calculation of Sub-surface Stress Increase:

- Sub-surface stress increase -

    1. Under Point/Concentrated Loading
    2. Under Uniformly Loaded Circular Area
    3. Under Line Load
    4. Under Strip Load
    5. Uniformly Loaded Rectangular Area

Stress Distribution

Sub-surface Stress Increase Under Point Load:

Boussinesq Assumptions:

i. The soil mass is an elastic media

ii. The soil is homogeneous

iii. The soil is isotropic

iv. The soil mass is semi-infinite

Stress Distribution

Derivation:

The polar radial stress-

$$\sigma_R = \frac{3}{2}\frac{Q}{\pi}\frac{\cos\beta}{R^2}$$

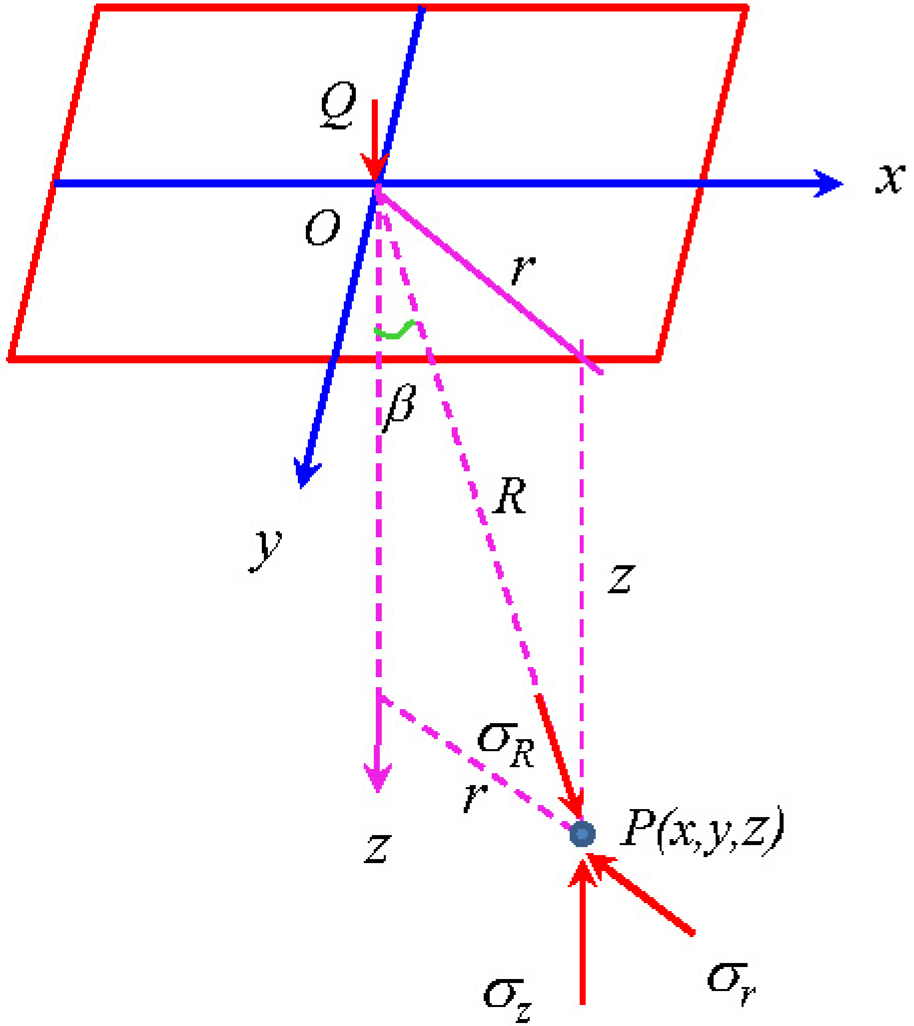
Obviously

$$R = \sqrt{r^2 + z^2}$$
$$= \sqrt{x^2 + y^2 + z^2}$$

$$\sin\beta = \frac{r}{R} \quad \text{and} \quad \cos\beta = \frac{z}{R}$$

The vertical stress-

$$\sigma_z = \sigma_R \cos^2\beta \qquad (i)$$



Stress Distribution

Putting the values of σ_R in (i)

$$\sigma_z = \sigma_R \cos^2\beta \qquad (i)$$

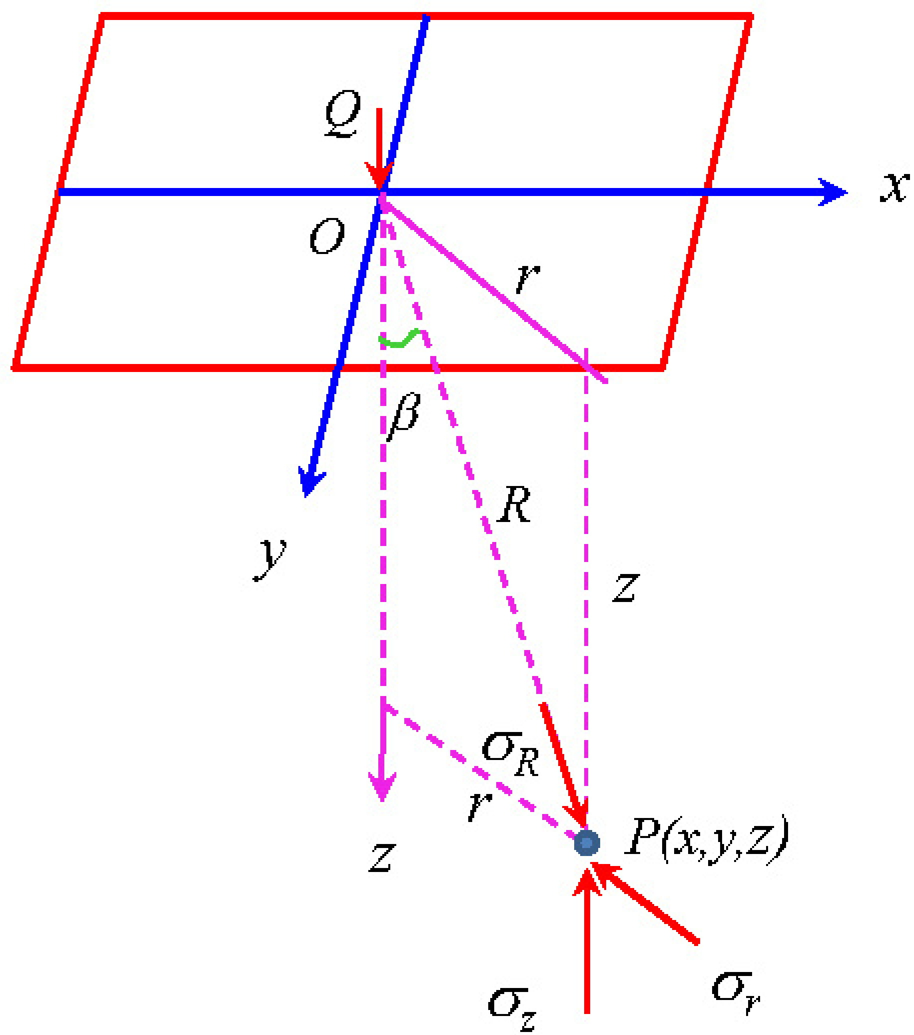
$$= \frac{3}{2}\frac{Q}{\pi}\frac{\cos\beta}{R^2}\cos^2\beta$$

$$= \frac{3}{2}\frac{Q}{\pi}\frac{\cos^3\beta}{R^2}$$

$$= \frac{3}{2}\frac{Q}{\pi}\frac{(z/R)^3}{R^2}$$

$$= \frac{3Q}{2\pi}\times\frac{z^3}{R^5}$$

$$= \frac{3Q}{2\pi}\times\frac{1}{z^2}\left[\frac{z^5}{\left(r^2+z^2\right)^{5/2}}\right]$$



Stress Distribution

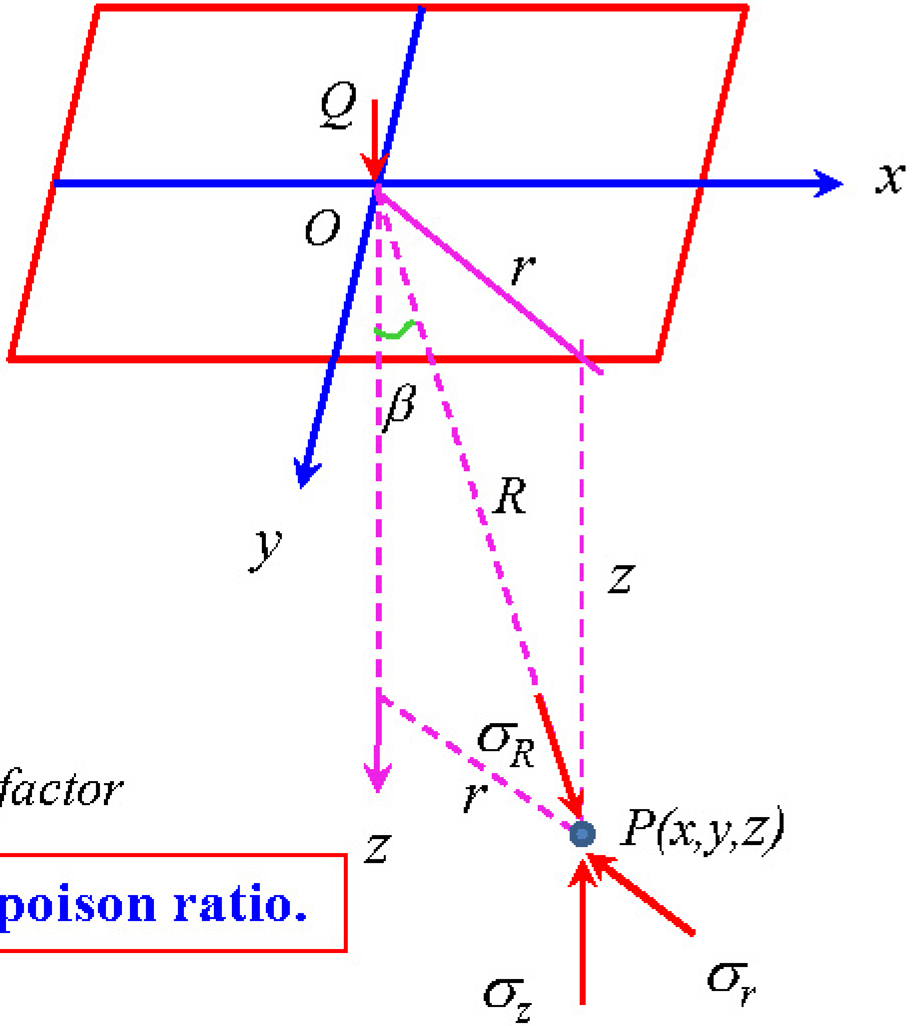
$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \left[\frac{z^5}{z^5 \{1 + (r/z)^2\}^{5/2}} \right]$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \left[\frac{1}{1 + (r/z)^2} \right]^{5/2}$$

$$= \frac{Q}{z^2} \times K_B$$

where, $K_B = \frac{3}{2\pi} \times \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$

$= Boussinesq\ influence\ factor$



Note, σ_z does not depend on E and poison ratio.

Stress Distribution

The tangential stress-

$$\tau_{rz} = \frac{1}{2}\sigma_R \sin 2\beta$$

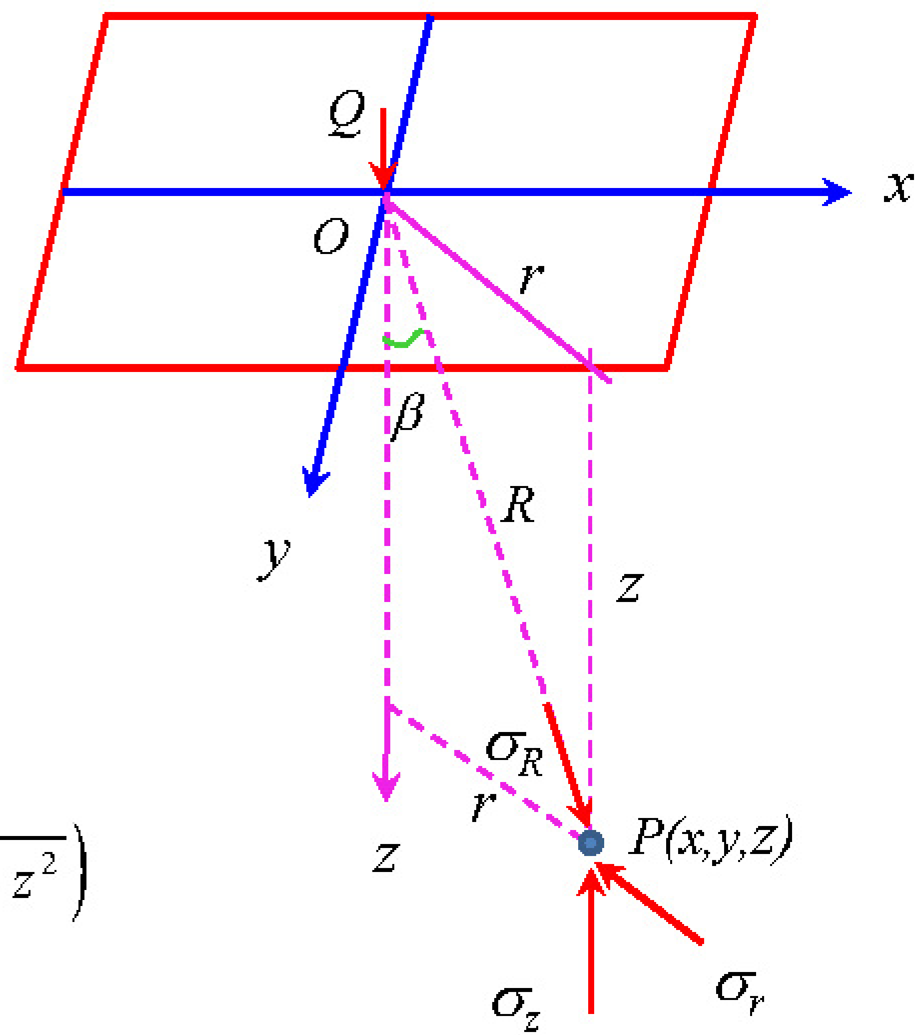
$$= \frac{1}{2} \times \frac{3Q\cos\beta}{2\pi R^2} \times 2\sin\beta\cos\beta$$

$$= \frac{3Q}{2\pi} \times \frac{\cos^2\beta\sin\beta}{R^2}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{R^2} \times \left(\frac{z}{R}\right)^2 \times \left(\frac{r}{R}\right)$$

$$= \frac{3Q}{2\pi} \times \frac{z^2 r}{R^5}$$

$$= \frac{3Q}{2\pi} \times \frac{z^2 r}{\left(r^2 + z^2\right)^{5/2}} \qquad \left(\because R = \sqrt{r^2 + z^2}\right)$$



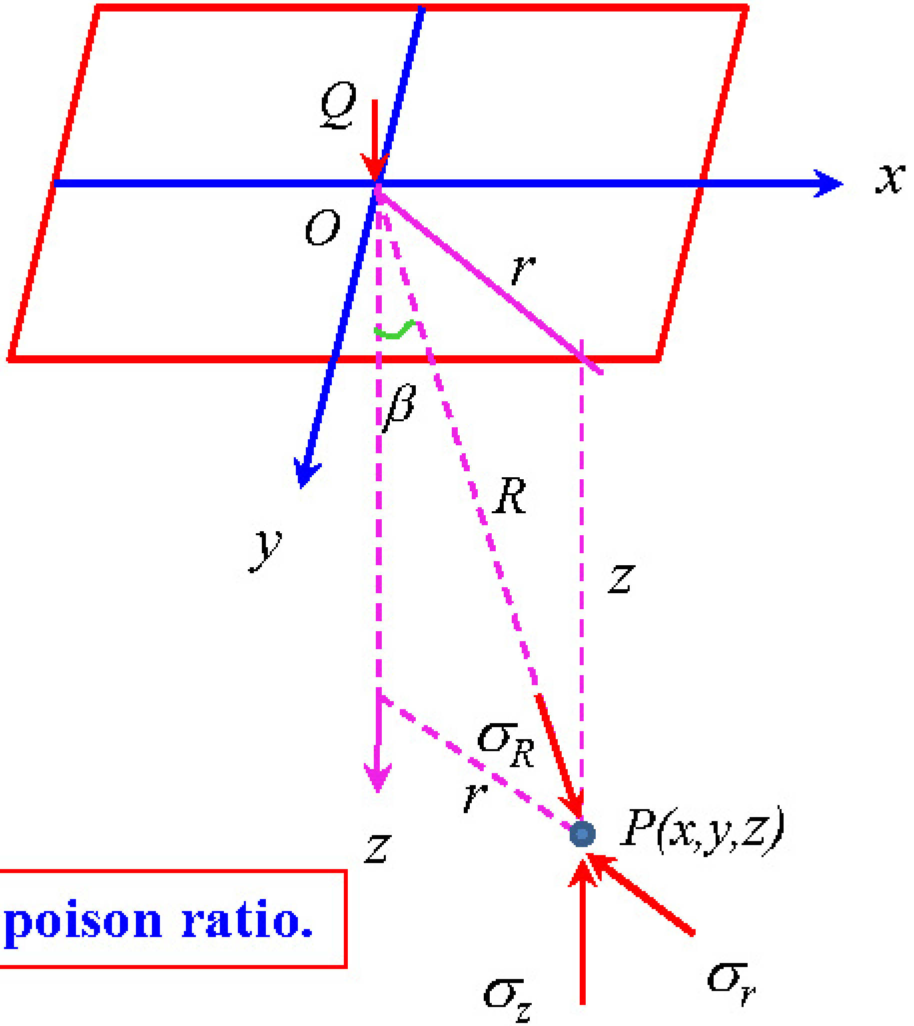
Stress Distribution

$$\tau_{rz} = \frac{3Q}{2\pi} \times \frac{1}{z^3} \times \frac{z^5 r}{\left(r^2 + z^2\right)^{5/2}}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^3} \times \frac{z^5 r}{z^5 \left\{1 + \left(r/z\right)^2\right\}^{5/2}}$$

$$= \frac{3Qr}{2\pi z^3} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$$= \frac{Qr}{z^3} K_B$$



Note, τ_{rz} does not depend on E and poison ratio.

Stress Distribution

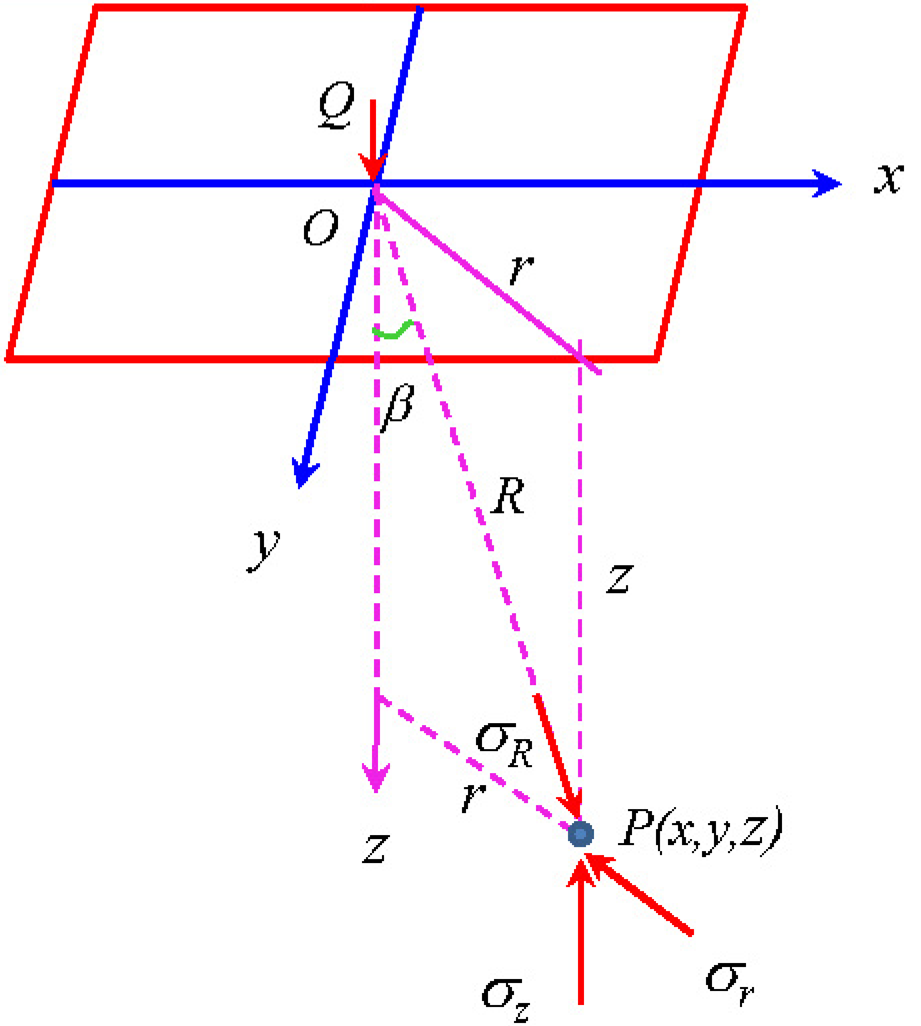
σ_z directly below the point load (i.e., r=0)

$$\sigma_z = \frac{3Q}{2\pi} \times \frac{1}{z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$\sigma_z = \frac{3Q}{2\pi} \times \frac{1}{z^2}\left[\frac{1}{1+0}\right]^{5/2}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2}$$

$$= \frac{0.4775Q}{z^2}$$



Stress Distribution

Vertical Pressure Distribution Diagrams

1. Stress isobar or isobar diagram
2. Vertical pressure distribution on a horizontal plane
3. Vertical pressure distribution on vertical plane

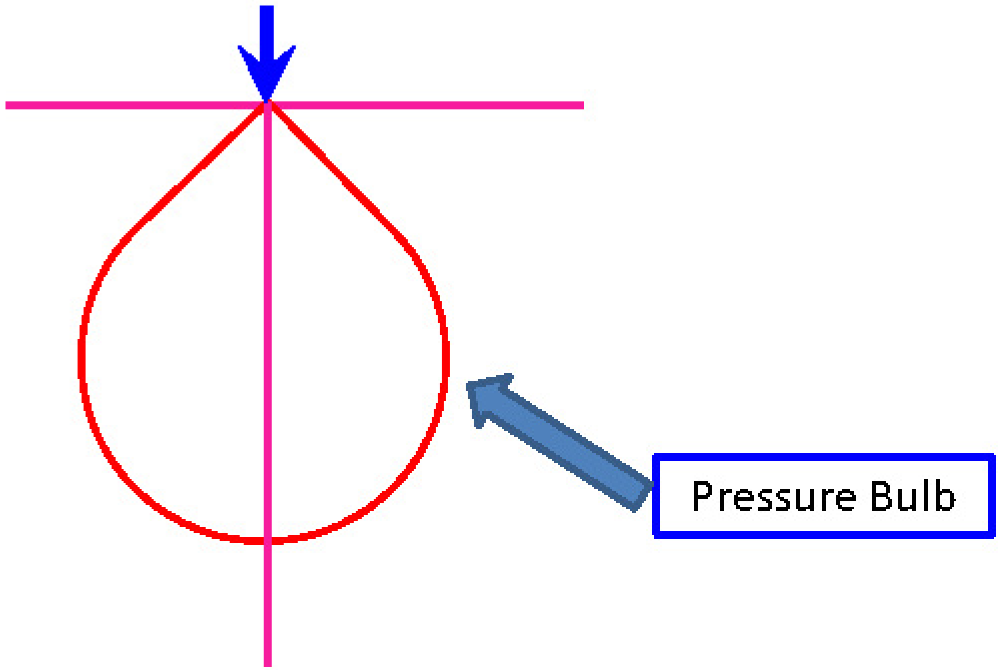
Stress Distribution

1. Isobar Diagram:

An isobar is **a curve or contour** joining the points of equal vertical stress below ground surface.

Its shape is like an **electric bulb.**

The zone bounded by an isobar is called a **pressure bulb.**



Stress Distribution

Drawing An Isobar Diagram:

Let us draw an isobar of σ_z=0.25Q

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$\sigma_z = \frac{Q}{z^2} \times K_B$$

$$\Rightarrow 0.25Q = \frac{Q}{z^2} \times K_B$$

$$K_B = 0.25z^2$$

z	K_B	r/z	r
0.2	**0.0100**	**1.92**	**0.38**
0.4	0.0400	1.30	0.52
0.6	0.0900	0.97	0.58
0.8	0.1600	0.74	0.59
1.0	0.2500	0.54	0.54
1.2	0.3600	0.34	0.41
1.38	0.4775	0.00	0.00

For maximum z,

$$\sigma_z = \frac{0.4775Q}{z^2}$$

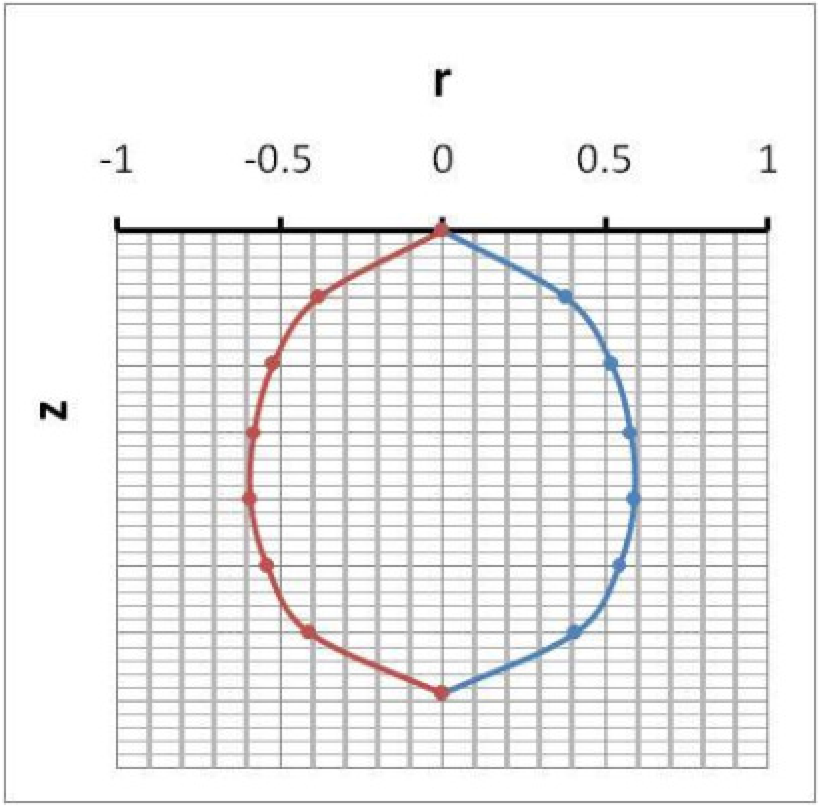
$$\Rightarrow 0.25Q = \frac{0.4775Q}{z^2}$$

$$\Rightarrow z = 1.38$$

Stress Distribution

Drawing An Isobar Diagram:

r	z
0.38	0.2
0.52	0.4
0.58	0.6
0.59	0.8
0.54	1.0
0.41	1.2
0.00	1.38



Assignment:

Draw isobar for σ_z=0.5Q, 0.75Q, Q, 2Q and check whether the isobar is getting smaller or larger.

Stress Distribution

Vertical pressure distribution on a horizontal plane:

Let us determine the stresses at a depth z=2 unit. Therefore

$$\sigma_z = \frac{Q}{z^2} \times K_B$$

$$\Rightarrow \sigma_z = \frac{Q}{2^2} \times K_B$$

$$\Rightarrow \sigma_z = 0.25 K_B Q$$

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

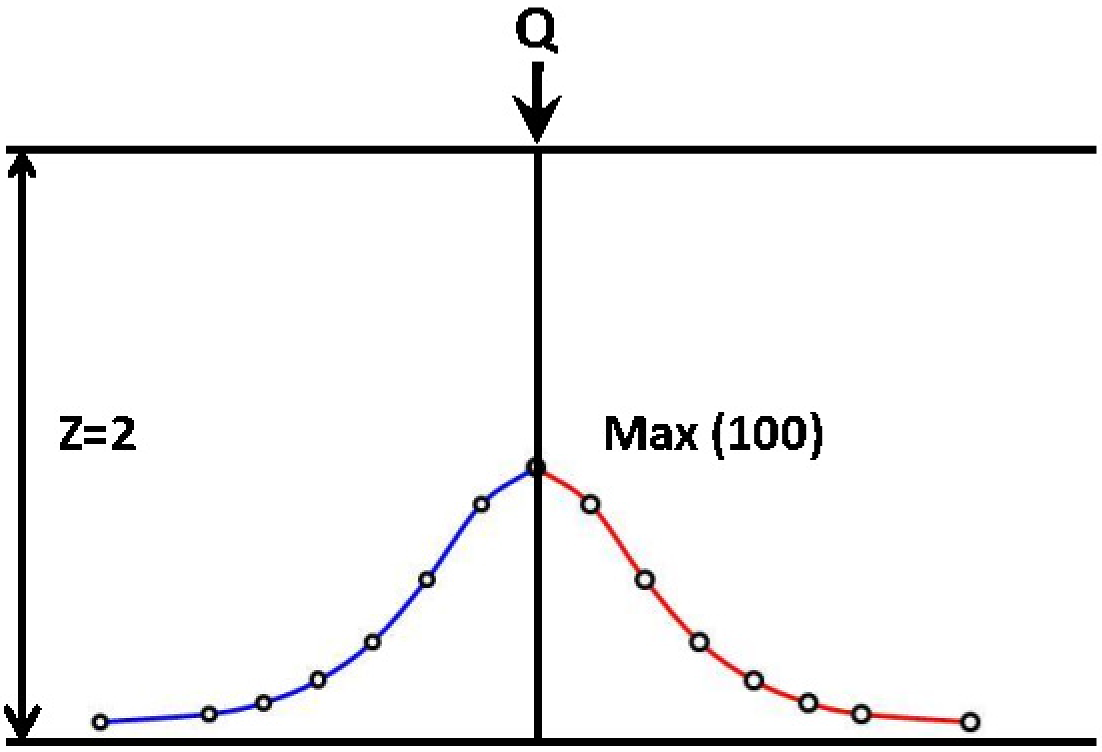
r	r/z	K_B	σ_z	%
0.0	**0.00**	**0.4775**	**0.1194Q**	**100**
0.5	0.25	0.4103	0.1026Q	86
1.0	0.50	0.2733	0.0683Q	57
1.5	0.75	0.1565	0.0390Q	32.7
2.0	1.00	0.0844	0.0211Q	17.7
2.5	1.25	0.0454	0.0113Q	9.5
3.0	1.50	0.0251	0.0063Q	5.2
4.0	2.00	0.0085	0.0021Q	1.8

When, **the horizontal distance= 2 x depth**, the vertical pressure due to point load is **negligible**.

Stress Distribution

Vertical pressure distribution on a horizontal plane:

r	%
0.0	100
0.5	86
1.0	57
1.5	32.7
2.0	17.7
2.5	9.5
3.0	5.2
4.0	1.8



Stress Distribution

Influence Diagram:

An influence diagram is the vertical stress distribution diagram on a horizontal plane at a given depth due to **a unit concentrated load.**

It is helpful to determine vertical stress at any point on that horizontal plane due to **number of concentrated load.**

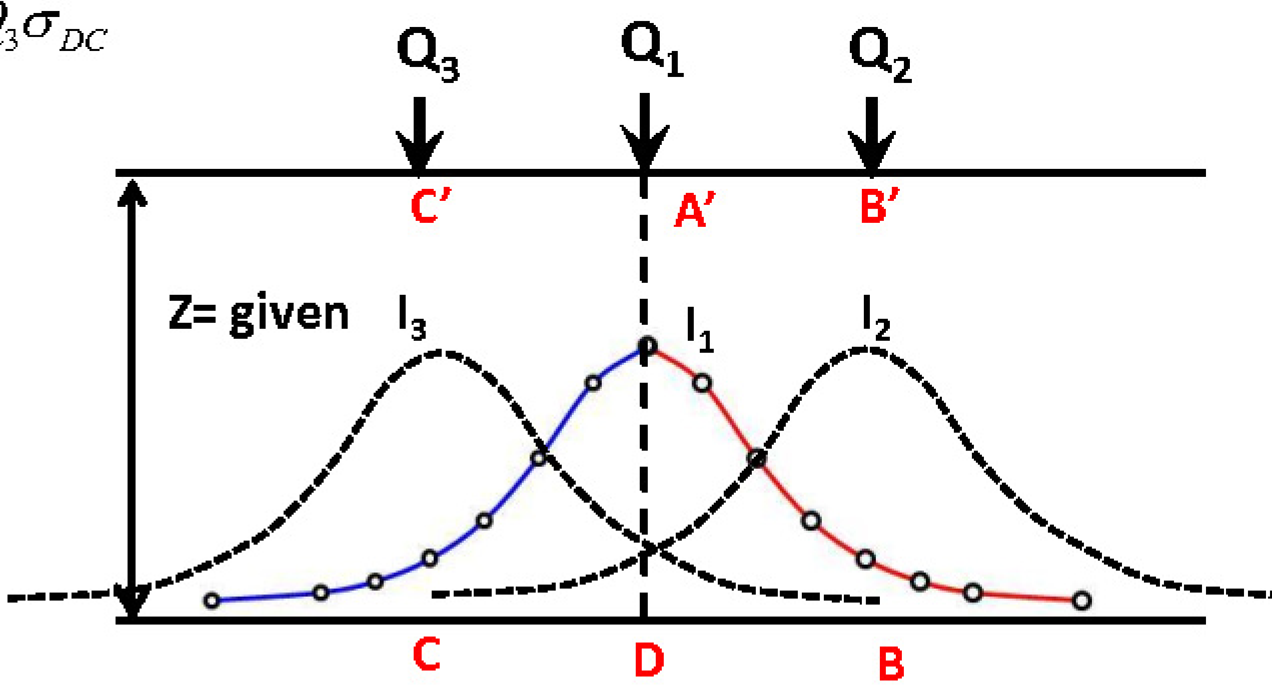
Let us determine stress at point D

$$(\sigma_z)_D = Q_1\sigma_{DD} + Q_2\sigma_{DB} + Q_3\sigma_{DC}$$

σ_{DD} = **V. Stress at D due to unit load at A'**

σ_{DB} = **V. Stress at D due to unit load at B'**

σ_{DC} = **V. Stress at D due to unit load at C'**



Stress Distribution

Vertical Stress Distribution on a Vertical Plane:

Let us determine the stresses at a radial distance r=1 unit. Therefore

$$\sigma_z = \frac{Q}{z^2} \times K_B$$

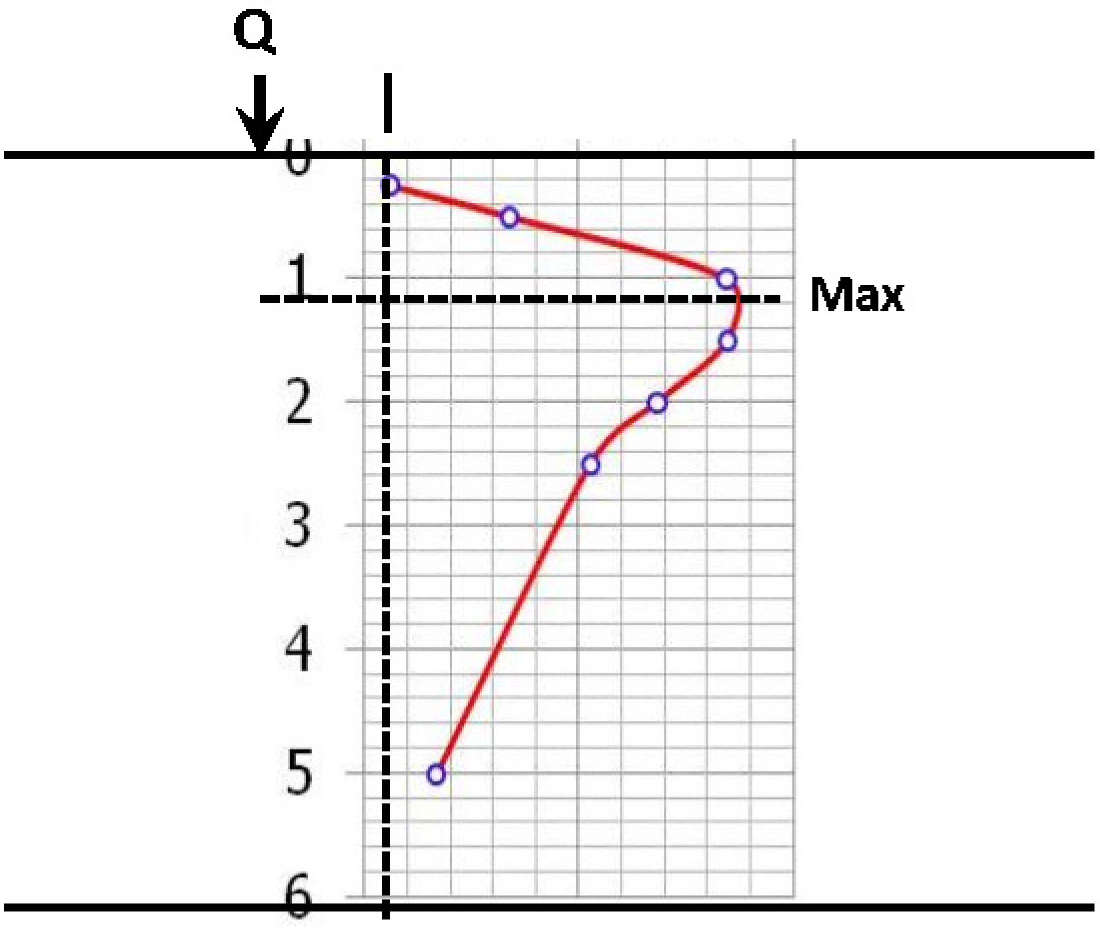
$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

z	r/z	K_B	σ_z
0.25	**4.0**	**0.0004**	**0.0064Q**
0.50	2.0	0.0085	0.0340Q
1.00	1.0	0.0844	0.0844Q
1.50	0.667	0.1904	0.0845Q
2.00	0.50	0.2733	0.0683Q
2.50	0.40	0.3294	0.0527Q
5.00	0.20	0.4329	0.017Q

Stress Distribution

Vertical pressure distribution on a vertical plane:

z	σ_z
0.25	0.0064
0.50	0.0340
1.00	0.0844
1.50	0.0845
2.00	0.0683
2.50	0.0527
5.00	0.017



Stress Distribution

Vertical Stress Distribution on a Vertical Plane:

Prove that the maximum vertical stress on a vertical line at a constant radial distance r from the axis of a vertical load is induced at the point of intersection of the vertical line with a radial line at β=39°15′

We know, $$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

For maxima, $$\frac{d\sigma_z}{dz} = 0$$

$$\Rightarrow \frac{d}{dz}\left[\frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}\right] = 0$$

$$\frac{r}{z} = 0.817 = \tan\beta \qquad \Rightarrow \beta = 39^\circ 15'$$

Stress Distribution

2. Vertical stress under uniformly loaded circular area

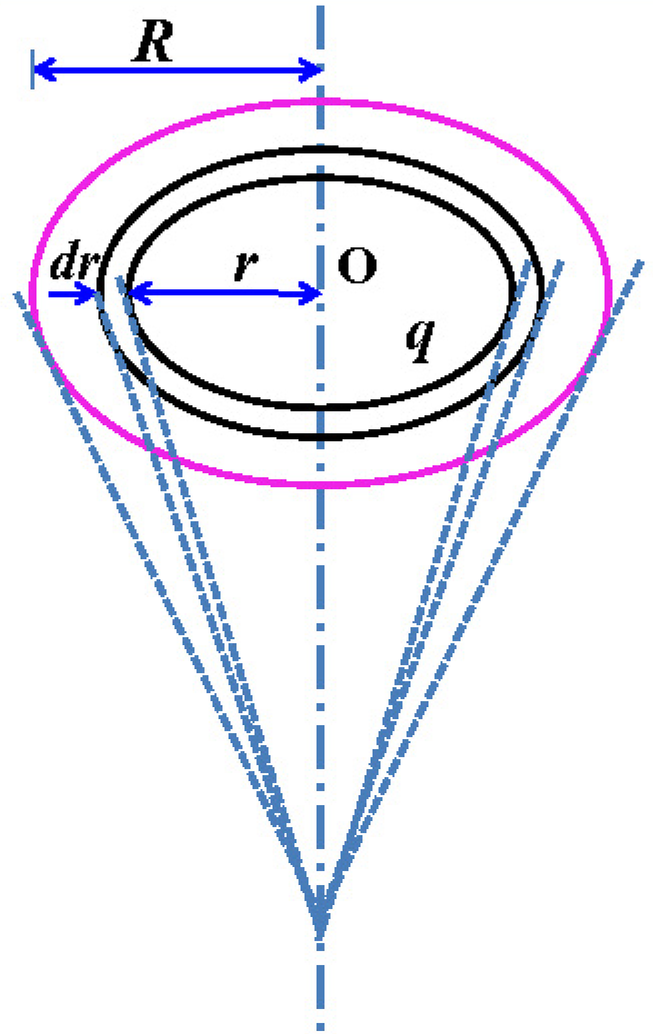
q= intensity of load per unit area

R= radius of loaded area

r= radius of elementary ring

dr= width of elementary ring

The load on elementary ring= $q \times 2\pi r dr$



We know,

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

Following this equation

$$\Delta\sigma_z = \frac{3(q \times 2\pi r dr)}{2\pi} \times \frac{1}{z^2} \times \frac{1}{[1+(r/z)^2]^{5/2}}$$

Stress Distribution

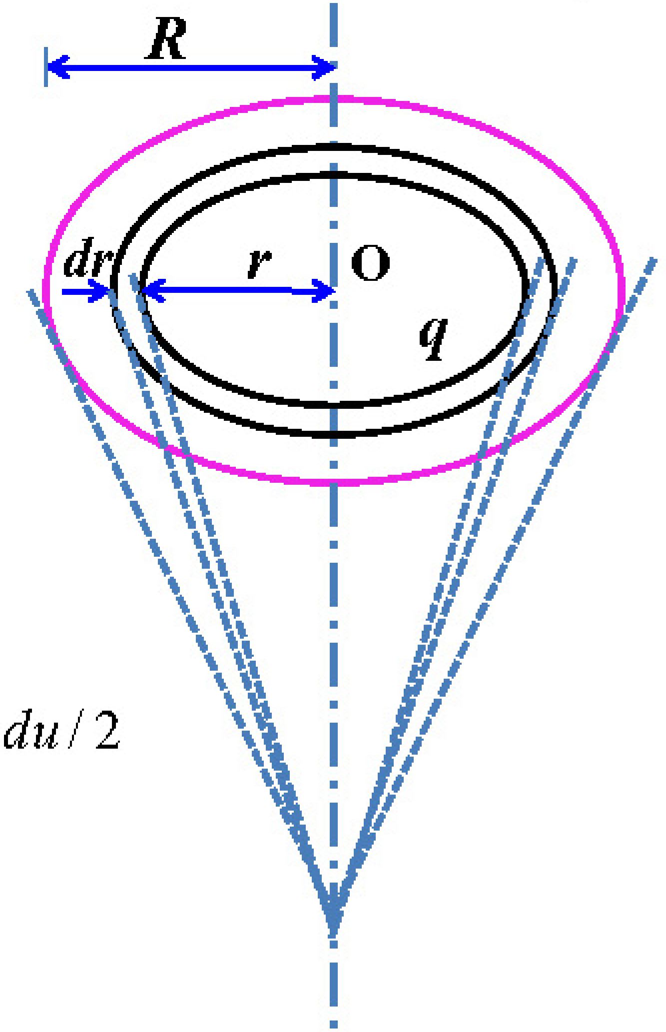
Vertical stress under uniformly loaded circular area

$$\Delta\sigma_z = \frac{3qrdr}{z^2} \times \frac{z^5}{[r^2 + z^2]^{5/2}}$$

$$\Delta\sigma_z = 3qz^3 \times \frac{rdr}{[r^2 + z^2]^{5/2}}$$

Vertical load due to entire load is given by

$$\sigma_z = 3qz^3 \times \int_0^R \frac{rdr}{[r^2 + z^2]^{5/2}} \qquad (i)$$



$$Let \quad r^2 + z^2 = u \quad \therefore \quad 2rdr = du \quad \Rightarrow \quad rdr = du/2$$

$$When \quad r = 0, \ u = z^2 \quad and \quad r = R, \ u = R^2 + z^2$$

Stress Distribution

Vertical stress under uniformly loaded circular area

Therefore equation (i) becomes

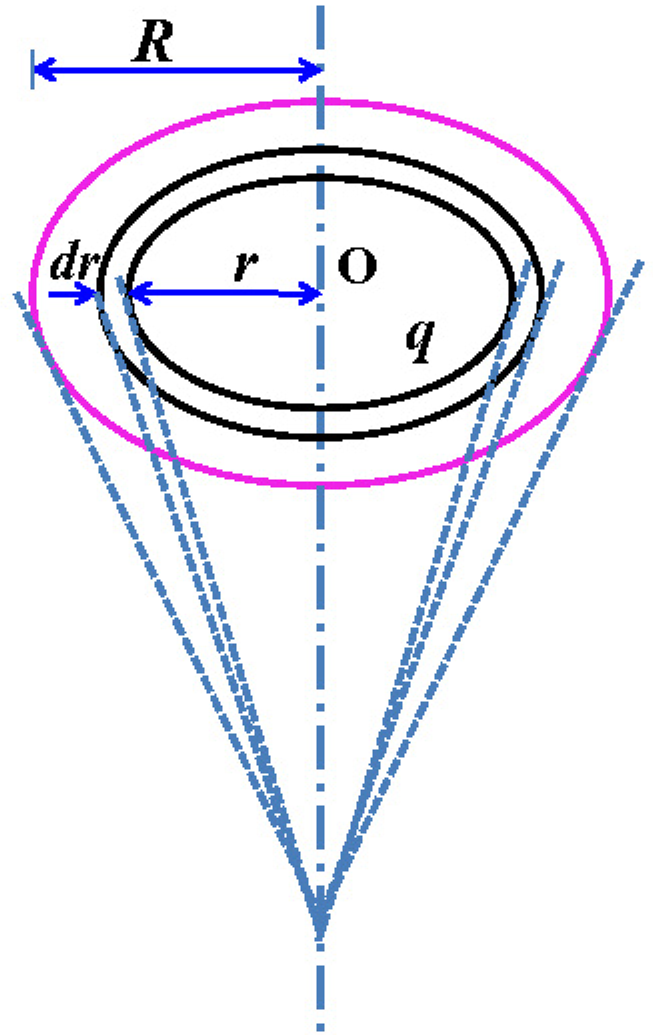
$$\sigma_z = 3qz^3 \times \int_{z^2}^{(R^2+z^2)} \frac{du}{2[u]^{5/2}}$$

$$= \frac{3}{2} qz^3 \left(-\frac{2}{3} \right) \left[u^{-3/2} \right]_{z^2}^{R^2+z^2}$$

$$= -qz^3 \left[\frac{1}{\left(R^2+z^2\right)^{3/2}} - \frac{1}{\left(z^2\right)^{3/2}} \right]$$

$$= qz^3 \left[\frac{1}{z^3} - \frac{1}{\left(R^2+z^2\right)^{3/2}} \right]$$

$$= q \left[1 - \left\{ \frac{1}{1+(R/z)^2} \right\}^{3/2} \right] = q \times K_C$$



Stress Distribution

3. Vertical stress under a Line Load

q′= intensity of line load per unit length

Let us consider the load acting on a small length δy

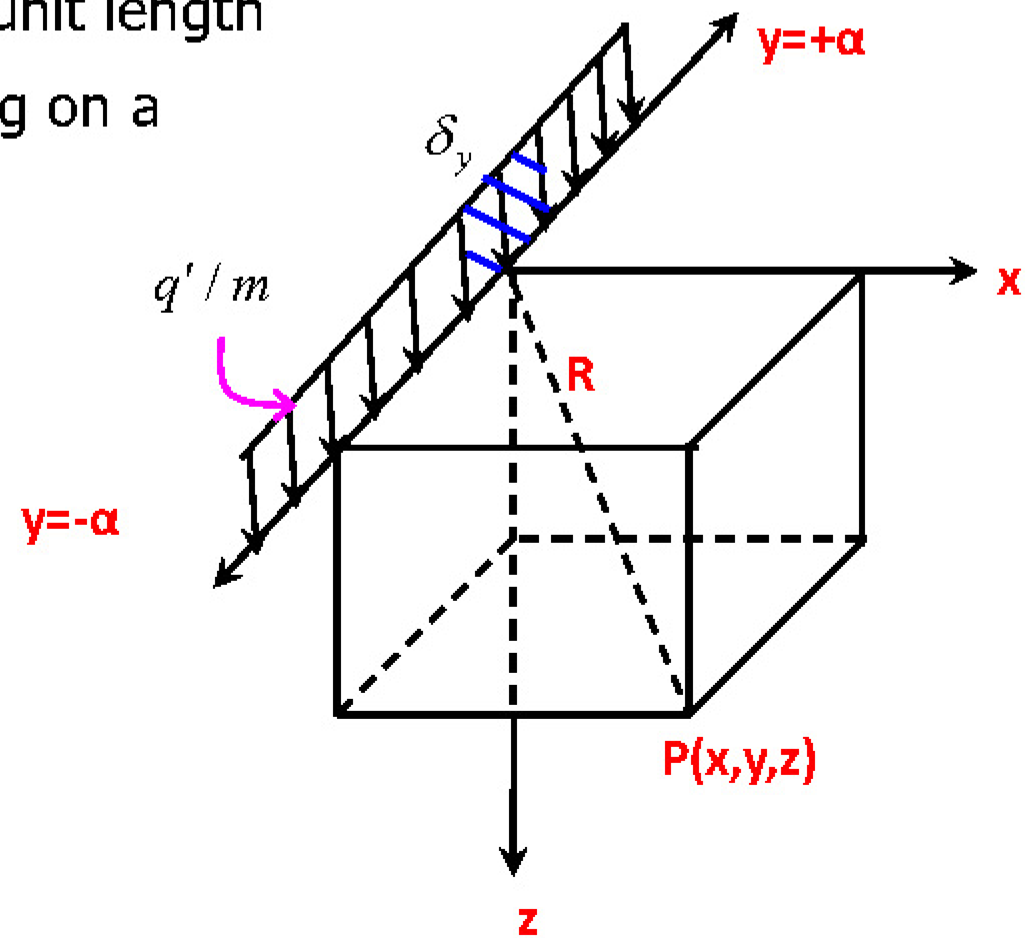
Point load=q′ δy

We know,

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

Following this equation

$$\Delta\sigma_z = \frac{3(q'\delta y)}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$



Stress Distribution

$$\Delta\sigma_z = \frac{3(q'\delta y)}{2\pi} \times \frac{z^3}{(r^2+z^2)^{5/2}}$$

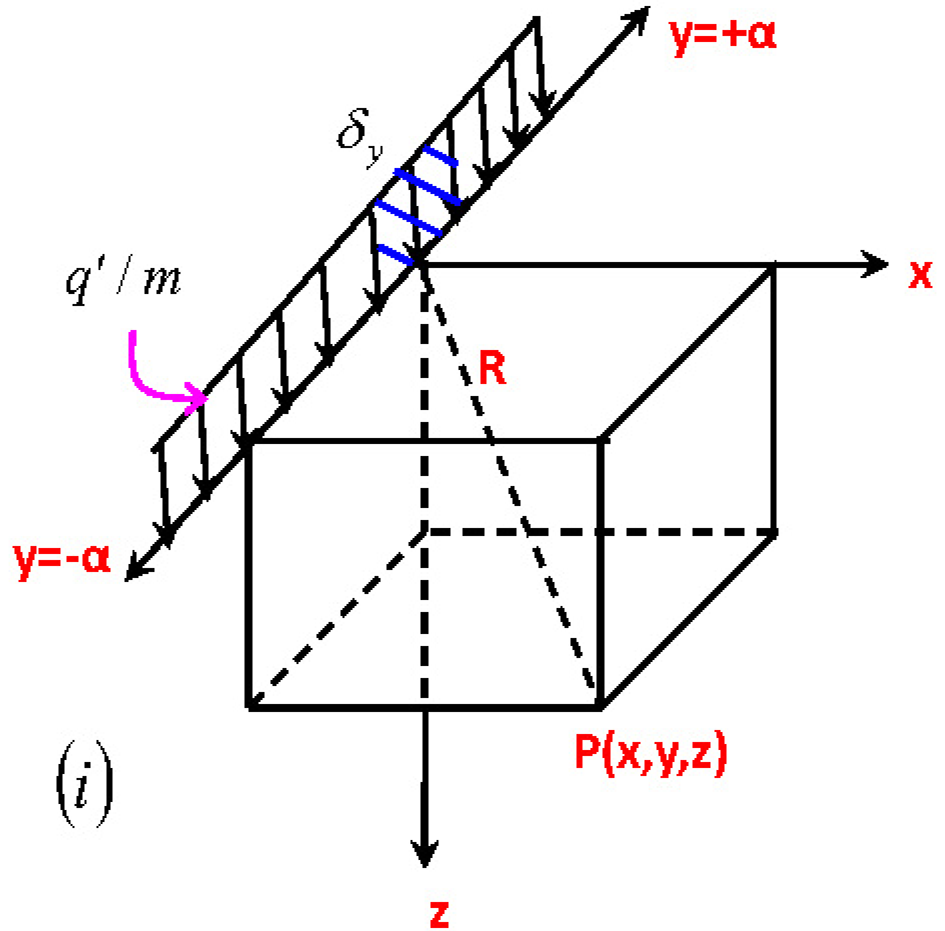
Integrating,

$$\sigma_z = \frac{3q'z^3}{2\pi} \times \int_{-\alpha}^{+\alpha} \frac{dy}{(r^2+z^2)^{5/2}}$$

$$= \frac{3q'z^3}{2\pi} \times \int_{-\alpha}^{+\alpha} \frac{dy}{(x^2+y^2+z^2)^{5/2}}$$

$$= \frac{3q'z^3}{2\pi} \times 2\int_{0}^{+\alpha} \frac{dy}{(x^2+y^2+z^2)^{5/2}} \qquad (i)$$

$$Let \quad x^2+z^2 = u^2$$



Stress Distribution

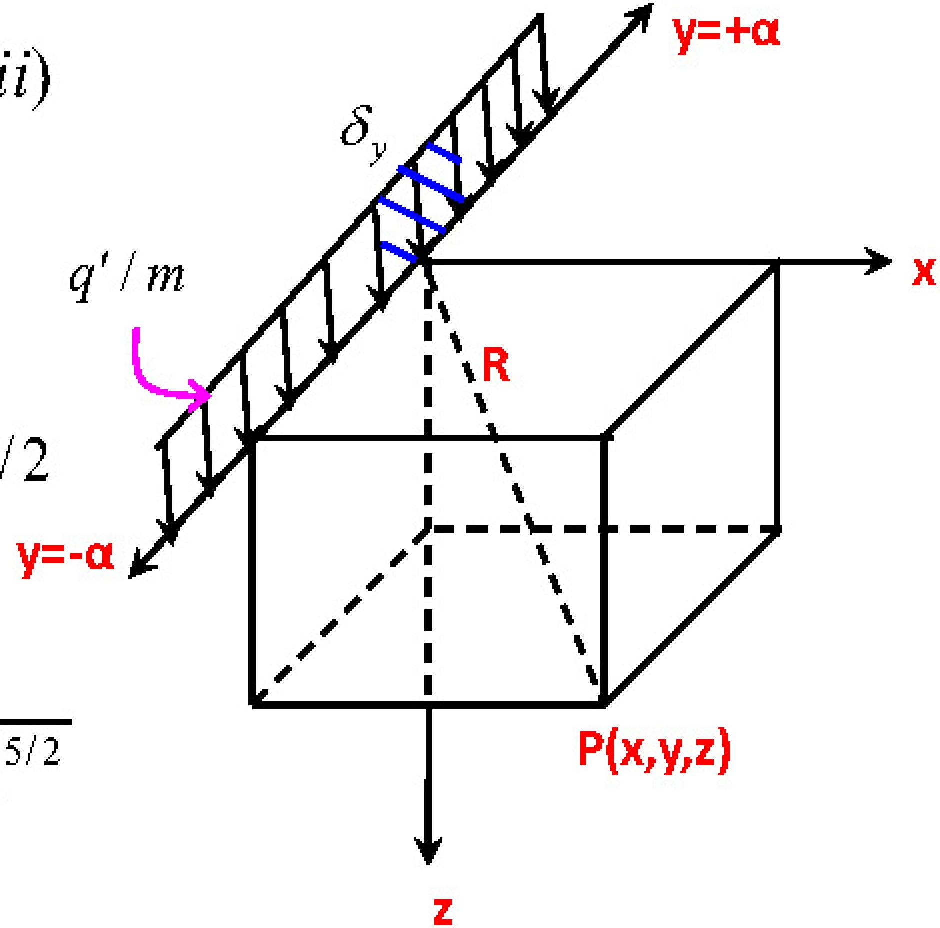
Equation (i) becomes

$$\sigma_z = \frac{3q'z^3}{\pi} \times \int_0^{+\alpha} \frac{dy}{(u^2 + y^2)^{5/2}} \qquad (ii)$$

$$Let \quad y = u \tan\theta$$

$$Therefore \quad dy = u \sec^2\theta \, d\theta$$

$$When \quad y = 0,\ \theta = 0 \quad and \quad y = \alpha,\ \theta = \pi/2$$



Equation (ii) becomes

$$\sigma_z = \frac{3q'z^3}{\pi} \times \int_0^{\pi/2} \frac{u \sec^2\theta d\theta}{(u^2 + u^2 \tan^2\theta)^{5/2}}$$

$$= \frac{3q'z^3}{\pi} \times \int_0^{\pi/2} \frac{u \sec^2\theta \, d\theta}{u^5 (1 + \tan^2\theta)^{5/2}}$$

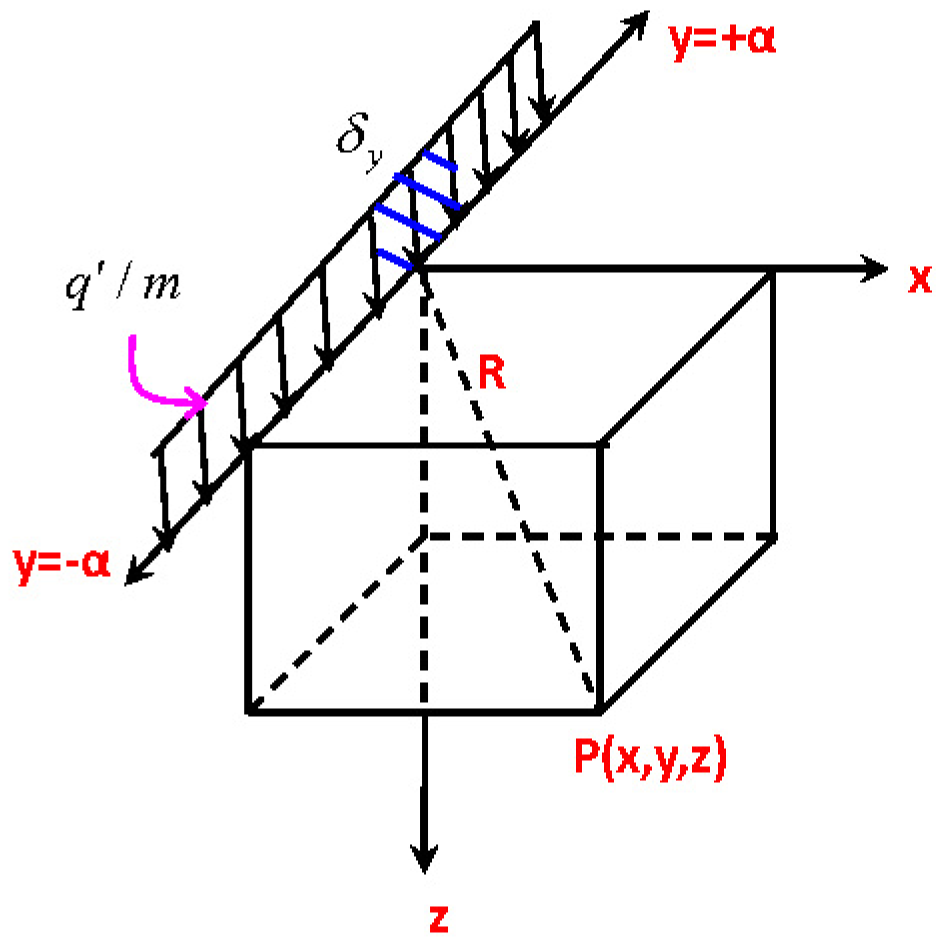
Stress Distribution

$$\sigma_z = \frac{3q'z^3}{\pi} \times \int_0^{\pi/2} \frac{\sec^2\theta \, d\theta}{u^4(\sec^2\theta)^{5/2}}$$

$$= \frac{3q'z^3}{\pi} \times \int_0^{\pi/2} \frac{\sec^2\theta \, d\theta}{u^4 \sec^5\theta}$$

$$= \frac{3q'z^3}{\pi} \times \int_0^{\pi/2} \frac{d\theta}{u^4 \sec^3\theta}$$

$$= \frac{3q'z^3}{\pi u^4} \times \int_0^{\pi/2} \cos^3\theta \, d\theta$$



Stress Distribution

$$\sigma_z = \frac{3q'z^3}{\pi u^4} \times \int_0^{\pi/2} \cos^2\theta \cos\theta \, d\theta$$

$$= \frac{3q'z^3}{\pi u^4} \times \int_0^{\pi/2} \left(1 - \sin^2\theta\right) \cos\theta \, d\theta \quad (iii)$$

$$Let \quad t = \sin\theta$$

$$Therefore \quad dt = \cos\theta \, d\theta$$

$$When \quad \theta = 0, \; t = 0 \quad and \quad \theta = \pi/2, \; t = 1$$

Equation (iii) becomes

$$\sigma_z = \frac{3q'z^3}{\pi u^4} \times \int_0^1 \left(1 - t^2\right) dt$$

$$= \frac{3q'z^3}{\pi u^4} \times \left[t - t^3/3\right]_0^1$$

$$= \frac{3q'z^3}{\pi u^4} \times \frac{2}{3}$$

$$= \frac{2q'z^3}{\pi\left(x^2 + z^2\right)^2} \left[\because u^2 = x^2 + z^2\right]$$

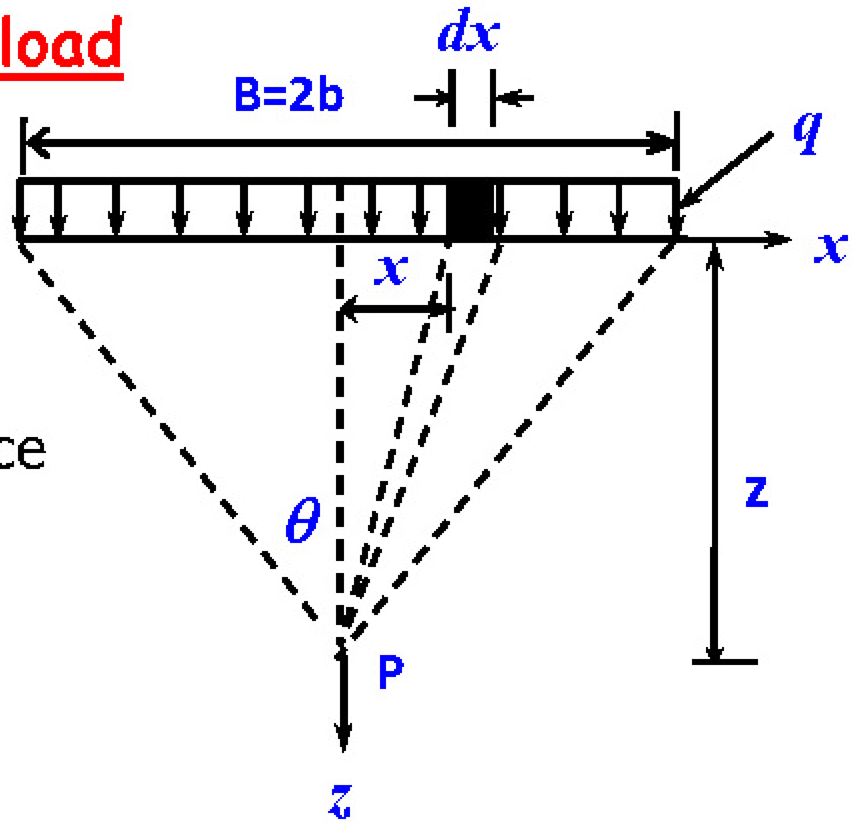
$$= \frac{2q'}{\pi z} \left[\frac{1}{1 + (x/z)^2}\right]^2$$

Stress Distribution

4. Vertical stress under a strip load

B=2b=width of strip load

q=intensity of load



Let us consider the load acting on a small elementary width *dx* at a distance x from center.

q′=Line load=qdx

We know,

$$\sigma_z = \frac{2q'}{\pi z}\left[\frac{1}{1+(x/z)^2}\right]^2$$

Following this equation,

$$\Delta\sigma_z = \frac{2qdx}{\pi z}\left[\frac{1}{1+(x/z)^2}\right]^2$$

Stress Distribution

The stress due to entire strip

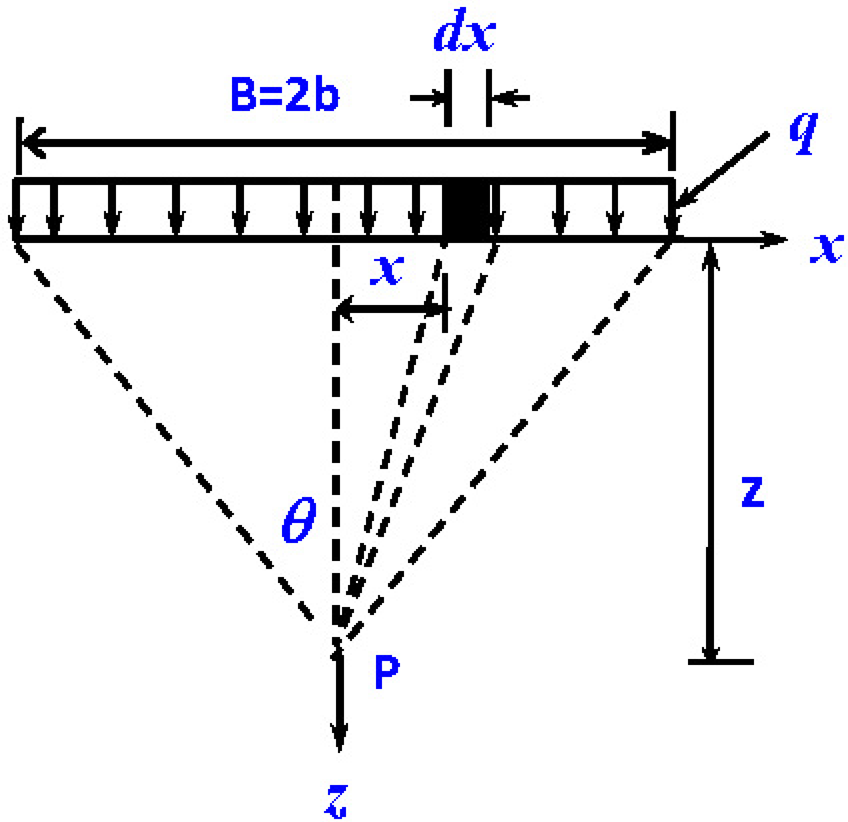
$$\sigma_z = \frac{2q}{\pi z}\int_{-b}^{+b}\left[\frac{1}{1+(x/z)^2}\right]^2 dx$$

$$= 2\times\frac{2q}{\pi z}\int_{0}^{+b}\left[\frac{1}{1+(x/z)^2}\right]^2 dx \qquad (i)$$

$$Let \quad x/z = \tan u$$

$$Therefore \quad dx = z\sec^2 u\, du$$

$$When \quad x=0, u=0 \quad and \quad x=b,\ u=\theta$$



Equation (i) becomes

$$\sigma_z = 2\times\frac{2q}{\pi z}\int_{0}^{\theta}\frac{z\sec^2 u}{(1+\tan^2 u)^2}du$$

Stress Distribution

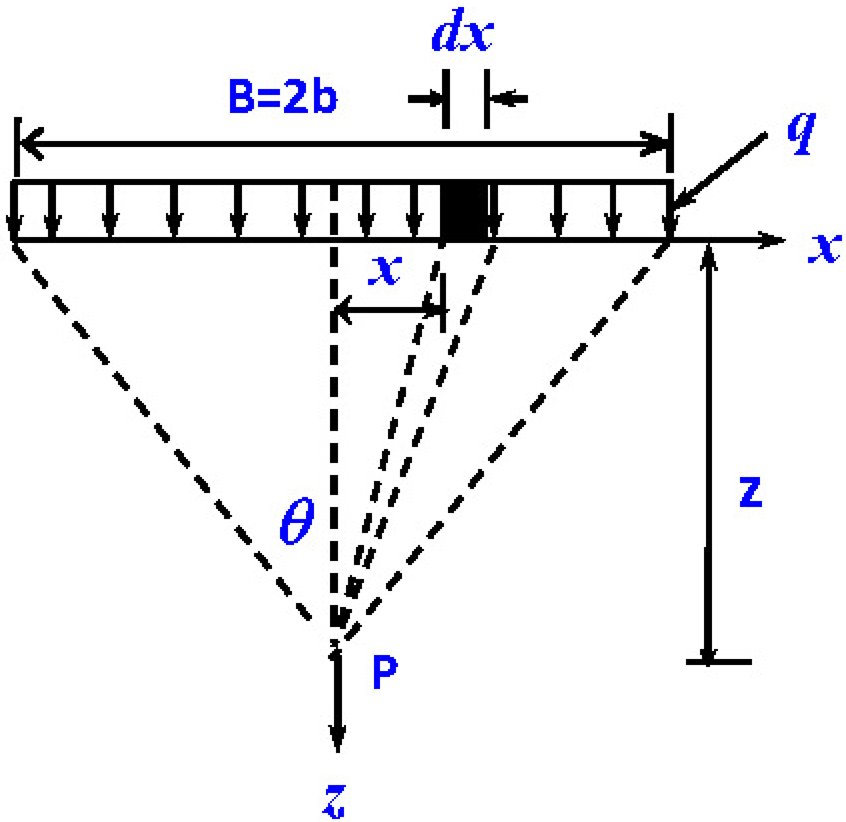
$$\sigma_z = \frac{4q}{\pi}\int_0^{\theta}\frac{\sec^2 u}{\sec^4 u}du$$

$$= \frac{4q}{\pi}\int_0^{\theta}\frac{1}{\sec^2 u}du$$

$$= \frac{4q}{\pi}\int_0^{\theta}\cos^2 u\,du$$

$$= \frac{4q}{\pi}\int_0^{\theta}\frac{1}{2}\left(1+\cos 2u\right)du$$

$$= \frac{2q}{\pi}\left[u + \sin 2u/2\right]_0^{\theta}$$



$$= \frac{2q}{\pi}\left[\theta + \sin 2\theta/2\right] \quad = \frac{2q}{\pi}\times\frac{\left[2\theta + \sin 2\theta\right]}{2} \quad = \frac{q}{\pi}\times\left[2\theta + \sin 2\theta\right]$$

Stress Distribution

5. Vertical Stress at a Corner of a Uniformly Loaded Rectangular Area

q=intensity of load

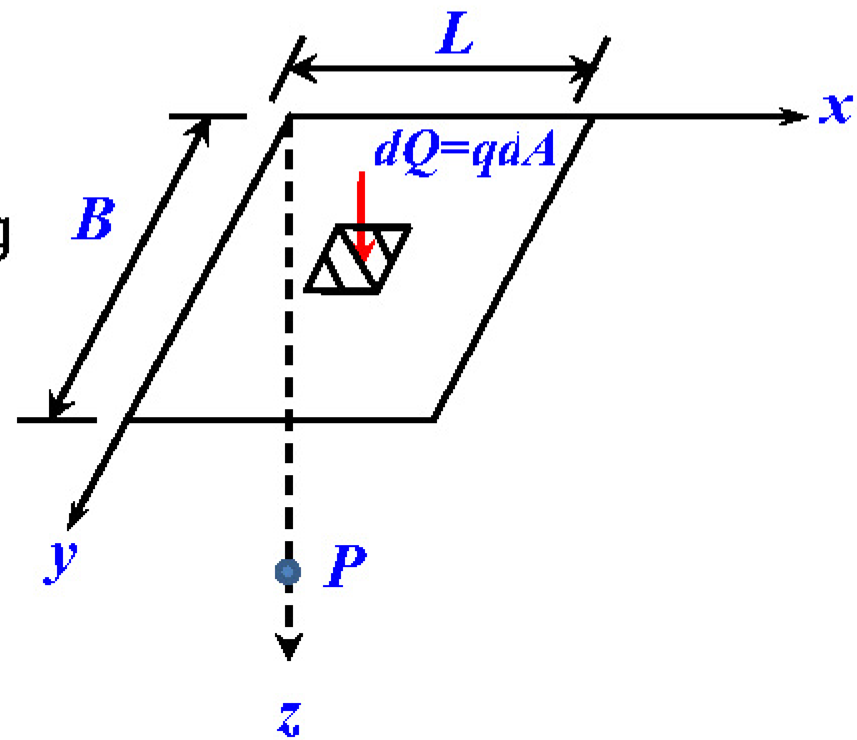
The stress at depth z is given by taking

$$dQ = qdA = qdxdy$$

$$\Delta\sigma_z = \frac{3(qdxdy)z^3}{2\pi} \times \frac{1}{(x^2 + y^2 + z^2)^{5/2}}$$

Integrating

$$\sigma_z = \frac{3qz^3}{2\pi} \times \int_0^L \int_0^B \frac{dxdy}{(x^2 + y^2 + z^2)^{5/2}}$$



Stress Distribution

Newmark's solution of the above integration is as follows:

$$\sigma_z = \frac{q}{2\pi}\left[\frac{mn}{\sqrt{m^2+n^2+1}}\times\frac{m^2+n^2+2}{m^2+n^2+m^2n^2+1}+\sin^{-1}\left(\frac{mn}{m^2+n^2+m^2n^2+1}\right)\right]$$

$$= \frac{q}{2\pi}\left[\frac{mn}{\sqrt{f}}\times\frac{f+1}{f+m^2n^2}+\sin^{-1}\left(\frac{mn}{f+m^2n^2}\right)\right] = K_N q$$

where, $f = m^2+n^2+1$ and $m = B/z$ $n = L/z$

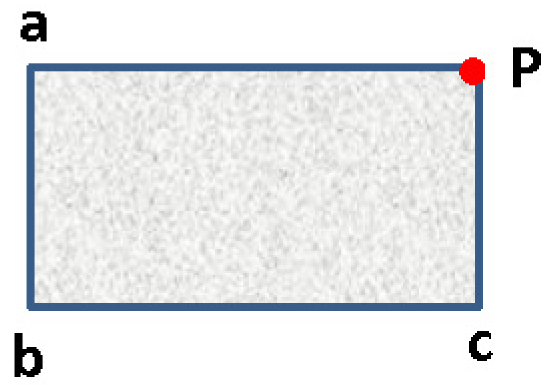
$$K_N = \frac{1}{2\pi}\left[\frac{mn}{\sqrt{f}}\times\frac{f+1}{f+m^2n^2}+\sin^{-1}\left(\frac{mn}{f+m^2n^2}\right)\right]$$

Alternatively, $\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\sqrt{f}}{f+m^2n^2}\times\frac{f+1}{f}+\tan^{-1}\left(\frac{2mn\sqrt{f}}{f-m^2n^2}\right)\right]$

Stress Distribution

Vertical stress at any Point Under a Rectangular Area

Case-01:

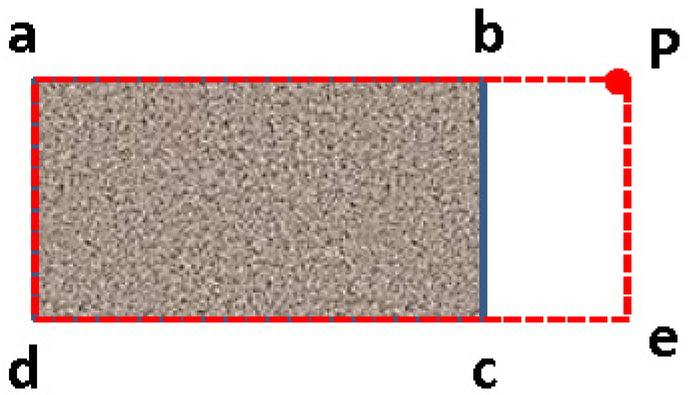


Vertical stress at depth z is:

$$\sigma_z = K_N q$$

K_N = Newmark's influence factor for area abcp

Case-02:



Vertical stress at depth z is:

$$\sigma_z = q\left(K_{N1} - K_{N2}\right)$$

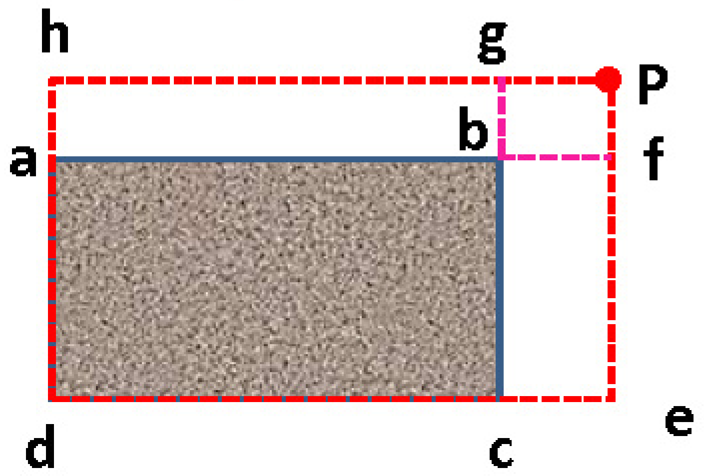
K_{N1} = Newmark's influence factor for area adep

K_{N2} = Newmark's influence factor for area bcep

Stress Distribution

Vertical stress at any Point Under a Rectangular Area

Case-03:



Vertical stress at depth z is:

$$\sigma_z = q\left(K_{N1} - K_{N2} - K_{N3} + K_{N4}\right)$$

K_{N1} = Newmark's influence factor for area hdep

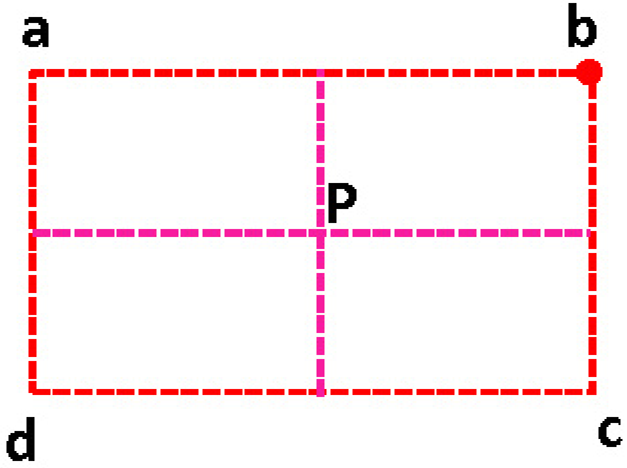
K_{N3} = Newmark's influence factor for area gcep

K_{N2} = Newmark's influence factor for area hafp

K_{N4} = Newmark's influence factor for area gbfp

Stress Distribution

Equivalent Point Load Method



The entire area is divided into a number of small area units.
The distributed load over the unit area is replaced by a point load of the same magnitude acting at the centroid of the area unit

$$\sigma_z = \frac{1}{z^2}\left(Q_1 K_{B1} + Q_2 K_{B2} + \ldots + Q_n K_{Bn}\right) \qquad K_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

Stress Distribution

Newmark's Influence Charts

Sometimes, σ_z under q for other shapes is required.

In such case, Newmark's influence chart is extremely useful.

Drawing Newmark's Influence Charts

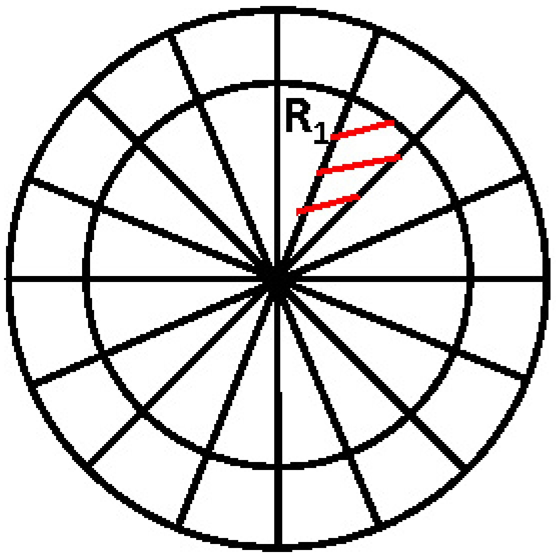
Let us consider a concentric circle of radius= R_1.

Let the circle be divided into 20 equal sector.

Load on each sector (red hatched area) =q/20

Vertical stress at the center is:

$$\sigma_z = \frac{q}{20}\left[1-\left\{\frac{1}{1+(R_1/z)^2}\right\}^{3/2}\right] \qquad (i)$$

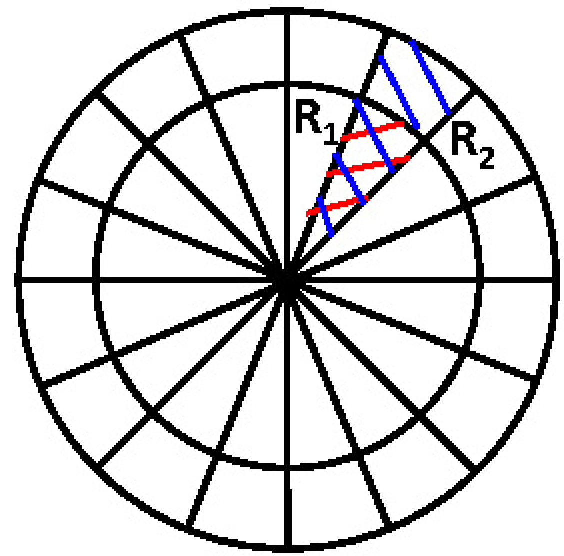


Stress Distribution

$Let \quad \sigma_z = 0.005q$

Eq. (i) becomes

$$0.005q = \frac{q}{20}\left[1 - \left\{\frac{1}{1+(R_1/z)^2}\right\}^{3/2}\right] \qquad (ii)$$



Solving equation (ii), we get

$$R_1/z = 0.27 \quad \Rightarrow R_1 = 0.27z$$

Let us consider a concentric circle of radius= R_2.

Let the circle be divided into 20 equal sector.

Load on each sector (blue hatched area) =q/20

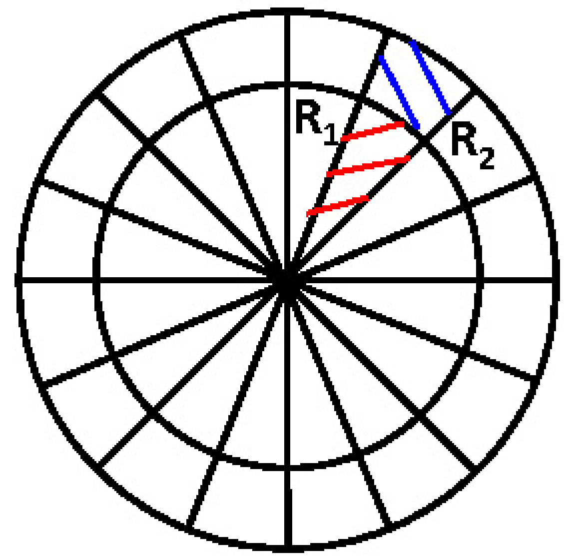
Stress Distribution

Let σ_z for green hatched area =0.005q

Thus, σ_z for total 1/20 sector=2x0.005q

Therefore,

$$2\times 0.005q = \frac{q}{20}\left[1-\left\{\frac{1}{1+\left(R_2/z\right)^2}\right\}^{3/2}\right] \qquad (iii)$$



Solving equation (iii), we get

$$R_2/z = 0.40 \quad \Rightarrow R_2 = 0.40z$$

Likewise, the radii for 3rd to 9th circle can be drawn.

However, the radii for 10th circle, we get,

$$R_{10} = \alpha$$

Stress Distribution

Use of Newmark's Influence Charts

- A plan of the loaded area is drawn on a tracing paper

- The scale is chosen such unit length in Newmark's chart =depth (z) at point P below the surface

- The traced plan of the loaded area is placed over Newmark's chart such that point P coincides with the centre of the chart.

- Count the number of small unit area, n covered by the traced plan. Fractions of unit area should also be counted. Then

$$\sigma_z = I \times n \times q$$

Here, I=Influenced coefficient (=0.005)

Stress Distribution

Contact Pressure Distribution

The upward pressure due to soil on the underside of the footing is termed contact pressure.

So far we assumed that the footing is flexible and contact pressure distribution is uniform.

However, actual footings are not flexible as assumed.

Dependence of Contact Pressure

- Elastic properties of the footing materials
- Thickness of the footing
- Relative rigidity (K_r) of footing-soil system

Stress Distribution

Relative rigidity (K_r) is given by

$$K_r = \frac{1}{6} \times \frac{(1-\nu_s^2)}{(1-\nu_f^2)} \left(\frac{E_f}{E_s} \right) \left(\frac{t}{b} \right)$$

ν_s, ν_f = Poisson's ratios for soil and footing materials

E_s, E_f = Moduli of elasticity for soil and footing material

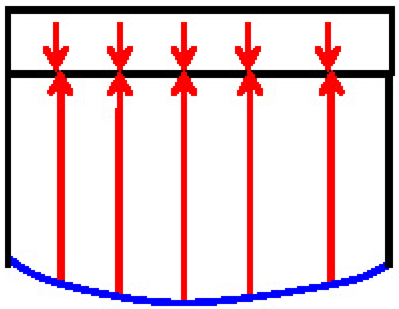
2b= Width of footing

t= Thickness of footing

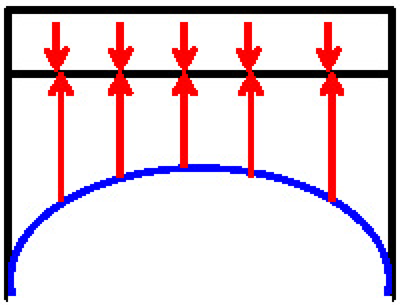
- If, Kr=0, it is a purely flexible footing
- If, Kr=α,it is a perfectly rigid footing

Stress Distribution

Contact Pressure on Saturated Clay

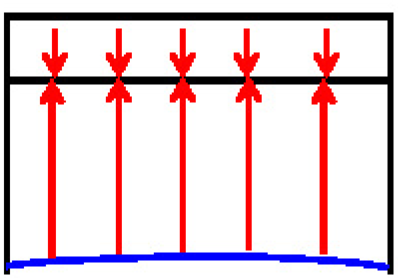


Flexible Footing

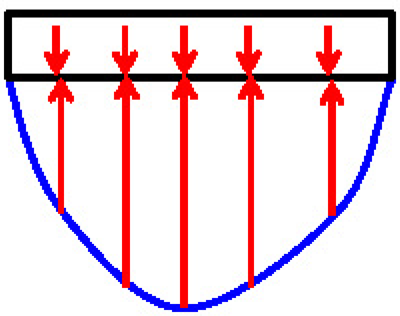


Rigid Footing

Contact Pressure on Sand



Flexible Footing

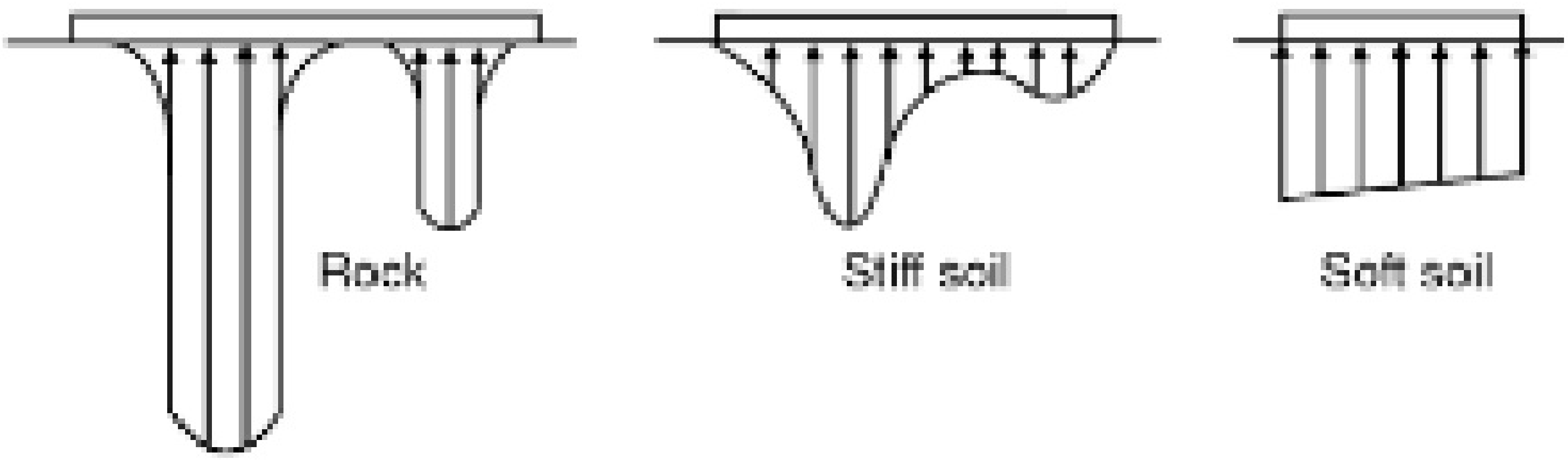


Rigid Footing

Stress Distribution

Contact Pressure on Mat Foundation

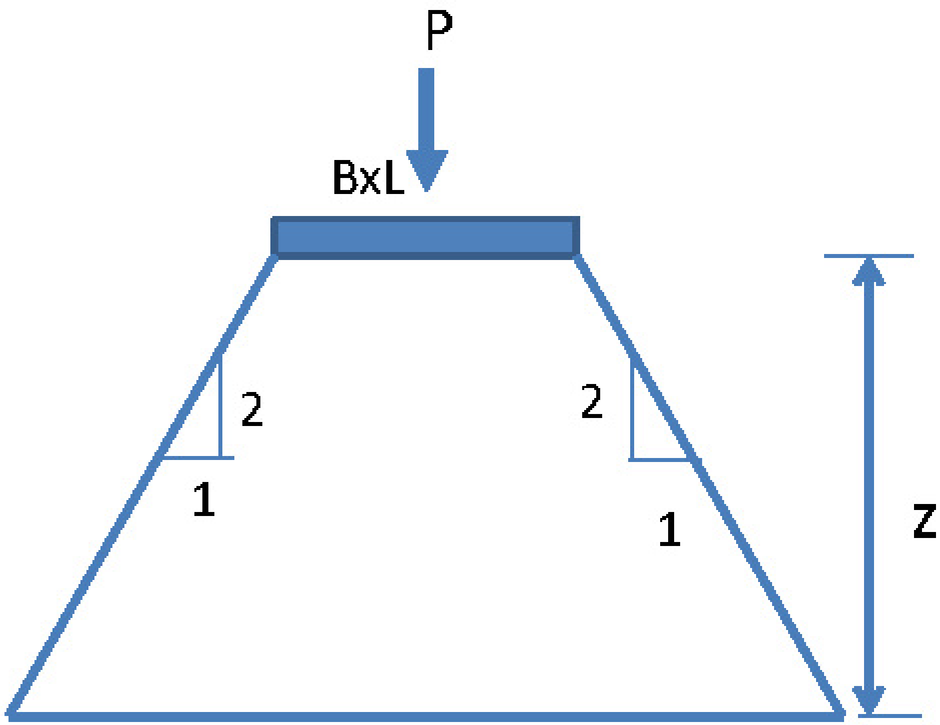
- The contact pressure distribution for raft or mat footing are quite different from those with spread footings
- Usually mat foundations have a much smaller thickness to width ratio and thus more flexible than spread footing.
- Therefore the assumption of rigidity is no longer valid.
- Also the assumptions of linear contact pressure distribution is erroneous.



Stress Distribution

Concept of Linear Dispersion

Two-to-one Load Distribution Method



Stress Distribution

Concept of Linear Dispersion

Two-to-one Load Distribution Method

- The vertical stress at depth z for a footing of size B x L is given by-

$$\sigma_z = \frac{q(B \times L)}{(B+z) \times (L+z)}$$

- For square Area,

$$\sigma_z = \frac{qB^2}{(B+z)^2}$$

- For strip Area (L>>B),

$$\sigma_z = \frac{qB}{(B+z)}$$

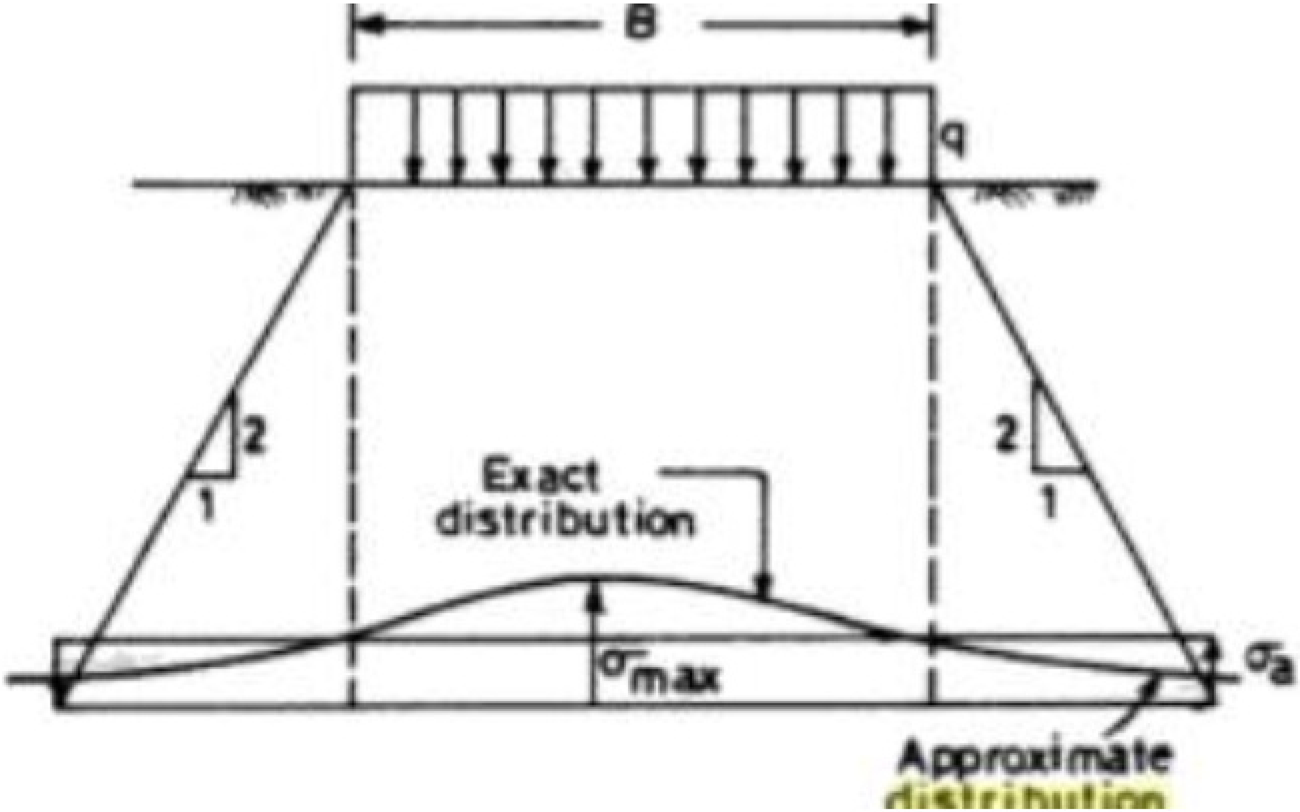
- For Circular Area, (D=diameter)

$$\sigma_z = \frac{qD^2}{(D+z)^2}$$

Stress Distribution

Concept of Linear Dispersion

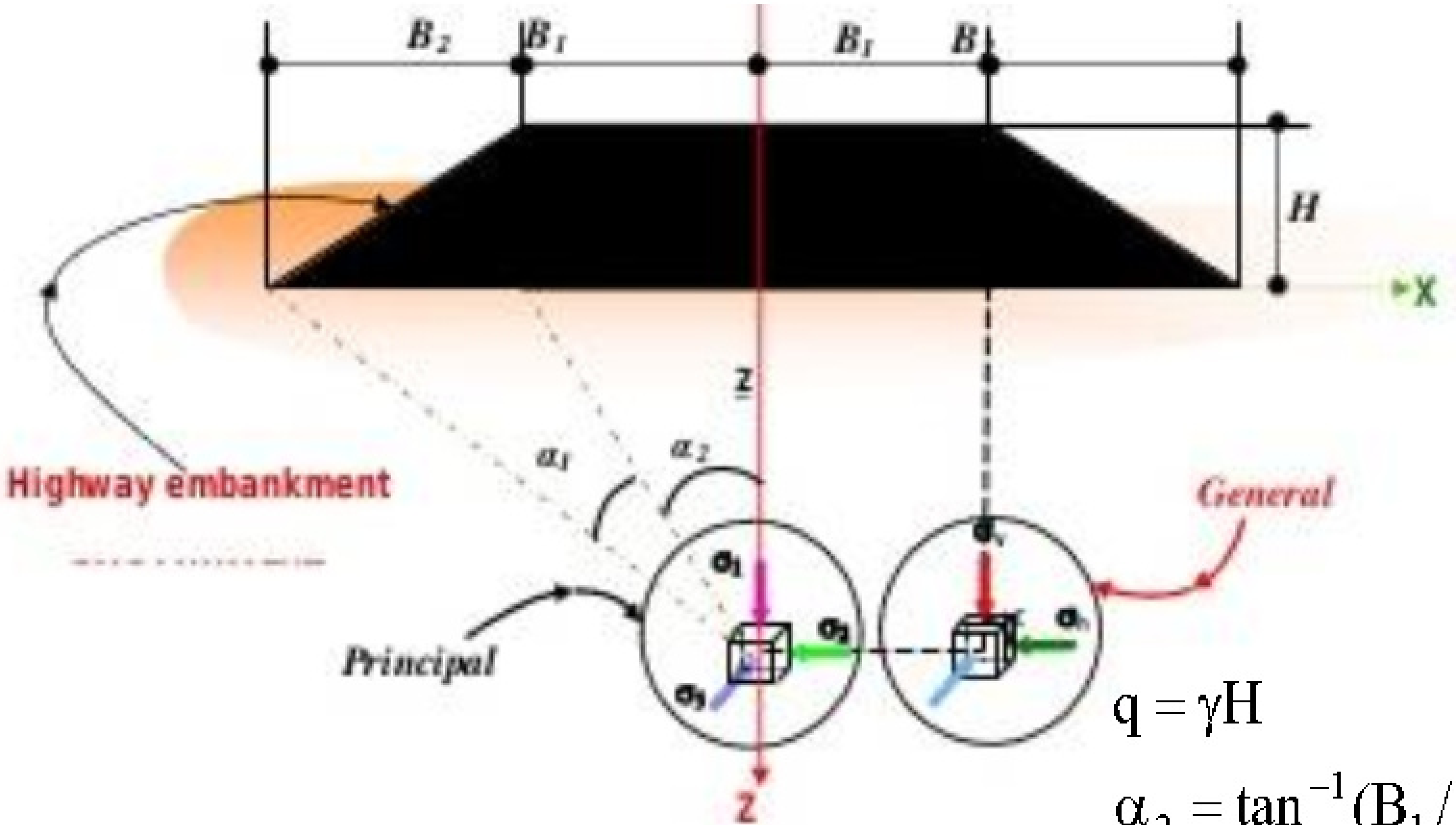
Two-to-one Load Distribution Method



This method gives fairly accurate values of the average vertical stress if the depth z is less than 2.5 times the width of loaded area

Stress Distribution

Vertical stress Due to Embankment Loading



$$q = \gamma H$$

$$\alpha_2 = \tan^{-1}(B_1 / z)$$

$$\alpha_1 = \tan^{-1}[(B_1 + B_2)/z] - \alpha_2$$

$$\sigma_z = \frac{q}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}\alpha_2\right]$$

α_1 and β_1 are in radian

Stress Distribution

Westergaard's Solution

- The vertical stress at depth z below concentrated load Q is given by-

$$\sigma_z = \frac{c/2\pi}{\left[c^2 + (r/cz)^2\right]^{3/2}} \times \frac{Q}{z^2}$$

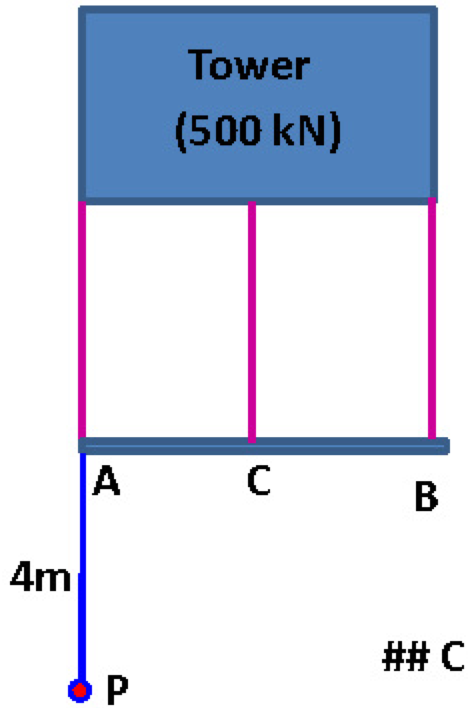
where c depends on Poisson ratio (ν) and given by

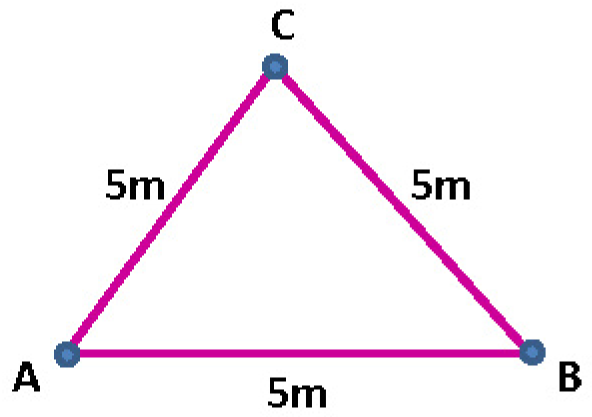
$$c = \sqrt{\frac{1-2\nu}{2-2\nu}}$$

For elastic materials, the value of n varies from 0.0 to 0.50.

Stress Distribution

Problem-01:

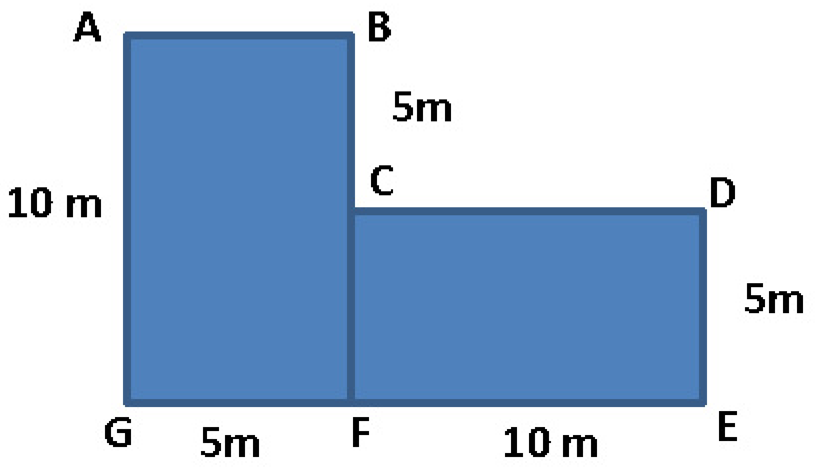




Compute the increase in the vertical stress at point P.

Stress Distribution

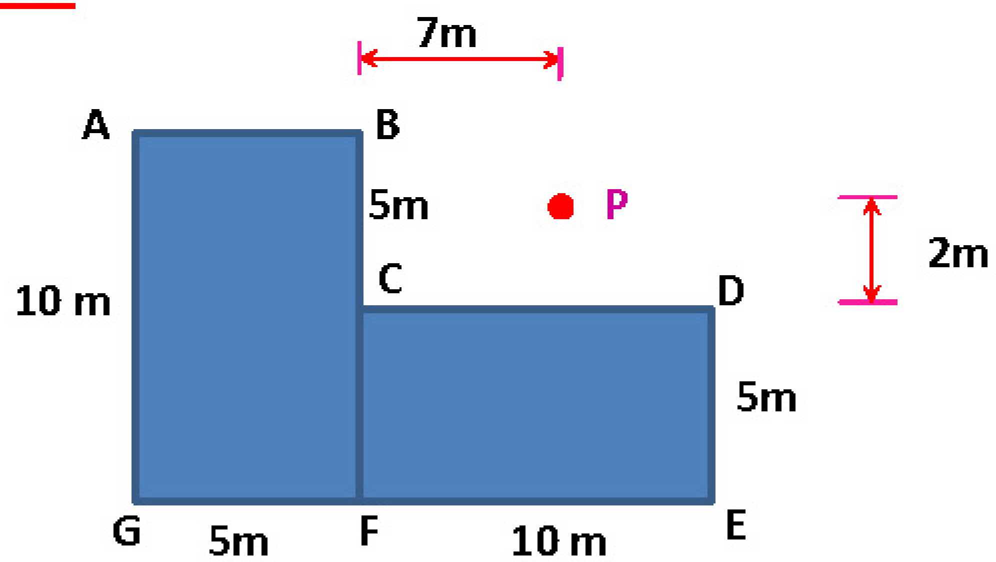
Problem-02:



A L-shaped building in plan exerts a pressure of 75 kN/m² on the soil. Determine the vertical stress increment at a depth of 5 m below the point C.

Stress Distribution

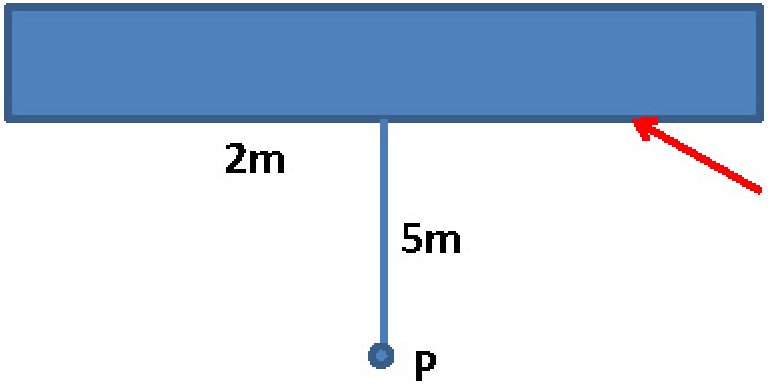
Problem-03:



A L-shaped building in plan exerts a pressure of 80 kN/m^2 on the soil. Determine the vertical stress increment at a depth of 5 m below the point P. Determine also the vertical stress increment at a depth of 5 m below the point P using the equivalent point load method.

Stress Distribution

Problem-04:

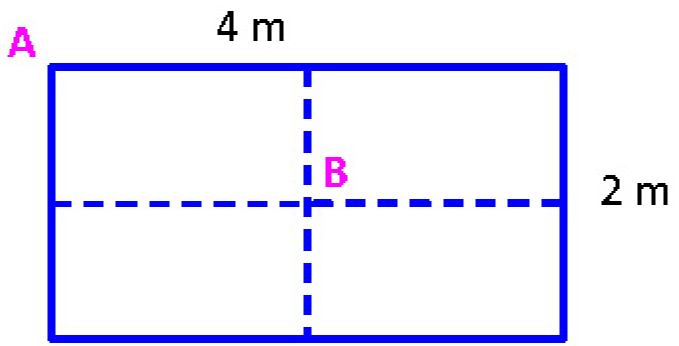


A strip footing is shown in figure above. Determine the vertical stress increment at point C.

Stress Distribution

Problem-05:

A rectangular area 4m x 2m caries a uniform load of 70 kN/m2 at the ground surface. Find the vertical stress at 5 m below the centre (B) and corner of the loaded area (A) using Newmark's Influence chart.



Stress Distribution

Problem-06:

A railway embankment is shown in Figure 8.16. Assume the unit weight of the soil to be 20 kN/m^3. Compute the increase in vertical stress under the centre line at depths of 2.5 m and 5 m.