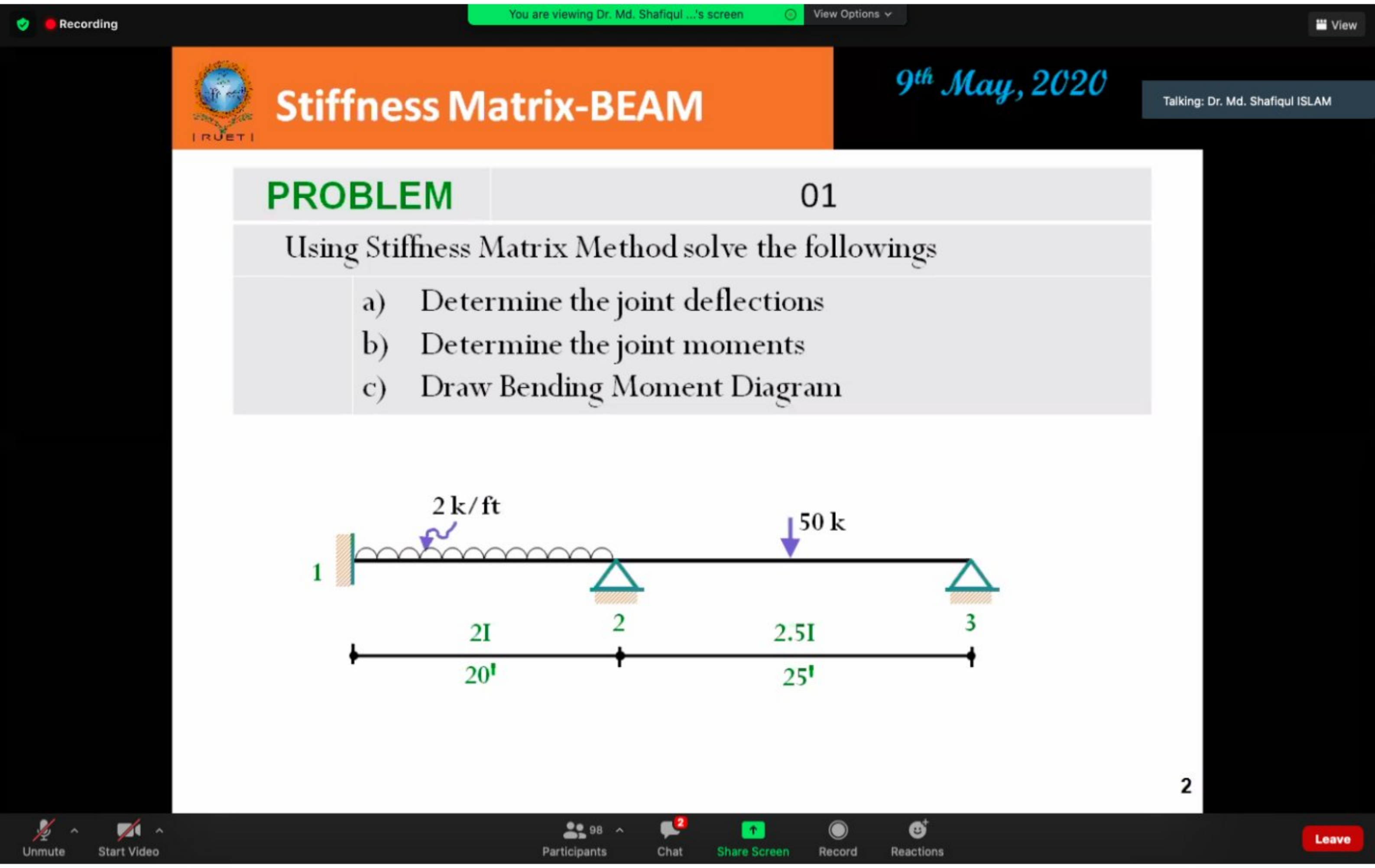


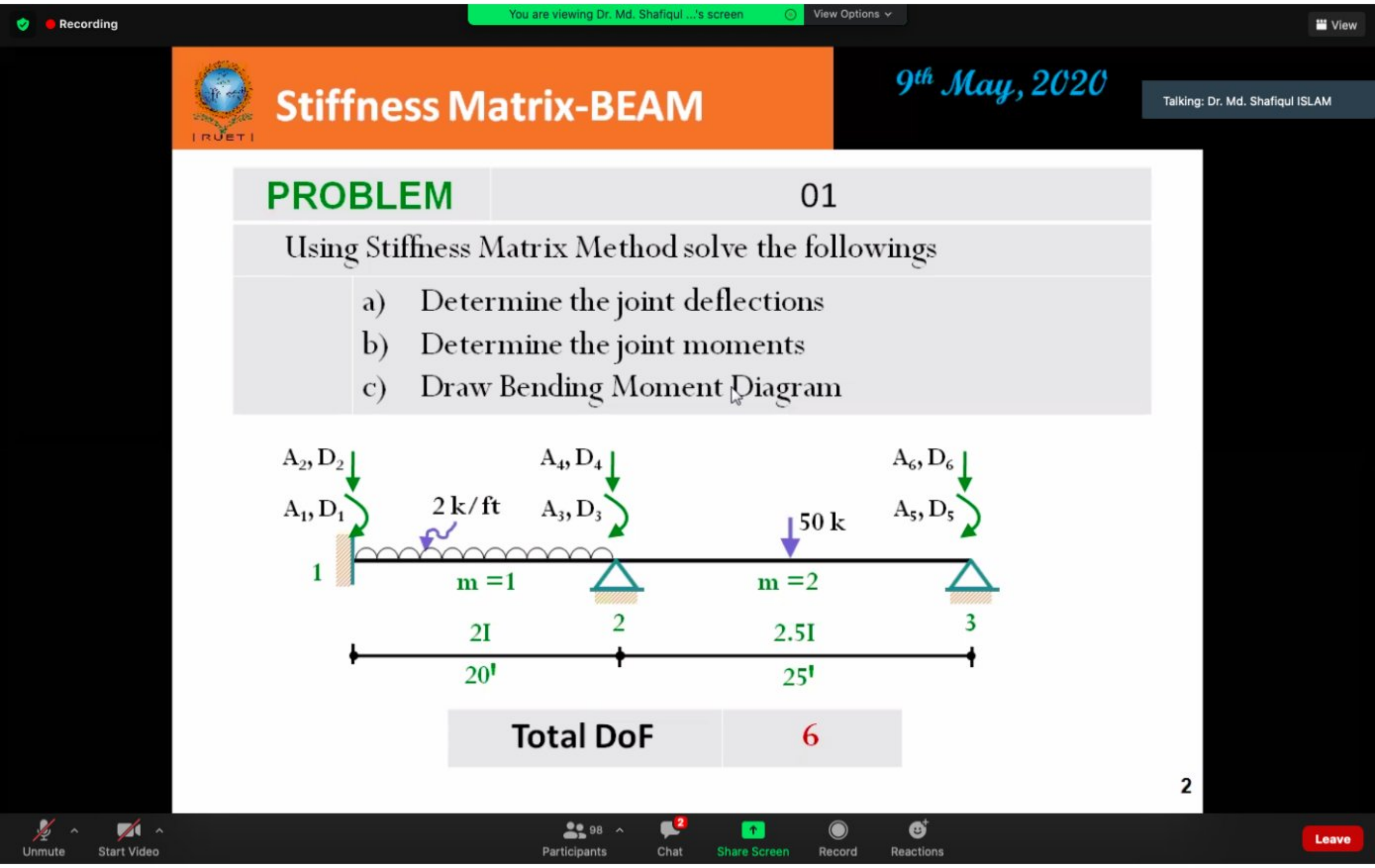
Stiffness Matrix-BEAM

9th May, 2020

PROBLEM 01

Using Stiffness Matrix Method solve the followings

a) Determine the joint deflections
b) Determine the joint moments
c) Draw Bending Moment Diagram



Stiffness Matrix-BEAM

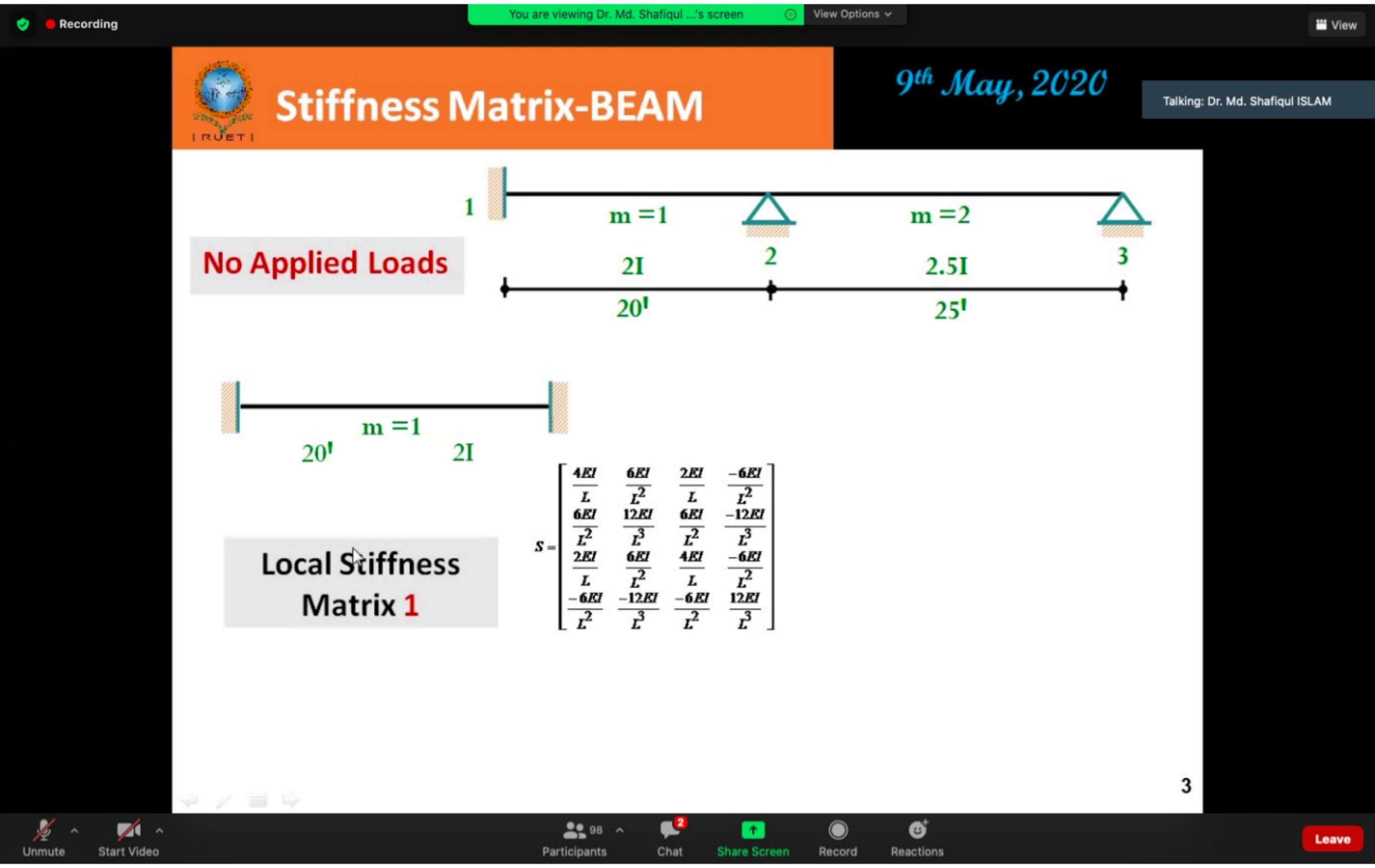
9th May, 2020

PROBLEM	01

Using Stiffness Matrix Method solve the followings

a) Determine the joint deflections
b) Determine the joint moments
c) Draw Bending Moment Diagram

Total DoF	6



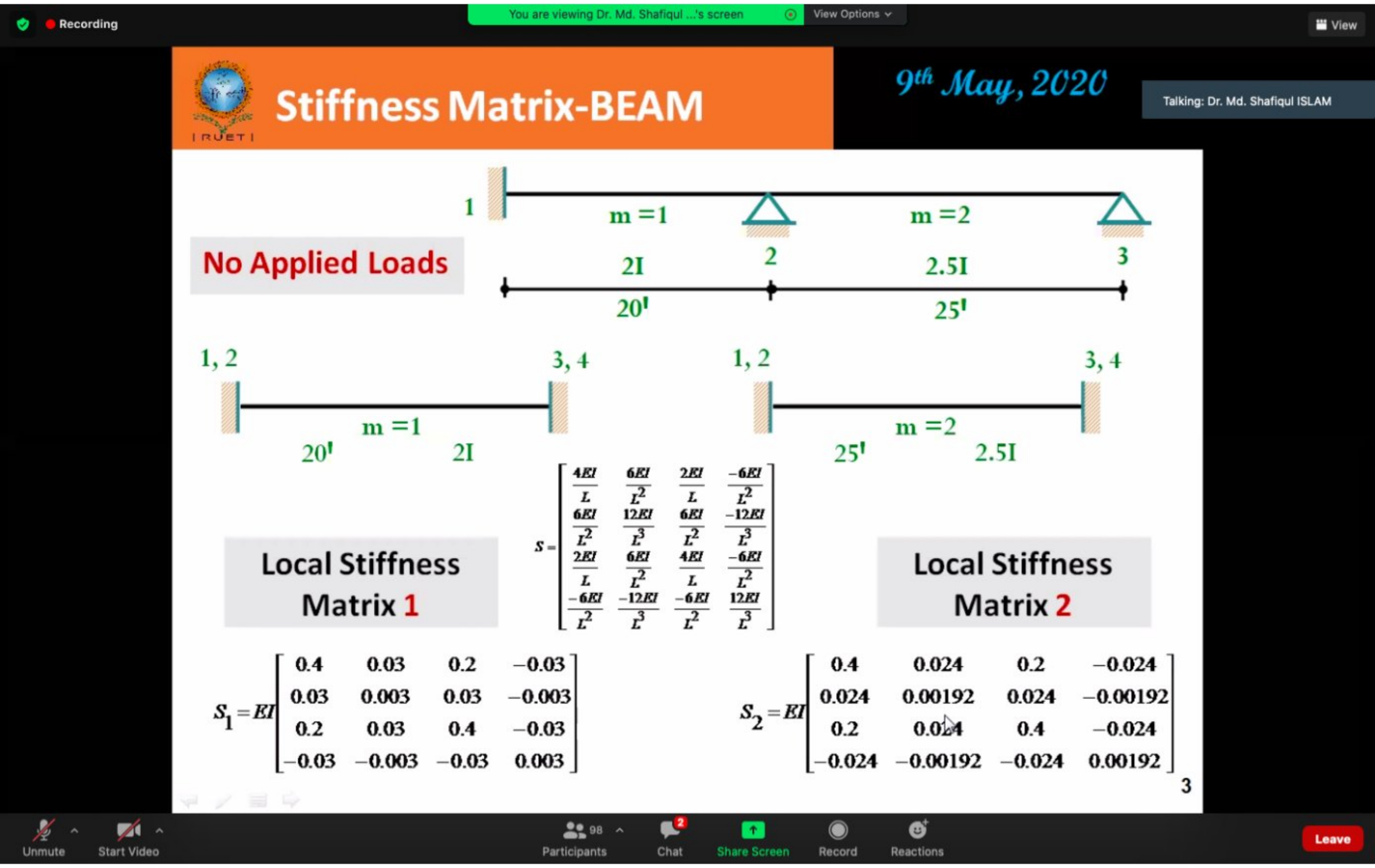
Stiffness Matrix-BEAM

9th May, 2020

No Applied Loads

Local Stiffness Matrix 1

$$S = \begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} \\ \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$$



Stiffness Matrix-BEAM

9th May, 2020

No Applied Loads

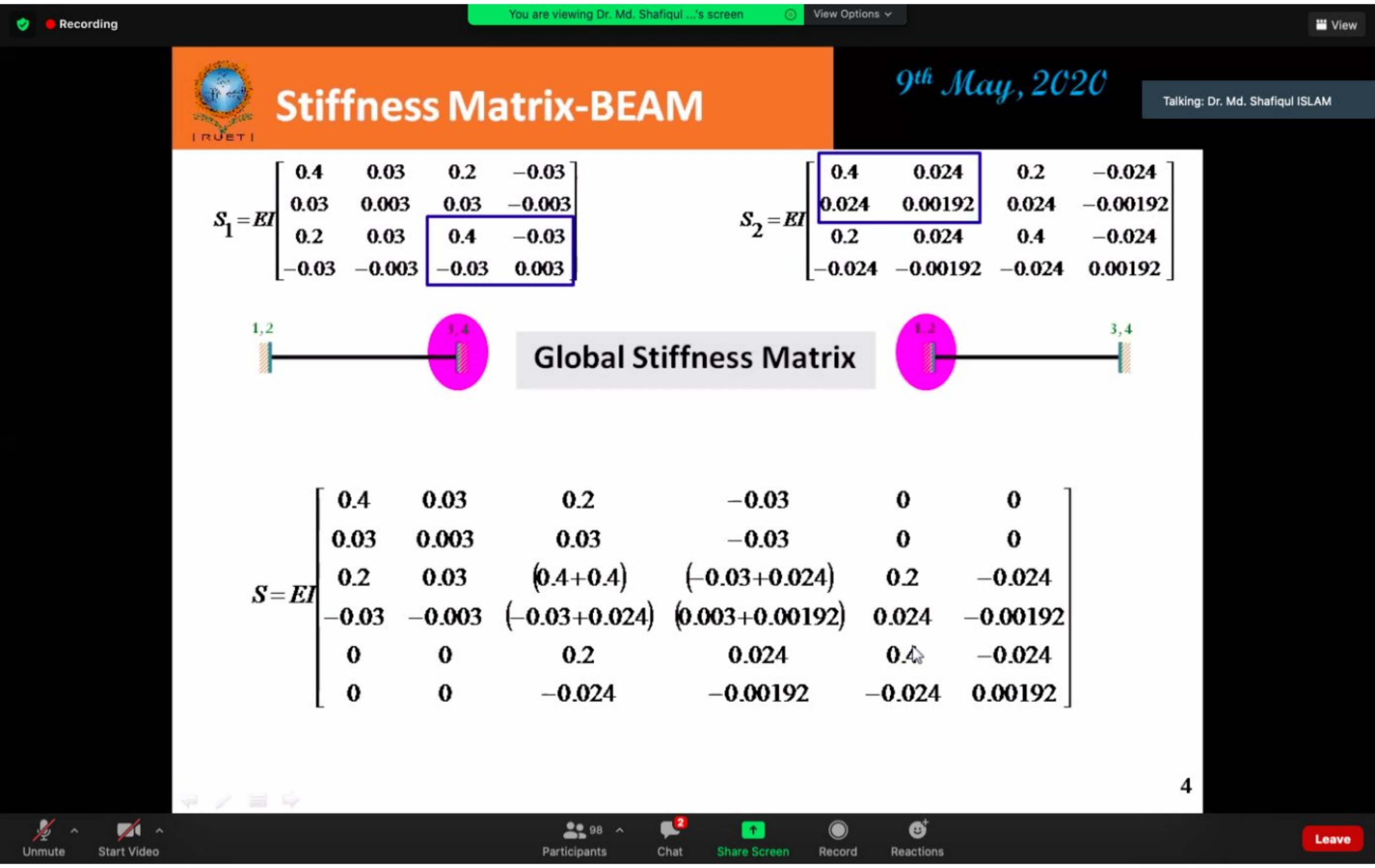
$$S = \begin{bmatrix} \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} \\ \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} \\ \frac{-6EI}{L^2} & \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}$$

Local Stiffness Matrix 1

Local Stiffness Matrix 2

$$S_1 = EI \begin{bmatrix} 0.4 & 0.03 & 0.2 & -0.03 \\ 0.03 & 0.003 & 0.03 & -0.003 \\ 0.2 & 0.03 & 0.4 & -0.03 \\ -0.03 & -0.003 & -0.03 & 0.003 \end{bmatrix}$$

$$S_2 = EI \begin{bmatrix} 0.4 & 0.024 & 0.2 & -0.024 \\ 0.024 & 0.00192 & 0.024 & -0.00192 \\ 0.2 & 0.024 & 0.4 & -0.024 \\ -0.024 & -0.00192 & -0.024 & 0.00192 \end{bmatrix}$$



Stiffness Matrix-BEAM

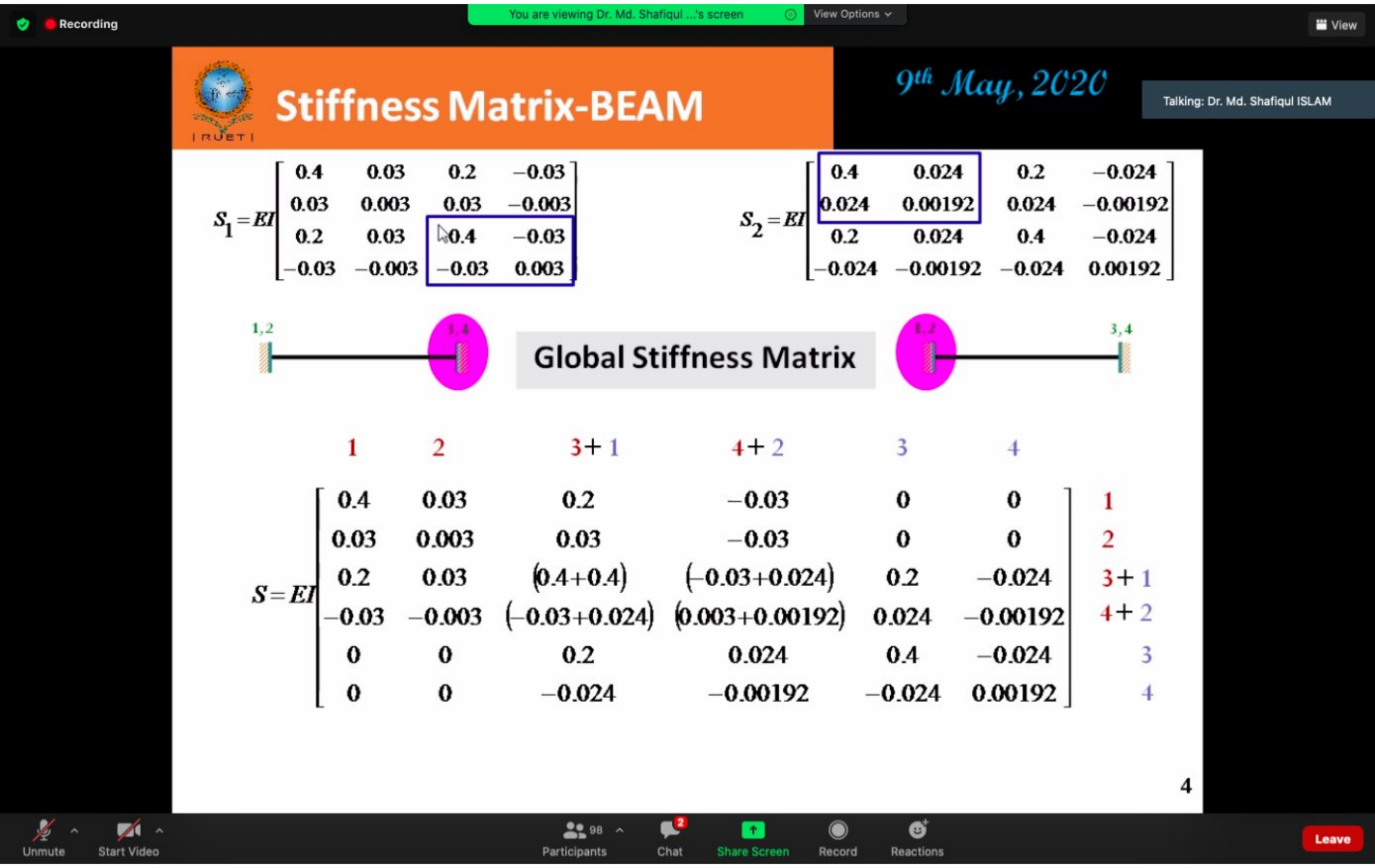
9th May, 2020

$$S_1 = EI\begin{bmatrix} 0.4 & 0.03 & 0.2 & -0.03 \\ 0.03 & 0.003 & 0.03 & -0.003 \\ 0.2 & 0.03 & 0.4 & -0.03 \\ -0.03 & -0.003 & -0.03 & 0.003 \end{bmatrix}$$

$$S_2 = EI\begin{bmatrix} 0.4 & 0.024 & 0.2 & -0.024 \\ 0.024 & 0.00192 & 0.024 & -0.00192 \\ 0.2 & 0.024 & 0.4 & -0.024 \\ -0.024 & -0.00192 & -0.024 & 0.00192 \end{bmatrix}$$

Global Stiffness Matrix

$$S = EI\begin{bmatrix} 0.4 & 0.03 & 0.2 & -0.03 & 0 & 0 \\ 0.03 & 0.003 & 0.03 & -0.03 & 0 & 0 \\ 0.2 & 0.03 & (0.4+0.4) & (-0.03+0.024) & 0.2 & -0.024 \\ -0.03 & -0.003 & (-0.03+0.024) & (0.003+0.00192) & 0.024 & -0.00192 \\ 0 & 0 & 0.2 & 0.024 & 0.4 & -0.024 \\ 0 & 0 & -0.024 & -0.00192 & -0.024 & 0.00192 \end{bmatrix}$$



Stiffness Matrix-BEAM

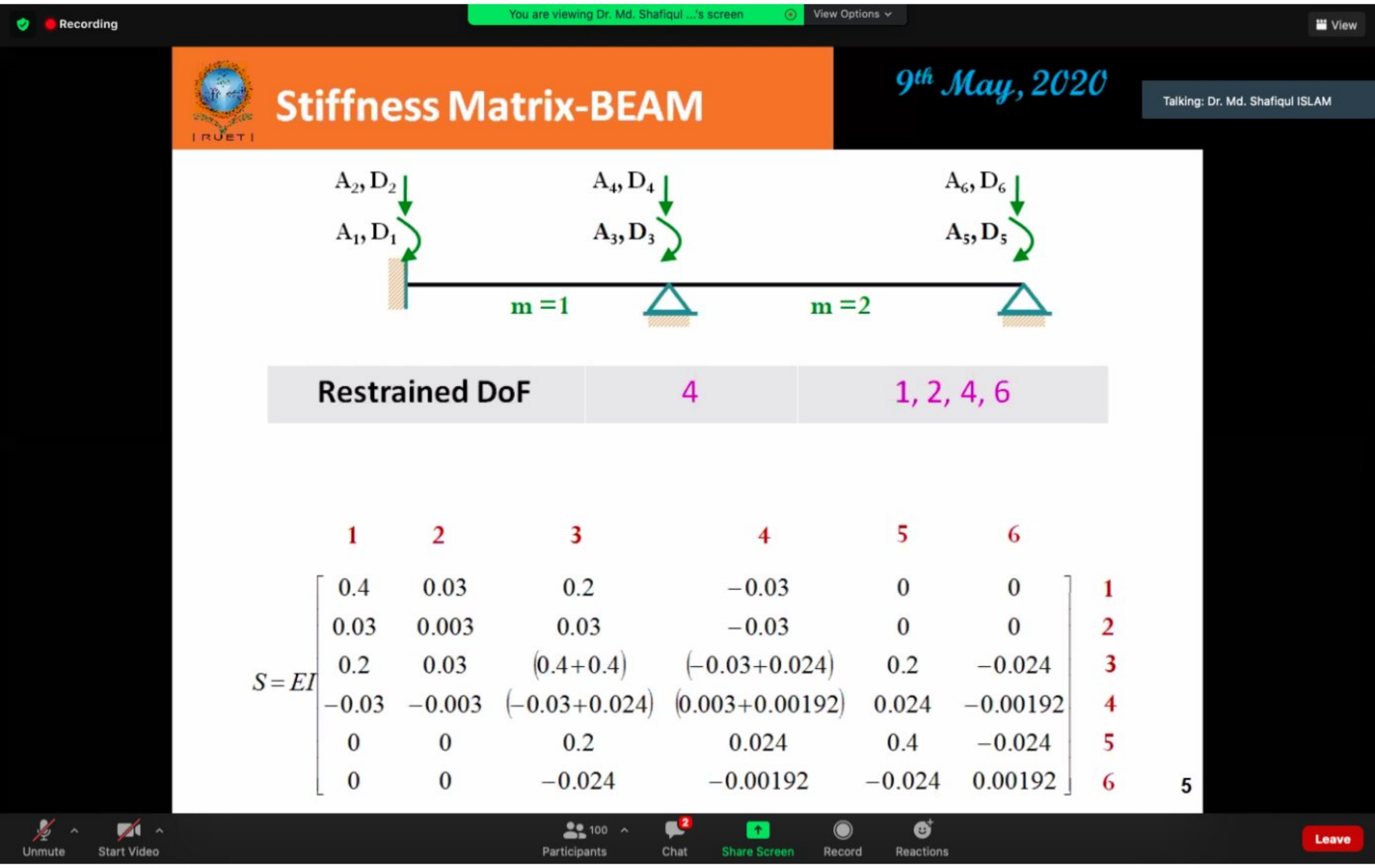
9th May, 2020

$$S_1 = EI\begin{bmatrix} 0.4 & 0.03 & 0.2 & -0.03 \\ 0.03 & 0.003 & 0.03 & -0.003 \\ 0.2 & 0.03 & \boxed{\begin{matrix} 0.4 & -0.03 \\ -0.03 & 0.003 \end{matrix}} \end{bmatrix}$$

$$S_2 = EI\begin{bmatrix} \boxed{\begin{matrix} 0.4 & 0.024 \\ 0.024 & 0.00192 \end{matrix}} & \begin{matrix} 0.2 & -0.024 \\ 0.024 & -0.00192 \end{matrix} \\ \begin{matrix} 0.2 & 0.024 \\ -0.024 & -0.00192 \end{matrix} & \begin{matrix} 0.4 & -0.024 \\ -0.024 & 0.00192 \end{matrix} \end{bmatrix}$$

Global Stiffness Matrix

$$S = EI\begin{array}{cccccc|c} 1 & 2 & 3+1 & 4+2 & 3 & 4 & \\ \hline 0.4 & 0.03 & 0.2 & -0.03 & 0 & 0 & 1 \\ 0.03 & 0.003 & 0.03 & -0.03 & 0 & 0 & 2 \\ 0.2 & 0.03 & (0.4+0.4) & (-0.03+0.024) & 0.2 & -0.024 & 3+1 \\ -0.03 & -0.003 & (-0.03+0.024) & (0.003+0.00192) & 0.024 & -0.00192 & 4+2 \\ 0 & 0 & 0.2 & 0.024 & 0.4 & -0.024 & 3 \\ 0 & 0 & -0.024 & -0.00192 & -0.024 & 0.00192 & 4 \end{array}$$

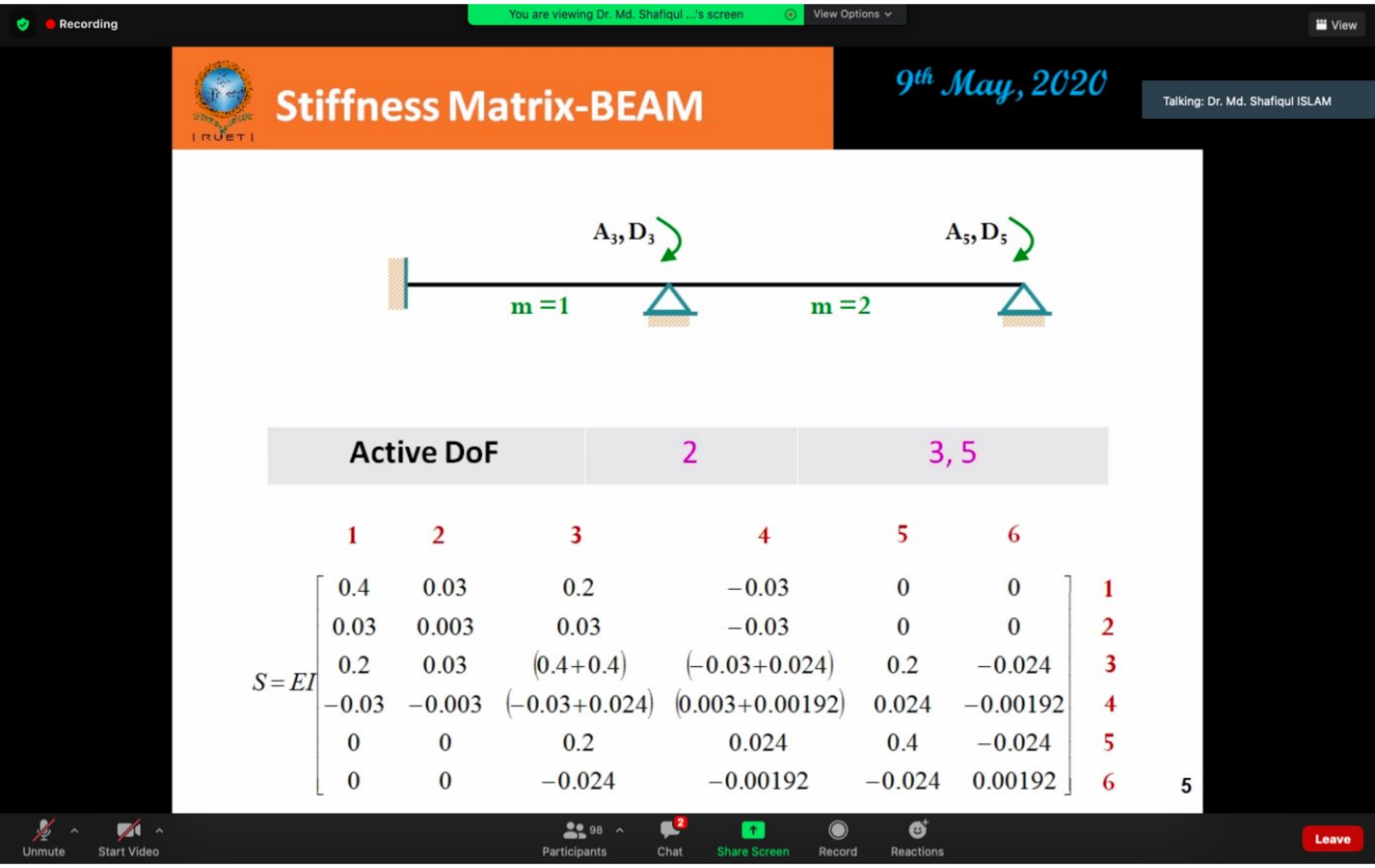


Stiffness Matrix-BEAM

9th May, 2020

Restrained DoF	4	1, 2, 4, 6

$$S = EI \begin{array}{c|cccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline & 0.4 & 0.03 & 0.2 & -0.03 & 0 & 0 & 1 \\ & 0.03 & 0.003 & 0.03 & -0.03 & 0 & 0 & 2 \\ & 0.2 & 0.03 & (0.4+0.4) & (-0.03+0.024) & 0.2 & -0.024 & 3 \\ & -0.03 & -0.003 & (-0.03+0.024) & (0.003+0.00192) & 0.024 & -0.00192 & 4 \\ & 0 & 0 & 0.2 & 0.024 & 0.4 & -0.024 & 5 \\ & 0 & 0 & -0.024 & -0.00192 & -0.024 & 0.00192 & 6 \end{array}$$

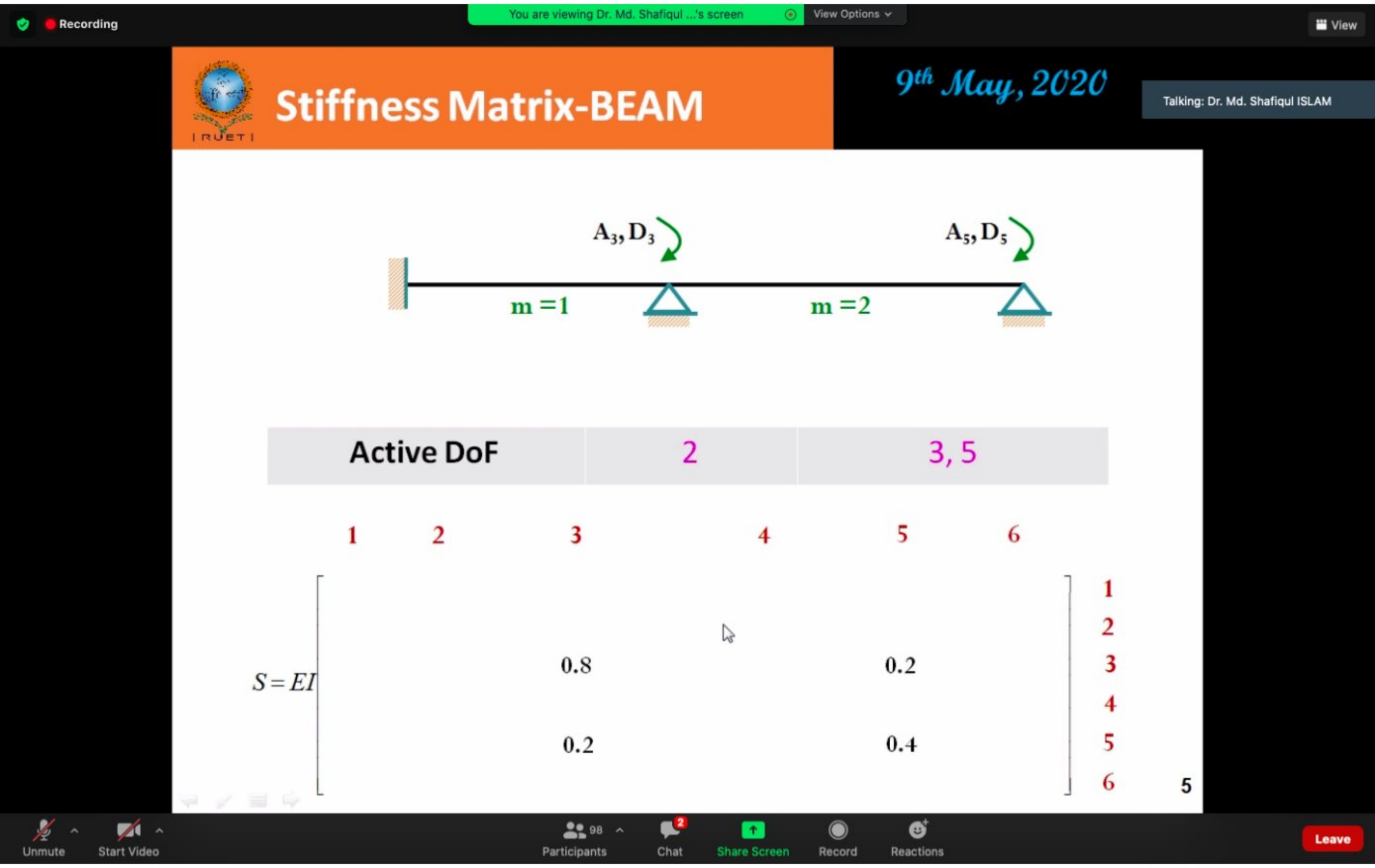


Stiffness Matrix-BEAM

9th May, 2020

Active DoF	2	3, 5

$$S = EI\begin{array}{c|cccccc|c} & 1 & 2 & 3 & 4 & 5 & 6 & \\ \hline & 0.4 & 0.03 & 0.2 & -0.03 & 0 & 0 & 1 \\ & 0.03 & 0.003 & 0.03 & -0.03 & 0 & 0 & 2 \\ & 0.2 & 0.03 & (0.4+0.4) & (-0.03+0.024) & 0.2 & -0.024 & 3 \\ & -0.03 & -0.003 & (-0.03+0.024) & (0.003+0.00192) & 0.024 & -0.00192 & 4 \\ & 0 & 0 & 0.2 & 0.024 & 0.4 & -0.024 & 5 \\ & 0 & 0 & -0.024 & -0.00192 & -0.024 & 0.00192 & 6 \end{array}$$



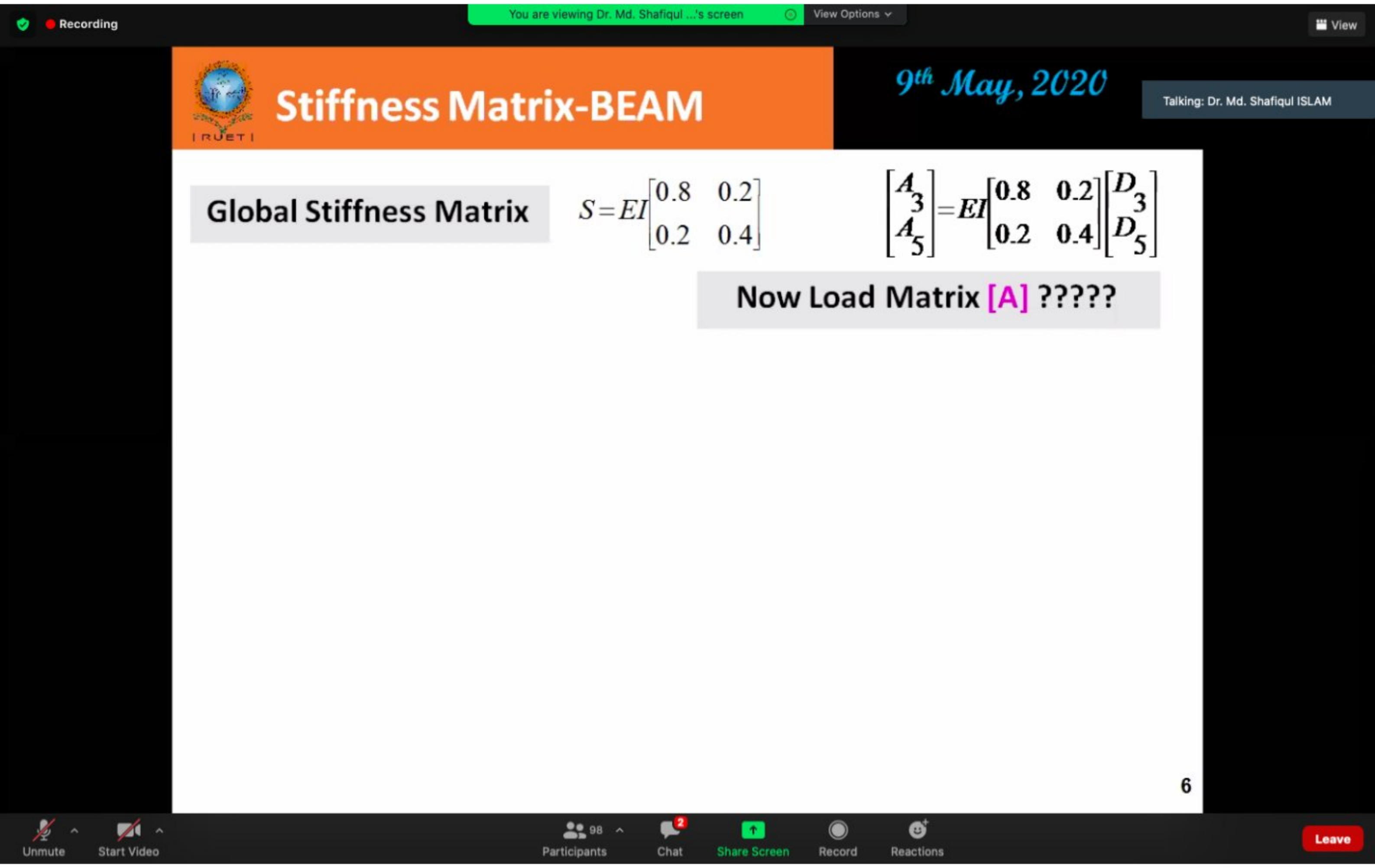
Stiffness Matrix-BEAM

9th May, 2020

Active DoF	2	3, 5

$$S = EI\left[\begin{array}{cccccc} &&&&& \\ &&&&& \\ &&0.8&&0.2& \\ &&&&& \\ &&0.2&&0.4& \\ &&&&& \end{array}\right]$$

with columns labelled 1, 2, 3, 4, 5, 6 and rows labelled 1, 2, 3, 4, 5, 6



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9th May, 2020

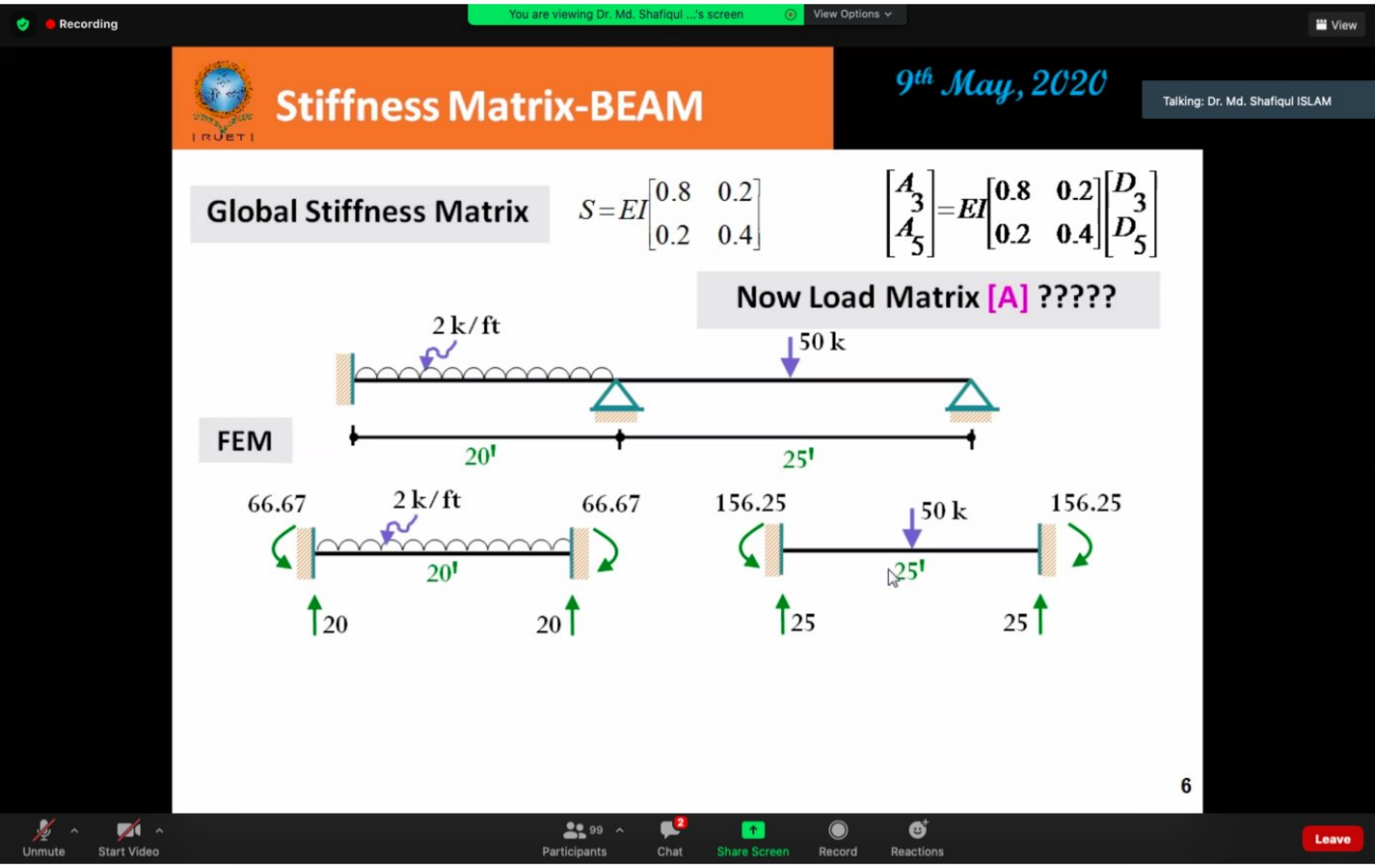
Stiffness Matrix-BEAM

Global Stiffness Matrix

$$S = EI\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} A_3 \\ A_5 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} D_3 \\ D_5 \end{bmatrix}$$

Now Load Matrix [A] ?????



Stiffness Matrix-BEAM

9th May, 2020

Global Stiffness Matrix

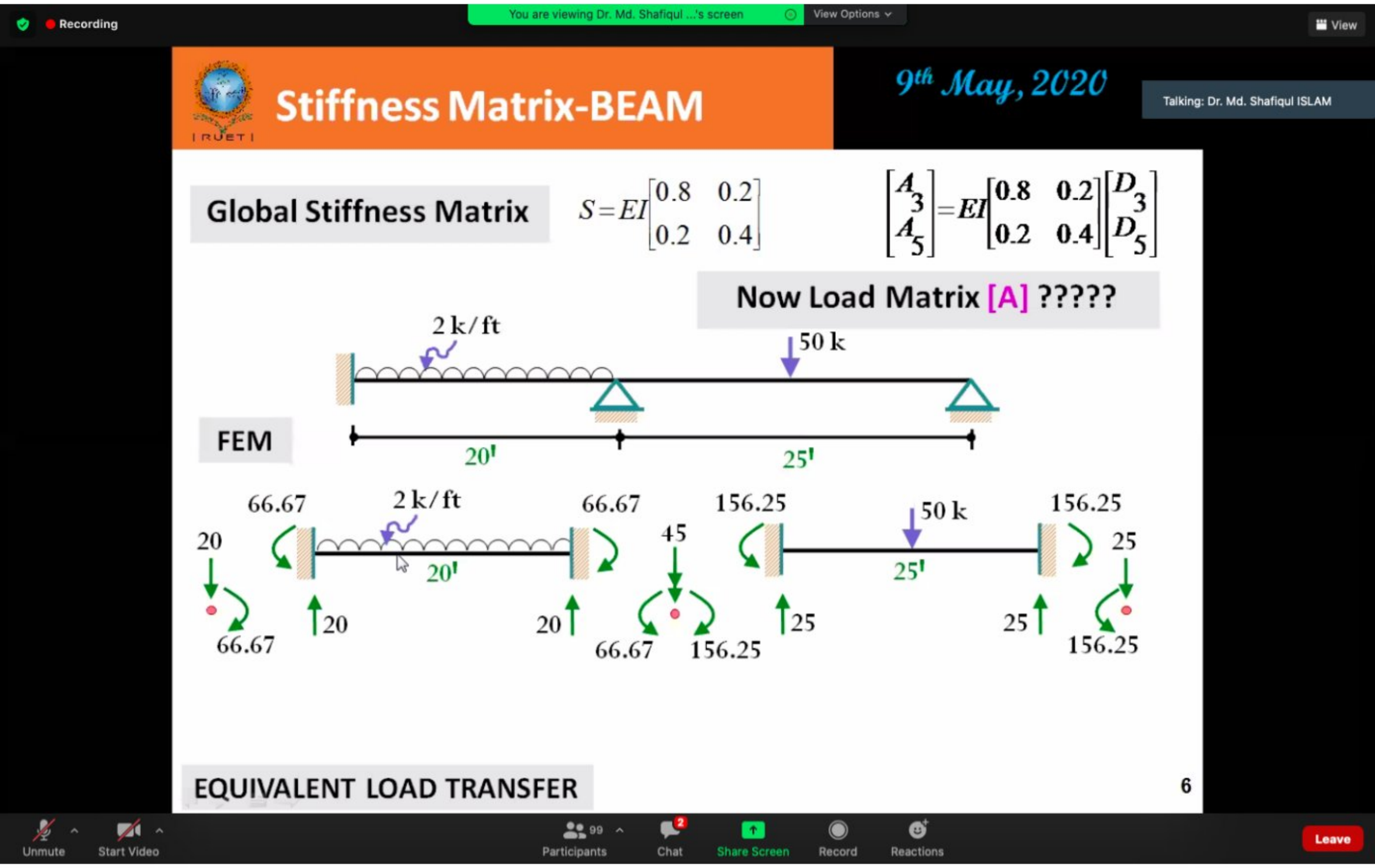
$$S = EI\begin{bmatrix}0.8 & 0.2\\ 0.2 & 0.4\end{bmatrix}$$

$$\begin{bmatrix}A_3\\ A_5\end{bmatrix} = EI\begin{bmatrix}0.8 & 0.2\\ 0.2 & 0.4\end{bmatrix}\begin{bmatrix}D_3\\ D_5\end{bmatrix}$$

Now Load Matrix [A] ?????

FEM

6



Stiffness Matrix-BEAM

9th May, 2020

Global Stiffness Matrix

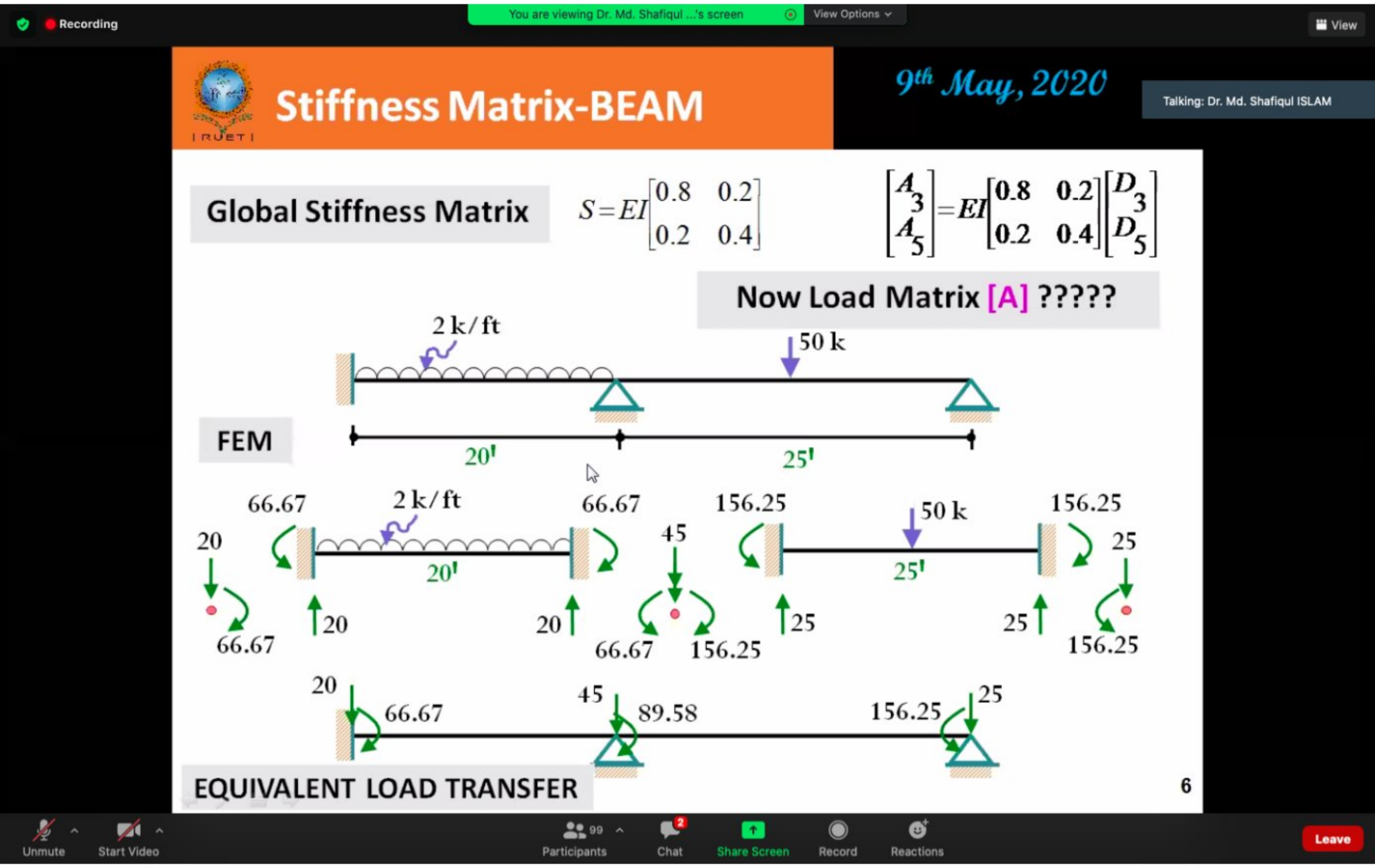
$$S = EI\begin{bmatrix}0.8 & 0.2\\ 0.2 & 0.4\end{bmatrix}$$

$$\begin{bmatrix}A_3\\ A_5\end{bmatrix} = EI\begin{bmatrix}0.8 & 0.2\\ 0.2 & 0.4\end{bmatrix}\begin{bmatrix}D_3\\ D_5\end{bmatrix}$$

Now Load Matrix [A] ?????

FEM

EQUIVALENT LOAD TRANSFER



Stiffness Matrix-BEAM

9th May, 2020

Global Stiffness Matrix

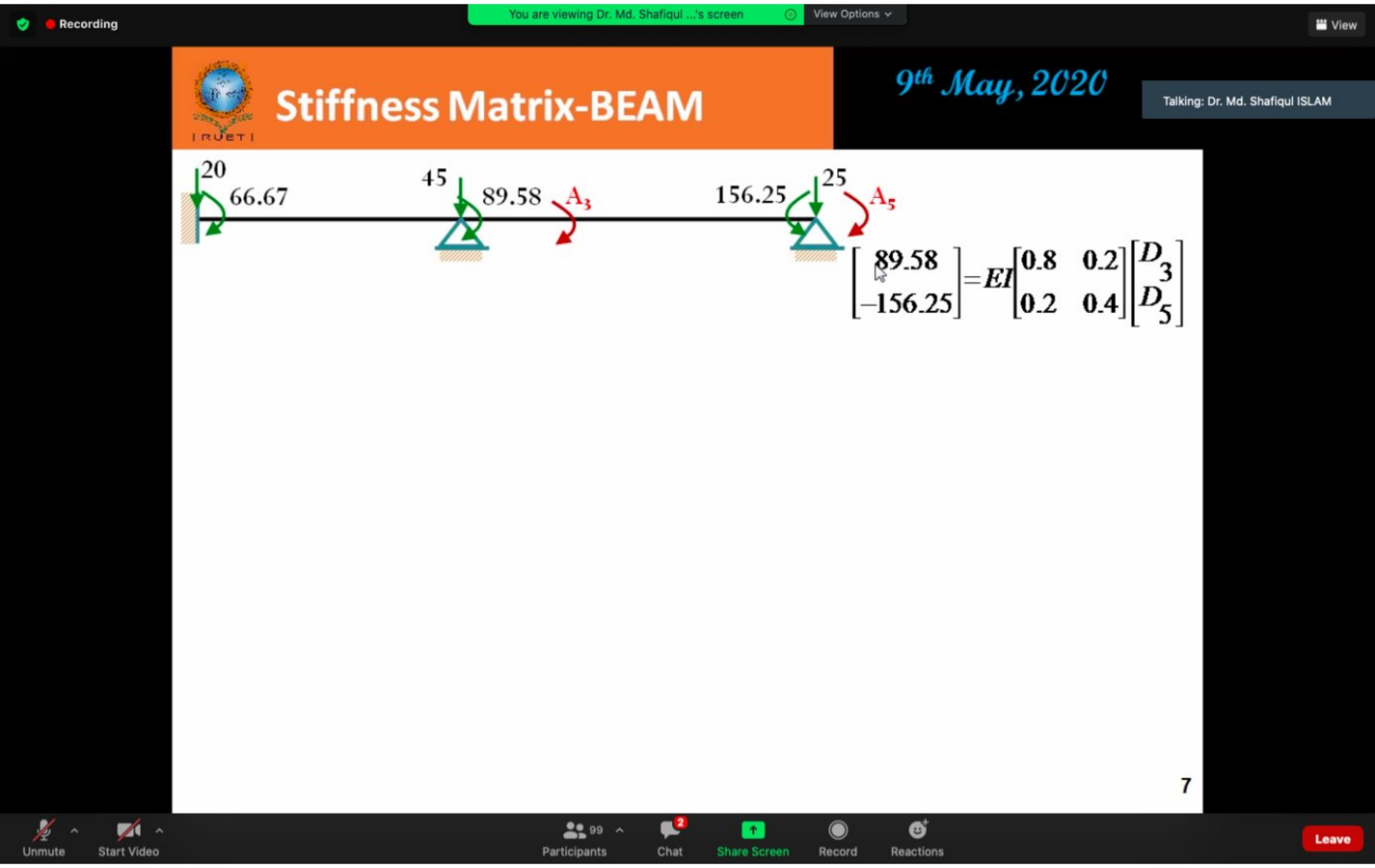
$$S = EI\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} A_3 \\ A_5 \end{bmatrix} = EI\begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}\begin{bmatrix} D_3 \\ D_5 \end{bmatrix}$$

Now Load Matrix [A] ?????

FEM

EQUIVALENT LOAD TRANSFER



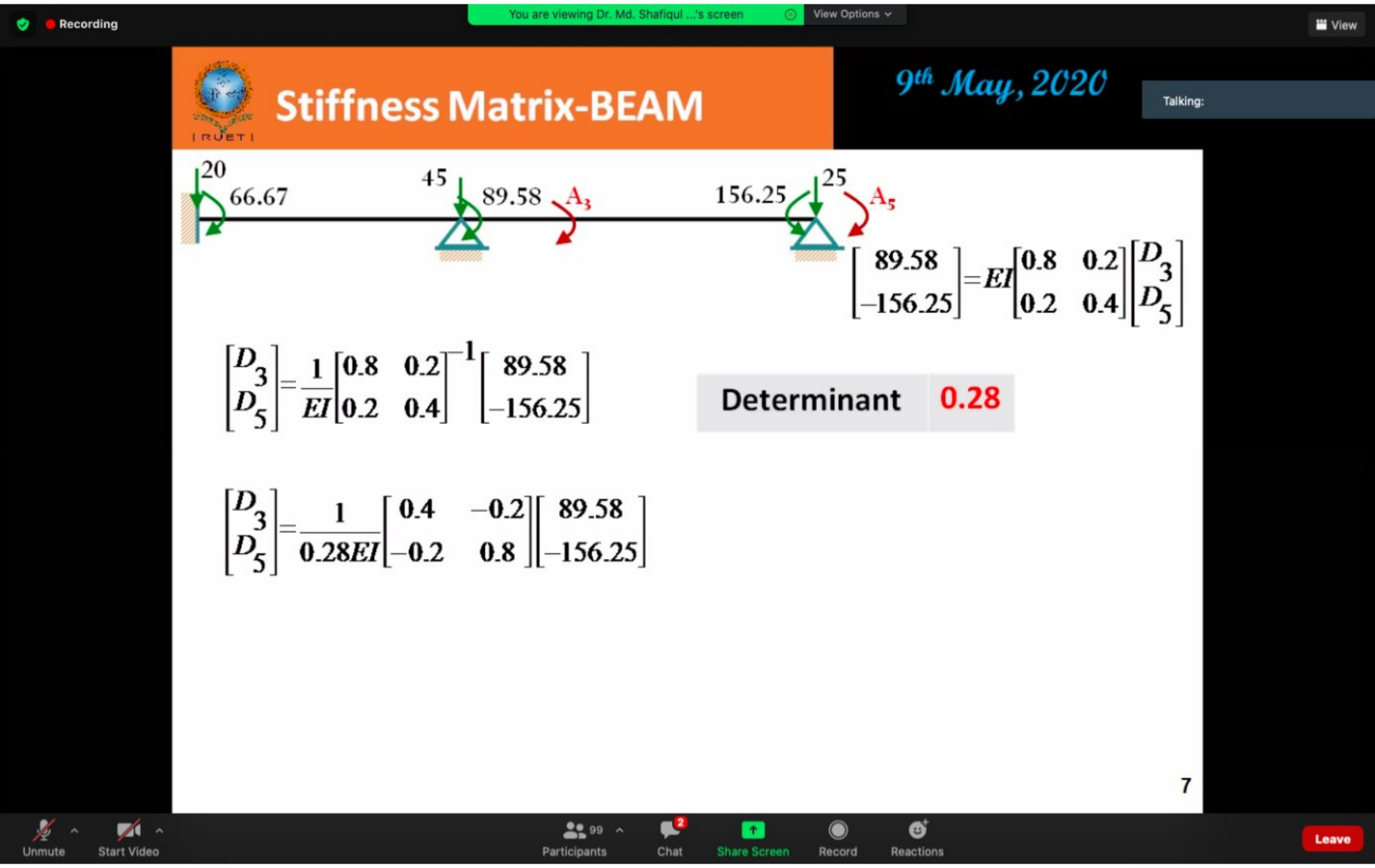
You are viewing Dr. Md. Shafiqul ...'s screen

Talking: Dr. Md. Shafiqul ISLAM

Stiffness Matrix-BEAM

9th May, 2020

$$\begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix} = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} D_3 \\ D_5 \end{bmatrix}$$



Stiffness Matrix-BEAM

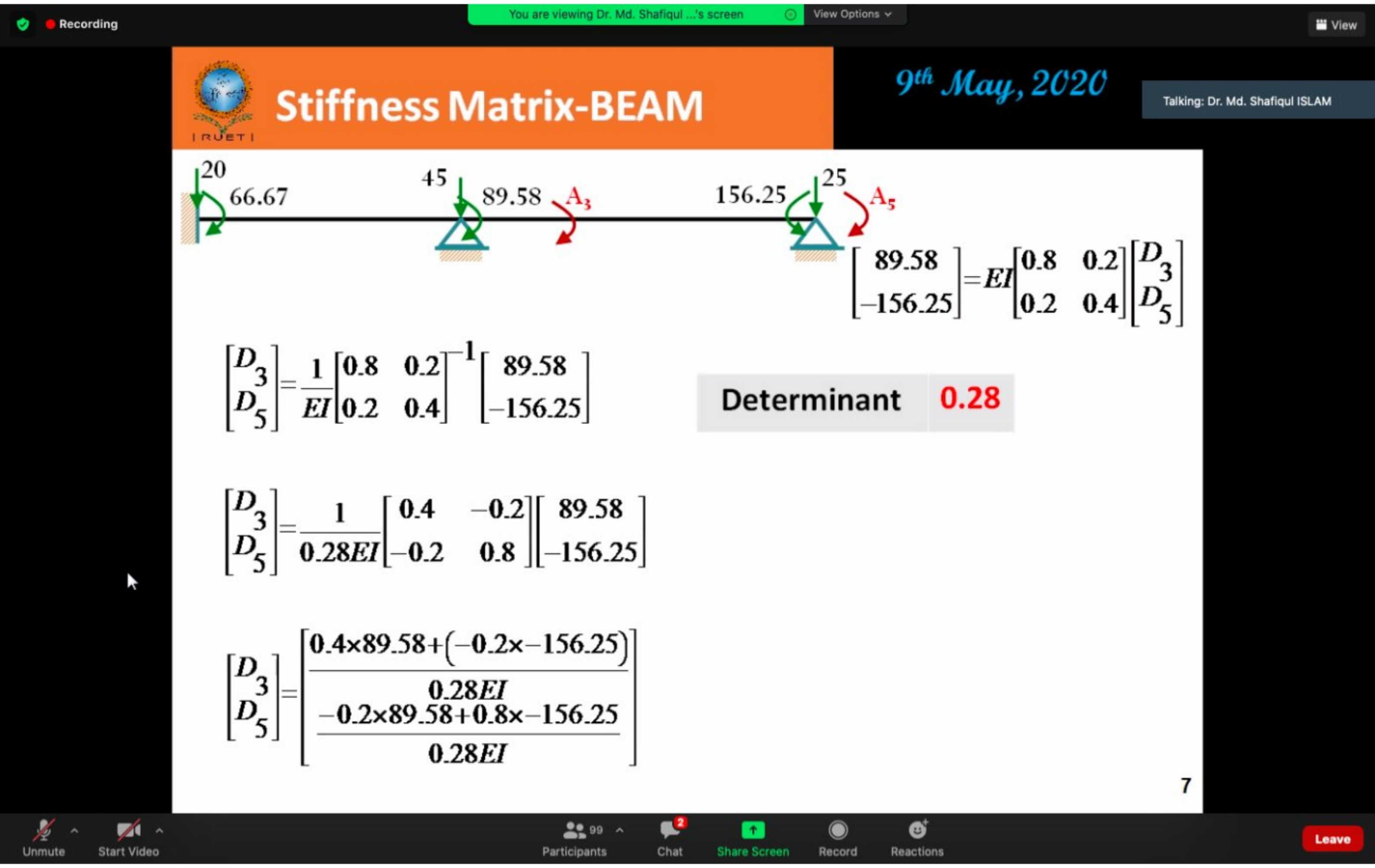
9th May, 2020

$$\begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix} = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} D_3 \\ D_5 \end{bmatrix}$$

$$\begin{bmatrix} D_3 \\ D_5 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}^{-1} \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

Determinant 0.28

$$\begin{bmatrix} D_3 \\ D_5 \end{bmatrix} = \frac{1}{0.28EI} \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$



Stiffness Matrix-BEAM

9th May, 2020

$$\begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix} = EI \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} D_3 \\ D_5 \end{bmatrix}$$

$$\begin{bmatrix} D_3 \\ D_5 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}^{-1} \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

Determinant	0.28

$$\begin{bmatrix} D_3 \\ D_5 \end{bmatrix} = \frac{1}{0.28EI} \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 89.58 \\ -156.25 \end{bmatrix}$$

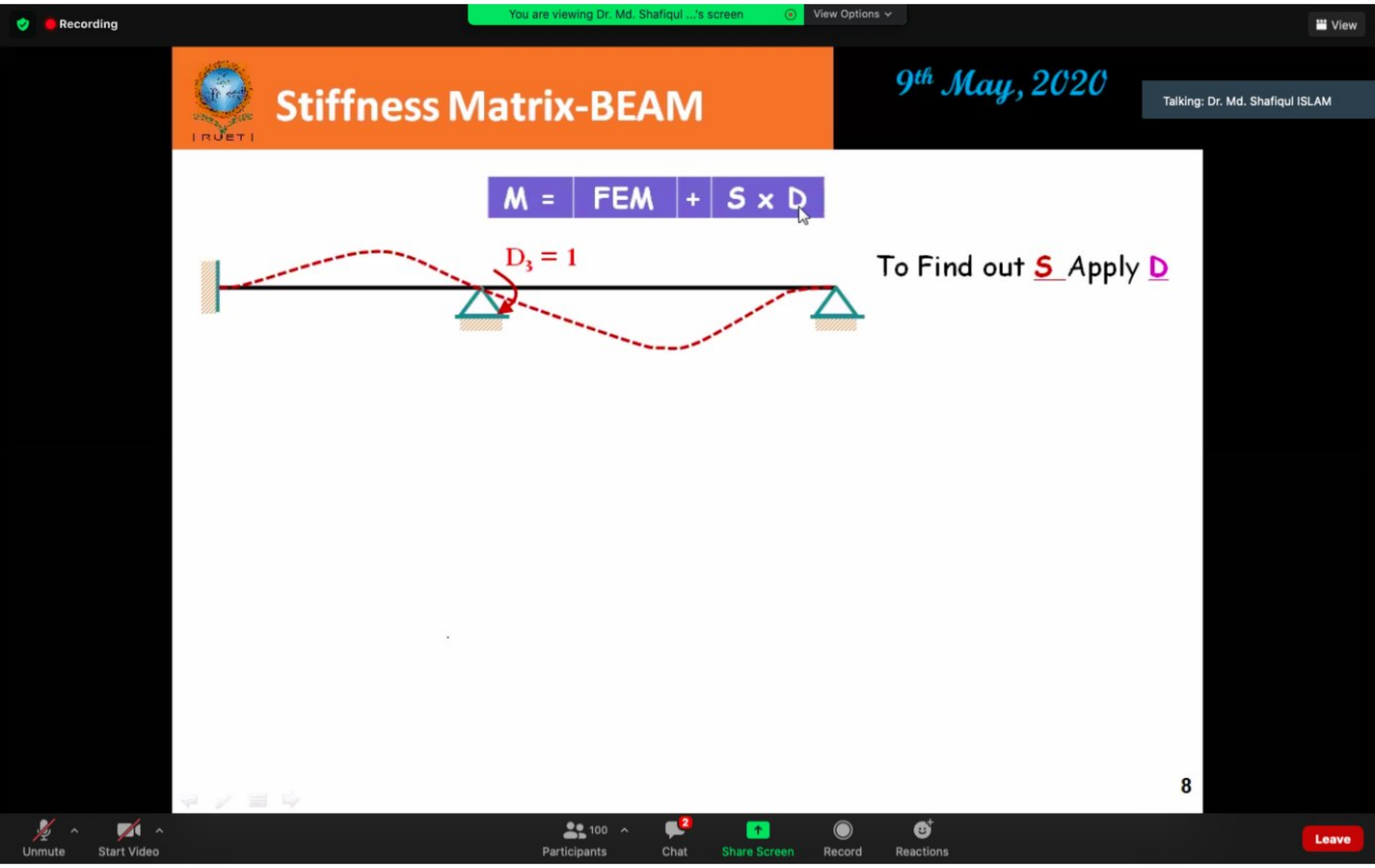
$$\begin{bmatrix} D_3 \\ D_5 \end{bmatrix} = \begin{bmatrix} \dfrac{0.4 \times 89.58 + (-0.2 \times -156.25)}{0.28EI} \\ \dfrac{-0.2 \times 89.58 + 0.8 \times -156.25}{0.28EI} \end{bmatrix}$$

7

Stiffness Matrix-BEAM

9th May, 2020

M = FEM + S x D

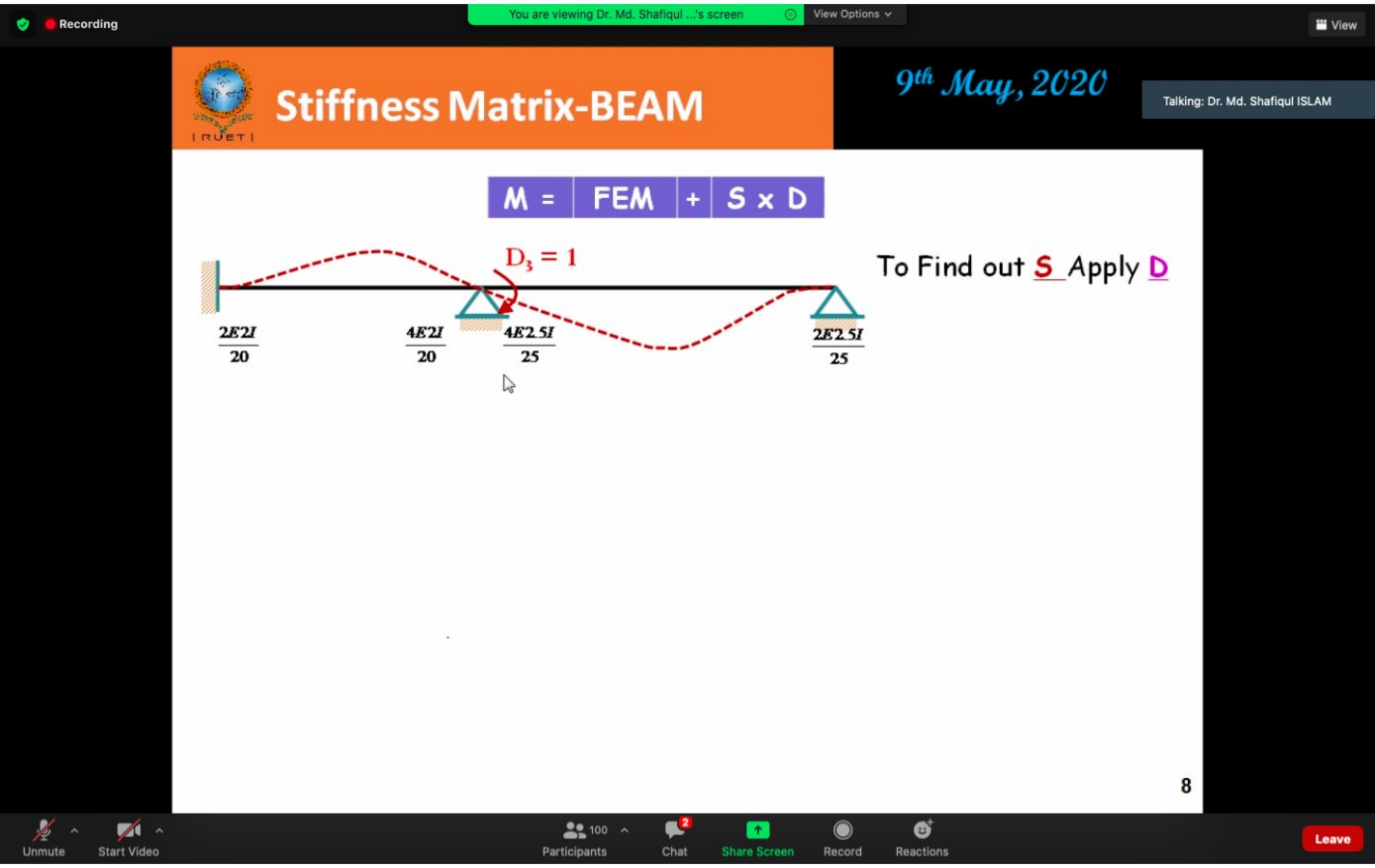


Stiffness Matrix-BEAM

9th May, 2020

M = FEM + S x D

To Find out S Apply D

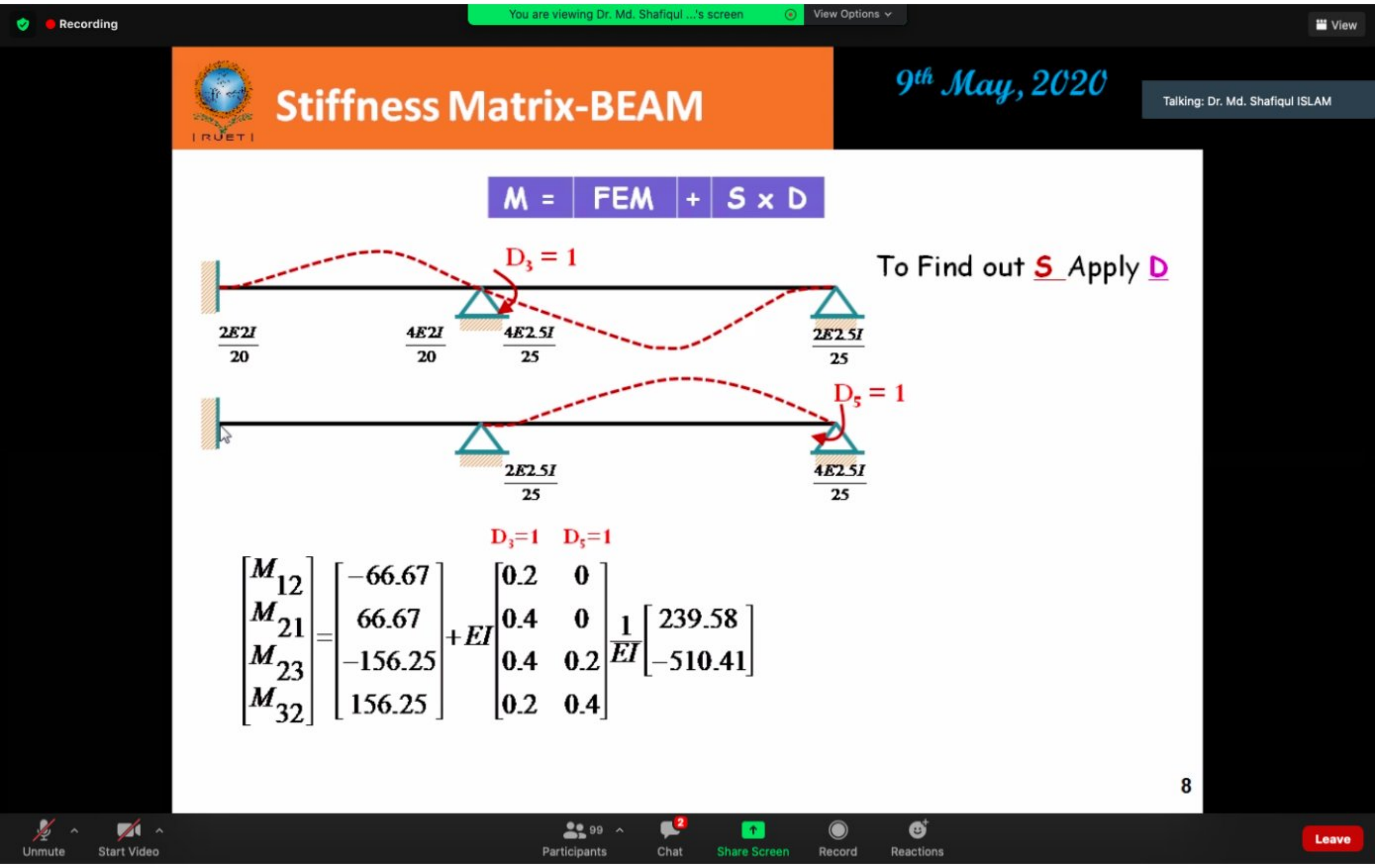


Stiffness Matrix-BEAM

9th May, 2020

M = FEM + S x D

To Find out S Apply D



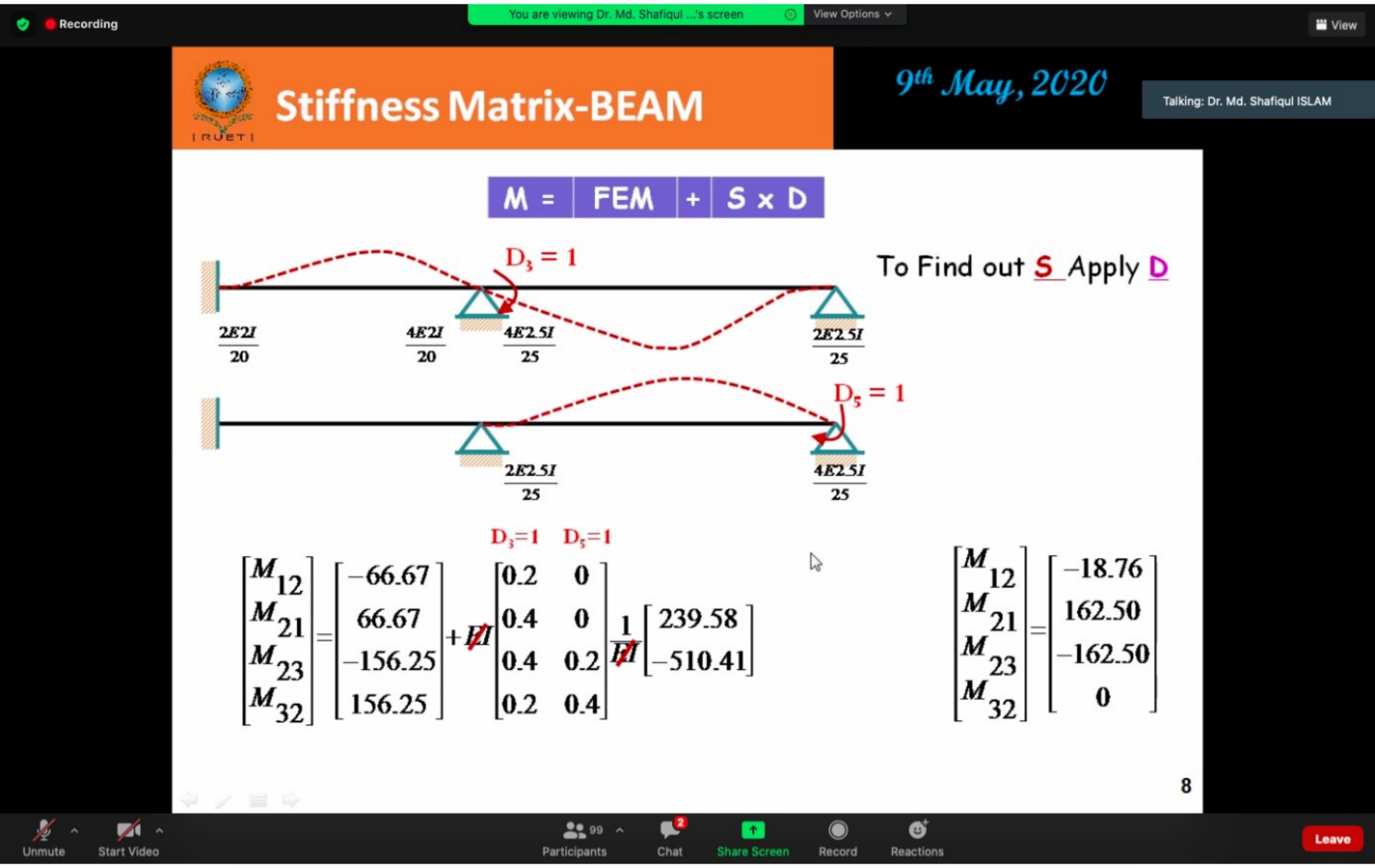
Stiffness Matrix-BEAM

9th May, 2020

M = FEM + S x D

To Find out **S** Apply **D**

$$\begin{bmatrix} M_{12} \\ M_{21} \\ M_{23} \\ M_{32} \end{bmatrix} = \begin{bmatrix} -66.67 \\ 66.67 \\ -156.25 \\ 156.25 \end{bmatrix} + EI \begin{array}{cc} D_3=1 & D_5=1 \\ \left[\begin{matrix} 0.2 & 0 \\ 0.4 & 0 \\ 0.4 & 0.2 \\ 0.2 & 0.4 \end{matrix}\right. & \left.\right] \end{array} \frac{1}{EI} \begin{bmatrix} 239.58 \\ -510.41 \end{bmatrix}$$



Stiffness Matrix-BEAM

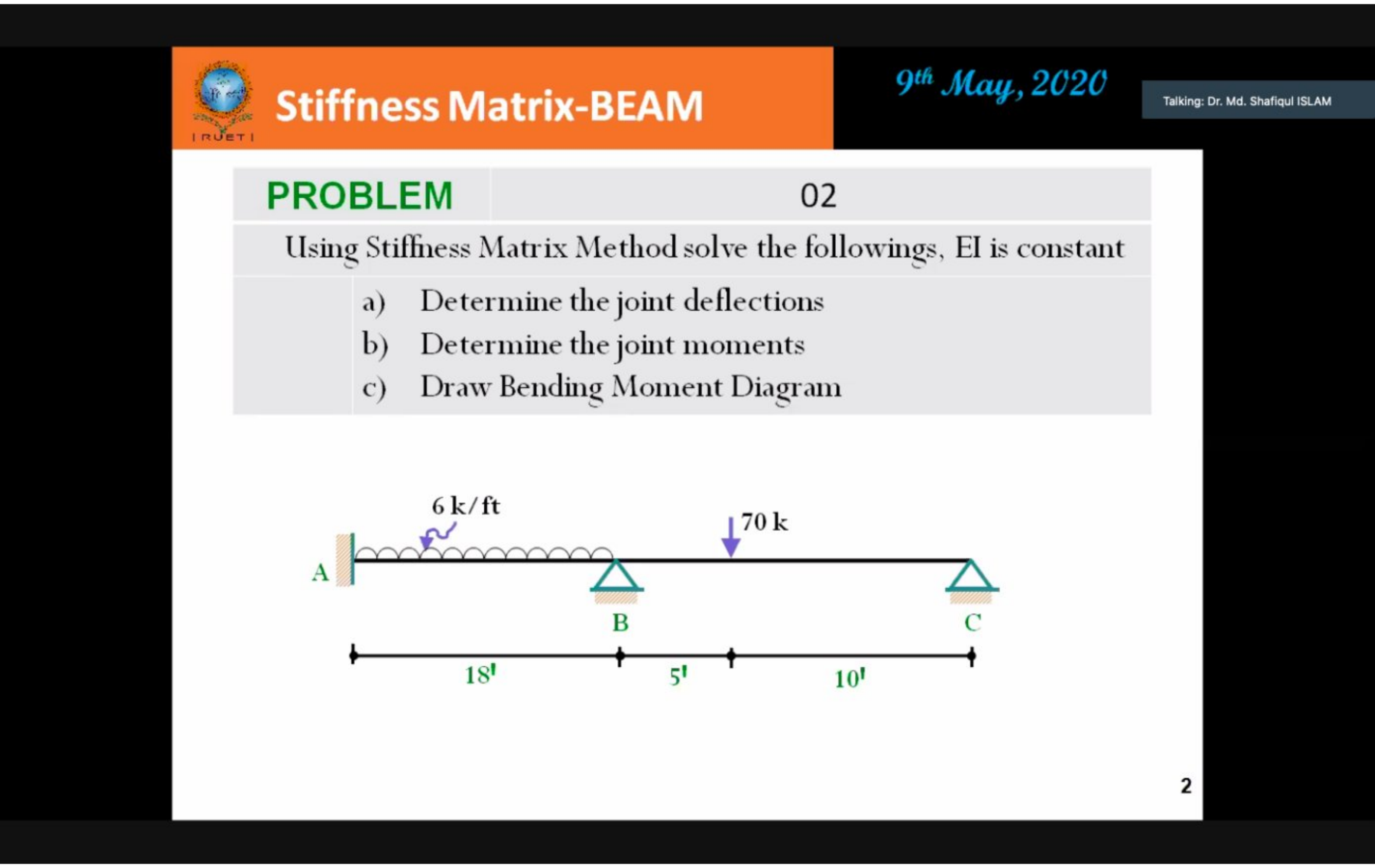
9th May, 2020

M = FEM + S x D

To Find out **S** Apply **D**

$$\begin{bmatrix}M_{12}\\M_{21}\\M_{23}\\M_{32}\end{bmatrix}=\begin{bmatrix}-66.67\\66.67\\-156.25\\156.25\end{bmatrix}+\cancel{E}I\begin{array}{cc} D_3=1 & D_5=1 \\ \left[\begin{matrix}0.2 & 0\\0.4 & 0\\0.4 & 0.2\\0.2 & 0.4\end{matrix}\right. & \left.\right]\end{array}\frac{1}{\cancel{E}I}\begin{bmatrix}239.58\\-510.41\end{bmatrix}$$

$$\begin{bmatrix}M_{12}\\M_{21}\\M_{23}\\M_{32}\end{bmatrix}=\begin{bmatrix}-18.76\\162.50\\-162.50\\0\end{bmatrix}$$



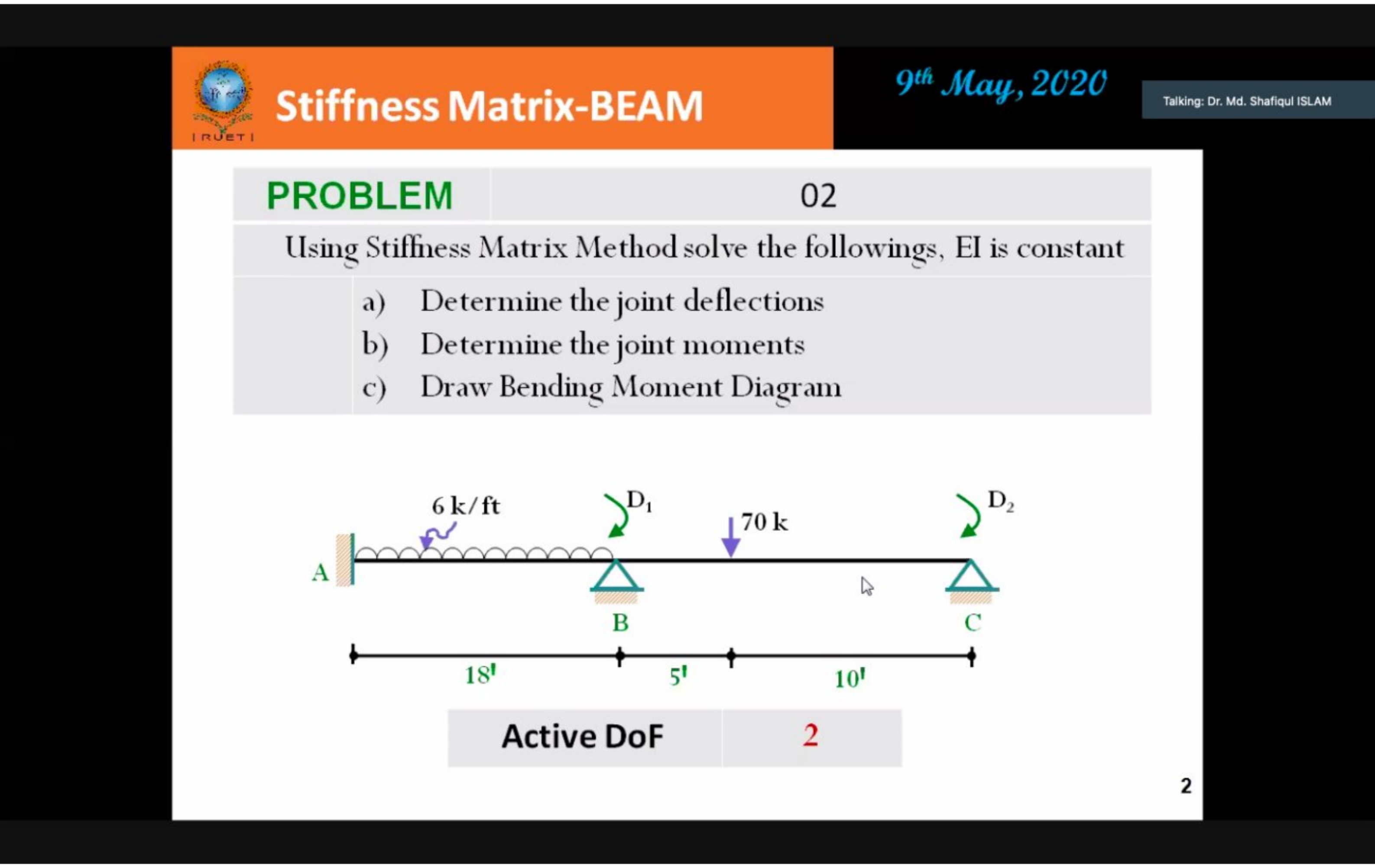
Stiffness Matrix-BEAM

9th May, 2020

PROBLEM	02

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the joint deflections
b) Determine the joint moments
c) Draw Bending Moment Diagram



Stiffness Matrix-BEAM

9th May, 2020

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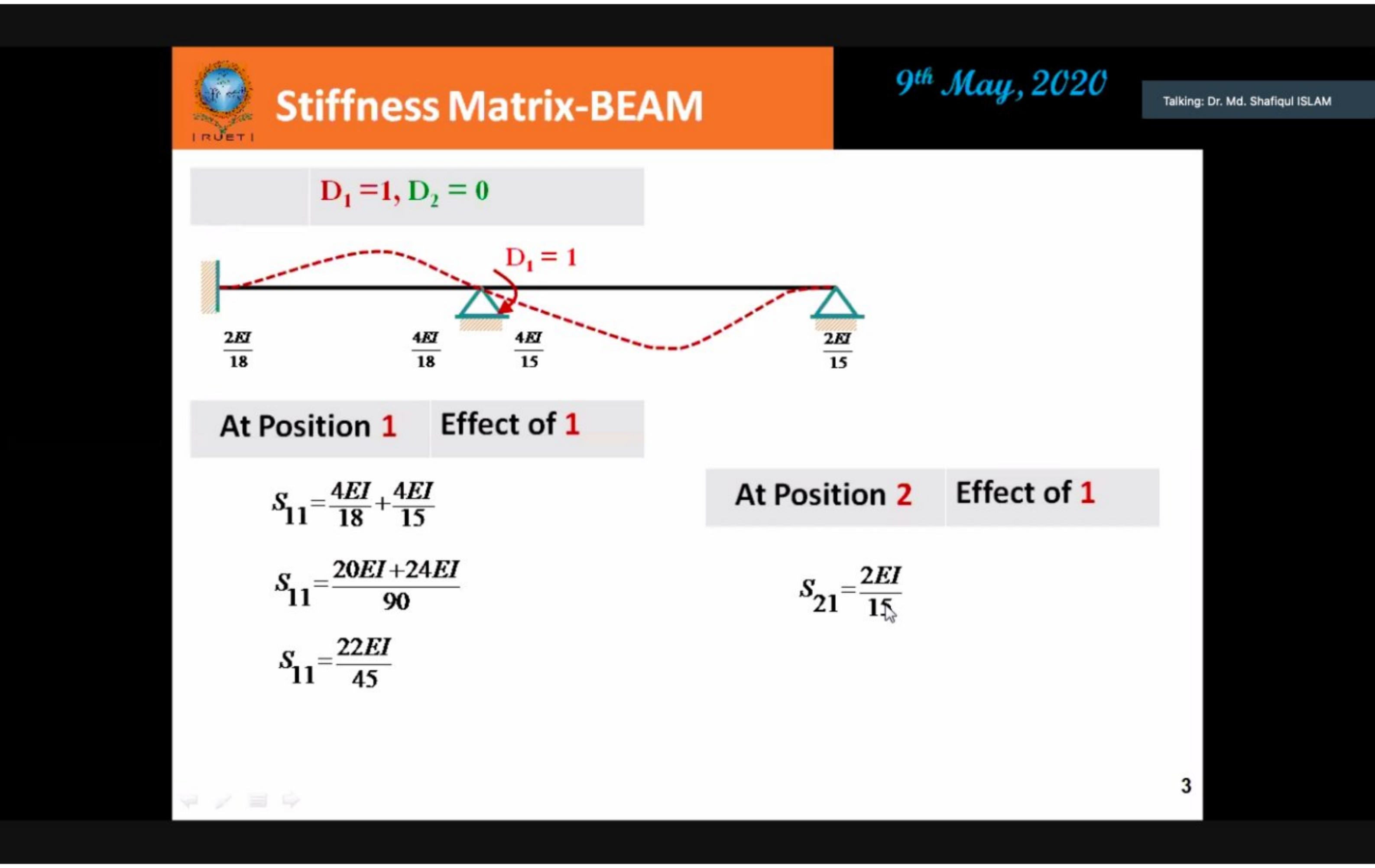
PROBLEM	02

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the joint deflections
b) Determine the joint moments
c) Draw Bending Moment Diagram

Active DoF	2



Stiffness Matrix-BEAM

9th May, 2020

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$D_1 = 1, D_2 = 0$

At Position 1 | **Effect of 1**

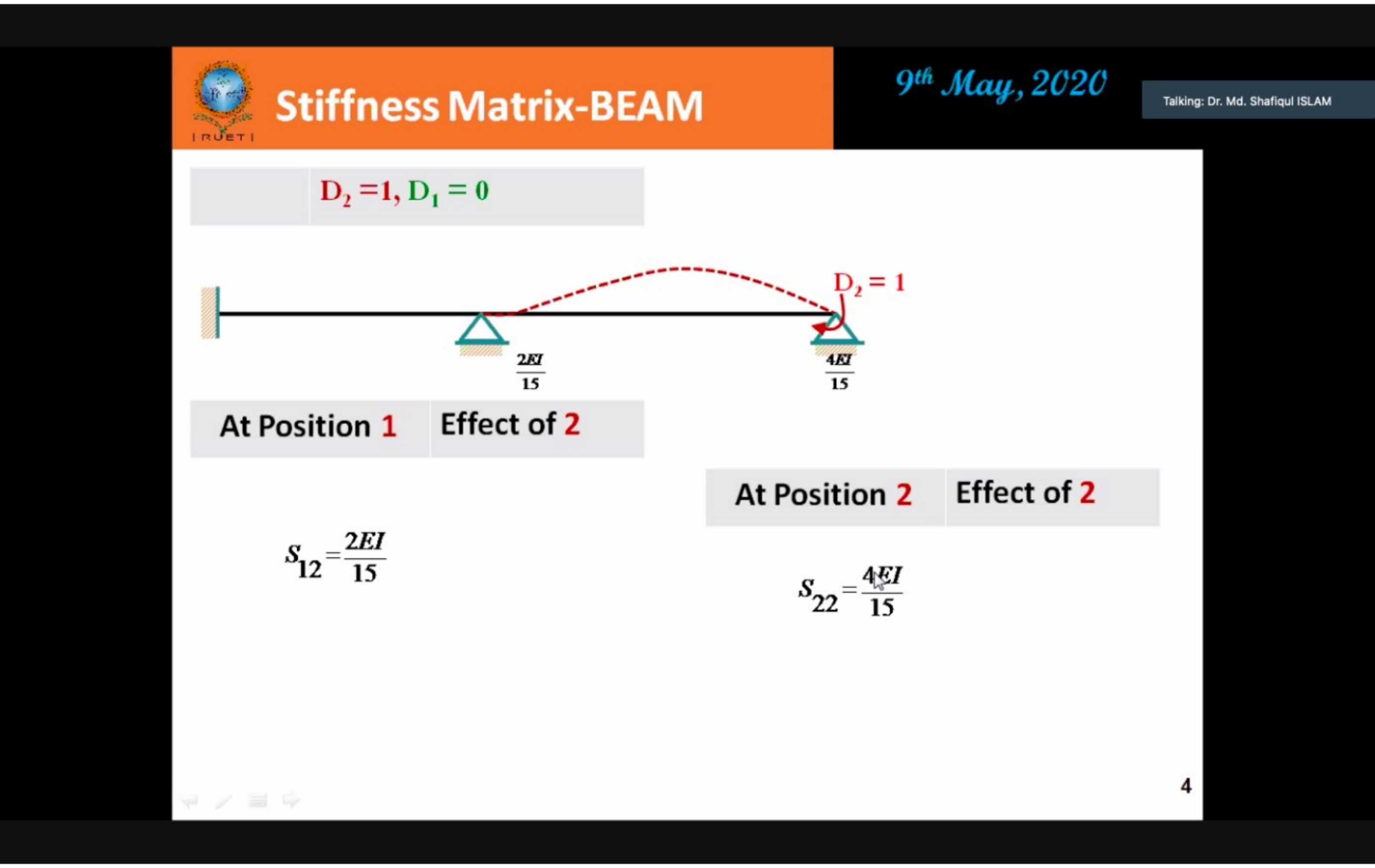
$$S_{11} = \frac{4EI}{18} + \frac{4EI}{15}$$

$$S_{11} = \frac{20EI + 24EI}{90}$$

$$S_{11} = \frac{22EI}{45}$$

At Position 2 | **Effect of 1**

$$S_{21} = \frac{2EI}{15}$$



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Stiffness Matrix-BEAM

9th May, 2020

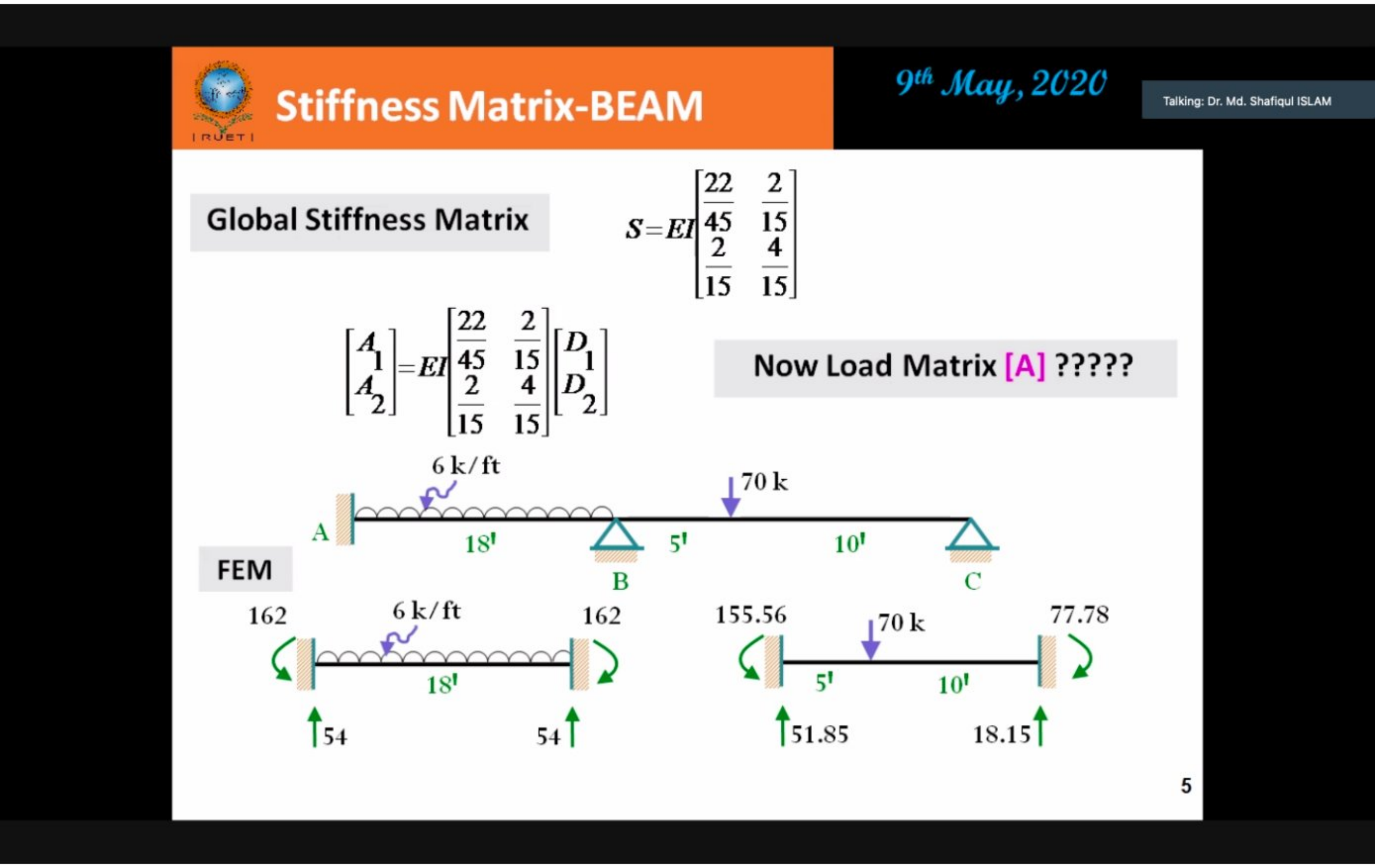
$D_2 = 1, D_1 = 0$

At Position 1 **Effect of 2**

$$S_{12} = \frac{2EI}{15}$$

At Position 2 **Effect of 2**

$$S_{22} = \frac{4EI}{15}$$



Stiffness Matrix-BEAM

9th May, 2020

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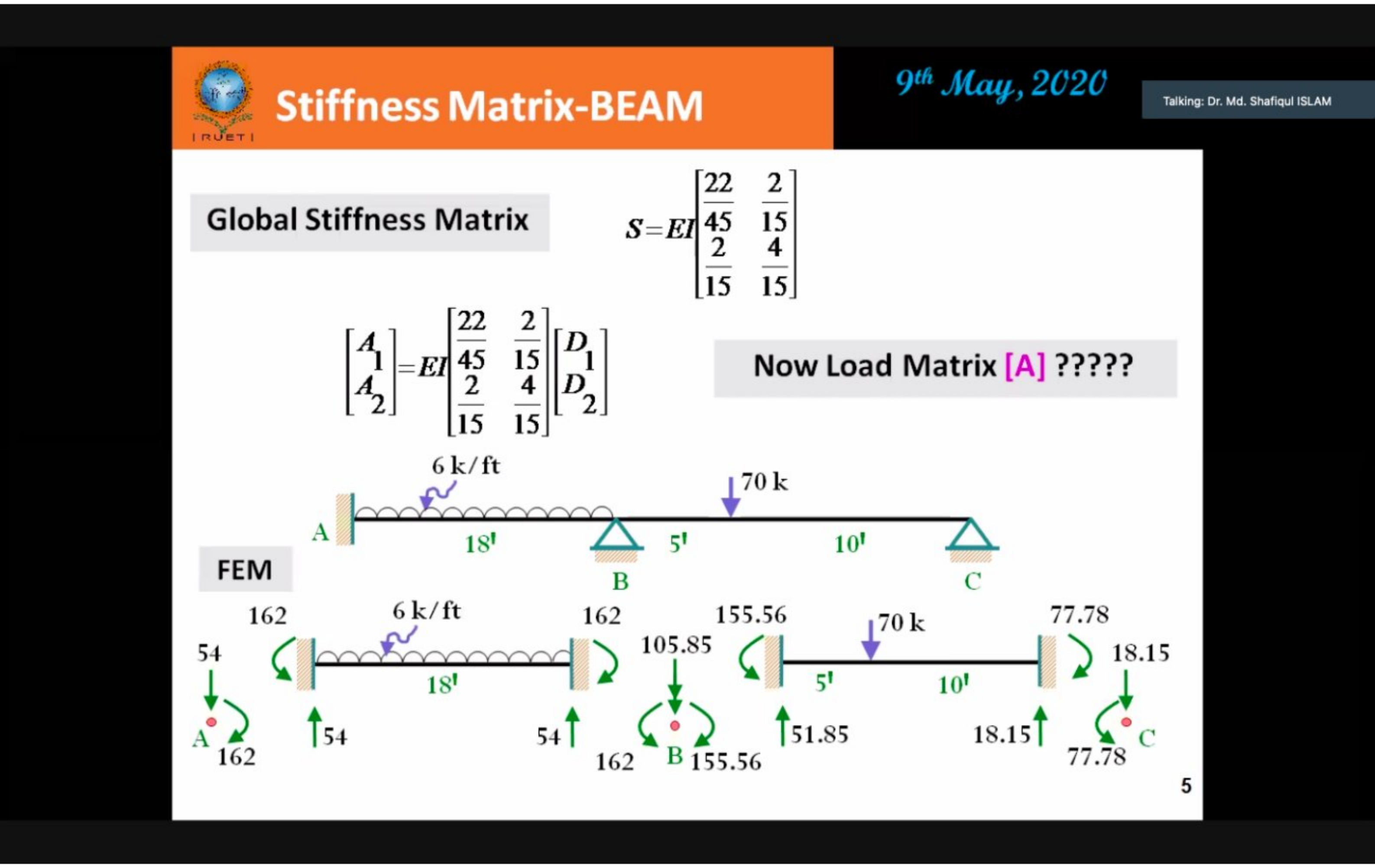
Global Stiffness Matrix

$$S = EI\begin{bmatrix} \frac{22}{45} & \frac{2}{15} \\ \frac{2}{15} & \frac{4}{15} \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = EI\begin{bmatrix} \frac{22}{45} & \frac{2}{15} \\ \frac{2}{15} & \frac{4}{15} \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

Now Load Matrix [A] ?????

FEM



Stiffness Matrix-BEAM

9th May, 2020

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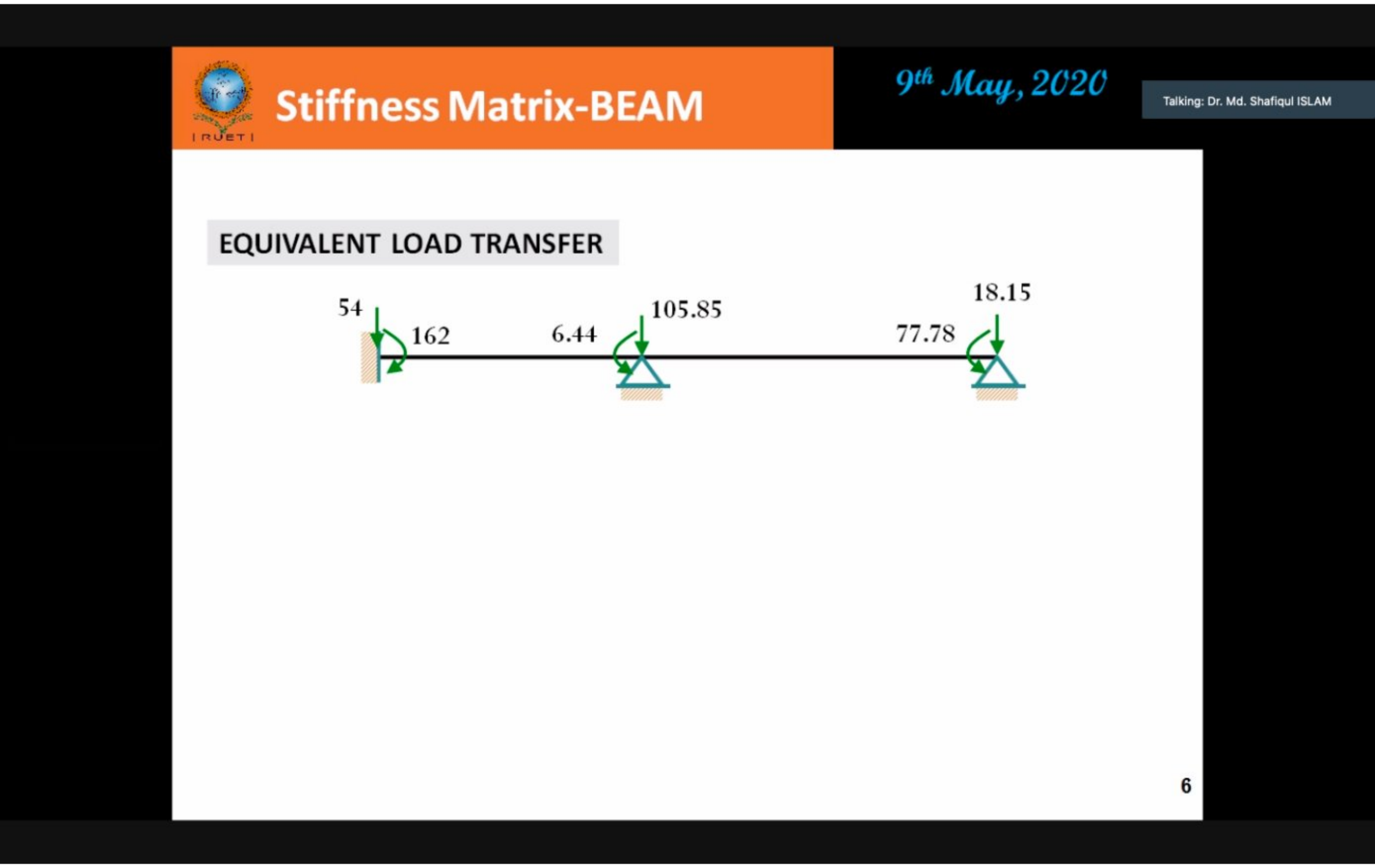
Global Stiffness Matrix

$$S = EI\begin{bmatrix} \frac{22}{45} & \frac{2}{15} \\ \frac{2}{15} & \frac{4}{15} \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = EI\begin{bmatrix} \frac{22}{45} & \frac{2}{15} \\ \frac{2}{15} & \frac{4}{15} \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

Now Load Matrix [A] ?????

FEM



Stiffness Matrix-BEAM

9th May, 2020

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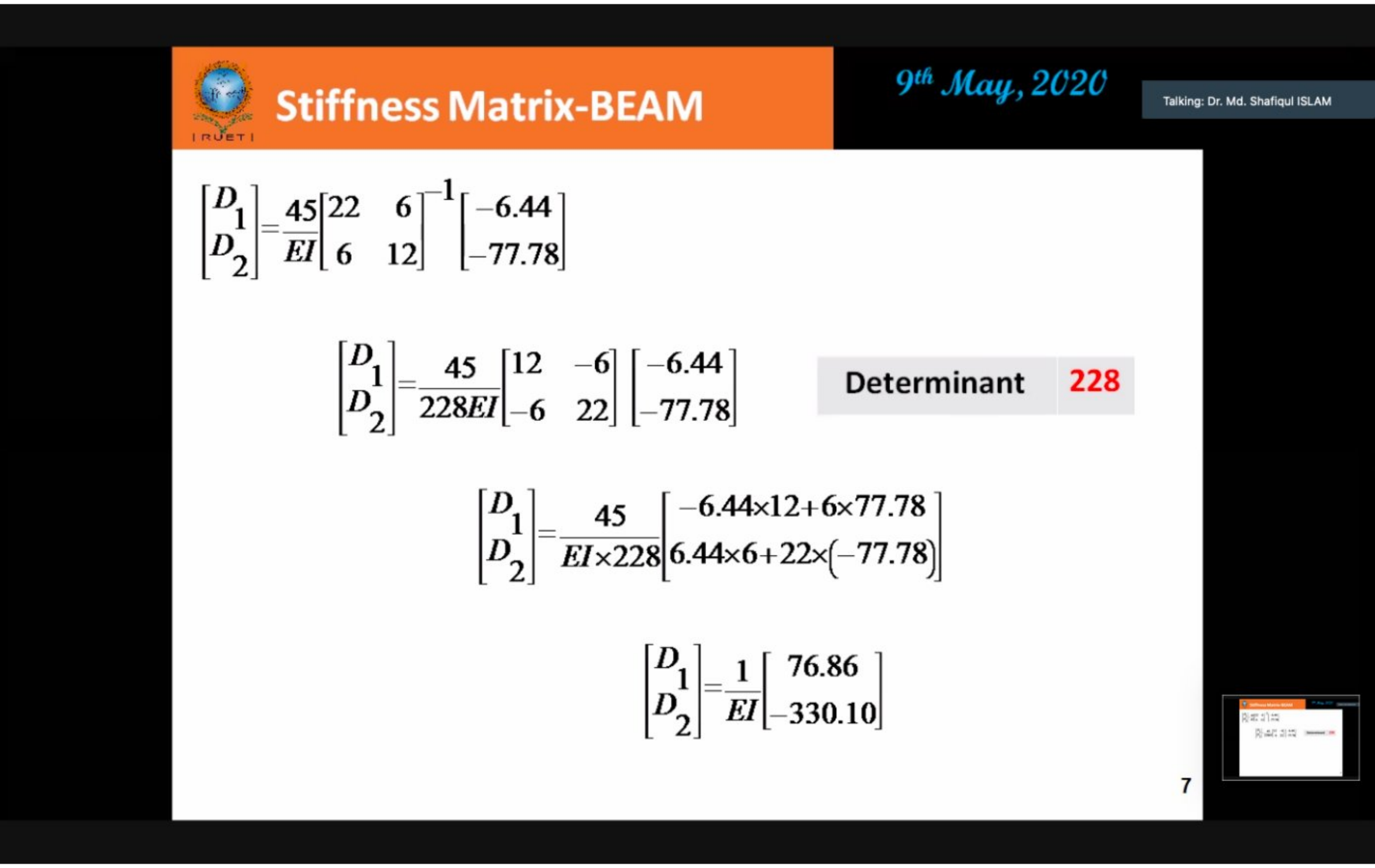
EQUIVALENT LOAD TRANSFER



Stiffness Matrix-BEAM

9th May, 2020

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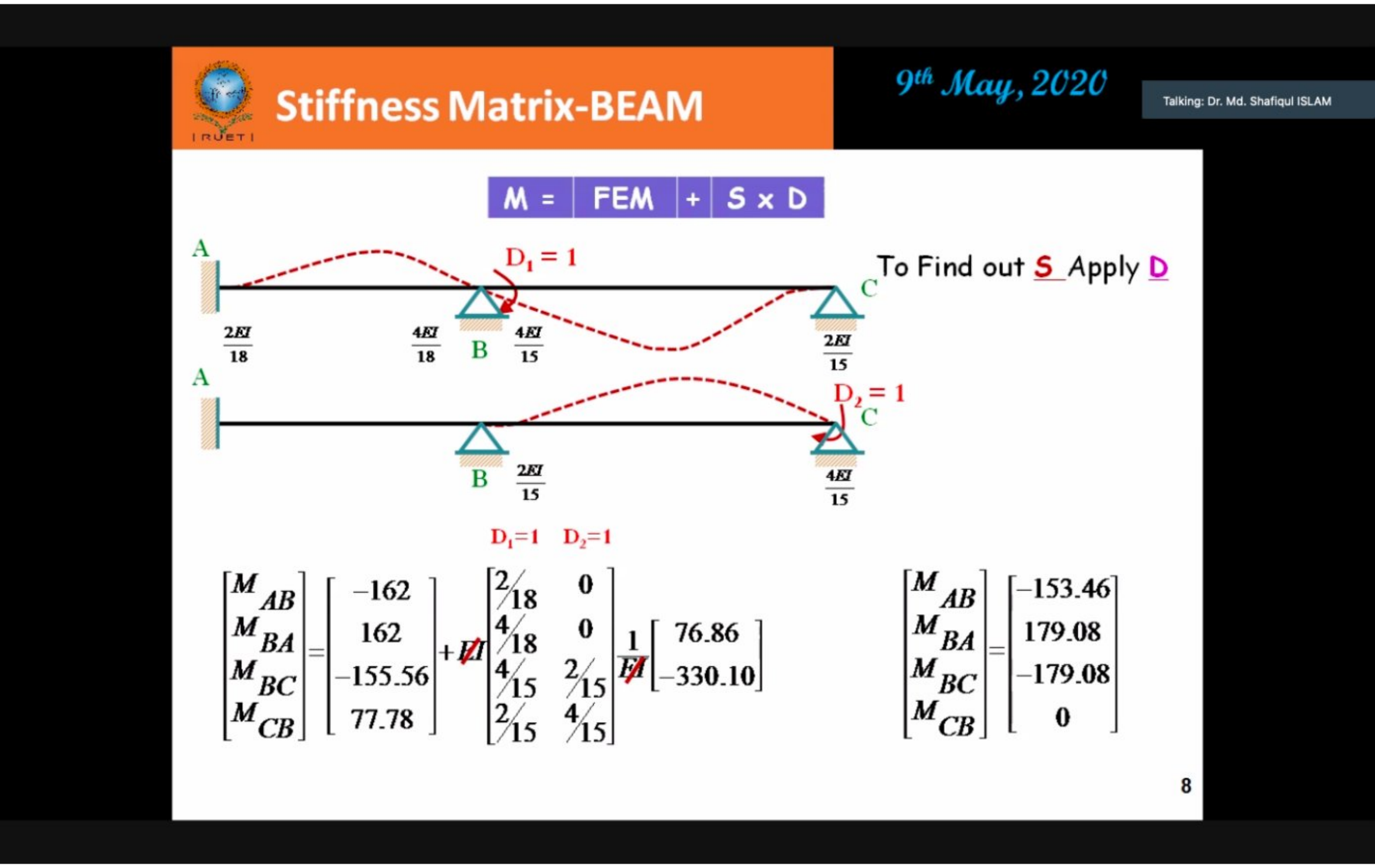
$$\begin{bmatrix}D_1\\D_2\end{bmatrix}=\frac{45}{EI}\begin{bmatrix}22&6\\6&12\end{bmatrix}^{-1}\begin{bmatrix}-6.44\\-77.78\end{bmatrix}$$

$$\begin{bmatrix}D_1\\D_2\end{bmatrix}=\frac{45}{228EI}\begin{bmatrix}12&-6\\-6&22\end{bmatrix}\begin{bmatrix}-6.44\\-77.78\end{bmatrix}$$

Determinant	228

$$\begin{bmatrix}D_1\\D_2\end{bmatrix}=\frac{45}{EI\times 228}\begin{bmatrix}-6.44\times 12+6\times 77.78\\6.44\times 6+22\times(-77.78)\end{bmatrix}$$

$$\begin{bmatrix}D_1\\D_2\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}76.86\\-330.10\end{bmatrix}$$



9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

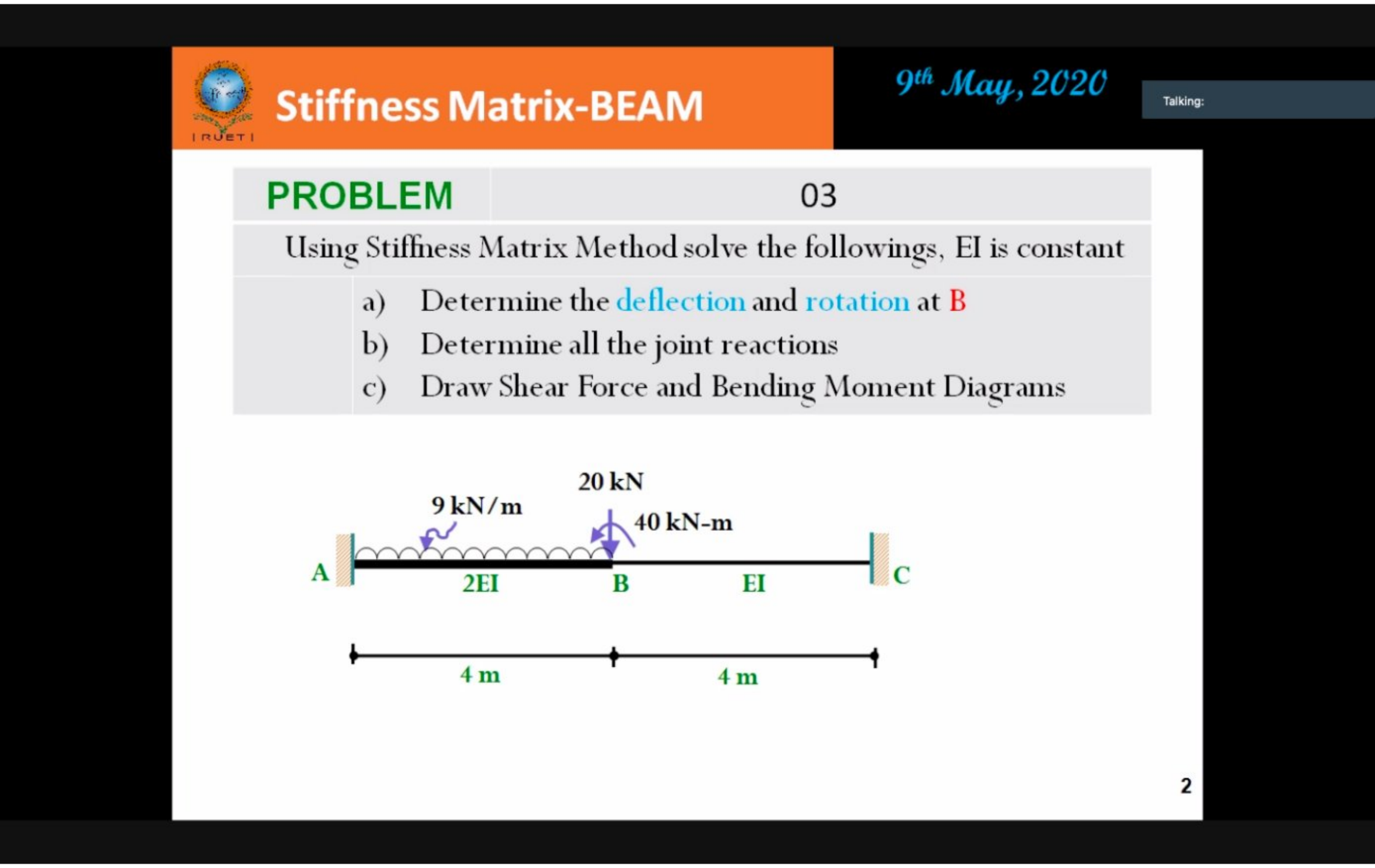
Stiffness Matrix-BEAM

M = FEM + S x D

To Find out **S** Apply **D**

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -162 \\ 162 \\ -155.56 \\ 77.78 \end{bmatrix} + \cancel{EI} \begin{array}{c} \begin{array}{cc} D_1=1 & D_2=1 \end{array} \\ \begin{bmatrix} 2/18 & 0 \\ 4/18 & 0 \\ 4/15 & 2/15 \\ 2/15 & 4/15 \end{bmatrix} \end{array} \frac{1}{\cancel{EI}} \begin{bmatrix} 76.86 \\ -330.10 \end{bmatrix}$$

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -153.46 \\ 179.08 \\ -179.08 \\ 0 \end{bmatrix}$$



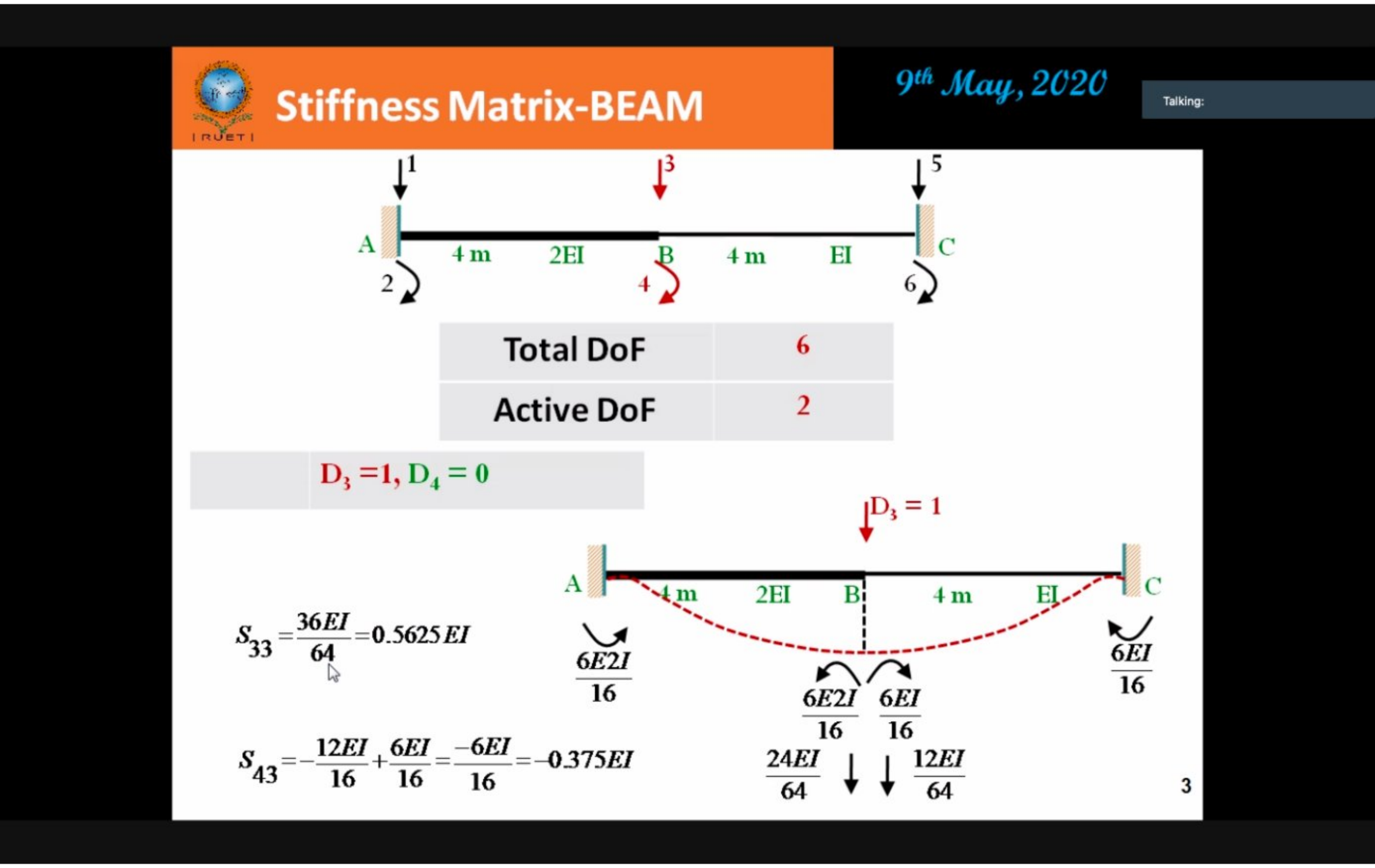
Stiffness Matrix-BEAM

9th May, 2020

PROBLEM	03

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the deflection and rotation at B
b) Determine all the joint reactions
c) Draw Shear Force and Bending Moment Diagrams



Stiffness Matrix-BEAM

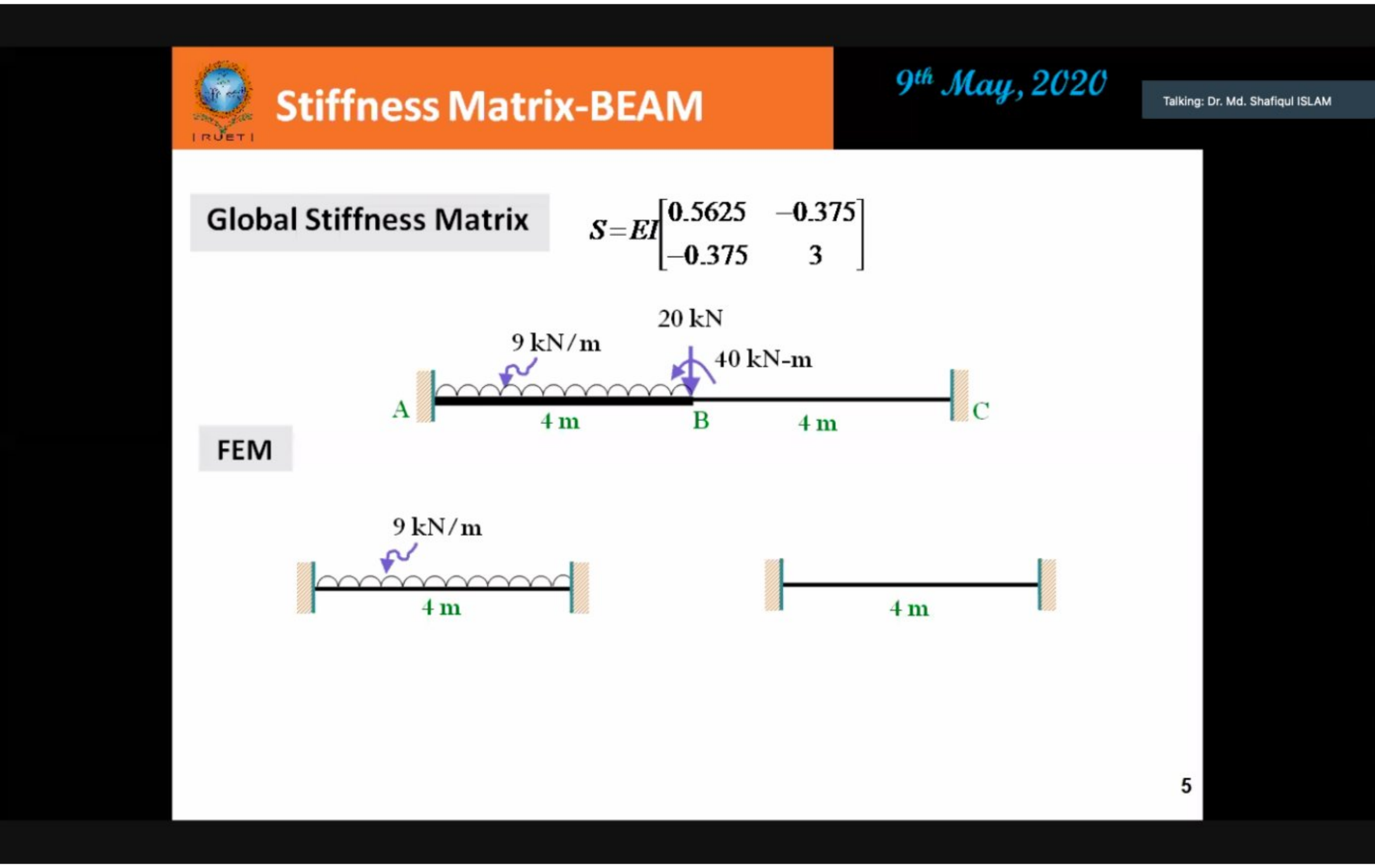
9th May, 2020

Total DoF	6
Active DoF	2

$D_3 = 1, D_4 = 0$

$$S_{33} = \frac{36EI}{64} = 0.5625EI$$

$$S_{43} = -\frac{12EI}{16} + \frac{6EI}{16} = \frac{-6EI}{16} = -0.375EI$$



Stiffness Matrix-BEAM

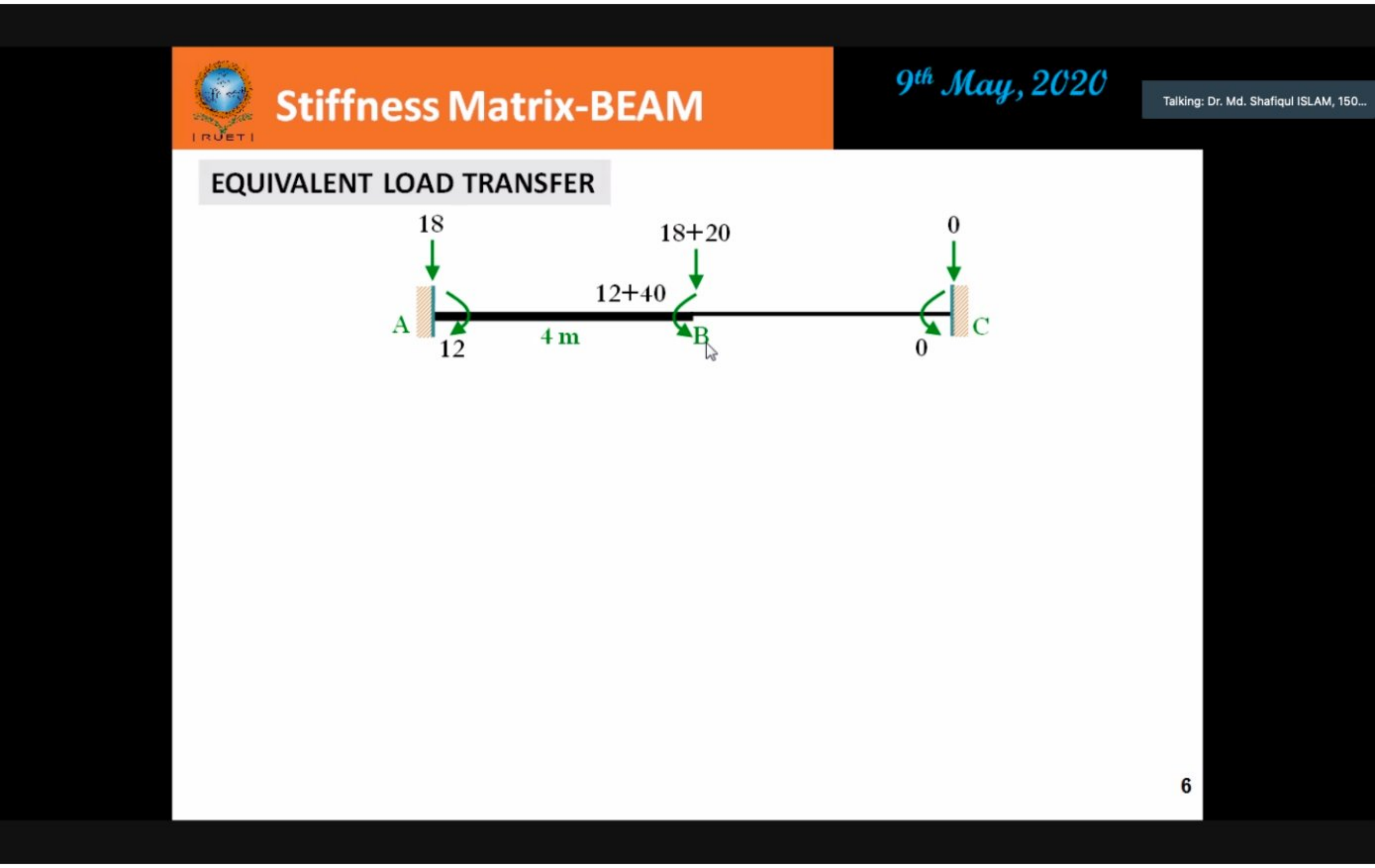
9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Global Stiffness Matrix

$$S = EI\begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix}$$

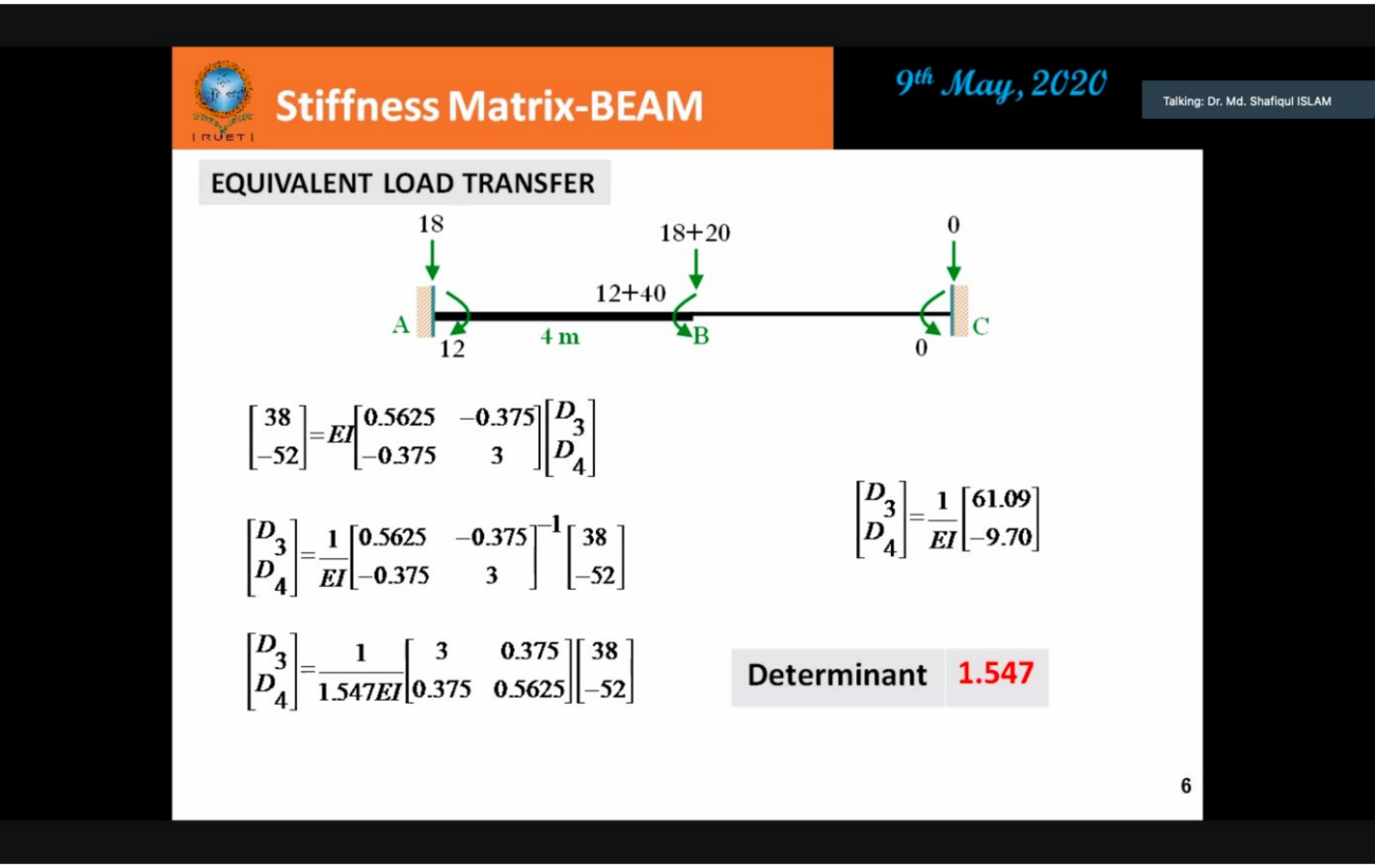
FEM



9th May, 2020

Stiffness Matrix-BEAM

EQUIVALENT LOAD TRANSFER



Stiffness Matrix-BEAM

9th May, 2020

EQUIVALENT LOAD TRANSFER

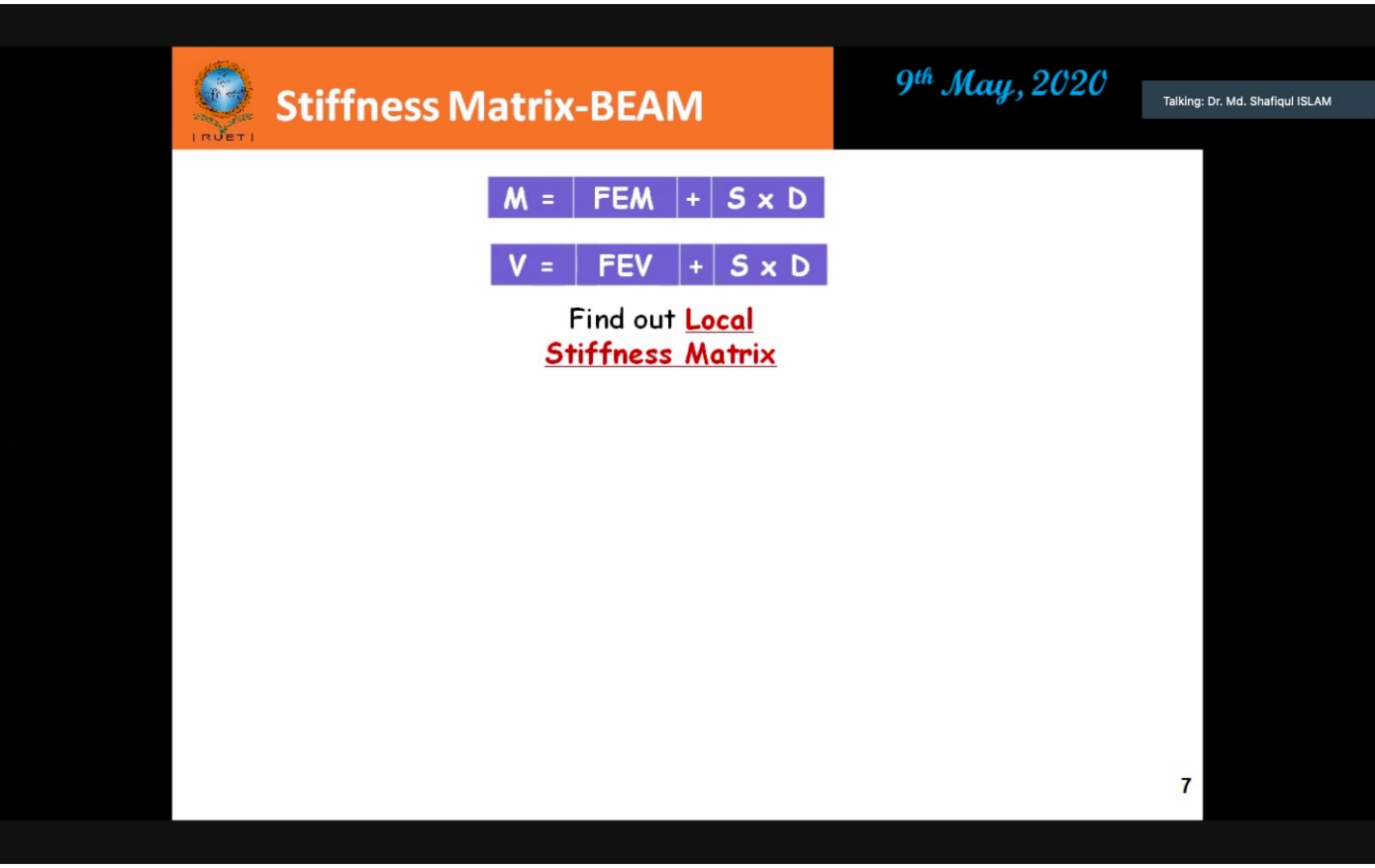
$$\begin{bmatrix} 38 \\ -52 \end{bmatrix} = EI \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix} \begin{bmatrix} D_3 \\ D_4 \end{bmatrix}$$

$$\begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 0.5625 & -0.375 \\ -0.375 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 38 \\ -52 \end{bmatrix}$$

$$\begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{1.547EI} \begin{bmatrix} 3 & 0.375 \\ 0.375 & 0.5625 \end{bmatrix} \begin{bmatrix} 38 \\ -52 \end{bmatrix}$$

$$\begin{bmatrix} D_3 \\ D_4 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 61.09 \\ -9.70 \end{bmatrix}$$

Determinant	1.547



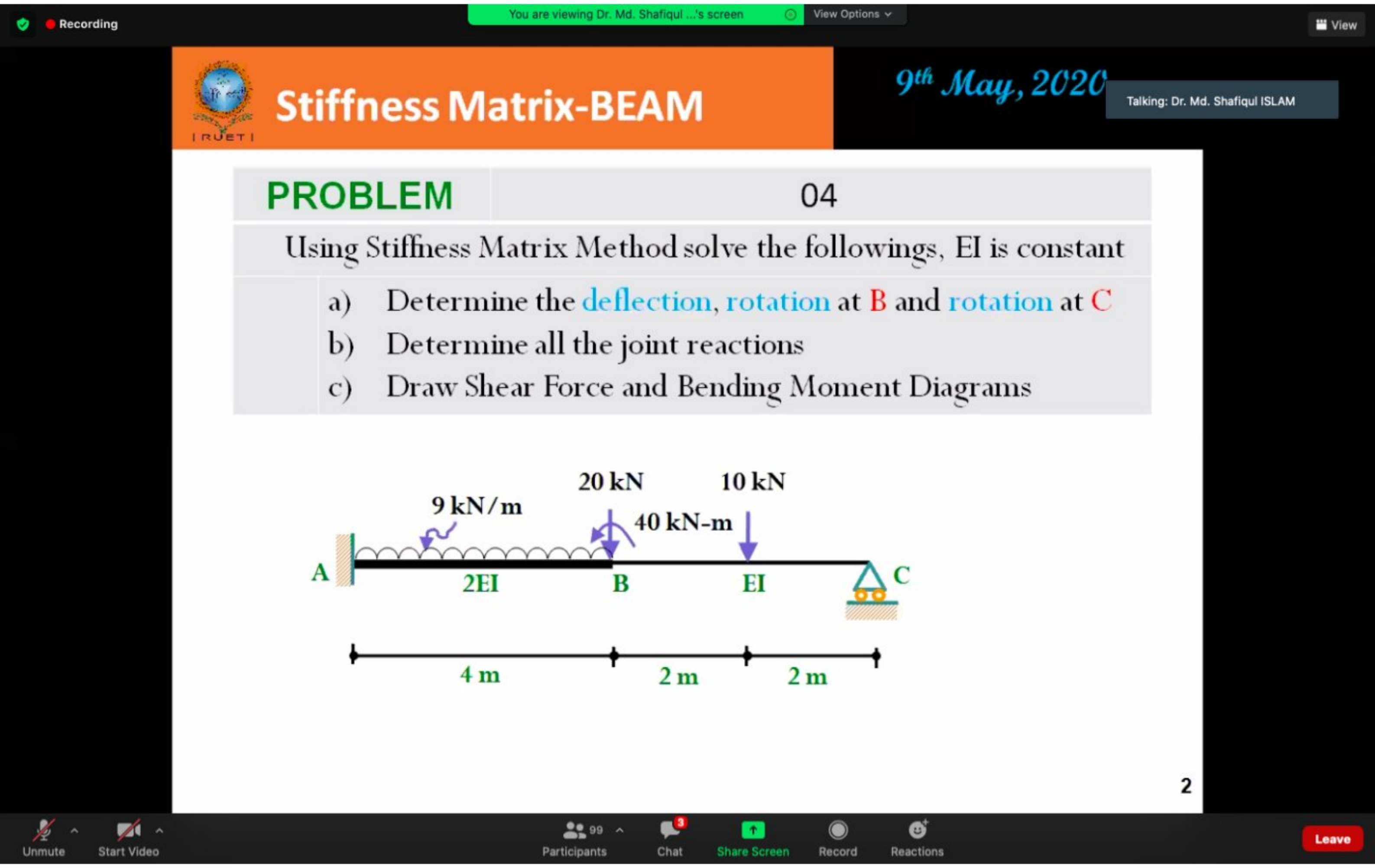
Stiffness Matrix-BEAM

9th May, 2020

M =	FEM	+	S x D

V =	FEV	+	S x D

Find out **Local Stiffness Matrix**



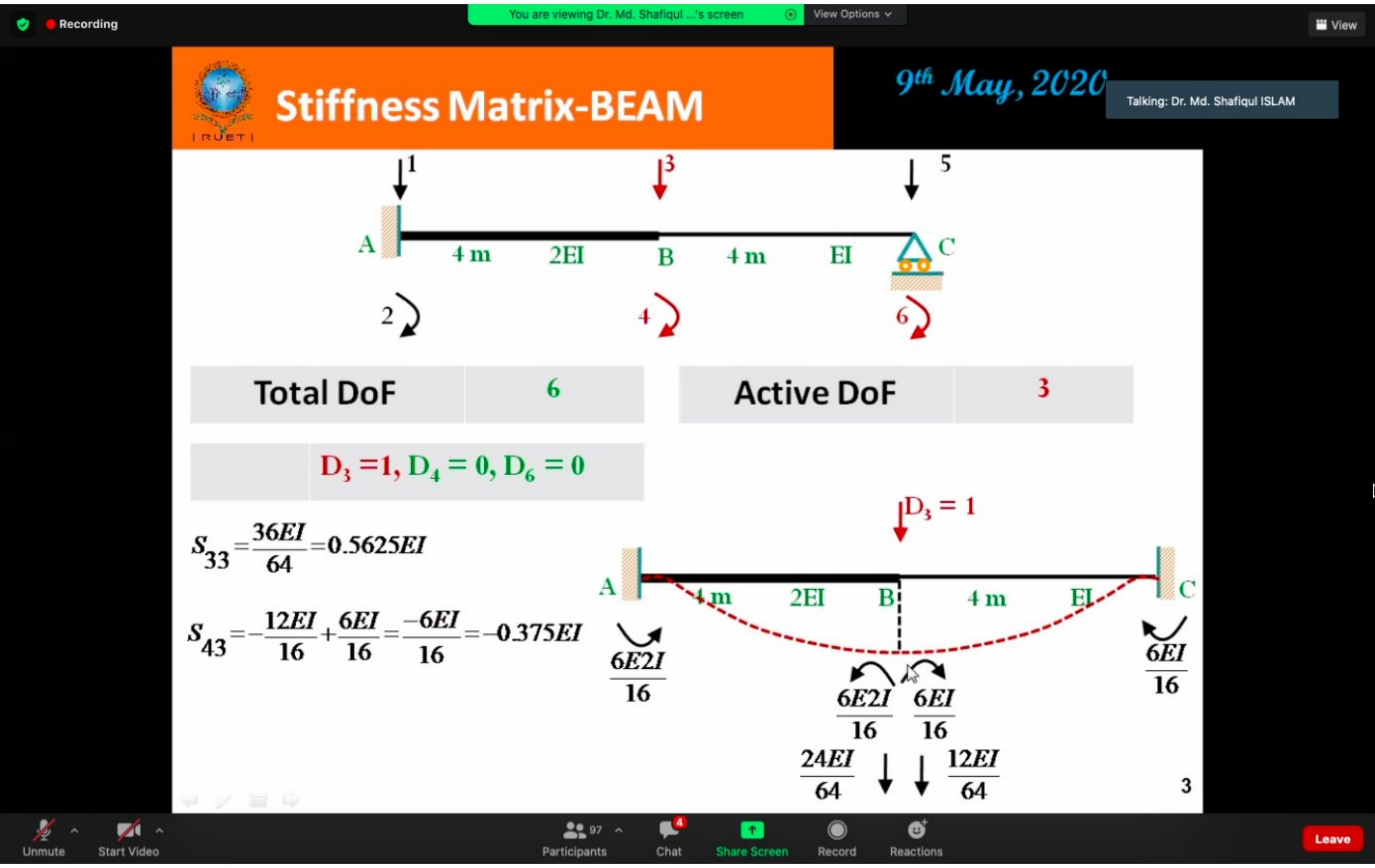
Stiffness Matrix-BEAM

9th May, 2020

PROBLEM 04

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the deflection, rotation at B and rotation at C
b) Determine all the joint reactions
c) Draw Shear Force and Bending Moment Diagrams



Stiffness Matrix-BEAM

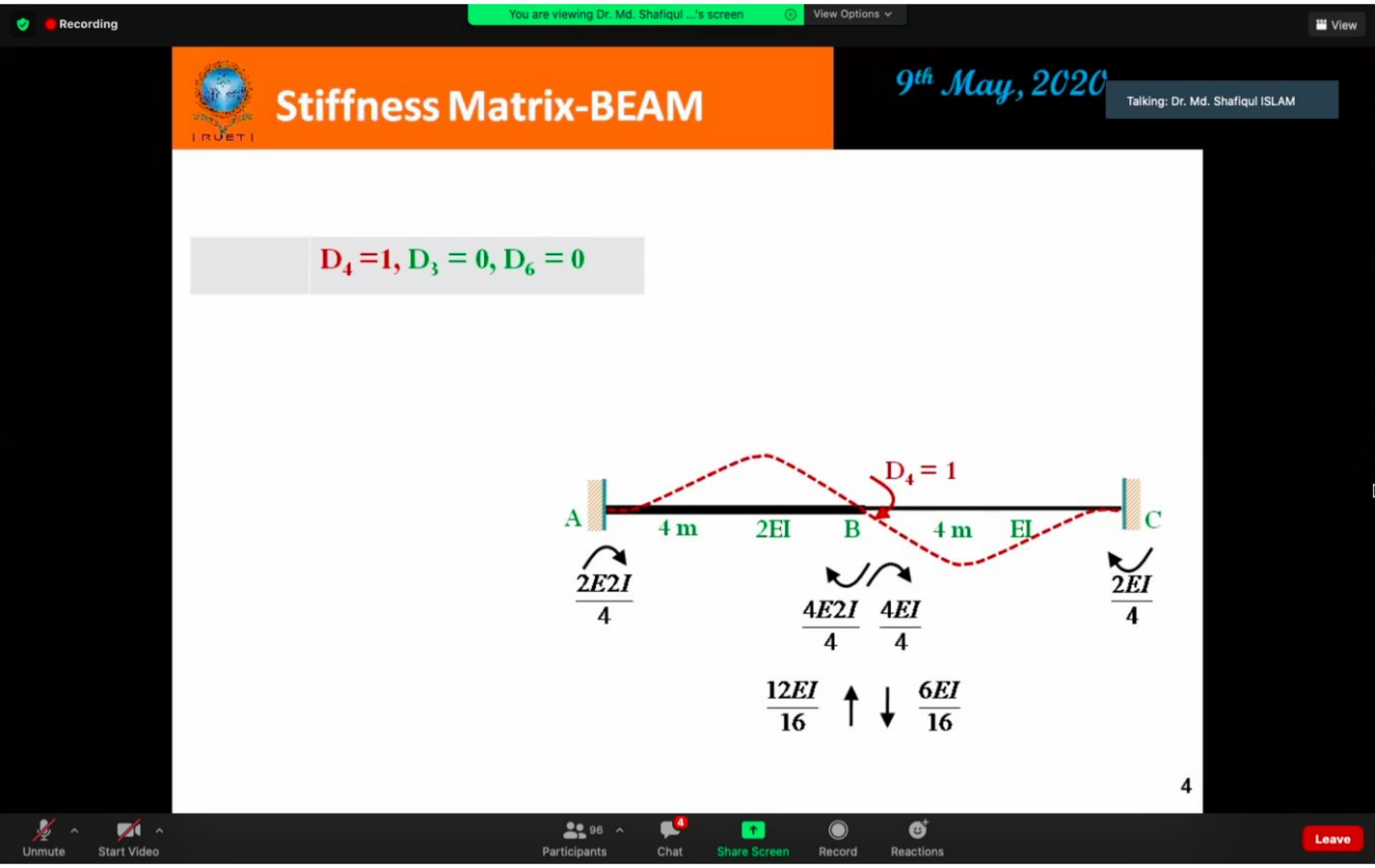
9th May, 2020

Total DoF	6	Active DoF	3

$D_3 = 1, D_4 = 0, D_6 = 0$

$$S_{33} = \frac{36EI}{64} = 0.5625EI$$

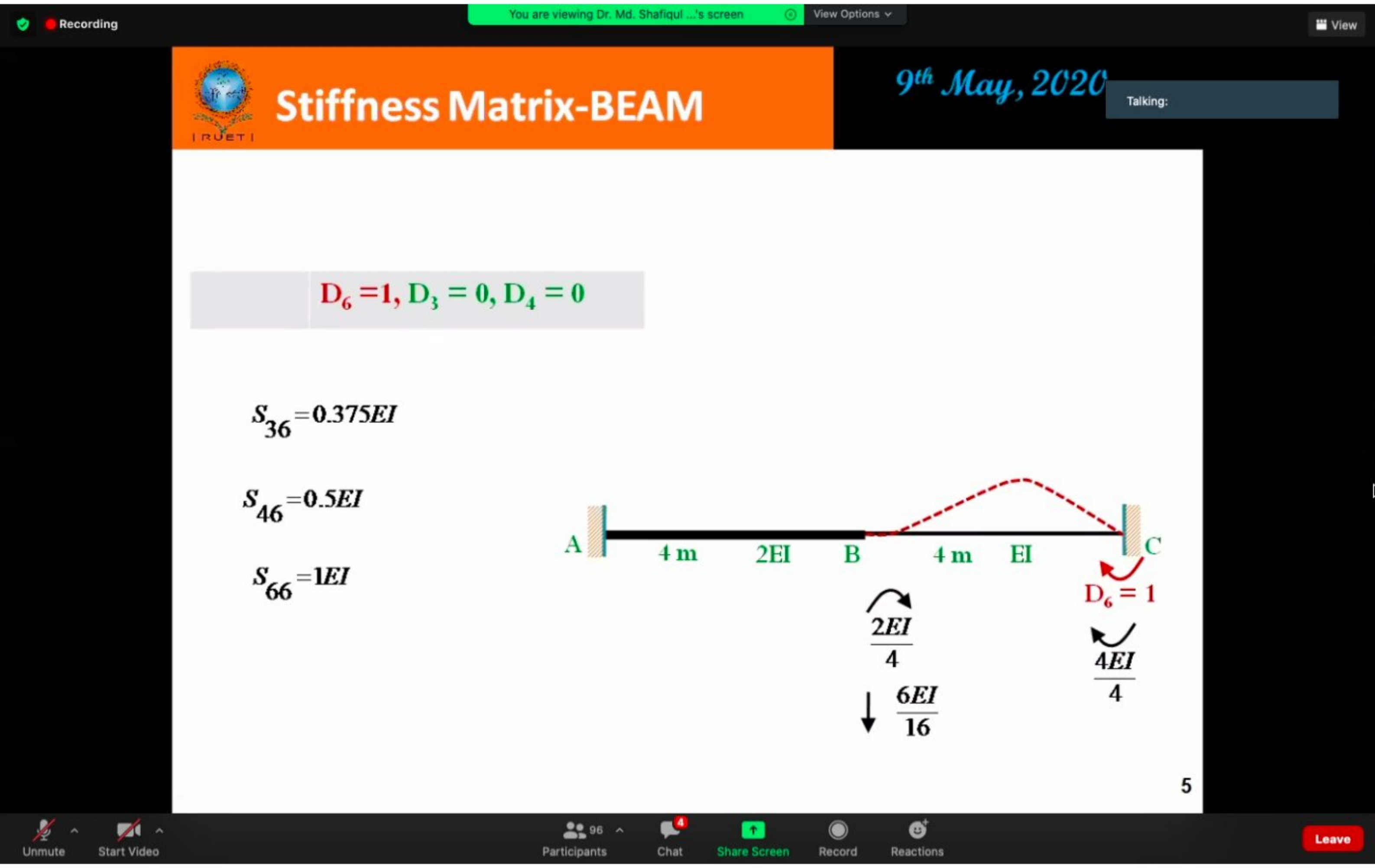
$$S_{43} = -\frac{12EI}{16} + \frac{6EI}{16} = \frac{-6EI}{16} = -0.375EI$$



Stiffness Matrix-BEAM

9th May, 2020

$D_4 = 1, D_3 = 0, D_6 = 0$



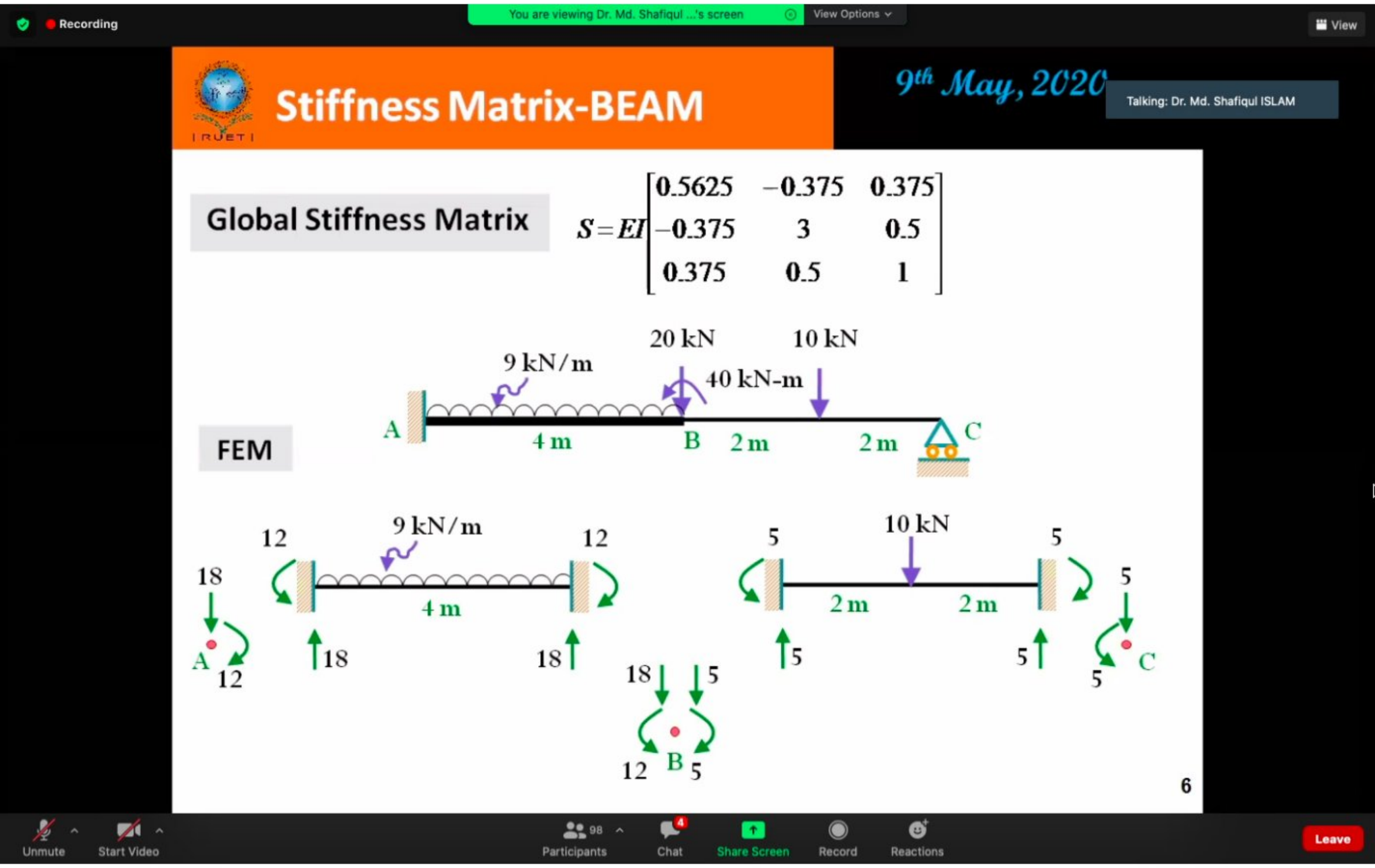
Stiffness Matrix-BEAM

$D_6 = 1, D_3 = 0, D_4 = 0$

$S_{36} = 0.375EI$

$S_{46} = 0.5EI$

$S_{66} = 1EI$



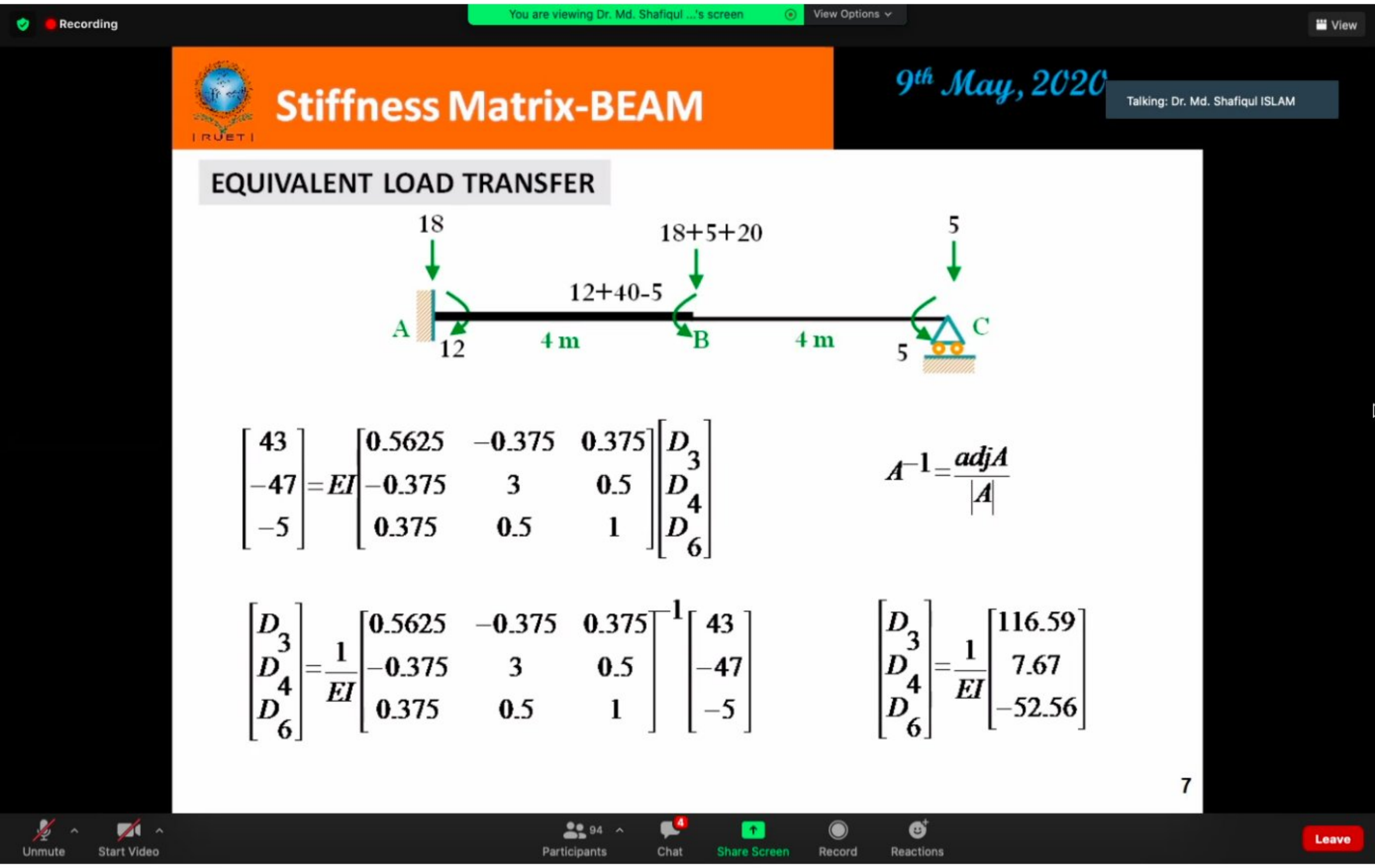
Stiffness Matrix-BEAM

9th May, 2020

Global Stiffness Matrix

$$S = EI\begin{bmatrix} 0.5625 & -0.375 & 0.375 \\ -0.375 & 3 & 0.5 \\ 0.375 & 0.5 & 1 \end{bmatrix}$$

FEM



Stiffness Matrix-BEAM

9th May, 2020

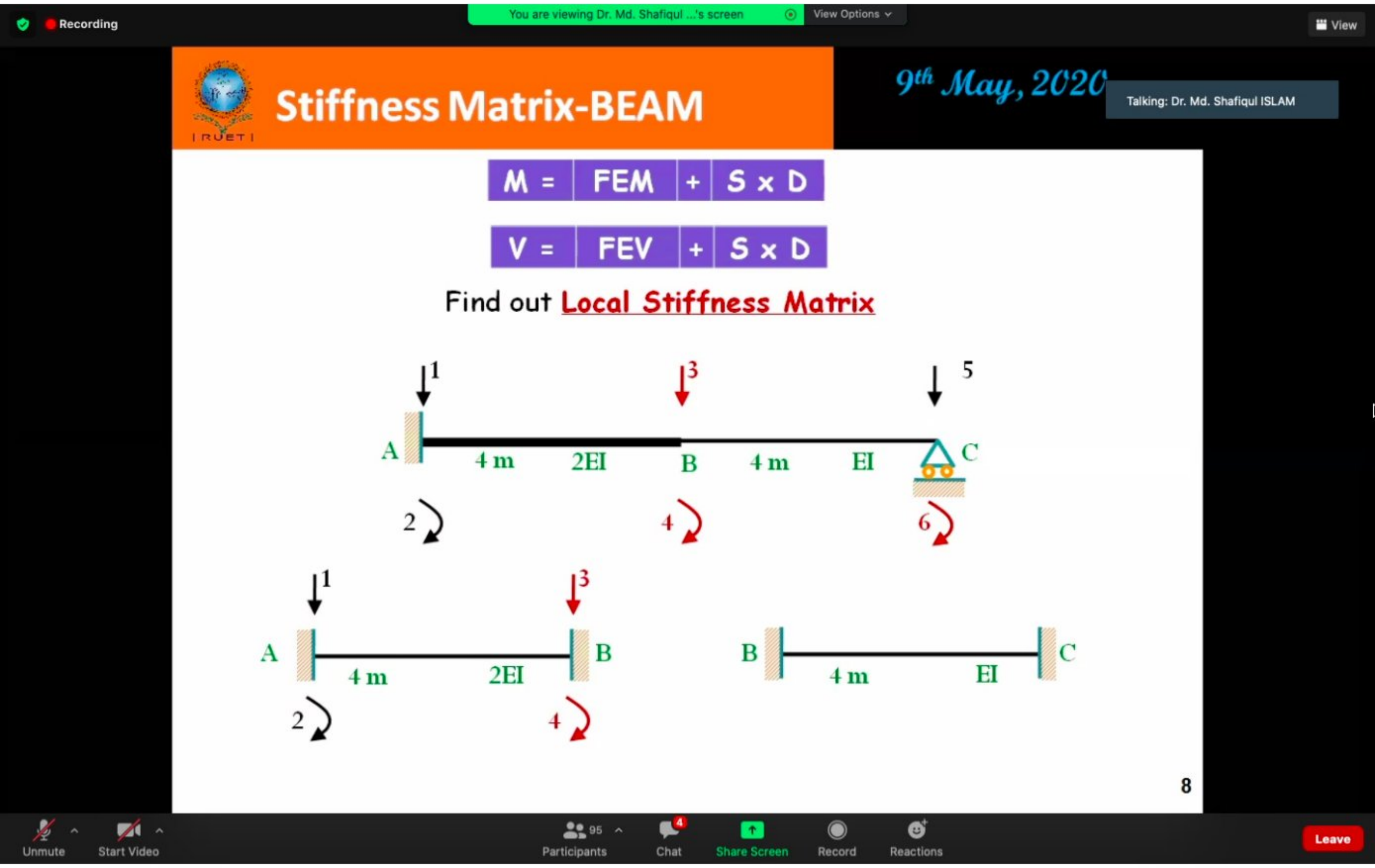
EQUIVALENT LOAD TRANSFER

$$\begin{bmatrix}43\\-47\\-5\end{bmatrix}=EI\begin{bmatrix}0.5625 & -0.375 & 0.375\\-0.375 & 3 & 0.5\\0.375 & 0.5 & 1\end{bmatrix}\begin{bmatrix}D_3\\D_4\\D_6\end{bmatrix}$$

$$A^{-1}=\frac{adjA}{|A|}$$

$$\begin{bmatrix}D_3\\D_4\\D_6\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}0.5625 & -0.375 & 0.375\\-0.375 & 3 & 0.5\\0.375 & 0.5 & 1\end{bmatrix}^{-1}\begin{bmatrix}43\\-47\\-5\end{bmatrix}$$

$$\begin{bmatrix}D_3\\D_4\\D_6\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}116.59\\7.67\\-52.56\end{bmatrix}$$



9th May, 2020

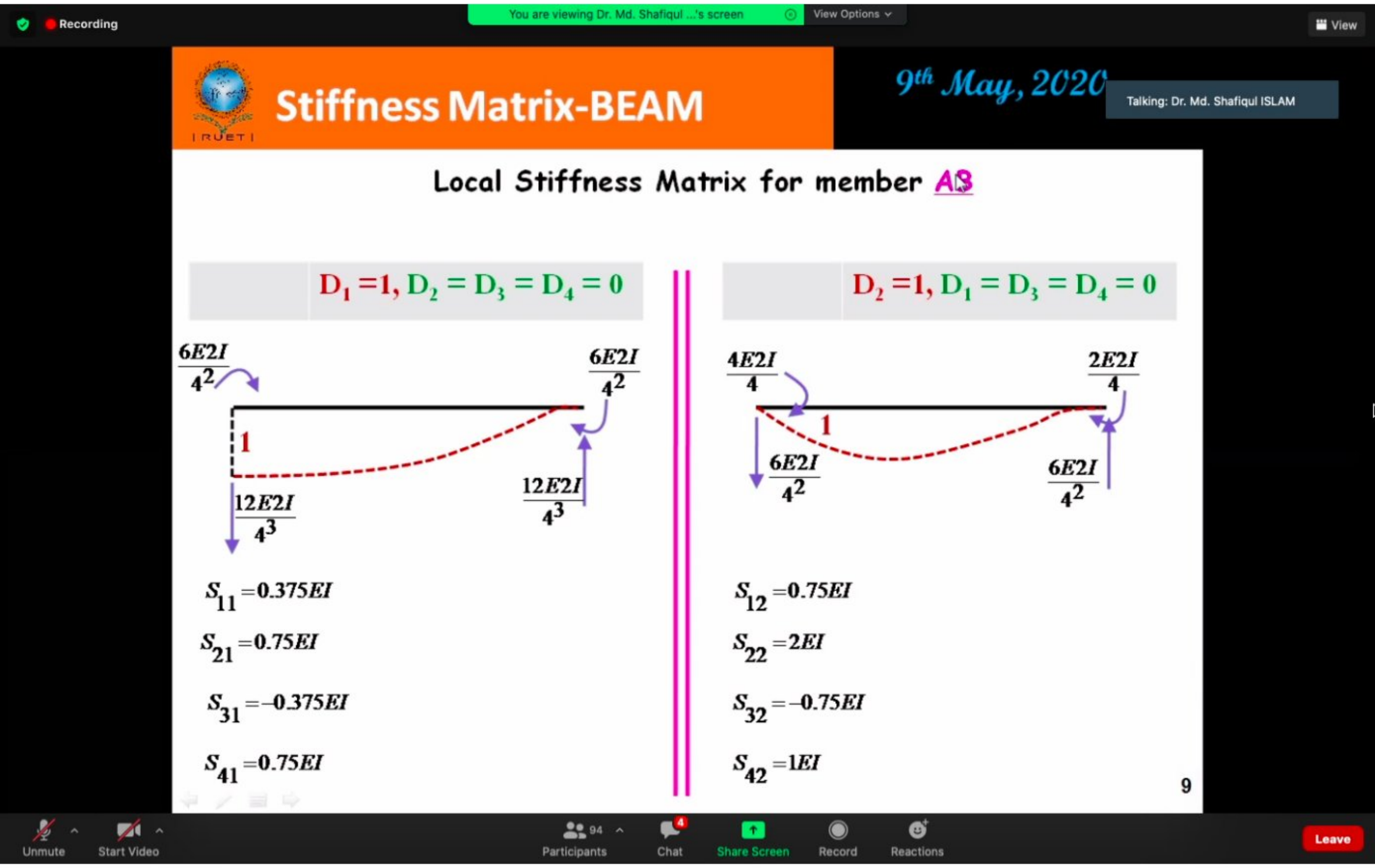
Stiffness Matrix-BEAM

M = FEM + S x D

V = FEV + S x D

Find out **Local Stiffness Matrix**

8



Stiffness Matrix-BEAM

9th May, 2020

Local Stiffness Matrix for member AB

$D_1 = 1, D_2 = D_3 = D_4 = 0$

$S_{11} = 0.375EI$

$S_{21} = 0.75EI$

$S_{31} = -0.375EI$

$S_{41} = 0.75EI$

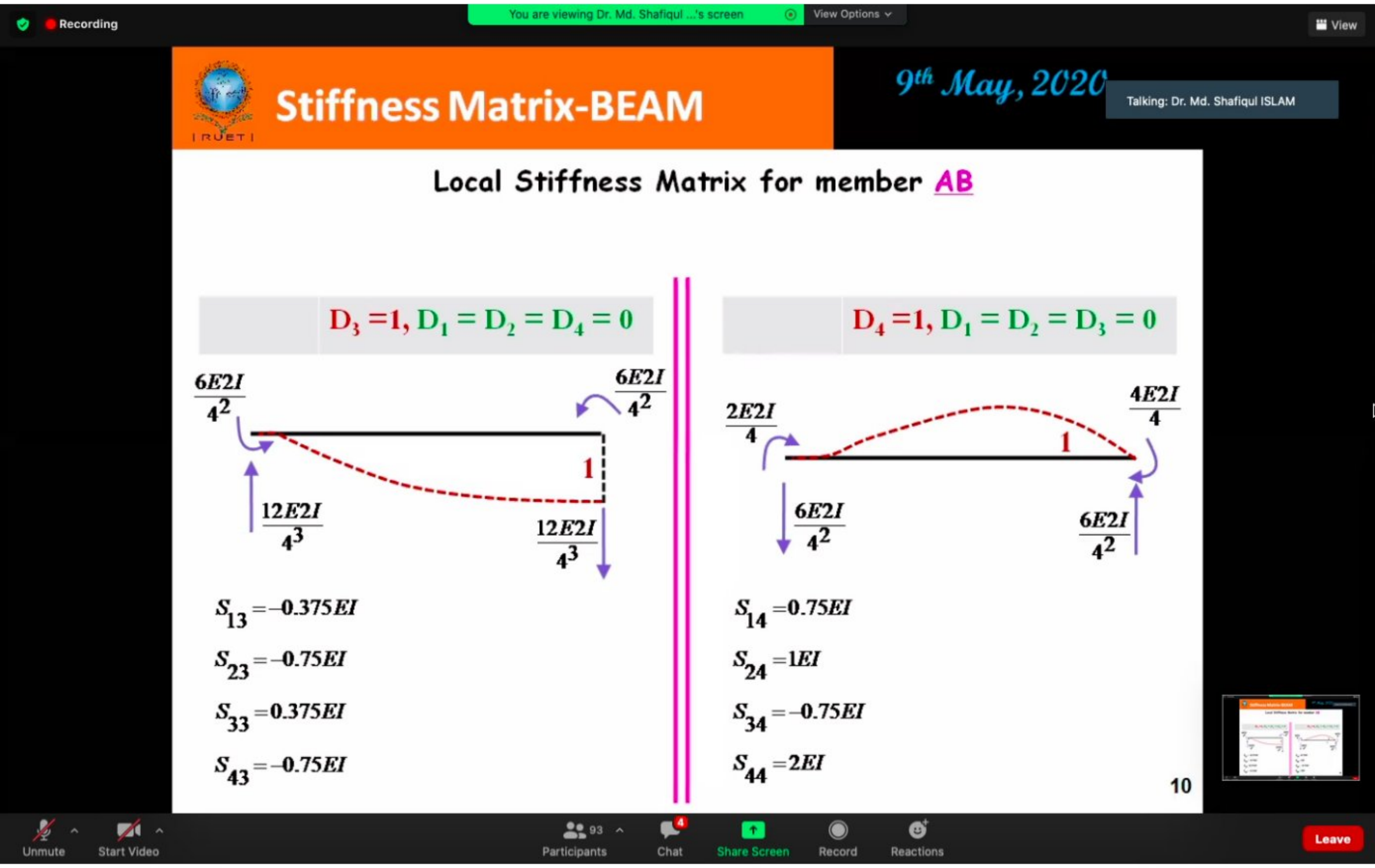
$D_2 = 1, D_1 = D_3 = D_4 = 0$

$S_{12} = 0.75EI$

$S_{22} = 2EI$

$S_{32} = -0.75EI$

$S_{42} = 1EI$



9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Stiffness Matrix-BEAM

Local Stiffness Matrix for member AB

$D_3 = 1, D_1 = D_2 = D_4 = 0$

$S_{13} = -0.375EI$

$S_{23} = -0.75EI$

$S_{33} = 0.375EI$

$S_{43} = -0.75EI$

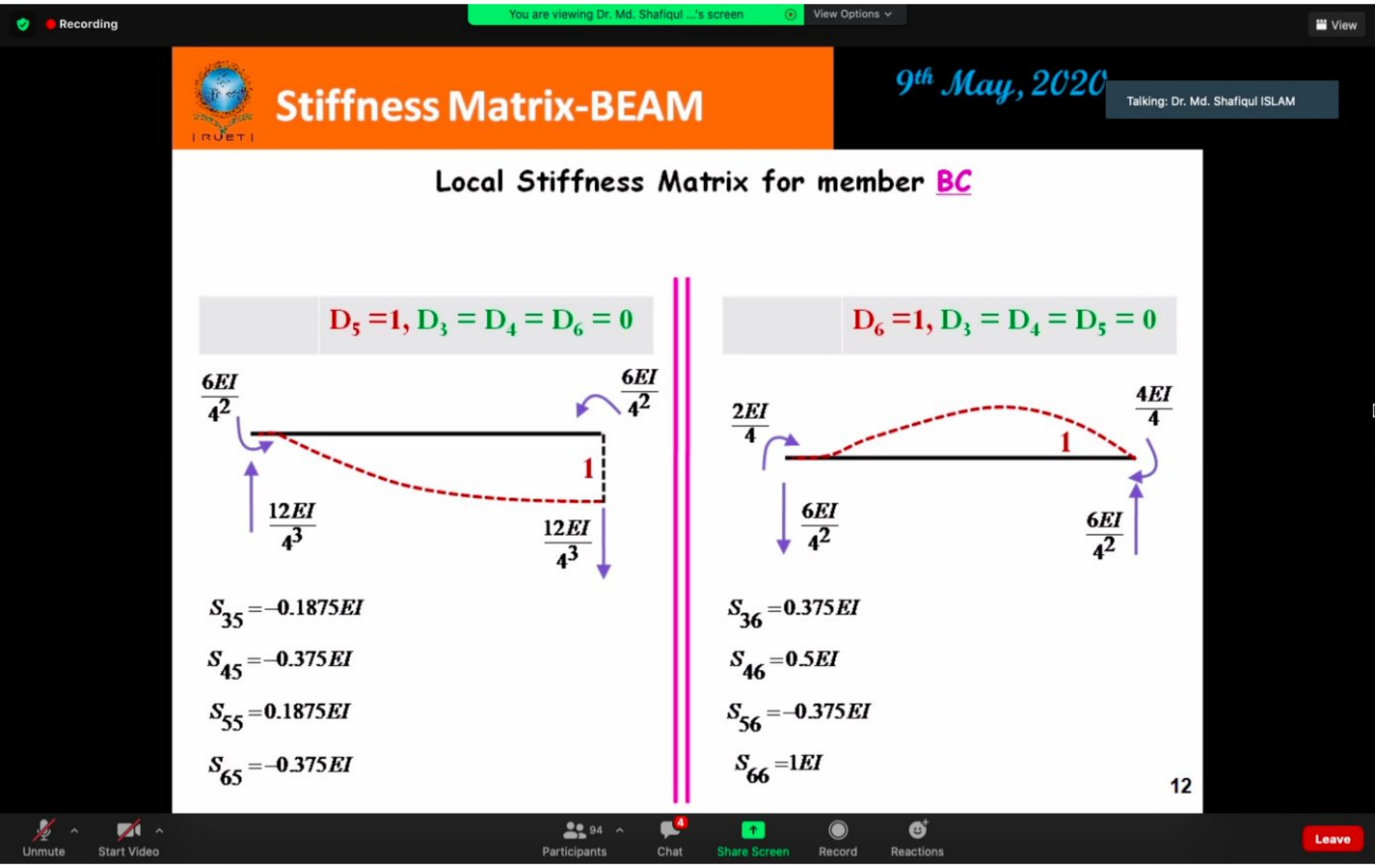
$D_4 = 1, D_1 = D_2 = D_3 = 0$

$S_{14} = 0.75EI$

$S_{24} = 1EI$

$S_{34} = -0.75EI$

$S_{44} = 2EI$



Stiffness Matrix-BEAM

9th May, 2020

Local Stiffness Matrix for member BC

$D_5 = 1, D_3 = D_4 = D_6 = 0$

$S_{35} = -0.1875EI$

$S_{45} = -0.375EI$

$S_{55} = 0.1875EI$

$S_{65} = -0.375EI$

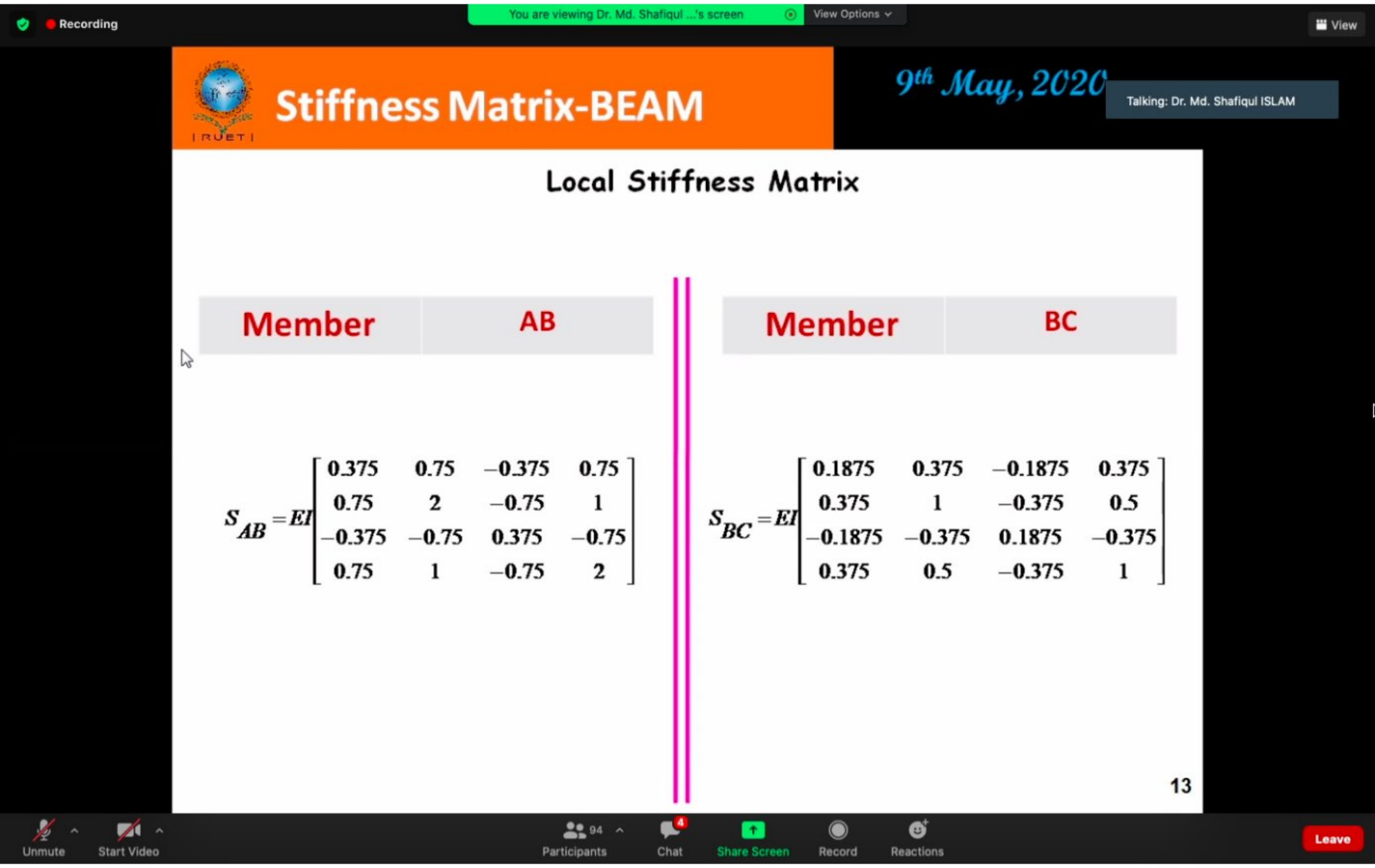
$D_6 = 1, D_3 = D_4 = D_5 = 0$

$S_{36} = 0.375EI$

$S_{46} = 0.5EI$

$S_{56} = -0.375EI$

$S_{66} = 1EI$



Stiffness Matrix-BEAM

9th May, 2020

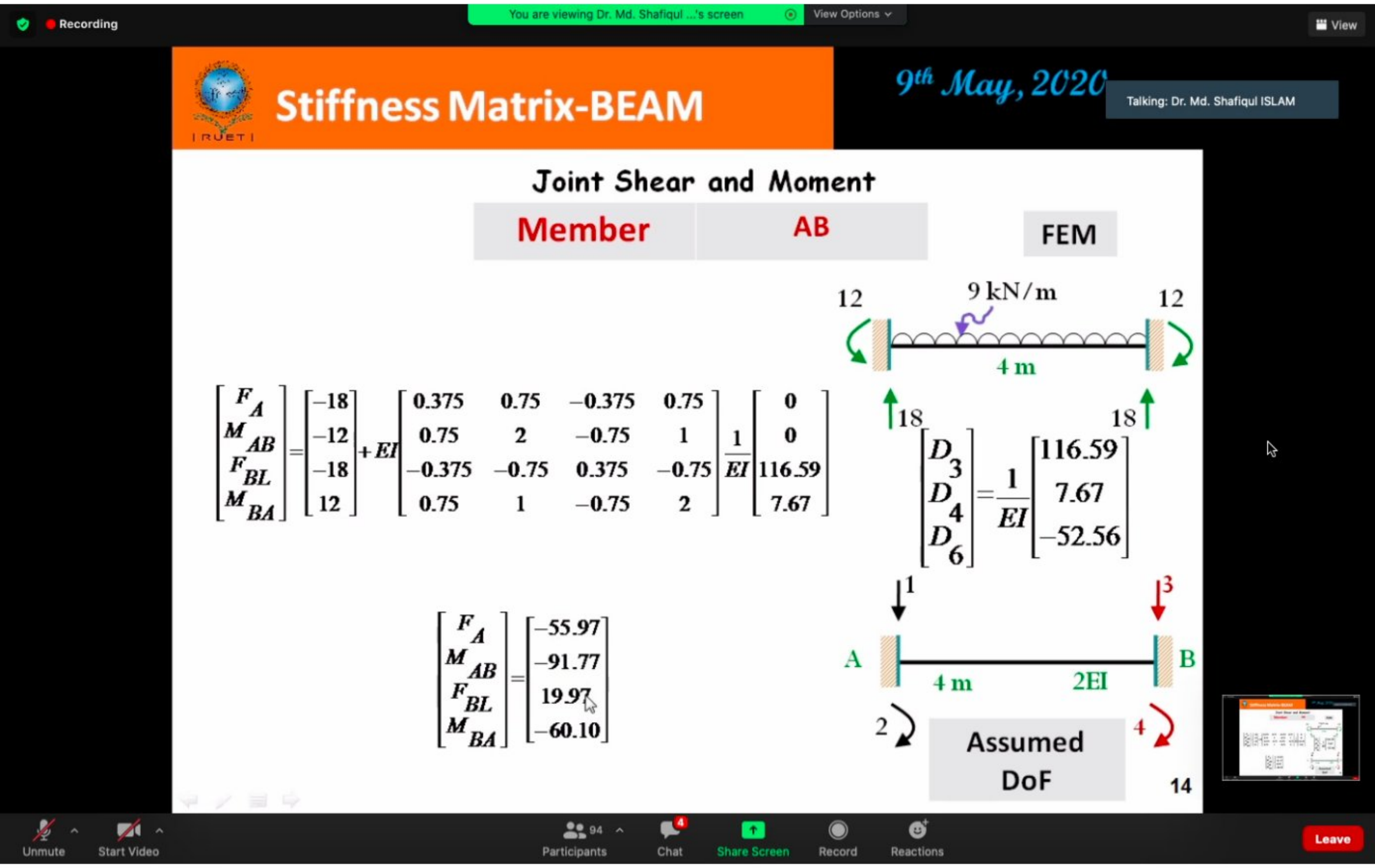
Local Stiffness Matrix

Member	AB

$$S_{AB} = EI\begin{bmatrix} 0.375 & 0.75 & -0.375 & 0.75 \\ 0.75 & 2 & -0.75 & 1 \\ -0.375 & -0.75 & 0.375 & -0.75 \\ 0.75 & 1 & -0.75 & 2 \end{bmatrix}$$

Member	BC

$$S_{BC} = EI\begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{bmatrix}$$



Stiffness Matrix-BEAM

9th May, 2020

Joint Shear and Moment

Member	AB

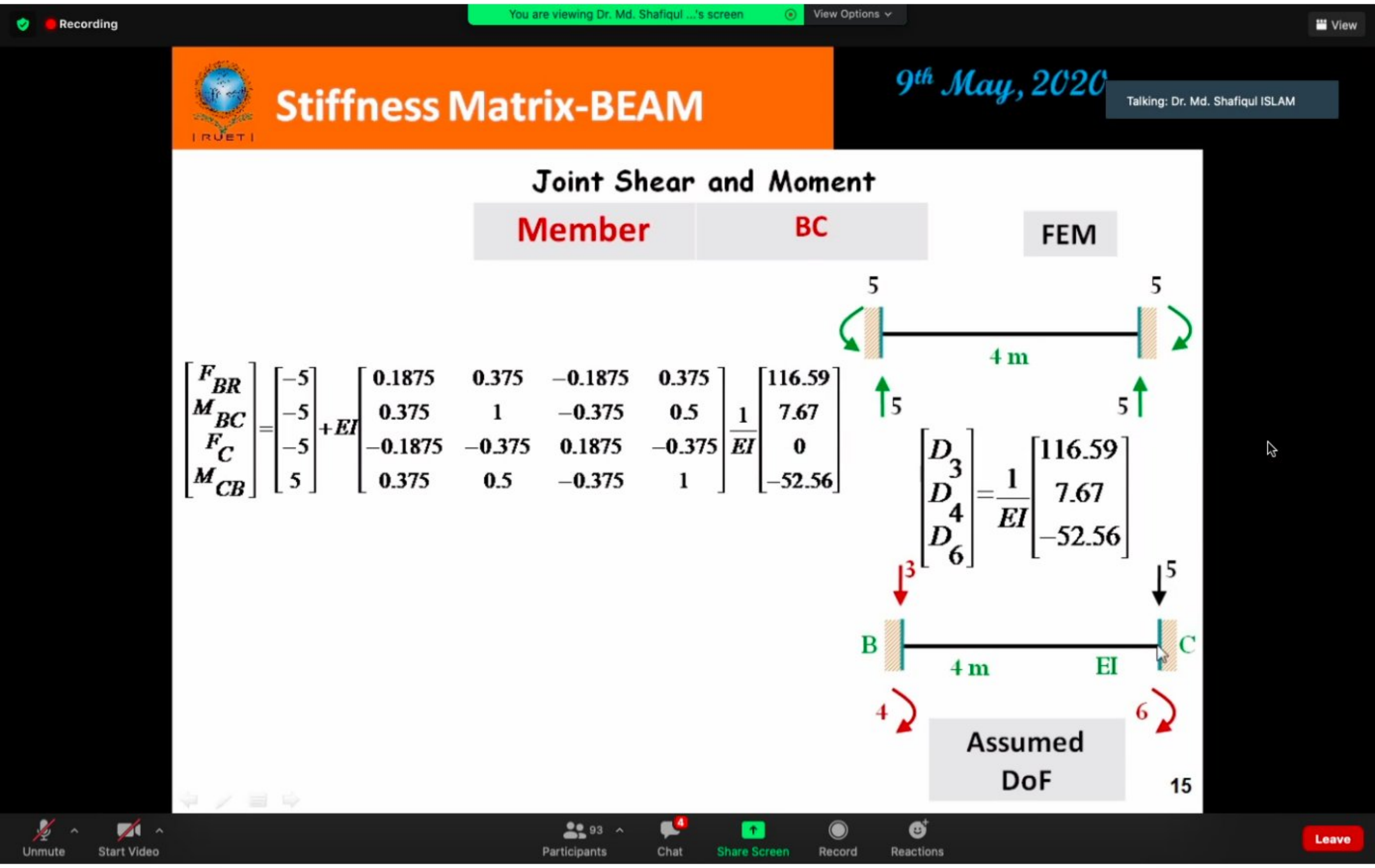
FEM

$$\begin{bmatrix}F_A\\M_{AB}\\F_{BL}\\M_{BA}\end{bmatrix}=\begin{bmatrix}-18\\-12\\-18\\12\end{bmatrix}+EI\begin{bmatrix}0.375&0.75&-0.375&0.75\\0.75&2&-0.75&1\\-0.375&-0.75&0.375&-0.75\\0.75&1&-0.75&2\end{bmatrix}\frac{1}{EI}\begin{bmatrix}0\\0\\116.59\\7.67\end{bmatrix}$$

$$\begin{bmatrix}D_3\\D_4\\D_6\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}116.59\\7.67\\-52.56\end{bmatrix}$$

$$\begin{bmatrix}F_A\\M_{AB}\\F_{BL}\\M_{BA}\end{bmatrix}=\begin{bmatrix}-55.97\\-91.77\\19.97\\-60.10\end{bmatrix}$$

Assumed DoF



Stiffness Matrix-BEAM

9th May, 2020

Joint Shear and Moment

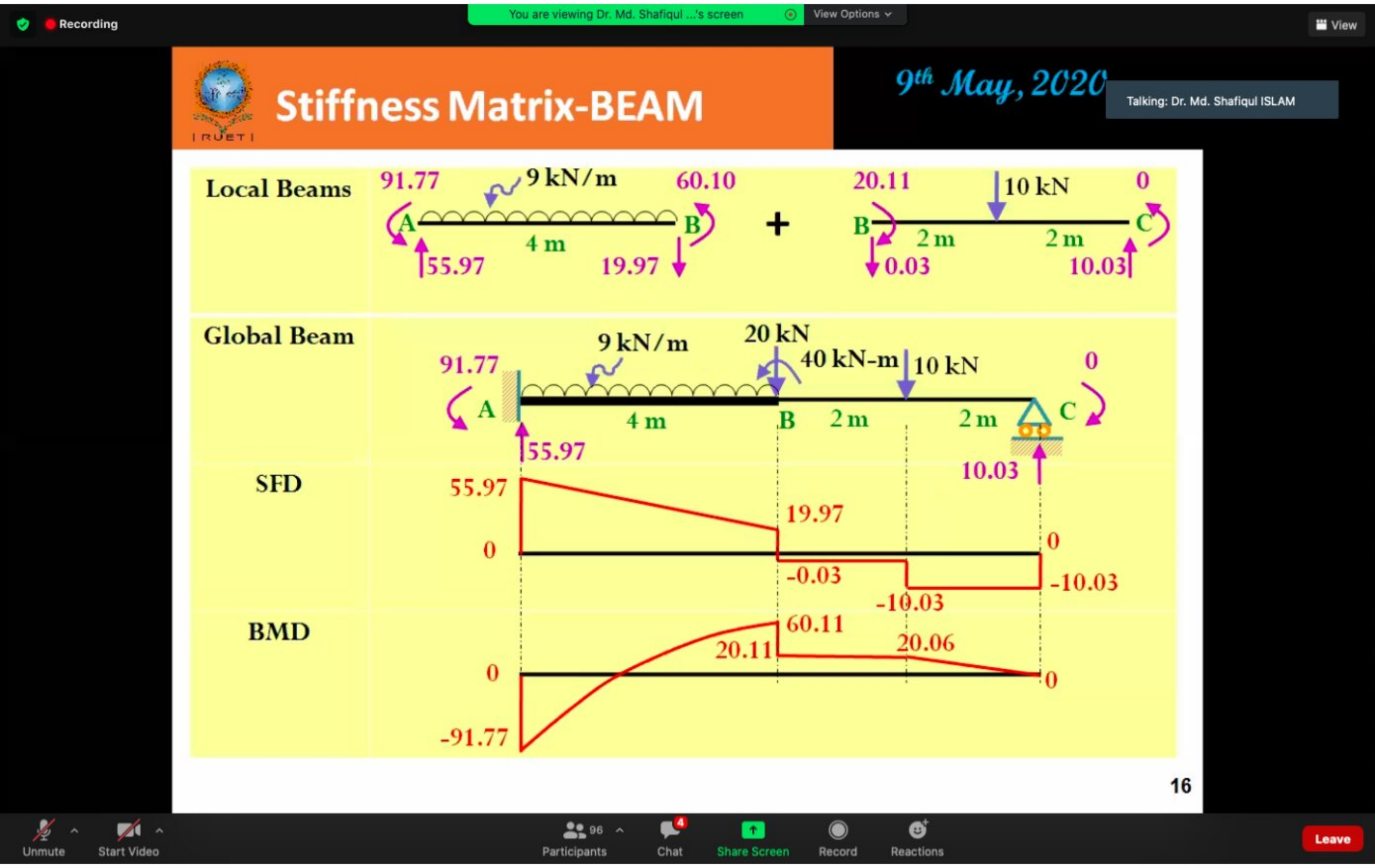
Member	BC

FEM

$$\begin{bmatrix} F_{BR} \\ M_{BC} \\ F_C \\ M_{CB} \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ -5 \\ 5 \end{bmatrix} + EI \begin{bmatrix} 0.1875 & 0.375 & -0.1875 & 0.375 \\ 0.375 & 1 & -0.375 & 0.5 \\ -0.1875 & -0.375 & 0.1875 & -0.375 \\ 0.375 & 0.5 & -0.375 & 1 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 116.59 \\ 7.67 \\ 0 \\ -52.56 \end{bmatrix}$$

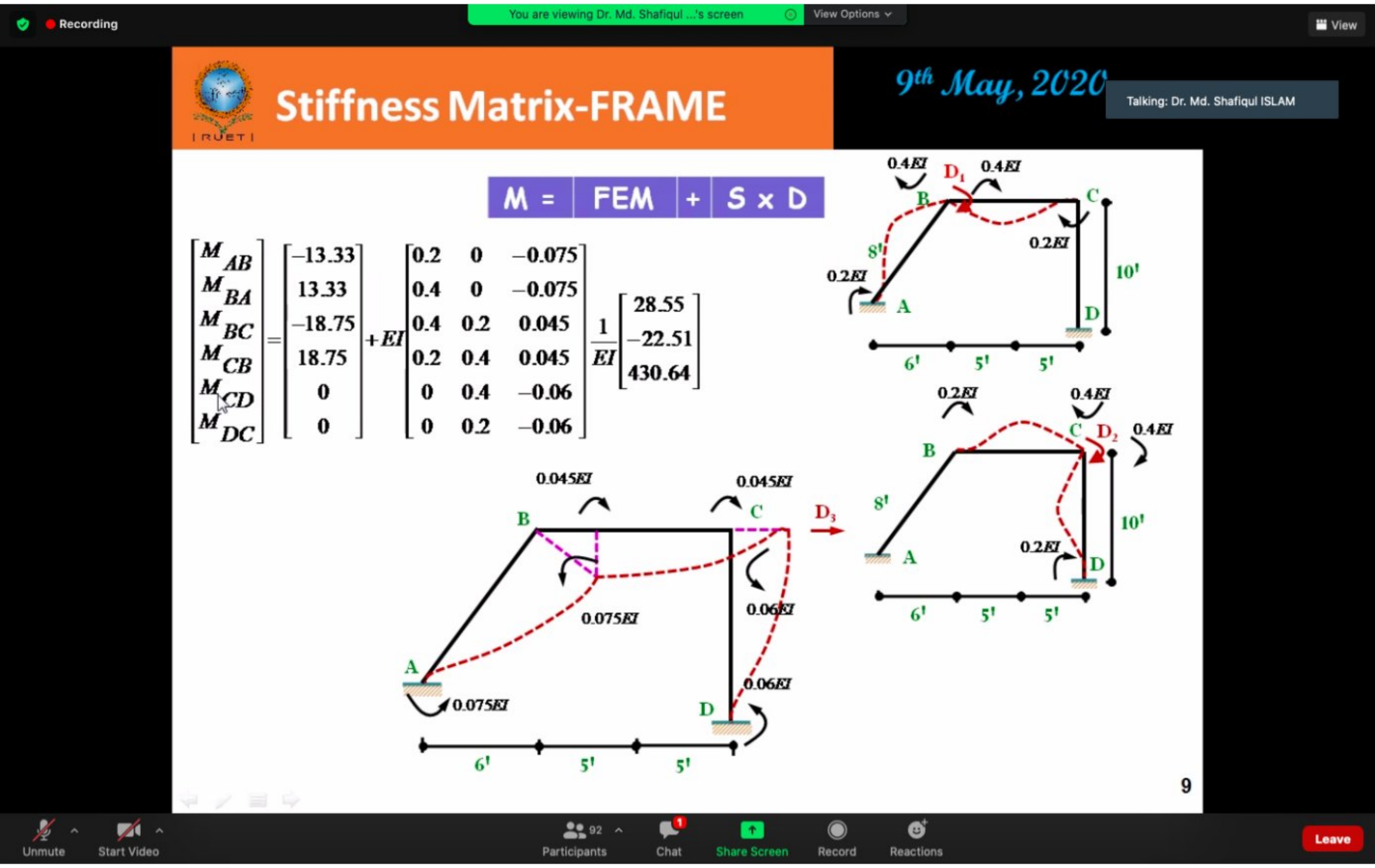
$$\begin{bmatrix} D_3 \\ D_4 \\ D_6 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 116.59 \\ 7.67 \\ -52.56 \end{bmatrix}$$

Assumed DoF



Stiffness Matrix-BEAM

9th May, 2020

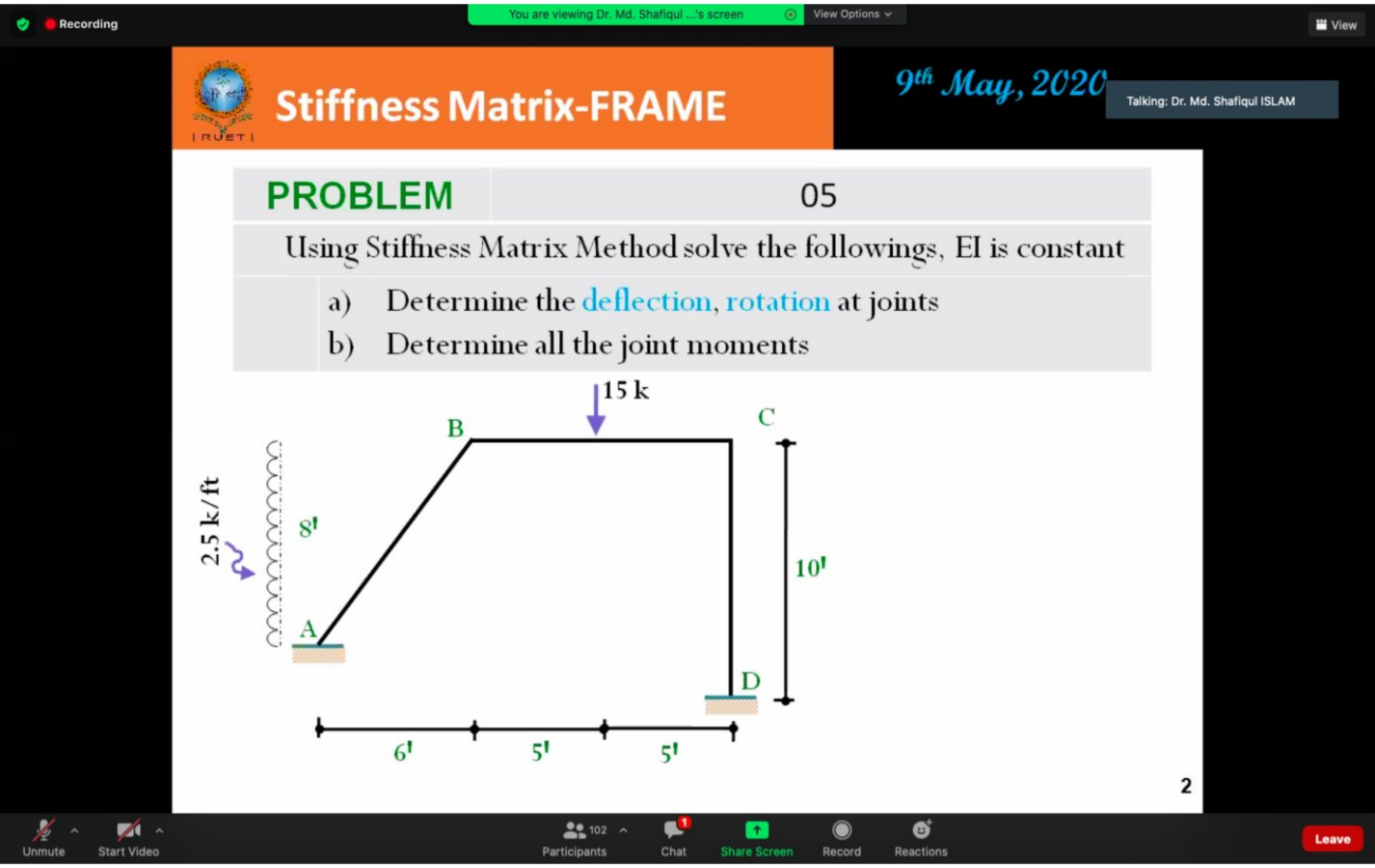


Stiffness Matrix-FRAME

9th May, 2020

M = FEM + S x D

$$\begin{bmatrix} M_{AB}\\ M_{BA}\\ M_{BC}\\ M_{CB}\\ M_{CD}\\ M_{DC}\end{bmatrix} = \begin{bmatrix}-13.33\\ 13.33\\ -18.75\\ 18.75\\ 0\\ 0\end{bmatrix} + EI\begin{bmatrix}0.2 & 0 & -0.075\\ 0.4 & 0 & -0.075\\ 0.4 & 0.2 & 0.045\\ 0.2 & 0.4 & 0.045\\ 0 & 0.4 & -0.06\\ 0 & 0.2 & -0.06\end{bmatrix}\frac{1}{EI}\begin{bmatrix}28.55\\ -22.51\\ 430.64\end{bmatrix}$$



Stiffness Matrix-FRAME

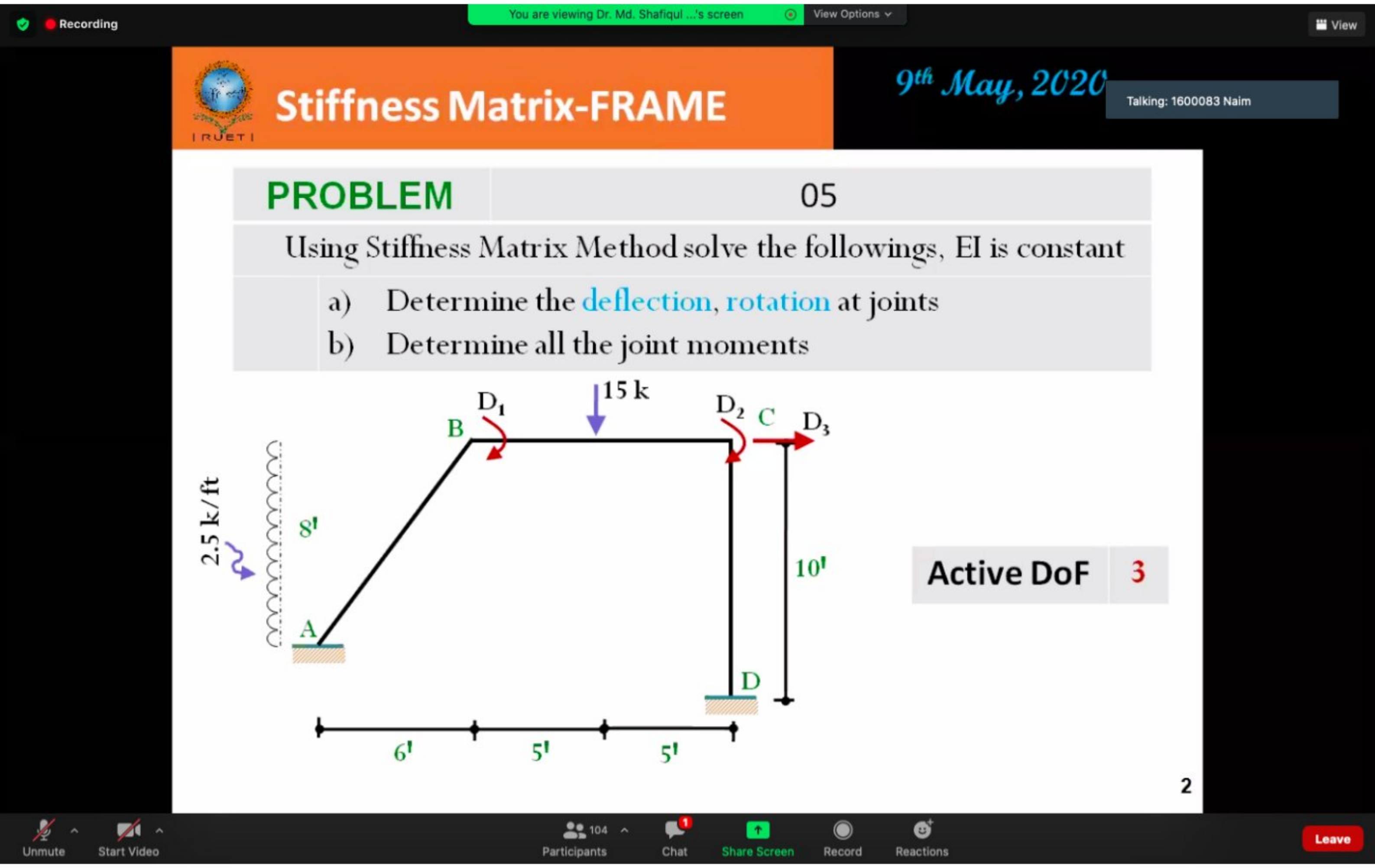
9th May, 2020

PROBLEM 05

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the deflection, rotation at joints

b) Determine all the joint moments



Stiffness Matrix-FRAME

9th May, 2020

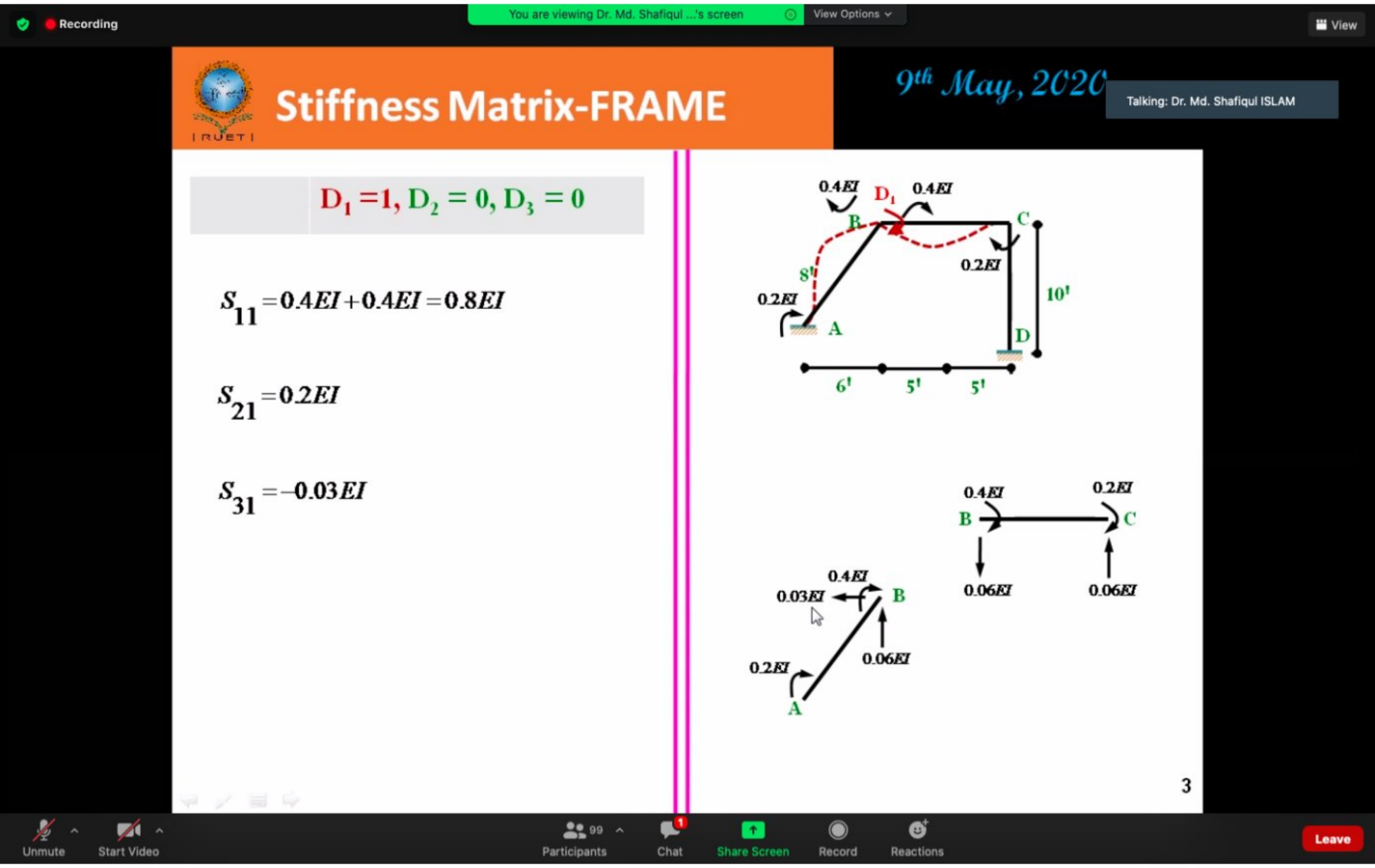
PROBLEM	05

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the deflection, rotation at joints

b) Determine all the joint moments

Active DoF	3



Stiffness Matrix-FRAME

9th May, 2020

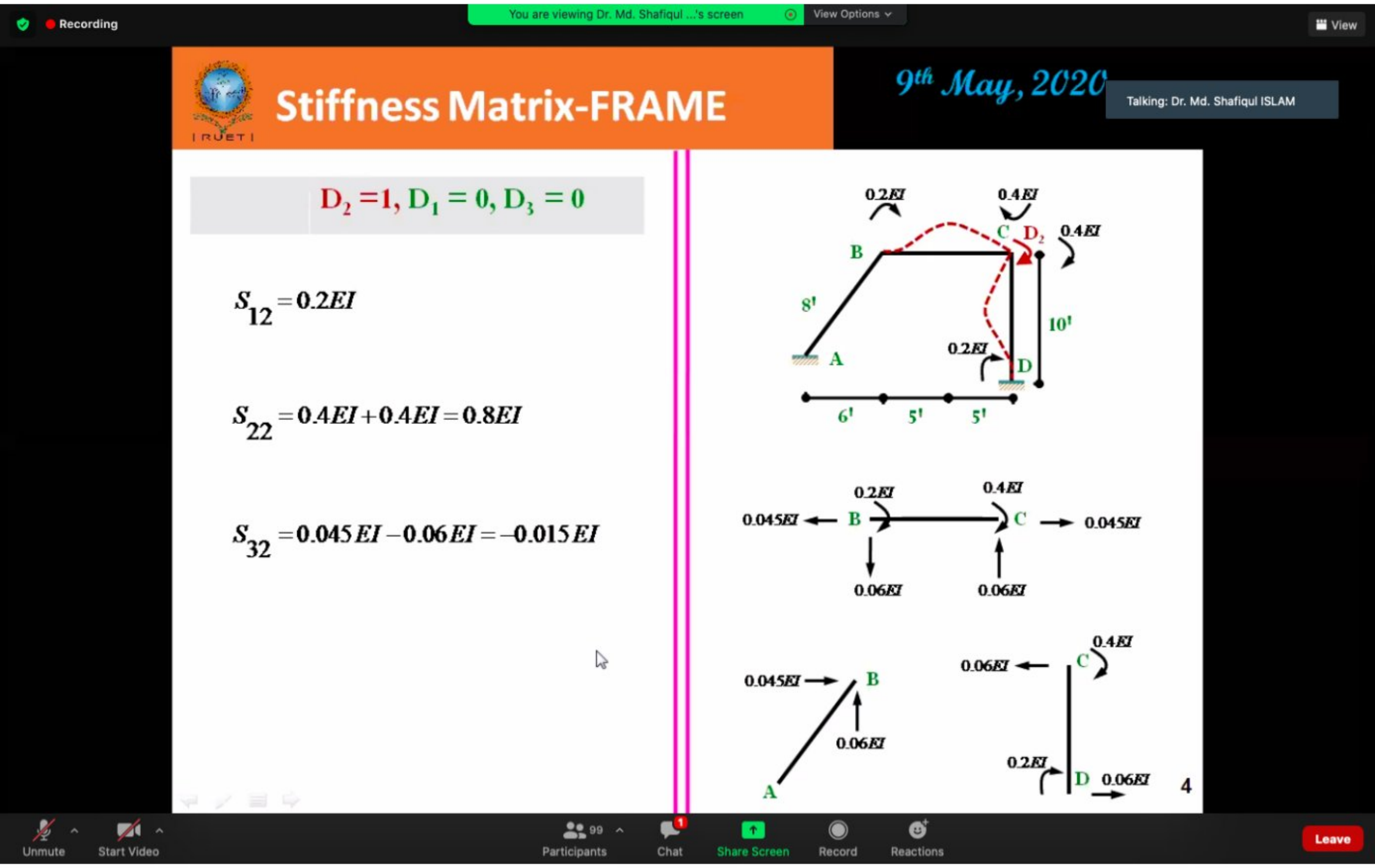
$D_1 = 1, D_2 = 0, D_3 = 0$

$S_{11} = 0.4EI + 0.4EI = 0.8EI$

$S_{21} = 0.2EI$

$S_{31} = -0.03EI$

3



Stiffness Matrix-FRAME

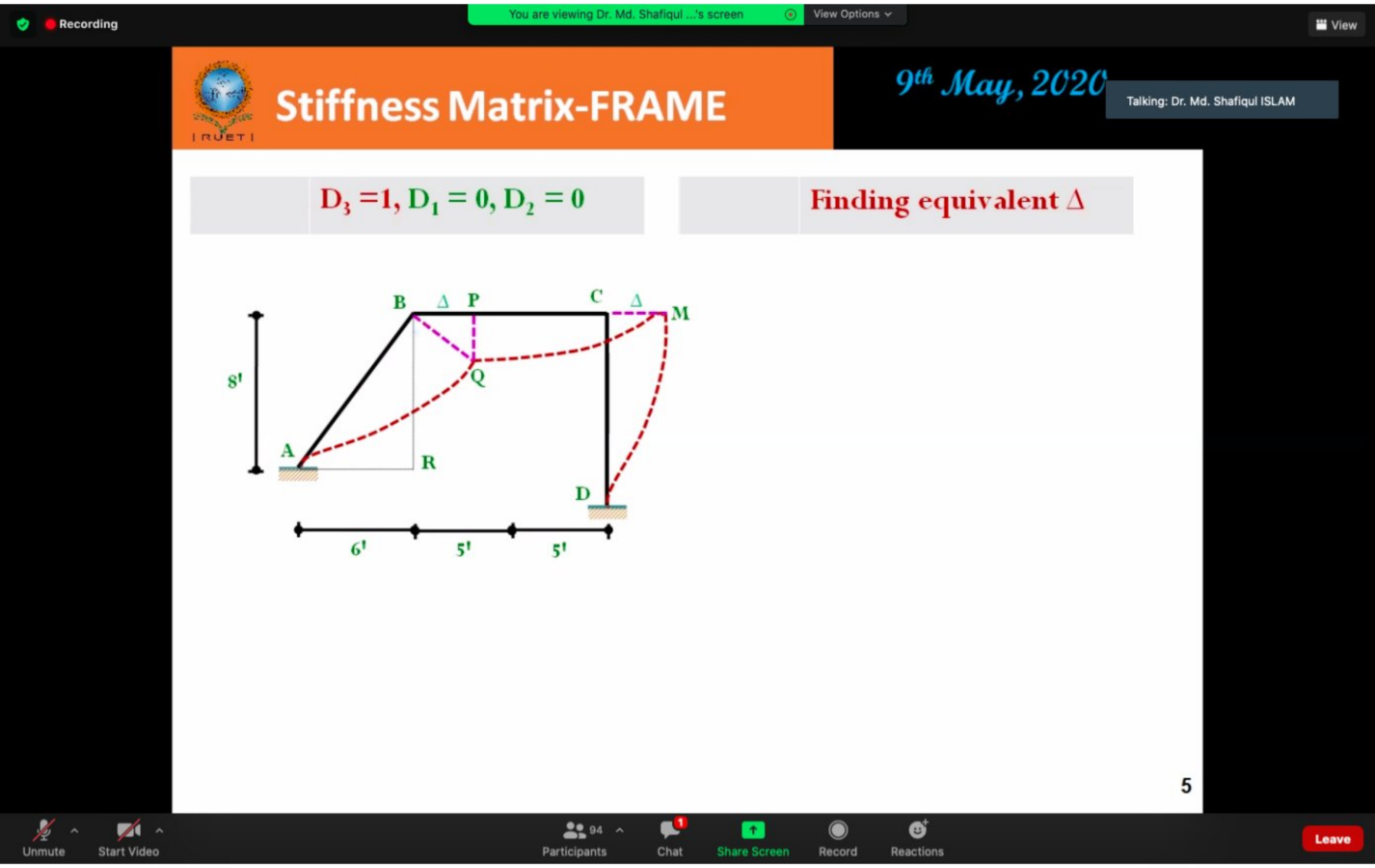
9th May, 2020

$D_2 = 1, D_1 = 0, D_3 = 0$

$$S_{12} = 0.2EI$$

$$S_{22} = 0.4EI + 0.4EI = 0.8EI$$

$$S_{32} = 0.045EI - 0.06EI = -0.015EI$$

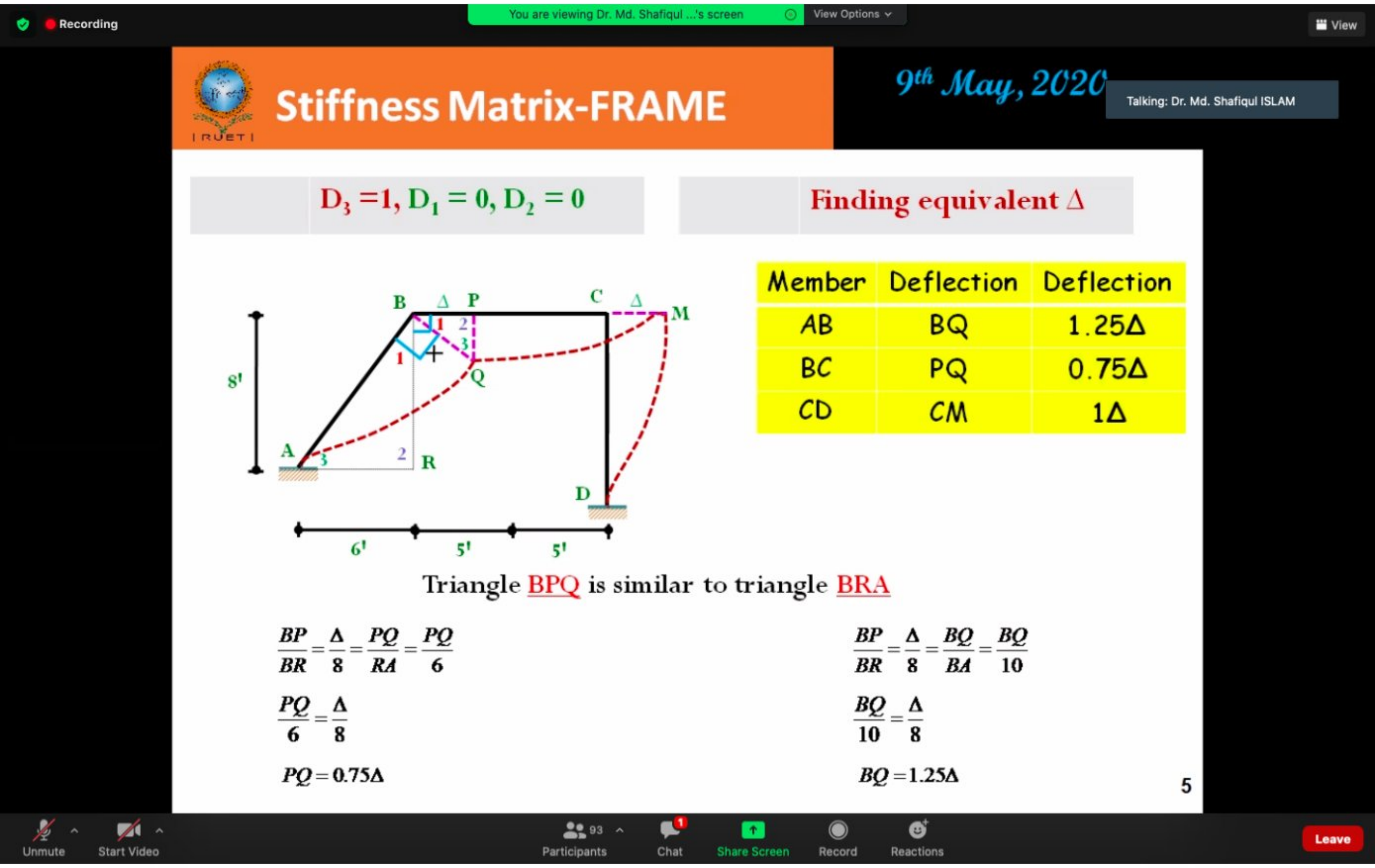


Stiffness Matrix-FRAME

9th May, 2020

$D_3 = 1, D_1 = 0, D_2 = 0$

Finding equivalent Δ



Stiffness Matrix-FRAME

9th May, 2020

$D_3 = 1, D_1 = 0, D_2 = 0$

Finding equivalent Δ

Member	Deflection	Deflection
AB	BQ	1.25Δ
BC	PQ	0.75Δ
CD	CM	1Δ

Triangle BPQ is similar to triangle BRA

$$\frac{BP}{BR} = \frac{\Delta}{8} = \frac{PQ}{RA} = \frac{PQ}{6}$$

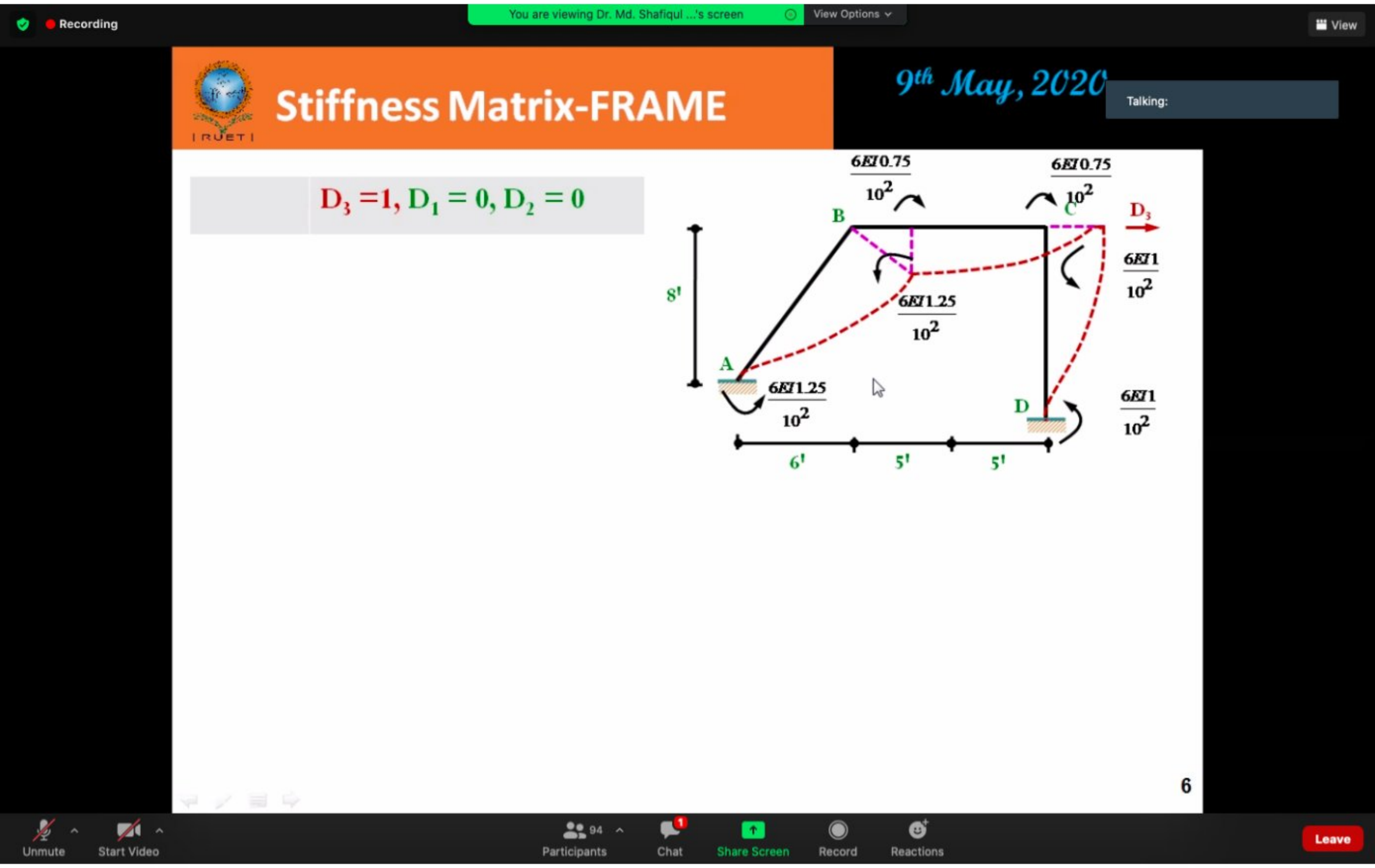
$$\frac{PQ}{6} = \frac{\Delta}{8}$$

$$PQ = 0.75\Delta$$

$$\frac{BP}{BR} = \frac{\Delta}{8} = \frac{BQ}{BA} = \frac{BQ}{10}$$

$$\frac{BQ}{10} = \frac{\Delta}{8}$$

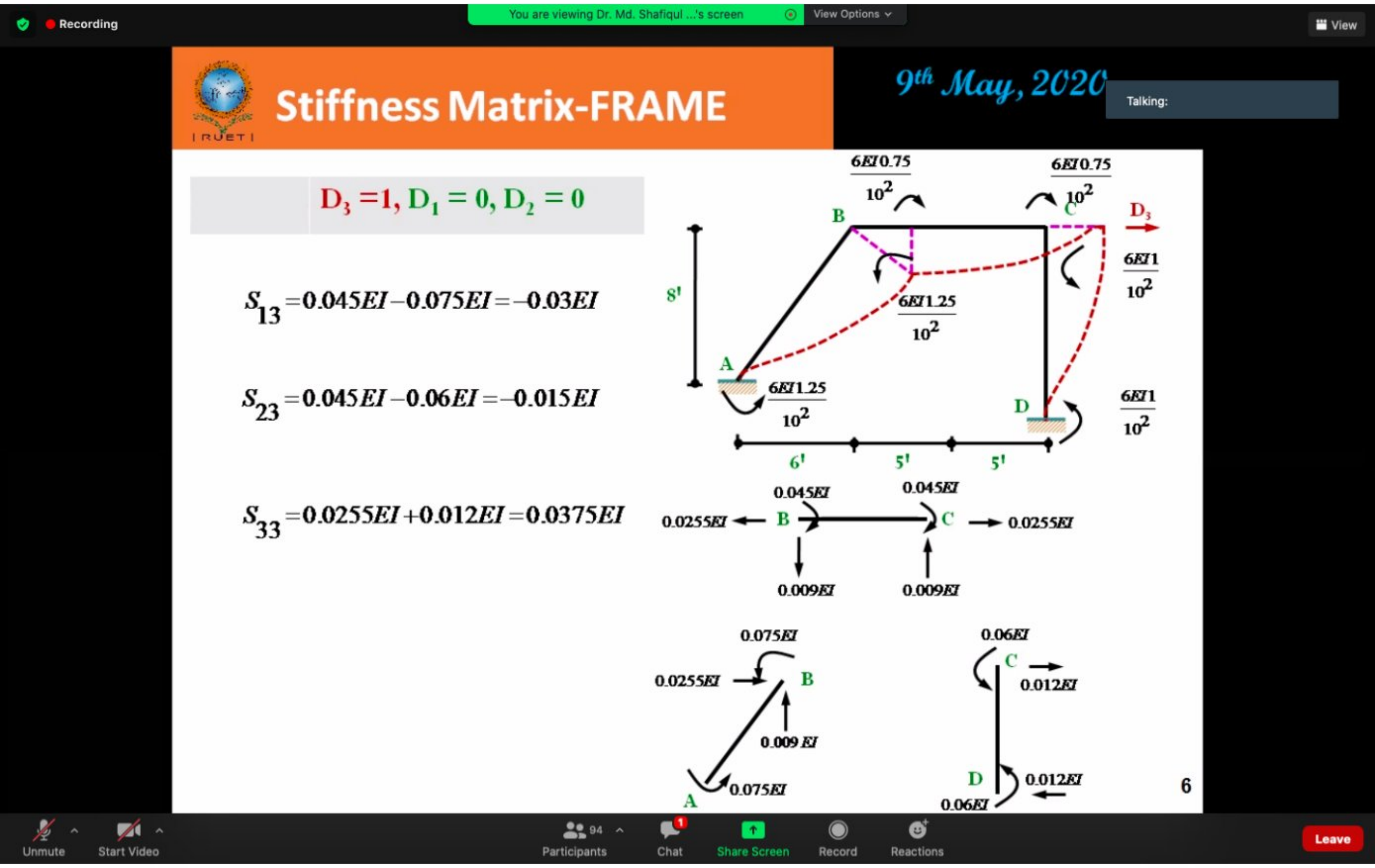
$$BQ = 1.25\Delta$$



Stiffness Matrix-FRAME

9th May, 2020

$D_3 = 1, D_1 = 0, D_2 = 0$



Stiffness Matrix-FRAME

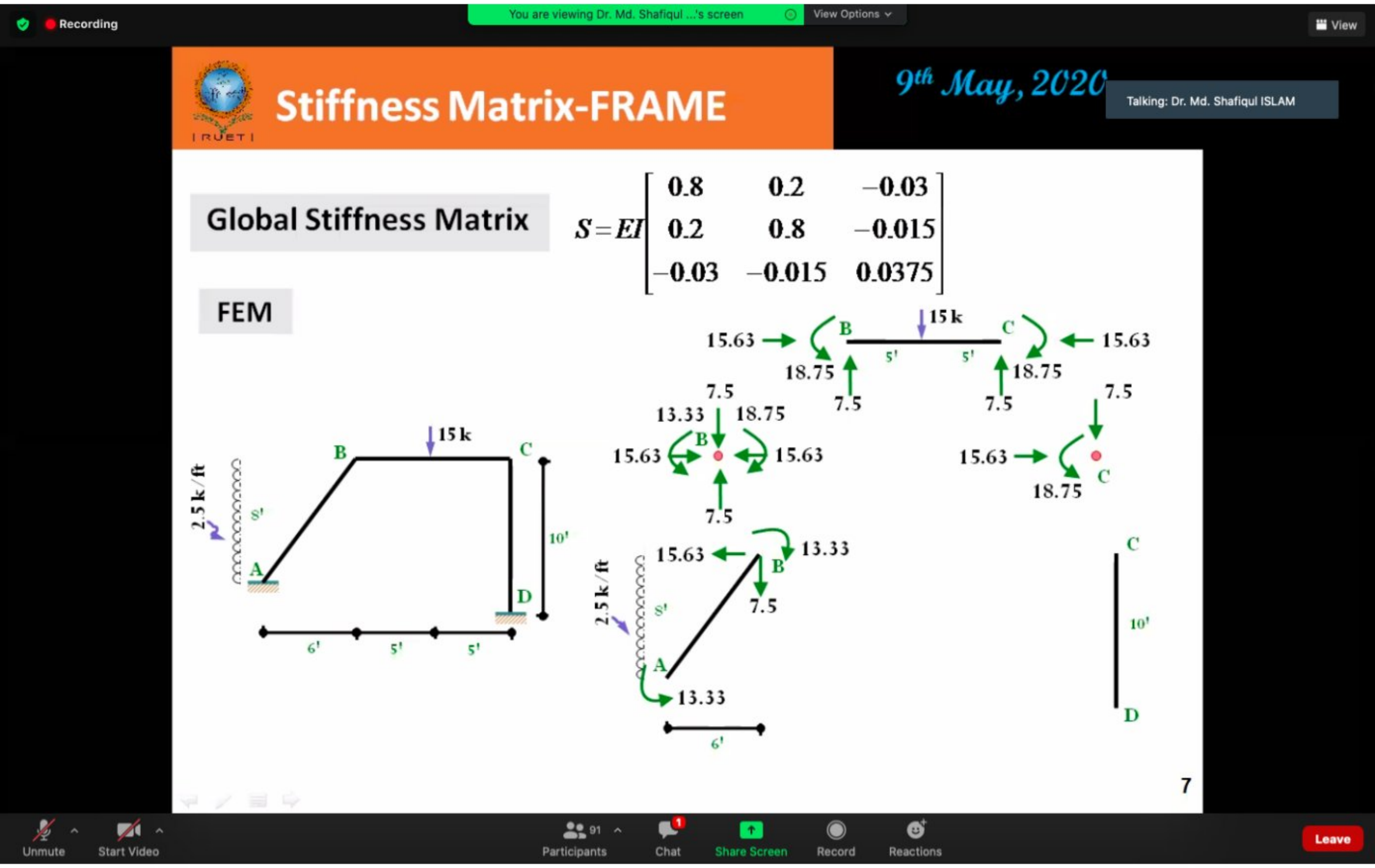
9th May, 2020

$D_3 = 1, D_1 = 0, D_2 = 0$

$$S_{13} = 0.045EI - 0.075EI = -0.03EI$$

$$S_{23} = 0.045EI - 0.06EI = -0.015EI$$

$$S_{33} = 0.0255EI + 0.012EI = 0.0375EI$$



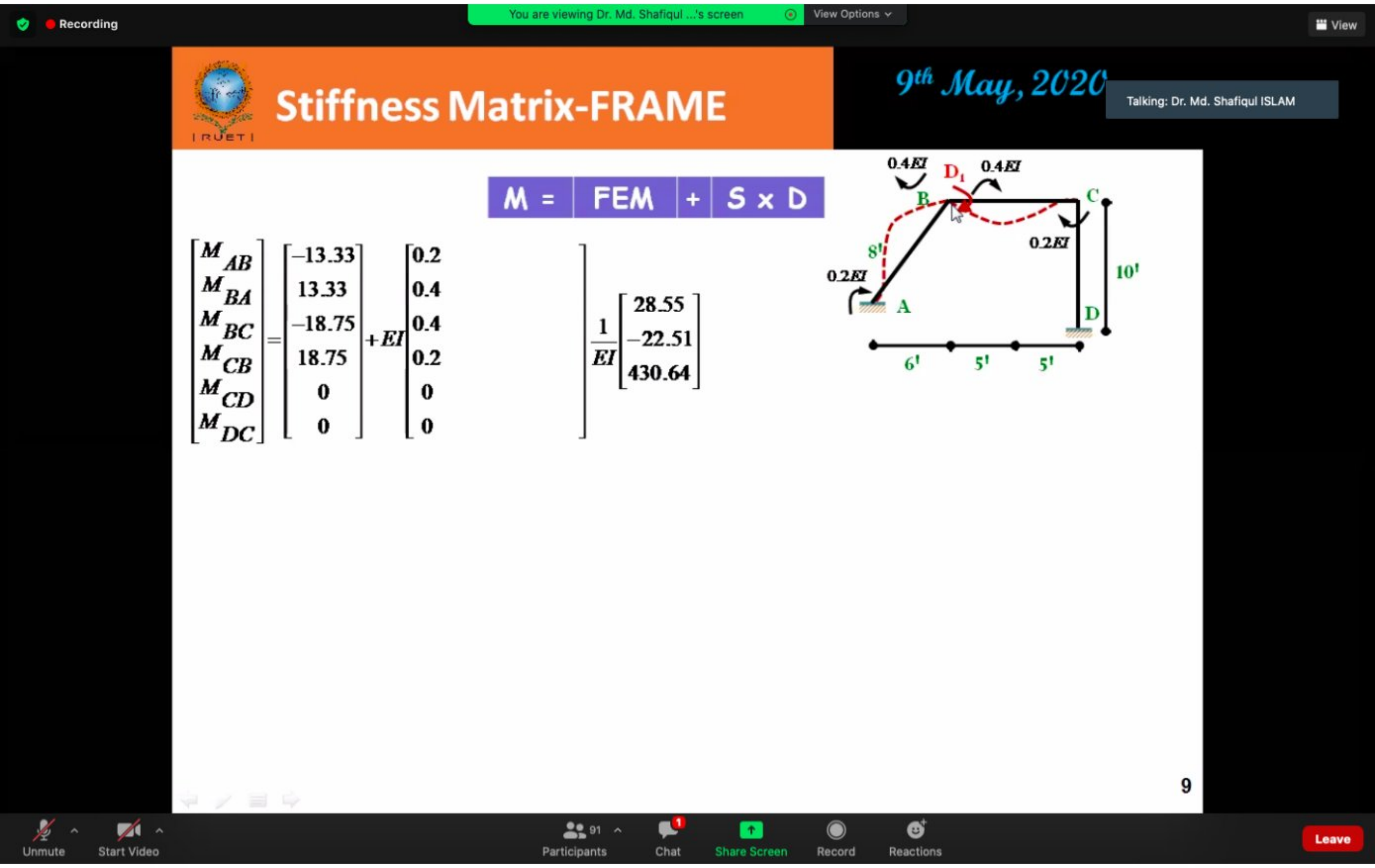
Stiffness Matrix-FRAME

9th May, 2020

Global Stiffness Matrix

$$S = EI\begin{bmatrix} 0.8 & 0.2 & -0.03 \\ 0.2 & 0.8 & -0.015 \\ -0.03 & -0.015 & 0.0375 \end{bmatrix}$$

FEM

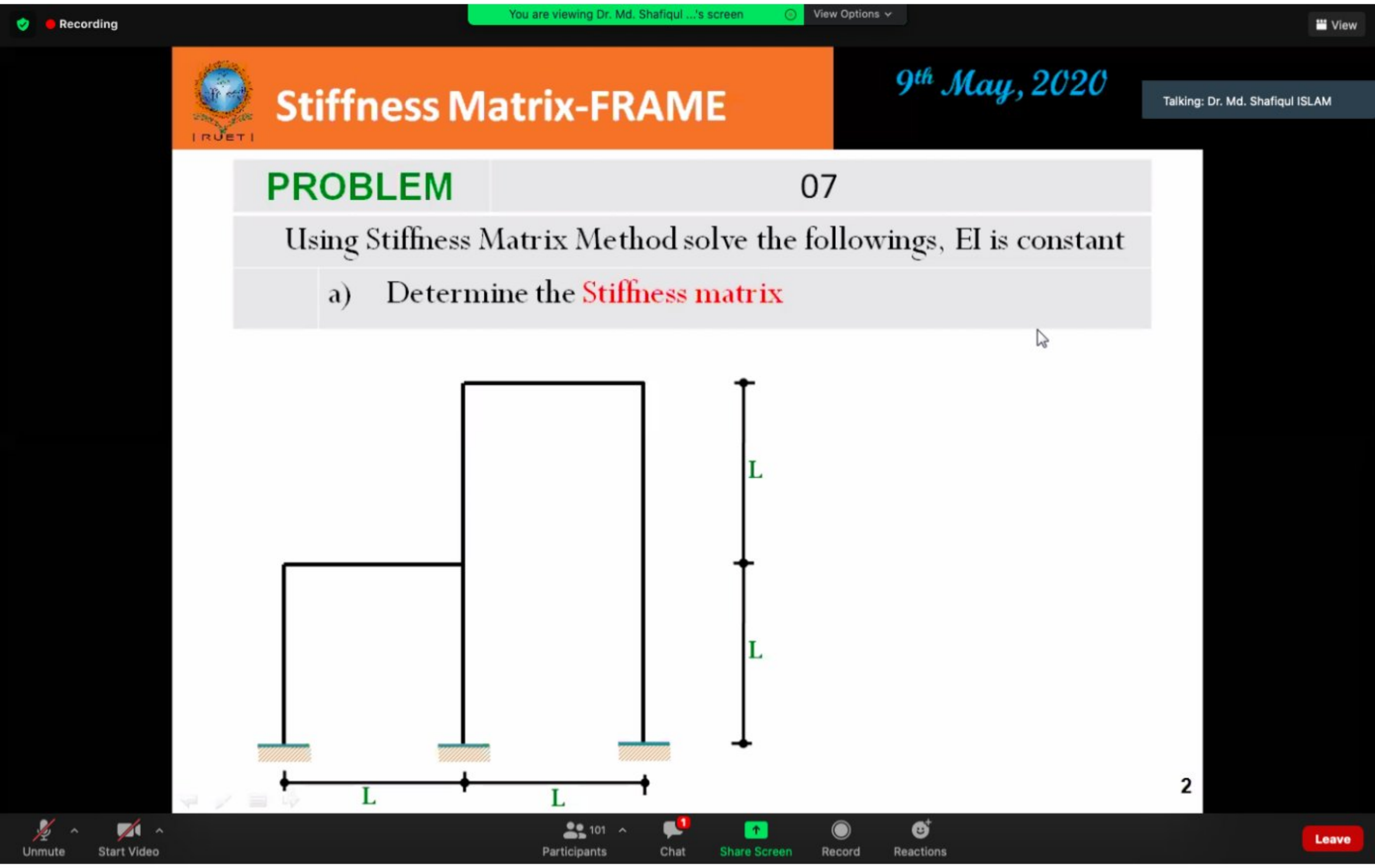


Stiffness Matrix-FRAME

9th May, 2020

M = FEM + S x D

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{bmatrix} = \begin{bmatrix} -13.33 \\ 13.33 \\ -18.75 \\ 18.75 \\ 0 \\ 0 \end{bmatrix} + EI \begin{bmatrix} 0.2 & & \\ 0.4 & & \\ 0.4 & & \\ 0.2 & & \\ 0 & & \\ 0 & & \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 28.55 \\ -22.51 \\ 430.64 \end{bmatrix}$$



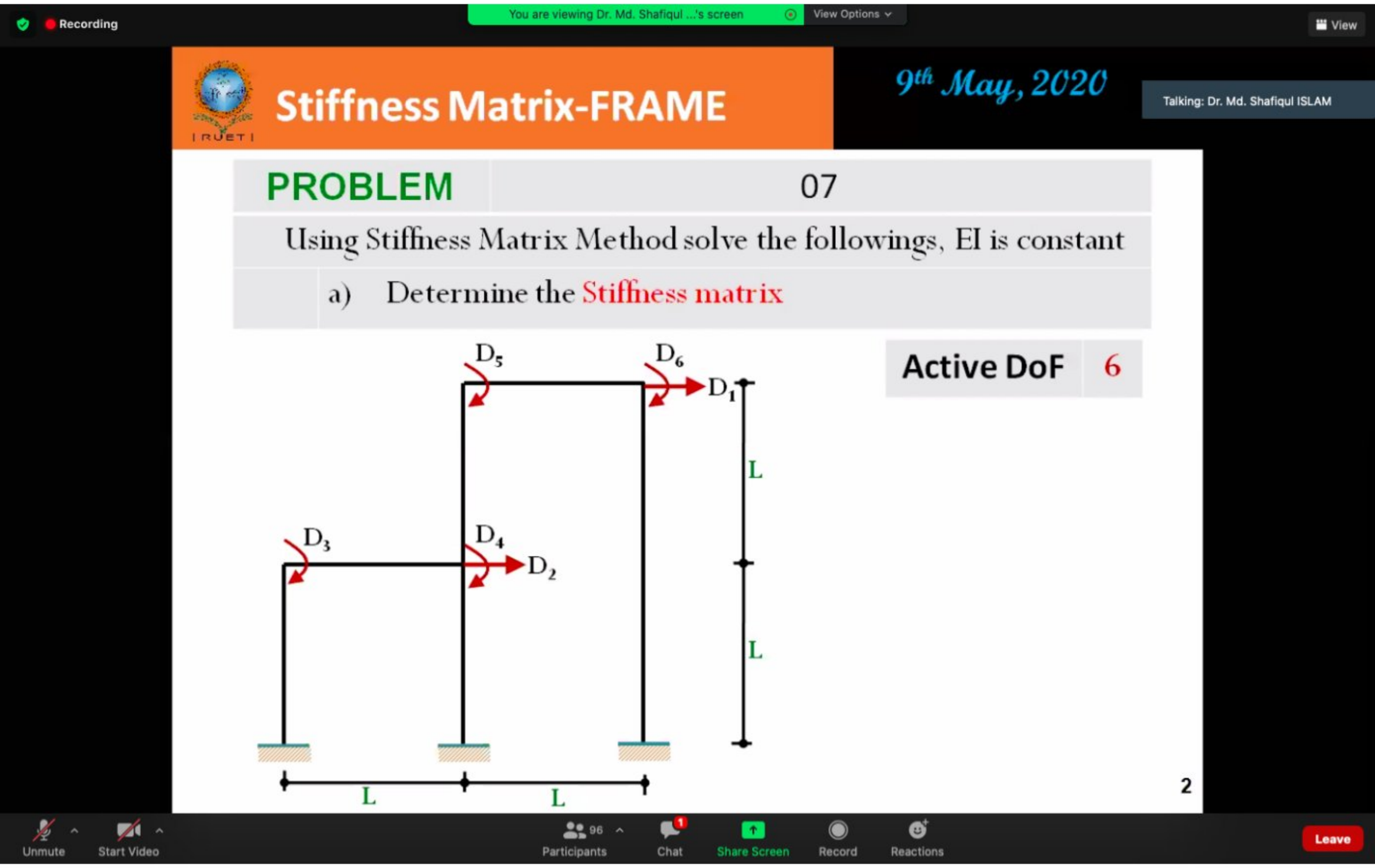
Stiffness Matrix-FRAME

9th May, 2020

PROBLEM 07

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the Stiffness matrix



Stiffness Matrix-FRAME

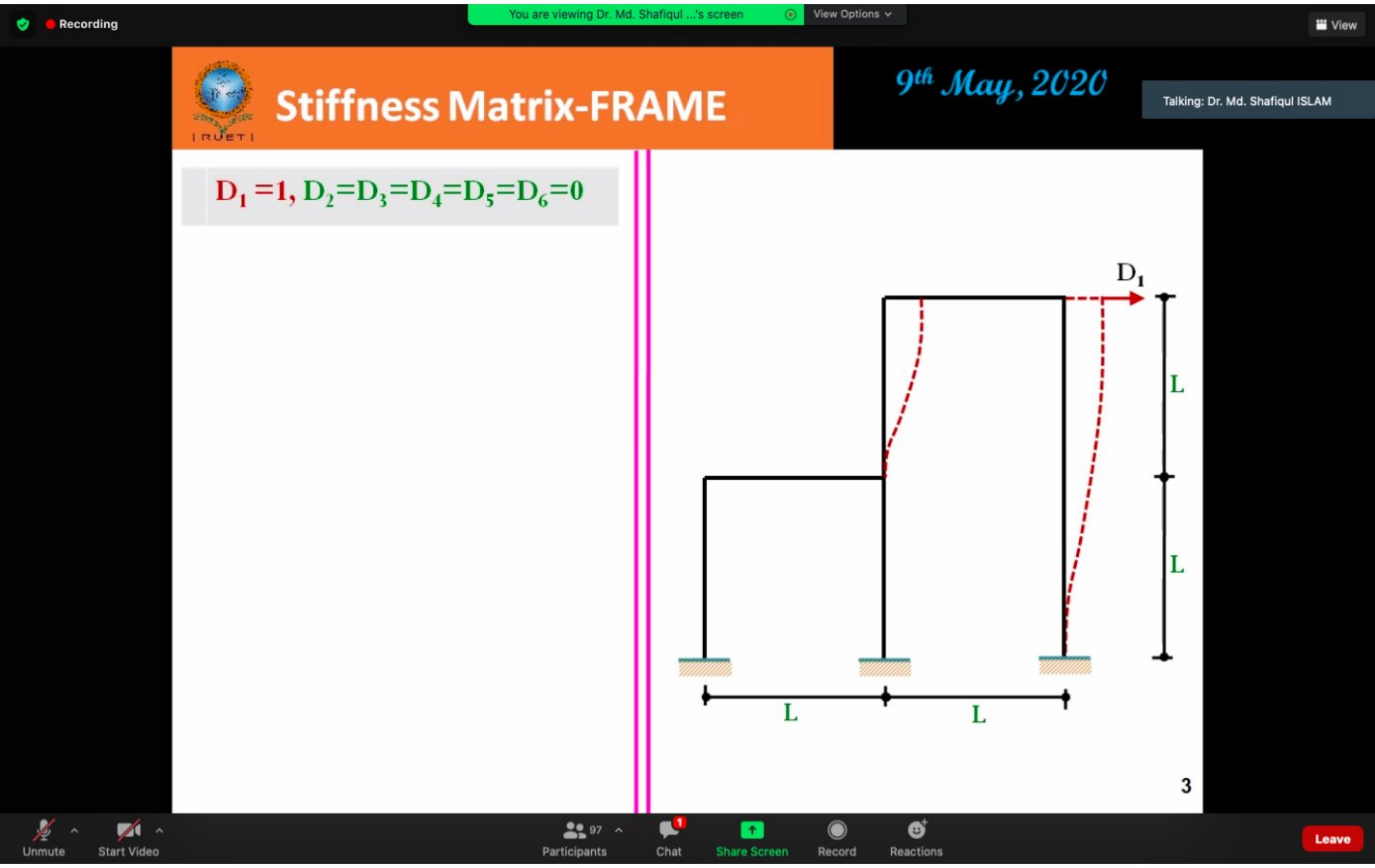
9th May, 2020

PROBLEM	07

Using Stiffness Matrix Method solve the followings, EI is constant

a) Determine the Stiffness matrix

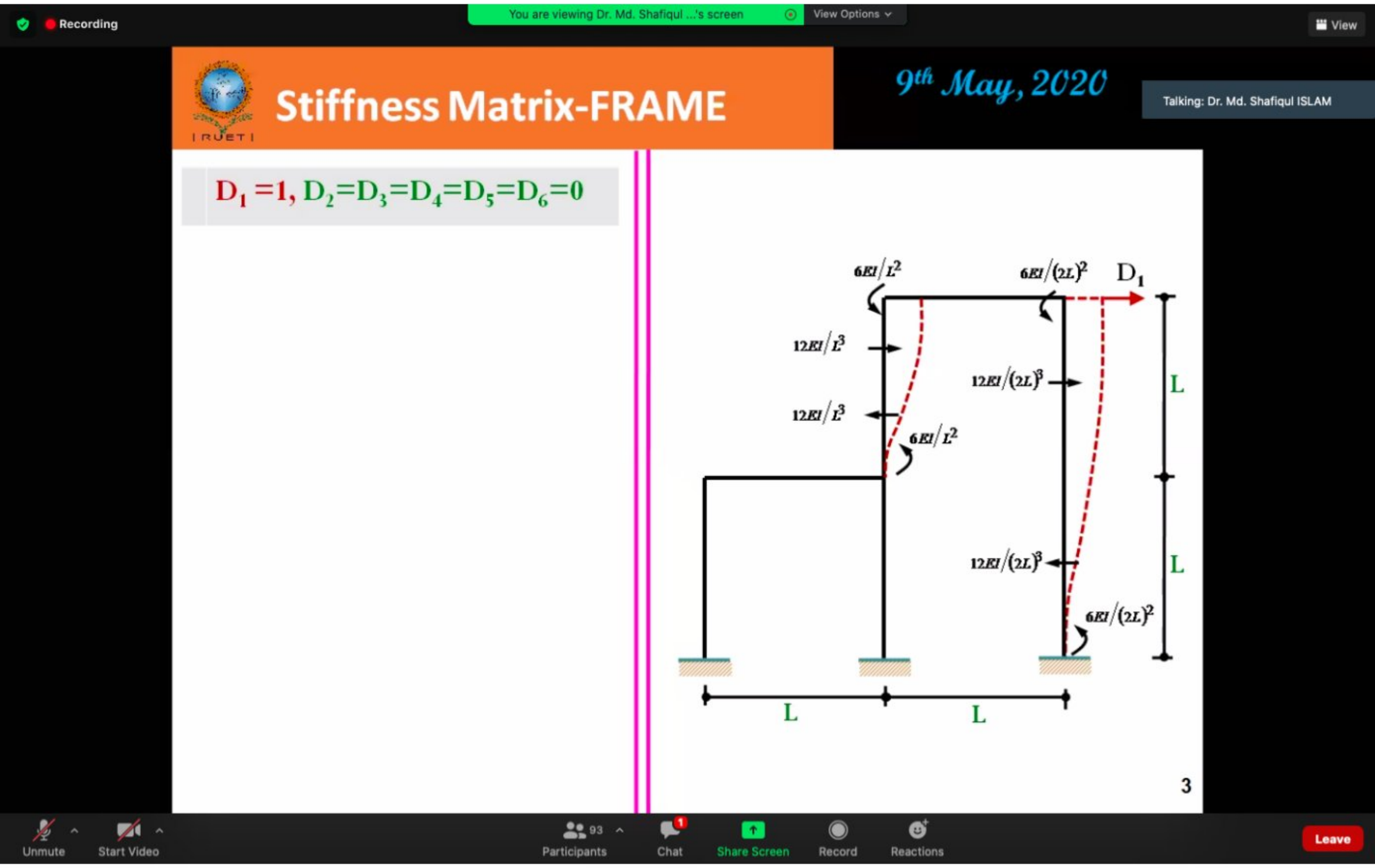
Active DoF	6



Stiffness Matrix-FRAME

9th May, 2020

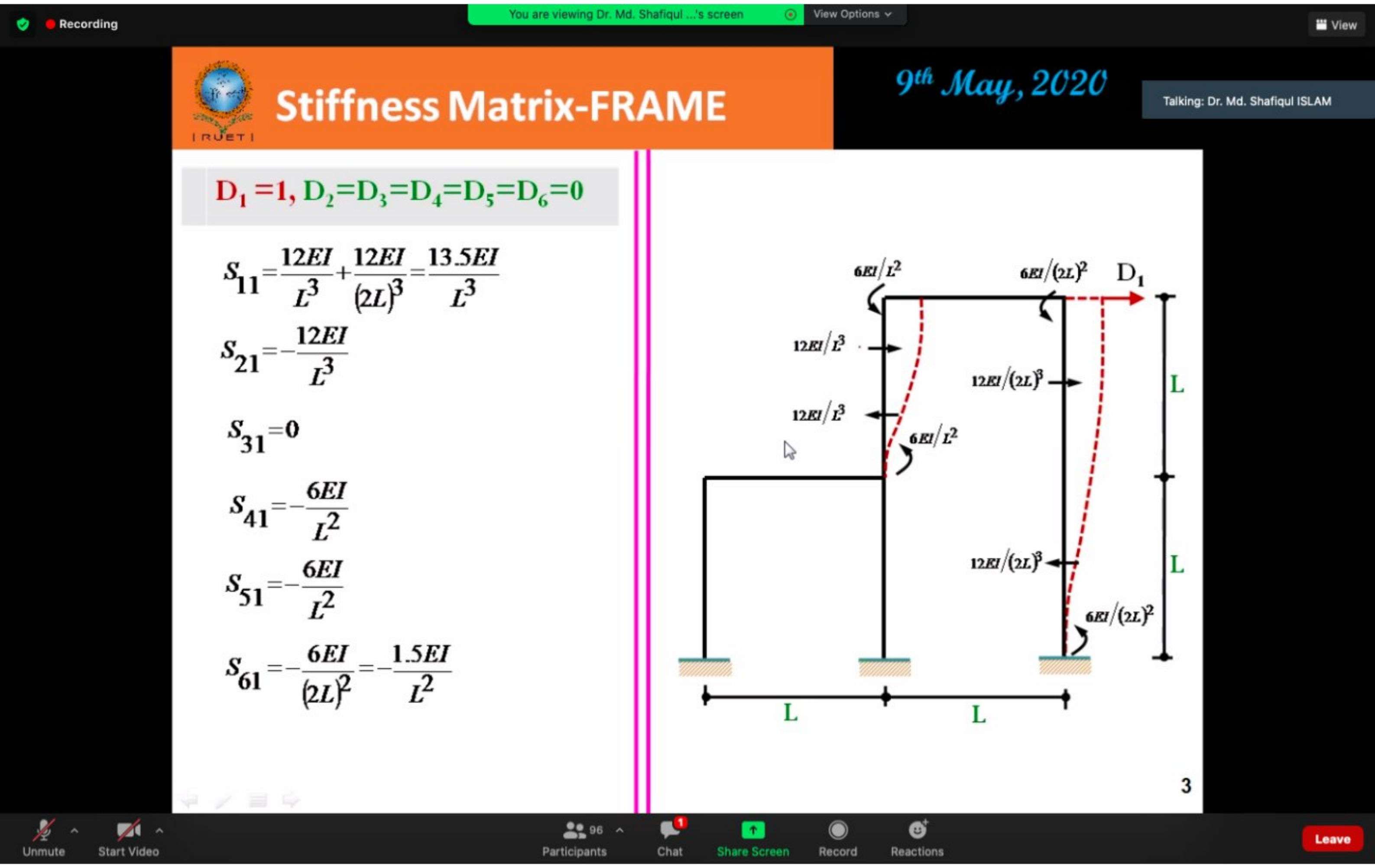
$D_1 = 1$, $D_2 = D_3 = D_4 = D_5 = D_6 = 0$



Stiffness Matrix-FRAME

9th May, 2020

$D_1 = 1, D_2 = D_3 = D_4 = D_5 = D_6 = 0$



Stiffness Matrix-FRAME

9th May, 2020

$D_1 = 1, D_2 = D_3 = D_4 = D_5 = D_6 = 0$

$$S_{11} = \frac{12EI}{L^3} + \frac{12EI}{(2L)^3} = \frac{13.5EI}{L^3}$$

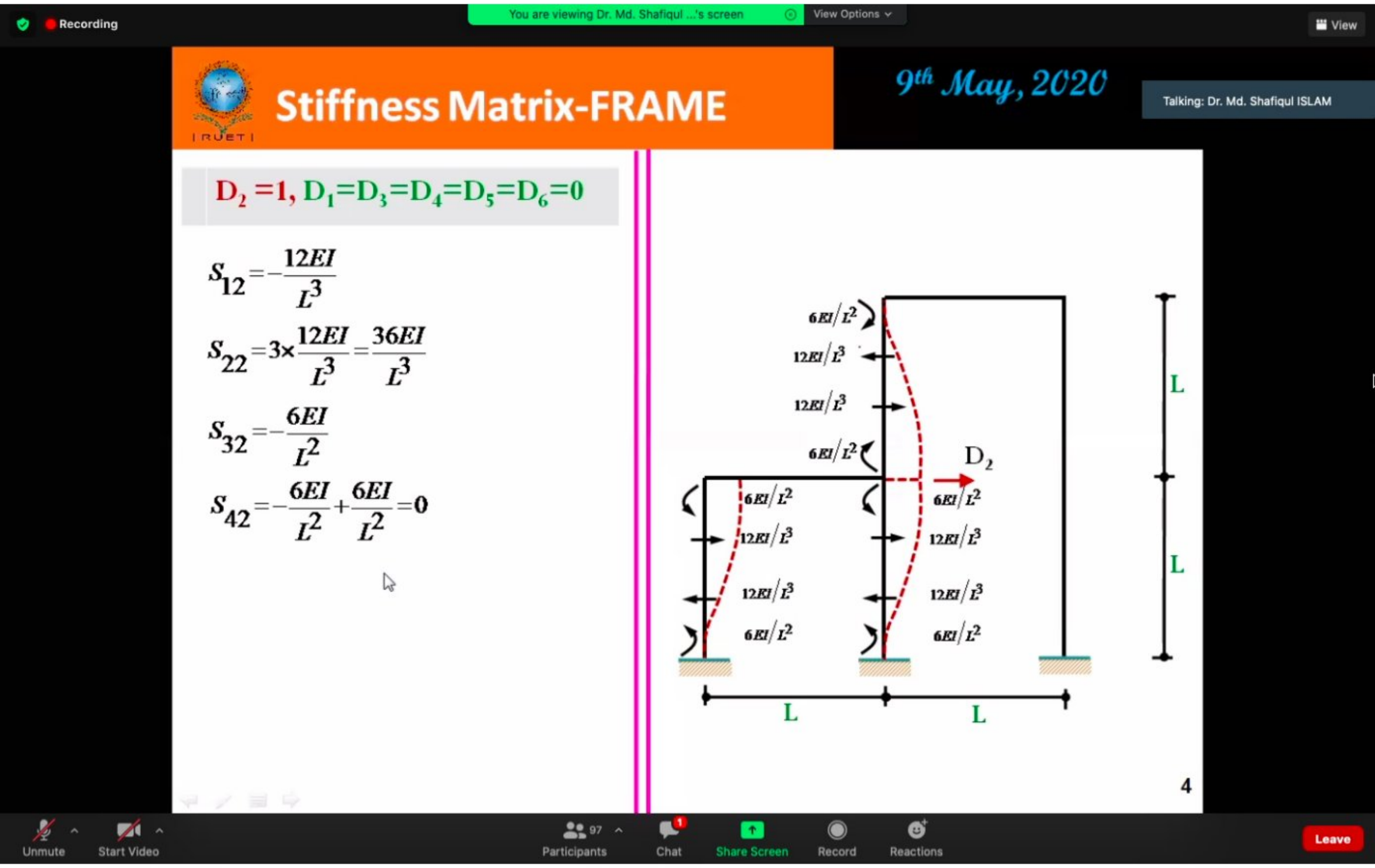
$$S_{21} = -\frac{12EI}{L^3}$$

$$S_{31} = 0$$

$$S_{41} = -\frac{6EI}{L^2}$$

$$S_{51} = -\frac{6EI}{L^2}$$

$$S_{61} = -\frac{6EI}{(2L)^2} = -\frac{1.5EI}{L^2}$$



Stiffness Matrix-FRAME

9th May, 2020

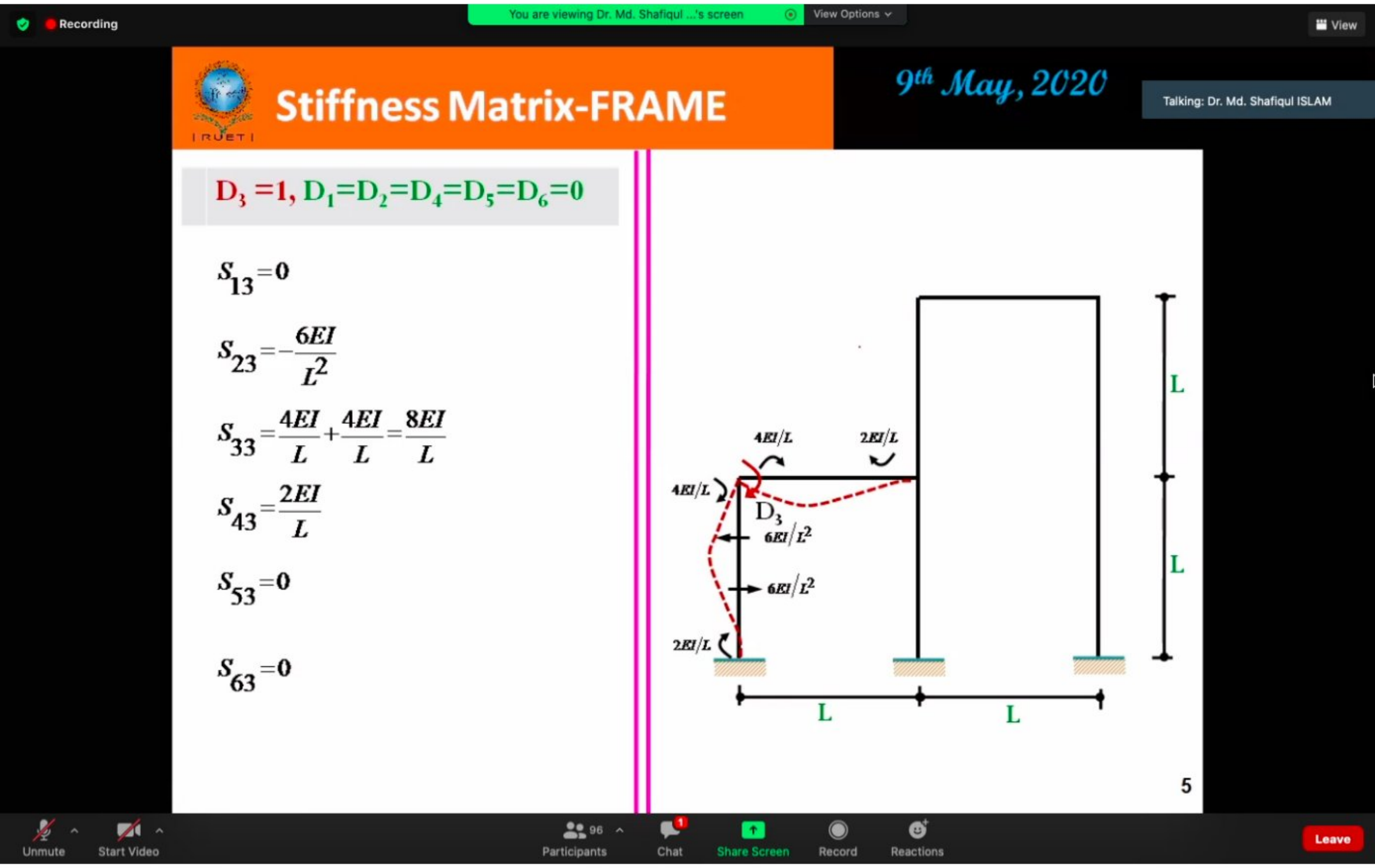
$D_2 = 1,\ D_1 = D_3 = D_4 = D_5 = D_6 = 0$

$$S_{12} = -\frac{12EI}{L^3}$$

$$S_{22} = 3 \times \frac{12EI}{L^3} = \frac{36EI}{L^3}$$

$$S_{32} = -\frac{6EI}{L^2}$$

$$S_{42} = -\frac{6EI}{L^2} + \frac{6EI}{L^2} = 0$$



Stiffness Matrix-FRAME

9th May, 2020

$D_3 = 1,\ D_1 = D_2 = D_4 = D_5 = D_6 = 0$

$$S_{13} = 0$$

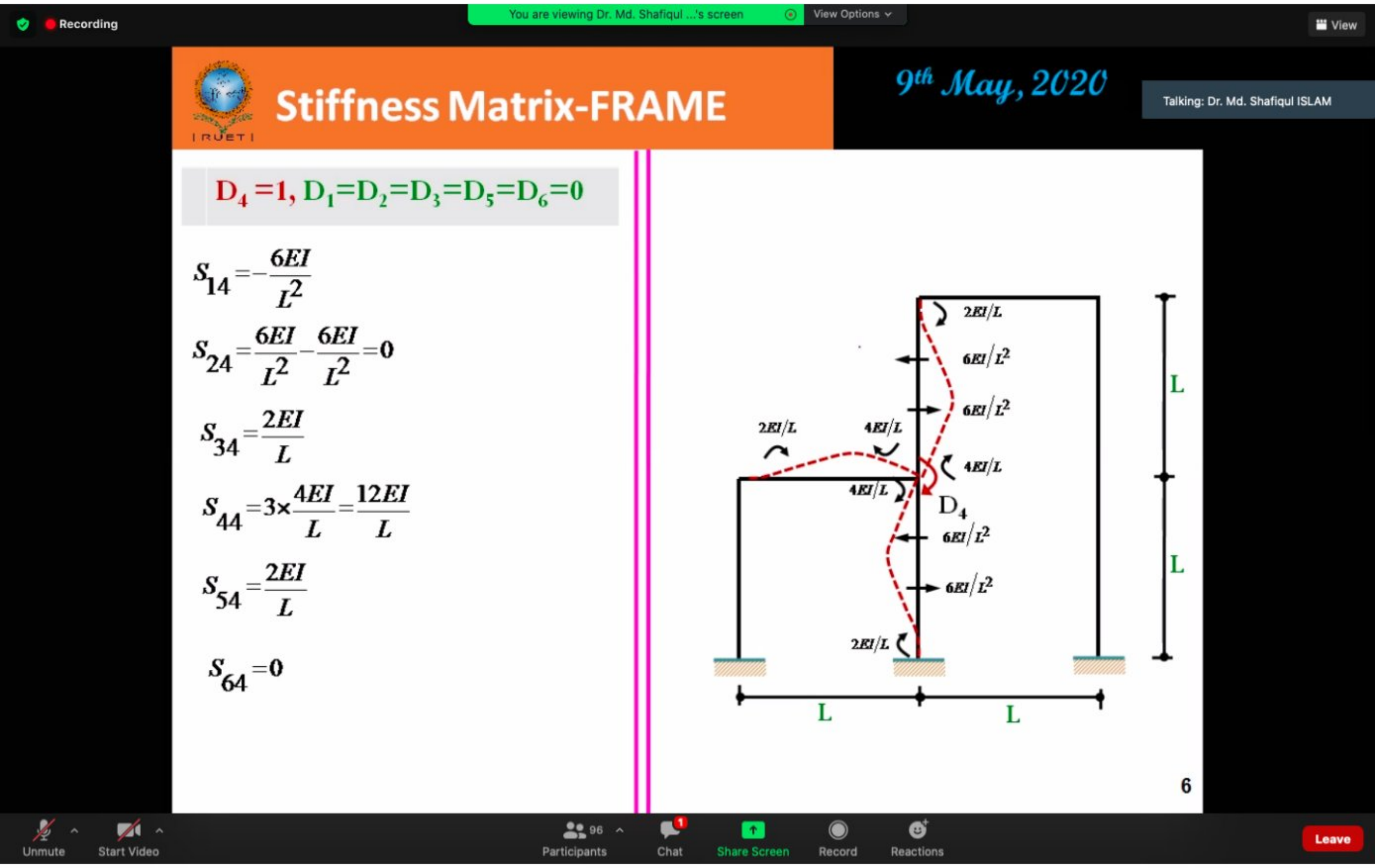
$$S_{23} = -\frac{6EI}{L^2}$$

$$S_{33} = \frac{4EI}{L} + \frac{4EI}{L} = \frac{8EI}{L}$$

$$S_{43} = \frac{2EI}{L}$$

$$S_{53} = 0$$

$$S_{63} = 0$$



Stiffness Matrix-FRAME

9th May, 2020

$D_4 = 1,\ D_1 = D_2 = D_3 = D_5 = D_6 = 0$

$$S_{14} = -\frac{6EI}{L^2}$$

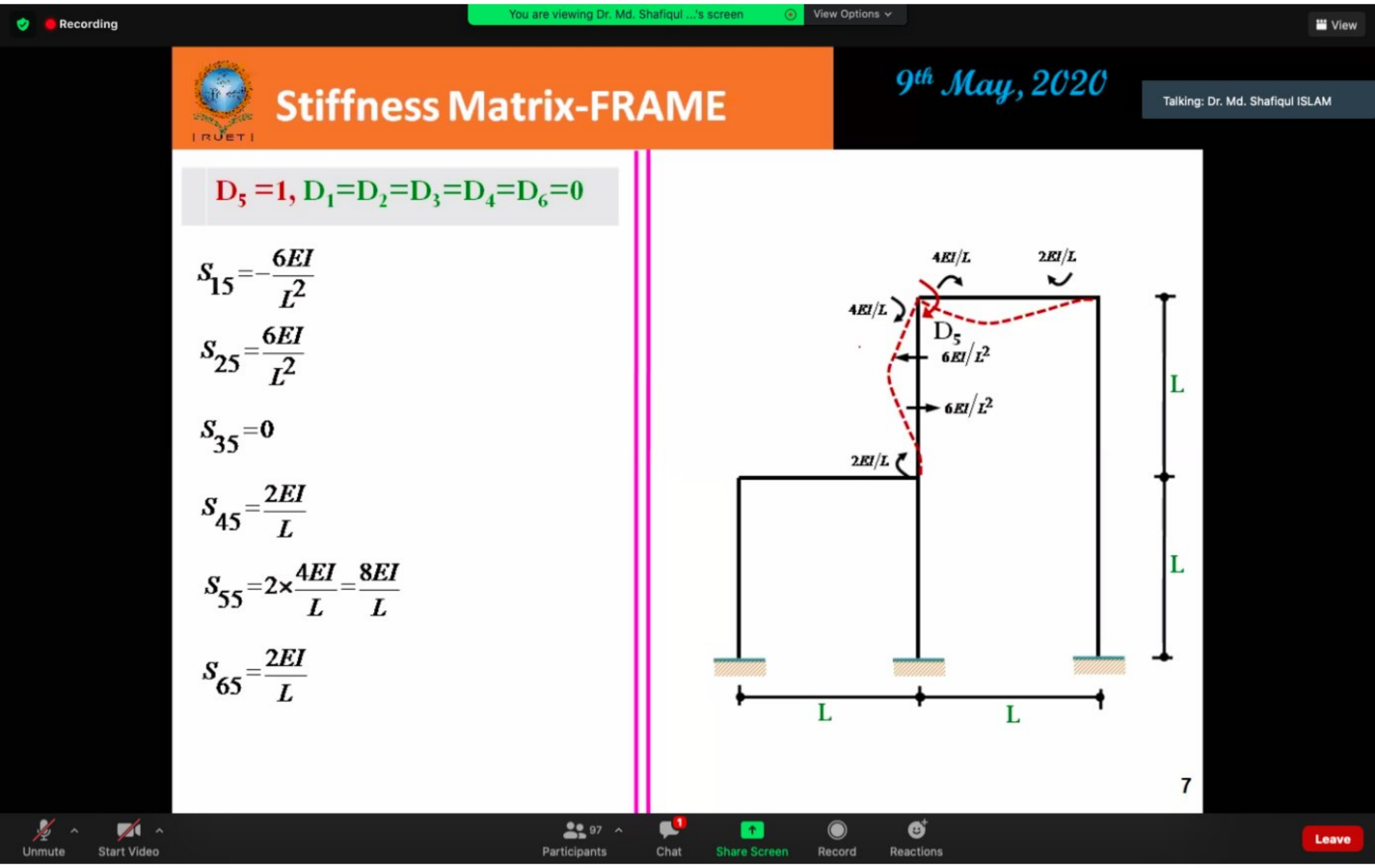
$$S_{24} = \frac{6EI}{L^2} - \frac{6EI}{L^2} = 0$$

$$S_{34} = \frac{2EI}{L}$$

$$S_{44} = 3\times\frac{4EI}{L} = \frac{12EI}{L}$$

$$S_{54} = \frac{2EI}{L}$$

$$S_{64} = 0$$



Stiffness Matrix-FRAME

9th May, 2020

$D_5 = 1,\ D_1 = D_2 = D_3 = D_4 = D_6 = 0$

$$S_{15} = -\frac{6EI}{L^2}$$

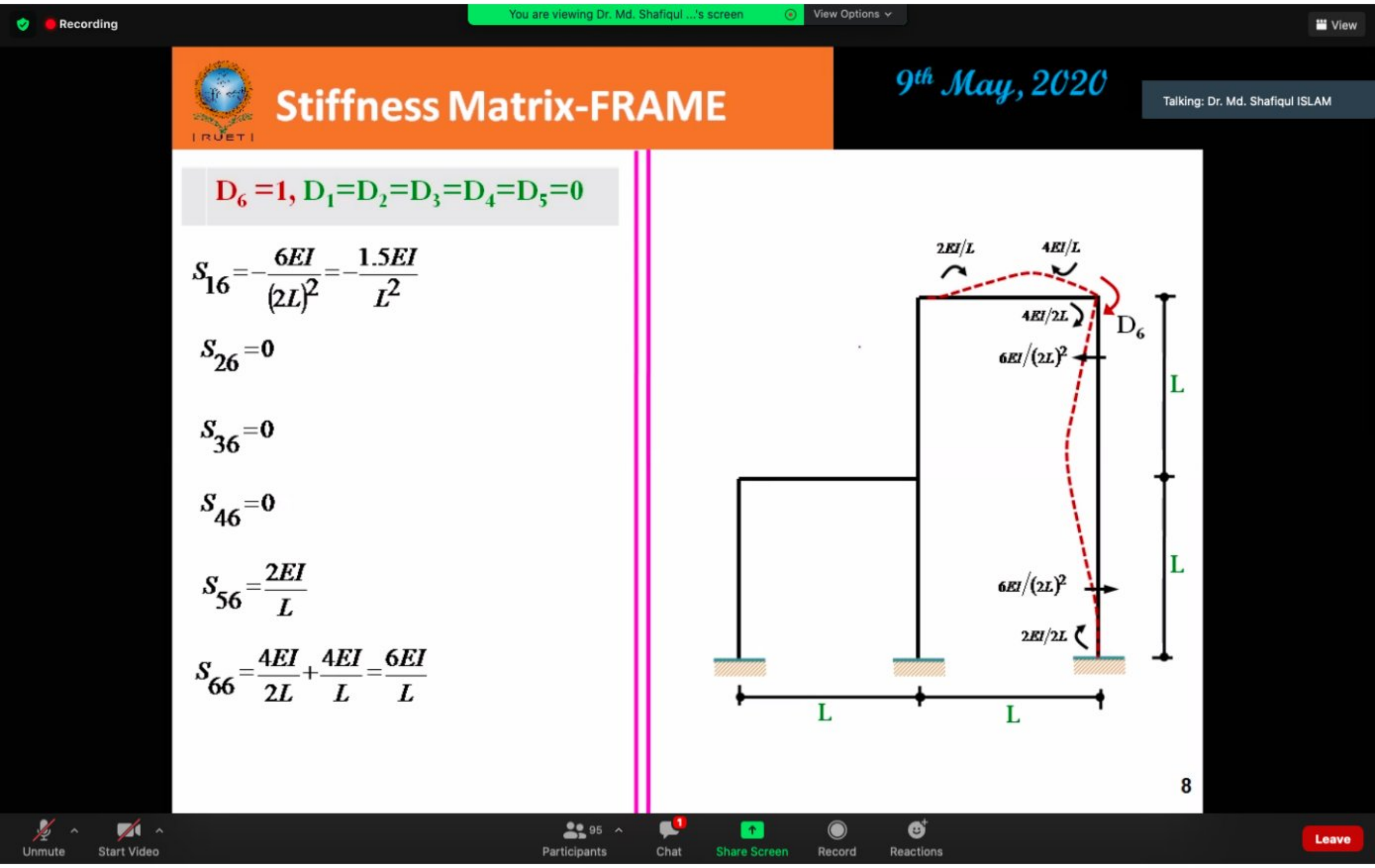
$$S_{25} = \frac{6EI}{L^2}$$

$$S_{35} = 0$$

$$S_{45} = \frac{2EI}{L}$$

$$S_{55} = 2\times\frac{4EI}{L} = \frac{8EI}{L}$$

$$S_{65} = \frac{2EI}{L}$$



Stiffness Matrix-FRAME

9th May, 2020

$D_6 = 1, D_1 = D_2 = D_3 = D_4 = D_5 = 0$

$$S_{16} = -\frac{6EI}{(2L)^2} = -\frac{1.5EI}{L^2}$$

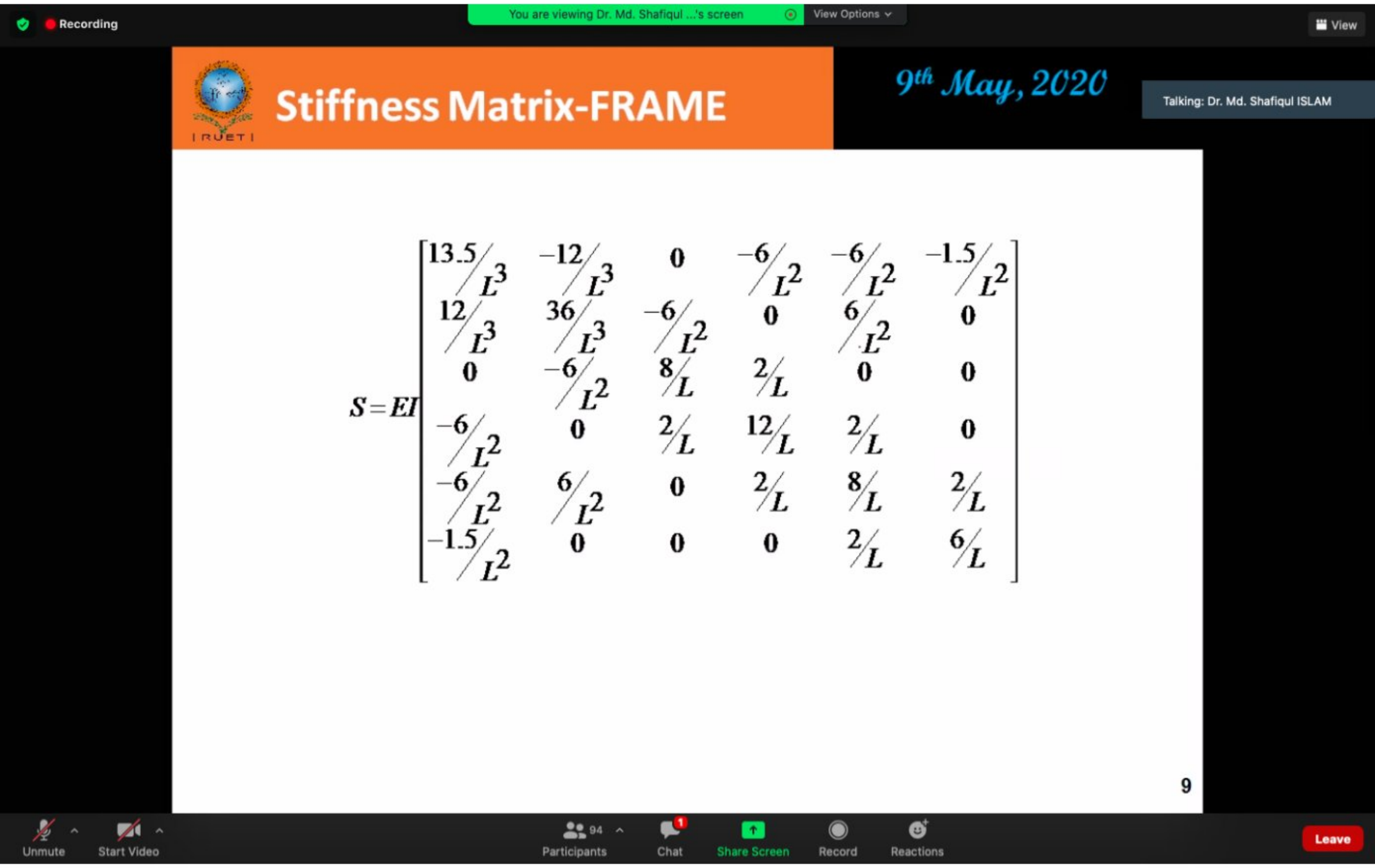
$$S_{26} = 0$$

$$S_{36} = 0$$

$$S_{46} = 0$$

$$S_{56} = \frac{2EI}{L}$$

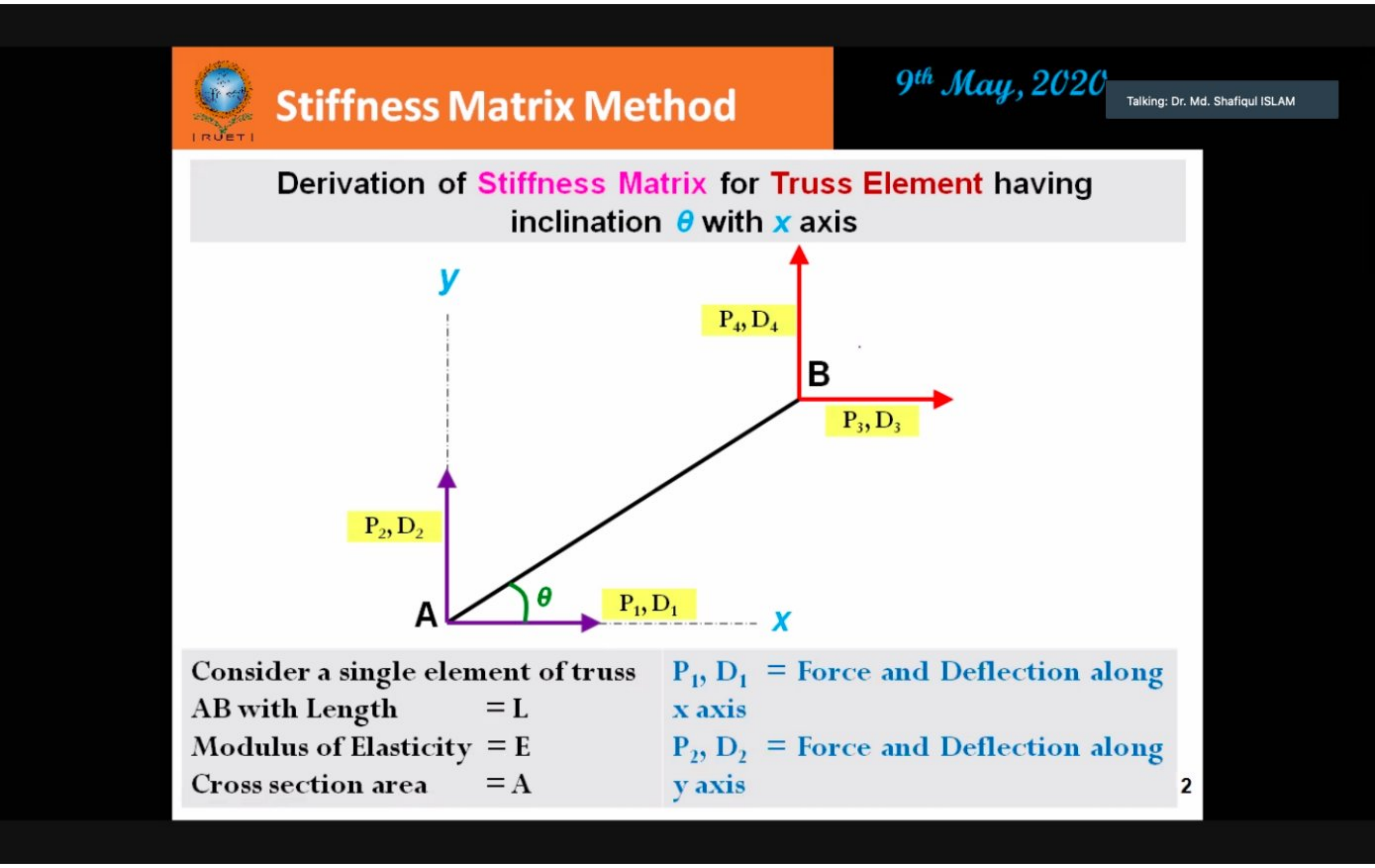
$$S_{66} = \frac{4EI}{2L} + \frac{4EI}{L} = \frac{6EI}{L}$$



Stiffness Matrix-FRAME

9th May, 2020

$$S = EI\begin{bmatrix} 13.5/L^3 & -12/L^3 & 0 & -6/L^2 & -6/L^2 & -1.5/L^2 \\ 12/L^3 & 36/L^3 & -6/L^2 & 0 & 6/L^2 & 0 \\ 0 & -6/L^2 & 8/L & 2/L & 0 & 0 \\ -6/L^2 & 0 & 2/L & 12/L & 2/L & 0 \\ -6/L^2 & 6/L^2 & 0 & 2/L & 8/L & 2/L \\ -1.5/L^2 & 0 & 0 & 0 & 2/L & 6/L \end{bmatrix}$$



Stiffness Matrix Method

9th May, 2020

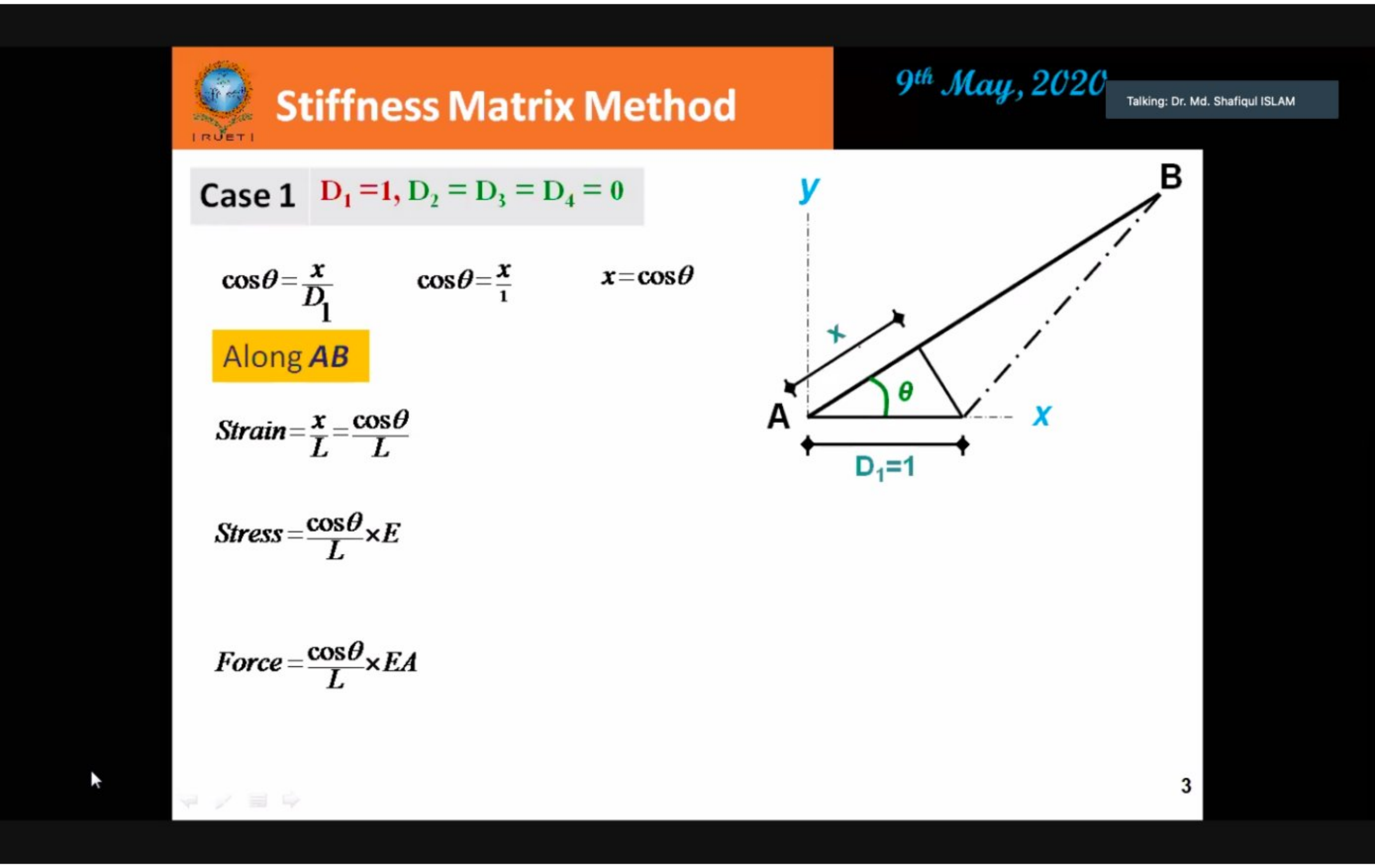
Talking: Dr. Md. Shafiqul ISLAM

Derivation of Stiffness Matrix for Truss Element having inclination θ with x axis

Consider a single element of truss
AB with Length = L
Modulus of Elasticity = E
Cross section area = A

P_1, D_1 = Force and Deflection along x axis

P_2, D_2 = Force and Deflection along y axis



9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Stiffness Matrix Method

Case 1 $D_1 = 1, D_2 = D_3 = D_4 = 0$

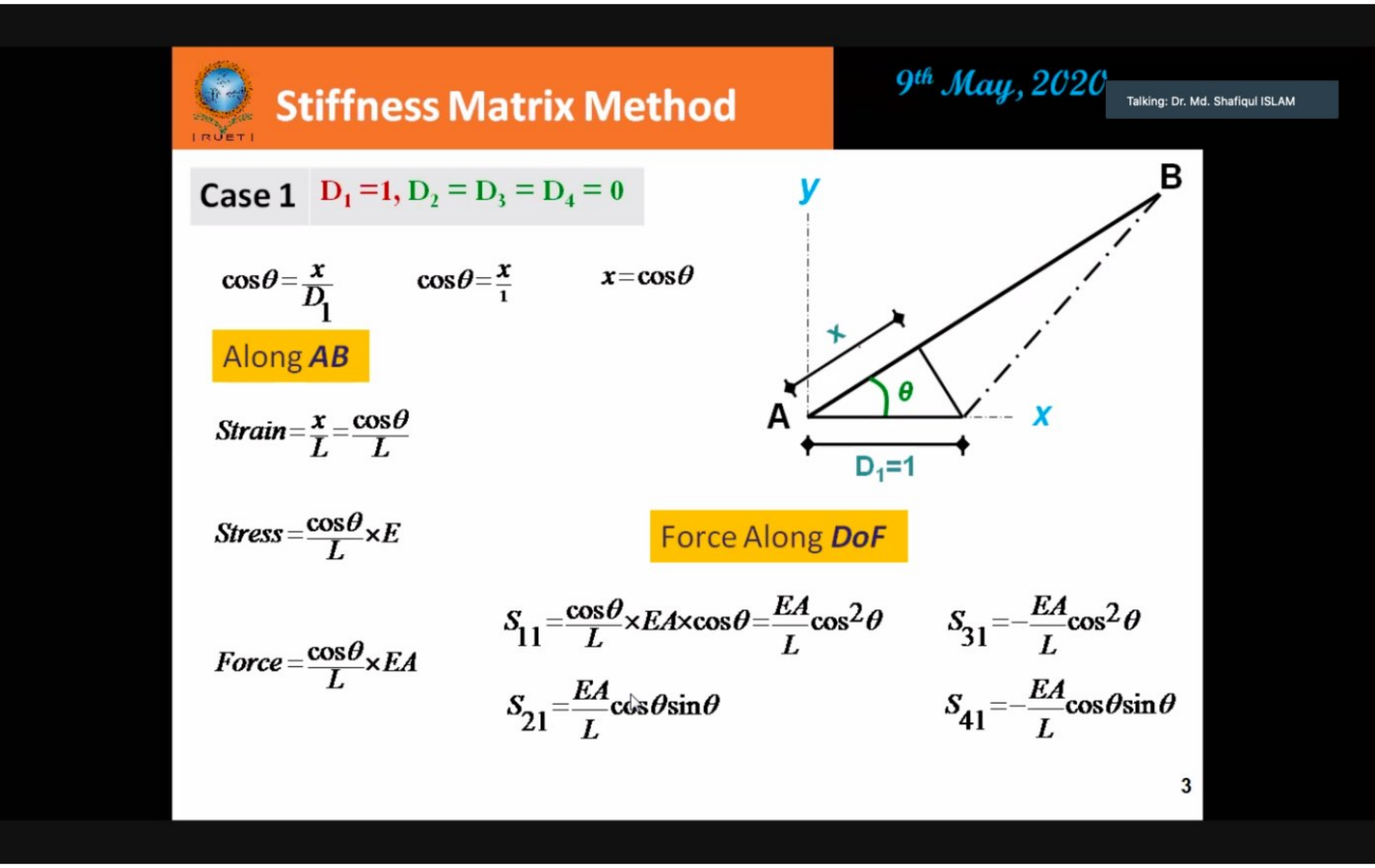
$\cos\theta = \frac{x}{D_1}$ $\cos\theta = \frac{x}{1}$ $x = \cos\theta$

Along *AB*

$$Strain = \frac{x}{L} = \frac{\cos\theta}{L}$$

$$Stress = \frac{\cos\theta}{L} \times E$$

$$Force = \frac{\cos\theta}{L} \times EA$$



Stiffness Matrix Method

9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Case 1 $D_1 = 1, D_2 = D_3 = D_4 = 0$

$\cos\theta = \frac{x}{D_1}$ $\quad$ $\cos\theta = \frac{x}{1}$ $\quad$ $x = \cos\theta$

Along ***AB***

$$Strain = \frac{x}{L} = \frac{\cos\theta}{L}$$

$$Stress = \frac{\cos\theta}{L} \times E$$

$$Force = \frac{\cos\theta}{L} \times EA$$

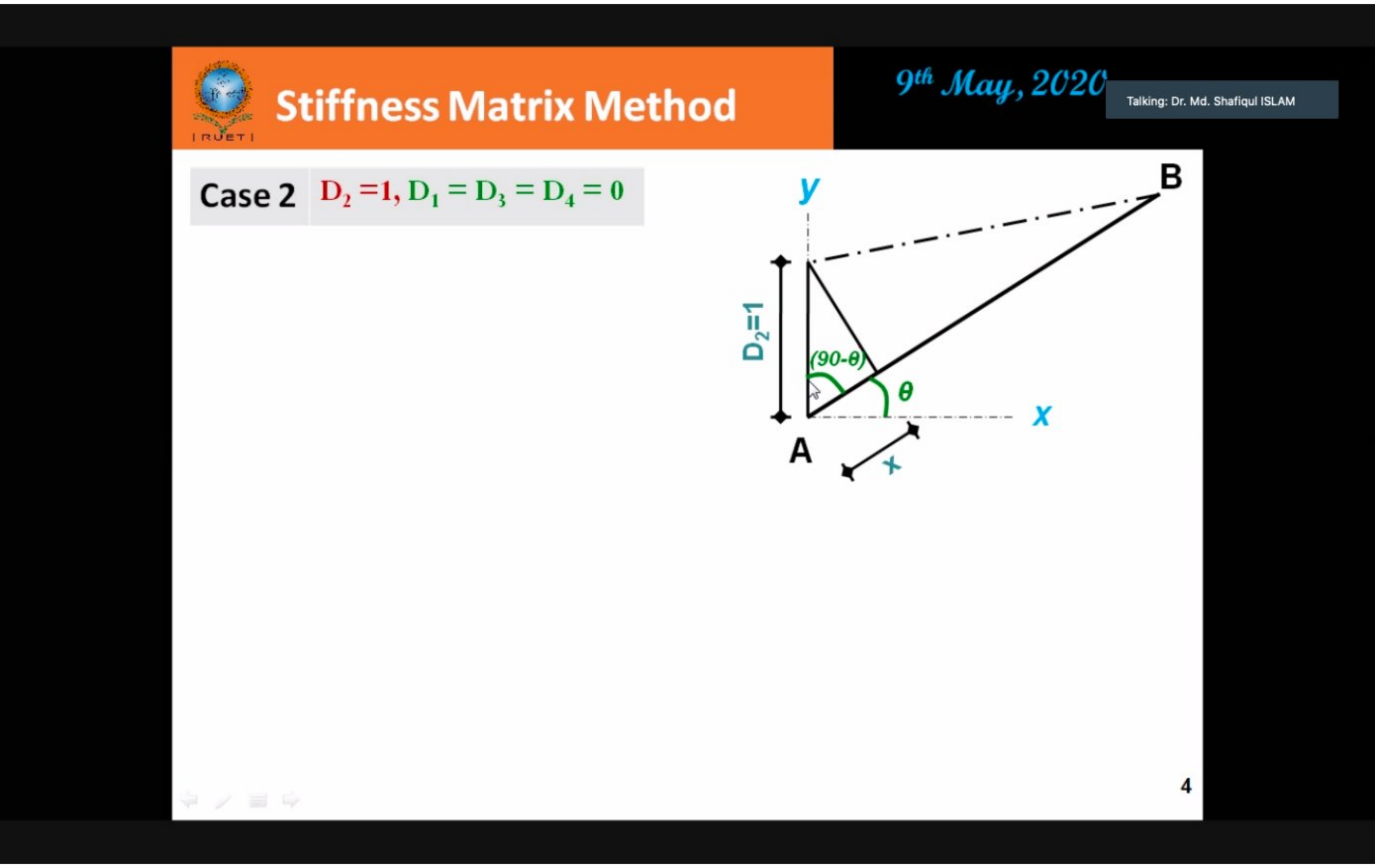
Force Along ***DoF***

$$S_{11} = \frac{\cos\theta}{L} \times EA \times \cos\theta = \frac{EA}{L}\cos^2\theta$$

$$S_{31} = -\frac{EA}{L}\cos^2\theta$$

$$S_{21} = \frac{EA}{L}\cos\theta\sin\theta$$

$$S_{41} = -\frac{EA}{L}\cos\theta\sin\theta$$

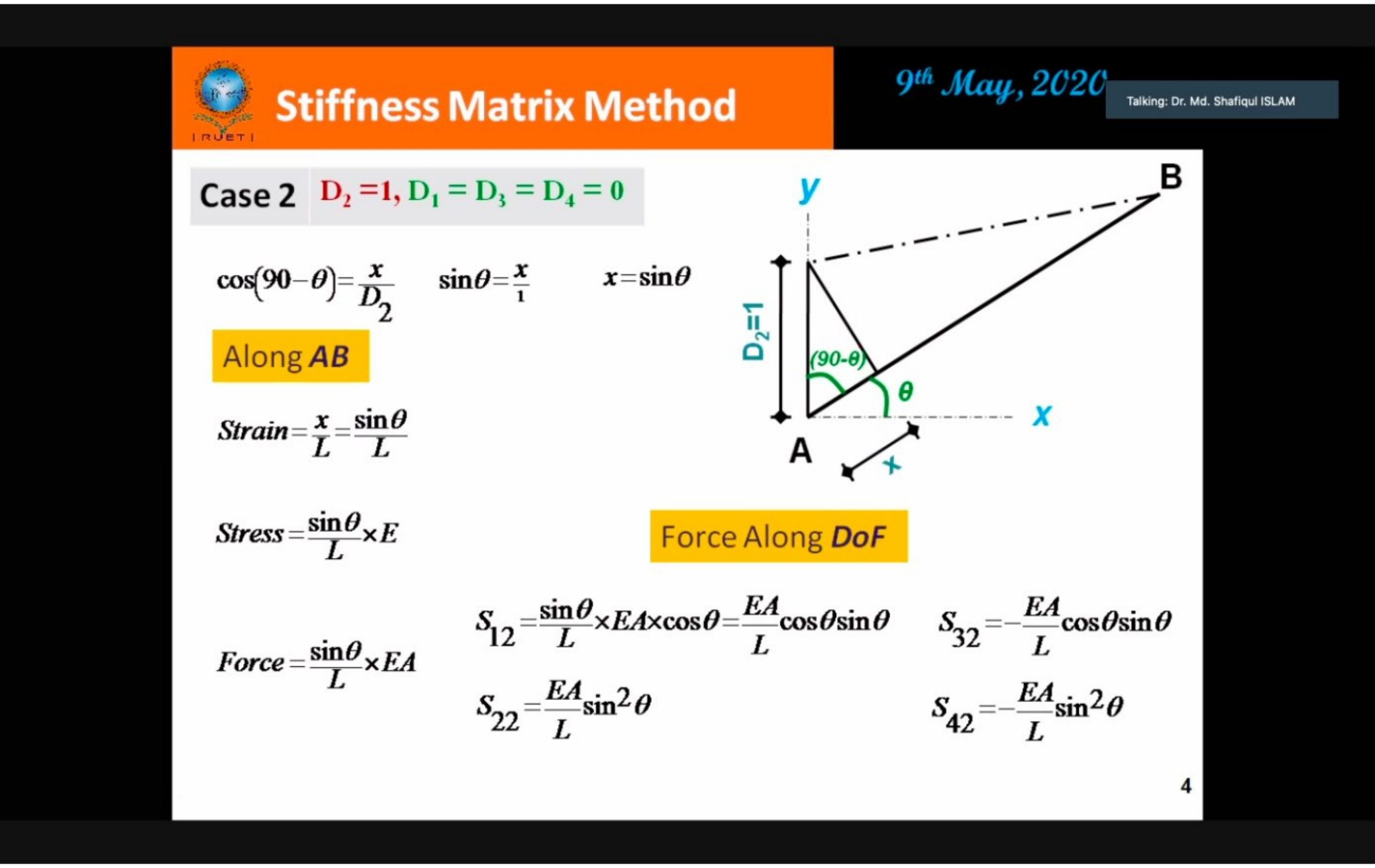


Stiffness Matrix Method

9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Case 2 $D_2 = 1, D_1 = D_3 = D_4 = 0$



Stiffness Matrix Method

9th May, 2020

Case 2 $D_2=1, D_1=D_3=D_4=0$

$\cos(90-\theta)=\frac{x}{D_2}$ $\sin\theta=\frac{x}{1}$ $x=\sin\theta$

Along *AB*

$$Strain=\frac{x}{L}=\frac{\sin\theta}{L}$$

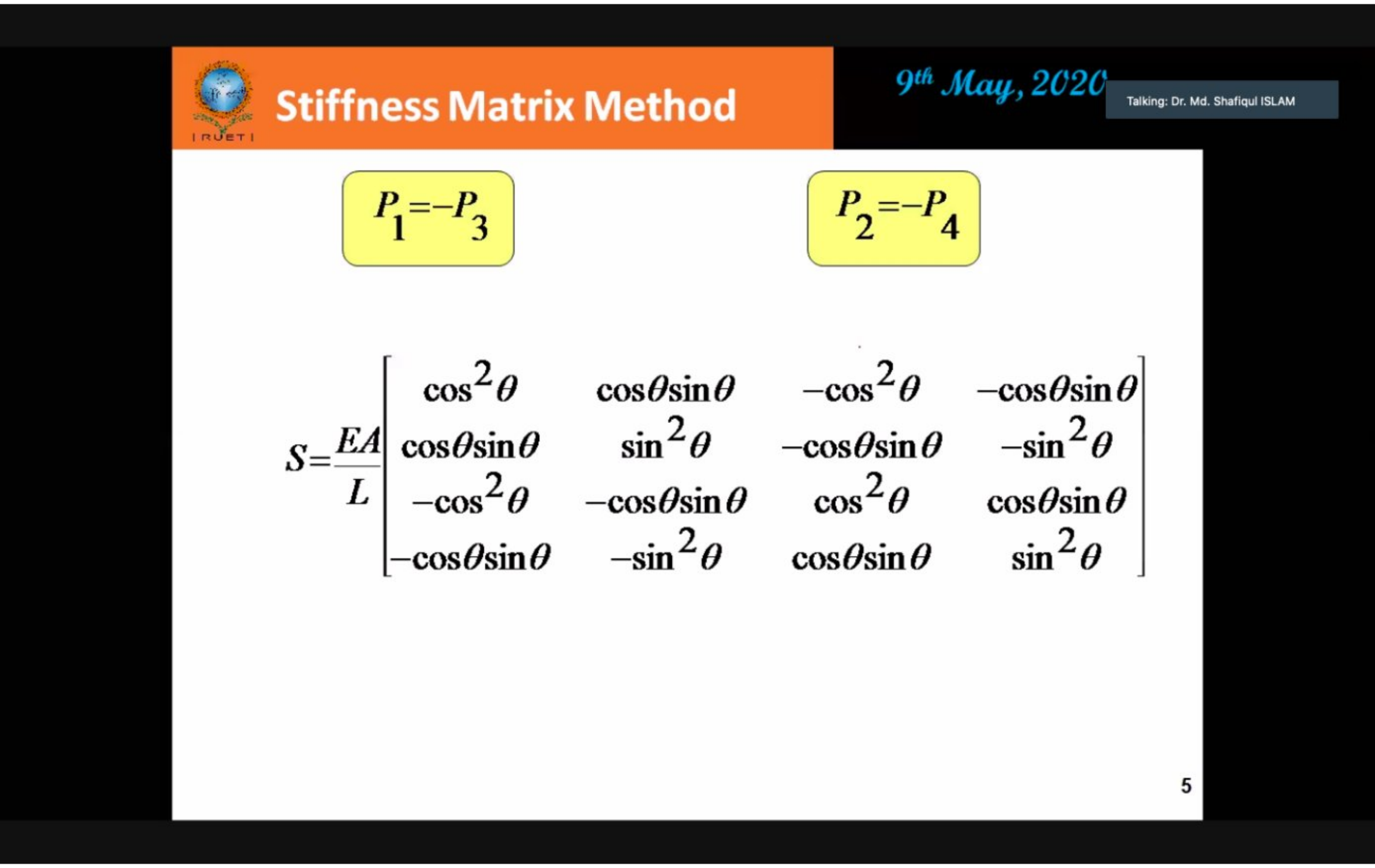
$$Stress=\frac{\sin\theta}{L}\times E$$

$$Force=\frac{\sin\theta}{L}\times EA$$

Force Along *DoF*

$$S_{12}=\frac{\sin\theta}{L}\times EA\times\cos\theta=\frac{EA}{L}\cos\theta\sin\theta \qquad S_{32}=-\frac{EA}{L}\cos\theta\sin\theta$$

$$S_{22}=\frac{EA}{L}\sin^2\theta \qquad S_{42}=-\frac{EA}{L}\sin^2\theta$$



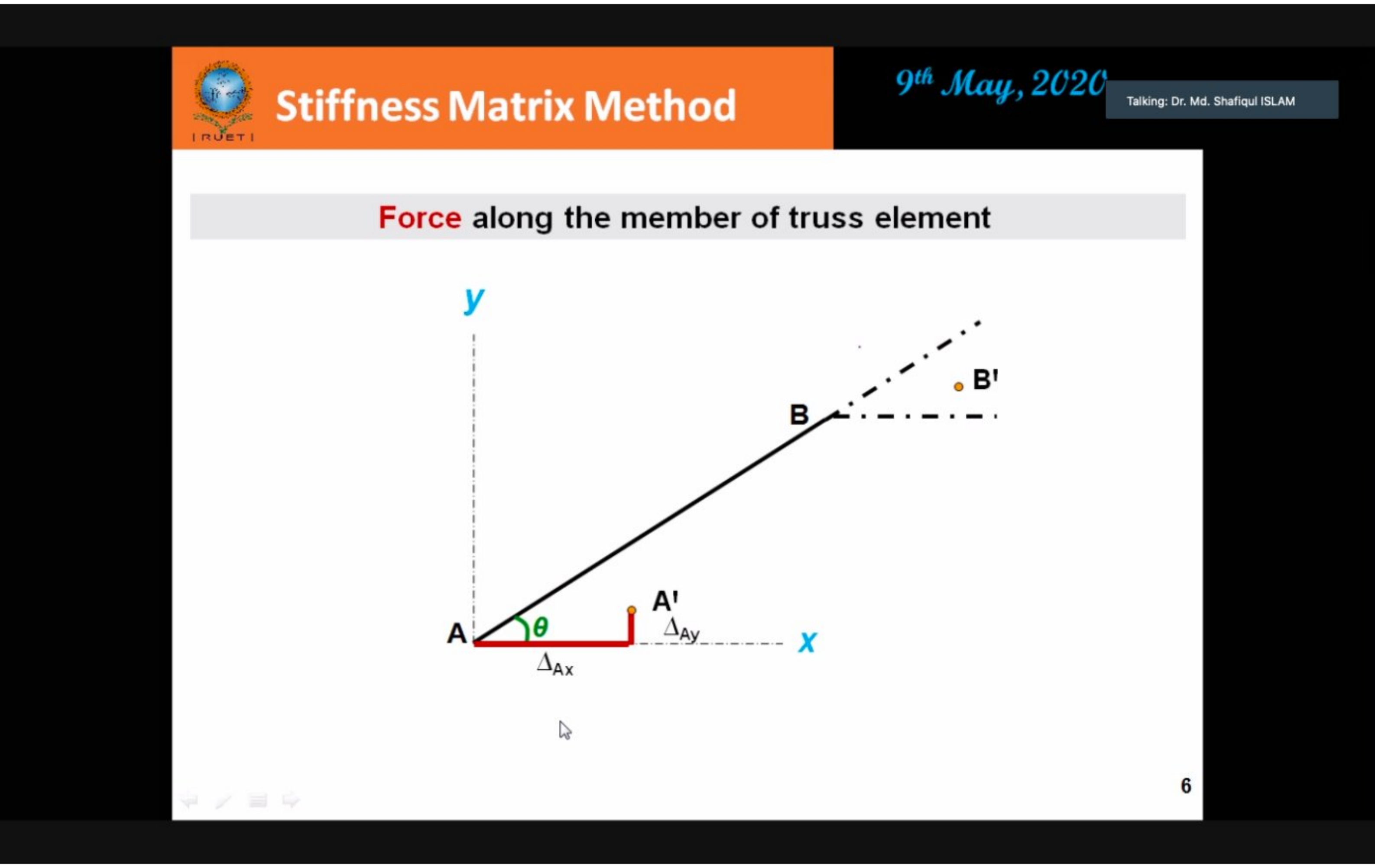
Stiffness Matrix Method

9th May, 2020

$P_1 = -P_3$

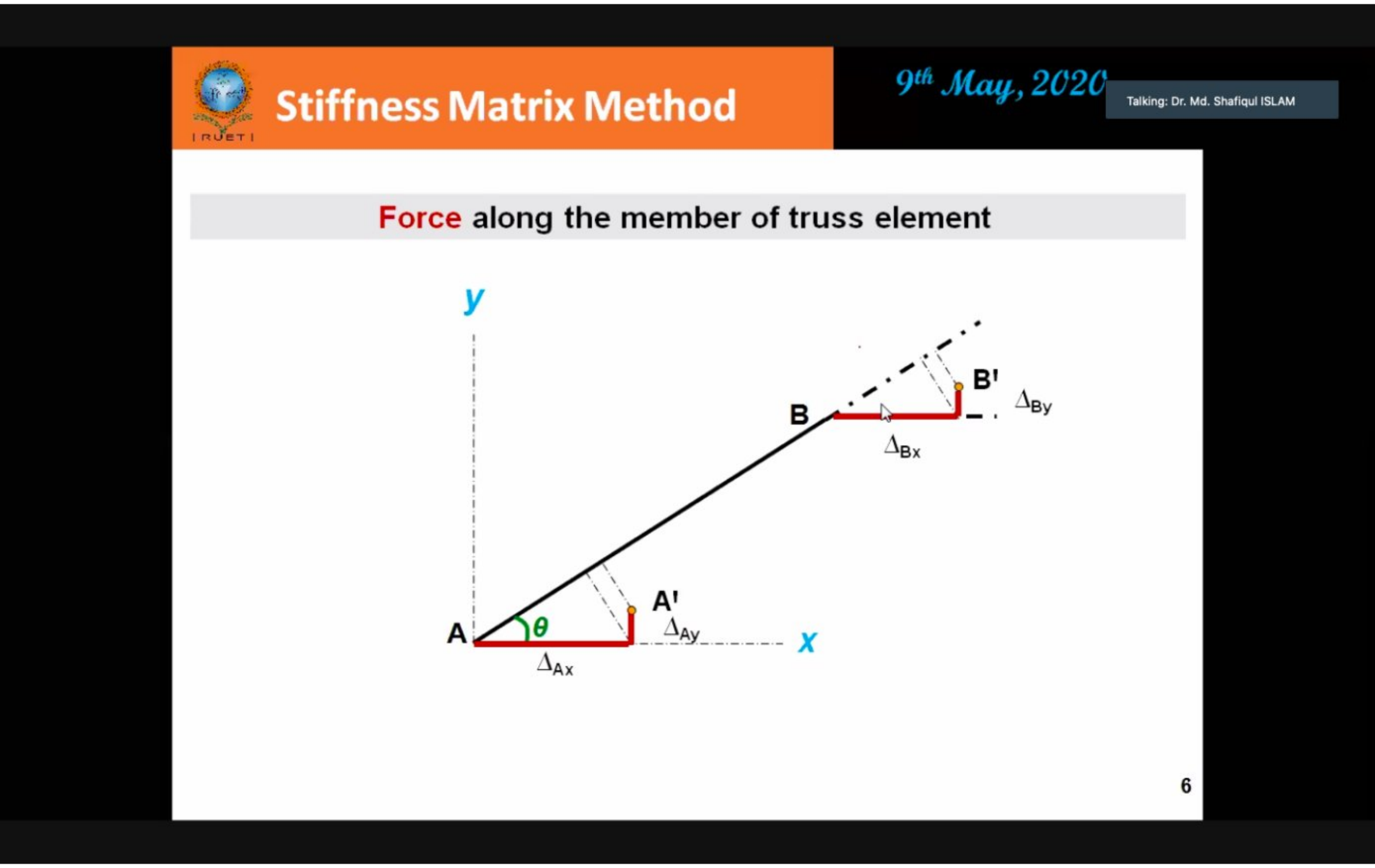
$P_2 = -P_4$

$$S = \frac{EA}{L}\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$



Stiffness Matrix Method

Force along the member of truss element

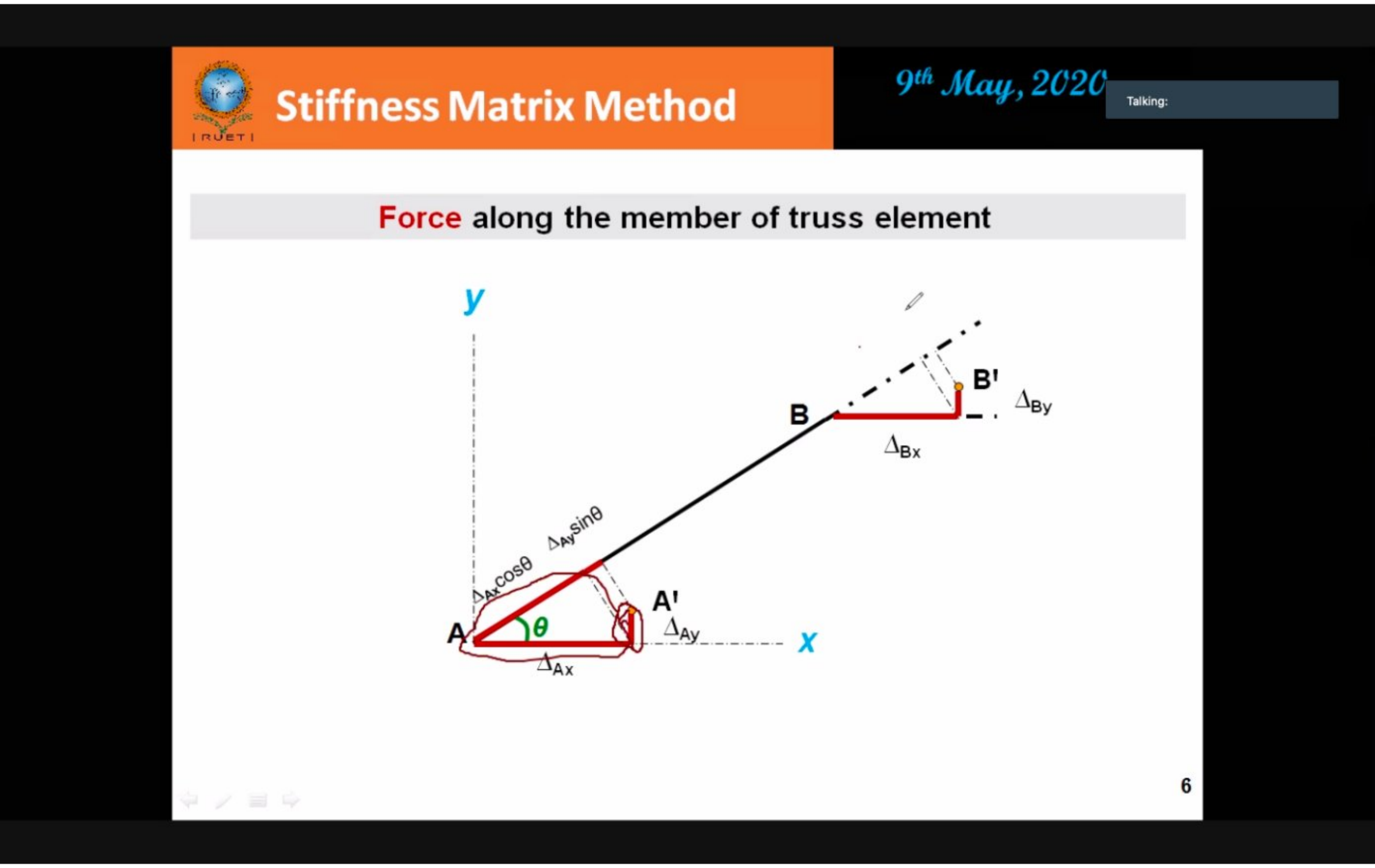


Stiffness Matrix Method

9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Force along the member of truss element

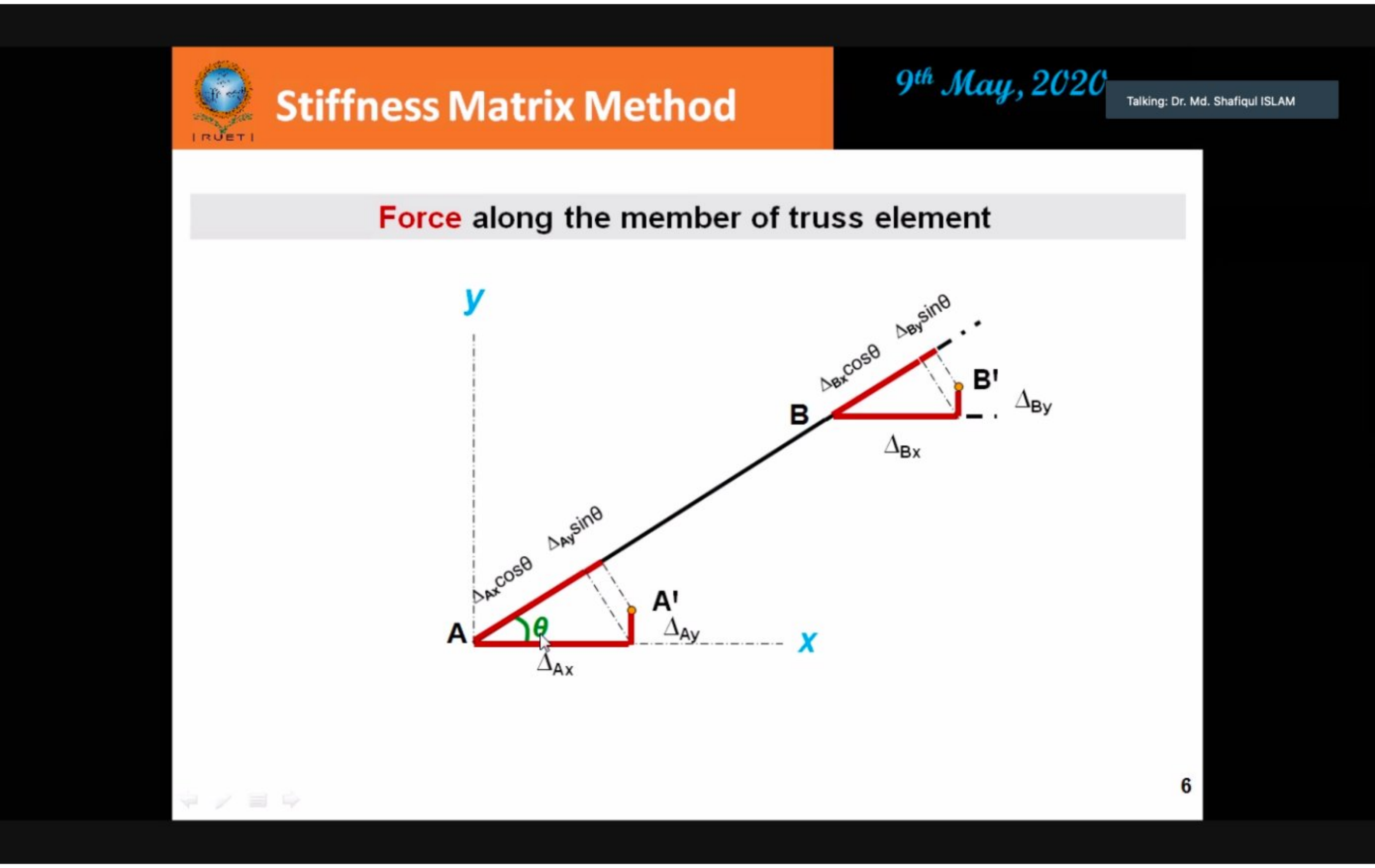


Stiffness Matrix Method

9th May, 2020

Talking:

Force along the member of truss element

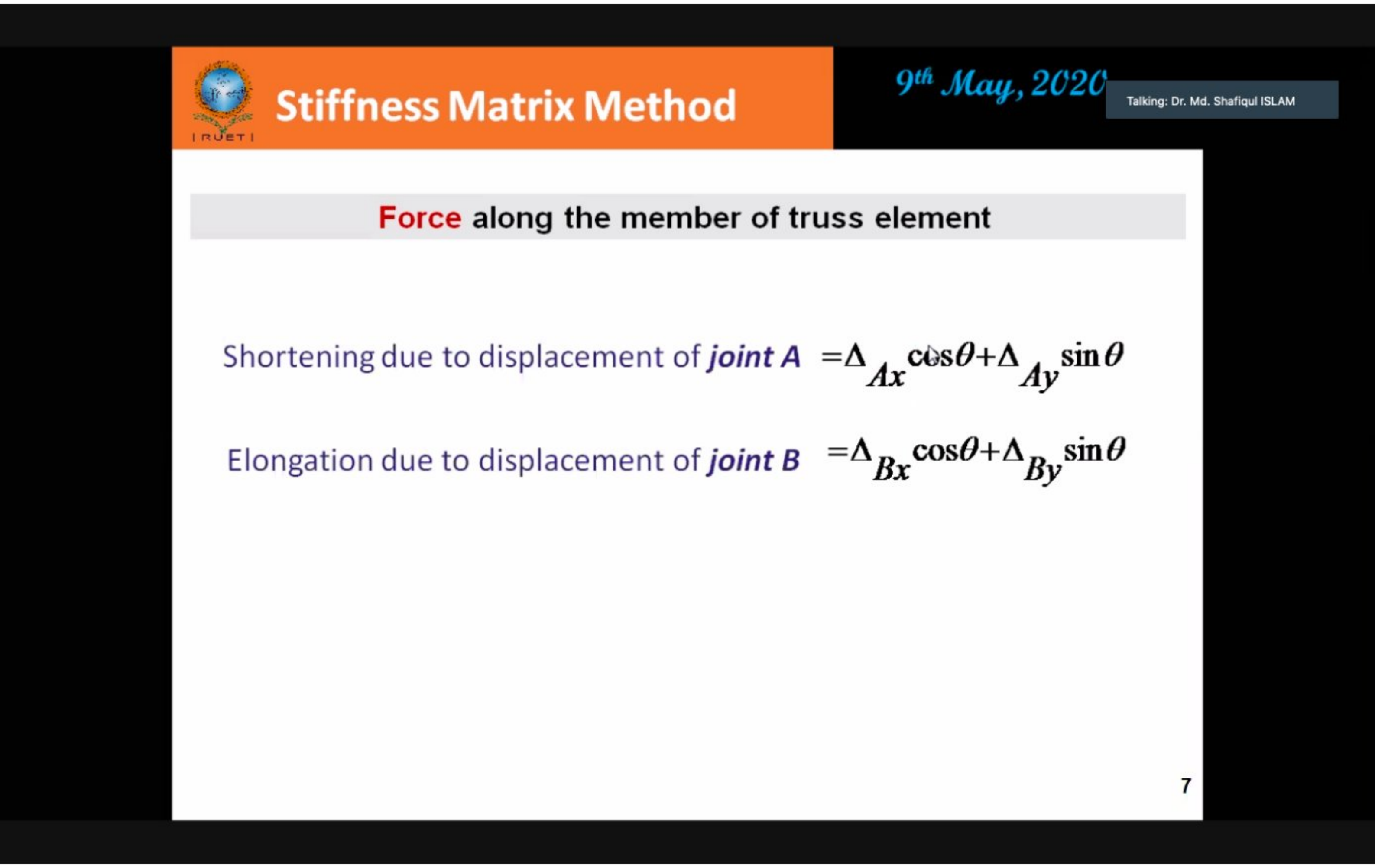


Stiffness Matrix Method

9^{th} May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Force along the member of truss element



Stiffness Matrix Method

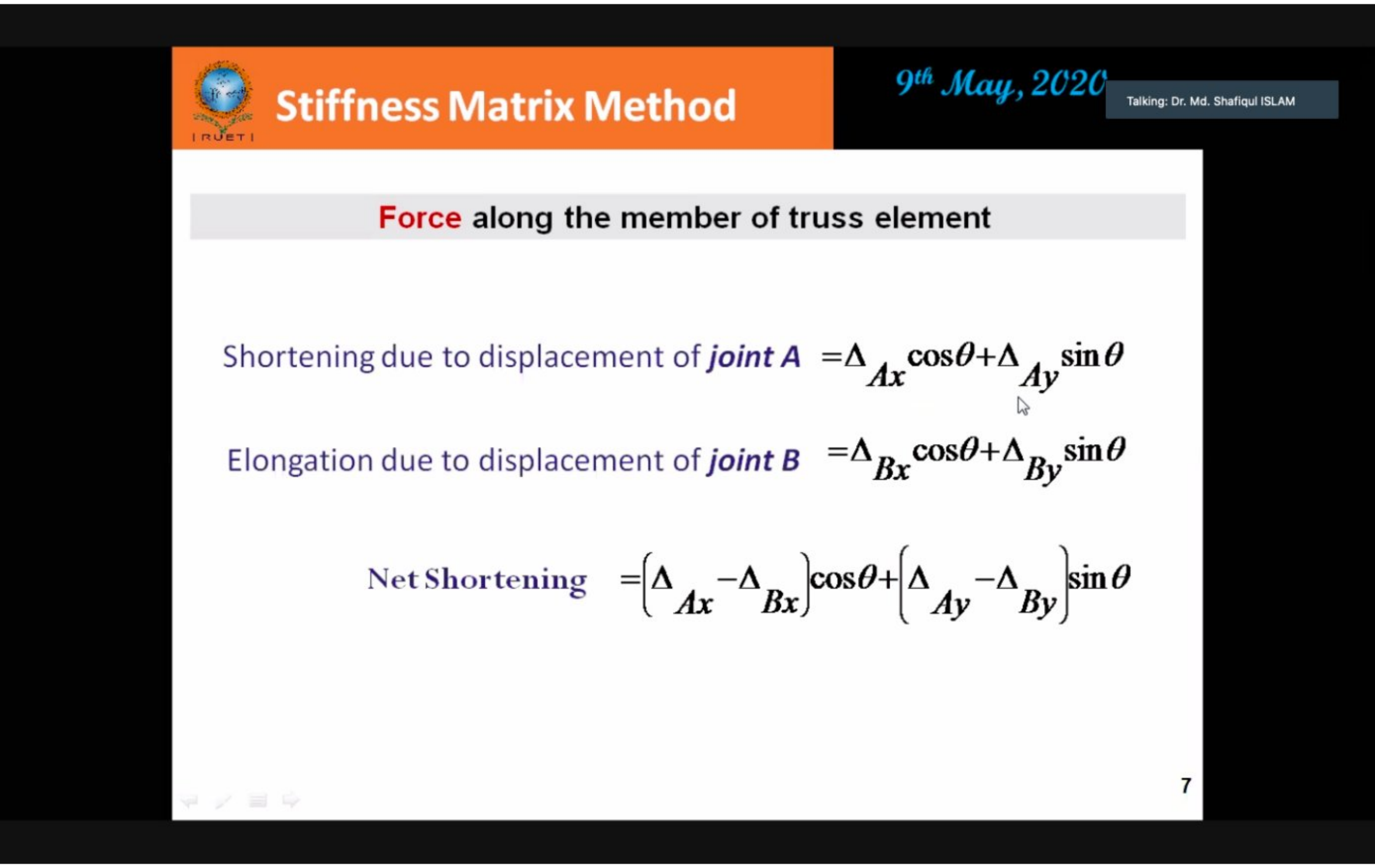
9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

Force along the member of truss element

Shortening due to displacement of ***joint A*** $=\Delta_{Ax}\cos\theta+\Delta_{Ay}\sin\theta$

Elongation due to displacement of ***joint B*** $=\Delta_{Bx}\cos\theta+\Delta_{By}\sin\theta$



Stiffness Matrix Method

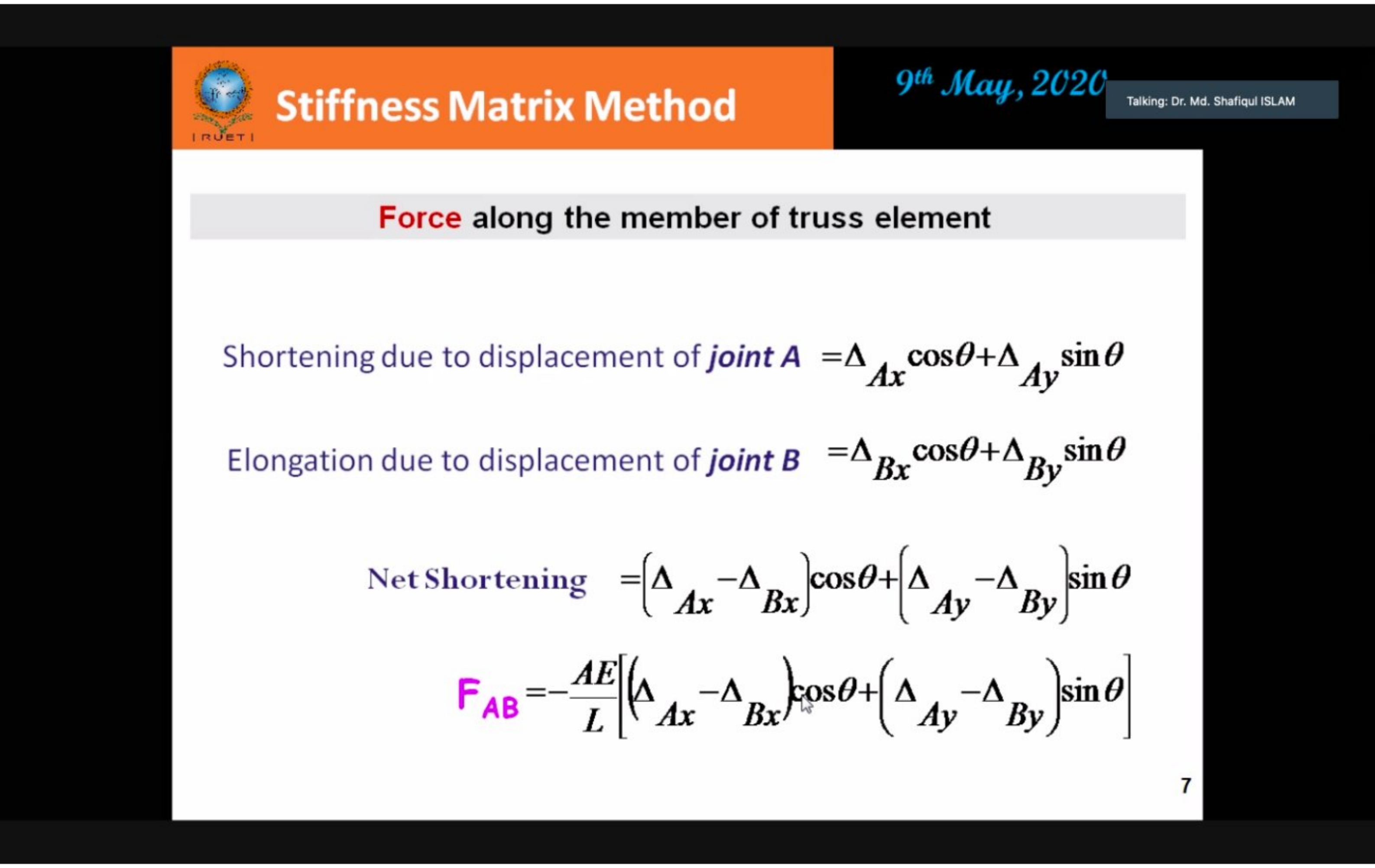
9th May, 2020

Force along the member of truss element

Shortening due to displacement of ***joint A*** $=\Delta_{Ax}\cos\theta+\Delta_{Ay}\sin\theta$

Elongation due to displacement of ***joint B*** $=\Delta_{Bx}\cos\theta+\Delta_{By}\sin\theta$

Net Shortening $=\left(\Delta_{Ax}-\Delta_{Bx}\right)\cos\theta+\left(\Delta_{Ay}-\Delta_{By}\right)\sin\theta$



Stiffness Matrix Method

9th May, 2020

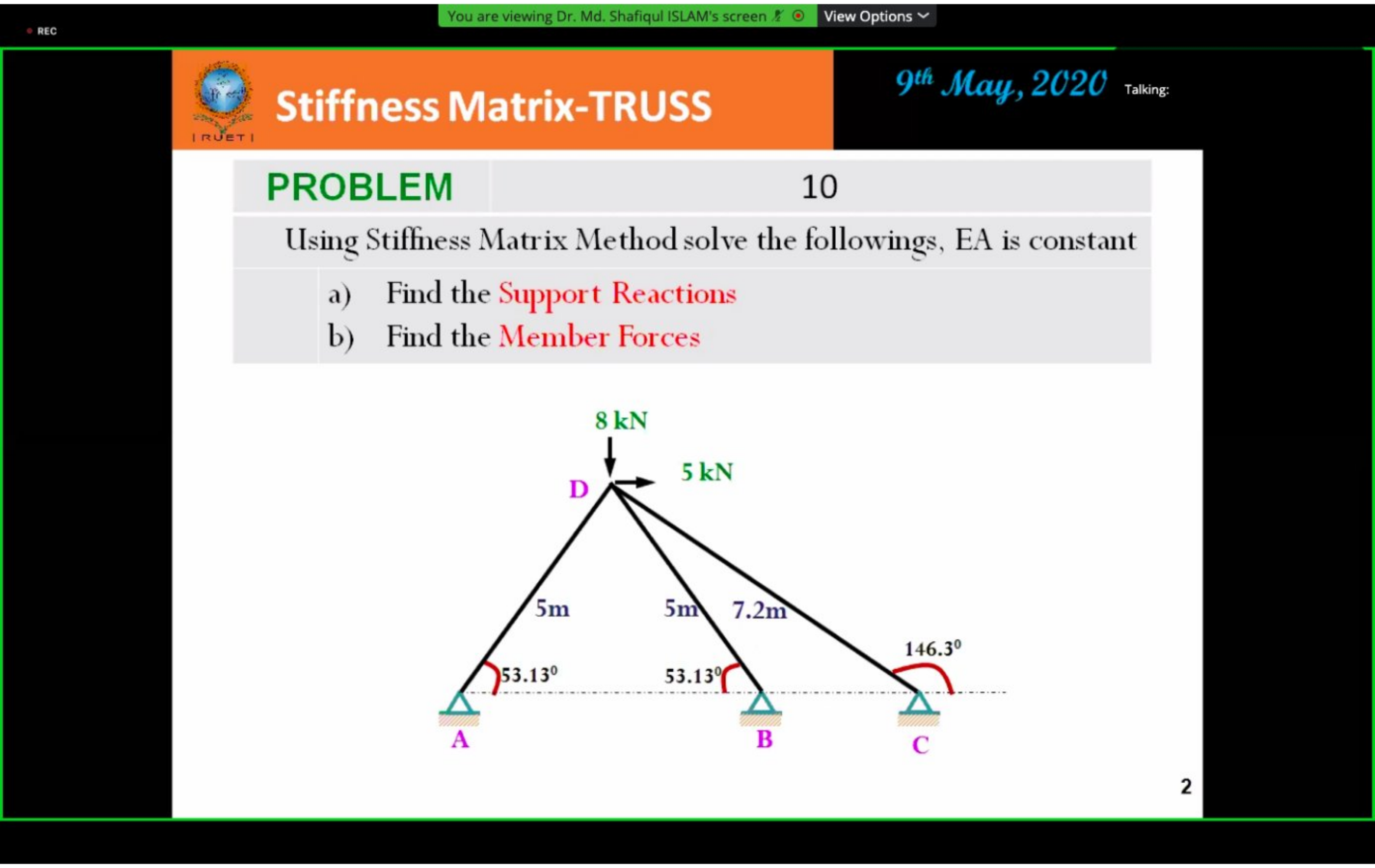
Force along the member of truss element

Shortening due to displacement of ***joint A*** $=\Delta_{Ax}\cos\theta+\Delta_{Ay}\sin\theta$

Elongation due to displacement of ***joint B*** $=\Delta_{Bx}\cos\theta+\Delta_{By}\sin\theta$

Net Shortening $=\left(\Delta_{Ax}-\Delta_{Bx}\right)\cos\theta+\left(\Delta_{Ay}-\Delta_{By}\right)\sin\theta$

$$F_{AB}=-\frac{AE}{L}\left[\left(\Delta_{Ax}-\Delta_{Bx}\right)\cos\theta+\left(\Delta_{Ay}-\Delta_{By}\right)\sin\theta\right]$$



Stiffness Matrix-TRUSS

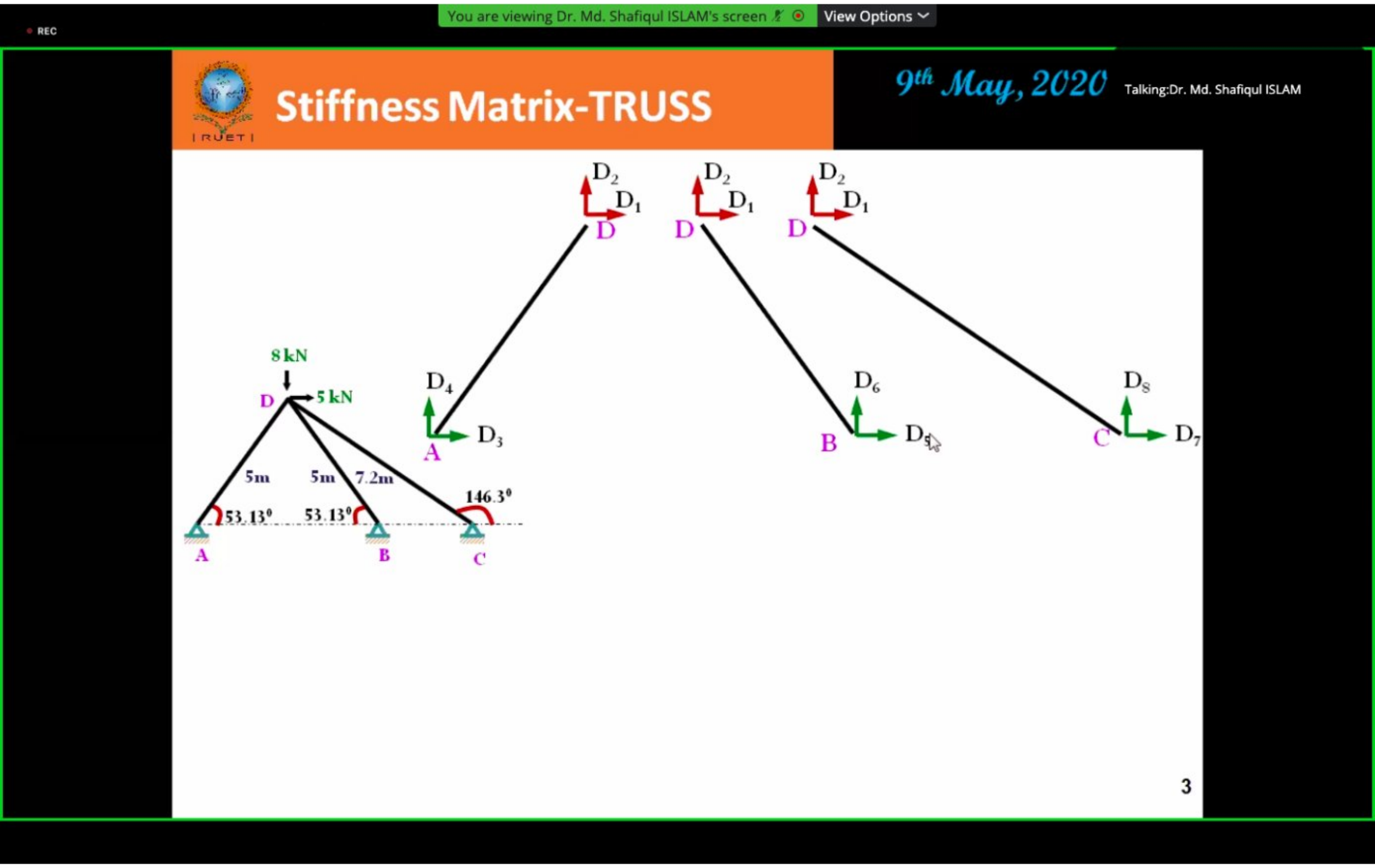
9th May, 2020

PROBLEM 10

Using Stiffness Matrix Method solve the followings, EA is constant

a) Find the Support Reactions

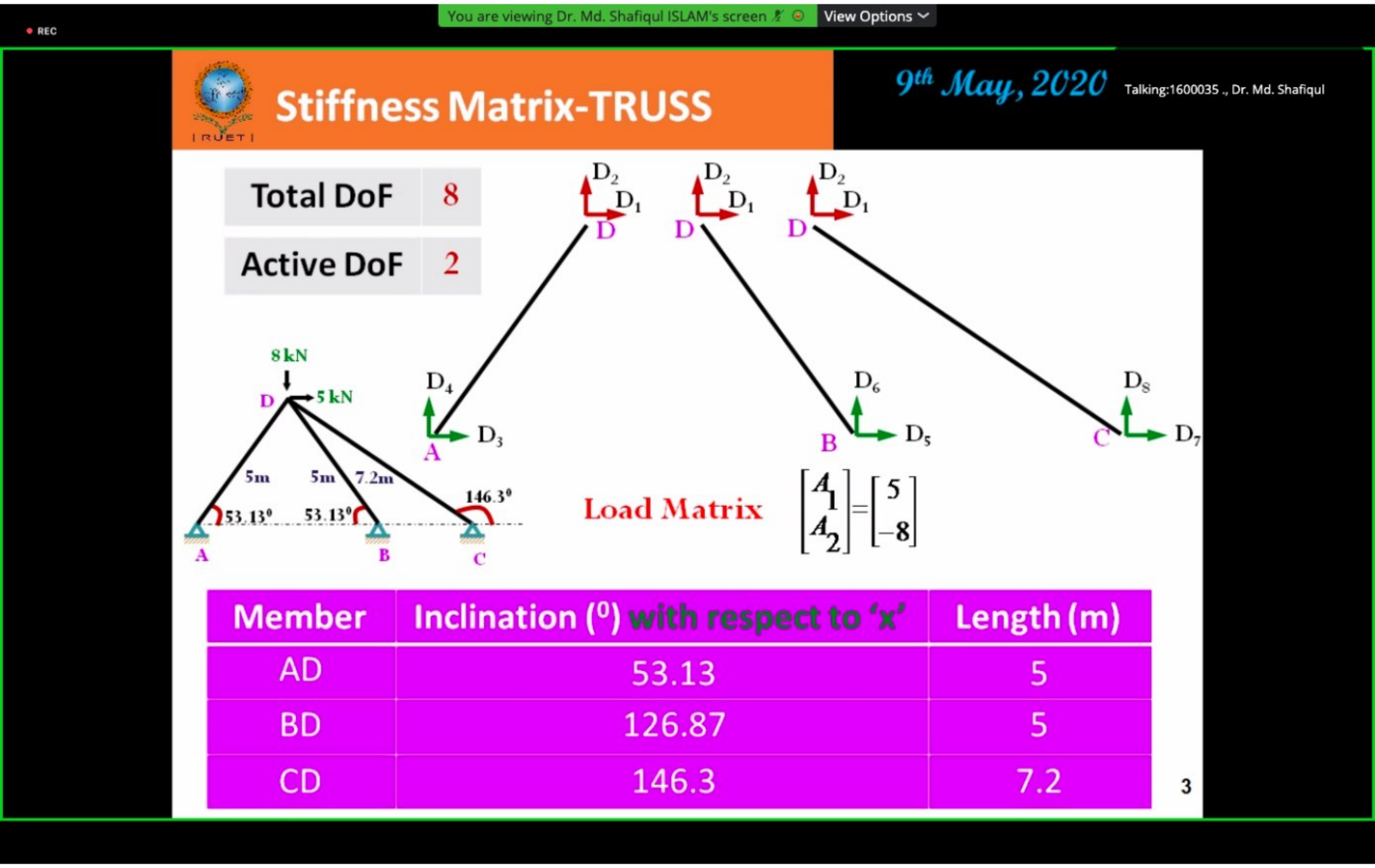
b) Find the Member Forces



Stiffness Matrix-TRUSS

9th May, 2020

Talking:Dr. Md. Shafiqul ISLAM



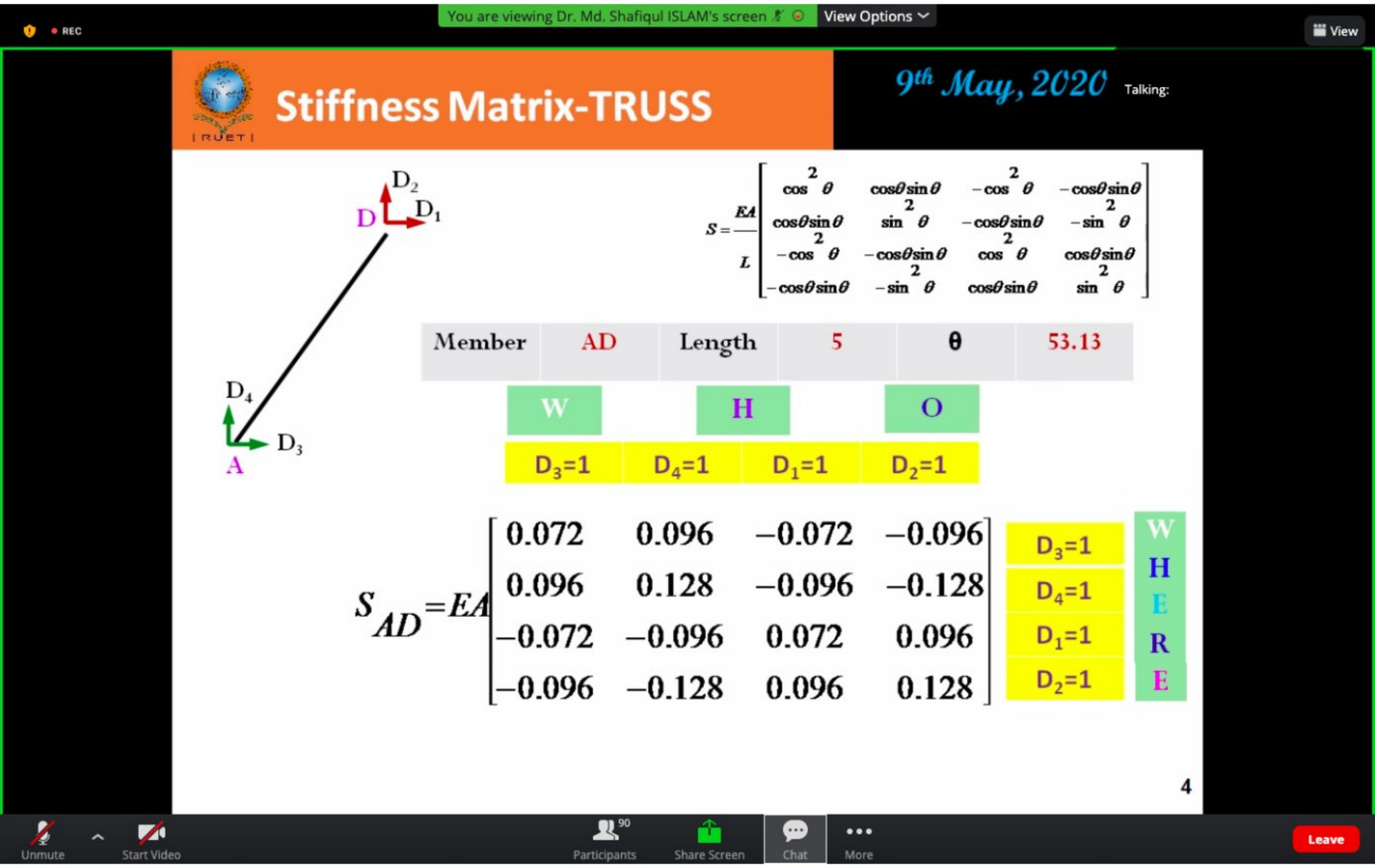
Stiffness Matrix-TRUSS

9th May, 2020

Total DoF	8
Active DoF	2

Load Matrix $\begin{bmatrix}A_1\\A_2\end{bmatrix}=\begin{bmatrix}5\\-8\end{bmatrix}$

Member	Inclination (^{0}) with respect to 'x'	Length (m)
AD	53.13	5
BD	126.87	5
CD	146.3	7.2



Stiffness Matrix-TRUSS

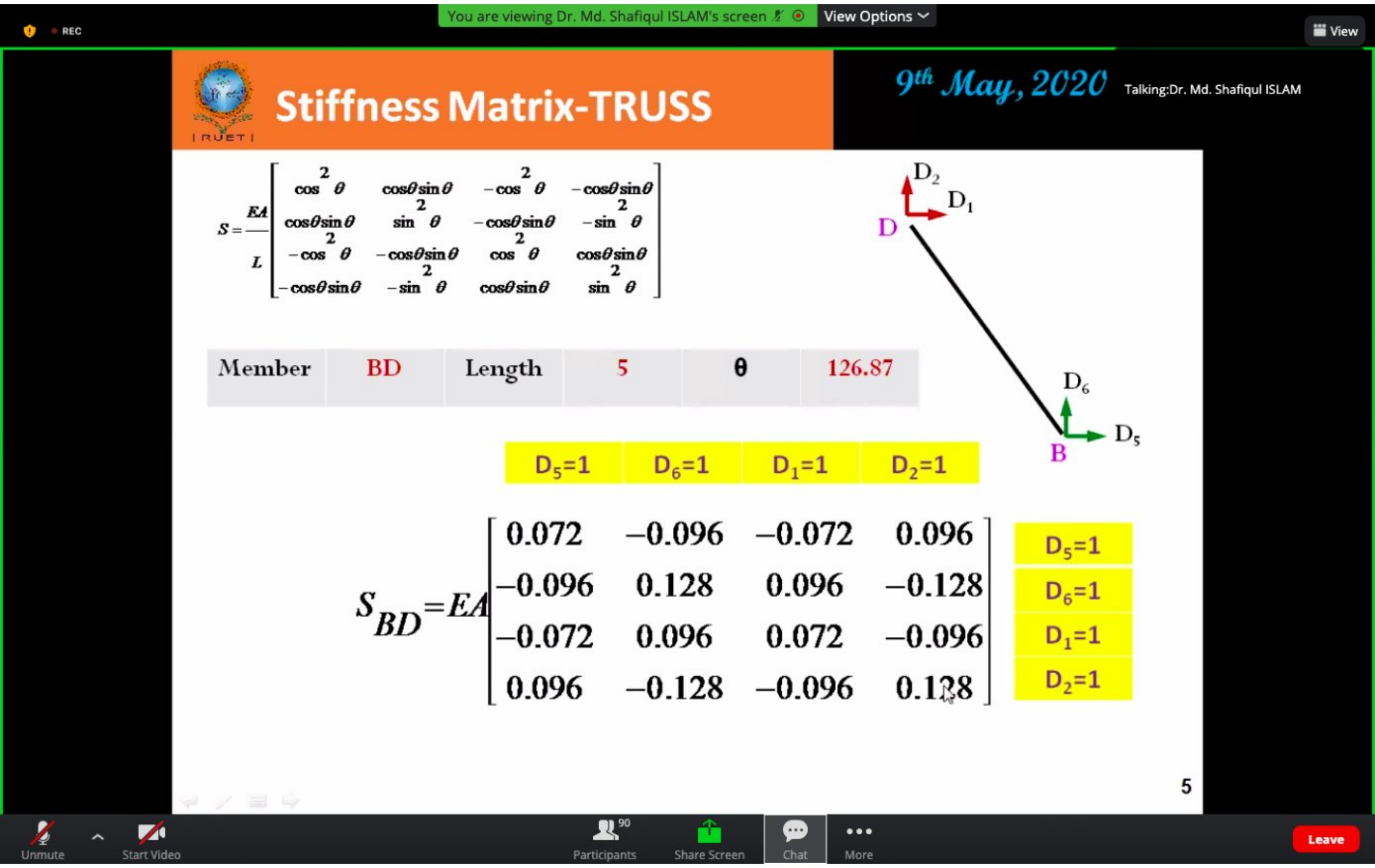
9th May, 2020

$$S=\frac{EA}{L}\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

Member	AD	Length	5	θ	53.13

W H O

$$S_{AD}=EA\begin{array}{c} \begin{array}{cccc} D_3=1 & D_4=1 & D_1=1 & D_2=1 \end{array} \\ \begin{bmatrix} 0.072 & 0.096 & -0.072 & -0.096 \\ 0.096 & 0.128 & -0.096 & -0.128 \\ -0.072 & -0.096 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0.096 & 0.128 \end{bmatrix} \end{array} \begin{array}{cc} D_3=1 & W \\ D_4=1 & H \\ D_1=1 & E \\ D_2=1 & R \\ & E \end{array}$$



Stiffness Matrix-TRUSS

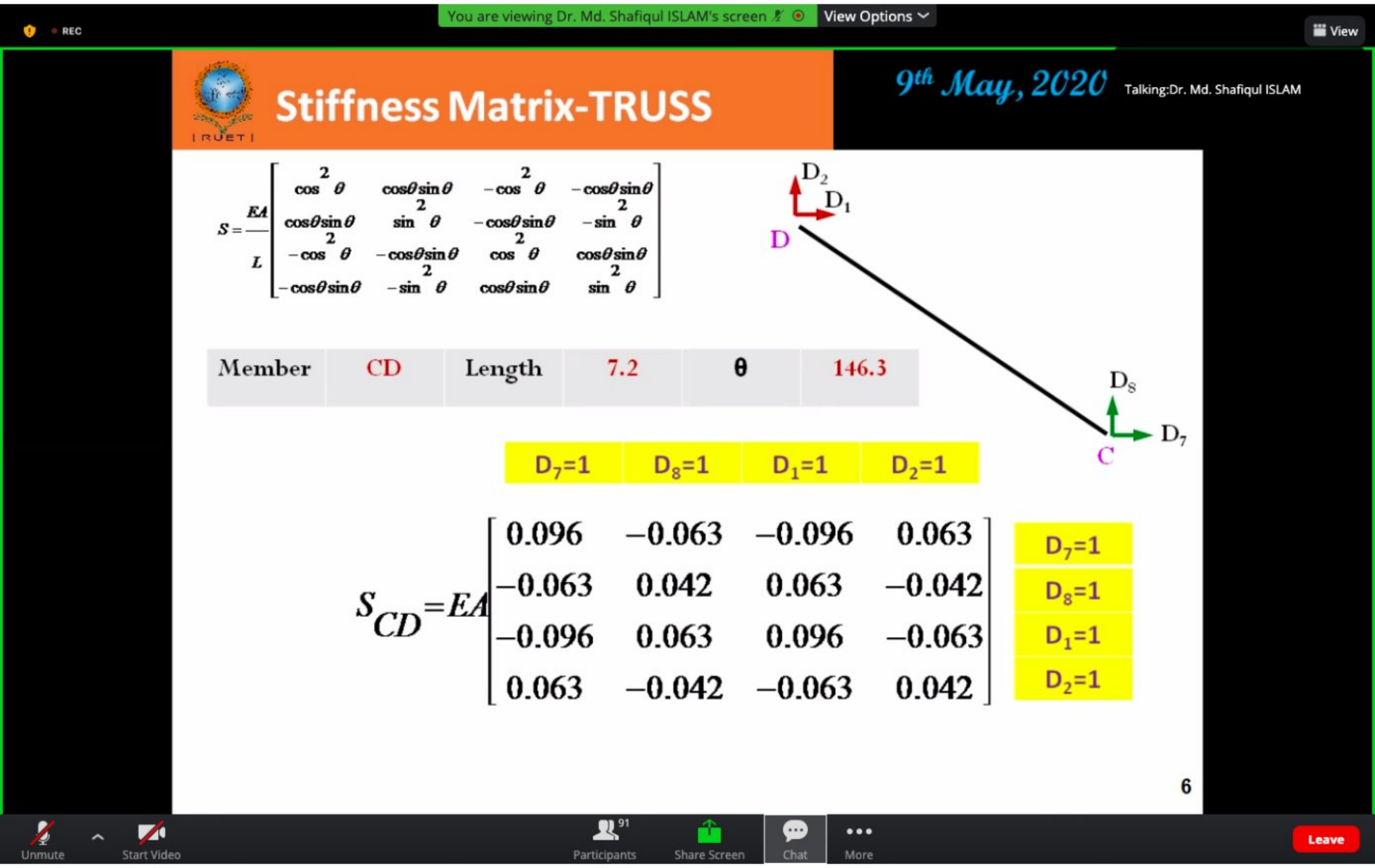
9th May, 2020

Talking:Dr. Md. Shafiqul ISLAM

$$S=\frac{EA}{L}\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta\\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$$

Member	BD	Length	5	θ	126.87

$$S_{BD}=EA\begin{array}{cccc|c} D_5=1 & D_6=1 & D_1=1 & D_2=1 & \\ 0.072 & -0.096 & -0.072 & 0.096 & D_5=1\\ -0.096 & 0.128 & 0.096 & -0.128 & D_6=1\\ -0.072 & 0.096 & 0.072 & -0.096 & D_1=1\\ 0.096 & -0.128 & -0.096 & 0.128 & D_2=1\end{array}$$



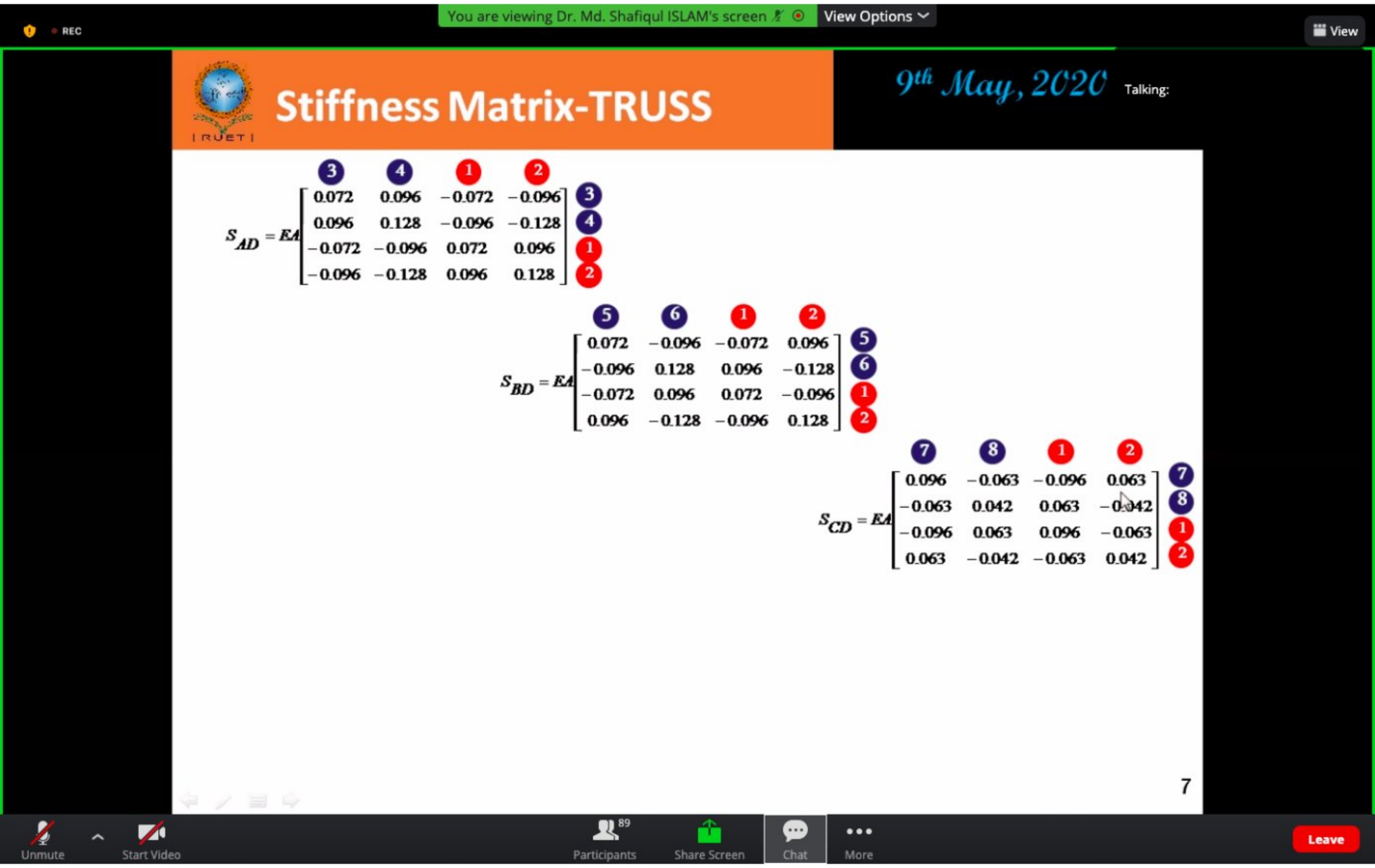
9th May, 2020 Talking:Dr. Md. Shafiqul ISLAM

Stiffness Matrix-TRUSS

$$S=\frac{EA}{L}\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

Member	CD	Length	7.2	θ	146.3

$$\begin{array}{cc} & \begin{array}{cccc} D_7=1 & D_8=1 & D_1=1 & D_2=1 \end{array} \\ S_{CD}=EA & \begin{bmatrix} 0.096 & -0.063 & -0.096 & 0.063 \\ -0.063 & 0.042 & 0.063 & -0.042 \\ -0.096 & 0.063 & 0.096 & -0.063 \\ 0.063 & -0.042 & -0.063 & 0.042 \end{bmatrix} \begin{array}{c} D_7=1 \\ D_8=1 \\ D_1=1 \\ D_2=1 \end{array} \end{array}$$



Stiffness Matrix-TRUSS

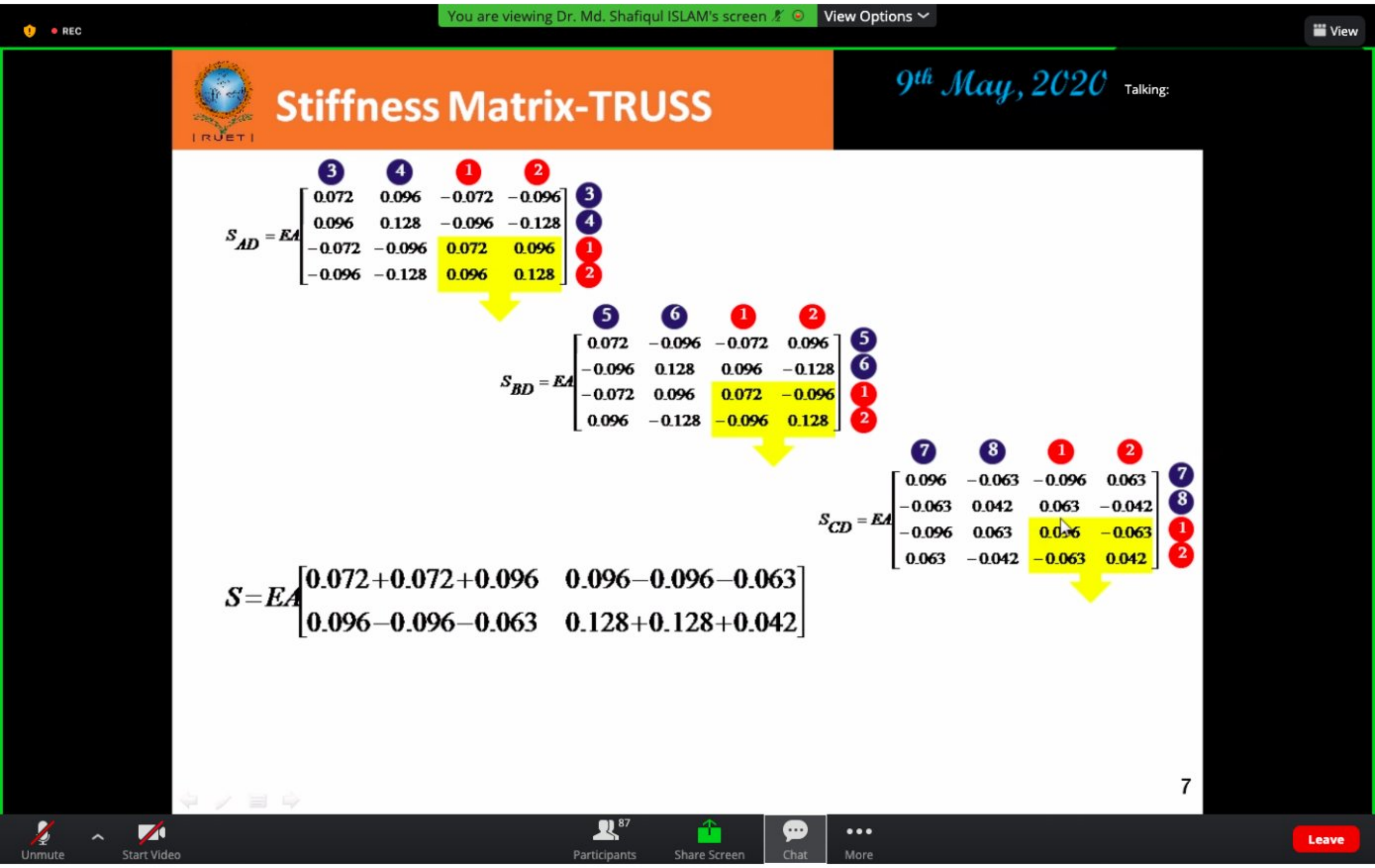
9th May, 2020

Talking:

$$S_{AD} = EA\begin{array}{c} \begin{array}{cccc} ③ & ④ & ① & ② \end{array} \\ \begin{bmatrix} 0.072 & 0.096 & -0.072 & -0.096 \\ 0.096 & 0.128 & -0.096 & -0.128 \\ -0.072 & -0.096 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0.096 & 0.128 \end{bmatrix} \end{array} \begin{array}{c} \\ ③ \\ ④ \\ ① \\ ② \end{array}$$

$$S_{BD} = EA\begin{array}{c} \begin{array}{cccc} ⑤ & ⑥ & ① & ② \end{array} \\ \begin{bmatrix} 0.072 & -0.096 & -0.072 & 0.096 \\ -0.096 & 0.128 & 0.096 & -0.128 \\ -0.072 & 0.096 & 0.072 & -0.096 \\ 0.096 & -0.128 & -0.096 & 0.128 \end{bmatrix} \end{array} \begin{array}{c} \\ ⑤ \\ ⑥ \\ ① \\ ② \end{array}$$

$$S_{CD} = EA\begin{array}{c} \begin{array}{cccc} ⑦ & ⑧ & ① & ② \end{array} \\ \begin{bmatrix} 0.096 & -0.063 & -0.096 & 0.063 \\ -0.063 & 0.042 & 0.063 & -0.042 \\ -0.096 & 0.063 & 0.096 & -0.063 \\ 0.063 & -0.042 & -0.063 & 0.042 \end{bmatrix} \end{array} \begin{array}{c} \\ ⑦ \\ ⑧ \\ ① \\ ② \end{array}$$



Stiffness Matrix-TRUSS

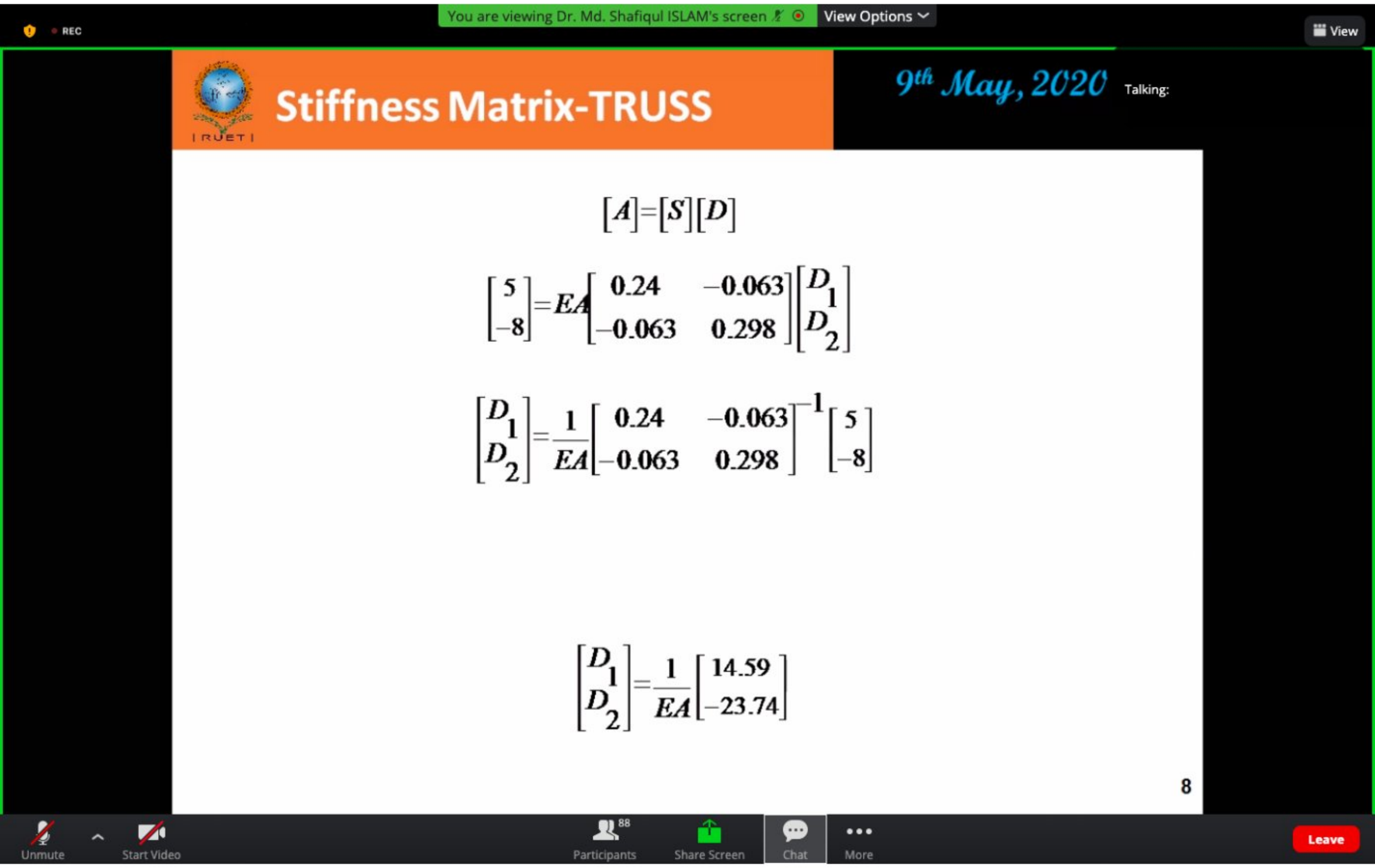
9th May, 2020

$$S_{AD} = EA\begin{array}{c|cccc|c} & ③ & ④ & ① & ② & \\ \hline & 0.072 & 0.096 & -0.072 & -0.096 & ③ \\ & 0.096 & 0.128 & -0.096 & -0.128 & ④ \\ & -0.072 & -0.096 & 0.072 & 0.096 & ① \\ & -0.096 & -0.128 & 0.096 & 0.128 & ② \end{array}$$

$$S_{BD} = EA\begin{array}{c|cccc|c} & ⑤ & ⑥ & ① & ② & \\ \hline & 0.072 & -0.096 & -0.072 & 0.096 & ⑤ \\ & -0.096 & 0.128 & 0.096 & -0.128 & ⑥ \\ & -0.072 & 0.096 & 0.072 & -0.096 & ① \\ & 0.096 & -0.128 & -0.096 & 0.128 & ② \end{array}$$

$$S_{CD} = EA\begin{array}{c|cccc|c} & ⑦ & ⑧ & ① & ② & \\ \hline & 0.096 & -0.063 & -0.096 & 0.063 & ⑦ \\ & -0.063 & 0.042 & 0.063 & -0.042 & ⑧ \\ & -0.096 & 0.063 & 0.096 & -0.063 & ① \\ & 0.063 & -0.042 & -0.063 & 0.042 & ② \end{array}$$

$$S = EA\begin{bmatrix} 0.072+0.072+0.096 & 0.096-0.096-0.063 \\ 0.096-0.096-0.063 & 0.128+0.128+0.042 \end{bmatrix}$$



Stiffness Matrix-TRUSS

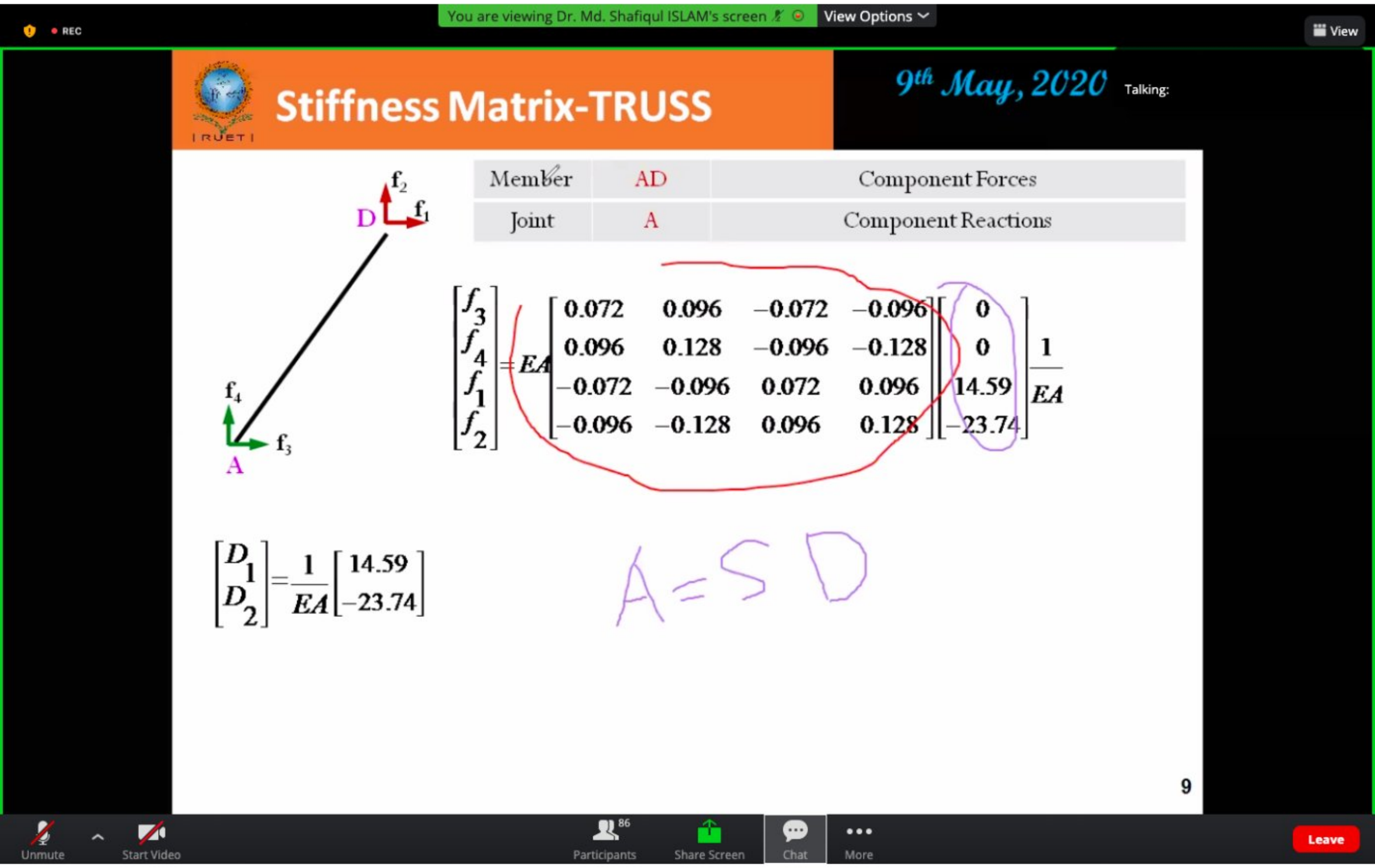
9th May, 2020

$$[A]=[S][D]$$

$$\begin{bmatrix} 5 \\ -8 \end{bmatrix} = EA\begin{bmatrix} 0.24 & -0.063 \\ -0.063 & 0.298 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} 0.24 & -0.063 \\ -0.063 & 0.298 \end{bmatrix}^{-1}\begin{bmatrix} 5 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA}\begin{bmatrix} 14.59 \\ -23.74 \end{bmatrix}$$



Stiffness Matrix-TRUSS

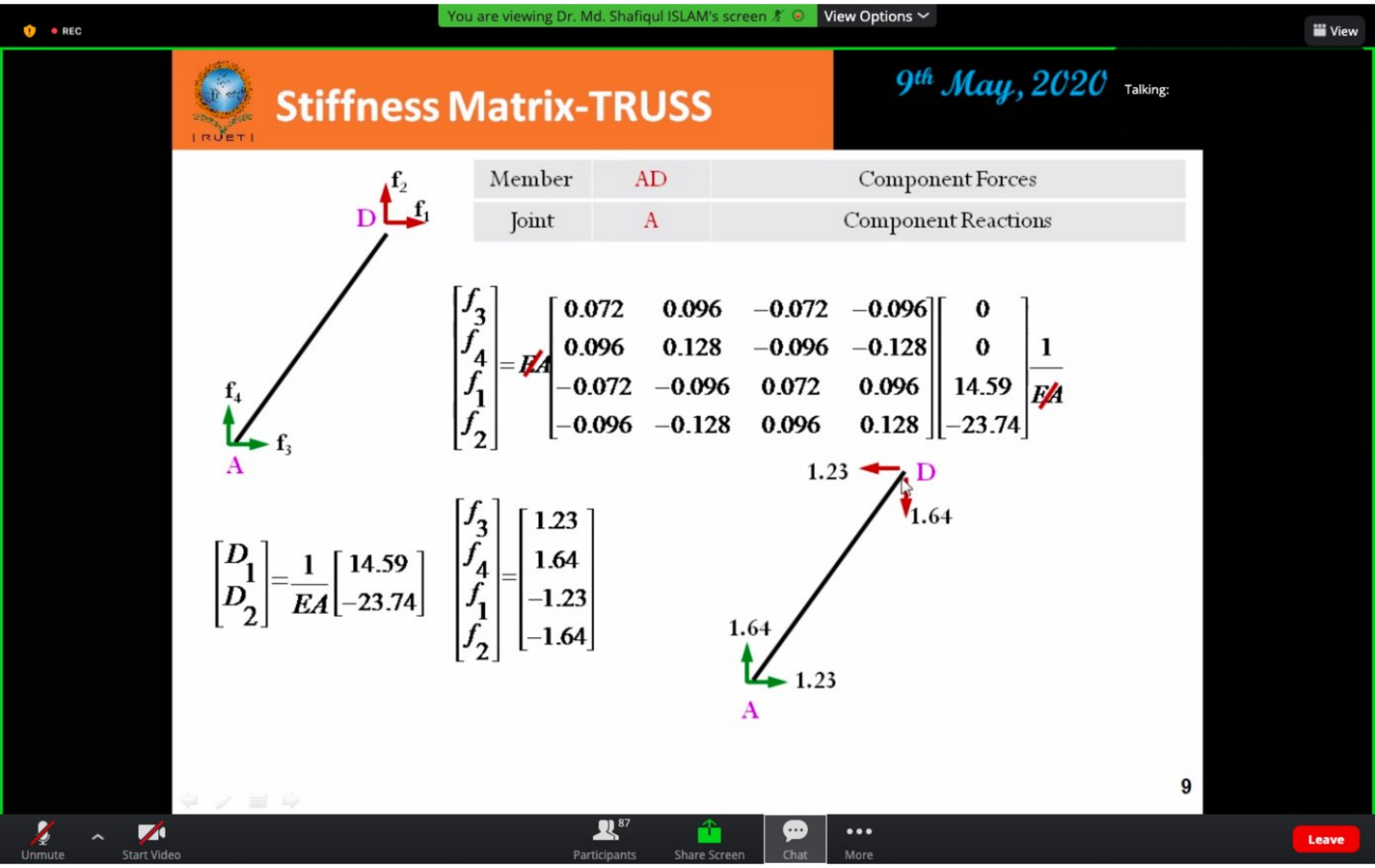
9th May, 2020

Member	AD	Component Forces
Joint	A	Component Reactions

$$\begin{bmatrix} f_3 \\ f_4 \\ f_1 \\ f_2 \end{bmatrix} = EA \begin{bmatrix} 0.072 & 0.096 & -0.072 & -0.096 \\ 0.096 & 0.128 & -0.096 & -0.128 \\ -0.072 & -0.096 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0.096 & 0.128 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 14.59 \\ -23.74 \end{bmatrix} \frac{1}{EA}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 14.59 \\ -23.74 \end{bmatrix}$$

$A = S D$



Stiffness Matrix-TRUSS

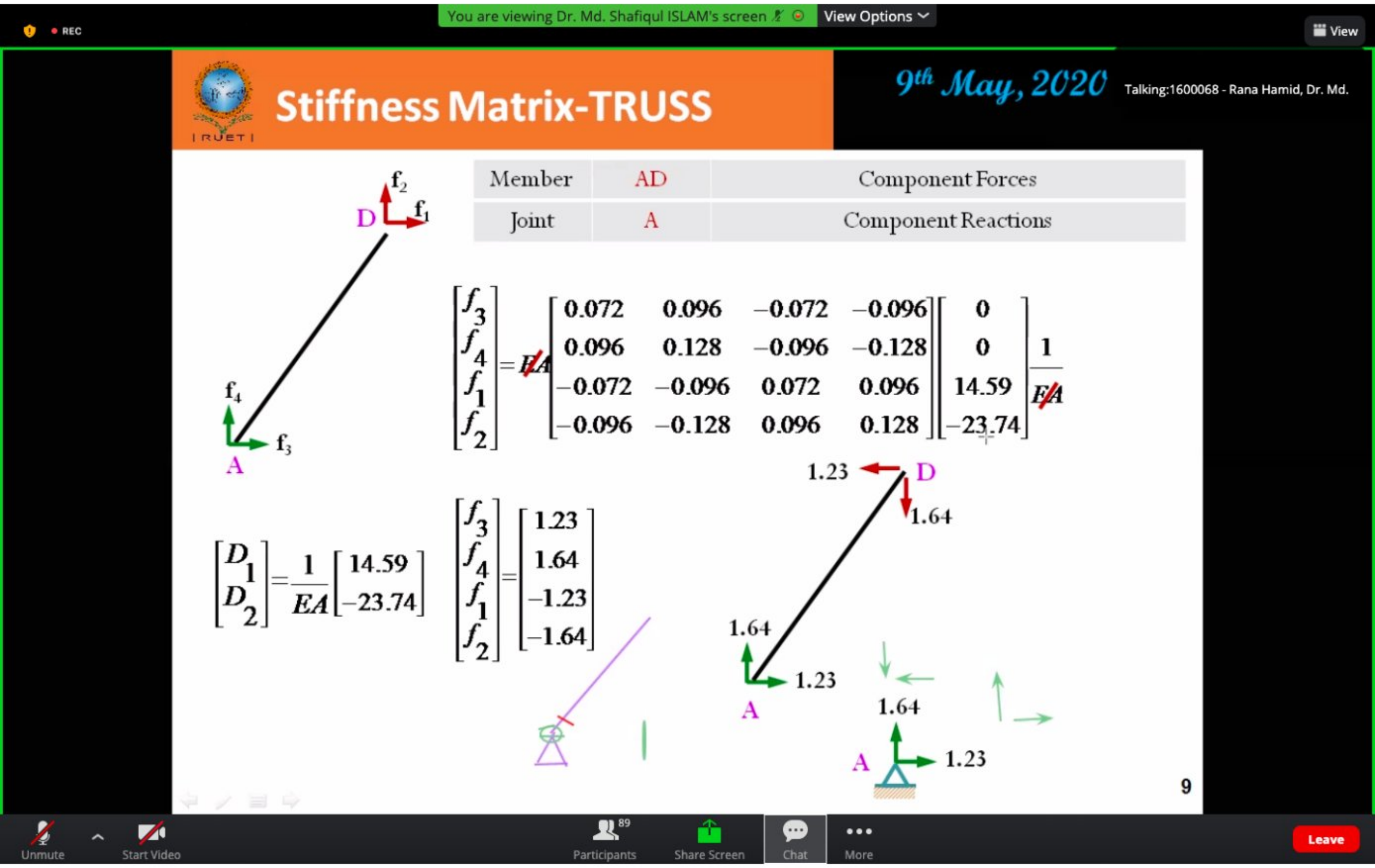
9th May, 2020

Member	AD	Component Forces
Joint	A	Component Reactions

$$\begin{bmatrix} f_3 \\ f_4 \\ f_1 \\ f_2 \end{bmatrix} = \cancel{EA} \begin{bmatrix} 0.072 & 0.096 & -0.072 & -0.096 \\ 0.096 & 0.128 & -0.096 & -0.128 \\ -0.072 & -0.096 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0.096 & 0.128 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 14.59 \\ -23.74 \end{bmatrix} \frac{1}{\cancel{EA}}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{EA} \begin{bmatrix} 14.59 \\ -23.74 \end{bmatrix}$$

$$\begin{bmatrix} f_3 \\ f_4 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} 1.23 \\ 1.64 \\ -1.23 \\ -1.64 \end{bmatrix}$$



Stiffness Matrix-TRUSS

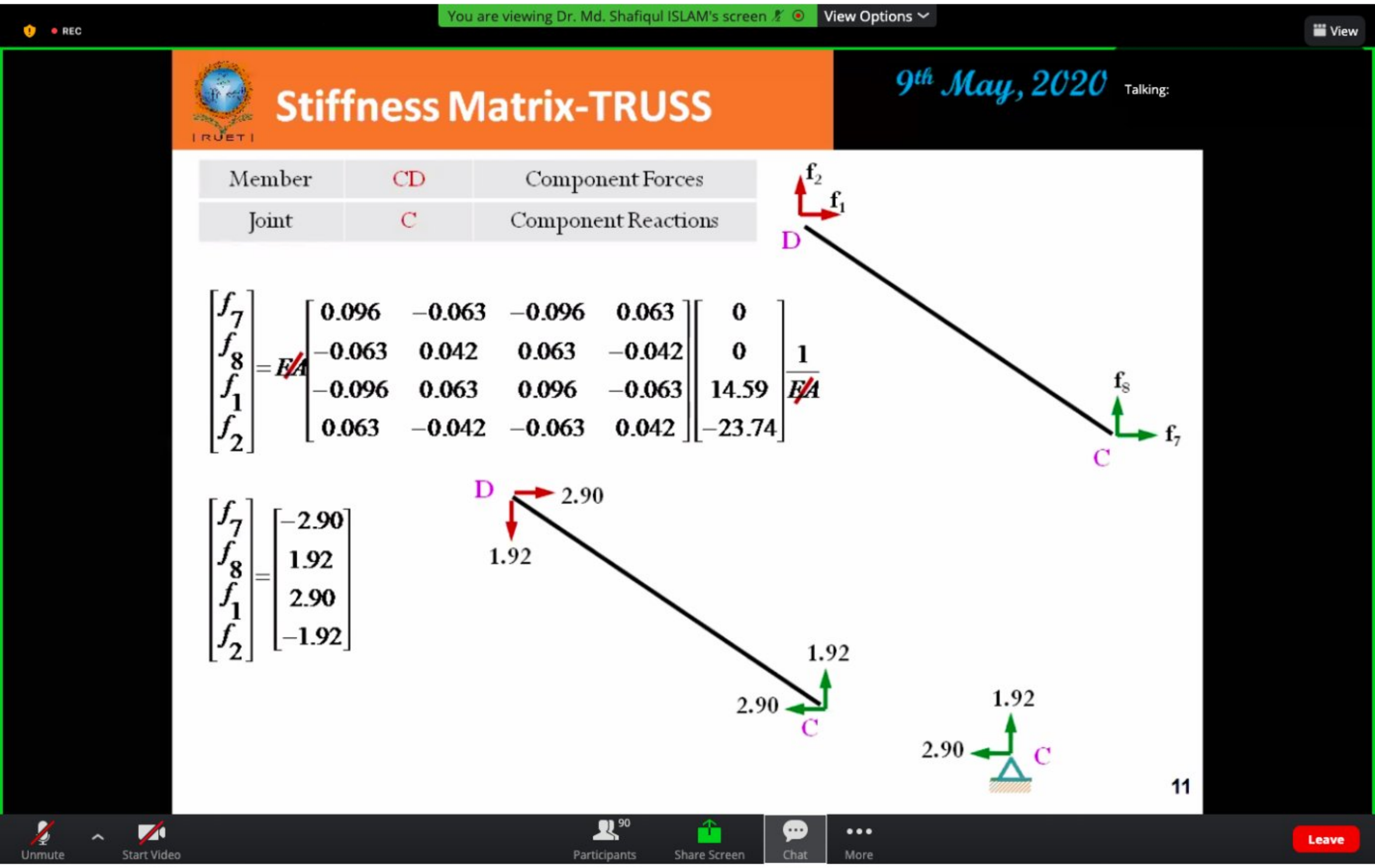
9th May, 2020

Member	AD	Component Forces
Joint	A	Component Reactions

$$\begin{bmatrix} f_3\\ f_4\\ f_1\\ f_2\end{bmatrix} = \cancel{EA}\begin{bmatrix}0.072 & 0.096 & -0.072 & -0.096\\ 0.096 & 0.128 & -0.096 & -0.128\\ -0.072 & -0.096 & 0.072 & 0.096\\ -0.096 & -0.128 & 0.096 & 0.128\end{bmatrix}\begin{bmatrix}0\\ 0\\ 14.59\\ -23.74\end{bmatrix}\frac{1}{\cancel{EA}}$$

$$\begin{bmatrix} D_1\\ D_2\end{bmatrix} = \frac{1}{EA}\begin{bmatrix}14.59\\ -23.74\end{bmatrix}$$

$$\begin{bmatrix} f_3\\ f_4\\ f_1\\ f_2\end{bmatrix} = \begin{bmatrix}1.23\\ 1.64\\ -1.23\\ -1.64\end{bmatrix}$$



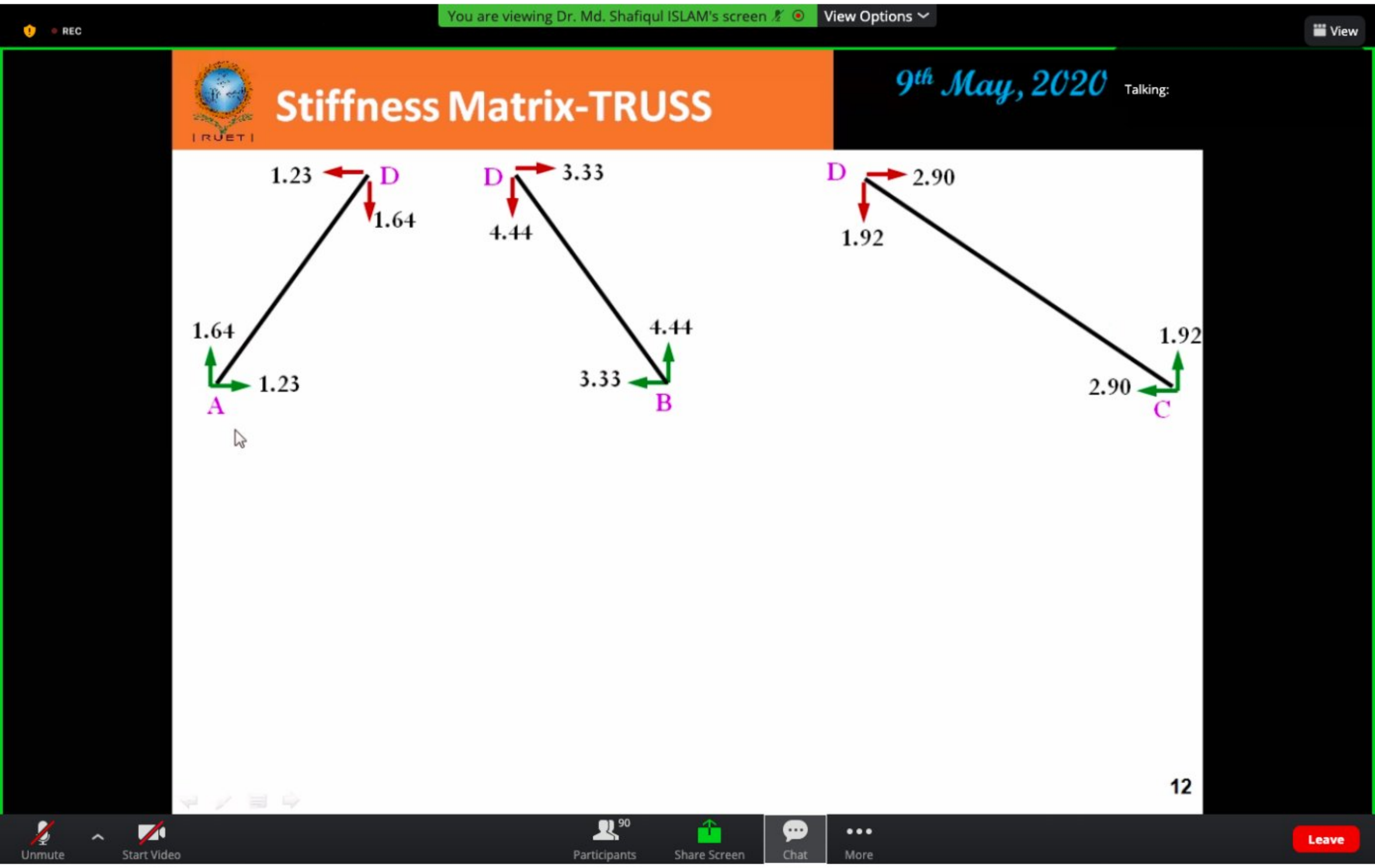
Stiffness Matrix-TRUSS

9th May, 2020

Member	CD	Component Forces
Joint	C	Component Reactions

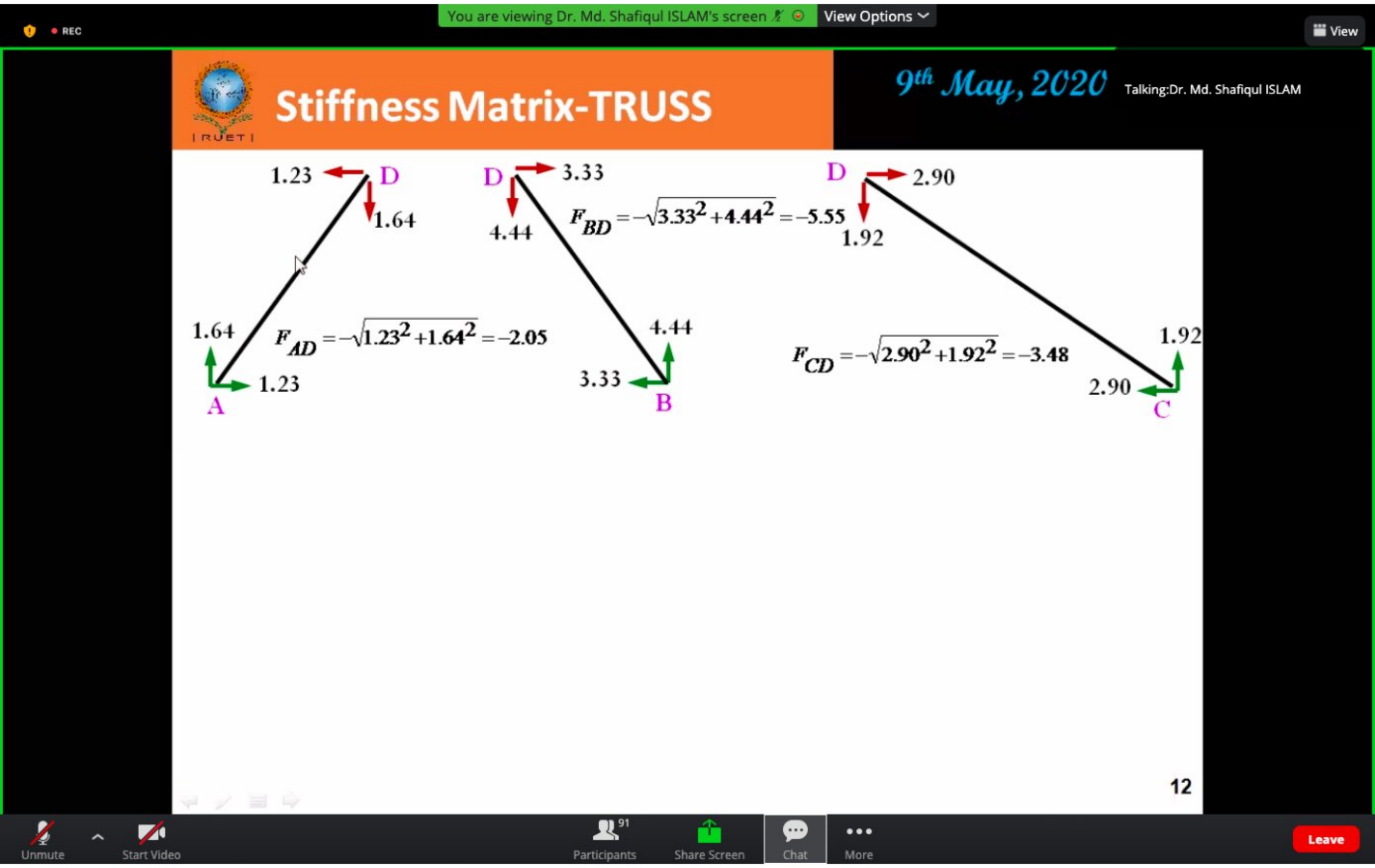
$$\begin{bmatrix} f_7\\ f_8\\ f_1\\ f_2\end{bmatrix} = \cancel{EA}\begin{bmatrix}0.096 & -0.063 & -0.096 & 0.063\\ -0.063 & 0.042 & 0.063 & -0.042\\ -0.096 & 0.063 & 0.096 & -0.063\\ 0.063 & -0.042 & -0.063 & 0.042\end{bmatrix}\begin{bmatrix}0\\ 0\\ 14.59\\ -23.74\end{bmatrix}\frac{1}{\cancel{EA}}$$

$$\begin{bmatrix} f_7\\ f_8\\ f_1\\ f_2\end{bmatrix} = \begin{bmatrix}-2.90\\ 1.92\\ 2.90\\ -1.92\end{bmatrix}$$



Stiffness Matrix-TRUSS

9th May, 2020



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Stiffness Matrix-TRUSS

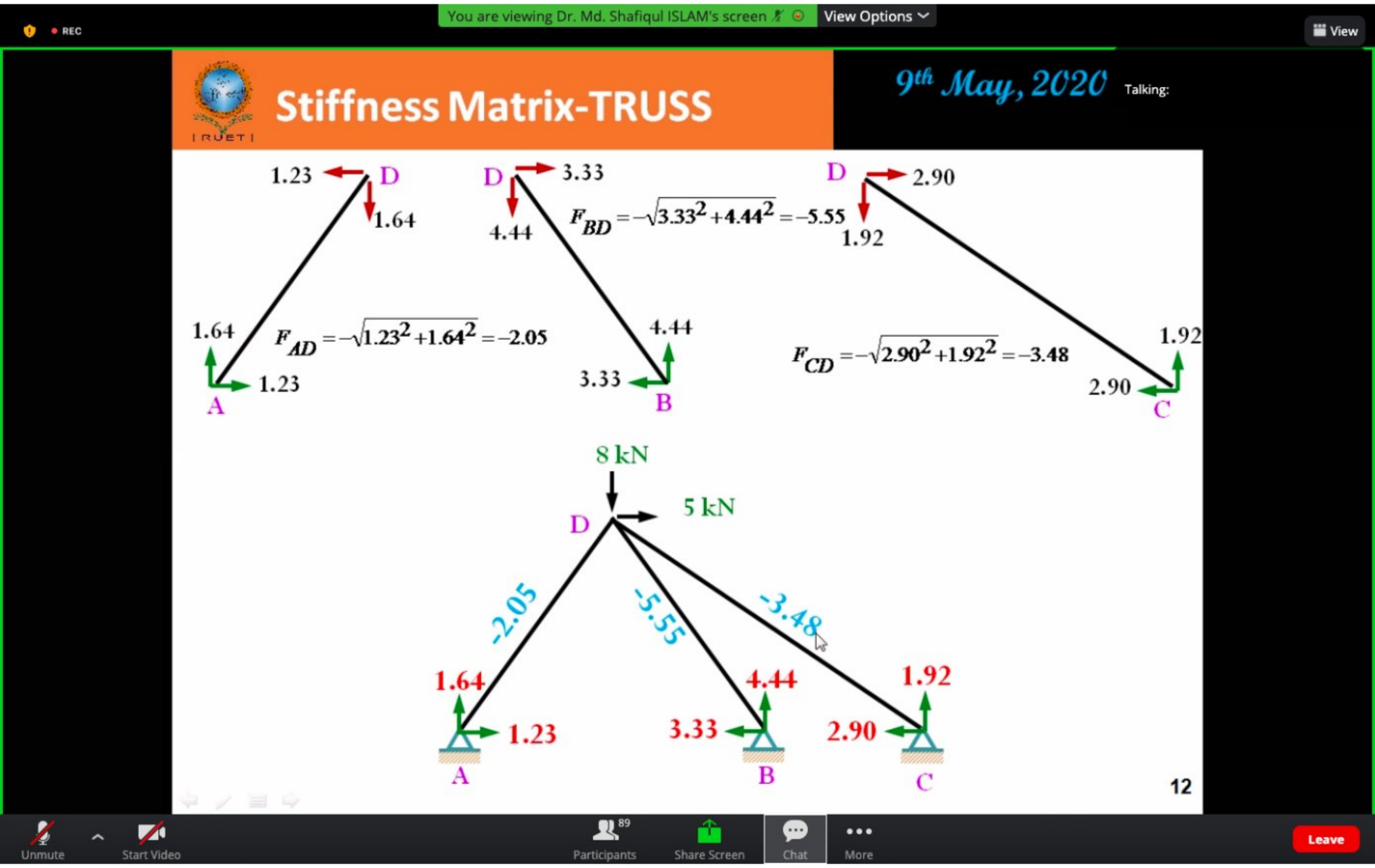
9th May, 2020

Talking:Dr. Md. Shafiqul ISLAM

$$F_{AD} = -\sqrt{1.23^2 + 1.64^2} = -2.05$$

$$F_{BD} = -\sqrt{3.33^2 + 4.44^2} = -5.55$$

$$F_{CD} = -\sqrt{2.90^2 + 1.92^2} = -3.48$$



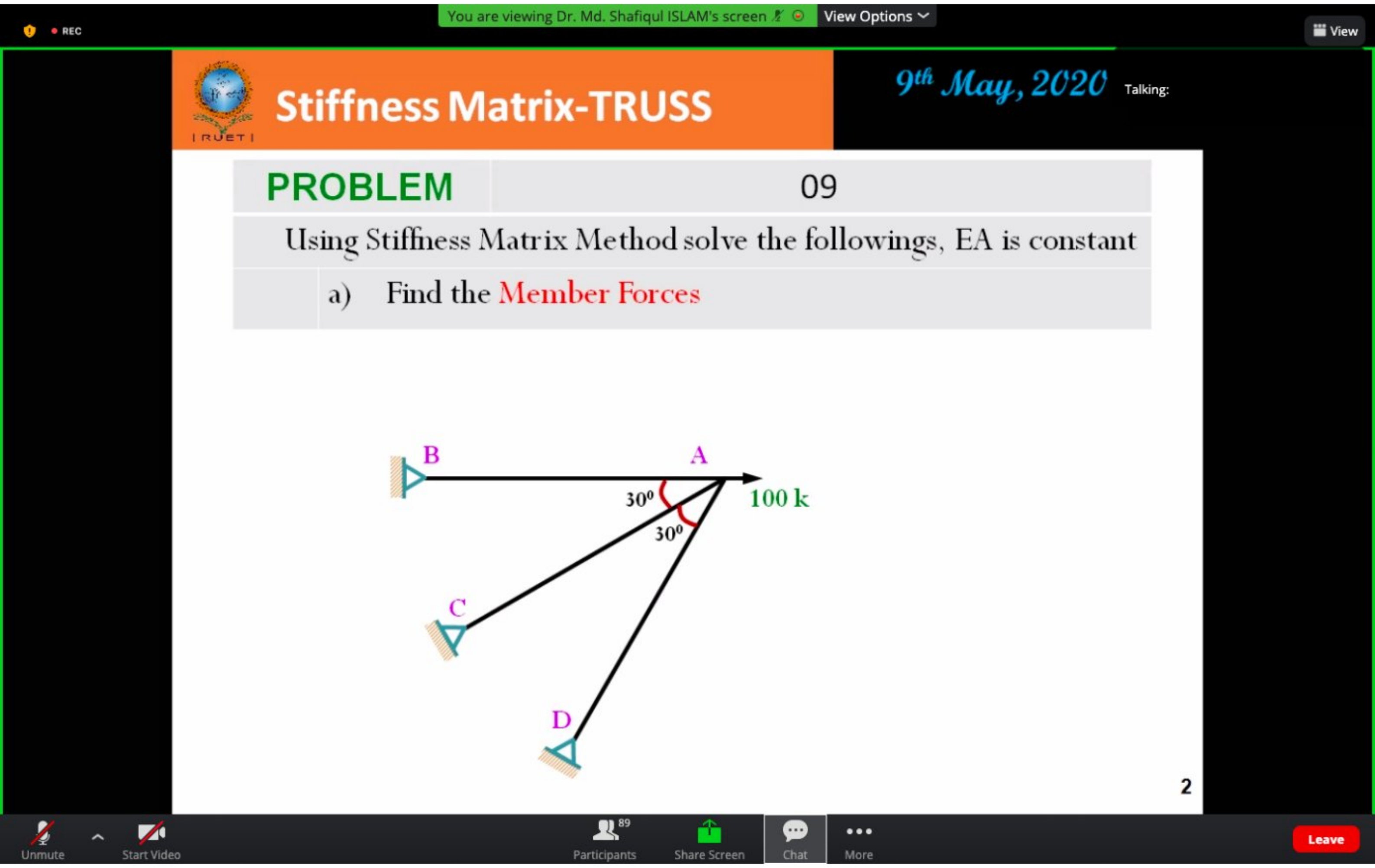
Stiffness Matrix-TRUSS

9th May, 2020

$$F_{BD} = -\sqrt{3.33^2 + 4.44^2} = -5.55$$

$$F_{AD} = -\sqrt{1.23^2 + 1.64^2} = -2.05$$

$$F_{CD} = -\sqrt{2.90^2 + 1.92^2} = -3.48$$



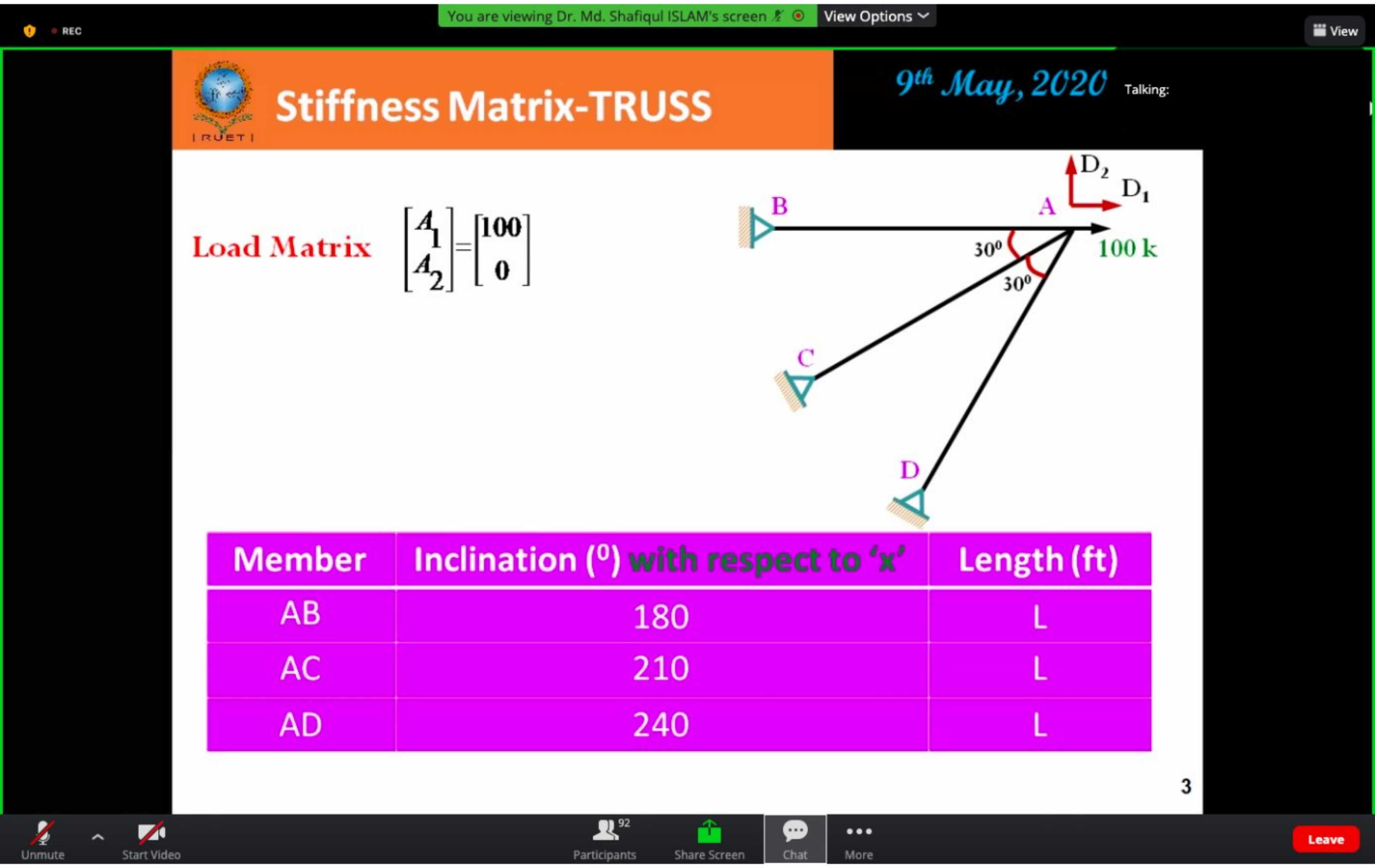
Stiffness Matrix-TRUSS

9th May, 2020

PROBLEM	09

Using Stiffness Matrix Method solve the followings, EA is constant

a) Find the Member Forces



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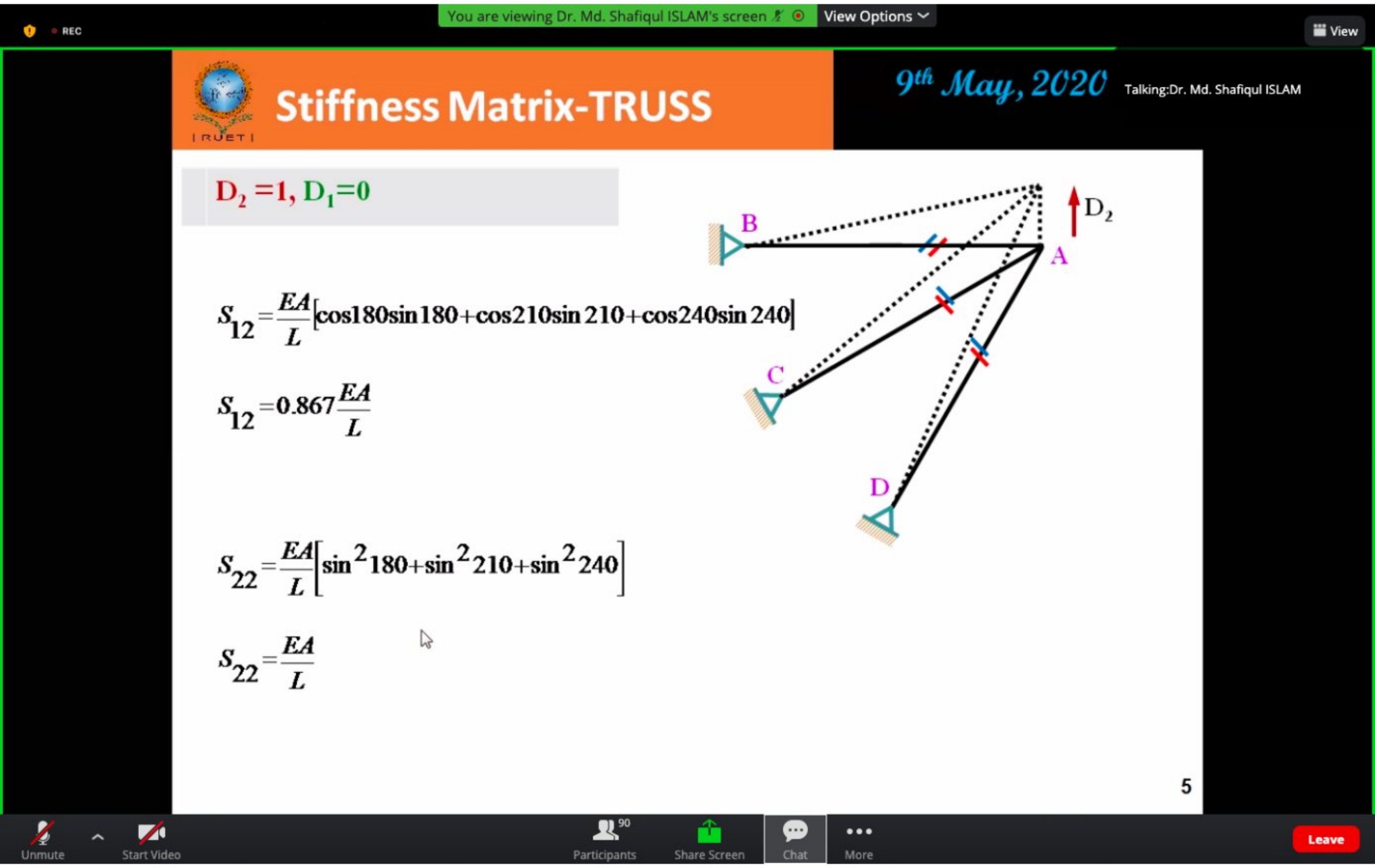
Stiffness Matrix-TRUSS

9th May, 2020

Load Matrix $\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$

Member	Inclination (^{0}) with respect to 'x'	Length (ft)
AB	180	L
AC	210	L
AD	240	L



Stiffness Matrix-TRUSS

9th May, 2020

Talking:Dr. Md. Shafiqul ISLAM

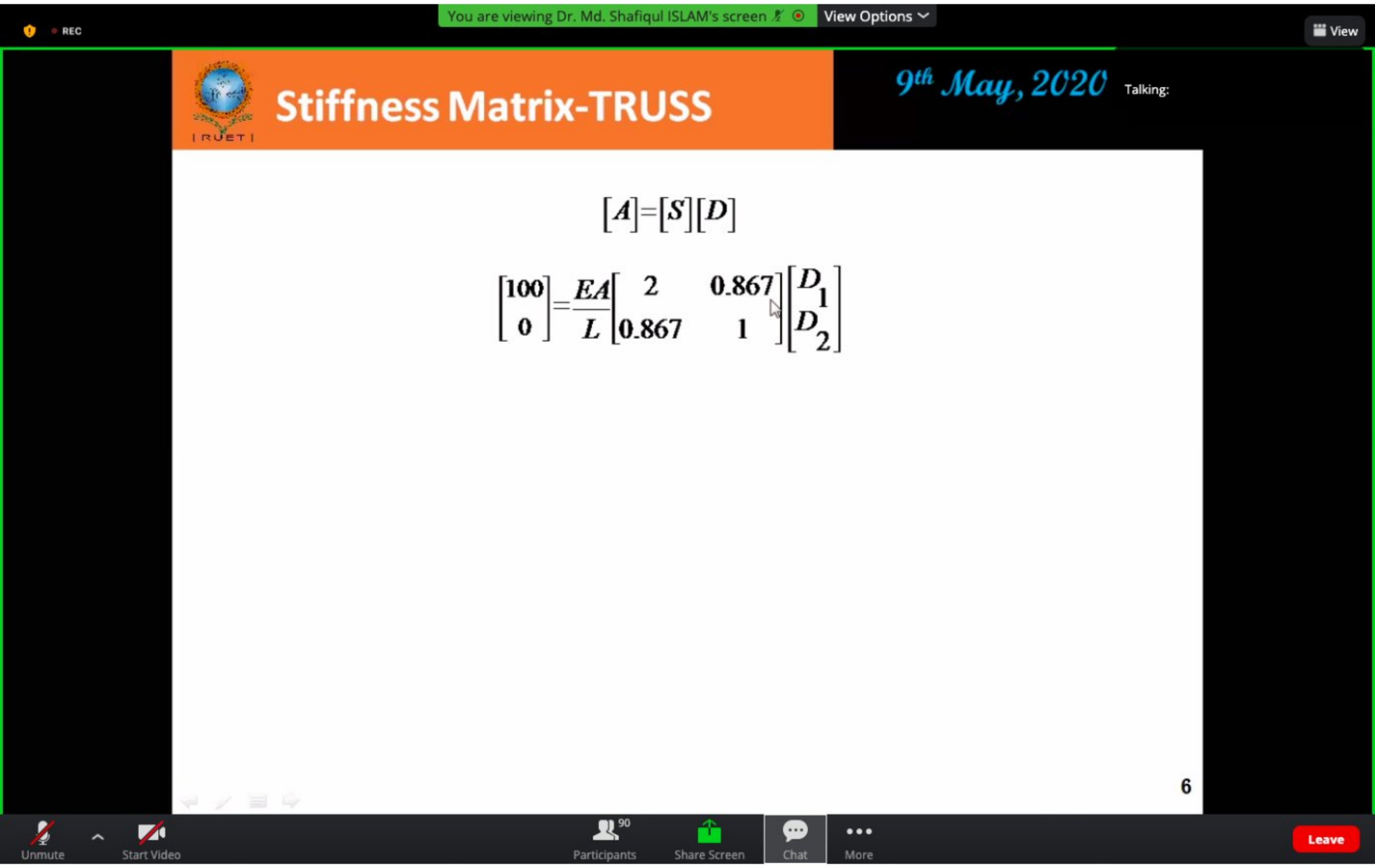
$D_2 = 1$, $D_1 = 0$

$$S_{12} = \frac{EA}{L}\left[\cos 180 \sin 180 + \cos 210 \sin 210 + \cos 240 \sin 240\right]$$

$$S_{12} = 0.867\frac{EA}{L}$$

$$S_{22} = \frac{EA}{L}\left[\sin^2 180 + \sin^2 210 + \sin^2 240\right]$$

$$S_{22} = \frac{EA}{L}$$

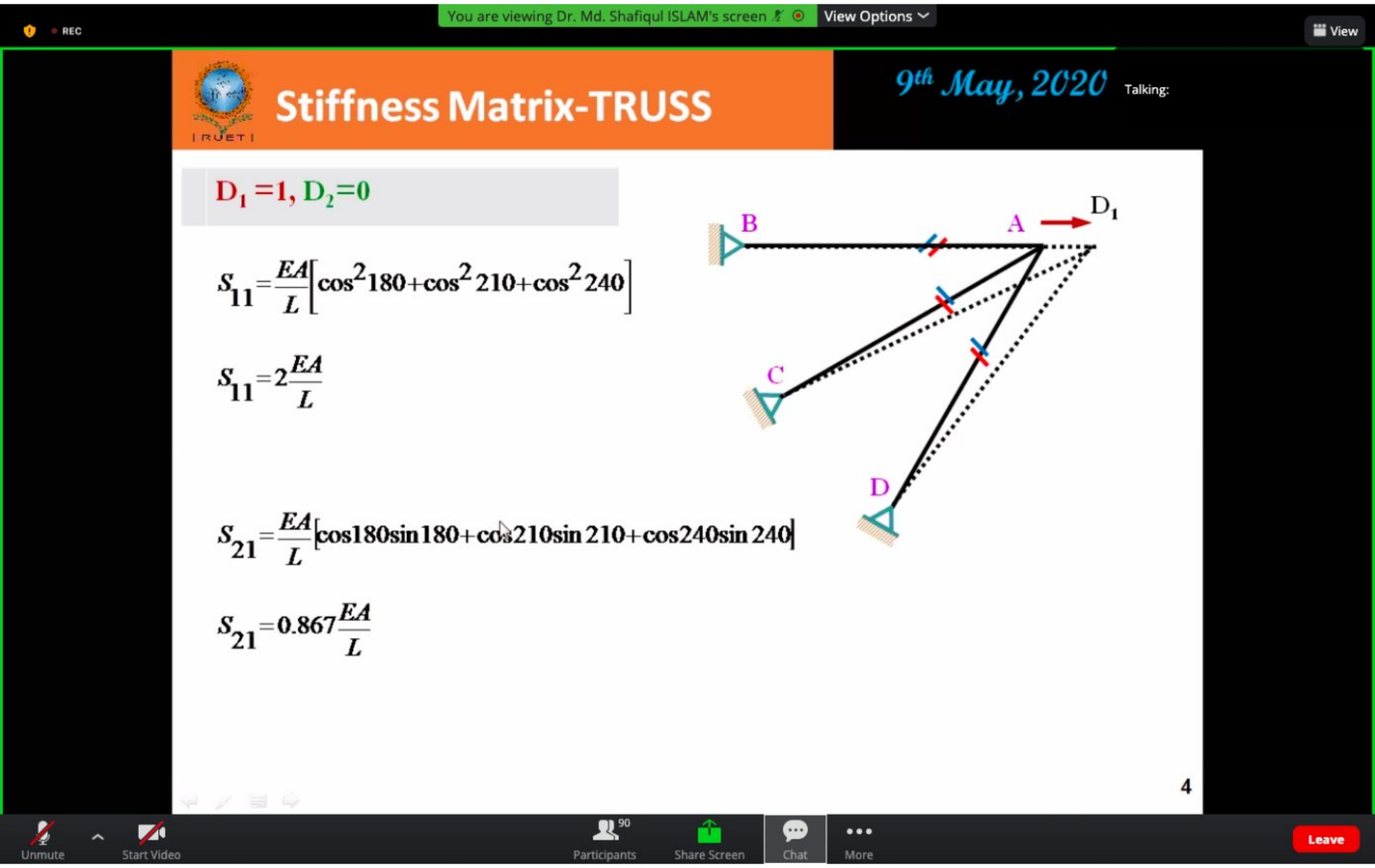


Stiffness Matrix-TRUSS

9th May, 2020

$$[A]=[S][D]$$

$$\begin{bmatrix}100\\0\end{bmatrix}=\frac{EA}{L}\begin{bmatrix}2 & 0.867\\0.867 & 1\end{bmatrix}\begin{bmatrix}D_1\\D_2\end{bmatrix}$$



Stiffness Matrix-TRUSS

9th May, 2020

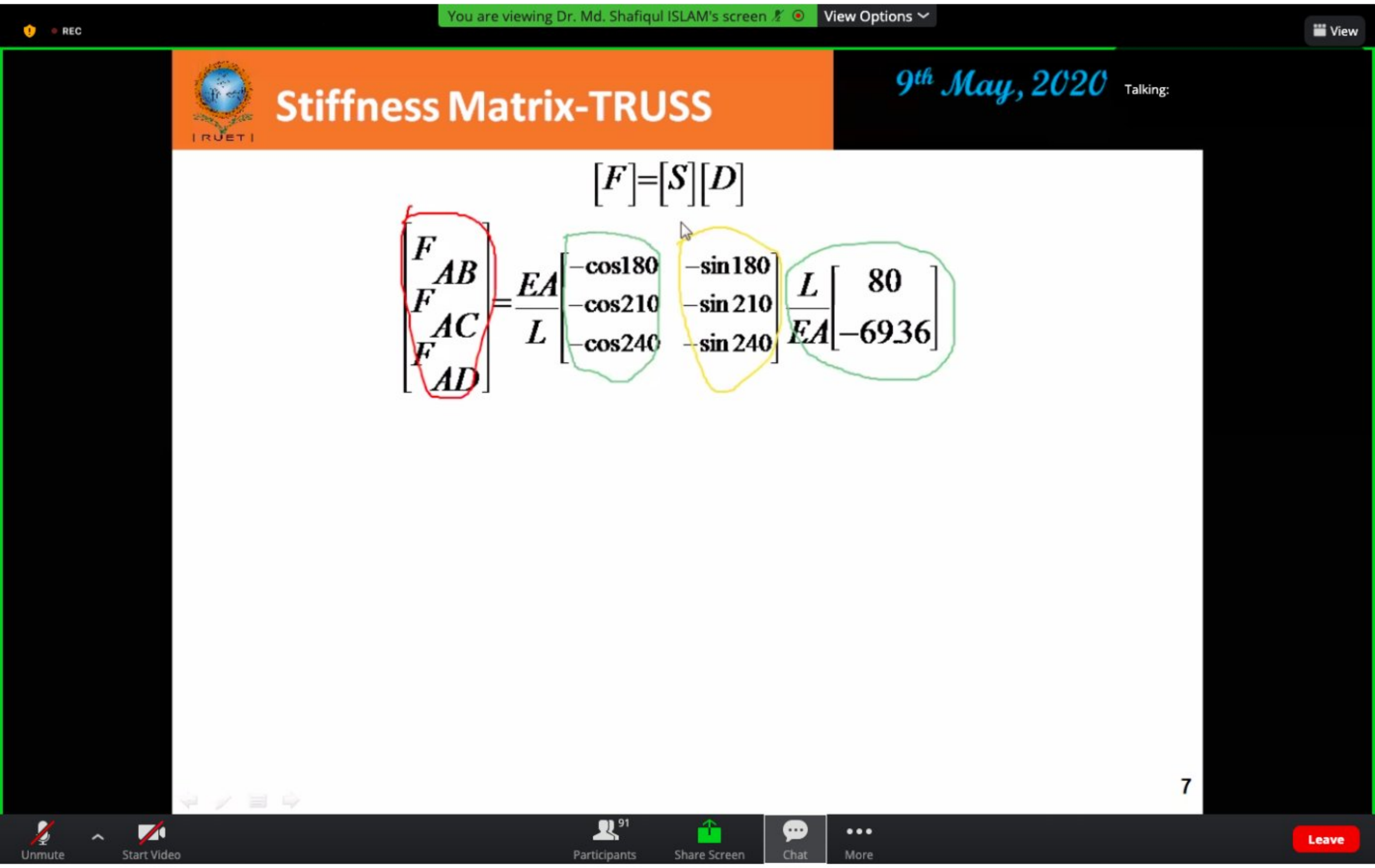
$D_1 = 1, D_2 = 0$

$$S_{11} = \frac{EA}{L}\left[\cos^2 180 + \cos^2 210 + \cos^2 240\right]$$

$$S_{11} = 2\frac{EA}{L}$$

$$S_{21} = \frac{EA}{L}\left[\cos 180 \sin 180 + \cos 210 \sin 210 + \cos 240 \sin 240\right]$$

$$S_{21} = 0.867\frac{EA}{L}$$

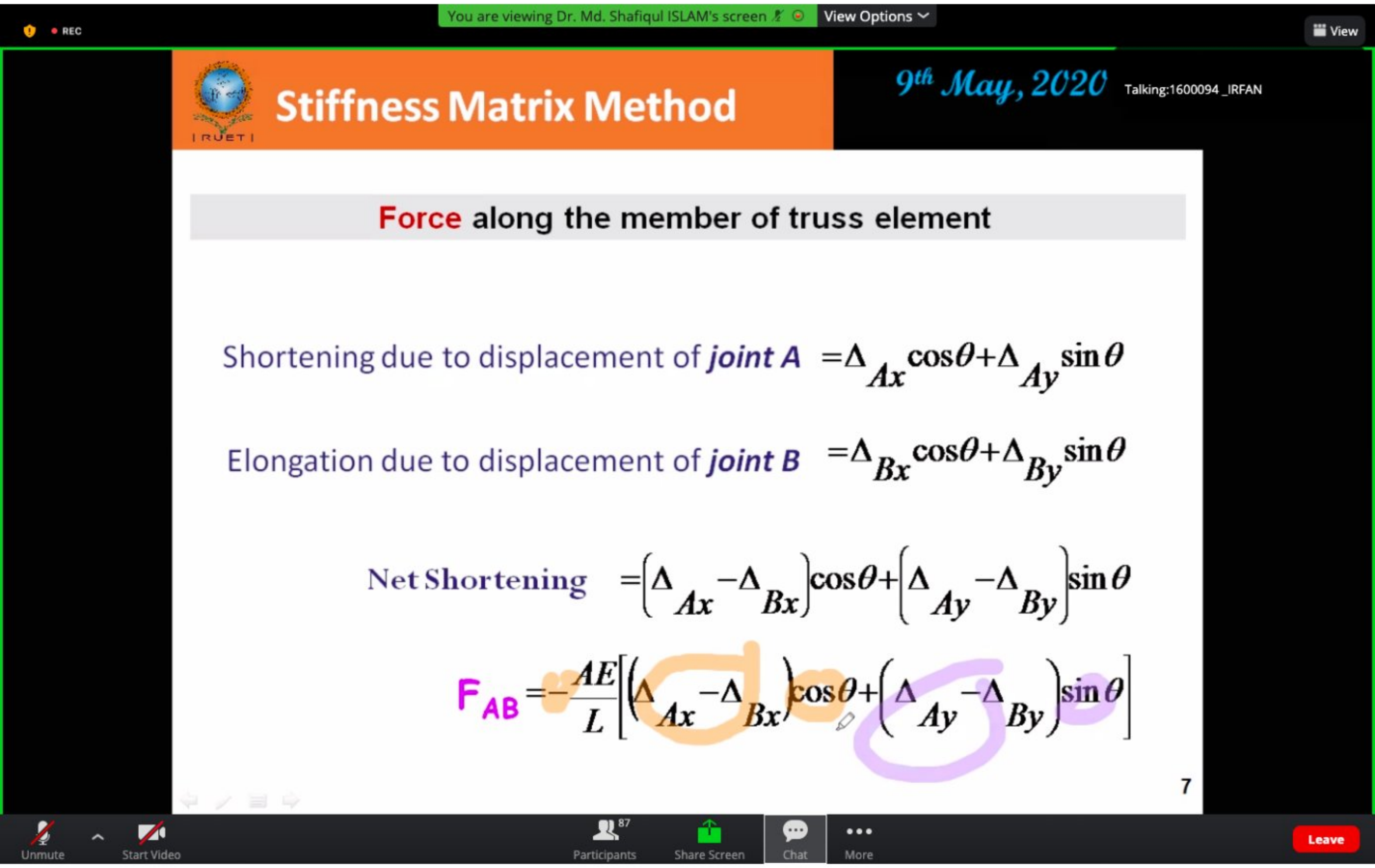


Stiffness Matrix-TRUSS

9th May, 2020

$$[F]=[S][D]$$

$$\begin{bmatrix} F_{AB} \\ F_{AC} \\ F_{AD} \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} -\cos 180 & -\sin 180 \\ -\cos 210 & -\sin 210 \\ -\cos 240 & -\sin 240 \end{bmatrix} \frac{L}{EA}\begin{bmatrix} 80 \\ -6936 \end{bmatrix}$$



Stiffness Matrix Method

9th May, 2020

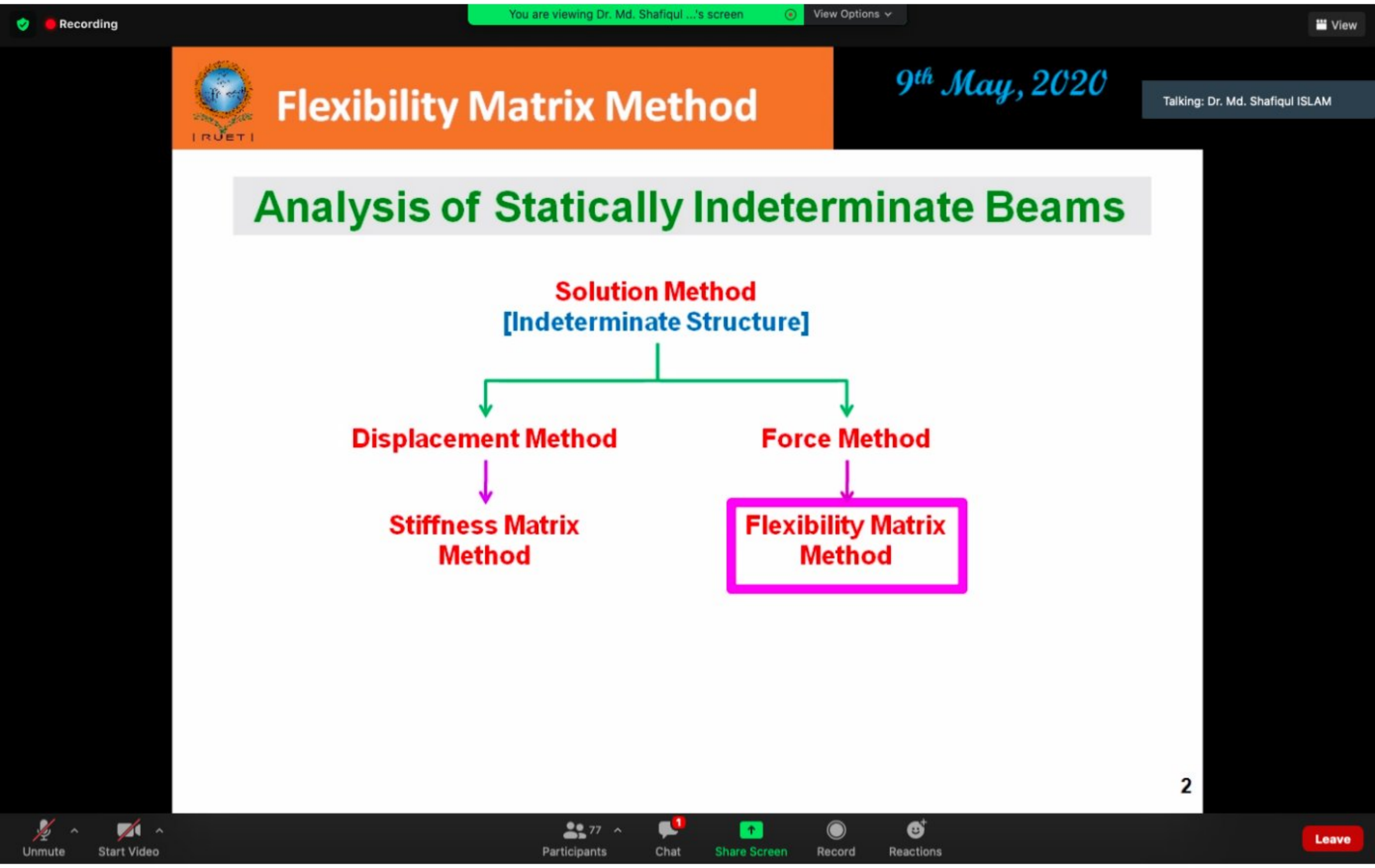
Force along the member of truss element

Shortening due to displacement of ***joint A*** $=\Delta_{Ax}\cos\theta+\Delta_{Ay}\sin\theta$

Elongation due to displacement of ***joint B*** $=\Delta_{Bx}\cos\theta+\Delta_{By}\sin\theta$

Net Shortening $=\left(\Delta_{Ax}-\Delta_{Bx}\right)\cos\theta+\left(\Delta_{Ay}-\Delta_{By}\right)\sin\theta$

$$F_{AB}=-\frac{AE}{L}\left[\left(\Delta_{Ax}-\Delta_{Bx}\right)\cos\theta+\left(\Delta_{Ay}-\Delta_{By}\right)\sin\theta\right]$$



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Talking: Dr. Md. Shafiqul ISLAM

Flexibility Matrix Method

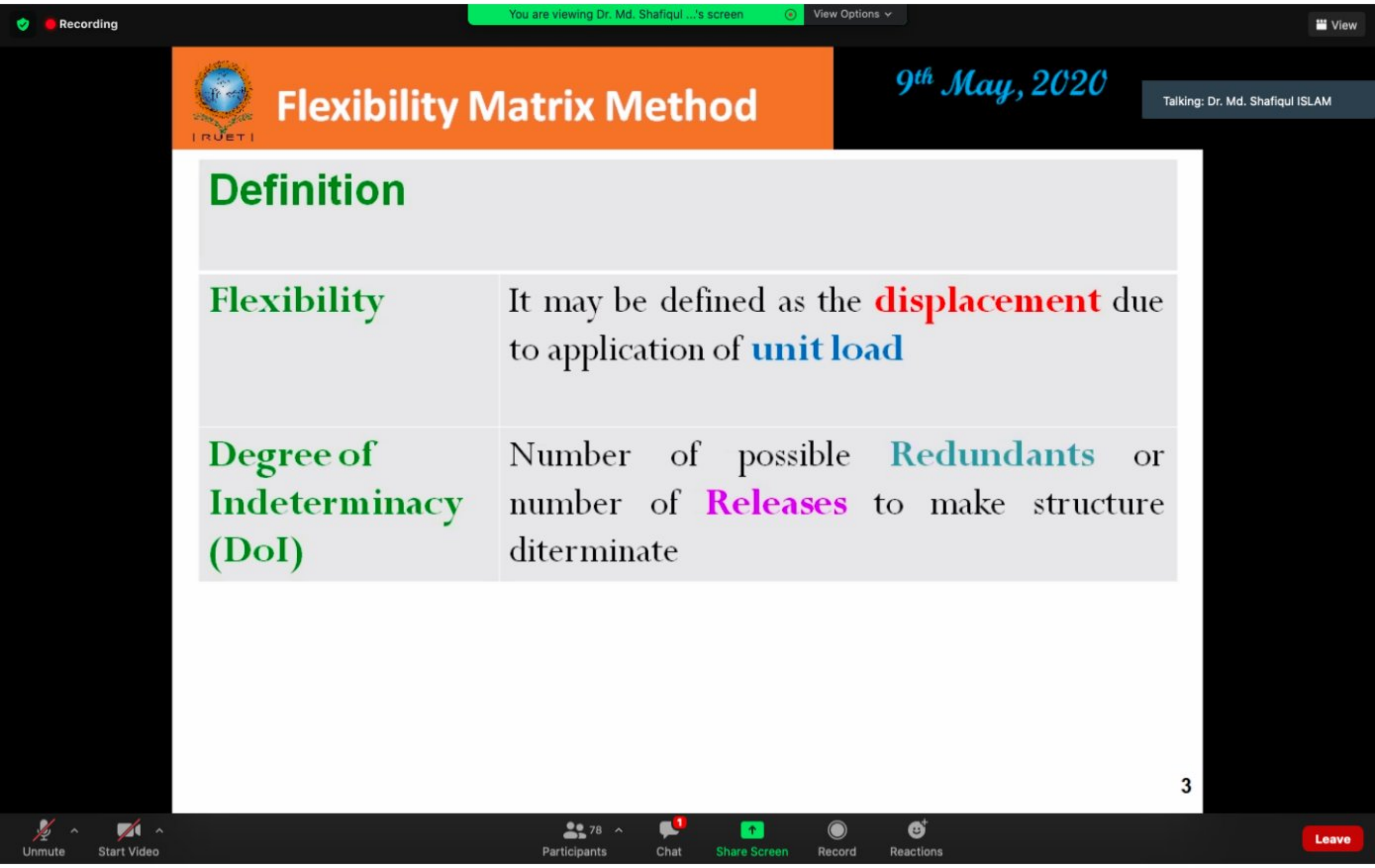
9th May, 2020

Analysis of Statically Indeterminate Beams

Solution Method
[Indeterminate Structure]

Displacement Method → **Stiffness Matrix Method**

Force Method → **Flexibility Matrix Method**

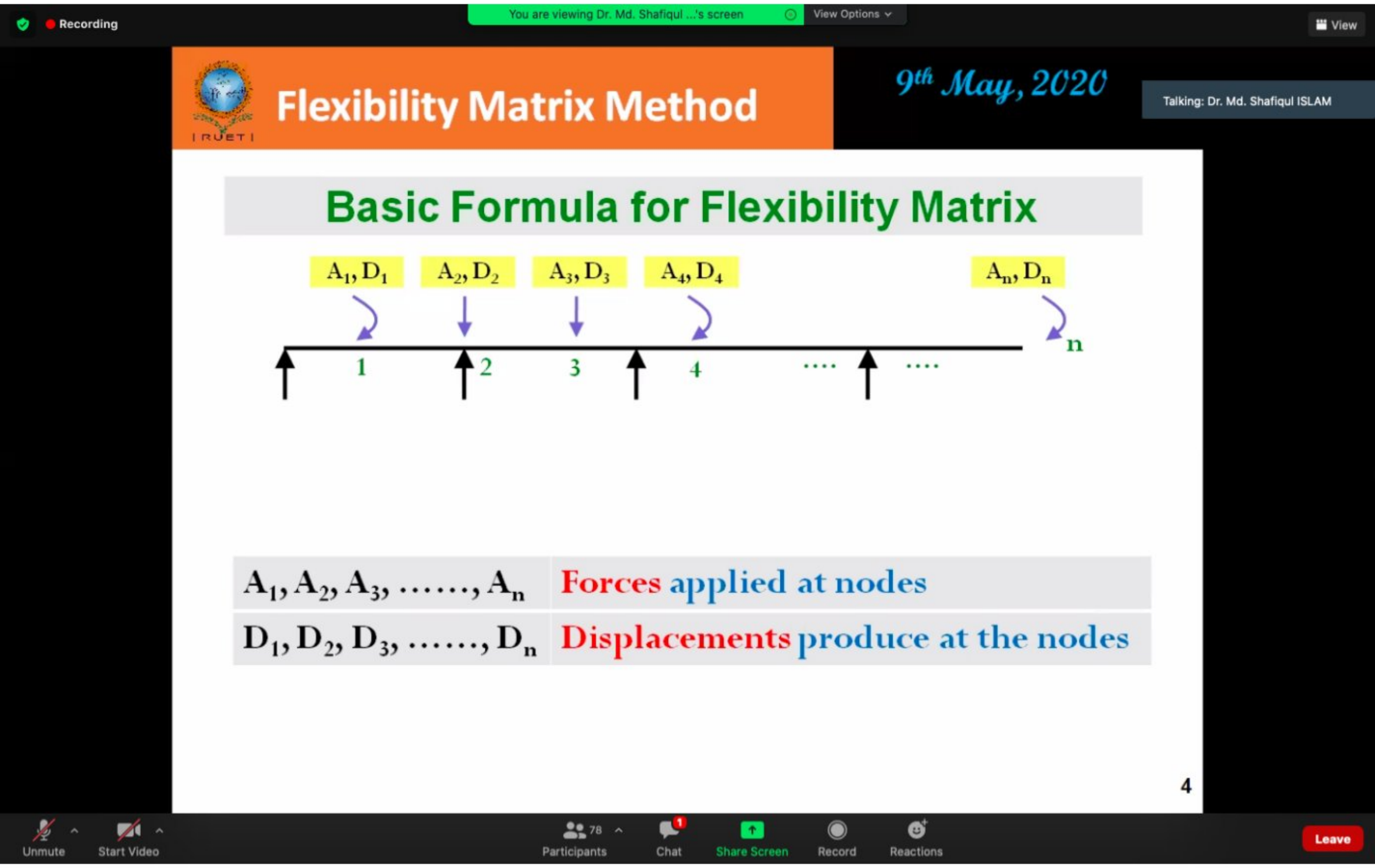


Flexibility Matrix Method

9th May, 2020

Definition

Flexibility	It may be defined as the **displacement** due to application of **unit load**
Degree of Indeterminacy (DoI)	Number of possible **Redundants** or number of **Releases** to make structure diterminate

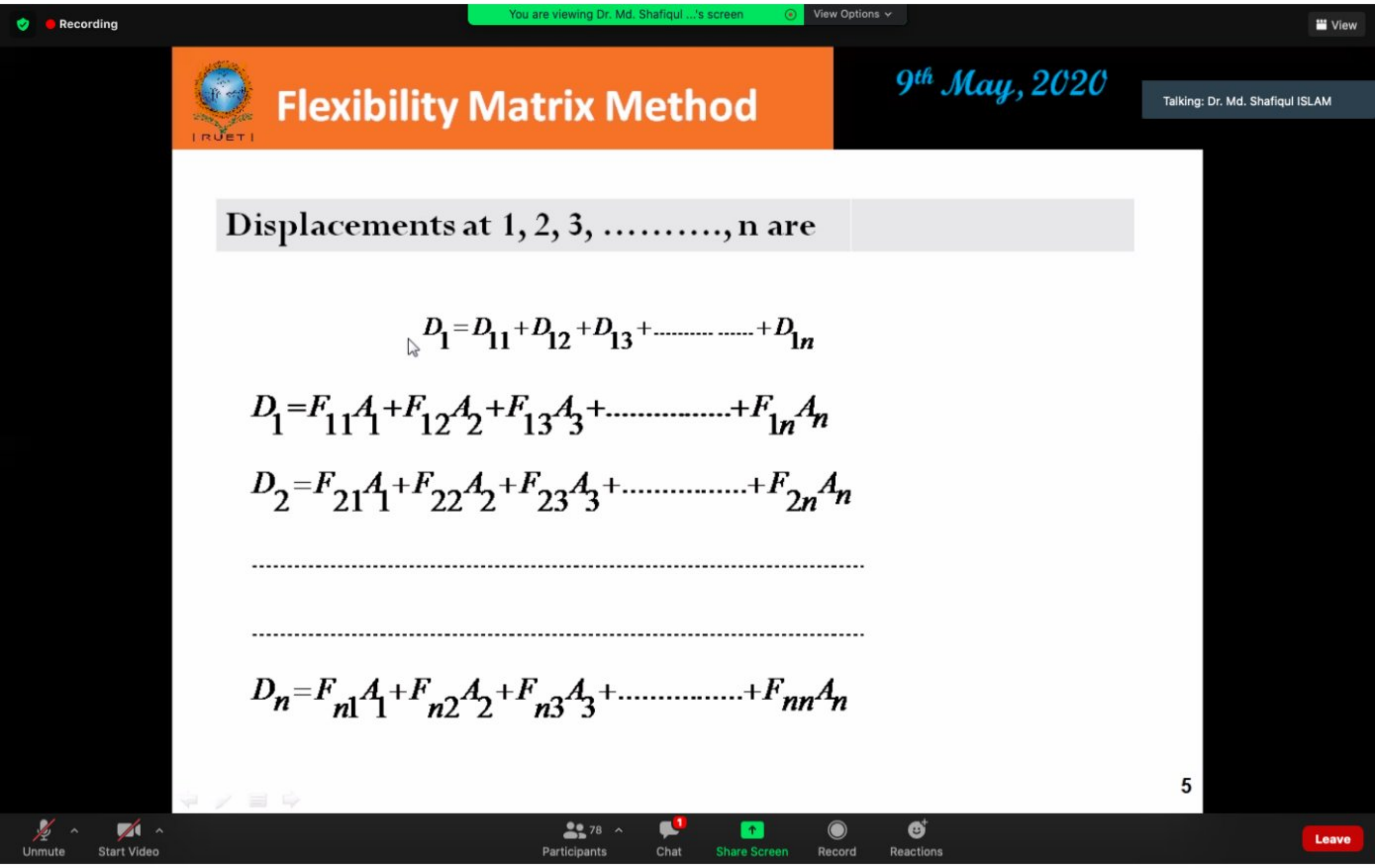


Flexibility Matrix Method

9th May, 2020

Basic Formula for Flexibility Matrix

$A_1, A_2, A_3, \ldots\ldots, A_n$	Forces applied at nodes
$D_1, D_2, D_3, \ldots\ldots, D_n$	Displacements produce at the nodes



Flexibility Matrix Method

9th May, 2020

Displacements at 1, 2, 3, ………, n are

$$D_1 = D_{11} + D_{12} + D_{13} + \cdots\cdots + D_{1n}$$

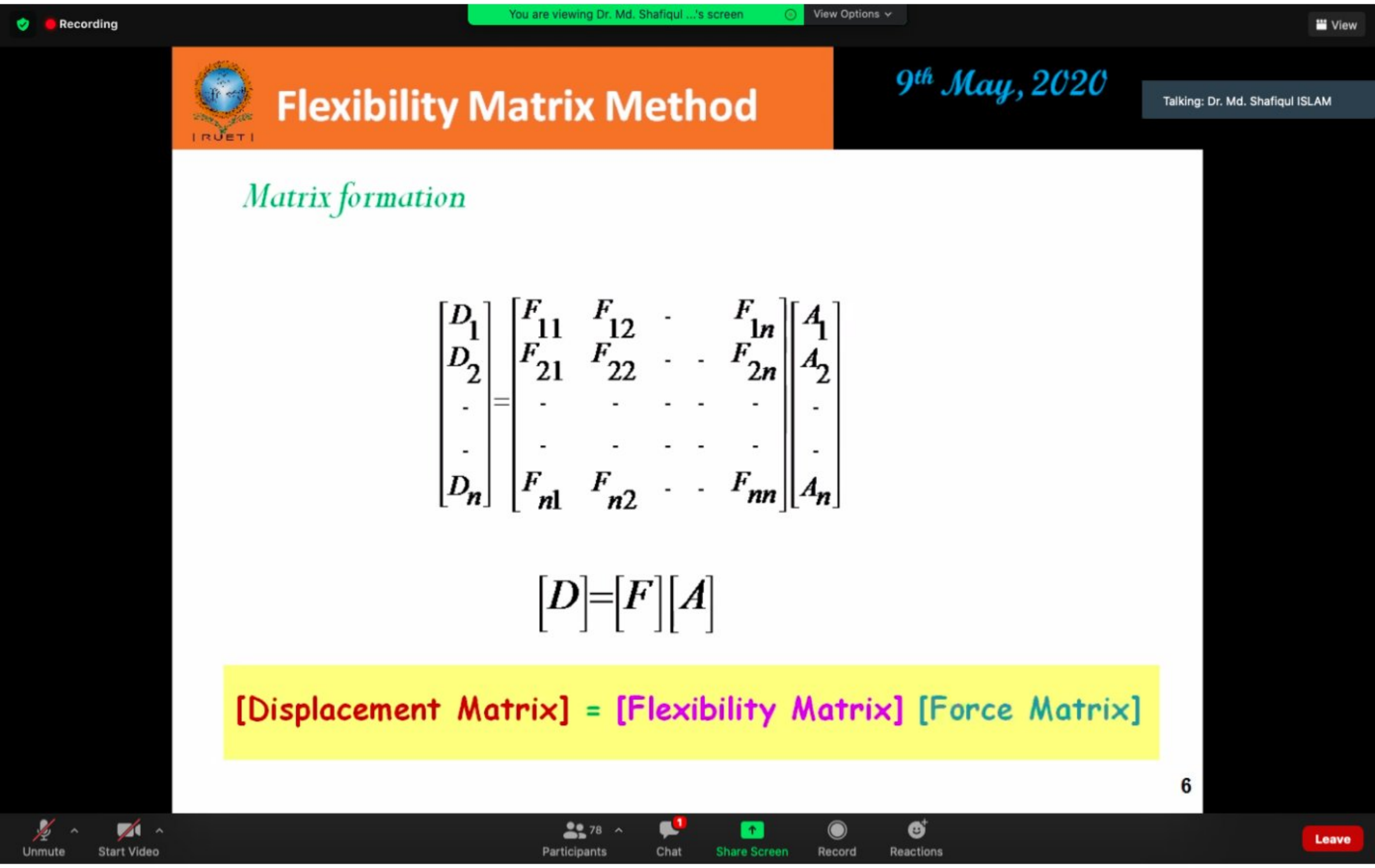
$$D_1 = F_{11}A_1 + F_{12}A_2 + F_{13}A_3 + \cdots\cdots + F_{1n}A_n$$

$$D_2 = F_{21}A_1 + F_{22}A_2 + F_{23}A_3 + \cdots\cdots + F_{2n}A_n$$

....

....

$$D_n = F_{n1}A_1 + F_{n2}A_2 + F_{n3}A_3 + \cdots\cdots + F_{nn}A_n$$



Flexibility Matrix Method

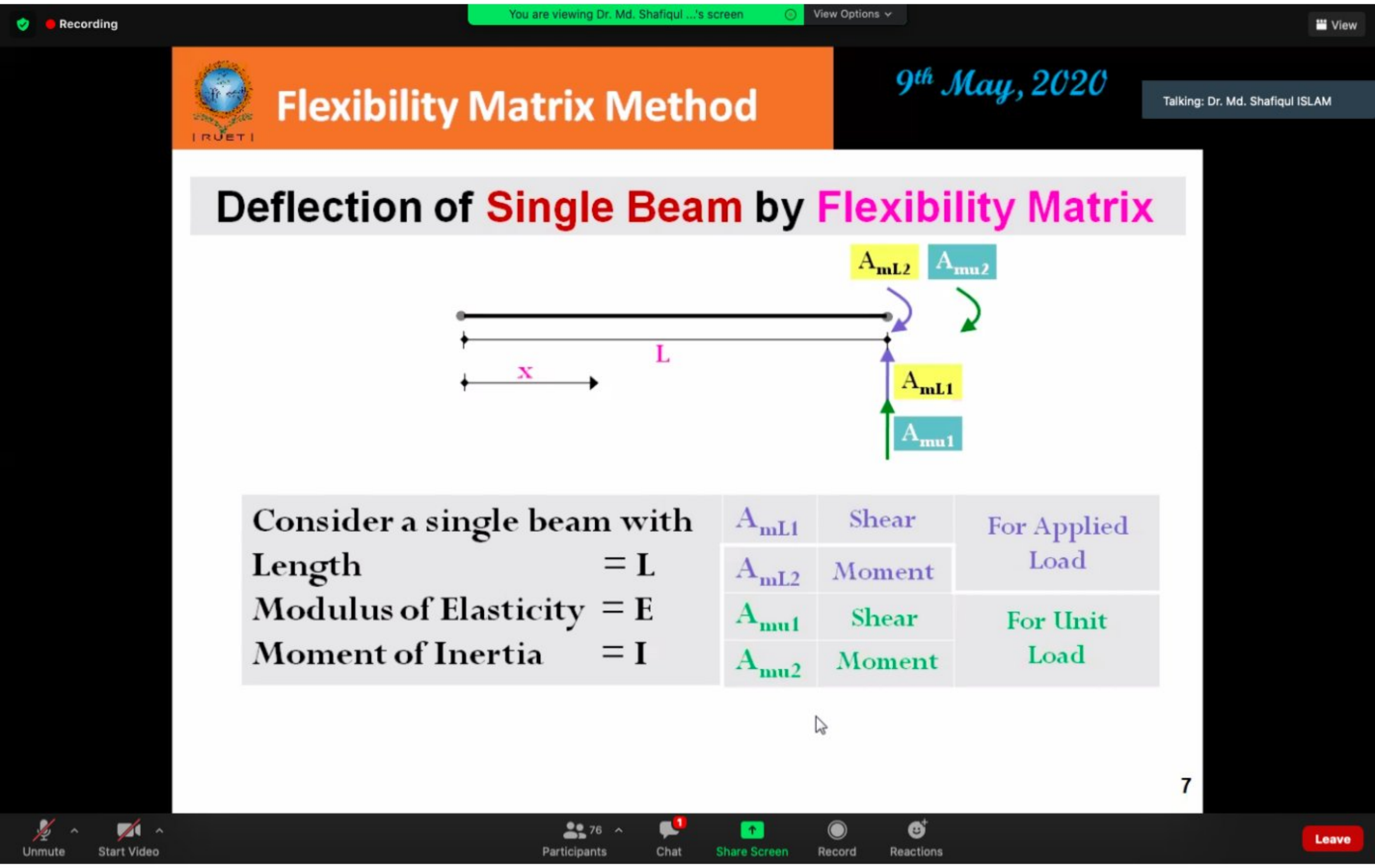
9th May, 2020

Matrix formation

$$\begin{bmatrix} D_1 \\ D_2 \\ \cdot \\ \cdot \\ D_n \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} & \cdot & & F_{1n} \\ F_{21} & F_{22} & \cdot & \cdot & F_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ F_{n1} & F_{n2} & \cdot & \cdot & F_{nn} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ A_n \end{bmatrix}$$

$$[D] = [F][A]$$

[Displacement Matrix] = [Flexibility Matrix] [Force Matrix]



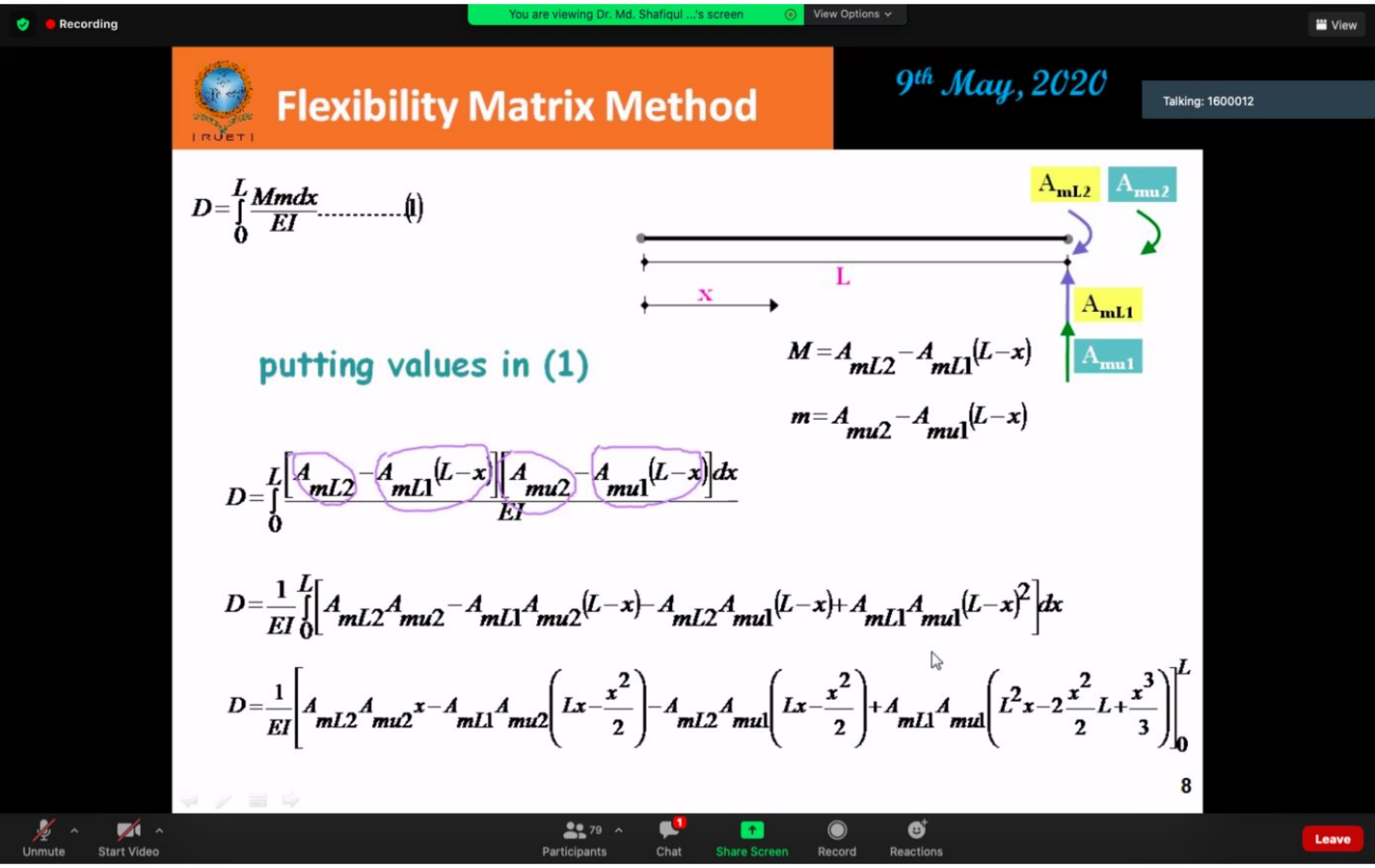
Flexibility Matrix Method

9th May, 2020

Deflection of Single Beam by Flexibility Matrix

Consider a single beam with
Length = L
Modulus of Elasticity = E
Moment of Inertia = I

A_{mL1}	Shear	For Applied Load
A_{mL2}	Moment	
A_{mu1}	Shear	For Unit Load
A_{mu2}	Moment	



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Flexibility Matrix Method

9th May, 2020

$$D=\int_0^L \frac{Mmdx}{EI}\text{..........}(1)$$

$$M=A_{mL2}-A_{mL1}(L-x)$$

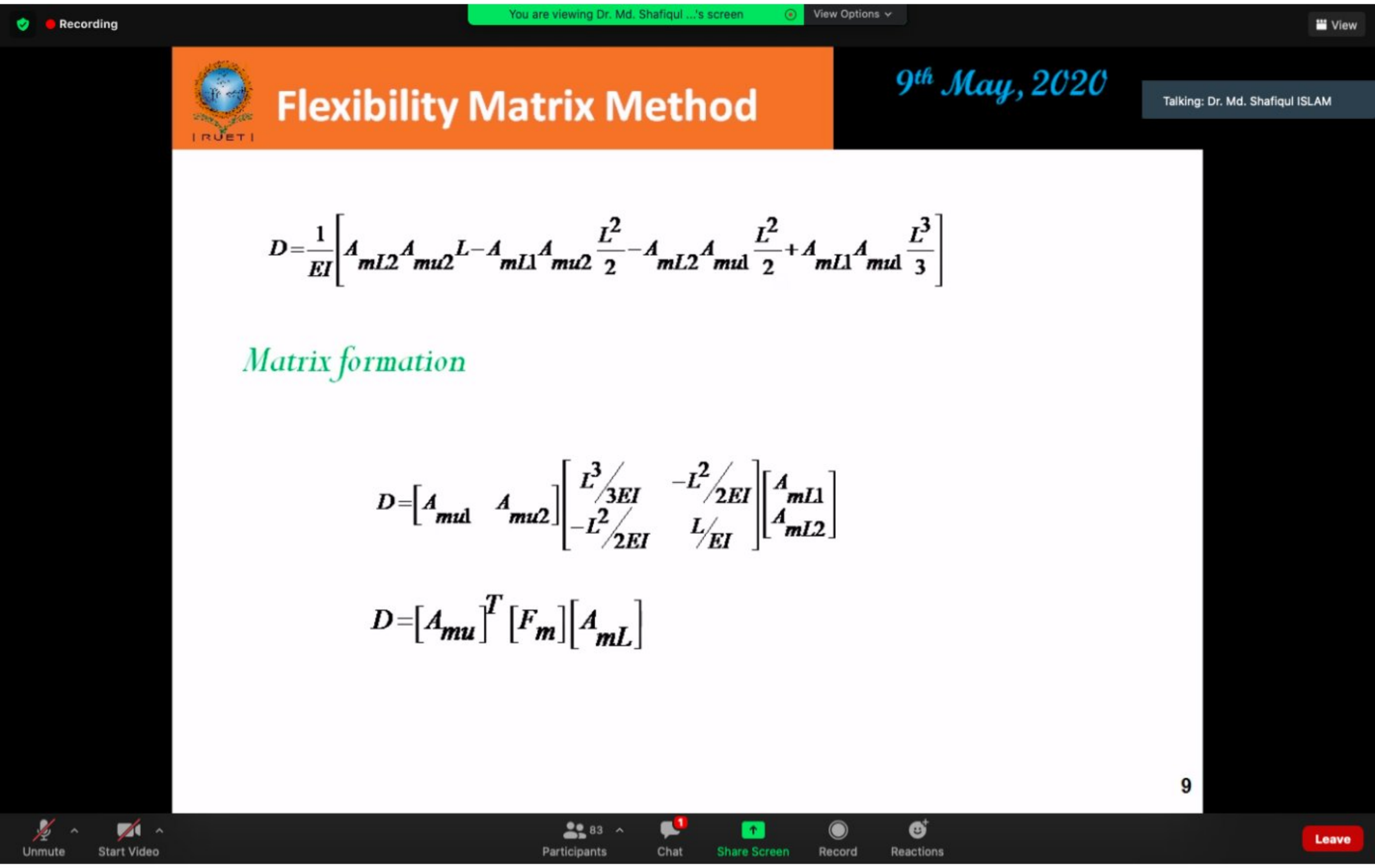
$$m=A_{mu2}-A_{mu1}(L-x)$$

putting values in (1)

$$D=\int_0^L \frac{\left[A_{mL2}-A_{mL1}(L-x)\right]\left[A_{mu2}-A_{mu1}(L-x)\right]dx}{EI}$$

$$D=\frac{1}{EI}\int_0^L\left[A_{mL2}A_{mu2}-A_{mL1}A_{mu2}(L-x)-A_{mL2}A_{mu1}(L-x)+A_{mL1}A_{mu1}(L-x)^2\right]dx$$

$$D=\frac{1}{EI}\left[A_{mL2}A_{mu2}x-A_{mL1}A_{mu2}\left(Lx-\frac{x^2}{2}\right)-A_{mL2}A_{mu1}\left(Lx-\frac{x^2}{2}\right)+A_{mL1}A_{mu1}\left(L^2x-2\frac{x^2}{2}L+\frac{x^3}{3}\right)\right]_0^L$$



Flexibility Matrix Method

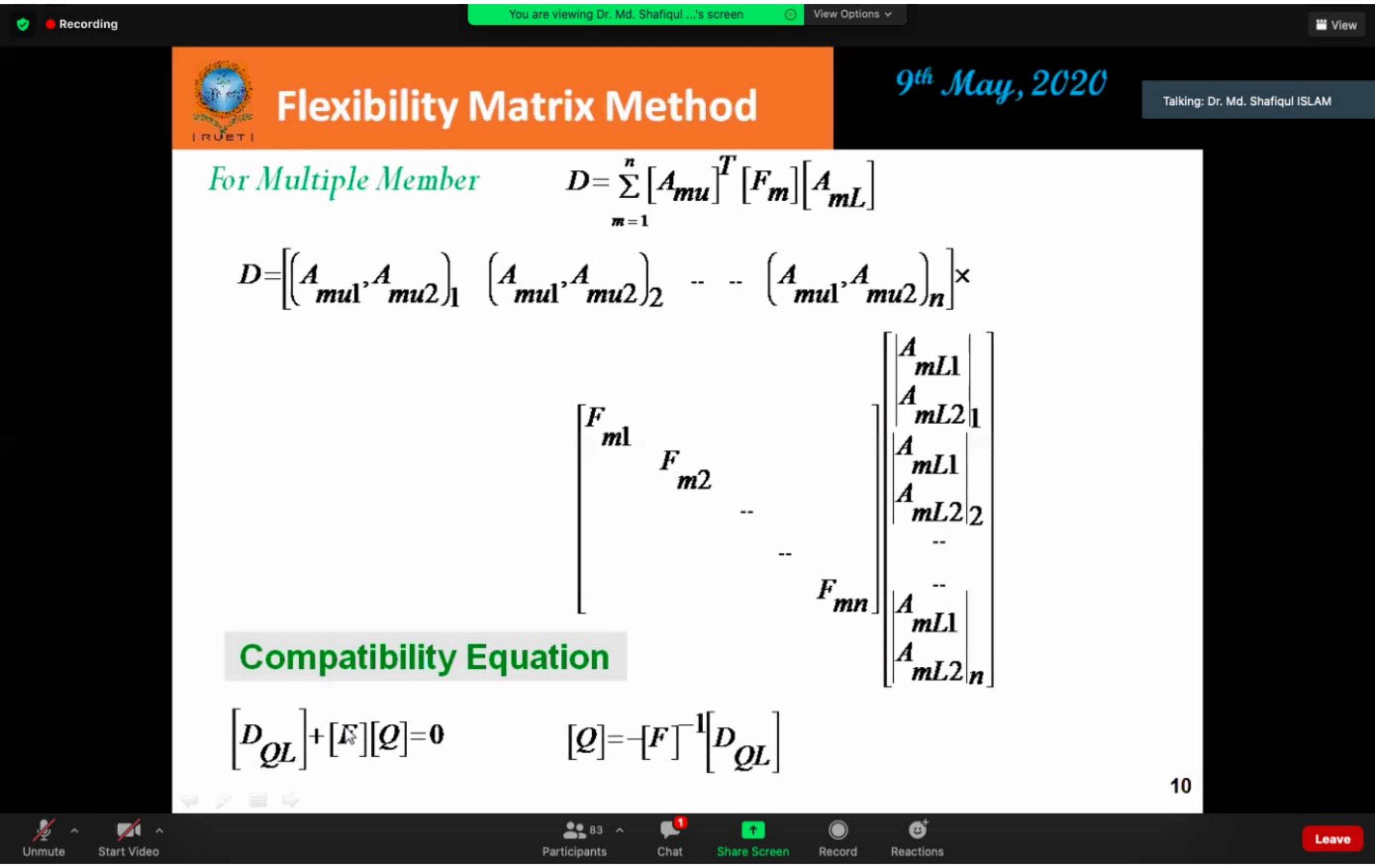
9th May, 2020

$$D=\frac{1}{EI}\left[A_{mL2}A_{mu2}L-A_{mL1}A_{mu2}\frac{L^2}{2}-A_{mL2}A_{mu1}\frac{L^2}{2}+A_{mL1}A_{mu1}\frac{L^3}{3}\right]$$

Matrix formation

$$D=\begin{bmatrix}A_{mu1} & A_{mu2}\end{bmatrix}\begin{bmatrix}L^3/3EI & -L^2/2EI\\ -L^2/2EI & L/EI\end{bmatrix}\begin{bmatrix}A_{mL1}\\ A_{mL2}\end{bmatrix}$$

$$D=\left[A_{mu}\right]^T\left[F_m\right]\left[A_{mL}\right]$$



Flexibility Matrix Method

9th May, 2020

For Multiple Member

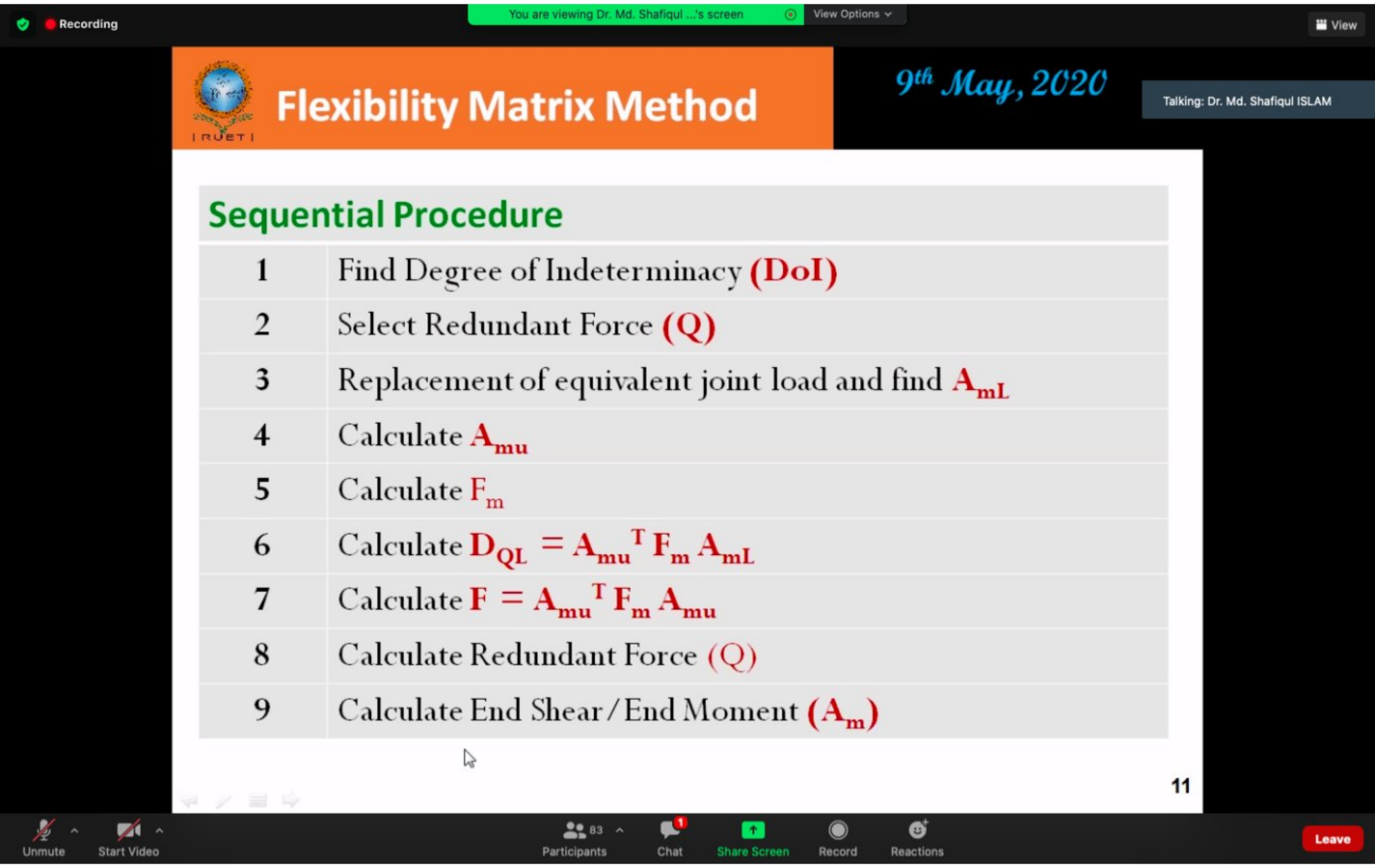
$$D=\sum_{m=1}^{n}[A_{mu}]^T[F_m][A_{mL}]$$

$$D=\left[\left(A_{mu1},A_{mu2}\right)_1\quad\left(A_{mu1},A_{mu2}\right)_2\quad--\quad--\quad\left(A_{mu1},A_{mu2}\right)_n\right]\times$$

$$\begin{bmatrix}F_{m1}&&&&\\&F_{m2}&&&\\&&--&&\\&&&--&\\&&&&F_{mn}\end{bmatrix}\begin{bmatrix}\begin{vmatrix}A_{mL1}\\A_{mL2}\end{vmatrix}_1\\\begin{vmatrix}A_{mL1}\\A_{mL2}\end{vmatrix}_2\\--\\--\\\begin{vmatrix}A_{mL1}\\A_{mL2}\end{vmatrix}_n\end{bmatrix}$$

Compatibility Equation

$$[D_{QL}]+[F][Q]=0 \qquad [Q]=-[F]^{-1}[D_{QL}]$$

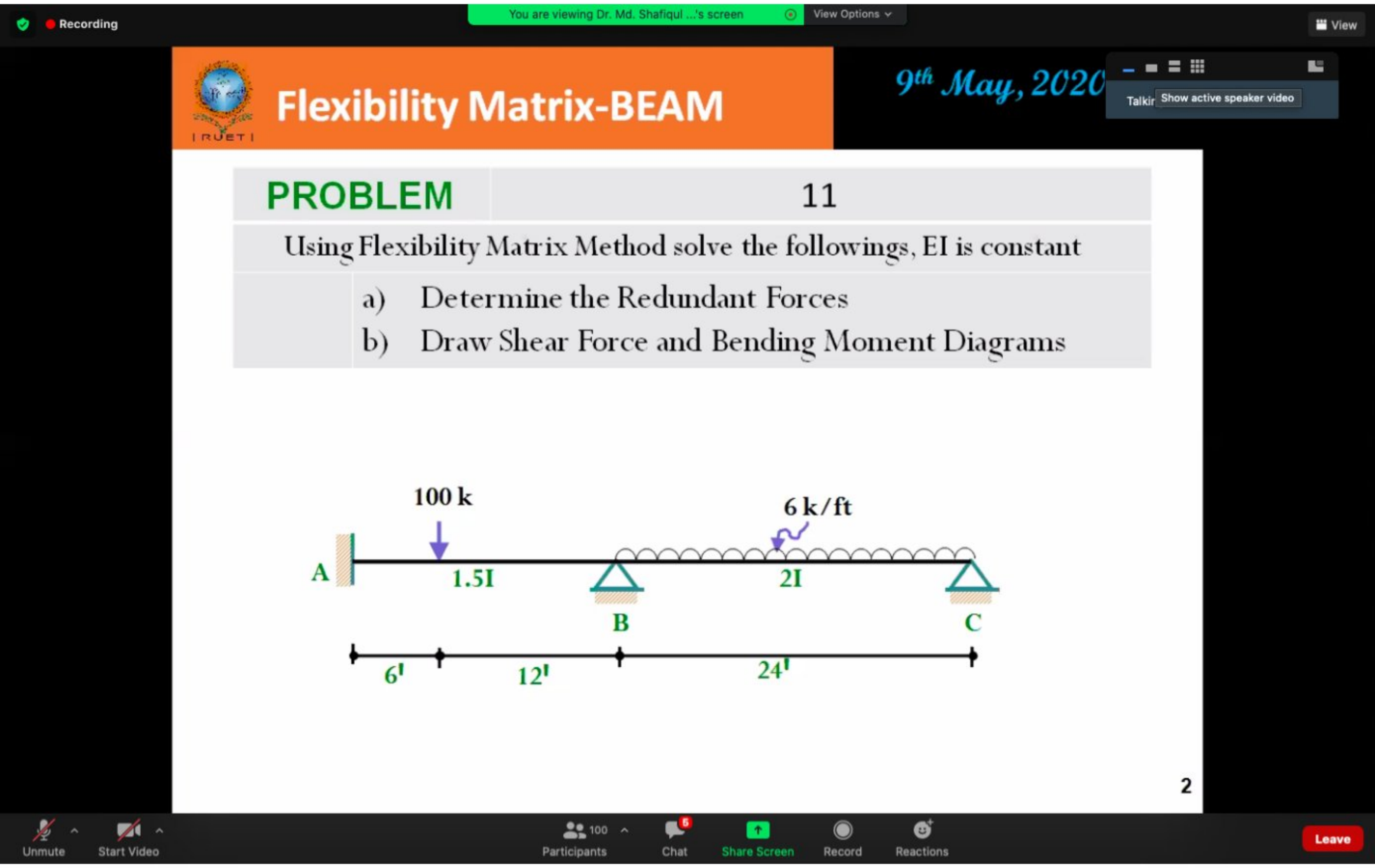


Flexibility Matrix Method

9th May, 2020

Sequential Procedure

1	Find Degree of Indeterminacy **(DoI)**
2	Select Redundant Force **(Q)**
3	Replacement of equivalent joint load and find $\mathbf{A_{mL}}$
4	Calculate $\mathbf{A_{mu}}$
5	Calculate F_m
6	Calculate $\mathbf{D_{QL} = A_{mu}{}^T F_m A_{mL}}$
7	Calculate $\mathbf{F = A_{mu}{}^T F_m A_{mu}}$
8	Calculate Redundant Force (Q)
9	Calculate End Shear / End Moment **($\mathbf{A_m}$)**



Flexibility Matrix-BEAM

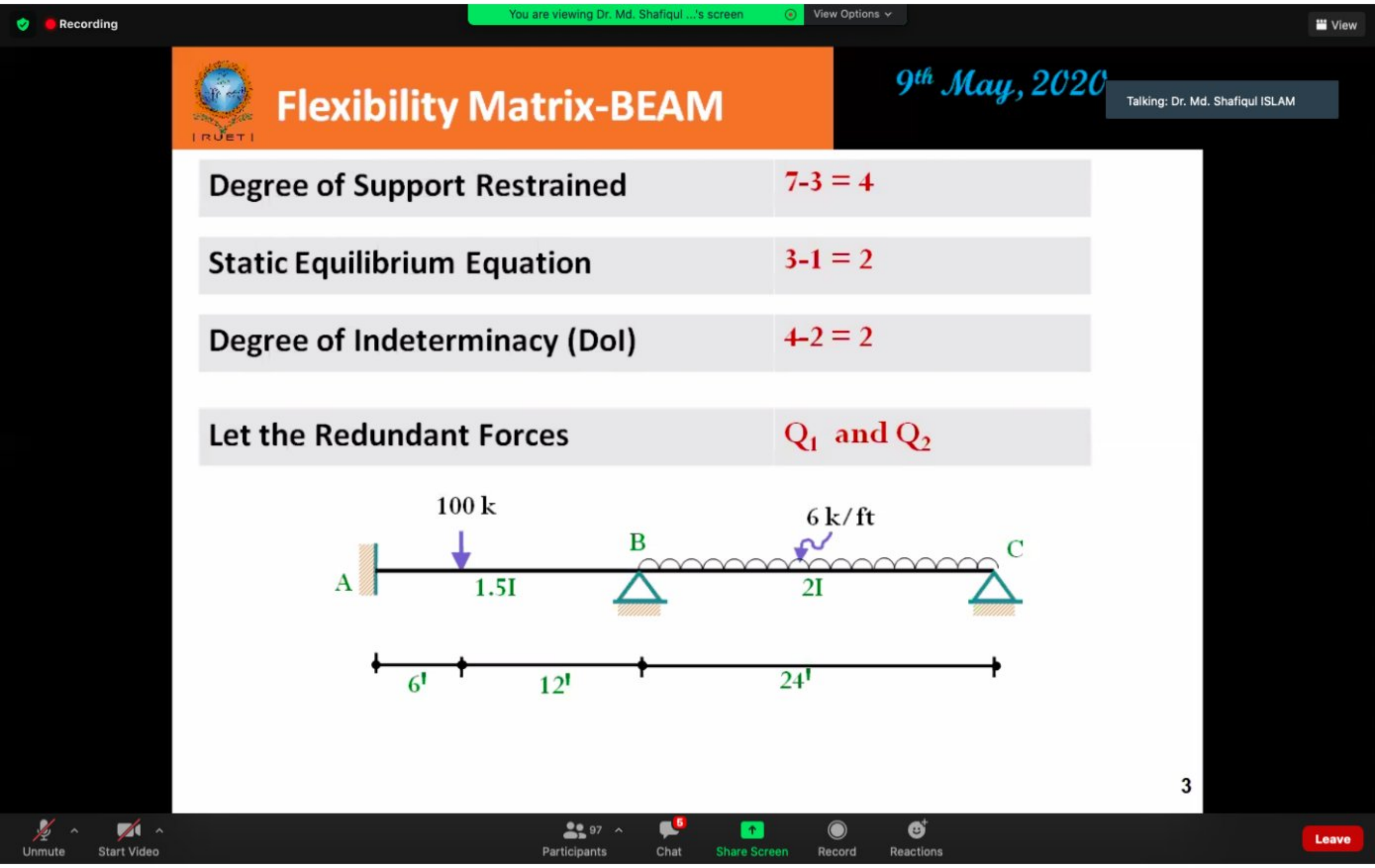
9th May, 2020

PROBLEM 11

Using Flexibility Matrix Method solve the followings, EI is constant

a) Determine the Redundant Forces

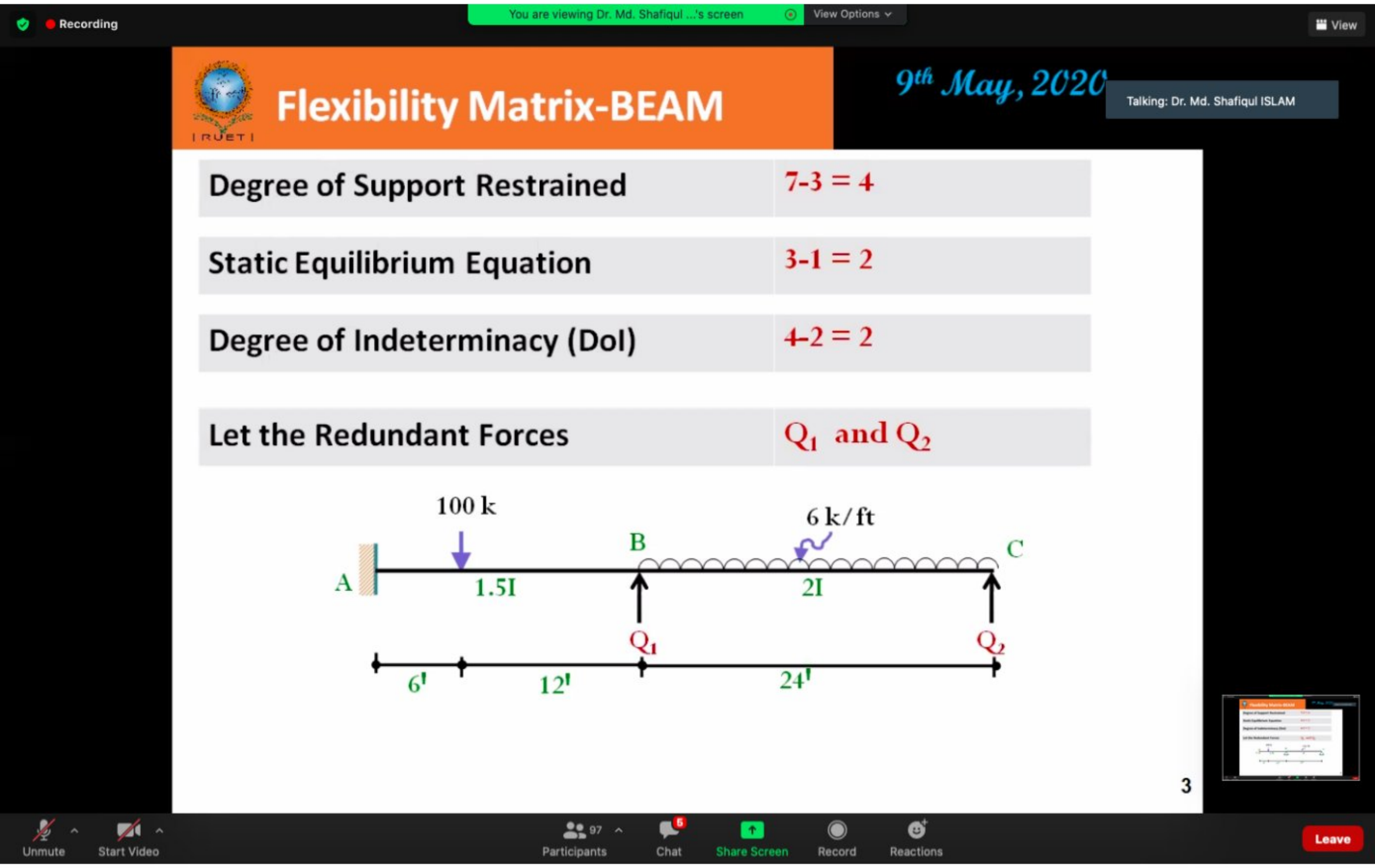
b) Draw Shear Force and Bending Moment Diagrams



Flexibility Matrix-BEAM

9th May, 2020

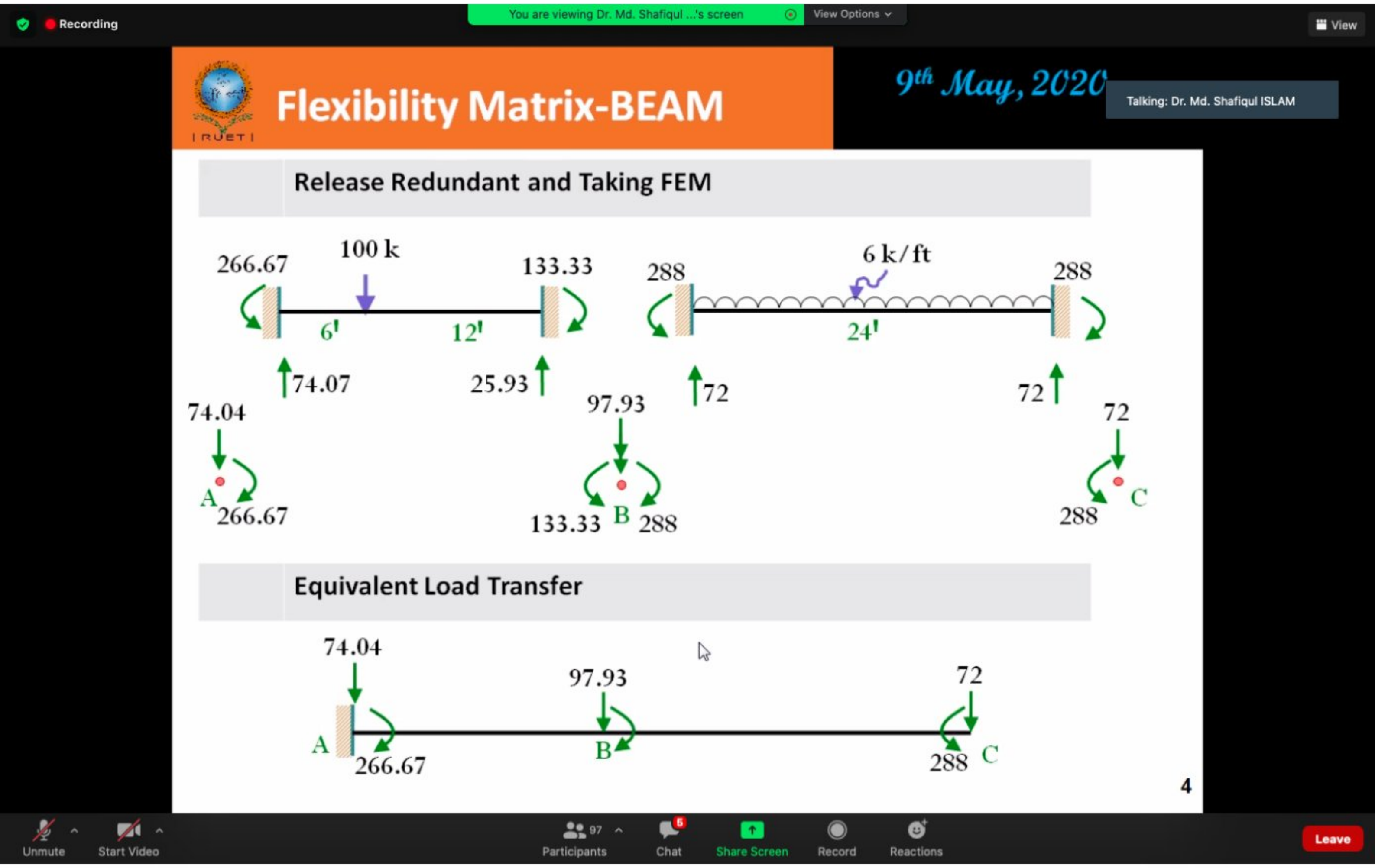
Degree of Support Restrained	7-3 = 4
Static Equilibrium Equation	3-1 = 2
Degree of Indeterminacy (DoI)	4-2 = 2
Let the Redundant Forces	Q_1 and Q_2



Flexibility Matrix-BEAM

9th May, 2020

Degree of Support Restrained	7-3 = 4
Static Equilibrium Equation	3-1 = 2
Degree of Indeterminacy (DoI)	4-2 = 2
Let the Redundant Forces	Q_1 and Q_2

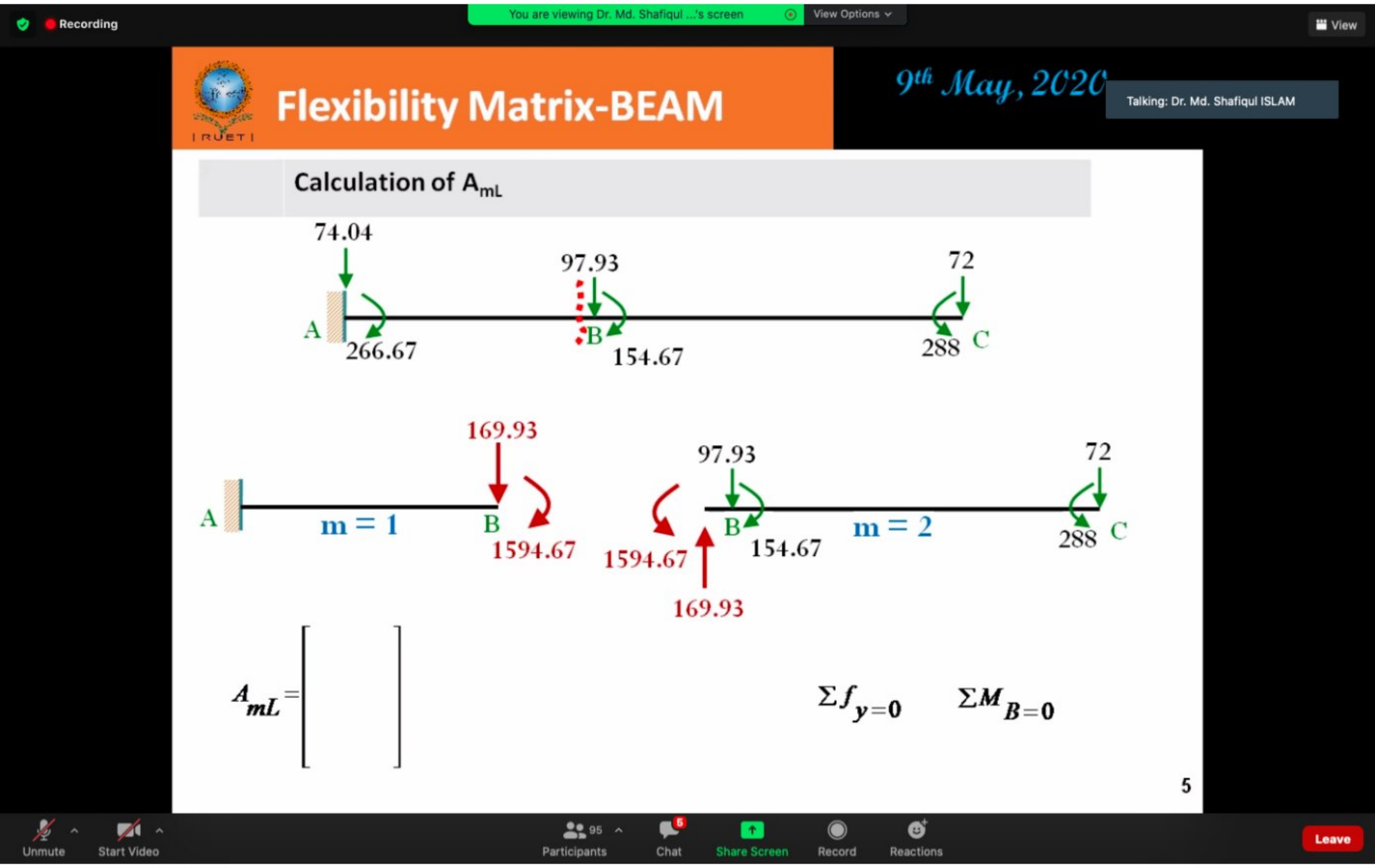


Flexibility Matrix-BEAM

9th May, 2020

Release Redundant and Taking FEM

Equivalent Load Transfer



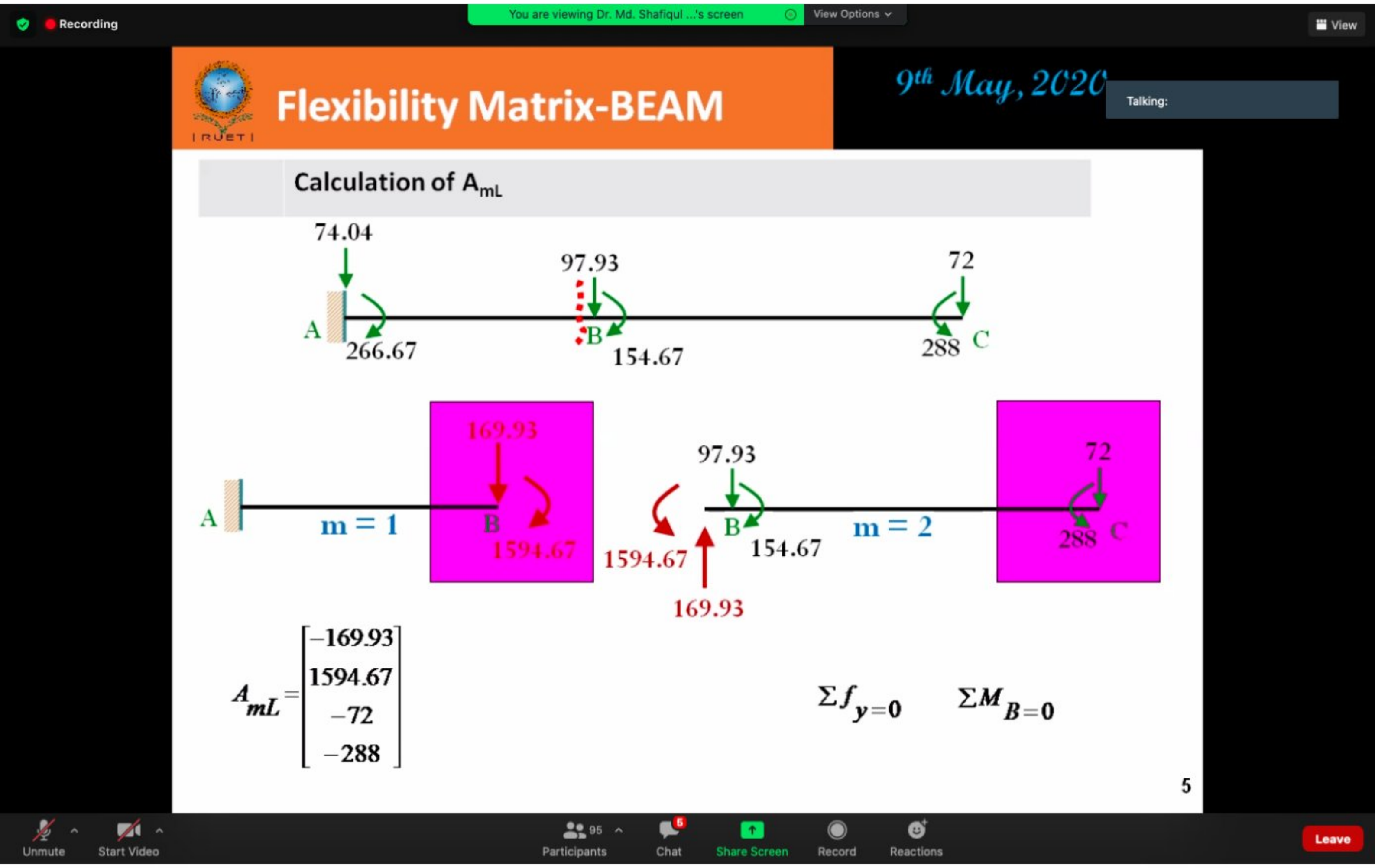
9th May, 2020

Flexibility Matrix-BEAM

Calculation of A_{mL}

$$A_{mL} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

$\Sigma f_{y=0}$ $\Sigma M_{B=0}$



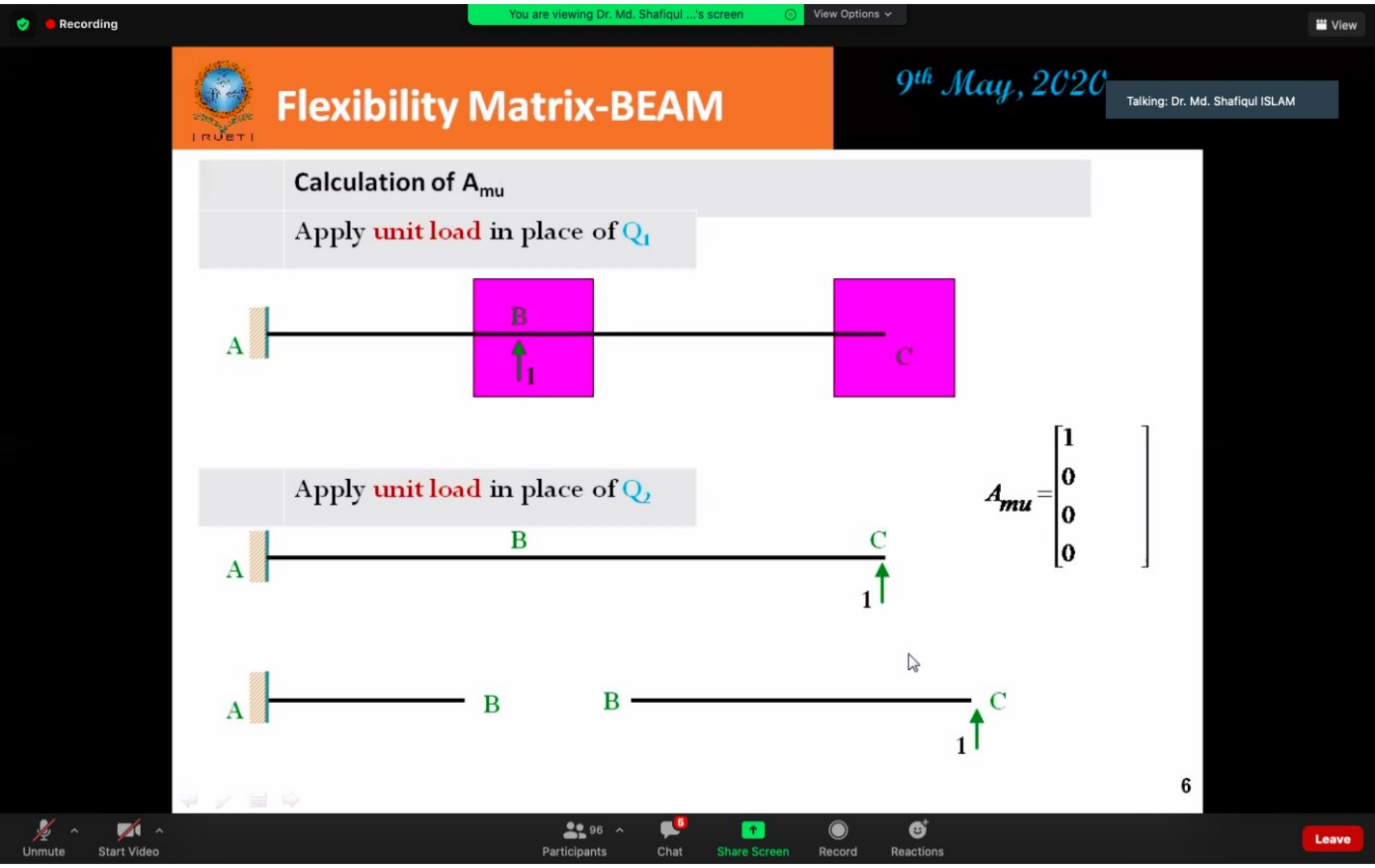
Flexibility Matrix-BEAM

9th May, 2020

Calculation of A_{mL}

$$A_{mL} = \begin{bmatrix} -169.93 \\ 1594.67 \\ -72 \\ -288 \end{bmatrix}$$

$\Sigma f_{y=0}$ $\Sigma M_{B=0}$



Flexibility Matrix-BEAM

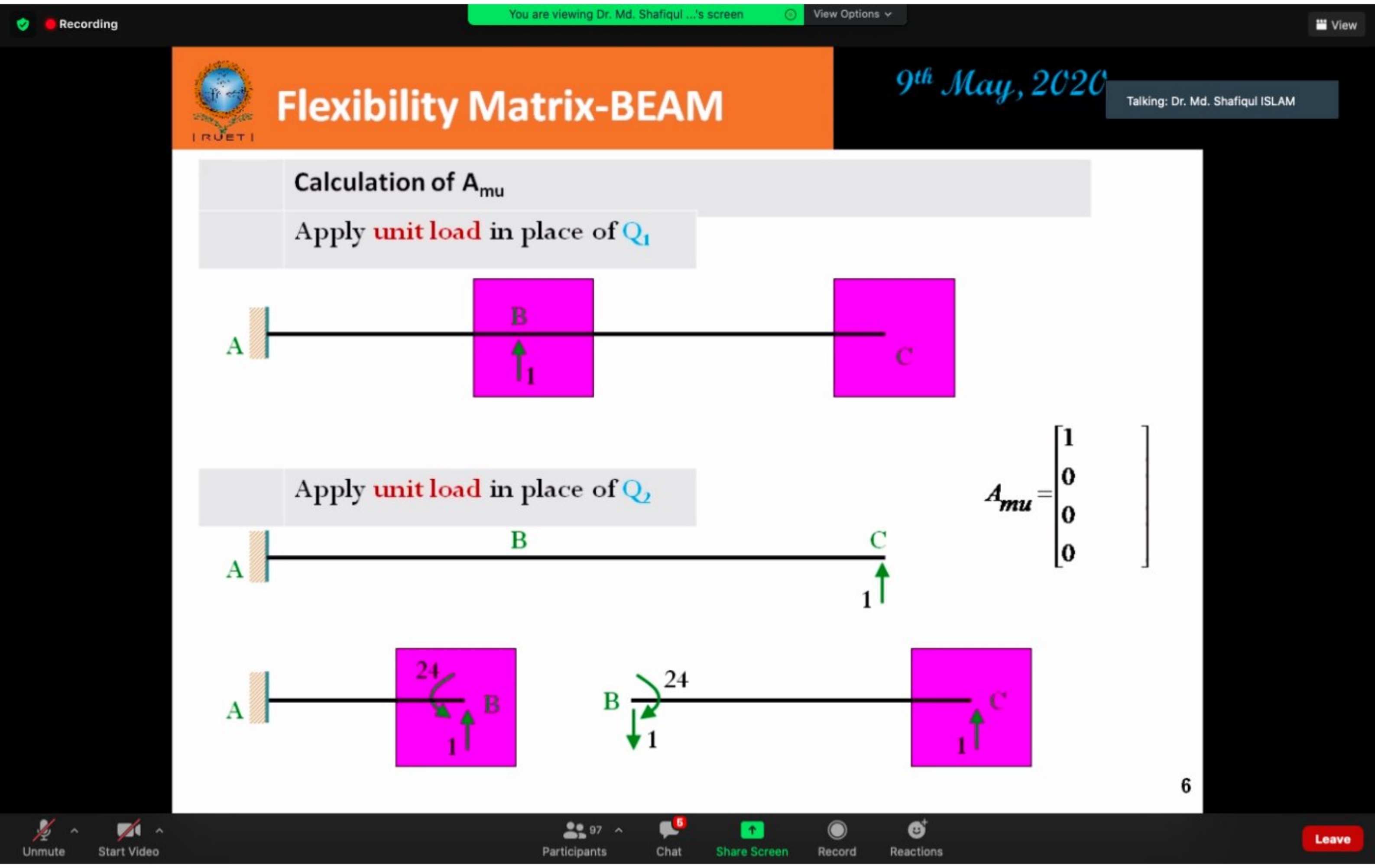
9th May, 2020

Calculation of A_{mu}

Apply unit load in place of Q_1

Apply unit load in place of Q_2

$$A_{mu} = \begin{bmatrix} 1 & \\ 0 & \\ 0 & \\ 0 & \end{bmatrix}$$



Flexibility Matrix-BEAM

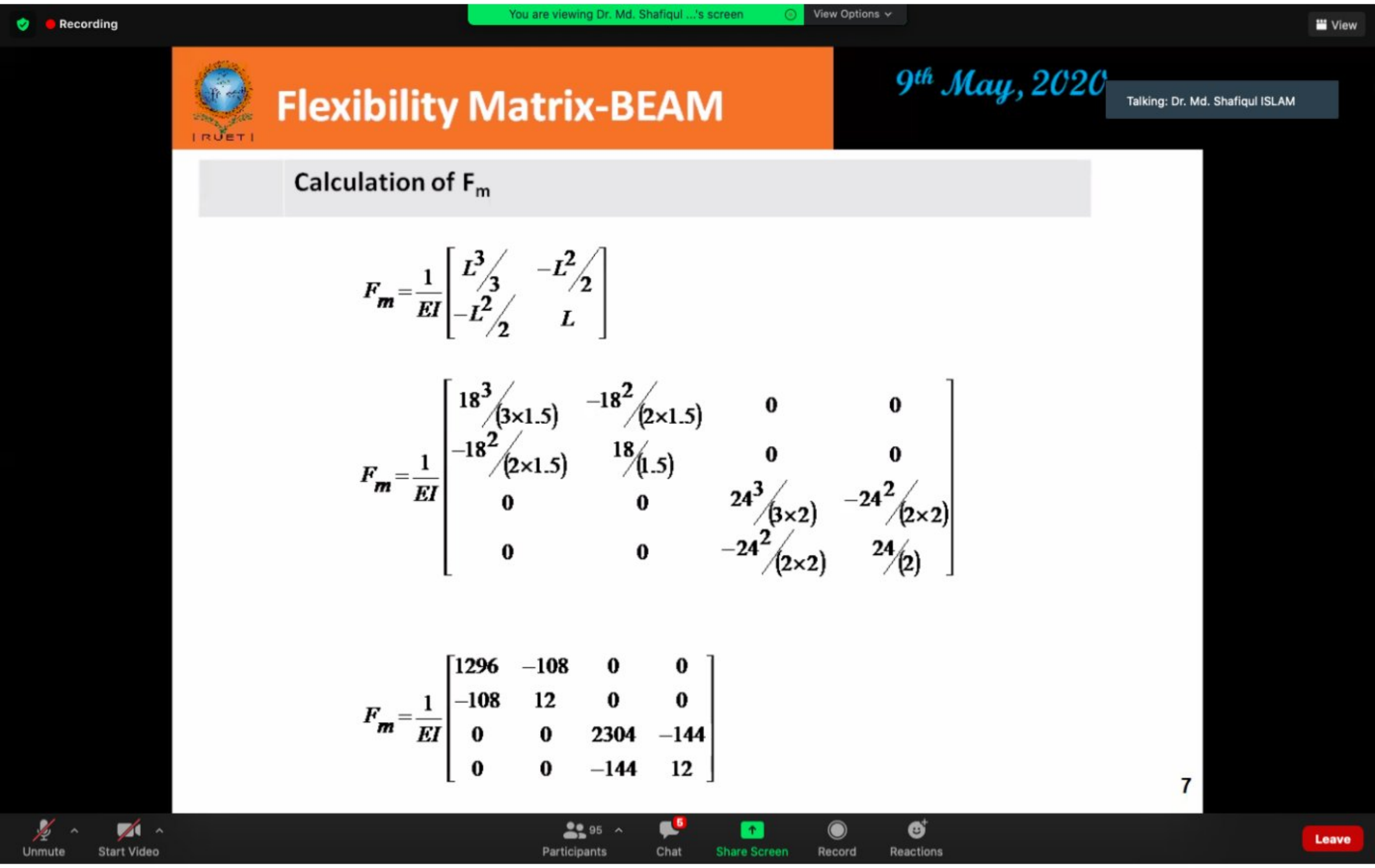
9th May, 2020

Calculation of A_{mu}

Apply unit load in place of Q_1

Apply unit load in place of Q_2

$$A_{mu} = \begin{bmatrix} 1 & \\ 0 & \\ 0 & \\ 0 & \end{bmatrix}$$



Flexibility Matrix-BEAM

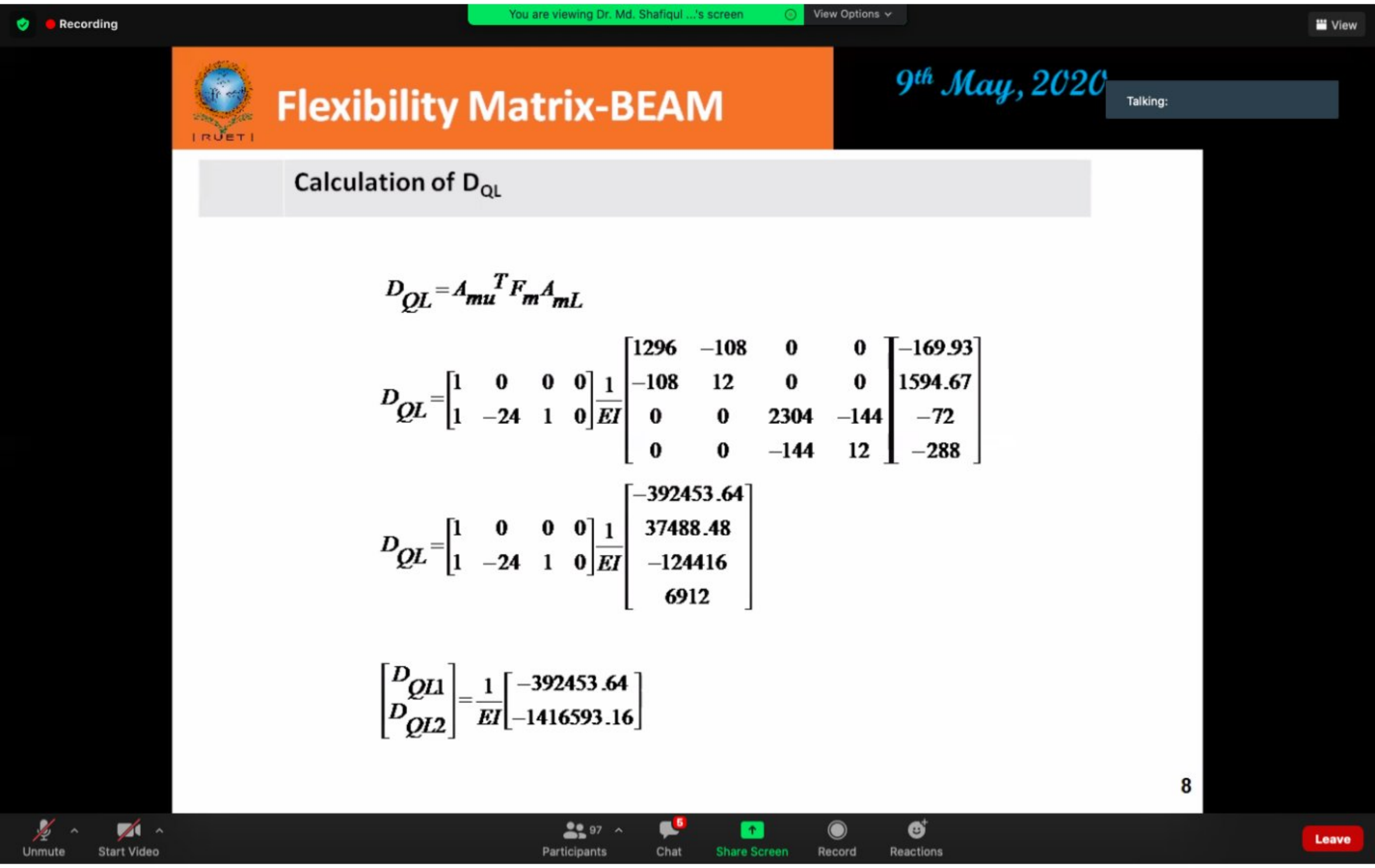
9th May, 2020

Calculation of F_m

$$F_m = \frac{1}{EI}\begin{bmatrix} L^3/3 & -L^2/2 \\ -L^2/2 & L \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 18^3/(3\times1.5) & -18^2/(2\times1.5) & 0 & 0 \\ -18^2/(2\times1.5) & 18/(1.5) & 0 & 0 \\ 0 & 0 & 24^3/(3\times2) & -24^2/(2\times2) \\ 0 & 0 & -24^2/(2\times2) & 24/(2) \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 1296 & -108 & 0 & 0 \\ -108 & 12 & 0 & 0 \\ 0 & 0 & 2304 & -144 \\ 0 & 0 & -144 & 12 \end{bmatrix}$$



Flexibility Matrix-BEAM

9th May, 2020

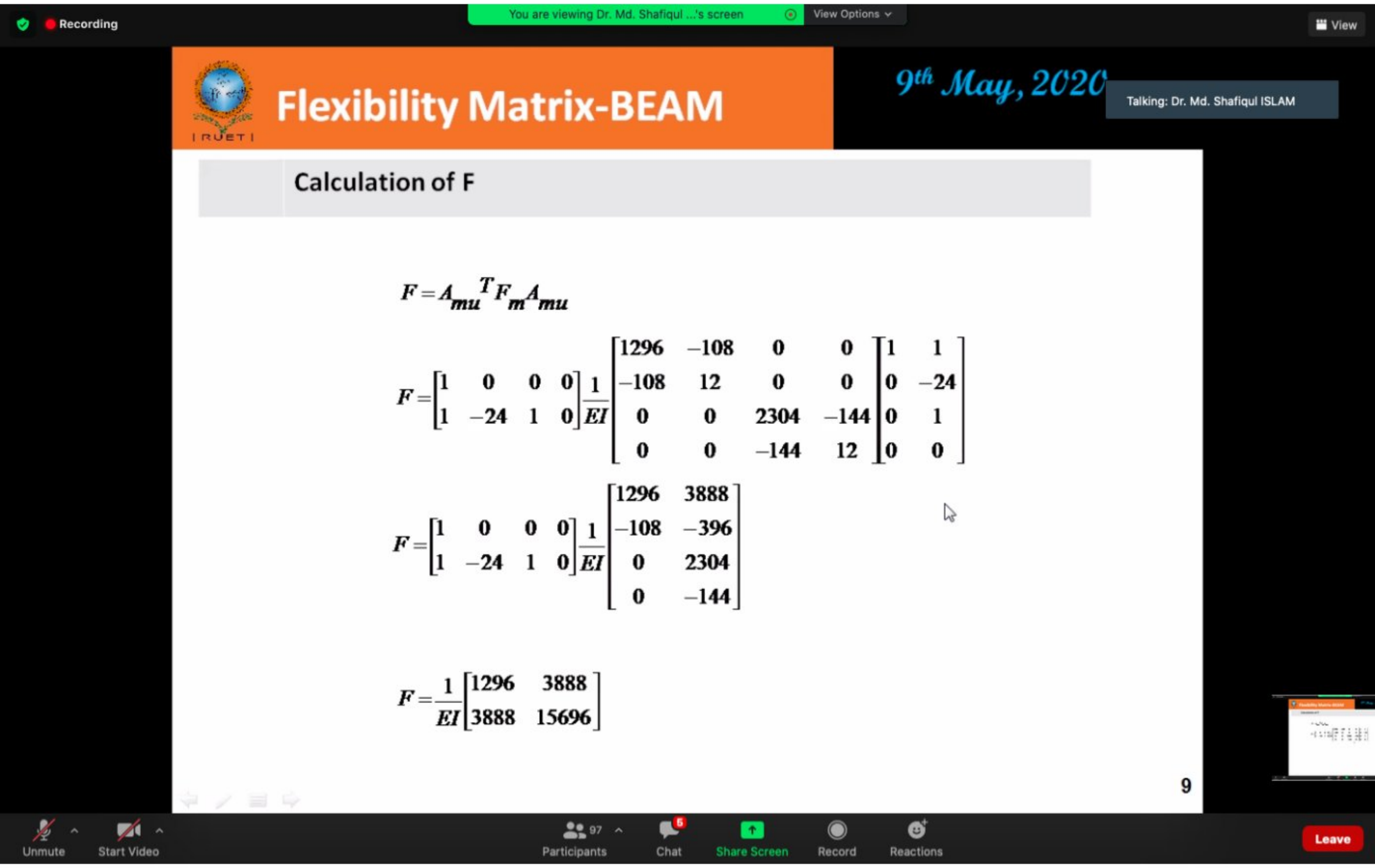
Calculation of D_{QL}

$$D_{QL} = A_{mu}{}^{T} F_m A_{mL}$$

$$D_{QL} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -24 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1296 & -108 & 0 & 0 \\ -108 & 12 & 0 & 0 \\ 0 & 0 & 2304 & -144 \\ 0 & 0 & -144 & 12 \end{bmatrix} \begin{bmatrix} -169.93 \\ 1594.67 \\ -72 \\ -288 \end{bmatrix}$$

$$D_{QL} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -24 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} -392453.64 \\ 37488.48 \\ -124416 \\ 6912 \end{bmatrix}$$

$$\begin{bmatrix} D_{QL1} \\ D_{QL2} \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -392453.64 \\ -1416593.16 \end{bmatrix}$$



Flexibility Matrix-BEAM

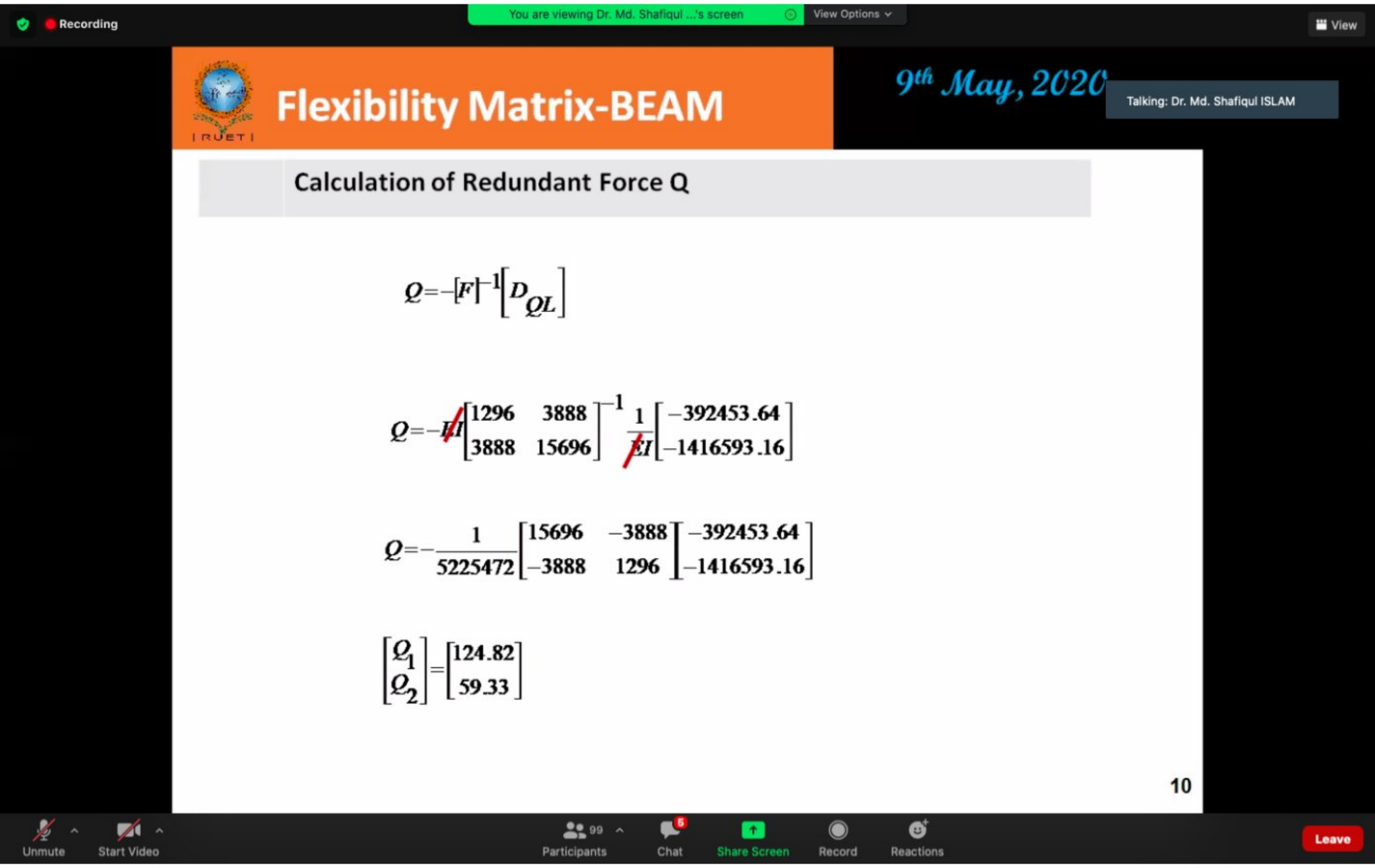
Calculation of F

$$F = A_{mu}{}^{T} F_{m} A_{mu}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -24 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1296 & -108 & 0 & 0 \\ -108 & 12 & 0 & 0 \\ 0 & 0 & 2304 & -144 \\ 0 & 0 & -144 & 12 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -24 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -24 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1296 & 3888 \\ -108 & -396 \\ 0 & 2304 \\ 0 & -144 \end{bmatrix}$$

$$F = \frac{1}{EI} \begin{bmatrix} 1296 & 3888 \\ 3888 & 15696 \end{bmatrix}$$



Flexibility Matrix-BEAM

9th May, 2020

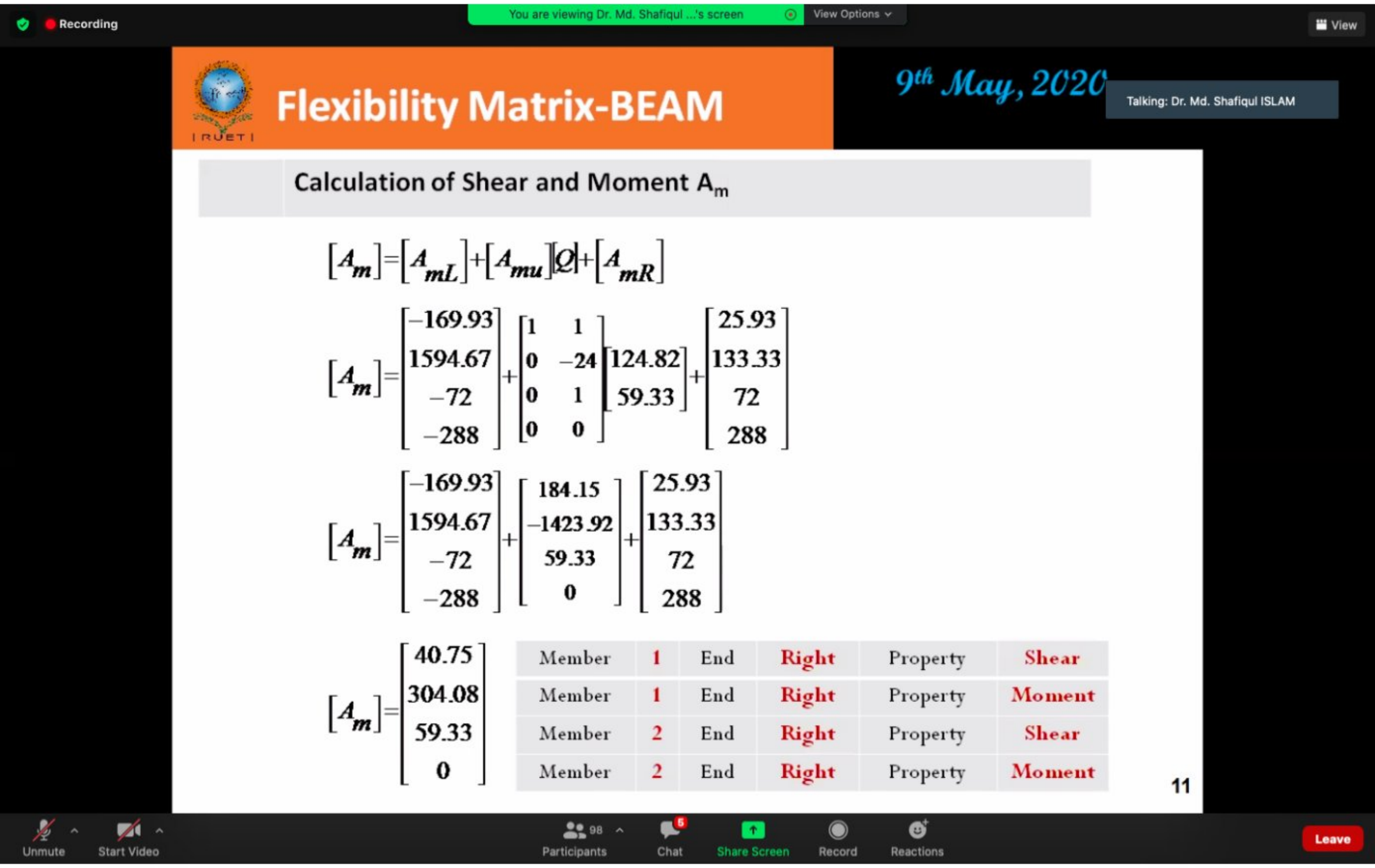
Calculation of Redundant Force Q

$$Q = -[F]^{-1}[D_{QL}]$$

$$Q = -\cancel{EI}\begin{bmatrix} 1296 & 3888 \\ 3888 & 15696 \end{bmatrix}^{-1} \frac{1}{\cancel{EI}}\begin{bmatrix} -392453.64 \\ -1416593.16 \end{bmatrix}$$

$$Q = -\frac{1}{5225472}\begin{bmatrix} 15696 & -3888 \\ -3888 & 1296 \end{bmatrix}\begin{bmatrix} -392453.64 \\ -1416593.16 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 124.82 \\ 59.33 \end{bmatrix}$$



Flexibility Matrix-BEAM

9th May, 2020

Calculation of Shear and Moment A_m

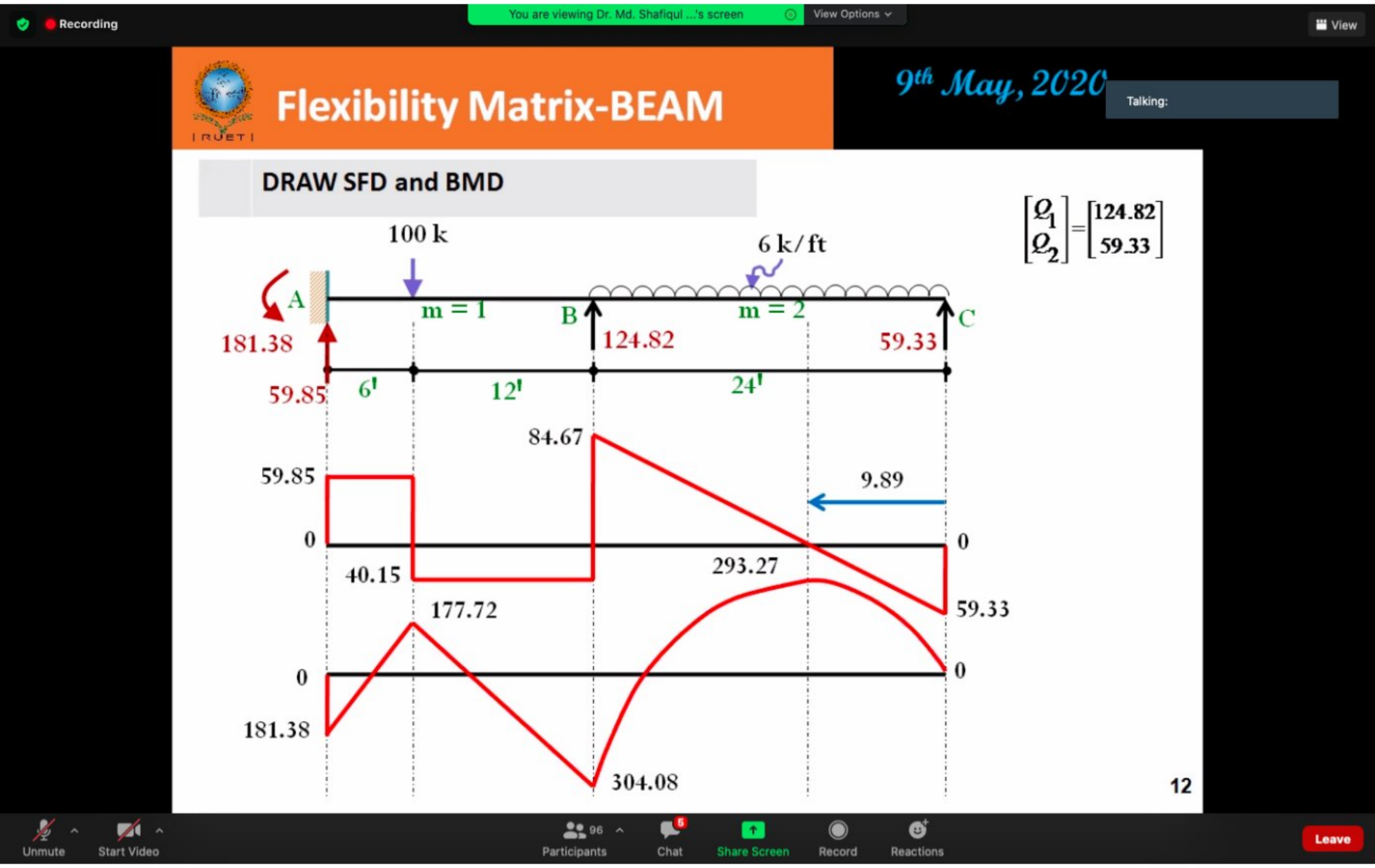
$$[A_m]=[A_{mL}]+[A_{mu}][Q]+[A_{mR}]$$

$$[A_m]=\begin{bmatrix}-169.93\\1594.67\\-72\\-288\end{bmatrix}+\begin{bmatrix}1&1\\0&-24\\0&1\\0&0\end{bmatrix}\begin{bmatrix}124.82\\59.33\end{bmatrix}+\begin{bmatrix}25.93\\133.33\\72\\288\end{bmatrix}$$

$$[A_m]=\begin{bmatrix}-169.93\\1594.67\\-72\\-288\end{bmatrix}+\begin{bmatrix}184.15\\-1423.92\\59.33\\0\end{bmatrix}+\begin{bmatrix}25.93\\133.33\\72\\288\end{bmatrix}$$

$$[A_m]=\begin{bmatrix}40.75\\304.08\\59.33\\0\end{bmatrix}$$

Member	1	End	Right	Property	Shear
Member	1	End	Right	Property	Moment
Member	2	End	Right	Property	Shear
Member	2	End	Right	Property	Moment



Flexibility Matrix-BEAM

9th May, 2020

DRAW SFD and BMD

$$\begin{bmatrix}Q_1\\Q_2\end{bmatrix}=\begin{bmatrix}124.82\\59.33\end{bmatrix}$$

CE 4111

9th May, 2020

STATICALLY INDETERMINATE STRUCTURE

Dr. Md. Shafiqul ISLAM

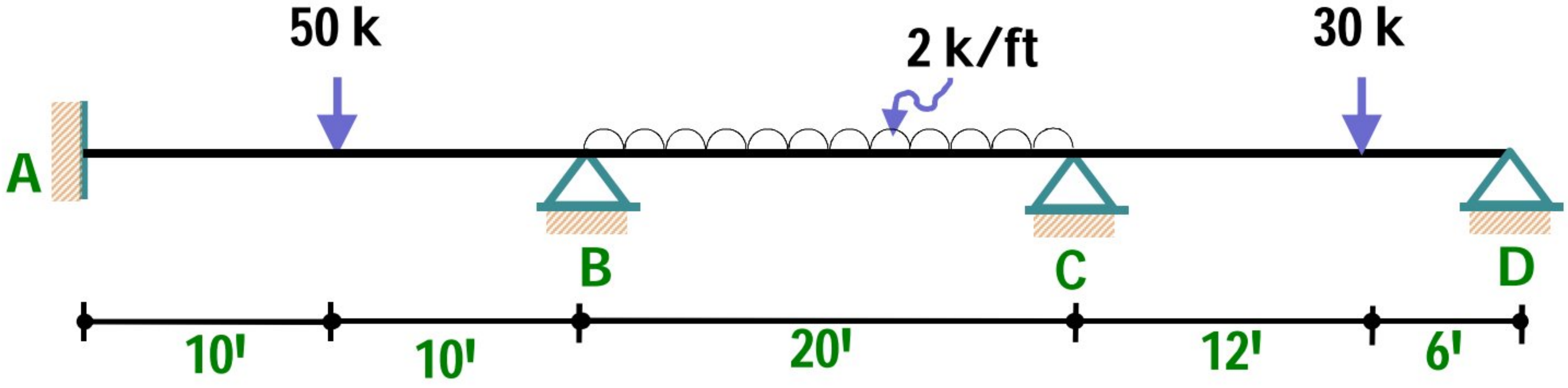
Professor, CE, RUET

Flexibility Matrix-BEAM

9th May, 2020

PROBLEM	12
Using Flexibility Matrix Method solve the followings, EI is constant	

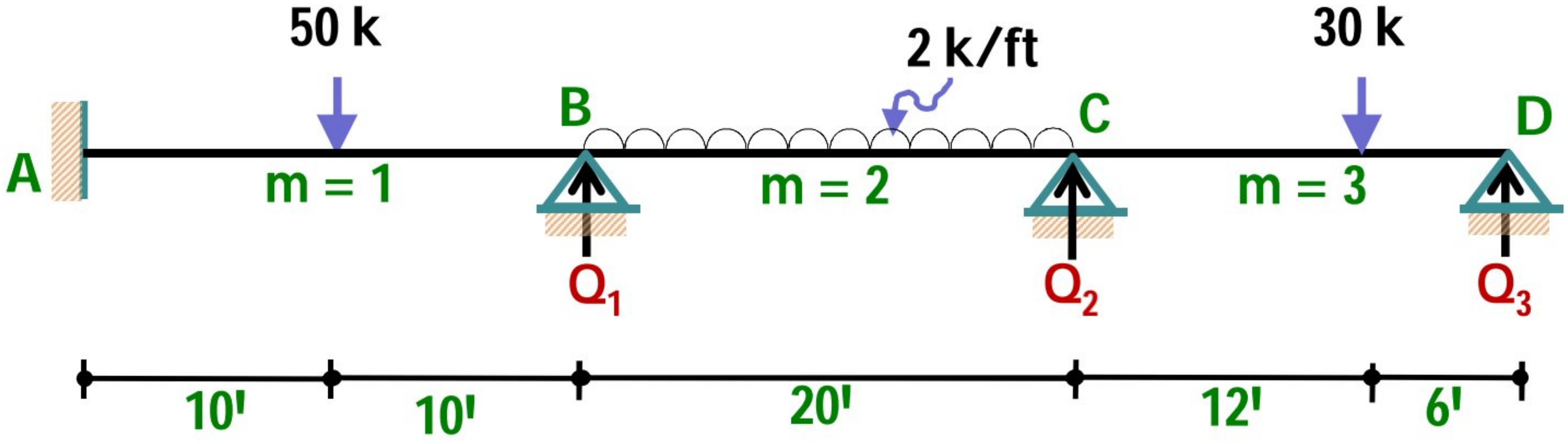
a) Determine the Redundant Forces
b) Draw Shear Force and Bending Moment Diagrams



Flexibility Matrix-BEAM

9th May, 2020

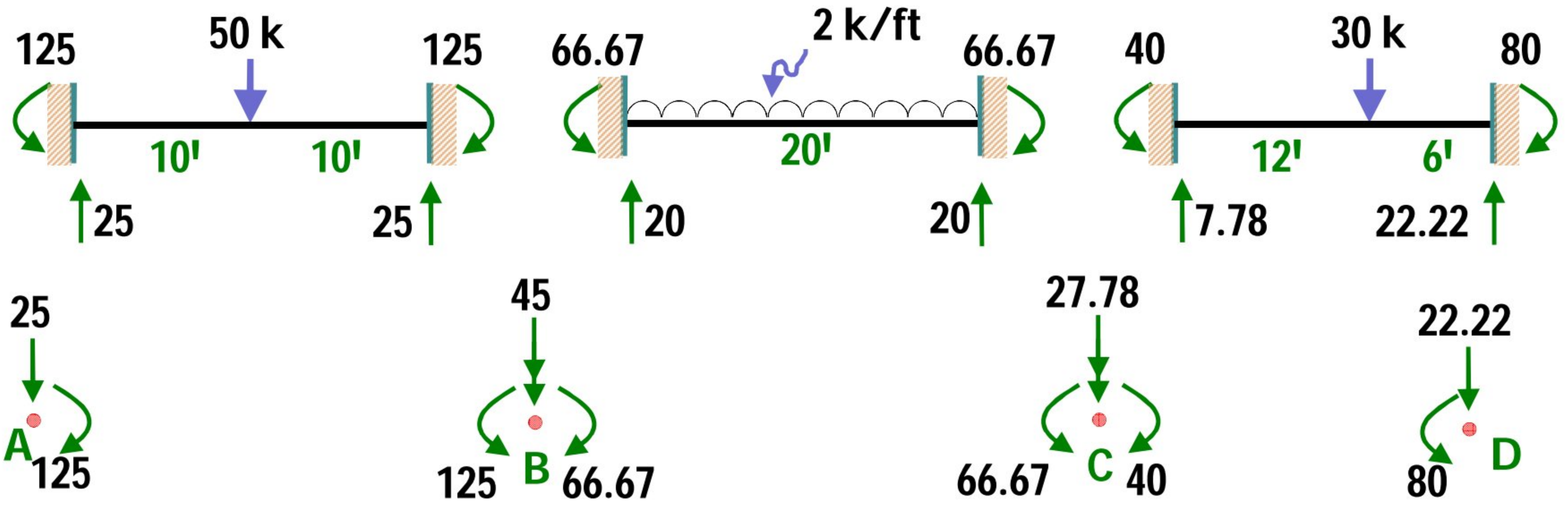
Degree of Support Restrained	9-4 = 5
Static Equilibrium Equation	3-1 = 2
Degree of Indeterminacy (DoI)	5-2 = 3
Let the Redundant Forces	Q_1 , Q_2 and Q_3



Flexibility Matrix-BEAM

9th May, 2020

Release Redundant and Taking FEM

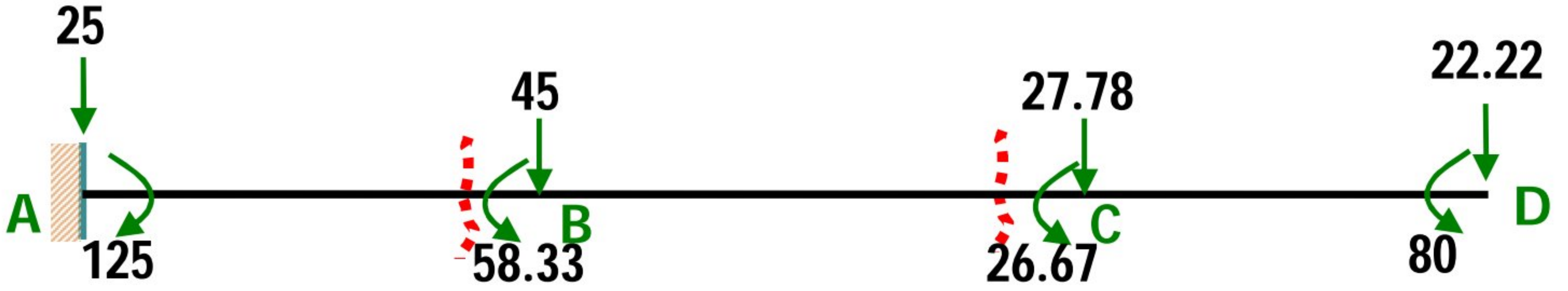


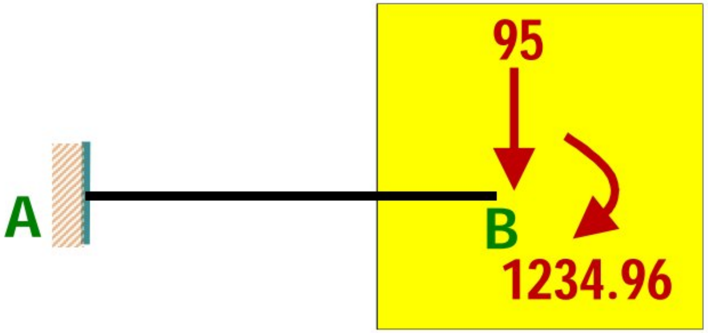
Equivalent Load Transfer

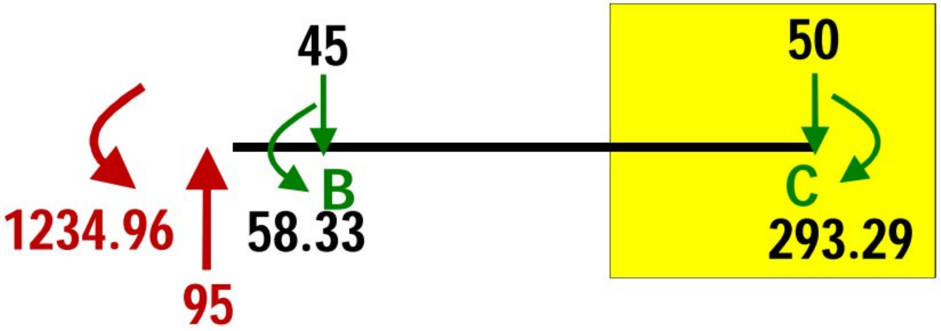
Flexibility Matrix-BEAM

9th May, 2020

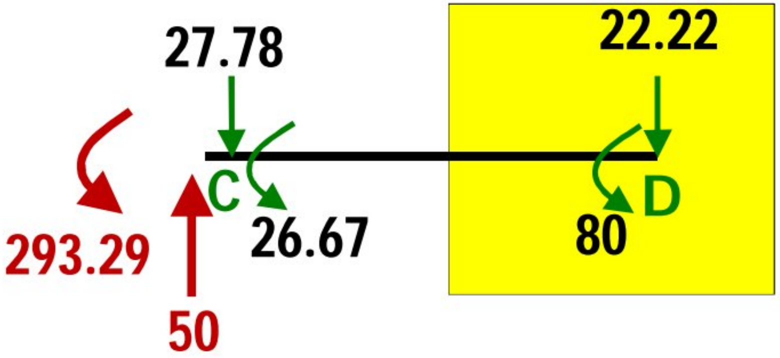
Calculation of A_{mL}







$$A_{mL} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

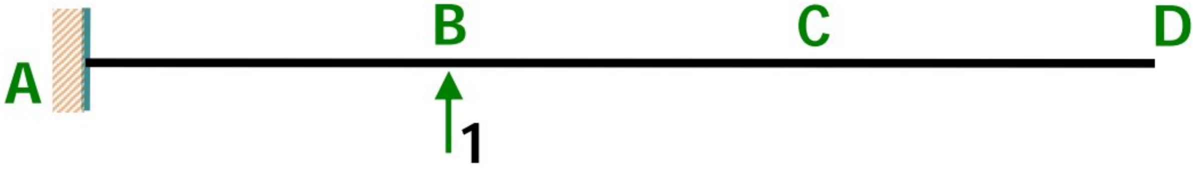


Flexibility Matrix-BEAM

9th May, 2020

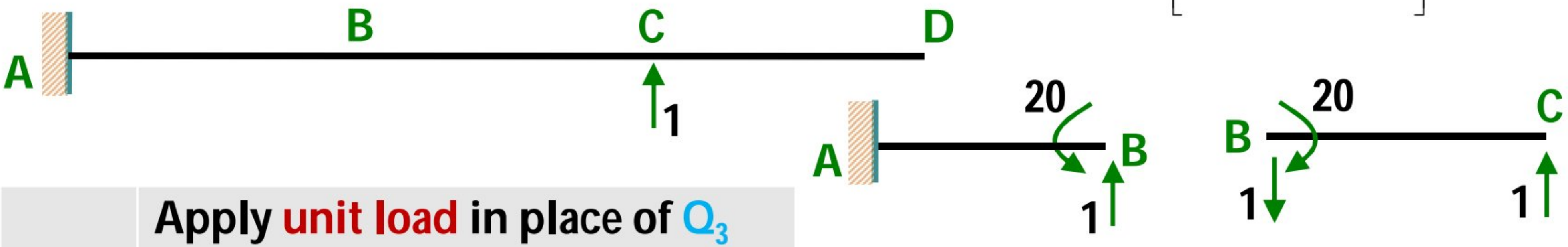
Calculation of A_{mu}

Apply **unit load** in place of Q_1

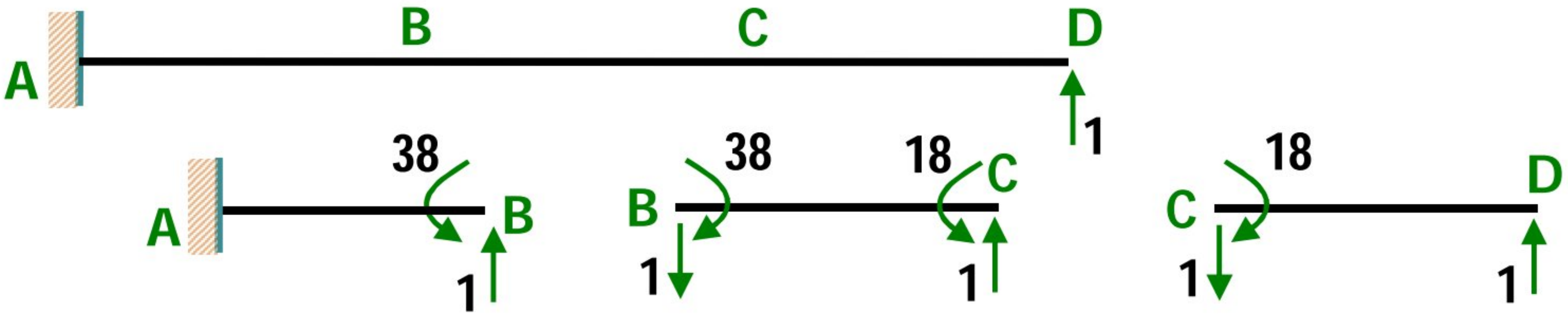


$$A_{mu} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Apply **unit load** in place of Q_2



Apply **unit load** in place of Q_3



Flexibility Matrix-BEAM

Calculation of F_m

$$F_m = \frac{1}{EI}\begin{bmatrix} L^3/3 & -L^2/2 \\ -L^2/2 & L \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 20^3/3 & -20^2/2 & 0 & 0 & 0 & 0 \\ -20^2/2 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20^3/3 & -20^2/2 & 0 & 0 \\ 0 & 0 & -20^2/2 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 18^3/3 & -18^2/2 \\ 0 & 0 & 0 & 0 & -18^2/2 & 18 \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 2666.67 & -200 & 0 & 0 & 0 & 0 \\ -200 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2666.67 & -200 & 0 & 0 \\ 0 & 0 & -200 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1944 & -162 \\ 0 & 0 & 0 & 0 & -162 & 18 \end{bmatrix}$$

Flexibility Matrix-BEAM

9th May, 2020

Calculation of D_{QL}

$$D_{QL} = A_{mu}{}^{T} F_m A_{mL}$$

$$D_{QL} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -20 & 1 & 0 & 0 & 0 \\ 1 & -38 & 1 & -18 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 2666.67 & -200 & 0 & 0 & 0 & 0 \\ -200 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2666.67 & -200 & 0 & 0 \\ 0 & 0 & -200 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1944 & -162 \\ 0 & 0 & 0 & 0 & -162 & 18 \end{bmatrix} \begin{bmatrix} -95 \\ 1234.96 \\ -50 \\ 293.29 \\ -22.22 \\ -80 \end{bmatrix}$$

$$D_{QL} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -20 & 1 & 0 & 0 & 0 \\ 1 & -38 & 1 & -18 & 1 & 0 \end{bmatrix} \begin{bmatrix} -500325.65 \\ 43699.20 \\ -191991.50 \\ 15865.80 \\ -30235.68 \\ 2159.64 \end{bmatrix}$$

$$\begin{bmatrix} D_{QL1} \\ D_{QL2} \\ D_{QL3} \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -500325.65 \\ -1566301.15 \\ -2668706.83 \end{bmatrix}$$

Flexibility Matrix-BEAM

Calculation of F

$$F = A_{mu}{}^{T} F_{m} A_{mu}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -20 & 1 & 0 & 0 & 0 \\ 1 & -38 & 1 & -18 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 2666.67 & -200 & 0 & 0 & 0 & 0 \\ -200 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2666.67 & -200 & 0 & 0 \\ 0 & 0 & -200 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1944 & -162 \\ 0 & 0 & 0 & 0 & -162 & 18 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -20 & -38 \\ 0 & 1 & 1 \\ 0 & 0 & -18 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -20 & 1 & 0 & 0 & 0 \\ 1 & -38 & 1 & -18 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 2666.67 & 6666.67 & 10266.67 \\ -200 & -600 & -960 \\ 0 & 2666.67 & 6266.67 \\ 0 & -200 & -560 \\ 0 & 0 & 1944 \\ 0 & 0 & -162 \end{bmatrix}$$

$$F = \frac{1}{EI} \begin{bmatrix} 2666.67 & 6666.67 & 10266.67 \\ 6666.67 & 21333.34 & 35733.34 \\ 10266.67 & 35733.34 & 65037.34 \end{bmatrix}$$

Flexibility Matrix-BEAM

9th May, 2020

Calculation of Redundant Force Q

$$Q = -[F]^{-1}\left[D_{QL}\right]$$

$$Q = -\cancel{EI}\begin{bmatrix} 2666.67 & 6666.67 & 10266.67 \\ 6666.67 & 21333.34 & 35733.34 \\ 10266.67 & 35733.34 & 65037.34 \end{bmatrix}^{-1} \frac{1}{\cancel{EI}}\begin{bmatrix} -500325.65 \\ -1566301.15 \\ -2668706.83 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.002341 & -0.001413 & 0.000407 \\ -0.001413 & 0.001440 & -0.000568 \\ 0.000407 & -0.000568 & 0.000263 \end{bmatrix}\begin{bmatrix} 500325.65 \\ 1566301.15 \\ 2668706.83 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 44.24 \\ 32.69 \\ 15.84 \end{bmatrix}$$

Flexibility Matrix-BEAM

Calculation of Shear and Moment A_m

$$\left[A_m\right]=\left[A_{mL}\right]+\left[A_{mu}\right][Q]+\left[A_{mR}\right]$$

$$A_m=\begin{bmatrix}-95\\1234.96\\-50\\293.29\\-22.22\\-80\end{bmatrix}+\begin{bmatrix}1&1&1\\0&-20&-38\\0&1&1\\0&0&-18\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}44.24\\32.69\\15.84\end{bmatrix}+\begin{bmatrix}25\\125\\20\\66.67\\22.22\\80\end{bmatrix}$$

$$\left[A_m\right]=\begin{bmatrix}22.77\\104.24\\18.53\\74.84\\15.84\\0\end{bmatrix}$$

Member	**1**	End	**Right**	Property	**Shear**
Member	**1**	End	**Right**	Property	**Moment**
Member	**2**	End	**Right**	Property	**Shear**
Member	**2**	End	**Right**	Property	**Moment**
Member	**3**	End	**Right**	Property	**Shear**
Member	**3**	End	**Right**	Property	**Moment**

Flexibility Matrix-BEAM

9th May, 2020

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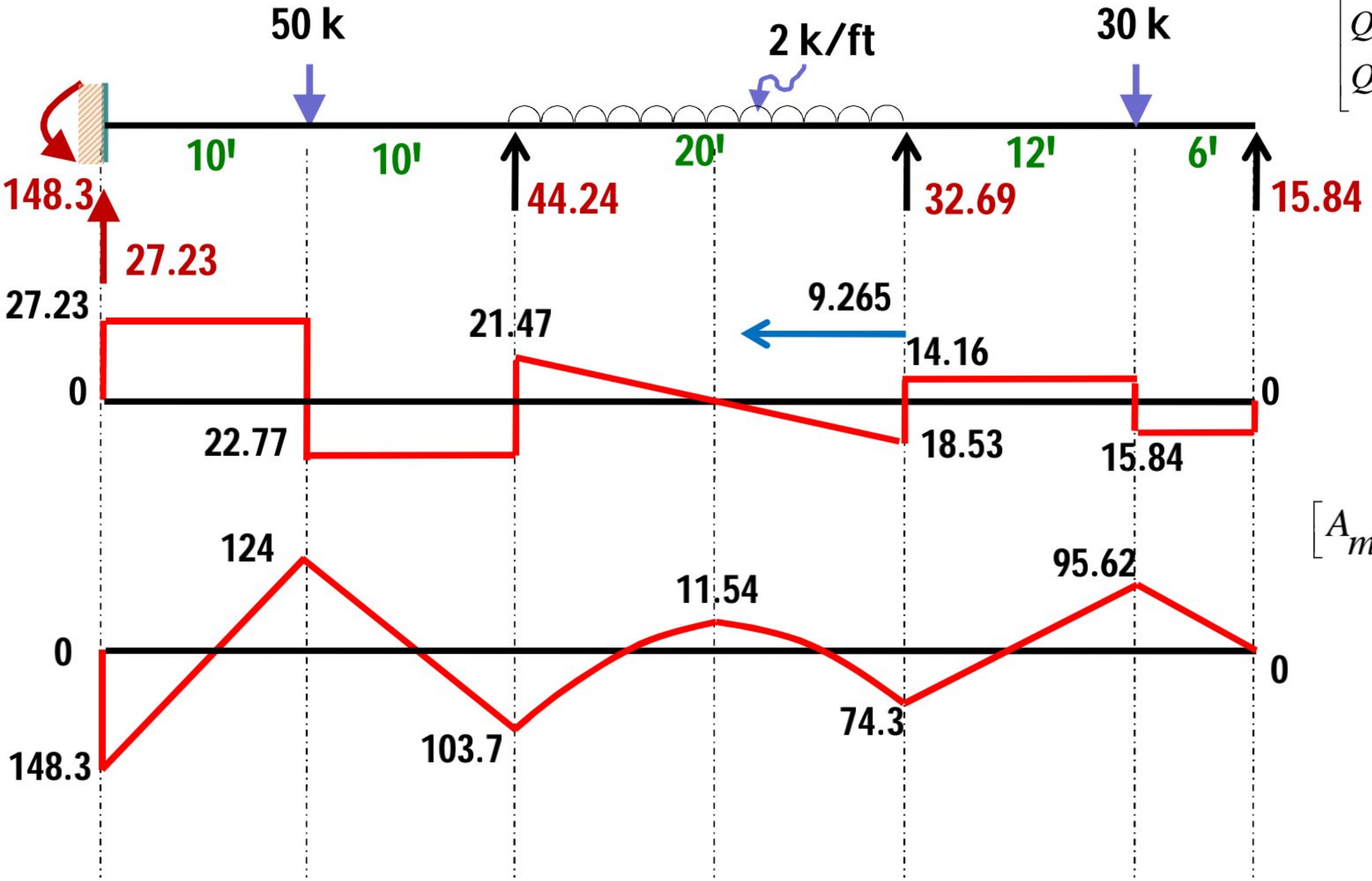
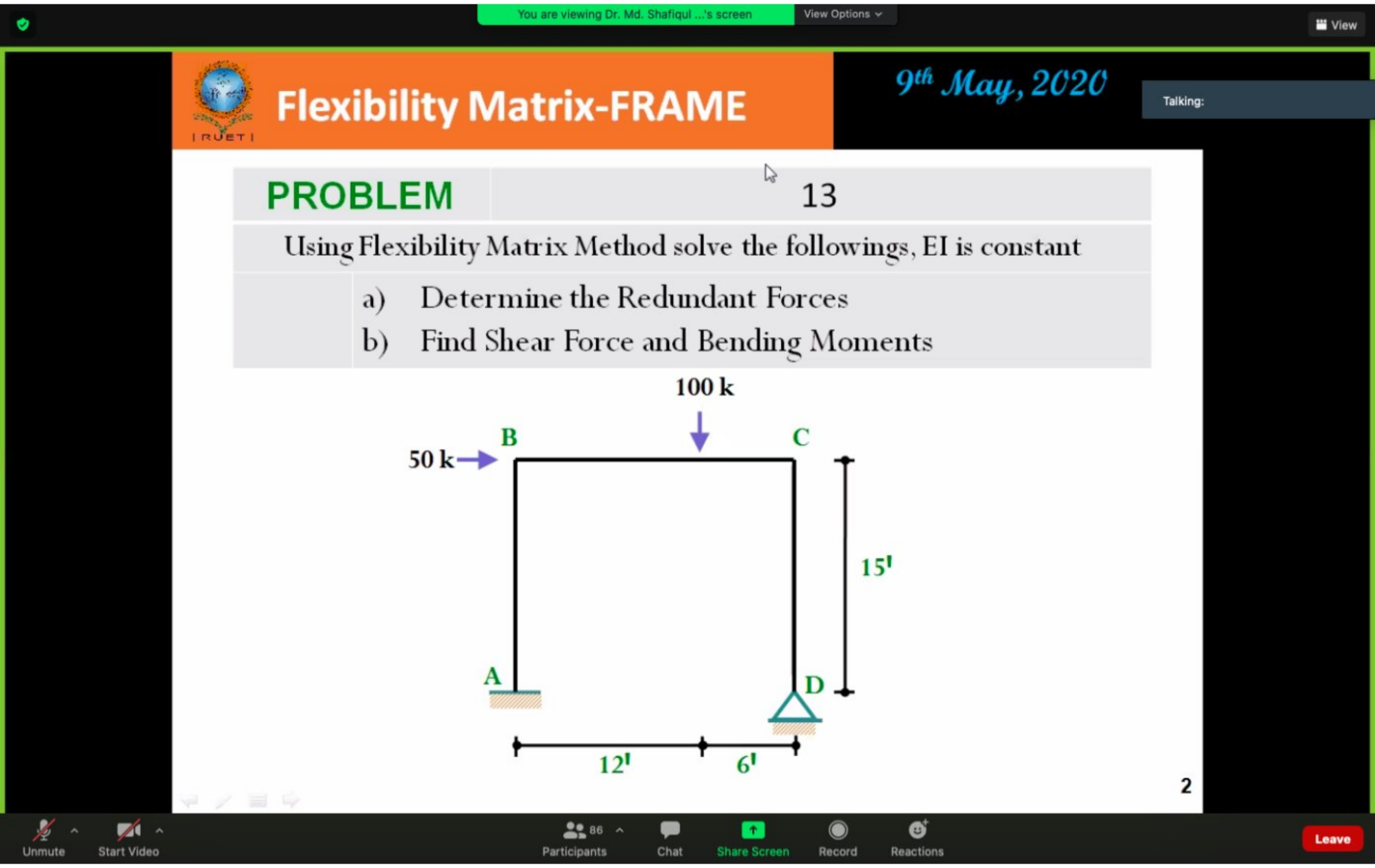
$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 44.24 \\ 32.69 \\ 15.84 \end{bmatrix}$$



$$[A_m] = \begin{bmatrix} 22.77 \\ 104.24 \\ 18.53 \\ 74.84 \\ 15.84 \\ 0 \end{bmatrix}$$

Students/CE/ RUET

Professor/CE/ RUET



Flexibility Matrix-FRAME

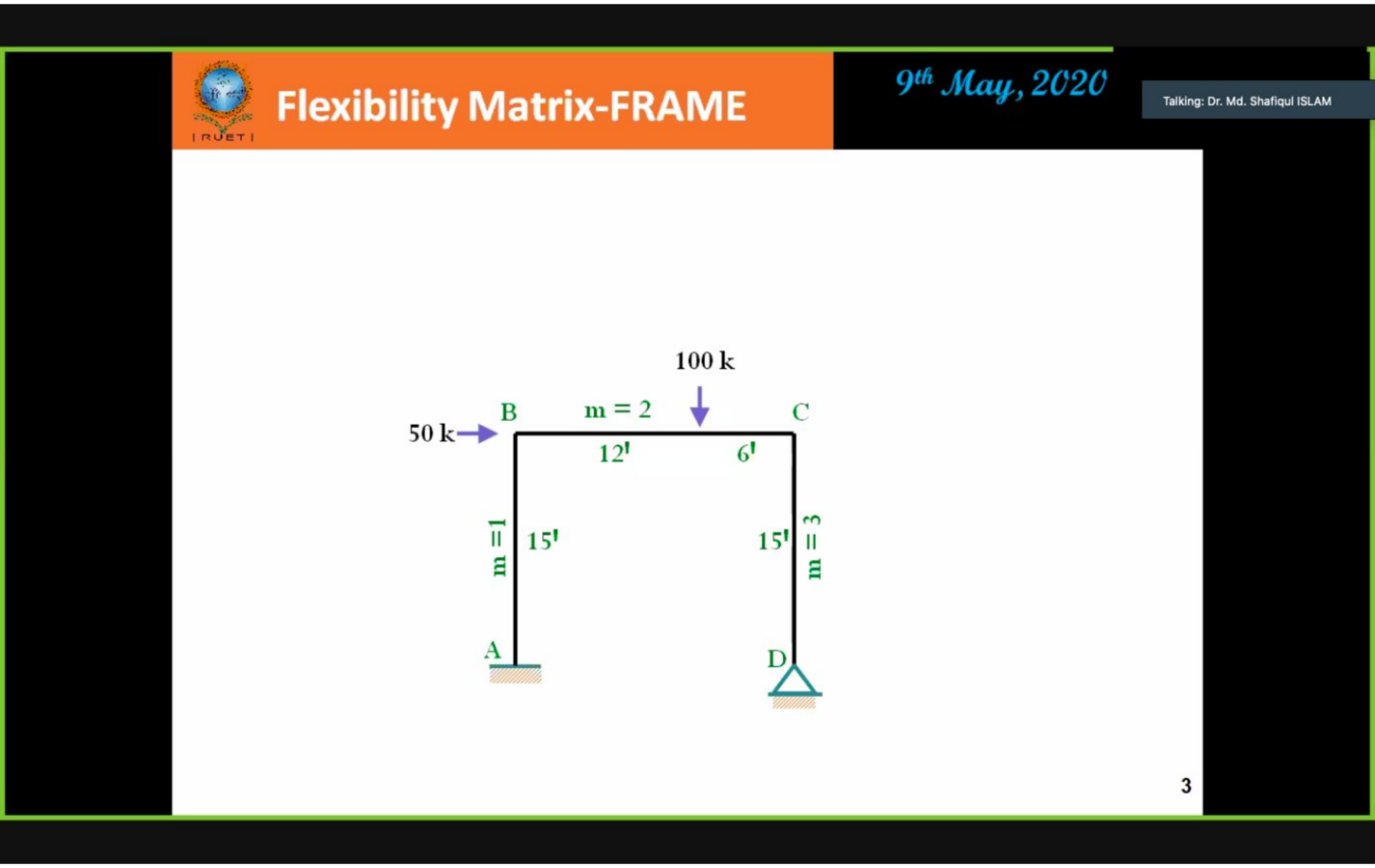
9th May, 2020

PROBLEM 13

Using Flexibility Matrix Method solve the followings, EI is constant

a) Determine the Redundant Forces

b) Find Shear Force and Bending Moments



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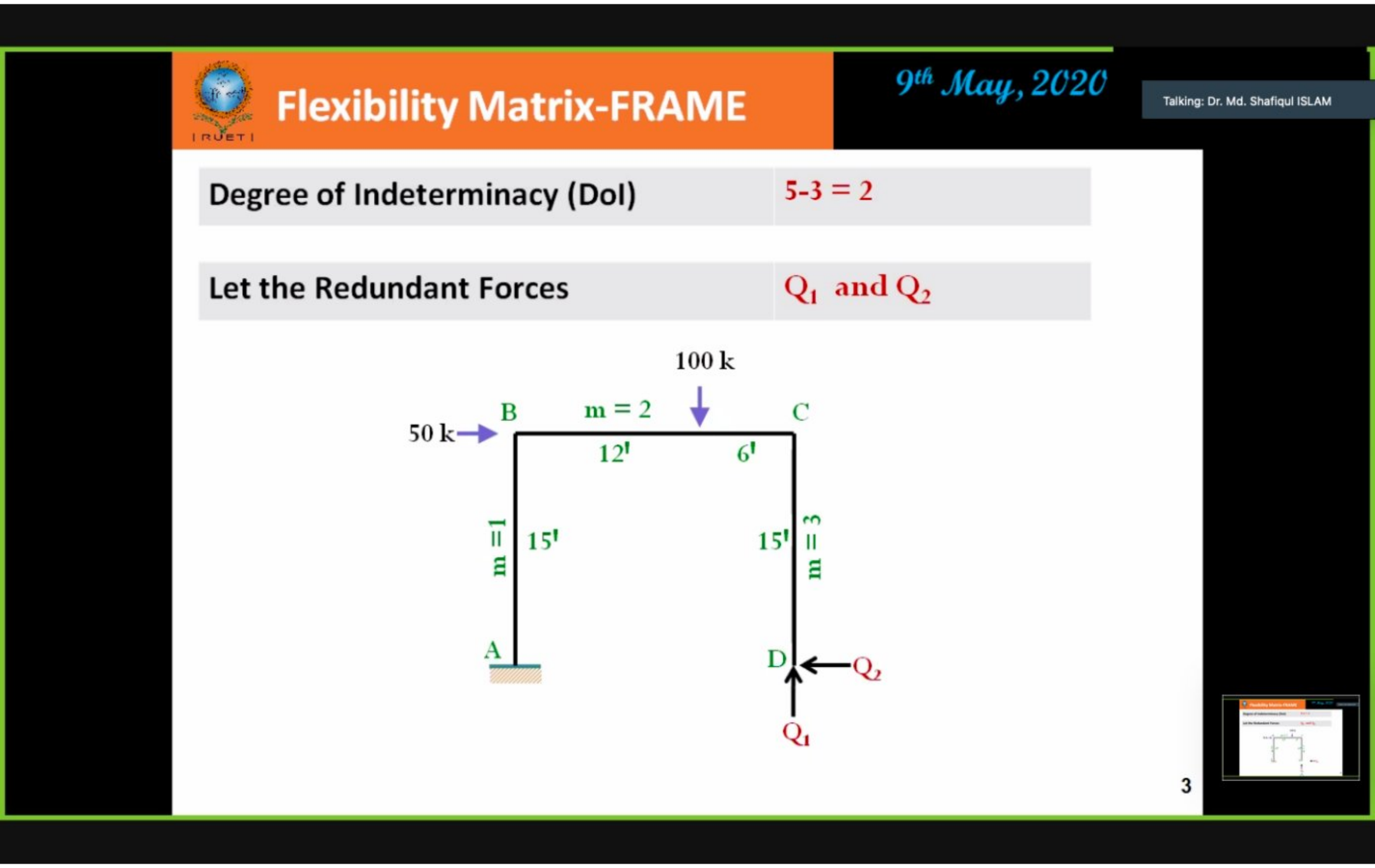
Flexibility Matrix-FRAME

9th May, 2020



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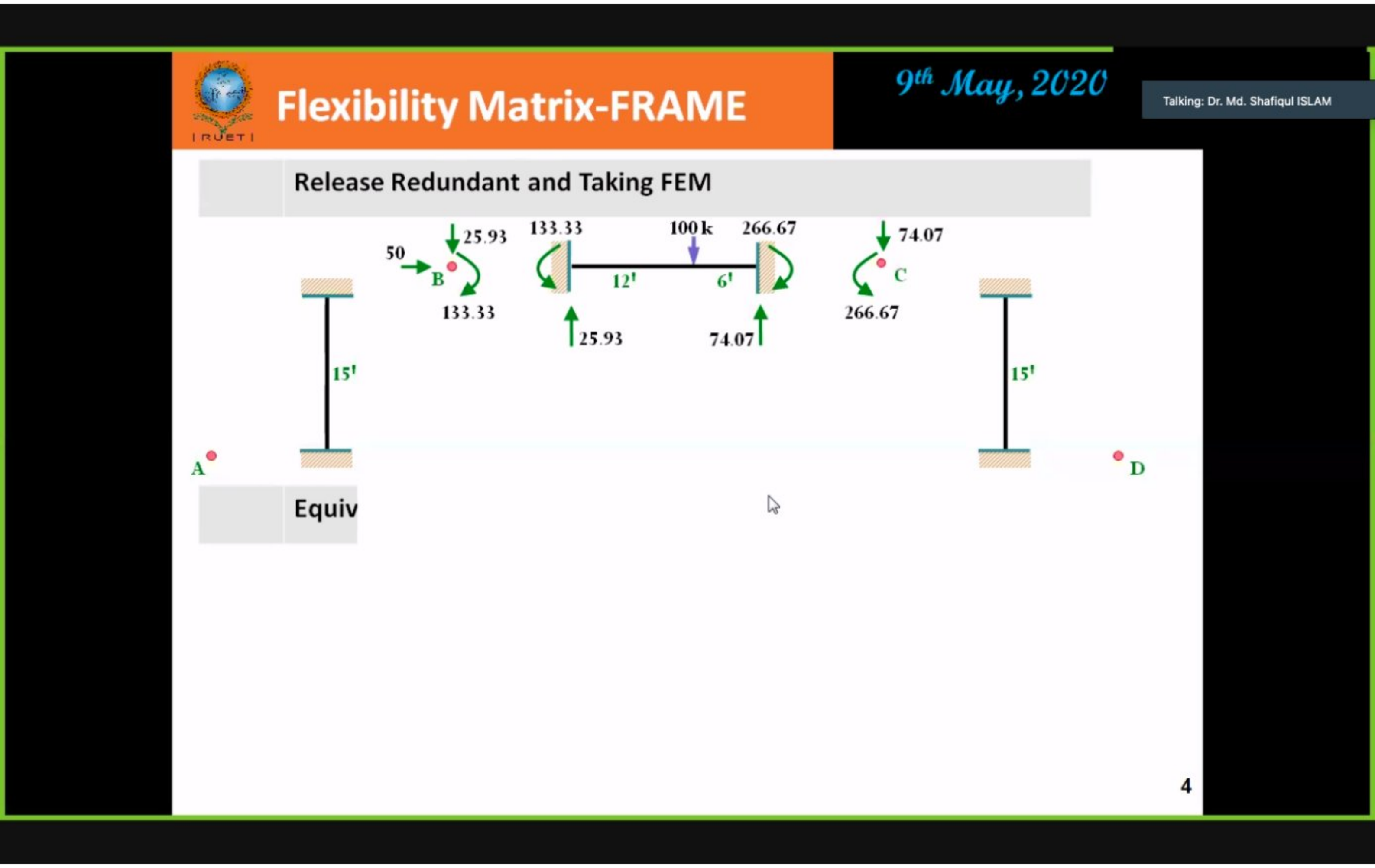
Flexibility Matrix-FRAME

9th May, 2020

Degree of Indeterminacy (DoI)	5-3 = 2
Let the Redundant Forces	Q_1 and Q_2



9th May, 2020

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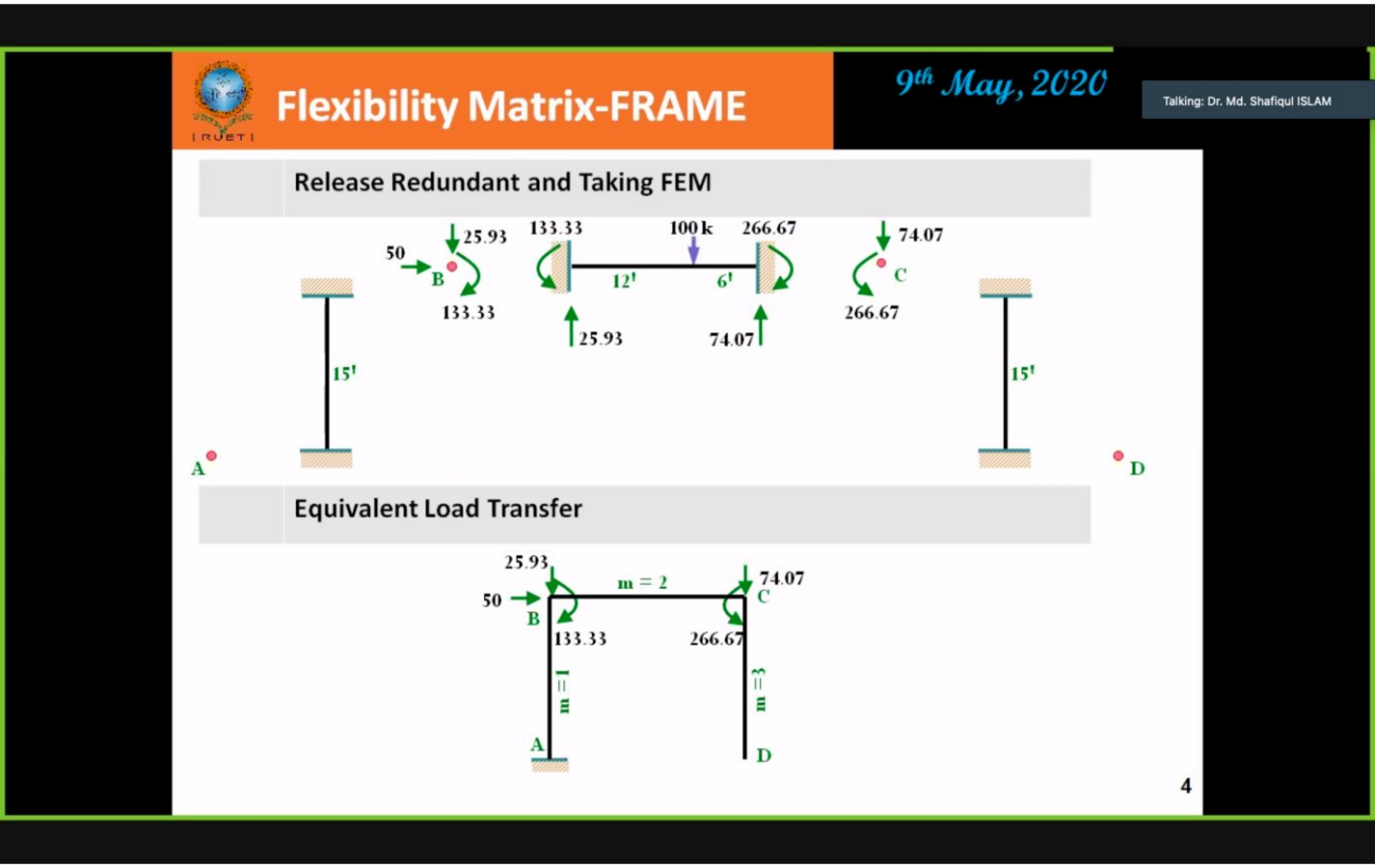
Flexibility Matrix-FRAME

Release Redundant and Taking FEM

Equiv



Flexibility Matrix-FRAME

9th May, 2020

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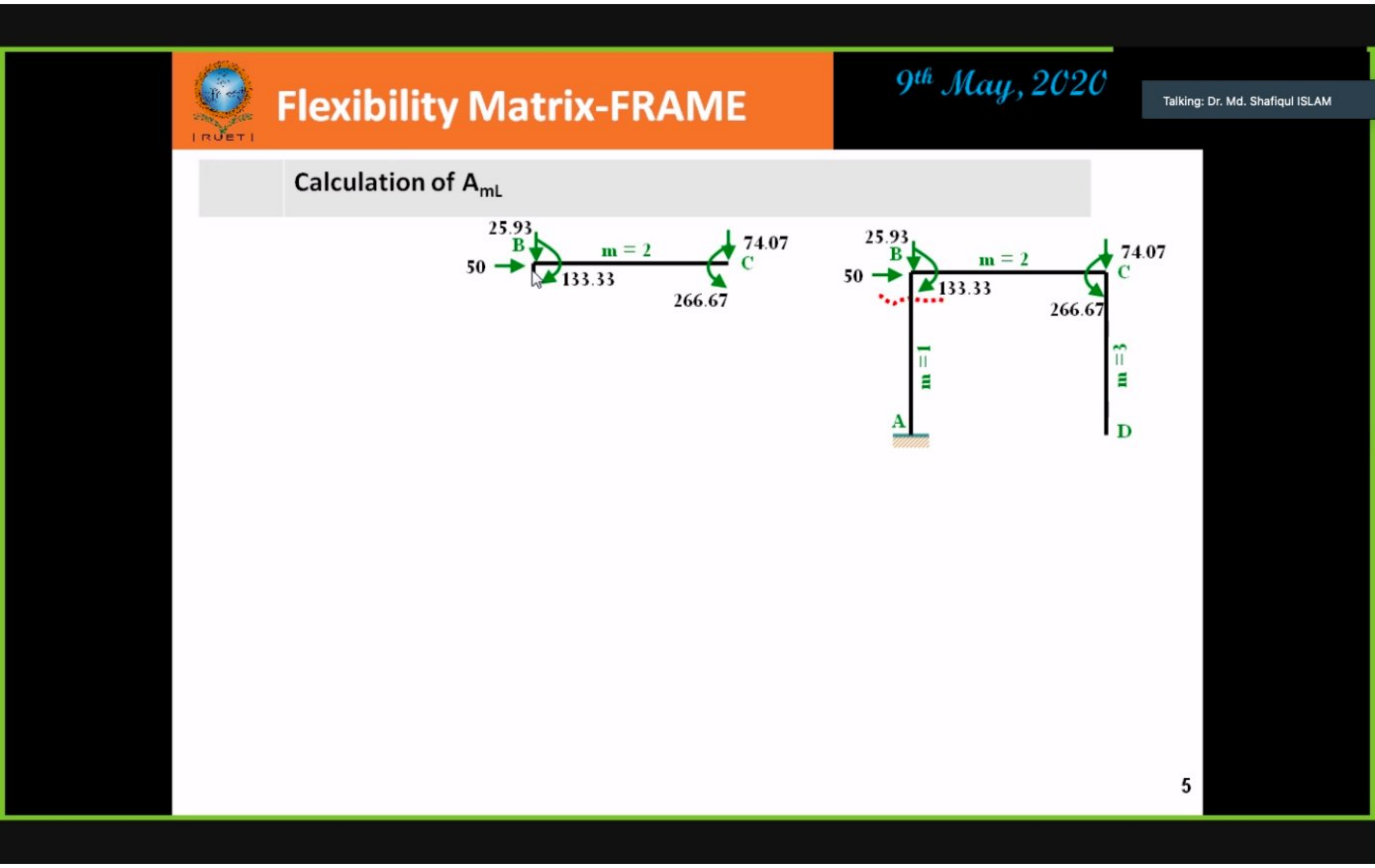
Release Redundant and Taking FEM

Equivalent Load Transfer



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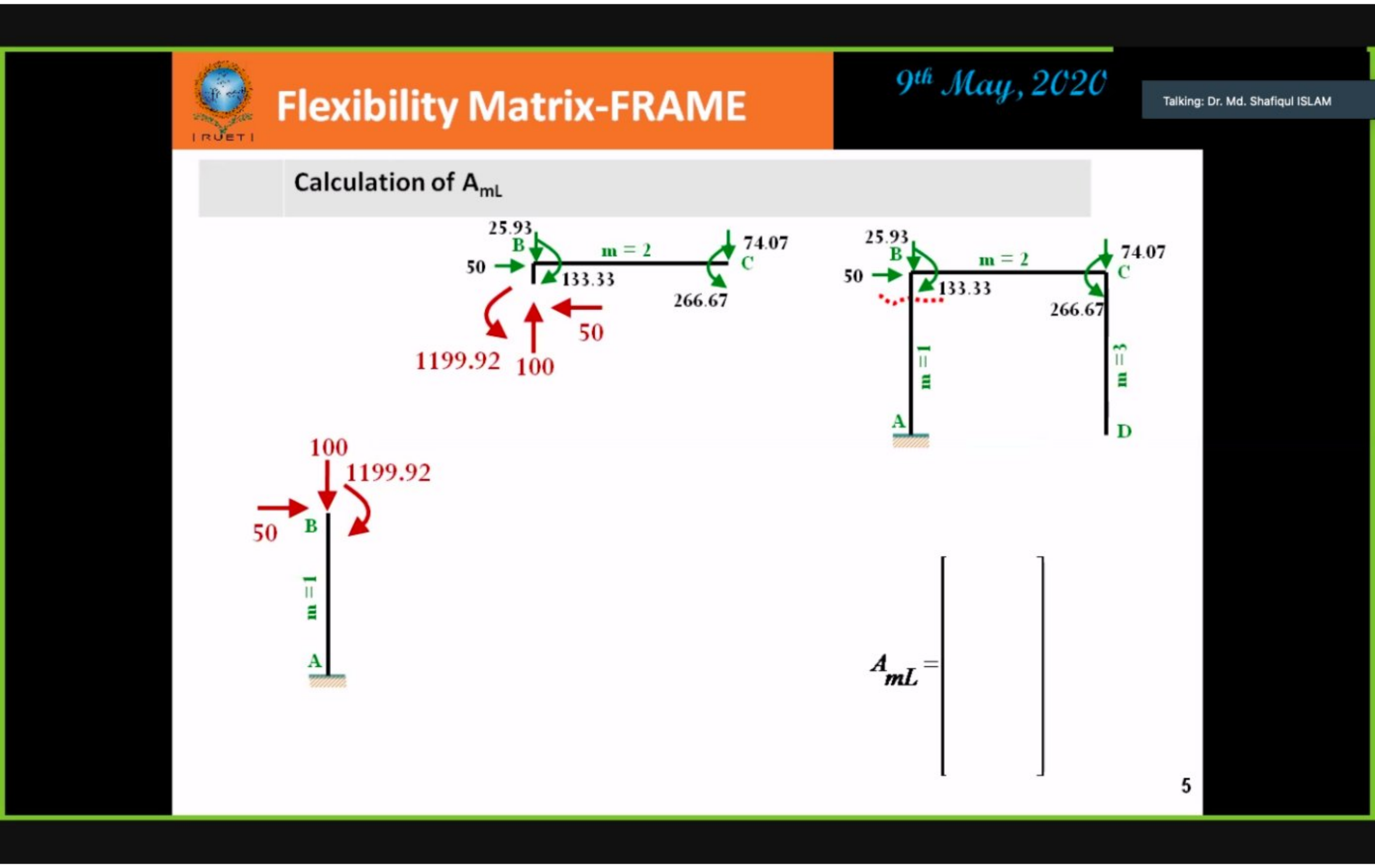
Flexibility Matrix-FRAME

9th May, 2020

Calculation of A_{mL}



Flexibility Matrix-FRAME

9th May, 2020

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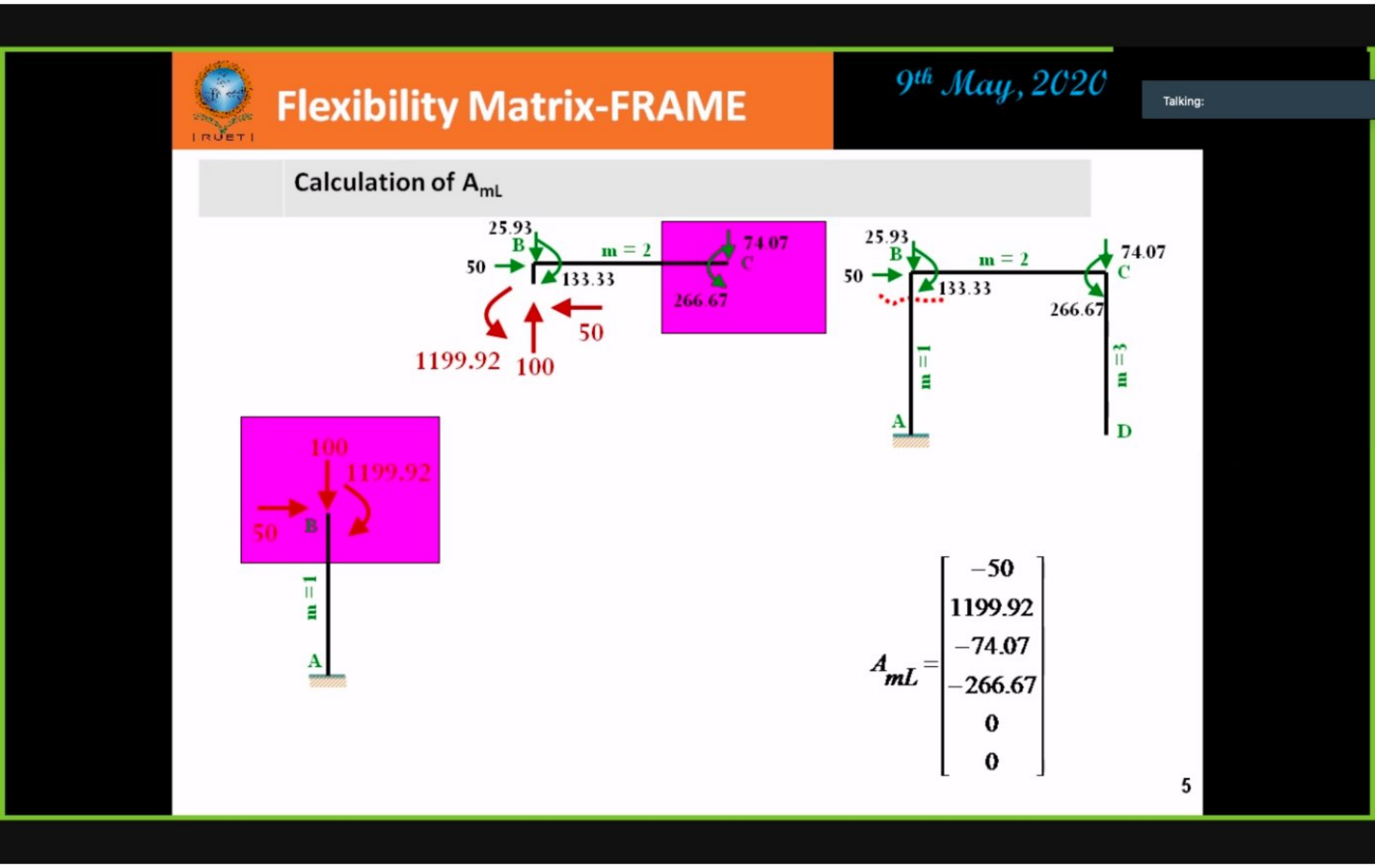
Calculation of A_{mL}

$$A_{mL} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

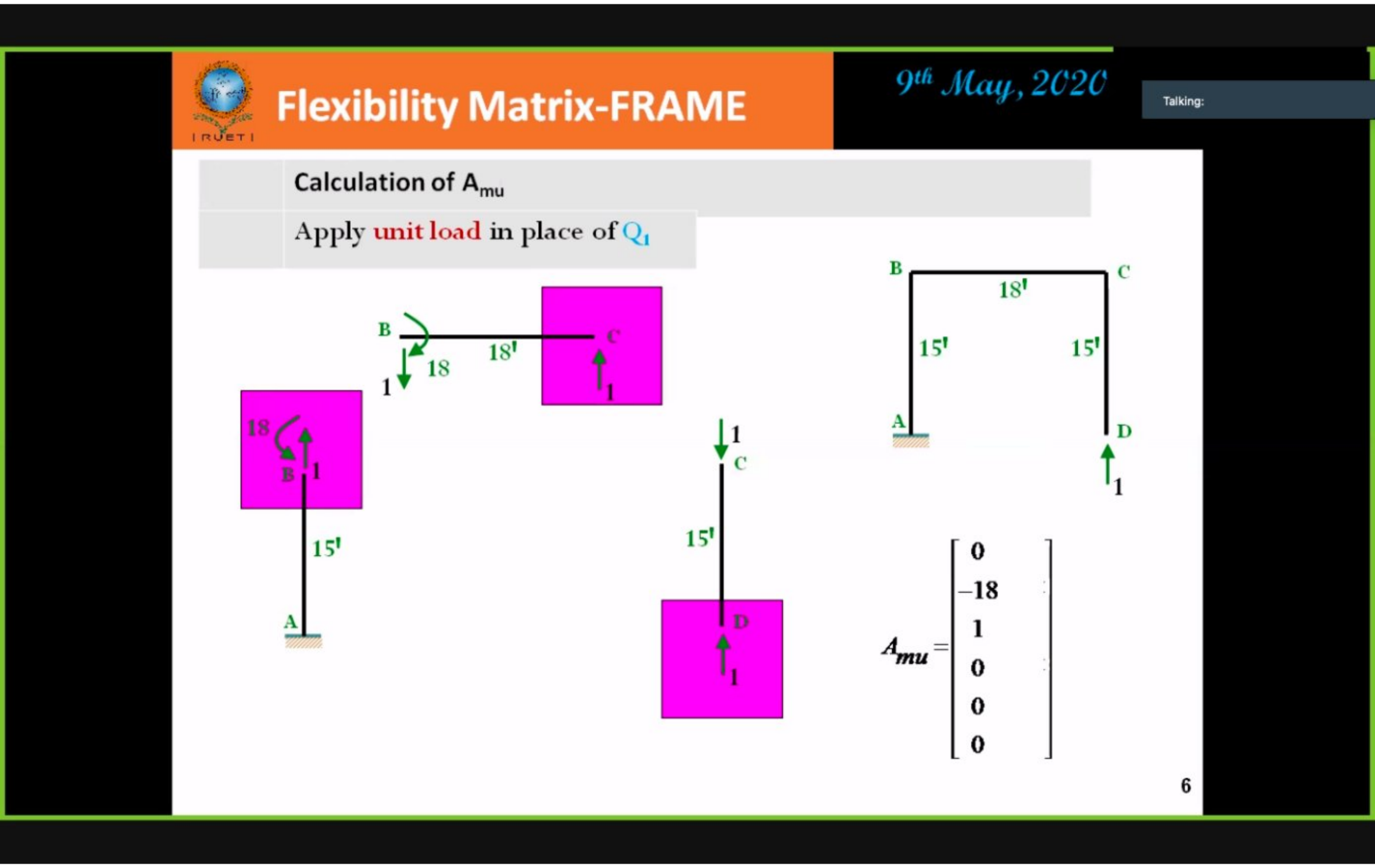


Flexibility Matrix-FRAME

9th May, 2020

Calculation of A_{mL}

$$A_{mL} = \begin{bmatrix} -50 \\ 1199.92 \\ -74.07 \\ -266.67 \\ 0 \\ 0 \end{bmatrix}$$



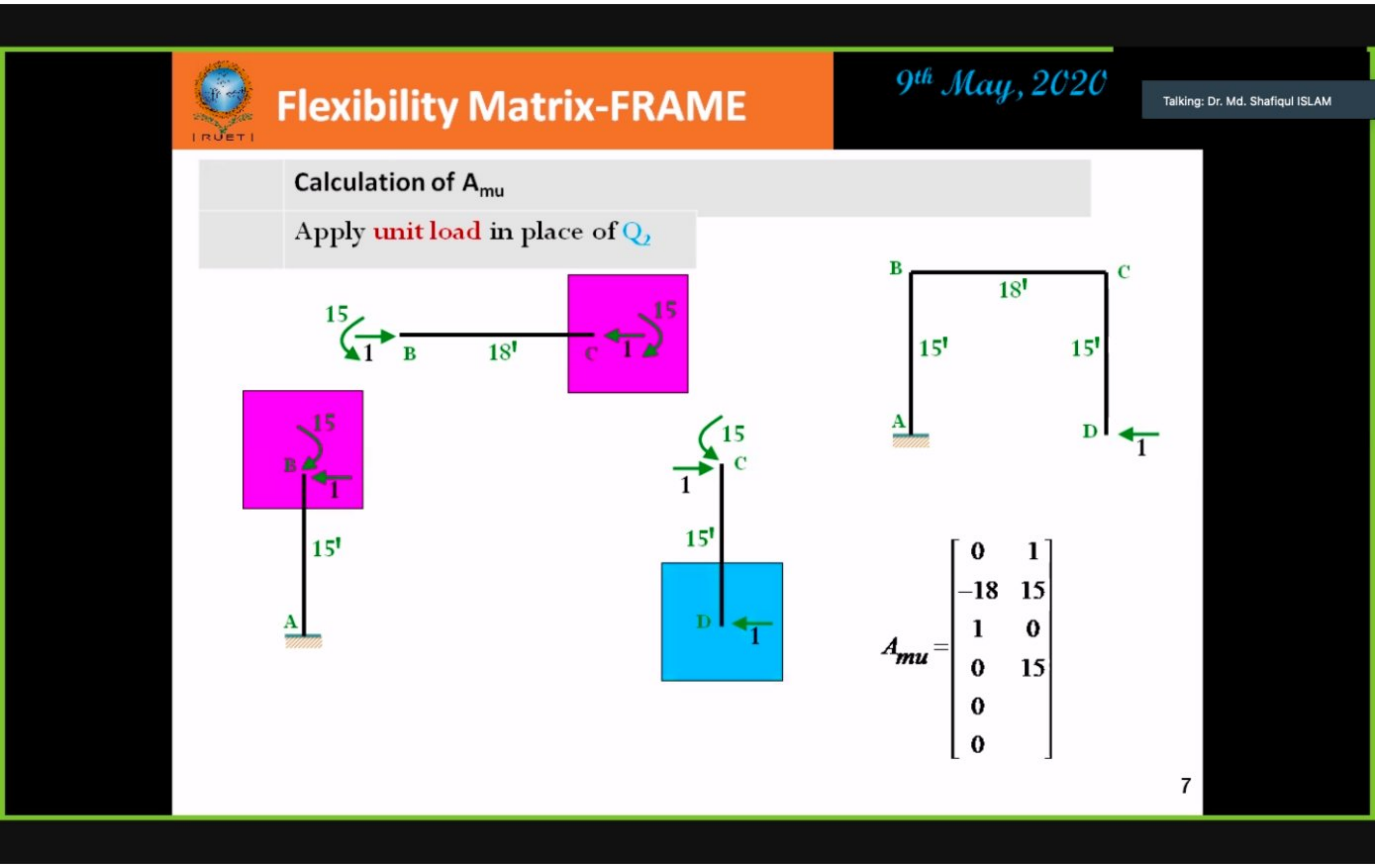
Flexibility Matrix-FRAME

9th May, 2020

Calculation of A_{mu}

Apply unit load in place of Q_1

$$A_{mu} = \begin{bmatrix} 0 \\ -18 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Flexibility Matrix-FRAME

9th May, 2020

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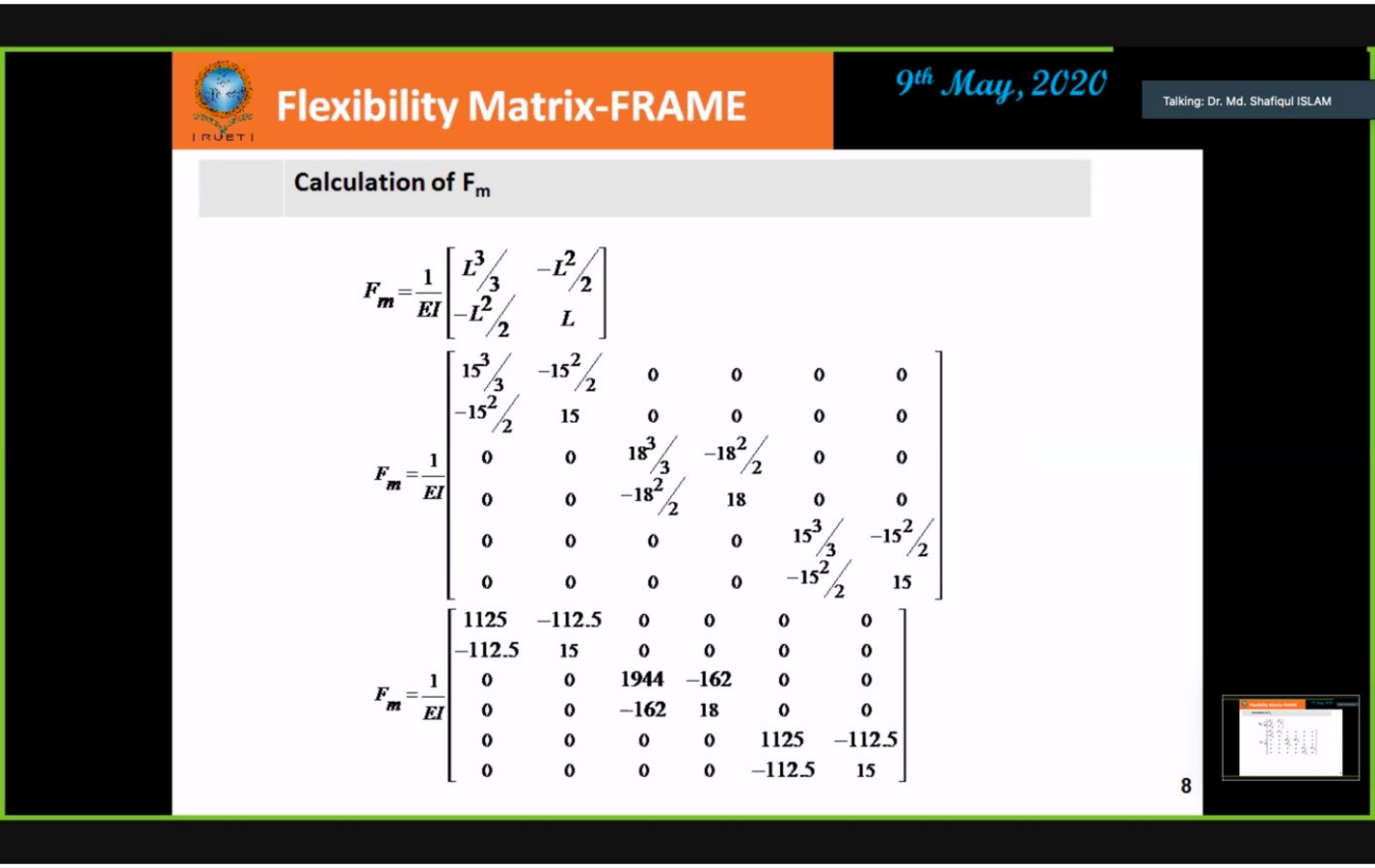
Calculation of A_{mu}

Apply unit load in place of Q_2

$$A_{mu} = \begin{bmatrix} 0 & 1 \\ -18 & 15 \\ 1 & 0 \\ 0 & 15 \\ 0 & \\ 0 & \end{bmatrix}$$



Flexibility Matrix-FRAME

9th May, 2020

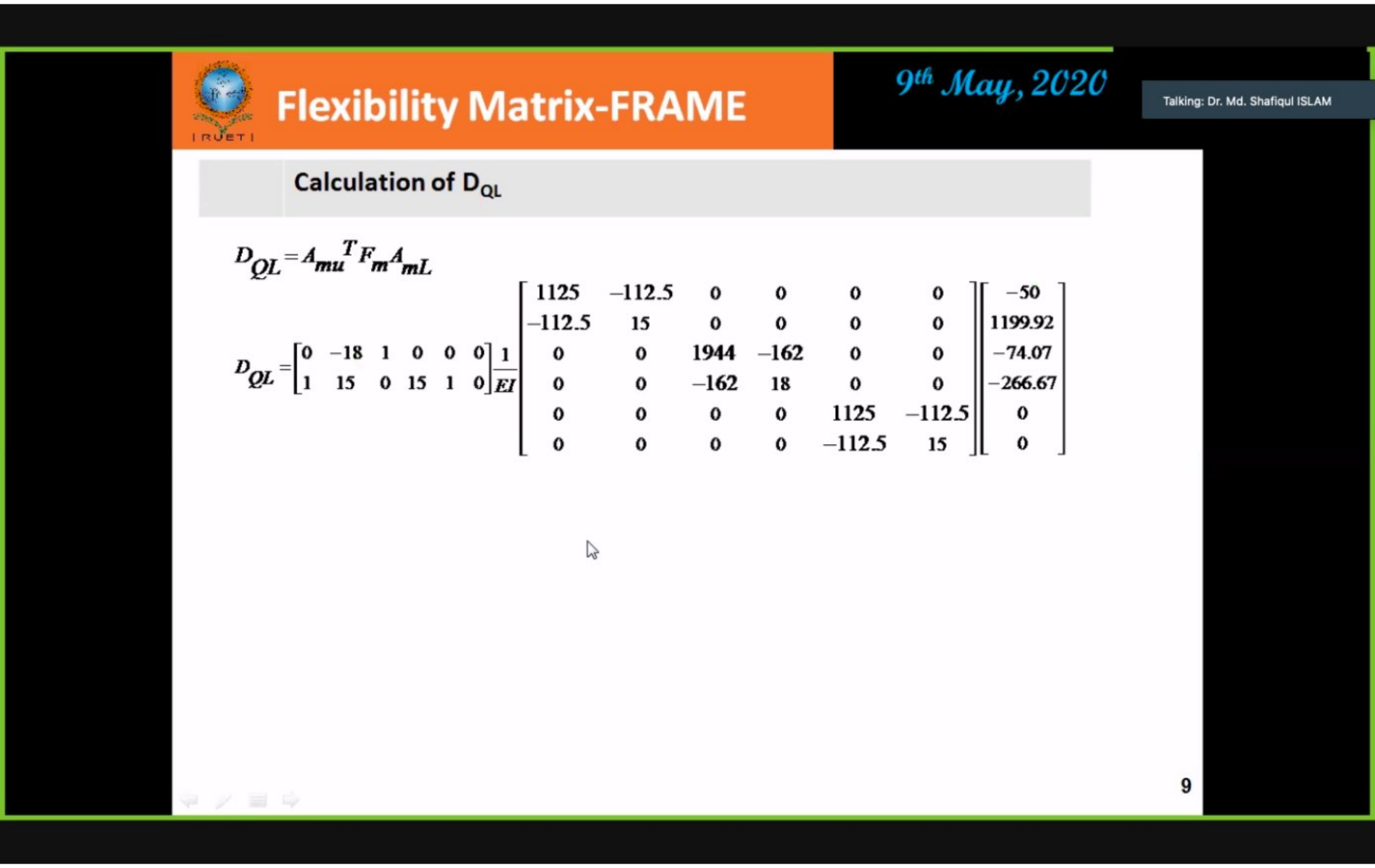
Talking: Dr. Md. Shafiqul ISLAM

Calculation of F_m

$$F_m = \frac{1}{EI}\begin{bmatrix} L^3/3 & -L^2/2 \\ -L^2/2 & L \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 15^3/3 & -15^2/2 & 0 & 0 & 0 & 0 \\ -15^2/2 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 18^3/3 & -18^2/2 & 0 & 0 \\ 0 & 0 & -18^2/2 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 15^3/3 & -15^2/2 \\ 0 & 0 & 0 & 0 & -15^2/2 & 15 \end{bmatrix}$$

$$F_m = \frac{1}{EI}\begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1944 & -162 & 0 & 0 \\ 0 & 0 & -162 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1125 & -112.5 \\ 0 & 0 & 0 & 0 & -112.5 & 15 \end{bmatrix}$$



Flexibility Matrix-FRAME

9th May, 2020

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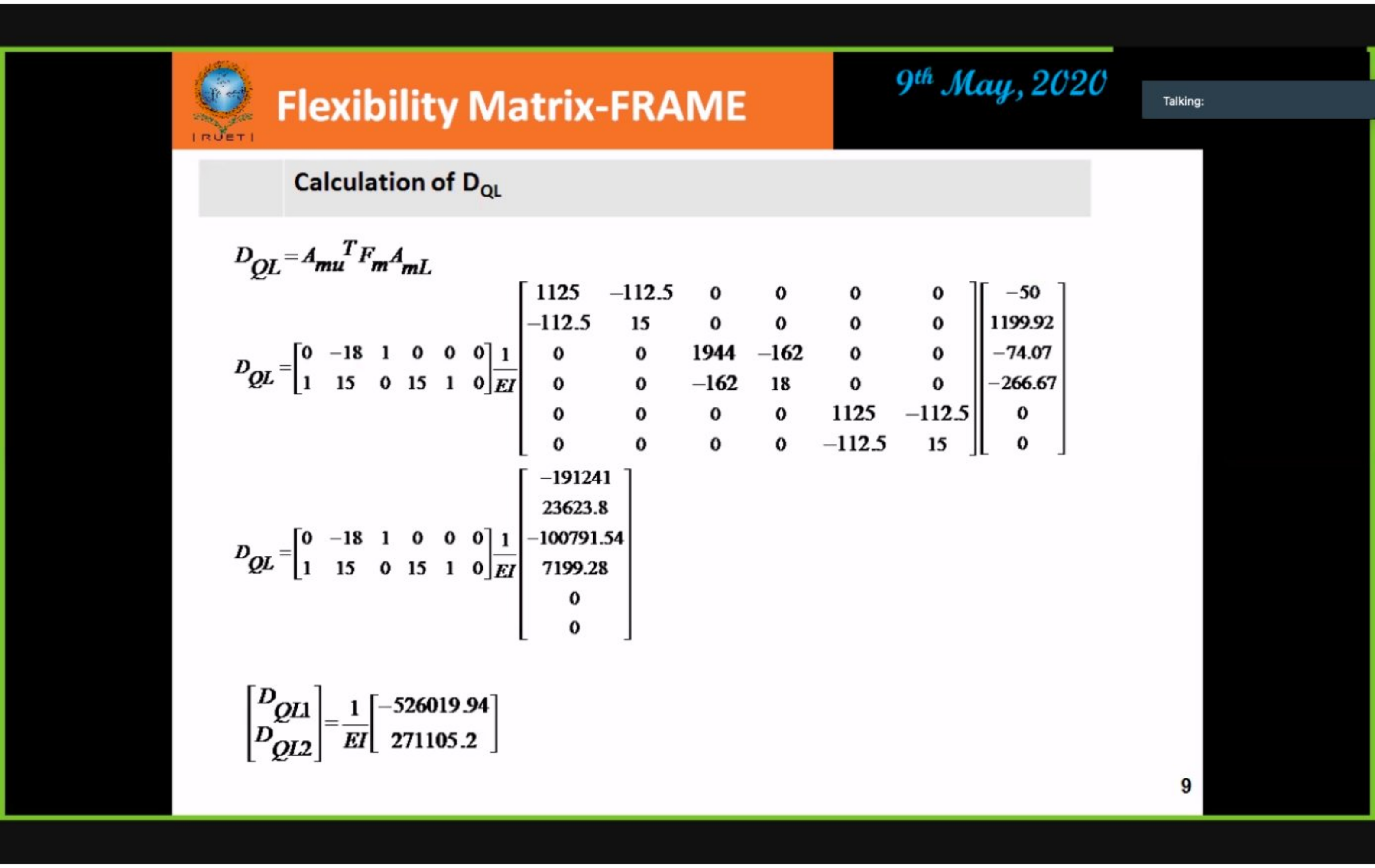
Calculation of D_{QL}

$$D_{QL} = A_{mu}{}^{T} F_{m} A_{mL}$$

$$D_{QL} = \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 15 & 0 & 15 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1944 & -162 & 0 & 0 \\ 0 & 0 & -162 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1125 & -112.5 \\ 0 & 0 & 0 & 0 & -112.5 & 15 \end{bmatrix} \begin{bmatrix} -50 \\ 1199.92 \\ -74.07 \\ -266.67 \\ 0 \\ 0 \end{bmatrix}$$



Flexibility Matrix-FRAME

9th May, 2020

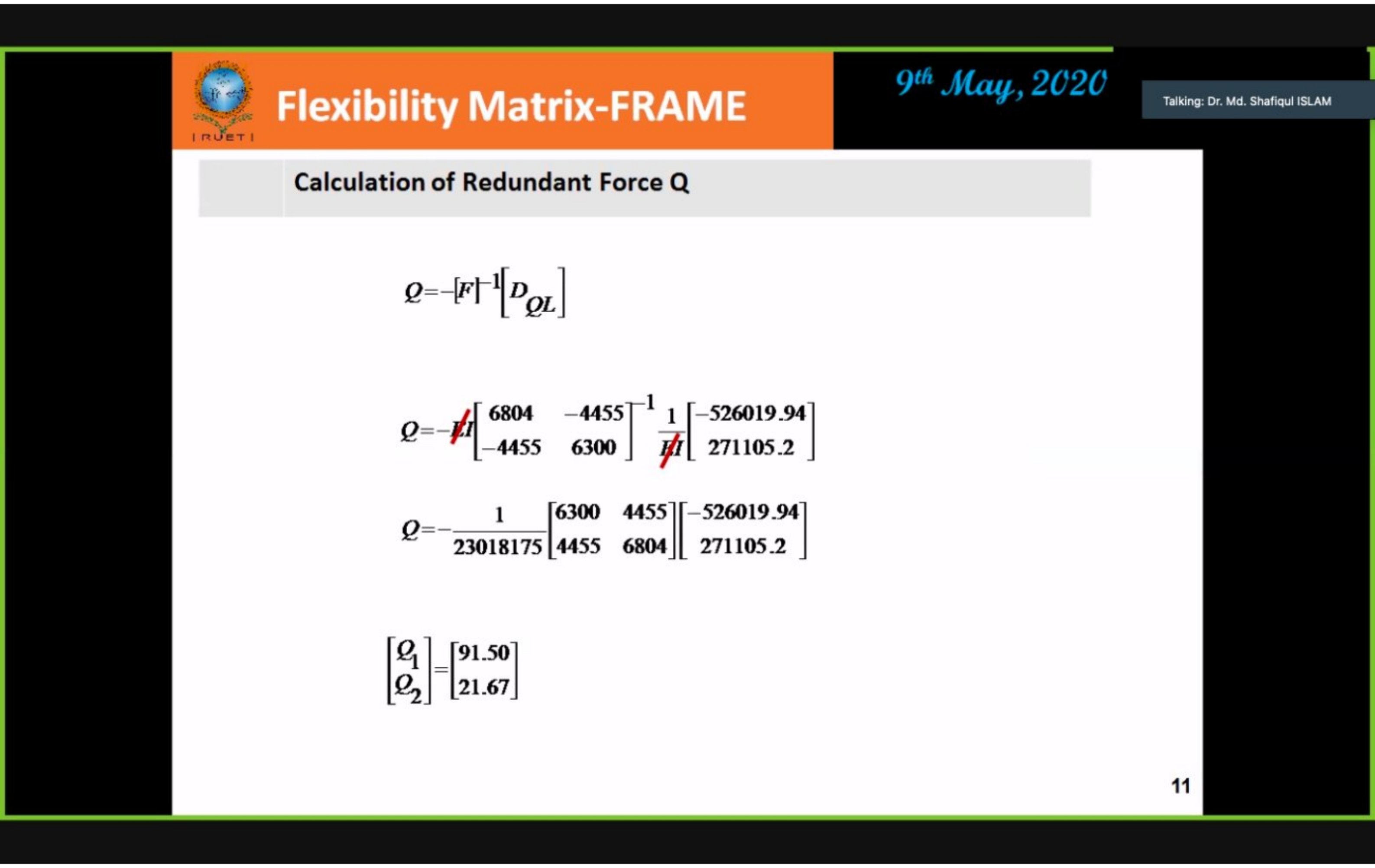
Calculation of D_{QL}

$$D_{QL} = A_{mu}^{T} F_m A_{mL}$$

$$D_{QL} = \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 15 & 0 & 15 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} 1125 & -112.5 & 0 & 0 & 0 & 0 \\ -112.5 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1944 & -162 & 0 & 0 \\ 0 & 0 & -162 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1125 & -112.5 \\ 0 & 0 & 0 & 0 & -112.5 & 15 \end{bmatrix} \begin{bmatrix} -50 \\ 1199.92 \\ -74.07 \\ -266.67 \\ 0 \\ 0 \end{bmatrix}$$

$$D_{QL} = \begin{bmatrix} 0 & -18 & 1 & 0 & 0 & 0 \\ 1 & 15 & 0 & 15 & 1 & 0 \end{bmatrix} \frac{1}{EI} \begin{bmatrix} -191241 \\ 23623.8 \\ -100791.54 \\ 7199.28 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} D_{QL1} \\ D_{QL2} \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -526019.94 \\ 271105.2 \end{bmatrix}$$



Flexibility Matrix-FRAME

9th May, 2020

Talking: Dr. Md. Shafiqul ISLAM

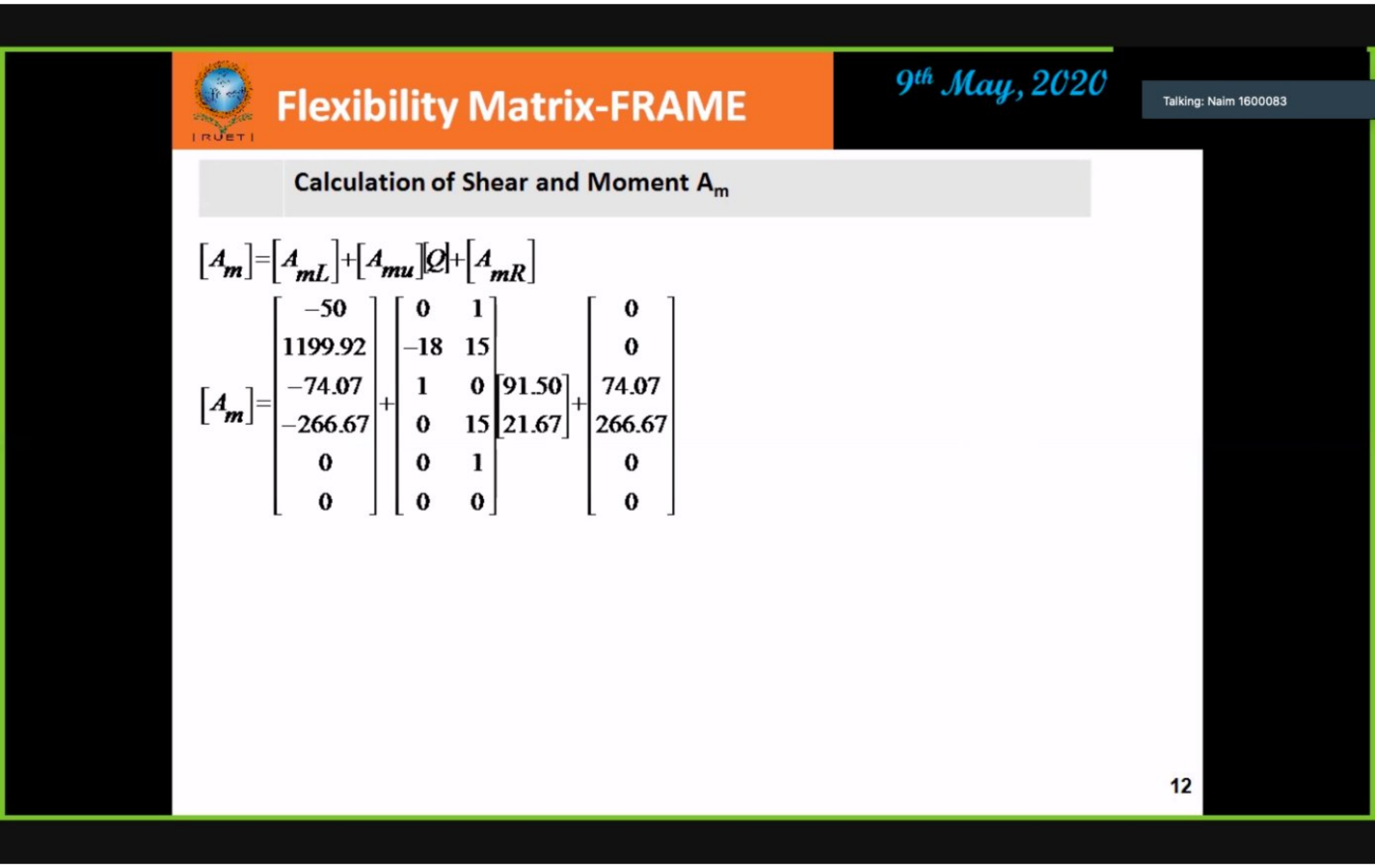
Calculation of Redundant Force Q

$$Q=-[F]^{-1}\left[D_{QL}\right]$$

$$Q=-\cancel{E}I\begin{bmatrix}6804 & -4455\\ -4455 & 6300\end{bmatrix}^{-1}\frac{1}{\cancel{E}I}\begin{bmatrix}-526019.94\\ 271105.2\end{bmatrix}$$

$$Q=-\frac{1}{23018175}\begin{bmatrix}6300 & 4455\\ 4455 & 6804\end{bmatrix}\begin{bmatrix}-526019.94\\ 271105.2\end{bmatrix}$$

$$\begin{bmatrix}Q_1\\ Q_2\end{bmatrix}=\begin{bmatrix}91.50\\ 21.67\end{bmatrix}$$



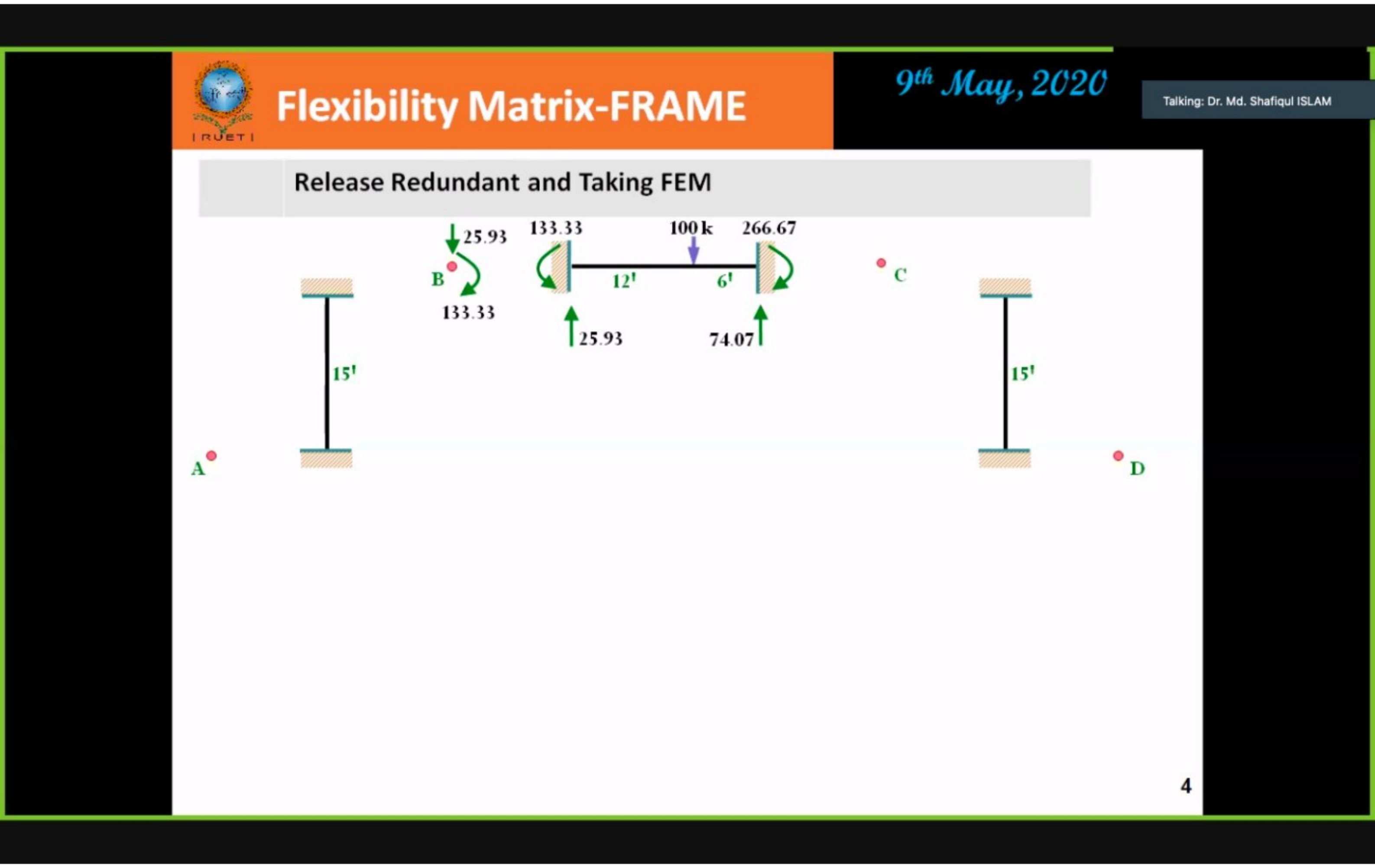
Flexibility Matrix-FRAME

9th May, 2020

Calculation of Shear and Moment A_m

$$[A_m]=[A_{mL}]+[A_{mu}][Q]+[A_{mR}]$$

$$[A_m]=\begin{bmatrix}-50\\1199.92\\-74.07\\-266.67\\0\\0\end{bmatrix}+\begin{bmatrix}0&1\\-18&15\\1&0\\0&15\\0&1\\0&0\end{bmatrix}\begin{bmatrix}91.50\\21.67\end{bmatrix}+\begin{bmatrix}0\\0\\74.07\\266.67\\0\\0\end{bmatrix}$$

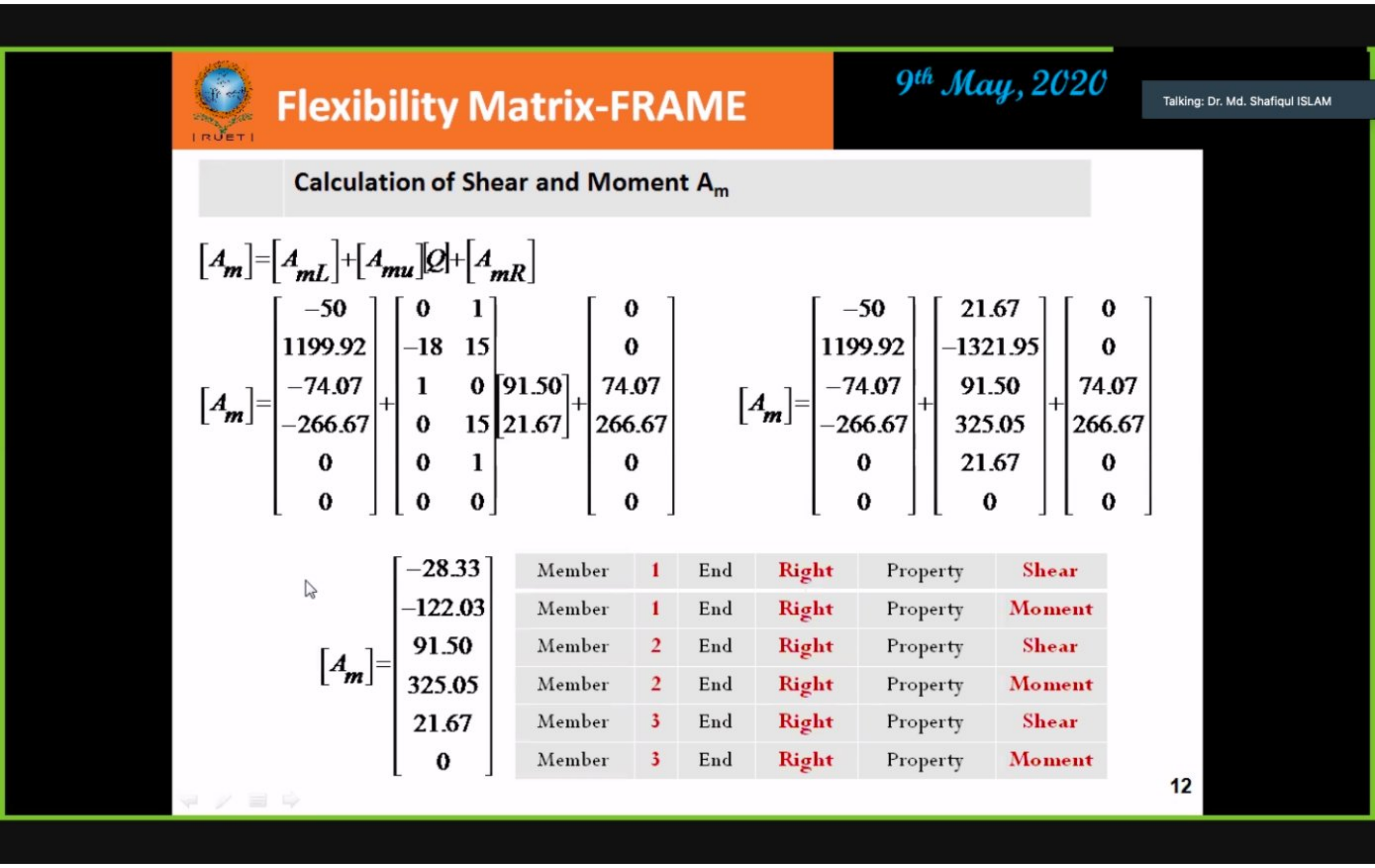


Talking: Dr. Md. Shafiqul ISLAM

9th May, 2020

Flexibility Matrix-FRAME

Release Redundant and Taking FEM



Flexibility Matrix-FRAME

9th May, 2020

Calculation of Shear and Moment A_m

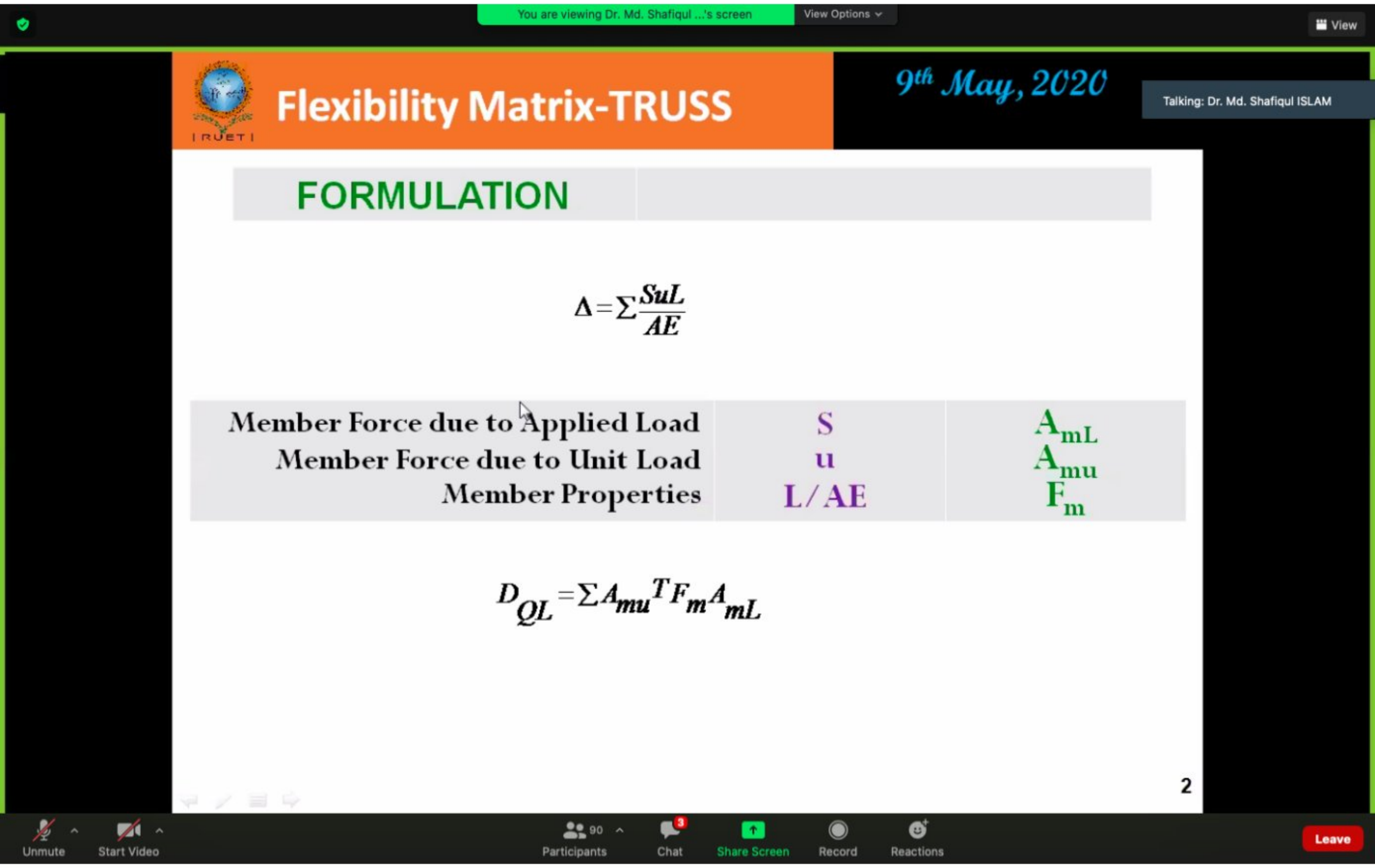
$$[A_m]=[A_{mL}]+[A_{mu}][Q]+[A_{mR}]$$

$$[A_m]=\begin{bmatrix}-50\\1199.92\\-74.07\\-266.67\\0\\0\end{bmatrix}+\begin{bmatrix}0&1\\-18&15\\1&0\\0&15\\0&1\\0&0\end{bmatrix}\begin{bmatrix}91.50\\21.67\end{bmatrix}+\begin{bmatrix}0\\0\\74.07\\266.67\\0\\0\end{bmatrix}$$

$$[A_m]=\begin{bmatrix}-50\\1199.92\\-74.07\\-266.67\\0\\0\end{bmatrix}+\begin{bmatrix}21.67\\-1321.95\\91.50\\325.05\\21.67\\0\end{bmatrix}+\begin{bmatrix}0\\0\\74.07\\266.67\\0\\0\end{bmatrix}$$

$$[A_m]=\begin{bmatrix}-28.33\\-122.03\\91.50\\325.05\\21.67\\0\end{bmatrix}$$

Member	1	End	Right	Property	Shear
Member	1	End	Right	Property	Moment
Member	2	End	Right	Property	Shear
Member	2	End	Right	Property	Moment
Member	3	End	Right	Property	Shear
Member	3	End	Right	Property	Moment



Flexibility Matrix-TRUSS

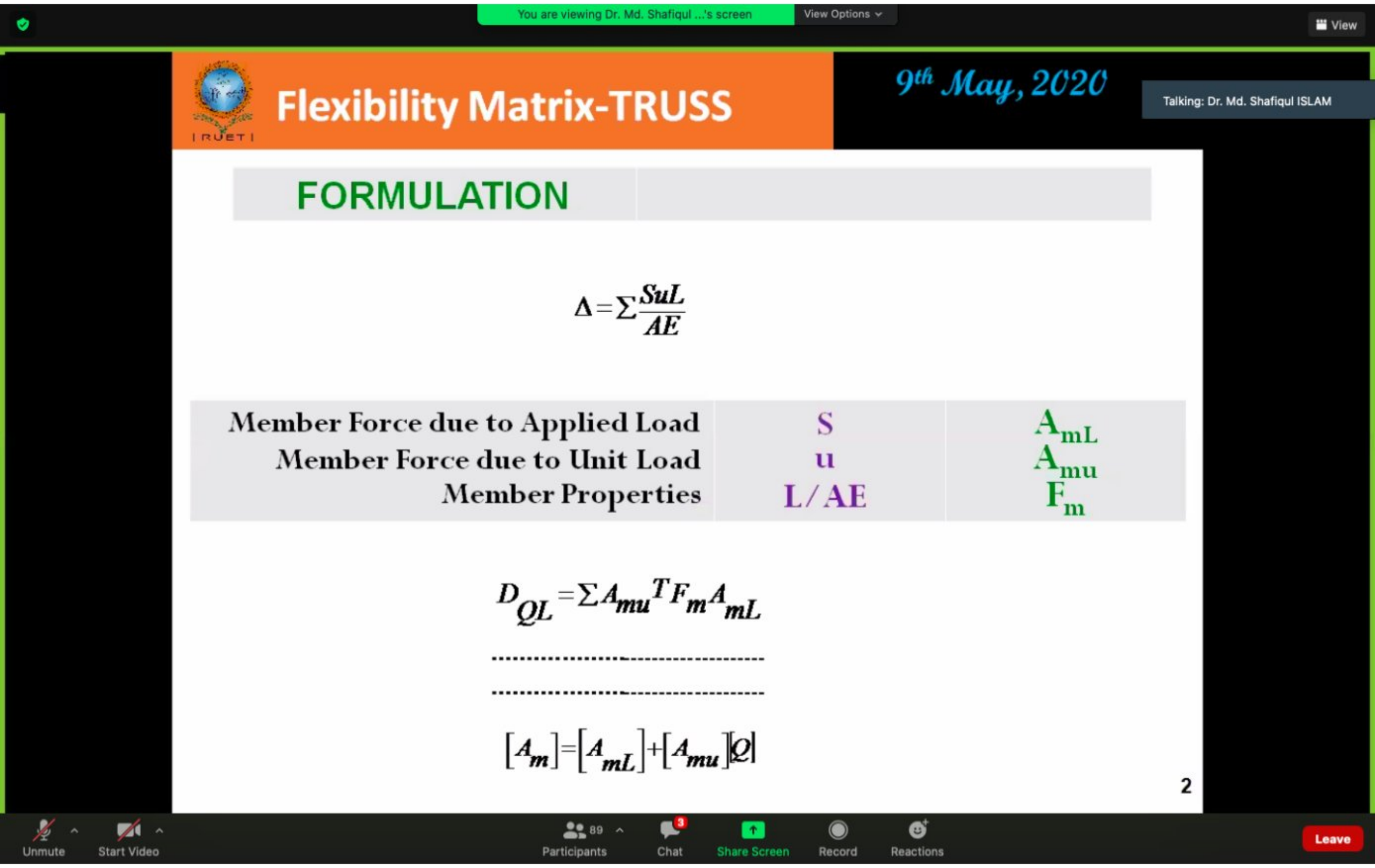
9th May, 2020

FORMULATION

$$\Delta = \sum \frac{SuL}{AE}$$

Member Force due to Applied Load	S	A_{mL}
Member Force due to Unit Load	u	A_{mu}
Member Properties	L/AE	F_m

$$D_{QL} = \sum A_{mu}{}^{T} F_m A_{mL}$$



Flexibility Matrix-TRUSS

9th May, 2020

FORMULATION

$$\Delta = \Sigma \frac{SuL}{AE}$$

Member Force due to Applied Load	S	A_{mL}
Member Force due to Unit Load	u	A_{mu}
Member Properties	L / AE	F_m

$$D_{QL} = \Sigma A_{mu}{}^{T} F_m A_{mL}$$

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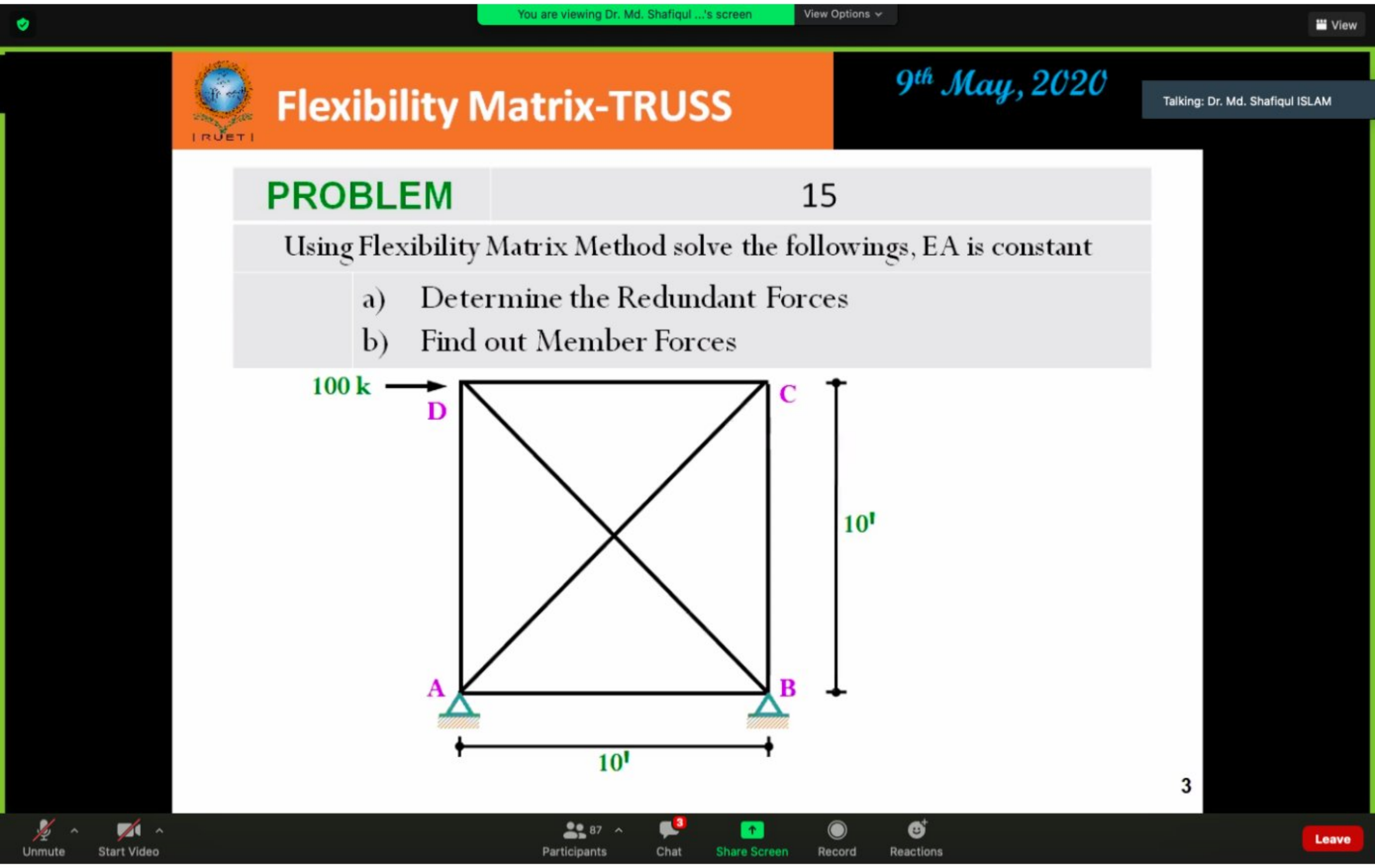
$$[A_m] = [A_{mL}] + [A_{mu}][Q]$$



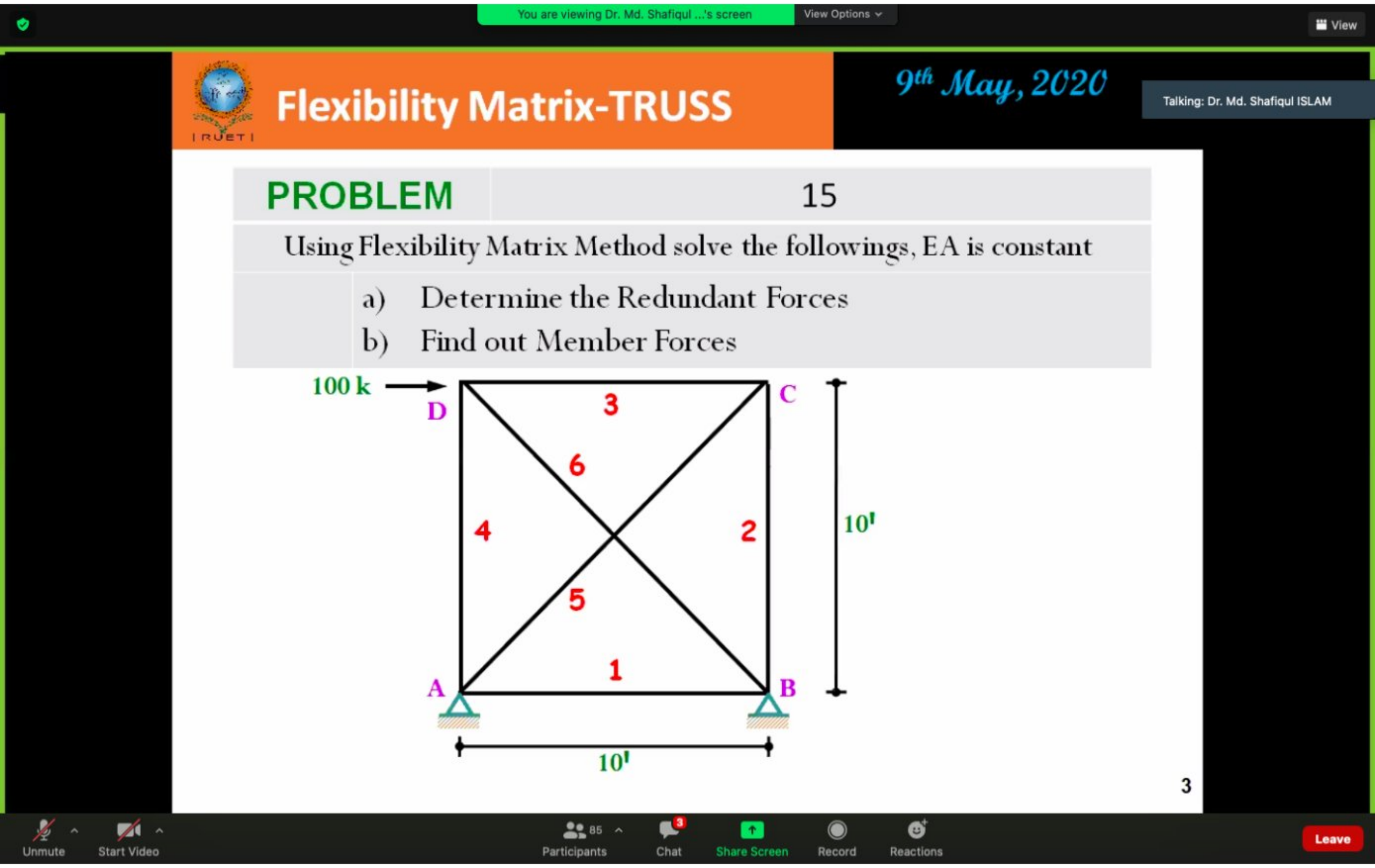
Flexibility Matrix-TRUSS

9th May, 2020

PROBLEM	15

Using Flexibility Matrix Method solve the followings, EA is constant

a) Determine the Redundant Forces
b) Find out Member Forces



Flexibility Matrix-TRUSS

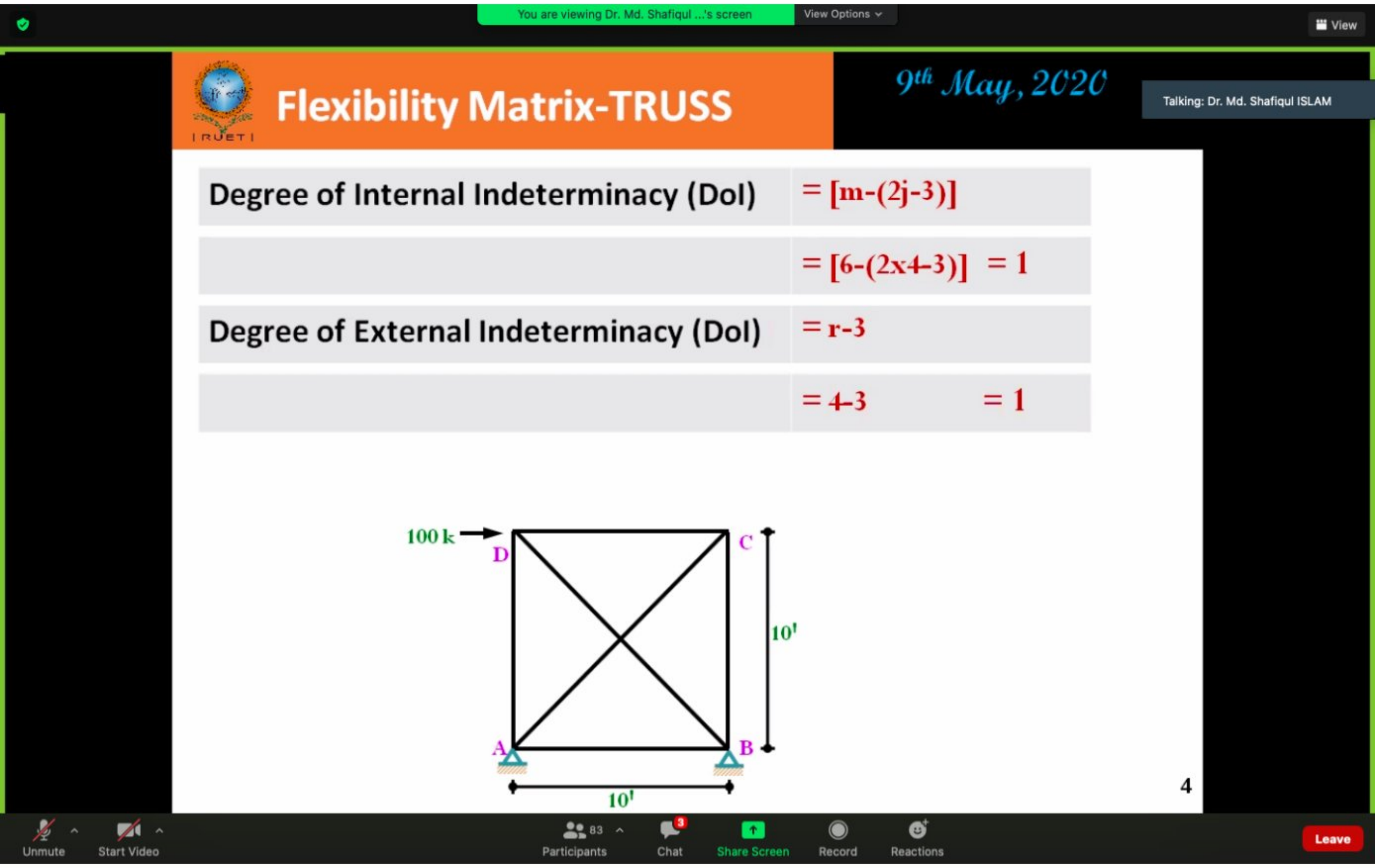
9th May, 2020

PROBLEM 15

Using Flexibility Matrix Method solve the followings, EA is constant

a) Determine the Redundant Forces

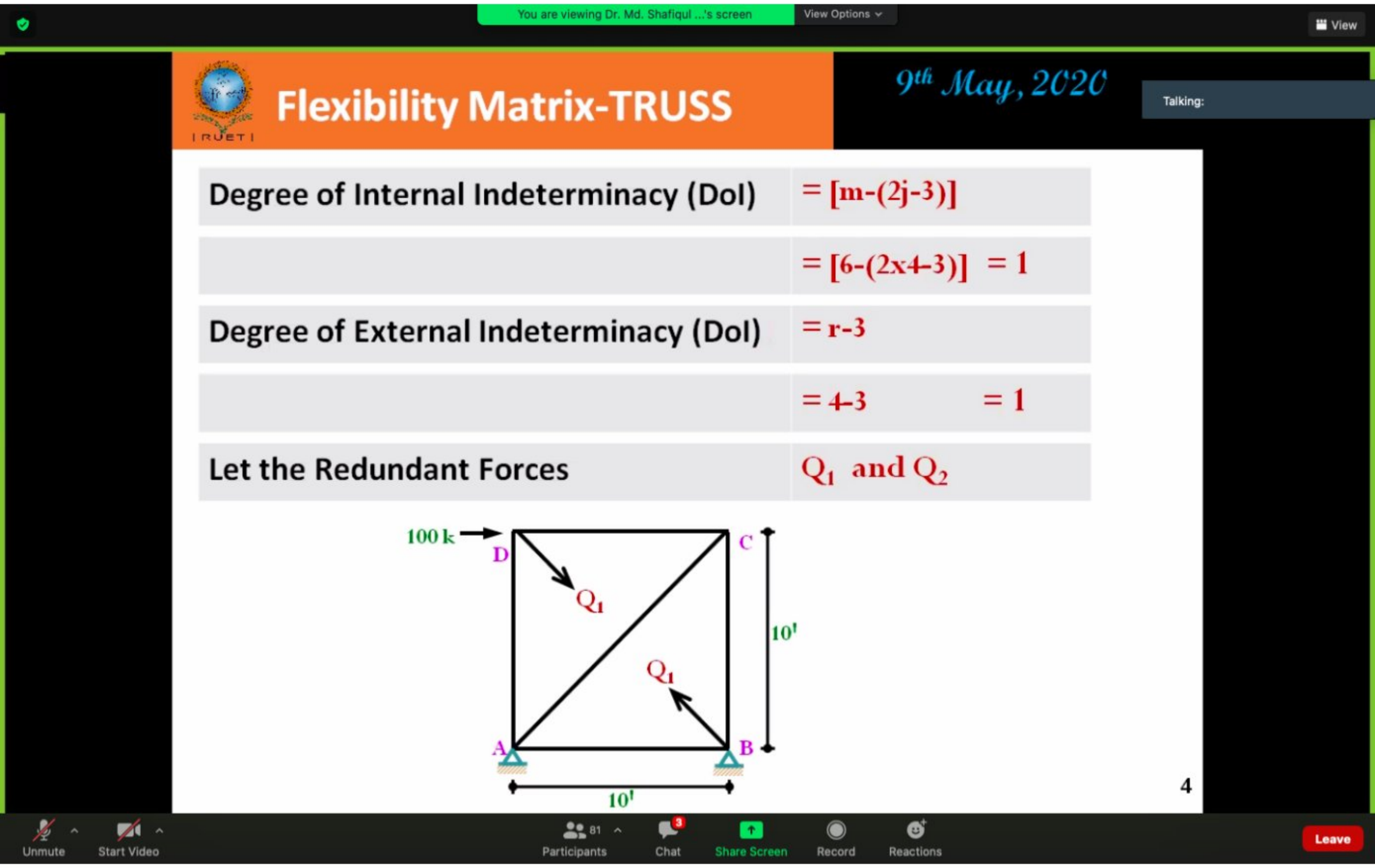
b) Find out Member Forces



Flexibility Matrix-TRUSS

9th May, 2020

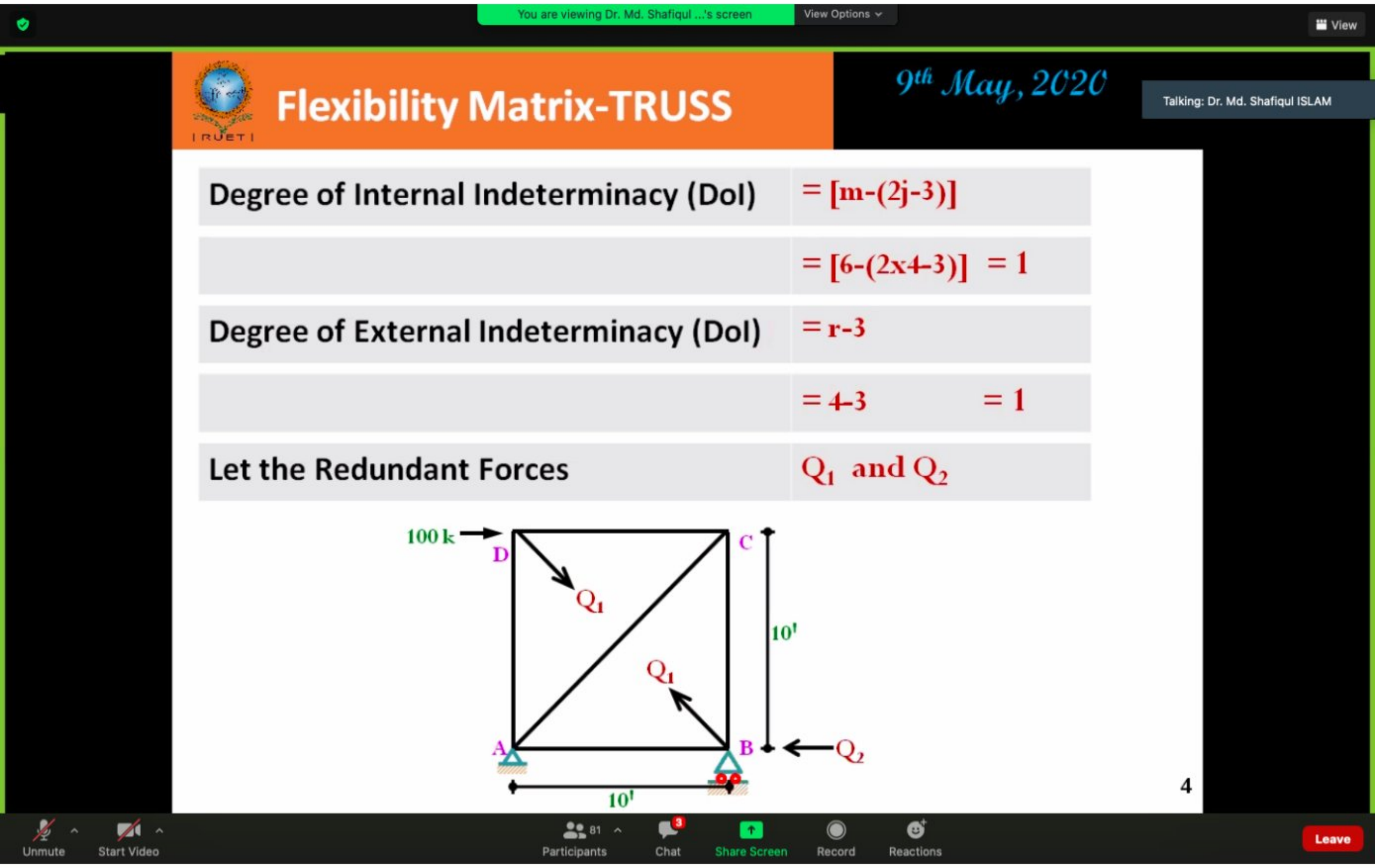
Degree of Internal Indeterminacy (DoI)	$= [m-(2j-3)]$
	$= [6-(2\times4-3)] \quad = 1$
Degree of External Indeterminacy (DoI)	$= r-3$
	$= 4-3 \qquad = 1$



Flexibility Matrix-TRUSS

9th May, 2020

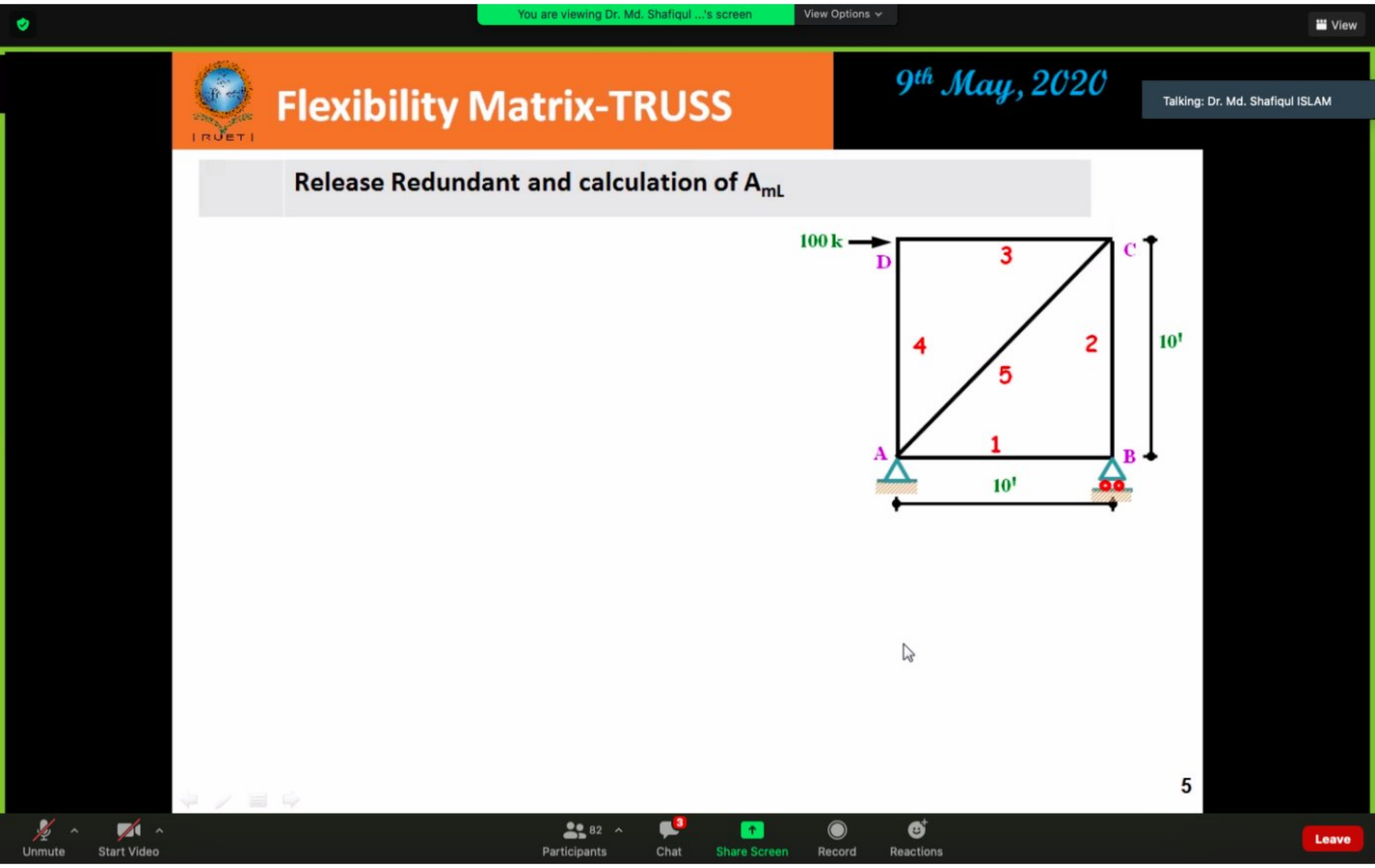
Degree of Internal Indeterminacy (DoI)	$= [m-(2j-3)]$
	$= [6-(2\text{x}4-3)] = 1$
Degree of External Indeterminacy (DoI)	$= r-3$
	$= 4-3 \qquad = 1$
Let the Redundant Forces	Q_1 and Q_2



Flexibility Matrix-TRUSS

9th May, 2020

Degree of Internal Indeterminacy (DoI)	$= [m-(2j-3)]$
	$= [6-(2x4-3)] \quad = 1$
Degree of External Indeterminacy (DoI)	$= r-3$
	$= 4-3 \qquad = 1$
Let the Redundant Forces	Q_1 and Q_2

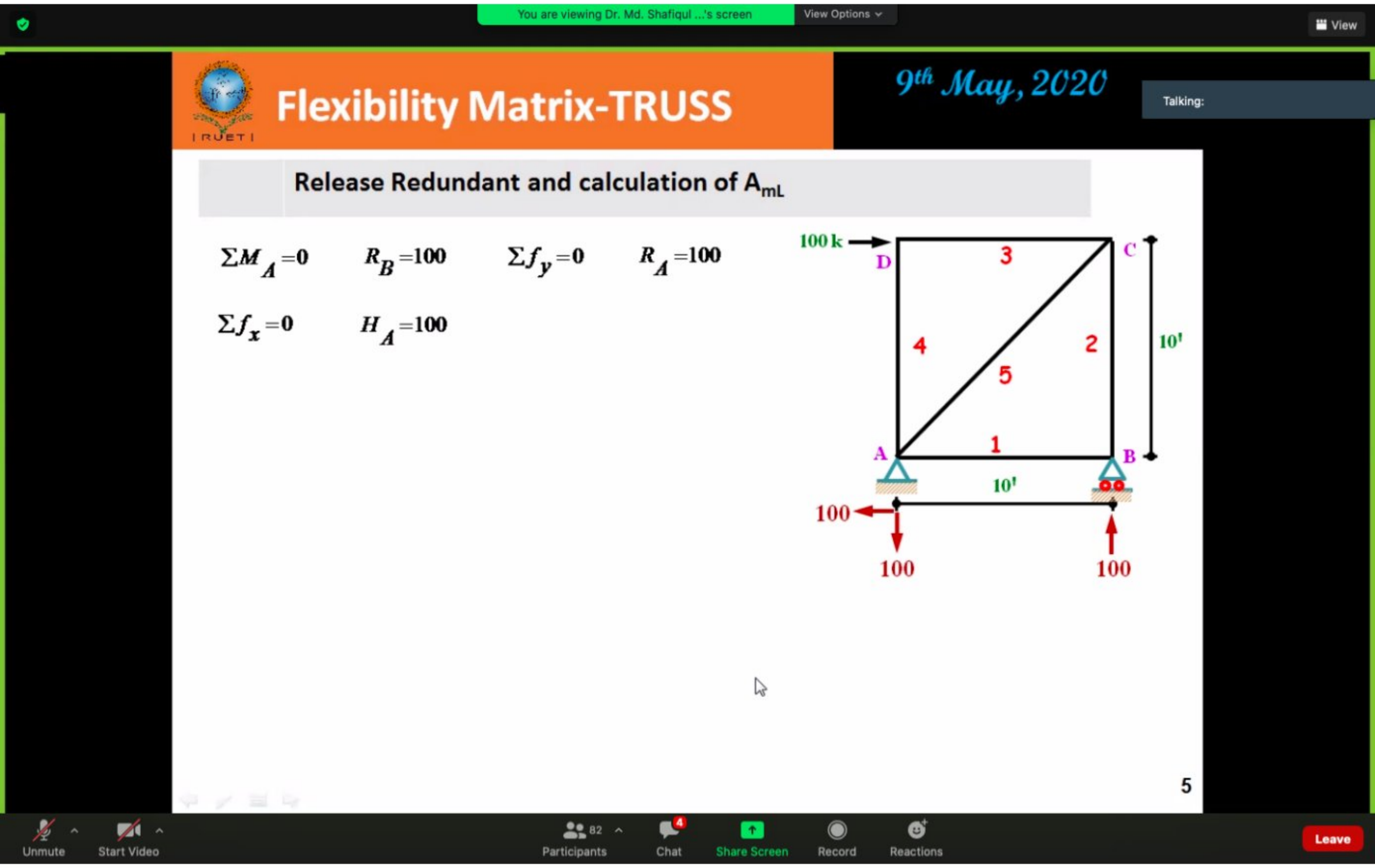


Flexibility Matrix-TRUSS

9th May, 2020

Release Redundant and calculation of A_{mL}

5



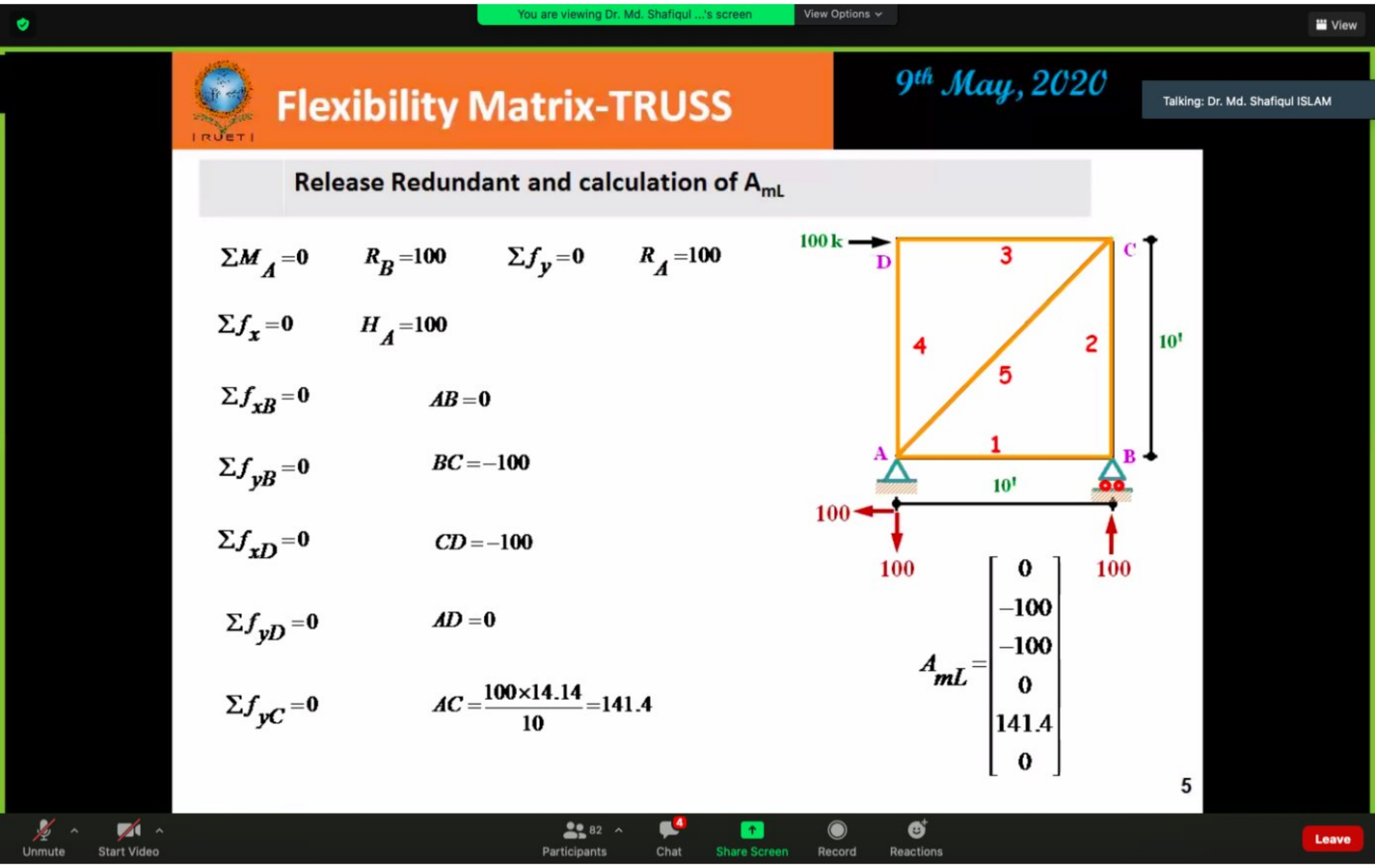
Flexibility Matrix-TRUSS

9th May, 2020

Release Redundant and calculation of A_{mL}

$\Sigma M_A = 0$ $R_B = 100$ $\Sigma f_y = 0$ $R_A = 100$

$\Sigma f_x = 0$ $H_A = 100$



Flexibility Matrix-TRUSS

9th May, 2020

Release Redundant and calculation of A_{mL}

$\Sigma M_A = 0$ $R_B = 100$ $\Sigma f_y = 0$ $R_A = 100$

$\Sigma f_x = 0$ $H_A = 100$

$\Sigma f_{xB} = 0$ $AB = 0$

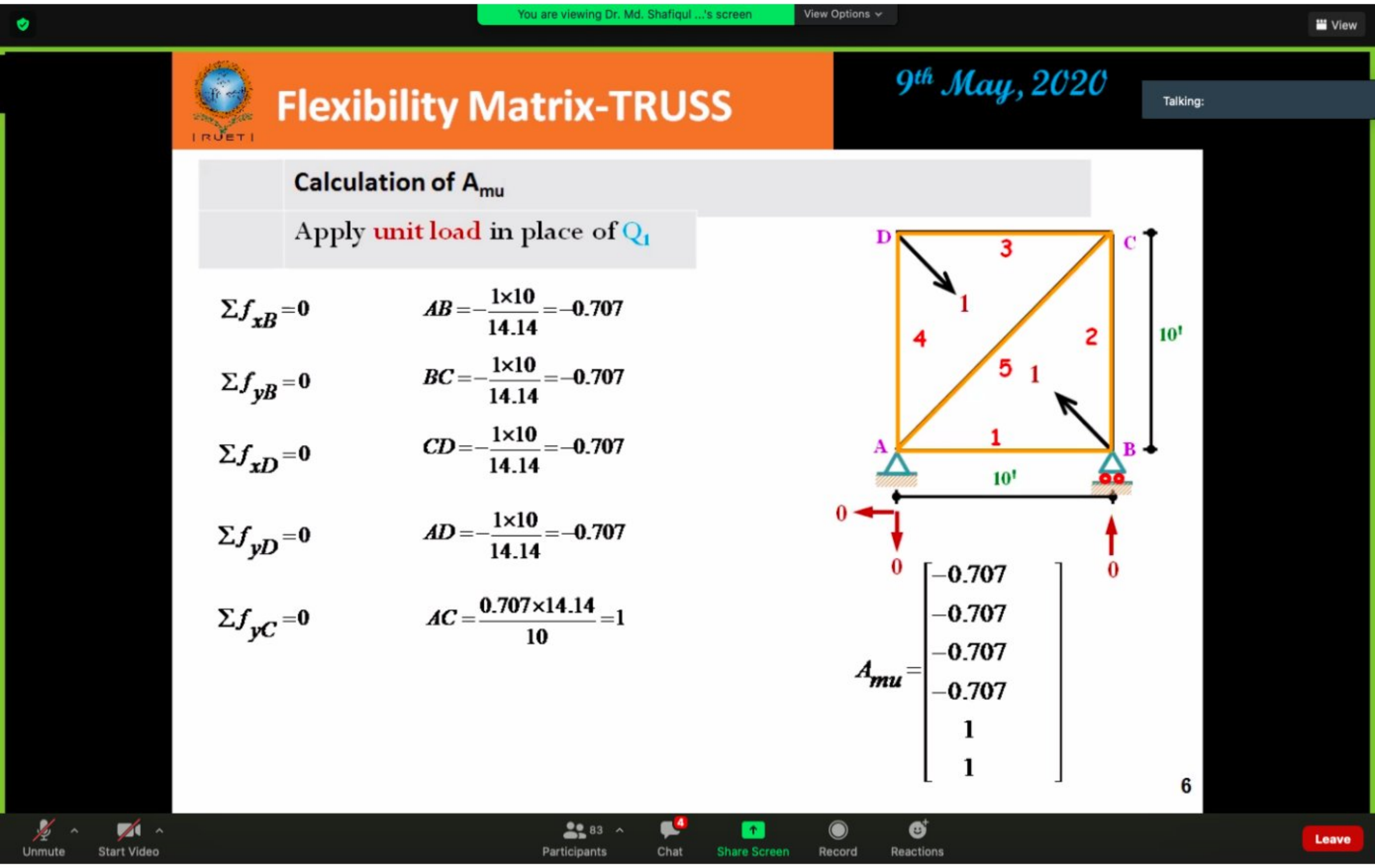
$\Sigma f_{yB} = 0$ $BC = -100$

$\Sigma f_{xD} = 0$ $CD = -100$

$\Sigma f_{yD} = 0$ $AD = 0$

$\Sigma f_{yC} = 0$ $AC = \frac{100 \times 14.14}{10} = 141.4$

$$A_{mL} = \begin{bmatrix} 0 \\ -100 \\ -100 \\ 0 \\ 141.4 \\ 0 \end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

Calculation of A_{mu}

Apply unit load in place of Q_1

$$\Sigma f_{xB}=0 \qquad AB=-\frac{1\times10}{14.14}=-0.707$$

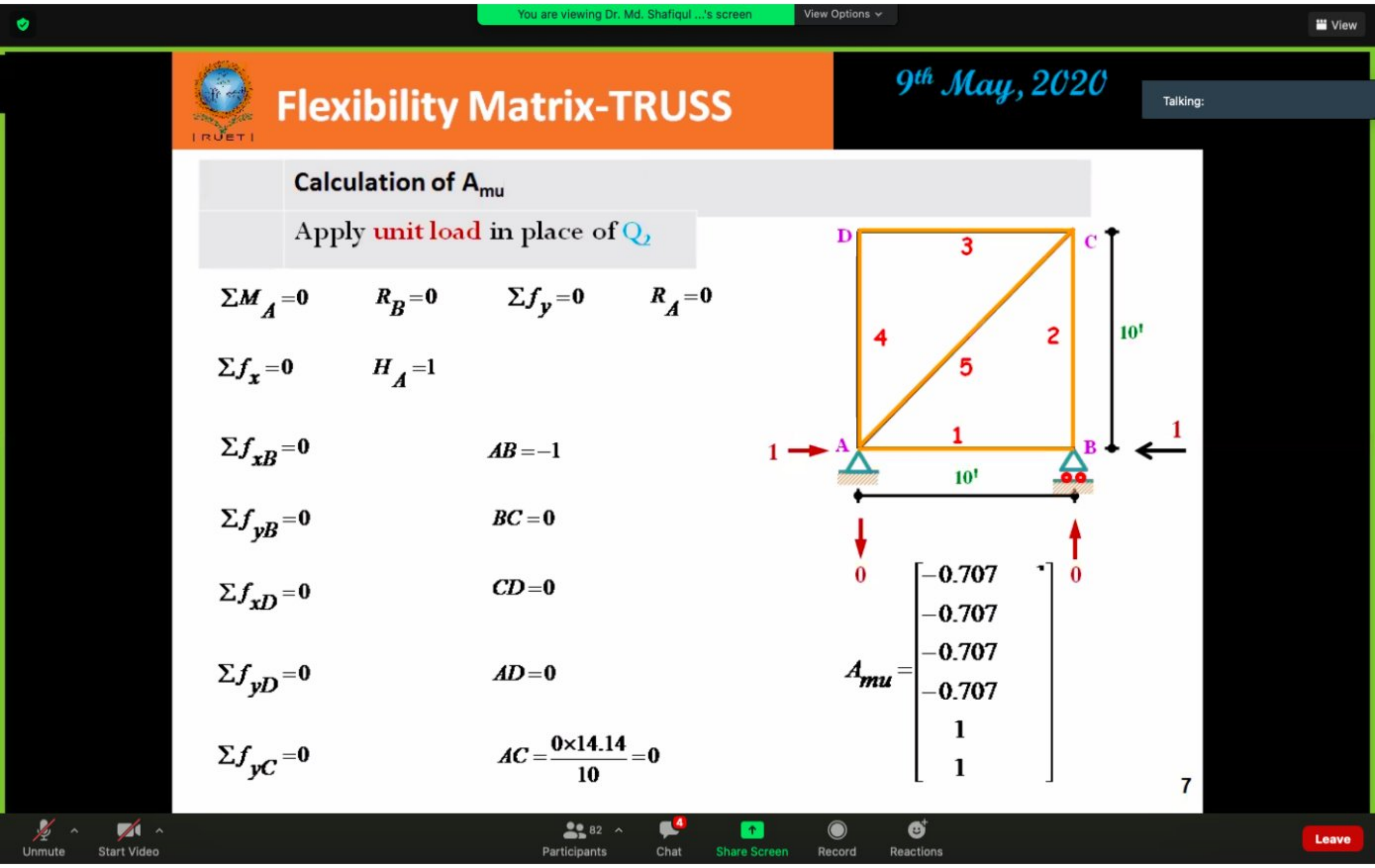
$$\Sigma f_{yB}=0 \qquad BC=-\frac{1\times10}{14.14}=-0.707$$

$$\Sigma f_{xD}=0 \qquad CD=-\frac{1\times10}{14.14}=-0.707$$

$$\Sigma f_{yD}=0 \qquad AD=-\frac{1\times10}{14.14}=-0.707$$

$$\Sigma f_{yC}=0 \qquad AC=\frac{0.707\times14.14}{10}=1$$

$$A_{mu}=\begin{bmatrix}-0.707\\-0.707\\-0.707\\-0.707\\1\\1\end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

Calculation of A_{mu}

Apply unit load in place of Q_2

$\Sigma M_A = 0$ $R_B = 0$ $\Sigma f_y = 0$ $R_A = 0$

$\Sigma f_x = 0$ $H_A = 1$

$\Sigma f_{xB} = 0$ $AB = -1$

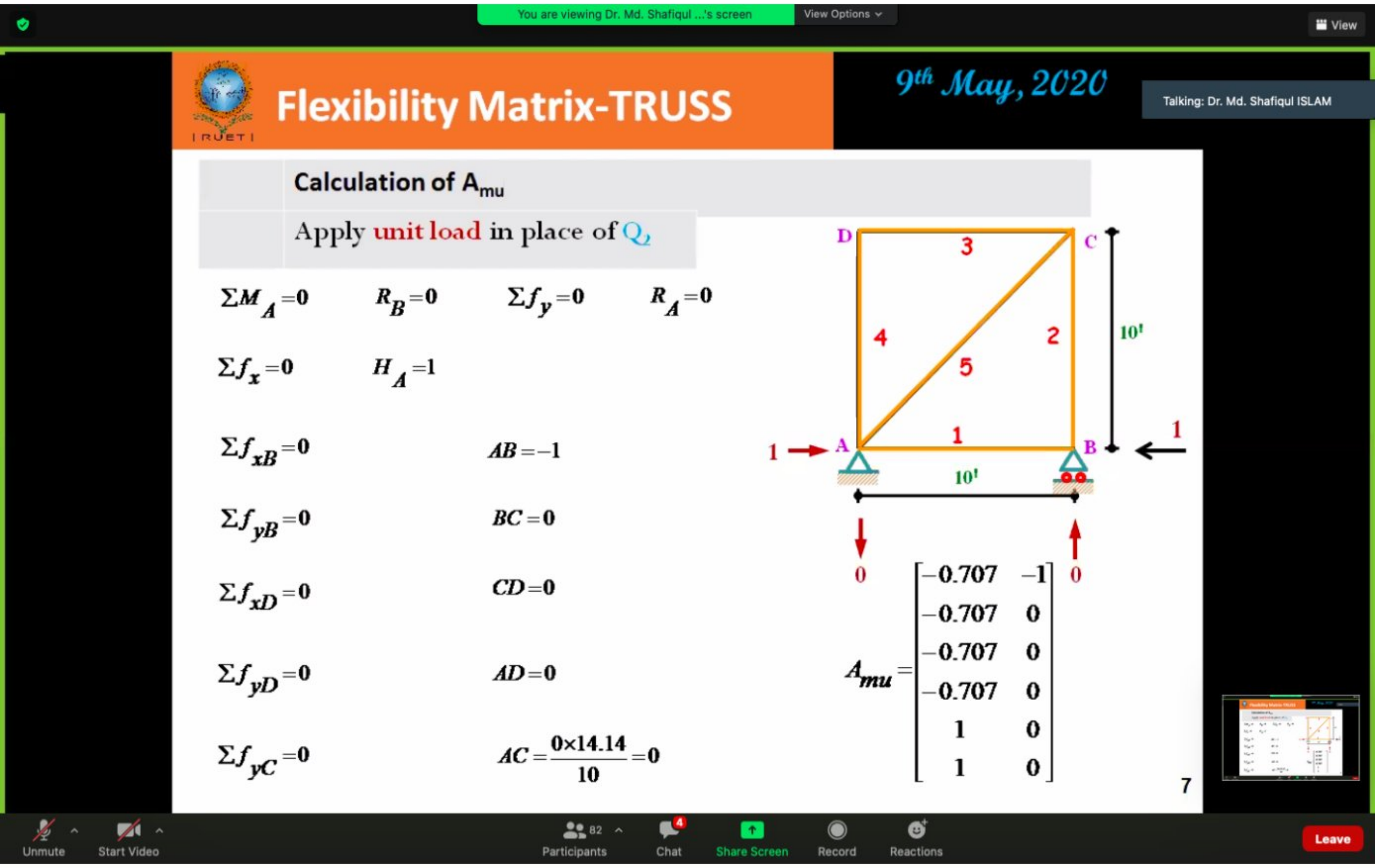
$\Sigma f_{yB} = 0$ $BC = 0$

$\Sigma f_{xD} = 0$ $CD = 0$

$\Sigma f_{yD} = 0$ $AD = 0$

$\Sigma f_{yC} = 0$ $AC = \frac{0 \times 14.14}{10} = 0$

$$A_{mu} = \begin{bmatrix} -0.707 \\ -0.707 \\ -0.707 \\ -0.707 \\ 1 \\ 1 \end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

Calculation of A_{mu}

Apply unit load in place of Q_2

$\Sigma M_A = 0$ $R_B = 0$ $\Sigma f_y = 0$ $R_A = 0$

$\Sigma f_x = 0$ $H_A = 1$

$\Sigma f_{xB} = 0$ $AB = -1$

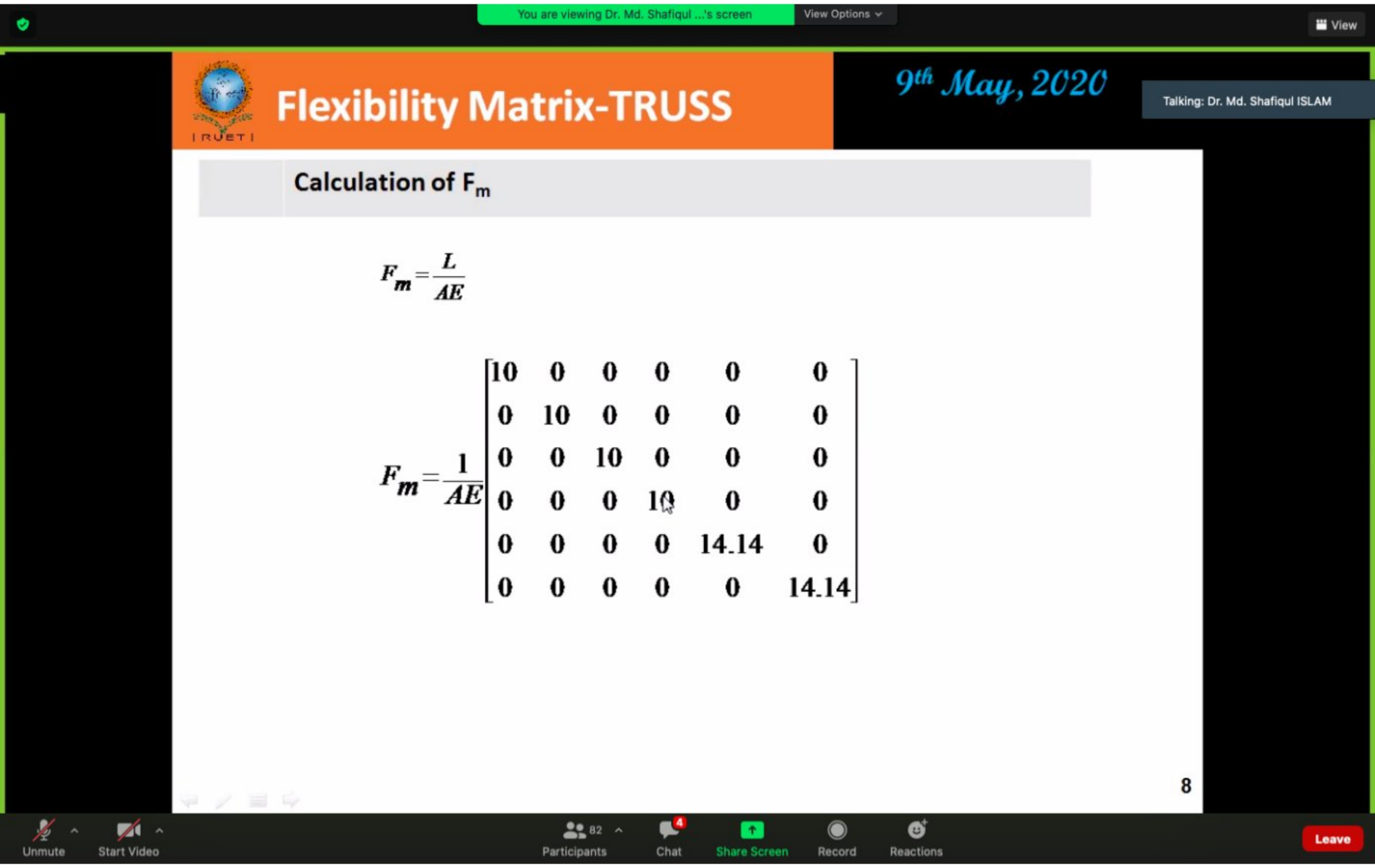
$\Sigma f_{yB} = 0$ $BC = 0$

$\Sigma f_{xD} = 0$ $CD = 0$

$\Sigma f_{yD} = 0$ $AD = 0$

$\Sigma f_{yC} = 0$ $AC = \frac{0 \times 14.14}{10} = 0$

$$A_{mu} = \begin{bmatrix} -0.707 & -1 \\ -0.707 & 0 \\ -0.707 & 0 \\ -0.707 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$



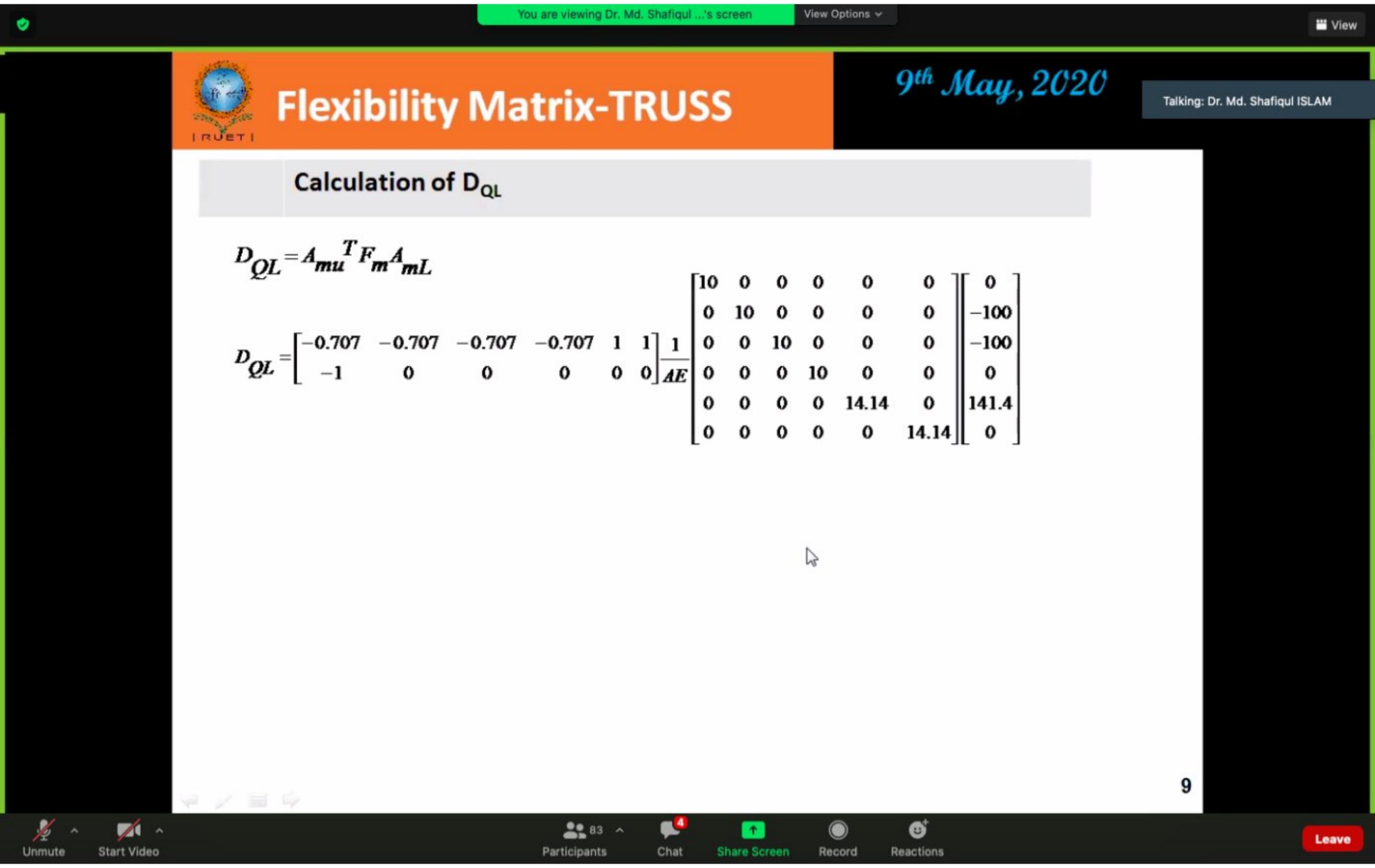
Flexibility Matrix-TRUSS

9th May, 2020

Calculation of F_m

$$F_m = \frac{L}{AE}$$

$$F_m = \frac{1}{AE}\begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14.14 \end{bmatrix}$$



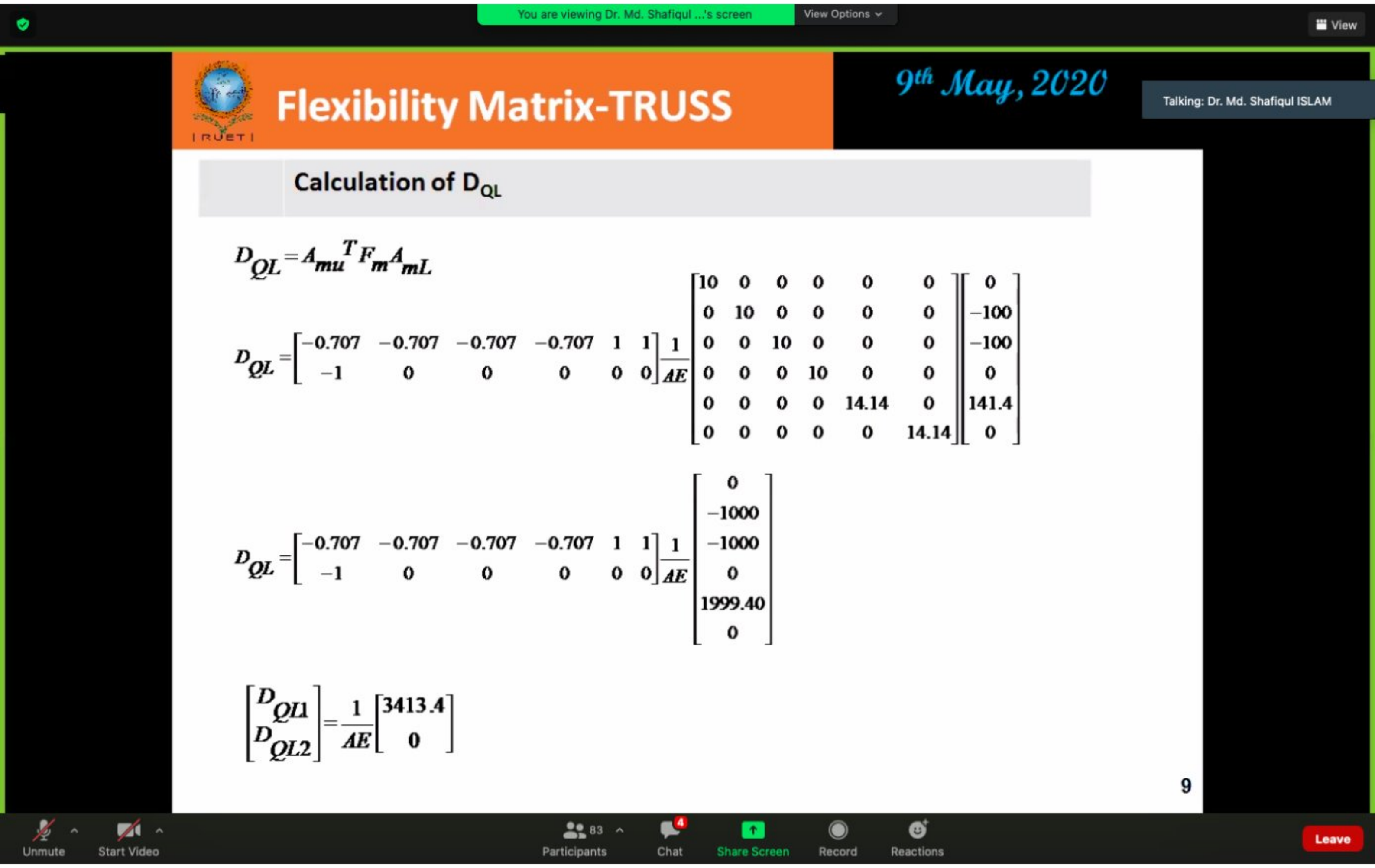
Flexibility Matrix-TRUSS

9th May, 2020

Calculation of D_{QL}

$$D_{QL} = A_{mu}{}^{T} F_m A_{mL}$$

$$D_{QL} = \begin{bmatrix} -0.707 & -0.707 & -0.707 & -0.707 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14.14 \end{bmatrix} \begin{bmatrix} 0 \\ -100 \\ -100 \\ 0 \\ 141.4 \\ 0 \end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

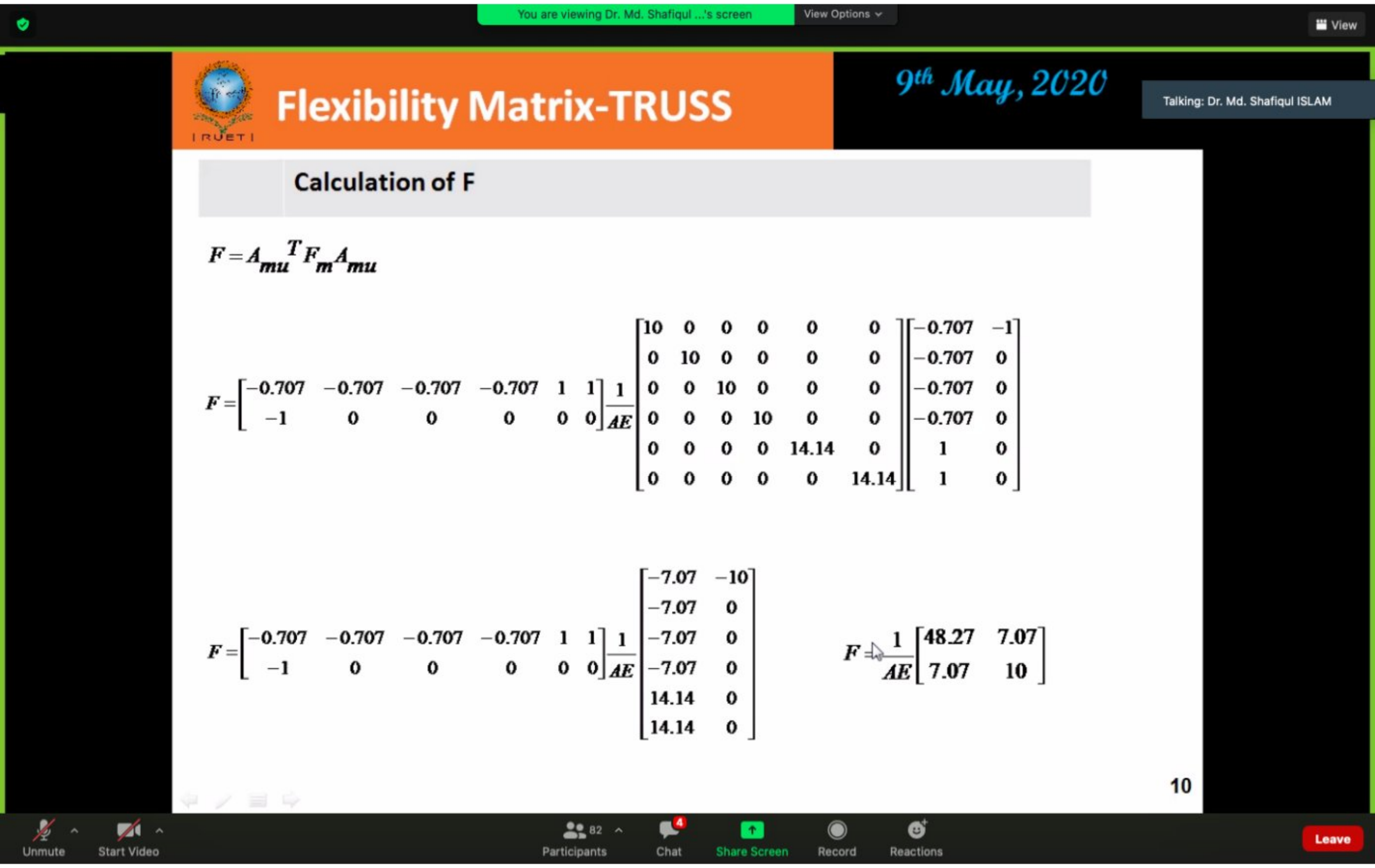
Calculation of D_{QL}

$$D_{QL} = A_{mu}{}^{T} F_m A_{mL}$$

$$D_{QL} = \begin{bmatrix} -0.707 & -0.707 & -0.707 & -0.707 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14.14 \end{bmatrix} \begin{bmatrix} 0 \\ -100 \\ -100 \\ 0 \\ 141.4 \\ 0 \end{bmatrix}$$

$$D_{QL} = \begin{bmatrix} -0.707 & -0.707 & -0.707 & -0.707 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 0 \\ -1000 \\ -1000 \\ 0 \\ 1999.40 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} D_{QL1} \\ D_{QL2} \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} 3413.4 \\ 0 \end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

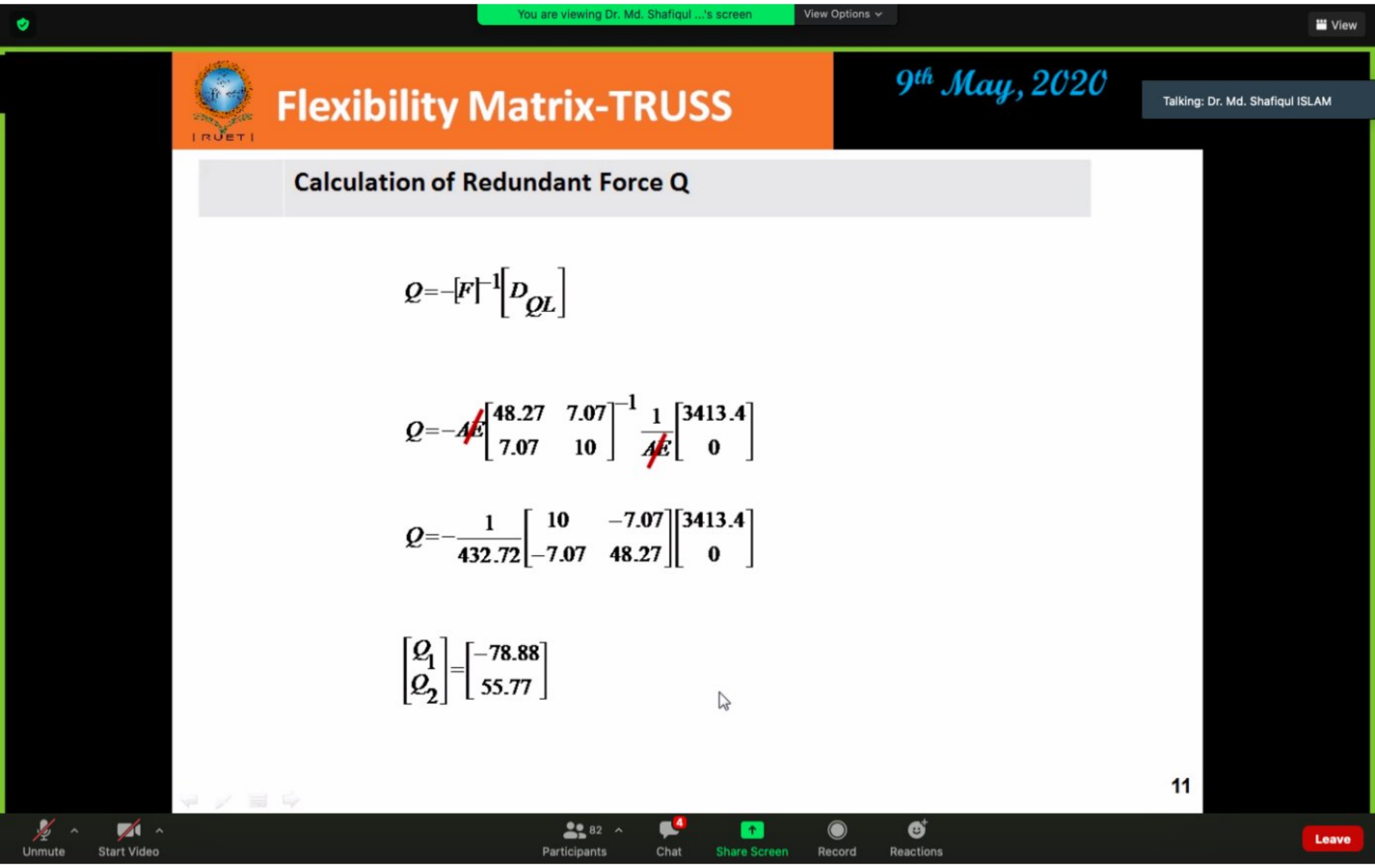
Calculation of F

$$F = A_{mu}{}^{T} F_{m} A_{mu}$$

$$F = \begin{bmatrix} -0.707 & -0.707 & -0.707 & -0.707 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 14.14 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14.14 \end{bmatrix} \begin{bmatrix} -0.707 & -1 \\ -0.707 & 0 \\ -0.707 & 0 \\ -0.707 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} -0.707 & -0.707 & -0.707 & -0.707 & 1 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} -7.07 & -10 \\ -7.07 & 0 \\ -7.07 & 0 \\ -7.07 & 0 \\ 14.14 & 0 \\ 14.14 & 0 \end{bmatrix}$$

$$F = \frac{1}{AE} \begin{bmatrix} 48.27 & 7.07 \\ 7.07 & 10 \end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

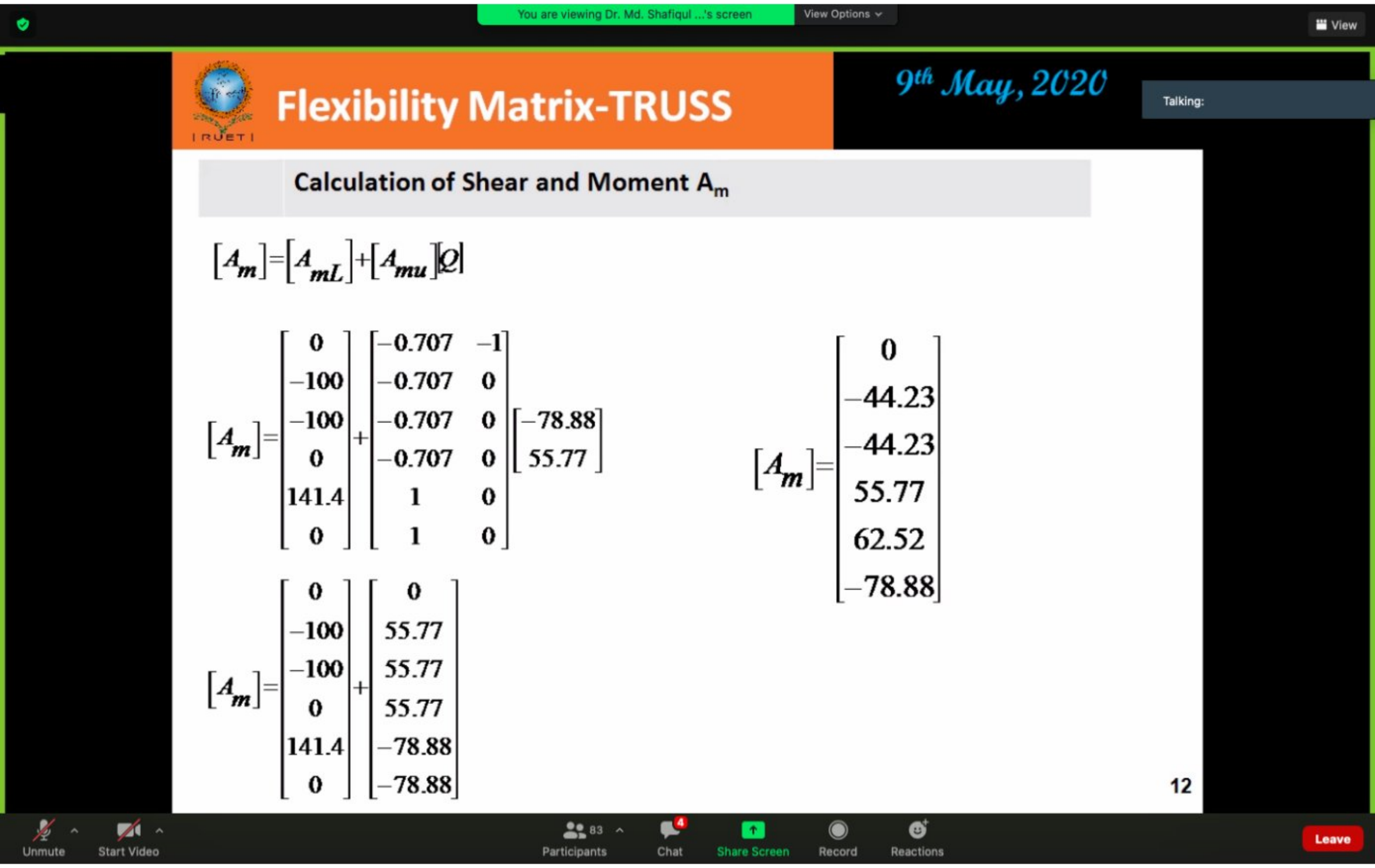
Calculation of Redundant Force Q

$$Q = -[F]^{-1}\left[D_{QL}\right]$$

$$Q = -\cancel{AE}\begin{bmatrix} 48.27 & 7.07 \\ 7.07 & 10 \end{bmatrix}^{-1} \frac{1}{\cancel{AE}}\begin{bmatrix} 3413.4 \\ 0 \end{bmatrix}$$

$$Q = -\frac{1}{432.72}\begin{bmatrix} 10 & -7.07 \\ -7.07 & 48.27 \end{bmatrix}\begin{bmatrix} 3413.4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} -78.88 \\ 55.77 \end{bmatrix}$$



Flexibility Matrix-TRUSS

9th May, 2020

Calculation of Shear and Moment A_m

$$[A_m]=[A_{mL}]+[A_{mu}][Q]$$

$$[A_m]=\begin{bmatrix}0\\-100\\-100\\0\\141.4\\0\end{bmatrix}+\begin{bmatrix}-0.707 & -1\\-0.707 & 0\\-0.707 & 0\\-0.707 & 0\\1 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}-78.88\\55.77\end{bmatrix}$$

$$[A_m]=\begin{bmatrix}0\\-100\\-100\\0\\141.4\\0\end{bmatrix}+\begin{bmatrix}0\\55.77\\55.77\\55.77\\-78.88\\-78.88\end{bmatrix}$$

$$[A_m]=\begin{bmatrix}0\\-44.23\\-44.23\\55.77\\62.52\\-78.88\end{bmatrix}$$